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Distributed Adaptive Fault-Tolerant Output Regulation of Heterogeneous Multi-Agent Systems With Coupling Uncertainties and Actuator Faults

Chao Deng, Weinan Gao, and Weiwei Che

Abstract—In this paper, we consider the distributed adaptive fault-tolerant output regulation problem for heterogeneous multiagent systems with matched system uncertainties and mismatched coupling uncertainties among subsystems under the influence of actuator faults. First, distributed finite-time observers are proposed for all subsystems to observe the state of the exosystem. Then, a novel fault-tolerant controller is designed to compensate for the influence of matched system uncertainties and actuator faults. By using the linear matrix inequality technique, a sufficient condition is provided to guarantee the solvability of the considered problem in the presence of mismatched coupling uncertainties. Moreover, it is shown that the system in closed-loop with the developed controller can achieve output regulation by using the Lyapunov stability theory and cyclic-small-gain theory. Finally, a numerical example is given to illustrate the effectiveness of the obtained result.

Index Terms—Adaptive control, cyclic-small-gain, fault-tolerant control, multi-agent systems (MASs).

I. INTRODUCTION

THE application of multi-agent systems (MASs) [1]–[6] is wide, including smart grid systems [1], intelligent transportation systems [2] and multi-robot systems [3]. In the last decade, rapid development has achieved in MASs. The cooperative output regulation (COR), one of the important research topics on MASs, has received considerable attention. The control objective of COR is to design the control protocol

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C. Deng is with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore (e-mail: cdeng@ntu.edu.sg).

W. N. Gao is with the Department of Electrical and Computer Engineering, Allen E. Paulson College of Engineering and Computing, Georgia Southern University, Statesboro, GA 30460 USA (e-mail: wgao@georgiasouthern.edu).

W. W. Che is with the State Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, and also with the Institute of Complexity Science, Qingdao University, Qingdao 266071, China (e-mail: cwwemail1980@126.com).

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of subsystems to achieve the disturbance rejection and trajectory tracking for each of them. Some effective COR methods have been designed for linear heterogeneous MASs to achieve its control objective [7]–[13]. In particular, a distributed control approach is proposed in [7] to solve the robust output regulation problem. In [8], the authors develop a new robust COR method by relaxing the assumption that circles are allowed in the network topology. In [9], the adaptive distributed observer approach is proposed to solve the COR problem assuming that the neighbours of the exosystem know the system matrix of the exosystem. In [10], a novel data-driven solution is given to solve the COR problem under switching network topology.

Practically, actuator faults often occur owing to the long-running operation of the systems. The presence of actuator faults deteriorate the performance or even destabilize the system [14]–[16]. For the sake of reliability and safety, some valuable distributed fault-tolerant control methods have been proposed to compensate for the influence of actuator faults. In [17], an effective distributed fault estimator is designed for MASs under the directed network topology. In [18], the authors concern with the output tracking consensus for a class of pure-feedback nonlinear MASs with actuator failures. In [19], the fixed-time fault-tolerant controller is designed for second-order nonlinear MASs. In [20], a distributed adaptive fuzzy fault-tolerant controller is proposed for the high-order nonlinear MASs with actuator bias faults. A distributed adaptive event-triggered fault-tolerant method is designed in [21] to reduce the communication burden among the communication networks. In [22], the authors design a circuit implementation method for the developed fault-tolerant consensus controller and apply to a multiple coupled nonlinear forced pendulum system. However, it is still difficult to solve the cooperative fault-tolerant output regulation problem directly through the existing methods. The major challenges are: 1) to find an effective way to ensure the COR problem is solvable under the influenced of actuator faults; 2) how to present a fault-tolerant controller to compensate for actuator faults. In [23], a sufficient condition is established to ensure that the regulator equations are solvable and then a distributed fault-tolerant controller is designed. In [24], the authors solve the COR for linear heterogeneous MASs under the influence of denial-of-service attacks and actuator faults. However, only nominal and certain MASs are considered in the aforementioned results.

Another practical issue on MASs controller design is to

address the effect of uncertainties, including both system uncertainty and coupling uncertainty. In [25], the COR problems of MASs with coupling uncertainties among subsystems are studied. However, only the matched coupling uncertainties are considered. It is well known that mismatched coupling uncertainties are more general in practical applications. Moreover, the existing cyclic-small-gain theorem method in [25] cannot be directly used to settle this problem. Therefore, to develop a fault-tolerant and robust distributed control method for heterogeneous MASs with system uncertainties and mismatched coupling uncertainties to resist the influence of actuator faults is an open and interesting problem.

In this paper, we will solve the distributed adaptive fault-tolerant output regulation problem for heterogeneous MASs with system uncertainty and mismatched coupling uncertainties under the influence of actuator faults. Compared with existing results, the contributions are threefold.

1) A novel distributed adaptive fault-tolerant control method is proposed to solve the fault-tolerant output regulation problem for heterogeneous MASs with matched system uncertainties and mismatched coupling uncertainties among subsystems. To the best of our knowledge, this paper is the first attempt to integrate cyclic-small-gain techniques, COR theory and distributed fault-tolerant method.

2) Different from the existing distributed fault-tolerant control result [23], a more general directed network topology is considered in this paper. To observe the state of the exosystem, novel distributed finite-time observers are designed. In particular, a new variable is introduced in observers to identify which subsystem accurately estimates the state of the exosystem. Besides, we develop a novel distributed fault-tolerant control method that is able to handle unknown matched and bounded uncertainties. Besides, it is shown that all subsystems can achieve trajectory tracking while rejecting nonvanishing disturbances.

3) Different from our previous work [25], in which only the matched coupling uncertainties are considered, the more general unmatched coupling uncertainties are considered in this paper. To deal with it, a novel sufficient condition with cyclic-small-gain condition is proposed by using the linear matrix inequality technique.

Notations: For vectors $x_i \in \mathbb{R}^m$, the vector $[x_1^T, \dots, x_N^T]^T$ is denoted by $\text{col}\{x_1, \dots, x_N\}$. I and 0 represent respectively the identity matrix and zero matrix (or vector). A matrix with matrices $N_1, \dots, N_m$ on its principal diagonal is symbolized by $\text{diag}\{N_1, \dots, N_m\}$. $\mathbf{1} = \text{col}\{1, \dots, 1\} \in \mathbb{R}^N$. $\text{sign}(x)$ denotes the sign function of a scalar x .

II. PROBLEM FORMULATION

A. Systems Dynamics

Consider the heterogeneous uncertain MASs generalized from [25]

$$\dot{v}(t) = S v(t) \quad (1)$$

$$\dot{x}_i(t) = (A_i + \Delta A_i)x_i(t) + B_i u_i(t) + H_i \Phi_i(e, v) + E_i v(t) \quad (2)$$

$$e_i(t) = C_i x_i(t) + F_i v(t), \quad i = 1, \dots, N \quad (3)$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$ and $e_i(t) \in \mathbb{R}^{p_i}$ are the state, control input and regulated output vectors, respectively. $v(t) \in \mathbb{R}^q$ is the exogenous signal. S , A_i , B_i , C_i , E_i and F_i are system matrices with proper dimensions. The matched system uncertainty satisfies $\Delta A_i = B_i G_i(t)$. The function $\Phi_i(e, v)$ denotes unknown interconnections satisfying $|\Phi_i(e, v)| \leq d_i |e|$, where $e = \text{col}\{e_1, \dots, e_N\}$ and d_i ($i = 1, 2, \dots, N$) are known constants.

We introduce the following assumptions.

Assumption 1: (A_i, B_i) is stabilizable for $i = 1, 2, \dots, N$.

Assumption 2: There exist solution matrices X_i and U_i such that the following regulator equation are satisfied

$$\begin{cases} X_i S - A_i X_i - E_i = B_i U_i \\ C_i X_i + F_i = 0, \quad i = 1, \dots, N. \end{cases} \quad (4)$$

Remark 1: As discussed in [9] and [26], Assumptions 1 and 2 are necessary to ensure that the COR problem is solvable.

In this paper, we consider a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{S})$, which consists of nodes $\mathcal{N} = \{v_1, v_2, \dots, v_N\}$ and edges $\mathcal{S} = \{(v_i, v_j) | v_i, v_j \in \mathcal{N}\}$. If node v_i can receive information from node v_j , then $(v_i, v_j) \in \mathcal{S}$. Node v_j is the neighbor of node v_i if $(v_i, v_j) \in \mathcal{S}$. The neighbor set of the node v_i is given by $\mathcal{N}_i = \{v_j \in \mathcal{N} : (v_i, v_j) \in \mathcal{S}\}$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is denoted as $a_{ii} = 0$, $a_{ij} = 0$ if $(v_i, v_j) \notin \mathcal{S}$ and $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{S}$. The definition of Laplacian matrix $\mathcal{L}$ is $\mathcal{L} = \text{diag}\{\phi_1, \dots, \phi_N\} - \mathcal{A}$ with $\phi_i = \sum_{j=1}^N a_{ji}$. Let $\mathcal{M} = \text{diag}\{a_{10}, \dots, a_{N0}\}$, where $a_{i0} = 1$ represents that the exosystem can send information to subsystem i and $a_{i0} = 0$ otherwise. Define $\mathcal{H} = \mathcal{L} + \mathcal{M}$.

Assumption 3: The directed graph $\mathcal{G}$ contains a directed spanning tree and the exosystem is the root.

B. Fault Model

Similar to [27], this paper considers the following actuator faults. $u_{ij}(t)$ and $u_{ij}^{Fh}(t)$ represent respectively the input and the output under the fault model h ($h = 1, \dots, H$) for the actuator j ($j = 1, \dots, m_i$) of subsystem i ($i = 1, \dots, N$). It is defined by

$$u_{ij}^{Fh}(t) = \rho_{ij}^h u_{ij}(t) + \omega_{ij}^h(t) \quad (5)$$

where H represents the total of fault models. The unknown constant ρ_{ij}^h satisfies $0 \leq \underline{\rho}_{ij}^h \leq \rho_{ij}^h \leq \bar{\rho}_{ij}^h \leq 1$. $\omega_{ij}^h(t)$ is a bounded time-varying function.

We can summarize the model (5) as follows: 1) $\rho_{ij}^h = \bar{\rho}_{ij}^h = 1$ and $\omega_{ij}^h(t) = 0$ mean the fault-free; 2) $0 < \underline{\rho}_{ij}^h \leq \bar{\rho}_{ij}^h < 1$ and $\omega_{ij}^h(t) = 0$ signify the loss-of-effectiveness fault; 3) $\rho_{ij}^h = 0$ and $\omega_{ij}^h(t) = 0$ represent the outage fault; 4) $\rho_{ij}^h = 0$ and $\omega_{ij}^h(t) \neq 0$ represent the bias fault.

Denote

$$u_i^{Fh}(t) = \Lambda_i^h u_i(t) + \omega_i^h(t)$$

where $u_i^{Fh}(t) = \text{col}\{u_{i1}^{Fh}(t), \dots, u_{im_i}^{Fh}(t)\}$, $u_i(t) = \text{col}\{u_{i1}(t), \dots, u_{im_i}(t)\}$, $\Lambda_i^h = \text{diag}\{\rho_{i1}^h, \dots, \rho_{im_i}^h\}$ and $\omega_i^h(t) = \text{col}\{\omega_{i1}^h(t), \dots, \omega_{im_i}^h(t)\}$. Finally, the fault model is presented as

$$u_i^F(t) = \Lambda_i u_i(t) + \omega_i(t) \quad (6)$$

where $\Lambda_i = \text{diag}\{\rho_{i1}, \dots, \rho_{im_i}\}$, $\Lambda_i \in \{\Lambda_i^1, \dots, \Lambda_i^H\}$ and $\omega_i \in \{\omega_i^1, \dots, \omega_i^H\}$.

Assumption 4: The uncertain parameter $G_i(t)$ and actuator bias fault signal $\omega_i(t)$ are bounded, i.e., there exist $\bar{g}_i > 0$ and $\bar{w}_i > 0$ such that $\|G_i(t)\| \leq \bar{g}_i$ and $\|\omega_i(t)\| \leq \bar{w}_i$.

Assumption 5: $\text{rank}(B_i \Lambda_i) = \text{rank}(B_i)$ for any $\Lambda_i \in \{\Lambda_i^1, \dots, \Lambda_i^H\}$, $i = 1, \dots, N$. $\text{rank}(B_i)$ represents the rank of matrix B_i .

Lemma 1 [27]: If Assumption 5 holds, there exists positive constants $\mu_1, \mu_2, \dots$, and μ_N such that

$$B_i \Lambda_i B_i^T \geq \mu_i B_i B_i^T, \quad i = 1, 2, \dots, N.$$

Lemma 2 [23]: If Assumptions 1, 2 and 5 are satisfied, then there exist matrices X_i and fault-dependent matrices $U_i^{\Lambda_i}$ satisfy the following regulator equations

$$\begin{cases} X_i S - A_i X_i - E_i = B_i \Lambda_i U_i^{\Lambda_i} \\ C_i X_i + F_i = 0, \quad i = 1, \dots, N. \end{cases} \quad (7)$$

Lemma 3 [28]: Consider the system:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= u. \end{aligned}$$

Let $l_1, l_2, \dots, l_r$ be positive constants such that $s^n + l_r s^{r-1} + \dots + l_2 s + l_1$ is Hurwitz. There exists positive constant $\varepsilon < 1$ such that, for any $\kappa \in (1 - \varepsilon, 1)$, the above system is globally finite-time stability under the following controller

$$u = -l_1 \text{sign}(z_1)|z_1|^{\kappa_1} - \dots - l_r \text{sign}(z_r)|z_r|^{\kappa_r}$$

where κ_i satisfies

$$\kappa_i = \frac{\kappa_{i+1} \kappa_{i+2}}{2\kappa_{i+2} - \kappa_{i+1}}, \quad i = r-1, \dots, 1$$

with $\kappa_{r+1} = 1$ and $\kappa_r = \kappa$.

Remark 2: Assumption 4 has been used in some existing fault-tolerant control results [23] and [27]. From Lemma 4 in [23], we know that a sufficient condition, which ensures that the cooperative fault-tolerant output regulation problem is solvable, is that Assumption 4 holds.

C. Cooperative Fault-Tolerant Output Regulation Problem

The control objective is to propose a distributed adaptive robust fault-tolerant control method for MASs (1)–(3) with system uncertainties and mismatched coupling uncertainties among subsystems under the influence of actuator faults (6) such that

1) The regulated outputs satisfy $\lim_{t \rightarrow \infty} e_i(t) = 0$, for $i = 1, 2, \dots, N$.

2) The closed-loop MASs are asymptotically stable when $v = 0$.

Remark 3: Different from the existing cooperative fault-tolerant results [18] and [19], the dynamics of the MASs considered in this paper contain nonvanishing signal $v(t)$ and the unknown interconnection $\Phi_i(e, v)$ in (2). To eliminate the influence of the signal $v(t)$ and $\Phi_i(e, v)$, a framework of COR theory and cyclic-small-gain techniques is introduced in this paper. Besides, compared with [18] and [19], the outage faults,

which may bring difficulties to theoretical analysis and often occur in practical applications, is considered in this paper.

III. MAIN RESULTS

A. Distributed Finite-Time Observers Design

In order to estimate the state of exosystem (1), we design the distributed finite-time observers for subsystems $i = 1, 2, \dots, N$

$$\dot{\eta}_i = \begin{cases} 0, & \text{if } t < t_i \\ \bar{S} \eta_{i(t)} + \bar{T} v_i(t), & \text{else} \end{cases} \quad (8)$$

where $\bar{S} = T S T^{-1}$, $\bar{T} = \text{diag}\{T_1, \dots, T_r\}$, and T is given in Lemma 2 of [23]. t_i is the time constant, which is given by

$$t_i = \min_{j \in N_i} \{t_{ij}\} \quad (9)$$

with

$$t_{ij} = \arg \min_{t \in \mathbb{R}, j \in N_i} \|v_j(t)\| \neq 0 \quad (10)$$

$v_i(t) = \text{col}\{v_{i1}(t), \dots, v_{ir}(t)\} \in \mathbb{R}^r$ is the input vector which is given by

$$v_{ij}(t) = - \sum_{k=1}^r \alpha_{jk} \zeta_{ik}(t) - \sum_{k=1}^{p_j} \beta_{ij}^k \text{sign}(\zeta_{ij}^k(t)) |\zeta_{ij}^k(t)|^{\theta_{ij}^k} \quad (11)$$

where parameters α_{jk} are given in Lemma 2 of [23] and positive scalars β_{ij}^k and θ_{ij}^k are given in Lemma 4 and $\zeta_{ij}(t) = \text{col}\{\zeta_{ij}^1(t), \dots, \zeta_{ij}^{p_j}(t)\} \in \mathbb{R}^{p_j}$. The dynamic of $\zeta_i(t) = \text{col}\{\zeta_{i1}(t), \dots, \zeta_{ir}(t)\}$ is described by

$$\dot{\zeta}_i(t) = \bar{S} \zeta_i(t) - c \bar{T} \bar{T}^T R \varphi_i(t) - \iota \text{sign}(R \varphi_i(t)) + \bar{T} v_i(t) \quad (12)$$

where $\varphi_i(t) = \sum_{j=0}^N a_{ij} c_{ij}(t) [(\zeta_i(t) - \zeta_j(t)) - (\eta_i(t) - \eta_j(t))]$ and $\eta_0(t) = T v(t)$ and

$$c_{ij}(t) = \begin{cases} 1, & \text{if } j = i_{j_0} \text{ and } t \geq t_i \\ 0, & \text{else} \end{cases} \quad (13)$$

with

$$i_{j_0} = \arg \min_{j \in N_i} \{t_{ij}\} \quad (14)$$

ι is an arbitrary positive constant and $c \geq 1/\lambda_1$. $\bar{T}$ is given in the proof of Lemma 2 of [23]. $R > 0$ satisfies a Riccati equation $R\bar{S} + \bar{S}^T R - 2R\bar{T}\bar{T}^T R = -Q_1$, where Q_1 is a any positive definite matrix.

One can have the following Lemma 4 to ensure the finite-time estimation.

Lemma 4: Choose $\beta_{ij}^1, \dots, \beta_{ij}^{p_j}$ such that the polynomial $s^{p_j} + \beta_{ij}^{p_j} s^{p_j-1} + \dots + \beta_{ij}^2 s + \beta_{ij}^1$ is Hurwitz. If θ_{ij}^k is chosen as

$$\theta_{ij}^k = \frac{\theta_{ij}^{k+1} \theta_{ij}^{k+2}}{2\theta_{ij}^{k+2} - \theta_{ij}^{k+1}}, \quad k = p_j - 1, \dots, 1, \quad j = 1, \dots, r$$

with $\theta_{ij}^{p_j+1} = 1$, $\theta_{ij}^{p_j} = \theta_{ij}$ and $\theta_{ij} \in (1 - \varepsilon, 1)$ (ε is given in Lemma 3). Then, the state $v_i(t)$ with $v_i(t) = T^{-1} \eta_i(t)$ converges to $v(t)$ after finite-time T_0 , i.e., $v_i(t) \equiv v(t)$ for any $t \geq T_0$.

Proof: See Appendix. ■

Remark 4: Different from the existing finite-time observers

in [23], the developed distributed finite-time observers can deal with more general directed network topology, which contains the undirected network topology as a special case. Interestingly, a new variable is introduced in the observers to identify which subsystem accurately estimates the state of the exosystem.

Remark 5: In the distributed finite-time observers (8), the matrix S of the exosystem is known for all subsystems. However, it is only assumed that the neighbors of the exosystem know this matrix in [24]. To solve the problem, a data-driven distributed learning approach in [29] can be used to estimate the matrix S for all subsystems.

Remark 6: Different from [30], in which the distributed cooperative filters are designed for MASs to estimate the state, the objective of designing distributed finite-time observers is to estimate the trajectories of the exosystem. Based on the developed observers, the fault-tolerant controller will be designed in next subsection to eliminate the influence of the nonvanishing signal $v(t)$ for each subsystem.

B. The Fault-Tolerant Controller Design

Incorporating finite-time observers (8), the fault-tolerant controller is designed as follows

$$u_i(t) = u_{i1}(t) + u_{i2}(t) + u_{i3}(t) \quad (15)$$

where

$$u_{i1}(t) = \hat{K}_i(t)\bar{x}_i(t) + \hat{U}_i(t)v_i(t) \quad (16)$$

$$u_{i2}(t) = -\frac{B_i^T P_i \bar{x}_i(t) \hat{w}_i^2(t)}{\|\bar{x}_i^T(t) P_i B_i\| \hat{w}_i(t) + e^{-t}} \quad (17)$$

$$u_{i3}(t) = -\frac{\hat{g}_i^2(t) B_i^T P_i \bar{x}_i(t) \|x_i\|^2}{\hat{g}_i(t) \|\bar{x}_i^T(t) P_i B_i\| \cdot \|x_i\| + e^{-t}} \quad (18)$$

$\bar{x}_i(t) = x_i(t) - X_i v_i(t)$, $\hat{K}_i(t) = \text{col}\{\hat{K}_i^1(t), \dots, \hat{K}_i^{m_i}(t)\}$ and $\hat{U}_i(t) = \text{col}\{\hat{U}_i^1(t), \dots, \hat{U}_i^{m_i}(t)\}$. $\hat{K}_i^j(t) \in \mathbb{R}^{1 \times n_i}$ and $\hat{U}_i^j(t) \in \mathbb{R}^{1 \times q}$ are updated by

$$\dot{\hat{K}}_i^j(t) = -\gamma_{ij1} b_{ij}^T P_i \bar{x}_i(t) \bar{x}_i^T(t) \quad (19)$$

$$\dot{\hat{U}}_i^j(t) = -\gamma_{ij2} b_{ij}^T P_i \bar{x}_i(t) v_i^T(t) \quad (20)$$

where b_{ij} is the j th column of matrix B_i . γ_{ij1} and γ_{ij2} are arbitrary positive constants. $\hat{w}_i$ and $\hat{g}_i$ are the estimation of $\bar{w}_i$ and $\bar{g}_i$ and is updated by

$$\dot{\hat{w}}_i(t) = \gamma_i \|\bar{x}_i^T(t) P_i B_i\| \quad (21)$$

$$\dot{\hat{g}}_i(t) = \gamma_{gi} \|\bar{x}_i^T(t) P_i B_i\| \cdot \|x_i(t)\| \quad (22)$$

where $\hat{w}_i(0)$ and $\hat{g}_i(0)$ are nonnegative and γ_i and γ_{gi} are arbitrary positive constants. Besides, $P_i > 0$ is determined by

$$\begin{bmatrix} P_i A_i + A_i^T P_i - 2P_i B_i B_i^T P_i + (1 + \frac{1}{\beta_i})I & P_i H_i \\ H_i^T P_i & -\frac{1}{d_i^2 \pi} I \end{bmatrix} < 0 \quad (23)$$

where $\pi = \max_{i=1,2,\dots,N} \|C_i^T C_i\|$, $\beta_i > 0$ is a design parameter.

Remark 7: Different from the existing cooperative fault-tolerant output regulation result [23], the more general

heterogeneous MASs with actuator bias fault $\omega_i(t)$ and system uncertainty ΔA_i are considered in this paper. To solve the problem, compensation terms $u_{i2}(t)$ and $u_{i3}(t)$ with adaptive parameters $\hat{w}_i(t)$ and $\hat{g}_i(t)$ are introduced, respectively.

C. Stability Analysis

Let $\bar{x}_i(t) = x_i(t) - X_i v(t)$. From the MASs (1)–(3) and the fault model (6), we have

$$\dot{\bar{x}}_i = (A_i + \Delta A_i)x_i + B_i(\Lambda_i u_i(t) + \omega_i) + H_i \Phi_i(e, v) + E_i v - X_i S v$$

Using $\Delta A_i = B_i G_i$, we further have

$$\dot{\bar{x}}_i = A_i x_i + B_i(\Lambda_i u_i + G_i x_i + \omega_i) + H_i \Phi_i(e, v) + E_i v - X_i S v.$$

According to the regulator equations (7), it yields

$$\dot{\bar{x}}_i = A_i \bar{x}_i - B_i \Lambda_i U_i^{\Lambda_i} v + B_i(\Lambda_i u_i + G_i x_i + \omega_i) + H_i \Phi_i(e, v).$$

Define $\bar{A}_i = A_i + B_i \Lambda_i K_i$. It is derived that

$$\begin{aligned} \dot{\bar{x}}_i &= \bar{A}_i \bar{x}_i - B_i \Lambda_i K_i \bar{x}_i - B_i \Lambda_i U_i^{\Lambda_i} v \\ &\quad + B_i(\Lambda_i u_i + G_i x_i + \omega_i) + H_i \Phi_i(e, v). \end{aligned} \quad (24)$$

Then, a sufficient condition on the COR for the MASs (1)–(3) with actuator faults (6) is presented in the following Theorem.

Theorem 1: For given $\beta_i > 0$ where β_i satisfies the following cyclic-small-gain condition

$$\sum_{j=1}^{N-1} j \sum_{1 \leq i_1 < i_2 < \dots < i_{j+1} \leq N} \beta_{i_1} \beta_{i_2} \dots \beta_{i_{j+1}} < 1. \quad (25)$$

If there exists $P_i > 0$ such that (23) is satisfied, then the cooperative fault-tolerant output regulation problem can be solved by using the finite-time observer (8) and fault-tolerant controller (15) with adaptive laws (19)–(21).

Proof: Construct the Lyapunov functional

$$V(t) = \sum_{i=1}^N \sigma_i \beta_i V_i(t)$$

where

$$\begin{aligned} V_i &= \bar{x}_i^T P_i \bar{x}_i + \sum_{j=1}^m \rho_{ij} \left(\frac{1}{\gamma_{ij1}} \tilde{K}_i^j (\tilde{K}_i^j)^T + \frac{1}{\gamma_{ij2}} \tilde{U}_i^j (\tilde{U}_i^j)^T \right) \\ &\quad + \frac{\mu_i}{\gamma_i} \bar{w}_i^2 + \frac{\mu_i}{\gamma_{gi}} \bar{g}_i^2 \end{aligned}$$

and

$$\sigma_i = \frac{\prod_{j=1, j \neq i}^N (1 + \beta_j)}{1 - \sum_{j=1}^{N-1} j \sum_{1 \leq i_1 < i_2 < \dots < i_{j+1} \leq N} \beta_{i_1} \beta_{i_2} \dots \beta_{i_{j+1}}}. \quad (26)$$

$\tilde{K}_i^j = \hat{K}_i^j - K_i^j$, $\tilde{U}_i^j = \hat{U}_i^j - U_i^j$, $\bar{w}_i = \hat{w}_i - \bar{w}_i/\mu_i$ and $\bar{g}_i = \hat{g}_i - \bar{g}_i/\mu_i$. $K_i = [K_i^1, \dots, K_i^{m_i}] = -\frac{1}{\mu_i} B_i^T P_i$ and $U_i = [U_i^1, \dots, U_i^{m_i}] = U_i^{\Lambda_i}$. The definition of μ_i is given in Lemma 1. From Lemma 3, we know $v_i \equiv v$ for $t \geq T_0$. Thus, we discuss the case for $t \geq T_0$ in the following. The derivative of V_i along (24) is given as

$$\begin{aligned}\dot{V}_i &= 2\bar{x}_i^T P_i (A_i \bar{x}_i - B_i \Lambda_i U_i^{\Lambda_i} v + B_i (\Lambda_i u_i + G_i x_i + \omega_i) \\ &\quad + H_i \Phi_i(e, v)) + 2 \sum_{j=1}^m \rho_{ij} \left(\frac{1}{\gamma_{ij1}} \dot{\tilde{K}}_i^j (\tilde{K}_i^j)^T \right. \\ &\quad \left. + \frac{1}{\gamma_{ij2}} \dot{\tilde{U}}_i^j (\tilde{U}_i^j)^T \right) + 2 \frac{\mu_i}{\gamma_i} \dot{\tilde{w}}_i \tilde{w}_i + 2 \frac{\mu_i}{\gamma_{gi}} \dot{\tilde{g}}_i \tilde{g}_i. \quad (27)\end{aligned}$$

By using the fault-tolerant controller (15), we further have

$$\begin{aligned}\dot{V}_i &= 2\bar{x}_i^T P_i (\bar{A}_i \bar{x}_i + B_i \Lambda_i (\tilde{K}_i x_i + \tilde{U}_i v_i) + B_i (G_i x_i + \omega_i) \\ &\quad + H_i \Phi_i(e, v)) - 2 \frac{\bar{x}_i^T P_i B_i \Lambda_i B_i^T P_i \bar{x}_i \hat{w}_i^2}{\|\bar{x}_i^T P_i B_i\| \hat{w}_i + e^{-t}} \\ &\quad + 2 \sum_{j=1}^m \rho_{ij} \left(\frac{1}{\gamma_{ij1}} \dot{\tilde{K}}_i^j (\tilde{K}_i^j)^T + \frac{1}{\gamma_{ij2}} \dot{\tilde{U}}_i^j (\tilde{U}_i^j)^T \right) \\ &\quad + 2 \frac{\mu_i}{\gamma_i} \dot{\tilde{w}}_i \tilde{w}_i + 2 \frac{\mu_i}{\gamma_{gi}} \dot{\tilde{g}}_i \tilde{g}_i. \quad (28)\end{aligned}$$

Using the Young's inequality, it yields that

$$2\bar{x}_i^T P_i H_i \Phi_i(e, v) \leq d_i^2 \pi \bar{x}_i^T P_i H_i H_i^T P_i \bar{x}_i + \frac{1}{d_i^2 \pi} \Phi_i^T(e, v) \Phi_i(e, v). \quad (29)$$

From $|\Phi_i(e, v)| \leq d_i |e|$ and using the regulator equation (7), it yields that

$$\frac{1}{d_i^2 \pi} \Phi_i^T(e, v) \Phi_i(e, v) \leq \frac{1}{\pi} e^T e \leq \bar{x}^T \bar{x} \quad (30)$$

where $\bar{x} = \text{col}\{\bar{x}_1, \bar{x}_2, \dots, \bar{x}_N\}$. Besides, the fact $e_i = C_i x_i + F_i v = C_i \bar{x}_i$ is used. Substituting (29) and (30) into (28), we have

$$\begin{aligned}\dot{V}_i &\leq \bar{x}_i^T (P_i \bar{A}_i + \bar{A}_i^T P_i + d_i^2 \pi P_i H_i H_i^T P_i) \bar{x}_i + \bar{x}^T \bar{x} \\ &\quad + 2\bar{x}_i^T P_i (B_i \Lambda_i (\tilde{K}_i x_i + \tilde{U}_i v_i) + B_i (G_i x_i + \omega_i)) \\ &\quad - 2 \frac{\bar{x}_i^T P_i B_i \Lambda_i B_i^T P_i \bar{x}_i \hat{w}_i^2}{\|\bar{x}_i^T P_i B_i\| \hat{w}_i + e^{-t}} + 2 \frac{\mu_i}{\gamma_i} \dot{\tilde{w}}_i \tilde{w}_i \\ &\quad + 2 \sum_{j=1}^m \rho_{ij} \left(\frac{1}{\gamma_{ij1}} \dot{\tilde{K}}_i^j (\tilde{K}_i^j)^T + \frac{1}{\gamma_{ij2}} \dot{\tilde{U}}_i^j (\tilde{U}_i^j)^T \right). \quad (31)\end{aligned}$$

Using Assumption 4 and the adaptive updated law (21), we have

$$\begin{aligned}2\bar{x}_i^T P_i B_i \omega_i - 2 \frac{\bar{x}_i^T P_i B_i \Lambda_i B_i^T P_i \bar{x}_i \hat{w}_i^2}{\|\bar{x}_i^T P_i B_i\| \hat{w}_i + e^{-t}} + 2 \frac{\mu_i}{\gamma_i} \dot{\tilde{w}}_i \tilde{w}_i \\ \leq 2 \|\bar{x}_i^T P_i B_i\| \hat{\omega}_i - 2 \frac{\mu_i \|\bar{x}_i^T P_i B_i\|^2 \hat{w}_i^2}{\|\bar{x}_i^T P_i B_i\| \hat{w}_i + e^{-t}} + 2 \frac{\mu_i}{\gamma_i} \dot{\tilde{w}}_i \tilde{w}_i \\ = 2\mu_i \|\bar{x}_i^T P_i B_i\| \hat{\omega}_i - 2 \frac{\mu_i \|\bar{x}_i^T P_i B_i\|^2 \hat{w}_i^2}{\|\bar{x}_i^T P_i B_i\| \hat{w}_i + e^{-t}} \\ = 2 \frac{\mu_i e^{-t} \|\bar{x}_i^T P_i B_i\| \hat{w}_i}{\|\bar{x}_i^T P_i B_i\| \hat{w}_i + e^{-t}} \\ \leq 2\mu_i e^{-t}. \quad (32)\end{aligned}$$

Using Assumption 4 and the adaptive updated law (22), we have

$$\begin{aligned}2\bar{x}_i^T P_i B_i G_i x_i - 2 \frac{\bar{x}_i^T P_i B_i \Lambda_i B_i^T P_i \bar{x}_i \hat{g}_i^2 \|x_i\|^2}{\|\bar{x}_i^T P_i B_i\| \hat{g}_i \|x_i\| + e^{-t}} + 2 \frac{\mu_i}{\gamma_{gi}} \dot{\tilde{g}}_i \tilde{g}_i \\ \leq 2 \|\bar{x}_i^T P_i B_i\| \hat{g}_i \|x_i\| - 2 \frac{\mu_i \|\bar{x}_i^T P_i B_i\|^2 \hat{g}_i^2 \|x_i\|^2}{\|\bar{x}_i^T P_i B_i\| \hat{g}_i \|x_i\| + e^{-t}} + 2 \frac{\mu_i}{\gamma_{gi}} \dot{\tilde{g}}_i \tilde{g}_i \\ = 2\mu_i \|\bar{x}_i^T P_i B_i\| \hat{g}_i \|x_i\| - 2 \frac{\mu_i \|\bar{x}_i^T P_i B_i\|^2 \hat{g}_i^2 \|x_i\|^2}{\|\bar{x}_i^T P_i B_i\| \hat{g}_i \|x_i\| + e^{-t}} \\ = 2 \frac{\mu_i e^{-t} \|\bar{x}_i^T P_i B_i\| \hat{g}_i \|x_i\|}{\|\bar{x}_i^T P_i B_i\| \hat{g}_i \|x_i\| + e^{-t}} \\ \leq 2\mu_i e^{-t}. \quad (33)\end{aligned}$$

Using the adaptive laws (19) and (20), it yields

$$\begin{aligned}2\bar{x}_i^T P_i (B_i \Lambda_i (\tilde{K}_i x_i + \tilde{U}_i v_i) + 2 \sum_{j=1}^m \rho_{ij} \left(\frac{1}{\gamma_{ij1}} \dot{\tilde{K}}_i^j (\tilde{K}_i^j)^T \right. \\ \left. + \frac{1}{\gamma_{ij2}} \dot{\tilde{U}}_i^j (\tilde{U}_i^j)^T \right)) \\ = 2 \sum_{j=1}^m \rho_{ij} (\bar{x}_i^T P_i b_{ij} (\tilde{K}_i^j x_i + \tilde{U}_i^j v_i) + \frac{1}{\gamma_{ij1}} \dot{\tilde{K}}_i^j (\tilde{K}_i^j)^T \\ + \frac{1}{\gamma_{ij2}} \dot{\tilde{U}}_i^j (\tilde{U}_i^j)^T) \\ = 0. \quad (34)\end{aligned}$$

Substituting (32) – (34) into (31), we have

$$\dot{V}_i \leq \bar{x}_i^T (P_i \bar{A}_i + \bar{A}_i^T P_i + d_i^2 \pi P_i H_i H_i^T P_i) \bar{x}_i + \bar{x}^T \bar{x} + 4\mu_i e^{-t}. \quad (35)$$

From the definition of $\bar{A}_i = A_i + B_i \Lambda_i K_i$ and $K_i = -\frac{1}{\mu_i} B_i^T P_i$, we have

$$\begin{aligned}P_i \bar{A}_i + \bar{A}_i^T P_i \\ = P_i (A_i + B_i \Lambda_i K_i) + (A_i + B_i \Lambda_i K_i)^T P_i \\ = P_i (A_i - B_i \Lambda_i \frac{1}{\mu_i} B_i^T P_i) + (A_i - B_i \Lambda_i \frac{1}{\mu_i} B_i^T P_i)^T P_i \\ = P_i A_i + A_i^T P_i - 2 \frac{1}{\mu_i} P_i B_i \Lambda_i B_i^T P_i \\ \leq P_i A_i + A_i^T P_i - 2 P_i B_i B_i^T P_i. \quad (36)\end{aligned}$$

By applying the Schur complement lemma to (23), we have

$$P_i A_i + A_i^T P_i - 2 P_i B_i B_i^T P_i + d_i^2 \pi P_i H_i H_i^T P_i + (1 + \frac{1}{\beta_i}) I < 0. \quad (37)$$

Substituting (36) and (37) into (35), we have

$$\dot{V}_i \leq -(1 + \frac{1}{\beta_i}) \bar{x}_i^T(t) \bar{x}_i(t) + \bar{x}^T \bar{x} + 4\mu_i e^{-t}. \quad (38)$$

Hence,

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^N \sigma_i \beta_i \left(-\left(1 + \frac{1}{\beta_i}\right) \bar{x}_i^T(t) \bar{x}_i(t) + \bar{x}^T \bar{x} + 4\mu_i e^{-t} \right) \\
&= \sum_{i=1}^N \sigma_i \left(-(\beta_i + 1) \bar{x}_i^T(t) \bar{x}_i(t) + \beta_i \bar{x}^T \bar{x} + 4\mu_i \beta_i e^{-t} \right) \\
&= \sum_{i=1}^N \sigma_i \left(-\bar{x}_i^T(t) \bar{x}_i(t) + \beta_i \sum_{j=1, j \neq i}^N \bar{x}_j^T(t) \bar{x}_j(t) + 4\mu_i \beta_i e^{-t} \right) \\
&= \begin{bmatrix} \|x_1(t)\|^2 \\ \|x_2(t)\|^2 \\ \vdots \\ \|x_N(t)\|^2 \end{bmatrix}^T \begin{bmatrix} -1 & \beta_2 & \dots & \beta_N \\ \beta_1 & -1 & \dots & \beta_N \\ \vdots & \vdots & \ddots & \vdots \\ \beta_1 & \beta_2 & \dots & -1 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{bmatrix} \\
&\quad + \sum_{i=1}^N 4\mu_i \sigma_i \beta_i e^{-t}.
\end{aligned}$$

Using the cyclic-small-gain condition (25), we have

$$\dot{V}(t) \leq -\bar{x}^T \bar{x} + \sum_{i=1}^N 4\mu_i \sigma_i \beta_i e^{-t}. \quad (39)$$

By using the Barbalat's lemma in [31], it is deduced that $\lim_{t \rightarrow \infty} \|\bar{x}(t)\| = 0$. By using the regulator equations (7), we further show that

$$\begin{aligned}
\lim_{t \rightarrow \infty} e_i &= \lim_{t \rightarrow \infty} (C_i x_i + F_i v) \\
&= \lim_{t \rightarrow \infty} C_i (x_i - X_i v) + \lim_{t \rightarrow \infty} (C_i X + F_i) v \\
&= 0.
\end{aligned} \quad (40)$$

Remark 8: Different from our previous work [25], in which only the matched coupling uncertainties are considered, the more general unmatched coupling uncertainties are considered in this paper. The major difficulties met when deriving the current results can be summarized: 1) whether the cooperative fault-tolerant output regulation problem can be solved under the more general unmatched coupling uncertainties? 2) if 1) is solved, then how to obtain a new circle-small-gain condition? To overcome the difficulties, a novel sufficient condition (23) with cyclic-small-gain condition (25) is proposed.

Remark 9: In the inequality condition (23), the Lyapunov matrix P_i cannot be solved directly because the term $P_i B_i B_i^T P_i$ is a nonlinear term. To solve the problem, the linear matrix inequality (41) can be achieved by using the following mathematical derivation: 1) multiplying $\text{diag}\{X_i, I\}$ ($X_i = P_i^{-1}$) on the left and right in (23); 2) using the well known Schur complement lemma.

$$\begin{bmatrix} A_i X_i + X_i A_i^T - 2B_i B_i^T & H_i & X_i \\ H_i^T & -\frac{1}{d_i^2} I & 0 \\ X_i & 0 & -\frac{\beta_i}{1 + \beta_i} I \end{bmatrix} < 0. \quad (41)$$

Remark 10: In the fault-tolerant controller (17), when t goes to infinity, the function e^{-t} will tend to zero, which means that the chattering phenomenon of u_i may occur in simulation. To eliminate chattering, the well-known boundary layer approach in [32] and [33] is introduced as follows

$$u_{i2} = \begin{cases} -\frac{B_i^T P_i \bar{x}_i(t) \hat{w}_i^2(t)}{\|\bar{x}_i^T(t) P_i B_i\| \hat{w}_i(t) + e^{-t}}, & \text{if } e^{-t} > \varepsilon \\ -\frac{B_i^T P_i \bar{x}_i(t) \hat{w}_i^2(t)}{\|\bar{x}_i^T(t) P_i B_i\| \hat{w}_i(t) + \varepsilon}, & \text{if } e^{-t} \leq \varepsilon \end{cases} \quad (42)$$

where ε is a sufficiently small constant. This method can be also used for u_{i3} .

IV. SIMULATION RESULTS

In this section, we present an example from [25] to demonstrate the efficiency of the proposed method. The network topology graph $\mathcal{G}$ is depicted in Fig. 1. Considering the following MASs (1) – (3) with

Fig. 1. The network topology.

$$\begin{aligned}
S &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad A_i = \begin{bmatrix} 0 & 1 \\ -0.2i & -0.1i \end{bmatrix} \\
B_i &= H_i = \begin{bmatrix} 0 \\ i \end{bmatrix}, \quad E_i = \begin{bmatrix} 0 & i \\ 0 & 0 \end{bmatrix}, \quad C_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T \\
F_i &= \begin{bmatrix} -1 & 0 \end{bmatrix}, \quad \Phi_i(e, v) = v_1 \sum_{j=1}^3 e_j.
\end{aligned}$$

The considered system uncertainties are chosen as $\Delta A_i = B_i \sin(t)$ for $i = 1, 2, 3, 4$. In this simulation, subsystems 1–3 are fault-free and the actuator fault of the 4th subsystem is set as

$$\Lambda_4 = \begin{cases} 1, & 0 \leq t \leq 50 \\ 0.01, & t > 50 \end{cases}$$

which implies that the actuator is loss-of-effectiveness at $t = 50$ s.

The initial states of the systems and the adaptive distributed observer are set as $v(0) = [1 \ -1]^T$, $v_i(0) = i \cdot [1 \ -1]^T$, $x_i(0) = i \cdot [0.1 \ 0.1]^T$, $\hat{K}_i^1(0) = \hat{K}_i^2(0) = i$, $\hat{U}_i^1(0) = \hat{U}_i^2(0) = i$, $\hat{w}_i(0) = i$, $i = 1, \dots, 4$.

The regulated outputs $e_i(t)$ with the developed method and the method in [25] are shown in Figs. 2 and 3, which illustrates that, in the presence of actuator faults, the regulated outputs converge to zero by using the developed method, while they cannot converge to zero by using the method in [25]. The trajectories of adaptive parameters $\hat{K}_i^1(t)$, $\hat{K}_i^2(t)$, $\hat{U}_i^1(t)$, $\hat{U}_i^2(t)$, $\hat{w}_i(t)$, and $\hat{g}_i(t)$ are given in Figs. 4–9, which demonstrate that all these signals are bounded by using the developed method.

V. CONCLUSION

This paper has proposed a novel distributed adaptive robust

Fig. 2. Trajectories of e_i under the proposed method. Fig. 6. Trajectories of $\hat{U}_i^1$. Fig. 3. Trajectories of e_i under the method in [25]. Fig. 7. Trajectories of $\hat{U}_i^2$. Fig. 4. Trajectories of $\hat{K}_i^1$. Fig. 8. Trajectories of adaptive parameters $\hat{w}_i$. Fig. 5. Trajectories of $\hat{K}_i^2$. Fig. 9. Trajectories of adaptive parameters $\hat{g}_i$.

fault-tolerant control method to deal with the COR problem for heterogeneous MASs with matched system uncertainties

and mismatched coupling uncertainties among subsystems under the influence of actuator faults. A new fault-tolerant

controller has been proposed to compensate for the influence of matched system uncertainties and actuator faults. Further, a sufficient condition based on linear matrix inequality technique has been provided to guarantee the solvability of the considered problem under the influence of mismatched coupling uncertainties. It has been shown that the COR problem can be solved via the developed method by using the Lyapunov stability theory and cyclic-small-gain theorem.

APPENDIX PROOF OF LEMMA 4

Now we prove via mathematical induction.

1) Consider the subsystems which can receive the information from the exosystem. Define $\tau_i = \zeta_i - \tilde{\eta}_i$, where $\tilde{\eta}_i = \text{col}\{\tilde{\eta}_{i1}, \dots, \tilde{\eta}_{ir}\} = \eta_i - \eta_0$ is the observation error. From (1) and (8), the dynamics of $\tilde{\eta}_i$ are

$$\dot{\tilde{\eta}}_i = \bar{S} \tilde{\eta}_i + \bar{T} v_i. \quad (43)$$

According to (12) and (43), we have

$$\dot{\tau}_i = \bar{S} \tau_i - c \bar{T} \bar{T}^T R (\zeta_i - \tilde{\eta}_i) - \iota \text{sign}(\zeta_i - \tilde{\eta}_i)$$

where $\tilde{\eta}_0 = 0$. Let $\tau = [\tau_1, \dots, \tau_N]^T$. Accordingly,

$$\dot{\tau} = (I_N \otimes \bar{S}) \tau - c (\mathcal{L}_1 \otimes \bar{T} \bar{T}^T R) \tau - \iota \text{sign}((\mathcal{L}_1 \otimes R) \tau).$$

Let $W = \tau_i^T R \tau_i$. The derivative of W is given by

$$\dot{W} = 2\tau_i^T R \bar{S} \tau_i - 2c\tau_i^T R \bar{T} \bar{T}^T R \tau_i - 2\iota \|R \tau_i\|_1.$$

Applying Theorem 4.2 in [34], it can be shown that τ_i converges to zero after finite-time.

From (43) and the definitions of $\tilde{\eta}_i$, $\bar{S}$, $\bar{T}$ and v_i , one obtains

$$\dot{\tilde{\eta}}_{ij} = \bar{S}_{jj}^n \tilde{\eta}_{ij} + T_i (v_{ij} + \sum_{k=1}^r \alpha_{jk} \tilde{\eta}_{ik}). \quad (44)$$

Substituting the value of v_{ij} in (11) into (44), it shows that

$$\begin{aligned} \dot{\tilde{\eta}}_{ij} = & \bar{S}_{jj}^n \tilde{\eta}_{ij} + T_i \left(- \sum_{k=1}^{p_j} \beta_{ij}^k \text{sign}(\zeta_{ij}^k) |\zeta_{ij}^k|^{\theta_{ij}^k} \right. \\ & \left. - \sum_{k=1}^r \alpha_{jk} (\zeta_{ik} - \tilde{\eta}_{ik}) \right). \end{aligned} \quad (45)$$

Therefore, after finite-time, (45) will reduce to

$$\dot{\tilde{\eta}}_{ij} = \bar{S}_{ii}^n \tilde{\eta}_{ij} - T_i \sum_{k=1}^{p_j} \beta_{ij}^k \text{sign}(\zeta_{ij}^k) |\zeta_{ij}^k|^{\theta_{ij}^k}. \quad (46)$$

According to Lemma 3, we have that $\tilde{\eta}_{ij}$ converges to zero after finite time, which together with the conclusion of that τ_i converges to zero after finite-time yields that η_i converges to η_0 in finite-time.

2) Consider the j th subsystem which can obtain the information from the i -th subsystem, which converges to η_0 in finite-time. By using the similar method, we can obtain that η_j converges to η_0 in finite-time.

Summing up 1) and 2), we can complete the proof. ■

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Chao Deng received the Ph.D. degree in control engineering from Northeastern University, China, in 2018. He is currently a Postdoctoral Research Fellow at the School of Electrical and Electronic Engineering in Nanyang Technological University, Singapore. His research interests include distributed fault-tolerant control, event-triggered control, adaptive fuzzy control and cyber-physical systems.

Weinan Gao received the B.Sc. degree in automation from Northeastern University, Shenyang, China, in 2011, the M.Sc. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2013, and the Ph.D. degree in electrical engineering from New York University, Brooklyn, NY, USA in 2017. He is an Assistant Professor of electrical and computer engineering at Allen E. Paulson College of Engineering and Computing, Georgia Southern University since 2017, and a Visiting Professor of Mitsubishi Electric Research Laboratory (MERL) at Cambridge, MA, USA during the summer of 2018. His research interests include reinforcement learning, adaptive dynamic programming (ADP), optimal control, cooperative adaptive cruise control (CACC), intelligent transportation systems, sampled-data control systems, and output regulation theory. He is the recipient of the Best Paper Award in IEEE International Conference on Real-time Computing and Robotics (RCAR) in 2018, and David Goodman Research Award at New York University in 2019. Dr. Gao is currently an Associate Editor of *IEEE/CAA Journal of Automatica Sinica*, *Neurocomputing*, a member of Editorial Board of Neural Computing and Applications, and a Technical Committee Member in IEEE Control Systems Society on Nonlinear Systems and Control. He was also the Web Chair in the International Workshop on the Security, Privacy, and Digital Forensics of Mobile Systems and Networks in IEEE International Conference on Computer Communications (INFOCOM) in both 2018 and 2019.

Weiwei Che received the B.S. degree in mathematics and applied mathematics in 2002 from Jinzhou Normal University, China; the M.S. degree in applied mathematics in 2005 from Bohai University, China; the Ph.D. degree in control engineering from Northeastern University, China, in 2008. She joined the EEE of Nanyang Technological University from October 2008 to October 2009 as a Postdoctoral Fellow. And she joined the Department of Mechanical Engineering, the University of Hong Kong, Hong Kong, China, from January 2015 to April 2015 as a Research Associate. She is currently a Professor at the Institute of System Science, Qingdao University. Her research interest includes non-fragile control, quantization control as well as fault tolerant control and their applications to NCSs, MASs and CPSs design.