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**ESSAYS ON BEHAVIORAL HETEROGENEITY
AND ASSET PRICING**

LI CHANGTAI

SCHOOL OF SOCIAL SCIENCES

2020

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ASSET PRICING**

LI CHANGTAI

SCHOOL OF SOCIAL SCIENCES

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

2020

Statement of Originality

I certify that all work submitted for this thesis is my original work. I declare that no other person's work has been used without due acknowledgement. Except where it is clearly stated that I have used some of this material elsewhere, this work has not been presented by me for assessment in any other institution or University. I certify that the data collected for this project are authentic and the investigations were conducted in accordance with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from one paper accepted at conferences in which I am listed as an author.

Part of the content of Chapter 3 was presented as Li Changtai. “Investor Sentiment, Behavioral Heterogeneity and Stock Market Dynamics” at *The 25th Computing in Economics and Finance Conference* (Ottawa) in June 2019, *The Allied Social Sciences Associations (ASSA) Annual Meeting* (San Diego) in January 2020.

8 July 2020

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Date

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Executive Summary

Recent empirical studies have confirmed the importance of investor behavior in asset pricing. This thesis aims to explore the effect of behavioral heterogeneity on asset pricing with focuses on investor sentiment and trading volume. Other related issues on financial markets, including stylized facts, fundamental value, steady states, chart patterns and the formation of sudden, smooth and disturbing crises are also investigated. I mainly explore these issues by using a heterogeneous agent model methodology.

In recent years, asset pricing models based on Efficient Market Hypothesis (EMH) fail to explain several ubiquitous financial regularities, such as fat tail of returns, excess volatility and systemic under- or over-valuation of stock prices relative to their intrinsic values. It gives rise to the alternative behavioral finance theory which aims to provide rationale behind the unexplained market anomalies. Asset pricing models in behavioral finance usually incorporate the psychological factors into the decision making process and investigate the role of these factors in various market dynamics. As one of the representative factor in psychology, sentiment has attracted much attention from both academia and industry. Empirical evidences have been found that sentiment is highly related to assets returns and many market dynamics. Investors have attempted to construct sentiment index and develop sentiment-based strategies to invest in financial markets. To better understand the role of sentiment in asset pricing, a solid behavioral finance model should be built. Among the new behavioral approaches, heterogeneous agent model (HAM) is one of the best to fit the financial data and explain the market stylized facts. Most of the HAMs include two important classes of investors: fundamentalists and chartists. Fundamentalists make trading decision based upon market fundamentals, such as dividends,

earnings, macroeconomic factors etc. They believe that irrational price fluctuation is temporary and price is mean-reverting to fundamental level. In contrast, chartists look for simple patterns, e.g. trends, in past prices and base their investment decisions upon extrapolation of these patterns. More complicated HAMs consider the interaction of different agents, and agents could switch their trading strategies. HAMs with interacting agents have the ability to explain most of the stylized facts observed in financial time series. Combining with sentiment, HAMs could be more powerful to demonstrate the role of investor's behavior in asset pricing.

This thesis will mainly focus on four questions: (1) How to model sentiment in HAM and what is the format of market equilibrium steady state with sentiment? (2) What is role of sentiment in stylized facts and different types of crisis in financial market? (3) How to estimate the two-market HAM with sentiment? (4) How to use HAMs to explain another important indicator, trading volume? To answer the first question, I develop a heterogeneous agent model in Chapter 2, and a sentiment index is constructed by taking into consideration of sentiment memory, social interaction of investors and sentiment shocks. From the steady state analysis of the market equilibrium, I find that sentiment models could generate both fundamental steady states featured with neutral sentiment and non-fundamental steady states with polarized sentiment. Chapter 3 applies the three-agent sentiment model proposed in Chapter 2 to conduct numerical analysis. I find the sentiment model is more powerful to explain the stylized facts in market comparing with model without sentiment. By further exploring the role of sentiment in the crisis formation, I conclude that investor sentiment could contribute to the frequency and magnitude of financial crisis. Chapter 4 examines a two-market sentiment HAM with a focus on estimation. I attempt to estimate two-market model by Bayesian approach with extended Kalman filter. Estimation results from US and UK data show the evidence of market switching behavior of internationalist group, which provides a channel to explain crisis contagion through sentiment-induced market switching of international traders. Jumping out from sentiment, Chapter 5 investigates another important indicator, trading volume, under a three-agent HAM. The model is proven to have capability of explaining the styl-

ized facts on both market prices and trading volumes. It also provides an option to theoretically explore the co-evolution of prices, volumes and beliefs in financial markets.

The sentiment models and volume model developed in this thesis provide new angles to understand the financial markets. Both sentiment-based factors and market-based factors are important for asset pricing. With the support of these theoretical studies, one can build relevant trading strategies and investment portfolios in practice.

Chapter 1

Introduction

1.1 Background

For a long time, Efficient Market Hypothesis (EMH) has been the cornerstone and mainstream belief of modern asset pricing theory. In recent years, however, skepticism against validity of EMH has grown in light of its failure to explain several ubiquitous financial regularities, such as excess volatility and systemic under- or over-valuation of stock prices relative to their intrinsic values. This gives rise to the alternative behavioral finance theory which aims to provide rationale behind the unexplained market anomalies.

The new behavioral approach, particularly heterogeneous agent model (HAM), fits the financial data well. HAM shifts the paradigm of representative agent hypothesis with rational expectation to the boundedly rational agents with heterogeneous expectations. It mainly uses computational and numerical simulation methods in the analysis. Such models provide rich and satisfying depiction of market behavior dynamic and they are capable of explaining a wide range of stylized facts in financial markets, such as fat tails of return distribution, excessive volatility, volatility clustering, bubbles and crashes (see for instances, Lux, 1995, 1998; Brock and Hommes, 1998; Chiarella and He, 2003; He and Li, 2007; Dieci and Westerhoff, 2010; Huang et al., 2010; Chen, 2012). Rather than investigating large-scale HAM with lots of agent types, many HAMs are restricted to a small number of agent types. The most popular HAM framework distinguishes two types

of agent, fundamentalists and chartists. Fundamentalists make trading decision based on fundamental information of the market and economic conditions, such as dividends, earnings, macroeconomic factors, etc. They believe the irrational price fluctuation is temporary and price is mean-reverting to fundamental value. So they will buy (sell) the asset when the current price is below (above) the fundamental value. In contrast, chartists form their expectations on future prices upon the technical rules, which are mainly derived from the observed historical patterns of market prices. For example, chartists form their expectations by extrapolating observed patterns, such as price trends, support and resistance price lines. Some HAMs further distinguish chartists to momentum traders and contrarian traders according to the specific technical rules they adopt. The empirical evidences of fundamentalists and chartists have been found in literature, such as Frankel and Froot (1990), Allen and Taylor (1990), Menkhoff (2010) and Dick and Menkhoff (2013). In practice, fundamentalists are viewed as "rational" investors, they attempt to look to the reasons behind the price move upon the fundamental information. Usually, institutional investors act as fundamentalists in financial markets. On the other hand, chartists see the market price as embodying all aspects of the market. They use basic chart analysis involves visually identifying recurring patterns in financial time series. For example, certain 'reversal patterns' are taken to indicate the imminent reversal of a trend. Chartists usually consist of individual traders and fund managers who employ technique analysis to build their trading strategies.

In this chapter, I review three pioneering and influential models in HAM literature, namely, Day and Huang (1990), Lux (1995) and Brock and Hommes (1997). These three models are widely adopted by the later research and they laid the cornerstones for subsequent HAM studies.

1.2 Three pioneering HAMs

Day and Huang (1990) HAMs have started to gain popularity since the early 1990s. Day and Huang (1990) is one of the earliest seminal contributors in this area. They

develop a nonlinear dynamic model of the interaction between α investor (fundamentalist) and β investor (chartist). By introducing some specific types of nonlinearity in the model, they are able to simulate several patterns of market dynamics, such as boom-bust cycles and chaotic fluctuation of stock prices. In more detail, their model is formalized as follows:

They assume that asset prices are set by a market maker, who adjusts prices as a function of excess demand:

$$p_{t+1} = p_t + \lambda(D_{f,t} + D_{c,t}) \quad (1.1)$$

where λ denotes the speed of price adjustment by market maker. Excess demand is aggregated by fundamentalists demand $D_{f,t}$ and chartists demand $D_{c,t}$. They further assume that fundamentalists believe with more confidence that price is mean-reverting when further deviating from its fundamental value within a restrict zone. This behavior of fundamentalists could be captured by a nonlinear function as

$$D_{f,t} = a(p_t - \mu_t)f(p_t - \mu_t) \quad (1.2)$$

$$f(p_t - \mu_t) = \frac{1}{(p_t - \mu_t - m_1)^d (m_2 - p_t - \mu_t)^d} \quad (1.3)$$

where a is the strength of fundamentalists' investment demand and μ_t denotes the fundamental value of asset. $f(p_t - \mu_t)$ is a bimodal probability density with peaks of m_1 and m_2 . This is the only nonlinear component in their model, and the $f(\cdot)$ function reveals the nonlinear behavior of fundamentalist. The demand of chartists in their model is given by

$$D_{c,t} = b(p_t - v_t) \quad (1.4)$$

where b is the extrapolating rate of chartists and v_t is the long-term investment value perceived by chartists.

In their simulations, asset prices move erratically between bull market regime with prices above the fundamental value and bear market regime with prices below the funda-

mental value. Day and Huang (1990, p.312) also note that “*numerical market simulation does exhibit a salient feature of real stock market data, namely, irregular fluctuations around a shifting level, that is, the appearance of randomly switching bear and bull markets*”.

Lux (1995) In Lux’s (1995) model, he describes a financial market with a fixed number of N investors, which could be divided into fundamentalist group of size n_t^f and chartist group of size n_t^c at time t , where $n_t^f + n_t^c = N$. The chartist group is further subdivided into n_t^+ as being optimistic investor and n_t^- as being pessimistic investor, where $n_t^+ + n_t^- = n_t^c$. The contagion behavior of chartists in Lux (1995) is inspired by the ant model as in Kirman (1993), in which speculators tend to switch their beliefs depending on the majority opinion and prevailing price trend. The opinion formation of chartists is based on the social interaction within group. Lux constructs an opinion index by computing the relative difference between size of optimistic traders and pessimistic traders, which is given as

$$x = \frac{n^+ - n^-}{n^c} \quad (1.5)$$

where $x \in [-1, 1]$ and $x = 1$ ($x = -1$) represents the case that chartists are all optimists (pessimists). The switching behavior of two groups of chartist is determined by the opinion index x and the price trend which is denoted as $\dot{p} = dp/dt$. The transition probabilities in the switching process can be written as

$$\pi_{+-} = \nu \exp(a_1 \dot{p} / \nu + a_2 x) \quad (1.6)$$

$$\pi_{-+} = \nu \exp(-a_1 \dot{p} / \nu - a_2 x) \quad (1.7)$$

where a_1 and a_2 represent sensitivity of traders to the opinion index, and ν denotes the frequency of this type of transition. Lux (1995) follows the market maker mechanism for price formation. In a continuous form, it can be expressed as

$$\frac{dp}{dt} = \lambda (D_f + D_c) \quad (1.8)$$

where λ is the speed of adjustment. The corresponding demand of fundamentalists is written in a linear form as

$$D_f = \gamma n^f (\mu - p) \quad (1.9)$$

$\gamma > 0$ measures fundamentalist's speed of reaction to price deviation from fundamental value. As the chartist group comprises two subgroups with opposite beliefs, the demand of chartists is given as

$$D_c = (n^+ - n^-) t^c \quad (1.10)$$

where t^c denotes a fixed amount of the asset bought or sold by two groups of chartists. Using this setup, Lux (1995) proves that the opinion contagion process may lead to existence of (positive and negative) bubbles. In Lux's following papers such as Lux (1998) and Lux and Marchesi (1999, 2000), they show that the opinion contagion model can successfully explain several stylized facts observed in financial market within a single framework.

Brock and Hommes (1997) Among the early literature on HAMs, Brock and Hommes (1997) are the first to introduce adaptive belief system to the model. They discuss evolutionary selection of expectation rules by investors as a discrete choice problem in a traditional asset pricing model with mean-variance utility maximization. Because of that, their model is more complicated than models in Day and Huang (1990) and Lux (1995). The demand per trader of type h for risky asset is expressed as

$$D_{h,t} = \frac{E_{h,t}[x_{t+1}] - (1+r)x_t}{\alpha \sigma^2} \quad (1.11)$$

where $x_t = p_t - \mu_t$ is the price deviation from fundamental value μ_t , r is the risk-free interest rate, α denotes the risk aversion of investors and σ^2 captures the conditional variance of excess return. Again, their model divides investors into fundamentalists and chartists based on traders' heterogeneous expectation on future prices of risky asset with a form of

$$E_{h,t} = E(\mu_t) + f_h(x_{t-1}, \dots, x_{t-L}) \quad (1.12)$$

f_h denotes the forecasting rule based on technical analysis. For fundamentalists, $f_t(\cdot) = 0$, as they only care about the fundamental value, while chartists who have positive or negative $f_t(\cdot)$ are defined as biased optimists and pessimists, respectively. In Brock and Hommes (1997), all investors update their trading rules over time based on the realized profits. The evolutionary switching process is modeled by a multinomial logit probability function as

$$n_{h,t} = \frac{\exp(\beta U_{h,t-1})}{\sum_{h=1}^H \exp(\beta U_{h,t-1})} \quad (1.13)$$

$U_{h,t-1}$ is the fitness measure of trading rule h and is calculated as realized profits. β is intensity of choice measuring the speed of switching between different beliefs. They assume zero supply of outside share with an equilibrium asset-pricing framework, and the asset price in two-agent type (fundamentalists and chartists) model is determined as

$$p_t = \frac{1}{1+r} (n_{f,t} E_{f,t}[x_{t+1}] + n_{c,t} E_{c,t}[x_{t+1}]) \quad (1.14)$$

The most important feature of Brock and Hommes's (1997) model is the adaptive process with evolutionary selection of expectation rules or trading strategies. It provides a novel way to analyze financial markets by viewing them as a complex adaptive system. The subsequent papers following Brock and Hommes (1997) have proven that the interactions of individuals with adaptive learning process at the micro-level could explain rich market structures and stylized facts at macro-level.

These three models discussed above all assume the heterogeneity of investors in financial markets. The interaction of heterogeneous investors could cause the chaotic price pattern. By introducing a nonlinear fundamentalist demand function into their model, Day and Huang (1990) find the evidence that belief heterogeneity could be related to market bubbles and crashes. The fractions of fundamentalists and chartists are assumed constant over time in Day and Huang (1990), while some survey studies like those of Taylor and Allen (1992) find that most of the investors are familiar with both fundamental and chartism trading strategies, and only minor fraction of the investors relies exclusively on one trading strategy. In other words, investors may switch their strategies

under some circumstances. The fixed fraction assumption is released in Lux (1995) and Brock and Hommes (1997). The dynamics of their models are derived by the fact that investors switch between different trading strategies. In Lux (1995) model, belief switching heavily depends on the social interactions between investors. For instance, traders may herd together on one strategy. While Brock and Hommes (1997) assume the selection of strategies is based on past realized profits, and they find that belief switching could be a significant source of chaotic price trend. All these three models and their subsequent models are able to produce chaotic motion, and the simulations mimic the behavior of real financial data quite closely.

1.3 Organisation of the Thesis

Over the past three decades, HAM study has been extended to many areas, such as financial markets, macroeconomy, sociology and psychology. A comprehensive review of HAM study can be found in Hommes and LeBaron (2018). In recent years, more and more researchers have recognized the important role of investor sentiment in financial markets and many abnormal market phenomena are proven to be related to sentiment. The empirical evidences in Stambaugh et al. (2012) show that sentiment could provide explanation for 11 asset pricing “anomalies”. Although empirical analyses of sentiment effect on financial markets have been extensively explored, theoretical studies on how the sentiment affects asset prices are still lacking. In this thesis, I mainly discuss the sentiment effect on financial markets with a HAM framework.

Chapter 2 proposes a HAM with a sentiment effect. The market consists of fundamentalists and chartists. Fundamentalists believe market price is mean-reverting to the fundamental value, which is determined by the fundamental factor in the market and macroeconomy. To more accurately capture the behavior of fundamentalist, I use a non-linear excess demand function similar to that in Day and Huang (1990) for fundamentalists, and it could represent the time-varying confidence level in response to different levels of price deviation from fundamental value. Meanwhile, chartists are assumed to

form their price expectation based on the price trend. Instead of using moving average of prices as price trend, I follow the setup in Huang et al. (2010) in which price trends are formed and updated as series of psychological trading regimes. Chartists group can be further divided into two subgroups, namely momentum traders and contrarian traders. The momentum traders tend to follow the price trend while the contrarian traders go against the trend. Similar to fundamentalists, chartists also have time-varying extrapolation rates, but their trading behaviors are susceptible to the market sentiment. Momentum traders and contrarian traders are assumed to response differently to the same market sentiment due to their opposite beliefs. The sentiment is constructed by considering three main sources including memory of sentiment, investor mood from social interaction and sentiment shock. I further introduce evolutionary switching mechanism as in Brock and Hommes (1997) to this model, in which all agents are free to switch their beliefs among different expectation rules. The market price is determined by the aggregate excess demand of all agents within a market maker framework. I conduct in-depth analysis on two-type and three-type sentiment HAMs by focusing on the sentiment-related steady states of the market equilibrium. The main finding in this chapter is that sentiment-HAM could produce both fundamental steady state featured with neutral sentiment as well as non-fundamental steady states with polarized sentiment.

Chapter 3 aims to further evaluate the three-agent-sentiment-HAM by focusing on its capability to replicate stylized facts and explain different types of crises in financial markets. By comprising sentiment effect in the model, HAM appears very successful in duplicating the stylized facts including fat tails of return, long memory of return, clustering of volatility, and excessive volatility in the US stock market. By comparing simulation results between model with sentiment and model without sentiment, I find the former is more powerful in explaining these market features, which indirectly proves that sentiment could be the source of these market dynamics. I further investigate the role of sentiment in the crisis formation. By excluding other nonlinear components in the model, I find sentiment could contribute to the formation of different types of crises, which include smooth crisis, sudden crisis and disturbing crisis.

Chapter 4 extends one-market model system to two-market model system. It examines a two-market-sentiment-HAM with a focus on estimating the model parameters. In this two-market model, the demand of agents is derived from traditional asset pricing framework as in Brock and Hommes (1997). In each market, there are three types of investors, fundamentalists, chartists and internationalists. These two markets are essentially linked by internationalists who build their international portfolios based on the market sentiment. Fundamentalists and chartists could switch their strategies based on the realized profits but they can only trade in one market, whereas internationalists can freely switch their trading markets based on the sentiment difference of two markets. Specifically, internationalists are assumed to adopt chartist strategy and tend to invest in the market with higher sentiment. To reduce the difficulty of estimation, I simplify the model by eliminating some nonlinear components in the model. For example, the demand functions of fundamentalists and chartists are simplified to linear forms and endogenous sentiment index is removed. Nonlinearities of this model are mainly manifested in the belief switching and market switching mechanisms. Instead of estimating the model using nonlinear least square or maximum likelihood approach, I convert the model to a state-space model then estimate it by a novel Bayesian approach with extended Kalman filter. Real stock market data of the US and UK are used in the estimation. The result shows the existence of behavior heterogeneity in international financial markets. Fundamentalists and chartists switch their beliefs less aggressively in US and UK stock markets comparing with US and Japan markets, while their fractions are more evenly distributed in UK market. Internationalists are assumed to adopt chartism strategy, and the market switching behavior of internationals is found in the estimation. By adding the market switching mechanism, this model is able to detect more significant investor heterogeneity. Through the market switching, internationalists are also shown to be related to market instability, prices co-movement and boom-and-bust cycles in stock markets.

As argued by Baker and Wurgler (2006), trading volume can be used as a simple proxy for the sentiment index. Moreover, volume is an important indicator to reflect investors' behavior in financial market. Chapter 5 investigates the trading volume with

three-type HAM with the similar model setup to that in Chapter 2, but without sentiment factor. The model is proven to have a great potential to explain the stylized facts on both price and volume. It also provides a method to explore the co-evolvement of prices, volumes and beliefs in financial markets. I also show that volume could play an important information rule when investors predict the asset price by conducting chart pattern analysis. Finally, I simulate three different crises as in Chapter 3, and the model with volume provides a supplementary explanation to the formation of crises.

Chapter 6 summarizes the findings and contribution of this thesis. It also points out the caveats and future research directions.

1.4 Contributions of the Thesis

There are several contributions in this thesis. Firstly, it proposes a novel method to construct sentiment index. By including three main sources of investor sentiment, I successfully build a sentiment index for the simulations. From perspective of empirical studies, this measurement of sentiment can also be achieved upon the availability of data. Secondly, I model heterogeneous effect of sentiment under HAM framework and find that the steady states of dynamic models are related to sentiment parameters. Thirdly, through the simulation of sentiment HAM, I find investor sentiment is a significant source of market volatility. Moreover, the sentiment channel provides an explanation to the mechanism underlying different types of financial crisis. Fourthly, I propose a Bayesian approach with extended Kalman filter to estimate two-market HAM. The estimation results show that sentiment-sensitive internationalists are related to market instability, price co-movement and boom-and-bust cycles in stock markets. Finally, using a three-agent HAM with trading volumes, I find how price-volume-belief relationship evolves in stock market especially during financial crisis periods.

Chapter 2

Modelling Sentiment Effect with Heterogeneous Agents

2.1 Introduction

Behavioral finance challenges the fundamental assumption of the EMH, in that investors are assumed to be boundedly rational and human psychology plays a crucial role in investment decisions. In fact, even before behavioral paradigm came into the limelight in finance and economics, investor sentiment has been perceived as a common phenomenon by financial analysts and market participants. As Shiller (2003) mentioned, perhaps one of the oldest financial theories expressed long ago in nonacademic papers is the price-to-price feedback theory. The feedback theory suggests that an increase in speculative prices is further propagated into bubble when financial successes of some investors are envied by the others and lead to public enthusiasm towards the speculative asset despite such upward spiral in prices is unsustainable. While conventional wisdom largely supports the idea that investor sentiment may overcome rational thoughts in trading behavior, the sentiment analysis has only started gaining recognition in financial academic research in the past two decades.

One of the pioneering works that formalizes the role of investor sentiment in financial market is the noise trader model proposed by De Long et al. (1990). In their model,

uninformed noise traders are susceptible to the influence of sentiment that is in part unpredictable, while rational investors are wary of the noise trader risk and refrain from aggressive arbitrage, thus contributing to prolonged mispricing in financial market. The subsequent related studies conduct more in-depth analyses on specific channels of investor sentiment. Lux (1995, 1998) explicitly model market mood contagion through social interaction among agents to provide a behavioral explanation for bubbles and crashes. Daniel et al. (1998) and Barberis et al. (1998) construct models of investor sentiment based on psychological evidence to reconcile the empirical findings of over-reaction and under-reaction of stock prices to news. In particular, Daniel et al. (1998) attribute sentiment to overconfidence and self-attribution, whereas Barberis et al. (1998) concentrate on conservatism and representativeness heuristic. More recently, Benhabib et al. (2015) and Benhabib et al. (2016) introduce sentiment into DSGE and OLG models and they find that sentiment-based asset prices with informational feedback drive multiple equilibria and sunspot equilibrium. I am motivated by their studies to explore non-fundamental and sentiment-driven equilibria.

There are a large empirical literature that has documented that investor sentiment in financial markets can affect asset prices. Baker and Wurgler (2006, 2007) develop a “top down” approach to behavioral finance, by first forming a composite sentiment index and then empirically testing the effects of the sentiment index on different types of stocks. They find that low sentiment can predict higher returns for a subset of stocks. By investigating the interactions between daily media context from *Wall Street Journal* column and stock market from 1984-1994, Tetlock (2007) finds that media content is linked to the behavior of individual investors rather than serving as a proxy for new information about fundamental asset values or proxy for market volatility. Furthermore, he finds that high media pessimism predicts downward pressure on market price and increased market volatility. To date, there have been many studies, both theoretical and empirical, that prove evidence of investor sentiment effects in financial market (See for examples Brown and Cliff, 2004; Da et al., 2015; Lee et al., 2002; Neal and Wheatley, 1998; Stambaugh et al., 2012). In view of that, several experimental studies have brought this into labora-

tory environment and find that investor's psychology, specifically over-optimism (Hüsler et al., 2013), friendship network (Makarewicz, 2017) and induced positive mood (Lahav and Meer, 2012), can amplify market price oscillations. Most empirical and experimental studies what determines sentiment and how sentiment could be use in financial analysis, very few studies track the roots of sentiment formation in behavioral trading and model the sentiment formation process in dynamic systems.

In this chapter, I try to fill this research gap by investigating formation of investor sentiment and its role on asset price dynamics and market equilibrium within the framework of heterogeneous agent model (HAM). HAM is a burgeoning framework under behavioral finance which incorporates interacting agents with heterogeneous trading beliefs. Instead of following standard representative agent assumption, HAM assumes the differentiation of traders, especially with respect to their expectations or beliefs on future price. A strand of HAM literature which based on a dichotomy of fundamentalist and chartist beliefs has been proven useful in accommodating market features that seem not easily reconcilable under the traditional financial market paradigm. These features include fat tail, volatility clustering, bubbles and crises (see pioneering works by Day and Huang, 1990; Lux, 1995; Brock and Hommes, 1998; Chiarella and He, 2003; He and Westerhoff, 2005). While HAM has gained increased popularity in recent years, only a small number of HAM studies have taken into account investor sentiment (Chiarella et al., 2017; Lux, 2012), let alone a rigorous research on the role of sentiment in financial market. To fill this gap in the literature, I propose a HAM model with sentiment indicator that captures memory of sentiment, social interaction and sentiment shock. The idea of social interaction is inspired by the work of Lux (1995, 1998) such that agents are not isolated units. Speculators, who are also commonly known as chartists, will rely on both actual price movements as well as the behavior of their competitors in forming their expectations. As such, I suggest that sentiment may not affect everyone in a homogeneous way. This conjecture is founded on the abundant evidence of behavioral heterogeneity between individual traders and institutional traders, in which the former lacks access to insider information and thus is more susceptible to market sentiment (see

De Long et al., 1990; Kumar and Lee, 2006). I take advantage of the merits in all three pioneer HAMs mentioned in Chapter 1. More specifically, I include nonlinear fundamentalists and market maker as in Day and Huang (1990); I borrow the idea from Lux (1995) that investor sentiment is generated from investors' social interaction; I also follow Brock and Hommes (1998) evolutionary switching mechanism and allow agents to switch their trading rules. Under my HAM setting, fundamentalists represent rational arbitrageurs that possess information about fundamental asset value, whereas chartists are ill-informed speculators extrapolating on market trend. I thus discriminate between these two groups, in which the chartists' expectations are liable to market sentiment. An endogenous mechanism between sentiment and agent's belief switching is developed with investors switching between fundamentalist and chartist beliefs according to past performance while the sentiment index is contingent on the fraction of adopted beliefs in the market.

By analyzing the equilibria of deterministic models, I find the existence of sentiment-related non-fundamental steady states in the systems. Through explicitly modelling the sentiment in three-agent model, I show that sentiment related parameters are able to affect the stability of fundamental steady state. Furthermore, complementing findings of Huang and Zheng (2012), the sentiment channel provides an explanation to the mechanism of regime switching.

This chapter is organized as follows. Section 2 defines investor sentiment and explains the formation of market sentiment. Section 3 presents the sentiment model. Section 4 explores the dynamic of different deterministic skeletons with focuses on stability conditions and sentiment role in two-type and three-type HAMs. Section 5 concludes.

2.2 Sentiment

Classical finance theory neglects the role of investor sentiment as investors are supposed to be rational. Rational investor assumption is the key for many classical models and market is also assumed to be perfectly efficient. Even if some investors are not rational,

arbitrageurs can exploit their irrational behavior, thus causing prices to reflect the fundamental information of the market. The assumption of the rational investor is generally quite useful in modeling, but it misses many important exceptions. John Maynard Keynes was one of the first economists to explore the relationship between mental processes and economic activity. He argued that human psychology played a crucial role in driving economic activity. As he stated, *“Our decisions to do something positive. . . can only be taken as the result of animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities”*. Keynes’s notion of animal spirits highlight the role of psychological factors in fueling economic and investment activity. With the emergence of behavioral economics and finance, role of psychological factors have been widely investigated in financial markets. As a common concept in psychology, sentiment has attracted much attention in behavioral finance. In psychology, sentiment is understood as human emotion. Human experience a broad range of emotions, including joy, fear, stress, anger, gloom and so forth. These emotions could influence the decision making of human. Hence, it is important to explore cognitive or emotional factors in decision making. There are so many emotions involved in investment behavior, to measure the aggregate sentiment for investors in financial markets, I focus on the sources of sentiment. In other words, I care about what factors cause the change of sentiment in financial market. By reviewing literature in psychology and behavioral finance, I find three important sources for sentiment formation in financial markets, which are memory of sentiment, social interaction and sentiment shocks (news, policies and events etc.).

Memory of sentiment One important phenomenon documented by psychologists is sentiment persistence. Human brain stores information with different sentiment. This memory of sentiment may last for a few days or weeks. In terms of trading behavior, investors’ sentiment is persistent if no arrival of new information. If investors are optimistic in the investment, they will keep this sentiment for periods until new information change their mood. Many psychological experiments have shown that individuals are

slow to change their beliefs even in the face of new information. The sentiment model developed in Barberis et al. (1998) is based on the persistence of sentiment and conservatism behavior of investors. Empirical evidences of sentiment persistence in financial markets are also found in Brown and Cliff (2005), Schmeling (2007) and Heiden et al. (2013).

Social interaction Humans are inherently social. The social interaction is an important way to form opinion, sentiment or beliefs. People who regularly interact with each other tend to think and behave similarly. As mentioned in Shiller (1995), there are two main strands literature dealing with this herding behavior. The first group focuses on information cascades model proposed by Banerjee (1992). He argues that people acquire information in sequences by observing the actions of other individuals who precede them in the sequence. In financial markets, the trading activity of one investor conveys information to other investor that can cause latter to react, leading to a cascade of trading. Another strand of literature focuses on interpersonal communication of investors. The conversation and social interaction is more relevant to describe mass behavior than informational cascades. Conversation not only serves to exchange information, it also reinforces memories of pieces of information. With these social interaction, sentiment may also contagion within the group. Suppose the majority in a social group hold the optimistic opinion on stock market, this bullish sentiment may spread in the group through social interaction. With the help of social media, the sentiment may spread even faster.

Sentiment shocks Sentiment shocks refer to exogenous sources that can shift the sentiment. An example is pseudo signal which is also called noise. It may be news, policies, rumors or advice of a financial guru. The noise could shift the current sentiment of investors and lead to noising trading. As stated in Black (1986), noise is often a causal factor much more powerful than a small number of large events can be. As news and noise information are so important to measure market sentiment, many studies are attempt to develop the sentiment index based on media news. Soo (2018) finds that media

sentiment has significant predictive power in housing market.

2.3 The Model

In this section, I set up an asset pricing model with a single risky asset to characterize time series momentum and investor sentiment in the financial market. The modelling approach follows closely the current HAM framework by incorporating bounded rationality, belief heterogeneity, sentiment formation and adaptive learning process.

In the HAM literature, the market fractions of different types of traders play a pivotal role in determining the market price behavior. According to Lux (1998), the time-varying market fraction of investors is the source of market mood or market sentiment, which may introduce complicated dynamics in the financial market. Based on both theoretical and empirical evidence, my model extends early models by introducing investor sentiment into the decision making process of agents. In each trading period, population of agents is assumed to be distributed among three groups, each relying upon different behavioral rules. These include fundamentalists who trade based on fundamental analysis as well as momentum traders and contrarian traders who trade differently according to historical price trend. In particular, I postulate that sentiment affects different types of agents in a heterogeneous way. I assume that fundamentalist group is more rational and is immune from market sentiment, while the chartists, both momentum traders and contrarian traders, are susceptible to market sentiment. Moreover, momentum and contrarian traders also react differently with respect to positive and negative sentiment. An endogenous mechanism between sentiment and agents' belief switching is developed, in that investors are allowed to switch their beliefs according to past performance while the sentiment index is contingent on the fraction of adopted beliefs in the market. As in Day and Huang (1990), market price in each period is determined by a market maker who adjusts market by considering aggregate excess demand of agents.

Fundamentalists I assume that fundamentalists have more knowledge about the economy and have a notion about fundamental price. Hence, fundamentalists make decision

based on fundamental price, μ_t . They believe that market price p_t is mean-reverting to the fundamental price and hence will buy (sell) the stock when the current price is below (above) the fundamental price of the stock. They estimate the fundamental price based on various types of fundamental information, such as expected dividends, earnings, price-earnings ratios, economic growth and so forth. In each period, the fundamental price is updated with new information arrival which is accessible to the public. The prior of μ_t is governed by:

$$\mu_t = \mu + e_t \quad (2.1)$$

where μ is the mean of fundamental value and the noise term e_t is independently and normally distributed with mean 0 and standard deviation σ . Instead of deriving the demand functions from expected utility maximization, I adopt simple demand functions for all the three types of agents. As shown in some literature, such as Beja and Goldman (1980), Chiarella et al. (2006), and Day and Huang (1990), these seemingly ‘ad hoc’ assets demand functions can be reconciled with the underlying utility maximization. The excess demand of fundamentalists $D_{f,t}$ is based on the spread between the latest market price p_t and the fundamental price μ_t , which can be written as:

$$D_{f,t} = A(x_t)(\mu_t - p_t) \quad (2.2)$$

The reaction function $A(x_t)$ captures the behavior of the fundamentalists when price is around the fundamental price. It is assumed to be a nonlinear smooth function of price deviation from fundamental value. Let $x_t = p_t - \mu_t$ denotes this price deviation, then:

$$A(x_t) = \frac{ax_t^2}{1 + bx_t^4} \quad (2.3)$$

$A(x_t)$ could mimic the change in confidence of fundamentalists. I assume their confidence continuously increases with absolute price deviation $|x_t|$ in a reasonable zone $x_t \in (-Z_t, Z_t)$, and the range is determined by parameter b . Within this reasonable zone, fundamentalists firmly hold the fundamental strategy, and the rise of price misalignment

makes them feel more confident that price will revert to fundamental value soon, so they should grasp the opportunity to buy or sell stocks to maximize their gain. As suggested by Day and Huang (1990), such behavior is justified by increasing profit opportunities within a reasonable zone. However, if the misalignment further increases and exceeds the reasonable zone, with high uncertainty in the market, fundamentalists may wrongly predict the trend of asset price and gradually lose confidence. The decreasing power of fundamentalist with increasing price deviation has been proven theoretically and empirically in Frankel et al. (1986), Frankel and Froot (1990) and Westerhoff and Reitz (2003). The properties of the function A are discussed in Appendix A.

To understand the role of fundamentalists, I first focus on the case where fundamentalists are the representative agents in the financial market. This assumption is consistent with EMH. In fact, the nonlinear demand function of fundamentalists alone could generate some price dynamics in the financial market. With the market maker mechanism and adaptive expectation, price function can be written as:

$$p_{t+1} = p_t + \gamma A(x_t)(\mu_t - p_t) \quad (2.4)$$

Assuming $\mu_{t+1} = \mu_t$, it can be further converted to functions of x_t such that:

$$x_{t+1} = x_t - \gamma \frac{ax_t^3}{1 + bx_t^4} \quad (2.5)$$

$$f(x) = \dot{x} = -\gamma \frac{ax^3}{1 + bx^4} \quad (2.6)$$

where γ is the speed of price adjustment to excess demand of market maker. To investigate whether fundamentalists stabilize or destabilize the market, I conduct a simple stability analysis. In a deterministic model, $p_t = \mu$ or $x_t = 0$ is the unique steady state of Equation (2.4). For the stability of this fundamental steady state, non-hyperbolic fixed point with high power increases the difficulty of analysis. According to Dannan et al. (2003) and Murakami (2005), this kind of fixed point is asymptotically stable if condition $f(x) = 0, f'(x) = 0, f^{(2)}(x) = 0, f^{(3)}(x) < 0$ holds for Equation (2.6). As can be

Figure 2.1: Confidence function for fundamentalist

seen in Figure 2.1, when the initial value is close to zero, price eventually converges to the fundamental steady state $p_t = \mu$.

However, local stability is not necessarily global stability. If the initial value is far away from zero, with some parameter settings, there are also unstable regions. Within these regions, the system exhibits period doubling followed by a chaotic regime (see Figure 2.2).

Although fundamentalist group is always regarded as a stabilizing force in the financial market, they can also serve as destabilizing traders with specific conditions as in Figure 2.2. The chaos originates from nonlinear function $A(x_t)$ and time-varying confidence behavior of fundamentalists. As shown in Figure 2.3, the phase diagram of x_t indicates the unstable region of price deviation.

Chartists There are two types of chartists in the financial market, namely momentum traders and contrarian traders. Unlike the fundamentalists, both momentum traders and contrarian traders focus only on their short-term estimated market value, v_t , albeit different strategies are adopted by both parties. Similar to fundamentalists, chartists also have time-varying extrapolation rate, but their trading behaviors or confidence levels are sensitive to market sentiment.

Momentum traders When the current market price is above the short-term value, momentum traders expect future market price to rise and choose to take a long position; conversely, they take a short position. Excess demand of momentum traders is assumed

Figure 2.2: Chaotic price dynamic in unstable region: (a) chaotic time series of x_t ; (b) graphical analysis of x_t trajectory; (c) bifurcation diagram; (d) Lyapunov exponent plot. $a = 0.56, b = 0.01, \gamma = 1, x_0 = 2.7$

Figure 2.3: Phase diagram of x_t with parameter values $a = 0.56, b = 0.01, \gamma = 1, x_0 = 2.7$

to evolve over time based on the current short-term value by:

$$D_{mo,t} = \beta_1 m_t (p_t - v_t) \quad (2.7)$$

where $\beta_1 m_t$ is the time-varying extrapolation rate of price trend. $\beta_1 > 0$, represents the base extrapolation rate without sentiment effect. m_t is the time-varying sentiment factor, which is updated each period and will affect the trading decision of investors for next period. The sentiment factor is constructed as:

$$m_t = 1 + h_1 \tanh[\kappa(p_t - v_t)] S_t \quad (2.8)$$

where the sentiment index S_t is derived from social interaction of different types of agents and random sentiment-related information such as news and policies. $S_t \in [-1, 1]$ and the construction of sentiment index will be introduced later. $h_1 \in [0, 1]$ measures sensitivity of momentum traders to market sentiment. If $h_1 = 0$, it means investors are totally immune to sentiment, and the extrapolation rate is only determined by β_1 as $m_t = 1$. On the contrary, $h_1 = 1$ indicates that traders are very sensitive to sentiment that they perceive in financial markets.

Both positive and negative sentiments are expected to stimulate opposite impacts on chartists' long position and short position. More specifically, positive sentiment can reinforce the confidence of momentum traders in long position and raise their cautiousness in short position, while negative sentiment can bolster confidence of momentum traders on short position and weaken it on long position. If the price is above v_t while current market sentiment is positive, momentum traders will feel more confident to follow the price trend and buy in. Hence, their confidence level increases with further price deviation given positive market sentiment. On the other hand, if price trend is upward but market sentiment is negative, momentum traders will still follow the trend but with less confidence. This contradiction of momentum traders' prediction against market sentiment makes them more cautious and reduce their demand for the speculative asset. Besides the market sentiment, the price deviation from the short-term value $(p_t - v_t)$ also exerts

an impact on the investor's sentiment. To standardize price deviation $(p_t - v_t)$ within the range of $(-1, 1)$, I introduce a tanh function with a scaling factor κ . Thus, the range of m_t is $[0, 2]$.

Let's look more closely at the short-term asset value, v_t . I assume that all chartists, both momentum traders and contrarian traders, hold on to an identical short-term asset value. As in Huang et al. (2010) and Huang and Zheng (2012), I assume that chartists adopt the adaptive belief mechanism where they update their expectations on short-term asset value according to different price regimes. They believe in support and resistance levels which are derived from common rules of technical analysis. Accordingly, chartists divide price domain $P = [P_{\min}, P_{\max}]$ into n regimes such that:

$$\mathbb{P} = \bigcup_{j=1}^n P_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \dots \cup [\bar{p}_{n-1}, \bar{p}_n] \quad (2.9)$$

where $\bar{p}_j$ for $j = 1, 2, \dots, n$ represents the different support and resistance levels set by the chartists. The short-term asset value can be simply extrapolated as the average of the top and the bottom threshold prices:

$$v_t = (\bar{p}_{j-1} + \bar{p}_j) / 2, \quad \text{if } p_t \in [\bar{p}_{j-1}, \bar{p}_j) \quad (2.10)$$

When price fluctuates within the current regime, there are enough reasons for chartists to believe that the short-term asset value will remain unchanged for next period. However, once the price breaks through either the support or resistance lines, chartists will adjust their expectation on the short-term asset value according to Equation (2.10). This regime dependent phenomenon is commonly found in stock market with chartist's beliefs evolve with regime switching. According to Huang et al. (2010), the short-term asset value for each period is estimated as:

$$v_t = \left(\lfloor \frac{p_t}{\lambda} \rfloor + \lceil \frac{p_t}{\lambda} \rceil \right) \cdot \frac{\lambda}{2}, \quad \text{if } p_t \in [\bar{p}_{j-1}, \bar{p}_j) \quad \text{and} \quad j = 1, 2, \dots, n \quad (2.11)$$

Contrarian traders Unlike momentum traders, contrarian investors trade stock based on the hypothesis of market overreaction. Specifically, when current price is higher than short-term value v_t , they believe that future market price will drop and therefore take a short position; conversely, they take the long position. I assume contrarian traders use the same method as momentum traders to calculate short-term value v_t , hence the demand function of contrarian traders can be expressed as:

$$D_{co,t} = \beta_2 c_t (p_t - v_t) \quad (2.12)$$

Similarly, $\beta_2 c_t$ is the time-varying extrapolation rate of price trend for contrarian traders. $\beta_2 < 0$, represents the base extrapolation rate without sentiment effect and c_t is the time-varying sentiment factor for contrarian traders, which can be written as:

$$c_t = 1 - h_2 \tanh[\kappa(p_t - v_t)] S_t \quad (2.13)$$

where h_2 is sensitivity of contrarian traders to market sentiment with a range $[0, 1]$. Although contrarian traders adopt trading strategy opposite to that of momentum traders, they are affected by market sentiment in the same way. When market price is above v_t , contrarian traders expect price to decline. If the market sentiment is negative, contrarian traders will be more confident to take the short position. However, positive market sentiment will decrease the confidence level of contrarian investors. Besides the market sentiment, price deviation is another element that contributes to contrarian traders' sentiment factor c_t .

Belief switching regime One of the important features underlying the model is the belief switching regime of agents, which has become widely adopted since it was first proposed by Brock and Hommes (1997). Experimental evidence of beliefs switching behavior has been found in recent studies by Anufriev and Hommes (2012), Anufriev et al. (2013), Anufriev et al. (2016) and Anufriev et al. (2019). This regime switching behavior assumes that, at the end of each trading period, agents may switch their prediction

strategies or beliefs conditional on the past performance of three rules. Specifically, the performance measure depends on realized profitability, which is defined as:

$$\pi_{n,t} = (p_t - p_{t-1}) D_{n,t-1} \quad (2.14)$$

For simplicity, I assume that the gross interest rate between period $t - 1$ and period t is one and there is no cost for fundamentalists to acquire additional information. I further introduce additional memory into the measurement of performance. The performance can be calculated as the geometrically declining weighted average of the realized profits, given by:

$$U_{n,t} = \varphi U_{n,t-1} + \pi_{n,t} \quad (2.15)$$

where $0 \leq \varphi \leq 1$ represents the strength of memory put into the last-period performance, n denotes different types of agents.

I use $\omega_{n,t}$ to denote market fraction of three different types of investors. The fractions of three groups vary endogenously over time according to the choice model introduced by Manski et al. (1981) as well as Brock and Hommes (1997, 1998):

$$\omega_{n,t} = \frac{\exp(\rho U_{n,t})}{\sum_{h=1}^3 \exp(\rho U_{h,t})} \quad (2.16)$$

Note that, the new fractions of traders are determined on the basis of the most recent performance measure $U_{n,t}$. The parameter $\rho > 0$ is the intensity of choice. It measures the sensitivity of agents to the difference of past performance. The higher is ρ , the quicker agents will respond to difference in performance by switching to the most profitable strategy. For finite ρ , $\omega_{n,t}$ is always positive which implies that not all agents are going for strategy that gives highest profit.

Sentiment index As pointed out by previous section, there are three important sources of sentiment, namely memory of sentiment(last-period sentiment), investor mood from social interaction, and sentiment shock such as news, policies, firm innovations and so

forth. I develop the sentiment index by simply combining these three sources. Although it is only used to explain the formation of sentiment in my model. One can also use this method to construct sentiment indexes for the real financial markets. The index function can be written as:

$$S_t = \eta_1 S_{t-1} + \eta_2 SI_t + \eta_3 \varepsilon_t \quad (2.17)$$

where η_1 , η_2 , η_3 are weights assigned to different factors, such that $\eta_1 + \eta_2 + \eta_3 = 1$. ε_t is the sentiment shock, which I assume to follow a uniform distribution with range $[-1, 1]$. SI_t is social interaction of different types of investors, which influences the current market mood. The idea is inspired by majority opinion formation in Kirman (1993), Lux (1995), and Lux and Marchesi (1999), in which the majority opinion index is computed as the difference between optimistic and pessimistic individuals. Different from Lux and Marchesi (1999), I assume that the opinion index not only contains the opinion of optimistic and pessimistic chartists, but also includes the opinion of fundamentalists. The social interaction should exist among all types of investors, including both fundamentalist and chartist groups. Fundamentalists are optimistic (pessimistic) investors when market price is below (above) their fundamental values. Two different types of chartist may change their opinion under different market states. For instance, when current market price is above the short-term value v_t , momentum traders believe price will continue to go up and thus they belong to the optimistic group. In contrast to momentum traders, contrarian traders believe price trend will reverse and hence they belong to the pessimistic group. However, in the next period, if the market price falls below the short-term value v_{t+1} , momentum traders will become pessimistic while contrarian traders will switch to optimistic opinion. Besides the fractions of different agents, the price deviations from fundamental value μ_t and short-term value v_t could also affect the opinion index. Large price deviation could make investor opinion more convincing in social interaction. By considering both fractions of agents and magnitude of price deviation, I construct the social interaction index as:

$$SI_t = \tanh[\kappa(\mu_t - p_t)] \omega_{f,t} + \tanh[\kappa(p_t - v_t)] (\omega_{mo,t} - \omega_{co,t}) \quad (2.18)$$

where $\omega_{f,t}$, $\omega_{mo,t}$, $\omega_{co,t}$ are fractions of fundamentalists, momentum traders and contrarian traders in financial market, respectively. $\tanh$ function and κ are used to scale the price deviation, so both the range of SI_t and the range of sentiment index S_t can be constrained to $[-1, 1]$.

Market clearing mechanism Instead of using Walrasian auctioneer clearing mechanism, I adopt market maker mechanism, which is akin to the role of specialists in New York Stock Exchange. I assume net zero supply of the risky asset, and market price is set by a market maker who considering excess demand of all agents. The aggregate market's excess demand is weighted by population fraction. Hence, for a three-agent model (including fundamentalists, momentum traders and contrarian traders), the price p_{t+1} is set by market maker according to the aggregate excess demand, that is:

$$p_{t+1} = p_t + \gamma(\omega_{f,t}D_{f,t} + \omega_{mo,t}D_{mo,t} + \omega_{co,t}D_{co,t}) \quad (2.19)$$

where γ measures the speed of price adjustment by the market maker.

2.4 Analysis of model equilibrium conditions

2.4.1 Market stability analysis

The core of the HAM framework is the existence of heterogeneous agents with different trading strategies. Comparing with standard representative agent models, HAMs are more complicated due to heterogeneous agents' interaction. It would be more interesting to explore how market achieves its steady states and what are the necessary conditions for stable equilibria. By removing the perturbation of noise, the stability analysis could provide an insight into the effect of trading activities as well as interaction among the different types of investors on market stability. Specifically, I focus on market stability with investor sentiment effect in this chapter.

In this section, I give an extensive exploration of two-agent (2-agent) and three-agent (3-agent) deterministic models by focusing on the local stability of the fundamental

steady states (denoted by $\bar{\mu}$ and $\bar{v}$). I also discuss the conditions for the existence of fundamental steady states and non-fundamental steady states for both the single regime and multiple-regime cases¹.

2.4.2 Fundamentalists versus momentum traders

I first consider a simple two-agent model with market populated only by fundamentalists and momentum traders. This scenario has been investigated by many HAM studies within the standard fundamentalist-chartist framework. Fundamentalists make the trading decision on the basis of fundamental value μ_t . Momentum traders adopt the strategies based on the hypothesis of market under-reaction to price trend, and they aim to exploit the opportunities of market continuity. Sentiment index is formed from social interaction of fundamentalists and momentum traders, but it only affects the behavior of momentum traders in the process of making trading decisions. In this case, the system can be modelled as a three-dimensional nonlinear equation system such that:

$$\begin{aligned} p_{t+1} &= p_t + \gamma[\omega_{f,t}A_t(\mu_t - p_t) + \beta_1\omega_{mo,t}m_t(p_t - v_t)] \\ U_{t+1} &= \phi U_t + (p_{t+1} - p_t)[A_t(\mu_t - p_t) - \beta_1m_t(p_t - v_t)] \\ S_{t+1} &= \eta_1 S_t + \eta_2 [\tanh(\kappa(\mu_t - p_{t+1}))\omega_{f,t+1} + \tanh(\kappa(p_{t+1} - v_{t+1}))\omega_{mo,t+1}] \end{aligned} \quad (2.20)$$

where

$$\begin{aligned} A_t &= \frac{a(\mu_t - p_t)^2}{1 + b(\mu_t - p_t)^4} \\ \omega_{f,t} &= \frac{\exp(\rho U_t)}{\exp(\rho U_t) + 1} \quad \text{and} \quad \omega_{mo,t} = \frac{1}{\exp(\rho U_t) + 1} \\ m_t &= 1 + h_1 \tanh[\kappa(p_t - v_t)]S_t \\ U_t &= U_{f,t} - U_{mo,t} \end{aligned}$$

The dynamics in Equation (2.20) are stochastic and there are two sources of noise. The first source, e_t is the noise term of the fundamental value. The second source, θ_t is the stochastic component of market sentiment derived from random news, innovations and policies. When both noise terms are zero, I am able to study the stability of the system,

¹Regime here refers to the price window set by chartists as in Huang and Zheng (2012). One regime case means short-run asset price is constant, and support and resistance levels approach infinity. Multiple-regime case means is updated in each period.

and I adopt the same way to generate deterministic skeletons for the following scenarios. I first begin by investigating the occurrence of the steady state of Equation (2.20) in one regime case, in which short-term value v_t is assumed as constant $\bar{v}$ over time. The system has multiple steady states with different stability properties.

Proposition 1. *The system has*

1. *an unstable fundamental steady state (FSS) with $(p^*, U^*, S^*) = (\mu, 0, 0)$ if $\mu = \bar{v}$; two types of non-fundamental steady states (NFSS) with the form $(p, U, S) = (p_1^*, 0, 0)$, $(p, U, S) = (p_2^*, 0, 0)$ and $p_1^* < \mu$, $p_2^* > \mu$ if $\mu = \bar{v}$.*
2. *two types of non-fundamental steady states (NFSS) with the form $(p^*, U^*, S^*) = (p_1^*, 0, S_1^*)$, $(p, U, S) = (p_2^*, 0, S_2^*)$ and $p_1^* < \mu$, $p_2^* > \mu$ if $\mu \neq \bar{v}$.*

Proof. See Appendix B.

In line with previous literature, I find the fundamental steady state $p^* = \mu$ with a sufficient condition $\mu = \bar{v}$, and it is proven to be an unstable steady state. With the introduction of sentiment, I also find that when market sentiment is allowed to influence decision-making process of momentum traders, the system can be driven towards several steady states with either smaller (p_1^*) or greater (p_2^*) price than fundamental price. These complex attractors are characterized by persistently polarized levels of positive sentiment, negative sentiment and zero sentiment. **Lemma 1** shows the conditions for these sentiment-persistent steady states.

Lemma 1. *For non-fundamental steady states, the system could achieve positive sentiment equilibria ($S^* > 0$) if $\mu - \bar{v} > 0$, negative sentiment equilibria ($S^* < 0$) if $\mu - \bar{v} < 0$, zero sentiment equilibria ($S^* = 0$) if $\mu - \bar{v} = 0$ and $\beta_1 \leq \frac{1}{2}ab^{-\frac{1}{2}}$.*

Proof. See Appendix B.

The high degree of nonlinearity of the equations makes it difficult to explore the stability of the non-fundamental steady states. But from the structure of Jacobean matrix corresponding to this 3D system, we can infer that investor's feature, encompassed in parameters a , b , β_1 and sentiment-related parameters h_1 , η_1 , η_2 do not only foster the

emergence of non-fundamental steady states, but also significantly affect their number and positions. In this chapter, I focus on features of these sentiment-related steady states rather than the stability conditions.

When condition $\mu - \bar{v} > 0$ is satisfied, only positive sentiment equilibria are achievable. There are two possible positive sentiment cases in terms of position of equilibrium price. For low-price case, equilibrium price is below the fundamental price μ and short-term value $\bar{v}$ ($p_1^* < \bar{v} < \mu$), so fundamentalists hold the optimistic opinion while momentum traders are pessimist. As the equilibrium configuration can solely exist in an even distribution of fundamentalists and momentum traders, so $\omega_{f,t} = \omega_{mo,t} = \frac{1}{2}$ in steady state. As mentioned in previous section, the sentiment index is determined by both fraction of different types of investor and price deviations from fundamental value μ and short-term value $\bar{v}$. With $p_1^* < \bar{v} < \mu$, price and short-term value are lower than fundamental value, which indicates bear market state and price deviates further from μ than $\bar{v}$. The persistent optimism sentiment is sourced from the fundamentalist group. To achieve this positive sentiment steady state, confidence level of fundamentalists must be lower than the extrapolation rate of momentum traders, that is, $A(p_1^*) < \beta_1 m^*$. Conversely, for bull market case with $\bar{v} < \mu < p_2^*$, momentum traders' optimistic opinion will dominate to form the positive market sentiment. The relation between the confidence level of fundamentalists and extrapolation rate of momentum traders must satisfy $A(p_2^*) > \beta_1 m^*$.

If $\mu - \bar{v} < 0$, only negative sentiment equilibria are achievable. For negative sentiment equilibria, momentum traders' pessimistic opinion dominates the market with $A(p_1^*) > \beta_1 m^*$ for bear market case ($p_1^* < \mu$), and pessimism from fundamentalists sweeps the market with $A(p_2^*) < \beta_1 m^*$ for bull market case ($p_2^* > \bar{v}$). When $\mu = \bar{v}$ holds in one regime case, both fundamental steady state and non-fundamental steady states are possible. NFSS is featured by neutral sentiment and must satisfy the condition $A(p^*) = \beta_1$, which suggests fundamentalists have the same confidence level as the extrapolation rate of momentum traders. In this case, the number of NFSS is determined by parameter a , b and β_1 . Specifically, if $\beta_1 < \frac{1}{2}ab^{-\frac{1}{2}}$, four NFSSs exists; If $\beta_1 = \frac{1}{2}ab^{-\frac{1}{2}}$,

number of NFSS reduces to two; If $\beta_1 > \frac{1}{2}ab^{-\frac{1}{2}}$, no NFSS exists.

For multiple-regime case, I allow short-term value to be updated in each period. Price could fluctuate within one regime or among different regimes. The regime switching behavior has been analyzed in previous section through different types of chartists, and one can further explore this feature in Huang et al. (2010) and Huang and Zheng (2012). As discussed in Huang and Zheng (2012), there are many equilibria with multiple regimes, which make it rather complicated to investigate the stability of the system if regime switching of price is considered. Hence, I only focus on the possible price trajectory to form the steady states for multiple-regime case. There are three possible scenarios:

Scenario 1. If $\mu > v_1$ and they are located at different price regimes, one NFSS is possible to emerge in the first regime (v_1 regime) with $p_1^* < v_1$ and $A(p_1^*) < \beta_1 m(p_1^*)$. For price higher than v_1 in the first regime, price would increase within this regime, and finally escape from the old regime to a new one. Other NFSSs are possible to exist in a higher price regime with $\mu > v_t > p_t^*$ and $A(p_t^*) < \beta_1 m(p_t^*)$. Or the case is converted to **Scenario 2** or **3** in next period.

Scenario 2. If $\mu < v_1$ and they belong to different price regimes, one NFSS exists in the first regime with $p_1^* < v_1$ and $A(p_1^*) < \beta_1 m(p_1^*)$. For the case when price is lower than v_1 in the first regime, price would decrease in this regime until escaping from the old regime to a new one. Other NFSSs are possible in a lower price regime with $p_t^* > v_t > \mu$ and $A(p_t^*) < \beta_1 m(p_t^*)$. Or it may go to **Scenario 1** or **3** in next period.

Scenario 3. If $\mu = v_1$, occurrence and positions of NFSSs are determined by parameter a , b and β_1 as well as λ . NFSS can only emerge in the first regime given $A(p^*) = \beta_1$ and $|p^* - \mu| \leq \lambda$. Otherwise, price will move downward to lower regimes or upward to higher regimes. Then **Scenarios 1** and **2** are possibilities that can happen in next period.

2.4.3 Fundamentalists versus contrarian traders

This design has been explored in Chiarella and He (2003), He and Li (2015), and they confirm the stabilizing role of contrarian traders. I also adopt similar two-agent design here. Unlike these studies, I incorporate investor sentiment into the system. I assume

that only contrarian traders are sensitive to market sentiment, and the three-dimensional nonlinear system can be written as:

$$\begin{aligned}
p_{t+1} &= p_t + \gamma[\omega_{f,t}A_t(\mu_t - p_t) + \beta_2\omega_{co,t}c_t(p_t - v_t)] \\
U_{t+1} &= \phi U_t + (p_{t+1} - p_t)[A_t(\mu_t - p_t) - \beta_2c_t(p_t - v_t)] \\
S_{t+1} &= \eta_1 S_t + \eta_2[\tanh(\kappa(\mu_t - p_{t+1}))\omega_{f,t+1} - \tanh(k(p_{t+1} - v_{t+1}))\omega_{co,t+1}]
\end{aligned} \tag{2.21}$$

where

$$\begin{aligned}
A_t &= \frac{a(\mu_t - p_t)^2}{1 + b(\mu_t - p_t)^4} \\
\omega_{f,t} &= \frac{\exp(\rho U_t)}{\exp(\rho U_t + 1)} \quad \text{and} \quad \omega_{co,t} = \frac{1}{\exp(\rho U_t + 1)} \\
c_t &= 1 - h_2 \tanh(p_t - v_t) S_t \\
U_t &= U_{f,t} - U_{co,t}
\end{aligned}$$

Also, I start by investigating the stability of the system from one regime case.

Proposition 2. *The system has*

1. *a unique fundamental steady state with $(p^*, U^*, S^*) = (\mu, 0, 0)$ if $\mu = \bar{v}$. Fundamental steady state is asymptotically stable for $-4 < \gamma\beta_2 < 0$, and it undergoes a flip bifurcation for $\gamma\beta_2 = -4$.*
2. *three types of non-fundamental steady states with the form $(p^*, U^*, S^*) = (p_1^*, 0, S_1^*)$, $(p^*, U^*, S^*) = (p_2^*, 0, S_2^*)$, $(p^*, U^*, S^*) = (p_3^*, 0, S_3^*)$ and $p_1^* < \frac{1}{2}(\mu + \bar{v})$, $p_2^* > \frac{1}{2}(\mu + \bar{v})$, $p_1^* = \frac{1}{2}(\mu + \bar{v})$ if $\mu \neq \bar{v}$.*

Proof. See Appendix C.

I find a unique fundamental steady state if $\mu = \bar{v}$, and it is asymptotically stable for $-4 < \gamma\beta_2 < 0$. The flip bifurcation may occur bifurcation if the extrapolation parameter of the contrarian rule increases. If the extrapolation power of contrarains is too big, they may overreact to the short-term price deviation with the sentiment bias. Price will fluctuate around fundamental value but not converge to it. For $\mu \neq \bar{v}$, I find three different types of NFSS, and these complex attractors are characterized by persistently polarized levels of positive, negative and neutral sentiment. **Lemma 2** shows these sentiment-persistent steady states in detail.

Lemma 2. *For non-fundamental steady states, the system achieves positive sentiment equilibria ($S^* > 0$) if $p^* < \frac{1}{2}(\mu + \bar{v})$, negative sentiment equilibria ($S^* < 0$) if $p^* > \frac{1}{2}(\mu + \bar{v})$, and neutral sentiment equilibria ($S^* = 0$) if $p^* = \frac{1}{2}(\mu + \bar{v})$ and $\beta_1 \leq \frac{1}{2}ab^{-\frac{1}{2}}$.*

Proof. See Appendix C.

Different from the previous case, equilibrium price of NFSS can only emerge between μ and $\bar{v}$, and type of NFSS depends on the position of p^* . When p^* is below the average of μ and $\bar{v}$, that is $p^* < \frac{1}{2}(\mu + \bar{v})$, positive sentiment equilibria could be achieved. If $\mu > \bar{v}$, this persistent positive sentiment state is sourced from fundamentalists' optimistic opinion with $A(p^*) < -\beta_2 c^*$. If $\mu < \bar{v}$, the positive sentiment state is derived from the optimistic belief of contrarian traders with $A(p^*) > -\beta_2 c^*$. When p^* is above the average of μ and $\bar{v}$ negative sentiment equilibria appear. Opposite to positive sentiment steady state, pessimistic opinion of contrarian traders dominates with $A(p^*) > -\beta_2 c^*$ to generate negative sentiment state if $\mu > \bar{v}$, while pessimistic opinion of fundamentalists will take in charge with $A(p^*) < -\beta_2 c^*$ to generate negative sentiment if $\mu < \bar{v}$. A special case is NFSS at $p^* = \frac{1}{2}(\mu + \bar{v})$ with neutral sentiment state as in FSS, and the sufficient condition for the occurrence of this NFSS is $A(p^*) = -\beta_2 c^*$. The price zone above the $\max(\mu, \bar{v})$ is declining zone and below the $\min(\mu, \bar{v})$ is rising zone, so price drops in these zones would be attracted to $[\min(\mu, \bar{v}), \max(\mu, \bar{v})]$. Similarly, parameters a , b , β_2 and sentiment related parameters h_2 , η_1 and η_2 affect the number and position of NFSS.

For multiple-regime case, price will rise in the first regime under bear market ($\mu > v_1$) with $p_1 < v_1$. The steady state can be achieved in the first regime or price could escape to higher price regimes, and final equilibrium will be reached between fundamental value μ and the latest short-term value v_t . Otherwise, if the initial market is bullish ($\mu < v_1$) with $p_1 > v_1$, price will decrease in the first regime and equilibrium could be achieved in the same regime with lower price or price continuously goes down until equilibrium is achieved in a lower price regime. This kind of price dynamics illustrates the stabilizing role of fundamentalists and contrarian traders. However, the stabilizing forces of these two types of investor do not always go with the same direction. For instance,

price initially drops into a window between μ and v_t . If fundamentalists and contrarian traders work on the different directions of price movement, equilibrium may be achieved between latest short-term value and fundamental value. This could provide a possible explanation for why price could fluctuate above or below fundamental price for a long time in real financial markets. In the regime switching process, investor sentiment plays an important role. It speeds up the regime switching process and expands the magnitude of price change, which could partially explain the existence of excess volatility and even bubbles and crashes in financial markets.

2.4.4 Momentum traders versus contrarian traders

This two-agent model has been rarely explored within HAM framework. In this case, market only consists of two types of chartist, momentum traders and contrarian traders. This rare case may occur during extreme market state, for instance, during long-lasting bubbles or crises, fundamentalists gradually lose their confidence in financial market, so they decide not to enter the market or switch to chartism strategies. Another possible situation is that the market is highly speculative or it is difficult to access the fundamental value of the asset (such as Bitcoin market), then most of the investors would choose chartism strategies. By only including momentum traders and contrarian traders, I model the system as:

$$\begin{aligned} p_{t+1} &= p_t + \gamma(\beta_1 \omega_{mo,t} m_t + \beta_2 \omega_{co,t} c_t)(p_t - v_t) \\ U_{t+1} &= \varphi U_t + (\beta_1 m_t - \beta_2 c_t)(p_{t+1} - p_t)(p_t - v_t) \\ S_{t+1} &= \eta_1 S_t + \eta_2 \tanh(\kappa(p_{t+1} - v_{t+1})) G_{t+1} \end{aligned} \quad (2.22)$$

where

$$\begin{aligned} \omega_{mo,t} &= \frac{\exp(\rho U_t)}{\exp(\rho U_t + 1)} \quad \text{and} \quad \omega_{co,t} = \frac{1}{\exp(\rho U_t + 1)} \\ m_t &= 1 + h_1 \tanh(p_t - v_t) S_t \\ c_t &= 1 - h_2 \tanh(p_t - v_t) S_t \\ U_t &= U_{mo,t} - U_{co,t} \\ G_t &= \omega_{mo,t} - \omega_{co,t} = \tanh\left\{\frac{\rho \kappa}{2}[\varphi U_t + (\beta_1 m_t - \beta_2 c_t)(p_{t+1} - p_t)(p_t - v_t)]\right\} \end{aligned}$$

I begin by analyzing the one regime case. As momentum traders have belief opposite to that of contrarian traders, the heterogeneous extrapolation rate of these two chartist groups may affect the price dynamic and stability of steady state.

Proposition 3. *The system (2.22) has a unique steady state $(p^*, U^*, S^*) = (\bar{v}, 0, 0)$. The steady state is locally and asymptotically stable provided that $-4 < \gamma(\beta_1 + \beta_2) < 0$, and it undergoes a flip bifurcation for $\gamma(\beta_1 + \beta_2) = -4$.*

Proof. See Appendix D.

The unique steady state in this case is very similar to FSS in the fundamentalists versus contrarian traders design. The stability of the equilibrium depends on γ , β_1 and β_2 . To satisfy the stability condition, we must have $(\beta_1 + \beta_2) < 0$. It suggests that contrarian traders have stronger extrapolating power to pull price back to short-term value $\bar{v}$. For a special case where $\beta_1 = -\beta_2$, price would have no fluctuation given the same initial utility and fraction for two types of chartist. As their demands will offset each other, the initial price will be the steady state price for this case.

I further explore the multiple-regime case in this design. If momentum traders have stronger extrapolating force ($\beta_1 > -\beta_2$), v_1 can divide the first regime into rising zone (above v_1) and declining zone (below v_1). When price drops in the rising zone, it will increase in next period. Otherwise, price will further decrease in the declining zone. If $\beta_1 < -\beta_2$, contrarian traders have stronger force to pull price toward the short-term value v_t . If the difference of their extrapolating force is small, price will converge to initial regime short-term value v_1 . On the contrary, if the difference is large, price may fluctuate among different price regimes. Hence, the regime switching behavior is determined by the extrapolation rate. For a model without sentiment factor, low extrapolation rates or small strength difference between them may lead price to fluctuate within two adjacent regimes. However, if investor sentiment is considered, time-varying extrapolation rate could make both within regime fluctuation and regime switching behavior available as shown in Figure 2.4. Hence, sentiment is an important factor to affect price trajectory in a multiple-regime design.

Figure 2.4: Price trajectory without (left) and with (right) sentiment effect, $\beta_1 = 1.3$ and $\beta_2 = -1.2$.

2.4.5 Fundamentalists, momentum traders and contrarian traders

For the three-agent design, I include all the three types of investors, fundamentalists, momentum traders and contrarian traders into the system. This design has been widely investigated in literature. These include the work of Brock and Hommes (1998), Chiarella and He (2003), and He and Li (2015). This scenario can be treated as a combination of the previous two models and hence is expected to generate richer dynamic features. As this model is closer to the real financial market, I use this model as a benchmark for the numerical analysis in Chapter 3. The system can be modelled to a five-dimensional dynamic map such that:

$$\begin{aligned}
 p_{t+1} &= p_t + \gamma [\omega_{f,t} A_t (\mu_t - p_t) + (\beta_1 \omega_{mo,t} m_t + \beta_2 \omega_{co,t} c_t) (p_t - v_t)] \\
 U_{f,t+1} &= \phi U_{f,t} + (p_{t+1} - p_t) A_t (\mu_t - p_t) \\
 U_{mo,t+1} &= \phi U_{mo,t} + (p_{t+1} - p_t) \beta_1 m_t (p_t - v_t) \\
 U_{co,t+1} &= \phi U_{co,t} + (p_{t+1} - p_t) \beta_2 c_t (p_t - v_t) \\
 S_{t+1} &= \eta_1 S_t + \eta_2 [\tanh(\kappa(\mu_{t+1} - p_{t+1})) \omega_{f,t+1} + \tanh(\kappa(p_{t+1} - v_{t+1})) (\omega_{mo,t+1} - \omega_{co,t+1})]
 \end{aligned} \tag{2.23}$$

where

$$\begin{aligned}
 A_t &= \frac{a(\mu_t - p_t)^2}{1 + b(\mu_t - p_t)^4} \\
 \omega_{h,t} &= \frac{\exp(\rho U_{h,t})}{\sum_{h=1}^2 \exp(\rho U_{h,t})} \\
 m_t &= 1 + h_1 \tanh[\kappa(p_t - v_t)] S_t \\
 c_t &= 1 - h_2 \tanh[\kappa(p_t - v_t)] S_t
 \end{aligned}$$

Let's start by investigating the stability of the system for one regime case.

Proposition 4. *The system has*

1. *a unique fundamental steady state with $(p^*, U_f^*, U_{mo}^*, U_{co}^*, S^*) = (\mu, 0, 0, 0, 0)$ if $\beta_1 = -\beta_2$. The Jacobean matrix of this system has five eigenvalues with $\lambda_1 = 1$, $\lambda_2 = \varphi$. Fundamental steady state is asymptotically stable for $|\lambda_3|, |\lambda_4|, |\lambda_5| < 1$.*
2. *a fundamental steady state with $(p^*, U_f^*, U_{mo}^*, U_{co}^*, S^*) = (\mu, 0, 0, 0, 0)$ if $\beta_1 \neq -\beta_2$ and $\mu = \bar{v}$. FSS is asymptotically stable for $-6 < \gamma(\beta_1 + \beta_2) < 0$; Two types of non-fundamental steady states with the form $(p^*, U_f^*, U_{mo}^*, U_{co}^*, S^*) = (p_1^*, 0, 0, 0, S_1^*)$, $(p^*, U_f^*, U_{mo}^*, U_{co}^*, S^*) = (p_2^*, 0, 0, 0, S_2^*)$, and $p_1^* < \mu, S_1^* > 0$; $p_2^* > \mu, S_2^* < 0$ if $\beta_1 \neq -\beta_2$ and $\mu \neq \bar{v}$.*

Proof. See Appendix E.

As all the three types of agents can be regarded to have equal forces in financial market, their behavior, especially the extrapolating power of them, may influence the existence and stability of steady state. When $\beta_1 = -\beta_2$, only fundamental steady state exists. In this case, demands of momentum traders and contrarian traders offset each other if I do not consider sentiment effect, and demand of fundamentalists solely determines the market price. With sentiment effect, the price dynamics generated by this model would become richer, but there is a unique steady state with $p = \mu$. To analyze the stability of this system, I use the Jacobean matrix. I find that $\lambda_1 = 1, \lambda_2 = \varphi$ and $\lambda_3, \lambda_4, \lambda_5$ are functions of standard parameters such as γ, ρ, φ and sentiment related parameters η_1, η_2, h_1, h_2 . Fundamental steady state is asymptotically stable if $|\lambda_3|, |\lambda_4|, |\lambda_5| < 1$, so sentiment is a factor to influence the stability of the steady state. When $\beta_1 \neq -\beta_2$, both fundamental steady state and non-fundamental steady states exist under some specific conditions. The sufficient condition for fundamental steady state is $\mu = \bar{v}$, and this FSS is asymptotically stable for $-6 < \gamma(\beta_1 + \beta_2) < 0$, which means that contrarian traders need to have a stronger extrapolation power to stabilize the market. I find have two types of non-fundamental steady state for $\beta_1 \neq -\beta_2$ and $\mu \neq \bar{v}$, the features of them are shown in **Lemma 3**.

Lemma 3. *For non-fundamental steady states, the system can achieve both positive and negative sentiment equilibria. If $\beta_1 > -\beta_2$, positive (negative) sentiment equilibrium exists at $p^* < (>) \mu$ and $p^* < (>) \bar{v}$. If $\beta_1 < -\beta_2$, positive (negative) sentiment equilibrium exists at $p^* < (>) \mu$ and $p^* > (<) \bar{v}$.*

Proof. See Appendix E.

For non-fundamental steady states, the position of p^* with respect to μ determines the types of equilibrium. The sentiment generated by two types of chartists will be offset by each other and their effects are cancelled out, so the sentiment of fundamentalists is the main source of equilibrium sentiment. If momentum traders extrapolate stronger (weaker), a positive (negative) sentiment steady state would emerge at $p^* < \bar{v}$. On the contrary, if contrarian traders extrapolate stronger (weaker), a positive (negative) sentiment steady state would occur at $p^* > \bar{v}$. Concerning the multiple-regime case, the regime switching behavior is similar to the two-agent case, and sentiment is still an important factor that affects this behavior. Here, I skip the analysis of regime switching for three-agent case.

2.5 Conclusion

In this chapter, I set up sentiment models constituted with memory strength of sentiment, social interaction and sentiment shock components in financial markets. I conjecture that investor sentiment has heterogeneous influence on different types of investors and the sentiment effect is transmittable through interaction among agents. I then carry out an in-depth analysis on the role of sentiment under 2-agent and 3-agent HAM frameworks, especially on the steady state conditions of deterministic models.

The key findings of this chapter are in three respects. First, fundamentalists could serve as destabilizing force in the market even though they are trying to pull market prices back to fundamental value at the first place. The destabilizing force stems from nonlinear behavior of fundamentalists. Second, I find both fundamental steady state featured with neutral sentiment corresponding to that of standard paradigm as well as non-fundamental

steady states with polarized sentiment (either positive or negative) in two-agent and three-agent models. Third, in three-agent model, sentiment related parameters are able to affect the stability of fundamental steady state when two types of chartists have equal extrapolating power ($\beta_1 = -\beta_2$).

Appendix A

Confidence function for fundamentalists

$$A(x_t) = \frac{ax_t^2}{1 + bx_t^4}$$

$A(x_t)$ is a nonlinear smooth and symmetric function of price deviation x_t . The fundamentalists react to the positive and negative trading signals equally and they are less confident when price deviation x_t is extremely large. The shape of $A(x_t)$ function is shown in Figure 2.5, and it follows some general properties:

Property 1

$$A(0) = 0 \quad \text{and} \quad A(x_t) > 0 \quad \forall x_t \neq 0$$

When there are price deviations, the fundamentalists will have positive confidence in their mean-reverting strategy.

Property 2 If $x_t > 0$, $A(x_t)' > 0$ for $x_t < b^{-\frac{1}{4}}$ and $A(x_t)' < 0$ for $x_t > b^{-\frac{1}{4}}$; If $x_t < 0$, $A(x_t)' < 0$ for $x_t > -b^{-\frac{1}{4}}$ and $A(x_t)' > 0$ for $x_t < -b^{-\frac{1}{4}}$.

Range of increasing confidence level is $[0, A_{\max}]$, $A_{\max} = A(b^{-\frac{1}{4}}) = A(-b^{-\frac{1}{4}}) = \frac{1}{2}ab^{-\frac{1}{2}}$. The reasonable zone $(-Z_t, Z_t) = (-b^{-\frac{1}{4}}, b^{-\frac{1}{4}})$. The confidence level of fundamentalists increases with $|x_t|$ within reasonable zone $(-b^{-\frac{1}{4}}, b^{-\frac{1}{4}})$, and decreases with $|x_t|$ outside this zone. The maximum confidence of fundamentalists is achieved when $x_t = \pm b^{-\frac{1}{4}}$, which is $\frac{1}{2}ab^{-\frac{1}{2}}$.

Figure 2.5: Confidence function for fundamentalist

Property 3

$$\lim_{x_t \rightarrow \pm\infty} A(x_t) = 0$$

If the price deviation from fundamental value is too large, the fundamentalists will lose confidence and exit the market.

Appendix B

Proof of Proposition 1. A steady state (p^*, U^*, S^*) of the deterministic system Equation (2.20) in one regime case must satisfy the following equations:

$$A * (\mu - p^*) + \beta_1 m^* (p^* - \bar{v}) = 0$$

$$S^* = \frac{\eta_2}{2(1 - \eta_1)} [\tanh(\kappa(\mu - p^*)) + \tanh(\kappa(p^* - \bar{v}))]$$

$$m^* = 1 + h_1 \tanh(\kappa(p^* - \bar{v})) S^*$$

If $\mu = \bar{v}$, I find one fundamental steady state $p^* = \mu = \bar{v}$. In this case, $S^* = 0$ and $m^* = 1$.

The stability of this fundamental steady state can be inferred from Jacobian matrix J of the system Equation (2.20) as

$$J = \begin{pmatrix} 1 + \frac{1}{2}\gamma\beta_1, & 0 & 0 \\ 0 & \varphi & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}$$

The eigenvalues of diagonal matrix lie on the main diagonal of matrix, so

$$\lambda_1 = 1 + \frac{1}{2}\gamma\beta_1, \lambda_2 = \varphi, \lambda_3 = \eta_1$$

The fundamental steady state is asymptotically stable when all the eigenvalues of Jacobian matrix at the steady state are less than one. Hence, fundamental steady state is unstable due to $\lambda_1 > 0$ (both γ and β_1 are positive values).

Non-fundamental steady states may also coexist with fundamental steady state when $\mu = \bar{v}$. At the NFSS, $S^* = 0$ and $m^* = 1$, the first equation in Appendix B can be simplified as $(A - \beta_1)(\mu - p^*) = 0$. In this case, $\mu \neq p^*$ and the solutions are from $A - \beta_1 = 0$. According to shape of A , the possible number of solutions is 0, 2, 4. If β_1 is larger than $A_{\max}$ which is $\frac{1}{2}ab^{-\frac{1}{2}}$, there is no solution; If $\beta_1 = A_{\max}$, there are two symmetric solutions around fundamental value μ (positive x_t and negative x_t). If $\beta_1 < A_{\max}$, four steady states occur with two above μ and the other two below μ .

Proof of Lemma 1. If $\mu \neq \bar{v}$, non-fundamental steady states could also emerge with persistent positive or negative sentiment. p^* must be lower or higher than both μ and $\bar{v}$ to satisfy the first equation of system (2.20). According to the positions of μ and $\bar{v}$, we can also derive the market sentiment equilibria well as the relation between the confidence level of fundamentalists and momentum traders' extrapolation rate at steady state. When $\mu > \bar{v}$, S^* must be positive due to $\tanh(\kappa(\mu - p^*)) + \tanh(\kappa(p^* - \bar{v})) > 0$ for both lower and higher p^* , so we have a positive sentiment equilibrium. At the same time, $A(p_1^*) < \beta_1 m^*$ ($A(p_2^*) > \beta_1 m^*$) must be satisfied to get lower (higher) equilibrium p_1^* (p_2^*). Conversely, when $\mu < \bar{v}$, S^* is always negative and we have negative sentiment equilibria. Lower (higher) equilibrium p_1^* (p_2^*) is associated with $A(p_1^*) > \beta_1 m^*$ ($A(p_2^*) < \beta_1 m^*$).

Appendix C

Proof of Proposition 2. Besides the heterogeneous extrapolation rates for momentum traders and contrarian traders, they also react differently to market sentiment. The steady state (p^*, U^*, S^*) of the deterministic system (2.21) in one regime case must satisfy the following equations:

$$\begin{aligned} A^*(\mu - p^*) + \beta_2 c^*(p^* - \bar{v}) &= 0 \\ S^* &= \frac{\eta_2}{2(1 - \eta_1)} [\tanh(\kappa(\mu - p^*)) - \tanh(\kappa(p^* - \bar{v}))] \\ c^* &= 1 - h_1 \tanh(\kappa(p^* - \bar{v})) S^* \end{aligned}$$

If $\mu = \bar{v}$, there is an unique fundamental steady state $p^* = \mu = \bar{v}$. In this case, $S^* = 0$ and $c^* = 1$. The stability of this fundamental steady state can be inferred from Jacobian matrix J of the system (2.21) as

$$J = \begin{pmatrix} 1 + \frac{1}{2}\gamma\beta_2 & 0 & 0 \\ 0 & \varphi & 0 \\ -\eta_2(1 + \frac{1}{2}\gamma\beta_2) & 0 & \eta_1 \end{pmatrix}$$

The eigenvalues of Jacobian matrix are

$$\lambda_1 = 1 + \frac{1}{2}\gamma\beta_2, \lambda_2 = \varphi, \lambda_3 = \eta_1$$

As $0 < \varphi < 1$ and $0 < \eta_1 < 1$, the stability of the system is determined by λ_1 . It is possible to have λ_1 inside the unit circle due to $\beta_2 < 0$. When $|\lambda_1| < 1$ or $-4 < \gamma\beta_2 < 0$, the fundamental steady state is locally stable. Note that setting $\gamma\beta_2 = -4$ gives us the flip bifurcation as $\lambda_1 = -1$.

Proof of Lemma 2. If $\mu \neq \bar{v}$, three types of non-fundamental steady states may emerge with persistent positive, negative sentiment or zero sentiment. The equilibrium p^* must be between the fundamental value μ and short-term value $\bar{v}$. If price is lower or higher than both μ and $\bar{v}$, it will be attracted to the zone constructed by $\min(\mu, \bar{v})$ and $\max(\mu, \bar{v})$.

According to the position of p^* , we can derive three different market steady states with positive, negative or neutral sentiment. As the equilibrium price is within $[\min(\mu, \bar{v}), \max(\mu, \bar{v})]$, the distances of p^* to μ and $\bar{v}$ decide the sign of equilibrium sentiment S^* . If $p^* < \frac{1}{2}(\mu + \bar{v})$, $S^* > 0$ and a positive sentiment equilibrium is achieved. To know the source of this positive market sentiment, we need to analyze the relative position of μ and $\bar{v}$. If $\mu > \bar{v}$ and $p_1^* < \frac{1}{2}(\mu + \bar{v})$, we need to have $A(p_1^*) < -\beta_2 c^*$ to satisfy the first equation of system (2.21), and market sentiment is mainly from fundamentalists' optimistic beliefs. Otherwise, if $\mu < \bar{v}$ and $p_1^* < \frac{1}{2}(\mu + \bar{v})$, $A(p_1^*) > -\beta_2 c^*$ must be satisfied to get the positive sentiment equilibrium, and optimism is derived from relatively aggressive contrarian traders. Another negative sentiment steady state is featured with $p_2^* > \frac{1}{2}(\mu + \bar{v})$. If $\mu > \bar{v}$, equilibrium could be achieved with $A(p_2^*) > -\beta_2 c^*$. If $\mu < \bar{v}$, equilibrium could emerge with $A(p_2^*) < -\beta_2 c^*$. A special case is when $p_3^* = \frac{1}{2}(\mu + \bar{v})$, then $S^* = 0$, and equilibrium is achievable if $A(p_3^*) = -\beta_2$. It means we must have $\beta_2 = -\frac{4a(\mu + \bar{v})^2}{16 + b(\mu + \bar{v})^4}$ to get this steady state.

Appendix D

Proof of Proposition 3. The steady state (p^*, U^*, S^*) of the deterministic system (2.22) in one regime case must satisfy the following equations:

$$(\beta_1 m^* + \beta_2 c^*)(p^* - \bar{v}) = 0$$

$$S^* = 0$$

$$m^* = c^* = 1$$

If $\beta_1 \neq -\beta_2$, we have unique steady state $p^* = \bar{v}$. The corresponding Jacobian matrix can be derived as

$$J = \begin{pmatrix} 1 + \frac{1}{2}\gamma(\beta_1 + \beta_2), & 0 & 0 \\ 0 & \varphi & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}$$

The eigenvalues of Jacobian matrix are

$$\lambda_1 = 1 + \frac{1}{2}\gamma(\beta_1 + \beta_2), \lambda_2 = \varphi, \lambda_3 = \eta_1$$

As $0 < \varphi < 1$ and $0 < \eta_1 < 1$, the stability of the system is determined by λ_1 . For absolute value of λ_1 to be less than 1, we must have $-4 < \gamma(\beta_1 + \beta_2) < 0$ to make the system locally stable. The setting $\gamma(\beta_1 + \beta_2) = -4$ gives us the flip bifurcation as $\lambda_1 = -1$.

Appendix E

Proof of Proposition 4. The relationship between β_1 and β_2 could determine the number and type of steady states. If $\beta_1 = -\beta_2 = \beta$, only fundamental steady state is possible with the following equations:

$$\omega_f^* A^* (\mu - p^*) + \beta (\omega_{mo}^* m^* - \omega_{co}^* c^*) (p^* - \bar{v}) = 0$$

$$\omega_f^* = \omega_{mo}^* = \omega_{co}^* = \frac{1}{3}$$

$$U_f^* = U_{mo}^* = U_{co}^* = 0$$

$$S^* = 0$$

$$m^* = c^* = 1$$

The corresponding Jacobian matrix with $\beta_1 = -\beta_2 = \beta$ can be derived as

$$J = \begin{pmatrix} 1 & 0 & B & -B & C \\ 0 & \varphi & 0 & 0 & 0 \\ 0 & 0 & \varphi + \beta \delta B & -\beta \delta B & \beta \delta C \\ 0 & 0 & -\beta \delta B & \varphi + \beta \delta B & \beta \delta C \\ 0 & 0 & D & -D & E \end{pmatrix}$$

where

$$\begin{aligned}\delta &= \mu - \bar{v}, B = \frac{1}{3}\gamma\rho\beta(\mu - \bar{v}), C = \frac{1}{3}\gamma\beta(\mu - \bar{v})(h1 + h2)\tanh(\mu - \bar{v}) \\ D &= -\frac{1}{9}\rho\eta_2[\gamma\beta(\mu - \bar{v}) - 3\varphi\tanh(\kappa(\mu - \bar{v}))] \\ E &= \eta_1 - \frac{1}{9}\eta_2\gamma\beta(\mu - \bar{v})(h1 + h2)\tanh(\kappa(\mu - \bar{v}))\end{aligned}$$

There are five eigenvalues for this Jacobian matrix, which are

$$\begin{aligned}\lambda_1 &= 1, \lambda_2 = \varphi, \lambda_3 = \varphi + \beta\delta B, \lambda_4 = \frac{\varphi^2 + 2\varphi\beta\delta B}{\varphi + \beta\delta B}, \\ \lambda_5 &= E - \frac{2\beta\delta CD}{\varphi} + \frac{4\beta\delta CD(\varphi + \beta\delta B)}{\varphi^2 + 2\varphi\beta\delta B} - \frac{2\varphi\beta\delta CD(\varphi + \beta\delta B)}{(\varphi^2 + 2\varphi\beta\delta B)(\varphi + \beta\delta B)}\end{aligned}$$

To make the system locally stable, we need to have $|\lambda_3|, |\lambda_4|, |\lambda_5| < 1$. For a special case $\mu = \bar{v}$, the Jacobian matrix could be simplified to

$$J = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \varphi & 0 & 0 & 0 \\ 0 & 0 & \varphi & 0 & 0 \\ 0 & 0 & 0 & \varphi & 0 \\ 0 & 0 & D & 0 & \eta_1 \end{pmatrix}$$

The corresponding five eigenvalues for this diagonal matrix are

$$\lambda_1 = 1, \lambda_2 = \varphi, \lambda_3 = \varphi, \lambda_4 = \varphi, \lambda_5 = \eta_1$$

As $0 < \varphi < 1$ and $0 < \eta_1 < 1$, the system is always stable for this special case.

If $\beta_1 \neq -\beta_2$, both FSS and NFSS are possible, and steady state $(p^*, u_f^*, u_{mo}^*, u_{co}^*, S^*)$ of the deterministic system (2.23) in one regime case must satisfy the following equa-

tions:

$$\omega_f^* A^* (\mu - p^*) + (\omega_{mo}^* \beta_1 m^* + \omega_{co}^* \beta_2 c^*) (p^* - \bar{v}) = 0$$

$$\omega_f^* = \omega_{mo}^* = \omega_{co}^* = \frac{1}{3}$$

$$U_f^* = U_{mo}^* = U_{co}^* = 0$$

$$S^* = \frac{\eta_2}{3(1 - \eta_1)} \tanh(\kappa(\mu - p^*))$$

$$m^* = 1 + h_1 \tanh(k(p^* - \bar{v})) S^*$$

$$c^* = 1 - h_2 \tanh(k(p^* - \bar{v})) S^*$$

For fundamental steady state, we have $p^* = \mu = \bar{v}$, and Jacobian matrix can be derived as

$$J = \begin{pmatrix} 1 + \frac{1}{3}\gamma(\beta_1 + \beta_2), & 0 & 0 & 0 & 0 \\ 0 & \varphi & 0 & 0 & 0 \\ 0 & 0 & \varphi & 0 & 0 \\ 0 & 0 & 0 & \varphi & 0 \\ -\frac{1}{3}\eta_2 & 0 & 0 & 0 & \eta_1 \end{pmatrix}$$

Five eigenvalues for this Jacobian matrix will be

$$\lambda_1 = 1 + \frac{1}{3}\gamma(\beta_1 + \beta_2), \lambda_2 = \varphi, \lambda_3 = \varphi, \lambda_4 = \varphi, \lambda_5 = \eta_1$$

We can have a locally stable system provided that $-6 < \gamma(\beta_1 + \beta_2) < 0$.

For NFSS, I focus on the features of the equilibria. There are two types of steady states, positive and negative sentiment steady states. From the sentiment function, it is clear that when $p^* < \mu$, $S^* > 0$ and vice versa. But p^* must satisfy the first equation of system (2.23).

$$A^* (\mu - p^*) + (\beta_1 + \beta_2) (p^* - \bar{v}) + (\beta_1 h_1 - \beta_2 h_2) [S^* \tanh(\kappa(p^* - \bar{v}))] (p^* - \bar{v}) = 0$$

If $p^* < \mu$, we have S^* and the first and third term of this equation are positive, so the second term must be negative. When $\beta_1 > -\beta_2$ ($\beta_1 < -\beta_2$), we have $p^* < \bar{v}$ ($p^* > \bar{v}$). For negative sentiment non-fundamental steady state $S^* < 0$, we have $p^* > \mu$ and $p^* > \bar{v}$ ($p^* < \bar{v}$) when $\beta_1 > -\beta_2$ ($\beta_1 < -\beta_2$).

Chapter 3

Investor Heterogeneity and Financial Crises - The Role of Sentiment

3.1 Introduction

While the consensus is that investor sentiment can affect asset prices, the question of its importance on these prices remains. More specifically, can sentiment explain financial crisis which traditional financial models have failed to rationalize? How does sentiment work in the formation of financial crisis? These are particularly pertinent questions to ask given that we are living in an era with more frequent financial crises. According to Bordo et al. (2001), crisis frequency since 1973 is twice as many compared to that of the Bretton Woods and classical gold standard periods. Although we can see repeated formation of market crashes, much of the underlying mechanism are still not well understood. Comparing to the large volume of published works on investor sentiment, few studies have directly linked sentiment to market crises. Among these are Siegel (1992) and Baur et al. (1996) that focus on U.S. stock market crash of 1987 and Zouaoui et al. (2011) who use a panel data of international stock markets to find the contribution of investor sentiment in raising the probability of crises within one-year horizon.

In this chapter, I aim to study the role of sentiment for asset pricing, mainly for financial crises, in a market with heterogeneous investors. The most important question to ask is whether, and if so, how sentiment could contribute to the financial crises. The theoretical framework in this chapter is same as the three-agent sentiment HAM in Chapter 2. Although this sentiment model is more complicated than the pioneering HAMs and other succeeding models, it incorporates most of the merits from three pioneering models that I discuss in Chapter 1. Most importantly, it includes endogenous sentiment formation process, which could clearly explain how behavioral factors affect the decision making process of investors. Another novelty of this model is consideration of heterogeneity of sentiment effect. The heterogeneity exists between fundamentalists and chartists as well as within two chartist groups. To be specific, I assume there are three types of agent in the market, namely fundamentalists, momentum traders and contrarian traders. Fundamentalists are immune to sentiment, but they have time-varying confidence. While momentum and contrarian traders are sentiment traders, they have heterogeneous responses to market sentiment. The detail of the model framework and market equilibrium have been discussed in Chapter 2. Here I focus on the simulation of the three-agent model. To study more features in the financial market such as fat tails, negative skewness, and long memory of the distributions of returns, I run simulations based on stochastic models with a disturbing term in price formation in Equation (3.9). Some highlights of the simulation findings include: (1) HAM with sentiment is able to replicate different types of crises, including sudden crisis, disturbing crisis and smooth crisis as in Huang et al. (2010) and simulation results show that sentiment sensitivity of investors is positively correlated with the frequency and the magnitude of crisis; (2) sentiment contributes to stylized facts such as fat tails, volatility clustering and long memory dependence of daily returns that are commonly observed in actual stock market; (3) market volatility increases with the presence of sentiment and investor sentiment may help to explain the excess volatility puzzle.

Following the framework of three-agent model with sentiment, I begin this chapter by briefly reviewing some of the stylized facts in real financial market. Next, I conduct

simulation practice on stochastic models with and without sentiment separately, thereby enable us to compare the fitness between both models to real financial market in terms of its capability to generate the well-documented stylized facts. More importantly, I am able to explore the role of sentiment effect on these stylized facts. In addition, I also investigate whether sentiment is the source of excess market volatility and the different types of crises, and how sentiment could influence the emergence of crises. Finally, I run simulations with different forms of demand function to show that sentiment is main source of excess volatility and financial crises.

3.2 Framework of three-agent HAM with sentiment

I follow the setup for three-agent HAM with sentiment in the previous chapter. Agents in the market include fundamentalists, momentum traders and contrarian traders. Their demand functions for risky assets can be written as:

$$D_{f,t} = A(x_t)(\mu_t - p_t) \quad (3.1)$$

$$D_{mo,t} = \beta_1 m_t (p_t - v_t) \quad (3.2)$$

$$D_{co,t} = \beta_2 c_t (p_t - v_t) \quad (3.3)$$

Where nonlinear smooth function $A(x_t)$ captures the confidence level of fundamentalists to their strategies. Let $x_t = p_t - \mu_t$ denotes price deviation, $A(x_t)$ can be expressed as:

$$A(x_t) = \frac{ax_t^2}{1 + bx_t^4} \quad (3.4)$$

The m_t and c_t are the time-varying factors that determine the influence of sentiment

on chartists. They are smooth functions with a range of $[0, 1]$ and can be written as:

$$m_t = 1 + h_1 \tanh[\kappa(p_t - v_t)] S_t \quad (3.5)$$

$$c_t = 1 - h_2 \tanh[\kappa(p_t - v_t)] S_t \quad (3.6)$$

where S_t is the sentiment index which has been discussed in Chapter 2. $h_1 \in [0, 1]$ measures sensitivity of momentum traders to market sentiment. The construction of the sentiment index is one of the innovation of this thesis. I construct this index by considering three sources, last period sentiment index, investor mood from social interaction, and information shocks. The sentiment index can be expressed as:

$$S_t = \eta_1 S_{t-1} + \eta_2 SI_t + \eta_3 \varepsilon_t \quad (3.7)$$

where η_1, η_2, η_3 are weights assigned to different factors, such that $\eta_1 + \eta_2 + \eta_3 = 1$. ε_t is the sentiment shock, which I assume to follow a uniform distribution with range $[-1, 1]$. SI_t is social interaction of different types of investors, which influences the current market mood. SI_t is derived from the fractions of each types of agent as:

$$SI_t = \tanh[\kappa(\mu_t - p_t)] \omega_{f,t} + \tanh[\kappa(p_t - v_t)] (\omega_{mo,t} - \omega_{co,t}) \quad (3.8)$$

where $\omega_{f,t}, \omega_{mo,t}, \omega_{co,t}$ are fractions of fundamentalists, momentum traders and contrarian traders in financial market, respectively. $\tanh$ function and κ are used to scale the price deviation, so both the range of SI_t and the range of sentiment index S_t can be constrained to $[-1, 1]$.

If I model the behavioral heterogeneity and sentiment index within the HAM frame-

work, I can get a five-dimensional dynamic map such that:

$$\begin{aligned}
p_{t+1} &= p_t + \gamma[\omega_{f,t}A_t(\mu_t - p_t) + (\beta_1\omega_{mo,t}m_t + \beta_2\omega_{co,t}c_t)(p_t - v_t)] \\
U_{f,t+1} &= \phi U_{f,t} + (p_{t+1} - p_t)A_t(\mu_t - p_t) \\
U_{mo,t+1} &= \phi U_{mo,t} + (p_{t+1} - p_t)\beta_1m_t(p_t - v_t) \\
U_{co,t+1} &= \phi U_{co,t} + (p_{t+1} - p_t)\beta_2c_t(p_t - v_t) \\
S_{t+1} &= \eta_1S_t + \eta_2[\tanh(\kappa(\mu_{t+1} - p_{t+1}))\omega_{f,t+1} + \tanh(\kappa(p_{t+1} - v_{t+1}))(\omega_{mo,t+1} - \omega_{co,t+1})]
\end{aligned} \tag{3.9}$$

This model is used to simulate the stylized facts and different types of crises in financial market for the following sections.

3.3 Stylized facts in financial market

I calibrate the sentiment model such that it mimics some well-documented stylized facts of financial markets following the practice of existing literature (See for example, Schmitt and Westerhoff, 2017). As summarized by Westerhoff and Dieci (2006), the five salient characteristics of real-world speculative prices include (1) price distortions in the forms of bubbles and crashes; (2) excess price volatility; (3) leptokurtic distribution of returns (characterized by kurtosis exceeding 3); (4) negligible autocorrelation of daily returns; and (5) strong autocorrelation of absolute daily returns. These patterns can be seen in Figure 3.1, which depicts the dynamics of US stock prices and returns based on daily S&P500 index from Jan 3, 2000 to Feb 12, 2019. As seen from the top panel, the evolution of daily stock prices shows both strong price appreciations and crashes in some periods. The second panel displays daily log returns which show evidence of volatility clustering. Distribution of returns in the third panel indicates the presence of fat tails. The last panel plots the autocorrelation functions (ACF) using both raw returns and absolute returns. We can observe the absence of autocorrelations in raw returns and the slow-dampening autocorrelations in absolute returns, implying long memory of daily returns. I conduct simulation analysis on the basis of the standard parameter setting as shown in Table 3.1. Although parameter setting do affect the simulation results, they are not

Figure 3.1: The dynamics of daily S&P500 index between Jan 3, 2000 and Feb 12, 2019. The panel shows (a) the evolution of the stock price index (b) the returns, (c) the histogram of returns overlaid by normal curve and (d) the autocorrelation function of raw returns (red line) together with the autocorrelation function of absolute returns (blue line).

the key. After a tedious trial-and-error calibration exercise, I find sentiment model with this base group parameters could consistently generate most of stylized facts in stock market. Further experiments revealed that the model performance does not react overly sensitively with respect to changes in the parameter setting. With the following in-depth Monte Carlo simulations, I can conclude that this calibrated model systematically replicates the main statistical properties in financial markets. However, future work could try to estimate the parameters for such a complicated nonlinear model. For the explanatory purposes, the model's performance already appears to be sufficient.

To examine the effect of sentiment on price movements, I compare model with sentiment to model without sentiment by setting the sentiment sensitivity parameter, h , from 1 to 0, while holding all other parameter values constant. Simulation results of 17,000 periods (daily) price trajectories are shown in Figure 3.2. Table 3.2 summarizes the de-

Table 3.1: Standard Parameter setting

Parameter	Value	Definition
μ	1014	Mean of fundamental prices
σ	1	SD of fundamental prices
β_1	1.75	Momentum extrapolation rate
β_2	-1.25	Contrarian extrapolation rate
φ	0.1	Performance memory strength
ρ	0.5	Intensity of choice
γ	0.845	Speed of price adjustment
η_1	0.4	Last-period sentiment weight
η_2	0.5	Social Interaction weight
η_3	0.1	Sentiment shock weight
λ	12	Support and resistance level
a	1.11×10^{-5}	Confident function factor
b	1×10^{-8}	Confident function factor
κ	1000	Scaling factor
$h = h_1 = h_2$	0/1	Without sentiment/with sentiment

Table 3.2: Summary statistics of returns

Variable	Kurtosis	Skewness	Mean	SD	Min	Max
Actual Market						
S&P 500	11.481	-0.215	0.000	0.012	-0.095	0.110
Artificial Market						
With sentiment	6.677	-0.099	0.000	0.010	-0.095	0.064
Without sentiment	2.140	-0.017	0.000	0.005	-0.010	0.010

scriptive statistics of both actual and artificial market returns.

Overall, I find the simulated series with investor sentiment exhibiting much more realistic statistical properties as compared to series without investor sentiment. As shown by the top left panel of Figure 3.2, the model with sentiment is capable to produce more volatile prices (indicated by dark line) relative to rather stable fundamental values (indicated by blue line). Unlike the case without sentiment where prices mostly fluctuate closely around the fundamentals, prolonged bubbles and crashes can be generated when investor sentiment is included in the framework. Meanwhile, by allowing for sentiment, the return trajectories show volatility clustering similar to the actual S&P 500 returns, while the distribution of returns from the model with sentiment demonstrates leptokurtic behavior given the presence of fat tails. From Table 3.2, the kurtosis and skewness for model with sentiment are 6.68 and -0.099, respectively; whereas model without senti-

Figure 3.2: The dynamics of the models with sentiment (left) and without sentiment (right). The panels show, from top to bottom, the evolution of the stock prices, the returns, the histogram of returns overlaid by normal curve and the ACF of raw returns (red line) together with the ACF of absolute returns (blue line), respectively. The simulation run is based on 17,000 observations.

ment have underestimated kurtosis and skewness of 2.14 and -0.017, respectively.

To check whether there exists long memory dependence in daily returns, I plot ACFs for both raw daily returns (blue line) and absolute daily returns (red line). Without sentiment, the absolute returns have fast-decaying ACF, suggesting there is no long-range dependence for daily returns. However, with sentiment, ACF of absolute returns is strong and persistent even after 50 lags. This finding matches the stylized facts of real stock return, that cross-correlation should be weak for raw returns but strong for absolute returns. This qualitative finding holds for simulation of deterministic model as well and I further exclude the contribution of stochastic noise to these statistical properties. I also conduct robustness check by evaluating several statistical properties of simulated prices and returns for 1000 simulation runs across a range of sentiment sensitivity parameter values. More details can be found in Table 3.3.

3.4 Sentiment and excess volatility

Prices of financial assets are typically more volatile than can be justified within standard asset pricing model. This result, known as excess volatility, has been documented in many studies, such as Campbell and Shiller (1987), LeRoy and Porter (1981), Shiller (1980). Recently, some studies attempt to use behavioral approach to tackle excess volatility puzzle. (see for examples, Dumas et al., 2009; Li, 2007; Lof, 2015; Xiong, 2013). In order to study the effect of sentiment on excess volatility, I use the standard deviation of the market prices from the fundamental values $SD_{p-\mu}$ as a quantitative measure of market volatility. Standard deviation $SD_{p-\mu}$ is computed as:

$$SD_{p-\mu} = \sqrt{\frac{1}{T} \sum_{t=1}^T (p_t - \mu_t)^2} \quad (3.10)$$

Keeping the same standard parameter set, the deviation of the market prices from the fundamental values $SD_{p-\mu}$ is 358.95 for the model with sentiment and 58.46 for the model without sentiment. This implies that the market is more volatile when sentiment effect is included. In general, I find that $SD_{p-\mu}$ becomes larger as sentiment sensitivity

Figure 3.3: The effect of sentiment sensitivity on market volatility with 1,000 simulations.

parameter h increases from 0 to 1.

To check the robustness of the result, I run the three-agent model 1,000 times by using Monte Carlo simulation using the standard parameter values with different levels of sentiment sensitivity h from 0 to 1 with an interval of 0.2. After that I calculate the standard deviation using eq. (3.10) for each simulation. Then average standard deviation is calculated based on 1000 simulation for each sentiment sensitivity level. Figure 3.3 shows the average standard deviations $SD_{p-\mu}$ for 1,000 simulation runs at different h values. As we can see, the average value of $SD_{p-\mu}$ increases with the sentiment sensitivity, suggesting that the volatility of market is positively related to the sensitivity of investors to market sentiment.

3.5 Sentiment and crises

Financial crisis is the main aspect I am interested to investigate under the HAM framework. Many researchers have proven and indicated that standard economic theory failed to envisage the occurrence of financial crises, and heterogeneous beliefs and interaction of heterogeneous agents should be taken into account to understand financial crises. Some literature has documented that models with heterogeneous beliefs have extraordinary power in explaining financial crises, and these studies include but not limit to

De Jong et al. (2009), Huang et al. (2010), Lux and Westerhoff (2009) and Xiong (2013). In this section, I investigate different financial crises under the HAM framework as in Huang et al. (2010), but I focus on the role of sentiment on the formation of crises.

In Huang et al. (2010), they have successfully replicated three typical types of crises, namely sudden crisis, smooth crisis and disturbing crisis, by using HAM. They find that endogenous price dynamic from agents' interaction might be the reason of these crises, and both fundamentalists and chartists could potentially contribute to the financial crises. Following their logic, I also generate the same three typical types of financial crises using three-agent model with fundamentalists, momentum traders and contrarian traders. For more accurate replication of crisis patterns, I set $\mu = 200, \rho = 0.7, \gamma = 1.8, \lambda = 10, b = (1/150)^4$ while keeping other parameters in line with the standard values.

In a sudden crisis, price abruptly drops from the peak (or near peak) straight down to bottom within a short timeframe. According to Huang et al. (2010), when the price is at the peak, it is highly overvalued. Observing opportunities for profit, investors switch to fundamental strategy excessively and execute great selling forces that cause the steep fall in price. Contrary to their results, I find no contribution of fundamentalists before and during the sudden crisis. As illustrated in Figure 3.4, there is a dramatic price fall between $t=50$ and $t=60$, during which, the market is dominated by momentum traders without any switching to fundamental strategy. It is the strong negative sentiment that drives the market crash. Due to either exogenous cause or regime switching, price starts to go down at around $t=45$, which makes the momentum traders change from buy to sell and create a bearish sentiment. The strong negative sentiment further accelerates selling forces of the momentum traders and subsequently, leads to market panic and causes a sharp decline in price.

In a smooth crisis, price declines moderately but persistently over a period of time without a visible crash. As shown from Figure 3.5, the downward trend starts somewhere between $t = 30$ and $t = 40$ with an increase in the fraction of fundamentalists. After the first selling from fundamentalists, there are some counter-movements in price caused by contrarian traders. When the downward trend becomes more observable, investors cluster

Figure 3.4: Sudden crisis modelling. The panels show, from top to bottom (a) comparison between simulated price with S&P 500 from 1987/8/3 to 1987/12/22, (b) simulated sentiment, and (c) fractions of 3 types of investors.

to momentum trading strategy and execute more selling forces to push the price further down. During the decline period, sentiment switches between bullish and bearish with more negative values overall. From $t = 30$ to $t = 70$, the average value of sentiment is -0.3751.

A disturbing crisis is characterized by volatile fluctuations with a downward trend and possible moderate crashes in price. The period of disturbing crisis is somewhere between sudden crisis and smooth crisis. Figure 3.6 shows a simulation of disturbing crisis in the model. Price fluctuates disturbingly before starting to drop sharply at around $t = 37$ due to significant negative sentiment. From $t = 45$ to $t = 55$, price becomes volatile again as the sentiment fluctuates between positive and negative. After that, the downward trend continues with another strong bearish sentiment. Similar to the other types of crisis, momentum traders are responsible for the downward trend. However, during the crisis, some investors shift to contrarian strategy because of high market volatility.

The common feature of these three figures is that crises is always accompanied by

Figure 3.5: Smooth crisis modelling. The panels show, from top to bottom, (a) comparison between simulated price with S&P 500 from 1932/1/20 to 1932/6/13, (b) simulated sentiment and (c) fractions of 3 types of investors.

Figure 3.6: Disturbing crisis modelling. The panels show, from top to bottom, (a) comparison between simulated price with S&P 500 from 1929/8/1 to 1929/12/26 (b) simulated sentiment and (c) fractions of 3 types of investors.

Figure 3.7: Scatter plots of sentiment and asset return for different types of crises.

negative sentiment. During crises, bearish sentiment dominates the market, in turn the negative sentiment may amplify the crises. To explore the correlation between sentiment and asset return, I calculate the correlation coefficients between these two variable for whole simulated sample as well as crises samples. All the results are positive, which indicate that investor sentiment is positively correlated with return. The correlation coefficient is 0.41 for the whole sample, while coefficients for three crises samples are all above 0.8. The scatter plots for sentiment-return relation for three crises are shown in Figure 3.7. Sentiment index seems to have stronger predictive power on asset price during crisis periods.

To further investigate the effect of sentiment on crises, I compare the frequency and magnitude of crises in simulated series from model with and without sentiment effect in 1000 simulations. Following Patel and Sarkar (1998) and Zouaoui et al. (2011), I define the magnitude of a crisis as the percentage drop from the peak to the trough. The date of the peak is the month when price reaches its maximum value over T-period window prior to the crisis identification, and the date of the trough is month when price reaches its minimum during the crisis. To identify the crisis in financial market, crisis/crash index

is usually constructed based on firm level data as in Hutton et al. (2009) or market index data as in Zouaoui et al. (2011). I adopt a crisis indicator called CMAX used in Zouaoui et al. (2011) with some adjustments. In this method, CMAX is a ratio calculated by dividing current value by the maximum price over the previous T periods, usually T is one to two years.

$$CMAX_t = \frac{P_t}{\max(P_{t-T} \cdots P_t)} \quad (3.11)$$

where P_t is the stock market index at time t . CMAX equals one if price rise over the period considered, indicating a bullish market. If price declines over a period, CMAX goes less than one, and crisis is detected each time CMAX drops below a threshold set at the mean of CMAX minus two standard deviations. Both mean and standard deviation are calculated on the whole sample. However, this method may mistakenly identify the bubble correction as a crisis. To fix this problem and make it more suitable for the simulated data, I add one compulsory condition to detect the crisis, which is the current price must be lower than fundamental prices for a certain threshold value. Therefore, the crisis indicator C_t is defined as following:

$$C_t = \begin{cases} 1, & \text{if } CMAX_t < \overline{CMAX} - 2\sigma \text{ and } P_t < \tau * \mu_t \\ 0, & \text{otherwise} \end{cases} \quad (3.12)$$

where σ is the standard deviation of whole sample CMAX, and $0 < \tau < 1$ is the threshold to determine the minimum magnitude to be defined as a crisis. To check the efficiency of this method, I use the S&P 500 monthly data from 1950m1 and 2018m12 and corresponding fundamental value constructed from monthly dividend to detect the occurrence crisis in US stock market. In accordance to Hommes et al. (2017), fundamental value is estimated using S&P 500 data from 1950 onwards. Both the real S&P 500 prices and dividends on monthly frequency are provided by Shiller (2015). More specifically, I calculate fundamental value using static Gordon growth model with constant discount rate r and growth rate g of the dividend flows D_t , for which

Figure 3.8: Crises detected from 1950 using real S&P 500 data.

$$P_t^* = \frac{1+g}{r-g} D_t \quad (3.13)$$

where P_t^* denotes the fundamental price of the stock index. I set $T = 12 \text{ months}$ and $\tau = 0.9$.

As shown in Figure 3.8, five periods of crises are identified during the period 1950-2018. The first crash occurs in 1962 known as Kennedy Slide, followed by the second tech-stock crash in 1970 and third in 1973-1974 after the end of Bretton Woods monetary system and oil crisis. Most recent two market crashes are detected in 1987 known as Black Monday and during 2008-2009 global financial crisis.

As CMAX indicator has been proven reliable in identifying crisis in real financial market, I attempt to use it in the simulated data. To be consistent with case of real financial market, I convert the simulated daily data to monthly data and choose $T = 12 \text{ month}$ and $\tau = 0.9$ to detect crisis. To obtain a visual impression on effect of sentiment on crisis, I try to detect the crisis in the previous two simulated time series with and without sentiment, respectively. From Figure 3.9, I find more crises in with-sentiment case

Figure 3.9: Crisis identified in simulated monthly data with sentiment (left) and without sentiment (right), $\tau = 0.9$

than without-sentiment case. To check the robustness of the result and investigate the magnitude of crisis in these two cases, I again conduct Monte Carlo simulation with the three-agent model using the standard parameter values across different levels of sentiment sensitivity with interval of 0.2. As shown in Figure 3.10, number of crisis increases with sentiment sensitivity h , which illustrates the significant positive relationship between sentiment sensitivity and frequency of crisis occurrence. I further investigate whether sentiment affects the magnitude of crisis. As expected, the average magnitude of crisis rises with higher h which corroborates the effect of sentiment on depth of crisis. Hence, I conclude that sentiment has a significant effect on crisis in terms of frequency and magnitude in the simulations. The result is consistent with findings in Zouaoui et al. (2011), who find investor sentiment positively influences the probability of the occurrence of stock market crises by using panel data of 16 countries.

In summary, I have demonstrated that investor sentiment contributes to more realistic stylized facts, The simulation results in Table 3.3 also show that the market will be more volatile if investors are very sensitive to market sentiment. More importantly, I successfully replicate different crises, and find that sentiment could be an important source of crisis formation. Through 1000 Monte Carlo simulations, I find the evidence that sentiment could amplify both the frequency and magnitude of financial crises.

Figure 3.10: The effect of sentiment sensitivity on frequency of crisis (left) and average magnitude of crises (right) with 1000 simulations

Table 3.3: Simulated market dynamics with different sentiment sensitivities (average for 1000 simulations)

	h=0	h=0.2	h=0.4	h=0.6	h=0.8	h=1
kurtosis	2.155	2.075	3.396	4.615	4.73	5.804
skewness	-0.01	-0.015	-0.026	-0.036	-0.037	-0.04
AC r_1	0.01	0.041	0.166	0.219	0.189	0.166
AC r_5	-0.006	-0.004	0.004	0.001	0.007	0.004
AC r_{10}	-0.003	-0.004	-0.002	-0.003	-0.002	-0.003
AC r_{20}	-0.003	-0.002	-0.002	-0.001	-0.001	-0.001
AC $ r_1 $	-0.014	-0.003	0.172	0.344	0.372	0.412
AC $ r_5 $	0.07	0.075	0.179	0.252	0.253	0.298
AC $ r_{10} $	0.039	0.046	0.167	0.241	0.243	0.29
AC $ r_{20} $	0.015	0.025	0.158	0.231	0.235	0.282
$SD_{p-\mu}$	57.796	71.944	217.539	270.436	291.671	339.284
# of crisis	3.618	4.929	6.365	6.728	6.759	6.655
magnitude of crisis (%)	14.703	17.141	30.961	38.443	39.213	43.804

3.6 Robustness analysis

The goal of this section is to perform robustness checks by relaxing several nonlinearity assumptions that I previously imposed on the investor behavior. By excluding possible influences from other factors, such as nonlinear confidence of fundamentalist and regime-switching trading rules of chartist, I further confirm the significance of sentiment in contributing to the formations of stylized facts and crises in the model. In Section 3.5.1, I first consider different setups for fundamentalist's demand function, in the forms of both linear as in Westerhoff and Dieci (2006) and nonlinear as in Day and Huang (1990). In Section 3.5.2, I set different trading rules for chartists to examine the model's robustness to chartist's strategy. The main outcomes of sentiment model still hold even when I simplify these assumptions albeit the benchmark model is still the best in terms of replicating the stylized facts and crises.

3.6.1 Linear and nonlinear behaviors of fundamentalist

Sentiment model is robust to the alternative assumptions of fundamentalist behavior. Specifically, the main results hold even if I change the confidence function in the benchmark model into two commonly used forms in the extant HAM literature, which are the linear function as in Westerhoff and Dieci (2006) and the nonlinear monotonous function as in Day and Huang (1990). The linear and nonlinear functions are shown in Equations (3.14) and (3.15), respectively.

$$D_{f,t} = a_f(\mu_t - p_t) \quad (3.14)$$

$$D_{f,t} = (p_t - a_1\mu_t)^d(a_2\mu_t - p_t)^d(\mu_t - p_t) \quad (3.15)$$

In Equation (3.14), a_f is a positive constant and fundamentalists are assumed to react with the same speed of mean reversion given any price deviation. Equation (3.15) is generalized version of demand function for fundamentalist following Day and Huang (1990), in which $d < 0, 0 \leq a_1 < 1, a_2 > 1$. The function implies that the more price deviates

from its fundamental value, the more quickly the price trend is going to reverse within the range $(a_1\mu_t, a_2\mu_t)$. As can be seen from Table 3.4, after replacing the chance function with linear or nonlinear monotonous function, sentiment model can still replicate most of the stylized facts, and the amplification effect of sentiment on volatility and crises remains. It suggests that the exact specification of fundamental demand component is not crucial for the model's ability to explain the stylized facts and crises in the financial markets. However, the nonlinear confidence function of fundamentalist is preferred as it can generate more realistic stylized facts with a wider range of sentiment sensitivity. For instance, the heavy-tail presents starting from $h = 0.4$ under the benchmark model, whereas under the linear chance function, heavy-tail only becomes evident at $h = 1$. In addition, it also helps to explain why the boom and bust cycles in the actual financial markets last longer than expected, since the fundamentalists may gradually lose their confidence when price deviation exceeds a certain threshold.

3.6.2 Different technical rules of chartists

Finally, I check the robustness of sentiment model dynamic regarding the technical trading rule of chartists. Instead of deriving the short-run value from supporting and resistance levels, I assume chartists use moving average to calculate their reference price. Following Chiarella et al. (2007) with an exponential moving average, the short-run value can be written as:

$$v_t = kp_t + (1 - k)v_{t-1} \quad (3.16)$$

Another common trading strategy is trend extrapolative rule, the chartists base their orders on the most recent price change as in Westerhoff and Dieci (2006). The larger the price trend, the stronger the demand. Take the momentum traders for example, the excess demand can be calculated as:

$$D_{mo,t} = \beta_1 m_t (p_t - p_{t-1}) \quad (3.17)$$

Table 3.5 presents the simulated results for the two alternative trading strategies with a linear demand function of the fundamentalists. It shows that parts of stylized facts can still be replicated. More importantly, the impact of sentiment on excess volatility, leptokurtosis of returns, volatility clustering and magnitude of crisis still persists.

Table 3.4: Simulated market dynamics with different sentiment sensitivities using linear and Day and Huang (1990) setups for fundamentalist demand function (average for 1000 simulations)

	Linear demand function						Day and Huang (1990) demand function					
	h=0	h=0.2	h=0.4	h=0.6	h=0.8	h=1	h=0	h=0.2	h=0.4	h=0.6	h=0.8	h=1
kurtosis	2.378	2.312	2.657	2.931	3.053	3.377	2.537	2.551	3.442	4.397	4.807	5.309
skewness	-0.018	-0.02	-0.057	-0.091	-0.088	-0.109	-0.009	-0.012	-0.043	-0.077	-0.086	-0.106
AC r_1	0.013	0.041	0.159	0.218	0.188	0.165	0.022	0.05	0.183	0.225	0.191	0.166
AC r_5	-0.004	0.000	0.006	0.002	0.007	0.004	-0.003	0.000	0.003	-0.002	0.002	-0.002
AC r_{10}	-0.001	0	-0.001	-0.001	-0.001	-0.002	0.000	0.000	-0.002	-0.002	-0.002	-0.003
AC r_{20}	0.000	0.000	-0.001	-0.002	-0.001	-0.001	0.000	0.000	-0.001	-0.001	-0.002	-0.001
AC $ r_1 $	0.035	0.063	0.172	0.299	0.332	0.352	0.087	0.123	0.248	0.37	0.403	0.418
AC $ r_5 $	0.103	0.126	0.183	0.209	0.214	0.238	0.134	0.165	0.231	0.276	0.291	0.315
AC $ r_{10} $	0.074	0.097	0.166	0.195	0.202	0.227	0.086	0.116	0.194	0.25	0.27	0.298
AC $ r_{20} $	0.058	0.084	0.158	0.188	0.196	0.223	0.058	0.088	0.175	0.235	0.259	0.29
$SD_{p-\mu}$	141.601	164.813	239.047	266.797	277.417	293.9	145.606	164.613	234.051	281.749	304.278	336.214
# of crisis	5.56	5.872	6.774	7.376	7.268	7.353	5.452	5.837	6.59	6.76	6.842	6.8
magnitude of crisis (%)	20.166	22.44	30.059	35.723	36.732	38.883	20.927	23.511	33.006	38.715	40.851	42.916

Note: I set $a_f = 0.01$ for linear demand function; $d = -0.4, a_1 = 0$ and $a_2 = 2.4$ for Day and Huang (1990) demand function

Table 3.5: Simulated market dynamics with different sentiment sensitivities using exponential moving average and extrapolative trend strategy for chartists demand function (average for 1000 simulations)

	Exponential moving ($k = 0.5$) average						Extrapolative trend strategy					
	h=0	h=0.2	h=0.4	h=0.6	h=0.8	h=1	h=0	h=0.2	h=0.4	h=0.6	h=0.8	h=1
kurtosis	3.5	3.549	3.617	3.723	3.913	4.259	3.557	3.593	3.651	3.748	3.939	4.375
skewness	-0.002	-0.003	-0.004	-0.006	-0.01	-0.017	-0.006	-0.007	-0.01	-0.014	-0.02	-0.033
AC r_1	0.113	0.129	0.148	0.17	0.196	0.226	0.095	0.111	0.131	0.155	0.185	0.221
AC r_5	0.021	0.029	0.039	0.051	0.066	0.086	0.002	0.007	0.014	0.022	0.034	0.049
AC r_{10}	0	0.001	0.003	0.006	0.01	0.017	-0.002	-0.002	-0.002	-0.002	-0.001	0.001
AC r_{20}	-0.003	-0.003	-0.004	-0.004	-0.005	-0.006	-0.002	-0.003	-0.003	-0.004	-0.005	-0.006
AC $ r_1 $	0.07	0.079	0.09	0.105	0.124	0.15	0.106	0.121	0.14	0.164	0.194	0.232
AC $ r_5 $	0.055	0.059	0.063	0.07	0.08	0.096	0.059	0.061	0.064	0.069	0.077	0.09
AC $ r_{10} $	0.053	0.056	0.06	0.065	0.072	0.084	0.057	0.059	0.062	0.066	0.072	0.082
AC $ r_{20} $	0.051	0.054	0.058	0.062	0.069	0.079	0.056	0.058	0.06	0.064	0.069	0.078
$SD_{p-\mu}$	169.035	172.429	176.441	181.499	188.128	196.864	176.808	178.924	181.32	184.624	189.657	197.421
# of crisis	9.162	9.286	9.364	9.413	9.494	9.596	9.078	9.203	9.287	9.403	9.487	9.564
magnitude of crisis (%)	39.815	41.13	42.697	44.513	46.699	49.425	40.234	41.41	42.838	44.543	46.671	49.301

Note: I set $a_f = 0.01$ for linear demand function and change the parameter of chartists as $\beta_1 = 0.35, \beta_2 = -0.25$.

3.7 Conclusion

In this chapter, I run the simulation for three-agent sentiment model built in Chapter 2. The simulations are trying to mimic real financial market features, including stylized facts, excess volatility and three different types of crises. The analysis mainly focuses on the sentiment roles in the dynamic formation.

The key findings of this chapter are in two respects. First, reconciling three-agent model with actual US stock market, I discover that by supplementing the standard model with investor sentiment, it dramatically improves the model's ability to replicate stylized facts in financial markets. In particular, numerous key indicators, such as negative skewness, leptokurtosis and long memory of returns, market volatility, as well as number of crisis detected increase with sentiment sensitivity. Second, I also try to deduce theoretical underpinning of different types of crises. The key implication that differentiates my finding from the extant HAM literature is that financial crisis can be triggered even without mean-reverting action of fundamentalist. With presence of investor sentiment, just the momentum traders alone can initiate sudden crisis when there is abrupt downward pressure in market sentiment. In other words, the sentiment channel provides an additional explanation underlying different types of financial crises.

Chapter 4

Estimating Two-market HAM with Sentiment

4.1 Introduction

A substantial literature has proven the existence of return comovement across markets (See, King et al., 1990; Lin et al., 1994; Longin and Solnik, 1995; Karolyi and Stulz, 1996; Brooks and Negro, 2006; Forbes and Rigobon, 2002; Rua and Nunes, 2009). However, the cause of comovement cannot be fully explained by traditional financial theory, as several empirical studies have found that asset returns comove more than what can be justified by the correlation of their fundamentals (Kallberg and Pasquariello, 2008; Greenwood, 2008; Pindyck and Rotemberg, 1993; Brealey et al., 2010). Instead, recent research has placed a role in nonfundamental factor to explain phenomenon of excess comovement (See for examples, Barberis et al., 2005; Chen et al., 2019; Connolly and Wang, 2003; Chen et al., 2016). In this chapter, I estimate a two-market asset pricing model that incorporates two elements of behavioral finance, namely investor sentiment and heterogeneous expectation, to establish the comovement across international equity markets.

Investor sentiment can be defined as *fluctuating general attitudes toward investment categories* (Hirshleifer, 2015), or *belief about future cash flows and investment risks that*

is not justified by the facts at hand (Baker and Wurgler, 2007). A prolific literature has confirmed the importance of investor sentiment in financial market but most of these studies are conducted within domestic context. Recently, there is a growing body of literature contributing to the investigation of sentiment spillover in international stock markets. For instances, Hudson and Green (2015) examine sentiment spillover between the US and the UK stock markets and find that US investor sentiment is highly significant in predicting the UK equity returns. Study of Sayim and Rahman (2015) documents similar evidence in that the US investor sentiment has significant spillover effects on Istanbul Stock Market (ISE) return and volatility. Aissia (2016) discovers both foreign and home sentiments are strong contrarian predictors of French stock returns. Empirical finding of Frijns et al. (2017) shows investor sentiment as the main driver for international stock return comovements. All these studies suggest that investor sentiment serves as an alternative channel to fundamental factors that underlies the comovement of asset returns across different markets.

A second strand of literature on HAM posits that agents' investment beliefs are contingent on the state of market (Day and Huang, 1990; Brock and Hommes, 1998; Lux, 1998; LeBaron et al., 1999; Hommes, 2006). Under HAM setting, market pricing is jointly determined by heuristic expectations of different groups, commonly represented by fundamentalists and chartists that form their beliefs based on fundamental and technical analyses, respectively. Against backdrop of rising financial market integration, more recent HAM has developed multi-market setup to account for interaction between markets. Comovement across market is usually introduced via cross-market agents that condition their behavior on the returns of foreign market. There are several theoretical works of multi-market HAM. Westerhoff and Dieci (2006) develop a chartist-fundamentalist model for two speculative markets and study effects of the imposition of transaction tax. Chiarella et al. (2007) focus on setup with multiple assets to investigate portfolio diversification. Dieci and Westerhoff (2010) construct a three-market model that links stock markets in two countries via foreign exchange market. Schmitt and Westerhoff (2014) simulate a number of stylized facts of two interacting stock markets. Huang and Chen

(2014) model regional financial linkage with two-market HAM model and prove in theory the existence of price comovement. Hommes and Vroegop (2019) develop a two market HAM and assume markets are mutually dependent through biased chartists. They find bias of chartists could destabilise spillover effects between two markets and lead to unpredictable price dynamics with bubbles and crashes. Further, De Jong et al. (2009) pave the ground for empirical application of multi-market HAM, in which they explore the role of shift contagion through estimating a model with presence of internationalist, switching beliefs, and two equity markets between Hong Kong and Thailand. In a similar vein, Chen et al. (2016) focus on French and the US stock markets with two-regime switching process and confirm evidence of comovement between the two markets. Due to the complex and nonlinear feature of HAMs, most of the papers in this area mainly focus on theoretical explanations and numerical simulation. Chapter 2, 3 and 5 continue the study in this direction. Recent years, more and more econometric approaches are explored and developed to contribute to the estimation of HAMs. Alfarano et al. (2005, 2006) are among the first in the direction. They estimate a simple agent based model by using maximum likelihood (ML) method. After this, ML is widely used in estimating different types of HAMs, these studies include but not limit to De Jong et al. (2009), Ter Ellen and Zwinkels (2010), Chiarella et al. (2014) and Kouwenberg and Zwinkels (2014). Reitz and Westerhoff (2007) build HAM in forms of STAR-GARCH models, which can be estimated by Quasi maximum likelihood approach. An important contribution in the estimation of HAMs is by Boswijk et al. (2007). They combine nonlinear least squares (NLS) estimation with a multinomial switching rule to empirically validate a HAM with the S&P 500 data. They are also the first to estimate a full-fledged HAM with belief switching framework in Brock and Hommes (1998). Chiarella et al. (2012) and Chia et al. (2014) attempt to apply a Markov regime switching model to estimate HAMs, and their results match well with volatility in both boom and bust states. With increasing popularity of moment based methods, Generalized Method of Moments (GMM) and the Simulated Method of Moments (SMM) are used in HAM estimation. Franke and Westerhoff (2012, 2016) have applied SMM estimation to a variety of HAM models and it

has been proven to be a more accurate estimation method. By employing GMM, Ghonghadze and Lux (2016) estimate HAM on cross section of stock markets, exchange rates and commodity data. In the most recent papers by Grazzini et al. (2017) and Lux (2018, 2020), Bayesian estimation for state space model is proposed to estimate the complex nonlinear systems like HAM. It paves a new direction for HAM estimation with richer Bayesian tools. This chapter is also devoted to this quantitative strand of estimation and extended Kalman filter approach is used to conduct the filtering and estimation.

Although some estimation works have been done for two market HAM framework (see Li et al. (2016), Chen et al. (2018) and Frijns and Zwinkels (2018)), the investigation of investor sentiment under a two-market HAM framework is very rare. Lux (2012) and Chen and Lux (2018) attempt to estimate sentiment effect within a HAM framework, but sentiment in these models is endogenously formed by herding behavior and their models are single market models. Sentiment spillover between markets is rarely investigated in HAM literature. This chapter aims to contribute to this gap of the extant literature, through developing a two-market HAM with fundamentalists, chartists, and internationalists. In the model, internationalists can choose to invest domestically or internationally, subjecting to the condition of market sentiment across two markets. The Bayesian approach allows the model to incorporate more market information like fundamentals and sentiment. It improves the efficiency of estimation for such a complicated model. Using monthly data in the US and UK stock markets, I find fundamentalists, chartists and internationalists are present in the two-market system. Market switching behavior of internationalists exists, and it provides a channel to explain crisis contagion through sentiment-induced market switching of international traders.

The remainder of the chapter is organized as follows. In Section 4.2, I develop the two-country heterogeneous agent model with sentiment. Section 4.3 describes the data used for empirical estimation. Section 4.4 is devoted to the estimation method and in Section 4.5 I present the results. Section 4.6 runs robustness check. Section 6 concludes.

4.2 The model

In this section I develop a simple version two-market heterogeneous agent model. The model consists of three main blocks. First, I assume all agents are utility maximizing individuals. They build optimal portfolio using a mean-variance utility framework. Second, agents make forecasts about the stock market prices based on simple but different rules. In this second block I introduce discrete choice model established in Brock and Hommes (1998) and assume that fundamentalists and chartists are able to evaluate and switch their strategies based on realized profits. Third, I assume there is a certain amount of international investors who invest in either market A or market B, and they can freely switch investment markets by comparing the difference of sentiment in two markets.

Assuming myopic investors with a mean-variance utility function and a common risk-aversion parameter $\alpha > 0$. The agents' utility function can be represented by the following equation

$$U(W_{h,t+1}) = E_{h,t}(W_{h,t+1}) - \frac{1}{2}\alpha V_{h,t}(W_{h,t+1}) \quad (4.1)$$

where $W_{h,t+1}$ is the wealth of agent of type h at time $t + 1$. $E_{h,t}$ and $V_{h,t}(W_{h,t+1})$ denote the conditional expectation and variance of agent h with wealth W_{t+1} . For analytical tractability, all investors are assumed to have same conditional variance of excess returns, such that $V_{h,t}(p_{t+1} + y_{t+1} - Rp_t) = \sigma^2$. I consider an economy with one risky asset and one risk-free asset. I assume that the gross return of risk-free asset $R > 1$. The single risky asset with price per share p_t pays a (assume commonly known) stochastic dividend y_t . The wealth of investor h then evolves according to

$$W_{h,t+1} = RW_{h,t} + (p_{t+1} + y_{t+1} - Rp_t)z_{h,t} \quad (4.2)$$

where z_t is the number of share of risky asset at time t . Assume each investor type is a

myopic mean variance maximizer such that each investor h solves

$$\underset{z_{h,t}}{\text{Max}} \{E_{h,t}[W_{h,t+1}] - \frac{\alpha}{2} V_{h,t}[W_{h,t+1}]\} \quad (4.3)$$

for his optimal amount of shares, which yields

$$z_{h,t} = \frac{E_{h,t}(p_{t+1} + y_{t+1} - Rp_t)}{\alpha \sigma^2} \quad (4.4)$$

Although investors may have heterogeneous expectations on price, they share the same belief on dividends. Let g be the average growth rate of dividends. The expected dividend in time $t + 1$ can be expressed as

$$E_t(y_{t+1}) = (1 + g)y_t \quad (4.5)$$

In turn, fundamental value of risky asset can be derived from static Gordon growth model (see Gordon, 1959). The details about calculation of fundamental price are discussed in Section 4.3.

In order to get a benchmark notion of the rational expectation ‘fundamental solution’ p_t^* , I consider the equation

$$Rp_t^* = E_t(p_{t+1}^* + y_{t+1}) \quad (4.6)$$

where E_t is expectation conditional on available information. It is then convenient to write the model in terms of price deviation from fundamental price, that is

$$x_t = p_t - p_t^* \quad (4.7)$$

The expected excess return in terms of price deviations of type h investor is then given as

$$E_{h,t}(R_{t+1}) = E_{h,t}(x_{t+1}) - Rx_t \quad (4.8)$$

Let us now turn to the expectation formation behavior of investors in markets. Inspired by Brock and Hommes (1998), I assume investors select between competing expectation rules to forecast future stock prices. All agents know the fundamental price p_t^* , but disagree about how the next period's price will deviate from p_{t+1}^* . Two types of expectation rules are prevailing in the stock markets, which are fundamentalism rule and chartism rule. As my interest lays at the interaction between multiple markets, I consider three types of agent, namely fundamentalist, chartist and internationalist. Fundamentalists and chartists are only active in their 'own' market, but internationalists can invest in both markets. Fundamentalists adopt fundamentalism rule when forecast the future prices, while chartists and internationalists use the chartism rules.

Let us define two separate markets, market A and market B. Since the markets are theoretical symmetric, I focus on one market first and integrate the markets into a single system later.

Fundamentalists The fundamentalists believe in mean-reversion of the stock price and they expect the market price to converge to its fundamental, and thus the deviation of the market price x_t reverts to zero. The expectations of the fundamentalists can thus be written as

$$E_{f,t}(x_{t+1}) = ax_t \quad (4.9)$$

where $a > 0$ measures the speed of mean reversion of the market price toward the fundamental value. Combining with Equation (4.8), I rewrite Equation (4.9) as

$$E_{f,t}(R_{t+1}) = (a - R)x_t \quad (4.10)$$

Chartists The chartists group, on the other hand, focuses on changes in past market prices. For simplicity, chartists will only use the information in time $t-1$ and t . As such, the expectations of chartists can be expressed as

$$E_{c,t}(x_{t+1}) = x_t + b(x_t - x_{t-1}) \quad (4.11)$$

The parameter b measures the direction and degree to which chartists respond to change of price deviation (price trend). If $b > 0$, chartist is trend follower and pushes the price deviation further away from past value; $b < 0$, on the other hand, implies contrarian trader and the market price is expected to move toward its past value. Taking Equation (4.12) into account, the expected returns of chartists can be written as

$$E_{c,t}(R_{t+1}) = (1 - R)x_t + b(x_t - x_{t-1}) \quad (4.12)$$

Internationalists Inspired by De Jong et al. (2009), I introduce a new group of traders, called internationalists. They can choose to invest in either market A or market B. As argued in Kim and Wei (2002), international traders commonly adopt positive and negative feedback strategies (trend following and contrarian) and ignore information about fundamentals. Hence, I assume internationalists believe in same chartism trading rule in both country A and country B, thus they share the same form of expected returns with chartists, which can be written as

$$E_{i,t}(R_{t+1}) = (1 - R)x_t + c(x_t - x_{t-1}) \quad (4.13)$$

where parameter c is extrapolating speed of internationalists and it has same economic meaning with parameter b .

As all types of agents are mean variance optimizers, I derive the optimal demand of three heterogeneous agents for the risky asset by using Equation (4.4), such as

$$z_{f,t} = \frac{(a - R)x_t}{\alpha \sigma^2} \quad (4.14)$$

$$z_{c,t} = \frac{(1-R)x_t + b(x_t - x_{t-1})}{\alpha\sigma^2} \quad (4.15)$$

$$z_{i,t} = \frac{(1-R)x_t + c(x_t - x_{t-1})}{\alpha\sigma^2} \quad (4.16)$$

Market maker Following Day and Huang (1990) and Chiarella (1992), the price of the risky asset is decided by a market maker according to the aggregate demand for the risky asset. The aggregate demand is calculated as weight average demand of fundamentalists, chartists and internationalists. I assume no new entry of agent in both markets and total population of domestic investors (fundamentalists and chartists) is N in each country. The size of internationalists is ϕN and ϕ is less than one. The market maker takes an offsetting position so as to clear the market. He increases or decreases prices in response to excess aggregate demand with a speed of price adjustment μ . A stochastic dynamic system describing the behavior of the market maker in market A can be written as

$$x_{t+1}^A = x_t^A + \frac{\mu}{1+\phi} (\omega_{f,t}^A z_{f,t}^A + \omega_{c,t}^A z_{c,t}^A + \phi \omega_{i,t}^A z_{i,t}^A) + \varepsilon_{t+1} \quad (4.17)$$

where $\omega_{f,t}^A$ and $\omega_{c,t}^A$ are the fractions of fundamentalists and chartists in domestic market A. $\omega_{i,t}^A$ denotes the fraction of internationalists in market A. As I am developing a stochastic model, ε_{t+1} follows IID and represents noisy demand of any other types of traders in market A as well as the arrival of trivial new information.

Belief switching mechanism Domestic agents, fundamentalists and chartists, are allowed to switch their beliefs. They can update their expectation conditional on the profitability of the two strategies based on multinomial logit probabilities as introduced by Manski et al. (1981) and by Brock and Hommes (1998) in HAM literature. In market A, the fraction of fundamentalists and chartists can be expressed as

$$\omega_{h,t}^A = \frac{e^{\beta\pi_{h,t}^A}}{\sum_{h=f,c} e^{\beta\pi_{h,t}^A}} \quad (4.18)$$

where parameter β is the intensity of choice measuring the speed of switching to a different trading rule. $\pi_{h,t}^A$ is the realized profit for strategy h , $h = f$ or c in market A. It provides a motivation for investors to adjust their strategies and investors tend to cluster to strategy that produces higher realized profit. This profit can be calculated as optimal demand for the risky asset multiplied by the actual price change, that is

$$\pi_{h,t}^A = (x_t^A - R x_{t-1}^A) z_{h,t-1}^A \quad (4.19)$$

Market switching mechanism As mentioned earlier, internationalists can choose to invest in either market A or market B and they are allowed to switch market without any cost. The switching behavior of internationalists is determined by the difference of sentiment in two markets. In other words, high market sentiment is able to attract more international fund inflow. This argument has been proved in literature, such as Froot and Ramadorai (2001) and Brown et al. (2003). The market sentiment in this chapter could be understood as market mood, bull and bear. It may consist of the accumulated sentiment in the market, investors' mood and sentiment related news. For detailed construction of market sentiment, one can refer to Chapter 2. Similar to belief switching, I assume internationalists switch investing market conditional on the sentiment of two markets. The switching is modeled by discrete choice probabilities, and fraction of internationalists in market A can be written as

$$\omega_{i,t}^A = \frac{e^{\lambda s_t^A}}{\sum_{k=A,B} e^{\lambda s_t^k}} \quad (4.20)$$

Where s_t^k with $k = A, B$ is the market sentiment perceived by internationalists in market A and market B at time t . λ is the intensity of market choice measuring how fast internationalists switch between market A and market B. The higher is λ , the quicker internationalists will respond to a difference in market sentiment by switching to the other market. At $\lambda = 0$, there is no switching and internationalists are distributed equally across markets. At the other extreme, when $\lambda \rightarrow \infty$, it means that the entire mass of

internationalists switches to the market had highest sentiment in the previous period.

To simplify the notation, I let $m_t = \omega_{f,t} - \omega_{c,t}$, thus in market A

$$m_t^A = \frac{e^{\beta\pi_{f,t}^A} - e^{\beta\pi_{c,t}^A}}{e^{\beta\pi_{f,t}^A} + e^{\beta\pi_{c,t}^A}} = \tanh\left[\frac{\beta}{2}(x_t^A - Rx_{t-1}^A)(z_{f,t-1}^A - z_{c,t-1}^A)\right] \quad (4.21)$$

Similarly, I use n_t to denote the fraction difference of internationalists in market A and market B, such that $n_t = \omega_{i,t}^A - \omega_{i,t}^B$. It is also a linkage of two market dynamic systems. It can be derived as

$$n_t = \frac{e^{\lambda s_t^A} - e^{\lambda s_t^B}}{e^{\lambda s_t^A} + e^{\lambda s_t^B}} = \tanh\left[\frac{\lambda}{2}(s_t^A - s_t^B)\right] \quad (4.22)$$

In turn, I rewrite $\omega_{f,t}^A = \frac{1+m_t^A}{2}$, $\omega_{c,t}^A = \frac{1-m_t^A}{2}$ and $\omega_{i,t}^A = \frac{1+n_t}{2}$ according to Equations (4.21) and (4.22). I further rewrite Equation (4.17) as

$$x_{t+1}^A = x_t^A + \frac{\mu}{2(1+\phi)}[(1+m_t^A)z_{f,t}^A + (1-m_t^A)z_{c,t}^A + \phi(1+n_t)z_{i,t}^A] + \varepsilon_{t+1} \quad (4.23)$$

Now, I can derive a stochastic dynamic system for one market by combining Equations (4.14) to (4.17), Equation (4.21) and Equation (4.22). I further extend the system to two-market system by incorporating two mirrored markets linked by sentiment-driven internationalists.

$$\textbf{Market A} \begin{cases} x_{t+1}^A = x_t^A + \frac{\mu}{2(1+\phi)\alpha\sigma^2} \{ (a-R)x_t^A(1+m_t^A) + (1-m_t^A)[(1-R+b)x_t^A - bx_{t-1}^A] \\ \quad + \phi(1+n_t)[(1-R+c)x_t^A - cx_{t-1}^A] \} + \varepsilon_{t+1} \\ m_{t+1}^A = \tanh\left\{ \frac{\beta}{2\alpha\sigma^2} (x_{t+1}^A - Rx_t^A)[(a-1)x_t^A - b(x_t^A - x_{t-1}^A)] \right\} \end{cases} \quad (4.24)$$

$$\textbf{Linkage } n_t = \tanh\left[\frac{\lambda}{2}(s_t^A - s_t^B)\right] \quad (4.25)$$

$$\text{Market B} \begin{cases} x_{t+1}^B = x_t^B + \frac{\mu}{2(1+\phi)\alpha\sigma^2} \{ (d-R)x_t^B(1+m_t^B) + (1-m_t^B)[(1-R+e)x_t^B - ex_{t-1}^B] \\ \quad + \phi(1-n_t)[(1-R+f)x_t^B - fx_{t-1}^B] \} + \zeta_{t+1} \\ m_{t+1}^B = \tanh\{ \frac{\gamma}{2\alpha\sigma^2} (x_{t+1}^B - Rx_t^B)[(d-1)x_t^B - e(x_t^B - x_{t-1}^B)] \} \end{cases} \quad (4.26)$$

The model outlined in this section will be estimated and evaluated in the remainder of the chapter.

4.3 Data

US and UK stock markets are two most important international financial markets in the world, and international assets investment and trading between these two markets are quite common due to close economic and political tie between US and UK. In this chapter, I mainly use US and UK data to conduct the estimations. Data from Japan market is also collected and used in the robustness check. Monthly data of stock prices and earnings for Standard & Poor's 500 (S&P500) is obtained from Shiller's online database, while data for Financial Times Stock Exchange 100 (FTSE100) and Tokyo Stock Exchange 1st Section (TSE 1st) are collected from CEIC database. Subjecting to data availability, the US and Japanese data covers period between Jun 1980 and Dec 2019, while the UK data covers period between Jul 1993 and Mar 2020. Consumer price index acquired from International Financial Statistics is used to calculate real prices and earnings. Following the existing literature, I proxy investor sentiment using Consumer Confidence Index (CCI) from OECD database. The correlations of real market indexes and sentiment indexes in three markets are calculated and presented in Table 4.1. The CCIs in all three countries have high correlation with real stock prices, which could provide a preliminary evidence that sentiment plays a role in asset pricing for both domestic and foreign markets. The table also presents high correlation between US and UK stock prices and US and UK sentiment values, which suggest possible price co-movement and sentiment spillover for these two markets. The correlation coefficient between US and UK sentiment indexes is 0.74, and the high correlation indicates strong transmitting channel through which in-

Figure 4.1: Sentiment indexes for US and UK markets

formation or sentiment can work on both markets. To show the sentiment co-movement between US and UK, I plot the CCIs for these two countries from year 1993 in Figure 4.1. From the graph, we can see the co-movement of these two sentiment indexes especially during “turbulent” periods. To investigate time-varying relationship between US and UK sentiment indexes, I simply divide the sample into non-crisis periods and crisis periods. The crisis periods only consider the large market crashes including 2000-2002 dot-com bubble and crash, 2007-2009 global financial crisis and 2015 US stock market crash. The scatter plots are generated to show the time-varying correlation between two sentiment indexes in Figure 4.2. The correlation coefficients are 0.73 and 0.86 for non-crisis periods and crisis periods, respectively. The results suggest sentiment indexes are highly correlated during crisis periods, in turn, more correlated sentiment may cause large market fluctuation.

Fundamental stock prices are derived based on the static Gordon growth model as given below:

$$f_t = \frac{1+g}{r-g} Y_t$$

where Y_t is the earning flows, g is the growth rate of earnings and r is the discount rate. Table 4.2 presents parameter values used to estimate the Gordon growth model.

Figure 4.2: Sentiment correlations for US and UK markets during non-crisis and crisis periods

Table 4.1: Correlation matrix

	US stock	UK stock	Japan stock	US CCI	UK CCI	Japan CCI
US stock	1.000					
UK stock	0.629	1.000				
Japan stock	0.362	0.342	1.000			
US CCI	0.379	0.491	0.550	1.000		
UK CCI	0.312	0.492	0.390	0.740	1.000	
Japan CCI	0.1074	0.099	0.537	0.354	0.220	1.000

Figure 4.3: Real market and fundamental prices for the US, UK and Japan stock markets

Table 4.2: Estimation of the stock fundamental values

	e/p	g	r	δ^*
S&P500	0.23	1.13	1.36	447.41
FTSE100	0.54	0.39	0.93	187.31
TSE 1st	0.47	0.42	0.90	211.80

Notes: Values used for estimating the static Gordon growth model $\delta^* = (1 + g)/(r - g)$: e/p is the average earnings yield E_t/P_{t-1} , g denotes the average growth rate of earnings, $r = e/p + g$ is the discount rate calculated as sum of risk free rate and required risk premium. I use monthly data from Jun 1980 to Dec 2019 for estimation.

4.4 Estimation method

The main concern of this section is to estimate the two-market model. There is no straightforward way to estimate heterogeneous agent models because of large number of unknown parameters. Furthermore, the nonlinearity of the HAMs increases the difficulty of empirical validation. Existing literature provides variety of methods for HAM estimation, such as maximum likelihood, nonlinear least square and moment-based estimations. For the maximum likelihood method, it will only be possible if the HAM can be represented by a reduced form equation or a system of equations. The standard ML estimation has very limited power to deal with nonlinear models. Meanwhile, the crucial full maximum likelihood for models with heterogeneous agents require availability of closed-form solutions for transient density of the process. As most of HAMs are very complicated, such information will hardly ever be available. One way to solve this problem is to use the numerical and simulated approaches to approximate the transient density used in the likelihood function. As shown in Kukacka and Barunik (2017), the simulated maximum likelihood estimation is reliable in HAMs. Other popular tools to estimate complex models are the Generalized Method of Moments (GMM) and Simulated Method of Moments (SMM). These moments-based approaches have a long legacy of applications in economics and finance, but they have limitations. One of these is the lack of continuity of many moments when varying certain parameters. Most studies on moments-based estimation use the statistical moments of the underlying data and stylized facts in financial market, but few of them include other market statistics such as volume. Moments-based

estimation might be biased if moments are not continuous and lack of important information. Instead of using frequentist methods to estimate two-market model, I adopt a Bayesian approach. More specifically, I relate the nonlinear dynamic model to the framework of state-space models, so the estimation of HAMs could benefit substantially from the well-developed toolbox for such models. Comparing with moment-based estimators, Bayesian approaches with Kalman filter do not only allow inference on model parameters, but also filter off time variation of unobserved state variables which are often at the center of interest in HAMs, such as agents' beliefs, opinions or attitudes. To my best knowledge, very few HAMs have been estimated by Bayesian approaches, with the exception of papers by Grazzini et al. (2017) and Lux (2018, 2020). In their papers, all implementations of estimators are based on an approximation of the likelihood using the concept of a particle filter (or sequential Monte Carlo method). However, particle filter approach requires a high number of simulations and is computationally expensive. To overcome this drawback, I use a computational efficient extended Kalman filter in my estimation.

Since HAM is nested as a particular case of state-space model, I provide a brief introduction of this model then explain methods for parameter estimation via extended Kalman filter. A state-space model can be defined by the stochastic evolution in time a vector of states X_t , and the vector of observable measurements Y_t . In a general case, a state-space model can be written as

$$\text{State: } X_t = F(X_{t-1}) + V_t \quad (4.27)$$

$$\text{Measurement: } Y_t = G(X_t) + W_t \quad (4.28)$$

where V_t and W_t are noise processes for state equation and measurement equation, respectively. $F(\cdot)$ and $G(\cdot)$ are stochastic processes. If Equations (4.27) and (4.28) are linear equation systems with Gaussian noises, the optimal approach to estimate parameters and filter unobserved variables is the well-known Kalman filter. For nonlinear systems with Gaussian noises, various extensions like extended Kalman filter and unscented Kalman filter could be used (see e.g. Hartikainen et al. 2011). To estimate the two-market HAM,

I convert HAM to state-space model by considering asset price p_t as observed variable and x_t, m_t, n_t as state variables. As mentioned in Lux and Zwinkels (2018), HAMs usually have too many free parameters, that could not all be estimated at the same time with the limited information available from the price dynamics. In particular, the coefficient for the market maker μ cannot be estimated directly as it serves as scaling factor in price formation.

Chiarella et al. (2014) provide a indirect numerical optimization method to estimate μ . I instead simply set $\mu = 1$ and further assume $R = 1, \frac{1}{2\alpha\sigma^2} = 1$ without loss of generality. As μ and $\frac{1}{2\alpha\sigma^2}$ only work as scaling factors in the dynamic system, the elimination does not affect the precision of the estimation. For parameter ϕ , I refer to Boyer and Zheng (2009) who suggest 11% investors are foreign investors in US market. I set $\phi = 0.12$ in the model. For the nonlinear estimation by extended Kalman filter, I focus on six crucial parameters for each market. For instance, I estimate the reaction parameters of chartists, fundamentalists, and internationalists, a, b and c , speed of belief switching β , speed of market switching λ , and the noise component in price equation, σ_ε^2 . These parameters can be denoted by a vector $\theta^A = \{a, b, c, \beta^A, \lambda, \sigma_\varepsilon^2\}$ for market A, $\theta^B = \{d, e, f, \beta^B, \lambda, \sigma_\varepsilon^2\}$ for market B. The two-market system in Equations (4.24) to (4.26) can be converted to a reduced-form system, which is written as

$$\begin{cases} x_{t+1}^A = x_t^A + \frac{\mu}{1+\phi} [(a-1)(1+m_t^A)x_t^A + b(1-m_t^A)(x_t^A - x_{t-1}^A) + \phi c(1+n_t)(x_t^A - x_{t-1}^A)] + \varepsilon_{t+1} \\ m_{t+1}^A = \tanh\{\beta(x_{t+1}^A - x_t^A)[(a-1)x_t^A - b(x_t^A - x_{t-1}^A)]\} \\ n_t = \tanh[\frac{\lambda}{2}(s_t^A - s_t^B)] \\ x_{t+1}^B = x_t^B + \frac{\mu}{1+\phi} [(d-1)(1+m_t^B)x_t^B + e(1-m_t^B)(x_t^B - x_{t-1}^B) + \phi f(1-n_t)(x_t^B - x_{t-1}^B)] + \zeta_{t+1} \\ m_{t+1}^B = \tanh\{\beta(x_{t+1}^B - x_t^B)[(d-1)x_t^B - e(x_t^B - x_{t-1}^B)]\} \end{cases} \quad (4.29)$$

For the detailed Bayesian extended Kalman filter estimation process, one can refer to Bar-Shalom et al. (2004) and Chow et al. (2007). I summarize the process to four steps:

- **Step 1:** Nonlinear dynamic functions are linearized or approximated by Taylor

series expansion retaining only the first-order terms.

- **Step 2:** At period t , linearized state-space system is used to predict latent state at period $t + 1$ conditional on the observed measurements up to period t .
- **Step 3:** The prediction is updated based on the measurement equation and the actual measurements from the next time period.
- **Step 4:** These predictions and errors are used to construct a log-likelihood function known as the *prediction error decomposition function*. This function is optimized to yield maximum-likelihood estimates of all the parameters in the free parameter vector θ .

Parameter estimation is usually sensitive to initial values. To assure optimization procedure in **Step 4** does not converge to a local optimum, I undertake the following procedures by following Chiarella et al. (2014). First, I estimate a static version of one market model without switching between fundamentalists and chartists and switching of markets. The static model can be estimated by OLS without issue of initial conditions. Second, I perform a grid-search over different initial values for reaction parameters of fundamentalists and chartists as well as intensity of market switching parameter λ .

The next section presents estimation results of the model by extended Kalman filter approach.

4.5 Main Results

This section presents the estimation results of two-market HAM. The estimation method is the Bayesian approach with extended Kalman filter as outlined in Section 4.4. I use US and UK stock and sentiment data to conduct estimations to investigate possible behavioral heterogeneity and market switching. The estimation is conducted country by country with considering the link of markets. In the estimations, I focus on two cases: with and without market switching mechanism. By doing so, I attempt to examine the role of internationalists in the market, i.e. whether they are the significant source of price

Table 4.3: Estimation results for US and UK stock markets

	Without market switching		With market switching	
	US	UK	US	UK
a(d)	0.997* (0.002)	1.000 (0.016)	0.998* (0.002)	1.000 (0.017)
b(e)	-38.12* (1.065)	54.46* (28.43)	-40.99*** (0.297)	67.85*** (5.392)
c(f)	-2.728*** (0.053)	6.696 (13.42)	152.1*** (0.968)	120.1** (40.72)
β	98.6*** (3.196)	3.305 (4.855)	177*** (0.659)	2.555 (2.589)
λ			1.986*** (0.014)	
σ	3.59e-7** (1.341e-7)	0.003 (0.003)	6.49e-4*** (4.061e-5)	3.147e-10 (7.687e-9)
AIC	-1621.14	-629.00	-1074.81	-637.17
Obs	316	316	316	316

Note. This table presents the estimated coefficients and standard errors of the system in Equation (4.29), without(left two columns) and with (right two columns) market switching mechanism, for US and UK. *, **, *** represents significance at the 10%, 5% and 1% level, respectively. AIC measures goodness of fit of likelihood function; Obs denotes number of observations.

fluctuations in both markets. The characteristics and evolvement of different agents' weights $\omega_{h,t}$ are investigated with real stock price in US and UK markets.

4.5.1 Estimation results

Table 4.3 shows the estimation results for US and UK markets. The left two columns exhibit estimation results by model without market switching mechanism. In terms of model, without switching means $\lambda = 0$ and sentiment is not included in the model. With switching implies the market switching parameter λ is estimated in the two-market system. As the model is estimated country by country, the estimated λ is directly used in the second estimation.

The results of model estimation without market switching are qualitatively different with market switching model. Concentrating on the non-switching case first, three types of agents significantly present in US market. The sign of fundamentalists' coefficient in US market is positive and its value is less than one, which imply stabilizing role of fundamentalists. The sign of chartists coefficient is negative for US market, indicating

contrarian traders. The large magnitude of the coefficient implies the price changes are mainly driven by these technical traders. The internationalists are significant traders in US market, and they tend to adopt contrarian strategies with negative extrapolation rate. The magnitude of internationalists' extrapolation rate is smaller than that of chartists, which suggests internationalists are not so powerful in stock markets and they are not the determining force of price fluctuation. Belief switching behavior exists between fundamentalists and chartists as β is significant. For UK market, only chartists coefficient is significant. Negative sign suggests momentum traders dominate US stock market. The belief switching between different types of investors does not exist based on the estimation. Model without market switching fails to detect the behavioral heterogeneity in UK market, which also suggests that it is especially the market switching that is important for model fit.

The results of model estimation without market switching are shown in the left panel of Table 4.3. I observe significant evidence for existence of all three types of agent in US market. The sign of fundamentalists' coefficient is positive and its value is less than one. Chartists still adopt contrarian strategies, while internationalists change to momentum strategies instead of contrarian strategies. Momentum rules are more popular in international stock markets. Given sign and magnitude of intensity of choice, the belief switching mechanisms operate as a positive rule in US markets, and fundamentalists and chartists are more flexible to switch toward the more profitable strategy. Comparing to β , market switching coefficient λ has a smaller but significant value in the estimation. It further provides evidence of market switching behavior of internationalists, but the switching speed is relative low. It may be caused by the information asymmetry, additional cost, regulation and other investment barriers in the international investment. In UK market, chartists' dominance still exists with momentum rules and internationalists prefer trend-following strategies in international investment.

Although the estimation results for model without market switching are comparable to the results of market switching model, but the coefficients of chartists and internationalists are more significant in market switching model. The AIC is also smaller in market

switching case for UK. The number of estimated coefficients are different for US case, so it makes no sense to compare AICs for two estimations. Generally, the market switching model is relative strong in terms of number of significant coefficient and fitness of estimation.

4.5.2 Switching between strategies

To get a better understanding of belief switching and market switching mechanisms of the model, I investigate the estimated weights of agents and their characteristics. Figure 4.4 displays the evolution of m_t versus real stock price and distribution of estimated weight m_t in US and UK markets. The left hand figure clearly indicates an intensive belief switching between fundamentalists and chartists in US market, but the switching is not so aggressive in UK market. The figures also indicate strong relation between stock price and m_t especially during bubble and crash periods. As m_t measures fraction difference of fundamentalists and chartists, m_t closing to one means fundamentalists dominate the market and m_t approaching zero indicates chartism is dominant. In US market, investors either tend to be fully fundamentalists or fully chartists, and chartists are dominating more periods than fundamentalists for the whole sample period. While these two types of agent seem to split the market more evenly in UK market. So far, I have not discussed weights of internationalists. As the size of internationalists is assumed to be fixed as 11% of total population of fundamentalists and chartists groups, I separately investigate the weights dynamics and features of internationalists in next subsection.

4.5.3 Switching between markets

The evolvments of internationalist market fractions and stock prices for two-market system are displayed in Figure 4.5. The wights of internationalists refer to the fractions of internationalists in the whole internationalists group. Hence, we have $\omega_t^A + \omega_t^B = 1$. As shown in the diagram, internationalists are frequently switching their investing markets. This is mainly because of the close linkages between US and UK financial markets. Fraser and Oyefeso (2005) argue that the real stock prices in US and UK are

Figure 4.4: Estimated weights and real stock prices in US and UK markets

Figure 4.5: Stock prices and estimated internationalists' weights in US and UK stock markets

linked by a common stochastic trend and therefore are perfectly correlated in the long-term. The correlation coefficient between US and UK stock prices is 0.629 as shown in Table 4.1, which suggests the high co-movement of US and UK stock markets. The intensive switching in US and UK markets might be also explained by the similar culture, language and economic policies. Market mood or sentiment is easier to be perceived by international investors in these two countries. The fraction of internationalists is high during bubble periods and low during crisis periods in US market, while it is exactly opposite in UK market. As shown in Figure 4.1, sentiment in UK market is a little bit higher than that in US market during crisis periods. As most of the financial crises started from US market, UK sentiment might be less affected. Internationalists will switch to UK market which has relative high sentiment and low risk. The market switching behavior of internationalists could lead to much severer crisis in US market, as they may sell off the US equities to reduce risk during crises. However, UK may also experience crisis due to sentiment contagion from US crisis.

4.6 Robustness check

To check the robustness of the estimation results in Section 4.5, I further estimate two-market model by employing US and Japan data. Following the same logic as for US-UK analysis, I show the results of model estimations with and without market switching in

Table 4.4: Estimation results for US and Japan stock markets

	Without market switching		With market switching	
	US	Japan	US	Japan
a(d)	0.999 (0.002)	0.971*** (3.522e-4)	0.983*** (2.855e-5)	0.973*** (0.001)
b(e)	39.1*** (2.292)	26.52*** (0.008)	41.15*** (0.001)	23.159*** (0.01)
c(f)	-148.5*** (19.86)	3.125*** (3.945e-4)	-2.632*** (9.642e-5)	1.561*** (0.001)
β	193.7*** (31.46)	65.48*** (0.01)	63.7*** (0.002)	64.472*** (0.059)
λ			2.292*** (9.244e-5)	
σ	0.001** (1.014e-4)	0.47*** (5.008e-5)	0.003*** (6.9e-7)	0.46*** (2.784e-4)
AIC	-1616.5	944.64	-1334.14	944.87
Obs	452	452	452	452

Note. This table presents the estimated coefficients and standard errors of the system in Equation (4.29), without(left two columns) and with (right two columns) market switching mechanism, for US and Japan. *, **, *** represents significance at the 10%, 5% and 1% level, respectively. AIC measures goodness of fit of likelihood function; Obs denotes number of observations

Table 4.4. Estimation results of market switching model are shown in the right panel. Three types of agents significantly present in both US and Japan markets. Fundamentalists are found in both market with coefficients less than one. The coefficients for chartists in two markets are both positive, suggesting chartists are mainly momentum traders during the sample period. The internationalists are also significant traders in both US and Japan markets. In US market, internationalists tend to adopt contrarian strategies with negative extrapolation rate, while momentum strategies are used by them in Japan stock market. As intensities of choice in two markets are very close, the belief switching mechanisms operate similarly in both markets, and fundamentalists and chartists are more flexible to switch toward the more profitable strategy. The significant λ indicates the market switching behavior of internationalists. The estimation results of model without sentiment is presented in left panel of the diagram. The model fails to detect heterogeneity of investors' beliefs in US market.

To further explore the belief switching and market switching behaviors in US and Japan markets, I plot the similar figures as in Section 4.5. Figure 4.6 displays the evolu-

tion of m_t versus real stock price and distribution of estimated weight m_t in US and Japan markets. In the Japan market, m_t appears to be driven by extremes in that the weight is often either equal to negative one or to one, chartism seems prevailing in the market during market crashes, for instance, m_t hovers around -1 during 2003 dot-com bust and 2008 global financial crisis. The histograms on the right sides also indicate the dominance of chartism. The fractions of internationalists in US-Japan system are shown in Figure 4.7. The internationalists' behavior is a little bit different in US-Japan system. Market switching between US and Japan markets is more smoothing comparing with belief switching. There is a strong positive relation between stock prices and internationalists' weights. For example, internationalists are more concentrating on Japanese stock market from 1987 to 1992. This period is exactly corresponding to the Japanese asset price bubble in the history. At the same time, US stock market is experiencing the crash period related to Black Monday in 1987. Japanese assets are more attractive than US assets, and internationalists are fully switching to Japan market. While this Japanese bubble bust in 1992 following by "Lost Decade" of Japanese economy. The Asian financial crisis 1997 further hit financial markets in Japan, so internationalists switch back to US market during this time period. After dot-com bubble in 2003, international investors head back to Japan again because of recovering Japanese economy and stock market boom. During the global financial crisis 2008-09, internationalists switch aggressively between US and Japan markets to earn the speculative profits. After crisis, there is a prolonged stock market boom in US, so internationalists tend to put more stake in US market. In general, internationalists in US-Japan system are more prone to flock to markets with bubbles and higher sentiment. These "hot money" from international investors may not play the determinant role in market crash, but they indeed increase the market volatility and accelerate the boom-bust cycles of markets.

Figure 4.6: Estimated weights and real stock prices in Japan market

Figure 4.7: Stock prices and estimated internationalists' weights in US and Japan stock markets

4.7 Conclusion

In this chapter, I mainly develop a two-market HAM with sentiment. A new type of agent internationalist is introduced in the model, and they can optionally switch investment market based on two markets' sentiment levels. To this end, I estimate the two-market dynamic model by applying a novel Bayesian approach with extended Kalman filter. By estimating market switching model using US and UK stock market data and sentiment data, I find the existence of behavior heterogeneity in international financial markets. Fundamentalists and chartists switch their beliefs less aggressively in US and UK stock markets comparing with US and Japan markets, while their fractions are more evenly distributed in UK market. Internationalists are assumed to adopt chartism strategy, and the market switching behavior of internationals is found in the estimation. By adding the market switching mechanism, this model is able to detect more significant investor heterogeneity. Through the market switching, internationalists are also shown to be related to market instability, prices co-movement and boom-and-bust cycles in stock markets.

Future research in this field can spread into different directions. First, the two-market model can be extended to multiple-market model with the similar framework. Second, extended Kalman filter approach can be improved or replaced by other filter method like particle filter to estimation multiple market HAM. Third, a new sentiment index specialized in international financial markets can be constructed to better capture the inducement of market switching behavior.

Chapter 5

Investor Heterogeneity and Trading Volume in Stock Market

5.1 Introduction

In the analysis of financial market, much attention has been drawn to the study of asset prices. Another important indicator of financial activities, trading volume, is ignored in many studies. Volume represents the total amount of transactions in a risky asset or entire market during a specific period of time. In the technical analysis, volume is used to measure the relative worth of market move. A high volume during the price move always implies a strong market. The information roles of volume have been discovered by practitioners. As the old Wall Street adage asserts, “it takes the volume to move prices”.

The characteristics of trading volume have been commonly found in financial markets. To better understand these characteristics, I investigate stylized facts of trading volume first. Some of these stylized facts can only be observed visually, and others can be examined statistically. One visualized stylized fact concerning volume is that a dramatic change in prices is always accompanied by significant volumes. This feature is frequently found in crisis periods. Figure 5.1 is the illustration of this phenomenon in 2008 financial crisis. We can clearly see that the Dow Jones Index experienced a sharp

Figure 5.1: Dow Jones Index and trading volume during 2008 Global Financial Crisis

decline in the mid-2008 and fell to the bottom subsequently in the early 2009. At the same time the volume also surged and hit to an unprecedented level. Other similar examples are Black Monday in 1987 and Asian Financial crisis in 1997. Another visualized stylized fact is that trading volume is an important confirmation signal of price action in technical analysis. For example, if investors observe a sudden increase in trading volume, they can confirm the breakout of price to the trend-line. (See Murphy, 1999 and Bulkowski, 2011). Figure 5.2 illustrates this fact in shares of Google. Its stock price fluctuated below the resistance line (around 575) for a long time until suddenly jumped up to around 660 in July 17, 2015. Meanwhile, the volume was almost five times the average value. After that the price moved up and down above the previous resistance level. The enormous volume can be recognized as the signal to confirm that price breaks out the old regime and enters a new one.

In addition, the statistical relationship between price and volume has received considerable attention in a large amount of empirical research. Karpoff (1987) points out that volume could correlate positively with the elements of both the absolute change of price and the price change *per se*. Although an early empirical study by Granger and Mor-

Figure 5.2: Google stock and trading volume (2014-2015)

genstern (1963) fails to find a correlation between price index and aggregated volume by using US stock market data. Succeeding studies in this direction have proven the positive correlation between these two terms. Since 1990s, the dynamic (causal) correlation between price and volume has drawn a lot of attention. Vector autoregressive (VAR) models and Granger causality tests are used to investigate the price-volume relation. Saatcioglu and Starks (1998) find evidence that volume leads to price change. Statman et al. (2006) employ monthly data from NYSE and find the positive relation between trading activities and lagged return. By using S&P 500 data from 1973 to 2008, Chen (2012) finds no causality from trading volume to stock return, but return Granger causes volume. Apart from the linear model, Hiemstra and Jones (1994) apply nonlinear Granger causality tests to investigate the price-volume relationship in the US market. They find evidence of significant bidirectional nonlinear causality between returns and volumes. Moreover, Diks and Panchenko (2006) modify the method of nonlinear causality test to improve the performance, and find weak nonlinear causality between returns and volumes.

Although much literature has proven that Granger causality exist between volumes and returns, the significance and directions are still controversial. To have a clearer view

on price-volume relation, I reexamine the linear and nonlinear Granger causality between returns and volume changes. I use both S&P 500 and Dow Jones index daily data, the range of the sample is from 1/1/2010 to 12/31/2016. The methods of linear and nonlinear Granger causality tests are provided in Appendix A and the results are reported in Table 5.7 and Table 5.8, respectively. I find evidence that returns Granger cause volumes in the linear test, which is consistent with the results in Chen (2012). For the nonlinear test, I find weak Granger causality between specific lagged returns and volume changes in S&P 500 data, but fail to find the causality relation in Dow Jones index.

Besides the price-volume casual relation, it is also worth investigating the correlation between volume and volatility. Clark (1973) shows that the squared price changes are positively related to volumes. Daigler and Wiley (1999) find positive volume and volatility relation in the future market, and highlight that this relation is driven by the existence of different types of investors. In practice, the positive relation between volume and volatility can be easily found in stock markets. A recent example is the obvious positive correlation of volatility and volume in US stock market during 2008 financial crisis. In practice, Chicago Board Options Exchange (CBOE) Volatility Index is frequently used to measure the stock market volatility. As shown in Figure 5.3, the fluctuation of Volatility Index (VIX) is followed by the similar movement of S&P 500 trading volume. Just by eye-balling the trends of trading volumes and VIX in Figure 5.3, there appears to be a strong inter-temporal correlation. To further check this correlation, I plot the correlation chart and present it in Appendix C. I find a strong positive relation between volume and VIX with a significant coefficient correlation 0.725.

While ample studies focus on the detection of price-volume relationship, very few of them have explained why such patterns exist and how the relation evolves in the market. Even some theoretical studies attempt to explain this relationship, few models are able to address all the stylized facts as mentioned above. Due to the complexity of price-volume dynamic, nonlinear dynamic models have been drawn attention in the past decades. Granger (2014) argues that univariate and multivariate nonlinear models represent the proper way to model a real world that is almost certainly nonlinear. Since the

Figure 5.3: S&P 500 volume and the VIX

sudden crisis in 1987, many studies have worked on the nonlinear models to detect and explain the well known stylized facts in financial markets. At the same time, behavioral finance with bounded rationality assumption rises to compete with traditional financial models with Efficient Market Hypothesis (EMH). As argued by Thaler (2005), behavior finance is a new approach to explain some abnormal regularities of financial markets, in response to the difficulties faced by traditional paradigm. The heterogeneous agent model (HAM) is one of the nonlinear behavioral models that has been widely used to explain complexity of financial markets, and it has been proven to be very powerful on asset pricing, replicating the stylized facts and explaining different features of financial market, such as crises, crashes and bubbles.

In this chapter, I aim to build a HAM with trading volume, and mainly investigate how the prices, volumes and beliefs co-evolves in the stock market within fundamentalist-chartist framework. To evaluate the fitness of the model to real financial markets, I examine its capability of replicating the well known stylized facts in the market as in Chapter 2. As shown in the current literature, most of the HAMs have the ability to explain stylized facts concerning price or return. These stylized facts and strand of literature include: non-stationary price and stationary return, fat tail, volatility clustering, leverage effect, asymmetric returns and the power law of return. Regrettably, little attention has been paid so far to volume except Brock and LeBaron (1995) and Chen and Liao (2005).

Brock and LeBaron (1995) build an adaptive beliefs model, which is able to reproduce the slowly decaying autocorrelation function of volatility and trading volume, but other stylized facts and co-evolution of price and volume have not been fully investigated. Chen and Liao (2005) develop an agent-based stock markets (ABMs) model to simulate the price-volume series. They also find significant and nonlinear Granger causality relation between the price and volume, but the simulation results are unpersuasive.

To fill the gap of HAM on volume, I develop a simple HAM with trading volume. Comparing to previous HAMs, this model has greater potential to simulate the features of financial market. The contribution of this chapter is threefold: First, the model is able to generate most of stylized facts both on price and volume. Second, co-evolution of prices, volumes and beliefs can be simulated by this model. Third, the model is able to explain the formation of different chart patterns, bubbles and crises, which provide theoretical support for technical analysis.

The rest of the chapter is organized as follows. Section 5.2 describes the model. Section 5.3 presents the simulation results and verifies model's capability to explain market dynamics, including stylized facts, price dynamics, volume dynamics, bubbles and crises formation and the co-evolution of price changes, volumes and beliefs. Section 5.4 concludes.

5.2 Model

5.2.1 Heterogeneous investors

Following the standard HAM literature, I consider a market with only one risky asset and two types of agents, namely fundamentalists and chartists. I assume that these agents are boundedly rational and hold heterogeneous beliefs on future price trend. While fundamentalists form their expectations on future price and adopt trading strategies based on market fundamental factors, such as dividends, profits and economic growth, chartists focus on technical analysis. They use historical price trends or chart patterns as essential elements to form their expectations and hence trading strategies. A market maker who

adjusts market price in response to aggregate demand from fundamentalists and chartist is also included in the model.

Fundamentalists Fundamentalists are assumed to have full information about fundamental value μ_t . They believe that asset price will not deviate from this value for long, and they expect price to fluctuate within a reasonable zone of (m_t, M_t) due to some short term external disturbances. The price will eventually converge to its fundamental value. As a result, fundamentalists buy asset when the price is below its fundamental value and sell otherwise. Accordingly, their demand function is written as:

$$D_{f,t} = \begin{cases} (\mu_t - p_{t-1})A(\mu_t, p_{t-1}), & p_t \in \mathbb{Q} \\ 0, & p_t \notin \mathbb{Q} \end{cases} \quad (5.1)$$

where $\mathbb{Q} = [m_t, M_t]$ is the reasonable price zone, m_t and M_t are the minimum and maximum of the price boundaries, respectively and $A(\cdot)$ is a chance function with respect to μ_t and p_{t-1} . This function demonstrates the psychological behavior of investors as in Day and Huang (1990). It shows that when price is closer to the upper boundary of the price zone M_t , the chance of losing the existing gains increases. Likewise, when price is closer to the lower boundary of the price zone m_t , the chance of missing a capital gain by failing to buy is high¹.

Fundamentalists are assumed to update information Ω_t in each period to obtain more accurate fundamental value of the risky asset. Instead of using a constant fundamental value, I assume that they adjust the value according to the following rule:

$$\mu_{t+1} = g(t)\mu_t \quad (5.2)$$

¹More details about the chance function can be found in Appendix B

where $g(t)$ is the simulated business cycle such that:

$$g(t) = \begin{cases} g & t \in [4(i-1) \cdot s, (4i-1)s) \\ -\frac{g}{2} & t \in [(4i-1)s, 4i \cdot s) \end{cases} \quad i = 1, 2, 3 \dots \quad (5.3)$$

where g is the economic growth rate. I assume that a business cycle consists of $4s$ periods. During economic boom, the economy expands at an average growth rate of g percent during the first $3s$ periods. Then a recession kicks in and lasts for s periods with a growth rate of $-g/2$. Although it is not a rigorous assumption, with this setting up, macroeconomics factors are able to be incorporated into the model and hence improves the ability of the model in explain the relationship between business cycle and stock market cycle.

Chartists Unlike fundamentalists who make their trading decisions based on fundamental values, chartists trade mostly based on the past price trend of the risky asset. As a result, they estimate the short-term fundamental value v_t based on historical price and then extrapolate the next period market price by the deviation of price from its short-term fundamental value. This deviation is also known as estimation bias. Accordingly, chartists form their expectation based on the following rule:

$$E_{c,t-1}(p_t) = p_{t-1} + \beta(p_{t-1} - v_{t-1}) \quad (5.4)$$

where $\beta \neq 0$ measures the sensitivity of price expectation to the latest bias which is denoted by $(p_{t-1} - v_{t-1})$. Thus, the demand function of chartists is given by:

$$D_{c,t} = \eta[E_{c,t-1}(p_t) - p_{t-1}] = \eta\beta(p_{t-1} - v_{t-1}) \quad (5.5)$$

where η denotes the elasticity of the demand function of chartists. For simplicity, I further assume that all chartists share the same belief on the short-term fundamental value v_t . However, due to opinion differences, chartists may choose to follow or against the price trend. Such differences between chartists are captured by the sign of β . For those

who believe that the market price p_t will deviate further away from p_{t-1} , $\beta = \beta_1 > 0$ and the price trend will persist. They are said to be trend followers (momentum traders) who hold on to positive bias. Accordingly, their excess demand can be expressed as:

$$D_{c,t}^1 = \eta_1 \beta_1 (p_{t-1} - v_{t-1}) \quad (5.6)$$

This equation means that when $p_{t-1} > v_{t-1}$, trend followers with $\beta_1 > 0$ would believe that the price trend will continue and hence buy in ($D_{c,t}^1 > 0$).

On the contrary, for chartists who believe that the price trend will reverse in the next period, then $\beta = \beta_2 < 0$. These chartists are known as contrarian traders as they take negative bias in making their trading decision. As a result, their excess demand function is taken as:

$$D_{c,t}^2 = \eta_2 \beta_2 (p_{t-1} - v_{t-1}) \quad (5.7)$$

Thus, when $p_{t-1} > v_{t-1}$, contrarians with $\beta_2 < 0$ would think that the price trend will reverse and choose to sell ($D_{c,t}^2 < 0$).

Let's look more closely at the short-term asset value, v_t . I assume that all chartists, both trend followers and contrarians, hold on to an identical short-term assets value. As in Huang et al. (2010) and Huang and Zheng (2012), I assume that chartists adopt the adaptive belief mechanism where they update their expectation on short-term asset value. They believe in support and resistance levels which are derived from common rules of technical analysis. I assume that chartists divide price domain $\mathbb{P} = [p_{min}, p_{max}]$ into n regimes such that:

$$\mathbb{P} = \bigcup_{j=1}^n \mathbb{P}_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \dots \cup [\bar{p}_{n-1}, \bar{p}_n] \quad (5.8)$$

where $\bar{p}_j$ for $j = 1, 2, \dots, n$ represents the different support and resistance levels chartists set.

The short-term fundamental asset value can be simply extrapolated as the average of

the top and the bottom threshold prices:

$$v_t = (\bar{p}_{j-1} + \bar{p}_j)/2 \quad \text{if } p_t \in [\bar{p}_{j-1}, \bar{p}_j) \text{ and } j = 1, 2, \dots, n \quad (5.9)$$

When price fluctuates within the current regime, there are enough reasons for chartists to believe that the short-term fundamental asset value will remain unchanged. However, once price breaks through either the support line or the resistance line, chartists will adjust their expectation on the short-term fundamental asset value according to Equation (5.9). This regime switching phenomenon is commonly found in stock market with chartist's beliefs evolve with regime switching. According to Huang et al. (2010), assuming that $\bar{p}_j - \bar{p}_{j-1} = \lambda$, the short-term fundamental value for each period is estimated as:

$$v_t = (\lfloor p_t/\lambda \rfloor + \lceil p_t/\lambda \rceil) \cdot \frac{\lambda}{2} \quad \text{if } p_t \in [\bar{p}_{j-1}, \bar{p}_j) \text{ and } j = 1, 2, \dots, n \quad (5.10)$$

5.2.2 Evolutionary belief switch

In the model, I assume that agents can switch their strategies and the switching behavior is driven by discounted expected profit which is denoted by π . In each period, market then updates according to the switching between strategies by different types of agents. Specifically, the discounted expected profit functions of fundamentalist and chartists (trend followers and contrarians) are written as:

$$\begin{aligned} \pi_{c,t}^1 &= s(p_t) |\beta_1(p_t - v_t) - rp_t| \\ \pi_{c,t}^2 &= s(p_t) |\beta_2(p_t - v_t) - rp_t| \\ \pi_{f,t} &= s(p_t) \left| (u_t^f - (1+r)p_t) \right| - C \end{aligned} \quad (5.11)$$

where $\pi_{c,t}^1$, $\pi_{c,t}^2$ and $\pi_{f,t}$ are expected profit of trend followers, contrarians and fundamentalists, respectively. r is the interest rate between period $t-1$ and period t . C is the information cost which fundamentalists have to incur to acquire additional information. $s(p_t)$ is the discount factor. For chartists, as they are only interested in one-period returns and capitalize gains or losses immediately, I assume that they have a discount factor of 1.

While fundamentalists value assets according to the cash flows that the asset is expected to generate. These agents can be thought of as following a buy-and-hold strategy and they take more time to capitalize their gains or losses, so the discount factor for them is assumed in the form of $s(p_t) = |(u_{f,t} - p_t)/3u_{f,t}|$.

I use $\omega_{i,t}$ to denote market fraction of the three different types of investors. The fractions of the three groups vary endogenously over time according to choice model with multinomial logit probabilities (Brock and Hommes, 1998):

$$\omega_{i,t}(p_t) = \frac{\exp(\rho \pi_{i,t}(p_t))}{\sum_k \exp(\rho \pi_{k,t}(p_t))} \quad (5.12)$$

where the parameter ρ is the intensity of choice which measures the speed of transition between different beliefs. A high value of ρ means that more investors will switch between strategies. $\omega_{i,t}$ is always positive which implies that not all agents are going for strategy with high discounted expected profit.

5.2.3 Price determination

Another participant is market maker who mediates transactions in the market to provide liquidity. The market marker collects orders from all traders, namely trend followers, contrarians and fundamentalists, then sets the price. The market maker supplies from his inventory when there is a positive excess demand and do the reverse otherwise. In each period, market price is updated adaptively according to the following adjustment rule:

$$p_t = p_{t-1} + \gamma(\omega_{1,t}D_{c,t}^1 + \omega_{2,t}D_{c,t}^2 + \omega_{3,t}D_{f,t}) \quad (5.13)$$

where γ is the speed of price adjustment to excess demand.

5.2.4 Volume formation

Besides price, volume is another important indicator in stock market. The total number of all shares transacted in the market is known as trading volume. As mentioned by Grossman and Miller (1988), in the market maker framework, market maker will provide the

liquidity services and equate the market demand and supply using the inventory in hand. I assume that chartists with different expectation on future price trade with each other first, then together they generate either a positive or a negative excess demand. Compared to chartists, fundamentalists hold totally different strategy and they may have either a positive or a negative excess demand. Putting fundamentalist and chartists together, we can foresee two scenarios. First, if fundamentalists and chartists share the same opinion on the future price trend, then both groups should trade directly with market maker. Trading volume is the absolute value of aggregate demand. Second, if fundamentalists and chartists hold different opinion on the future price trend, they will trade with each other first. Market maker will only come in to satisfy the remaining excess demand. In this case, trading volume is the maximum of the absolute value of the excess demand for each group. As a result, trading volume is defined as:

$$V(p) = \begin{cases} \min(|\tilde{D}_{c,t}^1|, |\tilde{D}_{c,t}^2|) + |\tilde{D}_{f,t} + \tilde{D}_{c,t}^1 + \tilde{D}_{c,t}^2| & \text{if } \tilde{D}_{f,t} \cdot (\tilde{D}_{c,t}^1 + \tilde{D}_{c,t}^2) > 0 \\ \min(|\tilde{D}_{c,t}^1|, |\tilde{D}_{c,t}^2|) + \max(|\tilde{D}_{f,t}|, |\tilde{D}_{c,t}^1 + \tilde{D}_{c,t}^2|) & \text{if } \tilde{D}_{f,t} \cdot (\tilde{D}_{c,t}^1 + \tilde{D}_{c,t}^2) < 0 \end{cases} \quad (5.14)$$

where $\tilde{D}_{c,t}^1$, $\tilde{D}_{c,t}^2$ and $\tilde{D}_{f,t}$ are the weighted excess demand for trend followers, contrarians and fundamentalists, respectively. $\tilde{D}_{c,t}^1 = \omega_{1,t}D_{c,t}^1$, $\tilde{D}_{c,t}^2 = \omega_{2,t}D_{c,t}^2$ and $\tilde{D}_{f,t} = \omega_{3,t}D_{f,t}$.

5.3 Simulation results

In this section, I discuss simulation results from the HAM framework with an additional feature of trading volume that I have elaborated earlier. By putting together prices, trading volumes and agents' beliefs, I am able to generate rich simulation results from the model. I first evaluate the fitness of the model in capturing the complex dynamics of stock market by exploring the power of the model in reproducing some of the well-documented stylized facts that were mentioned in Section 5.1, keeping in mind that no single model can, or should, fit most aspects of the data but recognize instead that some consistency can still be useful. Besides the price related stylized facts, I also investigate whether the simulated dynamics of the model are consistent with the stylized facts on trading vol-

ume. Specifically I look at (1) the correlation between volume and volatility, (2) Granger causality between volume and returns, (3) information role of volume and (4) volume and chart patterns. I then examine both price and trading volume dynamics and see how these variables co-evolve with agents' beliefs under different chart patterns and financial crises.

In order to maintain consistency and unity, I simulate price and volume series using a uniform set of parameters throughout the paper unless otherwise specified. Specifically, I set $\mu_1 = 50$, $d_1 = d_2 = -0.3$, $k = 2$, $s = 25$, $\lambda = 13.1787$, $C = 3$, $r = 10^{-4}$, $g = 0.008$, $a = 1$, $\beta_1 = 1.2$, $\beta_2 = -0.7$, $\eta_1 = 0.833$, $\eta_2 = 3.214$, $\rho = 0.9$ and $\gamma = 1$. To see how fit the simulated data is in matching the actual financial time series in terms of its statistical and qualitative properties, I take the daily price and trading volume of S&P 500 and Dow Jones Index as benchmark. The time range of these dataset is from 1 January 2010 to 31 December 2016. Stock returns are expressed in percentage and hence taken as $\log(p_t/p_{t-1}) \times 100$ and percentage change in trading volume is taken as $\log(v_t/v_{t-1}) \times 100$. As some stylized facts may be sensitive to the selection of sample period, I also refer to other literature in finance to verify the explanatory power of the model.

Before exploring the simulated stylized facts, let's first look at the simulated price, returns and volume dynamics of the model in Figure 5.4. Generally, the simulated prices are similar to the real market price up-and-down. It's return fluctuates around zero and tend to cluster. The largest positive and negative returns demonstrate different magnitudes, implying that there exists asymmetry in stock returns. Trading volume changes with price and returns and demonstrates occasional spikes which suggest abnormal large trading volume at times.

The top panel of Figure 5.5 displays the simulated prices and market fundamental value which is computed based on Equation (5.2). I simply classify periods with market prices above (below) its fundamental values as bubbles (busts). The remaining panels show the corresponding fractions of agents in the market with ω_1 , ω_2 and ω_3 indicate fraction of trend followers, contrarians and fundamentalists in the market, respectively.

Figure 5.4: Simulated time series of price (top), returns (middle) and volume (bottom)

The value of fraction ranges between 0 and 1. When $\omega_{i,t}$ approaches 1, agents i are said to dominate the market at time t . When market prices deviate away from its fundamental values, we see some endogenous self correction where agents switch between different types of trading strategies.

5.3.1 Price dynamics

The model I set up is a nonlinear model. The nonlinearity in the model is mainly from the setup of the chance function $A(\mu_t, p_{t-1})$ and the way chartists compute the market's short-term fundamental values. The chaotic system developed has the capability of generating relatively rich price dynamic patterns which are commonly observed in stock market. For more insights, I show a typical phase diagram of the simulated prices in Figure 5.6. The 45 degree line shows that $p_t = p_{t-1}$. From the figure, chaotic multi-phase switching of prices is observed. Prices fluctuate across the 45 degree line several times suggest that there exists multiple equilibria. As introduced by Huang and Zheng (2012), these price patterns can be classified into a rising zone and a declining zone. When prices are above the 45 degree line, it would rise in the next period. Likewise, if the prices

Figure 5.5: Simulated prices and fundamental values (top) and agents' fractions (bottom)

are below the 45 degree line, it is expected to fall. By combining the market conditions and investigating further into two step-wise dynamics, Huang and Zheng (2012) further classify the declining zones into sudden, smooth and disturbing declining zones, and discuss the mechanisms and conditions of how prices continue to stay within the same regime or escape from one regime to another.

As bifurcation diagrams are frequently used to illustrate the dynamic properties of a nonlinear system, I also run a number of numerical simulations to illustrate how the local dynamics of the model vary with some important parameters, such as the speed of price adjustment (γ), the intensity of choices (ρ) and other exogenous parameters which include the sensitivity of fundamentalists when price moves close to the boundaries of m_t and M_t , the sensitivity of the price expectation of trend followers to the bias (β_1), the sensitivity of the price expectation of contrarians to the bias (β_2), and information cost of fundamentalists (C). Unless otherwise stated, in all bifurcation diagrams, bifurcation parameter is increased in 2000 discrete steps and other parameters are kept constant.

As discussed earlier, the model has multiple equilibria due to multiple-regimes setup for chartists. Unlike Huang and Zheng (2012) who look at regime-switching dynamics, my discussion focuses on within-regime dynamics. In doing so, I set the short-term fundamental value v_t to 50 and growth rate to 0 in the simulations. I begin by examining

Figure 5.6: The phase diagram of price

the effect of the speed of price adjustment (γ) on the price dynamics when the intensity of choice, $\rho = 0.85$ as in panel (a) and $\rho = 1.85$ as in panel (b), respectively. In panel (a) of Figure 5.7, I set ρ to 0.85 and vary γ from 0 to 3. The bifurcation evolves from a stable steady state to a chaotic price fluctuation when γ increases from 0 to 3. The primary bifurcation toward instability is a period-doubling bifurcation at which the steady state becomes unstable and a stable 2-cycle emerges. This happens when the value of γ is approaching 1.7. When γ continues to increase, further bifurcations occur and the price dynamics become increasingly more complicated. Likewise, in panel (b) of Figure 5.7, I use ρ of 1.85 and vary γ from 0 to 3 to explore the effect of the intensity of choice on the price dynamics. With a larger ρ , the primary bifurcation occurs at a much lower value of γ of around 1. Besides, price fluctuations are more complicated with higher price adjustment speed, i.e. larger value of γ .

Panels (c) and (d) are results from my second analysis where I study the effect of the intensity of choice ρ on the price dynamics when the speed of price adjustment is set at 1 and 2, respectively. The bifurcation in panel (c) is very similar to that of panel (a) which suggests that an increase in the intensity of choice will trigger a rational route to randomness. In panel (d), I repeat these simulations for $\gamma = 2$ and find that the primary bifurcation toward instability occurs at low intensity of choice of 0.7 in panel (d) as

Figure 5.7: Dynamics of the model. (a) Bifurcation diagram for speed of price adjustment with $\rho = 0.85$. (b) Bifurcation diagram for speed of price adjustment with $\rho = 1.85$. (c) Bifurcation diagram for intensity of choice with $\gamma = 1$. (d) Bifurcation diagram for intensity of choice with $\gamma = 2$.

compared to 1.6 in panel (c). The price dynamic shows higher amplitude fluctuation with higher intensity of choice.

The bifurcation diagrams in Figure 5.8 show how the asymptotic dynamics vary with other exogenous parameters including the sensitivity of fundamentalists when price moves close to the boundaries of m_t and M_t (panel (a)), the sensitivity of the price expectation of trend followers to the bias (panel (b)), the sensitivity of the price expectation of contrarians to the bias (panel (c)), and information cost of fundamentalists (panel (d)).

In Figure 5.8(a), the sensitivity of fundamentalists when price moves close to the boundaries (a) is used as a bifurcation parameter, the price dynamics are stable for $a \leq 1.3$. However, when a goes beyond this value, the equilibrium begins to lose its stability

with a stable period-2 cycle. When a increases further, chaos takes place through a cascade of infinite sequence of period-doubling bifurcation. When a approaches 4.8, the price dynamics become stable again and a reversed flip bifurcation is observed.

Figure 5.8(b) shows a similar periodic bifurcation when the sensitivity of the price expectation of trend followers to past estimation bias (β_1) is used. The price dynamics are stable when the value of β_1 is small and cascade of flip bifurcation occurs before it leads to chaotic motion. The dynamics will eventually stabilize when β_2 goes beyond 16. Figure 5.8(c) is the bifurcation diagram when the sensitivity of the price expectation of contrarians to past estimation bias (β_2) is used as the parameter of the analysis. As β_2 holds on to a negative value, β_2 decreases from 0 means that the sensitivity of contrarians to past estimation bias increases. Again we see that when contrarians are more sensitive to past estimation bias, i.e. β_2 is more negative, a period-2 cycle first emerges followed by a period-4 cycle before it generates chaos.

Panel(b) illustrates a similar periodic bifurcation for sensitiveness of trend follower, with the increase in parameter β_1 , cascade of flip bifurcations leads to chaotic motion, and eventually stable price appears in system when β_2 is beyond 16. Panel (c) is the bifurcation diagram for sensitiveness of contrarian β_2 . As β_2 is negative value, β_2 decreases from 0 implies the rise of sensitiveness. When β_2 falls, period-2 cycle emerges and the period-4 cycle occurs for a specific interval of β_2 . Then again the price dynamics turn to 2-cycle and further decrease in β_2 generates chaos. Information cost may affect the model dynamics through the expected profit function of fundamentalist. As shown in panel (d), if information cost C is small then there exists a stable period-2 cycle. If C increases, this cycle becomes unstable and flip bifurcations occur. After infinitely many flip bifurcations the price behaviour becomes chaotic, as C increases. When C further increases after a cascade of period halving bifurcations there exists stable stationary equilibrium.

5.3.2 Stylized facts on price

Unit root As been proven in the finance literature, the series of stock prices are not stationary but returns and volumes are usually stationary. To examine whether the model

(a)

(b)

(c)

(d)

Figure 5.8: Dynamics of the model. (a) Bifurcation diagram for sensitiveness of fundamentalist. (b) Bifurcation diagram for sensitiveness of trend follower. (c) Bifurcation diagram for sensitiveness of contrarian. (d) Bifurcation diagram for information cost of fundamentalist.

Table 5.1: Unit root test

Statistic	Price		Return		Volume	
	ADF	p-value	ADF	p-value	ADF	p-value
S&P 500	-0.661	0.857	-44.010	0.000	-18.742	0.000
Dow Jones Index	-0.675	0.853	-44.037	0.000	-21.444	0.000
My simulation	-2.158	0.222	-37.604	0.000	-31.645	0.000

Notes: ADF critical value are:-3.43(1%), -2.86(5%), -2.57(10%)

is compatible with this stylized fact, I check the unit root of SP&500 Dow Jones Index data as well as the simulated data, and the results are displayed in the Table 5.1. The augmented Dickey-Fuller(ADF) are used to test the existence of unit root. The ADF for three price time series are -0.661,-0.675 and -2.158, which are significantly greater than the critical value at 10% significant level. Their p-values are all zero implying that all three price statistic are unit root process. In the return and volume series, the ADF tests significantly reject the null hypothesis of non-stationary at 1% level for all series. It indicates that the model simulation match the actual financial data well in the terms of stationary for price, return and volume.

Fat tails Fat-tailed distributions of financial asset returns are well documented in empirical studies (see Cont, 2001 and Chakraborti et al., 2011). The fat tail of return suggests that extreme returns appear more frequently than what are predicted by the normal distribution. To investigate the fat tails of the simulated return, I calculate the skewness and kurtosis values and compare them with that of actual financial data. Table 5.2 summarizes the statistic of fat tail tests on my simulation, S&P 500 and Dow Jones Index. The kurtosis (the fourth moments) for my simulation is 10.514, which is close to the value for actual financial time series. The kurtosis for all three series are positive and greater than the benchmark value 3 when returns are normally distributed. It is consistent with finding in literature that distribution of returns displays a fat tail with positive excess kurtosis. The skewness (the third moments) of simulated returns is -0.440, which is very close to the skewness of the well-known indexes. The negative value of skewness implies that the price falls more than it rises on average.

Table 5.2: Fat tails

Statistic	Skewness	Kurtosis
S&P 500	-0.424	7.202
Dow Jones Index	-0.385	6.526
My simulation	-0.440	10.514

Volatility clustering Another robust statistical property of financial market is the existence of volatility clustering. The volatility clustering phenomenon refers to significant changes of prices tend to cluster together, resulting in persistence of the amplitudes of price changes. Cont (2007) reveals that the different behavior of agents is the reason causes the volatility clustering. In order to check whether the model is able to account for this stylized fact, I plot the autocorrelation functions (ACF) for both artificial data and S&P 500 data.

The returns show almost insignificant autocorrelations for both actual data and artificial data in Figure 5.9. No linear correlation does not mean independence of return. A different picture emerges when one takes non-linear functions of return into account, such as absolute or squared returns. As shown in Figure 5.9, the ACFs of simulated absolute returns and squared returns are slowly decaying as time lag increases. The persistent positive correlation is a quantitative feature of volatility clustering, meanwhile it also implies a long range dependence that one typically finds in financial time series. The slowly decaying patterns in the ACFs of artificial data are analogous to that for S&P 500 data, which demonstrates that my model performs quite well in generating the important stylized facts in financial market.

Asymmetric returns Asymmetric returns is another stylized fact in financial market. It is also known as negative skewness and excess kurtosis. In the simulated sample, the most positive return is 44% while the most negative return is -61%, which matches the documented asymmetry in returns. To test the asymmetry of the statistic, I run the Shapiro-Wilk test for normality. The null hypothesis for this test is that return is normally distributed. The result in the Table 5.3 significantly rejects the null hypothesis, and the distribution of simulated returns does not follow a normal distribution. I also run the

Figure 5.9: Autocorrelation functions for raw return, absolute return and squared return

Table 5.3: Shapiro-Wilk test

Variable	Observations	W	V	z	Prob>z
Return	2000	0.929	83.850	11.266	0.000

symmetric plot for the simulated return, and some points departing from the 45 degree line in Figure 5.10 strongly implies that the returns are asymmetric.

The power law of returns The tail distribution of returns can be well approximated by the power law, which has been found and investigated in many literature (Gabaix et al., 2003, He and Li, 2007 and Schmitt and Westerhoff, 2017). In particular, the distribution of returns is found to decay according to

$$P(|r_t| > x) \sim X^{-\alpha} \quad (5.15)$$

where α is a constant parameter of the distribution known as scaling parameter or tail index. To detect the power law distribution of return, I follow the method as in Clauset et al. (2009), and the method involves maximum-likelihood fitting methods with goodness-of-

Figure 5.10: Symmetric plot of return

Table 5.4: The power law of return

Statistic	X_{min}	α	p-value
S&P 500(weekly)	3.45	3.83	0.77
Dow Jones Index(weekly)	4.07	4.44	0.82
My simulation	0.10	3.90	0.13

Test hypothesis:
 H_0 : data is generated from a power law distribution.
 H_1 : data is not generated from a power law distribution.

fit tests based on the Kolmogorov-Smirnov statistic and likelihood ratios. As the power law distribution of return is sensitive to the frequency of sample and time range, I test the daily data (2010/1/1-2016/12/31) and weekly data (1983w1-2016w52) for both S&P 500 and Dow Jones Index, and find that the weekly data of these two indexes follow the power law distribution. To investigate whether the model could generate data with the power law distribution, I test the simulated data and find that p-value equals to 0.13, which suggests one can not reject the null hypothesis that data is generated from a power law distribution. The details of the results are shown in Table 5.4.

Figure 5.11: Statistics of simulated volume

5.3.3 Volume dynamic

Before we turn to the stylized facts on volume, I explore the statistical properties of the simulated volumes. Figure 5.11 turns to the statistics of simulated volumes. The top panel of the diagram characterizes the distribution of simulated volumes and normally distributed volumes. As can be seen, the distributions of simulated volumes deviate significantly from normal distribution and possess fat tails. The bottom panel of Figure 5.11 exemplifies the autocorrelation function of simulated volumes, and the persistence of ACF goes out for a long range and the rate of decaying is very low, which quantifies the long memory behavior of trading volumes.

Another statistical feature in financial market is convex or V-shaped relationship between price and trading volume. As reported by Karpoff (1987), a V-shape has been found by virtually all empirical investigation of the price-volume relation in equity market. This V-shape curve has also been widely documented in price-volume relation studies, such as Gallant et al. (1992), Blume et al. (1994) and Puri and Philippatos (2008). By building a model for price and trading volume in financial market, Gallant et al. (1992) found that the dispersion of the information affects this V-shape relation, but in this model the effect of the disagreement of investor's beliefs is perhaps even more interesting. I plot

the price-volume outcomes for simulated data from the model, and I explored the possible effect of disagreement of belief from two chartists groups (sensitiveness disagreement is measured by $\beta_1 - \beta_2$). Figure 5.12 depicts the resulting price-volume equilibrium outcomes for three different level of disagreements of chartists. What is most interesting is that while the greater disagreement of chartists increases the disagreement of the points (as from panel (a) to (c)), it does not change the general V-shape of the relation. Indeed, the graph suggests when the disagreement of the chartists' belief is small enough, the price-volume relation is characterized by a simple V-shape. As volume also contains information of investor's beliefs and volume statistic provides information to the market that is not conveyed by price, observing price and volume together is more informative than observing price alone.

5.3.4 Stylized facts on volume

As mentioned before, most of the HAMs have the ability to generate the stylized facts on price, but few have investigated the performance of HAM on generating volume related stylized facts. In this section, I test the model about the capability of replicating some stylized facts on trading volume, which have been mentioned in Section 5.1. While some of them can be detected by statistic approaches, others will rely on the visualized analysis.

Correlation between volume and volatility As shown in Section 5.1, there exists a strong positive linear relation between volume and VIX. Using the same method, I test the correlation between the simulated trading volume and volatility. Squared returns serve as the proxy of volatility. The correlation coefficient for the test is 0.337 (Figure 5.13), which suggests a weak positive correlation. Although the correlation between simulated series is not as strong as that in actual dataset, the significant positive correlation is consistent with the literature. In the past decades, the mechanism of positive correlation between volume and volatility has been always discussed by many researchers, the heterogeneous beliefs of agents in my model could provide a reasonable explanation for this question. As stated in Shalen (1993), the dispersion of beliefs is a factor contributing to

(a)

(b)

(c)

Figure 5.12: The relation of price and volume for different disagreements of chartists belief. (a) Disagreements of chartists belief $\beta_1 - \beta_2 = 2$. (b) Disagreements of chartists belief $\beta_1 - \beta_2 = 3$. (c) Disagreements of chartists belief $\beta_1 - \beta_2 = 4$.

Figure 5.13: Correlation between volume and volatility

the positive correlation between volume and volatility.

Granger causality between return and volume change The linear and nonlinear relations between price and volume have been found in different markets and countries. To further examine the validity of the model, I conduct the linear and nonlinear Granger causality tests to investigate whether the causality relations exist in the simulation. The methods I use to test the linear and nonlinear Granger causality are the same with those in Section 5.1. The results are reported in Table 5.5 and Table 5.6, respectively. In the linear Granger causality test, four lags for dependent and independent variables are considered in the VAR model. The Granger tests show strong evidence of unidirectional causality from return to volume changes. In particular, the Wald statistic value equals 17.829, and suggests one can confidently reject the null hypothesis of no causality at 1% significant level. On the other hand, Granger noncausality from volume change to stock return can not be rejected, as the p-value of the test 0.919 is great enough. Hence, unidirectional causality from stock return to trading volume exists in the simulation.

To check whether the nonlinear relation exists in the artificial series, I further conduct the nonlinear Granger causality tests for return and volume change. I follow the setup

Table 5.5: Linear Granger causality test

Dependent varibale	(1) R_t	(2) V_t
Constant	0.005 (0.009)	1.438*** (0.074)
R_{t-1}	0.237*** (0.022)	0.183 (0.167)
R_{t-2}	0.065 (0.023)	0.565*** (0.172)
R_{t-3}	-0.002 (0.023)	-0.182 0.172
R_{t-4}	-0.145*** (0.022)	0.076 (0.168)
V_{t-1}	-0.002 (0.003)	0.337*** (0.022)
V_{t-2}	-0.001 (0.003)	0.045 (0.024)
V_{t-3}	0.002 (0.003)	-0.050*** (0.024)
V_{t-4}	-0.001 (0.003)	0.021 (0.022)
Wald-stat	0.937	17.829***
p-value	0.919	0.005

Notes: (1) The entries in brackets are the standard errors. The Wald-stat and p-value are tests of Granger causality. (2)* denotes rejection at the 10% level, ** rejection at the 5% level, ***rejection at the 1% level.

Table 5.6: Nonlinear Granger causality test

Lags	H_0 : Stock Returns Do not Cause Volume Changes		H_0 : Volume Changes Do not Cause Stock Returns	
	T-statistic	p-value	T-statistics	p-value
Lx=Ly				
1	1.654	0.049	3.393	0.000
2	1.733	0.042	2.990	0.001
3	1.976	0.024	3.303	0.000
4	2.180	0.015	2.211	0.014
5	2.190	0.014	2.348	0.009
6	1.806	0.035	2.177	0.015
7	1.895	0.029	1.678	0.047
8	1.818	0.034	1.811	0.035

Note: Test critical value for T statistics are 2.326 (1%), 1.645 (5%) and 1.282 (10%).

for lags and other parameters in Section 5.1, and the results in Table 5.6 show that all the t-statistics are larger than 1.645 (5% significant level), which implies there are strong evidence that one can reject the null hypothesis. Bidirectional nonlinear Granger causality between stock returns and volume changes is found in the simulation. The linear and nonlinear test results I get from the simulation are roughly consistent with the results using above S&P 500 and Dow Jones index dataset. Referring to the literature, the results are exactly consistent with the findings in Hiemstra and Jones (1994) both in linear and nonlinear Granger causality tests. The results also justify that my model is capable of simulating the relationships between returns and volume changes in stock market.

Significant volume with large price change There are some visualized stylized facts on trading volume which have drawn much attention from practitioners and researchers. The first one I intend to simulate is that significant volume is always associated with price jump. Besides the statistical relationship I have tested using econometric approaches, the visualized relationship between prices fluctuation and volumes change during abnormal periods is also an important topic in the stock market. As shown in Figure 5.1, the price jumps suddenly with significant volume during the crisis periods, which reflects a crucial characteristic of stock market. In the simulation, this kind of relation can be straightforwardly observed. As illustrated in the Figure 5.14, the dramatic changes of

Figure 5.14: Significant volume with large price change

price (either positive or negative) are found with high trading volumes. When the price suddenly increases or decreases, the volumes almost double the daily average volume.

Informational role of trading volume As one of informational tools, trading volume is widely adopted by technicians. Investors identify specific signal from the volume to confirm the future price trend and discover the selling and buying opportunities. In efficient market, price movement is attributed to the available of new piece of information. The previous example of Google stock demonstrates the the informational role of trading volume to predict the future price trend. In Figure 5.15, I find similar price and volume pattern as in the real world. The trading volume of periods before $t = 436$ is less than 20, and prices fluctuate in a window from 30 to 100. When the price suddenly jumps up to the next level at $t = 436$, the trading volume is almost five times the daily average, which confirms the breaking-out of price to the resistance line. After jumping up, prices move above the $p = 100$. This price level also becomes a new support line in technical analysis.

Figure 5.15: Informational role of trading volume

5.3.5 Asset prices in bubbles and crises

Bubbles and crises have been the hot topic in financial market studies. As shown in overall price patterns in Figure 5.4, the model can generate essential features of bubbles and crises, namely, growing and significant overvaluation and undervaluation of the risky asset. Overvaluation (undervaluation) here means that the price exceeds (under) the fundamental value. To further explore the explanatory power of the model in bubbles and crises, I check whether large overvaluation and undervaluation hold for different parameter values. Figure 5.16 illustrates this. The four panels (a)-(d) correspond to four important parameters: adjustment speed of market maker γ ; intensity of choice ρ ; sensitiveness of fundamentalist a ; and disagreements of chartists belief $\beta_1 - \beta_2$. In each panel, the upper diagram plots the maximum deviation of the asset price, and the bottom diagram shows the corresponding trading volumes at the moment of peak deviation. For each panel, I generate maximum price deviations and volumes by varying the value of the parameter on the horizontal axis while keeping all other parameter values at the benchmark levels. The figure confirms that the model generates a large deviation from fundamental value for a wide range of parameter values. Panel (a) and (c) of the figure indicate that low values of γ and a increase the magnitude of deviation. In other words, bubbles or crises are more likely to occur with small price adjustment speed of market

maker and low sensitiveness of fundamentalist. In financial markets, when fundamentalists are more active and the trend followers are relatively less active, an increase in γ stabilizes the market price to the fundamental price. It provides a evidence of stabilizing force of market maker. This finding is consistent with argument by Zhu et al. (2009) that when adjustment speed of market maker is small, he/she may act as a destabilizing force in the market. In addition, trading volumes keep the same trend with price deviation in panel (a) but present an opposite trend in panel (c). With the rise of sensitiveness of fundamentalists, the stabilizing role of fundamentalists will be amplified, and prices are more likely to be drew back to fundamental values, but the trading volume will be scaled up with the increasing demand from fundamentalists. In panel (b), increase of intensity of choice means that agents are able to quickly switch their trading strategies. With high clustering to one strategy of investors in the market, the possibility of bubbles and crises is very large. Most interestingly, the large disagreement of chartist's beliefs ($\beta_1 - \beta_2$) also generates large bubbles and crises with enormous trading volumes.

5.3.6 Prices, volumes and beliefs co-evolve in different chart patterns and crises

Besides simply looking at bubbles and crises, I am also interested in the co-evolvement of prices, volumes and investor beliefs in financial market. Some literature on HAM have investigated the co-movement of assets prices and agents beliefs, such as Boswijk et al. (2007) and Huang et al. (2010). Nevertheless, few of them have included the trading volumes into the analysis. In this section, I investigate the stock market from three dimensional viewpoints, and co-evolvement of prices, volumes and beliefs can be observed in the simulation. Furthermore, I introduce the chart patterns and crises into analysis. As expected, the model has the ability to provide the reasonable explanation for the formation of different chart patterns and crises patterns.

Chart pattern is one of the most popular strategies for technicians, and they have summarized many patterns in the price series. Although not all of them appear during a specific period, some patterns such as double tops, double bottoms, head-and-shoulders

(a)

(b)

(c)

(d)

Figure 5.16: The peak deviation of prices from fundamental values and corresponding volumes. (a) Variation of adjustment speed of market maker γ . (b) Variation of intensity of choice ρ . (c) Variation of sensitiveness of fundamentalist a . (d) Disagreements of chartists belief $\beta_1 - \beta_2$.

Figure 5.17: Prices, volumes and beliefs co-evolve in double tops pattern

and V tops are found in stock market frequently. I select some chart patterns from the simulation, and the co-evolution of prices, volumes and beliefs within these pattern periods are displayed in Figure 5.17 and Figure 5.18. A double tops pattern is shown in the top of Figure 5.17. At the first stage of the pattern, the fundamentalists are in charge of the market as the price is lower than their fundamental value. The great excess demand of fundamentalists pushes up the price, and when the price hikes gradually to the first top, the trend followers dominate the market. The gradual bubble occurs with large trading volume. At the next stage, when the price reaches a high level, the fraction of contrarians increases. It follows that price falls with more chartists chasing the falling trend. At the bottom, the price is lower than the fundamental value, and the fundamentalists take over the market again. Huge demand from fundamentalists leads to large trading volume, and the price rises again with the increasing number of chartist. Finally, the second top appears. Double bottoms pattern is reversal pattern of double tops. As shown in Figure 5.18, the bottoms are formed with a sudden or smooth falling of price, and trading volume is also significant at the same time. The fundamentalists dominate the market at bottoms' period, and chartists take over during other periods.

HAMs have been widely used to explain the financial crises in many ways. Huang et al. (2010) have found that switches between trading strategies lead to price dynamic and cause different types of financial crises, such as sudden crisis, smooth crisis and dis-

Figure 5.18: Prices, volumes and beliefs co-evolve in double bottoms pattern

turbing crisis. I examine the sudden crisis and smooth crisis in the simulation, and the co-evolution of prices, volumes and beliefs during crisis periods are demonstrated in Figure 5.19 and Figure 5.20. In the smooth crisis, the price falls smoothly from 300 to 100, and trading volume also decreases accordingly. When the price declines to a low level, fundamentalists replace the chartists to dominate the market. The lasting descending trading volume could be regarded as signal of smooth crisis. During smooth crisis, chartists dominate the market and induce the slow decline of price. The existence of contrarians slow down the speed of price falling. Once price falls below the fundamental value, fundamentalists begin to increase and take over the market. In a sudden crisis, the price plunges from the peak precipitately to the bottom, and trading volume surges at the same time. In the Figure 5.20, the sudden crisis occurs with the spike shape volumes, and the number of fundamentalists suddenly increases and fundamentalists become the majority in the market.

5.4 Conclusion

In this chapter, I develop a HAM with trading volume to replicate qualitative and quantitative features commonly observed in the stock market. Under the framework of market maker, fundamentalists and chartists hold heterogeneous beliefs on future price of

Figure 5.19: Prices, volumes and beliefs co-evolve in a smooth crisis

Figure 5.20: Prices, volumes and beliefs co-evolve in a sudden crisis

risky assets. Agents are allowed to update their expected prices based on different behaviors: the fundamentalists set their fundamental value referring to the costly internal information and economy growth rate, while chartists update their expected short-term fundamental value according to a series of psychological windows. To fit the real life case well, I introduce the adaptive evolutionary regime and agents freely switch to other group and choose the strategies that would optimize their discounted expected profit. The interaction between the fundamentalists and chartists could generate the price fluctuation and price-volume relationship. Meanwhile, the adaptive switching behavior of agents also increases market volatility both in price and volume. I am able to explore the rich dynamics of price and trading volume by building such a simple model, such as price bifurcations, V-shape price volume relation and features of bubbles and crises with variation of parameters

Although I keep the model as simple as possible, it is also capable of generating a wide range of stylized facts both on price and volume simultaneously. As documented in literature, many HAMS are capable of generating stylized facts on price or return, such as unit root process in prices, fat tails, asymmetric and volatility clustering returns. The HAM in Huang and Zheng (2012) even has the ability to simulate the strict power-law distribution of return. After successfully simulating all the “standard” stylized facts above, I further explore the potential of the model in generating the stylized facts on volume. The deterministic model performs well in reproducing the stylized facts like stationary volume, positive correlation between volume and volatility, Granger causality between return and volume change. In addition, this model also closely mimics some visualized stylized facts which are commonly found in financial market, such as significant volume with dramatic price change and information role of trading volume. To demonstrate the power of the model in explaining different patterns in stock market, I identify different chart patterns and crises pattern that are frequently documented in technical analysis and literature, then analyze the formation of these patterns. One focus of this chapter is using tridimensional analysis approach to investigate the co-evolution of prices, volumes and beliefs in these patterns. The co-evolution of these three elements

comprehensively reflects the trading activities and the investors' behavior in financial market, which could give us a thorough view on financial market. By analyzing the chart patterns, the model could also provide theoretical underpinning for technical analysis.

Appendix A

Linear Granger causality definition Two stationary time series X_t and Y_t , let $F(X_t|\Omega_{t-1})$ be the conditional probability distribution of x_t given the bivariate information set Ω_{t-1} consisting of an L_x -length lagged vector of X_t and L_y -length lagged vector of Y_t . If:

$$F(X_t|\Omega_{t-1}) = F(X_t|(\Omega_{t-1} - Y^{L_y})), t = 1, 2, \dots \quad (5.16)$$

Given lags L_x and L_y , the time series Y_t does not strictly Granger causality cause X_t . If the equality does not hold, Y is said to strictly Granger cause X . In plain words, X_t is said to Granger-cause Y_t if X cannot help predict future Y .

To test for Granger causality between stock return and volume change, we conduct the following vector autoregressive (VAR) model:

$$R_t = A + B(L)R_t + C(L)V_t + U_t \quad (5.17)$$

$$V_t = D + E(L)R_t + F(L)V_t + V_t \quad (5.18)$$

Where, R_t is stock return and V_t is percentage change of volume. $B(L)$, $C(L)$, $E(L)$ and $F(L)$ are lag polynomials of R_t and V_t .

Nonlinear Granger causality definition Consider two strictly stationary and weakly dependent time series $X_t, Y_t, t = 1, 2, 3, \dots$, We then denote the m -length lead vector of X_t by X_t^m and the L_x -length and L_y -length lag vectors of X_t and Y_t , respectively, by $X_{t-L_x}^{L_x}$ and $Y_{t-L_y}^{L_y}$. For given values of m, L_x , and $L_y \geq 1$ and for $e \geq 0$, Y does not strictly Granger-

cause X if

$$\begin{aligned} Pr(\|X_t^m - X_s^m\| \leq e \mid \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| \leq e, \|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| \leq e) \\ = Pr(\|X_t^m - X_s^m\| \leq e \mid \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| \leq e) \end{aligned} \quad (5.19)$$

Where $Pr(\cdot)$ denotes probability and $\|\cdot\|$ denotes the maximum norm. In order to transform Equation (5.19) into a testable form, we denote the joint and marginal probabilities by:

$$\begin{aligned} C1(m+Lx, Ly, e) &\equiv Pr(X_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx} \leq e, \|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| \leq e) \\ C2(Lx, Ly, e) &\equiv Pr(X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx} \leq e, \|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| \leq e) \\ C3(m+Lx, e) &\equiv Pr(X_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx} \leq e) \\ C4(Lx, e) &\equiv Pr(X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx} \leq e) \end{aligned}$$

The strict Granger noncausality condition in Equation (5.19) can be expressed as

$$\frac{C1(m+Lx, Ly, e)}{C2(Lx, Ly, e)} = \frac{C3(m+Lx, e)}{C4(Lx, e)} \quad (5.20)$$

The null hypothesis for Y_t strictly Granger-causing X_t in Equation (5.20) is

$$\sqrt{n} \left(\frac{C1(m+Lx, Ly, e, n)}{C2(Lx, Ly, e, n)} - \frac{C3(m+Lx, e, n)}{C4(Lx, e, n)} \right) \sim N(0, \sigma^2(m, Lx, Ly, e)) \quad (5.21)$$

After getting two estimated residual series U_t and V_t from the linear VAR estimation, i use the modified HJ test(see Diks and Panchenko, 2006) to investigate the nonlinear Granger causality between stock return and trading volume. The results of linear and nonlinear Granger causality tests are shown in Table 5.7 and Table 5.8, respectively.

Table 5.7: Linear Granger causality test

Dependent variable	S&P 500		Dow Jones Index	
	(1) R_t	(2) V_t	(3) R_t	(4) V_t
Constant	-0.660 (2.664)	5.321*** (0.478)	-1.301 (1.143)	3.168*** (0.361)
R_{t-1}	-0.047* (0.024)	-0.011*** (0.004)	-0.048** (0.024)	-0.026*** (0.008)
R_{t-2}	0.020 (0.024)	-0.003 (0.004)	0.031 (0.024)	-0.016** (0.008)
R_{t-3}	-0.066*** (0.024)	-0.002 (0.004)	-0.042* (0.024)	-0.006 (0.008)
R_{t-4}	-0.005 (0.024)	-0.010 (0.004)	-0.015 (0.024)	-0.004 (0.008)
V_{t-1}	-0.104 (0.133)	0.471*** (0.024)	-0.011 (0.075)	0.378*** (0.024)
V_{t-2}	0.252* (0.147)	0.456*** (0.026)	0.134* (0.080)	0.188*** (0.025)
V_{t-3}	-0.247* (0.147)	0.051* (0.026)	-0.084 (0.081)	0.152*** (0.025)
V_{t-4}	0.131 (0.133)	0.079*** (0.024)	0.032 (0.076)	0.113*** (0.024)
Wald-stat	7.923	11.340***	4.218	15.678***
p-value	0.295	0.023	0.377	0.003

Notes: (1) The entries in brackets are the standard errors.

The Wald-stat and p-value are tests of Granger causality.

(2)* denotes rejection at the 10% level,** rejection at the 5% level, ***rejection at the 1% level.

Table 5.8: Nonlinear Granger causality test

Lags	H_0 : Stock Returns Do not Cause Volume Changes		H_0 : Volume Changes Do not Cause Stock Returns	
Lx=Ly	T-statistic	p-value	T-statistics	p-value
S&P 500				
1	3.027	0.001	1.059	0.145
2	2.237	0.013	0.960	0.169
3	1.975	0.024	0.403	0.343
4	2.076	0.019	1.241	0.101
5	1.175	0.120	1.115	0.132
6	0.748	0.227	1.372	0.085
7	0.966	0.167	1.924	0.027
8	0.566	0.286	1.575	0.058
Dow Jones Index				
1	1.026	0.153	-0.613	0.730
2	0.874	0.191	-0.575	0.717
3	0.128	0.449	0.248	0.402
4	0.249	0.401	-0.041	0.516
5	0.609	0.271	-0.086	0.534
6	-0.436	0.668	0.118	0.453
7	-0.422	0.663	0.239	0.405
8	-0.222	0.588	0.872	0.192

Note: Test critical value for T statistics are 2.326 (1%), 1.645 (5%) and 1.282 (10%).

Appendix B

The chance function shows the chance of lost opportunity either to buy when the assets price is low or fail to sell when the price is high. It can be expressed as

$$A(\mu_t, p_t) = a(p_t - m(\mu_t))^d (M(\mu_t) - p_t)^d \quad (5.22)$$

where a and d are the parameters that describe the sensitiveness of fundamentalists when the price move close to the boundaries and $a > 0$, $d < 0$. Assuming $\mu_{f,t}$ as constant, the chance function can be simply illustrated as Figure 5.21.

I define the boundaries as below

$$M_t = k\mu_t \text{ and } m_t = \frac{1}{k}\mu_t \quad (5.23)$$

As defined in Black (1986), price fluctuates within a reasonable bond in efficient market.

$k > 1$ is a pre-selected factor and μ_t is the fundamental value of risky assets.

Figure 5.21: The chance function

Figure 5.22: Correlation between volume and the VIX

Appendix C

Scatter plotting of S&P 500 trading volume and VIX between 1/1/2010 and 12/31/2016

Chapter 6

Conclusion

For long, the link between asset pricing and investor sentiment has been in the spotlight in financial economics. As argued by Keynes (1936), markets can fluctuate wildly under the influence of investors' "animal spirits" which move prices in a way unrelated to fundamentals. With the emergence of behavioral economics, the studies of sentiment role in financial market provide a better explanation to market anomalies comparing with traditional asset pricing models. This thesis supports such argument by modelling sentiment effect in a HAM framework. The behavioral heterogeneous agents approach challenges the traditional representative rational agent assumption. Through incorporating interaction of different types of agents, HAM has been proven potent in explaining the rich stylized facts and market features. I further introduce sentiment in the HAM framework to investigate whether the sentiment acting on micro-level could affect the market dynamics on macro-level.

The stochastic models developed in Chapter 2 and the deterministic model in Chapter 5 are both able to generate various stylized facts, such as fat tails and long memory of returns, excessive volatility and volatility clustering in the stock market. Chapter 2 presents a sentiment-HAM. In this model, both fundamentalists and chartists response to behavioral terms. Fundamentalists' trading behavior subject to their own time-varying confidence levels, while chartists' behavior is sensitive to the market sentiment. The sentiment index consists of both endogenous and exogenous components. The endogenous

component is derived as the opinion of majority, which is the dominant trading strategy resulted from social interaction of heterogeneous investors. Exogenous part of sentiment is modelled as information shocks or sentiment-related news. With consideration of two types of chartists, momentum traders and contrarian traders, this model is able to capture the heterogeneous responses to sentiment between the two chartist groups. The dynamic HAM in Chapter 2 is a highly nonlinear system, and it could generate a wide range of dynamic behaviors. By evaluating the equilibria in 2-type and 3-type HAM, sentiment effect is found in the steady states. Polarized sentiment non-fundamental steady states are possible comparing with neutral sentiment fundamental steady state. The sentiment related parameters are able to affect the stability of fundamental steady states with some specific conditions. In other words, sentiment could be the source of chaotic price process.

Chapter 3 analyses three-type sentiment model by using computational and numerical simulation methods. This model can replicate a wide variety of statistical and qualitative features as in the real financial markets. The comparison between sentiment-HAM and baseline HAM without sentiment is conducted through simulation practice, in which I find evidence that sentiment is closely related to some of the stylized facts witnessed in the actual financial markets. Specifically, I show that sentiment could help explain different types of crises, including smooth crisis, sudden crisis and disturbing crisis. In a sudden crisis, for instance, the negative sentiment is found to accelerate selling forces of momentum traders and subsequently leads to market panic and sharp decline in price. Put simply, sentiment may work as inducing or reinforcing factor of crises. This can also explain why crises are more frequently triggered by sentiment. Through conducting the 1000 Monte Carlo simulation, I also prove that sentiment could amplify both frequency and magnitude of financial crises.

In Chapter 4, I develop a two-market HAM with sentiment. A new type of agent, internationalist, is introduced in the model. The internationalists can optionally switch investing market based on two markets' sentiment level. This chapter attempts to apply econometric method to estimate HAM on real market data. In particular, I estimate

the two market dynamic model by applying a novel Bayesian approach with extended Kalman filter. Using the US, Japan and the UK stock market data and sentiment indexes, I find the existence of behavior heterogeneity in international financial markets. Fundamentalists and chartists switch their beliefs less aggressively in US and UK stock markets comparing with US and Japan markets, while their fractions are more evenly distributed in UK market. Internationalists are assumed to adopt chartism strategy, and the market switching behavior of internationalists is found in the estimation. By adding the market switching mechanism, this model is able to detect more significant investor heterogeneity. Through the market switching, internationalists are also shown to be related to market instability, prices co-movement and boom-and-bust cycles in stock markets.

Chapter 5 investigates another important indicator, trading volume, under a three-type HAM. The model setup is similar to that in Chapter 2, but without sentiment factors. I analyse the model's stability by examining the bifurcations generated by different parameter values. The model is proven to have potential to explain the stylized facts on both prices and volumes. It also provides an approach to theoretically explore the co-evolvement of prices, volumes and beliefs in financial markets. I also show that volume could play an important information rule when investors predict the asset price by conducting chart pattern analysis. Finally, I simulate different crises as in Chapter 3, and the model with volumes provides a good explanation to the formation of crises.

6.1 Caveats and Extensions

This thesis aims to investigate the sentiment effect on financial markets within a HAM framework. I develop three models with focus on sentiment-driven chartists, sentiment linkage of two markets and a sentiment proxy volume. Although these models are useful in explaining theoretically and replicating numerically stylized facts and different types of crises, they are subject to some limitations.

First, the sentiment model in Chapter 2 has too many nonlinear components, which include confidence function for fundamentalists, short-term value for chartists v_t , discrete

choice function and sentiment index. All these confounding factors could be the source of stylized facts and crises. Meanwhile, large number of parameters further complicate this problem. Although I have excluding some of the possible factors, the role of sentiment on market pricing still needs more evidence.

Second, foreign exchange market is missing in the two-market model in Chapter 4. Besides, to facilitate the estimation, the setup of internationalists is too simple. In real world, international investor may follow different trading strategies, and the cost of international information is also neglected this model.

Finally, the HAM with volume in Chapter 5 is able to duplicate many stylized facts on prices and volumes. As the statistical feature of volume is not a common knowledge, the analysis on simulated volume is only partially representative.

6.2 Future Research

Three main strands of this thesis can be further explored in the near future.

First, real financial data could be used to calibrate or estimate the sentiment model. With the availability of stock market data and sentiment data, model calibration could provide a more accurate explanation to stylized facts and crises.

Second, two-market HAM model estimation can be extended to multiple markets. Extended Kalman filter approach can be improved or replaced by other filter method like particle filter to estimate multiple market HAM. A new sentiment index specialized on international financial markets can be construct to better capture the inducement of market switching of investors.

Third, as trading volume is an important indicator and proxy of sentiment in financial market, it can be further modelled as an endogenous variable rather than only the output of the model.

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