Coupled acoustic resonator systems with novel acoustic properties

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**COUPLED ACOUSTIC RESONATOR SYSTEMS WITH
NOVEL ACOUSTIC PROPERTIES**

YANG YAHUI

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

2020

**COUPLED ACOUSTIC RESONATOR SYSTEMS WITH
NOVEL ACOUSTIC PROPERTIES**

YANG YAHUI

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for
the degree of Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me except where otherwise stated in this thesis. The thesis work has not been submitted for a degree or professional qualification to any other university or institution. I declare that this thesis is written by myself and is free of plagiarism and of sufficient grammatical clarity to be examined. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from 3 paper(s) published in the following peer-reviewed journal(s) / from papers accepted at conferences in which I am listed as an author.

Chapter 2 is published as Y. Yang, Z. Yang, and B. Zhang, Acoustic valley edge states in a graphene-like resonator system. *Journal of Applied Physics* **123(9)**, 091713.1-091713.4 (2018). DOI: 10.1063/1.5009626.

The contributions of the co-authors are as follows:

- Prof. Zhang provided the initial project direction and edited the manuscript drafts.
- I prepared the manuscript drafts. The manuscript was revised by Dr. Yang.
- I co-designed the study with Dr. Yang performed all the simulations. I also analyzed the results.
- Dr. Yang provided guidance in the calculation of Berry curvature.

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The contributions of the co-authors are as follows:

- Prof. Zhang and Prof. Chong supervised the project and edited the manuscript drafts.
- I performed the simulations for the original acoustic resonator system and built up the experimental setup which can be used to measure acoustic properties.
- I prepared samples and performed the laboratory work with Mr. Xue.
- Mr. Xue and Dr. Gao designed the work and performed simulations to verify the topological nature of the designed structure lattice.
- Mr. Xue wrote the drafts of the manuscript. The manuscript was revised with Dr. Gao.

Chapter 4 contains material from paper published as H. Xue, Y. Yang, G. Liu, F. Gao,

Y. Chong, and B. Zhang, Realization of an acoustic third-order topological insulator. Physical Review Letters 122, 244301.1-244301.6 (2019). DOI: 10.1103/PhysRevLett.122.244301

The contributions of the co-authors are as follows:

- Prof. Zhang and Prof. Chong suggested the area and edited the manuscript drafts.
- I built up the experimental setup which can be used to measure acoustic properties and performed the laboratory work with Mr. Xue.
- Mr. Xue prepared the drafts of the manuscript. He also performed simulations with Dr. Gao and Mr. Liu.

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Date

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Abstract

The discovery of topological phases is a remarkable development with profound impact in condensed matter physics. Such systems host robust unidirectional edge states immune to backscattering from local defects or disorders and there exist topological invariants characterizing the global properties. Inspired by the potential applications based on their unique properties, tremendous explorations to extend the topological concept to acoustic systems have been witnessed in recent years. Analogues of topological phases of matter have been rapidly developed in acoustic systems, which acts as an excellent platform to study the topological phenomena. Among various kinds of acoustic systems to explore novel acoustic properties, coupled acoustic resonator systems have drawn much attention. The acoustic resonator systems are always of large size in space, making the sample fabrications and experimental measurements easier, and they could be analyzed with tight-binding model in theoretical investigations. In this thesis, three projects to investigate novel properties in sound waves based on coupled acoustic resonator systems are discussed.

At first, the topological valley Hall edge states for sound waves are demonstrated. In our design, a two-dimensional periodic acoustic resonator system which directly simulates a gapped graphene monolayer is adopted. The inversion symmetry of the lattice is broken by differing the heights of two resonators within a unit cell, leading to the analogue of valley Hall effect. Similar to a gapped graphene, gapless topological valley edge states are shown at a zigzag domain wall separating different domains with opposite valley Chern numbers, while an armchair domain wall hosts no gapless edge states.

Next, the observation of acoustic pseudo-Landau levels is demonstrated. Our system consists of the acoustic resonators specifically arranged according to a triaxial strain field, which can effectively generate a uniform magnetic field in the acoustic system. The acoustic transmission spectra for different strain strengths have been measured, which exhibit transmission gaps near the Dirac frequency due to the formation of acoustic Landau levels. The pressure profiles in the resonators are displayed to demonstrate the changes between spreading and localizing, as the frequency is tuned continuously among different discrete spectral components.

Thirdly, the experimental realization of acoustic two-dimensional higher-order topological insulators in acoustic resonator systems is introduced. A second-order triangular-shaped topological insulator on kagome lattice is demonstrated. The nontrivial bulk topology for the lattice is characterized by quantized Wannier centers. Through measuring transmission spectra and acoustic pressure distributions, the topological corner states at three corners are observed. Then, the lattice was extended to three-dimensional and a third-order topological insulator on an anisotropic diamond lattice was implemented, leading to topological corner states at two corners of a rhombohedron-like sample.

The work in this thesis has theoretically studied and experimentally investigated topological related novel properties in acoustic resonator systems, ranging from the strain field induced magnetic-like effects, to the bulk polarization. Introducing topological concepts to acoustic systems benefits both two fields; as shown in our work, acoustic resonators system acts as a highly tunable platform for exploring the topological phases, which build on larger scales and thus pose only little fabrication challenges in practical compared to photonic and electronic systems. Meanwhile,

topological properties also contribute to the robust control of sound waves, which is desirable for practical wave transport applications and may prompt innovative sound device such as sound lasing.

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List of Abbreviations

QHE	quantum Hall effect
QSHE	quantum spin Hall effect
QVHE	quantum valley Hall effect
P	parity inversion
TR	time-reversal
TI	topological insulator
HOTI	higher-order topological insulator
NN	nearest-neighboring
0D	zero-dimensional
1D	one-dimensional
2D	two-dimensional
3D	two-dimensional
SSH	Su-Schrieffer-Heager

Chapter 1

Introduction

The discovery of topological concepts, such as quantum Hall effect (QHE) and topological insulators is a remarkable development with profound impact in condensed matter physics and its historical significance has been recognized by the 2016 Nobel Prize in Physics. Compared to the electronic systems, acoustic systems possess the advantages of being easy to fabricate and measure, which offer possibilities to further explore topological effects. Incorporating the concept of topological states which were originally discovered and explored in condensed matter physics, topological acoustics have attracted tremendous attentions and rapidly emerged as an exciting field of research in recent years. Here we will cover the essential concepts of topological phases and recent realizations in topological acoustics.

1.1 Topological phases of matter

During a quite long time, symmetry was considered as a guiding principle to characterize all possible phases of matter. It was thought that different phases of matter correspond to different symmetries and a continuous phase transition must be associated with symmetry breaking. For example, a liquid follows continuous translation and rotation symmetries while a crystal follows only a discrete translation symmetry; when a liquid solidifies into a crystal, it suffers symmetry breaking where these continuous symmetries are reduced to the discrete symmetry. This description of phases of matter was completely accepted and widely used until failure of attempts to identify different chiral spin states in XY-model (1, 2).

A prototypical example of the phases of matter outside symmetry breaking is

the quantum Hall effect (QHE) (3). In the early 1980s, Klaus von Klitzing found that the energy levels of a two-dimensional (2D) electron gas sample under low temperature and a strong perpendicular uniform magnetic field are quantized into degenerate bands known as Landau levels (Fig. 1.1a). The electrons at the edge forms a one-way conductive channel immune to backscattering. Near the edges of the sample, the Landau levels bend up and intersect with the Fermi level, and then the system exhibits quantized conductivity. The quantized Hall conductance equals to integer multiples of the constant e^2/h , regardless of sample size and impurities. D. J. Thouless and others later introduced the concepts of topology to explain the phenomenon and demonstrated that this distinct type of phase is characterized by topological order (4, 5), which was first proposed in 1972 (6); the integer coefficient of the unit e^2/h could be regarded as a topological invariant denoted as the Chern number, which describes the topological nature of the band structure (5, 7-9). The discovery of the QHE was a milestone in the understanding of both phases of matter and electronic behaviors (10-17), which opens the door to the study of topological properties of energy bands (18-23).

Topology is a concept in mathematics and it concerns the global properties of geometric objects which keep unchanged under continuous deformations. For example, the geometry of a ball can be continuously changed into a bowl but not a coffee cup, because both ball and bowl have no hole while a coffee cup has one hole in its surface. Then we can regard that a ball is topologically equivalent to a bowl but not equivalent to a coffee cup. Here, the topological invariants to characterize the topology of the objects is the number of the holes, which remain constant under continuous deformations. The concept was introduced into physics to identify phases of matter and the topological invariant in physics is related to the collective behaviors of the wavefunctions over the entire band (5).

To understand the topological invariant, we start from the electronic band structure, which shows the range of energy levels of the electrons in a periodical lattice. The wavefunctions for such electrons could be described with the Schrodinger equation: $E\psi_{n,\mathbf{k}}(\mathbf{r}) = \left[-\frac{1}{2m}\nabla^2 + V(\mathbf{r})\right]\psi_{n,\mathbf{k}}(\mathbf{r})$ (n is the band index and $\mathbf{k}$ is the crystal momentum), where $V(\mathbf{r}) = V(\mathbf{r} + \mathbf{R})$ due to the periodicity of the lattice. The solution to the equation satisfies Bloch theorem and can be expressed as $\psi_{n,\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{n,\mathbf{k}}(\mathbf{r})$. For each band n , we can find an eigen value $E_{n,\mathbf{k}}$ as a function of $\mathbf{k}$, forming the band structure. The geometric properties of the band could be characterized by Berry phase, which relates to the change of phase obtained over an adiabatically motion along a closed path (24-26). In momentum space, Berry connection A_n and Berry curvature $\mathcal{F}_n(\mathbf{k})$ are defined, give as $A_n = i\langle u_n(\mathbf{k})|\nabla_{\mathbf{k}}|u_n(\mathbf{k})\rangle$ and $\mathcal{F}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times A_n(\mathbf{k})$, respectively. Berry connection describes the local change in the phase; it is gauge-dependent and it can be regarded as the counterpart of vector potential of a magnetic field. Then the Berry curvature, defined as curl on the Berry connection, is a gauge invariant and similar to a magnetic field in real space. The Chern number for band n in 2D system is evaluated by integrating the Berry curvature over the first Brillouin zone

$$C_n = \frac{1}{2\pi} \oint \mathcal{F}_n(\mathbf{k}) \cdot d\mathbf{s}, \quad (1.1)$$

The integral part in Eq. (1.1), known as Berry flux, is quantized in units of 2π and then the Chern number always takes an integer value. The Chern number of a gap is the sum of the Chern numbers for all bulk bands below the gap. A band gap with non-zero gap Chern number corresponds to a topological non-trivial phase, while that with a zero gap Chern number indicates a topological trivial phase.

Figure 1.1: a. The quantum Hall effect. Applying a perpendicular magnetic field B to a 2D electro gas system give rise to QHE. Quantized Landau levels (right panel) and one-way current flow at the edge of the system (left panel) are formed. This figure was reproduced from ref (27). **b. Topological phase diagram with symmetry operations.** In 2D periodic system, when either P or TR symmetry is broken, the Dirac cones open. If P symmetry breaks, the Chern number of the bulk bands equals zero and the band gap is topologically trivial. When TR symmetry is broken, the system becomes topologically nontrivial and one-way edge mode exists. This figure was reproduced from ref (28).

Now we discuss how symmetry operations effect the Chern number. The Berry curvature $\mathcal{F}(\mathbf{k})$ is a pseudovector field, it is odd under time-reversal symmetry (TR) and even under parity inversion (P), which means $T\mathcal{F}(\mathbf{k}) = -\mathcal{F}(-\mathbf{k})$ and $P\mathcal{F}(\mathbf{k}) = \mathcal{F}(-\mathbf{k})$. In the system preserving both T symmetry and P symmetry, $\mathcal{F}(\mathbf{k}) = 0$ and thus the Chern number is zero. As shown in Fig. 1.1b, we assume a 2D periodic system with a pair of Dirac cones (28). When either P or T symmetry is broken, the degeneracies at the Dirac cones will be lifted and a Berry flux with π of each bulk band will be generated. For P symmetry breaking case (lower left panel in Fig. 1.1b), the Chern number is still zero. Only when T symmetry is broken and P symmetry is preserved, just as that of the QHE, a total Berry flux of 2π is generated and a non-zero Chern number (equals 1) is acquired, indicating that the system is topological nontrivial.

Topological equivalent systems can be directly connected without phase transition, while a topological phase transition must take place at the interface between two topological inequivalent systems (Fig. 1.2a). The phase transition requires the closing and reopening of bandgap to neutralize the Chern numbers, which indicates the existence of gapless states at the interface. The states are topological protected by the difference of the topological phases between both sides; the number of the topologically protected gapless states is determined by the change of the gap Chern number across the interface (Fig. 1.2b). It is known as bulk-edge correspondence (29-31).

Figure 1.2: Topological phase transition. **a.** Topological phase transition takes place at the interface between systems with different through closing and reopening of the band gap. **b.** Dispersion diagrams of the waveguide. The change in Chern number of the corresponding band across the waveguide ΔC determines the number of unidirectional and gapless interfacial modes. This figure was reproduced from ref (28).

The discovery of topological description of matter inspired numerous studies in the topology physics of electronic systems. Questions such as whether there are other methods to realize similar phenomena and obtain non-zero Chern number have drawn much attentions. In 1988, Haldane proposed that similar phenomenon can be achieved in a 2D periodic lattice model by breaking of TR symmetry without any external magnetic fields, while the TR symmetry is broken by the complex second nearest hopping strength (32). As shown in Fig. 1.3a, in Haldane's model, a honeycomb lattice system of spinless electrons is used and hopping between the nearest neighboring (NN) t_1 / second NN sites t_2 is considered, where it is supposed that second nearest hopping between **a** carries a phase $e^{i\phi}$ and for **b** the phase is $e^{-i\phi}$. To break TR symmetry, a

periodic local magnetic-flux which vanishes per unit cell is added along the perpendicular direction. Then the system exhibits nonzero Hall quantization and a non-zero topological invariant, called as the quantum anomalous Hall effect.

The realizations of topological phases introduced above are all based on TR symmetry breaking. It motivated researchers to explore approaches to obtain topological non-trivial phase with TR symmetry intact and it was achieved by Kane (12) and Zhang et al (13) at around 2005. They discovered a new class of topological phases - the quantum spin Hall effect (QSHE) or topological insulators (TI), which preserves the T symmetry. It was found that spin-orbital coupling could open a band gap and a pair of robust gapless edge states propagating along opposite directions is supported in the insulating band gap (Fig. 1.3b). This topological phase can be regarded as a combination of two QHE under opposite magnetic fields: two states carry conjugate electronic spins. The spin-dependent edge states are protected by the TR symmetry. Due to the TR symmetry, the total Hall conductance and the Chern number in QSHE systems is zero. However, the spin Hall conductance is non-zero and the wavefunction can be characterized by a new topological invariant, the spin Chern number ($C_{\pm}$). The spin Chern number is defined as

$$C_{\pm} = \frac{1}{2\pi} \oint \mathcal{F}_{\pm}(\mathbf{k}) \cdot d\mathbf{s}, \quad (1.2)$$

where the subscript $+$ and $-$ indicates the spin up and spin down states respectively.

Figure 1.3: a. Haldane's model. Open and solid points mark the a and b sublattice sites, respectively. Arrows indicate hopping between second-neighbor sites under TR symmetry breaking. This figure was cited by *ref (32)* **b. QSHE.** A pair of spin-dependent edge states propagating along opposite directions connects the valence band to the conduction band.

Similarly, another binary degree of freedom, namely the valley, has recently been proposed to realize a new topological phase under TR symmetry, known as the quantum valley Hall effect (QVHE) (33-42). The concepts originate from graphene. For a graphene, there exist two valleys and they refer to momentum with a local energy extremum. By reducing the band structure for graphene to a low energy Hamiltonian, $H(k) = v(\tau_z k_x \sigma_x + k_y \sigma_y)$, where σ_i (i can be x, y) are Pauli matrices related to degrees of freedom for the valley, $\tau_z = \pm 1$ labels the two valleys (1 for valley K and -1 for K') and v is the Fermi velocity, it is noticed that the Hamiltonian for the two valleys are different, indicating the existence of valley contrasting properties. In 2007, Yao et al. report that by breaking P symmetry, the Berry curvature will have a nonzero value concentrated at the valleys and they carry opposite signs at each valley (33, 43). Thus, a topological invariant defined by integrating the Berry curvature over half of the Brillouin zone could be obtained, known as the valley Chern number. Then for the system with P broken, topological valley phases exist and edge transport are supported

at interfaces between media with distinct valley Chern number, which are in opposite directions at different valleys.

Since then, theoretical and experimental investigations to explore different topological phases of matter in condensed-matter physics surged further (44-50). However, it is quite difficult to realize such topological phases in electronic systems due to the inevitable material defects and the assumed electron interaction. Researchers then turn their attentions to classical systems for feasible sample fabrication and easier measurements in these systems benefiting from their large scale.

Research on classical analogs of condensed-matter problems started from 1980s. Many fascinating problems in condensed-matter physics involve quantum mechanics and the Schrödinger equation: $-\frac{\hbar^2}{2m}\nabla^2\psi + V'(\vec{r})\psi = E\psi$. ψ is eigenfunction which describes the behavior of an electron of energy E under the potential $V'(\vec{r})$. The equation can be written as $\nabla^2\psi + [q^2 - V(\vec{r})]\psi = 0$, where q is an eigenvalue parameter (equals to $\sqrt{2mE/\hbar^2}$). The governed equation for classical waves is $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$, where c is the propagation speed of the wave. Through Fourier transform, the equation can be rewritten as $\nabla^2\psi + [q^2 - V(\vec{r})]\psi = 0$ ($q = \frac{\omega}{c_0}$, ω is the eigenfrequency and c_0 is the mean wave propagation speed for the system), which is in the similar form of the quantum system. So in many cases, the classical system can be regarded as a very accurate model for the quantum system. Thus it is possible to design macroscopic systems governed by a classical wave equation to model salient features of quantum-mechanical systems and then measurements of features and tests of phenomena which are virtually inaccessible on the microscopic for condensed-matter systems are permitted. Using such acoustic analog systems, significant research results have been obtained, such as Anderson localization, the behavior of nonlinear pulses and

continuous waves in a disordered system.(51-54)

The topological properties of a system are defined on the band structure, depends on the phase information solved from the eigenvalue equations, which are seen in both the electronic and classical systems. So the classical analog of topological physics in condensed matter should be accessible, although there are fundamental differences between the electrons and classical waves. There have been increasing attempts to transfer the topology physics to the classical realms and a wide range of significant phenomena has been found (38, 55-76). Here, we will introduce several achievements as the counterparts of the non-trivial topological phases in classical systems, especially in acoustic systems.

1.2 Acoustic analogue of quantum Hall effect

In 2008, Haldane and Raghu firstly proposed the analogue of the QHE in photonic systems theoretically (66). In their seminal work, they considered a 2D gyromagnetic photonic crystal under a static magnetic field which breaks the TR symmetry. It is showed that non-zero topological invariant above the bandgap could be obtained and thus protected one-way edge states were predicted. These edge states are expected to propagate along the edge of the system and immune to defects and scattering. Also, the group velocity of an edge state should be in only one direction, determined by the applied magnetic field. Several months later, Haldane and Raghu's idea was verified by Wang et al, who were the first to experimentally realize the classical analogue of the QHE (62). They implemented the experiment using 2D photonic crystal constructed by square lattice of rods in air and the TR symmetry is broken by strong magnetic-optic response (Fig. 1.4a). The observation of the unidirectional propagating edge modes at

microwave regime was reported (Fig. 1.4c) and it is showed that the edge modes are robust against scattering (Fig. 1.4d). The realization of the QHE in photonic crystals opens a new chapter for the study of topological phases by providing strong support for the observations of topological phenomena in classical systems and enables the promising applications of the novel and efficient transmission method related to the protected edge modes.

Figure 1.4: Photonic analogue of QHE **a. Experimental design.** Schematic of the photonic crystal slab used in experiment. Square lattice comprised of ferrite rods are indicated as blue dots. The antennas A and B acts as source/probe. **b. Band structure.** Calculated band structure (blue and grey areas) and the supported edge mode (red curve) for the photonic crystal. Then Chern number for each band is given. **c. Propagation of edge state.** Edge state excited by antenna A only propagates to the right. The propagation direction is indicated by black arrow. **d. Robustness of the edge state.** When a large metallic scatterer is inserted, the edge mode keeps forward transmission and wraps around it with no reflection induced. These figures are reproduced from *ref* (62).

When it comes to acoustics, due to the fact that acoustic waves inherently do not respond to magnetic field, achieving analogue of the QHE - which requires broken TR symmetry - becomes much more challenging. Inspired by the method to achieve nonreciprocal propagation of electromagnetic waves based on Zeeman effect through a magnetic bias, a magnetic-free approach to break TR symmetry using a resonant ring cavity biased by a circulating fluid was proposed (67). As shown in Fig. 1.5a, the angular momentum vector introduced by the circular motion of the fluid in the ring cavity, which acts as the magnetic bias in electronic case, splits the degeneracy of the counter-propagating resonant modes and thus realizes a nonreciprocal linear acoustic isolator. Fig. 1.5b shows the schematic of an acoustic meta-atom designed in this work. The atom couples to each other through three waveguides and acoustic waves propagates along the path 1-3-2-1 given the specified circulation of fluid. The proposal was also experimentally realized in this work (Fig. 1.5c-e). The circulating fluid bias was generated by a fan in the cavity and the velocity of the fluid was simply controlled through the input current. Non-reciprocal propagating was clearly observed under bias while full reciprocity was exhibited without bias. The effect of introducing the circulating airflow was further discussed from the sound wave master equation. For sound wave in circulating airflow with velocity $\vec{V} = v\vec{e}_\theta$, the governing equation can be represented as

$$\left[(\nabla - i\vec{A}_{eff})^2 + V(x, y) \right] \Psi = 0, \quad (1.3)$$

where $V(x, y) = \omega^2/c^2 + (\nabla\rho/2\rho)^2 - \nabla^2\rho/(2\rho)$ is the scalar potential and $\vec{A}_{eff} = -\omega\vec{V}/c^2$ indicates the effective vector potential induced by circulating air flow. The non-zero effective vector potential results in an effective magnetic field which

breaks the TR symmetry and reciprocity of sound waves.

Figure 1.5: Acoustic nonreciprocity **a.** Acoustic analogue of Zeeman effect induced by angular momentum bias. **b.** Schematic of an acoustic meta-atom. **c.** Experimental device. **d.** Measured transmission spectrum under no bias. Full reciprocity was showed. **e.** Measured transmission spectrum under bias. Nonreciprocal propagating was clearly observed. These figures are reproduced from *ref* (67).

This method of using circulating airflow as an external bias to break TR symmetry for sound waves was commonly used in subsequently theoretical studies to realize acoustic analogue of the QHE in phononic crystals. Yang et al. were the first to realize the topological non-trivial phase in acoustics (70). In their work, they considered

a triangular lattice as shown in Fig. 1.6a, one solid cylinder with radius r_1 located at the central of each unit cell and it was surrounded by fluid-filled cylindrical region with radius r_2 . The circulating of the surrounding fluid broke the TR symmetry and lifting the Dirac-like degeneracies. The band structure for the lattice was shown in Fig. 1.6b, the introduction of the circulating flow opened the band gap and the obtained non-zero Chern numbers for the bands indicated that the topological non-trivial nature of the designed crystal. Then according to the bulk-edge correspondence, there must exist two unidirectional acoustic edge states within the band gap, which were expected to propagate along the boundary of the finite lattice adjacent to trivial materials. In the same year, two other designs of topological phononic crystal consisting of honeycomb lattice were also proposed (68, 69). Ni et al. used the acoustic meta-atom as shown in Fig. 1.6c, each unit cell consists of a such air-flow circulating ring resonator couples to other rings with three waveguides and air flow circulates within the ring. In Khanikaev's work, the graphene lattice is constructed by resonators interconnected by tubes. Different from previous designs, two resonators rather than one are consisted in each unit cell and the air rotation within the resonators are opposite – one is clockwise while the other is counterclockwise (Fig. 1.6d). After constructing the topological non-trivial sonic crystals, the edge modes predicted from the principle of bulk-correspondence were further studied. Fig. 1.6e-g showed the propagation of the edge states in a finite lattice. The topological robustness of the modes was demonstrated by introducing different kinds of lattice imperfections. As shown in Fig. 1.6e, the edge mode transports along the external boundary of the lattice unidirectionally and neither the acoustic cavity or sharp bends occurring in its path induce backscattering. In Ni's work, lattice defects caused by the change of the number/position/size/shape of one ring were introduced, and it was obvious that the mode is highly robust to the cases (Fig.

1.6f). In addition to edge modes propagating along the external boundary with different cuts of the honeycomb lattice (zigzag, armchair, bearded) which have been discussed in the other two works, mode confined at the interface between two domains with reversed angular momentum bias was also studied in Khanikaev's work (Fig. 1.6g).

Figure 1.6: Designs to realize acoustic analogue of the QHE. In **a**, **c**, **d**, three designs of 2D lattice to realize acoustic QHE are presented. **b**. Acoustic band structure for the lattice shown in **a** with(blue)/without(red) circulating air flow. **e-f**. Topological robustness of the edge states. The figures are reproduced from: **a,b,e** *ref (70)* **c,f** *ref (69)* **d, g** *ref (68)*.

These works successfully extend the topological concepts to classical acoustic realm and surged further explorations of topological phenomenon in acoustic systems. The topologically protected nature of the edge mode promises the feasibility of applications to manipulate sound waves such as controllable reflection-free sound signal processing device. Though the underlying mechanism to realize the acoustic

QHE using circulating fluid is obvious and understandable, it is still challenging in practice due to the requirements of stability, high speed and synchronicity of the rotation over the whole lattice. The experimental demonstration has not been achieved until a recent work. Zhu et al. adopted a high Q-factor resonator to construct the honeycomb lattice, which overcame the high rotation speed requirement (77). Additionally, attempts using active liquids (75) which can flow spontaneously to break TR symmetry have also been seen as a promising method to ease the experiment requirements.

1.3 Acoustic analogue of quantum spin Hall effect

With the development of classical analogue of QHE, attempts to realize the QSHE in classical systems also emerged. QHSE is characterized by topological invariant $C_{\pm} \bmod 2$ and features by TR symmetry protected gapless helical edge states existing in the strong spin-orbit coupling induced gap. In electronic systems, TR symmetry is maintained to obtain the Kramer degeneracy, which plays a vital role in the QSHE. However, it is noticed that the presence of TR symmetry could not guarantee the Kramer degeneracy in classical systems. Photons and phonons are bosons whose TR operator $\mathcal{T}$ satisfies $\mathcal{T}^2 = +1$, while $\mathcal{T}$ for electrons is fermionic and satisfies $\mathcal{T}^2 = -1$. Thus, the key to realize the direct QSHE analogue in classical systems is to construct a pseudo fermion-like TR symmetry which satisfies $\mathcal{T}^2 = -1$ and under which a pair of pseudospins can form Kramer pairs. Photonic counterpart of the QSHE was first theoretically demonstrated in a hexagonal lattice comprised of permittivity/permeability-matched bianisotropic metamaterials (57). In their work, metamaterial with $\epsilon = \mu$ (ϵ :electric permittivity; μ :magnetic permeability), which gives same responds to the electric and magnetic field components is designed. For the

metamaterial, linear combinations of transverse magnetic (TM) mode and transverse electric (TE) mode $E_z \pm H_z$ are doubly degenerate and could be seen as pseudo spin states (Fig. 1.7a). Then giant spin-orbit coupling was synthesized through strong bianisotropy to break the degeneracy and open a non-trivial topological bandgap (Fig. 1.7b). The system exhibits a zero net Chern number due to the preserved TR symmetry while a non-zero spin Chern number $C_{spin} = C_{up} - C_{down} = 2$, indicating that it is an analogue of the QSHE. The existence of the spin-polarized edge modes localized at the boundaries/interfaces was verified through simulations (Fig. 1.7c). The spin-up mode with a positive group velocity propagates forward while the spin-down mode is backward propagating. The propagation of the helical edge states was further studied by introducing different types of interface disorders and it was shown that the states were immune to backscattering from the disorders. Inspired by the bianisotropic metamaterials approach above, W. J. Chen et al. experimentally realized the photonic topological insulator (78). The difference was that they used symmetry reduction to introduce strong naturally modes coupling in waveguides rather than bianisotropic coupling which is always weak in metamaterials. Though the photonic QSHE analogue have been achieved and protected helical edge states have been observed, what protects the photonic topological insulator still remains unclear. In a later work, researchers broke the real bosonic TR symmetry while keeping the fermionic-like pseudo TR symmetry intact and validated that it is the fermionic-like pseudo TR symmetry rather than the real bosonic TR symmetry which protects the topological phases in the photonic topological insulators (79).

Figure 1.7: The first theoretical demonstration of photonic topological insulator.

a. Pseudospins are constructed by linear combinations of TE and TM modes **b.** Band structure for the designed metacrystal with (dashed black lines)/without introducing spin-orbit coupling. **c.** Dispersion of the pseudo spin up (green) and spin down (red) states. **d.** Edge modes are robust to disorders in the interface. These figures are reproduced from *ref* (78).

However, due to the intrinsic longitudinally propagation nature of airborne sound, the approaches to realize photonic QSHE analogue which requires different polarizations and spin-orbit coupling are hard to implement in acoustics. Several early attempts adopted coupled resonators which supports degenerated clockwise and anticlockwise circulating acoustic waves to mimic pseudo spins and form the Kramers doublet (55, 71, 73, 80). But the complexity of sample fabrications poses great challenges to the practical implements of this method. To solve the problem, two novel

approaches were proposed nearly around the same time, one is zone folding mechanism(81) and the other is accidental degeneracy technique (82). The concept of zone folding was first proposed in photonics by Hu et al (81). When nearly all realizations of photonic topological states were based on specially designed gyromagnetic materials or bianisotropic metamaterials, their design are based on two Bloch modes rather than two different polarizations and can be implemented in purely conventional dielectric material. In their work, they deformed a honeycomb lattice into a triangular lattice by taking larger hexagonal unit cells rather than primitive rhombic unit cells (Fig. 1.8a). Therefore, the Dirac cones at the K and K' for the original honeycomb lattice folded back to the Γ point for the triangular lattice and thus a doubly degenerate Dirac-like cone with diploes and quadrupoles modes formed at the Γ point. While maintaining the newly expanding unit cell and C_6 lattice symmetry, the degeneracy can be lifted and topological phase transitions can be observed by tuning the geometric parameters of the crystal. The zone folding mechanism which uses Bloch modes to construct pseudo spins and induces band inversion through expanding or contracting the unit cell makes the realization of QSHE analogue for all kinds of waves possible and the low fabrication requirements makes the practical applications feasible. This approach was then soon numerically (76) and experimentally (83) demonstrated in acoustics. The lattice structure considered was similar to that used in photonics and the process to construct double Dirac cones was shown in Fig. 1.8b. Then by contracting or expanding the lattice to affect the pseudospin-orbit coupling, the double Dirac cone degeneracy was lifted and band inversion took place through the process. As shown in Fig. 1.8c-d, when the lattice changed from contracting to expanding, order for dipole modes and quadrupole modes inverted, which was associated with topological phase transition.

Figure 1.8: Zone folding mechanism. **a.** Schematic of designed honeycomb lattice **b.** Zone folding process to construct double degenerate Dirac cone. **c.** Topological transition with tuning geometric parameters **d.** Band inversion and intensity circulation for pseudospins. These figures are reproduced from a,b,d *ref (81)*, c *ref (84)*.

Figure 1.9: Accidental degeneracy technique. Tuning the filling ration of lattice to obtain an accidental double Dirac cone and induce band inversion. The figure is cited from *ref (82)*.

The accidental degeneracy approach (72, 82) was the other method which can realize acoustic topological insulators in a static flow-free structure. Based on the approach, He was the first to realize the acoustic QSHE analogue experimentally (82). In their work, a honeycomb lattice comprised of metallic rods in air was adopted. Possessing 2D irreducible representations, two degenerate modes - dipole modes and quadrupole modes - exist at the Brillouin zone center for the lattice with C_6 symmetry and hybridization between the modes constructed the pseudo spin states. By choosing specific filling ratio of the lattice, an accidental double Dirac cone can be formed and the two-fold degenerate states above and below the gap could be inverted, accompanying with topological phase transitioning from trivial to non-trivial (Fig. 1.9). A pair of acoustic spin edge states propagating in opposite directions exists at the interface separated topological trivial and non-trivial region. They also experimentally observed the one-way pseudospin-dependent propagation of the edge states (Fig. 1.10a) and demonstrated the robustness of the edge states (Fig. 1.10b). These approaches to demonstrate phononic topological insulators based on lattice symmetry and band inversion, having the advantages of flow-free and simple fabrication requirements, are of great significance for further studies in topological phenomena and applications such as manipulating sound transport and other acoustic devices.

Figure 1.10: Acoustic analogue of QSHE. **a.** One-way spin-locked transport for the edge states. **b.** The edge states were robust against several defects. Figures are reproduced from *ref* (82).

1.4 Higher-order topological insulator

In recent two years, a new branch of topological phases of matter known as higher-order topological insulators (HOTI) emerged and developed quickly. Different from standard topological insulators that obey conventional bulk-boundary correspondence principle (29, 85) discussed above, which exhibit gapless topological edge states at one lower dimension, the generalized n th-order ($n \geq 2$) topological insulators in a d -dimensional ($d \geq n$) system support $d-n$ dimensional $[(d-n)D]$ topological states. For example, 0D topological corner states rather than 1D edge states exist in 2D second-order topological insulators. Similarly, for 3D systems, 0D corner states are supported in third-order topological insulators and 1D hinge states are supported in second-order

topological insulators. Since several early theoretical predictions based on quantized quadrupole moments (86-90), HOTIs have been quickly implemented in different types of platforms (91-93).

Benalcazar et al. first proposed that systems with quantized higher multipole moments could exhibit gapless topological corner states instead of edge or surface states (86). In their study, second-order topological insulators and third-order topological insulators with 0D topologically protected corner states were demonstrated, based on 2D square insulator with a nonzero quadrupole moment (Fig. 1.11a) and 3D cubic insulator with a nonzero octupole moment (Fig. 1.11b) respectively. “Nested” Wilson loops were then introduced to obtain the bulk topology index to characterize this kind of insulators. In the work, a 2D lattice with fully periodic boundary conditions along y direction and open boundaries along x direction are considered. The Wilson loop along x direction from point k over the 2D BZ (Fig. 1.11c), with N occupied bands considered, could be written as,

$$\mathcal{W}_{x,k} = F_{x,k+2\pi} F_{x,k+2\pi-\delta k_x} \dots F_{x,k+\delta k_x} F_{x,k}, \quad (1.4)$$

where $[F_{x,k}]^{m,n} = \langle u_{k+\delta k_x}^m | u_k^n \rangle$ and $|u_k^n\rangle$ is the Bloch function for the nth energy band. Through diagonalization of the Wilson loop as $\mathcal{W}_{x,k} = \sum_{j=\pm} |v_{x,k}^j\rangle e^{2\pi i v_x^j(k_y)} \langle v_{x,k}^j|$, where $|v_{x,k}^j\rangle$ are the eigenstates for Wannier Hamiltonian, two Wannier bands which do not touch over the 1D BZ along y direction are constructed (Fig. 1.11d). The gap between these two bands indicates that the bands may could be characterized by topological invariants. For the Wannier bands subspaces $|\mathcal{W}_{y,k_x}^\pm\rangle = \sum |u_k^n\rangle [v_{x,k}^\pm]^n$, the nested Wilson loops along y direction are invented,

$$\tilde{\mathcal{W}}_{y,k_x}^\pm = F_{y,k+2\pi}^\pm F_{y,k+2\pi-\delta k_y}^\pm \dots F_{y,k+\delta k_y}^\pm F_{y,k}^\pm,$$

(1.5)

where $F_{y,k}^{\pm} = \langle w_{x,k+\delta k_y}^{\pm} | w_{x,k}^{\pm} \rangle$. Then by summing the nested Wilson loop over k_x , the polarization for Wannier subspace $v_x^{\pm}$ is obtained, which is quantized to 0 or 1/2. The polarization of 1/2 suggests that the edges for the lattice considered here are nontrivial topological insulators. Following this theoretical prediction, experimental realizations based on quadrupole quantization in square and cubic lattice have quickly been reported in mechanics, microwave circuits and photonic crystals.

Figure 1.11: **a.** 2D insulator with quantized bulk quadrupole moment, γ, λ : hopping strengths. **b.** 3D insulator with quantized bulk octupole moment. **c.** Wilson loop $\mathcal{W}_{x,k}$ along x direction starting from point $\mathbf{k}$ over the 2D BZ. **d.** Wannier bands with a gap along y direction over 1D BZ are constructed thorough diagonalization of the Wilson loop. Figures are reproduced from *ref (86)*.

Several months later, M. Ezawa extended the HOTIs to the breathing Kagome lattice (94). In this work, a second-order topological insulator is constructed by triangles made of the Kagome lattice and three topological boundary states at the corner of the triangle is found. As shown in Fig. 1.12a-c, the NN hopping parameters for the upward and downward triangles are t_a and t_b , respectively. Through tight-binding analysis, the change of the energy spectrum with tuning t_a/t_b is given in Fig. 1.12d. When $-1 < t_a/t_b < 1/2$, zero-energy states emerge and the states are well localized at the corners (Fig. 1.12e). Considering the case in which $t_a = 0$, the boundary for the lattice exhibits as the Su-Schrieffer-Heeger (SSH) model (95), which describes a TI and the bulk topology is characterized by the quantized polarization protected by lattice symmetry. The lattice could be regarded as an extension of the classic SSH model. Then the bulk topology could be described by the polarization, which is an integral of the Berry connection $A_i = -i\langle u | \partial k_i | u \rangle$ with $i = x, y$, expressed as,

$$P_i = -\frac{1}{S} \iint_{BZ} A_i d^2k. \quad (1.6)$$

The polarization (P_x, P_y) is identical to the Wannier centre. When the C_3 and mirror symmetries are intact, the Wannier centre will be quantized to two positions within each unit cell, which are relates to two different topological phases respectively. It is only dependent on t_a/t_b and it will not change until the band gap open or reclose with the change of t_a/t_b . In a following work (96), M. Ezawa generalized the quantized Wannier centre to anisotropic diamond lattice in 3D and predicted the existence of two corner states in a rhombohedron made of such diamond lattice (Fig. 1.12f). Possessing the possibility to strongly confine states in any dimensions, the HOTIs attracts much attention in both scientific research and practical applications.

Figure 1.12: Wannier-type HOTIs. **a-c.** Triangles made of the Kagome lattice with different hopping energy. t_a and t_b are hopping energy for upward and downward triangles, respectively. **d.** Energy spectrum for the triangle model as a function of t_a/t_b . **e.** A second-order TI with zero corner states localized at three corners of the triangle. **f.** A third-order TI with two zero states localized at the two corners of the rhombohedron. The figures are reproduced from: a-e *ref (94)* f *ref (96)*.

1.5 Motivation

Acoustics is an ancient field of physics, whose origin can be dated back to 300 BC when an ancient Greek philosopher Aristotle proposed that sound could be understood as vibrations of air, which describes the nature of wave motion. The field was flourishing around the period of Scientific Revolution and physical understanding towards the acoustics evolved with the developments in wave equation and continuum physics. Acoustics is such a classical field that the acoustical processes are so familiar to us, even a little child could talk something about the properties of sound waves. It is

amazing that the field was reinvigorated by introducing concepts in quantum theory and novel properties continue to be discovered in recent three decades. Acoustic waves in periodic systems have been shown to possess similar band properties to that in electronics. Numerous studies based on the periodic structures to explore new acoustic phenomena such as topological analogues have been witnessed.

Over the past decades, topology in broad areas of physics has received significant attention for its related novel properties. It has been quickly extended to classical realm from condensed matter. Topological acoustics emerges as a new subfield of acoustics in recent years, where we study structures whose modes of sound waves are allowed to form band structures analogues to that of topological phases of matter in condensed matter physics. However, compared to electrons and photons, there does not exist much freedoms to control sound. For example, for electronic systems, there are charges and spins, and they could respond to magnetic field; for photonic systems, there are polarizations to simulate spins and magneto-optical material could be used to let them respond to magnetic field. But for sound, air-borne sound waves are longitudinal waves which do not have spin or polarizations and they are magnetically inert. As a result, at that time when I started my PhD study, the development of the field of topological acoustics lagged far behind that in other systems.

Driven partly by the gap between the developments of topological phases for sound and that for electronic/photonic system, but also by the topological-related novel properties and its potential applications, we focus on the realization of novel topological-related properties in acoustic systems. The work included in this thesis are all based on coupled acoustic resonator system. The controllable parameters for resonators and their coupled waveguides provide us with much room to find out our desired structures, and also make it convenient for both theoretical and experimental

investigations. It was first proposed by C. T. Chan that the system could be used to realize acoustic topological phases. The resonator carries some sound modes with a pressure profile varying only along the cylinder axis, similar to the atomic orbitals. The system has been used to study acoustic SSH chain and Weyl points (97-99). Based on the acoustic resonator system, we have explored several novel properties of sound waves.

During the period of my PhD study, the study of topological phases in acoustic systems developed quickly. Although numerous efforts have been done, there still exist interesting questions that were worth exploring. The previous realizations of topological phases in acoustic were focused on analogues of QHE and QSHE. The emerging topological phase known as QVHE which relies on the inversion symmetry breaking, has not been well studied in acoustics. There were demonstrations for acoustic waves which adopted a mirror symmetry breaking mechanism (38, 74) rather than simple inversion symmetry breaking. So we first demonstrated a direct acoustic counterpart of the topological valley Hall phases (33, 35, 37) in graphene; Landau quantization in acoustic systems is absent: we noticed that the attempts to realize QHE in the field are all based on Haldane's model, they consider periodic systems and no external magnetic field. It is known that uniform magnetic field could not be generated in that periodic systems. So for the studies, although they obtained nonzero topological invariant and exhibited one-way edge transport as that in QHE in condensed matter systems, no Landau levels could be observed due to the lack of effective uniform magnetic field. The fact motivates us to attempt to realize Landau quantization in acoustic systems, which is a key concept in the mechanism of QHE and the subsequent introduction of topology into quantum physics; In recent two years, a new type of topological phase called a higher-order topological phase that does not obey the

traditional bulk-boundary correspondence was discovered (86-94, 96). The novel properties stimulated researchers to further explorations and it obtained fruitful results in microwaves, which motivates us to demonstrate it in acoustics. So study to realize acoustic HOTIs and to observe the higher-order corner states was conducted.

In general, these three questions guide my work and novel topological properties for sound waves are explored. The work in the thesis show that: the megascopic scale acoustic resonator system acts as an excellent platform for exploring topological features, because most conditions in the system could be precisely controlled or measured; also, the topological phases provide a robust way to control sound waves. The topological robust and defect insensitive properties of the states my find use in applications such as on-chip filters in mobile phones, sound lasing, and biochemical sensing.

Chapter 2

Acoustic Valley Edge States in a Graphene-like Resonator System

2.1 Introduction

Since the discovery of topological phases of matter in condensed matter physics, tremendous efforts have been devoted to realize their classical analogues and it has soon matured into a novel field. Some early attempts to construct topological phases in acoustic systems aimed at the QHE and relied on breaking time-reversal (T) symmetry. Due to the difficulty of breaking T symmetry, later studies largely emphasized on the analogue of QSHE for acoustic waves, which support two effective “spin” states over a topological band gap with intact T symmetry.

Recently, the valley degree of freedom (DOF) has drawn intense research interest as a route to realize a new topological state called quantum valley Hall effect (QVHE) (33-37, 43). Valley labels the two energy extrema at the corners of the Brillouin zone where non-zero opposite topological charge is found in systems with broken inversion symmetry. The conservation of valley DOF is protected by the large separation between two valleys in momentum space that leads to negligible intervalley interaction. This concept of valley manipulation has attracted much interest in valleytronics, similar to the use of spin in spintronics. Inspired by the exciting achievements in valleytronics, the valley DOF has recently been extended to acoustic crystals for novel manipulation of air-borne sound (38, 74). However, the previously demonstrated acoustic crystal adopted a mirror-symmetry-breaking mechanism (74), thus lacking a direct counterpart in condensed matter systems.

In this chapter, we investigate a 2D periodic acoustic resonator system that can

be directly described with a tight-binding model, as an acoustic counterpart of a gapped graphene monolayer. Similar to a gapped graphene, gapless topological valley edge states can be found at a zigzag domain wall separating different domains with opposite valley Chern numbers, while an armchair domain wall hosts no gapless edge states. Besides, the valley vortex states in the previously demonstrated valley acoustic crystal exist in the space among adjacent sites (artificial atoms), but in our resonator system, the valley vortex states rotate around the six sites as a π orbit in graphene.

2.2 Acoustic honeycomb lattice

We start with an acoustic honeycomb lattice, which consists of two acoustic resonators (marked as A and B) only coupled to its NN resonators by thin cylindrical coupling waveguides (Fig. 2.1a). The lattice is adopted to simulate a monolayer graphene. It can be described by the coupled mode equation $i\partial_t p_n = \sum_m \kappa p_m$, where p_n refers the amplitude of the mode for the n th resonator, κ is the coupling strength between two resonators and the summation is taken over the NN resonators. Considering only the lowest two acoustic eigenmodes, the solution to coupled-mode equation can be viewed as the one to the tight-binding eigenvalue problem $H\psi = E\psi$. Then our system could be described by a tight-binding model, the Bloch Hamiltonian H of this system can be written as,

$$\begin{aligned}
 H &= \begin{pmatrix} V_a & H_{12}(\vec{k}) \\ H_{12}(\vec{k})^* & V_b \end{pmatrix} \\
 &= \begin{pmatrix} 0 & H_{12}(\vec{k}) \\ H_{12}(\vec{k})^* & 0 \end{pmatrix} + \frac{V_a+V_b}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{V_a-V_b}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
 \end{aligned} \tag{2.1}$$

where

$$H_{12}(\vec{k}) = -t[\exp(-i\vec{k} \cdot \vec{e}_1) + \exp(-i\vec{k} \cdot \vec{e}_2) + \exp(-i\vec{k} \cdot \vec{e}_3)] \quad (2.2)$$

Here $V_a(V_b)$ is on site energy for resonator A(B), which can be tuned by the size of the resonators and t represents the hopping terms. $\vec{e}_1 = (0, a)$, $\vec{e}_2 = \left(-\frac{\sqrt{3}}{2}a, -\frac{a}{2}\right)$ and $\vec{e}_3 = \left(\frac{\sqrt{3}}{2}a, -\frac{a}{2}\right)$ define the positions of NN bound atoms and $\mathbf{k} = (k_x, k_y, k_z)$ is the Bloch wave vector. For the lattice, there will be a two-fold Dirac degeneracy at each corner of the first BZ when $V_a = V_b$. When inversion symmetry for the system is broken by differing the two acoustic resonators ($V_a \neq V_b$), the degeneracies in the band structure will be lifted and a band gap occurs.

The Hamiltonian near the Dirac points can be expressed as $H = \frac{\sqrt{3}}{2}at(\delta k_x \sigma_x \tau_z + \delta k_y \sigma_y) + \frac{\Delta}{2}\sigma_z$, where a is the lattice constant, σ_i (i can be x, y, z) are Pauli matrices related to degrees of freedom for the valley, $\tau_z = \pm 1$ labels the two valleys (1 for valley K and -1 for K') and $\Delta = V_a - V_b$. Then the Berry curvature $\Omega_n(k) = \nabla_k \times \langle u_n(k) | i \nabla_k | u_n(k) \rangle$ in the first band for the lattice can be obtained from the Hamiltonian: $\Omega_n(k) = \frac{3a^2 t^2 \Delta}{2(\Delta^2 + 3a^2 t^2 \delta k^2)^{3/2}}$, which shows distributions with opposite signs concentrated in the vicinity of the K and K' points. By integrating the Berry curvature over an individual valley, the valley Chern number can be obtained as $C_v = \text{sgn}(\Delta) \tau_z / 2$.

2.3 Band structures from full wave simulations

To verify the above theoretical predictions from tight-binding model, we performed first principle simulation for the lattice with COMSOL Multiphysics. The model is shown in Fig. 10a, two acoustic resonators connected by thin cylindrical coupling waveguides exist in each unit cell. The NN distance for the lattice is chosen as $a = 0.01$ m and then the lattice constant is $a_0 = \sqrt{3}a$. For the two cylindrical resonators A and B within each unit cell, they are of the same radius $r = 0.3a$ and have different heights $h_A = h + \Delta h$, $h_B = h - \Delta h$, where $h = \frac{5}{6}a$ and Δh is tunable. The band structures for the cases $\Delta h = 0$ and $\Delta h = 0.15h$ were calculated, with the blue surfaces are treated as sound hard-wall boundaries and the white surfaces are applied with periodic boundary conditions.

Figure 2.1: **a.** Top and side views of the unit cell of acoustic honeycomb lattice. Two resonators playing the role of artificial atoms are connected by thin coupling waveguides. **b.** Band diagram when a unit cell contains identical resonators (blue lines) and when the resonators are of different heights with $\Delta h = 0.15h$ (red lines). **c.** Pressure field distribution for eigenstates K_1 and K_2 . **d.** Topological transition as the height difference Δh changes from negative to positive.

As shown in Fig. 2.1b, the obtained band structure exhibits a Dirac degeneracy at K point when $\Delta h=0$ (blue lines) and the degeneracy is lifted when a non-zero heights difference $\Delta h = 0.15h$ (red lines) is chosen, which breaks the inversion symmetry. The resulted band gap is of the range from 4273 Hz to 5045 Hz. To further investigate the acoustic Poynting vectors (the direction and amplitude of local energy flow) in the system, which could be shown as the vortex revolution for the lattice, the pressure fields for eigenstates K_1 and K_2 (marked by black dots in Fig. 2.1b) was shown. The top panel in Fig. 2.1c displays the distribution of real and imaginary part of pressure for K_1 state in the higher band and the bottom panel displays that for K_2 state in the lower band. We notice that the higher resonators (marked as B in Fig. 2.1c) coupled to each other through the static shorter resonators (marked as A in Fig. 2.1c) for K_1 state and the situation is reversed for K_2 state. The vortex revolution along the hexagon ring (counterclockwise for K_1 and clockwise for K_2) plays the role of the valley DOF. Different from previous study where the vortices exist among adjacent sites (38), it circulates around the six sites as a π orbit in graphene.

The tuning of the phase transition at K point by varying Δh (while keeping another parameters constant) is shown in Fig. 2.1d. It is shown that these two states will cross each other when Δh transits from negative to positive, with the degeneracy occurring at $\Delta h = 0$. It indicates that when the acoustic resonators A and B interchange

their heights (i.e., when Δh changes its sign), band inversion will occur and the vortex chirality will be inverted, although the bandgap appears unchanged. When $\Delta h > 0$, the vortex chirality of the lower (upper) state is anticlockwise (clockwise), which appears exactly inverted when $\Delta h < 0$.

2.4 Acoustic valley edge states

The Berry curvature of the higher band (that of the lower band is of opposite sign) is displayed in Fig. 2.2, which shows distributions with opposite signs in the vicinity of the K and K' points. The valley Chern number is then obtained by integrating the Berry curvature the half of Brillouin zone the around the valley $2\pi C_v = \int \Omega_n(k) d^2k$. The Chern number of the higher band is $\frac{1}{2}(-\frac{1}{2})$ at valley K (K') for a domain with $\Delta h > 0$ (denoted as “domain A”) and $-\frac{1}{2}(\frac{1}{2})$ at valley K (K') for a domain with $\Delta h < 0$ (denoted as “domain B”). Then if a proper domain wall is constructed between domain A and domain B, the difference of the valley-projected Chern numbers across the interface is equal to ± 1 , indicating the existence of a chiral edge state per valley at the interface between two domains according to the bulk-edge correspondence.

To verify the existence of such topological edge states at the domain wall, we performed full-wave simulations on a supercell which consists of 6 unit cells for each of the two domains ($\Delta h = \pm 0.15h$) separated by a domain wall. At first, we consider a zigzag domain wall. As shown in Fig. 2.3a, a zigzag domain wall between domain A (the upper domain) and domain B (the lower domain) is constructed. Red circles denote shorter resonators and blue circles are higher resonators. The domain wall is formed with the higher resonators from different domains connected to each other along the interface between two domains. The x -projection of valley K' and K onto this

domain wall are $\frac{k_x a_0}{\pi} = -\frac{2}{3}$ and $\frac{k_x a_0}{\pi} = \frac{2}{3}$, respectively. Simulation results in Fig. 2.3b exhibit a gapless edge state at valley K (K') within the bulk bandgap. Due to the change of the valley Chern number across the interface at the two valleys are opposite, the group velocity of the edge state at the two valleys are opposite, which indicates that the valley pseudospin of the state is locked to the propagation direction. The pressure distribution of this edge state at K point is shown in Fig. 2.3c and we can find that it is highly localized at the interface.

Figure 2.2: Berry curvature of the upper band over the first Brillouin zone.

Figure 2.3: States at zigzag interface. **a.** Zigzag domain wall between domain A as the upper domain and domain B as the lower domain. Red circles represent shorter resonators while blue are higher resonators. **b.** Band diagram for the structure. **c.** Pressure field of the edge state shown in **b** at K.

Next, we consider an armchair domain wall along the y direction (Fig. 2.4a). The band diagram for this case is shown in Fig. 2.4b. A bandgap exists and there are two trivial edge states within the bulk bandgap. At $k_y = 0$, there is a small gap between these two states and the two states tend to have zero group velocity at valley K (K'). For this domain wall, the y -projection of valley K' coincides with that of valley K , and then the topological charge annihilation will occur at $k_y = 0$, which verifies that the band gap is topologically trivial. In Fig. 2.4c, the pressure fields for these two states are shown. It is noted that the sound waves could not be confined at the interface and it

will dissipate into the domains. Now we can conclude that the propagation of such kind valley edge states along the domain wall is only supported at a zigzag domain wall, as predicted in gapped graphene (100). There was a work where valley Hall edge states were found along armchair domain walls (101). Actually, there will be a gap between the edge states for armchair interfaces, but with some specifically geometry design, the mix between the projections of the valleys K and K' on the interface might be such weak that the gap between the edge states is negligible, leading to the propagating edges states along the armchair interfaces. However, the case is quite dependent on the geometry design and it could not be a general description at all.

Figure 2.4: States at armchair interface. **a.** Armchair domain wall between domain A as the left domain and domain B as the right domain. Red circles represent shorter resonators while blue are higher resonators. **b.** Band diagram for this structure. **c.** Pressure fields for two states within the bandgap, as marked by red points in **b**.

Then we constructed a finite lattice comprising 26×15 unit cells and consists a zigzag interface to investigate the propagation of the edge states along the interface. As shown in Fig. 2.5, the lattice consists of two domains: domains A (left domain) and B (right domain) separated by a Z-shaped zigzag interface (marked by a blue dotted line). There are two sharp bends in the interface, which are set to see whether there will be backscattering. We performed simulations on the plate; we excited the edge state by a point source (indicated by red arrow in Fig. 2.5) operating at 4600 Hz, which is within the band gap. The obtained acoustic pressure field distribution is plotted in Fig. 2.5. We notice that the acoustic pressures along the interface are much higher than the remaining region; sound waves propagate along the interface and it is immune to backscattering from the sharp bends, verifying the robustness of the edge states.

Figure 2.5: Acoustic pressure field for a finite honeycomb lattice comprising 26×15 unit cells with a Z-shaped zigzag interface. Red arrow indicates the position of the point source.

The discussion above only considered the propagation for sound waves along zigzag interface. Here we adjusted the lattice to include both the zigzag interface and the armchair interface. As shown in Fig. 2.6, different from the previous case, the domain wall here contains two vertical zigzag interfaces while the middle horizontal part is armchair interface. Then we performed simulation on the plate. We put a point source at the edge of the lattice (indicated by a red arrow in Fig. 2.6) and excited the plate with operating frequency at 4500 Hz, which locates within the small gap of edge states in Fig. 2.3b. The simulated field pattern is given in Fig. 2.6; when the domain wall transits from zigzag to armchair, the edge state stops propagating. This result is consistent with the previous band structure analysis for the interface, which indicates that gapless topological edge state is not supported at the armchair domain wall.

Figure 2.6: Acoustic pressure field for a finite honeycomb lattice comprising 1794 unit cells. A domain wall with 90° bents separates the two domains, as indicated by a blue dotted line. The vertical parts of the domain wall are zigzag while the horizontal part is armchair. Red arrow indicates the position of a point source.

2.5 Conclusions

In this Chapter, we present the design of an acoustic resonator system that works as the acoustic analogue of quantum valley Hall topological insulator phase. Different from the previously demonstrated valley acoustic crystal which lacks a direct counterpart in condensed matter systems, our acoustic resonator system directly simulates a gapped graphene monolayer. Gapless topological valley edge states can be found at a zigzag domain wall separating different domains with opposite valley Chern numbers, while an armchair domain wall hosts no gapless edge states. The similarity between our system and gapped graphene provides a unique platform to simulate novel valley phenomena predicted in gapped graphene and other 2D materials with classical acoustic waves.

Chapter 3

Observation of Acoustic Landau Levels

3.1 Introduction

Landau energy quantization is one of the most fascinating quantum phenomena that exhibits rich physics by simply applying a magnetic field on a 2D electronic system. The quantized energy levels, known as Landau levels, is a key concept in the mechanism of quantum Hall effect and the subsequent introduction of topology into quantum physics. In a recent theoretical proposal (98), we have shown that such Landau energy quantization can be realized in a 2D acoustic resonator system for sound, although the airborne acoustic waves intrinsically carry no charge or spin and do not respond to magnetic fields. Besides their scientific interest, this kind of acoustic pseudo-Landau levels for classical phonons also hold promising application prospects in the phononic lasers, couplers and rectifiers (102).

In this chapter, we report the first observation of acoustic Landau levels. The underlying mechanism is the strain engineering developed in graphene physics (103, 104) that firstly produced electronic Landau levels in graphene under an effective 300-Tesla pseudomagnetic field in 2010 (105). Two years later, similar pseudomagnetic-field-induced electronic Landau levels were also observed at a metal surface covered by carbon monoxide molecules (106). Then in the following year the same approach of strain engineering was introduced into a 3D photonic waveguide structure (107). However, this photonic demonstration utilized the z axis as the role of time, and thus demonstrated the momentum quantization in the spatial spectrum, rather than the energy quantization in the temporal spectrum. How to remove the reliance on z axis

and demonstrate a similar phenomenon in 2D geometry remains as one of the “fundamental questions” (107) (a recent demonstration of photonic Landau levels utilized spatial curvature and required to be confined in a single resonator (108)). It is also noticed that another work on acoustic Landau quantization has been reported recently (109). The work is conducted on a graphene lattice with linearly varying vector potential along one direction, and thus it is markedly different from those realized in our work, in which the pseudo-magnetic field is produced by applying a triaxial strain to deform the lattice.

We have predicted the exist of acoustic pseudo-Landau levels in a system that consists of acoustic resonators specifically arranged according to the triaxial strain field in our previous theoretical work. Recently we experimentally observed such acoustic pseudo-Landau levels in the system. The acoustic transmission spectra for different strain strengths have been measured, which exhibit transmission gaps near the Dirac cone region due to the formation of acoustic Landau levels. The pressure profiles in the resonators are displayed to demonstrate the changes between spreading and localizing, as the frequency is tuned continuously among different discrete spectral components.

3.2 Acoustic lattice design

Inspired by previous studies in the graphene systems (103-105), which have shown that a designed triaxial strain field can give rise to a uniform pseudo-magnetic field and subsequently lead to discrete spectral components, we want to introduce the same strain engineering into acoustic lattice and demonstrate similar phenomenon in our familiar acoustic resonator system. To simulate the case in graphene, we adopt a honeycomb lattice consists of acoustic resonators and apply the triaxial strain field to our lattice by

displacing the resonators from their original positions with specific displacement $(d_x, d_y) = q(2xy, x^2 - y^2)$, where x, y represent the original coordinates for each resonator and q describes the strength of the strain. The strain field will give rise to an effective gauge field with vector potential $A(r) = \frac{4\beta q}{a_0}(y, -x)$ and then generate an effective uniform magnetic field $B = 8\beta q/a_0$; here β describes the decay rate of the coupling strength. With this effective magnetic field, discrete pseudo-Landau levels away from the Dirac frequency with $\Delta\omega \propto \text{sign}(n)\sqrt{|nB|}$ will be generated; the gap size between adjacent levels should be proportional to $\sqrt{|nB|}$, where n represents the order of the discrete level (98).

Figure 3.1: Applying a triaxial strain field to a honeycomb lattice. a. Displacement of a disk with radius $20a_0$ after strain engineering of strength $q=0.004/a$. **b.** Energy spectrum of unstrained disk near the Dirac region, landau levels with gaps are shown.

To verify whether the strain filed work for our system, we first modelled a honeycomb disk with radius $20a_0$ and engineer it with strain of strength $q=0.004/a$ (Fig. 3.1a). Tight-binding calculations are performed on the model and the obtained energy spectrum is shown in Fig. 3.1b. There exist significant flat energy levels for the strain engineered lattice, which confirms the effect of strain engineering to our lattice: pseudo-Landau quantization occurs for this honeycomb lattice after strain engineering.

Figure 3.2: Geometry of the honeycomb lattice of acoustic resonators **a.** Photo of the lower half of an unstrained sample. The sample consists of a honeycomb lattice of air holes (resonators) drilled in a slab of aluminum. Neighboring resonators are connected by a thin cylindrical coupling waveguide. Positions for the acoustic source and the microphone in measuring the transmission spectrum are indicated. **b.** Unit cell for the lattice. The NN distance is $a = 2$ cm. Each resonator has radius $r = 0.6$ cm. The coupling waveguide has radius $r_c = 0.2$ cm. By combining two such samples face-to-face tightly, a complete structure can be formed. The complete height of each resonator is $h = 1.33$ cm.

To observe the pseudo-Landau levels in experiment, we fabricated a honeycomb

lattice of acoustic (Helmholtz) resonators, as shown in Fig. 3.2a (note that Fig. 3.2a shows only half of the structure). There are in total 254 resonators within a circular disk with radius $6a_0$. These resonators are fabricated by drilling cylindrical holes in a slab of aluminium with numerical control machining. The unit cell of the lattice is marked by red rhombus and the detailed structure is shown in Fig. 3.2 b. There are both coupling tubes and resonators exits at the edges when we cut the sample from a whole lattice and it does not affect the states of the whole bulk. For the lattice, the NN distance is $a = 2$ cm and the lattice constant is $a_0 = \sqrt{3} a$. Each unit cell contains two identical resonators with radius $r = 0.3a$ and height $h = \frac{2}{3}a$ (the sample shown in Fig. 3.2a has height of $h/2$). Each pair of neighboring resonators is connected through a cylinder coupling waveguide with radius $r_c = 0.1a$. For the honeycomb lattice, Dirac cones located at the K (K') valley at the frequency 2519 Hz can be found in the band diagram (98). In our experiments, the whole structure is divided into upper/lower halves for convenience of fabrication, and each piece is made in a separate aluminum slab. The complete system of the acoustic lattice is formed by combing the two pieces face-to-face, and then it is sandwiched between two pieces of organic glass to form sound hard boundaries. In this acoustic lattice, each resonator plays the role of an artificial “atom”. The acoustic waves can hop into the neighboring “atoms” through evanescent-wave coupling.

3.3 Experimental demonstration: transmission spectra

In our work, we considered three cases with different strength of strain to show the effect of the strain and observe the resulted pseudo Landau levels. As schematically shown in Figs. 3.3a-c, we adopt a lattice without strain engineering and two other

lattices strained with $q=0.015/a$ and $q=0.030/a$ respectively. Before experimental demonstration, we first performed acoustic eigenmode simulations to show the frequency eigenvalues for the samples. The simulations in this work are performed by pressure acoustics module in a commercial finite element software (COMSOL Multiphysics). In the simulations, the parameters are the same with those in the real samples; the surfaces are treated as sound hard boundaries and the material within the system is air. The simulated eigenvalue spectra are shown in Figs. 3.3d-f. It can be seen that discrete levels centering at the Dirac frequency 2519 Hz emerge for the strained disks while the unstrained sample exhibits continuous spectrum, indicating the occurrence of Landau quantization for the lattices with strain engineering. The states within each discrete level are nearly degenerate and the levels are separated by band gaps: one gap is between the -1^{st} Landau level and the 0^{th} Landau level, another one between the 0^{th} Landau level and the 1^{st} Landau level. The states within the gaps are the edge states of the lattice. It is noticed that the gap size gets increased when the strain strength increases, which agrees with previous theoretical predictions. Note that for the strain of $q = 0.030/a$, higher-order Landau levels such as the -2^{nd} Landau level have started to emerge, yet its gap is too small to observe in experiment.

Figure 3.3: Eigenvalue spectra of unstrained and strained samples. **a-c.** Schematics of three samples with different strain strengths $q = 0.000/a$ **a**, $q = 0.015/a$ **b**, and $q = 0.030/a$ **c**. **d-f.** Simulated eigenvalues in terms of the state number for the above three samples.

Now we proceed to the experimental demonstration. Apart from the sample with $q = 0$, we fabricated two strained samples with $q = 0.015/a$ and $q = 0.030/a$, respectively, as shown in Figs. 3.4a and d. Here, we measured the acoustic transmission spectra for the samples. In the measurement, an acoustic source (Visaton K64WP speaker) is positioned at Port 1 (marked in Fig. 3.2a). This acoustic excitation is driven by a Tektronix AFG3102C function generator, which covers the frequency range from 2 to 3.2 kHz. The signals are detected at another edge of the sample (marked as Port 2 in Fig. 3.2a) with a microphone (ICP 130E20) connected to an oscilloscope (Rohde and Schwarz RTE1024).

Figure 3.4: Acoustic transmission measurement. **a** and **d**. Top views of fabricated samples with strain strength $q = 0.015/a$ **a** and $q = 0.030/a$ **d**, respectively. **b** and **c**. Simulated **b** and measured **c** acoustic transmission spectra for the sample with strain strength $q = 0.015/a$. The transmission spectrum for the unstrained sample is plotted as grey curves for comparison. **e** and **f**. Simulated **e** and measured **f** acoustic transmission spectra for the sample with strain strength $q = 0.039/a$.

The transmission spectra for the strained samples are shown in the second (simulation) and the third (experiment) columns in Fig. 3.4. The transmission spectra for the unstrained samples are plotted as grey curves in these figures for comparison. Due to acoustic resonances in the finite samples (98), the unstrained sample shows fluctuation maximum to -50dB in the simulated and measured transmission spectra. For the strained disks, the peaks centered at the Dirac frequency corresponds to the 0th discrete frequency level, and the other two peaks are related to the -1st and 1st levels. These levels separated by significant gaps indicates a typical phenomenon of Landau quantization in our system. To differ the resonances and transmission gaps arising from

Landau quantization, we adopt the criteria of -50 dB (maximum value of resonances) such that only those gaps with transmission below -50 dB are considered as gap between adjacent Landau levels.

Compared to the unstrained sample with small fluctuations in the transmission spectra, two significant gaps can be observed in both the simulated and measured spectra for the two strained samples. The measured transmission dips for the sample with strength of strain $q = 0.015/a$ are located around (2250, 2500) Hz and (2655, 2875) Hz (see Fig. 3.4(c)). When the strain increases to $q = 0.030/a$, the gaps now cover the frequency range of (2190, 2565) Hz and (2640, 2970) Hz (see Fig. 3.4f), the size of which are roughly 1.5 times those for the previous $q = 0.015/a$ case. The change of the gap size is consistent with the principle - energy gaps of Landau levels are proportional to $\sim\sqrt{Bn}$. It is noticed that for the sample with $q = 0.030/a$, there is another small gap below -40 dB around 2100 Hz in simulation (Fig. 3.4e). This is consistent with the negligible discontinuity at the same frequency appeared in the eigen-frequency spectrum (Fig. 3.3f), which results from the limited size of the sample; yet the effect is too weak to be measured in the experiment. In general, the measured spectra are in good agreement with the simulations. The two large gaps emerged in the transmission spectra indicate the gaps between adjacent Landau levels; the 0th discrete level is shown as a transmission peak between the gaps.

3.4 Experimental demonstration: spatial distribution

Here we map out the spatial patterns of the pressure fields for the three samples. The operating frequency was fixed at 2704 Hz, which falls into the gap between the 0th order and the 1st order Landau levels for the two strained samples. In our experiment, we

excited the samples with a speaker (diameter of 1.2cm) connected to a generator module (B&K Type 3109); The speaker was inserted into one resonator near the edge of the sample. Then we measured the acoustic pressure at each resonator with a movable microphone (diameter ~ 0.7 cm, B&K Type 4187); the signals were processed with an analyzer system (B&K Type 3560C). The simulated acoustic pressure distributions for the three samples are shown in Figs. 3.5a-c, and that measured in the experiments are plotted in Figs. 3.5d-f.

For the unstrained sample, we observed a spreading pattern originating from the sound source, which resulted from the excited bulk states (Fig. 3.5a and d). However, from the comparison between the simulated and experimental results, we noticed that sound waves suffer a lot from loss while propagating within the system. It is reasonable to infer that the loss occurs due to the tiny gaps between two halves of the sample when combining them together and the thermal and viscous losses when the sound waves propagating through the structures with small dimensions.

For the two strained samples, only localized edge states were supported at the operating frequency of 2704 Hz; sound waves were not be able to propagate in the samples and will be confined within a small region around the sound source, as depicted in Figs. 3.5b and c. From the simulated results, we can also find that the strength of confinement is proportional to the strength of strain. Figs. The measured pressure distributions for the two strained samples are plotted in Figs. 3.5e and f; compared with the unstrained case in Fig. 3.5d, we can conclude that the sound waves were highly confined. To further compare the confinement, we plotted the contour of the acoustic pressure with the ratio of 10% (dotted curves in Figs. 3.5e and 3.5f). The criteria of 10% is adopted because the pressure field decays to around 10% for the unstrained case when the acoustic wave reaches at the opposite edge of the sample. From the contour, it can

be seen that the confinement effect is significant and the pressure distribution shrinks to a smaller region when the strain strength increases, which agreed well with the simulated results.

Figure 3.5: Acoustic pressure patterns at operating frequency of 2704 Hz. The simulation results are shown in **a-c** and the measured results are shown in **d-f**. The acoustic source is indicated by the black arrow. The dotted curves in **e-f** are the contours of acoustic pressure at the ratio of 10%.

To further demonstrate the acoustic pseudo-Landau levels in the frequency spectrum, we considered the sample with strength of strain $q = 0.015/a$ and excited it with operating frequency sweeping from the -1st level to the 1st level. The acoustic pressure distributions for the sample at four different frequencies are given (Fig. 3.6); the operating frequencies were chosen as 2300 Hz (within the gap between the -1st Landau level and the 0th Landau level), 2520 Hz (at the 0th Landau level), 2704 Hz (within the gap between the 0th Landau level and the 1st Landau level) and 2900 Hz (at

the 1st Landau level), respectively. As shown in Figs. 3.6a and d, a confined acoustic pressure distribution was observed when the operating frequency falls in the first gap. When the operating frequency shifts to the 0th (Figs. 3.6b and e) level, spreading pressure pattern similar to that for the unstrained sample occurred. As the operating continues to increase and fell into the region of the second gap, confined pattern could be observed again (Figs. 3.6c and g). Also, we observed a spreading pattern at the second level (Figs. 3.6d and h with frequency at 2900 Hz). For the spreading case, there are some differences between the simulated and experimental results, where sound waves suffer a lot from loss in the experiments. It is resulted from the tiny gaps between two halves of the sample when combining them together and the thermal and viscous losses when the sound waves propagating through the structures with small dimensions. Therefore, with the increase of the frequency, the acoustic pressure distribution evolves from localization (Fig. 3.6e) to spreading (Fig. 3.6f), localization (Fig. 3.6g), and then back to spreading (Fig. 3.6h). This evolution clearly demonstrates the existence of acoustic pseudo-Landau levels and the energy gaps between them.

Figure 3.6: Acoustic pressure patterns at different operating frequencies with

strength of strain $q = 0.015/a$. The simulation results are shown in **a-d** and the measured results are shown in **e-h**. The acoustic source is indicated by the black arrow. The operating frequencies are 2300 Hz (**a** and **e**), 2520 Hz (**b** and **f**), 2704 Hz (**c** and **g**), and 2900 Hz (**d** and **h**) respectively.

3.5 Conclusion

In conclusion, we introduced strain gauge field into our 2D acoustic resonator systems in the shape of honeycomb and thus induce an effective perpendicular uniform magnetic field in the lattice, which enables the emergence of acoustic pseudo-Landau levels in the energy spectrum. To demonstrate the existence of pseudo-Landau levels, we measured the transmission spectra for acoustic resonator systems with different strength of strain engineering and plotted the acoustic pressure field distributions for the samples at different operating frequencies. From the results, we successfully observed the pseudo-Landau levels as well as the gaps between them. The pressure profiles in the resonators are displayed to demonstrate the changes between spreading and localizing, as the frequency is tuned continuously among different discrete spectral components. Our study provides a platform of aperiodic acoustic structure to explore many magnetic-like effects that are inaccessible in previous periodic structures. The observation of acoustic pseudo-Landau levels may find use in acoustic sensing and amplification, and offer more opportunities in topological acoustics and phonon lasing.

Chapter 4

Realization of Acoustic Higher-order Topological Insulators

4.1 Introduction

Higher-order Topological insulators (HOTIs) (86-93) are a new class of topological materials supporting a generalization of the bulk-boundary correspondence principle, in which topological states are guaranteed to exist along boundaries two or more dimensions lower than that of the bulk. For n th-order ($n \geq 2$) topological insulators in a d -dimensional ($d \geq n$) system, $d-n$ dimensional $[(d-n)D]$ topological states is supported. For example, a 2D second-order topological insulator does not exhibit gapless 1D topological edge states, like a standard 2D topological insulator, but instead has topologically protected 0D corner states. Likewise, a third-order topological insulator in 3D supports 0D topological states. This kind of topological insulators was first predicted and quickly demonstrated in mechanical (91), electromagnetic (92), and electrical metamaterials (93). These realizations utilized square lattices with topological properties based on the quantization of quadrupole moments. Later, another type of HOTIs based on quantized Wannier centers was proposed (94, 96), which has zero quadrupole polarization but a non-trivial bulk topology characterized by quantized Wannier centres.

In this chapter, we first report on the experimental realization of a 2D HOTI in an acoustic resonator system, based on a ‘breathing’ kagome lattice. Whereas the topological phases of previous higher-order topological insulators were characterized by quantized lattice quadrupole moments, the present lattice exhibits quantized dipole moments. Exhibiting similar quantized dipole polarizations, the lattice can be regarded

as a 2D generalization of the classic 1D Su-Schrieffer-Heeger (SSH) model (95). Just as in the SSH case, the eigenmode polarizations are quantized by lattice symmetries, and the Wannier centers are pinned to high-symmetry points; the mismatch between the Wannier centers and lattice truncations gives rise to charge fractionalization and hence lower-dimensional topological boundary states. Through measuring transmission spectra and acoustic pressure distributions, we observe the topological corner states in a triangular-shaped structure and show that the states exist at acute-angled corners of the kagome lattice but not at obtuse-angled corners in a parallelogram-shaped structure. Then, we extend the Wannier-type second-order TIs to 3D and implemented a third-order TI on an anisotropic diamond lattice, observing topological corner states at two corners of a rhombohedron-like sample.

4.2 Theoretical investigation

In our study, we start with a kagome lattice, which have been well studied in previous work and theoretically predicted as a platform to realize 2D second-order TIs. A unit cell for the lattice is shown in Fig. 4.1a, there are three atoms within each unit cell and the NN couplings in the upward and downward-pointing triangles are t_1 and t_2 , respectively. We implemented this kagome lattice model using acoustic resonators, shown schematically in Fig. 4.1b. Each resonator is an air-filled cylindrical cavity with metal walls, with height $H = 41$ mm and radius $r = 20$ mm. The surfaces of the cavity are acoustic hard boundaries. For an isolated resonator, the resonant acoustic mode of interest is shown in Fig. 4.1c; the acoustic pressure varies sinusoidally in the axial (z) direction and is homogeneous in the xy plane. The coupling between each pair of NN resonators is provided by two identical thin cylindrical connecting waveguides, placed

at heights $H/4$ and $3H/4$. For this model, two connecting waveguides rather than one as that in previous work are used to increase the coupling the strength. The coupling strength is tunable by varying the radius of the connecting waveguides, with radius rc_1 (rc_2) corresponding to the coupling strength t_1 (t_2) in Fig. 4.1a. For $rc_1 = rc_2 = 5.2$ mm and lattice constant $a = 108$ mm, numerical simulations produce the bulk band structure shown in Fig. 4.1d, which has two dispersive bands that meet at linear band-crossing points, with an additional flat band above.

Figure 4.1: Kagome lattice and its acoustic implementation. **a.** Tight-binding model for the kagome lattice. The dashed hexagon denotes the unit cell. **b.** Unit cell of the acoustic kagome lattice. **c.** Real part of the acoustic eigen pressure field for a single acoustic resonator at 4,185.4 Hz. **d.** Numerically computed bulk bands for the acoustic kagome lattice shown in b, at the critical point $rc_1 = rc_2 = 5.2$ mm.

The bulk Hamiltonian of the acoustic kagome lattice can be written as:

$$H = \begin{pmatrix} f_0 & h_{12} & h_{13} \\ h_{12}^* & f_0 & h_{23} \\ h_{13}^* & h_{23}^* & f_0 \end{pmatrix},$$

with $h_{12} = t_1 + t_2 e^{i\left(\frac{k_x}{2} + \frac{\sqrt{3}k_y}{2}\right)a}$, $h_{13} = t_1 + t_2 e^{i\left(-\frac{k_x}{2} + \frac{\sqrt{3}k_y}{2}\right)a}$ and $h_{23} = t_1 + t_2 e^{-ik_x a}$, where f_0 is the resonant frequency. Then we calculate the eigenfrequency spectra as a function of t_1/t_2 for a triangular-shaped structure consists of the Kagome lattice with side length $L = 19.5a$. Note that t_1/t_2 can only achieve positive value in our acoustic resonators systems just by tuning the radius of the connecting waveguides, so we only consider the cases where $t_1/t_2 > 0$. As shown in Fig. 4.2a, for the triangular-shaped structure, three degenerate corner states exist (Fig. 4.2b) at f_0 within the band gap when $0 < t_1/t_2 < 1/2$. The results is calculated based on the tight-binding model. For the resonant frequency f_0 , the band gap will then close and reopen till t_1/t_2 reaches 2 and there does not exist state within the band gap when $t_1/t_2 > 2$. The bulk topology for the lattice can be characterized by the polarization which can be computed using equation (1.6) and the polarization (P_x, P_y) is identical to the Wannier centre. Previous theoretical work showed that (P_x, P_y) is only dependent on t_1/t_2 and it will not change until the band gap close and reopen with the change of t_1/t_2 . Mirror symmetries restrict the Wannier centre to two positions within each unit cell, which correspond to the two topologically distinct phases of the bulk. We refer to these as topologically ‘trivial’ and ‘nontrivial’ phases. We first set $t_1/t_2 = 10$, the calculated polarizations $(P_x, P_y) = (0, 0)$, which correspond to the situation where the Wannier centres are located at the centre of upward triangles. Then t_1/t_2 is set to be 0.1 and the calculated polarizations $(P_x, P_y) = (-1/2, -1/2\sqrt{3})$, which correspond to the situation where the Wannier centres are located at the centre of downward

triangles. In conclusion, for $t_1/t_2 > 2$, the system is topologically trivial and the Wannier centre lies at the centre of upward triangles; for $0 < t_1/t_2 < 1/2$, the system is topologically non-trivial, and the Wannier centre lies at the centre of the downward triangles.

Figure 4.2: Triangular-shaped acoustic structure. **a.** Spectrum for a triangular-shaped lattice with $L=19.5a$. **b.** Local density of states for lattice in **a** at resonant frequency when $t_1/t_2 = 0.3$. **c.** Lattice schematic in which red stars indicate the Wannier centre positions in the topological non-trivial phase $t_1/t_2 < 1/2$. Red and green dashed lines indicate the edges for finite triangular and parallelogram-shaped samples. **d.** Numerically computed eigenfrequencies for a triangular sample cut along the red dashed lines in **c**.

To realize a second-order TI, we varied the radius of the connecting waveguides rc_1 (rc_2) to tune the coupling strength t_1 (t_2). On decreasing rc_1 to 2.08 mm and increasing rc_2 to 8.32 mm, we achieved $t_1/t_2 = 0.1$ (estimated by fitting simulation results to the tight-binding model). In this phase, the Wannier centres are located at the centres of the downward-pointing triangles, marked by red stars in Fig. 4.2c. When a large triangular-shaped section is cut from the lattice, along the three red dashed lines depicted in Fig. 4.2c, the boundary runs through the downward-pointing triangles, and hence induces a separation of the charge associated with the Wannier centre. We therefore expect the corners of the large triangular section to host corner states. By the same token, the charges associated with the Wannier centres along the edges also experience separation, which gives rise to edge states. We performed full-wave simulations on our acoustic structure in this shape using Pressure Acoustics module in finite element solver COMSOL Multiphysics. The numerically calculated eigenfrequencies are shown in Fig. 4.2d. It is shown that bulk states exist at around 3,950 Hz and 4,400 Hz; edge states could be found at around 4,080 Hz and three degenerate corner states exist at 4,197.3 Hz, indicating that the system is in the HOTI phase.

4.3 Observation of corner states in the 2D acoustic second-order TI

To experimentally observe the topological corner states predicted above, we built up an experimental setup shown in Fig. 4.3a. All experimental samples consist of seven parts: three pieces of aluminium plates with holes which build up the main resonator structures and four pieces of organic glass on the bottom and top to form the closed acoustic resonators. There is a small hole (radius=5mm) on the top (bottom) organic

glass to insert the probe (source). The acoustic measurement was conducted by exciting a resonator through a small hole in the bottom organic glass sheet, and then measuring the acoustic pressure of another resonator through a small hole in the upper organic glass sheet. In the measurements, sound is launched through a narrow tube ($r \approx 2$ mm) and detected by a microphone (B&K type4939). The measured signal is recorded by an analyzer (B&K type3560 C).

Figure 4.3: Experimental setup. a. All the samples consist of seven parts: three pieces of aluminum plates which build up the main resonator structures and four pieces of organic glass on the bottom and top to form the closed acoustic resonators **b.** Photograph of the fabricated triangle-shaped structure.

At first, we measured the bulk transmission by exciting resonator marked as ‘1’ and measured the response at the resonator marked as ‘2’ in Fig. 4.3b. The results are shown as the black curve in Fig. 4.4a. Two peaks are observed at 3,950 Hz and 4,400 Hz, which correspond to the two bulk bands separated by a bandgap observed in the simulations of Fig. 4.2d. The 4,400 Hz peak is higher because of a higher density of

states. We then measured the edge transmission spectrum by exciting and measuring the resonators marked as ‘3’ and ‘4’ in Fig. 4.3b. The measured transmission, indicated by the blue curve in Fig. 4.4a, shows a peak at around 4,080 Hz, which corresponds to the edge states (simulation in Fig. 4.2d). There is another peak at around 4,400 Hz, coincident with the higher bulk band, but no peak that corresponds to the lower bulk band was observed. This seems to be because the bulk states in the upper band have a significant spatial overlap with the lattice edges, whereas those in the lower band have a negligible spatial overlap. The interference was confirmed by following measured acoustic pressure distributions (Fig. 4.4c, e, f), which were obtained by exciting each resonator and measuring the acoustic pressure of the same resonator. It can be seen that the states are well localized at the edges (Fig. 4.4d) when the operating frequency is at around 4,065 Hz; the spatial distribution for the upper band (Fig. 4.4e) overlap with the edges, while the edges are not excited at the lower band modes (Fig. 4.4f). The results agree well with the simulated band structure for the lattice and the previous interference,

Next, we measured the response of the lower left corner resonator by exciting and measuring from the same resonator to obtain the transmission spectrum for the corner state. As shown by the red curve in Fig. 4.4a, the resulting spectrum shows a strong peak at around 4,200 Hz, consistent with the frequency of the corner states predicted in the simulation of Fig. 4.2b. To demonstrate the robustness of the corner states, we introduced bulk disorder into the system. Small aluminium rods, with radius $r_p = 6mm$ and height h_p uniformly distributed between [0mm,14mm], are placed into all resonators (see Fig. 4.4b) except three corners of the sample. The measured spectrum for the same lower-left corner resonator (green curve in Fig. 4.4a) exhibits a similar resonance peak at around 4,200 Hz, which verifies the robustness of the corner states against bulk disorder. To further demonstrate the corner states, we plotted the spatial

distribution for the acoustic pressure at around 4,200 Hz, as shown in Fig. 4.4c. We could see that the measured acoustic pressures at the three corners are much higher than at other points of the lattice.

Figure 4.4: Observation of topological corner states in the triangle-shaped finite acoustic structure. **a.** Measured transmission spectra. Top: measured bulk and edge transmission spectra. Bottom: measured corner spectra for lattice with and without disorder. **b.** Illustration of the disorder introduced in experiments. **c–f.** Measured acoustic pressure distributions at typical frequencies for the corner **c**, edge **d** and bulk (**e** and **f**) states. a.u., arbitrary units.

For the triangular sample discussed above, only the acute-angled corners

located at the Wannier centres are included. To investigate the topological nature of other kinds of corners, we constructed an additional parallelogram-shaped sample that has three different types of corners, denoted by A , $B_{1,2}$ and C in Fig. 4.5a. The structure can be regarded as being cut from the infinite lattice through the green dashed lines in Fig. 4.2c. As we can see, the left and edges do not pass through Wannier centres, so we expect no corner states at C , $B_{1,2}$, and no edge states on the left and top edges. The corner A and its two adjacent edges (right and bottom edges) are similar to that of the previous triangular sample; the charges associated with the Wannier centres experience separation and hence corner state and edge states are expected at A and the two adjacent edges. The numerically calculated eigenstates are plotted in Fig. 4.5b. There is a single non-degenerate corner state localized at corner A , confirming the reasoning above.

To verify the theoretical and numerical predictions, we implemented the same protocol described in the previous paragraph. The fabricated parallelogram-shaped acoustic structure is shown in Fig. 4.5a. Same to previous triangular-shaped sample, the sample here was fabricated in aluminium plates using mechanical machining. In experiment, the structure was sandwiched by two glasses plates to form sound hard boundaries. At first, we measured the acoustic spectra for the four corners by exciting the resonator and measuring the response at the same resonator. As shown in Fig. 4.5d, corners $B_{1,2}$ exhibit a peak around 4,080 Hz because of the edge states, and another peak around 4,450 Hz, which resulted from the higher-frequency bulk states. The corner C only has two peaks around 4,000 Hz and 4,450 Hz, for the two bulk bands. Over the whole frequency range of interest, only corner A possesses a peak around 4,200 Hz, which corresponds to the corner state predicted in Fig. 4.5b and observed in Fig. 4.5c. Then we show the spatial distributions for the lattice at around 4,200 Hz (top) and 4,080

Hz (bottom) in Fig. 4.5c. Well localized corner state at A and edge states along its two adjacent edges are observed. For the edge states, the acoustic pressure at corner B_2 is much higher than that of B_1 , which agrees well with the transmission spectra in which the peak of B_2 is higher.

Figure 4.5: Observation of topological corner states in a parallelogram-shaped acoustic structure. **a.** Photograph of the fabricated parallelogram-shaped structure. **b.** Numerically computed eigenfrequencies of the structure. There is a single non-degenerate corner state, localized at corner A. **c.** Measured acoustic pressure distributions at 4,170 Hz (top) and 4,066 Hz (bottom). **d.** Measured spectra for the four corners A, B₁, B₂ and C.

The above results demonstrate the acoustic analogue of a second-order

topological insulator on a Kagome lattice and show the shape-dependence of the corner state; With respect to alternative approaches to demonstrate higher-order topological insulators in other systems, we rely on the use of Wannier centres as a topological invariant to predict and characterize them. Our structure is simple to realize and can serve as a basis for further studies. For example, the acoustic structure can be extended to three dimensions to build higher-order topological insulators with corner or hinge states.

4.4 An extension of the corner state to 3D: realization of an acoustic third-order TI

Inspired by the realization of second-order Wannier-type acoustic TI, here we extend the structure to 3D; we realize a third-order TI in the 3D acoustic metamaterial, observing topological states at the corners of a rhombohedron-like sample. A third-order d -dimensional HOTI is a system which possesses $(d-n)$ -dimensional boundary states. For example, a 3D HOTI possesses 0D corner states. The acoustic metamaterial considered here is based on an anisotropic diamond lattice, with cubic unit cell shown in Fig. 4.6a. The lattice constant is $a/\sqrt{2}$ and the three primitive lattice vectors are $\mathbf{a}_1 = (a/2, a/2, 0)$, $\mathbf{a}_2 = (0, a/2, a/2)$, $\mathbf{a}_3 = (a/2, 0, a/2)$, where a is the side length of the cubic cell. The two sublattice atoms are located at $(0,0,0)$ and $(a/4, a/4, a/4)$. There are two sets of NN couplings. The couplings along $[111]$ [plotted in red in Fig. 4.6a] have strength t_2 , and those along other directions (plotted in blue) have strength t_1 . The theoretical tight-binding analysis of this lattice has been performed by Ezawa (96); he predicted that when $|t_1/t_2| < 1/3$, the lattice is a higher-order TI of the Wannier type. For $|t_1/t_2| < 1/3$, the Wannier centres are fixed at

$(1/2, 1/2, 1/2)$, corresponding to the centers of the red bonds in Fig. 4.6a. According to the theoretical predictions, when cutting through these bonds to form a finite sample, fractional charges reside at the boundaries, similar to the SSH chain and 2D generalizations studied above. In the present 3D case, the same principle predicts the existence of topological states at certain corners.

Figure 4.6: Structure of acoustic anisotropic diamond lattice. **a.** Schematic of the cubic cell of an anisotropic diamond lattice. Red (blue) bonds indicate couplings with strength $t_{2(1)}$. **b.** The unit cell of the acoustic anisotropic diamond lattice.

Then we implemented an acoustic metamaterial realization of the above model using coupled acoustic resonators, as shown in Fig. 4.6b. The two identical thick cylindrical resonators correspond to the two sublattice atoms in Fig. 4.6a, connected and coupled by other thinner cylindrical waveguides. The entire structure is filled with air, and the walls are regarded as hard boundaries. The three lattice vectors are the same as in the tight-binding model of Fig. 4.6a, with $a = 175 \text{ mm}$. The height (H) and

radius (r) of each cylindrical resonator are 60 and 20 mm, respectively. The two sets of coupling strength are realized by tuning the radius of the connecting waveguides. The larger connecting waveguides, with radius rc_1 , correspond to t_2 bonds in Fig. 4.6a; the others, with radius rc_2 , correspond to t_1 bonds. All connecting waveguides are located at $h = 8.125$ mm, measured from either the top or bottom of each resonator, as indicated in Fig. 4.6b. Here we set $rc_1 = 2$ mm and $rc_2 = 7.8$ mm, where $t_1/t_2 \approx 0.076$, to achieve the HOTI phase.

Figure 4. 7: Design of a rhombohedron-like sample made of acoustic resonators. a. Schematic of the rhombohedron-like structure. **b.** $(\bar{1}\bar{1}1)$ surface atoms of the structure shown in **a**. **c.** Simulated eigenfrequencies of the lattice. Black and red dots represent bulk and corner states, respectively. **d and e.** Simulated eigenmode profiles of the two corner states. **f.** Photograph of the fabricated rhombohedron-like sample containing 52 resonators.

A third-order TI is characterized by the existence of topological corner states at certain corners of a finite 3D sample. M. Ezawa theoretically predicted the existence of two corner states in a rhombohedron made of diamond lattice introduced above. Then we consider the same rhombohedron like structure [Fig. 4.7a] containing three rhombus-shaped surfaces [see Fig. 4.7b for the $(\bar{1}\bar{1}1)$ surface]. Corner states should occur at the two corner resonators, labelled “A” and “B” in Fig. 4.7a, because all atoms are connected to red bonds (where the Wannier centres lie) except those at A and B. To check this prediction, we performed acoustic simulations on a structure containing 52 resonators, shown in Figs. 4.7d–e. The results, shown in Fig. 4.7c, revealed two in-gap modes at around 2,891 Hz, between the upper bulk band around 3000 Hz and the lower bulk band around 2800 Hz. The eigenmode patterns for the two in-gap modes were plotted in Figs. 4.7d–e. It can be seen that the acoustic pressure is highly concentrated on the two corners, verifying the existence of the third-order topological corner states.

To experimentally observe this phenomenon, we fabricated a sample [see Fig. 4.7f] through stereo-lithography 3D printing, with the same parameters as in the preceding simulations. The samples are made of photosensitive resin, fabricated using a stereo lithography apparatus. The thickness of the resonator walls is about 5mm. There are two small holes ($r = 1\text{ mm}$) at the two sides of each resonator, allowing for excitation and detection. When not in use, the holes are closed with stoppers. Other parts shown in Fig. 4.7f are frame structures to support the main part. In our experiment, we first measured the bulk transmission by exciting the lattice at the resonator labelled “C” and detecting the response at the resonator labelled “D” in Fig. 4.7a. As shown in Fig. 4.8a, the spectrum exhibits two clear peaks corresponding to the upper and lower

bulk bands. Next, we measured the local acoustic response at the two corner resonators, A and B. For both resonators, we observed a peak at around 2900 Hz, corresponding to the corner states. Finally, we repeated the acoustic response measurement for all resonators to show the acoustic pressure distribution for the two modes within the band gap. The results of these measurements, conducted at the peak frequency of the corner resonances (2900 Hz), are plotted in Figs. 4.8c–d from different view angles. Fig. 4.8c is the result viewed from front and Fig. 4.8d is viewed from back. The illustration of the structure is simplified for clarity. The balls correspond to the cylindrical resonators and the grey bonds indicate the coupling waveguides. The colour of each ball represents the measured acoustic pressure at that site, as measured at the top of the cylindrical resonator. From the results we could find that the acoustic pressure is indeed highly concentrated at the two relevant corners *A* and *B*, while the response at other sites are negligible, revealing that the lattice is in the third-order TI phase.

Figure 4.8: Experimental observation of corner states in a rhombohedron-like acoustic structure. **a.** Measured bulk transmission spectrum. **b.** Measured spectra at the two corner sites. **c, d.** Measured pressure distribution at 2900 Hz, viewed from the front (c) and back (d).

4.5 Conclusion

In conclusion, we demonstrate the acoustic analogue of HOTIs. Second-order TI on Kagome lattice made of acoustic resonators are realized and 0D corner states are observed. With respect to alternative approaches to demonstrate higher-order topological insulators in other systems, we rely on the use of Wannier centres as a topological invariant to predict and characterize them. Then we extend the acoustic structure to 3D and implement a third-order TI on an acoustic anisotropic diamond lattice. Topological corner states are observed in a rhombohedron-like sample. Unlike electronic systems, the absence of a Fermi level in acoustic structures makes the entire spectrum easily and make the sample fabrications simple. Our study may stimulate more experimental studies into the implementation of higher order TIs and high-order topological semimetals in higher dimensions. Besides, the highly localized corner states may have potential use in applications such as acoustic manipulation and sensing.

Chapter 5

Summary

In this thesis, we discussed several novel acoustic properties related to topological phases of matter in acoustic resonator systems. They answered several interesting problems in the field remained unsolved when I started my PhD study: the QVHE which relies on the inversion symmetry breaking, has not been well studied in acoustics; Landau quantization, which is a key concept in the mechanism of QHE and the subsequent introduction of topology into quantum physics, is absent in acoustic systems; A new type of topological phase called a higher-order topological phase has not been demonstrated in acoustics.

At first, an acoustic honeycomb lattice was adopted to simulate a monolayer graphene and the inversion symmetry was broken to obtain the topological valley Hall phases. The quantized non-zero difference in Chern number across the zigzag interface ensures the existence of gapless topological valley edge states at this kind of interfaces; the valley contrasting maintains the valley-dependent nature of the edge states. However, due to that the valley Chern number vanishes at the armchair interface, the topological valley edge states are not supported at the interfaces. Through this work, it was confirmed that our model works well, indicating that acoustic resonator system acted as a good platform to explore topological properties for sound waves.

So when I moved on to generate an effective uniform pseudo-magnetic field and realize Landau quantization as that in graphene, I adopted the same acoustic honeycomb lattice again. A triaxial strain gauge field was applied to the acoustic resonator system by displacing the resonators with specifically displacements. Transmission gaps emerged near the Dirac frequency due to the formation of acoustic Landau levels. We

successfully observed acoustic pseudo Landau levels, although the airborne acoustic waves intrinsically carry no charge or spin and do not respond to magnetic fields. In this work, we did experiments and showed that sample fabrication was not challenging at all. The transmission spectrum and pressure fields measurements could be easily achieved. However, it was also noticed that there were non-negligible leaky losses in the system, which may affect the measurements spatial distribution.

Then in the next project, where the newly emerging topological phases – HOTIs was realized in acoustics, some improvements have been done. The connecting waveguides were increased to two between each atom to increase the coupling waveguides; we had the acoustic resonators enlarged (the lattice constant was 4 times longer and the heights/radius of the resonators were around 3 times longer) and then the pressure fields were obtained by exciting and measuring at a same resonator rather than exciting at a fixed site and measuring the response at all other sites. The improvements lower the impact of the leaky loss effectively. In this work, the wannier-type HOTIs phases, which are characterized by quantized wannier centres, have been demonstrated in an acoustic kagome lattice. It was achieved by tuning the radius of connected waveguides, which corresponds to coupling strength. A 2D second-order triangular-shaped TI based on the lattice was constructed and 0D corner states was observed at three corners, charactering the second-order topological phases. Then it was extended to 3D based on diamond lattice and construct a rhombohedron-like structure, which possess 0D corner states at two corners of the structure, indicating the third-order topological phases.

These three work in the thesis indicates that: the megascopic scale acoustic resonator system can serve as a good platform to investigate nontrivial topological physics, with all the parameters being easily controlled or measured; also, the

introduction of the novel topological properties into acoustic systems provides a robust way to control sound waves. The topological robust and defect insensitive properties of the states may find use in applications such as on-chip filters in mobile phones, sound lasing, and bio-chemical sensing.

In recent years, the field of topological acoustics develops rapidly. A lot of other interesting properties have been found and there still remains much unknown to be explored. There are also some potential projects that can be implemented in our acoustic resonator systems, such as 4D QHE – QHE where particles move in four dimensions. The concept of 4D QHE we discussed here is based on synthetic dimensions (56, 110-113) and it can be implemented in 2D lattice. Propagation of particles along the samples over higher-dimensional momenta space in two fictitious coordinates perpendicular to the physical dimensions are considered, thus realizing a 2D topological pump. Incorporating two originally physical dimensions, the two pumping parameters extend the original 2D system to a synthetic four dimensions, which corresponds to a 4D quantum Hall system. For a finite lattice, the 4D topological bulk response is carried by localized edges modes that cross the sample as a function of the fictitious momentum. The 4D QHE have been experimentally demonstrated in cold atom (114) and photonic systems (5) through tuning coupling strength.

For a 2D square lattice, 4D QHE could be realized by introducing 2D pump with pumping parameters ϕ_x , ϕ_y , which correspond to momenta in the two fictitious perpendicular directions. The 2D pump is introduced to the system by modulating the on-site energy. The tight-binding Hamiltonian for the lattice should be given as,

$$\mathcal{H}(\phi_x, \phi_y) = \sum_x t_x a_{x,y}^\dagger a_{x+1,y} + t_y a_{x,y}^\dagger a_{x,y+1} + h.c. + [\lambda_x \cos(2\pi b_x x + \phi_x) + \lambda_y \cos(2\pi b_y y + \phi_y)] a_{x,y}^\dagger a_{x,y}, \quad (5.1)$$

where $a_{x,y}^\dagger$ ($a_{x,y}$) is the creation (annihilation) operator of a particle at site (x, y), t_x (t_y) is the hopping amplitude in the x (y) direction, λ_x (λ_y) is the amplitude of on-site potential modulated along x (y) direction with the pumping frequency b_x (b_y). In our acoustic resonator systems, the proposal could be easily implemented by tuning the heights of the acoustic resonators as,

$$H(\phi_x, \phi_y) = dH_x * \cos(2\pi b_x x + \phi_x) + dH_y * \cos(2\pi b_y y + \phi_y), \quad (5.2)$$

here dH_x (dH_y) represents the pumping amplitude.

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