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Bayesian Filtering with Unknown Sensor Measurement Losses

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Abstract—This work studies the state estimation problem of a stochastic nonlinear system with *unknown* sensor measurement losses. If the estimator knows the sensor measurement losses of a linear Gaussian system, the minimum variance estimate is easily computed by the celebrated intermittent Kalman filter (IKF). However, this will no longer be the case when the measurement losses are unknown and/or the system is nonlinear or non-Gaussian. By exploiting the binary property of the measurement loss process and the IKF, we design three suboptimal filters for the state estimation, i.e., BKF-I, BKF-II and RBPF. The BKF-I is based on the MAP estimator of the measurement loss process and the BKF-II is derived by estimating the conditional loss probability. The RBPF is a particle filter based algorithm which marginalizes out the loss process to increase the efficiency of particles. All the proposed filters can be easily implemented in recursive forms. Finally, a linear system, a target tracking system and a quadrotor's path control problem are included to illustrate their effectiveness, and show the tradeoff between computational complexity and estimation accuracy of the proposed filters.

Index Terms—Stochastic systems, networked estimation, intermittent Kalman filter, sensor measurement losses, particle filter.

I. INTRODUCTION

THE Kalman filter (KF) has a simple structure in optimally correcting propagated state estimates with the sensor measurement of the observed system, and has been successfully applied in countless guidance, navigation and control (GNC) related applications. While in some practical applications, the estimator may not always access the true sensor measurement [1]–[9]. For example, a sensor fault results in that the estimator only receives a pure noise [4]–[6], which does not contain any information of the estimated system. In networked systems, the channel traffic congestion will also lead to sensor measurement losses [7]. Similar issues commonly exist in the following important applications.

- In a target tracking system, the position, distance, and velocity of the target to the base station is measured by a radar or Global Positioning System (GPS). The sensor information is sent to a missile or an unmanned aircraft via resource-limited wireless channels where communications between devices are power constrained and therefore limited in range and reliability [7]–[9]. If the true sensor measurement is lost, the base station only receives error data, e.g., channel noises.

- GPS is a common way for positioning, which however requires signals to be detected precisely. Thus any slight inaccuracy in the signal's reception or some external disturbances can lead to an error in the measured location [2]. Environmental factors such as trees, valleys, buildings, or even heavy cloud cover can impact or even interrupt the transmission of the GPS signal between the receiver and the satellites. Besides, urban environments surrounded by tall buildings also cause inaccurate GPS signals.
- A vehicle or aircraft is usually equipped with many sensors. For instance, a quadrotor may have an inertial measurement unit (IMU), a GPS receiver, barometers, and magnetometers. Large aircrafts even use more sensor units. It is possible that some error measurements occur from time to time due to unexpected faults in the system [3]–[5].

In the above situations, the KF or extended KF can only intermittently access the true sensor measurements. If an wrong measurement or pure noise is used for updating the state estimate, the estimation performance will significantly degrade, and the filter may even diverge.

Then, a natural question is how to handle issues induced by occasional measurement errors or losses. If the measurement loss is *known* to the estimator, the minimum variance estimate (MVE) of a linear Gaussian system is provided by an intermittent Kalman filter (IKF) [10], and a vast body of literature, see e.g. [11], [12] and references therein, focuses on the stability of the IKF. For instance, it is shown in [10] that there exists a critical measurement loss probability beyond which the IKF may diverge. In [13], this critical value for certain type of systems is explicitly obtained under Markovian measurement losses. More cases can be found in [14].

The problem is further complicated if the measurement loss is *unknown* to the estimator. In real applications, the estimator receives a data packet which may be a pure noise or fake sensor measurement. A faulty sensor might also return “wrong” measurement data to the estimator. Thus, it is of practical importance to study the state estimation problem under *unknown* sensor measurement losses, which is the main focus of this work. Clearly, the unknown measurement loss results in a non-Gaussian and nonlinear system where the IKF is no longer applicable. Although there are some generic estimation methods for nonlinear and/or non-Gaussian systems, e.g., extended KF (EKF), unscented KF [15], [16] and particle filter (PF) [17], they do not particularly explore the unique feature of the current problem, in which the measurement loss or error results in a discontinuous measurement

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equation. To resolve this issue, we model the process of sensor measurement losses as a binary sequence $\{\gamma_k\}$. Particularly, $\gamma_k = 1$ means that the true sensor measurement is received and $\gamma_k = 0$ indicates the occurrence of a sensor measurement loss. Our objective is to design practical suboptimal filters to accommodate the unknown intermittent measurement losses in a nonlinear stochastic system.

Motivated by the optimality of the IKF, our idea is that we first estimate the binary sequence $\{\gamma_k\}$ under the maximum a posteriori probability (MAP) estimation criterion. Following the principle of certainty equivalence, we then use the MAP estimate to replace the unknown measurement loss γ_k in the IKF for the linearized systems. This results in our Bayesian Kalman filter I (BKF-I). We also derive a Bayesian Kalman filter II (BKF-II) by estimating the conditional measurement loss probability in the linearized system, which is a compromise between the KF and the IKF. Clearly, both filters are very different from the existing nonlinear extended KF, unscented KF or PF, and will reduce to the intermittent extended Kalman filter (IEKF) if the measurement loss process $\{\gamma_k\}$ is known to the estimator.

Another method to address the non-Gaussianity and/or non-linearity is the PF which samples particles to approximate the conditional density of state. However, the amount of computations required for the PF in the high-dimensional state space is usually large. To increase the sampling efficiency, one can marginalize out some of the states and use the standard algorithms such as the KF to estimate them. Then, the PF is applied to estimate the rest of state variables, which is called Rao-Blackwellised particle filter (RBPF). The implementation and comparison between the standard PF and RBPF are well documented in [15], [18]–[20]. In this work, we adopt this idea by using the PF to estimate the conditional distribution of the measurement loss process $\{\gamma_k\}$, which is a binary process and requires only a few number of particles to approximate its posterior distribution. As in the BKF-I, the state of the linearized system is then estimated by the IKF. Compared to the standard PF, the efficiency of using particles is significantly improved, no matter how large the dimension of the system state is.

We compare the computational complexities and the estimation performance of the proposed suboptimal filters. Both the BKF-I and BKF-II have similar computational complexities to the IEKF. The computational complexity of the RBPF is essentially proportional to the number of particles. The larger number of particles is adopted in the RBPF, the better estimation accuracy is expected in general. Moreover, the RBPF weakly converges to an optimal MVE for a linear stochastic system with unknown measurement losses if the number of particles tends to infinity. Although the estimation accuracies of the BKF-I and BKF-II cannot be ensured, they are usually easier to implement. Thus, the choice of the three filters depends on the problem on hand.

Finally, we use the proposed filters to solve the state estimation problem of a linear stochastic system, a target tracking problem and a quadrotor's path control problem, all of which are subject to unknown measurement losses. In these examples, the proposed filters well complete their

estimation tasks under different levels of measurement losses, and their performance even comes close to the case with *known* measurement losses, showing that the measurement loss process $\{\gamma_k\}$ is correctly estimated. For the RBPF, we also illustrate how the estimation accuracy is improved by increasing the number of particles in the linear system. This explains the tradeoff between computational complexity and the estimation accuracy of the RBPF.

It should be noted that the conference version of this paper [21] only studies linear stochastic systems. In comparison, this work focuses on nonlinear systems and provides detailed comparisons of the proposed three filters in terms of estimation accuracy and computational complexity, see Section V and Section VI-A. Besides, this paper includes the quadrotor's path control problem in Section VI-C to validate the effectiveness of our filters and provides a fast algorithm for implementing the RBPF in Section IV-C.

The rest of this paper is organized as follows. In Section II, we formulate the estimation problem with unknown measurement losses. In Section III, we design the BKF-I and the BKF-II based on the Bayes' theorem and the technique of EKF. In Section IV, we derive the RBPF to address the estimate of the process of unknown measurement losses. We compare the three filters both in terms of computational complexity and estimation accuracy in Section V. Simulation is performed in Section VI to show the effectiveness of the proposed filters and compare their performance. Finally, we draw some concluding remarks in Section VII.

II. PROBLEM FORMULATION

In this section, we formulate the state estimation problem with unknown measurement losses and introduce the celebrated intermittent KF, whose optimality is ensured for a linear Gaussian system with known measurement losses.

A. State Estimation with Unknown Measurement Losses

Consider a discrete dynamical system with measurement losses and the additive noise of the form:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) + w_k \\ y_k &= \tilde{\gamma}_k \cdot h(x_k) + v_k \end{aligned} \quad (1)$$

where $x_k \in \mathbb{R}^n$ and $y_k \in \mathbb{R}^m$ are the vector state and measurement, respectively. The random vectors $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are independent white Gaussian noises with zero means and covariance matrices $Q \geq 0$ and $R \geq 0$, respectively. The initial state x_0 is assumed to be a random Gaussian vector with mean $\bar{x}_0$ and covariance matrix $\Sigma_0 > 0$. $\tilde{\gamma}_k$ is a diagonal matrix with its i -th diagonal element γ_{ik} , $i \in \{1, 2, \dots, m\}$, representing the i -th sensor measurement loss at time step k . In particular, $\gamma_{ik} = 1$ indicates that the true sensor measurement y_{ik} is successfully received by the estimator while $\gamma_{ik} = 0$ means that the estimator only receives a fake measurement of y_{ik} , i.e., the pure noise v_{ik} .

In many real systems, all entries of y_k are obtained from the same sensor and are transmitted as a single packet, e.g., the GPS signal usually consists of position and velocity measurements. Then, all γ_{ik} , $i \in \{1, \dots, m\}$ are identical, and

$\bar{\gamma}_k$ is reduced to a random variable. For brevity, this paper mainly studies this case and focuses on the following nonlinear stochastic system

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) + w_k \\ y_k &= \gamma_k \cdot h(x_k) + v_k \end{aligned} \quad (2)$$

where γ_k is a binary random variable that represents the sensor measurement loss at time step k . Thus, $\gamma_k = 1$ indicates that the true sensor measurement y_k is contained in the arrival data packet while $\gamma_k = 0$ means that the estimator only receives pure noise.

The goal of this work is to propose recursive filters to estimate the state of the system in (2) under unknown measurement loss process $\{\gamma_k\}$.

B. Intermittent Extended KF (IEKF)

For a linear Gaussian model, the IKF in [10] assumes that $\{\gamma_k\}$ is known to the estimator. To revise it for the nonlinear system in (2), define $\Gamma_k = \{\gamma_0, \dots, \gamma_k\}$, $U_k = \{u_0, \dots, u_k\}$ and $Y_k = \{y_0, \dots, y_k\}$, and the conditional minimum variance estimate and error covariance matrices are given by

$$\begin{aligned} \hat{x}_{k|k-1} &= \mathbb{E}[x_k | Y_{k-1}, \Gamma_{k-1}, U_{k-1}] \\ \hat{x}_{k|k} &= \mathbb{E}[x_k | Y_k, \Gamma_k, U_{k-1}] \\ \Sigma_{k|k-1} &= \mathbb{E}[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^T | Y_{k-1}, \Gamma_{k-1}, U_{k-1}] \\ \Sigma_{k|k} &= \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T | Y_k, \Gamma_k, U_{k-1}] \\ \hat{y}_{k|k-1} &= \mathbb{E}[y_k | Y_{k-1}, \Gamma_{k-1}, U_{k-1}]. \end{aligned}$$

Denote $A_k = \frac{\partial f(\hat{x}_{k|k}, u_k)}{\partial x}$ and $C_k = \frac{\partial h(\hat{x}_{k|k-1})}{\partial x}$, this enables us to approximate (2) by a linearized system

$$\begin{aligned} x_{k+1} &= A_k x_k + w_k + b_k \\ y_k &= \gamma_k \cdot C_k x_k + v_k + z_k \end{aligned} \quad (3)$$

where b_k and z_k are computed online from the equations

$$b_k = f(\hat{x}_{k|k}, u_k) - A_k \hat{x}_{k|k}, z_k = \gamma_k (h(\hat{x}_{k|k-1}) - C_k \hat{x}_{k|k-1}).$$

If γ_k is known to the estimator, the measurement update for the nonlinear system (2) can be obtained by applying the IKF [10] to the linearized system (3), which leads to the Intermittent Extended KF (IEKF), i.e.,

$$\begin{aligned} \hat{x}_{k|k} &= \hat{x}_{k|k-1} + \gamma_k K_k (y_k - h(\hat{x}_{k|k-1})) \\ \Sigma_{k|k} &= \Sigma_{k|k-1} - \gamma_k K_k C_k \Sigma_{k|k-1} \end{aligned} \quad (4)$$

and the time update is the same as the EKF, i.e.,

$$\begin{aligned} \hat{x}_{k+1|k} &= f(\hat{x}_{k|k}, u_k), \\ \Sigma_{k+1|k} &= A_k \Sigma_{k|k} A_k^T + Q, \end{aligned} \quad (5)$$

where the Kalman gain $K_k = \Sigma_{k|k-1} C_k^T (C_k \Sigma_{k|k-1} C_k^T + R)^{-1}$ and $\hat{x}_{0|-1} = \bar{x}_0$, $\Sigma_{0|-1} = \Sigma_0$.

In the present situation, $\{\gamma_k\}$ is *unknown* to the estimator, which renders the above IEKF inapplicable. However, it is still very helpful in designing an effective filter in a recursive form.

III. BAYESIAN KALMAN FILTERS

In this section, we propose two suboptimal recursive filters to solve the filtering problem with unknown sensor measurement losses.

A. Bayesian Kalman Filter I

We design a nonlinear filter called Bayesian Kalman filter I (BKF-I) to recursively compute the state estimate with unknown measurement losses. An intuitive idea, which is motivated by the principle of certainty equivalence, is that we first estimate the measurement losses Γ_k , based on which the IEKF (4) is then applied to compute the state estimate. We shall elaborate it in this subsection.

Since γ_k is binary, it is natural to adopt the maximum a posteriori probability (MAP) estimate, and the MAP estimate of Γ_k is given as follows:

$$\hat{\Gamma}_k = \operatorname{argmax}_{\Gamma_k} p(\Gamma_k | Y_k)$$

where $\hat{\Gamma}_k = \{\hat{\gamma}_0, \dots, \hat{\gamma}_k\}$ and for notional simplicity, we directly use $p(x)$ to denote either the probability density or mass function of a random vector X .

Substitute $\hat{\Gamma}_k$ into (4), i.e., use the estimate $\hat{\gamma}_k$ to replace unknown measurement loss γ_k in (4), we obtain the BKF-I, see Algorithm 1. The remaining problem reduces to the derivation of the MAP estimate of Γ_k . To solve it, we use the Bayes' formulas and obtain that

$$\begin{aligned} p(\Gamma_k | Y_k) &= p(\gamma_k, \Gamma_{k-1} | Y_k) \\ &= \frac{p(\gamma_k, \Gamma_{k-1}, y_k | Y_{k-1}) p(Y_{k-1})}{p(Y_k)}. \end{aligned}$$

To recursively compute the above, we note that

$$\begin{aligned} p(\gamma_k, \Gamma_{k-1}, y_k | Y_{k-1}) &= p(y_k | \gamma_k, \Gamma_{k-1}, Y_{k-1}) p(\gamma_k, \Gamma_{k-1} | Y_{k-1}) \\ &= p(y_k | \gamma_k, \Gamma_{k-1}, Y_{k-1}) p(\gamma_k | \Gamma_{k-1}, Y_{k-1}) p(\Gamma_{k-1} | Y_{k-1}). \end{aligned}$$

This implies that

$$\begin{aligned} p(\Gamma_k | Y_k) &= \frac{p(y_k | \gamma_k, \Gamma_{k-1}, Y_{k-1}) p(\gamma_k | \Gamma_{k-1}, Y_{k-1})}{p(y_k | Y_{k-1})} \\ &\quad \times p(\Gamma_{k-1} | Y_{k-1}). \end{aligned} \quad (6)$$

To compute $p(\Gamma_k | Y_k)$, it requires to consider all possible values of Γ_k , which grows unboundedly with respect to the time steps. Therefore, it is impossible to recursively obtain the MAP estimate of Γ_k . In real applications, recursive algorithms are essential. Thus, we consider to approximately compute $p(\Gamma_k | Y_k)$ in a recursive way. This is achieved by using the estimate $\hat{\Gamma}_{k-1}$ rather than the unknown Γ_{k-1} to estimate γ_k .

By substituting Γ_{k-1} with $\hat{\Gamma}_{k-1}$, it follows from (6) that

$$\begin{aligned} p(\Gamma_k | Y_k) &\approx p(\gamma_k, \hat{\Gamma}_{k-1} | Y_k) \\ &\propto p(y_k | \gamma_k, \hat{\Gamma}_{k-1}, Y_{k-1}) p(\gamma_k | \hat{\Gamma}_{k-1}, Y_{k-1}) p(\hat{\Gamma}_{k-1} | Y_{k-1}). \end{aligned} \quad (7)$$

Since $\hat{\Gamma}_{k-1}$ is already obtained at time step k , our objective is to find γ_k to maximize the approximated posterior probability $p(\gamma_k, \hat{\Gamma}_{k-1} | Y_k)$ in (7). As γ_k is a binary variable, this is easily solved by letting

$$\begin{aligned} \hat{\gamma}_k &= \operatorname{argmax}_{\gamma_k} p(\gamma_k, \hat{\Gamma}_{k-1} | Y_k) \\ &= \begin{cases} 1, & \text{if } p(\gamma_k = 1, \hat{\Gamma}_{k-1} | Y_k) > p(\gamma_k = 0, \hat{\Gamma}_{k-1} | Y_k), \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (8)$$

In view of (7), we further obtain that

$$\begin{aligned} & \frac{p(\gamma_k = 1, \hat{\Gamma}_{k-1}|Y_k)}{p(\gamma_k = 0, \hat{\Gamma}_{k-1}|Y_k)} \\ &= \frac{p(y_k|\gamma_k = 1, \hat{\Gamma}_{k-1}, Y_{k-1})p(\gamma_k = 1|\hat{\Gamma}_{k-1}, Y_{k-1})}{p(y_k|\gamma_k = 0, \hat{\Gamma}_{k-1}, Y_{k-1})p(\gamma_k = 0|\hat{\Gamma}_{k-1}, Y_{k-1})}. \end{aligned} \quad (9)$$

Then, we compute $p(y_k|\gamma_k, \hat{\Gamma}_{k-1}, Y_{k-1})$ and $p(\gamma_k|\hat{\Gamma}_{k-1}, Y_{k-1})$. To obtain $p(y_k|\gamma_k, \hat{\Gamma}_{k-1}, Y_{k-1})$, we denote the probability density function of the Gaussian distribution with mean μ and covariance matrix σ^2 by $\mathcal{N}(\mu, \sigma^2)$. Once $\hat{\Gamma}_k$ is known, it follows from the IEKF that

$$\begin{aligned} & p(y_k|\gamma_k, Y_{k-1}, \hat{\Gamma}_{k-1}) \\ & \approx \begin{cases} \mathcal{N}(h(\hat{x}_{k|k-1}), C_k \Sigma_{k|k-1} C_k^T + R), & \gamma_k = 1, \\ \mathcal{N}(0, R), & \gamma_k = 0, \end{cases} \end{aligned} \quad (10)$$

where $\hat{x}_{k|k-1}$ and $\Sigma_{k|k-1}$ are computed in (4) by replacing $\hat{\Gamma}_{k-1}$ with $\hat{\Gamma}_{k-1}$.

If the prior probability distribution of γ_k is also available, we are able to easily compute $p(\gamma_k|Y_{k-1}, \hat{\Gamma}_{k-1})$. Two common cases are illustrated below.

(a) $\{\gamma_k\}$ is a Bernoulli process with parameter θ , then

$$p(\gamma_k = 1|Y_{k-1}, \hat{\Gamma}_{k-1}) = p(\gamma_k = 1) = \theta.$$

(b) $\{\gamma_k\}$ is a Markov process with the transition probability matrix [22]

$$p(\gamma_k = j|\gamma_{k-1} = i) = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}, i, j \in \{0, 1\},$$

then $p(\gamma_k|Y_{k-1}, \hat{\Gamma}_{k-1}) = p(\gamma_k|\hat{\gamma}_{k-1})$.

Finally, we can compute $p(\gamma_k, \hat{\Gamma}_{k-1}|Y_k)$, and thus obtain the MAP estimate $\hat{\gamma}_k$ by (8) and (9). The estimate $\hat{x}_{k|k}$ is then updated from (4) by replacing γ_k with $\hat{\gamma}_k$.

Overall, the BKF-I is summarized in Algorithm 1. Compared with the IEKF, we need to further compute the MAP estimate of γ_k . The good news is that its computational complexity is still comparable to that of the IEKF.

Algorithm 1 Bayesian Kalman filter I

- 1) **Initialization:** Let $\hat{x}_{0|-1} = \bar{x}_0, \Sigma_{0|-1} = \Sigma_0$.
 - 2) **MAP estimate of γ_k :** Given $y_k, \hat{x}_{k|k-1}, \Sigma_{k|k-1}$ and $\hat{\Gamma}_{k-1}$, compute (9) and approximately obtain the MAP estimate of γ_k by (8).
 - 3) **Measurement update:** Given y_k and $\hat{\gamma}_k$, update the state estimate and its estimation error covariance matrix via (4) by replacing γ_k with $\hat{\gamma}_k$.
 - 4) **Time update:** Update the state prediction via (5).
-

B. Bayesian Kalman Filter II

In this subsection, we design the second suboptimal filter by approximately computing the posterior distribution of the measurement loss process $\{\gamma_k\}$.

Consider the minimum variance estimate, which is given by the conditional expectation

$$\hat{x}_{k|k} = \mathbb{E}[x_k|Y_k] = \int x_k p(x_k|Y_k) dx_k.$$

Since the system is nonlinear and non-Gaussian, the EKF cannot be applied to compute the optimal estimate. Since the posterior distribution $p(x_k|Y_k)$ is not tractable, the optimal filter is unavailable or cannot be implemented recursively. Note that a recursive suboptimal filter is necessary in practice. To this end, we adopt the following approximation

$$p(x_k|Y_k) \approx p(x_k|y_k, \hat{x}_{k|k-1}).$$

That is, we use $\hat{x}_{k|k-1}$ to synthesize the information contained in Y_{k-1} to compute the posterior distribution. Then, it follows that

$$\begin{aligned} \hat{x}_{k|k} &= \mathbb{E}[x_k|Y_k] \approx \mathbb{E}[x_k|y_k, \hat{x}_{k|k-1}], \\ \Sigma_{k|k} &= \mathbb{E}[(x_k - \mathbb{E}(x_k|Y_k))(x_k - \mathbb{E}(x_k|Y_k))^T|Y_k] \\ &\approx \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T|\hat{x}_{k|k-1}, y_k]. \end{aligned}$$

Besides, one can derive that

$$\begin{aligned} & p(x_k|\hat{x}_{k|k-1}, y_k) \\ &= \sum_{i=0}^1 p(x_k|\hat{x}_{k|k-1}, \gamma_k = i, y_k) p(\gamma_k = i|\hat{x}_{k|k-1}, y_k) \\ &= \lambda_k p(x_k|\gamma_k = 1, \hat{x}_{k|k-1}, y_k) \\ &\quad + (1 - \lambda_k) p(x_k|\gamma_k = 0, \hat{x}_{k|k-1}, y_k) \end{aligned}$$

where λ_k denotes the probability of event $\{\gamma_k = 1\}$ conditioned on $\hat{x}_{k|k-1}$ and y_k , and is given by

$$\begin{aligned} \lambda_k &= p(\gamma_k = 1|\hat{x}_{k|k-1}, y_k) \\ &= \frac{p(y_k|\gamma_k = 1, \hat{x}_{k|k-1})p(\gamma_k = 1|\hat{x}_{k|k-1})}{\sum_{j=0}^1 p(y_k|\gamma_k = j, \hat{x}_{k|k-1})p(\gamma_k = j|\hat{x}_{k|k-1})}, \end{aligned} \quad (11)$$

where $p(\gamma_k = 1|\hat{x}_{k|k-1})$ is approximated by λ_{k-1} .

Now we compute $p(y_k|\gamma_k, \hat{x}_{k|k-1})$. Consider a Gaussian approximation $p(x_k|\hat{x}_{k|k-1}) \approx \mathcal{N}(\hat{x}_{k|k-1}, \Sigma_{k|k-1})$, it yields that

$$\begin{aligned} & p(y_k|\gamma_k, \hat{x}_{k|k-1}) \\ & \approx \begin{cases} \mathcal{N}(h(\hat{x}_{k|k-1}), C_k \Sigma_{k|k-1} C_k^T + R), & \gamma_k = 1, \\ \mathcal{N}(0, R), & \gamma_k = 0. \end{cases} \end{aligned} \quad (12)$$

Adopting the extended Kalman filtering technique [23], we obtain that

$$p(x_k|\gamma_k, \hat{x}_{k|k-1}, y_k) \approx \begin{cases} \mathcal{N}(\mu_1, \sigma_1^2), & \gamma_k = 1, \\ \mathcal{N}(\mu_2, \sigma_2^2), & \gamma_k = 0, \end{cases}$$

where the mean and covariance are respectively given by

$$\begin{aligned} \mu_1 &= \hat{x}_{k|k-1} + K_k(y_k - h(\hat{x}_{k|k-1})), \\ \sigma_1^2 &= \Sigma_{k|k-1} - \Sigma_{k|k-1} C_k^T (C_k \Sigma_{k|k-1} C_k^T + R)^{-1} C_k \Sigma_{k|k-1}, \\ \mu_2 &= \hat{x}_{k|k-1}, \\ \sigma_2^2 &= \Sigma_{k|k-1}. \end{aligned} \quad (13)$$

Combining the above, the measurement update for the minimum variance estimate is approximately given as

$$\begin{aligned}
\hat{x}_{k|k} &\approx \int x_k p(x_k|y_k, \hat{x}_{k|k-1}) dx_k \\
&= \lambda_k \int x_k p(x_k|\gamma_k = 1, \hat{x}_{k|k-1}, y_k) dx_k \\
&\quad + (1 - \lambda_k) \int x_k p(x_k|\gamma_k = 0, \hat{x}_{k|k-1}, y_k) dx_k \\
&= \lambda_k \mu_1 + (1 - \lambda_k) \mu_2 \\
&= \hat{x}_{k|k-1} + \lambda_k K_k (y_k - h(\hat{x}_{k|k-1})).
\end{aligned}$$

where the last equality follows from (13).

To run the algorithm recursively, we still need to update $\Sigma_{k|k}$ as well, which is derived below.

$$\begin{aligned}
\Sigma_{k|k} &\approx \mathbb{E}[(x_k - \mathbb{E}(x_k))(x_k - \mathbb{E}(x_k))^T | \hat{x}_{k|k-1}, y_k] \\
&= \mathbb{E}[x_k x_k^T | \hat{x}_{k|k-1}, y_k] - \mathbb{E}[x_k | \hat{x}_{k|k-1}, y_k] \mathbb{E}[x_k | \hat{x}_{k|k-1}, y_k]^T \\
&\approx \mathbb{E}[x_k x_k^T | \hat{x}_{k|k-1}, y_k] - \hat{x}_{k|k} \hat{x}_{k|k}^T.
\end{aligned}$$

The first term of $\Sigma_{k|k}$ is further written as

$$\begin{aligned}
\mathbb{E}[x_k x_k^T | \hat{x}_{k|k-1}, y_k] &= \int x_k x_k^T p(x_k | \hat{x}_{k|k-1}, y_k) dx_k \\
&= \lambda_k \int x_k x_k^T p(x_k | \gamma_k = 1, \hat{x}_{k|k-1}, y_k) dx_k + \\
&\quad (1 - \lambda_k) \int x_k x_k^T p(x_k | \gamma_k = 0, \hat{x}_{k|k-1}, y_k) dx_k \\
&= \lambda_k \mathbb{E}[x_k x_k^T | \gamma_k = 1, \hat{x}_{k|k-1}, y_k] \\
&\quad + (1 - \lambda_k) \mathbb{E}[x_k x_k^T | \gamma_k = 0, \hat{x}_{k|k-1}, y_k] \\
&= \lambda_k (\mu_1 \mu_1^T + \sigma_1^2) + (1 - \lambda_k) (\mu_2 \mu_2^T + \sigma_2^2).
\end{aligned}$$

Finally, $\Sigma_{k|k}$ is updated by the following formula

$$\begin{aligned}
\Sigma_{k|k} &\approx \mathbb{E}[x_k x_k^T | \hat{x}_{k|k-1}, y_k] - \hat{x}_{k|k} \hat{x}_{k|k}^T \\
&= \lambda_k (\mu_1 \mu_1^T + \sigma_1^2) + (1 - \lambda_k) (\mu_2 \mu_2^T + \sigma_2^2) \\
&\quad - [\lambda_k \mu_1 + (1 - \lambda_k) \mu_2][\lambda_k \mu_1 + (1 - \lambda_k) \mu_2]^T \\
&= \lambda_k (1 - \lambda_k) (\mu_1 - \mu_2) (\mu_1 - \mu_2)^T + \lambda_k \sigma_1^2 + (1 - \lambda_k) \sigma_2^2 \\
&= \Sigma_{k|k-1} - \lambda_k K_k C_k \Sigma_{k|k-1} + \lambda_k (1 - \lambda_k) \\
&\quad \times [K_k (y_k - h(\hat{x}_{k|k-1})) (y_k - h(\hat{x}_{k|k-1}))^T K_k^T].
\end{aligned}$$

Overall, the BKF-II is summarized in Algorithm 2. In the BKF-II, we use λ_k to represent the uncertainty induced by the measurement losses. By (11), it is clear that if γ_k is known to the estimator, then $\lambda_k = \gamma_k$, which reduces to the IEKF. As γ_k is *unknown* in our case, λ_k plays the role of estimating γ_k .

Note again that approximation is adopted to derive the recursive filter. However, from the storage and computation points of view, this suboptimal filter may be better than an optimal one in applications, since an optimal minimum variance estimate conditioned on Y_k is generally not tractable.

Algorithm 2 Bayesian Kalman filter II

- 1) **Initialization:** Let $\hat{x}_{0|-1} = \bar{x}_0, \Sigma_{0|-1} = \Sigma_0$.
- 2) **Measurement update:** The state estimate and its approximated error covariance matrix are updated as follows.

$$\begin{aligned}
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + \lambda_k K_k (y_k - h(\hat{x}_{k|k-1})), \\
\Sigma_{k|k} &= \Sigma_{k|k-1} - \lambda_k K_k C_k \Sigma_{k|k-1} + \lambda_k (1 - \lambda_k) \\
&\quad [K_k (y_k - h(\hat{x}_{k|k-1})) (y_k - h(\hat{x}_{k|k-1}))^T K_k^T]
\end{aligned}$$

where λ_k is given in (11).

- 3) **Time update:** Update the state prediction via (5).
-

IV. RAO-BLACKWELLISED PARTICLE FILTER

Clearly, both the BKF-I and BKF-II are not the minimum variance estimate and their performance cannot be guaranteed due to the use of approximation. In this section, a numerical method called Rao-Blackwellised particle filter (RBPf) is applied to approximately compute the minimum variance estimate. While the RBPf is computationally more demanding than the BKF-I and BKF-II, its estimation performance can be improved by increasing the number of particles.

A. RB Particle Filter

Recall that a minimum variance filter of x_k is expressed by

$$\hat{x}_{k|k} = \int x_k p(x_k | Y_k) dx_k = \iint x_k p(x_k, \Gamma_k | Y_k) dx_k d\Gamma_k. \quad (14)$$

Since $p(x_k, \Gamma_k | Y_k)$ is not Gaussian, it is impossible to be analytically obtained, even for linear systems. Then, the integral is not computable and we have to resort to a numerical approach.

In this section, we choose a particle based algorithm to approximate the integral in (14). The particle filter (PF) is a powerful sampling based method to approximate any probability distribution, and address the non-linearity and non-Gaussianity problem. However, it is acknowledged that the number of particles grows dramatically with the dimension of the underlying random vector, which substantially increases the computation load.

In our case, if the particles are directly used to approximate $p(x_k, \Gamma_k | Y_k)$, it will result in a significant waste of particles since the unconditional distribution of the state for the linearized model in (3) is deemed to be Gaussian, and the non-linearity and/or non-Gaussianity only appear in the measurement equation, which results from the unknown measurement losses. To well exploit this observation, we shall adopt the Rao-Blackwellised particle filter (RBPf) and use all the particles to approximate the conditional distribution of the binary variable γ_k . Intuitively, a few particles are sufficient to accomplish it. Furthermore, it follows from [19] that the RBPf leads to better estimation performance than the standard PF.

To exposit it, it follows from the conditional probability definition that

$$p(x_k, \Gamma_k | Y_k) = p(x_k | \Gamma_k, Y_k) p(\Gamma_k | Y_k)$$

where $p(x_k | \Gamma_k, Y_k)$ is an approximated Gaussian density and is recursively computed via the IEKF.

However, $p(\Gamma_k|Y_k)$ is difficult to obtain. As Γ_k is a binary sequence, we use N particles $\{\Gamma_k^i\}_{i=1}^N$ that are drawn from an importance density $q(\Gamma_k|Y_k)$ to approximate $p(\Gamma_k|Y_k)$, i.e.,

$$p(\Gamma_k|Y_k) \approx \sum_{i=1}^N \omega_k^i \delta(\Gamma_k - \Gamma_k^i), \quad (15)$$

where $\delta(\cdot)$ is the standard Dirac delta function and ω_k^i is the normalized particle weight associated with Γ_k^i , i.e.,

$$\omega_k^i \propto \frac{p(\Gamma_k^i|Y_k)}{q(\Gamma_k^i|Y_k)}.$$

Inserting (15) into (14), we obtain that

$$\begin{aligned} \hat{x}_{k|k} &= \iint x_k p(x_k, \Gamma_k|Y_k) dx_k d\Gamma_k \\ &= \iint x_k p(x_k|\Gamma_k, Y_k) dx_k p(\Gamma_k|Y_k) d\Gamma_k \\ &\approx \sum_{i=1}^N \omega_k^i \int x_k p(x_k|\Gamma_k^i, Y_k) dx_k \\ &= \sum_{i=1}^N \omega_k^i \mathbb{E}[x_k|\Gamma_k^i, Y_k] = \sum_{i=1}^N \omega_k^i \hat{x}_{k|k}^i, \end{aligned} \quad (16)$$

where $\hat{x}_{k|k}^i = \mathbb{E}[x_k|\Gamma_k^i, Y_k]$.

Similarly, let $\Sigma_{k|k}^i = \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T|\Gamma_k^i, Y_k]$, then the estimation error covariance matrix is given as

$$\begin{aligned} \Sigma_{k|k} &\approx \sum_{i=1}^N \omega_k^i \mathbb{E}[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T|\Gamma_k^i, Y_k] \\ &= \sum_{i=1}^N \omega_k^i \Sigma_{k|k}^i. \end{aligned} \quad (17)$$

It should be noted from (3) that both (16) and (17) can be recursively computed by using the IEKF. Specifically, we obtain that

$$\begin{aligned} \hat{x}_{k|k}^i &= \hat{x}_{k|k-1}^i + \gamma_k^i K_k^i (y_k - h(\hat{x}_{k|k-1}^i)), \\ \Sigma_{k|k}^i &= \Sigma_{k|k-1}^i - \gamma_k^i K_k^i C_k \Sigma_{k|k-1}^i. \end{aligned}$$

The remaining problem is how to recursively generate particles $\{\Gamma_k^i\}$ and compute their associated weights $\{\omega_k^i\}$. This is resolved in next subsection.

B. Importance Density

If an importance density is chosen to factorize such that

$$q(\Gamma_k|Y_k) = q(\gamma_k|\Gamma_{k-1}, Y_k) q(\Gamma_{k-1}|Y_{k-1}), \quad (18)$$

one can obtain particles $\Gamma_k^i \sim q(\Gamma_k|Y_k)$ by augmenting each existing particle $\Gamma_{k-1}^i \sim q(\Gamma_{k-1}|Y_{k-1})$ with the new state $\gamma_k^i \sim q(\gamma_k|\Gamma_{k-1}^i, Y_k)$ and obtain a recursive filter algorithm. To elaborate it, we express $p(\Gamma_k|Y_k)$ in the following form

$$p(\Gamma_k|Y_k) \propto p(y_k|Y_{k-1}, \Gamma_k) p(\gamma_k|\Gamma_{k-1}, Y_{k-1}) p(\Gamma_{k-1}|Y_{k-1}).$$

Jointly with (18), it implies that

$$\begin{aligned} w_k^i &\propto \frac{p(y_k|Y_{k-1}, \Gamma_k^i) p(\gamma_k^i|\Gamma_{k-1}^i, Y_{k-1}) p(\Gamma_{k-1}^i|Y_{k-1})}{q(\gamma_k^i|\Gamma_{k-1}^i, Y_k) q(\Gamma_{k-1}^i|Y_{k-1})} \\ &= w_{k-1}^i \frac{p(y_k|Y_{k-1}, \Gamma_k^i) p(\gamma_k^i|\Gamma_{k-1}^i, Y_{k-1})}{q(\gamma_k^i|\Gamma_{k-1}^i, Y_k)}. \end{aligned} \quad (19)$$

A nice feature of the above is that $p(y_k|Y_{k-1}, \Gamma_k^i)$ is approximately a conditional Gaussian density and is recursively computed by IEKF, i.e.,

$$\begin{aligned} p(y_k|Y_{k-1}, \Gamma_k^i) &\approx \begin{cases} \mathcal{N}(h(\hat{x}_{k|k-1}^i), C_k \Sigma_{k|k-1}^i C_k^T + R), & \gamma_k^i = 1, \\ \mathcal{N}(0, R), & \gamma_k^i = 0. \end{cases} \end{aligned}$$

To alleviate the degeneracy problem, which is key to the success of PFs, there are two good choices of the importance density $q(\gamma_k|\Gamma_{k-1}^i, Y_k)$ in the literature [17]. One is $p(\gamma_k|\Gamma_{k-1}^i, Y_k)$, which minimizes a suitable measure of the degeneracy of the algorithm, i.e.,

$$N_{\text{eff}} = \frac{N}{1 + \text{var}(w_k^{*i})}$$

where $w_k^{*i} = p(\gamma_k^i|Y_k)/q(\gamma_k^i|\Gamma_{k-1}^i, Y_k)$ is referred to as the true importance weights.

The other is $p(\gamma_k|\gamma_{k-1}^i)$, which makes it easy to draw particles and compute the important weights. Since $p(\gamma_k|\Gamma_{k-1}, Y_k)$ is usually difficult to access, we choose $q(\gamma_k|\Gamma_{k-1}, Y_k) = p(\gamma_k|\gamma_{k-1})$ as the importance density in this work. That is, the new particle is generated from the following distribution

$$\gamma_k^i \sim p(\gamma_k|\gamma_{k-1}^i).$$

If $\{\gamma_k\}$ is a Bernoulli process or Markov process, the update equation for the particle weight in (19) becomes particularly simple. Specifically,

$$\begin{aligned} \omega_k^i &\propto \frac{p(\Gamma_k^i|Y_k)}{q(\Gamma_k^i|Y_k)} = \omega_{k-1}^i \frac{p(y_k|Y_{k-1}, \Gamma_k^i) p(\gamma_k^i|\gamma_{k-1}^i)}{q(\gamma_k^i|\Gamma_{k-1}^i, Y_k)} \\ &= \omega_{k-1}^i p(y_k|Y_{k-1}, \Gamma_k^i). \end{aligned}$$

In the current situation, the number of particles is very small since γ_k is binary. The RBPF with a resampling step is summarized in Algorithm 3.

C. Fast Implementation

The major computation of the RBPF is taken in the **Output** step in Algorithm 3, which requires an IEKF iteration for each triple $(\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i)$. Thus, the computational complexity essentially is proportional to the number of particles. When the number of particles is very large, one may observe that there are many duplicate triples after the **Resampling** c), i.e., there may be only $m \ll N$ triples that take different values. We can take advantage of this observation to reduce the computational complexity of RBPF.

Instead of directly executing an IEKF iteration for each triple $(\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i)$, we first remove the duplicate triples from the list and obtain a set of triples that take different values, see step a) in Algorithm 4, and then do an IEKF iteration for each of them, see step b) in Algorithm 4. Finally, we set the updated value for the whole list, see step c).

Clearly, Algorithm 4 returns the same estimate and prediction as those of RBPF in Algorithm 3. It may significantly reduce the computational complexity of the RBPF, especially when the number of particles is large. Besides, the reduction of the computational complexity reflects the effectiveness

Algorithm 3 Rao-Blackwellised particle filter

- 1) **Initialization:** For $i = 1, \dots, N$, draw N particles γ_0^i from the prior $p(\gamma_0)$ and let $\hat{x}_{-1|0}^i = x_0, \Sigma_{-1|0}^i = \Sigma_0$.
- 2) **Importance sampling:**
 - a) For $i \in \{1, \dots, N\}$ and $k > 0$, draw N particles from the importance distribution $\gamma_k^i \sim p(\gamma_k | \gamma_{k-1}^i)$.
 - b) For $i \in \{1, \dots, N\}$ and given y_k , update the normalized importance weights $\omega_k^i \propto \omega_{k-1}^i p(y_k | Y_{k-1}, \Gamma_k^i)$.
- 3) **Resampling:**
 - a) Compute an estimate of the effective number of particles

$$\hat{N}_{\text{eff}} = \frac{1}{\sum_{i=1}^N (\omega_k^i)^2}.$$
 - b) If $\hat{N}_{\text{eff}} < N_T$, which is a prescribed threshold, perform resampling. Take N new samples $(\gamma_k^{i*}, \hat{x}_{k|k-1}^{i*}, \Sigma_{k|k-1}^{i*})$ with replacement from the list $(\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i)$, $i \in \{1, \dots, N\}$ according to the probability distribution that $\Pr\{\gamma_k^{i*} = \gamma_k^i\} = \omega_k^i$.
 - c) Let $(\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i) = (\gamma_k^{i*}, \hat{x}_{k|k-1}^{i*}, \Sigma_{k|k-1}^{i*})$ and $\omega_k^i = 1/N$ for all $i \in \{1, \dots, N\}$.
- 4) **Output**
 - a) For each triple $(\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i)$ and y_k , do

$$\begin{aligned} K_k^i &= \Sigma_{k|k-1}^i C_k^T (C_k \Sigma_{k|k-1}^i C_k^T + R)^{-1} \\ \hat{x}_{k|k}^i &= \hat{x}_{k|k-1}^i + \gamma_k^i K_k^i (y_k - h(\hat{x}_{k|k-1}^i)) \\ \Sigma_{k|k}^i &= \Sigma_{k|k-1}^i - \gamma_k^i K_k^i C_k \Sigma_{k|k-1}^i. \end{aligned}$$

- b) Measurement update

$$\hat{x}_{k|k} = \sum_{i=1}^N \omega_k^i \hat{x}_{k|k}^i, \quad \Sigma_{k|k} = \sum_{i=1}^N \omega_k^i \Sigma_{k|k}^i.$$

- c) Update the state prediction by (5), and set

$$\hat{x}_{k+1|k}^i = f(\hat{x}_{k|k}^i, u_k), \quad \Sigma_{k+1|k}^i = A_k \Sigma_{k|k}^i A_k^T + Q.$$

- 5) **Let** $k = k + 1$.
-

of particles to some extent. However, when the number of particles is small, the fast implementation may be not efficient as it further requires to remove duplicate triples. This will be illustrated in Section VI-A by an numerical example.

V. COMPARISONS OF THE THREE FILTERS

This section provides some comparisons of the proposed filters in terms of the estimation accuracy and computational complexity. We shall further verify the major results of this section by numerical examples in the next section.

A. Estimation Accuracy

If both f and h in (2) are linear, any filter cannot outperform the optimal IKF, which however requires $\{\gamma_k\}$ is known. The performance of the RBPF can approach that of the IKF arbitrarily well by increasing the number of particles, and hence has a predictable performance. This cannot be guaranteed for the BKF-I and BKF-II due to approximations in deriving the recursive filters. However, if $y_k - h(\hat{x}_{k|k-1})$ is very different

Algorithm 4 Fast RBPF

The **Output** step in Algorithm 3 is replaced as follows.

- a) For the list of triples $(\gamma_k^j, \hat{x}_{k|k-1}^j, \Sigma_{k|k-1}^j)$, $j \in \{1, \dots, N\}$, remove duplicate triples and obtain an index set $\mathcal{I}$ such that there is no duplicate triple in the set $\{(\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i), i \in \mathcal{I}\}$.
- b) For each triple $(\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i)$, $i \in \mathcal{I}$ and y_k , do

$$\begin{aligned} K_k^i &= \Sigma_{k|k-1}^i C_k^T (C_k \Sigma_{k|k-1}^i C_k^T + R)^{-1} \\ \hat{x}_{k|k}^i &= \hat{x}_{k|k-1}^i + \gamma_k^i K_k^i (y_k - h(\hat{x}_{k|k-1}^i)) \\ \Sigma_{k|k}^i &= \Sigma_{k|k-1}^i - \gamma_k^i K_k^i C_k \Sigma_{k|k-1}^i \\ \hat{x}_{k+1|k}^i &= f(\hat{x}_{k|k}^i, u_k), \\ \Sigma_{k+1|k}^i &= A_k \Sigma_{k|k}^i A_k^T + Q. \end{aligned}$$

- c) For each $j \in \{1, \dots, N\}$, set

$$\begin{aligned} (\hat{x}_{k+1|k}^j, \hat{x}_{k|k}^j, \hat{\Sigma}_{k+1|k}^j, \hat{\Sigma}_{k|k}^j) \\ = (\hat{x}_{k+1|k}^i, \hat{x}_{k|k}^i, \hat{\Sigma}_{k+1|k}^i, \hat{\Sigma}_{k|k}^i) \end{aligned}$$

if $(\gamma_k^j, \hat{x}_{k|k-1}^j, \Sigma_{k|k-1}^j) = (\gamma_k^i, \hat{x}_{k|k-1}^i, \Sigma_{k|k-1}^i)$ for some $i \in \mathcal{I}$.

- d) Do measurement update

$$\hat{x}_{k|k} = \sum_{i=1}^N \omega_k^i \hat{x}_{k|k}^i, \quad \Sigma_{k|k} = \sum_{i=1}^N \omega_k^i \Sigma_{k|k}^i,$$

and update the state prediction by (5).

from 0, the measurement loss γ_k can be estimated correctly with high probability. Then, both the BKF-I and BKF-II are likely to approach the IKF. Compared to the BKF-II, the BKF-I is more intuitive and easier to implement. The BKF-II appears to be more stable as λ_k retains the probability information of $\{\gamma_k = 1\}$. Since the system in (2) is nonlinear under the unknown measurement losses, these observations are impossible to prove in theory. We shall provide numerical examples for validation in the next section.

If either f or h in (2) is nonlinear, our filters first linearize the nonlinear function and then applies the corresponding algorithms to the system in (3). Due to the approximation in the linearization step, the IEKF is not optimal even it uses the true Γ_k , and all our filters might have better performance than the IEKF. Note that even when there is no measurement loss, i.e., $\gamma_k = 1$ for all k , the estimation accuracy of the EKF cannot be guaranteed. By using a sufficiently large number of particles, the estimation accuracy of the RBPF may even outperform that of the IEKF. Since approximation is used twice in both the BKF-I and BKF-II, it is expected that the IEKF and RBPF usually outperform the BKF-I and BKF-II.

B. Computational Complexity

We provides Table I to compare the computational complexities of the three proposed filters. The computation for linearization in deriving the system in (3) is omitted since all the proposed filters require it.

TABLE I
COMPUTATIONAL COMPLEXITIES IN ONE ITERATION

Filters	IEKF	RBPF	BKF-I	BKF-II
Number of times of Gaussian PDF evaluations	0	$2N$	2	2
Number of times of matrix multiplications	7	$O(N)$	7	9
Computational complexity	$O(n^3)$	$O(Nn^3)$	$O(n^3)$	$O(n^3)$

^a N is the number of particles in RBPF.

^b n is the dimension of state vectors.

The computational complexity can be quantified by the number of times of evaluating the Gaussian probability density function (PDF) and the matrix multiplications in one iteration. It is apparent that the RBPF has the highest computational complexity which is proportional to the number of particles and is far more than that of BKF-I and BKF-II. When N is large, the fast RBPF might significantly reduce the computational complexity, and will be illustrated by numerical examples in the next section.

The BKF-I and BKF-II have similar computational complexities, and both require additionally two Gaussian PDF evaluations than that of the IEKF. Generally speaking, the computational complexity of evaluating a Gaussian PDF is $O(n^3)$, where n is the dimension of the random vector and is the dimension of x_k here. Since the computational complexity of multiplication of two $n \times n$ matrices is also $O(n^3)$ in general, both the BKF-I and BKF-II have the same computational complexity $O(n^3)$, and are similar to that of the IEKF and EKF.

C. Filter Selection

The filter selection depends on many factors, and usually requires to find a good tradeoff between the estimation accuracy and the computational complexity. The tradeoff between the estimation accuracy and the computational complexity is illustrated in Section VI-A. If the computation resource is abundant, it is recommended to use the RBPF with a sufficient number of particles to obtain a good estimation accuracy. If not, we may consider to use the BKF-I or BKF-II. Note that whether one filter is always superior to the other two filters is not conclusive.

VI. APPLICATION EXAMPLES

In this section we adopt the proposed filters to solve three estimation problems. The first one is a state estimation problem of a linear system with randomly unknown measurement losses. In this example, it does not need to do linearization as in (3). Then, the approximation in both BKF-I and BKF-II is solely used to handle the measurement loss process $\{\gamma_k\}$ in a recursive way. We compare the performance of proposed filters and show the relations between the number of particles, the computation time, and the root-mean-square error (RMSE) of the RBPF.

The second one is a target tracking problem, which is a nonlinear system with measurement losses, and illustrates the

effectiveness of our filters. We also provide a case where our filters do not work very well and give some explanations of it in this example. The last example is a quadrotor's path control problem with random measurement losses, where the quadrotor's states estimated by our filters are used in feedback to control a quadrotor. This requires precise state estimates, otherwise the system may diverge quickly. The result shows the quadrotor can complete the task well under all our filters.

A. Linear System with Random Measurement Losses

Consider the following system

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 0.6 & 0.4 \\ 0.1 & 0.9 \end{bmatrix} x_k + w_k, \\ y_k &= \gamma_k \begin{bmatrix} 1 & -2 \end{bmatrix} x_k + v_k, \end{aligned} \quad (20)$$

where w_k and v_k are independent Gaussian noise with zero means, and the covariance matrices are both identity matrices. $\{\gamma_k\}$ is an i.i.d. Bernoulli process with $Pr\{\gamma_k = 0\} = p$. The initial state is also a Gaussian random vector with zero mean and identity covariance matrix. Since the open-loop poles of the system (20) are $\lambda_1 = 0.5$ and $\lambda_2 = 1$, it follows from [13] that the optimal IKF is expected to be mean square stable if the measurement loss level p is strictly less than one.

To validate the proposed filters, we use the same sequence of noisy observations Y_{200} for each filter, and the RBPF is implemented by using 20 particles. We also compare them with the optimal IKF, which requires to know the measurement loss, i.e., we simply set $\gamma_k = 1$ for all k in (4) and (5), which is denoted as 'KF' in Fig. 1. To compute the RMSE of each estimate where the RMSE of an estimate $\hat{x}_{k|k}$ at time k is defined as $(E[\|x_k - \hat{x}_{k|k}\|^2])^{1/2}$, we adopt the Monte Carlo method with 500 independent experiments. Fig. 1 illustrates that the estimation accuracy of the proposed three filters comes close to that of the optimal IKF, and is much better than that of the 'KF'. Moreover, the larger probability of the measurement losses, the better of the improvement of the estimation accuracy over the 'KF'. This clearly shows the advantage of considering the measurement losses in the filter design.

In Fig. 2, we illustrate how the number of particles affects the computational complexity and the estimation accuracy of the RBPF. One can easily observe that the computational complexity per iteration of the RBPF is proportional to the number of particles, which validates the result for the RBPF in Table I. It also verifies that the fast implementation of RBPF in Algorithm 4 can greatly reduce the computation load when N is large, while it is slightly slower than the RBPF for a small number of particles. The reason is that the fast RBPF further requires 'find and set' steps, see steps a) and c) in Algorithm 4, which are dominated by the computation for executing the IEKF iteration if the number of particles is large. Moreover, Fig. 2 reveals that the sum of RMSE reduces quickly when N is smaller, and slowly when N is large. This suggests the existence of a critical number of particles that is vital to the tradeoff between the estimation accuracy and the computational complexity. Note that the critical number is also related to the levels of random measurement losses.

Fig. 1. The sum of RMSE over all the 200 time steps under different levels of measurement losses.

Fig. 2. Computation time per iteration and RMSE of the RBPF with respect to the number of particles. The time is measured using Matlab.

B. Target Tracking

In this example, we test our filters on a target tracking problem, which further requires the linearization in (3). Consider a 2D target tracking problem where a radar is positioned on the ground to measure the distance and the angle of a moving target to the base station. The radar measurements are then sent to an unmanned tracking aircraft via a lossy network, which is subject to unknown measurement losses.

The motion and measurement equations are described by

$$\begin{aligned} x_{k+1}^{(i)} &= Fx_k^{(i)} + u_k^{(i)} + w_k^{(i)}, i \in \{1, 2\} \\ y_k &= \gamma_k \begin{bmatrix} r \\ \phi \end{bmatrix} + \begin{bmatrix} v_k^r \\ v_k^\phi \end{bmatrix} \end{aligned} \quad (21)$$

where $x_k^{(i)} = [p_k^{(i)}, \dot{p}_k^{(i)}, \ddot{p}_k^{(i)}]^T$ denotes the target state at time step k , including the target position, speed and acceleration in the i -th dimension. The system input $u_k^{(i)}$ is given as a prior, and the system matrices are

$$F = \begin{bmatrix} 1 & \tau & \tau^2/2 \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} r \\ \phi \end{bmatrix} = \begin{bmatrix} \sqrt{(p_k^{(1)})^2 + (p_k^{(2)})^2} \\ \arctan \frac{p_k^{(2)}}{p_k^{(1)}} \end{bmatrix},$$

TABLE II
PARAMETERS IN THE TARGET TRACKING SYSTEM

Parameter	Description	Value
τ	Sampling period	0.01s
σ_m	Variance of target acceleration	4m/s ²
α	Reciprocal of the maneuver (acceleration) time constant	1
σ_ϕ	Standard deviation of angular measurement noise	5°
σ_r	Standard deviation of distance measurement noise	5m
p	Probability of measurement losses	0.1, 0.3, 0.5, 0.7

where τ is the sampling period.

Here the measurement loss process $\{\gamma_k\}$ is also a Bernoulli process, which represents the loss of the radar measurement. The input noise $w_k^{(i)}$ and the output noise $v_k = [v_k^r, v_k^\phi]^T$ are independently Gaussian distributed with zero mean. As in [24] and [20], the covariance matrix of w_k is set to

$$Q = 2\alpha\sigma_m^2 \begin{bmatrix} \tau^5/20 & \tau^4/8 & \tau^3/6 \\ \tau^4/8 & \tau^3/3 & \tau^2/2 \\ \tau^3/6 & \tau^2/2 & \tau \end{bmatrix}$$

and the covariance matrix of v_k is given as

$$R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}.$$

The initial state $x_0^{(i)}$ is a Gaussian random vector with mean $\bar{x}_0^{(i)} = [10, 0, 0]^T$, $i \in \{1, 2\}$ and covariance

$$\Sigma_0 = \begin{bmatrix} \sigma_r^2 & \sigma_r^2/\tau & 0 \\ \sigma_r^2/\tau & 2\sigma_r^2/\tau^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

As in (3), we obtain the linearized measurement equation

$$y_k = \gamma_k \begin{bmatrix} \frac{p_{k|k-1}^{(1)}}{r_{k|k-1}} & \frac{p_{k|k-1}^{(2)}}{r_{k|k-1}} \\ -\frac{p_{k|k-1}^{(2)}}{r_{k|k-1}^2} & \frac{p_{k|k-1}^{(1)}}{r_{k|k-1}^2} \end{bmatrix} \begin{bmatrix} p_k^{(1)} \\ p_k^{(2)} \end{bmatrix} + v_k + z_k,$$

where z_k is a function of $p_{k|k-1}$. All the parameters are of the same as those in [24], [25], see Table II, and the initial estimate is set to $\hat{x}_{0|0}^{(i)} = [10, 0, 0]^T$, $i \in \{1, 2\}$.

To track the system, we use the proposed three filters and the IEKF, which requires to know the measurement loss, and run 1500 independent experiments for each filter to compute the RMSE of the position per time step. The RBPF adopts 200 particles.

From Fig. 3 and Fig. 4, one can observe that the RMSE of the RBPF is generally smaller than that of the BKF-I and BKF-II, and the BKF-I appears to outperform the BKF-II in this example. However, we cannot conclude that the BKF-I is better than the BKF-II. Note that the computational complexities of these two filters come close to each other. Clearly, the larger the probability of random measurement losses, the larger the RMSE as expected.

It is interesting to observe that the estimation accuracy of the RBPF is better than that of the IEKF in many cases, especially

Fig. 3. The RMSE of position under different levels of random measurement losses.

Fig. 4. The RMSE of position under different levels of random measurement losses, e.g., $p = 0.1$, $p = 0.5$ and $p = 0.8$.

when the probability of measurement losses p is large. It is known that IKF is optimal when both f and h in (2) are linear functions and γ_k is known. Under this case, the proposed filter cannot perform better than the IKF. However, the IEKF is not optimal if either f or h is nonlinear. In such cases, the RBPF may achieve a better performance than the IEKF due to the non-linearity in the measurement equation (21).

Fig. 5a and Fig. 5b show the true and estimated trajectories in one experiment when $p = 0.5$ and $p = 0.7$ respectively. We can observe that our filters track the target well, especially under the low probability of measurement losses.

Fig. 5c indicates that the proposed filters perform unsatisfactorily under $p = 0.9$ and the poor initial estimate, i.e., $\hat{x}_{0|0}^{(i)} = [200, 0, 0]^T$, $i \in \{1, 2\}$. In this case, all the filters track the trajectory badly, including the IEKF. This is due to the non-linearity of the measurement equation and the poor initial state estimate. Once the filters have an inaccurate estimate at time k , the linearization step will further introduce estimation errors and tends to a bad estimate. Similar to the standard EKF, a good initial estimate is essential for nonlinear systems.

To summarize, the performance of the proposed filters varies

TABLE III
PARAMETERS IN THE QUADROTOR'S PATH CONTROL

Parameter	Description	Value
m	Quadrotor mass	1.023kg
J_{xx}, J_{yy}	Moments of inertia	9.5g·m ²
$\sigma_\phi, \sigma_\theta, \sigma_\psi$	Standard deviation of angular measurement noise	0.1°
$\sigma_X, \sigma_Y, \sigma_Z$	Standard deviation of position measurement noise	2m
T	Control frequency	100Hz
p	Measurements' error probability	0.2

from different systems because of the non-linearity of the systems. One can not conclude that one filter always performs better than another. Usually, the RBPF outperforms the other filters by increasing the number of particles.

C. Quadrotor's Path Control

Finally, we test our filters on a quadrotor's path control problem where the state estimate is used for the feedback design. A quadrotor is a very maneuverable unmanned aircraft with four rotors, and has been popular both in research and real applications over last years. The structure of quadrotors is shown in Fig. 6 [26], where $[X, Y, Z]$ and $[\phi, \theta, \psi]$ are position and angular of the quadrotor in the inertia frame, respectively. $[U, V, W]$ and $[P, Q, R]$ are position and angular velocities in the body frame. Due to page limitation, we omit the dynamical model of quadrotors, see [27]–[31] for more details.

The control architecture is illustrated in Fig. 7. The IMU and GPS block provides the noisy measurements of $[\phi, \theta, \psi]$, $[P, Q, R]$ and $[X, Y, Z]$, $[U, V, W]$ respectively. Due to bad GPS data and/or the unstable IMU, an estimator may only receive the true measurements intermittently. To this end, the proposed filters are applied to this situation and the estimated state are further sent to the outer and inner loop PID controller to control both the quadrotor's position and attitude respectively. This is realized by building a Matlab Simulink model based on a Matlab package called Quadcopter Dynamic Modeling and Simulation [26].

Our goal is to control the quadrotor to follow a given triangle path in space, see Fig. 8 (black line), and the projected trajectories in X, Y and Z directions are shown in black dashed lines in Fig. 9. We consider that the IMU and GPS measurements are sent as a single packet, and they will be either simultaneously lost or received by the estimator. Moreover, the probability of measurement losses is set to be $p = 0.2$. Parameters of the PID controller are selected to be the same as those in [26], which however does not consider the problem of measurement losses, and are provided in Table III.

The trajectories of the closed-loop system are shown in Fig. 8 and Fig. 9, where the reference path represents the desired trajectory. The trajectory of 'EKF' denotes the controlled path of using the standard EKF without any measurement loss, and 'IEKF' is that of using the IEKF to estimate the state, which relies on the true value of γ_k at each time step. This is different from 'BKF-I', 'BKF-II', and 'RBPF' where the packet loss

Fig. 5. One run trajectory.

Fig. 6. The structure of a quadrotor [26].

Fig. 7. The control architecture of the quadrotor's path control system.

process $\{\gamma_k\}$ is unknown to the estimator. For comparison, all filters are tested under the same input noise, measurement loss process $\{\gamma_k\}$, initial conditions and covariance matrices.

In view of Fig. 8 and Fig. 9, it is not difficult to observe that all the proposed filters can fulfill the tracking task and their performance comes close to that of the standard EKF without any measurement loss. In contrast, we are unable to control the quadrotor to follow the desired trajectory if we directly apply the EKF update to all time steps without considering the measurement losses, see Fig. 10, where the tracking error diverges.

VII. CONCLUSION

In this paper, we have designed three suboptimal filters to deal with the state estimation problem with unknown

Fig. 8. The trajectory of the controlled quadrotor in space under different filters. The red triangle mark is the end point of the trajectories.

measurement losses. Among these filters, the BKF-I and the BKF-II were established by using the Bayesian point of view and the IEKF. Some approximations were made to obtain recursive forms. The RBPF is a particle filter based numerical method with a relatively small number of particles. All these proposed filters were applied to three different application problems, showing their effectiveness. Future work will focus on the theoretically convergence of the proposed filters.

REFERENCES

- [1] R. Teo, D. M. Stipanovic, and C. J. Tomlin, "Decentralized spacing control of a string of multiple vehicles over lossy datalinks," *IEEE Transactions on Control Systems Technology*, vol. 18, no. 2, pp. 469–473, 2010.
- [2] M. N. Soltani, R. Izadi-Zamanabadi, and R. Wisniewski, "Reliable control of ship-mounted satellite tracking antenna," *IEEE Transactions on Control Systems Technology*, vol. 19, no. 1, pp. 221–228, 2011.
- [3] D. Choukroun and J. L. Speyer, "Mode estimation via conditionally linear filtering: Application to gyro failure monitoring," *Journal of Guidance, Control, and Dynamics*, vol. 35, no. 2, pp. 632–644, 2012.
- [4] X. Yu and J. Jiang, "Hybrid fault-tolerant flight control system design against partial actuator failures," *IEEE Transactions on Control Systems Technology*, vol. 20, no. 4, pp. 871–886, 2012.

Fig. 9. The trajectory of the controlled quadrotor under different filters in X, Y, and Z directions.

Fig. 10. Tracking error of the EKF without considering the measurement losses.

[5] M. Joerger and B. Pervan, "Kalman filter-based integrity monitoring against sensor faults," *Journal of Guidance, Control, and Dynamics*, vol. 36, no. 2, pp. 349–361, 2013.

[6] R. C. Avram, X. Zhang, and J. Muse, "Nonlinear adaptive fault-tolerant quadrotor altitude and attitude tracking with multiple actuator faults," *IEEE Transactions on Control Systems Technology*, 2017.

[7] D. Zhang and D. Ionescu, "A new method for measuring packet loss probability using a Kalman filter," *IEEE Transactions on Instrumentation and Measurement*, vol. 58, no. 2, pp. 488–499, 2009.

[8] N. Kottenstette, J. F. Hall, X. Koutsoukos, J. Szatpanovits, and P. Antsaklis, "Design of networked control systems using passivity," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 3, pp. 649–665, 2013.

[9] B. L. Reed and F. S. Hover, "JLS-PPC: A jump linear system framework for networked control," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 3, pp. 924–939, 2017.

[10] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan,

and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1453–1464, 2004.

[11] J. Wu, G. Shi, B. D. Anderson, and K. H. Johansson, "Kalman filtering over Gilbert-Elliott channels: Stability conditions and critical curve," *IEEE Transactions on Automatic Control*, 2017.

[12] K. You, N. Xiao, and L. Xie, *Analysis and Design of Networked Control Systems*. Springer, 2015.

[13] K. You, M. Fu, and L. Xie, "Mean square stability for Kalman filtering with Markovian packet losses," *Automatica*, vol. 47, no. 12, pp. 2647–2657, 2011.

[14] E. R. Rohr, D. Marelli, and M. Fu, "Kalman filtering with intermittent observations: On the boundedness of the expected error covariance," *IEEE Transactions on Automatic Control*, vol. 59, no. 10, pp. 2724–2738, 2014.

[15] R. Van Der Merwe, A. Doucet, N. De Freitas, and E. Wan, "The unscented particle filter," in *Conference on Neural Information Processing Systems*, vol. 2000. Denver, CO, USA, 2000, pp. 584–590.

[16] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," *Proceedings of the IEEE*, vol. 92, no. 3, pp. 401–422, 2004.

[17] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, 2002.

[18] A. Doucet, N. De Freitas, and N. Gordon, "An introduction to sequential Monte Carlo methods," in *Sequential Monte Carlo Methods in Practice*. Springer, 2001, pp. 3–14.

[19] A. Doucet, N. De Freitas, K. Murphy, and S. Russell, "Rao-Blackwellised particle filtering for dynamic Bayesian networks," in *Proceedings of the Sixteenth Conference on Uncertainty in Artificial Intelligence*. Morgan Kaufmann Publishers Inc., 2000, pp. 176–183.

[20] F. Gustafsson, F. Gunnarsson, N. Bergman, U. Forssell, J. Jansson, R. Karlsson, and P.-J. Nordlund, "Particle filters for positioning, navigation, and tracking," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 425–437, 2002.

[21] J. Zhang and K. You, "Kalman filtering with unknown sensor measurement losses," *IFAC-PapersOnLine*, vol. 49, no. 22, pp. 315–320, 2016.

[22] M. Huang and S. Dey, "Stability of Kalman filtering with Markovian packet losses," *Automatica*, vol. 43, no. 4, pp. 598–607, 2007.

[23] B. D. Anderson and J. B. Moore, *Optimal Filtering*. Courier Corporation, 2012.

[24] R. A. Singer, "Estimating optimal tracking filter performance for manned maneuvering targets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 6, no. 4, pp. 473–483, 1970.

[25] G. R. Curry, "Radar system performance modeling," *Artech House*, 2005.

[26] "Quadcopter dynamic modeling and simulation," <https://github.com/dch33/Quad-Sim/blob/master/README.md>.

[27] D. W. Kun and I. Hwang, "Linear matrix inequality-based nonlinear adaptive robust control of quadrotor," *Journal of Guidance, Control, and Dynamics*, pp. 996–1008, 2015.

[28] J. L. Crassidis and F. L. Markley, "Three-axis attitude estimation using rate-integrating gyroscopes," *Journal of Guidance, Control, and Dynamics*, pp. 1–14, 2016.

[29] J. J. Dougherty, H. El-Sherief, and D. S. Hohman, "Use of the Global Positioning System for evaluating inertial measurement unit errors," *Journal of Guidance, Control, and Dynamics*, vol. 17, no. 3, pp. 435–441, 1994.

[30] R. Mahony, V. Kumar, and P. Corke, "Multirotor aerial vehicles: Modeling, estimation, and control of quadrotor," *IEEE Robotics Automation Magazine*, vol. 19, no. 3, pp. 20–32, Sept 2012.

[31] T. Bresciani, "Modelling, identification and control of a quadrotor helicopter," *MSc Theses*, 2008.

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