Non-orthogonal multiple access in wireless communication networks : transmission scheme design and performance analysis

Cheng, Yanyu

2021

Cheng, Y. (2021). Non-orthogonal multiple access in wireless communication networks : transmission scheme design and performance analysis. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/146356>

<https://doi.org/10.32657/10356/146356>

This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License (CC BY-NC 4.0).

Downloaded on 26 Apr 2023 17:21:48 SGT

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**NON-ORTHOGONAL MULTIPLE ACCESS IN WIRELESS
COMMUNICATION NETWORKS: TRANSMISSION SCHEME
DESIGN AND PERFORMANCE ANALYSIS**

CHENG YANYU

SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

2021

**Non-Orthogonal Multiple Access in Wireless
Communication Networks: Transmission Scheme
Design and Performance Analysis**

Cheng Yanyu

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2021

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

12/08/2020

.....

Date

Cheng Yanyu

.....

Cheng Yanyu

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

12/08/2020

.....
Date

.....
Li Kwok Hung

Authorship Attribution Statement

This thesis contains material from five papers published in or submitted to the following peer-reviewed journals in which I am listed as an author.

Chapter 3 is originated from **Y. Cheng**, K. H. Li, K. C. Teh, and S. Luo, “Joint user pairing and subchannel allocation for multisubchannel multiuser nonorthogonal multiple access systems,” *IEEE Trans. Veh. Technol.*, vol. 67, no. 9, pp. 8238-8248, Sep. 2018.

The contributions of the co-authors are as follows:

- Prof. Li and Prof. Teh provided the initial project direction. Dr. Luo and I determined the topic of this paper.
- I conducted all the design, analysis, testing, and simulation.
- I prepared the manuscript draft. The manuscript was revised by Prof. Li, Prof. Teh, and Dr. Luo.

Chapter 4 is originated from **Y. Cheng**, K. H. Li, K. C. Teh, S. Luo, and W. Wang, “Two-step user pairing for OFDM-based cooperative NOMA systems,” *IEEE Commun. Lett.*, vol. 24, no. 4, pp. 903-906, Apr. 2020.

The contributions of the co-authors are as follows:

- Prof. Li and Prof. Teh provided the initial project direction. I determined the topic of this paper.
- I conducted all the design, analysis, testing, and simulation.
- I prepared the manuscript draft. The manuscript was revised by Prof. Li, Prof. Teh, Dr. Luo, and Dr. Wang.

Chapter 5 is originated from **Y. Cheng**, K. H. Li, K. C. Teh, S. Luo, and W. Wang, “Performance analysis of cooperative NOMA systems with adaptive mode selection and subchannel allocation,” *IEEE Trans. Veh. Technol.*, vol. 68, no. 11, pp. 10981-10990, Nov. 2019.

The contributions of the co-authors are as follows:

- Prof. Li and Prof. Teh provided the initial project direction. I determined the topic of this paper.
- I conducted all the design, analysis, testing, and simulation.
- I prepared the manuscript draft. The manuscript was revised by Prof. Li, Prof. Teh, Dr. Luo, and Dr. Wang.

Chapter 6 is originated from **Y. Cheng**, K. H. Li, Y. Liu, K. C. Teh, and H. V. Poor, “Downlink and uplink intelligent reflecting surface aided networks: NOMA and OMA,” *IEEE Trans. Wireless Commun.*, to be published.

The contributions of the co-authors are as follows:

- Prof. Liu provided the initial project direction. I determined the topic of this paper.
- I conducted all the design, analysis, testing, and simulation.
- I prepared the manuscript draft. The manuscript was revised by Prof. Li, Prof. Liu, Prof. Teh, and Prof. Poor.

Chapter 7 is originated from **Y. Cheng**, K. H. Li, Y. Liu, K. C. Teh, and G. K. Karagiannidis, “Non-orthogonal multiple access (NOMA) with multiple intelligent reflecting surfaces,” *IEEE Trans. Wireless Commun.*, to be published.

The contributions of the co-authors are as follows:

- Prof. Li, Prof. Liu, and Prof. Teh provided the initial project direction. I determined the topic of this paper.
- I conducted all the design, analysis, testing, and simulation.
- I prepared the manuscript draft. The manuscript was revised by Prof. Li, Prof. Liu, Prof. Teh, and Prof. Karagiannidis.

17/01/2021

.....
Date

Cheng Yanyu

.....
Cheng Yanyu

Dedication

This thesis is dedicated to my beloved family.

Acknowledgments

First and foremost, I would like to express my sincere gratitude to my respectable supervisor, Dr. Li Kwok Hung, for providing me with this valuable opportunity to pursue my Ph.D. degree at Nanyang Technological University (NTU). His constructive guidance and continuous support have inspired me throughout my Ph.D. period. It was my privilege and honor to work and study under his supervision. Without him, I could never conduct my doctoral research successfully. Then, I would also like to express my deep gratitude to my co-supervisor, Dr. Teh Kah Chan, for his illuminating instruction and endless patience. His amiability and encouragement always guided me forward. Furthermore, I would extend my sincere gratitude to Dr. Liu Yuanwei for hosting my visit at Queen Mary University of London (QMUL). His insightful comments and professional suggestions have benefited me from the method of thinking.

I would like to thank my two senior fellow apprentices, Dr. Luo Sheng and Dr. Wang Wei. Their kindness and friendliness facilitated my assimilation into the environment, and their solid research skills greatly helped me conduct my research.

Particular thanks would be recorded to my labmates in CR1 and INFINITUS at NTU, including Dr. Li Beibei, Dr. Zhang Heng, Dr. Bi Hui, Dr. Ma Lijia, Dr. Wang Nanshuo, Dr. Bo En, Dr. Tran Tuong Xuan, Dr. Luo Jiaqi, Dr. Sun Meng, Dr. Gao Yumeng, Ms. Liu Lin, Ms. Xiong Daiqi, and Mr. Li Xiang. Following that, special thanks would be given to my labmates in CSR at QMUL, including Dr. Yi Wenqiang, Ms. Zou Yixuan, Ms. Hao Yanling, Mr. Liu Xiao, Mr. Hou

Tianwei, Mr. Yang Zhong, Mr. Tang Zhiqing, Mr. Zhang Chao, and Mr. Xu Jiaqi. My appreciations also go to my friends, including, but not limited to, Dr. Pang Yipeng, Ms. Qiu Wei, Ms. Wu Keyu, Ms. Mao Shangbo, Ms. Wang Chenlu, Ms. Li Dian, Ms. Lu Meiyi, Ms. Ge Yuting, Ms. Wang Yuanyuan, Mr. Fang Zhou, Mr. Cheng Jiayang, Mr. Cao Zhen, Mr. Wu Zhenyu, Mr. Liu Guangda, Mr. Ye Dexian, Mr. Weng Quanchi, Mr. Tang Mutian, Mr. Guo Fengyi, Mr. Li Jiabin, and Mr. Liu Jinliang.

Last and most importantly, my most heartfelt gratitude goes to my family, especially my parents and grandmother, who always unconditionally support me. Besides, special and grateful thanks are given to my girlfriend, Ms. Wang Jingxi, for her love and encouragement. Without her company, I would be very lonely along the way.

Abstract

Non-orthogonal multiple access (NOMA) has been regarded as a promising technique for next-generation wireless communication networks due to its potential capability on spectrum enhancement. It can meet the demand of high spectral efficiency and massive connectivity by allocating multiple users to a single resource block, which is different from the conventional orthogonal multiple access (OMA). This thesis provides a systematic discourse of this newly emerging technology including the transmission scheme design and the performance analysis. More specifically, it investigates multi-carrier NOMA, multi-carrier cooperative NOMA (C-NOMA), intelligent reflecting surface (IRS)-aided NOMA, and NOMA assisted by multiple IRSs with discrete phase shifts.

First, a problem related to user pairing and subcarrier allocation for multi-carrier NOMA systems is investigated. Correspondingly, a joint user pairing and subcarrier allocation scheme is proposed, and the closed-form expression for the outage probability of the worst-performance user in the worst case is derived. To gain further insight into the performance, the diversity order is obtained by asymptotic analysis. It is revealed that the proposed scheme can achieve the optimal diversity order.

Second, multi-carrier C-NOMA systems are studied, where cell-edge users cannot communicate with the base station (BS) directly and need assistance from cell-center users. A two-step scheme pairing cell-center and cell-edge users is designed. To measure the performance of the proposed scheme, the closed-form expression for

the upper bound of outage probability is derived. By asymptotic analysis, we find that our proposed scheme achieves the optimal performance in terms of diversity order.

Third, multi-carrier C-NOMA systems are explored, in which all users are able to communicate the BS directly. Specifically, each NOMA pair can work in two modes: non-cooperative NOMA (NC-NOMA) and C-NOMA, where the mode selection is based on users' channel conditions. Correspondingly, a scheme involving user pairing, subcarrier allocation, and mode selection is proposed. The closed-form expression for the upper bound of outage probability is derived. Following that, the achievable diversity order is obtained by asymptotic analysis, which is proved to be optimal.

Fourth, since IRSs can enhance the channel quality, the combination of NOMA and IRS can further improve system performance, and IRS-aided NOMA systems are investigated, in which an IRS is deployed to intensify the coverage by assisting a cell-edge user device (UD) to communicate with the BS. To characterize system performance, new channel statistics of the BS-IRS-UD link with Nakagami- m fading are investigated. For both downlink and uplink transmissions, the closed-form expressions for the outage probability and ergodic rate are derived. To gain further insight, the diversity order and the high signal-to-noise ratio (SNR) slope are obtained according to asymptotic approximations in the high-SNR regime. It is demonstrated that the diversity order is affected by the number of IRS reflecting elements and Nakagami- m fading parameters, but the high-SNR slope is not related to those parameters. It is also revealed that the IRS outperforms the full-duplex (FD) decode-and-forward (DF) relay in the high-SNR regime.

Last, NOMA systems assisted by multiple IRSs with discrete phase shifts are investigated, in which each UD is served by an IRS to improve the quality of the received signal. Two scenarios are considered according to whether there is a direct link between the BS and each UD, and the outage performance is analyzed for each

of them. Specifically, the asymptotic expressions for the upper and lower bounds of the outage probability in the high-SNR regime are derived. Following that, the diversity order is obtained. It is shown that the use of discrete phase shifts does not degrade diversity order. More importantly, simulation results reveal that a 3-bit resolution for discrete phase shifts is sufficient to achieve near-optimal outage performance. Simulation results also imply the superiority of IRSs over FD DF relays.

Table of Contents

Dedication	i
Acknowledgments	iii
Abstract	v
Table of Contents	ix
List of Figures	xv
List of Tables	xix
List of Abbreviations	xxi
List of Symbols	xxv
1 Introduction	1
1.1 Background	1
1.1.1 Overview of 5G	1
1.1.2 Non-Orthogonal Multiple Access	3
1.2 Motivation	6
1.3 Contributions	7
1.4 Literature Review	10
1.4.1 Single-Carrier NOMA	10
1.4.2 Multi-Carrier NOMA	12

1.4.3	Cooperative NOMA	13
1.4.4	Intelligent Reflecting Surface	14
1.4.5	IRS-Aided NOMA	16
1.5	Outline of Thesis	16
2	Fundamentals	19
2.1	Key Technologies Enabling NOMA	19
2.1.1	Superposition Coding	20
2.1.2	Successive Interference Cancellation	20
2.2	Basic Principles	20
2.2.1	Downlink NOMA	20
2.2.2	Uplink NOMA	23
2.2.3	Cooperative NOMA	23
2.2.4	Intelligent Reflecting Surface	25
3	Joint User Pairing and Subcarrier Allocation for Multi-Carrier NOMA	29
3.1	Introduction	29
3.2	System Model	31
3.3	User Pairing in a Single Subcarrier	34
3.4	Benchmark Scheme	37
3.4.1	Description of the Benchmark Scheme	37
3.4.2	Performance Analysis of the Benchmark Scheme	38
3.5	JUP-SA Scheme	39
3.5.1	Description of the JUP-SA Scheme	39
3.5.2	Feasibility of the JUP-SA Scheme	40
3.5.3	Performance Analysis of the JUP-SA Scheme	41
3.6	Numerical Results	42
3.6.1	User Pairing in a Single Subcarrier	42

3.6.2	Comparisons of Benchmark and JUP-SA Schemes	43
3.7	Summary	47
4	Two-Step Pairing of Cell-Center and Cell-Edge Users for Multi-Carrier C-NOMA	49
4.1	Introduction	49
4.2	System Model	50
4.3	User Pairing in a Single Subcarrier	53
4.4	Benchmark Scheme	56
4.4.1	Description of the Benchmark Scheme	56
4.4.2	Performance Analysis of the Benchmark Scheme	56
4.5	Two-Step User Pairing Scheme	58
4.5.1	Description of the Two-Step User Pairing Scheme	58
4.5.2	Feasibility of the Two-Step User Pairing Scheme	59
4.5.3	Performance Analysis of the Two-Step User Pairing Scheme	60
4.6	Numerical Results	62
4.6.1	User Pairing in a Single Subcarrier	62
4.6.2	Comparisons of Benchmark and Two-Step User Pairing Schemes	63
4.7	Summary	65
5	Resource Allocation for Multi-Carrier C-NOMA	67
5.1	Introduction	67
5.2	System Model	68
5.3	Impact of Mode Selection in a Single Subcarrier	73
5.3.1	Performance Analysis of Mode I	74
5.3.2	Performance Analysis of Mode II	75
5.3.3	Performance Analysis after Mode Selection	76
5.4	Benchmark Scheme	78
5.4.1	Description of the Benchmark Scheme	78

5.4.2	Performance Analysis of the Benchmark Scheme	79
5.5	AMS-UPSA Scheme	81
5.5.1	Description of the AMS-UPSA Scheme	81
5.5.2	Performance Analysis of the AMS-UPSA Scheme	83
5.6	Numerical Results	85
5.6.1	Performance Improvement through Mode Selection	85
5.6.2	Comparisons of Benchmark and AMS-UPSA Schemes	87
5.7	Summary	91
6	IRS-Aided NOMA	93
6.1	Introduction	93
6.2	System Model	95
6.2.1	Channel Model	95
6.2.2	Signal Model	96
6.3	New Channel Statistics of the BS-IRS-F Link	98
6.3.1	Parameters of the IRS	98
6.3.2	New Channel Statistics	99
6.4	Performance Analysis for Downlink Transmission	101
6.4.1	NOMA	101
6.4.2	OMA	106
6.5	Performance Analysis for Uplink Transmission	110
6.5.1	NOMA	110
6.5.2	OMA	114
6.5.3	Summary of All Results	115
6.6	Numerical Results	115
6.6.1	Downlink Systems	116
6.6.2	Uplink Systems	120
6.7	Summary	122

7	Multi-IRS Assisted NOMA	123
7.1	Introduction	123
7.2	System Model	124
7.2.1	Channel Model	126
7.2.2	Signal Model	126
7.3	Scenario I (Without Direct Link)	128
7.3.1	IRS Parameters for Scenario I	128
7.3.2	Channel Statistics without Direct Link	130
7.3.3	Performance Analysis of NOMA	131
7.3.4	Performance Analysis of OMA	133
7.4	Scenario II (With Direct Link)	134
7.4.1	IRS Parameters for Scenario II	134
7.4.2	Channel Statistics with Direct Link	135
7.4.3	Performance Analysis of NOMA	137
7.4.4	Performance Analysis of OMA	138
7.4.5	Summary of Diversity Orders	139
7.5	Numerical Results	140
7.5.1	Scenario I	140
7.5.2	Scenario II	143
7.6	Summary	146
8	Conclusion and Future Work	147
8.1	Conclusion	147
8.2	Future Work	149
8.2.1	Optimal Power Allocation for NOMA	149
8.2.2	Imperfect CSI	150
8.2.3	MIMO-NOMA	150
8.2.4	User Mobility	150
8.2.5	Other Performance Indicators	150

Appendix A Proof of Proposition in Chapter 5	151
A.1 Proof of Proposition 5.1	151
Appendix B Proofs of Lemmas and Theorems in Chapter 6	153
B.1 Proof of Lemma 6.1	153
B.2 Proof of Lemma 6.2	154
B.3 Proof of Theorem 6.2	156
B.4 Proof of Theorem 6.6	157
Appendix C Proofs of Lemmas in Chapter 7	159
C.1 Proof of Lemma 7.2	159
C.2 Proof of Lemma 7.4	161
Author's Publications	165
Bibliography	181

List of Figures

1.1	Three service scenarios of 5G.	2
1.2	Evolution of multiple access techniques in wireless communication networks.	3
1.3	Comparisons of OMA and power-domain NOMA.	4
2.1	The downlink NOMA system model with one BS and two users. . .	21
2.2	The uplink NOMA system model with one BS and two users. . . .	22
2.3	The system model of a form of C-NOMA with one BS and two users, where there is no direct link between the BS and the far user. . . .	24
2.4	The IRS-aided system model.	26
3.1	The system model of multi-carrier NOMA when $K = 2$	31
3.2	Outage probabilities in a single subcarrier when $M = 4, i = 1, j = 2$ and $M = 6, i = 3, j = 5$	43
3.3	Outage probabilities of the benchmark scheme in the multi-carrier NOMA system when $K = 2$	44
3.4	Outage probability of the worst-performance user in the worst case of the JUP-SA scheme in the multi-carrier NOMA system when $K = 2$	44
3.5	Outage probabilities of the benchmark scheme in the multi-carrier NOMA system when $K = 3$	46
3.6	Outage probability of the worst-performance user in the worst case of the JUP-SA scheme in the multi-carrier NOMA system when $K = 3$	46

4.1	The system model of multi-carrier C-NOMA, where cell-edge users cannot communicate with the BS directly.	51
4.2	Outage probabilities in a single subcarrier when $i = 1, j = 3, Q_N = Q_F = 3$ and $i = 3, j = 2, Q_N = Q_F = 3$	62
4.3	Outage probabilities of the benchmark scheme in the multi-carrier C-NOMA system when $K = 3$	64
4.4	Outage probabilities of the two-step user pairing scheme in the multi-carrier C-NOMA when $K = 3$	64
5.1	The system model of multi-carrier C-NOMA when $K = 2$, where all users can communicate with the BS directly. Mode I is used by pair 1, and Mode II is used by pair 2.	69
5.2	Outage probabilities in a single subcarrier when $K = 1$, including Mode I, Mode II, and the proposed mode selection strategy.	86
5.3	Outage probabilities in a single subcarrier when $M = 4, i = 1, j = 3$ and $M = 6, i = 3, j = 6$	86
5.4	Outage probabilities of the benchmark scheme in the multi-carrier C-NOMA system when $K = 2$	88
5.5	Outage probabilities of the AMS-UPSA scheme in the multi-carrier C-NOMA system when $K = 2$	88
5.6	Outage probabilities of the benchmark scheme in the multi-carrier C-NOMA system when $K = 3$	90
5.7	Outage probabilities of the AMS-UPSA scheme in the multi-carrier C-NOMA system when $K = 3$	90
6.1	The System model of IRS-aided NOMA.	95
6.2	The PDF and CDF of X derived by the CLT when $K = 30, m_G = 3$, and $m_g = 2$	99
6.3	Outage probabilities in downlink systems.	117

6.4	Ergodic rates in downlink systems when $K = 10$	119
6.5	Outage probabilities in uplink NOMA systems when $K = 8$	121
6.6	Ergodic rates in uplink NOMA systems when $K = 10$	122
7.1	The system models of multi-IRS assisted NOMA with discrete phase shifts for two scenarios.	125
7.2	The ratio of the average equivalent channel gain with discrete phase shifts to that with continuous phase shifts when $m_G = 2$, $m_g = 1$, and $\beta = 0.9$ in Scenario I.	129
7.3	The ratio of the average equivalent channel gain with discrete phase shifts to that with continuous phase shifts when $m_G = 2$, $m_g = 1$, $m_h = 1$, and $\beta = 0.9$ in Scenario II.	136
7.4	Outage probabilities versus SNR in Scenario I when $N = 2$, $K = 2$, $m_G = 2$, $m_g = 1$, $\alpha_1 = 0.9$, $\alpha_2 = 0.1$, and $b = 3$	141
7.5	Outage probability of U_1 versus SNR for NOMA in Scenario I when $m_G = 2$, $m_g = 1$, and $b = 3$	142
7.6	Outage probability of U_1 versus K for NOMA in Scenario I when $\rho = 3$ dB and 6 dB. Here, $N = 3$, $m_G = 2$, $m_g = 1$, $\alpha_1 = 0.7$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$	143
7.7	Outage probabilities versus SNR in Scenario II when $N = 2$, $K = 2$, $m_G = 2$, $m_g = 1$, $m_h = 1$, $\alpha_1 = 0.9$, $\alpha_2 = 0.1$, and $b = 3$	144
7.8	Outage probability of U_1 versus SNR for NOMA in Scenario II when $m_G = 2$, $m_g = 1$, $m_h = 1$, and $b = 3$	145
7.9	Outage probability of U_1 versus K for NOMA in Scenario II when $\rho = 3$ dB and 6 dB. Here, $N = 3$, $m_G = 2$, $m_g = 1$, $m_h = 1$, $\alpha_1 = 0.7$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$	145

List of Tables

3.1	Example of channel gains of the multi-carrier NOMA system when $K = 2$	40
6.1	Diversity order ($\mathcal{D}$) and high-SNR slope ($\mathcal{S}$) for each scenario . . .	115
6.2	Setting of parameters	116
7.1	Diversity orders of U_n (the n th UD) for all cases	139
7.2	Setting of parameters	140

List of Abbreviations

1G	First-Generation
2G	Second-Generation
3G	Third-Generation
3GPP	3rd Generation Partnership Project
4G	Fourth-Generation
5G	Fifth-Generation
AMS-UPSA	Adaptive Mode Selection, User Pairing, and Subcarrier Allocation
AR	Augmented Reality
AWGN	Additive White Gaussian Noise
BS	Base Station
C-NOMA	Cooperative Non-Orthogonal Multiple Access
C-RAN	Cloud Radio Access Networks
CDF	Cumulative Distribution Function
CDMA	Code-Division Multiple Access
CLT	Central Limit Theorem
CSI	Channel State Information
D2D	Device-to-Device
DF	Decode-and-Forward

E2E	End-to-End
eMBB	Enhanced Mobile Broadband
FD	Full-Duplex
FDMA	Frequency-Division Multiple Access
FDR	Full-Duplex Decode-and-Forward Relay
FDS	Frequency-Domain Spreading
GOCA	Group Orthogonal Coded Access
HD	Half-Duplex
IDMA	Interleave-Division Multiple Access
IGMA	Interleave-Grid Multiple Access
i.i.d.	Independent and Identically Distributed
IoT	Internet of Things
IRS	Intelligent Reflecting Surface
ITU	International Telecommunication Union
JUP-SA	Joint User Pairing and Subcarrier Allocation
LCRS	Low Code Rate Spreading
LDS	Low Density Spreading
LDS-SVE	Low Density Spreading with Signature Vector Extension
LoS	Line-of-Sight
LIS	Large Intelligent Surface
LPMA	Lattice Partition Multiple Access
LSSA	Low Code Rate and Signature Based Shared Access
LT	Laplace Transform

LTE	Long-Term Evolution
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
mMTC	Massive Machine-Type Communications
mmWave	Millimeter Wave
MRC	Maximum-Ratio Combining
MUSA	Multi-User Shared Access
MUST	Multi-User Superposition Transmission
NCMA	Non-Orthogonal Coded Multiple Access
NC-NOMA	Non-Cooperative Non-Orthogonal Multiple Access
NLoS	Non-Line-of-Sight
NOCA	Non-Orthogonal Coded Access
NOMA	Non-Orthogonal Multiple Access
NP	Non-Deterministic Polynomial-Time
OFDM	Orthogonal Frequency-Division Multiplexing
OFDMA	Orthogonal Frequency-Division Multiple Access
OMA	Orthogonal Multiple Access
PDF	Probability Density Function
PDMA	Pattern Division Multiple Access
PLS	Physical Layer Security
QoS	Quality of Service
RDMA	Repetition Division Multiple Access
RIS	Reconfigurable Intelligent Surface

RSMA	Resource Spread Multiple Access
SC	Superposition Coding
SCMA	Sparse Code Multiple Access
SIC	Successive Interference Cancellation
SINR	Signal-to-Interference-plus-Noise Ratio
SISO	Single-Input Single-Output
SNR	Signal-to-Noise Ratio
TDMA	Time-Division Multiple Access
UD	User Device
URLLC	Ultra-Reliable Low-Latency Communications
V-BLAST	Vertical-Bell Laboratories Layered Space-Time
VLC	Visible Light Communication
VR	Virtual Reality

List of Symbols

Scalars are written in italic letters. Vectors and matrices are written in boldface letters.

$\log_a(\cdot)$	Logarithm to base a
$\ln(\cdot)$	Natural logarithm
$\lg(\cdot)$	Common logarithm
$\max\{\cdot\}$	Return the maximum element
$\min\{\cdot\}$	Return the minimum element
$\arg(\cdot)$	Argument of complex numbers
$ \cdot $	Absolute value
$\lfloor \cdot \rfloor$	Floor function
$n!$	Factorial
$\binom{n}{k}$	Binomial coefficient
$(\cdot)^T$	Transpose operation
$\text{diag}(\cdot)$	Diagonal matrix
$\Gamma(x)$	Gamma function
$\gamma(a, x)$	Lower gamma function
$\Gamma(a, x)$	Upper gamma function
$I_\nu(x)$	Modified Bessel function of the first kind

$K_v(x)$	Modified Bessel function of the second kind
$Q_M(a, b)$	Marcum Q-function
$D_p(z)$	Parabolic cylinder function
$\text{Ei}(z)$	Exponential integral
$F(\alpha, \beta; \gamma; z)$	Hypergeometric series
$\mathbb{E}(\cdot)$	Statistical expectation
$\text{P}_r(\cdot)$	Probability of events occurring
E_c	Euler constant
$\mathbb{C}^{x \times y}$	The space of $x \times y$ complex-valued matrices
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with mean μ and variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution, where the real and imaginary parts are i.i.d. and follow $\mathcal{N}(\mu/2, \sigma^2/2)$

Chapter 1

Introduction

In this chapter, we first provide an overview of the fifth-generation (5G) wireless communication networks, followed by a brief introduction to the evolvement of multiple access techniques. Then, the motivation and contributions of this thesis are illustrated. Next, the state-of-the-art literature review is presented. Finally, we present the outline of this thesis.

1.1 Background

1.1.1 Overview of 5G

Wireless communications have become an indispensable technology for modern society, which has evolved from the first-generation (1G) to 5G [1–3]. As the latest generation wireless networks, 5G networks are to meet the explosive growth of traffic demand caused by the emergence and development of the internet of things (IoT) and new applications, such as tactile internet, high-resolution video streaming, remote medicine, remote surgery, smart transportation, and real-time control [4–7].

Based on different demands from users, the international telecommunication union (ITU) has categorized 5G networks into three different generic service sce-

Figure 1.1: Three service scenarios of 5G.

narios, enhanced mobile broadband (eMBB), massive machine-type communications (mMTC), and ultra-reliable low-latency communications (URLLC) as shown in Figure 1.1 [8]. These three scenarios correspond to three demands, which are peak data rate, connectivity density, and latency, respectively [9]. Note that not all demands need to be fulfilled simultaneously [8]. Specifically, these three main scenarios are illustrated as follows:

- eMBB provides stable connections with very high peak data rates, as well as moderate rates for cell-edge users [10–12]. It aims to provide services with high demand for bandwidth, such as virtual reality (VR), augmented reality (AR), and high-resolution video streaming [13].
- mMTC provides massive connectivity for a further developed society and aims to focus on services such as IoT, smart city, and smart transportation [14–16].
- URLLC supports low-latency transmissions of small payloads with very high

Figure 1.2: Evolvment of multiple access techniques in wireless communication networks.

reliability and aims to provide latency-sensitive services, such as automated driving, remote surgery, and real-time control [17–19].

To support the aforementioned three scenarios, 5G networks are envisioned to achieve 1000 times higher system capacity, 10 times higher spectral efficiency, 10 times lower end-to-end (E2E) latency than the fourth-generation (4G) networks [20–22]. To realize the vision of 5G networks, fundamentally new technologies have been proposed by researchers [23, 24], such as ultra-densification [25], millimeter wave (mmWave) [26], massive multiple-input multiple-output (MIMO) [27], non-orthogonal multiple access (NOMA), full-duplex (FD) communication [28], device-to-device (D2D) communication [29], visible light communication (VLC) [30], cloud radio access networks (C-RAN) [31], etc.

1.1.2 Non-Orthogonal Multiple Access

As a fundamental technology of wireless communications, multiple access has also experienced significant development with the evolvement of wireless communications [32–34]. As shown in Figure 1.2, it is frequency-division multiple access (FDMA) for 1G, time-division multiple access (TDMA) mostly for the second-generation (2G), code-division multiple access (CDMA) for the third-generation (3G), and orthogonal FDMA (OFDMA) for 4G [35–37]. All of these multiple access techniques belong to orthogonal multiple access (OMA) [38–40]. In these

Figure 1.3: Comparisons of OMA and power-domain NOMA.

conventional multiple access schemes, different users are allocated to different orthogonal resource blocks (time, frequency, or code domain) to eliminate inter-user interference [41, 42].

To further improve system performance, NOMA has been proposed by both academia and industry for 5G and beyond [43–45]. It has been demonstrated that NOMA outperforms conventional OMA from the aspects of spectral efficiency, connection density, and user fairness [46]. Based on the domain of multiplexing, NOMA techniques can be divided into two categories, i.e., power-domain NOMA and code-domain NOMA. In particular, code-domain NOMA includes sparse code multiple access (SCMA) [47, 48], pattern division multiple access (PDMA) [49, 50], low density spreading (LDS) [51], lattice partition multiple access (LPMA) [52], etc. The key idea of these techniques is the same, that is to allocate multiple users to an orthogonal resource block, e.g., a time slot, a frequency band, a spreading code, or an orthogonal spatial degree of freedom, which is different from OMA [53–55]. For example, the difference between OMA and power-domain NOMA is illustrated in Figure 1.3, where the power domain is exploited as a new dimension for multiplexing.

From the viewpoint of standardization, the study of NOMA in the 3rd Generation Partnership Project (3GPP) started in Long-Term Evolution (LTE) Release 13, which is termed multi-user superposition transmission (MUST) and mainly fo-

cuses on downlink transmission [56, 57]. There are three categories for MUST [58]. Specifically, in category 1, coded bits of two or more users are independently mapped to component constellation symbols, where the composite constellation does not have Gray mapping [57]. In category 2, coded bits of two or more users are jointly mapped to component constellation symbols, where the composite constellation has Gray mapping [57]. In category 3, coded bits of two or more users are directly mapped to the symbols of a composite constellation [57]. It is worth noting that the MUST may be designed to be compatible with the existing LTE structure. In other words, NOMA can serve two or more users in an orthogonal frequency-division multiplexing (OFDM) subcarrier without changing the current structure [59].

For the uplink NOMA standardization, 15 NOMA schemes have been proposed in 3GPP Release 14, which are listed as follows [57, 60]:

- Power-domain NOMA [61],
- SCMA [47, 48],
- Multi-user shared access (MUSA) [62],
- PDMA [49, 50],
- Low code rate and signature based shared access (LSSA) [57],
- Resource spread multiple access (RSMA) [63],
- Interleave-grid multiple access (IGMA) [64],
- Interleave-division multiple access (IDMA) [65],
- Non-orthogonal coded multiple access (NCMA) [66],
- Non-orthogonal coded access (NOCA) [67],
- Group orthogonal coded access (GOCA) [68],

- LDS with signature vector extension (LDS-SVE) [69],
- Frequency-domain spreading (FDS) [70],
- Low code rate spreading (LCRS) [71],
- Repetition division multiple access (RDMA) [57].

Although the final decision on the scheme selection has not been done yet due to the limited study period, comprehensive simulations of link level and system level have been conducted to demonstrate the performance gain of NOMA over OMA [57].

1.2 Motivation

NOMA, as a candidate technique for 5G and beyond networks, has numerous advantages in improving system performance, which motivates us to study NOMA in different systems. Specifically, they are listed as follows [72].

- High spectral efficiency: Spectral efficiency is a widely used performance metric in wireless communications. Since multiple users can share one resource block, NOMA offers higher spectral efficiency, which is important to eMBB [73–75].
- Massive connectivity: Future networks are envisioned to support billions of devices. The characteristic of NOMA allows the number of users to be not strictly limited by the number of available resource blocks [76]. Therefore, NOMA can accommodate substantially more users than OMA [53]. This conduces to the realization of mMTC.
- Low latency: Latency is an important metric for wireless communication networks. Since NOMA allows that multiple users can be allocated to the same resource block, these users can avoid waiting for serving even if there are

not sufficient orthogonal resource blocks available [53]. As such, the latency can be reduced, which contributes to the URLLC.

- Good compatibility: Since NOMA exploits a new dimension of power, it can combine with any existing OMA techniques as an “add-on” technology [59]. Besides, due to the development of superposition coding (SC) and successive interference cancellation (SIC), the compatibility performance of NOMA and existing OMA techniques can be guaranteed.

Due to the aforementioned advantages, NOMA has been widely studied by both academia and industry. It is important to fully take advantage of NOMA to improve system performance, which motivates us to investigate NOMA in different systems. Also, although code-domain NOMA provides a coding gain to improve system capacity, power-domain NOMA requires fewer changes in physical-layer operations at the transmitter [77]. Moreover, power-domain NOMA has shown superiority in improving system performance, e.g., spectral efficiency [78,79], energy efficiency [80, 81], and user fairness [82,83]. Therefore, this thesis provides a systematic discourse of power-domain NOMA¹ including transmission scheme design and performance analysis.

1.3 Contributions

The research in this thesis focuses on the system design and performance analysis related to NOMA, including multi-carrier NOMA, multi-carrier cooperative NOMA (C-NOMA), intelligent reflecting surface (IRS)-aided NOMA, and multi-IRS assisted NOMA with discrete phase shifts. The main contributions are summarized as follows.

- In Chapter 3, we design a joint user pairing and subcarrier allocation (JUP-

¹Since this thesis mainly focuses on power-domain NOMA, we use NOMA to indicate power-domain NOMA in the following for simplicity of notation.

SA) scheme for multi-carrier NOMA systems. Moreover, we derive the closed-form expression for the outage probability of the worst-performance user in the worst case. Following that, we determine the diversity order based on the asymptotic analysis at a high signal-to-noise ratio (SNR). It is demonstrated that the optimal diversity order can be achieved by using this proposed scheme as compared with the exhaustive search scheme.

- In Chapter 4, we propose a two-step scheme for pairing cell-center and cell-edge users for multi-carrier C-NOMA systems, where there is no direct link between the base station (BS) and cell-edge users. We derive the closed-form expression for the upper bound of the outage probability and approximate it in the high-SNR regime to obtain the diversity order. It is revealed that the proposed scheme achieves the optimal performance in terms of the diversity order.
- In Chapters 5, we further consider multi-carrier C-NOMA systems, where all users can communicate with the BS directly. In particular, a pair of users can work in two modes, NC-NOMA and C-NOMA. We design an adaptive mode selection, user pairing, and subcarrier allocation (AMS-UPSA) scheme to adaptively select the mode, pair users, and allocating subcarriers. Also, we derive the closed-form expression for the upper bound of the outage probability and obtain the diversity order. It is revealed that the proposed scheme achieves the optimal diversity order.
- In Chapter 6, We analyze the performance of downlink and uplink IRS-aided NOMA systems, where an IRS is deployed to enhance the coverage by assisting a cell-edge user device (UD) to communicate with the BS. In particular, we investigate two aspects for each scenario, namely outage probability and ergodic rate. Correspondingly, we derive new channel statistics for the BS-IRS-UD link based on the central limit theorem (CLT). Since the CLT-based

channel statistics are inaccurate when the channel gain is near 0, we further derive exact channel statistics for channel gain near 0 by utilizing the Laplace transform (LT). Then, we derive the closed-form expressions for the outage probability and ergodic rate for each scenario. To gain further insight, we derive the asymptotic approximations of the outage probability and ergodic rate in the high-SNR regime to obtain the diversity order and high-SNR slope, respectively. We demonstrate that the number of IRS reflecting elements and Nakagami- m fading parameters affect the diversity order but have no influence on the high-SNR slope. Finally, we compare the performance of IRS-aided NOMA systems with that of FD relay aided NOMA systems. Simulation results reveal the superiority of the IRS over FD relay in the high-SNR regime.

- In Chapter 7, we apply multiple IRSs with discrete phase shifts to NOMA systems. To be practical, we consider two scenarios based on whether there are direct links between the BS and UDs. For each scenario, we derive the probability density function (PDF) and the cumulative distribution function (CDF) of the ordered channel gain by utilizing the LT. We derive the high-SNR asymptotic expressions for the upper and lower bounds of the outage probability for each scenario. To gain further insight, we obtain the diversity order for each scenario. We demonstrate that discrete phase shifts do not jeopardize diversity order as compared with continuous phase shifts. Simulation results further reveal that a 3-bit resolution for phase shifts is sufficient to achieve near-optimal outage performance.

1.4 Literature Review

1.4.1 Single-Carrier NOMA

The concept of NOMA traces back to the 1990s [84], which lays the foundation of the power-domain NOMA. The performance of NOMA was first explored by simulations with different power allocation schemes, i.e., proportional fairness scheduler [85], tree-search based transmission power allocation [86], and fractional transmission power allocation [87]. More specifically, Otao *et al.* demonstrated that NOMA can enhance system-level throughput than conventional OMA [85]. Saito *et al.* showed that the NOMA can improve both the system capacity and the cell-edge user capacity [86]. Furthermore, Saito *et al.* considered key link adaptation functionalities of the LTE radio interface and demonstrated the superiority of NOMA over OMA on the throughput [87]. In [88], Ding *et al.* evaluated the performance of a downlink NOMA system with randomly roaming users and showed that NOMA has better performance than OMA on the ergodic sum rate. Following that, they explored the impact of user pairing on the sum rate of two NOMA systems, i.e., NOMA with fixed power allocation and cognitive-radio-inspired NOMA [78]. It was found that NOMA with fixed power allocation prefers to select the users with distinctive channel conditions to enlarge the performance gain of NOMA over conventional OMA. On the other hand, cognitive-radio-inspired NOMA prefers to pair two users with the best channel conditions. In [89], Oviedo *et al.* demonstrated that NOMA using a fair power allocation approach can always outperform OMA in terms of the capacity for each user, regardless of channel conditions. In [90], Xu *et al.* proposed a new evaluation criterion to analyze the performance gain of NOMA over OMA. In [91], Yang *et al.* considered partial channel state information (CSI) and evaluated the outage probability degradation and the ergodic sum rate of NOMA systems. It is revealed that NOMA with the second-order statistical CSI has similar performance to that with the perfect CSI in the low-SNR regime.

The aforementioned works focus on single-antenna NOMA systems, to further improve spectral efficiency, NOMA has been applied to multi-antenna NOMA systems. In [79], Hanif *et al.* developed a minorization-maximization method to maximize the downlink sum rate of a multiple-input single-output (MISO)-NOMA system. In [92], Chen *et al.* proposed the quasi degradation based precoding for MISO-NOMA systems. In [93], Higuchi *et al.* designed a less restrictive beamforming method than zero-forcing to maximize the sum rate for both downlink and uplink MIMO-NOMA systems. They also proposed a novel scheme with random beamforming and intra-beam SIC for MIMO-NOMA systems [94]. However, random beamforming may cause the degradation of system performance. To tackle this problem, Ding *et al.* designed precoding and decoding matrices for MIMO-NOMA systems. As such, the MIMO-NOMA system can be decomposed into several independent single-input single-output (SISO)-NOMA systems [95]. In [96], Ali *et al.* proposed dynamic user clustering and power allocation schemes for downlink NOMA in MIMO systems. To strike a balance between data rate and energy consumption, Zhang *et al.* proposed an energy-efficient user grouping and power allocation scheme for the heterogeneous NOMA networks [81]. They further investigated resource allocation algorithms to maximize energy efficiency by using the Lyapunov optimization method [97].

Security provisioning is always crucial for wireless networks. Since a user needs to decode the signals of other users to perform SIC, there is a security risk for NOMA systems. For scenarios with external eavesdroppers, transmission security can be improved by combining with physical layer security (PLS) [53]. In particular, as the power allocation is designed based on the legitimate users' channel conditions, the SIC at eavesdroppers may not be realized and the eavesdropping can be suppressed [98–100]. For scenarios with internal eavesdroppers, by designing the beamforming at the BS for NOMA systems with multicasting and unicasting streaming, the secrecy data rate can be increased [101].

1.4.2 Multi-Carrier NOMA

The previous subsection introduces works focusing on single-carrier NOMA. However, it is not practical to assign all users in one resource block to form a NOMA group as the strong user has to perform SIC, i.e., demodulate and decode the signals of weaker users, which incurs high computational complexity and decoding delay [53]. Hence, multi-carrier NOMA that can be regarded as a mixed scheme of NOMA and OMA was proposed as an effective approach to strike a balance between system performance and computational complexity. For instance, a relatively small number of users form a NOMA group in one subcarrier and different NOMA groups are allocated to different subcarriers.

For multi-carrier NOMA systems, it is crucial to practically design the scheme of user grouping, subcarrier allocation, and power allocation to ensure performance gain [53]. In [102], Hojeij *et al.* minimized the amount of bandwidth by optimizing the power allocation, and they proposed a hybrid scheme that adaptively switches the transmission mode between NOMA and OMA. In [103], Di *et al.* investigated the problem related to subcarrier assignment and power allocation to maximize the sum rate, which is a non-convex optimization problem, resulting from the binary constraint and the existence of the interference term in the objective function. To overcome this issue, they designed a suboptimal algorithm by adopting matching theory and water-filling power allocation. In [104], Lei *et al.* formulated, analyzed, and solved the problems of subcarrier and power allocation for multi-carrier NOMA systems. Since the problem is non-deterministic polynomial-time (NP)-hard, they further proposed a suboptimal algorithm using Lagrangian duality and dynamic programming. In [105], Sun *et al.* explored problems related to the power, subcarrier allocation, and user pairing, to maximize the sum system throughput. To reduce the complexity, low-complexity suboptimal resource allocation schemes that have near-optimal performance were also provided. In [106], Zhu *et al.* investigated the power and subcarrier allocation of NOMA systems by using a matching algo-

rithm along with the optimal power allocation. In [107], Al-Abbasi *et al.* optimized power and subcarrier allocation by maximizing sum rate under the constraint of total power and proportional rate. In [108], Chen *et al.* presented a minimum Euclidean distance precoder for MIMO-NOMA systems.

1.4.3 Cooperative NOMA

Cooperative transmission can enhance the performance of NOMA by introducing a cooperative gain [109,110]. The key idea of the C-NOMA is to regard strong NOMA users as decode-and-forward (DF) relays to assist weak users. The motivation of considering the cooperation transmission for NOMA are as follows: 1) There is redundant information in NOMA networks, which can be used for cooperative transmission. This is because the strong user needs to perform SIC by detecting weak users' messages. 2) Since the weak user experiences poor channel quality and co-channel interference, it is necessary to improve the weak user's performance. 3) Introducing cooperative transmission can also provide better fairness for NOMA systems [53].

In [111], Ding *et al.* proposed a C-NOMA protocol, where two time phases are used for data transmission. Specifically, For a two-user C-NOMA system, in the first phase, the BS broadcasts the superimposed signal, and in the second phase, the near user acting as a relay forwards the far user's message to the far user. It is shown that the C-NOMA outperforms the cooperative OMA due to its inherent advantage that the signal of the far user is decoded at the near user. In [112], NOMA was introduced to D2D aided cooperative relaying systems to improve spectral efficiency. In [113–115], the FD relaying is considered, i.e., the near user receives the signal from the BS and forwards the signal to the far user simultaneously. As such, it can avoid using two time phases for transmission. In [116], two optimal relay selection schemes were designed for downlink C-NOMA systems. In the aforementioned papers, it is assumed that there is no direct link

between the BS and the weak user. In contrast, it is assumed that the weak users are able to communicate with the BS directly in [117–120]. In [117], a DF relay was introduced to a two-user C-NOMA system to enhance outage performance and average throughput. In [118], an incremental C-NOMA protocol for a two-user NOMA network was proposed to efficiently use the spectrum, which can adaptively switch between the NC-NOMA transmission and the C-NOMA transmission. In [119] and [120], FD relaying is adopted for C-NOMA systems by considering direct links.

1.4.4 Intelligent Reflecting Surface

IRSs are envisioned to provide reconfigurable wireless environments for future communication networks, which are also named reconfigurable intelligent surfaces (RISs) [121, 122] and large intelligent surfaces (LISs) [123]. Specifically, an IRS consists of a large number of reconfigurable passive elements, and each element can induce a change of amplitude and phase for the incident signal [124, 125]. By appropriately adjusting amplitude-reflection coefficients and phase-shift variables, it can improve the link quality and enhance the coverage significantly [126–128]. Compared with the conventional communication assisting techniques, such as relays, IRSs consume less energy due to passive reflection and can operate in FD mode without self-interference [129]. Therefore, IRSs have been proposed as a cost-effective solution to enhance the spectral and energy efficiency of future wireless communication networks [130].

IRS with Continuous Phase Shifts

IRSs have attracted intensive research interest in both academia and industry [131, 132]. Accurate channel estimation is crucial for the application of IRSs. For multi-user MISO networks assisted by IRSs, a channel estimation protocol was designed in [133]. Since the training overhead is proportional to the size of the IRS, it is

necessary to design a channel estimation method with affordable training overhead. To shorten the training sequences, a channel estimation framework was proposed for downlink IRS-aided MIMO networks in [134]. A message-passing based algorithm was designed for uplink IRS-aided MIMO networks in [135]. In [136], a three-phase pilot-based channel estimation scheme was proposed for MISO networks assisted by IRSs to reduce the channel estimation time by utilizing the redundancy in the UD-IRS-BS reflected channels of different UDs. In [137], the superiority of an IRS was indicated as compared with a half-duplex (HD) DF relay. In [138], the performance of a system aided by multiple IRSs was evaluated, and it was revealed that the IRS-aided system outperforms the FD DF relay (FDR)-aided system when the number of IRSs exceeds a certain value. Passive-beamforming design for IRSs is critical to system performance. In [139], an IRS was deployed in a MISO system, where the fixed maximum-ratio transmission was adopted at the BS and the phase shifts of the IRS were optimized by minimizing the outage probability. In [140,141], active and passive beamforming was jointly optimized for IRS-aided MISO systems with different objectives.

IRS with Discrete Phase Shifts

The aforementioned works assume that IRSs have continuous phase shifts. On the other hand, discrete phase shifts have attracted considerable attention due to low complexity. In [142], the performance of an IRS-aided SISO system was evaluated from the data rate perspective. In [143], a MISO IRS-aided system with discrete phase shifts was considered, and the active beamforming at the BS and the passive beamforming at the IRS were jointly optimized to maximize the weighted sum rate. In [144], the uplink transmission for an IRS-aided system was studied. Specifically, the problems of the channel estimation and passive beamforming for the IRS with discrete phase shifts were investigated. In [145], the relation between the reflection amplitude and the phase shift was characterized, and the transmit

power was minimized by designing active and passive beamforming. In [146], the transmit power was minimized by designing the continuous transmit beamforming at the BS and the discrete passive beamforming at the IRS. In [147], the symbol error rate was minimized by optimizing the precoding at the BS and the phase shifts at the IRS.

1.4.5 IRS-Aided NOMA

Since both IRS and NOMA are promising candidate techniques for future wireless communication networks, their combination has been widely investigated recently. In [148], an IRS was deployed to improve the coverage by assisting a cell-edge UD in data transmission, where this cell-edge UD is paired with a cell-center UD under the NOMA scheme. The impact of random phase shifting and coherent phase shifting for an IRS-aided NOMA system was further investigated in [149]. In [150] and [151], the beamforming vectors of the BS and IRS were optimized for an IRS-assisted NOMA system. The aforementioned works assumed that the BS-IRS-UD channel is non-line-of-sight (NLoS). Since the IRS can be pre-deployed, the path between the BS and the IRS may have line-of-sight (LoS) [152–154]. In [152], Yang *et al.* assumed the BS-IRS-UD link to be LoS and optimized the active and passive beamforming vectors for a NOMA system aided by an IRS. In [153], Mu *et al.* studied an IRS-assisted NOMA system by considering ideal and non-ideal IRSs. In [154], the IRS's parameters were designed for a prioritized UD in an IRS-assisted NOMA network.

1.5 Outline of Thesis

The remainder of this thesis is organized as follows: Chapter 2 introduces fundamental knowledge related to this thesis. Chapter 3 proposes a JUP-SA scheme for multi-carrier NOMA systems and analyzes the outage performance of the pro-

posed scheme. Chapter 4 designs a scheme of pairing cell-center and cell-edge users for multi-carrier C-NOMA systems, where cell-edge users cannot communicate with the BS directly. Chapter 5 investigates multi-carrier C-NOMA systems and proposes an AMS-UPSA scheme. In Chapter 6, IRS-aided NOMA systems are investigated, and the system performance is characterized, including outage probability and ergodic rate. In Chapter 7, multi-IRS aided NOMA systems with discrete phase shifts are studied, and the corresponding outage performance is characterized. Chapter 8 concludes this thesis and discusses some promising research directions for future work.

Chapter 2

Fundamentals

In this chapter, the technical knowledge used in this thesis is presented. Specifically, the key technologies of NOMA are first introduced. Then, the basic principles related to NOMA, including downlink and uplink transmissions, are provided. Next, the principle of C-NOMA is illustrated. Lastly, the technical guidelines of IRSs are described.

2.1 Key Technologies Enabling NOMA

The fundamental idea of NOMA is to exploit the power domain as a new dimension to realize multiplexing. To enable the application of NOMA, two principal techniques, namely SC and SIC, are introduced. In particular, SC is to communicate two messages simultaneously by encoding them into a single message in two layers [155], and SIC is to decode the desired message from the superimposed message. With the development and maturity of these two techniques, NOMA can be applied without considering the implementation difficulty.

2.1.1 Superposition Coding

The fundamental concept of SC is that it encodes the signal of a weaker user at a low rate, and then superimposes the signal of a stronger user on it [77]. SC was first introduced by Cover in 1972 [156]. Following that, the SC scheme proposed by Cover was applied to a general degraded broadcast channel by Bergmans [157]. From the perspective of information theory, SC has been applied to different channels [77], e.g., interference channels [158], wiretap channels [159], relay channels [160], and multiple access channels [161]. With the development of the theory, SC has been investigated for practical applications [162, 163].

2.1.2 Successive Interference Cancellation

The basic concept of SIC is that a SIC receiver detects its own message after detecting other users' messages [164]. The decoding order is according to the received signal powers, where the user with a higher received power has a higher priority to be decoded. After decoding the signal of one user, the receiver subtracts it from its received signal. This process is repeated until the receiver decodes its desired signal [165]. It has been demonstrated that SIC can achieve the region boundary of Shannon Capacity. Moreover, SIC has the advantage of low hardware complexity. Due to these advantages, SIC has been widely studied and implemented in practical systems, e.g., CDMA and vertical-bell laboratories layered space-time (V-BLAST) [166].

2.2 Basic Principles

2.2.1 Downlink NOMA

NOMA can serve multiple users in the same resource block by allocating different power levels to different users. Let us consider a downlink NOMA system as shown

Figure 2.1: The downlink NOMA system model with one BS and two users.

in Figure 2.1, where a BS simultaneously serves two users in a subcarrier. The channels between the BS and user i are denoted by h_i ($i \in \{1, 2\}$). Without loss of generality, we assume that user 1 and user 2 are the weaker and stronger users, respectively, i.e., $|h_1| \leq |h_2|$. The BS transmits the superimposed signal of two users, which is given by

$$x = \sqrt{\alpha_1 P} s_1 + \sqrt{\alpha_2 P} s_2, \quad (2.1)$$

where P is the transmit power of the BS, s_i is the transmitted signals to user i , and α_i is the power allocation coefficient for user i ($i \in \{1, 2\}$). Since the key idea of power-domain NOMA is allocating more power to the weaker user, we have $\alpha_1 > \alpha_2$ and $\alpha_1 + \alpha_2 = 1$. The received signals at user i is given by

$$y_i = h_i x + n_i, \quad (2.2)$$

where n_i is the noise at user i with the same variance σ_n^2 . User 1 decodes its own signal by regarding the signal of user 2 as noise. The corresponding achievable rate

Figure 2.2: The uplink NOMA system model with one BS and two users.

is given by

$$R_1 = \log_2 \left(1 + \frac{\alpha_1 \rho |h_1|^2}{\alpha_2 \rho |h_1|^2 + 1} \right), \quad (2.3)$$

where $\rho = P/\sigma_n^2$ is the transmit SNR. On the other hand, user 2 needs to perform SIC. It first detects the message of user 1 by regarding its own signal as noise with the data rate

$$R_{2 \leftarrow 1} = \log_2 \left(1 + \frac{\alpha_1 \rho |h_2|^2}{\alpha_2 \rho |h_2|^2 + 1} \right). \quad (2.4)$$

Then, it removes this message and detects its own message with the achievable rate that is given by

$$R_2 = \log_2 (1 + \alpha_2 \rho |h_2|^2). \quad (2.5)$$

2.2.2 Uplink NOMA

An uplink NOMA system with one BS and two users is illustrated in Figure 2.2. Similar to the downlink, we assume that user 1 and user 2 are the weaker and stronger users, respectively. The channel between the BS and user i is also denoted by h_i ($i \in \{1, 2\}$). Both users simultaneously transmit the signal to the BS in a subcarrier. The observation at the BS is given by

$$y = h_1\sqrt{P}s_1 + h_2\sqrt{P}s_2 + n, \quad (2.6)$$

where P is the transmit power of each user, s_i is the transmitted signal by user i , and n is the noise with the variance σ^2 . At the BS, the signal of user 2 is decoded first, and its achievable rate is given by

$$R_2 = \log_2 \left(1 + \frac{\rho|h_2|^2}{\rho|h_1|^2 + 1} \right), \quad (2.7)$$

where $\rho = P/\sigma^2$ is the transmit SNR. Following this, the message of user 1 is detected after subtracting the message of user 2 with the achievable rate

$$R_1 = \log_2 (1 + \rho|h_1|^2). \quad (2.8)$$

2.2.3 Cooperative NOMA

The key idea of the C-NOMA is to regard the stronger users as a DF relay to assist the weaker user. We introduce a form of C-NOMA that the cell-edge user cannot communicate with the BS directly, which is used in this thesis. As shown in Figure 2.3, the BS simultaneously serves two users, where user 1 and user 2 are the far and near users, respectively. The channel response between the BS and user 2 is denoted by h_2 , while the channel response from user 2 to user 1 is denoted by $h_{2 \rightarrow 1}$. For C-NOMA, two time phases are required in a unit time, where we assume

Figure 2.3: The system model of a form of C-NOMA with one BS and two users, where there is no direct link between the BS and the far user.

that two phases are equal without loss of generality.

In the first time phase, the BS broadcasts the superimposed signal that is given by

$$x = \sqrt{\alpha_1 P} s_1 + \sqrt{\alpha_2 P} s_2, \quad (2.9)$$

where P is the transmit power of the BS, s_i is the transmitted signals to user i , and α_i is the power allocation coefficient for user i ($i \in \{1, 2\}$). The received signal at user 2 is given by

$$y_2 = h_2 x + n_2, \quad (2.10)$$

where n_i is the noise at user i ($i \in \{1, 2\}$). User 2 first decodes the signal of user 1 and then decodes its own signal. The signal-to-interference-plus-noise ratio (SINR) at user 2 decoding the signal of user 1 is given by

$$\gamma_{2 \leftarrow 1} = \frac{\alpha_1 \rho |h_2|^2}{\alpha_2 \rho |h_2|^2 + 1}, \quad (2.11)$$

where ρ is the transmit SNR at the BS. The SNR at user 2 decoding its own signal

is given by

$$\gamma_2 = \alpha_2 \rho |h_2|^2. \quad (2.12)$$

The achievable data rate of user 2 is given by

$$R_2 = \frac{1}{2} \log_2 (1 + \gamma_2). \quad (2.13)$$

During the second time slot t_2 , user 2 forwards the decoded signal to user 1 via the DF relaying protocol. The observation at user 1 is given by

$$y_1 = h_1 \sqrt{P_u} s_1 + n_1, \quad (2.14)$$

where P_u is the transmit power of user 2. The SNR at user 1 decoding its own signal is given by

$$\gamma_1 = \rho' |h_{2 \rightarrow 1}|^2, \quad (2.15)$$

where ρ' is the transmit SNR at user 2. The achievable data rate of user 1 is given by

$$R_1 = \frac{1}{2} \log_2 (1 + \min\{\gamma_{2 \leftarrow 1}, \gamma_1\}). \quad (2.16)$$

2.2.4 Intelligent Reflecting Surface

IRSs can enhance link quality and coverage. To be more specific, consider an IRS-aided system as shown in Figure 2.4, where the IRS has K reflection elements. The reflection-coefficient matrix is denoted by $\mathbf{\Theta} = \text{diag}(\beta_1 e^{j\theta_1}, \beta_2 e^{j\theta_2}, \dots, \beta_K e^{j\theta_K})$ ($j = \sqrt{-1}$), where $\beta_k \in [0, 1]$ is the amplitude-reflection coefficient and $\theta_k \in [0, 2\pi)$ is the phase-shift variable of the k th element that can be adjusted by the IRS

Figure 2.4: The IRS-aided system model.

($k = 1, 2, \dots, K$). The channel response from the source to the IRS is denoted by $\mathbf{G} \in \mathbb{C}^{1 \times K}$, and the channel response from the IRS to the destination is denoted by $\mathbf{g} \in \mathbb{C}^{K \times 1}$. Particularly, they are $\mathbf{G} = [G_1, G_2, \dots, G_K]$ and $\mathbf{g} = [g_1, g_2, \dots, g_K]^T$, respectively. The equivalent channel response is given by $\mathbf{G}\mathbf{\Theta}\mathbf{g} = \sum_{k=1}^K \beta_k G_k g_k e^{j\theta_k}$. Hence, by adjusting β_k and θ_k , we can reconfigure the wireless environment. Phase shifts can be categorized into two types, i.e., continuous phase shifts and discrete phase shifts.

IRS with Continuous Phase Shifts

For the IRS with continuous phase shifts, there is more than one solution to maximize the equivalent channel gain by adjusting phase-shift variables, and the generalized solution is given by

$$\bar{\theta}_k = \tilde{\theta} - \arg(G_k g_k), \quad k = 1, 2, \dots, K, \quad (2.17)$$

where $\tilde{\theta}$ is an arbitrary constant ranging in $[0, 2\pi)$. After adopting $\{\bar{\theta}_k\}$, we can derive the equivalent channel gain as

$$|\mathbf{G}\mathbf{\Theta}\mathbf{g}| = \sum_{k=1}^K \beta_k |G_k| |g_k|. \quad (2.18)$$

IRS with Discrete Phase Shifts

We assume that each discrete phase shift has a b -bit resolution with $L = 2^b$ levels. By uniformly quantizing the interval $[0, 2\pi)$, we have the set of discrete phase-shift variables, $\mathcal{D} = \left\{ \frac{\Delta}{2}, \frac{3\Delta}{2}, \dots, \frac{(2L-1)\Delta}{2} \right\}$, where $\Delta = 2\pi/L$. For the IRS with discrete phase shifts, we can adjust the phase-shift variable of each element to be close to the optimal continuous phase-shift variable. Denote the optimal discrete phase-shift variable by $\hat{\theta}_k$, and we have

$$\hat{\theta}_k = \Delta \left(\left\lfloor \frac{\bar{\theta}_k}{\Delta} \right\rfloor + \frac{1}{2} \right), \quad k = 1, 2, \dots, K, \quad (2.19)$$

where $\lfloor \cdot \rfloor$ is the floor function. The quantization error at the k th element is given by $\theta_k^{er} = \hat{\theta}_k - \bar{\theta}_k$, which is uniformly distributed over $\left[-\frac{\Delta}{2}, \frac{\Delta}{2} \right)$. As such, after adopting $\{\hat{\theta}_k\}$, we can obtain the equivalent channel gain as

$$|\mathbf{G}\mathbf{\Theta}\mathbf{g}| = \left| \sum_{k=1}^K \beta_k G_k g_k e^{j\hat{\theta}_k} \right| = \left| \sum_{k=1}^K \beta_k |G_k| |g_k| e^{j\theta_k^{er}} \right|. \quad (2.20)$$

Chapter 3

Joint User Pairing and Subcarrier Allocation for Multi-Carrier NOMA

This chapter is organized as follows. The main contributions of this work are first introduced in Section 3.1. Then, in Section 3.2, the system model is described. Next, the performance of user pairing in a single subcarrier is evaluated in Section 3.3. Following that, the benchmark scheme is presented in Section 3.4 with the performance analysis. Furthermore, the JUP-SA scheme is proposed in Section 3.5 with the performance analysis. Numerical and simulation results are provided in Section 3.6. Last, our summary is drawn in Section 3.7.

3.1 Introduction

Single-carrier NOMA systems have been widely considered [78, 85–91]. This motivates us to investigate multi-carrier NOMA systems. For multi-carrier NOMA systems, it is crucial to design a JUP-SA scheme. Most of the current works mainly focus on designing different schemes for adaptive-rate systems [102–108]. For adaptive-rate transmission, the transmitter adaptively selects an appropriate

coding and modulation scheme based on the channel quality. To realize that, a feedback process, by which the receiver feeds the channel estimates back to the transmitter, needs to be performed in real time. It results in high computational complexity and cost. On the other hand, for fixed-rate transmission, the transmitter always works with a fixed coding and modulation scheme, and there is no need to feed back the CSI in real time. Hence, although the fixed-rate transmission cannot make use of the channel efficiently, it reduces the overhead and complexity of the system. The fixed-rate transmission is also widely used in some practical systems.

For a fixed-rate multi-carrier NOMA system, the outage probability is a commonly used metric for performance evaluation as it measures the ability of the system to meet the quality of service (QoS). However, it is a challenging task to find the optimal JUP-SA scheme that minimizes the outage probability of every user. To address this challenge, we use diversity order as the performance metric. It reflects how fast the outage probability decreases as the SNR increases. Compared with outage probability, diversity order makes our JUP-SA scheme design more tractable. By using diversity order, we design the JUP-SA scheme by maximizing the minimum achievable diversity order. The contributions of this chapter are summarized as follows:

- We explore the JUP-SA problem for downlink multi-carrier NOMA systems, where the fixed-rate transmission is adopted.
- We first investigate the impact of user pairing on the diversity order in a single subcarrier. It is shown that the diversity order is only determined by the position in sort.
- We propose a JUP-SA scheme for improving the diversity order. The closed-form expression for the outage probability of the worst-performance user in the worst case is derived.

Figure 3.1: The system model of multi-carrier NOMA when $K = 2$.

- To obtain the diversity order, we derive the high-SNR approximation of the outage probability. It is demonstrated that the proposed scheme achieves the optimal diversity order which is the same as that of the exhaustive search scheme.

3.2 System Model

We consider a multi-carrier NOMA system with a BS and $2K$ users as shown in Figure 3.1. These users are denoted by $U_1, U_2, \dots, U_{2K}$. Note that each node has only one antenna. There are K subcarriers, which are denoted by $S_1, S_2, \dots, S_K$. In each subcarrier, two users form a NOMA pair to communicate with the BS. Different pairs occupy different subcarriers without inter-pair interference. Hence, we combine NOMA with OFDMA, which is also called partial NOMA with overall orthogonality [167]. The channel response of U_i in S_m is denoted by $h_{m,i}$ and assumed to experience independent and identically distributed (i.i.d.) Rayleigh fading, i.e., $h_{m,i} \sim \mathcal{CN}(0, 1)$. We assume that the CSI of all users in all channels is perfectly known by the BS.

Remark 3.1. In multi-carrier NOMA systems, each subcarrier may be used by a NOMA group with more than two users. Generally, the multi-user interference increases with the number of users that are multiplexed on the same subcarrier, which degrades the performance of individual users. In addition, as NOMA requires SIC detection at the receivers, the user with strong channel response has to demodulate and decode the signals of the users with weak channel response. Thus, the processing complexity and decoding delay of the strong user increase dramatically with the number of users multiplexed in the same subcarrier. In light of these reasons, we assume that there are two users in each subcarrier to mitigate the multi-user interference and ensure the low processing complexity and decoding delay.

Without loss of generality, we assume that U_j and U_i are allocated to S_m , where U_j and U_i are the stronger and weaker users, respectively, i.e., $|h_{m,j}| \geq |h_{m,i}|$. The BS transmits x_m in S_m , which is given by

$$x_m = \sqrt{\alpha_n P} s_j + \sqrt{\alpha_f P} s_i, \quad (3.1)$$

where P is the transmit power of the BS, s_j and s_i are the messages transmitted to U_j and U_i , respectively, α_n and α_f are the power allocation coefficients for the stronger and weaker users, respectively. Note that we use fixed power allocation, and set $\alpha_n < \alpha_f$ ($\alpha_n + \alpha_f = 1$) for user fairness. The observations at U_j and U_i are given by

$$y_j = h_{m,j} x_m + n_j, \quad (3.2)$$

and

$$y_i = h_{m,i} x_m + n_i, \quad (3.3)$$

respectively, where n_j and n_i denote additive white Gaussian noise (AWGN) components at U_j and U_i , respectively.

At U_j (the stronger user), SIC is performed. It decodes the signal of U_i first due to $\alpha_n \ll \alpha_f$, and the corresponding SINR is given by

$$\gamma_{j,i} = \frac{\alpha_f \rho |h_{m,j}|^2}{\alpha_n \rho |h_{m,j}|^2 + 1}, \quad (3.4)$$

where ρ is the transmit SNR. Then, the signal of U_j will be decoded after removing the signal of U_i , and the SNR is expressed as

$$\gamma_j = \alpha_n \rho |h_{m,j}|^2. \quad (3.5)$$

For U_i (the weaker user), it decodes its own signal directly by regarding the signal of U_j as interference, and the corresponding SINR is given by

$$\gamma_i = \frac{\alpha_f \rho |h_{m,i}|^2}{\alpha_n \rho |h_{m,i}|^2 + 1}. \quad (3.6)$$

The outage probabilities of U_j and U_i are given by

$$\mathbb{P}_j = 1 - \text{Pr}(\gamma_{j,i} > \tilde{\gamma}_i, \gamma_j > \tilde{\gamma}_j), \quad (3.7)$$

and

$$\mathbb{P}_i = \text{Pr}(\gamma_i < \tilde{\gamma}_i), \quad (3.8)$$

respectively, where $\tilde{\gamma}_j = 2^{\tilde{R}_j} - 1$ and $\tilde{\gamma}_i = 2^{\tilde{R}_i} - 1$ with $\tilde{R}_j$ and $\tilde{R}_i$ being the target rates U_j and U_i , respectively.

3.3 User Pairing in a Single Subcarrier

To design the JUP-SA scheme for multi-carrier NOMA systems, we first evaluate the performance of a pair of users in a single subcarrier. We assume that there are M users, and we sort these users in ascending order according to their channel gains in one subcarrier. We assume that the i th user and j th user are selected to form a pair ($1 \leq i < j \leq M$). Without loss of generality, we assume that U_i and U_j are the i th and j th users, respectively. The closed-form expressions for the outage probabilities of U_j and U_i are derived in the following theorem.

Theorem 3.1. *In a single subcarrier, if the j th and i th users from M users are selected to form a pair, the closed-form expressions for outage probabilities of these two users are given by*

$$\mathbb{P}_j = C_1 \sum_{l=0}^{j-1} \binom{j-1}{l} \frac{(-1)^l}{\xi_1} (1 - e^{-\xi_1 \tilde{\rho}_m}), \quad (3.9)$$

and

$$\mathbb{P}_i = C_2 \sum_{l=0}^{i-1} \binom{i-1}{l} \frac{(-1)^l}{\xi_2} (1 - e^{-\xi_2 \tilde{\rho}}), \quad (3.10)$$

respectively, where $C_1 = \frac{M!}{(j-1)!(M-j)!}$, $C_2 = \frac{M!}{(i-1)!(M-i)!}$, $\xi_1 = l + M - j + 1$, $\xi_2 = l + M - i + 1$, $\tilde{\rho}_m = \max\{\tilde{\gamma}/((\alpha_f - \tilde{\gamma}\alpha_n)\rho), \tilde{\gamma}/(\alpha_n\rho)\}$, $\tilde{\rho} = \tilde{\gamma}/((\alpha_f - \tilde{\gamma}\alpha_n)\rho)$, and $\tilde{\gamma} = 2^{\tilde{R}} - 1$ with $\tilde{R} = \tilde{R}_i = \tilde{R}_j$.

Proof. According to order statistics [168], the PDF of the k th ($k \in \{j, i\}$) channel gain is given by

$$f_M^k(x) = \frac{M!}{(k-1)!(M-k)!} f(x) (F(x))^{k-1} (1 - F(x))^{M-k}, \quad (3.11)$$

where $f(x) = e^{-x}$ for $x \geq 0$ and $F(x) = 1 - e^{-x}$ for $x \geq 0$ are the PDF and CDF of a single channel gain, respectively. Hence, we can derive the outage probability

of the j th user as

$$\mathbb{P}_j = 1 - \text{Pr}(\gamma_{j,i} > \tilde{\gamma}_i, \gamma_j > \tilde{\gamma}_j) = \int_0^{\tilde{\rho}_{1,m}} f_M^j(x) dx. \quad (3.12)$$

Then, by applying the binomial expansion, we have

$$\mathbb{P}_j = \frac{M!}{(j-1)!(M-j)!} \sum_{l=0}^{j-1} (-1)^l \binom{j-1}{l} \int_0^{\tilde{\rho}_{1,m}} e^{-\xi_1 x} dx. \quad (3.13)$$

Hence, we can derive (3.9) after calculating the integral of the exponential function.

On the other hand, the i th user's outage probability is given by

$$\mathbb{P}_i = \text{Pr}(\gamma_i < \tilde{\gamma}_i) = \int_0^{\tilde{\rho}_1} f_M^i(x) dx. \quad (3.14)$$

Similarly, we can derive (3.10). This completes the proof. ■

To provide insight into the performance, the diversity order is considered, which is defined as $d = -\lim_{\rho \rightarrow \infty} \frac{\lg \mathbb{P}(\rho)}{\lg \rho}$, where $\mathbb{P}(\rho)$ is the outage probability and ρ is the transmit SNR [169]. To obtain it, the high-SNR approximation of the outage probability is derived in the following proposition.

Proposition 3.1. *In the high-SNR regime, the outage probabilities of the j th and i th users can be approximated as*

$$\mathbb{P}_j^\infty = A_1 \tilde{\rho}_m^j, \quad (3.15)$$

and

$$\mathbb{P}_i^\infty = A_2 \tilde{\rho}^i, \quad (3.16)$$

respectively, where $A_1 = \frac{M!}{j!(M-j)!}$ and $A_2 = \frac{M!}{i!(M-i)!}$.

Proof. In $\mathbb{P}_j$, the exponential function can be expanded as

$$e^{-\xi_1 \tilde{\rho}_m} = \sum_{p=0}^{\infty} \frac{(-1)^p \xi_1^p}{p!} \tilde{\rho}_m^p. \quad (3.17)$$

After substituting (3.17) into (3.9), we observe that the zero-order coefficient of $\mathbb{P}_j$ is 0, and the p th-order term ($p \geq 1$) of $\mathbb{P}_j$ is

$$-\frac{M!}{(j-1)!(M-j)!} \frac{(-1)^p}{p!} \sum_{l=0}^{j-1} \binom{j-1}{l} (-1)^l \xi_1^{p-1} \tilde{\rho}_m^p. \quad (3.18)$$

Based on [170, eqs. (0.154.5) and (0.154.6)], we find that

$$\sum_{l=0}^{j-1} \binom{j-1}{l} (-1)^l \xi_1^{p-1} = \begin{cases} 0, & 1 \leq p \leq j-1, \\ (-1)^{j-1} (j-1)!, & p = j. \end{cases} \quad (3.19)$$

Hence, for the Taylor series expansion of $\mathbb{P}_j$, the coefficient of the p th-order term ($0 \leq p \leq j-1$) is 0, and the coefficient of the j th-order term is not 0. Finally, after extracting the j th-order term, we can derive (3.15). On the other hand, we can also derive the high-SNR approximation of $\mathbb{P}_i$ as (3.16) by following similar steps. This completes the proof. $\blacksquare$

Based on Proposition 3.1, the diversity order is obtained in the following corollary.

Corollary 3.1. *The diversity orders of the j th and i th users are j and i , respectively.*

Proof. Since $\tilde{\rho}_m$ and $\tilde{\rho} \propto \rho^{-1}$, we can determine the diversity order according to (3.15) and (3.16). This completes the proof. $\blacksquare$

3.4 Benchmark Scheme

3.4.1 Description of the Benchmark Scheme

Based on Corollary 3.1, we need to select two users with the largest channel gains to maximize the diversity order. It is natural to consider a scheme that pairs users subcarrier by subcarrier, which is called the benchmark scheme as illustrated in Algorithm 3.1.

Algorithm 3.1 Benchmark scheme for multi-carrier NOMA systems

- 1: **Initialize:** $\mathcal{U} = \{U_1, U_2, \dots, U_{2K}\}$, $\mathcal{U}^i = \emptyset$ ($i = 1, 2, \dots, K$), $m = 1$;
 - 2: **while** $m \leq K$ **do**
 - 3: Sort users in $\mathcal{U}$ in ascending order based on their channel gains in S_m ;
 - 4: Select the last two users to form a pair $\mathcal{U}_m$ in S_m ;
 - 5: Update the user set $\mathcal{U} \leftarrow \mathcal{U} \setminus \mathcal{U}_m$;
 - 6: $m \leftarrow m + 1$;
 - 7: **end while.**
-

Explanation: To clearly illustrate this benchmark scheme, we explain it in a step-by-step manner. The set of users that have not been allocated and the set of users that are allocated to S_m are denoted by $\mathcal{U}$ and $\mathcal{U}^m$, respectively. In each step, the users that have not been paired are first sorted in ascending order according to their channel gains in the subcarrier. Then, the last two users are chosen to form a pair. For instance, we first choose two users with the best channel conditions to form a pair in S_1 . After excluding two users selected for S_1 , we sort the remaining $2K - 2$ users in S_2 , and choose the $(2K - 2)$ th and $(2K - 3)$ th users to form a pair in S_2 . Generally, in S_m , there are $2K - 2m + 2$ users to choose, and we select the $(2K - 2m + 2)$ th and $(2K - 2m + 1)$ th users.

3.4.2 Performance Analysis of the Benchmark Scheme

The closed-form expressions for the outage probabilities of two users in S_m are provided in the following theorem.

Theorem 3.2. *In the considered NOMA system with K subcarriers and $2K$ users, after using the benchmark scheme, the stronger and weaker users in S_m have the outage probabilities as*

$$\mathbb{P}_{s,m}^b = C_3 \sum_{l=0}^{2K-2m+1} \binom{2K-2m+1}{l} \frac{(-1)^l}{l+1} (1 - e^{-(l+1)\tilde{\rho}_m}), \quad (3.20)$$

and

$$\mathbb{P}_{w,m}^b = C_4 \sum_{l=0}^{2K-2m} \binom{2K-2m}{l} \frac{(-1)^l}{l+2} (1 - e^{-(l+2)\tilde{\rho}}), \quad (3.21)$$

respectively, where $C_3 = 2K - 2m + 2$ and $C_4 = (2K - 2m + 2)(2K - 2m + 1)$.

Proof. In the m th subcarrier, there are $2K - 2m + 2$ users to choose, and we select the $(2K - 2m + 2)$ th and $(2K - 2m + 1)$ th users. After substituting $M = 2K - 2m + 2$, $j = 2K - 2m + 2$, and $i = 2K - 2m + 1$ into (3.9) and (3.10), we can derive (3.20) and (3.21). This completes the proof. $\blacksquare$

The high-SNR approximation of the outage probability is derived as shown in the following proposition.

Proposition 3.2. *In the high-SNR regime, $\mathbb{P}_{s,m}^b$ and $\mathbb{P}_{w,m}^b$ can be approximated as*

$$\mathbb{P}_{s,m}^{b,\infty} = \tilde{\rho}_m^{2K-2m+2}, \quad (3.22)$$

and

$$\mathbb{P}_{w,m}^{b,\infty} = (2K - 2m + 2)\tilde{\rho}^{2K-2m+1}, \quad (3.23)$$

respectively.

Proof. After substituting $M = 2K - 2m + 2$, $j = 2K - 2m + 2$, and $i = 2K - 2m + 1$ into (3.15) and (3.16), we can derive (3.22) and (3.23). This completes the proof. $\blacksquare$

Then, the diversity order is obtained as shown in the following corollary.

Corollary 3.2. *By using the benchmark scheme, the diversity orders of the stronger and weaker users paired in S_m are $2K - 2m + 2$ and $2K - 2m + 1$, respectively, i.e., the diversity orders from the first user to the last user are $2K, 2K - 1, \dots, 1$, respectively.*

Proof. Since $\tilde{\rho}_m$ and $\tilde{\rho} \propto \rho^{-1}$, the diversity orders of the stronger and weaker users in the m th pair are $2K - 2m + 2$ and $2K - 2m + 1$, respectively. This completes the proof. $\blacksquare$

It can be observed that in this scheme, the users in the latter formed pairs experience poor performance. To overcome this drawback, the JUP-SA scheme is proposed.

3.5 JUP-SA Scheme

We design the JUP-SA scheme by jointly considering all users' channel gains in all subcarriers. It ensures that each user is allocated to a subcarrier in which the channel gain is relatively large.

3.5.1 Description of the JUP-SA Scheme

In the considered multi-carrier NOMA system with K subcarriers and $2K$ users, we can pair users in the following steps:

- Sort the $K \times 2K = 2K^2$ channel gains ($|h_{m,i}|$, $m \in \{1, 2, \dots, K\}$ and $i \in \{1, 2, \dots, 2K\}$) in ascending order.

Table 3.1: Example of channel gains of the multi-carrier NOMA system when $K = 2$

	U ₁	U ₂	U ₃	U ₄
S ₁	$h_{1,1}$	$h_{1,2}$	$h_{1,3}$	$h_{1,4}$
S ₂	$h_{2,1}$	$h_{2,2}$	$h_{2,3}$	$h_{2,4}$

- Delete the smallest $K - 1$ channel gains. Note that if $|h_{m,i}|$ is deleted, U_i cannot be allocated to S_m .
- Pair all users randomly on the constraints that there are two users in each subcarrier and each user is only allocated to one subcarrier.

Example: We consider a 2-subcarrier and 4-user system and the channel gains are shown in Table 3.1. We assume that the ascending order is $|h_{21}| < |h_{13}| < |h_{23}| < |h_{24}| < |h_{11}| < |h_{12}| < |h_{22}| < |h_{14}|$. Then we delete the smallest channel gain $|h_{21}|$. Finally, we can pair 4 users based on the constraint that there are two users in each subcarrier and each user is only allocated to one subcarrier. There are many choices for us to choose from. For instance, U_1 and U_2 are in S_1 , and U_3 and U_4 are in S_2 as shown in Table 3.1. Another choice is that U_1 and U_4 are in S_1 , and U_2 and U_3 are in S_2 . Within all these choices, we only need to choose one randomly.

3.5.2 Feasibility of the JUP-SA Scheme

For this scheme to work well, we need to ensure that there is at least one pairing scheme after deleting $K - 1$ channel gains. The following lemma is introduced to ensure that.

Lemma 3.1. *For a $K \times 2K$ matrix, after deleting $K - 1$ elements randomly, we can always find a scheme that picks two elements in each row on the constraint that only one element is picked in each column.*

Proof. We can prove it by contradiction. Assuming that there is no way to pick two elements in each row on the constraint that there is only one element is picked in each column. The deleted elements must contain a full column or a row of $2K - 1$ elements. Hence, the number of deleted elements must be greater than or equal to K . As we only delete $K - 1$ elements, this event cannot happen. This completes the proof. ■

Remark 3.2. There is at least one feasible scheme allocating users after deleting $K - 1$ channel gains. Here, we provide a solution to select users after the channel-gain deletion. First, we count the number of deleted channel gains in each subcarrier. Then, we select the two strongest users for the largest-number subcarrier first and for the smallest-number subcarrier last. Note that for each subcarrier, we cannot select the user who has been allocated to other subcarriers or whose channel gain has been deleted.

3.5.3 Performance Analysis of the JUP-SA Scheme

Since the outage probability of each user in the proposed JUP-SA scheme is intractable to derive, we aim to derive the outage probability of the worst-performance user in the worst case, which is provided in the following theorem.

Theorem 3.3. *In the considered NOMA system, after using the JUP-SA scheme, the closed-form expression for the outage probability of the worst-performance user in the worst case is given by*

$$\mathbb{P}_{ub} = C_5 \sum_{l=0}^{K-1} \binom{K-1}{l} \frac{(-1)^l}{\xi_3} (1 - e^{-\xi_3 \bar{\rho}}), \quad (3.24)$$

where $C_5 = (2K^2)! / ((K-1)!(2K^2 - K)!) and $\xi_3 = l + 2K^2 - K + 1$.$

Proof. In the considered system with K subcarriers and $2K$ users, after deleting the smallest $K - 1$ channel gains, there is more than one scheme to pair them. The

worst case is that the K th channel gain is selected, i.e., the user corresponding to the K th smallest channel gain is paired with a user in the subcarrier corresponding to the K th smallest channel gain. This user must be the weaker user in this pair. By substituting $M = 2K^2$ and $i = K$ into (3.10), we can derive (3.24). This completes the proof. ■

Proposition 3.3. *In the high-SNR regime, $\mathbb{P}_{ub}$ can be approximated as*

$$\mathbb{P}_{ub}^{\infty} = A_3 \tilde{\rho}^K, \quad (3.25)$$

where $A_3 = \frac{(2K^2)!}{K!(2K^2-K)!}$.

Proof. After substituting $M = 2K^2$ and $i = K$ into (3.16), we can derive (3.25). This completes the proof. ■

Corollary 3.3. *By using the proposed JUP-SA scheme, the achievable diversity order is K .*

Proof. Since $\tilde{\rho} \propto \rho^{-1}$, the diversity order of the weaker user is K . This completes the proof. ■

3.6 Numerical Results

The numerical and simulation results are presented in this section. We assume that the target data rate is $\tilde{R} = 1$ bps/Hz, the power allocation coefficients are $\alpha_l = 0.3$ and $\alpha_s = 0.7$, the transmit SNR ranges from 0 dB to 25 dB.

3.6.1 User Pairing in a Single Subcarrier

We plot outage probabilities for $M = 4, i = 1, j = 2$ and $M = 6, i = 3, j = 5$ in Figure 3.2 to validate our analysis of user pairing in single subcarrier. Specifically, there are M users available to select. After sorting these users in ascending order

Figure 3.2: Outage probabilities in a single subcarrier when $M = 4, i = 1, j = 2$ and $M = 6, i = 3, j = 5$.

based on channel gains, we select the i th and j th users to form a NOMA pair. We observe that the simulation points match their corresponding theoretical results derived from (3.9) and (3.10), which validates our analysis. It is also observed that the derived high-SNR approximations that are derived from (3.15) and (3.16) are accurate. We observe that the i th and j th users have the diversity orders of 1 and 2, respectively, when $M = 4, i = 1, j = 2$; they have the diversity orders of 3 and 5, respectively, when $M = 6, i = 3, j = 5$. This phenomenon validates the insight from Corollary 3.1.

3.6.2 Comparisons of Benchmark and JUP-SA Schemes

The outage probabilities of the benchmark scheme when $K = 2$ are plotted in Figure 3.3. We first observe that all simulation results match the corresponding theoretical results that are derived from (3.20) and (3.21), which validates our analysis. It is also observed that the high-SNR approximations that are derived from (3.22) and (3.23) are accurate. Hence, it can be used to obtain the diversity

Figure 3.3: Outage probabilities of the benchmark scheme in the multi-carrier NOMA system when $K = 2$.

Figure 3.4: Outage probability of the worst-performance user in the worst case of the JUP-SA scheme in the multi-carrier NOMA system when $K = 2$.

order. We observe that the diversity orders of four users are 4, 3, 2, and 1, respectively, which is consistent with Corollary 3.2. Furthermore, in Figure 3.3, the outage probability curve of the exhaustive search scheme is plotted, which is the optimal solution. In this scheme, all possible pairing candidate schemes are carried out and the candidate scheme with the least outage users is selected to be the final pairing scheme. Since all users in this scheme have the same outage probability statistically, only one outage probability curve is presented for this scheme. We observe that the exhaustive search scheme achieves the diversity order of 2, while the worst-performance user in the benchmark scheme has the diversity order of 1. Therefore, the benchmark scheme cannot achieve optimal performance. This is the disadvantage of the benchmark scheme, and the JUP-SA scheme is proposed to address it.

In Figure 3.4, we plot the outage probability of the worst-performance user in the worst case with the proposed JUP-SA scheme when $K = 2$. We observe that all simulation points match the analytical curve that is derived from (3.24), which validates our analysis. By comparing the outage probabilities of the worst-performance user in the benchmark and proposed schemes, we observe that our proposed scheme has much better performance. Furthermore, it is also observed that the asymptotic analysis is accurate, which is derived from (3.25). By observing the slopes in the high-SNR regime, we find that the diversity order is 2 when $K = 2$, which is consistent with Corollary 3.3. It can be observed that the outage probability curve of the proposed JUP-SA scheme is in parallel with that of the exhaustive search scheme in the high-SNR regime, which implies that the proposed scheme achieves the optimal performance in terms of the diversity order.

Next, we increase the value of K to 3 and plot the outage probabilities of six users in the benchmark scheme in Figure 3.5. It is observed that the simulation points match the analytical results. It is observed that asymptotic analysis is accurate, and the diversity orders of six users in the benchmark scheme are 6, 5, 4,

Figure 3.5: Outage probabilities of the benchmark scheme in the multi-carrier NOMA system when $K = 3$.

Figure 3.6: Outage probability of the worst-performance user in the worst case of the JUP-SA scheme in the multi-carrier NOMA system when $K = 3$.

3, 2, and 1, respectively. On the other hand, the diversity order of the exhaustive search scheme is 3.

In contrast, the outage probabilities of the worst-performance user in the worst case with the proposed JUP-SA scheme when $K = 3$ are plotted in Figure 3.6. It is observed that the simulation points match the analytical results. Also, it is observed that the asymptotic analysis is accurate, and the diversity order is 3 when $K = 3$. By comparing the outage probability curves of the proposed scheme and the exhaustive search scheme, we find that the proposed scheme achieves the optimal diversity order.

3.7 Summary

In this chapter, the JUP-SA problem for multi-carrier NOMA systems has been studied. For the considered system, a JUP-SA scheme improving the diversity order has been proposed. The closed-form expression for the outage probability of the worst-performance user in the worst case has been derived. Then, for further insight, the diversity order has been obtained by analyzing the high-SNR approximation. It has been shown that the proposed scheme achieves the optimal performance in terms of the diversity order with low computational complexity.

Chapter 4

Two-Step Pairing of Cell-Center and Cell-Edge Users for Multi-Carrier C-NOMA

The organization of this chapter is as follows. The main contributions are first introduced in Section 4.1. Then, the system model is illustrated in Section 4.2. Next, the impact of user pairing in a single subcarrier is investigated in Section 4.3. Following that, the benchmark scheme is described in Section 4.4. Furthermore, a two-step user pairing scheme is designed in Section 4.5. In Section 4.6, numerical and simulation results are provided with discussions. Last, the summary is given in Section 4.7.

4.1 Introduction

Multi-carrier NOMA systems have been investigated in Chapter 3. On the other hand, Cooperative transmission can enhance the performance of NOMA by introducing a cooperative gain [109, 110]. This motivates us to investigate multi-carrier C-NOMA systems. Specifically, we first explore the scenario that cell-edge users cannot communicate with the BS directly. Correspondingly, we study the

user pairing and subcarrier allocation problem for the considered system. Similar to [117–120], we also adopt a fixed-rate transmission and use outage probability as the performance metric. To make the design of user pairing and subcarrier allocation more tractable, we utilize the diversity order to measure the system performance and design the scheme. The contributions of this chapter can be summarized as follows:

- We investigate the problem related to user pairing and subcarrier allocation for multi-carrier C-NOMA systems, where there are no direct links between the BS and cell-edge users.
- We first explore the impact of user pairing in a single subcarrier on the diversity order, where a cell-center user and a cell-edge user are selected.
- Based on the results of the performance analysis in a single subcarrier, we propose a two-step user pairing scheme. The closed-form expression for the upper bound of the outage probability is derived.
- To obtain the diversity order, we derive the high-SNR approximation of the upper bound of the outage probability. It is demonstrated that the proposed scheme achieves the optimal performance in terms of the diversity order.

4.2 System Model

We consider a downlink multi-carrier C-NOMA system with a BS and K cell-center users $\{N_i\}$ ($i = 1, 2, \dots, K$) and K cell-edge users $\{F_j\}$ ($j = 1, 2, \dots, K$) as shown in Figure 4.1, in which each node has only one antenna. There are no direct links between the BS and cell-edge users. To assist in transmitting the data from the BS to cell-edge users, cell-center users act as HD DF relays [171]. There are one cell-center user and one cell-edge user allocated to each subcarrier, and each user can only be allocated to a subcarrier. There are K subcarriers $\{S_m\}$

Figure 4.1: The system model of multi-carrier C-NOMA, where cell-edge users cannot communicate with the BS directly.

($m = 1, 2, \dots, K$). The channel response from the BS to the cell-center user N_i in subcarrier S_m is denoted by $h_{N_i}^m$ and the channel response from the cell-center user N_i to the cell-edge user F_j in subcarrier S_m is denoted by $g_{N_i \rightarrow F_j}^m$. Note that all channels experience i.i.d. Rayleigh fading, i.e., $h_{N_i}^m \sim \mathcal{CN}(0, 1)$, $g_{N_i \rightarrow F_j}^m \sim \mathcal{CN}(0, 1)$. All CSI is assumed to be perfectly known by the BS.

Without loss of generality, we assume that N_i and F_j are allocated to S_m . There are two equal time phases in a normalized period. During the first time phase, the BS sends the message x_m to N_i in S_m , which is given by

$$x_m = \sqrt{\alpha_n P_b} s_{N_i} + \sqrt{\alpha_f P_b} s_{F_j}, \quad (4.1)$$

where P_b is the transmit power at the BS, s_{N_i} and s_{F_j} are the messages for N_i and F_j , respectively, α_n and α_f are the power allocation coefficients for cell-center and cell-edge users, respectively. Note that we adopt fixed power allocation, and

$\alpha_n < \alpha_f$, $\alpha_n + \alpha_f = 1$. The observation at N_i is given by

$$y_{N_i} = h_{N_i}^m x_m + n_{N_i}, \quad (4.2)$$

where n_{N_i} is the AWGN at N_i with variance $\sigma_{n_{N_i}}^2$. SIC is implemented at N_i where the perfect SIC is assumed [117, 118]. Specifically, N_i first detects the signal of F_j and then removes that signal to decode its own signal. At N_i , the SNR to decode x_{F_j} is given by

$$\gamma_{N_i}^{F_j} = \frac{\alpha_f \rho |h_{N_i}^m|^2}{\alpha_n \rho |h_{N_i}^m|^2 + 1}, \quad (4.3)$$

where $\rho = P_b / \sigma_{n_{N_i}}^2$ is the transmit SNR. The received SNR to decode x_{N_i} at N_i is expressed as

$$\gamma_{N_i} = \alpha_n \rho |h_{N_i}^m|^2. \quad (4.4)$$

During the second time slot, N_i forwards the signal of F_j to F_j after detecting it, which is given by

$$y_{F_j} = g_{N_i \rightarrow F_j}^m \sqrt{P_r} s_{F_j} + n_{F_j}, \quad (4.5)$$

where P_r is the transmit power of N_i as a DF relay and n_{F_j} is the AWGN at F_j with variance $\sigma_{n_{F_j}}^2$. We further assume that $P_b = P_r$ and $\sigma_{n_{N_i}}^2 = \sigma_{n_{F_j}}^2$. At F_j , the received SNR to detect its own message is expressed as

$$\gamma_{F_j} = \rho |g_{N_i \rightarrow F_j}^m|^2. \quad (4.6)$$

The outage probabilities of N_i and F_j are given by

$$\mathbb{P}_{N_i} = 1 - \Pr\left(\gamma_{N_i}^{F_j} > \tilde{\gamma}_F, \gamma_{N_i} > \tilde{\gamma}_N\right) \quad (4.7)$$

and

$$\mathbb{P}_{F_j} = 1 - \Pr\left(\gamma_{N_i}^{F_j} > \tilde{\gamma}_F, \gamma_{F_j} > \tilde{\gamma}_F\right), \quad (4.8)$$

respectively, where $\tilde{\gamma}_N = 2^{2\tilde{R}_N} - 1$ and $\tilde{\gamma}_F = 2^{2\tilde{R}_F} - 1$ with $\tilde{R}_N$ and $\tilde{R}_F$ being the target rate of cell-center and cell-edge users, respectively.

4.3 User Pairing in a Single Subcarrier

In this section, the outage performance of a pair of users in a single subcarrier is evaluated. First, we assume that the i th of Q_N cell-center users and the j th of Q_F cell-edge users are paired in a single subcarrier, where users are sorted in ascending order based on channel gains. Without loss of generality, we assume that N_i and F_j are the i th and j th users, respectively. The outage probabilities of these two users are provided in the following theorem.

Theorem 4.1. *In a single subcarrier, the outage probabilities of N_i and F_j are given by*

$$\mathbb{P}_{N_i} = W_{Q_N}^i(\tilde{\rho}_{N,F,m}), \quad (4.9)$$

and

$$\mathbb{P}_{F_j} = 1 - \left(1 - W_{Q_N}^i(\tilde{\rho}_{N,F})\right) \left(1 - W_{Q_F}^j(\tilde{\rho}_F)\right), \quad (4.10)$$

respectively, where $\tilde{\rho}_{N,F,m} = \max\{\tilde{\gamma}_F/((\alpha_f - \tilde{\gamma}_F\alpha_n)\rho), \tilde{\gamma}_N/(\alpha_n\rho)\}$, $\tilde{\rho}_F = \tilde{\gamma}_F/\rho$,

$\tilde{\rho}_{N,F} = \tilde{\gamma}_F / ((\alpha_f - \tilde{\gamma}_F \alpha_n) \rho)$. Importantly, $W_Q^k(\tilde{\rho})$ is a function that is given by

$$W_Q^k(\tilde{\rho}) = \frac{Q!}{(k-1)!(Q-k)!} \sum_{l=0}^{k-1} \binom{k-1}{l} \frac{(-1)^l}{l+Q-k+1} (1 - e^{-(l+Q-k+1)\tilde{\rho}}). \quad (4.11)$$

Proof. Based on order statistics [168], the PDF of the k th ($k \in \{i, j\}$) of Q channel gains ($Q \in \{Q_N, Q_F\}$) is given by

$$f_Q^k(x) = \frac{Q!}{(k-1)!(Q-k)!} f(x) (F(x))^{k-1} (1 - F(x))^{Q-k}, \quad (4.12)$$

where $f(x) = e^{-x}$ for $x \geq 0$ and $F(x) = 1 - e^{-x}$ for $x \geq 0$ are the PDF and CDF of a single channel gain, respectively. Hence, we can transform (4.7) and (4.8) into

$$\mathbb{P}_{N_i} = \int_0^{\tilde{\rho}_{N,F,m}} f_{Q_N}^i(x) dx, \quad (4.13)$$

and

$$\mathbb{P}_{F_j} = 1 - \left(1 - \int_0^{\tilde{\rho}_{N,F}} f_{Q_N}^i(x) dx\right) \left(1 - \int_0^{\tilde{\rho}_F} f_{Q_F}^j(y) dy\right), \quad (4.14)$$

respectively. We notice that both (4.13) and (4.14) contain the integral term of

$$\int_0^{\tilde{\rho}} f_Q^k(x) dx = \frac{Q!}{(k-1)!(Q-k)!} \sum_{l=0}^{k-1} (-1)^l \binom{k-1}{l} \int_0^{\tilde{\rho}} e^{-(l+Q-k+1)x} dx = W_Q^k(\tilde{\rho}). \quad (4.15)$$

After substituting (4.15) into (4.13) and (4.14), we can derive (4.9) and (4.10). This completes the proof. ■

The approximated expression for the outage probability in high SNR regime is required to derive the diversity order.

Proposition 4.1. *In the high-SNR regime, the outage probabilities of N_i and F_j*

are approximated as

$$\mathbb{P}_{N_i}^\infty = V_{Q_N}^i \tilde{\rho}_{N,F,m}^i, \quad (4.16)$$

and

$$\mathbb{P}_{F_j}^\infty = V_{Q_N}^i \tilde{\rho}_{N,F}^i + V_{Q_F}^j \tilde{\rho}_F^j, \quad (4.17)$$

respectively, where $V_Q^k = \frac{(-1)^{2k} Q!}{k!(Q-k)!}$.

Proof. Both $\mathbb{P}_{N_i}$ and $\mathbb{P}_{F_j}$ consist of the algebraic expression $W_Q^k(\tilde{\rho})$, and its approximation for $\rho \rightarrow \infty$ can be derived as follows. We first rewrite $W_Q^k(\tilde{\rho})$ as

$$W_Q^k(\tilde{\rho}) = \frac{Q!}{(k-1)!(Q-k)!} \sum_{l=0}^{k-1} \binom{k-1}{l} \frac{(-1)^l \left(1 - \sum_{p=0}^{\infty} \frac{(-1)^p (l+Q-k+1)^p}{p!} \tilde{\rho}^p \right)}{l+Q-k+1}, \quad (4.18)$$

by using the Taylor series expansion. It is observed that the coefficients of the p th ($0 \leq p \leq k-1$) and k th order are zero and non-zero, respectively, since

$$\sum_{l=0}^{k-1} \binom{k-1}{l} (-1)^l (l+Q-k+1)^{p-1} = \begin{cases} 0, & 1 \leq p \leq k-1, \\ (-1)^{k-1} (k-1)!, & p = k, \end{cases} \quad (4.19)$$

which is obtained from [170, eqs. (0.154.5) and (0.154.6)]. Hence, $W_Q^k(\tilde{\rho})$ can be approximated as

$$W_Q^k(\tilde{\rho}) \approx V_Q^k \tilde{\rho}^k. \quad (4.20)$$

After substituting (4.20) into (4.9) and (4.10), we can obtain (4.16) and (4.17). This completes the proof. ■

According to Proposition 4.1, the diversity order is obtained in the following corollary.

Corollary 4.1. *The diversity orders of N_i and F_j are i and $\min\{i, j\}$, respectively.*

4.4 Benchmark Scheme

4.4.1 Description of the Benchmark Scheme

Based on Corollary 4.1, to maximize the diversity order, we need to select the strongest cell-center user and the strongest cell-edge user in each subcarrier. It is natural to consider a benchmark scheme pairing users subcarrier by subcarrier, as illustrated in Algorithm 4.1.

Algorithm 4.1 Benchmark scheme

```

1: Initialize:  $\mathcal{N}=\{N_1, N_2, \dots, N_K\}$ ,  $\mathcal{F}=\{F_1, F_2, \dots, F_K\}$ ,  $\mathcal{N}^i = \emptyset$  ( $i = 1, 2, \dots, K$ ),  $\mathcal{F}^j = \emptyset$  ( $j = 1, 2, \dots, K$ ),  $m = 1$ ;
2: while  $m \leq K$  do
3:   Sort cell-center users in  $\mathcal{N}$  in ascending order based on channel gains in  $S_m$ ;
4:   Allocate the last cell-center user to  $S_m$ , and add this user to the set  $\mathcal{N}^m$ ;
5:   Update the user set  $\mathcal{N} \leftarrow \mathcal{N} \setminus \mathcal{N}^m$ ;
6:   Sort cell-edge users in  $\mathcal{F}$  in ascending order based on channel gains from the cell-center user allocated to  $S_m$  to cell-edge users in  $\mathcal{F}$  in  $S_m$ ;
7:   Allocate the last cell-edge user to  $S_m$ , and add this user to the set  $\mathcal{F}^m$ ;
8:   Update the user set  $\mathcal{F} \leftarrow \mathcal{F} \setminus \mathcal{F}^m$ ;
9:    $m \leftarrow m + 1$ ;
10: end while.

```

4.4.2 Performance Analysis of the Benchmark Scheme

The closed-form expressions for the outage probabilities of two users in S_m are provided in the following theorem.

Theorem 4.2. *In the considered downlink C-NOMA system, after using the benchmark scheme, the stronger and weaker users of the m th pair have the outage probabilities as*

$$\mathbb{P}_{N,m}^b = W_\xi^\xi(\tilde{\rho}_{N,F,m}), \quad (4.21)$$

and

$$\mathbb{P}_{F,m}^b = 1 - \left(1 - W_\xi^\xi(\tilde{\rho}_{N,F})\right) \left(1 - W_\xi^\xi(\tilde{\rho}_F)\right), \quad (4.22)$$

respectively, where $\xi = K - m + 1$.

Proof. In the m th subcarrier, there are ξ cell-center users to choose from, and we select the ξ th user. Then, there are ξ cell-edge users to choose, and we select the ξ th user paired with the selected cell-center user. After substituting $Q_N = \xi$, $Q_F = \xi$, $j = \xi$, and $i = \xi$ into (4.9) and (4.10), we can derive (4.21) and (4.22). ■

Proposition 4.2. *In the high-SNR regime, $\mathbb{P}_{N,m}^b$ and $\mathbb{P}_{F,m}^b$ can be approximated as*

$$\mathbb{P}_{N,m}^{b,\infty} = V_\xi^\xi \tilde{\rho}_{N,F,m}^\xi, \quad (4.23)$$

and

$$\mathbb{P}_{F,m}^{b,\infty} = V_\xi^\xi \tilde{\rho}_{N,F}^\xi + V_\xi^\xi \tilde{\rho}_F^\xi, \quad (4.24)$$

respectively.

Proof. Substituting $Q_N = \xi$, $Q_F = \xi$, $j = \xi$, and $i = \xi$ into (4.16) and (4.17), we can derive (4.23) and (4.24). This completes the proof. ■

Corollary 4.2. *By using the benchmark scheme, both diversity orders of the stronger and weaker users paired in the m th subcarrier are $K - m + 1$, i.e., the diversity*

orders from the first pair to the last pair are $K, K - 1, \dots, 1$, respectively.

It can be observed that in this scheme, the users in the latter formed pairs experience poor performance. To overcome this drawback, a two-step user pairing scheme is proposed.

4.5 Two-Step User Pairing Scheme

We design the two-step user pairing scheme by jointly considering all users' channel gains in all subcarriers. It ensures that each user is allocated to a subcarrier in which the channel gain is relatively large.

4.5.1 Description of the Two-Step User Pairing Scheme

The proposed scheme has two steps that are described as follows.

The First Step:

- Sort K^2 channel gains in $\{|h_{N_i}^m|\}$ ($m \in \{1, 2, \dots, K\}$, $i \in \{1, 2, \dots, K\}$) in ascending order.
- Delete the first $K - 1$ channel gains. Note that if $|h_{N_i}^m|$ is deleted, N_i cannot be allocated to S_m .
- Based on the remaining channel gains, select a cell-center user for each subcarrier under the constraint that each subcarrier selects only one cell-center user and each cell-center user can only be selected once. Assume that $N_{i_1}, N_{i_2}, \dots, N_{i_K}$ are allocated to $S_1, S_2, \dots, S_K$, respectively.

The Second Step:

- Sort K^2 channel gains from cell-center users to cell-edge users in corresponding subcarriers, i.e., $\{|g_{N_{i_m} \rightarrow F_j}^m|\}$ ($m \in \{1, 2, \dots, K\}$, $j \in \{1, 2, \dots, K\}$) in ascending order.

- Delete the first $K - 1$ channel gains. Note that if $|g_{N_{im} \rightarrow F_j}^m|$ is deleted, F_j cannot be allocated to S_m .
- Based on the remaining channel gains, select a cell-edge user for each subcarrier under the constraint that each subcarrier selects only one cell-edge user and each cell-edge user can only be selected once. Assume that $F_{j_1}, F_{j_2}, \dots, F_{j_K}$ are allocated to $S_1, S_2, \dots, S_K$, respectively.
- Finally, N_{i_1} and F_{j_1} , N_{i_2} and F_{j_2} , $\dots$, N_{i_K} and F_{j_K} are paired in $S_1, S_2, \dots, S_K$, respectively.

4.5.2 Feasibility of the Two-Step User Pairing Scheme

We need to guarantee that there is at least one scheme allocating cell-center users after deleting $K - 1$ channel gains in $\{|h_{N_i}^m|\}$ and at least one scheme allocating cell-edge users after deleting $K - 1$ channel gains in $\{|g_{N_{im} \rightarrow F_j}^m|\}$. Correspondingly, we can refer to the following lemma.

Lemma 4.1. *There is a $K \times K$ matrix and $K - 1$ elements are randomly deleted. There is at least one scheme to select an element for each row under the constraint that only a selected element is located in each column.*

Proof. This can be proved by contradiction. Assuming that there is no method to select an element for each row under the constraint that K selected elements are in K different columns. Then, a row or a column of elements in the matrix must be deleted, i.e., more than or exactly K elements are deleted. Since only $K - 1$ elements are deleted, there is at least one scheme to allocate all users. This completes the proof. ■

Remark 4.1. There is at least one feasible scheme allocating cell-center users after deleting $K - 1$ channel gains. Here, we provide a solution to select cell-center users after the channel-gain deletion. First, we count the number of deleted channel gains

in each subcarrier. Then, we select the strongest cell-center user for the largest-number subcarrier first and for the smallest-number subcarrier last. Note that for each subcarrier, we can only select the user who has not been allocated and whose channel gain is not deleted. The allocation of cell-edge users follows a similar rule.

4.5.3 Performance Analysis of the Two-Step User Pairing Scheme

The closed-form expressions for upper bounds of outage probabilities of cell-center and cell-edge users are presented in the following theorem.

Theorem 4.3. *In the considered system, after using the proposed two-step user pairing scheme, the upper bounds of outage probabilities of cell-center and cell-edge users are given by*

$$\mathbb{P}_N^{ub} = W_{K^2}^K(\tilde{\rho}_{N,F,m}), \quad (4.25)$$

and

$$\mathbb{P}_F^{ub} = 1 - (1 - W_{K^2}^K(\tilde{\rho}_{N,F})) (1 - W_{K^2}^K(\tilde{\rho}_F)), \quad (4.26)$$

respectively.

Proof. In the proposed scheme, the first step is to allocate cell-center users. Assume that the K th channel gain is $|h_{N_{i_w}}^w|$. It is the worst case when N_{i_w} is allocated to S_w since the smallest $K - 1$ channel gains are deleted. By referring to (4.13), the corresponding outage probability, i.e., the upper bound for outage probabilities of cell-center users, is given by

$$\mathbb{P}_N^{ub} = \mathbb{P}_{N_{i_w}} = W_{K^2}^K(\tilde{\rho}_{N,F,m}), \quad (4.27)$$

where F_{j_w} denotes the cell-edge user paired with N_{i_w} in S_w . Similarly, for the cell-edge user, the worst case is that the channel gain from N_{i_w} to F_{j_w} in S_w is the K th channel gain coincidentally (the smallest $K - 1$ ones are deleted). Based on (4.14), the outage probability of F_{j_w} in this case, i.e., the upper bound for outage probabilities of cell-edge users, is given by

$$\mathbb{P}_F^{ub} = \mathbb{P}_{F_{j_w}} = 1 - (1 - W_{K^2}^K(\tilde{\rho}_{N,F})) (1 - W_{K^2}^K(\tilde{\rho}_F)). \quad (4.28)$$

This completes the proof. ■

Then, the high-SNR approximations of $\mathbb{P}_{N_{i_w}}$ and $\mathbb{P}_{F_{j_w}}$ are presented in the following proposition.

Proposition 4.3. *In the high-SNR regime, the approximations of $\mathbb{P}_N^{ub}$ and $\mathbb{P}_F^{ub}$ are given by*

$$\mathbb{P}_N^{ub,\infty} = V_{K^2}^K \tilde{\rho}_{N,F,m}^K, \quad (4.29)$$

and

$$\mathbb{P}_F^{ub,\infty} = V_{K^2}^K \tilde{\rho}_{N,F}^K + V_{K^2}^K \tilde{\rho}_F^K, \quad (4.30)$$

respectively.

Proof. Substitute $Q_N = K^2$ and $i = K$ into (4.16), and we can obtain (4.29). Similarly, after substituting $Q_F = K^2$ and $j = K$ into (4.17), and we can obtain (4.30). This completes the proof. ■

Based on Proposition 4.3, we can obtain the diversity order as shown in the following corollary.

Corollary 4.3. *Both achievable diversity orders of the cell-center and cell-edge users are K .*

Figure 4.2: Outage probabilities in a single subcarrier when $i = 1, j = 3, Q_N = Q_F = 3$ and $i = 3, j = 2, Q_N = Q_F = 3$.

4.6 Numerical Results

The performance of the proposed two-step pairing scheme is evaluated in this section. We assume that the target data rates are $\tilde{R}_{N_i} = \tilde{R}_{F_j} = 1$ bps/Hz; the power allocation coefficients are $\alpha_n = 0.1$ and $\alpha_f = 0.9$; the transmit SNR is ranged from 0 dB to 30 dB.

4.6.1 User Pairing in a Single Subcarrier

Figure 4.2 is to validate the performance analysis for a single subcarrier. It depicts the outage probabilities of N_i and F_j in a single subcarrier with $i = 1, j = 3, Q_N = Q_F = 3$ and $i = 3, j = 2, Q_N = Q_F = 3$, respectively. The theoretical curves for N_i and F_j are derived from (4.9) and (4.10), respectively. It is observed that the simulation points and the corresponding theoretical results show the precise agreement, which validates our analysis. It is also observed that the derived high-SNR approximations, i.e., (4.16) and (4.17), are accurate. Based on the slope in

the high-SNR regime, we observe that N_i and F_j achieve the same diversity order of 1 when $i = 1, j = 3, Q_N = Q_F = 3$, and they achieve the diversity orders of 3 and 2, respectively, when $i = 3, j = 2, Q_N = Q_F = 3$. This observation is consistent with Corollary 4.1.

4.6.2 Comparisons of Benchmark and Two-Step User Pairing Schemes

The outage probabilities of the benchmark scheme when $K = 3$ are plotted in Figure 4.3. In this figure, we first observe that all simulation results match the corresponding theoretical results that are derived from (4.21) and (4.22), which validates our analysis. It is also observed that the high-SNR approximations that are derived from (4.23) and (4.24) are accurate. Hence, it can be used to obtain the diversity order. We observe that the diversity orders of the first pair, second pair, and third pair are 3, 2, and 1, respectively, which is consistent with Corollary 4.2. Furthermore, in Figure 4.3, the outage probability curve of the exhaustive search scheme is plotted, which is the optimal solution. In this scheme, all possible pairing candidate schemes are carried out and the candidate scheme with the least outage users is selected to be the final pairing scheme. Since all users in this scheme have the same outage probability statistically, only one outage probability curve is presented for this scheme. We observe that the exhaustive scheme achieves the diversity order of 3, while the smallest diversity order in the benchmark scheme is 1. Therefore, the benchmark scheme cannot achieve optimal performance.

In Figure 4.4, we plot the outage probabilities of cell-center and cell-edge users by using the proposed two-step user pairing scheme for $K = 3$. Statistically, three cell-center users have the same outage probability and three cell-edge users also have the same outage probability. Hence, there is only a set of simulation points for cell-center and cell-edge users, respectively. It is observed that the simulation points for cell-center and cell-edge users are below their respective analytical upper

Figure 4.3: Outage probabilities of the benchmark scheme in the multi-carrier C-NOMA system when $K = 3$.

Figure 4.4: Outage probabilities of the two-step user pairing scheme in the multi-carrier C-NOMA when $K = 3$.

bounds which are derived from (4.25) and (4.26), which validates our analysis. By comparing the outage probabilities of the benchmark and the proposed scheme, we observe that the proposed scheme can provide better performance to both cell-center and cell-edge users. The asymptotic curves of the upper bounds for cell-center and cell-edge users, which are given by (4.29) and (4.30), are plotted in this figure as well. It can be observed that the upper-bound curves for cell-center and cell-edge users are close to their respective asymptotic curves in the high-SNR region. By observing the slopes, both diversity orders of the center-center and cell-edge users are 3, which validates our analysis in Corollary 4.3. Furthermore, we observe that the slope of outage probability in the high-SNR regime for the proposed scheme is the same as that for the exhaustive search scheme. Hence, the proposed scheme can achieve the optimal diversity order with low computational complexity.

4.7 Summary

In this chapter, we have studied the user pairing problem for a multi-carrier C-NOMA system, where cell-edge users cannot communicate with the BS directly. In particular, a two-step user pairing scheme has been proposed to allocate all users. The closed-form expression for the outage probability's upper bound has been derived. Furthermore, the diversity order has been determined by using the high-SNR approximation of the outage probability. It has been demonstrated that the proposed scheme can achieve the optimal performance in terms of diversity order but with lower computational complexity as compared with the exhaustive search scheme.

Chapter 5

Resource Allocation for Multi-Carrier C-NOMA

This chapter is organized as follows. The main contributions are first introduced in Section 5.1. Then, the system model is illustrated in Section 5.2 with the mode selection strategy. Next, the impact of user selection is investigated in Section 5.3. Following that, the benchmark scheme is presented in Section 5.4. Furthermore, the AMS-UPSA scheme is proposed in Section 5.5. In Section 5.6, numerical and simulation results are provided with discussions. Last, the summary is given in Section 5.7.

5.1 Introduction

In Chapter 4, there is no direct link between the BS and each cell-edge user in multi-carrier C-NOMA systems. This motivates us to investigate more complicated multi-carrier C-NOMA systems in this chapter, where all users can communicate with the BS directly. We adopt a fixed-rate transmission and use outage probability as the performance metric. However, it is a challenging task to obtain the optimal user pairing, subcarrier allocation, and mode selection scheme that minimizes the outage probability of each user. The reason is that the outage probability of each

user for a certain scheme is nontrivial to obtain. To address this challenge, we use diversity order as the performance metric and design the AMS-UPSA scheme. The proposed scheme pairs users in different subcarriers with mode selection for each pair, which maximizes the achievable diversity order. The contributions of this chapter are summarized as follows:

- For the downlink of a multi-carrier C-NOMA system, the problem related to user pairing, subcarrier allocation, and mode selection is investigated. For each pair, a mode selection strategy is proposed to improve the achievable diversity order.
- The impact of the proposed mode selection strategy on the achievable diversity order is investigated. Assume that there are M users in a C-NOMA system and these users are sorted in ascending order based on channel gains in subcarrier S_m . If the i th and j th users are allocated to S_m ($1 \leq i < j \leq M$), by using the proposed mode selection strategy, their achievable diversity orders are $i + 1$ and j , respectively.
- The AMS-UPSA scheme that can improve the achievable diversity order for the considered C-NOMA system is presented. The achievable diversity order is derived by analyzing the upper bound for the outage probability. In a K -subcarrier and $2K$ -user C-NOMA system, by using the proposed AMS-UPSA scheme, the diversity orders achieved by all users are $2K$.
- The AMS-UPSA scheme achieves the optimal performance in terms of diversity order by comparing it with the exhaustive search scheme.

5.2 System Model

We consider a downlink multi-carrier C-NOMA system with a BS and $2K$ users $\{U_i\}$ ($i \in \{1, 2, \dots, 2K\}$) as shown in Figure 5.1, in which each node has only

Figure 5.1: The system model of multi-carrier C-NOMA when $K = 2$, where all users can communicate with the BS directly. Mode I is used by pair 1, and Mode II is used by pair 2.

one antenna. There are K subcarriers $\{S_m\}$ ($m = 1, 2, \dots, K$). All $2K$ users can communicate with the BS directly. Each user can only be allocated to one subcarrier, and hence, there are two users in each subcarrier. Each user is able to act as an HD DF relay. As shown in Figure 5.1, a pair of users in each subcarrier has two modes that can be selected to communicate with the BS, where Mode I is the NC-NOMA and Mode II is the C-NOMA. It is observed that in Figure 5.1, pair 1 selects Mode I, and pair 2 selects Mode II.

All channel responses are i.i.d., and the channel response of U_i in subcarrier S_m is denoted by $h_{m,i}$, which follows the Rayleigh fading model, i.e., $h_{m,i} \sim \mathcal{CN}(0, 1)$. Each user is also able to act as an HD DF relay [118]. The channel response from U_i to U_j in S_m is denoted by $g_{m,ij}$, where $g_{m,ij} \sim \mathcal{CN}(0, 1)$. We assume that all CSI is perfectly known by the BS.

We assume that U_i and U_j are allocated to subcarrier S_m . Without loss of generality, we further assume that U_i and U_j are the weaker user and the stronger

user, respectively, i.e., $|h_{m,i}| \leq |h_{m,j}|$. The BS sends x_m in S_m , which is given by

$$x_m = \sqrt{\alpha_f P_b} s_i + \sqrt{\alpha_n P_b} s_j, \quad (5.1)$$

where P_b denotes the power transmitted on each subcarrier, which is assumed to be equal and constant, s_i and s_j are the signals transmitted to U_i and U_j , respectively, and α_f and α_n are power allocation coefficients for U_i and U_j , respectively. Note that we use fixed power allocation, and $\alpha_f > \alpha_n$, $\alpha_f + \alpha_n = 1$.

Signal Model of Mode I

Mode I is the NC-NOMA scheme. The signals received at U_j and U_i are given by

$$y_j^N = h_{m,j} x_m + n_j, \quad (5.2)$$

and

$$y_i^N = h_{m,i} x_m + n_i, \quad (5.3)$$

respectively, where n_j and n_i are AWGN components at U_j and U_i , respectively.

At U_j , the SIC is performed. The signal of U_i is decoded first as $\alpha_f > \alpha_n$, and the corresponding achievable data rate is given by

$$R_{j,i}^N = \log_2 \left(1 + \frac{\alpha_f \rho |h_{m,j}|^2}{\alpha_n \rho |h_{m,j}|^2 + 1} \right), \quad (5.4)$$

where $\rho = P_b/\sigma_n^2$ is the transmit SNR ($\sigma_{n_i}^2 = \sigma_{n_j}^2 = \sigma_n^2$). After removing the signal of U_i , the signal of U_j is decoded and the data rate achieved by U_j is expressed as

$$R_j^N = \log_2 (1 + \alpha_n \rho |h_{m,j}|^2). \quad (5.5)$$

At U_i , it decodes its own signal by regarding the signal of U_j as interference and the data rate achieved by U_i is given by

$$R_i^N = \log_2 \left(1 + \frac{\alpha_f \rho |h_{m,i}|^2}{\alpha_n \rho |h_{m,i}|^2 + 1} \right). \quad (5.6)$$

The outage probabilities of U_j and U_i in Mode I are given by

$$\mathbb{P}_j^N = 1 - \Pr \left(R_{j,i}^N > \tilde{R}_i, R_j^N > \tilde{R}_j \right), \quad (5.7)$$

and

$$\mathbb{P}_i^N = \Pr \left(R_i^N < \tilde{R}_i \right), \quad (5.8)$$

respectively. Note that $\tilde{R}_j$ and $\tilde{R}_i$ are the target rates of U_j and U_i , respectively.

Signal Model of Mode II

Mode II is the C-NOMA scheme used by pair 2 in Figure 5.1. Since U_j acts as an HD DF relay, two time slots are required. The BS sends the signal x_m in the first time slot and the signal received at U_j is given by

$$y_j^R = h_{m,j} \left(\sqrt{\alpha_f P_b} s_i + \sqrt{\alpha_n P_b} s_j \right) + n_j. \quad (5.9)$$

At U_j , the signal of U_i is decoded first. Then the signal of U_i is forwarded to U_i in the second time slot, and the signal received at U_i is expressed as

$$y_i^R = g_{m,ji} \sqrt{P_r} s_i + n_i, \quad (5.10)$$

where P_r is the transmit power of U_j as a relay and it is assumed to be equal to P_b .

At U_j , the signal of U_i is decoded first and the achievable data rate of U_i is

given by

$$R_{j,i}^R = \frac{1}{2} \log_2 \left(1 + \frac{\alpha_f \rho |h_{m,j}|^2}{\alpha_n \rho |h_{m,j}|^2 + 1} \right), \quad (5.11)$$

where the factor of $1/2$ is resulting from that two time slots are used. Then the signal of U_j is detected after canceling the signal of U_i and the achievable data rate of U_j is given by

$$R_j^R = \frac{1}{2} \log_2 (1 + \alpha_n \rho |h_{m,j}|^2). \quad (5.12)$$

At U_i , the achievable data rate of U_i is expressed as

$$R_i^R = \frac{1}{2} \log_2 (1 + \rho_r |g_{m,ji}|^2), \quad (5.13)$$

where $\rho_r = P_r / \delta_n^2 = \rho$. The outage probabilities of U_j and U_i in Mode II are given by

$$\mathbb{P}_j^R = 1 - \text{P}_r \left(R_{j,i}^R > \tilde{R}_i, R_j^R > \tilde{R}_j \right), \quad (5.14)$$

and

$$\mathbb{P}_i^R = 1 - \text{P}_r \left(R_{j,i}^R > \tilde{R}_i, R_i^R > \tilde{R}_i \right), \quad (5.15)$$

respectively.

Mode Selection Strategy

The data rate of the stronger user is halved in Mode II compared with that in Mode I, and its performance will be degraded after carrying out mode selection. However, since its channel condition is good enough, the performance is still acceptable even in Mode II. On the other hand, due to the poor channel condition of the weaker user, its performance needs to be improved by mode selection. Hence, we design the mode selection strategy by only considering the weaker user. For each pair of

users, the mode selection strategy is stated as follows:

- When $|h_{m,i}| \geq |g_{m,ji}|$, Mode I is selected. Otherwise, Mode II is selected.

Remark 5.1. This mode selection strategy is proposed due to the following reasons.

1) Since there are direct and relay links for the weaker user, the weaker user can combine these two signals to improve its reception reliability by applying maximum-ratio combining (MRC). However, this will increase the complexity of the weaker user, and the computational complexity of MRC may be too high for a hand-held device in reality. In addition, both MRC and our proposed mode selection strategies achieve the same diversity order. Due to the aforementioned reasons, we adopt mode selection in this chapter. 2) For the mode selection strategy, we can also use the method by comparing the data rates of the weaker users in Mode I and Mode II, i.e., R_i^N and $\min\{R_{j,i}^R, R_i^R\}$. The mode with a higher data rate is selected. As compared with our proposed strategy, this approach may achieve a lower outage probability. However, both strategies achieve the same diversity order that is adopted to measure the system performance in this chapter. Hence, to carry out the complicated performance analysis, we adopt the strategy of comparing the channel gains for analytical simplicity since it does not degrade the diversity order.

5.3 Impact of Mode Selection in a Single Subcarrier

In this section, the impact of the proposed mode selection strategy on the achievable diversity order in a single subcarrier is investigated. Assume that there are M users available to select. After sorting these users in ascending order based on the channel gains in subcarrier S_m (these channel gains form an array $\mathcal{A}$), the i th and j th users are allocated to S_m ($1 \leq i < j \leq M$). Without loss of generality, assume that U_i and U_j are the i th and j th users, respectively. Based on order statistics [168], the

PDF of the i th user's channel gain is given by

$$f_M^i(x) = \frac{M!}{(i-1)!(M-i)!} f(x) (F(x))^{i-1} (1-F(x))^{M-i}, \quad (5.16)$$

where $f(x) = e^{-x}$ for $x \geq 0$ and $F(x) = 1 - e^{-x}$ for $x \geq 0$ are the PDF and CDF of the under ordered channel gain, respectively.

5.3.1 Performance Analysis of Mode I

When $|h_{m,i}| \geq |g_{m,ji}|$, Mode I is selected.

The j th User (The Stronger User)

At the j th user, the i th user's signal is decoded first. Following this, the j th user's signal is detected after subtracting the i th user's signal. Since $|h_{m,j}| \geq |h_{m,i}| \geq |g_{m,ji}|$, $|h_{m,j}|$ will be the $(j+1)$ th element after inserting $|g_{m,ji}|$ to the array $\mathcal{A}$. Hence, by defining

$$C_M^j(\tilde{\rho}) = \frac{M!}{(j-1)!(M-j)!} \sum_{l=0}^{j-1} \binom{j-1}{l} \frac{(-1)^l}{l+M-j+1} (1 - e^{-(l+M-j+1)\tilde{\rho}}), \quad (5.17)$$

we can derive that

$$\mathbb{P}_j^N = 1 - \text{Pr} \left(R_{j,i}^N > \tilde{R}_i, R_j^N > \tilde{R}_j \right) = \int_0^{\tilde{\rho}_{1,m}} f_{M+1}^{j+1}(x) dx = C_{M+1}^{j+1}(\tilde{\rho}_{1,m}), \quad (5.18)$$

where $\tilde{\rho}_{1,m} = \max\{\tilde{\gamma}_1/((\alpha_i - \tilde{\gamma}_1\alpha_j)\rho), \tilde{\gamma}_1/(\alpha_j\rho)\}$ and $\tilde{\gamma}_1 = 2^{\tilde{R}_i} - 1 = 2^{\tilde{R}_j} - 1$.

The i th User (The Weaker User)

At the i th user, its own signal is decoded directly as treating the signal of the j th user as interference. Since $|h_{m,i}| \geq |g_{m,ji}|$, $|h_{m,i}|$ will be the $(i+1)$ th element after

inserting $|g_{m,ji}|$ to the array $\mathcal{A}$. The i th user's outage probability is given by

$$\mathbb{P}_i^N = \int_0^{\tilde{\rho}_1} f_{M+1}^{i+1}(x)dx = C_{M+1}^{i+1}(\tilde{\rho}_1), \quad (5.19)$$

where $\tilde{\rho}_1 = \tilde{\gamma}_1 / ((\alpha_f - \tilde{\gamma}_1 \alpha_n) \rho)$.

5.3.2 Performance Analysis of Mode II

When $|h_{m,i}| < |g_{m,ji}|$, Mode II is selected.

The j th User (The Stronger User)

At the j th user, its own signal is decoded after decoding the i th user's signal. Since the relation between $|g_{m,ji}|$ and $|h_{m,j}|$ is uncertain, $|h_{m,j}|$ will be the j th element at least after inserting $|g_{m,ji}|$ to array $\mathcal{A}$. The j th user's outage probability is bounded by

$$\mathbb{P}_j^R = 1 - \text{Pr} \left(R_{j,i}^R > \tilde{R}_i, R_j^R > \tilde{R}_j \right) \leq \int_0^{\tilde{\rho}_{2,m}} f_{M+1}^j(x)dx = C_{M+1}^j(\tilde{\rho}_{2,m}), \quad (5.20)$$

where $\tilde{\rho}_{2,m} = \max\{\tilde{\gamma}_2 / ((\alpha_i - \tilde{\gamma}_2 \alpha_j) \rho), \tilde{\gamma}_2 / (\alpha_j \rho)\}$ and $\tilde{\gamma}_2 = 2^{2\tilde{R}_i} - 1 = 2^{2\tilde{R}_j} - 1$.

The i th User (The Weaker User)

The signal of the i th user is decoded by the j th user first. Since $|h_{m,j}|$ is the j th element at least after inserting $|g_{m,ji}|$ to array $\mathcal{A}$, the probability that the j th user cannot decode the i th user's signal is bounded by

$$\text{Pr} \left(R_{j,i}^R < \tilde{R}_i \right) \leq \int_0^{\tilde{\rho}_2} f_{M+1}^j(x)dx = C_{M+1}^j(\tilde{\rho}_2), \quad (5.21)$$

where $\tilde{\rho}_2 = \tilde{\gamma}_2 / ((\alpha_f - \tilde{\gamma}_2 \alpha_n) \rho)$. Then the signal is forwarded to the i th user. Since $|g_{m,ji}|$ will be the $(i+1)$ th element at least after inserting $|g_{m,ji}|$ to array $\mathcal{A}$, the upper bound for the probability that the i th user cannot detect its own signal

successfully is expressed as

$$\Pr \left(R_i^R < \tilde{R}_i \right) \leq \int_0^{\tilde{\rho}_{2,r}} f_{M+1}^{i+1}(x) dx = C_{M+1}^{i+1}(\tilde{\rho}_{2,r}), \quad (5.22)$$

where $\tilde{\rho}_{2,r} = \tilde{\gamma}_2/\rho_r = \tilde{\gamma}_2/\rho$. According to (5.21) and (5.22), the i th user's outage probability is bounded by

$$\mathbb{P}_i^R = 1 - \Pr \left(R_{j,i}^R > \tilde{R}_i, R_i^R > \tilde{R}_i \right) \leq 1 - \left(1 - C_{M+1}^j(\tilde{\rho}_2) \right) \left(1 - C_{M+1}^{i+1}(\tilde{\rho}_{2,r}) \right). \quad (5.23)$$

5.3.3 Performance Analysis after Mode Selection

The outage probabilities of the j th and i th users after adopting the mode selection strategy are derived as shown in the following theorem.

Theorem 5.1. *In a single subcarrier, if the j th and i th users from M users are selected to form a pair, the upper bounds of outage probabilities after mode selection are given by*

$$\mathbb{P}_j^{ub} = (1 - \Psi(i, M)) C_{M+1}^{j+1}(\tilde{\rho}_{1,m}) + \Psi(i, M) C_{M+1}^j(\tilde{\rho}_{2,m}), \quad (5.24)$$

and

$$\begin{aligned} \mathbb{P}_i^{ub} = & (1 - \Psi(i, M)) C_{M+1}^{i+1}(\tilde{\rho}_1) + \Psi(i, M) \left(C_{M+1}^{i+1}(\tilde{\rho}_{2,r}) + C_{M+1}^j(\tilde{\rho}_2) \right. \\ & \left. - C_{M+1}^{i+1}(\tilde{\rho}_{2,r}) C_{M+1}^j(\tilde{\rho}_2) \right), \end{aligned} \quad (5.25)$$

respectively, where

$$\Psi(i, M) = \frac{M!}{(i-1)!(M-i)!} \sum_{l=0}^{i-1} \binom{i-1}{l} \frac{(-1)^l}{l+M-i+2}. \quad (5.26)$$

Proof. The outage probability of the j th user can be derived as

$$\mathbb{P}_j = \text{P}_r(|h_{m,i}| \geq |g_{m,ji}|) \mathbb{P}_j^N + \text{P}_r(|h_{m,i}| < |g_{m,ji}|) \mathbb{P}_j^R. \quad (5.27)$$

We denote $\text{P}_r(|h_{m,i}| < |g_{m,ji}|)$ by $\Psi(i, M)$ and have

$$\Psi(i, M) = \int_0^\infty f(y) \int_0^y f_M^i(x) dx dy = \frac{M!}{(i-1)!(M-i)!} \sum_{l=0}^{i-1} \binom{i-1}{l} \frac{(-1)^l}{l+M-i+2}. \quad (5.28)$$

Then, based on (5.18) and (5.20), we can obtain the upper bound of $\mathbb{P}_j$ as (5.24).

On the other hand, the outage probability of the i th user can be derived as

$$\mathbb{P}_i = \text{P}_r(|h_{m,i}| \geq |g_{m,ji}|) \mathbb{P}_i^N + \text{P}_r(|h_{m,i}| < |g_{m,ji}|) \mathbb{P}_i^R. \quad (5.29)$$

Following that, by substituting (5.19) and (5.23) into (5.29), we can obtain the upper bound of $\mathbb{P}_i$ as (5.25). This completes the proof. $\blacksquare$

To derive the diversity order, we further derive the high-SNR approximations of both users as shown in the following proposition.

Proposition 5.1. *In the high-SNR regime, the upper bounds for the outage probabilities of the j th and i th users are approximated as*

$$\mathbb{P}_j^{ub,\infty} = \Psi(i, M) A_{M+1}^j \tilde{\rho}_{2,m}^j, \quad (5.30)$$

and

$$\mathbb{P}_i^{ub,\infty} = (1 - \Psi(i, M)) A_{M+1}^{i+1} \tilde{\rho}_1^{i+1} + \Psi(i, M) \left(A_{M+1}^{i+1} \tilde{\rho}_{2,r}^{i+1} + A_{M+1}^j \tilde{\rho}_2^j \right), \quad (5.31)$$

respectively.

Proof. See Appendix A.1. $\blacksquare$

The diversity orders of the i th and j th users are obtained as shown in the following corollary.

Corollary 5.1. *In an M -user C-NOMA system in which users are sorted in ascending order based on channel gains in subcarrier S_m , assume that the i th and j th users are allocated to S_m ($1 \leq i < j \leq M$). By using the proposed mode selection strategy, the diversity orders achieved by the i th and j th users are $i + 1$ and j , respectively.*

5.4 Benchmark Scheme

Based on Corollary 5.1, in each subcarrier, the two strongest users should be paired to maximize the achievable diversity order. As this strategy maximizes the diversity orders of both users in each pair, it can be applied to the considered C-NOMA system and the benchmark scheme is derived.

5.4.1 Description of the Benchmark Scheme

Based on the analysis in a single subcarrier, it is natural to design a benchmark scheme pairing users subcarrier by subcarrier, which is illustrated in Algorithm 5.1.

Explanation: The set of users that have not been allocated and the set of users that are allocated to S_m are denoted by $\mathcal{U}$ and $\mathcal{U}^m$, respectively. In each step, we first sort users that have not been allocated in ascending order based on channel gains and then pair the last two users in the corresponding subcarrier. For instance, in S_1 , there are $2K$ users that are sorted in ascending order, and the last two users are selected. After excluding these two users, in S_2 , there are $2K - 2$ users in ascending order, and the last two users are selected. The user pairing in other subcarriers follows the similar rule. For each pair, the mode selection strategy is applied to select either Mode I or Mode II.

Algorithm 5.1 Benchmark scheme

- 1: **Initialize:** $\mathcal{U} = \{U_1, U_2, \dots, U_{2K}\}$, $\mathcal{U}^i = \emptyset$ ($i = 1, 2, \dots, K$), $m = 1$;
 - 2: **while** $m \leq K$ **do**
 - 3: Sort users in $\mathcal{U}$ in ascending order based on channel gains of these users in S_m ;
 - 4: Allocate the last two users to subcarrier S_m and these two users form a user set $\mathcal{U}^m$;
 - 5: Select a mode for this pair based on the mode selection strategy;
 - 6: Update the user set $\mathcal{U} \leftarrow \mathcal{U} \setminus \mathcal{U}^m$;
 - 7: $m \leftarrow m + 1$;
 - 8: **end while.**
-

5.4.2 Performance Analysis of the Benchmark Scheme

In the benchmark scheme, the outage probability of each user is given in the following theorem.

Theorem 5.2. *In the considered multi-carrier C-NOMA system, by using the benchmark scheme, the stronger and weaker users in S_m have the outage probabilities that are bounded by*

$$\mathbb{P}_{b,m,s}^{ub} = (1 - \Psi(a, a + 1)) C_{a+2}^{a+2}(\tilde{\rho}_{1,m}) + \Psi(a, a + 1) C_{a+2}^{a+1}(\tilde{\rho}_{2,m}), \quad (5.32)$$

and

$$\begin{aligned} \mathbb{P}_{b,m,w}^{ub} = & (1 - \Psi(a, a + 1)) C_{a+2}^{a+1}(\tilde{\rho}_1) + \Psi(a, a + 1) \\ & \times \left(C_{a+2}^{a+1}(\tilde{\rho}_{2,r}) + C_{a+2}^{a+1}(\tilde{\rho}_2) - C_{a+2}^{a+1}(\tilde{\rho}_{2,r}) C_{a+2}^{a+1}(\tilde{\rho}_2) \right), \end{aligned} \quad (5.33)$$

respectively, where $a = 2K - 2m + 1$.

Proof. In S_m , there are $a + 1$ remaining users sorted in ascending order, and we choose the last two users to form the m th pair. By substituting $M = a + 1$, $i = a$,

and $j = a+1$ into (5.24) and (5.25), we can derive (5.32) and (5.33). This completes the proof. $\blacksquare$

Then, the high-SNR approximations of outage probabilities in the benchmark scheme are given in the following proposition.

Proposition 5.2. *In the high-SNR regime, the outage probabilities of the stronger and weaker users in subcarrier S_m are given by*

$$\mathbb{P}_{b,m,s}^{ub,\infty} = \Psi(a, a+1) A_{a+2}^{a+1} \tilde{\rho}_{2,m}^{a+1}, \quad (5.34)$$

and

$$\mathbb{P}_{b,m,w}^{ub,\infty} = (1 - \Psi(a, a+1)) A_{a+2}^{a+1} \tilde{\rho}_1^{a+1} + \Psi(a, a+1) \left(A_{a+2}^{a+1} \tilde{\rho}_{2,r}^{a+1} + A_{a+2}^{a+1} \tilde{\rho}_2^{a+1} \right), \quad (5.35)$$

respectively.

Proof. By substituting $M = a+1$, $i = a$, and $j = a+1$ into (5.30) and (5.31), we can derive (5.34) and (5.35). This completes the proof. $\blacksquare$

Following that, the diversity orders of users in the benchmark scheme are obtained as shown in the following corollary.

Corollary 5.2. *In a K -subcarrier and $2K$ -user C-NOMA system, by using the benchmark scheme, a pair of users allocated to subcarrier S_m have the achievable diversity orders of $2K - 2m + 2$, i.e., the achievable diversity orders are $2K, 2K - 2, \dots, 2$ from the first pair to the last pair, respectively.*

It is observed that the diversity order decreases from $2K$ to 2 as each pair is formed and the benchmark scheme cannot guarantee every pair a large diversity order. Therefore, an AMS-UPSA scheme is proposed to improve the achievable diversity order.

5.5 AMS-UPSA Scheme

We design the AMS-UPSA scheme by improving the performance of the worst-performance user.

5.5.1 Description of the AMS-UPSA Scheme

The AMS-UPSA scheme is to pair users in different subcarriers with mode selection for each pair in the considered K -subcarrier and $2K$ -user C-NOMA system. It is able to maximize the achievable diversity order, which is illustrated in Algorithm 5.2.

Explanation: This scheme includes K different candidate schemes. Specifically, the first candidate scheme is the same as the benchmark scheme, and other candidate schemes are similar to it. In each candidate scheme, there are K pairs that are formed in different subcarriers. The difference between these candidate schemes is that the same pair is formed in different subcarrier in different candidate schemes. In the κ th candidate scheme, the n th pair is formed in S_m ($1 \leq n \leq K$), where

$$m = \begin{cases} \kappa + n - 1 \bmod K, & \text{if } (\kappa + n - 1 \bmod K) \neq 0, \\ K, & \text{if } (\kappa + n - 1 \bmod K) = 0. \end{cases} \quad (5.36)$$

For instance, in the first candidate scheme, we form the first, second, $\dots$, K th pairs in $S_1, S_2, \dots, S_K$, respectively. The second candidate scheme is different from the first one. We form the first, second, $\dots$, $(K-1)$ th, K th pairs in $S_2, S_3, \dots, S_K, S_1$, respectively. The other candidate schemes follow the same rule. For each pair, mode selection is implemented based on the proposed mode selection strategy.

Example: To make our scheme easy to follow, the illustration of the AMS-UPSA scheme when $K = 2$ is presented in a step-by-step manner, which is the simplest case.

- When $K = 2$, there are two subcarriers and four users so that there are two

Algorithm 5.2 AMS-UPSA scheme

```

1: Initialize:  $\kappa = 1$ ;
2: while  $\kappa \leq K$  do
3:   Initialize:  $\mathcal{U} = \{U_1, U_2, \dots, U_{2K}\}$ ,  $\mathcal{U}^i = \emptyset$  ( $1 \leq i \leq K$ ),  $m = \kappa$ ;
4:   repeat
5:     Sort users in  $\mathcal{U}$  in ascending order based on channel gains of these
       users in subcarrier  $S_m$ ;
6:     Allocate the last two users in subcarrier  $S_m$  and these two users form
       a user set  $\mathcal{U}^m$ ;
7:     Select a mode for this pair based on the mode selection strategy;
8:     Update the user set  $\mathcal{U} \leftarrow \mathcal{U} \setminus \mathcal{U}^m$ ;
9:     if  $m = K$  then
10:        $m = 0$ ;
11:     end if
12:      $m \leftarrow m + 1$ ;
13:   until  $m = \kappa$ ;
14:   Calculate  $\mu(\kappa)$ , where  $\mu(\kappa)$  is the number of users in outage in the  $\kappa$ th
       candidate scheme;
15:    $\kappa \leftarrow \kappa + 1$ ;
16: end while
17: Select the  $\hat{\kappa}$ th candidate scheme as the final AMS-UPSA scheme, where  $\hat{\kappa} =$ 
     $\arg \min_{1 \leq \kappa \leq K} \mu(\kappa)$ .

```

candidate schemes.

- For the first candidate scheme, we first allocate two strongest users in S_1 and select a mode for this pair based on the mode selection strategy. Then we pair the remaining two users in S_2 after excluding two users of the first pair and conduct the mode selection. Following that, we compute the number of outage users, $\mu(1)$, for the first candidate scheme.
- The second candidate scheme is different from the first candidate scheme. We first allocate the two strongest users to S_2 and select a mode for this pair. Next, we exclude two users of the first pair and pair the remaining two users in S_1 with implementing the mode selection. The number of outage users in the second candidate scheme, $\mu(2)$, is calculated as well.
- Finally, the candidate scheme with the least outage users is the AMS-UPSA scheme, i.e., if $\mu(1) \leq \mu(2)$, the first candidate scheme is selected; otherwise, the second candidate scheme is selected.

5.5.2 Performance Analysis of the AMS-UPSA Scheme

In this subsection, we will analyze the outage performance of the proposed AMS-UPSA scheme.

Theorem 5.3. *The upper bounds of the outage probabilities of all stronger and all weaker users are given by*

$$\mathbb{P}_s^{ub} = \left((1 - \Psi(1, 2K)) C_{2K+1}^3(\tilde{\rho}_{1,m}) + \Psi(1, 2K) C_{2K+1}^2(\tilde{\rho}_{2,m}) \right)^K, \quad (5.37)$$

and

$$\begin{aligned} \mathbb{P}_w^{ub} = & \left((1 - \Psi(1, 2K)) C_{2K+1}^2(\tilde{\rho}_1) + \Psi(1, 2K) \left(C_{2K+1}^2(\tilde{\rho}_{2,r}) + C_{2K+1}^2(\tilde{\rho}_2) \right. \right. \\ & \left. \left. - C_{2K+1}^2(\tilde{\rho}_{2,r}) C_{2K+1}^2(\tilde{\rho}_2) \right) \right)^K, \end{aligned} \quad (5.38)$$

respectively.

Proof. In each candidate scheme, the stronger and weaker users in the K th pair (the last pair) have the highest outage probabilities, respectively. For the K th pair, it is the worst case that in all K candidate schemes, the users in this pair are always the weakest users among $2K$ users. In one candidate scheme, based on (5.24) and (5.25), the upper bounds for outage probabilities of the stronger and weaker users in the K th pair for the worst case are given by

$$\mathbb{P}_{s,one}^{worst} = (1 - \Psi(1, 2K)) C_{2K+1}^3(\tilde{\rho}_{1,m}) + \mathbb{P}_{h < g}^{1,2K} C_{2K+1}^2(\tilde{\rho}_{2,m}), \quad (5.39)$$

and

$$\begin{aligned} \mathbb{P}_{w,one}^{worst} = & (1 - \Psi(1, 2K)) C_{2K+1}^2(\tilde{\rho}_1) + \Psi(1, 2K) \left(C_{2K+1}^2(\tilde{\rho}_{2,r}) + C_{2K+1}^2(\tilde{\rho}_2) \right. \\ & \left. - C_{2K+1}^2(\tilde{\rho}_{2,r}) C_{2K+1}^2(\tilde{\rho}_2) \right), \end{aligned} \quad (5.40)$$

respectively. Then, the upper bounds of the stronger and weaker users in the proposed scheme can be derived as $\mathbb{P}_s^{ub} = (\mathbb{P}_{s,one}^{worst})^K$ and $\mathbb{P}_w^{ub} = (\mathbb{P}_{w,one}^{worst})^K$, respectively.

This completes the proof. ■

Then, the high-SNR approximations of upper bounds are given in the following proposition

Proposition 5.3. *In the high-SNR regime, $\mathbb{P}_s^{ub}$ and $\mathbb{P}_w^{ub}$ can be approximated as*

$$\mathbb{P}_s^{ub,\infty} = (\Psi(1, 2K) A_{2K+1}^2)^K \tilde{\rho}_{2,m}^{2K}, \quad (5.41)$$

and

$$\mathbb{P}_w^{ub,\infty} = \left((1 - \Psi(1, 2K)) A_{2K+1}^2 \tilde{\rho}_1^2 + \Psi(1, 2K) \left(A_{2K+1}^2 \tilde{\rho}_{2,r}^2 + A_{2K+1}^2 \tilde{\rho}_2^2 \right) \right)^K, \quad (5.42)$$

respectively.

Proof. By substituting $M = 2K$, $i = 1$, and $j = 2$ into (5.30) and (5.31), we can derive the high-SNR approximations of $\mathbb{P}_{s,one}^{worst}$ and $\mathbb{P}_{w,one}^{worst}$. Furthermore, we can derive (5.41) and (5.42). This completes the proof. ■

Following that, the diversity orders of users in the AMS-UPSA scheme are obtained as shown in the following corollary.

Corollary 5.3. *In a K -subcarrier and $2K$ -user C-NOMA system, by using the proposed AMS-UPSA scheme, the diversity orders achieved by all users are $2K$.*

Compared with the benchmark scheme, the AMS-UPSA scheme achieves much better outage performance by sacrificing the computational complexity properly.

5.6 Numerical Results

In this section, theoretical and simulation results are presented for the performance evaluation. In the considered system, it is assumed that the power allocation coefficients are $\alpha_f = 0.9$, $\alpha_n = 0.1$ and the target data rate is $\tilde{R}_i = \tilde{R}_j = 1$ bps/Hz which is similar to the settings adopted in some relevant works [114].

5.6.1 Performance Improvement through Mode Selection

When $K = 1$, there is one subcarrier and two users. In Figure 5.2, we plot two users' outage probabilities of Mode I, Mode II, and using the proposed mode selection strategy. This figure reveals the agreement between simulation results and the corresponding analytical curves in Mode I and Mode II. We obtain that the stronger and weaker users have the diversity orders of 2 and 1, respectively, in both modes by observing the outage probability curves' slopes in the high-SNR regime. For the proposed mode selection strategy, the simulation points of the stronger and weaker users are below their analytical upper bounds, respectively. Since both

Figure 5.2: Outage probabilities in a single subcarrier when $K = 1$, including Mode I, Mode II, and the proposed mode selection strategy.

Figure 5.3: Outage probabilities in a single subcarrier when $M = 4, i = 1, j = 3$ and $M = 6, i = 3, j = 6$.

users' diversity orders are 2, the weaker user's diversity order can be improved by employing mode selection.

When there are M users that are sorted in ascending order based on their channel gains in subcarrier S_m , the i th and j th users are allocated to S_m ($1 \leq i < j \leq M$). These two users' outage probabilities when $M = 4$, $i = 1$, $j = 3$ and $M = 6$, $i = 3$, $j = 6$ are plotted in Figure 5.3. It is observed that all simulation points are below the corresponding analytical upper bounds that are derived from (5.24) and (5.25). It is also observed that the analyses of high-SNR approximations are accurate, which are derived from (5.30) and (5.31). By observing the slopes in the high-SNR regime, we find that the i th and j th users' diversity orders are 2 and 3, respectively, when $M = 4$, $i = 1$ and $j = 3$; they are 4 and 6, respectively, when $M = 6$, $i = 3$ and $j = 6$. This phenomenon validates the insight from Corollary 5.1.

5.6.2 Comparisons of Benchmark and AMS-UPSA Schemes

In Figure 5.4, the outage probabilities of the benchmark scheme are depicted when $K = 2$. It can be observed that the simulation points of all users are close to and below their respective upper bounds that are derived from (5.32) and (5.33). Furthermore, it is observed that the upper bounds of the stronger and weaker users approach their respective asymptotic curves that are derived from (5.34) and (5.35), respectively. Based on the slope in the high-SNR regime, the diversity orders of users in pair 1 and pair 2 are 4 and 2, respectively, which validates Corollary 5.2. For comparisons, the exhaustive search scheme is employed to obtain optimal performance. In this scheme, we search all possible candidate schemes of subcarrier allocation and user pairing, while the mode selection is carried out for each pair based on the proposed strategy. Then, we select the candidate scheme that has the least outage users as the final scheme. In the exhaustive search scheme, all users have the same outage probabilities statistically, and hence there is only one

Figure 5.4: Outage probabilities of the benchmark scheme in the multi-carrier C-NOMA system when $K = 2$.

Figure 5.5: Outage probabilities of the AMS-UPSA scheme in the multi-carrier C-NOMA system when $K = 2$.

curve for this scheme. It is observed that pair 1 has the same diversity order as the exhaustive search scheme since their outage probability curves are parallel in the high-SNR regime, while pair 2 has a smaller diversity order than the exhaustive search scheme. It is the drawback of the benchmark scheme, and the AMS-UPSA scheme is proposed to overcome it.

In Figure 5.5, the outage probabilities of the AMS-UPSA scheme are depicted when $K = 2$. As expected, the analytical upper bounds of all stronger users and all weaker users are above the corresponding simulation curves, respectively, which are derived from (5.37) and (5.38), respectively. By comparing the outage probabilities of the benchmark and proposed schemes, we observe that our proposed scheme has much better performance. Then, it is observed that two upper bounds approach two asymptotic curves, respectively, which are derived from (5.41) or (5.42). By observing the slope in the high-SNR regime, the diversity orders of all four users are 4. This phenomenon is consistent with Corollary 5.3. Importantly, it is observed that the AMS-UPSA scheme achieves the same diversity order as the exhaustive search scheme, which demonstrates that our proposed scheme achieves the optimal performance in terms of diversity order.

Then, we increase K to 3 and plot the six users' outage probabilities in the benchmark scheme in Figure 5.6. It is observed that the simulation points match the analytical results. It is observed that asymptotic analysis is accurate, and the diversity orders of pair 1, pair 2, and pair 3 are 6, 4, and 2, respectively. Besides, pair 1 has the same diversity order as the exhaustive search scheme. The diversity orders of pair 2 and pair 3 are less than that of the exhaustive search scheme.

For comparison, the outage probabilities of six users in the AMS-UPSA scheme for $K = 3$ are plotted in Figure 5.7. It is observed that the simulation points match the analytical results. Also, it is observed that they all achieve the same diversity orders of 6. Moreover, the proposed scheme and the exhaustive search scheme have the same diversity order, which demonstrates that our proposed scheme achieves

Figure 5.6: Outage probabilities of the benchmark scheme in the multi-carrier C-NOMA system when $K = 3$.

Figure 5.7: Outage probabilities of the AMS-UPSA scheme in the multi-carrier C-NOMA system when $K = 3$.

the optimal diversity order.

5.7 Summary

In this chapter, multi-carrier C-NOMA systems have been studied. For the considered system, the AMS-UPSA scheme has been designed. The closed-form expressions of the stronger and weaker users' outage probabilities have been derived. Besides, we have determined the diversity order by analyzing the high-SNR approximation of the outage probability. It has been revealed that the proposed AMS-UPSA scheme achieves the optimal diversity order but with lower computational complexity as compared with the exhaustive search scheme.

Chapter 6

IRS-Aided NOMA

This chapter is organized as follows. The main contributions of this work are first introduced in Section 6.1. In Section 6.2, the model of the IRS-aided NOMA system is described. New channel statistics of the BS-IRS-UD link are presented in Section 6.3. The performance analysis for the downlink is conducted in Section 6.4, followed by the analysis for the uplink in Section 6.5. Furthermore, numerical and simulation results are presented in Section 6.6. Finally, the summary is given in Section 6.7.

6.1 Introduction

Due to the introduction of IRSs, the performance analysis of the IRS-aided system becomes much more difficult than that of the conventional system. The channel statistics of the BS-IRS-UD link are crucial for performance evaluation. After adjusting the parameters of the IRS to be optimal, it is a challenging task to derive the equivalent channel statistics of the BS-IRS-UD link. Furthermore, most of the existing works have assumed that the BS-IRS-UD link is NLoS, such as [148–151]. Some works have assumed that the BS-IRS-UD link is LoS but without deriving exact channel statistics, e.g., [152–154]. This motivates us to consider the BS-IRS-UD link to be either LoS or NLoS, and derive new channel statistics for

further analysis. As compared with the aforementioned works, we provide a more comprehensive analysis including both downlink and uplink IRS-aided NOMA and OMA systems. In addition, since the aim of deploying IRSs is to improve the service coverage and NOMA prefers to pair users with distinctive channel conditions for system performance improvement [78, 172], we consider a NOMA system in which a cell-center UD is paired with a cell-edge UD that cannot communicate with the BS directly and needs assistance from an IRS. The contributions of this chapter can be summarized as follows:

- We provide a comprehensive study on IRS-aided systems, including both downlink and uplink transmissions for NOMA and OMA. In particular, we investigate two aspects for each scenario, namely outage probability and ergodic rate.
- We adopt the Nakagami- m fading model for the BS-IRS-UD link so that this link can be either LoS or NLoS. Correspondingly, we derive new channel statistics for this link based on the CLT. Since the CLT-based channel statistics are inaccurate when the channel gain is near 0, we further derive exact channel statistics for the channel gain near 0 by utilizing the LT.
- We derive the closed-form expressions for the outage probability and ergodic rate of each scenario. To gain further insight, we derive the asymptotic approximations of the outage probability and ergodic rate in the high-SNR regime to obtain the diversity order and high-SNR slope, respectively. We demonstrate that the number of IRS reflecting elements and Nakagami fading parameters affect the diversity order but have no influence on the high-SNR slope.
- Finally, we compare the performances of IRS-aided and FDR-aided systems. Simulation results reveal the superiority of the IRS over FDR in the high-SNR regime.

Figure 6.1: The System model of IRS-aided NOMA.

6.2 System Model

We consider an IRS-aided NOMA system with a single-antenna BS and two single-antenna UD, denoted by N and F, respectively, as shown in Figure 6.1. More specifically, N is the cell-center UD that can communicate with the BS directly, while F is the cell-edge UD that needs help from an IRS for communications since there is no direct link between the BS and F due to long distance and blocking objects. The IRS has K reflecting elements, and its reflection-coefficient matrix is denoted by $\mathbf{\Theta} = \text{diag}(\beta_1 e^{j\theta_1}, \beta_2 e^{j\theta_2}, \dots, \beta_K e^{j\theta_K})$, where $\beta_k \in [0, 1]$ is the amplitude-reflection coefficient and $\theta_k \in [0, 2\pi)$ is the phase-shift variable of the k th element that can be adjusted by the IRS ($k = 1, 2, \dots, K$).

6.2.1 Channel Model

All channels experience quasi-static flat fading, and the CSI of all channels is assumed to be perfectly known by the BS [148–151]. In particular, several channel estimation schemes for IRS-assisted systems have been proposed to acquire accurate

CSI [133–136]. The link between the BS and N is assumed to be NLoS as it is a link between the BS and the ground UD. Hence, the small-scale fading between the BS and N follows the Rayleigh fading model and is denoted by $h \sim \mathcal{CN}(0, 1)$, where $\mathcal{CN}(\cdot, \cdot)$ is the complex Gaussian distribution. The BS-IRS and IRS-F links can be either LoS or NLoS for different scenarios. Due to the severe path loss, the signals that are reflected by the IRS twice or more times are ignored. The small-scale fading vector between the BS and the IRS is denoted by $\mathbf{G} \in \mathbb{C}^{1 \times K}$. The small-scale fading vector between the IRS and F is denoted by $\mathbf{g} \in \mathbb{C}^{K \times 1}$. Particularly, they are $\mathbf{G} = [G_1, G_2, \dots, G_K]$ and $\mathbf{g} = [g_1, g_2, \dots, g_K]^T$, respectively. All elements in $\mathbf{G}$ and $\mathbf{g}$ follow the Nakagami- m fading model with fading parameters, m_G and m_g , respectively. In particular, it is NLoS for $m_g = 1$ and is LoS for $m_g > 1$ ($\mathcal{G} \in \{G, g\}$).

6.2.2 Signal Model

Downlink

The BS transmits the signal $x = \sqrt{\alpha_1 P_b} s_1^d + \sqrt{\alpha_2 P_b} s_2^d$, where P_b denotes the transmit power of the BS, s_1^d and s_2^d denote the transmitted signals to N and F, respectively, and α_1 and α_2 denote the power allocation coefficients for N and F, respectively ($\alpha_1 + \alpha_2 = 1$). Note that we assume the fixed power sharing between two UDs and set $\alpha_1 < \alpha_2$ for user fairness [148, 149, 154]. The received signals at N and F are given by

$$y_N = h d_N^{-\frac{\alpha_h}{2}} x + n_1, \quad (6.1)$$

and

$$y_F = \left(\mathbf{G} \mathbf{\Theta} \mathbf{g} d_{F1}^{-\frac{\alpha_G}{2}} d_{F2}^{-\frac{\alpha_g}{2}} \right) x + n_2, \quad (6.2)$$

respectively, where d_N and d_{F1} denote the distances from the BS to N and the IRS, respectively, d_{F2} denotes the distance between the IRS and F, α_h , α_G , and α_g denote the path loss exponents for BS-N, BS-IRS, and IRS-F links, respectively, and n_1 and n_2 denote the AWGNs at N and F, respectively, with the same variance σ_n^2 .

At N, the signal of F is detected first, and the corresponding SINR is given by

$$\text{SINR}_{N,F}^d = \frac{|h|^2 d_N^{-\alpha_h} \alpha_2}{|h|^2 d_N^{-\alpha_h} \alpha_1 + 1/\rho}, \quad (6.3)$$

where $\rho = P_b/\sigma_n^2$ denotes the transmit SNR of the BS. After implementing the SIC, the signal of N is decoded, and the corresponding SNR is given by

$$\text{SNR}_N^d = |h|^2 d_N^{-\alpha_h} \alpha_1 \rho. \quad (6.4)$$

At F, its signal is decoded directly by regarding N's signal as interference, and its SINR is given by

$$\text{SINR}_F^d = \frac{|\mathbf{G}\mathbf{\Theta}\mathbf{g}|^2 d_{F1}^{-\alpha_G} d_{F2}^{-\alpha_g} \alpha_2}{|\mathbf{G}\mathbf{\Theta}\mathbf{g}|^2 d_{F1}^{-\alpha_G} d_{F2}^{-\alpha_g} \alpha_1 + 1/\rho}. \quad (6.5)$$

Uplink

The received signal at the BS is given by

$$y = h d_N^{-\frac{\alpha_h}{2}} \sqrt{P_u} s_1^u + \mathbf{G}\mathbf{\Theta}\mathbf{g} d_{F1}^{-\frac{\alpha_G}{2}} d_{F2}^{-\frac{\alpha_g}{2}} \sqrt{P_u} s_2^u + n, \quad (6.6)$$

where P_u denotes the transmit power of each UD, s_1^u and s_2^u denote the transmitted signals from N and F, respectively, and n denotes the AWGN at the BS with variance σ^2 .

At the BS, the signal of N is decoded first by regarding the signal from F as

interference, and the corresponding SINR is given by

$$\text{SINR}_N^u = \frac{|h|^2 d_N^{-\alpha_h}}{|\mathbf{G}\mathbf{\Theta}\mathbf{g}|^2 d_{F1}^{-\alpha_G} d_{F2}^{-\alpha_g} + 1/\rho'}, \quad (6.7)$$

where $\rho' = P_u/\sigma^2$ denotes the transmit SNR of each UD. After carrying out the SIC, F's signal is detected with the SNR given by

$$\text{SNR}_F^u = |\mathbf{G}\mathbf{\Theta}\mathbf{g}|^2 d_{F1}^{-\alpha_G} d_{F2}^{-\alpha_g} \rho'. \quad (6.8)$$

6.3 New Channel Statistics of the BS-IRS-F Link

In this section, we will present the optimized IRS's parameters and the channel statistics of the BS-IRS-F link.

6.3.1 Parameters of the IRS

For the BS-IRS-F link, we aim to provide the best channel quality to F by adjusting the parameters of the IRS. That is to maximize $|\mathbf{G}\mathbf{\Theta}\mathbf{g}| = \left| \sum_{k=1}^K \beta_k G_k g_k e^{j\theta_k} \right|$, where G_k and g_k are the k th element of $\mathbf{G}$ and $\mathbf{g}$, respectively. This can be achieved by intelligently adjusting the phase-shift variable θ_k for each element, i.e., the phases of all $G_k g_k e^{j\theta_k}$ are set to be the same. Therefore, there is not only one solution for $\{\theta_k\}$ ($k = 1, 2, \dots, K$), and the generalized solution is given by $\theta_k = \tilde{\theta} - \arg(G_k g_k)$, where $\tilde{\theta}$ is an arbitrary constant ranging in $[0, 2\pi)$. After adopting the optimal $\{\theta_k\}$, we have

$$|\mathbf{G}\mathbf{\Theta}\mathbf{g}|^2 = \beta^2 \left(\sum_{k=1}^K |G_k| |g_k| \right)^2, \quad (6.9)$$

where we assume that $\beta_k = \beta$, $\forall k$ without loss of generality.

Figure 6.2: The PDF and CDF of X derived by the CLT when $K = 30$, $m_G = 3$, and $m_g = 2$.

6.3.2 New Channel Statistics

New channel statistics can be obtained by using the CLT as shown in the following lemma, which has been verified by Monte Carlo simulations as shown in Figure 6.2.

Lemma 6.1. *Denote that*

$$X = \frac{\left(\sum_{k=1}^K |G_k| |g_k|\right)^2}{K(1 - \xi)}, \quad (6.10)$$

where $\xi = \frac{1}{m_G m_g} \left(\frac{\Gamma(m_G + \frac{1}{2})}{\Gamma(m_G)}\right)^2 \left(\frac{\Gamma(m_g + \frac{1}{2})}{\Gamma(m_g)}\right)^2$. When the number of reflecting elements K is large,² X tends to follow a noncentral chi-square distribution as $X \sim \chi_1'^2(\lambda)$, where $\lambda = \frac{K\xi}{1-\xi}$. Its PDF and CDF are given by

$$f_X(x) = \frac{\lambda^{\frac{1}{4}}}{2} e^{-\frac{x+\lambda}{2}} x^{-\frac{1}{4}} I_{-\frac{1}{2}}\left(\sqrt{\lambda x}\right) = e^{-\frac{x+\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i x^{i-\frac{1}{2}}}{i! 2^{2i+\frac{1}{2}} \Gamma\left(i + \frac{1}{2}\right)}, \quad (6.11)$$

²In general, $K = 30$ is large enough. The precision is also acceptable to set $K = 8$, which will be validated by simulations in the following.

and

$$F_X(x) = 1 - Q_{\frac{1}{2}}\left(\sqrt{\lambda}, \sqrt{x}\right) = e^{-\frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i \gamma\left(i + \frac{1}{2}, \frac{x}{2}\right)}{i! 2^i \Gamma\left(i + \frac{1}{2}\right)}, \quad (6.12)$$

for $x \geq 0$, respectively, where $I_v(\cdot)$ is the modified Bessel function of the first kind, $Q_v(\cdot, \cdot)$ is the Marcum Q -function, $\Gamma(\cdot)$ is the gamma function, and $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function.

Proof. See Appendix B.1. ■

Remark 6.1. The CLT-based PDF, $f_X(x)$, is not accurate for $x \rightarrow 0^+$, which will result in that the derived outage probability for downlink transmission is inaccurate in the high-SNR regime [149].

To address the issue stated in Remark 6.1, we further derive exact channel statistics for the channel gain near 0 without using the CLT as shown in the following lemma.

Lemma 6.2. Denote that

$$Z = \sum_{k=1}^K |G_k| |g_k|. \quad (6.13)$$

When $m_G \neq m_g$, the PDF and the CDF of Z for $z \rightarrow 0^+$ are given by

$$f_Z^{0+}(z) = \frac{\tilde{m}^K}{\Gamma(2m_s K)} z^{2m_s K - 1} e^{-2\sqrt{m_s m_l} z}, \quad (6.14)$$

and

$$F_Z^{0+}(z) = \frac{\tilde{m}^K (4m_s m_l)^{-m_s K}}{\Gamma(2m_s K)} \gamma(2m_s K, 2\sqrt{m_s m_l} z), \quad (6.15)$$

for $z \geq 0$, respectively, where $\tilde{m} = \frac{\sqrt{\pi} 4^{m_s - m_l + 1} (m_s m_l)^{m_s} \Gamma(2m_s) \Gamma(2m_l - 2m_s)}{\Gamma(m_s) \Gamma(m_l) \Gamma(m_l - m_s + \frac{1}{2})}$ with $m_s = \min\{m_G, m_g\}$ and $m_l = \max\{m_G, m_g\}$.

Proof. See Appendix B.2. ■

6.4 Performance Analysis for Downlink Transmission

In this section, we will investigate the performance of downlink NOMA and OMA systems. In particular, for each scenario, the outage probability for the fixed-rate transmission and the ergodic rate for the adaptive-rate transmission are derived.

6.4.1 NOMA

Outage Probability

For the fixed-rate transmission system, outage probability is a widely used metric to measure system performance. The outage probabilities of N and F are given by

$$\mathbb{P}_N^d = 1 - \text{Pr}(\text{SINR}_{N,F}^d \geq \tilde{\gamma}_F, \text{SNR}_N^d \geq \tilde{\gamma}_N), \quad (6.16)$$

and

$$\mathbb{P}_F^d = \text{Pr}(\text{SINR}_F^d < \tilde{\gamma}_F), \quad (6.17)$$

respectively, where $\tilde{\gamma}_N = 2^{\tilde{R}_N} - 1$ and $\tilde{\gamma}_F = 2^{\tilde{R}_F} - 1$ with $\tilde{R}_N$ and $\tilde{R}_F$ being the target rates of N and F, respectively.

Based on the new channel statistics in Lemma 6.1, the outage probabilities of N and F are derived as shown in the following theorem.

Theorem 6.1. *In the considered IRS-aided NOMA system, the outage probabilities of N and F for the downlink are given by*

$$\mathbb{P}_N^d = 1 - e^{-\tilde{\rho}_m}, \quad (6.18)$$

and

$$\mathbb{P}_F^d \approx e^{-\frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i \gamma \left(i + \frac{1}{2}, \frac{\tilde{\rho}}{2}\right)}{i! 2^i \Gamma\left(i + \frac{1}{2}\right)}, \quad (6.19)$$

respectively, where $\tilde{\rho}_m = \max \left\{ \frac{\tilde{\gamma}_F}{(\alpha_2 - \alpha_1 \tilde{\gamma}_F) a \rho}, \frac{\tilde{\gamma}_N}{a \alpha_1 \rho} \right\}$ and $\tilde{\rho} = \frac{\tilde{\gamma}_F}{(\alpha_2 - \alpha_1 \tilde{\gamma}_F) b \rho}$ with $a = d_N^{-\alpha_h}$ and $b = K \beta^2 (1 - \xi) d_{F1}^{-\alpha_g} d_{F2}^{-\alpha_g}$. Note that when we set power allocation coefficients, we need to ensure that $\alpha_2 - \alpha_1 \tilde{\gamma}_F > 0$.

Proof. First, we denote that $Y = |h|^2$, and its PDF and CDF are given by $f_Y(y) = e^{-y}$ for $y \geq 0$ and $F_Y(y) = 1 - e^{-y}$ for $y \geq 0$, respectively. Then, $\mathbb{P}_N^d$ can be derived as

$$\mathbb{P}_N^d = 1 - \text{Pr} \left(\frac{a \alpha_2 Y}{a \alpha_1 Y + 1/\rho} \geq \tilde{\gamma}_F, a \alpha_1 \rho Y \geq \tilde{\gamma}_N \right) = F_Y(\tilde{\rho}_m). \quad (6.20)$$

On the other hand, $\mathbb{P}_F^d$ can be derived as

$$\mathbb{P}_F^d = \text{Pr} \left(\frac{b \alpha_2 X}{b \alpha_1 X + 1/\rho} < \tilde{\gamma}_F \right) \approx F_X(\tilde{\rho}). \quad (6.21)$$

This completes the proof. ■

Remark 6.2. According to Remark 6.1, the outage probability of F is accurate in the low-SNR regime but is inaccurate in the high-SNR regime. Because the outage probability of F for $\rho \rightarrow \infty$ is derived by using $\int_0^{0+} f_X(x) dx$.

To solve the aforementioned problem, we can use Lemma 6.2 to derive the high-SNR approximation of F's outage probability as shown in the following proposition.

Proposition 6.1. *In the high-SNR regime, $\mathbb{P}_N^d$ can be approximated as*

$$\mathbb{P}_N^{d,\infty} = \tilde{\rho}_m. \quad (6.22)$$

When $m_G \neq m_g$, the high-SNR approximation of $\mathbb{P}_F^d$ is given by

$$\mathbb{P}_F^{d,\infty} = \frac{\tilde{m}^K \tilde{c}_1^{2m_s K}}{\Gamma(2m_s K + 1)} \rho^{-m_s K}, \quad (6.23)$$

where $\tilde{c}_1 = \sqrt{\frac{\tilde{\gamma}_F}{c(\alpha_2 - \alpha_1 \tilde{\gamma}_F)}}$ with $c = \beta^2 d_{F1}^{-\alpha_G} d_{F2}^{-\alpha_g}$.

Proof. By expanding the exponential function in (6.18) and extracting the leading-order term, we can obtain (6.22). On the other hand, by expanding the lower incomplete gamma function in (6.15), we have

$$F_Z^{0+}(z) = \tilde{m}^K \sum_{l=0}^{\infty} \frac{(2\sqrt{m_s m_l})^l z^{l+2m_s K}}{\Gamma(l + 2m_s K + 1)} e^{-2\sqrt{m_s m_l} z}. \quad (6.24)$$

Then, by using the expansion of the exponential function, we have

$$F_Z^{0+}(z) = \tilde{m}^K \sum_{l=0}^{\infty} \frac{(2\sqrt{m_s m_l})^l z^{l+2m_s K}}{\Gamma(l + 2m_s K + 1)} \sum_{i=0}^{\infty} \frac{(-2\sqrt{m_s m_l})^i z^i}{i!}. \quad (6.25)$$

Meanwhile, the outage probability of F for $\rho \rightarrow \infty$ can be derived as

$$\mathbb{P}_F^{d,\infty} = \text{Pr} \left(\frac{c\alpha_2 Z^2}{c\alpha_1 Z^2 + 1/\rho} < \tilde{\gamma}_F \right) = F_Z^{0+} \left(\tilde{c}_1 \rho^{-\frac{1}{2}} \right). \quad (6.26)$$

Finally, by substituting $z = \tilde{c}_1 \rho^{-\frac{1}{2}}$ into (6.25) and extracting the leading-order term, we can approximate $\mathbb{P}_F^{d,\infty}$ as (6.23). This completes the proof. ■

Corollary 6.1. *In the considered IRS-aided NOMA system, the diversity orders of N and F for the downlink are given by $\mathcal{D}_N^d = 1$ and $\mathcal{D}_F^d = m_s K$, respectively.*

Proof. Based on Proposition 6.1, we can obtain that the diversity order of F is $m_s K$ when $m_G \neq m_g$. For the case of $m_G = m_g$, it is also $m_s K$ according to the limit. This completes the proof. ■

Remark 6.3. The diversity order of F for the downlink NOMA is affected by the number of IRS reflecting elements and Nakagami fading parameters.

Ergodic Rate

For the adaptive-rate transmission system, ergodic rate is a commonly used metric to measure system performance. The ergodic rates of N and F are given by

$$R_N^d = \mathbb{E} \left(\log_2 \left(1 + \text{SNR}_N^d \right) \right), \quad (6.27)$$

and

$$R_F^d = \mathbb{E} \left(\log_2 \left(1 + \text{SINR}_F^d \right) \right), \quad (6.28)$$

respectively.

Remark 6.4. For the NOMA with adaptive-rate transmission, the SINR at F is $\min \{ \text{SINR}_{N,F}^d, \text{SINR}_F^d \}$. However, in the considered system, F has a much more severe path loss than N due to a much larger distance to the BS. Therefore, we have $\min \{ \text{SINR}_{N,F}^d, \text{SINR}_F^d \} \approx \text{SINR}_F^d$.

After some mathematical manipulations, the ergodic rates of N and F are obtained as presented in the following theorem.

Theorem 6.2. *In the considered IRS-aided NOMA system, the ergodic rates of N and F for the downlink are, respectively, given by*

$$R_N^d = -\frac{e^{\frac{1}{a\alpha_1\rho}}}{\ln(2)} \text{Ei} \left(-\frac{1}{a\alpha_1\rho} \right), \quad (6.29)$$

and

$$R_F^d \approx \log_2(1 + \tilde{\alpha}) - \frac{e^{-\frac{\lambda}{2}}}{\ln(2)} \sum_{i=0}^{\infty} \frac{\lambda^i}{i!2^i\Gamma(i + \frac{1}{2})} \sum_{l=1}^{u_1} \omega_{1,l} \mathcal{J}_1(t_l), \quad (6.30)$$

where $\text{Ei}(\cdot)$ is the exponential integral, $\tilde{\alpha} = \frac{\alpha_2}{\alpha_1}$, u_1 is the number of nodes for the Chebyshev-Gauss quadrature and determines the approximation precision, $\omega_{1,l} = \frac{\pi}{u_1}$

is the weight, $t_l = \cos\left(\frac{2l-1}{2u_1}\pi\right)$, and

$$\mathcal{J}_1(t) = \gamma \left(i + \frac{1}{2}, \frac{\tilde{\alpha}(1+t)}{4b\rho\alpha_2 - 2b\rho\alpha_1\tilde{\alpha}(1+t)} \right) \frac{\sqrt{1-t^2}}{1 + \frac{2}{\tilde{\alpha}} + t}. \quad (6.31)$$

Proof. See Appendix B.3. ■

Remark 6.5. The ergodic rate of F is accurate as the calculation is not related to $\int_0^{0+} g(x)f_X(x)dx$. Because of the principle of downlink NOMA, the ergodic rate of N increases for the increase of the SNR, while the ergodic rate of F approaches a ceiling when $\rho \rightarrow \infty$.

To provide insight into the performance, the high-SNR slope is considered, which is defined as $S = \lim_{\rho \rightarrow \infty} \frac{R(\rho)}{\log_2(\rho)}$ [173]. To obtain it, the asymptotic expression for N's ergodic rate and the ceiling for F's ergodic rate are derived in the following proposition.

Proposition 6.2. *In the high-SNR regime, R_N^d can be approximated as*

$$R_N^{d,\infty} = \log_2(a\alpha_1\rho) - \frac{E_c}{\ln(2)}, \quad (6.32)$$

where E_c denotes the Euler constant. On the other hand, the ceiling for R_F^d in the high-SNR regime is given by

$$R_F^{d,\infty} = \log_2(1 + \tilde{\alpha}). \quad (6.33)$$

Proof. By using $\lim_{x \rightarrow 0} e^x = 1$ and $\text{Ei}(-x) \approx \ln(x) + E_c$ for $x \rightarrow 0$ [170, eq. (8.214.2)], we can approximate (6.29) as (6.32) when $\rho \rightarrow \infty$. On the other hand, since we have $\lim_{\rho \rightarrow \infty} \frac{b\alpha_2 X}{b\alpha_1 X + 1/\rho} = \tilde{\alpha}$, we can approximate (6.28) as (6.33) when $\rho \rightarrow \infty$. This completes the proof. ■

Corollary 6.2. *In the considered IRS-aided NOMA system, the high-SNR slopes of N and F for the downlink are given by $\mathcal{S}_N^d = 1$ and $\mathcal{S}_F^d = 0$, respectively.*

Proof. We have $\mathcal{S}_N^d = \frac{dR_N^{d,\infty}}{d\log_2(\rho)} = \ln(2)\rho \frac{dR_N^{d,\infty}}{d\rho} = 1$. This completes the proof. ■

Remark 6.6. The high-SNR slope of F for the downlink NOMA is not related to the number of IRS reflecting elements and Nakagami fading parameters.

6.4.2 OMA

OMA can be regarded as a special case of NOMA. When N and F work under the OMA scheme, their data rates are given by $\frac{1}{2} \log_2 (1 + |h|^2 d_N^{-\alpha_h} \rho)$ and $\frac{1}{2} \log_2 \left(1 + \beta^2 \left(\sum_{k=1}^K |G_k| |g_k| \right)^2 d_{F1}^{-\alpha_G} d_{F2}^{-\alpha_g} \rho \right)$, respectively. Here, we assume that each UD shares half of the resource block for a fair comparison.

Outage Probability

The outage probabilities of N and F for the downlink are presented in the following theorem.

Theorem 6.3. *When N and F are under the downlink OMA scheme, their outage probabilities are given by*

$$\mathbb{P}_N^{do} = 1 - e^{-\frac{\tilde{\gamma}_N^o}{a\rho}}, \quad (6.34)$$

and

$$\mathbb{P}_F^{do} \approx e^{-\frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i \gamma \left(i + \frac{1}{2}, \frac{\tilde{\gamma}_F^o}{2b\rho} \right)}{i! 2^i \Gamma \left(i + \frac{1}{2} \right)}, \quad (6.35)$$

respectively, where $\tilde{\gamma}_N^o = 2^{2\tilde{R}_N} - 1$ and $\tilde{\gamma}_F^o = 2^{2\tilde{R}_F} - 1$.

Proof. $\mathbb{P}_N^{do}$ and $\mathbb{P}_F^{do}$ can be derived as follows:

$$\mathbb{P}_N^{do} = \mathbb{P}_r(a\rho Y < \tilde{\gamma}_N^o) = F_Y \left(\frac{\tilde{\gamma}_N^o}{a\rho} \right), \quad (6.36)$$

and

$$\mathbb{P}_F^{do} = \mathbb{P}_r(b\rho X < \tilde{\gamma}_F^o) \approx F_X\left(\frac{\tilde{\gamma}_F^o}{b\rho}\right). \quad (6.37)$$

This completes the proof. ■

Remark 6.7. According to Remark 6.1, $\mathbb{P}_F^{do}$ is accurate in the low-SNR regime but is inaccurate in the high-SNR regime. Because $\mathbb{P}_F^{do}$ for $\rho \rightarrow \infty$ is derived by using $\int_0^{0+} f_X(x)dx$.

To address the problem in Remark 6.7, we use Lemma 6.2 and obtain the high-SNR approximation of $\mathbb{P}_F^{do}$ that is shown in the following proposition.

Proposition 6.3. *In the high-SNR regime, $\mathbb{P}_N^{do}$ can be approximated as*

$$\mathbb{P}_N^{do,\infty} = \frac{\tilde{\gamma}_N^o}{a} \rho^{-1}. \quad (6.38)$$

When $m_G \neq m_g$, the high-SNR approximation of $\mathbb{P}_F^{do}$ is given by

$$\mathbb{P}_F^{do,\infty} = \frac{\tilde{m}^K \tilde{c}_2^{2m_s K}}{\Gamma(2m_s K + 1)} \rho^{-m_s K}, \quad (6.39)$$

where $\tilde{c}_2 = \sqrt{\frac{\tilde{\gamma}_F^o}{c}}$.

Proof. Similar to the proof of Proposition 6.1. ■

Corollary 6.3. *When N and F are under the downlink OMA scheme, the diversity orders of N and F are given by $\mathcal{D}_N^{do} = 1$ and $\mathcal{D}_F^{do} = m_s K$, respectively.*

Remark 6.8. The diversity order of F for the downlink OMA is affected by the number of IRS reflecting elements and Nakagami fading parameters. Moreover, the diversity orders of N and F for the downlink OMA are the same as these for the downlink NOMA, respectively.

Ergodic Rate

The ergodic rates of N and F for OMA are presented in the following theorem.

Theorem 6.4. *When N and F are under the downlink OMA scheme, their ergodic rates are, respectively, given by*

$$R_N^{do} = -\frac{e^{\frac{1}{a\rho}}}{2\ln(2)} \text{Ei}\left(-\frac{1}{a\rho}\right), \quad (6.40)$$

and

$$R_F^{do} \approx \frac{\lambda^{\frac{1}{4}}}{4} e^{-\frac{\lambda}{2}} \sum_{l=1}^{u_2} \omega_{2,l} \mathcal{J}_2(x_{2,l}), \quad (6.41)$$

where u_2 is the number of nodes for the Gauss-Laguerre quadrature and determines the approximation precision, $x_{2,l}$ is the l th root of Laguerre polynomial $L_{u_2}(x)$, $\omega_{2,l} = \frac{x_{2,l}}{(u_2+1)^2 (L_{u_2+1}(x_{2,l}))^2}$ is the weight, and

$$\mathcal{J}_2(x) = x^{-\frac{1}{4}} e^{\frac{x}{2}} \log_2(1 + b\rho x) I_{-\frac{1}{2}}(\sqrt{\lambda x}). \quad (6.42)$$

Proof. The analysis for N is similar to that in the proof of Theorem 6.2. For the ergodic rate of F, it can be expressed as

$$\begin{aligned} R_F^{do} &\approx \int_0^\infty \frac{1}{2} \log_2(1 + b\rho x) f_X(x) dx \\ &= \frac{\lambda^{\frac{1}{4}}}{4} e^{-\frac{\lambda}{2}} \underbrace{\int_0^\infty x^{-\frac{1}{4}} e^{-\frac{x}{2}} \log_2(1 + b\rho x) I_{-\frac{1}{2}}(\sqrt{\lambda x}) dx}_{J_2}. \end{aligned} \quad (6.43)$$

Next, J_2 can be approximated by using the Gauss-Laguerre quadrature. As such, we have

$$J_2 \simeq \sum_{l=1}^{u_2} \omega_{2,l} \mathcal{J}_2(x_{2,l}). \quad (6.44)$$

This completes the proof. ■

Remark 6.9. The ergodic rate of F is accurate as the calculation is not related to $\int_0^{0+} g(x)f_X(x)dx$. On the other hand, both ergodic rates of N and F increase as the SNR increases.

Then, the approximations in the high-SNR regime are given in the following proposition.

Proposition 6.4. *In the high-SNR regime, the approximations of R_N^{do} and R_F^{do} are given by*

$$R_N^{do,\infty} = \frac{1}{2} \left(\log_2(a\rho) - \frac{E_c}{\ln(2)} \right), \quad (6.45)$$

and

$$R_F^{do,\infty} = \frac{1}{2} \log_2(b\rho(1 + \lambda)), \quad (6.46)$$

respectively.

Proof. The analysis for N is similar to that in the proof of Proposition 6.2. For F, we have $\mathbb{E}(X) = 1 + \lambda$ according to the property of the noncentral chi-square distribution. Then, since $\frac{1}{2} \log_2(1 + b\rho X)$ with respect to X is concave, with the aid of Jensen's inequality [174], we have

$$R_F^{do} = \mathbb{E} \left(\frac{1}{2} \log_2(1 + b\rho X) \right) \leq \frac{1}{2} \log_2(1 + b\rho \mathbb{E}(X)) = \frac{1}{2} \log_2(1 + b\rho(1 + \lambda)). \quad (6.47)$$

Since X is mainly distributed in a relatively large range, the second derivative of $\frac{1}{2} \log_2(1 + b\rho X)$ with respect to X is approximately equal to 0. As such, based on the property of Jensen's inequality, we have $R_F^{do} \approx \frac{1}{2} \log_2(1 + b\rho(1 + \lambda))$. When $\rho \rightarrow \infty$, R_F^{do} can be approximated as (6.46). This completes the proof. ■

Corollary 6.4. *When N and F are under the downlink OMA scheme, their high-SNR slopes are given by $\mathcal{S}_N^{do} = 0.5$ and $\mathcal{S}_F^{do} = 0.5$, respectively.*

Proof. Similar to the proof of Corollary 6.2. ■

Remark 6.10. The high-SNR slope of N for the downlink OMA is half of that for the downlink NOMA. On the other hand, the high-SNR slope of F in downlink OMA systems, which is not related to the number of IRS reflecting elements and Nakagami fading parameters, is larger than that in downlink NOMA systems.

6.5 Performance Analysis for Uplink Transmission

In this section, we will investigate the performance of the uplink transmission. The analyses of outage probability and ergodic rate will be presented in the following.

6.5.1 NOMA

Outage Probability

The outage probabilities of N and F are given by

$$\mathbb{P}_N^u = \Pr(\text{SINR}_N^u < \tilde{\gamma}_N), \quad (6.48)$$

and

$$\mathbb{P}_F^u = 1 - \Pr(\text{SINR}_N^u \geq \tilde{\gamma}_N, \text{SNR}_F^u \geq \tilde{\gamma}_F), \quad (6.49)$$

respectively.

Then, the closed-form expressions for outage probabilities can be derived, and the results are shown in the following theorem.

Theorem 6.5. *In the considered IRS-aided NOMA system, the outage probabilities of N and F for the uplink are given by*

$$\mathbb{P}_N^u \approx 1 - e^{-\frac{\tilde{\gamma}_N}{a\rho'} - \frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i}{i! 2^{2i+\frac{1}{2}} \left(\frac{b\tilde{\gamma}_N}{a} + \frac{1}{2}\right)^{i+\frac{1}{2}}}, \quad (6.50)$$

and

$$\mathbb{P}_F^u \approx 1 - e^{-\frac{\tilde{\gamma}_N}{a\rho'} - \frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i \Gamma\left(i + \frac{1}{2}, \frac{\tilde{\gamma}_N \tilde{\gamma}_F}{a\rho'} + \frac{\tilde{\gamma}_F}{2b\rho'}\right)}{i! 2^{2i+\frac{1}{2}} \left(\frac{b\tilde{\gamma}_N}{a} + \frac{1}{2}\right)^{i+\frac{1}{2}} \Gamma\left(i + \frac{1}{2}\right)}, \quad (6.51)$$

respectively, where $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function.

Proof. The outage probability of N can be transformed into

$$\begin{aligned} \mathbb{P}_N^u &= \text{Pr} \left(\frac{aY}{bX + 1/\rho'} < \tilde{\gamma}_N \right) \approx \int_0^\infty \int_0^{\frac{b\tilde{\gamma}_N x}{a} + \frac{\tilde{\gamma}_N}{a\rho'}} f_Y(y) dy f_X(x) dx \\ &= 1 - e^{-\frac{\tilde{\gamma}_N}{a\rho'} - \frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i}{i! 2^{2i+\frac{1}{2}} \Gamma\left(i + \frac{1}{2}\right)} \int_0^\infty e^{-\left(\frac{b\tilde{\gamma}_N}{a} + \frac{1}{2}\right)x} x^{i-\frac{1}{2}} dx. \end{aligned} \quad (6.52)$$

Furthermore, by referring to [170, eq. (3.381.4)], (6.50) can be derived. Similarly, the outage probability of F can be first transformed into

$$\begin{aligned} \mathbb{P}_F^u &= 1 - \text{Pr} \left(\frac{aY}{bX + 1/\rho'} \geq \tilde{\gamma}_N, b\rho'X \geq \tilde{\gamma}_F \right) \approx 1 - \int_{\frac{\tilde{\gamma}_F}{b\rho'}}^\infty \int_{\frac{b\tilde{\gamma}_N x}{a} + \frac{\tilde{\gamma}_N}{a\rho'}}^\infty f_Y(y) dy f_X(x) dx \\ &= 1 - e^{-\frac{\tilde{\gamma}_N}{a\rho'} - \frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i}{i! 2^{2i+\frac{1}{2}} \Gamma\left(i + \frac{1}{2}\right)} \int_{\frac{\tilde{\gamma}_F}{b\rho'}}^\infty e^{-\left(\frac{b\tilde{\gamma}_N}{a} + \frac{1}{2}\right)x} x^{i-\frac{1}{2}} dx. \end{aligned} \quad (6.53)$$

Then, by referring to [170, eq. (3.381.3)], we can derive the closed-form expression as (6.51). This completes the proof. $\blacksquare$

Remark 6.11. The outage probability of N is accurate since it does not involve the integral of $\int_0^{0+} g(x) f_X(x) dx$. Although the calculation of F's outage probability is related to $\int_0^{0+} g(x) f_X(x) dx$, it only has a small bias in the medium-SNR regime.

This is because F's outage probability must be higher than N's outage probability and converges to a floor in the high-SNR regime with N's outage probability due to the principle of uplink NOMA.

The floor for outage probabilities of both UDs is presented in the following proposition.

Proposition 6.5. *In the high-SNR regime, $\mathbb{P}_N^u$ and $\mathbb{P}_F^u$ approach the same floor as*

$$\mathbb{P}_{both}^{u,\infty} = 1 - e^{-\frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i}{i! 2^{2i+\frac{1}{2}} \left(\frac{b\tilde{\gamma}_N}{a} + \frac{1}{2}\right)^{i+\frac{1}{2}}}. \quad (6.54)$$

Proof. We have that $\lim_{\rho' \rightarrow \infty} e^{-\frac{\tilde{\gamma}_N}{a\rho'}} = 1$ and $\lim_{\rho' \rightarrow \infty} \Gamma\left(i + \frac{1}{2}, \frac{\tilde{\gamma}_N \tilde{\gamma}_F}{a\rho'} + \frac{\tilde{\gamma}_F}{2b\rho'}\right) = \Gamma\left(i + \frac{1}{2}\right)$. Thus, the limit of both $\mathbb{P}_N^u$ and $\mathbb{P}_F^u$ for $\rho' \rightarrow \infty$ can be obtained. This completes the proof. ■

Corollary 6.5. *In the considered IRS-aided NOMA system, both diversity orders of N and F for the uplink are 0, i.e., $\mathcal{D}_N^u = 0$ and $\mathcal{D}_F^u = 0$.*

Remark 6.12. The diversity order of F for the uplink NOMA is not related to the number of IRS reflecting elements and Nakagami fading parameters.

Ergodic Rate

The ergodic rates of the N and F are given by

$$R_N^u = \mathbb{E}(\log_2(1 + \text{SINR}_N^u)), \quad (6.55)$$

and

$$R_F^u = \mathbb{E}(\log_2(1 + \text{SNR}_F^u)), \quad (6.56)$$

respectively.

Then, the ergodic rates of N and F can be obtained, and the results are provided in the following theorem.

Theorem 6.6. *In the considered IRS-aided NOMA system, the ergodic rate of N for the uplink is given by*

$$R_N^u \approx -\frac{\lambda^{\frac{1}{4}}}{2\ln(2)} e^{\frac{1}{a\rho'} - \frac{\lambda}{2}} \sum_{l=1}^{u_3} \omega_{3,l} \mathcal{J}_3(x_{3,l}), \quad (6.57)$$

where u_3 is the number of nodes for the Gauss-Laguerre quadrature and determines the approximation precision, $x_{3,l}$ is the l th root of Laguerre polynomial $L_{u_3}(x)$,

$\omega_{3,l} = \frac{x_{3,l}}{(u_3+1)^2 (L_{u_3+1}(x_{3,l}))^2}$ is the weight, and

$$\mathcal{J}_3(x) = x^{-\frac{1}{4}} e^{\left(\frac{b}{a} + \frac{1}{2}\right)x} \text{Ei} \left(-\frac{b}{a}x - \frac{1}{a\rho'} \right) I_{-\frac{1}{2}} \left(\sqrt{\lambda x} \right). \quad (6.58)$$

On the other hand, the ergodic rate of F for the uplink is given by

$$R_F^u \approx \frac{\lambda^{\frac{1}{4}}}{2} e^{-\frac{\lambda}{2}} \sum_{l=1}^{u_4} \omega_{4,l} \mathcal{J}_4(x_{4,l}), \quad (6.59)$$

where u_4 is the number of nodes for the Gauss-Laguerre quadrature and determines the approximation precision, $x_{4,l}$ is the l th root of Laguerre polynomial $L_{u_4}(x)$,

$\omega_{4,l} = \frac{x_{4,l}}{(u_4+1)^2 (L_{u_4+1}(x_{4,l}))^2}$ is the weight, and

$$\mathcal{J}_4(x) = x^{-\frac{1}{4}} e^{\frac{x}{2} \log_2(1 + b\rho'x)} I_{-\frac{1}{2}} \left(\sqrt{\lambda x} \right). \quad (6.60)$$

Proof. See Appendix B.4. ■

Remark 6.13. Both ergodic rates are accurate as they are not related to the integral of $\int_0^{0+} g(x) f_X(x) dx$. Due to the principle of uplink NOMA, the ergodic rate of N approaches a ceiling when $\rho' \rightarrow \infty$, while the ergodic rate of F increases as the SNR increases.

The ceiling for N and the high-SNR approximation for F are presented in the following proposition.

Proposition 6.6. *In the high-SNR regime, R_N^u approaches a ceiling that is given by*

$$R_N^{u,\infty} \approx -\frac{\lambda^{\frac{1}{4}}}{2\ln(2)} e^{-\frac{\lambda}{2}} \sum_{l=1}^{u_3} \omega_{3,l} \mathcal{J}_3^\infty(x_{3,l}), \quad (6.61)$$

where

$$\mathcal{J}_3^\infty(x) = x^{-\frac{1}{4}} e^{\left(\frac{b}{a} + \frac{1}{2}\right)x} \text{Ei}\left(-\frac{b}{a}x\right) I_{-\frac{1}{2}}\left(\sqrt{\lambda x}\right). \quad (6.62)$$

On the other hand, the high-SNR approximation of R_F^u is given by

$$R_F^{u,\infty} = \log_2(b\rho'(1+\lambda)). \quad (6.63)$$

Proof. For N, when $\rho' \rightarrow \infty$, we have $\frac{1}{a\rho'} \rightarrow 0$. Hence, based on (6.57) and (6.58), we can obtain (6.61). The analysis for F is similar to that in the proof of Proposition 6.4. ■

Corollary 6.6. *In the considered IRS-aided NOMA system, the high-SNR slopes of N and F for the uplink are given by $\mathcal{S}_N^u = 0$ and $\mathcal{S}_F^u = 1$, respectively.*

Proof. Similar to the proof of Corollary 6.4. ■

Remark 6.14. The high-SNR slope of F for the uplink NOMA is not affected by the number of IRS reflecting elements and Nakagami fading parameters.

6.5.2 OMA

For OMA, the performance analyses for the uplink and the downlink are similar, and we can obtain all results for the uplink by referring to Subsection 6.4.2 in a

Table 6.1: Diversity order ($\mathcal{D}$) and high-SNR slope ($\mathcal{S}$) for each scenario

Multiple-access scheme	UD	Downlink		Uplink	
		$\mathcal{D}$	$\mathcal{S}$	$\mathcal{D}$	$\mathcal{S}$
NOMA	N	1	1	0	0
	F	$m_s K$	0	0	1
OMA	N	1	0.5	1	0.5
	F	$m_s K$	0.5	$m_s K$	0.5

simple method, i.e., replacing the transmit SNR of the BS (ρ) with the transmit SNR of the UD (ρ').

6.5.3 Summary of All Results

After completing all analyses for downlink and uplink systems, all results related to diversity order and high-SNR slope are summarized in Table 6.1 for ease of reference.

Remark 6.15. We observe that OMA has higher diversity orders than NOMA in uplink systems. It only reflects that OMA outperforms NOMA in the high-SNR regime for uplink systems with the fixed-rate transmission. In the low-SNR regime, NOMA has better performance than OMA [175].

6.6 Numerical Results

In this section, numerical results are presented for the performance evaluation of the considered system. Meanwhile, Monte Carlo simulations are conducted to verify the accuracy. After referring to some relevant works [148, 149, 154], the parameters are set as shown in Table 6.2.

For comparisons, we regard an FDR-aided NOMA system as the benchmark. Specifically, an FDR under the classic protocol is deployed at the place of the IRS to help F to communicate with the BS. The FDR works under a realistic assumption that is the same as [114]. Specifically, since the relay is aware of its

Table 6.2: Setting of parameters

Bandwidth	$B = 1$ MHz
Amplitude-reflection coefficient of the IRS	$\beta = 0.9$
Distances	$d_N = 10$ m, $d_{F1} = 40$ m, and $d_{F2} = 10$ m
Path-loss exponents	$\alpha_h = 3.5$, $\alpha_G = 2.5$, and $\alpha_g = 2.5$
Nakagami- m fading parameters	$m_G = 3$ and $m_g = 1.5$
Target data rates for the fixed-rate transmission	$\tilde{R}_N = \tilde{R}_F = 0.1$ Mbps
Number of points for Chebyshev-Gauss quadratures	$u_1 = 100$
Number of points for Gauss-Laguerre quadratures	$u_2 = u_3 = u_4 = 100$

own transmitted symbol, self-interference cancellation can be applied. we assume that the self-interference channel experiences the Nakagami- m fading and the self-interference cancellation is imperfect. Since the reflection at the IRS is passive without consuming the energy, for fairness, we assume that the transmit power at the BS and the FDR is $P_b^r = 0.5P_b$ for the downlink, and the transmit power at F and the FDR is $P_u^r = 0.5P_u$ for the uplink.

6.6.1 Downlink Systems

In Figure 6.3(a), the outage probabilities versus the transmit SNR in IRS-aided NOMA, IRS-aided OMA, and FDR-aided NOMA systems of downlink are plotted. First, the analyses of N for both NOMA and OMA are accurate as all simulation results coincide with the corresponding analytical results that are derived from (6.18) and (6.34). For F, the analyses under these two schemes that are derived from (6.19) and (6.35) are accurate in the low-SNR regime but are inaccurate in the high-SNR regime, which results from the use of the CLT-based channel statistics. As a benchmark, the outage probability curves for the FDR-aided system are plotted for comparisons. We observe that N in the IRS-aided NOMA system always has better performance than that in the FDR-aided NOMA system since the transmit power of the BS in the former system is twice that in the latter. For F, the FDR-aided system has better performance than the IRS-aided system in the low-SNR regime, since the IRS transmission experiences severe path loss, which has

(a) Outage probabilities derived from the CLT-based channel statistics when $K = 8$, $\alpha_1 = 0.1$, and $\alpha_2 = 0.9$.

(b) High-SNR approximations of outage probabilities derived from the LT-based channel statistics when $K = 2$.

Figure 6.3: Outage probabilities in downlink systems.

been pointed out in [149]. Nevertheless, in the high-SNR regime, the IRS-aided system has much better performance. One reason is that the outage probability of F in the FDR-aided system converges to a floor in the high-SNR regime due to the residual self-interference. On the other hand, F in the IRS-aided system has a large diversity order, which is the advantage of the IRS over FDR.

Since the theoretical results of F are not accurate in the high-SNR regime in Figure 6.3(a), we further plot the high-SNR approximation curves when $\alpha_1 = 0.1$ ($\alpha_2 = 0.9$) and $\alpha_1 = 0.3$ ($\alpha_2 = 0.7$) in Figure 6.3(b). It is observed that the outage probabilities of N and F for NOMA and OMA gradually approach their respective asymptotic curves derived from (6.22), (6.23), (6.38), and (6.39), which validates our analysis. We also observe that a larger α_1 ($\alpha_2 = 1 - \alpha_1$) incurs a lower outage probability of N and a higher outage probability of F. Furthermore, by observing slopes, we find that both diversity orders of N under NOMA and OMA schemes are 1 when $K = 2$. For F, the diversity orders under NOMA and OMA schemes are the same and equal 3 for $K = 2$, which is consistent with Corollary 6.1 and Corollary 6.3. Therefore, the fixed power allocation coefficients affect the outage probability but have no effect on the diversity order.

In Figure 6.4(a), the ergodic rate curves for different downlink systems are depicted. First, it is observed that the simulation points for IRS-aided NOMA and OMA systems match their respective analytical results that are derived from (6.29), (6.30), (6.40), and (6.41). As a benchmark, the ergodic rate curves of the FDR-aided NOMA are plotted for comparisons. It is observed that N in the IRS-aided system always has a higher ergodic rate than that in the FDR-aided system since the transmit power of the BS in the former system is twice that in the latter. On the other hand, for F, the ergodic rate in the IRS-aided system is lower than that in the FDR-aided system in the low-SNR regime. This is because the IRS transmission experiences severe path loss, which has been pointed out in [149]. As the SNR increases, the ergodic rate of F in the IRS-aided system approaches

(a) Ergodic rates derived from the CLT-based channel statistics when $\alpha_1 = 0.1$ and $\alpha_2 = 0.9$.

(b) High-SNR approximations of ergodic rates.

Figure 6.4: Ergodic rates in downlink systems when $K = 10$.

a higher ceiling than that in the FDR-aided system due to the existence of the residual self-interference when using the FDR, which demonstrates the superiority of IRS again.

Furthermore, the asymptotic analysis is validated when $\alpha_1 = 0.1$ ($\alpha_2 = 0.9$) and $\alpha_1 = 0.2$ ($\alpha_2 = 0.8$) in Figure 6.4(b). It is observed that the high-SNR approximations that are derived from (6.32), (6.33), (6.45), and (6.46) are accurate. We also observe that a larger α_1 ($\alpha_2 = 1 - \alpha_1$) incurs a higher ergodic rate of N and a lower ergodic rate of F. Also, we observe that the high-SNR slope of N for NOMA is 1,³ while that for OMA is 0.5. Due to the principle of downlink NOMA, the ergodic rate of F approaches a ceiling in the high-SNR regime, i.e., the high-SNR slope of F for NOMA is 0. Meanwhile, F has a high-SNR slope of 0.5 under the OMA scheme. These observations coincide with Corollary 6.2 and Corollary 6.4. Therefore, the fixed power allocation coefficients affect the ergodic rate but have no effect on the high-SNR slope.

6.6.2 Uplink Systems

In Figure 6.5, the outage probabilities versus the transmit SNR in uplink NOMA systems are plotted. For the IRS-aided NOMA system, it is observed that the simulation points of N match the analytical results that are derived from (6.50). The simulation results of F have a small bias with the analytical results derived from (6.51) in the medium-SNR regime (e.g., 42 dB), which is consistent with our explanation in Remark 6.11. Then, we observe that both outage probabilities of N and F decrease as the increase of the transmit SNR and approach a floor gradually that is derived from (6.54). This is because of the principle of uplink NOMA that F's signal is regarded as interference to decode N's signal. Thus, the diversity orders of both UDs are 0, which is consistent with Corollary 6.5. Finally, it is observed that the outage probabilities for the FDR-aided NOMA system also

³The high-SNR slope of 1 corresponds to the slope of $0.1 \log_2 10 \approx 0.33$ in Figure 6.4(b).

Figure 6.5: Outage probabilities in uplink NOMA systems when $K = 8$.

approach a floor. In Figure 6.5, the floor for the IRS-aided system is lower than that for the FDR-aided system. However, the former will be higher than the latter if we increase K . Meanwhile, the floor for the FDR-aided system is also determined by the efficiency of the self-interference cancellation. Thus, for uplink IRS-aided systems, the contribution of the IRS to outage probability is not obvious.

In Figure 6.6, the ergodic rate curves for different uplink systems are depicted. For the IRS-aided NOMA system, we observe that the simulated results coincide with the corresponding analytical results that are derived from (6.57) and (6.59). In addition, the approximations in the high-SNR regime that are derived from (6.61) and (6.63) are also asymptotic exact. Following that, it is observed that the high-SNR slopes of N and F are 0 and 1, respectively, which is consistent with Corollary 6.6. Lastly, we observe that the ergodic rate of N in the FDR-aided NOMA system also converges to a ceiling. For the ergodic rate of F , it remains increasing with the increase of the transmit SNR in the IRS-aided system, while it has a ceiling in the FDR-aided system due to the residual self-interference, which reveals the advantage of the IRS.

Figure 6.6: Ergodic rates in uplink NOMA systems when $K = 10$.

6.7 Summary

In this chapter, we have characterized the system performance of IRS-aided NOMA and OMA systems for downlink and uplink transmissions. We have demonstrated that the use of IRS can significantly enhance system performance, especially improving the diversity order. We consider the fixed power allocation for downlink NOMA, which affects the outage probability and ergodic rate but has no influence on the diversity order and high-SNR slope. Furthermore, simulation results have demonstrated that IRS outperforms FDR in the high-SNR regime.

Chapter 7

Multi-IRS Assisted NOMA

This chapter is organized as follows. The main contributions of this work are first introduced in Section 7.1. In Section 7.2, the system model of multi-IRS assisted NOMA with discrete phase shifts is described. The performance analysis of Scenario I is conducted in Section 7.3, followed by the analysis of Scenario II in Section 7.4. Furthermore, numerical and simulation results are presented in Section 7.5. Finally, the summary is drawn in Section 7.6.

7.1 Introduction

In Chapter 6, IRSs with continuous phase shifts were studied. However, it is difficult to realize continuous phase shifts in practice. Moreover, the complexity and cost will be extremely high, especially when there are a large number of reflecting elements. Thus, it is more practical to consider discrete phase shifts for each reflecting element [142–147]. On the other hand, a system assisted by multiple IRSs is also practical and is worthy of investigation. Hence, in this chapter, we investigate multi-IRS assisted NOMA systems with discrete phase shifts, and the contributions are summarized as follows:

- We characterize the performance of multi-IRS aided NOMA networks with

discrete phase shifts. More specifically, we consider two scenarios based on whether there are direct links between the BS and UDs.

- We adopt the Nakagami- m fading model for the BS-IRS-UD link so that this link can be either LoS or NLoS. Correspondingly, for each scenario, we derive the PDF and CDF of the ordered channel gain for the channel gain near 0 by utilizing the LT.
- We derive the high-SNR asymptotic expressions for the upper and lower bounds of the outage probability for each scenario.
- To gain further insight, we obtain the diversity order for each scenario. We demonstrate that discrete phase shifts do not jeopardize diversity order as compared with continuous phase shifts. Simulation results further reveal that a 3-bit resolution for phase shifts is sufficient to achieve near-optimal outage performance.

7.2 System Model

We consider a NOMA network assisted by multiple IRSs with discrete phase shifts. Specifically, there is a single-antenna BS and N IRSs, denoted by $R_1, R_2, \dots, R_N$. All IRSs can be pre-deployed and have a sufficient distance from each other. Therefore, each IRS has a service coverage without overlapping with other IRSs' service coverages, and the interference signals reflected from neighboring IRSs can be neglected due to severe path loss [148]. Within the service area of each IRS, we select a single-antenna UD for this IRS. All N selected UDs, denoted by $U_1, U_2, \dots, U_N$, form a NOMA group, and each UD is assisted by the corresponding IRS to communicate with the BS. Based on the positions of all UDs, we consider two scenarios, Scenario I and Scenario II. More specifically, in Scenario I, all UDs are far away from the BS, and there is no direct link between the BS and each UD as shown

Figure 7.1: The system models of multi-IRS assisted NOMA with discrete phase shifts for two scenarios.

in Fig. 7.1(a), while in Scenario II, all UDs can communicate with the BS directly as shown in Fig. 7.1(b). Each IRS has K discrete reflecting elements. The reflection-coefficient matrix of R_n , denoted by $\mathbf{\Theta}_n = \text{diag}(\beta_1^n e^{j\theta_1^n}, \beta_2^n e^{j\theta_2^n}, \dots, \beta_K^n e^{j\theta_K^n})$, can be intelligently adjusted, where $\beta_k^n \in [0, 1]$ and θ_k^n are the amplitude-reflection coefficient and phase-shift variable of the k th element, respectively. To be practical, we consider discrete phase shifts which have a b -bit resolution with $L = 2^b$ levels. By uniformly quantizing the interval $[0, 2\pi)$, we have the set of discrete phase-shift variables, $\mathcal{D} = \left\{ \frac{\Delta}{2}, \frac{3\Delta}{2}, \dots, \frac{(2L-1)\Delta}{2} \right\}$, where $\Delta = 2\pi/L$.

Remark 7.1. In the system model, each IRS serves a UD in the NOMA group. It can also simultaneously serve other UDs that are not in this NOMA group within the service coverage. Each IRS's parameters are optimized for the UD in the NOMA group, i.e., the prioritized UD. For other UDs served by this IRS, it is a scenario of random phase shifts, and the corresponding performance has been analyzed [149]. Hence, in this chapter, we only analyze the performance of

prioritized UDs. Moreover, since IRSs can significantly improve channel quality, the performance of prioritized UDs can be guaranteed. For each IRS, we can select the UD with the highest quality-of-service requirement to be the prioritized UD.

7.2.1 Channel Model

For both scenarios, all channels experience quasi-static flat fading, and the CSI of all channels is assumed to be perfectly known at the BS. As IRSs can be pre-deployed, both BS-IRS and IRS-UD links can be either LoS or NLoS. To realize this assumption, we adopt the Nakagami- m fading model [154]. The channel between the BS and R_n is denoted by $\mathbf{G}_n \in \mathbb{C}^{1 \times K}$. The channel between R_n and U_n is denoted by $\mathbf{g}_n \in \mathbb{C}^{K \times 1}$. Particularly, they are $\mathbf{G}_n = [G_1^n, G_2^n, \dots, G_K^n]$ and $\mathbf{g}_n = [g_1^n, g_2^n, \dots, g_K^n]^T$, respectively. All elements in $\mathbf{G}_n$ have a fading parameter of m_G , and all elements in $\mathbf{g}_n$ have a fading parameter of m_g . For Scenario II, besides BS-IRS-UD links, there are direct links between the BS and UDs. The channel between the BS and U_n is denoted by h_n , which also follows the Nakagami- m fading model with a fading parameter of m_h . In particular, it is NLoS for $m_G = 1$ and it is LoS for $m_G > 1$ ($\mathcal{G} \in \{G, g, h\}$).

7.2.2 Signal Model

Scenario I

In Scenario I, there is no direct link between the BS and each UD. Without loss of generality, assume that $|\mathbf{G}_1 \mathbf{\Theta}_1 \mathbf{g}_1| \leq |\mathbf{G}_2 \mathbf{\Theta}_2 \mathbf{g}_2| \leq \dots \leq |\mathbf{G}_N \mathbf{\Theta}_N \mathbf{g}_N|$. The BS broadcasts the signal x which is given by

$$x = \sum_{n=1}^N \sqrt{\alpha_n P} s_n, \quad (7.1)$$

where P is the transmit power, s_n is the signal transmitted to U_n , and α_n is the power allocation coefficient of U_n ($\sum_{n=1}^N \alpha_n = 1$). Note that we assume the fixed power allocation sharing between all UDs and set $\alpha_1 > \alpha_2 > \dots > \alpha_N$ for user fairness [148, 149, 154]. The received signal at U_n is expressed as

$$y_n^1 = \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n x + n_n, \quad (7.2)$$

where n_n is the AWGN at U_n . Based on the SIC principle, U_n needs to detect the signals of U_l ($l \leq n$) in a successive manner. The SINR of detecting U_l 's signal at U_n ($l \leq n < N$ or $l < n = N$) is given by

$$\Psi_{n \leftarrow l}^1 = \frac{|\mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n|^2 \alpha_l}{|\mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n|^2 \sum_{i=l+1}^N \alpha_i + \frac{1}{\rho}}, \quad (7.3)$$

where $\rho = P/\sigma_n^2$. As a special case, when $l = n = N$, the SNR of decoding U_N 's signal at U_N is given by

$$\Psi_{N \leftarrow N}^1 = |\mathbf{G}_N \mathbf{\Theta}_N \mathbf{g}_N|^2 \alpha_N \rho. \quad (7.4)$$

Scenario II

As compared with Scenario I, the difference is that there are direct links between the BS and all UDs in Scenario II. Without loss of generality, we assume that $|h_1 + \mathbf{G}_1 \mathbf{\Theta}_1 \mathbf{g}_1| \leq |h_2 + \mathbf{G}_2 \mathbf{\Theta}_2 \mathbf{g}_2| \leq \dots \leq |h_N + \mathbf{G}_N \mathbf{\Theta}_N \mathbf{g}_N|$. The received signal at U_n is given by

$$y_n^2 = (h_n + \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n)x + n_n. \quad (7.5)$$

The SINR of detecting U_l 's signal at U_n ($l \leq n < N$ or $l < n = N$) is given by

$$\Psi_{n \leftarrow l}^2 = \frac{|h_n + \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n|^2 \alpha_l}{|h_n + \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n|^2 \sum_{i=l+1}^N \alpha_i + \frac{1}{\rho}}. \quad (7.6)$$

For the case of $l = n = N$, the SNR of decoding U_N 's signal at U_N is given by

$$\Psi_{N \leftarrow N}^2 = |h_N + \mathbf{G}_N \mathbf{\Theta}_N \mathbf{g}_N|^2 \alpha_N \rho. \quad (7.7)$$

7.3 Scenario I (Without Direct Link)

In this section, we will analyze the outage performance of Scenario I.

7.3.1 IRS Parameters for Scenario I

We aim to provide the best channel quality to each UD by adjusting the parameters of each IRS. For the BS-IRS- U_n link, it is to maximize $|\mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n| = \left| \sum_{k=1}^K \beta_k^n G_k^n g_k^n e^{j\theta_k^n} \right|$. This can be achieved by intelligently adjusting θ_k^n , the phase-shift variable of each element. The optimal discrete phase-shift variables are derived below.

Lemma 7.1. *Denote the optimal discrete phase-shift variable by $\hat{\theta}_k^{1,n}$. Then, we have*

$$\hat{\theta}_k^{1,n} = \Delta \left(\left\lfloor \frac{\bar{\theta}_k^{1,n}}{\Delta} \right\rfloor + \frac{1}{2} \right), \quad k = 1, 2, \dots, K, \quad (7.8)$$

where $\bar{\theta}_k^{1,n} = \tilde{\theta}_n - \arg(G_k^n g_k^n)$ and $\tilde{\theta}_n$ is an arbitrary constant ranging in $[0, 2\pi)$.

Proof. By adjusting phase-shift variables to make all phases of $G_k^n g_k^n e^{j\theta_k^n}$ to be the same, we can obtain the optimal continuous phase-shift variables. There is more than one solution for $\{\theta_k^n\}$, and the generalized solution is $\bar{\theta}_k^{1,n}$. Since we consider discrete phase shifts, we can adjust the phase-shift variable of each element to be

Figure 7.2: The ratio of the average equivalent channel gain with discrete phase shifts to that with continuous phase shifts when $m_G = 2$, $m_g = 1$, and $\beta = 0.9$ in Scenario I.

close to the optimal continuous phase-shift variable and obtain the optimal discrete phase-shift variable as (7.8). This completes the proof. ■

The equivalent channel gain after adopting optimal continuous phase shifts $\{\bar{\theta}_k^{1,n}\}$ is given by $|\mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n| = \beta \left| \sum_{k=1}^K G_k^n g_k^n e^{j\bar{\theta}_k^{1,n}} \right| = \beta \sum_{k=1}^K |G_k^n| |g_k^n|$, where we assume that $\beta_k^n = \beta$, $\forall n, k$, without loss of generality. For discrete phase shifts, the quantization error at the k th element is given by $\theta_k^{1,n,er} = \hat{\theta}_k^{1,n} - \bar{\theta}_k^{1,n}$, which is uniformly distributed in $[-\frac{\Delta}{2}, \frac{\Delta}{2})$. As such, after adopting the optimal discrete phase-shift variables $\{\hat{\theta}_k^{1,n}\}$, we can obtain the equivalent channel gain as

$$|\mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n| = \beta \left| \sum_{k=1}^K G_k^n g_k^n e^{j\hat{\theta}_k^{1,n}} \right| = \beta \left| \sum_{k=1}^K |G_k^n| |g_k^n| e^{j\theta_k^{1,n,er}} \right|. \quad (7.9)$$

To verify our analysis of the optimal discrete phase shifts, we have conducted Monte Carlo simulations, and the results are shown in Fig. 7.2. It is observed that the ratio of the equivalent channel gain with discrete phase shifts to that with continuous phase shifts increases as the value of b increases, and gradually

approaches 1.

7.3.2 Channel Statistics without Direct Link

The derivation of channel statistics for IRS-aided systems is a challenging task. In this chapter, we take into account the discrete phase shifts, which further increases the difficulty of the derivation. It is proved that the CLT-based channel statistics of the BS-IRS-UD link are inaccurate when the channel gain is near 0 [149]. As we aim to obtain the diversity order, we cannot use the CLT to derive channel statistics. On the other hand, the LT can be used to derive the accurate channel statistics for the channel gain near 0 [149]. Therefore, we directly use it to derive new channel statistics of ordered UD. Consequently, we can gain some insightful results, such as diversity order, although the closed-form expression for the outage probability cannot be derived. The channel statistics for Scenario I are shown in the following lemma.

Lemma 7.2. *Denote that $Y_n = \beta \left| \sum_{k=1}^K |G_k^n| |g_k^n| e^{j\theta_k^{1,n,er}} \right|$ which is the n th channel gain in ascending order. Based on order statistics, when $m_G \neq m_g$ and $b \geq 2$, the CDF of Y_n 's lower bound for $y \rightarrow 0^+$ is given by*

$$F_{Y_n, low}^{0+}(y) = \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_1^{n+i} y^{2m_s K(n+i)}, \quad (7.10)$$

where $a = \beta \cos\left(\frac{\pi}{2^b}\right)$, $m_s = \min\{m_G, m_g\}$, $m_l = \max\{m_G, m_g\}$, $\xi_1 = \zeta_1/a^{2m_s K}$, $\zeta_1 = \frac{(\sqrt{\pi} 4^{m_s - m_l + 1} (m_s m_l)^{m_s} \Gamma(2m_s) \Gamma(2m_l - 2m_s))^K}{2m_s K \Gamma(2m_s K) (\Gamma(m_s) \Gamma(m_l) \Gamma(m_l - m_s + \frac{1}{2}))^K}$, and $\Gamma(\cdot)$ is the gamma function.

Note that when $b \rightarrow \infty$, it is the case of continuous phase shifts. The CDF of Y_n for $y \rightarrow 0^+$ when $b \rightarrow \infty$ is given by

$$F_{Y_n}^{0+, b \rightarrow \infty}(y) = \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_2^{n+i} y^{2m_s K(n+i)}, \quad (7.11)$$

where $\xi_2 = \zeta_1/\beta^{2m_s K}$.

Proof. See Appendix C.1. ■

7.3.3 Performance Analysis of NOMA

The outage probability of U_n under the NOMA scheme is given by

$$\mathbb{P}_n^1 = 1 - \Pr(\Psi_{n \leftarrow 1}^1 \geq \tilde{\gamma}_1, \Psi_{n \leftarrow 2}^1 \geq \tilde{\gamma}_2, \dots, \Psi_{n \leftarrow n}^1 \geq \tilde{\gamma}_n), \quad (7.12)$$

where $\tilde{\gamma}_i = 2^{\tilde{R}_i} - 1$ with $\tilde{R}_i$ being the target rate of U_i . Furthermore, $\mathbb{P}_n^1$ can be transformed into

$$\begin{aligned} \mathbb{P}_n^1 &= 1 - \Pr(Y_n^2 \geq \tilde{\rho}_{n \leftarrow 1}, Y_n^2 \geq \tilde{\rho}_{n \leftarrow 2}, \dots, Y_n^2 \geq \tilde{\rho}_{n \leftarrow n}) \\ &= \Pr(Y_n^2 < \tilde{\rho}_{n, \max}), \end{aligned} \quad (7.13)$$

where $\tilde{\rho}_{n, \max} = \max\{\tilde{\rho}_{n \leftarrow 1}, \tilde{\rho}_{n \leftarrow 2}, \dots, \tilde{\rho}_{n \leftarrow n}\}$ and

$$\tilde{\rho}_{n \leftarrow l} = \begin{cases} \frac{\tilde{\gamma}_l}{\rho(\alpha_l - \tilde{\gamma}_l \sum_{i=l+1}^N \alpha_i)}, & l \leq n < N \text{ or } l < n = N, \\ \frac{\tilde{\gamma}_N}{\rho \alpha_N}, & l = n = N. \end{cases} \quad (7.14)$$

Note that we need to ensure $\alpha_l - \tilde{\gamma}_l \sum_{i=l+1}^N \alpha_i > 0$ for $1 \leq l \leq N - 1$. This is because the ceiling of (7.3) is $\alpha_l / (\sum_{i=l+1}^N \alpha_i)$, which needs to be greater than $\tilde{\gamma}_l$ so that it is possible to implement the SIC successfully.

Based on Lemma 7.2, we can analyze the outage performance in the high-SNR regime as shown in the following proposition.

Proposition 7.1. *Under the NOMA scheme in Scenario I, when $b \geq 2$, the high-SNR asymptotic expressions for the upper and lower bounds of U_n 's outage probability are as follows:*

$$\mathbb{P}_n^{1, up, \infty} = \frac{N!}{(N-n)!n!} \xi_1^n \tilde{\rho}_{n, \max}^{nm_s K}, \quad (7.15)$$

$$\mathbb{P}_n^{1,low,\infty} = \frac{N!}{(N-n)!n!} \xi_2^n \tilde{\rho}_{n,max}^{nm_s K}. \quad (7.16)$$

Note that (7.16) is also the outage probability for the case of continuous shifts.

Proof. For the upper bound, we have

$$\mathbb{P}_n^{1,up,\infty} = F_{Y_n,low}^{0+} \left(\sqrt{\tilde{\rho}_{n,max}} \right) = \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_1^{n+i} \tilde{\rho}_{n,max}^{m_s K(n+i)}. \quad (7.17)$$

Then, by extracting the lowest-order term, we can obtain (7.15).

It is the case of continuous shifts when $b \rightarrow \infty$, and the corresponding outage probability is the lower bound. We have

$$\begin{aligned} \mathbb{P}_n^{1,low,\infty} &= F_{Y_n}^{0+,b \rightarrow \infty} \left(\sqrt{\tilde{\rho}_{n,max}} \right) \\ &= \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_2^{n+i} \tilde{\rho}_{n,max}^{m_s K(n+i)}. \end{aligned} \quad (7.18)$$

Next, after extracting the lowest-order term, we can obtain (7.16). This completes the proof. ■

Corollary 7.1. *In Scenario I, the diversity orders of the n th UD for both discrete and continuous phase shifts under the NOMA scheme are $nm_s K$.*

Proof. Since $\tilde{\rho}_{n,max} \propto \rho^{-1}$, the diversity orders can be obtained based on (7.15) and (7.16). This completes the proof. ■

Remark 7.2. For Scenario I, the proposed system with b -bit discrete phase shifts achieves the same diversity order as that with continuous phase shifts when $b \geq 2$, which implies that the application of discrete phase shifts does not jeopardize the diversity order. The diversity order is related to the number of IRS reflecting elements, the Nakagami fading parameters of the BS-IRS-UD link, and the order of the channel gain.

7.3.4 Performance Analysis of OMA

The comparison between NOMA and OMA has been well investigated in existing works, such as [88–90]. Hence, we provide the results of OMA as a special case of NOMA. We assume that all UDs occupy a resource block under the OMA scheme for fair comparisons, and each UD is allocated $1/N$ resource block [88–90]. Since there is no ordering for the channel gains in the OMA scheme, all UDs have the same outage probability, which is given by

$$\mathbb{P}_{OMA}^1 = \text{P}_r(\Psi_{OMA}^1 < \tilde{\gamma}_{OMA}), \quad (7.19)$$

where $\Psi_{OMA}^1 = \rho |\mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n|^2$ and $\tilde{\gamma}_{OMA} = 2^{N\tilde{R}} - 1$ with $\tilde{R}$ being the target rate of each UD. Furthermore, $\mathbb{P}_{OMA}^1$ can be transformed into

$$\mathbb{P}_{OMA}^1 = \text{P}_r\left(Y^2 < \frac{\tilde{\gamma}_{OMA}}{\rho}\right), \quad (7.20)$$

where Y is the unordered equivalent channel gain.

Proposition 7.2. *Under the OMA scheme in Scenario I, when $b \geq 2$, the high-SNR asymptotic expressions for the upper and lower bounds of U_n 's outage probability are as follows:*

$$\mathbb{P}_{OMA}^{1,up,\infty} = \xi_1 \tilde{\gamma}_{OMA}^{m_s K} \rho^{-m_s K}, \quad (7.21)$$

$$\mathbb{P}_{OMA}^{1,low,\infty} = \xi_2 \tilde{\gamma}_{OMA}^{m_s K} \rho^{-m_s K}. \quad (7.22)$$

Note that (7.22) is also the outage probability for the case of continuous shifts.

Proof. The CDF of Y 's lower bound for $y \rightarrow 0^+$ has been derived as (C.1.10) in the proof of Lemma 7.2. Other steps are similar to the proof of Proposition 7.1. ■

Corollary 7.2. *In Scenario I, the diversity orders of each UD for both discrete*

and continuous phase shifts under the OMA scheme are $m_s K$.

Remark 7.3. For the OMA case of Scenario I, the diversity order with discrete phase shifts is the same as that with continuous phase shifts. The diversity order is related to the number of IRS reflecting elements and the Nakagami fading parameters of the BS-IRS-UD link.

7.4 Scenario II (With Direct Link)

In this section, we will analyze the outage performance of Scenario II. To derive the outage probability, the channel statistics are required. As compared with Scenario I, Scenario II is more complicated to analyze due to the existence of the direct link. Specifically, the equivalent channel gain is the sum of direct and reflected components that do not follow the same distribution, which increases the difficulty of analysis.

7.4.1 IRS Parameters for Scenario II

When there is a direct link between the BS and each UD, we need to maximize the equivalent channel gain between the BS and U_n , i.e., $|h_n + \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n| = \left| h_n + \sum_{k=1}^K \beta_k^n G_k^n g_k^n e^{j\theta_k^n} \right|$. The optimal discrete phase-shift variables are derived below.

Lemma 7.3. *Denote the optimal discrete phase-shift variable by $\hat{\theta}_k^{2,n}$. Then, we have*

$$\hat{\theta}_k^{2,n} = \Delta \left(\left\lfloor \frac{\bar{\theta}_k^{2,n}}{\Delta} \right\rfloor + \frac{1}{2} \right), \quad k = 1, 2, \dots, K, \quad (7.23)$$

where $\bar{\theta}_k^{2,n} = \arg(h_n) - \arg(G_k^n g_k^n)$.

Proof. The optimal continuous phase shifts can be obtained by setting the phases of all $G_k^n g_k^n e^{j\theta_k^n}$ to be the same as h_n . For continuous phase shifts, there is only one

solution that is given by $\bar{\theta}_k^{2,n} = \arg(h_n) - \arg(G_k^n g_k^n)$. After considering discrete phase shifts, we can adjust the phase-shift variable of each element to be close to the optimal continuous phase-shift variable and obtain the optimal discrete phase-shift variable as (7.23). This completes the proof. ■

After adopting the optimal continuous phase shifts $\{\bar{\theta}_k^{2,n}\}$, we have $|h_n + \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n| = |h_n| + \beta \sum_{k=1}^K |G_k^n| |g_k^n|$. For discrete phase shifts, the quantization error is given by $\theta_k^{2,n,er} = \hat{\theta}_k^{2,n} - \bar{\theta}_k^{2,n}$, which is uniformly distributed in $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$. As such, after adopting the optimal discrete phase-shift variables $\{\hat{\theta}_k^{2,n}\}$, we can obtain the equivalent channel gain as

$$\begin{aligned} |h_n + \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n| &= \left| |h_n| e^{j\arg(h_n)} + \beta \sum_{k=1}^K |G_k^n| |g_k^n| e^{j\arg(G_k^n g_k^n)} e^{j\bar{\theta}_k^{2,n}} e^{j\theta_k^{2,n,er}} \right| \\ &= \left| |h_n| + \beta \sum_{k=1}^K |G_k^n| |g_k^n| e^{j\theta_k^{2,n,er}} \right|. \end{aligned} \quad (7.24)$$

Due to the introduction of direct link, Scenario II has a better channel quality than Scenario I, which will result in better performance.

To verify our analysis of the optimal discrete phase shifts, we have conducted Monte Carlo simulations, and the results are shown in Fig. 7.3. It is observed that the ratio of the equivalent channel gain with discrete phase shifts to that with continuous phase shifts increases as the value of b increases, and gradually approaches 1.

7.4.2 Channel Statistics with Direct Link

For Scenario II, the new channel statistics for channel gain near 0 are shown in the following lemma.

Lemma 7.4. *Denote that $Z_n = \left| |h_n| + \beta \sum_{k=1}^K |G_k^n| |g_k^n| e^{j\theta_k^{2,n,er}} \right|$ which is the n th channel gain in the ascending order. Based on order statistics, when $m_G \neq m_g$ and*

Figure 7.3: The ratio of the average equivalent channel gain with discrete phase shifts to that with continuous phase shifts when $m_G = 2$, $m_g = 1$, $m_h = 1$, and $\beta = 0.9$ in Scenario II.

$b \geq 2$, the CDF of Z_n 's lower bound for $z \rightarrow 0^+$ is given by

$$F_{Z_n, low}^{0+}(z) = \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_3^{n+i} z^{(2m_h+2m_sK)(n+i)}, \quad (7.25)$$

where $\xi_3 = \zeta_2/a^{2m_sK}$ and $\zeta_2 = \frac{(2m_h)^{m_h} \Gamma(2m_h) (\sqrt{\pi} 4^{m_s-m_l+1} (m_s m_l)^{m_s} \Gamma(2m_s) \Gamma(2m_l-2m_s))^K}{(2m_h+2m_sK) \Gamma(m_h) \Gamma(2m_h+2m_sK) (\Gamma(m_s) \Gamma(m_l) \Gamma(m_l-m_s+\frac{1}{2}))^K}$.

Note that when $b \rightarrow \infty$, it is the case of continuous phase shifts. The CDF of Z_n for $z \rightarrow 0^+$ when $b \rightarrow \infty$ is given by

$$F_{Z_n}^{0+, b \rightarrow \infty}(z) = \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_4^{n+i} z^{(2m_h+2m_sK)(n+i)}, \quad (7.26)$$

where $\xi_4 = \zeta_2/\beta^{2m_sK}$.

Proof. See Appendix C.2. ■

7.4.3 Performance Analysis of NOMA

In Scenario II, the outage probability of U_n under the NOMA scheme is given by

$$\mathbb{P}_n^2 = 1 - \text{P}_r(\Psi_{n \leftarrow 1}^2 \geq \tilde{\gamma}_1, \Psi_{n \leftarrow 2}^2 \geq \tilde{\gamma}_2, \dots, \Psi_{n \leftarrow n}^2 \geq \tilde{\gamma}_n). \quad (7.27)$$

Furthermore, $\mathbb{P}_n^2$ can be transformed into

$$\begin{aligned} \mathbb{P}_n^2 &= 1 - \text{P}_r(Z_n^2 \geq \tilde{\rho}_{n \leftarrow 1}, Z_n^2 \geq \tilde{\rho}_{n \leftarrow 2}, \dots, Z_n^2 \geq \tilde{\rho}_{n \leftarrow n}) \\ &= \text{P}_r(Z_n^2 < \tilde{\rho}_{n, \max}). \end{aligned} \quad (7.28)$$

Based on Lemma 7.4, we can characterize the outage performance in the high-SNR regime as shown in the following proposition.

Proposition 7.3. *Under the NOMA scheme in Scenario II, when $b \geq 2$, the high-SNR asymptotic expressions for the upper and lower bounds of U_n 's outage probability are as follows:*

$$\mathbb{P}_n^{2, up, \infty} = \frac{N!}{(N-n)!n!} \xi_3^n \tilde{\rho}_{n, \max}^{n(m_h + m_s K)}, \quad (7.29)$$

$$\mathbb{P}_n^{2, low, \infty} = \frac{N!}{(N-n)!n!} \xi_4^n \tilde{\rho}_{n, \max}^{n(m_h + m_s K)}. \quad (7.30)$$

Note that (7.30) is also the outage probability for the case of continuous shifts.

Proof. For the upper bound, we have

$$\begin{aligned} \mathbb{P}_n^{2, up, \infty} &= F_{Z_n, low}^{0+} \left(\sqrt{\tilde{\rho}_{n, \max}} \right) \approx \frac{N!}{(N-n)!(n-1)!} \\ &\quad \times \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_3^{n+i} \tilde{\rho}_{n, \max}^{(m_h + m_s K)(n+i)}. \end{aligned} \quad (7.31)$$

Then, by extracting the lowest-order term, we can derive (7.29).

It is the case of continuous shifts when $b \rightarrow \infty$, and the corresponding outage

probability is the lower bound. We have

$$\begin{aligned} \mathbb{P}_n^{2,low,\infty} &= F_{Z_n}^{0+,b \rightarrow \infty} \left(\sqrt{\tilde{\rho}_{n,max}} \right) = \frac{N!}{(N-n)!(n-1)!} \\ &\times \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \xi_4^{n+i} \tilde{\rho}_{n,max}^{(m_h+m_s K)(n+i)}. \end{aligned} \quad (7.32)$$

Next, after extracting the lowest-order term, we can obtain (7.30). This completes the proof. $\blacksquare$

Corollary 7.3. *In Scenario II, the diversity orders of the n th UD for both discrete and continuous phase shifts under the NOMA scheme are $n(m_h + m_s K)$.*

Proof. Since $\tilde{\rho}_{n,max} \propto \rho^{-1}$, the diversity orders can be obtained based on (7.29) and (7.30). This completes the proof. $\blacksquare$

Remark 7.4. For Scenario II, the proposed system with b -bit discrete phase shifts achieves the same diversity order as that with continuous phase shifts when $b \geq 2$, i.e., the application of discrete phase shifts does not jeopardize the diversity order. The diversity order is related to the number of IRS reflecting elements, the Nakagami fading parameters of the direct and BS-IRS-UD links, and the order of the channel gain.

7.4.4 Performance Analysis of OMA

Since there is no ordering for the channel gains in the OMA scheme, all UDs have the same outage probability. The outage probability of U_n is given by

$$\mathbb{P}_{OMA}^2 = \text{P}_r(\Psi_{OMA}^2 < \tilde{\gamma}_{OMA}), \quad (7.33)$$

where $\Psi_{OMA}^2 = \rho |h_n + \mathbf{G}_n \mathbf{\Theta}_n \mathbf{g}_n|^2$. Furthermore, $\mathbb{P}_{OMA}^2$ can be transformed into

$$\mathbb{P}_{OMA}^2 = \text{P}_r \left(Z^2 < \frac{\tilde{\gamma}_{OMA}}{\rho} \right), \quad (7.34)$$

Table 7.1: Diversity orders of U_n (the n th UD) for all cases

MA scheme	NOMA		OMA	
Phase shifts	Discrete	Continuous	Discrete	Continuous
Scenario I	nm_sK	nm_sK	m_sK	m_sK
Scenario II	$n(m_h + m_sK)$	$n(m_h + m_sK)$	$m_h + m_sK$	$m_h + m_sK$

where Z is the unordered equivalent channel gain.

Proposition 7.4. *Under the OMA scheme in Scenario II, when $b \geq 2$, the high-SNR asymptotic expressions for the upper and lower bounds of U_n 's outage probability are as follows:*

$$\mathbb{P}_{OMA}^{2,up,\infty} = \xi_3 \tilde{\gamma}_{OMA}^{m_h+m_sK} \rho^{-(m_h+m_sK)}, \quad (7.35)$$

$$\mathbb{P}_{OMA}^{2,low,\infty} = \xi_4 \tilde{\gamma}_{OMA}^{m_h+m_sK} \rho^{-(m_h+m_sK)}. \quad (7.36)$$

Note that (7.36) is also the outage probability for the case of continuous shifts.

Proof. The CDF of Z has been derived as (C.2.11) in the proof of Lemma 7.4.

Other steps are similar to the proof of Proposition 7.1. ■

Corollary 7.4. *In Scenario II, the diversity orders of each UD for both discrete and continuous phase shifts under the OMA scheme are $m_h + m_sK$.*

Remark 7.5. For the OMA case of Scenario II, the diversity order with discrete phase shifts is the same as that with continuous phase shifts. The diversity order is related to the number of IRS reflecting elements and the Nakagami fading parameters of the direct and BS-IRS-UD links.

7.4.5 Summary of Diversity Orders

After completing all analyses for both scenarios, all results are summarized in Table 7.1 for ease of reference. First, we observe that the discrete phase shifts can achieve the same diversity order as the continuous phase shifts. On the other hand, discrete

Table 7.2: Setting of parameters

Number of IRSs and UDs	$N = 2, 3, \text{ and } 4$
Number of IRS reflecting elements	$K = 1, 2, 3, 4, \text{ and } 5$
Number of resolution bits	$b = 2, 3, 4, \text{ and } \infty$
Amplitude-reflection coefficient	$\beta = 0.9$
Nakagami fading parameters	$m_G = 2, m_g = 1, \text{ and } m_h = 1$
Target data rates	$\tilde{R}_i = \tilde{R} = 1 \text{ bps/Hz}, i = 1, 2, \dots, N$
Power allocation coefficients	$N = 2: \alpha_1 = 0.9, \alpha_2 = 0.1$ $N = 3: \alpha_1 = 0.7, \alpha_2 = 0.2, \text{ and } \alpha_3 = 0.1$ $N = 4: \alpha_1 = 0.6, \alpha_2 = 0.25, \alpha_3 = 0.1, \text{ and } \alpha_4 = 0.05$

phase shifts have lower hardware complexity and cost, which is the advantage of discrete phase shifts. We also notice that the diversity order is improved with the introduction of direct link. Furthermore, in both scenarios, the lowest diversity order under the NOMA scheme is the same as the diversity order of each UD under the OMA scheme. Hence, NOMA outperforms OMA in terms of diversity order.

7.5 Numerical Results

In this section, numerical results are presented for the performance evaluation of the considered network. Monte Carlo simulations are conducted to verify the accuracy. All N IRSs are pre-deployed so that their service coverages do not overlap, and each IRS selects a UD within its coverage. The parameters are set by referring to some relevant works [88, 154], which are shown in Table 7.2.

7.5.1 Scenario I

In Fig. 7.4, two UDs' outage probabilities versus the transmit SNR in Scenario I when $N = 2$, $K = 2$, and $b = 3$ are plotted. First, we observe that the outage probability of each UD for NOMA gradually approaches the interval between the upper and lower bounds in the high-SNR regime, which are derived from (7.15) and (7.16), respectively. Then, by observing the slopes, the diversity orders of U_1 and U_2 are 2 and 4, respectively, which is consistent with Corollary 7.1. For OMA,

Figure 7.4: Outage probabilities versus SNR in Scenario I when $N = 2$, $K = 2$, $m_G = 2$, $m_g = 1$, $\alpha_1 = 0.9$, $\alpha_2 = 0.1$, and $b = 3$.

the outage probability points are also located between the upper and lower bounds in the high-SNR regime, which are derived from (7.21) and (7.22), respectively. There is only one curve for OMA, as both UDs have the same outage probability statistically. It is observed that the diversity order for OMA is 2, which validates Corollary 7.2. This observation demonstrates the superiority of NOMA over OMA from the diversity order perspective. For comparisons, we regard a multi-FDR-assisted NOMA network as the benchmark. Specifically, FDRs under the classic protocol are deployed at the places of IRSs to help their respective UDs. The FDR works under a realistic assumption that is the same as [114, 148]. Specifically, since the relay is aware of its own transmitted symbol, self-interference cancellation can be applied. We assume that the self-interference cancellation is imperfect and the channel of residual self-interference experiences the Nakagami- m fading. Since the reflection at the IRS is passive without consuming the energy, for a fair comparison, we assume that the transmit power at the BS and the FDR is $P_r = 0.5P$. We observe that the outage probabilities of both UDs for FDR-NOMA

Figure 7.5: Outage probability of U_1 versus SNR for NOMA in Scenario I when $m_G = 2$, $m_g = 1$, and $b = 3$.

are higher than those for IRS-NOMA. Moreover, the outage probability of each UD for FDR-NOMA gradually converges to a floor as the transmit SNR increases, due to the self-interference. These two phenomenons show the disadvantage of FDRs as compared with IRSs.

We further plot the outage probabilities of U_1 with different values of N and K in Fig. 7.5. First, we observe that the outage probability of U_1 gradually approaches the interval between the analytical upper and lower bounds in the high-SNR regime. The observation demonstrates that the diversity orders of U_1 are 2, 3, and 4 when $N = 4$, $K = 2$, $N = 3$, $K = 3$, and $N = 2$, $K = 4$, respectively, which further validates our analytical results.

In Fig. 7.6, the outage probability of U_1 versus the number of IRS reflecting elements in Scenario I is plotted. First, we observe that the outage probability decreases as K increases. Then, we observe that the outage probability for $\rho = 6$ dB is lower than that for $\rho = 3$ dB. Next, we observe that for each specific transmit SNR, the outage probability decreases as b increases. More importantly, when

Figure 7.6: Outage probability of U_1 versus K for NOMA in Scenario I when $\rho = 3$ dB and 6 dB. Here, $N = 3$, $m_G = 2$, $m_g = 1$, $\alpha_1 = 0.7$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$.

$b \geq 3$, the outage probability is close to the outage probability of continuous phase shifts. It reveals that a 3-bit resolution for discrete phase shifts is sufficient to achieve near-optimal performance.

7.5.2 Scenario II

In Fig. 7.7, two UD's outage probabilities versus the transmit SNR in Scenario II when $N = 2$, $K = 2$, and $b = 3$ are plotted. First, we observe that the outage probability of each UD for NOMA gradually approaches the interval between the upper and lower bounds that are derived from (7.29) and (7.30), respectively. Then, by observing the slopes, the diversity orders of U_1 and U_2 are 3 and 6, respectively, which is consistent with Corollary 7.3. For OMA, the outage probability points are also located between the upper and lower bounds in the high-SNR regime, which are derived from (7.35) and (7.36), respectively. There is only one curve for OMA, as both UD's have the same outage probability statistically. It is observed that the diversity order for OMA is 3, which validates Corollary 7.4. This obser-

Figure 7.7: Outage probabilities versus SNR in Scenario II when $N = 2$, $K = 2$, $m_G = 2$, $m_g = 1$, $m_h = 1$, $\alpha_1 = 0.9$, $\alpha_2 = 0.1$, and $b = 3$.

vation demonstrates the superiority of NOMA over OMA from the diversity order perspective.

We further plot the outage probabilities of U_1 with different values of N and K in Fig. 7.8. First, we observe that the outage probability of U_1 gradually approaches the interval between the analytical upper and lower bounds in the high-SNR regime. We observe that the diversity orders of U_1 are 3, 4, and 5 when $N = 4$, $K = 2$, $N = 3$, $K = 3$, and $N = 2$, $K = 4$, respectively, which further validates our analysis. Due to the existence of direct links, it is observed that the scenario with direct link has a larger diversity order than the scenario without direct link.

In Fig. 7.9, the outage probability of U_1 versus the number of IRS reflecting elements in Scenario II is plotted. First, we observe that the outage probability decreases as K increases. Then, we observe that the outage probability for $\rho = 6$ dB is lower than that for $\rho = 3$ dB. Next, we observe that for each specific transmit SNR, the outage probability decreases with the increase of b . Importantly, when

Figure 7.8: Outage probability of U_1 versus SNR for NOMA in Scenario II when $m_G = 2$, $m_g = 1$, $m_h = 1$, and $b = 3$.

Figure 7.9: Outage probability of U_1 versus K for NOMA in Scenario II when $\rho = 3$ dB and 6 dB. Here, $N = 3$, $m_G = 2$, $m_g = 1$, $m_h = 1$, $\alpha_1 = 0.7$, $\alpha_2 = 0.2$, and $\alpha_3 = 0.1$.

$b \geq 3$, the outage probability is close to the outage probability of continuous phase shifts. It is revealed that when direct links are considered, a 3-bit resolution is also sufficient for discrete phase shifts to achieve near-optimal performance.

7.6 Summary

In this chapter, we have studied NOMA systems assisted by multiple IRSs with discrete phase shifts. We have determined the diversity order by deriving the high-SNR asymptotic expressions for the upper and lower bounds of the outage probability. It has been demonstrated that the use of discrete phase shift does not degrade diversity order. Furthermore, it has been shown that a 3-bit resolution is sufficient for discrete phase shifts to achieve the near-optimal performance. Therefore, IRSs with discrete phase shifts strike a balance between system performance and hardware cost.

Chapter 8

Conclusion and Future Work

In this chapter, we conclude the thesis and propose some promising research directions for future work.

8.1 Conclusion

The research in this thesis focuses on the system design and performance analysis related to NOMA. We conclude this thesis by summarizing our main contributions as follows.

- In Chapter 1, we have presented the background, the motivation, main contributions, the literature review, and the outline of this thesis. In Chapter 2, we have provided some fundamental concepts that are useful in later chapters.
- In Chapter 3, we have designed a JUP-SA scheme for multi-carrier NOMA systems. Specifically, we first investigated the impact of user pairing in a single carrier on the diversity order. Then, based on the obtained result, we designed the JUP-SA scheme. Next, we derived the closed-form expression for the outage probability of the worst-performance user in the worst case. Following that, we determined the diversity order based on the asymptotic analysis in the high-SNR regime. It has been demonstrated that the optimal

diversity order can be achieved by using this proposed scheme as compared with the exhaustive search scheme.

- In Chapter 4, we have proposed a two-step scheme for pairing cell-center and cell-edge users for a multi-carrier C-NOMA system, where there are no direct links between the BS and cell-edge users. We derived the closed-form expression for the upper bound of the outage probability and approximated it in the high-SNR regime to obtain the diversity order. It has been revealed that the proposed scheme achieves the optimal performance in terms of the diversity order.
- In Chapters 5, we further considered multi-carrier C-NOMA systems, where all users can communicate with the BS directly. In particular, a pair of users can work in two modes, NC-NOMA and C-NOMA. We have designed an AMS-UPSA scheme to adaptively select the mode, pair users, and allocating subcarriers. Also, we derived the closed-form expression for the upper bound of the outage probability and obtained the diversity order. It has been revealed that the proposed scheme achieves the optimal diversity order.
- In Chapter 6, We have analyzed the performance of downlink and uplink IRS-aided NOMA systems. In particular, we investigated two aspects for each scenario, namely outage probability the ergodic rate. Correspondingly, we derived new channel statistics of the BS-IRS-UD link based on the CLT. Since the CLT-based channel statistics are inaccurate when the channel gain is near 0, we further derived the exact channel statistics for channel gain near 0 by utilizing the LT. Then, we derived the closed-form expressions for the outage probability and ergodic rate for each scenario. To gain further insight, we derived the asymptotic approximations of the outage probability and ergodic rate in the high-SNR regime to obtain the diversity order and high-SNR slope, respectively. We have demonstrated that the number of IRS

reflecting elements and Nakagami- m fading parameters affect the diversity order but have no influence on the high-SNR slope. Finally, we compared the performance of IRS-aided NOMA systems with that of FDR-aided NOMA systems. Simulation results have revealed the superiority of the IRS over FDR in the high-SNR regime.

- In Chapter 7, we have applied multiple IRSs with discrete phase shifts to NOMA systems. To be practical, we considered two scenarios based on whether there are direct links between the BS and UDs. For each scenario, we derived the PDF and CDF of the ordered channel gain by utilizing the LT. Then, we derived the high-SNR asymptotic expressions for the upper and lower bounds of the outage probability for each scenario. To gain further insight, we obtained the diversity order for each scenario. We have demonstrated that discrete phase shifts do not jeopardize diversity order as compared with continuous phase shifts. Simulation results have revealed that a 3-bit resolution for phase shifts is sufficient to achieve near-optimal outage performance.

8.2 Future Work

Our studies have provided a systematic discourse of this newly emerging technology including the transmission scheme design and the performance analysis. There are still some potential directions of NOMA, which are worthy of studying in the future.

8.2.1 Optimal Power Allocation for NOMA

Power allocation is always a crucial problem for NOMA. In Chapters 3 to 7, fixed power allocation is adopted. Optimal power allocation can further enhance the performance of NOMA, which is an interesting research topic.

8.2.2 Imperfect CSI

Channel estimation is important for the application of NOMA. The perfect CSI is assumed in Chapters 3 to 7. Due to various factors such as the imperfect design of channel estimation algorithms, the CSI is imperfect in practice, which is an obstacle to realize the performance gain of NOMA [53]. The related study is a promising direction for future work.

8.2.3 MIMO-NOMA

We have studied SISO-NOMA in Chapters 3 to 7. As MIMO can enhance system performance by providing a spatial degree of freedom, which has been used in practice, MIMO-NOMA is worth investigating in the future. For example, for IRS-aided MIMO-NOMA, the design of active and passive beamforming is a meaningful research topic.

8.2.4 User Mobility

In this thesis, user mobility has not been considered, which affects the resource allocation for NOMA. In the future, we can consider the scenarios taking into account user mobility and conduct the related research.

8.2.5 Other Performance Indicators

In this thesis, we mainly use the performance indicators of outage probability and data rate. For wireless networks, there are other key performance indicators, such as latency, connectivity, spectral efficiency, energy efficiency, and bit error rate, which can be performance metrics for the design of NOMA in the future.

Appendix A

Proof of Proposition in Chapter 5

A.1 Proof of Proposition 5.1

Since both $\mathbb{P}_j^{ub}$ and $\mathbb{P}_i^{ub}$ consist of the function $C_M^j(\tilde{\rho})$, we first approximate the function $C_M^j(\tilde{\rho})$ in the high-SNR regime. By expanding the exponential function, we have

$$C_M^j(\tilde{\rho}) = \frac{M!}{(j-1)!(M-j)!} \sum_{l=0}^{j-1} \binom{j-1}{l} \frac{(-1)^l}{\xi} \left(1 - \sum_{p=0}^{\infty} \frac{(-1)^p \xi^p}{p!} \tilde{\rho}^p \right), \quad (\text{A.1.1})$$

where $\xi = l + M - j + 1$. It is noted that the zero-order coefficient of (A.1.1) is 0 and the p th-order term ($p \geq 1$) of (A.1.1) is

$$-\frac{M!}{(j-1)!(M-j)!} \frac{(-1)^p}{p!} \sum_{l=0}^{j-1} \binom{j-1}{l} (-1)^l \xi^{p-1} \tilde{\rho}^p. \quad (\text{A.1.2})$$

Based on [170, eqs. (0.154.5) and (0.154.6)], we find that

$$\sum_{l=0}^{j-1} \binom{j-1}{l} (-1)^l \xi^{p-1} = \begin{cases} 0, & 1 \leq p \leq j-1, \\ (-1)^{j-1} (j-1)!, & p = j. \end{cases} \quad (\text{A.1.3})$$

Hence, based on (A.1.1), (A.1.2), and (A.1.3), we observe that the coefficient of the p th-order term ($0 \leq p \leq j-1$) in the Taylor series expansion of $C_M^j(\tilde{\rho})$ is 0, and the coefficient of the j th-order term is not 0. Finally, after extracting the j th-order term, we can derive

$$C_M^j(\tilde{\rho}) \approx A_M^j \tilde{\rho}^j, \quad (\text{A.1.4})$$

where

$$A_M^j = \frac{(-1)^{2j} M!}{j(j-1)!(M-j)!}. \quad (\text{A.1.5})$$

Based on Lemma (A.1.4), $C_{M+1}^{j+1}(\tilde{\rho}_{1,m})$, $C_{M+1}^j(\tilde{\rho}_{2,m})$, $C_{M+1}^{i+1}(\tilde{\rho}_1)$, $C_{M+1}^j(\tilde{\rho}_2)$, and $C_{M+1}^{i+1}(\tilde{\rho}_{2,r})$ can be simplified as well. Hence, after substituting these simplified expressions into (5.24) and (5.25), we have

$$\mathbb{P}_j^{ub} \approx (1 - \Psi(i, M)) A_{M+1}^{j+1} \tilde{\rho}_{1,m}^{j+1} + \Psi(i, M) A_{M+1}^j \tilde{\rho}_{2,m}^j, \quad (\text{A.1.6})$$

and

$$\begin{aligned} \mathbb{P}_i^{ub} \approx & (1 - \Psi(i, M)) A_{M+1}^{i+1} \tilde{\rho}_1^{i+1} + \Psi(i, M) \left(A_{M+1}^{i+1} \tilde{\rho}_{2,r}^{i+1} + A_{M+1}^j \tilde{\rho}_2^j \right. \\ & \left. - A_{M+1}^{i+1} A_{M+1}^j \tilde{\rho}_{2,r}^{i+1} \tilde{\rho}_2^j \right). \end{aligned} \quad (\text{A.1.7})$$

Since $\tilde{\rho}_{1,m}$, $\tilde{\rho}_{2,m}$, $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\tilde{\rho}_{2,r} \propto \rho^{-1}$, and $i+1 \leq j$, by extracting the lowest-order term, we can derive high-SNR approximations of $\mathbb{P}_j^{ub}$ and $\mathbb{P}_i^{ub}$ as (5.30) and (5.31), respectively. This completes the proof.

Appendix B

Proofs of Lemmas and Theorems in Chapter 6

B.1 Proof of Lemma 6.1

Based on the property of the Nakagami- m fading model, the expectation and variance of $|\mathcal{G}_k|$ are $\mu_{\mathcal{G}} = \left(\frac{1}{m_{\mathcal{G}}}\right)^{\frac{1}{2}} \frac{\Gamma(m_{\mathcal{G}} + \frac{1}{2})}{\Gamma(m_{\mathcal{G}})}$ and $\text{Var}_{\mathcal{G}} = 1 - \frac{1}{m_{\mathcal{G}}} \left(\frac{\Gamma(m_{\mathcal{G}} + \frac{1}{2})}{\Gamma(m_{\mathcal{G}})}\right)^2$, respectively, where $\mathcal{G} \in \{G, g\}$. Then, the expectation and variance of $|G_k||g_k|$ are $\mu_p = \mu_g \mu_G = \sqrt{\xi}$ and $\text{Var}_p = (\text{Var}_g + \mu_g^2)(\text{Var}_G + \mu_G^2) - \mu_g^2 \mu_G^2 = 1 - \xi$, respectively. Next, since all $|G_k||g_k|$ ($k = 1, 2, \dots, K$) are i.i.d., based on the CLT, $\sum_{k=1}^K |G_k||g_k|$ tends towards a Gaussian distribution as

$$\sum_{k=1}^K |G_k||g_k| \sim \mathcal{N}(K\mu_p, K\text{Var}_p). \quad (\text{B.1.1})$$

Furthermore, we unify the variance and have

$$\frac{\sum_{k=1}^K |G_k||g_k|}{\sqrt{K\text{Var}_p}} \sim \mathcal{N}\left(\frac{\sqrt{K}\mu_p}{\sqrt{\text{Var}_p}}, 1\right). \quad (\text{B.1.2})$$

Hence, $X = \frac{(\sum_{k=1}^K |G_k| |g_k|)^2}{K \text{Var}_p}$ follows a noncentral chi-square distribution $\chi_1'^2(\lambda)$ with a PDF given by

$$f_X(x) = \frac{\lambda^{\frac{1}{4}}}{2} e^{-\frac{x+\lambda}{2}} x^{-\frac{1}{4}} I_{-\frac{1}{2}}(\sqrt{\lambda x}), \quad (\text{B.1.3})$$

where $\lambda = \frac{K \mu_p^2}{\text{Var}_p}$ and

$$I_{-\frac{1}{2}}(\sqrt{\lambda x}) = \left(\frac{\sqrt{\lambda x}}{2} \right)^{-\frac{1}{2}} \sum_{i=0}^{\infty} \frac{\left(\frac{\lambda x}{4} \right)^i}{i! \Gamma(i + \frac{1}{2})}. \quad (\text{B.1.4})$$

The CDF of X is given by

$$F_X(x) = 1 - Q_{\frac{1}{2}}(\sqrt{\lambda}, \sqrt{x}), \quad (\text{B.1.5})$$

where

$$Q_{\frac{1}{2}}(\sqrt{\lambda}, \sqrt{x}) = 1 - e^{-\frac{\lambda}{2}} \sum_{i=0}^{\infty} \left(\frac{\lambda}{2} \right)^i \frac{\gamma(i + \frac{1}{2}, \frac{x}{2})}{i! \Gamma(i + \frac{1}{2})}. \quad (\text{B.1.6})$$

This completes the proof.

B.2 Proof of Lemma 6.2

Denote that $Q_k = |G_k| |g_k|$. According to [176], the PDF of Q_k (the product of two Nakagami- m random variables) is given by

$$f_{Q_k}(q) = \frac{4(m_s m_l)^{\frac{m_s + m_l}{2}}}{\Gamma(m_s) \Gamma(m_l)} q^{m_s + m_l - 1} K_{m_s - m_l}(2\sqrt{m_s m_l} q), \quad (\text{B.2.1})$$

for $q \geq 0$, where $K_v(\cdot)$ is the modified Bessel function of the second kind. The LT of f_{Q_k} is derived as

$$\mathcal{L}_{f_{Q_k}}(s) = \frac{4(m_s m_l)^{\frac{m_s+m_l}{2}}}{\Gamma(m_s)\Gamma(m_l)} \int_0^\infty q^{m_s+m_l-1} e^{-sq} K_{m_s-m_l}(2\sqrt{m_s m_l} q) dq. \quad (\text{B.2.2})$$

Furthermore, by referring to [170, eq. (6.621.3)], we have

$$\begin{aligned} \mathcal{L}_{f_{Q_k}}(s) = & \phi(s + 2\sqrt{m_s m_l})^{-2m_s} \\ & \times F\left(2m_s, m_s - m_l + \frac{1}{2}; m_s + m_l + \frac{1}{2}; \frac{s - 2\sqrt{m_s m_l}}{s + 2\sqrt{m_s m_l}}\right), \end{aligned} \quad (\text{B.2.3})$$

where $\phi = \frac{\sqrt{\pi} 4^{m_s-m_l+1} (m_s m_l)^{m_s} \Gamma(2m_s) \Gamma(2m_l)}{\Gamma(m_s) \Gamma(m_l) \Gamma(m_s+m_l+\frac{1}{2})}$ and $F(\cdot, \cdot; \cdot; \cdot)$ is the hypergeometric series.

When $s \rightarrow \infty$, since $m_s < m_l$ satisfies the condition of [170, eq. (9.122.1)], we have

$$\mathcal{L}_{f_{Q_k}}^\infty(s) = \tilde{m}(s + 2\sqrt{m_s m_l})^{-2m_s}. \quad (\text{B.2.4})$$

Since all Q_k ($k = 1, 2, \dots, K$) are i.i.d., the LT of the PDF of $Z = \sum_{k=1}^K Q_k$ for $s \rightarrow \infty$ is given by

$$\mathcal{L}_{f_Z}^\infty(s) = \prod_{k=1}^K \mathcal{L}_{f_{Q_k}}^\infty(s) = \tilde{m}^K (s + 2\sqrt{m_s m_l})^{-2m_s K}. \quad (\text{B.2.5})$$

Thus, by conducting the inverse LT for (B.2.5), the PDF of Z for $z \rightarrow 0^+$ can be derived as (6.14) based on [170, eq. (17.13.3)]. Following that, according to [170, eq. (3.351.1)], the CDF of Z for $z \rightarrow 0^+$ can be derived as (6.15). This completes the proof.

B.3 Proof of Theorem 6.2

First, we have

$$R_N^d = - \int_0^\infty \log_2(1 + a\alpha_1\rho y) d(1 - F_Y(y)) = \frac{1}{\ln(2)} \int_0^\infty \frac{e^{-y}}{y + \frac{1}{a\alpha_1\rho}} dy. \quad (\text{B.3.1})$$

Then, by referring to [170, eq. (3.352.4)], we can derive (6.29). To obtain R_F^d , we denote that $\tilde{X} = \frac{b\alpha_2 X}{b\alpha_1 \tilde{X} + 1/\rho}$, and the corresponding CDF is given by

$$F_{\tilde{X}}(\tilde{x}) = F_X\left(\frac{\tilde{x}}{b\rho(\alpha_2 - \tilde{x}\alpha_1)}\right) = e^{-\frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i \gamma\left(i + \frac{1}{2}, \frac{\tilde{x}}{2b\rho(\alpha_2 - \tilde{x}\alpha_1)}\right)}{i! 2^i \Gamma\left(i + \frac{1}{2}\right)}. \quad (\text{B.3.2})$$

Hence, we have

$$\begin{aligned} R_F^d &\approx - \int_0^{\tilde{\alpha}} \log_2(1 + \tilde{x}) d(1 - F_{\tilde{X}}(\tilde{x})) = \frac{1}{\ln(2)} \int_0^{\tilde{\alpha}} \frac{1 - F_{\tilde{X}}(\tilde{x})}{1 + \tilde{x}} d\tilde{x} \\ &= \log_2(1 + \tilde{\alpha}) - \frac{1}{\ln(2)} e^{-\frac{\lambda}{2}} \sum_{i=0}^{\infty} \frac{\lambda^i}{i! 2^i \Gamma\left(i + \frac{1}{2}\right)} \underbrace{\int_0^{\tilde{\alpha}} \frac{\gamma\left(i + \frac{1}{2}, \frac{\tilde{x}}{2b\rho(\alpha_2 - \tilde{x}\alpha_1)}\right)}{1 + \tilde{x}} d\tilde{x}}_{J_1}, \end{aligned} \quad (\text{B.3.3})$$

where $\tilde{\alpha} = \alpha_2/\alpha_1$. To further transform J_1 , we denote that $t = 2\tilde{x}/\tilde{\alpha} - 1$. Then, we have

$$J_1 = \int_{-1}^1 \frac{\gamma\left(i + \frac{1}{2}, \frac{\tilde{\alpha}(1+t)}{2b\rho(2\alpha_2 - \alpha_1\tilde{\alpha}(1+t))}\right)}{1 + \frac{2}{\tilde{\alpha}} + t} dt. \quad (\text{B.3.4})$$

Next, by using the Chebyshev-Gauss quadrature, we can approximate J_1 as

$$J_1 \simeq \sum_{l=1}^{u_1} \omega_{1,l} \mathcal{J}_1(t_l). \quad (\text{B.3.5})$$

This completes the proof.

B.4 Proof of Theorem 6.6

First, the ergodic rate of N can be transformed into

$$\begin{aligned} R_N^u &\approx \int_0^\infty \left(- \int_0^\infty \log_2 \left(1 + \frac{ay}{bx + 1/\rho} \right) d(1 - F_Y(y)) \right) f_X(x) dx \\ &= \frac{1}{\ln(2)} \int_0^\infty \left(\int_0^\infty \frac{e^{-y}}{y + \frac{b}{a}x + \frac{1}{a\rho}} dy \right) f_X(x) dx. \end{aligned} \quad (\text{B.4.1})$$

Then, by referring to [170, eq. (3.352.4)], (B.4.1) can be rewritten as

$$\begin{aligned} R_N^u &\approx -\frac{1}{\ln(2)} \int_0^\infty e^{\frac{b}{a}x + \frac{1}{a\rho}} \text{Ei} \left(- \left(\frac{b}{a}x + \frac{1}{a\rho} \right) \right) f_X(x) dx \\ &= -\frac{\lambda^{\frac{1}{4}}}{2\ln(2)} e^{\frac{1}{a\rho} - \frac{\lambda}{2}} \underbrace{\int_0^\infty x^{-\frac{1}{4}} e^{(\frac{b}{a} - \frac{1}{2})x} \text{Ei} \left(-\frac{b}{a}x - \frac{1}{a\rho} \right) I_{-\frac{1}{2}}(\sqrt{\lambda x}) dx}_{J_3}. \end{aligned} \quad (\text{B.4.2})$$

Next, by using the Gauss-Laguerre quadrature, we have

$$J_3 \simeq \sum_{l=1}^{u_3} \omega_{3,l} \mathcal{J}_3(x_{3,l}). \quad (\text{B.4.3})$$

For the ergodic rate of F, it can be expressed as

$$\begin{aligned} R_F^u &\approx \int_0^\infty \log_2(1 + b\rho x) f_X(x) dx \\ &= \frac{\lambda^{\frac{1}{4}}}{2} e^{-\frac{\lambda}{2}} \underbrace{\int_0^\infty x^{-\frac{1}{4}} e^{-\frac{x}{2}} \log_2(1 + b\rho x) I_{-\frac{1}{2}}(\sqrt{\lambda x}) dx}_{J_4}. \end{aligned} \quad (\text{B.4.4})$$

Next, J_4 also can be approximated by adopting the Gauss-Laguerre quadrature.

As such, we have

$$J_4 \simeq \sum_{l=1}^{u_4} \omega_{4,l} \mathcal{J}_4(x_{4,l}). \quad (\text{B.4.5})$$

This completes the proof.

Appendix C

Proofs of Lemmas in Chapter 7

C.1 Proof of Lemma 7.2

Unordered channel gain: First, we need to derive the PDF of the unordered channel gain after adopting the optimal discrete phase shifts $\{\hat{\theta}_k^*\}$,⁴ which is denoted by Y . We have

$$Y = \beta \left| \sum_{k=1}^K |G_k^*| |g_k^*| e^{j\theta_k^{1,*,er}} \right| \geq \beta \sum_{k=1}^K |G_k^*| |g_k^*| \cos(\theta_k^{1,*,er}) \geq a \sum_{k=1}^K |G_k^*| |g_k^*|, \quad (\text{C.1.1})$$

where the equality holds when $b \rightarrow \infty$. Next, we denote $Q = \sum_{k=1}^K Q_k$, where $Q_k = |G_k^*| |g_k^*|$. Since all Q_k ($k = 1, 2, \dots, K$) are i.i.d., according to [176], the PDF of single Q_k (the product of two Nakagami- m random variables) is given by

$$f_{Q_k}(q) = \frac{4(m_s m_l)^{\frac{m_s+m_l}{2}}}{\Gamma(m_s)\Gamma(m_l)} q^{m_s+m_l-1} K_{m_s-m_l}(2\sqrt{m_s m_l} q), \quad (\text{C.1.2})$$

for $q \geq 0$. The LT of $f_{Q_k}(q)$ is derived as

$$\mathcal{L}_{f_{Q_k}}(s) = \frac{4(m_s m_l)^{\frac{m_s+m_l}{2}}}{\Gamma(m_s)\Gamma(m_l)} \int_0^\infty q^{m_s+m_l-1} e^{-sq} K_{m_s-m_l}(2\sqrt{m_s m_l} q) dq. \quad (\text{C.1.3})$$

⁴We use the superscript $*$ instead of n to indicate non-ordering in this section.

Furthermore, by referring to [170, eq. (6.621.3)], we have

$$\begin{aligned} \mathcal{L}_{f_{Q_k}}(s) = & \phi_1 (s + 2\sqrt{m_s m_l})^{-2m_s} \\ & \times F\left(2m_s, m_s - m_l + \frac{1}{2}; m_s + m_l + \frac{1}{2}; \frac{s - 2\sqrt{m_s m_l}}{s + 2\sqrt{m_s m_l}}\right), \end{aligned} \quad (\text{C.1.4})$$

where $\phi_1 = \frac{\sqrt{\pi} 4^{m_s - m_l + 1} (m_s m_l)^{m_s} \Gamma(2m_s) \Gamma(2m_l)}{\Gamma(m_s) \Gamma(m_l) \Gamma(m_s + m_l + \frac{1}{2})}$. When $s \rightarrow \infty$, since $m_s < m_l$ satisfies the condition of [170, eq. (9.122.1)], we have

$$\mathcal{L}_{f_{Q_k}}^\infty(s) = \phi_2 s^{-2m_s}, \quad (\text{C.1.5})$$

where $\phi_2 = \frac{\sqrt{\pi} 4^{m_s - m_l + 1} (m_s m_l)^{m_s} \Gamma(2m_s) \Gamma(2m_l - 2m_s)}{\Gamma(m_s) \Gamma(m_l) \Gamma(m_l - m_s + \frac{1}{2})}$. Since all Q_k are i.i.d., we have

$$\mathcal{L}_{f_Q}^\infty(s) = \prod_{k=1}^K \mathcal{L}_{f_{Q_k}}^\infty(s) = \phi_2^K s^{-2m_s K}. \quad (\text{C.1.6})$$

Thus, by performing the inverse LT for (C.1.6) and referring to [170, eq. (17.13.3)], the PDF of Q for $q \rightarrow 0^+$ can be derived as

$$f_Q^{0+}(q) = \frac{\phi_2^K}{\Gamma(2m_s K)} q^{2m_s K - 1}. \quad (\text{C.1.7})$$

Following that, according to [170, eq. (3.351.1)], the CDF of Q for $q \rightarrow 0^+$ can be derived as

$$F_Q^{0+}(q) = \frac{\phi_2^K}{\Gamma(2m_s K) 2m_s K} q^{2m_s K}. \quad (\text{C.1.8})$$

Finally, the PDF and CDF of Y 's lower bound for $y \rightarrow 0^+$ can be derived as follows:

$$f_{Y,low}^{0+}(y) = \frac{1}{a} f_Q^{0+}\left(\frac{y}{a}\right) = \phi_3 y^{2m_s K - 1}, \quad (\text{C.1.9})$$

$$F_{Y,low}^{0+}(y) = F_Q^{0+}\left(\frac{y}{a}\right) = \xi_1 y^{2m_s K}, \quad (\text{C.1.10})$$

where $\phi_3 = \frac{\phi_2^K}{\Gamma(2m_s K)a^{2m_s K}}$ and $\xi_1 = \frac{\phi_3}{2m_s K}$.

Ordered channel gain: Based on order statistics [168], the CDF of Y_n 's lower bound for $y \rightarrow 0^+$ is given by

$$F_{Y_n, low}^{0+}(y) = \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \left(F_{Y, low}^{0+}(y)\right)^{n+i}. \quad (\text{C.1.11})$$

For the case of $b \rightarrow \infty$, the quantization error $\theta_k^{1,n,er} \rightarrow 0$, and the equality holds in (C.1.1) with $a = \beta \cos\left(\frac{\pi}{2b}\right) \rightarrow \beta$. Therefore, the derived CDF is for Y_n , not Y_n 's lower bound. Furthermore, we can obtain (7.11) after substituting a by β in (7.10). This completes the proof.

C.2 Proof of Lemma 7.4

Unordered channel gain: First, we need to derive the PDF of the unordered channel gain which is denoted by Z . We have⁵

$$Z \geq |h_*| + \beta \sum_{k=1}^K |G_k^*| |g_k^*| \cos(\theta_k^{2,*,er}) \geq |h_*| + a \sum_{k=1}^K |G_k^*| |g_k^*|, \quad (\text{C.2.1})$$

where the equality holds when $b \rightarrow \infty$. Since h_* follows the Nakagami- m fading model, the PDF of $|h_*|$ is given by

$$f_{|h_*|}(x) = \frac{2m_h^{m_h}}{\Gamma(m_h)} x^{2m_h-1} e^{-m_h x^2}. \quad (\text{C.2.2})$$

The LT of $f_{|h_*|}(x)$ is given by

$$\mathcal{L}_{f_{|h_*|}}(s) = \frac{2m_h^{m_h}}{\Gamma(m_h)} \int_0^\infty x^{2m_h-1} e^{-m_h x^2} e^{-sx} dx. \quad (\text{C.2.3})$$

⁵We use the subscript/superscript $*$ instead of n to indicate non-ordering in this section.

Then, by referring to [170, eq. (3.462.1)], we can derive that

$$\mathcal{L}_{f_{|h_*|}}(s) = \frac{\Gamma(2m_h)}{\Gamma(m_h)} e^{\frac{s^2}{8m_h}} D_{-2m_h} \left(\frac{s}{\sqrt{2m_h}} \right), \quad (\text{C.2.4})$$

where $D_p(\cdot)$ is the Parabolic cylinder function. Furthermore, based on $D_p(s) \approx e^{-\frac{s^2}{4}} s^p$ [170, eq. (9.246.1)], we approximate (C.2.4) for $s \rightarrow \infty$ as

$$\mathcal{L}_{f_{|h_*|}}^\infty(s) = \frac{\Gamma(2m_h)(2m_h)^{m_h}}{\Gamma(m_h)} s^{-2m_h}. \quad (\text{C.2.5})$$

On the other hand, we denote $Q' = \sum_{k=1}^K Q'_k$ with $Q'_k = a|G_k^*||g_k^*| = aQ_k$ and have

$$f_{Q'_k}(q') = \frac{1}{a} f_{Q_k} \left(\frac{q'}{a} \right). \quad (\text{C.2.6})$$

Then, the LT of $f_{Q'_k}(q')$ is derived as

$$\mathcal{L}_{f_{Q'_k}}(s) = \mathcal{L}_{f_{Q_k}}(as). \quad (\text{C.2.7})$$

Since all Q'_k are i.i.d., we have

$$\mathcal{L}_{f_{Q'}}(s) = \prod_{k=1}^K \mathcal{L}_{f_{Q'_k}}(s) = \left(\mathcal{L}_{f_{Q_k}}(as) \right)^K. \quad (\text{C.2.8})$$

Furthermore, since $|h_*|$ and Q' are independent, by referring to (C.1.5), we have

$$\begin{aligned} \mathcal{L}_{f_{Z,low}}^\infty(s) &= \mathcal{L}_{f_{|h_*|}}^\infty(s) \mathcal{L}_{f_{Q'}}^\infty(s) = \mathcal{L}_{f_{|h_*|}}^\infty(s) \left(\mathcal{L}_{f_{Q_k}}^\infty(as) \right)^K \\ &= \frac{\Gamma(2m_h)(2m_h)^{m_h} \phi_2^K a^{-2m_s K}}{\Gamma(m_h)} s^{-2m_h - 2m_s K}, \end{aligned} \quad (\text{C.2.9})$$

for $s \rightarrow \infty$. After carrying out the inverse LT, we can derive the PDF and CDF of Z 's lower bound for $z \rightarrow 0^+$ as follows:

$$f_{Z,low}^{0+}(z) = \phi_5 z^{2m_h+2m_sK-1}, \quad (\text{C.2.10})$$

$$F_{Z,low}^{0+}(z) = \xi_3 z^{2m_h+2m_sK}, \quad (\text{C.2.11})$$

where $\phi_5 = \frac{\Gamma(2m_h)(2m_h)^{m_h} \phi_2^K a^{-2m_sK}}{\Gamma(m_h)\Gamma(2m_h+2m_sK)}$ and $\xi_3 = \frac{\phi_5}{2m_h+2m_sK}$.

Ordered channel gain: Based on order statistics [168], the CDF of Z_n 's lower bound for $z \rightarrow 0^+$ is given by

$$F_{Z_n,low}^{0+}(z) = \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} \binom{N-n}{i} \frac{(-1)^i}{n+i} \left(F_{Z,low}^{0+}(z) \right)^{n+i}. \quad (\text{C.2.12})$$

For the case of $b \rightarrow \infty$, it is similar to the proof of Lemma 7.2, and we can derive (7.26). This completes the proof.

Author's Publications

Journal Papers:

1. **Y. Cheng**, K. H. Li, K. C. Teh, S. Luo, and B. Li, "Two-tier NOMA-based wireless powered communication networks," *IEEE Syst. J.*, major revision.
2. **Y. Cheng**, K. H. Li, Y. Liu, K. C. Teh, and G. K. Karagiannidis, "Non-orthogonal multiple access (NOMA) with multiple intelligent reflecting surfaces," *IEEE Trans. Wireless Commun.*, to be published.
3. **Y. Cheng**, K. H. Li, Y. Liu, K. C. Teh, and H. V. Poor, "Downlink and uplink intelligent reflecting surface aided networks: NOMA and OMA," *IEEE Trans. Wireless Commun.*, to be published.
4. **Y. Cheng**, K. H. Li, K. C. Teh, S. Luo, and W. Wang, "Two-step user pairing for OFDM-based cooperative NOMA systems," *IEEE Commun. Lett.*, vol. 24, no. 4, pp. 903-906, Apr. 2020.
5. **Y. Cheng**, K. H. Li, K. C. Teh, S. Luo, and W. Wang, "Performance analysis of cooperative NOMA systems with adaptive mode selection and subchannel allocation," *IEEE Trans. Veh. Technol.*, vol. 68, no. 11, pp. 10981-10990, Nov. 2019.
6. **Y. Cheng**, K. H. Li, K. C. Teh, and S. Luo, "Joint user pairing and subchannel allocation for multisubchannel multiuser nonorthogonal multiple access

systems,” *IEEE Trans. Veh. Technol.*, vol. 67, no. 9, pp. 8238-8248, Sep. 2018.

Conference Papers:

1. **Y. Cheng**, K. H. Li, Y. Liu, and K. C. Teh, “Outage performance of downlink IRS-assisted NOMA systems,” in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Taipei, Taiwan, China, Dec. 2020, pp. 1-6.
2. **Y. Cheng**, K. H. Li, K. C. Teh, S. Luo, and W. Wang, “Joint user clustering and subcarrier allocation for downlink non-orthogonal multiple access systems,” in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Abu Dhabi, United Arab Emirates, Dec. 2018, pp. 1-6.

Bibliography

- [1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. Soong, and J. C. Zhang, “What will 5G be?” *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.
- [2] M. Vaezi, R. Schober, Z. Ding, and H. V. Poor, “Non-orthogonal multiple access: Common myths and critical questions,” *IEEE Wireless Commun.*, vol. 26, no. 5, pp. 174–180, Oct. 2019.
- [3] K. Yang, N. Yang, N. Ye, M. Jia, Z. Gao, and R. Fan, “Non-orthogonal multiple access: Achieving sustainable future radio access,” *IEEE Commun. Mag.*, vol. 57, no. 2, pp. 116–121, Feb. 2019.
- [4] Y. Liu, H. Xing, C. Pan, A. Nallanathan, M. Elkashlan, and L. Hanzo, “Multiple-antenna-assisted non-orthogonal multiple access,” *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 17–23, Apr. 2018.
- [5] D. Wan, M. Wen, F. Ji, H. Yu, and F. Chen, “Non-orthogonal multiple access for cooperative communications: Challenges, opportunities, and trends,” *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 109–117, Apr. 2018.
- [6] B. Di, L. Song, Y. Li, and Z. Han, “V2X meets NOMA: Non-orthogonal multiple access for 5G-enabled vehicular networks,” *IEEE Wireless Commun.*, vol. 24, no. 6, pp. 14–21, Dec. 2017.
- [7] X. Chen, G. Liu, Z. Ma, X. Zhang, P. Fan, S. Chen, and F. R. Yu, “When full duplex wireless meets non-orthogonal multiple access: Opportunities and challenges,” *IEEE Wireless Commun.*, vol. 26, no. 4, pp. 148–155, Aug. 2019.
- [8] S. E. Elayoubi, S. B. Jemaa, Z. Altman, and A. Galindo-Serrano, “5G RAN slicing for verticals: Enablers and challenges,” *IEEE Commun. Mag.*, vol. 57, no. 1, pp. 28–34, Jan. 2019.
- [9] X. Foukas, G. Patounas, A. Elmokashfi, and M. K. Marina, “Network slicing in 5G: Survey and challenges,” *IEEE Commun. Mag.*, vol. 55, no. 5, pp. 94–100, May 2017.
- [10] P. Popovski, K. F. Trillingsgaard, O. Simeone, and G. Durisi, “5G wireless network slicing for eMBB, URLLC, and mMTC: A communication-theoretic view,” *IEEE Access*, vol. 6, pp. 55 765–55 779, Oct. 2018.

- [11] A. A. Zaidi, R. Baldemair, V. Molés-Cases, N. He, K. Werner, and A. Cedergren, "OFDM numerology design for 5G new radio to support IoT, eMBB, and MBSFN," *IEEE Commun. Standards Mag.*, vol. 2, no. 2, pp. 78–83, Jun. 2018.
- [12] L. Wan, Z. Guo, Y. Wu, W. Bi, J. Yuan, M. Elkashlan, and L. Hanzo, "4G/5G spectrum sharing: Efficient 5G deployment to serve enhanced mobile broadband and internet of things applications," *IEEE Veh. Technol. Mag.*, vol. 13, no. 4, pp. 28–39, Dec. 2018.
- [13] G. Liu, X. Hou, J. Jin, F. Wang, Q. Wang, Y. Hao, Y. Huang, X. Wang, X. Xiao, and A. Deng, "3-D-MIMO with massive antennas paves the way to 5G enhanced mobile broadband: From system design to field trials," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1222–1233, Jun. 2017.
- [14] N. H. Mahmood, M. Lauridsen, G. Berardinelli, D. Catania, and P. Mogensen, "Radio resource management techniques for eMBB and mMTC services in 5G dense small cell scenarios," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Montreal, QC, Canada, Sep. 2016, pp. 1–5.
- [15] S.-Y. Lien, S.-C. Hung, D.-J. Deng, and Y. J. Wang, "Efficient ultra-reliable and low latency communications and massive machine-type communications in 5G new radio," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Singapore, Dec. 2017, pp. 1–7.
- [16] T. Salam, W. U. Rehman, and X. Tao, "Cooperative data aggregation and dynamic resource allocation for massive machine type communication," *IEEE Access*, vol. 6, pp. 4145–4158, Jan. 2018.
- [17] I. Parvez, A. Rahmati, I. Guvenc, A. I. Sarwat, and H. Dai, "A survey on low latency towards 5G: RAN, core network and caching solutions," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 4, pp. 3098–3130, 4th Quarter 2018.
- [18] J. Tang, B. Shim, and T. Q. Quek, "Service multiplexing and revenue maximization in sliced C-RAN incorporated with URLLC and multicast eMBB," *IEEE J. Sel. Areas Commun.*, vol. 37, no. 4, pp. 881–895, Apr. 2019.
- [19] O. Semiari, W. Saad, M. Bennis, and M. Debbah, "Integrated millimeter wave and sub-6 GHz wireless networks: A roadmap for joint mobile broadband and ultra-reliable low-latency communications," *IEEE Wireless Commun.*, vol. 26, no. 2, pp. 109–115, Apr. 2019.
- [20] C.-X. Wang, F. Haider, X. Gao, X.-H. You, Y. Yang, D. Yuan, H. M. Aggoune, H. Haas, S. Fletcher, and E. Hepsaydir, "Cellular architecture and key technologies for 5G wireless communication networks," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 122–130, Feb. 2014.
- [21] Q. Wang, R. Zhang, L.-L. Yang, and L. Hanzo, "Non-orthogonal multiple access: A unified perspective," *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 10–16, Apr. 2018.

-
- [22] W. Wang, Y. Liu, Z. Luo, T. Jiang, Q. Zhang, and A. Nallanathan, "Toward cross-layer design for non-orthogonal multiple access: A quality-of-experience perspective," *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 118–124, Apr. 2018.
 - [23] V. W. Wong, *Key Technologies for 5G Wireless Systems*. Cambridge, UK: Cambridge Univ. Press, 2017.
 - [24] F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski, "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 74–80, Feb. 2014.
 - [25] J. Feng, Z. Feng, Z. Wei, W. Li, and S. Roy, "Optimal base station density in ultra-densification heterogeneous network," in *Proc. IEEE Wireless Commun. and Netw. Conf. (WCNC)*, New Orleans, LA, USA, Mar. 2015, pp. 1452–1457.
 - [26] M. Mezzavilla, M. Zhang, M. Polese, R. Ford, S. Dutta, S. Rangan, and M. Zorzi, "End-to-end simulation of 5G mmWave networks," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 3, pp. 2237–2263, 3rd Quarter 2018.
 - [27] C.-N. Chuah, D. N. C. Tse, J. M. Kahn, and R. A. Valenzuela, "Capacity scaling in MIMO wireless systems under correlated fading," *IEEE Trans. Inf. Theory*, vol. 48, no. 3, pp. 637–650, Mar. 2002.
 - [28] H. Ju and R. Zhang, "Optimal resource allocation in full-duplex wireless-powered communication network," *IEEE Trans. Commun.*, vol. 62, no. 10, pp. 3528–3540, Oct. 2014.
 - [29] K. Doppler, M. Rinne, C. Wijting, C. B. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-advanced networks," *IEEE Commun. Mag.*, vol. 47, no. 12, pp. 42–49, Dec. 2009.
 - [30] S. Rajagopal, R. D. Roberts, and S.-K. Lim, "IEEE 802.15. 7 visible light communication: Modulation schemes and dimming support," *IEEE Commun. Mag.*, vol. 50, no. 3, pp. 72–82, Mar. 2012.
 - [31] A. Checko, H. L. Christiansen, Y. Yan, L. Scolari, G. Kardaras, M. S. Berger, and L. Dittmann, "Cloud RAN for mobile networks — A technology overview," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 1, pp. 405–426, 1st Quarter 2015.
 - [32] L. Dai, B. Wang, Y. Yuan, S. Han, I. Chih-Lin, and Z. Wang, "Non-orthogonal multiple access for 5G: Solutions, challenges, opportunities, and future research trends," *IEEE Commun. Mag.*, vol. 53, no. 9, pp. 74–81, Sep. 2015.
 - [33] M. Xu, F. Ji, M. Wen, and W. Duan, "Novel receiver design for the cooperative relaying system with non-orthogonal multiple access," *IEEE Commun. Lett.*, vol. 20, no. 8, pp. 1679–1682, Aug. 2016.

- [34] Y. Cai, Z. Qin, F. Cui, G. Y. Li, and J. A. McCann, "Modulation and multiple access for 5G networks," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 1, pp. 629–646, 1st Quater 2017.
- [35] F. Zhou, Y. Wu, R. Q. Hu, Y. Wang, and K. K. Wong, "Energy-efficient NOMA enabled heterogeneous cloud radio access networks," *IEEE Netw.*, vol. 32, no. 2, pp. 152–160, Mar. 2018.
- [36] M. Shirvanimoghaddam, M. Dohler, and S. J. Johnson, "Massive non-orthogonal multiple access for cellular IoT: Potentials and limitations," *IEEE Commun. Mag.*, vol. 55, no. 9, pp. 55–61, Sep. 2017.
- [37] Z. Ding, Y. Liu, J. Choi, Q. Sun, M. ElKashlan, I. Chih-Lin, and H. V. Poor, "Application of non-orthogonal multiple access in LTE and 5G networks," *IEEE Commun. Mag.*, vol. 55, no. 2, pp. 185–191, Feb. 2017.
- [38] P. Wang, J. Xiao, and L. Ping, "Comparison of orthogonal and non-orthogonal approaches to future wireless cellular systems," *IEEE Veh. Technol. Mag.*, vol. 1, no. 3, pp. 4–11, Sep. 2006.
- [39] G. Ma, B. Ai, F. Wang, X. Chen, Z. Zhong, Z. Zhao, and H. Guan, "Coded tandem spreading multiple access for massive machine-type communications," *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 75–81, Apr. 2018.
- [40] W. Shin, M. Vaezi, B. Lee, D. J. Love, J. Lee, and H. V. Poor, "Non-orthogonal multiple access in multi-cell networks: Theory, performance, and practical challenges," *IEEE Commun. Mag.*, vol. 55, no. 10, pp. 176–183, Oct. 2017.
- [41] C. Zhong, X. Hu, X. Chen, D. W. K. Ng, and Z. Zhang, "Spatial modulation assisted multi-antenna non-orthogonal multiple access," *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 61–67, Apr. 2018.
- [42] Y. Liu, Z. Qin, Y. Cai, Y. Gao, G. Y. Li, and A. Nallanathan, "UAV communications based on non-orthogonal multiple access," *IEEE Wireless Commun.*, vol. 26, no. 1, pp. 52–57, Feb. 2019.
- [43] L. Song, Y. Li, Z. Ding, and H. V. Poor, "Resource management in non-orthogonal multiple access networks for 5G and beyond," *IEEE Netw.*, vol. 31, no. 4, pp. 8–14, Jul. 2017.
- [44] M. Mohammadi, X. Shi, B. K. Chalise, Z. Ding, H. A. Suraweera, C. Zhong, and J. S. Thompson, "Full-duplex non-orthogonal multiple access for next generation wireless systems," *IEEE Commun. Mag.*, vol. 57, no. 5, pp. 110–116, May 2019.
- [45] B. Di, L. Song, and Y. Li, "Trellis coded modulation for non-orthogonal multiple access systems: Design, challenges, and opportunities," *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 68–74, Apr. 2018.

-
- [46] J. Men, J. Ge, and C. Zhang, "Performance analysis of nonorthogonal multiple access for relaying networks over Nakagami- m fading channels," *IEEE Trans. Veh. Technol.*, vol. 66, no. 2, pp. 1200–1208, Feb. 2017.
 - [47] H. Nikopour and H. Baligh, "Sparse code multiple access," in *Proc. IEEE Int. Symp. Pers., Indoor, and Mobile Radio Commun. (PIMRC)*, London, UK, Sep. 2013, pp. 332–336.
 - [48] M. Taherzadeh, H. Nikopour, A. Bayesteh, and H. Baligh, "SCMA codebook design," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Las Vegas, NV, US, Sep. 2014, pp. 1–5.
 - [49] S. Chen, B. Ren, Q. Gao, S. Kang, S. Sun, and K. Niu, "Pattern division multiple access — A novel nonorthogonal multiple access for fifth-generation radio networks," *IEEE Trans. Veh. Technol.*, vol. 66, no. 4, pp. 3185–3196, Apr. 2017.
 - [50] X. Dai, Z. Zhang, B. Bai, S. Chen, and S. Sun, "Pattern division multiple access: A new multiple access technology for 5G," *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 54–60, Apr. 2018.
 - [51] A.-I. Mohammed, M. A. Imran, R. Tafazolli, and D. Chen, "Performance evaluation of low density spreading multiple access," in *Proc. Int. Wireless Commun. and Mobile Comput. Conf. (IWCMC)*, Limassol, Cyprus, Aug. 2012, pp. 383–388.
 - [52] D. Fang, Y.-C. Huang, Z. Ding, G. Geraci, S.-L. Shieh, and H. Claussen, "Lattice partition multiple access: A new method of downlink non-orthogonal multiuser transmissions," in *Proc. IEEE Global Commun. Conf. (GLOBE-COM)*, Washinton DC, US, Dec. 2016, pp. 1–6.
 - [53] Z. Ding, X. Lei, G. K. Karagiannidis, R. Schober, J. Yuan, and V. K. Bhargava, "A survey on non-orthogonal multiple access for 5G networks: Research challenges and future trends," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 10, pp. 2181–2195, Oct. 2017.
 - [54] J. Montalban, P. Scopelliti, M. Fadda, E. Iradier, C. Desogus, P. Angueira, M. Murrioni, and G. Araniti, "Multimedia multicast services in 5G networks: Subgrouping and non-orthogonal multiple access techniques," *IEEE Commun. Mag.*, vol. 56, no. 3, pp. 91–95, Mar. 2018.
 - [55] L. Lv, J. Chen, Q. Ni, Z. Ding, and H. Jiang, "Cognitive non-orthogonal multiple access with cooperative relaying: A new wireless frontier for 5G spectrum sharing," *IEEE Commun. Mag.*, vol. 56, no. 4, pp. 188–195, Apr. 2018.
 - [56] H. Lee, S. Kim, and J.-H. Lim, "Multiuser superposition transmission (MUST) for LTE-A systems," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Kuala Lumpur, Malaysia, May 2016, pp. 1–6.

- [57] Y. Chen, A. Bayesteh, Y. Wu, B. Ren, S. Kang, S. Sun, Q. Xiong, C. Qian, B. Yu, Z. Ding, S. Wang, S. Han, X. Hou, H. Lin, R. Visoz, and R. Razieh, "Toward the standardization of non-orthogonal multiple access for next generation wireless networks," *IEEE Commun. Mag.*, vol. 56, no. 3, pp. 19–27, Mar. 2018.
- [58] Y. Yuan, Z. Yuan, G. Yu, C.-h. Hwang, P.-k. Liao, A. Li, and K. Takeda, "Non-orthogonal transmission technology in LTE evolution," *IEEE Commun. Mag.*, vol. 54, no. 7, pp. 68–74, Jul. 2016.
- [59] Y. Liu, Z. Qin, and Z. Ding, *Non-Orthogonal Multiple Access for Massive Connectivity*. Cham, Switzerland: Springer, 2020.
- [60] L. Dai, B. Wang, Z. Ding, Z. Wang, S. Chen, and L. Hanzo, "A survey of non-orthogonal multiple access for 5G," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 3, pp. 2294–2323, 3rd Quater 2018.
- [61] F. Fang, H. Zhang, J. Cheng, and V. C. Leung, "Energy-efficient resource allocation for downlink non-orthogonal multiple access network," *IEEE Trans. Commun.*, vol. 64, no. 9, pp. 3722–3732, Sep. 2016.
- [62] Z. Yuan, G. Yu, W. Li, Y. Yuan, X. Wang, and J. Xu, "Multi-user shared access for internet of things," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Nanjing, China, May 2016, pp. 1–5.
- [63] Y. Cao, H. Sun, J. Soriaga, and T. Ji, "Resource spread multiple access - A novel transmission scheme for 5G uplink," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Toronto, ON, Canada, Sep. 2017, pp. 1–5.
- [64] S. Hu, B. Yu, C. Qian, Y. Xiao, Q. Xiong, C. Sun, and Y. Gao, "Nonorthogonal interleave-grid multiple access scheme for industrial internet of things in 5G network," *IEEE Trans. Ind. Informat.*, vol. 14, no. 12, pp. 5436–5446, Dec. 2018.
- [65] K. Kusume, G. Bauch, and W. Utschick, "IDMA vs. CDMA: Analysis and comparison of two multiple access schemes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 78–87, Jan. 2012.
- [66] G. S. Rajan and B. S. Rajan, "A non-orthogonal cooperative multiple access (NCMA) protocol and low ML decoding complexity codes," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Kowloon, Hong Kong, China, Mar. 2007, pp. 885–890.
- [67] Q. Wang, Z. Zhao, D. Miao, Y. Zhang, J. Sun, M. Liu, and Z. Zhong, "Non-orthogonal coded access for contention-based transmission in 5G," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Toronto, ON, Canada, Sep. 2017, pp. 1–6.
- [68] L. Tian, C. Yan, W. Li, Z. Yuan, W. Cao, and Y. Yuan, "On uplink non-orthogonal multiple access for 5G: Opportunities and challenges," *China Commun.*, vol. 14, no. 12, pp. 142–152, Dec. 2017.

-
- [69] J. Zhang, X. Wang, X. Yang, and H. Zhou, "Low density spreading signature vector extension (LDS-SVE) for uplink multiple access," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Toronto, ON, Canada, Sep. 2017, pp. 1–5.
- [70] R. Chrabieh and S. Soliman, "Enhanced frequency domain spreading for WiMedia UWB," in *Proc. Int. Symp. Commun. and Inf. Technol.*, Sydney, NSW, Australia, Oct. 2007, pp. 1247–1251.
- [71] S. D. Sosnin, G. Xiong, D. Chatterjee, and Y. Kwak, "Non-orthogonal multiple access with low code rate spreading and short sequence based spreading," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Toronto, ON, Canada, Sep. 2017, pp. 1–5.
- [72] D. Wan, M. Wen, X. Cheng, S. Mumtaz, and M. Guizani, "A promising non-orthogonal multiple access based networking architecture: Motivation, conception, and evolution," *IEEE Wireless Commun.*, vol. 26, no. 5, pp. 152–159, Oct. 2019.
- [73] R. Kassab, O. Simeone, and P. Popovski, "Coexistence of URLLC and eMBB services in the C-RAN uplink: An information-theoretic study," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*. Abu Dhabi, United Arab Emirates: IEEE, Dec. 2018, pp. 1–6.
- [74] Z. Wu, K. Lu, C. Jiang, and X. Shao, "Comprehensive study and comparison on 5G NOMA schemes," *IEEE Access*, vol. 6, pp. 18 511–18 519, Mar. 2018.
- [75] N. Wang, L. Jiao, A. Alipour-Fanid, M. Dabaghchian, and K. Zeng, "Pilot contamination attack detection for NOMA in 5G mm-wave massive MIMO networks," *IEEE Trans. Inf. Forensics Security*, vol. 15, pp. 1363–1378, Sep. 2019.
- [76] M. Liaqat, K. A. Noordin, T. A. Latef, and K. Dimyati, "Power-domain non orthogonal multiple access (PD-NOMA) in cooperative networks: An overview," *Wireless Netw.*, vol. 26, no. 1, pp. 181–203, Jan. 2020.
- [77] S. R. Islam, N. Avazov, O. A. Dobre, and K.-S. Kwak, "Power-domain non-orthogonal multiple access (NOMA) in 5G systems: Potentials and challenges," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 2, pp. 721–742, 2nd Quater 2017.
- [78] Z. Ding, P. Fan, and H. V. Poor, "Impact of user pairing on 5G nonorthogonal multiple-access downlink transmissions," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6010–6023, Aug. 2016.
- [79] M. F. Hanif, Z. Ding, T. Ratnarajah, and G. K. Karagiannidis, "A minorization-maximization method for optimizing sum rate in the downlink of non-orthogonal multiple access systems," *IEEE Trans. Signal Process.*, vol. 64, no. 1, pp. 76–88, Jan. 2016.

- [80] Y. Zhang, H.-M. Wang, T.-X. Zheng, and Q. Yang, “Energy-efficient transmission design in non-orthogonal multiple access,” *IEEE Trans. Veh. Technol.*, vol. 66, no. 3, pp. 2852–2857, Jun. 2016.
- [81] H. Zhang, F. Fang, J. Cheng, K. Long, W. Wang, and V. C. Leung, “Energy-efficient resource allocation in NOMA heterogeneous networks,” *IEEE Wireless Commun.*, vol. 25, no. 2, pp. 48–53, Apr. 2018.
- [82] Y. Liu, M. ElKashlan, Z. Ding, and G. K. Karagiannidis, “Fairness of user clustering in MIMO non-orthogonal multiple access systems,” *IEEE Commun. Lett.*, vol. 20, no. 7, pp. 1465–1468, Apr. 2016.
- [83] S. Timotheou and I. Krikidis, “Fairness for non-orthogonal multiple access in 5G systems,” *IEEE Signal Process. Lett.*, vol. 22, no. 10, pp. 1647–1651, Mar. 2015.
- [84] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. Hoboken, NJ, USA: Wiley, 2006.
- [85] N. Otao, Y. Kishiyama, and K. Higuchi, “Performance of non-orthogonal access with SIC in cellular downlink using proportional fair-based resource allocation,” in *Proc. Int. Symp. Wireless Commun. Syst. (ISWCS)*, Paris, France, Aug. 2012, pp. 1–6.
- [86] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, “Non-orthogonal multiple access (NOMA) for cellular future radio access,” in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Dresden, Germany, Jun. 2013, pp. 1–5.
- [87] Y. Saito, A. Benjebbour, Y. Kishiyama, and T. Nakamura, “System-level performance evaluation of downlink non-orthogonal multiple access (NOMA),” in *Proc. IEEE Int. Symp. Pers., Indoor, and Mobile Radio Commun. (PIMRC)*, London, UK, Sep. 2013, pp. 611–615.
- [88] Z. Ding, Z. Yang, P. Fan, and H. V. Poor, “On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users,” *IEEE Signal Process. Lett.*, vol. 21, no. 12, pp. 1501–1505, Dec. 2014.
- [89] J. A. Oviedo and H. R. Sadjadpour, “A fair power allocation approach to NOMA in multi-user SISO systems,” *IEEE Trans. Veh. Technol.*, vol. 66, no. 9, pp. 7974–7985, Sep. 2017.
- [90] P. Xu, Z. Ding, X. Dai, and H. V. Poor, “A new evaluation criterion for non-orthogonal multiple access in 5G software defined networks,” *IEEE Access*, vol. 3, pp. 1633–1639, Sep. 2015.
- [91] Z. Yang, Z. Ding, P. Fan, and G. K. Karagiannidis, “On the performance of non-orthogonal multiple access systems with partial channel information,” *IEEE Trans. Commun.*, vol. 64, no. 2, pp. 654–667, Dec. 2015.

-
- [92] Z. Chen, Z. Ding, X. Dai, and G. K. Karagiannidis, "On the application of quasi-degradation to MISO-NOMA downlink," *IEEE Trans. Signal Process.*, vol. 64, no. 23, pp. 6174–6189, Dec. 2016.
 - [93] K. Higuchi and A. Benjebbour, "Non-orthogonal multiple access (NOMA) with successive interference cancellation for future radio access," *IEICE Trans. Commun.*, vol. 98, no. 3, pp. 403–414, Mar. 2015.
 - [94] K. Higuchi and Y. Kishiyama, "Non-orthogonal access with random beamforming and intra-beam SIC for cellular MIMO downlink," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Las Vegas, NV, USA, Sep. 2013, pp. 1–5.
 - [95] Z. Ding, R. Schober, and H. V. Poor, "A general MIMO framework for NOMA downlink and uplink transmission based on signal alignment," *IEEE Trans. Wireless Commun.*, vol. 15, no. 6, pp. 4438–4454, Jun. 2016.
 - [96] S. Ali, E. Hossain, and D. I. Kim, "Non-orthogonal multiple access (NOMA) for downlink multiuser MIMO systems: User clustering, beamforming, and power allocation," *IEEE Access*, vol. 5, pp. 565–577, Mar. 2017.
 - [97] H. Zhang, B. Wang, C. Jiang, K. Long, A. Nallanathan, V. C. Leung, and H. V. Poor, "Energy efficient dynamic resource optimization in NOMA system," *IEEE Trans. Wireless Commun.*, vol. 17, no. 9, pp. 5671–5683, Sep. 2018.
 - [98] Y. Zhang, H.-M. Wang, Q. Yang, and Z. Ding, "Secrecy sum rate maximization in non-orthogonal multiple access," *IEEE Commun. Lett.*, vol. 20, no. 5, pp. 930–933, May 2016.
 - [99] Y. Liu, Z. Qin, M. ElKashlan, Y. Gao, and L. Hanzo, "Enhancing the physical layer security of non-orthogonal multiple access in large-scale networks," *IEEE Trans. Wireless Commun.*, vol. 16, no. 3, pp. 1656–1672, Mar. 2017.
 - [100] B. He, A. Liu, N. Yang, and V. K. Lau, "On the design of secure non-orthogonal multiple access systems," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 10, pp. 2196–2206, Oct. 2017.
 - [101] Z. Ding, Z. Zhao, M. Peng, and H. V. Poor, "On the spectral efficiency and security enhancements of NOMA assisted multicast-unicast streaming," *IEEE Trans. Commun.*, vol. 65, no. 7, pp. 3151–3163, Jul. 2017.
 - [102] M.-R. Hojeij, J. Farah, C. A. Nour, and C. Douillard, "New optimal and sub-optimal resource allocation techniques for downlink non-orthogonal multiple access," *Wireless Pers. Commun.*, vol. 87, no. 3, pp. 837–867, May 2015.
 - [103] B. Di, S. Bayat, L. Song, and Y. Li, "Radio resource allocation for downlink non-orthogonal multiple access (NOMA) networks using matching theory," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, San Diego, CA, USA, Dec. 2015, pp. 1–6.

- [104] L. Lei, D. Yuan, C. K. Ho, and S. Sun, "Power and channel allocation for non-orthogonal multiple access in 5G systems: Tractability and computation," *IEEE Trans. Wireless Commun.*, vol. 15, no. 12, pp. 8580–8594, Dec. 2016.
- [105] Y. Sun, D. W. K. Ng, Z. Ding, and R. Schober, "Optimal joint power and subcarrier allocation for full-duplex multicarrier non-orthogonal multiple access systems," *IEEE Trans. Commun.*, vol. 65, no. 3, pp. 1077–1091, Mar. 2017.
- [106] J. Zhu, J. Wang, Y. Huang, S. He, X. You, and L. Yang, "On optimal power allocation for downlink non-orthogonal multiple access systems," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 12, pp. 2744–2757, Dec. 2017.
- [107] Z. Q. Al-Abbasi and D. K. So, "Resource allocation in non-orthogonal and hybrid multiple access system with proportional rate constraint," *IEEE Trans. Wireless Commun.*, vol. 16, no. 10, pp. 6309–6320, Oct. 2017.
- [108] Z. Chen and X. Dai, "MED precoding for multiuser MIMO-NOMA downlink transmission," *IEEE Trans. Veh. Technol.*, vol. 66, no. 6, pp. 5501–5505, Jun. 2017.
- [109] Z. Ding, H. Dai, and H. V. Poor, "Relay selection for cooperative NOMA," *IEEE Wireless Commun. Lett.*, vol. 5, no. 4, pp. 416–419, Jun. 2016.
- [110] Y. Zhou, V. W. Wong, and R. Schober, "Dynamic decode-and-forward based cooperative NOMA with spatially random users," *IEEE Trans. Wireless Commun.*, vol. 17, no. 5, pp. 3340–3356, May 2018.
- [111] Z. Ding, M. Peng, and H. V. Poor, "Cooperative non-orthogonal multiple access in 5G systems," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1462–1465, Jun. 2015.
- [112] J.-B. Kim, I.-H. Lee, and J. Lee, "Capacity scaling for D2D aided cooperative relaying systems using NOMA," *IEEE Wireless Commun. Lett.*, vol. 7, no. 1, pp. 42–45, Sep. 2018.
- [113] Z. Zhang, Z. Ma, M. Xiao, Z. Ding, and P. Fan, "Full-duplex device-to-device-aided cooperative nonorthogonal multiple access," *IEEE Trans. Veh. Technol.*, vol. 66, no. 5, pp. 4467–4471, Aug. 2016.
- [114] C. Zhong and Z. Zhang, "Non-orthogonal multiple access with cooperative full-duplex relaying," *IEEE Commun. Lett.*, vol. 20, no. 12, pp. 2478–2481, Dec. 2016.
- [115] L. Zhang, J. Liu, M. Xiao, G. Wu, Y.-C. Liang, and S. Li, "Performance analysis and optimization in downlink NOMA systems with cooperative full-duplex relaying," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 10, pp. 2398–2412, Jul. 2017.

-
- [116] P. Xu, Z. Yang, Z. Ding, and Z. Zhang, "Optimal relay selection schemes for cooperative NOMA," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7851–7855, Aug. 2018.
 - [117] H. Liu, Z. Ding, K. J. Kim, K. S. Kwak, and H. V. Poor, "Decode-and-forward relaying for cooperative NOMA systems with direct links," *IEEE Trans. Wireless Commun.*, vol. 17, no. 12, pp. 8077–8093, Dec. 2018.
 - [118] G. Li, D. Mishra, and H. Jiang, "Cooperative NOMA with incremental relaying: Performance analysis and optimization," *IEEE Trans. Veh. Technol.*, vol. 67, no. 11, pp. 11 291–11 295, Nov. 2018.
 - [119] Z. Zhang, Z. Ma, M. Xiao, Z. Ding, and P. Fan, "Full-duplex device-to-device-aided cooperative nonorthogonal multiple access," *IEEE Trans. Veh. Technol.*, vol. 66, no. 5, pp. 4467–4471, May 2017.
 - [120] T. N. Do, D. B. Da Costa, T. Q. Duong, and B. An, "Improving the performance of cell-edge users in NOMA systems using cooperative relaying," *IEEE Trans. Commun.*, vol. 66, no. 5, pp. 1883–1901, May 2018.
 - [121] E. Basar, "Reconfigurable intelligent surface-based index modulation: A new beyond MIMO paradigm for 6G," *IEEE Trans. Commun.*, vol. 68, no. 5, pp. 3187–3196, May 2020.
 - [122] W. Tang, M. Z. Chen, X. Chen, J. Y. Dai, Y. Han, M. Di Renzo, Y. Zeng, S. Jin, Q. Cheng, and T. J. Cui, "Wireless communications with reconfigurable intelligent surface: Path loss modeling and experimental measurement," *IEEE Trans. Wireless Commun.*, vol. 20, no. 1, pp. 421–439, Jan. 2021.
 - [123] S. Hu, F. Rusek, and O. Edfors, "Beyond massive MIMO: The potential of positioning with large intelligent surfaces," *IEEE Trans. Signal Process.*, vol. 66, no. 7, pp. 1761–1774, Apr. 2018.
 - [124] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, "Reconfigurable intelligent surfaces for energy efficiency in wireless communication," *IEEE Trans. Wireless Commun.*, vol. 18, no. 8, pp. 4157–4170, Aug. 2019.
 - [125] H. Shen, W. Xu, S. Gong, Z. He, and C. Zhao, "Secrecy rate maximization for intelligent reflecting surface assisted multi-antenna communications," *IEEE Commun. Lett.*, vol. 23, no. 9, pp. 1488–1492, Sep. 2019.
 - [126] G. Zhou, C. Pan, H. Ren, K. Wang, and A. Nallanathan, "A framework of robust transmission design for IRS-aided MISO communications with imperfect cascaded channels," *IEEE Trans. Signal Process.*, vol. 68, pp. 5092–5106, 2020.
 - [127] L. Dong and H.-M. Wang, "Secure MIMO transmission via intelligent reflecting surface," *IEEE Wireless Commun. Lett.*, vol. 9, no. 6, pp. 787–790, Jun. 2020.

- [128] K. Feng, Q. Wang, X. Li, and C.-K. Wen, “Deep reinforcement learning based intelligent reflecting surface optimization for MISO communication systems,” *IEEE Wireless Commun. Lett.*, vol. 9, no. 5, pp. 745–749, May 2020.
- [129] Q. Wu and R. Zhang, “Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network,” *IEEE Commun. Mag.*, vol. 58, no. 1, pp. 106–112, Jan. 2020.
- [130] M. Cui, G. Zhang, and R. Zhang, “Secure wireless communication via intelligent reflecting surface,” *IEEE Wireless Commun. Lett.*, vol. 8, no. 5, pp. 1410–1414, Oct. 2019.
- [131] W. Shi, X. Zhou, L. Jia, Y. Wu, F. Shu, and J. Wang, “Enhanced secure wireless information and power transfer via intelligent reflecting surface,” *IEEE Commun. Lett.*, vol. 25, no. 4, pp. 1084–1088, Apr. 2021.
- [132] Ö. Özdogan, E. Björnson, and E. G. Larsson, “Intelligent reflecting surfaces: Physics, propagation, and pathloss modeling,” *IEEE Wireless Commun. Lett.*, vol. 9, no. 5, pp. 581–585, May 2020.
- [133] D. Mishra and H. Johansson, “Channel estimation and low-complexity beamforming design for passive intelligent surface assisted MISO wireless energy transfer,” in *Proc. IEEE Int. Conf. Acoust., Speech and Signal Process. (ICASSP)*, Brighton, UK, May 2019, pp. 4659–4663.
- [134] Z.-Q. He and X. Yuan, “Cascaded channel estimation for large intelligent metasurface assisted massive MIMO,” *IEEE Wireless Commun. Lett.*, vol. 9, no. 2, pp. 210–214, Feb. 2020.
- [135] H. Liu, X. Yuan, and Y.-J. A. Zhang, “Matrix-calibration-based cascaded channel estimation for reconfigurable intelligent surface assisted multiuser MIMO,” *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2621–2636, Nov. 2020.
- [136] Z. Wang, L. Liu, and S. Cui, “Channel estimation for intelligent reflecting surface assisted multiuser communications: Framework, algorithms, and analysis,” *IEEE Trans. Wireless Commun.*, vol. 19, no. 10, pp. 6607–6620, Oct. 2020.
- [137] E. Björnson, Ö. Özdogan, and E. G. Larsson, “Intelligent reflecting surface versus decode-and-forward: How large surfaces are needed to beat relaying?” *IEEE Wireless Commun. Lett.*, vol. 9, no. 2, pp. 244–248, Feb. 2020.
- [138] J. Lyu and R. Zhang, “Spatial throughput characterization for intelligent reflecting surface aided multiuser system,” *IEEE Wireless Commun. Lett.*, vol. 9, no. 6, pp. 834–838, Jun. 2020.
- [139] C. Guo, Y. Cui, F. Yang, and L. Ding, “Outage probability analysis and minimization in intelligent reflecting surface-assisted MISO systems,” *IEEE Commun. Lett.*, vol. 24, no. 7, pp. 1563–1567, Jul. 2020.

-
- [140] X. Yu, D. Xu, and R. Schober, "MISO wireless communication systems via intelligent reflecting surfaces," in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, Changchun, China, Aug. 2019, pp. 735–740.
 - [141] Q. Wu and R. Zhang, "Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming," *IEEE Trans. Wireless Commun.*, vol. 18, no. 11, pp. 5394–5409, Nov. 2019.
 - [142] J. Xu, W. Xu, and A. L. Swindlehurst, "Discrete phase shift design for practical large intelligent surface communication," in *Proc. IEEE Pacific Rim Conf. Commun., Comput. and Signal Process. (PACRIM)*, Victoria, BC, Canada, Aug. 2019, pp. 1–5.
 - [143] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, "Weighted sum-rate maximization for reconfigurable intelligent surface aided wireless networks," *IEEE Trans. Wireless Commun.*, vol. 19, no. 5, pp. 3064–3076, May 2020.
 - [144] C. You, B. Zheng, and R. Zhang, "Channel estimation and passive beamforming for intelligent reflecting surface: Discrete phase shift and progressive refinement," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2604–2620, Nov. 2020.
 - [145] S. Abeywickrama, R. Zhang, Q. Wu, and C. Yuen, "Intelligent reflecting surface: Practical phase shift model and beamforming optimization," *IEEE Trans. Commun.*, vol. 68, no. 9, pp. 5849–5863, Sep. 2020.
 - [146] Q. Wu and R. Zhang, "Beamforming optimization for wireless network aided by intelligent reflecting surface with discrete phase shifts," *IEEE Trans. Commun.*, vol. 68, no. 3, pp. 1838–1851, Mar. 2019.
 - [147] J. Ye, S. Guo, and M.-S. Alouini, "Joint reflecting and precoding designs for SER minimization in reconfigurable intelligent surfaces assisted MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 19, no. 8, pp. 5561–5574, Aug. 2020.
 - [148] Z. Ding and H. V. Poor, "A simple design of IRS-NOMA transmission," *IEEE Commun. Lett.*, vol. 24, no. 5, pp. 1119–1123, May 2020.
 - [149] Z. Ding, R. Schober, and H. V. Poor, "On the impact of phase shifting designs on IRS-NOMA," *IEEE Wireless Commun. Lett.*, vol. 9, no. 10, pp. 1596–1600, Oct. 2020.
 - [150] J. Zhu, Y. Huang, J. Wang, K. Navaie, and Z. Ding, "Power efficient IRS-assisted NOMA," *IEEE Trans. Commun.*, vol. 69, no. 2, pp. 900–913, Feb. 2021.
 - [151] M. Fu, Y. Zhou, and Y. Shi, "Intelligent reflecting surface for downlink non-orthogonal multiple access networks," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Waikoloa, HI, USA, Dec. 2019, pp. 1–6.

- [152] G. Yang, X. Xu, and Y.-C. Liang, "Intelligent reflecting surface assisted non-orthogonal multiple access," in *Proc. IEEE Wireless Commun. and Netw. Conf. (WCNC)*, Seoul, South Korea, May 2020, pp. 1–6.
- [153] X. Mu, Y. Liu, L. Guo, J. Lin, and N. Al-Dhahir, "Exploiting intelligent reflecting surfaces in NOMA networks: Joint beamforming optimization," *IEEE Trans. Wireless Commun.*, vol. 19, no. 10, pp. 6884–6898, Oct. 2020.
- [154] T. Hou, Y. Liu, Z. Song, X. Sun, Y. Chen, and L. Hanzo, "Reconfigurable intelligent surface aided NOMA networks," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2575–2588, Nov. 2020.
- [155] L. Wang, E. Şaşıoğlu, B. Bandemer, and Y.-H. Kim, "A comparison of superposition coding schemes," in *Proc. IEEE Int. Symp. Inf. Theory*, Istanbul, Turkey, Jul. 2013, pp. 2970–2974.
- [156] T. Cover, "Broadcast channels," *IEEE Trans. Inf. Theory*, vol. 18, no. 1, pp. 2–14, Jan. 1972.
- [157] P. Bergmans, "Random coding theorem for broadcast channels with degraded components," *IEEE Trans. Inf. Theory*, vol. 19, no. 2, pp. 197–207, Mar. 1973.
- [158] A. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, vol. 24, no. 1, pp. 60–70, Jan. 1978.
- [159] I. Csiszár and J. Körner, "Broadcast channels with confidential messages," *IEEE Trans. Inf. Theory*, vol. 24, no. 3, pp. 339–348, May 1978.
- [160] T. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, Sep. 1979.
- [161] A. J. Grant, B. Rimoldi, R. L. Urbanke, and P. A. Whiting, "Rate-splitting multiple access for discrete memoryless channels," *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 873–890, Mar. 2001.
- [162] S. Vanka, S. Srinivasa, Z. Gong, P. Vizi, K. Stamatiou, and M. Haenggi, "Superposition coding strategies: Design and experimental evaluation," *IEEE Trans. Wireless Commun.*, vol. 11, no. 7, pp. 2628–2639, May 2012.
- [163] R. Zhang and L. Hanzo, "A unified treatment of superposition coding aided communications: Theory and practice," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 3, pp. 503–520, Jul. 2010.
- [164] P. Patel and J. Holtzman, "Analysis of a simple successive interference cancellation scheme in a DS/CDMA system," *IEEE J. Sel. Areas Commun.*, vol. 12, no. 5, pp. 796–807, Jun. 1994.
- [165] M. Kobayashi, J. Boutros, and G. Caire, "Successive interference cancellation with SISO decoding and EM channel estimation," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 8, pp. 1450–1460, Aug. 2001.

-
- [166] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," in *Proc. URSI Int. Symp. Signals, Syst., and Electron.*, Pisa, Italy, Oct. 1998, pp. 295–300.
 - [167] H. Zhang, D.-K. Zhang, W.-X. Meng, and C. Li, "User pairing algorithm with SIC in non-orthogonal multiple access system," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Kuala Lumpur, Malaysia, May 2016, pp. 1–6.
 - [168] H. A. David and H. N. Nagaraja, *Order Statistics*, 3rd ed. Hoboken, NJ, USA: Wiley, 2004.
 - [169] W. Wang, K. C. Teh, and K. H. Li, "Relay selection for secure successive AF relaying networks with untrusted nodes," *IEEE Trans. Inf. Forensics Security*, vol. 11, no. 11, pp. 2466–2476, Nov. 2016.
 - [170] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Amsterdam, The Netherlands: Elsevier, 2007.
 - [171] W. Wang, K. C. Teh, and K. H. Li, "Generalized relay selection for improved security in cooperative DF relay networks," *IEEE Wireless Commun. Lett.*, vol. 5, no. 1, pp. 28–31, Feb. 2016.
 - [172] Y. Liu, Z. Ding, M. ElKashlan, and H. V. Poor, "Cooperative non-orthogonal multiple access with simultaneous wireless information and power transfer," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 4, pp. 938–953, Apr. 2016.
 - [173] A. Lozano, A. M. Tulino, and S. Verdú, "High-SNR power offset in multi-antenna communication," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4134–4151, Dec. 2005.
 - [174] Y. Liu, Z. Qin, M. ElKashlan, A. Nallanathan, and J. A. McCann, "Non-orthogonal multiple access in large-scale heterogeneous networks," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 12, pp. 2667–2680, Dec. 2017.
 - [175] Z. Wei, L. Yang, D. W. K. Ng, J. Yuan, and L. Hanzo, "On the performance gain of NOMA over OMA in uplink communication systems," *IEEE Trans. Commun.*, vol. 68, no. 1, pp. 536–568, Jan. 2019.
 - [176] N. Bhargav, C. R. N. da Silva, Y. J. Chun, É. J. Leonardo, S. L. Cotton, and M. D. Yacoub, "On the product of two κ - μ random variables and its application to double and composite fading channels," *IEEE Trans. Wireless Commun.*, vol. 17, no. 4, pp. 2457–2470, Apr. 2018.