Cold atoms laboratory in a hollow core fiber

Leong, Wui Seng

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SINGAPORE

COLD ATOMS LABORATORY IN A HOLLOW CORE FIBER

LEONG WUI SENG

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

2020

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SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2020

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Statement of Originality

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28 December 2020

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28 December 2020

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Prof. Lan Shau-Yu

Authorship Attribution Statement

This thesis contains material from 0 paper published in the following peer-reviewed journal where I was the first and/or corresponding author and 2 papers published in the following peer-reviewed journal where I was the second author. However, there are 2 manuscripts submitted in the following peer-reviewed journal where I was the first and/or corresponding author.

[Chapter 7](#) was published as M. Xin, W.S. Leong, Z. Chen and S.-Y. Lan. An atom interferometer inside a hollow-core photonic crystal fiber. *Science Advances* **4**, e1701723 (2018). DOI: [10.1126/sciadv.1701723](#). The contributions of the co-authors are as follows:

- Prof. Lan Shau-Yu provided the initial project direction and edited the manuscript drafts.
- Dr. Xin Mingjie, Dr. Chen Zilong and I prepared and built the experimental apparatus and setup.
- Dr. Xin Mingjie, Dr. Chen Zilong and I prepared and performed all the laboratory work at the School of Physical and Mathematical Sciences.
- Dr. Xin Mingjie, Dr. Chen Zilong and I collected and analyzed the experimental data.

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- Dr. Xin Mingjie, Dr. Chen Zilong and I collected and analyzed the experimental data.

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- Prof. Lan Shau-Yu provided the initial project direction and edited the manuscript drafts.
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- Dr. Xin Mingjie, Dr. Chen Zilong and I prepared and performed all the laboratory work at the School of Physical and Mathematical Sciences.
- Dr. Xin Mingjie, Dr. Chen Zilong and I collected and analyzed the experimental data.
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- Dr. Xin Mingjie, Dr. Chen Zilong and I prepared and performed all the laboratory work at the School of Physical and Mathematical Sciences.
- Dr. Xin Mingjie, Dr. Chen Zilong and I collected and analyzed the experimental data.
- I wrote the simulation algorithm for DD in light storage and Dr. Chen Zilong put it into text as shown in [§9.6](#).

28 December 2020

Date

Leong Wui Seng

Abstract

Coherent manipulation and interaction between atoms and light are the core ingredients in modern quantum science and technology. Various applications such as quantum computation, quantum memories and quantum sensors which are promising in future technologies, are currently under exploration. However, due to the diffraction nature of light, miniaturizing quantum systems involving atom-light interactions has been highly limited. To avoid diffraction, I load cold atoms into a hollow-core fiber (HCF) and demonstrate four major experiments inside the fiber: inertia-sensitive Mach-Zehnder atom interferometer, long-lived quantum spin coherence, light storage and creation of an array of Schrödinger cat states.

In this thesis, I show the theoretical framework of each experiment and demonstrate experimentally a cold atoms quantum laboratory realized a HCF. Cold ^{85}Rb atoms of temperature $\sim 6\mu\text{K}$ are loaded into a Kagome type photonic crystal hollow-core fiber (HCF) through an optical trapping beam coupled into the HCF. I first demonstrate an inertia-sensitive Mach-Zehnder atom interferometer using two counter-propagating Raman beams. However, the short quantum spin coherence time of $\sim 100\mu\text{s}$ due to the differential AC stark shift from the trapping beam is a major obstacle to reaching high sensitivity of the optically trapped atom interferometer inside the HCF. By introducing a vector light shift from the dipole trap to cancel the differential scalar light shift, the quantum spin coherence time is increased to hundreds of milliseconds.

The long quantum spin coherence time opens up possibilities for various quantum experiments in HCF. Here, I demonstrate light storage using atoms as the storage medium in the HCF. A maximum storage efficiency of 3% and storage time of over 20 ms is shown in this thesis. With the implementation of dynamical decoupling, the storage time is improved to $\sim 100\text{ms}$. Moreover, I create a 1D array of Schrödinger cats states in facilitated by an optical lattice in the HCF and study its potential for applications in quantum sensing and quantum simulation in HCF. Both odd and even cat states are validated and their coherence properties are studied.

This thesis realized the state-of-the-art quantum experiments that are miniaturized into a HCF. The results signify a promise in future quantum sciences and technologies with compact fiber-based devices.

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Mum, I love you.

Dedicated to my parents

Leong Chee Ming and Wong Ai Yet.

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Optical Drawing Convention

Below are the convention of optical drawing used in this thesis. The value of x in AOM and EOM indicates the frequency shift in MHz caused by them.

Mirror
Gold Mirror
Polarizing beam splitter (PBS)
Non-polarizing beam splitter (NPBS)
Half waveplate
Quarter waveplate
Acousto-optic modulator (AOM)
Electro-optic modulator (EOM)
Etalon
Photodetector
Optical isolator

Chapter 1

General Introduction

1.1 Introduction

The rapid development of cooling and trapping atoms in 80s and 90s has opened up a new field in quantum science and multiple interesting applications with cold atoms, accompanied by 1997 Nobel prize in Physics "for development of methods to cool and trap atoms with laser light" [1] awarded to Steven Chu [2–5], Claude Cohen-Tannoudji [6–8] and William D. Phillips [9–11]. For instance, optical lattice clocks have been demonstrated with accuracy at the level of 10^{-18} [12–15]. Development of cold atom interferometry created the field of high precision inertia sensing for measuring the Earth's gravitational acceleration [16–18], gravitational gradient [19, 20], gravitational curvature [21], microgravity [22], the Earth's Coriolis force [23] and rotation [24, 25]. On the fundamental Physics side, the value of fundamental constants such as universal Gravitational constant [26, 27] and fine structure constant [28, 29] were determined by means of atom interferometry. Among others, the theory of relativity [30, 31] and the equivalence principle [32, 33] were tested while the permanent electric dipole moments of atoms and molecules have been evaluated [34, 35]. Furthermore, there was a pioneering approach to link time to a particle's mass that could potentially alter the definition of mass [36]. Needless to say, cold atoms setup is at the heart of quantum information [37], quantum computation [38] and quantum memory [39, 40].

Despite the great success of cold atoms quantum technologies, all free-space experiments encounter various challenges. Due to the diffraction nature of light, the scalability is limited by the length scale of diffraction. In atom interferometers, in order to increase the integration length for better sensitivity, cumbersome and power-consuming designs are compulsory, leading small-scale sensors to be impossible. For example, a 10 m long interferometer tower with an interferometer beam waist of 2 cm requires few tens of W of optical power at 30 GHz detuning from resonance [41]. A large beam waist aims to minimize Gouy phase and for uniform atom light interaction, limiting device miniaturization. Moreover, wavefront distortion from imperfect optical surfaces has always been an issue which creates phase noise into the measurement.

Since the discovery of the law of total internal reflection in the nineteenth century, index

guiding optical waveguide started to become popular [42]. In 1960s, Kuen Charles Kao suggested that optical fiber could be low loss and suitable for long-distance communication [43]. The era of optical communication then emerged after the demonstration of low attenuation of 0.2 dB/km in 1979 [44], which leads to 2009 Nobel prize in physics "for groundbreaking achievements concerning the transmission of light in fibers for optical communication" [45]. Unlike index guiding optical fibers, the hollow core fiber (HCF) contains air as its core and does not depend on total internal reflection. Its invention began from the improvement of military radar system in World War II using metal hollow core waveguide to transmit microwave for telecommunication [46]. Low microwave attenuation below 1 dB/km [47] inspired researchers to extend the wavelength to optical regime with around 1 dB/km loss [48]. At the same time, dielectric HCF was studied [49] but due to the difficulty of HCF fabrication with the technology of that age, the research progress is curtailed until the appearance of hollow-core photonic bandgap (HCPBG) fiber near the twentieth century [50] proposed by Eli Yablonovitch [51] and Sajeev John [52]. The term "bandgap" is inherited from "electronic bandgap" in solid-state physics, which is the range of energy of a solid with no electronic state. Since then, multiple groundworks have been done in improving light transmission [53–56]. In 2000s, a new type of HCF called the "Kagome" fiber was invented [57] with the advantage of large optical frequency coverage from the infrared to the visible spectrum. Its name comes from the periodical Kagome lattice structure in the cladding. Unlike HCPBG fiber, Kagome fibers experience more attenuation from the absence of photonic bandgap, which could be resolved by enlarging the dimension of core [58].

Since the invention of HCF, the study of the behavior of atoms in HCF became an interest to physicists. Different types of atomic vapour such as Rubidium [59–64] and Cesium [65], are loaded into HCF, and the loading mechanism are studied. Among others, hot Rydberg atoms and molecules loading experiments were also carried out [57, 66]. One of the main applications was to build a gain medium in HCF and create a fiber-based laser [67, 68] and frequency comb [69]. For instance, a sub-picosecond pulse laser at infrared regime with an average power over 100 W has been generated from Argon filled Kagome fiber [70]. Other applications include fiber-based frequency standard [71–73], saturation absorption spectroscopy [74–76], quantum memory [77] and quantum computation [78] contributing towards modern technologies. Apart from that, the study of electromagnetically induced transparency (EIT) of various species in vapour has also been examined [79–81]. On the other hand, groundworks of cold atoms and cold Rydberg atoms loading into HCF have been done by several pioneers with distinct directions [82–91]. Peters' group focus on EIT related researches [92] such as light storage [93] and quantum memory [94]. Lukin's and Bajcsy's groups are towards slow light and optical switch [95–100], whereas superradiance is carried out by Katori's group [85]. Other than cold atom in HCF, there are research contributions to the atoms around nanofibers [101–105] and optical cavity [106, 107].

Despite these groundbreaking efforts, no research has been done on the quantum control of atoms using optical waveguide modes in HCF. Internal and external coherence related experiments with atoms inside HCF have not been carried out. With optical fiber as the beam mode filter, the homogeneity of the wavefront can be maintained throughout the

Figure 1.1: (a) When cold atoms are in a hollow core fiber, the dipole trapping beam coupled into the fiber could provide a platform for long guiding distance along the fiber. (b) The optical surface of optics in general can cause the wavefront distortion in free space.

fiber length. Since the beam waist remains unchanged in the fiber, the Gouy phase as one of the error sources is eliminated. Therefore, putting atom into HCF as the new cold atom laboratory could become promising as miniaturized sensors and devices in future. In this thesis, I demonstrate a state-of-the-art system in performing various quantum control experiments in HCF and prove its potential in future quantum technologies.

1.2 Why Inside Hollow Core Fiber?

There are several advantages of cold atoms in hollow core fiber over free space cold atoms. One of them is that hollow core fiber provides an environment that has a long guiding distance with a dipole trapping beam coupling into the fiber. The trap depth is stronger when the beam waist of dipole beam is smaller (Detail in §2.4). In free space, due to the diffraction limit, the beam waist of a Gaussian beam will vary in distance (shown in equation (2.20)), resulting a varying trap depth in space. A fiber can provide an environment such that the beam waist is constant along the fiber, resulting a homogeneous trap depth along the fiber. Therefore, a fiber could provide a platform for long guiding distance for atoms, basically limited by the length of the fiber.

Moreover, due to the same diffraction nature of light in free space, the space of strong light-matter interaction of cold atoms in free space is highly limited by the Rayleigh length. For example, in order to enhance strong light-matter interaction, the Gaussian beam needs to be tightly focused to small space at a beam waist of several micrometers. At this scale, the space of interaction is limited by its Rayleigh length at the scale of hundred micrometers. On the other hand, putting them into hollow core fiber with the same beam waist allow it to have large space of strong light-matter interaction, only limited by the length of the fiber.

Other than that, the Gaussian beam in free space would provide the Gouy phase which acts as one of the major systematic error to quantum measurement. For example, Guellati-Khélifa has perform the measurement of fine structure constant [108] in 2020 with a great precision at the level of 8.1×10^{-11} . In her measurement, the major source of systematic error is the Gouy phase at the level of 1.1×10^{-9} . On the other hand, Gouy phase does not exist in fiber as the beam waist will remain unchanged throughout the fiber.

Furthermore, the optical surface of typical optics such as mirrors and lens are not in ideal case, meaning that there will be some surface roughness that will cause the wavefront distortion in free space. This wavefront distortion can be a systematic error in a system and this cannot be eliminated in free space. As the fiber acts as a mode filter, any wavefront distortion will be eliminated in the fiber.

Apart from that, changing from free space cold atom devices to fiber-based devices can help improving compactness of quantum system. For example, a typical mobile quantum sensor such as mobile quantum gravimeter takes the size at the scale of a meter with large set of vacuum system. If it were to be setup into a hollow core fiber, it is possible to make a miniaturized and compact mobile quantum device for future technology.

In this thesis, I am going to demonstrate the possibilities of cold atom laboratory inside a hollow core fiber. There will be 4 major experimental projects to show its feasibility. The first project is the demonstration of atom interferometer, which is typically used as the quantum sensor. In this this thesis, I am going to show that the fiber-based cold atom can be used to measure the gravity. The second project is regarding the long spin coherence time measurements, which fully utilizes the advantage of long guiding distance. Here, I have demonstrated the transportation of long lived quantum spin coherence in a photonic crystal fiber at macroscopic distance, which proves its robustness in the fiber. Moreover, long spin coherence time allows various coherence dependent projects to be carried out. For example, a fiber-based atomic clock device would be an important design in the future technology. The last two projects, light storage and creation of a large array of Schrödinger cat states, are experiments to demonstrate the quantum manipulation of internal and external state, respectively, in the fiber. As a result, these cold atom experiments in hollow fiber provide promising result into better quantum technologies.

1.3 Organization of the thesis

This thesis comprises of three major parts. The first part touches on the theories of laser cooling techniques and atomic physics required in the thesis. [Chapter 2](#) explains the basic of two-level system such as the Rabi flopping, spin echo, AC Stark shift, dipole trapping, light shift cancellation, etc. [Chapter 3](#) illustrates the fundamental theory of the experiments.

The second part describes the details of the apparatus for the experiments. [Chapter 4](#) shows the preparation of the laser alignment and electronics in laser locking. It also presents the data acquisition system and the electronics for radio frequency generation. [Chapter 5](#) delineates the hardware in the science chambers. For example, the optical properties of the HCF used are examined in this chapter. The ultra-high vacuum system and magnetic coils are also explained here in detail.

The third part comprises of the experimental results and analysis of the four major experiments, each in a chapter, i.e. demonstration of atom interferometer, long spin coherence time measurements, light storage and creation of a large array of Schrödinger cat states are in [Chapter 7](#), [Chapter 8](#), [Chapter 9](#) and [Chapter 10](#), respectively. Finally, [Chapter 11](#) summarizes the results and provides a conclusion and outlook.

Part I

Theory

Chapter 2

Light-Atom Interaction

This chapter provides a simple theoretical background required for the experimental analysis in part II of the thesis.

2.1 Two-Level System

An atom of two-level system can be described by the eigenstates $|g\rangle := \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ and $|e\rangle := \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ from an unperturbed Hamiltonian

$$\hat{H}_0 = \begin{bmatrix} \hbar\omega_0 & 0 \\ 0 & 0 \end{bmatrix} \quad (2.1)$$

in the absence of light field. Here we set the ground state $|g\rangle$ to be the zero point energy state, and $\hbar\omega_0$ to be the energy difference between the states. In the presence of monochromatic light field with frequency ω , an extra dipolar interaction Hamiltonian $\hat{H}_{\text{int}} = -\mathbf{E} \cdot \mathbf{d}$, or

$$\hat{H}_{\text{int}} = -\mathcal{E}_0 \hat{\epsilon} \cdot \mathbf{d} \cos(\omega t + \phi) \quad (2.2)$$

is added as a perturbation, so that the full Hamiltonian is $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$. Here,

$$\mathbf{d} = \begin{bmatrix} 0 & \langle e | \mathbf{d} | g \rangle \\ \langle g | \mathbf{d} | e \rangle & 0 \end{bmatrix}$$

is the dipole matrix element which determines the strength of atom-light interaction. Its magnitude depends on the type of atom and transition used. $\hat{\epsilon}$ is a unit vector called the *polarization*, originated from the electric field from light. Practically, it is decomposed into the linear combination of π , σ^+ and σ^- polarization, so that the dot product in equation (2.2) is simpler for $\mathbf{d}$ used from the Zeeman levels, which is measured and calculated in [109] (For ^{85}Rb). The rest of the notations are originated from the electromagnetic field

$$\mathbf{E} = \mathcal{E}_0 \hat{\epsilon} \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi) \approx \frac{\mathcal{E}_0}{2} \hat{\epsilon} (e^{-i\omega t} e^{-i\phi} + e^{i\omega t} e^{i\phi}). \quad (2.3)$$

The approximation in equation (2.3) assumes the size of atom wave packet at position $\mathbf{r} = \mathbf{0}$ to be much smaller than the wavelength of the light field λ , such that the electric field experienced is homogeneous. This approximation is called the *dipole approximation*. This is true when the atom temperature T is high enough such that its wave packet size, deduced from uncertainty principle $\Delta x \cdot m \Delta v \approx \frac{\hbar}{2}$, is much smaller than the wavelength of light field, i.e. $\lambda \gg \frac{\hbar}{2mv_{\text{rms}}}$, where v_{rms} is the rms speed of atom, determined from the temperature using the equivalence principle $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_B T$. At a typical wavelength of light field, $\lambda \approx 780 \text{ nm}$, which deduces that the atom temperature should be $T \gg 0.78 \text{ nK}$. This is true as we are normally working at minimum few μK of temperature environment. Evaluating equation (2.2) gives

$$\hat{H}_{\text{int}} = -\frac{\mathcal{E}_0}{2} (e^{-i\omega t} e^{-i\phi} + e^{i\omega t} e^{i\phi}) \begin{bmatrix} 0 & \hat{\epsilon} \cdot \langle e | \mathbf{d} | g \rangle \\ \hat{\epsilon} \cdot \langle g | \mathbf{d} | e \rangle & 0 \end{bmatrix}. \quad (2.4)$$

Since $\langle e | \mathbf{d} | g \rangle \sim e^{i\omega_0 t}$ and $\langle g | \mathbf{d} | e \rangle \sim e^{-i\omega_0 t}$ from the free evolution of $|e\rangle$ under $\hat{H}_0$, $\hat{H}_{\text{int}}$ will have $e^{\pm i(\omega - \omega_0)t}$ and $e^{\pm i(\omega + \omega_0)t}$ term. As long as the laser frequency detuning $\delta = \omega - \omega_0$ is small compared to the sum of frequencies, i.e. $|\delta| \ll \omega + \omega_0$ and we are looking at a time scale slower than the fast oscillation term, we can claim that this term oscillation is too fast which averages to zero and safely neglect it. In practice, the resonance wavelength/frequency of ^{85}Rb is 780 nm or 384 THz, and the furthest laser wavelength used is nearer to resonance than 900 nm or 333 THz, with a detuning of $|\delta| < 51 \text{ THz}$ which is small¹ compared to the frequencies sum of $\omega + \omega_0 \approx 717 \text{ THz}$. This neglect approximation is call the *rotating wave approximation*. Therefore,

$$\hat{H}_{\text{int}} \approx \hbar \begin{bmatrix} 0 & \frac{\Omega}{2} e^{-i\omega t} \\ \frac{\Omega^*}{2} e^{i\omega t} & 0 \end{bmatrix} \quad (2.5)$$

where²

$$\Omega := -\frac{\mathcal{E}_0 e^{-i\phi} \hat{\epsilon} \cdot \langle e | \mathbf{d} | g \rangle}{\hbar} \quad (2.6)$$

is called the *Rabi frequency*. As a result, the full Hamiltonian finally becomes

$$\hat{H} = \hbar \begin{bmatrix} \omega_0 & \frac{\Omega}{2} e^{-i\omega t} \\ \frac{\Omega^*}{2} e^{i\omega t} & 0 \end{bmatrix}. \quad (2.7)$$

Consider atom with state

$$|\psi\rangle = b_g(t) |g\rangle + b_e(t) |e\rangle, \quad (2.8)$$

¹In fact, most of the laser are working at detuning of less than a few GHz.

²Here, we can choose the complex phase of $\hat{\epsilon} \cdot \langle e | \mathbf{d} | g \rangle$ to be zero so that it is a real number to simplify the formulation later.

using time dependent Schrödinger equation $\hat{H} = i\hbar \frac{\partial}{\partial t}$, we can deduce that

$$\dot{b}_e(t) = -i\omega_0 b_e - i\frac{\Omega}{2} e^{-i\omega t} b_g \quad (2.9a)$$

$$\dot{b}_g(t) = -i\frac{\Omega^*}{2} e^{i\omega t} b_e. \quad (2.9b)$$

These equations look complicated because we are looking it from the lab frame. With the substitution

$$c_g(t) = b_g(t) \text{ and } c_e(t) = b_e(t) e^{i\omega t} \quad (2.10)$$

i.e. we use the laser frame to $c_i(t)$, $i = g, e$, then the equations can be simplified to

$$\dot{c}_e(t) = i\delta c_e - i\frac{\Omega}{2} c_g \quad (2.11a)$$

$$\dot{c}_g(t) = -i\frac{\Omega^*}{2} c_e. \quad (2.11b)$$

In fact, the frame changing in equation (2.10) is equivalent to transforming $\hat{H}$ by unitary operator

$$\hat{U} = \begin{bmatrix} e^{i\omega t} & 0 \\ 0 & 1 \end{bmatrix} \quad (2.12)$$

into the new Hamiltonian

$$\hat{H}_{\text{laser frame}} = U H U^\dagger + i\hbar (\partial_t U) U^\dagger = \hbar \begin{bmatrix} -\delta & \frac{\Omega}{2} \\ \frac{\Omega^*}{2} & 0 \end{bmatrix} \quad (2.13)$$

which results equation (2.11) under Schrödinger equation.

2.2 Rabi Flopping and Bloch Sphere Picture

The general solution for equation (2.11) is

$$c_g(t) = e^{i\delta t/2} \left[c_g(0) \cos\left(\frac{1}{2}\Omega_G t\right) - \frac{i}{\Omega_G} [\delta c_g(0) + \Omega^* c_e(0)] \sin\left(\frac{1}{2}\Omega_G t\right) \right] \quad (2.14a)$$

$$c_e(t) = e^{i\delta t/2} \left[c_e(0) \cos\left(\frac{1}{2}\Omega_G t\right) + \frac{i}{\Omega_G} [\delta c_e(0) - \Omega c_g(0)] \sin\left(\frac{1}{2}\Omega_G t\right) \right], \quad (2.14b)$$

where $\Omega_G = \sqrt{\delta^2 + |\Omega|^2}$ is called the *generalized Rabi frequency*. Considering atom initially at $|g\rangle$, i.e. $c_g(0) = 1, c_e(0) = 0$, the population at ground state over time is

$$|c_g|^2 = \frac{|\Omega|^2}{\delta^2 + |\Omega|^2} \cos^2\left(\frac{1}{2}\sqrt{\delta^2 + |\Omega|^2} t\right),$$

which is shown in Figure 2.1a. One can see that as δ increases, the oscillation frequency increases but amplitude decreases. In order to visualize the flopping system, we can use the *Bloch sphere* picture by starting with its density matrix element $\rho_{ij} = c_i c_j^*$, $i, j \in \{e, g\}$.

Figure 2.1: (a) Sketch of $|c_g|^2$ over time at different δ in Rabi flopping. (b) Bloch sphere representation for Rabi flopping. The rotation axis is shown in red arrow and the Bloch vector evolves from cyan to blue arrow.

With the help of equation (2.11) and the fact that $\rho_{gg} + \rho_{ee} = 1$, we can get the equation of motion³ for the *excited population*

$$\begin{aligned}\dot{\rho}_{ee}(t) &= c_e^* \dot{c}_e + c_e \dot{c}_e^* \\ &= i\delta \rho_{ee} - i\frac{\Omega}{2} c_e^* c_g + c.c. \\ &= -i\frac{\Omega}{2} (\rho_{eg} + \rho_{ge}),\end{aligned}$$

the *ground state population*

$$\begin{aligned}\dot{\rho}_{gg}(t) &= -\dot{\rho}_{ee} \\ &= i\frac{\Omega}{2} (\rho_{eg} + \rho_{ge}),\end{aligned}$$

and the *coherence*

$$\begin{aligned}\dot{\rho}_{ge}(t) &= c_g \dot{c}_e^* + c_e^* \dot{c}_g \\ &= c_g \left(-i\delta c_e^* + i\frac{\Omega^*}{2} c_g^* \right) + c_e^* \left(-i\frac{\Omega^*}{2} c_e \right) \\ &= -i\delta \rho_{ge} + i\frac{\Omega^*}{2} \rho_{gg} - i\frac{\Omega^*}{2} \rho_{ee} \\ &= -i\delta \rho_{ge} - i\frac{\Omega^*}{2} (\rho_{ee} - \rho_{gg}) \\ \dot{\rho}_{eg}(t) &= \dot{\rho}_{ge}^* \\ &= i\delta \rho_{eg} + i\frac{\Omega}{2} (\rho_{ee} - \rho_{gg}).\end{aligned}$$

³This is also equivalent to the *Schrödinger-von Neumann equation*.

By defining the *Bloch vector* $\mathbf{b} = \mathbf{b}_x \hat{\mathbf{x}} + \mathbf{b}_y \hat{\mathbf{y}} + \mathbf{b}_z \hat{\mathbf{z}} = \begin{bmatrix} \mathbf{b}_x & \mathbf{b}_y & \mathbf{b}_z \end{bmatrix}^T$, where $\mathbf{b}_x = 2\text{Re}(\rho_{ge}) = \rho_{eg} + \rho_{ge}$, $\mathbf{b}_y = 2\text{Im}(\rho_{ge}) = i(\rho_{eg} - \rho_{ge})$ represent the coherence and $\mathbf{b}_z = \rho_{ee} - \rho_{gg}$ represents the population, one can show that $|\mathbf{b}| = 1$ and its time evolution is always on the surface of a unit sphere called the *Bloch sphere*. In addition, the population of atom is related to the projection of $\mathbf{b}$ to z axis or $\rho_{gg} = \frac{1}{2}(1 - \mathbf{b}_z)$. Furthermore,

$$\frac{\partial}{\partial t} \mathbf{b} = \boldsymbol{\Omega} \times \mathbf{b}$$

which shows that the evolution of $\mathbf{b}$ is just a rotation with the rotation vector $\boldsymbol{\Omega} = |\Omega| \cos \phi \hat{\mathbf{x}} + |\Omega| \sin \phi \hat{\mathbf{y}} + \delta \hat{\mathbf{z}}$ and the “speed” of rotation is $|\boldsymbol{\Omega}| = \sqrt{\delta^2 + |\Omega|^2}$ which is equivalent to equation (2.14). The evolution process is summarized in Figure 2.1b. If the light is resonance and has a pulse time of $t_\pi = \frac{\pi}{|\Omega|}$, $\mathbf{b}$ rotates at half of the period and population flips. We called it the π pulse. Furthermore, a pulse time of $t_{\pi/2} = \frac{\pi}{2|\Omega|}$ is called the $\pi/2$ pulse [110].

2.3 Ramsey Interferometry and Spin Echo

Ramsey interferometry is a common technique in measuring the resonance frequency and coherence of a system. It starts with the ground state $|g\rangle$ and applying two $\pi/2$ pulses (denote as $\boldsymbol{\Omega}_{\pi/2}$) with time separation of T (denote as $\boldsymbol{\delta}_T$) doing nothing, i.e. the operation $\boldsymbol{\Omega}_{\pi/2} \cdot \boldsymbol{\delta}_T \cdot \boldsymbol{\Omega}_{\pi/2}$. The first pulse $\boldsymbol{\Omega}_{\pi/2}$ rotates $\mathbf{b}$ to equator⁴. When the light has a detuning δ , $\boldsymbol{\delta}_T$ has a rotation vector parallel to $\hat{\mathbf{z}}$ that will rotate $\mathbf{b}$ along the equator at an angle of δT . Therefore, the second $\boldsymbol{\Omega}_{\pi/2}$ results $\mathbf{b}$ at different position of the sphere (see Figure 2.2). One can show that the excited population after Ramsey interferometer is

$$\rho_{ee} = \cos^2 \left(\frac{\delta T}{2} \right) = \frac{1}{2} (1 + \cos \delta t). \quad (2.15)$$

By measuring the population ρ_{ee} through scanning T , one can determine the detuning of the light.

However, in practice, the system is the environment where the detuning δ is inhomogeneous. This could be coming from the differential AC Stark shift of the beam, or from inhomogeneous magnetic field, that cause atoms “seeing” different δ at different location. As a result, the Bloch vector will spread out throughout the equator (see Figure 2.4). That will cause equation (2.15) to have amplitude (after integrated over a distribution of δ) to be less than 1/2. This effect is called the *dephasing effect* and it is one of the sources of decoherence. A typical way of measuring the coherence is by defining the contrast C to be the ratio of amplitude to average, or $C \equiv \frac{\max - \min}{\max + \min}$ where $\max = \max(\rho_{ee})$ and $\min = \min(\rho_{ee})$. In ideal case, $C = 1$.

Spin echo is a powerful technique to reverse the effect of dephasing from the inhomogeneous detuning of the system. The method is to provide a π pulse in the midpoint of Ramsey interferometry, so that the operation is $\boldsymbol{\Omega}_{\pi/2} \cdot \boldsymbol{\delta}_{T/2} \cdot \boldsymbol{\Omega}_\pi \cdot \boldsymbol{\delta}_{T/2} \cdot \boldsymbol{\Omega}_{\pi/2}$, with the light being on

⁴Here, we assume $|\Omega| \gg |\delta|$ so the rotation vector has negligible z component.

Figure 2.2: Pictorial demonstration of Ramsey interferometry for resonance case ($\delta = 0$). Red arrow represents the rotation axis under the labelled operation. $\mathbf{b}$ rotates to the equator after the first $\pi/2$ pulse of Ramsey interferometry and continues to rotate to north pole at the second $\pi/2$ pulse without detuning during δ_T operation. Then, the second $\pi/2$ pulse will rotate $\mathbf{b}$ towards north pole. Here, I assume $\Omega \gg \delta$ so that $\Omega_{\pi/2}$ is at the equator.

Figure 2.3: Pictorial demonstration of Ramsey interferometry for non-resonance case ($\delta \neq 0$). Red arrow represents the rotation axis under the labelled operation. $\mathbf{b}$ rotates to the equator after the first $\pi/2$ pulse of Ramsey interferometry. When the light is not on resonance, δ_T rotates $\mathbf{b}$ along the equator. As a result, the second $\pi/2$ pulse will rotate $\mathbf{b}$ away from north pole.

Figure 2.4: Pictorial demonstration of Ramsey interferometry for inhomogeneous detuning δ . Red arrow represents the rotation axis under the labelled operation. $\mathbf{b}$ rotates to the equator after the first $\pi/2$ pulse of Ramsey interferometry. When the light has an inhomogeneous detuning, δ_T rotates $\mathbf{b}$ along the equator at different rate. As a result, the second $\pi/2$ pulse will only rotate the “scattered” $\mathbf{b}$ around the rotation axis.

resonance to the system. The sequences is shown in Figure 2.5. Hence, in the presence of inhomogeneous detuning, $\mathbf{b}$ only “spreads” out along the equator during the first evolution time $T/2$ after the first $\pi/2$ pulse. Then, a π pulse flips $\mathbf{b}$ to the other side of the sphere before the second evolution time $T/2$ closes the spread. Finally, a $\pi/2$ pulse put $\mathbf{b}$ to the south pole, or ground state, without dephasing effect.

Spin echo is popular for many applications [111–113] because it can cancel the dephasing effect regardless how bad is it. However, there are some effects that cannot be reversed by spin echo. For example, when atoms experience inhomogeneous detuning that is changing over time, or in the presence of scattering effect that reduces the coherence, spin echo cannot cancel the effect because it is *irreversible*.

2.4 AC Stark Shift and Dipole Trap

Except coupling between the states in two-level system, the presence of light also provides an energy shift of [114]

$$U = \frac{\hbar |\Omega|^2}{4} \cdot \frac{\omega - \omega_0}{(\omega - \omega_0)^2 + \Gamma^2/4}, \quad (2.16)$$

which is called the *AC Stark shift*. Here, $\Gamma = \frac{\omega_0^3}{3\pi\epsilon_0\hbar c^3} \frac{2J+1}{2J'+1} |\langle g | \hat{\mathbf{e}} \cdot \mathbf{d} | e \rangle|^2$ is the *scattering rate*. J and J' are the *total electron angular momentum quantum number* of $|g\rangle$ and $|e\rangle$ respectively. In the case of Rubidium 85, $J' = 1/2$ ($3/2$) for D1 (D2) line and $J = 1/2$ for both line. In the far detuned limit, i.e. $|\delta| = |\omega - \omega_0| \gg \Gamma$, the energy shift can be simplified

Figure 2.5: Pictorial demonstration of spin echo. Red arrow represents the rotation axis under the labelled operation. $\mathbf{b}$ rotates to the equator after the first $\pi/2$ pulse and “spreads” out on the equator under $\delta_{T/2}$ with inhomogeneous detuning. Then, the π pulse flips $\mathbf{b}$ to the other side of Bloch sphere and $\mathbf{b}$ “un-spreads” back under $\delta_{T/2}$ with the same inhomogeneous detuning. Finally, $\mathbf{b}$ rotates to the south pole after the last $\pi/2$ pulse.

to

$$U \approx \frac{\hbar |\Omega|^2}{4\delta} = \frac{\hbar \Gamma^2}{8\delta} \cdot \frac{I}{I_{\text{sat}}}, \quad (2.17)$$

where $I_{\text{sat}} := \frac{c\epsilon_0 \Gamma^2 \hbar^2}{4|\langle g|\hat{\epsilon} \cdot \mathbf{d}|e\rangle|^2}$ is called the *saturation intensity*, $I = (\epsilon_0 c/2) \mathcal{E}_0^2$ is the intensity, and ϵ_0 is the *vacuum permittivity*. Note that when the light is red detuned, $\delta < 0$ and the potential is negative. This provides a force to atoms towards high intensity if the intensity has a spatial distribution, i.e. $I = I(\mathbf{r})$. The commonly used light beam is the Gaussian beam. For the light propagating along z direction, its electric field distribution⁵ is

$$\mathcal{E}(r, z) = \mathcal{E}_0 \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w(z)^2}\right) \exp\left(-ik\left[z + \frac{r^2}{2z(1 + z_R^2/z^2)}\right]\right) \exp\left(i \arctan \frac{z}{z_R}\right), \quad (2.18)$$

and the intensity distribution is

$$I(r, z) = I_{\text{max}} \left(\frac{w_0}{w(z)}\right)^2 \exp\left(-\frac{2r^2}{w(z)^2}\right), \quad (2.19)$$

where

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2} \quad (2.20)$$

and $w_0 = w(0)$ is called the *beam waist*, representing the characteristic “radius” of the beam. $z_R = \frac{\pi n w_0^2}{\lambda}$, called the *Rayleigh range*, is the characteristic “length” of the beam. n is the refractive index and it is normally put as unity. At $z = 0$, the intensity takes $I(r, 0) = I_{\text{max}} \exp\left(-\frac{2r^2}{w_0^2}\right) \approx I_{\text{max}} - \frac{2r^2}{w_0^2} I_{\text{max}}$ at small r . Ignoring the constant offset and combining with equation (2.17), a parabolic potential can be formed in which it is useful for Raman sideband cooling (in §10.3) and creating Schrödinger Cat (in §10.5).

2.5 Light Shift Cancellation and Scattering

In most of the case, there are many transitions in an atom. For Rubidium 85, a hyperfine ground state $5^2S_{1/2}$, $F = 2$ can have transition to $5^2P_{1/2}$, $F = 2, 3$ of D1 line and $5^2P_{3/2}$, $F = 1, 2, 3$ of D2 line. Another hyperfine ground state $5^2S_{1/2}$, $F = 3$ can have transition to $5^2P_{1/2}$, $F = 2, 3$ of D1 line and $5^2P_{3/2}$, $F = 2, 3, 4$ of D2 line. Each transition will

⁵The last exponent term is called the *Gouy phase*. It only depends on z which is the source of phase error in Mach-Zehnder atom interferometer. See §3.1.4.

Figure 2.6: Energy level of hyperfine ground states under AC Stark shift from a Gaussian beam. Both states are shifted by different amount due to different CG coefficients and detuning, and it is position dependence. The differential AC Stark shift between them is position dependence (e.g. red and blue arrow have different energy difference) and it is a source of decoherence.

contribute a dipole potential that add equation (2.17) to

$$\begin{aligned}
 U &= \sum_{\text{transition}} \frac{\hbar \Gamma^2}{8\delta} \cdot \frac{I}{I_{\text{sat}}} \\
 &= \sum_{|e\rangle} \frac{|\langle g | \hat{\epsilon} \cdot \mathbf{d} | e \rangle|^2}{2\delta c \epsilon_0 \hbar} \cdot I \\
 &= \frac{I}{2c \epsilon_0 \hbar} \sum_{|e\rangle} \frac{|\langle g | \hat{\epsilon} \cdot \mathbf{d} | e \rangle|^2}{\delta},
 \end{aligned} \tag{2.21}$$

where the value of $\langle g | \hat{\epsilon} \cdot \mathbf{d} | e \rangle$ depends on the light polarization $\hat{\epsilon}$ and it is directly proportional to the *Clebsch–Gordan* (CG) coefficients $\text{CG}_{D1/D2, (F, m) \rightarrow (F', m')}$ which details are shown in [Appendix A](#).

Since $\langle g | \hat{\epsilon} \cdot \mathbf{d} | e \rangle$ and detuning δ are different for every transition, both hyperfine ground states of Rubidium will have *AC Stark shift* by different amount. This difference, called the *differential AC Stark shift*, is one of the main sources of decoherence. This is the main issue of the decoherence effect when the project of atom interferometer is carried out in [Chapter 7](#).

In order to solve the decoherence problem, Derevianko has shown that in the presence of Zeeman and Stark perturbation, it is possible to generate extra shift from vector light shift [[115](#), [116](#)] can be used to cancel the differential Stark effect on Zeeman *clock states* ($m = 0$). The main idea starts with two hyperfine ground states (We use Rubidium 85 for example) $|2\rangle \equiv |F=2\rangle$ and $|3\rangle \equiv |F=3\rangle$. The Hamiltonian involves the unperturbed Hamiltonian $H_0 = h\nu_0 |3\rangle \langle 3|$, Zeeman Hamiltonian $H_B = -\mu_B B (|2\rangle \langle 3| + |3\rangle \langle 2|)$ and Stark Hamiltonian $H_U = \sum_{F, F'} -\alpha_{FF'} \frac{\mathcal{E}_0^2}{4} |F\rangle \langle F'|$. In practice, the energy shift by both perturbed Hamiltonian is small compared to the energy difference between both states, i.e. $\mu_B B \ll h\nu_0$ and $\alpha_{FF'} \mathcal{E}_0^2 \ll h\nu_0$. Therefore, we only evaluate the eigenenergy up to second order.

Without Stark Hamiltonian, $H_0 + H_B$ will shift the energy to $h\nu_0 + \delta E_B \equiv h\nu_0 + \frac{2(\mu_B B)^2}{h\nu_0}$. This is known as *second order Zeeman shift* and it is only true for clock state due to zero diagonal element of H_B . With Stark Hamiltonian, $H_0 + H_B + H_U$ will shift the energy to

$h\nu_0 + \delta E_B + \delta E_U$ where [117]

$$\delta E_U = \beta_1 U_a + \beta_2 B U_a + \beta_4 U_a^2, \quad (2.22)$$

with the definition of notation

$$\begin{aligned} \beta_1 &= \frac{\alpha_{33} - \alpha_{22}}{\alpha_{5s}} \\ \beta_2 &= -\frac{2A\mu_B\alpha_{5s}^v}{h\nu_0\alpha_a} \\ \beta_4 &= \frac{A^2}{2h\nu_0} \cdot \left(\frac{\alpha_{5s}^v}{\alpha_a}\right)^2 = \frac{\nu_0}{8h} \left(\frac{\beta_2}{\mu/h}\right)^2 \\ U_a &= -\alpha_{5s} \frac{\mathcal{E}_0^2}{4}. \end{aligned}$$

Here, A is the light polarization ellipticity ranging from 0 (linear) to 1 (circular), $\alpha_{5s}^v = \frac{2}{A}\alpha_{23}$ is the vector polarizability of the ground states and α_{5s} is the dominant dynamic ground-state polarizability.

At small U_a , the quadratic term is small compare to the linear term. It is possible to tune A and B experimentally to cancel the Stark shift as shown in Chapter 8. This light shift cancellation technique is mainly used and the experimental detail is shown in Chapter 8.

Other than *AC Stark shift*, the presence of light field also contributes a scattering effect from arbitrary state $|g_1\rangle$ to $|g_2\rangle$ at a rate of [118]

$$\Gamma_{\text{sc}}(g_1 \rightarrow g_2) = \frac{3\pi c^2 \omega^3 I}{2\hbar \mu^4} \left| \sum_{|e\rangle} \frac{\Gamma(g_2|\hat{\epsilon} \cdot \mathbf{d}|e) \langle e|\hat{\epsilon} \cdot \mathbf{d}|g_1\rangle}{\omega_0^3 \delta} \right|^2. \quad (2.23)$$

The *Raman scattering* is defined as the scattering rate towards other states $\Gamma_{\text{sc}}^{\text{Ram}}(g_1) \equiv \sum_{g_2 \neq g_1} \Gamma_{\text{sc}}(g_1 \rightarrow g_2)$. The *Rayleigh scattering* is the scattering back to itself $\Gamma_{\text{sc}}^{\text{Ray}}(g_1) \equiv \Gamma_{\text{sc}}(g_1 \rightarrow g_1)$ and it contributes most of the total scattering rate $\Gamma_{\text{sc}}^{\text{tot}}(g_1) = \Gamma_{\text{sc}}^{\text{Ram}}(g_1) + \Gamma_{\text{sc}}^{\text{Ray}}(g_1)$. Table 2.1 shows the parameters of beams used in all experiments in the thesis and its optical properties. In the experimental design, the time scale of the scattering rate, of any beam must be small compared to the duration of the beam in experiment. For example, the scattering rate of dipole beam for atom interferometer in Chapter 7 is 45 Hz or 22 ms when the duration of the interferometer is at most ~ 100 ms. On the other hand, the scattering rate of cat beams for creation of Schrödinger cat states in Chapter 10 is 10^5 Hz or 100 μs whereas the pulse duration of the cat beam during experiment is at most ~ 10 μs .

⁶For dipole/lattice, since the detuning is far enough, the scattering rate for $F=2$ and $F=3$ are roughly the same. However, the case of 744 MHz is not. Here, only the higher scattering rate at $F=3$ is stated.

Wavelength/ Detuning	Single Beam Power	U/k_B (μK)	$\Delta U/h$ (kHz)	$\Gamma_{\text{sc}}^{\text{tot}}/2\pi$ (Hz) ⁶	Role	Used in
808 nm	280 mW	514	4.24	45	Dipole	Chapter 6 Chapter 7
875 nm	300 mW	537	0.90	7.8	Lattice	Chapter 8
875 nm	156 mW	279	0.47	4.1	Lattice	Chapter 8
875 nm	100 mW	179	0.30	2.6	Lattice	Chapter 8
821 nm	200 mW	219	0.96	9.9	Dipole	Chapter 9
821 nm	100 mW	438	1.92	19.9	Lattice	Chapter 9
744 MHz (Figure 4.1)	13 μW	-	1736	10^5	Cat beams	Chapter 10

Table 2.1: List of trapping potential U , differential AC Stark shift ΔU and scattering rate of various beam used in experiment. These calculations are based on maximum intensity.

2.6 Two-Photon Raman Transition

2.6.1 Two-Level System Analogy

In most of the case, the energy between $|g\rangle$ and $|e\rangle$ is related to microwave frequency. For example, the species we used, Rubidium-85, has two hyperfine ground states with frequency splitting of $\omega_0 = 2\pi \times (3.035\,732\,439\text{ GHz})$. There are some cases we need the coupling between them using optical frequency in order to transfer more momentum to the atom [[119](#)]. In addition, the polarization of optical light is easier to control compared to microwave. A common method to couple $|g\rangle$ and $|e\rangle$ using a pair of optical beams, called the *two-photon Raman transition*, is through the third intermediate state $|\iota\rangle$. The energy diagram scheme is shown in [Figure 2.7](#). Using the convention $|g\rangle := \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$, $|e\rangle := \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ and $|\iota\rangle := \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$, we can obtain the full Hamiltonian similar to two-level system in equation ([2.7](#))

$$\hat{H} = \hbar \begin{bmatrix} \omega_{\iota g} & \frac{\Omega_2}{2} e^{-i\omega_2 t} & \frac{\Omega_1}{2} e^{-i\omega_1 t} \\ \frac{\Omega_2^*}{2} e^{i\omega_2 t} & \omega_0 & 0 \\ \frac{\Omega_1^*}{2} e^{i\omega_1 t} & 0 & 0 \end{bmatrix} \quad (2.24)$$

with two light beams coupled to respective transition. Using the rotation frame

$$\hat{U} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\omega_2 t} & 0 \\ 0 & 0 & e^{-i\omega_1 t} \end{bmatrix}, \quad (2.25)$$

Figure 2.7: Energy diagram for two-photon Raman transition. If the Raman beams are counter-propagating, the states need to include the momentum state, which is discussed in §2.6.2.

Figure 2.8: The direction of momentum transferred during Raman "absorption" and "stimulated emission". The sphere represents the atom, and the red arrows represent Raman beams. The Raman beams are counter-propagating to each other, so both momentums are transferred in the same direction.

the Hamiltonian becomes $|\psi\rangle = c_g(t)|g\rangle + c_e(t)|e\rangle + c_l(t)|l\rangle$, then solving the time dependent Schrödinger equation $\hat{H} = i\hbar \frac{\partial}{\partial t}$ gives a set of three coupled equations

$$i\dot{c}_l(t) = -\Delta c_l + \frac{1}{2}(\Omega_2 c_e + \Omega_1 c_g) \quad (2.26a)$$

$$i\dot{c}_e(t) = -\delta c_e + \frac{\Omega_2^*}{2} c_l \quad (2.26b)$$

$$i\dot{c}_g(t) = \frac{\Omega_1^*}{2} c_l. \quad (2.26c)$$

Here, we set $\Delta \gg \Omega_1, \Omega_2, \delta$ so that single photon transition to $|l\rangle$ is suppressed. In this case, $\dot{c}_l(t)$ oscillates much faster than $\dot{c}_g(t)$ and $\dot{c}_e(t)$. Therefore, in this time scale, fast oscillation of $\dot{c}_l(t)$ can be averaged to zero in equation (2.26a), leaving $c_l \approx \frac{1}{2\Delta}(\Omega_2 c_e + \Omega_1 c_g)$. This is called the *adiabatic elimination*. After the elimination, the three coupled equations reduced to two equations

$$i\dot{c}_e(t) = -\delta c_e + \frac{\Omega_2^*}{4\Delta}(\Omega_2 c_e + \Omega_1 c_g) \quad (2.27a)$$

$$i\dot{c}_g(t) = \frac{\Omega_1^*}{4\Delta}(\Omega_2 c_e + \Omega_1 c_g). \quad (2.27b)$$

Defining $\Lambda_i = \frac{|\Omega_i|^2}{4\Delta}$, $i = 1, 2$ and

$$\Omega_{2p} = \frac{\Omega_1 \Omega_2^*}{2\Delta}, \quad (2.28)$$

the coupled equations become equivalent to the two-level system with Hamiltonian

$$\hat{H}_{\text{Raman}} = \hbar \begin{bmatrix} -\delta + \Lambda_2 & \frac{\Omega_{2p}}{2} \\ \frac{\Omega_{2p}^*}{2} & \Lambda_1 \end{bmatrix}. \quad (2.29)$$

The energy shift of $\hbar\Lambda_i, i = 1, 2$ at both states, respectively, are the *AC Stark shift* identical to equation (2.17). The effective shift between the states $\Lambda_2 - \Lambda_1$, known as *differential AC Stark shift*, is one of the sources of dephasing effect. Ignoring the shift⁷, $\hat{H}_{\text{Raman}}$ is equivalent to two-level system with detuning δ and Rabi frequency of Ω_{2p} , called the *two-photon Rabi frequency*. Note that from equation (2.6), we have the phase relation $\Omega \sim e^{-i\phi}$. Therefore, from equation (2.28), the two-photon Rabi phase for Raman transition is $\phi_{2p} = \phi_1 - \phi_2$, where ϕ_1 and ϕ_2 are the laser phases of both Raman beams [120].

2.6.2 Momentum Transfer

If the Raman beams are counter propagating, the atom will experience maximum momentum transfer. The idea of momentum transfer is illustrated in Figure 2.8. An atom in state $|g\rangle$ with momentum p absorbs the first Raman photon of energy $\hbar\omega_1$ to $|\iota\rangle$ state and receives momentum transfer to $p + \hbar k_1$. Then, it emits a photon of energy $\hbar\omega_2$ together with the second Raman photon (like “stimulated emission”) to state $|e\rangle$ and the momentum changes to $p + 2\hbar k_{\text{eff}}$, where $\mathbf{k}_{\text{eff}} := \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ is known as the *effective wavevector*⁸. Since the Raman beams are counter propagating, $k_{\text{eff}} = \frac{1}{2}|\mathbf{k}_1 - \mathbf{k}_2| = \frac{1}{2}(k_1 + k_2)$ is the average of both wavevector. In summary, the energy diagram in Figure 2.7 can be updated with the replacement of

$$|g\rangle \rightarrow |g, \mathbf{p}\rangle \quad (2.30a)$$

$$|\iota\rangle \rightarrow |\iota, \mathbf{p} + \hbar \mathbf{k}_1\rangle \quad (2.30b)$$

$$|e\rangle \rightarrow |e, \mathbf{p} + 2\hbar \mathbf{k}_{\text{eff}}\rangle. \quad (2.30c)$$

Since the energy of $|g\rangle, |\iota\rangle, |e\rangle$ becomes $\frac{p^2}{2m}, \hbar\omega_{\iota g} + \frac{(p + \hbar k_1)^2}{2m}, \hbar\omega_0 + \frac{(p + 2\hbar k_{\text{eff}})^2}{2m}$, respectively, a shift of $-\frac{p^2}{2m}$ in Hamiltonian results an extra replacement of

$$\hbar\omega_{\iota g} \rightarrow \hbar\omega_{\iota g} + \frac{(p + \hbar k_1)^2}{2m} - \frac{p^2}{2m} \quad (2.31a)$$

$$\hbar\omega_0 \rightarrow \hbar\omega_0 + \frac{(p + 2\hbar k_{\text{eff}})^2}{2m} - \frac{p^2}{2m} \quad (2.31b)$$

in Figure 2.7. As a result, we can obtain equation (2.29) under the convention $|g, p\rangle := \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ and $|e, p + 2\hbar k_{\text{eff}}\rangle := \begin{bmatrix} 1 & 0 \end{bmatrix}^T$.

⁷One may also shift the Hamiltonian in equation (2.29) by $-\Lambda_1$ and absorb the differential AC Stark shift into δ to make it the actual detuning.

⁸The intention of defining k_{eff} is simply just for convenience in simplification because in most of the time, $k_1 \approx k_2 \approx k_{\text{eff}}$.

Chapter 3

State Manipulation of Atom

This chapter uses the knowledges of the previous chapter to lay the theoretical background for the three major experiments in this thesis: the inertia-sensitive Mach–Zehnder atom interferometer (AI), light storage by electromagnetically induced transparency and Schrödinger cat experiment. In the derivation of the atom light interaction experiment, equations in the previous chapter would be reused multiple times.

3.1 Inertia-sensitive Mach–Zehnder Atom Interferometer

3.1.1 Introduction

Mach–Zehnder atom interferometer is a common tool [32, 121, 122] in precision gravity measurement. It is similar to light Mach–Zehnder interferometer, where the light beam is split into two beams by a beam splitter, reflected by a mirror and recombined by another beam splitter. A phase difference between two paths will result in interference fringes. In the atom interferometer, atom wave packet replaces the light beam, and the Raman $\pi/2$ and π pulses are analogous to the beam splitter and mirror respectively that are used to alter the atomic path.

As shown in Figure 3.1, the wave packet is split into two different momentum states by a $\pi/2$ pulse at time $t = 0$ as a beam splitter. After a separation time T , the Raman π pulse flips the state like a mirror. Finally, at $t = 2T$, another $\pi/2$ pulse combines them together and the phase difference results in interference that can be measured. Note that the phases of the atomic wave packets on each path are determined by their local phases from the Raman beam, denoted by $\phi_{2p}^1, \phi_{2p}^{2a}, \phi_{2p}^{2b}$ and ϕ_{2p}^3 , which are determined by their position. This is the key of gravity measurement.

Figure 3.1: Schematic diagram of Mach-Zehnder atom interferometer. The vertical axis represents the space where the gravity is point in the direction of $-z$. The horizontal axis shows the time. The atom is prepared in the spin state $|g\rangle$ (marked blue). The Raman $\pi/2$ pulse will bring the atom to the superposition of the spin state $|g\rangle$ and $|e\rangle$ (marked green). Like the light interferometer, the Raman $\pi/2$ pulse is analogous to the beam splitter and π pulse is analogous to the mirror. At last $\pi/2$ pulse, the atom will recombine and interfere together. The dashed line represents the trajectory of atom in the absence of gravity. With gravity, the phase of the Raman pulses ϕ_{2p}^1 , ϕ_{2p}^{2a} , ϕ_{2p}^{2b} and ϕ_{2p}^3 imprints onto the atoms are different, and they increase with the separation time T between the pulses. As a result, the Mach-Zehnder phase is $\phi_{MZ} = -k_{\text{eff}}gT^2$ where $k_{\text{eff}} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ is the effective wavevector.

3.1.2 Mathematical Framework

Mathematically, we can recall the replacement by two photon Raman transition $\Omega \rightarrow \Omega_{2p}$, $\phi \rightarrow \phi_{2p}$ and let $\delta = 0$ into equation (2.14)

$$c_g(t) = \cos\left(\frac{1}{2}|\Omega_{2p}|t\right)c_g(0) - ie^{i\phi_{2p}}\sin\left(\frac{1}{2}|\Omega_{2p}|t\right)c_e(0) \quad (3.1a)$$

$$c_e(t) = \cos\left(\frac{1}{2}|\Omega_{2p}|t\right)c_e(0) - ie^{-i\phi_{2p}}\sin\left(\frac{1}{2}|\Omega_{2p}|t\right)c_g(0) \quad (3.1b)$$

or in matrix form

$$\begin{bmatrix} c_e \\ c_g \end{bmatrix}(t) = \begin{bmatrix} \cos\left(\frac{1}{2}|\Omega_{2p}|t\right) & -ie^{-i\phi_{2p}}\sin\left(\frac{1}{2}|\Omega_{2p}|t\right) \\ -ie^{i\phi_{2p}}\sin\left(\frac{1}{2}|\Omega_{2p}|t\right) & \cos\left(\frac{1}{2}|\Omega_{2p}|t\right) \end{bmatrix} \begin{bmatrix} c_e \\ c_g \end{bmatrix}(0). \quad (3.2)$$

One can then find at a $\pi/2$ pulse ($t_{\pi/2} = \pi/2|\Omega_{2p}|$) and π pulse ($t_\pi = \pi/|\Omega_{2p}|$) will result the state changing and phase difference

$$\begin{bmatrix} c_e \\ c_g \end{bmatrix}(t_{\pi/2}) = \sqrt{\frac{1}{2}} \begin{bmatrix} 1 & e^{-i(\phi_{2p}+\pi/2)} \\ e^{-i(-\phi_{2p}+\pi/2)} & 1 \end{bmatrix} \begin{bmatrix} c_e \\ c_g \end{bmatrix}(0) \quad (3.3a)$$

$$\begin{bmatrix} c_e \\ c_g \end{bmatrix}(t_\pi) = \begin{bmatrix} 0 & e^{-i(\phi_{2p}+\pi/2)} \\ e^{-i(-\phi_{2p}+\pi/2)} & 0 \end{bmatrix} \begin{bmatrix} c_e \\ c_g \end{bmatrix}(0). \quad (3.3b)$$

In short, the pulses not only change the atomic state, they also imprint the phase of the states. For example, using the short form notation $|g; \phi\rangle \equiv e^{-i\phi} |g, \mathbf{p}\rangle$ and $|e; \phi\rangle \equiv e^{-i\phi} |e, \mathbf{p} + 2\hbar\mathbf{k}_{\text{eff}}\rangle$, a $\pi/2$ pulse with local phase ϕ convert the states

$$|g; \phi_0\rangle \xrightarrow[\phi]{\pi/2} \sqrt{\frac{1}{2}} |g; \phi_0\rangle + \sqrt{\frac{1}{2}} |e; \phi_0 - \phi + \pi/2\rangle \quad (3.4a)$$

$$|e; \phi_0\rangle \xrightarrow[\phi]{\pi/2} \sqrt{\frac{1}{2}} |g; \phi_0 + \phi + \pi/2\rangle + \sqrt{\frac{1}{2}} |e; \phi_0\rangle \quad (3.4b)$$

and a π pulse with local phase ϕ convert the states

$$|g; \phi_0\rangle \xrightarrow[\phi]{\pi} |e; \phi_0 - \phi + \pi/2\rangle \quad (3.5a)$$

$$|e; \phi_0\rangle \xrightarrow[\phi]{\pi} |g; \phi_0 + \phi + \pi/2\rangle. \quad (3.5b)$$

Let the atom being prepared at state $|g; \phi_0\rangle$ initially. The three pulses imprint the phase on the lower arm as

$$\begin{aligned} |g; \phi_0\rangle &\xrightarrow[\phi_{2p}^1]{\pi/2} \sqrt{\frac{1}{2}} |g; \phi_0\rangle \xrightarrow[\phi_{2p}^{2b}]{\pi} \sqrt{\frac{1}{2}} |e; \phi_0 - \phi_{2p}^{2b} + \pi/2\rangle \\ &\xrightarrow[\phi_{2p}^3]{\pi/2} \frac{1}{2} (|g; \phi_0 - \phi_{2p}^{2b} + \phi_{2p}^3 + \pi\rangle + |e; \phi_0 - \phi_{2p}^{2b} + \pi/2\rangle) \end{aligned} \quad (3.6)$$

and on the upper arm as

$$\begin{aligned} |g; \phi_0\rangle &\xrightarrow[\phi_{2p}^1]{\pi/2} \sqrt{\frac{1}{2}} |e; \phi_0 - \phi_{2p}^1 + \pi/2\rangle \xrightarrow[\phi_{2p}^{2a}]{\pi} \sqrt{\frac{1}{2}} |g; \phi_0 - \phi_{2p}^1 + \phi_{2p}^{2a} + \pi\rangle \\ &\xrightarrow[\phi_{2p}^3]{\pi/2} \frac{1}{2} (|g; \phi_0 - \phi_{2p}^1 + \phi_{2p}^{2a} + \pi\rangle + |e; \phi_0 - \phi_{2p}^1 + \phi_{2p}^{2a} - \phi_{2p}^3 + 3\pi/2\rangle). \end{aligned} \quad (3.7)$$

After superposing together, the final state is

$$\begin{aligned} |\psi\rangle = &\frac{1}{2} e^{-i\phi_0} \left\{ - \left[e^{-i(-\phi_{2p}^{2b} + \phi_{2p}^3)} + e^{-i(-\phi_{2p}^1 + \phi_{2p}^{2a})} \right] |g, \mathbf{p}\rangle \right. \\ &\left. + e^{-i\pi/2} \left[e^{i\phi_{2p}^{2b}} - e^{-i(-\phi_{2p}^1 + \phi_{2p}^{2a} - \phi_{2p}^3)} \right] |e, \mathbf{p} + 2\hbar\mathbf{k}_{\text{eff}}\rangle \right\} \end{aligned} \quad (3.8)$$

and the probability of finding the atom in $|g, \mathbf{p}\rangle$ is

$$\frac{1}{2} + \frac{1}{2} \cos(\phi_{2p}^1 - \phi_{2p}^{2a} - \phi_{2p}^{2b} + \phi_{2p}^3) \quad (3.9)$$

and in $|e, \mathbf{p} + 2\hbar\mathbf{k}_{\text{eff}}\rangle$ is

$$\frac{1}{2} - \frac{1}{2} \cos(\phi_{2p}^1 - \phi_{2p}^{2a} - \phi_{2p}^{2b} + \phi_{2p}^3),$$

which is the interference pattern at a phase

$$\phi_{\text{MZ}} = \phi_{2p}^1 - \phi_{2p}^{2a} - \phi_{2p}^{2b} + \phi_{2p}^3. \quad (3.10)$$

Note that ϕ_{MZ} does not depend on the initial phase ϕ_0 , indicating that a large atomic assemble size, even cover over a wavelength or 2π phase, will not affect the measurement result. Moreover, since ϕ_{MZ} only measure the relative phase between the pulses, the difference in initial laser phase at every measurement does not matter. These robust properties make Mach-Zehnder atom interferometer a useful tool in precision measurement.

3.1.3 Gravity Measurement

In general, Mach-Zehnder atom interferometers are sensitive to any perturbation that changes the relative phase of two interferometer arms. In the thesis, I only focus on the measurement of gravitational acceleration g . We set the gravity pointing at $-z$ direction and let atom position at the first $\pi/2$ pulse be $z(t=0) = 0$ (Hence, $\phi_{2p}^1 = 0$) and velocity of $v(t=0) = v_0$. Then, the positions of the two interferometer arms are

$$z_{\text{lower arm}}(T) = v_0 T - \frac{1}{2} g T^2 \quad (3.11a)$$

$$z_{\text{upper arm}}(T) = (v_0 + v_r) T - \frac{1}{2} g T^2 \quad (3.11b)$$

$$\begin{aligned} z(2T) &= z_{\text{lower arm}}(T) + v_{\text{lower arm}}(t=T) \cdot T - \frac{1}{2} g T^2 \\ &= \left[v_0 T - \frac{1}{2} g T^2 \right] + [v_0 - gT + v_r] \cdot T - \frac{1}{2} g T^2 \\ &= (2v_0 + v_r) T - 2gT^2, \end{aligned} \quad (3.11c)$$

where v_r is the velocity added after the momentum transfer. Hence, the Mach-Zehnder phase is

$$\begin{aligned} \phi_{\text{MZ}} &= k_{\text{eff}} [0 - z_{\text{lower arm}}(T) - z_{\text{upper arm}}(T) + z(2T)] \\ &= -k_{\text{eff}} g T^2, \end{aligned} \quad (3.12)$$

which only depends on g and other known variables. Here, $\mathbf{k}_{\text{eff}} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ is the effective wavevector determined by the wavevector of each beam $\mathbf{k}_1$ and $\mathbf{k}_2$. To determine ϕ_{MZ} experimentally, we scan the laser phase of ϕ_{2p}^3 for every T and perform a sinusoidal fit (see §7.2).

3.1.4 Advantages in Hollow Core Fiber

Mach-Zehnder atom interferometers have been used for gravity measurement for cold atoms in free space. However, all free space experiments share the common error phase source from the foundational property of a Gaussian beam: the *Gouy phase*. From the mathematical framework, one can see that ϕ_{MZ} is independent of absolute position of atom, assuming the beam being a plane wave. However, a Gaussian beam adds a *Gouy phase* (see equation (2.18)) of $-\arctan(z/z_R)$ into the system. Furthermore, the intensity distribution in equation (2.19) shows changes axially as the beam waist changes at the length scale of z_R , suggesting the Raman pulse might not provide the prefect pulse. These systematic errors

could be removed if the atom interferometer happens in the hollow core fiber, where the beam distribution has an axial symmetry. Furthermore, any beam in free space, in general, will have a phase distortion due to imperfection of optical surface, contributing an extra systematic phase error to the measurement. Optical fiber has a beam profile cleaning role that could help to remove the error source. In addition, the *effective wavevector* $\mathbf{k}_{\text{eff}} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2)$ highly depends on the direction of $\mathbf{k}_1$ and $\mathbf{k}_2$. It is challenging to make sure both $\mathbf{k}_1$ and $\mathbf{k}_2$ to be perfectly counter-propagating in free space experimentally. This difficulty is fully removed when coupled into fiber. Furthermore, there are also advantages of compact and small power consumption. Due to tight confinement of light inside the fiber, strong light-atom interaction allows lower optical power which could reduce the power consumption. This compact design provides a promising future in quantum science and technologies.

3.2 Electromagnetically Induced Transparency

3.2.1 Introduction

Electromagnetically induced transparency (EIT) [123] is a nonlinear effect in a medium where a narrow transparent spectral window for an optical beam is created by another beam [123, 124]. A simple three level system with Λ configuration [125] is shown in Figure 3.2a that is identical to Figure 2.7 except with a few notation replacements $|g\rangle \rightarrow |1\rangle$, $|e\rangle \rightarrow |2\rangle$, $|\iota\rangle \rightarrow |3\rangle$, $\omega_1 \rightarrow \omega_p$, $\omega_2 \rightarrow \omega_c$, $\Omega_1 \rightarrow \Omega_p$, $\Omega_2 \rightarrow \Omega_c$, $\Delta \rightarrow \Delta_p$ and $\delta \rightarrow \Delta_p - \Delta_c$. Therefore, the Hamiltonian can be inherited from equation (2.24) and (2.25)

$$\hat{H}_{\text{EIT}} = \frac{\hbar}{2} \begin{bmatrix} -2\Delta_p & \Omega_c & \Omega_p \\ \Omega_c^* & 2(\Delta_c - \Delta_p) & 0 \\ \Omega_p^* & 0 & 0 \end{bmatrix}. \quad (3.13)$$

The two beams, labelled c and p , are called the control and probe beam, respectively. In the case of EIT, we require probe beam to be weak compared to the control beam. In dressed state, we assume $\Delta_p \approx \Delta_c = \Delta$ to simplify the calculation and the eigenenergies determined by $\det[\hat{H}_{\text{EIT}} - \lambda \hat{1}] = 0$ yields

$$\lambda_0 = 0 \quad (3.14a)$$

$$\lambda_+ \equiv \hbar\omega_+ = \frac{\hbar}{2} \left(\Delta + \sqrt{\Delta^2 - (|\Omega_c|^2 + |\Omega_p|^2)} \right) \quad (3.14b)$$

$$\lambda_- \equiv \hbar\omega_- = \frac{\hbar}{2} \left(\Delta - \sqrt{\Delta^2 - (|\Omega_c|^2 + |\Omega_p|^2)} \right) \quad (3.14c)$$

Figure 3.2: (a) Energy diagram for EIT. (b) The plot of $\tilde{\rho}_{13}$ from equation (3.21), which is proportional to the susceptibility of the EIT medium. The real part determines the dispersion property and the imaginary part shows the absorption coefficient.

with the corresponding unnormalized eigenstates

$$|0\rangle = \frac{\Omega_c |1\rangle + \Omega_p |2\rangle}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}} \quad (3.15a)$$

$$|+\rangle = \frac{\Omega_p^* |1\rangle + \Omega_c^* |2\rangle}{\Delta + \sqrt{\Delta^2 + |\Omega_c|^2 + |\Omega_p|^2}} + |3\rangle \quad (3.15b)$$

$$|-\rangle = -\frac{\Omega_p^* |1\rangle + \Omega_c^* |2\rangle}{\Delta - \sqrt{\Delta^2 + |\Omega_c|^2 + |\Omega_p|^2}} + |3\rangle. \quad (3.15c)$$

We will find that the state $|0\rangle$ has no composition of $|3\rangle$, meaning that atoms that fall into this state will no longer be in excited state with decaying property, and hence, are dark to the probe beam. This is called the *dark state*. In the case of a weak probe beam, or $|\Omega_p| \ll |\Omega_c|$, the dark state will approach to the bare-atom state $|1\rangle$. You will find it interesting that the probe beam is transparent to the medium even it is on resonance!

3.2.2 Absorption Spectrum

One of the usages of EIT is its large dispersion can be used to slow the group velocity of light. In order to determine its dispersion, we will need to evaluate its electric susceptibility from the density matrix element by solving the *Von Neumann equation* $\dot{\tilde{\rho}} = -\frac{i}{\hbar} [\hat{H}_{\text{EIT}}, \tilde{\rho}] - \frac{1}{2} \{\Gamma, \tilde{\rho}\}$, where

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_3 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.16)$$

represents the decay from spontaneous emission. I put $\gamma_1 = 0$ because it is impossible for the lowest state $|1\rangle$ to decay anymore. Then part of the *Von Neumann equation* yields

$$\dot{\rho}_{12} = \dot{\rho}_{21}^* = \left[-\frac{\gamma_2}{2} + i(\Delta_c - \Delta_p) \right] \tilde{\rho}_{12} + \frac{i}{2} \Omega_c \tilde{\rho}_{13} - \frac{i}{2} \Omega_p^* \tilde{\rho}_{32} \quad (3.17a)$$

$$\dot{\rho}_{13} = \dot{\rho}_{31}^* = \left[-\frac{\gamma_3}{2} - i\Delta_p \right] \tilde{\rho}_{13} - \frac{i}{2} \Omega_p^* (\tilde{\rho}_{33} - \tilde{\rho}_{11}) + \frac{i}{2} \Omega_c^* \tilde{\rho}_{12} \quad (3.17b)$$

$$\dot{\rho}_{23} = \dot{\rho}_{32}^* = \left[-\frac{\gamma_2 + \gamma_3}{2} - i\Delta_c \right] \tilde{\rho}_{23} - \frac{i}{2} \Omega_c^* (\tilde{\rho}_{33} - \tilde{\rho}_{22}) + \frac{i}{2} \Omega_p^* \tilde{\rho}_{21}. \quad (3.17c)$$

Here, I assume that since the probe beam is much weaker than the control beam and the atom is initially in state $|1\rangle$, the steady state is most in $|1\rangle$, or $\tilde{\rho}_{11} \approx 1$ and $\tilde{\rho}_{22} \approx \tilde{\rho}_{33} \approx 0$. Furthermore, at steady state, the left hand side of equation (3.17) will be zero. Hence, $\dot{\rho}_{23} = 0$ lead to

$$\tilde{\rho}_{23} = \frac{i\Omega_p^*}{\gamma_2 + \gamma_3 + 2i\Delta_c} \tilde{\rho}_{21}. \quad (3.18)$$

Simplifying equation (3.17) leads to

$$\begin{bmatrix} \dot{\rho}_{12} \\ \dot{\rho}_{13} \end{bmatrix} = \begin{bmatrix} -\frac{\gamma_2}{2} + i(\Delta_c - \Delta_p) & \frac{i}{2} \Omega_c \\ \frac{i}{2} \Omega_c^* & -\frac{\gamma_3}{2} - i\Delta_p \end{bmatrix} \begin{bmatrix} \tilde{\rho}_{12} \\ \tilde{\rho}_{13} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{i}{2} \Omega_p^* \end{bmatrix}, \quad (3.19)$$

which is in the form of $\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{X}_0$. In the steady state, $\dot{\mathbf{X}} = \mathbf{0}$ and $\mathbf{X} = -\mathbf{A}^{-1}\mathbf{X}_0$, which solves the density matrix elements. One of them is

$$\tilde{\rho}_{13} = \frac{-4\frac{i}{2}\Omega_p^* \left[-\frac{\gamma_2}{2} + i(\Delta_c - \Delta_p) \right]}{-[\gamma_2 + 2i(\Delta_c - \Delta_p)](\gamma_3 + 2i\Delta_p) + |\Omega_c|^2}. \quad (3.20)$$

In the experiment of this thesis, the state $|2\rangle$ is one of the long-lived hyperfine ground state with $\gamma_2 \approx 0$, therefore, the real and imaginary part of simplified $\tilde{\rho}_{13}/\Omega_p^*$ are

$$\text{Re} \left(\frac{\tilde{\rho}_{13}}{\Omega_p^*} \right) = -\frac{1}{\gamma_3} \cdot \frac{x(x_c^2 - 2x^2)}{(x_c^2 - 2x^2)^2 + x^2} \quad (3.21a)$$

$$\text{Im} \left(\frac{\tilde{\rho}_{13}}{\Omega_p^*} \right) = \frac{1}{\gamma_3} \cdot \frac{x^2}{(x_c^2 - 2x^2)^2 + x^2}, \quad (3.21b)$$

where $x = \Delta_p/\gamma_3$ and $x_c = |\Omega_c|/\sqrt{2}\gamma_3$ are related to probe detuning and control Rabi frequency, respectively.

The density matrix elements $\tilde{\rho}$ are calculated in the rotating frame from the unitary

operator from equation (2.25). After the change of notation,

$$\hat{U} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\omega_c t} & 0 \\ 0 & 0 & e^{-i\omega_p t} \end{bmatrix}. \quad (3.22)$$

The original density matrix elements can be converted back through $\rho = \hat{U}^\dagger \tilde{\rho} \hat{U}$, or

$$\begin{bmatrix} \rho_{33} & \rho_{32} & \rho_{31} \\ \rho_{23} & \rho_{22} & \rho_{21} \\ \rho_{13} & \rho_{12} & \rho_{11} \end{bmatrix} = \begin{bmatrix} \tilde{\rho}_{33} & \tilde{\rho}_{32}e^{-i\omega_c t} & \tilde{\rho}_{31}e^{-i\omega_p t} \\ \tilde{\rho}_{23}e^{i\omega_c t} & \tilde{\rho}_{22} & \tilde{\rho}_{21}e^{-i\omega_p t}e^{i\omega_c t} \\ \tilde{\rho}_{13}e^{i\omega_p t} & \tilde{\rho}_{12}e^{-i\omega_c t}e^{i\omega_p t} & \tilde{\rho}_{11} \end{bmatrix}. \quad (3.23)$$

The electric susceptibility is defined from the polarization in classical electrodynamics

$$\begin{aligned} P &= \epsilon_0 \chi E \\ &= \epsilon_0 \chi \frac{\mathcal{E}_p}{2} \left(e^{i(\omega_p t + \phi_p)} + e^{-i(\omega_p t + \phi_p)} \right) + \epsilon_0 \chi \frac{\mathcal{E}_c}{2} \left(e^{i(\omega_c t + \phi_c)} + e^{-i(\omega_c t + \phi_c)} \right) \end{aligned} \quad (3.24)$$

and

$$\begin{aligned} P &= N \langle \hat{\epsilon} \cdot \mathbf{d} \rangle \\ &= N (\rho_{13}d_{31} + \rho_{31}d_{13} + \rho_{23}d_{32} + \rho_{32}d_{23}) \\ &= N (d_{31}\tilde{\rho}_{13}e^{i\omega_p t} + d_{13}\tilde{\rho}_{31}e^{-i\omega_p t} + d_{32}\tilde{\rho}_{23}e^{i\omega_c t} + d_{23}\tilde{\rho}_{32}e^{-i\omega_c t}) \end{aligned} \quad (3.25)$$

from the dipole moment operator

$$\mathbf{d} = \begin{bmatrix} 0 & d_{32} & d_{31} \\ d_{23} & 0 & 0 \\ d_{13} & 0 & 0 \end{bmatrix}. \quad (3.26)$$

By comparing the coefficient of $e^{i\omega_p t}$, we can get

$$\chi = -\frac{2N}{\hbar\epsilon_0} |d_{13}|^2 \cdot \frac{\tilde{\rho}_{13}}{\Omega_p^*} \quad (3.27)$$

where $\Omega_p = -\frac{\mathcal{E}_p e^{-i\phi_p} d_{31}}{\hbar}$ is the Rabi frequency of probe beam. The refractive index $n := \sqrt{1 + \chi} \approx 1 + \frac{\chi}{2}$ can then be written as the real and imaginary parts

$$n_{\text{Re}} = 1 - \frac{N}{\hbar\epsilon_0\gamma_3} |d_{13}|^2 \cdot \text{Re} \left(\frac{\tilde{\rho}_{13}}{\Omega_p^*} \right) \quad (3.28a)$$

$$n_{\text{Im}} = -\frac{N}{\hbar\epsilon_0\gamma_3} |d_{13}|^2 \cdot \text{Im} \left(\frac{\tilde{\rho}_{13}}{\Omega_p^*} \right). \quad (3.28b)$$

We can also determine the dispersion and absorption from the probe field

$$\mathbf{E}_p = \frac{\mathcal{E}_p}{2} e^{i(\omega_p t - n \frac{\omega_p}{c} z)} + \text{cc} = e^{-\frac{\alpha}{2} z} \frac{\mathcal{E}_p}{2} e^{i(\omega_p t - n_{\text{Re}} \frac{\omega_p}{c} z)} + \text{cc}, \quad (3.29)$$

where ω_p is the probe frequency and

$$\alpha(\omega_p) = -2 \frac{\omega_p}{c} n_{\text{Im}} = \frac{2N\omega_p}{\hbar\epsilon_0 c \gamma_3} |d_{13}|^2 \cdot \text{Im} \left(\frac{\tilde{\rho}_{13}}{\Omega_p^*} \right) \quad (3.30)$$

is the absorption coefficient as the intensity goes like $I \sim e^{-\alpha z}$. It is interesting to notice that at non-zero control beam power, although most atom population is at state $|1\rangle$, a resonance probe will fully transmit the medium. Moreover, the width of the transmission window increases with control power (see Figure 3.2b). A negative dispersion also appears in transmission window for light dragging effect.

3.2.3 Slow Light and Light Storage

For light storage experiment, a Gaussian probe pulse is sent into the medium with time-width of σ_t , i.e. $\frac{\mathcal{E}_p}{2} e^{-\frac{t^2}{4\sigma^2}} e^{-i\omega_0 t} + \text{cc}$, where ω_0 is the resonance probe frequency¹. In frequency domain, the Fourier transform of the field is $\tilde{\mathcal{E}} e^{-\frac{(\omega_p - \omega_0)^2}{4(1/2\sigma)^2}}$ where $\tilde{\mathcal{E}}$ is some normalization constant. After passing the medium with length L , the field acquires phase to $\tilde{\mathcal{E}} e^{-\frac{(\omega_p - \omega_0)^2}{4(1/2\sigma)^2}} e^{in \frac{\omega_p}{c} L} + \text{cc}$ where n has real and imaginary part. Ignoring the imaginary part that has a role of attenuating the amplitude, the phase $n_{\text{Re}}(\omega_p) \frac{\omega_p}{c} L$ is non-linear and can be approximated through Taylor expansion, and with $n_{\text{Re}}(\omega_0) = 1$,

$$\begin{aligned} n_{\text{Re}}(\omega_p) \frac{\omega_p}{c} &\approx \frac{n_{\text{Re}}(\omega_0) \omega_0}{c} + \frac{n_{\text{Re}}(\omega_0) + n'_{\text{Re}}(\omega_0) \omega_0}{c} \Delta_p + \frac{2n'_{\text{Re}}(\omega_0) + n''_{\text{Re}}(\omega_0) \omega_0}{2c} \Delta_p^2 \\ &= \frac{\omega_0}{c} + \frac{1 + n'_{\text{Re}}(\omega_0) \omega_0}{c} \Delta_p + \frac{2n'_{\text{Re}}(\omega_0) + n''_{\text{Re}}(\omega_0) \omega_0}{2c} \Delta_p^2 \\ &\equiv k_p(\omega_0) + k'_p(\omega_0) \Delta_p + \text{GVD}(\omega_0) \Delta_p^2. \end{aligned} \quad (3.31)$$

Here, GVD is the *group velocity dispersion*, measuring how the medium affect the pulse. Performing the inverse Fourier transform will return the resultant electric field after medium.

$$\mathbf{E}_p^{\text{drag}} = \frac{\mathcal{E}_p}{2} \cdot \frac{\sigma}{\sqrt{\sigma^2 - i\text{GVD}}} \exp \left[-\frac{(t - k'_p(\omega_0) L)^2}{4(\sigma^2 - i\text{GVD})} \right] e^{-i(\omega_0 t - \frac{\omega_0}{c} L)} + \text{cc}. \quad (3.32)$$

This equation consists of two interesting effects:

- **Slow light:** In the absent of the medium, light should travel at the speed of c and the time propagating equation should be in the form of $t - \frac{L}{c}$ after traveling at a distance L . However, equation (3.32) shows the form of $t - k'_p(\omega_0) L$ which indicates that the group speed of light in the medium is $1/k'_p(\omega_0) = \frac{c}{1 + n'_{\text{Re}}(\omega_0) \omega_0}$. Thanks to the opening transparent windows, positive gradient of n_{Re} (or negative gradient of $\text{Re}(\tilde{\rho}_{13}/\Omega_p^*)$ as shown in Figure 3.2b) clearly reduces the speed of light in the medium. If the gradient is large enough, this speed can be slow down so much such that is almost stopped and can be “stored” into the medium.

- **Pulse shape distortion:** The presence of group velocity dispersion can change the pulse

¹This means that $\Delta_p = \omega_p - \omega_0$.

shape. In this Gaussian pulse case, the pulse is lengthened with the variance width of the pulse to be

$$\sigma_{\text{GVD}}^2 = \sigma^2 + \left(\frac{\text{GVD}(\omega_0)}{\sigma} \right)^2. \quad (3.33)$$

3.3 Schrödinger Cat States

3.3.1 Displacement Operator and Coherent States

The Schrödinger cat state, a thought experiment formulated by Erwin Schrödinger in 1935, illustrates the boundary between classical and quantum physics [126]. It has been demonstrated with various quantum systems. Here, I demonstrate the first Schrödinger cat state formed by neutral atoms in an optical potential. The alive and dead states are defined by the coherent states created by the *displacement operator*

$$\hat{D}(\alpha) := \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}) \quad (3.34)$$

from the ground state $|0\rangle$ in the Fock state basis. Here, α is a complex number, representing the “strength” of the displacement operator. $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operator. There are several properties of the *displacement operator*:

- *Kermack-McCrae identity*: $\hat{D}(\alpha) = e^{-\frac{1}{2}|\alpha|^2} e^{\alpha \hat{a}^\dagger} e^{-\alpha^* \hat{a}}$;
- *Baker-Campbell-Hausdorff formula*: $\hat{D}(\alpha) \hat{D}(\beta) = e^{(\alpha\beta^* - \alpha^*\beta)/2} \hat{D}(\alpha + \beta)$; and
- $\hat{D}(\alpha)$ is equivalent to shift the state in position and momentum space, as being named. To prove the statement, we use the properties of creation and annihilation operator $\hat{a}^\dagger = \frac{1}{\sqrt{2}} \left(\frac{\hat{x}}{x_0} + i \frac{\hat{p}}{p_0} \right)$, $\hat{a} = \frac{1}{\sqrt{2}} \left(\frac{\hat{x}}{x_0} - i \frac{\hat{p}}{p_0} \right)$ where $x_0 = \sqrt{\frac{\hbar}{m\omega}}$, $p_0 = \sqrt{\hbar m\omega}$ are the characteristic size and momentum of vibrational ground state $|0\rangle$, respectively. Furthermore, by Taylor series,

$$\psi(x + \delta x) = \left[\sum_{n=0}^{\infty} \delta x^n \frac{\partial^n}{\partial x^n} \right] \psi(x) = \left[\sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i}{\hbar} \delta x \cdot \hat{p} \right)^n \right] \psi(x) = e^{\frac{i}{\hbar} \delta x \hat{p}} \psi(x) \quad (3.35a)$$

$$\psi(p + \delta p) = \left[\sum_{n=0}^{\infty} \delta p^n \frac{\partial^n}{\partial p^n} \right] \psi(x) = \left[\sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{i}{\hbar} \delta p \cdot \hat{x} \right)^n \right] \psi(x) = e^{-\frac{i}{\hbar} \delta p \hat{x}} \psi(x) \quad (3.35b)$$

shows that $\exp\left(\frac{i}{\hbar} \delta x \hat{p}\right)$ is the translation operator in space and $\exp\left(-\frac{i}{\hbar} \delta p \hat{x}\right)$ in momentum. The *displacement operator*

$$\begin{aligned} \hat{D}(\alpha) &= \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}) \\ &= \exp \left[\frac{1}{\sqrt{2}} \left[(\text{Re}(\alpha) + i \text{Im}(\alpha)) \left(\frac{\hat{x}}{x_0} + i \frac{\hat{p}}{p_0} \right) - (\text{Re}(\alpha) - i \text{Im}(\alpha)) \left(\frac{\hat{x}}{x_0} - i \frac{\hat{p}}{p_0} \right) \right] \right] \\ &= \exp \left[i \sqrt{2} \left(\frac{\text{Im}(\alpha)}{x_0} \hat{x} + \frac{\text{Re}(\alpha)}{p_0} \hat{p} \right) \right] \end{aligned} \quad (3.36)$$

then has a translation of $\delta x = \sqrt{2\hbar} \frac{\text{Re}(\alpha)}{p_0}$ in space and $\delta p = \sqrt{2\hbar} \frac{\text{Im}(\alpha)}{x_0}$ in momentum.

From the above properties, $\hat{D}(\alpha)$ can be represented by the vector α in its complex plane where the real part represents δx and imaginary part represents δp , up to a scale factor. The final state $|\alpha\rangle = \hat{D}(\alpha)|0\rangle$ is called the *coherent state* because it is a coherent superposition of the Fock states

$$\begin{aligned} |\alpha\rangle &= e^{-\frac{1}{2}|\alpha|^2} e^{\alpha \hat{a}^\dagger} |0\rangle \\ &= e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} (\hat{a}^\dagger)^n |0\rangle \\ &= e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \end{aligned} \quad (3.37)$$

Since the Fock states will acquire a phase over time $|n\rangle \rightarrow e^{-in\omega t} |n\rangle$, one may also notice that $|\alpha\rangle$ has the time evolution of $|\alpha\rangle(t) = |\alpha e^{-i\omega t}\rangle$. In the complex plane representation, $|\alpha\rangle$ will rotate clockwise about the origin at a frequency of trapping frequency ω . Furthermore, the average vibrational level of $|\alpha\rangle$ is

$$\begin{aligned} \langle \hat{n} \rangle &= \langle \alpha | \hat{a}^\dagger \hat{a} | \alpha \rangle \\ &= e^{-|\alpha|^2} \sum_{m=0}^{\infty} \frac{(\alpha^*)^m}{\sqrt{m!}} \langle m | \hat{a}^\dagger \hat{a} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \\ &= e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} n \\ &= e^{-|\alpha|^2} \sum_{j=0}^{\infty} \frac{|\alpha|^{2j+2}}{j!} \\ &= e^{-|\alpha|^2} |\alpha|^2 \sum_{j=0}^{\infty} \frac{|\alpha|^{2j}}{j!} \\ &= e^{-|\alpha|^2} |\alpha|^2 e^{|\alpha|^2} \\ &= |\alpha|^2. \end{aligned} \quad (3.38)$$

Note that the *coherent states* form an overcomplete basis

$$\begin{aligned} \langle \alpha | \beta \rangle &= \langle 0 | \hat{D}^\dagger(\alpha) \hat{D}(\beta) | 0 \rangle \\ &= \langle 0 | \hat{D}(-\alpha) \hat{D}(\beta) | 0 \rangle \\ &= \langle 0 | e^{\frac{1}{2}(\alpha^* \beta - \alpha \beta^*)} \hat{D}(\beta - \alpha) | 0 \rangle \\ &= e^{\frac{1}{2}(\alpha^* \beta - \alpha \beta^*)} e^{-\frac{1}{2}|\beta - \alpha|^2} \sum_{n=0}^{\infty} \left\langle 0 \left| \sum_{n=0}^{\infty} \frac{(\beta - \alpha)^n}{\sqrt{n!}} \right| n \right\rangle \\ &= e^{\frac{1}{2}(\alpha^* \beta - \alpha \beta^*)} e^{-\frac{1}{2}(|\alpha|^2 + |\beta|^2 - \alpha^* \beta - \alpha \beta^*)} \\ &= e^{-\frac{1}{2}(|\alpha|^2 + |\beta|^2 - 2\alpha^* \beta)} \neq \delta_{\alpha, \beta}. \end{aligned} \quad (3.39)$$

Figure 3.3: Complex plane of α which represents the *coherence state*. $|0\rangle$ lies on the origin of the plane (in green). After translation by $\hat{D}(\alpha)$, the state becomes $|\alpha\rangle$ in red.

Figure 3.4: Schematic of Raman beams in harmonic trap as displacement operator. The red arrow represents one of the Raman beams and the blue represent another one.

3.3.2 Displacement Operator by Raman Beams

Displacement operator $\hat{D}(\alpha)$ can be created experimentally by many methods. Here, I use a pair of Raman beams to generate the operation $\hat{D}(\alpha)$. The source of the ladder operator in equation (3.34) comes from the position operator $\hat{x} \equiv x_0 (\hat{a} + \hat{a}^\dagger)$ in extra position dependence phase $\phi = \mathbf{k} \cdot \mathbf{x}$ for Rabi frequency in equation (2.6). Here, $x_0 = \sqrt{\hbar / (2m\omega_{\text{vib}})}$ is the rms width of the vibrational state in the trap with vibrational frequency ω_{vib} . When putting into two photon Raman equation (2.28), an extra term of $\exp[(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{x}]$ appears in the two photon Rabi frequency. If the beams are counter-propagating, i.e. $\mathbf{k}_1 \approx -\mathbf{k}_2$, then the extra term will be $e^{i2k_{\text{eff}}\hat{x}} \approx 1 + i2k_{\text{eff}}\hat{x} \equiv 1 + i\eta(\hat{a} + \hat{a}^\dagger)$ where $k_{\text{eff}} = \frac{k_1 + k_2}{2}$. The higher order of the exponent term $i\eta(\hat{a} + \hat{a}^\dagger)$ has to be considered because the coupling between the vibrational states are involved. Since $\langle n-1 | \hat{a} | n \rangle = \langle n | \hat{a}^\dagger | n-1 \rangle = \sqrt{n}$, a factor of $i\sqrt{n}\eta$ should be added onto the Rabi frequency for the transition involved between them. In this derivation, we assume

$$\eta = kx_0 = \sqrt{\hbar k^2 / 2m} / \omega_{\text{vib}} = \sqrt{\omega_{\text{rec}} / \omega_{\text{vib}}} \quad (3.40)$$

to be small so that we can omit any higher order term than η . This *Lamb-Dicke regime* is valid in practice because we want the trap to be deep such that ω_{vib} is much higher than the recoil frequency ω_{rec} .

Without loss of generality, I consider a system with two internal states of energy separation of $\hbar\omega'$ (the upper state acts as the intermediate state for Raman transition) and three vibrational ground states (This number of state can eventually be increased to infinity) as shown in Figure 3.4. The ground vibrational states are labelled as $|n\rangle$ and excited states are labelled as $|n'\rangle$. The Raman beams are prepared with large single photon detuning Δ and small two-photon detuning that is equal to the vibrational frequency ω_{vib} . We use the

convention such that the unperturbed Hamiltonian is ($\circ = 0$)

$$\hat{H}_0 = \hbar \begin{bmatrix} 0 & \circ & \circ & \circ & \circ \\ \circ & \omega_{\text{vib}} & \circ & \circ & \circ \\ \circ & \circ & 2\omega_{\text{vib}} & \circ & \circ \\ \circ & \circ & \circ & \omega' & \circ \\ \circ & \circ & \circ & \circ & \omega' + \omega_{\text{vib}} \end{bmatrix}. \quad (3.41)$$

Here we only consider the transition $|n\rangle \leftrightarrow |n'\rangle \leftrightarrow |n+1\rangle$. In the presence of Raman lasers with frequency $\omega_1 = \omega' + \Delta$ and $\omega_2 = \omega' + \Delta - \omega_{\text{vib}}$, the Hamiltonian is

$$\hat{H} = \hat{H}_0 + \frac{\hbar}{2} \begin{bmatrix} \circ & \circ & \circ & \Omega_1^* e^{i\omega_1 t} & \circ \\ \circ & \circ & \circ & -i\sqrt{1}\eta\Omega_2^* e^{i\omega_2 t} & \Omega_1^* e^{i\omega_1 t} \\ \circ & \circ & \circ & \circ & -i\sqrt{2}\eta\Omega_2^* e^{i\omega_2 t} \\ \Omega_1 e^{-i\omega_1 t} & i\sqrt{1}\eta\Omega_2 e^{-i\omega_2 t} & \circ & \circ & \circ \\ \circ & \Omega_1 e^{-i\omega_1 t} & i\sqrt{2}\eta\Omega_2 e^{-i\omega_2 t} & \circ & \circ \end{bmatrix}. \quad (3.42)$$

Note that a factor of $i\sqrt{n}\eta$ is added accordingly for vibrational level jump. By using the unitary operator

$$\begin{aligned} \hat{U} = & 0|0\rangle\langle 0| + e^{-i(\omega_1 - \omega_{\text{vib}})}|1\rangle\langle 1| + e^{-i(\omega_1 - 2\omega_{\text{vib}})}|2\rangle\langle 2| \\ & + |0'\rangle\langle 0'| + e^{\omega_{\text{vib}} t}|1'\rangle\langle 1'| + e^{2\omega_{\text{vib}} t}|2'\rangle\langle 2'| \end{aligned} \quad (3.43)$$

and offsetting of $-\hbar(\Delta + \omega')$ to it, the transformed Hamiltonian becomes

$$\hat{H} = \frac{\hbar}{2} \begin{bmatrix} \circ & \circ & \circ & \Omega_1^* & \circ & \circ \\ \circ & \circ & \circ & -i\sqrt{1}\eta\Omega_2^* & \Omega_1^* & \circ \\ \circ & \circ & \circ & \circ & -i\sqrt{2}\eta\Omega_2^* & \Omega_1^* \\ \Omega_1 & i\sqrt{1}\eta\Omega_2 & \circ & -2\Delta & \circ & \circ \\ \circ & \Omega_1 & i\sqrt{2}\eta\Omega_2 & \circ & -2\Delta & \circ \\ \circ & \circ & \Omega_1 & \circ & \circ & -2\Delta \end{bmatrix}. \quad (3.44)$$

The derivation is similar to §2.6.1. Letting the wavefunction be $|\psi\rangle = c_0|0\rangle + c_1|1\rangle + c_2|2\rangle +$

$C_0 |0'\rangle + C_1 |1'\rangle + C_2 |2'\rangle$, the Schrödinger equation shows that

$$i\dot{C}_0 = \frac{\Omega_1^*}{2} C_0 \quad (3.45a)$$

$$i\dot{C}_1 = -i\sqrt{1}\eta \frac{\Omega_2^*}{2} C_0 + \frac{\Omega_1^*}{2} C_1 \quad (3.45b)$$

$$i\dot{C}_2 = -i\sqrt{2}\eta \frac{\Omega_2^*}{2} C_1 + \frac{\Omega_1^*}{2} C_2 \quad (3.45c)$$

$$i\dot{C}_0 = \frac{\Omega_1}{2} c_0 + i\sqrt{1}\eta \frac{\Omega_2}{2} c_1 - \Delta C_0 \quad (3.45d)$$

$$i\dot{C}_1 = \frac{\Omega_1}{2} c_0 + i\sqrt{1}\eta \frac{\Omega_2}{2} c_1 - \Delta C_1 \quad (3.45e)$$

$$i\dot{C}_2 = \frac{\Omega_1}{2} c_2 - \Delta C_2. \quad (3.45f)$$

After the *adiabatic elimination* on C_n , the new Hamiltonian of only state $|n\rangle$ becomes

$$\begin{aligned} \hat{H} &= \frac{\hbar |\Omega_1|^2}{4\Delta} \hat{I} + \frac{\hbar}{4\Delta} \eta \begin{bmatrix} \circ & i\sqrt{1}\Omega_1^* \Omega_2 & \circ \\ -i\sqrt{1}\Omega_1 \Omega_2^* & \circ & i\sqrt{2}\Omega_1^* \Omega_2 \\ \circ & -i\sqrt{2}\Omega_1 \Omega_2^* & \circ \end{bmatrix} \\ &= \frac{\hbar |\Omega_1|^2}{4\Delta} \hat{I} + i\eta \frac{\hbar}{2} (\Omega_{2p} \hat{a} - \Omega_{2p}^* \hat{a}^\dagger) \end{aligned} \quad (3.46)$$

where $\Omega_{2p} = \frac{\Omega_1 \Omega_2^*}{2\Delta}$ is the two-photon Rabi frequency. The first term corresponds to the AC Stark shift from one of the Raman beams. One can show that the Stark shift from another beam appear when we include the transition $|n\rangle \leftrightarrow |(n+1)'\rangle \leftrightarrow |n+1\rangle$. The second term is the source of displacement operator as the time evolution operator is

$$\hat{U}(t) = \exp\left(-\frac{it}{\hbar} \hat{H}\right) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}) = \hat{D}(\alpha)$$

where

$$\alpha = -\eta \Omega_{2p} t / 2. \quad (3.47)$$

3.3.3 Schrödinger Cat States

The cat state is created by entangling coherent state with spin (internal state), or $|s, \alpha\rangle \equiv |s\rangle \otimes |\alpha\rangle$ where $s = \uparrow, \downarrow$ is the state of a two-level system. The full sequence is shown in [Figure 3.5](#). First, the system is prepared to $|\downarrow, 0\rangle$ state by Raman sideband cooling and optical pumping (refer to (§10.3)). Then, a $\pi/2$ pulse is sent to create a superposition of $|\uparrow, 0\rangle$ and $|\downarrow, 0\rangle$ before $\hat{D}(\alpha)$ operate it to $|\uparrow, \alpha\rangle$. After that, a π pulse flip the spin to $|\uparrow, 0\rangle$ and $|\downarrow, \alpha\rangle$ before another $\hat{D}$ operation. The second $\hat{D}$ has a phase shift ϕ relative to the first one which is controllable by user, so that both of the states are $|\uparrow, \alpha e^{i\phi}\rangle$ and $|\downarrow, \alpha\rangle$. When another $\pi/2$ pulse is applied, a superposition will occur. To express the wavefunction mathematically, note that the π and $\pi/2$ pulses contribute phase that follow equations (3.4)

Figure 3.5: Sequence of Schrödinger cat interferometer. The $\pi/2$ and π pulses change the internal states $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$ and the displacement operator $\hat{D}$ only operates on $|\uparrow\rangle$ state.

and (3.5) with $\phi = 0$. Therefore, the first $\pi/2$ pulse and $\hat{D}(\alpha)$ change the wavefunction to

$$|\downarrow, 0\rangle \xrightarrow{\pi/2} \sqrt{\frac{1}{2}} |\downarrow, 0\rangle - i\sqrt{\frac{1}{2}} |\uparrow, 0\rangle \xrightarrow{\hat{D}(\alpha)} \sqrt{\frac{1}{2}} |\downarrow, 0\rangle - i\sqrt{\frac{1}{2}} |\uparrow, \alpha\rangle. \quad (3.48)$$

Then, the π pulse and $\hat{D}(\alpha)$ change the $|\downarrow, 0\rangle$ to

$$|\downarrow, 0\rangle \xrightarrow{\pi} -i|\uparrow, 0\rangle \xrightarrow{\hat{D}(\alpha e^{i\phi})} -i|\uparrow, \alpha e^{i\phi}\rangle \quad (3.49)$$

and $-i|\uparrow, \alpha\rangle$ to

$$-i|\uparrow, \alpha\rangle \xrightarrow{\pi} -|\downarrow, \alpha\rangle, \quad (3.50)$$

leading the function before the last $\pi/2$ pulse to be

$$-i\sqrt{\frac{1}{2}} |\uparrow, \alpha e^{i\phi}\rangle - \sqrt{\frac{1}{2}} |\downarrow, \alpha\rangle. \quad (3.51)$$

The last $\pi/2$ pulse converts $-i|\uparrow, \alpha e^{i\phi}\rangle$ to

$$-i|\uparrow, \alpha e^{i\phi}\rangle \xrightarrow{\pi/2} -\sqrt{\frac{1}{2}} |\downarrow, \alpha e^{i\phi}\rangle - i\sqrt{\frac{1}{2}} |\uparrow, \alpha e^{i\phi}\rangle \quad (3.52)$$

and $-\sqrt{\frac{1}{2}} |\downarrow, \alpha\rangle$ to

$$-\sqrt{\frac{1}{2}} |\downarrow, \alpha\rangle \xrightarrow{\pi/2} -\sqrt{\frac{1}{2}} |\downarrow, \alpha\rangle + i\sqrt{\frac{1}{2}} |\uparrow, \alpha\rangle. \quad (3.53)$$

The final wavefunction $|\psi_{\text{cat}}\rangle = c_{\uparrow} |\uparrow\rangle + c_{\downarrow} |\downarrow\rangle$ or

$$\begin{aligned} |\psi_{\text{cat}}\rangle &= -\frac{1}{2} |\downarrow, \alpha e^{i\phi}\rangle - i\frac{1}{2} |\uparrow, \alpha e^{i\phi}\rangle - \frac{1}{2} |\downarrow, \alpha\rangle + i\frac{1}{2} |\uparrow, \alpha\rangle \\ &= \frac{i}{2} (|\uparrow, \alpha\rangle - |\uparrow, \alpha e^{i\phi}\rangle) - \frac{1}{2} (|\downarrow, \alpha\rangle + |\downarrow, \alpha e^{i\phi}\rangle) \end{aligned} \quad (3.54)$$

will be the superposition of both equations above, resulting the coefficient to be

Figure 3.6: The population of the state $|\uparrow\rangle$ vs the relative phase ϕ between the displacement operation in Schrödinger cat experiment. The function is shown in equation (3.57).

$$\begin{aligned}
 c_{\uparrow} &= \frac{i}{2} (|\alpha\rangle - |\alpha e^{i\phi}\rangle) \\
 &= \frac{i}{2} e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (1 - e^{in\phi}) |n\rangle
 \end{aligned} \tag{3.55}$$

and

$$\begin{aligned}
 c_{\downarrow} &= -\frac{1}{2} (|\alpha\rangle + |\alpha e^{i\phi}\rangle) \\
 &= -\frac{1}{2} e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} (1 + e^{in\phi}) |n\rangle.
 \end{aligned} \tag{3.56}$$

The population of the state $|\uparrow\rangle$ is (using the property in equation (3.39))

$$\begin{aligned}
 |c_{\uparrow}|^2 &= \frac{1}{4} (2 - \langle \alpha | \alpha e^{i\phi} \rangle - \langle \alpha e^{i\phi} | \alpha \rangle) \\
 &= \frac{1}{2} - \frac{1}{4} e^{-\frac{1}{2}|\alpha|^2} (2|\alpha|^2 - 2|\alpha|^2 e^{i\phi}) - \frac{1}{4} e^{-\frac{1}{2}|\alpha|^2} (2|\alpha|^2 - 2|\alpha|^2 e^{-i\phi}) \\
 &= \frac{1}{2} - \frac{1}{4} e^{-|\alpha|^2} (1 - \cos \phi - i \sin \phi) - \frac{1}{4} e^{-|\alpha|^2} (1 - \cos \phi + i \sin \phi) \\
 &= \frac{1}{2} - \frac{1}{4} e^{-|\alpha|^2} (1 - \cos \phi) (e^{i|\alpha|^2 \sin \phi} + e^{-i|\alpha|^2 \sin \phi}) \\
 &= \frac{1}{2} - \frac{1}{2} e^{-|\alpha|^2 (1 - \cos \phi)} \cos (|\alpha|^2 \sin \phi)
 \end{aligned} \tag{3.57}$$

and of the $|\downarrow\rangle$ state is $|c_{\downarrow}|^2 = 1 - |c_{\uparrow}|^2$.

3.3.4 Odd and Even Cat States

The odd and even cat states are defined by

$$|\psi_{\text{odd}}\rangle(\phi) = |\alpha\rangle - |\alpha e^{i\phi}\rangle \quad (3.58a)$$

$$|\psi_{\text{even}}\rangle(\phi) = |\alpha\rangle + |\alpha e^{i\phi}\rangle. \quad (3.58b)$$

In fact, the general cat state is

$$|\psi_{\phi_M}\rangle(\phi) = |\alpha\rangle - e^{i\varphi} |\alpha e^{i\phi}\rangle. \quad (3.59)$$

This can be built by changing the phase ϕ_M of the last $\pi/2$ pulse, which will change equation (3.52) and (3.53) to

$$-i|\uparrow, \alpha e^{i\phi}\rangle \xrightarrow[\phi_M]{\pi/2} -e^{-i\phi_M} \sqrt{\frac{1}{2}} |\downarrow, \alpha e^{i\phi}\rangle - i\sqrt{\frac{1}{2}} |\uparrow, \alpha e^{i\phi}\rangle \quad (3.60)$$

and

$$-|\downarrow, \alpha\rangle \xrightarrow[\phi_M]{\pi/2} -\sqrt{\frac{1}{2}} |\downarrow, \alpha\rangle + ie^{i\phi_M} \sqrt{\frac{1}{2}} |\uparrow, \alpha\rangle. \quad (3.61)$$

The final wavefunction will then be

$$\begin{aligned} |\psi_{\text{cat}}\rangle &= -e^{-i\phi_M} \frac{1}{2} |\downarrow, \alpha e^{i\phi}\rangle - i\frac{1}{2} |\uparrow, \alpha e^{i\phi}\rangle - \frac{1}{2} |\downarrow, \alpha\rangle + ie^{i\phi_M} \frac{1}{2} |\uparrow, \alpha\rangle \\ &= \frac{i}{2} e^{i\phi_M} (|\uparrow, \alpha\rangle - e^{-i\phi_M} |\uparrow, \alpha e^{i\phi}\rangle) - \frac{1}{2} (|\downarrow, \alpha\rangle + e^{-i\phi_M} |\downarrow, \alpha e^{i\phi}\rangle) \\ &= \frac{i}{2} e^{i\phi_M} |\psi_{\varphi}\rangle(\phi) \otimes |\uparrow\rangle - \frac{1}{2} |\psi_{\phi_M+\pi}\rangle(\phi) \otimes |\downarrow\rangle \end{aligned} \quad (3.62)$$

and the population in state $|\uparrow\rangle$ is

$$|c_{\uparrow}|^2 = \frac{1}{2} - \frac{1}{2} e^{-|\alpha|^2(1-\cos\phi)} \cos(\phi_M + |\alpha|^2 \sin\phi). \quad (3.63)$$

Experimentally, φ can be changed easily to convert to odd and even cat, which is shown in §10.5.

Part II

Experimental Apparatus

Chapter 4

Optics and Electronics

4.1 Laser Setup and Configuration

The lasers used for the experiment are summarized in Table 4.1. The Cooling TA, which is injected from the Cooling Laser after double passed AOM, is used for all variety of beams that come from hyperfine ground state of $F = 3$ in D2 line: MOT beams, probe beam, depump beam and blow beam, shown in Figure 4.1. The double passed AOM¹ is used to control the laser frequency of Cooling TA to match the required frequency (See Figure 4.1). The Cooling Laser is locked to $F = 3 \rightarrow F' = 2 - 4$ crossover, or -92 MHz detuned from the *cycling transition* ($F = 3 \rightarrow F' = 4$). The double passed AOM increases the frequency in the range of $2 \times (+80 \pm 20)$ MHz such that the laser frequency range of the Cooling TA is from $+28$ MHz to $+108$ MHz detuned from the *cycling transition*. Then, $(-80, -80, -55)$ MHz AOM is used to red detune the beam to *cyclic transition* for MOT beams, blow beam and probe beam respectively. A -200 MHz AOM is used for depump beam in $F = 3 \rightarrow F' = 3$ transition. Repump Laser is locked at $F = 2 \rightarrow F' = 2 - 4$ crossover and it is switched by a -80 MHz AOM. These AOMs are also used for the switch and power modulation.

The Dipole Laser is sourced from a 5W Ti Sapphire laser, split into two beams by PBS and switched by two identical AOMs and eventually coupled into the hollow core fiber at about 100 mW each. Its tunable wavelength is typically set in the range of 808-875 nm, or 6-34 THz red detuned from the nearest transition (D1 line) since it is mainly used for dipole trap, lattices, and Raman beams. The frequency drifts a few hundreds of MHz over time measured by a wavelength meter (HighFinesse: WS6-200) which is negligible compared to the detuning.

The AI, Optical Pump, EIT and Cat Lasers (Used in Chapter 7, 8, 9 and 10, respectively) are set or removed depending on each experimental needs. The details of the setup is shown in Figure 4.2. EIT Laser is locked to $F = 3 \rightarrow F' = 2 - 3$ cross over of D1 line so that it is 182 MHz detuned from the transition $F = 3 \rightarrow F' = 3$, which is one of the required beams for EIT, called the EIT control beam. Before performing frequency shift, part of EIT laser

¹ All AOM in this thesis uses Gooch & Housego: 3080-122 for RF frequency less than 100 MHz, Brimrose: TEM-110-25-780 for 110 MHz and Brimrose: TEM-200-50-780 for frequency between 150 and 250 MHz.

Name	Laser Type	Brand/ Model No	Transition locked & Locking mechanism
Cooling Laser	Cateye diode laser	Moglabs/ CEL002	D2 line, $F = 3 \rightarrow F' = 2 - 4$ CO, Doppler free spectroscopy
Cooling TA	Laser amplifier	Toptica/ BoosTA	Injected from Cooling Laser
Repump Laser	ECDL	Toptica/ DL100	D2 line, $F = 2 \rightarrow F' = 1 - 2$ CO, Doppler free spectroscopy
EIT Laser	ECDL	Moglabs/ ECD004	D1 line, $F = 3 \rightarrow F' = 2 - 3$ CO, Doppler free spectroscopy
Dipole Laser	Ti Sapphire laser	M Squares/ SolsTiS	Self etalon lock. Will drift by few 100 MHz over time.
AI Laser	ECDL	Moglabs/ ECD004	D2 line of ^{87}Rb atom, $F = 2 \rightarrow F' = 2 - 3$ CO, Doppler free spectroscopy
Optical Pump Laser	ECDL	Moglabs/ ECD004	D1 line, $F = 2 \rightarrow F' = 2 - 3$ CO, Doppler free spectroscopy
Cat Laser	Cateye diode laser	Moglabs/ CEL002	A few GHz <i>frequency offset lock</i> to Optical Pump Laser
Phase Locked Laser	DBR laser	Vescent/ D2-100-DBR	3.035 GHz <i>phase lock</i> (by Vescent: D2-135) to Cat Laser

Table 4.1: Technical information of the lasers used in this project. All lasers are locked by ^{85}Rb atom unless otherwise stated. Here, F refers to the hyperfine ground state of both D1 and D2 line ($5^2S_{1/2}$), and F' refers to the hyperfine excited state of D1 ($5^2P_{1/2}$) or D2 ($5^2P_{3/2}$) line depending on the laser wavelength used. AI Laser and Optical Pump Laser are the same laser but locked differently under different usage. CO = crossover. ECDL = External cavity diode laser. DBR = Distributed Bragg reflector.

Figure 4.1: Laser frequency diagram from Rubidium-85 (not to scale).

beam is taken for frequency reference for frequency offset lock of Cat Laser, and another part of it is used to make the EIT probe beam. To build EIT control beam, a double passed AOM of -91 MHz is used. On the other hand, the EIT probe beam is 400 MHz blue shifted by another 200 MHz double passed AOM before sending to a fiber EOM (EOSPACE: PM-0K5-10-PFU-PFU-780) whose frequency is generated by a synthesized RF signal generator (Hewlett Packard: 8648C). After EOM, the beam is filtered using an etalon (QuantaSER: FPE002B).

AI beams are required to be far detuned from resonance because they are used for two-photon process. Therefore, AI Laser is locked to $F = 2 \rightarrow F' = 2 - 3$ cross over of D2 line of ^{87}Rb atom, generating an AI beam that is 1.276 GHz red shifted from the nearest $F = 3 \rightarrow F' = 2$ transition of ^{85}Rb D2 line after a -200 MHz AOM. Another AI beam is generated by a fiber EOM (Photline: NIR-MPX800-LN-10-P-P-FC-FC) where the RF source is stated in §4.3.

Cat Laser is locked to EIT Laser at tunable frequency, usually several GHz away. The locking mechanism is described in §4.2. The beams are then split into two for two photon process, each frequency is shifted by different AOM where the frequency difference can be manipulated. Then, they are coupled into different fibers and sent to the chamber from both

Figure 4.2: The schematic drawing of laser alignment. The value in AOM and EOM indicates the frequency shift in MHz.

Figure 4.3: Typical beam alignment near vacuum chamber. Dashed octagon represents the vacuum chamber. Three pairs of orthogonal MOT beams enter the chamber, where the third pair is not shown in the drawing.

top and bottom direction.

Figure 4.3 shows the typical beam alignment near vacuum chamber, where it may be modified for each experiment needs. Three pairs of orthogonal MOT beams enter the chamber, building cold atom cloud at the center of the chamber. The MOT beams are collimated by a 75 mm lens with optical power of ~ 15 mW per cooling beam and ~ 2 mW per repump beam. A 4-cm-long hollow core fiber is placed vertically below the atom cloud. To perform the coupling, two lenses with 150 mm of focal length are placed at both ends of hollow core fiber but outside the chamber. Since all other fibers are from Thorlabs (P3-780PM-FC-x series), aspheric lens of focal length 18.4 mm are used for mode matching purpose.

The probe beam enters the fiber from top to measure atom number in the fiber by absorption method. It is then passed to avalanche photodiode (Hamamatsu: C10508-01) for detection. Although the depump beam follows the same path, it would not be turned on during detection. Dipole beams are sent from both directions to form an optical lattice to trap atom vertically. Most of the time, only the bottom dipole beam is turned on for the radial trapping and guidance of atom loading.

The blow beam with maximum optical intensity of ~ 90 mW/cm² is sent above the fiber tip to purposely blow unwanted atoms away from fiber tip. On the other hand, the optical pump beam is hit directly on the fiber wall to perform optical pump process in the fiber. The polarization is set to be parallel to fiber direction so that any scattering beam polarization is also on the same fiber direction (same as to magnetic field). Moreover, a microwave antenna with radio frequency of 3.035 GHz is prepared to couple the hyperfine ground state of ⁸⁵Rb atom.

Figure 4.5: (a) Sketch and (b) Image of frequency offset lock signal vs δ_{bs} . The delay line here is long for illustration purpose. In practice, delay line of 1 m is used for minimum zero crossing signal.

the frequency offset lock. One can include them to complete the formalism. The result of the mixer is the product of the signal $V_b V_s \propto e^{i(\omega_b - \omega_s)t} + e^{i(\omega_b + \omega_s)t} + \text{c.c.}$ which consists of the sum and difference of both frequencies. Passing it through an appropriate low pass filter (Minicircuit: BLP-100+) to filter the high frequency above 100 MHz will remain the signal $e^{i\delta_{bs}t} + \text{c.c.}$ where³ $\delta_{bs} = \omega_b - \omega_s$. This signal $V_1 \equiv e^{i\delta_{bs}t} + \text{c.c.}$ is split into two by RF splitter (Minicircuit: ZFSC-2-1+) with different path length difference L to introduce phase shift to $V_2 = e^{i(\delta_{bs}t + \phi)} + \text{c.c.}$ before sending into the phase detector (Minicircuit: ZRPD-1+). This phase $\phi = \delta_{bs}\tau$ depends on the time delay in the path difference $\tau = \frac{L}{v}$ where $v \approx 2 \times 10^8 \text{ m s}^{-1}$ is the speed of signal in cable. In the current setup, $L = 1 \text{ m}$ coaxial cable is used as the delay line so that $\tau \approx 5 \text{ ns}$. Phase detector multiplies the two signal and returns $V_1 V_2 = e^{i\phi} + e^{i(2\delta_{bs}t + \phi)} + \text{c.c.}$ and the high frequency component is removed by another low pass filter (Minicircuit: BLP-1.9+). Therefore, the final signal from frequency offset lock V_{FOL} is

$$V_{\text{FOL}} = V_{\text{FOL}}^{\max} \cos \phi = V_{\text{FOL}}^{\max} \cos \delta_{bs}\tau \quad (4.1)$$

where the amplitude of the signal $V_{\text{FOL}}^{\max}$ depends on the RF amplifier and low pass filter. Note that the final signal V_{FOL} is an oscillating signal if you scan the laser frequency, or δ_{bs} with the “period” of $\frac{1}{\tau} \approx 200 \text{ MHz}$. This signal consists of zero-crossing that can be used for laser locking feedback. In order to make this signal having only one zero crossing for robustness, $V_{\text{FOL}}^{\max} = V_{\text{FOL}}^{\max}(\delta_{bs})$ is adjusted such that it has an envelope of $\sim 100 \text{ MHz}$ “bandwidth” which is purposely set by the BLP-100+ low pass filter. After setting the locking setup, the locked frequency is stable to within 1 MHz in an hour measured by a RF detector (Startek: ATH-65).

³Since the laser will eventually lock at $|\delta_{bs}| \approx 2\pi \times (48 \text{ MHz})$, I choose Minicircuit BLP-100+ which cuts off at about 100 MHz.

4.3 Data Acquisition System and RF Sources

The experiments in this thesis require multiple switches and frequency controls for different beam. Therefore, high number of digital outputs are needed. Here, all our data acquisition hardware is used from National Instrument. Three analog/digital (A/D) modules (PXIe-6363, PXI-6733, PXIe-6356) are used and mounted into chassis (PXIe-1073) which is connected to a PC by LabVIEW⁴. To synchronize these modules, a digital PFI output port from PXIe-6363 is connected to digital PFI trigger input port of every other module to make sure they run the sequence at the same time. In addition, PXIe-6363 is triggered by 50 Hz AC line to prevent the noise from the random phase of AC line. To protect the module, the AC line is stepped down and passed to shunt regulator to 5 V before triggering the A/D module.

PXIe-6356 is mainly used for analog input with sampling rate of 1 MS s^{-1} , typically used to measure voltage from APD in Figure 4.3. On the other hand, all three modules are used for analog output due to limited total number of channels. These channels are used for controlling the current of every magnetic coil of the setup, all of which are at the sampling rate of 1 MS s^{-1} . PXIe-6363 is mainly used for digital outputs which controlled AOMs and coil TTL switches, as well as direct digital synthesizer (DDS) command, at the sampling rate of 4 MS s^{-1} . These three modules contain internal clock with an accuracy of few tens of ppm, which is equivalent to maximum time accuracy of few tens of ps. All three modules are inside the chassis and extended to USB connection block in a 19-inch rack, which also contains DDS and single sideband (SSB) modulator. Since the rack is some distance away from the optical setup, a cluster of long BNC cables are built to connect them through a BNC panel.

Most RFs for AOMs are sourced from DDS (Analog Devices: AD9959) which can send RF frequency up to 250 MHz and they are fully programmable in amplitude, frequency and phase at the resolution of 10, 32 and 14 bits, respectively, and the accuracy of the system clock is 10 ppm. To create a fully controlled RF frequency at 3.035 GHz for EOM in Figure 4.2c and antenna in Figure 4.3, a 2.9 GHz RF generated by Hewlett Packard: E4422B is mixed with 135 MHz RF from DDS using a quadrature single-sideband (SSB) modulator (Analog Device: ADL5375). The advantage of SSB modulator over an ordinary mixer is that it will provide only one sideband and suppress the other by interference.

⁴We thank Prof. James Thompson's group at JILA for allowing us to adapt their data acquisition program to our experiment.

Figure 4.6: Circuit diagram of shunt regulator by Zener diode as a trigger source of data acquisition modules.

Figure 4.7: Image of data acquisition setup. All A/D modules are inside the chassis and are connected to BNC connection blocks which are mounted on the 19 inch rack for user to plug and play.

Chapter 5

Science Chamber Parts

5.1 Hollow Core Fibers

To choose the type of hollow core fiber (HCF), it is required to have a large hollow core size so that more atoms can be loaded into the fiber. Moreover, the fiber needs to be able to maintain the Gaussian profile in the fiber and having high laser damage threshold so that high power dipole beam can be coupled at sub-Watt level. The fiber also needs to have a broad spectral coverage to include transition wavelength of Rubidium (780 nm and 795 nm) and dipole beams of wavelength from 795 to 900 nm.

Figure 5.1: (a) Far field and (b) Near field beam profile image from HCF. (c) Image of HCF under microscope.

Figure 5.2: (a) A fit of beam waist w vs axial distance z from fiber tip measured using equation (2.20). The resulting fitting parameter is $w_0 = 23.3(5) \mu\text{m}$ and $\theta = 0.0208(4)$. (b) A typical fit for beam waist by knife-edge measurement at position $z = 5 \text{ mm}$. The equations in both figures indicate the fitting equations.

Therefore, the HCF from Glophotonics with model number PMC-C-TiSa_Er-7C was chosen. It was the only fiber available commercially in the market with the properties mentioned. According to the datasheet, it has total diameter of $300(1) \mu\text{m}$, core diameter of $63(1) \mu\text{m}$ and mode field diameter ($1/e^2$) of $44(1) \mu\text{m}$. The beam waist is measured to be $23.3(5) \mu\text{m}$ by knife edge measurement¹. It had a coupling efficiency of around 70% while maintaining its Gaussian profile both at near and far field. To measure far field image, beam profiler from Newport (LBP2-VIS) is used to image the collimated beam after HCF. Near field image is taken using imaging lens system by a pair of lenses with focal lengths of 150 mm and 1000 mm so that the beam profiler placed at the focal point obtained an image of the fiber tip at a magnification of 6.7. Large magnification is chosen because the large pixel size of $9.9 \mu\text{m}$ of the beam profiler is not enough resolve the near field beam.

5.2 Vacuum System

The vacuum design is shown in Figure 5.3a. The custom-made aluminum fiber holder is screwed into the spherical octagon vacuum chamber (Kimball Physics: MCF800-SphOct-G2C8). Three 4-cm-long HCFs² are then glued onto the fiber holder by vacuum epoxy. The fibers are placed such that the fiber tips are about half centimeter below the center of the chamber, where cold atoms are created. The pressure in the vacuum chamber is mainly measured by an ion gauge and maintained by a 20 Ls^{-1} ion pump (both from Agilent) between 10^{-8} to 10^{-9} torr. The Rubidium source in the chamber came from Rubidium ampule from Espi Metals, and its holder is connected to a right-angle valve.

¹This work is credit to Iap Yap Chang Jie in his final year project.

²Multiple HCF placed are meant for backup fiber only. Therefore, only one HCF are used.

Figure 5.3: (a) Diagram of hollow core fiber mounted in vacuum chamber. (b) Image of science chamber setup.

5.3 Magnetic Coils

Magnetic coil system is important in magneto-optical trap (MOT) setup and controlling the magnetic field environment for atoms. In order to control the ambient magnetic field, three pairs of Helmholtz compensation coils are aligned along three different axes (name as x , y , and z coils, direction refer to [Figure 5.3a](#)). Each compensation coil has ~ 15 turns of diameter range between 20 to 30 cm due to space availability. Then, x and y coils are connected to 10 A power supply (Elektro-Automatik: EA-PS 3016-10 B). On the other hand, z coils are separated into upper z and lower z coils which are connected to 20 A power supply (Elektro-Automatik: EA-PS 3016-20 B) to make sure atoms along HCF do not experience magnetic field gradient. These power supplies are set to be constant current mode which are voltage controlled by the analog outputs from data acquisition modules.

To build the MOT, a pair of anti-Helmholtz coils, called MOT coils are required to provide a magnetic field gradient around 10 G cm^{-1} . Here, two identical coils are built with 80 turns and diameter $2r$ of $\sim 17 \text{ cm}$, separated by a distance d of $\sim 17 \text{ cm}$. Under this design, a field gradient of 10 G cm^{-1} could be created at a current I of 15 A. [Figure 5.4](#) shows the fit of magnetic field B across the coils by the equation

$$B(z) = \frac{\mu_0 I}{2r} \left(\frac{1}{\left(1 + \left(\frac{z}{r} + \frac{d}{2r}\right)^2\right)^{3/2}} - \frac{1}{\left(1 + \left(\frac{z}{r} - \frac{d}{2r}\right)^2\right)^{3/2}} \right) \quad (5.1)$$

where $\mu_0 = 0.4\pi \text{ G cm A}^{-1}$ is magnetic permeability. However, the MOT coils generate a power over 100 W at a typical current of 17 A, resulting the whole vacuum chamber's temperature to be over 60°C . Constant high chamber temperature creates fluctuating refractive index of the surrounding air due to heat convection process, resulting the HCF coupling efficiency to have more than 50 % fluctuation.

Therefore, aluminum coil mounts with water cooling function are designed such that the whole MOT coils are immersed into flowing water. To prevent any possible electrical contact between coils and water, the coils are painted with a thermal conducting resin (Loctite: Stycast 2850 MT) and its catalyst. After putting the coil into the mount, the cover is welded into the mount to withstand high water pressure.

The current of the MOT coils needs to be switched on and off in the experiments. During the switch, inductance from the coil creates back emf of several times of the typical voltage. To protect the circuit in the data acquisition module, an IGBT module from Semikron (SKM75GB12T4) and its driver (SKHI 10/17 R) with isolation function are used as switch.

Figure 5.4: Measurement of magnetic field of anti-Helmholtz MOT coils. The marker shows the experimental data and the lines are fitted result. The fitted magnetic gradient at the center are 6.62(1), 9.94(2), 11.30(2) and 13.09(2) G cm^{-1} respectively.

(a)

(b)

Figure 5.5: (a) Image of resin painted coils. (b) Schematic design of coil mount and its cover.

Chapter 6

Atom Loading into Hollow Core Fiber

6.1 Loading Sequence

Before running the experiment, cold atoms loading into the HCF is necessary. The loading sequence is summarized in [Figure 6.1](#). Cold atoms are prepared and trapped by magneto-optical trap for 500 ms with cooling beam red detuned by 18 MHz. The compensation coils are used to tune the ambient magnetic field $\mathbf{B}_{\text{MOT}}$ that affects the position of atom cloud. Therefore, $\mathbf{B}_{\text{MOT}}$ is changed to optimize the loading efficiency. Then, MOT coil is turned off and the eddy current is allowed to decay in 6 ms. During this period, it is realized that the atom cloud position is pushed due to the presence of MOT beams in slow changing position of zero magnetic field. Therefore, the repump beam is switched off in this period.

Then, the atom cloud is cooled down further by polarization gradient cooling (PGC). The cooling and repump beam power are changed from 20 to 4 mW, and 2 to 0.3 mW, respectively. Cooling beam is red detuned by 62 MHz and the compensation coil is set such that the atoms experience almost zero magnetic field $\mathbf{B}_{\text{PGC}} \approx \mathbf{0}$. This process lasts for 25 ms before the cooling beam is turned off. At this time, the atomic cloud started to fall freely, and I define this time to be $t_{\text{seq}} = 0$. After PGC, the repump beam is continued on for 0.9 ms to make sure all the atoms are in the state $F = 3$. At $t_{\text{seq}} = 1$ ms, the compensation field is set to be along z direction $\mathbf{B}_{\text{rst}} \parallel \hat{z}$ for experiment during loading process. For experiment shown in this chapter, I set $|\mathbf{B}_{\text{rst}}| = 3.6$ G.

6.2 Time of Flight Method

During PGC, all the parameters such as magnetic field, beam power and detuning are optimized to the lowest atom temperature. To determine the atom temperature, an imaging camera (Basker: acA1300-60gmNIR and Navitar: NMV-50M1) is used to capture the ther-

Figure 6.1: Atom loading sequence. Frequency in bracket shows the detuning of cooling from *cycling transition*.

mal expansion of atom cloud. This method is called the *time of flight* (ToF) method. During ToF measurement, the dipole beam is always off so that the cloud does not experience extra force. Without loss of generality, I let the system to be in one dimension. Assuming the cloud follow the Gaussian distribution in position and momentum phase space initially

$$\rho(x, p, t = 0) = \frac{1}{2\pi\sigma_{x0}\sigma_p} e^{-\left(\frac{x^2}{2\sigma_{x0}^2} + \frac{p^2}{2\sigma_p^2}\right)} \quad (6.1)$$

where σ_{x0} represents the initial atom cloud size and σ_p is the momentum width that determines the cloud temperature from equipartition theorem $\frac{1}{2m}\sigma_p^2 = \frac{1}{2}k_B T$. Since there is no force acting on the cloud during free fall (except gravity, which is ignore without changing the concept of ToF method), $\dot{p} = 0$ and $\dot{x} = p/m$. Liouville equation shows that

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho}{\partial x} \dot{x} - \frac{\partial \rho}{\partial p} \dot{p} = -\frac{\partial \rho}{\partial x} \frac{p}{m}. \quad (6.2)$$

With equation (6.1) as initial condition, the time evolution solution equation would be

$$\rho(x, p, t) = \frac{1}{2\pi\sigma_{x0}\sigma_p} e^{-\left(\frac{(x-pt/m)^2}{2\sigma_{x0}^2} + \frac{p^2}{2\sigma_p^2}\right)}. \quad (6.3)$$

Therefore, from the camera point of view, the cloud distribution over time becomes

$$\rho(x, t) = \int_{-\infty}^{\infty} \rho(x, p, t) dp = \frac{1}{\sqrt{2\pi\sigma_x^2(t)}} e^{-\frac{x^2}{2\sigma_x^2(t)}}, \quad (6.4)$$

which is a Gaussian distribution with expanding atom cloud size

$$\sigma_x^2(t) = \sigma_{x0}^2 + \frac{\sigma_p^2}{m^2} t^2 = \sigma_{x0}^2 + \frac{k_B}{m} T t^2. \quad (6.5)$$

Figure 6.2: (a) A fit of cloud size at different time. (b) The relationship between w and t_{seq} is fitted by equation (6.5) with gradient of $k_B T/m = 6.04(3) \text{ cm}^2 \text{ s}^2$, which corresponds to temperature of $T = 6.17(3) \text{ } \mu\text{K}$.

Figure 6.2 shows the fit of experimental measurement on the atomic cloud after PGC by equation (6.4) and (6.5). The fit shows that the cloud temperature to be $6.17(3) \text{ } \mu\text{K}$.

Figure 6.3: Typical images taken by imaging camera during ToF measurement. The time in the images shows the value of t_{seq} during image taking.

Figure 6.4: Image of the configuration when taking atom number measurement.

6.3 Atom Number Measurement

The atom number N in the MOT is estimated by measuring the fluorescence from the atoms. I assume the atoms emit photon at a rate of $s\Gamma/2$ where

$$s = \frac{I/I_{\text{sat}}}{1 + I/I_{\text{sat}} + (2\delta/\Gamma)^2} \approx 0.58 \quad (6.6)$$

is the saturation parameter calculated from the total intensity from 6 MOT cooling beams ($I/I_{\text{sat}} \approx 6 \times 3.895$, $\delta/\Gamma \approx 1.978$), and $\Gamma = 2\pi \times (6.067 \text{ MHz})$ is the decay rate. A lens with diameter $D = 5 \text{ cm}$ is put at the distance of $L = 30 \text{ cm}$ away from MOT to collect the photon at the solid angle fraction of $f = \frac{1}{2} \int_0^{\tan^{-1} D/2L} \sin \theta d\theta \approx 3.45 \times 10^{-3}$.

I put a photodetector (Hamamatsu: S2386-8K) at the image point to convert optical power into electrical current that passes through an $R = 100 \text{ k}\Omega$ resistor at a voltage of $V = 41 \text{ mV}$. Knowing that the photosensitivity of the photodiode at wavelength of $\lambda = 780.241 \text{ nm}$ is $S = 0.55 \text{ A W}^{-1}$, the number of atom is found out to be $N \approx 4.78 \times 10^8$ from the equation $\frac{V}{RS} = Nf \frac{hc}{\lambda} s\Gamma/2$.

Figure 6.5: Sequence in measuring optical depth vs time.

6.4 Optical Depth in HCF

After loading the atoms into the HCF, the probe beam is used to measure the atom number by looking at its absorption A . The probing sequence is shown in Figure 6.5. In this sequence, two identical probe pulses are sent: First pulse is for the absorption and the second pulse is for the reference without absorption. Therefore, the dipole beam is turned off before probe because the atoms need to be disappeared at the second pulse. Furthermore, since there is a pickup noise of the APD reading by 50 Hz AC line¹, time interval between two pulses is set to be 20 ms to match any error source from the phase of the 50 Hz AC line.

With absorption A , the optical depth (OD) is defined as $OD = -\ln(1 - A)$ which depends on the probe beam detuning δ as

$$OD = \frac{OD_{\max}}{1 + I/I_{\text{sat}} + (2\delta/\Gamma)^2} \quad (6.7)$$

where $OD_{\max}$ is the maximum OD at probe beam resonance. probe beam power is chosen such that I/I_{sat} is as small as possible but not too small for good signal-to-noise ratio from the photodetector. Therefore, the probe beam power² is chosen to be 3 nW which is 1/10 of saturation power (30 nW). The OD measurement in Figure 6.6 is taken purposely with the probe beam blue detuned to $\delta = 8.3$ MHz because high absorption at resonance would lead to large uncertainty of OD reading. Furthermore, OD is then related to the atom number through

$$OD = \sigma n L \quad (6.8)$$

where σ is the absorption cross-section (see §6.6 for detail), n is atomic volume density and L is the length of atomic cloud. However, this equation holds only for constant atomic density

¹The noise amplitude is ~ 5 mV and the signal amplitude is ~ 50 mV.

²This is the optical power in HCF. There will be power cut off by half from the NPBS before entering the photodetector. See Figure 4.3.

Figure 6.6: Optical depth vs time with and without the presence of blow beam. It is clear that the atom enters the HCF at $t_{\text{prb}} \approx 20$ ms.

n , which is not the case for atoms trapped by the optical trap inside the HCF. Therefore, the atom distribution is assumed to be a Gaussian or $n(r) = n_0 \exp(-r^2/r_0^2)$, and the total number of atom is

$$N_{\text{atom}} = n_0 \pi r_0^2 L. \quad (6.9)$$

Besides, the beam intensity also has a normalized Gaussian distribution of $\frac{2}{\pi w^2} \exp(-2r^2/w^2)$ that acts like a “weightage” of the optical depth as

$$\begin{aligned} \text{OD} &= \frac{2}{\pi w^2} \int_0^R \sigma n_0 \exp(-r^2/r_0^2) L \cdot \exp(-2r^2/w^2) \cdot 2\pi r dr \\ &= \sigma n_0 L \cdot \frac{1}{1 + \frac{w^2}{2r_0^2}} \left[1 - \exp\left(-\frac{2R^2}{w^2} \left(1 + \frac{w^2}{2r_0^2}\right)\right) \right]. \end{aligned} \quad (6.10)$$

Here, $w = 22 \mu\text{m}$ and the upper limit of the integral changes to the core radius $R = 31.5 \mu\text{m}$ of HCF to limit the atom distribution in HCF within the core, even though the result is very similar to infinity limit (Since $\frac{2R^2}{w^2} = 4.1$ result a small contribution for the exponent term). From equation (6.9) and (6.10), the number of atom per OD is estimated as

$$\frac{\text{OD}}{N_{\text{atom}}} \approx \frac{\sigma/\pi}{r_0^2 + w^2/2}. \quad (6.11)$$

Experimentally, OD is measured while scanning the time of probe t_{prb} across. In order to distinguish atoms between inside and outside HCF, a blow beam is introduced horizontally (in x direction) to push atoms outside HCF away from optical axis. By comparing OD vs t_{prb} graph with and without the presence of blow beam (shown in Figure 6.6), it is clear to see that the atoms enter HCF at $t_{\text{prb}} \approx 20$ ms. In this experiment, the dipole beam is a single 280 mW beam with wavelength of 808 nm, resulting the calculated trap depth to be

Figure 6.7: Sequence of capture and recapture technique in measuring the radial temperature.

$$U_0/k_B = 0.51 \text{ mK}.$$

6.5 Radial Temperature of Atoms in HCF

When atoms enter HCF, the presence of dipole beam acts as a funnel to “squeeze” the atom distribution in space. Since the atom density in phase space is conserved, the momentum distribution becomes broaden. To characterize it, the radial temperature of the atomic cloud in HCF is measured.

However, ToF is no longer useful because it is challenging to image atom in HCF. Therefore, a technique, called the release and recapture, is used. First, the atoms are considered lost when they hit the fiber wall. It is possible that they are stucked and coated onto the wall. It is also assumed that the radial distribution of atomic cloud has come to a steady state in the dipole trap so that the equipartition theorem can be used later. Experimentally, the dipole beam is switched off for an amount of time t_{rls} to release the atoms from trap and allow the atomic cloud to expand freely. The atoms with higher radial speed will hit the wall first and disappear. Then, the dipole beam is turned back on to recapture the remaining atoms before probing them.

The expansion of the atom cloud follows equation (6.5) with replacement of $r_0^2 \rightarrow r_0^2 + \sigma_v^2 t^2$ in equation (6.10), where velocity distribution σ_v obeys the equipartition theorem $\frac{1}{2} m \sigma_v^2 = k_B T$. Furthermore, the cloud size is assumed to be small enough such that the Gaussian dipole trap can be approximated to a harmonic trap

$$U(r) = -U_0 \exp(-2r^2/w^2) \approx -U_0 + \frac{1}{2} k r^2 \quad (6.12)$$

where $k \equiv 4 \frac{U_0}{w^2}$ acts like the “spring constant” of the trap. Since the cloud is in steady state, the cloud size r_0 can be further related to beam size w by equipartition theorem

$$\frac{1}{2} k r_0^2 = k_B T \Rightarrow r_0^2 = \frac{k_B T}{2U_0} w^2. \quad (6.13)$$

Figure 6.8: (a) The radial temperature fitted by capture and recapture technique using equation (6.14). (b) The exponential fit of radial temperature vs t_{prb} with the rise time of $\tau = 4.88(4)$ ms.

After the formalism, equation (6.10) becomes

$$\begin{aligned} \text{OD} &= \frac{\sigma n_0 L}{1 + \frac{1}{AT + BT^2}} \left[1 - \exp \left(-C \left(1 + \frac{1}{AT + BT^2} \right) \right) \right] \\ &\approx \frac{\sigma n_0 L}{1 + \frac{1}{AT + BT^2}} \end{aligned} \quad (6.14)$$

where³ $A \equiv \frac{k_B}{U_0} = 0.0019 \mu\text{K}^{-1}$, $B \equiv \frac{k_B}{mw^2} = 0.79 \mu\text{K}^{-1} \text{ms}^{-2}$ and $C \equiv \frac{2R^2}{w^2} = 4.1$ are all constants and the temperature T is the only parameter to fit with. The radial temperature measurements are performed at different time t_{prb} as shown in Figure 6.8. It is noticed that the radial temperature rises at a time scale of $\tau = 4.88(4)$ ms, which is faster than the dipole scattering time scale (from Table 2.1) of $2\pi/\Gamma_{\text{sc}} \approx 25.2$ ms. It is believed that the other heating mechanism comes from dipole intensity noise [129]. This intensity noise can be minimized by implementing an intensity feedback system to dipole power.

6.6 Atom Number Measurement in HCF

For a simple two-level system, the absorption cross-section for a cycling transition is

$$\sigma_0 = \frac{3\lambda^2}{2\pi} \quad (6.15)$$

where λ is the transition wavelength. For a system with multiple Zeeman states, the complete absorption cross-section is the sum of individual absorption cross-section weighted by its

³In the current experiment, the dipole beam is at a wavelength of 808 nm with power of 290 mW that lead the dipole potential to be $U_0/k_B = 0.53$ mK.

Clebsch–Gordan coefficient

$$\sigma = \sigma_0 \sum_{m,m'} \frac{f_{mm'}}{1 + I/I_{\text{sat}} + 4(\delta\nu_{mm'} + \delta)^2/\Gamma^2} \quad (6.16)$$

where the summation is over all the σ^+ and σ^- transitions, $I/I_{\text{sat}} = 0.1$, δ is the laser detuning w.r.t. magnetic field insensitive transition and $\delta\nu_{mm'}$ is the energy shift due to magnetic field. $f_{mm'}$ is the weightage defined by

$$f_{mm'} = \frac{|\text{CG}_{(3,m) \rightarrow (4,m')}|^2}{|\text{CG}_{(3,3) \rightarrow (4,4')}|^2}. \quad (6.17)$$

Combining with equation (6.13) and (6.11) as shown in [Appendix B.1](#), It is calculated that at magnetic field of 3.6 G and cloud radial temperature of $T = 30 \mu\text{K}$, OD of 1 corresponds to atom number of 2×10^4 at detuning of $\delta = 8.3 \text{ MHz}$ or 1.59×10^4 on resonance.

Part III

Experiments in Hollow Core Fiber

Chapter 7

Inertia-Sensitive Atom Interferometer

After atom loading process, an example of inertia-sensitive atom interferometer, the Mach-Zehnder atom interferometer (AI) which is sensitive to the gravity, is constructed in HCF as the first demonstration of cold atoms application in hollow core fiber. The hyperfine ground states $|g\rangle = |F = 2, m = 0\rangle$ and $|e\rangle = |F = 3, m = 0\rangle$ are used in §3.1 as the states for AI in the two interferometer arms for the AI.

7.1 Sequence and Setup

The sequence of AI is shown in Figure 7.1. When the atoms are inside HCF, a 1 ms depump beam is sent to prepare the atom to the state $F = 2$. A 2 ms time window ($28 \text{ ms} < t_{\text{seq}} < 30 \text{ ms}$) is then opened for AI (sequence of $\frac{\pi}{2} - T - \pi - T - \frac{\pi}{2}$), even though the time used is less than that. This is to ensure that the atoms are always probed at the same time $t_{\text{seq}} = 30 \text{ ms}$ for different T . During the AI process, the compensation magnetic field is set to $\mathbf{B}_{\text{rst}} = 3.6 \text{ G}$ along z direction.

The optical alignment is shown in Figure 7.2. The AI beam and probe beam are combined and separated by a PBS whereas the dipole beam at 808 nm and other beams are by a dichroic mirror (Semrock: FF791-SDi01). Since the atoms contain an initial velocity before the counter-propagating AI beam pulses, the frequencies of AI beams are set such that it compensates the Doppler shift. Furthermore, the polarizations of the AI beams are designed to be linear but orthogonal to each other. This is to suppress the co-propagating Raman transition from the individual AI beam that consists of carrier and sideband frequencies after EOM (from Figure 4.2c). There are four sets of two-photon Raman transition, with the intermediate states $|F', m'\rangle = |2, -1\rangle, |3, -1\rangle$ by σ^- polarized beam, and $|2, 1\rangle, |3, 1\rangle$ by σ^+ polarized beam as shown in Figure 7.3. The Rabi frequency is the sum of equation

Figure 7.1: Sequence of Mach-Zehnder atom interferometer. A 2 ms time window is opened of AI. ϕ is the phase of the last $\pi/2$ pulse relative to the previous two pulses.

Figure 7.2: Optical setup for Mach-Zehnder atom interferometer experiment.

Figure 7.3: Clebsch–Gordan coefficients for all transition for AI. The products of the coefficients for the transitions are $-\sqrt{1/324}$ (σ^- , blue), $-\sqrt{1/81}$ (σ^- , green), $\sqrt{1/324}$ (σ^+ , red) and $\sqrt{1/81}$ (σ^+ , orange).

(2.28) as

$$\Omega_{2p} \sim \sum_{F',m'} S_{(2,0) \rightarrow (F',m')} S_{(3,0) \rightarrow (F',m')} \frac{\text{CG}_{(2,0) \rightarrow (F',m')} \text{CG}_{(3,0) \rightarrow (F',m')}}{\Delta_{F'}} \quad (7.1)$$

where $\text{CG}_{(F,m) \rightarrow (F',m')}$ is the Clebsch–Gordan coefficients of transitions (see [Appendix A](#)) and $\Delta_{F'}$ is the single photon detuning that mostly depends the hyperfine excited state. Its dependency on m' due to Zeeman effect is neglected because it is relatively small. S is the sign of the beam depending on the polarization.

For co-propagating Raman, the polarizations of both Raman fields are the same and linear $\sigma^+ + \sigma^-$ or $\sigma^+ - \sigma^-$. Therefore, the term $S_{(2,0) \rightarrow (F',m')} S_{(3,0) \rightarrow (F',m')}$ will be always positive because the Raman transition encounter either $(\sigma^+)(\sigma^+)$ or $(\pm\sigma^-)(\pm\sigma^-)$. Since the product of CG for both polarization are equal in magnitude but opposite in sign in [Figure 7.3](#), equation (7.1) ends up to be zero by cancellation. For counter-propagating Raman, the AI beams are orthogonally polarized, i.e. one $\sigma^+ + \sigma^-$ with another $\sigma^+ - \sigma^-$ polarized, then the product of the sign term is positive for σ^+ but negative for σ^- . Therefore, instead of cancellation in equation (7.1), it adds up to a maximum magnitude. In summary, for co-propagating Raman, the phase of σ^+ Raman coupling destructively interferes with σ^- Raman coupling whereas for counter-propagating Raman, it constructively interferes.

Since the AI beams are created from an EOM, there are multiple frequencies per beam. The Doppler shift of atoms is used to distinguish the direction of effective wavevector $\mathbf{k}_{\text{eff}}$ as shown in [Figure 7.4](#). Thanks for initial speed before AI, the gravity measurement could be performed with both directions of $\mathbf{k}_{\text{eff}}$ separately by setting the frequency of the EOM.

Figure 7.4: (a) When the atom is stationary, the counter-propagating AI beam (direction shown in black arrows) that have multiple frequencies can have two directions of $\mathbf{k}_{\text{eff}}$. (b) When the atom is in motion, the beam frequency needs to be shifted to compensate the Doppler effect. In this case, only one $\mathbf{k}_{\text{eff}}$ is used, and this method can be used for AI at different direction of $\mathbf{k}_{\text{eff}}$.

7.2 Gravity Measurement

Figure 7.5 shows the interference fringes when scanning the phase of the last $\pi/2$ pulse ϕ at different separation time T . The phase information is then extracted by fitting to equation (3.12) to data as shown in Figure 7.6. The experiment is repeated with different direction of $\mathbf{k}_{\text{eff}}$. With the value of $|\mathbf{k}_{\text{eff}}| = 2\pi/(780.2\text{nm})$, the measured gravitational acceleration are $9.2(7)\text{ms}^{-2}$ and $9.7(6)\text{ms}^{-2}$ with 95% certainty, compared to the local gravity in Singapore from International Gravity Formula 1980 (IGF80) or $g = 9.78\text{ms}^{-2}$. To verify that the gravity measurement remains unchanged at different position of atoms in the fiber, the experiment is repeated at different probe times, i.e. the replacement of $t_{\text{seq}} = 30 \rightarrow t_{\text{prb}}, 28 \rightarrow t_{\text{prb}} - 2, 25 \rightarrow t_{\text{prb}} - 5$ in Figure 7.1. Figure 7.7 shows the result of $T = 100\mu\text{s}$ at different t_{prb} , where the fitted phase shifts agree within a standard deviation.

7.3 Contrast Analysis

There are many sources of systematic errors that need to be investigated. For example, the tilt of HCF may change the value of g measurement. The presence of magnetic field gradient may cause a magnetic force onto atom through second order Zeeman shift. Before these investigations, improvement of the sensitivity is required and, hence, T needs to be increased. However, the signal reduces due to loss of atom in the absence of dipole trap. It is clear in Figure 6.8a that the atoms only survive in $\sim 100\mu\text{s}$, which limits the maximum T imposed.

When the dipole beam is turned on during AI, the differential AC Stark shift causes decoherence. The decoherence is characterized by the decay of its contrast which is defined as the ratio of amplitude to offset of the fitted sinusoidal function, i.e. for the signal $A_0 \sin(\phi - \phi_0) + A_1$, the contrast is defined as

$$\chi := \frac{A_0}{A_1}. \quad (7.2)$$

Figure 7.5: Interference fringes of Mach-Zehnder atom interferometer at different separation time T . Each data point is an average of 60 runs with the standard error.

Figure 7.6: Quadratic fit of phase shift of the Mach-Zehnder atom interferometer vs separation time T for different direction of $\mathbf{k}_{\text{eff}}$ at 95% confidence. The dashed line shows the theoretical values with $g = 9.78 \text{ ms}^{-2}$.

Figure 7.7: Optical depth vs phase at different probe times t_{prb} for $T = 100 \mu\text{s}$, fitted to a sinusoidal function. Each data point is an average of 60 runs with the standard error. The phase shift ϕ_0 agrees to each other within one standard deviation.

Figure 7.8: Contrast of AI signal vs T with and without the presence of dipole beam.

Figure 7.8 shows the decay of contrast with and without the presence of dipole beam.

Moreover, when a timing asymmetry δT between three AI pulses is introduced so that the wave packets do not overlap completely as shown in Figure 7.9a, the size of the wave packet could be obtained by the contrast. Then, the axial temperature could be determined from the decay of the contrast. Considering the Gaussian distribution of the wave packet $\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$ with the rms width of σ_z , the contrast of the interference follows the convolution of the wave packet overlapping

$$\chi(\delta T) \sim \int_{-\infty}^{\infty} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \exp\left(-\frac{(z - v \cdot \delta T)^2}{2\sigma_z^2}\right) dz \sim \exp\left(-\frac{(\delta T)^2}{2\sigma_{\delta T}^2}\right)$$

where $\sigma_{\delta T} = \sqrt{2}\sigma_z/v$. Here, $v = \frac{2\hbar k_{\text{eff}}}{m}$ is the relative speed of the overlapping wave packet due to the momentum difference of $2\hbar k_{\text{eff}}$. From the uncertainty principle $m\sigma_z\sigma_v \approx \hbar/2$, $\sigma_{\delta T}$ can be related to the atomic velocity width

$$\sigma_v = \frac{1}{\sqrt{8}k_{\text{eff}}\sigma_{\delta T}}. \quad (7.3)$$

By equilibrium principle $\frac{1}{2}m\sigma_v^2 = \frac{1}{2}k_B\tilde{T}$, the axial temperature can be obtained

$$\tilde{T} = \frac{mk_{\text{eff}}^2}{8k_B\sigma_{\delta T}^2}. \quad (7.4)$$

From Figure 7.9, the velocity width for $T = 10 \mu\text{s}$ is $\sigma_v = 2.2(3) \text{ cm s}^{-1}$ and axial temperature is $\tilde{T} = 5(2) \mu\text{K}$ which is consistent to the temperature taken by ToF method. However, the velocity width for $T = 20 \mu\text{s}$ is $\sigma_v = 4.8(7) \text{ cm s}^{-1}$ and axial temperature is $\tilde{T} = 23(6) \mu\text{K}$, suggesting that there is some heating mechanism in axial direction.

Figure 7.9: (a) Skewed sequence of the Mach-Zehnder AI in Figure 7.1 for axial temperature measurement. (b) Axial temperature measurement from asymmetry separation time δT . The fitted standard deviations for $T = 10 \mu\text{s}$ and $20 \mu\text{s}$ are $\sigma_{\delta T} = 2.0(2) \mu\text{s}$ and $0.9(1) \mu\text{s}$ respectively.

7.4 Discussion

In this project, I have demonstrated the quantum sensor with cold atom in a hollow core fiber, by measuring the gravitational acceleration as an example. In general, anything that can affect the phase of Mach-Zehnder atom interferometer can also be measured, suggesting the wide contribution of the quantum sensor in a fiber-based device.

Although the current accuracy of gravitational acceleration (at the level of $10^{-1}g$) is low, the time taken for the measurement is low (at the level of $10 \mu\text{s}$) due to insufficient coherence time. Since the Mach-Zehnder phase is directly proportional to the square of the separation time T as shown in equation (3.12), the accuracy of measurement is directly proportional to T^2 . If the coherence time can be improved to sub-second time scale which is typical in free space atom interferometry [32], the accuracy can be improved to the level of $10^{-9}g$. In the next chapter, I will demonstrate the improvement of coherence time to the level of 100 ms, an improvement by 4 orders of magnitude.

Chapter 8

Long-lived Quantum Spin Coherence

This chapter describes an upgrade system aiming to improve the coherence time of atomic spin states in HCF and, at the same time, the OD is improved by optically pumping the atoms that distributed across all Zeeman states to clock state. Furthermore, the lifetime of atoms in HCF is also studied.

8.1 Optical Alignment

The upgraded system is shown in [Figure 8.1](#). The dichroic mirrors that are used to combine and separate dipole beam from other beams, are replaced with a PBS. This is because the reflected beam from the dichroic mirror is distorted after about 2 m of traveling distance, consistent with Semrock's flatness classification of "Standard Epi-fluorescence" with radius of curvature of ~ 6 m [[130](#)]. It is found that the distorted beam did not give good HCF coupling as expected. A solution is to replace it with a better dichroic mirror or in this case, I just simply use PBS.

Furthermore, the optical pump beam is coupled into HCF with linear polarization set along x direction. We will discuss in [§8.3](#) that during optical pump sequence, the magnetic field is required to switch to x direction and then back to z direction for the experiment which requires extra time for eddy current to decay. This is different from the setup in [Figure 4.3](#) which is the best configuration up to date because it is realized with optical pump sent through HCF from the side with polarization along z direction without affecting the optical pump result. Therefore, optical pump beam coupled into HCF only applies for this chapter.

8.2 Atom Loading and Characterization in Lattice

The loading sequence is very similar to [Figure 6.5](#) except blow beam pulse time is 2 ms and the dipole beam is replaced by lattice beams which are composed by a pair of $\lambda = 875$ nm

Figure 8.1: Optical setup for Mach-Zehnder atom interferometer experiment.

dipole beams. The speed of the moving lattice, similar to a conveyor belt, is controlled by the frequency difference of the individual AOMs in Figure 4.2c. A frequency difference of $\delta f = 50 \text{ kHz}$ has lattice speed of $v = \delta f \lambda / 2 = 2.2 \text{ cm s}^{-1}$. The lattice beam is turned off before $t_{\text{seq}} = 0$ and on until $t_{\text{seq}} = t_{\text{prb}}$ before the probing sequence. Figure 8.2 shows the OD as a function of lattice traveling time at different lattice speed. The earlier OD peak represents the OD of free-falling atom without held by the lattice beams. In general, the faster the lattice speed, the higher the OD decay rate.

After taking OD vs probe time, fluorescence images of atoms in moving lattice is taken by EMCCD camera (Andor: iXon Ultra 897) with the probe beam used as the fluorescence imaging beam. Figure 8.3 shows the fluorescence images at the lattice speed of 100 kHz or 4.4 cm s^{-1} . The inset shows the double Gaussian fit of atoms distribution horizontally. The larger Gaussian distribution is from the atom not being trap by the lattice.

The lifetime of the atoms in a stationary lattice is also measured. This is done by switching the moving lattice at 50 kHz or 2.2 cm s^{-1} to stationary lattice after the blow sequence, holding for some time t_{st} before measuring the OD. Figure 8.4 shows the exponential decay fit of the OD as a function of t_{st} at different $t_{\text{prb}} = 500, 700, 900$ and 1100 ms which corresponds to 6.0, 10.0, 14.8 and 19.2 mm from the upper tip of the fiber, respectively. The fit in the Figure shows that the $1/e$ decay times are 1.87(4), 2.59(7), 2.33(5) and 1.74(7) s, respectively. The result suggests that there is no significant loss of atoms due to heating inside HCF. The minimum lifetime of 1.7 s shows that the pressure in HCF is lower than 10^{-8} torr [85] and this sets the upper bound of our coherence time.

Moreover, the technique in § 6.5 is used to measure the radial temperature of atom

Figure 8.2: OD vs t_{prb} at different lattice speed. Lattice frequency difference of 50 kHz corresponds to the lattice speed of 2.2 cm s^{-1} .

Figure 8.3: The fluorescence images of atoms during loading process in a 100 kHz moving lattice.

Figure 8.4: OD vs t_{st} at different atom position in the fiber.

transported by 50 kHz moving lattice. Figure 8.5 shows the radial temperature at different position. The result indicates that the radial temperature of $\sim 80 \mu\text{K}$ does not vary significantly during transportation, suggesting the heating effect is negligible at this time scale. In §6.2, the temperature before HCF is less than $10 \mu\text{K}$, which is less than temperature in HCF, and it is partly due to the *parametric heating*. The other heating mechanism is due to the increase of the momentum distribution in phase space when atoms are squeezed in the position space during loading, the atom phase space density remains unchanged. Squeezing in the position space during loading meaning expanding in the momentum space, leading to heating effect.

8.3 Optical Pump

After the loading process, internal states of atoms are distributed evenly over the Zeeman states whereas the Mach-Zehnder AI is done only on the Zeeman clock state¹. In order to gain the signal to noise ratio, optical pump sequence is used to prepare the atoms in the hyperfine ground state $|F = 2, m = 0\rangle$. A depump beam pumping all $|F = 3\rangle$ state to $|F = 2\rangle$ and a π -polarized optical pump beam are combined to make $|F = 2, m = 0\rangle$ the dark state.

The sequence is shown in Figure 8.6. The moving lattice is turned on at $t_{seq} = 0$ to control the speed of atom loading in HCF. At $t_{seq} = t_{prb}$, the moving lattice is stopped to hold the atoms from falling. Then, the state of atom is prepared by a 1 ms optical pump process after the blow. During this period, the magnetic field is set from $\mathbf{B}_{PGC} \approx \mathbf{0}$ to $\mathbf{B}_{OP} = 1.4 \text{ G}\hat{z}$ along the direction of optical pump polarization. It is then changed to $\mathbf{B}_{rst} = 0.6 \text{ G}\hat{z}$ and a 20 ms time gap is set between optical pump and microwave to allow the eddy current to decay. I do not wait $\mathbf{B}_{OP}$ to stabilize because optical pump process only depends on the

¹So as for light storage and Schrödinger cat experiment in future.

Figure 8.5: Radial temperature of atoms at different t_{prb} or position in the fiber.

direction of magnetic field, not its magnitude. During the transition from $\mathbf{B}_{\text{OP}}$ to $\mathbf{B}_{\text{rst}}$, the magnetic field is made sure not to go through zero, which might create degeneracy of the Zeeman distribution.

After state preparation, a microwave pulse is used scan the Zeeman spectrum by transferring Zeeman states of $|F = 2\rangle$ to $|F = 3\rangle$ before measured by probe beam. Figure 8.7 shows the Zeeman spectrum with and without optical pump process. The result shows almost 100% pumping efficiency to clock state. Note that the signal is relative weak on the 0th, $\pm 2^{\text{nd}}$ and $\pm 4^{\text{th}}$ order. These signals correspond to the transfer of signal through π transition $|F = 2, m\rangle \rightarrow |F = 3, m\rangle$ whereas other signal has both σ_+ and σ_- transition $|F = 2, m - 1\rangle \rightarrow |F = 3, m\rangle$ and $|F = 2, m\rangle \rightarrow |F = 3, m - 1\rangle$ respectively. The $\pm 5^{\text{th}}$ order only contains single $\sigma_{\pm}$ transition $|F = 2, m = \pm 2\rangle \rightarrow |F = 3, m = \pm 3\rangle$, explaining the relatively weak signal.

The microwave pulse power and time can be altered depending on the needs. Strong power with short pulse duration provides power broadening effect, allowing us to look for the spectrum easily with low scanning resolution. On the other hand, weak power and long pulse duration narrows the width, allowing us to determine magnetic field accurately and analyze any magnetic field broadening effect. In fact, I use this method to calibrate the compensation magnetic field currents for zeroing ambient magnetic field.

8.4 Light Shift Cancellation

The key of improvement on atomic coherence is to minimize the differential light shift (DLS) by means of vector light shift. Its theory has been discussed in § 2.5. Experimentally, a quarter waveplate is added to create elliptical polarized lattice beams to induce the vector light shift for cancellation. In practice, the polarization ellipticity is small and magnetic field is large (i.e. small A but high B in equation (2.22)) because most of the probe beam is

Figure 8.6: Optical pump sequence in the lattice.

Figure 8.7: Typical Zeeman spectrum with and without presence of optical pump.

Figure 8.8: Sequence for quantum spin coherence time measurement by spin echo technique.

required to pass through a PBS for detection (see optical alignment in Figure 8.1). However, the magnetic field cannot be too strong since that will increase its sensitivity on polarization ellipticity. Therefore, the magnetic field is set to be $\mathbf{B}_{\text{rst}} = 2.08 \text{ G}$ along $\hat{z}$ direction and attached both waveplates on high precision rotation mounts (Thorlabs: PRM1/M) for better control.

The coherence time is investigated by microwave Ramsey $\Omega_{\pi/2} \cdot \delta T \cdot \Omega_{\pi/2}$ operation and spin echo $\Omega_{\pi/2} \cdot \delta T_{/2} \cdot \Omega_{\pi} \cdot \delta T_{/2} \cdot \Omega_{\pi/2}$ operation (see §2.3). By scanning the phase of last $\pi/2$ pulse, the information of the contrast defined in equation (7.2) can be extracted. Figure 8.9 shows the coherence time measurement results with $t_{\text{prb}} = 200 \text{ ms}$ after transported by a 50 kHz or 2.2 cm s^{-1} lattice. The lattice single beam power of 300 mW creates a trap depth of $537 \text{ } \mu\text{K}$. The results show that spin echo can reverse the decoherence compared to Ramsey technique (decay time increases from $3.0(1) \text{ ms}$ to $37.2(7) \text{ ms}$) and DLS cancellation can improve the coherence of atom (decay time increases from $37.2(7) \text{ ms}$ to $60.1(1) \text{ ms}$).

To investigate the effect of the trap depth on the DLS cancellation, the spin echo experiment is repeated with different trap depth. The atom position is set at $t_{\text{prb}} = 250 \text{ ms}$ with lattice speed unchanged. Figure 8.10 indicates that the decay time increases from $T_c = 101(2) \text{ ms}$ to $252(8) \text{ ms}$ when trap depth decreases from $279 \text{ } \mu\text{K}$ to $179 \text{ } \mu\text{K}$ ². This suggest that the limitation of the coherence decay is from the imperfect AC Stark cancellation by the vector light shift. This is due to the presence of non-linear term in equation (2.22). Therefore, the small the trap depth, the better the coherence in general.

Moreover, the dependency of atoms' position on its coherence is also studied. The trapping potential is fixed to $279 \text{ } \mu\text{K}$ and varied the positions of atom to 0.5 , 7.0 and 11.4 mm

²This equivalent to lattice single beam power reduces from 156 mW to 100 mW

Figure 8.9: Contrast χ vs separation time T by Ramsey and spin echo operation, with and without optimized light shift cancellation. The error bars are the standard error after four experimental runs. The solid line is fitted with exponential function $\chi_0 \exp(-T/T_c)$. The inset shows the exemplary Ramsey fringe for separation time $T = 3$ ms. The decay times from the fit are 3.0(1), 37.2(7) and 60.1(1) ms.

Figure 8.10: Contrast χ vs separation time T by spin echo measurement. The solid line is fitted with exponential function $\chi_0 \exp(-T/T_c)$.

Figure 8.11: Spin echo measurement at different atom position. The solid line is fitted with exponential function $\chi_0 \exp(-T/T_c)$. The $1/e$ lifetime for $t_{\text{prb}} = 250, 550$ and 750 ms is $T_c = 106(2), 108(2)$ and $106(2)$ ms respectively.

from HCF upper tip ($t_{\text{prb}} = 250, 550$ and 750 ms). Figure 8.11 shows that the coherence is independent to the atom position.

8.5 Coherence in Motion

To study the coherence time of atoms in motion, the lattice is moved continuously at 50 kHz or 2.2 cm s^{-1} after $t_{\text{seq}} = t_{\text{prb}}$ during measurements instead of switching to a stationary lattice. This is repeated with switching off one of the lattice beams at $t_{\text{seq}} = t_{\text{prb}}$, leaving only one dipole beam on for atoms to fall freely under gravity. In these cases, I set $t_{\text{prb}} = 250$ ms. The trap depth in lattice is $537 \mu\text{K}$ whereas single dipole is $134 \mu\text{K}$. The results shown in Figure 8.12 indicate that the contrast can still be observed after tens of ms of atomic motion in a macroscopic distance at mm scale. This result is three orders of magnitude better than previous study in Mach-Zehnder AI.

8.6 Discussion

In principle, the improvement of the coherence time can increase the time of measurement (for example, T in Figure 7.8 for Mach-Zehnder atom interferometer), hence, improve the sensitivity of measurement. The long quantum state transportation distance in HCF demonstrated is already an order of magnitude longer than the free space scheme [131]. As shown in Figure 8.10, the coherence is better when the atom is not in motion. When it is in motion, the main source of decoherence is from inhomogeneity of local magnetic field in HCF. The inhomogeneity magnetic field is measured to be 11 mG by Zeeman spectrum method, which can cause broadening on clock state by a few tens of hertz through second-order Zeeman shift. This makes imperfect DLS when it is in motion.

Figure 8.12: Coherence time measurement on moving atoms in the fiber under (a) the moving lattice by spin echo; and (b) free falling in the dipole trap by both Ramsey and spin echo technique. R=Ramsey, SE=Spin Echo.

Although the compensation coils along z direction are controlled by two independent current sources to remove the magnetic field gradient along HCF. It is believed that there is some higher order field gradient that only allows us to reduce the broadening to 11 mG. This is probably due to the magnet of the ion pump of about 1 m. Moreover, it is realized that the stainless-steel screws that are used to tighten the fiber holder in vacuum chamber are magnetic. Furthermore, the magnetic hysteresis effect is found at a mG level.

Moreover, the limitation of the AC Stark shift cancellation is also from the presence of non-linear term in the vector light shift. However, this limitation will be the same if the cold atoms were setup in the free space instead. Therefore, it can be claimed that there is no difference for the coherence environment in free space and in hollow core fiber, except that the fiber-based system can be more compact device for the future technologies.

Chapter 9

Demonstration of Long Light Storage Time

Long quantum spin coherence time in the previous chapter inspires us to investigate one of the quantum technologies: light storage. It is an attractive technology in building long distance quantum communication networks [132–137]. Although silica-core optical fibers are excellent in transmitting broadband optical signals, manipulating group velocity of light through altering its dispersive property remains challenging. Hollow-core fibers loaded with tunable dispersive medium is promising in constructing high performance optical delay line with low loss [77, 93, 94, 138–142].

Light storage is an important project for the cold atom laboratory in hollow core fiber as this is a demonstration in quantum manipulation of atomic internal state in the fiber. In this chapter, I demonstrate slow-light effect and light storage in HCF loaded with cold Rb atomic vapour by electromagnetically induced transparency (EIT). The theory of EIT is stated in §3.2, where the hyperfine ground $|F, m = 2, 0\rangle \rightarrow |1\rangle$, $|F, m = 3, 0\rangle \rightarrow |2\rangle$ and excited states $|F', m' = 3, \pm 1\rangle \rightarrow |3\rangle$ of Rubidium 85 D1 line are used as the three-level system for EIT.

9.1 Experimental Configuration

The experimental setup is shown in Figure 9.1. The optical pump beam is shined sideways through HCF as it is realized in Figure 8.3 that light beam can penetrate HCF without significant distortion of the polarization. A 500 mm focal length cylindrical lens is used outside vacuum chamber to focus the optical pump beam to HCF about half meter away. The polarization of optical pump is set along HCF (i.e. in z direction) to make sure the polarization of the reflected light from HCF fiber holder (in which the surface is vertical) remains unchanged. The main advantage of the upgrade is that the direction of compensation magnetic field $\mathbf{B}_{\text{rst}} = 2.08 \text{ G}$ can be kept to z direction, without the requirement of waiting extra time for the eddy current to decay. It is found out that the optical pump efficiency after the upgrade is similar to Figure 8.7.

Figure 9.1: Optical setup for light storage experiment.

The EIT beams (control and probe) are designed to be co-propagating in HCF. As shown in Figure 4.2a, their polarization are set to be linear and orthogonal to each other for two reasons: First, both beam can be mostly separated by a PBS after HCF; Second reason is similar to Mach-Zehnder AI, where orthogonal polarization can boost the total Rabi frequency as $\sigma_+\sigma_+$ transition has a strength opposite sign to $\sigma_-\sigma_-$, as shown in Figure 9.2.

In this experiment, the atomic ensemble of $10\mu\text{K}$ is loaded into an optical trap guided by the fiber. The optical dipole beam of wavelength 821 nm with 250 mW power is used as the guide and the atomic ensemble with resonant OD of 40 is prepared before optical pump sequence. This can be converted into atom number density of $n = 1.3 \times 10^{11}\text{ cm}^{-3}$ using the atomic length of $L = 5\text{ mm}$ using equations in §6.4. The detection probe beam (not EIT probe beam) is guided by a flipped mirror (not shown in Figure 9.1) to skip the etalon and enter the detector. After preparing the atomic state after the optical pump sequence, the EIT control beam is sent for about 1 ms to ensure the atoms in state $|1\rangle$. Then, either long EIT probe pulse for measuring the EIT spectroscopy or short Gaussian pulse for light delay and storage experiments, is sent. The control and probe beams power inside the HCF are set to be 300 and 40 nW , respectively, and the frequencies and the power of both beams are controlled by two independent and phase coherent AOMs shown in Figure 4.2c.

The intensity modulation of the EIT probe beam for light delay and storage experiment is controlled by the AOM frequency source from Moglabs: Agile RF Synthesizer which contains amplitude modulation function. The Gaussian pulse is generated by a function generator (Tektronix: AFG2051C) which is triggered by a delay generator (Stanford Research Systems: DG535) in order to control the pulse timing at ns level beyond the resolution of our data acquisition board. Since high time resolution sampling is required when taking data, an

Figure 9.2: Clebsch–Gordan coefficients for all transition for light storage experiment. The products of the coefficients for the transitions are $1/9$ (σ^- , blue), $2/9$ (σ^- , green), $-1/9$ (σ^+ , red) and $-2/9$ (σ^+ , orange).

Figure 9.3: Timing sequence for light storage experiment. The EIT probe beam is set to be either long square pulse for EIT spectroscopy or short Gaussian pulse for light delay and storage experiment.

Figure 9.4: EIT spectrum of atoms in the HCF.

oscilloscope (Tektronix: TDS 2024C) is used to collect the data, which is then sent to PC for analysis.

9.2 Experimental Results

When the atoms are under free fall inside the HCF, an EIT spectrum is measured, as shown in Figure 9.4, to pinpoint to the resonance frequency for the EIT probe beam. Then, the group delay of the Gaussian probe pulse of FWHM width of $T_p = 304\text{ ns}$, is measured, as shown in Figure 9.5. A Gaussian function in equation (9.1) is fitted to the transmitted probe pulse to determine the group delay of 174 ns with 24% efficiency of transmission. A simulation is performed to compare the experimental results, which is discussed in the next section. In the simulation, I assume the distribution of the atomic cloud, the dipole beam, and the EIT beams are all Gaussian in the HCF.

9.3 Numerical Calculation of Slowed Light

Hsiao et al. have performed a detailed analytical calculation on the slowed light in supplemental material of [143]. In the derivation, for Gaussian temporal probe pulse of intensity

$$I(t) = I_0 e^{-4 \ln 2 \cdot \left(\frac{t}{T_p}\right)^2} \quad (9.1)$$

where T_p is the FWHM electric field duration, the probe Rabi frequency is in the form of $\Omega_p(t) = \Omega_{p0} e^{-2 \ln 2 \cdot \left(\frac{t}{T_p}\right)^2}$ and its Fourier transform is

$$\tilde{\Omega}_p(\omega, z=0) = \frac{\Omega_{p0} T_p}{2\sqrt{\ln 2}} e^{-\frac{T_p^2 \omega^2}{8 \ln 2}}. \quad (9.2)$$

Figure 9.5: Measured slowed light when the control beam is permanently on. The input pulse (black squares) is fitted by a Gaussian function with the FWHM of 304 ns, and the slowed pulse (blue circles) is fitted by a Gaussian function with the FWHM of 415 ns. The peak of the fitted Gaussian function of slowed light is delayed by 173 ns.

After propagating through the medium with optical depth of OD of length L , the Rabi frequency becomes

$$\tilde{\Omega}_p(\omega, z = L) = \tilde{\Omega}_p(\omega, z = 0) e^{-i\frac{\omega L}{c}} e^{-f(\omega)}. \quad (9.3)$$

The first exponential term shows the phase accumulated due to traveling beam even without medium. The second term is a complex term from the EIT media response function

$$f(\omega) = \frac{\text{OD}\Gamma}{4} \cdot \frac{i\omega - \gamma_{21}}{(i\omega - \gamma_{31})(i\omega - \gamma_{21}) + |\Omega_c|^2/4} \quad (9.4)$$

where Γ is the decay rate, $\gamma_{\alpha 1}$ is the decoherence from state $|\alpha\rangle$ to $|1\rangle$. Presence of $|\Omega_c|^2$ shows that $f(\omega)$ relates to the optical intensity of control beam. Taking the power series of $f(\omega)$ and keeping the dispersion relation up to the ω^2 term results it to have the form of $f(\omega) \approx A + iB\omega + C\omega^2$ where

$$A = \text{OD} \cdot \Gamma \cdot \frac{\gamma_{21}}{4\gamma_{21}\gamma_{31} + |\Omega_c|^2} \quad (9.5a)$$

$$B = \text{OD} \cdot \Gamma \cdot \frac{4\gamma_{21}^2 - |\Omega_c|^2}{\left(4\gamma_{21}\gamma_{31} + |\Omega_c|^2\right)^2} \quad (9.5b)$$

$$C = 4\text{OD} \cdot \Gamma \cdot \frac{(2\gamma_{21} + \gamma_{31})|\Omega_c|^2 - 4\gamma_{21}^3}{\left(4\gamma_{21}\gamma_{31} + |\Omega_c|^2\right)^3}. \quad (9.5c)$$

Note that if the probe beam is CW, the EIT media response function is just $f(0) = A$ and it only contributes to the transmission loss by absorption from the medium. Putting equation (9.4)-(9.5c) into equation (9.3) and performing inverse Fourier transform results

$$\Omega_p(t, z = L) = \frac{\Omega_{p0}}{\beta} e^{-A} e^{-2 \ln 2 \cdot \frac{(t-B)^2}{\beta T_p}} \text{ or}$$

$$I_p(t, z = L) = \frac{I_0}{\beta^2} e^{-2A} e^{-4 \ln 2 \cdot \left(\frac{t-B}{\beta T_p}\right)^2} \quad (9.6)$$

where

$$\beta = \sqrt{1 + \frac{C \cdot 8 \ln 2}{T_p^2}} = \sqrt{1 + \frac{\text{OD} \cdot \Gamma \cdot 32 \ln 2}{T_p^2} \cdot \frac{(2\gamma_{21} + \gamma_{31}) |\Omega_c|^2 - 4\gamma_{21}^3}{(4\gamma_{21}\gamma_{31} + |\Omega_c|^2)^3}}. \quad (9.7)$$

One can see that the factor B is the main factor of light delay effect and C increases the pulse width at a factor of β .

Since the spatial distribution of the control and probe beams are Gaussian, the replacements $I_0 \rightarrow I_0 \exp(-2r^2/w^2)$ and $|\Omega_c|^2 \rightarrow |\Omega_c|^2 \exp(-2r^2/w^2)$ are made and the probe signal is obtained by performing the numerical integration similar to §6.4

$$\int_0^R 2\pi r \cdot I_p(t, z = L) dr. \quad (9.8)$$

Furthermore, OD in the integrand is also dependent on r through equation (6.10).

In this simulation, I set $\Gamma = 2\pi \times (5.75 \times 10^6 \text{ Hz})$, $\gamma_{31} = \Gamma/2$, and $\gamma_{21} = 2\pi \times (1 \times 10^3 \text{ Hz})$ due to the inhomogeneous differential ac Stark shift of the control and dipole beams (see Table 2.1). Figure 9.6 compares the group delay of the probe pulse in the fiber with free space scenario of uniform control beam, probe beam, and atoms density profiles. The inhomogeneous profiles of the control and the atom density in HCF result in more than a factor of two in reduction of the group delay and the efficiency. The result shows similar trend to the experimental result in Figure 9.5 if non-uniform profile of the control beam, the probe beam, and the atom density in HCF is considered.

9.4 Light Storage

Since the group velocity of the probe pulse in the medium is slowed down, it is possible to store the information of the probe pulse in the medium. This is done by turning off the control beam when the probe pulse is propagating inside the medium and turning on the control beam again after a storage time T . In Figure 9.7, the retrieved pulse has a FWHM duration of 160 ns, which corresponds to about 1 MHz bandwidth. The storage efficiency is defined by the ratio of retrieved signal to signal without medium though the pulse area. Here, I can store and retrieve 10 % of the input probe pulse in the fiber.

To study the storage time of probe pulse in the HCF, the motion of the ensemble is ceased by replacing the single dipole beam by a pair of counter-propagating 821 nm lattice beams of 100 mW each. This is to avoid loss and decoherence of the atoms due to their motion in the fiber. The moving lattice at frequency difference of 50 kHz moves at the speed of 2.05 cm s^{-1} , guiding the atoms into the HCF. About 10 % of the atoms are transported

Figure 9.6: Numerical calculations of slowed pulses. The blue circles are calculations of the slowed pulse with non-uniform profile of the control beam, the probe beam, and the atom density in the fiber. The green squares are the calculations of the slowed light with uniform EIT beam profiles and atom density distribution.

Figure 9.7: The measured temporal profiles of the leaked and delayed light pulses. The control beam is switched off when the probe is propagating inside the atomic medium. The bandwidth of the delayed pulse is defined from its FWHM of 160 ns.

Figure 9.8: Efficiency as a function of storage time T . (a) The grey circles are the measurements of the delay efficiency without light shift cancellation with fitted $T_c = 823(10) \mu\text{s}$. For comparison, the curve in purple represents the $0.03 \text{ dB } \mu\text{s}^{-1} = 0.15 \text{ dB km}^{-1}$ loss of 1550 nm light propagating in a solid core fiber as a delay line. (b) Measurements of the efficiency with light shift cancellation (orange circles). An exponential decay function is fitted to the data with a $1/e$ time of $20(1) \text{ ms}$. The curve in purple is the same as in (a). The error bars for both results are the standard errors of 40 experimental runs.

into HCF compared to the loading with just a single optical dipole beam. The dominant loss is due to the atoms trapped in the higher energy bands of the optical lattice, which leads to escape from the trapping potential. Then, it is changed to a stationary lattice and the storage efficiency measurement as a function of storage time T is performed.

Figure 9.8a shows the efficiency of the delayed light as a function of T . It is fitted with the function $A[1 + (T/T_c)]^{-3/2}$, where A is the efficiency at $T = 0$. The decay characteristic time is fitted to be $T_c = 823(10) \mu\text{s}$, and it is limited by the differential ac Stark shift of the dipole potential on the atomic spin coherence. It is compared with the plot of the efficiency of 1550 nm light propagating in a solid-core fiber loop [144]. The result exceeds the performance of 1550 nm light after 600 μs of delay time.

The performance is further improved by minimizing the differential ac Stark shift of $|1\rangle$ and $|2\rangle$ states which is the primary decoherence source of the atomic spin coherence [145] by the light shift cancellation technique in § 8.4. Figure 9.8b shows the storage efficiency as a function of the delay time. An exponential decay function is fitted to the data with a $1/e$ time of $20(1) \text{ ms}$, which is limited by the inhomogeneous stray magnetic field of the experimental apparatus [145].

9.5 Dynamic Decoupling

One way to prolong the delay time is to execute the dynamic decoupling (DD) method [146, 147], a series of N microwave population inverting π pulses separated by cycling time T_{dd} is applied to the atoms during the storage period to rapidly rephase the spin coherence,

Figure 9.9: Normalized efficiency versus delay time with different dynamic decoupling (DD) pulse separation time. The black squares represent an average of 40 experimental runs of light delay efficiency and the error bars indicate the standard errors. The red circles are the best least square regression fit of the simulation to the experimental data. The lines are used to guide the eyes. The efficiencies are normalized to the first points for each T_{dd} .

where N is an even integer. The DD method is studied under the fiber delay line condition experimentally and theoretically.

Figure 9.9 shows the normalized delay efficiency versus time measurements with $T_{dd} = 1, 2.5$ and 5 ms. The oscillation of the efficiency over time is observed due to the frequency difference Δ between the microwave frequency and the two-photon detuning between the control and the probe pulses. The oscillation is confirmed by simulating the rotation of the Bloch vectors of the atoms under DD.

In the simulation, the residual inhomogeneous broadening of the dipole beam $\delta = 2\pi \times (50 \text{ Hz})$ is set from the data obtained in Figure 9.8b and introduce α as the ratio of Gaussian waists of the atomic ensemble to the control beam in radial direction and set it as a free parameter together with Δ . The simulation to the data is fitted and the parameters α and Δ are found for each T_{dd} . The decay of the efficiency is due to the spatial inhomogeneity of control beam propagating in the fiber, which is determined by α . Further details can be found in §9.6.

In the Bloch sphere picture, the inhomogeneous control pulse prepares Bloch vectors on the Bloch sphere with a distribution for the DD sequence. To verify it, the same DD sequence is used on microwave Ramsey interferometer, replacing the EIT probe and control pulses with microwave pulses which is assumed to be homogeneous across the whole atomic ensemble. It is found that the $1/e$ decay time τ of the contrast of pure microwave sequence is two times longer than the storage time under the same DD, see Figure 9.10.

9.6 Simulation Algorithm

The details of simulations of the light storage efficiency under dynamical decoupling are provided here. The simulation is run by MATLAB with the code shown in Appendix C.

Figure 9.10: The measurement of contrast versus time using microwave Ramsey interferometer with DD sequence. The curve is an exponential fit to the experimental data, with the decay rate of $\tau = 102(8)$ ms.

9.6.1 Assumptions

In the simulation, I assume:

- The effective Rabi frequency $\Omega_R(r)$ of the EIT control and probe pulse for an atom at radial position r is Gaussian distributed according the fundamental mode field profile of the hollow-core fiber as $\Omega_R(r) = \Omega_R^{\max} e^{-2r^2/R^2}$ where $R = 22 \mu\text{m}$ is the $1/e^2$ mode field radius and $\Omega_R^{\max} = \pi/(500 \text{ ns})$.
- The EIT fields are two-photon resonant on the unperturbed atomic transition.
- The Rabi frequency Ω_M of the microwaves is uniform across the ensemble with a π -pulse time of $37 \mu\text{s}$.
- The microwave detuning from the unperturbed transition frequency is Δ . Δ is used as one of the fit parameters to the data.
- The inhomogeneous broadening $\delta(r)$ caused by the residual optical lattice differential ac Stark shift is also Gaussian distributed as $\delta(r) = \delta_{\max} e^{-2r^2/R^2}$ where $\delta_{\max} = 2\pi \times (50 \text{ Hz})$ from [Figure 9.8b](#).
- Initially, at $t = 0$, the phase of the microwaves is the same as the EIT fields, i.e. over time t , the microwave phase relative to an atom at position r is $(\Delta + \delta(r))t$.
- The atoms are Gaussian distributed according to $\rho(r) = \exp(-r^2/2R_{\text{atom}}^2)/R_{\text{atom}}^2$ where R_{atom} is the rms radius of the atomic ensemble. The probability distribution is normalized: $\int_0^\infty \rho(r)rdr = 1$. In the simulation, the atom positions are fixed for the duration of the dynamical decoupling sequence. The ratio α of the ensemble size to the mode field radius, i.e. $\alpha \equiv R_{\text{atom}}/R$ is used as the other fit parameter to the data.

9.6.2 Simulation on a single atom

The goal is to determine the collective state of the atoms after the EIT pulse and microwave dynamical decoupling pulses with atoms evolving phase in the optical lattice.

Beginning with an atom at position r in the ground state, represented by the Bloch vector $\mathbf{u}_0 = (0, 0, -1)$, I model the EIT pulse as a two-photon Raman $\pi/2$ -pulse acting on the Bloch vector. The rotation matrix representing $\pi/2$ pulse takes the inhomogeneous Rabi frequency $\Omega_R(r)$ and detuning $\delta(r)$ into account. In the Bloch sphere picture, the inhomogeneous EIT pulse prepares Bloch vectors on the Bloch sphere with a distribution for the DD sequence.

After the EIT pulse, the atom evolves phase $\delta(r)T_{\text{dd}}$ in the optical lattice for DD cycling time T_{dd} . In the Bloch sphere picture, the Bloch vector rotates about the z -axis by angle $\delta(r)T_{\text{dd}}$. A microwave π -pulse is then applied, with Rabi frequency Ω_M , detuning $\Delta + \delta(r)$ and phase $(\Delta + \delta(r))t$ where t is the time at which the π -pulse is applied. Again, the radial dependence encodes the inhomogeneity of the ensemble with respect to the microwaves. The DD sequence repeats the elementary sequence [free-evolution T_{dd} – π -pulse] for an even number of times. The Bloch vector of the atom $\mathbf{J} = (J_x, J_y, J_z)$ is numerically determined after each repeating unit of the DD sequence.

9.6.3 Weighting over ensemble

The ensemble has a radial distribution across the fiber mode profile leading to a distribution of Rabi frequency $\Omega_R(r)$ and detuning $\delta(r)$. As there is a one-to-one mapping between the radial position r and the normalized fiber mode intensity $I(r) = e^{-2r^2/R^2}$, the single atom simulation $\mathbf{J}$ is actually a function $\mathbf{J}(I)$ of the normalized mode intensity I .

To simulate the effect of this inhomogeneous distribution, I weight the simulated single atom Bloch vectors $\mathbf{J}(I)$ by the intensity distribution $\rho(I)$ that the ensemble samples as in the following

$$\langle \mathbf{J} \rangle \equiv \int_0^1 \mathbf{J}(I) \rho(I) dI. \quad (9.9)$$

This defines the collective Bloch vector $\langle \mathbf{J} \rangle$ that I use to calculate the retrieval efficiency. Using the Gaussian atom distribution provided in the assumptions, it can be shown that the equivalent intensity distribution is given by

$$\rho(I) = \frac{1}{4\alpha^2} I^{\frac{1}{4\alpha^2}-1}. \quad (9.10)$$

For the numerical simulations, I approximate the integral using the trapezium rule.

9.6.4 Retrieval Efficiency

The retrieval efficiency is determined by the collective coherence of the ensemble, which corresponds to the component of the collective Bloch vector in the equatorial plane $J_{\perp}^2 \equiv \langle J_x \rangle^2 + \langle J_y \rangle^2$ where $\langle J_x \rangle$ is the x -component of the collective Bloch vector $\langle \mathbf{J} \rangle$, similarly for $\langle J_y \rangle$. The normalized efficiency is then $(J_{\perp}/J_{\perp,0})^2$ where $J_{\perp,0}$ is the collective coherence of the ensemble right after the EIT storage process.

9.6.5 Fitting simulation to data

To fit the simulation to the data, I use the method of least squares. Specifically, I define the the sum squares deviation between the simulation and experimental data points as

$$SS = \sum_i \frac{(\text{simulation}(\Delta, \alpha)_i - \text{data}_i)^2}{\text{error}_i^2} \quad (9.11)$$

where error_i is the standard error of the i -th experimental data point. I then numerically minimize SS over the free parameters Δ and α to obtain fits to the data sets.

9.7 Discussion

Direct optical fiber delay line using light storage in HCF shows three orders of magnitude slower decay rate than the propagation loss of light in the solid core fiber. Moreover, the delay time could be varied precisely by merely turning on and off the control pulse as an optical switch without modifying any hardware as in the solid-core fiber loop delay line. The low initial delay efficiency in the experiment compared to free space case is due to the inhomogeneous profile of the control beam and atom density inside the fiber.

In general, I have achieved the maximum storage efficiency of $\sim 3\%$ with maximum storage time of a few tens of ms under dynamic decoupling with cold atom in HCF [148]. This can be compared with the free space light storage by Hsiao [143] with storage efficiency of 92 % but storage time of 325 μs and Zhang [149] with storage time of 1 s but storage efficiency of 0.03 %.

I have achieved the time-bandwidth product (TBP) to about 5×10^4 , a figure of merit for quantifying the performance of the optical storage capacity. The solid-core fiber systems [134–137] have achieved TBP to about 800 with a few percent of efficiency and atomic systems [77, 93, 94, 140–142] have achieved TBP to about 10 with about 30 % efficiency. The efficiency can be improved by increasing the optical depth [93, 143] or using other light storage methods such as Autler–Townes splitting to ease the demand of high optical depth [150]. The near-infrared wavelength of the light can be converted into telecom band in the same medium using the higher level of atomic transitions [151, 152] and be extended to single photons for quantum network applications [153].

Chapter 10

Creation of Schrödinger Cat States

The previous chapter demonstrates the quantum manipulation of cold atoms in the internal state as it is regarding the quantum storage into the coherence between two internal states $|F, m = 2, 0\rangle$ and $|F, m = 3, 0\rangle$. In this chapter, I demonstrate another example of quantum manipulation, but this in on the external state of cold atoms: the creation of Schrödinger cat states. It is a project inspired by Raman sideband cooling of atoms to the vibrational ground in HCF for improving the performance of Mach-Zehnder atom interferometer. The Schrödinger cat state, a thought experiment formulated by Erwin Schrödinger in 1935, illustrates the boundary between classical and quantum physics and forms the foundation of quantum information science.

It has been created in a variety of systems, such as ions, photons, and neutral atoms, but only a few at a time [154, 155]. However, there are limited experiment performed in a waveguide other than in cavity [156] while various theories in waveguide have been investigated [157, 158]. Departing from free space quantum engineering toolbox towards photonic system interfaces could provide new paradigms, such as increased interaction strength, scalability of the devices, and novel design of functionalities. For instance, atoms trapped by the evanescent fields of nanofibers and photonic crystal slabs have been used to study long-range atom-atom interactions mediated by the coupling of superradiant emission into the structure [159, 160]. In quantum communication applications, single collective excitation [142] and light storage [77, 94] have been demonstrated in nanofibers and hollow-core fibers. Moreover, tight and diffraction-free confinement of atoms in photonic structures is used for optical switching [95, 161]. In metrology, cold atoms in hollow-core fibers have also shown potential applications in timekeeping and sensing [85, 145, 162]. Regardless of the achievements, quantum control and manipulation at the single quantum state level in terms of both external and internal degrees of freedom of atoms such as Schrödinger cat creation using photonic waveguide modes has not been realized.

10.1 Experimental Configuration

The optical setup is shown in [Figure 10.1](#). The Raman beams (RBs) are sourced from the 821 nm lattice beams and sent in counter-propagating manner with linear polarization orthogonal to each other. RBs are meant for pumping state $|F = 2, m = 0, n\rangle$ to $|F = 3, m = 0, n - 1\rangle$ during Raman sideband cooling process (detail in [§10.3](#)) and should not be confused with the cat beams. The reason of orthogonal polarization is identical to the second reason of EIT beams in [§9.1](#).

The cat beams are sourced from a diode laser (see [§4.1](#)) and are designed as the displacement operator explained in [§3.3.2](#). The lasers are at 795 nm and 744 MHz red-detuned from the $|F = 3\rangle$ to $|F' = 2\rangle$ transition (see [Figure 4.1](#)), with approximately 13 μW per beam. They are sent in counter-propagating manner with linear polarization parallel to each other and drive same the transition $|F = 2, m = 0, n\rangle \leftrightarrow |F = 2, m = 0, n + 1\rangle$, unlike RBs, leading the Clebsch–Gordan coefficients for both Raman transitions to be the same as shown in [Figure 10.3](#). Therefore, the polarization needs to be parallel to boost the transition strength.

Similar to light storage experiment, the π -polarized optical pump beam incident from the side of the fiber can efficiently prepare atoms in the $F = 2, m = 0$ state with more than 90 % efficiency.

10.2 Vibrational Spectrum

10.3 Raman Sideband Cooling

The time sequence for Raman sideband cooling (RSC) is shown in [Figure 10.4](#). After atom loading and transportation by an optical conveyor belt using a moving optical lattice at a speed of 2 cm s^{-1} , the optical lattice is stopped and the atoms are prepared to magnetic-field-insensitive state $|F = 2, m = 0\rangle$ by optical pump. The vibrational frequency is $\omega_{\text{vib}} = 2\pi \times (400 \text{ kHz})$ in the axial direction and $2\pi \times (3.5 \text{ kHz})$ in the radial direction. Then, it is cooled down to axial vibrational ground state of the optical harmonic potential by RSC, which consists of two steps of process:

RBs Cooling The pair of RB1 and RB2 beams pump the atomic state from $|F = 2, m = 0, n\rangle$ to $|F = 3, m = 0, n - 1\rangle$, reducing the vibrational number by one; and

Optical Pump The depump beam pump the state $|F = 3, m = 0, n - 1\rangle$ to $|F = 2, m, n - 1\rangle$ through spontaneous emission. To ensure atoms accumulate in $m = 0$ state after cooling, a π -polarized optical pump beam is also used in the process.

To prevent heating by spontaneous emission, operating in the Lamb-Dicke regime is required where the vibrational frequency is much greater than recoil frequency $\omega_{\text{vib}} \gg \omega_{\text{rec}}$. Since the recoil frequency of 821 nm RB1 and RB2 beams is $\omega_{\text{rec}} = 2\pi \times (14 \text{ kHz})$, the Lamb-Dicke parameter is $\eta = \sqrt{\omega_{\text{rec}}/\omega_{\text{vib}}} \approx 0.19$ (see equation (3.40)).

The RSC is optimized with 120 cooling cycles of 250 μs RBs cooling pulse and 25 μs optical pump, a total of 33 ms cooling time. The vibrational spectrum is measured by

Figure 10.1: Optical setup for the Schrödinger cat states experiment.

Figure 10.2: Illustration of the Schrödinger cat experiment. Each lattice site in the fiber represents a harmonic oscillator potential for atoms (red filled circles). The colored Gaussian wave packets in the potential illustrate the coherent harmonic oscillator states that are entangled with the spin states. PBS: polarizing beam splitter. NPBS: non-polarizing beam splitter. NF: notch filter. APD: avalanche photodiode.

Figure 10.3: Clebsch–Gordan coefficients of transitions for cat experiment. Since the internal ground states are same, the Clebsch–Gordan coefficients for both Raman transitions are same. The products of the coefficients for the transitions are $1/9$ (σ^- , blue), $2/9$ (σ^- , green), $1/9$ (σ^+ , red) and $2/9$ (σ^+ , orange).

Figure 10.4: Timing sequence for RSC. There are 120 cooling cycle during RSC, not fully shown here. δ_{RB} is the detuning of RB1 and RB2 beams to be scanned for vibrational spectrum. RS = Raman sideband.

Figure 10.5: Energy levels for RSC. The energy levels represent the vibrational energy splitting of the optical lattice potential in the axial direction. The wavy yellow arrow denotes the spontaneous emission of atoms.

Figure 10.6: Vibrational spectra with and without RSC. The y axis indicates the OD for $F = 3$.

scanning the two-photon detuning¹ δ_{RB} of RBs in Figure 10.4, as shown in Figure 10.6. The axial temperature is determined by measuring the mean vibrational quantum number $\langle n \rangle$ as

$$k_B T = \left(\langle n \rangle + \frac{1}{2} \right) \hbar \omega_{\text{vib}} \quad (10.1)$$

from Figure 10.6 through comparing the area of the first blue sideband A_{bsb} and red sideband A_{rsb} as $R := \frac{A_{\text{rsb}}}{A_{\text{bsb}}} = \frac{\langle n \rangle}{\langle n \rangle + 1}$ or

$$\langle n \rangle = \frac{R}{1 - R}. \quad (10.2)$$

Here, R is taken by taking the ratio of the areas A_{bsb} and A_{rsb} in Figure 10.6. I achieve $\langle n \rangle_0 = 0.25$ after cooling from $\langle n \rangle = 3.3$ before cooling, which corresponds to the temperature drop from 73 μK to 14 μK .

10.4 Quantum Coherent Harmonic Oscillator States

To study the coherence of the Fock states in the system after RSC, resonance RB1 and RB2 ($\delta_{\text{RB}} = 0$) are used to excite a two-photon transition between $|\downarrow, n\rangle \equiv |F = 2, m = 0, n\rangle$ to $|\uparrow, n - 1\rangle \equiv |F = 3, m = 0, n - 1\rangle$ through a $\pi/2$ - π - $\pi/2$ spin-echo sequence. 37% and 22% contrasts are observed at 0.1 ms and 0.9 ms separation times between the $\pi/2$ and π pulses, respectively, as shown in Figure 10.7.

Since the spin coherence time of the two spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ measured by a microwave $\pi/2$ - π - $\pi/2$ spin-echo sequence exceeds tens of milliseconds in § 8.4, it is believed that the decay of the contrast for the Fock states is mostly due to the amplitude noise of the optical lattice and the radial motion of atoms, or motional dephasing.

¹ $\delta_{\text{RB}} = 0$ is defined as the RB1 and RB2 beams resonance to the coupling $|F = 2, m = 0, n\rangle \leftrightarrow |F = 3, m = 0, n - 1\rangle$ as in Figure 10.5.

Figure 10.7: Coherence of the Fock states. A $\pi/2$ - π - $\pi/2$ spin-echo sequence is implemented on the atoms before sideband cooling. The duration of the $\pi/2$ pulses is $7.5 \mu\text{s}$. The OD of $|\uparrow, n-1\rangle$ is measured with varying phase of the final $\pi/2$ pulse. The curves are sinusoidal fits to the data.

The amplitude of the coherent state $|\alpha|$ versus the laser driving time t is also measured as shown in Figure 10.8. To avoid the effect of the radial motion while creating a coherent state, the driving lasers in the short pulse regime $t \ll \omega_{\text{vib}}^{-1}$ are operated. I also consider the case if there is two photon detuning δ_{cat} for the Cat beams such that equation (3.47) is generalized to

$$\alpha = \int_0^t \frac{\eta |\Omega_{2p}| e^{-i\delta_{\text{cat}} t'} t'}{2} dt' = \frac{\eta |\Omega_{2p}|}{\delta_{\text{cat}}} \sin \frac{\delta_{\text{cat}} t}{2} e^{-i\delta_{\text{cat}} t} \quad (10.3)$$

and the data show a linear trend suggesting that δt is small enough to use equation (3.47).

However, this amplitude measurement does not reveal information about the coherence between the superposition of Fock states in the coherent state [163]. The coherence in the coherent state is demonstrated by applying two consecutive driving pulses with phase difference ϕ_{cat} . The mean phonon number after the application of two successive displacement operators $\hat{D}(\alpha)$ and $\hat{D}(\alpha e^{i\phi_{\text{cat}}})$ is

$$\langle n \rangle = \langle n \rangle_0 + 2|\alpha|^2 + 2|\alpha|^2 \cos \phi_{\text{cat}} \quad (10.4)$$

from equation (3.38) and by *Baker-Campbell-Hausdorff formula* discussed in §3.3.1, as shown in Figure 10.9. The mean phonon number returning to the initial value $\langle n \rangle_0$ when the phase ϕ_{cat} is π indicates the coherence between different Fock states in the coherent state. The fitted data show the coherent state with an amplitude $|\alpha| = 0.36$.

Figure 10.8: Creation of the coherent state. The amplitude of the coherent state is determined by measuring $\langle n \rangle$ with vibrational spectroscopy, similar to [Figure 10.6](#). (a) Vibrational spectrum at difference pulse duration. (b) Plot of $|c|$ vs pulse duration. Each data point corresponds to one vibrational spectroscopy measurement. The straight line is a linear fit to the data with a fixed intercept at zero.

Figure 10.9: Coherence in the coherent state. The coherence in the coherent state is examined by applying two consecutive driving pulses with phase difference ϕ_{cat} . After the two driving pulses, $\langle n \rangle$ is measured from the vibrational spectra and plot it for different phases between the first and second displacement pulses. The curve is a sinusoidal fit to the data.

10.5 Schrödinger Cat States

The time sequence for Schrödinger Cat (SC) experiment is shown in Figure 10.10 and protocol of the SC state preparation is shown in Figure 10.11. After the atom preparation in HCF, a microwave $\pi/2$ pulse of 7.5 μs brings the atoms from the vibrational ground state $|\downarrow, n=0\rangle$ into a superposition state of spin up $|\uparrow, n=0\rangle$ and spin down $|\downarrow, n=0\rangle$. The detail in atomic state is explained in §3.3.3. The coherent state $|\uparrow, \alpha\rangle$ is prepared by the driving lasers, ideally leaving $|\downarrow, n=0\rangle$ unchanged. The internal spin states are then flipped by a microwave π pulse, followed by another driving laser pulse with a different phase ϕ to create a coherent state. The resulting SC state can then be written as equation (3.51). It is an entanglement between a superposition of coherent states and spin states. To confirm this, the interference of the two separated coherent states is measured by applying another microwave $\pi/2$ pulse with a phase shift ϕ_M relative to the first two microwave pulses, and the state becomes

$$i|\uparrow\rangle \left(\frac{|\alpha\rangle - e^{i\phi_M} |\alpha e^{i\phi}\rangle}{2} \right) - |\downarrow\rangle \left(\frac{|\alpha\rangle + e^{i\phi_M} |\alpha e^{i\phi}\rangle}{2} \right). \quad (10.5)$$

The population of atoms in state $|\uparrow\rangle$ is then determined by the overlap between the two coherent states in phase space. The population of the atoms in state $|\uparrow\rangle$ is detected by measuring OD with varying phase ϕ_{cat} . The probability of finding the atoms in $|\uparrow\rangle$ is similar to equation (3.57).

$$|c_{\uparrow}|^2 = \frac{1}{2} \left[1 - e^{-|\alpha|^2(1-\cos\phi)} \cos(\phi_M + |\alpha|^2 \sin\phi) \right]. \quad (10.6)$$

Figure 10.12 shows the experimental data of SC state interference fringes for different pulse durations. The population of the atoms in $|\uparrow\rangle$ is normalized to 0.5 when the driving lasers are off, and the relative phase is set at $\phi_M = \pi/2$. When the microwave phase is set to $\phi_M = 0$ (π), the SC state is termed the odd (even) cat state. The odd (even) cat states only contain odd (even) Fock states, similar to squeezed vacuum states. This approach is used to verify the validity of the SC state created in my experiment. Figure 10.13 shows a comparison of the SC states when $\phi_M = 0$ and π , and it clearly confirms equation (10.6) for odd (even) cat states.

10.6 Imperfect Schrödinger Cat States

My results deviate from the ideal case of equation (10.6) due to the imperfect conditions of the experiment. Equation (10.6) is modified to fit the data in Figure 10.12 by introducing the effects of the residual excitation of the driving lasers on the $|\downarrow\rangle$ state, contrast and imperfect initial ground state preparation.

In ideal case, $\hat{D}(\alpha)$ only act state $|\uparrow\rangle$. Consider it also acts on $|\downarrow\rangle$ with strength $\epsilon\alpha$, then the Schrödinger cat sequence with a change of the wavefunction before the final $\pi/2$ pulse

Figure 10.10: Sequence for Schrödinger cat experiment. RS = Raman sideband.

Figure 10.11: Protocol for creating the SC state in phase space representation. The top and bottom columns represent $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively. The circles in red indicate the uncertainty in the ground state and the coherent state.

Figure 10.12: Interference fringes of the SC state interferometer versus relative phase of the two driving pulses with different pulse durations. Each data point is an average of four experimental runs.

Figure 10.13: Even and odd SC states. The driving pulse duration is 0.45 μs . Each data point is an average of four experimental runs.

as

$$\begin{aligned}
 |\downarrow, 0\rangle &\xrightarrow{\pi/2} \sqrt{\frac{1}{2}} |\downarrow, 0\rangle - i\sqrt{\frac{1}{2}} |\uparrow, 0\rangle \\
 &\xrightarrow{\hat{D}(\alpha)} \sqrt{\frac{1}{2}} |\downarrow, \epsilon\alpha\rangle - i\sqrt{\frac{1}{2}} |\uparrow, \alpha\rangle \\
 &\xrightarrow{\pi} -i\sqrt{\frac{1}{2}} |\uparrow, \epsilon\alpha\rangle - \sqrt{\frac{1}{2}} |\downarrow, \alpha\rangle \\
 &\xrightarrow{\hat{D}(\alpha e^{i\phi})} e^{i\epsilon|\alpha|^2 \sin \phi} \left[-i\sqrt{\frac{1}{2}} |\uparrow, (e^{i\phi} + \epsilon)\alpha\rangle - \sqrt{\frac{1}{2}} |\downarrow, (1 + \epsilon e^{i\phi})\alpha\rangle \right] \quad (10.7)
 \end{aligned}$$

before the final $\pi/2$ pulse, where the global phase comes from *Baker-Campbell-Hausdorff* formula

$$\hat{D}(\alpha e^{i\phi}) |\epsilon\alpha\rangle = e^{i\epsilon|\alpha|^2 \sin \phi} |(e^{i\phi} + \epsilon)\alpha\rangle \quad (10.8a)$$

$$\hat{D}(\epsilon\alpha e^{i\phi}) |\alpha\rangle = e^{i\epsilon|\alpha|^2 \sin \phi} |(1 + \epsilon e^{i\phi})\alpha\rangle. \quad (10.8b)$$

After the final $\pi/2$ pulse (conversion is shown in equation (3.52) and (3.53)), the final

Figure 10.14: The population of the state $|\uparrow\rangle$ in imperfect Schrödinger cat experiment. The function is shown in equation (10.12).

wavefunction (global phase excluded) is

$$\begin{aligned} |\psi_{\text{cat}}\rangle &= \frac{i}{2} (|\uparrow, (1 + \epsilon e^{i\phi}) \alpha\rangle - |\uparrow, (e^{i\phi} + \epsilon) \alpha\rangle) - \frac{1}{2} (|\downarrow, (1 + \epsilon e^{i\phi}) \alpha\rangle + |\downarrow, (e^{i\phi} + \epsilon) \alpha\rangle) \\ &\equiv \frac{i}{2} (|\uparrow, \beta\rangle - |\uparrow, \beta e^{i\theta}\rangle) - \frac{1}{2} (|\downarrow, \beta\rangle + |\downarrow, \beta e^{i\theta}\rangle) \end{aligned} \quad (10.9)$$

which is similar to equation (3.54) where

$$\beta = (1 + \epsilon e^{i\phi}) \alpha \quad (10.10)$$

and

$$\begin{aligned} e^{i\theta} &= \frac{e^{i\phi} + \epsilon}{1 + \epsilon e^{i\phi}} \\ &= \frac{e^{i\phi} + 2\epsilon + \epsilon^2 e^{-i\phi}}{1 + \epsilon^2 + 2\epsilon \cos \phi} \\ \Rightarrow \cos \theta + i \sin \theta &= \frac{2\epsilon + (1 + \epsilon^2) \cos \phi}{1 + \epsilon^2 + 2\epsilon \cos \phi} + i \frac{(1 - \epsilon^2) \sin \phi}{1 + \epsilon^2 + 2\epsilon \cos \phi}. \end{aligned} \quad (10.11)$$

Therefore, the population at $|\uparrow\rangle$ is

$$|c_{\uparrow}|^2 = \frac{1}{2} - \frac{1}{2} e^{-|\beta|^2(1-\cos \theta)} \cos(|\beta|^2 \sin \theta) \quad (10.12)$$

where $|\beta|^2$, $\cos \theta$ and $\sin \theta$ can be expressed from equation (10.10) and (10.11). If the atom has an initial mean vibrational number $\langle n \rangle_0$, with the phase of last microwave $\pi/2$ pulse ϕ_M , the population at $|\uparrow\rangle$ becomes

$$|c_{\uparrow}|^2 = \frac{1}{2} - \frac{1}{2} \chi e^{-(2\langle n \rangle_0 + 1)|\beta|^2(1-\cos \theta)} \cos(\phi_M + |\beta|^2 \sin \theta) \quad (10.13)$$

where χ is the contrast used as the fitting parameter. Moreover, ϕ is replaced to $\phi - \phi_f$ where $\phi_f = \omega' t$ is the phase advance of the driving lasers due to the increase in the pulse time and it is another fitting parameter. For the residual driving, since the nearest single photon

Figure 10.15: Fitting parameters of equation (10.13) for the experimental data in Figure 10.12.

detuning for $|\uparrow\rangle$ and $|\downarrow\rangle$ is 744 MHz and 3782 MHz with the ratio of 0.19:1, the parameter $\epsilon = 0.19$ is used to consider the off-resonant dipole force on $|\downarrow\rangle$. The imperfect ground state preparation is taken into account by adding a factor of $2\langle n \rangle_0 + 1 = 1.5$ into the exponent of equation (10.13). The fitting parameters $|\alpha|$, χ and ϕ_f are displayed in Figure 10.15.

The phase advance ϕ_f is linear in time, as expected, due to the free evolution of the phase of the beating of the driving lasers. For $t = 0.45 \mu\text{s}$, the amplitude of the coherent state is $\alpha = 1.42$, which corresponds to a maximum separation of the two coherent wave packets of $2\sqrt{2}\alpha z_0 = 54 \text{ nm}$, where $z_0 = \hbar / (2m\omega_{\text{vib}})$ is the variance in the ground state wave packet size. The loss of contrast suggests phase noise during the operation [164], which also widens the narrow interference feature. The increase of the interference width leads to the saturation of $|\alpha|$ after $t = 0.45 \mu\text{s}$ for the fitting of equation (10.13).

10.7 Radial Motion Study

The effect of radial motion during the free evolution between driving pulses by inserting an additional free evolution time τ is studied, as shown in Figure 10.16. The sequence follows $\pi/2 - \hat{D}(\alpha) - \pi - \hat{D}(\alpha e^{i\phi}) - \pi/2$. The data shows that the radial motion does not significantly degrade the contrast at my experimental time scale. I also numerically integrate equation (10.13) over the radial distribution of atoms with radial position-dependent parameters of

Figure 10.16: (a) Measurements of the $|\uparrow\rangle$ population in cat interferometer operation versus phase with additional free evolution time τ . The total free evolution time between the two driving pulses for $\tau = 1 \mu\text{s}$ is $16 \mu\text{s}$. (b) Simulation of the temperature effect on the SC state interferometer. The simulation uses the fitted parameters for $\tau = 0.45 \mu\text{s}$ in Figure 10.15. The $1/e$ Gaussian radius of the atomic cloud in the radial direction is $0.9 \mu\text{m}$ and $8 \mu\text{m}$ for $2 \mu\text{K}$ and $150 \mu\text{K}$, respectively.

$\eta(r)$, $\Omega_{2p}(r)$, and $\delta_{\text{cat}}(r)$, as

$$\frac{\int_0^\infty 2\pi r \cdot \exp\left(-\frac{r^2}{r_0^2}\right) |c_\uparrow|^2(r) dr}{\int_0^\infty 2\pi r \cdot \exp\left(-\frac{r^2}{r_0^2}\right) dr} \quad (10.14)$$

where $r_0 = \sqrt{w^2 k_B T_{\text{rad}} / (2U)}$ is the $1/e$ radius of the spatial distribution of the atomic cloud in the radial direction, $w = 22 \mu\text{m}$ is the $1/e^2$ mode field radius, T_{rad} is the radial atom temperature and U is the radial trapping potential.

The results indicate that the radial distribution of atoms is not the main loss mechanism for the interference contrast. The anharmonicity of the optical lattice also causes deviation of the vibrational energy splitting between $|n\rangle$ and $|n+1\rangle$ from the pure harmonic potential as $\omega_{\text{anh}} = \omega_{\text{vib}} - \omega_{\text{rec}}(1+n)$. This gives an approximately few percent deviation at the center of the fiber [165]. Other decoherence mechanisms, such as single photon scattering of the driving lasers and amplitude fluctuation of the lattice potential [166], are 10 and 100 times slower than the coherent interaction between the driving lasers and the atoms.

10.8 Discussion

The results show the coherent interaction of single quantum harmonic oscillator states of matter with the light guided in a fiber. I use the fundamental waveguide mode to create entanglement between coherent states and spin states inside a HCF. Such a state can enable quantum simulation of a truly one-dimensional system with the assistance of ground-state cooling in the radial direction [167, 168]. Other than the fundamental importance, the plethora of photonic structures of fibers [169] provides fruitful ways for manipulating atoms. For example, the higher-order modes can be tailored to create multiple lattice sites

and different populations in the transverse plane, creating a three-dimensional atom array in a cylindrical symmetric photonic structure [170]. Strong atom–light interactions can be achieved by trapping atoms in the cladding modes, which can be patterned by engineering the geometry and thickness of the cladding structure [171]. As silica fiberglass is transparent to light in the visible to near-infrared range, light propagating from the side into the fiber can also be used for controlling the internal and external degrees of freedom of atoms [172]. Other photonic crystal structures also possess similar degrees of freedom to control atoms at the single quantum state level. This experiment paves the way for the realization of quantum devices of photonic structures with the matter for promising applications in quantum-information science.

Part IV

Outlook

Chapter 11

Outlook

In this thesis, I have demonstrated the atom loading mechanism into HCF and the cooling inside HCF. Atoms with axial and radial temperature at tens of μK , are measured through release-and-recapture method and asymmetry atom interferometry technique. Then, four main experimental projects were carried out, all of each is to show the capability and feasibility for the cold atom quantum system inside a HCF.

First, the Mach-Zehnder atom interferometer, which is typically used for inertia sensing application, was built inside the fiber. It is proven that quantum sensor can be built inside a HCF. As the example, the state-of-the-art gravimeter in HCF is shown in this thesis with gravitational acceleration measured to be $9.2(7) \text{ ms}^{-2}$ and $9.7(6) \text{ ms}^{-2}$ with 95 % certainty from both directions of the Raman beams. In general, not only gravity, anything that contribute to the change of Mach-Zehnder phase can be used to make the corresponding sensor such as rotation sensor/gyroscope, gravitational gradient sensor, or even motion sensor by measuring its acceleration. Instead of the free space quantum sensor which takes up at a meter in size, the fiber-based quantum sensor will be a more compact device the future technologies.

In the gravitational acceleration measurement, the low accuracy is due to short coherence time of around 10-100 μs mainly from differential AC Stark shift. Therefore, light shift is cancelled by introducing vector light shift and with the help of spin echo, the coherence time is increased to sub-second level. At the meantime, optical pump is introduced by shining the light from the side of the fiber to enhance the optical depth. In this part of the project, I have shown that the quantum coherence can be preserved in the fiber. The limitation of the coherence time is from the inhomogeneous magnetic field and the existence of non-linear term in the vector light shift shown in equation (2.22). This limitation is the same if the cold atom were to be placed in the free space instead. In conclusion, putting cold atom into HCF will not lose its coherence but it could be a promising direction to quantum sensing devices.

With long quantum spin coherence time, I have demonstrated an example of the quantum manipulation of the internal state of cold atom in the HCF. As an example, a light storage with storage time of 50 ms in the fiber has been achieved and the result is equivalent to

$1.4 \times 10^{-5} \text{ dB } \mu\text{s}^{-1}$ of propagation loss in the conventional fiber. This demonstration could be used for buffering and regulating classical and quantum information flow between remote networks, and hence, the fiber-based quantum information can be realized.

Next, a large array of Schrödinger cat states is created along the fiber as a demonstration of quantum manipulation of the external state of cold atom in the HCF. While other group successfully created the Schrödinger cat states with quantity near the unity, I have created 15000 Schrödinger cat states inside the fiber, which presents the advantage of long guiding distance in fiber. It demonstrates the first coherent quantum control of the atoms with the photonic structure and provide an essential step for quantum information and simulation with a wide range of photonic waveguide systems. Furthermore, this demonstration shows that cold atoms setup in HCF open a wide field of quantum simulation under a fact of long guiding distance inside the fiber.

Of course, the experimental results in this thesis are not perfect and can still be improved in the future work. For example, the improved quantum spin coherence time can be used to measure gravitational acceleration with a better precision, where it has not been done in this thesis. Furthermore, a series of systematic errors and noise sources in the measurement can be determined to fully understand the atom laboratory in HCF. Moreover, to fully understand the study of cold atom laboratory with photonic modes in HCF, other types of coherent manipulation could be further explored. For example, large array of spin-motion entanglement states [173] can be evaluated and investigated in HCF. In addition, atom interferometry inside the HCF open a field of miniaturized sensors such as magnetic field sensors and rotation sensors with high precision. It can also be a promising path to fundamental science study such as determining the Newtonian gravitational constant with greater precision.

In conclusion, cold atom laboratory in a hollow core fiber open a new direction to quantum science and technologies without losing its coherence. Many applications which face difficulty in free space could be solved in the fiber. In the field of quantum technology, a compact quantum sensor and devices can be built in future, similar concept to the history of computer, where thousands of transistor in a big room can be miniaturized into trillions of transistors at the size of a palm. In addition, the concept of cold atom in HCF can still be applied on other atomic species or molecules, which could be an interesting direction. In general, the current state-of-the-art system in HCF has demonstrated various quantum control experiments and proved its potential in the future quantum technologies.

Part V

Appendix

Appendix A

Clebsch–Gordan Coefficients

This appendix shows all Clebsch–Gordan (CG) coefficients that form dipole matrix elements for D1 and D2 lines of Rubidium 85. The diagrams below are self-explanatory. These CG coefficients are used to evaluate the dipole matrix element $\hat{\epsilon} \cdot \langle e | \mathbf{d} | g \rangle$ in many equations (e.g. Equation (2.6)). For Rubidium 85,

$$\hat{\epsilon} \cdot \langle e | \mathbf{d} | g \rangle|_{\text{D1/D2}, (F, m) \rightarrow (F', m')} = d_{\text{D1/D2}} \text{CG}_{\text{D1/D2}, (F, m) \rightarrow (F', m')} \quad (\text{A.1})$$

where [109] $d_{\text{D1}} = 2.5377 \times 10^{-29} \text{ C m}$ and $d_{\text{D2}} = 3.58425 \times 10^{-29} \text{ C m}$ depending the transition used and CG is taken from the diagrams below depending on the polarization $\hat{\epsilon}$.

Notice that the CG coefficients for the π polarized transition with $F' = F$, $m' = m$ are zeros, leading the corresponding Rabi frequencies to be zero, i.e.

$$\Omega|_{(2,2) \rightarrow (2',2')} = \Omega|_{(3,3) \rightarrow (3',3')} = 0. \quad (\text{A.2})$$

Therefore, we can use this property to optical pump the all the states to the clock state ($m = 0$) because it is dark to the beam (See §4.1)

A.1 D2 Line $F = 3$

Figure A.1: Rubidium 85 D2 ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 3 \rightarrow F' = 4$.

Figure A.2: Rubidium 85 D2 ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 3 \rightarrow F' = 3$.

Figure A.3: Rubidium 85 D2 ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 3 \rightarrow F' = 2$.

A.2 D2 Line $F = 2$

Figure A.4: Rubidium 85 D2 ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 2 \rightarrow F' = 3$.

Figure A.5: Rubidium 85 D2 ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 2 \rightarrow F' = 2$.

Figure A.6: Rubidium 85 D2 ($5^2S_{1/2} \rightarrow 5^2P_{3/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 2 \rightarrow F' = 1$.

A.3 D1 Line $F = 3$

Figure A.7: Rubidium 85 D1 ($5^2S_{1/2} \rightarrow 5^2P_{1/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 3 \rightarrow F' = 3$.

Figure A.8: Rubidium 85 D1 ($5^2S_{1/2} \rightarrow 5^2P_{1/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 3 \rightarrow F' = 2$.

A.4 D1 Line $F = 2$ Figure A.9: Rubidium 85 D1 ($5^2S_{1/2} \rightarrow 5^2P_{1/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 2 \rightarrow F' = 3$. Figure A.10: Rubidium 85 D1 ($5^2S_{1/2} \rightarrow 5^2P_{1/2}$) CG coefficients for σ^+ (blue), σ^- (red) and π (green) transitions of $F = 2 \rightarrow F' = 2$.

Appendix B

Mathematica Calculation

B.1 Number of Atom

This is the calculation on the number of atom per OD for §6.6.

```

In[ ]:= $Assumptions = {w0 ∈ Reals, r0 ∈ Reals};
BeamProfile =  $\frac{2}{\pi w0^2} e^{-2r^2/w0^2}$ ; (*normalized Gaussian beam profile*)
NumberDensity =  $\frac{N_{total}}{\pi r0^2 L} e^{-r^2/r0^2}$ ; (*Atom density*)
ODperAtom =  $\frac{1}{N_{total}} \int_0^\infty \sigma * NumberDensity * L * BeamProfile * 2\pi r dr$ ; (*OD per atom,  $\sigma$ =absorption cross section*)
 $\sigma0 = \frac{3\lambda^2}{2\pi}$ ; (*Maximum absorption cross section on (F,m)=(3,3)→(4',4')*)
CGsquare = { $\frac{1}{56}, \frac{3}{56}, \frac{3}{28}, \frac{5}{28}, \frac{15}{56}, \frac{3}{8}, \frac{1}{2}$ }; (*CG coeff. square of  $\sigma^+$  transition, since  $\sigma^-$  has same result,
the average remains unchange*)
m = {-3, -2, -1, 0, 1, 2, 3}; (*m for F=3*)
MhzperGF3 = 0.47; (*Zeeman shift on F=3, MHz/G*)
MhzperGF4 = 0.7; (*Zeeman shift on F'=4, MHz/G*)
 $\Gamma = 6.0666$ ; (*Decay rate, MHz*)
CGFactorRes =  $\sum_{i=1}^{Length[CGsquare]} \frac{CGsquare[[i]] / Max[CGsquare]}{1.1 + \left( \frac{8 (m[[i]] MhzperGF3 - (m[[i]] + 1) MhzperGF4)}{\Gamma/2} \right)^2} / Length[CGsquare]$  /. B → 3.6;
(*average over all Zeeman transitions for probe resonant on F=3, mF=0 to F' = 4, mF'=0,
linearly polarized light, assume 3.6 Gauss bias field which cause Zeeman shifts*)
CGFactor =  $\sum_{i=1}^{Length[CGsquare]} \frac{CGsquare[[i]] / Max[CGsquare]}{1.1 + \left( \frac{8 (m[[i]] MhzperGF3 - (m[[i]] + 1) MhzperGF4) + 8.3}{\Gamma/2} \right)^2} / Length[CGsquare]$  /. B → 3.6;
(*average over all Zeeman transitions for 8.3 MHz probe detuned on F=3, mF=0 to F' = 4, mF'=0,
linearly polarized light, assume 3.6 Gauss bias field which cause Zeeman shifts*)
ODperAtom * CGFactorRes /. {r0 →  $\sqrt{\frac{T}{2U} w0}$ } /. {w0 → 22,  $\sigma$  →  $\sigma0$ } /. { $\lambda$  → 0.780241, T → 30, U → 514}
(*Resonance OD per atom, assuming 3.6 Gauss bias field*)
1 / % (*Number of atoms corresponding to OD=1*)
ODperAtom * CGFactor /. {r0 →  $\sqrt{\frac{T}{2U} w0}$ } /. {w0 → 22,  $\sigma$  →  $\sigma0$ } /. { $\lambda$  → 0.780241, T → 30, U → 514}
(*OD per atom @ 8.3 MHz detuned, assuming 3.6 Gauss bias field*)
1 / % (*Number of atoms corresponding to OD=1*)

Out[ ]:= 0.0000628395

Out[ ]:= 15913.6

Out[ ]:= 0.0000498781

Out[ ]:= 20048.9

```

B.2 Atom Properties Under Beam

This is the calculation of all atom light interactions that generates [Table 2.1](#) which are commonly used in the experiments of the thesis. Most of the constants are based on Daniel Steck [\[109\]](#).

Define ClebschGordonCoefficient function for 85Rb CG[F, mF, Ep, mFp, Line]

```
In[ ]:= Off[ClebschGordan::phy];
Off[ClebschGordan::tri];
Off[SixJSymbol::tri];
RbNumbersMatE1[F_, mF_, Fp_, mFp_, q_, J_, Jp_, I_] :=
  (-1)^(Fp-1+mF) Sqrt[2 F+1] ThreeJSymbol[{Fp, mFp}, {1, q}, {F, -mF}] (-1)^(Fp+J+1+I) Sqrt[(2 Fp+1) (2 J+1)]
  SixJSymbol[{J, Jp, 1}, {Fp, F, I}];
MatE1D2[F_, mF_, Fp_, mFp_, q_] := RbNumbersMatE1[F, mF, Fp, mFp, q, 1/2, 3/2, 5/2];
MatE1D1[F_, mF_, Fp_, mFp_, q_] := RbNumbersMatE1[F, mF, Fp, mFp, q, 1/2, 1/2, 5/2];
MatD2New[F_, mF_, Fp_, mFp_] := (MatE1D2[F, mF, Fp, mFp, 1] + MatE1D2[F, mF, Fp, mFp, 0] + MatE1D2[F, mF, Fp, mFp, -1]);
MatD1New[F_, mF_, Fp_, mFp_] := (MatE1D1[F, mF, Fp, mFp, 1] + MatE1D1[F, mF, Fp, mFp, 0] + MatE1D1[F, mF, Fp, mFp, -1]);
CG[F_, mF_, Fp_, mFp_, Line_] := If[Line == 1, MatD1New[F, mF, Fp, mFp], If[Line == 2, MatD2New[F, mF, Fp, mFp], 0]];
(*allow the D1 and D2 lines to be selected with 1,
or 2*)
```

Define Constants

```
In[ ]:= Co = 2.99792458 * 10^8 (*m/sec*);
eo = 8.854188 * 10^-12 (*F/m*);
h = 1.0545718 * 10^-34 (*J*sec*);
h = 2 * Pi * h (*J*sec*);
mu0 = 4 * Pi * 10^-7 (*N/Ampere^2*);
mrb = 1.409993199 * 10^-25 (*Rb85 mass in kg*);
kg = 1.38 * 10^-23 (*Boltzmann's constant J/K*);
```

Define Rb and Lattice Constants

```
In[ ]:= Based[Line_] := {377.107.385690, 384.230.406373}[[Line]]; (*D1, D2 splitting in GHz*)
Grnd[F_] := {-1.7708439228, +1.2648885163}[[F-1]]; (*F2, F3 HFS in GHz*)
Excited[Fp_, Line_] := { {0, -0.210923, 0.150659, 0}, {-0.113208, -0.083835, -0.020435, 0.100205} }[[Line]][[Fp]];
(*D1, D2 excited state HFS in GHz*)
d[Line_] := {2.5377 * 10^-29, 3.58425 * 10^-29}[[Line]];
(*Coulomb x meter*)
```

Input Parameters

```
In[ ]:= w = 22 * 10^-6 (*waist size in [m]*);
```

Optical Calculation

Calculation Formulae (All in SI Unit)

```
In[ ]:= k[lambda_] := 2 * Pi / lambda; (*Wavenumber*)
f[lambda_] := Co / lambda; (*Laser frequency, Hz*)
w[lambda_] := 2 * Pi * f[lambda]; (*Laser frequency, 2*Pi Hz*)
wtran[Fp_, Line_] := 2 * Pi * (Based[Line] + Excited[Fp, Line] - Grnd[F]) * 10^9; (*Transition frequency, 2*Pi Hz*)
delta[lambda_, Fp_, Line_] := w[lambda] - wtran[Fp, Line]; (*Detuning, 2*Pi Hz*)
wo[Line_] := 2 * Pi * Based[Line] * 10^9;
rnat[Line_] := If[Line == 1, 1, If[Line == 2, 1/2, 0]] * (wo[Line]^3 / (3 * Pi * eo * h * Co^3)) * d[Line]^2; (*2*Pi Hz*)
ERcoil[lambda_] := h^2 / (2 * mrb) * k[lambda]^2;
RayleighLength[lambda_] := Pi * w^2 / lambda;
```

```

PeakIntensity = If[IsLattice, 4, 1] *  $\frac{2 \text{OpticalPow}}{\pi w^2}$ ;
QRabi[F_, mF_, Fp_, mFp_, Line_] := - $\frac{1}{h} \sqrt{\frac{2 \text{PeakIntensity}}{c_0 e_0}} d[\text{Line}] * \text{CG}[F, mF, Fp, mFp, \text{Line}];$ 
F3shifts[m_, pol_,  $\lambda$ ] :=  $\frac{h}{4} \sum_{Fp=1}^4 \sum_{line=1}^2 \text{QRabi}[3, m, Fp, m + \text{pol}, \text{line}]^2 * \frac{\delta[\lambda, 3, Fp, \text{line}]}{\delta[\lambda, 3, Fp, \text{line}]^2 + \text{rnat}[\text{line}]^2 / 4}$ ;
F2shifts[m_, pol_,  $\lambda$ ] :=  $\frac{h}{4} \sum_{Fp=1}^4 \sum_{line=1}^2 \text{QRabi}[2, m, Fp, m + \text{pol}, \text{line}]^2 * \frac{\delta[\lambda, 2, Fp, \text{line}]}{\delta[\lambda, 2, Fp, \text{line}]^2 + \text{rnat}[\text{line}]^2 / 4}$ ;
DiffShift[m_, pol_,  $\lambda$ ] := F3shifts[m, pol,  $\lambda$ ] - F2shifts[m, pol,  $\lambda$ ];
AvgShift[m_, pol_,  $\lambda$ ] :=  $\frac{\text{F3shifts}[m, \text{pol}, \lambda] + \text{F2shifts}[m, \text{pol}, \lambda]}{2}$ ;
DiffScaShift[ $\lambda$ ] := DiffShift[0, 0,  $\lambda$ ];
AvgScaShift[ $\lambda$ ] := AvgShift[0, 0,  $\lambda$ ];
DiffVecShift[ $\lambda$ ] := DiffShift[1, 1,  $\lambda$ ] - DiffScaShift[ $\lambda$ ]; (*Vector ligh shift per m on  $\sigma$ *)
AvgVecShift[ $\lambda$ ] := AvgShift[1, 1,  $\lambda$ ] - AvgScaShift[ $\lambda$ ];
rsc[Fini_, mFini_, Ffin_, mFfin_, pol_,  $\lambda$ ] :=
 $\frac{3 \pi c_0^2 \omega[\lambda]^3 \text{PeakIntensity}}{2 h d[2]^4 \text{CG}[3, 3, 4, 4, 2]^4}$ 
 $\left( \sum_{line=1}^2 \sum_{Fp=1}^4 \frac{\text{rnat}[\text{line}] / \omega_{\text{tran}}[Fini, Fp, \text{line}]^3}{\delta[\lambda, Fini, Fp, \text{line}]} * d[\text{line}]^2 * \text{CG}[Fini, mFini, Fp, mFini + \text{pol}, \text{line}] * \right.$ 
 $\left. \text{CG}[Ffin, mFfin, Fp, mFini + \text{pol}, \text{line}] \right)^2$ ;
rscTotal[F_, mF_, pol_,  $\lambda$ ] :=  $\sum_{Ffin=2}^3 \sum_{mFfin=-Ffin}^{Ffin} \text{rsc}[F, mF, Ffin, mFfin, \text{pol}, \lambda]$ ;
rscRay[F_, mF_, pol_,  $\lambda$ ] := rsc[F, mF, F, mF, pol,  $\lambda$ ];
rscRam[F_, mF_, pol_,  $\lambda$ ] := rscTotal[F, mF, pol,  $\lambda$ ] - rsc[F, mF, F, mF, pol,  $\lambda$ ];
Fflip[F_] := 5 - F;
rscHypFlip[F_, mF_, pol_,  $\lambda$ ] :=  $\sum_{mFfin=-Fflip[F]}^{Fflip[F]} \text{rsc}[F, mF, Fflip[F], mFfin, \text{pol}, \lambda]$ ;
RadialFreq[pol_,  $\lambda$ ] :=  $\frac{1}{2 \pi} \sqrt{\text{Abs}\left[\frac{4 * \text{AvgShift}[0, \text{pol}, \lambda]}{m_{\text{th}} w^2}\right]}$ ;
AxialFreq[pol_,  $\lambda$ ] := If[IsLattice, k[ $\lambda$ ],  $\frac{1}{\text{RayleighLength}[\lambda]} * \frac{1}{2 \pi} \sqrt{\text{Abs}\left[\frac{2 \text{AvgShift}[0, \text{pol}, \lambda]}{m_{\text{th}}}\right]}$ ];
(*AxialFreq calculate trap freq within Rayleigh length distance if without lattice*)
rscTest[F_, mF_, pol_,  $\lambda$ ] :=
 $\frac{2 \omega[\lambda]^3 \text{PeakIntensity}}{3 \pi h^3 c_0^4 e_0^2}$ 
 $\left( \sum_{line=1}^2 \sum_{Fp=1}^4 \text{If}[\text{line} = 1, 1, \text{If}[\text{line} = 2, \frac{1}{2}, 0]] \frac{\omega_0[\text{line}] * d[\text{line}]^2 * \text{CG}[F, mF, Fp, mF + \text{pol}, \text{line}]^2}{\omega[\lambda]^2 - \omega_0[\text{line}]^2} * \frac{d[\text{line}]^2}{d[2]^2 / 2} \right)$ ;

```

Create Table of results

```

In[ ] := table = TableForm[


|                                  |                                                                                                                                               |                        |                                                                           |        |
|----------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|------------------------|---------------------------------------------------------------------------|--------|
| "Peak Intensity"                 | PeakIntensity * 10 <sup>-4</sup>                                                                                                              | "[W/cm <sup>2</sup> ]" | Null                                                                      | Null   |
| "F2 Stark"                       | F2shifts[0, pol, $\lambda * 10^{-9}$ ] / h * 10 <sup>-3</sup>                                                                                 | "[kHz]"                | 10 <sup>3</sup> / (F2shifts[0, pol, $\lambda * 10^{-9}$ ] / h)            | "[ms]" |
| "F3 Stark"                       | F3shifts[0, pol, $\lambda * 10^{-9}$ ] / h * 10 <sup>-3</sup>                                                                                 | "[kHz]"                | 10 <sup>3</sup> / (F3shifts[0, pol, $\lambda * 10^{-9}$ ] / h)            | "[ms]" |
| "ΔStark"                         | DiffShift[0, pol, $\lambda * 10^{-9}$ ] / h * 10 <sup>-3</sup>                                                                                | "[kHz]"                | 10 <sup>3</sup> / (DiffShift[0, pol, $\lambda * 10^{-9}$ ] / h)           | "[ms]" |
| "Avg Stark"                      | AvgShift[0, pol, $\lambda * 10^{-9}$ ] / h * 10 <sup>-6</sup>                                                                                 | "[MHz]"                | AvgShift[0, pol, $\lambda * 10^{-9}$ ] / k <sub>B</sub> * 10 <sup>3</sup> | "[mK]" |
| "ΔStark <sub>scalar</sub> "      | DiffScaShift[ $\lambda * 10^{-9}$ ] / h * 10 <sup>-3</sup>                                                                                    | "[kHz]"                | 10 <sup>3</sup> / (DiffScaShift[ $\lambda * 10^{-9}$ ] / h)               | "[ms]" |
| "Avg Stark <sub>scalar</sub> "   | AvgScaShift[ $\lambda * 10^{-9}$ ] / h * 10 <sup>-6</sup>                                                                                     | "[MHz]"                | AvgScaShift[ $\lambda * 10^{-9}$ ] / k <sub>B</sub> * 10 <sup>3</sup>     | "[mK]" |
| "ΔStark <sub>vector</sub> "      | DiffVecShift[ $\lambda * 10^{-9}$ ] / h * 10 <sup>-3</sup>                                                                                    | "[kHz]"                | Null                                                                      | Null   |
| "Avg Stark <sub>vector</sub> "   | AvgVecShift[ $\lambda * 10^{-9}$ ] / h * 10 <sup>-3</sup>                                                                                     | "[kHz]"                | AvgVecShift[ $\lambda * 10^{-9}$ ] / k <sub>B</sub> * 10 <sup>6</sup>     | "[μK]" |
| "QWP angle to cancel ΔStark"     | $\frac{\text{ArcSin}\left[\frac{\text{DiffScaShift}[\lambda * 10^{-9}]}{\text{DiffVecShift}[\lambda * 10^{-9}]}\right]}{2} * \frac{180}{\pi}$ | "[deg]"                | Null                                                                      | Null   |
| "Γ <sub>sc</sub> for F=2"        | rscTotal[2, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                       | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscTotal[2, 0, 0, $\lambda * 10^{-9}$ ]                 | "[ms]" |
| "Γ <sub>sc</sub> for F=3"        | rscTotal[3, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                       | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscTotal[3, 0, 0, $\lambda * 10^{-9}$ ]                 | "[ms]" |
| "Γ <sub>sc</sub> (Ray) for F=2"  | rscRay[2, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                         | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscRay[2, 0, 0, $\lambda * 10^{-9}$ ]                   | "[ms]" |
| "Γ <sub>sc</sub> (Ray) for F=3"  | rscRay[3, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                         | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscRay[3, 0, 0, $\lambda * 10^{-9}$ ]                   | "[ms]" |
| "Γ <sub>sc</sub> (Ram) for F=2"  | rscRam[2, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                         | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscRam[2, 0, 0, $\lambda * 10^{-9}$ ]                   | "[ms]" |
| "Γ <sub>sc</sub> (Ram) for F=3"  | rscRam[3, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                         | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscRam[3, 0, 0, $\lambda * 10^{-9}$ ]                   | "[ms]" |
| "Γ <sub>sc</sub> (Flip) for F=2" | rscHypFlip[2, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                     | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscHypFlip[2, 0, 0, $\lambda * 10^{-9}$ ]               | "[ms]" |
| "Γ <sub>sc</sub> (Flip) for F=3" | rscHypFlip[3, 0, 0, $\lambda * 10^{-9}$ ]                                                                                                     | "[s <sup>-1</sup> ]"   | 10 <sup>3</sup> / rscHypFlip[3, 0, 0, $\lambda * 10^{-9}$ ]               | "[ms]" |
| "Recoil Energy"                  | ERecoil[ $\lambda * 10^{-9}$ ] / h * 10 <sup>-3</sup>                                                                                         | "[kHz]"                | 10 <sup>3</sup> / (ERecoil[ $\lambda * 10^{-9}$ ] / h)                    | "[ms]" |
| "Radial HO Freq"                 | RadialFreq[0, $\lambda * 10^{-9}$ ] * 10 <sup>-3</sup>                                                                                        | "[kHz]"                | 10 <sup>3</sup> / RadialFreq[0, $\lambda * 10^{-9}$ ]                     | "[ms]" |
| "Axial HO Freq"                  | AxialFreq[0, $\lambda * 10^{-9}$ ] * 10 <sup>-3</sup>                                                                                         | "[kHz]"                | 10 <sup>3</sup> / AxialFreq[0, $\lambda * 10^{-9}$ ]                      | "[ms]" |



```

/. { λ → If[DetuningQ, $\frac{c_0}{(\omega_{\text{tran}}[3, 2, 1] / (2 \pi) - \text{Detuning} * 10^9)} 10^9, \lambda]$, pol → polar}, TableDirections → Column,
TableAlignments → Left];

```


```

Tabulate Results

$$\begin{aligned}
 F2 \text{ shift} &= F2_{\text{scalar}} - m * F2_{\text{vector}}; \\
 F3 \text{ shift} &= F3_{\text{scalar}} + m * F3_{\text{vector}}; \\
 \Delta \text{Stark} &= F3 \text{ shift} - F2 \text{ shift} = (F3_{\text{scalar}} - F2_{\text{scalar}}) + m * (F3_{\text{vector}} + F2_{\text{vector}}) = \Delta \text{Stark}_{\text{scalar}} + m * \Delta \text{Stark}_{\text{vector}}; \\
 \text{Avg Stark} &= \frac{F3 \text{ shift} + F2 \text{ shift}}{2} = \frac{F3_{\text{scalar}} + F2_{\text{scalar}}}{2} + m * \frac{F3_{\text{vector}} + F2_{\text{vector}}}{2} = \text{Avg Stark}_{\text{scalar}} + m * \text{Avg Stark}_{\text{vector}};
 \end{aligned}$$

```

In[ ]:= Manipulate[Evaluate[table],
  {{λ, 808, "λ [nm]"}, 700, 1700},
  {{Detuning, 0.744, "Detuning from D1 3-2' [GHz]"}, -1000, 1000},
  {{OpticalPow, 0.000013, Dynamic[If[IsLattice, "Single Beam Power [W]", "Dipole Trap Power [W]"]], 0, 10},
  {{polar, 0, "Polarization"}, {-1, 0, 1}},
  {{IsLattice, False, "Lattice?"}, {True, False}},
  {{DetuningQ, True, "Detuning?"}, {True, False}}]

```

Out[]:=

λ [nm]	<input type="text" value="808"/>	☐
Detuning from D1 3-2' [GHz]	<input type="text" value="0.744"/>	☐
Dipole Trap Power [W]	<input type="text" value="0.000013"/>	☐
Polarization	<input type="radio" value="-1"/> -1 <input type="radio" value="0"/> 0 <input type="radio" value="1"/> 1	
Lattice?	☐	
Detuning?	☒	

Peak Intensity	1.70993	[W/cm ²]	Null	Null
F2 Stark	-380.707	[kHz]	-0.00262669	[ms]
F3 Stark	-2117.08	[kHz]	-0.000472349	[ms]
ΔStark	-1736.37	[kHz]	-0.000575914	[ms]
Avg Stark	-1.24889	[MHz]	-0.0599656	[mK]
ΔStark _{scalar}	-1736.37	[kHz]	-0.000575914	[ms]
Avg Stark _{scalar}	-1.24889	[MHz]	-0.0599656	[mK]
ΔStark _{vector}	1224.81	[kHz]	Null	Null
Avg Stark _{vector}	477.641	[kHz]	22.934	[μK]
QWP angle to cancel ΔStark	-45. + 25.348 i	[deg]	Null	Null
Γ _{sc} for F=2	3334.37	[s ⁻¹]	0.299907	[ms]
Γ _{sc} for F=3	103 316.	[s ⁻¹]	0.00967904	[ms]
Γ _{sc} (Ray) for F=2	1114.03	[s ⁻¹]	0.897644	[ms]
Γ _{sc} (Ray) for F=3	34 453.	[s ⁻¹]	0.0290251	[ms]
Γ _{sc} (Ram) for F=2	2220.34	[s ⁻¹]	0.450381	[ms]
Γ _{sc} (Ram) for F=3	68 863.	[s ⁻¹]	0.0145216	[ms]
Γ _{sc} (Flip) for F=2	1480.23	[s ⁻¹]	0.675571	[ms]
Γ _{sc} (Flip) for F=3	22 954.3	[s ⁻¹]	0.0435647	[ms]
Recoil Energy	3.71786	[kHz]	0.268972	[ms]
Radial HO Freq	1.10843	[kHz]	0.902174	[ms]
Axial HO Freq	0.0090153	[kHz]	110.923	[ms]

Appendix C

Simulation Algorithm for DD in Light Storage

This chapter shows the actual MATLAB programs used to simulate the DD for light storage in [Chapter 9](#). The detail of the algorithm is shown in [§9.6](#).

RawData.mat

All experimental raw data are stored in **RawData.mat**. It contains the following variables:

T_{dd}	1.0 ms	2.5 ms	3.0 ms	5.0 ms
Time (ms)	Raw10t	Raw25t	Raw30t	Raw50t
Experimental signal result	Raw10r	Raw25r	Raw30r	Raw50r
Experimental signal error	Raw10e	Raw25e	Raw30e	Raw50e
Normalized result	Raw10rN	Raw25rN	Raw30rN	Raw50rN
Normalized error	Raw10eN	Raw25eN	Raw30eN	Raw50eN

Normalized data are data divided by the first data point.

selectRawData.m

This is to choose raw data from RawData.mat. Because 1.0 ms DD case has skipped data (number of π pulses = 4, 8, 12, ...). skipDD is created to remove simulation data to match experiment. Therefore, the result from **inhomoResult.m** has the identical time array as experiment.

```

1 load('RawData.mat');
2 if tauDD == 1.0e-3
3     if isPiNumOdd
4         RawT = Raw100t;
5         RawR = Raw100rN;
6         RawE = Raw100eN;
7         RawP = Raw100p;
8     else

```

```

9         RawT = Raw10t;
10        RawR = Raw10rN;
11        RawE = Raw10eN;
12        RawP = Raw10p;
13    end
14    elseif tauDD == 2.5e-3
15        RawT = Raw25t;
16        RawR = Raw25rN;
17        RawE = Raw25eN;
18        RawP = Raw25p;
19    elseif tauDD == 3.0e-3
20        RawT = Raw30t;
21        RawR = Raw30rN;
22        RawE = Raw30eN;
23        RawP = Raw30p;
24    elseif tauDD == 5.0e-3
25        RawT = Raw50t;
26        RawR = Raw50rN;
27        RawE = Raw50eN;
28        RawP = Raw50p;
29    else
30        error('Incorrect tauDD');
31    end
32    Raw=[RawT,RawR,RawE,RawP];

```

main.m

This is to perform simulation across 3D parameters set by users and gives best plot comparison between experiment and simulation.

```

1  clear all
2  clc
3
4  % Define and calculations of paramters
5  tauDD      = 5e-3;
6  epsilon    = 1;
7  delta      = 2*pi*50;
8  tauDecay   = 0;
9  MicrWPiT   = 37e-6;
10 RamanPiT   = 250e-9;
11 OmegaRMax  = pi/RamanPiT;
12 isPiNumOdd = false;
13 selectRawData;
14
15 % Prepare array of parameters
16 DeltaArr    = 2*pi*(5934+(-2:0.01:2));
17 epsilonArr  = (1:0.05:1);
18 alphaArr    = 0.14+(-0.01:0.0001:0.01);
19 DeltaN      = length(DeltaArr);
20 epsilonN    = length(epsilonArr);
21 alphaN      = length(alphaArr);
22

```

```

23 % Calculate and plot result
24 SSArr = zeros(alphaN,DeltaN,epsilonN);
25 cl = clock;
26 cls = [num2str(cl(4))',' ',num2str(cl(5))',' ',num2str(cl(6))',' '];
27 fprintf(['Current time: ',cls,'\n']);
28 parfor alphaI = 1:alphaN
29     for DeltaI = 1:DeltaN
30         for epsilonI = 1:epsilonN
31             alpha = alphaArr(alphaI);
32             Delta = DeltaArr(DeltaI);
33             epsilon = epsilonArr(epsilonI);
34             param = [epsilon,delta,Delta,tauDecay,OmegaRMax,RamanPiT,
                       MicrWPiT,tauDD];
35             result = inhomResult_JPerp(param,alpha,Raw);
36             SS = SumSquare(result,RawR,RawE);
37             SSArr(alphaI,DeltaI,epsilonI) = SS;
38         end
39     end
40     cl = clock;
41     cls = [num2str(cl(4))',' ',num2str(cl(5))',' ',num2str(cl(6))',' '];
42     fprintf([cls,'[alpha]=[',num2str(alpha),']\n']);
43 end
44
45 drawColorMap
46 drawOptimizedGraph

```

main2.m

This is to perform simulation on the parameter set by users and plot comparison between experiment and simulation.

```

1 clear all
2 clc
3
4 % Define and calculations of paramters
5 tauDD = 5e-3;
6 epsilon = 1;
7 delta = 2*pi*50;
8 Delta = 2*pi*5934.11;
9 tauDecay = 0/1000;
10 alpha = 0.1391;
11 MicrWPiT = 37e-6;
12 RamanPiT = 250e-9;
13 OmegaRMax = pi/RamanPiT;
14 isPiNumOdd = false;
15 selectRawData;
16
17 % Calculate and plot result
18 param = [epsilon,delta,Delta,tauDecay,OmegaRMax,RamanPiT,MicrWPiT,tauDD];
19 result = inhomResult_JPerp(param,alpha,Raw);
20 plotSingleAtomDD(param,alpha,Raw)
21 pause(0.1)

```

plotSingleAtomDD.m

This is to plot the simulated result compared with the raw data, as shown in [Figure 9.9](#).

```

1 function [] = plotSingleAtomDD(param,alpha,Raw)
2 % Retrieve paramters
3 epsilon    = param(1);
4 delta      = param(2);
5 Delta      = param(3);
6 tauDecay   = param(4);
7 OmegaR     = param(5);
8 RamanPiT   = param(6);
9 MicrWPiT   = param(7);
10 tauDD      = param(8);
11 RawT       = Raw(:,1);
12 RawR       = Raw(:,2);
13 RawE       = Raw(:,3);
14
15 % Calculation
16 result = inhomResult_JPerp(param,alpha,Raw);
17 SS      = SumSquare(result,RawR,RawE);
18
19 % Plot result
20 plot(RawT,result,'o-')
21 hold on
22 plot(RawT,RawR,'rx--');
23 hold off
24 legend('simulation','experimental')
25 title([num2str(tauDD*1000),' ms DD time, \delta=',num2str(delta/2/pi),' Hz, \
        Delta=',num2str(Delta/2/pi),' Hz, \epsilon=', num2str(epsilon),' , \alpha='
        , num2str(alpha), ', SS=',num2str(SS)]);
26 end

```

drawColorMap.m

This is to draw color map of sum square for users to have a view of local minima.

```

1 figure(1)
2 caxisMax    = 4;
3 pauseTime   = 0.5;
4 contourline = linspace(min(min(min(SSArr))),caxisMax,10);
5 for i=1:epsilonN
6     contourf(DeltaArr/2/pi,alphaArr,SSArr(:,:,i),contourline)
7     title(['Sum Squared: ',num2str(tauDD*1000),' ms DD time, \tau = ',num2str(
            tauDecay*1000),' ms, \epsilon = ',num2str(epsilonArr(i))]);
8     colorbar
9     xlabel('\Delta (Hz)')
10    ylabel('\alpha')
11    caxis([min(min(min(SSArr))),caxisMax])
12    pause(pauseTime)
13 end

```

drawOptimizedGraph.m

This is to plot the best simulated result compared with the raw data, as shown in [Figure 9.9](#).

```

1 figure(2)
2
3 % Get optimized parameters
4 [alphaI,DeltaI,epsilonI] = ind2sub(size(SSArr),find(SSArr==min(min(SSArr))
   ));
5 alpha    = alphaArr(alphaI);
6 Delta    = DeltaArr(DeltaI);
7 epsilon  = epsilonArr(epsilonI);
8 param    = [epsilon,delta,Delta,tauDecay,OmegaRMax,RamanPiT,MicrWPiT,tauDD];
9
10 % Plot result
11 plotSingleAtomDD(param,alpha,Raw)

```

IntensityDistribution.m

This is the function $\rho(I)$ in equation (9.10).

```

1 function [output] = IntensityDistribution(alpha,normInt)
2     temp = 1./(4*alpha.^2);
3     output = (normInt.^(temp)).*temp./normInt;
4 end

```

InvIntensityDistribution.m

This is the inverse function of $\rho(I)$.

```

1 function [output] = InvIntensityDistribution(alpha,y)
2     temp = alpha.^2;
3     temp2 = -4*temp./(4*temp-1);
4     output = (4.^temp2).*((y.*temp).^temp2);
5 end

```

RabiFlop.m

This is the rotation matrix for Bloch vector under Rabi flopping for an angle of θ under Rabi frequency Ω , detuning δ and phase ϕ . I denote the function as $\mathbf{R}_{\Omega,\delta}^{\phi}(\theta)$.

```

1 function [output] = RabiFlop(theta,Omega,delta,phi)
2 % Temporary variables
3 delta2    = delta.*delta;
4 Omega2    = Omega.*Omega;
5 GenOmega2 = delta2+Omega2;
6 GenOmega  = sqrt(GenOmega2);
7 thetaact  = theta.*GenOmega./Omega;
8 Sphi      = sin(phi);
9 Cphi      = cos(phi);

```

```

10 Sphi2      = Sphi.*Sphi;
11 Cphi2      = Cphi.*Cphi;
12 Sth        = sin(thetaact);
13 Cth        = cos(thetaact);
14 SthHalf    = sin(thetaact/2);
15 SthHalf2   = SthHalf.*SthHalf;
16 CphiDoub   = cos(2*phi);
17 temp12a    = delta.*Sth./GenOmega;
18 temp12b    = Omega2.*(Cth-1).*Cphi.*Sphi./GenOmega2;
19 temp13a    = Omega.*2*delta.*Cphi.*SthHalf2./GenOmega2;
20 temp13b    = Omega.*Sth.*Sphi./GenOmega;
21 temp23a    = Omega.*Cphi.*Sth./GenOmega;
22 temp23b    = 2*delta.*Omega.*SthHalf2.*Sphi./GenOmega2;
23
24 % Calculate matrix
25 output = zeros(3);
26 output(1,1) = (Omega2+delta2.*Cth).*Cphi2./GenOmega2 + Cth.*Sphi2;
27 output(1,2) = -temp12a-temp12b;
28 output(1,3) = temp13a+temp13b;
29 output(2,1) = temp12a - temp12b;
30 output(2,2) = (0.5*Cth.*(2*delta2+Omega2+Omega2.*CphiDoub)+Omega2.*Sphi2)./
    GenOmega2;
31 output(2,3) = -temp23a+temp23b;
32 output(3,1) = temp13a-temp13b;
33 output(3,2) = temp23a+temp23b;
34 output(3,3) = (delta2+Omega2.*Cth)./GenOmega2;
35 end

```

RabiPi.m

Subset of **RabiFlop.m**. This is the rotation matrix of π pulse. I denote the function as $\Pi_{\Omega,\delta}(\phi) = \mathbf{R}_{\Omega,\delta}^{\phi} \left(\pi \times \frac{\sqrt{\delta^2 + \Omega^2}}{\Omega} \right)$. This is because the atoms at inhomogeneous detuning environment will see different π pulse and I assume the π pulse is a good pi pulse if there is no detuning.

```

1 function [output] = RabiFlop(theta,Omega,delta,phi)
2 % Temporary variables
3 delta2      = delta.*delta;
4 Omega2      = Omega.*Omega;
5 GenOmega2   = delta2+Omega2;
6 GenOmega    = sqrt(GenOmega2);
7 thetaact    = theta.*GenOmega./Omega;
8 Sphi        = sin(phi);
9 Cphi        = cos(phi);
10 Sphi2       = Sphi.*Sphi;
11 Cphi2       = Cphi.*Cphi;
12 Sth         = sin(thetaact);
13 Cth         = cos(thetaact);
14 SthHalf     = sin(thetaact/2);
15 SthHalf2    = SthHalf.*SthHalf;
16 CphiDoub    = cos(2*phi);

```

```

17 temp12a = delta.*Sth./GenOmega;
18 temp12b = Omega2.*(Cth-1).*Cphi.*Sphi./GenOmega2;
19 temp13a = Omega.*2*delta.*Cphi.*SthHalf2./GenOmega2;
20 temp13b = Omega.*Sth.*Sphi./GenOmega;
21 temp23a = Omega.*Cphi.*Sth./GenOmega;
22 temp23b = 2*delta.*Omega.*SthHalf2.*Sphi./GenOmega2;
23
24 % Calculate matrix
25 output = zeros(3);
26 output(1,1) = (Omega2+delta2.*Cth).*Cphi2./GenOmega2 + Cth.*Sphi2;
27 output(1,2) = -temp12a-temp12b;
28 output(1,3) = temp13a+temp13b;
29 output(2,1) = temp12a - temp12b;
30 output(2,2) = (0.5*Cth.*(2*delta2+Omega2+Omega2.*CphiDoub)+Omega2.*Sphi2)./
    GenOmega2;
31 output(2,3) = -temp23a+temp23b;
32 output(3,1) = temp13a-temp13b;
33 output(3,2) = temp23a+temp23b;
34 output(3,3) = (delta2+Omega2.*Cth)./GenOmega2;
35 end

```

FreeEvol.m

Subset of **RabiFlop.m**. This is the free evolution matrix. I denote the function as $\mathbf{T}_\delta(\tau) = \mathbf{R}_{0,\delta}^0(\delta\tau)$.

```

1 function [output] = FreeEvol(deltat)
2 Cdt = cos(deltat);
3 Sdt = sin(deltat);
4 output = eye(3);
5 output(1,1) = Cdt;
6 output(1,2) = -Sdt;
7 output(2,2) = Cdt;
8 output(2,1) = Sdt;
9 end

```

SingleAtomDD_JPerp.m

This is the calculation of atomic state after DD pulses of a single atom. Detail steps are discussed in §9.6.2.

```

1 function [v] = SingleAtomDD_JPerp(param,Raw)
2 % Retrieve paramters
3 epsilon = param(1);
4 delta = param(2);
5 Delta = param(3);
6 tauDecay = param(4);
7 OmegaR = param(5);
8 RamanPiT = param(6);
9 MicrWPiT = param(7);

```

```

10 tauDD      = param(8);
11 RawP       = Raw(:,4);
12 OmegaM     = pi/MicrWPiT;
13 deltaM     = delta+Delta;
14
15 % Calculation
16 u0         = [0;0;-1];
17 vini       = RabiFlop(epsilon*pi/2,OmegaR,delta,0)*u0;
18 t0         = tauDD + epsilon*RamanPiT;
19 N          = RawP(end);
20 v          = zeros(3,N+1);
21 v(:,1)     = FreeEvol(delta*tauDD)*vini;
22 for n=1:N
23     t       = t0+(n-1)*(tauDD+MicrWPiT);
24     M       = FreeEvol(delta*tauDD)*RabiPi(OmegaM,deltaM,deltaM*t);
25     v(:,n+1) = M*v(:,n);
26 end
27
28 % Output result vector
29 v(:,1)     = [];
30 v          = v(:,RawP);
31 v(:,end+1) = vini;
32 end

```

inhomoResult_JPerp.m

This is the calculation of atomic state after DD pulses of an ensemble of atoms. Detail steps are discussed in §9.6.3.

```

1 function [output] = inhomoResult_JPerp(param,alpha,Raw)
2 % Retrieve paramters
3 epsilon      = param(1);
4 deltaMax     = param(2);
5 Delta        = param(3);
6 tauDecay     = param(4);
7 OmegaRMax    = param(5);
8 RamanPiT     = param(6);
9 MicrWPiT     = param(7);
10 tauDD        = param(8);
11
12 % Initialization
13 nItsty       = 100;
14 nRaw         = size(Raw,1);
15 JxIni        = zeros(nItsty,1);
16 JyIni        = zeros(nItsty,1);
17 JxFin        = zeros(nItsty,nRaw);
18 JyFin        = zeros(nItsty,nRaw);
19 Itsty        = linspace(1/nItsty,1,nItsty);
20 if alpha<0.353553
21     Itsty     = InvIntensityDistribution(alpha,Itsty*IntensityDistribution(alpha,1));
22 end

```



```

23 Density = IntensityDistribution(alpha,Itsty);
24
25 % Calculation
26 for iItsty=1:nItsty
27     delta      = deltaMax*Itsty(iItsty);
28     OmegaR      = OmegaRMax*Itsty(iItsty);
29     v           = SingleAtomDD_JPerp([epsilon,delta,Delta,tauDecay,OmegaR,
        RamanPiT,MicrWPiT,tauDD],Raw);
30     JxIni(iItsty) = v(1,end);
31     JyIni(iItsty) = v(2,end);
32     JxFin(iItsty,:) = v(1,1:nRaw);
33     JyFin(iItsty,:) = v(2,1:nRaw);
34 end
35 AvgJxIni = trapz(Itsty,JxIni.*Density');
36 AvgJyIni = trapz(Itsty,JyIni.*Density');
37 AvgJxFin = trapz(Itsty,JxFin.*(Density'*ones(1,nRaw)));
38 AvgJyFin = trapz(Itsty,JyFin.*(Density'*ones(1,nRaw)));
39 AvgJperpIni = (AvgJxIni.*AvgJxIni+AvgJyIni.*AvgJyIni);
40 AvgJperpFin = (AvgJxFin.*AvgJxFin+AvgJyFin.*AvgJyFin);
41
42 % Output result
43 output = AvgJperpFin/AvgJperpIni;
44 output = output/output(1);
45
46 end

```

SumSquare.m

This is the calculation of the square sum to be minimized. The definition is shown in §9.6.5.

```

1 function [output] = SumSquare(result,RawR,RawE)
2     output = sum(((result'-RawR).^2)./RawE./RawE)/(length(RawR)-3);
3 end

```


Appendix D

Ultrasonic Warning to Viewport

If your vacuum viewport consists of AR coating, it is highly recommended not to put it into ultrasonic bath for cleaning process. I realized that the AR coating wore off when I put it into the ultrasonic bath. When I look for the information online, I found that there are multiple sources warned about the ultrasonic bath. Therefore, it is only recommended to use only the optical tissue to clean the viewport when it is dirty.

Figure D.1: The AR coating of the viewport wore off after ultrasonic bath.

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