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Augmented Empirical Mode Decomposition for Complex-Valued Univariate Signals

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Abstract: In this paper, we propose an efficient extension of the standard empirical mode decomposition (EMD) for complex-valued univariate signal decomposition. The key idea of the extension is to convert a complex-valued univariate signal into a longer real-valued signal by augmenting the real part with the flipped imaginary part, and then to decompose it into intrinsic mode functions (IMFs) using the EMD once only. The bivariate IMFs are then retrieved from the obtained IMFs. Our empirical results on synthetic data show that the proposed method significantly outperforms the traditional bivariate EMD (BEMD) method in terms of the computational efficiency while producing a comparable extraction error. Moreover, the proposed method shows better micro-Doppler signature analysis performance on physically measured continuous-wave radar data than that of the BEMD.

1 Introduction

The empirical mode decomposition (EMD) [1] is a powerful tool for time-frequency signal analysis. It decomposes a real-valued univariate signal into a set of amplitude and frequency modulated (AM-FM) functions called the intrinsic mode functions (IMFs). Different from the Fourier analysis, the basis function for EMD decomposition is not predefined. Instead, the EMD removes the lower frequency component from the input signal adaptively and locally at each iteration. Due to these properties, the EMD has successfully been deployed in analysis of nonlinear and nonstationary signals (e.g., speech signal [2, 3] and biomedical signal [4, 5]).

One of the main research streams on the EMD is to extend its capability to deal with bivariate (including complex-valued univariate) [6–10], trivariate [11, 12] or even multivariate [13–18] signals. For the case of bivariate signals, several methods have been recently proposed namely, complex EMD (CEMD) [6], rotation-invariant EMD (RI-EMD) [7], bivariate EMD (BEMD) [8], nonuniformly sampled BEMD (NS-BEMD) [9] and dynamically-sampled BEMD (DS-BEMD) [10]. In the CEMD, the complex values are firstly converted into an analytic form of signal based on the Hilbert transform [1]. The standard EMD is then applied to the positive and negative frequency components of the analytic form. Due to the possibility of having an inconsistent number of IMFs for each channel [10, 16, 19], the application of the CEMD is limited. Moreover, such channel-wise decomposition cannot guarantee that the same-indexed IMFs contain similar scales across data channels [16, 19].

Different from the CEMD, both the RI-EMD and BEMD are full bivariate extensions of the EMD method [10]. A bivariate/complex-valued univariate signal is firstly projected onto a set containing multiple directions. A tube-like 3D signal envelope is then computed by interpolating the extrema of these projections. The local mean of the envelope is finally computed similar to that of the standard EMD. The number of directions taken to project the bivariate signal is the main difference between the RI-EMD and the BEMD methods [10]. The RI-EMD takes two directions while the BEMD can take any number of directions. Because of the flexibility, the BEMD is more appropriate for real-world signals than the RI-EMD. The last two methods, NS-BEMD and DS-BEMD, are extensions of the BEMD method in terms of the selection process of the directions for projection.

Although the BEMD has shown promising performance in decomposing the bivariate signals [20, 21], it is well known that the BEMD suffers from high computational cost [14, 15]. This is due to

the nature of projecting bivariate signals, where the number of directions becomes a tradeoff between the decomposition accuracy and the computational cost [10].

In this paper, we propose an efficient extension of the standard EMD for complex-valued univariate signal decomposition. The key idea is to augment the real part with the flipped imaginary part of the input complex-valued univariate signal, and then use the EMD on the resultant longer real-valued signal once only. The IMFs obtained from the EMD decomposition are then converted to bivariate IMFs (BIMFs). We note here that the proposed method is different from the channel-wise EMD decomposition proposed in [6] and mentioned in [16]. Particularly, the same number of mode functions for each real and imaginary channel is guaranteed since only one EMD is performed. Moreover, the same-indexed BIMF over the real and the imaginary parts will have a similar scale.

The main contributions of this paper can be enumerated as follows: i) proposal of an efficient extension of the standard EMD method for complex-valued univariate signal decomposition; ii) provision of extensive empirical results to validate the proposed method; iii) provision of analysis results showing that some unique signal characteristics may not appear on both the real and the imaginary channels simultaneously. In such a case, the proposed treatment of signal features can have a better resolution than that of the conventional approach.

The paper is organized as follows: the background knowledge on the EMD and the BEMD methods is provided in Section 2 for immediate reference. In Section 3, we introduce the proposed extension of the standard EMD method for complex-valued univariate signal decomposition. Section 4 presents our experimental settings and numerical results with discussions followed by experimental results on physically measured radar signals in Section 5. Some concluding remarks are presented in section 6.

2 A brief review of EMD and BEMD

The main idea of the EMD is to treat a real-valued univariate signal as fast oscillations being superimposed on slow oscillations. As an extension of the notion, the BEMD treats a bivariate signal as fast rotations being superimposed on slow rotations. The EMD and the BEMD methods are briefly reviewed in this section.

2.1 EMD

Given a real-valued univariate signal $\mathbf{x} \in \mathbb{R}^{1 \times d}$, the standard EMD decomposes it into a set of oscillating waveforms centered at zero [1] as: $\mathbf{x} = \sum_{l=1}^L \mathbf{m}_l + \mathbf{q}_L$, where $\mathbf{m}_l \in \mathbb{R}^{1 \times d}$, $l = 1, \dots, L$, denotes the l -th IMF and $\mathbf{q}_L \in \mathbb{R}^{1 \times d}$ is the residual resulted after the L -th IMF is computed. Every IMF satisfies the following two conditions [1]: (i) The number of local extrema and zero-crossings are either equal or differ by at most one; and (ii) at any point of time, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. The process for computing a set of IMFs is called a ‘sifting process’ which is summarized in Algorithm 1.

Algorithm 1 Empirical mode decomposition

```

1: Input: A real-valued univariate signal  $\mathbf{x} \in \mathbb{R}^{1 \times d}$ 
2: Initialize:  $\tilde{\mathbf{x}} \leftarrow \mathbf{x}$  and  $l \leftarrow 1$ 
3: do
4:   Identify the entire local extrema of  $\tilde{\mathbf{x}}$ 
5:   Construct a lower envelope  $\tilde{\mathbf{x}}_{\min}$  (cf. upper envelope  $\tilde{\mathbf{x}}_{\max}$ )
   by interpolating between the adjacent minima (cf. maxima)
6:   Compute the envelope mean:  $\tilde{\mathbf{x}}_{\text{mean}} \leftarrow (\tilde{\mathbf{x}}_{\min} + \tilde{\mathbf{x}}_{\max})/2$ 
7:   if  $\mathbf{d} \leftarrow \tilde{\mathbf{x}} - \tilde{\mathbf{x}}_{\text{mean}}$  satisfies a stopping criterion [1] then
8:      $\mathbf{m}_l \leftarrow \mathbf{d}$  becomes the  $l$ -th IMF
9:   else
10:    Go to Line #4 with  $\tilde{\mathbf{x}} \leftarrow \mathbf{d}$ 
11:   end if
12:    $\mathbf{q}_l \leftarrow \tilde{\mathbf{x}} - \mathbf{m}_l$  becomes the  $l$ -th residual
13:    $\tilde{\mathbf{x}} \leftarrow \mathbf{q}_l$  and  $l \leftarrow l + 1$ 
14: while the residual  $\mathbf{q}_l$  becomes a monotonic function

```

Note: ‘A $\leftarrow$ B’ indicates that ‘B’ is assigned to ‘A’.

2.2 BEMD

Similar to the EMD method, the BEMD decomposes a real-valued bivariate signal $\mathbf{s} \in \mathbb{R}^{2 \times d}$ (or a complex-valued univariate signal $\mathbf{s} \in \mathbb{C}^{1 \times d}$ where its real and imaginary part respectively forms an individual data channel) into a set of rotational components namely, BIMFs $\mathbf{c}_k$, $k = 1, \dots, K$, as [8]: $\mathbf{s} = \sum_{k=1}^K \mathbf{c}_k + \mathbf{p}_K$, where $\mathbf{p}_K$ indicates the residual obtained after the K -th BIMF is computed. As shown in Algorithm 2, once the mean of the envelope curve is obtained (see Lines #7 to #14 of Algorithm 2) the similar sifting process to that of the EMD process is performed.

Algorithm 2 Bivariate EMD

```

1: Input: A complex-valued univariate signal  $\mathbf{s} \in \mathbb{C}^{1 \times d}$  (or real-
   valued bivariate signal  $\mathbf{s} \in \mathbb{R}^{2 \times d}$ )
2: Initialize:  $\tilde{\mathbf{s}} \leftarrow \mathbf{s}$  and  $k \leftarrow 1$ 
3: do
4:   Project  $\tilde{\mathbf{s}}$  onto a set of directions  $\theta_v = (\frac{2\pi v}{V})$ ,  $v = 1, \dots, V$ :
    $p_{\theta_v}(t) = \text{Re}(e^{-i\theta_v} \tilde{\mathbf{s}}(t))$ 
5:   Extract the locations  $\{t_i^v\}$  of the extrema of  $p_{\theta_v}(t)$  for  $\forall v$ 
6:   Interpolate the set  $\{(t_i^v, s(t_i^v))\}$  to obtain the envelope curve
   in direction  $p_{\theta_v}(t)$ 
7:   Compute the mean:  $\tilde{\mathbf{s}}_{\text{mean}} \leftarrow \frac{1}{V} \sum_k e(t)$ 
8:   if  $\mathbf{d} \leftarrow \tilde{\mathbf{s}} - \tilde{\mathbf{s}}_{\text{mean}}$  satisfies a stopping criterion [1] then
9:      $\mathbf{c}_k \leftarrow \mathbf{d}$  becomes the  $k$ -th BIMF
10:  else
11:    Go to Line #4 with  $\tilde{\mathbf{s}} \leftarrow \mathbf{d}$ 
12:  end if
13:   $\mathbf{p}_k \leftarrow \tilde{\mathbf{s}} - \mathbf{c}_k$  becomes the  $k$ -th residual
14:   $\tilde{\mathbf{s}} \leftarrow \mathbf{p}_k$  and  $k \leftarrow k + 1$ 
15: while the residual  $\mathbf{p}_k$  becomes a monotonic function

```

Note: ‘A $\leftarrow$ B’ indicates that ‘B’ is assigned to ‘A’.

3 Proposed method

In this section, we propose an extension of the EMD method namely, an augmented EMD (AEMD) for complex-valued univariate signal decomposition. The input complex-valued signal is converted into a longer real-valued signal by concatenating the real and the imaginary parts, and then decompose the concatenated signal into a set of IMFs using the standard EMD once only. The BIMFs are then retrieved from the obtained IMFs. The following subsections provide details of the proposed method.

3.1 Augmented EMD

3.1.1 Signal augmentation: Let $\mathbf{s} = [s_1, s_2, \dots, s_d] \in \mathbb{C}^{1 \times d}$ be a complex-valued univariate signal to be decomposed into a set of BIMFs. The proposed method starts with constructing an exchange matrix $\mathbf{E} \in \mathbb{N}^{d \times d}$ as [22]:

$$\mathbf{E}_{(a,b)} = \begin{cases} 1, & \text{if } b = d - a + 1 \\ 0, & \text{if } b \neq d - a + 1 \end{cases} \quad (1)$$

where (a, b) denotes a matrix coordinate. Note that the $\mathbf{E}$ is a column-reversed version of the identity matrix [22]. Using this exchange matrix, the input signal $\mathbf{s}$ is converted to $\tilde{\mathbf{s}}$ as follows:

$$\tilde{\mathbf{s}} = [\text{Re}(\mathbf{s}), (\text{Im}(\mathbf{s}) + \eta)\mathbf{E}], \quad (2)$$

where $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times 2d}$, the operation $[\cdot, \cdot]$ indicates a row vector concatenation, $\text{Re}(\mathbf{s})$ and $\text{Im}(\mathbf{s})$ respectively denote the real and the imaginary part of $\mathbf{s}$. Here, $(\text{Im}(\mathbf{s}) + \eta)\mathbf{E} \in \mathbb{R}^{1 \times d}$ indicates the time (i.e., left-right) flipped $\text{Im}(\mathbf{s})$ after shifting by a constant η , which is defined as follows:

$$\eta = \begin{cases} \frac{|\overline{\text{Re}(\mathbf{s})} - \overline{\text{Im}(\mathbf{s})}|}{-|\text{Re}(\mathbf{s}) - \text{Im}(\mathbf{s})|}, & \text{if } \overline{\text{Re}(\mathbf{s})} \geq \overline{\text{Im}(\mathbf{s})} \\ \frac{|\overline{\text{Re}(\mathbf{s})} - \overline{\text{Im}(\mathbf{s})}|}{|\text{Re}(\mathbf{s}) - \text{Im}(\mathbf{s})|}, & \text{if } \overline{\text{Re}(\mathbf{s})} < \overline{\text{Im}(\mathbf{s})} \end{cases}, \quad (3)$$

where $\overline{\text{Re}(\mathbf{s})}$ denotes the mean of $\text{Re}(\mathbf{s})$ and $|\cdot|$ denotes the magnitude operation. This step is to reduce the mean difference of the real and the imaginary parts while maintaining the local signal trend.

3.1.2 Signal discontinuity detection: As mentioned earlier in this section, the augmented signal $\tilde{\mathbf{s}}$ obtained using (2) is to be decomposed into a set of IMFs using the standard EMD method. However, it is worth noting here that the augmented signal may or may not contain a large signal discontinuity. This is because the $\tilde{\mathbf{s}}$ is a simple concatenation of the real part and the flipped imaginary part. As illustrative examples, Fig. 1 (a) and (b) respectively show the augmented signal obtained using synthetic signals s_{21} (9) and s_{22} (10).

In Fig. 1 (a), the ending point of the real part and the starting point of the imaginary part look quite close to each other. Moreover, both the real and the flipped imaginary parts are of the same frequency. In such a case, the proposed augmentation may not cause a large signal discontinuity. On the other hand, the concatenated signal shown in Fig. 1 (b) contains a large discontinuity in the amplitude as well as in the frequency. Since such discontinuity may affect the extracted IMFs, we propose to remove it. Particularly, if the input signal $\mathbf{s}$ satisfies either of the following two conditions, we proceed to Section 3.1.3, otherwise to Section 3.1.4.

- Is $|f_0 - f_1| \geq \frac{\max[f_0, f_1]}{10}$? Here, the symbols f_0 and f_1 respectively indicate the estimated instant frequency [23] at the edge point of $\text{Re}(\mathbf{s})$ and $\text{Im}(\mathbf{s})$.
- Is $|\text{Re}(\mathbf{s}_d) - \text{Im}(\mathbf{s}_d)| \geq \frac{\max[\text{Re}(\mathbf{s}), \text{Im}(\mathbf{s})]}{10}$? Recall that $\mathbf{s}_d$ denotes the last element of $\mathbf{s} \in \mathbb{C}^{1 \times d}$.

3.1.3 (Conditional step) Removal of the signal discontinuity based on an interpolation using a chirp signal: Signal smoothing in time-/frequency-domain [24, 25] and linear/nonlinear interpolation [25] are among the most popular choices to handle the

Fig. 1: An illustration of the proposed signal augmentation and the potential issue of a signal discontinuity.

(a) Obtained using a synthetic signal s_{21} shown in (9)
(b) Obtained using a synthetic signal s_{22} shown in (10)

discontinuity issue. However, the former is an inappropriate option to be incorporated with the AEMD because of the locality and adaptivity properties of the EMD method (see Section 2.1 for details). The main reason is that the smoothing changes the original signal around the edge where the two data channels are joined together. Since the decomposition error in an IMF is propagated to the following IMF with a large impact (see Section 3.2 for our empirical analysis), touching the original data triggers a significant error in the extracted IMFs.

Our proposed resolution belongs to the latter approach, interpolation, but different from those existing techniques in the literature. Essentially, we propose to interconnect one end of both real and imaginary parts using a frequency modulated sinusoidal wave (a.k.a. a chirp signal) $\mathbf{v}$. It is emphasized here that padding a sinusoidal signal is a natural approach to EMD since the core idea of EMD is to treat a signal as fast oscillations being superimposed on slow oscillations. The sinusoidal padding can thus be regarded as bringing the zero-padding effect as in other signal processing techniques (e.g., filtering) to the extracted IMFs. Moreover, the frequency modulation plays an important role in compensating the frequency differences existing between the real and the imaginary parts. The chirp signal $\mathbf{v} = [v(1), v(2), \dots, v(P)]^T$ is defined as follows:

$$v(t) = \{A \cos(2\pi f(t)t)\}_{t=1}^P, \quad (4)$$

where $f(t) = \frac{k}{2}t + f_0$ denotes the time-varying frequency, $k = \frac{f_1 - f_0}{P}$ indicates the changing rate of the frequency, f_0 and f_1 respectively indicate the frequencies at the beginning (i.e., $t = 1$) and at the end (i.e., $t = P$) of the signal. The P is fixed to an empirical value 1,000 if the sampling rate $f_s \geq 1\text{KHz}$, otherwise $P = f_s$. This P setting is observed to contain an enough number of oscillations. The amplitude A is defined as $A = \max[|s_{\max}|, |s_{\min}|]$ where $s_{\max} = \max[\max(Re(s)), \max(Im(s))]$, $s_{\min} = \min[\min(Re(s)), \min(Im(s))]$.

Both ends of the generated chirp signal $\mathbf{v}$ are then adjusted to ensure the local signal continuity as detailed in Algorithm 3 and shown in Fig. 2. The main idea here is to represent the signal continuity by locally computed signal slopes (e.g., m_a , m_v and m_b shown

in Lines #2, #4, #11 and #14 of Algorithm 3). We then segment $\tilde{\mathbf{v}} \in \mathbb{R}^{1 \times P}$, which is adjusted to both $Re(s)$ and $(Im(s) + \eta)E$, from $\mathbf{v} \in \mathbb{R}^{1 \times P}$ where $p \leq P$ (see Line #23 of Algorithm 3 for detail). As a result of this boundary adjustment step, the two signal discontinuities observed from the left panel of Fig. 2 are effectively alleviated as shown in the right panel of Fig. 2, respectively. With the segmented $\tilde{\mathbf{v}}$, (2) is now re-defined as follows:

$$\tilde{\mathbf{s}} = [Re(s), \tilde{\mathbf{v}}, (Im(s) + \eta)E], \quad (5)$$

where $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times (2d+p)}$.

Recall that this interpolation is a conditional step. As aforementioned, if the concatenated signal $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times 2d}$ by (2) does not contain a large discontinuity, then it is utilized for computing the BIMFs as described in the following subsection. Otherwise, the BIMFs are computed using the signal $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times (2d+p)}$ obtained by (5).

Algorithm 3 EdgeAdjust($Re(s)$, $\mathbf{v}$, $(Im(s) + \eta)E$)

```

1: Input:  $\mathbf{a} \leftarrow Re(s) \in \mathbb{R}^{1 \times d}$ ,  $\mathbf{v} \in \mathbb{R}^{1 \times P}$ , and  $\mathbf{b} \leftarrow (Im(s) + \eta)E \in \mathbb{R}^{1 \times d}$ 
   ▷ For simplicity,  $Re(s)$  and  $(Im(s) + \eta)E$  are respectively
   denoted as  $\mathbf{a} \in \mathbb{R}^{1 \times d}$  and  $\mathbf{b} \in \mathbb{R}^{1 \times d}$  within this algorithm
2:  $m_a = \mathbf{a}(1, d) - \mathbf{a}(1, d-1)$  ▷ Compute the slope of  $\mathbf{a}$ 
3: for  $i \leftarrow 2$  to  $P$  step 1 do
4:    $m_v = \mathbf{v}(1, i) - \mathbf{v}(1, i-1)$  ▷ Compute the slope of  $\mathbf{v}$ 
5:   if  $m_a, m_v \geq 0$  &&  $\mathbf{v}(1, i) > \mathbf{a}(1, d)$  then
6:      $i_{from} \leftarrow i$ ; BREAK
7:   else if  $m_a, m_v < 0$  &&  $\mathbf{v}(1, i) < \mathbf{a}(1, d)$  then
8:      $i_{from} \leftarrow i$ ; BREAK
9:   end if
10: end for
11:  $m_b = \mathbf{b}(1, 2) - \mathbf{b}(1, 1)$  ▷ Compute the slope of  $\mathbf{b}$ 
12:  $diff \leftarrow 100$  ▷ Variable initialization
13: for  $i \leftarrow P$  to  $i_{from}$  step -1 do
14:    $m_v = \mathbf{v}(1, i) - \mathbf{v}(1, i-1)$  ▷ Compute the slope of  $\mathbf{v}$ 
15:   if  $m_b, m_v \geq 0$  &&  $diff > \text{abs}(\mathbf{b}(1, 1) - \mathbf{v}(1, i))$  &&  $\mathbf{v}(1, i) < \mathbf{b}(1, 1)$  then
16:      $i_{to} \leftarrow i$ 
17:      $diff \leftarrow \text{abs}(\mathbf{b}(1, 1) - \mathbf{v}(1, i))$ 
18:   else if  $m_b, m_v < 0$  &&  $diff > \text{abs}(\mathbf{v}(1, i) - \mathbf{b}(1, 1))$  &&  $\mathbf{v}(1, i) > \mathbf{b}(1, 1)$  then
19:      $i_{to} \leftarrow i$ 
20:      $diff \leftarrow \text{abs}(\mathbf{v}(1, i) - \mathbf{b}(1, 1))$ 
21:   end if
22: end for
23: Output:  $\tilde{\mathbf{v}} \in \mathbb{R}^{1 \times P} \leftarrow \mathbf{v}(1, i_{from} : i_{to})$ 

```

Note 1: ' $A \leftarrow B$ ' indicates that ' B ' is assigned to ' A '.

Note 2: $\mathbf{a}(1, x)$ indicates a vector coordinate (e.g., the x -th element of the vector $\mathbf{a}$).

3.1.4 BIMFs retrieval from IMFs: As a result of the signal augmentation shown in (2) and (5), the complex-valued signal $\mathbf{s}$ now becomes a real-valued signal $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times (2d+p)}$ by (5) (or $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times 2d}$ by (2)). The augmented signal $\tilde{\mathbf{s}}$ is then input to the standard EMD, where L number of IMFs $\mathbf{M} = [\mathbf{m}_1^T, \dots, \mathbf{m}_L^T]^T \in \mathbb{R}^{L \times (2d+p)}$ if $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times (2d+p)}$ by (5) (or $\mathbf{M} \in \mathbb{R}^{L \times 2d}$ if $\tilde{\mathbf{s}} \in \mathbb{R}^{1 \times 2d}$ by (2)) is obtained. Here, the first d columns of $\mathbf{M}$ are the IMFs obtained using the real part of $\mathbf{s}$, while the middle p columns and the last d columns are the EMD results obtained using the padded chirp signal and the imaginary part, respectively. To separate these three sets of IMFs, $\mathbf{M}$ is divided into three subsets $\mathbf{M}^{real}$, $\mathbf{M}^{chirp}$ and $\mathbf{M}^{imag}$ as follows:

$$\begin{aligned}
\mathbf{M}^{real} &= [\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_d] \in \mathbb{R}^{L \times d}, \\
\mathbf{M}^{chirp} &= [\mathbf{M}_{d+1}, \mathbf{M}_{d+2}, \dots, \mathbf{M}_{d+p}] \in \mathbb{R}^{L \times p}, \\
\mathbf{M}^{imag} &= [\mathbf{M}_{(d+p)+1}, \mathbf{M}_{(d+p)+2}, \dots, \mathbf{M}_{2d+p}] \in \mathbb{R}^{L \times d},
\end{aligned} \quad (6)$$

Fig. 2: An illustration of the proposed signal augmentation and discontinuity removal steps on a synthetic signal s_{22} in (10).

where $M_i \in \mathbb{R}^{L \times 1}$ denotes the i -th column vector of M . Note that the M^{chirp} is not obtained if $\tilde{s} \in \mathbb{R}^{1 \times 2d}$ by (2) is decomposed (see Lines #14–#16 of Algorithm 4). The BIMFs of the proposed AEMD are then subsequently obtained as follows:

$$C = M^{real} + j \cdot (M^{imag} \cdot E) \in \mathbb{C}^{L \times d}, \quad (7)$$

where $j = \sqrt{-1}$ and the l -th row vector of C corresponds to the l -th BIMF. The M^{imag} is multiplied by the exchange matrix E again to retrieve the original time order of the imaginary part, which was flipped over in (2) and (5). Note that the M^{chirp} is not used in computing the BIMFs.

As aforementioned, it is worth emphasizing here that the proposed method is different from decomposing the real and the imaginary parts of a complex-valued signal individually. Essentially, as pointed out by [16, 19], such channel-wise EMD decomposition cannot guarantee an equal number of IMFs over the data channels. Moreover, there is no guarantee that the same-indexed IMFs contain similar scales of information across the data channels. Different from the channel-wise decomposition, the proposed method is free from such concerns.

Apart from the channel-wise decomposition issue, adopting the standard EMD to decompose the augmented signal $\tilde{s}$ may raise several concerns such as the mode-mixing problem, the over sifting problem and the signal boundary issue. These are well-known limitations of the standard EMD method. However, as aforementioned, the main objective of this work is to extend the standard EMD for efficient decomposition of complex-valued univariate signals. The proposed method shall be improved to handle these problems in our future work. The processing steps of the proposed AEMD method is summarized in Algorithm 4.

3.2 Local decomposition property of EMD

Recall that in (2) and (5) the real part and the time flipped imaginary part of s are concatenated with continuity checking to $\tilde{s}$. The EMD is then used to decompose the $\tilde{s}$ into a set of IMFs. We note here that our proposed idea is valid since the EMD decomposes the signal locally and adaptively. In this subsection, we empirically show the validity as well as the limitation of the proposed idea on a physically measured radar echo signal.

Let x be the real part of a complex-valued radar echo signal s returned from a helicopter mini-UAV (50 ms long, see Section 5 for

Algorithm 4 Augmented EMD

- 1: **Input:** A complex-valued univariate signal $s \in \mathbb{C}^{1 \times d}$
- 2: Construct an exchange matrix E using (1)
- 3: Calculate the mean difference η using (3)
- 4: $\tilde{s} \in \mathbb{R}^{1 \times 2d} \leftarrow [Re(s), (Im(s) + \eta)E]$ $\triangleright$ Signal augmentation
- 5: **if** the $\tilde{s}$ contains a large discontinuity (see Section 3.1.2) **then**
- 6: Generate a chirp signal v using (4)
- 7: $\tilde{v} \in \mathbb{R}^{1 \times p} \leftarrow \text{EdgeAdjust}(Re(s), v, (Im(s) + \eta)E)$ in Algo. 3
- 8: $\tilde{s} \in \mathbb{R}^{1 \times (2d+p)} \leftarrow [Re(s), \tilde{v}, (Im(s) + \eta)E]$
- 9: $M = [m_1^T, \dots, m_L^T]^T \in \mathbb{R}^{L \times (2d+p)} \leftarrow \text{EMD}(\tilde{s})$ in Algo. 1
- 10: $M^{real} \in \mathbb{R}^{L \times d} \leftarrow [M_1, \dots, M_d]$ $\triangleright M_i$ denotes the i -th column vector of M
- 11: $M^{chirp} \in \mathbb{R}^{L \times p} \leftarrow [M_{d+1}, \dots, M_{d+p}]$
- 12: $M^{imag} \in \mathbb{R}^{L \times d} \leftarrow [M_{(d+p)+1}, \dots, M_{2d+p}]$
- 13: **else**
- 14: $M = [m_1^T, \dots, m_L^T]^T \in \mathbb{R}^{L \times 2d} \leftarrow \text{EMD}(\tilde{s})$ in Algo. 1
- 15: $M^{real} \in \mathbb{R}^{L \times d} \leftarrow [M_1, \dots, M_d]$
- 16: $M^{imag} \in \mathbb{R}^{L \times d} \leftarrow [M_{d+1}, \dots, M_{2d}]$
- 17: **end if**
- 18: **Output:** $C \in \mathbb{C}^{L \times d} \leftarrow M^{real} + j \cdot (M^{imag} \cdot E)$ $\triangleright$ The i -th row vector of C corresponds to the i -th BIMF

Note: 'A $\leftarrow$ B' indicates that 'B' is assigned to 'A'.

more details). The EMD is performed on i) the entire 50ms long signal, denoted as $x^{1:50}$, and ii) a 20ms long segment (from 15ms to 35ms) of the signal, denoted as $x^{15:35}$, respectively. The two sets of IMFs, $m_l^{1:50}$ obtained from $x^{1:50}$ and $m_l^{15:35}$ obtained from $x^{15:35}$, are then directly compared to each other. Particularly, the absolute distance between $m_l^{15:35}$ and a (15ms to 35ms) subset of $m_l^{1:50}$ is computed for $l = 1, \dots, 4$, as shown in Fig. 3. Note that a zero-padding is applied to the first 15ms and the last 15ms in Fig. 3 to increase the readability of the results.

In Fig. 3, some differences (i.e., non-zero values) are observed around the signal boundaries (e.g., 15ms and 35ms). Note here that the boundaries can be regarded as time points where two signals are joined together. The differences indicate the error of extracting mode functions. As expected, no such difference is observed if no such boundary exists (e.g., around 20ms to 30ms in the figure). It is also observed from the figure that the difference in an IMF is propagated to the subsequent IMF with a large impact. However, the differences

Fig. 3: Absolute distance between $m_l^{15:35}$ and a (15ms to 35ms) subset of $m_l^{1:50}$ for $l = 1, \dots, 4$. A zero-padding is applied to the first and last 15ms.

Table 1 Computational complexity for processing one sifting iteration. Symbol $m_{l,r}$ indicates the l -th IMF computed at the r -th iteration, L denotes the number of IMFs, $M_{EMD}(m_{l,r})$ and $M_{BEMD}(m_{l,r}, v)$ respectively denote the number of extrema detected in $m_{l,r}$ using the standard EMD and the BEMD and V indicates the number of directions for projection.

Method	Numerical complexity $F(m_{l,r+1})$ for processing one sifting iteration
EMD	$18d + 15M_{EMD}(m_{l,r})$
BEMD	$d(11V + 2) + 15 \sum_{v=1}^{V/2} M_{BEMD}(m_{l,r}, v)$
AEMD	$18(2d) + 15M_{EMD}(m_{l,r})$ if $\tilde{s} \in \mathbb{R}^{1 \times 2d}$ $18(2d + p) + 15M_{EMD}(m_{l,r})$ if $\tilde{s} \in \mathbb{R}^{1 \times (2d+p)}$

(extraction error) can be ignored as they are insignificant in real-world applications (e.g., radar signal processing) and appear only at one end of the signal. These results and observations lead us to the conclusion that decomposing an augmented signal using the EMD method produces similar IMFs to those by decomposing the signals individually without augmentation.

3.3 Numerical complexity analysis

The main advantage of the proposed AEMD over the BEMD is its high computational efficiency. To gain a better understanding about the advantage, in this section, the standard EMD, BEMD and AEMD methods are analyzed in terms of their numerical complexity for processing one sifting iteration [15]. Particularly, as shown in Table 1, the numerical complexity of the EMDs is defined in terms of the number of floating point operations in which only the multiplications are considered [15].

It is observed from Table 1 that the complexity of the BEMD is significantly affected by the number of projection directions V on top of the signal length d and the number of extrema $M_{BEMD}(m_{l,r}, v)$ detected in the l -th IMF at the r -th iteration. Since the standard EMD is free from such projection operation, its numerical complexity is only influenced by the length d and the number of extrema $M_{EMD}(m_{l,r})$. Recall that the main idea of the AEMD is to decompose the augmented signal $\tilde{s}$ using the standard EMD. Thus, its complexity is equal to that of the EMD except for the signal length which becomes either $2d$ or $2d + p$ as shown in the last two rows of Table 1. Though the AEMD requires two more processing steps (e.g., signal augmentation and BIMFs retrieval) than the BEMD, their computational loads are negligible compared to the sifting process.

The total computational cost of these EMDs is defined over the entire number of IMFs L and the number of iterations R for a sifting process as follows [15]:

$$C = \sum_{l=1}^L \sum_{r=1}^R F(m_{l,r}). \quad (8)$$

4 Empirical performance evaluation on synthetic signals

The main goal of this section is to verify the effectiveness and efficiency of the proposed AEMD method in extracting BIMFs

from complex-valued univariate signals. To achieve this goal, two complex-valued synthetic signals are utilized in this experimental study. For performance comparison, the obtained evaluation results are compared with those by the conventional BEMD method. The AEMD method is then compared with two multivariate extensions of the EMD namely, the turning tangent EMD (2T-EMD) [14] and the eXtended EMD (X-EMD) [15] in terms of the best BIMFs extraction error. The following subsections detail the data, the evaluation setup and the obtained results.

4.1 Synthetic data: complex-valued univariate signals

As aforementioned, two complex-valued synthetic signals, s_{21} and s_{22} adopted from [15], are utilized in this experimental study. The former signal is defined as follows [15]:

$$s_{21} = d_1^{(21)} + d_2^{(21)} = \left\{ e^{\frac{j\pi}{4}} e^t \sin(80\pi t + 1.5) \right\} + \left\{ e^{-\frac{j\pi}{4}} \sin\left(2\pi[2.5(1+t)]^2\right) \right\}, \quad (9)$$

where $d_1^{(21)}$ and $d_2^{(21)}$ are two AM-FM planar components which are orthogonal to each other, and t indicates the time index. Similarly, the latter signal (see Fig. 5 (a)) is defined as follows [15]:

$$s_{22} = d_1^{(22)} + d_2^{(22)} + d_3^{(22)} = \{0.3 \cos(56\pi t) + 0.8j \sin(54\pi t)\} + \{1.7 \cos(20\pi t) + 3.2j \sin(22\pi t)\} + \left\{ e^{0.23t-0.46} \left[2.5 \cos(\pi(0.82t + 4.49)^2) + 1.1j \sin(\pi(0.82t + 4.49)^2) \right] \right\}, \quad (10)$$

where $d_1^{(22)}$, $d_2^{(22)}$ and $d_3^{(22)}$ are three different AM-FM components rotating elliptically as respectively shown in Fig. 5 (b), (c) and (d) (see the red dashed line).

The individual components, $d_l^{(21)}$ where $l \in \{1, 2\}$ shown in (9) and $d_l^{(22)}$ where $l \in \{1, 2, 3\}$ shown in (10), are the groundtruth BIMFs. Following the protocols on the synthetic signals reported in [10, 15], both signals s_{21} and s_{22} are defined on a time interval $T = [0; 1]$ and sampled at $f_s = 1\text{kHz}$.

4.2 Experimental settings

The proposed AEMD method has an adjustable parameter namely, the number of iterations R as a stopping criterion of the EMD sifting process (see Algorithm 1). Since this R setting is closely related to the mode function extraction accuracy, the proposed method shall be evaluated over various $R \in \{10, 25, 50, 75, 100\}$ settings. The BEMD method is evaluated at different number of projection directions $V \in \{4, 8, 16, 32, 64\}$ as in [10]. Similar to the proposed method, the BEMD method shall also be evaluated over $R \in \{10, 25, 50, 75, 100\}$.

The BIMFs extraction performance of the AEMD and BEMD methods is evaluated quantitatively and qualitatively. Under the former evaluation, the BIMFs extraction accuracy is computed by the following quadratic error [15]:

$$\epsilon_l = \sum_{i=1}^L \frac{\|c_l - \hat{c}_l\|^2}{\|c_l\|^2}, \quad (11)$$

where $\|\cdot\|$ indicates the Euclidean norm, c_l and $\hat{c}_l$ respectively indicate the l -th groundtruth and the estimated BIMF. We then compute the numerical computational complexity for processing one sifting iteration. These obtained numerical quantities are then investigated in a qualitative manner in which the groundtruth (see Section 4.1 for

Fig. 4: Comparison between the AEMD and BEMD methods in terms of the BIMFs extraction error
(a) Quadratic error ϵ_l evaluated on signal s_{21} and (b) Quadratic error ϵ_l evaluated on signal s_{22}

details) and the obtained BIMFs are visually compared. All experiments are performed on a PC of 3.2GHz CPU with 16GByte RAM under the Matlab platform [26]. The Matlab source code for EMD and BEMD used in our experiments is obtained from [27].

4.3 Results and Discussions

4.3.1 Quantitative performance evaluation: Fig. 4 (a) and (b) show the BIMFs extraction error of the compared methods evaluated on signals s_{21} and s_{22} , respectively. From these two subplots, it is observed that the BIMFs extraction error of the AEMD method decreases as the number of iterations R increases. For example, among the evaluated $R \in \{10, 25, 50, 75, 100\}$ settings, the AEMD method achieves the highest (i.e., the worst) and lowest (i.e., the best) errors at $R = 10$ and 100, respectively.

From Fig. 4 (a) and (b), it is also observed that the AEMD method produces comparable (see Fig. 4 (a)) or even lower (see Fig. 4 (b)) BIMFs extraction errors than that of the compared BEMD method. The lowest error achieved by the AEMD method (at $R = 75$ and 100) is about $2.6e-4$ higher than that of the BEMD method (at $V = 8$ and at $R = 100$) on the s_{21} signal (see Fig. 4 (a)). As shown in Fig. 4 (b) obtained on the s_{22} signal, the AEMD method achieves the lowest error $3.62e-2$ at $R = 100$, which is about $5.41e-2$ lower than that of the best performance of the BEMD method (at $V = 16$ and at $R = 10$).

Besides the errors discussed above, as shown in the last column of Table 2, the proposed AEMD method significantly outperforms the BEMD method in terms of the computational complexity (8). As

mentioned, the high computational complexity of the BEMD is due to its nature of projecting samples onto V projection directions (see Section 3.3 and Table 1 for details).

4.3.2 Qualitative performance evaluation: From our quantitative evaluation study provided in the previous subsection, the proposed method appears to have minor extraction errors (see Fig. 4). To gain a better understanding on the obtained quantities, it is worth exploring into from where the error occurs and how severe the error is. In this subsection, we evaluate the BIMFs extraction performance of the AEMD method measured on the s_{22} signal, by means of comparing the extracted BIMFs and the groundtruth BIMFs visually. Due to space constraint, the qualitative analysis result on the s_{21} signal is not provided here.

Fig. 5 (b), (c) and (d) respectively show the groundtruth BIMFs (dashed red line) and the extracted BIMFs (solid blue line) by the proposed method on $d_l^{(22)}$ where $l \in \{1, 2, 3\}$. Based on the results shown in Fig. 4 (b), the stopping criterion for the AEMD is set to $R = 100$ in which the best extraction accuracy was achieved.

As shown in Fig. 5, the AEMD produces almost no error for the time interval $T \in [0; 0.9]$. Only a minor discrepancy between the estimated BIMFs and the groundtruth BIMFs is observed from the remaining time interval $T \in [0.9; 1]$. The main source of such errors is decomposing the augmented signal $\tilde{s}$ (see (2) and (5)) using a single EMD. Even though a discontinuity alleviated sinusoidal signal is padded in (5), joining signals still triggers a minor difference. This trend of results is also congruent to our case study results provided in Section 3.2. However, we emphasize here that such error is minor and tolerable. Particularly, in real-world scenarios (see our results and discussion from Section 5), such error is negligible.

4.3.3 Analysis on SNR: Both the quantitative evaluation (see Section 4.3.1) and the qualitative evaluation (Section 4.3.2) assume that the signals are noise-free. In this sub-section, both the BEMD and AEMD methods are evaluated in terms of their sensitivity to noise using the synthetic signal s_{21} . Particularly, the white Gaussian noise is added to s_{21} to make its signal-to-noise ratio (SNR) $\in \{5, 10, \dots, 50\}$ dB. As shown in Fig. 6 (a) and (b), the BIMFs extraction error of the BEMD and AEMD methods are represented in a form of box plot over the SNRs. Note that the boxplot shown in Fig. 6 (a) is obtained over combinations of $V \in \{4, 8, 16, 32, 64\}$ and $R \in \{10, 25, 50, 75, 100\}$ while the plot shown in Fig. 6 (b) is obtained over $R \in \{10, 25, 50, 75, 100\}$. This is because the parameter settings which produce the best ϵ_l vary over SNRs.

It is observed from the two subplots that both methods work well at high SNRs (e.g., ≥ 40 dB). The proposed AEMD method produces almost no error while the BIMFs obtained by the BEMD contains minor errors. At SNR = 35 dB, the gap in error between the BEMD and AEMD methods increases significantly. To gain a better understanding about the results, we analyze the extracted $d_1^{(21)}$ (see Fig. 6 (c) and (d)) and $d_2^{(21)}$ (Fig. 6 (e) and (f)) by the BEMD and AEMD, respectively. The BEMD result as shown in Fig. 6 (c) and (e) contain severe error in the time interval of $T \in [0; 0.8]$. Contrarily, the proposed method (Fig. 6 (d) and (f)) shows such error only in the earlier time interval of $T \in [0; 0.2]$. The estimated $d_2^{(21)}$ (Fig. 6 (e) and (f)) contain more error than that of $d_1^{(21)}$ (Fig. 6 (c) and (d)) which is congruent to the results of Fig. 3. At low SNRs (e.g., ≤ 30 dB), both methods seem to be largely affected by the noise.

4.3.4 Summary of the results and observations: A summary of the results shown in Fig. 4 is provided in Table 2. To increase the representational clarity of Table 2, only the best BIMFs extraction errors of both AEMD and BEMD methods are tabulated together with the corresponding computational complexity C obtained using (8). Besides the results summary, our observations are summarized as follows:

- The proposed AEMD method showed comparable BIMFs extraction error to that of the BEMD method (see Fig. 4). However, the AEMD has significantly outperformed the BEMD in terms of the computational efficiency (see the last column of Table 2).

Fig. 5: The extracted BIMFs (solid blue line) by the proposed AEMD method. The groundtruth (dashed red line) is also shown in the subplots for qualitative performance evaluation

(a) Signal s_{22} , (b) Estimated $d_1^{(22)}$, (c) estimated $d_2^{(22)}$ and (d) estimated $d_3^{(22)}$ by the proposed AEMD

Table 2 Summary of the BIMFs extraction error in terms of the best extraction accuracy and the computational complexity C calculated using (8)

Method	The best BIMFs extraction error, ε_l		Computational complexity C computed on s_{21}
	computed on s_{21}	computed on s_{22}	
BEMD	$1.17\text{e-}3$ ($V = 8, R = 100$)	$9.03\text{e-}2$ ($V = 16, R = 10$)	$1.90\text{e+}7$ ($V = 8, R = 100$)
AEMD	$1.43\text{e-}3$ ($R = 75, 100$)	$3.62\text{e-}2$ ($R = 100$)	$0.57\text{e+}7, 0.76\text{e+}7$ ($R = 75, 100$)

- Our qualitative evaluation study showed that the BIMFs extraction error of the AEMD method was mainly occurred from one end of the signal (see Fig. 5). This was due to the nature of decomposing a concatenated signal (see Section 3.2).
- At high SNRs (e.g., $\geq 35\text{dB}$), the AEMD appeared to be less sensitive to noise than that of the BEMD. However, both methods were largely affected by the noise when the SNR is low (e.g., $\leq 30\text{dB}$).
- Table 2 showed the sensitivity of the BEMD method to both parameters settings (i.e., the stopping criterion R and the number of projection directions V). On the contrary, the proposed method appeared to be less sensitive to the parameter setting.

4.4 Comparison with multivariate extensions of the EMD method: 2T-EMD and X-EMD

As shown in Table 3, the BIMFs extraction error of the proposed method is now compared with that of the 2T-EMD and the X-EMD. Though both methods are extensions of the standard EMD for multivariate signals, only the bivariate case is considered in

Table 3 Comparison with multivariate variants of EMD: 2T-EMD [14] and X-EMD [15]

Sig- nal	ε_l with time interval $T = [0; 1]$			ε_l with $T = [-1; 0] \cup [1; 2]$		
	2T-EMD*	X-EMD*	Proposed	2T-EMD*	X-EMD*	Proposed
s_{21}	$4.20\text{e-}3$	$4.46\text{e-}3$	$6.32\text{e-}3$	$4.76\text{e-}1$	$5.52\text{e-}1$	$5.04\text{e-}1$
s_{22}	$4.35\text{e-}3$	$3.94\text{e-}3$	$1.86\text{e-}1$	$4.25\text{e-}2$	$4.21\text{e-}2$	$1.45\text{e-}1$

* The BIMFs extraction error ε_l of 2T-EMD and X-EMD are directly taken from [15]. These error values were obtained under similar evaluation settings with those of ours.

this comparison study. Since both 2T-EMD and X-EMD methods were evaluated on the synthetic signals s_{21} (9) and s_{22} (10) under the similar experimental settings (see [15] for details) with those of ours, their quadratic error ε_l (11) shown in Table 3 are directly taken from [15]. According to [15], for a fair comparison, the quadratic errors of the proposed method shown in Table 3 are obtained using a modified Cauchy-like criterion as a stopping rule of the sifting process. Particularly, the sifting process of the EMD method is stopped if 90% of values $\|m_{l,r+1}(t) - m_{l,r}(t)\| / \|m_{l,r}(t)\|$ for $t = 1, 2, \dots, 2d$, are lower than 10^{-2} [15]. Different from the results provided in Section 4.3.1, Table 3 contains one more set of results (see the last three columns of the table) obtained at time interval $T = [-1; 0] \cup [1; 2]$. This is a more difficult scenario than that of the continuous time interval $T = [0; 1]$.

It is observed from Table 3 that the quadratic errors of the proposed method are comparable to those of the 2T-EMD and X-EMD for signal s_{21} . For signal s_{22} , the AEMD achieves slightly higher errors for both time intervals than those of the 2T-EMD and X-EMD. It is worth noting here that the physical meaning of the differences

Fig. 6: Effects of the SNR on the BIMFs extraction error ϵ_i obtained using the signal s_{21}

(a) BIMFs extraction accuracy over SNRs by the BEMD, (b) BIMFs extraction accuracy over SNRs by the AEMD, (c) estimated $d_1^{(21)}$ by the BEMD, (d) estimated $d_1^{(21)}$ by the AEMD, (e) estimated $d_2^{(21)}$ by the BEMD, and (f) estimated $d_2^{(21)}$ by the AEMD.

in error are insignificant. As discussed in Section 4.3.2, the majority of the error of the AEMD method comes from the signal boundaries, which can be ignored in real-world applications.

5 m-DS analysis on measured radar echo data from a mini-UAV

Both the BEMD and AEMD methods are now evaluated under the radar micro-Doppler signature (m-DS) analysis platform. A radar echo signal (of 50ms long) returned from a helicopter mini-UAV

is utilized in this study. The radar echo signal, which is complex-valued, was collected using an X-band continuous wave radar operating at 9.7GHz radio frequency and at 192KHz sampling rate. To reduce its sampling rate, the signal is decimated by a factor of two. For the stopping criteria, we set $R = 50$ iterations in which both methods produce similar BIMF extraction error as shown in Fig. 4 (a). For the BEMD, $V = 32$ directions are utilized.

The goal of this m-DS analysis study is to observe whether the blade flash effect [28] can be clearly captured by the BIMFs. The blade flash effect is a unique characteristic of the radar m-DS returned from rotating blades [28]. Since several physical parameters (e.g., the number of blades and its length, rotation speed etc.) of the

Fig. 7: The m-DS analysis result (in spectrogram) of the BEMD. For the spectrogram, the short time Fourier transform (STFT) is computed using a sliding Hamming window of length 2^8 with 90% overlapping.

(a) The original signal, (b) The second BIMF extracted by the BEMD, (c) The third BIMF extracted by the BEMD, (d) The fourth BIMF extracted by the BEMD

Fig. 8: The m-DS analysis result (in spectrogram) of the AEMD. See the Fig. 7 caption for the spectrogram parameters.

(a) The original signal, (b) The second BIMF extracted by the AEMD, (c) The third BIMF extracted by the AEMD, (d) The fourth BIMF extracted by the AEMD

aerial target can be retrieved from the blade flash patterns [29, 30], it is important to extract complete blade flashes clearly from the radar signal. We use the spectrogram [28] to visualize the m-DS analysis performance as shown in Fig. 7 (a) and Fig. 8 (a) which are obtained using the original radar signal (i.e., no EMD). Note that both sub-figures show four blade flashes (which appear as vertical bars of orange color) returned from a helicopter rotor with two blades while it rotates for two cycles.

Figs. 7 and 8 show two sets of spectrograms obtained using the BEMD and the AEMD, respectively. Particularly, spectrograms shown in (b), (c) and (d) of Figs. 7 and 8 respectively correspond to the second, the third and the fourth BIMF. As shown in Figs. 7 and 8, the blade flashes are captured at different orders of the BIMF. Particularly, the BEMD captures all the four blade flashes at the fourth BIMF (see Fig. 7 (d)) while the AEMD captures them at the third BIMF (see Fig. 8 (c)). It is noted from Fig. 7 that, however, the BEMD results contain the mode mixing problem. For example, the result shown in Fig. 7 (c) contains two of the four blade flashes shown in Fig. 7 (d).

We now analyze the m-DS results of the AEMD method. As shown in the first row of Fig. 9, the real part of the input radar signal contains relatively weak blade flash responses comparing with that of the imaginary part. Due to such a weak response, as shown in the middle row of Fig. 9, the left most blade flash response has been missed from the third IMF obtained using the real part signal. However, since the third IMF of the imaginary part as shown in the middle row contains all the four responses clearly, the resulted BIMF3 (see the bottom row of Fig. 9) and the spectrogram (Fig. 8 (b)) lose no blade flash information. These results and observations show that the proposed treatment of signal features by AEMD can handle weak responses either in real or imaginary part better than the conventional bivariate treatment of signal features. Moreover, the AEMD extracts the BIMFs at only 0.36s while the BEMD takes 20.66s, which is about 57 times slower than the AEMD.

6 Conclusion

In this paper, we proposed a novel extension of the standard EMD method, called the AEMD, for complex-valued **univariate** signal

Fig. 9: The m-DS analysis performance of the proposed AEMD method.

decomposition. Essentially, a conversion of the **complex-valued** input signal to an augmented **real-valued** signal was performed by concatenating the time flipped imaginary part to the real part of the original signal. To reduce the effect of amplitude gap triggered by the difference in the edges of the two signals, a sinusoidal signal was appended in between the real and the imaginary parts for discontinuity compensation. The augmented signal was then decomposed into a set of IMFs using the standard EMD method once only. The BIMFs were subsequently obtained by combining both the real and the imaginary parts of the IMFs. From our empirical experiments conducted on both synthetic and real radar measurement data, the proposed method was observed to outperform the standard BEMD method in terms of the computational complexity while producing the comparable BIMFs extraction error.

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