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2019

Seetoh, I., Markandan, K., & Lai, C. Q. (2019). Effect of reinforcement bending on the elastic properties of interpenetrating phase composites. *Mechanics of Materials*, 136, 103070-.
doi:10.1016/j.mechmat.2019.103071

<https://hdl.handle.net/10356/144394>

<https://doi.org/10.1016/j.mechmat.2019.103071>

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This Article Has Been Accepted and Published in

Mech. Mater. **136**, 103071 (2019).

DOI: 10.1016/j.mechmat.2019.103071

Effect of Reinforcement Bending on the Elastic Properties of Interpenetrating Phase Composites

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Keywords: Interpenetrating Phase Composites, Octet Truss, Kelvin Cell, Stiffness, Modulus, Bending Dominated, Stretch Dominated, 3D Printing

ABSTRACT

The elastic modulus of interpenetrating phase composites (IPCs) was analyzed through a theoretical model that accounted for bending deformation of the reinforcement phase. The model was validated against literature data, as well as simulation and experimental results of IPCs that were constructed from 3D-printed polymeric reinforcements embedded in a polydimethylsiloxane (PDMS) matrix. The reinforcements were in the form of Octet Truss and Kelvin Cell lattices, which are known to exhibit very different degrees of bending during elastic deformation. When the matrix modulus was relatively low, the model was able to explain how the bending of reinforcement struts caused the overall IPC modulus to be much lower than those predicted by other theoretical models. As the matrix modulus increased to beyond 20% that of the reinforcement material, however, the different lattice designs were found to have no significant influence on the IPC modulus. Further increase in matrix modulus pushed the elastic response of IPCs towards the isostrain limit, as the matrix helped to distribute the load more evenly and suppress the bending of struts, especially for lower density lattices. The model was able to account for a wide range of different constituent moduli and was also applicable to IPCs

which utilized stochastic foams for reinforcement. The insights derived in this study is expected to be particularly useful for designing polymer-based IPCs where the elastic moduli of the reinforcement and matrix can differ over several orders of magnitude.

1. INTRODUCTION

Interpenetrating phase composites (IPCs) consist of solid phases that each form a continuous network throughout the material. This contrasts with fiber and particle composites, where the reinforcement material is made up of discrete isolated phases. Materials for IPCs typically adopt metal-ceramic combinations [1–9], the most common being Al-Al₂O₃ composites, as well as metal-metal (*e.g.* stainless steel/bronze) [10,11], metal-polymer, ceramic-polymer and polymer-polymer pairings (*e.g.* polystyrene/poly(ether-ester)) [12–18]. They are generally fabricated by infiltrating a molten matrix into stochastic foams or 3D printed scaffolds, which serve as the reinforcement phase [5,9] [11].

Because of the interconnectivity of the individual phases, IPCs were postulated to provide important multifunctional characteristics (*e.g.* electrical, magnetic, structural) [19], and have been shown to possess superior mechanical stiffness and strength over particle composites [4,12,16,20]. In addition, they were also observed to be more stable under off-axis loading than fiber and lamella composites [16] and have better resistance to crack propagation during fracture failure, which improved its energy dissipation properties [16]. These characteristics make IPCs highly attractive for applications in aviation, protective materials and medical implants.

When designing an IPC for use in engineering applications, it is important to consider how its elastic modulus is influenced by the relative proportion, as well as the inherent mechanical properties, of the matrix and reinforcement materials. Existing theoretical models, including the widely used Hashin and Shtrikman (H&S) bounds [21] and those developed by dividing unit cells into parallel/series or series/parallel segments with respect to the loading direction [22] [23], generally do not give good agreement with experimental results when the moduli of the matrix and the reinforcement are very different. In such cases, the gap between the upper and lower bound tends to be too wide for the model to yield useful predictions [20].

On the other hand, models that can successfully predict the elastic moduli of IPCs with vastly different matrix and reinforcement moduli (*e.g.* effective medium approximation [7,8]) can be rather complex, often relying on shape-dependent strain relation tensors and numerical methods, which makes it difficult to glean intuitive insights.

Furthermore, current models generally assume that the reinforcement phase is always under compression/ tension, neglecting the contributions of bending and buckling deformations. While this assumption is most likely true when the matrix is much stiffer than the reinforcement, it may not necessarily hold up in the reverse case, where the reinforcement material is much stiffer than the matrix material. This is because, in such cases, the IPC would essentially resemble a foam/ lattice, the elastic response of which can be dominated by bending or buckling deformations, rather than compression/ tension [24].

The aim of the current study, therefore, is to examine the effect of reinforcement bending on the elastic modulus of an IPC. We start by investigating the elastic response of lattice-based IPCs with respect to the proportion and mechanical properties of the reinforcement and matrix materials through analytical, numerical and experimental means. The use of orderly lattice designs allowed us to modulate the amount of bending in the reinforcement phase. Octet Truss and Kelvin Cell (tetrakaidecahedron) designs, which are well-known stretch/ compression-dominated and bending-dominated topologies respectively [24,25], were chosen for the present study. To further validate our analysis, we have also compared our analytical and numerical results to experimental data on foam-based IPCs reported in the literature.

2. METHODS

2.1 Generation of 3D models

3D models of reinforcement lattices were constructed using SOLIDWORKS software (Fig. 1). Two types of lattices were constructed: i) Octet Truss (stretch/ compression-dominated) and ii) Kelvin Cell (bending-dominated). The lattices are symmetrical about the X-, Y- and Z-axes, have solid fractions ranging from 0.01 to 0.4 and are made up of trusses with circular cross-sections. Here, we focus on the low volume fraction regime, which is generally applicable to most cellular materials such as foams.

Fig. 1: Unit cells of (a) Octet Truss and (b) Kelvin Cell.

2.2 Experimental: Fabrication and Quasistatic Compression Tests

The Octet Truss and Kelvin Cell lattices (4 x 4 x 4 unit cells) were fabricated using a commercial stereolithography-based 3D printer (Form 2, Formlabs Inc., USA) with a print resolution of 0.025 mm (Fig. 2a and 2b). The photopolymer used was Grey resin (compressive modulus 530 ± 19 MPa), which is mainly made up of methacrylated monomers and oligomers. After polymerization, the lattices were soaked in isopropyl alcohol (IPA) for 5 minutes to remove the excess resin. The dimensions of the lattices are listed in Table I. To obtain lattice IPCs, a mixture of polydimethylsiloxane (PDMS) prepolymer and its curing agent (Sylgard

184, Dow Corning, US) was prepared with a ratio 10:1 and infiltrated into the lattices. The PDMS (compressive modulus 1.56 ± 0.06 MPa) was then left inside a vacuum desiccator for 1.5 hours to de-gas and ensure the removal of gas bubbles, before being cured at 80°C for 2 hours (Fig. 2c and 2d).

Quasistatic compression testing on the lattices and lattice-IPCs was conducted using Shimadzu Autograph AG-500A (Shimadzu Corporation, Kyoto, Japan) with a 50 kN load cell and at a strain rate of ~ 0.01 /s. For strains up to 20%, no significant delamination of the PDMS from the Grey resin lattices was observed.

Table I: Dimensions of Kelvin Cell and Octet Truss lattices (4 x 4 x 4 unit cells) fabricated experimentally.

Design	Solid fraction	Length of truss (mm)	Diameter of truss (mm)	Length/ Width/ Height of lattice (mm)
Kelvin Cell	0.12	2.0	1	27
	0.17	1.6	1	22
	0.21	2.9	2	40
	0.30	2.7	2.5	40
Octet truss	0.11	7.1	1	40
	0.17	5.6	1	32
	0.24	4.7	1	27
	0.30	4.0	1	23

Fig. 2: (a) Kelvin Cell (b) Octet Truss lattices fabricated by 3D-printing using Formlabs Grey resin. (c) Kelvin Cell and (d) Octet Truss lattices infiltrated with PDMS.

2.3 Simulations

Finite element analysis was performed using COMSOL Multiphysics to estimate the elastic modulus of lattice-based IPCs. This was based on CAD files generated from the SOLIDWORKS lattice models. In the geometry settings of COMSOL, the respective lattice models were then added to a matrix so that IPC cubes were obtained. Tetrahedral meshes were generated with 200,000 elements or more, while ensuring mesh convergence. These correspond to more than 1,000,000 degree-of-freedom. The model was defined using linear elastic materials in the “Solid Mechanics” module in COMSOL and solved with the stationary solver

employing the direct MUMPS method. Since the present study focuses only on the small, elastic deformations, the “nonlinear geometry” option was not necessary and therefore, not included in the simulations, so that computational time can be shortened significantly.

The IPC cubes were compressed to 1% strain along the z-axis, and the resultant z-component reaction force was calculated via integration of the top surface to calculate the effective stress and overall modulus. The elastic modulus of the matrix relative to the struts, E_m/E_s , was varied over several orders of magnitude for the various IPC models to study its influence on the overall modulus of the IPC.

3. ANALYTICAL MODEL

The main theoretical models that are often used to predict the elastic moduli of IPCs are summarized below, before the presentation of our analytical model that accounts for reinforcement bending. The IPC modulus (E) and the matrix modulus (E_m) were normalized by the modulus of the reinforcement/ scaffold material (E_s), as E/E_s and E_m/E_s respectively, while f_s refers to the fraction of volume in the IPC occupied by the reinforcement material.

3.1 Isostrain

The isostrain limit represents the theoretical upper limit for composite modulus and is achieved when the composite behaves as if its constituents are arranged in parallel i.e. each constituent is subjected to the same strain. It is also referred to as the “rule of mixtures” and can be described by the following equation:

$$\frac{E}{E_s} = f_s + \frac{E_m}{E_s}(1 - f_s) \quad (1)$$

3.2 Isostress

The isostress limit is typically the lowest bound for composite modulus and is achieved when the composite behaves as if its constituents are arranged such that each constituent is subjected to the same stress. It is represented by the following equation:

$$\frac{E}{E_s} = 1/[f_s + \frac{E_s}{E_m}(1 - f_s)] \quad (2)$$

3.3 Hashin and Shtrikman (H&S) bounds

The H&S bounds are widely used in literature to predict the modulus of composites. They are based on variational principles in the linear theory of elasticity and derived after a rigorous analysis of elastic strain energy. However, bending deformation was not considered in the model. Assuming isotropic materials with Poisson's ratio of 1/3, the H&S bounds can be expressed as [21,25]:

$$\text{H\&S upper bound: } \frac{E}{E_s} = 1 + (1 - f_s) / \left(\frac{1}{\frac{E_m}{E_s} - 1} + \frac{2}{3} f_s \right) \quad (3)$$

$$\text{H\&S lower bound: } \frac{E}{E_s} = \frac{E_m}{E_s} + f_s / \left[\frac{1}{1 - \frac{E_m}{E_s}} + \frac{2}{3} (1 - f_s) \left(\frac{E_s}{E_m} \right) \right] \quad (4)$$

3.4 Reinforcement Bending and Compression Model

Fig. 3: (a) Geometric model for a lattice-based IPC under uniaxial compression. (b) Bending of the horizontal struts during compression.

To investigate the elastic response of an IPC where the reinforcements can potentially bend, we first define the variables using a generic schematic model shown in Fig. 3. It is important to note that our analysis is not limited to this geometry; it is simply to provide a visual aid to the following discussion. Here, we have assumed a unit cell of IPC with a volume proportional to $L \times L \times L$, made up of reinforcement trusses with a cross-sectional area proportional to $t \times t$. A compressive force, F , is applied on the unit cell.

The modulus of the IPC can be interpreted as the contribution of the i) compression of the struts and matrix, and the ii) bending of the horizontal beam surrounded by the matrix. Assuming that these two mechanisms are coupled in an isostress configuration, the overall strain (ϵ_{total}) of the unit cell and therefore, the entire composite can be expressed as

$$\epsilon_{total} \approx \epsilon_{comp} + 2\delta/L. \quad (5)$$

where ε_{comp} and $2\delta/L$ refer to the strain contributed by compression and bending respectively.

Regarding (i), the contribution from segment A can be neglected, which is a reasonable assumption for low reinforcement volume fractions, where $t \ll L$. The modulus of the unit cell due to compression is then largely contributed by the compression of segment B, which consists of the vertical beams and the matrix arranged in an isostrain configuration (i.e. $E_{comp} = E_s \left(\frac{t}{L}\right)^2 + E_m \left[1 - \left(\frac{t}{L}\right)^2\right]$, as shown earlier in Eq. 1). The corresponding strain ε_{comp} , can then be expressed as

$$\varepsilon_{comp} = \frac{\alpha F/L^2}{E_{comp}} = \frac{\alpha F/L^2}{E_s \beta' \left(\frac{t}{L}\right)^2 + E_m \left[1 - \beta' \left(\frac{t}{L}\right)^2\right]}, \quad (6)$$

where α and β' are constants added to account for the geometry of the reinforcement phase.

The deflection due to bending, δ , of the horizontal beams can be expressed as

$$\delta \propto (\alpha F - F_m) \frac{L^3}{E_s I} \propto (\alpha F - F_m) \frac{L^3}{E_s t^4}, \quad (7)$$

where I is the second moment of inertia and is proportional to t^4 [24]. F_m is the reaction force from the matrix material as a result of being compressed by the bent beam, which can be derived as

$$F_m \propto E_m \left(\frac{\delta}{L}\right) L^2. \quad (8)$$

Eq.s 5 to 8 can be combined to give the following expression for the overall elastic modulus of the IPC, E , as

$$\frac{E}{E_s} = \frac{F/L^2}{\varepsilon_{total}} \left(\frac{1}{E_s}\right) = \frac{(2+k)/k}{\frac{1}{\beta f_s + \frac{E_m}{E_s}(1-\beta f_s)} + \frac{2\gamma}{f_s^2 + \gamma k \frac{E_m}{E_s}}}, \quad (9)$$

where β , γ , and k are constants related to the geometry of the lattice. Details on the derivation of Eq. 9 can be found in the appendix.

The first term $\left(\frac{1}{\beta f_s + \frac{E_m}{E_s}(1-\beta f_s)}\right)$ in the denominator accounts for compression/tension, while the second term $\left(\frac{2\gamma}{f_s^2 + \gamma k \frac{E_m}{E_s}}\right)$ accounts for bending. These two mechanisms are arranged in series relative the loading direction (*i.e.* isostress configuration), resulting in the reciprocal form. Due to assumptions related to the simplification of f_s , the model represented by Eq. 9 is limited to describing IPCs with low reinforcement volume fraction (*i.e.* $f_s < 0.3$).

For reinforcement structures where the deformation is bending-dominated without the matrix (*e.g.* Kelvin Cell), $\frac{E}{E_s} = f_s^2$ when $E_m = 0$ [24]. Applying this limiting condition in Eq. 9 and ignoring the first term in the denominator, we get $k = \frac{2}{2\gamma-1}$. The model then simplifies to the following,

$$\frac{E}{E_s} = \frac{\frac{2\gamma}{1}}{\frac{1}{\beta f_s + \frac{E_m}{E_s}(1-\beta f_s)} + \frac{2\gamma}{f_s^2 + \frac{E_m}{E_s}\left(\frac{2\gamma}{2\gamma-1}\right)}} \quad . \quad (10)$$

Similarly, when the deformation of the reinforcement structure is compression-dominated, $\frac{E}{E_s} \propto f_s$ when $E_m = 0$ [24]. For the Octet Truss design, the proportionality constant is 0.2 [25]. By imposing this condition on Eq. 9 and ignoring the second term in the denominator, we get $\beta = 0.2$ so that Eq. 9 for Octet Truss can be simplified to

$$\frac{E}{E_s} = \frac{\frac{(2+k)/k}{1}}{\frac{1}{0.2 f_s + \frac{E_m}{E_s}(1-0.2 f_s)} + \frac{2\gamma}{f_s^2 + \gamma k \frac{E_m}{E_s}}} \quad . \quad (11)$$

4. RESULTS

4.1 Effect of Matrix Modulus

Fig. 4: Relative elastic modulus of interpenetrating phase composite, E/E_s , as a function of relative matrix modulus, E_m/E_s , for (a) $f_s = 0.03$ (b) $f_s = 0.17$ (c) $f_s = 0.30$. Simulation data from Octet Truss and Kelvin Cell IPCs were compared to the models from Eq. 10 (red line) and 12 (green line), as well as other analytical models (dashed curves). (d) Simulated data on the ratio of Kelvin Cell IPC modulus to Octet Truss IPC modulus, E_{KC}/E_{OT} , and its trend with respect to the ratio of the matrix modulus to reinforcement modulus, E_m/E_s .

Table II: Values of fitting parameters (without *) from Eq. 10 and Eq. 11 obtained through least-square fitting with simulation results.

Parameter	Kelvin Cell (Eq. 10)	Octet Truss (Eq. 11)
β	4.7	0.2*
γ	1.75	0.02
k	0.8*	4.5

*Calculated from limiting condition of $E_m = 0$ (*i.e.* not fitted). Added for comparison purposes.

Fig. 5: Relative contributions of compressive and bending mechanisms to the relative elastic modulus of Kelvin Cell and Octet Truss IPCs. The bold curves are the theoretical predictions shown earlier in Fig. 4a-b, while the dotted and dashed curves are the respective compression and bending contributions.

The relative elastic modulus E/E_s of the lattice IPCs was plotted against the relative matrix modulus E_m/E_s at various lattice (*i.e.* reinforcement) volume fractions, f_s , and presented in Fig. 4a-c. The data was taken from simulation results of IPCs with Octet Truss and Kelvin Cell reinforcement lattices and fitted with the theoretical models from Eq. 10 (bending-dominated deformation) and Eq. 11 (compression-dominated deformation). The values of the

fitted parameters are listed in Table II. It can be observed that β and γ are significantly larger for Kelvin Cell compared to Octet Truss, indicating that reinforcement bending has a greater influence on the elastic response of Kelvin Cell IPC compared to Octet Truss IPC. This is illustrated in Fig. 5, where the fitted models are plotted again together with the corresponding contributions from bending and compression mechanisms. The ‘compression only’ curve is obtained by setting the second term in the denominators of Eq.s 10 and 11 to zero, while the ‘bending only’ curve is obtained by setting the corresponding first terms to zero. Due to the reciprocal form of Eq.s 10 and 11, the mechanism with the lower magnitude tend to exert a greater influence on the overall behavior. In the plots, we can see that the behavior in Kelvin Cell and Octet Truss IPCs are dominated by reinforcement bending and axial compression respectively, especially at low E_m/E_s .

Importantly, the models described the simulation data well over the entire range of E_m/E_s for the various volume fractions f_s , as shown in Fig. 4. In contrast, the H&S upper bound and the isostrain bound were only able to account for the data at higher E_m/E_s . The H&S lower bound and isostress bound were not particularly useful as the simulation data points resided far above their predictions.

The elastic behavior of the IPCs can generally be separated into three regimes:

- i) Lattice/ Reinforcement-dominated ($E_m/E_s \sim 0$)

When E_m/E_s is very small, E/E_s did not change appreciably with E_m/E_s , resulting in the data lining up horizontally on the left-side of Fig.s 4a-c. The modulus of Kelvin Cell IPC approached that of a bending-dominated lattice where $\frac{E}{E_s} = f_s^2$ [24], while that of Octet Truss IPC behaved like a compression-dominated lattice where $\frac{E}{E_s} = 0.2f_s$ [25]. In both cases, E/E_s is a function of f_s only and insensitive to E_m/E_s . Under such conditions, the matrix is not stiff

enough to influence the deformation behavior of the struts and the lattice structure is free to deform freely as if there is no matrix.

For the Kelvin Cell IPC, its lattice design enabled a bending-dominated deformation behavior which caused its modulus to be far below those predicted by other models (*e.g.* H&S (upper bound) and isostrain), which do not account for the bending behavior. The Octet Truss IPCs performed fairly close to the H&S upper bound and were stiffer than Kelvin Cell IPCs at low volume fractions due to its compression-dominated deformation. This can be visualized from the images in Figs 6-7, which depict the spatial stress distributions for different E_m/E_s . When the matrix modulus is low (*i.e.* $E_m/E_s < 0.01$), the Kelvin Cell deformed almost entirely through bending, with the stress concentrated at the joints (see Fig. 6). For Octet Truss, the stress was distributed more uniformly distributed along the length of the struts, although some bending of the trusses can also be observed, in line with observations made previously [24,26].

ii) Isostrain ($E_m/E_s \sim 1$)

At large E_m/E_s , E/E_s approached the isostrain condition (Eq. 1), which is theoretically the highest elastic modulus a composite can achieve. This is because the matrix is so strong that the struts are unable to bend, and the strain on the IPC becomes purely compression-dominated.

iii) Transition

This regime lies between the “lattice/ reinforcement-dominated” and “isostrain” regimes and is well-described by Eq. 10 and Eq. 11, while the H&S and isostrain models are useful only at the upper E_m/E_s portion. With increasing E_m/E_s , the matrix in both Octet Truss and Kelvin Cell lattices help distribute the stress more evenly throughout the struts (Figs 6-7), resulting in the significant increase of the IPC modulus towards the isostrain limit (Figs 4a-c).

Fig. 6: Distribution of von Mises stress across Kelvin Cell-based IPCs at increasing E_m/E_s obtained from COMSOL simulations. The images were taken from IPCs with $f_s = 0.03$ that were compressed to 1% strain. The matrix phase was excluded from the images and the deformation was scaled by 30X for clarity.

Fig. 7: Distribution of von Mises stress across Octet Truss-based IPCs at increasing E_m/E_s obtained from COMSOL simulations. The images were taken from IPCs with $f_s = 0.03$ that were compressed to 1% strain. The matrix phase was excluded from the images and the deformation was scaled by 50X for clarity.

The “elbow” of the curves in Fig.s 4a-c shifts towards higher E_m/E_s when f_s is increased, indicating that denser lattices required stiffer matrices for the overall modulus to increase significantly. This is because denser lattices have proportionally thicker struts that require stronger resistance to reduce bending. Fig. 4d shows the effect of E_m/E_s on the ratio of Kelvin Cell IPC modulus to that of Octet Truss IPCs (*i.e.* E_{KC}/E_{OT}). It was found that Kelvin Cell and Octet Truss IPCs behave similarly when $E_m/E_s > 0.2$, which implies that the lattice/reinforcement geometry is not particularly important to the overall IPC modulus when the matrix is very rigid, since the deformation of the IPC would essentially be dictated by the matrix geometry instead. On the other hand, when $E_m/E_s < 0.2$, the low f_s Octet Truss tend to be stiffer than Kelvin Cell, as its deformation is compression-dominated, while that of Kelvin Cell is bending-dominated.

To predict the deformation behavior of an IPC with a given E_m/E_s and f_s , Eq. 10 and Eq. 11 were used to construct a chart that demarcates the boundaries between the reinforcement-dominated, transition and isostrain regimes (Fig. 8). Here, the elastic response of the IPC is considered “reinforcement-dominated” if the modulus predicted by Eq. 10 or Eq. 11 is less than 110% that of the reinforcement structure without matrix. In other words,

$$\frac{E}{E_s} < 1.1 \left(\frac{E}{E_s} \right)_{E_m=0} . \quad (12)$$

On the other hand, the elastic response would be considered to be “isostrain” if the modulus predicted by Eq. 10 and 11 is greater than 90% of the isostrain bound, that is,

$$\frac{E}{E_s} > 0.9 \left[f_s + \frac{E_m}{E_s} (1 - f_s) \right] . \quad (13)$$

Based on these definitions, we can therefore establish the criteria for each regime of elastic response as:

$$\text{Reinforcement-Dominated: } \frac{E_m}{E_s} < 0.074 f_s^{2.0} \quad (14)$$

$$\text{Isostrain: } \frac{E_m}{E_s} > 0.57 f_s^{0.6} \quad (15)$$

for a bending-dominated reinforcement design and

$$\text{Reinforcement-Dominated: } \frac{E_m}{E_s} < 0.025 f_s^{1.1} \quad (16)$$

$$\text{Isostrain: } \frac{E_m}{E_s} > 0.60 f_s^{0.4} \quad (17)$$

for a compression-dominated reinforcement design. The above framework is expected to be useful in guiding material and design selection for construction of IPCs with the desired elastic response.

Fig. 8: Chart of E_m/E_s vs. f_s showing the “isostrain”, “transition”, and “reinforcement-dominated” regimes for Kelvin Cell and Octet Truss IPCs.

4.2 Comparison of Analytical Model and Simulation Results with Experimental Data

The analytical, simulation and experimental values of the relative moduli, E/E_s , of different IPCs were plotted as a function of f_s for different relative matrix moduli, E_m/E_s , in Fig. 9. The experimental data for Kelvin-Cell- and Octet-Truss- reinforced IPCs were obtained in this present study, while moduli values of other IPCs (mostly foam-based) reported in the literature were also included for comparison in Figs 9b-d.

i) $E_m/E_s = 0$ (Fig. 9a, no matrix)

The experimental data points for the Kelvin Cell design was well-described by the bending-dominated model from Eq. 10, which reduces to a parabolic dependence on f_s when $E_m/E_s = 0$ [24]. The compression-dominated model from Eq. 11 slightly overestimated simulation and experimental data from Octet Trusses. The Octet Truss data also lay below the H&S upper bound with a similar slope, which was also observed previously [25].

ii) $E_m/E_s = 0.0004$ to 0.02 (Fig. 9b, *e.g.* polymer-based IPCs)

This dataset corresponds to IPCs with low E_m/E_s , which are typically observed in polymer-based IPCs. Experimental data were taken from Octet Truss and Kelvin Cell-based IPCs with $E_m/E_s \sim 0.003$, along with the corresponding set of analytical and simulation results. Also included were polystyrene/poly(ether-ester) polymer blends ($E_m/E_s \sim 0.02$) from Veenstra *et. al.* and gyroid-IPCs fabricated by 3D printing with dual polymer sources ($E_m/E_s \sim 0.0004$) from Al-Ketan *et. al.*

Under these conditions, the Kelvin Cell and Octet Truss data were aligned closer to each other than when $E_m/E_s = 0$, as the matrix restricted the bending of Kelvin Cells and caused the structure to increasingly compress/ stretch similarly to that in Octet Trusses. The higher volume fraction region remained inside or close to the “reinforcement-dominated” regime, where the IPC modulus is relatively insensitive to E_m/E_s . This explains why most of the

experimental data lay around the regions described by Eq. 10 and 12, despite the different conditions of E_m/E_s in these samples. Kelvin Cell IPCs with low f_s (< 0.1) are expected to experience more significant increase in modulus due to greater influence experienced from the suppression of bending by the matrix. This is evident from the red curves tapering horizontally towards low f_s .

iii) $E_m/E_s = 0.18$ (Fig. 9c, *e.g.* $\text{Al}_2\text{O}_3/\text{Al}$ systems)

When the matrix modulus was ‘moderately-high’, simulation and experimental data from Octet Truss, Kelvin Cell, and $\text{Al}_2\text{O}_3/\text{Al}$ IPCs converged into a linear distribution. The data generally agreed well with the H&S upper bound, Eq. 10 (bending-dominated) and Eq. 11 (compression-dominated). Lower f_s IPCs achieved the isostrain condition, while higher f_s IPCs remained at upper regions of transition regime.

iv) $E_m/E_s = 0.5$ (Fig. 9d, *e.g.* $\text{Al}_2\text{O}_3/\text{Ni}$ systems)

When the matrix modulus was ‘very high’, all IPCs effectively behaved at the isostrain condition, which is expected from the discussion above.

Overall, the experimental results generally agreed well with the predictions from simulations and theory. The model represented by Eq. 10 for IPCs with bending-dominated lattices also described IPCs with disordered phases well, which demonstrates its usefulness beyond orderly lattices. This is not particularly surprising since disordered reinforcement phases such as foams can have a characteristic length scale and fairly consistent unit cell geometry, which can be successfully described by the Kelvin Cell design [14].

Fig. 9: (a-d) IPC relative elastic modulus E/E_s as a function of volume fraction f_s at different relative matrix moduli E_m/E_s . Simulation and experimental results are compared to various analytical models. Some experimental results were obtained from the indicated literature references. In (b), the models and simulations were obtained using $E_m/E_s = 0.003$. (e) Chart shown earlier in Fig. 8, with the range of E_m/E_s and f_s investigated in (b) to (d) indicated with dotted lines.

5. DISCUSSION

Bending deformations are important considerations for understanding the elastic response of IPCs, especially since they are the dominant deformation mode in foams, which are widely used as the reinforcement phase. Eq. 10 contains a bending contribution term that was needed to predict the elastic modulus of IPCs when the matrix modulus was much lower than that of the reinforcement. In such cases, the bending of lattice struts caused the modulus of Kelvin Cell IPCs to be lower than that of Octet Truss IPCs, and much lower than those predicted by other theoretical models. The discrepancy occurred because these models did not consider bending of the reinforcement and were unable to account for the behavior under such conditions. In addition, Eq. 10 described the evolution of the elastic response of pure lattices to isostrain composites very well. It started with Ashby's parabolic law for bending-dominated lattices at low E_m/E_s and merged with the H&S (upper) and then the isostrain bounds as E_m/E_s increased. In contrast, the H&S upper bound over-estimated at low matrix moduli, while the predictions from the H&S lower bound and isostress bound were much too low.

From our analysis, we also observed that it is very difficult for cellular materials to achieve the isostrain limit at low volume fractions. Even compression-dominated structures like the Octet Truss generally do not bear load uniformly among their struts, resulting in an effective modulus that is a fraction of what is predicted by the rule-of-mixtures. Structures like honeycombs do perform at the isostrain limit, but only when loaded along one axis; if the loading direction deviates from this axis, the mechanical properties rapidly deteriorate.

Bending-dominated structures have very low moduli at low volume fractions, as described previously by Ashby. In an IPC configuration, the matrix material can help prevent bending and distribute the load more evenly among the struts, bringing the elastic response close to an isostrain behavior. In this regard, the framework presented in Fig. 8 helps predict whether the modulus is expected to behave in a i) reinforcement-dominated regime where the

struts are able to bend freely, ii) isostrain regime where the struts are unable to bend, or in a iii) transition regime that falls in between the reinforcement-dominated and isostrain regimes. This is particularly useful for IPCs with low-density reinforcements exhibiting bending-dominated behavior, which other than Kelvin Cells, also include polymer-based, $\text{Al}_2\text{O}_3/\text{Al}$ and $\text{Al}_2\text{O}_3/\text{Ni}$ IPCs with disordered-phases, whose moduli have also been included in the current analysis.

In lattice form, Octet Trusses are stiffer than Kelvin Cells for $f_s < 0.2$ (and vice versa) due to different deformation mechanisms (stretch/ compression-dominated vs. bending-dominated). When the matrix modulus increased to $E_m/E_s > 0.2$, design was no longer important as both Octet Truss and Kelvin Cell IPCs were observed to have similar moduli. Under such conditions, the matrix severely restricted bending in Kelvin Cell IPCs and caused it to deform like Octet Truss in an increasingly compression-dominated behavior.

The framework shown in Fig. 8 was developed by assuming low f_s lattices in the derivation of Eq. 10 and Eq. 11. At higher f_s , qualitatively, the matrix phase will function as the lattice phase instead and vice versa. Accordingly, the isostrain/transition and lattice-dominated/ transition lines (Fig. 8) are expected to dip down as f_s was increased further. This is supported by experimental results elsewhere [7,8], where $\text{Al}_2\text{O}_3/\text{Al}$ systems have been observed to progress towards isostrain conditions at higher f_s .

6. CONCLUSIONS

The elastic response of lattice-based IPCs with Kelvin Cell and Octet Truss reinforcements were studied. They were fabricated by infiltrating 3D-printed polymeric lattices with PDMS. Their elastic behavior was also simulated using finite element analysis. Focusing on low reinforcement volume fractions, an analytical model was developed to describe the bending behavior of the lattice scaffolds in the IPC.

When the matrix modulus was relatively low, the bending of lattice/ reinforcement struts caused the overall modulus to be much lower than those predicted by other theoretical models. Higher matrix modulus helped suppress the bending of struts and distributed the load more evenly throughout the lattices. This was most significant in Kelvin Cell IPCs with low reinforcement volume fractions. As the matrix modulus increased to above 20% that of the reinforcement material, no significant influence from lattice design was found as the matrix effectively inhibited bending deformations, forcing both Kelvin Cells and Octet Truss IPCs to deform similarly in an increasingly compression-dominated behavior.

A framework was developed to predict the whether the elastic modulus of an IPC is expected to behave in a i) reinforcement-dominated regime, ii) isostrain regime, or in a iii) transition regime that lies in between. The analytical model developed here was able to predict the elastic modulus of the IPC in the transition regime and described how the system evolved from lattices into an isostrain composite along with the increase in the matrix modulus. In addition, the model was able to describe experimental data obtained from IPCs with stochastic reinforcement structures (foam-based) made from polymers, $\text{Al}_2\text{O}_3/\text{Al}$, and $\text{Al}_2\text{O}_3/\text{Ni}$ systems. Given that large differences in moduli between matrix and reinforcement tend to occur in polymer-based IPCs, the analysis is particularly useful for these types of composites. The framework and analyses presented here are expected to be useful for material selection and design of future IPCs.

ACKNOWLEDGEMENTS

The authors would like to acknowledge funding for this project by the Temasek Research Fellowship.

APPENDIX**A1: Derivation of equation (9)**

Eq.s 6-8 are from section 3.4 and are restated here as follows:

$$\varepsilon_{comp} = \frac{\alpha F/L^2}{E_{comp}} = \frac{\alpha F/L^2}{E_s \beta' \left(\frac{t}{L}\right)^2 + E_m \left[1 - \beta' \left(\frac{t}{L}\right)^2\right]} \quad , \quad (6)$$

$$\delta \propto (\alpha F - F_m) \frac{L^3}{E_s t^4} = \gamma' (\alpha F - F_m) \frac{L^3}{E_s t^4} \quad , \quad (7)$$

$$F_m \propto E_m \left(\frac{\delta}{L}\right) L^2 = k' E_m \delta L \quad , \quad (8)$$

where α , β' , γ' and k' are the proportionality constants related to the geometry of the lattice.

Combining Eq.s 7 and 8, we get the following relationship:

$$\delta = \gamma' \alpha F \frac{L^3}{E_s t^4} - \gamma' k' E_m \delta \frac{L^4}{E_s t^4} \quad . \quad (A1)$$

Rearranging to move δ to the left-hand side, we get:

$$\delta \left(1 + \frac{\gamma' k' E_m L^4}{E_s t^4}\right) = \frac{\gamma' \alpha F L^3}{E_s t^4} \quad . \quad (A2)$$

The strain contributed by bending is therefore

$$\varepsilon_{bend} = \frac{2\delta}{L} = \frac{2\gamma' \alpha F L^2}{E_s t^4 \left(1 + \frac{\gamma' k' E_m L^4}{E_s t^4}\right)} \quad . \quad (A3)$$

Since the total strain is given by $\varepsilon_{total} \approx \varepsilon_{comp} + 2\delta/L$, by combining Eq. 6 and Eq. A3 we get

$$\frac{E}{E_s} = \frac{F/L^2}{\varepsilon_{total}} \left(\frac{1}{E_s}\right) = \frac{F/L^2}{\varepsilon_{comp} + 2\delta/L} \left(\frac{1}{E_s}\right) = \frac{F/L^2}{E_s} \left\{ \frac{1}{\frac{\alpha F/L^2}{E_s \beta' \left(\frac{t}{L}\right)^2 + E_m \left[1 - \beta' \left(\frac{t}{L}\right)^2\right]} + \frac{2\gamma' \alpha F L^2}{E_s t^4 \left(1 + \frac{\gamma' k' E_m L^4}{E_s t^4}\right)}} \right\}$$

$$\frac{E}{E_s} = \frac{1}{\frac{\alpha}{\beta' \left(\frac{t}{L}\right)^2 + \frac{E_m}{E_s} \left[1 - \beta' \left(\frac{t}{L}\right)^2\right]} + \frac{2\gamma' \alpha (L/t)^4}{1 + \frac{\gamma' k' E_m (L/t)^4}{E_s}}} \quad . \quad (A4)$$

For lattices with low reinforcement volume fraction, f_s can be approximated as $f_s \propto \left(\frac{t}{L}\right)^2$ [24].

Therefore, Eq. A4 can be simplified further to obtain.

$$\frac{E}{E_s} = \frac{\frac{1/\alpha}{\frac{1}{\beta f_s + \frac{E_m}{E_s}(1-\beta f_s)} + \frac{2\gamma}{f_s^2 + \gamma k \frac{E_m}{E_s}}}} \quad , \quad (\text{A5})$$

where α , β , γ , and k are constants related to the geometry of the lattice.

Taking the limit where $\frac{E}{E_s} = \frac{E_m}{E_s}$ when $f_s = 0$ (i.e. no matrix), therefore, it can be shown

in Eq. A5 that $\alpha = \frac{k}{2+k}$. Substituting this into Eq. (A5), we get the following general solution

represented by Eq. 9 earlier in the main text:

$$\frac{E}{E_s} = \frac{\frac{(2+k)/k}{\frac{1}{\beta f_s + \frac{E_m}{E_s}(1-\beta f_s)} + \frac{2\gamma}{f_s^2 + \gamma k \frac{E_m}{E_s}}}} \quad , \quad (9)$$

REFERENCES

- [1] H.A. Bruck, B.H. Rabin, Evaluating microstructural and damage effects in rule-of-mixtures predictions of the mechanical properties of Ni-Al₂O₃ composites, *J. Mater. Sci.* 34 (1999) 2241–2251. doi:10.1023/A:1004509220648.
- [2] D.E. Aldrich, Z. Fan, P. Mummery, Processing, microstructure, and physical properties of interpenetrating Al₂O₃/Ni composites, *Mater. Sci. Technol.* 16 (2000) 747–752. doi:10.1179/026708300101508748.
- [3] H.X. Peng, Z. Fan, J.R.G. Evans, Bi-continuous metal matrix composites, *Mater. Sci. Eng. A.* 303 (2001) 37–45.
- [4] M. Kouzeli, D. Dunand, Effect of reinforcement connectivity on the elasto-plastic behavior of aluminum composites containing sub-micron alumina particles, *Acta Mater.* 51 (2003) 6105–6121.
- [5] C. San Marchi, M. Kouzeli, R. Rao, J.A. Lewis, D.C. Dunand, Alumina–aluminum interpenetrating-phase composites with three-dimensional periodic architecture, *Scr. Mater.* 49 (2003) 861–866. doi:10.1016/S1359-6462(03)00441-X.
- [6] J. Zeschky, F. Goetz-Neunhoffer, J. Neubauer, S.H. Jason Lo, B. Kummer, M. Scheffler, P. Greil, Preceramic polymer derived cellular ceramics, *Compos. Sci. Technol.* 63 (2003) 2361–2370. doi:10.1016/S0266-3538(03)00269-0.
- [7] R.J. Moon, M. Tilbrook, M. Hoffman, A. Neubrand, Al–Al₂O₃ Composites with Interpenetrating Network Structures: Composite Modulus Estimation., *J. Am. Ceram. Soc.* 88 (2005) 666–674.
- [8] M.T. Tilbrook, R.J. Moon, M. Hoffman, On the mechanical properties of alumina–epoxy composites with an interpenetrating network structure, *Mater. Sci. Eng. A.* 393 (2005) 170–178.
- [9] M.L. Young, R. Rao, J.D. Almer, D.R. Haefner, J.A. Lewis, D.C. Dunand, Load partitioning in Al₂O₃–Al composites with three-dimensional periodic architecture, *Acta Mater.* 57 (2009) 2362–2375. doi:10.1016/j.actamat.2009.01.019.
- [10] L.D. Wegner, L.J. Gibson, The mechanical behaviour of interpenetrating phase composites – II: a case study of a three-dimensionally printed material, *Int. J. Mech. Sci.* 42 (2000) 943–964. doi:10.1016/S0020-7403(99)00026-0.
- [11] A.E. Pawlowski, Z.C. Cordero, M.R. French, T.R. Muth, J. Keith Carver, R.B. Dinwiddie, A.M. Elliott, A. Shyam, D.A. Splitter, Damage-tolerant metallic composites via melt infiltration of additively manufactured preforms, *Mater. Des.* 127 (2017) 346–351. doi:10.1016/j.matdes.2017.04.072.
- [12] H. Veenstra, P.C. Verkooijen, B.J. van Lent, J. van Dam, A.P. de Boer, A.P.H. Nijhof, On the mechanical properties of co-continuous polymer blends: experimental and modelling, *Polymer.* 41 (2000) 1817–1826.
- [13] L.D. Wegner, L.J. Gibson, The mechanical behaviour of interpenetrating phase composites — III: resin-impregnated porous stainless steel, *Int. J. Mech. Sci.* 43 (2001) 1061–1072. doi:10.1016/S0020-7403(00)00035-7.
- [14] R. Jhaver, H. Tippur, Processing, compression response and finite element modeling of syntactic foam based interpenetrating phase composite (IPC), *Mater. Sci. Eng. A.* 499 (2009) 507–517.
- [15] N. Dukhan, N. Rayess, J. Hadley, Characterization of aluminum foam–polypropylene interpenetrating phase composites: Flexural test results, *Mech. Mater.* 42 (2010) 134–141. doi:10.1016/j.mechmat.2009.09.010.
- [16] L. Wang, J. Lau, E.L. Thomas, M.C. Boyce, Co-continuous composite materials for stiffness, strength, and energy dissipation., *Adv. Mater. Deerfield Beach Fla.* 23 (2011) 1524–1529. doi:10.1002/adma.201003956.

- [17] A. Coldea, M.V. Swain, N. Thiel, Mechanical properties of polymer-infiltrated-ceramic-network materials, *Dent. Mater.* 29 (2013) 419–426. doi:10.1016/j.dental.2013.01.002.
- [18] O. Al-Ketan, R.K.A. Al-Rub, R. Rowshan, Mechanical Properties of a New Type of Architected Interpenetrating Phase Composite Materials, *Adv. Mater. Technol.* 2 (2017) 1600235. doi:10.1002/admt.201600235.
- [19] D.R. Clarke, Interpenetrating Phase Composites., *J. Am. Ceram. Soc.* 75 (1992) 739.
- [20] L.D. Wegner, L.J. Gibson, The mechanical behaviour of interpenetrating phase composites – I: modelling, *Int. J. Mech. Sci.* 42 (2000) 925–942.
- [21] Z. Hashin, S. Shtrikman, A variational approach to the theory of the elastic behaviour of multiphase materials, *J. Mech. Phys. Solids.* 11 (1963) 127–140.
- [22] L.I. Tuchinskii, Elastic constants of pseudoalloys with a skeletal structure, *Powder Metall. Met. Ceram.* 22 (1983) 588.
- [23] K.S. Ravichandran, Deformation behavior of interpenetrating-phase composites, *Compos. Sci. Technol.* 52 (1994) 541–549.
- [24] M.. Ashby, The properties of foams and lattices, *Philos. Trans. R. Soc. Math. Phys. Eng. Sci.* 364 (2006) 15–30. doi:10.1098/rsta.2005.1678.
- [25] V.S. Deshpande, N.A. Fleck, M.F. Ashby, Effective properties of the octet-truss lattice material, *J Mech Phys Solids.* (2001) 23.
- [26] Z. He, F. Wang, Y. Zhu, H. Wu, H.S. Park, Mechanical properties of copper octet-truss nanolattices, *J. Mech. Phys. Solids.* 101 (2017) 133–149. doi:10.1016/j.jmps.2017.01.019.