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2020

Sharma, S., Madhukumar, A. S., & Swaminathan, R. (2020). Performance optimisation for dual-hop hybrid FSO/RF system with selection combining. IET Optoelectronics, 14(6), 422-433. doi:10.1049/iet-opt.2020.0059

<https://hdl.handle.net/10356/144833>

<https://doi.org/10.1049/iet-opt.2020.0059>

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Performance Optimization for Dual-Hop Hybrid FSO/RF System with Selection Combining

Shubha Sharma, A.S. Madhukumar, and Swaminathan R

Abstract—The present paper investigates the performance of a decode-and-forward-relay-based relaying-based hybrid free space optical (FSO)/radio frequency (RF) system with selection combining (SC) at the destination. The proposed hybrid system considers an FSO sub-system as a default transmission option and RF sub-system serves as a backup when the FSO sub-system is in outage. For the proposed system, the outage threshold signal-to-noise ratio (SNR) is an important parameter and the optimum value for the same has been calculated to achieve the target average symbol error rate (SER) and high capacity. The presented model considers the effect of beam waist at the transmitter output, detector size, and jitter variance explicitly for the FSO link. Beam waist at the transmitter output and detector size are optimized to minimize the average SER considering various weather conditions and link distances. The exact and asymptotic outage probability and average symbol error rate (SER) expressions are derived and the diversity order is determined. Large coding gains are observed for optimum beam waist compared to any arbitrary beam waist in the average SER. The obtained analytical results are verified with the Monte-carlo simulations, which show that the proposed hybrid system improves the performance significantly compared to the individual FSO and RF systems.

Index Terms—Decode and forward relaying, Hybrid FSO/RF systems, Pointing errors, Selection combining

I. INTRODUCTION

FREE space optics (FSO) communication has garnered attention for high data rate point-to-point communication using unlicensed and unregulated optical bandwidth. FSO system transmits using laser beams through the atmosphere, where the beams get affected by atmospheric turbulence, weather attenuation and pointing errors. These effects fade and attenuate the signal, which lead to restriction in the transmission range and degradation in the system performance [1], [2]. Millimeter-wave (MMW) radio frequency (RF) establishes a reliable link, as it is not affected by the same effects, however, the data rate is lower than that of the FSO link. Due to operating wavelengths, FSO and MMW RF signals get severely attenuated by fog and rain, respectively. However, the MMW RF signal is not affected by fog. Therefore, the combination of FSO and MMW RF has the potential to provide reliable high data rate communication [2].

It is well known that both FSO and MMW RF establish short-range links. Therefore to increase the coverage, relay-based mixed RF-FSO systems have been studied extensively, where RF and FSO links are used in first and second hops, respectively. In [3] and [4], outage probability and average bit

error rate (BER) expressions are presented for the mixed RF-FSO system. The analysis has been extended in [5] by deriving unified expressions considering generalized distribution for RF link. Authors in [6] and [7], further extended the system by considering outdated channel state information (CSI) and co-channel interference to RF link at the relay, respectively. In [8] and [9], authors explored multi-user scenario in the first hop using RF links and the user with the highest received signal-to-noise ratio (SNR) is selected at the relay. Moreover, in [10], a mobile user with multiple antennas is considered in the first hop. In [11], energy harvesting from the FSO signal is also included in a mixed RF-FSO system. In the aforementioned mixed RF-FSO systems, the major limitations are as follows: a) the achievable data rate is limited to the data rate of RF links and b) degradation in the BER performance due to the usage of unreliable FSO link.

To improve the BER performance, hybrid FSO/RF systems are investigated in [12], [13], and [14] for single-hop (SH), dual-hop (DH) and multi-hop (MH) scenarios, respectively, where RF link transmits simultaneously with the FSO link in each hop. However, the achievable data rate is still restricted to that of the RF link. This is due to the fact that the data rate of FSO link needs to be reduced to RF link, in order to combine the data at the destination. Therefore, to improve both data rate and BER performance, in [15] and [16], SH switching-based hybrid FSO/RF system is considered, where FSO link initiates the transmission and when it is in outage, the system switches to RF link. In [17], the data rate and link availability analyses are presented for the SH hybrid FSO/RF system. The idea has been extended in [18] for a DH system to increase the coverage and diversity. However, the above-mentioned hybrid FSO/RF systems do not consider the misalignment condition between the transmitter and receiver apertures and the effects of transmit beam waist, detector size, and jitter variance, which play a critical role in degrading the performance of the system. Therefore, for more realistic simulation of the FSO link, these effects must be considered. Switching-based system is also presented for DH scenario in [19] using maximal ratio combining, but the derived expressions are too complex to comprehend. Therefore, simpler expressions are required to understand the system behavior and it becomes inevitable to analyse the performance of the system using selection combining (SC), which is the simplest combining scheme. Another key point is the selection of the switching threshold SNR, which poses considerable impact on the performance of the hybrid system. As per authors knowledge, the same has never been obtained for the hybrid FSO/RF system under the combined effect of atmospheric turbulence, weather attenuation, and pointing errors. Therefore, it is important to obtain

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the optimum threshold SNR value to achieve the target symbol error rate (SER) performance and high data rate.

The misalignment between the transmitter and receiver apertures causes pointing errors that degrades the system performance to a great extent [20] and [21]. Misalignment can be avoided by using large transmit beam waist. But large beam waist causes geometric signal loss at the receiver, which reduces the power levels and degrades the system performance [22]. Therefore to minimize the effect of pointing errors, optimization of beam waist at the transmitter output and detector size are important and the same has never been investigated for the hybrid FSO/RF systems considering SC scheme in the current literature.

This paper proposes a decode-and-forward (DF)-relay-based hybrid FSO/RF system using SC. The hybrid system comprises of FSO and RF sub-systems, where FSO sub-system is considered as a default transmission option and RF sub-system serves as a backup when the FSO sub-system is in outage. The FSO links are characterized by Gamma-Gamma atmospheric turbulence together with pointing errors and weather attenuation. The effect of beam waist at the transmitter output, detector size, and jitter variance are considered explicitly for FSO link. The RF links are characterized using Nakagami- m distribution [15]-[19], as it can model wide variety of line-of-sight (LOS) and non-LOS channel scenarios by varying the values of m together with different path loss factors. The major contributions of our work are the following:

A. Contributions

- Closed-form mathematical expressions are derived for exact outage probability and average symbol error rate (SER) for the proposed DF-relay-based hybrid FSO/RF system using SC at the destination. The average SER expressions are calculated for the non-outage period, since the sub-systems are in stand-by mode during outage period.
- Simple asymptotic expressions for outage probability and average SER are derived to analyze the system behavior. Diversity order of the system is determined from these expressions.
- Optimum beam waist and detector size are obtained for different weather conditions (clear air, light fog and heavy rain) and link distances, to minimize the average SER.
- The optimal threshold SNR, which is a critical parameter for the proposed system, is also determined to achieve the target SER and high capacity.

II. SYSTEM AND CHANNEL MODEL

The proposed hybrid FSO/RF system comprises of source (S), relay (R), and destination (D). The FSO and RF sub-systems employ SC to combine the data of the two phase transmissions at the destination. A one-bit feedback signaling [20] is applied to activate and deactivate the sub-systems. The atmospheric turbulence induced fading channels over S-R and R-D FSO links, which are denoted by I^{SR} and I^{RD} , are modeled using Gamma-Gamma distribution with pointing errors. The corresponding norm of the flat faded RF channel

gains of S-R and R-D links, which are denoted as h_{SR} and h_{RD} , respectively, are modeled using Nakagami- m distribution. Path loss attenuation in different weather conditions is also considered for both FSO and RF channels [2]. Table I gives the constants and notations used in the paper.

A. FSO Channel Model

The FSO channel is modeled using Gamma-Gamma distribution with pointing errors and path loss for which the probability distribution function (PDF) of I^j is given by [6, Eq.(13)]

$$f_{I^j}(I) = \frac{\xi_j^2 \alpha_j \beta_j}{A_0^j I_l^j \Gamma(\alpha_j) \Gamma(\beta_j)} G_{1,3}^{3,0} \left(\frac{\alpha_j \beta_j}{A_0^j I_l^j} I \mid \xi_j^2 - 1, \alpha_j - 1, \beta_j - 1 \right), \quad (1)$$

where $j \in \{SD, SR, RD\}$, α_j and β_j are the small and large scale parameters of the scattering environment of j^{th} link as given in [18, Eq.(8)], ξ_j is the pointing errors coefficient, which is given by [21, Eq. (2.3)]:

$$\xi_j = \frac{w_{Leqj}}{2\sigma_s^j}, w_{Leqj}^2 = \frac{w_{Lj}^2 \sqrt{\pi} \text{erf}(v_j)}{2v_j \exp(-v_j^2)}, w_{Lj} \approx w_0 \sqrt{1 + \Theta_{0j} \left(\frac{\lambda L_j}{\pi w_0^2} \right)^2}, \quad (2)$$

where σ_s^j is the jitter standard deviation, w_{Leqj} is the equivalent beam waist, $\Theta_{0j} = 1 + \frac{2w_0^2}{\rho_0^2(L_j)}, \rho_0(L_j) = (0.55 C_n^2 k^2 L_j)^{-3/5}$, L_j is the j^{th} link distance in meters, w_{Lj} is the beam waist at L_j , w_0 is the beam waist at $L_j = 0$, C_n^2 is the refractive index structure parameter, $k = 2\pi/\lambda$ is the optical wave number, λ is the wavelength, A_0^j is the fraction of the collected power without any pointing errors and is given by $A_0^j = |\text{erf}(v_j)|^2$, $\text{erf}(\cdot)$ is the error function, $v_j = \frac{\sqrt{\pi} a_j}{\sqrt{2} w_{Lj}}$, and a_j is the detector radius size. The atmospheric path loss of the FSO link is determined by Beers-Lambert law as $I_l^j = \exp(-\alpha_1 L)$, where α_1 is a weather dependent attenuation coefficient [6, Eq.(12)]. The instantaneous and average SNR at the output of the FSO receiver, respectively, are given by

$$\gamma_j^f = [I^j]^2 (P_j^f)^2 \eta_j^2 / \sigma_j^2, \bar{\gamma}_j^f = [(P_j^f)^2 \eta_j^2 \kappa_j^2 (A_0^j)^2 I_l^2] / \sigma_j^2, \quad (3)$$

where P_j^f is the transmitted optical power and $E[I^j] = 1$, where $E[\cdot]$ is the expectation operator. The σ_j^2 is the variance of additive white Gaussian noise (AWGN) with zero-mean, η_j is the optical-to-electrical conversion coefficient of j^{th} link and $\kappa_j = \xi_j^2 / (\xi_j^2 + 1)$. Using power transformation of random variables, the PDF of instantaneous SNR of FSO link is given by

$$f_{\gamma_j^f}(\gamma) = \mathcal{K}_1^j \gamma^{-1} G_{1,3}^{3,0} \left(\frac{\alpha_j \beta_j \kappa_j}{\sqrt{\gamma_j^f}} \sqrt{\gamma} \mid \xi_j^2 + 1, \mathcal{N}_1^j \right), \quad (4)$$

where $\mathcal{K}_1^j$ and $\mathcal{N}_1^j$ are given in Table I. Using (4) and [23, Eq.(07.34.21.0084.01)], the cumulative distribution function (CDF) of γ_j^f is given by

$$F_{\gamma_j^f}(\gamma) = 2\mathcal{K}_1^j G_{2,4}^{3,1} \left(\frac{\alpha_j \beta_j \kappa_j}{\sqrt{\gamma_j^f}} \sqrt{\gamma} \mid 1, \xi_j^2 + 1, \mathcal{N}_1^j, 0 \right). \quad (5)$$

TABLE I
LIST OF CONSTANTS

$\mathcal{K}_1^j = \frac{\xi_j^2}{2\Gamma(\alpha_j)\Gamma(\beta_j)}$	$C_1^j = \frac{csc(\pi(\alpha_j - \xi_j^2))csc(\pi(\beta_j - \xi_j^2))}{\Gamma(1)}$	$q_{1,n}^j = \gamma_{th}^{\Omega_{g,n}^j}; q_{2,n}^j = -\frac{2}{\sqrt{\pi}} \sum_{n_3=0}^{\infty} \mathcal{K}_{n_3} \gamma_{th}^{\Omega_{g,n}^j + n_3 + \frac{1}{2}}$	$\mathcal{N}_{7,g,n}^{j,i} \triangleq \mathcal{N}_3^i, -\Omega_{g,n}^j, 0, \frac{1}{2}$
$v_j \triangleq \frac{(\alpha_j \beta_j \kappa_j)^2}{16\gamma_j^p}$	$C_2^j = \frac{csc(\pi(\xi_j^2 - \alpha_j))csc(\pi(\beta_j - \alpha_j))}{\Gamma(\xi_j^2 + 1 - \alpha_j)}$	$X_{3,n}^j \triangleq \frac{C_3^j(\beta_j - \xi_j^2)_n (v_j^f)^{\beta_j + n}}{n! \Gamma(n+1 - \xi_j^2 + \beta_j) \Gamma(n+1 - \alpha_j + \beta_j)}; \Omega_{1,n}^j = \frac{\xi_j^2 + n}{2}$	$\mathcal{N}_{11,g}^{j,i,n} \triangleq 1 - \Omega_{g,n}^j, \frac{1}{2}, \mathcal{N}_2^i$
$\kappa_j = \frac{\xi_j^2}{\xi_j^2 + 1}, w_j \triangleq \frac{m_j}{\gamma_j^r}$	$C_3^j = \frac{csc(\pi(\xi_j^2 - \beta_j))csc(\pi(\alpha_j - \beta_j))}{\Gamma(\xi_j^2 + 1 - \beta_j)}$	$\mathcal{N}_1^j = \xi_j^2, \alpha_j, \beta_j; \mathcal{N}_2^j = 1, \frac{\xi_j^2 + 1}{2}, \frac{\xi_j^2 + 2}{2}; \Omega_{2,n}^j = \frac{\alpha_j + n}{2}$	$\mathcal{N}_{12,g}^{j,i,n} \triangleq 1 - \Omega_{g,n}^j + n_3 + \frac{1}{2}, \frac{1}{2}, \mathcal{N}_2^i$
$\mathcal{K}_2^j = \frac{\xi_j^2 2^{\alpha_j + \beta_j}}{8\pi \Gamma(\alpha_j) \Gamma(\beta_j)}$	$X_{1,n}^j \triangleq \frac{C_1^j(0)_n (v_j^f)^{\xi_j^2 + n}}{n! \Gamma(n+1 - \alpha_j + \xi_j^2) \Gamma(n+1 - \beta_j + \xi_j^2)}$	$\mathcal{N}_3^j = \frac{\xi_j^2}{2}, \frac{\xi_j^2 + 1}{2}, \frac{\alpha_j}{2}, \frac{\alpha_j + 1}{2}, \frac{\beta_j}{2}, \frac{\beta_j + 1}{2}; \Omega_{3,n}^j = \frac{\beta_j + n}{2}$	$\mathcal{N}_{21,g}^{j,i,n} \triangleq \mathcal{N}_3^i, 0, \frac{1}{2}, -\Omega_{g,n}^j$
$\mathcal{K}_{n_3} = \frac{(-1)^{n_3} D^{2n_3+1}}{n_3! (2n_3+1)}$	$X_{2,n}^j \triangleq \frac{C_2^j(\alpha_j - \xi_j^2)_n (v_j^f)^{\alpha_j + n}}{n! \Gamma(n+1 - \xi_j^2 + \alpha_j) \Gamma(n+1 - \beta_j + \alpha_j)}$	$\mathcal{N}_{6,g,n}^{j,i} \triangleq 1, 1 - \Omega_{g,n}^j, 1 - \Omega_{g,n}^j - \frac{1}{2}, \frac{\xi_j^2 + 1}{2}, \frac{\xi_j^2 + 2}{2}$	$\mathcal{N}_{22,g}^{j,i,n} \triangleq \mathcal{N}_3^i, 0, \frac{1}{2}, -\Omega_{g,n}^j + n_3 + \frac{1}{2}$

It is noticed from (4) that the PDF of the FSO channel contains Meijer-G function of γ . It is infeasible to derive the exact outage and average SER expressions for the proposed system, as it is required to deal with multiple such Meijer-G functions. Therefore, by using [24], [23, Eq.(07.32.02.0001.01)], and [23, Eq.(06.10.02.0001.01)], the Meijer G function in (4) can be written in simple terms as

$$f_{\gamma_j^f}(\gamma) = \mathcal{K}_1^j \pi^2 \sum_{n=0}^{\infty} \sum_{g=1}^3 X_{g,n}^j \gamma^{\Omega_{g,n}^j - 1}, \quad (6)$$

where $X_{g,n}^j$ and $\Omega_{g,n}^j$ are given in Table I.

B. RF channel modeling

The instantaneous received SNR (γ_j^r) and average SNR ($\bar{\gamma}_j^r$) of any given RF link can be expressed, respectively, as

$$\gamma_j^r = \bar{\gamma}_j^r |h_j|^2, \quad \bar{\gamma}_j^r = \frac{P_j^r g_j^r}{\sigma_{rj}^2} \quad (7)$$

where P_j^r is the transmitted power, h_j is the RF channel fading with $E[|h_j|^2] = 1$. The norm of the fading channel gain i.e. $|h_j|$ is modeled as Nakagami- m distribution, where m indicates the fading severity parameter. The σ_{rj}^2 is the variance of AWGN with zero mean. Assuming RF carrier frequency of 60 GHz, the average path attenuation is given by [2]

$$g_j^r[\text{dB}] = G_t + G_r - 20 \log_{10}(4\pi L / \lambda_r) - (\alpha_{oxy} + \alpha_{rain})L, \quad (8)$$

where G_t and G_r denote the transmit and receive antenna gains, respectively, λ_r is the wavelength of the RF channel, and α_{oxy} and α_{rain} are the attenuation caused by oxygen absorption and rain, respectively. The noise variance in the RF link is given by $\sigma_{rj}^2[\text{dBm}] = B + N_0 + N_F$ [2], [12], where B is the RF bandwidth (in dBMHz), N_0 is the noise power spectral density (in dBm/MHz), and N_F is the noise figure of the receiver. The PDF of γ_j^r , which follows Gamma distribution, is given by

$$f_{\gamma_j^r}(\gamma) = w_j^{m_j} \frac{\gamma^{m_j-1}}{\Gamma(m_j)} e^{-w_j \gamma}. \quad (9)$$

where w_j is given in Table I. By using [25, Eq. (3.351.1)], the CDF of γ_j^r can be expressed as

$$F_{\gamma_j^r}(\gamma) = \frac{1}{\Gamma(m_j)} \gamma(m_j, w_j \gamma), \quad (10)$$

where $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function [25, Eq. (8.350.1)].

III. OUTAGE ANALYSIS

In the proposed hybrid FSO/RF system, FSO sub-system is considered as a default transmission option if $\gamma_{SC}^f > \gamma_{th}$, where γ_{SC}^f is the received FSO SNR after two phases by employing SC at the destination and γ_{th} is the threshold SNR. If $\gamma_{SC}^f < \gamma_{th}$, the FSO sub-system gets deactivated and the backup RF sub-system gets activated. The system is declared to be in outage, if the received SNRs of both FSO and RF sub-systems are below γ_{th} . The expression for outage probability is given by

$$P_{SC} = F_{\gamma_{SC}^f}(\gamma_{th}) F_{\gamma_{SC}^r}(\gamma_{th}), \quad (11)$$

where $F_{\gamma_{SC}^f}(\gamma_{th})$ and $F_{\gamma_{SC}^r}(\gamma_{th})$ are the CDF of the FSO and RF sub-systems, respectively, and are given by

$$F_{\gamma_{SC}^T}(\gamma) = F_{\gamma_{SD}^T}(\gamma) \left[F_{\gamma_{SR}^T}(\gamma) + F_{\gamma_{RD}^T}(\gamma) - F_{\gamma_{SR}^T}(\gamma) F_{\gamma_{RD}^T}(\gamma) \right]. \quad (12)$$

where $F_{\gamma_j^T}(\gamma)$ is given by (5) and (10) for FSO and RF sub-systems, respectively. Note that $T \in \{f, r, f^\infty\}$, where f , r , and f^∞ stand for FSO, RF and asymptotic FSO expressions, respectively.

IV. AVERAGE SER ANALYSIS

The average SER during non-outage period can be calculated in terms of the average SER of individual FSO and RF sub-systems and is given by

$$\overline{SER}_{SC} = \frac{B_{SC}^f(\gamma_{th}) + F_{\gamma_{SC}^f}(\gamma_{th}) B_{SC}^r(\gamma_{th})}{1 - P_{SC}}, \quad (13)$$

where P_{SC} is given by (11), $F_{\gamma_{SC}^f}(\gamma_{th})$ is given by (12), and $B_{SC}^T(\gamma_{th})$, when $\gamma_{SC}^T > \gamma_{th}$, is given by (14). In (14), $f_{\gamma_{SC}^T}(\gamma)$ is the PDF of FSO and RF sub-systems, which is obtained by differentiating (12) with respect to γ and $p(e/\gamma)$ is the conditional SER for MPSK signaling conditioned on the instantaneous SNR of any given link γ and is given by [15, Eq. 11]

$$p(e/\gamma) = \frac{A}{2} \text{erfc}(\sqrt{\gamma} D), \quad (15)$$

where $A = 1$ for modulation order $M = 2$, $A = 2$ for $M > 2$, $D = \sin(\frac{\pi}{M})$ and $\text{erfc}(\cdot)$ is the complementary error function. The detailed derivations of each term in (14) for $B_{SC}^f(\gamma_{th})$ and $B_{SC}^r(\gamma_{th})$ are given in the following sections.

$$\begin{aligned}
B_{SC}^T(\gamma_{th}) &= \int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{SC}^T}(\gamma) d\gamma = \underbrace{\int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{SR}^T}(\gamma) F_{\gamma_{SD}^T}(\gamma) d\gamma}_{B_{SR,SD}^T(\gamma_{th})} + \underbrace{\int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{SD}^T}(\gamma) F_{\gamma_{SR}^T}(\gamma) d\gamma}_{B_{SD,SR}^T(\gamma_{th})} + \underbrace{\int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{RD}^T}(\gamma) F_{\gamma_{SD}^T}(\gamma) d\gamma}_{B_{RD,SD}^T(\gamma_{th})} \\
&+ \underbrace{\int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{SD}^T}(\gamma) F_{\gamma_{RD}^T}(\gamma) d\gamma}_{B_{SD,RD}^T(\gamma_{th})} - \underbrace{\int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{SR}^T}(\gamma) F_{\gamma_{RD}^T}(\gamma) F_{\gamma_{SD}^T}(\gamma) d\gamma}_{B_{SR,RD,SD}^T(\gamma_{th})} - \underbrace{\int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{RD}^T}(\gamma) F_{\gamma_{SR}^T}(\gamma) F_{\gamma_{SD}^T}(\gamma) d\gamma}_{B_{RD,SR,SD}^T(\gamma_{th})} - \underbrace{\int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{SD}^T}(\gamma) F_{\gamma_{SR}^T}(\gamma) F_{\gamma_{RD}^T}(\gamma) d\gamma}_{B_{SD,SR,RD}^T(\gamma_{th})}
\end{aligned} \tag{14}$$

A. Average SER of FSO sub-system

To derive the average SER of the FSO sub-system, $B_{SC}^f(\gamma_{th})$ can be expressed as

$$B_{SC}^f(\gamma_{th}) = \int_{\gamma_{th}}^{\infty} p(e|\gamma) f_{\gamma_{SC}^f}(\gamma) d\gamma = B_{SC}^f - B_{SC}^{f,out}(\gamma_{th}), \tag{16}$$

where B_{SC}^f and $B_{SC}^{f,out}(\gamma_{th})$, respectively, are given by

$$B_{SC}^f = \int_0^{\infty} p(e|\gamma) f_{\gamma_{SC}^f}(\gamma) d\gamma, B_{SC}^{f,out}(\gamma_{th}) = \int_0^{\gamma_{th}} p(e|\gamma) f_{\gamma_{SC}^f}(\gamma) d\gamma. \tag{17}$$

where $p(e|\gamma)$ can be represented in terms of Meijer G function as in [23, Eq.(07.34.03.0619.01)] and in terms of MacLaurin series as in [25, Eq. (3.321)], respectively, are given by

$$p(e|\gamma) = \frac{A}{2\sqrt{\pi}} G_{1,2}^{2,0} \left(D^2 \gamma \left| 0, \frac{1}{2} \right. \right), \frac{A}{2} \left[1 - \frac{2}{\sqrt{\pi}} \sum_{n_3=0}^{\infty} \mathcal{K}_{n_3} \gamma^{\frac{2n_3+1}{2}} \right], \tag{18}$$

where $\mathcal{K}_{n_3} = \frac{(-1)^{n_3} D^{2n_3+1}}{n_3! (2n_3+1)}$. Each term in (14) is solved using (16). To solve $B_{j,i}^f(\gamma_{th})$ in (14), where $j \in \{SR, SD, RD\}$ and $i \in \{SD, SR, RD\}$, we first calculate $B_{j,i}^f$ by substituting (5), (6), $p(e|\gamma)$ in terms of Meijer G function from (18) into (17) and by using [23, Eq. (07.34.21.0013.01)], $B_{j,i}^f$ is given by

$$B_{j,i}^f = \frac{A\pi^2 \mathcal{K}_1^i \mathcal{K}_2^j}{2\sqrt{\pi}} \sum_{g=1}^3 \sum_{n=0}^{\infty} \frac{X_{g,n}^j}{D^{2\Omega_{g,n}^j}} G_{6,4}^{6,4} \left(\frac{v_i}{D^2} \left| \frac{1}{2}, \mathcal{N}_{6,g,n}^{j,i} \right. \right) \tag{19}$$

where $\mathcal{K}_2^j$, $\mathcal{N}_{6,g,n}^{j,i}$, and $N_{7,g,n}^{j,i}$ are given in Table I. Then, we calculate $B_{j,i}^{f,out}(\gamma_{th})$ by substituting (5), (6), and (18) into (17) and by using [23, Eq. (07.34.21.0084.01)], the exact expression for $B_{j,i}^{f,out}(\gamma_{th})$ is given by

$$B_{j,i}^{f,out}(\gamma_{th}) = \frac{A\mathcal{K}_1^j \mathcal{K}_2^i \pi^2}{2} \sum_{n=0}^{\infty} \sum_{l=1}^2 \sum_{g=1}^3 q_{l,n}^j X_{g,n}^j G_{5,9}^{6,3} \left(v_i \gamma_{th} \left| \frac{11g}{2lg}, \mathcal{N}_{2lg}^{j,i,n} \right. \right) \tag{20}$$

where $q_{l,n}^j$, $\mathcal{N}_{11g}^{j,i,n}$, and $\mathcal{N}_{2lg}^{j,i,n}$ are given in Table I. Finally, by substituting (19) and (20) into (16), $B_{j,i}^f(\gamma_{th})$ is obtained. In order to solve $B_{j,i,p}^f(\gamma_{th})$ in (14), where $j \in \{SR, RD, SD\}$,

$i \in \{RD, SR\}$ and $p \in \{SD, RD\}$, we rewrite an alternate expression for (5) as

$$F_{\gamma_j^p}(\gamma) = \mathcal{K}_2^j G_{3,7}^{6,1} \left(v_j \gamma \left| \mathcal{N}_{3,0}^j \right. \right), \tag{21}$$

Now, to solve $B_{j,i,p}^f(\gamma_{th})$, we first calculate $B_{j,i,p}^f$ by substituting (6), (15), and (21) into (17) and use [25, Eq.(9.301)] to expand the Meijer G function and the remaining integral is solved using [25, Eq. (4.1.11)]. Using [26], a generalized Meijer G function of two variables can be obtained and the final expression for $B_{j,i,p}^f$ is given by (22), where $G_{p_1,q_1;p_2,q_2;p_3,q_3}^{m_1,0;m_2,m_2;n_3,m_3}(\cdot)$ is the extended generalized bi-variate Meijer G-function (EGBMGF) [26]. Then we calculate $B_{j,i,p}^{f,out}(\gamma_{th})$ by substituting (6), (18), and (21) into (17) and use [25, Eq.(9.301)] to expand the Meijer G functions and the integral is solved using [25, Eq.(3.191.1)]. With similar approach used in (22), the final expression for $B_{j,i,p}^{f,out}(\gamma_{th})$ is given by (23). Finally, by substituting (22) and (23) into (16), $B_{j,i,p}^f(\gamma_{th})$ is obtained. Upon substituting $B_{j,i}^f(\gamma_{th})$ and $B_{j,i,p}^f(\gamma_{th})$ into (14), the average SER of the FSO sub-system is obtained.

B. Average SER of RF sub-system

To derive the average SER of the RF sub-system, we define the following function as

$$\begin{aligned}
N(\lambda, \Omega, \phi, a) &= \int_{\phi}^{\infty} \text{erfc}(a\sqrt{Y}) Y^{\lambda-1} e^{-\Omega Y} dY = \text{erfc}(a\sqrt{\phi}) \\
&\quad - \frac{\Gamma(\lambda, \Omega\phi)}{\Omega^{\lambda}} - \frac{a}{\sqrt{\pi}} \sum_{r=0}^{\lambda-1} \frac{\Omega^r \Gamma(r+\frac{1}{2}, (a^2 + \Omega)\phi)}{r! (a^2 + \Omega)^{r+\frac{1}{2}}},
\end{aligned} \tag{24}$$

where the expression in (24) is solved using integration by parts as in [15, Eq.(18)] using [25, Eq. (2.33.10) and (2.33.11)], $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function [25, Eq. (8.350.2)]. To solve $B_{j,i}^r(\gamma_{th})$, where $j \in \{SR, SD, RD\}$ and $i \in \{SD, SR, RD\}$, we substitute (9), (10), and (15) into (14). Using [25, Eq.(8.352.6)] and (24), $B_{j,i}^r(\gamma_{th})$ is given by

$$\begin{aligned}
B_{j,i}^r(\gamma_{th}) &= \frac{Aw_j^{m_j}}{2\Gamma(m_j)} \left[N(m_j, w_j, \gamma_{th}, D) \right. \\
&\quad \left. - \sum_{k_2=0}^{m_i-1} \frac{w_i^{k_2}}{k_2!} N(m_j + k_2, w_{j,i}, \gamma_{th}, D) \right]
\end{aligned} \tag{25}$$

$$B_{j,i,p}^f = \frac{AK_1^j K_2^i K_2^p \pi^2}{2\sqrt{\pi}} \sum_{n=0}^{\infty} \sum_{g=1}^3 \frac{X_{g,n}^j}{D^{2\Omega_{g,n}^j}} G_{2,1:3,7:3,7}^{2,0:6,1:6,1} \left(\begin{matrix} \Omega_{g,n}^j + \frac{1}{2}, \Omega_{g,n}^j \\ \Omega_{g,n}^j + 1 \end{matrix} \middle| \begin{matrix} \mathcal{N}_{3,0}^i \\ \mathcal{N}_{3,0}^i \end{matrix} \middle| \begin{matrix} \mathcal{N}_{3,0}^p \\ \mathcal{N}_{3,0}^p \end{matrix} \middle| \begin{matrix} v_i \\ D^2 \end{matrix}, \begin{matrix} v_p \\ D^2 \end{matrix} \right) \quad (22)$$

$$B_{j,i,p}^{P_{out}}(\gamma_{th}) = \frac{AK_1^j K_2^i K_2^p \pi^2}{2} \sum_{n=0}^{\infty} \sum_{g=1}^3 \sum_{l=1}^2 X_{g,n}^j q_{l,n}^j G_{1,1:3,7:3,7}^{1,0:6,1:6,1} \left(\begin{matrix} \Omega_{g,n}^j \\ \Omega_{g,n}^j + 1 \end{matrix} \middle| \begin{matrix} \mathcal{N}_{3,0}^i \\ \mathcal{N}_{3,0}^i \end{matrix} \middle| \begin{matrix} \mathcal{N}_{3,0}^p \\ \mathcal{N}_{3,0}^p \end{matrix} \middle| v_i \gamma_{th}, v_p \gamma_{th} \right) \quad (23)$$

where $w_{j,i} \triangleq w_j + w_i$. With similar approach used for solving (25), $B_{j,i,p}^r(\gamma_{th})$, where $j \in \{SR, RD, SD\}$, $i \in \{RD, SR\}$ and $p \in \{SD, RD\}$, is given by

$$B_{j,i,p}^r(\gamma_{th}) = \frac{A}{2} \frac{w_j^m}{\Gamma(m_j)} \left[N(m_j, w_j, \gamma_{th}, D) - \sum_{k_2=0}^{m_i-1} \frac{1}{k_2!} \right. \\ \times N(k_2 + m_j, w_{j,i}, \gamma_{th}, D) - \sum_{s_2=0}^{m_p-1} \frac{1}{s_2!} N(s_2 + m_j, w_{j,i}, \gamma_{th}, D) \\ \left. + \sum_{k_2=0}^{m_i-1} \frac{1}{k_2!} \sum_{s_2=0}^{m_p-1} \frac{1}{s_2!} N(k_2 + s_2 + m_j, w_{j,i,p}, \gamma_{th}, D) \right], \quad (26)$$

where $w_{j,i,p} \triangleq w_j + w_i + w_p$. Finally, by substituting (25) and (26), into (14), the exact closed-form expression for average SER of the RF sub-system is obtained.

V. ASYMPTOTIC OUTAGE ANALYSIS

It is assumed that the average electrical SNR of the FSO link ($\bar{\gamma}_{SC}^f$) goes to infinity for a fixed value of average SNR of RF links. The asymptotic outage probability is given by

$$P_{SC}^{\infty} = F_{\gamma_{SC}^f}(\gamma_{th}) F_{\gamma_{SC}^r}(\gamma_{th}), \quad (27)$$

where $F_{\gamma_{SC}^f}(\gamma_{th})$ and $F_{\gamma_{SC}^r}(\gamma_{th})$ are given by (12). Applying [23, Eq. (07.34.06.0044.01)] in (4), the CDF in the asymptotic regime is given as

$$F_{\gamma_j^f}(\gamma_{th}) \simeq 2 \sum_{k=1}^3 \mathcal{K}_1^j \Lambda_{1,k}^j \left[\sqrt{\gamma_j^f} \right]^{-d_j}, \quad (28)$$

where $\Lambda_{1,k}^j$ is given in Table II, $d_j = \min\{\frac{\xi_j^2}{2}, \frac{\alpha_j}{2}, \frac{\beta_j}{2}\}$. By substituting (28) into (12), we obtain $F_{\gamma_{SC}^f}(\gamma_{th})$. It is inferred from (12) that the diversity order depends on either $F_{\gamma_{SD}^f}^{\infty}(\gamma_{th}) F_{\gamma_{SR}^f}^{\infty}(\gamma_{th})$ or $F_{\gamma_{SD}^f}^{\infty}(\gamma_{th}) F_{\gamma_{RD}^f}^{\infty}(\gamma_{th})$. To calculate the diversity order, we assume $\bar{\gamma}_{SD}^f = \bar{\gamma}_{SR}^f = \bar{\gamma}_{RD}^f = \bar{\gamma}^f$. Therefore, the diversity order is given by $G_{op} = d_j + d_i$, where $j = SD$ and $i \in \{SR, RD\}$. It can be written as $G_{op} = \min\{\frac{\xi_j^2}{2}, \frac{\alpha_j}{2}, \frac{\beta_j}{2}\} + \min\{\frac{\xi_i^2}{2}, \frac{\alpha_i}{2}, \frac{\beta_i}{2}\}$. Since $\alpha > \beta$ [18, Eq.(8)], the diversity order can be simplified as $G_{op} = \min\{\frac{\xi_j^2}{2}, \frac{\beta_j}{2}\} + \min\{\frac{\xi_i^2}{2}, \frac{\beta_i}{2}\}$. If $\beta = \beta_j = \beta_i$ and $\xi = \xi_j = \xi_i$, then the diversity order is given by $G_{op} = \min\{\xi^2, \beta\}$.

VI. ASYMPTOTIC SER ANALYSIS

The asymptotic SER during non-outage period is given by

$$\overline{SER}_{SC}^{\infty} = \frac{B_{SC}^{f\infty}(\gamma_{th}) + F_{\gamma_{SC}^f}(\gamma_{th}) B_{SC}^r(\gamma_{th})}{1 - P_{SC}^{\infty}}, \quad (29)$$

where $B_{SC}^{f\infty}(\gamma_{th})$ and $B_{SC}^r(\gamma_{th})$ are given by (14), $F_{\gamma_{SC}^f}(\gamma_{th})$ and P_{SC}^{∞} are given by (12) and (27), respectively. Each

TABLE II
CONSTANTS ASYMPTOTIC RESULTS

$\Lambda_{1,k}^j = \frac{\prod_{q=1, q \neq k}^3 \Gamma(\mathcal{N}_{1,q}^j - \mathcal{N}_{1,k}^j) \Gamma(\mathcal{N}_{1,k}^j)}{\Gamma(1 + \xi_j^2 - \mathcal{N}_{1,k}^j) \Gamma(1 + \mathcal{N}_{1,k}^j)} (\alpha_j \beta_j \kappa_j \sqrt{\gamma_{th}})^{\mathcal{N}_{1,k}^j}$
$\Lambda_{4,k_3}^j = \sum_{k_3=1}^3 \frac{\prod_{q=1, q \neq k_3}^3 \Gamma(\mathcal{N}_{1,q}^j - \mathcal{N}_{1,k_3}^j)}{\Gamma(1 + \xi_j^2 - \mathcal{N}_{1,k_3}^j)} (\alpha_j \beta_j \kappa_j)^{\mathcal{N}_{1,k_3}^j}$
$\Lambda_{x\{.\}} \triangleq (\gamma_{th})^{x\{.\}} G_{2,3}^{3,0} \left(\begin{matrix} 1, 1 - x\{.\} \\ -x\{.\}, 0, \frac{1}{2} \end{matrix} \middle D^2 \gamma_{th} \right)$
$x_{\{1,k_3,k\}} \triangleq \left(\frac{\mathcal{N}_{1,k}^j}{2} + \frac{\mathcal{N}_{1,k_3}^j}{2} \right), x_{\{2,k_3,k\}} \triangleq \left(\frac{\mathcal{N}_{1,k}^j}{2} + \frac{\mathcal{N}_{1,k_3}^j}{2} + \frac{\mathcal{N}_{1,k}^p}{2} \right)$

term in (14) is simplified using [23, Eq. (07.34.06.0040.01)] in (4) and (5), which expands the Meijer-G function in the asymptotic regime. The resultant expression and (18) are substituted into (14). The remaining integral is solved using [23, Eq. (07.34.21.0085.01)]. The closed-form expression for $B_{j,i}^{f\infty}(\gamma_{th})$, where $j \in \{SR, SD, RD\}$ and $i \in \{SD, SR, RD\}$, is given by

$$B_{j,i}^{f\infty}(\gamma_{th}) = \sum_{k=1}^3 \sum_{k_3=1}^3 \frac{AK_1^j K_1^i \Lambda_{4,k_3}^j \Lambda_{1,k}^i \Lambda_{x\{1,k_3,k\}}}{\sqrt{\pi}} \left[\bar{\gamma}_j^f \right]^{-v_j} \left[\bar{\gamma}_i^f \right]^{-v_i}, \quad (30)$$

where $\Lambda_{x\{1,k_3,k\}}$ and $x_{\{1,k_3,k\}}$ are given in Table II, and the closed-form expression of $B_{j,i,p}^{f\infty}(\gamma_{th})$, where $j \in \{SR, RD, SD\}$, $i \in \{RD, SR\}$ and $p \in \{SD, RD\}$, is given by

$$B_{j,i,p}^{f\infty}(\gamma_{th}) = \sum_{k=1}^3 \sum_{k_3=1}^3 \frac{2AK_1^j K_1^i K_1^p \Lambda_{4,k_3}^j \Lambda_{1,k}^i \Lambda_{1,k}^p}{\sqrt{\pi}} \Lambda_{x\{2,k_3,k\}} \left[\bar{\gamma}_j^f \right]^{-v_j} \left[\bar{\gamma}_i^f \right]^{-v_i} \left[\bar{\gamma}_p^f \right]^{-v_p}, \quad (31)$$

where $\Lambda_{x\{2,k_3,k\}}$ and $x_{\{2,k_3,k\}}$ are given in Table II. Upon substituting (30) and (31) into (14), the asymptotic average SER of the FSO sub-system $B_{SC}^{f\infty}(\gamma_{th})$ is obtained. It is inferred from (14) that the diversity order depends on $B_{j,i}^{f\infty}(\gamma_{th})$. Using similar arguments while calculating G_{op} , the diversity order from asymptotic SER is given by $G_{SER} = \min\{\xi^2, \beta\}$.

VII. NUMERICAL OPTIMIZATION, RESULTS AND DISCUSSIONS

This section presents the optimization of outage threshold SNR, transmit beam waist, and detector size, which is followed by numerical and simulation results. The values of FSO and RF link parameters are given in Table III. The values of α_j and β_j are calculated using [18, Eq.(8)] and ξ_j is calculated from (2). The value of refractive index parameter for strong turbulence conditions is considered as $C_n^2 = 1 \times 10^{-13}$, unless and otherwise stated. It is considered that R is in the middle of S and D (i.e. $L_{SR} = L_{RD} = 0.5L_{SD}$). This implies that

Fig. 1. Average SER and Capacity for Optimum threshold detection

TABLE III
PARAMETER OF FSO AND RF SUB-SYSTEMS [2], [12]

FSO/RF Parameter	Value	FSO/RF Parameter	Value
FSO Wavelength λ	1.55 μ m	RF Carrier Frequency f_c	60 GHz
FSO Power P_j^f	40 mW	Tx Antenna Gain G_t	44 dBi
RF Power P_j^r	10 mW	Rx Antenna Gain G_r	44 dBi
Responsivity η	0.5 $\frac{A}{W}$	Attenuation oxygen α_{oxy}	15.1 dB/km
Noise variance σ_1^2	$10^{-14} \frac{A}{Hz}$	Noise PSD N_0	-114 $\frac{dBm}{MHz}$
jitter std. deviation σ_s^j	30 cm	Receiver Noise Figure N_f	5 dB
detector radius size a_j	10 cm	RF Fading parameter m_j	5

TABLE IV
OPTIMUM VALUES OBTAINED FOR TRANSMIT BEAM WAIST (w_{0opt}) AND DETECTOR SIZE (a_{jopt})

L_{SD} (m)	a_{jopt}	w_{0opt}	$w_{L_{SD}}$	$w_{L_{SR}} = w_{L_{RD}}$	ξ_{SD}	$\xi_{SR} = \xi_{RD}$
500	8cm	1.5cm	2.6cm	1.7cm	8	11
1000	12cm	2cm	5.2cm	2.7cm	3.06	8
2000	27cm	3cm	13cm	5.3cm	3.29	10.2
3000	55cm	4cm	24cm	9cm	4.5	11.3

$L_{SD} = 1000$ m, $a_{jopt} = 12$ cm				
Weather	α_1 (dB/km)	α_{rain} (dB/km)	C_n^2 ($m^{-2/3}$)	w_{0opt} (cm)
Clear air	0.43	0	5×10^{-14}	2
Light Fog	20	0	3×10^{-15}	3.5
Heavy rain	9.2	10.2	4×10^{-15}	4.2

$\alpha_{SR} = \alpha_{RD}$, $\beta_{SR} = \beta_{RD}$, and $\xi_{SR} = \xi_{RD}$. However, the results can also be plotted for any other placement of R. It is assumed that $m_j = 5$ for which Nakagami- m distribution perfectly models fading over MMW RF channel [15]–[19]. The expressions in (6), (20), (23) and subsequent equations using these expressions for computing the exact average SER values are truncated to $n = n_3 = 50$.

A. Optimum Outage Threshold SNR

To obtain the optimum outage threshold SNR, we have plotted Fig. 1a and Fig. 1b, which show the average SER performance as a function of threshold SNR for different values of FSO and RF average SNR, respectively. It is shown in both the figures that the target SER of 10^{-3} is achieved at the threshold SNR $\gamma_{th} = 5.75$ dB. The threshold SNR γ_{th} is chosen considering average SNR of FSO and RF values where the worst average SER performance is observed. Another important parameter to be noted is the value of the target SER, which is chosen to be 10^{-3} , however, the lower SER value can give more reliable link and the same can be achieved at a higher threshold value. But higher threshold SNR value raises the concern that the system will be in outage with higher probability, which leads to degradation in the achievable

capacity. Fig. 1c shows that high capacity is achieved for the threshold SNR below 6 dB. Therefore, in order to keep the trade off between reliability and capacity, optimal threshold SNR is calculated considering a worst case scenario for the target SER of 10^{-3} .

B. Optimum Transmit Beam waist and Detector Size

The fundamental design criterion for FSO sub-system followed in this paper is to select the transmit beam waist w_{0opt} and detector size a_{jopt} , which minimize the average SER. According to [22], the feasible values of transmit beam waist w_0 and detector size a_j should be in terms of few centimeters. The average SER versus w_0 for different values of a_j and link distance L_j is shown in Fig. 2. The optimum values of w_0 and a_j were found through extensive simulations for each case. However, for the sake of clarity only few plots have been shown for each case. Notice that for $L_{SD} = 500$ m, the average SER performance gets highly affected with the change in the value of w_0 . As the distance increases, the average SER does not change abruptly with respect to w_0 . This is due to the fact that for large distances, the beam waist at the receiver is large and requires large detector size due to which the misalignment effect gets reduced. However, at small distances accurate control of beam waist is critically important as it can give significant gains with the proper selection of the beam waist. The observed values of w_{0opt} and a_{jopt} for different L_j are tabulated in Table IV. Table IV, also shows the the values of received beam waist (w_{L_j}) and ξ_j with respect to the values of w_0 . The hybrid system performance deteriorates for both large and small values of w_0 compared to $w_0 = w_{0opt}$. This is because, using large beam sizes reduces the beam intensity and power levels at the receiver due to its geometric properties and too small beam sizes causes significant diffraction. This observation confirms that the optimum beam waist is a critical parameter to be considered while designing the FSO sub-system to achieve the best performance. Apart from L_j , different weather conditions also affect the FSO link to a great extent. Hence, the analysis to choose optimum beam waist for different weather conditions is given below.

Fig. 3 shows the average SER with respect to FSO transmit power P_j^f for different values of transmit beam waist w_0 and different weather conditions i.e. clear air, light fog and rain. The weather attenuation parameters and the observed values of w_{0opt} for different weather conditions are given in Table IV. The optimum beam waist w_{0opt} was found for each case

Fig. 2. Average SER versus w_0 for different a_j and L_j

Fig. 3. Average SER versus transmitted power (P_j^f) of a hybrid FSO/RF system for Clear air, Light fog, and Heavy Rain

through an exhaustive search over discrete set of values of w_0 . It is to be noted that $a_{jopt} = 12$ cm with $L_{SD} = 1000$ m is assumed for plotting the SER performance curves for different weather conditions. For clarity of depiction, only few sets of w_0 are shown in the figure. It is also noted that further reduction in the value of w_0 does not help in reducing the average SER due to diffraction in the small beam. For clear air, light fog, and heavy rain scenarios, link margins of around 13 dB, 5 dB, and 4.9 dB optical power are realized by optimizing the beam waist for average SER = 10^{-4} compared to the arbitrary value of $w_0 = 7$ cm, respectively. Another point to be noted here is that to achieve average SER = 10^{-4} , the required values of P_j^f after optimizing the beam waist are -6 dBm, 8 dBm, and 18 dBm for clear air, light fog and rain conditions, respectively. This behavior of the system shows that the signals get highly attenuated for light fog and heavy rain scenarios.

the individual FSO and RF sub-systems are also plotted. The figure shows that for $\bar{\gamma}^r = 10$ dB, significant performance improvement is observed in the low SNR region due to utilization of strong RF links by the hybrid system. At high SNR, the average SER performance of hybrid FSO/RF system, which is a function of $\bar{\gamma}^f$ and $\bar{\gamma}^r$ (i.e. $\bar{\gamma}^f = \bar{\gamma}^r$), merges with the average SER performance of hybrid FSO/RF system with fixed $\bar{\gamma}_j^f = 10$ dB. This indicates that at high SNR, the slope of the hybrid system does not depend on the increasing value of the SNR of RF links but only on the FSO links, which justifies the proposed asymptotic analysis. It is important to note that the hybrid FSO/RF system performs better than the RF sub-system in the low SNR region, however, the RF sub-system outperforms hybrid FSO/RF system in the high SNR region. This is due to the fact that with the increase in the transmit power of the RF sub-system, it becomes more reliable, where as the FSO sub-system suffers from atmospheric turbulence induced fading and pointing errors. Another key point to be noted here is that to achieve the SER = 5×10^{-5} , the SNR gains of 2.5 dB and 7 dB are observed for FSO sub-system compared to hybrid system, for low pointing errors scenario (i.e. w_{0opt}) and high pointing errors scenario (i.e. $w_0 = 25$ cm), respectively. By comparing the coding gains in both the scenarios, it can be assessed that the backup RF sub-system is definitely improving the system performance for the case when the pointing error severity is high.

Fig. 4. Average SER performance for different values of w_0 for Hybrid FSO/RF system, FSO sub-system, and RF sub-system

C. Numerical Results

Fig. 4 shows the average SER curves for the hybrid FSO/RF system with respect to the average SNR of FSO/RF links ($\bar{\gamma}^f$ and $\bar{\gamma}^r$). The plot also shows the performance of the hybrid system with respect to $\bar{\gamma}^f$ for a fixed value of $\bar{\gamma}_j^r = 10$ dB. In order to compare the performance, the average SER of

TABLE V
DIVERSITY PERFORMANCE OF THE HYBRID FSO/RF SYSTEM

$L_{SD} = 3000$ m, $C_n^2 = 1 \times 10^{-13} \text{ m}^{-2/3}$, $\alpha_{SD} = 2.18$				
$\beta_{SD} = 1.18$, $\alpha_{SR} = \alpha_{RD} = 2.31$, and $\beta_{SR} = \beta_{RD} = 1.84$				
a_j (cm)	w_0 (cm)	ξ_{SD}	$\xi_{SR} = \xi_{RD}$	$G_{op} = G_{SER}$
20	40	0.85	0.77	$\frac{\xi_{SD}^2}{2} + \frac{\xi_{SR}^2}{2} = 0.65$
35	40	1.06	1.02	$\frac{\xi_{SD}^2}{2} + \frac{\xi_{SR}^2}{2} = 1.08$
55	4	4.5	11.3	$\frac{\beta_{SD}}{2} + \frac{\beta_{SR}}{2} = 2.34$

In Fig. 5a and Fig. 5b, the diversity performances of

Fig. 5. Analytical, asymptotic and simulated performance plots

the hybrid FSO/RF system. The derived asymptotic outage probability and average SER expressions are sufficiently tight with the exact analytical outage and SER expressions at high SNR. The values of diversity gain are tabulated in Table V. It is observed from the figures and Table V that large pointing errors severity with small values of ξ_j adversely affect the diversity gain. However, for small pointing errors, the value of ξ_j is large enough to become less effective and the atmospheric turbulence influence the diversity gain.

VIII. CONCLUSIONS

This paper presented a DF-relaying-based hybrid FSO/RF system with SC. The exact and asymptotic expressions are presented in closed-form and the diversity order is obtained, which is given by $d_j = \min\{\xi^2, \beta\}$. The value of outage threshold SNR is calculated to achieve a target SER. The values of w_{0opt} and a_{jopt} , which minimize the average SER, are also calculated. It is inferred from the plots that the proposed hybrid system improves the performance significantly for the case when pointing error severity is high due to backup RF sub-system. It is also inferred from the plots that the proposed system outperforms the individual FSO and RF systems.

REFERENCES

- [1] M. A. Khalighi and M. Uysal, "Survey on free space optical communication: A communication theory perspective," *IEEE Commun. Surveys Tuts.*, vol. 16, pp. 2231–2258, 2014.
- [2] B. He and R. Schober, "Bit-interleaved coded modulation for hybrid RF/FSO systems," *IEEE Trans. Commun.*, vol. 57, no. 12, pp. 3753–3763, December 2009.
- [3] H. Samimi and M. Uysal, "End-to-end performance of mixed RF/FSO transmission systems," *IEEE/OSA J. Opt. Commun. Netw.*, vol. 5, no. 11, pp. 1139–1144, Nov 2013.
- [4] S. Anees and M. R. Bhatnagar, "Performance of an amplify-and-forward dual-hop asymmetric RF-FSO communication system," *IEEE/OSA J. Opt. Commun. Netw.*, vol. 7, no. 2, pp. 124–135, Feb. 2015.
- [5] J. Zhang, L. Dai, Y. Zhang, and Z. Wang, "Unified performance analysis of mixed radio frequency/free-space optical dual-hop transmission systems," *J. Lightw. Technol.*, vol. 33, no. 11, pp. 2286–2293, June 2015.
- [6] G. T. Djordjevic, M. I. Petkovic, A. M. Cvetkovic, and G. K. Karagiannis, "Mixed RF/FSO Relaying With Outdated Channel State Information," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 9, pp. 1935–1948, Sep. 2015.
- [7] E. Balti and M. Guizani, "Mixed RF/FSO cooperative relaying systems with co-channel interference," *IEEE Trans. Commun.*, vol. 66, no. 9, pp. 4014–4027, Sep. 2018.
- [8] A. M. Salhab, F. S. Al-Qahtani, R. M. Radaydeh, S. A. Zummo, and H. Alnuweiri, "Power allocation and performance of multiuser mixed RF/FSO relay networks with opportunistic scheduling and outdated channel information," *J. Lightw. Technol.*, vol. 34, no. 13, pp. 3259–3272, Jul 2016. [Online]. Available: <http://jlt.osa.org/abstract.cfm?URI=jlt-34-13-3259>
- [9] L. Yang, M. O. Hasna, and I. S. Ansari, "Unified performance analysis for multiuser mixed $\eta - \mu$ and $\mathcal{M}$ - distribution dual-hop RF/FSO systems," *IEEE Trans. Commun.*, vol. 65, no. 8, pp. 3601–3613, Aug 2017.
- [10] N. Varshney and P. Puri, "Performance analysis of decode-and-forward-based mixed MIMO-RF/FSO cooperative systems with source mobility and imperfect CSI," *J. Lightw. Technol.*, vol. 35, no. 11, pp. 2070–2077, Jun 2017.
- [11] J. Chen, L. Yang, W. Wang, H. Yang, Y. Liu, M. O. Hasna, and M. Alouini, "A novel energy harvesting scheme for mixed FSO-RF relaying systems," *IEEE Trans. Veh. Technol.*, vol. 68, no. 8, pp. 8259–8263, Aug 2019.
- [12] N. D. Chatzidiamantis, G. K. Karagiannis, E. E. Kriezis, and M. Matthaiou, "Diversity combining in hybrid RF/FSO systems with PSK modulation," in *Proc. IEEE ICC*, June 2011, pp. 1–6.
- [13] M. Najafi, V. Jamali, and R. Schober, "Optimal relay selection for the parallel hybrid RF/FSO relay channel: Non-buffer-aided and buffer-aided designs," *IEEE Trans. Commun.*, vol. 65, no. 7, pp. 2794–2810, July 2017.
- [14] B. Makki, T. Svensson, M. Brandt-Pearce, and M. Alouini, "On the performance of millimeter wave-based RF-FSO multi-hop and mesh networks," *IEEE Trans. Wireless Commun.*, vol. 16, no. 12, pp. 7746–7759, Dec 2017.
- [15] M. Usman, H. Yang, and M. Alouini, "Practical switching-based hybrid FSO/RF transmission and its performance analysis," *IEEE Photon. J.*, vol. 6, no. 5, pp. 1–13, Oct 2014.
- [16] T. Rakia, H. Yang, M. Alouini, and F. Gebali, "Outage analysis of practical FSO/RF hybrid system with adaptive combining," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1366–1369, Aug 2015.
- [17] M. M. Abadi, Z. Ghassemlooy, S. Zvanovec, M. R. Bhatnagar, and Y. Wu, "Hard switching in hybrid FSO/RF link: Investigating data rate and link availability," in *Proc. IEEE ICC Workshops*, May 2017, pp. 463–468.
- [18] S. Sharma, A. S. Madhukumar, and R. Swaminathan, "Switching-based cooperative decode-and-forward relaying for hybrid FSO/RF networks," *IEEE/OSA J. Opt. Commun. Netw.*, vol. 11, no. 6, pp. 267–281, June 2019.
- [19] —, "Effect of pointing errors on the performance of hybrid FSO/RF networks," *IEEE Access*, vol. 7, pp. 131 418–131 434, 2019.
- [20] M. R. Bhatnagar and Z. Ghassemlooy, "Performance analysis of gamma gamma fading FSO MIMO links with pointing errors," *J. Lightw. Technol.*, vol. 34, no. 9, pp. 2158–2169, May 2016.
- [21] A. A. Farid and S. Hranilovic, "Outage Capacity Optimization for Free-Space Optical Links With Pointing Errors," *J. Lightw. Technol.*, vol. 25, no. 7, pp. 1702–1710, July 2007.
- [22] A. E. Willner, G. Xie, L. Li, Y. Ren, Y. Yan, N. Ahmed, Z. Zhao, Z. Wang, C. Liu, A. J. Willner, N. Ashrafi, S. Ashrafi, M. Tur, and A. F. Molisch, "Design challenges and guidelines for free-space optical communication links using orbital-angular-momentum multiplexing of multiple beams," *J. Opt.*, vol. 18, no. 7, p. 074014, Jun 2016.
- [23] W. R. Inc., "Mathematica, Version 8," champaign, Illinois, 2010.
- [24] K. Roach, "Meijer-G function representations," in *ACM Int. Conf. Symbolic Algebraic Comput.*, Jul. 1997, pp. 205–211.
- [25] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 7th ed. San Diego, CA, USA: Academic, 2007.
- [26] B. L. Sharma and R. F. A. Abiodun, "Generating Function for Generalized Function of Two Variables," *Proc. American Mathematical Society*, vol. 46, no. 1, pp. 69–72, Oct. 1974.