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Relative Docking via Range-only Measurements

Kun Cao, Zhirong Qiu, and Lihua Xie

Abstract—This paper studies the problem of range-based relative docking of a single robot. In particular, we propose an integrated localization and navigation scheme for a robot to navigate itself to a desired relative displacement w.r.t. a fixed landmark at an unknown position, where the proposed controllers only require distance and odometry measurements. Instead of using a single range measurement as an indicator of the proximity to the desired docking point in existing works, the main idea of this work is to construct additional artificial anchors along the robot's motion trajectory first and then adopt the measurements corresponding to these anchors as the indicator. It is proved that the proposed controller will navigate the robot to the desired docking position asymptotically given proper parameter settings. Two simulation examples are presented to demonstrate the effectiveness of our theoretical results.

I. INTRODUCTION

Recent decade has witnessed a dramatic surge of robots which enjoy wide application in civil, industrial and military areas, such as service [1], rehabilitation [2], search [3], rescue, environmental monitoring [4] and surveillance [5]. Two fundamental problems, localization and navigation, are investigated separately in robotics, control and signal processing communities, see [6]–[11] and the references therein.

As a subtopic of navigation, the docking problem has extensive applications in unmanned systems, such as UAV precise landing and UGV autonomous parking. Docking to a known fixed landmark, when either the absolute positions of the agent itself and the landmark are available or the relative position between them can be measured directly, is relatively easy since a naive proportional controller works. However, docking to a fixed landmark at an unknown position is challenging. Therefore, some recent works have investigated the integrated localization and navigation problem. In [12], the target pursuit problem with a fixed unknown landmark was addressed by adopting a diminishing persistent excitation to reconcile the localization and position tracking problems, where range measurements and global positions of the agent itself are required. A similar idea was further applied in [13] to tackle the station-keeping problem. However, the GPS signal may be not precise enough (meter accuracy, RTK-GPS can enjoy centimeter-level accuracy but is expensive) or even unavailable (e.g., indoor environment and urban environment with high buildings), and other positioning systems (e.g., UWB, VICON and WiFi) require extra infrastructures (e.g.,

anchors or cameras which are expensive), extensive labors and costs (e.g., deployment, calibration and maintenance). Recently, the authors in [14] introduced an integrated localization and navigation scheme as a discrete-time counterpart of [12], where the designed controller is bounded and only requires range and self-displacement (odometry) measurements. However, docking to a position with a known displacement w.r.t. a fixed unknown landmark, referred as relative docking in this work, is more practical since in many cases, for example, say a firefighting task, a beacon cannot be put exactly on top of blazing sites but at somewhere safer while is still able to guide the robot (fire engine or firefighting aircraft) to a prescribed location.

Motivated by the above observations, this work aims to generalize the distance-based docking [14] to distance-based relative docking. This generalization is non-trivial since the distance measurement in the former can directly indicate whether the mission has been completed, which is not the case for the latter. Inspired by the multi-lateration in localization, we propose a novel motion controller which is triggered at several time instants (number of triggering depends on the dimension and required accuracy) such that the distances measured at those time instants can signal the completion of a mission. Furthermore, the convergence of the entire system under the proposed controller is established via discrete-time LaSalle's invariance principle [15] provided that the triggering positions satisfy some mild condition.

The remainder of this paper is structured as follows. Section II introduces some preliminaries and formally formulates the problem. Section III presents an integrated localization and navigation scheme for relative docking of a single robot. Simulation results are given in Section IV and the conclusion is finally drawn in Section V.

II. PRELIMINARIES

A. Notations

Throughout the paper, $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{R}_{\geq 0}$ denote the sets of real numbers, positive real numbers and nonnegative real numbers, respectively. Let $\mathbb{Z}_+$ and $\mathbb{Z}_{\geq 0}$ represent the sets of positive integers and nonnegative integers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ denote the sets of n -dimensional real vectors and $m \times n$ real matrices, respectively. $|c|$ is the absolute value of $c \in \mathbb{R}$. Let $\|x\|$ and $\|x\|_\infty$ denote the 2 norm and infinity norm for a vector $x \in \mathbb{R}^n$, respectively. Given a matrix $A \in \mathbb{R}^{n \times n}$, $A^\top$, A^{-1} and $\|A\|$ are its transpose, inverse and induced 2 norm, respectively. Let $I_n \in \mathbb{R}^{n \times n}$ be the n -dimensional identity matrix.

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Fig. 1: An illustration of the control diagram for relative docking. d_k and ϕ_k are from range sensor and odometer, respectively. d_{k_χ} is range measurement at some triggering time instant, which will be elaborated in subsection III-A.

B. Problem Statement

Given a fixed landmark at the unknown position $p^* \in \mathbb{R}^d$ ($d = 2, 3$), we aim to dock a robot to the relative position $x^* \in \mathbb{R}^d$, i.e.,

$$\lim_{k \rightarrow \infty} p_k = p^* + x^*, \quad (1)$$

where $p_k \in \mathbb{R}^d$ denotes the position of the robot at time instant k , whose motion is modeled by a single-integrator with bounded input:

$$p_{k+1} = p_k + T\bar{u}_k, \|\bar{u}_k\|_\infty \leq U, \quad (2)$$

where T and U denote the sampling period and maximal velocity, respectively.

The problem can be formally defined as follows:

Problem II.1. Design an adaptive estimator for the desired docking position $p^* + x^*$ and a motion controller to navigate the robot to this position based on the distance measurements $d_k = \|p_k - p^*\|$ and self displacements $\phi_k = T\bar{u}_k$.

III. INTEGRATION OF LOCALIZATION AND NAVIGATION FOR RELATIVE DOCKING

In this section, we present an integrated localization and navigation scheme to tackle Problem II.1. Specifically, as shown in Fig. 1, we design an adaptive estimator for estimating the desired docking position with distance measurements from range sensor and self displacements from odometer. Then this estimate is used in the proposed motion controller along with a periodic signal for navigating the robot to the desired docking position.

A. Adaptive Estimator

Let $q_k = p_k - (p^* + x^*)$ and $\hat{q}_k$ be its estimate. By (2),

$$q_{k+1} = q_k + \phi_k, \quad (3)$$

where $\phi_k = T\bar{u}_k$. On the other hand, in view of the equality

$$\begin{aligned} d_{k+1}^2 - d_k^2 &= \|p_{k+1} - p^*\|^2 - \|p_k - p^*\|^2 \\ &= \|q_{k+1} + x^*\|^2 - \|q_k + x^*\|^2 \\ &= \|\phi_k\|^2 + 2\phi_k^\top(q_k + x^*), \end{aligned} \quad (4)$$

the following adaptive updating law can be designed:

$$\hat{q}_{k+1} = \hat{q}_k + \phi_k + \Gamma\phi_k\epsilon_{k+1}, \quad (5)$$

where $\epsilon_{k+1} = \psi_{k+1} - \phi_k^\top(\hat{q}_k + x^*)$ with $\psi_k = \frac{1}{2}(d_{k+1}^2 - d_k^2 - \|\phi_k\|^2) = \phi_k^\top(q_k + x^*)$, $0 < \Gamma < \gamma I$.

B. Motion Controller

We first introduce a function f and a periodic signal σ_k that will be used in our proposed motion controller.

$f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is a function satisfying the following assumption:

Assumption III.1. $f(0) = 0$, and $0 < f(x) \leq x$, $\forall x > 0$.

σ_k is an external signal generated from an autonomous system:

$$\begin{aligned} \rho_{k+1} &= \Pi(\rho_k), \\ \sigma_k &= \Sigma(\rho_k), k \in \mathbb{Z}_{\geq 0}, \end{aligned} \quad (6)$$

where $\Pi(\cdot)$ and $\Sigma(\cdot)$ are two continuous mappings to be chosen. The autonomous system (6) is introduced for reconciling two conflicting objectives: adaptive estimation of docking position and navigation. In other words, the adaptive estimator (5) requires a persistent excitation while the motion controller tends to gradually immobilize the robot. Thus, the autonomous system keeps “perturbing” the robot motion and the perturbation will decrease with the decrease of proximity and disappear when the target is reached. Specifically, σ_k is required to be constructed such that the following assumption holds:

Assumption III.2. 1) ρ_k is bounded, $\forall k \in \mathbb{Z}_{\geq 0}$, and $\sup_{k \in \mathbb{Z}_{\geq 0}} \|\sigma_k\| \leq 1$.
2) $\exists T_0 \in \mathbb{Z}_+$ such that $\sigma_{k+T_0} = -\sigma_k$, $\forall k \in \mathbb{Z}_{\geq 0}$ and $\text{Span}\{\sigma_0, \dots, \sigma_{T_0-1}\} = \mathbb{R}^d$.

Remark III.3. 1) Assumption III.1 can be easily fulfilled by selecting $f(x) = x$ or choosing other options which appeared in [12] and references therein;

2) For 2D space, Assumption III.2 can be satisfied by letting $\rho_0 \in \mathbb{R}^2$ with $\|\rho_0\| = 1$, $\Sigma = I_2$ and $\Pi = \begin{bmatrix} \cos(2\pi/a) & -\sin(2\pi/a) \\ \sin(2\pi/a) & \cos(2\pi/a) \end{bmatrix}$ with the period $a \geq 4$ being an even integer. For 3D space, ρ_0 and Π remain the same while $\Sigma([\rho_{0,1} \ \rho_{0,2}]^\top) = [a_1\rho_{0,1} \ a_1\rho_{0,2} \ a_2(4\rho_{0,1}^3 - 3\rho_{0,2})]^\top / (a_1^2 + a_2^2)$, $\rho_0 = [\rho_{0,1} \ \rho_{0,2}]^\top$, see [16] for more details.

After designing f and σ_k carefully, the following bounded motion controller is proposed:

$$\bar{u}_k = \text{proj}_U u_k, u_k = -\beta\hat{q}_k + Cf(\xi_k)\sigma_k, \quad (7)$$

where $\text{proj}_U(\cdot)$ is a projection operator defined by

$$\text{proj}_U u_k = \frac{U}{\max(U, \|u_k\|)} u_k := s_k u_k \quad (8)$$

and

$$\begin{aligned} \xi_k &= \max(\zeta_k, |d_k - \|x^*\||), \\ \zeta_k &= \max_{\chi \in \{1, \dots, \bar{d}\}} (|\|x^* - \sum_{l=k_\chi}^k \phi_l\| - d_{k_\chi}|), \bar{d} \geq d, \end{aligned} \quad (9)$$

with k_χ being the time instant triggering the running sum term $\sum_{l=k_\chi}^k \phi_l$ and d_{k_χ} being the distance measurement at the same time w.r.t. the landmark. Notice that $\sum_{l=k_\chi}^k \phi_l = p_k - p_{k_\chi}$ and p_{k_χ} should be chosen properly, whose condition

Fig. 2: An illustration of triggering in 2-D.

will be given later. We first give some intuition of our controller design. Suppose $\bar{d} = d = 2$, the three constraints in $\xi_k = 0$ form three circles, see Fig. 2. It can be seen that $\xi_k = 0$ implies $p_k = p^* + x^*$. This idea is closely related to multilateration approaches in localization literature, where we can regard p^* , $p_{k_1} + x^*$ and $p_{k_2} + x^*$ as positions of three transmitters and $p^* + x^*$ as that of a receiver. However, whether $p^* + x^*$ can be uniquely determined depends on the choice of p_{k_χ} and hence we require the following assumption.

Assumption III.4. *There exists d points $\{p_{k_\chi}\}_{\chi=1}^d$ in $\{p_{k_\chi}\}_{\chi=1}^{\bar{d}}$ such that $[(p^* - x^*)^\top, p_{k_1}^\top, \dots, p_{k_d}^\top]^\top$ is generic, i.e., the coordinates $p^* - x^*$, p_{k_1} , $\dots$, p_{k_d} are algebraically independent over rational numbers [17].*

Remark III.5. *Assumption III.4 can be easily satisfied if the agent starts from anywhere except the ball $\mathcal{B}(p^*, \|x^*\|) = \{p : \|p - p^*\| = \|x^*\|\}$ or from the ball $\mathcal{B}(p^*, \|x^*\|)$ with $\hat{q} \neq 0$. For example, the agent can move $d-1$ random step(s) and record the self-displacement(s) and range measurement(s) at an initialization stage. For the case when the agent starts from the ball $\mathcal{B}(p^*, \|x^*\|)$ with $\hat{q} = 0$, a perturbation either on $\hat{q}$ or $\bar{u}$ will suffice to fulfill Assumption III.4. Notice that p_{k_χ} is not necessarily chosen at the initialization stage, where the running sum term will accumulate error w.r.t time in real implementation. To this end, we may use (2) with controller (7) in [14] to approach the ball $\mathcal{B}(p^*, \|x^*\|)$ first and choose p_{k_χ} near $\mathcal{B}(p^*, \|x^*\|)$. In real implementation, the running sum is not obtained by summing up self displacements, alternatively, this can be done by subtracting the buffered fused estimates of position at triggering time instants from the current fused estimates.*

Remark III.6. *This idea shares some similarity with the station keeping problem [13]; actually, our method can be thought as putting $\bar{d}$ artificial landmarks along the motion trajectory in addition to the fixed one at the docking point p^* . However, our method is more practical by considering discrete-time implementation, and only requires self-*

displacement measurements instead of GPS measurements. The strategy of a mobile sensor triplet unit (MSTU) in [18] can be regarded as a special case of (9), where we can replace the terms $\sum_{l=k_\chi}^k \phi_l$ and d_{k_χ} with a known spacing between receivers and the distance measurement of the auxiliary receiver.

C. Stability Analysis

In this subsection, we shall establish the convergence results of our proposed controller in the integrated scheme. To this end, the following results should be provided first.

Lemma III.7. *Under Assumption III.4, $\xi_k = 0 \Rightarrow p_k = p^* + x^*$.*

Proof. That $[(p^* - x^*)^\top, p_{k_1}^\top, \dots, p_{k_d}^\top]^\top$ is generic implies that $[p^{*\top}, (p_{k_1} + x^*)^\top, \dots, (p_{k_d} + x^*)^\top]^\top$ is generic. The barycentric coordinate of $p^* + x^*$ can be computed via Cayley-Menger determinant [19] if $p^* + x^*$ lies in convex hull of $\{p^*\} \cup \{p_{k_\chi} + x^*\}_{\chi=1}^{\bar{d}}$ or calculated through multidimensional scaling [20], [21] otherwise. In both cases, only the inter-node distances are used. According to the uniqueness of barycentric coordinates, we conclude that the solution to $\xi_k = 0$ is unique and thus $p_k = p^* + x^*$. ■

Remark III.8. *The condition given in Lemma III.7 is sufficient but not necessary since $\xi_k = 0 \Rightarrow p_k = p^* + x^*$ still holds if $p^* + x^*$ lies on the surface of simplex formed by at most $d+1$ elements in $\{p^*\} \cup \{p_{k_\chi} + x^*\}_{\chi=1}^{\bar{d}}$. For example, $x^* = 0$ directly implies $\xi_k = 0 \Rightarrow p_k = p^* + x^*$ and (7) can be reduced to (7) in [14] by letting $\zeta_k = 0$.*

Now, the conditions of control parameters for guaranteeing the boundedness of some state vectors are presented.

Proposition III.9. *$\tilde{q}_k := \hat{q}_k - q_k \in \ell_\infty$ and $\epsilon_k \in \ell_\infty \cap \ell_2$ if condition*

$$\gamma(TU)^2 < 2 \quad (10)$$

holds and adaptive estimator (5) is adopted.

Proof. It follows from (3), (5) and $\epsilon_{k+1} = -\phi_k^\top \tilde{q}_k$ that

$$\tilde{q}_{k+1} = \tilde{q}_k + \Gamma \phi_k \epsilon_{k+1} = (I - \Gamma \phi_k \phi_k^\top) \tilde{q}_k. \quad (11)$$

Consider the Lyapunov candidate $V_k = \tilde{q}_k^\top \Gamma^{-1} \tilde{q}_k$ and its difference $\Delta V_k = V_{k+1} - V_k$, one has

$$\Delta V_k = -\epsilon_{k+1}^2 (2 - \phi_k^\top \Gamma \phi_k). \quad (12)$$

If $\gamma(TU)^2 < 2$, $\phi_k^\top \Gamma \phi_k = \gamma \|\phi_k\|^2 \leq \gamma(TU)^2 < 2$, which is equivalent to $\Delta V_k \leq 0$. Thus, one has $V_k \leq V_{k-1} \leq \dots \leq V_0$, i.e., $\tilde{q}_k$ is bounded. On the other hand, it follows from the boundedness of ϕ_k that ϵ_{k+1} is also bounded. By $\sum_{k=0}^\infty \Delta V_k = -\sum_{k=0}^\infty \epsilon_{k+1}^2 (2 - \gamma \|\phi_k\|^2) = \lim_{k \rightarrow \infty} V_{k+1} - V_0 \leq 0$, one has $\epsilon_{k+1} \in \ell_2$. ■

Proposition III.10. *There exists a constant M such that $\limsup_{k \rightarrow \infty} \|q_k\| \leq M$ if conditions (10) and*

$$C < \beta < 1/T \quad (13)$$

hold and motion controller (7) is adopted.

Proof. Let $M_k = \|q_k\|^2$ and its difference $\Delta M_k = M_{k+1} - M_k$, one has

$$\begin{aligned}\Delta M_k &= (q_k + \phi_k)^\top (q_k + \phi_k) - q_k^\top q_k \\ &= s_k T (s_k T u_k^\top u_k + 2u_k^\top q_k),\end{aligned}\quad (14)$$

and hence $\Delta \tilde{M}_k = T u_k^\top u_k + 2u_k^\top q_k \geq \Delta M_k / (s_k T)$ by noting that $s_k \in (0, 1]$. By substituting (7) into $\Delta \tilde{M}_k$, one has

$$\begin{aligned}\Delta \tilde{M} &= T\beta^2 \|\hat{q}\|^2 + TC^2 f^2 \|\sigma\|^2 - 2T\beta C f \sigma^\top \hat{q} \\ &\quad - 2\beta \hat{q}^\top q + 2C f \sigma^\top q \\ &= (T\beta^2 - 2\beta) \|q_k\|^2 + TC^2 f^2 \|\sigma\|^2 \\ &\quad + (2 - 2T\beta) C f \sigma^\top q + (2T\beta^2 - 2\beta) \tilde{q}^\top q \\ &\quad - 2T\beta C f \sigma^\top q + T\beta^2 \|\tilde{q}\|^2 \\ &\leq [T\beta^2 + TC^2 - 2\beta + (2 - 2T\beta)C] \|q\|^2 \\ &\quad + 2\beta[(1 - T\beta) + TC] \|q\| \|\tilde{q}\| + T\beta^2 \|\tilde{q}\|^2,\end{aligned}\quad (15)$$

where the subscripts $(\cdot)_k$ are omitted for the sake of compactness and $f = f(\xi_k)$. The second equality follows from $\hat{q} = \tilde{q} + q$. The last inequality follows from Cauchy-Schwarz inequality, Assumption III.2, the reverse triangular inequalities

$$\begin{aligned}\|d_k - \|x^*\|\| &= \| \|p_k - p^*\| - \|x^*\| \| \\ &\leq \| (p_k - p^*) - x^* \| \\ &= \|q_k\|,\end{aligned}$$

$$\begin{aligned}\| \|x^* - \sum_{l=k_\chi}^k \phi_l \| - d_{k_\chi} \| &= \| \|x^* - (p_k - p_{k_\chi})\| - \|p_{k_\chi} - p^*\| \| \\ &\leq \| (x^* - (p_k - p_{k_\chi})) - (p_{k_\chi} - p^*) \| \\ &= \|q_k\|,\end{aligned}\quad (16)$$

and $1 - T\beta > 0$ by (13). Moreover, $\beta > C$ implies that

$$\begin{aligned}T\beta^2 + TC^2 - 2\beta + (2 - 2T\beta)C &= (\beta - C)[T(\beta - C) - 2] \\ &\leq (C - \beta)(1 + TC) < 0,\end{aligned}$$

and hence there exists some constant M' such that $\Delta \tilde{M}_k < 0$ if $\|q_k\| \geq M'$ as $\|\tilde{q}_k\|$ is bounded from Proposition III.9. Then one has $\Delta M_k \leq s_k T \Delta \tilde{M}_k < 0$, which completes the proof. ■

Remark III.11. It can be found that the conditions for control parameters are the same with that in [14] while the key difference lies in analysis.

Now, we are ready to establish the convergence result of the relative docking.

Theorem III.12. Under Assumptions III.1, III.2 and III.4, the relative docking problem (1) with dynamic model (2) can be solved by adopting adaptive estimator (5) and motion controller (7) with gains satisfying (10) and (13).

Proof. The overall system under controller (3), (5), (7) and

(6) can be written as

$$\begin{aligned}q_{k+1} &= q_k + \phi_k, \\ \tilde{q}_{k+1} &= (I - \gamma \phi_k \phi_k^\top) \tilde{q}_k, \\ \rho_{k+1} &= \Pi(\rho_k).\end{aligned}\quad (17)$$

In view of LaSalle's invariance principle [15], all trajectories of states in (17) will converge to the largest invariant set Ω included in $\Delta V_k = 0$, $k \in \mathbb{Z}_{\geq 0}$, where V_k is the Lyapunov function defined in the proof of Proposition III.9. It suffices to show that $q_k \equiv 0$ holds for any trajectory $\{[q_k^\top, \tilde{q}_k^\top, \rho_k^\top] : k \in \mathbb{Z}_{\geq 0}\}$ in Ω .

It follows from (12) that $\Delta V_k \equiv 0$ iff $\epsilon_{k+1} = -\phi_k^\top \tilde{q}_k \equiv 0$, which also follows from (11) that $\tilde{q}_k \equiv \tilde{q}^{ss}$. It suffices to prove that $\phi_k^\top \tilde{q}_k \equiv 0$ is equivalent to $q^{ss} = 0$. The following three cases should be considered:

1) $\phi_k \neq 0$ or $\tilde{q}^{ss} \neq 0$. It follows from

$$0 \equiv \phi_k^\top \tilde{q}_k = (q_{k+1} - q_k)^\top \tilde{q}^{ss} = (\hat{q}_{k+1} - \hat{q}_k)^\top \tilde{q}^{ss}$$

that $\hat{q}_k^\top \tilde{q}^{ss} \equiv m$ for some constant $m \in \mathbb{R}$. On the other hand, $0 \equiv \phi_k^\top \tilde{q}^{ss}$ also implies that $0 \equiv \phi_k^\top \tilde{q}^{ss} = [-\beta \hat{q}_k + C f(\xi_k) \sigma_k]^\top \tilde{q}^{ss}$, or equivalently $\beta m \equiv \beta \hat{q}_k^\top \tilde{q}^{ss} = C f(\xi_k) \sigma_k^\top \tilde{q}^{ss}$. Specifically, we have $f(\xi_k) \sigma_k^\top \tilde{q}^{ss} = f(\xi_{k+T_0}) \sigma_{k+T_0}^\top \tilde{q}^{ss}$. Next we establish $f(\xi_k) > 0$ by contradiction. Suppose $f(\xi_k) = 0$, then $\xi_k = \max(\zeta_k, \|d_k - \|x^*\|\|)$, or equivalently $d_k = \|x^*\|$ and $\|x^* - \sum_{l=k_\chi}^k \phi_l\| = d_{k_\chi}$ for $i \in \{1, \dots, \bar{d}\}$, which implies $p_k = p^* + x^*$ by Assumption III.4 and Lemma III.7. Then it follows from $u_k = -\beta \hat{q}_k = -\beta \tilde{q}^{ss}$, one has $\phi_k^\top \tilde{q}^{ss} = T s_k u_k = -\beta T s_k \|\tilde{q}^{ss}\|^2 \neq 0$, a contradiction. Therefore, one has $f(\xi_k) > 0$, $\forall k \in \mathbb{Z}_{\geq 0}$. On the other hand, it follows from $\sigma_{k+T_0} = -\sigma_k$ that

$$[f(\xi_k) + f(\xi_{k+T_0})] \sigma_k^\top \tilde{q}^{ss} \equiv 0 \quad (18)$$

and hence $\sigma_k^\top \tilde{q}^{ss} \equiv 0$. By Assumption III.2, one has $\|\tilde{q}^{ss}\|^2 = \tilde{q}^{ss\top} \sum_{l=0}^{T_0-1} \tau_l \sigma_l = 0$, another contradiction.

2) $\phi_k \equiv 0$. It follows from $u_k = -\beta \hat{q}_k + C f(\xi_k) \sigma_k = 0$ that $q_k \equiv q^{ss}$, $\hat{q}_k \equiv \hat{q}^{ss}$, and $\xi_k \equiv \xi^{ss}$. On the other hand,

$$\begin{aligned}u_k &= -\beta \hat{q}^{ss} + C f(\xi^{ss}) \sigma_k = 0, \\ u_{k+T_0} &= -\beta \hat{q}^{ss} + C f(\xi^{ss}) \sigma_{k+T_0} = 0,\end{aligned}\quad (19)$$

which hold only if $\hat{q}^{ss} = 0$ and $\xi^{ss} = 0$. By Assumption III.4 and Lemma III.7, $p^{ss} = p^* + x^*$, i.e., $q^{ss} = 0$.

3) $\tilde{q}^{ss} = 0$. In this case, one has

$$q_{k+1} = q_k + T s_k (-\beta q_k + C f(\xi_k) \sigma_k), \quad (20)$$

and hence $\Delta \tilde{M}_k \leq [T\beta^2 + TC - 2\beta + (2 - 2T\beta)C] \|q_k\|^2$, which implies that $q^{ss} = 0$ is a globally stable equilibrium.

This completes the proof. ■

Fig. 3: The trajectory of robot in 2-D.

Fig. 4: The states of robot in 2-D.

IV. SIMULATION EXAMPLES

In this section, two simulation examples in both 2-D and 3-D spaces are presented to demonstrate the effectiveness of our proposed controller. According to Theorem III.12, we choose the following parameters: $U = 0.75\text{m/s}$, $T = 0.1\text{s}$, $\gamma = 10$, $\beta = 2$, $C = 1$. Π and Σ are chosen according to Remark III.3 and let $a_1 = a_2 = 1$, $a = 24$, $\rho_0 = [1 \ 0]^T$. p^* and x^* are set as $5\mathbf{1}_2$ (resp. $5\mathbf{1}_3$) and $[5\sqrt{3} \ -5]^T$ (resp. $\sqrt{2}[5\sqrt{3} \ -5 \ 10]^T/2$) for 2-D (resp. 3-D) space. Let $\hat{q}_0 = 0$. Fig. 3 and Fig. 5 show the trajectories of the robot in 2-D and 3-D, respectively. The states of the robot in 2-D are recorded in Fig. 4. It can be found that the robot is moving towards the desired docking position $p^* + x^*$ gradually, which confirms our theoretical results. The triggering positions p_{k_χ} in both simulations are selected at the initialization stage. However, this is not necessary as we have indicated in Remark III.5, i.e., p_{k_χ} can be selected near $\mathcal{B}(p^*, \|x^*\|)$.

V. CONCLUSION

In this paper, we have addressed the range-based relative docking problem for a single robot, which is a nontrivial generalization of the original range-based docking problem. This has been tackled by incorporating multiple constraints constructed from range measurements at several proper triggering time instants along the robot's motion trajectory. The

Fig. 5: The trajectory of robot in 3-D.

convergence of the overall system has been theoretically established and validated by two simulation examples.

Future work will be on the experimental validation of the proposed scheme. Applying this idea to multi-robot formation control [22]–[24] is of another particular interest and is currently under investigation.

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