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Capacity Characterization of Uplink NOMA in Multi-UAV Networks

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Abstract—In this paper, non-orthogonal multiple access (NOMA) is investigated as a potential solution to address spectrum scarcity in unmanned aerial vehicle (UAV) communications. In particular, the ergodic capacity of uplink (UL) NOMA is characterized within a stochastic geometry framework for a multi-UAV network comprising a ground station (GS) and multiple UAVs. Our analysis reveals that UL NOMA enables fairness, in terms of ergodic capacity, to be achieved in the multi-UAV network. When compared against conventional UL orthogonal multiple access (OMA), we show that the ergodic capacity and ergodic sum capacity is higher for the proposed UL NOMA transmissions. Thus, the proposed scheme enables higher throughput to be achieved in multi-UAV networks while simultaneously improving spectrum efficiency.

Index Terms—Unmanned Aerial Vehicle, Ergodic Capacity, Rician Fading, Stochastic Geometry.

I. INTRODUCTION

With the forthcoming roll out of fifth generation (5G) networks, unmanned aerial vehicles (UAVs) are highly anticipated to take on many prominent roles. Already, use cases for multi-UAV networks to provide temporary cellular communications in disaster areas, and geographical surveying, have been envisaged [1]. However, the shortage of spectrum, i.e., spectrum scarcity, available for UAV communications is a major hurdle towards enabling UAVs a greater role in 5G networks [1], [2]. In particular, spectrum bands that have been allocated for UAV communications are also shared with other existing systems [1], [2].

To this end, power-domain non-orthogonal multiple access (NOMA) has been extensively studied, in the context of cellular systems, to improve spectrum efficiency [3], [4]. As opposed to conventional orthogonal multiple access (OMA) techniques where transmissions occur over separate time-frequency resource blocks, users in power-domain NOMA communicate over shared time-frequency resource blocks. For uplink (UL) NOMA transmissions in multi-UAV networks, UL UAVs simultaneously transmit the desire message over the same time-frequency resource block to a ground station (GS) by employing unequal power allocation [5]. Then, the GS recovers the desired messages by employing successive interference cancellation (SIC) on the received composite signal [1], [4]–[9].

Already, NOMA has been extensively studied in the literature for cellular systems, i.e., NOMA-aided cellular systems. For instance, analysis pertaining to the reliability [4], [6], [9] and ergodic capacity [4], [8]–[10] of NOMA-aided cellular systems over Rayleigh fading channels have been noted. Nonetheless, despite the above related studies, the resulting conclusions are not directly applicable for UAV communications. In particular, studies on NOMA-aided cellular systems have largely assumed Rayleigh fading channels, e.g., [4], [6]–[10]. Furthermore, when NOMA-aided cellular systems are evaluated within a stochastic geometry framework, a Poisson point process (PPP) is commonly assumed for the spatial locations of the users, e.g., [7], [9]. In contrast, it has since been shown that Rician fading and the binomial point process (BPP) are more suitable for the modeling of the UAV channel model [1], [2] and UAV spatial locations [11], respectively.

Evidently, a suitable evaluation framework for UL NOMA ergodic capacity in UAV communications is lacking in the literature. Towards this endeavor, this paper characterizes the UL NOMA ergodic capacity of multi-UAV networks over Rician fading channels within a BPP-based stochastic geometry framework. The major contributions of this paper are thus summarized as follows:

- By considering a BPP-based stochastic geometry model, we derive semi-analytical ergodic capacity expressions for both UL NOMA and UL OMA transmissions over Rician fading channels in a multi-UAV network.
- It is shown that fairness in the multi-UAV network, in terms of ergodic capacity, can be achieved by UL NOMA.
- It is also demonstrated that UL NOMA enables the ergodic capacity of each UAV to be increased while improving spectrum efficiency. Furthermore, at low-to-moderate transmit power regimes, the ergodic sum capacity of UL NOMA is higher than UL OMA in the multi-UAV network.

The remainder of this paper is organized as follows. The system model is introduced in Section II, with ergodic capacity expressions presented in Section III. Thereafter, numerical results are discussed in Section IV before the conclusion of the paper in Section V.

Fig. 1. A multi-UAV network with uplink NOMA, where UL UAVs operate at different altitudes.

II. SYSTEM MODEL

A multi-UAV network with UL NOMA, comprising N_U single antenna UL UAVs and a single antenna GS in a suburban setting, is depicted in Fig. 1. To model the suburban UAV channel, it is assumed that NOMA transmissions occur over Rician fading channels [2], with Doppler shift assumed to be compensated. Through a BPP, the UL UAVs are assumed to be uniformly distributed around a disc centered above the origin O , with altitude H_i , radius r_a , and angle $[0, 2\pi)$ [11]. The Euclidean distance (km) between UL UAV- i and the GS is denoted as:

$$d_i = \sqrt{D_i^2 + (H_i + \lambda_i)^2}, \quad (1)$$

where D_i is the Euclidean distance between the projection of UL UAV- i onto the ground plane and the GS, λ_i represents a separation altitude between the UL UAVs and the GS, and $\lambda_i < \lambda_{i+1}$. Using [11, eq. (3)], the probability density function (PDF) of d_i is:

$$f_{d_i}(w) = \frac{2w}{r_a^2}, \quad (2)$$

where $L_{m,i} \leq w \leq L_{p,i}$, $L_{m,i} = H_i + \lambda_i$ and $L_{p,i} = \sqrt{(H_i + \lambda_i)^2 + r_a^2}$. Through H_i and λ_i , the distribution of the UL UAVs can be made to conform to operating altitudes that are regulatory compliant.

At the GS, the detection of the signal-of-interest (SOI) starts at UL UAV- i . Specifically, the SOI from UL UAV- i is detected in the presence of multi-user interference (MUI) from the remaining $N_U - i$ UL UAVs. Thereafter, the GS subtracts UL UAV- i from the received composite signal via successive interference cancellation (SIC) [4]–[7]. The GS then repeats the SIC detection process for the remaining $N_U - i$ UL UAVs.

Let P_t be the normalized transmit power, h_i be the channel between UL UAV- i and the GS, α_i be the power allocation factor for UL UAV- i , and $\sum_{i=1}^{N_U} \alpha_i = 1$. Then, the instantaneous signal-to-interference-plus-noise ratio (SINR) of UL UAV- i at the GS is:

$$\text{SINR}_i = \frac{X_i}{1 + \sum_{j=1}^{i-1} Y_j + \sum_{k=i+1}^{N_U} Y_k}, \quad (3)$$

where n is the pathloss exponent, $X_i = P_i |h_i|^2$, $Y_j = P_j |h_j|^2$, $Y_k = P_k |h_k|^2$, $P_i = P_t \alpha_i d_i^{-n}$, $P_j = P_t \alpha_j \beta_j d_j^{-n}$, $P_k = P_t \alpha_k d_k^{-n}$, and $0 \leq \beta_j \leq 1$ is the strength of the residual MUI after SIC [1], [4], [8]. Conditioned on d_x^{-n} , $x \in \{i, j, k\}$, the variables X_i , Y_j , and Y_k are non-centered Chi-squared distributed random variables (RVs) with Rician K factor K_x .

At high P_t regimes, (3) becomes interference-limited. Thus, SINR_i can be simplified into the following instantaneous signal-to-interference ratio (SIR):

$$\text{SIR}_i = \frac{\tilde{X}_i}{\sum_{j=1}^{i-1} \tilde{Y}_j + \sum_{k=i+1}^{N_U} \tilde{Y}_k}, \quad (4)$$

where $\tilde{X}_i = \tilde{P}_i |h_i|^2$, $\tilde{Y}_j = \tilde{P}_j |h_j|^2$, $\tilde{Y}_k = \tilde{P}_k |h_k|^2$, $\tilde{P}_i = \alpha_i d_i^{-n}$, $\tilde{P}_j = \alpha_j \beta_j d_j^{-n}$, and $\tilde{P}_k = \alpha_k d_k^{-n}$. Similar to (3), $\tilde{X}_i$, $\tilde{Y}_j$, and $\tilde{Y}_k$ are non-centered Chi-squared distributed RVs with Rician K factor $\tilde{K}_x$, $x \in \{i, j, k\}$ when conditioned on d_x^{-n} . For the case of OMA transmissions, the instantaneous signal-to-noise ratio (SNR) of UL UAV- i at the GS is simply $\text{SNR}_i = \bar{X}_i$, where $\bar{X}_i = P_t d_i^{-n} |h_i|^2$ is a non-centered Chi-squared distributed RV with Rician K factor $\bar{K}_i$.

As the SOI detection of UL UAV- i is performed in the presence of MUI, with $\lambda_i < \lambda_{i+1}$, the power allocation factor α_i is heuristically defined as:

$$\alpha_i = \frac{\lambda_{N_U+1-i}}{\sum_{k=1}^{N_U} \lambda_k}, \quad (5)$$

to allow $\alpha_i > \alpha_{i+1}$ so that UL UAVs that are nearer to the GS are allocated higher transmit powers. Through such an approach, SIC at the GS is enabled while ensuring fairness amongst the UL UAVs [9].

III. ERGODIC CAPACITY DERIVATIONS

Based on [12] and [13, Lemma 1], we characterize in this section, the ergodic capacity of UL NOMA in multi-UAV networks within a stochastic geometry framework. Specifically, we present NOMA ergodic capacity expressions for UL UAV- i in the multi-UAV network. For comparison, we also present the OMA ergodic capacity expression for UL UAV- i .

A. Uplink NOMA Ergodic Capacity

The ergodic capacity for UL UAV- i (C_i^{NOMA}) is defined as $C_i^{\text{NOMA}} = E \left\{ \ln \left(1 + \text{SINR}_i \right) \right\}$, where $E \{\bullet\}$ is the statistical expectation operator. In the following theorem, a semi-analytical expression is derived for C_i^{NOMA} .

Theorem 1: The NOMA ergodic capacity expression for UL UAV- i is:

$$C_i^{\text{NOMA}} \approx \int_{L_{m,i}}^{L_{p,i}} \int_0^\infty \sum_{q=0}^{K_{ir}} \frac{(K_i)^q \exp(-K_i)}{\Gamma(q+1)^2} g_{q+1}(z) \times \exp(-m_i z) \left(\prod_{j=1}^{i-1} \tau_j(m_i z) \right) \left(\prod_{k=i+1}^{N_U} \tau_k(m_i z) \right) f_{d_i}(w_i) dz dw_i \quad (6)$$

where $\Gamma(\bullet)$ is the Gamma function, the function $g_{q+1}(z)$ is defined as

$$g_{q+1}(z) = \frac{\Gamma(q+2)}{z(q+1)} \left(1 - \frac{1}{(1+z)^{q+1}} \right), \quad (7)$$

$m_i = \frac{K_i+1}{P_i}$, and K_{tr} is the truncation order. The function $\tau_x(z), x \in \{j, k\}$ is defined as:

$$\tau_x(z) = \int_{L_{m,x}}^{L_{p,x}} M_{Y_x}(z) f_{d_x}(w_x) dw_x, \quad (8)$$

where $M_{Y_x}(z), x \in \{j, k\}$ is the moment generating function (MGF) of the RV Y_x , which is given as [14, Table. I]:

$$M_{Y_x}(z) = \frac{1 + K_x}{1 + K_x + P_x z} \exp\left(\frac{-K_x P_x z}{1 + K_x + P_x z}\right), \quad (9)$$

Proof: The proof is provided in Appendix A. ■

At high P_t regimes, asymptotic ergodic capacity becomes relevant as the UL NOMA transmissions become interference-limited. For UL UAV- i , the asymptotic ergodic capacity for UL UAV- i ($C_{i,\infty}^{NOMA}$) is defined as $C_{i,\infty}^{NOMA} = E\left\{\ln(1 + SINR_i)\right\}$. A semi-analytical expression is presented in the following theorem for C_i^{NOMA} .

Theorem 2: The asymptotic NOMA ergodic capacity expression for UL UAV- i is:

$$C_{i,\infty}^{NOMA} \approx \int_{L_{m,i}}^{L_{p,i}} \int_0^\infty \frac{1}{z} \left[1 - M_{\bar{X}_i}(z)\right] \times \left(\prod_{j=1}^{i-1} \mu_j(z)\right) \left(\prod_{k=i+1}^{N_U} \mu_k(z)\right) f_{d_i}(w_i) dz dw_i, \quad (10)$$

where $\mu_x(z), x \in \{j, k\}$ is defined as:

$$\mu_x(z) = \int_{L_{m,x}}^{L_{p,x}} M_{\bar{Y}_x}(z) f_{d_x}(w_x) dw_x. \quad (11)$$

Proof: The proof is provided in Appendix B. ■

B. OMA Ergodic Capacity

For OMA transmissions, the SOI of UL UAV- i is detected in the absence of MUI at the GS. Thus, the OMA ergodic capacity of UL UAV- i (C_i^{OMA}) is defined as $C_i^{OMA} = E\left\{\frac{1}{N_U} \ln(1 + SNR_i)\right\}$ [6, eq. (19)], where the factor of $\frac{1}{N_U}$ is due to OMA transmissions occurring over separate time-frequency resource blocks. Following the same approach in Appendix A, the OMA ergodic capacity expression for UL UAV- i is:

$$C_i^{OMA} \approx \frac{1}{N_U} \int_{L_{m,i}}^{L_{p,i}} \int_0^\infty \sum_{q=0}^{K_{tr}} \frac{(\bar{K}_i)^q \exp(-\bar{K}_i)}{\Gamma(q+1)^2} g_{q+1}(z) \times \exp(-\bar{m}_i z) f_{d_i}(w_i) dz dw_i, \quad (12)$$

where $\bar{m}_i = \frac{\bar{K}_i+1}{P_i d_i^{-n}}$. Thus, (12) can be used to provide a fair comparison against UL NOMA in the multi-UAV network within a stochastic geometry framework.

IV. NUMERICAL RESULTS

We present numerical and simulation results for both UL NOMA and OMA in the multi-UAV network. The simulation parameters are provided in Table I, with Monte Carlo simulations conducted with 10^5 samples.

Observation 1: Ergodic capacity fairness can be achieved for UL NOMA in the multi-UAV network.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Rician K Factors	10 dB [2, Table V]
Normalized Transmit Power	$0 \text{ dB} \leq P_t \leq 80 \text{ dB}$
Radius	$r_a = 10 \text{ km}$
Altitude	$H_i = 0.2 \text{ km}$
Separation Altitude	$\lambda_i = 1 + \frac{i-1}{2} \text{ km}$
Residual UL Interference	$\beta_j = (0.02)^2$ [8]
Pathloss Exponent	$n = 2$ [2, Table III]

Fig. 2. Comparison of the UL NOMA ergodic capacity in the multi-UAV network.

Observation 2: At low-to-moderate P_t regimes, higher ergodic capacity is achieved by UL UAVs nearer to the GS. In contrast, at high P_t regimes, higher ergodic capacity is achieved by UL UAVs that are further from the GS.

The UL NOMA ergodic capacity for the multi-UAV network is seen in Fig. 2 for $N_U = 5$. At very low P_t regimes, i.e., $P_t < 5 \text{ dB}$, it is seen that the UL UAVs exhibit similar ergodic capacities as $SINR_i, i \in \{1, \dots, N_U\}$ is noise-limited. As P_t increases, i.e., $5 \text{ dB} \leq P_t < 22 \text{ dB}$, UL UAV- i achieves higher ergodic capacity than UL UAV- $(i+1)$ for $i \in \{1, \dots, N_U\}$ due to weak MUI at the GS. When $23 \text{ dB} \leq P_t \leq 25 \text{ dB}$, i.e., high P_t regimes, it is observed that the UL UAVs achieve similar ergodic capacities due to stronger MUI. Specifically, the ergodic capacity of UL UAV- $i, i \in \{1, 2\}$ begins to plateau as $SINR_i$ becomes MUI-limited. Finally, for $P_t \geq 26 \text{ dB}$, the ergodic capacity of UL UAV- $(i+1)$ is higher than UL UAV- i for $i \in \{1, \dots, N_U\}$ due to an MUI-limited $SINR_i$. Furthermore, UL UAV- $i, i \in \{1, 2, 3\}$ is shown the approach the asymptotic capacity for $P_t > 35 \text{ dB}$.

Therefore, Fig. 2 shows that fairness in terms of ergodic capacity can be achieved for UL NOMA in the multi-UAV network at low and high P_t regimes. Also, operating at low-to-moderate P_t regimes allows nearby UL UAVs to achieve higher ergodic capacities. In contrast, operating at high P_t regimes enable UL UAVs that are further from the GS to operate at higher ergodic capacities than nearby UL UAVs. Thus, the operating P_t regime for UL NOMA in the multi-

Fig. 3. Impact of height on UL NOMA asymptotic ergodic capacity in the multi-UAV network for $H_i = H_U$.

Fig. 4. Comparison of the UL NOMA and UL OMA ergodic capacity in the multi-UAV network for $N_U = 2$.

UAV network can be chosen based on Quality-of-Service (QoS) requirements.

The impact of height on the UL NOMA asymptotic ergodic capacity is plotted in Fig. 3 for $H_i = H_U$. It is observed that increasing H_U slightly decreases the asymptotic ergodic capacity of UL UAV- $i, i \in \{1, \dots, 4\}$. The decrease in asymptotic ergodic capacity is due to a weaker SOI in $SIR_i, i \in \{1, \dots, 4\}$. On the contrary, UL UAV-5 is not impaired by any MUI. As such, increasing H_U leads to weaker residual SIC interference which results in slightly higher asymptotic ergodic capacity. However, unlike UL UAV- $i, i \in \{1, \dots, 4\}$, changes in β can greatly decrease the asymptotic ergodic capacity of UL UAV-5. Thus, Fig. 3 shows that varying H_U and β only greatly affects UL UAV-5 in the multi-UAV network.

Observation 3: In multi-UAV networks with UL NOMA, ergodic capacity at each UL UAV is increased while improving spectrum efficiency.

In Fig. 4, a comparison between UL NOMA and UL OMA in the multi-UAV network is shown for $N_U = 2$. Starting

Fig. 5. Comparison of the UL NOMA and UL OMA ergodic sum capacity in the multi-UAV network for $N_U = 5$.

with UL OMA, it is observed that the UL UAVs achieve similar ergodic capacities at low, moderate, and high, P_t regimes. In contrast, for $0 \text{ dB} \leq P_t \leq 15 \text{ dB}$, NOMA at UL UAV-1 and UL UAV-2 respectively attains higher and lower ergodic capacity than OMA at UL UAV- $i, i \in \{1, 2\}$. When $16 \text{ dB} \leq P_t \leq 20 \text{ dB}$, NOMA achieves higher ergodic capacity than OMA at UL UAV- $i, i \in \{1, 2\}$. As P_t increases beyond 20 dB , i.e., $21 \text{ dB} \leq P_t \leq 40 \text{ dB}$, it is observed that NOMA at UL UAV-1 and UL UAV-2 respectively attains lower and higher ergodic capacity than OMA at UL UAV- $i, i \in \{1, 2\}$. Thus, Fig. 4 shows that a multi-UAV network with UL NOMA can achieve greater ergodic capacity at the individual UAVs than UL OMA while improving the spectrum efficiency of UAV communications.

Observation 4: A multi-UAV network with UL NOMA achieves ergodic sum capacity that is higher, if not equivalent, than UL OMA at low-to-moderate P_t regimes.

The ergodic sum capacity comparison between UL NOMA and UL OMA in the multi-UAV network is shown in Fig. 5 for $N_U = 5$. It can be seen that at low P_t regimes, i.e., $0 \text{ dB} \leq P_t \leq 10 \text{ dB}$, both UL NOMA and UL OMA achieves similar ergodic sum capacity. As P_t increases, i.e., $10 \text{ dB} < P_t \leq 44 \text{ dB}$, the ergodic sum capacity of UL NOMA is higher than UL OMA as the SINRs of UL UAVs that are further away are not MUI-limited. However, when $P_t \geq 46 \text{ dB}$, the ergodic sum capacity of UL NOMA approaches the asymptotic ergodic sum capacity due to the SINRs of all UL UAVs becoming MUI-limited. Although UL OMA attains higher ergodic sum capacity than UL NOMA at asymptotic P_t regimes, the increase in throughput is at the cost of more spectrum utilization since MUI is absent in OMA transmissions. In contrast, UL NOMA is able to achieve ergodic sum capacity that is the same, if not higher, as compared to UL OMA at low-to-moderate P_t regimes. Thus, UL NOMA in multi-UAV networks is a promising solution to improve throughput and spectrum utilization in multi-UAV networks.

V. CONCLUSION

To address spectrum scarcity in UAV communications, the ergodic capacity multi-UAV networks with UL NOMA is analyzed within a BPP-based stochastic geometry framework in this paper. Extensive analysis shows that ergodic capacity fairness in the multi-UAV network is achievable in UL NOMA transmissions. Furthermore, higher UL NOMA ergodic capacity and UL NOMA ergodic sum capacity is achieved when compared against UL OMA transmissions. Therefore, we demonstrate that higher throughput and improved spectrum efficiency is achievable when UL NOMA is implemented in multi-UAV networks.

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APPENDIX A PROOF OF THEOREM 1

To begin, the PDF of X_i can be expressed as [15, Table I]:

$$\begin{aligned} f_{X_i}(x_i) &= m_i \exp(-K_i - m_i x_i) I_0(2\sqrt{K_i m_i x_i}) \\ &\stackrel{(a)}{=} \sum_{q=0}^{K_{tr}} \frac{(K_i)^q \exp(-K_i)}{\Gamma(q+1)^2} m_i^{q+1} x_i^q \exp(-m_i x_i), \end{aligned} \quad (13)$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind with zero order [16, eq. (9.6.10)] and (a) is due to [16, eq. (9.6.10)].

Next, let $g(z) = \ln(1+z)$ and $g_k(z) = \frac{d^k}{dz^k} g(z) z^q$, where it is easily shown that:

$$g_k(0) = \begin{cases} 0, & \text{for } k < q \\ g(0), & \text{for } k = q. \end{cases} \quad (14)$$

When $k = q+1$, $g_{q+1}(z)$ is evaluated to be

$$\begin{aligned} g_{q+1}(z) &= \frac{d^{q+1}}{dz^{q+1}} z^q \ln(1+z) \\ &\stackrel{(a)}{=} \frac{d^{q+1}}{dz^{q+1}} z^{q+1} {}_2F_1(1, 1; 2; -z) \\ &\stackrel{(b)}{=} \frac{\Gamma(q+2)}{z(q+1)} \left[1 - \frac{1}{(1+z)^{q+1}} \right], \end{aligned} \quad (15)$$

where (a) is due to [16, eq. (15.1.3)], (b) is due to [16, eq. (15.2.3)] and [16, eq. (15.3.1)], and ${}_2F_1(\bullet, \bullet; \bullet; \bullet)$ is the Gauss hypergeometric function [16, eq. (15.1.1)].

Then, $E\{g(\text{SINR}_i)\}$ conditioned on $Y_j, Y_k, d_x^{-n}, x \in \{i, j, k\}$, is expressed as [12, eq. (2)]:

$$\begin{aligned} E\{g(\text{SINR}_i)\} &\approx \int_0^\infty \sum_{q=0}^{K_{tr}} \frac{(K_i)^q \exp(-K_i)}{\Gamma^2(q+1)} \\ &\quad \times (m_i[1+y])^{q+1} g(z) z^q \exp(-m_i z[1+y]) dz \\ &\stackrel{(a)}{\approx} \int_0^\infty \sum_{q=0}^{K_{tr}} \frac{(K_i)^q \exp(-K_i)}{\Gamma^2(q+1)} g_{q+1}(z) \exp(-m_i z[1+y]) dz, \end{aligned} \quad (16)$$

where (a) is due to applying integration by parts $q+1$ times [12, eq. (6)], $z = \frac{x_i}{1+y}$, and $y = \sum_{j=1}^{i-1} Y_j + \sum_{k=i+1}^{N_U} Y_k$.

Finally, averaging over the PDFs of $Y_j, Y_k, d_x^{-n}, x \in \{i, j, k\}$ yields (6), which completes the proof.

APPENDIX B PROOF OF THEOREM 2

Conditioned on $d_x^{-n}, x \in \{i, j, k\}$, $E\left\{\ln\left(1 + \text{SIR}_i\right)\right\}$ can be expressed as [13, eq. (5)]:

$$\begin{aligned} &E\left\{\ln\left(1 + \text{SIR}_i\right)\right\} \\ &= \int_0^\infty \frac{1}{z} \left[1 - M_{\bar{X}_i}(z)\right] \left(\prod_{j=1}^{i-1} M_{\bar{Y}_j}(z)\right) \left(\prod_{k=i+1}^{N_U} M_{\bar{Y}_k}(z)\right) dz. \end{aligned} \quad (17)$$

Thereafter, (10) is obtained by averaging over the PDFs of $d_x^{-n}, x \in \{i, j, k\}$, which completes the proof.

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