Nonlinear optics of coupled nanophotonic resonators

Banerjee, Rimi

2020

Banerjee, R. (2020). Nonlinear optics of coupled nanophotonic resonators. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/146234>

<https://doi.org/10.32657/10356/146234>

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SINGAPORE

**NONLINEAR OPTICS OF COUPLED
NANOPHOTONIC RESONATORS**

RIMI BANERJEE

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

2020

NONLINEAR OPTICS OF COUPLED NANOPHOTONIC RESONATORS

RIMI BANERJEE

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for the
degree of Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me except where otherwise stated in this thesis. The thesis work has not been submitted for a degree or professional qualification to any other university or institution. I declare that this thesis is written by myself and is free of plagiarism and of sufficient grammatical clarity to be examined. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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8 Jan 2021

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Date

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Asst. Prof. Timothy C. H. Liew

Authorship Attribution Statement

This thesis contains material from four published papers in the following peer-reviewed journals in which I am listed as an author.

Parts of Chapter 2 have been published as R. Banerjee, T. C. H. Liew, and O. Kyriienko, “Realization of Hofstadter’s butterfly and a one-way edge mode in a polaritonic system”, Phys. Rev. B **98**, 075412 (2018).

[DOI:10.1103/PhysRevB.98.075412](https://doi.org/10.1103/PhysRevB.98.075412).

The contributions of the co-authors are as follows:

- Prof. Kyriienko and Prof. Liew conceived the project and formulated the problem.
- I performed all the numerical calculations, simulations with inputs from them.
- I prepared the manuscript draft.
- Prof. Liew and Prof. Kyriienko edited the manuscript draft and supervised the project.

Parts of Chapter 3 have been published as R. Banerjee, S. Mandal, and T. C. H. Liew, “Coupling between Exciton-Polariton Corner Modes through Edge States”, Phys. Rev. Lett. **124**, 063901 (2020). [DOI:10.1103/PhysRevLett.124.063901](https://doi.org/10.1103/PhysRevLett.124.063901).

The contributions of the co-authors are as follows:

- I, Mr. Mandal, and Prof. Liew conceived the project and formulated the problem.
- I performed the numerical calculations of the spectrum, parametric instability, $|C_n(t)|^2$, linear fluctuations in the eigenbasis. Mr. Mandal showed the realization of corner mode in the continuous Gross–Pitaevskii model and the overlap in the absence of any nonlinear terms. I and Mr. Mandal performed the numerical simulations to demonstrate the transfer of binary information with and without continuous pump at $C1$, respectively. I also performed the calculation showing the advantage of topological corner states over regular square lattice.
- I and Mr. Mandal wrote the manuscript.
- Prof. Liew edited the manuscript and supervised the project.

Chapter 4 is published as published as S. Mandal, R. Banerjee, and T. C. H. Liew, “One-Way Reflection-Free Exciton- Polariton Spin-Filtering Channel”, Phys. Rev. Applied **12**, 054058 (2019). [DOI:10.1103/PhysRevApplied.12.054058](https://doi.org/10.1103/PhysRevApplied.12.054058).

The contributions of the co-authors are as follows:

- I, Mr. Mandal and Prof. Liew conceived the project and provided the model.
- At the initial stage I calculated the spectrum in the tight binding calculation and predicted the parameters. Mr. Mandal calculated the band structures of the polaritons in the topological and spin filtering regime and calculated their spin-polarization degree in the continuous Gross–Pitaevskii model. I and Mr. Mandal showed the robust pulse propagation through the system. Mr. Mandal also calculated the dispersion using different diameters micropillars. I also showed the effect of TE-TM splitting on dispersion and the effect of lifetime on the propagation distance.
- I and Mr. Mandal wrote the manuscript.
- Prof. Liew edited the manuscript and supervised the project.

Parts of Chapter 5 have been published as R. Banerjee and T. C. H. Liew, “Artificial Life in an Exciton-Polariton Lattice”, New J. Phys **22**, 103062 (2020). [DOI:10.1088/1367-2630/abc290](https://doi.org/10.1088/1367-2630/abc290).

The contributions of the co-authors are as follows:

- Prof. Liew conceived the project and provided the model.
- I carried out numerical calculations with inputs from Prof. Liew.
- I prepared the manuscript draft.
- Prof. Liew edited the manuscript and supervised the project.

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Abstract

Excitons are bound hydrogen-like states of electrons and holes, typically appearing in semiconductor quantum wells. Their electric dipole moment allows them to couple with light in resonating structures (e.g. microcavities, micropillars and microring resonators, etc.). In the strong coupling limit they form hybrid states called exciton-polaritons. Being mixed states, exciton-polaritons adopt some properties from excitons and some properties from photons, such as: strong nonlinearity, sensitivity to electric/magnetic fields, light effective mass, long dephasing time, rich spin dynamics, etc. These astonishing properties make them suitable for spin-dependent devices and low-power ultrafast switching based optoelectronics devices. Besides, due to their effective mass they can also form Bose-Einstein condensates at relatively high temperatures, even at room temperature.

This thesis concerns four different theoretical proposals in exciton-polariton systems. First, we introduce a scheme to obtain Hofstadter's butterfly with polarization splitting, which generates an artificial gauge field in the system. This field breaks the time reversal symmetry in the system and enables us to achieve topologically protected one-way edge modes. Second, we study second-order topological modes in exciton-polariton square lattice. In particular, we propose coupling of topological corner modes with the help of nonlinearity to achieve robust memory-based devices as well as communication between them via edge states. Third, we propose theoretically a feedback-free optical spin filter in a honeycomb lattice geometry, where the system allows only one of the pure spins in the system to propagate along one of the edges, while propagation in the opposite direction is restricted even considering the opposite edge. In the considered system all the edge states split in energy and there exists an energy window where one edge state has a pure spin and is surrounded by bulk states of opposite spin, which enable propagation of spin unhampered even in a remarkably thin strip. Fourth, we introduce a theoretical scheme for cellular automata (CAs) in the exciton-polariton system, which automatically give rise to a variety of solitons. Due to the Turing completeness of the CAs, this scheme will be able to reproduce arbitrary logic circuits.

Acknowledgements

First and foremost, I would like to thank my supervisor Asst. Prof. Timothy Liew for his dedicated guidance and constant support during the last four years. His advises throughout this journey, specially during the tough phases have helped me immensely. He was always patient and ready to hear each one of my queries. I am indebted and deeply grateful for his invaluable time and expert mentorship. During our countless discussions, I have learned a lot about physics as well as life, which I hope to carry with me for the rest of my life.

I would also like to express my sincere gratitude to Dr. Oleksandr Kyriienko who has helped me to understand the concepts of the topological physics during the initial phase. I am extremely thankful to all the alumni and current members of our group: Dr. Sanjib Ghosh, Dr. Kevin Dini, Dr. Tanjung Krisnanda, Mr. Huawen Xu, Dr. Wijnand Broer, Dr. Rongchun Ge, Dr. Liu Tao, Dr. Jing Li, and Dr. Kristin Arnardottir from whom I have learned a lot specially during the group meetings. I would like to thank my thesis advisory committee members Asst Prof. Justin Song and Prof. Michal Matuszewski for their support during PhD. Special thanks to Subhaskar Mandal for encouraging me, cheering me up from the depression, celebrating with me when things went well and for making this journey a real joy.

Most importantly, I want to thank my parents, sister, and my big amazing family for always being with me, backing me, believing in me and their unconditional love. Without them this would have been meaningless. I dedicate this thesis to them. Last but not least, I want to thank Nanyang Technological University (NTU) and Singapore for all the opportunities and unforgettable memories.

Introduction

Modern electronic devices and integrated circuits are limited by interconnected bandwidth density inside machines, which results in a high heating and time delay between components. Although the smaller transistor reduces the required energy per bit, the energy to send the signals inside electronic machines is not reduced at the same scale. Here optics offers unique opportunities; it allows high densities of interconnections [1], which can operate at a significantly low energy with a high operation speed. This situation further boosts up by making an efficient interaction between optical modes carrying the signals. It can be achieved using material nonlinearities by confining light into resonating structures (e.g. microring resonators, microcavities, micropillars, etc.) and coupling to matter. In particular, we consider an exciton as a particle of matter that is a bound electron-hole pair. Due to the strong coupling between excitons in a quantum well with cavity photons, we get highly nonlinear exciton-polariton modes, which possess some properties of excitons and some properties of photons [2,3]. From photons, the exciton-polaritons have light effective mass and long dephasing time, meaning they can preserve a coherence for a reasonable time. At the same time, via their excitonic part, polaritons show strong nonlinearity ($\chi^{(3)}$) and sensitivity to electric and magnetic field. As a result, they can show quantum confined Stark effect in the presence of electric field and magnetic moments inducing Zeeman shift in the presence of magnetic field. Due to strong nonlinearity between polaritons, when a polariton mode is excited by a laser with a slight blue detuning, it exhibits the phenomenon of bistability, where the mode must exist in one of two distinct stable states, which have different intensities. The intensity of polaritons in a particular mode can then naturally encode a binary information. This bistability operates at low optical energy (pico-joule) and allows switching between the bistable states on ultrafast (picosecond) timescale. This is why logical circuits based on polaritons are promising for future applications. Significant improvements have been made in the field of optoelectronics by implementing different optical logic gates [4-9], polariton switches [10], ultrafast memories [11], transistors [12,13], optical diodes [14,15], polariton-neurons [16,17], optical routers [18], etc. Recent works on reservoir computing in an exciton-polariton system show a promising result to do tasks such as image and speech recognition [19], realization of Toffoli gates, and error-correction [20]; the former task has been realized experimentally by Ballarini et al. [21].

The thesis is organized as follows: in **chapter 1**, we give an overview of exciton-polaritons. We briefly discuss excitons, dispersion of cavity photons, and the formation of exciton-polaritons in the strong coupling limit. We discuss their properties, schemes of excitation to create them, and the theoretical model to describe them in the mean field approximation. Then we introduce the effects arising from the nonlinear dynamics of polaritons.

In **chapter 2**, we present theoretically a scheme with elliptical exciton-polariton micropillars, which effectively gives rise to a phase dependent hopping, corresponding to an artificial gauge field. This field breaks the time reversal symmetry and shows Hofstadter's butterfly, which is an archetypal model of a quantum Hall sys-

tem. We observe a unidirectional edge mode for a finite sample, which can be used for robust polariton transfer.

In **chapter 3**, we consider theoretically the realization of a second order topological polariton bandstructure, which gives rise to 0D localized corner states in a polariton lattice. Due to the topological nature, information can be trapped in the corner even in the presence of disorder. We show that in the presence of polariton-polariton scattering, polaritons can scatter from a pumped corner state into an edge state, which again scatters back to another adjacent corner. In this way, we find that as a nonlinear driven-dissipative system exciton-polaritons offer a unique opportunity for realizing spatially localized topological states that can be coupled together.

In **chapter 4**, we propose theoretically a feedback-free optical spin filter based on the exciton-polariton system, where the system allows only one of the pure spins in the system to propagate along one of the edges, while propagation in the opposite direction is prohibited even with the consideration of the other edge. In the considered system the edge modes are split in energy and within a particular energy window, there is only one edge state with a pure spin, which is surrounded by bulk states having opposite spin. Since within the window of interest there is no other state having the same spin to backscatter to, the propagation through the edge state is robust and feedback-free.

In **chapter 5**, we show theoretically in a lattice of exciton-polaritons that when a series of laser pulses is applied to the system, it behaves as lifelike cellular automata (CAs), i.e., a form of artificial life. A polariton cellular automaton allows access to a variety of kinds of solitons (such as oscillating solitons, dissipative solitons, rotating solitons, soliton trains, etc.) and patterns simultaneously. We show that the CAs that we obtain are universally complete. They are furthermore robust against realistic level of disorder and do not decay despite the finite polariton lifetime.

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Chapter 1

Overview of exciton-polaritons

1.1 Excitons

During an interband transition in a semiconductor, when a photon is absorbed, an electron is created in a conduction band and a hole in a valence band. If E_g is the bandgap of the semiconductor, then the energy of the electrons can be expressed under parabolic approximation near the band edge as

$$E_e(k) = E_g + \frac{\hbar^2 k^2}{2m_e}, \quad (1.1)$$

where m_e is the effective mass of an electron having wavevector k . Similarly the energy of the holes are given by,

$$E_h(k) = -\frac{\hbar^2 k^2}{2m_h}, \quad (1.2)$$

where m_h is the effective mass of the hole. For low momentum, i.e., $k \approx 0$, the Coulomb interaction makes them a bound electron-hole pair, known as an exciton [22, 23]. There are mainly two types of excitons: Wannier-Mott excitons, also known as free excitons; and Frenkel excitons, also known as tightly bound excitons.

Wannier-Mott excitons are typically found in semiconductors having high dielectric constants and small bandgap. The strong dielectric screening in semiconductors decreases the Coulomb interaction between electrons and holes, which results in low binding energy (~ 0.01 eV) for such excitons. These kinds of excitons are only observed clearly at cryogenic temperatures except in a few wide bandgap inorganic materials, such as ZnO and GaN. Wannier-Mott excitons have a large Bohr radius (~ 10 - 100 Å), which encompasses many atoms as shown in Fig. 1.1(a). These excitons are delocalized states, which can move freely in the crystal. To get an idea of the energy spectrum of Wannier-Mott excitons, we follow the effective mass approximation and modify the Schrödinger equation for an electron in a hydrogen atom. We replace the free electron mass by the reduced mass of the exciton of the form $\mu = m_e m_h / (m_e + m_h)$ and the dielectric constant of free space ε_0 with the dielectric constant of the crystal $\varepsilon \varepsilon_0$. The Schrödinger

equation takes the form

$$\left(-\frac{\hbar^2 \nabla^2}{2\mu} - \frac{e^2}{4\pi\epsilon\epsilon_0 r} \right) \Psi(r) = E\Psi(r), \quad (1.3)$$

where r is the relative distance between the electron and the hole. The ground state (1s) takes the form

$$\Psi(r) = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a_B}, \quad (1.4)$$

where a_B is called the Bohr radius given by

$$a_B = \frac{4\pi\epsilon\epsilon_0 \hbar^2}{\mu e^2}. \quad (1.5)$$

The eigen-energy of the state which corresponds to the binding energy of the excitons is given by

$$E_B = \frac{2\mu e^4}{(8\pi\hbar\epsilon\epsilon_0)^2} = \frac{\hbar^2}{2\mu a_B^2}. \quad (1.6)$$

In contrast, Frenkel excitons can be observed in organic materials having relatively small dielectric constant. As a result, the Coulomb interaction is strong and thus the Bohr radius tends to be smaller ($\sim 5 \text{ \AA}$, which is comparable to the size of the unit cell, see Fig. 1.1(b)). Consequently, Frenkel excitons possess larger binding energies (of the order 0.1-1 eV), which stabilize them at room temperature. Unlike Wannier-Mott excitons, Frenkel excitons are highly localized to specific atoms or molecules and move through the crystal by hopping.

Figure 1.1: Schematic of the two types of excitons. (a) Wannier-Mott excitons and (b) Frenkel excitons. Wannier-Mott excitons have large Bohr radius compared to Frenkel excitons. Here e denotes an electron and h denotes a hole, whereas black-dots are atoms or molecules of the respective material.

1.2 Semiconductor quantum wells

The quantum confinement effect makes the optical properties in a small crystal size dependent and gives rise to interesting physics. If the confinement of a particle along the x axis is Δx , then according to Heisenberg's uncertainty principle its uncertainty in the momentum space becomes $\Delta p_x \sim \hbar/\Delta x$. For mass m , this confinement induces an additional kinetic energy in the x direction of magnitude $E_0 = \frac{(\Delta p_x)^2}{2m} \sim \frac{\hbar^2}{2m(\Delta x)^2}$. When this confinement energy is comparable or greater than the thermal motion, i.e., $E_0 \sim \frac{\hbar^2}{2m(\Delta x)^2} > \frac{1}{2}k_B T$, which implies that

$$\Delta x \sim \sqrt{\frac{\hbar^2}{mk_B T}}, \quad (1.7)$$

the quantum confinement effects start to become significant. The simplest model of a quantum well (QW) can be thought of as a 2D layer of semiconductor, with a typical thickness comparable to the bulk exciton Bohr radius, which is sandwiched between two barrier layers characterized by a much larger band gap on top of a thicker substrate crystal. The structure is mainly fabricated in two ways: one is molecular beam epitaxy MBE and another one is metal-organic chemical vapour deposition MOCVD. A commonly used quantum well structure is based on a GaAs/AlGaAs structure as shown in Fig. 1.2(a). The GaAs layer of thickness d (in agreement with Eq. (1.7)) is sandwiched between AlGaAs layers with thickness greater than d . The larger bandgap of AlGaAs creates a potential barrier in which the carriers in the GaAs layer get trapped. The confinement in d quantizes the motion of the carriers along the crystal growth direction, which we take as 'z', while they are free in the other two axes in the plane of the layer, which we take as 'x' and 'y'. Thus we effectively have a two dimensional system. To increase the absorption, people often use a stack of quantum wells instead of a single quantum well as shown in Fig. 1.2(b). The idea is to place a stack of GaAs quantum wells of width d separated from each other by a AlGaAs layer of thickness b . Depending on the value of b , it is either called a Multiple Quantum well (MQW) or a superlattice. In MQWs, b has large values isolating the quantum wells from each other. As a result, the properties of the MQWs remain the same as the single quantum well but with a significant optical density, which is useful for optical applications. On the other hand, for a superlattice b is comparatively small meaning it is made of much thinner barriers and as a result the quantum wells are now coupled by tunneling which forms new extended state in the 'z' direction. These superlattices have additional properties than of individual quantum wells.

1.2.1 Quantum-well Exciton

Due to confinement in the quantum well, the overlap of electron and hole wavefunctions increases. As a result, the exciton oscillator strength and binding energy increase. In particular, due to confinement, the exciton binding energy increases by a factor of four and its oscillator strength is increased by a factor of eight.

Figure 1.2: (a) Schematic of the GaAs/AlGaAs QW grown on a GaAs substrate. The typical thickness of a single quantum well is around 10 nm. The GaAs has a smaller bandgap than the AlGaAs; as a result the states of the conduction and valence band of GaAs are trapped. As a result, we have quantized motion in the z direction and free motion in the other two directions x, y . Thus QW excitons behave as two-dimensional quasiparticles. (b) Diagram of a multiple quantum well or superlattice. For a multiple quantum well structure, b is larger compared to the case of a superlattice.

These effects have been thoroughly investigated in Refs. [24–28]. The Schrödinger equation for an exciton in a QW is given by

$$\left(-\frac{\hbar^2 \nabla_e^2}{2m_e} - \frac{\hbar^2 \nabla_h^2}{2m_h} + V_e(z_e) + V_h(z_h) - \frac{e^2}{4\pi\epsilon\epsilon_0|r_e - r_h|} \right) \Psi = E\Psi, \quad (1.8)$$

where the first two terms represent the kinetic energies of the electrons and holes, respectively. $V_e(V_h)$ describe the quantum confinement of electrons (holes) along the growth axis of the structure, i.e., z -axis, and the last term corresponds to the Coulomb potential. We apply the method of separation of variables of the form

$$\Psi(r_e, r_h) = F(R)f(\rho)U_e(z_e)U_h(z_h), \quad (1.9)$$

where

$$R = \frac{m_e r_e + m_h r_h}{m_e + m_h} \quad \text{and} \quad \rho = \rho_e - \rho_h. \quad (1.10)$$

R is the exciton centre of mass coordinate, ρ is the relative in-plane radius-vector of electron and hole and in general $r = (\rho, z)$. The four functions (F, f, U_e, U_h) follow:

$$\begin{aligned} \int_0^\infty |F(R)|^2 2\pi R dR &= 1, & \int_0^\infty |f(\rho)|^2 2\pi \rho d\rho &= 1, \\ \int |U_e(z_e)|^2 dz_e &= 1, & \int |U_h(z_h)|^2 dz_h &= 1. \end{aligned} \quad (1.11)$$

The ground state (1s) for the two dimensional exciton is given by [25]

$$f(\rho) = \sqrt{\frac{2}{\pi}} \frac{1}{a_B^{2D}} e^{-\rho/a_B^{2D}}, \quad \text{where } a_B^{2D} = \frac{a_B}{2}. \quad (1.12)$$

The binding energy of the two dimensional exciton is given by $E_B^{2D} = 4E_B$. Thus confined excitons are much more robust and optically active than the bulk excitons.

The spin structure of excitons depends on the spins of the electrons and holes, which furthermore depends on the crystal structure. For light based devices we often rely on the direct gap semiconductor materials such as GaAs, CdTe, GaN, ZnO, etc, which are so called Zinc-Blend crystals. In such materials the lowest conduction band is a two-fold degenerate s orbital ($l = 0, s = 1/2, m_l = 0, m_s = \pm 1/2$) and the uppermost valence band is a six-fold degenerate p orbital ($l = 1, s = 1/2, m_l = 0, \pm 1, m_s = \pm 1/2$) [29], where l and s are the orbital and spin quantum numbers, respectively, and m_l and m_s are their projections on the z axis, respectively. The spin-orbit coupling of the material lifts this degeneracy as shown in Fig. 1.3(a) and forms three p like parabolic dispersions (see Fig. 1.3(b)), where the state with ($j = 3/2, m_j = \pm 3/2$) forms the so-called heavy hole band. j is the total angular momentum and m_j is its projection on the z axis. The eigenstates with ($j = 3/2, m_j = \pm 1/2$) form the light hole band and the band with ($j = 1/2, m_j = \pm 1/2$) is known as the spin-orbit split-off (so) valence band [30]. At $k = 0$ the heavy hole and the light hole bands are degenerate. So far in the spin description of excitons we have not considered the confinement effect, hence the above (spin description) is only applicable for bulk excitons. Due to the confinement in quantum wells, there is a reduced symmetry in the ‘ z ’ direction and the energy band is discontinued at the interfaces, which lifts the degeneracy between heavy-holes and light-holes. Heavy holes lie typically at the uppermost level of the valence band, higher than the light holes, as shown in Fig. 1.3(c) [23, 25, 31]. A significant band mixing takes place and an anti-crossing occurs between these two bands. This manifests in the states with heavier effective mass in the growth direction having lighter effective mass in the transverse direction and vice versa, which is commonly known as *mass reversal* (Fig. 1.3(d)) [29, 30, 32]. Thus the ground state of an quantum-well-exciton is formed by an electron and a heavy-hole having projection of spin on the growth axis (z axis) is $\pm 3/2$. As a result, the projections of the total angular momentum of exciton along the growth axis is $J_z = \pm 1$ and $J_z = \pm 2$ (see Fig. 1.3(e)). Since photons have angular momentum ± 1 , excitons with $J_z = \pm 1$ can couple to light and known as bright excitons. On the other hand, due to the conservation of angular momentum, excitons with $J_z = \pm 2$ do not couple with photons and known as dark excitons [25].

1.3 Microcavities (MC)

The main motivation of the field of semiconductor microcavities is to have an effective interaction between excitons and photons and study their fascinating properties. In this case, we confine excitons and photons in a small volume for sufficiently long time. The most common technique to trap photons is to use a

Figure 1.3: (a) The top of the valence band and bottom of the conduction band without (left) and with (right) spin-orbit coupling in Zinc-Blend crystal near $k = 0$. (b) Near $k = 0$, there is an s like parabolic conduction band (C. B.) and three p like parabolic valence bands: heavy hole (hh) and light hole (lh) bands, which are degenerate at $k = 0$ and, the so-called split-off (so) valence band. The spin orbit splitting between holes at $k = 0$ is given by Δ and E_g is the band gap. (c) Due to confinement in quantum wells, the energy levels of the heavy holes lie lower than that of the light holes. The lowest energy excitation in a quantum well is an exciton composed of a heavy-hole and electron. (d) Qualitative sketch of a heavy hole and light hole anti-crossing as a result of band mixing. The solid curves are depicted from the Luttinger framework whereas the dashed curves are without bandmixing. (e) Schematic diagram of the origin of the bright and dark exciton.

planar Fabry-Perot (FP) cavity, which is bounded by distributed Bragg reflector (DBR) pairs.

1.3.1 Distributed Bragg reflector

A DBR is a periodic structure of dielectric layers with alternating high and low refractive indices of thickness $\lambda/(4n)$ (see Fig. 1.4(a)), which satisfies the Bragg condition, where λ is the wavelength of the incident light and n is the refractive index of the respective material. In this structure reflected light from each interface interferes constructively and the whole structure acts as a high-reflective mirror (see Fig. 1.4(b)). It creates a broad spectral range called the ‘stop-band’, for any wavelength of the incident light within this band is forbidden to transmit through the structure (see Fig. 1.4(c)). The reflectivity of the DBR increases with the number of the layer pairs (see Fig. 1.4(d)) and also with the increment of the difference between the high and low refractive indices. The reflectivity of the stop-band near the band center for a large number of dielectric pairs (N) is given by [33]

$$R = 1 - 4 \frac{n_o}{n_i} \left(\frac{n_2}{n_1} \right)^{2N}, \quad (1.13)$$

where n_i and n_o are the refractive indices of the originating medium (i.e., incident medium) and terminating medium (i.e., substrate), respectively. Here n_1 and n_2 are the refractive indices of the two alternating dielectric layers where $n_1 > n_2$. We choose λ as the middle of the stop band (for which the DBR is designed) and known as the Bragg wavelength. The width of this stopband is given by [32, 34]

$$\Delta\lambda_{\text{stopband}} = \frac{2\lambda(n_1 - n_2)}{\pi n_{\text{eff}}}, \quad (1.14)$$

where n_{eff} is the effective refractive index of the DBR, which is given by the following expressions:

$$n_{\text{eff}} = 2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)^{-1}. \quad (1.15)$$

High reflectivity around 0.999 could be achieved in a DBR using 30 pairs of AlAs/GaAs ($n_{\text{GaAs}} = 3.54$, $n_{\text{AlAs}} = 2.96$). To calculate the optical mode profile of this kind of structure, we use the transfer matrix method (TMM), which is discussed in detail in Ref. [35].

1.3.2 Bragg mirror microcavity

To form a cavity, a photonic defect layer (such as GaAs) of optical thickness integer time $\lambda/2$ is sandwiched between the two highly reflective DBRs as shown in Fig. 1.4(e). The length of the cavity (L_c) is ideally given by

$$L_c = p \frac{\lambda}{2n_c}, \quad (1.16)$$

where p is an integer and n_c is the refractive index of the defect layer. It creates a dip at the center of the the stop-band in the reflectivity spectrum, which allows

Figure 1.4: (a) Schematic representation of a DBR. (b) Alternating layers create a high-reflectance mirror. (c) The reflectivity spectrum of an AlAs/GaAs DBR, which creates a stopband around $\lambda = 840$ nm. (d) Dependence of the reflectivity on the number of layer pairs, N . (e) Schematic diagram of a microcavity. (f) The reflectivity spectrum of an AlAs/GaAs based microcavity at normal incidence. (g) Electric field profile in the microcavity.

a transmission corresponding to the Bragg wavelength (see Fig. 1.4(f)). From the width of this reflectivity dip, we determine the value of the quality factor Q , which describes the average numbers of round trips that photons make before escaping from the cavity, by the following expression [2, 36]

$$Q = \frac{\lambda}{\Delta\lambda} = \frac{\omega}{\Delta\omega} \simeq \frac{\pi(R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/2}}, \quad (1.17)$$

where ω is the frequency of the light corresponding to wavelength λ and $\Delta\lambda$ is the width (FWHM) of the resonance called the linewidth of the cavity. R_1 and R_2 are the reflectivity of the two DBRs. The lifetime of photons (τ) in the cavity is given by

$$\tau = Q/\omega. \quad (1.18)$$

For an ideal cavity $Q = \infty$ while realistically by state-of-the-art epitaxy we have reached Q upto 10^5 or higher in (Al)GaAs DBR based microcavities [37–39] which corresponds to the cavity photon lifetime $\sim 1 - 130$ ps. The electric field intensity distribution ($|E(z)|^2$) is shown in Fig. 1.4(g). The field is intense around the center of the cavity and this is why we put QWs at the center of the cavity: to have strong light-matter coupling. The enhancement of the field amplitude inside the cavity is ~ 20 times compared to the field in free space [2] and decays exponentially inside the DBR mirrors [25]. Therefore the effective cavity length is given by the sum of the physical cavity length (L_c) and penetration depth inside the DBR-mirror (L_{DBR})

$$L_{\text{eff}} = L_c + L_{\text{DBR}}, \quad (1.19)$$

where

$$L_{\text{DBR}} = \frac{\lambda}{2n_c} \frac{n_1 n_2}{|n_1 - n_2|}. \quad (1.20)$$

In a planar DBR-cavity, photons are strongly confined in the growth direction ('z' axis) and can freely propagate in the other two directions, which modifies the dispersion of cavity-photons in contrast to photons in (3D) vacuum. The confinement (for the cavity spacer of length L_c and refractive index n_c) forces photons to have discrete values of the wave vector along the 'z' axis, which can be written as

$$k_{\perp} = p \frac{\pi}{L_c} = p \left(n_c \frac{2\pi}{\lambda} \right), \quad (1.21)$$

where p is an integer. The total wavevector can be decomposed into two terms: longitudinal component $k_{\perp}$ and in-plane (transverse) component $k_{\parallel}$. Then the energy dispersion of the cavity-photons can be expressed as

$$E_c = \frac{\hbar c}{n_c} \sqrt{k_{\parallel}^2 + k_{\perp}^2}. \quad (1.22)$$

Since $k_{\perp}$ is much larger than $k_{\parallel}$, we have

$$\begin{aligned} E_c &\approx \frac{\hbar c}{n_c} k_{\perp} \left(1 + \frac{k_{\parallel}^2}{2k_{\perp}^2} \right) \\ &= E_c(k_{\parallel} = 0) + \frac{\hbar^2 k_{\parallel}^2}{2m_{\text{cav}}}, \end{aligned} \quad (1.23)$$

where $E_c(k_{\parallel} = 0) = \hbar c k_{\perp} / n_c$. Here m_{cav} is the effective mass of the cavity-photon, which is given by

$$m_{\text{cav}} = \frac{E_c(k_{\parallel} = 0)}{c^2 / n_c^2}. \quad (1.24)$$

The typical value of m_{cav} is $10^{-5}m_e$, where m_e is the free electron mass. Eq. (1.23) shows that there are two terms that contribute to the dispersion: confinement energy ($E_c(k_{\parallel} = 0)$) and in-plane kinetic energy (second term of Eq. (1.23)). Thus the cavity photon shows a parabolic dispersion with high curvature due to extremely light effective mass.

Similarly, the excitons, which have effective mass comparable to the electrons are also confined in the plane of the QWs. They have parabolic dispersion but with smaller curvature. In the relevant momentum space region for photons, the exciton dispersion looks nearly flat. For the rest of the thesis we will use k as the simplified notion for $k_{\parallel}$.

1.4 Exciton-polaritons

When QWs are placed at the maximum of the photon electric field (i.e., antinodes) inside the DBR-cavity with a high quality factor (see Fig. 1.5), there is an exchange of energy happening back and forth between the cavity photon and the resonantly coupled exciton [40]. QWs absorb photons and create excitons, which due to radiative recombination recreates photons. If the rate of this reversible and coherent energy exchange process is much faster than any other decay (and decoherence) rates in the system, then the system enters to the *strong coupling* regime and creates new quasiparticles called exciton-polaritons (also known as cavity polaritons)[1]. Such a regime has been first observed in the pioneering work by Weisbuch et al. [40]. These new excitations are no longer excitons or photons but are both partly light and partly matter.

1.4.1 The Hamiltonian

The system of this photon-dressed exciton in the linear regime can be written in the second quantization form as

$$\hat{H} = \sum_k \hat{H}_k = \sum_k \left[E_x(k) \hat{\chi}_k^{\dagger} \hat{\chi}_k + E_c(k) \hat{\phi}_k^{\dagger} \hat{\phi}_k + V(\hat{\chi}_k^{\dagger} \hat{\phi}_k + \hat{\phi}_k^{\dagger} \hat{\chi}_k) \right], \quad (1.25)$$

¹Throughout the thesis we will use the terms ‘exciton-polaritons’ and ‘polaritons’ interchangeably.

Figure 1.5: Schematic diagram shows that the excitonic wave-function χ of the quantum well is interacting with photonic wave-function ϕ inside a microcavity, which forms a new quasi-particle called exciton-polariton. χ and ϕ are introduced in subsection [1.4.1](#).

where $\hat{\chi}_k^\dagger$ and $\hat{\phi}_k^\dagger$ are creation field operators of the excitons and photons, respectively, which are coupled by the interaction coupling constant V . In general the interaction coupling constant, V , is related to the dipolar matrix element of the exciton and depends on k [\[41\]](#). For simplicity, here we take the interaction coupling constant as k independent. The field operators satisfy the standard Bose commutation rules $[\hat{\chi}_k, \hat{\chi}_{k'}] = 0$, $[\hat{\chi}_k, \hat{\chi}_{k'}^\dagger] = \delta_{k,k'}$, $[\hat{\phi}_k, \hat{\phi}_{k'}] = 0$, and $[\hat{\phi}_k, \hat{\phi}_{k'}^\dagger] = \delta_{k,k'}$, where $\delta_{k,k'}$ is the Kronecker Delta. E_c is the cavity-mode energy, which is given by Eq. [1.23](#). $E_x(k)$ corresponds to the in-plane dispersion of excitons is given by

$$E_x = E_x(0) + \frac{\hbar^2 k^2}{2m_{\text{exc}}}, \quad (1.26)$$

where m_{exc} is the free in-plane exciton mass. As exciton-polaritons are the normal modes of the exciton-photon coupled system, the exciton-polariton operator at a momentum k can be written as the superposition of the excitons and cavity photons as

$$\hat{\Psi}_k = A_k \hat{\chi}_k + B_k \hat{\phi}_k, \quad (1.27)$$

where

$$|A_k|^2 + |B_k|^2 = 1. \quad (1.28)$$

Now, with the exciton-polariton operator, the above Eq. (1.25) can be diagonalized in the following form

$$\hat{H} = \sum_k E_k \hat{\Psi}_k^\dagger \hat{\Psi}_k = \sum_k \hat{H}_k, \quad (1.29)$$

$$\therefore [\hat{\Psi}_k, \hat{H}_k] = E_k \hat{\Psi}_k = E_k (A_k \hat{\chi}_k + B_k \hat{\phi}_k). \quad (1.30)$$

From Eqs. (1.25) and (1.27) we have

$$[\hat{\Psi}_k, \hat{H}_k] = A_k E_x(k) \hat{\chi}_k + A_k V \hat{\phi}_k + B_k E_c(k) \hat{\phi}_k + B_k V \hat{\chi}_k. \quad (1.31)$$

Physically Eqs. (1.30) and (1.31) are the same and they can be also expressed in the following matrix form

$$\begin{pmatrix} E_x(k) - E_k & V \\ V & E_c(k) - E_k \end{pmatrix} \begin{pmatrix} A_k \\ B_k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (1.32)$$

which gives the eigen-mode of this coupled cavity-exciton system as

$$E_k = \frac{1}{2} \left(E_c(k) + E_x(k) \pm \sqrt{(E_c(k) - E_x(k))^2 + 4V^2} \right). \quad (1.33)$$

It states that we have two new normal modes in the systems: the states corresponding to higher energy is known as the upper polariton branch (UP) and the states corresponding to the lower energy is called the lower polariton brunch (LP). From Eq. (1.32) we get

$$(E_x(k) - E_k) A_k = -V B_k = -V \sqrt{1 - |A_k|^2}, \quad (1.34)$$

$$\therefore |A_k|^2 = \frac{V^2}{V^2 + (E_x(k) - E_k)^2} = \frac{1}{2} \left(1 + \frac{\Delta E(k)}{\sqrt{\Delta E(k)^2 + 4V^2}} \right), \quad (1.35)$$

which is called the exciton fraction and similarly, the photon fraction is given by

$$|B_k|^2 = \frac{V^2}{V^2 + (E_c(k) - E_k)^2} = \frac{1}{2} \left(1 - \frac{\Delta E(k)}{\sqrt{\Delta E(k)^2 + 4V^2}} \right), \quad (1.36)$$

where $\Delta E(k) = E_c(k) - E_x(k)$. The quantities $|A_k|^2$ and $|B_k|^2$ are also known as Hopfield coefficients [42]. The wave-functions of the two branches are given by

$$\begin{aligned} \hat{\Psi}_{LP,k} &= A_k \hat{\chi}_k + B_k \hat{\phi}_k, \\ \hat{\Psi}_{UP,k} &= -B_k \hat{\chi}_k + A_k \hat{\phi}_k. \end{aligned} \quad (1.37)$$

In Fig. 1.6 we plot energy (E) as a function of the transverse wavevector (k) of the exciton, cavity-photon, LP and UP modes. The Hopfield coefficients

Figure 1.6: Dispersion of the cavity, exciton, lower polariton (LP) and upper polariton (UP) are plotted in (a,c,e) for different detuning, δ and their Hopfield coefficients corresponding to the LP branch are plotted in (b,d,f) where $|A_k|^2$ is the exciton fraction and $|B_k|^2$ is the photon fraction. Parameters used for effective mass of cavity and exciton are $5 \times 10^{-5}m_e$ and $0.3m_e$, respectively, where m_e is the free electron mass. $V = 20$ meV. $\delta = -10$ meV in (a) and (b); $\delta = 0$ in (c) and (d); $\delta = 10$ meV in (e) and (f). In (g), the arrow indicates the Rabi splitting at resonance and the upper and lower polariton dispersions show anti-crossing behaviour, which is the characteristic of the strong coupling.

corresponding to the LP branch are also shown in the right panel of Fig. 1.6. The LP branch near $k = 0$ is photon-like and has extremely light effective mass whereas for large k , it has a flatter dispersion, which is exciton-like with relatively heavy effective mass. The exciton-photon detuning is given by

$$\delta = \Delta E(k = 0) = E_c(k = 0) - E_x(k = 0). \quad (1.38)$$

Depending on the sign of δ , we can define three cases; red detuned ($\delta < 0$), zero detuned ($\delta = 0$), and blue detuned ($\delta > 0$). Fig. 1.6(b) shows that for red detuning, the photon fraction of the lower polariton branch is dominant (near $k = 0$), while for blue detuning (see Fig. 1.6(f)) the exciton fraction is larger. For zero detuning, i.e., $\delta = 0$, $|A_k|^2 = |B_k|^2 = 1/2$ and the LP and UP are exactly half photon and half exciton like (at $k = 0$). When the bare cavity and exciton are at resonance ($E_c = E_x$), the splitting in the energy of the upper polariton and lower polariton modes is $2V$ (from Eq. 1.33), which is known as the Rabi splitting (see Fig. 1.6(c,g)). Due to coupling between the exciton and photons, new modes anticross, which is the signature of the strong coupling regime. This regime is analogous to that of classical coupled harmonic oscillators (see Appendix A).

1.4.2 The effective mass of the polariton

Similar to the cavity modes, we can also define the effective mass of the upper and lower polaritons [2, 43]. For the small wavevectors k , which follows $\hbar^2 k^2 / (2m_{\text{cav}}) \ll 2V$, the dispersion of Eq. (1.33) can be approximated to a parabolic dispersion by expanding in a Taylor series up to the second-order term as the following

$$E_k \simeq E_k(0) + \frac{\hbar^2 k^2}{2m_{\text{LP/UP}}}, \quad (1.39)$$

where m_{LP} and m_{UP} are the effective masses of the lower polariton and upper polariton, respectively, which are given by the weighted harmonic mean of the exciton and photon parts

$$\begin{aligned} \frac{1}{m_{\text{LP}}} &= \frac{|A|^2}{m_{\text{exc}}} + \frac{|B|^2}{m_{\text{cav}}}, \\ \frac{1}{m_{\text{UP}}} &= \frac{|B|^2}{m_{\text{exc}}} + \frac{|A|^2}{m_{\text{cav}}}, \end{aligned} \quad (1.40)$$

where $|A|^2$ and $|B|^2$ are the Hopfield coefficients given by Eqs. (1.35, 1.36). To recall, m_{exc} is the effective mass of the exciton center-of-mass motion, and m_{cav} is the effective mass of the cavity-photon. Since the effective mass of the exciton is much larger than that of the photon $m_{\text{exc}} \approx 10^4 m_{\text{cav}}$, the polariton masses can be approximated by

$$\begin{aligned} m_{\text{LP}}(k \sim 0) &= \frac{m_{\text{cav}}}{|B|^2} \sim 10^{-4} m_{\text{exc}}, \\ m_{\text{UP}}(k \sim 0) &= \frac{m_{\text{cav}}}{|A|^2}. \end{aligned} \quad (1.41)$$

The extremely small effective mass of lower polaritons (LP) near $k \sim 0$ helps to achieve Bose-Einstein condensation [44–46] at temperatures up to room temperature. For very large k , where $E_c(k) - E_x(k) \gg V$, the upper and lower polariton dispersions are very similar to the photon and exciton dispersions, respectively (see Fig. 1.6) and $m_{\text{LP}}(k \gg 0) \sim m_{\text{exc}}$ meaning that the effective mass of the LP is increased by four orders of magnitude.

1.4.3 Polariton decay

So far we have neglected the finite lifetime of the cavity photon and QW exciton. In available samples the lifetime of the exciton is ~ 1 ns, whereas lifetime of photon is approximately 1 – 100 ps. As a result, polariton lifetime is mainly dominated by the photon part. Polaritons decay by emitting photons from the cavity and the emitted photons carry the same properties as the internal polariton states, such as: frequency, wavevector, polarization, and phase. The typical lifetime of polariton is 1 – 100ps [47,48]. Recently much experimental effort was devoted to the implementation of long lifetime polaritons 180 – 250ps [39,49]. Due to the polariton lifetime, the Eq. 1.33 for LP branch modifies to

$$E_k = \frac{1}{2} \left(E_c(k) + E_x(k) - i\frac{\Gamma}{2} - \sqrt{\left[E_x(k) - E_c(k) - i\frac{\Gamma}{2} \right]^2 + 4V^2} \right), \quad (1.42)$$

where Γ is the linewidth of polariton, which is inversely proportional to polariton lifetime. The Rabi splitting is given by

$$\Omega_{\text{Rabi}} = \sqrt{4V^2 - \frac{\Gamma^2}{4}}. \quad (1.43)$$

When $V > \Gamma/4$, coherent energy exchange happens between photon and exciton and the system enters in the strong coupling regime. As a result, it shows anti-crossing in the new eigenmodes, which we call exciton-polaritons. In the opposite limit, $V < \Gamma/4$ is known as weak-coupling regime, where the eigen modes of the system remain cavity-photon and exciton.

1.4.4 The properties of polaritons

The properties of polaritons are inherited from their constituents (excitons and cavity photons). Due to their excitonic component, polaritons possess electric and magnetic dipole moments, which make them sensitive to electric and magnetic fields, respectively, showing effects such as the quantum confined Stark effect, Zeeman shift, etc., [50–53]. Due to their photonic component, they have low effective mass $\sim 10^{-4} - 10^{-5}m_e$ where m_e is the free electron mass. The small effective mass and the bosonic nature of polaritons allow to have Bose-Einstein condensation (BEC) at a relatively high temperature (around liquid helium temperature) compared to atomic condensates [44–46]. BEC is a quantum phenomena predicted by Satyendra Nath Bose and Albert Einstein where bosons have a macroscopic

occupation of the lowest energy state below the critical temperature, T_c , which is given by the following expression

$$T_c = \left(\frac{n}{\zeta(3/2)} \right)^{2/3} \frac{2\pi\hbar^2}{mk_B} \approx 3.3125 \frac{\hbar^2 n^{2/3}}{mk_B}, \quad (1.44)$$

where m is the mass of the boson, n is the density of the bosons, k_B is the Boltzmann constant, and ζ is the Riemann zeta function. Atomic BEC is observed around 1 nK $\sim$ 1 μ K [54–56]. While conventionally BEC is thought of as a phenomena where bosons macroscopically occupy the ground state in thermal equilibrium, several theoretical proposals have predicted non-ground state condensation obtained by resonant excitation [57, 58]. As a typical polariton mass is eight orders of magnitude lower than that of an atom, we can get BEC even up to room temperature [59–63] using different materials having relatively strong exciton binding energy, such as ZnO [64], GaN [59, 60], organic systems [61, 65], and organic-inorganic perovskites [63]. Such exciton-polariton condensates emit light spontaneously and have all the properties of laser light, such as coherence, polarization, and narrow spectrum [2]. In contrast to conventional lasing systems, polariton lasing is spontaneous emission based on stimulated scattering and does not require population inversion [66]. It has been demonstrated in the pioneering work in Refs. [67, 68]. The other consequence of the presence of the photonic component is that polaritons have short lifetime as we already discussed in subsec. 1.4.3 which makes the system non-equilibrium. To maintain a steady state of the polaritons, we need to pump the system continuously.

1.4.5 Resonant excitation

One of the popular methods of excitation is to excite the lower polariton branch with a laser in resonance (or quasi-resonant). Using this technique, we can directly inject polaritons with a particular energy set by the resonant pump and also by controlling the angle of the incident pump-laser we can tune the momentum of the injected polaritons. This scheme can directly map the phase of the pump to the phase of the created coherent state. Under the resonant excitation, in the second quantization formalism, the Hamiltonian can be written as

$$\hat{H} = \sum_k E_k \hat{\Psi}_k^\dagger \hat{\Psi}_k + \sum_{\mathbf{x}} V_{\mathbf{x}} \hat{\Psi}_{\mathbf{x}}^\dagger \hat{\Psi}_{\mathbf{x}} + \sum_k (F_k e^{-i\omega t} \hat{\Psi}_k^\dagger + F_k e^{i\omega t} \hat{\Psi}_k), \quad (1.45)$$

where E_k is the real part of the bare polariton dispersion (see Eq. 1.33) and the imaginary part, representing decay in subsec. 1.4.3, will instead be introduced as a non-Hamiltonian term following standard quantum optics [69]. The second term represents the two dimensional spatially dependent external potential where $\mathbf{x} = (x, y)$. Through the last term the coherent resonant excitation is considered with a laser of energy $\hbar\omega$. An alternative case of non-resonant excitation is considered later in subsec. 1.4.9. The field operators $\hat{\Psi}_k$ and $\hat{\Psi}_{\mathbf{x}}$ are related by the discrete Fourier transform as

$$\hat{\Psi}_k = \frac{1}{\sqrt{N}} \sum_{\mathbf{x}} \hat{\Psi}_{\mathbf{x}} e^{-ik \cdot \mathbf{x}}, \quad \hat{\Psi}_{\mathbf{x}} = \frac{1}{\sqrt{N}} \sum_k \hat{\Psi}_k e^{ik \cdot \mathbf{x}}. \quad (1.46)$$

We have discretized our space into N points, upon which we have the commutation relation $[\hat{\Psi}_{\mathbf{x}}, \hat{\Psi}_{\mathbf{x}'}^\dagger] = \delta_{\mathbf{x}, \mathbf{x}'}$. We note that our results should be independent of the choice of N ; provided sufficiently large N is chosen, our final result will converge. Now, to include the dissipation, we add a Lindblad term in the generalized Von Neumann equation as

$$i\hbar \frac{\partial \rho}{\partial t} = [\hat{H}, \rho] + \frac{i\Gamma}{2} \sum_k \left(2\hat{\Psi}_k \rho \hat{\Psi}_k^\dagger - \hat{\Psi}_k^\dagger \hat{\Psi}_k \rho - \rho \hat{\Psi}_k^\dagger \hat{\Psi}_k \right), \quad (1.47)$$

where ρ is the density matrix and Γ is the decay rate of polaritons. This equation is also known as the Liouville-Von Neumann equation (or Master equation), widely used in quantum optics [69]. Next, we define the mean-field by taking the trace of the product of the field operator and density matrix as the following

$$\langle \Psi_k \rangle = \text{tr} \left\{ \rho \hat{\Psi}_k \right\}. \quad (1.48)$$

The equation in the mean field approximation takes the form

$$i\hbar \frac{\partial \langle \Psi_k \rangle}{\partial t} = \text{tr} \left\{ [\hat{H}, \rho] \hat{\Psi}_k \right\} + \frac{i\Gamma}{2} \text{tr} \left\{ \sum_{k'} (2\hat{\Psi}_{k'} \rho \hat{\Psi}_{k'}^\dagger - \hat{\Psi}_{k'}^\dagger \hat{\Psi}_{k'} \rho - \rho \hat{\Psi}_{k'}^\dagger \hat{\Psi}_{k'}) \hat{\Psi}_k \right\}. \quad (1.49)$$

Note that, $\text{tr}\{ABC\} = \text{tr}\{BCA\} = \text{tr}\{CAB\}$, where A , B , and C are operators. Next we did some mathematics and used the expression $V_{\mathbf{x}} = (1/\sqrt{N}) \sum_{k'} V_{k'} e^{ik' \cdot \mathbf{x}}$, which takes us to the following expression

$$i\hbar \frac{\partial \langle \Psi_k \rangle}{\partial t} = \left(E_k - \frac{i\Gamma}{2} \right) \langle \Psi_k \rangle + \frac{1}{\sqrt{N}} \sum_{k'} V_{k'} \langle \Psi_{k-k'} \rangle + F_k e^{-i\omega t}. \quad (1.50)$$

In real space, this can be written as

$$i\hbar \frac{\partial \langle \Psi_{\mathbf{x}} \rangle}{\partial t} = \frac{1}{\sqrt{N}} \sum_{\mathbf{x}'} E_{\mathbf{x}'} \langle \Psi_{\mathbf{x}-\mathbf{x}'} \rangle + \left(V_{\mathbf{x}} - \frac{i\Gamma}{2} \right) \langle \Psi_{\mathbf{x}} \rangle + F_{\mathbf{x}} e^{-i\omega t}, \quad (1.51)$$

where $F_{\mathbf{x}} = \frac{1}{\sqrt{N}} \sum_k F_k e^{ik \cdot \mathbf{x}}$ is the pump profile in real space. The first term can be written as $\frac{1}{\sqrt{N}} \sum_{\mathbf{x}'} E_{\mathbf{x}'} \langle \psi_{\mathbf{x}-\mathbf{x}'} \rangle = \hat{E} \langle \psi_{\mathbf{x}} \rangle$. By replacing $\langle \Psi_{\mathbf{x}} \rangle = \psi(\mathbf{x})$, the above equation can be written in a continuous space as

$$i\hbar \frac{\partial \psi(\mathbf{x})}{\partial t} = \left(\hat{E} - \frac{i\Gamma}{2} + V(\mathbf{x}) \right) \psi(\mathbf{x}) + F(\mathbf{x}) e^{-i\omega t}. \quad (1.52)$$

Since the polariton dispersion near $k \sim 0$ is parabolic, we take the energy operator $\hat{E} = -\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} = -\frac{\hbar^2}{2m_{\text{LP}}} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$.

1.4.6 Polariton pseudospin

To recall, photons have angular momentum ± 1 , corresponding to left and right circularly polarized light. Since in the photoabsorption process, the spin is conserved [70], bright excitons with $J_z = \pm 1$ couple with photons of the same spin,

which furthermore results in polaritons with the same spin, $\sigma_{\pm} = \pm 1$. Linearly polarized light excites linear combinations of $J_z = \pm 1$ excitons and creates linearly polarized exciton-polaritons [71]. Thus with the polarized optical excitation, we can control the spin orientation of the polaritons [72]. Since the dark excitons are optically inactive, we neglect them in this thesis, which allows us to treat exciton-polaritons as a two level system in the pseudospin formalism [73]. A polariton state with a given wavevector k can be described by the 2×2 density matrix ρ_k analogous to the case of electron spins as the following [74, 75]

$$\rho_k = \frac{N_k}{2} I + S_k \cdot \sigma, \quad (1.53)$$

where N_k is the total number of polaritons having wavevector k and I is the identity matrix. $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (1.54)$$

S_k is the pseudospin of polaritons with wavevector k , which determines its spin state. The pseudospin component corresponds directly to the Stokes parameters of the polaritons [75]. The polarization of the coherent polariton state, which is directly connected to the pseudospin components, is fully represented by a point on the Poincaré sphere (see Fig. 1.7). Any point inside the sphere corresponds to an unpolarized polariton state, while points on the surface of the sphere corresponds to a definite polarization. For example, points that lie on the equator of the sphere are linearly polarized, the upper and lower poles correspond to the right-handed and the left-handed circularly polarized states ($\sigma_{\pm}$), respectively, and any other point on the surface of the sphere corresponds to the general case of elliptical polarization. When the orientation of the pseudospin is parallel/anti-parallel to the x-axis of the sphere, it corresponds to linearly X-/Y- (also represented as H/V) polarized polaritons, which are characterized by the quantity s_x . The pseudospin vector orientated along the y-axis corresponds to diagonal (D) or antidiagonal (A) polarized polaritons, is described by the quantity s_y , while orientation of pseudospin along the z-axis corresponds to circular polarization which can be expressed by s_z . If $I_{H,V}$ represents intensity of the coherent polariton state in horizontal (H) and vertical (V) polarizations; $I_{D,A}$ is the intensity in the diagonal (D) and antidiagonal (A) polarizations, and I_{σ_+, σ_-} represents intensity corresponding to the two circular polarization components σ_+ and σ_- , then the Stokes parameters are given by

$$s_x = \frac{I_H - I_V}{I_H + I_V}; \quad s_y = \frac{I_D - I_A}{I_D + I_A}; \quad s_z = \frac{I_{\sigma_+} - I_{\sigma_-}}{I_{\sigma_+} + I_{\sigma_-}}, \quad (1.55)$$

where $s_x = S_x/N_k$; $s_y = S_y/N_k$; $s_z = S_z/N_k$ are the normalized polarization degrees [73]. Furthermore, it should be noted that even though the subscript k notation is dropped when writing S_x , S_y , and S_z , they can still be different for different k .

Figure 1.7: The polarization of a coherent polariton state which is directly connected to pseudospin vector lies on the surface of the Poincaré sphere, as shown by the blue arrow. Black arrows correspond to different polariton polarizations: linearly polarized states lie on the equatorial plane whereas circular polarized states ($\sigma_{\pm}$) lie on the two poles.

1.4.7 TE-TM splitting

In microcavities, there exists TE-TM splitting, i.e., splitting between states with polarization parallel to wavevector $\vec{k}$ (transverse magnetic TM modes) and states with polarization perpendicular to $\vec{k}$ (transverse electric TE modes) due to different reasons :

- In two dimensional microcavities, the TE modes have polarization lying always in the ‘layer’s plane’, i.e., (x,y) plane (see Fig. 1.8 (a)), while TM modes have only part of their polarization projected on the layer’s plane with an angle θ for oblique incidence. Due to different boundary conditions, the TE and TM polarized cavity modes have different reflectivity, which ultimately results in an energy splitting between these modes [76]. This effect can be incorporated by defining different effective cavity mass for TE and TM modes as

$$E_c^{\text{TE/TM}} = E_c(0) + \frac{\hbar^2 k^2}{2m_c^{\text{TE/TM}}}. \quad (1.56)$$

The splitting is zero at $k = 0$ and grows quadratically with k as shown in Fig. 1.8 (b). This splitting of the cavity TE-TM modes results in a splitting of the TE-TM modes of the polaritons.

- Another origin is the long range electron-hole exchange interaction, which splits the longitudinal and transverse (LT) mode of the exciton inside the microcavity [77, 78]. Excitons having dipole moments perpendicular (TE)

and parallel (TM) to the wavevector have different energy. This splitting is also k dependent [79].

- The exciton-photon coupling strength, which depends on in-plane wavevector k (therefore incident angle) also plays a role here. Since the TM mode projects on the layer plane with $\cos \theta$, where θ is the angle of incidence, the exciton-photon coupling strength for the TE mode is larger than that for the TM mode [76].

We can write down the effective Hamiltonian of the lower polariton branch in the parabolic approximation in the linear polarization basis (ψ_{TM}, ψ_{TE}) by defining different effective mass for TE and TM modes of the polaritons due to the above mentioned combined effects as the following

$$H_{lin} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m_{TM}} & 0 \\ 0 & \frac{\hbar^2 k^2}{2m_{TE}} \end{pmatrix}. \quad (1.57)$$

If ϕ is the in-plane polar angle as shown in Fig. 1.8 (c), then the transformation matrix between the linear and circular polarization basis is given by

$$\begin{pmatrix} \psi_{TM} \\ \psi_{TE} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} \psi_{\sigma_+} \\ \psi_{\sigma_-} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\phi} & e^{-i\phi} \\ -ie^{i\phi} & ie^{-i\phi} \end{pmatrix} \begin{pmatrix} \psi_{\sigma_+} \\ \psi_{\sigma_-} \end{pmatrix}. \quad (1.58)$$

Using this, the Hamiltonian in the circular polarization basis can be represented by

$$H_{\sigma} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m_{LP}} & \Delta_T k^2 e^{-2i\phi} \\ \Delta_T k^2 e^{2i\phi} & \frac{\hbar^2 k^2}{2m_{LP}} \end{pmatrix}, \quad (1.59)$$

where m_{LP} is the updated polariton effective mass taking into account the TE-TM splitting and Δ_T is the TE-TM coupling strength, which are given by

$$\frac{1}{m_{LP}} = \frac{1}{2} \left(\frac{1}{m_{TM}} + \frac{1}{m_{TE}} \right) \quad \text{and} \quad \Delta_T = \frac{\hbar^2}{4} \left(\frac{1}{m_{TM}} - \frac{1}{m_{TE}} \right). \quad (1.60)$$

The Hamiltonian in Eq. 1.59 can be written as $H_{\sigma} = \frac{\hbar^2 k^2}{2m_{LP}} I + \hbar \sigma \cdot \mathbf{\Omega}_{\mathbf{k}}$, where σ is the Pauli vector. $\mathbf{\Omega}_{\mathbf{k}}$ is identified as an effective magnetic field, which lies in the plane of the microcavity and is given by

$$\mathbf{\Omega}_{\mathbf{k}} = \Omega_0 k^2 (\cos 2\phi, \sin 2\phi) = \Omega_0 (k_x^2 - k_y^2, 2k_x k_y), \quad (1.61)$$

where $\Delta_T/\hbar = \Omega_0$, $\cos \phi = \frac{k_x}{k}$ and $\sin \phi = \frac{k_y}{k}$. Finally, the Hamiltonian takes the form

$$\begin{aligned} H_{\sigma} &= \frac{\hbar^2 k^2}{2m_{LP}} I + \hbar \Omega_0 \begin{pmatrix} 0 & k_x^2 - k_y^2 - i2k_x k_y \\ k_x^2 - k_y^2 + i2k_x k_y & 0 \end{pmatrix} \\ &= \frac{\hbar^2 k^2}{2m_{LP}} I + \hbar \Omega_0 \begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix}. \end{aligned} \quad (1.62)$$

Figure 1.8: (a) At oblique incidence, TE polarized modes are polarized parallel to the ‘layer’s plane’, i.e., (x,y) plane and TM polarized modes make an angle θ with the layer’s plane. (b) TE-TM splitting of the cavity modes arises due to polarization dependent reflection in the DBRs. (c) A schematic diagram where the TE-TM mode is rotated by an polar angle ϕ with respect to a frame of reference fixed in the microcavity plane. (d) Orientation of the effective magnetic field in reciprocal space caused by TE-TM splitting is shown by red arrows, which depends on the in-plane wavevector. (e) The polarization of the polaritons without the affecting magnetic field is shown by green arrows. All polaritons are linear polarized along the x axis. The red arrows are the orientation of the effective magnetic field. (f) Due to the effective magnetic field, the initial linearly polarized pseudospin converted into σ_+ and σ_- polaritons in alternative quadrants. Fig (f) is taken from Ref. [81]

The orientation of the effective magnetic field, $\mathbf{\Omega}_k$, in reciprocal space is shown in Fig. 1.8 (d) which corresponds to Eq. 1.61. In presence of this effective magnetic field $\mathbf{\Omega}_k$, the dynamics of the polariton pseudospin (S_k , discussed in Eq. 1.53) can be described by [73]

$$\frac{dS_k}{dt} = S_k \times \mathbf{\Omega}_k. \quad (1.63)$$

If we start with linear horizontal polarization (indicated by green arrows) as shown in Fig. 1.8 (e), then due to the precession of the polariton pseudospin around this effective magnetic field, we obtain the separation of the polariton polarizations in space, which is known as the optical spin Hall effect (see Fig. 1.8 (f)). From Fig. 1.8 (e) it is clear that for $\phi = (0, \pi/2, \pi, 3\pi/2)$ the effective magnetic field has no effect on the pseudospins as they are (in the considered case) initiated parallel or anti-parallel. But for $\phi = (\pi/4, 3\pi/4, \pi, 5\pi/4, 7\pi/4)$ pseudospins are perpendicular to the effective magnetic field, which allows them to align parallel with the z axis in the first and third quadrants and anti parallel with the z axis in the second and fourth quadrants. So we have the emission of alternating σ_+ or σ_- circularly polarized polaritons in the microcavity. The optical spin Hall effect was first proposed by Kavokin et al. [80] and experimentally demonstrated in 2007 by Leyder et al. [81]. Including the TE-TM splitting the Eq. 1.52 will be modified as

$$i\hbar \frac{\partial \psi_{\sigma_{\pm}}(\mathbf{x})}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{LP}} - \frac{i\Gamma}{2} + V(\mathbf{x}) \right) \psi_{\sigma_{\pm}}(\mathbf{x}) + \hbar\Omega_0 (i\sigma_{\pm} \partial_x + \partial_y)^2 \psi_{\sigma_{\mp}}(\mathbf{x}) + F_{\sigma_{\pm}}(\mathbf{x}) e^{-i\omega t}, \quad (1.64)$$

where $\hbar\Omega_0 = \Delta_T$, is the strength of the TE-TM splitting. $\psi_{\sigma_{\pm}}$ are the wave-functions for $\sigma_{\pm}$ spin polarized polaritons. $F_{\sigma_{\pm}}$ denotes the coherent driving field with different spin components. We have used this equation in Chapter 4 to obtain a feedback-free optical spin filter.

1.4.8 Nonlinearity: polariton-polariton interactions

Classically the polarizability of a medium is given by

$$P(t) = \epsilon_0 [\chi^{(1)} E(t) + \chi^{(2)} E^2(t) + \chi^{(3)} E^3(t) + \dots], \quad (1.65)$$

where E is the applied electric field and ϵ_0 is the permittivity of free space. $\chi^{(1)}$ is the linear response of the medium whereas other higher-order terms are the nonlinear response. Here we focus on Zinc Blende structures (such as GaAs, InGaAs, etc.) which are centrosymmetric crystals possessing $\chi^{(2)} = 0$. Then the first non-vanishing nonlinear term is $\chi^{(3)}$, known as Kerr nonlinearity [82].

The main origin of the polariton-polariton interaction is the exciton-exciton scattering due to Coulomb interaction [83], which involves the exchange of an electron or a hole, given by [84]

$$\hat{H}_{\text{exc-exc}} = \frac{1}{2} \sum_{k, k', q} v_{\text{exc-exc}}(k, k', q) \hat{\chi}_{k+q}^{\dagger} \hat{\chi}_{k'-q}^{\dagger} \hat{\chi}_{k'} \hat{\chi}_k. \quad (1.66)$$

Here two excitons exchange momentum is $\hbar q$ and typically the momentum exchange wavevector q is much smaller than the inverse of the two-dimensional exciton radius ($qa_B^{2D} \ll 1$). In the zero-momentum limit, the interaction term is given by [84, 85]

$$v_{\text{exc-exc}} = \frac{6E_B^{2D}(a_B^{2D})^2}{S}, \quad (1.67)$$

where a_B^{2D} and E_B^{2D} are the two dimensional exciton Bohr radius and the binding energy, respectively. S is the occupied surface by the excitons. For polaritons, this interaction strength is modified by the excitonic fraction

$$v_{\text{LP}} = |A|^4 v_{\text{exc-exc}}. \quad (1.68)$$

In the mean field approximation, nonlinearity modifies the Eq. 1.52 as the following (see appendix B)

$$i\hbar \frac{\partial \psi(\mathbf{x})}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} - \frac{i\Gamma}{2} + V(\mathbf{x}) + \alpha_1 |\psi(\mathbf{x})|^2 \right) \psi(\mathbf{x}) + F(\mathbf{x}) e^{-i\omega t}, \quad (1.69)$$

where α_1 is the interaction constant between polaritons and given by $\alpha_1 = v_{\text{LP}} S$. This equation is known as the Gross-Pitaevskii equation (GPe), or the non-linear Schrödinger equation [86, 87]. The theoretical prediction of the polariton-polariton interaction is $\sim 3\mu\text{eV}\mu\text{m}^2$ [84] and experimentally estimated in micropillars $\sim 9\mu\text{eV}\mu\text{m}^2$ [88]. A recent study showing interaction strength around $0.1 \mu\text{eV}\mu\text{m}^2$ has also appeared [89].

In the paper by Cuiti et al. [84], it was shown that the scattering between excitons is spin dependent: the interaction between excitons of parallel spins is repulsive, on the other hand the scattering between anti-parallel excitons is attractive in nature. In the spinor basis (i.e., circular polarization basis $\sigma_{\pm}$) the Eq. 1.69 takes the form

$$i\hbar \frac{\partial \psi_{\sigma_{\pm}}(\mathbf{x})}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} - \frac{i\Gamma}{2} + V(\mathbf{x}) + \alpha_1 |\psi_{\sigma_{\pm}}(\mathbf{x})|^2 + \alpha_2 |\psi_{\sigma_{\mp}}(\mathbf{x})|^2 \right) \psi_{\sigma_{\pm}}(\mathbf{x}) + F_{\sigma_{\pm}}(\mathbf{x}) e^{-i\omega t}. \quad (1.70)$$

To recall, $\psi_{\sigma_{\pm}}$ correspond to the exciton-polariton wave-functions having two circular polarization components $\sigma_{\pm}$. The interaction strength between polaritons of parallel spin is represented by α_1 and α_2 represents the strength of interactions between polaritons with antiparallel spin. Usually the attractive interaction is weaker than the repulsive one [73, 83] which makes the polariton a repulsive interactive fluid. The work by Vladimirova et al. has shown that the ratio of α_2/α_1 depends on the detuning of exciton and photon modes and they reach to the regime where $\alpha_2/\alpha_1 \lesssim -1$ [90].

Polaritons show a number of nonlinear effects such as: superfluid behaviour [91–93], formation of quantized vortices [94–96], solitons [97–102], spatial nontrivial patterns [103–107], bistability [108–112], optical parametric scattering [113–116], etc.

1.4.9 Non-resonant excitation

In the previous section [1.4.5](#), a coherent excitation of polaritons was considered corresponding to their direct excitation by a near resonant laser. An alternative, and commonly used method of excitation is non-resonant excitation. Initially the energy of the excitation (which can be done either by optically or by electrical injection [\[117,118\]](#)) is set at a higher energy than dispersion of the LP branch (~ 100 meV), which creates free electron-hole pairs in the QWs (see Fig. [1.9\(a\)](#)). These free carriers then scatter with longitudinal-optical (LO) phonons (< 1 ps) [\[119,120\]](#) followed by slower acoustic phonons (~ 1 ns) and relax down toward high momentum exciton states on the exciton dispersion [\[121\]](#). Here Coulomb interaction binds them and forms an exciton reservoir. Since their momentum is larger than the momentum of light in the medium, they do not couple to light and are long-lived [\[121\]](#). Further, through the emission of acoustic phonons, excitons then relax down from the reservoir to still high wavevectors on the lower polariton branch. Now they enter into the lightcone and couple with light to form polaritons. These polaritons populate the LP branch of large wavevector and cannot further relax down to bottom of the LP dispersion by emitting acoustic phonons because of the *bottleneck effect*, which occurs due to the dramatic change of the effective mass of polaritons [\[122,123\]](#). In this region the effective mass of the polariton changes from nearly the exciton mass to twice the photon mass as discussed in subsec. [1.4.2](#), which decreases the density of states of the LP branch by four orders of magnitude and changes the lifetime from nearly that of the exciton to a value closer to the photon lifetime (~ 10 ps) [\[124\]](#). In contrast, scattering with acoustic phonons needs ~ 50 ps, which is longer than the polariton lifetime for decreasing k [\[25\]](#) and eventually polaritons accumulate in the high momentum LP dispersion [\[85\]](#). But this effect can be overcome by increasing the excitation power, which increases the polariton density in the bottleneck region and inter-particle scattering effects such as exciton-exciton scatterings become significant [\[124,125\]](#). In this (2-body) elastic interaction, one polariton loses energy and momentum and relaxes down toward the ground state while another one gains energy and momentum and scatters to a higher energy state in the LP dispersion [\[124\]](#). This is similar to the optical parametric oscillations (OPO) scheme (discussed in subsection [1.4.10](#)). This excited polariton then relaxes down via phonon emission [\[47\]](#). When the polariton population in the ground state of the LP branch exceeds an order of unity, the relaxation is enhanced by bosonic stimulation, as polaritons are bosonic in nature [\[85,124\]](#). When the scattering rate is higher than the escape rate of polaritons from the cavity, they occupy macroscopically the ground state. These polaritons show long range coherence and behave as a Bose-Einstein condensate (see Fig. [1.9\(b\)](#)). Such condensate is typically out of thermal equilibrium as it forms via a balance of the excitons relaxing toward condensate and photons escaping from the cavity. Several groups reported polariton BEC [\[45\]](#), even up to room temperature in wide bandgap organic and inorganic semiconductors, which exhibit strong exciton binding energy [\[59,61,126\]](#). To describe the dynamics of the polaritons, we couple the mean field equation for polariton condensate to a rate equation for the reservoir of excitons as follows (here for simplicity we neglect

the spin degree of freedom, equivalent to exciting a single spin component) [127]

$$\begin{aligned}
i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t} = & \left(-\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} + V(\mathbf{x}) + \alpha_1 |\psi(\mathbf{x}, t)|^2 + 2g_R n_R(\mathbf{x}, t) \right. \\
& \left. + \frac{i\hbar}{2} (R n_R(\mathbf{x}, t) - \gamma) \right) \psi(\mathbf{x}, t), \\
\frac{\partial n_R(\mathbf{x}, t)}{\partial t} = & P_R(\mathbf{x}) - (\gamma_R + R |\psi(\mathbf{x}, t)|^2) n_R(\mathbf{x}, t),
\end{aligned} \tag{1.71}$$

where the exciton-density in the reservoir is given by n_R and $P_R(\mathbf{x})$ represents the non resonant pump, which gives the pumping rate of the reservoir. α_1 is the strength of the polariton polariton interaction. The interaction between polaritons and the exciton-reservoir is given by g_R , which is repulsive in nature. R determines the exciton-polariton condensation growth rate, i.e., the rate at which polaritons relax down from reservoir to condensate by stimulated scattering. γ and γ_R are the decay rates of the condensate of polaritons and the reservoir, respectively. In the limit of fast reservoir evolution, the condensate sees a reservoir with a constant density n_R given by

$$n_R = \frac{P_R}{\gamma_R + R |\psi|^2} = \frac{P_R}{\gamma_R (1 + \frac{R}{\gamma_R} |\psi|^2)} \sim \frac{P_R}{\gamma_R} - \frac{P_R R}{\gamma_R^2} |\psi|^2. \tag{1.72}$$

Substituting n_R in the equation for polariton condensate in Eq. [1.71], we obtain

$$\begin{aligned}
i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t} = & \left(-\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} + V(\mathbf{x}) + \alpha_1 |\psi(\mathbf{x}, t)|^2 + 2g_R \left\{ \frac{P_R}{\gamma_R} - \frac{P_R R}{\gamma_R^2} |\psi(\mathbf{x}, t)|^2 \right\} \right. \\
& \left. + \frac{i\hbar}{2} \left[R \left\{ \frac{P_R}{\gamma_R} - \frac{P_R R}{\gamma_R^2} |\psi(\mathbf{x}, t)|^2 \right\} - \gamma \right] \right) \psi(\mathbf{x}, t).
\end{aligned} \tag{1.73}$$

Considering the following transformation

$$\frac{2g_R P_R}{\gamma_R} \rightarrow GP, \quad \alpha_1 - \frac{2g_R P_R R}{\gamma_R^2} \rightarrow \alpha_1, \quad \frac{\hbar R P_R}{2\gamma_R} \rightarrow P, \quad \frac{\hbar P_R R^2}{2\gamma_R^2} \rightarrow \alpha_{NL}, \tag{1.74}$$

the Eq. [1.73] reduces to the following equation

$$\begin{aligned}
i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t} = & \left(-\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} - \frac{i\Gamma}{2} + V(\mathbf{x}) + \alpha_1 |\psi(\mathbf{x}, t)|^2 \right) \psi(\mathbf{x}, t) + iP(\mathbf{x})\psi(\mathbf{x}, t) \\
& - i\alpha_{NL} |\psi(\mathbf{x}, t)|^2 \psi(\mathbf{x}, t) + GP(\mathbf{x})\psi(\mathbf{x}, t),
\end{aligned} \tag{1.75}$$

where $P(\mathbf{x})$ is the effective non resonant pump applied to the system, added as a non Hermitian term to represent the gain in the system. α_{NL} corresponds to the gain saturation caused by depletion of the excitonic reservoir [128]. The last term corresponds to the additional pump-induced shift by the reservoir [129]. To recall $\Gamma = \hbar\gamma$ corresponds to dissipation of polaritons.

Figure 1.9: (a) Incoherent pumping scheme. Initially the system is subjected to an incoherent excitation, which is tuned at a very high energy and injects free carriers. These carriers then relax down to lower polariton states, possibly the ground state, by phonon emission and stimulated scattering. (b) Theoretical images of the polariton condensation observed with increasing density of the exciton reservoir. Polaritons are distributed in energy and momentum below threshold. Above threshold the polaritons start to occupy the ground state macroscopically and forms a condensate. Fig (a) is taken from [130] and (b) is taken from [131].

1.4.10 Optical Parametric Scattering

Due to the non-parabolic behavior of the lower polariton (LP) branch, when polaritons are continuously injected at the inflection point (the point in the LP dispersion from which the effective mass starts to change its sign) in the LP branch, known as ‘magic angle’, they undergo coherent scattering. Two polaritons from this pump state having energy and wavevector (E_p, k_p) scatter elastically, one of them goes to a signal state (E_s, k_s) by losing energy and momentum, while the other one by gaining energy and momentum scatters to the idler state (E_i, k_i) as shown Fig. 1.10(a). This scattering process is known as polariton parametric scattering and it arises due to the polariton-polariton interactions. It conserves both energy and momentum (giving a phase-matching condition) that can be written as

$$2k_p = k_s + k_i, \quad (1.76)$$

$$2E_p = E_s + E_i. \quad (1.77)$$

This process was first experimentally observed in an InGaAs/GaAs based microcavity in 2000 by Savvidis et al. [113] in a pump probe experiment known as

optical parametric amplification (OPA). In this experiment, polaritons are excited resonantly with magic angle and a probe pulse was directed normally to the plane as shown in Fig. 1.10 (b-c). The result showed that the intensity of the normally reflected light increases by two orders of magnitude compared to the case when there is no probe pulse. This corresponds to a significant gain in the polariton parametric scattering. This can be explained as follows: due to the polariton parametric scattering, one polariton goes to the ground state (where the probe pulse is directed) while the other one goes to a higher state with $k_i = 2k_p$ (k_p is the wavevector of the pump) and due to the bosonic nature of the polaritons, the probe pulse triggers the scattering. This experiment is the evidence of the stimulated scattering process and more generally of the presence of nonlinear polariton-polariton interactions. These results were theoretically modeled shortly after by Ciuti et al. [132]. When the pump power is strong enough, i.e., above the threshold, which depends on the polariton sample [133], a new regime appears called optical parametric oscillator (OPO) where signal and idler modes are macroscopically occupied even in the absence of a probe pulse and emit coherent signal and idler beams. The OPO regime in microcavities was experimentally reported by several groups: Baumberg et al. [115], Stevenson et al. [134], Tartakovskii et al. [135]. In general, in the OPO regime three states (pump, signal, idler) are getting populated [116, 136]. Later in some experiments and numerical simulations some additional peaks are also observed [137–139] as schematically shown in Fig. 1.10(d). These satellite states arise due to further parametric scattering; they are equally spaced in energy and momentum [3, 137–140]. If the system is subjected to the pump of frequency ω_p , while the signal and idler emissions are occurring at ω_s and ω_i respectively, then the first satellite states (s_2, i_2) will be visible at frequency $\omega_{s_2} = 2\omega_s - \omega_p$ and $\omega_{i_2} = 2\omega_i - \omega_p$ due to secondary parametric scattering. Above the threshold, the population of the signal and idler modes are not negligible, as a result further parametric scattering takes place from signal (idler) states to the second-signal s_2 (second-idler i_2) satellite state and pump state p i.e., $2s \rightarrow s_2 + p$ ($2i \rightarrow i_2 + p$). Note that with the increasing order of the parametric scattering, the population of the satellite states are becoming smaller (i.e., s_2 population is much weaker than s). A more comprehensive review can be found in Ref. [140].

Recent studies showed that optical parametric scattering is also possible under different scattering configurations without injecting polaritons near the inflection point, for example, in a triple-microcavity structure (see Fig. 1.10 (e)), where the middle energy mode was pumped and a strong signal appeared in the higher and lower energy modes [141]. In Leyder et al., the cavity was pumped by two continuous wave lasers having the same energy but opposite in-plane wavevectors and the emission was observed in the orthogonal direction as shown in Fig. 1.10 (f) [8].

Later in the Chapter 3 we will use the effect of multiple parametric scattering to show the coupling between two topologically protected corner modes.

Figure 1.10: (a) Schematic diagram of polariton parametric scattering where two pump polaritons scatter into a signal and a idler state by conserving energy and momentum. (b) The dispersion of polariton for incidence angle of pump θ at zero detuning. The probe polariton (○) stimulating the parametric scattering of the pump polariton (●). (c) The scheme of the experiment of the Ref. [113]. (d) OPO spectrum above the threshold. Dispersions of the exciton and cavity photon are plotted in gray whereas dispersion of polaritons are plotted in blue. Polaritons scatter to signal and idler which further scatter to additional satellite states s_2 and i_2 . (e) Triple-microcavity structure. (f) Schematic diagram of elastic scattering between two pumped polaritons having equal energy but opposite wavevector which scatter to the orthogonal directions as shown in the figure. Figs. (b-c), (e), (f) are reproduced from [113], [141], and [8], respectively.

1.4.11 Bistability

An important feature of the quasi-resonant excitation is bistability where the mode must exist in one of two distinct stable states, which have different intensities [73, 142]. If we drive the system with a coherent continuous-wave pump of the form $F e^{-i\omega_p t}$ (i.e., normal incidence), then in absence of any external potential, the nonlinear Gross-Pitaevskii equation can be written as (neglecting spin)

$$i\hbar \frac{\partial \psi}{\partial t} = \left(E_0 - \frac{i\Gamma}{2} + \alpha_1 |\psi|^2 \right) \psi + F e^{-i\omega_p t}, \quad (1.78)$$

where E_0 is the bare lower polariton energy. Inserting the ansatz $\psi = \Psi e^{-i\omega_p t}$, the Eq. 1.78 takes the form

$$i\hbar \frac{\partial \Psi}{\partial t} = \left(E_0 - \hbar\omega_p - \frac{i\Gamma}{2} + \alpha_1 |\Psi|^2 \right) \Psi + F = \left(-\Delta - \frac{i\Gamma}{2} + \alpha_1 |\Psi|^2 \right) \Psi + F, \quad (1.79)$$

where $\Delta = \hbar\omega_p - E_0$ defines the energy detuning between the the pump and the polariton mode. Assuming spatial homogeneity, the stationary solution of this equation gives rise to a cubic equation

$$\left[(-\Delta + \alpha_1 n)^2 + \frac{\Gamma^2}{4} \right] n = P_0, \quad (1.80)$$

where $n = |\Psi|^2$ and $P_0 = |F|^2$ are the density of the polaritons and intensity of the pump, respectively. In Fig 1.11(a-c) we have plotted the dependence of polariton intensities (n) on the pump intensity (P_0) for three cases ($\Delta < 0$, $\Delta = 0$, $\Delta > 0$) and it clearly shows that the last case ($\Delta > 0$) yields bistability in the system. To get more insight, we calculate the condition $\frac{\partial P_0}{\partial n} = 0$, which gives

$$n = n_t = \frac{4\Delta \pm \sqrt{4\Delta^2 - 3\Gamma^2}}{6\alpha_1}. \quad (1.81)$$

Here n_t denotes the turning points in the curve. To yield bistability, the system should satisfy the condition

$$\Delta > \sqrt{3}\Gamma/2. \quad (1.82)$$

When the laser is excited at (or below) the bare polariton energy (see Fig 1.11(d)), initially the population of the polariton mode increases monotonically with the increase of the pump intensity. However with the increase of the pump intensity, more polaritons are added into the system and the nonlinearity induces a blueshift [143] of the polariton mode which moves the LP dispersion further away from resonance with the pump (see dashed curve in Fig 1.11(d)). As a result, it makes harder to inject polaritons into the system and starts to show a sub-linear growth of the polariton population (see Fig 1.11(a-b)); this effect is known as *optical limiter*.

This becomes a bit more interesting if we start with a slightly blue detuned laser as shown in Fig 1.11(e). In this case, initially the LP mode is out of resonance, and it is hard to add polaritons into the systems. However, if the intensity of the pump goes beyond a certain critical value, the blueshift induced by the nonlinearity takes the polariton mode in resonance with the pump. This results in a sudden jump in the polariton population at the first turning point (higher pumping power). Similarly, if we start from the upper branch and decrease the pump intensity, the population decreases and suddenly we observe a drop in the population near the second turning point (lower pumping power). This is characterized by the S shaped curve (see Fig. 1.11(c)) [116, 144]. The central branch (dashed curve in

Figure 1.11: (a) Hysteresis curve of the polariton population for (a) $\Delta = -0.5$ meV, (b) $\Delta = 0$, and (c) $\Delta = 0.5$ meV. The former two regimes are known as *optical limiter* whereas the last case shows bistability. (d-e) The schematic diagram where laser excites the polariton mode with (d) zero detuning i.e., $\Delta = 0$ (e) negative detuning i.e., $\Delta > 0$. The solid curve represents the lower polariton (LP) branch and the dashed curve is the nonlinear shift of the LP. Parameters: $\Gamma = 3$ ps and $\alpha_1 = 6 \times 10^{-3} \text{ meV } \mu\text{m}^2$.

Fig 1.11 (c) with negative slope is dynamically unstable which can be understood from the following Bogoliubov analysis.

After getting a steady-state of polaritons, to assess stability, we can study the effects of small fluctuations around the steady state and this can be done using Bogoliubov theory [145,146] of the generalized Gross-Pitaevskii equation

$$i\hbar \frac{\partial \Psi}{\partial t} = \left(-\Delta - \frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} - \frac{i\Gamma}{2} + \alpha_1 |\Psi|^2 \right) \Psi + F, \quad (1.83)$$

where m_{LP} is the mass of the polariton. We consider the case where signal and idler states are perturbatively populating from the pump state via parametric scattering processes (see Fig 1.12(a)). The three-mode ansatz for the lower polariton field can be written as [116,136]

$$\Psi = \Psi_0 + u e^{i(k \cdot \mathbf{x} - \omega t)} + v^* e^{i(-k \cdot \mathbf{x} + \omega^* t)}, \quad (1.84)$$

where u and v are the amplitudes of fluctuations and Ψ_0 is the stationary solution of Eq. 1.83. ω and k is the frequency and wavevector of the fluctuations. Here we took the in-plane wavevector of the pump zero. Substituting Eq. 1.84 into Eq. 1.83 and keeping terms up to first order in u and v , we get the following eigenvalue

equations

$$\hbar\omega \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -\Delta - \frac{i\Gamma}{2} + \frac{\hbar^2 k^2}{2m_{\text{LP}}} + 2\alpha|\Psi_0|^2 & \alpha\Psi_0^2 \\ -\alpha(\Psi_0^2)^* & \Delta - \frac{i\Gamma}{2} - \frac{\hbar^2 k^2}{2m_{\text{LP}}} - 2\alpha|\Psi_0|^2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}. \quad (1.85)$$

The dynamical stability is guaranteed by $\text{Im}(\omega) < 0$; otherwise the system shows instability, i.e., the solution grows exponentially with time. The real and imaginary parts for different steady states as shown in Fig 1.12(b) are plotted in Figs 1.12(c-i). For very low polariton populations (see Fig 1.12(c)), imaginary values of all the energies are less than zero and the states are stable. At relatively large population (see Fig 1.12(d)), fluctuations near $k = 0$ are stable while for large wavevectors instability appears, which corresponds to the fact that pump polaritons scatter to the higher wavevector states (see zoomed view Fig 1.12(e)). Note that the states corresponding to positive imaginary energy have real part of energy zero, which suggests that the parametric scattering happens to the states that have the same energy as the pump, as shown in Fig 1.12(a). At the middle branch (see Fig 1.12(f-g)) the system is unstable near $k = 0$ fluctuations. At the upper branch, the hysteresis curve is always stable for any finite wavevector fluctuations (see Fig 1.12(h-i)).

In Fig. 1.12(b) we get a region of pump intensities where two stable states exist (A and B) for the same input pump. The stable and unstable solutions meet in the two turning points are called saddle node bifurcations. The system chooses its actual state not only based on the current intensity of the pump but also depends on its history of the system (analogous to ferromagnetic materials), i.e., the system exhibits hysteresis. If we start from a vacuum state (corresponding to zero mean field) and apply a continuous pump F_a (see Fig 1.12(b)) then we will end up with a steady state ‘A’. By applying additional optical pulses one can switch between states. Bistability has been observed experimentally by different groups [12, 14, 109, 147]. Experiments report the threshold power to observe bistability as low as 2.8 mW [109].

Bogoliubov theory is also used in the assessment of superfluidity where the polariton fluid flows without friction such that the scattering with defects disappears [92]; such an effect was also shown taking into account the polarization dependence [148]. In contrast to the spinless nonlinear case, the interplay between nonlinearity and polarization dependent interaction (α_2) gives rise to polarization multistability where the intensity and polarization of the pump controls the polariton polarization and the number of stable states [149]. Finally, although in this thesis we will consider only mechanisms of bistability under resonant pumping, it is worth mentioning that mechanisms of bistability under non-resonant pumping also exist [111, 150–152]. In chapter 3 we show robust binary transformation based on bistability. In chapter 5 we will also use bistability to realize cellular automata in exciton-polariton lattices.

Figure 1.12: (a) Schematic diagram of the optical parametric scattering which can be described by Bogoliubov theory in the perturbative limit. (b) Hysteresis curve of the polariton population where the corresponding dots represent the stationary state $|\Psi_0^2|$ used for the calculation of Eq. [1.85](#). The real (solid curves) and imaginary parts (dotted curves) of the eigenenergy, $E = \hbar\omega$ corresponding the Bogoliubov spectrum is plotted in (c-i). (e) is the zoomed view of (d). The states corresponding to $\text{Im}(E) > 0$ are dynamically unstable. The gray horizontal line corresponds to $E = 0$. The color coding for different spectrum correspond to different point on the bistability curve in (b).

1.4.12 Potential engineering

Various techniques have been developed over the years to create traps in order to confine polaritons spatially. Here we briefly discuss some of the commonly used techniques; an illustrated discussion can be found in Ref [36]. In general, the potential for the polaritons can be arranged in two ways: either by trapping the photonic parts or by trapping the excitonic parts of the polaritons.

Potential induced by trapping the excitonic component.— In the following we discuss methods to localize polaritons via their excitonic part. One of the techniques is application of local strain by a sharp metal tip with a radius of $\sim 50 \text{ } \mu\text{m}$ [153]. It creates a local parabolic potential minimum for QW excitons (thus polaritons are also trapped), where the potential-depth can be several tens of meV [153, 154]. Experiments showed that after creating polaritons using non-resonant pump away from the trap ($\sim 20 \text{ } \mu\text{m}$), polaritons flow down to the potential minimum from the excitation spot and the leads to Bose-Einstein condensation of exciton-polaritons. However, this technique is not useful for having complex potential landscapes.

Figure 1.13: (a) Deep etching through top DBR, QWs, and bottom DBR to form a micropillar. The overlap controls the coupling between the adjacent pillars. (b) Micropillars are coupled through channel.

The second technique relies on Surface Acoustic Waves (SAW), where sound waves propagate along the sample [155]. It also affects the photonic component due to the change of the refractive index and physical length of the cavity but the effect is weaker than that for the exciton [155, 156].

Another popular technique is to induce a potential optically by a nonresonant pump, which creates a high energy excitonic reservoir, providing an additional

potential barrier due to strong repulsive interactions between polaritons and excitonic reservoir [13, 157]. This technique can be extended to create potentials all optically, for example ring shaped confinement can be generated using spatial light modulators [158, 159] or multiple pump spots can be used to create a 2D potential [160, 161].

Potential induced by trapping the photonic component.— Polariton localization can be created by depositing metal films on the sample surface [162]. The boundary condition of the electromagnetic field changes where metal layers are deposited with respect to the semiconductor-air interface and create an optical-node, leading to the shift in the energy of the order of 0.1-1 meV [46, 163, 164].

The most powerful technique of creating polariton traps is deep etching of microcavities. With this technique, it is possible to create arbitrary potential profiles [165–167]: from micropillars and channels to complicated structures like honeycomb lattices, Lieb lattices, etc. The typical diameter of such micropillars is 1-6 μm [36, 63, 168–170]. The high refractive index contrast between the structure ($n_{\text{GaAs}} \sim 3.5$) and its surrounding ($n_{\text{air}} \sim 1$) guarantees the confinement of the photonic mode inside the micropillars [88]. By controlling the distance between the pillars, we can control the coupling between the adjacent micropillars (see Fig. 1.13(a)). More complicated structures such as honeycomb lattices [171], Lieb lattices [172] of coupled micropillars have been developed in recent years. We can control the coupling between the micropillars also by introducing channels [173] with different dimensions as shown schematically in Fig. 1.13(b). This scheme has been extended to observe BEC of polaritons in one dimensional chains [5, 63, 174].

1.4.13 Solitons

Solitons are self-reinforcing wave-packets that propagate with a constant speed maintaining their shape [175, 176] due to the balance between the effects of dispersion and nonlinearity of the medium [177]. Solitons were first documented by John Scott Russell in a water canal in 1863 [178, 179]. The conservative form of the Gross-Pitaevskii equation (neglected driving and dissipation terms) [180],

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2 \nabla^2}{2m} \psi + g|\psi|^2 \psi, \quad (1.86)$$

admits soliton solutions [180–182] where m is the mass of the considered particles (e.g., a condensate of cold atoms), ψ is the wave-function of the condensate, and g is the strength of particle interactions. Depending on the character of interaction g , we can get *dark* (for $g > 0$, i.e., repulsive interactions) and *bright* (for $g < 0$, i.e., attractive interactions) solitons. Dark solitons are featured as a lack of local intensity with respect to the background and show a tanh wave-function profile [183, 184]. Solitons were predicted theoretically in the context of Bose-Einstein condensates (BECs) [185], and observed in non-linear optics [186] and cold-atom BECs [180]. In contrast, bright solitons are characterized by a localized peak above the background and a wave packet with sech wave-function profile [187, 188]. Since the polariton nonlinearity is much (3-4 orders of magnitude)

larger than the photon-photon interaction, solitons are very popular in exciton-polariton system. A lot of work has been done to realize dark solitons [97, 98, 189]. The lower polariton dispersion branch exhibits changing of positive effective mass of polaritons to negative effective mass around the inflection point [25, 190]. To obtain a bright soliton, polaritons are excited above the inflection point, which corresponds to negative effective mass. The interplay between negative effective mass and repulsive interaction favours the formation of bright solitons, i.e., the broadening of the wavepacket due to (repulsive) nonlinearity is compensated by negative kinetic energy [191, 192]. Another kind of soliton, known as dissipative soliton (or cavity soliton), also popular in the community, which is a spatially self-localized wave-packet, maintains its shape by compensating the loss with an external pump [193]. The overall motivation of studying these solitons has been that they are expected to be useful for information processing and building blocks for polaritonic circuits [101, 194]. The polariton-polariton interaction also gives rise to formation of patterns [103, 105, 107, 159], soliton trains [195], solitons that emit radiation in the backward direction [196], etc. These patterns are also expected to play a key role in future polaritonic circuits [159, 195, 196]. In chapter 5 we have introduced a new scheme to obtain solitons that are capable of universal computing.

Chapter 2

Realization of Hofstadter's butterfly in a polaritonic system

In this chapter we propose theoretically a scheme of exciton-polariton micropillars with polarization splitting where the coupling between sites is spin-dependent. The system can be reduced to an effective model corresponding to an effective phase dependent hopping to induce an artificial gauge field. We further show related effects such as the appearance of Hofstadter's butterfly and chiral edge states. The Hofstadter model is a prototypical model of a quantum Hall system, best-known for observation of topological effects. The unidirectional edge mode has a privilege in the optical technology. As a starting point, we review topological states. We then introduce a linear polarization splitting of exciton polaritons and later discuss our main results, which are reproduced from R. Banerjee, T. C. H. Liew, and O. Kyriienko, Phys. Rev. B **98**, 075412 (2018) [197].

2.1 Topology

The observation of the integer quantum Hall effect by Klitzing et al. in 1980 [198], where two dimensional (2D) electrons in a uniform strong perpendicular magnetic field exhibit² quantized (integer multiples of e^2/h) Hall conductance as a function of magnetic field regardless of the sample size, purity and composition, was the beginning of the new era of the topological insulators [199–201]. Later the works by Thouless et al. [202] and Kohmoto [203] relate this integer that appears in the Hall conductance to a topological invariant of the system, called the Chern number, which is an integer-valued quantity defined by the total Berry flux in the Brillouin zone. For Bloch wavefunctions (of the n th band) $|u_{n,\mathbf{k}}\rangle$, the Berry connection can be defined as

$$\mathcal{A}_n(k) = i \langle u_{n,\mathbf{k}} | \nabla_k | u_{n,\mathbf{k}} \rangle, \quad (2.1)$$

²Landau levels, quantized cyclotron orbits of different energies.

and for adiabatic displacement around a closed loop we can define the Berry phase as [204]

$$\gamma = \oint \mathcal{A}_n(k) \cdot d\mathbf{k}. \quad (2.2)$$

This can be expressed as a surface integral of the Berry flux (also called Berry curvature) $\Omega_n(k) = \nabla_k \times \mathcal{A}_n(k)$. The Chern number is given by [205]

$$C_n = \frac{1}{2\pi} \int_{\text{BZ}} d^2k \Omega_n(k_x, k_y). \quad (2.3)$$

Systems having different topological invariants (Chern numbers) belong to different topological classes. The two objects will be topologically equivalent if they can be deformed into each other continuously [200]. For example a doughnut and a coffee cup are related by a continuous deformation and so belong to the same topological class (see Fig. 2.1(a)). In contrast a doughnut and a ball can not be continuously deformed into one another, hence they are not topologically equivalent. They have a different number of holes, which is also known as a genus, for example genus=1 for doughnut and genus=0 for ball. During a topological phase transition, the topological invariant changes through creating or removing a hole. Similarly in the context of the energy gap between valence and conduction band, two materials will be topologically equivalent if the Hamiltonians of the systems can deform continuously into each other without closing the gap. When we consider the interface of the two systems having different topology, a phase transition happens, which leads to the emergence of the edge state(s) at the interface linking the valence and conduction band (see Fig. 2.1(b)) [205]. The bulk-edge correspondence states that the number of gapless edge modes is equal to the difference of the bulk topological invariant across the interface [200]. These edge states are unidirectional, immune to backscattering and spatially separated, which suppresses scattering processes from one to another edge state [205]. If we take the finite size sample of a topologically nontrivial material, then the physical edge of the sample is the interface between the system of topological nonzero invariant and topologically trivial vacuum and it guarantees the existence of edge states on the boundary.

In 1988, a theoretical model was proposed by Haldane, which showed the quantum Hall physics without Landau levels in periodic systems, called the quantum anomalous Hall effect [206]. It is the first example a Chern insulator. This electronic model was then transferred to the realm of photonics in 2005 [207, 208], and later designed [209] and observed experimentally in Ref [210]. Such a system exhibits chiral³ edge state(s), which appear due to the breaking of time reversal symmetry (see Fig. 2.1(b)). Another popular topological phase of matter is the $\mathbb{Z}_2$ topological insulator or quantum spin Hall system, discovered by Kane and Mele in 2005 [211, 212]. Such a system consists of two copies of a Chern insulator, one for each spin. As a result the Chern number is zero and for this kind of system the nontrivial topology is defined by a $\mathbb{Z}_2$ topological invariant. The system shows

³Chiral edge states propagate only in one direction along the sample boundary.

helical edge states in the presence of time reversal symmetry (see Fig. 2.1(c)). In this thesis we will discuss the realization of a Hofstadter model [213, 214] in the exciton-polariton system, which shows physics like integer quantum Hall systems.

Figure 2.1: (a) A doughnut and a coffee cup are topologically equivalent having genus = 1, which is a topological invariant quantity defining the number of holes within a closed surface. A doughnut and a ball (genus = 0) belong to different topological classes. (b) Lower panel illustrates a chiral edge mode appearing at the interface between an insulator and a quantum Hall state. The cyclotron skipping orbits allow to have backscattering immune unidirectional propagation. n is the total Chern number, summed over all occupied bands, which is a topological invariant quantity determining the number of edge mode(s). Upper panel shows an edge state appearing, connecting valence band and the conduction band. (c) In lower panel, helical edge states appear at the interface between an ordinary insulator and a quantum spin Hall insulator. The dispersion of the Kane and Mele model in a graphene strip shows a propagation of down and up spins in opposite directions (upper panel). Fig. (a) is taken from [215].

2.2 Polarization splitting and elliptical micropillar

As discussed earlier in subsection 1.4.7, the energy splitting of TE and TM polarized modes, which acts as an effective magnetic field is k dependent. Here, we will discuss another kind of polarization splitting, which exists even for zero in-plane momentum and k independent. Such kind of splitting arises due to the asymmetry in the horizontal and vertical directions of structures like microwires, elliptical pillars, etc.

Añón et. al. [216] investigated the collective spin dynamics of polariton condensates in a quasi-1D microcavity ridge (see Fig. 2.2). The confinement in the y direction results in the splitting of the lower polariton branch into many subbands and a clear energy splitting was observed between two polarizations: parallel (H) and perpendicular (V) to the x axis of the order of 0.36 meV where the lowest

subband is H polarized. In this experiment, a σ_+ polarized nonresonant excitation was used to create polaritons at the center of the ridge. These polaritons propagate outside the pumping spot by transferring potential energy into kinetic energy. Scattering with the excitonic reservoir and through intra-branch scattering, polaritons show rich oscillating spin precession in the photoluminescence (PL) distribution in real space. Such effects are also reported in many recent papers [174, 217, 218].

Such kind of linear polarization splitting also exists inside a micropillar having an elliptical cross section [219, 220] (see Fig. 2.2). The ellipticity introduces a splitting of the fundamental modes, which are orthogonally linearly polarized. The splitting between the fundamental modes increases with the increasing ellipticity. Since lateral confinement of the optical mode increases with smaller diameter, for fixed ellipticity, the smaller pillar has larger splitting. The splitting was achieved up to ~ 3 meV [219].

In section 2.4 we will use the effect of polarization splitting arises in elliptical micropillar to get the phase dependent coupling between neighbours. Later in chapter 5 we have used the concept polarization splitting appears in 1D channel to realize cellular automaton in exciton-polariton lattices.

Figure 2.2: Upper panel: schematics of a 1D microcavity ridge. Lower panel: a circular micropillar does not show a splitting between fundamental modes whereas for an elliptical micropillar, splitting arises due to shape anisotropy.

2.3 Motivation

In a system of charged particles, the application of a magnetic field qualitatively changes its behavior, and usually is responsible for intriguing physical effects. For instance, consider electrons in a periodic potential, which in the absence of a magnetic field gives rise to Bloch bands. In the presence of perpendicular uniform magnetic field, the spectrum splits into highly degenerate Landau levels. It was predicted that the interplay between a periodic potential and magnetic field for

the two-dimensional electron gas leads to the emergence of a self-similar fractal energy spectrum, known as Hofstadter's butterfly [214]. It corresponds to the plot of the allowed and forbidden energies as a function of magnetic flux. For a rational value of the normalized magnetic flux $\beta = p/q$ (where p and q are coprime integers), each Bloch band splits into q subbands. These energy bands deform into Bloch bands only after closing the fractal gaps between them, admitting a topological characterization in the system [202]. This leads to the appearance of topologically protected edge states. Ultimately, the application of magnetic field to the strongly correlated system may give rise to topological order, leading to the fractional quantum Hall effect [221, 222].

While the described physics naturally emerges for electrons and other charged particles, systems of neutral particles do not exhibit the same behavior, limiting their possible scope of application. This posed the question of the possibility to generate an artificial gauge field [223], where the effect of the field is simulated by some means. For continuous systems, examples include the generation of magnetic field for cold atom gases using rotation [224] and laser illumination [223]. For lattice systems, different microwave [208], optical [225–227] and cold-atom [228–230] setups were considered recently, and recipes for artificial gauge field generations were proposed. Up to date, Hofstadter's butterfly was demonstrated using engineered superlattice structures with bilayer graphene [231], optical ring microresonators [225, 226], cold-atom lattices [232], etc. Finally, the Harper Hamiltonian was simulated using a chain of superconducting circuits [233], promising the way towards implementation of magnetism in the interacting systems.

The described systems and approaches to observe fractal Hofstadter butterfly behavior can be classified into several categories. First, the lattice models with phase-dependent hopping can be divided into systems where TRS is broken (integer quantum Hall effect type physics) and not broken (spin Hall effect type physics). In the former case, there is a unique chiral edge state associated to the boundary, while in the latter case two copropagating channels exist, albeit for different spin components. From the point of realization, the butterfly behavior may be posed by magneto-optical effects, time-dependent modulation of the tight-binding coupling [225], and phase-dependent hopping on a spinful lattice [226]. One of the promising platforms where the realization of artificial gauge fields and the associated Hofstadter's butterfly is missing is the system of exciton-polaritons.

In traditional condensed matter systems, as an atomic spacing is very small (of the order of an angstrom), one needs a large magnetic field (thousands of Tesla to get one flux quantum per unit cell) to observe Hofstadter's butterfly [234]. Alternatively, the self-similar pattern can be observed in semiconductor nanostructures hosting high-mobility two-dimensional electron gas, with the presence of a superlattice [235]. In the polaritonic system the periodicity is on the scale of microns, and from the experimental perspective the system favors the possibility to observe Hofstadter's butterfly.

Here we consider theoretically a system of coupled micropillars with polarization splitting at each site, along with spin-dependent coupling. This breaks

time-reversal symmetry even in the absence of a real magnetic field. The splitting is realized in a way that effectively couples different lattice sites i and i' with a phase factor $\phi_{i,i'}$. Controlling $\phi_{i,i'}$ on the square lattice, we show that the analogue of Hofstadter's butterfly for the spectrum of the system and chiral edge modes can be observed. The latter is demonstrated to avoid backscattering from disorder, potentially leading to unidirectional transport in the polaritonic system.

Finally, considering the driven-dissipative nature of exciton-polaritonic lattices, we show that the combination of the topological transport and gain in the system can lead to single-mode lasing associated to an edge mode. This corresponds to the recently discovered topological insulator lasing, which was described theoretically in Ref. [236] and experimentally confirmed in Ref. [237] for ring cavity arrays. We demonstrate that polariton lattices can host a lasing mode corresponding to a chiral edge state, which has a positive gain and is unhampered by disorder. This opens the way towards polaritonic lasers of largely increased efficiency and large gain.

2.4 Model

We consider an array of coupled exciton-polariton elliptical micropillars that form a square lattice, as shown in Fig. 2.3(a). We make use of the recent experimental development of micropillars with elliptical cross-section [219, 220, 238]. Each micropillar has two kinds of polarization ($\sigma_{\pm} = \pm 1$). We define the lattice in terms of two sets of modes corresponding to alternating spin polarizations in successive micropillars. The first set of modes [represented by filled circles in Fig. 2.3(a)] will be called the main *logical* modes; the second set of modes [represented by empty circles in Fig. 2.3(a)] will be called the *auxiliary* modes. The hopping between only the same polarization of two neighboring pillars is allowed. We consider modes with $\sigma_+ = +1$ tunneling horizontally, whereas modes with $\sigma_- = -1$ tunnel vertically. We note that this spin-dependent hopping corresponds to a breaking of time-reversal symmetry (see appendix C), which will later be shown to give rise to an artificial gauge field. This kind of potential landscape can be created by non-resonant excitation with a localized optical pump, which induces a spin-dependent potential that can selectively gate different spin polarizations as we discussed in sec. 2.2 [174]. Another work also shows that alternating patterns of spin polarization may form spontaneously under near-resonant excitation, offering a further alternative for engineering spin-dependent coupling for our lattice [218]. In the tight-binding approximation with nearest-neighbor coupling, the Hamiltonian can be written as

$$\begin{aligned} \hat{H} = \sum_{i,j,\sigma_{\pm}} \Big[& -\Delta_T e^{-i2\pi\phi_{i,j}\sigma_{\pm}} \hat{a}_{i,j,\sigma_{\pm}}^{\dagger} \hat{a}_{i,j,\sigma_{\mp}} + J \hat{a}_{i,j,\sigma_{\pm}}^{\dagger} \hat{a}_{(i-\delta_{1,\sigma_{\mp}}),(j-\delta_{1,\sigma_{\pm}}),\sigma_{\pm}} \\ & + J \hat{a}_{i,j,\sigma_{\pm}}^{\dagger} \hat{a}_{(i+\delta_{1,\sigma_{\mp}}),(j+\delta_{1,\sigma_{\pm}}),\sigma_{\pm}} + \text{h.c.} \Big], \end{aligned} \quad (2.4)$$

where the operators $\hat{a}_{i,j,\sigma_{\pm}}^{\dagger}$ ($\hat{a}_{i,j,\sigma_{\pm}}$) create (annihilate) an exciton-polariton at site (i, j) with circular polarization $\sigma_{\pm} = \pm 1$ and $\delta_{1,\sigma_{\pm}}$ is the Kronecker Delta. The

first term in Eq. 2.4 describes the coupling between the cross-polarized polaritons in the same micropillars, leading to the splitting between $\sigma_{\pm}$ components. We model elliptical micropillars by the linear polarization splitting Δ_T and an angle ϕ . The splitting Δ_T can be order of magnitude larger than the linewidth (~ 3 -4 meV) as reported in Refs. 219, 220. The next two terms describe the tunneling between nearest neighbors having the same polarization as drawn by solid lines in Fig. 2.3(a). Using the described model, we can write the equation of motion for the polariton field at site (i, j) as

$$i\hbar \frac{\partial \psi_{i,j,\sigma_{\pm}}}{\partial t} = -\Delta_T e^{-i2\pi\phi_{i,j}\sigma_{\pm}} \psi_{i,j,\sigma_{\mp}} + J\psi_{(i-\delta_{1,\sigma_{\mp}}),(j-\delta_{1,\sigma_{\pm}}),\sigma_{\pm}} + J\psi_{(i+\delta_{1,\sigma_{\mp}}),(j+\delta_{1,\sigma_{\pm}}),\sigma_{\pm}}, \quad (2.5)$$

where we observe the onsite coupling through polarization splitting (first term on r.h.s.), as well as intersite tunneling terms. Our goal is to generate an effective phase-dependent interpillar tunneling between main *logical* modes, which is mediated by the polarization splitting. For this, the *auxiliary* modes, chosen to be $\{\psi_{i,j,\sigma_{-}}; \psi_{i+1,j+1,\sigma_{-}}; \psi_{i,j+1,\sigma_{+}}; \psi_{i+1,j,\sigma_{+}}\}$, have to be eliminated.

The elimination can be done by taking the second derivative at the l.h.s. of Eq. (2.5) and substituting the evolution of the neighbors. Then, the evolution of site (i, j) can be described by a quadratic equation,

$$\begin{aligned} \frac{\partial^2 \psi_{i,j,\sigma_{\pm}}}{\partial t^2} = & \left(-\frac{\Delta_T^2 + 2J^2}{\hbar^2} \psi_{i,j,\sigma_{\pm}} + \frac{J\Delta_T e^{-i2\pi\sigma_{\pm}\phi_{i,(j-\delta_{1,\sigma_{\pm}})}}}{\hbar^2} \psi_{i,j-1,\sigma_{\mp}} \right. \\ & + \frac{J\Delta_T e^{-i2\pi\sigma_{\pm}\phi_{(i+\delta_{1,\sigma_{\mp}}),j}}}{\hbar^2} \psi_{(i+1),j,\sigma_{\mp}} + \frac{J\Delta_T e^{-i2\pi\sigma_{\pm}\phi_{i,(j+\delta_{1,\sigma_{\pm}})}}}{\hbar^2} \psi_{i,j+1,\sigma_{\mp}} \\ & + \frac{J\Delta_T e^{-i2\pi\sigma_{\pm}\phi_{(i-\delta_{1,\sigma_{\mp}}),j}}}{\hbar^2} \psi_{i-1,j,\sigma_{\mp}} - \frac{J^2}{\hbar^2} \psi_{(i-2\delta_{1,\sigma_{\mp}}),(j-2\delta_{1,\sigma_{\pm}}),\sigma_{\pm}} \\ & \left. - \frac{J^2}{\hbar^2} \psi_{(i+2\delta_{1,\sigma_{\mp}}),(j+2\delta_{1,\sigma_{\pm}}),\sigma_{\pm}} \right), \end{aligned} \quad (2.6)$$

and we note that no approximation was made so far. Equation 2.6 demonstrates that the system decouples into two separate parts, corresponding to the main *logical* and *auxiliary* modes (filled and empty circles shown in Fig. 2.3(a)). Considering the filled circles, the model is reduced to Fig. 2.3(b). The first term in Eq. 2.6 describes the onsite energy. The middle four terms describe the tunneling with four orthogonal sites in the reduced model, which are phase dependent. These are the plaquette terms which are required to generate an artificial magnetic field. Finally, the last two terms denote the interaction with the next-nearest neighbors (NNN) in the reduced model, which are inherent to our effective model, and are typically absent in the other geometries (e.g., cold-atomic lattices with modulated hopping 232). In the following we highlight the differences, and show the limit where NNN coupling can effectively be neglected.

As a result of polariton hopping from one site to another, the spatially-dependent complex tunneling leads to the accumulation of a non-trivial phase. This tunneling is directionally dependent. It means that when particles hop between two

Figure 2.3: (a) Full model: schematic diagram of a square lattice formed by coupled exciton-polariton elliptical micropillars that support two kinds of polarizations ($\sigma_{\pm} = \pm 1$) with the splitting Δ_T . Here, filled circles denote main *logical* modes, and empty circles denote *auxiliary* modes. The angle of the micropillar polarization axis at site (i, j) is $\phi_{i,j}$. The tunneling (overlap) between nearest micropillars is allowed only for polaritons of the same polarization. The two spin polarizations of a micropillar interact differently, one hopping horizontally and the other one vertically, while other channels are closed by the polarized potentials. The red lines describe the direct tunneling between neighbors of polarization $\sigma_+ = +1$, while blue lines denote the direct tunneling between neighboring pillars of polarization $\sigma_- = -1$, both described by the hopping amplitude J . (b) Reduced model, where only main *logical* modes are shown. The dashed lines represent the complex hopping associated with magnetic vector potential $\mathbf{A}$ (see the discussion in the text), which is responsible for the realization of an artificial magnetic field in the system. Polaritons can hop in four orthogonal directions, and pick up nonzero phases that are direction dependent. The phase per plaquette is uniform and equal to $2\pi\beta$.

sites clockwise, the value of the accumulated phase will be the negative of the value when it hops anticlockwise. Associating a gauge potential $\mathbf{A}$ to the phase $\phi_c = (q/h) \int_c \mathbf{A} \cdot d\mathbf{r}$, this introduces an effective artificial magnetic field in the system, which is given by $B = (1/a^2) \oint_{\text{plaquette}} \mathbf{A} \cdot d\mathbf{r}$, where a^2 is the area of the plaquette. Physically, the phases $\phi_{i,j}$ are dependent on the chosen polarization axis of the micropillars and we choose the phases in such a way that polaritons accumulate a uniform phase ($2\pi\beta$) after completing each plaquette. Therefore, the magnetic flux per lattice unit cell in units of the magnetic flux quantum is set to be β . As a result we can realize a uniform artificial magnetic field in the system.

2.5 Results

2.5.1 Hofstadter's butterfly

First, we consider the full model (see Fig. 2.3(a)) with a square lattice of (10×10) micropillars, corresponding to 200 polarization modes. Imposing the toric boundary conditions, we mimic the effect of an infinite lattice. Solving the eigenvalue equation for the system, the energy spectrum (ε) of the single particle as a function of normalized magnetic flux β is given by Fig. 2.4. Notably, due to the structure of dynamical equations, instead of solving the full model, by diagonal-

Figure 2.4: Hofstadter's butterfly spectrum. Energy ε (stated in meV) is shown as a function of dimensionless parameter β , which is the effective magnetic flux per lattice unit cell in terms of magnetic flux quantum. We consider a 10×10 lattice with periodic boundary condition to study the bulk properties, and effectively subject the system to a uniform magnetic field by making the flux per plaquette uniform ($2\pi\beta$). Here, we consider $\Delta_T = 4$ meV and $J = 0.1$ meV [169]. For each β , we calculate the density of states (DOS), find the maximum value (say M), and plot the points corresponding to DOS being greater than $0.36 \times M$. The choice of $\beta = 1/4$ is shown by the pink line.

izing the reduced model under periodic boundary condition, we can also obtain the same spectrum of two identical Hofstadter butterfly patterns, which are symmetric about the $\varepsilon = 0$ axis. The tunneling amplitude J controls the width of the spectrum ($\simeq 4J$), whereas the limits of the eigenvalues corresponding to each magnetic flux are symmetric around $\varepsilon = \pm\Delta_T$. At the same time, compared to the commonly described Hofstadter butterfly, the wings of each are skewed. The symmetry of the spectrum depends on the ratio of J to Δ_T , and close-to-symmetric butterflies can be observed in the limit $J/\Delta_T \rightarrow 0$. So, although the NNN terms

are present in the equations, by taking the ratio small ($J/\Delta_T = 0.025$) the effect is negligible and we observe the self-similar spectrum. From onwards, we focus on the region with $\beta = 1/4$ only.

2.5.2 Band structure

In the previous subsection, accounting for the effect of infinite lattice by taking the toric boundary condition, we obtain only the bulk states, which correspond to the energy bands separated by the band gaps. However, for the finite lattice, the breaking of translational symmetry leads to the appearance of distinct edge states. In the case of the lattice system subjected to the artificial magnetic field, the bulk states are connected by edge modes, which are propagating unidirectionally (clockwise or anticlockwise) along the edges. Here we take a finite rectangular lattice of dimension $N_x \times N_y$ (see Fig. 2.3(a)) corresponding to the $2 \times N_x \times N_y$ polarization modes and after diagonalizing we consider the modes with positive polarizations. Taking two dimensional discrete Fourier transforms of the corresponding eigenstates, and summing over k_y , we obtain the energy spectrum as a function of k_x (see Fig. 2.5(a)). We consider the case $\beta = 1/4$, so the acquired phase per plaquette is $2\pi\beta = \pi/2$ implying that the magnetic unit cell consists of four lattice unit cells and as a result the single particle Bloch band splits into four subbands. Fig. 2.5(b) shows the integrated intensity of the states marked by black and red spots in Fig. 2.5(a) in the y direction. Their spatial structure confirms the edge-type behavior for the states.

2.5.3 Edge state transport

Next, we simulate the transport properties of the system. To illustrate the propagation of the edge mode, we excite one of the edges by introducing a coherent pulse of the form

$$F = F_0 \exp[-((x - x_0)^2 + (y - y_0)^2)/L^2] \times \exp[-(t - t_0)^2/\tau^2] \exp[i(k_p x - \omega_p t)], \quad (2.7)$$

where F_0 is the amplitude of a Gaussian pulse of width L , which is centered on the edge at (x_0, y_0) . We choose k_p and ω_p in the gap, resonant with the edge mode dispersion (see small green dot in Fig. 2.5(a)). We add this pulse to the r.h.s of Eq. (2.5), considering also the dissipation of the condensate with rate Γ , given by

$$i\hbar \frac{\partial \psi_{x,y,\sigma_{\pm}}}{\partial t} = \left(-\Delta_T e^{-i2\pi\phi_{x,y}\sigma_{\pm}} \psi_{x,y,\sigma_{\mp}} + J\psi_{(x-\delta_{1,\sigma_{\mp}}),(y-\delta_{1,\sigma_{\pm}}),\sigma_{\pm}} + J\psi_{(x+\delta_{1,\sigma_{\mp}}),(y+\delta_{1,\sigma_{\pm}}),\sigma_{\pm}} - \frac{i\Gamma}{2} \psi_{x,y,\sigma_{\pm}} + F \right). \quad (2.8)$$

Here, we consider the full model as shown in Fig. 2.3(a) of the dimension $(N_x \times N_y)$ and solve the time dynamics to observe the evolution of the states at different times (see Fig. 2.6(a)). In this case the injected pulse is propagating in a single direction along the edges of the rectangular lattice with the absence of backscattering.

Figure 2.5: (a) Energy diagram of the finite rectangular lattice (200×20) formed by elliptical micropillars. As we consider $\beta = 1/4$, we have four bands and associated edge states. The upper edge state corresponds to the propagation clockwise along the edges, whereas the lower edge state corresponds to the propagation anti-clockwise along the edges. The energy and wavevector corresponding to the green dot is used in sec. 2.5.3. (b) Field intensity of the two edge modes indicated by the black and red dots in Fig. 2.5(a). The fields are localized on the two edges and decay exponentially into the bulk.

Now, to demonstrate the robustness of the edge states, we account for the presence of a defect by removing two micropillars along one edge of the rectangular lattice ($N_x \times N_y$) and exciting the edge mode by injecting the Gaussian pulse as given by Eq. (2.7). From the intensity distribution shown in Fig. 2.6(b), we observe that the edge state goes around the defect and maintains its unidirectionality, which demonstrates that the edge state propagation is robust against disorder.

To quantify the robustness we calculate the total intensity of the wavepacket, which shall be fully transferred from right to left. We consider the case with no dissipation and plot the total intensity of the sites situated to the right or left of the defect as a function of time (see Fig. 2.6(c)). Here $I_t(R)$ is the total intensity for the sites on the right side of the defect, and the total intensity of the rest is denoted by $I_t(L)$. We observe that in the large time limit $I_t(L)$ slowly reaches the intensity of the pulse before the defect, $I_t(R)$. Thus, an ultimate limiting factor then corresponds to the lifetime of the pulse.

Figure 2.6: (a) Propagation of the one-way edge mode. We consider a 200×20 lattice formed by elliptical micropillars and excite the edge by a coherent probe field focused at the lower edge. We consider $\beta = 1/4$, $L = 2$, $t_0 = 0$ ps, $\tau = 60$ ps, and choose k_p and ω_p from the dispersion curve of the edge states (green dot in Fig. 2.5(a)). The pulse is injected at $x_0 = 150$ in the lower edge, shown by the red box. The field profiles at different times indicate that the edge state is unidirectional. (b) Propagation of the one-way edge mode in the presence of a defect. We create the defect by removing two micropillars at the lower edge. Solving the dynamics with the same parameters as above, we plot intensity at different times showing that the edge state goes around the defect and maintains its unidirectionality which emphasizes that the edge state is robust against disorder. (c) Integrated intensity before and after the defect as a function of time, showing that the total intensity is totally transferred from right to left in the long time limit (here the dissipationless case is considered to isolate the effects of dispersion and potential scattering from the overall decay).

2.5.4 Topological insulator lasing mode

Finally, we make use of the intrinsic driven-dissipative nature of the polaritonic system, and study its lasing properties. It was shown recently that optical analogues of topological insulators can act as powerful single mode lasers, where the influence of disorder is suppressed [236, 237]. In this section, we show how topological lasing can be achieved in the polaritonic setup, and characterize its gain properties.

We consider the system to be driven with a non-resonant pump, which has a

non-zero value along the edges. The time evolution of the system is given by

$$i\hbar \frac{\partial \psi_{x,y,\sigma\pm}}{\partial t} = \left(-\Delta_T e^{-i2\pi\phi_{x,y}\sigma\pm} \psi_{x,y,\sigma\mp} + J\psi_{(x-\delta_{1,\sigma\mp}),(y-\delta_{1,\sigma\pm}),\sigma\pm} \right. \\ \left. + J\psi_{(x+\delta_{1,\sigma\mp}),(y+\delta_{1,\sigma\pm}),\sigma\pm} - \frac{i\Gamma}{2} \psi_{x,y,\sigma\pm} + iP\psi_{x,y,\sigma\pm} \right). \quad (2.9)$$

The gain and loss properties of the system are given by the solution of Eq. (2.9) in the frequency domain, which are represented by imaginary and real parts of ε . The results are shown in Fig. 2.7(a). We find that the eigenvalue with the largest imaginary part, marked by the blue dot in Fig. 2.7(a), corresponds to the edge state. Solving the time dynamics and taking the average over many iterations, we get a large occupation of the edge modes, as shown in Fig. 2.7(b). Similarly to the previous subsection, we observe that the edge states are robust against disorder and thus correspond to efficient lasing modes. In analogy to topological insulator lasers, topological polariton lasing could be realized using electrical injection [239] to provide the gain for the lasing mode [240, 241].

Figure 2.7: Topological polariton lasing. (a) Imaginary and real part of the modes of the system, calculated from Eq. (2.9) for the rectangular lattice (200×20) formed by elliptical micropillars. We consider the system with $\beta = 1/4$, $\Gamma = 200$ ps, and $P = 0.0075$ meV. The blue dot points the maximum value of the gain (imaginary part of energy), and corresponds to the edge mode. (b) Zoomed plot of the occupation calculated by evolving Eq. (2.9) in time, averaging over randomly chosen initial conditions. The bright spots correspond to a lasing mode.

2.6 Conclusions

We considered an array of elliptical polariton micropillars in the form of a square lattice. The splitting between the two polarizations of the micropillars, together with interpillar spin-dependent tunneling, creates a phase-dependent coupling between the neighboring micropillars. With the suitable choice of phases, we are

able to generate a uniform flux corresponding to each plaquette, which manifests an artificial uniform magnetic field in the system. This artificial magnetic field represents the breaking of the time-reversal symmetry in the system. Considering the toric boundary condition, we calculated the spectrum of the system, which shows fractal behavior akin to the Hofstadter's butterfly. Furthermore, using the Dirichlet boundary condition we have observed the edge states, which are unidirectional and robust against disorder. We show that with the nonresonant excitation we can obtain lasing in a topologically protected mode, being a unique feature of the driven-dissipative polaritonic system. This makes the system a promising candidate for practical applications, and puts the proposal on the road map for artificial gauge potential realization with exciton-polaritons.

Chapter 3

Coupling between exciton-polariton corner modes through edge states

In this chapter we consider theoretically the realization of a second order topological polariton bandstructure, which corresponds to the implementation of topological corner states in a polariton lattice. Our main interest has been on the possibility of coupling different topological states. As topological states are eigenstates of the system and robust against disorder they are typically thought of as completely isolated from one another. However, in the exciton-polariton system there is a significant effect from polariton-polariton scattering, which allows to go beyond the usual linear physics. We show that polaritons can scatter from a corner state into an edge state, which itself allows coupling to another corner state. In this way, we find that as a nonlinear driven-dissipative system exciton-polaritons offer a unique opportunity for realizing spatially localized topological states that can be coupled together. We begin with the review of the higher order topological insulators and later we discuss our scheme. This chapter is reproduced from R. Banerjee, S. Mandal, and T. C. H. Liew, Phys. Rev. Lett. **124**, 063901, (2020) [242].

3.1 Second order topology

Topological insulators (TIs) have attracted attention in the past decade due to their unique exotic property, namely the appearance of backscattering-immune edge states, which can propagate against perturbation without being backscattered. TIs have been explored in various systems including electronics [199, 201], photonics [207, 209, 210, 226, 243, 244], cold atoms [245], exciton-polaritons [246–250], acoustics [251], etc. In the previous chapter (see sec. 2.1), we have introduced the first order topology where 2D systems possess 1D edge states, or more generally for a D dimensional system (D-1) dimensional gapless topologically protected boundary modes appear [199]. Very recently, topological insulators (TIs) are extended to second order (and even higher) that go beyond the conventional

bulk-boundary correspondence. For example, in a two-dimensional second order topological insulator, zero dimensional (0D) topologically protected gapless corner modes appear along with one dimensional (1D) gapped edge states [252–259]. In general, a D dimensional r th order topological insulator has $(D-1)$ dimensional, $(D-2)$ dimensional, $(D-3)$ dimensional,..., $(D-r+1)$ dimensional gapped boundary states and $(D-r)$ dimensional gapless boundary states. The 0D corner states can be obtained using quantization of quadrupole moments in a 2D square lattice (see Fig. 3.1(a)). To understand this, let us consider a 1D Su-Schrieffer-Heeger (SSH) chain (see Fig. 3.1(b)), which exhibits quantized dipole moment [4]. However, such a dipole moment does not correspond to accumulation of charges at the bulk, but the contribution comes from non-vanishing monopoles of opposite sign at the boundaries. Similarly a 2D square lattice with quantized quadrupole moment corresponds to 1D boundary modes having non-vanishing dipole moments such that the total dipole moment taking all the 1D boundary modes into account cancels out. Then the 1D boundary modes can be thought as an SSH chain with monopoles at the 0D boundary, i.e., corner modes. This is depicted in Fig. 3.1(c-d). Later, the topological corner modes were also obtained in breathing Kagome lattices, where, instead of quadrupole moment, dipole moment is quantized [257, 260]. They have been realized in other systems as well, such as classical mechanical systems [253], acoustic metamaterials [256], electromagnetic metamaterials [254, 255], etc. The topological properties associated with these corner states making it a suitable candidate for information processing [261].

3.2 Motivation

Due to the topological properties of these corner states, information can be trapped at the corners of the system, which will be unhampered even in the presence of disorder, making it a potential candidate for information processing [261]. But corner states, as with other topological modes, are well isolated from each other even in the presence of disorder making it difficult for them to overlap (they are orthogonal eigenstates). Consequently, it is far from obvious whether there are ways in which different corner states can interact. Although information processing necessitates operating with a coupling of multiple modes, the coupling of multiple

⁴The topological dipole moment (and hence topological charges) has completely different interpretation to the usual dipole moment (for example exciton dipole moment) as discussed in the introduction. Unlike the usual dipole moments, topological dipole moment occurs in a nontrivial topological band structure corresponding to an underlying periodic lattice. In early 1990 it was realized that topological charges are related to Bloch waves and for 1D SSH chains it can be expressed in terms of the winding number,

$$\nu = i \oint_{BZ} \frac{dk}{\pi} \langle u(k) | \frac{\partial}{\partial k} | u(k) \rangle, \quad (3.1)$$

where, $u(k)$ is the Bloch wave function with wavevector k and the integral is performed over the whole Brillouin zone. For a topologically non-trivial SSH chain, $\nu = \pm 1$ and it corresponds to monopoles located at the edges for a finite chain [262]. On the other hand, for topologically trivial SSH chains $\nu = 0$ and it does not correspond to any charges.

Figure 3.1: (a) Tight-binding representation of a quadrupole topological insulator which consists of four different couplings J , J' , $-J$ (black dashed line) and $-J'$ (magenta dashed line). The system enters in the quadrupole topological regime for $J' > J$. This figure is reproduced from [252]. (b) Schematic diagram of the SSH model in the topological regime where two topologically protected localized modes at the ends of the chain appear and we obtain this regime by controlling the couplings in such a way that $t_2 > t_1$. (c) A finite size system having bulk dipole moment corresponding to two monopoles at the boundary. (d) A finite size 2D system as shown in (a) having bulk quadrupole moment corresponds to 1D edge dipoles and 0D corner monopoles. Here the two different colors represent positive and negative contributions. Figs. (c-d) are reproduced from [253].

topological modes is unexplored in the literature (especially for corner states).

Here we consider a second order topological polariton band structure, which is based on achieving hopping between sites with opposite sign. This can not be achieved just by varying the overlap between neighboring micropillars, but can be implemented by placing auxiliary micropillars between a main lattice of micropillars. This follows a generic scheme introduced in Ref. [263] for tight-binding lattices, which we verified starting from a particular polariton potential profile. Having established polariton corner states, we study the influence of polariton-polariton scattering, which allows them to couple to edge states. It is via edge states that polariton corner states can interact, where the excitation of one corner state causes excitation of its neighbor, which would not be possible in the linear regime. We have demonstrated a transfer of information encoded in a binary state from one corner to the next, which can occur even in the presence of a realistic

level of disorder. We find that the mechanism of information transfer proceeds both faster and with lower required power than the same mechanism considered in a regular non-topological square lattice.

3.3 Scheme

We consider a square lattice of coupled exciton-polariton micropillars as shown in Fig. 3.2(a). For simplicity, we neglect the spin degree of freedom in the system and consider a single mode of each micropillar, which evolves according to the Gross-Pitaevskii equation

$$i\hbar \frac{\partial \psi_i}{\partial t} = \left(\Delta - \frac{i\Gamma}{2} + iP \right) \psi_i + \sum_{\langle j \rangle} J_{ij} \psi_j + \alpha_1 |\psi_i|^2 \psi_i - i\alpha_{NL} |\psi_i|^2 \psi_i + F_i, \quad (3.2)$$

where Δ is the energy detuning between the polariton mode energy (onsite energy) and the laser energy, and Γ is the polariton dissipation. P is a nonresonant pump applied uniformly to all micropillars in the system and as a result the nonlinear loss term α_{NL} is inevitable (see sec. 1.4.9). α_1 is the strength of nonlinear interaction and F is a coherent driving field (i.e., laser). It is convenient to move to dimensionless units by making the following transformations: $t \rightarrow t\hbar/J$ and $\psi_i \rightarrow \psi_i \sqrt{(J/\alpha_1)}$, where J is the weakest hopping amplitude. With these choices Eq. (3.2) becomes

$$i \frac{\partial \psi_i}{\partial t} = \left(\Delta - \frac{i\Gamma}{2} + iP \right) \psi_i + \sum_{\langle j \rangle} J_{ij} \psi_j + (1 - i\alpha_{NL}) |\psi_i|^2 \psi_i + F_i, \quad (3.3)$$

where the polariton-polariton nonlinear interaction is scaled to unity. All the energy terms in the equation are normalized by a factor J , α_{NL} is normalized by α_1 , and $F_i \rightarrow F_i \sqrt{(\alpha_1/J^3)}$. We consider four different hopping terms (J , $-J$, J' , and $-J'$) realizing the potential described in Fig. 3.2(a). In writing Eq. (3.3), we have assumed a tight-binding approximation. Exciton-polaritons can also be modelled directly from a continuous model (see sec. 3.9). Although we operate with dimensionless units, we also provide typical real units, corresponding to setting the coupling $J = 1$ meV (corresponding to Ref. [63, 169, 264]).

Neglecting at first the excitation, decay, and nonlinear terms, the energy spectrum (E) as a function of eigenstate quantum number (n) is given in Fig. 3.2(b). The zero energy modes correspond to the corner states, which are well separated in energy from other modes of the system. This should be expected as, in the absence of nonlinear terms, the model is essentially the same as that applicable to coupled microwave resonators previously shown to exhibit the same topological corner states [255]. Our main aim here is to excite one of the corner modes coherently and couple to other corner modes without affecting the topological property of the system. In this way topological corner modes may be used to store information as well as support exchange of information among themselves. To do this we consider all terms as described in Eq. (3.3) and take F_i as proportional to

the amplitudes in each micropillar corresponding to one of the corner states, i.e., $F_i = f_s \psi_i^{(c)}$, where $\psi^{(c)}$ is the corner state eigenfunction. We choose the pump profile with the same spatial profile as the eigen state corresponding to $n = 1251$, shown in Fig. 3.2(c), and also for simplicity we fix $\Gamma/2 = P$. With proper choice of parameters, we consider parametric instability in the system where pairs of polaritons from this corner state can scatter to the edge modes while conserving energy. This regime of optical parametric oscillation (OPO) was previously discussed in sec. 1.4.10. Due to secondary parametric scattering processes we expect that the edge states will couple to another corner. In this way with the help of the edge states we can nonlinearly couple the two corners, as described by the schematic figure 3.2(d).

Figure 3.2: (a) Schematic diagram of a square lattice formed by coupled exciton-polariton micropillars with four different hoppings, J , $-J$, J' , and $-J'$, indicated by different colours. (b) Energy eigen-values of the system consisting of 50×50 micropillars, as a function of the quantum number n . The modes corresponding to $n = 1249 - 1252$ are the corner states appearing at $E = 0$, denoted by red. The bulk and edge states are shown in blue and green respectively. (c) Spatial profile of the pumped corner state corresponding to $n = 1251$. (d) Schematic diagram of the optical parametric scattering processes in the system. First polaritons from the pumped corner state scatter to edge states (black arrows) and then the polaritons from the edge states scatter back to the adjacent corner state (red arrows). Parameters: $J' = 5$ and $\Delta = 0$.

3.4 Parametric instability

To investigate parametric instability we first drive the system to a steady state, which is obtained by solving Eq. (3.3), and then study the behaviour of linear (Bogoliubov) fluctuations

$$\psi_i = \psi_{(0,i)} + u_i e^{-i\omega t} + v_i^* e^{i\omega^* t}. \quad (3.4)$$

ψ_0 is the stationary solution of Eq. (3.3), which essentially takes the form of the driven corner state; u and v are spatial functions of the fluctuations. $\psi_{(0,i)}$ is the value of the ψ_0 at the lattice site i . Similarly u_i and v_i are the amplitudes of fluctuations at lattice site i . Here ω is the frequency of the fluctuations, which is in general complex to encapsulate the instabilities of the system. Substituting Eq. (3.4) into Eq. (3.3), we obtain the following eigenvalue equations

$$\begin{aligned} \omega u_i &= (iP' + \Delta') u_i + \sum_{\langle j \rangle} J_{ij} u_j + (1 - i\alpha_{NL}) \psi_{(0,i)}^2 v_i, \\ \omega v_i &= (iP' - \Delta') v_i - \sum_{\langle j \rangle} J_{ij} v_j - (1 + i\alpha_{NL}) (\psi_{(0,i)}^*)^2 u_i, \end{aligned} \quad (3.5)$$

where $\Delta' = \Delta + 2|\psi_{(0,i)}|^2$ and $P' = P - \frac{\Gamma}{2} - 2\alpha_{NL}|\psi_{(0,i)}|^2$. The eigenvalues of Eq. (3.5) are plotted as a function of the quantum number l of the fluctuation in Fig. 3.3. The modes corresponding to $\text{Im}(\omega) > 0$ indicate instability in the system and the spatial profile of the intensity of the modes are plotted in Fig. 3.4, which correspond to edge states. Thus, polariton-polariton scattering induces coupling between the corner mode and edge modes.

Figure 3.3: Real and imaginary parts of the eigenvalues of the fluctuations as a function of quantum number l in (a) and (b) respectively. The states located at the corners are shown in red. The bulk and the edge states are represented by blue and green dots respectively. Note that the total eight states corresponding to the four corners are in the band gap in (a). The positive imaginary part implies instability in the system and in the real space those four states having $\text{Im}(\omega) > 0$ correspond to different edge states. This also indicates that the pumped corner state couples to more than one edge state. Parameters: $J' = 5$, $\Delta = -0.3$, $\alpha_{NL} = 0.2$, and $f_s = \sqrt{1.4}$.

Figure 3.4: Spatial profile of the intensity corresponding to the states whose $\text{Im}(\omega) > 0$ in Fig. 3.3(b).

3.5 Corner-corner coupling mediated via parametric interaction

Now instead of depending on the linear Bogoliubov theory we directly simulate the dynamics of the system described by Eq. 3.3 starting from an initial vacuum state corresponding to a zero mean field. At each time step, the solution can be expanded as a linear superposition of the eigenstates of the linear system as

$$\psi_i(t) = \sum_n C_n(t) \psi_{(n,i)}, \quad (3.6)$$

where $C_n(t) = \sum_i \psi_i(t) \psi_{(n,i)}^*$. Physically $|C_n(t)|^2$ represents the intensity (or overlap) of each eigenstate. A plot of $|C_n(t)|^2$ as a function of time is shown in Fig. 3.5 for n corresponding to the excited corner, edge states and another adjacent corner state. Their spatial profiles are shown in sec. 3.10. Fig. 3.5(a) shows that in a very short time (much faster than the chosen range in the time plotted), the excited corner reaches its steady state and starts to couple to the edge states as shown in Fig. 3.5(b). Note that the excited corner state couples with more than one edge state and they have different intensity profiles with time (here as an example we have plotted only one). In Fig. 3.5(c) the intensity profile of the adjacent corner is plotted with time and the nonzero value indicates that there is coupling between the two corners. Without the nonlinear terms, the coupling of the excited corner state to the edge states or its adjacent corner state vanishes (see sec. 3.10). In a later time we observe significant intensity corresponding to another edge state as shown in Fig 3.5(d), which eventually has no effect on the steady adjacent corner state.

The possibility that the nonlinear terms directly couple the corner states can not be discarded from the obtained results so far. Consequently, the analysis of the linear fluctuations will be repeated in the eigenbasis in the next section.

Figure 3.5: The overlap $|C_n(t)|^2$ as a function of time for (a) the excited corner, $n = 1251$, (b) the edge state corresponding to $n = 1208$, and (c) the adjacent corner, $n = 1249$. (d) In later time, due to higher order terms, another edge state corresponding to $n = 1238$ appears in the system. The nonzero value of intensity of the adjacent corner indicates that both the corners are coupled.

3.6 Proof of no direct coupling between corner states

To show that the nonlinear terms do not directly couple the corner states, we switch to the eigenstate basis, using the transformation

$$\tilde{\psi}_n = \sum_i (U^{-1})_{ni} \psi_i, \quad (3.7)$$

where U is the unitary matrix that diagonalizes the linear problem. Eq. (3.3) can be expressed in the eigen basis as

$$i \frac{\partial \tilde{\psi}_n}{\partial t} = E_n \tilde{\psi}_n + \sum_{n'n''n'''} \alpha_{nn'n''n'''} \tilde{\psi}_{n'}^* \tilde{\psi}_{n''} \tilde{\psi}_{n'''} + \tilde{F}_n, \quad (3.8)$$

where $\alpha_{nn'n''n'''} = (1 - i\alpha_{NL}) \sum_i U_{ni}^{-1} U_{in'}^* U_{in''} U_{in'''}$ represents the effective polariton-polariton interaction and nonlinear loss, which is no longer a local term (instead it is n dependent). Here $\tilde{F}_n = \sum_i (U^{-1})_{ni} F_i$ is the coherent excitation term in the eigen basis. We recall that the coherent pump F has the profile matching the corner state $n = 1251$, which we define as $n = c$. Consequently, $\tilde{F}_n = 0$ for $n \neq c$,

and the second term in the r.h.s of Eq. (3.8) will only survive if $n' = n'' = n''' = c$. The dynamics of the system is then governed by

$$i \frac{\partial \tilde{\psi}_c}{\partial t} = E_c \tilde{\psi}_c + \alpha_{ccc} |\tilde{\psi}_c|^2 \tilde{\psi}_c + \tilde{F}_c. \quad (3.9)$$

Here E_c is the energy detuning between the eigen-value corresponding to the corner and laser. Once the stationary state ($\tilde{\psi}_{0,c}$) from Eq. (3.9) is obtained, we can define the linear Bogoliubov fluctuations of the form

$$\tilde{\psi}_n = \tilde{\psi}_{0,c} \delta_{n,c} + \tilde{u}_n \delta_{n,m} e^{-i\omega t} + \tilde{v}_n^* \delta_{n,m'} e^{i\omega^* t}, \quad (3.10)$$

which demonstrate that a pair of polaritons from the stationary state can scatter to states $n = m$ and $n = m'$ conserving energy. Inserting Eq. (3.10) into Eq. (3.8):

$$\begin{aligned} \omega \tilde{u}_m &= \left(E_m + (\alpha_{mccm} + \alpha_{mcmc}) |\tilde{\psi}_{0,c}|^2 \right) \tilde{u}_m + \alpha_{mm'cc} \tilde{\psi}_{0,c}^2 \tilde{v}_{m'}, \\ \omega \tilde{v}_m' &= \left(-E_{m'}^* - (\alpha_{m'ccm'} + \alpha_{m'cm'c})^* |\tilde{\psi}_{0,c}|^2 \right) \tilde{v}_m' - \alpha_{m'mcc}^* (\tilde{\psi}_{0,c}^2)^* \tilde{u}_m. \end{aligned} \quad (3.11)$$

Physically Eq. (3.11) and Eq. 3.5 are the same although written in different bases. Solving the eigenvalues for all possible pairs of m, m' we obtain Fig. 3.6 where only the positive imaginary values are plotted. We see that there is no instability for m or m' corresponding to the corner states, which proves that the intensity dynamics plotted for the adjacent corner in Fig 3.5(c) is not due to the direct coupling between the corners. We can conclude that polaritons from the pumped

Figure 3.6: Zoomed view of the positive imaginary value of the energy of the Bogoliubov spectrum in the eigenstate basis calculated from Eq. (3.11). The green lines corresponds to $m = c$ and $m' = c$. The figure shows that there is no instability for m or m' corresponding to any corner states, which proves that there is no direct coupling between two corners. The instability points correspond to the edge states showing that they are responsible for intermediating the coupling between adjacent corner states.

corner first couple to the edge states, then the polaritons from the edge states couple to the adjacent corner resulting in a nonzero intensity at the adjacent corner. One of the indices corresponding to the maximum instability is $m = 1208$, whose overlap is plotted in Fig. 3.5(b). The state in Fig. 3.5(d) does not appear in Fig. 3.6 indicating that its significant intensity is due to higher order scattering processes not taken into account in the linear Bogoliubov theory. We checked that all the states having positive imaginary values in Fig. 3.6 are edge states and the intense modes (white and yellow ones) are plotted in Fig. 3.7. Their nonzero overlap, as calculated from Eq. 3.6, indicates the coupling with the excited corner state.

Figure 3.7: Spatial profile corresponding to modes m or m' whose $\text{Im}(\omega) > 0$ in Fig. 3.6.

3.7 Demonstration of transfer of binary information

Here we demonstrate that the coupling between corners mediated by parametric instability is sufficient to transfer binary information. Such demonstration is based on using near-resonant coherent laser fields at each corner to place them in a bistable regime, which forces each corner state to either be in a low or high intensity state. Switching the state of one corner results in a later switch of the adjacent

corner state, corresponding to a transfer of information. Remarkably, this can occur even in the presence of a realistic level of disorder.

To show bistability we consider Eq. (3.3) with the pump profile of the eigenstate $n = 1251$ and $n = 1249$ respectively, and slowly vary the intensity of the pump in time (over 9000 units ~ 6000 ps). The intensity corresponding to the corner sites ($C1$ and $C2$) as a function of pump intensity is plotted in Fig. 3.8(a), where the characteristic hysteresis loops show that bistability is present (different corners are non-identical due to the lack of symmetry of the underlying lattice, which is why the hysteresis curves are slightly different). Gradually increasing the pump intensity to the level marked by the vertical grey lines in Fig. 3.8(a), allows each corner state to be initialized in its lower intensity state. Next, we apply a coherent Gaussian shaped pulse at corner $C1$ of the form $F = F_0 \exp[-((x - x_0)^2 + (y - y_0)^2)/L^2 - (t - t_0)^2/\tau^2 - i\omega_p t]$, where F_0 is the amplitude of the pulse which is launched at (x_0, y_0) , the coordinates of $C1$; L and τ are the widths of the pulse in space and time. The time dynamics of both the corner sites in presence of the pumps and pulse is plotted in Fig. 3.8(b), which shows that the pulse switches $C1$ from its lower state to the upper state and then due to the parametric scattering, $C2$ also switches to the upper intensity state.

Figure 3.8: (a) Hysteresis curve of polariton density vs pump power for the two corner sites ($C1$ and $C2$). For the following plots we fix the value of the pump at each site to the intensity indicated by the vertical gray lines. (b) Time dynamics of the corner sites, $C1$ and $C2$ in the presence of a continuous pump at both corners and pulse at $C1$ (which is gradually turned on to avoid unwanted jumps in the initial stages). A pulse switches $C1$ from the lower to upper intensity state and then due to the parametric scattering, $C2$ is also switched. We consider that the frequencies of both pumps and the central pulse frequency are same. All the blue curves correspond to $C1$ and red curves correspond to $C2$. Parameters: $\Delta = -0.3$, $F_0 = 33$, $x_0 = 1$, $y_0 = 1$, $L = 1$, $t_0 = 1283$, and $\tau = 15$.

To show that the switching at corner $C2$ is due to the parametric scattering and the pulse at $C1$ can not alone induce switching at $C2$, we repeat 3.8(b) but without the continuous pump at $C1$. The time dynamics plotted in Fig. 3.9 shows that in this case the site $C2$ stays at the lower intensity state. In conclusion, the

switching does not occur in the absence of the pump at C1, that is, it is only when the first corner supports bistability that a binary signal can be transported and not just a direct effect of the applied pulse.

All the calculations in this section were performed considering also an on site disorder with uniform distribution and peak to peak magnitude of 0.03. This, physically corresponds to a disorder strength of $30 \mu\text{eV}$ (for $J = 1 \text{ meV}$), which has been recorded experimentally [265]. The most important parameters in our scheme to realize the coupling are the nonlinear interaction and loss terms. Here the nonlinear self-energy in the system becomes about 1.36 meV , which is within experimental limits [266]. We have taken the nonlinear loss coefficient as $\alpha_{NL} = 0.2$, where a similar value was used in Ref. [104].

Figure 3.9: Time dynamics of the two corner sites $C1$ and $C2$ in the presence of the same pulse at $C1$ and the continuous pump at $C2$, that is, same as in [3.8](b), but without the continuous pump at $C1$. The intensity in $C1$ is insignificant because there is no continuous pump but $C2$ is still in the lower state. It implies that the pulse alone cannot switch the adjacent corner. The blue curve corresponds to $C1$ and red curve corresponds to $C2$.

3.8 Advantage of topological corner states over regular square lattice

Here we show the advantage of information transfer with topological states over trivial states. We consider the same parameters and the same pump profile as the ones used in Fig. [3.8], for a simple square lattice with the same average coupling strength between sites. We observe that under this condition, the square lattice does not reach a steady state even with 5000 ps (see Figs. [3.10](a-b), where we plot the time dynamics of the same corner site ($C1$) and the whole system, respectively). In contrast, the topological corner modes reach their steady state around 700 ps (see the blue curve before the pulse was launched in Fig. [3.8](b)). It happens due to the fact that in the considered model the only dissipation present is the nonlinear decay ($\alpha_{NL}|\psi_i|^2\psi_i$), which should grow to compensate the pump P to form a steady state. For the square lattice, since there is no localized mode,

for a particular site, the nonlinear decay term is very weak and as a result it will reach steady state very slowly. On the other hand, the topological corner modes are well localized and as a result this problem does not arise. To resolve this, we add some linear decay (γ) in the system, which modifies Eq. 3.3 as,

$$i\frac{\partial\psi_i}{\partial t} = \left(\Delta - \frac{i\Gamma}{2} + iP - i\gamma\right)\psi_i + J_{ij} \sum_{\langle j \rangle} \psi_j + (1 - i\alpha_{NL})|\psi_i|^2\psi_i + F_i. \quad (3.12)$$

It indeed helps the system to reach the steady state quickly, but we do not observe bistability for the regular square lattice within the same window of pump intensity (or in fact a larger intensity window either, see Fig. 3.10(c)). But we still observe bistability for the corner modes even with the linear decay, shown in Fig. 3.10(d). In this case polaritons are localized and experience a stronger nonlinear interaction than the case of delocalized polaritons in a square lattice that automatically spread out over a wider area.

In other words, the advantage of the scheme with topological corner states over a regular lattice is that it operates faster and at lower power.

Figure 3.10: (a-b) Time dynamics of the same corner site $C1$ and the total intensity of the regular square lattice for the pump of intensity 1×10^{-3} respectively. Parameter: $\Delta = -0.3$. (c) Plot of the population of $C1$ of the regular square lattice as a function of pump intensity. (d) Hysteresis curve of the corner modes in presence of linear decay. In (c) and (d) a supplementary dissipation $\gamma = 0.05$ is included according to Eq. 3.12 to ensure that the regular square lattice reaches a stationary state in a reasonable time.

3.9 Continuous model

Here we describe the scheme to realize the four different couplings used in our model and by solving the Gross-Pitaevskii equation in a continuous space (without tight-binding approximation) of a coupled micropillars system we realize topologically protected corner states. We start by solving the time independent equation in the absence of nonlinear terms

$$-\frac{\hbar^2}{2m_{\text{LP}}}\nabla^2\psi(\mathbf{x}) + V(\mathbf{x})\psi(\mathbf{x}) = E\psi(\mathbf{x}), \quad (3.13)$$

where m_{LP} is the polariton mass and $V(\mathbf{x})$ is the potential profile of the system described in Fig. 3.11(a). In this scheme, the micropillars are coupled evanescently in which the strength of the hopping constant can be controlled by changing the center to center distance between the micropillars. The micropillars are shown in green in Fig. 3.11(a). The center to center distance between the micropillars is varied deliberately in order to realize two positive couplings in the corresponding tight-binding model, J and J' , where $J < J'$. In order to realize the negative couplings ($-J$ and $-J'$) we introduce another set of micropillars, which we call auxiliary pillars, in between the main micropillars. The auxiliary pillars are shown in blue in Fig. 3.11(a). If the auxiliary pillars have potential depth (V_a) greater than that of the main micropillars (V_0) then the negative hopping between the main micropillars can be realized [263]. In Fig. 3.11(b) the eigenmodes corresponding to a 14×14 system size are plotted. By proper choice of parameters, topologically protected corner modes can be realized. The spatial profile of the corner modes and

Figure 3.11: (a) The potential profile of the system consisting of 14×14 (main) micropillars. The green dots correspond to the main micropillars and the blue dots correspond to the auxiliary micropillars by which the negative hopping is realized. The center to center distance between the micropillars is varied to realize J and J' couplings. (b) Plot of the energy eigenvalues of the system. The red dots correspond to the topologically protected corner states, the green dots correspond to the edge states and the blue dots correspond to the bulk states.

Figure 3.12: Spatial profile of the corner states (a-d), and two (of many) edge states (e-f) obtained from the solution of Eq. [3.13](#).

edge modes are shown in Fig. [3.12](#). The diameter of all the micropillars was taken to be $3 \mu\text{m}$ [\[169\]](#), and the center to center distances between the micropillars were $10.8 \mu\text{m}$ and $5.04 \mu\text{m}$ to realize J and J' couplings, respectively. The auxiliary micropillars are placed exactly at the middle of the two main micropillars whenever the negative hopping between the main micropillars is needed. The potential depth of the main micropillars is $V_0 = 1.8 \text{ meV}$ and that of the auxiliary micropillars is $V_a = 50.6 \text{ meV}$. The mass of the polaritons $m_{\text{LP}} = 10^{-5}m_e$, where m_e is the free electron mass. The obtained band gap corresponding to the above mentioned parameters is around 0.2 meV , which is higher than the typical disorder strength in the exciton-polariton systems [\[265\]](#).

3.10 Overlap as a function of time in the absence of any nonlinear terms

To show that the parametric instability arises purely due to the nonlinearity in the system we evolve Eq. [3.3](#) in time, from the initial condition $\psi_i = 0$, in the absence of the nonlinear terms. We plot the overlap $|C_n(t)|^2$ corresponding to the edge state $n = 1208$, and for the adjacent corner state $n = 1249$ in Figs. [3.13](#)(a-b). The order of the overlap implies that without nonlinearity, the coupling of the excited corner state to the edge states or its adjacent corner state is insignificant. In Figs. [3.13](#)(c-e) we plot the spatial profile of the states corresponding to $n = 1208$, $n = 1249$, and $n = 1238$, respectively, which shows that they correspond to an edge state, adjacent corner state, and another edge state.

Figure 3.13: (a-b) Overlap $|C_n(t)|^2$ as a function of time corresponding to the same edge $n = 1208$ and adjacent corner $n = 1249$ in Fig. 3.5 but with no nonlinear terms. (c-e) Spatial profile corresponding to the edge state $n = 1208$, adjacent corner $n = 1249$, and another edge state $n = 1238$, respectively.

3.11 Conclusions

We considered the appearance of topologically protected corner states in a square array of coupled exciton-polariton micropillars. These systems can be distinguished from other topological photonic systems by the presence of significant nonlinearity arising from polariton-polariton scattering. Here we found that such processes allow corner states to be nonlinearly coupled to edge states, which can be further coupled to adjacent corner states. That is, in the nonlinear regime, edge states act as intermediaries between corners. It is generally speculated that topological modes can be relevant in information processing and we anticipate that the ability to couple multiple topological modes in a single system will be essential to such directions [12, 18].

Chapter 4

Exciton-polariton Spin filter

It is well known that topological states can act as robust information carriers. Topological polariton systems are characterized by the appearance of chiral edge states in a strip geometry, which have a specific polarization that mixes the two pure spins and consequently, they do not preserve the individual spins. Furthermore, chiral edge states always appear in pairs, which propagate in opposite directions on opposite edges of the strip. While there has been a very clear fundamental interest in these states, the suggestion that they can act as information carriers is hindered by this fact that they appear in pairs, as it means that in any system there will be an unwanted counterpropagating signal, potentially leading to detrimental feedback effects, unless the strip is made wide enough, which will lead to other problems in scalability.

In this chapter we propose theoretically a feedback-free optical spin filter based on the exciton-polariton system, where the system allows only one spin while the propagation through the other edge is prohibited. This chapter is reproduced from S. Mandal, R. Banerjee, and T. C. H. Liew, *Phys. Rev. Applied* **12**, 054058 (2019) [267].

4.1 Motivation

It has been recognized long ago that one of the crucial ingredients for optical circuits is the isolation of input and output [268]. Equivalently, circuits should operate in a one-way fashion, where feedback in the backward direction is suppressed. As Maxwell's equations are time reversal invariant, the realization of such a requirement is not obvious in many optical systems, and reflections must be considered carefully. With the emergence of topological photonics [200, 205], special techniques were introduced to artificially break time-reversal symmetry (e.g., helical waveguide arrays as in Floquet topological insulators [269]) or exotic material solutions employed at particular wavelengths (e.g., magnetically sensitive gyrotropic materials [210]). One of the well-known features of topological photonic lattices is the appearance of chiral edge states. As these propagate in only a certain direction and do not undergo backscattering, it has been natural to expect that such states are relevant for the robust propagation of optical signals.

Instead of breaking time-reversal symmetry directly in a photonic system, an alternative is to couple it to a system in which time-reversal symmetry is broken. Exciton-polaritons inherit a Zeeman splitting due to their excitonic component when placed in a magnetic field [52]. Since such micropillars can be arranged into lattices it has been possible to form topological polariton systems [247], accounting for the effect of TE-TM splitting (see sec. 1.4.7) [246, 249, 270].

As we know, exciton-polaritons also exhibit a significant nonlinearity, which in theory was predicted to itself induce topological behavior [271], antichiral behavior [272], topological solitons [273], and solving *NP*-hard problems [274]. Separate from topological physics, nonlinearity was shown to provide low-energy polaritonic switches [150, 275, 276] and realize other information processing elements such as transistors [12, 13, 276, 277], routers [18, 278, 279], and amplifiers [280, 281]. The possible use of topological polariton states at the edges of a strip to couple these elements suffers however from an obvious drawback: such states come in pairs, which propagate in opposite directions on the opposite edges of the strip [246, 249].

Furthermore, it has long been expected that polaritonic devices would make use of the spin degree of freedom [282], which can take one of two spin projections on the structure growth axis (denoted $\sigma_{\pm}$). Individual spin switches [283] and gates [174] have encoded information in this spin. Unfortunately though, topological polariton states have a specific polarization that mixes the σ_{+} and σ_{-} components [246, 249], which would not allow the preservation of individual σ_{+} or σ_{-} spin polarized wavepackets.

In this chapter, we aim to develop the recent advances in topological polariton physics to introduce a channel for connecting polaritons of a given spin. We aim for the channel to be reasonably thin and allow only one spin to propagate in a specific direction. To meet this aim, we consider a strip of honeycomb polariton lattice, which is composed of two interlocking sublattices, A and B (see Fig. 4.1(a)). The sites of the different sublattices are engineered with different onsite energy. Accounting for a magnetic field and TE-TM splitting, we find that the states at different edges are split in energy and become almost completely spin polarized. Furthermore, an edge state of a given spin can be embedded in an energy range where the only existing states have opposite spin polarization. As the scattering of polaritons with disorder tends to be spin conserving, this maintains a unidirectional propagating edge state for a specific spin, which is not possible with a regular topological polariton system (see Fig. 4.1(b)). A further key difference with existing topological polariton realizations is that the strip can be made remarkably thin (three lattice constants thick) and still maintain its unidirectional spin polarized behavior.

4.2 Scheme

We start by considering a planar semiconductor microcavity containing exciton-polaritons subjected to a resonant excitation, expressed by the Gross-Pitaevskii

Figure 4.1: (a) Schematic diagram of coupled polariton micropillars arranged in a honeycomb strip where two different sub-lattices are indicated by two colors. The considered system can act as a spin filter by allowing the polaritons with one of the spins in the system to propagate robustly from right to left while the propagation in the opposite direction is restricted. The red arrow indicates the perpendicularly applied magnetic field (H_z). (b) Potential application of the polariton spin filter (SF) as a channel that can be used to couple two polariton networks without feedback. The same can not be done using a polariton topological insulator (TI) where there exists a possibility of feedback via an opposite edge state (right).

equation,

$$\begin{aligned}
 i\hbar \frac{\partial \psi_{\sigma_{\pm}}(x, y)}{\partial t} = & \left[-\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} + V(x, y) - i\frac{\Gamma}{2} \pm \Delta_z \right] \psi_{\sigma_{\pm}}(x, y) \\
 & + \Delta_T \left(\frac{\partial}{\partial y} \pm i\frac{\partial}{\partial x} \right)^2 \psi_{\sigma_{\mp}}(x, y) + F_{\sigma_{\pm}}(x, y) e^{i(k_0 x - \omega_p t)}.
 \end{aligned} \tag{4.1}$$

This is the same equation as introduced in Eq. [1.64](#), although including the effect of an applied magnetic field perpendicular to the microcavity plane (Faraday configuration). To recall, here $\psi_{\sigma_{\pm}}(x, y)$ are the wave functions for $\sigma_{\pm}$ spin polarized polaritons; $V(x, y)$ is a potential describing a strip of honeycomb lattice having zigzag edges along the y direction; Γ is a uniform decay throughout the sample; $2\Delta_z$ and Δ_T are the Zeeman and TE-TM splittings, respectively (where the Zeeman splitting is the consequence of the applied magnetic field). $F_{\sigma_{\pm}}(x, y)$ are the circular polarization components of a resonant excitation with frequency ω_p . Each micropillar is modelled by placing a Gaussian of width σ at each site [\[248\]](#) and

the lattice has a periodicity a along the x direction, such that,

$$V(x, y) = V(x + a, y) = -(v_o \pm \delta) \sum_{x_n, y_n} \exp \left[-\frac{(x - x_n)^2 + (y - y_n)^2}{\sigma^2} \right]. \quad (4.2)$$

Here the summation runs over all sites, $(v_o + \delta)$ is the potential depth corresponding to the A sublattices, and $(v_o - \delta)$ is the potential depth corresponding to the B sublattices. In principle, the energy detuning, δ , between the A and B sublattices can be induced by locally adjusting the thickness of the cavity or by non-resonantly pumping one of the sublattices, where the excitonic reservoir will induce a blueshift locally [169]. Alternatively, micropillars corresponding to the A and B sublattices could be engineered with different diameters (see sec. 4.5).

In order to know the characteristics of the system we consider the linear band structure by setting $F_{\sigma\pm} = 0$ and $\Gamma = 0$ in Eq. (4.1) and after applying the Bloch theorem the wave function can be written as, $\psi_{\sigma\pm}(x, y) = u_{\sigma\pm}(x, y) \exp(ikx)$. Here $u_{\sigma\pm}(x, y)$ are the Bloch wave functions, which are periodic along the x direction and k is the wave-vector in the x direction. Upon substitution, Eq. (4.1) leads to an eigenvalue problem, which can be solved to obtain the band structure.

In Figs. 4.2 (a-b) and (c-d) we plot the dispersion without (i.e., $\delta = 0$) and with (i.e., $\delta \neq 0$) the onsite term, respectively. For $\delta = 0$, we obtain the band structure corresponding to the polariton topological insulator (TI) (see Fig. 4.2 (a)) where the time reversal symmetry is broken by the Zeeman splitting. Here, the bulk modes are gapped and in the band gap robust edge states appear. In this case, the edge states come in counter propagating pairs, meaning that within the band gap of each propagating edge state there is another one propagating in the opposite direction along the other edge.

The inclusion of δ (i.e., $\delta \neq 0$) breaks the inversion symmetry and the parameters can be chosen such that the band gap at one of the Dirac points gets closed resulting in a topologically trivial system. The dispersion of the bulk modes is similar to the ones in bilayer graphene with a perpendicular bias [284]. Although, in this case the edge states are completely separated in energy (see Fig. 4.2 (c)). Note that there is no counter propagating edge mode present in the energy window of interest. We have also plotted the localization of the edge states in Fig. 4.2(e), which shows that the edge states in the spin filter (SF) regime are more localized than the standard polariton TI.

To emphasize the interest of the obtained states, we plot the band structures and colour code them depending upon the contribution of the spins. In the case of a polariton TI (see Fig. 4.2 (b)) we find that within the bandgap the edge states have mixed spin.

On the other hand in the SF regime (Fig. 4.2 (d)) the edge states have almost pure spin and more importantly the top most edge state is surrounded by only the bulk modes with opposite spin polarization. We can then expect σ_+ spin polarized polaritons to propagate through the edge mode without backscattering, manifesting a robust SF.

Considering typical parameters, we have set: $a = 3 \mu\text{m}$; $m_{\text{LP}} = 2.5 \times 10^{-5} m_e$ with m_e the free electron mass; $v_o = 16.5 \text{ meV}$; $\delta = 0.54 \text{ meV}$; $\sigma = 0.45 \mu\text{m}$;

Figure 4.2: Band structure of the system under consideration of onsite energy $\delta = 0$ in (a-b), corresponds to standard polariton TI and $\delta \neq 0$ in (c-d), corresponds to polariton spin filter (SF). In (a) and (c) the states residing in bulk are shown in grey whereas blue and magenta correspond to the states located near the two edges. In (b) and (d) the same dispersions with spin contribution are plotted where red corresponds to the states having spin σ_+ and green corresponds to the states having spin σ_- . The dashed lines in (c-d) represent the energy window where the edge state is surrounded by the bulk states with opposite spins and also there is no counter propagating edge state. (e) comparison of the localization of the edge states between the polariton SF and polariton TI.

$\Delta_T = 0.15 \text{ meV}\mu\text{m}^2$; and $\Delta_Z = 0.18 \text{ meV}$. This gives an energy interval of the SF edge state of around 0.12 meV , which is well above the linewidth of modern

samples. Generally, the SF regime can be found for $\varepsilon_g \simeq 2\Delta_z \geq \Delta_T$, where ε_g is the trivial bandgap induced by the broken inversion symmetry (see sec. 4.6) and Δ_T determines the group velocity of the edge state. It should be stressed that instead of the magnetic field (Zeeman splitting), the nonresonant excitation can itself break the symmetry between the two spins [285, 286]. For the rest of the paper we will focus on the top most edge state and although here only σ_+ spin polarized polaritons are filtered, the same can be done for the σ_- spin polarized polaritons (see sec. 4.5).

4.3 Spin filtering

To validate our claim we excite the top most edge state resonantly with a continuous linearly polarized Gaussian shaped pump by setting $F_{\sigma_+} = F_{\sigma_-}$. The spin polarization degree is defined as,

$$s_z(x, y) = \frac{|\psi_{\sigma_+}(x, y)|^2 - |\psi_{\sigma_-}(x, y)|^2}{|\psi_{\sigma_+}(x, y)|^2 + |\psi_{\sigma_-}(x, y)|^2}, \quad (4.3)$$

which corresponds to the degree of circular polarization as discussed in sec. 1.4.6. The spatial distribution of $s_z(x, y)$ is plotted in Fig. 4.3(a) in the SF regime, which shows that although we have excited both spin states, only σ_+ spin polarized polaritons populate the top edge and propagate over 100 μm from the spot of excitation. Due to the finite lifetime the intensity decreases with the increase of propagation distance. σ_- spin polarized polaritons are instead scattered randomly into the bulk. To demonstrate that there is no state of σ_+ spin polarized polaritons to propagate in the reverse direction, we consider injecting both spins from the opposite corner of the top edge (see Fig. 4.3(c)). We observe that the σ_+ spin polarized polaritons can not propagate from left to right of the system; instead they are converted to σ_- polaritons and scatter into the bulk. This confirms that the system can act as a feedback free unidirectional spin filter. On the other hand, in a polariton TI the polaritons with mixed spins rotate through the edges of the sample in anticlockwise direction (See Fig. 4.3(b)). Although the exact contribution of the spins in $s_z(x, y)$ can be changed slightly depending upon the state that is being excited, there is always a significant contribution from both the spins.

In all our calculations, we have taken a lifetime around 60 ps, close to that reported in [169, 247]. The SF would also work with shorter lifetime, although the propagation distance would decrease (see Fig. 4.9). Higher lifetimes such as those reported in [39] are thus the ideal.

4.4 Robustness and strip width

Fig. 4.4 (a-d) considers the propagation of a σ_+ polarized pulse (obtained by multiplying the coherent excitation with a Gaussian in time) in the case where a micropillar is removed from the top edge. We observe that near the defect the

Figure 4.3: Spatial distribution of the spin degree defined in Eq. 4.3 for the polariton SF is shown in (a), where polaritons having both spins are pumped but only σ_+ spin polarized polaritons flow one-way. The same for polariton TI is shown in (b), where polaritons with mixed spins flow counter-clock wise through the edges of the sample. (c) Feedback suppression effect where the σ_+ spin polarized polaritons can not flow from left to right, instead they are converted to σ_- spin polarized polaritons and scatter randomly into the bulk. The red arrows indicate the position of the continuous pump. Parameters: pump width = $9 \mu\text{m}$; $\Gamma = 0.01 \text{ meV}$; $k_0 = 2.5/a$ and $\omega_p = -1.65 \text{ meV}/\hbar$ are used in (a) and (c); $k_0 = 2.35/a$ and $\omega_p = -1.98 \text{ meV}/\hbar$ are used in (b).

pulse spreads a little but does not backscatter and after some time the pulse passes the defect. This further proves the robustness of the edge state.

In real systems, disorder is present throughout. Fig. 4.4 (e) shows the propagation of a σ_+ polarized pulse in the presence of a continuous disorder potential, which is characterized by a root mean squared amplitude of $30 \mu\text{eV}$ and a correlation length of $3 \mu\text{m}$ (corresponding to typical experimental values [265]). Again, no backscattering of the σ_+ polarized wavepacket is observed.

In addition to supporting one-way propagation, a channel for connecting information processing elements should not be too wide, so as not to limit the footprint

Figure 4.4: (a-d) Propagation of a σ_+ coherent pulse in a honeycomb lattice strip with one of the sites removed. (e) Propagation of the same pulse in a honeycomb lattice strip with spatial disorder. The colour scale represents the σ_+ polarized intensity, integrated over the strip width (y -coordinate). The green curve indicates the average position of the wavepacket and the dashed lines the variance. These are defined as $x_m = \int x |\psi_{\sigma_+}|^2 dx dy / I_0$ and $x_v^2 = \int x^2 |\psi_{\sigma_+}|^2 dx dy / I_0 - x_m^2$, respectively, where $I_0 = \int |\psi_{\sigma_+}|^2 dx dy$. The parameters were the same as in Fig. 4.3(a).

Figure 4.5: Band structure corresponding to a very short strip consisting of three unit cells along the y direction for TI shown in (a) and the same for SF shown in (b). The edge states in the topological gap corresponding to the TI disappear. (c) Spatial distribution of the spin degree for SF. The red arrow indicates the position of continuous pump. Parameters were kept the same as those in Fig. 4.3(a).

area of future systems. In Figs. 4.5 (a-b) the band structure for a narrow strip having three unit cells along the y direction is plotted for a polariton TI and SF, respectively. Due to the lack of bulk states, the well spread edge states in the polariton TI (see Fig. 4.2(e)) vanish while comparatively more localized edge states in the polariton SF remain unaffected resulting in σ_+ spin propagation as shown in Fig. 4.5 (c).

4.5 Band structure of the spin filter using micropillars of different diameters

In this section, we consider micropillars of two different diameters to construct the honeycomb lattice. In the considered system two types of micropillars have different mode localization energies due to their different diameters, which automatically gives rise to the onsite detuning, δ . In Fig. 4.6(a) a schematic diagram of such a system of micropillars arranged in a honeycomb lattice is considered, which is infinite along the x direction and finite along the y direction having the zigzag edges. The band structure is calculated the same way as done in sec. 4.2. The diameters of the micropillars are considered to be $2.4 \mu\text{m}$ and $1.9 \mu\text{m}$ [247], which represent the A and B sublattices, respectively, and the depth of the micropillars are the same and taken 16.5 meV . In order to realize the coupling between the two neighboring sites, the micropillars have an overlap between them such that the unit cell size along the x direction is $3 \mu\text{m}$. In Fig. 4.6(b) the band structure of the system is plotted without taking into account the Zeeman splitting, $2\Delta_z$, and the TE-TM splitting, Δ_T . There is a trivial bandgap, ε_g , which is due to the breaking of the inversion symmetry. Since, the edges of the lattice are composed of different types of micropillars, the edge states located at the two edges of the lattice are no longer symmetric. There is no term ($\Delta_z = 0$) to break the symmetry between the σ_+ and σ_- spin polarized polaritons and as a result the band structure is degenerate for both spins. The effect of Δ_z is shown in Fig. 4.6(c), which lifts the degeneracy between the two spins and the band structure of the σ_+ and σ_- polarized polaritons gets shifted in energy by $2\Delta_z$. Figs. 4.6(d-e) show the SF band structure taking into account both Δ_z and Δ_T terms. The parameters are chosen such that the σ_- spin polarized edge state is surrounded by the σ_+ spin polarized bulk modes and hence one can show that the σ_- spin polarized polaritons will propagate robustly along the lower edge from left to right (determined by the group velocity of the edge state which is positive in this case) and the system will act as a SF where σ_- spin polarized polaritons will be filtered out. It should be noted that, by reversing the sign of the magnetic field (which changes Δ_z to $-\Delta_z$) one can easily change the spin polarization of the SF edge state (see Fig. 4.7).

Figure 4.6: (a) Schematic diagram of a honeycomb lattice having zigzag edges consisting of two different types of micropillars having different diameters. Band structure of the system under consideration for $\Delta_z = \Delta_T = 0$ in (b); for $\Delta_z = 0.18$ meV and $\Delta_T = 0$ meV μm^2 in (c); and for $\Delta_z = 0.18$ meV and $\Delta_T = 0.1$ meV μm^2 in (d-e). In (b) and (d) the bulk modes are represented in grey whereas blue and magenta represent the states located at the two edges. In (c) and (e) red and green represent the σ_+ and σ_- spin polarized modes, respectively. The mass of the polaritons is $2.5 \times 10^{-5} m_e$ with m_e the free electron mass.

4.6 Effect of TE-TM splitting on the band structure

In this section we obtain the band structure by solving Eqs. (4.1-4.2) corresponding to different Δ_T . In Fig. 4.8(a) the band structure of the system without Δ_z and Δ_T is plotted, which shows the trivial bandgap similar to Fig. 4.6(b). Fig. 4.8(b) shows the effect of Δ_z , which shifts the band structure for $\sigma_{\pm}$ spin polarized polaritons in energy. Figs. 4.8(c-d) show the band structures corresponding to $\Delta_T = 0.09$ meV μm^2 , showing that the SF regime can still exist even for a lower value of TE-TM splitting. However, the band structure starts to deviate from the SF regime for larger values of TE-TM splitting, $\Delta_T = 0.24$ meV μm^2 , as shown in 4.8(e-f). The SF edge state starts to be flat and also two edge states start to

Figure 4.7: Band structure of the system is shown in Fig. 4.6(a) for $\Delta_z = \Delta_T = 0$ in (a); for $\Delta_z = -0.18$ meV and $\Delta_T = 0$ meV μm^2 in (b); and for $\Delta_z = -0.18$ meV and $\Delta_T = 0.1$ meV μm^2 in (c-d). The colour codings are the same as those in Fig. 4.6. In this case, the SF works for σ_+ spin polarized polaritons.

appear within the same energy interval. To conclude, there is a great degree of freedom in choosing the parameters as long as $\varepsilon_g \simeq 2\Delta_z \geq \Delta_T$.

4.7 Effect of the polariton lifetime on the propagation distance

In this section, the propagation distance for different polariton lifetime is plotted using Eqs. (4.1), (4.2), and (4.3). For polaritons with lifetime 30 ps the propagation distance is about 60 μm . As expected, the propagation distance increases with polariton lifetime and the propagation distance reaches around 100 μm for 50 ps (see Fig. 4.9).

4.8 Discussion and Conclusion

Modern electronic devices and integrated circuits are limited by heating and time delay of communication between components. Optical devices seem promising candidates for fast interconnections and higher fidelity may reduce error cor-

Figure 4.8: Band structure of the system under consideration for $\Delta_z = \Delta_T = 0$ in (a), for $\Delta_z = 0.18$ meV and $\Delta_T = 0$ meV μm^2 in (b), for $\Delta_z = 0.18$ meV and $\Delta_T = 0.09$ meV μm^2 in (c-d), and for $\Delta_z = 0.18$ meV and $\Delta_T = 2.4$ meV μm^2 in (e-f). The colour codings are the same as those in Fig. 4.6.

rection overhead, resulting in less heating [1]. But optics needs to operate at lower threshold to be relevant, for which nonlinear optical systems are essential. Along with the nonlinearity, the spin degree freedom of exciton-polaritons make them extremely promising for nonlinear spintronics devices. The coupling between exciton-polariton condensates in lattices has been demonstrated but this coupling is always bidirectional and consequently, there will be unwanted feedback. Recently, in elliptical micropillars chain, feedback free polariton propagation was shown, although that does not conserve spin [287].

We have proposed theoretically a scheme to obtain an optical spin filter based

Figure 4.9: Spin propagation in real space for different values of Γ which corresponds to the polariton lifetime of 30 ps in (a), 40 ps in (b), 50 ps in (c), and 60 ps in (d). The red arrows indicate the position of the continuous pump. All other parameters were kept the same as those in Fig. 4.3(a).

on exciton-polaritons. The naturally present TE-TM splitting along with Zeeman splitting and an onsite detuning between sublattices in a polariton honeycomb lattice gives rise to a band structure where all the edge states of the system split in energy and there exists an energy window where one of the edge states with pure spin is surrounded by bulk states with opposite spin. Since there is no state with the same spin to backscatter to, polaritons having one of the spins in the system can propagate robustly through the edge state and thus spin filtering is achieved. The absence of the pairing backward propagating edge state reduces feedback in the system. Furthermore, the edge state in the polariton SF is more localized than those in a polariton TI, which allows to operate with very narrow channels. In contrast, regular polariton TI systems would require much wider

strips/lattices to support robust propagation and would cause mixing between spin polarized states. Although the proposed spin filter is not itself a device it can be an essential component of complete future circuits or networks.

Chapter 5

Artificial Life : Cellular automata

In this chapter we show theoretically that a lattice of exciton-polaritons can behave as a life-like cellular automaton, that is, a form of artificial life when simultaneously excited by a continuous wave coherent field and a time-periodic sequence of non-resonant pulses. This provides a mechanism of realizing a range of highly sought spatiotemporal structures under the same conditions, including: discrete, oscillating, and rotating solitons; breathers; soliton trains; guns; and chaotic behaviour. These structures can survive in the system indefinitely, despite the presence of dissipation and disorder, and allow universal computation. We start by introducing cellular automata, then we will discuss our results. This chapter is reproduced from R. Banerjee and T. C. H. Liew, [New J. Phys **22**, 103062 \(2020\)](#) [\[288\]](#).

5.1 Cellular automata

The origins of life have been questioned for longer than history can remember. One possible conjecture is that life emerged spontaneously, from a nonlinear system subjected to a particular stimulus. This has motivated the study of minimal models of artificial life, in particular cellular automata [\[289–297\]](#) such as Conway’s life. These show how remarkably complex behaviour associated to living organisms such as movement, growth, and replication, can appear from apparently simple nonlinear rules applied on a lattice. Here artificial life does not mean creating life artificially but to study the artificial systems that mimic the features of natural living systems.

A cellular automaton (CA) is a phenomenological model studied in different disciplines like physics, mathematics, theoretical biology, computability theory, complexity science, microstructure modelling, etc., to study the characteristics of natural living systems. It is comprised of a regular grid of cells of any dimensions but here we consider two dimensions. Each cell can have k -discrete states. For simplicity we consider two states: 0 (dead) and 1 (alive), as shown Fig. [5.1\(a\)](#). Each cell interacts with its neighboring cells in one of two ways: Von-Neumann interaction happens when there is influence of a cell’s vertical and horizontal neighbors, as shown in Fig. [5.1\(b\)](#), or there can be interaction via the Moore neighborhood, where each cell interacts with its eight neighbours (vertically, horizontally and

diagonally, see Fig. 5.1(c)). Later in this chapter we will consider only the Moore neighborhood (which gives rise to more interesting automata in simple lattices). Over time, the cells change state depending on their own state, the number of alive neighbouring states (independent of their relative position) and the automaton's rule they are following. More importantly all the cells change state at the same time. For example, consider the automata following the rule characterized by the behaviour that a dead cell is born if it has three alive neighbours and an alive cell survives if and only if it has two or three neighbours alive. Then if we start with an initial condition as shown in Fig. 5.1(d), after one iteration we will end up with Fig. 5.1(e). The explanation goes as follows: initially A is an alive state with only one alive neighbour and according to the rule it will die; B is a dead state with two alive neighbours so the state will still remain a dead state and so on⁵. Applying the rule one more time it updates to Fig. 5.1(f). This rule is known as Conway's life and we can compare the behavior of the automaton to real life. For example, the rule can be stated in the following way:

- Any live cell, if it is surrounded by less than two alive neighbors then it dies, as if by loneliness (*under-population*, a well known concept developed in biology).
- Any live cell, if it is surrounded by more than three neighbors then also it dies, as if by overcrowding (*over-population*)
- Any live cell for two or three alive neighbors, remains the same and goes to the next generation
- Any dead cell surrounded by exactly three neighbors will be born in the next generation, as if by *reproduction*.

Rules that define complexity of two-dimensional cellular automata are classified into four classes of increasing complexity [289, 293, 298]. In class 1 cellular automata, any initial condition is quickly driven to a homogeneous state. In class 2 automata, periodic patterns may emerge and be stable. Class 3 automata support chaotic states. Class 4 represents the most interesting class, reproducing patterns from the other classes with specific initial conditions, but also being the most complex. In particular, this class is typically universally (Turing) complete [299–303]. This implies that, in principle, any calculation or computer program can be simulated using this automaton. In other words they can actually reproduce any classical computer⁶. This class creates very interesting non-trivial patterns in the sequence space (evolution space). Such patterns can have a specific form and

⁵C is an alive state with three alive neighbours so it will survive. D is an alive state with two alive neighbours and will survive. E is a dead state with three alive neighbours so will be born.

⁶A Turing machine is a hypothetical machine proposed by Alan Turing, which consists of an infinitely long tape on which numbers can be written in the binary format and various binary operations can be performed on these numbers. The Turing machine is a primitive but a perfect classical computer, which can perform all mathematical operations. According to the Church-Turing thesis, a problem is solvable by a classical computer (with infinite resources) if and only if it is solvable on a Turing machine.

Figure 5.1: (a) Two dimensional cellular automata consist of a regular grid of cells, where each cell can have two possible states ‘0’ (dead) and ‘1’ (alive). Here 0 is indicated by gray and 1 is indicated by white. (b) The Von-Neumann neighbourhood, where each cell interacts with four neighbours (vertically and horizontally). (c) Moore neighborhood where interaction happens with eight neighbours. (d-f) Starting with an initial condition (d), we find a pattern evolving into (e) and (f) after subsequent updates.

even they can move along the sequence space. They show complex behaviour such as annihilations, reflections, fusions, etc., [304]. They have applications in image processing (e.g., restoration and edge detection) [305–307]. In this work we have found that cellular automata can also produce a variety of solitons which are universal complete.

Later in this chapter we will focus on two dimensional class 4 cellular automata mainly ‘Conway’s life’, ‘Life without death’ and a new rule (which has not appeared in the literature). The Life without Death cellular automata are characterized by the behaviour that a dead cell becomes alive if and only if it is surrounded by three alive neighbours and all alive cells remain alive always. According to the new rule (B3/S0123456, using standard notation of automata specifying the number of neighbours needed for a cell to be born/survive), a cell is born if it has three alive neighbours and survives if and only if it has less than seven neighbours alive. The truth table corresponding to these three rules is given in Fig. 5.2.

5.2 Motivation

Exciton-polaritons have been used to study a variety of fundamental nonlinear effects, with a general motivation of reaching optoelectronic devices [308]. For example, a large body of work was focused on the creation and control of polariton solitons [97–99, 101, 309–311], which were conjectured to play a role in devices [103, 195, 312] with some soliton logic gates constructed [313]. In parallel, polaritons

Rule	Initial state	Number of Alive Neighbours								
		0	1	2	3	4	5	6	7	8
Life without Death B3/S012345678	dead	0	0	0	1	0	0	0	0	0
	alive	1	1	1	1	1	1	1	1	1
Conway's Life B3/S23	dead	0	0	0	1	0	0	0	0	0
	alive	0	0	1	1	0	0	0	0	0
B3/S0123456	dead	0	0	0	1	0	0	0	0	0
	alive	1	1	1	1	1	1	1	0	0

Figure 5.2: Truth table of these three rules is given here. Here ‘0’ means dead and ‘1’ means alive state.

were found to form spatial patterns [104, 107, 159, 314, 315], where topologically stable structures were considered as a memory [100, 316] and pattern transitions could effectively compose switches [276]. There has also been a growing interest in studying polaritons in periodic potentials [165], which have allowed the routing of polaritons [18, 279], gap and lattice solitons [155, 193, 194], and the formation of spin-ordered patterns [151] for information processing.

While the aforementioned works are promising for polaritonic devices in principle, a complication in applying the aforementioned phenomena is that they have been found under different conditions in different parameter ranges. Here, we consider whether solitons, stable structures, and evolving patterns can co-exist *under the same conditions*, together with the analogues of a variety of other soliton-related structures studied separately in the literature, including: oscillating solitons and patterns [106, 218, 317]; guns [318] and soliton trains [99, 103, 195]; soliton explosions [102]; and backward radiation emitting solitons [196]. We also aim for polariton solitons and related structures to exist indefinitely [100, 319], beyond the finite polariton lifetime, and survive in the presence of noise and disorder. To reach these aims, we attempt to associate a polariton lattice to a cellular automaton.

There are reasons to expect that polaritons could operate as life-like automata. First, they can be confined in square lattices [155, 163, 320]. Second, under continuous near-resonant coherent excitation, polaritons exhibit bistability [99, 109], such that each site in a lattice would be in a high intensity (i.e., ‘alive’) or low intensity (i.e., ‘dead’) state for as long as the coherent excitation is maintained [11]. Furthermore the nonlinearity of polaritons suggests a potential for non-trivial behaviour, as the aforementioned works have demonstrated, however, it is a highly non-trivial question as to whether the nonlinearity can result in any complex automaton rules.

In addition to continuous near-resonant excitation, we consider the effect of non-resonant pulses applied to the system. Each pulse is found to initiate one update according to a life-like automaton rule, making use also of the spin-dependent

interactions between polaritons [321] and recently realized spin-orbit coupling in lattices [323, 324]. This provides a platform for merging many of the separately studied soliton-related polariton phenomena in microcavities and such combination shows most clearly their prospects for information processing: we find that *polariton solitons are universally complete*.

5.3 Scheme

Figure 5.3(a) shows a schematic illustration of a square lattice of polariton resonators (e.g., micropillars [165]). Each resonator represents a “cell” supporting two polariton spin components ($\sigma_{\pm}$), represented by the wavefunction $\psi_{n,\sigma_{\pm}}$, which evolves according to [149]:

$$i\hbar \frac{\partial \psi_{n,\sigma_{\pm}}}{\partial t} = \left(-\Delta + \alpha_1 |\psi_{n,\sigma_{\pm}}|^2 + \alpha_2 |\psi_{n,\sigma_{\mp}}|^2 + iP_{\sigma_{\pm}}(t) - \frac{i\Gamma}{2} \right) \psi_{n,\sigma_{\pm}} + J \sum_{\langle m \rangle} \psi_{m,\sigma_{\mp}} + F_{\sigma_{\pm}}, \quad (5.1)$$

where Δ represents an energy detuning between a driving laser field, with circularly polarized components $F_{\sigma_{\pm}}$, and the resonant polariton energy. To recall, spin-dependent interactions are accounted for, where the interaction strength between polaritons of parallel spin is α_1 and α_2 is strength of interactions between polaritons with antiparallel spin. $P_{\sigma_{\pm}}(t)$ represents a gain in the system [104], i.e., a non-resonant pulse with different spin components. The term $-i\Gamma/2$ represents polariton dissipation. It is convenient to move to dimensionless units by making the following transformations: $t \rightarrow t\hbar/\Gamma$ and $\psi_{n,\sigma_{\pm}} \rightarrow \psi_{n,\sigma_{\pm}} \sqrt{\Gamma/\alpha_1}$ which modifies the equation Eq. 5.1 to

$$i \frac{\partial \psi_{n,\sigma_{\pm}}}{\partial t} = \left(-\Delta + |\psi_{n,\sigma_{\pm}}|^2 + \alpha_2 |\psi_{n,\sigma_{\mp}}|^2 + iP_{\sigma_{\pm}}(t) - \frac{i}{2} \right) \psi_{n,\sigma_{\pm}} + J \sum_{\langle m \rangle} \psi_{m,\sigma_{\mp}} + F_{\sigma_{\pm}}. \quad (5.2)$$

The interaction strength between polaritons of parallel spin is scaled to unity. All the energy terms in the equation are rescaled by a factor Γ , α_2 is normalized by α_1 , and $F_{\sigma_{\pm}} \rightarrow F_{\sigma_{\pm}} \sqrt{\alpha_1/\Gamma^3}$.

We assume that each cell is coupled to its eight nearest neighbours (see Fig. 5.3(b)) via intermediate polariton channels. It is well established experimentally that etched polariton channels have an energy splitting between polaritons polarized parallel and perpendicular to the channel axis [217, 322]. This energy splitting is well-known to cause oscillations in the spin of polaritons as they propagate, as observed experimentally in polariton channels [216]. In principle, the parameters of the channel (such as its length and thickness) can be tuned so that a polariton travelling from one site to the next via the channel inverts its spin. We propose that this leads to the cross-spin type coupling in an effective tight-binding model, while we also provide the complete description with a continuous model. The

different lengths of diagonal and horizontal/vertical links are readily compensated by varying the thickness of the channel, which is known to cause a change in the polarization splitting. This motivates our choice of a spin-inverting type coupling in the tight-binding description represented by Eq. (5.2). We choose this model here for its simplicity, while we show that the same results can be reproduced with a continuous description, including explicit modelling of the channels with polarization splitting, in sec. 5.12.

It is instructive to consider first the single cell behaviour ($J = 0$). In the stationary limit (with no pulse; $P_{\sigma_{\pm}}(t)=0$), the stationary states of Eq. (5.2) excited by a σ_+ circularly polarized field ($F_{\sigma_-} = 0$) show a well-known S-shaped dependence [109] (see Fig. 5.3(c)) of the polariton intensity $|\psi_{\sigma_+}|^2$ on the driving intensity $|F_{\sigma_+}|^2$, given by $\left[(-\Delta + n_{\sigma_+} + \alpha_2 n_{\sigma_-})^2 + 1/4\right] n_{\sigma_+} = |F_{\sigma_+}|^2$, where $n_{\sigma_{\pm}} = |\psi_{\sigma_{\pm}}|^2$. We fix F_{σ_+} throughout so as to remain in the bistable regime, where ψ_{σ_+} in each cell must adopt either a high intensity (“alive”) or low intensity (“dead”) state in the stationary limit. The S-shaped curve depends on the detuning, Δ , which is effectively renormalized by the implantation of a population of σ_- polarized polaritons via the α_2 dependent term in Eq. (5.2) (see Fig. 5.3(c)).

In the limit of finite but small coupling between cells ($J \ll 1$), the coupling can be considered as a perturbation. It enters as an effective driving for σ_- polarized polaritons, where the field ψ_{n,σ_-} is driven by $J \sum_{\langle m \rangle} \psi_{m,\sigma_+}$, according to Eq. (5.2). It is important that this field depends on the state of neighbouring cells, such that each cell is influenced based on how many of its neighbours are dead or alive (see Fig. 5.3(b)). In the perturbative limit, stationary states of σ_- polarized polaritons are given by $\psi_{n,\sigma_-} = F_{\text{eff}} / (\Delta - |\psi_{n,\sigma_-}|^2 - \alpha_2 |\psi_{n,\sigma_+}|^2 + \frac{i}{2})$, where $F_{\text{eff}} = J \sum_{\langle m \rangle} \psi_{m,\sigma_+} + F_{\sigma_-}$. We focus first on realizing the Life without death cellular automaton. To recall, the rule is characterized by the behaviour that a dead cell becomes alive if and only if it has three alive neighbours whereas all alive cells remain alive always. Figure 5.3(d) (lower points) shows the stationary states of $\psi_{n,-}$ for the case where a cell is initially dead and has different numbers of alive neighbours under the condition $P_{\sigma_{\pm}}(t) = 0$ and $F_{\sigma_-} = 0$. Choosing now a finite F_{σ_-} will shift the stationary states. We choose $F_{\sigma_-} = -J(3\psi_{\text{alive}} + 5\psi_{\text{dead}})$, where ψ_{alive} and ψ_{dead} represent the expected wavefunction of alive and dead neighbouring cells. This field, which will be fixed throughout, shifts the intensity of σ_- polarized polaritons to zero when a given cell has exactly three neighbours alive and otherwise leaves a finite intensity (Fig. 5.3(d)). Recalling Fig. 5.3(c), the S-shaped curve describing stationary states of σ_+ polarized polaritons should now be shifted if a cell has any number of neighbours alive other than three.

Having modified the stationary states of a cell in a neighbour-dependent way, we now consider the action of a non-resonant pulse, $P_{\sigma_{\pm}}(t) = P_{\sigma_{\pm}} e^{-\Gamma_R t}$. Physically, the pulse excites an exciton reservoir and we take the pulse as a decaying exponential to represent decay of the reservoir (with decay rate Γ_R). The two spin components of the pulse serve different purposes. The σ_- component amplifies the up until now weak population of σ_- polaritons, so that their neighbour dependent intensity has a more significant effect. At the same time, the σ_+ component attempts to switch the state of σ_+ polarized polaritons to the alive state. Re-

Figure 5.3: (a) A square lattice of coupled cells formed by polariton resonators. Blue and grey denote alternating layers of refractive index, which trap polaritons in the cavity layer (red). The sample is etched in a way to leave channels that connect the resonators. (b) Each cell interacts with eight neighbours, which provide an effective driving, F_{eff} . This field depends on how many neighbours are alive and on the constant field F_{σ_-} . (c) Single cell stationary solutions under the condition $J = 0$, $P_{\sigma_{\pm}}(t) = 0$, and $F_{\sigma_-} = 0$ with and without the influence of ψ_{σ_-} . A non-zero population of σ_- particles raises the threshold intensity for reaching the higher ψ_{σ_+} state. Parameters: $\Delta = 3$ and $\alpha_2 = -1$ [90]. The grey line shows the fixed continuous driving for which there are two stable solutions (dashed curves correspond to unstable states). (d) Neighbour dependent values of the ψ_{σ_-} field caused by the effective driving, F_{eff} , with (upper points) and without (lower points) the driving field F_{σ_-} under the condition $P_{\sigma_{\pm}}(t) = 0$ for a cell initially in the lower intensity (dead) ψ_{σ_+} state. Colours represent different numbers of neighbours in the higher intensity (alive) ψ_{σ_+} state. Parameters: $J = 0.01$ and $\alpha_2 = -1$ [90].

markably, we find that for well-chosen parameters (see sec. 5.6) such switching is only possible when a cell has three alive neighbours, as shown in Fig. 5.4. For any number of neighbours different to three, the shift in the S-shaped curve has raised the threshold population needed to switch to the higher intensity state, such that the non-resonant pulse is insufficient.

While our scheme may appear complicated, involving different components of

the coherent driving field F_{σ_+} and F_{σ_-} as well as components of the non-resonant pulse P_{σ_+} and P_{σ_-} , we note that these just correspond to specific polarizations of a continuous wave laser and a pulse. Both can be spatially uniform with no specific site to site modulation. Such pulse induced switching of a system driven by a continuous wave driving field is well within the limits of current technology [10, 11, 151, 157, 325]. Each automaton update requires the application of just one pulse (this pulse is shown explicitly in sec. 5.10).

Figure 5.4: (a) Time evolution of ψ_{σ_+} after the application of a non-resonant pulse. The solid lines (violet color) represent the stable states. The dotted line represents the unstable state. The figure shows a switching to the higher intensity state if a dead cell has three neighbours alive, while otherwise it remains in the lower intensity state. (b) Plot of the σ_+ intensity vs the decaying non-resonant pulse intensity for different numbers of alive neighbours. Parameters: $\Gamma_R = 1$, $F_{\sigma_-} = -J(3\psi_{\text{alive}} + 5\psi_{\text{dead}})$, $\Delta = 3$, $J = 0.01$, and $\alpha_2 = -1$.

5.4 Conway's life and other cellular automata

We varied the parameters P_{σ_+} and P_{σ_-} and found that different automata rules were possible, for example, B3/S0123456 was possible (see sec. 5.6). This specific rule has not appeared in the literature but is also universally complete (see sec. 5.6). More importantly, the presence of this rule shows that it is possible to realize the behaviour of overpopulation, where a cell dies if too many of its neighbours are alive. This allows more complex automata.

Fig. 5.5 shows the numerically calculated probability for a cell to be born or to survive if it has a specific number of neighbours alive, where we have adjusted the field F_{σ_-} to $F_{\sigma_-} = -J(2.8\psi_{\text{alive}} + 5.2\psi_{\text{dead}})$. This gives a slight bias to allow the cell with two alive neighbours to survive and we find that for larger pulse intensity Conway's life is obtained (see, e.g., the red marker in Fig. 5.5). These results were obtained, as with all our numerical results, by propagating Eq. (5.2) in time using an Adams–Moulton–Bashforth error-corrector procedure. Fig. 5.6 shows the dynamics under a non-resonant pulsed excitation for different numbers of neighbours, confirming that Conway's life is attained in the system. In sec. 5.11

we have also shown that Conway's life can be achieved even in the presence of an additional polarization conserving coupling between neighbours.

Figure 5.5: Variation of automaton rules with P_{σ_+} and P_{σ_-} . Each plot shows whether a cell that is initially dead (upper nine plots) or initially alive (lower nine plots) finishes in an alive state after the application of a pulse for different numbers of neighbouring (initially) alive cells (given by the labels in the bottom-right corners of each plot). White corresponds to parameters for which a cell finishes in the alive state, while grey corresponds to parameters for which a cell finishes in the dead state. The red spot indicates a parameter choice for which Conway's life appears. Parameters: $F_{\sigma_-} = -J(2.8\psi_{\text{alive}} + 5.2\psi_{\text{dead}})$, $\Delta = 3$, $J = 0.01$, and $\alpha_2 = -1$.

Figure 5.6: Dynamics of $|\psi_{\sigma+}|^2$ under pulsed excitation for a cell initially dead (a) or alive (b). The different coloured curves correspond to different numbers of neighbours (initially) alive. Only configurations where a dead cell has three alive neighbours, or where a cell was initially alive and has two or three alive neighbours results in the cell being alive at the end of the pulse. The amplitude of the pulses were chosen according to the red spot in Fig. 5.5. Parameters: $F_{\sigma-} = -J(2.8\psi_{\text{alive}} + 5.2\psi_{\text{dead}})$, $\Delta = 3$, $J = 0.01$, and $\alpha_2 = -1$.

5.5 Solitons and Patterns

Having established the presence of complex cellular automata, we show examples of the structures that can be formed in the polariton system. Conway's life is known to support stable self-localized states (Fig. 5.7(a)) in analogy to the solitons considered in Ref. [193]. As we operate on a lattice, these can be considered as discrete solitons [326], which are also dissipative [319]. These solitons are permanent in the system so long as the near-resonant driving field is applied; they do not decay even accounting for the finite polariton lifetime. Another example is shown in Fig. 5.7(bi,bii), which illustrates a rotating soliton. In addition, since bistability at each lattice site exists for some range of detuning (Δ), these solitons and the cellular automaton behaviour in general survives in the presence of disorder (see sec. 5.9).

Conway's life also supports propagating oscillating solitons, known as gliders or spaceships (see Fig. 5.7(ci-cv)), which can be generated in a train using a glider gun (Fig. 5.7(di,dii)). Collisions between gliders give rise to the formation of a very wide variety of intricate structures, such as exploding patterns (Fig. 5.7(ei,eii), cf. [102]), puffer trains (Fig. 5.7(fi,fii), cf. [196]), and the possibility of ordered or chaotic patterns (see sec. 5.7, c.f. [218]). The proof of universal completeness of cellular automata can be established considering signals carried by "ladders" (see sec. 5.6). The sensitivity of our result in Fig. 5.5 shows that stable lasers with less than 1% intensity fluctuation will be required. We note that intensity fluctuations on the scale of 0.02% are used in polariton experiments [327].

Figure 5.7: Examples of configurations attained within Conway's life. (a) A 2×2 block maintains its form over time. (bi-bii) An oscillating soliton (blinker), with periodicity two. (ci-cv) A glider, that is, a soliton that translates diagonally in four timesteps. (di,dii) A glider gun, that is, starting with the configuration in (di), a train of solitons is emitted as shown in (dii). (ei,eii) A soliton explosion where an initial condition (ei) fills the whole space over time (eii). (fi,fi) An emitting soliton where the initial condition in (fi) moves upwards, while also leaving behind a dynamic pattern.

5.6 Life without death and completeness of the rule B3/S0123456

We varied P_{σ_+} and P_{σ_-} for $F_{\sigma_-} = -J(3\psi_{\text{alive}} + 5\psi_{\text{dead}})$ and plotted the probability of a cell to be alive or dead depending on their neighbours as shown in Fig. 5.8. We found different rules like the life without death (B3/S012345678) and the rule (B3/S0123456). In the case of the Life without death automaton, we take the parameters corresponding to the red spot in Fig. 5.8 while the green spot indicates the parameters for which we achieve the rule B3/S0123456.

To show that the rule B3/S0123456 is universally complete, we follow Ref. 300, which previously showed that the Life without death automaton is universally complete. The proof was based on considering signals in the form of repeating structures, known as “ladders”, that grow from a finite seed (Fig. 5.9(ai)) in a particular direction as shown in Fig. 5.9(aii). If ladders can be terminated, turned, and blocked by other ladders then they can be suitably combined into logic gates (“OR”, “AND”, “NOT”) to realize universal completeness 300. These situations are demonstrated in Figs. 5.9(bi-dii) for the rule B3/S0123456.

It should be noted that when one ladder is used to block another, the relative position of ladders is important. If ladders have the incorrect relative position, then their collision creates extra unnecessary fluctuations in the system, called “lava” 300, which may divert the propagation of the ladder into a different direction. As the horizontal periodicity of the considered ladders is four and the horizontal ladder can collide with an occupied or unoccupied portion of the vertical ladder, there are a total of eight possible relative shifts denoted by (h, v) , where $h \in \mathbb{Z}_4$ and $v \in \mathbb{Z}_2$ are the horizontal and vertical shifts respectively (note that in Ref. 300 these shifts are called phase shifts, but we avoid this terminology here so as to not confuse with an optical phase). In Fig. 5.9(di), two ladders incident from the left have phases (0,0) and (2,1) and are blocked by the ladder on the right. In Fig. 5.9(ei), the horizontal ladder will collide with the vertical ladder at the unoccupied portion of the vertical ladder, which will not result in the correct blocking behaviour. However, ladders can also be shifted using particular seeds. For example, the situation in Fig. 5.9(eii), causes the approaching ladder to shift in space so as to collide with the correct relative position with respect to the blocking ladder as shown in Fig. 5.9(eiii). In this way clean blocking can always be arranged by shifting ladders appropriately between collisions.

5.7 Chaos in cellular automata

As earlier, we have seen that different patterns emerge from Conway’s life, including stable, periodic, and moving patterns. Here we provide an example of chaotic behaviour. We start with the initial condition shown in Fig. 5.10(ai), which gives rise to Fig. 5.10(aii) after 59 updates. Changing the initial condition by just one cell to that of Fig. 5.10(bi), we arrive at a completely different pattern (see Fig. 5.10(bii)) after the same number of updates.

Figure 5.8: Probability for a cell to be in the alive state depending on the number of alive neighbours (marked in the bottom right corner of the plots) for $F_{\sigma_-} = -J(3\psi_{\text{alive}} + 5\psi_{\text{dead}})$, for cases when it was initially dead (upper nine plots) and alive (lower nine plots). For the parameters in the white region, the cell finishes in the alive state and for the grey region, it finishes in the dead state. The green dot marks a set of parameters for which the rule B3/S0123456 is obtained, while the red + marks a set of parameters for which Life without death (B3/S012345678) was obtained.

5.8 Materials

Two-dimensional polariton lattices have been intensely developed using GaAs [163, 320, 323, 324], so these are currently the most appropriate materials for our proposal. Of course real systems also come with an inevitable disorder, which in

Figure 5.9: Behaviour of ladders emerging from the rule (B3/S0123456). (ai) It is a finite seed that gives a periodic structure that moves in one direction, called a ladder shown in (aii). (bi,bii) The propagation of a ladder is ended by a single cell. (ci,cii) In the presence of a finite seed a ladder is turned so as to propagate in the perpendicular direction. (di,dii) Ladder can be blocked by other ladders. Here two ladders are incident from the left with relative phases $(0,0)$ and $(2,1)$. (ei-eiii) The relative displacement of ladders can be shifted, which helps to ensure clean blocking.

principle can destroy polariton solitons. In our system, while the stationary states plotted in Fig. 5.3(c) can be shifted in the presence of disorder, it is notable that

Figure 5.10: Example of chaotic behaviour in Conway's life. (ai,aii) The initial state (ai) evolves into the pattern (aii) after 59 updates. (bi,bii) The initial state (bi) differs from that of (ai) by only one cell, and gives rise to a completely different pattern (bii) after the same number of updates.

bistability survives over some range of detunings, Δ . Furthermore, introducing a distribution of values of Δ varying from site-to-site we found that cellular automaton rules could persist provided that the strength of disorder is below a finite threshold, estimated around $30 \mu\text{eV}$ (see sec. 5.9) in GaAs, which is comparable to the disorder of state-of-the-art lattices [265]. As our scheme is generic it could also be compatible with materials capable of operation at higher temperatures. For example, Te and Se-based micropillars are in development and could be assembled in the same way as their GaAs predecessors [328, 329].

5.9 Effect of disorder

In Fig. 5.11 we calculated the success rate in the presence of disorder, represented by variation in the detuning Δ at different lattice sites. The disorder in Δ is taken with a uniform distribution with peak-to-peak magnitude $\delta\Delta$. It is notable that when the disorder strength is below some limiting threshold, different automaton rules are obtained with 100% accuracy. Even above the limiting threshold the success rate remains high. It should also be noted that these curves only represent a lower bound on the success rate for a given task. If we consider the on-site disorder of the micropillar of the order of $30 \mu\text{eV}$, then with polariton lifetime 1.8 ps, the disorder is approximately 0.09 in a dimensionless unit (see the gray vertical line in Fig. 5.11). Even if some configurations of an automaton do not update with 100% accuracy, the configurations that do update correctly may be sufficient for a particular task, e.g., formation of ladder, creation of a propagating soliton or implementation of an imaging processing algorithm. We have separately verified

that in the presence of disorder at the level of 0.09 in our dimensionless units, all the structures required for universal computation are perfectly maintained.

Figure 5.11: Dependence of the success rate on disorder for different cellular automaton rules. Δ is chosen differently for different lattice sites, using a uniformly distributed random variable with peak-to-peak amplitude $\delta\Delta$. When disorder is below some threshold, cellular automata are still obtained perfectly with 100% success rate. Above this disorder threshold, the success rate slowly decreases.

5.10 Pulse sequence

As mentioned earlier, one non-resonant pulse of the form $P_{\sigma_{\pm}}(t) = P_{\sigma_{\pm}} e^{-\Gamma_R t}$ is needed for each automaton update. As an example we show how an oscillating soliton-blinker updates to the next state after applying the pulse. Initially we start with the configuration given by Fig. 5.12(j) and solve Eq. 5.2 with F_{σ_+} along with $F_{\sigma_-} = -J(2.8\psi_{\text{alive}} + 5.2\psi_{\text{dead}})$ for time unit, $t = (0, 20)$. Then we apply the non-resonant pulse to all lattice sites (whose amplitude is indicated by red dot in Fig. 5.5) as shown by dotted curves in Fig. 5.12. The steady state updates to Fig. 5.12(k), which we expect if the system follows Conway's life as depicted in Fig. 5.7 (bi-bii).

5.11 Effect of same polarization coupling

To check the effect of the same polarization coupling with the neighbours we add a term $J_P \sum_{\langle m \rangle} \psi_{m, \sigma_{\pm}}$ with Eq. 5.2 such that the dynamics of the micropillars are

Figure 5.12: Time evolution of oscillating soliton-blinker which emerges from Conway's life. After applying the pulse the initial state (j) updates to final state (k). The grey region corresponds to a dead state while white to an alive state. Here we plotted the time dynamics only for the centered 3×3 lattice sites indicated by indices 1, 2, 3..., 9 (top-left). It shows that initially we start with an alive state in cells (2, 5, 8) and after the pulse two new cells are born (4 and 6) and 5 remains alive. The pulses are indicated by the dotted curves where black represents $P_{\sigma_+}(t)$ and red represents $P_{\sigma_-}(t)$. The green dotted vertical lines represent the time at which the pulses are injected. The amplitude of the pulses is taken from the red dot in Fig. [5.5](#).

governed by the following equation

$$\begin{aligned}
 i \frac{\partial \psi_{n,\sigma_{\pm}}}{\partial t} = & \left(-\Delta + |\psi_{n,\sigma_{\pm}}|^2 + \alpha_2 |\psi_{n,\sigma_{\mp}}|^2 + iP_{\sigma_{\pm}}(t) - \frac{i}{2} \right) \psi_{n,\sigma_{\pm}} + J_P \sum_{\langle m \rangle} \psi_{m,\sigma_{\pm}} \\
 & + J \sum_{\langle m \rangle} \psi_{m,\sigma_{\mp}} + F_{\sigma_{\pm}}.
 \end{aligned} \tag{5.3}$$

Next we perform all the steps as described earlier. Indeed, the Conways life can be achieved by choosing proper values of the incoherent pulses (see Fig. 5.13). In the calculation we have chosen $J_P = J = 0.01$, however, we have checked that the Conways life still exists for $J_P > J$ as long as J_P is small ($J_P \ll 1$).

Figure 5.13: Variation of automaton rules with P_{σ_+} and P_{σ_-} in the tight binding model according to Eq. (5.3) where polarization conserving couplings with neighbours are also considered. The parameter indicated by the red dot corresponds to the Conways life. The blue dot corresponds to the parameter corresponding to the Conways life shown in Fig 5.5 where $J_P = 0$. Parameters: $J_P = J = 0.01$, $F_{\sigma_-} = -J(2.8\psi_{\text{alive}} + 5.2\psi_{\text{dead}})$ and all other parameters are kept the same as those in Fig. 5.5.

5.12 Continuous model

In this section we model the system with a continuous wave-function to represent the coupled exciton-polariton micropillars shown in Fig. 5.14(a). The diameter of the micropillars is chosen to be $5.3 \mu\text{m}$ [168] and the separation between the micropillars is $8 \mu\text{m}$ along the vertical and horizontal directions. To realize the diagonal couplings with the same strength as the horizontal and vertical ones we take the diagonal channels wider. Such a structure with channel connecting two pillars was studied in Refs. [63, 173]. The width of the channels along the vertical and horizontal direction is taken as $0.5 \mu\text{m}$ and those along the diagonal and anti-diagonal direction is taken as $2 \mu\text{m}$. The depths of the micropillars as well as the channels are taken to be around 4.5 meV . The polariton mass and lifetime are taken as $3 \times 10^{-5} m_e$, where m_e is the free electron mass, and 3 ps , respectively. Due to the shape anisotropy of the channels a splitting between modes polarized parallel and perpendicular to the channel naturally occurs inside them as discussed in sec. 2.2. We take the polarization splitting inside channels as $V_T = V_{avg}/\Delta L$, where V_{avg} is the average splitting in the system which is taken as $0.75 \text{ meV } \mu\text{m}$ and ΔL is the width of the respective channels. Since different channels have different orientations, a phase factor $\exp(2i\phi)$ appears in V_T , where ϕ is the angle of the channels, to ensure that the polarization splitting is always in the correct direction (this phase factor is readily derived from first writing a splitting between x and y directions and performing a rotation). The phase factors are shown in Fig. 5.14(b). Due to the symmetry of the unit cell it is clear that there will be no polarization splitting inside the pillars. The spatial profiles of the real and imaginary parts of the polarization splitting are shown in 5.14(c-d), respectively. Now that all the ingredients are ready, we start by solving the following Gross-Pitaevskii equation:

$$i\hbar \frac{\partial \psi_{\sigma_{\pm}}(\mathbf{x})}{\partial t} = \left[-\frac{\hbar^2}{2m_{\text{LP}}} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + V(\mathbf{x}) - \Omega_0 - \frac{i\Gamma}{2} + \alpha_1 |\psi_{\sigma_{\pm}}(\mathbf{x})|^2 \right. \\ \left. + \alpha_2 |\psi_{\sigma_{\mp}}(\mathbf{x})|^2 + iP_{\sigma_{\pm}}(t) \right] \psi_{\sigma_{\pm}}(\mathbf{x}) + V_T(\mathbf{x}, \phi) \psi_{\sigma_{\mp}}(\mathbf{x}) + F_{\sigma_{\pm}}(\mathbf{x}). \quad (5.4)$$

Here $V(\vec{x})$ is the potential profile represented in Fig. 5.14(a) and Ω_0 is the energy of the resonant pump, which we take 1.1 meV . To find the suitable parameters easily, we move to dimensionless units with the following transformations $t \rightarrow \hbar t/\Gamma$, $\mathbf{x} \rightarrow \hbar \mathbf{x}/\sqrt{2m\Gamma}$, $\psi \rightarrow \psi \sqrt{\Gamma/\alpha_1}$

$$i \frac{\partial \psi_{\sigma_{\pm}}(\mathbf{x})}{\partial t} = \left[-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + V(\mathbf{x}) - \Omega_0 - \frac{i}{2} + |\psi_{\sigma_{\pm}}(\mathbf{x})|^2 + \alpha_2 |\psi_{\sigma_{\mp}}(\mathbf{x})|^2 \right. \\ \left. + iP_{\sigma_{\pm}}(t) \right] \psi_{\sigma_{\pm}}(\mathbf{x}) + V_T(\mathbf{x}, \phi) \psi_{\sigma_{\mp}}(\mathbf{x}) + F_{\sigma_{\pm}}(\mathbf{x}), \quad (5.5)$$

where all the energy scales are normalized by Γ , α_2 by α_1 , and $F \rightarrow F \sqrt{\alpha_1/\Gamma^3}$. Initially, we vary the pump very slowly to achieve the hysteresis curve given by Fig. 5.14(e). The characteristic of the hysteresis curves are the same for all the micropillars. Next we fix F_{σ_+} indicated by the vertical gray line of Fig. 5.14(e),

which corresponds to $F_0 = 0.22$. Then we apply the F_{σ_-} pump of the form $F_{\sigma_-} = -\eta(2.8\psi_{\text{alive}} + 5.2\psi_{\text{dead}})$, where $\eta = 0.01$ in the dimensionless form, ψ_{alive} corresponds to the steady state solution of Eq. (5.5) with all the sites alive, and ψ_{dead} corresponds to the steady state solution with all the sites are dead in presence of $F_{\sigma_+} = F_0$. The effect of F_{σ_-} can be seen clearly in Fig. 5.14(f) where the configuration corresponding to three alive neighbours has the minimum $|\psi_{\sigma_-}|^2$. This is equivalent to Fig. 5.3(d). As discussed earlier, this is a very crucial step, which ensures that with the choice of proper parameter of $P_{\sigma_{\pm}}$ the states which are initially dead having three alive neighbours become alive, whereas all other initially dead cells with different neighbours remain dead. Next we vary P_{σ_+} and P_{σ_-} simultaneously in presence of F_{σ_+} and F_{σ_-} using the periodic boundary conditions. Indeed, by scanning the space spanned by P_{σ_+} and P_{σ_-} , Conway's life can be obtained, which is indicated by the red dot in Fig. 5.15. Further in Fig. 5.16 we have shown that the four configurations lying at the border in Fig. 5.15 can survive with the 0.02% fluctuation of $P_{\sigma_{\pm}}$ [327].

5.13 Conclusion

We predict that when a particular stimulus is applied to a polariton lattice, namely a series of identical non-resonant excitation pulses, artificial life can appear in the form of a cellular automaton. This allows the realization of a variety of fundamental nonlinear optical structures under the same conditions, such as solitons, oscillating solitons, breathers, and various patterns. The complexity that arises from the combination of these phenomena is encapsulated within the simple update rules of the automaton. This further shows that polariton solitons are universally complete, even in the presence of dissipation and disorder.

Figure 5.14: (a) An example of the potential profile of a 5×5 lattice. Each micropillar is connected to its eight neighbours through the channels. (b) Due to their orientations the horizontal and vertical channels have opposite polarization splittings. The polarization splitting inside the diagonal channels have only imaginary parts due to their $\pi/4$ orientation with respect to the x axis and those inside the antidiagonal channels are opposite to the diagonal ones. (c-d) The real and imaginary parts of the polarization splitting, respectively. (e) The bistability curve of the micropillars where the gray horizontal line indicates the $|F_{\sigma_+}|^2$ value used in the calculation. (f) The effect of F_{σ_-} on the initially dead configurations with different number of alive neighbours. After applying F_{σ_-} the configuration corresponding to three alive neighbours has the minimum $|\psi_{\sigma_-}|^2$, similar to Fig. 5.3(d).

Figure 5.15: Variation of automaton rules with P_{σ_+} and P_{σ_-} in the continuous model under the periodic boundary condition is depicted here. White corresponds to parameters for which a cell finishes in an alive state, while grey corresponds to parameters for which a cell finishes in a dead state. Corresponding to the nine upper plots, the states were initially dead and alive for the lower nine plots. The red dot indicates a parameter choice for which Conway's life appears. Parameter: $\alpha_2 = -1$.

Figure 5.16: Zoomed view of the 2o, 4o, 6o and 6n configurations, respectively, from the previous figure which show that the chosen parameter can tolerate the 0.02% variation of the incoherent pulses. Here ‘o’ and ‘n’ indicate initially dead and alive states, respectively.

Conclusion

Exciton-polaritons are quasiparticles typically formed in microcavities where light hybridizes with excitons contained in quantum wells. As a result of this hybridization, exciton-polaritons have been used to study a variety of fundamental nonlinear effects, with a general motivation of reaching optoelectronic devices including solitons, logic gates, optical transistors, optical diodes, etc. On the other hand the interplay between the driven-dissipative nature, TE-TM splitting, Zeeman splitting, polarization splitting (due to shape anisotropy), and spin dependent interactions give rise to exotic phenomena. In this thesis we have investigated the dynamics of polaritons both in the linear and nonlinear regime.

In **chapter 2** we presented a scheme to generate an artificial gauge field for the system of neutral bosons, represented by polaritons in elliptical micropillars arranged into a square lattice. The splitting between the two polarizations of the micropillars breaks the time-reversal symmetry (TRS) and results in the effective phase-dependent hopping between pillars. This can allow for engineering a nonzero flux on the plaquette, corresponding to an artificial magnetic field. Changing the phase, we observed a characteristic Hofstadter's butterfly pattern and the appearance of chiral edge states for a finite-size structure. For long-lived polaritons, we showed that the propagation of wave packets at the edge is robust against disorder. Moreover, given the inherent driven-dissipative nature of polariton lattices, we found that the system can exhibit topological lasing, recently discovered for active ring cavity arrays. The results point to a static way to realize artificial magnetic field in neutral spinful systems, avoiding the periodic modulation of the parameters or strong spin-orbit interaction. Ultimately, the described system can allow for high-power topological single-mode lasing, which is robust to imperfections.

In **chapter 3** we considered a second order topological polariton bandstructure that gives rise to zero-dimensional localized corner modes, which reside within the band gap of the system along with edge modes. Thanks to the topological nature, information can be trapped at the corners of these systems, which will be unhampered even in the presence of disorder. Being localized at the corners, the exchange of information among the corner states is an issue. We showed that the nonlinearity in an exciton polariton system can allow the coupling between the different corners through the edge states based on optical parametric scattering, realizing a system of multiple connectible topological modes.

In **chapter 4** we considered theoretically exciton-polaritons in a strip of honeycomb lattice with zigzag edges and showed that the interplay among the TE-TM

splitting, Zeeman splitting, and an on-site detuning between sublattices can give rise to a band structure where all the edge states of the system split in energy. Within an energy interval, one of the spin-polarized edge states resides with the gapless bulk that has opposite spin. Being surrounded by opposite spin, and the absence of the backward-propagating edge state, ensures both reflection-free and feedback-suppressed one-way flow of polaritons with one particular spin in the system. The edge states in this system are more localized than those in the standard topological-polariton systems and are fully spin polarized. This paves the way for feedback-free spin-selective polariton channels for transferring information in polariton networks.

In **chapter 5** we considered an array of generic driven-dissipative nonlinear cavities and showed that simultaneously excited by a continuous wave coherent field and a time-periodic sequence of non-resonant pulses, they behave as a cellular automaton, a form of artificial life. This makes a system that supports a wide range of spatial-temporal structures *under the same conditions*, including discrete solitons, oscillating solitons, breathers, and guns. These structures can survive in the system indefinitely, despite the presence of dissipation and disorder. Furthermore, such a system represents a form of universal classical computation.

List of publications

- **R. Banerjee**, S. Mandal, and T. C. H. Liew, ‘Optically Induced Topological Spin-Valley Hall Effect for Exciton-Polaritons’, under review in Phys. Rev. Lett.
- S. Mandal, **R. Banerjee**, E. A. Ostrovskaya, and T. C. H. Liew, “Nonreciprocal transport of exciton polaritons in a non-Hermitian chain”, Phys. Rev. Lett. **125**, 123902 (2020).
[DOI:10.1103/PhysRevLett.125.123902](https://doi.org/10.1103/PhysRevLett.125.123902).
- **R. Banerjee** and T. C. H. Liew, “Artificial Life in an Exciton-Polariton Lattice”, New J. Phys **22**, 103062 (2020).
[DOI:10.1088/1367-2630/abc290](https://doi.org/10.1088/1367-2630/abc290).
- **R. Banerjee**, S. Mandal, and T. C. H. Liew, “Coupling between Exciton-Polariton Corner Modes through Edge States”, Phys. Rev. Lett. **124**, 063901 (2020).
[DOI:10.1103/PhysRevLett.124.063901](https://doi.org/10.1103/PhysRevLett.124.063901).
- S. Mandal, **R. Banerjee**, and T. C. H. Liew, “One-Way Reflection-Free Exciton- Polariton Spin-Filtering Channel”, Phys. Rev. Applied **12**, 054058 (2019).
[DOI:10.1103/PhysRevApplied.12.054058](https://doi.org/10.1103/PhysRevApplied.12.054058).
- **R. Banerjee**, T. C. H. Liew, and O. Kyriienko, “Realization of Hofstadter’s butterfly and a one-way edge mode in a polaritonic system”, Phys. Rev. B **98**, 075412 (2018).
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Appendix A

Two strongly coupled modes

We consider two oscillators of mass m are connected by a spring of spring constant g_0 and connected to the walls with spring constants k_1 and k_2 on a frictionless surface as shown in Fig. [A.1\(a\)](#). In the absence of the middle spring, each oscillator follows simple harmonic motion with frequency $\sqrt{(k_1/m)}$ and $\sqrt{(k_2/m)}$ respectively. If x_1 and x_2 are the displacements of the oscillators from their respective equilibrium positions, then the Lagrangian of the system can be written as

$$\mathcal{L} = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2 - \frac{1}{2}g_0(x_1 - x_2)^2. \quad (\text{A.1})$$

Then the system is driven by the Euler-Lagrange equations of the following form

$$\begin{aligned} m\ddot{x}_1 + k_1x_1 + g_0(x_1 - x_2) &= 0 \\ m\ddot{x}_2 + k_2x_2 - g_0(x_1 - x_2) &= 0. \end{aligned} \quad (\text{A.2})$$

Assume the trial solutions of the above second order differential equation are $x_i = A_ie^{i\omega t}$ where ω is the normal mode frequency, which can be obtained by the eigen values of the following matrix (substituting trial solution to Eq. [A.2](#))

$$\begin{pmatrix} \omega_a^2 - \omega^2 & -g_0/m \\ -g_0/m & \omega_b^2 - \omega^2 \end{pmatrix} = 0, \quad (\text{A.3})$$

where $\omega_a = \sqrt{(k_1 + g_0)/m}$ and $\omega_b = \sqrt{(k_2 + g_0)/m}$. The eigen-frequencies of the system are given by

$$\omega^2 = \frac{1}{2} \left(\omega_a^2 + \omega_b^2 \pm \sqrt{(\omega_a^2 - \omega_b^2)^2 + \frac{4g_0^2}{m^2}} \right). \quad (\text{A.4})$$

For $k_1 = k_2$ (say = k_0) i.e., $\omega_a = \omega_b$ (say = ω_0), the above expression takes the form

$$\omega^2 = \omega_0^2 \pm \frac{g_0}{m} = k_0/m, (k_0 + 2g_0)/m. \quad (\text{A.5})$$

The eigen states corresponding to the $\omega = \sqrt{k_0/m}$ are in-phase whereas for $\omega = \sqrt{(k_0 + 2g_0)/m}$, the two oscillators are moving *out-of phase*. In Figs. [A.1\(b-c\)](#) we

plot the eigen frequencies of the system in the absence and presence of the middle spring and anti-crossing behaviour is reflected in the later figure.

Figure A.1: (a) Two oscillators of mass m are coupled by an ideal spring of spring constant g_0 and are connected to the wall by springs of spring constant k_1 and k_2 , respectively. (b-c) The normal modes of the system in the absence (left) and presence (right) of the middle spring as a function of ratio of two spring constants k_2/k_1 . Anti-crossing, the effects of classical strong coupling, is prominent in the later figure.

Appendix B

Gross-Pitaevskii equation

The coherent state of polaritons can be well described by the Gross-Pitaevskii equation. In the second quantization formalism, the Hamiltonian can be written as

$$\begin{aligned} \hat{H} = & \sum_k E_k \hat{\Psi}_k^\dagger \hat{\Psi}_k + \sum_{\mathbf{x}} V_{\mathbf{x}} \hat{\Psi}_{\mathbf{x}}^\dagger \hat{\Psi}_{\mathbf{x}} + \frac{\alpha_1}{2} \sum_{\mathbf{x}} \hat{\Psi}_{\mathbf{x}}^\dagger \hat{\Psi}_{\mathbf{x}}^\dagger \hat{\Psi}_{\mathbf{x}} \hat{\Psi}_{\mathbf{x}} \\ & + \sum_k (F_k e^{-i\omega t} \hat{\Psi}_k^\dagger + F_k e^{i\omega t} \hat{\Psi}_k), \end{aligned} \quad (\text{B.1})$$

where E_k is the real part of the energy spectrum of the polaritons and later we will consider polariton dissipation through Lindblad term, well known approach in quantum optics [69]. To recall, second term represents the two-dimensional spatially dependent potential (discussed in subsec. 1.4.12). α_1 describes Kerr type nonlinearity of the bosonic modes and the last term represents the coherent pumping in the system with frequency ω . To recall, the field operator $\hat{\psi}_k$ and $\hat{\psi}_{\mathbf{x}}$ are related by the discrete Fourier transform as following

$$\hat{\Psi}_k = \frac{1}{\sqrt{N}} \sum_{\mathbf{x}} \hat{\Psi}_{\mathbf{x}} e^{-ik \cdot \mathbf{x}}, \quad \hat{\Psi}_{\mathbf{x}} = \frac{1}{\sqrt{N}} \sum_k \hat{\Psi}_k e^{ik \cdot \mathbf{x}}, \quad (\text{B.2})$$

We have discretized our space into N points, upon which we have the commutation relation $[\hat{\Psi}_{\mathbf{x}}, \hat{\Psi}_{\mathbf{x}'}^\dagger] = \delta_{\mathbf{x}, \mathbf{x}'}$. Now to include the dissipation of the system, we add Lindblad dissipation term in the von Neumann equation as

$$i\hbar \frac{\partial \rho}{\partial t} = [\hat{H}, \rho] + \frac{i\Gamma}{2} \sum_k \left(2\hat{\Psi}_k \rho \hat{\Psi}_k^\dagger - \hat{\Psi}_k^\dagger \hat{\Psi}_k \rho - \rho \hat{\Psi}_k^\dagger \hat{\Psi}_k \right), \quad (\text{B.3})$$

where ρ is the density matrix and Γ is the decay rate of polaritons. As earlier, we define the mean-field by taking the trace of the product of the field operator and density matrix as the following

$$\langle \Psi_k \rangle = \text{tr} \left\{ \rho \hat{\Psi}_k \right\}. \quad (\text{B.4})$$

The equation in the mean field approximation takes the form

$$i\hbar \frac{\partial \langle \Psi_k \rangle}{\partial t} = \text{tr} \{ [\hat{H}, \rho] \hat{\Psi}_k \} + \frac{i\Gamma}{2} \text{tr} \left\{ \sum_{k'} (2\hat{\Psi}_{k'} \rho \hat{\Psi}_{k'}^\dagger - \hat{\Psi}_{k'}^\dagger \hat{\Psi}_{k'} \rho - \rho \hat{\Psi}_{k'}^\dagger \hat{\Psi}_{k'}) \hat{\Psi}_k \right\}. \quad (\text{B.5})$$

Following the steps used in sec. [1.4.5](#) and putting the expression $V_{\mathbf{x}} = (1/\sqrt{N}) \sum_{k'} V_{k'} e^{ik' \cdot \mathbf{x}}$, we get the following expression

$$\begin{aligned} i\hbar \frac{\partial \langle \Psi_k \rangle}{\partial t} = & (E_k - \frac{i\Gamma}{2}) \langle \Psi_k \rangle + \frac{1}{\sqrt{N}} \sum_{k'} V_{k'} \langle \Psi_{k-k'} \rangle \\ & + \frac{\alpha}{N} \sum_{k_1, k_2} \langle \hat{\Psi}_{k_1}^\dagger \hat{\Psi}_{k_2} \hat{\Psi}_{k_1+k_2-k} \rangle + F_k e^{-i\omega t}. \end{aligned} \quad (\text{B.6})$$

In real space this is written as

$$i\hbar \frac{\partial \langle \Psi_{\mathbf{x}} \rangle}{\partial t} = \frac{1}{\sqrt{N}} \sum_{\mathbf{x}'} E_{\mathbf{x}'} \langle \Psi_{\mathbf{x}-\mathbf{x}'} \rangle + (V_{\mathbf{x}} - \frac{i\Gamma}{2}) \langle \Psi_{\mathbf{x}} \rangle + \alpha \langle \Psi_{\mathbf{x}}^\dagger \Psi_{\mathbf{x}} \Psi_{\mathbf{x}} \rangle + F_{\mathbf{x}} e^{-i\omega t}, \quad (\text{B.7})$$

where $F_{\mathbf{x}} = \frac{1}{\sqrt{N}} \sum_k F_k e^{ik \cdot \mathbf{x}}$ is the pump profile in real space. The first term can be written as $\frac{1}{\sqrt{N}} \sum_{\mathbf{x}'} E_{\mathbf{x}'} \langle \Psi_{\mathbf{x}-\mathbf{x}'} \rangle = \hat{E} \langle \Psi_{\mathbf{x}} \rangle$. At higher densities of the condensates, $\langle \Psi_{\mathbf{x}}^\dagger \Psi_{\mathbf{x}} \Psi_{\mathbf{x}} \rangle \approx \langle \Psi_{\mathbf{x}}^\dagger \Psi_{\mathbf{x}} \rangle \langle \Psi_{\mathbf{x}} \rangle$ and the above equation can be written for a continuous 2D space ($\mathbf{x} = (x, y)$) as

$$i\hbar \frac{\partial \psi(\mathbf{x})}{\partial t} = \left(\hat{E} - \frac{i\Gamma}{2} + V(\mathbf{x}) + \alpha |\psi(\mathbf{x})|^2 \right) \psi(\mathbf{x}) + F(\mathbf{x}) e^{-i\omega t}, \quad (\text{B.8})$$

where $\psi(\mathbf{x}) = \langle \Psi_{\mathbf{x}} \rangle$. Since the polariton dispersion near $k \sim 0$ is parabolic, we take the energy operator $\hat{E} = -\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} = -\frac{\hbar^2}{2m_{\text{LP}}} (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})$.

Appendix C

The Hamiltonian in sec. 2.4 is TRs broken

Let us recall our system of dimension 4×5 for simplicity formed by coupled exciton-polariton elliptical micropillars, where each micropillar has two kind of polarizations ($\sigma_{\pm} = \pm 1$) and the linear polarization splitting Δ_T as shown in Fig. C.1. Now if we want to write down the equations for $(8+, 8-)$ modes which is placed at an angle ϕ_8 with the chosen polarization axis of the micropillars, then following the Eq. 2.5, we have

Figure C.1: The schematic diagram of Fig. 2.3 for system size 4×5 , where orientation angles of the elliptical micropillars are chosen such that polaritons accumulate a uniform phase ($2\pi\beta$) after completing each plaquette.

$$i\hbar \frac{\partial \psi_{8,\sigma_+}}{\partial t} = -\Delta_T e^{-2i\phi_8} \psi_{8,\sigma_-} + J\psi_{7,\sigma_+} + J\psi_{9,\sigma_+}, \quad (\text{C.1})$$

and

$$i\hbar \frac{\partial \psi_{8,\sigma_-}}{\partial t} = -\Delta_T e^{2i\phi_8} \psi_{8,\sigma_+} + J\psi_{3,\sigma_-} + J\psi_{13,\sigma_-}. \quad (\text{C.2})$$

To check whether the Hamiltonian breaks Time Reversal symmetry (TRs) or not, we can apply the time reversal operator $T: t \rightarrow -t, i \rightarrow -i, \sigma_{\pm} \rightarrow \sigma_{\mp}$ and if the Hamiltonian is invariant, then the system preserves TRs. Applying T operator on Eq. [C.1](#), we have

$$i\hbar \frac{\partial \psi_{8,\sigma_-}}{\partial t} = -\Delta_T e^{2i\phi_8} \psi_{8,\sigma_+} + J\psi_{7,\sigma_-} + J\psi_{9,\sigma_-} \quad (\text{C.3})$$

which is clearly not the same as Eq. [C.2](#) and indeed our system is TRs broken. The underlying reason for TRs broken is the spin dependent coupling, where we assume that the σ_+ spins tunnel horizontally, whereas σ_- spins tunnel vertically.

This type of coupling can be engineered by proper choice of circularly polarized nonresonant pump [\[169\]](#). For example, we can place σ_- polarized nonresonant pump in between the pillars (horizontal), which will create stronger potential barrier for σ_- polarized polaritons due to the nonlinear interaction between polaritons and excitonic reservoir. Thus hopping of σ_- polarized polaritons along horizontal is restricted and we can have tunnelling of σ_+ spins horizontally. In the same way we can limit the interpillar tunnel coupling for σ_+ spins vertically by placing σ_+ polarized nonresonant pump in between the pillars (vertical). This kind of spin-dependent coupling can also be possible following recent works [\[174, 218\]](#).

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