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**TRANSPORT AND LASING IN TOPOLOGICAL PHOTONIC
SYSTEMS**

UDVAS CHATTOPADHYAY

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

2020

TRANSPORT AND LASING IN TOPOLOGICAL PHOTONIC SYSTEMS

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SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for the
degree of Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me except where otherwise stated in this thesis. The thesis work has not been submitted for a degree or professional qualification to any other university or institution. I declare that this thesis is written by myself and is free of plagiarism and of sufficient grammatical clarity to be examined. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from 3 papers published in the following peer-reviewed journals in which I am listed as an author.

Contents of Chapter 2 and Appendix A are published as Yongquan Zeng, Udvash Chattopadhyay, Bofeng Zhu, Bo Qiang, Jinghao Li, Yuhao Jin, Lianhe Li, Alexander Giles Davies, Edmund Harold Linfield, Baile Zhang, Yidong Chong, and Qi Jie Wang, “Electrically pumped topological laser with valley edge modes” [Nature 578, 246 \(2020\)](#).

The contribution of the co-authors are as follows

- The idea was originated by Assoc. Prof. Yidong Chong.
- I co-designed the structure with Assoc. Prof. Yidong Chong.
- I, Dr. Zeng Yongquan and Dr. Bofeng Zhu did the numerical simulations.
- Dr. Lianhe Li, Prof. Alexander Giles Davies and Prof. Edmund Harold Linfield performed QCL wafer growth.
- Dr. Zeng Yongquan and Dr. Bo Xiang fabricated the laser devices.
- Dr. Zeng Yongquan, Mr. Jinghao Li and Mr. Jin Yuhao performed device characterization.
- I, Dr. Zeng Yongquan, Dr. Bofeng Zhu, Assoc. Prof. Yidong Chong, Assoc. Prof. Baile Zhang and Prof. Qi Jie Wang performed theoretical analysis and contributed to manuscript preparation.

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The contribution of the co-authors are as follows

- Dr. Alexander M. Dubrovkin conceived the idea of the work.
- Dr. Alexander M. Dubrovkin, Assoc. Prof. Yidong Chong, Prof. Nikolay I. Zheludev and I discussed the strategy of the experiment.
- I did the theoretical structure design and numerical simulations.

- Dr. Alexander M. Dubrovkin did sample fabrication, experiments and prepared the manuscript.
- Dr. Qiang Bo and Dr. Oleksandr Buchnev contributed to the data analysis and sample fabrication respectively.
- Assoc. Prof. Yidong Chong, Prof. Qi Jie Wang and Prof. Nikolay I. Zheludev supervised the project. All co-authors took part in manuscript revision.

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The contribution of the co-authors are as follows

- The idea was conceived by me and Assoc. Prof. Yidong Chong.
- I did numerical calculations with help from Assoc. Prof. Yidong Chong.
- I and Dr. Li-Kun Shi did analytical calculations.
- I wrote the manuscript with Assoc. Prof. Yidong Chong. All co-authors took part in discussions and manuscript revision.

06/08/2020

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Udas Chattopadhyay

Udas CHATTOPADHYAY

For my family and friends

Abstract

This thesis concerns the study of novel topological transport phenomena in photonic systems, particularly in valley photonic crystals, Weyl semimetals and photonic Chern insulators; and lasing in a valley photonic crystal.

In the first part of the thesis, I present our study of an electrically pumped quantum cascade laser based on the topological edge states of a valley photonic crystal operating in THz frequency regime. The topological protection of lasing modes allows for irregular laser cavity design with robust mode spacings arising from the running wave nature of topological lasing modes that resist the formation of localized standing-waves.

In the second part, I present our work on near field imaging of topological edge modes of a valley photonic crystal slab operating in infrared regime.

The third part of the thesis describes our study of beam displacements in a Weyl semimetal. Beams in Weyl semimetals undergo both lateral (Goos-Hänchen shift) and transverse (Imbert-Fedorov shift) displacements upon reflecting off a gapped medium. We show that the displacement forms a half-vortex structure in momentum space. The center of the half-vortex is determined by the Fermi arc which provides a way to use bulk transport to probe the topological characteristics of a Weyl semimetal.

In the last part of the thesis, I present our study of mode delocalization in a photonic Chern insulator. In contrast to trivial two-dimensional insulators, where all modes are localized when disorder is present, a single extended mode survives in Chern insulators up to a critical disorder strength. We study this phenomena in a photonic Chern insulator platform which is promising for direct experimental probing.

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Contents

1	Introduction	17
1.1	Topology in band structures	17
1.2	Valley photonic crystals	22
1.3	Weyl semimetals	26
2	Electrically Pumped Topological Laser using Valley Photonic Crystal	29
2.1	Topological laser	29
2.2	Quantum Cascade Lasers	31
2.3	Design of the topological laser	31
2.4	Experimental results	41
2.5	Comparison with a non-topological laser	42
2.6	Probing the extended topological edge states	46
2.7	Conclusion and future prospects	51
3	Near Field Imaging of Topological Edge State in Suspended Valley Photonic Crystal Slab	55
3.1	On chip topological photonics	55
3.2	VPC on a suspended slab	56
3.3	Experimental set-up and results	59
3.4	Conclusion and future prospects	63

4	Novel Vortex Structure in Weyl Beam Shifts	64
4.1	Spatial shift of wavepackets in Weyl medium	64
4.1.1	Plane waves in Weyl medium	66
4.1.2	Beams in Weyl medium and spatial shift	67
4.2	Surface states and shifts	69
4.2.1	Single cone model	69
4.2.2	Paired Weyl cones model	78
4.3	Beam shift in a thin-film geometry	81
4.4	Shifts in presence of a third band	84
4.5	Conclusion and future prospects	87
5	Mode Delocalization in Disordered Photonic Chern Insulator	90
5.1	Effects of disorder on Chern insulator states	90
5.2	Model	93
5.3	Results	96
5.4	Conclusion	102
6	Conclusion and Future Prospects	103
	Appendices	106
A	Fabrication and characterization of topological QCL	106
B	Derivation of Gaussian beam shifts	108
C	Non-Gaussian beams and gauge invariance	111
	References	115

Chapter 1

Introduction

1.1 Topology in band structures

The discovery of the integer *quantum Hall effect* (QHE) in a 2-dimensional electron gas (2DEG) by Von Klitzing *et. al.* in 1980 [1] started a new era in condensed matter physics in which topology, a branch of mathematics that studies quantities conserved under continuous deformations, became intimately combined with the concept of band structures in solids [2]. Band structure theory turned out to contain more rich physics than previously thought, involving phase transitions described by a discontinuous change of quantized topological quantities, in contrast to the continuous change of order parameters associated with conventional phase transitions.

By applying linear response theory to a non-interacting single particle band structure, one can show that the Hall conductance of a two dimensional insulator (i.e. when Fermi level resides in the band gap) reads [2]

$$\sigma_{xy} = -\frac{e^2}{h} \sum_n C_n, \quad (1.1)$$

where C_n is called the *Chern number* of the n^{th} band, and the sum is taken over the

bands below the gap. The quantity $C = \sum_n C_n$ is called the gap Chern number of the particular gap. The Chern number is a topological quantity that can be calculated from the Bloch wavefunctions $u_{n,\vec{k}}(\vec{r})$ as

$$C_n = \frac{1}{2\pi} \int_{BZ} d\vec{S}_k \cdot \vec{\Omega}_n(\vec{k}) \quad (1.2)$$

$$\text{where } \vec{\Omega}_n(\vec{k}) = \nabla_{\vec{k}} \times \vec{A}_n(\vec{k}) \quad (1.3)$$

$$\text{where } \vec{A}_n(\vec{k}) = i \langle u_{n,\vec{k}} | \nabla_{\vec{k}} | u_{n,\vec{k}} \rangle. \quad (1.4)$$

Here, $\vec{A}_n(\vec{k})$ is called the Berry connection and $\vec{\Omega}_n(\vec{k})$ is called the Berry curvature. Mathematically, the Berry connection $\vec{A}_n(\vec{k})$ is analogous to an electromagnetic vector potential and the Berry curvature $\vec{\Omega}_n(\vec{k})$ is analogous to a magnetic field in momentum space. Therefore the Chern number C_n is sometimes called a topological charge by analogy with magnetic charge. It can be shown that the Chern number is an integer [3] and hence is insensitive to small perturbations, which is why the Hall conductance is robust against disorder. Within the gaps with non-zero gap Chern number, there exist chiral states localized at the edge of the sample [4]. These edge states are chiral meaning that they are allowed to move along one direction only and experience negligible backscattering [5–7]. As long as the states residing at opposite edges (that have opposite chirality) do not overlap in real space, the edge states can bypass any disorder or defect in its path since there is no allowed mode in the bulk to scatter into (as the bulk is gapped). Insulators with non-zero gap Chern number ($\sum_n C_n$) are called *Chern insulators*. Generally, materials that are insulating in the bulk but have topological edge states are called *topological insulators*.

The existence of chiral edge states follows from the non-trivial topology of the bulk bands [8, 9]. Formally this is known as the bulk-edge correspondence principle. In the context of the quantum Hall effect and other Chern insulators, this states

that the number of topologically protected edge states at a boundary between two insulators (for the Quantum Hall system mentioned earlier, the second insulator is air, which can be considered a trivial insulator) is equal to the difference between their gap Chern numbers.

In time-reversal invariant systems, it can be shown that the Berry curvature is an odd function of the crystal momentum (i.e., $\Omega(-k) = -\Omega(k)$) and hence the Chern number, which is obtained by integrating the Berry curvature over the Brillouin zone, is necessarily zero [10]. The presence of a magnetic field breaks time reversal symmetry explicitly and hence the Landau levels in a 2DEG acquire non-zero Chern numbers that result in the quantum Hall effect. In the context of tight-binding models, breaking time reversal symmetry is accomplished by using complex hoppings, which was used by F.D.M. Haldane to construct a Chern insulator model with zero net magnetic field, in a famous theoretical model known as the Haldane model [11].

A topological insulator without breaking time reversal symmetry was first proposed by Kane and Mele in 2005 [12, 13]. This model can be regarded as two copies of a Chern insulator, one for each spin; the corresponding spin-locked edge states are called helical edge states. Even in the presence of some spin-flip processes, it can be shown that a pair of edge states survives and spans the gap as long as time reversal symmetry is preserved [14–18]. Subsequently, the concept of symmetry protected topological phases was proposed; here, the edge states are protected as long as one or more given symmetries of the system are preserved [19–25]. A related class of time-reversal invariant topological insulators is the valley Hall insulators, which will be discussed in the next section.

Though all the phenomena described above were first proposed and observed in fermionic (mainly electronic) systems, most of them do not depend on the particle statistics but arise solely from band structure theory. In periodic arrangements of

dielectric materials, solutions to Maxwell's equations can be written using Bloch's theorem and one can derive photonic band structures similar to electronic band structures, with frequency playing the role of energy [26, 27]. Analogously, one can calculate Chern numbers for photonic bands, and any boundary between two topologically different media hosts topologically protected edge modes of light.

In 2005, Haldane and Raghu proposed that a quantum Hall-like state for classical light can be created using gyroelectric or gyromagnetic materials, which effectively break the time-reversal symmetry of Maxwell's equations by utilizing the Faraday effect [28, 29]. This was soon experimentally realized using a square lattice of gyromagnetic rods operating at microwave frequencies, as shown in Fig. 1.1 [30]. Subsequently, similar gyromagnetic photonic crystals were realized by different groups based on different types of band structure engineering [31–39]. Photonic bands with large Chern numbers were also predicted and observed [40, 41]. Most of these designs are two dimensional and use the magneto-optic effect which is weak at frequencies higher than the microwave regime. Three dimensional waveguide arrays were also used to realize photonic topological insulators, where the third dimension is treated as the time axis and the helical shape of the waveguides effectively acts like a magnetic field [42–45]. One-way edge modes also inspired several practical applications such as tunable delay lines [30], reflectionless waveguide bends and splitters [46–49], directional filters [34] and couplers [50, 51], terahertz circuits [52] and slow-light waveguides [53], etc.

The first proposal for a photonic analogue of a spin Hall-like state came in 2011, based on a 2-dimensional array of ring resonators in which the degenerate clockwise and anti-clockwise circulating modes of a ring resonator lattice act as two decoupled Chern insulators [54–56]. In 2015, a purely dielectric scheme, constructed from cylinders in a honeycomb lattice, was proposed which was experimentally observed in 2017 [57, 58].

Figure 1.1: (a) Experimental set up for measuring the one-way edge state of a gyromagnetic photonic crystal with 2D TM modes. The inset is a picture of the ferrite rods that form a photonic crystal of lattice period $a = 4$ cm. (b) The bandstructure hosts one-way gapless edge state between the second and third TM bands, which have non-zero Chern numbers. (c) Simulated field propagation of the one-way mode and its topological protection against a long metallic scatterer. (d) The measured robust one-way transmission characteristics of the edge waveguide. [Figure excerpted from Ref. [59]]

1.2 Valley photonic crystals

Another important class of topological systems are based on the Dirac points that occur in honeycomb-type 2D lattices. The tight binding Hamiltonian for these systems is equivalent to that of p_z electrons in graphene. Let us consider a honeycomb lattice with two sublattices A and B distinguished by different on-site potential (mass) m_A and m_B , and only nearest neighbour coupling present. The system is time-reversal symmetric and when $m_A = m_B$ (i.e. inversion symmetry is preserved), the band structure hosts degeneracies at the corners of the hexagonal Brillouin zone (the K and K' points). These degeneracies are protected by the time-reversal and inversion symmetry and the dispersion near these points can be described by effective 2D Dirac Hamiltonians

$$H \approx \hbar v_D (\tau q_x \sigma_x + q_y \sigma_y). \quad (1.5)$$

Hence, the degeneracies are called Dirac points. Here v_D is the group velocity of the states, $\vec{q} = (q_x, q_y)$ are crystal momenta measured from the degeneracy point, (σ_x, σ_y) are Pauli matrices and $\tau = +(-)$ corresponds to $K(K')$. Note that the Pauli matrices do not correspond to the intrinsic spin of the electron.

To lift the degeneracy, one must add a term proportional to σ_z which can be achieved by either breaking time-reversal symmetry or inversion symmetry. Time-reversal symmetry can be broken by adding complex next-nearest neighbour hopping terms in the tight binding Hamiltonian, as was shown by Haldane [11]. This lifts the degeneracy and the two bands acquire opposite Chern numbers, giving rise to chiral edge states. Inversion symmetry breaking (which can be achieved by breaking the sublattice symmetry e.g. by assigning $m_A = +M, m_B = -M$) also lifts the degeneracy of the Dirac points but the resulting bands do not acquire nonzero Chern numbers (since time-reversal symmetry is preserved). Hence, the

system has no chiral edge states.

However, the states near K and K' can be viewed as topologically inequivalent systems. The Berry curvature is mostly concentrated near K and K' , but with opposite signs. These points are referred to as valleys. As the system is time-reversal symmetric, the effect of inversion breaking is the same for both valleys and the effective Hamiltonian near $K(K')$ point for this case becomes

$$H = \hbar v_D(\tau q_x \sigma_x + q_y \sigma_y) + v_D M \sigma_z, \quad (1.6)$$

The eigenstates for the lower band (with eigenvalue $E_- = -\sqrt{M^2 + q^2}$) are given by

$$\psi_- = \frac{1}{\sqrt{1 + \frac{(M - \sqrt{M^2 + q^2})^2}{q^2}}} \begin{bmatrix} \frac{M - \sqrt{M^2 + q^2}}{\tau q_x + i q_y} \\ 1 \end{bmatrix}, \quad (1.7)$$

and the Berry connection then reads

$$\vec{A} = \langle \psi | \nabla_k \psi \rangle = \frac{1 + \frac{M}{\sqrt{M^2 + q^2}}}{2q^2} (-q_y \hat{x} + \tau q_x \hat{y}). \quad (1.8)$$

Hence, the Berry curvature has out-of-plane component given by

$$\Omega_\tau = [\vec{\Omega}_\tau]_z = [\nabla_k \times \vec{A}]_z = \tau \frac{M}{2(q^2 + M^2)^{3/2}}. \quad (1.9)$$

Note that the Berry curvature is concentrated near the degeneracy point ($q = 0$) and becomes negligible away from the degeneracy point ($\Omega \approx \frac{1}{q^3}$ for $q > M$). The opposite sign of Berry curvature for the two valleys gives rise to a valley dependent motion of electrons leading to a valley dependent Hall effect and similar to spin current, pure valley current can be generated [60]. One can define a topological

index for a particular valley τ as

$$C_\tau = \frac{1}{2\pi} \int_{HBZ} \vec{\Omega}_\tau(\vec{k}) \cdot d\vec{S}_k, \quad (1.10)$$

where the integration is carried over half of the Brillouin zone around K or K' point (note that most of the contribution comes from states close to K or K' point) and C_τ is called the valley Chern number. For the Berry curvature given by Eq. (1.9), the valley Chern number can be calculated by expanding the denominator using the binomial theorem for $\frac{q}{M} < 1$ and the result is $C_\tau = \tau \frac{1}{2} \text{sgn}(M)$, i.e. the valley Chern number takes values $\pm \frac{1}{2}$ with opposite sign for the K and K' valleys. Now, if inversion symmetry is broken the other way ($m_A = -M, m_B = +M$), the valley Chern numbers switch (M is replaced by $-M$ and vice versa). Hence, an interface between two domains with oppositely broken inversion symmetry is expected to host topologically protected edge state for each valley (see Fig. 1.2). However, since time reversal symmetry is explicitly not broken, the edge states of the two valleys have opposite group velocities, and topological protection is limited to particular interfaces that do not cause strong scattering between the valleys. Note that states belonging to a particular valley can be assigned a quantum number (in particular, τ) specific to the valley.

A photonic crystal with valley protected edge states was first proposed by Ma and Shvets [61], using a triangular photonic crystal of dielectric pillars. Subsequently, many other valley photonic crystal (VPC) structures have been proposed and implemented [62–70]. In most of the works, one starts from a triangular photonic crystal that hosts Dirac points at Brillouin zone corners and then break inversion symmetry within the unit cell to realize the valley Hall phase. Though valley edge states are not robust for arbitrary defects, it has attracted a lot of attention due to its simple all-dielectric implementations; it does not require gyro-

Figure 1.2: (a) Tight binding model for a valley photonic crystal, infinite along x and finite along y . The unit-cell is highlighted. (b) Projected band structure with valley protected edge states (shown in red).

magnetic or gyroelectric materials. We have used a VPC to design an electrically pumped quantum cascade topological laser, as described in Chapter 2. We have also directly observed the valley edge states in infrared-regime near field measurements, as described in Chapter 3.

It is worth mentioning that valley Hall edge states has been explored recently as a platform for high speed data transfer at THz frequencies [71]. Also, on-chip routing and sensor applications are being explored, with important potential applications in photonic devices [69, 70, 72].

1.3 Weyl semimetals

A Weyl point is a degeneracy point of a three dimensional crystal where states near the degeneracy point have linear dispersion and are described by the Weyl Hamiltonian

$$H_w = \hbar(v_x k_x \sigma_x + v_y k_y \sigma_y + v_z k_z \sigma_z), \quad (1.11)$$

where $\vec{k} = (k_x, k_y, k_z)$ is the quasi-momentum measured from the degeneracy point, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices and (v_x, v_y, v_z) are the effective group velocities along (x, y, z) directions. In the following, we will assume the same group velocity ($v_x = v_y = v_z = v$) in all directions and set $\hbar = 1$ for simplicity.

The Weyl point can be thought as a generalization of the Dirac point to three dimensions but offers several special features. One thing to note immediately is that the Weyl Hamiltonian cannot be gapped by adding any small perturbation since the Hamiltonian (1.11) uses up all three Pauli matrices; any small perturbation can only move the Weyl point in momentum space without gapping it out. Further, the Weyl Hamiltonian commutes with the helicity operator ($\hat{h} = \frac{\vec{\sigma} \cdot \vec{k}}{k}$), and therefore its eigenstates have definite chirality given by $C = \text{sgn}(v)$.

The stability of a Weyl point can also be connected to a topological charge. As before, we calculate the Berry curvature for the Hamiltonian (1.11):¹

$$\vec{\Omega}(\vec{k}) = \frac{\text{sgn}(v)}{2} \frac{\vec{k}}{k^3}, \quad (1.12)$$

which leads to $\nabla_k \cdot \vec{\Omega} = \text{sgn}(v) \delta(\vec{k})$. Using the *divergence theorem*, the topological charge is calculated:

$$N = \frac{1}{2\pi} \oint \vec{\Omega}(\vec{k}) \cdot d\vec{S}_k = \text{sgn}(v), \quad (1.13)$$

¹The problem is mathematically equivalent to the problem of electronic spin in magnetic field. For the generic 2-band Hamiltonian $H = d_0(\vec{k})I + \vec{d}(\vec{k}) \cdot \vec{\sigma}$ it is well known that the Berry curvature for the valence band is given by $\Omega_j(\vec{k}) = \epsilon_{jlm} \frac{1}{|\vec{d}(\vec{k})|^3} \vec{d}(\vec{k}) \cdot \partial_{k_l} \vec{d}(\vec{k}) \times \partial_{k_m} \vec{d}(\vec{k})$.

where the integral is carried over a simple closed surface enclosing the Weyl point. Hence, a Weyl point can be regarded as a monopole source of Berry curvature in momentum space, carrying a topological charge equal to its chirality C . The topological charge can only be altered by annihilation with another Weyl point of opposite chirality [73].

Either parity or time-reversal symmetry should be broken for Weyl points to exist, since, in the presence of both time reversal (which implies $\vec{\Omega}(-\vec{k}) = -\vec{\Omega}(\vec{k})$) and parity (which implies $\vec{\Omega}(-\vec{k}) = \vec{\Omega}(\vec{k})$) the total Berry curvature must be zero at each point of momentum space. Also, according to the *fermion doubling theorem* [74, 75], Weyl points must occur in pairs of opposite chirality.

A system in which the Fermi level (operating energy for photonic systems) resides near the Weyl point (so that quasiparticle excitations are governed by the Weyl equation) is called a Weyl semimetal or Weyl medium. The possibility of Weyl semimetal was proposed by Murakami in 2007 [76, 77] and the term Weyl semimetal was coined by Wan *et. al.* who reported the first experimental finding of Weyl semimetal in a condensed matter system [73]. Since then, Weyl semimetals have been reported in a number of condensed matter systems [78–85]. Photonic analogue of Weyl semimetal was also realized using an inversion broken double-gyroid photonic crystal [86, 87] and consequently in other photonic geometries [88, 89]. Weyl points have also been observed in 2D photonic crystal geometries where a synthetic degree of freedom is used to simulate the third dimension [90, 91].

Similar to other topological materials, the surface of a Weyl semimetal hosts edge states (surface state in this case) known as the Fermi arcs. In the reciprocal space of a 3D Weyl semimetal with a surface, a Fermi arc connects two surface-projected Weyl points of opposite Chirality (see Fig. 1.3) and are guaranteed to exist as long as the Weyl points do not annihilate by neutralization of topological charge. In Chapter 4 we discuss how the Fermi arc influence the particle trajectory

Figure 1.3: Fermi arcs connecting surface projections of Weyl points of opposite chiralities. [Figure excerpted from Ref. [92]]

of a Weyl semimetal in addition to chirality.

Chapter 2

Electrically Pumped Topological Laser using Valley Photonic Crystal

2.1 Topological laser

One of the most important device applications for topological photonics is the use of topological properties of light in lasers. The immunity of topological modes against disorder and perturbations could make laser operation more robust or more efficient. However, topological lasers are challenging to study since lasers are fundamentally non-hermitian (i.e., non-conservative) and non-linear, whereas topological edge states arises from band structure theory which is based on linear and Hermitian systems [93, 94]. The introduction of non-Hermiticity to band topology thus raises fundamental theoretical questions which have been actively investigated in recent years [95–99].

Topological lasers have been realized in one-dimensional (1D) Su–Schrieffer–Heeger (SSH)-like systems [95, 96] where edge states act as high-Q (quality factor) nanocav-

ity modes that lase under suitable gain. However, the edge states of 1D lattices are practically localized defect states and do not support protected transport, and we will not discuss them further in this thesis. For two-dimensional (2D) lattices, realizing photonic topological edge states typically requires some means of simulating the effect of breaking time-reversal (T) symmetry [100], as discussed in the previous chapter. Though a topological laser based on magnetic materials has been reported [101], the need for using strong magnetic field and the relatively small band gap limits its practical applications. Another recent demonstration of 2D topological lasing [98, 102] used an array of ring resonators in which the clockwise (CW) or counterclockwise (CCW) circulation of light in the resonators act as a photonic pseudospin; staggered inter-resonator couplings generate an effective magnetic field and hence a T-broken band structure with non-trivial topology is realized for each pseudospin [54]. One limitation of this design is that it requires large-scale structural features (ring resonators) far exceeding the operating wavelength.

In this chapter, we describe an electrically pumped THz Quantum Cascade Laser (QCL) using the topological edge states of a Valley Photonic Crystal (VPC) described briefly in previous chapter. As discussed in Chapter 1, VPCs host topological edge states protected by a valley degree of freedom established by the underlying lattice symmetry. In 2D materials, the valley degree of freedom can function similarly to spin in a spintronic device but does not require strong spin-orbit coupling. Likewise, VPCs can provide robust light transport in highly compact structures with periodicity of the order of the wavelength without the need for magnetic materials or the complex construction of photonic pseudospins [61, 66]. They are therefore promising for the implementation of compact topological photonic crystal lasers.

2.2 Quantum Cascade Lasers

Quantum cascade lasers (QCLs) are electrically pumped semiconductor lasers based on intersubband electron transitions within multiple quantum wells in semiconductors [103, 104]. They are among the most important sources of mid-infrared and terahertz (THz) radiation owing to their compactness, electrical pumping performance and high efficiency [105]. Their practical applications include telecommunication [106], THz signal processing [107], imaging [108], sensing and spectroscopy.

As with any laser, the emission characteristics of a THz QCL depend on the design of the photonic cavity and are generally strongly affected by the cavity shape [109, 110]. Here, we aim to create laser modes based on valley Hall edge states. Unlike conventional waves, such edge states resist the formation of localized standing-wave modes, which is helpful for suppressing the spatial hole-burning effect [111, 112]. This is a particularly important consideration for QCLs because their gain recovery processes are faster than the carrier diffusion, unlike in traditional semiconductor lasers [113].

2.3 Design of the topological laser

Our design consists of a triangular lattice of quasi-hexagonal holes drilled through the active medium of a THz QCL wafer. The lattice is similar to the VPC first proposed by Ma and Shvets [61], but with air and dielectric regions inverted to account for the transverse-magnetic polarization required for QCLs. The 2D unit cell is shown in the inset of Fig. 2.1(a).

First let us consider the band structure along the irreducible Brillouin zone of a 2D triangular photonic crystal with hexagonal air-hole motifs, as shown in Fig. 2.1(a). The two lowest bands touch at the Brillouin zone corners (K and K'). Near these band crossing points (K and K'), the Bloch states can be described by

Figure 2.1: (a) Photonic band structure for the TM modes of a 2D triangular photonic crystal of hexagonal air holes in dielectric (refractive index 3.6), with unbroken inversion symmetry. The unit cell and Brillouin zone are shown inset. (b) Band structure after breaking inversion symmetry by setting $d_1 \neq d_2$. Inset, unit cell, with $d_1 = 0.58a, d_2 = 0.26a$ with $a = 19.5 \mu\text{m}$. The Dirac points at K and K' are lifted by the inversion symmetry breaking. (c), (d) Plots of the absolute value of the out-of-plane electric field $|E_z|$ (colour maps) and Poynting vector (white arrows) within each unit cell at the K and K' points. For both the lower band (c) and upper band (d), the modes in the two valleys are time-reversed counterparts, as shown by the opposite circulations of electromagnetic power.

an effective 2D Dirac Hamiltonian as shown in Eq. (1.5) [64].

Setting $d_1 \neq d_2$ breaks the inversion (C_{3v}) symmetry of the photonic crystal, and lifts the degeneracy of the Dirac points as shown in Fig. 2.1(b) opening a 8% band gap. In Fig. 2.1(c,d) we plot the absolute values of the out-of-plane electric field $|E_z|$ and Poynting vectors within each unit cell at the K and K' points for both the lower band and upper band. The modes in the two valleys are time-reversed counterparts, as shown by the opposite circulations of electromagnetic power. The effect of the inversion symmetry breaking can be modelled as a mass term added to the effective 2D Dirac Hamiltonian as shown in (1.6) [64]. Inversion breaking results in non-zero valley Chern numbers as described in Section 1.2.

Here we compute the Berry curvature and valley Chern number for the VPC from numerical simulations [64, 114] as

$$C_{K/K'} = \frac{1}{2} \int_{\text{HBZ}} \Omega_{K/K'} dS_k, \quad (2.1)$$

where the integration is performed over half of the Brillouin zone (HBZ) containing K or K' . Here $\Omega_{K/K'}(\vec{k})$ is the z -component of Berry curvature defined as $\Omega = [\nabla_k \times \vec{A}(\vec{k})]_z$, where $\nabla_k = (\frac{\partial}{\partial k_x}, \frac{\partial}{\partial k_y})$. $\vec{A}(\vec{k})$ represents the Berry connection, that is,

$$\vec{A}(\vec{k}) = \int_{\text{unit cell}} d^2r u_{\vec{k}}^*(\vec{r}) \nabla_r u_{\vec{k}}(\vec{r}), \quad (2.2)$$

where $\nabla_r = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ and $u_{\vec{k}}(\vec{r})$ represents the Bloch wavefunction of lowest band that can be calculated from numerical simulations using COMSOL.

Fig. 2.2(a,b) shows the numerically calculated Berry curvature near the K and K' points. The integration over each HBZ gives rise to opposite valley Chern numbers, $C_{K'} = 1/2$ and $C_K = -1/2$. Rotating the quasi-hexagonal motif by 180° is equivalent to flipping the sign of the mass parameter M , which flips the sign

Figure 2.2: Berry curvatures calculated numerically using 2D Bloch wavefunctions for the lowest TM band. (a) Near the K' valley. (b) Near the K valley. (c) as a colormap for the whole Brillouin zone.

of the Berry curvature and thereby flips the signs of the valley Chern numbers ($C'_{K'} = -1/2$ and $C'_K = 1/2$). Thus, for a straight boundary between domains of opposite hole orientations (Fig. 2.3(a)), the projected band diagram has a gap spanned by edge states with opposite group velocities in each valley, as shown in Fig. 2.3. The Chern number difference between two domains are

$$\Delta C_K = C_K - C'_{K'} = -1; \Delta C_{K'} = C_{K'} - C'_K = +1. \quad (2.3)$$

Based on the topological bulk-boundary correspondence principle, there must be one forward-propagating edge state at K' and one backward-propagating edge state at K . This is verified by the projected band structure of the semi-infinite lattice, which is shown in Fig. 2.3(a,b). The field plots of the edge states show that they are indeed strongly localized to the domain wall that is between the two domains with opposite Chern numbers, as shown in Fig. 2.3(c,d). These states are topologically protected provided inter-valley scattering is negligible. As explained in Section 1.2, this limitation is due to the overall T symmetry of the VPC, and similar limitations apply to other photonic topological edge states, at THz or other frequencies, that do not rely on magnetic materials.

So far, we have considered 2D VPCs. In the actual experiment, the VPC is 3D, patterned onto a THz QCL wafer in a metal-semiconductor-metal configuration.

Figure 2.3: Edge states of the 2D VPC. (a) Supercell comprising two inequivalent VPC domains separated by a domain wall (highlighted by a red box). (b) Projected band diagram for the supercell. The red (blue) curve indicates the valley edge mode for the K (K') valley. (c, d) Out-of-plane electric field $|E_z|$ (colour maps) and Poynting vector (white arrows) for the edge modes at K, K' .

The active medium is $10\,\mu\text{m}$ thick, sandwiched between two metal plates to ensure subwavelength vertical confinement of the TM polarized lasing waves within the active layer. We performed numerical results for the 3D structure, as shown in Fig. 2.4. Compared to the 2D case, bands are shifted to lower frequencies but otherwise remain qualitatively similar. We have chosen lattice constant $a = 19.5\,\mu\text{m}$ so that the photonic band gap overlaps with the gain bandwidth of the QCL wafer ($2.95 - 3.45\,\text{THz}$). Near the domain wall, the electric fields are concentrated in the active medium (see Fig. 2.3(c,d) and Fig. 2.5(b)) which is favourable for lasing.

Lasing requires a resonator which is achieved by using a domain wall that forms a triangular loop, as shown in Fig. 2.5(a). This triangular resonator is very different from conventional smoothly shaped optical cavities. However, the topological protection of the edge state ensures a reflection-free propagation though any bending that respects the crystal symmetry resulting in negligible coupling

Figure 2.4: (a) Unit cell of VPC in 2D. (b) Unit cell of 3D VPC containing a quasi-hexagonal hole perforated through the top metal and the semiconductor layer in a metal-semiconductor-metal structure. The lattice constant is $a = 19.5 \mu\text{m}$. (c) Band structure calculated by 3D finite element simulation. (d) Projected band structure for a straight domain wall separating two domains with opposite hole orientations with 10 holes on each side.

between the K and K' valleys. In this case, the 120° bending at the corner or the triangular cavity respects crystal symmetry and hence a valley edge mode travels smoothly around the bend, as shown in Fig. 2.5(b).

In the triangular cavity, these running wave modes form discrete high Q-factor whispering gallery modes [115] as shown in Fig. 2.5(c). The modes are equispaced in frequency and the separation or Free Spectral Range (FSR) can be estimated from the condition that running waves should interfere constructively over each round trip. Constructive interference requires

$$\Delta k = \frac{2\pi}{L}, \quad (2.4)$$

where k denotes the wavenumber of the running-wave-like envelope function corresponding to any given edge state, and L is the total path length (circumference of the triangular loop). The edge states have an approximately linear dispersion relation $\Delta\omega = v\Delta k$, where ω is the angular frequency detuning relative to the mid-gap and v is the group velocity. Hence, the FSR is

$$\Delta f = \frac{v}{L}. \quad (2.5)$$

For the structure, $L = 1257 \mu\text{m}$ and we estimate $v = 4.53 \times 10^7 \text{ ms}^{-1}$ from numerical calculations (Fig. 2.4). This yields $\Delta f \approx 0.036 \text{ THz}$ which matches closely with both the simulation results ($\Delta f \approx 0.035 \text{ THz}$ in Fig. 2.5) and experimental results ($\Delta f \approx 0.033 \text{ THz}$) to be presented below.

Owing to the time-reversal symmetry of the VPC, each running-wave mode has a degenerate counterpart with opposite circulation direction. Modes can be constructed from superpositions of clockwise (CW) and counter-clockwise (CCW) running waves. Numerical solvers typically do not return the CW and CCW solutions, but rather the superpositions of the two running waves. CW and CCW modes can be reconstructed by taking suitable superpositions of the degenerate solutions returned by the numerical solver as shown in Fig. 2.6. The CW and CCW valley edge modes form two orthogonal basis modes and thus each topological lasing mode is a superposition of CW and CCW valley edge modes.

To determine the superpositions, we use the framework of coupled-mode theory [116]: the field of a set of coupled modes (for example, a lattice of resonators) can be written as a linear combination of the individual eigenmodes as

$$\vec{E}(\vec{r}) = \sum_j \psi_j \vec{E}_j(\vec{r}); \quad (2.6)$$

where individual eigenmodes have the form

$$\vec{E}_j(\vec{r}, t) = \vec{E}_j(\vec{r}) \exp[-i(\Omega_j - i\gamma_j)t] \quad (2.7)$$

where Ω_j is the eigenfrequency of j^{th} mode and γ_j is the resonance linewidth proportional to the material and radiative loss. The amplitudes (or Fourier coefficients) ψ_j can be written as a matrix of the form $H'\psi = \omega\psi$, where the Hamiltonian H' contains diagonal terms $H'_{ij} = (\Omega_j - i\gamma_j)\delta_{ij}$ and off-diagonal terms $H_{ij}(i \neq j) = \kappa_{ij}$ as coupling strength between modes i and j ; ω is the operating frequency of the

Figure 2.5: (a) SEM image of the THz QCL, whose optical cavity consists of an in-plane triangular loop of side length $21a$. The yellow shaded area is pumped by electrical injection, while the other parts are passive. The green dashed line indicates the domain wall. The black rectangle indicates a defect ($39\text{ }\mu\text{m} \times 33.5\text{ }\mu\text{m}$) etched right through the active medium of the THz QCL. Inset, cross-sectional schematic and magnified view of the domain wall. (b) Simulated electric field distribution ($|E_z|$) (top view and cross-section view) of a transmission mode in a topological waveguide with a 120° corner. The white dashed line indicates the position of the cross-section view. (c) Calculated Q factors of the structures eigenmodes, with realistic material absorption losses (approximately 20 cm^{-1}) within the passive region. The shaded area indicates the photonic bandgap of the valley Hall lattice (d) Typical eigenmode electric field ($|E_z|$) profiles at around 323 THz.

Figure 2.6: Quiver plots of Poynting vectors for two degenerate modes in a topologically non-trivial triangular loop cavity. a, b, Starting from two degenerate eigenmodes returned by the numerical solver, denoted by ψ_1 and ψ_2 , the plotted modes are (a) $\psi_1 + i\psi_2$ and (b) $\psi_1 - i\psi_2$. These have CCW and CW characteristics, respectively.

coupled system; and $\delta\omega_j = \omega - \Omega_j$. Now, lasing can be modelled by adding saturable gain in the coupled equations [117]. The resulting equations read

$$\left[H_{ij} - \delta\omega_j - i\gamma_j + i\frac{g_j}{1 + |\psi_j|^2} \right] \psi_j = 0, \quad (2.8)$$

where the pump strength g_j is assumed to be independent of frequency (which is valid when the operating frequency is close to the center of the gain-frequency curve).

Let us represent the states of the laser by

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \quad (2.9)$$

where ψ_1 and ψ_2 are the CW and CCW mode amplitudes. As the CW and CCW modes of the topological laser are degenerate, we can set $\delta\omega_1 = \delta\omega_2 = \delta\omega$. The CW and CCW modes are topologically protected running waves that have the same intensity distribution, and therefore should experience the same rates of gain

($g_1 = g_2 = g$) and loss ($\gamma_1 = \gamma_2 = \gamma$). There is a weak coupling between CW and CCW modes, induced for example by symmetry-breaking defects in the VPC. The coupling is symmetric ($\kappa_{1,2} = \kappa_{2,1} = \kappa$), and hence, the condition for steady state lasing becomes

$$H_0\psi + i\left(\frac{g}{1+|\psi|^2} - \gamma\right)\psi = \delta\omega\psi \quad (2.10)$$

where

$$H_0 = \begin{bmatrix} 0 & -\kappa \\ -\kappa & 0 \end{bmatrix} \quad (2.11)$$

At the lasing threshold, regardless of the non-hermitian terms, the solutions of the coupled-mode equations are

$$\psi \propto \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{for } \delta\omega = +\kappa \quad (2.12)$$

$$\psi \propto \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{for } \delta\omega = -\kappa. \quad (2.13)$$

In other words, the CW and CCW modes should contribute equally to the steady state lasing mode. The overall amplitude can be determined by setting the imaginary part of the eigenproblem to zero.

These results hold not only at the lasing threshold, but also in the above-threshold regime where gain saturation is in effect. Above threshold, provided κ is not too large, a single steady-state lasing mode is spontaneously chosen from one of the two possible solutions solved above, and the other solution is suppressed (that is, its amplitude is pinned to zero) by gain competition.

The above analysis rests on the idea that the underlying CW and CCW modes are counter-propagating topological modes. It does not apply if the modes experience different gain/loss rates (so that the non-Hermitian term is non-diagonal), or

if they are non-degenerate as is the case in the non-topological cavity.

2.4 Experimental results

The experiment sample is shown in Fig. 2.5(a). Note that electrical pumping is applied only to the nearest three lattice periods on each side of the domain wall, to avoid supplying gain to bulk modes and to keep the total pump current low. The in-plane modes are vertically outcoupled by scattering through the air holes drilled into the QCL active region, as well as through the additional defects described below.

The upper panel of Fig. 2.7(b) (labelled 'No defect') shows the experimentally measured emission spectra for this structure at two representative pump currents. Multimode lasing is observed with regularly spaced peaks at 3.192 THz, 3.224 THz, 3.258 THz and 3.288 THz (vertical grey lines); the average free spectral range (FSR) is comparable to the FSR in the eigenmode simulations as discussed in the previous section. The intensities are fairly low, owing to poor vertical outcoupling: the valley edge modes lie near K and K' , below the light cone, so outcoupling occurs only by air-hole scattering.

To improve the optical outcoupling efficiency (as well as to probe the robustness of the regular spacing), we deliberately introduce a small rectangular defect, about $2a$ long and $\sqrt{3}a$ wide, drilled through the top metal plate and the active medium in the irregular cavity loop. Numerical simulations show that the defect has negligible effects on the field distributions (Fig. 2.7(a)) regardless of whether it is placed on an arm or a corner of the triangle. The resulting experimental lasing spectra exhibit substantially stronger peaks, with intensities enhanced by 10–20 times while the emission peaks still maintain a regular spacing and have negligible frequency shifts relative to the original device (middle and bottom panels of Fig. 2.7(b)).

The light–current–voltage characteristics of the topological lasers without an outcoupling defect, with a side defect, and with a corner defect show clearly the laser threshold and the ‘roll-over’ position of the QCL (see Fig. 2.8).

The preservation of the peak frequencies indicates that the defect does not spoil the running-wave character of the lasing modes. With increasing pump current, we observe variations in the relative peak intensities. This ‘mode-hopping’ effect can be attributed to mode competition as well as to band structure realignment in the QCL wafer with the increase in the pump current; this is also observed in a conventional ridge laser fabricated on the same wafer (Fig. 2.9(a)).

2.5 Comparison with a non-topological laser

For comparison, we fabricated a THz QCL with the same VPC design, but replaced the topological waveguide with a photonic crystal waveguide of size-graded holes, with all holes having the same orientation (Fig. 2.10(a)). As before, a defect is introduced to improve the outcoupling efficiency.

With a side defect on the arm of the triangular cavity, the experimental spectra exhibit multiple irregularly spaced lasing peaks between 3.20 THz and 3.38 THz (Fig. 2.10(d)). When the defect position is moved to a corner of the triangular cavity, a completely new set of emission peaks is observed.

Numerical simulations reveal numerous eigenmodes distributed over the upper half of the bandgap with a range of Q factors, no evident regular spacing patterns, and with modal intensities localized on different parts of the triangle (Fig. 2.10(c)). This reflects the tendency of conventional waveguide modes to undergo localization, unlike the valley edge modes.

Figure 2.7: Typical eigenmode electric field ($|E_z|$) profiles at around 3.23 THz, with no outcoupling defect, with a side defect, and with a corner defect. (b) Emission spectra for the QCL with no outcoupling defect (top), with a side defect (middle), and with a corner defect (bottom). Grey vertical lines indicate the peak frequencies of the defect-free QCL, which correspond closely to those of the QCL with a defect. For clarity, the emission spectra are vertically offset with increasing pumping currents. a.u., arbitrary units.

Figure 2.8: Light-current-voltage curves of the topological laser with different designs. (a) The topological laser without an outcoupling defect. (b) The topological laser with a side defect. (c) The topological laser device with a corner defect. The corresponding device emission spectra are shown in Fig. 2.7(b). All intensities in three sub-figures are measured with the same intensity scale. It can be inferred from these curves that the emission power is greatly enhanced by the outcoupling defect.

Figure 2.9: Emission characteristics of a conventional ridge laser fabricated on the quantum cascade wafer. (a) Emission spectra at different pump currents. (b) Light-current-voltage curves of the ridge laser.

Figure 2.10: Topologically trivial laser with triangular loop cavity formed by a conventional photonic crystal waveguide. a, SEM image of the fabricated structure. Inset, close-up view of the waveguide with single hole orientation, which consists of five rows of size-graded holes (with size scale factors $s_1 = 0.77$, $s_2 = 0.87$, $s_3 = 1$). A defect ($39 \mu\text{m} \times 33.5 \mu\text{m}$) is included to improve outcoupling efficiency. b, Calculated eigenmode Q factors for the structure with a side defect. The shaded area indicates the photonic bandgap of the valley Hall lattice. c, Electric field (E_z) plots for typical calculated eigenmodes of the trivial cavity. The white square indicates the position of the side defect. d, Emission spectra of the topologically trivial lasers with a side defect (top panel) and corner defect (bottom panel) at different pump currents. The spectra are vertically offset for clarity. The emission peaks of two lasers are different and do not present a clear and regularly spaced pattern in frequency space.

2.6 Probing the extended topological edge states

As discussed in Section 2.4, the regular spacing of the extended eigenmodes is a signature of running modes circulating around the triangular loop, analogous to whispering-gallery modes in a disk or a ring cavity. This is the most striking feature imparted by the non-trivial topology of the VPC. We saw in Section 2.5 that a trivial cavity cannot support such modes due to strong back-reflection at the corners, which localizes the electromagnetic field at various portions of the cavity.

This phenomenon can be quantified by calculating the Inverse Participation Ratio (IPR) along the domain wall. The IPR is widely used to characterize the localization of modes, and is defined as

$$\text{IPR}(\omega) = \frac{\int_L |E_z(\omega, \xi)|^4 d\xi}{[\int_L |E_z(\omega, \xi)|^2 d\xi]^2} L, \quad (2.14)$$

where ξ is the coordinate parametrizing the 1D curve of length L and the denominator ensures normalization. For a mode confined to length L_0 , the IPR goes as L/L_0 , whereas for completely delocalized modes $L_0 \approx L$, which leads to $\text{IPR} \approx 1$. With increasing localization, L_0 decreases and the IPR increases.

The numerical results for the triangular loop cavity are shown in Fig. 2.11. As expected, the topological modes have substantially smaller IPR than the non-topological modes.

To probe the spatial distributions of the topological lasing modes and verify their running-wave nature, we fabricated another set of lasers that included an array of rectangular outcoupling defects arranged in a larger triangle enclosing the topological cavity (Fig. 2.12(a)). The defects are separated by a distance of several wavelengths (4λ) away from the domain wall and hence couple evanescently to the topological cavity lasing modes. We refer to the set of defects along each arm of the triangle as an ‘emission channel’. By selectively blocking these emission

Figure 2.11: IPR for trivial and topologically non-trivial modes. (a,b) Schematics showing the topologically non-trivial (a) and trivial (b) cavities. The 1D interfaces along which the IPR is calculated are indicated by red and blue lines. For the design of the trivial cavity, see Fig. 2.10(a). (c) IPR versus frequency for eigenmodes in the band gap for each type of cavity. The topological cavity's eigenmodes have consistently lower IPR, indicating that they are more uniformly extended along the loop. (d–f), Intensity distributions for three representative eigenmodes of the trivial cavity. For comparison, eigenmodes of the topological cavity are shown in Fig. 2.7(a).

channels (that is, covering the defects along certain arms), we can indirectly probe the spatial distributions of the lasing modes.

When all emission channels are open, we observe regularly spaced emission peaks corresponding to topological lasing modes (Fig. 2.12(b)). Next, we sequentially cover two emission channels and measure the emission spectra from the remaining channel (Fig. 2.12(a)). In all three cases, the lasing spectra and the relative peak intensities under different pump currents are essentially the same (Fig. 2.12(c–e)), indicating that the lasing modes have equal intensities on the three arms of the triangular loop cavity.

As discussed theoretically in Section 2.3, the topological edge states form degenerate CW and CCW pairs, which have same intensity distributions, gain and vertical outcoupling rates. Coupled-mode theory predicts that each topological lasing mode is composed of an equal-weight superposition of a CW and CCW pair. The coexistence of CW and CCW modes also explains why the defect along the cavity does not spoil the running wave character, even in presence of backscattering induced by the defect. To test this, we fabricated a sample with an additional straight topological waveguide located just below the triangular laser cavity (Fig. 2.13). Each CW (CCW) cavity mode evanescently couples to the straight waveguide, propagates to the right (left) and then outcouples via a second-order grating.

The sample is found to support three topological lasing modes with frequencies near 3.2 THz. By selectively covering the left or the right side of the device, we observe that each lasing mode emits with approximately equal intensities from the two facets (Fig. 2.13(c) and Fig. 2.14(a)), indicating that the CW and CCW cavity modes have equal weights. For comparison, we observe that the same sample also supports non topological lasing modes in a neighbouring frequency range, just above the photonic bandgap (around 3.4 THz), at high pumping currents, for

Figure 2.12: Topological laser with an array of evanescent outcouplers. (a) Schematic of the structure. A triangular loop cavity (green triangle) hosting topological edge states is surrounded by an array of outcoupling defects (blue rectangles) distributed around the perimeter of a larger triangle. The defects are eight lattice periods away from the topological interface, allowing for evanescent outcoupling. The inset shows different defect-covering configurations for the spectral measurements. b, Emission spectra at different pump currents (vertically shifted for clarity), with all defects uncovered. (c–e) Emission spectra at various pump currents for the three different defect-covering configurations shown in the inset of (a).

Figure 2.13: Topological laser in a directional outcoupling configuration. (a) Schematic of the structure. A straight valley edge-state waveguide is introduced below the bottom arm of the triangular loop cavity (topological interfaces are indicated by green lines), with outcoupling gratings on the left and right ends. The output facets are selectively covered to observe the directionality of the lasing modes. (b) Intensity distribution for a typical topological eigenmode obtained via a 3D numerical calculation. (c, d) Emission spectra for the topological lasing modes (c) and non-topological lasing modes (d) with left and right output facets covered. For the topological lasing modes, the spectra have similar peak intensities, whereas for the nontopological lasing modes the spectra are completely different. The two sets of lasing peaks are measured under different pump currents.

Figure 2.14: Lasing peak intensity curves for topological and non-topological lasing modes in the same laser device in a directional outcoupling configuration. The schematic of the device is shown in Fig. 2.13(a). Here, peak intensities are plotted versus pump current for the topological modes (a) and non-topological modes (b) of the same sample. P1, P2 and so on represent different emission peaks. Solid (dashed) curves correspond to the measurement with left (right) side of the device covered. Emission spectra at two representative pump currents are shown in Fig. 2.13(c-d). For the topological lasing modes, the spectra from two output facets have comparable peak intensities, whereas for the non-topological lasing modes the peaks differ in intensity and frequency in the two cases.

example at 2.96 A. The non-topological lasing modes are observed to emit with very different intensities from the two output facets (Fig. 2.13(d) and Fig. 2.14(b)). This demonstrates a qualitative difference in behaviour between topological and non-topological lasing modes in a single device.

2.7 Conclusion and future prospects

In summary, we have implemented electrically pumped lasers based on the topological edge states of a valley photonic crystal, operating in the THz frequency regime. By investigating several different device configurations, we have established a chain of evidence demonstrating the running-wave features of the topological lasing modes. The most noteworthy observation is the regular mode spacing, which arises because the modes have running-wave characteristics despite the sharp

Figure 2.15: Out-coupling of valley-protected edge states into vacuum with different terminations. (a), (b) Zigzag terminations of the QVH-PTI waveguide. (a), (b) Efficient out-coupling for zigzag terminations. (c), (d) Same as (a) and (b) but with armchair terminations. White arrows: transmitted waves, red arrows: reflected waves. [Figure excerpted from Ref. [61]]

corners of the cavity and various other disturbances.

Looking ahead, there are further opportunities in using the valley degree of freedom in other active photonic devices, and the realization of an electrically pumped topological laser points the way towards incorporating topological protection into practical device applications. Apart from promising applications as a robust THz light source, this QCL platform may find immediate use in exploring the dynamical and nonlinear features of topological laser modes.

One limitation of photonic crystal devices is that they are usually designed for a particular operating frequency or a small bandwidth. Hence, one important future direction of research is to develop active control of topological edge states. For example, one often requires a tunable laser to sweep through a large range of frequencies in experiments. Active control of the lasing frequency could make

Figure 2.16: (Left) 100% out-coupling at a zigzag interface for the VPC described in previous sections. The outcoupling produces two beams with 7 : 3 ratio of intensity (right).

Figure 2.17: A and B denotes VPCs with opposite valley Chern numbers. Light wave launched at the top interface can couple to the bottom interface for particular frequencies supported by the rhombic cavity.

topological lasers more promising for practical uses. One interesting property of valley edge states is that they can out-couple with nearly 100% efficiency at zigzag terminations as shown in the original work of Ma and Shvets (see 2.15). This has also been experimentally verified by Gao *et. al.* [66]. This phenomena could be used to control the resonance frequency of a cavity, as shown in Fig. 2.16. For the particular VPC we use, Fig. 2.16 shows that there exist two out-coupling beams with intensity ratio of 7 : 3. It may be possible to use a reflector to reflect one beam back to the valley Hall waveguide and thereby create a laser cavity; the other beam can then be used as the laser output. Changing the position of the reflector will change the total path length of light, which in turn will change the lasing frequency, enabling active control from outside of the device.

Another possibility under exploration is the re-routing of selective frequencies using a cavity as shown in Fig. 2.17. Apart from lasing, it can also have applications in on-chip photonic circuits for selectively transmitting light to particular ports.

Chapter 3

Near Field Imaging of Topological Edge State in Suspended Valley Photonic Crystal Slab

3.1 On chip topological photonics

Integrated waveguides are a key technology in on-chip photonics [118]. They enable optical interconnects for optoelectronic circuits and information processing chips. Since early works utilizing the principle of total internal reflection in dielectrics [119, 120], several alternative approaches have been developed to address the demand for unperturbed light propagation and miniaturization of photonic circuits. These include the implementation of advanced concepts in the structuring of dielectrics (e.g., photonic crystals [26, 27] and subwavelength integrated photonics[121]) and the use of plasmonics [122] and other advanced materials [123, 124]. Although the latter approach holds considerable potential for making arbitrarily shaped [125] and compact waveguides [126–130], its applicability remains limited due to considerable optical losses. The use of appropriately structured dielectrics, on the other hand,

enables both compact designs and long-range light propagation.

Photonic crystal waveguides [131] can significantly reduce device footprints compared to conventional ridge-based architecture. This advantage has been developed even further with the invention of topological photonics, which allows for topologically protected edge states that support one-way light propagation.

As we have seen, Valley Photonic Crystals (VPCs) have high promise for on-chip photonics in the telecom and THz spectral range owing to its all-dielectric design and not having the need to design suitable photonic pseudospin. VPC-based topological waveguides support propagating edge states inside the light cone, which have been studied via detection of transmitted radiation. However, the direct imaging of light flow in VPC slab waveguides at optical frequencies had not previously been explored, due to the challenge of subwavelength-scale field imaging in photonic crystals. In this chapter, we introduce the use of scattering-type scanning near-field optical microscopy (s-SNOM) to directly map VPC waveguides, supporting transverse-magnetic (TM) topological modes in the telecom range.

3.2 VPC on a suspended slab

The 2D VPC used in this work is similar to the VPC described in the previous chapter (see Fig. 2.1 and 2.3 for the 2D band structure). The key difference is that the photonic crystal is fabricated on a semiconductor heterostructure slab consisting of a high-index ($n \approx 4.3$) functional layer (germanium) and a low-index ($n \approx 2$) supporting membrane (silicon nitride) suspended in air (the 3D unit cell is shown in Fig. 3.1), in contrast to a metal-semiconductor-metal design used for the topological laser. Ge films were fabricated by thermal deposition in high vacuum (10^{-7} mTorr base pressure) using a Univex 250 system (Leybold); commercially available Si_3N_4 membranes, suspended over a window in a bulk silicon frame, were

obtained from Norcada.

For a suspended slab photonic crystal, though the modes are not strictly TE or TM, one can still classify them as TE-like and TM-like [132, 133]. The band structures for the TM-like modes are shown in Fig. 3.1(c). We find that the mid-gap frequency and gap-midgap ratio depends strongly on the thickness of the Ge layer, as shown in Fig. 3.3. Also, for a fixed thickness, the gap-midgap ratio changes with the lattice constant. The optimized values found from COMSOL simulations are $a = 470$ nm for a 260 nm thick Ge layer to set the midgap frequency at the operating frequency of 193 THz for our s-SNOM setup.

As before, to construct the topological waveguide, we combine two VPCs with opposite valley Chern indices by flipping the orientation of the triangular air holes in one crystal with respect to the other. An SEM image of such a structure is shown in Fig. 3.2(a), where the two topologically distinct domains are highlighted in red and yellow. This configuration enables topological waveguiding in a frequency range corresponding to the bulk bandgap of the VPC, by means of edge states at the boundary between the topologically distinct domains. A 3D numerical simulation (via COMSOL) of topological waveguiding (optimized for transmission via the topological edge state) is shown in Fig. 3.2(b,c). In this simulation, the excitation of the photonic structure (from the right-hand side to the left) was accomplished using a rectangular port, mimicking the slit-launcher used in the experiment (see next section). The resulting field distribution indicates that the proposed platform provides suitable conditions for coupling into and transmitting along the VPC waveguide and the waveguide mode is strongly confined to the interface between the distinct VPC domains. As the band gap at $K(K')$ point lies below the light line, the edge state confined at the interface of crystals with opposite hole orientations will be confined in the semiconductor heterostructure by total internal reflections in the third dimension.

Figure 3.1: (a) Schematic of the unit cell of slab-VPC. (b) Top view of unit cell (blue colour represent Ge and grey represent SiN). (c) Band structure of the VPC calculated for a 260 nm thick lossless Ge layer on the 100 nm thick half-perforated Si_3N_4 slab, with parameters $a = 470$ nm, $d_1 = 0.6a$, and $d_2 = 0.26a$. The TM-like gap is highlighted in yellow.

Figure 3.2: (a) SEM image of a typical VPC structure consisting of two topologically distinct domains with opposite valley Chern indices (highlighted with red and yellow pseudo colors); the scale bar is $3\mu\text{m}$. (b) Simulated TM polarized field profile showing transmission through the waveguide (from the right to the left), calculated for a 260 nm thick lossless Ge layer on a 100 nm thick half-perforated Si_3N_4 slab at 193.5 THz. The color bar corresponds to the magnitude of the vertical component of the electric field in the middle of the slab, (c) Simulated TM polarized field profile showing transmission (from the right to the left) through the zigzag valley photonic crystal (VPC) slab waveguide. All the parameters are same as (b).

Figure 3.3: Variation of (a) gap-midgap ratio and (b) midgap frequency with changing thickness of Ge layer calculated for lattice constant $a = 470$ nm and half perforated 100 nm Si_3N_4 layer.

3.3 Experimental set-up and results

In the past decade, s-SNOM has been used to map a wide variety of nanoscale waveguide modes, benefiting from its wavelength independent optical resolution, extending down to the 10 nm scale. Since the technique is based on atomic-force microscopy combined with the tip-scattered light detection, it requires relatively flat surfaces for correct operation. To account for this, we perform the experiment using the configuration schematically illustrated in Fig. 3.4(a).

The Ge layer is fabricated on the bottom side of the membrane, i.e., inside the supporting frame (not shown). The near-field measurements are carried out on the flat top surface of the device. We use a commercial transmission-mode s-SNOM setup (Neaspec), which operates at a laser wavelength of $1.55 \mu\text{m}$ and maintains a synchronized sample-illumination regime. In this mode, the illumination spot is kept at the same position on the sample during the scanning process.

Laser radiation is focused at a slit (with orthogonal polarization) that launches a plane wave-like TM mode in the unpatterned area of the slab. This mode excites the topological waveguide structure, which is mapped by the s-SNOM tip [Arrow-NCpt (Nanoworld), with an oscillation frequency in the range of 245–275 kHz] via

Figure 3.4: (a) Schematic of the nanoimaging experiment. (b) SEM bottom-view image of a typical experimental sample; the yellow marks depict the part of slit-launched light propagating to the VPC waveguide; the scale bar is $10\ \mu\text{m}$.

a pseudo-heterodyne detection scheme. A bottom-view SEM image depicting a typical experimental configuration of the slit launcher and the topological waveguide structure is shown in Fig. 3.4(b). We utilize the tip-scattered signal demodulated at the second (s2) harmonic of the tip oscillation frequency, and the vertically polarized component of the tip-scattered light is selected using a polarizer.

Fig. 3.5(a) shows an experimentally obtained s-SNOM image of light propagation through the optimized VPC waveguide. The propagation direction is from the right to the left. The structure acts as a spatial filter, enabling transmission of the

Figure 3.5: . s-SNOM mapping of VPC waveguides. (a) Near-field distribution (optical amplitude s_2) for the structure with a period of $a = 711$ nm. (b) The corresponding vertical cross section (black curve), averaged along the region marked in red in subplot (a); the blue curve is the best fit Lorentzian profile. (c) and (d) Near-field distribution and averaged cross section for the structure with a period of $a = 613$ nm. Averaging is performed based on 25 adjacent vertical lines in the sampling region. Scale bars in panels (a) and (c) are $3 \mu\text{m}$.

incident light through the center channel (i.e., the boundary between topologically distinct domains), which is relevant for microwave near-field experiments and far-field observations on various slab topological platforms. Some of the incident light is coupled into the bulk VPC domains, but these mode intensities decay significantly with the propagation distance, whereas the mode confined to the topological interface remains clearly visible.

We observe that the waveguide mode is confined to within $1.4\ \mu\text{m}$ of the domain wall [Fig. 3.5(b)], estimated as the width at half maximum of a Lorentzian profile (blue curve) fitted to the averaged vertical cross section of the measured near field amplitude (black curve). The light transmitted via the interface outcouples at the opposite end of the VPC and thereafter diverges into the unstructured slab on the left side of the structure.

To find the VPC waveguides with the best waveguiding features in the actual experiments, we systematically explored structures with different lattice constants, using the theoretically optimized value as a reference starting point. The s-SNOM measurements were used to identify the optimized design supporting high confinement and transmission via the edge mode. All experimental measurements were carried out at the same excitation wavelength defined by the single frequency source of the s-SNOM apparatus. We found that the optimized experimental lattice constant shifted to larger values compared to the theoretical estimate, which may be ascribed to discrepancies between the materials parameters of the real materials and the simulation models and deviations of the fabricated structure geometry compared to the nominal shape.

An example of s-SNOM mapping over an off-tuned VPC waveguide is shown in Fig. 3.5(c). The structure has the same number of unit cells as the optimized waveguide and appears smaller in size due to the reduced lattice constant. In this case, light propagation is spread over the whole region of the top and bottom

VPC domains, without any enhanced transmission through the center channel. The measured intensity profile shows no strong central peak [Fig. 3.5(d)], unlike the sharp peak seen in Fig. 3.5(b).

3.4 Conclusion and future prospects

In summary, we have performed near-field optical mapping of a two-dimensional topological photonic structure using scattering-type scanning near-field optical microscopy in the telecom frequency range. The topological waveguide is hosted in a valley photonic crystal waveguide specifically designed to support a topological gap in the transverse magnetic spectrum with a suspended slab geometry, which enables near-field imaging on the flat side of the membrane. We have directly observed the coupling of light from the unpatterned slab medium into a guided mode in the topological photonic crystal structure.

The observed strong confinement of the optical field along the interface between topologically distinct domains, which have opposite valley Chern indices, agrees with theoretical predictions. The outcoupling of light from the topological waveguide occurs from a small area where the topological interface terminates.

This work provides a pathway for micron and nano scale imaging of waveguiding in topological nanophotonic structures. Although the present experiments were performed at telecom frequencies, similar near-field techniques can achieve deeply nanoscale resolutions and, thus, can be potentially applied to topological waveguides operating in the visible range.

Chapter 4

Novel Vortex Structure in Weyl Beam Shifts

4.1 Spatial shift of wavepackets in Weyl medium

One of the most interesting features of wave dynamics in both classical and quantum media is that wavepacket trajectories are not determined solely by the dispersion relation, but can also be influenced by the internal structure of the underlying wavefunctions. For instance, the equations of motion of a wavepacket in a periodic medium can include an “anomalous velocity” term tied to the Berry connection of the underlying Bloch functions [134–139].

Another example of nontrivial dynamics, originating from the field of optics, involves the displacement of a light beam as it reflects off a surface [140]. Interestingly, the possibility of a tiny spatial shift of a beam of light was conjectured as early as the 17th century by Isaac Newton. A lateral displacement i.e. a displacement along the plane of incidence was first observed experimentally by Von F. Goos and H. Hänchen in 1947, and is now called the Goos-Hänchen (GH) shift [141]. A transverse displacement, i.e. a displacement perpendicular to the

plane of incidence, is called an Imbert-Fedorov (IF) shift [142]. Both the GH and IF shift originate from the polarization degree of freedom of electromagnetic waves [143, 144]. Though these concepts originated from the field of optics, spatial shifts are general wave phenomena and can be observed in electronic or acoustic systems [25]. In electronic systems such as strained graphene, similar shifts can be induced by pseudospin degrees of freedom, and are predicted to have observable effects on transport in heterojunction devices [25, 145].

As introduced in Section 1.3, particles near a degeneracy point of a three-dimensional crystal with linear dispersion are governed by the 2×2 Weyl Hamiltonian

$$H_w = v(k_x\sigma_x + k_y\sigma_y + k_z\sigma_z), \quad (4.1)$$

where $\vec{k} = (k_x, k_y, k_z)$ is the quasi-momentum measured from the degeneracy point, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices and v is the effective group velocity assumed same along all directions. The degeneracy point, called the Weyl point, carries a topological charge equal to its chirality $C = \text{sgn}(v)$ and acts as a source of Berry curvature in momentum space. Previous studies into GH and IF shifts in Weyl media (in which quasi-particle excitations are governed by the Weyl Hamiltonian) dealt with *partial* reflection off a potential step separating two Weyl media [84, 146, 147]. When electrons reflect off a potential step, it can exhibit total reflection for certain wavevectors and it was found that the direction of Imbert-Fedorov shift (which was found to be non-zero only when the incident wave undergoes total internal reflection) depends on the chirality of the Weyl point. This phenomena was termed the *chirality Hall effect*.

In this chapter, we consider *total* reflection off an interface with a medium that is gapped (i.e., supporting no propagating waves at the operating energy). Unlike a potential step, an Weyl (quasi-)particle always undergoes total internal reflection

at such an interface and the interface also features a Fermi arc that extends outward from the Weyl cone in the two-dimensional (2D) surface momentum space [73]. We calculate the displacement of the reflected beam at the plane of reflection, and find that the displacement vector forms a half-vortex structure in the momentum space centered around the point where the Fermi arc touches the projected 2D surface momentum space of bulk states. This striking feature provides a way to use bulk transport to probe the topological characteristics of a Weyl medium.

4.1.1 Plane waves in Weyl medium

Let us consider the setup shown in Fig. 4.1(a), where a Weyl medium occupies the space $z > 0$ with a gapped medium in the space $z < 0$. A monochromatic beam of energy E is incident from the Weyl medium, and reflects off the surface at $z = 0$. The reflected beam can experience a displacement, denoted by a vector $\vec{\Delta}$ parallel to the x - y plane. Note that the beam always undergoes *total* reflection, as the $z < 0$ medium does not support propagating modes.

The eigenmodes of the bulk Weyl medium are described by the 2×2 Weyl Hamiltonian (4.1). For given $\vec{k}$, the modal eigenenergy (or eigenfrequency) is $E = v|\vec{k}|$, and the wavefunction is a superposition of two basis wavefunctions with coefficients given by the spinor components of the envelope function $\psi(\vec{k}, \vec{r}) = \Psi(\vec{k}) \exp(i\vec{k} \cdot \vec{r})$, where $\Psi(\vec{k})$ is an eigenvector of $H_w(\vec{k})$ and $\vec{r} \equiv (x, y, z)$.

For incident and reflected plane waves (not beams), the eigenvectors are taken to be

$$\Psi_i = \frac{1}{\sqrt{1 + \eta_-^2}} \begin{bmatrix} 1 \\ \eta_- e^{i\alpha} \end{bmatrix}, \quad \Psi_r = \frac{e^{i\phi}}{\sqrt{1 + \eta_+^2}} \begin{bmatrix} 1 \\ \eta_+ e^{i\alpha} \end{bmatrix}, \quad (4.2)$$

Figure 4.1: (a) Schematic of the reflection setup. A beam is incident from a Weyl medium in the space $z > 0$, and reflects off a gapped medium at $z < 0$. The vector $\vec{\Delta}$ denotes the total displacement of the reflected beam.

where

$$\alpha = \tan^{-1} \left(\frac{k_y}{k_x} \right), \quad \eta_{\pm} = \sqrt{\frac{E - vk_z^{\pm}}{E + vk_z^{\pm}}}, \quad (4.3)$$

$$k_z^{\pm} = \pm \sqrt{(E/v)^2 - |\vec{k}_{\perp}|^2}.$$

The $k_z < 0$ ($k_z > 0$) branch is chosen for the incident (reflected) wave, and $\vec{k}_{\perp}$ is the projection of $\vec{k}$ onto the k_x - k_y plane. The $\exp(i\phi)$ factor is a reflection coefficient, determined by the boundary condition at $z = 0$.

4.1.2 Beams in Weyl medium and spatial shift

A monochromatic Weyl beam of energy E can be described as a superposition of planar eigenmodes, with wavenumbers $\vec{k}$ constrained by $E = v|\vec{k}|$. The envelope

function has the form

$$\Psi(\vec{r}, \vec{K}_\perp) = \int d^2k_\perp g(k_x - K_x) g(k_y - K_y) \psi(\vec{k}, \vec{r}), \quad (4.4)$$

where $\vec{k}_\perp \equiv (k_x, k_y)$ denotes the in-plane wavevector of each eigenmode, $\vec{K}_\perp$ is the central (average) value for the in-plane wavevector of the beam, $g(k_j - K_j)$ is a k -space envelope function, and $\psi(\vec{k}, \vec{r})$ is the envelope function for a planar eigenmode. We let each g be a Gaussian function of unit area, zero mean, and standard deviation σ_k . For each E , the choice of sign for k_z is determined by the propagation direction of the beam.

We plug the incident and reflected eigenmodes [Eq. (4.2)] into Eq. (4.4), and expand the wavevector-dependent functions ϕ and α keeping only lowest-order terms, so as to compare the centers of the incident and reflected beams in the $z = 0$ plane. This results in the following formula for the displacement of the reflected beam:

$$\vec{\Delta}(\vec{K}_\perp) = \left[-\nabla_{k_\perp} \phi + \frac{\eta_-^2 - 1}{\eta_-^2 + 1} \nabla_{k_\perp} \alpha \right]_{\vec{k}_\perp = \vec{K}_\perp}. \quad (4.5)$$

Here, $\nabla_{k_\perp}$ denotes the two-dimensional in-plane k -space derivative, and $\vec{K}_\perp$ is the mean wave-vector for the incident beam. Note that although ϕ , $\eta_\pm$, and α are based on a specific eigenmode gauge choice [Eq. (4.2)], the shift (4.5) is gauge-independent. Details of the calculation are given in the Appendix B. It can also be shown that the beam shift formula remains same even if the Gaussian envelope approximation is relaxed (see Appendix C).

4.2 Surface states and shifts

Another phenomenon intimately linked to the internal structure of wavefunctions is the existence of topological surface states, which arise from subtle windings of the Bloch functions in momentum space [4]. For example, Weyl media in three dimensions (3D) feature linear band-crossing points called Weyl points that act as monopole sources of Berry curvature in momentum space [73], giving rise to a variety of non-trivial bulk dynamical effects [81, 84, 146–153]. At the same time, the net Berry flux between pairs of Weyl points guarantees the existence of “Fermi arcs” of surface states along any interface with a gapped medium [73]. In spite of this bulk-edge correspondence, the non-trivial bulk dynamics of Weyl media (a band geometric property) and Fermi arc surface states (a topological property) have largely been probed separately. The former has been studied using spectroscopic tools such as angle-resolved photoemission [82], whereas the latter has been studied through bulk transport effects such as negative magnetoresistance [81, 149, 152] and anomalous Hall conductivity [150].

Another interesting bulk phenomenon found in Weyl media involves the GH and IF shifts experienced by a wavepacket as it undergoes partial reflection off a potential step in the bulk [84, 146, 147, 153] leading to chirality Hall effect. These shifts are attributable to the Weyl medium’s spinor degree of freedom, and the direction of the IF shift has been shown to be determined by the sign of the Weyl point’s Berry flux. However, no direct connection to the Fermi arc surface states had previously been identified.

4.2.1 Single cone model

In this model, we consider a single Weyl point at the origin of momentum space described by the Hamiltonian (4.1). From the assumption that the real-space

Hamiltonian is Hermitian, it can be shown [154–156] that the boundary of a Weyl medium is characterizable by a single real angular parameter $\theta_b \in [0, 2\pi]$, such that

$$\begin{bmatrix} 1 & e^{-i\theta_b(\vec{k}_\perp)} \end{bmatrix} \psi_{\text{tot}} \Big|_{z=0} = 0, \quad (4.6)$$

where ψ_{tot} denotes the sum of the incident and reflected envelope functions given in (4.2). Note that θ_b may vary with $\vec{k}_\perp$. Using Eq. (4.6), we derive the reflection coefficient

$$r = e^{i\phi} = -\sqrt{\frac{1 + \eta_+^2}{1 + \eta_-^2}} \frac{1 + \eta_- e^{i\alpha} e^{-i\theta_b}}{1 + \eta_+ e^{i\alpha} e^{-i\theta_b}} \quad (4.7)$$

$$= -\frac{1 + \eta_- e^{i\beta}}{\eta_- + e^{i\beta}}, \quad (4.8)$$

where $\beta = \alpha - \theta_b$ and $\eta_- = 1/\eta_+$ is used. Eq. (4.6) also yields surface states, which have the form

$$\psi_s \propto e^{-\kappa z} \begin{bmatrix} e^{-i\theta_b} \\ -1 \end{bmatrix} \quad (4.9)$$

for $z > 0$, where $\kappa = k_x \sin \theta_b - k_y \cos \theta_b \geq 0$ is the penetration constant of the surface states in the bulk. Such surface states are found to exist along a “Fermi arc” [73] parameterized by

$$E = -k_x \cos \theta_b - k_y \sin \theta_b. \quad (4.10)$$

In this model, the Fermi arc extends to infinity, since there is only a single Weyl cone.

The resulting reflection phase behavior is displayed in Fig. 4.2, for two representative cases where θ_b is a constant independent of $\vec{k}_\perp$. The color map gives the values of ϕ within the circular domain $|\vec{k}_\perp| \leq E/v \equiv K_W$, which is a section

Figure 4.2: Map of the reflection phase ϕ experienced by an incident plane wave, versus the in-plane wavenumbers k_x and k_y (normalized to E/v). Results are shown for two different values of θ_b , which governs the boundary condition. The Fermi arc is denoted by dashes, and ϕ exhibits a -2π phase shift during a half-encirclement of the Fermi arc's touching-point (black arrow).

of the Weyl cone. The Fermi arc lies outside the cone and touches its boundary tangentially. The touching-point, and the orientation of the Fermi arc, depend on the choice of θ_b . We observe that ϕ winds by 2π during a half-encirclement around the Fermi arc touching-point. These features can be understood by considering the Weyl cone section's boundary, $|\vec{k}_\perp| = K_W$, which is parameterized by the polar angle α [Eq. (4.3)]. As we approach the boundary from the inside (i.e., real $k_z \rightarrow 0$), the expressions for Ψ_i and Ψ_r in Eq. (4.2) become linearly dependent, so that $\psi_{\text{tot}} \propto (1 + e^{i\phi})\Psi_i$ and $\eta_\pm \rightarrow 1$. Hence, the boundary condition (4.6) then implies

$$(1 + e^{i\phi})(1 + e^{i(\alpha - \theta_b)}) = 0, \quad (4.11)$$

which can be satisfied in two ways: (i) $\phi = \pi$, so that ψ_{tot} vanishes, or (ii) $\alpha - \theta_b(\vec{k}_\perp) = \pi \pmod{2\pi}$, so that the vectors in Eq. (4.6) are orthogonal. Case (ii) corresponds to the touching-point of the Fermi arc, and when it is satisfied ϕ is

Figure 4.3: Reflection phase ϕ versus α (the polar angle in the two-dimensional k -space of the incident beam), for $E = 1$, $\theta_b = \pi$, and different values of $|\vec{K}_\perp|$ close to the Weyl cone boundary. The Fermi arc touching-point occurs at $\alpha = 0$. Exactly at the boundary ($|\vec{K}_\perp| = E$), Eq. (4.11) states that $\phi = 0$ for all $\alpha \neq 0$ and is undefined at $\alpha = 0$.

undefined rather than being equal to π as in case (i). Evidently, this occurs at a minimum of $|m - 1|$ distinct values of α , where m is the winding number of the $\theta_b(\vec{k}_\perp)$ function. We deduce that $m = 0$, which includes the case where θ_b is a constant, describes the class of boundaries between a Weyl medium and a trivial gapped medium. Then there is a minimum of one touching-point, and by considering the points just inside the boundary we find that ϕ winds by 2π during a half-encirclement of the touching-point. To analytically derive the direction in which ϕ winds, consider momenta close to the boundary, such that $k_z^\pm = \pm \delta k_z$ where δk_z is a small positive number. Let $q = \delta k_z / E$, and expand η_- up to linear order in q :

$$\eta_- = \sqrt{\frac{1+q}{1-q}} \approx 1 + q, \quad \eta_-^2 \approx 1 + 2q. \quad (4.12)$$

Then the reflection coefficient is

$$\begin{aligned} r = e^{i\phi} &= -\frac{1 + \eta_- e^{i\beta}}{\eta_- + e^{i\beta}} \\ &= -1 - \frac{2iq \sin \beta}{1 + \cos \beta} + O(q^2), \end{aligned} \quad (4.13)$$

where $\beta = \alpha - \theta_b$. Hence, to leading order,

$$\sin \phi \approx -\frac{2q \sin \beta}{1 + \cos \beta}. \quad (4.14)$$

At exactly $\beta = \pm\pi$, the denominator diverges and the approximation breaks down. For small angle deviations, $\beta = \pi + \delta\beta$, we find that $\sin \phi \approx 2q \sin(\delta\beta)$, so that $\delta\phi$ switches sign with $\delta\beta$. Note that $q > 0$ for the upper cone, and $q < 0$ for the lower cone, so the two cones have opposite windings.

Fig. 4.4 shows the streamline and quiver plots of $\vec{\Delta}$ as a function of $\vec{K}_\perp$ calculated numerically using Eq. (4.5) with reflection phase given by Eq. (4.8). We find that the direction of $\vec{\Delta}$ winds by π during a half-encirclement of the Fermi arc touching point $\vec{K}_{\text{fa}}$ and moreover the magnitude scales as

$$|\vec{\Delta}| \sim \left| \vec{K}_\perp - \vec{K}_{\text{fa}} \right|^{-1/2}. \quad (4.15)$$

By analogy with the concept of free vortices from fluid mechanics, we say that $\vec{\Delta}$ exhibits a “half-vortex” structure centered at $\vec{K}_{\text{fa}}$. Note that the shift direction is not purely lateral (GH-like) or transverse (IF-like), but depends on the orientation of the incident beam relative to the Fermi arc. The scaling (4.15) is observed numerically for different directions stretching away from $\vec{K}_{\text{fa}}$ as shown in Fig. 4.5 and we can show that it comes from the $-\nabla_{k_\perp} \phi$ term in Eq. (4.5) as shown in Fig. 4.6.

The analysis also shows that the ϕ vortex at the k -space origin [Fig. 4.2], a

Figure 4.4: Maps of the beam shift $\vec{\Delta}$ versus the mean in-plane wavenumbers K_x and K_y (normalized to E/v) for the incident beam. Results are shown for two values of the boundary parameter: (a) $\theta_b = \pi/3$ and (b) $\theta_b = 2\pi/3$. In each case, the left panel shows a streamline plot indicating the direction of $\vec{\Delta}$ but not its magnitude; the right panel shows a quiver plot indicating both the direction and magnitude of $\vec{\Delta}$, over a region of K -space surrounding the Fermi arc touching-point. The dashes indicate the Fermi arc.

Figure 4.5: (a) Zoomed-in image of the shift vector $\vec{\Delta}$ close to the Fermi arc touching-point. (b) Log-log plot of the magnitude of the shift vector in different directions away from the Fermi arc touching-point. The straight lines show numerical least squares fits, which indicate that the magnitudes indeed scale as $|\vec{\Delta}| \sim |\delta\vec{K}_\perp|^{-1/2}$.

feature noted by several previous authors [147, 157, 158] does not yield a vortex in $\vec{\Delta}$, as the two terms in Eq. (4.5) have similar magnitudes and opposite vorticities and thus cancel. Near the Weyl cone, however, the $\nabla_{k_\perp} \alpha$ term is negligible and only the $-\nabla_{k_\perp} \phi$ term contributes.

The streamline plots in Fig. 4.4 (left panel) also show that $\vec{\Delta}$ winds around a point $\vec{K}_{\text{opp}}$ *opposite* to the Fermi arc touching-point. Around this point, the magnitude of the beam shift scales as $|\vec{\Delta}| \sim |\vec{K} - \vec{K}_{\text{opp}}|^{1/2}$. The vanishing of $\vec{\Delta}$ at $\vec{K}_{\text{opp}}$ means that deviations from ideal beam trajectories, due to internal spinor degrees of freedom, becomes negligible in this region of momentum space.

An analytical formula for the shift $\vec{\Delta}$ can be derived by noting that (in the following, we use $v = 1$ for brevity)

$$\nabla\phi = -i \frac{\nabla r(\vec{k}_\perp)}{r(\vec{k}_\perp)} \quad (4.16)$$

$$= \frac{i}{r} \frac{ie^{i\beta}(\eta_-^2 - 1)(\nabla\beta) + (e^{2i\beta} - 1)(\nabla\eta_-)}{(\eta_- + e^{i\beta})^2} \quad (4.17)$$

$$= \frac{(\eta_-^2 - 1)(\nabla\beta) + 2\sin\beta(\nabla\eta_-)}{1 + \eta_-^2 + 2\eta_- \cos\beta}, \quad (4.18)$$

Figure 4.6: Momentum dependence of the vector fields (a) $-\nabla\phi$ and (b) $\nabla\alpha$, for $\theta_b = \pi/2$. The half-vortex structure near the Fermi arc touching-point is evidently due to the gradient of ϕ . Note that $-\nabla\phi$ and $\nabla\alpha$ circulate in opposite directions around the origin.

and from Eq. (4.3) we have

$$\nabla\eta_- = \frac{-E}{\eta_-(E + k_z^-)^2} \frac{\vec{k}_\perp}{|k_z^-(\vec{k}_\perp)|}, \quad (4.19)$$

$$\nabla\beta = \nabla\alpha = (-k_y, k_x)/|\vec{k}_\perp|^2. \quad (4.20)$$

Putting everything together, we obtain the explicit formula

$$\begin{aligned} \vec{\Delta}(\vec{K}_\perp) = & \left[\frac{\eta_-^2(\vec{K}_\perp) - 1}{\eta_-^2(\vec{K}_\perp) + 1} + \frac{k_z^-(\vec{K}_\perp)}{E + |\vec{K}_\perp| \cos \beta(\vec{K}_\perp)} \right] \frac{1}{|\vec{K}_\perp|^2} (-K_y, K_x) \\ & + \frac{E \sin \beta(\vec{K}_\perp)}{|\vec{K}_\perp| E + |\vec{K}_\perp|^2 \cos \beta(\vec{K}_\perp)} \frac{\vec{K}_\perp}{|k_z(\vec{K}_\perp)|}. \end{aligned} \quad (4.21)$$

Now, the scaling can be verified analytically by expanding the gradient of reflection phase (4.18) about the Fermi arc touching-point. The latter is derived by requiring the penetration constant of the Fermi arc surface state to vanish, which yields

$$\vec{K}_{\text{fa}} = \left(-E \cos(\theta_b), -E \sin(\theta_b) \right). \quad (4.22)$$

We define $\vec{k}_\perp = \vec{K}_{\text{fa}} + \delta\vec{k}$, where $\delta\vec{k} = (\delta k_x, \delta k_y)$ is the wave-vector measured from

the Fermi arc touching-point. We can expand k_z^- in linear order as follows:

$$k_z^- \approx -\sqrt{2E[\delta k_x \cos(\theta_b) + \delta k_y \sin(\theta_b)]} = -Eq \quad (4.23)$$

where

$$q = \sqrt{\frac{2[\delta k_x \cos(\theta_b) + \delta k_y \sin(\theta_b)]}{E}}. \quad (4.24)$$

And from the definition of α we have

$$\cos(\alpha) \approx \frac{p \sin(\theta_b)}{E} - \cos(\theta_b), \quad \sin(\alpha) \approx -\frac{p \cos(\theta_b)}{E} - \sin(\theta_b), \quad (4.25)$$

where

$$p = \delta k_x \sin(\theta_b) - \delta k_y \cos(\theta_b). \quad (4.26)$$

and this gives

$$\sin \beta \approx -\frac{p}{E}, \quad \cos \beta \approx -1 \quad (4.27)$$

Now, from Eq. (4.18) we get

$$\begin{aligned} \nabla \phi \approx & -\frac{-q}{(1 - \sqrt{1 - q^2})} \frac{1}{E^2(1 - q^2)} (-\delta k_y + E \sin \theta_b, \delta k_x - E \cos \theta_b) \\ & - \frac{p}{E^2(\sqrt{1 - q^2} - (1 - q^2))} \frac{1}{Eq} (\delta k_x - E \cos \theta_b, \delta k_y - E \sin \theta_b). \end{aligned} \quad (4.28)$$

Expanding up to linear order in q , this simplifies to

$$\nabla \phi \approx -\frac{2}{E^2} \left(\frac{1}{q} + q \right) (-\delta k_y + E \sin \theta_b, \delta k_x - E \cos \theta_b) - \frac{2p}{E^3 q^3} (\delta k_x - E \cos \theta_b, \delta k_y - E \sin \theta_b). \quad (4.29)$$

Noting that $p/q^3 \sim 1/q$, it is readily seen that the leading order term scales as $1/q$.

4.2.2 Paired Weyl cones model

To show that the above results are not model-specific, we consider an alternative model described by the quadratic Hamiltonian [159, 160]

$$H = v \begin{bmatrix} -k_y & \gamma(k_x^2 - m) - ik_z \\ \gamma(k_x^2 - m) + ik_z & k_y \end{bmatrix}. \quad (4.30)$$

This has dispersion $E = \pm v \sqrt{\gamma^2(k_x^2 - m)^2 + k_y^2 + k_z^2}$, and exhibits either paired Weyl points or a complete bandgap, depending on the choice of m . In the region $z > 0$, we set $m = m_0 > 0$, so that there are Weyl points at $\vec{k} = [\pm\sqrt{m_0}, 0, 0]$; there are two Weyl cones for small $|E|$, which merge into a single band for larger $|E|$. In the region $z < 0$, we let $m = -m_1 < 0$, to ensure a complete gap for $|E| < v\gamma|m_1|$, and set E within this gap.

The form of the planar eigenmodes in the $z > 0$ region is similar to Eq. (4.2), and evanescent for $z < 0$. In the Weyl medium, for incident and reflected plane waves, the eigenvectors reads

$$\Psi_i = \frac{1}{\sqrt{1 + \eta^2}} \begin{bmatrix} e^{-i\alpha_-} \\ \eta \end{bmatrix}, \quad \Psi_r = \frac{e^{i\phi}}{\sqrt{1 + \eta^2}} \begin{bmatrix} e^{-i\alpha_+} \\ \eta \end{bmatrix}, \quad (4.31)$$

where

$$\alpha_{\pm} = \tan^{-1} \left(\frac{k_z^{\pm}}{\gamma(k_x^2 - m_0)} \right), \quad \eta = \sqrt{\frac{E + vk_y}{E - vk_y}}, \quad (4.32)$$

$$k_z^{\pm} = \pm \sqrt{(E/v)^2 - \gamma^2(k_x^2 - m_0)^2 - k_y^2},$$

where as before $k_z < 0$ ($k_z > 0$) branch chosen for the incident (reflected) wave.

Instead of using the boundary equation (4.6), the reflection amplitude $e^{i\phi}$ is calculated by matching the total incident and reflected envelope functions at $z = 0$

plane with the evanescent wave in the band-gap medium given by

$$\psi_t = \frac{t}{N_t} \left[\frac{E/v - k_y}{\gamma(k_x^2 + m_1) + \kappa} \right] e^{ik_x x + ik_y y + \kappa z}, \quad (4.33)$$

where $\kappa = \sqrt{\gamma^2(k_x^2 + m_1)^2 + k_y^2 - (E/v)^2}$ is the inverse decay length of the evanescent field, t is the transmission coefficient and N_t is a normalization factor. Equating $\psi_i + \psi_r = \psi_t$ at $z = 0$ for all x and y , the reflection coefficient $e^{i\phi}$ is calculated:

$$e^{i\phi} = \frac{\eta(E - vk_y) - [v\gamma(k_x^2 + m_1) + v\kappa]e^{-i\alpha_-}}{e^{-i\alpha_+}[v\gamma(k_x^2 + m_1) + v\kappa] - \eta(E - vk_y)}. \quad (4.34)$$

The spatial shift in the reflected beam can be calculated in a manner similar to the previous case, which gives

$$\vec{\Delta}(\vec{K}_\perp) = \left(-\nabla_{\vec{k}_\perp} \phi - \frac{2}{\bar{\eta}^2 + 1} \nabla_{\vec{k}_\perp} \alpha_- \right)_{\vec{k}_\perp = \vec{K}_\perp}. \quad (4.35)$$

The Fermi arc for this model is found to be the line segment $|k_x| < \sqrt{m_0}$, $k_y = E/v$ [159].

The resulting plots of $\vec{\Delta}$ versus $\vec{K}_\perp$ are shown in Fig. 4.7. The behavior is highly similar to the single-cone case, despite key differences in the calculation—not only in the Hamiltonian, but also the boundary condition implementation. There are now two Weyl cones; the cone boundaries are no longer circular, but nonetheless each cone has its own Fermi arc touching-point, and $\vec{\Delta}$ exhibits a half-vortex structure around each touching point. The winding direction is opposite in the two Weyl cones, consistent with their opposite chiralities, and a numerical fit shows that the $|\vec{\Delta}| \sim \left| \vec{K}_\perp - \vec{K}_{\text{fa}} \right|^{-1/2}$ scaling holds near the touching-points. The behaviour persists even at large values of E where the cones merge into a single band. These results indicate that these features of the beam shift are generic

Figure 4.7: Maps of the beam shift $\vec{\Delta}$, for the quadratic Hamiltonian (4.30) with $v = 1$, $m_0 = 1$ and $m_1 = 100$. The horizontal and vertical axes are the mean in-plane wavenumbers K_x and K_y (normalized to E/v) for the incident beam. Results are shown for (a) $E = 0.75$, where the Hamiltonian describes a pair of Weyl cones, and (b) $E = 1.5$, where the two Weyl cones have merged into a single band. The left panels show a streamline plot of $\vec{\Delta}$, while the right panels show a quiver plot of $\vec{\Delta}$ near the Fermi arc touching-points. The dashes indicate the Fermi arc.

to Weyl media, and are not qualitatively altered by model-specific bandstructure features.

4.3 Beam shift in a thin-film geometry

Consider a film of Weyl medium of thickness $2L$, within the space $|z| < L$ bounded above and below by a gapped medium. In this geometry, a beam in the Weyl medium will reflect repeatedly off the two parallel surfaces, much like a bouncing-ball trajectory within a waveguide, as shown in Fig. 4.8(a).

As the beam reflects consecutively off the bottom and top surfaces of the film, the beam displacements accumulate instead of cancelling. Fig. 4.8(b-c) plots the numerically-calculated beam shift over two consecutive reflections, using the quadratic Weyl Hamiltonian with continuity boundary conditions at $|z| = L$. The shift from the upper surface can be found by replacing $k_z^\pm \rightarrow k_z^\mp$ and $\kappa \rightarrow -\kappa$ in the previous equations.

In explicit terms, we take the Hamiltonian

$$H = \begin{bmatrix} -k_y & (k_x^2 - m) - ik_z \\ (k_x^2 - m) + ik_z & k_y \end{bmatrix}, \quad m = \begin{cases} m_0 > 0, & |z| \leq L \text{ (Weyl medium)} \\ -m_1, & |z| > L \text{ (gapped medium)} \end{cases} \quad (4.36)$$

Inside the Weyl semimetal, we look for states $\propto e^{i\vec{k}_\perp \cdot \vec{r}_\perp + ik_z^\pm z}$ with $\vec{k}_\perp = (k_x, k_y)$ and $\vec{r}_\perp = (x, y)$. We focus on a fixed energy E , whose bulk states have the form

$$\psi_\pm^w(\vec{k}_\perp, \vec{r}_\perp, z) = \frac{1}{\sqrt{1 + \eta^2}} \begin{bmatrix} e^{i\alpha_\mp} \\ \eta \end{bmatrix} e^{i\vec{k}_\perp \cdot \vec{r}_\perp + ik_z^\pm z}, \quad (4.37)$$

where

$$\begin{aligned}\eta &= \sqrt{\frac{E + k_y}{E - k_y}}, \\ \alpha_{\pm} &= \arctan\left(\frac{k_z^{\pm}}{k_x^2 - m_0}\right), \\ k_z^{\pm} &= \pm\sqrt{E^2 - (k_x^2 - m_0)^2 - k_y^2}.\end{aligned}\tag{4.38}$$

Here, $\psi_{\pm}^w(\vec{k}_{\perp}, \vec{r}_{\perp}, z)$ propagates to the upper/lower surface $z = \pm L$. We assume that the energy lies within the gap of the external medium. In the $z > L$ ($z < -L$) region, the solution has the decaying form

$$\psi_{\pm}^{\text{evs}}(\vec{k}_{\perp}, \vec{r}_{\perp}, z) \propto \begin{bmatrix} E - k_y \\ k_x^2 + m_1 \mp \kappa \end{bmatrix} e^{ik_x x + ik_y y \mp \kappa z}.\tag{4.39}$$

This wave function is not normalized, and due to energy conservation,

$$\kappa = \sqrt{(k_x^2 + m_1)^2 - (k_x^2 - m_0)^2}.\tag{4.40}$$

At the interface $z = \pm L$, the incident wave $\psi_{\pm}^w$, the reflected wave $\psi_{\mp}^w$, and the evanescent wave $\psi_{\pm}^{\text{evs}}$ must match:

$$\psi_{\pm}^w(\vec{k}_{\perp}, \vec{r}_{\perp}, \pm L) + r_{\pm}(\vec{k}_{\perp})\psi_{\mp}^w(\vec{k}_{\perp}, \vec{r}_{\perp}, \pm L) = t_{\pm}(\vec{k}_{\perp})\psi_{\pm}^{\text{evs}}(\vec{k}_{\perp}, \vec{r}_{\perp}, \pm L),\tag{4.41}$$

where $r_{\pm}(\vec{k}_{\perp})$ and $t_{\pm}(\vec{k}_{\perp})$ are reflection and transmission coefficients at $z = \pm L$. By multiplying $(k_x^2 + m_1 \mp \kappa, -E + k_y)$ to both sides of the equation, we obtain the total reflection coefficient

$$r_{\pm}(\vec{k}) = -\frac{e^{i\alpha_{\mp}}(k_x^2 + m_1 \mp \kappa) - \sqrt{E^2 - k_y^2}}{e^{i\alpha_{\pm}}(k_x^2 + m_1 \mp \kappa) - \sqrt{E^2 - k_y^2}} \equiv -\exp[i\phi_{\pm}(\vec{k}_{\perp})].\tag{4.42}$$

Figure 4.8: (a) Schematic of a beam bouncing within a thin film geometry. (b)–(c) Beam shift versus incident in-plane momentum, over two consecutive reflections in the thin film. The orange (blue) dashes show the Fermi arc on the bottom (top) surface.

For wave packets composed by $\psi_{\pm}^w$, the shift between the total reflected wave and the incident wave in real space at the interface $z = \pm L$ is

$$\vec{\Delta}_{\pm}(\vec{k}_{\perp}) = \vec{A}_{\mp}(\vec{k}_{\perp}) - \vec{A}_{\pm}(\vec{k}_{\perp}) - \nabla_{\vec{k}_{\perp}} \phi_{\pm}(\vec{k}_{\perp}), \quad (4.43)$$

where $\vec{A}_{\pm}(\vec{k}_{\perp}) = i\langle\psi_{\pm}^w(\vec{k}_{\perp})|\nabla_{\vec{k}_{\perp}}|\psi_{\pm}^w(\vec{k}_{\perp})\rangle$ is the Berry connection for the incident/reflected state (see Appendix C). Since $\vec{A}_{+}(\vec{k}_{\perp}) - \vec{A}_{-}(\vec{k}_{\perp})$ and $\vec{A}_{-}(\vec{k}_{\perp}) - \vec{A}_{+}(\vec{k}_{\perp})$ cancel, after two consecutive reflections between opposite surfaces $z = \pm L$, the total shift is

$$\vec{\Delta}_{\text{tot}}(\vec{k}_{\perp}) = \vec{\Delta}_{+}(\vec{k}_{\perp}) + \vec{\Delta}_{-}(\vec{k}_{\perp}) = -\nabla_{\vec{k}_{\perp}} \phi_{+}(\vec{k}_{\perp}) - \nabla_{\vec{k}_{\perp}} \phi_{-}(\vec{k}_{\perp}). \quad (4.44)$$

The resulting map of the shift is shown in Fig. 4.8. The displacement accumulates over successive reflections off two parallel surfaces, and thus affects the effective velocity of propagation in films of Weyl medium [145].

4.4 Shifts in presence of a third band

Consider a 3-band Hamiltonian describing a pair of Weyl cones along with a trivial band coupled to the Weyl part (by the term u)

$$H = \begin{bmatrix} -k_y & (k_x^2 - m) - ik_z & u \\ (k_x^2 - m) + ik_z & k_y & 0 \\ u & 0 & \beta(k_x^2 + k_y^2 + k_z^2) + V \end{bmatrix}. \quad (4.45)$$

In the Weyl medium ($z > 0$), $V = 0$ with $m > 0$; and in the gapped medium ($z < 0$), $V = V_0 > 0$ with $m = -m_1 < 0$. Taking $\det(H - E) = 0$ leads to

$$[\beta(k_x^2 + k_y^2 + k_z^2) + V - E] [E^2 - k_y^2 - k_z^2 - (k_x^2 - m)^2] + u^2(E - k_y) = 0. \quad (4.46)$$

This is quartic in k_z . We define

$$p = \beta(k_x^2 + k_y^2) + V - E, \quad q = E^2 - k_y^2 - (k_x^2 - m)^2, \quad v = u^2(E - k_y). \quad (4.47)$$

The equation for k_z reads

$$k_z^4 + k_z^2 \left(\frac{p}{\beta} - q \right) - \frac{v + pq}{\beta} = 0. \quad (4.48)$$

Let $\frac{p}{\beta} - q = B$ and $-\frac{v + pq}{\beta} = B'$. We then have

$$k_z^2 = \frac{1}{2} \left[-B \pm \sqrt{B^2 - 4B'} \right] = \frac{1}{2} \left[-B \pm \sqrt{F} \right], \quad (4.49)$$

where the term under square-root can be simplified to

$$F = B^2 - 4B' = \left(q + \frac{p}{\beta} \right)^2 + 4\frac{v}{\beta}. \quad (4.50)$$

In the Weyl medium (i.e. for both incident and reflected waves); written out explicitly,

$$\begin{aligned} B &= \frac{p}{\beta} - q = k_x^2 + 2k_y^2 + (k_x^2 - m)^2 - E^2 - \frac{E}{\beta}, \\ F &= \left(q + \frac{p}{\beta}\right)^2 + 4\frac{v}{\beta} = \left[E^2 + k_x^2 - (k_x^2 - m)^2 - \frac{E}{\beta}\right]^2 + \frac{4u^2(E - k_y)}{\beta}. \end{aligned} \quad (4.51)$$

For propagating waves in Weyl medium (which is assumed at $z > 0$ as before), we must have $F > 0$ and $\sqrt{F} > B$ (we have numerically verified that $B > 0$ for the regions of interest). Then the incident and reflected momentum along z is

$$\begin{aligned} k_z^i &= -\frac{1}{\sqrt{2}} \left(\sqrt{-B + \sqrt{F}} \right), & k_z^r &= +\frac{1}{\sqrt{2}} \left(\sqrt{-B + \sqrt{F}} \right) = -k_z^i \\ & \text{assert: } F > 0, \quad \sqrt{F} > B. \end{aligned} \quad (4.52)$$

For the transmitted wave, B and F are given by

$$\begin{aligned} B_t &= k_x^2 + (k_x^2 + m_1)^2 + 2k_y^2 - E^2 + \frac{V_0 - E}{\beta}, \\ F_t &= \left[E^2 + k_x^2 + \frac{V_0 - E}{\beta} - (k_x^2 + m_1)^2\right]^2 + 4\frac{u^2}{\beta} (E - k_y). \end{aligned} \quad (4.53)$$

A large gap for the Weyl part i.e. large m_1 and a relatively small gap for the third band i.e. relatively small but non-zero V_0 ensures that F_t is positive (this was numerically verified). We also require $-B_t + \sqrt{F_t} < 0$, which was likewise numerically verified in the regions of interest. There are two possible transmitted evanescent modes with

$$\begin{aligned} k_{z,+}^t &= -\frac{1}{\sqrt{2}} \left(\sqrt{-B_t + \sqrt{F_t}} \right), & k_{z,-}^t &= -\frac{1}{\sqrt{2}} \left(\sqrt{-B_t - \sqrt{F_t}} \right) \\ & \text{assert } -B_t + \sqrt{F_t} < 0, \end{aligned} \quad (4.54)$$

where each term in under square-root is then negative and describes evanescent

modes in the gapped medium.

The incident and reflected eigenvectors are given by

$$\psi_i = \frac{1}{1 + C^2 + D^2} \begin{bmatrix} e^{-i\alpha} \\ C \\ De^{-i\alpha} \end{bmatrix} \quad \psi_r = \frac{r}{1 + C^2 + D^2} \begin{bmatrix} e^{i\alpha} \\ C \\ De^{i\alpha} \end{bmatrix}, \quad (4.55)$$

where r is reflection coefficient and (using $k_z = k_z^i = -k_z^r \implies (k_z^i)^2 = (k_z^r)^2$)

$$\alpha = \tan^{-1} \left(\frac{k_z}{k_x^2 - m} \right), \quad C = \frac{\sqrt{(k_x^2 - m)^2 + k_z^2}}{E - k_y}. \quad (4.56)$$

$$D = \frac{u}{E - \beta(k_x^2 + k_y^2) - \beta k_z^2}$$

Two transmitted modes will have two different transmission amplitude t_+, t_- corresponding to $k_{z,+}^t, k_{z,-}^t$. We write the evanescent mode as (normalization is not required and they cancel out in the process of finding reflection amplitude):

$$\psi_{evs} = t_+ \begin{bmatrix} 1 \\ C_+ \\ D_+ \end{bmatrix} + t_- \begin{bmatrix} 1 \\ C_- \\ D_- \end{bmatrix}, \quad (4.57)$$

where

$$C_{\pm} = \frac{(k_x^2 + m_1) + k_{z,\pm}^t}{E - k_y}, \quad (4.58)$$

$$D_{\pm} = \frac{u}{E - V_0 - \beta(k_x^2 + k_y^2) - \beta(k_{z,\pm}^t)^2}.$$

Note that $C_{\pm}$ is real as $k_{z,\pm}^t$ is purely imaginary. Matching $\psi_i + \psi_r = \psi_{evs}$, we find the reflection coefficient r :

$$r = e^{i\phi} = \frac{(D_+ - D_-)(C - C_-e^{-i\alpha}) - e^{-i\alpha}(D - D_-)(C_+ - C_-)}{(De^{i\alpha} - D_-e^{i\alpha})(C_+ - C_-) - (D_+ - D_-)(C - C_-e^{i\alpha})}. \quad (4.59)$$

Figure 4.9: Beam shift map in presence of a third band. Parameters: $E = 0.75, m = 1.0, \beta = 5.0, m_1 = 100, V_0 = 100$ (a) $u = 5.0$, (b) $u = 0.5$.

Note that Wave-matching for 3-component spinors give 3 equations with 3 unknowns (r, t_+, t_-) which can be solved uniquely. Only one evanescent mode would lead to more equations than unknowns and usually requires a different boundary condition found by integrating the real space differential equation as in Ref. [161].

The beam displacement can be found using a similar procedure and is given by

$$\vec{\Delta}(k_x, k_y) = -\nabla_{k_\perp} \phi - \frac{2(1 + D^2)}{1 + C^2 + D^2} \nabla_{k_\perp} \alpha. \quad (4.60)$$

The resulting plot of $\vec{\Delta}$ is shown in Fig. 4.9. We see that the presence of a third band does not affect the half-vortex structure around the Fermi-arc-touching point.

4.5 Conclusion and future prospects

We have shown that when a beam in a Weyl medium reflects off an interface with a gapped medium, the real space displacement of the reflected beam ($\vec{\Delta}$) exhibits an anomalous half-vortex structure in momentum space. The real-space displacement of the reflected beam, $\vec{\Delta}$, varies with the direction of the incident beam, which is characterized by its average in-plane momentum $\vec{K}_\perp$ and $\vec{\Delta}(\vec{K}_\perp)$ circulates around $\vec{K}_{\text{fa}}$, the point on the boundary of the Weyl cone touched by the

Fermi arc. The magnitude of the displacement scales as an inverse square root, $|\vec{\Delta}| \sim |\vec{K}_\perp - \vec{K}_{\text{fa}}|^{-1/2}$. This behavior is observed in two different Hamiltonian models with distinct boundary condition implementations, indicating that it is generic. This effect provides a bulk probe of the topological Fermi arc at the reflecting surface.

We have also found that the displacement accumulates over successive reflections off two parallel surfaces, and thus affects the effective velocity of propagation [145] within a film of Weyl medium.

The easiest way to observe the predicted beam shift may be to use a classical Weyl medium, such as a microwave-scale photonic crystal of the sort implemented by Lu *et al.* [87]. In a microwave experiment, a phase array can be used to generate the incident beam, and a simple metal surface can serve as the reflecting surface [162]. In Ref. [87], the lattice constant is 13.4 mm and the Weyl points occur at frequency $f \approx 11.3$ GHz, with $v \approx 7 \times 10^7 \text{ ms}^{-1}$. If we operate 5% above the Weyl point frequency ($\delta f \approx 0.57$ GHz), the Weyl cone section has radius $K_W \approx 50 \text{ m}^{-1}$. Employing a beam of momentum-space width $\sigma_k \approx 5 \text{ m}^{-1}$ (real-space width ≈ 100 mm), with incident beam direction such that $|\vec{K}_\perp - \vec{K}_{\text{fa}}| \approx 15 \text{ m}^{-1}$, allows the envelope to have negligible overlap with the boundary of the Weyl cone. The model of Eqs. (4.2)–(4.5) then predicts $|\vec{\Delta}| \sim 46$ mm, which should be easily observable.

In solid state systems, these GH- and IF-like shifts may provide a probe for topological Fermi arc effects in Weyl semimetals and related materials. In a Weyl semimetal thin film bounded above and below by insulating media, a beam or traveling wavepacket undergoes repeated reflections off the two parallel surfaces; we have found that the shifts accumulate rather than canceling out, producing an anomalous boost to the in-plane motion. GH-shifts have been predicted to contribute to two-terminal conductance in graphene *p-n* interfaces [145], and likewise Fermi-arc induced shifts may be detectable via transport properties in thin films

of Weyl semimetals. Electrons in topological semimetals have been shown to have very high mobilities [81, 163], allowing multiple reflections to fit within a transport mean free path. Finally, although repeated reflections might change the beam profile, owing to higher order terms neglected in the above calculations, one can show that the beam shift formula (4.5) remains valid even if the beam profile is distorted, as long as it remains a real function with a single peak in real space (see Appendix C).

Chapter 5

Mode Delocalization in Disordered Photonic Chern Insulator

5.1 Effects of disorder on Chern insulator states

One of the features that made the quantum Hall effect an extremely important scientific discovery is the extraordinary accuracy of quantization in the Hall resistivity (approximately 1 part in 10^9). Most interestingly, it is the existence of disorder that leads to the existence of the Hall resistance plateaus in the first place.

Disorder in a crystal normally turns bulk Bloch states into localized states of the form

$$\psi(\vec{r}) \propto \exp\left(-\frac{\vec{r} - \vec{r}_0}{\xi}\right), \quad (5.1)$$

where ξ describes the extent of the wavefunction in real space and is termed the *localization length*. The seminal work of Anderson [164, 165] showed that the effect of disorder depends strongly on the dimensionality of the system. In one dimensional systems, arbitrarily small disorder localizes all states, whereas in three

dimensional systems there exist distinct energy regions of localized states (at low energies) and extended states (at high energies) with a well defined boundary called the *mobility edge*. The mobility edge can be moved by changing the disorder strength W . Hence, for a fixed Fermi level, a material can undergo a metal-insulator transition (Anderson localization transition) at some critical disorder W_c .

For two dimensional systems, it was found in subsequent works that the effect of disorder depends on the symmetry of the system. Following the usual classification of insulators in two dimensions, if discrete particle-hole and chiral (sublattice) symmetries are absent, then three “Wigner-Dyson classes” emerge [166]:

- *Unitary (class A)*: Time-reversal broken system (irrespective of spin rotation). Examples of this class are the quantum Hall system and the Haldane model.
- *Symplectic (class AII)*: Time-reversal invariant but spin-rotation broken system. One example is the Kane-Mele model.
- *Orthogonal (class AI)*: Both time-reversal and spin-rotation symmetries are preserved. Trivial insulators fall into this class.

For systems in the orthogonal class, disorder localizes all electronic states, similar to the one dimensional case [167]. However, this picture changes drastically in presence of topologically non-trivial bands. For both symplectic and unitary systems, the mechanism of localization is referred to as *levitation and annihilation*: for moderate disorder, states at the edges of both conduction and valence bands start to localize but states near the center of the bands remain extended. With increasing disorder, the band gap is populated by localized states and the extended states move towards each other and annihilate at some critical disorder strength resulting in an Anderson localization transition [168–170].

These localized states do not take part in net transport of particles, but allow the Fermi energy to take arbitrary values within a band gap. When a moderate amount of disorder is present, this leads to the Hall resistance plateaus observed in quantum Hall effect. The difference between unitary and symplectic systems is that, for symplectic systems, there exist a region of extended states with a well defined mobility edge, whereas for unitary systems there is only one extended state per topological band and hence no mobility edge can be defined [167, 171]. In the following study, we will consider only unitary systems (Chern insulators).

The localization length can be written as $\xi(E, W)$, where E is the energy and W characterizes the disorder strength. For a quasi-1D disordered system with length N_x and width N_y (where $N_x \gg N_y$), it can be shown that average transmittance along x (or Landauer conductance for condensed matter systems), T_{avg} , varies along the length of the sample as

$$T_{avg} = \exp(\langle \log(T) \rangle) \propto \exp\left(-\frac{2N_x}{\xi}\right) \implies \langle \log(T) \rangle \propto -\frac{2N_x}{\xi}. \quad (5.2)$$

Here $\langle \dots \rangle$ denotes averaging over an ensemble of independent microscopic disorder configurations. ξ can be calculated from a linear fit of $\langle \log(T) \rangle$ versus N_x [172, 173].

The normalized localization length $\xi_N(E, W)$ is defined as

$$\xi_N(E, W) = \frac{\xi(E, W)}{N_y}. \quad (5.3)$$

In the localized regime, ξ_N decreases with increasing N_y so that $\lim_{N_y \rightarrow \infty} \xi_N = 0$. For extended states, ξ_N increases with N_y , so that $\lim_{N_y \rightarrow \infty} \xi_N = \infty$. For the critical states at a mobility edge ξ_N is independent of N_y , so $\lim_{N_y \rightarrow \infty} \xi_N$ is finite. Hence, for a given disorder strength, one can locate extended states in a disordered system by calculating $\xi_N(E)$ for systems of different widths [169, 174–177].

Though the presence of the bulk extended state(s) in unitary systems (Chern in-

ulators) has been predicted by many theoretical and numerical calculations, there is thus far no direct experimental observation of their presence in Chern insulators without Landau levels. The main limitations are the necessity of using very large systems to calculate localization lengths, and competition between disorder and loss in realistic systems (both disorder and loss give rise to an exponential decay of transmission with increasing length). Though several phenomena (like the spikes in magnetoresistance in quantum Hall effect) in electronic systems are consistent with the existence of the bulk extended state, any more direct forms of observation in electronic systems is very challenging. In this regard, photonic Chern insulators are particularly promising as one can image the extended state using near-field measurements (see Chapter 3). In this chapter, we present a theoretical study using a photonic Chern insulator platform to observe experimental signatures of extended bulk states.

5.2 Model

We use a recently proposed reconfigurable photonic Chern insulator model consisting of a bipartite lattice of resonant site rings coupled via off-resonant link rings, as shown in Fig. 5.1 [178]. The lattice is positioned such a way that it facilitates both nearest-neighbour and next-nearest-neighbour hopping. The tight binding

Hamiltonian for this system reads

$$\begin{aligned}
 H &= \sum_{x,y} \hat{H}_a + \hat{H}_b + \hat{H}_{ab} + \hat{H}_{ab}^\dagger \\
 \text{where } \hat{H}_a &= \hat{a}_{x,y}^\dagger [(2J \cot(\phi/2) + M) \hat{a}_{x,y} + J \csc(\phi/2) \sum_{\pm} \hat{a}_{x,y\pm 1}] \\
 \hat{H}_b &= \hat{b}_{x,y}^\dagger [(2J \cot(\phi/2) - M) \hat{b}_{x,y} + J \csc(\phi/2) \sum_{\pm} \hat{b}_{x\pm 1,y}] \\
 \hat{H}_{ab} &= J \exp(i\phi/4) \csc(\phi/2) [\hat{a}_{x,y} (\hat{b}_{x,y} + \hat{b}_{x+1,y+1}) + \hat{b}_{x,y} (\hat{a}_{x-1,y} + \hat{a}_{x,y-1})].
 \end{aligned}$$

Here $\hat{a}^\dagger$ and $\hat{b}^\dagger$ are creation operators for the a and b sublattices, and (x, y) are integers indexing the lattice sites. For a fixed next-nearest neighbour coupling strength J and anti-resonant site and link rings ($\phi = \pi$), one can vary the detuning M to induce a topological phase transition as can be seen from Fig. 5.1(d). In the following, we choose $J = 1$ with $M = 0$ to study the topological case and $M = 3$ to study the trivial case.

The band structure for a semi-infinite lattice in topological phase ($M = 0$) with open boundary conditions along y , is shown in Fig. 5.2(a). The band gap is spanned by two counter-propagating edge states localized at opposite edges. These edge states are topologically protected and hence robust against disorder. To remove any contribution to transmission coming from these edge states, one can use periodic boundary condition along y ; the corresponding band structure is shown in Fig. 5.2(b). Note that the Chern number for the bulk bands is still non-zero.

This model has both topological and trivial phases with an easily accessible phase transition (by tuning resonator frequencies via thermal or electro-optic tuning) allowing for easy comparisons between topologically trivial and non-trivial behaviours. This reconfigurability may also be particularly useful for studying localization experimentally as different disorder realizations can be performed on a single experimental sample by applying different pumps.

Figure 5.1: (a) Square resonator lattice of site rings (blue, red) with relative detuning $\pm M$, coupled with strength J via offresonant link rings (black) detuned by $\Delta\nu$. All rings have intrinsic loss κ . Dashed line indicates the unit cell. (b) Detailed schematic centered on a single link ring, which mediates both nearest-neighbour and next-nearest-neighbour couplings between site rings, with coupling phases $\pm\phi/4$ and 0. (c) Tight binding model for the site ring resonances, consisting of a checkerboard lattice with staggered flux $\pm\phi = \pm 2\pi\Delta\nu/\text{FSR}$ respectively. (d) Bloch band energies $\delta\nu$ (shaded regions) versus site detuning M , calculated from the tightbinding model for anti-resonant site and link rings ($\phi = \pi$). Red/blue areas indicate the sublattice the modes are localized to in the large M limit. Gap Chern numbers C are indicated and topologically nontrivial link bands occur for $|M| < 2J \csc(\phi/2)$. [Figure excerpted from Ref. [178]]

Figure 5.2: Tight binding band structure for a semi-infinite lattice of the model shown in Fig. 5.1. The lattice is finite along y with (a) open boundary condition along y and (b) periodic boundary condition along y .

5.3 Results

We used the recursive Green function method to calculate transmission of a finite sample following the method proposed by Kramer and McKinnon [171]. The system geometry is shown in Fig. 5.3. We consider Anderson-type disorder characterized by a parameter W : each site has a random detuning drawn independently from a uniform distribution $[-W/2, W/2]$.

Using the recursive Green function method, starting from a given excitation at the first slice $N_x = 1$, one can calculate average transmission $\langle T(N_x; E, W) \rangle$ and $\langle \log(T) \rangle$ at any subsequent slice N_x where the averaging is performed over an ensemble of disorder configurations. A fitting of $\langle \log(T) \rangle$ versus N_x is used to extract localization length ξ according to Eq. (5.2). We consider an unit excitation at every site of the first slice.

To verify the presence of extended bulk states, we first use periodic boundary condition along y so that no effect from the robust topological edge state is present. We also consider samples that are very large along x ($N_x \approx 10^4$), so that the system is truly quasi-1D, and averaged over 500 ensembles. Fig. 5.4 shows the localization length $\xi_N(E, W)$ as a function of E for different disorder strengths W , and for

Figure 5.3: Schematic of the finite sample. A loss term $i\gamma$ is added to top and bottom sites when open boundary condition is used.

samples with different widths N_y . For most energies, samples of larger width have lower localization length, indicating that the all states of the system are localized. However, there exist a pair of energies for moderate disorders, one at each band, where localization length is actually larger for larger width which is a signature of extended states. With increasing disorder strength W these states move towards each other and annihilates near $W \approx 11$. This is consistent with the *levitation and annihilation* concept described previously. In contrast, a trivial phase with $M = 3$ do not show any signature of extended state as shown in Fig. 5.9.

Though in some cases there seems to be a region of energy where ξ_N increases with N_y , this region shrinks for larger N_y values (see Fig. 5.5), indicating the presence of a single extended state, as expected for a unitary system.

Next, we consider the possibility of detecting this extended bulk state in a realistic photonic lattice. In real photonic samples, realizing periodic boundary conditions along y is not easy, and therefore edge states are hard to remove completely. However, one can add loss at the sites residing at the edge of the

Figure 5.4: Normalized localization length ξ_N versus frequency detuning E for samples with length $N_x = 10000$, different widths N_y , different disorder strengths W , and periodic boundary condition along y . The ensemble size is 50. Red arrows indicate the critical behavior (ξ_N increasing or constant with N_y). For large W , levitation and annihilation of the critical states is observed, and the bulk states are completely localized for $W \geq 12$. The band gap of the ordered bulk system is highlighted.

Figure 5.5: The apparently extended region shrinks with increasing width. This indicates that there is a single de-localized state located at an isolated energy which is expected for unitary systems.

Figure 5.6: Normalized localization lengths versus frequency detuning E for samples of length $N_x = 50$ and fixed disorder $W = 6$, with hard boundary condition along Y , for different ensemble sizes. Red arrows indicate the signature of bulk state delocalization. A loss term $i\gamma$ with $\gamma = -1$ is added to the edge sites to suppress the edge state. A small ensemble size of 50 also shows signature of delocalization.

system, as shown in Fig. 5.3. Then after a few slices along x , the intensity of an edge state would become negligible. Also, for practicality, the sample size should not be too long.

Fig. 5.6 shows the results for a system of length $N_x = 50$, with open boundary conditions along y . A loss term $i\gamma$ with $\gamma = -1$ is added to the edge sites to suppress the edge state. We still see the signature of the extended states which suggests that this phenomena can be explored experimentally. Note that we have used an ensemble size of 500 here also; a smaller ensemble size makes the plots look less clean, but the key features are still observable, as shown in Fig. 5.7.

Another practical problem is the presence of material and radiative loss inherent to photonic systems. The presence of an intrinsic loss at all sites results in an exponential decay of transmission along the length of the sample, similar the decay due to disorder. However, we find that for a small overall loss, the signature of the bulk extended state is still visible, as shown in Fig. 5.8.

Figure 5.7: A smaller ensemble also shows the signature of extended bulk state.

Figure 5.8: In addition to high loss on edge sites, a small loss ($\kappa = 0.001$) is applied to all other lattice sites. The critical behavior is still observable at low values of κ (right panel). Here, the system length is $N_x = 50$ and the ensemble size is 500.

Figure 5.9: Similar to Fig. 5.4, but for the topological trivial lattice with $M = 3$.

5.4 Conclusion

We have proposed an experimentally feasible way to probe the localization behavior of disordered Chern insulators. This theoretical study was based on the model proposed by Leykam *et. al.* in Ref. [178]. The signature of bulk de-localization is still observable in reasonably small systems, and the results can be improved by using a sufficiently large ensemble.

Material losses must be sufficiently small to observe the delocalization in actual transmission measurements. Apart from determining the localization length, there are other theoretical methods like level-spacing statistics, but those appear to be hard to probe experimentally. Another quantity of interest is the Wigner time delay, which has previously been used to establish the extended nature of topological edge states in disordered topological photonic systems [179]. Interestingly, the Wigner time delay is insensitive to overall loss of the system. However, we have not been able to find any signature of delocalized bulk states in the statistics of Wigner time delays.

Chapter 6

Conclusion and Future Prospects

Topological photonics is an emerging field of physics which has grown exponentially in the last decade. Several interesting novel topological phenomena have been proposed and experimentally realized in various photonic platforms. Initial research in topological photonics was concentrated on realizing different topological phenomena, already explored in condensed matter systems, using suitably engineered photonic platforms. In recent years, photonics has also emerged as the potential platform for studying the interplay between topology and other interesting physical mechanisms which are out of the scope of band theory, such as nonlinearity and non-hermiticity [180–182]. The most fascinating development in this area of research has been the demonstration of topological lasers, lasers that utilize topological modes [95–99]. The robust photon transport enabled by topological states occurs in these devices notwithstanding the presence of strong non-hermiticity and non-linearity. The development of topological lasers also shows the potential of topological physics in practical applications.

In Chapter 2, we demonstrated an electrically pumped topological quantum cascade laser (QCL) based on a Valley Photonic Crystal (VPC) operating in THz frequency regime. This is the first topological laser using an electrical pumping

scheme, and also the first to utilize valley edge modes, which allows for an all-dielectric implementation with wavelength-scale system size and without an applied magnetic field. Robust multimode lasing was observed despite the existence of sharp corners in the laser cavity and other defects. We have also provided theoretical arguments and a chain of experimental evidence showing the uniform distribution of the topological lasing modes along the laser cavity. This work is expected to pave path for exploring other promising applications of VPCs, new types of topological lasers and fundamental research on dynamical and nonlinear features of topological lasing modes. The near-perfect out-coupling of valley edge modes is also promising for the development of externally tunable topological lasers [61, 66]. Future generations of telecommunication technologies are expected to use THz frequencies, and since QCLs are the most important source of THz radiation, our work may find a place in future communication technologies. Future studies may extend this topological design for selective on-chip re-routing, multiplexing/demultiplexing, and other functionalities. We have also been able to do a successful near-field imaging of valley edge modes at telecom frequencies, as described in Chapter 3.

In Chapter 4, we described a novel beam shift phenomena in Weyl media. Our study showed that the shift of an Weyl beam upon reflection in real space is strongly effected by the Fermi arc, the topological surface state of Weyl semimetals. In particular, the shift vector forms a half-vortex structure around the point where the Fermi arc touches the surface momentum space of a film of Weyl semimetal. This provides a way to probe the topological surface states via bulk transport. This finding demands further theoretical investigation for similar beam-shift-related phenomena in other types of materials. The in-plane boost in the motion of Weyl particles due to this shift should be an important phenomena to consider in the emerging field of valleytronics [183]. Like many other topological phenomena, we

have found that photonic platforms are most suitable for a direct experimental detection of this shift.

In Chapter 5, we proposed using a photonic platform to study the bulk delocalization phenomena in a disordered Chern insulator. Apart from the interesting physics at the edge, the non-trivial topology of the bands in a Chern insulator also manifests in its bulk properties, which are most prominent in presence of disorder. Contrary to the localization of bulk states in disordered 2D insulators, a single bulk state per topological band remains extended in Chern insulators up to a critical disorder strength; this phenomenon is predicted to occur even in Chern insulators without Landau levels, but has never been observed experimentally. Exploiting the ability of suitable disorder implementation via external pumps on a single device, we have shown that a photonic Chern insulator is an ideal platform for demonstrating this bulk de-localization phenomena and related physics. This study is expected to stimulate further experimental research to validate a number of theorized phenomena in disordered topological systems.

Appendix A

Fabrication and characterization of topological QCL

The fabrication process for the topological lasers described in Chapter 2 began with metal (Ti/Au, 20/700 nm) deposition by an electron-beam evaporator onto the THz QCL wafer and an n+-doped GaAs host substrate, followed by Au/Au thermo-compression wafer bonding. Wafer polishing and selective wet etching using $\text{NH}_3\cdot\text{H}_2\text{O}/\text{H}_2\text{O}_2/\text{H}_2\text{O}$ (3/57/120 ml) solution were sequentially conducted to remove the THz QCL substrate down to an etch-stop layer. The etchstop layer was then removed by 49% hydrofluoric acid solution, and the QCL active region was exposed for subsequent microfabrication. A 300-nm SiO_2 insulation layer was deposited onto the THz QCL wafer using plasma-enhanced chemical vapour deposition, followed by optical lithography and reactive-ion etching (RIE) to define the pumping area.

The photonic structure patterns were transferred onto the THz QCL wafer by optical lithography, with deposition and lift-off of the top metal layer (Ti/Au, 20/900 nm). With the top metal layer as a hard mask, the photonic structures were formed by reactive-ion dry etching through the active region with a gas mixture

of $\text{BCl}_3/\text{CH}_4 = 100/20$ standard cubic centimetres per minute. The top metal layer (remnant thickness approximately 300 nm) was retained as a top contact for current injection. The host substrate was covered by a Ti/Au (15/200 nm) layer as bottom contact. Finally, the device chip was cleaved, indium-soldered onto a copper heatsink, wire-bonded and attached to a cryostat cold finger for characterization.

The fabricated THz laser devices were characterized using a Bruker Vertex 70 Fourier-transform infrared spectrometer with a room-temperature deuterated-triglycine sulfate detector. Mounted in a helium gas-stream cryostat with temperature control at 9 K, the devices were driven by a pulser with repetition rate of 10 kHz and pulse width of 500 ns. The emission signal was captured by the detector in the vertical direction and Fourier-transformed into a spectrum, with the spectrometer scanner velocity of 1 kHz and spectrum resolution of 0.2 cm^{-1} . To measure the emission from different outcouplers, for example, the rectangular outcoupling defects or gratings, a thin metal sheet (approximately $100\text{ }\mu\text{m}$) coated with an absorptive PMMA layer (approximately $100\text{ }\mu\text{m}$) was used to cover the device emission surface partially. The absorption layer (single-pass absorption rate approximately 40%) was coated to reduce the light reflection from the metal sheet. The cover was positioned using a custom stage with a positional accuracy of about $20\text{ }\mu\text{m}$. The cover was placed very close to the device surface: the gap between the device surface and the metal sheet was smaller than $300\text{ }\mu\text{m}$.

Appendix B

Derivation of Gaussian beam shifts

In this Appendix, we derive Eq. (4.5) of Chapter 4.

Let f be a function of the in-plane momenta $\vec{k}_\perp = (k_x, k_y)$. We take a Gaussian beam of the form

$$\Psi = \frac{1}{2\pi\Delta_x\Delta_y} \int_{-\infty}^{\infty} dk_x dk_y e^{-\frac{(k_x-K_x)^2}{2\Delta_x^2}} e^{-\frac{(k_y-K_y)^2}{2\Delta_y^2}} e^{if(x,y,k_x,k_y)} e^{ik_x x + ik_y y}. \quad (\text{B.1})$$

To find the center of this beam, we expand $f(x, y, k_x, k_y)$ about the mean wave-vector $\vec{K}_\perp = (K_x, K_y)$ as

$$f(x, y, k_x, k_y) \approx f(x, y, K_x, K_y) + \left. \frac{\partial f}{\partial k_x} \right|_{\vec{K}_\perp} (k_x - K_x) + \left. \frac{\partial f}{\partial k_y} \right|_{\vec{K}_\perp} (k_y - K_y). \quad (\text{B.2})$$

Here, only terms up to first order are retained. The integral (B.1) can be written as

$$\Psi = C \int_{-\infty}^{\infty} dk_x \xi(k_x) e^{ik_x x} \int_{-\infty}^{\infty} dk_y \xi(k_y) e^{ik_y y} \quad (\text{B.3})$$

where

$$C = \frac{\exp[if(x, y, K_x, K_y)]}{2\pi\Delta_x\Delta_y}, \quad \xi(k_s) = \exp\left(-\frac{(k_s - K_s)^2}{2\Delta_s^2}\right) \exp\left(i\frac{\partial f}{\partial k_s}\Big|_{\vec{k}_\perp} (k_s - K_s)\right). \quad (\text{B.4})$$

Evaluating the integrals, we obtain

$$\Psi = \exp\left(if + iK_x x + iK_y y\right) \exp\left[-\frac{\Delta_x^2}{2} \left(x + \frac{\partial f}{\partial k_x}\right)^2\right] \exp\left[-\frac{\Delta_y^2}{2} \left(y + \frac{\partial f}{\partial k_y}\right)^2\right]. \quad (\text{B.5})$$

We find that the probability amplitude $|\Psi|^2$ is a Gaussian in real space, centered at

$$\vec{R} = \left[-\frac{\partial f}{\partial k_x}, -\frac{\partial f}{\partial k_y}\right]_{\vec{k}_\perp = \vec{K}_\perp}. \quad (\text{B.6})$$

For the particular case of Eq. (1) of the main text, the two components of the incident beam are centered at $(0, 0)$ and $(-\partial_{k_x}\alpha, -\partial_{k_y}\alpha)_{\vec{k}_\perp = \vec{K}_\perp}$ and the center of the beam is calculated by taking weighted averages over the pre-factors:

$$\vec{R}_i = \left(-\frac{\eta_-^2}{1 + \eta_-^2} \nabla_{\vec{k}_\perp} \alpha\right)_{\vec{k}_\perp = \vec{K}_\perp}, \quad (\text{B.7})$$

where, as in Eq. (4.3) of Chapter 4,

$$\alpha = \tan^{-1}\left(\frac{k_y}{k_x}\right), \quad \eta_\pm = \sqrt{\frac{E - vk_z^\pm}{E + vk_z^\pm}}, \quad (\text{B.8})$$

$$k_z^\pm = \pm \sqrt{(E/v)^2 - |\vec{k}_\perp|^2}. \quad (\text{B.9})$$

Similarly, the reflected beam is centered at

$$\vec{R}_r = \left(-\frac{1}{1 + \eta_+^2} \nabla_{\vec{k}_\perp} \phi - \frac{\eta_+^2}{1 + \eta_+^2} \nabla_{\vec{k}_\perp} (\alpha + \phi)\right)_{\vec{k}_\perp = \vec{K}_\perp}. \quad (\text{B.10})$$

The shift is given by their difference:

$$\vec{\Delta}(\vec{K}_\perp) = \left(-\nabla_{\vec{k}_\perp} \phi + \left(\frac{\eta_-^2}{1 + \eta_-^2} - \frac{\eta_+^2}{1 + \eta_+^2} \right) \nabla_{\vec{k}_\perp} \alpha \right)_{\vec{k}_\perp = \vec{K}_\perp}. \quad (\text{B.11})$$

Using the equality $\eta_+ = 1/\eta_-$ yields Eq. (4.5) of the main text.

Appendix C

Non-Gaussian beams and gauge invariance

In this Appendix, we show that the beam shift formulas described in Chapter 4 can be derived even if the Gaussian envelope approximation is relaxed. This derivation also shows that even though the reflection coefficient ϕ depends on the gauge choice used in the definition of the eigenfunctions [e.g., Eq. (4.2) of Chapter 4], the beam shift $\vec{\Delta}$ is a physical quantity that is gauge invariant. The beam shifts are derived in terms of geometrical connections defined using the spinors of the incident and reflected beams.

Let $g(\vec{k}_\perp)$ be a real valued function peaked at $\vec{k}_\perp = \vec{K}_\perp$. The incident and reflected beam can be written as

$$\Psi_i(\vec{r}, \vec{K}_\perp) = \int d\vec{k}_\perp g(\vec{k}_\perp - \vec{K}_\perp) \psi_i(\vec{k}_\perp) e^{i\vec{k}_\perp \cdot \vec{r}_\perp + ik_z^- z} \quad (\text{C.1})$$

$$\Psi_r(\vec{r}, \vec{K}_\perp) = \int d\vec{k}_\perp g(\vec{k}_\perp - \vec{K}_\perp) r(\vec{k}_\perp) \psi_r(\vec{k}_\perp) e^{i\vec{k}_\perp \cdot \vec{r}_\perp + ik_z^+ z}. \quad (\text{C.2})$$

Here, $r(\vec{k}_\perp) = e^{i\phi(\vec{k}_\perp)}$ is the reflection coefficient. To track the peak of the wavepacket,

we calculate the probability amplitude at the interface $z = 0$:

$$\left| \Psi_{i(r)}(\vec{K}_\perp, \vec{r}_\perp) \right|^2 = \int d\vec{k}_\perp d\vec{k}'_\perp g_F(\vec{k}_\perp, \vec{k}'_\perp, \vec{K}_\perp) e^{i\theta_{i(r)}(\vec{k}_\perp, \vec{k}'_\perp)} \quad (\text{C.3})$$

where

$$g_F(\vec{k}_\perp, \vec{k}'_\perp, \vec{K}_\perp) = g(\vec{k}_\perp, \vec{K}_\perp) g(\vec{k}'_\perp, \vec{K}_\perp), \quad (\text{C.4})$$

$$\theta_i(\vec{k}_\perp, \vec{k}'_\perp, \vec{r}) = -i \log \left\langle \psi_i(\vec{k}_\perp) \left| \psi_i(\vec{k}'_\perp) \right\rangle + (\vec{k}_\perp - \vec{k}'_\perp) \cdot \vec{r} \quad (\text{C.5})$$

$$\theta_r(\vec{k}_\perp, \vec{k}'_\perp, \vec{r}) = -i \log \left\langle \psi_r(\vec{k}_\perp) \left| \psi_r(\vec{k}'_\perp) \right\rangle + (\vec{k}_\perp - \vec{k}'_\perp) \cdot \vec{r} + \phi(\vec{k}_\perp) - \phi(\vec{k}'_\perp). \quad (\text{C.6})$$

We have used Dirac's bra-ket notation to express the various k -space integrals. The peak of the probability amplitude in real space is the stationary point $\vec{R}$ determined by

$$\nabla_{\vec{k}_\perp} \theta_{i(r)}(\vec{k}_\perp, \vec{k}'_\perp, \vec{R}_{i(r)}) \Big|_{\vec{K}_\perp} = 0 \quad (\text{C.7})$$

This gives the peaks of the incident and reflected beams:

$$\vec{R}_i = \vec{\mathcal{A}}_i(\vec{K}_\perp) \quad (\text{C.8})$$

$$\vec{R}_r = \vec{\mathcal{A}}_r(\vec{K}_\perp) - \nabla_{\vec{k}_\perp} \phi(\vec{k}_\perp) \Big|_{\vec{K}_\perp}. \quad (\text{C.9})$$

Here,

$$\vec{\mathcal{A}}_{i(r)}(\vec{k}_\perp) = i \left\langle \psi_{i(r)}(\vec{k}_\perp) \left| \nabla_{\vec{k}_\perp} \right| \psi_{i(r)}(\vec{k}_\perp) \right\rangle \quad (\text{C.10})$$

is the Berry connection for the incident (reflected) wave. By explicit calculation,

using the eigenstates of Weyl Hamiltonian, we obtain

$$\vec{\mathcal{A}}_i(\vec{k}_\perp) = -\frac{\eta_-^2}{\eta_-^2 + 1} \nabla_{\vec{k}_\perp} \alpha \quad (\text{C.11})$$

$$\vec{\mathcal{A}}_r(\vec{k}_\perp) = -\frac{\eta_+^2}{\eta_+^2 + 1} \nabla_{\vec{k}_\perp} \alpha. \quad (\text{C.12})$$

The shift is then given by

$$\vec{\Delta}(\vec{K}_\perp) = \vec{R}_r - \vec{R}_i = \vec{\mathcal{A}}_r(\vec{K}_\perp) - \vec{\mathcal{A}}_i(\vec{K}_\perp) - \nabla_{\vec{k}_\perp} \phi(\vec{k}_\perp)|_{\vec{K}_\perp} \quad (\text{C.13})$$

$$= \left(\frac{\eta_-^2 - 1}{\eta_-^2 + 1} \nabla_{\vec{k}_\perp} \alpha - \nabla_{\vec{k}_\perp} \phi \right)_{\vec{K}_\perp}. \quad (\text{C.14})$$

We have used the fact that $\eta_- = 1/\eta_+$.

Now consider boundary conditions of the form

$$\mathbf{M}(\vec{k}_\perp) [\psi_i(\vec{k}_\perp) + r(\vec{k}_\perp) \psi_r(\vec{k}_\perp)] = 0, \quad (\text{C.15})$$

where $\mathbf{M}(\vec{k}_\perp)$ is a 2×2 matrix characterizing the boundary. The reflection phase can be written as

$$\phi(\vec{k}_\perp) = i \log[\mathbf{M} \psi_r(\vec{k}_\perp)] - i \log[\mathbf{M} \psi_i(\vec{k}_\perp)]. \quad (\text{C.16})$$

The derivative is

$$\nabla \phi = i \frac{\nabla \mathbf{M} \psi_r}{\mathbf{M} \psi_r} - i \frac{\nabla \mathbf{M} \psi_i}{\mathbf{M} \psi_i}. \quad (\text{C.17})$$

We define two new objects

$$\psi_{i(r)}^\theta = \mathbf{M} \psi_{i(r)}, \quad (\text{C.18})$$

which are normalized as $\langle \psi_{i(r)}^\theta(\vec{k}_\perp) | \psi_{i(r)}^\theta(\vec{k}_\perp) \rangle = 1$. Then Eq. (C.17) reads

$$\nabla \phi = \left\langle \psi_r^\theta(\vec{k}) \left| i \nabla_{\vec{k}} \right| \psi_r^\theta(\vec{k}) \right\rangle - \left\langle \psi_i^\theta(\vec{k}) \left| i \nabla_{\vec{k}} \right| \psi_i^\theta(\vec{k}) \right\rangle, \quad (\text{C.19})$$

which is the difference between two geometric (non-Berry) connections. These geometric connections are related to the parallel transport determined by the boundary condition.

Although both $\vec{R}_i$ and $\vec{R}_r$ are gauge dependent (which amounts to a different choice of origin for the individual waves), their difference is gauge invariant. When

$$\psi_{i(r)}(\vec{k}_\perp) \rightarrow \psi_{i(r)}(\vec{k}_\perp) \exp(i\chi_{i(r)}(\vec{k}_\perp)), \quad (\text{C.20})$$

we find that

$$\vec{\mathcal{A}}_{i(r)}(\vec{k}_\perp) \rightarrow \vec{\mathcal{A}}_{i(r)}(\vec{k}_\perp) - \nabla_{\vec{k}_\perp} \chi_{i(r)}(\vec{k}_\perp) \quad (\text{C.21})$$

$$\nabla_{\vec{k}_\perp} \phi(\vec{k}_\perp) \rightarrow \nabla_{\vec{k}_\perp} \phi(\vec{k}_\perp) - \nabla_{\vec{k}_\perp} \chi_r(\vec{k}_\perp) + \nabla_{\vec{k}_\perp} \chi_i(\vec{k}_\perp). \quad (\text{C.22})$$

Hence, the shift given by Eq. (C.14) is gauge invariant.

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