Cascaded switching supervisory control of nonlinear underwater robotic vehicle for pipeline tracking

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**CASCADED SWITCHING SUPERVISORY CONTROL
OF NONLINEAR UNDERWATER ROBOTIC VEHICLE FOR
PIPELINE TRACKING**

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**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

2008

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of Nonlinear Underwater Robotic Vehicle for Pipeline Tracking**

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Abstract

In recent years, remotely operated vehicles (ROV) have experienced tremendous growth in underwater pipeline inspections where human diving is impractical. However, the ROV stabilization and pipeline tracking applications present several difficulties due to its inherent nonlinear state coupling, model uncertainties due to hydrodynamic forces, and in particular, and the vehicle possessing fewer actuators than the available degrees of freedom i.e. being underactuated. Most existing methods solve the problem in two-dimensional plane, very often by ignoring the nonlinear coupled terms and hydrodynamic uncertainties in the control system design, and treating stabilization and tracking separately.

This research project thus aims to develop a robust cascaded switched control and simulation software for the simultaneous stabilization and position tracking for an underactuated Research Robotic Centre (RRC) ROV. This is achieved through a combination of approaches. By considering only a few degrees of freedom at a time during the horizontal and vertical plane motions, and taking advantage of the vehicle inherent self-regulation in two directions during station-keeping mode, the problem of insufficient actuators can be avoided. A cascade structure and a proportional controller with nonlinear dynamics for states decoupling are implemented to control the velocities and positions simultaneously. To improve on the robustness and to handle the different motions in the desired planes, multiple controllers together with a supervisory controller are proposed. The supervisor is used to orchestrate the outer to inner loop switching based on the inner loop “energy” and the position tracking error, and the controllers switching using the ROV reference input requirements. Applying a dwell time during these switching results in a lower control effort and improves the asymptotic stability about equilibrium sub-manifold. Analysis and simulation of the proposed scheme showed that the system is stable and is able to achieve the desired goals. The advantages of the cascaded control scheme are its simple structure and its improved robustness.

A ROV Design and Analysis (RDA) package using MATLABTM scripting language and graphical block diagram simulation SIMULINKTM with virtual environment interface was developed to provide a systematic and an efficient platform for the control system simulations of the nonlinear ROV model with hydrodynamic perturbation. This nonlinear ROV model differs from existing models as it considers the hydrodynamic perturbations due to added mass computed by WAMITTM and ANSYSTM.

In conclusion, the control system was implemented for the RRC ROV and compared with other platforms such as JHU (Johns Hopkins University) and ROMEO ROV. The results for the RRC ROV showed a lower percent of thrust utilization, faster steady states response and robustness against hydrodynamic forces during the underwater pipeline tracking. Finally, experimental tests of the models and the cascaded switched control scheme were carried out in a swimming pool. The experimental results verified the efficacy of the scheme and compared well with the simulation results.

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*To my dearest wife Irene Ang Ai Wei
and my loving late daughter Giselle Chin Sze Kee*

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List of Symbols

Symbols	Descriptions
$\mathbf{I}$	Inertia tensor matrix
$\mathbf{M}_{\text{mass}}$	Mass matrix
$\mathbf{M}$	Total inertia mass matrix
$\mathbf{M}_{\text{RB}}$	Mass inertia matrix for rigid body
$\mathbf{M}_A$	Hydrodynamic added mass for mass inertia matrix
$\Delta \overline{\mathbf{M}}_A$	Upper bound for uncertainty $\Delta \mathbf{M}_A$ matrix
$\mathbf{M}_\eta$	Total mass inertia matrix in earth-fixed coordinate
$\mathbf{C}(\mathbf{v})$	Total Coriolis and centripetal matrix in body-fixed coordinate
$\mathbf{C}_{\text{RB}}(\mathbf{v})$	Coriolis and centripetal matrix for rigid body
$\mathbf{C}_A(\mathbf{v})$	Hydrodynamic added mass for Coriolis and centripetal matrix
$\Delta \overline{\mathbf{C}}_A$	Upper bound for uncertainty $\Delta \mathbf{C}_A$ matrix
$\mathbf{C}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})$	Total Coriolis and centripetal matrix in earth-fixed coordinate
$\mathbf{D}$	Linear hydrodynamic damping matrix in body-fixed coordinate
$\Delta \mathbf{D}$	Upper bound for uncertainty on linear hydrodynamic damping matrix
$\mathbf{D}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})$	Hydrodynamic damping matrix in earth-fixed coordinate
$\mathbf{G}_f(\boldsymbol{\eta})$	Gravitational and buoyancy forces vector in body-fixed coordinate
$\mathbf{G}_\eta(\boldsymbol{\eta})$	Gravitational and buoyancy forces vector in earth-fixed coordinate
$\mathbf{J}$	Euler transformation matrix
$\mathbf{J}_1$	Euler transformation matrix for linear motion
$\mathbf{J}_2$	Euler transformation matrix for angular motion
$\boldsymbol{\tau}_1$	Linear force vector, $[\tau_x \quad \tau_y \quad \tau_z]^T$
$\boldsymbol{\tau}_2$	Moment vector, $[\tau_\phi \quad \tau_\theta \quad \tau_\psi]^T$

τ	External forces and moments vector in body-fixed coordinate $[\tau_1 \quad \tau_2]^T$
τ_η	External forces and moments vector in earth-fixed coordinate
$\mathbf{v}$	Body-fixed velocity vector, $[\mathbf{v}_1 \quad \mathbf{v}_2]^T = [u \quad v \quad w \quad p \quad q \quad r]^T$
η	Earth-fixed position vector, $[\eta_1 \quad \eta_2]^T = [x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T$
$\eta_s, \mathbf{v}_s$	Earth and body-fixed statess vector for station-keeping dynamic
$\eta_v, \mathbf{v}_v$	Earth and body-fixed statess vector for vertical plane dynamic
$\eta_h, \mathbf{v}_h$	Earth and body-fixed statess vector for horizontal plane dynamic
η_Q	Observer states in earth-fixed coordinate
$\hat{\eta}_Q$	Estimated observer statess in earth-fixed coordinate
$\hat{\eta}_s$	Estimated observer statess in earth-fixed coordinate for station-keeping mode
$\hat{\eta}_o$	Observer error statess, $[\tilde{\eta}_s \quad \tilde{\eta}_Q]^T = [\eta_s - \hat{\eta}_s \quad \eta_Q - \hat{\eta}_Q]^T$
$\mathbf{u}$	Thrust output vector, $[\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3 \quad \mathbf{u}_4]^T$
$\bar{\mathbf{u}}$	Commanded input voltage vector
$\bar{\mathbf{u}}_I$	Inner control vector for station-keeping mode
$\bar{\mathbf{u}}_O$	Outer control vector for station-keeping mode
$\bar{\mathbf{u}}_{Iv}, \bar{\mathbf{u}}_{Ov}$	Inner loop control vector for vertical plane dynamic
$\bar{\mathbf{u}}_{Ih}, \bar{\mathbf{u}}_{Oh}$	Inner loop control vector for horizontal plane dynamic
$\bar{\mathbf{u}}_{I,dyn}$	Inner loop control vector for states decouple
$\bar{\mathbf{u}}_{I,pert}$	Inner loop control vector for perturbation attenuation
$\bar{\mathbf{u}}_{Ip}$	Inner loop control vector for proportional controller
$\bar{\mathbf{u}}_{O,dyn}$	Outer loop control input for states decouple
$\bar{\mathbf{u}}_{Op}$	Outer loop control input for proportional controller
$\mathbf{T}$	Thruster configuration matrix

f_T	Thruster's dynamic
$\mathbf{F}_T$	Thruster's dynamic matrix
$\mathbf{x}$	States vector, $\mathbf{x} = [\boldsymbol{\eta} \quad \mathbf{v}]^T$
$\mathbf{x}_{u1}$	Unactuated angular velocity of ROV, $[q \quad p]^T$
$\mathbf{x}_{u2}$	Unactuated angular angles of ROV, $[\theta \quad \phi]^T$
$\tilde{\mathbf{x}}_{2e}$	Velocity error in body-fixed frame, $[\tilde{u}_e \quad \tilde{v}_e \quad \tilde{w}_e \quad \tilde{p}_e \quad \tilde{q}_e \quad \tilde{r}_e]^T$
$\tilde{\mathbf{x}}_{1e}$	Position tracking error in body-fixed frame, $[\tilde{x}_e \quad \tilde{y}_e \quad \tilde{z}_e \quad \tilde{\phi}_e \quad \tilde{\theta}_e \quad \tilde{\psi}_e]^T$
$\tilde{\mathbf{x}}_{s2e}$	Velocity error in body-fixed frame for station keeping mode
$\tilde{\mathbf{x}}_{s1e}$	Position tracking error in body-fixed frame for station keeping mode
$\tilde{\mathbf{x}}_{se}$	Velocity and Position tracking error in body-fixed frame for station keeping mode $\tilde{\mathbf{x}}_{se} = [\tilde{\mathbf{x}}_{s1e} \quad \tilde{\mathbf{x}}_{s2e}]^T$
$\hat{\mathbf{x}}_1$	Estimated position states vector
$\hat{\mathbf{x}}_2$	Estimated velocity states vector
$\mathbf{x}_{1d}$	Actual position states
$\mathbf{x}_{2d}$	Actual velocity states
$\tilde{u}_e$	Surge velocity error during stabilization, $u_d - \hat{u}$
$\tilde{v}_e$	Sway velocity error during stabilization, $v_d - \hat{v}$
$\tilde{w}_e$	Heave velocity error during stabilization, $w_d - \hat{w}$
$\tilde{p}_e$	Roll rate error during stabilization, $p_d - \hat{p}$
$\tilde{q}_e$	Pitch rate error during stabilization, $q_d - \hat{q}$
$\tilde{r}_e$	Yaw rate error during stabilization, $r_d - \hat{r}$
$\mathbf{h}(\mathbf{x}, t)$	Nonlinear output dynamic
$\mathbf{f}(\mathbf{x}, t)$	Nonlinear dynamic, $\bar{\mathbf{f}}(\mathbf{x}, t) + \Delta\mathbf{f}(\mathbf{x}, t)$
$\bar{\mathbf{f}}(\mathbf{x}, t)$	Nominal nonlinear dynamic
$\Delta\mathbf{f}(\mathbf{x}, t)$	Perturbed nonlinear dynamic
$\mathbf{g}(\bar{\mathbf{u}}, t)$	Nonlinear input dynamic, $\bar{\mathbf{g}}(\bar{\mathbf{u}}, t) + \Delta\mathbf{g}(\bar{\mathbf{u}}, t)$
$\bar{\mathbf{g}}(\bar{\mathbf{u}}, t)$	Nominal nonlinear input dynamic

$\Delta \mathbf{g}(\bar{\mathbf{u}}, t)$	Perturbed nonlinear input dynamic
$\mathbf{M}_s, \mathbf{C}_s, \mathbf{D}_s, \boldsymbol{\tau}_s, \mathbf{T}_s$	Parameters used for station-keeping dynamic
$\mathbf{M}_h, \mathbf{C}_h, \mathbf{D}_h, \boldsymbol{\tau}_h, \mathbf{T}_h$	Parameters used for horizontal plane dynamic
$\mathbf{M}_v, \mathbf{C}_v, \mathbf{D}_v, \boldsymbol{\tau}_v, \mathbf{T}_v$	Parameters used for vertical plane dynamic
$\Delta \mathbf{f}_s(\mathbf{x}_s, t), \Delta \mathbf{g}_s(\bar{\mathbf{u}}, t)$	Perturbation dynamic for station-keeping mode
$\Delta \mathbf{f}_h(\mathbf{x}_h, t), \Delta \mathbf{g}_h(\bar{\mathbf{u}}, t)$	Perturbation dynamic for horizontal plane
$\Delta \mathbf{f}_v(\mathbf{x}_v, t), \Delta \mathbf{g}_v(\bar{\mathbf{u}}, t)$	Perturbation dynamic used for vertical plane
$\bar{\mathbf{f}}_s(\mathbf{x}_s, t), \bar{\mathbf{g}}_s(\bar{\mathbf{u}}, t)$	Nominal dynamic for station-keeping mode
$\bar{\mathbf{f}}_h(\mathbf{x}_h, t), \bar{\mathbf{g}}_h(\bar{\mathbf{u}}, t)$	Nominal dynamic for horizontal plane
$\bar{\mathbf{f}}_v(\mathbf{x}_v, t), \bar{\mathbf{g}}_v(\bar{\mathbf{u}}, t)$	Nominal dynamic for vertical plane
$\bar{\mathbf{f}}_{2e}(\tilde{\mathbf{x}}_{2e}, t), \bar{\mathbf{g}}_{2e}(\bar{\mathbf{u}}, \tilde{\mathbf{x}}_{2e}, t)$	Nominal dynamic for inner loop with observer
$\bar{\mathbf{f}}_{1e}(\tilde{\mathbf{x}}_{1e}, \tilde{\mathbf{x}}_{2e}, \bar{\mathbf{u}}_0, t)$	Nominal dynamic for outer loop with observer
l_m	Characteristic length of model
l_p	Characteristic length of prototype
ρ_m	Density of water around the model
ρ_p	Density of water around the prototype
u_m	Velocity of the model
u_p	Velocity of the prototype
$\mathbf{f}_G$	Weight vector of the ROV in earth-fixed coordinate
$\mathbf{f}_B$	Buoyancy force vector of the ROV in earth-fixed coordinate
W	Mass of the ROV in air
B	Buoyancy force of the ROV
∇	Volume displaced by submerged ROV
x_B, y_B, z_B	Cartesian coordinate of the center of buoyancy

x_G, y_G, z_G	Cartesian coordinate of the center of buoyancy
Q	Torque generated by the propeller
Ω	Rotational speed of the propeller
K_{Td}	Torque constant
K_{Qd}	Rotational speed constant
V_m	Armature voltage
I_t	Armature current
L_m	Armature inductance
R_m	Armature resistance
K_m	Motor torque constant
K_e	Motor back emf
V_{obs}	Lyapunov function involving observer dynamic
$\mathbf{Q}_o(\boldsymbol{\eta}_s)$	Coordinate transformation for observer design
$\mathbf{K}_{01}, \mathbf{K}_{02}$	Observer gains
V_I	Lyapunov function for inner loop
V_O	Lyapunov function for outer loop
$k_{px}, k_{py}, k_{pz}, k_{p\psi}$	Outer loop proportional controller constant
$k_{pu}, k_{pv}, k_{pw}, k_{pr}$	Outer loop proportional controller constant
$\sigma_L(t)$	Constant piecewise switching signal for loops switching
$\sigma_{co}(t)$	Constant piecewise switching signal for outer controllers switching
$\sigma_{ci}(t)$	Constant piecewise switching signal for inner controllers switching
$\boldsymbol{\rho}_s, \boldsymbol{\rho}_h, \boldsymbol{\rho}_v$	Reset signal
$\boldsymbol{\Omega}_C$	Closed position quadrant for multiple controllers
$\boldsymbol{\Omega}_L$	Closed position quadrant for inner loop switching
$\mathbf{R}_{10}, \mathbf{R}_1$	Regions for cascaded loops and inner loop respectively
$\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3$	Regions for different inner and outer controllers
I_C	Controller index
I_L	Loop index

Chapter 1

Introduction

1.1. Background of URV

In the last four decades, the use of underwater robotic vehicles (URV) has experienced tremendous growth. Many are used for underwater inspection of sub-sea cables and oil and gas installations like Christmas trees, structures and pipelines. They are essential at depths where the use of human divers is impractical. The deployments of URV present several difficulties as they are difficult to control remotely and autonomously. The design of autonomous control is a challenge due as the vehicle dynamics is highly nonlinear, with intrinsic uncertainties and subject to the influence of unknown external disturbances.

Traditionally, URV can be broadly classified as remotely operated vehicle (ROV) and autonomous underwater vehicle (AUV), depending on their designed tasks and modes of operations. As seen in the Fig. 1.1, a new class of URV called Hybrid ROV-URV (HROV) that have both the capabilities of a ROV and an AUV has been proposed and experimented with. Nevertheless, ROV are best suited for work that involves operating from a stationary point or cruising at relatively slow speeds such as pipeline inspection. Furthermore, for any tasks involving manipulation and requiring maneuverability, they are the most cost-effective platform. The following shows some of the common URV present in the literature.

For example, the ROV JASON II/MEDEA (see Fig. 1.2) system is designed by Wood Holes oceanographic's Institution of Deep Submergence Laboratory (IDSL) for scientific investigation of the deep ocean and seafloor. The ROV system includes tether management by the vehicle MEDEA that decouples Jason from wave motion. Detailed description can be found in the website cited as shown in Fig. 1.2.

Figure 1.1: Classification of underwater vehicle

SIRENE as shown in Fig. 1.3 is an AUV designed and built by the French Institute for the Sustainable Exploitation of the Sea, IFREMER. It has an open frame structure with a dry weight of 4000 kg and a maximum depth of 4000 m. It is used for environmental survey and monitoring. It is equipped with two back thrusters for surge and yaw motion control in the horizontal plane, one vertical thruster for heave control. Detailed description can be found in the website cited as shown in Fig. 1.3.

Figure 1.2: Sea trials of JASON II ROV (Photo cited from <http://robotics.me.jhu.edu>)

Figure 1.3: SIRENE AUV

(Picture cited from http://www.noc.soton.ac.uk/CASEE/CASEE2/pages/Masterclass/IFREMER_Vincent.pdf)

SWIMMER-PHENIX as shown in Fig. 1.4 is a HROV developed by IFREMER in collaboration with French partner, CYBERNETIX. It consists of the SWIMMER AUV which carries and delivers a standard work-class ROV PHENIX from the surface to a subsea docking station installed close to the equipment. The subsea docking station is connected to the surface production facilities through a preinstalled power or control umbilical. The ROV has a short 200 m umbilical that is wound round a self-contained winch mounted on the SWIMMER. Detailed description can be found in the website cited as shown in Fig. 1.4.

Figure 1.4: SWIMMER/PHENIX Hybrid ROV-AUV
(Photo cited from <http://www.ifremer.fr/offshore>)

Generally, the focus of URV (see Fig. 1.1) research has been centered on AUV or HROV, but recent experiences with commercial AUV have highlighted some of their limitations – the lost AUV and their lack of real-time communication with the surface vessels. In addition, current AUV designs are not able to replace ROV, for nearly four decades, in many operations such as pipelines inspection for the oil and gas industry, where the ROV hard wire link is still required. Hence in this thesis, the analysis and control system design on the ROV will be studied and applied for pipelines inspection operations. The goal is imbue the ROV with the capability of autonomous operations, freeing the operator to perform other tasks.

1.2 Overview of the RRC ROV and Basic Tasks Undertaken

The ROV designed by Robotics Research Centre (RRC) in Nanyang Technological University (NTU) known as the RRC ROV (see Fig. 1.5) is designed to perform underwater pipeline inspections such as locating pipe leakages or cracks. Since pipelines carry hydrocarbons such as natural gas or oil at relatively high pressures, a crack in the line can result in explosion as well as the accidental discharge of oil and/or gas with adverse environmental consequences. It is therefore desirable to inspect the pipelines periodically and to perform the tasks efficiently. This can be done with the operator focusing on inspection whilst the ROV automatically tracking the pipeline.

The twin “eye-ball” ROV depicted in Fig. 1.5 has an open-frame structure and is 1 m long, 0.9 m wide, and 0.9 m high. It has a dry weight of 115 Kg and a maximum operating depth of 100

m. Its designed tasks include inspections of underwater pipelines. The RRC ROV is underactuated as it has four thruster inputs for six degree of freedom (DoFs) (that is surge, sway, heave, roll, pitch and yaw velocity) with a high degree of cross coupling between them. The vehicle is equipped with two lateral thrusters mainly for surge, sway and yaw motion control in the horizontal plane, two vertical thrusters for heave in the vertical plane and a suite of sensors for position and velocity measurements (see Table 1.1). Roll and pitch motions are passive as the metacentric height (that is the distance between the center of gravity of a ROV and its metacenter) is sufficiently large to provide adequate static stability. A brief description of the component layout of the ROV RRC is given.

1. Four thrusters, each providing up to 70N of thrust;
2. Two cylindrical floats with four balancing steel weight;
3. Main pod (Pod 1) and sensors with navigational pod (Pod 2);
4. Two halogen lamps and external sensor such as altimeter (see Table 1.1).

The pre-defined underwater pipeline in sub-sea operation is shown in Fig. 1.6. In an actual vehicle deployment, the pre-defined pipeline locations are usually provided by the company and the vehicle uses external cameras on-board the ROV and navigation system to track it.

Figure 1.5: Schematic of RRC ROV

Figure 1.6: Proposed underwater pipeline tracking profile for RRC ROV

With the operational tasks defined, a system and control design can be undertaken. Upon completing the preliminary design, the MATLABTM[102] scripting language and graphical block diagram model, SIMULINKTM together with Virtual Reality Modeling Language (VRML) visualization were used to model the ROV dynamics, to refine controller designs, and to examine the feasibility of the overall control system design through execution of underwater pipeline tracking mission given in Fig. 1.6. The SIMULINKTM model consists of the followings: (a) pipeline tracking planner; (b) navigation sensors; (c) switching control system design and (d) ROV dynamic with hydrodynamic perturbation block diagram. In this thesis, the emphasis is on the ROV dynamics with hydrodynamic perturbations and the controller design algorithms. However for completeness, the pipeline tracking planner and navigation sensor used in the SIMULINKTM model are briefly explained.

The pipeline-tracking planner is the high level decision-maker on-board the ROV as shown in Fig. 1.7. It monitors all system states and issue commands to the controllers via a supervisor in the cascaded loop. The planner observes the states as well as flags from simulated emergency events such as collision avoidance (not included in the simulation) and system commands. Mainly, the planner decides the locations and the speed to go. During the simulation trials, the planner was set to command the ROV to descend to certain depth and then move to the first

location of the underwater pipeline in horizontal and vertical plane manner. The planner also issues a flag, which stops the simulation when the ROV complete the designated pipeline-tracking mission. Then, the ROV ascend to surface that is within sufficient capture range.

The navigation sensor block simulates the navigation system consisting of external sensors and a nonlinear observer for estimating the location and pose of the ROV with respect to its body-fixed frame. On the other hand, Euler's transformation maps the body-fixed frame to the earth fixed frame. From this block, the simulated system states are output to the pipeline tracking planner and controllers.

The six-DoFs ROV dynamic model is identified as the "ROV Dynamic". It is described by a set of differential equations derived using the Newton's Second law of motion. Modeling the dynamic equations of the ROV is usually the first step in developing an accurate simulation. To create a more realistic model, perturbations due to uncertain hydrodynamic forces such as damping and added mass on the ROV are included in the ROV model. These values [95,96,133] numerically determined by a hydrodynamic software package such as WAMITTM and ANSYSTM Computation Fluid Dynamic (CFD) package.

The controller design is proposed to enable the ROV to achieve the operating specifications given in Table 1.2. The control architecture representing the control algorithm is shown in Fig. 1.8. As seen in Fig. 1.8, a cascaded structure allows simultaneous stabilization and position tracking for the RRC ROV. To cater for the need of underwater pipeline tracking operation, the inner loop is designed for point stabilization while the cascaded loop is for tracking operation.

However, it can be seen switching from cascaded loop to inner loop allows the ROV to perform independent stabilization whilst in cascaded form, it allows simultaneously tracking and stabilization operation. To create a robust controller design against the uncertainty of the ROV model, several controllers are used in both loops. The decision to select which of these controllers from the "controller bank" to use is orchestrated by an interface called "supervisor" via a switching law. This supervisor also handles the switching between the cascaded loop to the inner loop.

Sensors	Purpose	Outputs	Functioning	Locations
Ultrasonic Altimeter	Depth	z	Yes	External
Magnetic Compass KVVH-C100	Heading	ψ	Yes	Pod 2
Argonaut DVL	Velocity	u, v, w	Yes	External

Table 1.1: Sensors used in RRC ROV

Figure 1.7: ROV Mission Planner decision algorithm

Figure 1.8: Block diagram of supervisory switching cascaded control system

1.3. Background of ROV Control

Similar to other marine vehicles such as a ship [35,66] and autonomous underwater vehicle (AUV) [37,39], the remotely-operated vehicle (ROV) [34,36,96,128] dynamics are nonlinear, highly coupled in motion and susceptible to hydrodynamic uncertainties. In addition, the RRC

ROV is underactuated (i.e. the number of DoFs exceeds the number of actuators [1]). The performance can suffer significantly due to lack of thruster for the unactuated DoFs when the vehicle is performing a pipeline inspection that involves both station-keeping and pipeline tracking at points on the pipeline that emphasize these nonlinearities and hydrodynamic uncertainties. This deteriorating performance can be observed when the ROV is operating in the open-loop condition. This causes the ROV to exhibit a whirling motion whereby the linear velocities (and its corresponding linear positions) are not asymptotically stable during stabilization. The problem of whirling ROV motion is due to the added mass components in the Coriolis and centripetal term creating a cross coupling effect on the ROV's motion – the Munk moments [132] – and arises from the change in the directions of the fluid as the ROV moves with a small pitch and roll angles.

Hence, a robust cascaded control system to control both the velocities and position simultaneously is required. Additionally, to reduce the coupling in motion, the number of DoFs for control action is to be reduced. An example of simultaneous stabilization and tracking of ROV (the ODIN ROV) using output feedback with backstepping method is given by Jiang et. al. [62,65]. However, a restriction of the ROV velocity [62,65], can be conservative and the controller design becomes complex, as solution to partial differential equations are required in the derivation of the control law. Moreover, their approach solves the problem in the two-dimensional space only. The measurement noises in the output measurements and the quadratic cross terms of the unmeasured velocities appearing in the Coriolis and centripetal matrix make the estimation of the velocities complicated. To avoid this, Jiang et. al. used coordinate transformation [62,65] to cancel the quadratic cross terms to simply design of the observer. Motivated by the above nonlinear dynamic and Jiang et. al. observer design on the ROV, a robust cascaded system with output feedback for both tracking and stabilization under the hydrodynamic uncertainties on the ROV operating in two and higher dimensional spaces is proposed.

However, it is known that a single-fixed controller cannot fully satisfy [47] the desired closed loop response as the performance can suffer significantly when the ROV is tasked to perform the pipeline inspection that involves both station-keeping and pipeline tracking operation at points on the pipeline that emphasize on these nonlinearities and hydrodynamic uncertainties. Hence, what is required is a cascaded control scheme that is robust to the hydrodynamic uncertainties and is capable to switch between station-keeping and tracking operations to achieve less coupling in motion and control efforts. Recently, a class of hybrid systems

namely, a switched system that could accomplish this has been reported in [45,46]. Only in recent years, have switching control [46,48,49,50] been applied extensively as many (if not most) of the dynamic systems encountered in practice involve both continuous and discrete dynamic e.g. an ROV that has an embedded system in which computer software interacts with physical devices such as sensors and thrusters. To improve on the robustness of the cascaded system, it is proposed that a multiple controllers' switching concept be adopted. On the other hand, switching between the station-keeping and tracking operation is accomplished by a cascaded to inner loop switching algorithm. The proposed cascaded switched system for simultaneous stabilization and position tracking under hydrodynamic uncertainties is used for the RRC ROV.

1.4. Objectives

Robust control for the underactuated nonlinear and highly coupled ROV under uncertainties in the hydrodynamic forces is always an open research area. To best of our knowledge, the work on the asymptotic stability of nonlinear Proportional-decoupled cascaded supervisory switching control system is not found in the present literature. In particular, simultaneous stabilization and tracking control to an equilibrium submanifold while operating in the decoupled planes under the hydrodynamic perturbation. With that, the following objective can be formulated.

- (1) to model the dynamic and the hydrodynamic forces acting on the RRC ROV;
- (2) to resolve the number of DoFs for control action, with as much decoupled motion as possible;
- (3) to solve the instability in open loop ROV system at equilibrium (due the added mass component in the Coriolis and centripetal component creates a cross coupling effect on ROV's motion);
- (4) to perform simultaneous stabilization and position tracking and to estimate velocity and position states;
- (5) to reduce the thrust forces required and improve the robustness and steady-state performance of the system despite of the hydrodynamic perturbations;
- (6) to achieve asymptotic stability at the equilibrium submanifold,

Besides the above objectives, the controller must be able to track the desired pipeline profile as given in Fig. 1.6 and at the same time regulate the output velocity (that is stabilization) as tabulated in Table 1.2. The RRC ROV is designed primarily for underwater inspection of pipes, pipe-lines and structures. It must be capable of being located near the target or of tracking pipe-lines.

The motivations behind the specifications (see Table 1.2) are to provide criteria to handle robustness of the model uncertainty and steady-state performance of the ROV system that has to achieve during the intended pipeline operation. Further experiment through sea trial can be incorporated to improve the specification. For a start, the summary of the ROV operation specifications are as follows.

- a) the input thrust must not exceed 70 N (or its equivalent voltage level);
- b) the maximum velocity must not exceed 0.6 m/s. The settling time must be less than 100 sec with maximum 10% overshoot;
- c) the earth-fixed position fluctuation should be $\pm 10\%$ of the desired position. The maximum operating length, $x = y = z = 100$ m. The settling time must be less than 100 sec with maximum 10% overshoot;
- d) the operating roll and pitch angles are to be small and should not exceed 30 degrees (or 0.5 radian);
- e) the mission time cannot exceed 3 hours (=10800 sec) using a portable electric generator;
- f) the controller design must be robust enough to withstand the worst case of 10 percent parameter variation in the ROV added mass and damping matrix.

Item	Performance Specifications	Requirements
1	Input thrust	≤ 70 N
2	Maximum body-fixed velocity	0.6 m/s, 0.6 rad/s
3	Maximum operating length in X, Y and Z directions	100 m
4	Settling time of body-fixed velocity and earth- fixed position	≤ 100 sec
5	Percent overshoot in earth- fixed position	≤ 10 %
6	Maximum roll and pitch angles	$\leq 30^\circ$
7	Operating or mission time	≤ 3 hours
8	Indication of robustness	Apply 10% parameter variation on ROV added mass and damping matrix.

Table 1.2: ROV operating specifications

1.5. Scope

The thesis covers the dynamic modeling of a prototype RRC ROV using the Newtonian method with focus on modeling and experimental verification on hydrodynamic damping and added mass using the computational Fluid Dynamic (CFD) software such as WAMITTM and ANSYSTM. Apart from the RRC ROV modeling, the cascaded switched system employed in this thesis work belongs to a class of hybrid systems that is designed primarily for simultaneous stabilization and tracking control during the pipeline inspection (see Fig. 1.6 for the typical pipeline profile) that operates in a least coupled manner in the following subsystems such as station-keeping mode, horizontal and vertical plane. The aforementioned objectives and ROV operating specifications for the pipeline inspection task have to be met. To facilitate the modeling and control system simulation, a ROV Design and Analysis (RDA) package was developed.

1.6. Organization

The thesis is organized as follows:

- (a) Chapter 3 describes the dynamic modeling of a prototype RRC ROV using the Newtonian [94] method. Parameters such as hydrodynamic damping and added mass were determined using the CFD software such as WAMITTM and ANSYSTM. A ROV Design and Analysis (RDA) package was designed and used to model the nonlinear ROV dynamics for the subsequent controller system designs.
- (b) Chapter 4 describes the nonlinear properties of the ROV that operates in either station-keeping mode and decoupled motion such as horizontal or vertical plane.
- (c) Chapter 5 describes the cascaded control system design with the nonlinear observer for each subsystem such as station-keeping mode, horizontal and vertical plane. The simulation result of the cascaded system was compared with other controller such as backstepping, PI controller and single-loop PID [33].

- (d) Chapter 6 describes the cascaded supervisory switched control systems for the cascaded system proposed in Chapter 5. The proposed cascaded switching law for both the cascaded to inner loops and multi-controllers switching were stated.
- (e) Chapter 7 describes the application of the proposed cascaded switched design and other controllers on two working ROVs, namely: JHU (Johns Hopkins University) ROV (from US) and ROMEO ROV (from Italy). Experimental result of the actual RRC ROV in pool was done.
- (f) The thesis concludes with a summary of results and recommendations for future works.

1.7. List of Publications

The followings are the Book chapter and Journal papers published during the course of Ph.D work. The papers used in the respective Chapters are marked in blue.

[95] Chin, C. S., Lau, M. W. S., Low, E. and Seet, G. L. (2006). Dynamic Modelling and Cascaded Controller Design of A Low-Speed Maneuvering ROV. Advanced Technologies: Research, Development & Application, Bojan Lalic (Ed.), Published by Advanced Robotic Systems International, ISBN 3-86611-197-5, Vienna, Austria, pp 159-186. (Chapter 3)

[133] Eng Y.H., Lau . S., Low E., Seet G. L. and Chin C. S. (2008) Estimation of the hydrodynamic coefficients of an ROV using Free Decay Pendulum Motion, Engineering Letters, 16(3), pp.326-331. (Chapter 3)

[102] Chin, C. S., Lau, M. W. S., Low, E. and Seet, G. L. (2006). Software for Modelling and Simulation of a Remotely Operated Vehicle. International Journal of Simulation Modelling, 5(3), pp 114-125. (Chapter 3, 4, 5, 6, 7)

[34] Chin, C. S., Lau, M. W. S., Low, E. and Seet, G. L. (2006). A Pipeline Tracking Control of an Underactuated Remotely Operated Vehicle. The Mediterranean Journal of Measurement and Control, 2(1), pp 22-34. *(Chapter 4, 5)*

[132] Chin, C. S., Lau, M. W. S., Low, E., and Seet, G. L. (2008). Robust and decoupled cascaded control system of underwater robotic vehicle for stabilization and pipeline tracking. Proceedings of the Institution of Mechanical Engineers, Part 1 (Journal of Systems and Control Engineering), 222, pp. 261-278. *(Chapter 5)*

[33] Chin, C. S., Lau, M. W. S., Low E., Seet G. G. (2006). Robust Controller Design Method and Stability Analysis of an Underactuated Underwater Vehicle. International Journal of Applied Mathematics and Computer Science, 16(3), pp. 101-112. *(Chapter 6)*

Chapter 2

Literature Review

This chapter presents the review of literature relevant to the thesis. This includes the control method available for the underactuated system, followed by a revisit of the types of control for stabilization and tracking application of the underactuated system. The subject most relevant to the thesis is the simultaneous stabilization and tracking of underactuated system and the various methods of determining the hydrodynamic coefficients. More importantly, in the various chapters that follow, references are made to literature directly relevant to the chapter material for as evidence to support assumptions or hypothesis and for the purpose of comparison.

2.1. Control Method for Underactuated Systems

As aforementioned, most ROV in particular the RRC ROV do not have an available control in roll and pitch and the problem of controlling these vehicles in six DoF is therefore an underactuated control problem. Linearization of the nonlinear underactuated system about its equilibrium points gives uncontrollable eigenvalues. This implies that a nonlinear analysis is necessary in order to characterize the controllability [4] and stabilizability properties of the system. Since the linearized system is not stabilizable, it cannot be exponentially stabilized to equilibrium points using any smooth feedback [3]. Thus, the underactuated systems do not satisfy Brockett's necessary condition [2] that showed the asymptotically stable to a desired equilibrium solution with any time- continuous invariant feedback cannot be achieved.

Firstly, the underactuated systems with non-integrable dynamics [4] cannot be asymptotically stabilized by a continuous time-invariant feedback law was shown. Fortunately, Reyhanoglu [5] first proved that the system is not asymptotically stable to a given equilibrium solution using a continuous time-invariant feedback, it is still strongly accessible and small-time locally

controllable at any equilibrium. Thus the system can be asymptotically stable to a desired configuration with exponential convergence rates using discontinuous time-invariant feedback laws. The same apply in the tracking control if the velocities become zero, the tracking problem become a stabilization problem that cannot be solved by using any time-invariant smooth feedback as mentioned.

However, the underactuated surface vessels couldn't be asymptotically stable to equilibrium by even the discontinuous time-invariant feedback laws [13]. It proposed a smooth time-varying feedback controller for an underactuated surface vessel that contained explicit time-periodic sinusoidal terms (similar in structure to [14]) to obtain local exponential regulation. A modified smooth time-varying feedback law [3,15] was used to locally exponentially regulate the position and orientation of an underactuated surface vessel. Several authors have shown that the underactuated problem can be overcome if the feedback laws such that the time varying discontinuous or hybrid control were used. For instance the survey paper [16] includes these control and numerous references therein.

In some cases [7,8,9,10] with the smooth time varying state feedback control on the underactuated vehicles having zero [6] gravitational fields are not asymptotically stable to single equilibrium. To resolve this problem, a smooth time-varying state feedback [7,8,9,10] law that asymptotically stabilized the systems to equilibrium manifold instead of the single equilibrium point was presented. As a result, a smooth time-varying state feedback law [11,12] that exponentially stabilizes a ship with non-zero gravitational term to an equilibrium manifold was published.

Although a smooth time-varying state feedback law can be applied to achieve asymptotic stability, it fails to meet the requirement of exponential convergence. Since exponential convergence is an important performance characteristic for practical applications. To date, several important steps have been made toward the design of a continuous time varying and/or discontinuous feedback law guaranteeing exponential regulation of nonholonomic systems in chained form [129, 17, 18, 133]. However, the feedback law and its stability proof were shown in the absence of disturbances and often limited to dynamic in the chained form.

From the above, there is several types of control methodology for underactuated systems for stabilization or position tracking control that guarantee some form of asymptotic stability. For clarity, they are classified into stabilization and/or position tracking control.

2.2. Control Method for Stabilization of Underactuated Systems

In literature, feedback stabilization of actuated systems has been most studied in the published literature. However, the stabilization of the underactuated system is sparse in the current literature. To have an overview of the existing technique on the stabilization of the underactuated system, the following methods using: (a) motion planning; (b) open-loop method; (c) discontinuous feedback; (d) linearization; (e) partitioned of system; (f) decoupled plane motions, and lastly (g) hybrid control were used.

1. The most intuitive method of overcoming the underactuated problem is to perform a motion planning [19], to avoid path that has nonholonomic constraints. In recent years there has been extensive research on motion planning for nonholonomic systems. A traditional problem in robot motion planning has been constructing an obstacle-avoiding path that is finding a path subject to constraints on the position. Therefore, this approach is often referred to as motion planning for nonholonomic systems. This problem has been extended to nonholonomic motion planning, that is motion planning subject to non-integrable constraints on the velocities. A path finding algorithm [20] for an embedded manifold rolling on a flat plane was shown, using the case where the embedded manifold is a unit ball.

Another approach to nonholonomic motion planning has been the use of a highly oscillatory control [21]. These are among several important contributions to nonholonomic motion planning which all have in common that they consider the kinematics of the nonholonomic system meaning a model with no drift vector field that contains the dynamic. For example the unicycle [16] may be steered from any initial configuration to any final configuration through the motion sequence. Rotate the unicycle so as to align the projection of its mid-plane with the straight line of the initial and final location. Roll forward until it reaches the final location and rotate to the final orientation. However, such a solution is not robust with respect to disturbance and prone to error due to the initial conditions and modelling inaccuracies.

2. Instead of performing a motion planning under the nonholonomic constraint, open loop strategies can be used. The most intuitive approach to open loop control for a nonholonomic system is to find a feasible path to achieve any desired configuration. Thus this problem can be viewed as one of path planning subject to constraints imposed by the

vehicle dynamics. Example of these approaches that use the sinusoid inputs [22, 23, 24] at integrally related frequencies at the kinematics level can be found. A small-amplitude, low frequency sinusoidal inputs [25] was used to steer an underactuated underwater vehicle from initial to a final, desired configuration. However, such a solution is not robust with respect to disturbance

3. A non-smooth feedback [11, 12] law that exponentially stabilizes a ship with non-zero gravitational term to an equilibrium manifold was proposed. Besides a discontinuous feedback [5] law utilized the sigma-process [26] to exponentially stabilize an underactuated ship was used. A discontinuous time-varying feedback [18] stabilizer for a nonholonomic system was applied on underactuated ships. Some local exponential stabilized results were reported [27] based on time-varying continuously control approach. Using a smooth time-varying state feedback that makes the origin of a model of an underactuated ship asymptotically stable was reported in the experiment [28].

The experimental result show that underactuated mechanical system is more susceptible to perturbations than fully actuated mechanical systems. This is caused by the fact that no actuator is available to directly compensate (part of) the perturbations acting on the unactuated degree of freedom. In most research [29,30] dealing with the control of underactuated mechanical systems such as ship, the influence of perturbations on the closed-loop dynamic has generally not been taken into account.

4. In linear control approach for trajectory-tracking of underactuated vehicles, utilizes local linearization and decoupling of the multi-variable model to steer the same number of DoFs as the number of available control inputs, which can be done using standard linear (or nonlinear) control method. Alternative approaches include the linearization of the vehicle error dynamic around the trajectories that lead to a time-invariant linear system (also known as trimming trajectory) combined with gain scheduling or linear parameters varying design method [31]. The basic limitation of these approaches is that stability is only guaranteed in a neighbourhood of the selected operating points. Moreover, performance suffers significantly when the vehicle executes manoeuvre that emphasizes its nonlinearity and cross coupling.
5. The actuated and unactuated states [1,4] of the nonlinear equations was partitioned [32] and a controller was designed for only the actuated state. This can be done as the restoring

forces (due to the nonzero gravitational field or potential energy [11] of the unactuated states) in the unactuated states allow these two motions to stabilize themselves effectively. With the unactuated states being self-stabilizable [33,34] and/or ROV moving in plane [33,34], the actuated closed-loop system is asymptotic stable to a small parameters perturbation. No control is required for the unactuated states as these are deemed to be self-stabilizable. Hence this reduces the DoF for the control system design.

6. On the other hand, in order to avoid dealing directly with a set of nonlinear equations, one may choose to work with sets of simpler equations (or linear) and to switch among these simpler models. One of the methods is to restrict the motion to a certain plane [36,37,38,39,40,41,42,43,44]. In each plane, the motions are decoupled from its six DoF and often the unactuated states do not exist due to decrease in DoF. For instance in horizontal plane, the vehicle can be steered at constant forward speed towards a straight line that passes through the desired target point with desired vehicle's orientation. At the end of this phase, the vehicle follows a constant heading with an altitude change known as vertical plane operation.

This approach avoids dealing with the problem of underactuation as the number of DoF matches the number of available control inputs. This leads to a more appealing method as the number of DoF is reduced for the controller design.

7. In addition, there is a class of hybrid systems, namely, a switched system [45,46] that have introduced. The need for switching stems [47] from the fact that typically no single controller can guarantee a desired behaviour when design has to be done with a poorly modelled process. Such switching schemes provide an alternative to more traditional continuously tuned adaptive control algorithm. Conventional continuous adaptation is not always capable of performing satisfactorily. This may be particularly true whenever the plant switches among different modes of operations or if the closed-loop signals are not sufficiently exciting. In both circumstances undesirable transience may typically arise due to slow adaptation. Moreover, many (if not most) of the dynamic systems encountered in practice involve both types of dynamics e.g. a ROV that is an embedded system in which computer software interacts with physical device such as sensors and actuators.

In recent years, switching control [46,48,49,50] has been applied extensively in many applications. The advantages of switching control include achieving stability and

improving transient response. A logic-based switching control for stabilizing a nonholonomic integrator was given in [46]. The hybrid controllers in the form of switching control architecture on chained and Chaplygin form were introduced to reduce the tracking problem to the stabilization problem in [49]. In addition, a switching control on extended power form consisting of a high level discrete event supervisor and a family of low-level feedback controllers were discussed in [50].

As observed in these papers, most of these hybrid controllers are based on either special models or special forms (power, chained or extended power form [48]). Extended application for systems with more than two control inputs for systems that are not in power or chained systems is an open problem [51]. With perturbations acting on the switched system, the robustness of the candidate controllers for the ROV model may not be asymptotically stable [52,29] – even then, these controllers are only kept in the loop for a short period of time.

2.3. Control Method for Tracking of Underactuated Systems

In addition to the stabilization or regulation problem (as seen in the Section 2.2), several controllers such as (a) continuous time-invariant state feedback and (b) Lyapunov's direct method and passivity approach have also been proposed for the tracking control problem.

1. Specifically, a continuous time-invariant state feedback [53] controller was used to achieve exponential position tracking. This is provided that the desired surge velocity is always positive. However, due to the control structure, the orientation of the surface vessel is not controlled. A tracking controller that achieved exponential practical stability (that is exponential stability of an arbitrarily small neighbourhood of the desired trajectory) of an underactuated surface vessel [54] was proposed. Pettersen et. al. [55] proposed a continuous time-invariant control law that enables semi-global exponential position and orientation tracking. They proved the semi-global exponential position and orientation tracking for a class of desired trajectories (that is a straight line or a sinusoidal trajectory cannot be tracked).
2. Additionally, a high-gain-based local tracking [56] scheme was proposed. Based on Lyapunov's direct method and passivity approach, two tracking solutions [57] were

presented. The simple tracking controller using cascaded [58] approach and linear time-varying system theory was proposed. It is noted that the yaw reference velocity is required to be nonzero [56,57, 58]. With that, the restrictive assumption [59] was then removed. The k-exponentially stable tracking problem [35] for an underactuated ship using two controls, namely surge force and yaw moment using a simple state-feedback control law were addressed. For tracking control of underactuated ship, a cascaded [60] structure with a weaker persistent excitation (PE) condition to obtain a k-exponential tracking condition was proposed.

2.4. Control Method for Simultaneous Stabilization and Tracking of Underactuated Systems

As shown above, the stabilization and tracking of ROVs or ships have been studied separately. It is not only recently that a new approach for simultaneously stabilization and tracking for underactuated systems using back stepping design methodology [61] was proposed. This is followed by the stabilization and tracking using output feedback for unicycle-type mobile robot [62], ship [63], nonlinear benchmark system [64] on rotational or translational actuator system (RTAC) were proposed. Furthermore, a practical result on stabilization and tracking was given using a dynamic feedback approach [66].

One classical application on a ROV is the stabilization and tracking control of an ODIN ROV [62, 65]. The control law required the tracking trajectories to satisfy the velocity constraint that preserved the stabilization property and at the same time using output feedback control by backstepping method to achieve asymptotically tracking. However, it is limited to motion in two-dimensional spaces, as the dynamic becomes less complicated for analysis and control systems design. Furthermore, the control system design is for unperturbed system and hence it is less robust against hydrodynamic uncertainties.

2.5 Modelling of ROV with Hydrodynamic Perturbation

It is observed that some of the ROV tasks would require the vehicle to hover over a certain set of coordinates while maintaining a fixed orientation in the presence of various types of disturbances. These disturbances can be parametric uncertainty due to inaccurate parameters such as imprecision on the mass properties, hydrodynamic added mass and damping

coefficient. Many hydrodynamic test facilities [67,68, 69,70,71,72,73,74,75] were set up to measure these hydrodynamic coefficients. They typically normally will include a planar motion mechanism (PMM) that would tow the test vessel according to prescribed motion trajectory. A suite of sensors on the PMM would measure all the six DoF motions .and forces acting on the test vessel. These are used then used to determine the hydrodynamic force coefficients. Such facilities are often expensive, sophisticated and time consuming to use. One of the hydrodynamic test facilities set up in John Hopkins University is shown in Fig. 2.1.

Figure 2.1: Hydrodynamic test facility at the John Hopkins University.

(photo cited from www.jhu.edu/news-info/news/home02/jhurov2.html)

For vessels of the size of ocean-going ships, the test facilities would use a suitable scaled model of vessel and then using laws of dynamic similitude, obtain the hydrodynamic force coefficients.

Alternatively, computational fluid dynamics, for computation of added mass and damping [83], and control systems software [76,77,78,79,80,81] provide a cost-effective and good approximation to carry out modelling and control system design. A crude approximation can be used to model critical components of the ROV by circular or rectangular cross sections and to apply nonlinear time-domain strip theory [82] and fluid dynamics equation to determine both the hydrodynamic force and drag coefficients. Such an approach could be acceptable for low-speed applications and for feedback control design. As errors in estimating these hydrodynamic coefficients for the ROV dynamic equations cannot be totally avoided, the ROV model is inherently inaccurate [84]. Greater trust on these values would reduce the

degree of uncertainty in controller design. Hence, these values should be validated by experiments.

As the RRC ROV is not a large piece of equipment, a low-cost approach was adopted. A scaled model of the ROV was used in a small water tank (where the model is stationary) to experimentally determined principally the diagonal elements of the added mass and drag matrices. These values are scaled up for the full size RRC ROV according to the laws of dynamic similitude to provide validation of the computationally determined values. The model derived and the principal parameters obtained by this approach will be reasonable for robust control system design. It comprised a nominal model [95,96] with a perturbation bound that is accurate enough to represent the ROV dynamic behaviour is discussed in Chapter 3.

2.6. Decoupled Models and Switching

The number of DoF to be controlled can be reduced by exploiting the physical design of the ROV. For example, the ROV roll and pitch motions may be self stabilizing because of the relative locations of the centre of buoyancy and centre of gravity. This implies that the roll and pitch can be left uncontrolled, so that less control effort is required. In fact, the systems will has less coupling at the input, as no thrusts are required on these unactuated states. Importantly, the ROV system becomes an actuated system for control and analysis. In addition, it is not desirable to have a coupled motion that lead to complicated control scheme. The motions can always be decoupled into vertical and horizontal plane [34,132] motion [47,52,85,86,87,88,89]. With that in mind, sets of simpler models that represent the different type of ROV motions in vertical and horizontal plane can be proposed. A station-keeping model to represent the ROV hovering at a point in the trajectory is defined. This provides a systematic navigation from one point to another with the least amount of states coupling.

This approach is most suitable for pipeline inspection as it involves position tracking and hovering at points on the pipeline (to locate the actual location of pipe's leakage or crack). A control system structure that can perform both tracking and stabilization simultaneously is required. As will be shown in Chapter 4, the open-loop ROV system is not asymptotically stable at zero equilibrium (or even during the point stabilization) due the added mass component in the Coriolis and centripetal component creates a cross coupling effect on ROV motion. Hence, a cascaded control structure that is able to perform stabilization using the inner

loop and position tracking using the outer loop at any equilibrium position on the pipeline is proposed.

One of the classical applications on simultaneously stabilization and tracking control was the output feedback with back stepping method on an underwater vehicle, ODIN by Jiang et. al. [62,65]. However the restriction in the ROV velocity is rather conservative and the controller design becomes quite complex, as partial differentiation equations are involved.

This method only solves the problem in the horizontal plane. Only few authors have tackled the control problem in three-dimensional space. The control system design is more involved because the ROV dynamics are far more complex with its nonlinearity, highly coupled motion, underactuation, and susceptibility to hydrodynamic uncertainties. Moreover, performance can suffer significantly when the vehicle executes manoeuvres that emphasize its nonlinearity and modes-couplings.

Furthermore, the dynamic equation did not take into account of the hydrodynamic uncertainty. Although in this paper, the observer and control law can be modified to account for the inaccurate knowledge of the ODIN parameters due to hydrodynamic uncertainty, which enter the dynamics, the back stepping control law cannot overcome this imprecise knowledge of the dynamic equation. This weak robustness is due to the fact that the hydrodynamic uncertainty cannot be compensated by the ODIN forward speed under the proposed back stepping controller.

It is hard to find a single-fixed controller that can fully satisfy the desired closed-loop response. Typically if a fixed gain nonlinear controller is used, the closed-loop performance could degrade [116] even to the point of process instability. To cope with these uncertainties would require a “robust” controller [85] that encompasses the strength of different controllers. To achieve that, a multi-controller supervisor may be required to orchestrate the switching among the various candidate controllers. Due to the finite switching, stability [62] is affected. With unknown switching time [62], the separate stabilization and tracking control approach as mentioned cannot be used. However with a piecewise stable switching signal with sufficient dwell time [46] and known switching sequence eliminate the instability shortcomings and improve stability.

2.7 Summary

The RRC ROV has to operate in an autonomous mode during pipeline inspection and it is necessary to understand its dynamics and hence to design appropriate controllers in an efficient manner. In its operation, simultaneous stabilization and tracking is required. The review of literature on various controllers design had shown the difficulty, especially in the presence of uncertainties due to the difficulties and possibilities in the variation of hydrodynamics coefficients and unknown disturbances. In order to design a robust control system, a reasonably accurate model and parameters is needed and to handle various operating modes, more than one type controllers may be required. Partial solution could be found in switching of a hybrid control system. A complete solution is attempted in the chapters that follow.

In Chapter 3, a ROV Design and Analysis (RDA) package, based on the scripting language MATLABTM and the graphical block diagram model, SIMULINKTM with VRML visualization, has been developed to evaluate the nonlinear model [102] with the hydrodynamic perturbation [95, 96]. The RDA provides a systematic and efficient platform for numerical computation in the form of multiple block diagrams. More importantly, it gives an insight to the behaviour of the ROV before the actual control system design and implementation.

In Chapter 4, a systematic analysis of the ROV subsystems model using various tools such as states coupling and Euclidean norm controllability test on higher dimensional spaces is discussed [34,132]. This forms the background to the design given.

Motivated by the work of Jiang et. al. [62,65], an extension is provided in Chapter 5. It is proposed [132] that: (a) a cascaded system design with output feedback control that uses a nonlinear P-decoupled controller (for simultaneously tracking and stabilization problem), and an exponential convergence observer (for both position and velocity); (b) with consideration of hydrodynamic perturbation for this underactuated ROV; (c) that is applicable in two and higher dimensional spaces. As given in Theorem 5.1, if the disturbances are bounded and there exist a control law that made the inner and outer loops asymptotically stable [92], then the entire cascaded system is also asymptotically stable. This provides an intuitive method to verify the stability of the cascaded nonlinear system with output feedback under the disturbances.

Moreover, to ensure that the above-mentioned controller design is asymptotically stable about the equilibrium sub-manifold in both output positions and velocity, a cascaded stability proof based on [90,91] is used. However, due to present of the nonlinear observer and hydrodynamic perturbation, the cascaded stability proof cannot be applied directly.

Lastly, to reduce the dependencies between the control loops that exists in the cascaded structure and enables independent stabilization (inner loop only) and tracking (cascaded loop) operation, a cascaded to inner loops switching is proposed in Chapter 6. A supervisor is tasked to switch between the cascaded and inner loop based on the inner loop “energy” and its switching condition utilizing the inner loop “energy” and the position tracking error. The resulted control system provides a definite separation of the inner and outer loops and allows option of independent operation of the loops with minimum interaction. For example, it is especially useful and efficient during the station-keeping condition whereby the ROV velocities are regulated to zero at a desired position on the trajectory. Thus, it results in a lower control effort as the inner loop (for velocity control) is activated in the cascaded system.

Chapter 3

Nonlinear RRC ROV Modeling

In this Chapter, the dynamic modeling of a nonlinear ROV with perturbation due to hydrodynamic forces is discussed. In general, there are basically two methods to derive the dynamic model: the Lagrange equations using the variation method and Newtonian's equation using Newton's Second Law of Motion. The latter method gives an equation using the angular velocities about the body-fixed axes; unlike the generalized variables used in the Lagrange equation, the angular velocities cannot be integrated to obtain angular displacements about these axes and are therefore unsatisfactory to describe fully the orientation of a rigid body in three-dimensional (3D) space.

However, the body-fixed angular velocities can be solved and transformed to give the orientation of the vehicle in 3D space by means of the Euler angles. The Euler's transformation (or commonly known as kinematics equation) provides the relationship between the dynamics derived in the earth-fixed and the body-fixed coordinates. As shown in [94], there are practical advantages to derive the dynamics in the body-fixed coordinate for the following reasons. Firstly, most sensors and actuators mounted on the ROV measure the body parameters such as the ROV velocities and provide propelling forces. Secondly, the dynamic equation is inherently parameterized and hence less complicated since this parameterized form is not Euler's angles dependent.

Unfortunately, due to modeling uncertainties, the model derived by either method can be inadequate for control system design. These uncertainties are mainly due to the hydrodynamic forces which are experimentally and theoretically difficult to obtain. Nevertheless, for control design purpose, there is a need to consider a simplified nominal model with additive perturbation bounds that are accurate enough to represent the ROV dynamic behavior.

The model developed and the parameters presented in this Chapter are integrated into a ROV Design and Analysis (RDA **102**) package developed by the author and is given in the Fig. A1.9 in Appendix A1. The package uses the MATLABTM scripting language and graphical block diagram model, SIMULINKTM to model the nonlinear ROV dynamic and kinematics equations with perturbation. It provides a systematic and efficient platform for numerical computation in the form of multiple block diagrams. In subsequent Chapter, the RDA package is used for controller system designs and to compare the proposed control system design with other controllers for the ROV in the execution of underwater pipeline tracking mission. An insight to the behavior of the ROV before the actual control system design can thus be available.

The Chapter is organized as follows. The body-fixed dynamic equation by Newtonian method was formulated in Section 3.1. The kinematics equations using Euler's angle was presented in Section 3.2. This is followed by the ROV's model as described in Section 3.1 was shown in Section 3.3. The perturbed ROV's model was included in Section 3.4 to model the perturbation due to the hydrodynamic forces and lastly Section 3.5 summarizes the work done in this Chapter.

3.1 Dynamic Equation using Newtonian Method

As in [120,121,122,123,124,125,126], the following assumptions can be made when deriving the ROV dynamic equation in order to simplify the effort in modeling. There are namely:

- (a) ROV is a rigid body and is fully submerged once in water;
- (b) Water is assumed to be ideal fluid that is incompressible, inviscid (frictionless) and irrotational;
- (c) ROV is slow moving for operation such as pipeline inspection;
- (d) The earth-fixed frame of reference is inertial;
- (e) Disturbance due to wave is neglected as it is fully submerged and underwater current is treated as a persistent perturbation in Chapter 6;
- (f) Tether dynamics attached to the ROV is not modeled (will be included in future work).

A rigid body ROV dynamic equation is commonly expressed in the body-fixed frame since the control forces and measurement devices are easily and intuitively related to this body frame of reference. Using the Newtonian approach, the motion of a rigid body with respect to the body-fixed reference frame at the origin (see Fig. 3.1) is given by the following set of equations [94]:

$$\mathbf{M}_{\text{mass}}[\dot{\mathbf{v}}_1 + \mathbf{v}_2 \times \mathbf{v}_1 + \dot{\mathbf{v}}_2 \times \mathbf{r}_G + \mathbf{v}_2 \times (\mathbf{v}_2 \times \mathbf{r}_G)] = \boldsymbol{\tau}_1 \quad (3.1)$$

$$\mathbf{I} \dot{\mathbf{v}}_2 + \mathbf{v}_2 \times (\mathbf{I} \mathbf{v}_2) + \mathbf{M}_{\text{mass}} \mathbf{r}_G \times (\dot{\mathbf{v}}_1 + \mathbf{v}_2 \times \mathbf{v}_1) = \boldsymbol{\tau}_2 \quad (3.2)$$

where $\mathbf{r}_G = [x_G \ y_G \ z_G]^T$ is the location of the centre of gravity, $\boldsymbol{\tau}_1 \in \mathfrak{R}^3$ and $\boldsymbol{\tau}_2 \in \mathfrak{R}^3$ are the external force and moment vector respectively, $\mathbf{v}_1 = [u \ v \ w]^T \in \mathfrak{R}^3$ is the linear velocity vector and $\mathbf{v}_2 = [p \ q \ r]^T \in \mathfrak{R}^3$ is the angular velocity vector. $\mathbf{M}_{\text{mass}} \in \mathfrak{R}^{3 \times 3}$ is the ROV mass matrix:

$$\mathbf{M}_{\text{mass}} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} = m\mathbf{I}_{3 \times 3}; \quad (3.3a)$$

$\mathbf{I}_{3 \times 3}$ is the identity matrix and $\mathbf{I} \in \mathfrak{R}^{3 \times 3}$ is the constant inertia tensor:

$$\mathbf{I} = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix}; \mathbf{I} = \mathbf{I}^T > 0, \dot{\mathbf{I}} = 0 \quad (3.3b)$$

I_x , I_y and I_z are the moments of inertia about the X , Y and Z axes respectively. As $\mathbf{I} = \mathbf{I}^T > 0$, the cross products of inertia $I_{xy} = I_{yx}$, $I_{xz} = I_{zx}$ and $I_{yz} = I_{zy}$.

Figure 3.1: Coordinate systems used in ROV

The rigid body equation (see Fig. A1.1 for SIMULINKTM block diagram in Appendix A1) consisting of the inertia forces and the Coriolis and centrifugal forces can be expressed in matrix form:

$$\mathbf{M}_{RB} \dot{\mathbf{v}} + \mathbf{C}_{RB}(\mathbf{v}) = \boldsymbol{\tau} \quad (3.4)$$

where $\mathbf{M}_{RB} \in \mathbb{R}^{6 \times 6}$ is the mass-inertia matrix, $\mathbf{C}_{RB}(\mathbf{v}) \in \mathbb{R}^{6 \times 6}$ is the Coriolis and centripetal matrix, $\boldsymbol{\tau} = [\boldsymbol{\tau}_1 \quad \boldsymbol{\tau}_2]^T \in \mathbb{R}^6$ is a vector of external forces and moments and $\mathbf{v} = [\mathbf{v}_1 \quad \mathbf{v}_2]^T \in \mathbb{R}^6$ is the linear and angular velocity vector.

The rigid body inertia matrix $\mathbf{M}_{RB}$ can be uniquely determined from (3.1) and (3.2) as:

$$\mathbf{M}_{RB} \dot{\mathbf{v}} = \begin{bmatrix} \mathbf{M}_{\text{mass}} \dot{\mathbf{v}}_1 + m \dot{\mathbf{v}}_2 \times \mathbf{r}_G \\ \mathbf{I} \dot{\mathbf{v}}_2 + m \mathbf{r}_G \times \dot{\mathbf{v}}_1 \end{bmatrix} \quad (3.5)$$

From the above, the positive $\mathbf{M}_{RB} = \mathbf{M}_{RB}^T > 0$ can be separated and defined as:

$$\mathbf{M}_{RB} = \begin{bmatrix} \mathbf{M}_{\text{mass}} & -m\mathbf{S}(\mathbf{r}_G) \\ m\mathbf{S}(\mathbf{r}_G) & \mathbf{I} \end{bmatrix} \quad (3.6)$$

where $\mathbf{S}(\mathbf{r}_G) = \begin{bmatrix} 0 & -z_G & y_G \\ z_G & 0 & -x_G \\ -y_G & x_G & 0 \end{bmatrix}$. Expanding the individual terms in the inertia matrix

in (3.6) gives:

$$\mathbf{M}_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_x & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{yx} & I_y & -I_{yz} \\ -my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \quad (3.7)$$

Similarly, the Coriolis and centripetal terms, (Figure A1.2 for SIMULINKTM block diagram) describing the angular motion of the ROV in (3.1)-(3.2), can be rewritten as:

$$\mathbf{C}_{RB}(\mathbf{v})\mathbf{v} = \begin{bmatrix} m[\mathbf{v}_2 \times \mathbf{v}_1 + \mathbf{v}_2 \times (\mathbf{v}_2 \times \mathbf{r}_G)] \\ \mathbf{v}_2 \times (\mathbf{I}\mathbf{v}_2) + m\mathbf{r}_G \times (\mathbf{v}_2 \times \mathbf{v}_1) \end{bmatrix} \quad (3.8)$$

Using the results $\mathbf{a} \times \mathbf{b} \times \mathbf{c} = \mathbf{S}(\mathbf{a})\mathbf{S}(\mathbf{b})\mathbf{c}$ and $\mathbf{S}(\mathbf{v}_1)\mathbf{v}_1 = 0$, (3.8) can be manipulated to give:

$$\mathbf{C}_{RB}(\mathbf{v})\mathbf{v} = \begin{bmatrix} -m\mathbf{S}(\mathbf{v}_1)\mathbf{v}_2 - m\mathbf{S}(\mathbf{v}_2)\mathbf{S}(\mathbf{r}_G)\mathbf{v}_2 \\ -m\mathbf{S}(\mathbf{v}_1)\mathbf{v}_1 + m\mathbf{S}(\mathbf{r}_G)\mathbf{S}(\mathbf{v}_2)\mathbf{v}_1 - \mathbf{S}(\mathbf{I}\mathbf{v}_2)\mathbf{v}_2 \end{bmatrix} \quad (3.9)$$

Separating $\mathbf{v}_1$ and $\mathbf{v}_2$ yields:

$$\mathbf{C}_{RB}(\mathbf{v}) = \begin{bmatrix} \mathbf{0}_{3 \times 3} & -m\mathbf{S}(\mathbf{v}_1) - m\mathbf{S}(\mathbf{v}_2)\mathbf{S}(\mathbf{r}_G) \\ -m\mathbf{S}(\mathbf{v}_1) + m\mathbf{S}(\mathbf{r}_G)\mathbf{S}(\mathbf{v}_2) & -\mathbf{S}(\mathbf{I}\mathbf{v}_2) \end{bmatrix} \quad (3.10)$$

Expanding the terms on the RHS of (3.10) gives:

$$\mathbf{C}_{\text{RB}}(\mathbf{v}) = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{C}_{12}(\mathbf{v}) \\ -\mathbf{C}_{12}^T(\mathbf{v}) & \mathbf{C}_{22}(\mathbf{v}) \end{bmatrix} \quad (3.11)$$

with

$$\mathbf{C}_{12}(\mathbf{v}) = \begin{bmatrix} m(y_G q + z_G r) & -m(x_G q - w) & -m(x_G r + v) \\ -m(y_G p + w) & m(z_G r + x_G p) & -m(y_G r - u) \\ -m(z_G p - v) & -m(z_G q + u) & m(x_G p + y_G q) \end{bmatrix} \quad (3.12)$$

$$\mathbf{C}_{22}(\mathbf{v}) = \begin{bmatrix} 0 & -I_{yz}q - I_{xz}p + I_z r & I_{yz}r + I_{xy}p - I_y q \\ I_{yz}q + I_{xz}p - I_z r & 0 & -I_{xz}r - I_{xy}q + I_x p \\ -I_{yz}r - I_{xy}p + I_y q & I_{xz}r + I_{xy}q - I_x p & 0 \end{bmatrix} \quad (3.13)$$

In (3.4), the external force and moment vector $\boldsymbol{\tau}$ includes the hydrodynamic forces and moments, $\boldsymbol{\tau}_H$ due to damping and inertial of surrounding fluid known as added mass, and the restoring force and moment, $\boldsymbol{\tau}_G$. These forces and moments tend to oppose the motion of the ROV and, other than the restoring force, are dependent on the velocities and accelerations of the vehicle and are hence expressed in the body-fixed frame. The restoring force and moment $\boldsymbol{\tau}_G$ due to the weight and buoyancy force have to be transformed to the body-fixed frame forces and moments $\mathbf{G}_f(\boldsymbol{\eta})$. The open-loop nonlinear ROV dynamic equations (see Fig. A1.3 for SIMULINKTM block diagram in Appendix A1) can be expressed according to:

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{\tau}_A + \boldsymbol{\tau}_H + \mathbf{G}_f(\boldsymbol{\eta}) = \boldsymbol{\tau}_A - \mathbf{M}_A - \mathbf{C}_A(\mathbf{v}) - \mathbf{D}(\mathbf{v})\mathbf{v} - \mathbf{G}_f(\boldsymbol{\eta}) \\ \mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{G}_f(\boldsymbol{\eta}) &= \boldsymbol{\tau}_A \end{aligned} \quad (3.14)$$

where $\mathbf{v} = [u \quad v \quad w \quad p \quad q \quad r]^T$ is the body-fixed velocity vector and $\boldsymbol{\eta} = [\boldsymbol{\eta}_1 \quad \boldsymbol{\eta}_2]^T$ is the earth-fixed vector, comprising the position vector $\boldsymbol{\eta}_1 = [x \quad y \quad z]^T \in \mathfrak{R}^3$ and the orientation vector of Euler angles, $\boldsymbol{\eta}_2 = [\phi \quad \theta \quad \psi]^T \in \mathfrak{S}^3$ (a torus of dimensions three) in Fig. 3.1; the angles are defined on the interval $[0, 2\pi)$. $\mathbf{M} = \mathbf{M}_{\text{RB}} + \mathbf{M}_A \in \mathfrak{R}^{6 \times 6}$ is the sum of the rigid body inertia mass and added fluid inertia mass matrix, $\mathbf{C}(\mathbf{v}) = \mathbf{C}_{\text{RB}}(\mathbf{v}) + \mathbf{C}_A(\mathbf{v}) \in \mathfrak{R}^{6 \times 6}$ is the sum of Coriolis and centripetal and the added mass forces and moments matrix, and $\mathbf{D}(\mathbf{v}) \in \mathfrak{R}^{6 \times 6}$ is the damping matrix due to the surrounding fluid. The input force and moment vector $\boldsymbol{\tau}_A = \mathbf{T}\mathbf{u} \in \mathfrak{R}^6$ relates the thrust output vector $\mathbf{u} = \mathbf{F}_T \bar{\mathbf{u}} \in \mathfrak{R}^4$ with the thruster configuration

matrix $\mathbf{T} \in \mathbb{R}^{6 \times 4}$, defined in Section 3.3.5; $\mathbf{F}_T \in \mathbb{R}^{4 \times 4}$ is the dynamics of each thruster and converts the input voltage command $\bar{\mathbf{u}} \in \mathbb{R}^4$ into thrusts to propel the vehicle. The details of these parameters used in the RRC ROV can be found in [95,96].

The following are the assumptions used in the ROV modeling. The ROV is designed to move very slowly and under assumption of an ideal fluid that is non-viscous liquid and the hydrodynamics forces are determined under the assumption of potential flow. Main fluid-body interactions are the hydrodynamic forces like added mass and damping. As the vehicle is operating in a least coupled in motion, the linear and diagonal hydrodynamic damping is used. Besides, with an additional balancing mass, the ROV is made to be neutrally buoyant with the X-Y coordinate of the Center of Gravity (CG) coincide with the Center of Buoyancy (CB).

3.2 Kinematics Equation and Earth-fixed frame equation using Euler's Angles

The transformation using the Euler angles provides an important transformation between the dynamics expressed in the body-fixed frame in (3.14) to the earth-fixed frame as seen in Fig. 3.1. As the accelerations of a point on the surface of the Earth can be neglected, for slow moving marine vehicles, the earth-fixed frame can be considered to be an inertial frame. The Kinematics equations [94] that represent the Euler's transformation can be written as:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\mathbf{v} \quad (3.15)$$

Alternatively, the Euler's transformation between the body and earth fixed frames $(\boldsymbol{\eta}, \mathbf{v})$ to $(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$ can be written as:

$$\begin{bmatrix} \boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{6 \times 6} & \mathbf{0}_{6 \times 6} \\ \mathbf{0}_{6 \times 6} & \mathbf{J}(\boldsymbol{\eta}_2) \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta} \\ \mathbf{v} \end{bmatrix} \quad (3.16)$$

where the Euler transformation matrix, $\mathbf{J}(\boldsymbol{\eta})$ (see Fig. A1.4 for SIMULINKTM block diagram in Appendix A1) is derived by successive rotation of the Euler's angles $(\boldsymbol{\eta}_2 = [\phi \ \theta \ \psi]^T)$ about Z, Y and X axes [94] in Fig. 3.1 to give:

$$\mathbf{J}(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{J}_1(\boldsymbol{\eta}_2) & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2(\boldsymbol{\eta}_2) \end{bmatrix} \quad (3.17)$$

where

$$\mathbf{J}_1(\boldsymbol{\eta}_2) = \begin{bmatrix} c(\psi)c(\theta) & -s(\psi)c(\phi) + c(\psi)s(\theta)s(\phi) & s(\psi)s(\phi) + c(\psi)c(\phi)s(\theta) \\ s(\psi)c(\theta) & c(\psi)c(\phi) + s(\phi)s(\theta)s(\psi) & -c(\psi)s(\phi) + s(\theta)s(\psi)c(\theta) \\ -s(\theta) & c(\theta)s(\phi) & c(\theta)c(\phi) \end{bmatrix};$$

$$\mathbf{J}_2(\boldsymbol{\eta}_2) = \begin{bmatrix} 1 & s(\phi)t(\theta) & c(\phi)t(\theta) \\ 0 & s(\phi) & -s(\phi) \\ 0 & \frac{s(\phi)}{c(\theta)} & \frac{c(\phi)}{c(\theta)} \end{bmatrix}$$

and $s = \sin(\cdot)$, $c = \cos(\cdot)$, $t = \tan(\cdot)$. The transformation is undefined for $\theta = \pm 90^\circ$. To overcome this singularity, a quaternion approach [94] must be considered. However, for the RRC ROV, this problem does not exist because the vehicle is neither designed nor required to operate at $\theta = \pm 90^\circ$.

Differentiating (3.16) and rearranging the equation gives:

$$\begin{bmatrix} \dot{\mathbf{v}} \\ \ddot{\boldsymbol{\eta}} \end{bmatrix} = \begin{bmatrix} \mathbf{J}^{-1}(\boldsymbol{\eta}_2) & \mathbf{0}_{6 \times 6} \\ \dot{\mathbf{J}}(\boldsymbol{\eta}_2)\mathbf{J}^{-1}(\boldsymbol{\eta}_2) & \mathbf{J}(\boldsymbol{\eta}_2) \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\mathbf{v}} \end{bmatrix} \quad (3.18)$$

where the body-fixed dynamic can be transformed into the earth-fixed frame (or vice versa) by substituting the $\mathbf{v}$ and $\dot{\mathbf{v}}$ as:

$$\mathbf{v} = \mathbf{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} \quad (3.19)$$

$$\begin{aligned} \dot{\mathbf{v}} &= \mathbf{J}^{-1}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} - \mathbf{J}^{-2}(\boldsymbol{\eta})\dot{\mathbf{J}}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} \\ &= \mathbf{J}^{-1}(\boldsymbol{\eta})[\ddot{\boldsymbol{\eta}} - \dot{\mathbf{J}}(\boldsymbol{\eta})\mathbf{J}^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}] \end{aligned} \quad (3.20)$$

into the body-fixed dynamic in (3.14), gives the ROV's dynamic in earth-fixed frame:

$$\mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \mathbf{C}_{\boldsymbol{\eta}}(\mathbf{v}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{D}_{\boldsymbol{\eta}}(\mathbf{v}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{G}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{\boldsymbol{\eta}} \quad (3.21)$$

where

$$\mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) = \mathbf{J}^{-T}(\boldsymbol{\eta})\mathbf{M}\mathbf{J}^{-1}(\boldsymbol{\eta}),$$

$$\mathbf{C}_\eta(\mathbf{v}, \eta) = \mathbf{J}^{-T}(\eta) [\mathbf{C}(\mathbf{v}) - \mathbf{M}\mathbf{J}^{-1}(\eta)\dot{\mathbf{J}}(\eta)]\mathbf{J}^{-1}(\eta)$$

$$\mathbf{D}_\eta(\mathbf{v}, \eta) = \mathbf{J}(\eta)^{-T} \mathbf{D}\mathbf{J}(\eta)^{-1}$$

$$\mathbf{G}_\eta(\eta) = \mathbf{J}^{-T}(\eta)\mathbf{G}_f(\eta)$$

$$\boldsymbol{\tau}_\eta = \mathbf{J}^{-T}(\eta)\boldsymbol{\tau}_A.$$

Hence, the dynamic in the earth-fixed frame is derived as seen in (3.21). Noticed that the earth-fixed dynamic in (3.21) is similar to that derived by Lagrangian method as shown in Appendix A2 and A3

3.3 RRC ROV Model

The RRC ROV dynamics and kinematics given in Section 3.1 and 3.2 are:

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v}) + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{G}_f(\eta) = \boldsymbol{\tau}_A \quad (3.22)$$

$$\dot{\eta} = \mathbf{J}(\eta)\mathbf{v} \quad (3.23)$$

These will be expressed in state-space form to facilitate the analysis and design of model based controllers.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) + \mathbf{g}(\bar{\mathbf{u}}, t), \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (3.24a)$$

The full-order states and functions in (3.24a) are as follows:

$$\begin{aligned} \mathbf{x} &= [\eta \quad \mathbf{v}]^T \\ \mathbf{f}(\mathbf{x}, t) &= \begin{bmatrix} \mathbf{0}_{6 \times 6} & \mathbf{J}(\eta) \\ \mathbf{0}_{6 \times 6} & -\mathbf{M}^{-1}[\mathbf{C}(\mathbf{v}) + \mathbf{D}(\mathbf{v})] \end{bmatrix} \begin{bmatrix} \eta \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{6 \times 1} \\ -\mathbf{M}^{-1}\mathbf{G}_f(\eta) \end{bmatrix} \\ \mathbf{g}(\bar{\mathbf{u}}, t) &= \begin{bmatrix} \mathbf{0}_{6 \times 1} \\ \mathbf{M}^{-1}\mathbf{T}\mathbf{F}_T\bar{\mathbf{u}} \end{bmatrix} \end{aligned}$$

The output equation can be expressed as:

$$\mathbf{y} = \mathbf{h}(\mathbf{x}, t) \quad (3.24b)$$

where

$$\mathbf{h}(\mathbf{x}, t) = \begin{bmatrix} \mathbf{I}_{6 \times 6} & \mathbf{0}_{6 \times 6} \\ \mathbf{0}_{6 \times 6} & \mathbf{0}_{6 \times 6} \end{bmatrix} \mathbf{x}$$

The state vector $\mathbf{f}(\mathbf{x}, t)$ is piecewise continuous in t and locally Lipschitz in $\mathbf{x}$, $\mathbf{g}(\bar{\mathbf{u}}, t)$ is piecewise continuous in t and locally Lipschitz in $\bar{\mathbf{u}}$, $\mathbf{h}(\mathbf{x}, t)$ is piecewise continuous in t and continuous in $\mathbf{x}$.

The parameters used in $\mathbf{f}(\mathbf{x}, t)$ are determined using CAD tools and experiments and the values are given in Appendix A4. They are observed to have the following properties expressed by Property 3.1 and are consistent with [94].

Property 3.1: Property of body-fixed matrices

The matrices in (3.22) satisfy the following property:

- a) $\mathbf{M} = \mathbf{M}^T > 0, \dot{\mathbf{M}} = 0$ where $\mathbf{M} = \mathbf{M}_{\text{RB}} + \mathbf{M}_{\text{A}}$
- b) $\mathbf{C}(\mathbf{v}) = -\mathbf{C}^T(\mathbf{v}), \mathbf{v} \in \mathfrak{R}^6$ where $\mathbf{C}(\mathbf{v}) = \mathbf{C}_{\text{RB}}(\mathbf{v}) + \mathbf{C}_{\text{A}}(\mathbf{v})$
- c) $s^T [\dot{\mathbf{M}} - 2\mathbf{C}(\mathbf{v})]s = 0, \forall s \in \mathfrak{R}^6$
- d) $\mathbf{D}(\mathbf{v}) > 0$

3.3.1 Rigid-Body Mass and Coriolis and Centripetal Matrix

The CAD software PRO/ENGINEER™ is used to determine the rigid-body mass [99] and Coriolis and centrifugal force parameters. The principal components as shown in Fig. 3.2 were included in the complete RRC ROV geometric model and using the density and mass values available in [99], the rigid-body mass and Coriolis and centripetal properties with respect to the ROV's center of gravity $\mathbf{r}_G$ can be determined.

Figure 3.2. ROV Components

A simple form of the $\mathbf{M}_{RB}$ can be obtained as the ROV body axes (origin O) coincide with the C.G. of the ROV. This is achieved by adding balancing weights at designated location in the ROV. This means that the locations of the center of gravity, $\mathbf{r}_G$ become zero. Hence, the rigid-body mass matrix in (3.7) can be simplified to:

$$\mathbf{M}_{RB} = \begin{bmatrix} 115.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 115.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 115.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.1000 & -0.00016 & -0.1850 \\ 0 & 0 & 0 & -0.00016 & 5.9800 & 0.0006 \\ 0 & 0 & 0 & -0.1850 & 0.0006 & 5.5170 \end{bmatrix} \quad (3.25)$$

where the parameters used in (3.7) are as follows

$$\begin{aligned}
m &= 115.00 \text{ kg} \\
I_x &= 6.1000 \text{ kg.m}^2 \\
I_y &= 5.9800 \text{ kg.m}^2 \\
I_z &= 5.5170 \text{ kg.m}^2 \\
I_{xy} &= I_{yx} = -0.0002 \text{ kg.m}^2 \\
I_{xz} &= I_{zx} = -0.1850 \text{ kg.m}^2 \\
I_{yz} &= I_{zy} = 0.0006 \text{ kg.m}^2
\end{aligned} \tag{3.26}$$

Recall in (3.11) to (3.13), the structure of the rigid-body Coriolis and Centripetal matrix can be rewritten as:

$$\mathbf{C}_{\mathbf{RB}}(\mathbf{v}) = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{C}_{12}(\mathbf{v}) \\ -\mathbf{C}_{12}^T(\mathbf{v}) & \mathbf{C}_{22}(\mathbf{v}) \end{bmatrix} \tag{3.27}$$

where $\mathbf{C}_{12}^T(\mathbf{v})$ and $\mathbf{C}_{22}(\mathbf{v})$ were substituted by the numerical values in (3.26) as seen in the followings matrices:

$$\mathbf{C}_{12}(\mathbf{v}) = \begin{bmatrix} 0 & 115.00w & -115.00v \\ -115.00w & 0 & 115.00u \\ 115.00v & -115.00u & 0 \end{bmatrix}$$

$$\mathbf{C}_{22}(\mathbf{v}) = \begin{bmatrix} 0 & -0.0006q + 0.1850p + 5.5170r & 0.0006r - 0.0002p - 5.9800q \\ 0.0006q - 0.1850p - 5.5170r & 0 & 0.1850r + 0.0002q + 6.1000p \\ -0.0006r + 0.0002p + 5.9800q & -0.1850r - 0.0002q - 6.1000p & 0 \end{bmatrix}$$

3.3.2 Hydrodynamic Forces: Added Mass terms

The motion of the surrounding body of fluid in response to the ROV motion manifests itself as hydrodynamic forces and moments that resist the vehicle motion. These forces and moments come about from the acceleration that the fluid particles experience when they encounter the vehicle. As shown below, their effect appears to be like “added” mass and inertia. For a fully submerged vehicle, the added mass and inertia are independent of the wave circular frequency.

Here, the Society of Naval Architects and Marine Engineers (SNAME) notation [94] is being used.

Hydrodynamic Added Mass Force X_A acting along the X-axis

The relationship between the hydrodynamic forces and moments and accelerations can be represented by coefficients known as hydrodynamics derivatives. For example, if there is acceleration $\ddot{u}$ in the X-direction, the hydrodynamic force X_A arising from that motion can be given as:

$$X_A = X_{\ddot{u}} \ddot{u} \quad (3.28)$$

where the hydrodynamic derivative $X_{\ddot{u}} = \frac{\partial X}{\partial \ddot{u}}$. The single DOF equation of motion to describe the ROV moving in the X-direction can be written as: $(m - X_{\ddot{u}})\ddot{u} = \tau_x + X_u u$; m is the mass, X_u is the linear damping and τ_x is the thrust. In this form, the hydrodynamic derivative $X_{\ddot{u}}$ can be considered as an added mass term on the left hand side of the expression.

□

As seen in (3.28), these forces and moments are proportional to the acceleration of the vehicle. In order to take this effect into consideration, the added mass inertia and Coriolis and centripetal matrices that are used in (3.22) are defined. Similar to the rigid-body case, hydrodynamic added mass term also consists of two parts; the added mass inertia component and the added mass Coriolis and centrifugal force component [94]. The hydrodynamic added mass matrix is written as:

$$\mathbf{M}_A = \begin{bmatrix} X_{\ddot{u}} & X_{\ddot{v}} & X_{\ddot{w}} & X_{\ddot{p}} & X_{\ddot{q}} & X_{\ddot{r}} \\ Y_{\ddot{u}} & Y_{\ddot{v}} & Y_{\ddot{w}} & Y_{\ddot{p}} & Y_{\ddot{q}} & Y_{\ddot{r}} \\ Z_{\ddot{u}} & Z_{\ddot{v}} & Z_{\ddot{w}} & Z_{\ddot{p}} & Z_{\ddot{q}} & Z_{\ddot{r}} \\ K_{\ddot{u}} & K_{\ddot{v}} & K_{\ddot{w}} & K_{\ddot{p}} & K_{\ddot{q}} & K_{\ddot{r}} \\ M_{\ddot{u}} & M_{\ddot{v}} & M_{\ddot{w}} & M_{\ddot{p}} & M_{\ddot{q}} & M_{\ddot{r}} \\ N_{\ddot{u}} & N_{\ddot{v}} & N_{\ddot{w}} & N_{\ddot{p}} & N_{\ddot{q}} & N_{\ddot{r}} \end{bmatrix} \quad (3.29)$$

On the other hand, the hydrodynamic added Coriolis and centripetal matrix $\mathbf{C}_A(\mathbf{v}) = -\mathbf{C}_A^T(\mathbf{v})$ is given by:

$$\mathbf{C}_A(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -a_3 & a_2 \\ 0 & 0 & 0 & a_3 & 0 & -a_1 \\ 0 & 0 & 0 & -a_2 & a_1 & 0 \\ 0 & -a_3 & a_2 & 0 & -b_3 & b_2 \\ a_3 & 0 & -a_1 & b_3 & 0 & -b_1 \\ -a_2 & a_1 & 0 & -b_2 & b_1 & 0 \end{bmatrix} \quad (3.30)$$

Where

$$\begin{aligned} a_1 &= X_{\dot{u}}u + X_{\dot{v}}v + X_{\dot{w}}w + X_{\dot{p}}p + X_{\dot{q}}q + X_{\dot{r}}r \\ a_2 &= X_{\dot{v}}u + Y_{\dot{v}}v + Y_{\dot{w}}w + Y_{\dot{p}}p + Y_{\dot{q}}q + Y_{\dot{r}}r \\ a_3 &= X_{\dot{w}}u + Y_{\dot{w}}v + Z_{\dot{w}}w + Z_{\dot{p}}p + Z_{\dot{q}}q + Z_{\dot{r}}r \\ b_1 &= X_{\dot{p}}u + Y_{\dot{p}}v + Z_{\dot{p}}w + K_{\dot{p}}p + K_{\dot{q}}q + K_{\dot{r}}r \\ b_2 &= X_{\dot{q}}u + Y_{\dot{q}}v + Z_{\dot{q}}w + K_{\dot{q}}p + M_{\dot{q}}q + M_{\dot{r}}r \\ b_3 &= X_{\dot{r}}u + Y_{\dot{r}}v + Z_{\dot{r}}w + K_{\dot{r}}p + M_{\dot{r}}q + N_{\dot{r}}r \end{aligned} \quad (3.31)$$

The hydrodynamic added mass matrix was computed through the used of the following assumptions commonly stated in the literature [94, 97, 98]:

Assumptions 3.1: Hydrodynamic Added Mass Matrix

1. To simplify the problem, the ideal fluid in [94] is used. It refers to fluid that is incompressible, inviscid (frictionless) and irrotational (fluid particles are not rotating). For a rigid body moving very slowly, the hydrodynamic system inertia matrix, $\mathbf{M}_A$ is positive and constant [97]. As shown in [98], the numerical values of the added mass derivatives in a real fluid are usually in good agreement with those obtained from ideal theory. Hence $\mathbf{M}_A > 0$ is a good approximation.
2. The effects of the off-diagonal elements in $\mathbf{M}_A$ on an underwater vehicle are small compared to the diagonal elements. For most low speed underwater vehicles, the off-

diagonal terms are often neglected. This approximation is found to hold true for many applications [94]. Hence, $\mathbf{M}_A$ has the simplified diagonal form as follows:

$$\mathbf{M}_A = -\text{diag} \{X_{\ddot{u}}, Y_{\ddot{v}}, Z_{\ddot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\} \quad (3.32)$$

3. As the off-diagonal elements in $\mathbf{M}_A$ are neglected, the Coriolis and centripetal added mass matrix $\mathbf{C}_A(\mathbf{v})$ is simplified to:

$$\mathbf{C}_A(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\ 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\ Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\ -Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0 \end{bmatrix} \quad (3.33)$$

Result 3.1: Hydrodynamic Added Mass Matrix (data from [96])

The added mass matrix for the ROV was computed using the CAD software, MULTISURFTM that converts the 3D geometric model into finite surface panels (see Fig. 3.3) and the resulting model is then used in the WAMITTM [76] software to compute the added mass of the ROV with zero wave circular frequency. Procedure of obtaining added mass of the RRC ROV and its result are given in [96]. Since the WAMITTM solve the ROV over finite panels, one of the most important issues is the convergence of the result. To maintain the accuracy and consistency of the results, the body of interests (that is the ROV) is divided into parts and solved incrementally. The analytical result of the added mass of a simple geometry body such as sphere is used to verify the program setup and the parameters used. The added mass parameters obtained and given in (3.34) seem reasonable for two reasons:

First, the computed added mass parameters give a always positive definite matrix, in agreement with the assumption made. Second, the data indicates that the added mass is smallest in the surge DOF and largest in the heave DOF. This is consistent with the fact that vehicle's cross-section area is smallest in the surge DOF and largest in heave DOF.

Figure 3.3: WAMITTM model used in added mass computation [96]

The added mass of the RRC ROV is computed as:

$$\mathbf{M}_A = \begin{bmatrix} 21.1403 & 0 & 0.0619 & 0 & -0.5748 & 0 \\ 0 & 51.7012 & 0 & -2.0928 & 0 & -0.3767 \\ 0.0917 & 0 & 92.4510 & 0 & 0.5871 & 0 \\ 0 & -2.0090 & 0 & 3.6191 & 0 & 0.0235 \\ -0.5237 & 0 & 0.5594 & 0 & 2.6427 & 0 \\ 0 & -0.3783 & 0 & 0.0275 & 0 & 2.3033 \end{bmatrix} \quad (3.34)$$

and $\mathbf{C}_A(\mathbf{v})$ becomes:

$$\mathbf{C}_A(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -a_3 & a_2 \\ 0 & 0 & 0 & a_3 & 0 & -a_1 \\ 0 & 0 & 0 & -a_2 & a_1 & 0 \\ 0 & -a_3 & a_2 & 0 & -b_3 & b_2 \\ a_3 & 0 & -a_1 & b_3 & 0 & -b_1 \\ -a_2 & a_1 & 0 & -b_2 & b_1 & 0 \end{bmatrix} \quad (3.35)$$

where

$$\begin{aligned}
a_1 &= 21.1403u + 0.0619w - 0.5748q \\
a_2 &= 51.7012v - 2.0928p - 0.3767r \\
a_3 &= 0.0619u + 92.4510w + 0.5871q \\
b_1 &= -2.0928v + 3.6191p + 0.0235r \\
b_2 &= -0.5748u + 0.5871w + 2.6427q \\
b_3 &= -0.3767v + 0.0235p + 2.3033r
\end{aligned}$$

Note that all the motions (both linear and angular motion) are defined according to the center of gravity. For example, the added mass matrix term $N_{\dot{r}}$ means that the hydrodynamic moment acts on the ROV as it rotates about the Z-axis. The $\mathbf{M}_A$ matrix is positive definite as all the eigenvalues (that is equal to 21.1403, 51.7012, 92.4510, 3.6191, 2.6427, 2.3033) of the added mass matrix is greater than zero.

As observed, the off-diagonal terms in $\mathbf{M}_A$ are small as compared to its diagonal components. Hence with the application of the Assumption 3.1.1 to 3.1.3, $\mathbf{M}_A$ can be approximated as:

$$\mathbf{M}_A = \begin{bmatrix} 21.1403 & 0 & 0 & 0 & 0 & 0 \\ 0 & 51.7012 & 0 & 0 & 0 & 0 \\ 0 & 0 & 92.4510 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.6191 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.6427 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.3033 \end{bmatrix} \quad (3.36)$$

where the Coriolis/centripetal added mass matrix becomes:

$$\mathbf{C}_A(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -92.4510w & 51.7012v \\ 0 & 0 & 0 & 92.4510w & 0 & -21.1403u \\ 0 & 0 & 0 & -51.7012v & 21.1403u & 0 \\ 0 & -92.4510w & 51.7012v & 0 & -2.3033r & 2.6427q \\ 92.4510w & 0 & -21.1403u & 2.3033r & 0 & -3.6191p \\ -51.7012v & 21.1403u & 0 & -2.6427q & 3.6191p & 0 \end{bmatrix} \quad (3.37)$$

3.3.3 Hydrodynamic Damping Matrix

The matrix $\mathbf{D}(\mathbf{v})$ is the hydrodynamic damping matrix consisting of both linear and quadratic terms. This hydrodynamic damping is caused by potential damping due to skin friction (linear

damping) drag and vortex shedding (quadratic damping). The sum of these individual components gives the overall hydrodynamic damping effect on the ROV (see Fig. A1.5 for SIMULINKTM block diagrams in Appendix A1).

With the assumptions commonly stated in [94], the hydrodynamic damping matrix can be simplified by using the following Assumption 3.2.

Assumptions 3.2: Hydrodynamic Damping Matrix

1. As the ROV is operating at around linear speed of 0.2 m/s (or angular speed of 0.2 rad/s), it is well within the linear damping region of maximum 2m/s (see circle region in Fig. 3.4). Hence, the linear hydrodynamic damping is used.
2. The off-diagonal elements in $\mathbf{D}$ on an underwater vehicle [94] are small compared to the diagonal elements, hence only the hydrodynamic damping in the diagonal form is used:

$$\mathbf{D} = -\text{diag}\{X_u, Y_v, Z_w, K_p, M_q, N_r\} \quad (3.38)$$

Figure 3.4: Hydrodynamic damping at low and high vehicle's velocity (cited from [94])

Result 3.2: Hydrodynamic Damping Matrix (data from [96])

Like the added mass terms, the hydrodynamic damping force and moment coefficients for the ROV are difficult to obtain experimentally without a full scale instrumented tow tank facility. The problem is circumvented using CFD and experimental study of a scaled model in small water tank (shown in Fig. 3.5), that is later translated to the full-scale model by using Similitude Law.

As seen in (3.39), the important parameters in the drag force equation are fluid density, length, velocity and hydrodynamic drag coefficient (that is a function of Reynolds Number). To use the Similitude Law, the prototype and the model must have geometric and kinematics similarity. This implies that they must have a constant length ratio (refers to as geometric similarity) and constant velocity ratio (refers to as kinematics similarity). For geometric similarity, the ratio of the frontal area for both prototype and the model has to be constant. Dimensional error in the model can affect the flow around the model, and if the scaling factor is large, the error will also scale up. In the thesis, accuracy in the model is quite good and the ratio is not very large. Hence the similarity in the geometric similarity can be achieved. Secondly, since water is used as the working fluid for both prototype and model, the density and viscosity can be made to be similar. However adjustments have to be made if the ROV is to be used in sea-water. In general, similarity in both kinematics and geometric can be achieved prior to the application of the Similitude Law.

The followings were the descriptions of the experiment study and application of the Similitude Law of a scaled model in small water tank. The scaled model of the ROV (prototype) was held onto the aluminum frame mounting in a water tunnel of size 45 cm by 45 cm by 45 cm as seen in the Fig. 3.5. A six-DOF load cell is used to measure the force and moment on the prototype in the water tunnel test. A computer is used to analyze the results. As observed in Fig. 3.5, the setup is used to measure the hydrodynamic forces in the longitudinal direction only. To account for other directions, it is necessary to rotate the scaled model such that the remaining velocity in earth fixed frame (contributed by the free stream water) can be simulated. This enables the hydrodynamic damping force in other directions to be determined. Since the free-stream velocity is referred to the earth-fixed frame, the Euler's coordinate transformation is used to obtain the ROV body-fixed velocity. Details can be found in [96]. Due to equipment constraint, the maximum steady speed of the water tank is limited to 0.15m/s. With the characteristics length of the scaled model to ROV at a ratio of 0.3:1, the water tank speed of

0.15 m/s means that the drag on the ROV can be estimated up to an operating speed of 0.05m/s.

Figure 3.5: RRC ROV prototype in water tank

Applying the Similitude Law, the drag force on the ROV (denotes as subscript m) can computed as:

$$\frac{X_{um}}{X_{up}} = \frac{\rho_m u_m^2 l_m^2}{\rho_p u_p^2 l_p^2} \Rightarrow X_{um} = \frac{l_m^2}{l_p^2} X_{up} \quad (3.39)$$

where $X_{um}(X_{up})$ is the hydrodynamic drag coefficient of the ROV (prototype) in surge direction, $\rho_m(\rho_p)$ are the density of the water used in ROV (prototype), and $l_m(l_p)$ is the characteristic length of the ROV (prototype): $l_m^2 / l_p^2 = 1 / 0.3^2 = 11.11$.

A numerical computation of the hydrodynamic damping using ANSYSTM[93] of a half-scaled model, shown in Fig 3.6 is also done. As shown Fig. 3.7a and 3.7b, the ANSYSTM computed results agree well with the experimental results.

Figure 3.6: ANSYS™ model used in hydrodynamic damping computation [96]

Figure 3.7a: Comparisons of theoretical and experimental values – linear direction

Figure 3.7b: Comparisons of theoretical and experimental values –angular direction

The experimentally or theoretically identified quadratic damping parameters indicate that the damping is the smallest in the surge DOF and largest in the heave DOF. This is reasonable given that the vehicle's cross-section area is smallest in the surge DOF and largest in heave DOF. However, the vehicle's linear damping behaves in the opposite manner with a smaller value. This is due to the skin friction being more dominant in the surge DOF than in the heave DOF.

On the Assumption 3.2.1 and 3.2.2, the linear hydrodynamic damping matrix is:

$$\mathbf{D} = - \begin{bmatrix} 3.7000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.9129 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.3756 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.6650 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.4560 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1099 \end{bmatrix} \quad (3.40)$$

3.3.4 Buoyancy and Gravitational Force Vector

The $\mathbf{G}_f(\boldsymbol{\eta})$ term is used to describe the gravitational and buoyancy [94] vector (Fig. A1.6 for SIMULINKTM block diagram in Appendix A1) exerted on the ROV in water. Transforming the restoring forces to the body-fixed coordinate system, the weight and buoyancy forces can be written as:

$$\mathbf{f}_G(\boldsymbol{\eta}) = \mathbf{J}_1^T(\boldsymbol{\eta}) \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix} \quad (3.41a)$$

$$\mathbf{f}_B(\boldsymbol{\eta}) = -\mathbf{J}_1^T(\boldsymbol{\eta}) \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix} \quad (3.41b)$$

where the weight of the ROV is given by $W = mg$ and g is the gravitational acceleration (positive downward) term. When fully submerged, the ROV's buoyancy is equal to the weight of water displaced, i.e. $B = \rho g \nabla$ where ρ is the fluid density and ∇ is the volume displaced by the submerged ROV. Here, $\mathbf{J}_1(\boldsymbol{\eta})$ is the Euler angle coordinate transformation matrix defined in (3.17). In the body-fixed coordinate system, the restoring force vector becomes:

$$\mathbf{G}_f(\boldsymbol{\eta}) = - \begin{bmatrix} \mathbf{f}_G(\boldsymbol{\eta}) + \mathbf{f}_B(\boldsymbol{\eta}) \\ \mathbf{r}_G \times \mathbf{f}_G(\boldsymbol{\eta}) + \mathbf{r}_B \times \mathbf{f}_B(\boldsymbol{\eta}) \end{bmatrix} \quad (3.42)$$

Expanding (3.42) gives:

$$\mathbf{G}_f(\boldsymbol{\eta}) = \begin{bmatrix} (W-B)\sin\theta \\ -(W-B)\cos\theta\sin\phi \\ -(W-B)\cos\theta\cos\phi \\ -(y_G W - y_B B)\cos\theta\cos\phi + (z_G W - z_B B)\cos\theta\sin\phi \\ (z_G W - z_B B)\sin\theta + (x_G W - x_B B)\cos\theta\cos\phi \\ -(x_G W - x_B B)\cos\theta\sin\phi - (y_G W - y_B B)\sin\theta \end{bmatrix} \quad (3.43)$$

The buoyancy and gravitational force hydrodynamic damping matrix, $\mathbf{G}_f(\boldsymbol{\eta})$ was computed using the following design rules.

1. In designing ROVs, it is desirable to make the ROV neutrally buoyant or slightly positive buoyant by adding additional float or balancing mass. With that, the RRC ROV becomes neutrally buoyant, $W = B$.
2. By placing additional mass on the ROV to make the X-Y coordinates of the C.B. coincide with the X-Y coordinate of the CG, that is $x_G = x_B = 0$, $y_G = y_B = 0$. Equation (3.43) becomes:

$$\mathbf{G}_f(\boldsymbol{\eta}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (z_G - z_B)W \cos \theta \sin \phi \\ (z_G - z_B)W \sin \theta \\ 0 \end{bmatrix} \quad (3.44)$$

Result 3.3: Buoyancy and Gravitational Force Matrix (cited from [99])

The buoyancy force and center of buoyancy was found via PRO/ENGINEERTM [99]. A CAD assembly, similar to the one used to generate the rigid-body parameters in Section 3.3.1 is created. The difference in this new assembly is that all the empty space in each of its parts is filled up. In order to generate the buoyancy parameters, the density of each component is equated to the density of water. Therefore, the model now represents the mass of water displaced by the ROV.

Step 1: Before adding additional float (or balancing mass), the buoyancy force and center of buoyancy (C.B) with respect to the center of gravity (C.G) (see Fig. 3.8a) is found to be:

$$\begin{aligned}
B &= 95.25 \times 9.81 = 934\text{N} \\
W &= 115 \times 9.81 = 1128\text{N} \\
x_G - x_B &= 217 - 231 = -14\text{mm} \\
y_G - y_B &= 0\text{mm} \\
z_G - z_B &= 253 - 283 = -30\text{mm}
\end{aligned} \tag{3.45}$$

where x_B , y_B , z_B are measured with respect to the CB. Notice that the gravitational force is greater than the buoyancy force ($W > B$), the ROV is negatively buoyant.

Figure 3.8a. RRC ROV side view (before addition of balancing mass)

Step 2: After adding additional float (or balancing mass), the ROV becomes neutrally buoyant. The gravitational force due to ROV weight equals the buoyancy force ($W = B$) due to water displaced by the submerged ROV. Additional mass is placed on the ROV to make the X-Y coordinate of the C.B. coincides with the X-Y coordinate of the C.G. i.e. $x_G = x_B = 0$, $y_G = y_B = 0$. Hence, the new buoyancy force and center of buoyancy with respect to the center of gravity (see Fig. 3.8b) is found to be:

$$\begin{aligned}
W &= B = 115 \times 9.81 = 1128\text{N} \\
x_G - x_B &= 0\text{mm} \\
y_G - y_B &= 0\text{mm} \\
z_G - z_B &= -48\text{mm}
\end{aligned} \tag{3.46}$$

where x_B , y_B , z_B are measured with respect to the center of gravity. The RRC ROV has been made neutrally buoyant ($W = B$) with center of gravity and buoyancy coincide in the X and Y-axes. This simply means the ROV would not sink and exhibit self-restoring forces in roll and pitch angular motion when deployed into the water.

Figure 3.8b. RRC ROV side view (after addition of balancing mass)

3.3.5 Thrusters' Configuration Matrix and its Model

The position of the thruster on the ROV (as seen in Fig. 3.9) is defined by the thrusters' configuration matrix. This is followed by establishing the thrust versus the input voltage relationship model. The details of the steady-state experiment performed on the thruster can be found in [100,101].

Figure 3.9: Thrusters position on the RRC ROV platform

As mentioned in the nonlinear ROV dynamic equation in (3.14), the left hand-side of the equations refers to the input forces and moments to the ROV. These input force and moments are determined based on summation of the force and moment equations in the six DOFs:

$$\begin{aligned}
 u_1 + u_2 &= \tau_x \\
 u_3 \sin \beta - u_4 \sin \beta &= \tau_y \\
 u_3 \cos \beta + u_4 \cos \beta &= \tau_z \\
 u_3 \delta - u_4 \delta &= \tau_\phi \\
 -u_1 \varepsilon - u_2 \varepsilon + u_3 \alpha \cos \beta - u_4 \alpha \cos \beta &= \tau_\theta \\
 u_1 \gamma - u_2 \gamma + u_3 \alpha \sin \beta - u_4 \alpha \sin \beta &= \tau_\psi
 \end{aligned} \tag{3.47}$$

where the $\alpha = 0.017\text{m}$, $\beta = 45^\circ$, $\gamma = 0.31\text{m}$, $\delta = 0.293\text{m}$, $\varepsilon = 0.016\text{m}$ are the geometrical parameters based on the thrusters' location on the ROV platform (see Fig. 3.9), $u_{\min} \leq u_i \leq u_{\max}$, $i = 1, 2, 3, 4$ are the minimum and maximum thrust output by each thrusters and $\tau_A = [\tau_x \ \tau_y \ \tau_z \ \tau_\phi \ \tau_\theta \ \tau_\psi]^T$ is the force and moment vector generated in the six DOFs.

As observed in (3.47), the roll and pitch motions (that is the fourth and fifth equations) are not fully actuated as the thrusters are positioned in the ROV such that the contributions to these motions are not dominant. The values of α and ε are small compared to the rest of the parameters. In this context, the ROV is underactuated since there are insufficient thrusters

(only four thrusters) to fully maneuver the ROV in the six DOFs. Fortunately, as shown in Section 4.1, the two unactuated DOFs of roll and pitch velocity are asymptotically stable and its angles are bounded. To simplify the (3.47) into matrix form:

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \sin \beta & -\sin \beta \\ 0 & 0 & \cos \beta & \cos \beta \\ 0 & 0 & \delta & -\delta \\ -\varepsilon & -\varepsilon & \alpha \cos \beta & -\alpha \cos \beta \\ \gamma & -\gamma & \alpha \sin \beta & -\alpha \sin \beta \end{bmatrix}}^{\mathbf{T}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}}_{\mathbf{u}} \quad (3.48)$$

$$\boldsymbol{\tau}_A = \mathbf{T} \mathbf{u}$$

$\mathbf{u} = \mathbf{F}_T \bar{\mathbf{u}} \in \Re^4$ with $\mathbf{F}_T = f_T \mathbf{I}_{4 \times 4}$, $\bar{\mathbf{u}} \in \Re^4$ the commanded voltage input to the thrusters and $\mathbf{I}_{4 \times 4}$ is an identity matrix of dimensions four. In summary, the relationship between these parameters can be shown in the block diagram in Fig. 3.10

Figure 3.10: Block diagram of the thrusters' parameters

As shown later, the thruster has a relatively shorter response time compared with the ROV and thus can be modeled as a gain term. This is similar to the approach found in [120,121,122,123,124,125,126] where a steady-state thruster model was considered due to its faster response time as compared to the ROV dynamic. An experiment was done to establish the thruster's steady-state model (f_T) between the thrust ($\mathbf{u}$) and the commanded voltage input ($\bar{\mathbf{u}}$).

In the thruster's experiment setup, a PWM servo amplifier was used to provide controlled voltage to drive the thruster. A force sensor (ATI Industrial Automation Force/Torque Sensor model Gamma) with a sensitivity of 0.1 N/count and a range of -130 to $+130$ N was utilized to measure the thrust exerted by the thruster. The thruster (see Fig. 3.11) was saddled on one end of a lever while the force sensor was mounted at the opposite end. The lever transmits the force exerted by the thruster to the sensor at a ratio of 1:1. In order to measure the rotational speed of the propeller, a water-resistance type optical-fiber speed sensor (Fuji Electric PH21A Series) was used together with a frequency counter.

Figure 3.11. Thruster Test Rig

From the steady-state experimental results [100,101], the lumped hydro-electromechanical dynamic model for both the DC motor shaft speed (assuming small electrical time constant as compared to the mechanical time constant) and propeller's dynamic can be written as:

$$J_m \dot{\Omega} + \frac{K_m K_e}{R_m} \Omega = \frac{K_m}{R_m} \bar{u}_i - Q, \quad i = 1, 2, 3 \text{ or } 4 \quad (3.49a)$$

$$\bar{u}_i = R_m I_t + L_m \frac{dI_t}{dt} + K_e \Omega, \quad i = 1, 2, 3 \text{ or } 4 \quad (3.49b)$$

The steady-state propeller's thrust and torque model can be written as:

$$u_i = K_{Td} \Omega^2, \quad i = 1, 2, 3 \text{ or } 4 \quad (3.49c)$$

$$Q = K_{Qd} \Omega^2 \quad (3.49d)$$

where $\bar{u}_i$ is the voltage input to thruster in V, R_m is the armature resistance in Ohms, I_t is the current to the armature in Ampere, L_m is the inductance in Henry, K_m is the motor torque constant N.m/A, K_e is the motor back emf in V.s/rad, J_m is the rotor moment of inertia in Nm.s²/rad, Ω is the rotational speed of the propeller in rad/s, Q is the propeller's torque in N.m, u_i is the thrust from each propeller i in Newton, K_{Td} is the thrust constant in N.s/rad and K_{Qd} is the torque constant in N.m.s/rad. (The SIMULINKTM block diagrams for (3.49a) to (3.49d) is given in Fig. A1.7 in Appendix A1).

By examining (3.49a) to (3.49d), an approximated linear relationship between the thrust (u_i) and the commanded voltage input ($\bar{u}_i$) for the thruster can be plotted as shown in the Fig. 3.12.

$$u_i = f_T \bar{u}_i, \quad i = 1, 2, 3 \text{ or } 4 \quad (3.50)$$

where $f_T = 0.92$ N/V (forward thrust) and $f_T = -0.61$ N/V (reverse thrust). The F_T in Fig. 3.10 for both forward and reverse thrust can be determined as follows:

$$F_T = \begin{bmatrix} 0.92 & 0 & 0 & 0 \\ 0 & 0.92 & 0 & 0 \\ 0 & 0 & 0.92 & 0 \\ 0 & 0 & 0 & 0.92 \end{bmatrix}, \quad (3.51a)$$

$$F_T = - \begin{bmatrix} 0.61 & 0 & 0 & 0 \\ 0 & 0.61 & 0 & 0 \\ 0 & 0 & 0.61 & 0 \\ 0 & 0 & 0 & 0.61 \end{bmatrix} \quad (3.51ba)$$

As shown in Fig. 3.12, there is a reasonable correlation between the two variables $\bar{u}_i$ and u_i as the typical value of R falls in the range 0.7 to 1. As seen in Fig. 3.12, the forward and linear thrusts were approximated by different linear functions. As shown in the time response of the thrust - theoretical and experimentation [100,101] in Fig. 3.13, there is a good match in the steady state response though the errors due to this approximation in the region less than 20 volts are quite high. As verified in Fig. 3.13, the thruster's time constant is about 0.5 second and is relatively shorter than the ROV response time (due to large inertia). Hence, the thruster dynamic (especially input up to 20 volts) can be ignored as it is not as dominant as compared to the ROV dynamic. The dead band does affect the position errors, while in practice, may not necessarily be zero. There is a possibility of limit cycle. This can be considered in future research.

Figure 3.12: Thrust vs voltage input to a thruster
(R^2 is the correlation factor)

Figure 3.13: Time response of the thrust - theoretical vs experimental results

In summary, the constants used in (3.49a) to (3.49d) can be tabulated in Table 3.1 while the details of obtaining these constants can be found in [100,101].

Constant	Forward Thrust	Reverse Thrust
K_m	12.562 N.m/A	8.739 N.m/A
R_m	2.4Ω	2.4Ω
K_e	0.245 V.s/rad	0.245 V.s/rad
K_{Td}	0.0022 N.s/rad	-0.0013N.s/rad
K_{Qd}	6.0×10^{-5} N.m.s/rad	-2.0×10^{-5} N.m.s/rad
L_m	0.491 H	0.491 H

Table 3.1. Summary of thruster constants

3.4 Perturbed RRC ROV's Model

The nonlinear equation with perturbation on (3.24a) can be expressed as:

$$\dot{\mathbf{x}} = \bar{\mathbf{f}}(\mathbf{x}, t) + \bar{\mathbf{g}}(\bar{\mathbf{u}}, t) + \Delta \mathbf{f}(\mathbf{x}, t) + \Delta \mathbf{g}(\bar{\mathbf{u}}, t), \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (3.52)$$

where the full-order states $\mathbf{x} = [\boldsymbol{\eta} \quad \mathbf{v}]^T$ and functions in (3.52) can be defined as:

$$\bar{\mathbf{f}}(\mathbf{x}, t) = \begin{bmatrix} \mathbf{0}_{6 \times 6} & \mathbf{J}(\boldsymbol{\eta}) \\ \mathbf{0}_{6 \times 6} & -\mathbf{M}_{\text{RB}}^{-1}[\mathbf{C}_{\text{RB}}(\mathbf{v}) + \mathbf{D}] \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta} \\ \mathbf{v} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{6 \times 1} \\ -\mathbf{M}_{\text{RB}}^{-1} \mathbf{G}_f(\boldsymbol{\eta}) \end{bmatrix}, \quad \bar{\mathbf{g}}(\bar{\mathbf{u}}, t) = \begin{bmatrix} \mathbf{0}_{6 \times 1} \\ -\mathbf{M}_{\text{RB}}^{-1} \mathbf{T} \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix}, \quad \bar{\mathbf{u}} \in \mathbb{R}^4 \quad (3.53a)$$

The perturbation terms in the $\Delta \mathbf{f}(\mathbf{x}, t), \Delta \mathbf{g}(\mathbf{x}, \bar{\mathbf{u}}, t)$ due to the hydrodynamic added mass and damping forces are defined as:

$$\Delta \mathbf{f}(\mathbf{x}, t) = \begin{bmatrix} \mathbf{0}_{6 \times 1} \\ \Delta \mathbf{M}_A^{-1}[\mathbf{C}_A(\mathbf{v}) + \Delta \mathbf{D}] \mathbf{v} \end{bmatrix}, \quad \Delta \mathbf{g}(\bar{\mathbf{u}}, t) = \begin{bmatrix} \mathbf{0}_{6 \times 1} \\ \Delta \mathbf{M}_A^{-1} \mathbf{T} \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix} \quad (3.53b)$$

The upper bound for the $\Delta \mathbf{M}_A$, $\Delta \mathbf{C}_A$ and $\Delta \mathbf{D}$ used in (3.53b) can be determined by the following Section 3.4.1 and 3.4.2.

3.4.1 Perturbation Bound on M and C Matrix

The uncertainties in the added mass computation can be treated as a perturbation on the nominal model in (3.53a). The sensitivity of the added mass matrix under various conditions can be considered to be due to:

- changes in the ROV's shape due to dimensional error and extra components [96]
- changes in the ROV's center of buoyancy [96]
- changes in fluid density around ROV [96]

In the first case, the ROV can be modeled without the extra components (such as a small cylinder) and with no change in the reference frame. The added mass is computed as:

$$\mathbf{M}_A = - \begin{bmatrix} 21.0981 & 0 & 0.0884 & 0 & -2.6127 & 0 \\ 0 & 51.0026 & 0 & 2.9200 & 0 & -1.0534 \\ 0.0692 & 0 & 91.837 & 0 & 1.8883 & 0 \\ 0 & 2.9199 & 0 & 3.5760 & 0 & -0.5795 \\ -2.5988 & 0 & 1.8884 & 0 & 6.7568 & 0 \\ 0 & -1.0537 & 0 & -0.5793 & 0 & 4.4147 \end{bmatrix} \quad (3.54)$$

By including those extra components and using the same reference frame, the added mass is:

$$\mathbf{M}_A = - \begin{bmatrix} 21.724 & 0 & -0.5683 & 0 & -2.8284 & 0 \\ 0 & 51.5244 & 0 & 3.3080 & 0 & -1.0988 \\ -0.0147 & 0 & 94.300 & 0 & 1.9858 & 0 \\ 0 & 3.3320 & 0 & 3.8803 & 0 & -0.7420 \\ -2.6862 & 0 & 1.9905 & 0 & 7.1148 & 0 \\ 0 & -1.1078 & 0 & -0.7329 & 0 & 4.7673 \end{bmatrix} \quad (3.55)$$

In the second case, with the extra components and a change in the reference frame to C.B, the added mass becomes:

$$\mathbf{M}_A = - \begin{bmatrix} 21.1398 & 0 & 0.0621 & 0 & 1.0083 & 0 \\ 0 & 51.7013 & 0 & -5.9705 & 0 & 1.588 \\ 0.0918 & 0 & 92.4511 & 0 & -2.9191 & 0 \\ 0 & -5.8866 & 0 & 4.2176 & 0 & -0.1719 \\ 1.0583 & 0 & -2.9489 & 0 & 2.7687 & 0 \\ 0 & 1.5863 & 0 & -0.171 & 0 & 2.3493 \end{bmatrix} \quad (3.56)$$

With the extra components and a change in the reference frame to C.G, the final M_A becomes:

$$\mathbf{M}_A = - \begin{bmatrix} 21.1403 & 0 & 0.0619 & 0 & -0.5748 & 0 \\ 0 & 51.7012 & 0 & -2.0928 & 0 & -0.3767 \\ 0.0917 & 0 & 92.4510 & 0 & 0.5871 & 0 \\ 0 & -2.0090 & 0 & 3.6191 & 0 & 0.0235 \\ -0.5237 & 0 & 0.5594 & 0 & 2.6427 & 0 \\ 0 & -0.3783 & 0 & 0.0275 & 0 & 2.3033 \end{bmatrix} \quad (3.57)$$

Note: equation (3.57) is being used in (3.34).

As can be observed from (3.54) to (3.57), changes in the off-diagonal components are quite small compared with the diagonal counterparts. Thus, the perturbation $\Delta\mathbf{M}_A$ can comprise only the diagonal terms, with all the maximum values as the upper bound for $\Delta\mathbf{M}_A$:

$$\Delta\bar{\mathbf{M}}_A = - \begin{bmatrix} 21.7240 & 0 & 0 & 0 & 0 & 0 \\ 0 & 51.7013 & 0 & 0 & 0 & 0 \\ 0 & 0 & 94.3000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.2176 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.1148 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.7673 \end{bmatrix} \quad (3.58)$$

In addition, as shown in Table 3.2, at different temperatures, the seawater density with 3.5% salinity (as compared to water at 1000kg/m³) varies. To account for differences in density at various temperatures, a factor is included in the perturbation $\Delta\mathbf{M}_A$ as followed:

$$\Delta\mathbf{M}_A \leq 1.028\Delta\bar{\mathbf{M}}_A \quad (3.59)$$

Medium	0°C	10°C	20°C	30°C
Seawater (3.5% salinity)	1028.48 kg/m ³	1026.91 kg/m ³	1024.85 kg/m ³	1021.81 kg/m ³

Table 3.2: Seawater density at various temperatures

Similarly, the upper bound on the uncertainties in the Coriolis and centripetal added mass matrix, $\mathbf{C}_A(\mathbf{v})$ can be expressed as:

$$\Delta\mathbf{C}_A(\mathbf{v}) \leq 1.028\Delta\bar{\mathbf{C}}_A(\mathbf{v}) \quad (3.60)$$

where

$$\Delta\bar{\mathbf{C}}_A(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -\Delta a_3^u & \Delta a_2^u \\ 0 & 0 & 0 & \Delta a_3^u & 0 & -\Delta a_1^u \\ 0 & 0 & 0 & -\Delta a_2^u & \Delta a_1^u & 0 \\ 0 & -\Delta a_3^u & \Delta a_2^u & 0 & -\Delta b_3^u & \Delta b_2^u \\ \Delta a_3^u & 0 & -\Delta a_1^u & \Delta b_3^u & 0 & -\Delta b_1^u \\ -\Delta a_2^u & \Delta a_1^u & 0 & -\Delta b_2^u & \Delta b_1^u & 0 \end{bmatrix} \quad (3.61)$$

$$\begin{aligned}
\Delta a_1^l &= 21.0981u, \Delta a_1^u = 21.7240u \\
\Delta a_2^l &= 51.0002v, \Delta a_2^u = 51.7013v \\
\Delta a_3^l &= 91.8370w, \Delta a_3^u = 94.3000w \\
\Delta b_1^l &= 3.5760p, \Delta b_1^u = 4.2176p \\
\Delta b_2^l &= 2.7687q, \Delta b_2^u = 7.1148q \\
\Delta b_3^l &= 2.4393r, \Delta b_3^u = 4.7673r
\end{aligned} \tag{3.62}$$

3.4.2 Perturbation Bound on $\mathbf{D}(\mathbf{v})$ Matrix

The uncertainties due to hydrodynamic damping can also be considered to account for the uncertainty in the linear damping due to skin friction and the dimensionless analysis that was used to predict the ROV damping coefficients. The perturbation on the linear damping $\Delta \mathbf{D}$ can be approximated as:

$$\Delta \mathbf{D} = - \begin{bmatrix} 0.370 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.291 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.237 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.166 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.145 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.011 \end{bmatrix} \tag{3.63}$$

$\Delta \mathbf{D}$ is taken to be about 10 percent of the $\mathbf{D}$ matrix as seen in [96]. The SIMULINKTM block diagram for $\mathbf{M}$, $\mathbf{C}(\mathbf{v})$ and $\mathbf{D}$ perturbation modeling are shown in Fig. A1.8 in Appendix A1; the perturbation is added onto the nominal model as an additive perturbation.

3.5 Summary

The dynamic and kinematics equations of the RRC ROV were formulated. The Mass inertia and buoyancy and gravitational force vector used in the dynamic equations were obtained through CAD software, while the hydrodynamic damping and added mass components were computed using various CFD software. Although experimental verifications for these hydrodynamic forces and moments parameters were difficult to obtain without a proper tow

tank facility, attempts at verification was done using a scaled model and dynamic similitude analysis. Experimental results from tests in a small water tank and the CFD values of the scaled model were compared and found to be in good agreement. A steady state model of thruster was also presented. To provide for the study of the ROV dynamic behavior for subsequent control design, a state space model using a simplified nominal model and additive perturbation model was defined.

The RDA package is first used to model and simulate the nonlinear ROV with perturbation. It uses the MATLABTM scripting language and graphical block diagram model, SIMULINKTM (together with VRML visualization as seen in Chapter 7) to simulate the control system designs through comparisons with others controllers using the underwater pipeline tracking mission as shown in Fig. 1.6 (in Chapter 1).

Chapter 4

RRC ROV Subsystems Analysis

This Chapter covers the analysis of the ROV's subsystems model for control systems design. As seen in Proposition 4.1, the roll and pitch velocities of the open-loop ROV system are shown to be asymptotically stable (and its corresponding roll and pitch angles are bounded). This suggests that the roll and pitch DOF can be left uncontrolled. This lessens the degree of input coupling as no thrusters are required to activate these unactuated states directly and with less DoF to be controlled, the ROV becomes an actuated system in some decoupled modes of operations such as vertical and horizontal plane motions.

If the ROV moves simultaneously in both vertical and horizontal planes, the complexity of the control algorithm can possibly increase. Intuitively, the proposed strategy is to move the ROV in either vertical or horizontal plane during maneuvering. This creates a systematic trajectory tracking from one point to another with the least amount of coupling in states. However, this creates a need to ensure that the unactuated dynamic is bounded during both the tracking and stabilization. In other words, the unactuated velocities (and its roll and pitch angles) of the ROV must be bounded and this condition is known as an equilibrium submanifold.

To understand the behavior of these subsystems, namely – the horizontal plane, vertical plane and station-keeping modes – several nonlinear system properties need to be examined. Properties such as the uniqueness, existence, and continuous dependency of parameters are demonstrated. To verify that the energy conservation principle is satisfied, the energy and passivity of the subsystems are examined. In addition, the subsystems are tested for state controllability so that controllers can be designed in Chapter 5. As the underactuation occurs in the vertical plane subsystem, properties such as stabilizability, strong accessibility (SA) and small-time locally controllability (STLC) are required to be known prior to controller design. In summary the nonlinear properties of the ROV are tabulated in Table 4.1.

Parameters	Full-Order Model	Station-keeping Model	Horizontal Plane Model	Vertical Plane Model
Uniqueness/Existence	YES	YES	YES	YES
Continuous dependence of states	YES	YES	YES	YES
Continuous dependence of thrust input, u	YES	YES	YES	YES
Controllability	NO	YES	YES	NO
$V > 0$	YES	YES	YES	YES
$\dot{V} \leq 0$	Not at all time	Not at all time	Not at all time	Not at all time
SA	-	-	-	YES
STLC	-	-	-	YES

Table 4.1: Summary of all nonlinear properties

The main contributions of this Chapter are: (a) the systematic analysis of ROV subsystems model using various tools such as states coupling and Euclidean norm controllability test on the higher dimensional spaces; (b) the RDA[102] written in MATLABTM scripting language and graphical block diagram simulation, SIMULINKTM (as seen in Fig. A1.9 in Appendix A1) with virtual environment interface are used to understand the nonlinear ROV dynamics. To aid in Lie Bracket calculation, mathematical software such as MAPLETM that offers a powerful symbolic method in solving differential equation is used.

The Chapter is organized as follows. The stability of the unactuated dynamic was described in Section 4.1. The nominal and perturbed station-keeping models were formulated in Section 4.2. The horizontal and vertical plane subsystem models were presented in Section 4.3. This is followed by the ROV's subsystem properties were included in Section 4.4. Lastly, Section 4.5 summarizes the work done in this Chapter.

4.1. Unactuated Dynamic Response

Recall, the roll and pitch motions are not fully actuated as the thrusters are positioned in the ROV such that the contributions to these motions are not dominant (as observed in the Fig. 4.5 and 4.6). In this context, the RRC ROV is underactuated since there are insufficient sets of thrusters (only four thrusters) to fully maneuver the ROV in the six DoFs.

However, as shown in Proposition 4.1, the unactuated roll and pitch rate are stable and they can be left uncontrolled in the subsequent control system design since they will not destabilize the system stability and nominal performance. This is mainly due to the meta-centric height being sufficiently large to provide adequate static stability to enable the gravitational and buoyancy force to act as self-restoring moments in these unactuated motions. Same phenomena can be observed in other working ROV such as JHU ROV [120] where the roll and pitch are also self-stabilizable. Intuitively, this reduces the number of DoF for control as the additional thrusters needed for these unactuated states are not required. Hence, it reduces significantly the computational complexity for subsequent control system analysis and design without affecting the generic dynamic of the ROV.

Assumption 4.1: Actuated and unactuated state conditions

The following conditions hold:

1. The surge velocity and its derivative satisfy: $0 < u_{\min} \leq |u(t)| \leq u_{\max} < \infty$ and $|\dot{u}(t)| \leq \dot{u}_{\max} < \infty, \forall 0 \leq t < \infty$.
2. The heave velocity is bounded: $|w(t)| \leq w_{\max} < \infty \quad \forall 0 \leq t < \infty$.
3. The yaw velocity is constant (and small) and its derivative is zero for ROV traveling via points on a straight line: $r(t) = \text{const}, \dot{r}(t) = 0$. The cross product of the angular velocities such as $r(t)^2, r(t)p(t)$ and $r(t)q(t)$ are small.
4. All the ROV parameters are known and obtained from Chapter 3.
5. The roll and pitch angles θ and ϕ and the derivatives of body rates $\dot{p}$ and $\dot{q}$ are small.

Assumption (4.1.1) means the forward speed of the ROV is nonzero and does not change its sign. This means that both forward and backward tracking are considered. Assumption (4.1.2) is always satisfied as long as the ROV does not overturn. Assumption (4.1.3) means that the ROV is moving in a straight-line path. This is reasonable since the ROV cannot track in smooth circle with arbitrarily small radius due to its large inertia. To have a less coupled motion, it is desired to operate the ROV in straight line segments.

Proposition 4.1: Stability of unactuated dynamics

By considering the following positive definite Lyapunov function:

$$V_1 = \frac{1}{2}(I_x - K_{\dot{p}})p^2 \quad (4.1)$$

$$V_2 = \frac{1}{2}(I_y - M_{\dot{q}})q^2 \quad (4.2)$$

where p and q refer to the roll and pitch angular velocity respectively.

Then the derivative of (4.1) and (4.2) becomes:

$$\|\dot{V}\|_2 \leq -D_u \|\mathbf{x}_{u1}\|_2 \quad (4.3)$$

where the $\mathbf{x}_{u1} = [p \ q]^T$, $D_u = \lambda_{\max} \left(\begin{bmatrix} K_p p & 0 \\ 0 & M_q q \end{bmatrix} \right)$ and $\lambda_{\max}(\cdot)$ is the maximum eigenvalues. The $\|\mathbf{x}_{u1}\|_2 = \sqrt{p^2 + q^2}$ and $\|\dot{V}\|_2 = \sqrt{\dot{V}_1^2 + \dot{V}_2^2}$ are the Euclidean norm of the unactuated state and Lyapunov function respectively. Equation (4.3) implies that the roll and pitch angles are small and bounded as:

$$\|\mathbf{x}_{u2}(t)\|_2 \leq J_{\max} \cdot \frac{\|\mathbf{x}_{u1}(t)\|_2^2}{2} + \|\mathbf{x}_{u2}(0)\|_2 \quad (4.4)$$

where the $\mathbf{x}_{u2} = [\phi \ \theta]^T$, $\|\mathbf{x}_{u2}\|_2 = \sqrt{\phi^2 + \theta^2}$ is the Euclidean norm and $J_{\max} \geq \sqrt{1/2} \left[1 + \phi^2 \theta + \phi^2 + \sqrt{-4\phi^2 + (-1 - \theta\phi^2 - \phi^2)^2} \right]^{1/2}$.

Proof:

The derivative of V_1 yields:

$$\begin{aligned} \dot{V}_1 &= (I_x - K_{\dot{p}})p\dot{p} \\ &= p[-(I_z - I_y)qr + (\dot{r} + pq)I_{xz} - (r^2 - q^2)I_{yz} - (pr - \dot{q})I_{xy} - K_p p + z_B B \cos \theta \sin \phi] \end{aligned} \quad (4.5)$$

where the roll dynamic defined in Chapter 3 as:

$$\begin{aligned}
& (I_x - K_{\dot{p}})\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} \\
& + (pr - \dot{q})I_{xy} + K_p p - z_B B \cos \theta \sin \phi = 0
\end{aligned} \tag{4.6}$$

By applying the Assumption (A4.1.3) and (A4.1.5) implies that $\dot{q}, \dot{p}$ are small. With additional balancing mass at the designated location in the ROV and with the symmetry in the X-Z plane, it is possible to have $I_{xy} = 0$ and $I_{yz} = 0$ (the values obtained for the RRC ROV are $I_{xy} = -0.00016$ and $I_{yz} = 0.0006$, which are almost equal to zero). Thus, equation (4.6) becomes:

$$\dot{V}_1 = p(pqI_{xz} - K_p p) \tag{4.7}$$

Similarly, the derivative of V_2 becomes:

$$\dot{V}_2 = (I_y - M_{\dot{q}})q\dot{q} = q[-p^2 I_{xz} - M_q q] \tag{4.8}$$

where the pitch dynamic is defined in Chapter 3 as:

$$\begin{aligned}
& (I_y - M_{\dot{q}})\dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{xz} \\
& + (qp - \dot{r})I_{yz} + M_q q - z_B B \sin \theta = 0
\end{aligned} \tag{4.9}$$

As seen in the Lyapunov function candidate in (4.1) and (4.2), it is obvious that V_1 and V_2 are positive definite and radially unbounded. To show that the derivative of the Lyapunov function is negative semi-definite, the (4.7) and (4.8) can be written into matrix form.

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} p & 0 \\ 0 & q \end{bmatrix} \cdot \begin{bmatrix} q & 0 \\ 0 & p \end{bmatrix} \cdot \begin{bmatrix} I_{xz} & 0 \\ -I_{xz} & 0 \end{bmatrix} \cdot \begin{bmatrix} p \\ q \end{bmatrix} - \begin{bmatrix} p & 0 \\ 0 & q \end{bmatrix} \cdot \begin{bmatrix} K_p & 0 \\ 0 & M_q \end{bmatrix} \cdot \begin{bmatrix} p \\ q \end{bmatrix} \tag{4.10}$$

Note that in some deviation, the function of time t is not shown for clarity. Next, applying the matrix norm (that is $\|\mathbf{A}\|_2 = \sqrt{\lambda_{\max}(\mathbf{A}^T \mathbf{A})}$ for $\mathbf{A}$ is a non-diagonal matrix) and triangular inequality on (4.10) yields:

$$\begin{aligned} \|\dot{\mathbf{V}}\|_2 &\leq \lambda_{\max} \left(\begin{bmatrix} pq & 0 \\ 0 & qp \end{bmatrix} \right) \lambda_{\max} \left(\begin{bmatrix} 2I_{xz}^2 & 0 \\ 0 & 0 \end{bmatrix} \right)^{\frac{1}{2}} \|\mathbf{x}_{u1}\|_2 - \lambda_{\max} \left(\begin{bmatrix} K_p p & 0 \\ 0 & M_q q \end{bmatrix} \right) \|\mathbf{x}_{u1}\|_2 \\ &\leq \left[\sqrt{2} I_{xz} \lambda_{\max}(pq) - \lambda_{\max} \left(\begin{bmatrix} K_p p & 0 \\ 0 & M_q q \end{bmatrix} \right) \right] \|\mathbf{x}_{u1}\|_2 \end{aligned} \quad (4.11)$$

where $\lambda_{\max}$ is the maximum eigenvalues of the matrix, $\mathbf{x}_{u1} = [p \ q]^T$ and $\|\mathbf{x}_{u1}\|_2 = \sqrt{p^2 + q^2}$. Since the first term in (4.11) is smaller than its second term, equation (4.11) can be bounded as:

$$\|\dot{\mathbf{V}}\|_2 \leq -D_u \|\mathbf{x}_{u1}\|_2 \quad (4.12)$$

where $D_u = \lambda_{\max} \left(\begin{bmatrix} K_p p & 0 \\ 0 & M_q q \end{bmatrix} \right) = K_p \cdot \lambda_{\max} \left(\begin{bmatrix} p & 0 \\ 0 & q \end{bmatrix} \right)$ (as seen in Chapter 3 that $K_p > M_q$).

As shown in (4.12), $\dot{V} \leq 0$ implies that $V(t) \leq V(0), \forall t \geq 0$ or the unactuated velocities, $\mathbf{x}_{u1}$ is bounded. By performing the derivative of $\ddot{V}$, it is bounded and thus $\dot{V}$ must be uniformly continuous. Using the Barbalat's lemma [94] shows that $\dot{V} \rightarrow 0$ that implies that $\mathbf{x}_{u1} \rightarrow 0$ as $t \rightarrow \infty$. Moreover as the V is radially unbounded, that is $\|V(\mathbf{x}_{u1})\|_2 \rightarrow \infty$ as $\|\mathbf{x}_{u1}\|_2 \rightarrow \infty$, the $\mathbf{x}_{u1}$ become globally uniformly asymptotically stable.

Next, the unactuated angle is shown to be bounded. Since the unactuated earth-fixed angle $\mathbf{x}_{u2} = [\phi \ \theta]^T$ is related to the unactuated body-fixed velocity $\mathbf{x}_{u1} = [p \ q]^T$ by the Euler's transformation matrix as seen in (3.17) for roll and pitch terms and given the small unactuated angles (as stated in Assumption (A4.1.5)), the following can be written as:

$$\begin{aligned} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \end{bmatrix} &= \begin{bmatrix} 1 & \phi\theta \\ 0 & \phi \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \\ \|\dot{\mathbf{x}}_{u2}\|_2 &\leq J_x \|\mathbf{x}_{u1}\|_2 \end{aligned} \quad (4.13)$$

where

$$\begin{aligned} J_x &= \left(\lambda_{\max} \begin{bmatrix} 1 & \phi\theta \\ \phi\theta & \phi^2\theta^2 + \phi^2 \end{bmatrix} \right)^{1/2} \\ &= \sqrt{1/2} \left[1 + \phi^2\theta + \phi^2 + \sqrt{-4\phi^2 + (-1 - \theta\phi^2 - \phi^2)^2} \right]^{1/2} \leq J_{\max} \end{aligned} \quad (4.14)$$

Substituting (4.14) into (4.13) and integrating (4.13) gives:

$$\begin{aligned}\|\mathbf{x}_{u2}(t)\|_2 &\leq J_{\max} \int_0^t \|\mathbf{x}_{u1}(t_1)\|_2 dt_1 + \|\mathbf{x}_{u2}(0)\|_2 \\ &\leq J_{\max} \frac{\|\mathbf{x}_{u1}(t)\|_2^2}{2} + \|\mathbf{x}_{u2}(0)\|_2\end{aligned}\quad (4.15)$$

Equation (4.15) implies that the unactuated angles $\mathbf{x}_{u2}$ are bounded and converge to the set defines as: $\mathbf{R}_{u2} = \{\mathbf{x}_{u2} : \|\mathbf{x}_{u2}\|_2 \leq x_{\max}\}$ and will not exceed $x_{\max}$ since the unactuated velocities $\mathbf{x}_{u1}$ are bounded in (4.12). In summary, the unactuated angles $[\phi \ \theta]^T$ are bounded if the unactuated velocities $[p \ q]^T$ are asymptotically stable. With that, they are left uncontrolled in the subsequent control system design since they will not destabilizing the system performance.

□

A simulation given in Fig. 4.1 shows that the roll and pitch motions are stable and bounded for input $\bar{\mathbf{u}} = 70$ volts (maximum voltage) to all thrusters. They exhibit a self-stabilizing behavior due to the physical construction of the vehicle. As long as the roll and pitch motions are not excessive, the buoyancy and gravitational force act as a restoring couple in the roll and pitch directions to restore these directions to an equilibrium state eventually. However, as observed from Fig. 4.1, the velocity u , v and w (and its corresponding position x , y , z) are not stable. These result in the ROV exhibiting a whirling motion, Fig 4.2, as the vehicle surges forward at a small initial pitch and roll angles. The whirling ROV motion is due to the added mass components in the Coriolis and centripetal term creating a cross coupling effect on the ROV's motion – the Munk moments arising from the change in the directions of the fluid as the ROV moves with a small pitch and roll angles. To address this problem, a cascaded control system is proposed in Chapter 5 to control the velocities and positions simultaneously.

Figure 4.1: Position (left) and velocity (right) response of ROV in open loop

Figure 4.2: 3D view of ROV in open loop

4.2. Nominal and Perturbed Station-Keeping Models

A station-keeping model is used to describe the ROV dynamic during the hovering or commonly known as stabilization about equilibrium submanifold [10]. As discussed in Section 5.5, the roll and pitch motions are unspecified in the equilibrium submanifold but they are asymptotically stable and bounded, they are neglected in the station-keeping mode. Thus, it mainly involves four DoFs such as: surge, sway, heave and yaw. In this Section, the nominal and perturbed models for the station-keeping dynamics are formulated.

The nominal nonlinear dynamic equation of the station-keeping model can be written as:

$$(m - X_{\ddot{u}})\dot{u} + X_{\ddot{u}}u + (Y_{\dot{v}} - m)vr = \tau_x \quad (4.16a)$$

$$(m - Y_{\dot{v}})\dot{v} + Y_{\dot{v}}v + (m - X_{\ddot{u}})ur = \tau_y \quad (4.16b)$$

$$(m - Z_{\dot{w}})\dot{w} + Z_{\dot{w}}w = \tau_z \quad (4.16c)$$

$$(I_z - N_{\dot{r}})\dot{r} + N_{\dot{r}}r + (-Y_{\dot{v}}u + X_{\ddot{u}}u)v = \tau_{\psi} \quad (4.16d)$$

The Kinematics equation can be written as:

$$\dot{x} = u \cos \psi - v \sin \psi \quad (4.17a)$$

$$\dot{y} = u \sin \psi + v \cos \psi \quad (4.17b)$$

$$\dot{z} = w \quad (4.17c)$$

$$\dot{\psi} = r \quad (4.17d)$$

In compact form, the nonlinear dynamic equation in (4.16a) to (4.16d) can be written as (Variables labeled with subscript 's'):

$$\dot{\mathbf{v}}_s = -\mathbf{M}_s^{-1}[\mathbf{C}_s(\mathbf{v}_s) + \mathbf{D}_s]\mathbf{v}_s + \mathbf{M}_s^{-1}\boldsymbol{\tau}_s \quad (4.18)$$

where $\mathbf{v}_s = [u \quad v \quad w \quad r]^T$, $\boldsymbol{\tau}_s = \mathbf{T}_s \mathbf{F}_T \bar{\mathbf{u}} = [\tau_x, \tau_y, \tau_z, \tau_{\psi}]^T$ in which $\bar{\mathbf{u}} \in \Re^4$, $\mathbf{F}_T \in \Re^{4 \times 4}$,

$$\mathbf{T}_s = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \sin \beta & -\sin \beta \\ 0 & 0 & \cos \beta & \cos \beta \\ \gamma & -\gamma & \alpha \sin \beta & -\alpha \sin \beta \end{bmatrix};$$

$$\mathbf{M}_s = \begin{bmatrix} m - X_{\ddot{u}} & 0 & 0 & 0 \\ 0 & m - Y_{\dot{v}} & 0 & 0 \\ 0 & 0 & m - Z_{\dot{w}} & 0 \\ 0 & 0 & 0 & I_r - N_{\dot{r}} \end{bmatrix}$$

$$\mathbf{D}_s = \begin{bmatrix} X_u & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 \\ 0 & 0 & Z_w & 0 \\ 0 & 0 & 0 & N_r \end{bmatrix}; \quad \mathbf{C}_s(\mathbf{v}_s) = \begin{bmatrix} 0 & (Y_v - m)r & 0 & 0 \\ 0 & 0 & 0 & (m - X_u)u \\ 0 & 0 & 0 & 0 \\ 0 & -Y_v u + X_u u & 0 & 0 \end{bmatrix}$$

The Kinematics equation in (4.17a) to (4.17d) can be written as compact form:

$$\dot{\boldsymbol{\eta}}_s = \mathbf{J}_s(\boldsymbol{\eta}_s) \mathbf{v}_s \quad (4.19)$$

$$\text{where } \boldsymbol{\eta}_s = [x \ y \ z \ \psi]^T \text{ and } \mathbf{J}_s(\boldsymbol{\eta}_s) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 & 0 \\ \sin \psi & \cos \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Equation (4.17) and (4.19) can be rewritten in state-space form:

$$\dot{\mathbf{x}}_s = \bar{\mathbf{f}}_s(\mathbf{x}_s, t) + \bar{\mathbf{g}}_s(\bar{\mathbf{u}}, t) \quad (4.20)$$

where $\mathbf{x}_s = [\boldsymbol{\eta}_s \ \mathbf{v}_s]^T$ and

$$\bar{\mathbf{f}}_s(\mathbf{x}_s, t) = \begin{bmatrix} \mathbf{J}_s(\boldsymbol{\eta}_s) \mathbf{v}_s \\ -\mathbf{M}_s^{-1}[\mathbf{C}_s(\mathbf{v}_s) + \mathbf{D}_s] \mathbf{v}_s \end{bmatrix} \quad (4.21)$$

and

$$\bar{\mathbf{g}}_s(\bar{\mathbf{u}}, t) = \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ [\bar{\mathbf{g}}_{s1} \ \bar{\mathbf{g}}_{s2} \ \bar{\mathbf{g}}_{s3} \ \bar{\mathbf{g}}_{s4}] \mathbf{T}_s \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix} \quad (4.22)$$

where

$$\bar{\mathbf{g}}_{s1} = \begin{bmatrix} \frac{1}{m - X_u} & 0 & 0 & 0 \end{bmatrix}^T; \quad \bar{\mathbf{g}}_{s2} = \begin{bmatrix} 0 & \frac{1}{m - Y_v} & 0 & 0 \end{bmatrix}^T; \quad (4.23)$$

$$\bar{\mathbf{g}}_{s3} = \begin{bmatrix} 0 & 0 & \frac{1}{m - Z_{\dot{w}}} & 0 \end{bmatrix}^T; \bar{\mathbf{g}}_{s4} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{I_r - N_{\dot{r}}} \end{bmatrix}^T \quad (4.24)$$

The corresponding perturbed model can be written in the form:

$$\dot{\mathbf{x}}_s = \bar{\mathbf{f}}_s(\mathbf{x}_s, t) + \bar{\mathbf{g}}_s(\bar{\mathbf{u}}, t) + \Delta \mathbf{d}_s(\mathbf{x}_s, \bar{\mathbf{u}}, t) \quad (4.25)$$

where

$$\begin{aligned} \Delta \mathbf{d}_s(\mathbf{x}_s, \bar{\mathbf{u}}, t) &= \Delta \mathbf{f}_s(\mathbf{x}_s, t) + \Delta \mathbf{g}_s(\bar{\mathbf{u}}, t) \\ &= \begin{bmatrix} \mathbf{0}_{4 \times 1} \\ \Delta \mathbf{M}_A^{-1} [\Delta \mathbf{D} + \Delta \mathbf{C}_A(\mathbf{v}_s)] \mathbf{v}_s \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{4 \times 1} \\ \Delta \mathbf{M}_A^{-1} \mathbf{T}_s \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix} \end{aligned} \quad (4.26)$$

are the system uncertainties in the hydrodynamic added mass and damping forces terms. The upper bound of $\Delta \mathbf{M}_A$, $\Delta \mathbf{C}_A(\mathbf{v}_s)$ and $\Delta \mathbf{D}$ can be obtained from (3.59), (3.60) and (3.63) respectively.

4.3. Horizontal and Vertical Plane Subsystem Models

From the ROV mass inertia, Centripetal and Corolis matrices, it can be seen that the nonlinear ROV motions are highly coupled, although the roll and pitch motions are shown to be bounded. To further reduce the motion coupling, it is suggested to adopt a decoupled (see Fig. 4.3) maneuver scheme: moving with a horizontal plane motion whereby the vehicle is first steered at constant forward speed along a straight line that passes through the desired target points at a certain vehicle orientation. At the end of this horizontal plane motion, the vehicle is switched to a vertical plane motion (see Fig. 4.3): it maintains a constant orientation (heading) and follows a straight line with a change in altitude.

Thus considering only few degrees of freedoms at a time decouples the motions. This reduces the number of controllable degree of freedoms involved in subsequent control system analysis and design. With that, the subsystems become actuated.

Figure 4.3: Horizontal (left) and vertical plane (right) motion

In summary, the ROV motions can be decoupled into two simple subsystems such as the:

- Horizontal plane subsystem: velocity states (u, v, r) and position states (x, y, ψ) .
- Vertical plane subsystem: velocity states (u, w, q) and position states (x, z, θ) .

These represent the ROV at different plane maneuvering conditions. In the literature, it is common to use decoupled subsystems as for the SIRENE ROV [103], NPS AUV II [104] and MARIUS vehicle [105].

The total mass inertial matrix can be written as:

$$\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A$$

$$= \begin{bmatrix} 93.8597 & 0 & 0 & 0 & 0 & 0 \\ 0 & 63.3000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 22.5490 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.4809 & -0.0002 & -0.1850 \\ 0 & 0 & 0 & -0.0002 & 3.3373 & 0.0006 \\ 0 & 0 & 0 & -0.1850 & 0.0006 & 7.2867 \end{bmatrix} \quad (4.27)$$

Extracting the row and column of the two subsystems DoF in (4.27), it gives the total mass inertia matrix of two subsystems as:

$$\mathbf{M}_h = \begin{bmatrix} 93.8597 & 0 & 0 \\ 0 & 63.3000 & 0 \\ 0 & 0 & 7.2867 \end{bmatrix} \quad (4.28)$$

$$\mathbf{M}_v = \begin{bmatrix} 93.8597 & 0 & 0 \\ 0 & 22.5490 & 0 \\ 0 & 0 & 3.3373 \end{bmatrix} \quad (4.29)$$

4.3.1 Nominal and Perturbed Horizontal Plane Models

The horizontal plane model describes the ROV dynamic during longitudinal motion that encompasses three DoFs such as: surge, sway and yaw. The roll, pitch and heave velocities are bounded as the ROV is following along the X-axis with z a constant (that is $w = \dot{w} = 0$) and at a constant surge velocity that defines the horizontal plane dynamic. The nominal horizontal plane dynamic can be written as:

$$(m - X_{\ddot{u}})\dot{u} + X_{\dot{u}}u + (Y_v - m)vr = \tau_x \quad (4.30a)$$

$$(m - Y_v)\dot{v} + Y_vv + (m - X_{\ddot{u}})ur = \tau_y \quad (4.30b)$$

$$(I_z - N_r)\dot{r} + N_rr + (-Y_vu + X_{\ddot{u}}u)v = \tau_\psi \quad (4.30c)$$

The dynamics in u and v are coupled with r respectively. The Kinematics equation in (3.23) becomes:

$$\dot{x} = u \cos \psi - v \sin \psi \quad (4.31a)$$

$$\dot{y} = u \sin \psi + v \cos \psi \quad (4.31b)$$

$$\dot{\psi} = r \quad (4.31c)$$

The dynamics of the horizontal plane (denoted by subscript 'h') subsystem as seen in (4.30a) to (4.30c) can be written in compact form:

$$\mathbf{M}_h \dot{\mathbf{v}}_h + \mathbf{C}_h(\mathbf{v}_h)\mathbf{v}_h + \mathbf{D}_h\mathbf{v}_h = \boldsymbol{\tau}_h \quad (4.32)$$

where $\boldsymbol{\eta}_h = [x \ y \ \psi]^T$, $\mathbf{v}_h = [u \ v \ r]^T$, $\boldsymbol{\tau}_h = \mathbf{T}_h \mathbf{F}_T \bar{\mathbf{u}} = [\tau_x \ \tau_y \ \tau_\psi]^T$ and

$$\mathbf{M}_h = \begin{bmatrix} m - X_{\ddot{u}} & 0 & 0 \\ 0 & m - Y_{\ddot{v}} & 0 \\ 0 & 0 & I_r - N_{\ddot{r}} \end{bmatrix}; \mathbf{C}_h(\mathbf{v}_h) = \begin{bmatrix} 0 & (Y_{\dot{v}} - m)r & 0 \\ 0 & 0 & (m - X_{\ddot{u}})u \\ 0 & -(Y_{\dot{v}} - X_{\ddot{u}})u & 0 \end{bmatrix};$$

$$\mathbf{D}_h = \begin{bmatrix} X_u & 0 & 0 \\ 0 & Y_v & 0 \\ 0 & 0 & N_r \end{bmatrix}; \mathbf{T}_h = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \sin \beta & -\sin \beta \\ \gamma & -\gamma & \alpha \sin \beta & -\alpha \sin \beta \end{bmatrix}$$

The kinematics equations of motion in the vertical plane (4.31a) to (4.31c) can be written as:

$$\dot{\boldsymbol{\eta}}_h = \mathbf{J}_h(\boldsymbol{\eta}_h) \mathbf{v}_h \quad (4.33)$$

where

$$\mathbf{J}_h(\boldsymbol{\eta}_h) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.34)$$

Equation (4.32) and (4.33) can be rewritten in state-space form:

$$\dot{\mathbf{x}}_h = \bar{\mathbf{f}}_h(\mathbf{x}_h, t) + \bar{\mathbf{g}}_h(\bar{\mathbf{u}}, t) \quad (4.35)$$

where $\mathbf{x}_h = [\boldsymbol{\eta}_h \ \mathbf{v}_h]^T$

$$\bar{\mathbf{f}}_h(\mathbf{x}_h, t) = \begin{bmatrix} \mathbf{J}_h(\boldsymbol{\eta}_h) \mathbf{v}_h \\ -\mathbf{M}_h^{-1}[\mathbf{C}_h(\mathbf{v}_h) + \mathbf{D}_h] \mathbf{v}_h \end{bmatrix}; \bar{\mathbf{g}}_h(\bar{\mathbf{u}}, t) = \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ [\bar{\mathbf{g}}_{h1} \ \bar{\mathbf{g}}_{h2} \ \bar{\mathbf{g}}_{h3}] \mathbf{T}_h \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix}; \quad (4.36)$$

$$\bar{\mathbf{g}}_{h1} = \begin{bmatrix} \frac{1}{m - X_{\ddot{u}}} & 0 & 0 \end{bmatrix}^T; \bar{\mathbf{g}}_{h2} = \begin{bmatrix} 0 & \frac{1}{m - Y_{\ddot{v}}} & 0 \end{bmatrix}^T; \bar{\mathbf{g}}_{h3} = \begin{bmatrix} 0 & 0 & \frac{1}{I_r - N_{\ddot{r}}} \end{bmatrix}^T$$

The corresponding perturbed model can be written in the form:

$$\dot{\mathbf{x}}_h = \bar{\mathbf{f}}_h(\mathbf{x}_h, t) + \bar{\mathbf{g}}_h(\bar{\mathbf{u}}, t) + \Delta \mathbf{d}_h(\mathbf{x}_h, \bar{\mathbf{u}}, t) \quad (4.37)$$

where

$$\begin{aligned}\Delta \mathbf{d}_h(\mathbf{x}_h, \bar{\mathbf{u}}, t) &= \Delta \mathbf{f}_h(\mathbf{x}_h, t) + \Delta \mathbf{g}_h(\bar{\mathbf{u}}, t) \\ &= \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \Delta \mathbf{M}_A^{-1} [\Delta \mathbf{D} + \Delta \mathbf{C}_A(\mathbf{v}_h)] \mathbf{v}_h \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \Delta \mathbf{M}_A^{-1} \mathbf{T}_h \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix}\end{aligned}$$

are the system uncertainties in the hydrodynamic added mass and damping forces terms. The upper bound of $\Delta \mathbf{M}_A, \Delta \mathbf{C}_A(\mathbf{v}_h)$ and $\Delta \mathbf{D}$ can be obtained from (3.59), (3.60) and (3.63) respectively.

4.3.2 Nominal and Perturbed Vertical Plane Models

The vertical plane model describes the ROV dynamic during altitudinal motion that involves three DoFs such as: surge, heave and pitch. Now, the roll, pitch and yaw velocities are bounded as the ROV is moving along the Z-axis with y a constant (i.e. $v = \dot{v} = 0$) defining the vertical plane motion dynamic. The nominal vertical plane dynamic for both the translational and rotational direction can be written as:

$$(m - X_{\ddot{u}})\dot{u} + X_{\dot{u}}u + (m - Z_{\dot{w}})wq = \tau_x \quad (4.38a)$$

$$(m - Z_{\dot{w}})\dot{w} + Z_w w + (X_{\ddot{u}} - m)uq = \tau_z \quad (4.38b)$$

$$(I_y - M_{\dot{q}})\dot{q} + M_q q + (Z_{\dot{w}} - X_{\ddot{u}})uw + (z_G - z_B)W \sin \theta = \tau_\theta \quad (4.38c)$$

The kinematics equation in (3.23) becomes:

$$\dot{x} = u \cos \theta + w \sin \theta \quad (4.39a)$$

$$\dot{z} = -u \sin \theta + w \cos \theta \quad (4.39b)$$

$$\dot{\theta} = q \quad (4.39c)$$

The dynamics of the vertical plane (denoted by subscript 'v') as seen in (4.38a)-(4.38c) can be written in compact form:

$$\mathbf{M}_v \dot{\mathbf{v}}_v + \mathbf{C}_v(\mathbf{v}_v) \mathbf{v}_v + \mathbf{D}_v \mathbf{v}_v + \mathbf{G}_v(\boldsymbol{\eta}_v) = \boldsymbol{\tau}_v \quad (4.40)$$

where

$\boldsymbol{\eta}_v = [x \quad z \quad \theta]^T$, $\mathbf{v}_v = [u \quad w \quad q]^T$, $\boldsymbol{\tau}_v = \mathbf{T}_v \mathbf{F}_T \bar{\mathbf{u}} = [\tau_x \quad \tau_z \quad \tau_\theta]^T$, and

$$\mathbf{M}_v = \begin{bmatrix} m - X_{\ddot{u}} & 0 & 0 \\ 0 & m - Z_{\ddot{w}} & 0 \\ 0 & 0 & I_y - M_{\ddot{q}} \end{bmatrix}, \mathbf{C}_v(\mathbf{v}_v) = \begin{bmatrix} 0 & (m - Z_{\dot{w}})q & 0 \\ 0 & 0 & (X_{\dot{u}} - m)u \\ 0 & (Z_{\dot{w}} - X_{\dot{u}})u & 0 \end{bmatrix},$$

$$\mathbf{D}_v = \begin{bmatrix} X_u & 0 & 0 \\ 0 & Z_w & 0 \\ 0 & 0 & M_q \end{bmatrix}, \mathbf{T}_v = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & \cos \beta & \cos \beta \\ -\varepsilon & -\varepsilon & \alpha \cos \beta & -\alpha \cos \beta \end{bmatrix}$$

$$\mathbf{G}_v = \begin{bmatrix} 0 \\ 0 \\ (z_G - z_B)W \sin \theta \end{bmatrix}$$

The kinematics equations of motion in the vertical plane (4.39a) to (4.39c) can be written as:

$$\dot{\boldsymbol{\eta}}_v = \mathbf{J}_v(\boldsymbol{\eta}_v) \mathbf{v}_v \quad (4.41)$$

where

$$\mathbf{J}_v(\boldsymbol{\eta}_v) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.42)$$

Equation (4.40) and (4.41) can be rewritten in state-space form:

$$\dot{\mathbf{x}}_v = \bar{\mathbf{f}}_v(\mathbf{x}_v, t) + \bar{\mathbf{g}}_v(\bar{\mathbf{u}}, t) \quad (4.43)$$

where $\mathbf{x}_v = [\boldsymbol{\eta}_v \quad \mathbf{v}_v]^T$;

$$\bar{\mathbf{f}}_v(\mathbf{x}_v, t) = \begin{bmatrix} \mathbf{J}_v(\boldsymbol{\eta}_v) \mathbf{v}_v \\ -\mathbf{M}_v^{-1}[(\mathbf{C}_v(\mathbf{v}_v) + \mathbf{D}_v) \mathbf{v}_v + \mathbf{G}_v(\boldsymbol{\eta}_v)] \end{bmatrix};$$

$$\bar{\mathbf{g}}_v(\bar{\mathbf{u}}, t) = \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ [\bar{\mathbf{g}}_{v1} \quad \bar{\mathbf{g}}_{v2} \quad \bar{\mathbf{g}}_{v3}] \mathbf{T}_v \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix}$$

$$\bar{\mathbf{g}}_{v1} = \begin{bmatrix} \frac{1}{m - X_{\dot{u}}} & 0 & 0 \end{bmatrix}^T; \bar{\mathbf{g}}_{v2} = \begin{bmatrix} 0 & \frac{1}{m - Z_{\dot{w}}} & 0 \end{bmatrix}^T; \bar{\mathbf{g}}_{v3} = \begin{bmatrix} 0 & 0 & \frac{1}{I_y - M_{\dot{q}}} \end{bmatrix}^T \quad (4.44)$$

The corresponding perturbed model can be written in the form:

$$\dot{\mathbf{x}}_v = \bar{\mathbf{f}}_v(\mathbf{x}_v, t) + \bar{\mathbf{g}}_v(\bar{\mathbf{u}}, t) + \Delta \mathbf{d}_v(\mathbf{x}_v, \bar{\mathbf{u}}, t) \quad (4.45)$$

where

$$\begin{aligned} \Delta \mathbf{d}_v(\mathbf{x}_v, \bar{\mathbf{u}}, t) &= \Delta \mathbf{f}_v(\mathbf{x}_v, t) + \Delta \mathbf{g}_v(\bar{\mathbf{u}}, t) \\ &= \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \Delta \mathbf{M}_A^{-1} [\Delta \mathbf{D} + \Delta \mathbf{C}_A(\mathbf{v}_v)] \mathbf{v}_v \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \Delta \mathbf{M}_A^{-1} \mathbf{T}_v \mathbf{F}_T \bar{\mathbf{u}} \end{bmatrix} \end{aligned}$$

are the system uncertainties in the hydrodynamic added mass and damping forces terms. The upper bound of $\Delta \mathbf{M}_A$, $\Delta \mathbf{C}_A(\mathbf{v}_v)$ and $\Delta \mathbf{D}$ can be obtained from (3.59), (3.60) and (3.63) respectively.

4.4. RRC ROV Subsystems Property

The properties of the station-keeping, horizontal and vertical plane model such as uniqueness and existence, continuous dependence of parameters, energy and passivity, stabilizability, strongly accessibility (SA) and small-time locally controllability (STLC) are analyzed.

As observed in the Section 4.3.2, the vertical plane subsystem can be underactuated if the control input $\tau_\theta = 0$ and the number of control inputs is less than the number of DoF. This violated the Brockett's necessary condition [2] that states an underactuated system is not asymptotically stabilizable to a desired equilibrium points using any time-invariant smooth feedback. However, from [5], if the system is strongly accessible (SA) and small-time locally controllable (STLC) at any equilibrium, the time-invariant discontinuous feedback laws [106,46] can be used. It guarantees exponential convergence towards an equilibrium point. Alternatively, the STLC subsystems can be locally stabilized by a time-varying smooth feedback to an equilibrium point (in a Lyapunov sense). Hence, the SA and STLC are important properties to verify for the underactuated subsystem.

To determine the above-mentioned properties, the RDA package in Fig. A1.10 (in Appendix A1) written in MATLABTM scripting language and graphical block diagram simulation, SIMULINKTM with virtual environment interface is used. This provides a numerical solution (that is less complex to solve analytically) to the first-order differential equation as seen in the state-space models in (4.20), (4.35) and (4.43). With the aid of MAPLETM, it provides a symbolic method of solving the differential equation in Lie-bracket.

4.4.1 ROV States Coupling

As seen in the Corolis and centripetal matrix of the ROV, the ROV is indeed nonlinear having cross-coupling in the velocities. Recalled the full-order nonlinear state equation in (3.24a), the nonlinearity and coupling effect come from the term $\mathbf{M}^{-1}[\mathbf{C}(\mathbf{v}) + \mathbf{D}]\mathbf{v}$ (in short **MCD**). In order to see the level of states coupling in the ROV, the sum of each off diagonal element of the **MCD** against the sum of its diagonal element for each row (see Table 4.2) for a typical ROV's operating velocity of $\mathbf{v} \in [0, 0.2]$ is computed. As observed, the system is highly coupled in states as the sum of the off diagonal is larger than the diagonal terms in the same row.

Row number	Row Sum of diagonal element	Row Sum of off-diagonal element
1	0.032	0.38
2	0.025	0.38
3	0.020	0.38
4	0.027	7.53
5	0.243	7.69
6	0.011	4.97

Table 4.2: Row sum of diagonal element vs diagonal term

4.4.2 ROV Uniqueness and Existence

The uniqueness and existence [107,119] of the ODEs can be verified by the Lipschitz condition in (4.46). For example, the ODEs with given initial condition for the station-keeping model at a given input $\bar{\mathbf{u}}$ is written as:

$$\dot{\mathbf{x}}_s = \bar{\mathbf{f}}_s(\mathbf{x}_s, t) + \bar{\mathbf{g}}_s(\bar{\mathbf{u}}) = \bar{\mathbf{f}}_s(\mathbf{x}_s, \bar{\mathbf{u}}, t), \quad \mathbf{x}_s(t_0) = \mathbf{x}_{s0} \quad (4.46)$$

for time interval $[t_0, t_f]$. To verify that (4.46) have a unique solution, it is sufficient that the right-hand side of (4.46) need to satisfy the following Lipschitz condition (where $L > 0$ is called the Lipschitz constant and can be varied for different subsystems):

$$\|\bar{\mathbf{f}}_s(\mathbf{x}_{s1}, t) - \bar{\mathbf{f}}_s(\mathbf{x}_{s2}, t)\|_2 \leq L \|\mathbf{x}_{s1} - \mathbf{x}_{s2}\|_2 \quad (4.47)$$

for all $(\mathbf{x}_{s1}, \bar{\mathbf{u}})$ and $(\mathbf{x}_{s2}, \bar{\mathbf{u}})$ from a neighborhood of a solution $\mathbf{x}_s : [t_0, t_f] \mapsto \mathfrak{R}^n$ of (4.46). Notice that the term $\bar{\mathbf{g}}_s(\bar{\mathbf{u}})$ does not appear in (4.47) since it is a constant and is independent of $\mathbf{x}_s$ for $[t_0, t_f]$. The equation (4.47) can be rewritten as:

$$\left\| \frac{\partial \bar{\mathbf{f}}_s(\mathbf{x}_s, t)}{\partial \mathbf{x}_s} \right\|_2 \leq L \quad (4.48)$$

where $\left\| \frac{\partial \bar{\mathbf{f}}_s(\mathbf{x}_s, t)}{\partial \mathbf{x}_s} \right\|_2 = \left(\lambda_{\max} \left(\left(\frac{\partial \bar{\mathbf{f}}_s(\mathbf{x}_s, t)}{\partial \mathbf{x}_s} \right)^T \frac{\partial \bar{\mathbf{f}}_s(\mathbf{x}_s, t)}{\partial \mathbf{x}_s} \right) \right)^{1/2}$ refers to the matrix norm, $\frac{\partial \bar{\mathbf{f}}_s(\mathbf{x}_s, t)}{\partial \mathbf{x}_s}$ is the Jacobian matrix and $L > 0$ (can vary for different subsystems). Hence if the maximum eigenvalue of $\frac{\partial \bar{\mathbf{f}}_s(\mathbf{x}_s, t)}{\partial \mathbf{x}_s}$ is bounded as in (4.48), then the ODE in (4.46) has a unique solution.

As observed, the Jacobian matrix in (4.48) contains function of states $\mathbf{x}_s$. To determine the maximum eigenvalue, the $\mathbf{x}_s$ ranges from 0.05 to 0.2 (maximum) and angle not exceeding 30 degrees at input $\bar{\mathbf{u}}_{\max} = 70$ volts (maximum) are substituted into (4.48). An example of equation (4.48) being realized is given by MAPLETM (see Appendix B1). In the MAPLETM, the following results at $t_f = 100$ seconds are tabulated in Table 4.3. As shown, the Lipschitz constant, L is positive for horizontal and vertical plane model. Hence, the subsystems including the full-order system have unique solution.

4.4.3 ROV Continuous Dependence on Parameters

Another essential factor in the validity of any mathematical model is the continuous dependence [107,119] of the parameters $(\mathbf{x}_s, \bar{\mathbf{u}})$. In this section, the sufficient conditions for continuous dependence on initial conditions and input $\bar{\mathbf{u}}$ are validated. For example, using the same station-keeping model in (4.46), and let $\mathbf{x}_s(t, \bar{\mathbf{u}}) : [t_0, t_f] \mapsto \mathfrak{R}^n$ be a solution of (4.46) with $\bar{\mathbf{u}}$.

For some $d > 0$, let $X^d = \{\bar{\mathbf{x}}_s \in \mathfrak{R}^n : \exists t \in [t_0, t_f] : |\bar{\mathbf{x}}_s - \mathbf{x}_s| < d\}$. The solution $\mathbf{x}_s(t, \bar{\mathbf{u}})$ is continuous on $\{(t, \bar{\mathbf{u}}) = [t_0, t_f] \times (0, \bar{\mathbf{u}}_{\max})$ if there exists $d_1 \in (0, d)$ such that the following Conditions (a) to (c) are satisfied:

(a) there exists $\varepsilon_f \in \mathfrak{R}$ such that:

$$|\bar{\mathbf{f}}_s(\bar{\mathbf{x}}_s, \bar{\mathbf{u}}, t)| \leq \varepsilon_f, \quad \forall \bar{\mathbf{x}}_s \in X^d, \quad t_0 \leq t \leq t_f, \quad \bar{\mathbf{u}} \in (0, \bar{\mathbf{u}}_{\max}); \quad (4.49)$$

(b) for every $\varepsilon_x > 0$ there exist $\delta_x > 0$ such that:

$$|\bar{\mathbf{x}}_s(0, \bar{\mathbf{u}}_1) - \bar{\mathbf{x}}_s(0, \bar{\mathbf{u}}_2)| \leq \varepsilon_x, \quad \forall \bar{\mathbf{u}}_1, \bar{\mathbf{u}}_2 \in (0, \bar{\mathbf{u}}_{\max}) : |\bar{\mathbf{u}}_1 - \bar{\mathbf{u}}_2| \leq \delta_x \quad (4.50a)$$

$$|\bar{\mathbf{f}}_s(\bar{\mathbf{x}}_s, \bar{\mathbf{u}}_1, t) - \bar{\mathbf{f}}_s(\bar{\mathbf{x}}_s, \bar{\mathbf{u}}_2, t)| \leq \varepsilon_x, \quad \forall \bar{\mathbf{u}}_1, \bar{\mathbf{u}}_2 \in (0, \bar{\mathbf{u}}_{\max}) : |\bar{\mathbf{u}}_1 - \bar{\mathbf{u}}_2| \leq \delta_x, \quad \bar{\mathbf{x}}_s \in X^d \quad (4.50b)$$

(c) there exists $L \in \mathfrak{R}$ such that:

$$\|\bar{\mathbf{f}}_s(\bar{\mathbf{x}}_{s1}, t) - \bar{\mathbf{f}}_s(\bar{\mathbf{x}}_{s2}, t)\|_2 \leq L \|\bar{\mathbf{x}}_{s1} - \bar{\mathbf{x}}_{s2}\|_2, \quad \forall \bar{\mathbf{u}}_1, \bar{\mathbf{u}}_2 \in (0, \bar{\mathbf{u}}_{\max}), \quad \forall \bar{\mathbf{x}}_{s1}, \bar{\mathbf{x}}_{s2} \in X^d, \quad t_0 \leq t \leq t_f \quad (4.51)$$

The Condition (a) means continuous dependence of equations and initial conditions on input $\bar{\mathbf{u}}$. Condition (b) simply bounds $\bar{\mathbf{f}}_s$ uniformly. Finally Condition (c) is the similar Lipschitz continuity requirement as seen in (4.47) in the previous section.

Modes	Item	$\mathbf{x}_s(t_f, \bar{\mathbf{u}}_{\max})$			Cond	Cond (b)		Cond
		$[u, v, w, p, q, r]$	$[x, y, z]$	$[\phi, \theta, \psi]$ in degree	(a) $\mathcal{E}_f$	$\mathcal{E}_x$	$\mathcal{D}_x$	(c) L
Full Order	1	0.05	0.05	5	1.56			4.94
	2	0.10	0.10	10	1.79			4.90
	3	0.15	0.15	20	2.56			4.83
	4	0.20	0.20	30	3.38	3.3	70	4.73
						3.4	70	
Station-keeping	1	0.05	0.05	5	4.64			1.21
	2	0.10	0.10	10	4.65			1.68
	3	0.15	0.15	20	4.66			2.26
	4	0.20	0.20	30	4.68	0.6	70	2.83
						4.7	70	
Horizontal Plane	1	0.05	0.05	5	1.49			1.21
	2	0.10	0.10	10	0.19			1.69
	3	0.15	0.15	20	0.34			2.26
	4	0.20	0.20	30	0.52	0.5	70	2.84
						1.6	70	
Vertical Plane	1	0.05	0.05	5	2.21			2.34
	2	0.10	0.10	10	2.17			3.55
	3	0.15	0.15	20	2.19			4.88
	4	0.20	0.20	30	2.40	1.7	70	6.28
						2.4	70	

Table 4.3: Continuous dependence on parameters and Lipschitz condition for all subsystems

As observed, the inequalities in (4.49) to (4.51) contains function of states $\mathbf{x}_s$ and constant $\bar{\mathbf{u}}$. To determine the constant in (4.49) to (4.51), the $\mathbf{x}_s$ ranges from 0.05 to 0.2 (maximum) and angle not exceeding 30 degrees at input $\bar{\mathbf{u}}_{\max} = 70$ volts (maximum) are used. The MAPLETM (see Appendix B1) is used to determine whether the Condition (a) to (c) are satisfied. The following results at $t_f = 100$ seconds are tabulated in Table 4.3. As seen in Table 4.3, the

constants $(\varepsilon_f, \varepsilon_x, \delta_x, L)$ are all positive for all the subsystems. Therefore, the systems are all continuous with respect to the parameters.

4.4.4 ROV State Controllability

Controllability [108] is an important property for control system design. A linear controllable system is defined as a system that can be steered to any state from the zero initial state. But for the nonlinear systems, the Lie-bracket (as shown in Appendix B2) will be useful. However, the fundamental of the state controllability to examine whether the ROV is controllable for a given input from an initial state can still be used. This is done by examining the full-order ROV's step response (for both position and velocity states) using the respective thrust input ranging from 0 to 70 (maximum). The Euclidean norm of the position and velocity states at each input configurations are plotted as shown in Fig. 4.4 to 4.5.

- $u_1(t)$, $u_2(t)$ for surge direction
- $u_3(t)$, $-u_4(t)$ for sway direction
- $u_3(t)$, $u_4(t)$ for heave direction
- $u_1(t)$, $-u_2(t)$, $u_3(t)$, $-u_4(t)$ for yaw direction

Figure 4.4: Position and velocities Euclidean norm in surge (left) and sway direction (right)

With the thrust input, as seen in Fig. 4.4 and 4.5, Euclidean norm of the roll and pitch angle and velocities are small as compared the rest of the DoFs. This implies that the input has the least effect on these states and hence they are uncontrollable in some sense. Thus, the ROV is

controllable for only four DoF (that is surge, sway, heave and yaw direction) and that the ROV is state controllable in the horizontal plane and station-keeping mode but not the vertical mode.

Figure 4.5: Position and velocities Euclidean norm in heave (left) and yaw direction (right)

Alternatively, the state controllability of the nonlinear system such as vertical plane subsystem can be determined using the repeated Lie-bracket [107,119]. As seen in Appendix B2, the nonlinear system is controllable if the rank of the following matrix:

$$\text{rank} [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \quad \mathbf{A}^3\mathbf{B} \quad \dots \quad \mathbf{A}^{n-1}\mathbf{B}] = n \quad (4.52)$$

where $\mathbf{B} = \bar{\mathbf{g}}_v$, $\mathbf{AB} = -[\bar{\mathbf{f}}_v \quad \bar{\mathbf{g}}_v]$, $\mathbf{A}^2\mathbf{B} = [\bar{\mathbf{f}}_v \quad [\bar{\mathbf{f}}_v \quad \bar{\mathbf{g}}_v]]$, $\mathbf{A}^3\mathbf{B} = -[\bar{\mathbf{f}}_v \quad [\bar{\mathbf{f}}_v \quad [\bar{\mathbf{f}}_v \quad \bar{\mathbf{g}}_v]]]$ and the rank of the subsystem must satisfy the order of the subsystems as follows: full-order system, $n = 12$, station-keeping model, $n = 8$, horizontal and vertical plane model, $n = 6$.

As indicated, the full-order and vertical subsystems are not controllable. For underactuated systems, locally strongly accessible (SA) and small-time locally controllable (STLC) as seen in Section 4.4.8 and 4.4.9 are required. From (4.52), it is obvious that the partial differentiation used in the Lie-bracket contains function of the position and velocity states variable. The controllability matrix at various conditions is determined as shown below.

(a) $\theta = 0^\circ$, $u = w = 0$ m/s, $q = 0$ rad/s (minimum),

(b) $\theta = 5^\circ$ to $\theta = 30^\circ$, $u = v = w = 0.05$ m/s to 0.20 m/s, $p = q = r = 0.05$ rad/s to 0.2 rad/s (maximum).

The details of each matrix can be seen in Appendix B3. The rank of the controllability matrix for the full-order model is less than 12. Similar observation can be seen in the vertical subsystem. Thus, the results coincide with the one obtained from the time-domain method. This implies that the ROV is state controllable in horizontal plane and station-keeping mode but not in the vertical mode (and definitely not for full-order model since the roll and pitch are not fully controllable).

4.4.5 ROV Energy

The ROV's energy can be derived as shown below. As the ROV's is neutrally buoyant [94] with no unstable restoring force affect the ROV motion, the ROV system is shown to exhibits only kinetic energy.

For example, using the station-keeping model in body-fixed frame as shown in (4.20), the Lyapunov function V can be chosen to represent the system's total mechanical energy consisting of kinetic and potential energy [94]. Consider the Lyapunov function candidate:

$$V(\mathbf{v}_s, \boldsymbol{\eta}_s) = \frac{1}{2} \mathbf{v}_s^T \mathbf{M}_s \mathbf{v}_s + \int_0^{\boldsymbol{\eta}_s} \mathbf{G}_f^T(z) dz \quad (4.53)$$

Since the ROV's submerged in water is neutrally buoyant that is $mg = \rho g \nabla$ (where ∇ is the volume displaced by the submerged ROV in m^3 , ρ is the density of the seawater in kg/m^3 , m is the mass of the ROV in kg), the effect of potential energy become negligible. Hence (4.53) becomes:

$$V(\mathbf{v}_s) = \frac{1}{2} \mathbf{v}_s^T \mathbf{M}_s \mathbf{v}_s \quad (4.54)$$

The ROV system only exhibits kinetic energy and this applies to all subsystems as well. To move the ROV, the input voltage of 70 volts (maximum) is applied to the designated thrusters for each plane maneuverings. The energy can be computed by running the SIMULINK™

model as shown in Fig. A1.11 in Appendix A1 for $0 < t \leq t_f (= 300 \text{ seconds})$. As observed in Fig. 4.6, the roll DoF of the full-order system is negative at transient response ($t \leq 3 \text{ seconds}$) but is small ($= -3 \times 10^{-6} \text{ kg.m}^2/\text{s}^2$) and settled to positive steady state value as time increases. However at subsystem level, all energy is positive as shown in Fig. 4.6 and 4.7.

Figure 4.6: Energy for each DOF – full-order (top) and station-keeping (bottom) subsystem

Figure 4.7: Energy for each DOF –vertical (top) and horizontal (bottom) subsystem

4.4.6. ROV Passivity

For example, the full-order model is said to be passive [94] if

$$\dot{V} \leq \mathbf{Y}^T(t) \boldsymbol{\tau}_t(t) - \mathbf{G}(t) \quad \forall t > 0 \quad (4.55)$$

with the total energy of the system $V \geq 0$ and $\dot{\mathbf{M}} - 2\mathbf{C}(\mathbf{v})$ is skew-symmetric. Here $\mathbf{Y}^T(t)$ refers to the system output, $\mathbf{G}(t)$ is the power loss function and $\boldsymbol{\tau}_t(t)$ is the input vector.

To prove that (4.54) is satisfied for our ROV, the derivative of V is semi-negative definite need to be shown. By differentiating the energy function in (4.54) that is $V > 0, \forall \mathbf{v} \neq 0$ along the system dynamics, it yields:

$$\begin{aligned}
 \dot{V} &= \mathbf{v}^T \dot{\mathbf{M}} \mathbf{v} + \frac{1}{2} \mathbf{v}^T \dot{\mathbf{M}} \mathbf{v} \\
 &= \mathbf{v}^T [\boldsymbol{\tau} - \mathbf{C}(\mathbf{v})\mathbf{v} - \mathbf{D}\mathbf{v}] \dot{\mathbf{v}} + \frac{1}{2} \mathbf{v}^T \dot{\mathbf{M}} \mathbf{v} \\
 &= \frac{1}{2} \mathbf{v}^T [\dot{\mathbf{M}} - 2\mathbf{C}(\mathbf{v})]\mathbf{v} + \mathbf{v}^T \boldsymbol{\tau} - \mathbf{v}^T \mathbf{D}\mathbf{v} \\
 &= \mathbf{v}^T \boldsymbol{\tau} - \mathbf{v}^T \mathbf{D}\mathbf{v} \Rightarrow \|\dot{V}\|_2 \leq \|\mathbf{v}^T\|_2 \|\boldsymbol{\tau}\|_2 - \|\mathbf{v}^T\|_2 \|\mathbf{D}\|_2 \|\mathbf{v}\|_2, \forall t \geq 0
 \end{aligned} \tag{4.56}$$

where $\|\cdot\|_2$ is a Euclidean norm.

Comparing this result with (4.55), the $\mathbf{v}^T \mathbf{D}\mathbf{v}$ corresponds to the power loss function $\mathbf{G}(t)$ for the system can be seen. The term $\int_0^{t_f} \mathbf{G}(t) dt$ can be interpreted as a power loss in the system during the period of $[0, t_f]$ that implies the system is dissipative. While the system input vector $\boldsymbol{\tau}$ corresponds to $\boldsymbol{\tau}_t(t)$ and the system output vector $\mathbf{v}$ corresponds to $\mathbf{Y}(t)$. Hence the underwater ROV system is passive if the inequality in (4.56) is satisfied for $[0, t_f]$.

To verify that the passivity in (4.56) is satisfied, the $\|\dot{V}\|$ (left hand side) is plotted against the $\|\mathbf{v}^T\|_2 \|\boldsymbol{\tau}\|_2 - \|\mathbf{v}^T\|_2 \|\mathbf{D}\|_2 \|\mathbf{v}\|_2$ (right-hand side) is computed by running the SIMULINKTM model in Fig. A1.11 in Appendix A1. To move the ROV, the input voltage of 70 volts (maximum) is applied to the designated thrusters for each plane maneuverings for $0 < t \leq t_f (= 300 \text{ seconds})$. As observed in Fig. 4.8 and Fig. 4.9, all subsystems including the

full-order system is passive except for the roll DoF. However the roll's magnitude is small as compared to the rest and it ranges from 2.8×10^{-8} to 3.5×10^{-6} kg.m²/s³.

Another observation on the asymptotical stability of the system can be seen in the $\dot{V}$ – plot in Fig. 4.8 and 4.9. The $\dot{V}$ for the roll DoF is not all negative definite especially at the transient response ($t \leq 3$ seconds) but they are small in magnitude as mentioned. Hence it proves that if the V is not positive definite (as shown in Fig. 4.6), the system is not asymptotically stable (in Fig. 4.8) during the transient only but settled to negative steady state value as time increases. On other hands, the sway DoF is decayed to negative steady state value as time increases.

Thus the open-loop subsystem including the full-order system is not asymptotically stable in Lyapunov sense. This phenomenon can be seen in the whirling of the ROV in Fig. 4.1 and 4.2.

Figure 4.8: $\dot{V}$ and $v^T \tau - v^T Dv$ for each DOF – full-order (left) and station-keeping (right) subsystem

Figure 4.9: $\dot{V}$ and $\mathbf{v}^T \boldsymbol{\tau} - \mathbf{v}^T \mathbf{D} \mathbf{v}$ for each DOF –vertical (left) and horizontal (right) subsystem

4.4.7 ROV Stabilizability

As shown in the proof in Appendix B4, the vertical plane model is stabilizable [11,12] at the equilibrium point as the RRC ROV has a non-zero gravitational field, $\mathbf{G}_v(\boldsymbol{\eta}_v)$ at the unactuated dynamic, as shown below:

$$\begin{bmatrix} \tau_x \\ \tau_z \\ -W(z_G - z_B) \sin \theta \end{bmatrix} = \begin{bmatrix} \alpha^1 \\ \alpha^2 \\ \beta \end{bmatrix} \quad (4.57)$$

However, it is a necessary condition but not sufficient condition for the ROV to be asymptotically stabilizable at the equilibrium point. For example, if $\theta = 90^\circ$, the pitch motion is not stabilizable as the gravitational field in (4.57) become zero.

Hence $\tau_\theta = 0$ occurred in the vertical plane, an underactuated subsystem evolved and thus no time-invariant continuous feedback can be used to stabilize the vertical subsystem at the equilibrium points (known as the Brockett Stabilization Theorem). Nevertheless, with the strongly accessible (SA) and small-time locally controllable (STLC) properties satisfied as shown in Section 4.4.8 and 4.4.9, the subsystem can be locally stabilized by a time-varying smooth feedback to an equilibrium point.

4.4.8. ROV Strongly Accessible

To examine the locally SA[5,19] and STLC[109] of the vertical plane subsystem, the Lie-bracket approach given in Appendix B3 is used. The vertical plane dynamic of the ROV described by (4.43) is locally SA[5,19] if the rank condition of the following vector fields $\bar{\mathbf{g}}_{v1}, \bar{\mathbf{g}}_{v2}, [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v1}], [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v2}], [\bar{\mathbf{g}}_{v2} [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v1}]], [\bar{\mathbf{g}}_{v2} [[\bar{\mathbf{g}}_{v1} \ \bar{\mathbf{f}}_v] \ \bar{\mathbf{f}}_v]]$ span a six-dimensional space at every $(\boldsymbol{\eta}_v, \mathbf{v}_v) \in \mathcal{R}^6$.

By computing the Lie bracket of $\bar{\mathbf{g}}_{v1}, \bar{\mathbf{g}}_{v2}, [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v1}], [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v2}], [\bar{\mathbf{g}}_{v2} [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v1}]]$, and $[\bar{\mathbf{g}}_{v2} [[\bar{\mathbf{g}}_{v1} \ \bar{\mathbf{f}}_v] \ \bar{\mathbf{f}}_v]]$, the vector fields of these Lie-brackets becomes function of the states as shown in (4.58) and (4.59). For example,

$$\begin{aligned} [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v1}] &= \frac{\partial \bar{\mathbf{g}}_{v1}}{\partial \mathbf{x}_v} \bar{\mathbf{f}}_v - \frac{\partial \bar{\mathbf{f}}_v}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v1} \\ &= \begin{bmatrix} \frac{\cos \theta}{m - X_u} & -\frac{\sin \theta}{m - X_u} & 0 & -\frac{X_u}{(m - X_u)^2} & -\frac{(X_u - m)q}{(m - X_u)(m - Y_v)} & -\frac{(Z_w - X_u)w}{(I_y - M_q)(m - X_u)} \end{bmatrix}^T \end{aligned} \quad (4.58)$$

$$\begin{aligned} [\bar{\mathbf{f}}_v \ \bar{\mathbf{g}}_{v2}] &= \frac{\partial \bar{\mathbf{g}}_{v2}}{\partial \mathbf{x}_v} \bar{\mathbf{f}}_v - \frac{\partial \bar{\mathbf{f}}_v}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v2} \\ &= \begin{bmatrix} \frac{\sin \theta}{m - Y_v} & \frac{\cos \theta}{m - Y_v} & 0 & -\frac{(m - Z_w)q}{(m - X_u)(m - Y_v)} & -\frac{Z_w}{(m - Y_v)^2} & -\frac{(Z_w - X_u)u}{(I_y - M_q)(m - Y_v)} \end{bmatrix}^T \end{aligned} \quad (4.59)$$

Since the Lie-brackets are functions of states variable as seen in the above example, the Lie-bracket at two extreme conditions is to be determined as:

(a) $\theta = 0^\circ, u = w = 0 \text{ m/s}, q = 0 \text{ rad/s}$ (minimum),

(b) $\theta = 5^\circ \text{ to } \theta = 30^\circ, u = v = w = 0.05 \text{ m/s to } 0.20 \text{ m/s}, p = q = r = 0.05 \text{ rad/s to } 0.2 \text{ rad/s}$ (maximum).

From the Lie-brackets computation in the Appendix B3, the Lie brackets span all DoFs and have a rank equals to six. Hence, the vertical plane subsystem at both condition (a) and (b) are SA.

4.4.9. ROV Small-Time Locally Controllable

Besides the SA of the vertical plane subsystem, the STLC [109, Corollary 7.2] of the subsystem need to be determined. The underactuated ROV dynamics described by (4.43) is said to be STLC at any equilibrium if it satisfies the Lie algebra rank condition such that the bracket $\bar{\mathbf{f}}_v$ vanishes at any equilibrium points and the bracket

$$[\bar{\mathbf{g}}_{v1} [\bar{\mathbf{f}}_v \quad \bar{\mathbf{g}}_{v1}]] = \frac{\partial}{\partial \mathbf{x}_v} \left(-\frac{\partial \bar{\mathbf{f}}_v}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v1} \right) \bar{\mathbf{g}}_{v1} \quad \text{and} \quad [\bar{\mathbf{g}}_{v2} [\bar{\mathbf{f}}_v \quad \bar{\mathbf{g}}_{v2}]] = \frac{\partial}{\partial \mathbf{x}_v} \left(-\frac{\partial \bar{\mathbf{f}}_v}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v2} \right) \bar{\mathbf{g}}_{v2} \text{ are identical zero}$$

vector fields.

As seen in Appendix B3, computation of the Lie-bracket contains functions of both the position and velocity states variable, the Lie-bracket at two extreme conditions is determined as:

(a) $\theta = 0^\circ, u = w = 0 \text{ m/s}, q = 0 \text{ rad/s}$ (minimum),

(b) $\theta = 5^\circ \text{ to } \theta = 30^\circ, u = v = w = 0.05 \text{ m/s to } 0.20 \text{ m/s}, p = q = r = 0.05 \text{ rad/s to } 0.2 \text{ rad/s}$ (maximum).

For example, the case (a) yields:

$$\bar{\mathbf{f}}_v = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \quad (4.60)$$

$$[\bar{\mathbf{g}}_{v1} [\bar{\mathbf{f}}_v \quad \bar{\mathbf{g}}_{v1}]] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \quad (4.61)$$

$$[\bar{\mathbf{g}}_{v2} [\bar{\mathbf{f}}_v \quad \bar{\mathbf{g}}_{v2}]] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \quad (4.62)$$

Similar results for case (b) can be obtained. Hence, the system is STLC the equilibrium conditions since the Lie-bracket evaluated at these conditions is zero. Thus, the STLC results imply the existence of piecewise analytic feedback laws [109], that could asymptotically stabilize the closed-loop system at the equilibrium point.

4.5. Summary

The unactuated dynamics (involving the roll and the pitch motions) were shown to be asymptotically stable. These simplify the use of decoupled models for station-keeping, horizontal and vertical plane maneuvering, which were defined. The roll and pitch dynamic are left uncontrolled and thus the ROV becomes an actuated system with respect to the remaining DoFs. The nonlinear behavior of those actuated subsystems (station-keeping and horizontal plane mode) such as the uniqueness, existence, and continuous dependency of parameters were demonstrated and instances of their behavior were verified using the RDA package written in MATLABTM scripting language and graphical block diagram simulation, SIMULINKTM. In addition, the total energy and passivity were determined. This was followed by demonstrating the state controllability using the Lie-bracket methods and Euclidean norm. As the vertical plane subsystem is underactuated, properties such as stabilizability, strongly accessibility (SA) and small-time locally controllability (STLC)) were satisfied so that an analytic feedback laws can be designed in Chapter 5.

The subsystem models are perturbed by the hydrodynamic uncertainties. This implies that the controllers must be robust in order to handle these perturbations during the stabilization and position tracking operation. Hence, the subsequent Chapter 5 involves the design of a robust controller in a cascaded system for simultaneous ROV's position tracking and stabilization application. This is followed by Chapter 6 using the switching methodology on the cascaded system to improve: (a) the percent of thrust utilization; (b) robustness and; (c) steady-state performance of the cascaded system.

Chapter 5

Cascaded Control System Design

Due to the nature of pipeline inspection that involves both position tracking and hovering at points on the pipeline (to locate the actual location of pipe's leakage or crack), a control system structure that is able to perform both tracking and stabilization at the same time is needed. As observed from Fig. 4.2, the ROV in open-loop condition exhibits a whirling motion whereby the velocity u , v and w (and its corresponding position x , y , z) are not asymptotically stable at zero equilibrium. This is due to the added mass component in the ROV's Coriolis and centripetal matrix that create a cross coupling effect on its motion. To address this, the cascaded control system to perform simultaneous stabilization (using the inner loop) and position tracking (using outer loop) is proposed.

One of the applications of simultaneous stabilization and tracking is given by Jiang *et al* [65] using global output feedback with backstepping method on the ODIN ROV. This method has a restriction that the ROV velocity can be conservative and the controller design complex, as the solution of partial differential equations are required in the control law. It solved only the horizontal plane motion, exploiting the three-plane symmetrical property of the ODIN. The control system design for full six-DoF operation becomes more involved as the ROV dynamics are nonlinear, highly coupled in motion, underactuated, susceptible to hydrodynamic uncertainties. In particular, performance can suffer significantly when the vehicle executes nonlinear and modes-couplings maneuvers.

In addition, the dynamic equations given in [62,65] did not take into account hydrodynamic uncertainties. Although, in the paper, the observer and control law can be modified to account for the inaccurate knowledge of the ODIN parameters due to the hydrodynamic uncertainties in the vehicle dynamic equations, the back stepping control law cannot overcome this imprecise knowledge in the dynamic equations. This apparent lack of robustness is due to the

fact that the hydrodynamic uncertainties cannot be compensated by the ODIN forward speed under the proposed backstepping controller.

Lastly, due to measurement noise in the output measurement and the quadratic cross terms of the unmeasured velocities appearing in the Coriolis and centripetal matrix, the design of the observer for velocities estimation become difficult. Hence a nonlinear observer design [62,65] is required. In this method, a coordinate transformation to earth-fixed frame cancels the quadratic cross terms before the observer is designed. Furthermore, the observer design approach solves the problem in the horizontal plane without considering the inherent hydrodynamic perturbation.

Motivated by the above considerations, an extension of the work by Jiang et. al. [65] as seen in the accepted publication [132] is provided. This Chapter proposed: (a) a cascaded system design (see Fig. 5.1) with output feedback control that uses a nonlinear Proportional-decoupled controller (for both tracking and stabilization) and an exponential convergence full-state observer (for both position and velocity); (b) with consideration of hydrodynamic perturbation for this underactuated ROV; (c) in three and higher dimensional spaces.

Figure 5.1: Cascaded system with output feedback control

In summary, the Chapter is organized as follows. A proposed cascaded structure of RRC ROV is described in Section 6.1. This is followed by the nonlinear observer design for all subsystems are defined in Section 6.2 to 6.4. A proposed nonlinear decoupled P-controller is

derived in Section 6.5. The proposed cascaded system was compared with the backstepping, PI and other controllers in Section 6.6. The simulated time responses were measured against the operating specification as stated in Table 1.2 (in Chapter 1) and to test for robustness, a 10 percent perturbation on the added mass and linear damping were applied on the nominal model and lastly Section 6.7 summarizes the work done in this Chapter.

5.1 Cascaded Structure of RRC ROV

To begin with, the cascaded structure for the RRC ROV systems need to be established. In the cascaded structure, there is an inner loop for stabilization and a cascaded loop for position tracking. The functions of each controller in each mode can be seen in Table 5.1. The following symbols from Chapter 3 – $\mathbf{x}$ for the full-order states of the single loop system – and from Chapter 4 – $\mathbf{x}_s$, $\mathbf{x}_h$ and $\mathbf{x}_v$ for to the station-keeping, horizontal and vertical plane subsystem states respectively in its single-loop structure (see Table 5.1) – will be used. For each subsystem, the inner loop is used for stabilization and hydrodynamic perturbation attenuation while the inner and outer loops are used for tracking purpose.

Since the full-order states are used in the simulation, the general cascaded structure for the full-order model have to be defined and $\mathbf{x}_2$ and $\mathbf{x}_1$ represent its inner (velocity) and outer (position) loop states respectively. Controllers ($\bar{\mathbf{u}}_1$ for inner loop and $\bar{\mathbf{u}}_0$ for outer loop) will be designed for each subsystem.

5.1.1 Inner loop Equation

The inner loop equation is used to represent the dynamics during the station-keeping operation whereby the ROV's body-fixed velocity is regulated about the desired velocity. The velocity error states, $\tilde{\mathbf{x}}_{2e} = [\tilde{u}_e \quad \tilde{v}_e \quad \tilde{w}_e \quad \tilde{p}_e \quad \tilde{q}_e \quad \tilde{r}_e]^T$ in the inner loop equation can be defined as:

$$\tilde{\mathbf{x}}_{2e} = \mathbf{x}_{2d} - \hat{\mathbf{x}}_2 \quad (5.1)$$

where $\mathbf{x}_{2d} = [u_d \quad v_d \quad w_d \quad p_d \quad q_d \quad r_d]^T$ is the desired velocity states vector and $\hat{\mathbf{x}}_2 = [\hat{u} \quad \hat{v} \quad \hat{w} \quad \hat{p} \quad \hat{q} \quad \hat{r}]^T$ is the estimate of velocity state vector $\mathbf{x}_2$. Using the same

method of deriving the station-keeping mode and its corresponding state-space representation as shown in (5.22) and (5.23b) respectively, the inner loop equations for the full-order system can be written as:

$$\dot{\tilde{\mathbf{x}}}_{2e} = \bar{\mathbf{f}}_{2e}(\tilde{\mathbf{x}}_{2e}, t) + \bar{\mathbf{g}}_{2e}(\bar{\mathbf{u}}_1, \tilde{\mathbf{x}}_{2e}, t) \quad (5.2)$$

where $\bar{\mathbf{u}}_1 \in \mathcal{R}^4$ is the inner-loop control law with roll and pitch velocity uncontrolled as seen in the block diagram (named “6 to 4 DOF”) in Fig. 5.3.

Modes /Subsystem	States	No. of States	Inner/Outer Loop Controller	Functions		
				Stabilization	Perturbation Attenuation	Tracking
Station-Keeping	Velocity, $\mathbf{x}_{s2} = \mathbf{v}_s$	4	$\bar{\mathbf{u}}_1$	✓	✓	
	Position, $\mathbf{x}_{s1} = \boldsymbol{\eta}_s$	4	$\bar{\mathbf{u}}_0$			
Horizontal Plane	Velocity, $\mathbf{x}_{h2} = \mathbf{v}_h$	3	$\bar{\mathbf{u}}_{1h}$	✓	✓	✓
	Position, $\mathbf{x}_{h1} = \boldsymbol{\eta}_h$	3	$\bar{\mathbf{u}}_{0h}$			
Vertical Plane	Velocity, $\mathbf{x}_{v2} = \mathbf{v}_v$	3	$\bar{\mathbf{u}}_{1v}$	✓	✓	✓
	Position, $\mathbf{x}_{v1} = \boldsymbol{\eta}_v$	3	$\bar{\mathbf{u}}_{0v}$			

Table 5.1: Function of inner and outer controllers for each mode

5.1.2 Outer-loop Equation

The outer loop equation, on the other hand, is used to represent the ROV dynamics during its pipeline tracking maneuver that requires the ROV to move in either horizontal or vertical plane.

Figure 5.2: Tracking error in body-fixed frame

In ROV tracking control, it is convenient to compute the tracking error in the body-fixed frame as shown in Fig. 5.2 as:

$$\tilde{\mathbf{x}}_{1e} = \mathbf{J}^{-1}(\mathbf{x}_1)(\mathbf{x}_{1d} - \hat{\mathbf{x}}_1) \quad (5.3)$$

where $\mathbf{x}_{1d} = [x_d \ y_d \ z_d \ \phi_d \ \theta_d \ \psi_d]^T$ is the desired position states, $\hat{\mathbf{x}}_1 = [\hat{x} \ \hat{y} \ \hat{z} \ \hat{\phi} \ \hat{\theta} \ \hat{\psi}]^T$ is the estimate of position state $\mathbf{x}_1$ and $(\mathbf{x}_{1d} - \hat{\mathbf{x}}_1)$ is the position tracking error in the earth-fixed frame. The $\mathbf{J}^{-1}(\mathbf{x}_1)$ is the inverse of the Euler's transformation defined in Chapter 3 (and its dependence of $\mathbf{x}_1$ is not shown in (5.4) for clarity). The tracking error dynamic, $\dot{\tilde{\mathbf{x}}}_{1e}$ becomes:

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_{1e} &= -\mathbf{J}^{-1}[\mathbf{J}^{-1}\dot{\mathbf{J}}(\mathbf{x}_{1d} - \hat{\mathbf{x}}_1)] + \mathbf{J}^{-1}(\dot{\mathbf{x}}_{1d} - \dot{\hat{\mathbf{x}}}_1) \\ &= -\mathbf{J}^{-1}\dot{\mathbf{J}} \tilde{\mathbf{x}}_{1e} + \mathbf{J}^{-1}(\dot{\mathbf{x}}_{1d} - \dot{\hat{\mathbf{x}}}_1) \end{aligned} \quad (5.4)$$

where $\dot{\mathbf{x}}_{1d} = \mathbf{J}(\mathbf{x}_1)\mathbf{x}_{2d}$, and $\dot{\hat{\mathbf{x}}}_1 = \mathbf{J}(\mathbf{x}_1)\hat{\mathbf{x}}_2$. Note that the components of the vector $\tilde{\mathbf{x}}_{1e}$ correspond to the error in the ROV longitudinal direction, the cross track error, the altitude error, roll, pitch and heading error. Using the same approach for the station-keeping mode as shown in (5.21) and its corresponding state-space representation in (5.23a), the outer loop equations for the full-order system can be written as:

$$\dot{\tilde{\mathbf{x}}}_{1e} = \bar{\mathbf{f}}_{1e}(\tilde{\mathbf{x}}_{1e}, \tilde{\mathbf{x}}_{2e}, \bar{\mathbf{u}}_0, t) \quad (5.5)$$

or alternatively as $\dot{\tilde{\mathbf{x}}}_{1e} = \bar{\mathbf{f}}_{1e}(\tilde{\mathbf{x}}_{2e}, t) + \bar{\mathbf{g}}_{1e}(\tilde{\mathbf{x}}_{1e}, \bar{\mathbf{u}}_0, t)$ where the $\bar{\mathbf{u}}_0 \in \mathcal{R}^4$ refers to the control law. Note that the former equation as it involved one less variable term, $\bar{\mathbf{g}}_{1e}(\tilde{\mathbf{x}}_{1e}, \bar{\mathbf{u}}_0, t)$ need to be used. Since the roll and pitch rate are not controlled, their angles are not controlled neither.

Figure 5.3. Closed-loop systems of cascaded RRC ROV

5.2 Nonlinear Observer Design for Station Keeping Mode

In [62,65], a coordinate transformation to earth-fixed frame is incorporated during the observer design for the ODIN ROV in station-keeping mode. The same method in [62,65] is used to cancel the quadratic cross terms appearing in the Coriolis and centripetal matrix. To

improve on the existing method, the observer is designed to solve the problem in three-dimensional spaces for both the velocity and the output positions under hydrodynamic perturbations. The followings steps are used in the nonlinear observer design for the station-keeping mode:

- Coordinate transformation
- Observer equation
- Exponential convergence proof of the observer states
- Full state observer error equations

Instead of using the states, $\mathbf{x}_{s2}$ and $\mathbf{x}_{s1}$ to denote the inner (velocity) and outer (position) loop states for station-keeping mode, the corresponding $\mathbf{v}_s = [u \ v \ w \ r]^T$ and $\boldsymbol{\eta}_s = [x \ y \ z \ \psi]^T$ for the velocity and position are used instead.

5.2.1 Coordinate Transformation

A coordinate transformation of $\mathbf{v}_s$ to earth-fixed frame is used to cancel the quadratic cross terms appearing in the Coriolis and centripetal matrix. This can be written as:

$$\boldsymbol{\eta}_Q = \mathbf{Q}_o(\boldsymbol{\eta}_s)\mathbf{v}_s \quad (5.6)$$

where $\mathbf{Q}_o(\boldsymbol{\eta}_s) \in \mathbb{R}^{4 \times 4}$, to be determined, is used to transform the body-fixed velocities $\mathbf{v}_s$ to $\boldsymbol{\eta}_Q \in \mathbb{R}^4$. The coordinate $(\boldsymbol{\eta}_s, \boldsymbol{\eta}_Q)$ can be obtained by substituting $\mathbf{v}_s = \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\dot{\boldsymbol{\eta}}_s$ into (5.6) to give:

$$\boldsymbol{\eta}_Q = \mathbf{Q}_o(\boldsymbol{\eta}_s)\mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\dot{\boldsymbol{\eta}}_s \Rightarrow \dot{\boldsymbol{\eta}}_s = \mathbf{J}_s(\boldsymbol{\eta}_s)\mathbf{Q}_o(\boldsymbol{\eta}_s)^{-1}\boldsymbol{\eta}_Q \quad (5.7)$$

Differentiating (5.7) and substituting $\mathbf{M}\dot{\mathbf{v}}_s + \mathbf{C}(\mathbf{v}_s)\mathbf{v}_s + \mathbf{D}\mathbf{v}_s = \boldsymbol{\tau}_s$ with $\mathbf{v}_s = \mathbf{Q}_o(\boldsymbol{\eta}_s)^{-1}\boldsymbol{\eta}_Q$ from (5.6), yields:

$$\dot{\boldsymbol{\eta}}_Q = [\dot{\mathbf{Q}}_o - \mathbf{Q}_o(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\mathbf{C}_s(\mathbf{v}_s)]\mathbf{Q}_o^{-1}\boldsymbol{\eta}_Q - \mathbf{Q}_o(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\mathbf{D}_s\mathbf{Q}_o^{-1}\boldsymbol{\eta}_Q + \mathbf{Q}_o\mathbf{M}_s^{-1}\boldsymbol{\tau}_s \quad (5.8)$$

The goal is to find $\mathbf{Q}_o(\boldsymbol{\eta}_s)$ such that the algebraic sum of the two terms in the first square bracket in (5.8) that contains the coupling term $\mathbf{C}_s(\mathbf{v}_s)$ is zero. By expanding:

$$\dot{\mathbf{Q}}_o(\boldsymbol{\eta}_s) - \mathbf{Q}_o(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\mathbf{C}_s(\mathbf{v}_s) = 0 \quad (5.9)$$

and substituting the solution as $\mathbf{Q}_o(\boldsymbol{\eta}_s) = \mathbf{J}_s(\boldsymbol{\eta}_s)$ back into (5.9), the ten partial differential equations(PDEs) in (5.9) can be satisfied as seen in Appendix C1, Section C1.4. Hence, $\mathbf{Q}_o(\boldsymbol{\eta}_s) = \mathbf{J}_s(\boldsymbol{\eta}_s)$ is indeed the solution for the PDEs. With that, substituting $\mathbf{Q}_o(\boldsymbol{\eta}_s) = \mathbf{J}_s(\boldsymbol{\eta}_s)$ into (5.7) and (5.8) yields:

$$\dot{\boldsymbol{\eta}}_s = \boldsymbol{\eta}_Q \quad (5.10)$$

$$\dot{\boldsymbol{\eta}}_Q = -\mathbf{D}_\eta(\boldsymbol{\eta}_s)\boldsymbol{\eta}_Q + \mathbf{J}_s(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\boldsymbol{\tau}_s \quad (5.11)$$

where $\mathbf{D}_\eta(\boldsymbol{\eta}_s) = \mathbf{J}_s(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\mathbf{D}_s\mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}$. The problem has now been transformed into the earth-fixed frame without the coupling term due to Coriolis and Centripetal matrix $\mathbf{C}_s(\mathbf{v}_s)$. The term $\dot{\boldsymbol{\eta}}_Q$ can be interpreted as the velocity along the earth-fixed frame.

5.2.2 Observer Equations

The full states observer equation for the position and velocity states in the station-keeping mode are to be determined. The observer gains $\mathbf{K}_{01}$ and $\mathbf{K}_{02}$ satisfying the following constraint are obtained as follows.

Equations (5.10)-(5.11) with the full-states observer become:

$$\dot{\hat{\boldsymbol{\eta}}}_s = \hat{\boldsymbol{\eta}}_Q + \mathbf{K}_{01}(\boldsymbol{\eta}_s - \hat{\boldsymbol{\eta}}_s) \quad (5.12)$$

$$\dot{\hat{\boldsymbol{\eta}}_Q} = -\mathbf{D}_{\boldsymbol{\eta}}(\boldsymbol{\eta}_s)\hat{\boldsymbol{\eta}}_Q + \mathbf{K}_{02}(\boldsymbol{\eta}_s - \hat{\boldsymbol{\eta}}_s) + \mathbf{J}_s(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\boldsymbol{\tau}_s \quad (5.13)$$

where $\hat{\boldsymbol{\eta}}_s$ and $\hat{\boldsymbol{\eta}}_Q$ are the estimates of $\boldsymbol{\eta}_s$ and $\boldsymbol{\eta}_Q$ respectively. The observer error dynamic in (5.12) and (5.13) becomes:

$$\dot{\tilde{\boldsymbol{\eta}}_s} = \tilde{\boldsymbol{\eta}}_Q - \mathbf{K}_{01}\tilde{\boldsymbol{\eta}}_s \quad (5.14a)$$

$$\dot{\tilde{\boldsymbol{\eta}}_Q} = -\mathbf{D}_{\boldsymbol{\eta}}(\boldsymbol{\eta}_s)\tilde{\boldsymbol{\eta}}_Q - \mathbf{K}_{02}\tilde{\boldsymbol{\eta}}_s \quad (5.14b)$$

where $\tilde{\boldsymbol{\eta}}_s = \boldsymbol{\eta}_s - \hat{\boldsymbol{\eta}}_s$, $\tilde{\boldsymbol{\eta}}_Q = \boldsymbol{\eta}_Q - \hat{\boldsymbol{\eta}}_Q$, $\mathbf{K}_{01}$ and $\mathbf{K}_{02}$ are the observer gains. To obtain these observer gains, the Lyapunov first method is used. A Lyapunov function $V_{obs} = \tilde{\boldsymbol{\eta}}_s^T \mathbf{P}_{01} \tilde{\boldsymbol{\eta}}_s + \tilde{\boldsymbol{\eta}}_Q^T \mathbf{P}_{02} \tilde{\boldsymbol{\eta}}_Q > 0$ is defined for all value of $\tilde{\boldsymbol{\eta}}_s \neq 0$, $\tilde{\boldsymbol{\eta}}_Q \neq 0$ and determining its derivative $\dot{V}_{obs}$ for all $\tilde{\boldsymbol{\eta}}_s$ and $\tilde{\boldsymbol{\eta}}_Q$. Then the $\mathbf{K}_{01}$ and $\mathbf{K}_{02}$ are obtained by ensuring that the $V_{obs} > 0$ and $\dot{V}_{obs} \leq 0$ (details of $\dot{V}_{obs}$ derivation can be seen in Appendix C1, Section C1.1) or simply the conditions can be written as:

$$\mathbf{K}_{01}^T \mathbf{P}_{01} + \mathbf{P}_{01} \mathbf{K}_{01} > \mathbf{0}, \quad \mathbf{P}_{01} = \mathbf{P}_{01}^T > \mathbf{0} \quad (5.15a)$$

$$\mathbf{P}_{01} - \mathbf{P}_{02} \mathbf{K}_{02} = \mathbf{0}, \quad \mathbf{P}_{02} = \mathbf{P}_{02}^T > \mathbf{0} \quad (5.15a)$$

$$\mathbf{D}_{\boldsymbol{\eta}}^T(\boldsymbol{\eta}) \mathbf{P}_{02} + \mathbf{P}_{02} \mathbf{D}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) > \mathbf{0} \quad (5.15c)$$

where $\mathbf{D}_{\boldsymbol{\eta}}(\boldsymbol{\eta})$ is positive definite.

5.2.3 Exponential Convergence of Observer States

The estimated state $\hat{\boldsymbol{\eta}}_s$ and $\hat{\boldsymbol{\eta}}_Q$ can be shown to be exponential convergence by means of the Lyapunov first method. For compactness in the equations, $\tilde{\boldsymbol{\eta}}_O(t) = [\tilde{\boldsymbol{\eta}}_s(t) \quad \tilde{\boldsymbol{\eta}}_Q(t)]^T$ is defined. Then, the V_{obs} in previous Section 5.2.2 becomes

$$V_{obs} = \tilde{\boldsymbol{\eta}}_O^T \begin{bmatrix} \mathbf{P}_{01} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{02} \end{bmatrix} \tilde{\boldsymbol{\eta}}_O = \tilde{\boldsymbol{\eta}}_O^T \mathbf{P}_O \tilde{\boldsymbol{\eta}}_O. \text{ Differentiating } V_{obs} \text{ along the solutions of (5.14)}$$

gives:

$$\begin{aligned}
\dot{V}_{obs} &= \dot{\tilde{\boldsymbol{\eta}}_o}^T \mathbf{P}_0 \tilde{\boldsymbol{\eta}}_o + \tilde{\boldsymbol{\eta}}_o^T \mathbf{P}_0 \dot{\tilde{\boldsymbol{\eta}}_o} \\
&= -[\tilde{\boldsymbol{\eta}}_s \quad \tilde{\boldsymbol{\eta}}_Q] \overbrace{\begin{bmatrix} 2\mathbf{P}_{01}\mathbf{K}_{01} & 2\mathbf{P}_{02}\mathbf{K}_{02} - \mathbf{P}_{01} + \mathbf{P}_{01}\mathbf{D}_\eta \\ -2\mathbf{P}_{01} + \mathbf{K}_{02}\mathbf{P}_{02} & -\mathbf{P}_{02} + 2\mathbf{P}_{02}\mathbf{D}_\eta \end{bmatrix}}^{\mathbf{H}_0} \begin{bmatrix} \tilde{\boldsymbol{\eta}}_s \\ \tilde{\boldsymbol{\eta}}_Q \end{bmatrix} \quad (5.16)
\end{aligned}$$

where $\tilde{\boldsymbol{\eta}}_s = \boldsymbol{\eta}_s - \hat{\boldsymbol{\eta}}_s$, $\tilde{\boldsymbol{\eta}}_Q = \boldsymbol{\eta}_Q - \hat{\boldsymbol{\eta}}_Q$ and hence implying that $\tilde{\boldsymbol{\eta}}_o = \boldsymbol{\eta}_o - \hat{\boldsymbol{\eta}}_o$. After some manipulation, $\tilde{\boldsymbol{\eta}}_o(t)$ becomes:

$$\|\tilde{\boldsymbol{\eta}}_o(t)\|_2^2 \leq \gamma_o \|\tilde{\boldsymbol{\eta}}_o(t_o)\|_2^2 e^{-\sigma_o(t-t_o)} \quad (5.17)$$

where $\gamma_o > 0, \sigma_o > 0$ are positive constant and $\|\cdot\|_2$ is the Euclidean norm. See Section C1.2 in Appendix C1 for the detail of obtaining (5.17). Expanding (5.17), the following can be obtained:

$$\|\tilde{\boldsymbol{\eta}}_s(t)\|_2^2 \leq \gamma_o \|\tilde{\boldsymbol{\eta}}_s(t_o)\|_2^2 e^{-\sigma_o(t-t_o)} \quad (5.18)$$

Since the velocity estimate error, $\tilde{\mathbf{v}}_s = \mathbf{v}_s - \hat{\mathbf{v}}_s$ can be transformed to the earth-fixed frame by means of $\tilde{\mathbf{v}}_s = \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1} \tilde{\boldsymbol{\eta}}_s$, the velocity estimate error is exponentially convergence as well.

5.2.4 Observer Error Equations

To include the observer into the cascaded system, the observer equations in (5.10) to (5.11) need to be expressed as functions of the position estimate error $\tilde{\boldsymbol{\eta}}_s$ and velocity estimate $\tilde{\mathbf{v}}_s$.

By equating (5.14b) to $\dot{\tilde{\boldsymbol{\eta}}_Q} = \dot{\mathbf{J}}_s(\boldsymbol{\eta}_s)\tilde{\mathbf{v}}_s + \mathbf{J}_s(\boldsymbol{\eta}_s)\dot{\tilde{\mathbf{v}}_s}$ and substituting (5.9) with $\mathbf{Q}_o(\boldsymbol{\eta}_s) = \mathbf{J}_s(\boldsymbol{\eta}_s)$ yields:

$$\dot{\tilde{\mathbf{v}}_s} = -\mathbf{M}_s^{-1}\mathbf{C}_s(\tilde{\mathbf{v}}_s)\tilde{\mathbf{v}}_s - \mathbf{M}_s^{-1}\mathbf{D}_s\tilde{\mathbf{v}}_s - \mathbf{K}_{02}\mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\tilde{\boldsymbol{\eta}}_s \quad (5.19)$$

where $\tilde{\boldsymbol{\eta}}_s = \boldsymbol{\eta}_s - \hat{\boldsymbol{\eta}}_s$ and $\tilde{\mathbf{v}}_s = \mathbf{v}_s - \hat{\mathbf{v}}_s$. The details of obtaining (5.19) are given in the Appendix C1.3. With $\tilde{\boldsymbol{\eta}}_Q = \mathbf{J}_s(\boldsymbol{\eta}_s)\tilde{\mathbf{v}}_s \Rightarrow \tilde{\mathbf{v}}_s = \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\tilde{\boldsymbol{\eta}}_Q$, (5.12) can be written as:

$$\dot{\tilde{\boldsymbol{\eta}}}_s = \mathbf{J}_s(\boldsymbol{\eta}_s)\tilde{\mathbf{v}}_s - \mathbf{K}_{01}\tilde{\boldsymbol{\eta}}_s \quad (5.20)$$

5.2.5 Observer Error Equation for Station-Keeping

The station-keeping model is used to describe the ROV motion during the stabilization about the equilibrium submanifold (see Section 5.3.1 for explanation) whereby the roll and pitch motions are unspecified and asymptotically stable. The parameters used in deriving the tracking and velocity error dynamic for the station-keeping mode has been defined in Section 4.2.

As seen in Table 5.1, there is no tracking requirement for the ROV during the station-keeping mode. However, the tracking errors dynamic in body-fixed dynamics still exist. The outer loop equation (5.5) for station-keeping mode with control law $\bar{\mathbf{u}}_0$ and the observer in (5.20) can be expressed as:

$$\dot{\tilde{x}}_e = u_d \cos \psi + v_d \sin \psi - \hat{u} + \tilde{r}\tilde{y}_e + \bar{u}_{Ox} - \Omega_{xe} \quad (5.21a)$$

$$\dot{\tilde{y}}_e = -u_d \sin \psi + v_d \cos \psi - \tilde{r}\tilde{x}_e - \hat{v} + \bar{u}_{Oy} - \Omega_{ye} \quad (5.21b)$$

$$\dot{\tilde{z}}_e = w_d - \hat{w} + \bar{u}_{Oz} - \Omega_{ze} \quad (5.21c)$$

$$\dot{\tilde{\psi}}_e = r_d - \hat{r} + \bar{u}_{O\psi} - \Omega_{\psi e} \quad (5.21d)$$

where $\tilde{x}_e, \tilde{y}_e, \tilde{z}_e, \tilde{\psi}_e$ are the position tracking errors in the body-fixed frame and $\Omega_{xe}, \Omega_{ye}, \Omega_{ze}, \Omega_{\psi e}$ are the first to third and last row of $\mathbf{K}_{01}\tilde{\boldsymbol{\eta}}_s$ in (5.20) respectively. The $\bar{u}_{Ox}, \bar{u}_{Oy}, \bar{u}_{Oz}$ and $\bar{u}_{O\psi}$ are the outer loop control law for the surge, sway, heave and yaw dynamic respectively.

The stabilization dynamic equation for station-keeping mode in (5.2) with the control law $\bar{\mathbf{u}}_1$ and observer in (5.19) can be expressed as:

$$\dot{\tilde{\mathbf{u}}}_e = -(m - X_{\dot{u}})^{-1} X_u \tilde{\mathbf{u}}_e - (m - X_{\dot{u}})^{-1} [(Y_{\dot{v}} - m) \tilde{v}_e \tilde{r}_e + \tau_x - \bar{\mathbf{u}}_{1x}] - \Omega_u \quad (5.22a)$$

$$\dot{\tilde{\mathbf{v}}}_e = -(m - Y_{\dot{v}})^{-1} Y_v \tilde{\mathbf{v}}_e - (m - Y_{\dot{v}})^{-1} [(m - X_{\dot{u}}) \tilde{\mathbf{u}}_e \tilde{r}_e + \tau_y - \bar{\mathbf{u}}_{1y}] - \Omega_v \quad (5.22b)$$

$$\dot{\tilde{\mathbf{w}}}_e = -(m - Z_{\dot{w}})^{-1} [Z_w \tilde{\mathbf{w}}_e + \tau_z - \bar{\mathbf{u}}_{1z}] - \Omega_w \quad (5.22c)$$

$$\dot{\tilde{\mathbf{r}}}_e = -(I_z - N_{\dot{r}})^{-1} N_r \tilde{\mathbf{r}}_e - (I_z - N_{\dot{r}})^{-1} [(-Y_{\dot{v}} \tilde{\mathbf{u}}_e + X_{\dot{u}} \tilde{\mathbf{u}}_e) \tilde{\mathbf{v}}_e + \tau_{\psi} - \bar{\mathbf{u}}_{1\psi}] - \Omega_r \quad (5.22d)$$

where $\tilde{\mathbf{u}}_e = \mathbf{u}_d - \hat{\mathbf{u}}$, $\tilde{\mathbf{v}}_e = \mathbf{v}_d - \hat{\mathbf{v}}$, $\tilde{\mathbf{w}}_e = \mathbf{w}_d - \hat{\mathbf{w}}$ and $\tilde{\mathbf{r}}_e = \mathbf{r}_d - \hat{\mathbf{r}}$ are the velocity errors. The $\Omega_u, \Omega_v, \Omega_w, \Omega_r$ are the first, second, third and last row of $\mathbf{K}_{02} \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1} \tilde{\boldsymbol{\eta}}_s = \mathbf{K}_{02} \tilde{\mathbf{v}}_s$ respectively. The $\bar{\mathbf{u}}_{1x}, \bar{\mathbf{u}}_{1y}, \bar{\mathbf{u}}_{1z}$ and $\bar{\mathbf{u}}_{1\psi}$ are the inner loop control law for the surge, sway, heave and yaw dynamic respectively. The $\tau_x, \tau_y, \tau_z, \tau_{\psi}$ are the thrust inputs to the ROV.

Now, the error dynamic for both tracking and stabilization for the cascaded system can be written in state-space form as:

$$\dot{\tilde{\mathbf{x}}}_{s1e} = \bar{\mathbf{f}}_{s1}(\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_0, t) \quad (5.23a)$$

$$\dot{\tilde{\mathbf{x}}}_{s2e} = \bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) + \bar{\mathbf{g}}_{s2}(\bar{\mathbf{u}}_1, \tilde{\mathbf{x}}_{s2e}, t) \quad (5.23b)$$

where $\tilde{\mathbf{x}}_{s2e} = [\tilde{u}_e \ \tilde{v}_e \ \tilde{w}_e \ \tilde{r}_e]^T$ is the velocity error $\tilde{\mathbf{x}}_{s1e} = [\tilde{x}_e \ \tilde{y}_e \ \tilde{z}_e \ \tilde{\psi}_e]^T$ is the position tracking errors in body-fixed frame and the remaining functions are defined as:

$$\bar{\mathbf{f}}_{s1}(\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_0, t) = \begin{bmatrix} \cos \psi & \sin \psi & 0 & 0 \\ -\sin \psi & \cos \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_{s2d} + \begin{bmatrix} -\hat{u} + \tilde{r} \tilde{y}_e \\ -\tilde{r} \tilde{x}_e - \hat{v} \\ -\hat{w} \\ -\hat{r} \end{bmatrix} + \bar{\mathbf{u}}_0 - \mathbf{K}_{01} \tilde{\mathbf{x}}_{s1}$$

$$\bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) = \begin{bmatrix} -(m - X_{\dot{u}})^{-1} X_u & -(m - X_{\dot{u}})^{-1} (Y_{\dot{v}} - m) \tilde{r}_e & 0 & 0 \\ 0 & -(m - Y_{\dot{v}})^{-1} Y_v & 0 & -(m - Y_{\dot{v}})^{-1} (m - X_{\dot{u}}) \tilde{u}_e \\ 0 & 0 & -(m - Z_{\dot{w}})^{-1} Z_w & 0 \\ 0 & -(I_z - N_{\dot{r}})^{-1} (-Y_{\dot{v}} + X_{\dot{u}}) \tilde{u}_e & 0 & -(I_z - N_{\dot{r}})^{-1} N_r \end{bmatrix} \tilde{\mathbf{x}}_{s2e}$$

$$\bar{\mathbf{g}}_{s2}(\bar{\mathbf{u}}_1, \tilde{\mathbf{x}}_{s2e}, t) = - \begin{bmatrix} (m - X_{\dot{u}})^{-1} & 0 & 0 & 0 \\ 0 & (m - Y_{\dot{v}})^{-1} & 0 & 0 \\ 0 & 0 & (m - Z_{\dot{w}})^{-1} & 0 \\ 0 & 0 & 0 & (I_z - N_{\dot{r}})^{-1} \end{bmatrix} (\bar{\mathbf{u}}_1 + \boldsymbol{\tau}_s) - \mathbf{K}_{02} \overbrace{\begin{bmatrix} \cos \psi & -\sin \psi & 0 & 0 \\ \sin \psi & \cos \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}}^{\tilde{\mathbf{x}}_{s2}} \tilde{\mathbf{x}}_{s1}$$

$\bar{\mathbf{u}}_1, \bar{\mathbf{u}}_0 \in \mathbb{R}^4$ are the inner and outer control law to be derived in Section 5.5 and $\mathbf{K}_{01}$, $\mathbf{K}_{02} \in \mathbb{R}^{4 \times 4}$ are the observer gains. With the observer designed for the station-keeping mode, the unmeasured states for a desired input $\mathbf{x}_{1d} = [5 \ 0 \ 3 \ 0 \ 0 \ 0]^T$ (for an example) can be estimated. The differences between the estimated and actual states can be seen in Fig. 5.4. As observed, the estimated states (dotted line) converged to the actual states (solid line) in less than ten seconds.

Figure 5.4a: Actual and estimated state for velocity of station-keeping mode

Figure 5.4b: Actual and estimated state for position of station-keeping mode

5.3 Observer Error Equation for Horizontal Plane Subsystem

By repeating the same method as seen in Section (5.2.5), the observer design for the horizontal plane subsystem as seen in Section 4.3.1 can be derived. The tracking dynamic in body-fixed frame can be obtained as:

$$\dot{\tilde{x}}_e = u_d \cos \psi + v_d \sin \psi - \hat{u} + \tilde{r}\tilde{y}_e + \bar{u}_{Ox} - \Omega_{xe} \quad (5.24a)$$

$$\dot{\tilde{y}}_e = -u_d \sin \psi + v_d \cos \psi - \tilde{r}\tilde{x}_e - \hat{v} + \bar{u}_{Oy} - \Omega_{ye} \quad (5.24b)$$

$$\dot{\tilde{\psi}}_e = r_d - \hat{r} + \bar{u}_{O\psi} - \Omega_{\psi e} \quad (5.24c)$$

where $\tilde{x}_e, \tilde{y}_e, \tilde{\psi}_e$ are the position tracking errors in body-fixed frame. And $\Omega_{xe}, \Omega_{ye}, \Omega_{\psi e}$ are the first, second, and last row of $\mathbf{K}_{01}\tilde{\eta}_h$ respectively. While the $\bar{u}_{Ox}, \bar{u}_{Oy}, \bar{u}_{O\psi}$ are the outer

loop control law for the surge, sway and yaw dynamic respectively. The stabilization dynamic can be expressed as:

$$\dot{\tilde{u}}_e = -(m - X_{\dot{u}})^{-1} X_u \tilde{u}_e - (m - X_{\dot{u}})^{-1} [(Y_{\dot{v}} - m) \tilde{v}_e \tilde{r}_e + \tau_x - \bar{u}_{I_x}] - \Omega_u \quad (5.25a)$$

$$\dot{\tilde{v}}_e = -(m - Y_{\dot{v}})^{-1} Y_v \tilde{v}_e - (m - Y_{\dot{v}})^{-1} [(m - X_{\dot{u}}) \tilde{u}_e \tilde{r}_e + \tau_y - \bar{u}_{I_y}] - \Omega_v \quad (5.25b)$$

$$\dot{\tilde{r}}_e = -(I_z - N_{\dot{r}})^{-1} N_r \tilde{r}_e - (I_z - N_{\dot{r}})^{-1} [(-Y_{\dot{v}} \tilde{u}_e + X_{\dot{u}} \tilde{u}_e) \tilde{v}_e + \tau_{\psi} - \bar{u}_{I_{\psi}}] - \Omega_r \quad (5.25c)$$

where $\tilde{u}_e = u_d - \hat{u}$, $\tilde{v}_e = v_d - \hat{v}$ and $\tilde{r}_e = r_d - \hat{r}$ are the velocity errors. And the $\Omega_u, \Omega_v, \Omega_r$ are the first, second, and last row of $\mathbf{K}_{02} \mathbf{J}_h^{-1}(\boldsymbol{\eta}_h) \tilde{\boldsymbol{\eta}}_h$ respectively. The $\bar{u}_{I_x}$, $\bar{u}_{I_y}$ and $\bar{u}_{I_{\psi}}$ are the inner loop control law for the surge, sway and yaw dynamic respectively. The $\tau_x, \tau_y, \tau_{\psi}$ are the thrust inputs to the ROV.

The error dynamic both tracking and stabilization in state-space form becomes:

$$\dot{\tilde{\mathbf{x}}_{h1e}} = \bar{\mathbf{f}}_{h1}(\tilde{\mathbf{x}}_{h1e}, \tilde{\mathbf{x}}_{h2e}, \bar{\mathbf{u}}_{Oh}, t) \quad (5.26a)$$

$$\tilde{\mathbf{x}}_{h2e} = \bar{\mathbf{f}}_{h2}(\tilde{\mathbf{x}}_{h2e}, t) + \bar{\mathbf{g}}_{h2}(\bar{\mathbf{u}}_{Ih}, \tilde{\mathbf{x}}_{h2e}, t) \quad (5.26b)$$

where $\tilde{\mathbf{x}}_{h2e} = [\tilde{u}_e \ \tilde{v}_e \ \tilde{r}_e]^T$ is the velocity error $\tilde{\mathbf{x}}_{h1e} = [\tilde{x}_e \ \tilde{y}_e \ \tilde{\psi}_e]^T$ is the position tracking error in body-fixed frame and the remaining functions are defined as:

$$\bar{\mathbf{f}}_{h1}(\tilde{\mathbf{x}}_{h1e}, \tilde{\mathbf{x}}_{h2e}, \bar{\mathbf{u}}_{Oh}, t) = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_{h2d} + \begin{bmatrix} -\hat{u} + \tilde{r} \tilde{y}_e \\ -\tilde{r} \tilde{x}_e - \hat{v} \\ -\hat{r} \end{bmatrix} + \bar{\mathbf{u}}_{Oh} - \mathbf{K}_{01} \tilde{\mathbf{x}}_{h1}$$

$$\bar{\mathbf{f}}_{h2}(\tilde{\mathbf{x}}_{h2e}, t) = \begin{bmatrix} -(m - X_{\dot{u}})^{-1} X_u & -(m - X_{\dot{u}})^{-1} (Y_{\dot{v}} - m) \tilde{r}_e & 0 \\ 0 & -(m - Y_{\dot{v}})^{-1} Y_v & -(m - Y_{\dot{v}})^{-1} (m - X_{\dot{u}}) \tilde{u}_e \\ 0 & -(I_z - N_{\dot{r}})^{-1} (-Y_{\dot{v}} + X_{\dot{u}}) \tilde{u}_e & -(I_z - N_{\dot{r}})^{-1} N_r \end{bmatrix} \tilde{\mathbf{x}}_{h2e}$$

$$\bar{\mathbf{g}}_{h2}(\bar{\mathbf{u}}_{1h}, \tilde{\mathbf{x}}_{h2e}, t) = - \begin{bmatrix} (m - X_{\ddot{u}})^{-1} & 0 & 0 \\ 0 & (m - Y_{\ddot{v}})^{-1} & 0 \\ 0 & 0 & (I_z - N_{\ddot{r}})^{-1} \end{bmatrix} \left((\bar{\mathbf{u}}_{1h} + \boldsymbol{\tau}_h) - \mathbf{K}_{02} \overbrace{\begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}}^{\tilde{\mathbf{x}}_{h2}} \tilde{\mathbf{x}}_{h1} \right)$$

$\bar{\mathbf{u}}_{1h}, \bar{\mathbf{u}}_{0h} \in \mathcal{R}^3$ are the inner and outer control law to be designed in Section 5.5 and $\mathbf{K}_{01}, \mathbf{K}_{02} \in \mathcal{R}^{3 \times 3}$ are the observer gains. With the observer designed on the horizontal plane, the unmeasured states for a desired input $\mathbf{x}_{1d} = [5 \ 0 \ 3 \ 0 \ 0 \ 0]^T$ (for an example) can be estimated. The differences between the estimated and actual states can be seen in Fig. 5.5. As observed, the estimated states (dotted line) converged to the actual states (solid line) in less than ten seconds.

Figure 5.5a. Actual and estimated state for velocity for horizontal plane

Figure 5.5b. Actual and estimated state for position for horizontal plane

5.4 Observer Error Equation for Vertical Plane Subsystem

By repeating the same method as seen in Section (5.2.5), the observer design for the vertical plane subsystem in Section 4.3.2 can be derived. The tracking dynamic in body-fixed frame can be expressed as:

$$\dot{\tilde{x}}_e = u_d \cos \theta + w_d \sin \theta - \hat{u} + \hat{q}\tilde{z}_e + \bar{u}_{Ox} - \Omega_{xe} \quad (5.27a)$$

$$\dot{\tilde{z}}_e = -u_d \sin \theta + w_d \cos \theta - \hat{q}\tilde{x}_e - \hat{w} + \bar{u}_{Oy} - \Omega_{ze} \quad (5.27b)$$

$$\dot{\tilde{\theta}}_e = q_d - \hat{q} + \bar{u}_{O\theta} - \Omega_{\theta e} \quad (5.27c)$$

where $\tilde{x}_e$, $\tilde{z}_e$, $\tilde{\theta}_e$ are the position tracking errors in body-fixed frame. And the Ω_{xe} , Ω_{ze} , $\Omega_{\theta e}$ are the first, second, and third row of $\mathbf{K}_{01}\tilde{\mathbf{n}}_v$ respectively. While the $\bar{u}_{Ox}$, $\bar{u}_{Oz}$ and $\bar{u}_{O\theta}$

are the outer loop control law for the surge, heave and pitch dynamic respectively. The stabilization dynamic can be defined as:

$$\dot{\tilde{u}}_e = -(m - X_{\dot{u}})^{-1} X_u \tilde{u}_e - (m - X_{\dot{u}})^{-1} [(m - Z_{\dot{w}}) \tilde{w}_e \tilde{q}_e + \tau_x - \bar{u}_{1x}] - \Omega_u \quad (5.28a)$$

$$\dot{\tilde{w}}_e = -(m - Z_{\dot{w}})^{-1} Z_w \tilde{w}_e - (m - Z_{\dot{w}})^{-1} [(X_{\dot{u}} - m) \tilde{u}_e \tilde{q}_e + \tau_z - \bar{u}_{1z}] - \Omega_w \quad (5.28b)$$

$$\begin{aligned} \dot{\tilde{q}}_e = & -(I_y - M_{\dot{q}})^{-1} M_q \tilde{q}_e - (I_y - M_{\dot{q}})^{-1} [(Z_{\dot{w}} - X_{\dot{u}}) \tilde{u}_e \tilde{w}_e + \tau_\theta - \bar{u}_{1\theta}] - \Omega_q \\ & + (z_G - z_B) W \sin \tilde{\theta}_e \end{aligned} \quad (5.28c)$$

where $\tilde{u}_e = u_d - \hat{u}$, $\tilde{w}_e = w_d - \hat{w}$, $\tilde{q}_e = q_d - \hat{q}$ are the velocity errors. And the Ω_u , Ω_w , Ω_q are the first, third, and fifth row of $\mathbf{K}_{02} \mathbf{J}^{-1}(\boldsymbol{\eta}_v) \tilde{\boldsymbol{\eta}}_v = \mathbf{K}_{02} \tilde{\mathbf{v}}_v$ respectively. The $\bar{u}_{1x}$, $\bar{u}_{1z}$ and $\bar{u}_{1\theta}$ are the inner loop control law for the surge, heave and pitch dynamic respectively. The τ_x , τ_z , τ_θ is the thrust inputs to the ROV.

The error dynamic for both tracking and stabilization in state-space form become:

$$\tilde{\mathbf{x}}_{v1e} = \bar{\mathbf{f}}_{v1}(\tilde{\mathbf{x}}_{v1e}, \tilde{\mathbf{x}}_{v2e}, \bar{\mathbf{u}}_{Ov}, t) \quad (5.29a)$$

$$\tilde{\mathbf{x}}_{v2e} = \bar{\mathbf{f}}_{v2}(\tilde{\mathbf{x}}_{v2e}, t) + \bar{\mathbf{g}}_{v2}(\bar{\mathbf{u}}_{Iv}, \tilde{\mathbf{x}}_{v2e}, t) \quad (5.29b)$$

where $\tilde{\mathbf{x}}_{v2e} = [\tilde{u}_e \quad \tilde{w}_e \quad \tilde{q}_e]^T$ is the velocity error $\tilde{\mathbf{x}}_{v1e} = [\tilde{x}_e \quad \tilde{z}_e \quad \tilde{\theta}_e]^T$ is the position tracking error in the body-fixed frame and the remaining functions are defined as:

$$\begin{aligned} \bar{\mathbf{f}}_{v1}(\tilde{\mathbf{x}}_{v1e}, \tilde{\mathbf{x}}_{v2e}, \bar{\mathbf{u}}_{Ov}, t) = & \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_{v2d} + \begin{bmatrix} -\hat{u} + \hat{q} \tilde{z}_e \\ -\hat{q} \tilde{x}_e - \hat{w} \\ -\hat{q} \end{bmatrix} + \bar{\mathbf{u}}_{Ov} - \mathbf{K}_{01} \tilde{\mathbf{x}}_{v1} \\ \bar{\mathbf{f}}_{v2}(\tilde{\mathbf{x}}_{v2e}, t) = & \begin{bmatrix} -(m - X_{\dot{u}})^{-1} X_u & -(m - X_{\dot{u}})^{-1} (m - Z_{\dot{w}}) \tilde{q}_e & 0 \\ 0 & -(m - Z_{\dot{w}})^{-1} Z_w & -(m - Z_{\dot{w}})^{-1} (X_{\dot{u}} - m) \tilde{u}_e \\ 0 & -(I_y - M_{\dot{q}})^{-1} (Z_{\dot{w}} - X_{\dot{u}}) \tilde{u}_e & -(I_y - M_{\dot{q}})^{-1} M_q \end{bmatrix} \tilde{\mathbf{x}}_{v2e} \end{aligned}$$

$$\bar{\mathbf{g}}_{v2}(\bar{\mathbf{u}}_{1v}, \tilde{\mathbf{x}}_{v2e}, t) = - \begin{bmatrix} (m - X_{\ddot{u}})^{-1} & 0 & 0 \\ 0 & (m - Z_{\ddot{w}})^{-1} & 0 \\ 0 & 0 & (I_y - M_{\ddot{q}})^{-1} \end{bmatrix} (\bar{\mathbf{u}}_{1v} + \boldsymbol{\tau}_v) - \mathbf{K}_{02} \overbrace{\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}}^{\tilde{\mathbf{x}}_{v2}} \tilde{\mathbf{x}}_{v1} + \begin{bmatrix} 0 \\ 0 \\ (z_G - z_B)W \sin \tilde{\theta}_e \end{bmatrix}$$

$\bar{\mathbf{u}}_{1v}, \bar{\mathbf{u}}_{0v} \in \mathcal{R}^3$ are the inner and outer control law to be defined in Section 5.5 and $\mathbf{K}_{01}, \mathbf{K}_{02} \in \mathcal{R}^{3 \times 3}$ are the observer gains. With the observer designed on the vertical plane, the unmeasured states for a desired input $\mathbf{x}_{1d} = [5 \ 0 \ 3 \ 0 \ 0 \ 0]^T$ (for an example) can be estimated. The differences between the estimated and actual states can be seen in Fig. 5.6. As observed, the estimated states (dotted line) converged to the actual states (solid line) in less than ten seconds.

Figure 5.6a: Actual and estimated state for velocity for vertical plane

Figure 5.6b: Actual and estimated state for position for vertical plane

As observed in the estimation for all subsystems, the observer transient will not limit the speed of response of the closed-loop system as the observer gains can be adjusted to respond faster. However, it will required some fine-tuning in the field as the result may vary due to unforeseen circumstances such as strong underwater current and waves. From the experimental pool test, the observer transients had no effect on the system response.

5.5. Robust Cascaded Controller Design

Once each subsystem observer has been designed, the next step is to design robust cascaded controllers. The proposed inner and outer controllers in this Section will be used for all subsystems. However, the cascaded systems with the observers are of a different form as those in [90,91,92]. This affects the direct application of the asymptotic stability proof for the cascaded system. Therefore, there is a need to identify the differences and to revise the asymptotic stability proof in order to make it applicable for the proposed cascaded system established here. Basically, the differences in the cascaded control system are as follows:

a) In [90,91,92], the perturbations in the hydrodynamic forces are not included in the equations. However, they are often required when the robustness of the control needs to be evaluated. In the cases presented here, the perturbations in the hydrodynamic damping and added mass terms are included. For example, the nonlinear dynamic for the station-keeping model with perturbation can be written as:

$$\dot{\tilde{\mathbf{x}}_{s1e}} = \bar{\mathbf{f}}_{s1}(\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_0, t) \quad (5.30a)$$

$$\dot{\tilde{\mathbf{x}}_{s2e}} = \bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) + \bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t) + \Delta \mathbf{d}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t) \quad (5.30b)$$

where the $\Delta \mathbf{d}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t) = \Delta \mathbf{f}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) + \Delta \mathbf{g}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t)$ is the perturbation in the hydrodynamic forces. The functions $\bar{\mathbf{f}}_{s1}(\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, t)$, $\bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t)$ and $\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t)$ are continuously differentiable in their arguments and locally Lipschitz as shown in the nonlinear properties (in the Section 4.4).

b) Also, the first equation is usually a function of its same states as seen in [90,91,92]. This is not true as (5.30a) contains two states. In our case, the inner loop controller $\bar{\mathbf{u}}_1$ is required to make $\tilde{\mathbf{x}}_{s2e}$ asymptotically stable such that $\tilde{\mathbf{x}}_{s1e}$ is asymptotically stable with the outer loop controller $\bar{\mathbf{u}}_0$.

c) In most cascaded stability proof, the system is asymptotic stability with respect to an equilibrium point. However in the present situation, the roll and pitch angles are not fully actuated and thus remain unspecified at all time. As a result, there is a need to use an equilibrium sub-manifold [10] instead of an equilibrium point to represent situation where the roll and pitch orientations of the ROV in 3D-space are unspecified. Fortunately, the roll and pitch angles are bounded (as shown in Theorem 4.1) and will not destabilize the ROV. To take this into account, the equilibrium sub-manifold for each subsystem is defined as follows:

$$\mathbf{E}_s = \{(u, v, w, r, x, y, z, \psi, p, q, \phi, \theta) | p, q = 0, \phi, \theta = \text{const}\} \quad (5.31a)$$

$$\mathbf{E}_h = \{(u, v, r, x, y, \psi) | p = q = 0, \phi = \theta = \text{const}\} \quad (5.31b)$$

$$\mathbf{E}_v = \{(u, w, q, x, z, \theta) | p = 0, \phi = \text{const}\} \quad (5.31c)$$

5.5.1 Cascaded Systems Stability

To ensure that the above-mentioned cascaded controller design for position tracking and stabilization is asymptotically stable (for the pipeline tracking), a cascaded stability proof similar to [90,91] is needed. Due to the above differences in the cascaded structure, there is a need to reformulate [Theorem 3 of 92] for the asymptotic stability of the nonlinear-cascaded system about the submanifold equilibrium.

In the Theorem 5.1, the conditions for the existence of the Lyapunov function under the converse Lyapunov theorem [107] are used. The Theorem provides sufficient conditions to guarantee the asymptotic stability of the nonlinear-cascaded system in (5.30). With the Proportional-decoupled controller used in the inner loop $\bar{\mathbf{u}}_1$, the sub-system involving the states $\tilde{\mathbf{x}}_{s2e}$ in (5.30b) is asymptotically stable. As the $\dot{V}_o(\tilde{\mathbf{x}}_{s1e}, t)$ for the outer loop is semi-negative definite with the Proportional-decoupled controller $\bar{\mathbf{u}}_0$, the $\tilde{\mathbf{x}}_{s1e}$ is also asymptotically stable. As the result, the entire cascaded system is asymptotically stable.

Theorem 5.1: Asymptotic stability of the cascaded system

The nonlinear time-varying system for the station-keeping mode in the (5.30a) and (5.30b) are used. The assumption used is that the inner and outer controllers can be designed such that the nonlinear coupled dynamic terms and hydrodynamic perturbations are cancelled. This results in some upper bound on the variables as seen in the (5.34) and (5.35).

With that, if the conditions (a) to (b) below are satisfied then the cascaded system defined by the equation (5.30a) and (5.30b) are said to be asymptotically stable.

a) With the Proportional-decoupled inner loop control law $\bar{\mathbf{u}}_1$, the system $\dot{\tilde{\mathbf{x}}}_{s2e}$ is asymptotically stable if the Lyapunov function $V_I(\tilde{\mathbf{x}}_{s2e}, t), V_I : \Re^4 \times \Re_{\geq 0} \rightarrow \Re_{\geq 0}$ satisfies the following inequality used in the converse Lyapunov theorem [107]:

$$\alpha_1(\|\tilde{\mathbf{x}}_{s2e}\|) \leq V_I(\tilde{\mathbf{x}}_{s2e}, t) \leq \alpha_2(\|\tilde{\mathbf{x}}_{s2e}\|) \quad (5.32a)$$

$$\frac{\partial V_I}{\partial t} + \frac{\partial V_I}{\partial \tilde{\mathbf{x}}_{s2e}} \bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) \leq -\alpha_3(\|\tilde{\mathbf{x}}_{s2e}\|) \quad (5.32b)$$

$$\left\| \frac{\partial V_I}{\partial \tilde{\mathbf{x}}_{s2e}} \right\| \|\tilde{\mathbf{x}}_{s2e}\| \leq \alpha_4(\|\tilde{\mathbf{x}}_{s2e}\|) \quad (5.32c)$$

for all $(\tilde{\mathbf{x}}_{s2e}, t) \in \mathcal{H}^4 \times [0, \infty)$ where $\alpha_1(\cdot), \alpha_2(\cdot), \alpha_3(\cdot), \alpha_4(\cdot)$ are class κ functions and there exists a positive constant b_1 such that the equation satisfies:

$$[d_1(\|\tilde{\mathbf{x}}_{s2e}\|) + \alpha_3(\|\tilde{\mathbf{x}}_{s2e}\|)]\alpha_4(\|\tilde{\mathbf{x}}_{s2e}\|)/\|\tilde{\mathbf{x}}_{s2e}\| \leq b_1 V_I(\tilde{\mathbf{x}}_{s2e}, t) \quad (5.33)$$

and the $\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t)$ and $\Delta \mathbf{d}_{s2}(\tilde{\mathbf{x}}_{s2e}, t)$ are bounded as follows:

$$\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) \leq -d_1(\|\tilde{\mathbf{x}}_{s2e}\|) \quad (5.34)$$

$$\Delta \mathbf{d}_{s2}(t) \leq d_4 \quad (5.35)$$

where $d_1(\cdot)$ is continuous function and d_4 is a positive constant.

b) With $\tilde{\mathbf{x}}_{s2e}$ asymptotically stable, $\tilde{\mathbf{x}}_{s1e}$ is asymptotically stable with the Proportional-decoupled outer loop controller $\bar{\mathbf{u}}_0$ as seen in Step (e) of Section 5.5.2.

Proof:

First, it is necessary to check if $\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t)$ is semi-negatively definite. The time derivative of $V_I(\tilde{\mathbf{x}}_{s2e}, t)$ in (5.33a) is first evaluated along the trajectories of $\tilde{\mathbf{x}}_{s2e}$:

$$\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t) = \frac{\partial V_I}{\partial t} + \frac{\partial V_I}{\partial \tilde{\mathbf{x}}_{s2e}} \bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) + \frac{\partial V_I}{\partial \tilde{\mathbf{x}}_{s2e}} \bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) \quad (5.36)$$

Using (5.32b) and (5.34), it is possible to write:

$$\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t) \leq -[d_1(\|\tilde{\mathbf{x}}_{s2e}\|) + \alpha_3(\|\tilde{\mathbf{x}}_{s2e}\|)] \left\| \frac{\partial V_I}{\partial \tilde{\mathbf{x}}_{s2e}} \right\| \quad (5.37)$$

Substituting (5.32c) into (5.37) yields:

$$\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t) \leq -[d_1(\|\tilde{\mathbf{x}}_{s2e}\|) + \alpha_3(\|\tilde{\mathbf{x}}_{s2e}\|)]\alpha_4(\|\tilde{\mathbf{x}}_{s2e}\|)/\|\tilde{\mathbf{x}}_{s2e}\| \quad (5.38)$$

Using the defined function, $[d_1(\|\tilde{\mathbf{x}}_{s2e}\|) + \alpha_3(\|\tilde{\mathbf{x}}_{s2e}\|)]\alpha_4(\|\tilde{\mathbf{x}}_{s2e}\|)/\|\tilde{\mathbf{x}}_{s2e}\| \leq b_1 V_I(\tilde{\mathbf{x}}_{s2e}, t)$ in (5.33), equation (5.38) becomes:

$$\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t) \leq b_1 V_I(\tilde{\mathbf{x}}_{s2e}, t) \quad (5.39)$$

The solution of $\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t)$ becomes:

$$V_I(\tilde{\mathbf{x}}_{s2e}, t) \leq e^{-b_1(t-t_0)} V_I(\tilde{\mathbf{x}}_{s2e}(t_0), t_0) \quad (5.40)$$

If the initial time, $t_0 = 0$, the expression can be simplified to become:

$$V_I(\tilde{\mathbf{x}}_{s2e}, t) \leq e^{-b_1 t} V_I(\tilde{\mathbf{x}}_{s2e}(0)) \quad (5.41)$$

where b_1 is a positive constants. Equation (5.41) shows that the $V_I(\tilde{\mathbf{x}}_{s2e}, t)$ is exponentially bounded and hence implying that the $\tilde{\mathbf{x}}_{s2e}$ is also exponentially bounded.

To show that the cascaded system is asymptotically stable, the following computer simulation on one particular trajectory was performed. This offers some appreciation on the validity of the proof in Theorem 5.1 as a complete simulation on all possible trajectories is too exhaustive and impracticable. With that in mind, the chosen reference position $\mathbf{x}_{s1d} = [5 \ 0 \ 3 \ 0]^T$ to position $\mathbf{x}_{s1d} = [5 \ 0 \ 10 \ 0]^T$ with zero input for roll and pitch angle are used. As observed in Fig. 5.7a, the velocity (and position) response converges to the steady-state value and the roll and pitch motions are small and bounded. Notice that the yaw angle converged to a non-zero value, however it is quite small(=0.57 degree) at the stipulated simulation time. Thus the cascaded system is asymptotically stable at the equilibrium submanifold. With the proposed cascaded system, the ROV's whirling due to corolis and centripetal forces (as seen in Fig. 4.2) are removed as seen in Fig. 5.7b.

Figure 5.7a: Position (left) and velocity (right) response of cascaded system using nonlinear Proportional-decoupled controller

Figure 5.7b: Position of ROV in 3D space

In the second part of the proof, it is obvious that the $\tilde{\mathbf{x}}_{s1e}$ (as seen in Fig. 5.7) becomes asymptotically stable as the $\tilde{\mathbf{x}}_{s2e}$ is asymptotically stable with the Proportional-decoupled inner loop controller (as shown in the first part of the proof). With that, the Proportional-decoupled

outer loop controller, $\bar{\mathbf{u}}_0$ is designed such that it would not destabilized the entire state $\tilde{\mathbf{x}}_{s1e}$ as seen in the computation of $\dot{V}_0$ that is semi-negative definite for the case of station-keeping model (see Step (e) of Section 5.5.2). With the roll and pitch bounded, the nonlinear cascaded system as defined in (5.30) is asymptotically stable about the equilibrium submanifold.

5.5.2 Proportional-decoupled Cascaded Controller Design for Station-Keeping

In this Section, the robust decoupling controllers for inner and outer loop are designed for the subsystem such that the closed-loop cascaded system satisfied the Theorem 5.1. The followings are the five steps used in the controller design.

a) **Design inner loop dynamic controller.** The inner control law $\bar{\mathbf{u}}_1$ to be designed will have three components such as:

$$\begin{aligned} \bar{\mathbf{u}}_1 &= \bar{\mathbf{u}}_{1,p} + \bar{\mathbf{u}}_{1,dyn} + \bar{\mathbf{u}}_{1,pert} \\ &= - \begin{bmatrix} k_{pu} & 0 & 0 & 0 \\ 0 & k_{pv} & 0 & 0 \\ 0 & 0 & k_{pw} & 0 \\ 0 & 0 & 0 & k_{pr} \end{bmatrix} \tilde{\mathbf{x}}_{s2e} + \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{m - X_{\dot{u}}} & 0 & 0 & 0 \\ 0 & \frac{m - Y_{\dot{v}} - \Delta Y_{\dot{v}}}{m - Y_{\dot{v}}} & 0 & 0 \\ 0 & 0 & \frac{m - Z_{\dot{w}} - \Delta Z_{\dot{w}}}{m - Z_{\dot{w}}} & 0 \\ 0 & 0 & 0 & \frac{I_z - N_{\dot{r}} - \Delta N_{\dot{r}}}{I_z - N_{\dot{r}}} \end{bmatrix} \begin{bmatrix} (Y_{\dot{v}} - m)\tilde{v}_e \tilde{r}_e \\ (m - X_{\dot{u}})\tilde{u}_e \tilde{r}_e \\ 0 \\ (-Y_{\dot{v}} + X_{\dot{u}})\tilde{u}_e \tilde{v}_e \end{bmatrix} \\ &\quad - \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{\Delta X_{\dot{u}}} & 0 & 0 & 0 \\ 0 & \frac{m - Y_{\dot{v}} - \Delta Y_{\dot{v}}}{\Delta Y_{\dot{v}}} & 0 & 0 \\ 0 & 0 & \frac{m - Z_{\dot{w}} - \Delta Z_{\dot{w}}}{\Delta Z_{\dot{w}}} & 0 \\ 0 & 0 & 0 & \frac{I_z - N_{\dot{r}} - \Delta N_{\dot{r}}}{\Delta N_{\dot{r}}} \end{bmatrix} \begin{bmatrix} (\Delta Y_{\dot{v}} - m)\tilde{v}_e \tilde{r}_e \\ (m - \Delta X_{\dot{u}})\tilde{u}_e \tilde{r}_e \\ 0 \\ (-\Delta Y_{\dot{v}} + \Delta X_{\dot{u}})\tilde{u}_e \tilde{v}_e \end{bmatrix} \end{aligned} \quad (5.42)$$

where the component $\bar{\mathbf{u}}_{1,p}$ ensure a faster response. The component $\bar{\mathbf{u}}_{1,dyn}$ is to cancel some nonlinear coupled dynamic behavior, for which there is a prior knowledge and to ensure robust stabilization with least amount of coupling. The component $\bar{\mathbf{u}}_{1,pert}$ is to attenuate the perturbation due to hydrodynamic uncertainties.

Hence, the inner loop controller makes the state velocity error to zero and keep it small until the ROV reaches the desired position in space.

b) Substitute $\bar{\mathbf{u}}_1$ into $\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t)$ and $\Delta \mathbf{d}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t)$, they are bounded as shown in (5.34) and (5.35) respectively.

By substituting the $\bar{\mathbf{u}}_1$ into $\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t)$, $\bar{\mathbf{u}}_1$ disappeared and becomes:

$$\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) = - \left[\overbrace{\begin{bmatrix} k_{pu} & 0 & 0 & 0 \\ 0 & k_{pv} & 0 & 0 \\ 0 & 0 & k_{pw} & 0 \\ 0 & 0 & 0 & k_{pr} \end{bmatrix}}^{\mathbf{K}_{pv}} \tilde{\mathbf{x}}_{s2e} - \mathbf{K}_{02} \overbrace{\begin{bmatrix} \cos \psi & \sin \psi & 0 & 0 \\ -\sin \psi & \cos \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{\tilde{\mathbf{x}}_{s2}} \tilde{\mathbf{x}}_{s1} \right] - \begin{bmatrix} (m - X_{\ddot{u}}) & 0 & 0 & 0 \\ 0 & (m - Y_{\ddot{v}}) & 0 & 0 \\ 0 & 0 & (m - Z_{\ddot{w}}) & 0 \\ 0 & 0 & 0 & (I_z - N_{\ddot{r}}) \end{bmatrix}^{-1} \boldsymbol{\tau}_s \quad (5.43)$$

Then, apply triangular inequality rule, equation (5.43) is bounded as:

$$\|\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t)\| \leq -d_1(\|\tilde{\mathbf{x}}_{s2e}\|) \quad (5.44)$$

where the input thrust $\boldsymbol{\tau}_s$ is finite and bounded.

By substituting $\bar{\mathbf{u}}_1$ into $\Delta \mathbf{d}_{s2}(t)$, it becomes:

$$\Delta \mathbf{d}_{s2}(t) = \begin{bmatrix} \frac{1}{X_{\ddot{u}}} & 0 & 0 & 0 \\ 0 & \frac{1}{Y_{\ddot{v}}} & 0 & 0 \\ 0 & 0 & \frac{1}{Z_{\ddot{w}}} & 0 \\ 0 & 0 & 0 & \frac{1}{N_{\ddot{r}}} \end{bmatrix} \boldsymbol{\tau}_s \quad (5.45)$$

Then, apply triangular inequality rule on $\Delta \mathbf{d}_{s2}(t)$ yields:

$$\|\Delta \mathbf{d}_{s2}(t)\| \leq d_4 \quad (5.46)$$

c) **Check whether the inner loop is asymptotically stable.** With $\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t)$ and $\Delta \mathbf{d}_{s2}(t)$ are bounded, $\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t)$ must be semi-negative definite. Let $V_I(\tilde{\mathbf{x}}_{s2e}, t) = \frac{1}{2} \tilde{\mathbf{x}}_{s2e}^T \mathbf{M}_s \tilde{\mathbf{x}}_{s2e}$ (with only K.E. term as shown in Section 4.4.5), it becomes obvious that it satisfies (5.32a) and (5.32c) as $V_I(\tilde{\mathbf{x}}_{s2e}, t)$ is positive definite. The remaining is to verify whether the inequality in (5.32b) is satisfied. By performing the time derivative of $V_I(\tilde{\mathbf{x}}_{s2e}, t)$, it becomes:

$$\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t) = \tilde{\mathbf{x}}_{s2e}^T \mathbf{M}_s \dot{\tilde{\mathbf{x}}}_{s2e} \leq -\mathbf{D}_s \|\tilde{\mathbf{x}}_{s2e}\|^2 - d_1 (\|\tilde{\mathbf{x}}_{s2e}\|) \|\tilde{\mathbf{x}}_{s2e}\| \quad (5.47)$$

In (5.48) $\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t)$ is semi-negative definite. $V_I(\tilde{\mathbf{x}}_{s2e}, t)$ is positive definite and the set $\mathbf{R}$ defined by $V_I(\tilde{\mathbf{x}}_{s2e}, t) = 0$ contain no trajectory of $\dot{\tilde{\mathbf{x}}}_{s2e}$ other than the trivial trajectory $\tilde{\mathbf{x}}_{s2e} = 0$ in a certain neighbor of the origin. This trivial trajectory $\tilde{\mathbf{x}}_{s2e} = 0$ that cause $\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t) = 0$ can be shown using Barbalat's[107] lemma. By applying the lemma, it shows that the $\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t) \rightarrow 0 \Rightarrow \tilde{\mathbf{x}}_{s2e} \rightarrow 0$ as $t \rightarrow \infty$. Hence, the inner closed loop system $\tilde{\mathbf{x}}_{s2e}$ is asymptotically stable.

d) **Design outer loop dynamic controller.** The outer control law $\bar{\mathbf{u}}_O$ to be designed will have two components satisfying:

$$\begin{aligned} \bar{\mathbf{u}}_O &= \bar{\mathbf{u}}_{O, \text{dyn}} + \bar{\mathbf{u}}_{O, \text{p}} \\ &= \begin{bmatrix} -u_d \cos \psi - v_d \sin \psi + \hat{u} - \tilde{r} \tilde{y}_e \\ u_d \sin \psi - v_d \cos \psi + \tilde{r} \tilde{x}_e + \hat{v} \\ -w_d + \hat{w} \\ -r_d + \hat{r} \end{bmatrix} - \overbrace{\begin{bmatrix} k_{px} & 0 & 0 & 0 \\ 0 & k_{py} & 0 & 0 \\ 0 & 0 & k_{pz} & 0 \\ 0 & 0 & 0 & k_{p\psi} \end{bmatrix}}^{\mathbf{K}_{px}} \tilde{\mathbf{x}}_{s1e} \end{aligned} \quad (5.48)$$

where the component $\bar{\mathbf{u}}_{O, \text{p}}$ is to ensure a faster response and $\bar{\mathbf{u}}_{O, \text{dyn}}$ is to cancel some nonlinear couple dynamic behavior for which there is a prior knowledge and also to ensure robust tracking to the desired trajectory with least amount of coupling.

e) **Check whether the outer loop is asymptotically stable.** Applying the same method in Theorem 5.1 for $\tilde{\mathbf{x}}_{s1e}$ and let $V_O(\tilde{\mathbf{x}}_{s1e}, t) = \frac{1}{2} \tilde{\mathbf{x}}_{s1e}^T \mathbf{M}_s \tilde{\mathbf{x}}_{s1e}$, it is obvious that both condition stated in (5.32a) and (5.32c) are satisfied as the $V_O(\tilde{\mathbf{x}}_{s1e}, t)$ is positive definite. The remaining is to check whether the inequality in (5.32b) is satisfied. By performing the time derivative of $V_O(\tilde{\mathbf{x}}_{s1e}, t)$, it becomes:

$$\begin{aligned} \dot{V}_O(\tilde{\mathbf{x}}_{s1e}, t) &= \tilde{\mathbf{x}}_{s1e}^T \mathbf{M}_s \dot{\tilde{\mathbf{x}}}_{s1e} \\ &= \tilde{\mathbf{x}}_{s1e}^T \mathbf{M}_s \left(- \begin{bmatrix} k_{px} & 0 & 0 & 0 \\ 0 & k_{py} & 0 & 0 \\ 0 & 0 & k_{pz} & 0 \\ 0 & 0 & 0 & k_{p\psi} \end{bmatrix} \tilde{\mathbf{x}}_{s1e} - \mathbf{K}_{01} \tilde{\mathbf{x}}_{s1} \right) \\ \|\dot{V}_O\| &\leq -d_5 (\|\tilde{\mathbf{x}}_{s1e}\|^2) \end{aligned} \quad (5.49)$$

where

$$\dot{\tilde{\mathbf{x}}}_{s1e} = - \begin{bmatrix} k_{px} & 0 & 0 & 0 \\ 0 & k_{py} & 0 & 0 \\ 0 & 0 & k_{pz} & 0 \\ 0 & 0 & 0 & k_{p\psi} \end{bmatrix} \tilde{\mathbf{x}}_{s1e} - \mathbf{K}_{01} \tilde{\mathbf{x}}_{s1} \quad (5.50)$$

Since the $\dot{V}_O(\tilde{\mathbf{x}}_{s1e}, t)$ is semi-negative definite, the outer-loop subsystem $\tilde{\mathbf{x}}_{s1e}$ is asymptotically stable as shown by the similar method in (5.47).

As shown in the first part of the proof, the inner loop controller $\bar{\mathbf{u}}_1$ resulted in $\bar{\mathbf{g}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t)$ and $\Delta \mathbf{d}_{s2}(t)$ being bounded and hence the subsystem $\tilde{\mathbf{x}}_{s2e}$ becomes asymptotically stable. Since the $\tilde{\mathbf{x}}_{s2e}$ is asymptotically stable and with the outer-loop controller $\bar{\mathbf{u}}_0$, the $\tilde{\mathbf{x}}_{s1e}$ is also asymptotically stable. In summary, the entire cascaded system is asymptotically stable. The SIMULINKTM block diagrams for the PI decoupled inner and outer controllers derived in (5.42) and (5.48) are shown in Fig. A1.12 and Fig. A1.13 in Appendix A1.

5.5.3 Proportional-decoupled Cascaded Control for Horizontal Plane Subsystem

Repeat the above controller design as shown in Step (a) to (e), the inner and outer loop controllers for the horizontal plane subsystem can be computed. Equation (5.42) becomes:

$$\begin{aligned}
\bar{\mathbf{u}}_{\text{th}} &= \bar{\mathbf{u}}_{\text{I,p}} + \bar{\mathbf{u}}_{\text{I,dyn}} + \bar{\mathbf{u}}_{\text{I,pert}} \\
&= - \begin{bmatrix} k_{pu} \tilde{u}_e \\ k_{pv} \tilde{v}_e \\ k_{pr} \tilde{r}_e \end{bmatrix} + \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{(m - X_{\dot{u}})} & 0 & 0 \\ 0 & \frac{m - Y_{\dot{v}} - \Delta Y_{\dot{v}}}{(m - Y_{\dot{v}})} & 0 \\ 0 & 0 & \frac{I_z - N_{\dot{r}} - \Delta N_{\dot{r}}}{(I_z - N_{\dot{r}})} \end{bmatrix} \begin{bmatrix} (Y_{\dot{v}} - m) \tilde{v}_e \tilde{r}_e \\ (m - X_{\dot{u}}) \tilde{u}_e \tilde{r}_e \\ (-Y_{\dot{v}} + X_{\dot{u}}) \tilde{u}_e \tilde{v}_e \end{bmatrix} \\
&\quad - \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{\Delta X_{\dot{u}}} & 0 & 0 \\ 0 & \frac{m - Y_{\dot{v}} - \Delta Y_{\dot{v}}}{\Delta Y_{\dot{v}}} & 0 \\ 0 & 0 & \frac{I_z - N_{\dot{r}} - \Delta N_{\dot{r}}}{\Delta N_{\dot{r}}} \end{bmatrix} \begin{bmatrix} (\Delta Y_{\dot{v}} - m) \tilde{v}_e \tilde{r}_e \\ (m - \Delta X_{\dot{u}}) \tilde{u}_e \tilde{r}_e \\ (-\Delta Y_{\dot{v}} + \Delta X_{\dot{u}}) \tilde{u}_e \tilde{v}_e \end{bmatrix} \quad (5.51)
\end{aligned}$$

Equation (5.48) becomes:

$$\begin{aligned}
\bar{\mathbf{u}}_{\text{Oh}} &= \bar{\mathbf{u}}_{\text{O,dyn}} + \bar{\mathbf{u}}_{\text{O,p}} \\
&= \begin{bmatrix} -u_d \cos \psi - v_d \sin \psi + \hat{u} - \tilde{r} \tilde{y}_e \\ u_d \sin \psi - v_d \cos \psi + \tilde{r} \tilde{x}_e + \hat{v} \\ -r_d + \hat{r} \end{bmatrix} - \begin{bmatrix} k_{px} \tilde{x}_e \\ k_{py} \tilde{y}_e \\ k_{p\psi} \tilde{\psi}_e \end{bmatrix} \quad (5.52)
\end{aligned}$$

where the selection of the controller terms are similar to the case of station-keeping mode.

5.5.4 Proportional-decoupled Cascaded Control for Vertical Plane Subsystem

Similarly repeat the above controller design as shown in Step (a) to (e), the inner and outer loop controllers for the vertical plane subsystems can be computed. Equation (5.42) becomes:

$$\begin{aligned}
\bar{\mathbf{u}}_{\text{Iv}} &= \bar{\mathbf{u}}_{\text{I,p}} + \bar{\mathbf{u}}_{\text{I,dyn}} + \bar{\mathbf{u}}_{\text{I,pert}} \\
&= - \begin{bmatrix} k_{pu} \tilde{u}_e \\ k_{pv} \tilde{v}_e \\ k_{pq} \tilde{q}_e \end{bmatrix} + \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{m - X_{\dot{u}}} & 0 & 0 \\ 0 & \frac{m - Z_{\dot{w}} - \Delta Z_{\dot{w}}}{m - Z_{\dot{w}}} & 0 \\ 0 & 0 & \frac{I_y - M_{\dot{q}} - \Delta M_{\dot{q}}}{I_y - M_{\dot{q}}} \end{bmatrix} \begin{bmatrix} (m - Z_{\dot{w}}) \tilde{w}_e \tilde{q}_e \\ (X_{\dot{u}} - m) \tilde{u}_e \tilde{q}_e \\ (Z_{\dot{w}} - X_{\dot{u}}) \tilde{u}_e \tilde{w}_e \end{bmatrix} \\
&\quad - \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{\Delta X_{\dot{u}}} & 0 & 0 \\ 0 & \frac{m - Z_{\dot{w}} - \Delta Z_{\dot{w}}}{\Delta Z_{\dot{w}}} & 0 \\ 0 & 0 & \frac{I_y - M_{\dot{q}} - \Delta M_{\dot{q}}}{\Delta M_{\dot{q}}} \end{bmatrix} \begin{bmatrix} (m - \Delta Z_{\dot{w}}) \tilde{w}_e \tilde{q}_e \\ (\Delta X_{\dot{u}} - m) \tilde{u}_e \tilde{q}_e \\ (\Delta Z_{\dot{w}} - \Delta X_{\dot{u}}) \tilde{u}_e \tilde{w}_e \end{bmatrix} \quad (5.53)
\end{aligned}$$

Equation (5.48) becomes:

$$\begin{aligned}\bar{\mathbf{u}}_{Ov} &= \bar{\mathbf{u}}_{O,dyn} + \bar{\mathbf{u}}_{O,pi} \\ &= \begin{bmatrix} -u_d \cos \theta - w_d \sin \theta + \hat{u} - \hat{q}\tilde{z}_e \\ u_d \sin \theta - w_d \cos \theta + \hat{q}\tilde{x}_e + \hat{w} \\ -q_d + \hat{q} \end{bmatrix} - \begin{bmatrix} k_{px}\tilde{x}_e \\ k_{pz}\tilde{z}_e \\ k_{p\theta}\tilde{\theta}_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ (z_G - z_B)W \sin \tilde{\theta}_e \end{bmatrix} \end{aligned} \quad (5.54)$$

where the selection of the terms are similar to the case of station-keeping mode.

5.6 Comparisons of Control Systems Design

5.6.1 Time Domain Response

Due to the steady-state time response requirement as stipulated in Table 1.2, the measurement in time response is only needed though in some cases frequency domain is used to understand the noise frequency impact on the ROV systems gain and phase. However, in this thesis, to synchronize with the pool test data in the time domain, the observer data were extracted to compare with the simulated time domain result.

With that in mind, time domain performance of the nonlinear decoupled controller was investigated and compared with other controllers such as backstepping (see Appendix C2), single loop PID controller [33] and other controllers such as P and PI controller. The controller parameters (as seen in Table 5.2) used in the simulation was obtained after some initial experimentation. The simulation was executed in 200 seconds using an ODE solver with variable step size and a relative tolerance of 0.001. As shown in Fig. A1.12 and A1.13 (in Appendix A1), the block diagrams of the nonlinear robust decoupled controller (for the inner and outer-loop) were built using available blocks in the RDA written in the MATLABTM scripting language and graphical block diagram simulation, SIMULINKTM. The added mass with off-diagonal term and quadratic damping are included in the simulation.

The desired position commands values, $\mathbf{x}_{1d} = [5 \ 0 \ 3 \ 0 \ 0 \ 0]^T$ are chosen to cater to the limited number of sensors currently fitted on the vehicle. The initial position is set as $\mathbf{x}_{10} = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$. To cater for both the stabilization and tracking operation, the ROV will be in the station-keeping mode after the desired position in the vertical plane is reached. The response of the

unactuated states, the angles and velocities for both roll and pitch DOF are plotted in Fig. 5.8 and 5.9 as well; both are asymptotically stable and bounded.

As observed in Fig. 5.8 and 5.9, the nonlinear Proportional-decoupled controller has a faster response with a shorter settling time. The output approaches the reference input, $x = 5\text{m}$ and $z = 3\text{m}$ with smaller steady state position and velocities error even under hydrodynamic perturbations. Besides, the Proportional-decoupled controller exhibits less oscillatory response in both position and velocity as compared to other type of controllers.

On the other hand, the back-stepping controller has the highest steady-state position and velocity errors. As observed, the single loop PID and cascaded PI-P are both oscillatory in their respective position and velocity responses. In addition, the settling times are longer as compared to the nonlinear Proportional-decoupled controller. As shown in Fig. 5.9, the PI and single loop PID exceeded the maximum velocity of 0.6m/s . From the responses plotted in Fig. 5.8 and 5.9, all controllers exhibit self-regulation in the roll and pitch motions. Alternatively, the time domain results are summarized in Table 5.3.

Controller Type	Components	Parameters
Decoupled P-P	Inner Controller	$k_{pu} = k_{pv} = k_{pw} = k_{pr} = 1$
	Outer Controller	$k_{px} = k_{py} = k_{pz} = k_{p\psi} = 1$
	Observer	$K_{01} = K_{02} = 0.1$
Backstepping P-P	Inner Controller	$k_{pu} = k_{pv} = k_{pw} = k_{pr} = 22$
	Outer Controller	$k_{px} = k_{py} = k_{pz} = k_{p\psi} = 1$
	Observer	$K_{01} = K_{02} = 0.1$
P-P	Inner Controller	$k_{pu} = k_{pv} = k_{pw} = k_{pr} = 1$
	Outer Controller	$k_{px} = k_{py} = k_{pz} = k_{p\psi} = 1$
	Observer	$K_{01} = K_{02} = 0.1$
PID	Controller	$k_{px} = k_{py} = k_{pz} = k_{p\psi} = 1$
	Observer	$K_{01} = K_{02} = 0.1$
PI-P	Inner Controller	$k_{pu} = k_{pv} = k_{pw} = k_{pr} = 1$
	Outer Controller	$k_{px} = k_{py} = k_{pz} = k_{p\psi} = 1$
	Observer	$K_{01} = K_{02} = 0.1$

Table 5.2: Controller parameters for simulation

Figure 5.8: Position response of all controllers

Figure 5.9: Velocity response of all controllers

5.6.2 Robustness and Stability Performance

A ten percent perturbation in the added mass and damping matrices was used to test the robustness of the proposed controller design. From the simulation and experimental results, it was found that the added mass matrix could vary around three percent. Hence a ten percent

perturbation is chosen to cover the worst case in [96]. This applies to the damping matrix as well. For the worst-case simulation, the upper limits of the perturbations in Section 3.4 were used. In addition to the time domain performances, addition performance indices such as the robustness and stability of the controllers shown in Table 5.3 are seen to be satisfied. Table 1.2 shows the performances of each controller against the given operating specifications.

The plot for each performance parameters in Table 5.3 can be found in Appendix C3. As observed, the input thrust for the nonlinear Proportional-decoupled controller is lower compared with the single-loop PID and PI controllers. This implies that the control effort used to maneuver the ROV is reduced. To check for the robustness of the cascaded system due to these perturbations, the sensitive of input to the position and velocity output was used. The proposed Proportional-decoupled controller has the lowest value in the sensitivity to the perturbation in both the position and velocity. This implies that the output is less susceptible or sensitive to model parameter changes.

For the cascaded-loop stability, $\dot{V}_I(\tilde{\mathbf{x}}_{s2e}, t)$ and $\dot{V}_O(\tilde{\mathbf{x}}_{s1e}, t)$ for the inner and outer loops are all semi-negative definite and thus satisfied the asymptotic stability criteria in Theorem 5.1. As observed, the Proportional-decoupled controllers meet the following condition:

- positive definite $V_I(\tilde{\mathbf{x}}_{s2e}, t) > 0$, $V_O(\tilde{\mathbf{x}}_{s1e}, t) > 0$ (see Appendix C3),
- semi-negative definite $\dot{V}_I(\tilde{\mathbf{x}}_{s1e}, t) \leq 0$, $\dot{V}_O(\tilde{\mathbf{x}}_{s1e}, t) < 0$ (see Table 5.3 or see Appendix C3),
- Barbalat's lemma: $V_I(\tilde{\mathbf{x}}_{s2e}, t) \rightarrow 0$, $\|\tilde{\mathbf{x}}_{s1e}\| \rightarrow 0$ and $V_O(\tilde{\mathbf{x}}_{s1e}, t) \rightarrow 0$, $\|\tilde{\mathbf{x}}_{s2e}\| \rightarrow 0$ (see Fig. C3.11 in Appendix C3).

Hence, the ROV is asymptotically stable at an equilibrium submanifold under hydrodynamic perturbations. Besides, the Proportional-decoupled controller is able to perform simultaneous stabilization and tracking operation as seen in the small steady state position error and velocity error (see Table 5.3). From Table 5.3, there is no single controller that can fully satisfy all the desired closed loop response specifications. This is due to the fact that each controller design has their strength and weakness.

However, it would be advantageous if each of these controllers (such as the Proportional-decoupled and P-P controllers) can be selectively used under appropriate condition that favors their respective advantages to improve upon the steady state response, to get a reduced thrust

level and an improved robustness to hydrodynamic perturbations. This would need a switched system, which switches between these controllers under certain switching condition. It would be presented in Chapter 6.

Item	Parameters	Proportional-decoupled	Backstepping	P-P	Single-PID	PI-P
1	Settling time-position (in seconds)	10	200	50	150	200
	Settling time-velocity(in seconds)	15	200	60	150	150
2	Percent overshoot-position	0	100	0	80	80
	Percent overshoot - velocity	35	20	25	60	60
3	Steady state position error	0.027	3.730	0.055	1.570	0.335
4	Steady state velocity error	0.002	0.002	0.002	0.220	0.286
5	Thrust- Euclidean norm	98.61	21.17	25.25	339.4	260.1
6	Stability- inner loop($\dot{V}_i(\tilde{\mathbf{x}}_{s2e}, t) \leq 0$)	-5675	-9837	-5302	-6487	-6229
	Stability- outer loop($\dot{V}_o(\tilde{\mathbf{x}}_{s1e}, t) \leq 0$)	-1925	-3968	-1760	-2038	-1928
7	Input perturbation to pos	147.0	327.1	173.7	753.1	148.1
	Input perturbation to vel	1797	1797	1797	1797	1797
8	Radially unbounded	YES	YES	YES	YES	YES
9	Simultaneous stabilization & tracking	YES	YES	YES	NO	YES

Table 5.3: Controller comparison against various parameters

5.7 Summary

An extension to the work on output feedback controller for stabilization and tracking [65] of a ROV for underwater pipeline inspection has been presented. The proposed cascaded structure of the RRC ROV had been described followed by the nonlinear full states observer design. For the robust cascaded controller design, a nonlinear Proportional-decoupled controller was designed for both the inner and outer loop to cancel the persistent velocities cross coupling, nonlinearity and hydrodynamic perturbation for different plane models. This provides a simultaneous stabilization and position tracking (or even independent stabilization as seen in Chapter 6) for different maneuvers. The controller design is asymptotically stable in both

output positions and velocity as shown in the cascaded stability proof of Theorem 5.1. Since the disturbances are bounded and control laws for each loop are asymptotically stable, then the cascaded system is said to be asymptotically stable. Applying this concept to our cascaded nonlinear system with output feedback under the hydrodynamic perturbations provides a systematic method to prove the stability of the proposed cascaded systems on RRC ROV.

The time domain responses using the nonlinear robust decoupled controllers were measured against the operating specification stated in Table 1.2. The simulations were done using the RDA package written in the MATLABTM scripting language and SIMULINKTM graphical block diagram libraries with VRML visualization capability (Chapter 7). As observed, the nonlinear Proportional-decoupled controller has a shorter settling time response. The steady state position error is smaller even under hydrodynamic perturbations. As observed in the simulation, the PI and single loop PID exceeded the maximum velocity of 0.6 m/s. Besides, the Proportional-decoupled controller is less oscillatory in both the position and velocity responses as compared with the other controllers. Hence, it seems to perform better in the time-domain specification as compared to other controllers.

To test for robustness, a ten percent perturbation on the added mass and linear damping was applied to the nominal model. The input thrust for the nonlinear Proportional-decoupled and P-P controllers were lower as compared with the single-loop PID and PI controllers. The proposed Proportional-decoupled controller had the lowest sensitivity value to perturbations in both the position and velocity. This implies that the output is less susceptible to model parameter changes. As the steady state position error and velocity errors are small, the Proportional-decoupled controller is able to achieve simultaneous stabilization and tracking despite the perturbations and is asymptotically stable to an equilibrium submanifold.

However, it is observed that the Proportional-decoupled controller has its strength and also its weakness – it exhibits a higher steady-state velocity error and a larger thrust is required compared with the back-stepping and P-P controllers. This shows that a single-fixed controller could not fully satisfy all the desired closed loop responses. To reduce the thrust required and to improve the steady state performance and robustness against perturbation, it is proposed in Chapter 6 to use a set of multiple controllers in the inner loop to stabilize the ROV and to use cascaded loops with multiple-controllers for simultaneous position tracking (in horizontal and vertical plane) and stabilization operation. The comparison results with other working ROV such as JHU and ROMEO ROV can be seen Chapter 7.

Chapter 6

Cascaded Supervisory Switching Control System Design

In the Chapter 5, it was observed that the nonlinear P-decoupled controller and other controllers have its strength and weakness. The simulation results show that a single-fixed controller cannot fully satisfy the desired closed loop response as the performance can suffer significantly when the ROV is tasked to perform the pipeline inspection that involves both station-keeping and pipeline tracking operation at points on the pipeline that emphasize on these nonlinearities and hydrodynamic uncertainties. Hence, what is required is a cascaded control scheme that is robust to the hydrodynamic uncertainties and is capable to switch between station-keeping and tracking operations to achieve less coupling in motion and control efforts.

Recently, a class of hybrid systems namely, a switched system that could fill this gap has been reported in [45,46]. Only in recent years, switching control [46,48,49,50] have been applied extensively. The need for switching [47] stems from the fact that typically no single controller can guarantee a desired behavior when connected to an uncertain e.g. ROV plant. Furthermore, many (if not most) of the dynamic systems encountered in practice involve both continuous and discrete dynamic such as the ROV that has an embedded system in which computer software interacts with physical devices such as sensors and thrusters.

To improve on the robustness of the cascaded system discussed in Chapter 5, it is proposed that a multiple controllers switching concept be adopted. The condition of the controllers switching is based on the different ROV maneuvering requirements defined by the reference pipeline input and the “energy” level of the system before the switching. This “energy” level refers to the Lyapunov Quadratic function [107]. On the other hand, the switching between the station-keeping and tracking operation is accomplished by a cascaded to inner loop switching

algorithm. This switching condition is based on the “energy” at the inner loop and the position tracking error.

The proposed cascaded switched system with inner loop controllers for the stabilization operation (or station keeping) and cascaded controllers for simultaneous position tracking (in the horizontal or vertical plane) can be seen in Fig. 6.1. With dwell time included in the switching, the asymptotic stability is improved.

To the best of my knowledge, the above-mentioned cascaded to inner loop switching using the switching condition based on the “energy” at the inner loop and the position tracking error for the three dimensional ROV system modeled with the hydrodynamic uncertainties has not been discussed in the literature.

Figure 6.1: Cascaded switched system

In summary, the Chapter is organized as follows. The cascaded switched system was modeled using the hybrid automaton described in Section 6.1. The proposed cascaded to inner loop switching method using a piecewise stable-switching signal (with sufficient dwell time) is mentioned in Section 6.2. Next, the cascaded to inner loop switching condition based on the “energy” at the inner loop and the position tracking error are defined in Section 6.3. The properties such as the existence and asymptotic stability at different controllers in the cascaded

switched systems were determined in Section 6.4. This is followed by the computer simulation of the proposed cascaded switched system to validate the cascaded switched system was shown in Section 6.5. Lastly, Section 6.6 summarizes the work done in this Chapter.

6.1 Cascaded Switched System Modeling

The hybrid automaton used for modeling the cascaded switched system of RRC ROV is first explained. For clarity, the definition used in the switched systems such as continuity, finite escape time, Zeno behavior, switching signal, switching condition and smooth function can be found in Appendix D1. This is followed by the derivation of the cascaded switched system dynamic using station keeping model as an example for the subsequent cascaded switched control system analysis and design.

6.1.1 Overview of Hybrid Automaton

Many models for hybrid systems in the literature are in fact for switched systems. For example, the Brockett's model [111], Koutsoukos' model and Pettersson's model [112] are actually switched systems. In [113], Antsaklis et al proposed a supervisory control framework for switched systems. Although Branicky's model [114] and Tittus' model [115] are more general, yet many examples and applications mentioned by them can be treated as switched systems. However, in this thesis work, a formal modeling framework using hybrid automaton for general hybrid systems, in particular the switched system is presented.

In the hybrid automaton modeling, switching usually takes place when an external event signal is received or an internal condition for the states and time evolution is satisfied. Switching triggered by an external event or external switching sequence is called signal externally forced switching (EFS), whereas switching triggered by an internal event signal is termed an internally forced switching (IFS). The combination of the EFS sequence and the IFS is called an overall switching sequence. It consists of piecewise continuous signals. Therefore, a switched system is a collection of dynamical subsystems related by a sequence of switching signals.

Cascaded switched systems are modeled as hybrid automata as shown in Fig. 6.2. They are represented by directed graphs with continuous (or discrete-time) dynamics in each of the node. The continuous flow evolves according to the differential equation specified in the current node of the graph. When conditions are fulfilled, a discrete transition may take place from one node to another connected by an edge. The continuous flow is then forced to satisfy the differential equation in the new node.

In this thesis, there are two switching scenarios in the cascaded switched system, namely: the cascaded to inner loops and inner and outer controllers switching.

(a) For the cascaded to inner switching in the RRC ROV, switching is triggered by the switching condition using the position tracking error and the “energy”. During the stabilization operation, only the inner loop is active whilst during both position tracking and stabilization operation, the cascaded loop is used. When the ROV approaches the desired position, the velocities become small. As a result, the outer loop can be disconnected and the inner loop controller is left to operate during the station keeping condition. This period provide a dwell time for the switching transient to diminish and hence improve the cascaded switched system stability. This dwell time can be inevitable as all electronic in controller, sensors and thrusters have some form of delay in response. However, to avoid infinite switching, the outer loop will not be switched back unless the next position command is issued. Additionally to enhance the flexibility of the switching, a “manual override” is used.

(b) On the other hand, the inner and outer controllers switching sequence is selected from a set of stabilizing controllers switching sequence based on the different phases of the vehicle positioning defined by the reference input to ROV. To improve on the stability of the cascaded switched system, the “energy” after the switching must be less than or equal to “energy” before the switching. As the dwell time is used during the inner and outer loop controllers’ switching, it allows the transience to diminish and thus improves upon the cascaded switched system stability. A simplify hybrid automaton to illustrate the above-mentioned switching mechanism is shown in Fig. 6.2.

Figure 6.2: Hybrid automaton for RRC ROV

6.1.2. Cascaded Switched System Dynamic Equation

The dynamic equation of the cascaded switched system for the station-keeping (represented by state, $\mathbf{x}_s$) model in (5.23) is formulated as equation (6.1) and (6.2). Similar method can be used to define the horizontal (represented by state, $\mathbf{x}_h$) and vertical (represented by state, $\mathbf{x}_v$) plane subsystems can be used, which are not shown here for brevity.

The station-keeping model in (5.23) with switching signal $\sigma(t)$ can be written as:

$$\dot{\tilde{\mathbf{x}}}_{s2e}(t) = \bar{\mathbf{f}}_{s2,\sigma(t)}(\tilde{\mathbf{x}}_{s2e}, t) + \bar{\mathbf{g}}_{s2,\sigma(t)}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_1, t) \quad (6.1)$$

$$\dot{\tilde{\mathbf{x}}}_{s1e}(t) = \bar{\mathbf{f}}_{s1,\sigma(t)}(\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_0, t) \quad (6.2)$$

where the $\tilde{\mathbf{x}}_{s2e} \in \mathbb{R}^4$ is the velocity error state, the $\tilde{\mathbf{x}}_{s1e} \in \mathbb{R}^3 \times S^1$ is the position tracking error in the body-fixed frame, the vector functions $\bar{\mathbf{f}}_{s1,\sigma(t)}, \bar{\mathbf{f}}_{s2,\sigma(t)}, \bar{\mathbf{g}}_{s2,\sigma(t)}$ are continuous and locally Lipschitz. The $\sigma(t) = \{\sigma_L(t), \sigma_{CO}(t), \sigma_{CI}(t)\}$ is the supervisory logic that is piecewise continuous from the right (see Appendix D1 for continuous from right) and the $\sigma_L(t) \in I_L \equiv \{0,1\}$ is the piecewise switching signal for the cascaded (=0) or inner loop(=1). On the other hands, $\sigma_{CI}(t) = \sigma_{CO}(t) \in I_C \equiv \{1,2,3\}$ is the piecewise switching signal for inner and outer controllers defined as:

$$\sigma_{CI}(t) = \sigma_{CO}(t) = \begin{cases} 1, & \text{if P - decoupled controller } (\bar{\mathbf{u}}_{i,1}), \text{ P - decoupled controller } (\bar{\mathbf{u}}_{o,1}); \\ 2, & \text{if P - decoupled controller } (\bar{\mathbf{u}}_{i,2}), \text{ P controller } (\bar{\mathbf{u}}_{o,2}); \\ 3, & \text{if P controller } (\bar{\mathbf{u}}_{i,3}), \text{ P - decoupled controller } (\bar{\mathbf{u}}_{o,3}); \end{cases} \quad (6.3)$$

where the P-decoupled controller (designed in Chapter 5) such as $\bar{\mathbf{u}}_{i,i}$ and $\bar{\mathbf{u}}_{o,i}$ used for the inner and outer loop consists of several components with intention to ensure a faster response, to cancel some nonlinear coupled dynamic behavior and to attenuate the perturbation due to hydrodynamic uncertainties. The control input used for inner or cascaded loop is defined as:

$$\sigma_L(t) = \begin{cases} \bar{\mathbf{u}}_{i,i}, & \text{if } \sigma_L(t) = 0 \\ \bar{\mathbf{u}}_{i,i}, \bar{\mathbf{u}}_{o,i}, & \text{if } \sigma_L(t) = 1 \end{cases} \quad (6.4)$$

at switching time t with $i \in I_C$. In addition, to describe the behavior of the state during the switching transition, the following expression is used. The inner loop switching with reset can be expressed as:

$$\tilde{\mathbf{x}}_{s2e}(t) = \mathbf{p}_s(\sigma_L(t), \sigma_L^-(t), \tilde{\mathbf{x}}_{s2e}^-(t)) \quad (6.5)$$

where $\mathbf{p}_s(\cdot)$ refers to the reset signal for the switching during the station-keeping mode and the $\sigma_L^-(t), \sigma_{CO}^-(t), \sigma_{CI}^-(t)$ are the known as previous switching signals.

The inner controller switching with reset can be expressed as:

$$\tilde{\mathbf{x}}_{s2e}(t) = \mathbf{p}_s(\sigma_{CI}(t), \sigma_{CI}^-(t), \tilde{\mathbf{x}}_{s2e}^-(t)) \quad (6.6)$$

The outer controller switching with reset can be expressed as:

$$\tilde{\mathbf{x}}_{s1e}(t) = \mathbf{p}_s(\sigma_{CO}(t), \sigma_{CO}^-(t), \tilde{\mathbf{x}}_{s1e}^-(t)) \quad (6.7)$$

6.1.3 Proposed Cascaded Supervisory Switching Law

As mentioned in Section 6.1.1, the switching of the inner loop is triggered by the “energy” in the inner loop and the position tracking error. On the other hand, the controllers switching sequence is selected from the set of stabilizing controllers based on different phase of vehicle’s positioning defined by the reference inputs.

Sufficient dwell time is introduced during the switching to inner loop and between controllers to improve the stability of the cascaded switched system. To orchestrate these switching, the supervisor is required to perform the switching of the cascaded to inner loop and controllers through the following proposed switching logic:

- Switching logic for the cascaded to inner loop switching
- Switching logic for the multiple inner and outer loop controllers switching

Fig. 6.3 summarizes the proposed switching concept in time graph. As shown in Fig. 6.3, the same controller is used for each input sequence. This input sequence represents the pipeline reference trajectory. As observed, each switching is governed by a set of switching conditions with a sufficient dwell time [117]. In this thesis work, finite switching is assumed since the ROV pipeline-tracking operation in Fig. 1.6 (in Chapter 1) has to be completed in finite time.

Figure 6.3: Proposed switching concept for cascaded to inner loop and controllers switching (assuming three input sequences are used)

6.2 Cascaded Supervisory Switching Law for Cascaded to Inner Loop Switching

The switching of the inner loop is triggered by the performance index determined by the “energy” in the inner loop and the position tracking error. To orchestrate these switching, the supervisor is required to perform the switching of the cascaded to inner loop through the following proposed switching logic in Section 6.2.1. This is followed by the classification of switching regions for the cascaded and inner loop in Section 6.2.2.

The cascaded to inner loop switching law for station-keeping (represented by state, $\mathbf{x}_s$) model in 5.23) will be formulated in this section. Since the concept is similar, for brevity, the horizontal (represented by state, $\mathbf{x}_h$) and vertical (represented by state, $\mathbf{x}_v$) plane subsystems are not shown.

6.2.1 Switching Logic for Cascaded to Inner Loop

The closed positive quadrant, $\Omega_L \subset \mathbb{R}^2$ for inner and outer loop can be defined as:

$$\Omega_L = \{[V_I(\tilde{\mathbf{x}}_{s2e}, t), V_O(\tilde{\mathbf{x}}_{s1e}, t)] \in \mathbb{R}^2 : V_I(\tilde{\mathbf{x}}_{s2e}, t), V_O(\tilde{\mathbf{x}}_{s1e}, t) \geq 0\} \quad (6.8)$$

where the subscript ‘L’ refers to the loops switching, $V_I(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_O(\tilde{\mathbf{x}}_{s1e}, t)$ are the Lyapunov function referred to the inner ($\tilde{\mathbf{x}}_{s2e}$) and outer ($\tilde{\mathbf{x}}_{s1e}$) loop states (denoted by subscript ‘I’ and ‘O’) respectively. The “energy” of the ROV during the stabilization is computed using:

$$V_I(\tilde{\mathbf{x}}_{s2e}, t) = \frac{1}{2} m \tilde{\mathbf{x}}_{s2e}^T \tilde{\mathbf{x}}_{s2e} \quad (6.9a)$$

while the “energy” during the tracking in either vertical or horizontal plane is determined by.

$$V_O(\tilde{\mathbf{x}}_{s1e}, t) = \frac{1}{2} m \tilde{\mathbf{x}}_{s1e}^T \tilde{\mathbf{x}}_{s1e} \quad (6.9b)$$

In the cascaded system, $V_I(\tilde{\mathbf{x}}_{s2e}, t) \geq V_O(\tilde{\mathbf{x}}_{s1e}, t)$ means that the inner loop responds faster [45] than the outer loop. The basic idea in the cascaded to inner loop switching is to deactivate the outer loop and leave the inner loop controller functioning when the desired position is reached. This is known as the station-keeping operation whereby the “energy” of the inner loop, $V_I(\tilde{\mathbf{x}}_{s2e}, t)$ is small. To avoid infinite switching, the outer loop will not be switched back unless the next position command is issued.

To verify whether the desired position at time, t_1 is within the threshold for the inner loop switching, it is proposed to use the switching condition consisting of the position tracking

error, $\tilde{\mathbf{x}}_{\text{se}}(t_1)$ and the “energy”, $V_I(\tilde{\mathbf{x}}_{\text{se}}, t_1)$. To satisfy $\sigma_L(t_1) = 0$, the $\tilde{\mathbf{x}}_{\text{se}}(t_1)$ must be smaller than the threshold specified and the $V_I(\tilde{\mathbf{x}}_{\text{se}}, t_1)$ must be close to zero at the switching instant t_1 . The condition for the cascaded to inner loop switching at t_1 can be expressed as:

$$\sigma_L(t_1) = \begin{cases} 0, & \text{if } \tilde{\mathbf{x}}_{\text{se}}(t_1) \leq 0.1\mathbf{x}_{\text{sd}}, V_I(\tilde{\mathbf{x}}_{\text{se}}, t_1) \equiv 0 \\ 1, & \text{if } \tilde{\mathbf{x}}_{\text{se}}(t_1) > 0.1\mathbf{x}_{\text{sd}}, V_I(\tilde{\mathbf{x}}_{\text{se}}, t_1) > 0 \end{cases} \quad (6.10)$$

where the $0.1\mathbf{x}_{\text{sd}}$ implies that the position must not exceed ten percents of the desired value and the $V_I(\tilde{\mathbf{x}}_{\text{se}}, t_1)$ is the “energy” at the switching instant, t_1 . However, to improve the flexibility of the switching condition, a manual override in the scenario that the switching is needed to start earlier or later is used. To illustrate the application of the cascaded to inner loop switching condition as stipulated in equation (6.10), Example 6.1 is used.

Example 6.1: Application of position tracking error and energy in loop switching

At $t = 0$, the cascaded system received a step input of $[0 \ 0 \ 15 \ 0 \ 0 \ 0]^T$ to move the ROV in heave direction. At $t_1 = 65$ sec, the inner loop is activated after the position tracking error and the “energy” in the heave direction satisfied the first switching condition as shown in (6.10). As seen in Fig. 6.4, the Euclidean norm of the error in heave direction is small at the point of switching. This indicates that the outer loop has to be deactivated to allow for inner loop operation only and hence the loop-switching signal $\sigma_L(t_1) = 0$. As seen in the Fig. 6.4, the duration of the inner loop activation can be treated as a dwell time, τ_D . This allows sufficient time for the transient (due to switching) to diminish before the switching to cascaded loop occurs for the next reference input.

The supervisory switching logic for the cascaded to inner loop switching can be written as an algorithm as shown in Algorithm 6.1. Note that the inner loop switching logic does not involved the selection of candidate inner and outer loop controllers, as the controllers’ switching logic is chosen in Section 6.3.2.

Figure 6.4: Cascaded to inner loop switching scenario for Example 6.1

Algorithm 6.1: Supervisory switching logic for the cascaded to inner loop switching

1. Let $\sigma_L(0) := 1; t := 0;$
2. $t := t + \Delta t; t \leq t_f;$
3. If $\tilde{\mathbf{x}}_{s1e}(t) \leq 0.1\mathbf{x}_{s1d}$ and $V_I(\tilde{\mathbf{x}}_{s2e}, t) \equiv 0;$
 then $\sigma_L(t) := 0$ and $t := t + \tau_D;$ go to 2;
 else $\sigma_L(t) := 1;$ go to 2;

where t_f is the allowable mission time in second.

6.2.2 Switching Regions for Inner and Cascaded Loop

To represent the two different scenarios namely: the cascaded and inner loop, the space $V_I(\tilde{\mathbf{x}}_{s2e}, t) \times V_O(\tilde{\mathbf{x}}_{s1e}, t)$ is partitioned into two regions namely: $\mathbf{R}_{IO}$ and $\mathbf{R}_I$ (where the subscript 'IO' and 'I' refer to the cascaded and inner loop respectively). These regions (see Fig. 6.5) can be formally defined as follows:

Figure 6.5: Regions $\mathbf{R}_I$ and $\mathbf{R}_{IO}$ in the $V_I(\tilde{\mathbf{x}}_{s2e}, t) \times V_O(\tilde{\mathbf{x}}_{s1e}, t)$ plane

1. The regions $\mathbf{R}_I$ and $\mathbf{R}_{IO}$ in the $V_I(\tilde{\mathbf{x}}_{s2e}, t) \times V_O(\tilde{\mathbf{x}}_{s1e}, t)$ plane have the following properties:

$$0 < \varepsilon_1 < \varepsilon_2; \quad (6.11)$$

and $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are bounded ;

2. Define two overlapping regions that constitute partitions of the closed positive $\Omega_L \subset \mathbb{R}^2$ in the form:

Inner Loop:

$$\mathbf{R}_I = \begin{cases} \{(V_I(t), V_O(t)) \in \Omega_L(t) : 0 \leq V_I(t) < \varepsilon_1\} \\ \{(V_I(t), V_O(t)) \in \Omega_L(t) : 0 \leq V_O(t) < \varepsilon_3\} \end{cases} \quad (6.12)$$

Inner and outer Loop:

$$\mathbf{R}_{IO} = \begin{cases} \{(V_I(t), V_O(t)) \in \Omega_L(t) : \varepsilon_1 \leq V_I(t) < \varepsilon_2\} \\ \{(V_I(t), V_O(t)) \in \Omega_L(t) : 0 \leq V_O(t) < \varepsilon_3\} \end{cases} \quad (6.13)$$

3. The inner and outer loop control laws for $\bar{\mathbf{u}}_{\mathbf{O},i}$ and $\bar{\mathbf{u}}_{\mathbf{I},i}$ for $i = 1, 2, 3$ are as follows:

$$\begin{aligned} \bar{\mathbf{u}}_{\mathbf{I},1} &= \bar{\mathbf{u}}_{\mathbf{I},2} \\ &= \begin{bmatrix} k_{pu} \tilde{u}_e \\ k_{pv} \tilde{v}_e \\ k_{pw} \tilde{w}_e \\ k_{pr} \tilde{r}_e \end{bmatrix} + \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{m - X_{\dot{u}}} & 0 & 0 & 0 \\ 0 & \frac{m - Y_{\dot{v}} - \Delta Y_{\dot{v}}}{m - Y_{\dot{v}}} & 0 & 0 \\ 0 & 0 & \frac{m - Z_{\dot{w}} - \Delta Z_{\dot{w}}}{m - Z_{\dot{w}}} & 0 \\ 0 & 0 & 0 & \frac{I_z - N_{\dot{r}} - \Delta N_{\dot{r}}}{I_z - N_{\dot{r}}} \end{bmatrix} \begin{bmatrix} (Y_{\dot{v}} - m) \tilde{v}_e \tilde{r}_e \\ (m - X_{\dot{u}}) \tilde{u}_e \tilde{r}_e \\ 0 \\ (-Y_{\dot{v}} + X_{\dot{u}}) \tilde{u}_e \tilde{v}_e \end{bmatrix} \\ &\quad - \begin{bmatrix} \frac{m - X_{\dot{u}} - \Delta X_{\dot{u}}}{X_{\dot{u}}} & 0 & 0 & 0 \\ 0 & \frac{m - Y_{\dot{v}} - \Delta Y_{\dot{v}}}{Y_{\dot{v}}} & 0 & 0 \\ 0 & 0 & \frac{m - Z_{\dot{w}} - \Delta Z_{\dot{w}}}{Z_{\dot{w}}} & 0 \\ 0 & 0 & 0 & \frac{I_z - N_{\dot{r}} - \Delta N_{\dot{r}}}{N_{\dot{r}}} \end{bmatrix} \begin{bmatrix} (\Delta Y_{\dot{v}} - m) \tilde{v}_e \tilde{r}_e \\ (m - \Delta X_{\dot{u}}) \tilde{u}_e \tilde{r}_e \\ 0 \\ (-\Delta Y_{\dot{v}} + \Delta X_{\dot{u}}) \tilde{u}_e \tilde{v}_e \end{bmatrix} \end{aligned} \quad (6.14)$$

$$\bar{\mathbf{u}}_{\mathbf{I},3} = \begin{bmatrix} k_{pu} \tilde{u}_e \\ k_{pv} \tilde{v}_e \\ k_{pw} \tilde{w}_e \\ k_{pr} \tilde{r}_e \end{bmatrix} \quad (6.15)$$

$$\bar{\mathbf{u}}_{\mathbf{O},1} = \bar{\mathbf{u}}_{\mathbf{O},3} = \begin{bmatrix} -u_d \cos \psi - v_d \sin \psi + \hat{u} - \hat{r} y_e \\ u_d \sin \psi - v_d \cos \psi + \hat{r} x_e + \hat{v} \\ -w_d + \hat{w} \\ -r_d + \hat{r} \end{bmatrix} - \begin{bmatrix} k_{px} \tilde{x}_e \\ k_{py} \tilde{y}_e \\ k_{pz} \tilde{z}_e \\ k_{p\psi} \tilde{\psi}_e \end{bmatrix} \quad (6.16)$$

$$\bar{\mathbf{u}}_{\mathbf{O},2} = - \begin{bmatrix} k_{px} \tilde{x}_e \\ k_{py} \tilde{y}_e \\ k_{pz} \tilde{z}_e \\ k_{p\psi} \tilde{\psi}_e \end{bmatrix} \quad (6.17)$$

4. The piecewise continuous signal, $\sigma_L(t)$ is defined to be continuous from the right at every point and the $\sigma_L(t)$ takes values from 0 or 1. It is defined as:

$$\sigma_L(t) = \begin{cases} 1 & \text{if } \mathbf{V}_L(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) \in \mathbf{R}_I \\ 0 & \text{if } \mathbf{V}_L(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) \in \mathbf{R}_{IO} \end{cases} \quad (6.18)$$

where

$$\mathbf{V}_L(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) = V_I(\tilde{\mathbf{x}}_{s2e}, t) \times V_O(\tilde{\mathbf{x}}_{s1e}, t) \quad (6.19)$$

and the first condition in (6.18) implies that the $\sigma_L(t)$ will remain as one if $\mathbf{V}_L(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t)$ remain in the shaded box in Fig. 6.5.

6.3 Cascaded Supervisory Switching Law for Multiple Inner and Outer Controllers

Since each controller design has its strength and weakness, switching amongst each of them increases the robustness of the control system. In fact, the performance using the hybrid controller should be no worse than what would be obtained by using any single feedback function without switching. In many cases, the performance using hybrid controllers is substantially better if the switching between controllers is done properly without causing large transients and instability [85].

In Section 6.3.1, the controllers switching sequence is selected from a set of stabilizing controllers switching sequences that are asymptotically stable at different phases of the ROV's maneuvering as defined by the reference inputs. To have a better understanding, the supervisory switching logic is explained in Section 6.3.2, followed by the classification of switching regions for each type of controller in Section 6.3.3.

The multiple inner and outer controllers switching law for station-keeping (represented by state, $\mathbf{x}_s$) model in (5.23) is formulated. Since the concept is similar, for brevity, the horizontal (represented by state, $\mathbf{x}_h$) and vertical (represented by state, $\mathbf{x}_v$) plane subsystems are not shown.

6.3.1 Conditions for Stabilizing Inner and Outer Controllers

As mentioned in Theorem 5.1 and as the simulation results show, the individual subsystems with P-decoupled controllers are asymptotically stable. However with different inner and outer loop controllers used in the cascaded switched system, the stability of the switched system can be affected due to the transience caused by the switching. But with sufficient dwell time imposed on the controllers switching, the cascaded switched systems remain asymptotically stable and independent of the controllers' switching sequence as shown in Theorem 6.1.

For all comparisons, the simulation model can be found in the RDA package written in the MATLABTM scripting language and graphical block diagram simulation, SIMULINKTM for the ROV control systems simulation. The ODE solver with variable step size and a relative tolerance of 0.001 were used. The pipeline profile as shown in Fig. 6.6 and its corresponding reference input in Table 6.1 are used for the simulations. The controllers and the inner or cascaded loop switching sequences for $t \in [0, 650)$ are shown in Fig. 6.7. The switching from cascaded to inner loop is dwelled for some times, $t_D = 10$ seconds before the next reference input is issued. And there is also sufficient dwell time for each controller's switching. Notice that the ROV is initially stationary at the launch point.

Table 6.2 and 6.3 show the different combinations of the inner and outer loop controllers used. The types of controllers $\bar{\mathbf{u}}_I, \bar{\mathbf{u}}_O$, used are labeled as 1, 2 or 3. As observed in Table 6.2, the maximum (worst case) Euclidean norm of the position and velocity errors with dwell time is smaller than the one without dwell time. However, the controllers' sequence $\{3, 2, 1\}$ without dwell time has a higher sway position error (due to the divergent sway response) and is more oscillatory in the roll and yaw angle compared to the one with dwell time. To avoid this case, the controllers' sequence $\{3, 2, 1\}$ will not be included in the controllers' bank.

Input Sequences	Decoupled Inputs
1	$[0 \ 0 \ 15 \ 0 \ 0 \ 0]^T$
2	$[15 \ 0 \ 15 \ 0 \ 0 \ 0]^T$
3	$[15 \ 0 \ 10 \ 0 \ 0 \ 0]^T$
4	$[25 \ 0 \ 10 \ 0 \ 0 \ 0]^T$
5	$[25 \ 0 \ 15 \ 0 \ 0 \ 0]^T$
6	$[25 \ 0 \ 15 \ 0 \ 0 \ 1.57]^T$
7	$[25 \ 15 \ 15 \ 0 \ 0 \ 0]^T$

Table 6.1: Input sequence for decoupled maneuvering

Figure 6.6: Schematic diagram of the simulated pipeline profile for the RRC ROV

Figure 6.7a: Time plots for the controller switching sequence 3, 2, and 1 with dwell time

Figure 6.7b: Time plots for the controller switching sequence 3, 2, and 1 without dwell time

As observed in Table 6.3, the minimum (worst case) Euclidean norm of the “energy” in the outer loop is higher than the one without dwelling. This large value on the “energy” in the outer loop indicates that it is more positive definite or stable than the one without dwell time. As shown in the time responses and “energy” plots in Appendix D2, $V_I(\tilde{\mathbf{x}}_{s2e}, t) \geq V_O(\tilde{\mathbf{x}}_{s1e}, t)$ implies that the inner loop responses faster than the outer loop. Importantly, the “energy” of the inner and outer loop after the switching should be smaller than the “energy” before the switching.

With the above observation, it is necessary to ensure that:

- (a) the “energy” for every pair of inner and outer loop controllers’ switching sequence must be non-decreasing. Reader can refer to Theorem 6.1 (in Section 6.4) for detail derivation on asymptotic stability of the cascaded switched system without dwell time;
- (b) the dwell time is used between each inner and outer loop controllers switching. Similarly, reader can refer to Proposition 6.1(in Section 6.4) for detail derivation on asymptotic stability of the cascaded switched system with dwell time.

Note that, the Theorem 6.1 and Proposition 6.1 are not mentioned here. Instead it will be covered in Section 6.4 as the conditions belong to one of the properties of the cascaded supervisory switch system.

Conditions	Position Ref. Input	Controller Seq.	Vel. Error $\max\ \mathbf{x}_{2e}(\cdot)\ _2$	Pos. Error $\max\ \mathbf{x}_{1e}(\cdot)\ _2$
Without Dwell	See Table 6.1	$i = 2,1,3$	3.10	5.49×10^2
		$i = 1,2,3$	3.20	5.18×10^2
		$i = 3,1,2$	3.10	5.23×10^2
		$i = 1,3,2$	3.20	5.18×10^2
		$i = 3,2,1$	13.4	5.34×10^2
With Dwell	See Table 6.1	$i = 2,1,3$	0.22	5.23×10^2
		$i = 1,2,3$	3.19	5.13×10^2
		$i = 3,1,2$	3.10	5.19×10^2
		$i = 1,3,2$	3.19	5.13×10^2
		$i = 3,2,1$	9.95	5.30×10^2

Table 6.2: Maximum Euclidean norm of position and velocity error without and with dwell time

Conditions	Position Ref. Input	Controller Seq.	Inner loop, $\min\ V_{I,i}(\cdot)\ _2$	Outer loop, $\min\ V_{O,i}(\cdot)\ _2$
Without Dwell	See Table 6.1	$i = 2,1,3$	2.04×10^5	2.97×10^4
		$i = 1,2,3$	2.89×10^5	2.81×10^4
		$i = 3,1,2$	2.04×10^5	2.97×10^4
		$i = 1,3,2$	2.89×10^5	2.81×10^4
		$i = 3,2,1$	1.69×10^5	1.02×10^6
With Dwell	See Table 6.1	$i = 2,1,3$	1.99×10^5	4.1×10^4
		$i = 1,2,3$	2.89×10^5	3.97×10^4
		$i = 3,1,2$	1.99×10^5	4.11×10^4
		$i = 1,3,2$	2.89×10^5	3.98×10^4
		$i = 3,2,1$	1.97×10^5	8.07×10^5

Table 6.3: Minimum Euclidean norm of “energy” without and with dwell time

6.3.2. Switching Logic for Multiple Inner and Outer Controllers Switching

The closed positive quadrant, $\Omega_{C,i} \subset \mathfrak{R}^2$ for different controllers is defined as:

$$\Omega_{C,i} = \{[V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t), V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)] \in \mathfrak{R}^2 : V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \geq 0, V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t) \geq 0\} \quad (6.20)$$

where the subscript ‘C’ refers to the controllers’ switching and the controller type, $i \in I_C$. Here $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ are the Lyapunov function associated with the inner velocity states ($\tilde{\mathbf{x}}_{s2e}$) and outer position states ($\tilde{\mathbf{x}}_{s1e}$) respectively. The $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)$ indicates the “energy” of the ROV during the stabilization and $V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ measure the “energy” during the tracking in either vertical or horizontal plane at a given controller, i .

The basic idea in controller switching is to ensure that $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \geq V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ and the “energy” in the loops must adhere to: $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \geq V_{I,i+1}(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t) \geq V_{O,i+1}(\tilde{\mathbf{x}}_{s1e}, t)$. When the inner loop is used in the stabilization operation, the “energy” has to be kept low. This ensures that the vehicle can stabilize to the desired point on the pipeline. However, during tracking operation, no switching is involved since both loops are used for simultaneous tracking and stabilization.

The supervisory switching logic for the controllers switching is described in Algorithm 6.2. In this algorithm, the condition on the non-decreasing “energy” as seen in Step (4) must be satisfied. With both Algorithms 6.1 and 6.2, the switching algorithm consists of the cascaded to inner loop and controller switching logic.

Algorithm 6.2: Supervisory switching logic for the inner and outer controllers switching

1. Let $\sigma_L(0) := 1; \sigma_{CO}(0) := 1; \sigma_{CI}(0); t := 0;$
 2. $t := t + \Delta t; t \leq t_f, \sigma_L(t) := 1;$
 3. If $t \geq \tau_D$, then go to 4;
 4. If $V_{I,\sigma_{CO}(t-1)}(t) \leq V_{I,\sigma_{CO}(t-1)}(t - \Delta t) - \rho_O(t - \Delta t)$ and
 $V_{O,\sigma_{CI}(t-1)}(t) \leq V_{O,\sigma_{CI}(t-1)}(t - \Delta t) - \rho_I(t - \Delta t)$
 then $\sigma_{CO}(t) := \{2,3\}; \sigma_{CI}(t) := \{2,3\};$ go to 2;
- else go to 2;

where $\rho_O(\cdot), \rho_I(\cdot)$ are positive values and t_f is the allowable mission time in second.

6.3.3. Switching Regions for Multiple Controllers

To indicate which inner and outer controllers, $i \in I_C = \{1,2,3\}$ are used at each time, it is necessary to represent each controller by its “energy” function in $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \times V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ space. These regions (see Fig. 6.8) can be formally defined as follows:

Figure 6.8: Regions $\mathbf{R}_1$, $\mathbf{R}_2$ and $\mathbf{R}_3$ in the $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \times V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ plane

1. The regions $\mathbf{R}_1$, $\mathbf{R}_2$ and $\mathbf{R}_3$ in the $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \times V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ plane have the following properties:

- a) $0 < \alpha_{2,i} \leq \alpha_{1,i}$;
- b) $\alpha_{1,i}$ and $\alpha_{2,i}$ are bounded.

where $i \in I_C$

2. Define three overlapping regions that constitute partitions of the closed positive $\Omega_{C,i} \subset \mathbb{R}^2$ in the form:

$$\mathbf{R}_1 = \begin{cases} \{(V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t), V_{O,1}(\tilde{\mathbf{x}}_{s1e}, t)) \in \Omega_{C,1}(t) : 0 \leq V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t) < \alpha_{2,1}\} \\ \{(V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t), V_{O,1}(\tilde{\mathbf{x}}_{s1e}, t)) \in \Omega_{C,1}(t) : 0 \leq V_{O,1}(\tilde{\mathbf{x}}_{s1e}, t) < \alpha_{1,1}\} \end{cases} \quad (6.21)$$

$$\mathbf{R}_2 = \begin{cases} \{(V_{I,2}(\tilde{\mathbf{x}}_{s2e}, t), V_{O,2}(\tilde{\mathbf{x}}_{s1e}, t)) \in \Omega_{C,2}(t) : 0 \leq V_{I,2}(\tilde{\mathbf{x}}_{s2e}, t) < \alpha_{2,2}\} \\ \{(V_{I,2}(\tilde{\mathbf{x}}_{s2e}, t), V_{O,2}(\tilde{\mathbf{x}}_{s1e}, t)) \in \Omega_{C,2}(t) : 0 \leq V_{O,2}(\tilde{\mathbf{x}}_{s1e}, t) < \alpha_{1,2}\} \end{cases} \quad (6.22)$$

$$\mathbf{R}_3 = \begin{cases} \{(V_{I,3}(\tilde{\mathbf{x}}_{s2e}, t), V_{O,3}(\tilde{\mathbf{x}}_{s1e}, t)) \in \Omega_{C,3}(t) : 0 \leq V_{I,3}(\tilde{\mathbf{x}}_{s2e}, t) < \alpha_{2,3}\} \\ \{(V_{I,3}(\tilde{\mathbf{x}}_{s2e}, t), V_{O,3}(\tilde{\mathbf{x}}_{s1e}, t)) \in \Omega_{C,3}(t) : 0 \leq V_{O,3}(\tilde{\mathbf{x}}_{s1e}, t) < \alpha_{1,3}\} \end{cases} \quad (6.23)$$

3. The inner and outer loop control laws for $\bar{\mathbf{u}}_{O,i}$ and $\bar{\mathbf{u}}_{I,i}$ for $i \in I_c$ can be obtained from (6.14) to (6.17).
4. The piecewise signal is defined to be continuous from the right at every point and is determined by;

$$\sigma_{CO}(t) = \sigma_{CI}(t) = \begin{cases} \sigma_{CO}(t) = \sigma_{CI}(t) = 1 & \text{if } \mathbf{V}_C(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) \in \mathbf{R}_1 \cap \mathbf{R}_2 \cap \mathbf{R}_3 \\ \sigma_{CO}(t) = \sigma_{CI}(t) = 1, & \text{if } \mathbf{V}_C(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) \in \mathbf{R}_1 \\ \sigma_{CO}(t) = \sigma_{CI}(t) = 2 & \text{if } \mathbf{V}_C(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) \in \mathbf{R}_2 \\ \sigma_{CO}(t) = \sigma_{CI}(t) = 3 & \text{if } \mathbf{V}_C(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) \in \mathbf{R}_3 \end{cases} \quad (6.25)$$

where $\mathbf{V}_C(\tilde{\mathbf{x}}_{s2e}, \tilde{\mathbf{x}}_{s1e}, t) = V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \times V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ and the first condition imply that the $\sigma_{CO}(t) = \sigma_{CI}(t)$ will remain as one by default.

6.4 Cascaded Supervisory Switched System Properties

With the cascaded switched system established, the properties such as the existence of solution, state convergence and asymptotic stability are discussed. The existence of the solution is shown using the Lipschitz continuous condition. A numerically-based argument using the Runge-Kutta numerical method based in MATLABTM is used to illustrate the convergence of states.

This is followed by the stability proof of the cascaded-switched system in the event of multiple inner and outer loop controllers switching. For the inner loop dynamics, the inner loop with different controllers is asymptotically stable as seen in the first part of the proof in Theorem 6.1. Thus, the second part is to show that the outer loop with different controllers is asymptotically stable as well.

The station-keeping (represented by states, $\mathbf{x}_s$) model in (5.23) is used in the stability proof. Since the concept is similar, for brevity, the horizontal (represented by states, $\mathbf{x}_h$) and vertical (represented by states, $\mathbf{x}_v$) plane subsystems are not shown. As observed in the observer design shown in Fig. 4.5, the estimated states converged to the actual states in less than ten seconds. Hence for conciseness, the estimated state $\hat{\mathbf{x}}_s$ is not shown in the subsequent plots.

6.4.1 Existence of Solution

In this Section, the existence of the solution will be shown using the Lipschitz continuous condition. The existence [107,119] of the ODEs can be verified by using the similar method as presented in (4.46) in Chapter 4.

Now, for each $i \in I_C$, the ODEs with given initial condition for the station-keeping model at a given input $\bar{\mathbf{u}}_{L,i}, \bar{\mathbf{u}}_{O,i}$ are written as:

$$\dot{\tilde{\mathbf{x}}}_{s2e}(t) = \bar{\mathbf{f}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t) + \bar{\mathbf{g}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_{L,i}, t) \quad (6.26a)$$

$$\dot{\tilde{\mathbf{x}}}_{s1e}(t) = \bar{\mathbf{f}}_{s1,i}(\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, \bar{\mathbf{u}}_{O,i}, t) \quad (6.26b)$$

for time interval $[t_0, t_f]$. To verify that (6.26) have a unique solution, it is sufficient that the following Lipschitz conditions in (6.27) and (6.28) are satisfied (where $L_1, L_2 > 0$ is called the Lipschitz constant and can be varied for different controller $\bar{\mathbf{u}}_{L,i}, \bar{\mathbf{u}}_{O,i}$:

$$\left\| \bar{\mathbf{f}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t_1) - \bar{\mathbf{f}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t_2) \right\|_2 \leq L_1 \left\| \tilde{\mathbf{x}}_{s2e}(t_1) - \tilde{\mathbf{x}}_{s2e}(t_2) \right\|_2 \Rightarrow \left\| \frac{\partial \bar{\mathbf{f}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial \tilde{\mathbf{x}}_{s2e}} \right\|_2 \leq L_1 \quad (6.27)$$

$$\left\| \bar{\mathbf{f}}_{s1,i}(\tilde{\mathbf{x}}_{s1e}, t_1) - \bar{\mathbf{f}}_{s1,i}(\tilde{\mathbf{x}}_{s1e}, t_2) \right\|_2 \leq L_2 \left\| \tilde{\mathbf{x}}_{s1e}(t_1) - \tilde{\mathbf{x}}_{s1e}(t_2) \right\|_2 \Rightarrow \left\| \frac{\partial \bar{\mathbf{f}}_{s1,i}(\tilde{\mathbf{x}}_{s1e}, t)}{\partial \tilde{\mathbf{x}}_{s1e}} \right\|_2 \leq L_2 \quad (6.28)$$

for all $(\tilde{\mathbf{x}}_{s1e}(t), \bar{\mathbf{u}}_{1,i}, \bar{\mathbf{u}}_{0,i})$ and $(\tilde{\mathbf{x}}_{s2e}(t), \bar{\mathbf{u}}_{1,i}, \bar{\mathbf{u}}_{0,i})$ from a neighborhood of a solution $\tilde{\mathbf{x}}_s(t) : [t_0, t_f] \mapsto \mathfrak{R}^n$ of (6.27) and (6.28) where $\tilde{\mathbf{x}}_s(t) = [\tilde{\mathbf{x}}_{s1e}(t) \ \tilde{\mathbf{x}}_{s2e}(t)]^T$. Notice that the term $\bar{\mathbf{g}}_{s2,i}(\cdot)$ does not appear in (6.27) and (6.28) since it is a constant and is independent of $\tilde{\mathbf{x}}_{s2e}$ for $[t_0, t_f]$.

In other words, if the maximum eigenvalue of $\frac{\partial \bar{\mathbf{f}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial \tilde{\mathbf{x}}_{s2e}}$ and $\frac{\partial \bar{\mathbf{f}}_{s1,i}(\tilde{\mathbf{x}}_{s1e}, t)}{\partial \tilde{\mathbf{x}}_{s1e}}$ are bounded as shown in (6.27a) and (6.27b) respectively, then the ODE in (6.26a) and (6.26b) have a unique solution. Since for each $i \in I_C$, the vector fields $\bar{\mathbf{f}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and $\dot{\tilde{\mathbf{x}}}_{s1e}(t) = \bar{\mathbf{f}}_{s1,i}(\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, t)$ are continuously differentiable with respect to $\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}$, implying that the (6.26) are bounded and hence the solution of (6.26) exist.

6.4.2 State Convergence

In the spirit of the work in [131], a numerically-based argument using the Runge-Kutta numerical method based in MATLABTM are used to shown that the initial conditions $\{\tilde{\mathbf{x}}_{se}(t_0), \sigma_{CO}^-(t_0), \sigma_{CI}^-(t_0)\}$ where the $\tilde{\mathbf{x}}_{se}(t)$ refers to $[\tilde{\mathbf{x}}_{s1e}(t) \ \tilde{\mathbf{x}}_{s2e}(t)]^T$ in the regions: $\{\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3\} \in \Omega_{c,i}$ converges. This provides an insight to the state convergence for each controller at different regions (during $0 \leq t < t_6$) namely:

1. Region, $\mathbf{R}_1$ during $t_0 \leq t < t_2$;
2. Region, $\mathbf{R}_3$ during $t_2 \leq t < t_4$;
3. Region, $\mathbf{R}_2$ during $t_4 \leq t < t_6$;

where $t_0 = 0, t_1 = 100, t_2 = 200, t_3 = 250, t_4 = 300, t_5 = 400, t_6 = 600$. To accomplish that, the time response plots of the states at each region defined by the respective controllers are plotted in Fig. 6.11a and Fig. 6.11b while the switching sequence for the controllers and loops are shown in the Fig. 6.10. The switching from cascaded to inner loop is dwelled for a while before the next reference input is issued and a sufficient dwell time is ensured between each controller's switching. To simplify the simulation and provide insight to the convergence

study, only one set of stabilizing controller sequence is used. As shown in Fig. 6.11a and Fig. 6.11b, at respective time frame, the $\mathbf{x}_{s2}, \mathbf{x}_{s1}$ settled to its steady state value before the next candidate controller is used.

As seen in Fig. 6.12a and Fig. 6.12b, $V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)$ is always greater than $V_{o,i}(\tilde{\mathbf{x}}_{s1e}, t)$. This is because that the inner loop operates faster than the outer loop. Exception is at time where the “energy” at the inner loop is small during station keeping. $V_{o,i}(\tilde{\mathbf{x}}_{s1e}, t)$ is small as the desired position is reached. It is observed that $V_{l,i+1}(\tilde{\mathbf{x}}_{s2e}, t) \leq V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_{o,i+1}(\tilde{\mathbf{x}}_{s1e}, t) \leq V_{o,i}(\tilde{\mathbf{x}}_{s1e}, t)$ for the transition from $\mathbf{R}_1$ to $\mathbf{R}_3$. Note that the plots of $V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_{o,i}(\tilde{\mathbf{x}}_{s1e}, t)$ show the “energy” of the individual DoFs and the argument $\tilde{\mathbf{x}}_{s1e}, \tilde{\mathbf{x}}_{s2e}, t$ in $V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_{o,i}(\tilde{\mathbf{x}}_{s1e}, t)$ are ignored for clarity.

Besides, the asymptotic stability with respect to different initial conditions (labeled as IC-1, IC-2 and etc in Fig. 6.10.) of the trajectory is discussed. From the Fig. D3.1 to Fig. D3.12 (in Appendix D3), it is observed to be convergence for different initial condition on the pipeline profiles. From the Fig. D3.13 to Fig. D3.24 (in Appendix D3), the $V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)$ is always greater than $V_{o,i}(\tilde{\mathbf{x}}_{s1e}, t)$ as the inner loop operates faster than the outer loop and the $V_{l,i+1}(\tilde{\mathbf{x}}_{s2e}, t) \leq V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_{o,i+1}(\tilde{\mathbf{x}}_{s1e}, t) \leq V_{o,i}(\tilde{\mathbf{x}}_{s1e}, t)$ is observed. Additionally, it is necessary to prove the asymptotic stability of the cascaded-switched system in the event of multiple inner and outer loop controllers switching. To illustrate the point, the cascaded switched system under different inner and outer loop controllers is shown to be asymptotically stable in the next Section.

Figure 6.9: Different initial conditions on the pipeline's segment

Figure 6.10: Time plots for the controller switching sequence 1, 3 and 2

 Figure 6.11a: Numerical solution of the $\mathbf{x}_{s1}$ for $0 \leq t < t_6$

 Figure 6.11b: Numerical solution of the $\mathbf{x}_{s2}$ for $0 \leq t < t_6$

 Figure 6.12a: $V_{I,i}$ of each individual DoFs during $0 \leq t < t_6$

 Figure 6.12b: $V_{O,i}$ for each individual DoFs during $0 \leq t < t_6$

6.4.3 Asymptotic Stability of Cascaded Switched System with and without Dwell Time

Next, it will be shown that the cascaded switched system is asymptotically stable about the equilibrium manifold. In most marine control literature such as for ships and ROVs, asymptotic stability refers mainly on three DoFs [13,15,35,54,62,65] with no perturbation. Hence, an extension of the asymptotic stability [110] to the cascaded switched system operating in three-dimensional space with model containing hydrodynamic uncertainties is proposed. In the proof, Lyapunov stability of the cascaded switched system via multiple Lyapunov functions will be discussed. The main concept is to impose some restrictions on the Lyapunov functions of each subsystem with different inner and outer loop controllers (using station-keeping as an example), so that the cascaded switched system is asymptotically stable.

As seen in Theorem 5.1, the nonlinear-cascaded system is asymptotically stable. For cascaded switched system, it will be shown that if both inner and outer loops can satisfy conditions (a) to (e) of Theorem 6.1 under different inner and outer loop controllers, then the entire cascaded switched systems is asymptotically stable. The sketch of the Theorem 6.1 is as follows:

(a) Firstly, it will be proved that the system is asymptotically stable about the equilibrium sub manifold in the case with no dwell time on the inner loop state $\tilde{\mathbf{x}}_{s2e}(t)$ under controller switching from i to j . Secondly, it is followed by the outer loop state $\tilde{\mathbf{x}}_{s1e}(t)$ using the same proof as shown in the first part. Note that, in each region $\Omega_{C,i}$, a scalar function $V_{O,i}(\tilde{\mathbf{x}}_{s1e}, t)$ and $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)$ where $i \in I_C$ is used to measure the “energy” of the outer and inner loop controller of the cascaded switched system.

(b) With some understanding on the cascaded switched system without dwelling, a dwell time [118, Lemma 2] between each inner and outer loop controllers switching is introduced to improve the asymptotic stability of the cascaded switched system. The asymptotic stability of the inner loop states is shown, followed by the outer loop state using the same method.

As seen in the Proposition 6.1, the dwell time can be determined analytically such that the convergence rate of the states before switching decays faster. With the dwell time, the asymptotic stability or in particular the exponential stability about the equilibrium sub

manifold of the nonlinear cascaded switched system can be improved as the explicit lower bound of the dwell time can be computed.

Theorem 6.1: Asymptotic Stability of the Cascaded Switched System without Dwell Time (for Controllers' Switching)

If there exists multiple Lyapunov scalar functions $V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t) : \Omega_{C,i} \rightarrow \mathfrak{R}$ and $V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)$ is differentiable with respect to $\tilde{\mathbf{x}}_{s2e}(t)$, $\forall \tilde{\mathbf{x}}_{s2e}(t) \in \Omega_{C,i}, i \in I_C$ and constants $\alpha_i, \beta_i, \gamma_i > 0$ such that:

$$(a) \quad \forall (\tilde{\mathbf{x}}_{s2e}, i) \in \Omega_{C,i}, \alpha_i \|\tilde{\mathbf{x}}_{s2e}\| \leq V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq \beta_i \|\tilde{\mathbf{x}}_{s2e}\| \quad (6.29)$$

$$(b) \quad \forall (\tilde{\mathbf{x}}_{s2e}, i) \in \Omega_{C,i}, \frac{\partial V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial t} + \frac{\partial V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial \tilde{\mathbf{x}}_{s2e}} \bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) \leq -\gamma_i \|\tilde{\mathbf{x}}_{s2e}\| \quad (6.30)$$

$$(c) \quad \forall (\tilde{\mathbf{x}}_{s2e}, i) \in \Omega_{C,ij}, V_{l,j}(\tilde{\mathbf{x}}_{s2e}, t) \leq V_{l,i}(\tilde{\mathbf{x}}_{s2e}, t); \quad i, j \in I_C \quad (6.31)$$

and with the input and perturbation terms are bounded as:

$$(d) \quad \forall (\tilde{\mathbf{x}}_{s2e}, i) \in \Omega_{C,i}, \bar{\mathbf{g}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq -d_1 \|\tilde{\mathbf{x}}_{s2e}\| \quad (6.32)$$

$$(e) \quad \forall (\tilde{\mathbf{x}}_{s2e}, i) \in \Omega_{C,i}, \Delta \mathbf{d}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq -d_4 \quad (6.33)$$

then the equilibrium submanifold $\tilde{\mathbf{x}}_{s2e}(t) = 0$ is asymptotically stable. Thus $\tilde{\mathbf{x}}_{s2e}(t)$ in region $\Omega_{C,i}$ becomes:

$$\|\tilde{\mathbf{x}}_{s2e}(t)\| \leq \delta_1 e^{-\delta_2(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}(t_0)\|, t \in [t_1, t_f] \quad (6.34)$$

where δ_1 and δ_2 are positive constants.

Proof:

Before the stability is proved, it will be first be shown that (6.34) holds. Assume that $\tilde{\mathbf{x}}_{s2e}(t)$ is in the region $\Omega_{C,i}$ for $t \in [t_0, t_f)$, where t_f may be infinite. With $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq \beta_i \|\tilde{\mathbf{x}}_{s2e}\|$,

$$\dot{V}_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq -\gamma_i \|\tilde{\mathbf{x}}_{s2e}\| \leq -\frac{\gamma_i}{\beta_i} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \quad (6.35)$$

Multiplying both sides with the positive function $e^{\frac{\gamma_i}{\beta_i} t}$, this is equivalent to:

$$\frac{d}{dt} e^{\frac{\gamma_i}{\beta_i} t} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq 0 \quad (6.36)$$

The function $e^{\frac{\gamma_i}{\beta_i} t} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)$ is obviously decreasing. Consequently for $t \in [t_0, t_f)$

$$\frac{d}{dt} e^{\frac{\gamma_i}{\beta_i} t} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq \frac{d}{dt} e^{\frac{\gamma_i}{\beta_i} t_0} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) \quad (6.37)$$

which is the same as:

$$V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq e^{-\frac{\gamma_i}{\beta_i}(t-t_0)} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) \quad (6.38)$$

Using this and $\alpha_i \|\tilde{\mathbf{x}}_{s2e}\| \leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq \beta_i \|\tilde{\mathbf{x}}_{s2e}\|$ implies that

$$\|\tilde{\mathbf{x}}_{s2e}(t)\| \leq \frac{1}{\alpha_i} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq \frac{1}{\alpha_i} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) e^{-\frac{\gamma_i}{\beta_i}(t-t_0)} \leq \frac{\beta_i}{\alpha_i} e^{-\frac{\gamma_i}{\beta_i}(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}, t_0\| \quad (6.39)$$

Next to prove asymptotic stability of (6.39), the two positive numbers δ_1, δ_2 have to be shown to exist, such that (6.39) can be expressed as:

$$\|\tilde{\mathbf{x}}_{s2e}(t)\| \leq \delta_1 e^{-\delta_2(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}(t_0)\|, \forall t \geq t_0 \quad (6.40)$$

with

$$\delta_1 = \max_{i \in 1, \dots, I_c} \frac{\beta_i}{\alpha_i}, \quad \delta_2 = \min_{i \in 1, \dots, I_c} \frac{\gamma_i}{\beta_i} \quad (6.41)$$

If $\tilde{\mathbf{x}}_{s2e}(t)$ never leaves $\Omega_{C,i}$ for $t \in [t_0, t_f)$, then the statement is proved since

$$\|\tilde{\mathbf{x}}_{s2e}(t)\| \leq \frac{\beta_i}{\alpha_i} e^{-\frac{\gamma_i}{\beta_i}(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}(t_0)\| \leq \delta_1 e^{-\delta_2(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}(t_0)\|, \forall t \geq t_0 \quad (6.42)$$

with the perturbation term $\Delta \mathbf{d}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and the input term $\bar{\mathbf{g}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t)$ are bounded as in (6.33) and (6.32) respectively.

Besides showing that $\tilde{\mathbf{x}}_{s2e}(t)$ is in region $\Omega_{C,i}$ with the i^{th} controller is asymptotically stable, the $\tilde{\mathbf{x}}_{s2e}$ will be shown to be asymptotically stable. If the trajectory $\tilde{\mathbf{x}}_{s2e}(t)$ hits $\tilde{\mathbf{x}}_{s2e}(t) \in \Omega_{C,j}$ at t_1 , then the switching from i^{th} to j^{th} controller (where $j > i$) commences and conditions (c), $V_{I,j}(\tilde{\mathbf{x}}_{s2e}, t) \leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)$ has to be met. In same way as shown (6.35) to (6.42) for $t \in [t_1, t_f)$, now:

$$V_{I,j}(\tilde{\mathbf{x}}_{s2e}, t) \leq V_{I,j}(\tilde{\mathbf{x}}_{s2e}, t_1) e^{-\frac{\delta_j}{\beta_j}(t-t_1)} \quad (6.43)$$

Using $V_{I,j}(\tilde{\mathbf{x}}_{s2e}, t_1) \leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_1)$ and $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) e^{-\frac{\delta_i}{\beta_i}(t-t_0)}$, it follows that for $t \in [t_1, t_f)$:

$$\begin{aligned} V_{I,j}(\tilde{\mathbf{x}}_{s2e}, t) &\leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_1) e^{-\frac{\delta_j}{\beta_j}(t-t_1)} \leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) e^{-\frac{\delta_i}{\beta_i}(t_1-t_0)} e^{-\frac{\delta_j}{\beta_j}(t-t_1)} \\ &\leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) e^{-\delta_2(t_1-t_0)} e^{-\delta_2(t-t_1)} = V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) e^{-\delta_2(t-t_0)} \end{aligned} \quad (6.44)$$

Using $\alpha_j \|\tilde{\mathbf{x}}_{s2e}(t_1)\| \leq V_{I,j}(\tilde{\mathbf{x}}_{s2e}, t)$ and $V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) \leq \beta_i \|\tilde{\mathbf{x}}_{s2e}(t_0)\|$ implies that:

$$\|\tilde{\mathbf{x}}_{s2e}(t)\| \leq \frac{\beta_i}{\alpha_j} e^{-\delta_2(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}(t_0)\| \leq \delta_1 e^{-\delta_2(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}(t_0)\|, t \in [t_1, t_f) \quad (6.45)$$

and the input $\bar{\mathbf{g}}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t)$ and the perturbation $\Delta \mathbf{d}_{s2,i}(\tilde{\mathbf{x}}_{s2e}, t)$ are bounded as in (6.33) and (6.32) respectively. Thus, in the whole time interval $t \in [t_0, t_f)$ with controller switching, the $\tilde{\mathbf{x}}_{s2e}(t)$ in region $\Omega_{C,j}$ is bounded as:

$$\|\tilde{\mathbf{x}}_{s2e}(t)\| \leq \delta_1 e^{-\delta_2(t-t_0)} \|\tilde{\mathbf{x}}_{s2e}(t_0)\|, t \in [t_1, t_f) \quad (6.46)$$

Hence, $\tilde{\mathbf{x}}_{s2e}(t)$ or the inner loop is asymptotically stable (or in fact exponentially stable) about the equilibrium submanifold $\tilde{\mathbf{x}}_{s2e}(t) = 0$.

By repeating the above-mentioned proof on the outer loop dynamic, $\tilde{\mathbf{x}}_{s1e}(t)$, the equilibrium submanifold $\tilde{\mathbf{x}}_{s1e}(t) = 0$ can be shown to be asymptotically stable. Thus, since both loops satisfy the condition (a) to (e) in Theorem 6.1 for different controllers, the cascaded switched systems are asymptotically stable.

□

Since there is a delay in the response for all electronic in controller, sensors and thrusters, it becomes inevitable to consider this delay as a form of dwell time (between any two consecutive switching times) in the cascaded switched system in particular. Hence, a dwell time [117] between each inner (and also the outer loop) controllers switching is introduced. As shown in Proposition 6.1, the asymptotic stability of the cascaded switched system can be improved through increasing the rate of “energy” decaying. With that, the proof in [117] is used and applied it on the proposed cascaded switched system.

Proposition 6.1 [117]. *Asymptotic Stability of the Cascaded Switched System with Dwell Time (for Controllers’ Switching)*

As seen in Theorem 6.1, the station-keeping subsystem with different controllers is asymptotically stable. For each inner controller $i \in I_C$, there exists a Lyapunov function

$V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)$ for positive constant $\alpha_i, \beta_i, \gamma_i$ satisfying:

$$\alpha_i \|\tilde{\mathbf{x}}_{s2e}\| \leq V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq \beta_i \|\tilde{\mathbf{x}}_{s2e}\| \quad (6.47)$$

$$\frac{\partial V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial t} + \frac{\partial V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial \tilde{\mathbf{x}}_{s2e}} \bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) \leq -\gamma_i \|\tilde{\mathbf{x}}_{s2e}\|, i \in I_C \quad (6.48)$$

Then, the asymptotic stability of the cascaded switched system during each controller's switching instant can be improved by a dwell time, τ_D that can be computed as:

$$\tau_D > \frac{1}{2 \left(\frac{\gamma_2}{\beta_1} + \frac{\gamma_1}{\beta_2} \right)} \log \frac{\gamma_1 \gamma_2}{\beta_2 \beta_1} \quad (6.49)$$

where $\gamma_1, \gamma_2, \beta_1, \beta_2$ are positive constant.

Proof:

With the input and perturbation terms bounded as expressed in (6.32) and (6.33) respectively, the trajectories of $\tilde{\mathbf{x}}_{s2e}(t)$ will decay exponentially and converge to the origin as shown in Theorem 6.1. Combining (6.47) and (6.48), the following expression can be obtained:

$$\frac{\partial V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial t} + \frac{\partial V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t)}{\partial \tilde{\mathbf{x}}_{s2e}} \bar{\mathbf{f}}_{s2}(\tilde{\mathbf{x}}_{s2e}, t) \leq -\frac{\gamma_i}{\beta_i} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \quad (6.50a)$$

$$\dot{V}_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \leq -\frac{\gamma_i}{\beta_i} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t) \quad (6.50b)$$

This imply that the solution of (6.50b) becomes,

$$V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0 + \tau_D) \leq e^{-\frac{\gamma_i}{\beta_i} \tau_D} V_{I,i}(\tilde{\mathbf{x}}_{s2e}, t_0) \quad (6.51)$$

for all $[t_0, t_0 + \tau_D)$. Let's consider the case where $i \in I_C$. For example, it takes on the value 1 on $[t_0, t_1)$ and 2 on $[t_1, t_2)$ and so on where $t_{j+1} - t_j \geq \tau_D, j = 0, 1$. From the above inequalities, the switching occurs at t_1 has to satisfy the following:

$$V_{I,2}(\tilde{\mathbf{x}}_{s2e}, t_1) \leq \frac{\gamma_2}{\beta_2} V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_1) \leq \frac{\gamma_2}{\beta_2} e^{-\frac{\gamma_2}{\beta_2} t} V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_0) \quad (6.52)$$

and similarly, the switching occurs at t_2 must satisfy:

$$V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_2) \leq \frac{\gamma_1}{\beta_1} V_{I,2}(\tilde{\mathbf{x}}_{s2e}, t_2) \leq \frac{\gamma_1}{\beta_1} e^{-\frac{\gamma_1}{\beta_1} t} V_{I,2}(\tilde{\mathbf{x}}_{s2e}, t_1) \quad (6.53)$$

From (6.52) and (6.53), it implies that:

$$V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_2) \leq \frac{\gamma_1 \gamma_2}{\beta_1 \beta_2} e^{-\left(\frac{\gamma_1}{\beta_1} + \frac{\gamma_2}{\beta_2}\right) t} V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_0) \quad (6.54)$$

Thus, Equation (6.54) shows that the $V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_2)$ is less than or equal to $V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_0)$ if the dwell time, τ_D at each switching instant is large enough for faster exponential convergence. See Fig D1.4 in Appendix D1 for the schematic diagram to illustrate the (6.54).

By manipulating the (6.54), the explicit lower bound on τ_D that guarantees the asymptotic stability (or in particular exponential stability) of the cascaded switched system can be computed as:

$$\tau_D > \frac{1}{2\left(\frac{\gamma_2}{\beta_1} + \frac{\gamma_1}{\beta_2}\right)} \log \frac{\gamma_1 \gamma_2}{\beta_2 \beta_1} \quad (6.55)$$

To verify that condition (6.54) is satisfied, a simulation using the same switching sequence as in Table 6.1 with the pipeline profile in Fig. 6.6 for different dwell times from ten to thirty seconds was done. As seen in Fig. 6.13, it is observed that the “energy” after the switching is smaller than the “energy” before the switching. Thus, it proves that by increasing the dwell time, the “energy” converges faster as the exponential term in (6.54) increases.

Figure 6.13: Effect of dwell time on the inner loop “energy”

Next, the above-mentioned proof for the outer loop dynamic $\tilde{\mathbf{x}}_{\text{slc}}(t)$ is repeated. Similarly, the equilibrium submanifold $\tilde{\mathbf{x}}_{\text{slc}}(t) = 0$ can be shown to be exponentially stable

□

In summary, the dwell time is only necessary to improve the asymptotic stability about the equilibrium sub manifold for the cascaded switched systems to ES. It was introduced to allow transience to decay before switching to the next mode. Theoretically, the dwell time that can be computed analytically in the (6.55). However, in the literature [117], there is no minimum or single dwell time imposed in application. This is mainly due to the limitation in systems hardware that consists of multiple sensors and actuators with different sampling rate, response time and electronics. However, for most cases, the dwell time must be greater than zero and less than 15 seconds to be effective to improve the stability

6.5 Simulation of Cascaded Supervisory Switched System

As shown in Chapter 5, the proposed cascaded system is used primarily to overcome the whirling motion whereby the velocity u , v and w (and its corresponding position x , y , z) are not asymptotically stable during the stabilizing operation. By using the proposed switching approach for the ROV moving in a sequence of plane-based motions, the steady-state performance, the robustness and the control effort of the cascaded system can be improved as demonstrated in the comparisons below.

Comparison One: Cascaded switched system versus cascaded system in Chapter 5.

Comparison Two: Cascaded switched system with constant hydrodynamic perturbations versus with persistent perturbations due to underwater current.

The same profile in Fig. 6.6 was used (except that the initial orientation of the ROV at the initial launch point was set to $[0 \ 0 \ 0 \ 0 \ 0 \ 1.57]^T$). For consistency, the same reference inputs in Table 6.1 and switching sequence in Fig. 6.7 are used. For the two comparisons, the simulation model can be found in the RDA package written in the MATLABTM scripting language and graphical block diagram simulation, SIMULINKTM to simulate the ROV control systems design.

The simulation time is set as 650 seconds. The ODE solver with variable step size and a relative tolerance of 0.001 were used. For each comparison, parameters such as steady-state time response, thrust output, “energy” functions are compared. In addition, the ROV’s operating specifications as stated in Table 1.2 are verified.

6.5.1 Comparison One: Cascaded Switched System vs Cascaded System

Using one the fixed controller given in Chapter 5, the difference between the cascaded-switched system and the cascaded system (without switching) can be compared. As seen in Fig. 6.14, both position tracking (in m and rad) and velocity (in m/s and rad/s) errors (using the maximum Euclidean norm) are smaller than the cascaded system. It can be observed that the Euclidean norm error is in the magnitude of thousand due to the accumulative nature of the sum of squared error at each time instant. Hence it is quite dependent on the step size of the time.

As observed, the roll and pitch angle are less oscillatory as compared to the cascaded system. In the simulated position and velocity of the two systems in Fig. 6.16a and Fig. 6.16b respectively, the cascaded system exhibits a larger sway error in the pipeline tracking. The oscillation is due to the instability of the cascaded system without switching during the pipeline tracking,

As seen in Fig. 6.15, the total thrust (as shown in Fig. D4.1 in Appendix D4) to move the ROV is lowered than the cascaded system as the inner loop is activated only after the targeted position is reached. With the dwell time included during the switching, the “energy” is generally smaller during the transient (from one reference input to another) as compared to the cascaded system. As shown in Fig. D4.2 to Fig. D4.4 in Appendix D4, the rate of change in the “energy” remains negatively definite and hence the cascaded switched system is asymptotically stable.

Figure 6.14: Position tracking (left) and velocity (right) error of cascaded switched system vs cascaded system (maximum Euclidean norm)

Figure 6.15: Total thrust to move ROV (maximum Euclidean norm)

Figure 6.16a: Position of the cascaded switched system and cascaded system

Figure 6.16b: Velocity of the cascaded switched system and cascaded system

6.5.2 Comparison Two: Cascaded Switched System with Constant Perturbations vs Persistent Perturbations

The next question is how robust is the proposed controller under perturbation due to hydrodynamic forces and underwater current effects. In Comparison Two, constant perturbations on the hydrodynamic damping and added mass terms are used and the

performance of the system under the persistent excitation of underwater current is also examined. The underwater current can be realized by using a sine wave with the frequency of 1 rad/s and pk-pk value of 0.2. The switching sequence used can be found in Fig. 6.10.

As shown in Fig. 6.17, the velocities (also see Fig. D5.1 to Fig. D5.2 in Appendix D5) due to the persistent excitation are quite oscillatory. As observed in Fig. 6.18, the Eulers' angle is quite oscillatory but the amplitude on these angles is quite small. As seen in the simulated trajectory of the two systems in Fig. 6.19, the cascaded switched system under both constant and persistent excitation is able to track the desired trajectory profile.

As shown in Fig. D5.3 to Fig. D5.5 in Appendix D5, the derivative of the “energy” function remains negatively definite and hence the cascaded switched system is said to be robust and asymptotically stable against the perturbation due to the underwater current effect and hydrodynamic forces.

Figure 6.17: Velocity of the cascaded switched system with constant and persistent perturbations

Figure 6.18: Positions of the cascaded switched system with constant and persistent perturbations

Figure 6.19: Resulting pipeline trajectory

6.6. Summary

As observed from the above two comparisons (in Appendix D4 and D5) using the RDA package, the proposed cascaded-switched system reduces the thrust required for ROV's and exhibits a smaller position tracking error (as the inner loop is used only). With the proposed switching conditions using the “energy” in the inner loop and the position tracking error, the inner loop is activated automatically once the stipulated switching conditions are satisfied. Besides, a multi-controller supervisor is tasked to orchestrate the switching among the

candidate controllers based on different phase of vehicle positioning defined by the reference input.

The cascaded switched system remains robust and asymptotically stable under the hydrodynamic forces and persistent underwater current perturbation. As observed in the time responses (see Table 1.2 for the operating specification), the roll and pitch angles become less oscillatory and the maximum velocities do not exceed 0.6 m/s. The position tracking and velocity error (Euclidean norm) are smaller in cascaded switched system. For all ROV positions, settling time of less than 100 seconds and a ten percent overshoot were achieved in all ROV positions

As noticed from the comparisons, the switching sequences used are similar. As will be shown in Chapter 7, the switching sequences can be different for different ROV such as JHU and ROMEO ROV. This is due to the different ROV used and hence the same switching sequence may not be applicable to all. Instead of using the three inner and outer controllers, only two of the inner and outer controllers (as shown in Chapter 7) were utilized. This yields less number of switching and a shorter mission time without adverse effect on the robustness of the systems. With the new-cascaded switched system evolved with the demonstrated advantages, the experimental and simulation results can be seen in Chapter 7.

Chapter 7

Case Study: An Application of Proposed Controller Design

In Chapter 6, the proposed cascaded switched control system design of the RRCROV was discussed and simulation results were given. In this Chapter, the pool test results of the cascaded switched system for the RRC ROV will be compared with the theoretical results given in Chapter 6. In addition, the performances of the proposed controller design for two other working ROVs under different control schemes were also compared. The ROV used in the case study are namely: JHU ROV (from US) and ROMEO ROV (from Italy). They were selected for the case study based on the experimentally validated ROV model. Sea trials were conducted for these ROV and numerous publications [120,121,122,123,124,125,126] were documented. As observed in the Table 7.1, they use different control architecture with different number of DC brushless thrusters and on-board sensors. Noticeably, these ROV's are different in size and hence the dynamic behaviors are expected to be different.

The dynamic models of JHU and ROMEO ROV were estimated using least-squares (LS) identification technique while other ROVs [94,120] used the adaptive identification technique. The LS can only be done offline and the adaptive identification is performed online and does not require acceleration sensor that can introduce noise in the data. Most of the ROVs' experimental data were collected in the surge, heave and yaw direction as there were no thrusters to provide pure lateral movement i.e. sway direction. In most cases, the position and heading of the vehicle are actively controlled and the vehicle is passively stable in the roll and pitch directions. The common practice for these models is to approximate the six DoF equations by neglecting the off-diagonal entries, coupling terms and tether dynamics, resulting in decoupled dynamical equation for each DoF. The papers cited seldom provide theoretical justification nor in the experimental verifications for these. However for the RRC ROV, the coupling terms are included in the ROV model, and even then, the roll and pitch motions are shown to be asymptotically stable.

For both the experiments on the JHU and ROMEO ROV, the thrusters' dynamics were modelled by static equations. This was due to the lack of more advanced thruster models at the time of experiments. In most cases, the thrusters dynamic would have much shorter response times compared with that of the ROV (due to its larger inertia) and it is reasonable to ignore thruster dynamics, except in very slow manoeuvres.

Chapter 7 is organized as follows. The JHU ROV and ROMEO ROV dynamic models and their experimental setups were described in Section 7.1 and 7.2. Next, in Section 7.3, the experimental results of the RRC ROV were compared with the theoretical results. Using the same control schemes for the JHU ROV and ROMEO ROV, it can be observed in Section 7.4 that they too were able to achieve compatible performance as the RRC ROV. The performance of the proposed control switching scheme is compared with different controllers [120,121,122,123,124,125,126] in Section 7.5. Lastly, Section 7.6 summarizes the work done in this Chapter.

Descriptions	JHU ROV	ROMEO ROV	RRC ROV
Designed	JHU	CNR- IAN	RRC-NTU
Dynamic Model	Body-fixed, Multiple Decoupled DOF Model	Body-fixed, Multiple Decoupled DOF Model	Body-fixed, Multiple Decoupled DOF Model
Mass	200 kg	450 kg	115 kg
Operating Depth	1000m	500m	300m
Size	1.5m x 1.0mx 0.6m	1.3m x 0.9mx 1.0m	1.0m x 0.9mx 0.5m
Number of Thrusters	4 brushless DC motors, electric	8 brushless DC motors, electric	4 brushless DC motors, electric
Stability	Passively stable in roll & pitch	Active controlled all DOF	Passively stable in roll & pitch
Buoyancy	Negatively buoyant	Negatively buoyant	Neutrally buoyant
Position Instrumentation	6 positions output with velocity and accel. by differentiation	6 positions and Savitzky–Golay polynomial filter for velocity, accel. by differentiation	6 velocity output, accel. by differentiation and position by integration
Thruster Model	Thrust proportional to current model, without motor shaft dynamic, peak thrust of 150N	Thrust proportional to square of input voltage, without motor shaft dynamic, peak thrust of 100N	Thrust proportional to square of angular speed with motor shaft dynamic, peak thrust of 70N
Sea Trial Location	U.S. Naval Academy's Hydrodynamics Lab.	CNR-IAN Pool in Italy	NTU Pool in Singapore
DOF examined	Surge ,heave & yaw direction	Surge ,heave & yaw direction	Surge, sway, heave & yaw direction

Table 7.1: Comparisons of two ROVs with RRC ROV

7.1 JHU ROV Experimental Setup and Model Parameters

The Johns Hopkins University (JHU) ROV, Fig. 7.1, is a tethered, remotely operated underwater robot designed and built by JHU in the U.S. The JHU ROV is powered by an isolated 10kW DC power supply. The mass of the vehicle is 200 kg and its dimensions are 1.5 m long, 1 m wide and 0.6 m high.

Figure 7.1: Physical Layout of JHU ROV

(Picture cited from http://robotics.me.jhu.edu/dsc/jhurov_main.htm)

The position and heading of the vehicle are actively controlled and the vehicle is passively stable in roll and pitch. Actuation is provided by five DC brushless, electric thrusters. A complete description of the JHU ROV was reported in [120]. The ROV is equipped for full six DOF-position measurements for this set of experiments. The vehicle's entire sensor suite is listed in Table 7.2. For these experiments, the vehicle was actuated by two longitudinal thrusters and one vertical thruster. There were no thrusters arranged to provide pure lateral movement. Free running experiments were conducted in the surge, heave and yaw degrees of freedom. Multiple trials in each degree of freedom were conducted with sinusoidal thrust profiles of varying magnitude and frequency.

Variable	Sensor	Precision	Update Rate
XYZ Position	300 kHz SHARPS Acoustic Transponder System	0.5 cm	10 Hz
Depth	Foxboro/ICT Model no. 15	2.5 cm	20 Hz
Heading	Litton LN200 IMU Gyro (USNA)	0.01 deg.	20 Hz
Roll and Pitch	KVH ADGC	0.1 deg.	10 Hz
Heading	KVH ADGC	1 deg	10 Hz

Table 7.2: Sensors Instrumentation for JHU ROV (cited from [120])

In many standard-maneuvering conditions, e.g., plane surge motion or vertical translation, and generally at low operating speeds, the coupling terms may be reasonably neglected without serious effect. As a consequence, on-board sensor-based identification experiments usually refer to a simplified decoupled model that can be deduced from (1) neglecting the off-diagonal elements of the added mass matrix, the Coriolis and centripetal kinematics, and drag coupling terms. This approximation relies on the fact that: (2) the off-diagonal elements of the added mass matrix of a rigid body having three symmetry planes; (3) the off-diagonal elements of the positive definite matrix are much smaller than their diagonal counterparts; and (4) the hydrodynamic damping coupling is negligible at low speeds. The resulting model structure for a single degree of freedom, $v_i(t)$ takes the form [120]:

$$\dot{v}_i(t) = \alpha_i \tau_i(t) + \beta_i v_i(t) |v_i(t)| + \mu_i v_i(t) + v_i(t) \quad (7.1)$$

where, for each degree of freedom i , $\tau_i(t)$ is the net thrust forces and moments, m_i is the inertia mass, $d_{Q_i} v_i(t) |v_i(t)|$ and $d_{L_i} v_i(t)$ represent the hydrodynamic drag, and b_i is the buoyancy. Equation (7.1) can be rewritten in vector form as

$$\dot{\mathbf{v}}_i(t) = \mathbf{\Phi}_i^T \mathbf{f}_i(t) \quad (7.2)$$

where $\mathbf{\Phi}_i = [\alpha_i \quad \beta_i \quad \mu_i \quad v_i]$ is the vector of lumped plant parameters and $\mathbf{f}_i(t) = [\tau_i(t) \quad v_i(t) |v_i(t)| \quad v_i(t) \quad 1]$ is a nonlinear vector of state and thrust input $\tau_i(t)$. The lumped parameters in (7.2) can be shown in Table 7.3.

Lumped Parameters	Physical Definition
α_i	m_i^{-1}
β_i	$-m_i^{-1}d_{Q_i}$
μ_i	$-m_i^{-1}d_{L_i}$
ν_i	$-m_i^{-1}b_i$

Table 7.3: Lumped parameters (cited from [120])

A static thruster model was used for these experiments to model the thrust produced by each thruster. The thrust produced is assumed to be proportional to the current commanded. The motor current was controlled using current mode amplifiers. In many applications, the servo velocity loop of the controlled thruster system has a negligible time constant with respect to the overall vehicle's time constant [127], and thus the thruster dynamics can be neglected with respect to the vehicle's dynamics. Although, it was indicated that the authors were aware of more advanced thruster models that provide a more precise dynamics model, dynamical thruster characterization was not available at the time of the experiments. The experiments reported in this paper were conducted using the JHU ROV at the United States Naval Academy's Hydrodynamics Laboratory, in Annapolis, MD.

The JHU employs the conventional least-square parameter identification to experimentally determine plant parameter values. Firstly, dynamic experiments were conducted in the surge, heave and yaw degrees of freedom. For each experiment, estimates for the plant parameters were determined using the least-squares method. Next numerical simulations with the identified parameters and the actual logged force profile from the experiment were conducted. The output state of the simulation was the velocity term of each of the direction, and this was compared with the respective actual logged experimental vehicle velocity. The error, for each degree of freedom was calculated as a mean error. The parameters that performed best (i.e. had the lowest "error") of all the experiments in numerical simulations are listed in Table 7.4. More detailed information about JHU ROV's mechanical, computer, software design and experimental results can be found in [120].

Degree of Freedom	α_i	β_i	μ_i	ν_i	Error e_i
1 (x_1)	0.003755	-4.406	0	0.007358	0.01664 m/s
1 (x_2)	0.002355	-4.9952	0	-0.004066	0.01607 m/s
3 (x_3)	6.237e-4	-1.102	0	0.01934	0.01652 m/s
4 (x_4)	0.0102	-1.9086	0	-0.001221	1.651 degrees/s

Degree of Freedom	Inertia	D_Q	D_L	Buoyancy	
1 (x_1)	266.3 [kg]	1173.4 [kg/m]	0 [kg/s]	-1.96 [N]	
2 (x_2)	424.6 [kg]	2120.9 [kg/m]	0 [kg/s]	1.73 [N]	
3 (x_3)	1603.3 [kg]	1766.84 [kg/m]	0 [kg/s]	-31.01 [N]	
4 (x_4)	98.04 [kg m ²]	187.1 [kg m ²]	0 [(kg m ²)/s]	0.1197 [N-m]	

Table 7.4: Experimental data for JHU ROV (cited from [120])

7.2 ROMEO ROV Experimental Setup and Model Parameters

ROMEO was developed by CNR-IAN in Italy for robotics research and scientific applications. It is about 0.9 m in height, 0.9 m in width, and 1.3 m in length, weighs about 450 kg and is intrinsically stable in pitch and roll. It is divided into three sections. A steel frame, which is 0.6 m in height, supports the two upper sections, the foam for buoyancy and the electronics and propulsion systems. The bottom section of the vehicle is equipped with two cylindrical canisters for batteries and payload electronics (see Fig. 7.2). The overall structure of the vehicle is symmetric with respect to both the XZ and YZ planes, and the eight thrusters are arranged two by two in the corners, with the horizontal ones parallel to the diagonals of the XY Section. This thruster configuration enables full controllability of the vehicle's motion and the possibility of distributing the propulsion and control forces according to constraints of a different nature.

An off-line Savitzky–Golay polynomial filter [128] was used to evaluate the velocity and then integrating numerically over time to obtain the position signal. The sampling rate of position is very low e.g. 3.3 Hz for echo sounders for the surge and sway and 10 Hz for the yaw. Experiments have shown that the estimated value is good enough to provide reliable and useful models for motion estimation and control purposes.

Neglecting the motor dynamics, the thruster force can be modeled as a function of the control voltage applied to the thruster servo-amplifiers. The LS procedure is used to identify the thruster model, with the actuator (motor, propeller, and nozzle) in a thrust tunnel and measuring the force as a function of a set of input voltages ($\bar{u}$) e.g. $Thrust = 1.02\bar{u}|\bar{u}|$ (forward) and $Thrust = 1.09\bar{u}|\bar{u}|$ (backward).

Figure 7.2: ROMEO ROV

(Picture cited from <http://www.robotlab.ian.ge.cnr.it/romeoing.htm>)

The CNR-IAN employed the conventional least-square parameter identification (identical to JHU and JASON II [122] ROV) to experimentally determine plant parameter values for the ROMEO ROV. Firstly, dynamic experiments were conducted in the surge, heave and yaw degrees of freedom. For each experiment, estimates for the plant parameters were determined using the least-squares method. Next numerical simulations were run on the identified parameters. The simulation was run using the actual logged force profile from the experiment. The output of the simulation was a velocity, which was compared to the actual logged experimental plant velocity. Table 7.5 shows the results from the reported paper [125,126]. More detailed information about ROMEO's mechanical, computer and software design can be found in [125,126].

	k_u^+ [Ns/m]	$k_{u u }^+$ [Ns ² /m ²]
$\hat{\theta}$	46.9	306.2
$\hat{\sigma}_\theta$	4.9	15.9

	k_w [Ns/m]	$k_{w w }$ [Ns ² /m ²]	W [N]	ΔW [N]
$\hat{\theta}$	44.7	430.3	-13.5	7.1
$\hat{\sigma}_\theta$	8.8	20.8	1.3	0.4

		k_r [Nms/rad]	$k_{r r }$ [Nms ² /rad ²]
Linear and quadratic yaw model	$\hat{\theta}$	20.5	49.5
	$\hat{\sigma}_\theta$	2.6	7.2
	$100 \hat{\sigma}_\theta / \hat{\theta} $	12.7 %	14.5 %

Table 7.5: Experimental data for surge, heave and yaw direction (cited from [125,126])

The ROMEO's dynamics in the surge, heave, and yaw DOF have been modeled and experimentally identified. In all the experiments, the required velocities for all the DOF have been estimated off-line on the basis of position measurements with a non-causal Savitzky–Golay polynomial filter [128] of fourth order with a symmetric moving window of different lengths according to the specific DOF.

7.3 RRC ROV Experimental Test Comparisons

The RRC ROV (as seen in Fig. 7.3) is tethered by a 100 m long cable that supplies 230 VAC electrical powers through an AC generator (powered by gasoline). The umbilical cable dispensed from the tether drum provides the communication and video signal lines that supports the RS-485 standard at a baud rate of 9600 via an RS-232C to RS-485 converter (see the Signal Converter in Fig. 7.3). The control system embedded inside the ROV consists of the industrial controller (incorporates an AMD 5x86-133 processor with 6 MB RAM), thrusters' driver, navigation module and two cameras that were hosted on a QNX real-time operating system (QNX-RT OS). The user-interface with this QNX-RT OS can be shown in the Main PC in Fig. 7.3

All control algorithms were coded in C language and that include the thrusters' driver for the thrusters' speed and direction control and the navigation module such as the velocity Doppler, magnetic compass and pressure sensors. On the other hands, the Graphic-User-Interface (GUI) with joystick provides the real-time command input to move the ROV. The joystick accepts motion commands for the URV much like a normal joystick while in the camera control mode; the joystick is used to direct the orientation of the pan-tilt mechanism of the camera via RS232 or LAN. The outputs from the navigation sensors were saved in the Main PC for offline analysis. Communications between the modules are achieved through the LAN 100Base-T crossed cable with RJ45 connectors at both ends. The actual pool test setup can be seen in Fig. 7.4.

Figure 7.3: RRC ROV control system setup for pool test

Figure 7.4: Actual setup for pool test

To compare the experimental with the theoretical results, the proposed cascaded switched control was applied on the RRC ROV. As seen in Fig. 7.5, a small pool measuring 10 m $\times$ 10 m $\times$ 5 m (depth) was used to test out the control system of the vehicle. As seen in the Fig. 7.6, due to the limitation of on-board sensors at the time of the test, only a simple test involving yaw and heave operation (pressure and magnetic compass sensor respectively) were compared with the measurement from the observer. The remaining unmeasured surge and sway positions were obtained through the nonlinear observers as seen in the Fig. 7.7.

Figure 7.5: RRC ROV in small pool tethered by umbilical cable

As shown in Fig 7.6, the observer (as designed in Chapter 5) is able to estimate the heading and altitude as acquired by the pressure and magnetic compass sensor. As seen in Fig. 7.6, the magnetic compass for yaw angle was initially at 360 degrees angle before correcting itself to the zero degree angles that corresponds to the ROV initial position. It took almost ten seconds for the correction to complete. As shown in Fig 7.7, the observer was able to estimate the desired squared profile with small estimated error. The ROV exhibits some oscillations in the surge and sway position as the vehicle maneuvered a 90-degreed turn. This can be contributed by the wave reflections from the sides of the pool walls.

Figure 7.6: Sensors and observer result for heading (left) and heave (right) DoF

Figure 7.7: Observer result (as compared with theoretical) for surge and sway DoF

To demonstrate the tracking and stabilization capability of the proposed cascade switched system (as designed in Chapter 6), the experimental results were compared with the theoretical results using the RDA simulation for a small segment of the actual ROV test as seen in Fig. 7.8. The ROV was moved from its launch point to the start point (as shown in Fig. 7.8) where the station-keeping began. For consistency, the starting point for comparisons was set at point $[-0.16 \ 2.07 \ 0.50 \ 0 \ 0 \ 2.30]^T$ (for station-keeping mode using inner velocity loop that is $\sigma_L(t) = 1$) that coincide with the ROV when it began to yaw (for horizontal plane tracking using cascaded loop that is $\sigma_L(t) = 0$) and move attitudinally to the required depth at point $[-0.16 \ 2.07 \ 3.80 \ 0 \ 0 \ 0.74]^T$ (for vertical plane tracking using cascaded loop). The results of the simulated data and the actual one in three-dimensional view can be seen in Fig. 7.8. The tracking profile matches the experimental result for the stipulated path as mentioned. To test the controller switching, the cascaded switched system with the switching signal $\sigma_{CI}(t) = \sigma_{CO}(t) = 1$ was used but at time $t = 30$ seconds, the controller was changed to $\sigma_{CI}(t) = \sigma_{CO}(t) = 2$. The dwell time between the switching is about 5 seconds.

Figure 7.8: 3D view of RRC ROV in pool test

The position and velocity response of the tracking and stabilization can be seen in Fig. 7.9. For the position tracking involving the heave and yaw DoF, the ROV is able to match the simulation results with small steady-state error. However, the outputs show some oscillations

in the surge and sway position due to the wall wave reflections as the ROV turns. But, the steady-state error is small. On the other hands, the actual steady-state velocities are quite closed to the simulated one except for the surge and sway motion caused by the wall wave reflections that will investigate in future research. With the ROV moving at a slow speed and the properly laid tether employed during the ROV maneuvering, the tethered effect did not affect the ROV position tracking. In summary, the proposed cascaded switched system is able to perform the position tracking and stabilization operation. Besides, the position and velocity responses are asymptotically stable to the steady state value without any divergent response.

Figure 7.9a: Velocity response of RRC ROV

Figure 7.9b: Position response of RRC ROV

7.4 ROVs Performance Comparison using Proposed Controller

To compare with other ROV using the proposed controller, computer simulation using the simulation software, RDA was performed. Since the pool test results in Section 7.3 compare well with the theoretical result for at least in the heave and yaw-DoF, the proposed cascaded-switched scheme is applied on the JHU and ROMEO ROV system for comparisons.

To standardize the comparison and for consistency, the following pipeline profile as seen in Fig. 6.6 and the same input sequence in the Table 7.1 are used. For each ROV systems, parameters such as steady-state time response, thrust output, stability and robustness are given in Table 7.6. But the switching sequences are different as the ROVs have different dynamics. Hence the same switching sequence may not necessarily be applicable. With a

faster settling time, the simulation time is set as 300 seconds. The integration solver, ODE with variable step size and a relative tolerance of 0.001 were used.

Parameters	JHU ROV	ROMEO	RRC ROV
Velocity error (Euclidean norm)	30	33	35
Position tracking error (Euclidean norm)	2016	1137	790
Thrust Output (Euclidean norm)	1528	754	306
$\dot{V}_O$ (Euclidean norm)	-8.4×10^4	-1.1×10^5	-1.2×10^5
$\dot{V}_I$ (Euclidean norm)	-1.5×10^7	-1.0×10^7	-3.5×10^6
V_O (Euclidean norm)	417	418	348
V_I (Euclidean norm)	3.8×10^5	1.6×10^4	1.8×10^4
Percent thrust- max	60	65	30

Table 7.6: Parameters for each ROVs using the proposed control scheme

For all cases, the RDA package written in the MATLABTM scripting language and graphical block diagram simulation, SIMULINKTM are used to simulate the ROV control systems design. The VRML visualization subsystems (designed using the Virtual RealityTM toolbox) in the control system design flow can be seen in Fig. 7.10. The CAD model for the RRC ROV is converted into VRML format and the axis for six DOF motions are then defined in the VRML editor. The VRML visualization subsystem allows for virtual world to be driven by the ROV's position and orientation given by the simulation results in the RDA. This is extremely useful as the formation as a whole can be visualized and the immediate user feedback on how the ROVs are moving with respect to the reference pipeline profile can be obtained. Figure 7.11 shows a 3D view of the RRC ROV maneuvering above the pipeline profile.

Figure 7.10: RDA with VRML visualization integration and flow chart for ROV simulation

Figure 7.11: VRML visualization using the Virtual Reality™ toolbox for ROVs

Figure 7.12: 3D tracking results for JHU, ROMEO and RRC ROV using proposed controller

As shown in Fig. 7.12, all ROVs were able to track the given pipeline trajectory. As seen in the time-domain response plots in Appendix E1, the operating specification as seen in the Table 1.2 was met without violation. Exception can be seen in JHU ROV as it was unable to complete the tracking profile in the pre-defined simulated time of 300 seconds. As shown in Fig. E1.13 (in Appendix E1), different switching sequences were used for the ROVs. The switching sequence was adjusted to adapt to the different ROV's systems. The position tracking Euclidean norm for the RRC ROV was considered low amongst the ROVs while the velocity Euclidean norm was quite comparable. As observed in velocity plots in Fig. E1.1 (in Appendix E1), the roll and pitch angles are stable for all ROVs. Though the RRC ROV and JHU ROV have four thrusters, the resultant roll and pitch angles are self-stabilizable as compared to the ROMEO ROV having eight thrusters.

As seen in Fig. E1.9 (in Appendix E1), the proposed cascaded-switched system has lower thrust utilization (as percentage of maximum allowable thrust for each ROV) of about 60 percent less as compared to the rest of the controller. With different initial conditions, all

ROVs exhibit asymptotical stability as seen in the negative value of the $\dot{V}_O$ and $\dot{V}_I$ (see Fig. E1.10) and the Lyapunov function for both inner and outer loop was positive definite (see Fig. E1.11 and Fig. E1.12) at all time. Hence all ROVs with the proposed cascaded switched controller design are asymptotically stable to an equilibrium submanifold.

7.5 ROVs Performance Comparison using Different Controllers

To compare with other ROV using the proposed controller, computer simulation using the simulation software, RDA was performed. The different model based controllers such as PD, nonlinear model based controller, adaptive nonlinear controller and sliding mode controllers were compared with the proposed cascaded-switched control as seen in Table 7.7.

As observed from the different controllers used to obtain Fig. E2.8 (in Appendix E2), the proposed cascaded-switched system has a lower thrust utilization (as percentage of maximum allowable thrust for each ROV) of about 60 percent less as compared to the rest of the controller. The response time of the proposed controller was faster by about 50 percents as the inner loop was activated only after the desired position has reached. This compared well with the JHU ROV using the adaptive control and ROMEO ROV, which were unable to complete the pipeline tracking within 300 seconds. For the JHU ROV using PD controller, the settling time for each desired position input was greater than 100 seconds (implies a longer response time to reach the desired steady state position). This may be due to the lack of integral control and nonlinear decoupled components to reduce the position tracking error. The simulated pipeline tracking profile of all ROVs were shown in Fig. 7.13.

With the decoupled controller and its input sequence, the tracking position error Euclidean norm was lower than most of the controllers and hence has the smallest tracking error. This improves the robustness against the hydrodynamic damping and added mass uncertainties. However the velocity error Euclidean norm was higher than the rest of the controllers except for the JHU ROV using PD controller. The maximum velocity in surge and heave direction exceeded 0.6m/s for the JHU ROV using PD controllers while similar observation can be found in the heave and surge direction.

As observed in Fig. E2.1, Fig E2.2 , Fig. E2.9 and Fig. E2.10 (in Appendix E2), all ROVs were asymptotically stable to an equilibrium point in space. The resultant roll and pitch angles

of the JHU and RRC ROV are self-stabilizable, and even for ROMEO ROV which has more than six thrusters for six DOF. With the proposed cascaded switched controller design, the RRC ROV was asymptotically stable to an equilibrium submanifold. Besides having lower thrust utilization (as seen in Fig. E2.8), the position tracking error (as seen in Fig. E2.4) are lowered despite of the hydrodynamic and persistent perturbations due to underwater current, hence the ROV is robust in some sense. As seen in the time domain response plots in Appendix E2, the operating specifications as stated in the Table 1.2 were achieved for the proposed controller.

Figure 7.13: 3D tracking results for JHU, ROMEO and RRC ROV using different controllers

7.6 Summary

The tracking and stabilization capabilities of the proposed cascade switched system have been demonstrated by comparing the experimental results with the theoretical results using the RDA simulation for a small segment of the actual ROV maneuvering. The actual position and velocity response of the tracking and stabilization have been shown to match the simulation results at steady state with small steady-state error. Hence, the ROV system is quite robust against the hydrodynamic forces such as the wave rebound from the wall of the pool. Besides,

the cascade switched system was able to perform the position tracking and stabilization operation, with the position and velocity responses are asymptotically stable without divergent responses.

In addition, the proposed cascaded-switched system was applied to two other ROVs dynamic models namely: the JHU ROV and ROMEO ROV. As observed from the comparisons (in Appendix E1), the two ROVs are able to achieve a compatible performance as compared to the RRC ROV. All the ROVs were able to track the given tracking profile and the operating specifications given in the Table 1.2 were met. Different switching sequences were used for the ROVs to adapt to the different ROV's systems. The position tracking Euclidean norm for the RRC ROV was considered the lowest amongst the ROVs while the velocity Euclidean norm was quite comparable. All the ROVs exhibited stable roll and pitch angles and with different initial conditions, the ROVs remained asymptotically stable. Besides, the Lyapunov functions for both inner and outer loop were all positive definite at all time and its rate of change in the Lyapunov functions were negative definite. Hence all ROVs with the proposed cascaded switched controller design are asymptotically stable to an equilibrium submanifold.

The proposed cascaded switched control scheme was compared with other controllers such as the PD, adaptive and sliding mode controller as given in Appendix E2. The proposed cascaded-switched system has lower thrust utilization (as percentage of maximum allowable thrust for each ROV) of about 60 percent less compared with the rest of the controllers. As seen in Fig. E2.2, the response time of the proposed controller is about 50 percents faster, since the inner loop is activated only after the desired position has been reached. With the decoupled controller and inputs sequence designed into the system, the tracking position error Euclidean norm was lowest amongst the other types of controllers and hence has the smallest tracking error. This improves the robustness against the hydrodynamic damping and added mass uncertainties. With the proposed cascaded switched controller design, the RRC ROV was asymptotically stable to an equilibrium submanifold. As observed in the time-domain responses (see Appendix E2), the operating specifications as stated in the Table 1.2 were achieved for the proposed controller.

Parameters	JHU ROV				RRC ROV	ROMEO	
	Nonlinear PD	PD	Adaptive	Sliding Mode	Proposed	P	PD
Velocity error (Euclidean norm)	29	45	29	28	35	23	22
Position tracking error (Euclidean norm)	1206	2031	790	1208	790	1505	1675
Thrust Output (Euclidean norm)	1789	579	1864	1801	306	504	493
$\dot{V}_o$ (Euclidean norm)	-9.4×10^4	-9.2×10^4	-1.0×10^5	-9.4×10^4	-1.2×10^5	-1.17×10^5	-1.17×10^5
$\dot{V}_I$ (Euclidean norm)	-2.6×10^7	-2.6×10^7	-1.7×10^7	-1.2×10^7	-3.5×10^6	-4.00×10^6	-3.88×10^6
V_o (Euclidean norm)	366	585	367	363	348	412	414
V_I (Euclidean norm)	142	15400	159	143	1.8×10^4	50	49
Percent thrust (max)	100	100	100	100	30	30	30

Table 7.7: Parameters for each ROV using different control schemes

Chapter 8

Conclusions and Recommendations for Future Works

8.1. Conclusions

This thesis presents a robust cascaded switched control law with full-states observer for simultaneous stabilizing and tracking of a desired pipeline profile for the underactuated RRC ROV. The main motivation for this problem comes out of the need to develop a robust controller and simulation software for an underactuated system to perform inspection operations. The objectives of the thesis are to:

- (1) model the dynamics and the hydrodynamic forces acting on the RRC ROV;
- (2) resolve the number of DOFs for control action, with as much decoupled motion as possible;
- (3) solve the open loop instability of the RRC ROV at equilibrium (due the added mass components in the Coriolis and centripetal components that create cross coupling effect on ROV motion);
- (4) perform simultaneous stabilization and position tracking and to estimate velocity and position states;
- (5) reduce the thrust forces required and improve the robustness and steady-state performance of the system despite the presence of hydrodynamic perturbations;
- (6) achieve asymptotic stability at the equilibrium submanifold.

The nonlinear dynamic model includes the hydrodynamic added mass and damping forces that were computed by WAMITTM and ANSYSTM CFD software respectively. With the unactuated velocity states asymptotically stable (see Proposition 4.1), the number of DOFs is reduced and the ROV (in the station-keeping mode, horizontal and vertical plane) becomes an actuated system with least decoupled in states for control.

To accomplish the above control objectives, a novel strategy is proposed for the control of the RRC ROV. The strategy uses a logic-based switched hybrid control approach that differs considerably from those previously proposed in the literature. The proposed control architecture for this logic-based switch hybrid system consists of an inner loop for station-keeping mode and the cascaded loops for tracking operation. The outer loop control uses the nonlinear Proportional-decoupled controller to compute the control commands to the vehicle's linear and rotational speeds for stabilizing the vehicle kinematics to a desired position. The inner loop control is achieved by another nonlinear Proportional-decoupled controller that evolves on a much faster time scale than the outer loop. It ensures that the commands for linear and rotational speeds issued by the outer loop controller are tracked with sufficient accuracy in the presence of hydrodynamic perturbation.

To address the whirling of the ROV motion due to the added mass components in the Coriolis and centripetal term, a cascaded control system is proposed in Chapter 5 to control the velocities and positions simultaneously. As shown in the Chapter 5, the nonlinear Proportional-decoupled controller exhibits a shorter settling time response. Besides, the Proportional-decoupled controller is less oscillatory in both the estimated position and velocity responses as compared with the other controllers. Hence, it seems to perform better in the time-domain specification as compared to other controllers. For robustness test, a ten percent perturbation on the added mass and linear damping was applied to the nominal model. The proposed Proportional-decoupled controller had the lowest sensitivity value to perturbations in both the position and velocity. The steady state position error is smaller under the perturbations. This implies that the output is less susceptible to model parameter changes. As the steady state position and velocity errors are small and the roll and pitch motion are asymptotically stable, the resulted cascaded system with Proportional-decoupled controller and the nonlinear observer is able to achieve simultaneous stabilization and tracking despite the perturbations and is asymptotically stable to an equilibrium submanifold (in Theorem 5.1).

To further improve the robustness of the cascaded system under hydrodynamic perturbation, multiple controllers' strategies are introduced. As a single-fixed controller could not fully satisfy the desired closed loop responses, it is proposed to switch between at least two controllers in the inner loop whilst the ROV is performing the stabilization operation and to switch between at least two controllers in the cascaded loop during simultaneous tracking (in either horizontal or vertical plane) and stabilization operation. As a result, the switched control system is evolved. The multiple-controllers switching is based on the different phases

of vehicle positioning while the cascaded to inner loop switching is based on the energy in the inner loop and the position tracking errors. Since the cascaded switched system stability (see Proposition 6.1) can be affected due to switching, dwell time is imposed during the switching as given in Proposition 6.2. This improved on the ROV's asymptotically stable to an equilibrium submanifold during pipeline inspection.

As shown in the Chapter 6, the proposed cascaded-switched system reduces the thrust required for ROV and the position tracking and velocity error (Euclidean norm) are smaller in cascaded switched system as a smaller position tracking error (as the inner loop is used only). The roll and pitch angles become less oscillatory and the maximum velocities do not exceed 0.6 m/s. For all ROV positions, settling time of less than 100 seconds and a ten percent overshoot were achieved in all ROV positions. Importantly, the cascaded switched system remains robust and asymptotically stable under the hydrodynamic forces and persistent underwater current perturbation.

As shown in the Chapter 7, the robustness of the ROV has improved as steady-state error is lowered despite of the ten percent hydrodynamic perturbation. In addition, it has lower thrust utilization (as percentage of maximum allowable thrust for each ROV) of about 60 percent less as compared to other types of controllers. The response time of the proposed controller is about 50 percents faster, since the inner loop is activated only after the desired position has been reached.

To validate the cascaded switched system and the nonlinear observer design, a pool test experiment was performed. For the position tracking involving the heave and yaw DoF, the ROV was able to match the simulation results with small steady-state errors. Hence, the ROV system is robust against the hydrodynamic forces such as the wave rebound from the wall of the pool. With the ROV moving at a slow speed and the properly laid tether employed during the ROV maneuvering, the proposed cascaded switched system was able to perform the position tracking and stabilization operation. Besides, the position and velocity responses are asymptotically stable as no divergent responses were noticed. Arising from the work done in this thesis, a new simulation software called the ROV Design and Analysis (RDA)[102] is developed. It is written in MATLABTM scripting language and graphical block diagram in SIMULINKTM environment with VR interface is used to model and simulate the nonlinear ROV control systems. The RDA helps the designer to understand the system dynamics and to design appropriate controllers for the inner and cascaded loops.

8.2. Recommendations for Future Works

Further work is required to improve on the cascaded-switched control system. The suggestions for the future works are shown as follows.

1. In current work, there is no explicit modelling of the tether and other underwater dynamics. The modeling of the ROV can therefore be enhanced. One of the most interesting improvements is to model the umbilical cable dynamics, which is typically characterized by partial differential equations. The inclusion of such disturbances into the control system design can further improve a sense of realism in maneuvering the vehicle.
2. Underwater vehicles are liable to faults or failures during underwater missions. It is necessary to embed fault-tolerant control (FTC) paradigms into ROV operating system to increase the reliability of the vehicles and enable them to execute complex missions. The FTC comprises fault detection, identification, and control reconfigurations for fault compensation. Thruster failures are one of the most common and critical sources of faults. In all but the most trivial cases the existence of a fault may lead to the cancellation of a mission. The implication of small faults could be very expensive and time consuming. Although good design practice tries to minimize the occurrence of faults and failures, there is a certain probability that faults will occur. Recognition that such events do occur enables system designers to develop strategies by which the effect they exert is minimized. For example fault detection and diagnosis can be accomplished by evaluating any deviation from the normal behaviour of the system by comparing the states, the parameters and other related quantities of the observed process with those of the normal and faulty processes. A large number of underwater vehicles have four horizontal thrusters for the motion in the horizontal plane in three degrees of freedom (surge, sway and yaw). For this class of overactuated vehicles, in the case of a partial or total fault in a horizontal thruster, it is possible to reconfigure the control system in an optimal manner, in order to maintain a high level of maneuverability of the faulty vehicle and complete the mission. However for the case of underactuated system, it may not be easy to reconfigure the control system in an optimal manner. Literature review shows that there have been no systematic methods for fault-tolerant control of underactuated ROV.
3. Although the RRC ROV has been employed for pipeline tracking, their range of operation is constrained by the length of the tether. Furthermore, the need for a support vessel and an

ROV operator adds to the cost of monitoring operation. One way to circumvent these problems is to render the vehicle autonomous, that is, to execute the task with minimal human intervention at depth beyond the maximum allowable tether length (that is 300 m) and use the tethered ROV when hard-link is needed (within the maximum allowable tether length). Thus it becomes a hybrid ROV system where both AUV and ROV are used in different operation need.

4. Control of underactuated systems is rather challenging even with knowledge in hydrodynamic uncertainties. However, it is important to formulate and address the case whereby there are unknown uncertainties due to external environmental factor such as strong underwater current, wave and obstacles. It is necessary to measure the changes in external environmental parameters such as velocity around the RRC ROV and capture images on external obstacles (may include FTC as mentioned in item 2) for feedback to the control system for more accurate control of the ROV.

5. As the focus of this thesis is on improving controller performance, little attention has paid to improve the current payload and sensors suit configuration. In order to build upon the work done so far, a Hardware-in-the-loop (HIL) system can be set up. The minimalist-configuration model shown in Fig. 8.1 can be used to simulate the actual RRC ROV using the supervisory cascaded-switched control system. The hardware required include two cameras and a microprocessor with DAC (such as xPC TargetBoxTM by MATLABTM) with output to the four thrusters as shown in Fig. 8.2. The xPC TargetBoxTM is an industrial PC system designed for rapid control prototyping applications. It is able to execute the generated code from Real-Time Workshop in MATLABTM for validating the control system design in real time.

Figure 8.1: Prototype RRC ROV

Figure 8.2: Prototype RRC ROV in small tank experiment

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APPENDICES

Appendix A1

SIMULINK™ Block Diagrams

Figure A1.1: SIMULINK™ block diagram for $\mathbf{M}_{RB} \dot{\mathbf{v}} + \mathbf{C}_{RB}(\mathbf{v}) = \boldsymbol{\tau}$

Figure A1.2: SIMULINK™ block diagram for Coriolis and centripetal matrix, $\mathbf{C}_{RB}(\mathbf{v})$

Figure A1.3: SIMULINKTM block diagram for $\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v}) + \mathbf{D}\mathbf{v} + \mathbf{G}_r(\boldsymbol{\eta}) = \boldsymbol{\tau}$ with added mass

Figure A1.4: SIMULINKTM block diagram for $\mathbf{J}(\boldsymbol{\eta})$

Figure A1.5: SIMULINKTM block diagram for hydrodynamic damping, **D** Figure A1.6: SIMULINKTM block diagram for **G_f(η)** Figure A1.7: SIMULINKTM block diagram for forward thrust

Figure A1.7: SIMULINK™ block diagram for forward thrust

Figure A1.8: SIMULINK™ block diagram for perturbed ROV model

Figure A1.9: RDA software designed using SIMULINK™

Figure A1.10: SIMULINK model for ROV's subsystems analysis
(see "ANALYSIS " in Fig. A1.9)

Figure A1.11: SIMULINK model for Energy and Passivity analysis (see “ANALYSIS “ in Fig. A1.10)

Figure A1.12: SIMULINK™ block diagram for P decoupled inner controller
(see “ROV CONTROLLERS “ in Fig. A1.9)

Figure A1.13: SIMULINK™ block diagram for P decoupled outer controller
(see “ROV CONTROLLERS “ in Fig. A1.9)

Figure A1.14: SIMULINK™ block diagram for controllers' comparison
(see "ROV CONTROLLERS " in Fig. A1.9)

Appendix A2

Earth-Fixed Model by Lagrangian Method

Lagrangian mechanics uses variation approach to describe the dynamic of a system. It permits the derivation of the equations of motion from only two scalar functions, the kinetic and potential energy. The basic tool for deriving the equations of motion consists of a set of general differential equations known as Lagrange's equations. The kinetic and potential energy can be expressed in terms of generalized coordinates (or the minimum number of degree of freedom to describe the motion of the system completely) and velocities. However every method has its advantages and disadvantages, in this case, the generalized coordinate may not have physical meaning and not necessarily unique.

First, we need to formulate a suitable expression for the vehicle's kinetic and potential energy denoted T and V respectively. Then we compute the Lagrangian L according to:

$$L = T - V \quad (\text{A.1})$$

Finally, we apply the Euler-Lagrange equation:

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{\boldsymbol{\eta}}} \right) - \left(\frac{\partial L}{\partial \boldsymbol{\eta}} \right) + \frac{\partial P_d}{\partial \dot{\boldsymbol{\eta}}} = \mathbf{J}^{-T}(\boldsymbol{\eta}) \boldsymbol{\tau} \quad (\text{A.2})$$

Here we have included an additional terms:

$$\frac{\partial P_d}{\partial \dot{\boldsymbol{\eta}}} = \mathbf{D}_{\boldsymbol{\eta}}(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta}) \dot{\boldsymbol{\eta}} \quad (\text{A.3})$$

to describe the dissipative forces. P_d can be interpreted as a power function. The Lagrangian for the submerged vehicles system is given by:

$$L = T_{RB} + T_A - V \quad (\text{A.4})$$

where T_{RB} is the rigid body kinetic energy, T_A is the fluid energy and V is the potential energy due to the gravitational and buoyancy effect of submerged ROV. The potential energy is defined implicit by:

$$\frac{\partial V}{\partial \boldsymbol{\eta}} = \mathbf{G}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) \quad (\text{A.5})$$

Hence the total kinetic energy can be expressed as:

$$\begin{aligned} T &= T_{RB} + T_A \\ &= \frac{1}{2} \dot{\boldsymbol{\eta}}^T \mathbf{J}^{-T} (\mathbf{M}_{RB} + \mathbf{M}_A) \mathbf{J}^{-1} \dot{\boldsymbol{\eta}} \\ &= \frac{1}{2} \dot{\boldsymbol{\eta}}^T \mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) \dot{\boldsymbol{\eta}} \end{aligned} \quad (\text{A.6})$$

Furthermore, we can compute:

$$\frac{\partial L}{\partial \dot{\boldsymbol{\eta}}} = \mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) \dot{\boldsymbol{\eta}} - \frac{\partial V}{\partial \dot{\boldsymbol{\eta}}} = \mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) \dot{\boldsymbol{\eta}} \quad (\text{A.7})$$

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{\boldsymbol{\eta}}} \right) = \mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) \ddot{\boldsymbol{\eta}} + \dot{\mathbf{M}}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) \dot{\boldsymbol{\eta}} \quad (\text{A.8})$$

The next step involves computing:

$$\frac{\partial L}{\partial \boldsymbol{\eta}} = \frac{\partial T}{\partial \boldsymbol{\eta}} - \frac{\partial V}{\partial \boldsymbol{\eta}} = \frac{1}{2} \dot{\boldsymbol{\eta}}^T \frac{\partial \mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} \dot{\boldsymbol{\eta}} - \mathbf{G}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) \quad (\text{A.9})$$

Using the results together with:

$$\dot{\mathbf{M}}_{\boldsymbol{\eta}}(\boldsymbol{\eta}) = \dot{\boldsymbol{\eta}}^T \frac{\partial \mathbf{M}_{\boldsymbol{\eta}}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} \quad (\text{A.10})$$

Implies that (A.2) can be written as:

$$\mathbf{M}_\eta(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \frac{1}{2}\dot{\mathbf{M}}_\eta(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{D}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{G}_\eta(\boldsymbol{\eta}) = \boldsymbol{\tau}_\eta \quad (\text{A.11})$$

where

$$\mathbf{M}_\eta(\boldsymbol{\eta}) = \mathbf{J}(\boldsymbol{\eta})^{-T}(\mathbf{M}_{\text{RB}} + \mathbf{M}_A)\mathbf{J}(\boldsymbol{\eta})^{-1} \quad (\text{A.12})$$

$$\mathbf{D}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta}) = \mathbf{J}(\boldsymbol{\eta})^{-T}\mathbf{D}\mathbf{J}(\boldsymbol{\eta})^{-1} \quad (\text{A.13})$$

$$\mathbf{G}_\eta(\boldsymbol{\eta}) = \mathbf{J}^{-1}(\boldsymbol{\eta})\mathbf{G}_f(\boldsymbol{\eta}) \quad (\text{A.14})$$

$$\boldsymbol{\tau}_\eta(\boldsymbol{\eta}) = \mathbf{J}^{-T}\boldsymbol{\tau} \quad (\text{A.15})$$

Since the skew symmetric property:

$$\begin{aligned} \mathbf{x}^T[\dot{\mathbf{M}}_\eta(\boldsymbol{\eta}) - 2\mathbf{C}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})]\mathbf{x} &= 0, \forall \mathbf{x} \\ \Rightarrow \frac{1}{2}\dot{\mathbf{M}}_\eta(\boldsymbol{\eta}) &= \mathbf{C}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta}) \end{aligned} \quad (\text{A.16})$$

From this expression, the definition of the Coriolis and Centripetal matrix is recognized as:

$\mathbf{C}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta}) = \frac{1}{2}\dot{\mathbf{M}}_\eta(\boldsymbol{\eta})$. By applying the Lagrange equations with $\boldsymbol{\eta} = [x, y, z, \phi, \theta, \psi]^T$, we have shown that,

$$\mathbf{M}_\eta(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \mathbf{C}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{D}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{G}_\eta(\boldsymbol{\eta}) = \boldsymbol{\tau}_\eta \quad (\text{A.17})$$

Defines $\mathbf{F}(\boldsymbol{\eta}) = \mathbf{J}^{-T}(\boldsymbol{\eta})\mathbf{T}$ with $\boldsymbol{\tau} = \mathbf{T}\bar{\mathbf{u}} \Rightarrow \boldsymbol{\tau}_\eta = \mathbf{J}^{-T}\mathbf{T}\bar{\mathbf{u}} = \mathbf{F}(\boldsymbol{\eta})\bar{\mathbf{u}}$ gives:

$$\mathbf{M}_\eta(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \mathbf{C}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{D}_\eta(\dot{\boldsymbol{\eta}}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{G}_\eta(\boldsymbol{\eta}) = \mathbf{F}(\boldsymbol{\eta})\bar{\mathbf{u}} \quad (\text{A.18})$$

where the matrix $\mathbf{F}(\boldsymbol{\eta}) \in \mathbb{R}^i$ with rank $\mathbf{F}(\boldsymbol{\eta}) = i$ defines the force fields corresponding to the input $\bar{\mathbf{u}} = [\bar{\mathbf{u}}_1, \bar{\mathbf{u}}_2]^T \in \mathbb{R}^i$. Notice that the system is underactuated as $i < n$.

Appendix A3

Quasi-Lagrangian Method

Integration of the velocity $\int_0^{t_1} \mathbf{v} dt$ where $\mathbf{v} = [u, v, w, p, q, r]^T$ with respect to time (in order to obtain the generalized coordinate) has no immediate physical interpretation. As a result, we cannot use the Lagrange equation directly to formulate the equation of motion in the body-fixed coordinate system. However, applying Kirchhoff's equation of motion or Quasi-Lagrangian approach can circumvent this problem. The former method considers the kinetic energy while later uses both kinetic and potential energy in the Lagrange equation.

Applying the Quasi-Lagrange equation, the ROV dynamic can be expressed as:

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \mathbf{v}_1} \right) + \mathbf{v}_2 \times \left(\frac{\partial L}{\partial \mathbf{v}_1} \right) + \frac{\partial P_d}{\partial \mathbf{v}_1} - \mathbf{J}_1^T(\boldsymbol{\eta}_2) \frac{\partial L}{\partial \boldsymbol{\eta}_1} = \boldsymbol{\tau}_1 \quad (\text{A.19})$$

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \mathbf{v}_2} \right) + \mathbf{v}_2 \times \left(\frac{\partial L}{\partial \mathbf{v}_2} \right) + \mathbf{v}_1 \times \left(\frac{\partial L}{\partial \mathbf{v}_1} \right) + \frac{\partial P_d}{\partial \mathbf{v}_2} - \mathbf{J}_2^T(\boldsymbol{\eta}_2) \frac{\partial L}{\partial \boldsymbol{\eta}_2} = \boldsymbol{\tau}_2 \quad (\text{A.20})$$

where

$$\begin{aligned} \mathbf{v}_1 &= [u, v, w]^T \\ \mathbf{v}_2 &= [p, q, r]^T \\ \boldsymbol{\eta}_1 &= [x, y, z]^T \\ \boldsymbol{\eta}_2 &= [\phi, \theta, \psi]^T \end{aligned}$$

Here we included additional term:

$$\frac{\partial P_d}{\partial \mathbf{v}_1} = \mathbf{D}_1 \mathbf{v}_1 \quad (\text{A.21a})$$

$$\frac{\partial P_d}{\partial \mathbf{v}_2} = \mathbf{D}_2 \mathbf{v}_2 \quad (\text{A.21b})$$

to describe the dissipative forces. P_d can be interpreted as a power function where $\mathbf{D}$ is the linear damping matrix of the ROV system:

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 \end{bmatrix} \quad (\text{A.22})$$

The Lagrangian for the submerged system is given by:

$$L = T_{RB} + T_A - V \quad (\text{A.23})$$

where T_{RB} is the rigid body kinetic energy, T_A is the fluid energy and V is the potential energy due to the gravitational and buoyancy effect of submerged ROV. The potential energy is defined implicit by:

$$\frac{\partial V}{\partial \boldsymbol{\eta}_1} = 0, \quad \frac{\partial V}{\partial \boldsymbol{\eta}_2} = \mathbf{g}_{\eta_2}(\boldsymbol{\eta}) \quad (\text{A.24})$$

where the first term of (A.24) is zero due to the neutral buoyant ROV ($mg = \rho g \nabla$ where ∇ is the fluid displaced by the ROV).

Furthermore, we can compute:

$$\frac{\partial L}{\partial \mathbf{v}_1} = \mathbf{M}_{11} \mathbf{v}_1 + \mathbf{M}_{12} \mathbf{v}_2; \quad \frac{\partial L}{\partial \mathbf{v}_2} = \mathbf{M}_{21} \mathbf{v}_1 + \mathbf{M}_{22} \mathbf{v}_2 \quad (\text{A.25})$$

$$\frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \mathbf{v}_1} \right) = \mathbf{M}_{11} \dot{\mathbf{v}}_1 + \mathbf{M}_{12} \dot{\mathbf{v}}_2; \quad \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \mathbf{v}_2} \right) = \mathbf{M}_{21} \dot{\mathbf{v}}_1 + \mathbf{M}_{22} \dot{\mathbf{v}}_2 \quad (\text{A.26})$$

The next step involves computing:

$$\frac{\partial L}{\partial \boldsymbol{\eta}_1} = \frac{\partial T}{\partial \boldsymbol{\eta}_1} - \frac{\partial V}{\partial \boldsymbol{\eta}_1} = 0 \quad (\text{A.27})$$

$$\frac{\partial L}{\partial \boldsymbol{\eta}_2} = \frac{\partial T}{\partial \boldsymbol{\eta}_2} - \frac{\partial V}{\partial \boldsymbol{\eta}_2} = -\mathbf{g}_{\eta_2}(\boldsymbol{\eta}) \quad (\text{A.28})$$

The total kinetic energy can be expressed as:

$$\begin{aligned} T &= T_{RB} + T_A \\ &= \frac{1}{2} \mathbf{v}^T (\mathbf{M}_{RB} + \mathbf{M}_A) \mathbf{v} \\ &= \frac{1}{2} \mathbf{v}^T \mathbf{M} \mathbf{v} \end{aligned} \quad (\text{A.29})$$

where $\mathbf{M}$ is the mass inertia matrix of the ROV system:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \quad (\text{A.30})$$

and then

$$T = \frac{1}{2} (\mathbf{v}_1^T \mathbf{M}_{11} \mathbf{v}_1 + \mathbf{v}_1^T \mathbf{M}_{12} \mathbf{v}_2 + \mathbf{v}_2^T \mathbf{M}_{21} \mathbf{v}_1 + \mathbf{v}_2^T \mathbf{M}_{22} \mathbf{v}_2) \quad (\text{A.31})$$

Equation (A.31) is expressed as the partitioned sub matrices that are related to $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively.

Theorem 2.1: See [94] on obtaining Coriolis & Centripetal matrix from inertia matrix

If the inertia matrix is given as (A.30) then we can always parameterize the Coriolis & Centripetal matrix such that $\dot{\mathbf{M}} - [\mathbf{C}(\mathbf{v}) + \mathbf{C}(\mathbf{v})^T] = 0$ (see Property 2.1) by defining:

$$\mathbf{C}(\mathbf{v}) = \begin{bmatrix} \mathbf{0}_{3 \times 3} & -S(\mathbf{M}_{11} \mathbf{v}_1 + \mathbf{M}_{12} \mathbf{v}_2) \\ -S(\mathbf{M}_{11} \mathbf{v}_1 + \mathbf{M}_{12} \mathbf{v}_2) & -S(\mathbf{M}_{21} \mathbf{v}_1 + \mathbf{M}_{22} \mathbf{v}_2) \end{bmatrix} \quad (\text{A.32})$$

where $S(\cdot)$ indicates a skew-symmetric matrix that is:

$$S(\lambda) = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix} = -S^T(\lambda) \quad (\text{A.33})$$

Proof: The kinetic energy T can be written as a quadratic form:

$$T = \frac{1}{2} \mathbf{v}^T \mathbf{M} \mathbf{v} \quad (\text{A.34})$$

expanding this expression yields:

$$T = \frac{1}{2} (\mathbf{v}_1^T \mathbf{M}_{11} \mathbf{v}_1 + \mathbf{v}_1^T \mathbf{M}_{12} \mathbf{v}_2 + \mathbf{v}_2^T \mathbf{M}_{21} \mathbf{v}_1 + \mathbf{v}_2^T \mathbf{M}_{22} \mathbf{v}_2) \quad (\text{A.35})$$

Here we obtain:

$$\frac{\partial T}{\partial \mathbf{v}_1} = \mathbf{M}_{11} \mathbf{v}_1 + \mathbf{M}_{12} \mathbf{v}_2 \quad (\text{A.36})$$

$$\frac{\partial T}{\partial \mathbf{v}_2} = \mathbf{M}_{21} \mathbf{v}_1 + \mathbf{M}_{22} \mathbf{v}_2 \quad (\text{A.37})$$

From Kirchhoff's equations, we recognize that:

$$\begin{aligned} \mathbf{C}(\mathbf{v}) \mathbf{v} &\equiv \begin{bmatrix} \mathbf{v}_2 \times \left(\frac{\partial L}{\partial \mathbf{v}_1} \right) \\ \mathbf{v}_2 \times \left(\frac{\partial L}{\partial \mathbf{v}_2} \right) + \mathbf{v}_1 \times \left(\frac{\partial L}{\partial \mathbf{v}_1} \right) \end{bmatrix} \\ &\equiv \begin{bmatrix} \mathbf{0}_{3 \times 3} & -S \left(\frac{\partial T}{\partial \mathbf{v}_1} \right) \\ -S \left(\frac{\partial T}{\partial \mathbf{v}_1} \right) & -S \left(\frac{\partial T}{\partial \mathbf{v}_2} \right) \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} \end{aligned} \quad (\text{A.38})$$

which after substitution of (A.36) and (A.37) proves (A.32). Using these results implies that (A.36) and (A.37) can be written as:

$$\mathbf{M}_{11} \dot{\mathbf{v}}_1 + \mathbf{M}_{12} \dot{\mathbf{v}}_2 + \mathbf{v}_2 \times (\mathbf{M}_{11} \mathbf{v}_1 + \mathbf{M}_{12} \mathbf{v}_2) + \mathbf{D}_1 \mathbf{v}_1 = \boldsymbol{\tau}_1 \quad (\text{A.39a})$$

$$\mathbf{M}_{21}\dot{\mathbf{v}}_1 + \mathbf{M}_{22}\dot{\mathbf{v}}_2 + \mathbf{v}_2 \times (\mathbf{M}_{21}\mathbf{v}_1 + \mathbf{M}_{22}\mathbf{v}_2) + \mathbf{D}_2\mathbf{v}_2 - \mathbf{J}_2^T(\boldsymbol{\eta}_2)\mathbf{G}_{\eta_2}(\boldsymbol{\eta}_2) = \boldsymbol{\tau}_2 \quad (\text{A.39b})$$

From this expression, there is a need to define the Coriolis & Centripetal matrix from the inertia matrix using the Theorem 2.1 and since $\mathbf{G}_{\eta_2}(\boldsymbol{\eta}_2) = \mathbf{J}(\boldsymbol{\eta}_2)^{-T}\mathbf{G}_2(\boldsymbol{\eta}_2)$ then:

$$\mathbf{J}_2^T(\boldsymbol{\eta}_2)\mathbf{G}_{\eta_2}(\boldsymbol{\eta}_2) = \mathbf{J}^T(\boldsymbol{\eta}_2)\mathbf{J}(\boldsymbol{\eta}_2)^{-T}\mathbf{G}_2(\boldsymbol{\eta}_2) = \mathbf{G}_2(\boldsymbol{\eta}_2) \quad (\text{A.40})$$

The dynamic of an underactuated system in (A.39) becomes:

$$\begin{aligned} \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}_1 \\ \dot{\mathbf{v}}_2 \end{bmatrix} + \begin{bmatrix} 0_{3 \times 3} & -\mathbf{S}(\mathbf{M}_{11}\mathbf{v}_1 + \mathbf{M}_{12}\mathbf{v}_2) \\ -\mathbf{S}(\mathbf{M}_{11}\mathbf{v}_1 + \mathbf{M}_{12}\mathbf{v}_2) & -\mathbf{S}(\mathbf{M}_{21}\mathbf{v}_1 + \mathbf{M}_{22}\mathbf{v}_2) \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} \\ + \begin{bmatrix} \mathbf{D}_1 & 0 \\ 0 & \mathbf{D}_2 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} + \begin{bmatrix} 0_{3 \times 3} \\ \mathbf{G}_2(\boldsymbol{\eta}_2) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \end{bmatrix} \end{aligned} \quad (\text{A.41})$$

Equation (A.41) can be expressed in a compact form:

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}\mathbf{v} + \mathbf{G}_f(\boldsymbol{\eta}) = \boldsymbol{\tau} \quad (\text{A.42})$$

where all parameters used in (A.42) can be found in Chapter 3.

□

Appendix A4

Nonlinear RRC ROV Parameters

Parameters	Values	Section
Mass inertia matrix (Rigid body)	$\mathbf{M}_{RB} = \begin{bmatrix} 115.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 115.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 115.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.1000 & -0.00016 & -0.1850 \\ 0 & 0 & 0 & -0.00016 & 5.9800 & 0.0006 \\ 0 & 0 & 0 & -0.1850 & 0.0006 & 5.5170 \end{bmatrix}$	3.3.1
Weight of ROV/Center of gravity	$W = 115 \text{ kg}$, $\mathbf{r}_G = [x_G, y_G, z_G]^T = [0, 0, 0]^T$	3.3.1
Mass inertia matrix (Added mass)	$\mathbf{M}_A = \begin{bmatrix} 21.1403 & 0 & 0 & 0 & 0 & 0 \\ 0 & 51.7012 & 0 & 0 & 0 & 0 \\ 0 & 0 & 92.4510 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.6191 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.6427 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.3033 \end{bmatrix}$	3.3.2
Coriolis and Centrifugal Matrix (Rigid body)	$\mathbf{C}_{12}(\mathbf{v}) = \begin{bmatrix} 0 & 115.0000w & -115.0000v \\ -115.00004w & 0 & 115.0000u \\ 115.00004v & -115.0000u & 0 \end{bmatrix}$ $\mathbf{C}_{22}(\mathbf{v}) = \begin{bmatrix} 0 & -0.0006q + 0.185p + 5.5170r & 0.0006r - 0.0002p - 5.9800q \\ 0.0006q - 0.185p - 5.5170r & 0 & 0.1850r + 0.0002q + 6.100p \\ -0.0006r + 0.0002p + 5.9800q & -0.1850r - 0.0002q - 6.100p & 0 \end{bmatrix}$	3.3.1
Coriolis and Centrifugal Matrix (Added mass)	$\mathbf{C}_A(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -92.4510w & 51.7012v \\ 0 & 0 & 0 & 92.4510w & 0 & -21.1403u \\ 0 & 0 & 0 & -51.7012v & 21.1403u & 0 \\ 0 & -92.4510w & 51.7012v & 0 & -2.3033r & 2.6427q \\ 92.4510w & 0 & -21.1403u & 2.3033r & 0 & -3.6191p \\ -51.7012v & 21.1403u & 0 & -2.6427q & 3.6191p & 0 \end{bmatrix}$	3.3.2
Hydrodynamic Damping Matrix	$\mathbf{D} = \begin{bmatrix} 3.7000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.9129 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.3756 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.6650 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.4560 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1099 \end{bmatrix}$	3.3.3
Thruster Configuration	$\mathbf{T} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0.707 & -0.707 \\ 0 & 0 & 0.707 & 0.707 \\ 0 & 0 & -0.293 & 0.293 \\ -0.016 & -0.016 & 0.012 & -0.012 \\ 0.31 & -0.31 & 0.012 & -0.012 \end{bmatrix}$	3.3.5
Center of buoyancy($\mathbf{C}_B$)	$[x_G - x_B, y_G - y_B, z_G - z_B]^T = [0, 0, -0.048]^T$	3.3.4

Table A4.1: Parameters used in Nonlinear RRC ROV dynamic

Appendix B1

Uniqueness/Existence and Continuous Dependence Verification using MAPLETM

Figure B1.1. MAPLETM used for computing the uniqueness/existence and continuous dependence on parameters

Example of MAPLE™ Script for Full-Order Model

Initialization

```
> restart; with(linalg):
```

Full order body-fixed coordinates

System Parameters

```
> M := matrix(6,6,[m,0,0,0,0,0, 0,m,0,0,0,0,
0,0,m,0,0,0, 0,0,0,Ixx,-Ixy,-Ixz, 0,0,0, -Ixy,Iyy,-Iyz,
0,0,0, -Ixz,-Iyz,Izz ]);
```

```
> C:=matrix(6,6,[0,0,0,0,m*w,-m*v, 0,0,0,-m*w,0,m*u,
0,0,0,m*v,-m*u,0, 0,m*w,-m*v,0,-Iyz*q-
Ixz*p+Izz*r,Iyz*r+Ixy*p-Iyy*q, -m*w,0,m*u,Iyz*q+Ixz*p-
Izz*r,0,-Ixz*r-Ixy*q+Ixx*p, m*v,-m*u,0,-Iyz*r-
Ixy*p+Iyy*q,Ixz*r+Ixy*q-Ixx*p,0]);
```

$$M := \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{xy} & I_{yy} & -I_{yz} \\ 0 & 0 & 0 & -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

$$C := \begin{bmatrix} 0, & 0, & 0, & 0, & m w, & -m v \\ 0, & 0, & 0, & -m w, & 0, & m u \\ 0, & 0, & 0, & m v, & -m u, & 0 \\ 0, & m w, & -m v, & 0, & -I_{yz} q - I_{xz} p + I_{zz} r, & I_{yz} r + I_{xy} p - I_{yy} q \\ -m w, & 0, & m u, & I_{yz} q + I_{xz} p - I_{zz} r, & 0, & -I_{xz} r - I_{xy} q + I_{xx} p \\ m v, & -m u, & 0, & -I_{yz} r - I_{xy} p + I_{yy} q, & I_{xz} r + I_{xy} q - I_{xx} p, & 0 \end{bmatrix}$$

```
> Dd:=diag(Xu,Yv,Zw,Kp,Mq,Nr);
```

$$Dd := \begin{bmatrix} Xu & 0 & 0 & 0 & 0 & 0 \\ 0 & Yv & 0 & 0 & 0 & 0 \\ 0 & 0 & Zw & 0 & 0 & 0 \\ 0 & 0 & 0 & Kp & 0 & 0 \\ 0 & 0 & 0 & 0 & Mq & 0 \\ 0 & 0 & 0 & 0 & 0 & Nr \end{bmatrix}$$

```
>
```

```
g:=matrix(6,1,[0,0,0,29.61*cos(theta)*sin(phi),29.61*sin(theta),0]);
```

$$g := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.61 \cos(\theta) \sin(\phi) \\ 29.61 \sin(\theta) \\ 0 \end{bmatrix}$$

```
> Tv:=matrix(6,4,[1,1,0,0, 0, 0, 0.707, -0.707,0, 0,
0.707, 0.707,0, 0, -0.293, 0.293, -0.016, -0.016,
0.017*0.707, -0.017*0.707 ,0.31, -0.31, 0.017*0.707, -
0.017*0.707 ]);
```

$$T_v := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0.707 & -0.707 \\ 0 & 0 & 0.707 & 0.707 \\ 0 & 0 & -0.293 & 0.293 \\ -0.016 & -0.016 & 0.012019 & -0.012019 \\ 0.31 & -0.31 & 0.012019 & -0.012019 \end{bmatrix}$$

```
> J2:=matrix(3,3,[1,
sin(phi)*tan(theta),cos(phi)*tan(theta),0, cos(phi),-
sin(phi), 0,sin(phi)/cos(theta), cos(phi)/cos(theta)]);
```

$$J_2 := \begin{bmatrix} 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix}$$

```
> J1:=matrix(3,3,[cos(psi)*cos(theta), -
sin(psi)*cos(phi)+cos(psi)*sin(theta)*sin(phi),
sin(psi)*sin(phi)+cos(psi)*cos(phi)*sin(theta), -
sin(psi)*cos(theta),
cos(psi)*cos(phi)+sin(phi)*sin(theta)*sin(psi), -
cos(psi)*sin(phi)+sin(theta)*sin(psi)*cos(phi),-
sin(theta),cos(theta)*sin(phi),cos(theta)*cos(phi) ]);
J1 :=
```

$$\begin{bmatrix} \cos(\psi) \cos(\theta), & -\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi), \\ \sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta) \\ -\sin(\psi) \cos(\theta), & \cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi), \\ -\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi) \\ -\sin(\theta), & \cos(\theta) \sin(\phi), \cos(\theta) \cos(\phi) \end{bmatrix}$$

```
> J:=diag(J1,J2);
```

$J :=$

$$\begin{aligned} & [\cos(\psi) \cos(\theta), -\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi), \\ & \sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta), 0, 0, 0] \\ & [-\sin(\psi) \cos(\theta), \cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi), \\ & -\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi), 0, 0, 0] \\ & [-\sin(\theta), \cos(\theta) \sin(\phi), \cos(\theta) \cos(\phi), 0, 0, 0] \\ & [0, 0, 0, 1, \sin(\phi) \tan(\theta), \cos(\phi) \tan(\theta)] \\ & [0, 0, 0, 0, \cos(\phi), -\sin(\phi)] \\ & \left[0, 0, 0, 0, \frac{\sin(\phi)}{\cos(\theta)}, \frac{\cos(\phi)}{\cos(\theta)} \right] \end{aligned}$$

System matrices, f, g, h

`> u_f:=matrix(4,1,[u1,u2,u3,u4]);`

$$u_f := \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \end{bmatrix}$$

`> vel_dot:=matrix(6,1,[udot,vdot,wdot,pdot,qdot,rdot]);`

$$vel_dot := \begin{bmatrix} udot \\ vdot \\ wdot \\ pdot \\ qdot \\ rdot \end{bmatrix}$$

`> vel:=matrix(6,1,[u,v,w,p,q,r]);`

$$vel := \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

`> eqn:=evalm(M&*vel_dot+C&*vel+Dd&*vel+g=Tv&*u_f);`
`eqn :=`

$$\begin{aligned} & [m \, udot + m \, w \, q - m \, v \, r + Xu \, u] \\ & [m \, vdot - m \, w \, p + m \, u \, r + Yv \, v] \\ & [m \, wdot + m \, v \, p - m \, u \, q + Zw \, w] \\ & [Ixx \, pdot - Ixy \, qdot - Ixz \, rdot + (-Iyz \, q - Ixz \, p + Izz \, r) \, q + (Iyz \, r + Ixy \, p - Iyy \, q) \, r] \end{aligned}$$

```

+ Kp p + 29.61 cos(θ) sin(φ)]
[−Ixy ṗ + Iyy q̇ − Iyz ṙ + (Iyz q + Ixz p − Izz r) p
+ (−Ixz r − Ixy q + Ixx p) r + Mq q + 29.61 sin(θ)]
[−Ixz ṗ − Iyz q̇ + Izz ṙ + (−Iyz r − Ixy p + Iyy q) p
+ (Ixz r + Ixy q − Ixx p) q + Nr r] =

$$\begin{bmatrix} u1 + u2 \\ 0.707 u3 - 0.707 u4 \\ 0.707 u3 + 0.707 u4 \\ -0.293 u3 + 0.293 u4 \\ -0.016 u1 - 0.016 u2 + 0.012019 u3 - 0.012019 u4 \\ 0.31 u1 - 0.31 u2 + 0.012019 u3 - 0.012019 u4 \end{bmatrix}$$

> J:=evalm(J&*vel);
J:=
[cos(ψ) cos(θ) u + (−sin(ψ) cos(φ) + cos(ψ) sin(θ) sin(φ)) v
+ (sin(ψ) sin(φ) + cos(ψ) cos(φ) sin(θ)) w]
[−sin(ψ) cos(θ) u + (cos(ψ) cos(φ) + sin(φ) sin(θ) sin(ψ)) v
+ (−cos(ψ) sin(φ) + sin(θ) sin(ψ) cos(φ)) w]
[−sin(θ) u + cos(θ) sin(φ) v + cos(θ) cos(φ) w]
[p + sin(φ) tan(θ) q + cos(φ) tan(θ) r]
[cos(φ) q − sin(φ) r]

$$\left[ \frac{\sin(\phi) q}{\cos(\theta)} + \frac{\cos(\phi) r}{\cos(\theta)} \right]$$

> f1:=evalm(−
inverse(M)&*(C&*vel+Dd&*vel+g)):f:=vector([J[1,1],J[2,1],J
[3,1],J[4,1],J[5,1],J[6,1],f1[1,1],f1[2,1],f1[3,1],f1[4,1]
,f1[5,1],f1[6,1]]);
f:=
$$\begin{bmatrix} \cos(\psi) \cos(\theta) u + (-\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi)) v \\ + (\sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta)) w, -\sin(\psi) \cos(\theta) u \\ + (\cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi)) v \\ + (-\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi)) w, \\ -\sin(\theta) u + \cos(\theta) \sin(\phi) v + \cos(\theta) \cos(\phi) w, \\ p + \sin(\phi) \tan(\theta) q + \cos(\phi) \tan(\theta) r, \cos(\phi) q - \sin(\phi) r, \frac{\sin(\phi) q}{\cos(\theta)} + \frac{\cos(\phi) r}{\cos(\theta)}, \\ -\frac{m w q - m v r + X u u}{m}, -\frac{-m w p + m u r + Y v v}{m}, -\frac{m v p - m u q + Z w w}{m}, - \\ (-Iyz^2 + Iyy Izz) \\ ((-Iyz q - Ixz p + Izz r) q + (Iyz r + Ixy p - Iyy q) r + Kp p + 29.61 \cos(\theta) \sin(\phi)) \\ / (Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz) - (Ixy Izz + Ixz Iyz) \\ ((Iyz q + Ixz p - Izz r) p + (-Ixz r - Ixy q + Ixx p) r + Mq q + 29.61 \sin(\theta)) / ($$


```

$$\begin{aligned}
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz} \\
& - \frac{(I_{xz} I_{yy} + I_{xy} I_{yz}) ((-I_{yz} r - I_{xy} p + I_{yy} q) p + (I_{xz} r + I_{xy} q - I_{xx} p) q + Nr r)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}}, \\
& - (I_{xy} I_{zz} + I_{xz} I_{yz}) \\
& ((-I_{yz} q - I_{xz} p + I_{zz} r) q + (I_{yz} r + I_{xy} p - I_{yy} q) r + Kp p + 29.61 \cos(\theta) \sin(\phi)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) - (I_{xx} I_{zz} - I_{xz}^2) \\
& ((I_{yz} q + I_{xz} p - I_{zz} r) p + (-I_{xz} r - I_{xy} q + I_{xx} p) r + Mq q + 29.61 \sin(\theta)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) \\
& - \frac{(I_{xx} I_{yz} + I_{xy} I_{xz}) ((-I_{yz} r - I_{xy} p + I_{yy} q) p + (I_{xz} r + I_{xy} q - I_{xx} p) q + Nr r)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}}, \\
& - (I_{xz} I_{yy} + I_{xy} I_{yz}) \\
& ((-I_{yz} q - I_{xz} p + I_{zz} r) q + (I_{yz} r + I_{xy} p - I_{yy} q) r + Kp p + 29.61 \cos(\theta) \sin(\phi)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) - (I_{xx} I_{yz} + I_{xy} I_{xz}) \\
& ((I_{yz} q + I_{xz} p - I_{zz} r) p + (-I_{xz} r - I_{xy} q + I_{xx} p) r + Mq q + 29.61 \sin(\theta)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) \\
& - \frac{(I_{xx} I_{yy} - I_{xy}^2) ((-I_{yz} r - I_{xy} p + I_{yy} q) p + (I_{xz} r + I_{xy} q - I_{xx} p) q + Nr r)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \Big]
\end{aligned}$$

```

> g1:=evalm(-inverse(M)*evalm(Tv&*u_f)):
g:=matrix(12,1,[0,0,0,0,0,0,  g1[1,1],g1[2,1],g1[3,1]
,g1[4,1],g1[5,1],g1[6,1] ]);
g:=
[0]
[0] [0] [ - 0.707 u3 - 0.707 u4 ]
[0] [ - u1 + u2 ] [ - 0.707 u3 + 0.707 u4 ]
[0] [ m ] [ m ]
[0]
[ - (-Iyz^2 + Iyy Izz) (-0.293 u3 + 0.293 u4) ]
[ Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz ]
[ (Ixy Izz + Ixz Iyz) (-0.016 u1 - 0.016 u2 + 0.012019 u3 - 0.012019 u4) ]
[ Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz ]
[ (Ixz Iyy + Ixy Iyz) (0.31 u1 - 0.31 u2 + 0.012019 u3 - 0.012019 u4) ]
[ Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz ]
[ (Ixy Izz + Ixz Iyz) (-0.293 u3 + 0.293 u4) ]
[ Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz ]
[ (Ixx Izz - Ixz^2) (-0.016 u1 - 0.016 u2 + 0.012019 u3 - 0.012019 u4) ]
[ Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz ]
[ (Ixx Iyz + Ixy Ixz) (0.31 u1 - 0.31 u2 + 0.012019 u3 - 0.012019 u4) ]
[ Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz ]

```

$$\begin{aligned}
& \left[-\frac{(I_{xz} I_{yy} + I_{xy} I_{yz}) (-0.293 u^3 + 0.293 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \right. \\
& \quad \left. -\frac{(I_{xx} I_{yz} + I_{xy} I_{xz}) (-0.016 u^1 - 0.016 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \right. \\
& \quad \left. -\frac{(I_{xx} I_{yy} - I_{xy}^2) (0.31 u^1 - 0.31 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \right] \\
& > \text{fg} := \text{evalm}(\text{f} + \text{g}); \\
& \text{fg} := \\
& \quad [\cos(\psi) \cos(\theta) u + (-\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi)) v \\
& \quad + (\sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta)) w] \\
& \quad [-\sin(\psi) \cos(\theta) u + (\cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi)) v \\
& \quad + (-\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi)) w] \\
& \quad [-\sin(\theta) u + \cos(\theta) \sin(\phi) v + \cos(\theta) \cos(\phi) w] \\
& \quad [p + \sin(\phi) \tan(\theta) q + \cos(\phi) \tan(\theta) r] \\
& \quad [\cos(\phi) q - \sin(\phi) r] \\
& \quad \left[\frac{\sin(\phi) q}{\cos(\theta)} + \frac{\cos(\phi) r}{\cos(\theta)} \right] \\
& \quad \left[-\frac{m w q - m v r + X u u}{m} - \frac{u^1 + u^2}{m} \right] \\
& \quad \left[-\frac{-m w p + m u r + Y v v}{m} - \frac{0.707 u^3 - 0.707 u^4}{m} \right] \\
& \quad \left[-\frac{m v p - m u q + Z w w}{m} - \frac{0.707 u^3 + 0.707 u^4}{m} \right] \\
& \quad \left[-(-I_{yz}^2 + I_{yy} I_{zz}) \right. \\
& \quad ((-I_{yz} q - I_{xz} p + I_{zz} r) q + (I_{yz} r + I_{xy} p - I_{yy} q) r + K p p + 29.61 \cos(\theta) \sin(\phi)) \\
& \quad / (I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) - (I_{xy} I_{zz} + I_{xz} I_{yz}) \\
& \quad ((I_{yz} q + I_{xz} p - I_{zz} r) p + (-I_{xz} r - I_{xy} q + I_{xx} p) r + M q q + 29.61 \sin(\theta)) / (\\
& \quad I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) \\
& \quad - \frac{(I_{xz} I_{yy} + I_{xy} I_{yz}) ((-I_{yz} r - I_{xy} p + I_{yy} q) p + (I_{xz} r + I_{xy} q - I_{xx} p) q + N r r)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& \quad - \frac{(-I_{yz}^2 + I_{yy} I_{zz}) (-0.293 u^3 + 0.293 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& \quad - \frac{(I_{xy} I_{zz} + I_{xz} I_{yz}) (-0.016 u^1 - 0.016 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& \quad \left. - \frac{(I_{xz} I_{yy} + I_{xy} I_{yz}) (0.31 u^1 - 0.31 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \right]
\end{aligned}$$

$$\begin{aligned}
& \left[- (I_{xy} I_{zz} + I_{xz} I_{yz}) \right. \\
& ((-I_{yz} q - I_{xz} p + I_{zz} r) q + (I_{yz} r + I_{xy} p - I_{yy} q) r + Kp p + 29.61 \cos(\theta) \sin(\phi)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) - (I_{xx} I_{zz} - I_{xz}^2) \\
& ((I_{yz} q + I_{xz} p - I_{zz} r) p + (-I_{xz} r - I_{xy} q + I_{xx} p) r + Mq q + 29.61 \sin(\theta)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) \\
& - \frac{(I_{xx} I_{yz} + I_{xy} I_{xz}) ((-I_{yz} r - I_{xy} p + I_{yy} q) p + (I_{xz} r + I_{xy} q - I_{xx} p) q + Nr r)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& - \frac{(I_{xy} I_{zz} + I_{xz} I_{yz}) (-0.293 u^3 + 0.293 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& - \frac{(I_{xx} I_{zz} - I_{xz}^2) (-0.016 u^1 - 0.016 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& - \frac{(I_{xx} I_{yz} + I_{xy} I_{xz}) (0.31 u^1 - 0.31 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \left. \right] \\
& \left[- (I_{xz} I_{yy} + I_{xy} I_{yz}) \right. \\
& ((-I_{yz} q - I_{xz} p + I_{zz} r) q + (I_{yz} r + I_{xy} p - I_{yy} q) r + Kp p + 29.61 \cos(\theta) \sin(\phi)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) - (I_{xx} I_{yz} + I_{xy} I_{xz}) \\
& ((I_{yz} q + I_{xz} p - I_{zz} r) p + (-I_{xz} r - I_{xy} q + I_{xx} p) r + Mq q + 29.61 \sin(\theta)) / (\\
& I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) \\
& - \frac{(I_{xx} I_{yy} - I_{xy}^2) ((-I_{yz} r - I_{xy} p + I_{yy} q) p + (I_{xz} r + I_{xy} q - I_{xx} p) q + Nr r)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& - \frac{(I_{xz} I_{yy} + I_{xy} I_{yz}) (-0.293 u^3 + 0.293 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& - \frac{(I_{xx} I_{yz} + I_{xy} I_{xz}) (-0.016 u^1 - 0.016 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \\
& - \frac{(I_{xx} I_{yy} - I_{xy}^2) (0.31 u^1 - 0.31 u^2 + 0.012019 u^3 - 0.012019 u^4)}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \left. \right]
\end{aligned}$$

Is f bounded for Input ubar_max =70? (continuous dependence 1)

Evaluate f at phi=theta=psi=5 deg, u = v = w = p = q = r = 0.05, u1=u2=u3=u4=70

```

> fs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,
u1=70,u2=70,u3=70,u4=70,

```



```
cos(psi)=0.996,sin(psi)=0.087,tan(psi)=0.087,cos(phi)=0.99
6,sin(phi)=0.087,tan(phi)=0.087,
cos(theta)=0.996,sin(theta)=0.087,tan(theta)=0.087,u=0.05,
v=0.05,w=0.05,p=0.05,q =0.05,r=0.05}, evalm(fg) );
```

$$fs := \begin{bmatrix} 0.05033885580 \\ 0.04134546135 \\ 0.04958340 \\ 0.05471105 \\ 0.04545 \\ 0.05436746988 \\ -1.219004347 \\ -0.001265217391 \\ -0.8617260870 \\ -0.4217244360 \\ -0.0572665473 \\ 0.01328596353 \end{bmatrix}$$

Vector Norm of f+g

```
>
eigen_values:=eval(sqrt(fs[1,1]^2+fs[2,1]^2+fs[3,1]^2+fs[4
,1]^2+fs[5,1]^2+fs[6,1]^2+fs[7,1]^2+fs[8,1]^2+fs[9,1]^2+fs
[10,1]^2+fs[11,1]^2+fs[12,1]^2)) ;
eigen_values := 1.557102857
```

**Evaluate f at phi=theta=psi=10 deg, u = v = w = p = q = r = 0.10,
u1=u2=u3=u4=70**

```
> fs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,cos(psi)=1,
sin(psi)=0,tan(psi)=0, u1=70,u2=70,u3=70,u4=70,
cos(psi)=0.984,sin(psi)=0.173,tan(psi)=0.176,
cos(phi)=0.984,sin(phi)=0.173,tan(phi)=0.176,
cos(theta)=0.984,sin(theta)=0.173,tan(theta)=0.176,
u=0.1,v=0.1,w=0.1,r=0.1,p=0.1,q=0.1}, evalm(fg) );
```

$$fs := \begin{bmatrix} 0.1024911424 \\ 0.0662419853 \\ 0.0965488 \\ 0.1203632 \\ 0.0811 \\ 0.1175813008 \\ -1.220617391 \\ -0.002530434783 \\ -0.8627565218 \\ -0.8282911376 \\ -0.4846520557 \\ 0.02630035695 \end{bmatrix}$$

Vector Norm of f+g

```
>
eigen_values:=eval(sqrt(fs[1,1]^2+fs[2,1]^2+fs[3,1]^2+fs[4,1]^2+fs[5,1]^2+fs[6,1]^2+fs[7,1]^2+fs[8,1]^2+fs[9,1]^2+fs[10,1]^2+fs[11,1]^2+fs[12,1]^2));
eigen_values := 1.793045327
```

**Evaluate f at phi=theta=psi=20 deg, u = v = w = p = q = r = 0.15,
u1=u2=u3=u4=70**

```
> fs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-0.185,Ixy=-0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.4510,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,u1=70,u2=70,u3=70,u4=70,cos(psi)=0.94,sin(psi)=0.342,tan(psi)=0.364,cos(phi)=0.94,sin(phi)=0.342,tan(phi)=0.364,cos(theta)=0.94,sin(theta)=0.342,tan(theta)=0.364,u=0.15,v=0.15,w=0.15,r=0.15,p=0.15,q=0.15}, evalm(fg) );
```

$$fs := \begin{bmatrix} 0.1636832040 \\ 0.0585881772 \\ 0.1294620 \\ 0.21999720 \\ 0.08970 \\ 0.2045744681 \\ -1.222230434 \\ -0.003795652174 \\ -0.8637869565 \\ -1.563427365 \\ -1.323490654 \\ 0.05056479393 \end{bmatrix}$$

Vector Norm of f+g

```
>
eigen_values:=eval(sqrt(fs[1,1]^2+fs[2,1]^2+fs[3,1]^2+fs[4,1]^2+fs[5,1]^2+fs[6,1]^2+fs[7,1]^2+fs[8,1]^2+fs[9,1]^2+fs[10,1]^2+fs[11,1]^2+fs[12,1]^2));
eigen_values := 2.565883810
```

**Evaluate f at phi=theta=psi=30 deg, u = v = w = p = q = r = 0.20,
u1=u2=u3=u4=70**

```
> fs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-0.185,Ixy=-0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.4510,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,cos(psi)=1,sin(psi)=0,tan(psi)=0,u1=70,u2=70,u3=70,u4=70,cos(psi)=0.866,sin(psi)=0.5,tan(psi)=0.577,cos(phi)=0.866,sin(phi)=0.5,tan(phi)=0.577,cos(theta)=0.866,sin(theta)=0.5,tan(theta)=0.577,u=0.2,v=0.2,w=0.2,r=0.2,p=0.2,q=0.2}, evalm(fg) );
```

$$fs := \begin{bmatrix} 0.23168680 \\ 0.0450912 \\ 0.1365912 \\ 0.3576364 \\ 0.0732 \\ 0.3154734411 \\ -1.223843478 \\ -0.005060869565 \\ -0.8648173913 \\ -2.105305428 \\ -2.108356584 \\ 0.06863159145 \end{bmatrix}$$

Vector Norm of f+g

```
>
eigen_values:=eval(sqrt(fs[1,1]^2+fs[2,1]^2+fs[3,1]^2+fs[4,1]^2+fs[5,1]^2+fs[6,1]^2+fs[7,1]^2+fs[8,1]^2+fs[9,1]^2+fs[10,1]^2+fs[11,1]^2+fs[12,1]^2)) ;
eigen_values := 3.381578180
```

Is f bounded for Different Input ubar? (continuous dependence 2)

Evaluate f at phi=theta=psi=30 deg, u = v = w = p = q = r = 0.20,
u1=u2=u3=u4=0

```
> fs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-0.185,Ixy=-0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.4510,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,u1=0,u2=0,u3=0,u4=0,cos(psi)=0.866,sin(psi)=0.5,tan(psi)=0.577,cos(phi)=0.866,sin(phi)=0.5,tan(phi)=0.577,cos(theta)=0.866,sin(theta)=0.5,tan(theta)=0.577,u=0.2,v=0.2,w=0.2,r=0.2,p=0.2,q=0.2}, evalm(fg) );
```

$$fs := \begin{bmatrix} 0.23168680 \\ 0.0450912 \\ 0.1365912 \\ 0.3576364 \\ 0.0732 \\ 0.3154734411 \\ -0.006452173913 \\ -0.005060869565 \\ -0.004121739130 \\ -2.105294356 \\ -2.482938528 \\ 0.06859048261 \end{bmatrix}$$

Vector Norm of f+g

```
>
eigen_values:=eval(sqrt(fs[1,1]^2+fs[2,1]^2+fs[3,1]^2+fs[4,1]^2+fs[5,1]^2+fs[6,1]^2+fs[7,1]^2+fs[8,1]^2+fs[9,1]^2+fs[10,1]^2+fs[11,1]^2+fs[12,1]^2));
eigen_values := 3.302906540
```

**Evaluate f at phi=theta=psi=30 deg, u = v = w = p = q = r = 0.20,
u1=u2=u3=u4=70**

```
> fs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-0.185,Ixy=-0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.4510,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,cos(psi)=1,sin(psi)=0,tan(psi)=0,u1=70,u2=70,u3=70,u4=70,cos(psi)=0.866,sin(psi)=0.5,tan(psi)=0.577,cos(phi)=0.866,sin(phi)=0.5,tan(phi)=0.577,cos(theta)=0.866,sin(theta)=0.5,tan(theta)=0.577,u=0.2,v=0.2,w=0.2,r=0.2,p=0.2,q=0.2}, evalm(fg) );
```

$$fs := \begin{bmatrix} 0.23168680 \\ 0.0450912 \\ 0.1365912 \\ 0.3576364 \\ 0.0732 \\ 0.3154734411 \\ -1.223843478 \\ -0.005060869565 \\ -0.8648173913 \\ -2.105305428 \\ -2.108356584 \\ 0.06863159145 \end{bmatrix}$$

Vector Norm of f+g

>

```
eigen_values:=eval(sqrt(fs[1,1]^2+fs[2,1]^2+fs[3,1]^2+fs[4,1]^2+fs[5,1]^2+fs[6,1]^2+fs[7,1]^2+fs[8,1]^2+fs[9,1]^2+fs[10,1]^2+fs[11,1]^2+fs[12,1]^2));
eigen_values := 3.381578180
```

Is f uniqueness/existence(lipschitz condition)? or (continuous dependence 3)

Jacobian [df/dx] only - since dg/dx =0

```
> f_diff:=(jacobian(f,[x,y,z,phi,theta,psi,u,v,w,p,q,r])):
simplify(f_diff[1,1]);simplify(f_diff[1,2]);simplify(f_diff[1,3]);simplify(f_diff[1,4]);simplify(f_diff[1,5]);simplify(f_diff[1,6]);simplify(f_diff[1,7]);simplify(f_diff[1,8]);simplify(f_diff[1,9]);simplify(f_diff[1,10]);simplify(f_diff[1,11]);simplify(f_diff[1,12]);
```

0

0

0

$$\begin{aligned} & v \sin(\psi) \sin(\phi) + v \cos(\psi) \cos(\phi) \sin(\theta) + w \sin(\psi) \cos(\phi) - w \cos(\psi) \sin(\theta) \sin(\phi) \\ & - \cos(\psi) \sin(\theta) u + \cos(\psi) \cos(\theta) \sin(\phi) v + \cos(\psi) \cos(\phi) \cos(\theta) w \\ & - \sin(\psi) \cos(\theta) u - v \cos(\psi) \cos(\phi) - v \sin(\phi) \sin(\theta) \sin(\psi) + w \cos(\psi) \sin(\phi) \\ & - w \sin(\theta) \sin(\psi) \cos(\phi) \end{aligned}$$

$$\cos(\psi) \cos(\theta)$$

$$-\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi)$$

$$\sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta)$$

0

0

0

>

```
simplify(f_diff[2,1]);simplify(f_diff[2,2]);simplify(f_diff[2,3]);simplify(f_diff[2,4]);simplify(f_diff[2,5]);simplify(f_diff[2,6]);simplify(f_diff[2,7]);simplify(f_diff[2,8]);simplify(f_diff[2,9]);simplify(f_diff[2,10]);simplify(f_diff[2,11]);simplify(f_diff[2,12]);
```

0

0

0

$$\begin{aligned}
& -v \cos(\psi) \sin(\phi) + v \sin(\theta) \sin(\psi) \cos(\phi) - w \cos(\psi) \cos(\phi) - w \sin(\phi) \sin(\theta) \sin(\psi) \\
& \sin(\psi) \sin(\theta) u + \sin(\phi) \cos(\theta) \sin(\psi) v + \cos(\theta) \sin(\psi) \cos(\phi) w \\
& -\cos(\psi) \cos(\theta) u - v \sin(\psi) \cos(\phi) + v \cos(\psi) \sin(\theta) \sin(\phi) + w \sin(\psi) \sin(\phi) \\
& + w \cos(\psi) \cos(\phi) \sin(\theta)
\end{aligned}$$

$$-\sin(\psi) \cos(\theta)$$

$$\cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi)$$

$$-\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi)$$

0

0

0

>

```
simplify(f_diff[3,1]);simplify(f_diff[3,2]);simplify(f_diff[3,3]);simplify(f_diff[3,4]);simplify(f_diff[3,5]);simplify(f_diff[3,6]);simplify(f_diff[3,7]);simplify(f_diff[3,8]);simplify(f_diff[3,9]);simplify(f_diff[3,10]);simplify(f_diff[3,11]);simplify(f_diff[3,12]);
```

0

0

0

$$\cos(\theta) \cos(\phi) v - \cos(\theta) \sin(\phi) w$$

$$-\cos(\theta) u - \sin(\theta) \sin(\phi) v - \sin(\theta) \cos(\phi) w$$

0

$$-\sin(\theta)$$

$$\cos(\theta) \sin(\phi)$$

$$\cos(\theta) \cos(\phi)$$

$$0$$

$$0$$

$$0$$

>

```
simplify(f_diff[4,1]);simplify(f_diff[4,2]);simplify(f_diff[4,3]);simplify(f_diff[4,4]);simplify(f_diff[4,5]);simplify(f_diff[4,6]);simplify(f_diff[4,7]);simplify(f_diff[4,8]);simplify(f_diff[4,9]);simplify(f_diff[4,10]);simplify(f_diff[4,11]);simplify(f_diff[4,12]);
```

$$0$$

$$0$$

$$0$$

$$-\frac{\sin(\theta)(-\cos(\phi)q + \sin(\phi)r)}{\cos(\theta)}$$

$$\frac{\sin(\phi)q + \cos(\phi)r}{\cos(\theta)^2}$$

$$0$$

$$0$$

$$0$$

$$0$$

$$1$$

$$\frac{\sin(\phi) \sin(\theta)}{\cos(\theta)}$$

$$\frac{\cos(\phi) \sin(\theta)}{\cos(\theta)}$$

>

```
simplify(f_diff[5,1]);simplify(f_diff[5,2]);simplify(f_diff[5,3]);simplify(f_diff[5,4]);simplify(f_diff[5,5]);simplify(f_diff[5,6]);simplify(f_diff[5,7]);simplify(f_diff[5,8]);simplify(f_diff[5,9]);simplify(f_diff[5,10]);simplify(f_diff[5,11]);simplify(f_diff[5,12]);
```

$$0$$

$$0$$

$$0$$

$$-\sin(\phi)q - \cos(\phi)r$$

$$0$$

$$\begin{aligned}
&0 \\
&0 \\
&0 \\
&0 \\
&0 \\
&\cos(\phi) \\
&-\sin(\phi)
\end{aligned}$$

>

```
simplify(f_diff[6,1]);simplify(f_diff[6,2]);simplify(f_diff[6,3]);simplify(f_diff[6,4]);simplify(f_diff[6,5]);simplify(f_diff[6,6]);simplify(f_diff[6,7]);simplify(f_diff[6,8]);simplify(f_diff[6,9]);simplify(f_diff[6,10]);simplify(f_diff[6,11]);simplify(f_diff[6,12]);
```

$$\begin{aligned}
&0 \\
&0 \\
&0 \\
&-\frac{\cos(\phi) q + \sin(\phi) r}{\cos(\theta)} \\
&\frac{\sin(\theta) (\sin(\phi) q + \cos(\phi) r)}{\cos(\theta)^2} \\
&0 \\
&0 \\
&0 \\
&0 \\
&0 \\
&\frac{\sin(\phi)}{\cos(\theta)} \\
&\frac{\cos(\phi)}{\cos(\theta)}
\end{aligned}$$

>

```
simplify(f_diff[7,1]);simplify(f_diff[7,2]);simplify(f_diff[7,3]);simplify(f_diff[7,4]);simplify(f_diff[7,5]);simplify(f_diff[7,6]);simplify(f_diff[7,7]);simplify(f_diff[7,8]);simplify(f_diff[7,9]);simplify(f_diff[7,10]);simplify(f_diff[7,11]);simplify(f_diff[7,12]);
```

$$0$$

$$0$$

0

0

0

0

$$-\frac{Xu}{m}$$

 r $-q$

0

 $-w$ v

>

```
simplify(f_diff[8,1]);simplify(f_diff[8,2]);simplify(f_diff[8,3]);simplify(f_diff[8,4]);simplify(f_diff[8,5]);simplify(f_diff[8,6]);simplify(f_diff[8,7]);simplify(f_diff[8,8]);simplify(f_diff[8,9]);simplify(f_diff[8,10]);simplify(f_diff[8,11]);simplify(f_diff[8,12]);
```

0

0

0

0

0

0

 $-r$

$$-\frac{Yv}{m}$$

 p w

0

 $-u$

>

```
simplify(f_diff[9,1]);simplify(f_diff[9,2]);simplify(f_diff[9,3]);simplify(f_diff[9,4]);simplify(f_diff[9,5]);simplify(f_diff[9,6]);simplify(f_diff[9,7]);simplify(f_diff[9,8]);simplify(f_diff[9,9]);simplify(f_diff[9,10]);simplify(f_diff[9,11]);simplify(f_diff[9,12]);
```

0

0

0

0

0

0

 q $-p$ $-\frac{Z_w}{m}$ $-v$ u

0

```
> f_diffs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109},
evalm(f_diff) ):
simplify(f_diffs[10,1]);simplify(f_diffs[10,2]);simplify(f
_diffs[10,3]);simplify(f_diffs[10,4]);simplify(f_diffs[10,
5]);simplify(f_diffs[10,6]);simplify(f_diffs[10,7]);simpli
fy(f_diffs[10,8]);simplify(f_diffs[10,9]);simplify(f_diffs
[10,10]);simplify(f_diffs[10,11]);simplify(f_diffs[10,12])
;
```

0

0

0

 $-4.859039890 \cos(\theta) \cos(\phi)$ $4.859039890 \sin(\theta) \sin(\phi) + 0.0001463559327 \cos(\theta)$

0

0

0

0

 $-0.03101907162 q + 0.00002583619708 r - 0.01641013134 - 0.6794700800 \cdot 10^{-7} p$ $0.0001951606918 q - 0.03101907162 p + 0.07496089765 r + 0.4942787326 \cdot 10^{-6}$

$$0.07496089765 \, q - 0.0001950927448 \, r + 0.00002583619708 \, p + 0.0005998012158$$

>

```
simplify(f_diffs[11,1]);simplify(f_diffs[11,2]);simplify(f
_diffs[11,3]);simplify(f_diffs[11,4]);simplify(f_diffs[11,
5]);simplify(f_diffs[11,6]);simplify(f_diffs[11,7]);simpli
fy(f_diffs[11,8]);simplify(f_diffs[11,9]);simplify(f_diffs
[11,10]);simplify(f_diffs[11,11]);simplify(f_diffs[11,12])
;
```

0

0

0

$$0.0001463559327 \cos(\theta) \cos(\phi)$$

$$-0.0001463559327 \sin(\theta) \sin(\phi) - 4.951505074 \cos(\theta)$$

0

0

0

0

$$-0.00009721777448 \, q - 0.09749162970 \, r + 0.4942787326 \, 10^{-6} + 0.06187290454 \, p$$

$$-0.5865369300 \, 10^{-10} \, q - 0.00009721777448 \, p - 0.00002564921414 \, r - 0.01672240822$$

$$-0.00002564921414 \, q - 0.06187290448 \, r - 0.09749162970 \, p - 0.2000385405 \, 10^{-5}$$

>

```
simplify(f_diffs[12,1]);simplify(f_diffs[12,2]);simplify(f
_diffs[12,3]);simplify(f_diffs[12,4]);simplify(f_diffs[12,
5]);simplify(f_diffs[12,6]);simplify(f_diffs[12,7]);simpli
fy(f_diffs[12,8]);simplify(f_diffs[12,9]);simplify(f_diffs
[12,10]);simplify(f_diffs[12,11]);simplify(f_diffs[12,12])
;
```

0

0

0

$$0.1629368257 \cos(\theta) \cos(\phi)$$

$$-0.1629368257 \sin(\theta) \sin(\phi) - 0.0005434074481 \cos(\theta)$$

0

0

0

0

$$0.02279109478 \, q + 0.00009728572151 \, r + 0.0005502763448 - 0.00005127128639 \, p$$

$$0.00005145826933 \, q + 0.02279109478 \, p + 0.03101907168 \, r - 0.1835215968 \, 10^{-5}$$

$$0.03101907168 \, q - 0.1869829440 \, 10^{-6} \, r + 0.00009728572151 \, p - 0.01977722755$$

>

Evaluate Maxtrix Norm of [df/dx] for phi=theta=psi=5 deg, u = v = w = p = q = r = 0.05

```
> f_diff:=(jacobian(f,[x,y,z,phi,theta,psi,u,v,w,p,q,r])):
> f_diffs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,
cos(psi)=0.996,sin(psi)=0.087,tan(psi)=0.087,cos(phi)=0.99
6,sin(phi)=0.087,tan(phi)=0.087,
cos(theta)=0.996,sin(theta)=0.087,tan(theta)=0.087,u=0.05,
v=0.05,w=0.05,p=0.05,q =0.05,r=0.05}, evalm(f_diff) );
f_diffs :=
[0,0,0,0.00864938340,0.04938506640,-0.05001066135,0.992016,
-0.079113276,0.093874392,0,0,0]
[0,0,0,-0.05358938895,0.00507065580,-0.04886274420,-0.086652,
0.992674503,-0.079113276,0,0,0]
[0,0,0,0.04526820,-0.05451105,0,-0.087,0.086652,0.992016,0,0,0]
[0,0,0,0.00395415,0.05455986135,0,0,0,0,1,0.007569,0.086652]
[0,0,0,-0.05415,0,0,0,0,0,0,0.996,-0.087]
[0,0,0,0.04563253012,0.004748965741,0,0,0,0,0,0.08734939759,
1.000000000]
[0,0,0,0,0,0,-0.03226086957,0.05,-0.05,0,-0.05,0.05]
[0,0,0,0,0,0,-0.05,-0.02530434783,0.05,0.05,0,-0.05]
[0,0,0,0,0,0,0.05,-0.05,-0.02060869565,-0.05,0.05,0]
[0,0,0,-4.820245315,0.03692384343,0,0,0,0,-0.01795979651,
0.002207343615,0.004339383271]
[0,0,0,0.0001451874270,-4.931700162,0,0,0,0,-0.001785302868,
-0.01672855158,-0.007971509555]
[0,0,0,0.1616359381,-0.001774502652,0,0,0,0,0.001692131806,
0.002691246021,-0.01822141903]
```

Matrix Norm of df/dx

```
> eigen_values:=evalf(sqrt(Eigenvals
(multiply(transpose(f_diffs),f_diffs) ))) ;
```

eigen_values := [0, 0, 0, 0.0000173370, 0.8344477967, 0.9278743851, 1.002731761, 1.006224143, 1.096570530, 1.180860422, 23.24172077, 24.36225903]^(1/2)

Evaluate Maxtrix Norm of [df/dx] for phi=theta=psi=10 deg, u = v = w = p = q = r = 0.10

```
> f_diffs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,
cos(psi)=0.984,sin(psi)=0.173,tan(psi)=0.176,
cos(phi)=0.984,sin(phi)=0.173,tan(phi)=0.176,
cos(theta)=0.984,sin(theta)=0.173,tan(theta)=0.176,
u=0.1,v=0.1,w=0.1,r=0.1,p=0.1,q=0.1}, evalm(f_diff) );
f_diffs :=
[0, 0, 0, 0.0338219152, 0.0950040192, -0.1002883853, 0.968256, -0.140781864,
0.197437288, 0, 0, 0]
[0, 0, 0, -0.1114215581, 0.0226887424, -0.0911600576, -0.170232,
0.973433717, -0.140781864, 0, 0, 0]
[0, 0, 0, 0.0798024, -0.1184161, 0, -0.173, 0.170232, 0.968256, 0, 0, 0]
[0, 0, 0, 0.0142736, 0.1192839232, 0, 0, 0, 0, 1, 0.030448, 0.173184]
[0, 0, 0, -0.1157, 0, 0, 0, 0, 0, 0, 0.984, -0.173]
[0, 0, 0, 0.08241869919, 0.02067232220, 0, 0, 0, 0, 0, 0.1758130081,
1.000000000]
[0, 0, 0, 0, 0, 0, -0.03226086957, 0.1, -0.1, 0, -0.1, 0.1]
[0, 0, 0, 0, 0, 0, -0.1, -0.02530434783, 0.1, 0.1, 0, -0.1]
[0, 0, 0, 0, 0, 0, 0.1, -0.1, -0.02060869565, -0.1, 0.1, 0]
[0, 0, 0, -4.704794527, 0.1455702191, 0, 0, 0, 0, -0.01950946168,
0.004414192950, 0.008078965327]
[0, 0, 0, 0.0001417100100, -4.872285373, 0, 0, 0, 0, -0.003571100015,
-0.01673469492, -0.01594101873]
[0, 0, 0, 0.1577645591, -0.005411249184, 0, 0, 0, 0, 0.002833987266,
0.005384327258, -0.01666561050]
```

Matrix Norm of df/dx

```
> eigen_values:=evalf(sqrt(Eigenvals
(multiply(transpose(f_diffs),f_diffs) ))) ;
eigen_values := [0, 0, 0, 0.0002507950, 0.6898813816, 0.8804880814, 1.008840145,
1.026810415, 1.213642516, 1.365837875, 21.94519480, 24.05888476](1/2)
```

Evaluate Maxtrix Norm of [df/dx] for phi=theta=psi=20 deg, u = v = w = p = q = r = 0.15

```
> f_diffs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,cos(psi)=1,
sin(psi)=0,tan(psi)=0,
cos(psi)=0.94,sin(psi)=0.342,tan(psi)=0.364,
cos(phi)=0.94,sin(phi)=0.342,tan(phi)=0.364,
cos(theta)=0.94,sin(theta)=0.342,tan(theta)=0.364,
u=0.15,v=0.15,w=0.15,r=0.15,p=0.15,q=0.15}, evalm(f_diff)
);
f_diffs :=
[0,0,0,0.03067740,0.1294620,-0.08970,0.94,0.116964,0.32148,0,0,0]
[0,0,0,-0.19230,0,-0.07523340,0,0.94,-0.342,0,0,0]
[0,0,0,0.0843180,-0.20676660,0,-0.342,0.32148,0.8836,0,0,0]
[0,0,0,0.03265080,0.2177789808,0,0,0,0,1,0.124488,0.34216]
[0,0,0,-0.19230,0,0,0,0,0,0,0.94,-0.342]
[0,0,0,0.09542553192,0.07443028520,0,0,0,0,0,0.3638297872,
1.000000000]
[0,0,0,0,0,0,-0.03226086957,0.15,-0.15,0,-0.15,0.15]
[0,0,0,0,0,0,-0.15,-0.02530434783,0.15,0.15,0,-0.15]
[0,0,0,0,0,0,0.15,-0.15,-0.02060869565,-0.15,0.15,0]
[0,0,0,-4.293447646,0.5684703163,0,0,0,0,-0.02105912685,
0.006621042287,0.01181854738]
[0,0,0,0.0001293201021,-4.654431888,0,0,0,0,-0.005356897162,
-0.01674083828,-0.02391052790]
[0,0,0,0.1439709792,-0.01956854588,0,0,0,0,0.003975842727,
0.008077408495,-0.01510980199]
```

Matrix Norm of df/dx

```
> eigen_values:=evalf(sqrt(Eigenvals
(multiply(transpose(f_diffs),f_diffs) ))) ;
eigen_values := [0,0,0,0.000244787,0.7963048385,0.9963981875,1.011184075,
1.083917005,1.127763332,1.523268376,17.31113041,23.35154087]^(1/2)
```

Evaluate Maxtrix Norm of [df/dx] for phi=theta=psi=30 deg, u = v = w = p = q = r = 0.20

Appendix B2

Lie-Bracket Definitions

First consider the driftless vertical subsystem where the dynamic in (4.43) is eliminated.

$$\dot{\mathbf{x}}_v = \bar{\mathbf{g}}_{v1} \mathbf{T}_{v1} \bar{\mathbf{u}}_1 + \bar{\mathbf{g}}_{v2} \mathbf{T}_{v2} \bar{\mathbf{u}}_2 \quad (\text{B.1})$$

where $\mathbf{x}_v = [\boldsymbol{\eta}_v, \mathbf{v}_v]^\top$ and $\mathbf{T}_{v1}$ and $\mathbf{T}_{v2}$ are the first two row of $\mathbf{T}_v$. The Lie-bracket of $\bar{\mathbf{g}}_{v1}$ and $\bar{\mathbf{g}}_{v2}$ can be expressed as:

$$[\bar{\mathbf{g}}_{v1}, \bar{\mathbf{g}}_{v2}](\mathbf{x}_v) = \frac{\partial \bar{\mathbf{g}}_{v2}}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v1}(\mathbf{x}_v) - \frac{\partial \bar{\mathbf{g}}_{v1}}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v2}(\mathbf{x}_v) \quad (\text{B.2})$$

provides the net motion of (B.2) as a result of the input sequence:

$$\bar{\mathbf{u}}(t) = \begin{cases} \bar{\mathbf{u}}_1(t) = +1, \bar{\mathbf{u}}_2(t) = 0, t \in [0, \varepsilon), \\ \bar{\mathbf{u}}_1(t) = 0, \bar{\mathbf{u}}_2(t) = +1, t \in [0, 2\varepsilon), \\ \bar{\mathbf{u}}_1(t) = -1, \bar{\mathbf{u}}_2(t) = 0, t \in [0, 3\varepsilon), \\ \bar{\mathbf{u}}_1(t) = 0, \bar{\mathbf{u}}_2(t) = -1, t \in [0, 4\varepsilon), \end{cases} \quad (\text{B.3})$$

where ε is an infinitesimal interval of time. Hence the solution of (B.1) at the time 4ε is obtained by following the flow of $\bar{\mathbf{g}}_{v1}$, the $\bar{\mathbf{g}}_{v2}$, then $-\bar{\mathbf{g}}_{v1}$ and finally $-\bar{\mathbf{g}}_{v2}$.

$$\begin{aligned} \mathbf{x}_v(4\varepsilon) &= \phi_\varepsilon^{-\bar{\mathbf{g}}_{v2}} \circ \phi_\varepsilon^{-\bar{\mathbf{g}}_{v1}} \circ \phi_\varepsilon^{\bar{\mathbf{g}}_{v2}} \circ \phi_\varepsilon^{\bar{\mathbf{g}}_{v1}}(\mathbf{x}_{v0}) \\ &= \mathbf{x}_{v0} + \varepsilon^2 \left[\frac{\partial \bar{\mathbf{g}}_{v2}}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v1}(\mathbf{x}_{v0}) - \frac{\partial \bar{\mathbf{g}}_{v1}}{\partial \mathbf{x}_v} \bar{\mathbf{g}}_{v2}(\mathbf{x}_{v0}) \right] + O(\varepsilon^3) \end{aligned} \quad (\text{B.4})$$

where $\phi_\varepsilon^{g_{v1}}$ is the flow of the vector field (analogous to the linear system $\dot{\mathbf{x}}_v = \mathbf{A}\mathbf{x}_v$, the flow is e^{At}), $\bar{\mathbf{g}}_{v1}$ that map each point $\mathbf{x}_v$, the solution at time t of the differential equation evolving from $\mathbf{x}_{v0}$ and 'o' is a composition operator. The geometrical representation of (B.4) can be seen in Fig. B2.1. By using more complicated input sequences, it is possible to obtain motion in the direction of higher-order brackets such as $[\bar{\mathbf{g}}_{v1}, [\bar{\mathbf{g}}_{v1}, \bar{\mathbf{g}}_{v2}]]$.

Figure B2.1: Lie bracket of $\bar{\mathbf{g}}_{v1}$ and $\bar{\mathbf{g}}_{v2}$

However, the vertical subsystem has drift term (or the dynamic components) and hence, the Lie-bracket operation should involve a mix of repeated $\bar{\mathbf{f}}_v$ and $\bar{\mathbf{g}}_v$ terms. In simplicity, using linear system $\dot{\mathbf{x}}_v = \mathbf{A}\mathbf{x}_v + \mathbf{B}\bar{\mathbf{u}}_v$ as an analogous, the Lie-bracket is written as follows:

$$\begin{aligned}
 -[\bar{\mathbf{f}}_v, \bar{\mathbf{g}}_v] &= \mathbf{A}\mathbf{B} \\
 [\bar{\mathbf{f}}_v, [\bar{\mathbf{f}}_v, \bar{\mathbf{g}}_v]] &= \mathbf{A}^2\mathbf{B} \\
 -[\bar{\mathbf{f}}_v, [\bar{\mathbf{f}}_v, [\bar{\mathbf{f}}_v, \bar{\mathbf{g}}_v]]] &= \mathbf{A}^3\mathbf{B} \\
 &\vdots
 \end{aligned} \tag{B.5}$$

where the drift $\bar{\mathbf{f}}_v = \mathbf{A}\mathbf{x}_v$ and input vector field $\bar{\mathbf{g}}_v = \mathbf{B}$. This is well-known Kalman's necessary and sufficient condition for controllability

$$\text{rank} [\mathbf{B} \quad \mathbf{A}\mathbf{B} \quad \mathbf{A}^2\mathbf{B} \quad \mathbf{A}^3\mathbf{B} \quad \dots \quad \mathbf{A}^{n-1}\mathbf{B}] = n \tag{B.6}$$

Hence the rank of the repeated Lie bracket can be useful in verifying the strongly accessible, STLC and controllability of nonlinear systems in Appendix B4.

Appendix B3

Controllability, SA and STLC Computation using MAPLE™

Figure B3.1. MAPLE™ used for computing the Lie Bracket

Example of MAPLE™ Script for Full-Order Model

Initialization

```
> restart; with(linalg):
```

Full order body-fixed coordinates

System Parameters

```
> M := matrix(6,6,[m,0,0,0,0,0,      0,m,0,0,0,0,
0,0,m,0,0,0,      0,0,0,Ixx,-Ixy,-Ixz,      0,0,0,-Ixy,Iyy,-Iyz,
0,0,0,-Ixz,-Iyz,Izz      ]);
```

```
> C:=matrix(6,6,[0,0,0,0,m*w,-m*v,      0,0,0,-m*w,0,m*u,
0,0,0,m*v,-m*u,0,      0,m*w,-m*v,0,-Iyz*q-
Ixz*p+Izz*r,Iyz*r+Ixy*p-Iyy*q,      -m*w,0,m*u,Iyz*q+Ixz*p-
Izz*r,0,-Ixz*r-Ixy*q+Ixx*p,      m*v,-m*u,0,-Iyz*r-
Ixy*p+Iyy*q,Ixz*r+Ixy*q-Ixx*p,0]);
```

$$M := \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{xy} & I_{yy} & -I_{yz} \\ 0 & 0 & 0 & -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

$$C := \begin{bmatrix} 0, & 0, & 0, & 0, & m w, & -m v \\ 0, & 0, & 0, & -m w, & 0, & m u \\ 0, & 0, & 0, & m v, & -m u, & 0 \\ 0, & m w, & -m v, & 0, & -I_{yz} q - I_{xz} p + I_{zz} r, & I_{yz} r + I_{xy} p - I_{yy} q \\ -m w, & 0, & m u, & I_{yz} q + I_{xz} p - I_{zz} r, & 0, & -I_{xz} r - I_{xy} q + I_{xx} p \\ m v, & -m u, & 0, & -I_{yz} r - I_{xy} p + I_{yy} q, & I_{xz} r + I_{xy} q - I_{xx} p, & 0 \end{bmatrix}$$

```
> Dd:=diag(Xu,Yv,Zw,Kp,Mq,Nr);
```

$$Dd := \begin{bmatrix} Xu & 0 & 0 & 0 & 0 & 0 \\ 0 & Yv & 0 & 0 & 0 & 0 \\ 0 & 0 & Zw & 0 & 0 & 0 \\ 0 & 0 & 0 & Kp & 0 & 0 \\ 0 & 0 & 0 & 0 & Mq & 0 \\ 0 & 0 & 0 & 0 & 0 & Nr \end{bmatrix}$$

```
>
```

```
g:=matrix(6,1,[0,0,0,29.61*cos(theta)*sin(phi),29.61*sin(theta),0]);
```

$$g := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.61 \cos(\theta) \sin(\phi) \\ 29.61 \sin(\theta) \\ 0 \end{bmatrix}$$

```
> Tv:=matrix(6,4,[1,1,0,0, 0, 0, 0.707, -0.707,0, 0,
0.707, 0.707,0, 0, -0.293, 0.293, -0.016, -0.016,
0.017*0.707, -0.017*0.707 ,0.31, -0.31, 0.017*0.707, -
0.017*0.707 ]);
```

$$Tv := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0.707 & -0.707 \\ 0 & 0 & 0.707 & 0.707 \\ 0 & 0 & -0.293 & 0.293 \\ -0.016 & -0.016 & 0.012019 & -0.012019 \\ 0.31 & -0.31 & 0.012019 & -0.012019 \end{bmatrix}$$

```
> J2:=matrix(3,3,[1,
sin(phi)*tan(theta),cos(phi)*tan(theta),0, cos(phi),-
sin(phi), 0,sin(phi)/cos(theta), cos(phi)/cos(theta)]);
```

$$J2 := \begin{bmatrix} 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \frac{\sin(\phi)}{\cos(\theta)} & \frac{\cos(\phi)}{\cos(\theta)} \end{bmatrix}$$

```
> J1:=matrix(3,3,[cos(psi)*cos(theta), -
sin(psi)*cos(phi)+cos(psi)*sin(theta)*sin(phi),
sin(psi)*sin(phi)+cos(psi)*cos(phi)*sin(theta), -
sin(psi)*cos(theta),
cos(psi)*cos(phi)+sin(phi)*sin(theta)*sin(psi), -
cos(psi)*sin(phi)+sin(theta)*sin(psi)*cos(phi),-
sin(theta),cos(theta)*sin(phi),cos(theta)*cos(phi) ]);
J1 :=
```

$$\begin{bmatrix} \cos(\psi) \cos(\theta), & -\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi), \\ \sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta)] \\ [-\sin(\psi) \cos(\theta), & \cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi), \\ -\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi)] \\ [-\sin(\theta), & \cos(\theta) \sin(\phi), \cos(\theta) \cos(\phi)] \end{bmatrix}$$

```
> J:=diag(J1,J2);
```

$J :=$

$$\begin{aligned} & [\cos(\psi) \cos(\theta), -\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi), \\ & \sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta), 0, 0, 0] \\ & [-\sin(\psi) \cos(\theta), \cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi), \\ & -\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi), 0, 0, 0] \\ & [-\sin(\theta), \cos(\theta) \sin(\phi), \cos(\theta) \cos(\phi), 0, 0, 0] \\ & [0, 0, 0, 1, \sin(\phi) \tan(\theta), \cos(\phi) \tan(\theta)] \\ & [0, 0, 0, 0, \cos(\phi), -\sin(\phi)] \\ & \left[0, 0, 0, 0, \frac{\sin(\phi)}{\cos(\theta)}, \frac{\cos(\phi)}{\cos(\theta)} \right] \end{aligned}$$

System matrices, f, g, h

$> u_f := \text{matrix}(4, 1, [u1, u2, u3, u4]);$

$$u_f := \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \end{bmatrix}$$

$> vel_dot := \text{matrix}(6, 1, [u_dot, v_dot, w_dot, p_dot, q_dot, r_dot]);$

$$vel_dot := \begin{bmatrix} u_dot \\ v_dot \\ w_dot \\ p_dot \\ q_dot \\ r_dot \end{bmatrix}$$

$> vel := \text{matrix}(6, 1, [u, v, w, p, q, r]);$

$$vel := \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

$> eqn := \text{evalm}(M * vel_dot + C * vel + Dd * vel + g = Tv * u_f);$
 $eqn :=$

$$\begin{aligned} & [m u_dot + m w q - m v r + Xu u] \\ & [m v_dot - m w p + m u r + Yv v] \\ & [m w_dot + m v p - m u q + Zw w] \\ & [Ixx p_dot - Ixy q_dot - Ixz r_dot + (-Iyz q - Ixz p + Izz r) q + (Iyz r + Ixy p - Iyy q) r \end{aligned}$$

$$\begin{aligned}
& + Kp p + 29.61 \cos(\theta) \sin(\phi)] \\
& [-Ixy \dot{p} + Iyy \dot{q} - Iyz \dot{r} + (Iyz q + Ixz p - Izz r) p \\
& + (-Ixz r - Ixy q + Ixx p) r + Mq q + 29.61 \sin(\theta)] \\
& [-Ixz \dot{p} - Iyz \dot{q} + Izz \dot{r} + (-Iyz r - Ixy p + Iyy q) p \\
& + (Ixz r + Ixy q - Ixx p) q + Nr r] = \\
& \begin{bmatrix} u1 + u2 \\ 0.707 u3 - 0.707 u4 \\ 0.707 u3 + 0.707 u4 \\ -0.293 u3 + 0.293 u4 \\ -0.016 u1 - 0.016 u2 + 0.012019 u3 - 0.012019 u4 \\ 0.31 u1 - 0.31 u2 + 0.012019 u3 - 0.012019 u4 \end{bmatrix} \\
& > J := \text{evalm}(J * \text{vel}); \\
& J := \\
& \begin{bmatrix} \cos(\psi) \cos(\theta) u + (-\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi)) v \\ + (\sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta)) w \\ -\sin(\psi) \cos(\theta) u + (\cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi)) v \\ + (-\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi)) w \\ -\sin(\theta) u + \cos(\theta) \sin(\phi) v + \cos(\theta) \cos(\phi) w \\ p + \sin(\phi) \tan(\theta) q + \cos(\phi) \tan(\theta) r \\ \cos(\phi) q - \sin(\phi) r \\ \frac{\sin(\phi) q}{\cos(\theta)} + \frac{\cos(\phi) r}{\cos(\theta)} \end{bmatrix} \\
& > f1 := \text{evalm}(- \\
& \text{inverse}(M) * (C * \text{vel} + Dd * \text{vel} + g)) : f := \text{vector}([J[1,1], J[2,1], J \\
& [3,1], J[4,1], J[5,1], J[6,1], f1[1,1], f1[2,1], f1[3,1], f1[4,1] \\
& , f1[5,1], f1[6,1]]); \\
& f := \begin{bmatrix} \cos(\psi) \cos(\theta) u + (-\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi)) v \\ + (\sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta)) w, -\sin(\psi) \cos(\theta) u \\ + (\cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi)) v \\ + (-\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi)) w, \\ -\sin(\theta) u + \cos(\theta) \sin(\phi) v + \cos(\theta) \cos(\phi) w, \\ p + \sin(\phi) \tan(\theta) q + \cos(\phi) \tan(\theta) r, \cos(\phi) q - \sin(\phi) r, \frac{\sin(\phi) q}{\cos(\theta)} + \frac{\cos(\phi) r}{\cos(\theta)}, \\ -\frac{m w q - m v r + Xu u}{m}, -\frac{-m w p + m u r + Yv v}{m}, -\frac{m v p - m u q + Zw w}{m}, - \\ (-Iyz^2 + Iyy Izz) \\ ((-Iyz q - Ixz p + Izz r) q + (Iyz r + Ixy p - Iyy q) r + Kp p + 29.61 \cos(\theta) \sin(\phi)) \\ / (Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz) - (Ixy Izz + Ixz Iyz) \\ ((Iyz q + Ixz p - Izz r) p + (-Ixz r - Ixy q + Ixx p) r + Mq q + 29.61 \sin(\theta)) / (\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz) \\
& - \frac{(Ixz \, Iyy + Ixy \, Iyz) ((-Iyz \, r - Ixy \, p + Iyy \, q) \, p + (Ixz \, r + Ixy \, q - Ixx \, p) \, q + Nr \, r)}{Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz}, \\
& - (Ixy \, Izz + Ixz \, Iyz) \\
& ((-Iyz \, q - Ixz \, p + Izz \, r) \, q + (Iyz \, r + Ixy \, p - Iyy \, q) \, r + Kp \, p + 29.61 \cos(\theta) \sin(\phi)) / (\\
& Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz) - (Ixx \, Izz - Ixz^2) \\
& ((Iyz \, q + Ixz \, p - Izz \, r) \, p + (-Ixz \, r - Ixy \, q + Ixx \, p) \, r + Mq \, q + 29.61 \sin(\theta)) / (\\
& Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz) \\
& - \frac{(Ixx \, Iyz + Ixy \, Ixz) ((-Iyz \, r - Ixy \, p + Iyy \, q) \, p + (Ixz \, r + Ixy \, q - Ixx \, p) \, q + Nr \, r)}{Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz}, \\
& - (Ixz \, Iyy + Ixy \, Iyz) \\
& ((-Iyz \, q - Ixz \, p + Izz \, r) \, q + (Iyz \, r + Ixy \, p - Iyy \, q) \, r + Kp \, p + 29.61 \cos(\theta) \sin(\phi)) / (\\
& Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz) - (Ixx \, Iyz + Ixy \, Ixz) \\
& ((Iyz \, q + Ixz \, p - Izz \, r) \, p + (-Ixz \, r - Ixy \, q + Ixx \, p) \, r + Mq \, q + 29.61 \sin(\theta)) / (\\
& Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz) \\
& - \frac{(Ixx \, Iyy - Ixy^2) ((-Iyz \, r - Ixy \, p + Iyy \, q) \, p + (Ixz \, r + Ixy \, q - Ixx \, p) \, q + Nr \, r)}{Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz} \Big] \\
& > g1:=evalm(-inverse(M)): g:=matrix(12,6,[0,0,0,0,0,0, \\
& 0,0,0,0,0,0, \quad 0,0,0,0,0,0, \quad 0,0,0,0,0,0, \quad 0,0,0,0,0,0, \\
& 0,0,0,0,0,0, \quad g1[1,1],g1[1,2],g1[1,3] \\
& ,g1[1,4],g1[1,5],g1[1,6], \quad g1[2,1],g1[2,2], \quad g1[2,3], \\
& g1[2,4],g1[2,5],g1[2,6],g1[3,1],g1[3,2],g1[3,3] \\
& ,g1[3,4],g1[3,5],g1[3,6] \quad ,g1[4,1],g1[4,2],g1[4,3] \\
& ,g1[4,4],g1[4,5],g1[4,6] \quad ,g1[5,1],g1[5,2],g1[5,3] \\
& ,g1[5,4],g1[5,5],g1[5,6] \quad ,g1[6,1],g1[6,2],g1[6,3] \\
& ,g1[6,4],g1[6,5],g1[6,6] \quad]); \\
& g:= \\
& \begin{bmatrix} [0,0,0,0,0,0] & [0,0,0,0,0,0] & \left[0,-\frac{1}{m},0,0,0,0\right] \\ [0,0,0,0,0,0] & [0,0,0,0,0,0] & \\ [0,0,0,0,0,0] & \left[-\frac{1}{m},0,0,0,0,0\right] & \left[0,0,-\frac{1}{m},0,0,0\right] \\ [0,0,0,0,0,0] & & \\ [0,0,0,0,0,0] & & \end{bmatrix} \\
& \begin{bmatrix} 0,0,0,-\frac{-Iyz^2 + Iyy \, Izz}{Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz}, \\ -\frac{Ixy \, Izz + Ixz \, Iyz}{Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz}, \\ -\frac{Ixz \, Iyy + Ixy \, Iyz}{Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz} \end{bmatrix} \\
& \begin{bmatrix} 0,0,0,-\frac{Ixy \, Izz + Ixz \, Iyz}{Iyy \, Izz \, Ixx - Iyy \, Ixz^2 - Iyz^2 \, Ixx - Ixy^2 \, Izz - 2 \, Iyz \, Ixy \, Ixz}, \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& -\frac{I_{xx} I_{zz} - I_{xz}^2}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}}, \\
& -\frac{I_{xx} I_{yz} + I_{xy} I_{xz}}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \Bigg] \\
& \left[0, 0, 0, -\frac{I_{xz} I_{yy} + I_{xy} I_{yz}}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}}, \right. \\
& -\frac{I_{xx} I_{yz} + I_{xy} I_{xz}}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}}, \\
& \left. -\frac{I_{xx} I_{yy} - I_{xy}^2}{I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}} \right]
\end{aligned}$$

Lie-Bracket- SA/ STLC g1, g2 , g3, g4

```

> g1:=vector([g[1,1],g[2,1],g[3,1],
g[4,1],g[5,1],g[6,1],g[7,1],g[8,1],g[9,1],
g[10,1],g[11,1],g[12,1]]);
g2:=vector([g[1,2],g[2,2],g[3,2],g[4,2],g[5,2],g[6,2],
g[7,2],g[8,2],g[9,2],g[10,2],g[11,2],g[12,2]]);
g3:=vector([g[1,3],g[2,3],g[3,3],g[4,3],g[5,3],g[6,3],
g[7,3],g[8,3],g[9,3],g[10,3],g[11,3],g[12,3]]);
g4:=vector([g[1,6],g[2,6],g[3,6],
g[4,6],g[5,6],g[6,6],g[7,6],g[8,6],g[9,6],
g[10,6],g[11,6],g[12,6]]);

g1:= [0,0,0,0,0,0,0,-1/m,0,0,0,0,0]

g2:= [0,0,0,0,0,0,0,0,-1/m,0,0,0,0]

g3:= [0,0,0,0,0,0,0,0,0,-1/m,0,0,0]

g4:= [0,0,0,0,0,0,0,0,0,-1/m,0,0,0,
- (Ixz Iyy + Ixy Iyz) / (Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz),
- (Ixx Iyz + Ixy Ixz) / (Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz),
- (Ixx Iyy - Ixy^2) / (Iyy Izz Ixx - Iyy Ixz^2 - Iyz^2 Ixx - Ixy^2 Izz - 2 Iyz Ixy Ixz)]

```

[f, g1],[f, g2],[f, g3],[f, g4]

```

> f_diff:=jacobian(f,[x,y,z,phi,theta,psi,u,v,w,p,q,r]):g1
_diff:=jacobian(g1,[x,y,z,phi,theta,psi,u,v,w,p,q,r]):
> Lfg1:=evalm(evalm(g1_diff*f)-evalm(f_diff*g1));

```

$$Lfg1 := \left[\frac{\cos(\psi) \cos(\theta)}{m}, -\frac{\sin(\psi) \cos(\theta)}{m}, -\frac{\sin(\theta)}{m}, 0, 0, 0, -\frac{Xu}{m^2}, -\frac{r}{m}, \frac{q}{m}, 0, 0, 0 \right]$$

> $g2_diff := jacobian(g2, [x, y, z, phi, theta, psi, u, v, w, p, q, r])$:

> $Lfg2 := evalm(evalm(g2_diff*f) - evalm(f_diff*g2))$;

$$Lfg2 := \left[\frac{-\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\theta) \sin(\phi)}{m}, \right. \\ \left. \frac{\cos(\psi) \cos(\phi) + \sin(\phi) \sin(\theta) \sin(\psi)}{m}, \frac{\cos(\theta) \sin(\phi)}{m}, 0, 0, 0, \frac{r}{m}, -\frac{Yv}{m^2}, -\frac{p}{m}, 0, 0, 0 \right]$$

> $g3_diff := jacobian(g3, [x, y, z, phi, theta, psi, u, v, w, p, q, r])$:

> $Lfg3 := evalm(evalm(g3_diff*f) - evalm(f_diff*g3))$;

>

$$Lfg3 := \left[\frac{\sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) \sin(\theta)}{m}, \right. \\ \left. \frac{-\cos(\psi) \sin(\phi) + \sin(\theta) \sin(\psi) \cos(\phi)}{m}, \frac{\cos(\theta) \cos(\phi)}{m}, 0, 0, 0, -\frac{q}{m}, \frac{p}{m}, -\frac{Zw}{m^2}, 0, 0, 0 \right]$$

> $g4_diff := jacobian(g4, [x, y, z, phi, theta, psi, u, v, w, p, q, r])$:

> $Lfg4 := evalm(evalm(g4_diff*f) - evalm(f_diff*g4))$:

[g2, [f, g1]]

>

$fg1_diff := jacobian(Lfg1, [x, y, z, phi, theta, psi, u, v, w, p, q, r])$

:

> $Lfg2_1 := evalm(fg1_diff*g2)$;

$$Lfg2_1 := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

[g3, [f, g4]]

>

$fg4_diff := jacobian(Lfg4, [x, y, z, phi, theta, psi, u, v, w, p, q, r])$

:

> $Lfg3_4 := evalm(fg4_diff*g3)$;

$$Lfg3_4 := \left[0, 0, 0, 0, 0, 0, \frac{I_{xx} I_{yz} + I_{xy} I_{xz}}{(I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) m}, \right. \\ \left. -\frac{I_{xz} I_{yy} + I_{xy} I_{yz}}{(I_{yy} I_{zz} I_{xx} - I_{yy} I_{xz}^2 - I_{yz}^2 I_{xx} - I_{xy}^2 I_{zz} - 2 I_{yz} I_{xy} I_{xz}) m}, 0, 0, 0, 0 \right]$$

[[g1 f], f]

> $Lg1_f := evalm(-Lfg1)$:

> $Lg1_ff := evalm(f_diff*Lg1_f - (-fg1_diff)*f)$:

[[g3 f], f]

```
> Lg3_f:=evalm(-
Lfg3):fg3_diff:=jacobian(Lfg3,[x,y,z,phi,theta,psi,u,v,w,p
,q,r]):
> Lg3_ff:=evalm(f_diff&*Lg3_f-(-fg3_diff)&*f):
```

[g2 [[g1 f], f]]

```
>
g1_ff_diff:=jacobian(Lg1_ff,[x,y,z,phi,theta,psi,u,v,w,p,q
,r]):
> Lg2g1ff:=evalm(g1_ff_diff&*g2);
Lg2g1ff := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
```

[g4 [[g3 f], f]]

```
>
g3_ff_diff:=jacobian(Lg3_ff,[x,y,z,phi,theta,psi,u,v,w,p,q
,r]):
> Lg4g3ff:=evalm(g3_ff_diff&*g4):
```

SA

```
> SA:=transpose(matrix(12,12,[g1,g2,g3,g4,Lfg1,Lfg2,Lfg3,L
fg4,Lfg2_1,Lfg3_4,Lg2g1ff,Lg4g3ff]))):
```

Evaluation (vary angle , vary speed)

Lie- Bracket [phi=theta=psi=0 deg, u = v = w = p = q = r = 0]

```
> SAs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
cos(psi)=1,sin(psi)=0,tan(psi)=0,
cos(phi)=1,sin(phi)=0,tan(phi)=0,
cos(theta)=1,sin(theta)=0,tan(theta)=0,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,u=0,v=0,w=0
,p=0,q=0,r=0}, evalm(SA) );
SAs :=
```

$$\begin{bmatrix} 0, 0, 0, 0, 0, \frac{1}{115}, 0, 0, 0, 0, 0, 0, -0.3191679945 \cdot 10^{-6} \\ 0, 0, 0, 0, 0, \frac{1}{115}, 0, 0, 0, 0, 0, 0, -0.00009570023388 \\ 0, 0, 0, 0, 0, \frac{1}{115}, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, -0.005502763448, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0.00001835215968, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0.1814424546, 0, 0, 0, 0 \end{bmatrix}$$

$$\begin{aligned}
& \left[\frac{-1}{115}, 0, 0, 0, -0.0002805293006, 0, 0, 0, 0, 0.1595839972 \cdot 10^{-6}, 0, \right. \\
& \left. 0.2588727483 \cdot 10^{-8} \right] \\
& \left[0, \frac{-1}{115}, 0, 0, 0, -0.0002200378072, 0, 0, 0, 0.00004785011694, 0, \right. \\
& \left. 0.4653750662 \cdot 10^{-6} \right] \\
& \left[0, 0, \frac{-1}{115}, 0, 0, 0, -0.0001792060492, 0, 0, 0, 0, 0 \right] \\
& [0, 0, 0, 0.005502763448, 0, 0, 0, 0.0001991304848, 0, 0, 0, 0] \\
& [0, 0, 0, -0.00001835215968, 0, 0, 0, -0.6725670427 \cdot 10^{-6}, 0, 0, 0, 0] \\
& [0, 0, 0, -0.1814424546, 0, 0, 0, -0.003591456785, 0, 0, 0, 0]
\end{aligned}$$

Lie- Bracket [phi=theta=psi=5 deg, u = v = w = p = q = r = 0.05]

```

> SAs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
cos(psi)=0.996,sin(psi)=0.087,tan(psi)=0.087,
cos(phi)=0.996,sin(phi)=0.087,tan(phi)=0.087,
cos(theta)=0.996,sin(theta)=0.087,tan(theta)=0.087,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,u=0.05,v=0.
05,w=0.05,r=0.05,p=0.05,q=0.05}, evalm(SA) );
SAs:=
[0,0,0,0,0.008626226087,-0.0006879415304,0.0008162990609,0,0,0,0,
0.7241305650 10-5]
[0,0,0,0,-0.0007534956522,0.008631952200,-0.0006879415304,0,0,0,0,
-0.00009554917571]
[0,0,0,0,-0.0007565217391,0.0007534956522,0.008626226087,0,0,0,0,
-0.8312432435 10-5]
[0,0,0,0,0,0,0,0.01021972703,0,0,0,0]
[0,0,0,0,0,0,0,-0.01576721479,0,0,0,0]
[0,0,0,0,0,0,0,0.1814440577,0,0,0,0]
[ -1/115,0,0,0,-0.0002805293006,0.0004347826087,-0.0004347826087,
0.009071205122,0,0.1595839972 10-6,0,0.00006400957966 ]
[ 0,-1/115,0,0,-0.0004347826087,-0.0002200378072,0.0004347826087,
-0.009347260902,0,0.00004785011694,0,0.00007338670937 ]

```

$$\begin{aligned}
& \left[0, 0, \frac{-1}{115}, 0, 0.0004347826087, -0.0004347826087, -0.0001792060492, \right. \\
& \left. 0.0002760557804, 0, 0, 0, 0.4769053294 \cdot 10^{-5} \right] \\
& [0, 0, 0, 0.005502763448, 0, 0, 0, 0.0008862173731, 0, 0, 0, 0] \\
& [0, 0, 0, -0.00001835215968, 0, 0, 0, -0.001436853166, 0, 0, 0, 0] \\
& [0, 0, 0, -0.1814424546, 0, 0, 0, -0.003315401005, 0, 0, 0, 0]
\end{aligned}$$

Lie- Bracket [phi=theta=psi=10 deg, u = v = w = p = q = r = 0.10]

```

> SAs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
cos(psi)=0.984,sin(psi)=0.17,tan(psi)=0.176,
cos(phi)=0.984,sin(phi)=0.17,tan(phi)=0.176,
cos(theta)=0.984,sin(theta)=0.17,tan(theta)=0.176,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,u=0.1,v=0.1
,w=0.1,r=0.1,p=0.1,q=0.1}, evalm(SA) );
SAs:=
[0, 0, 0, 0, 0.008419617391, -0.001207325217, 0.001682639304, 0, 0, 0, 0,
0.00001153053030]
[0, 0, 0, 0, -0.001454608696, 0.008462339131, -0.001207325217, 0, 0, 0, 0,
-0.00008837321889]
[0, 0, 0, 0, -0.001478260870, 0.001454608696, 0.008419617391, 0, 0, 0, 0,
-0.00001511419301]
[0, 0, 0, 0, 0, 0, 0, 0.02592071570, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, -0.03082715874, 0, 0, 0, 0]
[0, 0, 0, 0, 0, 0, 0, 0.1814456252, 0, 0, 0, 0]
[ -1
 115, 0, 0, 0, -0.0002805293006, 0.0008695652174, -0.0008695652174,
0.01814241024, 0, 0.1595839972 · 10-6, 0, 0.0001280165706 ]
[ 0, -1
 115, 0, 0, -0.0008695652174, -0.0002200378072, 0.0008695652174,
-0.01869452180, 0, 0.00004785011694, 0, 0.0001463080436 ]
[ 0, 0, -1
 115, 0, 0.0008695652174, -0.0008695652174, -0.0001792060492,
0.0005521115608, 0, 0, 0, 0.9538106589 · 10-5 ]
[0, 0, 0, 0.005502763448, 0, 0, 0, 0.001573304262, 0, 0, 0, 0]

```

```
[0,0,0,-0.00001835215968,0,0,0,-0.002873033766,0,0,0,0]
[0,0,0,-0.1814424546,0,0,0,-0.003039345224,0,0,0,0]
```

Lie- Bracket [phi=theta=psi=20 deg, u = v = w = p = q = r = 0.15]

```
> SAs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
cos(psi)=0.94,sin(psi)=0.34,tan(psi)=0.36,
cos(phi)=0.94,sin(phi)=0.34,tan(phi)=0.36,
cos(theta)=0.94,sin(theta)=0.34,tan(theta)=0.36,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,u=0.15,v=0.
15,w=0.15,r=0.15,p=0.15,q=0.15}, evalm(SA) );
SAs:=
[0,0,0,0,0.007683478261,-0.001834226087,0.003617600000,0,0,0,0,
0.00002003381372]
[0,0,0,0,-0.002779130435,0.008025252174,-0.001834226087,0,0,0,0,
-0.00009079788395]
[0,0,0,0,-0.002956521739,0.002779130435,0.007683478261,0,0,0,0,
-0.00003128415415]
[0,0,0,0,0,0,0,0.05589960948,0,0,0,0]
[0,0,0,0,0,0,0,-0.06167318352,0,0,0,0]
[0,0,0,0,0,0,0,0.1814490926,0,0,0,0]
[ $-\frac{1}{115}$ ,0,0,0,-0.0002805293006,0.001304347826,-0.001304347826,
0.02721361537,0,0.1595839972  $10^{-6}$ ,0,0.0001920235615]
[0, $-\frac{1}{115}$ ,0,0,-0.001304347826,-0.0002200378072,0.001304347826,
-0.02804178271,0,0.00004785011694,0,0.0002192293781]
[0,0, $-\frac{1}{115}$ ,0,0.001304347826,-0.001304347826,-0.0001792060492,
0.0008281673412,0,0,0,0.00001430715988]
[0,0,0,0.005502763448,0,0,0,0.002260391150,0,0,0,0]
[0,0,0,-0.00001835215968,0,0,0,-0.004309214365,0,0,0,0]
[0,0,0,-0.1814424546,0,0,0,-0.002763289445,0,0,0,0]
```

Lie- Bracket [phi=theta=psi=30 deg, u = v = w = p = q = r = 0.20]

```
> SAs:=subs( {m=115,Ixx=6.1,Iyy=5.98,Izz=5.517,Ixz=-
0.185,Ixy=-
```

```

0.00016,Iyz=0.0006,Xudot=21.1403,Yvdot=51.7012,Zwdot=92.45
10,Kpdot=3.6191,Mqdot=2.6427,Nrdot=2.3033,
cos(psi)=0.866,sin(psi)=0.5,tan(psi)=0.57,
cos(phi)=0.866,sin(phi)=0.5,tan(phi)=0.57,
cos(theta)=0.8669,sin(theta)=0.5,tan(theta)=0.57,
Xu=3.71,Yv=2.91,Zw=2.37,Kp=0.1,Mq=0.1,Nr=0.109,u=0.2,v=0.2
,w=0.2,r=0.2,p=0.2,q=0.2}, evalm(SA) );

```

SAs:=

```

[0,0,0,0,0.006528133913,-0.001882608696,0.005434591304,0,0,0,0,
0.00002351601656]
[0,0,0,0,-0.003769130435,0.007608313044,-0.001882608696,0,0,0,0,
-0.00009122754623]
[0,0,0,0,-0.004347826087,0.003769130435,0.006528133913,0,0,0,0,
-0.00004533008013]
[0,0,0,0,0,0,0,0.08406609136,0,0,0,0]
[0,0,0,0,0,0,0,-0.09070533433,0,0,0,0]
[0,0,0,0,0,0,0,0.1812646692,0,0,0,0]
[ -1/115,0,0,0,-0.0002805293006,0.001739130435,-0.001739130435,
0.03628482048,0,0.1595839972 10-6,0,0.0002560305524 ]
[ 0,-1/115,0,0,-0.001739130435,-0.0002200378072,0.001739130435,
-0.03738904360,0,0.00004785011694,0,0.0002921507124 ]
[ 0,0,-1/115,0,0.001739130435,-0.001739130435,-0.0001792060492,
0.001104223122,0,0,0,0.00001907621318 ]
[0,0,0,0.005502763448,0,0,0,0.002947478039,0,0,0,0]
[0,0,0,-0.00001835215968,0,0,0,-0.005745394962,0,0,0,0]
[0,0,0,-0.1814424546,0,0,0,-0.002487233665,0,0,0,0]

```

Appendix B4

Stabilizability of ROV

Consider the system (4.43), and suppose that $(\boldsymbol{\eta}_v, \mathbf{v}_v) = (0,0)$ is an equilibrium point of the system. If $\bar{\mathbf{g}}_v(\boldsymbol{\eta}_v)$ is zero, then there exists no continuous or discontinuous state feedback law that makes $(0,0)$ asymptotically stable.

Proof: Consider the mapping (4.43) and points of the form

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ \mathbf{M}_v^{-1} \begin{bmatrix} \alpha^1 \\ \alpha^2 \\ \beta \end{bmatrix} \end{bmatrix} \quad (\text{B.7})$$

where α^1, α^2 are arbitrary real numbers and β is an arbitrary nonzero real number. $\mathbf{J}_v(\boldsymbol{\eta}_v)$ has full rank, the equation $-\mathbf{M}_v^{-1}[(\mathbf{C}_v(\mathbf{v}_v) + \mathbf{D}_v)\mathbf{v}_v + \mathbf{g}_v(\boldsymbol{\eta}_v)] + [\bar{\mathbf{g}}_{v1} \quad \bar{\mathbf{g}}_{v2} \quad \bar{\mathbf{g}}_{v3}]^T \mathbf{T}_v \mathbf{F}_T \bar{\mathbf{u}} = \boldsymbol{\varepsilon}$ in (B.7) implies $\mathbf{v}_v = \mathbf{0}$. This in turn implies $\mathbf{C}_v(\mathbf{v}_v) = \mathbf{0}$ and consequently,

$$\begin{bmatrix} \tau_x \\ \tau_z \\ -\bar{\mathbf{g}}_v(\boldsymbol{\eta}_v)\tau_z \end{bmatrix} = \begin{bmatrix} \alpha^1 \\ \alpha^2 \\ \beta \end{bmatrix} \quad (\text{B.8})$$

which has no solution $(\mathbf{v}_v, \boldsymbol{\eta}_v, \boldsymbol{\tau}_v)$ as $\bar{\mathbf{g}}_v(\boldsymbol{\eta}_v)$ is zero and β is nonzero. Since $\mathbf{E}_v$ is constant and nonsingular, and α^1, α^2 are arbitrary, points of the form $\boldsymbol{\varepsilon}$ are contained in every neighborhood of zero in $\Re^6$. By theorem in [45], there does not exist a continuous state

feedback such that $(\boldsymbol{\eta}_v, \mathbf{v}_v) = (0,0)$ is an asymptotically stable equilibrium point. Hence the subsystem (4.43) cannot be asymptotically stabilized by continuous state or discontinuous state feedback. In other words, if $\bar{\mathbf{g}}_v(\boldsymbol{\eta}_v)$ is nonzero, there will exist a continuous or discontinuous feedback that stabilized the underactuated vertical plane model to the equilibrium point.

□

Appendix C1

Observer Design

C1.1 Observer Equation

Differentiating V_{obs} yields:

$$\begin{aligned}
 \dot{V}_{obs} &= \dot{\tilde{\eta}}^T P_{01} \tilde{\eta} + \tilde{\eta}^T P_{01} \dot{\tilde{\eta}} + \dot{\tilde{\eta}}_Q^T P_{02} \tilde{\eta}_Q + \tilde{\eta}_Q^T P_{02} \dot{\tilde{\eta}}_Q \\
 &= [\tilde{\eta}_Q^T - \tilde{\eta}^T K_{01}] P_{01} \tilde{\eta} + \tilde{\eta}^T P_{01} [\tilde{\eta}_Q - K_{01} \tilde{\eta}] \\
 &\quad + [-\tilde{\eta}_Q^T D_\eta(\eta) - \tilde{\eta}^T K_{02}] P_{02} \tilde{\eta}_Q + \tilde{\eta}_Q^T P_{02} [-D_\eta(\eta) \tilde{\eta}_Q - K_{02} \tilde{\eta}]
 \end{aligned} \tag{C.1}$$

C1.2. Exponential Convergence of Observer States

$$\begin{aligned}
 \dot{V}_{obs} &= \dot{\tilde{\eta}}_O^T P_0 \tilde{\eta}_O + \tilde{\eta}_O^T P_0 \dot{\tilde{\eta}}_O \\
 &= [\tilde{\eta}_Q - K_{01}(\eta_s - \hat{\eta}_s) \quad -D_\eta(\eta_s) \tilde{\eta}_Q - K_{02} \tilde{\eta}_s] \begin{bmatrix} P_{01} & 0 \\ 0 & P_{02} \end{bmatrix} \\
 &\quad \begin{bmatrix} \tilde{\eta}_s & \tilde{\eta}_Q \end{bmatrix} \begin{bmatrix} P_{01} & 0 \\ 0 & P_{02} \end{bmatrix} [\tilde{\eta}_Q - K_{01}(\eta_s - \hat{\eta}_s) \quad -D_\eta(\eta_s) \tilde{\eta}_Q - K_{02} \tilde{\eta}_s]^T \tag{C.3} \\
 &= -[\tilde{\eta}_s \quad \tilde{\eta}_Q] \overbrace{\begin{bmatrix} 2P_{01}K_{01} & 2P_{02}K_{02} - P_{01} + P_{01}D_\eta \\ -2P_{01} + K_{02}P_{02} & -P_{02} + 2P_{02}D_\eta \end{bmatrix}}^{H_0} \begin{bmatrix} \tilde{\eta}_s \\ \tilde{\eta}_Q \end{bmatrix}
 \end{aligned}$$

Hence,

$$\dot{V}_{obs} \leq -\lambda_{\min}(H_0) \|\tilde{\eta}_O\|_2^2 \tag{C.4}$$

Recalled,

$$V_{obs} = \tilde{\boldsymbol{\eta}}_{\mathbf{o}}^T \mathbf{P}_{\mathbf{o}} \tilde{\boldsymbol{\eta}}_{\mathbf{o}} \Rightarrow V_{obs} \leq \lambda_{\max}(\mathbf{P}_{\mathbf{o}}) \|\tilde{\boldsymbol{\eta}}_{\mathbf{o}}\|_2^2 \quad (\text{C.5})$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues respectively. Hence,

$$V_{obs}(t) \leq e^{-\sigma_o(t-t_0)} V_{obs}(t_0) \quad (\text{C.6})$$

where $\sigma_o = \frac{\lambda_{\min}(\mathbf{H}_{\mathbf{o}})}{\lambda_{\max}(\mathbf{P}_{\mathbf{o}})} > 0$ can be interpreted as the convergence rate. Substitute

$V_{obs}(t) = \tilde{\boldsymbol{\eta}}_{\mathbf{o}}^T \mathbf{P}_{\mathbf{o}} \tilde{\boldsymbol{\eta}}_{\mathbf{o}}$ and $V_{obs}(t_0) = \tilde{\boldsymbol{\eta}}_{\mathbf{o}}^T \mathbf{P}_{\mathbf{to}} \tilde{\boldsymbol{\eta}}_{\mathbf{o}}$ into (C.6) yields:

$$\begin{aligned} \tilde{\boldsymbol{\eta}}_{\mathbf{o}}^T(t) \mathbf{P}_{\mathbf{o}} \tilde{\boldsymbol{\eta}}_{\mathbf{o}}(t) &= e^{-\sigma_o(t-t_o)} \tilde{\boldsymbol{\eta}}_{\mathbf{o}}^T(t_o) \mathbf{P}_{\mathbf{to}} \tilde{\boldsymbol{\eta}}_{\mathbf{o}}(t_o) \\ \|\mathbf{P}_{\mathbf{o}}\|_2^2 \|\tilde{\boldsymbol{\eta}}_{\mathbf{o}}(t)\|_2^2 &\leq \|\mathbf{P}_{\mathbf{to}}\|_2^2 \|\tilde{\boldsymbol{\eta}}_{\mathbf{o}}(t_o)\|_2^2 e^{-\sigma_o(t-t_o)} \\ \|\tilde{\boldsymbol{\eta}}_{\mathbf{o}}(t)\|_2^2 &\leq \gamma_o \|\tilde{\boldsymbol{\eta}}_{\mathbf{o}}(t_o)\|_2^2 e^{-\sigma_o(t-t_o)} \end{aligned} \quad (\text{C.7})$$

where $\gamma_o = \frac{\|\mathbf{P}_{\mathbf{to}}\|_2^2}{\|\mathbf{P}_{\mathbf{o}}\|_2^2} > 0$ is a positive constant and $\|\cdot\|_2$ is the Euclidean norm.

C1.3. Observer Error Equations

By equating $\dot{\tilde{\boldsymbol{\eta}}}_{\mathbf{Q}} = -\mathbf{D}_{\boldsymbol{\eta}}(\boldsymbol{\eta}_s) \tilde{\boldsymbol{\eta}}_{\mathbf{Q}} - \mathbf{K}_{02}(\boldsymbol{\eta}_s - \hat{\boldsymbol{\eta}}_s)$ to $\dot{\tilde{\boldsymbol{\eta}}}_{\mathbf{Q}} = \mathbf{J}_s(\boldsymbol{\eta}_s) \tilde{\mathbf{v}}_s + \mathbf{J}_s(\boldsymbol{\eta}_s) \tilde{\mathbf{v}}_s$ and substitute

$\dot{\tilde{\boldsymbol{\eta}}}_{\mathbf{Q}} = -\mathbf{D}_{\boldsymbol{\eta}}(\boldsymbol{\eta}_s) \boldsymbol{\eta}_{\mathbf{Q}} + \mathbf{J}_s(\boldsymbol{\eta}_s) \mathbf{M}^{-1} \boldsymbol{\tau}_s$ with $\mathbf{Q}_o(\boldsymbol{\eta}_s) = \mathbf{J}_s(\boldsymbol{\eta}_s)$ into it yields:

$$\begin{aligned}
\dot{\mathbf{J}}_s(\tilde{\boldsymbol{\eta}}_s)\tilde{\mathbf{v}}_s + \mathbf{J}_s(\tilde{\boldsymbol{\eta}}_s)\dot{\tilde{\mathbf{v}}}_s &= -\mathbf{D}_\eta(\tilde{\boldsymbol{\eta}}_s)\tilde{\boldsymbol{\eta}}_Q - \mathbf{K}_{02}\tilde{\boldsymbol{\eta}}_s \\
\dot{\tilde{\mathbf{v}}}_s &= \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\mathbf{D}_\eta(\tilde{\boldsymbol{\eta}}_s)\tilde{\boldsymbol{\eta}}_Q - \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\mathbf{K}_{02}\tilde{\boldsymbol{\eta}}_s - \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\dot{\mathbf{J}}_s(\boldsymbol{\eta}_s)\tilde{\mathbf{v}}_s \\
&= \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}[\mathbf{J}_s(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\mathbf{D}_s\mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}]\tilde{\boldsymbol{\eta}}_Q - \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\mathbf{K}_{02}\tilde{\boldsymbol{\eta}}_s \\
&\quad - \mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}[\mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\mathbf{J}_s(\boldsymbol{\eta}_s)\mathbf{M}_s^{-1}\mathbf{C}_s(\tilde{\mathbf{v}}_s)]\tilde{\mathbf{v}}_s
\end{aligned} \tag{C.8}$$

Then, equation (C.8) becomes:

$$\dot{\tilde{\mathbf{v}}}_s = -\mathbf{M}_s^{-1}\mathbf{C}_s(\tilde{\mathbf{v}}_s)\tilde{\mathbf{v}}_s - \mathbf{M}_s^{-1}\mathbf{D}_s(\tilde{\mathbf{v}}_s)\tilde{\mathbf{v}}_s - \mathbf{K}_{02}\mathbf{J}_s(\boldsymbol{\eta}_s)^{-1}\tilde{\boldsymbol{\eta}}_s \tag{C.9}$$

C1.4. Derivation of $\mathbf{Q}_O(\boldsymbol{\eta}_s)$ for station-keeping mode

Assuming that the elements of $\mathbf{Q}_O(\boldsymbol{\eta}_s)$ are $q_{Oij}(\boldsymbol{\eta}_s)$, $i=1,2,3,4$, the equation (5.9) can be written as function of $\mathbf{v}_s$. This can be written as:

$$\dot{\mathbf{Q}}_O\mathbf{v}_s - \mathbf{Q}_O\mathbf{M}_s^{-1}\mathbf{C}_s(\mathbf{v}_s)\mathbf{v}_s = 0 \tag{C.10}$$

where $\mathbf{v}_s = \mathbf{Q}_O^{-1}\boldsymbol{\eta}_s$ and for brevity we omit the argument $\boldsymbol{\eta}_s$ of $q_{Oij}(\boldsymbol{\eta}_s)$. Expanding (C.10)

and grouping the same terms such as $u^2, v^2, w^2, r^2, uv, uw, vw, rw, vr, ur$ together gives the following PDE:

$$\begin{aligned}
&\left(\frac{\partial q_{oi1}}{\partial x}\cos\psi + \frac{\partial q_{oi1}}{\partial y}\sin\psi\right)u^2 + \left(-\frac{\partial q_{oi1}}{\partial x}\sin\psi + \frac{\partial q_{oi1}}{\partial y}\cos\psi + \frac{\partial q_{oi2}}{\partial x}\cos\psi + \frac{\partial q_{oi2}}{\partial y}\sin\psi\right. \\
&\quad \left.- q_{oi4}\frac{-Y_{\dot{v}} + X_{\dot{u}}}{I_r - N_{\dot{r}}}\right)uv + \left(-\frac{\partial q_{oi2}}{\partial x}\sin\psi + \frac{\partial q_{oi2}}{\partial y}\cos\psi\right)v^2 + \left(\frac{\partial q_{oi1}}{\partial z} + \frac{\partial q_{oi3}}{\partial x}\cos\psi + \frac{\partial q_{oi3}}{\partial y}\sin\psi\right)uw \\
&\quad + \left(-\frac{\partial q_{oi3}}{\partial x}\sin\psi + \frac{\partial q_{oi3}}{\partial y}\cos\psi + \frac{\partial q_{oi2}}{\partial z}\right)vw + \left(\frac{\partial q_{oi3}}{\partial z}\right)w^2 + \left(\frac{\partial q_{oi4}}{\partial \psi}\right)r^2 + \left(\frac{\partial q_{oi3}}{\partial \psi} + \frac{\partial q_{oi4}}{\partial z}\right)rw \\
&\quad + \left(\frac{\partial q_{oi2}}{\partial \psi} - \frac{\partial q_{oi4}}{\partial x}\sin\psi + \frac{\partial q_{oi4}}{\partial y}\cos\psi - q_{oi1}\frac{Y_{\dot{v}} - m}{m - X_{\dot{u}}}\right)vr \\
&\quad + \left(\frac{\partial q_{oi1}}{\partial \psi} + \frac{\partial q_{oi4}}{\partial x}\cos\psi + \frac{\partial q_{oi4}}{\partial y}\sin\psi - q_{oi2}\right)ur = 0
\end{aligned} \tag{C.11}$$

As observed in (C.11), they are ten PDEs to be solved. Now, we assumed the solution of the PDEs in (C.11) as:

$$q_{oi1} = \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} [(C_{i3}x + C_{i1})\sin\psi - (C_{i3}y - C_{i2})\cos\psi] \quad (C.12)$$

$$q_{oi2} = \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} [(C_{i3}x + C_{i1})\cos\psi + (C_{i3}y - C_{i2})\sin\psi] \quad (C.13)$$

$$q_{oi3} = C_{i4} \quad (C.14)$$

$$q_{oi4} = \frac{I_r - N_{\dot{r}}}{-Y_{\dot{v}} + X_{\dot{u}}} C_{i5} \quad (C.15)$$

where $C_{i1}, C_{i2}, C_{i3}, C_{i4}$ are arbitrary constant with the following values:

$$\begin{aligned} C_{13} = C_{11} = C_{14} = C_{15} = 0, \quad C_{12} = 1, \quad C_{23} = C_{22} = C_{24} = C_{25} = 0, \quad C_{21} = 1 \\ C_{33} = C_{31} = C_{32} = C_{35} = 0, \quad C_{34} = 1, \quad C_{43} = C_{41} = C_{42} = C_{45} = 0, \quad C_{44} = 1 \end{aligned} \quad (C.16)$$

substituting the solution $q_{oi1}, q_{oi2}, q_{oi3}, q_{oi4}$ into the ten PDEs in (C.11), the resulted PDEs in (C.11) are individually satisfied. The first PDEs in (C.11) gives:

$$\left(\frac{\partial q_{oi1}}{\partial x} \cos\psi + \frac{\partial q_{oi1}}{\partial y} \sin\psi \right) = 0 \Rightarrow (C_{i3} \sin\psi \cos\psi - C_{i3} \cos\psi \sin\psi) \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} = 0 \quad (C.17)$$

The second PDEs in (C.11) gives:

$$\begin{aligned} \left(-\frac{\partial q_{oi1}}{\partial x} \sin\psi + \frac{\partial q_{oi1}}{\partial y} \cos\psi + \frac{\partial q_{oi2}}{\partial x} \cos\psi + \frac{\partial q_{oi2}}{\partial y} \sin\psi - q_{oi4} \frac{-Y_{\dot{v}} + X_{\dot{u}}}{I_r - N_{\dot{r}}} \right) = 0 \\ \Rightarrow -\frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} (C_{i3} \sin^2\psi + C_{i3} \cos^2\psi) + \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} (C_{i3} \sin^2\psi + C_{i3} \cos^2\psi) - C_{i5} = 0 \end{aligned} \quad (C.18)$$

The third PDEs in (C.11) gives:

$$\left(-\frac{\partial q_{oi2}}{\partial x} \sin\psi + \frac{\partial q_{oi2}}{\partial y} \cos\psi \right) = 0 \Rightarrow -\frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} (-C_{i3} \sin\psi \cos\psi + C_{i3} \sin\psi \cos\psi) = 0 \quad (C.19)$$

The fourth PDEs in (C.11) gives:

$$\left(\frac{\partial q_{oi1}}{\partial z} + \frac{\partial q_{oi3}}{\partial x} \cos \psi + \frac{\partial q_{oi3}}{\partial y} \sin \psi \right) = 0 \quad (C.20)$$

The fifth PDEs in (C.11) gives:

$$\left(\frac{\partial q_{oi3}}{\partial x} \sin \psi + \frac{\partial q_{oi3}}{\partial y} \cos \psi + \frac{\partial q_{oi2}}{\partial z} \right) = 0 \quad (C.21)$$

The sixth PDEs in (C.11) gives:

$$\left(\frac{\partial q_{oi3}}{\partial z} \right) = 0 \quad (C.22)$$

The seventh PDEs in (C.11) gives:

$$\left(\frac{\partial q_{oi4}}{\partial \psi} \right) = 0 \quad (C.23)$$

The eighth PDEs in (C.11) gives:

$$\left(\frac{\partial q_{oi3}}{\partial \psi} + \frac{\partial q_{oi4}}{\partial z} \right) = 0 \quad (C.24)$$

The ninth PDEs in (C.11) gives:

$$\begin{aligned} \left(\frac{\partial q_{oi2}}{\partial \psi} - \frac{\partial q_{oi4}}{\partial x} \sin \psi + \frac{\partial q_{oi4}}{\partial y} \cos \psi - q_{oi1} \right) &\Rightarrow \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} [(C_{i3}x + C_{i1}) \sin \psi + (C_{i3}y - C_{i2}) \cos \psi] \\ &\quad - \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} [(C_{i3}x + C_{i1}) \sin \psi + (C_{i3}y - C_{i2}) \cos \psi] = 0 \end{aligned} \quad (C.25)$$

The tenth PDEs in (C.11) gives:

$$\begin{aligned} \left(\frac{\partial q_{oi1}}{\partial \psi} + \frac{\partial q_{oi4}}{\partial x} \cos \psi + \frac{\partial q_{oi4}}{\partial y} \sin \psi - q_{oi2} \right) &\Rightarrow \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} [(C_{i3}x + C_{i1}) \cos \psi + (C_{i3}y - C_{i2}) \sin \psi] \\ &\quad - \frac{m - X_{\dot{u}}}{Y_{\dot{v}} - m} [(C_{i3}x + C_{i1}) \cos \psi + (C_{i3}y - C_{i2}) \sin \psi] = 0 \end{aligned} \quad (C.26)$$

Appendix C2

Backstepping Controller Design

C3.1. Backstepping Controller Design for Station-Keeping

Define a virtual state for $\tilde{u}$,

$$z_1 = \tilde{u}_e - \alpha_1 \quad (\text{C.27})$$

Choose a CLF as:

$$V = \frac{1}{2}(m - X_{\ddot{u}} - \Delta X_{\ddot{u}})\tilde{u}_e^2 \quad (\text{C.28})$$

Imply $\dot{V}$:

$$\dot{V} = -\tilde{u}_e[-(X_u + \Delta X_u)\tilde{u}_e + (Y_{\dot{v}} + \Delta Y_{\dot{v}} - m)\tilde{v}_e\tilde{r}_e - \tau_x + \bar{u}_{\text{lx}}] \quad (\text{C.29})$$

where $(\Delta X_{\ddot{u}}, \Delta Y_{\dot{v}})$ and $(\Delta X_u, \Delta Y_v)$ are the hydrodynamic perturbation due to added mass and damping respectively.

Choose a virtual control as:

$$\alpha_1 = K_u \tilde{u}_e \quad (\text{C.30})$$

Virtual state equation becomes:

$$\dot{z}_1 = -\frac{(1-K_u)}{m-X_{\ddot{u}}-\Delta X_{\ddot{u}}}[(X_u+\Delta X_u)\tilde{u}_e-(Y_{\dot{v}}+\Delta Y_{\dot{v}}-m)\tilde{v}_e\tilde{r}_e+\tau_x-\bar{u}_{\text{Ix}}] \quad (\text{C.31})$$

Choose an augmented CLF as:

$$V_a = V + \frac{1}{2}z_1^2 \quad (\text{C.32})$$

Imply $\dot{V}_a$:

$$\begin{aligned} \dot{V}_a = & \frac{[\tilde{u}_e(m-X_{\ddot{u}}-\Delta X_{\ddot{u}})+(\tilde{u}_e-K_u\tilde{u}_e)(1-K_u)]}{m-X_{\ddot{u}}-\Delta X_{\ddot{u}}}[(X_u+\Delta X_u)\tilde{u}_e \\ & - (Y_{\dot{v}}+\Delta Y_{\dot{v}}-m)\tilde{v}_e\tilde{r}_e+\tau_x-\bar{u}_{\text{Ix}}] \end{aligned} \quad (\text{C.33})$$

Actual control $\bar{u}_{\text{Ix}}$:

$$\bar{u}_{\text{Ix}} = \frac{(m-X_{\ddot{u}}-\Delta X_{\ddot{u}})[1+(Y_{\dot{v}}+\Delta Y_{\dot{v}})\tilde{v}_e\tilde{r}_e]}{\tilde{u}_e(m-X_{\ddot{u}}-\Delta X_{\ddot{u}}+1-2K_u+K_u^2)} \quad (\text{C.34})$$

The $\dot{V}_a$ in (C.33) becomes

$$\dot{V}_a = -[X_u+\Delta X_u+\tilde{u}_e+1+\tau_x+\tilde{r}_e\tilde{v}_em] \quad (\text{C.35})$$

Repeat the same procedure in (C.27)-(C.35) for the other virtual state $\tilde{v}_e, \tilde{w}_e$ and $\tilde{r}_e$ using $\alpha_2 = K_v\tilde{v}_e$, $\alpha_3 = K_w\tilde{w}_e$ and $\alpha_4 = K_r\tilde{r}_e$ respectively. The actual inner controls for the virtual state $z_2 = \tilde{v}_e - \alpha_2$, $z_3 = \tilde{w}_e - \alpha_3$ and $z_4 = \tilde{r}_e - \alpha_4$ respectively becomes:

$$\bar{u}_{\text{Iy}} = \frac{(m-Y_{\dot{v}}-\Delta Y_{\dot{v}})(1+m\tilde{u}_e\tilde{r}_e)}{\tilde{v}_e(m-Y_{\dot{v}}-\Delta Y_{\dot{v}}+1-2K_v+K_v^2)} \quad (\text{C.36})$$

$$\bar{u}_{\text{Iz}} = \frac{m-Z_{\dot{w}}-\Delta Z_{\dot{w}}}{\tilde{w}_e(m-Z_{\dot{w}}-\Delta Z_{\dot{w}}+1-2K_w+K_w^2)} \quad (\text{C.37})$$

$$\bar{u}_{1\psi} = \frac{(I_z - N_{\dot{r}} - \Delta N_{\dot{r}})[1 + (X_{\dot{u}} + \Delta X_{\dot{u}})\tilde{u}_e \tilde{v}_e]}{\tilde{r}_e(I_z - N_{\dot{r}} - \Delta N_{\dot{r}} + 1 - 2K_r + K_r^2)} \quad (C.38)$$

C3.2. Backstepping Controller Design for Horizontal Plane

Repeat the same procedure in (C.27)-(C.35) for the virtual state $\tilde{u}_e, \tilde{v}_e$ and $\tilde{r}_e$ using $\alpha_1 = K_u \tilde{u}_e, \alpha_2 = K_v \tilde{v}_e$ and $\alpha_3 = K_r \tilde{r}_e$ respectively. The actual inner controls for the virtual state $z_1 = \tilde{u}_e - \alpha_1, z_2 = \tilde{v}_e - \alpha_2$, and $z_3 = \tilde{r}_e - \alpha_3$ respectively becomes:

$$\bar{u}_{1x} = \frac{(m - X_{\dot{u}} - \Delta X_{\dot{u}})[1 + (Y_{\dot{v}} + \Delta Y_{\dot{v}})\tilde{v}_e \tilde{r}_e]}{\tilde{u}_e(m - X_{\dot{u}} - \Delta X_{\dot{u}} + 1 - 2K_u + K_u^2)} \quad (C.39)$$

$$\bar{u}_{1y} = \frac{(m - Y_{\dot{v}} - \Delta Y_{\dot{v}})(1 + m\tilde{u}_e \tilde{v}_e)}{\tilde{v}_e(m - Y_{\dot{v}} - \Delta Y_{\dot{v}} + 1 - 2K_v + K_v^2)} \quad (C.40)$$

$$\bar{u}_{1\psi} = \frac{(I_z - N_{\dot{r}} - \Delta N_{\dot{r}})[1 + (X_{\dot{u}} + \Delta X_{\dot{u}})\tilde{u}_e \tilde{v}_e]}{\tilde{r}_e(I_z - N_{\dot{r}} - \Delta N_{\dot{r}} + 1 - 2K_r + K_r^2)} \quad (C.41)$$

C3.3. Backstepping Control System Design for Vertical Plane

Repeat the same procedure in (C.27)-(C.35) for the virtual state $\tilde{u}_e, \tilde{w}_e$ and $\tilde{q}_e$ using $\alpha_1 = K_u \tilde{u}_e, \alpha_2 = K_w \tilde{w}_e$ and $\alpha_3 = K_q \tilde{q}_e$ respectively. The actual inner controls for the virtual state $z_1 = \tilde{u}_e - \alpha_1, z_2 = \tilde{w}_e - \alpha_2$, and $z_3 = \tilde{q}_e - \alpha_3$ respectively becomes:

$$\bar{u}_{1x} = \frac{(m - X_{\dot{u}} + \Delta X_{\dot{u}})[1 + \tilde{w}_e \tilde{q}_e m]}{\tilde{u}_e(m - X_{\dot{u}} - \Delta X_{\dot{u}} + 1 - 2K_u + K_u^2)} \quad (C.42)$$

$$\bar{u}_{1z} = \frac{(-m + Z_{\dot{w}} + \Delta Z_{\dot{w}})[1 + \tilde{u}_e \tilde{q}_e (X_{\dot{u}} + \Delta X_{\dot{u}})]}{\tilde{w}_e(-m + Z_{\dot{w}} + \Delta Z_{\dot{w}} - 1 + 2K_w - K_w^2)} \quad (C.43)$$

$$\bar{\mathbf{u}}_{1\theta} = \frac{(I_y - M_{\dot{q}} - \Delta M_{\dot{q}})[1 + \tilde{u}_e \tilde{w}_e (Z_{\dot{w}} + \Delta Z_{\dot{w}})]}{\tilde{q}_e (I_y - M_{\dot{q}} - \Delta M_{\dot{q}} + 1 - 2K_q + K_q^2)} \quad (\text{C.44})$$

Appendix C3

Controllers' Robustness

Figure C3.1: Velocity error

Figure C3.2: Position tracking error

Figure C3.3: Velocity error (Euclidean norm)

Figure C3.4: Position tracking error (Euclidean norm)

Figure C3.5: Input thrust response

Figure C3.6: Input thrust (Euclidean norm)

Figure C3.7: $\dot{V}$ for outer and inner loop (Euclidean norm)

Figure C3.8: Sensitivity of perturbation to output position and velocity

Figure C3.9: Variation of $\dot{V}_O$

Figure C3.10: Variation of $\dot{V}_I$

Figure C3.11: $V_I(\tilde{\mathbf{x}}_{s2e}, t) \rightarrow 0, \|\tilde{\mathbf{x}}_{s2e}\| \rightarrow 0$ and $V_O(\tilde{\mathbf{x}}_{s1e}, t) \rightarrow 0, \|\tilde{\mathbf{x}}_{s1e}\| \rightarrow 0$ for decoupled controller

Appendix D1

Switching Definitions

Definition 5.1 (Continuity)

By convention, we will generally assume right continuity of the states as:

$$\mathbf{x}(t) = \mathbf{x}^+(t), \quad \forall t \geq 0 \quad (\text{D.1})$$

and left continuity of the time as:

$$t = t^- \quad (\text{D.2})$$

as shown in Fig. D1.1

Figure D1.1: Continuity of the signal

Definition 5.2 (Finite Escape Time)

Any solution that does not start at $\mathbf{x}(0) = 0$ is of the form:

$$f(t) = \frac{1}{T-t}, \quad t \in [0, T) \quad (\text{D.3})$$

and the solution $\mathbf{x}(t)$ tends to infinity in finite time $t \in [0, T)$ as shown in Fig. D1.2. Therefore the solution of the $f(t)$ must be bounded with bounded input or in other words it must be globally Lipschitz continuous.

Figure D1.2: Example of finite escape time

Definition 5.3 (Zeno Behavior)

System undergoes an unbounded number of discrete transitions in a finite time as seen in Fig. D1.3 is known as Zeno behavior. This Zeno behavior occurs, for example, when a controller unsuccessfully attempts to satisfy an invariance specification by switching the system among different configurations faster and faster. Therefore any switching law that causes Zeno behavior is regarded as unacceptable.

Figure D1.3: Example of Zeno behavior

Definition 5.4 (Switching Signal)

For a switched system in (6.1), if the supervisory control is adopted to switch the inner loop controller for example during the time $t \in [t_0, t_3]$, then the switching signal σ_{Cl} for the inner control can be defined as:

$$\sigma_{Cl} = ((t_0, \bar{\mathbf{u}}_{1,1}), (t_1, \bar{\mathbf{u}}_{1,2}), (t_2, \bar{\mathbf{u}}_{1,3})) \quad (\text{D.4})$$

with $t_0 \leq t_1 \leq t_2 \leq t_3$ and $\bar{\mathbf{u}}_1 = (\bar{\mathbf{u}}_{1,1}, \bar{\mathbf{u}}_{1,2}, \bar{\mathbf{u}}_{1,3})$ is the inner controller. Note that the piecewise constant switching signal is label as 1, 2 and 3 without the time reference.

For example, at the switching transition t_1 , the velocity error state of the inner loop is denoted by $\tilde{\mathbf{x}}_{s2e} = \rho(2, 1, \tilde{\mathbf{x}}_{s2e}^-)$. The inner loop is switched to the outer loop at time $t \in [t_3, t_4]$, then the switching signal σ_L can be defined as:

$$\sigma_L = ((t_3, 0), (t_4, 1)) \quad (\text{D.5})$$

with $t_3 \leq t_4$ and (0,1) refers to inner and outer loops respectively. In this thesis work, we defined $\sum_{[t_0, \infty)}$ to be the set of all valid switching sequences in $[t_0, \infty)$.

Definition 5.5: (*i* - *j* Switching Region)

The *i*-*j* switching region $\Omega_{CO}^{ij} = \Omega_{CI}^{ij}$ is the set of continuous states where the system is allowed to switch from *i*-th subsystem denotes by region $\Omega_{CO}^i = \Omega_{CI}^i$ to *j*-th subsystems denote by $\Omega_{CO}^j = \Omega_{CI}^j$ that consists of different controllers $i \neq j = 1, \dots, I_C$ where $I_C = 3$.

Definition 5.6: (Smooth Function)

Given a smooth positive function for controller switching $V_C^i : \mathcal{R}^n \rightarrow \mathcal{R}^n$, we define the following smooth regions $\Omega_{CO}^j = \Omega_{CI}^j$ and $\Omega_{CO}^{ij} = \Omega_{CI}^{ij}$ as:

$$\Omega_{CO}^i = \Omega_{CI}^i : \{\tilde{\mathbf{x}}_{s2} \in \mathcal{R}^n \mid \dot{V}_C^i \leq 0\} \quad (D.6)$$

$$\Omega_{CO}^{ij} = \Omega_{CI}^{ij} : \{\tilde{\mathbf{x}}_{s2} \in \mathcal{R}^n \mid V_C^i \geq V_C^j, j = i + 1\} \quad (D.7)$$

Definition 5.7: Schematic diagram on $V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_2)$ is less than or equal to $V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_0)$

Figure D1.4: Example of $V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_2)$ is less than or equal to $V_{I,1}(\tilde{\mathbf{x}}_{s2e}, t_0)$

Appendix D2

Set of Stabilizing Controller Responses

Figure D2.1: Position tracking response

Figure D2.2: Velocity response

Figure D2.3: Thrust output

Figure D2.4: "Energy" level for outer loop (position)

Figure D2.5: "Energy" level for inner loop (velocity)

Appendix D3

Difference Initial Conditions on Pipeline

Figure D3.1: Position in surge direction at various initial conditions

Figure D3.2: Position in sway direction at various initial conditions

Figure D3.3: Position in heave direction at various initial conditions

Figure D3.4: Position in roll direction at various initial conditions

Figure D3.5: Position in pitch direction at various initial conditions

Figure D3.6: Position in yaw direction at various initial conditions

Figure D3.7: Velocity in surge direction at various initial conditions

Figure D3.8: Velocity in sway direction at various initial conditions

Figure D3.9: Velocity in heave direction at various initial conditions

Figure D3.10: Velocity in roll direction at various initial conditions

Figure D3.11: Velocity in pitch direction at various initial conditions

Figure D3.12: Velocity in yaw direction at various initial conditions

Figure D3.13: Inner loop “energy” (in surge direction) at various initial conditions

Figure D3.14: Inner loop "energy" (in sway direction) at various initial conditions

Figure D3.15: Inner loop "energy" (in heave direction) at various initial conditions

Figure D3.16: Inner loop “energy” (in roll direction) at various initial conditions

Figure D3.17: Inner loop “energy” (in pitch direction) at various initial conditions

Figure D3.18: Inner loop “energy” (in yaw direction) at various initial conditions

Figure D3.19: Outer loop “energy” (in surge direction) at various initial conditions

Figure D3.20: Outer loop "energy" (in sway direction) at various initial conditions

Figure D3.21: Outer loop "energy" (in heave direction) at various initial conditions

Figure D3.22: Outer loop "energy" (in roll direction) at various initial conditions

Figure D3.23: Outer loop "energy" (in pitch direction) at various initial conditions

Figure D3.24: Outer loop “energy” (in yaw direction) at various initial conditions

Appendix D4

Chapter 5- Comparison One

Figure D4.1: Total thrust response

Figure D4.2: $\dot{V}$ in Euclidean norm for both loops

Figure D4.3: “Energy” level for outer loop (position)

Figure D4.4: “Energy” level for inner loop (velocity)

Appendix D5

Chapter 5- Comparison 2

Figure D5.1: Position tracking error in Euclidean norm

Figure D5.2: Velocity error in Euclidean norm

Figure D5.3: $\dot{V}$ in Euclidean norm for both loops

Figure D5.4: "Energy" level for outer loop (position)

Figure D5.5: "Energy" level for inner loop (velocity)

Appendix E1

Comparisons of ROVs using Proposed Controllers

Figure E1.1: Velocity response

Figure E1.2: Position tracking response

Figure E1.3: Velocity error in Euclidean norm

Figure E1.4: Position tracking error in Euclidean norm

Figure E1.5: Velocity error in Euclidean norm

Figure E1.6: Position tracking error in Euclidean norm

Figure E1.7: Thrust in Euclidean norm

Figure E1.8: Total thrust response

Figure E1.9: Percent of maximum thrust utilization in Euclidean norm

Figure E1.10: $\dot{V}$ in Euclidean norm

Figure E1.11: “Energy” level for outer loop (position)

Figure E1.12: “Energy” level for inner loop (velocity)

Figure E1.13: Switching signal for JHU (left), ROMEO (right) and RRC ROV (center)

Appendix E2

Comparisons of ROVs using Different Controllers

Figure E2.1: Velocity response

Figure E2.2: Position tracking response

Figure E2.3: Velocity error in Euclidean norm

Figure E2.4: Position tracking error in Euclidean norm

Figure E2.5: Velocity error in Euclidean norm

Figure E2.6: Position tracking error in Euclidean norm

Figure E2.7: Total thrust output in Euclidean norm

Figure E2.8: Percent of maximum thrust utilization in Euclidean norm

Figure E2.9: $\dot{V}$ for outer (top) and inner loop (bottom)

Figure E2.10: “Energy” level for both inner and outer loops