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Wireless Energy Harvesting Based Relaying: A Finite-SNR Diversity-Multiplexing Tradeoff Perspective

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Abstract—This paper presents an analytical framework to derive the closed-form expressions for diversity-multiplexing tradeoff (DMT) for wireless energy harvesting (WEH) based amplify-and-forward (AF) and decode-and-forward (DF) protocols in finite signal-to-noise (SNR) regime. The results of this investigation suggest that both AF and DF offer similar performance except for few nuances. At low multiplexing gains, DF offers marginally better performance in the low SNR regime, whereas AF performs better in the high SNR scenarios. However, in the higher multiplexing gain regime, though subtle, DF uniformly dominates AF across all SNRs. Furthermore, an analytical study is presented to evaluate the effect of the fraction of time devoted to WEH (time-sharing parameter, ϵ) on the finite SNR DMT (f-DMT). In addition, the impact of relay position on the outage performance is also presented. A distinguishing feature of the proposed work is the characterization of WEH-based f-DMT which reveals the complete interplay between the operating SNR and the time-sharing parameter (ϵ), which is of fundamental importance to system designers. Finally, Monte-Carlo simulations are provided to confirm the veracity of analytical solutions.

Index Terms—Wireless energy harvesting, relays, diversity-multiplexing tradeoff (DMT), amplify-and-forward (AF), decode-and-forward (DF), outage probability

I. INTRODUCTION

RELAYING is an elegant way of realizing spatial diversity gain in a distributed fashion [1], [2]. However, in practice, relay nodes are powered by fixed energy sources such as batteries with limited capacity [3]. A key issue which prohibits the continuous maintenance-free operation of such systems is the cost of battery replacement. This problem can be adequately addressed by energy harvesting (EH) where the relay nodes scavenge the energy from the environment. In recent times, among several alternatives, wireless energy harvesting (WEH) has emerged as a go-to solution because of its suitability in supporting applications which demand stringent quality-of-service (QoS) [4]. Since radio frequency (RF) waveforms used for wireless communications carries not only the information but also the energy, the energy needed to power the relay nodes can be harvested from the transmitted RF signal itself [5], [6]. This feature opens up many new fascinating and unprecedented possibilities in the realm of green communications.

Varshney, in his landmark paper [5], first discussed the problem of wireless information and power transfer. However, his approach considers a complex and challenging design where the receiver can harvest the energy and also decodes the information simultaneously. To overcome this deficiency, more usable receiver architectures based on power splitting (PS)

and time switching (TS) techniques are proposed in [6]. The practical receiver structures proposed in [6] have been adopted for a single-relay based system in [7]. Specifically, the authors considered WEH-based amplify-and-forward (AF) protocol in both delay-limited and non-delay-limited paradigms and characterized the system performance using outage probability and ergodic capacity. Moreover, the optimal values of energy harvesting interval and the power-splitting ratio has been obtained for TS and PS based relaying schemes. Finally, using system throughput as a performance metric, the supremacy of TS-based relaying has been established for the low signal-to-noise (SNR) regimes. As a consequence, the authors proposed an adaptive TS relaying where the energy harvest duration at the relay is optimized as per the source-to-relay channel quality [8]. For such a system, the analytical expressions for the achievable throughput have been obtained. In [9], the TS and PS based relaying schemes have been investigated for DF protocol. In addition, there are several works devoted to WEH-based relay networks [10]–[16]. However, all these works either dealt with the throughput or the outage probability. Notably, all these studies invariably employ approximations in deriving closed-form expressions for these metrics.

In [17], Zheng and Tse proposed an elegant performance metric, diversity-multiplexing tradeoff (DMT), which captures the divergence between the reliability and the data rate at high SNRs. Moreover, by using the exponential order analysis, Azarian *et al.* [18] showed that the DMT could be computed in closed-form even for more sophisticated wireless communication systems. As a result, DMT has been extensively used as a fundamental performance measure to evaluate various communication protocols in the high SNR regime [17]–[20]. While such DMT studies have been performed for traditional battery-powered systems, there are few parallel works related to WEH powered systems. The DMT of WEH-based DDF protocol for a single relay and multiple relay-based systems have been characterized, respectively, in [21] and in [22]. A closed-form expression for the DMT of WEH-based non-orthogonal amplify-and-forward (WEH-NAF) has been derived in [23]. In [24], the DMT is investigated for an interference-limited relay-aided user. The work in [25] focuses on the DMT of the multiple-access relay system.

The asymptotic DMT proposed in [17] provides a DMT upper-bound for a given system. Unfortunately, it does not reveal the trade-off between diversity and multiplexing gains at low to moderate SNRs. Thus, there is a need to characterize the DMT for finite SNR regimes to assess the complete picture

about the parameters involved. The framework to evaluate finite-SNR DMT (f-DMT) was first proposed for point-to-point MIMO links in [26] and now widely used as a performance metric in the literature [27]–[29]. However, to the best of our knowledge, only one previous study has considered f-DMT for WEH-based systems [30]. In their work, Chen *et al.* derived closed-form expressions for outage probability, ergodic capacity, and finite SNR DMT (f-DMT) for a two-way half-duplex WEH-based AF relaying channel using a PS-based receiver architecture.

In the current work, we analyze the f-DMT of WEH-based AF and DF protocols for a single half-duplex relay based system using TS-based relaying [7]. Even though, the primary focus of this work is f-DMT, closed-form expressions for outage probability, diversity order, and asymptotic DMT are also provided. The impact of relay position on the outage probability is also investigated. Furthermore, the influence of the time-sharing parameter (ϵ) on the chosen performance metrics is also analyzed. Finally, it is shown that the addition of a WEH-based relay node can improve the diversity gain even in the finite SNR regime. Unsurprisingly, this study also confirms the findings of our earlier studies related to asymptotic DMT [22], [23].

A. Contributions

The main contributions of this paper are summarized below:

- Derived closed-form expressions for outage probability, diversity order, f-DMT, and asymptotic DMT for WEH-based AF and DF protocols.
- Investigated the impact of relay position on the outage probability and concluded that the optimal performance is achieved when the relay is placed either near the source or the destination.
- Through extensive analytical and simulation studies, it is shown that both AF and DF offer similar diversity performance except for some marginal differences. At lower multiplexing gains, WEH-DF and WEH-AF protocols achieve better diversity gain in low SNR regime and high SNR regime, respectively. However, at higher multiplexing gains, DF outperforms AF across all SNRs. Moreover, as SNR approaches to infinity, it is demonstrated that both WEH-AF and WEH-DF achieve the same diversity gain that, predictably, coincide with the asymptotic DMT counterparts.
- Analyzed the impact of the time-sharing parameter (ϵ) on the achievable diversity gain. This study confirmed that the optimal value of ϵ depends on the operating SNR in low and moderate SNR regimes. However, at very high SNRs, infinitesimal ϵ value achieves maximum diversity gain. Once again, these results reaffirm the earlier observations related to asymptotic DMT [22], [23].

Organization: The rest of the paper is organized as follows. Section II introduces the system model and the definitions of diversity and multiplexing gains as functions of SNR. The performance analysis of AF and DF protocols are provided in Sections III and IV, respectively. Section V presents numerical results at finite SNRs for various time-sharing factors. A

discussion on the finite-SNR results and interpretations of the limiting diversity gain as the SNR approaches infinity is also given in this Section. Conclusions are given in Section VI.

Notation: (x, y) represent Cartesian coordinates, R_n^+ denotes the set of real n -vectors with non-negative elements, and $A^+ = A \cap R_n^+$. $(\cdot)^T$ is the transpose of a matrix (or a vector). $F_X(x)$ denotes the cumulative distribution function (CDF) of a random variable X . $\mathcal{N}(\mu, \sigma^2)$ means normally distributed with mean μ and variance σ^2 .

II. SYSTEM MODEL

As depicted in Fig. 1, a half-duplex relay channel where the source $\mathcal{S}$ transmits its data to the destination $\mathcal{D}$ aided by a WEH-based relay node $\mathcal{R}$ has been considered. The channel fading coefficients $h_{s,r}$, $h_{r,d}$, and $h_{s,d}$ are assumed to be zero-mean complex Gaussian random variables with unit variance. Moreover, the channel gains are assumed to be quasi-static over a coding block. The distances between $\mathcal{S} \rightarrow \mathcal{R}$, $\mathcal{R} \rightarrow \mathcal{D}$, and $\mathcal{S} \rightarrow \mathcal{D}$ are denoted by d_1 , d_2 , and d_3 , respectively. The source node transmits the energy as well as the information using TS-based relaying (TSR) [9].

Fig. 1. System Model

Now onwards, to keep the notation simple, we drop the prefix WEH while referring to proposed protocols WEH-AF and WEH-DF when it is clear from the context. For both AF and the DF protocols, it is assumed that the total energy per packet is $E (= P_s T)$, where T is the total number of channel uses and P_s is the average transmit power. As shown in Fig. 1, the TSR mechanism involves three disjoint phases. In the first phase, $\mathcal{S}$ transmits the power wirelessly to the relay $\mathcal{R}$ using $T_h = \epsilon T$ channel uses, where ϵ denotes the time sharing factor which is a fraction of the channel uses allocated for EH. In this phase, $\mathcal{R}$ harvests the energy using $\mathcal{S}$'s radio transmission. Subsequently, in the second phase, $\mathcal{S}$ transmits its data using $T_s = (1 - \epsilon)T/2$ channel uses. In this phase, both $\mathcal{R}$ and $\mathcal{D}$ listen to the transmitted signal due to the broadcast nature of the channel. Note that the source $\mathcal{S}$ consumes all the available energy during these first two phases. Therefore, the net transmit power $\hat{P}_s$ is given by:

$$\hat{P}_s = \frac{E}{\epsilon T + \frac{(1-\epsilon)T}{2}} = \psi P_s, \quad (1)$$

where $\psi = \frac{2}{1+\epsilon}$ is the power scaling factor. The received signals in the second phase at $\mathcal{R}$ and $\mathcal{D}$, respectively, are given as follows:

$$y_r = \sqrt{\psi \delta_1 P_s} h_{s,r} x_s + n_r \quad (2a)$$

$$y_{d,2} = \sqrt{\psi \delta_3 P_s} h_{s,d} x_s + n_{d,2}, \quad (2b)$$

where $n_r \sim \mathcal{N}(0, 1)$ and $n_{d,2} \sim \mathcal{N}(0, 1)$ are the received noise samples at the relay and the destination, respectively, and $\delta_i = \frac{1}{d_i^\alpha}$, $i \in \{1, 2, 3\}$ is the path loss, and α is the path loss exponent. Finally, during the third phase, the relay shifts to transmit mode and forwards the processed received data to the destination $\mathcal{D}$. In this model, it is assumed that the relay $\mathcal{R}$ is entirely powered by source $\mathcal{S}$'s RF radiation. Therefore, the WEH-based relay $\mathcal{R}$ expends all the harvested energy and also consumes the rest of the channel uses $T_r = (1 - \epsilon)T/2$ during re-transmission. The quantum of harvested energy during the first phase is given by

$$E_{\mathcal{R}} = \eta \delta_1 \epsilon T |h_{s,r}|^2 \hat{P}_s, \quad (3)$$

where $0 < \eta \leq 1$ is the RF-to-DC conversion efficiency [31]. The relay transmit power P_r is given by

$$P_r = \frac{2E_{\mathcal{R}}}{(1 - \epsilon)T} \quad (4)$$

which, together with (1) and (3), results in

$$P_r = \frac{2\eta \epsilon \psi \delta_1}{(1 - \epsilon)} |h_{s,r}|^2 P_s \quad (5)$$

and denoting $\bar{\eta} = \frac{2\eta \epsilon}{1 - \epsilon}$, (5) can be further rewritten as

$$P_r = \bar{\eta} \psi \delta_1 |h_{s,r}|^2 P_s \quad (6)$$

The third phase differs regarding processing at the relay between AF and DF protocols. The details of each protocol are described in the following subsections.

A. AF Protocol

In this protocol, the relay amplifies the signal received during the second phase and re-transmits the scaled version of the received data in the third phase. Thus, the transmitted signal from the relay x_r is given by

$$x_r = \frac{\sqrt{P_r} y_r}{\sqrt{\psi \delta_1 |h_{s,r}|^2 P_s + \sigma_{n_r}^2}}, \quad (7)$$

where $\sigma_{n_r}^2 (= 1)$ is the variance of the noise observed at the relay. Substitution of (6) in (7) yields the following:

$$x_r = \sqrt{\frac{\bar{\eta} \psi \delta_1 |h_{s,r}|^2 P_s}{\psi \delta_1 |h_{s,r}|^2 P_s + \sigma_{n_r}^2}} y_r \quad (8)$$

By denoting the amplification factor $a = \sqrt{\frac{\bar{\eta} \psi \delta_1 |h_{s,r}|^2 P_s}{\psi \delta_1 |h_{s,r}|^2 P_s + \sigma_{n_r}^2}}$ and by using (2a), the received signals at $\mathcal{D}$ during the third phase are given as follows:

$$\begin{aligned} y_{d,3}^{\text{AF}} &= a \sqrt{\delta_2 P_s} \left(\sqrt{\psi \delta_1 P_s} h_{s,r} x_s + n_r \right) + n_{d,3} \\ &= a \sqrt{\psi \delta_1 \delta_2 P_s} h_{r,d} h_{s,r} x_s + a \sqrt{\delta_2 P_s} n_r + n_{d,3}, \end{aligned} \quad (9)$$

where $n_{d,3} \sim \mathcal{N}(0, 1)$ denotes the noise samples at the destination during phase III.

B. DF Protocol

In the DF relaying protocol, the WEH-based relay decodes, re-encodes, and finally re-transmits the message. There are several variants of DF protocol in the literature: DF, DF with diversity combining, and selective DF [19]. In the case of DF protocol, the destination performs detection based only on the signal received from the relay in phase III. Whereas in the case of diversity combining, by exploiting the broadcast nature of the channel, the destination detects the source message by combining both the source-destination signal in phase II and the relay-to-destination signal in phase III using maximal ratio combiner (MRC). While the performance of diversity combining technique is better than the basic approach, the achievable rate is still limited by the quality of source-to-relay link. To overcome this deficiency, in this work, we consider selective DF in which the relay stays silent during the third phase if it fails to decode the source's message during the second phase. Therefore, the received signals at the destination $\mathcal{D}$ during the third phase are given as follows:

$$y_{d,3}^{\text{DF}} = \begin{cases} \sqrt{\bar{\eta} \psi \delta_1 \delta_2 P_s} h_{s,r} h_{r,d} x_s + n_{d,3}, & \text{successful decoding} \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

C. Preliminaries

In this subsection, we recall the definitions of diversity and multiplexing gains in the realm of finite SNR, ρ [32]. Traditionally, the DMT is defined for infinite SNR using asymptotic versions of multiplexing gain (r) and the diversity gain (d) and are given by $r = \lim_{\rho \rightarrow \infty} R/\log(\rho)$ and $d = -\lim_{\rho \rightarrow \infty} P_{\text{out}}/\log(\rho)$ [17]. In these expressions, P_{out} is the outage probability for the given target data rate of R bits per channel use (BPCU). It can be noticed that these definitions subsume the behavior at realistic SNRs. The focus of this paper is to characterize the DMT at finite SNRs and data rates to estimate the validity of asymptotic DMTs for WEH-based relay systems.

The f-DMT is expressed as [32]:

$$r = \frac{R}{\log(1 + \rho)} \quad (11)$$

$$d(r, \rho) = -\frac{d(\ln P_{\text{out}}(r, \rho))}{d(\ln \rho)} = \frac{-\rho}{P_{\text{out}}(r, \rho)} \frac{dP_{\text{out}}(r, \rho)}{d\rho} \quad (12)$$

where $P_{\text{out}}(r, \rho)$ is the outage probability, which is a function of multiplexing gain r and SNR ρ . In other words, in the finite SNR paradigm, for fixed multiplexing gain r and SNR ρ , the diversity gain $d(r, \rho)$ is defined as the negative slope of log-log plot of outage probability versus SNR.

Besides f-DMT, this work also characterizes the diversity order for a given SNR ρ and a fixed target transmission rate of R . It can be defined as follows:

$$d(\rho) = \frac{-\rho}{P_{\text{out}}(\rho)} \frac{dP_{\text{out}}(\rho)}{d\rho} \quad (13)$$

In the following two sections, using the above-described preliminaries, the detailed theoretical analysis for outage probability, diversity order, f-DMT, and asymptotic DMT for the proposed WEH-based AF and DF protocols has been presented.

III. WEH-BASED AMPLIFY-AND-FORWARD (WEH-AF) PROTOCOL

In AF protocol, the relay is not required to decode the source S 's data, implying reduced relay complexity. Consequently, we shall first develop the analysis for WEH-AF and then extend the discussion to WEH-DF protocol in the next section. As discussed earlier, to derive the f-DMT, first the outage probability and its derivative are to be computed.

A. Outage Probability of WEH-AF Protocol

The channel input-output relations described in (2b) and (9) can be compactly represented using matrices as:

$$\mathbf{y}_d = \mathbf{A}\mathbf{x}_s + \mathbf{B}\mathbf{z}, \quad (14)$$

where

$$\begin{aligned} \mathbf{y}_d &= \begin{bmatrix} y_{d,2} \\ y_{d,3} \end{bmatrix}, & \mathbf{A} &= \begin{bmatrix} \sqrt{\psi\delta_3\rho} h_{s,d} \\ a\sqrt{\psi\delta_1\delta_2\rho} h_{r,d} h_{s,r} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & a\sqrt{\delta_2} h_{r,d} & 1 \end{bmatrix}, & \mathbf{z} &= [n_{d,2} \quad n_r \quad n_{d,3}]^T \end{aligned} \quad (15)$$

Then, the mutual information of the relay channel using AF protocol (I_{AF}) is given by [19]

$$I_{AF} = \left(\frac{1-\epsilon}{2} \right) \log \det \left(I_2 + \mathbf{A}\mathbf{A}^\dagger (\mathbf{B}\mathbf{B}^\dagger)^{-1} \right) \quad (16)$$

Next, by substituting (15) in (16) yields the following:

$$I_{AF} = \left(\frac{1-\epsilon}{2} \right) \log \left(1 + \psi\delta_3\rho|h_{s,d}|^2 + \frac{a^2\psi\delta_1\delta_2\rho|h_{r,d}|^2|h_{s,r}|^2}{1 + a^2\delta_2|h_{r,d}|^2} \right) \quad (17)$$

Thus, the probability of outage for the given target rate of R can be expressed as

$$\begin{aligned} P_{out}^{AF} &= \Pr[I_{AF} < R] \\ &= \Pr \left[1 + \psi\delta_3\rho|h_{s,d}|^2 + \frac{a^2\psi\delta_1\delta_2\rho|h_{r,d}|^2|h_{s,r}|^2}{1 + a^2\delta_2|h_{r,d}|^2} < 2^{\frac{2R}{1-\epsilon}} \right] \\ &= \Pr \left[|h_{s,d}|^2 + \frac{|h_{r,d}|^2|h_{s,r}|^2}{\frac{\delta_3}{a^2\delta_1\delta_2} + \frac{\delta_3}{\delta_1}|h_{r,d}|^2} < \frac{2^{2\hat{R}} - 1}{\psi\delta_3\rho} \right] \end{aligned} \quad (18)$$

where $\hat{R} = \frac{R}{1-\epsilon}$. By denoting $X = |h_{s,d}|^2$, $Y = |h_{r,d}|^2$, $Z = |h_{s,r}|^2$, $c_1 = \frac{\delta_3}{a^2\delta_1\delta_2}$, $c_2 = \frac{\delta_3}{\delta_1}$, and $\zeta_3 = \frac{1}{\psi\delta_3\rho}[2^{2\hat{R}} - 1]$, (18) can be rewritten as

$$P_{out}^{AF} = \Pr \left[X + \frac{YZ}{c_1 + c_2Y} < \zeta_3 \right] \quad (19)$$

The following Lemma evaluates the CDF of a combination of exponential random variables which in turn helps us to evaluate the outage probability of AF protocol.

Lemma 1: The cumulative distribution function (CDF) $F_\Lambda(\lambda)$ with $\Lambda = X + \frac{YZ}{c_1 + c_2Y}$ is given by

$$F_\Lambda(\lambda) = \begin{cases} 1 - e^{-\lambda} \left(1 + \frac{1}{c_1} - 2\lambda K_2(2\sqrt{c_1\lambda}) \right), & c_2 = 1 \\ 1 - e^{-\lambda} \left(1 + \frac{1}{c_1} \sum_{m=0}^{\infty} \frac{2^{2m}}{m!} \left(\frac{1-c_2}{4c_1} \right)^m \right. \\ \left. \left[G_{1,3}^{2,1} \left(\sqrt{c_1\lambda}, \frac{1}{2} \middle| \begin{matrix} 1 \\ m+1, m+2, 0 \end{matrix} \right) \right] \right), & \text{otherwise} \end{cases} \quad (20)$$

where $K_2(\cdot)$ and $G(\cdot)$ are the second order modified Bessel function of the second kind and the Meijer-G function [33], respectively.

Proof: The proof is given in Appendix A. ■

A closed-form expression for the outage probability of WEH-based AF protocol is given in the following theorem.

Theorem 1: The outage probability of WEH-AF protocol is given by

$$P_{out}^{AF} = \begin{cases} 1 - e^{-\zeta_3} \left(1 + \frac{1}{c_1} - 2\zeta_3 K_2(2\sqrt{c_1\zeta_3}) \right), & c_2 = 1 \\ 1 - e^{-\zeta_3} \left(1 + \frac{1}{c_1} \sum_{m=0}^{\infty} \frac{2^{2m}}{m!} \left(\frac{1-c_2}{4c_1} \right)^m \right. \\ \left. \left[G_{1,3}^{2,1} \left(\sqrt{c_1\zeta_3}, \frac{1}{2} \middle| \begin{matrix} 1 \\ m+1, m+2, 0 \end{matrix} \right) \right] \right), & \text{otherwise} \end{cases} \quad (21)$$

Proof: By substituting $\lambda = \zeta_3$, the outage probability given in (21) follows from Lemma 1. ■

As shown in the preceding theorem, the outage probability expression is composed of two relay positioning scenarios. In the first case, the distance between the source to the relay and the distance between the source and the destination are considered to be equal. On the other hand, the second case captures the rest of the relay positioning scenarios. In the next subsection, we compute the diversity order for WEH-AF protocol for both these cases.

B. Finite SNR Diversity Order of WEH-AF Protocol

Diversity order signifies how fast the outage probability reduces with SNR. Traditionally, the diversity order is characterized at an asymptotically high SNR. However, in this work, the focus is on finite SNR version. It can be computed using the outage probability expressions derived in Theorem 1 and the definition of finite SNR diversity (13). The following Corollary summarizes this result.

Corollary 1: The diversity order of WEH-based AF protocol is given by

$$\tilde{d}_f^{AF}(\rho) = \begin{cases} \frac{\zeta_3 e^{-\zeta_3}}{P_{out}(\rho)} \left[1 + \frac{1}{c_1} - 2\zeta_3 K_2 \left(2\sqrt{c_1 \zeta_3} \right) - 2\sqrt{c_1 \zeta_3} K_1 \left(2\sqrt{c_1 \zeta_3} \right) \right], & \delta_3 = \delta_1 \\ \frac{\zeta_3 e^{-\zeta_3}}{P_{out}(\rho)} \left(1 + \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{1-c_2}{c_1} \right)^m \left[\frac{1}{c_1} G_{1,3}^{2,1} \left(\sqrt{c_1 \zeta_3} \right) - (c_1 \zeta_3)^m B_1 \left(2\sqrt{c_1 \zeta_3} \right) \right] \right), & \text{otherwise} \end{cases} \quad (22)$$

where $K_1(\cdot)$ is the first order modified Bessel function of the second kind, and $B_1(x) = xK_1(x)$.

Proof: The proof is given in Appendix B. ■

In the next subsection, the diversity order results have been extrapolated to derive the f-DMT which eventually captures the diversity gain across all the multiplexing gains and the SNRs.

C. f-DMT of WEH-AF Protocol

While the diversity order provides the performance at a fixed rate R , the f-DMT captures the diversity performance across the spectrum of transmission rates (through multiplexing gain r). In this subsection, we characterize the f-DMT for AF protocol by extending the results derived in Theorem 1 and Corollary 1.

Proposition 1: The probability of outage for the given target rate of $R = r \log(1 + \rho)$ is equivalent to the results given in Lemma 1 with $\lambda = \frac{1}{\psi \delta_3 \rho} \left[(1 + \rho)^{\frac{2r}{1-\epsilon}} - 1 \right]$.

Proof: The probability of outage for the given target rate of $R = r \log(1 + \rho)$ can be expressed as

$$\begin{aligned} P_{out}^{AF} &= \Pr[I_{AF} < r \log(1 + \rho)] \\ &= \Pr \left[|h_{s,d}|^2 + \frac{|h_{r,d}|^2 |h_{s,r}|^2}{\frac{\delta_3}{a^2 \delta_1 \delta_2} + \frac{\delta_3}{\delta_1} |h_{r,d}|^2} < \frac{1}{\psi \delta_3 \rho} \left[(1 + \rho)^{\frac{2r}{1-\epsilon}} - 1 \right] \right] \end{aligned} \quad (23)$$

The above problem formulation is equivalent to (19) with $\zeta_3 = \frac{1}{\psi \delta_3 \rho} \left[(1 + \rho)^{\frac{2r}{1-\epsilon}} - 1 \right]$. Therefore, the outage probability expression in (23) follows Lemma 1. ■

Based on the above proposition, the DMT for AF protocol is characterized in the following Corollary.

Corollary 2: The f-DMT of WEH-based AF protocol is

given by

$$d_f^{AF}(r, \rho) = \begin{cases} \frac{\Theta_3 e^{-\zeta_3}}{P_{out}(\rho)} \left[1 + \frac{1}{c_1} - 2\zeta_3 K_2 \left(2\sqrt{c_1 \zeta_3} \right) - 2\sqrt{c_1 \zeta_3} K_1 \left(2\sqrt{c_1 \zeta_3} \right) \right], & \delta_3 = \delta_1 \\ \frac{\Theta_3 e^{-\zeta_3}}{P_{out}(\rho)} \left(1 + \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{1-c_2}{c_1} \right)^m \left[\frac{1}{c_1} G \left(\sqrt{c_1 \zeta_3} \right) - (c_1 \zeta_3)^m B_1 \left(2\sqrt{c_1 \zeta_3} \right) \right] \right), & \text{otherwise} \end{cases} \quad (24)$$

where $\Theta_3 = -\rho \frac{d\zeta_3}{d\rho} = \zeta_3 - \frac{2\hat{r}}{\psi \delta_3} (1 + \rho)^{2\hat{r}-1}$, and $\hat{r} = \frac{r}{1-\epsilon}$.

Proof: The f-DMT is analogous to the finite SNR diversity order except for the formulation of outage probability given in Proposition 1. Therefore, the proof for this corollary follows the steps detailed in Appendix B. Thus, for brevity, omitted the detailed proof. ■

In the next subsection, we derive the asymptotic (or infinite SNR) DMT.

D. Asymptotic DMT of WEH-AF Protocol

The asymptotic DMT can be derived from the high SNR version of outage probability (19). The result is summarized in the following Corollary.

Corollary 3: The asymptotic diversity gain achieved by the WEH-based AF protocol is characterized by:

$$d^{AF}(r) = 2 \left(1 - \frac{2r}{1-\epsilon} \right)^+ \quad (25)$$

Proof: By using (11), (18) can be rewritten as

$$P_{out}^{AF} = \Pr \left[1 + \psi \delta_3 \rho |h_{s,d}|^2 + \frac{a^2 \psi \delta_1 \delta_2 \rho |h_{r,d}|^2 |h_{s,r}|^2}{1 + a^2 \delta_2 |h_{r,d}|^2} < (1 + \rho)^{2\hat{r}} \right] \quad (26)$$

Note that at asymptotically high SNR, the scaling factors and also the noise factors do not contribute towards the DMT [18]. Thus, defining v_s, v_r and u as the exponential orders [18] of $1/|h_{s,d}|^2$, $1/|h_{r,d}|^2$ and $1/|h_{s,r}|^2$, respectively, the outage region for AF can be defined as

$$O^+ = \{(v_s, v_r, u) \in \mathbb{R}^3 \mid \max \{(1 - (v_r + u))^+, (1 - v_s)^+\} < 2\hat{r}\} \quad (27)$$

By denoting $v'_r = v_r + u$, considering the case where $0 \leq v_s, v'_r \leq 1$, and using the Laplace integration technique, the diversity gain can be evaluated by solving the following optimization problem [23]:

$$\begin{aligned} d^{AF}(r) &= \inf_{(v_s, v'_r)} v_s + v'_r \\ \text{s.t. } \max \left\{ \left(1 - v'_r \right)^+, (1 - v_s)^+ \right\} &< 2\hat{r} \end{aligned} \quad (28a)$$

$$0 \leq v_s, v'_r \leq 1 \quad (28b)$$

In the above optimization problem, inf gives the infimum (largest lower bound) of a set. It is easy to show that for the

given constraints, $d^{AF}(r)$ is given by (25) and thus complete the proof. ■

Remark 1: Corollary 3 implies that for a multiplexing gain of zero ($r = 0$), the WEH-based AF protocol achieves full diversity, i.e., $d^{AF}(0) = 2$.

IV. WEH-BASED DECODE-AND-FORWARD (DF) PROTOCOL

As in the case of AF protocol, the performance metrics: outage probability, diversity order, f-DMT, and the asymptotic DMT have been considered for DF protocol as well. First, the outage probability analysis is presented.

The following lemma is needed to evaluate the outage probability of WEH-based DF protocol. Note that as defined earlier, X, Y , and Z are exponential random variables.

Lemma 2: The cumulative distribution function (CDF) $F_\Lambda(\lambda)$ with $\Lambda = X + cYZ$ is given by

$$F_\Lambda(\lambda) = 1 - c - e^{-\lambda} + 2\lambda K_2 \left(2\sqrt{\frac{\lambda}{c}} \right), \quad (29)$$

where c is a constant, and $K_2(\cdot)$ is the second order modified Bessel function of the second kind.

Proof: The proof is given in Appendix C. ■

A. Outage Probability of WEH-DF Protocol

The following theorem describes the closed-form expression for the outage probability.

Theorem 2: The outage probability of WEH-DF protocol is given by

$$P_{out}^{DF} = 1 - e^{-\zeta_3} - e^{-\zeta_1} \left(\kappa - 2\zeta_3 K_2 \left(2\sqrt{\frac{\zeta_3}{\kappa}} \right) \right), \quad (30)$$

where $\zeta_1 = \frac{1}{\psi\delta_1\rho} [2^{2\hat{R}} - 1]$, and $\kappa = \frac{\bar{\eta}\delta_1\delta_2}{\delta_3}$.

Proof: For DF protocol, the probability of outage for a given target rate R is given as [27]:

$$P_{out}^{DF} = P(E_1)P(E_3) + P(E_2) - P(E_1)P(E_2), \quad (31)$$

where E_1 represents the event that the message is detected in error at the relay, E_2 represents the event that the message is detected in error at the destination with the relay helping, and E_3 represents the event that the message is detected in error at the destination after the relay fails to decode the message. $P(E_i), i \in \{1, 2, 3\}$ are the respective probabilities for the outage error events $E_i, i \in \{1, 2, 3\}$. The outage events and the corresponding inequalities are listed below [27]:

$$\begin{aligned} E_1 &: \frac{1-\epsilon}{2} \log(1 + \psi\delta_1 P_s |h_{s,r}|^2) < R \\ E_2 &: \frac{1-\epsilon}{2} \log(1 + \psi\delta_3 P_s |h_{s,d}|^2 + \bar{\eta} \psi\delta_1 \delta_2 P_s |h_{s,r}|^2 |h_{r,d}|^2) < R \\ E_3 &: \frac{1-\epsilon}{2} \log(1 + \psi\delta_3 P_s |h_{s,d}|^2) < R \end{aligned} \quad (32)$$

The computation of closed-form expressions for $P(E_1)$ and $P(E_3)$ is straightforward since they are corresponding to point-to-point links and are given by:

$$\begin{aligned} P(E_1) &= \Pr \left[|h_{s,r}|^2 < \frac{1}{\psi\delta_1\rho} [2^{2\hat{R}} - 1] \right] \\ &= \Pr [|h_{s,r}|^2 < \zeta_1] = 1 - e^{-\zeta_1}, \end{aligned} \quad (33)$$

and

$$\begin{aligned} P(E_3) &= \Pr \left[|h_{s,d}|^2 < \frac{1}{\psi\delta_3\rho} [2^{2\hat{R}} - 1] \right] \\ &= \Pr [|h_{s,d}|^2 < \zeta_3] = 1 - e^{-\zeta_3}, \end{aligned} \quad (34)$$

where $\zeta_i = \frac{1}{\psi\delta_i\rho} [2^{2\hat{R}} - 1], i \in \{1, 3\}$.

Next, we compute the closed-form expression for $P(E_2)$.

$$\begin{aligned} P(E_2) &= \Pr \left[(1 + \psi\delta_3\rho |h_{s,d}|^2 + \bar{\eta} \psi\delta_1\delta_2\rho |h_{s,r}|^2 |h_{r,d}|^2) < 2^{2\hat{R}} \right] \\ &= \Pr \left[X + \kappa YZ < \frac{1}{\psi\delta_3\rho} [2^{2\hat{R}} - 1] \right] \\ &= \Pr [X + \kappa YZ < \zeta_3] \end{aligned} \quad (35)$$

By noting that the probability of error for event E_2 (i.e., $P(E_2)$) defined in (35) is equivalent to the CDF derived in Lemma 2. Thus, $P(E_2)$ is obtained by substituting λ and c with ζ_3 and κ , respectively, in Lemma 2. The resultant closed-form expression for $P(E_2)$ is shown as follows:

$$P(E_2) = 1 - \kappa - e^{-\zeta_3} + 2\zeta_3 K_2 \left(2\sqrt{\frac{\zeta_3}{\kappa}} \right) \quad (36)$$

Finally, by using (31), (33), (34), and (36), we get (30) and thus completes the proof. ■

In the next subsection, we focus on the diversity order.

B. Finite SNR Diversity Order of WEH-DF Protocol

The finite SNR diversity order of the DF protocol is computed using the closed-form expression for outage probability given in (30) and the definition of diversity order given in (13). The following corollary summarizes the result.

Corollary 4: The diversity order of WEH-based DF protocol is given by

$$\begin{aligned} \tilde{d}_f^{DF}(\rho) &= \frac{e^{-\zeta_1}}{P_{out}(\rho)} \left[\zeta_3 e^{-(\zeta_3 - \zeta_1)} + \zeta_1 \left(\kappa - 2\zeta_3 K_2 \left(2\sqrt{\frac{\zeta_3}{\kappa}} \right) \right) - \right. \\ &\quad \left. \zeta_3 \left(2\sqrt{\frac{\zeta_3}{\kappa}} K_1 \left(2\sqrt{\frac{\zeta_3}{\kappa}} \right) \right) \right] \end{aligned} \quad (37)$$

Proof: The proof is given in Appendix D. ■

C. f-DMT of WEH-DF Protocol

Here, we derive the f-DMT for DF protocol using the results obtained in the previous subsections. The following corollary is the DF counterpart of Corollary 2.

Corollary 5: The f-DMT of WEH-based DF protocol is characterized by:

$$d_f^{DF}(r, \rho) = \frac{e^{-\zeta_1}}{P_{out}(r, \rho)} \left[\Theta_3 e^{-(\zeta_3 - \zeta_1)} + \Theta_1 \left(\kappa - 2\zeta_3 K_2 \left(2\sqrt{\frac{\zeta_3}{\kappa}} \right) - \Theta_3 \left(2\sqrt{\frac{\zeta_3}{\kappa}} K_1 \left(2\sqrt{\frac{\zeta_3}{\kappa}} \right) \right) \right) \right], \quad (38)$$

where $\Theta_i = -\rho \frac{d\zeta_i}{d\rho} = \left[\zeta_i - \frac{2\hat{r}}{\psi\delta_i} (1+\rho)^{2\hat{r}-1} \right], \forall i \in \{1, 2, 3\}$.

Proof: We have omitted the detailed proof here, since similar to the arguments presented in Corollary 2, by replacing ζ_3 with $\frac{1}{\psi\delta_3\rho} [(1+\rho)^{2\hat{r}} - 1]$ in the outage expression for DF protocol given in (21), the f-DMT can be computed by following the steps detailed in Appendix D. ■

D. Asymptotic DMT of WEH-DF Protocol

The asymptotic DMT can be derived from the high SNR version of outage probability given in (30). The result is summarized in the following theorem.

Corollary 6: The asymptotic diversity gain achieved by the WEH-based DF protocol is characterized by:

$$d^{DF}(r) = 2 \left(1 - \frac{2r}{1-\epsilon} \right)^+ \quad (39)$$

Proof: We outline the proof for the sake of brevity since it is analogous to the earlier published work [22]. As detailed in Section III-D, the scaling factors disappear in the asymptotic DMT analysis. Thus, by using the notation introduced in Section III-D, the exponential order equivalents of the rate region definitions given in (33), (34), and (35) are represented by:

$$O^+ = \{(v_s, v_r, u) \in \mathbb{R}^{3+} \mid (1-u)^+ < 2\hat{r}, (1-v_s)^+ < 2\hat{r}, \max\{(1-(v_r+u))^+, (1-v_s)^+\} < 2\hat{r}\} \quad (40)$$

By considering the case where $0 \leq v_s, v_r, u \leq 1$, and using the Laplace integration technique, the diversity gain can be evaluated by solving the following optimization problem [23]:

$$d^{DF}(r) = \inf_{(v_s, v_r, u)} v_s + v_r + u \quad (41a)$$

$$\text{s.t. } \max\{(1-(v_r+u))^+, (1-v_s)^+\} < 2\hat{r} \quad (41b)$$

$$(1-v_s)^+ < 2\hat{r}, \quad (1-u)^+ < 2\hat{r} \quad (41c)$$

$$0 \leq v_s, v_r, u \leq 1$$

It is easy to show that for the given constraints, $d^{DF}(r)$ is given by (39) and thus complete the proof. ■

Remark 2: The asymptotic diversity performance of WEH-based DF protocol is same as that of WEH-based AF protocol. Therefore, as in the case of AF protocol, WEH-based DF protocol achieves the asymptotic diversity order of 2.

V. NUMERICAL RESULTS AND PROTOCOL COMPARISONS

In this section, simulation results are obtained to evaluate the performance of proposed WEH-based relaying protocols in terms of outage probability, diversity order, and f-DMT.

Fig. 2. AF protocol: Outage probability versus SNR for different multiplexing gains r and time sharing factors ϵ

Fig. 3. DF protocol: Outage probability versus SNR for different multiplexing gains r and time sharing factors $\epsilon \in \{0.01, 0.1, 0.2\}$

Without loss of generality, the following parameters have been considered: average transmit power $P_s = 1$ W, path loss exponent $n = 3.7$, energy conversion efficiency $\eta = 1$, and the target data rate $R = 0.2$ bit per channel use (BPCU). It is also assumed that the distance between the source $\mathcal{S}$ and the destination $\mathcal{D}$ is normalized to 1 and the relay is placed between the source ($x = 0, y = 0$) and the destination ($x = 1, y = 1$).

It has been made evident in the earlier Sections that the outage probability expressions derived through Theorem 1 and Theorem 2 form the basis for the derivation of the rest of the results. Therefore, first, numerical results are provided to analyze and also verify the accuracy of the derived analytical expressions for outage probabilities for both AF and DF protocols using Monte Carlo simulations. Fig. 2 and Fig. 3 plot the simulated (and analytical) outage probabilities for AF and DF protocols, respectively, for various time-sharing factors (ϵ) and multiplexing gains (r). Lines represent the results based

on simulation and the symbol markers are based on analytical closed-form expressions. Throughout this section, ϵ value is chosen from $\{0.01, 0.10, 0.20\}$ to elucidate the impact of TS. It is evident from Fig. 2 and Fig. 3 that the results based on analytical expressions presented in Theorem 1 and Theorem 2, respectively, matches with the simulation results for both the protocols. Furthermore, it can be seen that both of them offer similar outage performance. As illustrated in Fig. 2 and Fig. 3, in the higher multiplexing gain regime, the infinitesimal ϵ (e.g. $\epsilon = 0.01$) offers better outage performance. Whereas in low to medium multiplexing gain regimes, the infinitesimal ϵ is optimal only at lower SNRs. Another interesting revelation is that there is no virtue in allotting more than 20% of channel uses for energy harvesting. In other words, across the whole spectrum of operating SNRs, the optimum ϵ is always between 0 and 0.2.

Remark 3: In WEH-based protocols, the effect of the fraction of energy harvest duration (ϵ) manifests in two divergent effects. First, the increase in ϵ results in the increase in harvested energy which in turn reduces the outage probability. Second, the increase in ϵ reduces the forwarding interval thus aggravates the outage probability. Thus, there is a trade-off in arriving at optimum ϵ in finite SNR regime. However, at asymptotically high SNR, the reduction in co-operation phase plays a dominant role in comparison to the marginal increase in harvested energy [25].

Next, the results are presented to capture the impact of relay position on outage performance at both low and very high SNRs. In Fig. 4(a) and Fig. 4(b), the outage probability for AF protocol is plotted against the normalized relay position for low and very high SNRs, respectively. It can be observed from the figures that the system offers the best outage performance when the WEH-based relay is either near the source or the destination. These results are contrary to the earlier observations related to non-EH relay-based systems where the optimal position of the relay is near to the midpoint of the source and destination [34]. This intriguing result can be attributed to the wireless energy harvesting mechanism. For this reason, the subsequent simulations consider the relay position as $(x = 0.95, y = 0.01)$. As shown in Fig. 5(a) and Fig. 5(b), note that a similar trend can be observed for DF protocol as well.

In Fig. 6, the finite SNR diversity gain for both AF and DF protocols for different time-sharing parameters (ϵ) is presented. It is difficult to obtain insights from finite SNR diversity order expressions given in (22) and (37). However, as depicted in Fig. 6, employing the closed-form expressions assists in obtaining numerical results for different time-sharing parameters (ϵ). First, it can be observed in Fig. 6 that the DF offers marginally better performance in the low SNR regime, whereas AF performs better in the high SNR scenarios. Secondly, for both AF and DF, the optimum ϵ depends on the SNR. As an example, it can be observed from Fig. 6 that for SNRs between 10 and 20, $\epsilon = 0.1$ (among $\epsilon \in \{0.01, 0.1, 0.2\}$) offers best performance. Finally, as shown in the inset of the figure, the influence of ϵ is nominal in the high SNR regime.

In Fig. 7, we compare the f-DMT performance of proposed WEH-based AF and DF protocols at different operating SNRs. Without loss of generality, $\epsilon = 0.1$ has been chosen. In

Fig. 4. AF protocol: Outage probability versus relay position

Fig. 7, WEH-UB and nonEH-UB refer to asymptotic SNR bounds for WEH-based and traditional non-EH based systems, respectively [23]. It can be seen that the f-DMT performance improves with the increase in SNR and eventually converges to WEH asymptotic bound for both AF and DF protocols at asymptotically high SNRs. Moreover, the difference in finite SNR and asymptotic diversity gains is significant in the high multiplexing gain regime. Finally, note that the f-DMT performance of both AF and DF protocols exhibit the similar behavior across all the multiplexing gains. Even though DF offers better performance compared to AF, the difference is negligible. Thus, for brevity, the subsequent discussion focuses only on DF protocol.

The f-DMT performance of DF protocol is compared at low, moderate, high, and asymptotically high SNRs for various time-sharing parameters in Fig. 8. Note that in Fig. 8, 0 dB, 20 dB, 50 dB, and 100 dB reflect low, medium, high, and asymptotically high SNR regimes, respectively. As shown in Fig. 8, similar to the earlier observations, the optimum ϵ is a function of multiplexing gain and the SNR. However, at asymptotically high SNR, the infinitesimal ϵ value (i.e., $\epsilon = 0.01$) offers the highest diversity gain across all the multiplexing gains. This result agrees with our earlier work related to asymptotic DMT [22], [23]. Moreover, it is also observed that the infinitesimal ϵ offers the best diversity performance at higher multiplexing

Fig. 5. DF protocol: Outage probability versus relay position

gains across all SNRs. As highlighted in the figure, the transition point at which $\epsilon = 0.01$ turns optimal moves up with the increase in SNR. Therefore, comparing with the asymptotic DMT analysis, it is found that the f-DMT reveals the complete interplay between the time sharing parameter and the SNR.

VI. CONCLUSION

In this paper, the diversity-multiplexing tradeoffs (DMT) of wireless energy harvesting (WEH) based AF and DF relaying protocols in the finite SNR regime have been characterized. Unlike the traditional non-EH-based protocols, it was shown that both WEH-based protocols (WEH-AF and WEH-DF) offer a similar diversity performance across all SNRs. As conjectured in the literature, the diversity gain achieved by both the proposed protocols at low to moderate SNRs is significantly lower than the asymptotic estimate. Moreover, the investigation of results revealed the detailed information concerning the effect of energy harvesting factor (ϵ) on finite SNR DMT (f-DMT). While confirming the optimality of infinitesimal ϵ at very high SNRs, this work emphasizes the importance of finding optimal ϵ at arbitrary operating SNR. Besides, this study concluded that the best outage performance is achieved when the WEH-based relay is either close to the source or the destination. Finally, this work unveiled the complete interplay between the time-sharing parameter (ϵ) and the SNR besides

Fig. 6. WEH-based Protocols: finite-SNR diversity order for time sharing factors $\epsilon \in \{0.01, 0.1, 0.2\}$

Fig. 7. WEH-based Protocols: f-DMT for $\epsilon = 0.1$

providing results related to f-DMT. Further work may consider the extension to other relaying protocols such as dynamic DF (DDF) and compress-and-forward (CF) protocols.

APPENDIX A PROOF OF LEMMA 1

The proof follows in principle the one given by Liu *et al.* [28], i.e., we use the notion of removing the conditioning on random variables to arrive at the eventual CDF. The conditional CDF $F_{\Lambda|X,Y}(u)$ is given by

$$F_{\Lambda|X,Y}(u) = \Pr \left[Z < \left(c_2 + \frac{c_1}{Y} \right) (u - X) \right] \\ = \begin{cases} 1 - e^{-\left(c_2 + \frac{c_1}{Y} \right) (u - x)} & \text{for } u \geq x \\ 0 & \text{for } u < x \end{cases}$$

Fig. 8. DF Protocol: f-DMT for SNRs of 0 dB, 20dB, 50 dB, and 100 dB and for time sharing factors $\epsilon \in \{0.01, 0.1, 0.2\}$. The dotted curves represent f-DMT for asymptotically high SNR (100 dB)

For $u \geq x$, removing the conditioning on Y yields

$$F_{\Lambda|X}(u) = \int_0^\infty \left[1 - e^{-\left(c_2 + \frac{c_1}{y}\right)(u-x)} \right] e^{-y} dy \quad (42)$$

$$= 1 - e^{-c_2(u-x)} \int_0^\infty e^{-\left[\frac{4c_1(u-x)}{4y} - y\right]} dy \quad (43)$$

By using [35, Eq. (3.324.1)], (43) can be evaluated as

$$F_{\Lambda|X}(u) = 1 - e^{-c_2(u-x)} (2\sqrt{c_1(u-x)}) K_1(2\sqrt{c_1(u-x)}), \quad (44)$$

where $K_v(\cdot)$ is the modified Bessel function of second kind with order v . Now, by removing the conditioning on X yields

$$F_\Lambda(u) = 1 - e^{-u} - e^{-c_2 u} \int_0^u e^{-(1-c_2)x} (2\sqrt{c_1(u-x)}) K_1(2\sqrt{c_1(u-x)}) dx \quad (45)$$

It is easy to see that there are two solutions to the above problem in (45) based on the value of c_2 . Therefore, the CDF has to be computed for both the cases.

Case 1: $c_2 = 1$

By substituting $c_2 = 1$ in (45) and upon using [33, Eq.(7.40)] yields

$$F_\Lambda(u) = 1 - e^{-u} - e^{-u} \frac{1}{2c_1} \left[4c_1 u K_2(2\sqrt{c_1 u}) \Big|_{0^+} - 4c_1 u K_2(2\sqrt{c_1 u}) \right], \quad (46)$$

further by expanding the K_2 near zero using the small argument expansion [33, Eq.(7.95)] yields,

$$F_\Lambda(u) = 1 - e^{-u} - \frac{1}{c_1} e^{-u} [1 - 2c_1 u K_2(2\sqrt{c_1 u})] \quad (47)$$

Finally by substituting $u = \lambda$, we get the CDF for this case:

$$F_\Lambda(\lambda) = 1 - e^{-\lambda} \left(1 + \frac{1}{c_1} - 2\lambda K_2(2\sqrt{c_1 \lambda}) \right) \quad (48)$$

Case 2: $0 < c_2 < 1$

By setting $v = 2\sqrt{c_1(u-x)}$, and after some algebraic manipulation, (45) can be rewritten as

$$F_\Lambda(u) = 1 - e^{-u} - \frac{e^{-u}}{2c_1} \int_{2\sqrt{c_1 u}}^0 e^{\frac{1-c_2}{4c_1} v^2} v^2 K_1(v) dv \quad (49)$$

By denoting $\bar{c} = \frac{1-c_2}{4c_1}$, and upon expanding $e^{\bar{c}v^2}$ using Taylor series, (49) can be rewritten as:

$$F_\Lambda(u) = 1 - e^{-u} - \frac{e^{-u}}{2c_1} \sum_{m=0}^{\infty} \frac{(\bar{c})^m}{m!} \int_{2\sqrt{c_1 u}}^0 v^{2m+2} K_1(v) dv \quad (50)$$

The following identity [36] is needed to evaluate the integral in (50)

$$\int x^n K_1(x) dx = \left[G_{1,3}^{2,1} \left(\frac{x}{2}, \frac{1}{2} \middle| \frac{n}{2}, \frac{n+2}{2}, 0 \right) \right] + \text{constant} \quad (51)$$

where $G(\cdot)$ is the Meijer-G function [33]. Upon using (51), the CDF in (50) can be evaluated as

$$F_\Lambda(u) = 1 - e^{-u} - \frac{e^{-u}}{2c_1} \sum_{m=0}^{\infty} \frac{(\bar{c})^m}{m!} 2^{2m+1} \left[G_{1,3}^{2,1} \left(\sqrt{c_1 u}, \frac{1}{2} \middle| m+1, m+2, 0 \right) \right] \quad (52)$$

Finally, by substituting $u = \lambda$, and after some algebraic manipulation, we get

$$F_\Lambda(\lambda) = 1 - e^{-\lambda} \left(1 + \frac{1}{c_1} \sum_{m=0}^{\infty} \frac{(\bar{c})^m}{m!} 2^{2m} \left[G_{1,3}^{2,1} \left(\sqrt{c_1 \lambda}, \frac{1}{2} \middle| m+1, m+2, 0 \right) \right] \right) \quad (53)$$

By using (48), (53), we get (20) and which completes the proof. ■

APPENDIX B PROOF OF COROLLARY 1

As the outage probability of AF protocol is composed of two cases based on the value of c_2 , the diversity order also consists of two parts, which are detailed below:

Case 1: $c_2 = 1$

By denoting $x = (2\sqrt{c_1 \zeta_3})$, the outage probability (21) for this case can be rewritten as

$$P_{out}^{AF} = 1 - e^{-\zeta_3} \left(1 + \frac{1}{c_1} + B_2(x) \right), \quad (54)$$

where $B_2(x) = -\frac{1}{2c_1} x^2 K_2(x)$. By using the derivative for modified Bessel function of the second kind [36], the derivative of (54) can be evaluated as

$$\frac{dP_{out}^{AF}(r, \rho)}{d\rho} = e^{-\zeta_3} \cdot \frac{d\zeta_3}{d\rho} \left[1 + \frac{1}{c_1} - 2\zeta_3 K_2\left(2\sqrt{c_1\zeta_3}\right) - 2\sqrt{c_1\zeta_3} K_1\left(2\sqrt{c_1\zeta_3}\right) \right], \quad (55)$$

where $\frac{\partial \zeta_3}{\partial \rho} = -\frac{\zeta_3}{\rho}$. Consequently, by substituting (55) in (12), yields the desired result given in (22).

Case 2: $0 < c_2 < 1$

Upon using the derivative of Meijer-G function [36], the derivative of outage probability (21) for this case can be evaluated as:

$$\begin{aligned} \frac{dP_{out}^{AF}(r, \rho)}{d\rho} &= e^{-\zeta_3} \cdot \frac{d\zeta_3}{d\rho} \left(1 + \frac{1}{c_1} \sum_{m=0}^{\infty} \frac{(\bar{c})^m}{m!} 2^{2m} \right. \\ &\quad \left. \left[G_{1,3}^{2,1} \left(\sqrt{c_1\zeta_3}, \frac{1}{2} \middle| m+1, m+2, 0 \right) \right] \right) \\ &\quad - e^{-\zeta_3} \cdot \frac{d\zeta_3}{d\rho} \left(2\sqrt{c_1\zeta_3} \sum_{m=0}^{\infty} \frac{(4\bar{c}c_1\zeta_3)^m}{m!} K_1\left(2\sqrt{c_1\zeta_3}\right) \right), \end{aligned} \quad (56)$$

where $\bar{c} = \frac{1-c_2}{4c_1}$. After some algebraic manipulations, (56) can be rewritten as

$$\begin{aligned} \frac{dP_{out}^{AF}(r, \rho)}{d\rho} &= e^{-\zeta_3} \cdot \frac{d\zeta_3}{d\rho} \left(1 + \sum_{m=0}^{\infty} \frac{(4\bar{c})^m}{m!} \right. \\ &\quad \left. \left[\frac{1}{c_1} G(\sqrt{c_1\zeta_3}) - (c_1\zeta_3)^m B_1(2\sqrt{c_1\zeta_3}) \right] \right), \end{aligned} \quad (57)$$

where $B_1(x) = xK_1(x)$, and $G(x) = G_{1,3}^{2,1} \left(x, \frac{1}{2} \middle| m+1, m+2, 0 \right)$. Consequently, by substituting (57) in (12), yields the result given in (22). This completes the proof of Corollary 1. ■

APPENDIX C PROOF OF LEMMA 2

In this section, we compute the CDF of $F_\Lambda(\lambda)$ with $\Lambda = X + cYZ$. As in the proof of Lemma 1, for $u \geq x$, removing the conditioning on Y yields

$$F_{\Lambda|X}(u) = 1 - \int_0^\infty e^{-\frac{u-x}{cy} - y} dy \quad (58)$$

By using the table of integrals [35, Eq. (3.324.1)], (58) can be evaluated as

$$F_{\Lambda|X}(u) = 1 - 2\sqrt{\left(\frac{u-x}{c}\right)} K_1\left(2\sqrt{\left(\frac{u-x}{c}\right)}\right) \quad (59)$$

Next, removing the conditioning on X gives

$$F_\Lambda(u) = \int_0^u \left[1 - 2\sqrt{\left(\frac{u-x}{c}\right)} K_1\left(2\sqrt{\left(\frac{u-x}{c}\right)}\right) \right] e^{-x} dx$$

By setting $v = 2\sqrt{\frac{u-x}{c}}$, and upon using [35, Eq. (5.52.1)] yields

$$F_\Lambda(u) = 1 - e^{-u} - \frac{c}{2} \left[\frac{4u}{c} K_2\left(2\sqrt{\frac{u}{c}}\right) \Big|_{0^+} - \frac{4u}{c} K_2\left(2\sqrt{\frac{u}{c}}\right) \right] \quad (60)$$

By expanding the modified Bessel function of the second kind K_2 near zero using the small argument expansion [33, Eq.(7.95)] yields the following:

$$F_\Lambda(u) = 1 - c - e^{-u} + 2u K_2\left(2\sqrt{\frac{u}{c}}\right) \quad (61)$$

Finally, by substituting $u = \lambda$, we get the desired result in (29). This completes the proof of Lemma 2 ■

APPENDIX D PROOF OF COROLLARY 4

In this section, the closed-form expression for the finite-SNR diversity order of WEH-based DF protocol has been derived. First, as in Appendix B, by using the derivative for modified Bessel function of the second kind [36], the derivative of (30) has been evaluated below:

$$\begin{aligned} \frac{dP_{out}^{DF}(r, \rho)}{d\rho} &= e^{-\zeta_3} \frac{d\zeta_3}{d\rho} + e^{-\zeta_1} \frac{d\zeta_1}{d\rho} \left(\kappa - 2\zeta_3 K_2\left(2\sqrt{\frac{\zeta_3}{\kappa}}\right) \right) \\ &\quad - e^{-\zeta_1} \frac{d\zeta_3}{d\rho} \left(2\sqrt{\frac{\zeta_3}{\kappa}} K_1\left(2\sqrt{\frac{\zeta_3}{\kappa}}\right) \right), \end{aligned} \quad (62)$$

where $\frac{\partial \zeta_i}{\partial \rho} = -\frac{\zeta_i}{\rho}$, $\forall i \in \{1, 2\}$. Consequently, by substituting (62) in (12), we get the desired result given in (37). This completes the proof of Corollary 4 ■

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