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**DYNAMICS OF LIGHT-MATTER COUPLED
QUASIPARTICLES**

SUBHASKAR MANDAL

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

2020

**DYNAMICS OF LIGHT-MATTER COUPLED
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SUBHASKAR MANDAL

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for the
degree of Doctor of Philosophy

2020

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me except where otherwise stated in this thesis. The thesis work has not been submitted for a degree or professional qualification to any other university or institution. I declare that this thesis is written by myself and is free of plagiarism and of sufficient grammatical clarity to be examined. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

05 February 2021

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Subhaskar Mandal

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05 February 2021

Date

Asst. Prof. Timothy C. H. Liew

Authorship Attribution Statement

This thesis contains material from four papers published in the following peer-reviewed journals in which I am listed as an author.

Parts of Chapter 2 have been published as S. Mandal, K. Dini, O. V. Kibis , and T. C. H. Liew, “On the possibility of a terahertz light emitting diode based on a dressed quantum well”, Sci. Rep. **9**, 16320 (2019). [DOI:10.1038/s41598-019-52704-6](https://doi.org/10.1038/s41598-019-52704-6).

The contributions of the co-authors are as follows:

- Prof. Liew conceived the project from the discussion with Prof. Kibis.
- I and Prof. Liew formulated the problem and came up with the model.
- I performed the calculations of dressed spectrum, band bending of the quantum well inside a pn heterojunction, matrix elements of the terahertz transitions, rate equations, and the efficiency of the device. Dr. Dini helped me during the calculations of terahertz transitions and rate equations.
- I and Dr. Dini wrote the manuscript draft.
- Prof. Liew and Prof. Kibis edited the manuscript draft and supervised the project.

Parts of Chapter 3 have been published as S. Mandal, T. C. H. Liew, and O. V. Kibis, “Semiconductor quantum well irradiated by a two-mode electromagnetic field as a terahertz emitter”, Phys. Rev. A **97**, 043860 (2018).

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The contributions of the co-authors are as follows:

- Prof. Kibis and Prof. Liew conceived the project and formulated the problem.
- I performed all the numerical calculations in the single and multiple electron model with inputs from Prof. Liew and wrote the manuscript draft.
- Prof. Kibis and Prof. Liew edited the manuscripts draft and supervised the project.

Chapter 4 is published as S. Mandal, R. Ge, and T. C. H. Liew, “Antichiral edge states in an exciton polariton strip”, Phys. Rev. B **99**, 115423 (2019).

[DOI:10.1103/PhysRevB.99.115423](https://doi.org/10.1103/PhysRevB.99.115423).

The contributions of the co-authors are as follows:

- Prof. Liew conceived the project and provided the model.
- I performed the numerical calculations to find out the fluctuation spectrum of the antichiral edge states and robustness of the system in presence of disorder. I and Dr. Ge simulated the polariton propagation along the edge states of the system.
- I and Dr. Ge wrote the manuscript.
- Prof. Liew edited the manuscript and supervised the project.

Chapter 5 is published as S. Mandal, R. Banerjee, and T. C. H. Liew, “One-Way Reflection-Free Exciton- Polariton Spin-Filtering Channel”, Phys. Rev. Applied **12**, 054058 (2019). [DOI:10.1103/PhysRevApplied.12.054058](https://doi.org/10.1103/PhysRevApplied.12.054058).

The contributions of the co-authors are as follows:

- I, Ms. Banerjee and Prof. Liew conceived the project and provided the model.
- I calculated the pump and loss free dispersion of the polaritons in the Topological and spin filtering regime, performed the numerical simulations of the polariton spin filter for a wide and short strip. Ms. Banerjee predicted the parameters for the calculations through a tight-binding calculations. I and Ms. Banerjee performed the simulation of the robust pulse propagation through the system.
- I calculated the dispersion relations for a lattice with different sized micropillars. Ms. Banerjee showed the effect of lifetime on the propagation distance.
- I and Ms. Banerjee wrote the manuscript.
- Prof. Liew edited the manuscript and supervised the project.

05 February 2021

Date

Subhaskar Mandal

Subhaskar Mandal

Abstract

Our world is made of light and matter. While light can travel very fast, it interacts very weakly. On the other hand, matter interacts very strongly, but travels very slowly. The combination of light and matter can give rise to fascinating effects, which light or matter alone cannot show. Dressed electrons and exciton-polaritons are examples of such light-matter coupled systems where the matter is dressed by light to form new quasi particles with unique properties.

In chapters 2 and 3 of the thesis, we investigate the dressed electrons inside a quantum well and predict that such a system can be used to generate terahertz (THz) waves. This is particularly interesting due to the absence of THz bandgaps in state of the art QWs and it is only because of the dressing field that such a band gap opens up.

In chapters 4 and 5 of the thesis, we introduce schemes to obtain robust propagation of exciton-polaritons. Exciton-polaritons are predicted to play an important role in future opto-electronic devices. However, to realize complete devices, robust propagation of polaritons between different components is essential. We predict that the inherent nonlinear nature of the polaritons gives rise to antichiral edge states in a honeycomb lattice strip, where polaritons propagate robustly through the both edges of the strip in the same direction. Next, using the naturally present spin-orbit coupling in the microcavity along with external magnetic field, feedback free robust propagation of polaritons with a particular spin is also predicted. These results are motivated by the recent advance of topological photonics, although we will focus on non-topological systems. We find that in some cases there are trade-offs between having non-trivial or trivial topology in which even topologically trivial systems can partially display desirable effects, more commonly associated to topological systems, while having other advantages such as accessibility in light-matter coupled systems.

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Chapter 1

Introduction

With the advancement of nanofabrication techniques the field of light matter interaction has grown and spread immensely. The main motivation behind studying light matter interactions is to explore the fascinating effects leading to more compact and efficient signal transfer, communications, and many other future applications, which can not be achieved with systems limited to light or matter alone. A common example of a device based on the light matter interaction is the LASER, which stands for Light Amplification by Stimulated Emission of Radiation. A LASER produces coherent light using stimulated emission, a process where cavity photons (quanta of light) interact with the excited electrons, causing the electrons to relax to the ground state by emitting coherent photons. Today, there is a whole new field of research devoted towards the study of the light matter interactions where interactions of photons with electrons, excitons, phonons, etc., are considered.

In the last decade there has been rapid growth of interest in the generation of terahertz radiation because of its widespread practical applications such as cancer diagnosis, security imaging, etc. The lack of terahertz range bandgap in standard semiconductor materials, which are the foundation of most compact optoelectronic devices, makes the realization of terahertz sources harder. However, the electronic

band-structure can be modified if it is subjected to an electromagnetic field (which we call as a dressing field). In Chapters 2 and 3 we deal with such kind of a system, where a quantum well (QW) is subjected to a dressing field. There an electron is bound to a photon and due to this, the property of the QW changes significantly and a bandgap in the THz range appears, which plays an important role in generating THz waves.

When transistors and integrated circuits were first introduced, they consumed a lot of energy. Over the years they were optimized and presently, smaller transistors consume significantly less energy per logic operation. However, transferring information between the different components does not scale down the same way. In fact, the main source of energy loss in an electronic circuit is not due to the logic operations but is in the interconnections. This is where opto-electronics is promising due to their higher interconnect densities [1]. For example, optical fibers have revolutionized signal transfer over long distances. However, transferring signals efficiently in micro-nano scales still remains a challenge.

Exciton-polaritons are quasiparticles arising due to the strong light matter coupling of cavity photons and QW excitons [2]. Here the word “strong” does not signify that the intensity of the light is very high but rather signifies that the light is trapped for a long enough time. Being highly nonlinear, electrically sensitive, and ultrafast, exciton-polaritons have long been considered candidates for optoelectronic information processing. Although many works have developed concepts of logic gates, transistors [3–6], and switches [3, 7, 8], a key outstanding problem in the field lies in how to connect them robustly. In chapters 4 and 5 we present schemes to obtain robust propagation of exciton-polaritons which are in principle scalable and can be used to connect different components of a polariton network.

Figure 1.1: Schematic diagram of a GaAs layer sandwiched between two $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layers. Here x can be any number between 0 and 1. The wider bandgap of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ provides a potential barrier which confines charge carriers at the GaAs layer. If the thickness of the GaAs layer is small compared to the de-Broglie wavelength of the electrons then the structure can be called a quantum well.

1.1 Semiconductor heterostructures

Semiconductor heterostructures have turned out to be one of the most remarkable invention in the history of condensed-matter physics. They are at the heart of almost every modern electronic device, starting from semiconductor transistors used in daily life microprocessors to creating coherent light sources. Typically, they constitute of semiconductor blocks having different bandgaps (see Fig. 1.1). In 1951 Shockley patented the concept of obtaining one-way injection in the form of a wide-gap emitter with a homo-structure based pn junction transistor [11], which laid the foundation for the first heterostructure based transistor with more injection efficiency compared to homo-junction based transistors [12]. Here p represents the positive charge carrier doping concentration in a semiconductor with impurities having additional hole vacancies (e.g. Gallium) and n represents the negative charge carrier doping concentration in a semiconductor with impurities having additional electron vacancies (for e.g. Arsenic).

The field of optoelectronics announced its arrival with the introduction of semiconductor lasers [13]. Later, more works followed including the realization of stim-

Figure 1.2: Schematic diagram of a GaAs based Heterojunction laser device. The active layer is shown with green and the black layers represent electrodes from which charge carriers are injected.

ulated emission from a GaAs based pn junction [14], which laid the footprint of pn junction based light emitting diodes [15,16]. Although, a laser device was successfully realized, the low efficiency of such a device due to both optical and electrical losses limited the operation to low temperatures only. In 1963, Alferov and Kroemer independently came up with the proposal of using a double heterostructure to confine the charge carriers in the active region [17,18]. It was shown theoretically that both recombination and population inversion take place in the active middle layer. The potential barriers also reduced the direct electron (in the conduction band) and hole (in the valance band) currents significantly, which was the main reason behind the high electrical loss in homojunction based lasers. The difference in the permittivity between different layers also ensured that the emitted light was mainly concentrated in the active layer. Based on these principles the

first successful room temperature strip edge emitting laser source was realized (see Fig. 1.2).

1.1.1 Semiconductor quantum wells

Low dimensional systems such as quantum wells (where the motion of the electrons is confined in one direction), quantum wires (where the motion of the electrons is confined in two directions) , and quantum dots (where the motion of the electrons is confined in all three directions) are essential elements for future optoelectronic devices. This thesis mainly focuses on the interaction of a quantum well (QW) with light under different setups.

Optoelectronic properties of a semiconductor are mainly characterized by their band structure. In Fig. 1.3 the band structure of GaAs is shown, which has a bandgap of around 1.42 eV. Since the electrons in the conduction band and holes in the valence band (corresponding to the absence of electrons in the valence band) mostly occupy the states near the bottom of the conduction band and top of the valence band, respectively, the band structure can be approximated as parabolas ignoring other parts of the band structure (see Fig. 1.3(b)). In so-called direct gap semiconductors the bottom of the conduction band and the top of the valence band have the same wave vector. This has a very important consequence in optoelectronic systems. Let us consider an intrinsic (undoped) semiconductor subjected to an electromagnetic field having energy, $\hbar\omega_0 > \varepsilon_g$, where ε_g is the bandgap of the semiconductor. Under this condition an electron from the valence band can absorb a photon and make a transition to the conduction band creating a hole in the valence band. If the initial and final wave vectors of the electron are $\mathbf{k}_i$ and $\mathbf{k}_f$, respectively, then from the conservation of momentum one can write

$$\mathbf{k}_f = \mathbf{k}_i + \mathbf{k}_{ph}, \quad (1.1)$$

Figure 1.3: (a) Band structure of a bulk GaAs semiconductor at temperature 300 K. (b) Schematic diagram of the band structure of a GaAs quantum well under parabolic approximation with the first two valence and conduction sub-bands. Fig. (a) is taken from *Wikipedia*, https://en.wikipedia.org/wiki/Gallium_arsenide#/media/File:Bandstruktur_GaAs_en.svg

where $\mathbf{k}_{\text{ph}}$ is the photon wave vector. The de-Broglie wavelength of electrons is typically on the scale of nanometers, whereas the wavelength for optical photons is of the order of hundreds of nanometers (nm) to micrometers (μm). Due to this, $\mathbf{k}_i \gg \mathbf{k}_{\text{ph}}$ and $\mathbf{k}_f \simeq \mathbf{k}_i$. If the minimum of the conduction band and maximum of the valence band do not have the same $\mathbf{k}$ (such semiconductors are called indirect gap semiconductors), photons alone can not provide the momentum to the electrons to make an optical transition. However, in indirect gap semiconductors electrons can still make transitions from the valence band to the conduction band with the help of phonons.

Under the parabolic approximation, the conduction and valence band for a bulk semiconductor can be expressed as

$$\varepsilon^c = \frac{\varepsilon_g}{2} + \frac{\hbar^2}{2m_e} (k_x^2 + k_y^2 + k_z^2) \quad (1.2)$$

$$\varepsilon^v = -\frac{\varepsilon_g}{2} - \frac{\hbar^2}{2m_h} (k_x^2 + k_y^2 + k_z^2) \quad (1.3)$$

Here m_e and m_h are the effective electron and hole masses, respectively; $k_x, k_y,$

and k_z are the components of the wave vector along the three directions. If the dimension of a semiconductor along the z direction (l_z) is smaller than the de-Broglie wavelength of the electrons ($\lambda_B = 2\pi\hbar/\sqrt{2m_e k_B T}$, where $\hbar$ is Planck's constant, k_B is Boltzmann constant and T is the temperature), then k_z becomes quantized and the 3D semiconductor effectively becomes a 2D QW where the electrons can not move along the z direction, however they are free to move in the plane of the QW. For a QW, Eq. (1.2–1.3) can be rewritten as

$$\varepsilon^c = \frac{\varepsilon_g}{2} + \frac{n^2\pi^2\hbar^2}{2m_e l_z^2} + \frac{\hbar^2 k^2}{2m_e} \quad (1.4)$$

$$\varepsilon^v = -\frac{\varepsilon_g}{2} - \frac{n^2\pi^2\hbar^2}{2m_h l_z^2} - \frac{\hbar^2 k^2}{2m_h}. \quad (1.5)$$

Here n can be any integer and $k = (k_x, k_y)$ is the in plane wave vector. Due to the quantization of k_z , the valence and conduction bands are divided into subbands (the first two subbands are plotted in Fig. 1.3(b)).

So far we have introduced how the dispersion of the QW looks like. This does not give any idea about the total number of carriers (electrons or holes) available inside the QW. The total number of carriers is controlled by the density of states (DOS), which is defined as the total number of states per unit energy per unit area of the QW. If $\rho(\varepsilon^c)$ is the density of the electronic states in the conduction band and $\rho(\mathbf{k})d\mathbf{k}$ is the total number of states inside the area enclosed by the circles of radius $\mathbf{k}$ and $\mathbf{k} + d\mathbf{k}$ in the conduction band, then

$$\begin{aligned} \rho(\varepsilon^c)d\varepsilon^c &= 2\rho(\mathbf{k})d\mathbf{k} \\ \rho(\varepsilon^c) &= 2 \frac{2\pi k dk}{(2\pi)^2} \frac{1}{d\varepsilon^c} \\ &= \frac{k}{\pi} \left[\frac{d\varepsilon^c}{dk} \right]^{-1} = \frac{m_e}{\pi\hbar^2}. \end{aligned} \quad (1.6)$$

The factor 2 in the first step is introduced to take the spins into account. The DOS

Figure 1.4: Plot of the Fermi Dirac distribution for $T = 0$ and $T \neq 0$ as a function of energy.

for a QW is independent of energy. This depends on the effective dimensionality of the system, e.g.: for a bulk semiconductor (3D), $\rho(\varepsilon^c) \propto \sqrt{\varepsilon^c}$; for a quantum wire (1D), $\rho(\varepsilon^c) \propto 1/\sqrt{\varepsilon^c}$. The concept of the DOS has been used to calculate the terahertz transition rates in Chapter 2.

Another important factor in determining the total number of free carriers inside a semiconductor is the Fermi-Dirac distribution function $f(\varepsilon)$, which is the probability of a state being occupied by a carrier depending upon its energy (ε) and temperature (T):

$$f(\varepsilon) = \frac{1}{1 + \exp\left(\frac{\varepsilon - \varepsilon_F}{k_B T}\right)}. \quad (1.7)$$

Here ε_F is the Fermi energy of the system, which lies inside the bandgap for an intrinsic semiconductor (semiconductors without any impurity). In Fig. 1.4 $f(\varepsilon)$ is plotted for $T = 0$ and $T \neq 0$. For $T = 0$ all the states having energy below ε_F are completely filled and those having energy above ε_F are completely empty. As the temperature increases, states above ε_F have some finite probability to be filled. The total number of electrons (N_e) and holes (N_h) in a semiconductor at a

given temperature T can be expressed as

$$N_e = \int_{E_c^n}^{\infty} f(\varepsilon^c) \rho(\varepsilon^c) d\varepsilon^c \quad (1.8)$$

$$N_h = \int_{-\infty}^{E_c^v} [1 - f(\varepsilon^v)] \rho(\varepsilon^v) d\varepsilon^v, \quad (1.9)$$

where E_c^n and E_c^v are the minimum of the conduction band and the maximum of the valance band, respectively. At $T=0$, for an intrinsic semiconductor $f(\varepsilon^c) = 0$ and $f(\varepsilon^v) = 1$, which results in $N_e = N_h = 0$. However, as discussed in Chapter 2, ε_F can be changed by introducing impurity atoms in the semiconductor (known as doping) that give rise to additional free carriers.

1.1.2 Quantum well excitons

So far we have seen mainly two types of particles inside the semiconductors: the valence band holes and the conduction band electrons. During an optical transition an electron from the valence band absorbs a photon and moves to the conduction band creating a hole in the valence band, conserving both energy and momentum. The attractive Coulomb interaction between the electron and hole creates a new kind of quasiparticle similar to the hydrogen atom known as an exciton. Depending upon their Bohr radius excitons can be of two types: Wannier-Mott excitons or Frenkel excitons. Wannier-Mott excitons have large radius (for example around 12 nm for GaAs) compared to the crystal lattice constant, which makes Coulomb interaction between the electron and hole weak. Due to this reason the binding energy of the Wannier-Mott excitons is low (around 0.01 eV) compared to the bandgap, which makes them unstable given the typical 25meV ($k_B T$) thermal energy at room temperature (except in high bandgap materials, such as ZnO and GaN). On the other hand Frenkel excitons have the Bohr radius of the order of an atomic lattice constant (around 0.5 nm [19]), which ensures large binding

Figure 1.5: Schematic diagram of (a) Wannier-Mott excitons having large Bohr radius and (b) Frenkel excitons with comparatively smaller Bohr radius. (c) The lowest excitonic state in a direct bandgap semiconductor appears with a binding energy (ϵ_B) below the minimum of the conduction band.

energy (typically 0.1 - 1 eV) and stability at room temperature. In Fig. 1.5(a-b) schematic diagrams of the two types of excitons are shown. In this thesis, we focus on Wannier-Mott excitons, which are arguably more studied experimentally given the popularity of GaAs.

The Hamiltonian of a Wannier-Mott exciton can be expressed as

$$\hat{\mathcal{H}} = -\frac{\hbar^2 \nabla_e^2}{2m_e} - \frac{\hbar^2 \nabla_h^2}{2m_h} - \frac{e^2}{4\pi\epsilon_r\epsilon_0|\mathbf{r}_e - \mathbf{r}_h|}. \quad (1.10)$$

Here the first and the second term correspond to the kinetic energies of the electron and hole, respectively; while the third term takes into account the Coulomb attraction between electron and hole with ϵ_0 being the permittivity of vacuum, ϵ_r being the relative permittivity of the material, and e is the elementary charge. The above Hamiltonian can be written in a more compact form by moving into the center of mass frame,

$$\hat{\mathcal{H}} = -\frac{\hbar^2 \nabla_R^2}{2m_{ex}} - \frac{\hbar^2 \nabla_r^2}{2m_r} - \frac{e^2}{4\pi\epsilon_r\epsilon_0|\mathbf{r}|}. \quad (1.11)$$

Here $m_{ex} = m_e + m_h$ is the mass of the exciton, $m_r = m_e m_h / (m_e + m_h)$ is the reduced mass, $\mathbf{R}$ is the center of mass position of the exciton, and $\mathbf{r}$ is the electron-hole relative position:

$$\mathbf{R} = (m_e \mathbf{r}_e + m_h \mathbf{r}_h) / (m_e + m_h), \quad (1.12)$$

$$\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h. \quad (1.13)$$

In Eq. (1.11) the motion of the whole exciton is decided by the first term, whereas separately the rest of the terms determine the bound state wave functions and their corresponding energies [20]. For 2D systems the excitonic wave functions are analogous to the bound states of Hydrogen atoms:

$$\phi^{1s}(r) = \frac{\sqrt{2/\pi}}{r_B} e^{-r/r_B}, \quad (1.14)$$

$$\phi^{2s}(r) = \frac{\sqrt{4/3\pi}}{r_B} \left(1 - \frac{2r}{3r_B}\right) e^{-r/3r_B}, \quad (1.15)$$

$$\phi^{2p}(r, \varphi) = \frac{r\sqrt{4/3\pi}}{(3r_B)^2} e^{-r/3r_B} e^{\pm i\varphi}. \quad (1.16)$$

Here polar coordinate (r, φ) representation is used. The exciton Bohr radius r_B and the ground state binding energy ε_B can be expressed as

$$r_B = \frac{2\pi\hbar^2\epsilon_r\epsilon_0}{m_r e^2}, \quad (1.17)$$

$$\varepsilon_B = \frac{e^4 m_r}{8\pi^2 \hbar^2 \epsilon^2 \epsilon_0^2} \quad (1.18)$$

1.1.3 Bright and dark excitons

The holes in the valence band have p type orbitals, which correspond to the orbital angular quantum number $l = 1$. For typical Zinc Blende bulk semiconductors there exist two hole bands having different effective masses, which are degenerate at the maximum of the valence band. Depending upon their effective masses they are

called heavy holes ($m_{hh} = 0.45 m_o$ for GaAs where m_o is the free electron mass) and light holes ($m_{lh} = 0.062 m_o$ for GaAs). For 2D QWs ignoring the spin-orbit interaction the heavy holes correspond to $m_l = \pm 1$ and light holes correspond to $m_l = 0$, where m_l is the projection of the angular momentum on the z axis. However in reality the spin orbit interaction and the quantum confinement lifts the degeneracy for the hole bands (the bandgap between the heavy and light hole bands becomes around 0.3 meV for GaAs QW). Under the spin orbit interaction the heavy and light holes have total angular momentum $j = 3/2$ and $j = 1/2$, respectively, where $\mathbf{j} = \mathbf{l} + \mathbf{s}$, $\mathbf{s}$ being the spin quantum number. The projection of the total angular momentum on the z axis becomes $m_j = \pm 3/2$ for heavy holes and $m_j = \pm 1/2$ for light holes. Since the DOS of the heavy holes dominates near the maximum of the valence band we will not consider the light holes in this thesis.

Unlike the valence band, the conduction band near the band edge is made of s type orbitals ($l = 0$). The total angular momentum of the electrons becomes $j = 1/2$, which results in the total angular momentum of the excitons as $j_{ex} = \pm 1, \pm 2$ (where excitons of different spins are composed of combinations of electrons and heavy holes with particular spins). Due to the conservation of the angular momentum, excitons with $j_{ex} = \pm 2$ can not couple to light (the photon carries a spin angular momentum of ± 1) and are consequently known as “dark excitons”. On the other hand excitons with $j_{ex} = \pm 1$ are optically active and known as “bright excitons”. Unlike electrons and holes, excitons are bosonic in nature. Non-interacting bosons obey Bose-Einstein statistics (in thermal equilibrium), such that the probability of occupation of a state of energy ε is:

$$f_B(\varepsilon) = \frac{1}{\exp\left(\frac{\varepsilon - \mu}{k_B T}\right) - 1}. \quad (1.19)$$

Here μ is the chemical potential, which can be defined as the amount of energy released while removing one particle from the system. The key difference with the

Fermi-Dirac statistics in Eq. 1.7 is that here $f_B(\varepsilon)$ is not bound from 0 to 1 and can have any value greater than 0. The total number of particles in the system can be expressed as

$$n = \sum_i n_i = \sum_i \frac{1}{\exp\left(\frac{\varepsilon_i - \mu}{k_B T}\right) - 1}, \quad (1.20)$$

where there are n_i particles in energy level ε_i and all the energy levels of the system are assumed to be non-degenerate. The above equation can also be expressed as

$$n = n_0 + \sum_{i>0} n_i = \frac{1}{\exp\left(\frac{\varepsilon_0 - \mu}{k_B T}\right) - 1} + \sum_{i>0} \frac{1}{\exp\left(\frac{\varepsilon_i - \mu}{k_B T}\right) - 1}, \quad (1.21)$$

where n_0 is the particle number at the ground state energy ε_0 . If the chemical potential $\mu \rightarrow \varepsilon_0$, then n_0 diverges leading to Bose-Einstein condensation. At high temperatures relatively more particles occupy the higher energy states but as the temperature decreases particles start to occupy the ground state. Indeed, there exists a critical temperature T_c below which a massive occupation of the particles in the ground state is observed. For a non-interacting Bose gas having mass m and enclosed in a box of volume V , the critical temperature T_c can be expressed as [21]

$$T_c = \frac{2\pi\hbar^2}{mk_B} \left[\frac{n}{2.612V} \right]^{2/3}. \quad (1.22)$$

Since, excitons in semiconductors typically have an effective mass that is a fraction of the free electron mass ($0.22m_0$ in GaAs) and free electrons are considerably lighter than cold atoms, the Bose-Einstein condensation of excitons was reported at a temperature of 0.1K [22], which is considerably higher than in ultracold atom systems.

1.2 Light and matter interacting systems

With the advancement of laser sources, light has turned out to be one of the most useful tools to control various properties of matter. In the limit of strong interaction with matter, where the effect of light can not be treated as a perturbation, the properties of matter change significantly and give rise to many interesting effects. In this thesis we have considered QW electrons and excitons as “matter”. The first part of this section is devoted to the dressed electron (an electron bound with a photon), while in the later part we discuss another quasiparticle known as an exciton-polariton (an exciton bound with a photon).

1.2.1 Dressed electrons

Let us consider a QW having bandgap ε_g . The dispersion of the conduction band and valence band can be expressed as

$$\varepsilon^c(k) = \varepsilon_g/2 + \frac{\hbar^2 k^2}{2m} \quad (1.23)$$

$$\varepsilon^v(k) = -\varepsilon_g/2 - \frac{\hbar^2 k^2}{2m}. \quad (1.24)$$

Here $\mathbf{k} = (k_x, k_y)$ is the in plane wave vector and for simplicity we have kept the same effective mass for the valence band and conduction band (this should not have any qualitative effect on the present discussion, while later in the thesis, where it is important we will account for different masses of electrons and holes). The reference level of the energy is set at the middle of the bandgap. The QW is subjected to a monochromatic linearly polarized electromagnetic wave (which we will call a dressing field) propagating in the z direction having amplitude E_0 and energy ω_0 such that $\hbar\omega_0 > \varepsilon_g$. Under this condition, if the intensity of the dressing field is small, an electron simply absorbs a photon and transits to the conduction band. However if the intensity is strong enough, then the dressing field can not be

Figure 1.6: A two level system subjected to a linearly polarized electromagnetic wave. In the strong coupling limit, the two energy levels get repelled from each other when the energy of the dressing field is smaller than the band gap (shown in a) or they get attracted towards each other when the energy of the dressing field is larger than the band gap (shown in b). The energy levels without the dressing field are shown in black and the modified energy levels are shown in blue.

treated as a perturbation and one must consider the “electron+field” as a whole. In 1955 Autler and Townes showed that an intense dressing field can attract or repel two energy levels depending upon the energy of the dressing field [23]. In Fig. 1.6 the effect of a dressing field on the two level system is shown. This effect is also known as the dynamic Stark effect. In the following we will calculate the renormalized band structure of the QW in the presence of the dressing field.

The Hamiltonian corresponding to the electronic part can be represented in terms of the Pauli matrices as

$$\hat{H}_e = [\varepsilon^c(k) + \varepsilon^v(k)] \hat{I}/2 + [\varepsilon^c(k) - \varepsilon^v(k)] \hat{\sigma}_z/2, \quad (1.25)$$

where $\hat{I}$ is the identity matrix and $\hat{\sigma}_z$ is the z component of the Pauli matrix written in the basis of $[\psi_c(\mathbf{k}), \psi_v(\mathbf{k})]^T$. Here T represents the transpose and $\psi_c(\mathbf{k})$ ($\psi_v(\mathbf{k})$) represents the conduction (valence) band wavefunction. The interaction with the

dressng field under the dipole approximation can be expressed as

$$\hat{H}_{int} = -dE\hat{\sigma}_x, \quad (1.26)$$

where $d = \langle \psi^v | e\mathbf{r} | \psi^c \rangle$ is the interband dipole moment and $\hat{\sigma}_x$ is the x component of the Pauli matrix vector. Since the interaction with an electromagnetic wave is always parity conserving, both the diagonal terms of $\hat{H}_{int}$ are zero, $\langle \psi^c | e\mathbf{r} | \psi^c \rangle = \langle \psi^v | e\mathbf{r} | \psi^v \rangle = 0$. Following the concepts of quantum optics the electric field E can be replaced by the field operator $\hat{E} = i\sqrt{2\pi\hbar\omega_0/V}(\hat{a} - \hat{a}^\dagger)$, where $\hat{a}^\dagger$ and $\hat{a}$ are the photon creation and annihilation operator, respectively, and V is the quantization volume [24, 25]. After substituting $\hat{E}$ in Eq. 1.26 $\hat{H}_{int}$ becomes,

$$\begin{aligned} \hat{H}_{int} &= -id\sqrt{\frac{2\pi\hbar\omega_0}{V}}(\hat{a} - \hat{a}^\dagger)\hat{\sigma}_x \\ &= -id\sqrt{\frac{2\pi\hbar\omega_0}{V}}(\hat{a} - \hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-) \\ &= -id\sqrt{\frac{2\pi\hbar\omega_0}{V}}(\hat{a}\hat{\sigma}_+ - \hat{a}^\dagger\hat{\sigma}_- - \hat{a}^\dagger\hat{\sigma}_+ + \hat{a}\hat{\sigma}_-). \end{aligned} \quad (1.27)$$

Here we have used $\hat{\sigma}_\pm = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$, which corresponds to an electron transition from valence band to conduction band for $\hat{\sigma}_+$ and vice versa for $\hat{\sigma}_-$. The first two terms in Eq. 1.27 represent the electron transition from valence band to conduction band destroying a photon and the electron transition from conduction band to valence band creating a photon, respectively. However, there is no such process associated with the last two terms, which oscillate at a higher frequency $(\omega_0 + (\varepsilon^c(k) - \varepsilon^v(k))/\hbar)$ and average to (almost) zero. We make an approximation by removing the last two terms, which is very common in quantum optics and known as the rotating wave approximation [24, 25]. The total Hamiltonian of

the full system becomes

$$\begin{aligned}
\hat{H} &= \hat{H}_e + \hat{H}_{ph} + \hat{H}_{int} \\
&= [\varepsilon^c(k) + \varepsilon^v(k)] \hat{I}/2 + [\varepsilon^c(k) - \varepsilon^v(k)] \hat{\sigma}_z/2 + \hbar\omega_0 \hat{a}^\dagger \hat{a} \\
&\quad - id\sqrt{\frac{2\pi\hbar\omega_0}{V}} (\hat{a}\hat{\sigma}_+ - \hat{a}^\dagger\hat{\sigma}_-).
\end{aligned} \tag{1.28}$$

The above Hamiltonian is known as the Jaynes-Cummings Hamiltonian and it is very well known in the field of opto-electronics and opto-mechanics. In order to find the renormalized band structure of the QW, we need to diagonalize the above Hamiltonian, which can be done by introducing the joined electron photon space

$$|\psi_c(\mathbf{k}), N\rangle = |\psi_c(\mathbf{k})\rangle \otimes |N\rangle \tag{1.29}$$

$$|\psi_v(\mathbf{k}), N\rangle = |\psi_v(\mathbf{k})\rangle \otimes |N\rangle \tag{1.30}$$

Here $|N\rangle$ represents the state of the dressing field having N photons. The joined electron photon states satisfy the orthonormal conditions as

$$\langle\psi_c(\mathbf{k}), N|\psi_c(\mathbf{k}), N'\rangle = \delta_{N,N'} \text{ and } \langle\psi_c(\mathbf{k}), N|\psi_v(\mathbf{k}), N'\rangle = 0. \tag{1.31}$$

Additionally they satisfy the following relations

$$\begin{aligned}
\hat{\sigma}_+ |\psi_c(\mathbf{k}), N\rangle &= 0, \quad \hat{\sigma}_+ |\psi_v(\mathbf{k}), N\rangle = |\psi_c(\mathbf{k}), N\rangle \\
\hat{\sigma}_- |\psi_c(\mathbf{k}), N\rangle &= |\psi_v(\mathbf{k}), N\rangle, \quad \hat{\sigma}_- |\psi_v(\mathbf{k}), N\rangle = 0 \\
\hat{\sigma}_z |\psi_c(\mathbf{k}), N\rangle &= +|\psi_c(\mathbf{k}), N\rangle, \quad \hat{\sigma}_z |\psi_v(\mathbf{k}), N\rangle = -|\psi_v(\mathbf{k}), N\rangle \\
\hat{a} |\psi_c(\mathbf{k}), N\rangle &= \sqrt{N} |\psi_c(\mathbf{k}), N-1\rangle, \quad \hat{a} |\psi_v(\mathbf{k}), N\rangle = \sqrt{N} |\psi_v(\mathbf{k}), N-1\rangle \\
\hat{a}^\dagger |\psi_c(\mathbf{k}), N\rangle &= \sqrt{N+1} |\psi_c(\mathbf{k}), N+1\rangle, \quad \hat{a}^\dagger |\psi_v(\mathbf{k}), N\rangle = \sqrt{N+1} |\psi_v(\mathbf{k}), N+1\rangle.
\end{aligned} \tag{1.32}$$

Using the the above properties of the joined electron photon states, the Hamiltonian in Eq. (1.28) can be diagonalized analytically. The eigen values representing the two energy branches can be expressed as [26]

$$\varepsilon_N^c(\mathbf{k}) = N\hbar\omega_0 + \frac{\varepsilon^c(\mathbf{k}) + \varepsilon^v(\mathbf{k})}{2} + \frac{\hbar\omega_0}{2} + \frac{\alpha(\mathbf{k})}{2|\alpha(\mathbf{k})|}\hbar\Omega_+(\mathbf{k}) \quad (1.33)$$

$$\varepsilon_N^v(\mathbf{k}) = N\hbar\omega_0 + \frac{\varepsilon^c(\mathbf{k}) + \varepsilon^v(\mathbf{k})}{2} - \frac{\hbar\omega_0}{2} - \frac{\alpha(\mathbf{k})}{2|\alpha(\mathbf{k})|}\hbar\Omega_-(\mathbf{k}), \quad (1.34)$$

where $\alpha(\mathbf{k}) = \varepsilon^c(\mathbf{k}) - \varepsilon^v(\mathbf{k}) - \hbar\omega_0$ and

$\Omega_{\pm}(\mathbf{k}) = \sqrt{8d^2\pi\omega_0(N + 1/2 \pm 1/2)/\hbar V + \alpha^2(\mathbf{k})/\hbar^2}$. As the intensity of the dressing field increases as $N \rightarrow \infty$, the energy branches can be written as $N\hbar\omega_0 + \varepsilon(\mathbf{k})$, where $N\hbar\omega_0$ is the energy corresponding to the dressing field and $\varepsilon(\mathbf{k})$ corresponds to the renormalized bandstructure of the electrons, which can be expressed as

$$\varepsilon(\mathbf{k}) = \frac{\varepsilon^c(\mathbf{k}) + \varepsilon^v(\mathbf{k})}{2} \pm \frac{\hbar\omega_0}{2} \pm \frac{\alpha(\mathbf{k})}{2|\alpha(\mathbf{k})|}\sqrt{(\Delta\varepsilon)^2 + \alpha^2(\mathbf{k})}. \quad (1.35)$$

Here $\Delta\varepsilon = \hbar\Omega_0 = dE_0$ and $E_0 = \sqrt{8\pi N\hbar\omega_0/V}$. In Fig. 1.7 the renormalized band structure of the QW is plotted. At the resonance points of the $\mathbf{k}$ space where the energy of the dressing field equals the energy difference of the conduction and valence band, $\hbar\omega_0 = \varepsilon^c(\mathbf{k}) - \varepsilon^v(\mathbf{k})$, a band gap of amount $\Delta\varepsilon_g$ appears. These are called the Stark gaps, appearing due to the mixing of the valence and conduction bands. As already mentioned, the Stark gap depends on the dipole moment and the intensity of the dressing field

$$\Delta\varepsilon_g = dE_0 = d\sqrt{\frac{2I}{\epsilon_0 c}}, \quad (1.36)$$

where $I = \epsilon_0 c E_0^2/2$ is the intensity of the dressing field, with c being the velocity of light in free space. All the above expressions are valid for the strong coupling regime, which can be characterized by $\Delta\varepsilon_g > \hbar/\tau_e$, where τ_e is the average lifetime

Figure 1.7: The renormalized band structure of the QW under the effect of the dressing field is shown in blue. The band structure of the QW without the effect of the dressing field is shown in red. At the resonance points where $\hbar\omega_0 = \varepsilon^c(\mathbf{k}) - \varepsilon^v(\mathbf{k})$, Stark gaps $\Delta\varepsilon_g$ appear.

of the electronic states taking all the decaying effects into account. Depending upon the quality of the sample, in a typical GaAs QW, τ_e can range from 1 ps - 1 ns [27, 28]. In order to get a meV order Stark gap in GaAs QW, the intensity of the dressing field needs to be around kW/cm². The renormalized electronic states can be calculated by calculating the eigenstates of the Hamiltonian in Eq. (1.28) [26]

$$|\psi_N^c(\mathbf{k})\rangle = \sqrt{\frac{\Omega(\mathbf{k}) + |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_c(\mathbf{k}), N\rangle + i \frac{\alpha(\mathbf{k})}{|\alpha(\mathbf{k})|} \sqrt{\frac{\Omega(\mathbf{k}) - |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_v(\mathbf{k}), N+1\rangle \quad (1.37)$$

$$|\psi_N^v(\mathbf{k})\rangle = \sqrt{\frac{\Omega(\mathbf{k}) + |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_v(\mathbf{k}), N\rangle + i \frac{\alpha(\mathbf{k})}{|\alpha(\mathbf{k})|} \sqrt{\frac{\Omega(\mathbf{k}) - |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_c(\mathbf{k}), N-1\rangle. \quad (1.38)$$

It can be easily checked that by putting $d = 0$, the original (without the dressing field) band structure and electronic states are recovered. In chapters 2 and 3 we will make use of the above results to design terahertz sources of radiation using

single quantum wells.

1.2.2 Microcavities

When two mirrors are separated by some distance, the space between the mirrors forms a cavity. Microcavities, having micron size, aim to confine light with optical wavelengths (several hundreds of nanometers) such that only one or a few half wavelengths cover the whole cavity. In a microcavity the mirrors are made of distributed Bragg reflectors (DBRs). A DBR is a heterostructure of semiconductor layers having alternating refractive index placed periodically such that light within a particular range of wavelengths can not pass through the structure and gets reflected. In Fig. 1.8 (a) a schematic diagram of such a structure is presented. If n_1 is the refractive index of one layer having thickness w_1 and n_2 is the refractive index of the other layer having thickness w_2 , then the condition to obtain highest reflectivity at normal incidence for light with wavelength λ_B is

$$n_1 w_1 = n_2 w_2 = \frac{\lambda_B}{4}. \quad (1.39)$$

This condition allows light reflected at each interface of different refractive indices to interfere constructively yielding high overall reflectivity (see Fig. 1.8 (e)). For a sufficiently large number of pairs of layers N_c the reflectivity at λ_B is given by [30]

$$R(\lambda_B) = 1 - 4 \frac{n_i}{n_s} \left(\frac{n_1}{n_2} \right)^{2N_c}. \quad (1.40)$$

Here n_i is the refractive index of the incident media (which could be just air), n_s is the refractive index of the substrate, and $n_1 < n_2$. The wavelength range at which the DBR acts as a highly reflective mirror with reflectivity close to 100% is called the stop band, which is proportional to the refractive index difference ($n_2 - n_1$). In Fig. 1.8 (b) the reflectivity and transmittivity spectrum of a DBR made of AlAs

Figure 1.8: (a,d) Schematic diagram of a DBR and microcavity, respectively. (b) Reflectivity and transmittivity spectrum of a DBR having GaAs and AlAs as alternating layers. (c) Enhancement of reflectivity with the increase of the number of pairs of layers N_c . (e) Working principle of a Fabry-Perot cavity. (f) Reflectivity spectrum of a GaAs based microcavity with a sharp dip of reflectivity at the cavity mode. (g) Intensity profile of the electric field through the microcavity.

($n_1 = 2.96$) and GaAs ($n_2 = 3.54$) with $N_c = 15$ and $\lambda_B = 840$ nm is shown. With the increase of the number of pairs of the layers N_c the reflectivity of the DBR

also increases making them a very efficient mirror (see Fig. 1.8 (c)). Fig. 1.8(b,c,f) is prepared using the commercial FDTD software COMSOL Multiphysics.

A layer of thickness L_{cav} and refractive index n_{cav} can be sandwiched between two such DBR mirrors to realize a microcavity. In Fig. 1.8 (d) such a structure is depicted. To find out how a microcavity works let's consider a simple Fabry-Perot cavity with two mirrors having reflectivity coefficients R_1 , R_2 and transmittivity coefficients T_1 , T_2 , respectively (see Fig. 1.8 (e)). For a plane wave with complex amplitude E_i and angle of incidence α on the mirror 1, the angle of propagation β inside the cavity is given by Snell's law

$$n_1 \sin(\alpha) = n_{cav} \sin(\beta) \quad (1.41)$$

The reflectivity (and transmittivity) of the cavity depends on the multiple reflections of the incident wave inside the cavity. The phase gained by the incident wave due to one round trip inside the cavity is $\phi = 2 \times 2\pi n_{cav} L_{cav} \cos(\beta)/\lambda$, where λ is the wavelength of the incident wave in vacuum. The total transmitted field can be written as

$$\begin{aligned} E_T &= E_i T_1 T_2 \exp(i\phi/2) + E_i T_1 T_2 \exp(i\phi/2) R_2 R_1 \exp(i\phi) + \dots \\ &= E_i T_1 T_2 \exp(i\phi/2) \sum_{j=0}^{\infty} (R_2 R_1 \exp(i\phi))^j \\ &= E_i T_1 T_2 \exp(i\phi/2) \left[\frac{1}{1 - R_2 R_1 \exp(i\phi)} \right] \end{aligned} \quad (1.42)$$

The transmittivity and reflectivity, T and R , respectively, of the cavity can be obtained as

$$T = \frac{|E_T|^2}{|E_i|^2} = \frac{T_1^2 T_2^2}{1 - 2R_1 R_2 \cos(\phi) + R_1^2 R_2^2}, \quad (1.43)$$

$$R = 1 - T = \frac{2R_1 R_2 - 2R_1 R_2 \cos(\phi)}{1 - 2R_1 R_2 \cos(\phi) + R_1^2 R_2^2}. \quad (1.44)$$

In the expression of R , we have used $T_1 T_2 = 1 - R_1 R_2$, which corresponds to the Stokes relations. It can be easily seen that for $\cos(\phi) = 1$, $R = 0$. Alternatively, we can say that a Fabry-Perot cavity only transmits modes having $\phi = 2q\pi$, where q can be any integer. This condition can be expressed in terms of wavelength λ as

$$n_{cav} L_{cav} \cos(\beta) = \frac{q\lambda}{2} \quad (1.45)$$

For normal incidence ($\alpha, \beta = 0$), the energy difference between two consecutive modes ΔE and the full width at half maximum of a mode Γ_{cav} can be expressed by

$$\Delta E = \frac{\pi \hbar c}{L_{cav} n_{cav}}, \quad (1.46)$$

$$\Gamma_{cav} = \frac{\hbar c}{L_{cav} n_{cav}} \frac{1 - R_1 R_2}{\sqrt{R_1 R_2}}. \quad (1.47)$$

From the above two quantities the finesse F and quality factor Q of the cavity can be defined as

$$F = \frac{\Delta E}{\Gamma_{cav}} = \pi \frac{\sqrt{R_1 R_2}}{1 - R_1 R_2}, \quad (1.48)$$

$$Q = \frac{E}{\Gamma_{cav}} = qF = q\pi \frac{\sqrt{R_1 R_2}}{1 - R_1 R_2}. \quad (1.49)$$

Physically the quantity Q represents the number of bounces a photon makes before escaping from the cavity and the amount of time the photon stays inside the cavity is called the lifetime of the photons $\tau_{cav} = \hbar/\Gamma_{cav}$. Except the quality factor which depends upon the energy of the modes, all other quantities are the characteristics of the mirrors.

Now let's consider the case for oblique incidence ($\alpha \neq 0$). For oblique incidence the wave vector $\mathbf{k} = \mathbf{k}_z + \mathbf{k}_{||}$, where $\mathbf{k}_{||}$ is the in plane wave vector and $\mathbf{k}_z = k \cos(\beta) \hat{\mathbf{z}}$ is the z component of the $\mathbf{k}$ vector and the modulus of $\mathbf{k}$ is given by

$k = \sqrt{k_{\parallel}^2 + k_z^2} = 2n_{cav}\pi/\lambda$. Using the above relations the quantization condition in Eq. (1.45) can also be expressed as

$$k_z = \frac{q\pi}{L_{cav}}. \quad (1.50)$$

The dispersion of the confined optical modes then becomes

$$\begin{aligned} E_c(k_{\parallel}) &= \frac{\hbar c}{n_{cav}} \sqrt{k_{\parallel}^2 + \left(\frac{q\pi}{L_{cav}}\right)^2} \\ &= \frac{q\pi\hbar c}{L_{cav}n_{cav}} + \frac{\hbar^2 k_{\parallel}^2}{2m_{ph}}. \end{aligned} \quad (1.51)$$

The dispersion for the in plane motion has become parabolic with an effective mass $m_{ph} = q\pi\hbar n_{cav}/cL_{cav}$. Typically m_{ph} is on the scale of $10^{-5}m_o$, where m_o is the free electron mass.

In Fig. 1.8(f) a sharp dip in the reflectivity spectrum near λ_B is observed, which corresponds to the cavity mode for $n_{cav}L_{cav} = q\lambda_B/2$. Modern GaAs based microcavities can have quality factors around 10^5 or higher which corresponds to the photon lifetime around 50 ps. In Fig. 1.8(g) the amplitude of the electric field profile through the structure is shown, which is strongly confined within the cavity and decays exponentially in the DBRs.

1.2.3 Exciton-polaritons

In the previous section we have seen how a microcavity can confine the optical modes. In this section we consider a QW at the center of the microcavity where the intensity of the electric field is maximum. The whole structure is designed in such a way that the cavity mode is in resonance with the exciton modes. Since the exciton is an electron hole pair, it has a dipole moment and due to this dipole moment it can couple to light. Using second quantization theory the Hamiltonian

corresponding to a microcavity with QW at its center can be expressed as

$$\hat{H}_{pol} = \sum_k E_{cav}(k) \hat{a}_k^\dagger \hat{a}_k + E_{ex}(k) \hat{\xi}_k^\dagger \hat{\xi}_k + g_0 \left(\hat{a}_k^\dagger \hat{\xi}_k + \hat{a}_k \hat{\xi}_k^\dagger \right). \quad (1.52)$$

Here k represents the in plane wave vector, $E_{cav}(k)$ is the parabolic dispersion of the cavity photons and $E_{ex}(k)$ is the parabolic dispersion of the excitons. g_0 is the dipole interaction strength between the excitons and photons. The above Hamiltonian has a similar form to the one for dressed electrons in Eq. (1.28). $\hat{a}_k^\dagger$ ($\hat{\xi}_k^\dagger$) and $\hat{a}_k$ ($\hat{\xi}_k$) are the photon (exciton) creation and annihilation operators, respectively. Let's introduce a new operator $\hat{\Phi}_k$ that is the superposition of $\hat{a}_k$ and $\hat{\xi}_k$, and has the form

$$\hat{\Phi}_k = P_{1k} \hat{a}_k + P_{2k} \hat{\xi}_k, \quad (1.53)$$

where

$$|P_{1k}|^2 + |P_{2k}|^2 = 1 \quad (1.54)$$

In the transformed basis the Hamiltonian in Eq. 1.52 can be expressed as

$$\hat{H}_{pol}^D = \sum_k E_k^p \hat{\Phi}_k^\dagger \hat{\Phi}_k. \quad (1.55)$$

Since the two Hamiltonians expressed in Eq. 1.52 and Eq. 1.55 are equivalent, we get:

$$\left[\hat{\Phi}_k, \hat{H}_{pol} \right] = \left[\hat{\Phi}_k, \hat{H}_{pol}^D \right], \quad (1.56)$$

Figure 1.9: (a-c) Appearance of exciton-polariton branches due to the strong coupling between the excitons and cavity photons with detuning -10 meV, 0 meV, and 10 meV, respectively. (d-f) Variation of the exciton (black) and photon (blue) fractions as a function of k for the lower polariton branch.

or equivalently,

$$\begin{pmatrix} E_{cav}(k) - E_k^p & g_0 \\ g_0 & E_{ex}(k) - E_k^p \end{pmatrix} \begin{pmatrix} P_{1k} \\ P_{2k} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (1.57)$$

The above eigen value equation can be solved to find the eigen modes of the exciton photon coupled system, which corresponds to the dispersion of new quasiparticles known as exciton-polaritons,

$$E_k^p = \frac{1}{2} \left[E_{cav}(k) + E_{ex}(k) \pm \sqrt{(E_{cav}(k) - E_{ex}(k))^2 + 4g_0^2} \right] \quad (1.58)$$

The high energy branch is called the upper polariton branch and the low energy branch is called the lower polariton branch. In Fig. 1.9(a-c) the dispersion of the exciton-polariton branches is shown for different photon-exciton detuning ($E_{cav}(0) - E_{ex}(0)$). Since the exciton mass is orders (around 10^4) of magnitude larger than the photon mass the dispersion of the exciton appears to be flat when

plotted in the same scale with that of photons. The anti-crossing of the new modes is the signature of the formation of exciton-polaritons in the experiments. The exciton and the photon dispersion is repelled from each other by an amount $2g_0$, which is called the Rabi splitting Ω_R . The quantities $|P_{1k}|^2$ and $|P_{2k}|^2$ are called the photon and exciton fractions, respectively and can be expressed as

$$|P_{1k}|^2 = \frac{g_0^2}{g_0^2 + [E_{cav}(k) - E_k^p]^2} = \frac{1}{2} \left[1 - \frac{\Delta E(k)}{\sqrt{\Delta E(k)^2 + 4g_0^2}} \right], \quad (1.59)$$

$$|P_{2k}|^2 = \frac{g_0^2}{g_0^2 + [E_{ex}(k) - E_k^p]^2} = \frac{1}{2} \left[1 + \frac{\Delta E(k)}{\sqrt{\Delta E(k)^2 + 4g_0^2}} \right]. \quad (1.60)$$

where $\Delta E(k) = E_{cav}(k) - E_{ex}(k)$. The above two expressions are also known as the Hopfield coefficients [31] and they also determine the polariton field operator $\hat{\Phi}_k$, using Eq. 1.53. In Fig. 1.9(d-f) the photon and exciton fractions are plotted for different detuning, which shows that for small k the lower polaritons have significant photon fraction (depending upon the detuning), however as k increases the lower polariton becomes more exciton like.

Polaritons decay primarily due to the leakage of cavity photons through the DBR mirrors. While excitons also have a radiative decay, it corresponds to nanosecond lifetime and so is negligible compared to the much more significant photon decay, which corresponds to tens of picoseconds lifetime. If the decay of the polaritons $\Gamma \approx \Gamma_{cav} \gg \Gamma_{ex}$, where Γ_{cav} and Γ_{ex} represent the cavity and exciton decay, respectively, is included in Eq. (1.57), the eigen-energies of the system will be

$$E_k^p = \frac{1}{2} \left[E_{cav}(k) + E_{ex}(k) - i\frac{\Gamma}{2} \right] \pm \frac{1}{2} \sqrt{\left[E_{cav}(k) - E_{ex}(k) - i\frac{\Gamma}{2} \right]^2 + 4g_0^2}. \quad (1.61)$$

The Rabi splitting in this case becomes

$$\Omega_R = \frac{1}{2} \sqrt{4g_0^2 - \frac{\Gamma^2}{4}}. \quad (1.62)$$

It is clear from the above expression that Ω_R is real for $4g_0^2 > \Gamma^2/4$, that is when the exciton photon coupling strength exceeds the decay in the system. This is called the strong coupling regime. For the opposite case when $4g_0^2 < \Gamma^2/4$, Ω_R is imaginary. In this case the exciton photon dispersion is not modified but their respective linewidths change. This is called the weak coupling regime. For the rest of this thesis we will always work in the strong coupling regime.

Since exciton-polaritons are superpositions of excitons and photons, they have some properties inherited from excitons and some properties from photons. As it can be seen from the dispersion the mass of the lower polariton branch is similar to the photon's and typically ranges between $10^{-4} - 10^{-5}m_0$. Such a low effective mass enables exciton-polariton to undergo BEC in principle at room temperature. However, a further requirement is that excitons themselves are also stable. For this reason polariton condensation in GaAs systems still requires cryogenic cooling (to a few Kelvin). Still, organic systems, perovskites, and wide-bandgap inorganic semiconductors, in which the exciton binding energy is higher, allow polariton BEC at room temperature. In sec. 1.2.6 polariton BEC is discussed with more details.

Due to their excitonic component, unlike photons, polaritons can react to externally applied electric and magnetic fields giving rise to many interesting phenomena. To give a few example, polaritons can show a dynamic Stark effect in presence of red detuned electromagnetic field [32], there can be Zeeman shift of polaritons with different spins in presence of perpendicular magnetic field [33], etc. The excitonic component also enables the polaritons to interact with themselves nonlinearly. The main source of such an interaction is the exciton-exciton scatter-

ing due to the Coulomb interaction, which involves the exchange of an electron or hole:

$$\hat{H}_{exchange} = \frac{1}{2} \sum_{k,k',q_1} V_{exchange}(k,k',q_1) \hat{\xi}_{k+q_1}^\dagger \hat{\xi}_{k'-q_1}^\dagger \hat{\xi}_{k'} \hat{\xi}_k. \quad (1.63)$$

Here two excitons have exchanged momentum $\hbar q_1$ and $V_{exchange}$ is the strength of the interaction, which in the zero momentum limit ($q_1 \rightarrow 0$) can be expressed as [34]

$$V_{exchange} = \frac{6\varepsilon_B(r_B)^2}{S}, \quad (1.64)$$

where S is the surface area occupied by the excitons. The polariton-polariton interaction constant α_1 is proportional to the excitonic fraction and expressed as

$$\alpha_1 = |P_{2k}|^4 V_{exchange} S. \quad (1.65)$$

Typically the effective polariton-polariton nonlinear interaction coefficient ranges from 0.1-10 $\mu\text{eV}\mu\text{m}^2$ [35].

1.2.4 Mean field description of exciton-polaritons

In this section we derive the Gross-Pitaevskii equation using the mean field approach to describe the polaritons. Using the second quantization approach the Hamiltonian can be expressed as

$$\begin{aligned} \hat{H}_{pol} = & \sum_{\mathbf{k}} E_{\mathbf{k}} \hat{\Phi}_{\mathbf{k}}^\dagger \hat{\Phi}_{\mathbf{k}} + \sum_{\mathbf{r}} V_{\mathbf{r}} \hat{\Phi}_{\mathbf{r}}^\dagger \hat{\Phi}_{\mathbf{r}} + \frac{\alpha_1}{2} \sum_{\mathbf{r}} \hat{\Phi}_{\mathbf{r}}^\dagger \hat{\Phi}_{\mathbf{r}}^\dagger \hat{\Phi}_{\mathbf{r}} \hat{\Phi}_{\mathbf{r}} \\ & + \sum_{\mathbf{k}} \left(F_{\mathbf{k}} \exp(-i\omega t) \hat{\Phi}_{\mathbf{k}}^\dagger + F_{\mathbf{k}}^* \exp(i\omega t) \hat{\Phi}_{\mathbf{k}} \right). \end{aligned} \quad (1.66)$$

Here the first term represents the real part of the bare polariton dispersion. The imaginary part, representing decay in the previous section, will instead be introduced as a non-Hamiltonian term, as is standard in quantum optics. The second term indicates the external potential experienced by the polaritons, the third term is the nonlinear polariton-polariton interaction with strength α_1 , and the last term represents a coherent pump (i.e., resonant laser excitation) with energy $\hbar\omega$. $\hat{\Phi}_{\mathbf{r}}$ and $\hat{\Phi}_{\mathbf{k}}$ are the field operators in the real and reciprocal space, which are related by the following relations,

$$\hat{\Phi}_{\mathbf{k}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{r}} \hat{\Phi}_{\mathbf{r}} \exp(-i\mathbf{k} \cdot \mathbf{r}) \quad \text{and} \quad \hat{\Phi}_{\mathbf{r}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} \hat{\Phi}_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{r}). \quad (1.67)$$

Here N is the total number of sites and the field operators follow the following commutation relations

$$\left[\hat{\Phi}_{\mathbf{k}}, \hat{\Phi}_{\mathbf{k}'}^\dagger \right] = \delta_{\mathbf{k}, \mathbf{k}'}, \quad \text{and} \quad \left[\hat{\Phi}_{\mathbf{r}}, \hat{\Phi}_{\mathbf{r}'}^\dagger \right] = \delta_{\mathbf{r}, \mathbf{r}'}. \quad (1.68)$$

The finite lifetime of the polaritons is taken into account by adding the following Lindblad dissipation term in the master equation which is widely used in quantum optics [36],

$$i\hbar \frac{\partial \rho}{\partial t} = \left[\hat{H}_{pol}, \rho \right] + i \frac{\Gamma}{2} \sum_{\mathbf{k}} \left(2\hat{\Phi}_{\mathbf{k}} \rho \hat{\Phi}_{\mathbf{k}}^\dagger - \hat{\Phi}_{\mathbf{k}}^\dagger \hat{\Phi}_{\mathbf{k}} \rho - \rho \hat{\Phi}_{\mathbf{k}}^\dagger \hat{\Phi}_{\mathbf{k}} \right), \quad (1.69)$$

where ρ is the density matrix. The mean field can be defined as

$$\langle \Phi_{\mathbf{k}} \rangle = \text{Tr} \left\{ \rho \hat{\Phi}_{\mathbf{k}} \right\}, \quad (1.70)$$

where Tr represents the trace. Using Eq. (1.69-1.70) we can write

$$i\hbar \frac{\partial \langle \Phi_{\mathbf{k}} \rangle}{\partial t} = \left(E_{\mathbf{k}} - i\frac{\Gamma}{2} \right) \langle \Phi_{\mathbf{k}} \rangle + \frac{1}{\sqrt{N}} \sum_{\mathbf{k}'} V_{\mathbf{k}'} \langle \Phi_{\mathbf{k}-\mathbf{k}'} \rangle + \frac{\alpha_1}{N} \sum_{\mathbf{k}', \mathbf{k}''} \langle \hat{\Phi}_{\mathbf{k}'}^\dagger \hat{\Phi}_{\mathbf{k}''} \hat{\Phi}_{\mathbf{k}'+\mathbf{k}''-\mathbf{k}} \rangle + F_{\mathbf{k}} \exp(-i\omega t). \quad (1.71)$$

Here we have used $\text{Tr}\{ABC\} = \text{Tr}\{CAB\} = \text{Tr}\{BCA\}$, where A , B , and C are operators. Note that within the mean-field approximation, the effect of the dissipation term is the same as if we had included it as an imaginary term in the energy dispersion. $V_{\mathbf{k}}$ is related to $V_{\mathbf{r}}$ by

$$V_{\mathbf{r}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} V_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{r}). \quad (1.72)$$

Taking a Fourier transform, Eq. 1.71 can be written in real space as

$$i\hbar \frac{\partial \langle \Phi_{\mathbf{r}} \rangle}{\partial t} = \frac{1}{\sqrt{N}} \sum_{\mathbf{r}'} E_{\mathbf{r}'} \langle \Phi_{\mathbf{r}-\mathbf{r}'} \rangle + \left(V_{\mathbf{r}} - i\frac{\Gamma}{2} \right) \langle \Phi_{\mathbf{r}} \rangle + \alpha_1 \langle \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}} \Phi_{\mathbf{r}} \rangle + F_{\mathbf{r}} \exp\{-i\omega t\}. \quad (1.73)$$

In the high density limit, the interaction term in the above equation can be written as $\langle \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}} \Phi_{\mathbf{r}} \rangle \simeq \langle \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}} \rangle \langle \Phi_{\mathbf{r}} \rangle$ and by putting $\phi(\mathbf{r}) = \langle \Phi_{\mathbf{r}} \rangle$ the above equation can be written in the continuous space as

$$i\hbar \frac{\partial \phi(\mathbf{r})}{\partial t} = \left(\hat{E} - i\frac{\Gamma}{2} + V(\mathbf{r}) + \alpha_1 |\phi(\mathbf{r})|^2 \right) \phi(\mathbf{r}) + F(\mathbf{r}) \exp(-i\omega t). \quad (1.74)$$

The operator ($\hat{E}$) near $\mathbf{k} = 0$ can typically be approximated as parabolic such that $\hat{E} = -\hbar^2 \nabla^2 / 2m_{pol}$, where ∇^2 is the 2D Laplacian operator and m_{pol} is the effective polariton mass. The above equation is called the nonlinear Shrodinger equation (similar to that used in nonlinear optics with self-defocusing Kerr nonlinearity) or, equivalently, the Gross-Pitaevskii equation (commonly used in the study of cold

atom systems).

1.2.5 Polariton spins and spin-orbit interaction

As already mentioned in sec. (1.1.2), due to the conservation of angular momentum, excitons with spin ± 1 can couple to photons, which also have spin ± 1 (right and left circularly polarized light), and create polaritons with spin ± 1 (typically indicated with $\sigma_{\pm}$). Linearly polarized light creates a superposition of σ_+ and σ_- polaritons. The spin of polaritons is expressed using the spin density matrix ρ_k , analogous to electrons, as [37]

$$\rho_k = \frac{N_k}{2} \mathbf{I} + \mathbf{S}_k \cdot \boldsymbol{\sigma}, \quad (1.75)$$

where N_k is the number of polaritons having wave vector k , $\mathbf{I}$ is the identity matrix, $\boldsymbol{\sigma}$ is the pauli matrix and $\mathbf{S}_k$ is the three component pseudo-spin vector that represents the spin of the polaritons. The components of $\mathbf{S}_k$ are related to the Stokes parameters of the Poincare sphere as shown in Fig. 1.10(a). Each point on the surface of the sphere corresponds to a particular polariton polarization state, with the equator being linearly polarized, the poles of the sphere $\sigma_{\pm}$ polarized, and all other points on the surface of the sphere corresponding to elliptical polarization. Any point that lies inside the sphere corresponds to partially polarized polaritons, with the point in the center corresponding to the fully unpolarized state.

The Gross-Pitaevskii equation in (1.74) can be updated to the following to take the spins of the polaritons into account

$$\begin{aligned} i\hbar \frac{\partial \phi_{\sigma_{\pm}}(\mathbf{r})}{\partial t} = & \left(-\frac{\hbar^2 \nabla^2}{2m_{pol}} - i\frac{\Gamma}{2} + V(\mathbf{r}) + \alpha_1 |\phi_{\sigma_{\pm}}(\mathbf{r})|^2 + \alpha_2 |\phi_{\sigma_{\mp}}(\mathbf{r})|^2 \right) \phi_{\sigma_{\pm}}(\mathbf{r}) \\ & + F_{\sigma_{\pm}}(\mathbf{r}) \exp(-i\omega t). \end{aligned} \quad (1.76)$$

Here α_1 is the interaction between the polaritons with the same spin and α_2 is

the interaction between the opposite spin polaritons. Experiments have shown that polaritons with the same spin interact repulsively, whereas polaritons with the opposite spin interact attractively and typically $|\alpha_1| \geq |\alpha_2|$ [38–40].

An electromagnetic field propagating through a medium has two kinds of linearly polarization modes: TE (transverse electric, where the field is perpendicular to the plane of incidence) and TM (transverse magnetic, where the field is in the plane of incidence). A schematic diagram of such modes is shown in Fig. 1.10(b). At oblique incidences, due to their different boundary conditions TE and TM modes inside the microcavity have different reflectivity [41], which results in an energy splitting (TE-TM splitting) between those modes. Such a splitting in the cavity photon modes also results in TE-TM splitting of the polariton modes. Apart from this there are two more reasons behind TE-TM splitting of the polaritons. First is the long range electron and hole exchange interactions, which induce a splitting in the exciton modes having dipole moment parallel and perpendicular to the wave vector. However, such kind of splitting is very small (around a few μeVs) and has very little effect. On the other hand, the exciton-photon coupling strength g_0 is different for TE and TM modes. Since for oblique incidence TM modes make an angle θ with the QW plane, the exciton-photon coupling is larger for TE modes than that of TM modes. All the above three factors contribute in the final TE-TM splitting of the polaritons.

The TE-TM splitting can be modelled by defining different masses for TE and TM modes (see Fig. 1.10(b)). The Hamiltonian for the polaritons in the TE-TM basis can be expressed as,

$$\hat{H}_{TE-TM} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m_{TM}} & 0 \\ 0 & \frac{\hbar^2 k^2}{2m_{TE}} \end{pmatrix}. \quad (1.77)$$

Let ϕ be the in plane polar angle as shown in Fig. 1.10(d), then the transformation

matrix for going to the circular polarization basis can be written as

$$\begin{pmatrix} \psi_{TM} \\ \psi_{TE} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \exp(-i\phi) & \exp(i\phi) \\ -i \exp(-i\phi) & i \exp(i\phi) \end{pmatrix} \begin{pmatrix} \psi_{\sigma+} \\ \psi_{\sigma-} \end{pmatrix}. \quad (1.78)$$

The Hamiltonian in Eq. (1.77) can be written in the circular polarization basis as

$$\hat{H}_{\sigma\pm} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m_{pol}} & \Delta_T k^2 \exp(-2i\phi) \\ \Delta_T k^2 \exp(2i\phi) & \frac{\hbar^2 k^2}{2m_{pol}} \end{pmatrix}, \quad (1.79)$$

where

$$\frac{1}{m_{pol}} = \frac{1}{2} \left(\frac{1}{m_{TE}} + \frac{1}{m_{TM}} \right) \quad \text{and} \quad \Delta_T = \frac{\hbar^2}{4} \left(\frac{1}{m_{TM}} - \frac{1}{m_{TE}} \right). \quad (1.80)$$

Here m_{pol} is the (average) effective polariton mass and Δ_T is the effective TE-TM coupling strength. The Hamiltonian in Eq. (1.79) can also be expressed as $\hat{H}_{\sigma\pm} = \hbar^2 k^2 / 2m_e + \hbar \sigma \cdot \mathbf{\Omega}_{\mathbf{k}}$. In analogy with the Hamiltonian of the electrons in magnetic field, the term $\mathbf{\Omega}_{\mathbf{k}}$ is called the effective magnetic field with σ being the Pauli matrix vector. $\mathbf{\Omega}_{\mathbf{k}}$ can be expressed as

$$\begin{aligned} \mathbf{\Omega}_{\mathbf{k}} &= \Omega_0 k^2 (\cos(2\phi) \hat{x} + \sin(2\phi) \hat{y}) \\ &= \Omega_0 ((k_x^2 - k_y^2) \hat{x} + 2k_x k_y \hat{y}), \end{aligned} \quad (1.81)$$

where $\Omega_0 = \Delta_T / \hbar$, $\cos(\phi) = k_x / k$, and $\sin(\phi) = k_y / k$. The final form of the

Figure 1.10: (a) A schematic diagram of a Poincare sphere. (b) TE and TM mode orientations for oblique incidence. (c) TE and TM mode dispersions as a function of k . (d) TM mode oriented by an angle ϕ . (e) Orientation of the effective magnetic field on the $k_x k_y$ plane. (f-g) Optical spin Hall effect: initially all polaritons are created with x polarization (f), due to the effective magnetic field, the spin of the polaritons is determined depending upon the position in the k space and their precession about the effective magnetic field. (g) is taken from [43].

Hamiltonian becomes

$$\begin{aligned}\hat{H}_{\sigma_{\pm}} &= \frac{\hbar^2 k^2}{2m_c} \mathbf{I} + \hbar \Omega_0 \begin{pmatrix} 0 & k_x^2 - k_y^2 - i2k_x k_y \\ k_x^2 - k_y^2 + i2k_x k_y & 0 \end{pmatrix} \\ &= \frac{\hbar^2 k^2}{2m_c} \mathbf{I} + \hbar \Omega_0 \begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix}.\end{aligned}\quad (1.82)$$

The polariton pseudo spin $\mathbf{S}_{\mathbf{k}}$ under effect of the effective magnetic field $\mathbf{\Omega}_{\mathbf{k}}$ can be expressed as [42]

$$\frac{d\mathbf{S}_{\mathbf{k}}}{dt} = \mathbf{S}_{\mathbf{k}} \times \mathbf{\Omega}_{\mathbf{k}}. \quad (1.83)$$

In Fig. 1.10(e) the direction of $\mathbf{\Omega}_{\mathbf{k}}$ is plotted in the (k_x, k_y) plane. In Fig. 1.10(f-g) the effect of the effective magnetic field is depicted, where if we start with x polarized polaritons, the effective magnetic field changes the spins of the polaritons depending upon the wave vector of the polaritons. This is known as the optical spin Hall effect [43]. Finally, we write down the Gross-Pitaevskii equation including the effect of TE-TM splitting in the circular polarization basis

$$\begin{aligned}i\hbar \frac{\partial \phi_{\sigma_{\pm}}(\mathbf{r})}{\partial t} &= \left(-\frac{\hbar^2 \nabla^2}{2m_{pol}} - i\frac{\Gamma}{2} + V(\mathbf{r}) + \alpha_1 |\phi_{\sigma_{\pm}}(\mathbf{r})|^2 + \alpha_2 |\phi_{\sigma_{\mp}}(\mathbf{r})|^2 \right) \phi_{\sigma_{\pm}}(\mathbf{r}) \\ &\quad + \Delta_T \left(i\frac{\partial}{\partial x} \pm \frac{\partial}{\partial y} \right)^2 \phi_{\sigma_{\mp}}(\mathbf{r}) + F_{\sigma_{\pm}}(\mathbf{r}) \exp(-i\omega t).\end{aligned}\quad (1.84)$$

This equation has been used in Chapters 4 and 5 for studying the robust propagation of the polaritons in honeycomb lattice.

1.2.6 Non-resonant excitation

To generate polaritons in the microcavity, mainly two excitation schemes are used. The aforementioned equations 1.66, 1.76, and 1.84 account for a resonant coherent

excitation, corresponding to a laser tuned to have a frequency close to the polariton resonance frequency. An alternative is non-resonant excitation where free carriers are injected at much higher energy than the polariton branches and polaritons are formed via various relaxation processes. In this section we will focus on the non-resonant pumping.

Non-resonant injection of the carriers can be achieved either by electrical injection [44, 45] or optically where the excitation laser is tuned above the semiconductor bandgap (usually ~ 100 meV above the lower polariton dispersion), which creates electron-hole carriers in the microcavity at higher energy. Then the carriers relax down to the high momentum exciton states on the exciton dispersion through the scattering with Longitudinal-optical (LO) phonons (<1 ps) [46, 47] followed by scattering with slower acoustic phonons (<1 ns) and the formation of excitons [48]. These excitons at higher energy and wave vector form the so-called exciton reservoir. Since the wavevector of light in the medium is smaller than the wavevector of the exciton, these excitons do not couple to light [48]. These excitons relax down to the high momentum lower polariton dispersion (LP) through acoustic phonon and exciton-exciton scattering and enter the light cone where they form polaritons. These polaritons often accumulate at the LP branch with higher wave vector and due to the *bottleneck effect* cannot further relax down towards $k = 0$ [49]. This effect arises due to the steepness of the LP dispersion, which changes the effective polariton mass from nearly that of the exciton to twice the photon mass. This decreases the density of states by four orders of magnitude and the lifetime from ~ 500 ps to ~ 10 ps of the LP branch [50]. In contrast acoustic phonon mediated processes need about 50 ps, which is longer than the effective lifetime of the polariton in that region [51]. As a result polaritons are mainly populated to the high momentum LP dispersion [52]. However, by increasing the excitation power of the nonresonant pump, the density of the polaritons in the bottleneck region is sufficiently increased such that significant polariton-polariton

Figure 1.11: (a) Schematic diagram of the injection of free carriers at higher energy. After relaxation the free carriers form excitons and occupy the high momentum states, which is called the excitonic reservoir. (b-d) Momentum distribution of the polaritons as a function of reservoir density. As the density of the reservoir increases the polaritons occupy the ground state. (e) Electrical pumping scheme in a InGaAs based sample. (f) Polaritons occupying the ground state as the current density increases. Fig. (a) is taken from Ref. [57], Fig. (b-d) is taken from Ref. [58] and Fig. (e-f) is taken from Ref. [45].

scattering is achieved [50, 53]. In this process, a pair of polaritons scatter by conserving energy and momentum. One by gaining energy and momentum scatters to a higher state in the LP dispersion while the other by losing energy and scatters to a lower energy state.

Due to the bosonic nature of polaritons, when the density of polaritons in the ground state exceeds order unity, stimulated scattering occurs to the ground state [50, 52]. When the gain of the polariton exceeds losses, the polaritons macroscopically occupy a low energy state (possibly, but not necessarily the ground state), which is known as Bose Einstein condensation (BEC) of polaritons [54]. Such a condensate is a balance between loss of photons from the cavity and the gain of polaritons arising from the relaxation of electron-hole carriers to the condensate, i.e., it is an out of thermal equilibrium system. BEC of polaritons was experimentally reported at cryogenic temperatures [55, 56]. Later over the years different samples have been adopted to realize polariton condensation up to room temperature such as GaN [59, 60], ZnO [61–63], organic-inorganic perovskites [64], organic systems [65, 66]. Such condensates emit light that has all the properties similar to a laser light, i.e., it is coherent, monochromatic, and polarized. The main difference with a conventional laser is that the polariton laser is based on a spontaneous emission of polaritons, which are created via stimulated scattering whereas a conventional photon laser is based on stimulated emission. For a photon laser population inversion is necessary, while a polariton laser does not require population inversion [67], leading to a lower threshold [68–71]. The photons emitted from the cavity have all the information about the condensates such as wavevector, energy, polarization, phase, etc. To describe the dynamics using the mean field approximation, we can write down a coupled equation of the polariton condensate dynamics and a rate equation for the reservoir of excitons as follows

(neglecting the spin degree of freedom) [72]

$$i\hbar \frac{\partial \phi(\mathbf{x}, t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m_{pol}} + V(\mathbf{x}) + \alpha_1 |\phi(\mathbf{x}, t)|^2 + \left(g_R + i\hbar \frac{R}{2} \right) n_R(\mathbf{x}, t) - i\frac{\Gamma}{2} \right] \phi(\mathbf{x}, t) \quad (1.85)$$

$$\frac{\partial n_R(\mathbf{x}, t)}{\partial t} = P(\mathbf{x}) - (\gamma_R + R |\phi(\mathbf{x}, t)|^2) n_R(\mathbf{x}, t). \quad (1.86)$$

The equation for the polariton dynamics is similar to Eq. (1.74), although the terms representing resonant coherent excitation are replaced with the effect of the exciton reservoir generated now by a non-resonant excitation. The exciton-density in the reservoir is given by n_R and $P(\mathbf{x})$ is the non-resonant pump which gives the pumping rate of the reservoir. Let us recall that α_1 is the strength of the polariton-polariton interaction. The interaction between polaritons and the exciton-reservoir is given by g_R , which is repulsive in nature. R represents the exciton-polariton condensation growth rate, i.e., the rate at which polaritons relax down from reservoir to the condensate by stimulated scattering. γ_R is the decay rate of the the reservoir.

1.2.7 Elementary excitations of polariton condensates

So far we have described the mean field behaviour of polariton condensates, which can be used, for example, to define stationary polariton states. To assess though the stability of these condensate states it is important to consider their response to weak fluctuations. This can be done within the framework of the Bogoliubov theory, by considering weak fluctuations on top of the condensate of the form [73, 74]

$$\phi(\mathbf{r}, t) = e^{i(k_p \cdot \mathbf{r} - \omega_p t)} (\phi_0 + u e^{i(k \cdot \mathbf{r} - \omega t)} + v^* e^{i(-k \cdot \mathbf{r} + \omega^* t)}), \quad (1.87)$$

where ϕ_0 is the amplitude of the wavefunction corresponding to the steady state of the condensate, whereas u and v are the (weak) amplitudes of the fluctuations having momentum k and complex frequency ω . The positive imaginary part of ω signifies that the fluctuations will grow exponentially, leading to dynamical instability of the condensate. In contrast, the region with negative imaginary part of ω corresponds to the dynamical stability of the condensate. Inserting the ansatz Eq.(1.87) in the Gross-Pitaevskii equation subjected to a resonant pump F having energy and wave vector $\hbar\omega_p$ and k_p , respectively :

$$i\hbar \frac{\partial \phi(\mathbf{r}, t)}{\partial t} = \left(-\frac{\hbar^2 \nabla^2}{2m_{pol}} + \alpha_1 |\phi(\mathbf{r}, t)|^2 - \frac{i\Gamma}{2} \right) \phi(\mathbf{r}, t) + F e^{i(k_p \cdot \mathbf{r} - \omega_p t)}. \quad (1.88)$$

The presence of the resonant pump F results in a stationary state of the form $\phi_s = \phi_0 e^{ik_p r}$. Neglecting the higher-order terms in u and v we obtain the following eigen-value equation (also known as the Bogoliubov matrix):

$$\hbar\omega \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \frac{\hbar^2}{2m_{pol}}(k_p + k)^2 - \frac{i\Gamma}{2} + \xi & \alpha_1 \phi_0^2 \\ -\alpha_1 (\phi_0^2)^* & -\frac{\hbar^2}{2m_{pol}}(k_p - k)^2 - \frac{i\Gamma}{2} - \xi \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}, \quad (1.89)$$

where $\xi = 2\alpha_1 |\phi_0|^2 - \hbar\omega_p$. The nonzero value of k_p breaks the symmetry between the two eigen-values. For a specific excitation we can obtain an interesting regime as shown in Fig. 1.12. The polaritons are injected quasi-resonantly at $k = k_p$ as indicated by the gray point 'A' and due to the polariton-polariton interaction, the Bogoliubov spectra (red curve) is blue shifted with respect to the bare polariton dispersion (dotted curve). The important feature of the Bogoliubov spectra is that it is asymmetric and there exists a point 'A' for which there are no other states available at the same energy. As this scattering in presence of disorder is forbidden the behaviour of polaritons has been associated to polariton superfluidity [75–77] (even at room temperature [78]). In contrast, this feature is not present in the bare polariton dispersion, supporting scattering of the injected polaritons to states

Figure 1.12: Dispersion of the bare polaritons without the nonlinear interactions is shown with blue dashed curve. The solid red line shows the dispersion of the fluctuations. In the fluctuation dispersion there are no other states having the same energy as the state A to backscatter to, however, for the bare polaritons such a state exists.

having same the energy corresponding to the black circle (see Fig. 1.12). The key feature associated to superfluidity is the appearance of the linear dispersion in the vicinity of the pump wave-vector k_p . The sound velocity for the polariton fluid having polariton density $|\phi_c|^2$ is given by

$$c_s = \sqrt{\frac{\alpha_1 |\phi_c|^2}{m_{pol}}}, \quad (1.90)$$

and the flow velocity of the polariton fluid can be given by $v_p = \hbar k_p / m$. When a polariton fluid flows slower than the sound velocity (i.e. $v_p < c_s$), Landau's criterion for superfluidity is satisfied [79].

1.2.8 Potential landscape for exciton-polaritons

Instead of considering a planar microcavity, it is also possible to spatially pattern the microcavity such that the polaritons are provided with a potential pattern, in which they can become trapped. This can be done in two ways: trapping the excitonic or photonic component of the polaritons. The excitonic component of the polaritons can be trapped in many ways such as, by applying strain locally [80], using surface acoustic waves [81], using quantum confined Stark effect in presence of electric field [82], using optically induced excitonic reservoir [83], etc. Modulation of the thickness of the mirrors or applying a metal layer as a mask on top of the DBR [84] are among the most commonly used techniques. However, in all the above mentioned techniques the trapping potential for the polaritons is limited and often on the order of 1 meV (this is not much larger than the typical kinetic energy of polaritons and so results only in a weak trapping). An alternating technique to provide a large confinement potential is to etch away the unnecessary parts of the cavity leaving behind a 0D micropillar structure. The optical modes remain confined inside the micropillar due to the refractive index contrast inside ($n = 3.5$ for GaAs) and outside ($n = 1$) the micropillar. It is also possible to have multiple micropillars in a single sample. If the pillars have some spatial overlap, coupling can be engineered between them. The pillars can then be arranged to form a variety of shapes and geometries, including various lattices [183].

1.2.9 Band structure of exciton-polaritons in a honeycomb strip

In this section we provide the steps to calculate the linear band structure for polaritons in a periodic potential. Ignoring the spins, pump, decay and nonlinearity

the polaritons in a potential can be described as

$$\left[-\frac{\hbar^2}{2m_{pol}} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + V(x, y) \right] \psi(x, y) = E\psi(x, y). \quad (1.91)$$

Here the time dependence of the wave function is removed, which can be taken into account by assuming a stationary state $\phi(x, y) = \psi(x, y) \exp(-iEt/\hbar)$. $V(x, y)$ is the potential representing a strip, which is periodic with periodicity a in the x direction and finite in the y direction:

$$V(x + a, y) = V(x, y). \quad (1.92)$$

In Fig. 1.13(a-b) a honeycomb lattice strip of polariton micropillars, which can be considered as two sublattices (A and B), is shown, respectively. The typical diameters of the micropillars can range from 2-5 μm [85]. Next, we apply the Bloch theorem on the wave function ψ :

$$\psi(x, y) = u(x, y) \exp(ik_x x), \quad (1.93)$$

where $u(x, y) = u(x+a, y)$ is the Bloch mode with wave vector k_x . Being periodic, u and V can be written as

$$u(x, y) = \frac{1}{a} \sum_{G_x} \tilde{u}(G_x, y) \exp(iG_x x), \quad (1.94)$$

$$V(x, y) = \frac{1}{a} \sum_{G_x} \tilde{V}(G_x, y) \exp(iG_x x), \quad (1.95)$$

Figure 1.13: (a) A and B sublattices of the honeycomb lattice. (b) Micropillars arranged in a honeycomb lattice with zig-zag edges. The unit cell is indicated by the green rectangle. (c) Band structure corresponding to the above potential with the bulk modes shown in blue and the edge modes shown in red. (d) Spatial profile of the edge modes as a function of y , which shows the edge modes are located at the edges of the strip.

where $G_x = 2\pi n/a$ is the reciprocal lattice vector, with n being zero or any integer, and

$$\tilde{u}(G_x, y) = \int_{-a/2}^{a/2} u(x, y) \exp(-iG_x x) dx, \quad (1.96)$$

$$\tilde{V}(G_x, y) = \int_{-a/2}^{a/2} V(x, y) \exp(-iG_x x) dx. \quad (1.97)$$

Substituting Eqs. (1.93-1.97) in Eq. (1.91) we get the following eigen-value equation, which can be solved to obtain the band structure

$$\frac{\hbar^2}{2m_{pol}} \left[(k_x + G_x)^2 - \frac{\partial^2}{\partial y^2} \right] \tilde{u}(G_x, y) + \frac{1}{a} \sum_{G'_x} \tilde{V}(G_x - G'_x, y) \tilde{u}(G'_x, y) = E \tilde{u}(G_x, y). \quad (1.98)$$

In Fig. 1.13(c) the band structure of the polaritons in the honeycomb lattice with zigzag edges is shown. The states indicated in red are located at the edges of the strip (see Fig. 1.13(d)). The group velocity of the edge states can be expressed as

$$v_g = \frac{1}{\hbar} \frac{\partial E}{\partial k}. \quad (1.99)$$

Since the edge states are flat they have $v_g = 0$. The edge states became popular since the discovery of the quantum Hall effect where the electrons in a 2D electron gas under uniform perpendicular magnetic field go through cyclotron orbits resulting in quantized Landau levels [86]. In such systems the conductivity through the bulk is negligible whereas the main contribution comes from the edges. Later, Haldane came up with the model [87] showing quantum Hall physics without Landau levels. Since then, a new field of topological physics has emerged where the band structure of the system is manipulated in various ways to achieve the non-trivial phase where the bulk of the system is gapped and bridged by the edge states. Currently, topological phases are investigated in various systems: electronics [88, 89], photonics [90–93], polaritonics [94–99], acoustics [100, 101], etc. In Fig. 1.14 a non-trivial topological band structure for the polaritons is shown. The bulk bands (shown in gray) are gapped and the edge states (shown in red and blue) appear in the bandgap. Unlike the edge states in Fig. 1.13, here the group velocity of the edge states $v_g \neq 0$. Since the edge states are located at the two edges, they have negligible overlap. Due to this polaritons can propagate along the edges with out

Figure 1.14: Topological band structure of polaritons in a honeycomb strip. The bulk bands are represented in gray and the edge states located at the two edges of the strip are shown in blue and red, respectively. The figure is taken from Ref. [94].

being backscattered (because inside the bandgap there is no other state to back scatter to) even in presence of disorder. This is one of the main advantages of the 2D non-trivial topological systems. Such systems are often characterized by a non-zero Chern number [102]. In Chapters (4-5) we introduce schemes to achieve robust edge propagation of polaritons in topologically trivial systems with zero Chern number where the edge states reside together with the bulk.

Chapter 2

On the possibility of a terahertz light emitting diode based on a dressed quantum well

Depending upon the wavelength (or frequency) the electromagnetic spectrum can be divided into many regions. The terahertz (THz) band of the electromagnetic spectrum lies between the minimum of the wavelengths used for electronic communication and maximum of the wavelengths used for optical communication. Electromagnetic waves having frequency between 0.3 to 10 THz (1 THz= 10^{12} Hz) define the THz frequency band. This corresponds to nearly 1 mm to 30 μm in wavelengths. The whole family of the electromagnetic radiation is shown in Fig. 2.1. Although electromagnetic waves in other frequency ranges have developed significantly, THz waves have lagged behind due to absence of proper sources and devices to manipulate them. Consequently, the THz band is also known as the THz gap. There are relatively few commercial products available in the market operating at the THz range compared to the overwhelming number operating at other frequencies. Nevertheless, THz waves have widespread applications. Unlike X-rays, the non-ionizing nature of THz waves reduces the health risk during hu-

man imaging close to none [104]. THz waves can provide high resolution in 3D imaging [105]. Another important application of the THz waves is their ability to detect cancer cells [106]. Besides these, THz waves play important roles in security, other parts of Biology, Medicine, Astrophysics, etc [107].

Figure 2.1: Position of the THz radiation spectrum in the electromagnetic spectrum.

While semiconductors typically feature in the most compact optoelectronic systems, they are challenging to employ in the terahertz range at which electromagnetic transitions are usually absent. Exceptions have appeared in the use of transitions between exciton-polariton branches [108–110], direct-indirect exciton transitions [111, 112], and interband subband transitions [113], however, potential competition with larger quantum cascade lasers is undemonstrated. Cascade systems based on exciton-polaritons [116] and nanoparticles [117] were considered theoretically, but only operate at a fixed frequency. Ideally, a compact semiconductor system could be controlled by an external field to vary its operation frequency.

We have mentioned in sec. 1.2.1 that an intense enough electromagnetic field gives rise to a coupled light-matter object known as a dressed electron, which has been studied in various low dimensional systems, including: semiconductor quantum dots [118, 119], quantum rings [120, 121], quantum wells [26, 122, 123] etc. As the frequency of a dynamic Stark gap can be tuned into the THz range, it is natural to ask whether it can serve in the generation of THz radiation. This is

far from obvious as in a normal QW electromagnetic transitions across a dynamic Stark gap are forbidden as there is no change in the symmetry between electrons in the states at the top and bottom of the gap [119,124]. However, as we will show, the use of an asymmetric QW alleviates this problem. Furthermore, by considering a geometry where the QW is placed between p and n doped semiconductors, we predict that it is possible to realize a pn junction [125,126] operating at terahertz frequency. By considering the band-bending diagram and the transition matrix elements in the QW, we determine the relevant transition rates, and employ a simplified rate equation model to estimate the terahertz generation efficiency.

2.1 Band bending of a quantum well in a pn junction

In this section we will derive the potential felt by the electrons inside a pn junction. We start by deriving the characteristic equations starting from Gauss's law for a pn homo-junction and followed by a hetero-junction where a QW inside a pn junction is considered.

To start, let us consider a p type and n type semiconductor forming a pn junction. The doping concentration for the p side is represented with N_a and that for the n side is represented with N_d . Once the junction is formed electrons from the n side diffuse to the p side recombining with the holes, which forms a depletion region. The depletion region is free of the charge carriers (i.e. electrons or holes) and left with only the charged impurity atoms, which has positive charge at the n side and negative charge at the p side. Due to this, an electric field is formed, which is directed from the n side towards the p side in this region preventing further diffusion of the carriers (see Fig. 2.2 (a)). If z_p and z_n are the widths of the depletion region at the p and n side, respectively, then the charge density

profile can be expressed by [127]

$$\rho(z) = \begin{cases} 0, & \text{for } z < -z_p \\ -eN_a, & \text{for } -z_p < z < 0 \\ eN_d, & \text{for } 0 < z < z_n \\ 0, & \text{for } z > z_n \end{cases}, \quad (2.1)$$

where e is the electronic charge. The total charge inside the device needs to be zero, which is expressed as

$$\begin{aligned} -eN_az_pA + eN_dz_nA &= 0, \\ \therefore N_az_p &= N_dz_n. \end{aligned} \quad (2.2)$$

Here A is the cross section of the semiconductors. The gradient of the electric field can be expressed using Gauss's law,

$$\frac{dE}{dz} = \begin{cases} -eN_a/\epsilon, & \text{for } -z_p < z < 0 \\ eN_d/\epsilon, & \text{for } 0 < z < z_n \\ 0, & \text{elsewhere} \end{cases}, \quad (2.3)$$

where ϵ is the permittivity of the p and n type material. Using the continuity of E at the junction ($z = 0$) and integrating the above expression results in

$$E(z) = \begin{cases} -eN_a(z + z_p)/\epsilon, & \text{for } -z_p < z < 0 \\ eN_d(z - z_n)/\epsilon, & \text{for } 0 < z < z_n \\ 0, & \text{elsewhere} \end{cases}. \quad (2.4)$$

The expression in Eq. 2.2 can be recovered by equating the electric field at the two sides at $z = 0$. The potential profile for the electrons can be calculated using $d\phi(z)/dz = -E(z)$, which yields

Figure 2.2: (a) Schematic diagram of a pn homo-junction under forward bias. The built in electric field points from the n side to the p side, whereas the forward bias field is from the p side to the n side. (b) Band bending due to the built in voltage ϕ_i . The effective voltage across the depletion region decreases in the presence of V_a . This is expected as V_a injects more carriers in the system reducing the depletion region as well as the potential barrier. The black dashed lines represent the original bands of the p and n type of semiconductor materials before the junction is formed.

$$\phi(z) = \begin{cases} 0, & \text{for } z < -z_p \\ eN_a(z + z_p)^2/2\epsilon, & \text{for } -z_p < z < 0 \\ -eN_d(z^2 - 2zz_n)/2\epsilon + eN_az_p^2/2\epsilon, & \text{for } 0 < z < z_n \\ e(N_dz_n^2 + N_az_p^2)/2\epsilon, & \text{for } z_n < z \end{cases}. \quad (2.5)$$

In the above expression it can be seen that the total potential difference across the depletion region is given by

$$\phi_i - V_a = e(N_d z_n^2 + N_a z_p^2)/2, \quad (2.6)$$

where ϕ_i is the built in voltage across the depletion region and V_a is the externally applied forward bias voltage. Using the expression in Eq. 2.2 the depletion width parameters z_p and z_n can be expressed as functions of applied voltage as

$$z_p = \sqrt{\frac{2\epsilon}{e} \frac{N_d}{N_a(N_a + N_d)} (\phi_i - V_a)}, \quad (2.7)$$

$$z_n = \sqrt{\frac{2\epsilon}{e} \frac{N_a}{N_d(N_a + N_d)} (\phi_i - V_a)}, \quad (2.8)$$

$$w = z_p + z_n = \sqrt{\frac{2\epsilon}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) (\phi_i - V_a)}. \quad (2.9)$$

In Fig. 2.2(b) the band bending diagram is shown. As expected the forward bias voltage effectively decreases the built in potential across the depletion region allowing recombination of the electrons with the holes.

We have mentioned in the introduction of this thesis that homo-junction based devices are not very efficient and one major factor determining the efficiency is the carrier confinement, which is very low in such devices. This can be improved significantly by considering an intrinsic QW sandwiched between a p-n junction. The gradient of the electric field, E , along the growth direction, z , can be expressed using Gauss's law:

$$\frac{dE}{dz} = \begin{cases} -\frac{eN_a}{\epsilon}, & \text{for } -(z_p + l/2) \leq z \leq -l/2 \\ \frac{e(N_h - N_e)}{l\epsilon}, & \text{for } -l/2 \leq z \leq l/2 \\ \frac{eN_d}{\epsilon}, & \text{for } l/2 \leq z \leq z_n + l/2 \end{cases}. \quad (2.10)$$

Here l is the thickness of the QW; N_h and N_e are the number of holes and electrons

per unit area inside the QW. Integrating Eq. (2.10) and applying the continuous boundary conditions at the junctions yields

$$E(z) = \begin{cases} -\frac{eN_a}{\epsilon}(z + z_p + l/2), & \text{for } -(z_p + l/2) \leq z \leq -l/2 \\ \frac{e(N_h - N_e)}{l\epsilon}z - \frac{e(N_h - N_e)}{2\epsilon} - \frac{eN_d z_n}{\epsilon}, & \text{for } -l/2 \leq z \leq l/2 \\ \frac{eN_d z}{\epsilon} - \frac{eN_d z}{\epsilon}(z_n + l/2), & \text{for } l/2 \leq z \leq z_n + l/2 \end{cases} \quad (2.11)$$

The potential variation for the electrons throughout the double heterojunction can be expressed as

$$\phi(z) = \begin{cases} 0, & \text{for } z \leq -(z_p + l/2) \\ \frac{eN_a}{2\epsilon} \left[z^2 + 2z \left(z_p + \frac{l}{2} \right) + \left(z_p + \frac{l}{2} \right)^2 \right], & \text{for } -(z_p + l/2) \leq z \leq -l/2 \\ \phi_p + \frac{e(N_h - N_e)}{2\epsilon} \left[-\frac{z^2}{l} + z + \frac{3l}{4} \right] + \frac{eN_d z_n}{2\epsilon}(l + 2z), & \text{for } -l/2 \leq z \leq l/2 \\ \phi_p + \phi_{QW} + \frac{eN_d}{2\epsilon} \left[z^2 - 2 \left(z_n + \frac{l}{2} \right) z + z_n l + \frac{l^2}{4} \right], & \text{for } l/2 \leq z \leq z_n + l/2 \\ \phi_p + \phi_{QW} + \phi_n, & \text{for } z \geq z_n + l/2 \end{cases} \quad (2.12)$$

Here z_p and z_n are the widths of the depletion regions in the p and n side, respectively; $\phi_p + \phi_{QW} + \phi_n = \phi_i - V_a$, is the potential difference between the p side and the n side; ϕ_i is the total built in potential; $\phi_p = eN_a z_p^2 / 2\epsilon$, $\phi_{QW} = e(N_h - N_e)l / 2\epsilon + eN_d z_n l / \epsilon$, and $\phi_n = eN_d z_n^2 / 2\epsilon$ are the built in potentials in the p side, in the QW, and in the n side, respectively and V_a is the externally applied forward bias voltage. The electron and hole density in the QW can be expressed as

$$N_e = \int_{E_c^0}^{\infty} \rho(E) f(E) dE, \quad (2.13)$$

where $\rho(E) = m_e / \pi \hbar^2$ is the number of electronic states per unit area inside the

QW and $f(E) = 1 / \left[1 + \exp \left(\frac{E - F_n}{k_B T} \right) \right]$ is the Fermi distribution function. Substituting these inside Eq. 2.13 yields

$$N_e = \frac{m_e}{\pi \hbar^2} k_B T \ln \left[1 + \exp \left(\frac{F_n - E_c^0}{k_B T} \right) \right]$$

Following similar steps the expression for N_h can also be obtained as,

$$N_h = \frac{m_h}{\pi \hbar^2} k_B T \log \left[1 + \exp \left(\frac{|F_p| - |E_v^0|}{k_B T} \right) \right]. \quad (2.14)$$

Here m_e and m_h are the effective electron and hole masses in the QW. k_B is Boltzmann's constant and T is the temperature. E_c^0 and E_v^0 correspond to the minima and maxima of first electron and hole subbands of the QW, respectively. F_n and F_p are the quasi Fermi levels in the conduction band and in the valence band of the QW, respectively, which can be expressed as [127, 128]

$$\begin{aligned} F_n &= E_c^n - e(\phi_p + \phi_{QW} + \phi_n) \\ &\quad + k_B T \left[\ln \left(\frac{N_d}{N_c^n} \right) + \frac{1}{\sqrt{8}} \left(\frac{N_d}{N_c^n} \right) - \left(\frac{3}{16} - \frac{\sqrt{3}}{9} \right) \left(\frac{N_d}{N_c^n} \right)^2 + \dots \right], \quad (2.15) \\ F_p &= -E_v^p - k_B T \left[\ln \left(\frac{N_a}{N_v^p} \right) + \frac{1}{\sqrt{8}} \left(\frac{N_a}{N_v^p} \right) - \left(\frac{3}{16} - \frac{\sqrt{3}}{9} \right) \left(\frac{N_a}{N_v^p} \right)^2 + \dots \right]. \end{aligned} \quad (2.16)$$

Here $N_c^n = 2(m_e k_B T / 2\pi \hbar^2)^{3/2}$ is the effective density of states at the conduction band edge of the n type semiconductor; $N_v^p = 2(m_h k_B T / 2\pi \hbar^2)^{3/2}$ is the effective density of states at the valence band edge of the p type semiconductor; E_c^n is the minimum of the conduction band in the n side; E_v^p is the maximum of the valence band in the p side. Using Eqs. (2.12-2.16) the quasi Fermi levels of the QW can be controlled. The effect of $\phi(z)$ on the electron and hole potentials throughout the double heterojunction is plotted in Figure 2.3 where we consider GaAs and

AlGaAs as appropriate materials.

Figure 2.3: Potential variation in the double heterojunction consisting of an $\text{Al}_{0.15}\text{Ga}_{0.85}\text{As}$ p type layer, GaAs QW, and $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$ n type layer. The electric field inside the pn junction makes the QW asymmetric. The black dotted line indicates the potential of each layer before they are connected. Parameters: $N_a = 2 \times 10^{16} \text{ cm}^{-3}$, $N_d = 10^{16} \text{ cm}^{-3}$, $T = 4 \text{ K}$, $l = 8 \text{ nm}$, $\varepsilon_g = 1.4 \text{ eV}$, $m_e = 0.067m_0$, $m_h = 0.47m_0$, m_0 being the free electron mass, $F_n = 50 \text{ meV}$, $F_p = 2.5 \text{ meV}$, $\phi_p = 137 \text{ V}$, $\phi_n = 140 \text{ V}$, $E_c^n = 0.9 \text{ eV}$, $E_v^p = -0.8 \text{ eV}$. This figure is taken from Ref. [103].

2.2 QW energy spectrum under electromagnetic dressing

We consider now that the system is subjected to a linearly polarized high frequency monochromatic electromagnetic field, $E \cos \omega_0 t$, which is linearly polarized along the z direction (direction perpendicular to the QW plane) with the electric field amplitude E and the frequency ω_0 . For simplicity, we have assumed that the electric field is spatially homogeneous. Assuming that the photon energy is greater than the light-matter interaction characteristic energy ($\hbar\omega_0 \gg dE$, where d is the interband dipole moment), the corresponding Hamiltonian can be expressed

as [26, 129, 130]

$$\hat{H} = \hbar\omega_0\hat{a}^\dagger\hat{a} + \frac{\varepsilon^c(\mathbf{k}) + \varepsilon^v(\mathbf{k})}{2}\hat{I} + \frac{\varepsilon^c(\mathbf{k}) - \varepsilon^v(\mathbf{k})}{2}\hat{\sigma}_z - id\sqrt{\frac{2\pi\hbar\omega_0}{V}}(\hat{\sigma}_+\hat{a} - \hat{\sigma}_-\hat{a}^\dagger). \quad (2.17)$$

The superscripts c and v correspond respectively to the conduction and valence bands; $\hat{a}$ and $\hat{a}^\dagger$ are photon creation and annihilation operators; and $\hat{\sigma}_\pm$ are the electron interband transition operators.

We choose the dressing field frequency ω_0 such that $\hbar\omega_0 > \varepsilon_g$, where ε_g is the band gap of the QW. Under this condition the dressing field strongly mixes the valence band and the conduction band of the quantum well and the stationary state of the system shows energy gaps $\Delta\varepsilon$ at the resonant points of the Brillouin zone, k_0 , where the energy of the photons $\hbar\omega_0$ matches with the energy difference between the bands (see Fig. 2.4(a)) [131]. The modified energy spectrum $\varepsilon(\mathbf{k})$ that arises due to the mixing of the two bands $\varepsilon^c(\mathbf{k})$ and $\varepsilon^v(\mathbf{k})$ is expressed by [26]

$$\varepsilon(\mathbf{k}) = \frac{\varepsilon^c(\mathbf{k}) + \varepsilon^v(\mathbf{k})}{2} \pm \frac{\hbar\omega_0}{2} \pm \frac{\alpha(\mathbf{k})}{2|\alpha(\mathbf{k})|} \sqrt{(\Delta\varepsilon)^2 + \alpha^2(\mathbf{k})}. \quad (2.18)$$

Here $\mathbf{k} = (k_x, k_y)$ is the in plane electron wave vector; $\varepsilon^c(\mathbf{k}) = \hbar^2 k^2 / 2m_e + \varepsilon_g / 2$ is the first electron sub-band; $\varepsilon^v(\mathbf{k}) = -\hbar^2 k^2 / 2m_h - \varepsilon_g / 2$ is the first hole sub-band; $\alpha(\mathbf{k}) = \varepsilon^c(\mathbf{k}) - \varepsilon^v(\mathbf{k}) - \hbar\omega_0$ is the resonance detuning; $\Delta\varepsilon = dE$ is the dynamic Stark gap. It should be noted that the dressing field amplitude is chosen in such a way that it satisfies the strong light-matter coupling condition and consequently is not absorbed. This condition can be expressed as $\Omega_R \tau_e \gg 1$, where $\Omega_R = \Delta\varepsilon / \hbar$ is the Rabi frequency of interband electron transitions and τ_e is the average lifetime of the electron states. In Fig. 2.4 the modified spectrum of a GaAs based QW is shown. Since the effective mass of the holes is much lighter compared to the electrons the valence band appears flat.

Figure 2.4: Energy spectrum of the first electron and hole sub-band for a dressed GaAs QW. Since in GaAs based systems $m_h \gg m_e$ the upper dressed valence band appears flat. The two quasi Fermi levels, F_n and F_p , are shown with red dashed lines. The green and black arrows indicate the THz and allowed optical transitions, respectively. The black curved arrows indicate the possible direct optical transition from the top conduction band to the bottom valence band which is prevented by setting F_p at the Stark gap in the valence band. Parameters: $\hbar\omega_0 = \varepsilon_g + 15$ meV, $\Delta\varepsilon = 8$ meV. All other parameters are kept the same as those in Figure 2.3. This figure is taken from Ref. [103].

2.3 THz transition matrix element and transition rate

In order to estimate the THz rate we calculate the matrix element for the THz transition, defined as

$$M_T = \langle \psi_N^c(\mathbf{k}) | \mathbf{r} | \psi_N^v(\mathbf{k} + \mathbf{q}) \rangle. \quad (2.19)$$

Here $\mathbf{q}$ is the wave vector of the emitted photon, $|\psi_N^c(\mathbf{k})\rangle$ and $|\psi_N^v(\mathbf{k})\rangle$ are the dressed conduction and valence band states with photon number N , respectively

and expressed as [26]

$$|\psi_N^c(\mathbf{k})\rangle = \sqrt{\frac{\Omega(\mathbf{k}) + |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_c(\mathbf{k}), N\rangle + i \frac{\alpha(\mathbf{k})}{|\alpha(\mathbf{k})|} \sqrt{\frac{\Omega(\mathbf{k}) - |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_v(\mathbf{k}), N+1\rangle \quad (2.20)$$

$$|\psi_N^v(\mathbf{k})\rangle = \sqrt{\frac{\Omega(\mathbf{k}) + |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_v(\mathbf{k}), N\rangle + i \frac{\alpha(\mathbf{k})}{|\alpha(\mathbf{k})|} \sqrt{\frac{\Omega(\mathbf{k}) - |\alpha(\mathbf{k})|}{2\Omega(\mathbf{k})}} |\psi_c(\mathbf{k}), N-1\rangle. \quad (2.21)$$

Here $|\psi_c(\mathbf{k})\rangle$ and $|\psi_v(\mathbf{k})\rangle$ are the bare conduction and valence band states, respectively; and $\Omega(\mathbf{k}) = \sqrt{(\Delta\varepsilon)^2 + \alpha^2(\mathbf{k})}$. In a QW, the wave functions of the conduction and valence sub-bands can be written as

$$\psi_c(\mathbf{k}) = \psi_{1c}(z) u_c(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}} \quad (2.22)$$

$$\psi_v(\mathbf{k}) = \psi_{1v}(z) u_v(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}} \quad (2.23)$$

where $u_c(\mathbf{r})$ and $u_v(\mathbf{r})$ are the Bloch functions of conduction and valence bands in a bulk semiconductor material; $\psi_{1c}(z)$ and $\psi_{1v}(z)$ are the envelope wave functions of the first conduction and valence sub-band arising from the quantization of transverse motion of electrons and holes in the QW. We have to take into account that the Bloch functions, $u_{c,v}(\mathbf{r})$, oscillate with the crystal lattice period, a , whereas the characteristic scale of the wave functions $\psi_{1c,1v}(z)$ is the QW thickness, $l \gg a$. Taking the above mentioned conditions into account the interband dipole moment can be expressed as

$$d = \langle \psi_{1c}(z) | e z | \psi_{1v}(z) \rangle. \quad (2.24)$$

Since the dipole operator ($e\mathbf{r}$) will only act on the electronic part of the wave function and the photon states corresponding to different photon number are or-

thogonal, the THz matrix element in Eq. (2.19) can be approximated to

$$M_T \simeq \frac{1}{2} [\langle \psi_{1c}(z) | z | \psi_{1c}(z) \rangle - \langle \psi_{1v}(z) | z | \psi_{1v}(z) \rangle]. \quad (2.25)$$

This corresponds to the emitted photons having linear polarization along z axis. The matrix elements corresponding to photons having polarization in the other two directions (i.e. along x and y) vanish. For a symmetric QW both the terms in Eq. (2.25) will go to zero separately, but for an asymmetric QW the wave functions $\psi_{1c}(z)$ and $\psi_{1v}(z)$ do not have a definite parity resulting in nonzero M_T . Using our parameters we have found that $M_T \simeq 1$ nm, which is similar to the value obtained in Ref. [113]. The THz transition rate can be expressed as

$$W_T(\omega_T) = \frac{4e^2 |M_T|^2 \sqrt{\epsilon_r}}{\hbar \pi \epsilon_0 c^3} \rho_T(\omega_T) \omega_T^3. \quad (2.26)$$

Here, ϵ_r is the dielectric constant of the QW, ϵ_0 is the free space permittivity, c is the velocity of light in free space, and $\rho_T(\omega_T)$ is the total number of electronic states in the upper conduction band participating in the THz transition with frequency ω_T . In order to realize a monolithic THz source we consider placing the system inside a THz cavity. It is possible to use metal microcavities where the light is confined at sub-wavelength scales in the THz frequency range and the dispersion of the THz photons can be flat [132]. This also allows us to neglect THz transitions other than the one that is resonant with the cavity. The presence of the cavity modifies the three dimensional continuum of photonic modes to a two dimensional continuum of photonic modes and Eq. (2.26) becomes [133]

$$W_T(\omega_T) = \frac{4e^2 |M_T|^2}{\hbar \pi \epsilon_0 c^2 L_c} \rho_T(\omega_T) \omega_T^2. \quad (2.27)$$

Here L_c is the length of the cavity. Due to the presence of $\mathbf{q}$, we choose to calculate ρ_T numerically using,

$$\rho_T(\omega_T) = 2 \sum_{\mathbf{k}_i} \delta(\varepsilon_i(\mathbf{k}_i) - \varepsilon_f(\mathbf{k}_i + \mathbf{q}) - \hbar\omega_T) \quad (2.28)$$

where the summation runs over all the initial states $\mathbf{k}_i$. Each electron making the THz transition can also undergo an optical transition directly to the lowest valence band as shown with the dashed arrows in Fig. 2.4. This undesired transition can be stopped by setting F_p at the Stark gap (see Fig. 2.4). It should be noted that the transition from the top conduction band to the top valence band is restricted by momentum conservation. In order to estimate the efficiency of the device we solve the rate equations for different energy bands coupled with the THz photons inside the cavity,

$$\begin{aligned} \frac{\partial n_u}{\partial t} &= R_{in} - W_T n_u (n_t + 1) + W_T n_l n_t - \frac{n_u}{\tau_e} \\ \frac{\partial n_l}{\partial t} &= W_T n_u (n_t + 1) - W_T n_l n_t - \frac{B}{V} n_l p - \frac{n_l}{\tau_e} \\ \frac{\partial n_t}{\partial t} &= W_T n_u (n_t + 1) - W_T n_l n_t - \frac{n_t}{\tau_t} \end{aligned} \quad (2.29)$$

where n_u and n_l are the number of the electrons in the upper and lower conduction band of the QW, respectively; n_t and τ_t are the number of THz photons inside the cavity and their lifetime, respectively; B is the radiative recombination rate in usual QWs, which has the typical value $2 \times 10^{-10} \text{ cm}^3/\text{s}$ [134]; V is the volume of the QW; p is the number of holes in the QW, which is calculated from F_p ; τ_e is the average lifetime of the electrons, which we take as 0.1 ns; and R_{in} is the rate at which the electrons are injected into the QW. In principle, the THz transitions corresponding to $\hbar\omega_T > \Delta\varepsilon$ can be considered, which requires the probability of occupation for each state in the conduction band, however this is

Figure 2.5: Variation of the device efficiency, η (blue line), and output intensity, I_{out} (dashed red line), as a function of the emitted THz photon frequency from a dressed QW with sides 2.5 mm. Parameters: $L_c = 0.8 \mu\text{m}$, $\tau_e = 0.1 \text{ ns}$, $\tau_t = 0.5 \text{ ps}$, $B = 2 \times 10^{-10} \text{ cm}^3/\text{s}$. This figure is taken from Ref. [103].

Figure 2.6: Plot of the optimum THz photon frequency in (a) and its corresponding efficiency in (b), as functions of cavity length, L_c , and average electronic lifetime, τ_e . The values of L_c are taken from Ref. [132]. All the parameters are kept the same as those in Fig. 2.5. This figure is taken from Ref. [103].

beyond the scope of the present work. We limit ourselves to the THz transitions between the states that lie near the Stark gap such that $\hbar\omega_T \simeq \Delta\epsilon$. This allows us to assume that, due to the fast relaxation of the electrons, the lower energy states in the upper conduction band are filled and the higher energy states in the lower

conduction band are empty. Electron-electron scattering is likely to increase with the increase in temperature, which will put the system out of the strong coupling regime. To discard any temperature effect we consider that the system is in a low temperature environment such that $k_B T \ll \Delta\varepsilon$, which can be achieved using liquid ^4He . The input intensity (I_{in}) is defined as

$$I_{in} = \frac{eR_{in}V_a}{A} + \frac{1}{2}\varepsilon_0 c E^2. \quad (2.30)$$

Here $V_a = (F_n - F_p)/e$, and the first and second term in the right-hand side of Eq. (2.30) represent the intensity due to the electric current in the device and the dressing field intensity, respectively. The output intensity (I_{out}) is defined as

$$I_{out} = \frac{1}{A} (n_t/\tau_t) \hbar\omega_T, \quad (2.31)$$

where, the term n_t/τ_t represents the rate of emission of the THz photons from the cavity. In a GaAs based QW the interband dipole moment, $d \simeq 10$ D, and a Stark gap of meV order corresponds to $E \simeq 10^5$ V/cm [131]. The efficiency of the device, defined as, $\eta = I_{out}/I_{in}$, is calculated by finding the steady state of the rate equations in Eq. (2.29), which is obtained in a self consistent way where for each THz frequency the total number of electrons in the QW (defined by the F_n) is kept constant by varying R_{in} . In Fig. 2.5 the efficiency and output intensity are plotted as a function of THz frequency. The efficiency shows a maximum near 1.5 THz. This corresponds to the Pauli exclusion principle. Once all the electronic states in the lower conduction band are filled the population of the THz photons does not increase even after increasing ω_T . Although I_{out} increases linearly as a function of ω_T , I_{in} increases quadratically resulting in a maximum in the efficiency. It should be noted that ω_T can be easily tuned by changing the intensity of the dressing field. As shown in Fig. 2.6, the optimum frequency,

ω_T^{op} , of the THz emission can be controlled mainly by varying L_c with τ_e having very little effect on it. However, the optimum efficiency, η^{op} , decreases with the increase of L_c and τ_e . This is understandable, as from Eq. 2.27 it is clear that THz transition rate, W_T , decreases with the increase of L_c . On the other hand, the average electronic lifetime, τ_e , vary depending upon all the possible non-radiative scattering processes. Smaller τ_e creates vacancy in the lower conduction band increasing the THz transition probability, which results in higher η^{op} . The typical efficiency of the device, $\eta \simeq 7 \times 10^{-6}$, which is comparable to other state-of-the-art proposals for THz emission from Stark split bands [113]. The main energy cost in the considered system derives from the second term in Eq. (2.30) (right-hand side), that is, the energy needed to supply the dressing field. As this energy is not absorbed by the system, one could consider that this component of the input energy can be recycled in principle. In such an idealistic case, the efficiency would become $\eta \simeq 5 \times 10^{-3}$. One can also define the quantum efficiency, $\zeta = (n_t/\tau_t)/R_{in}$, which represents the number of THz photons emitted per injected electron. The typical quantum efficiency of the device, $\zeta \simeq 0.5$, which is very close to unity.

2.4 Discussion and conclusion

Quantum cascade lasers (QCL) are one of the best high power THz sources available, which operate approximately 0.8 - 5 THz frequency range [114]. Recently, the concept of QCL has been extended to the field of topological insulators, where topological valley Hall photonic crystal based cavity was demonstrated experimentally to work as a robust THz QCL [115]. Typically THz QCL have output power around a few Watts (W). Apart from QCL, the resonant tunnelling diodes can emit THz waves having power around 1 mW. The Bloch oscillation of the electrons in a superlattice under a DC electric field can also emit THz waves [162]. Although, the scheme presented in this chapter is estimated to emit THz waves

having intensity around 1 W/cm^2 , the efficiency is orders of magnitude smaller than the QCLs. This is because we need an intense dressing field to open the THz gap first in order to generate the THz waves.

The linewidth of the THz emission in the presented scheme is determined by the quality factor of the THz cavity. The THz cavities based on metallic mirrors are excellent in 1-9 THz frequency range and have linewidth around 0.1 THz [132]. Since in our scheme we have used such a cavity, we expect the linewidth of the THz emission to be around 0.1 THz.

To conclude, we have considered theoretically a THz source based on a dressed QW inside a highly doped p-n junction coupled to a THz cavity. By solving the rate equations we showed that the system subjected to a forward bias voltage can act as a THz emitting diode having energy efficiency of 7×10^{-6} and quantum efficiency of 0.5. The advantages of our scheme are that it is composed of only one quantum well, making it more compact and easier to fabricate, and the device is tuneable such that it can give different THz frequencies. In the next chapter we show that the dynamic Stark gaps in a dressed quantum well restrict the electron motion. This can be used to realize a frequency comb when excited with sub-THz frequency.

Chapter 3

Semiconductor quantum well irradiated by a two-mode electromagnetic field as a terahertz emitter

In recent time, lasers and microwave techniques are actively exploited to control the nonlinear optoelectronic properties of various systems with a strong high-frequency electromagnetic field (dressing field). The physical properties of dressed electrons can be described within the conventional Floquet theory of periodically driven quantum systems [136–139] and have been studied for various nanostructures, including semiconductor quantum wells [26, 122, 123, 140, 141], quantum dots [142, 143], quantum rings [120, 144–146], graphene [147–151], etc. Among a variety of nonlinear optical phenomena induced by a dressing field, the opening of dynamic Stark gaps in electron energy spectra of various condensed-matter structures should be noted especially (see, e.g., Refs. [26, 131, 144, 152, 153]). In the present chapter, we demonstrate theoretically that this nonlinear effect results in the emission of a terahertz frequency comb from a semiconductor quantum well

(QW) irradiated by a two-mode electromagnetic field.

3.1 Model

Let us consider a two-dimensional electron gas (2DEG) in a semiconductor QW irradiated by a linearly polarized two-mode electromagnetic wave, which propagates along the z axis and consists of a strong high-frequency dressing field, $\tilde{E} \cos \omega_0 t$, with the electric field amplitude, $\tilde{E}$, and the frequency, ω_0 , and a relatively weak and low-frequency driving field, $E \cos \omega t$, with the electric field amplitude, E , and the frequency, ω (see Fig. 3.1(a)). In what follows, we will assume that the dressing field frequency, ω_0 , satisfies the condition $\hbar\omega_0 > \varepsilon_g$, where ε_g is the band gap of the QW. In this case, the dressing field strongly mixes electron states from the valence and conduction bands and, therefore, substantially modifies their energy spectrum due to the dynamic Stark effect. Particularly, the dressing field opens energy gaps, $\Delta\varepsilon$, within the bands in the resonant points of the Brillouin zone, k_0 , where the photon energy, $\hbar\omega_0$, is equal to the energy interval between the bands [131]. This gap opening is shown schematically in Figs. 3.1 (b-c), where the first valence and conduction subbands of the QW are plotted in the Schrödinger and interaction pictures, correspondingly. For definiteness, we will restrict the following analysis by the Schrödinger picture. In the most general case, the energy spectrum of dressed electrons arising from any two bands mixed by the dressing field, $\varepsilon^+(\mathbf{k})$ and $\varepsilon^-(\mathbf{k})$, can be written in the form [26]

$$\varepsilon(\mathbf{k}) = \frac{\varepsilon^+(\mathbf{k}) + \varepsilon^-(\mathbf{k})}{2} \pm \frac{\hbar\omega_0}{2} \pm \frac{\alpha(\mathbf{k})}{2|\alpha(\mathbf{k})|} \sqrt{(\hbar\Omega_R)^2 + \alpha^2(\mathbf{k})}, \quad (3.1)$$

where $\alpha(\mathbf{k}) = \varepsilon^+(\mathbf{k}) - \varepsilon^-(\mathbf{k}) - \hbar\omega_0$ is the resonance detuning, $\mathbf{k}$ is the electron wave vector, $\Omega_R = d\tilde{E}/\hbar$ is the Rabi frequency of interband electron transitions, and d is the interband dipole moment. In the considered case of a semiconductor QW, these

two branches, $\varepsilon^+(\mathbf{k})$ and $\varepsilon^-(\mathbf{k})$, should be treated as the first electron subband, $\varepsilon^+(\mathbf{k}) = \hbar^2 k^2 / 2m_e + \varepsilon_g / 2$, and the first hole subband, $\varepsilon^-(\mathbf{k}) = -\hbar^2 k^2 / 2m_h - \varepsilon_g / 2$, where m_e and m_h are the electron and hole masses, respectively, and $\mathbf{k} = (k_x, k_y)$ is the electron wave vector in the QW plane. Since the dynamic Stark gaps, $\Delta\varepsilon = \hbar\Omega_R$, break the dispersion curves (3.1) only at the border wave vector, $|\mathbf{k}| = k_0$, the energy spectrum (3.1) around the ground state of the conduction band (see the red dot in Fig. 3.1(b)) is continuous and can be written as

$$\varepsilon_e(\mathbf{k}) = \frac{\hbar^2 k^2}{2m_-} + \frac{\hbar\omega_0}{2} - \frac{1}{2} \sqrt{(\Delta\varepsilon)^2 + \left[\hbar\omega_0 - \varepsilon_g - \frac{\hbar^2 k^2}{2m_+} \right]^2}, \quad (3.2)$$

where $m_{\pm} = m_e m_h / (m_e \pm m_h)$ is the reduced electron-hole mass, the electron wave vector, $\mathbf{k}$, lies within the range $|\mathbf{k}| \leq k_0$, and the border wave vector, k_0 , satisfies the condition $\hbar\omega_0 = \varepsilon_g + \varepsilon_0$ with the characteristic border electron energy, $\varepsilon_0 = \hbar^2 k_0^2 / 2m_+$. It should be stressed that the dressing field amplitude, $\tilde{E}$, is assumed to satisfy the strong light-matter coupling condition, $\Omega_R \tau \gg 1$, where τ is the mean free time of conduction electrons in the QW. In this regime, the interband absorption of the dressing field is absent (see, e.g., Ref. [26]) and, therefore, the dressing field does not generate electron-hole pairs.

As to the driving field, its frequency is relatively low, $\hbar\omega \ll \varepsilon_g$. Therefore, the off-resonant driving field does not mix almost electronic states from the conduction and valence bands and, correspondingly, does not change the energy spectrum (3.1) renormalized by the resonant dressing field. Thus, the driving field results only to the electron transitions between different states of the continuous spectrum (3.2), which can be described within the conventional semiclassical approach as follows. First of all, it should be stressed that the magnetic component of the driving field lies in the plane (x, y) . Since the electron velocities in the QW, $\mathbf{v} = (v_x, v_y)$, lie in the same plane, the Lorentz force caused by the magnetic field is directed perpendicularly to the QW. Therefore, it cannot affect electron dynamics in the

Figure 3.1: Sketch of the system under consideration: (a) Two-dimensional electron gas (2DEG) in a semiconductor quantum well (QW) irradiated by a two-mode electromagnetic field consisting of a dressing field with the electric field amplitude, $\tilde{E}$, and the frequency, ω_0 , and a driving field with the electric field amplitude, E , and the frequency, ω ; (b) Energy spectrum of the first conduction and valence subbands of the QW with the band gap, ϵ_g , in the presence of the dressing field (the blue lines) and without the field (the green lines) in the Schrödinger picture. Due to the gaps, $\Delta\epsilon$, induced by the dressing field, the driving field can move a conduction electron (the red dot) along the dispersion curve only between the states $-k_0$ and k_0 ; (c) Energy spectrum of the first conduction and valence subbands of the QW in the presence of the dressing field (the blue lines) and without the field (the green lines) plotted in the interaction picture; (d) Energy spectrum of the first electron subband in a GaAs-based QW irradiated by a dressing field with the frequency $\hbar\omega_0 = \epsilon_g + 1 \text{ meV}$ and the amplitude $\tilde{E} = 10^7 \text{ V/m}$. This figure is taken from Ref. [135].

QW plane. As a consequence, the dynamics of dressed electrons in the QW de-

depends only on the electric component of the driving field, $\mathbf{E} = (E \cos \omega t, 0)$. Within the continuous energy spectrum (3.2), the electron dynamics can be described by the energy conservation law, $e\mathbf{v}\mathbf{E} = \partial\varepsilon_e(\mathbf{k})/\partial t$, where the left part is the energy received by a dressed electron from the driving field per unit time. It follows from the exact solution of the electron-photon Schrödinger problem [26] that the averaged velocity of the dressed electron in the state with the wave vector $\mathbf{k}$ is given by the expression $\mathbf{v}(\mathbf{k}) = (1/\hbar)\partial\varepsilon_e(\mathbf{k})/\partial\mathbf{k}$, which coincides formally with the well-known classical Hamilton equation, $\mathbf{v}(\mathbf{p}) = \partial\varepsilon(\mathbf{p})/\partial\mathbf{p}$, describing the velocity of a particle with an energy $\varepsilon(\mathbf{p})$ and the generalized momentum $\mathbf{p} = \hbar\mathbf{k}$. Substituting this electron velocity, $\mathbf{v}(\mathbf{k})$, into the energy conservation law, we arrive at the conventional dynamics equation,

$$\hbar \frac{d\mathbf{k}}{dt} \Big|_{|\mathbf{k}| < k_0} = e\mathbf{E}. \quad (3.3)$$

Since the driving field is substantially weaker than the dressing field, a dressed electron cannot surmount the dynamic Stark gap, $\Delta\varepsilon$, at the border wave vector, $|\mathbf{k}| = k_0$ (see Fig. 3.1(b)). Taking this energy barrier into account, one has to complement the semiclassical dynamics equation (3.3) with the boundary condition,

$$\hbar \frac{d\mathbf{k}}{dt} \Big|_{|\mathbf{k}|=k_0} = \begin{cases} 0, & e\mathbf{k}\mathbf{E} > 0 \\ e\mathbf{E}, & e\mathbf{k}\mathbf{E} \leq 0 \end{cases}. \quad (3.4)$$

The three expressions (3.2), (3.3) and (3.4) give the complete description of the single-electron dynamics in the irradiated QW. For definiteness, let us apply them to consider an electron which is initially in the ground state, $\mathbf{k} = 0$ (see the red dot in Fig. 3.1(b)). In this case, the electron wave vector depends on time, t , as a

function $\mathbf{k}(t) = (k_x(t), 0)$, where

$$k_x(t) = \begin{cases} k_E \sin \omega t, & |k_E \sin \omega t| < k_0 \\ k_0, & k_E \sin \omega t \geq k_0 \\ -k_0, & k_E \sin \omega t \leq -k_0 \end{cases}. \quad (3.5)$$

and $k_E = eE/\hbar\omega$ is the amplitude of electron oscillations in k -space. The oscillating movement of the electron, which arises from the sines in Eq. (3.5), should result in the emission of electromagnetic waves from the QW. Restricting analysis of the emission by the dipole approximation, we can write the intensity of the emission at the frequency of the n -th harmonic, $\omega_n = n\omega$, in the conventional form (see, e.g., Ref. [154]),

$$I_n = \frac{4e^2}{3c^3} |\mathbf{a}_n|^2, \quad (3.6)$$

where $\mathbf{a}_n$ is the n -th coefficient of the Fourier expansion of the electron acceleration, $\mathbf{a} = (a_x, 0)$. It follows from Eq. (3.2) that the electron acceleration, $a_x = (\dot{v}_x, 0)$, can be written in the dimensionless form as

$$\begin{aligned} \frac{a_x}{a_0} = \frac{\dot{k}_x}{\omega k_0} & \left\{ 2 \left(\frac{m_+}{m_-} \right) + 2 \left(\left[\frac{\Delta\varepsilon}{\varepsilon_0} \right]^2 + \left[1 - \frac{k^2}{k_0^2} \right]^2 \right)^{-\frac{3}{2}} \right. \\ & \times \left[1 - \frac{k^2}{k_0^2} \right]^2 \frac{k_x^2}{k_0^2} + \left(\left[\frac{\Delta\varepsilon}{\varepsilon_0} \right]^2 + \left[1 - \frac{k^2}{k_0^2} \right]^2 \right)^{-\frac{1}{2}} \\ & \left. \times \left[1 - \frac{k^2}{k_0^2} - 2 \frac{k_x^2}{k_0^2} \right] \right\}, \end{aligned} \quad (3.7)$$

where $a_0 = \hbar k_0 \omega / 2m_+$ is the characteristic electron acceleration. The expressions (3.5)–(3.7) define the spectrum of electromagnetic emission from the QW, which is discussed below. For definiteness, we will proceed with the analysis of a GaAs-based QW ($\varepsilon_g = 1.39$ eV, $m_e = 0.067 m_0$, $m_h = 0.47 m_0$) which is a suitable system for experimental observation of the discussed effects (see the corresponding electron energy spectrum in Fig. 3.1(d)).

3.2 Results and discussion

The features of the electromagnetic emission from the QW originate from the dynamic Stark gaps, $\Delta\varepsilon$, which take place at the border electron wave vector, $|\mathbf{k}| = k_0$ (see Fig. 3.1(b)). Physically, these gaps confine electron oscillations induced by the driving field within the domain $-k_0 \leq k_x \leq k_0$ and, therefore, crucially affect the emission spectrum. It follows from Eq. (3.5) that the electron wave vector (3.5) depends on the time, t , as a purely harmonic function, $k_x(t) = k_E \sin \omega t$, if $k_E \leq k_0$. As a consequence, the electron acceleration, $\dot{a}_x \sim \dot{k}_x \sim \cos \omega t$, is also a harmonic function in this case. This results in the electromagnetic emission from the QW only with the frequency of the driving field, ω , if $k_E \leq k_0$. However, if the amplitude, k_E , satisfies the condition $k_E > k_0$, the electron oscillations (3.5) are stopped in k -space at the border wave vector, k_0 . This leads to the “clipped” oscillations of both the wave vector, $k_x(t)$, and the acceleration, $a_x(t)$, which are plotted in Fig. 3.2(a). The expansion of the clipped oscillations of the acceleration, $a_x(t)$, into the Fourier series leads to non-zero intensities (3.6) with multiple frequencies, $\omega_n = n\omega$. These intensities are plotted in Fig. 3.2(b) as I_n/I_0 , where $I_0 = (e\hbar\omega k_0/m^+c)^2/3c$ is the characteristic intensity. It should be noted that the Fourier expansion of the acceleration, $a_x(t)$, pictured in Fig. 3.2(a), has only odd harmonics with $n = 1, 3, 5, \dots$. Therefore, corresponding odd harmonics of the intensity, I_n , appear in the emission spectrum plotted in Fig 3.2(b). As expected, only the harmonic with $n = 1$ is significant in Fig 3.2(b) for low driving field amplitudes, $E/E_0 \ll 1$, where $E_0 = \hbar\omega k_0/e$ is the characteristic field strength. As we keep on increasing the driving field amplitude, E , other harmonics of the emission with $n > 1$ come into play in Fig 3.2(b). As a consequence, a frequency comb of the electromagnetic emission from the QW appears.

It follows from the aforesaid that the discussed mechanism of high-frequency generation arises from the nonparabolic area of the electron energy spectrum (3.2),

Figure 3.2: Scheme of the single-electron electromagnetic emission from the quantum well with the Stark gap $\Delta\varepsilon/\varepsilon_0 = 50$: (a) Oscillation dynamics of electron wave vector, k_x/k_0 (the dashed line), and electron acceleration, a_x/a_0 (the solid line) for the driving field amplitude $E/E_0 = 1.14$; (b) Intensity of the electromagnetic emission, I_n , with the different frequencies, $\omega_n = n\omega$, as a function of the driving field amplitude, E . This figure is taken from Ref. [135].

which takes place near the border electron wave vector, k_0 . A physically similar mechanism exists also in semiconductor QWs with strongly nonparabolic electron dispersion [155, 156] and superlattices [157]. However, in these systems the nonparabolic region lies very far from the band edge. Therefore, the driving field should be very strong for high-frequency generation. In contrast to this case,

the present effect arises from the dynamic Stark gaps which can be induced by a dressing field very close to the band edge (i.e., the border wave vector, k_0 , can be very small). As a consequence, the generation appears for a relatively weak driving field. The high-frequency generation in such a near-band-edge regime (which, particularly, corresponds to low electron densities) is the main advantage of the present mechanism. It should be noted that the minimal value of the border electron wave vector, k_0 , is restricted only by the Heisenberg uncertainty relation. Namely, the condition $k_0 l \gg 1$ should be satisfied, where l is the mean free path of electrons in the QW. Since the path, l , is macroscopically large in state-of-the-art QWs, the border wave vector, k_0 , can be very small.

To go from the considered single-electron model to the case of 2DEG in real semiconductor QWs (see, e.g., Ref. [158]), we have to make several assumptions. To neglect excitonic effects and intersubband electron dynamics, let us assume that the field frequencies are far from both the excitonic frequency and the frequencies corresponding to intersubband electron transitions. To avoid the destructive effect of scattering processes on the electron oscillations, we will assume that the driving field frequency, ω , is high enough to meet the condition $\omega\tau \gg 1$, where τ is the mean free time of conduction electrons restricted by the electron scattering with phonons and impurities. In modern QWs, this condition can be easily satisfied for driving field frequencies, ω , starting from the sub-terahertz range. It should be noted that the simple model that only accounts for intraband acceleration and ignores all scattering is already sufficient to successfully describe the high-harmonic generation in various nonlinear systems (see, e.g., Refs. [159, 160]).

Particularly, more advanced studies (e.g., based on the Bloch equations technique [161, 162]) do not dramatically change the results from the simple model. It should be noted also that the confinement potential of QW does not influence on the discussed effect since the polarization vector of the field lies in the plane of QW (see Fig. 3.1(a)). As to temperature, it should be less than the Stark gap to

Figure 3.3: Scheme of the multi-electron electromagnetic emission from the quantum well: (a) Sketch of electron oscillations under the driving field; (b) Intensity of the electromagnetic emission, I_n , with the different frequencies, $\omega_n = n\omega$, as a function of the driving field amplitude, E , for the Stark gap $\Delta\varepsilon/\varepsilon_0 = 50$ and the Fermi wave vector $k_F/k_0=0.7$; (c) Intensity of the electromagnetic emission, I_n , as a function of the Fermi wave vector, k_F , for the the Stark gap $\Delta\varepsilon/\varepsilon_0 = 50$ and the driving field amplitude $E/E_0 = 9.8$; (d) Intensity of the electromagnetic emission, I_n , from a GaAs-based QW with the planar dimensions $100 \times 100 \mu\text{m}$ as a function of the border wave vector, k_0 , for the the Stark gap $\Delta\varepsilon = 5 \text{ meV}$, the characteristic wave vector difference, $k_0 - k_F = 25 \mu\text{m}^{-1}$, and the driving field with the frequency $\omega = 10^{12} \text{ rad/s}$ and the irradiation intensity $I = 1 \text{ kW/cm}^2$. This figure is taken from Ref. [135].

avoid the thermal excitation of electrons over the gap. In typical semiconductor materials, the gap is of meV scale for the dressing field amplitudes $\tilde{E} \sim 10^7 \text{ V/m}$ [131].

To extend the single-electron model to the case of a multi-electron system in the QW, let us consider a degenerate 2DEG with the Fermi wave vector $k_F < k_0$, which fills electron states (3.2) at zero temperature. Physically, the electromagnetic emission from the 2DEG can be described within the same model (3.1)–(3.7), but complemented with the Pauli principle. The difference between the single-electron and the multi-electron models is illustrated schematically in Fig. 3.3(a). Namely, the distribution of the 2DEG in the k -space can be described by the Fermi circle (see the red line in Fig. 3.3(a)). Under the driving field, E , all electrons oscillate along the x axis with different amplitudes (see the green arrows in Fig. 3.3(a)), which are restricted by the Pauli principle and strongly depend on initial positions of the electrons in k -space (see the violet dots in Fig. 3.3(a)). As a result, in contrast to the single-electron model, not all electrons can reach the gap at the border wave vector k_0 (see the blue circle in Fig. 3.3(a)). Taking the Pauli principle into account, Eq. (3.5) for an electron from the 2DEG should be rewritten as

$$k_x(t) = \begin{cases} k_x(0) + k_E \sin \omega t, & k_x^- < k_E \sin \omega t < k_x^+ \\ k_x(0) + k_x^+, & k_E \sin \omega t \geq k_x^+ \\ k_x(0) + k_x^-, & k_E \sin \omega t \leq k_x^- \end{cases}, \quad (3.8)$$

where $k_x^\pm = \pm \left(\sqrt{k_0^2 - k_y^2} - \sqrt{k_F^2 - k_y^2} \right)$ and $k_x^2(0) + k_y^2 \leq k_F^2$. Substituting Eq. (3.8) into Eq. (3.7) and performing summation of the electron acceleration (3.7) over filled electron states, we arrive at the total acceleration of the multi-electron system, $\mathbf{a}$. Assuming the QW size to be less than the wavelength of the electromagnetic emission, the emission intensity at the frequency $\omega_n = n\omega$ can be described by Eq. (3.6). As a result, we arrive at the intensities of different

harmonics, I_n , which are plotted in Figs. 3.3(b–c). Due to the contribution from many electrons, the radiation emitted from the 2DEG is much stronger than the radiation emitted by a single electron (see Figs. 3.2(b) and 3.3(b)). It should be noted that the Fermi energy in GaAs-based QWs can be easily controlled by the gate voltage without doping. If we increase the Fermi wave vector, k_F , the intensities, I_n , also increase since the total number of electrons increase (see Fig. 3.3(c)). However, the intensities, I_n , start to decrease if the Fermi wave vector, k_F , comes very close to k_0 (see Fig. 3.3(c)). Indeed, in this case a large number of electrons have insufficient empty states in which to oscillate. It should be noted also that the Fermi energies in Figs. 3.3(b–c) lie in the sub-meV range. This corresponds to very low electron densities, such that electron-electron processes do not affect our results and can be neglected. Particularly, the characteristic energy of electron-electron interaction is also of sub-meV scale and, therefore, sufficiently less than the Stark gap induced by the dressing field. The emission power associated with each harmonic, I_n , as a function of the border wave vector, k_0 , is plotted in Fig. 3.3(d). Tuning the photon energy of the dressing field, $\hbar\omega_0$, we can easily control the border wave vector, k_0 . If the difference of the characteristic wave vectors, $\Delta k = k_0 - k_F$, is fixed, the increasing of the border wave vector, k_0 , results in increasing the total number of electrons. This leads to an enhancement of the output power (see Fig. 3.3(d)). But when the border wave vector, k_0 , becomes too large, many electrons cannot reach the Stark gaps. As a result, only the first few harmonics gain power and the power of higher harmonics starts to decrease.

The discussed mechanism of electromagnetic emission from the QW looks most attractive for the generation of frequency combs in the terahertz range. Indeed, applying a driving field with the sub-terahertz frequency, $\omega = 10^{12}$ rad/s, we can effectively produce the terahertz emission with the frequencies $\omega_n = n\omega$ ($n = 3, 5, 7, \dots$) and the intensities of a few W/cm² (see Fig. 3.3(d)). It should be noted that frequency combs are highly sought for making optical clocks, for

measuring fundamental constants accurately [163], for high resolution molecular spectroscopy [164], etc. Traditionally, frequency combs are generated using pulsed lasers [165] and quantum cascade lasers [166, 167]. In contrast to these systems, the present comb emitter is very compact. This is important since the search for compact reliable sources of terahertz emission is one of the exciting fields of modern applied physics. In particular, terahertz scattering processes between polariton branches [108, 109, 168], intersubband polaritons [124], dipolaritons [169], and different orbital states [170, 171] have been considered. Transitions between direct and indirect exciton states, where nonlinearity allows continuous oscillation and terahertz output as a result of parametric instability have also been studied [111, 112]. Moreover, bosonic cascade lasers in which multiple terahertz transitions can be chained have been proposed [116, 172, 173]. It should be noted that terahertz frequency combs from exciton-polariton systems have also been proposed, but their expected efficiency is largely unknown [174]. It follows from the aforesaid that the present theory describing the efficient terahertz emission from the strongly coupled electron-light system in a QW fits well current tendencies in terahertz physics.

3.3 Conclusion

In conclusion, we have demonstrated theoretically that a semiconductor QW irradiated by a two-mode electromagnetic field can serve as an effective source of a terahertz frequency comb. Physically, the discussed nonlinear optical effect arises from the dynamic Stark gaps in the electron energy spectrum of a QW, which strongly modify the electron dynamics in the QW and results in the terahertz emission. Since the parameters affecting the terahertz emission from the QW can be tuned by irradiation, the present theory paves a way to optically controlled QW-based terahertz emitters.

Chapter 4

Anti-chiral edge states in an exciton polariton strip

Topological phases of matter have received tremendous attention in the scientific community for the last few decades due to the presence of robust edge states in between gapped bulk states, which make the edges of finite samples conducting even with an insulating bulk. Topological electronic systems with broken time reversal (TR) symmetry [87] are characterized by counter propagating chiral edge states whereas TR invariant topological systems contain a pair of counter propagating helical edge states at each edge corresponding to different spins states and known as the quantum spin Hall effect [88]. The application of topology in bosonic systems has resulted in creation of one-way transport of photons [90, 91, 93, 175–177], phonons [100, 101], excitons [94, 178] and exciton-polaritons [94–99, 179–183]. Antichiral edge states have been proposed in the modified Haldane model, where modes corresponding to both the edges of a strip flow in the same direction, compensated by the conducting bulk modes that propagate in the opposite direction [186]. The concept of antichiral edge states has been extended to various condensed matter systems [187–193] and recently, such states have been realized experimentally in gyromagnetic photonic crystals [194] and in electrical cir-

cuits [195]. In this chapter we present an exciton polariton based scheme to obtain antichiral edge states in a photonic system.

As mentioned in the introduction, exciton polaritons are hybrid particles of excitons and cavity photons. Due to their excitonic fraction, polaritons are strongly interacting, which enables them to form Bose-Einstein condensates and to have quantum fluid nature [54]. Initial proposals for creating topological polaritons lied in the linear regime, not exploiting interactions between themselves, while interaction with a hot exciton reservoir was shown to provide a gain mechanism for topological polariton lasing [183]. Theoretically, nonlinearity may lead to interaction induced topology [97], solitons [196, 197], and bistable topological polaritons [198]. Here, we make use of the strong nonlinear polariton-polariton interaction in a honeycomb lattice with zigzag edges to realize anti-chiral edge states (see Fig. 4.1). Since the total number of right moving and total number of left moving modes must be equal, it is impossible to realize the antichiral edge states in a gapped band structure. But in a gapless band structure, where the counter propagating modes are provided by bulk modes, antichiral edge states become possible [186]. Due to the spatial separation of the edge modes from the bulk modes backscattering is significantly suppressed in these systems similar to the case in topological insulators.

4.1 Theoretical scheme

We consider polaritons under resonant excitation in the x and y linearly polarized basis. In order to move to the linear polarization basis, we make the following substitution in the spinor Gross-Pitaevskii equation expressed in Eq. (1.76)

$$\phi_{\sigma\pm} = \frac{1}{\sqrt{2}} (\phi_x \pm i\phi_y). \quad (4.1)$$

After the substitution the Gross-Pitaevskii equation in the linear polarization basis can be expressed as [202]

$$\begin{aligned}
i\hbar \frac{\partial \phi_{x,y}}{\partial t} = & \left[\varepsilon_{x,y} - \frac{\hbar^2 \nabla^2}{2m_{pol}} - i \frac{\Gamma_{x,y}}{2} + V(\mathbf{r}) + U_0 \left(|\phi_x|^2 \right. \right. \\
& \left. \left. + |\phi_y|^2 \right) \right] \phi_{x,y} - U_1 \left(|\phi_{x,y}|^2 \phi_{x,y} + \phi_{y,x}^2 \phi_{x,y}^* \right) \\
& + F_{x,y}(\mathbf{r}) e^{-i\omega_0 t},
\end{aligned} \tag{4.2}$$

where ε_x and ε_y are the energies of the x and y polarized polaritons with lifetime Γ_x and Γ_y respectively, ∇^2 is the Laplacian operator, m_{pol} is the effective mass of the polaritons and $V(\mathbf{r})$ is the potential. The nonlinear polariton-polariton interaction constants can be expressed in terms of those in the spinor basis by $U_0 = \alpha_1$ and $U_1 = (\alpha_1 - \alpha_2)/2$ [202]. It is well known that polaritons with the same spin interact repulsively making α_1 positive, whereas polaritons with opposite spin interact attractively making α_2 negative and typically $\alpha_1 \geq |\alpha_2|$ [203, 204]. $F_{x,y}$ is a polarization dependent resonant incident optical field with frequency ω_0 and $\phi_{x,y}$ are the polarization dependent polariton wave functions.

In what follows, we will excite the x polarized polaritons using the resonant excitation for which $F_y(\mathbf{r}) = 0$. In such a limit the population of x polarized polaritons is much stronger than that of y polarized polaritons. Nevertheless, we are interested in the dispersion of y polarized polaritons, which can still be weakly populated by fluctuations or additional pulses. We can further simplify the problem by choosing the strength of the polariton polariton interaction strengths to be equal, $U_0 = U_1$, which has been realized in the experiments [38–40]. In this limit terms $|\phi_{x,y}|^2 \phi_{x,y}$ vanish from Eq. (4.2). Given that y polarized polaritons are weakly populated, we neglect second order terms in ϕ_y (This assumption is particularly accurate as it is readily seen that $\phi_y = 0$ is a solution to Eq. (4.2) when $F_y(\mathbf{r}) = 0$ and stability analysis reveals that it is a stable solution). After

Figure 4.1: Schematic view of the system under consideration. A graphene like polariton strip with zigzag edges, which can be fabricated with different technologies [199–201], is subjected to an x linearly polarized optical field. y linearly polarized polaritons propagate at both the edges in the same direction compensated by counter propagating polaritons through the bulk. Since the edge and bulk modes are spatially separated, the edge states are robust against scattering with disorder. This figure is taken from Ref. [215].

taking all the above mentioned conditions into account Eq. (4.2) becomes:

$$\begin{aligned}
 i\hbar \frac{\partial \phi_x}{\partial t} &= \left[\varepsilon_x - \frac{\hbar^2 \nabla^2}{2m_{pol}} - i\frac{\Gamma_x}{2} + V(\mathbf{r}) \right] \phi_x + F_x(\mathbf{r}) e^{-i\omega_0 t} \\
 i\hbar \frac{\partial \phi_y}{\partial t} &= \left[\varepsilon_y - \frac{\hbar^2 \nabla^2}{2m_{pol}} - i\frac{\Gamma_y}{2} + V(\mathbf{r}) + U_0 |\phi_x|^2 \right] \phi_y - U_0 \phi_x^2 \phi_y^*
 \end{aligned} \tag{4.3}$$

We choose,

$$\begin{aligned}
 V(\mathbf{r}) = V_0 & \left[\cos \left(\frac{4\pi y}{\sqrt{3}a} \right) + \cos \left(-\frac{2\pi x}{a} - \frac{2\pi y}{\sqrt{3}a} \right) \right. \\
 & \left. + \cos \left(\frac{2\pi x}{a} - \frac{2\pi y}{\sqrt{3}a} \right) \right]
 \end{aligned} \tag{4.4}$$

which corresponds to a honeycomb lattice with periodicity a in the x direction

and zig-zag edges if a cut is made making the system finite in the y direction. Although we consider here a smooth potential, similar results can be obtained with a square well type potential at each lattice site. To simplify our analysis we focus on the slowly varying field $\psi_{x,y} = \phi_{x,y}e^{i\omega_0 t}$ and make the following substitutions to move to a dimensionless unit system $t \rightarrow (2m_{pol}a^2/\hbar)t$, $x \rightarrow ax$, $y \rightarrow ay$, $\varepsilon_{x,y} \rightarrow (\hbar^2/2m_{pol}a^2)\varepsilon_{x,y}$, $V_0 \rightarrow (\hbar^2/2m_{pol}a^2)V_0$, $\Gamma_{x,y} \rightarrow (\hbar^2/2m_{pol}a^2)\Gamma_{x,y}$, $\omega_0 \rightarrow (\hbar/2m_{pol}a^2)\omega_0$, $\psi_{x,y} \rightarrow (\sqrt{\hbar^2/2m_{pol}a^2})\psi_{x,y}/\sqrt{U_0}$ and $F_x \rightarrow F_x\sqrt{U_0}(2m_{pol}a^2/\hbar^2)^{\frac{3}{2}}$. In the dimensionless units, Eq. (4.3) takes the form

$$i\frac{\partial\psi_x}{\partial t} = \left[\varepsilon_x - \omega_0 - \nabla^2 - i\frac{\Gamma_x}{2} + V(\mathbf{r}) \right] \psi_x + F_x(\mathbf{r}) \quad (4.5)$$

$$i\frac{\partial\psi_y}{\partial t} = \left[\varepsilon_y - \omega_0 - \nabla^2 - i\frac{\Gamma_y}{2} + V(\mathbf{r}) + |\psi_x|^2 \right] \psi_y - \psi_x^2\psi_y^* \quad (4.6)$$

4.2 Band structure

As mentioned earlier our aim is to study the dispersion of the y polarized polaritons, which can be calculated by substituting $\psi_y = u(\mathbf{r})e^{i\omega t} + v^*(\mathbf{r})e^{-i\omega^* t}$ in Eq. (4.6), where $u(\mathbf{r})$ and $v^*(\mathbf{r})$ are spatial functions to be found and ω is the frequency, which is kept complex in order to capture potential instabilities [76] and we continue to work in the limit where the population of ψ_y is small. Upon substitution, Eq. (4.6) becomes two coupled equations in $u(\mathbf{r})$ and $v(\mathbf{r})$ which can be expressed in the following matrix form

$$\begin{bmatrix} \omega'(\mathbf{r}) & -\psi_x^2 \\ \psi_x^{*2} & -\omega'^*(\mathbf{r}) \end{bmatrix} \begin{bmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{bmatrix} = \omega \begin{bmatrix} u(\mathbf{r}) \\ v(\mathbf{r}) \end{bmatrix} \quad (4.7)$$

where $\omega'(\mathbf{r}) = \varepsilon_y - \omega_0 - \nabla^2 + |\psi_x|^2 - i\Gamma_y/2 + V(\mathbf{r})$. We know from sec. 1.2.9 that graphene with zigzag edges shows flat edge dispersion with zero group velocity. Our aim is to make the dispersion asymmetric such that the edge states gain non

zero group velocity. It was also shown in sec. 1.2.7 that a resonant excitation with non-zero in-plane wave vector can give rise to an asymmetric dispersion of excitations [76]. The first and principle ingredient of our scheme consists of a stationary field ψ_x with nonzero wave vector which tilts the dispersion of ψ_y . We choose $\psi_x = A(y)e^{ik_0x}$, where ψ_x vanishes at the boundaries $[-y_L, y_L]$. To take the boundary conditions into account we take (for $|y| < y_L$)

$$A(y) = A_0 \left[1 - e^{-(y-y_L)^2/2\zeta^2} \right] \times \left[1 - e^{-(y+y_L)^2/2\zeta^2} \right]. \quad (4.8)$$

This form of ψ_x can be easily imprinted by a specific choice of the driving field, which corresponds to a x polarized laser excitation with non-zero angle of incidence,

$$F_x(\mathbf{r}) = \left[-\varepsilon_x + \omega_0 + \nabla^2 + i\frac{\Gamma_x}{2} - V(\mathbf{r}) \right] A(y)e^{ik_0x}, \quad (4.9)$$

which can be found by substituting the form of ψ_x in Eq. (4.5) and solving for $F_x(\mathbf{r})$. To keep Eq. (4.7) periodic in the x direction the only nontrivial permitted values of k_0 are $(2n+1)\pi$, where n can be zero or any integer. Under this condition, we can apply Bloch theory on Eq. (4.7) to write down

$$u(\mathbf{r}) = e^{ik_x x} \tilde{u}_{k_x}(\mathbf{r}), \quad v(\mathbf{r}) = e^{ik_x x} \tilde{v}_{k_x}(\mathbf{r}). \quad (4.10)$$

where $\tilde{u}_{k_x}(\mathbf{r})$ and $\tilde{v}_{k_x}(\mathbf{r})$ are the Bloch wave functions of the fluctuations with wave vector k_x . Substituting Eq. (4.10) into Eq. (4.7) we get the following equations

which are used to calculate the bandstructure in Fig. 4.2

$$\begin{aligned}
& \left[(k_x + G)^2 - \frac{\partial^2}{\partial y^2} + \varepsilon_y + |A(y)|^2 - i\frac{\Gamma_y}{2} - \omega_0 - \omega \right] \tilde{u}(G, y) \\
& + \left[\sum_{G'} \tilde{V}(G - G', y) \tilde{u}(G', y) - \tilde{\psi}_x^{2*}(G' - G, y) \tilde{v}(G', y) \right] = 0 \\
& \left[-(k_x + G)^2 + \frac{\partial^2}{\partial y^2} - \varepsilon_y - |A(y)|^2 - i\frac{\Gamma_y}{2} + \omega_0 - \omega \right] \tilde{v}(G, y) \\
& - \left[\sum_{G'} \tilde{V}(G - G', y) \tilde{v}(G', y) - \tilde{\psi}_x^2(G - G', y) \tilde{u}(G', y) \right] = 0 \quad (4.11)
\end{aligned}$$

where $G = 2n\pi$ is the reciprocal lattice vector with n can be zero or any integer and using Fourier transformation the following expressions can be found

$$\begin{aligned}
\tilde{u}_{k_x}(\mathbf{r}) &= \sum_G \tilde{u}(G, y) e^{iGx}, \quad \tilde{u}(G, y) = \int_{-1/2}^{1/2} \tilde{u}_{k_x}(\mathbf{r}) e^{-iGx}, \\
\tilde{v}_{k_x}(\mathbf{r}) &= \sum_G \tilde{v}(G, y) e^{iGx}, \quad \tilde{v}(G, y) = \int_{-1/2}^{1/2} \tilde{v}_{k_x}(\mathbf{r}) e^{-iGx}, \\
V(\mathbf{r}) &= \sum_G \tilde{V}(G, y) e^{iGx}, \quad \tilde{V}(G, y) = \int_{-1/2}^{1/2} V(\mathbf{r}) e^{-iGx}, \\
\psi_x^2(\mathbf{r}) &= \sum_G \tilde{\psi}_x^2(G, y) e^{iGx}, \quad \tilde{\psi}_x^2(G, y) = \int_{-1/2}^{1/2} \psi_x^2(\mathbf{r}) e^{-iGx}. \quad (4.12)
\end{aligned}$$

Since it is a non-hermitian system the spectrum will be complex and only the real part of the spectrum near the Dirac points is plotted in Fig. 4.2. The spectrum corresponding to $\omega < 0$ is the image of that corresponding to $\omega > 0$ under transformation $k \rightarrow 2k_0 - k$ and $\omega \rightarrow 2\omega_0 - \omega$ [76]. In physical units, we have found that the energy shift between the two Dirac points can be around 0.1 meV, taking $m_{pol} = 7 \times 10^{-5} m_e$, where m_e is the free electron mass and $a = 2.2 \mu m$. This exceeds the decay rates in modern samples where polaritons with long lifetime are obtained [205]. The required nonlinear shift due to the polariton-polariton interaction to observe the effect is about 1.45 meV, which is within experimental limits [206–208]. Another requirement of this scheme is that the chosen energy

Figure 4.2: Dispersion of the y polarized polaritons calculated from Eq. (4.7) corresponding to the stable states. All the states marked in blue correspond to the bulk states and states marked in red correspond to the edge states. Parameters: $\omega_0 = 15$, $k_0 = \pi$, $\varepsilon_x = 0$, $\varepsilon_y = A_0^2$, $A_0 = 3.6$, $\Gamma_x = \Gamma_y = 0.23$, $V_0 = 17.7$, $\zeta = 0.1$. This figure is taken from Ref. [215].

and wave vector of the pump needs to be within the parabolic region of the lower polariton branch so that parametric instabilities are avoided [54, 184, 185]. Since these edge states are spatially separated from the bulk states the backscattering of the edge states should be significantly suppressed [186]. Nevertheless there is some overlap between the bulk and edge states and the suppression of backscattering is

Figure 4.3: Dispersion of the y polarized polaritons calculated from Eqs. (4.5 - 4.6) in the presence of white noise which corresponds to the photoluminescence spectrum obtainable in experiments. The blue dot corresponds to the state which is excited to probe the pulse propagation. This figure is taken from Ref. [215].

not expected to be as strong as in the topologically protected case.

4.3 Results and discussions

To verify our claim we evolve Eqs. (4.5–4.6) in time in the presence of an additional stochastic complex Langevin noise term (chosen as a white noise in space and time). Once ψ_x reaches its steady state one can obtain the dispersion corresponding to ψ_y as shown in Fig. 4.3. This corresponds to the photoluminescence spectrum that could be obtained experimentally. To have consistency, all the parameters are kept the same as the ones in Fig. 4.2. To illustrate the propagation and robustness of the edge states we introduced an impurity by slightly deforming the potential at the edges and then excited both the edges using a y polarized

coherent pulse of the form

$$\begin{aligned}
F_y = & F_0 \left[\exp\{ -[(x - x_0)^2 + (y - y_L)^2]/2\sigma^2 \} \right. \\
& \left. + \exp\{ -[(x - x_0)^2 + (y + y_L)^2]/2\sigma^2 \} \right] \\
& \times \exp\left[-(t - t_0)^2/\tau^2 \right] \exp[i(k_p x - \omega_p t)]
\end{aligned} \tag{4.13}$$

where F_0 , τ and ω_p are the amplitude, duration and frequency of the pulses respectively centered at (x_0, y_L) and $(x_0, -y_L)$ having wave vector k_p with width σ . The values of k_p and ω_p correspond to the blue dot shown in Fig. 4.3. Different stages of the pulse propagation are shown in Fig. 4.4. As expected both the pulses propagate in the same direction along the edges of the sample. Even in the presence of the impurities the pulses do not backscatter but flow around the impurities which prove the robustness of the edge states. However, once the pulses reach the left end of the sample they have no where to go but to couple to the counter propagating bulk modes (which propagate from left to right) and the intensity can be seen to be transferred towards the right end of the sample through the bulk. Bulk modes at the right edge couple to the edge states and the intensity is transferred back to the edges where they again start to propagate from right to left. To compare the robustness of an edge state to a bulk state we add disorder in the system by adding a Gaussian correlated disorder potential with the honeycomb lattice potential $V(\mathbf{r})$ and excite an edge state and a bulk state with a coherent Gaussian pulse. In Fig. 4.5 we plot the intensity distribution along the propagating axis with time by summing over the other axis. The edge state can be seen to propagate without being backscattered (Fig. 4.5(a)), whereas the bulk state spreads over the x axis with time indicating the presence of backscattering (Fig. 4.5(b)). The green line represents the mean position of the pulse and the dashed green lines represent the square root of variance around the mean position. We have checked that the unidirectional propagation of the antichiral edge states is unhampered for disorder strength less than $30 \mu eV$, which is consistent with

Figure 4.5: The propagation of an edge state (a) and a bulk state (b) in presence of disorder. Due to the absence of backscattering the edge state does not spread too much whereas the bulk state spreads indicating backscattering. The green line represents the mean position of the pulse and the dashed green lines represent the square root of variance. This figure is taken from Ref. [215].

the experimental observation [209]. Although the propagation length considered in Fig. 4.5 is long enough to show a clear advantage of anti-chiral edge states compared to bulk states, it is not enough to show the possible coupling of antichiral edge states to bulk states at the same energy. This can potentially be caused by disorder in the system, so we stress that we are not claiming that antichiral edge states are as robust as compared to the chiral edge states in topological systems. The relevant topological invariant associated with the antichiral edge states is the winding number. The winding number for a continuous system is given by

$$\nu_{k_x} = \frac{i}{\pi} \oint_{\text{BZ}} \left\langle u_{k_x}(k_y) \left| \frac{\partial u_{k_x}(k_y)}{\partial k_y} \right. \right\rangle dk_y. \quad (4.14)$$

Here $u_{k_x}(k_y)$ is the Bloch wave function at (k_x, k_y) for 2D infinite system of the u type fluctuations. However, numerically the momentum space becomes discrete and is no longer continuous. Following Ref. [210], the above can be approximated

for a discrete system as

$$\nu_{k_x} = \frac{1}{\pi} \text{Im} \left[\ln \prod_{i=1}^{n-1} \langle u_{k_x}(k_y^i) | u_{k_x}(k_y^{i+1}) \rangle \right], \quad (4.15)$$

where the Brillouin zone (BZ) along the y direction is divided into n parts and $u_{k_x}(k_y^i)$ is the Bloch wave function for a fixed k_x at k_y^i of the u type fluctuation. We obtain

$$\nu_{k_x} = \begin{cases} 1, & \text{for } \frac{2\pi}{3a} \leq k_x \leq \frac{4\pi}{3a} \\ 0, & \text{elsewhere} \end{cases} \quad (4.16)$$

Here a is the lattice period along the x direction. $k_x = 2\pi/3a$ and $k_x = 4\pi/3a$ correspond to the two Dirac points.

4.4 Discussion and conclusion

The anti-chiral edge states can be interesting in polaritonic systems for building polariton wires where information could be efficiently transferred along the edges due to the absence of backscattering. It is typically claimed that polariton gap solitons in nanowires and chiral edge states in lattices can be used in information processing polaritonic devices. However, criteria for photonic information processing were established long ago [211], where it was pointed out that it is essential for any feedback to be suppressed in the system. Polariton gap solitons can propagate in both forward and backward directions, while chiral edge states always come in forward and backward propagating pairs. In contrast antichiral edge states propagate in the same direction at both edges and thus correspond to a feedback suppression mechanism which make them an excellent candidate for information processing photonic devices [212].

In summary, we have discussed a theoretical scheme to obtain antichiral edge

states in a photonic system, making use of the polarization dependent interactions of polariton condensates, where the motion of an x linearly polarized polariton condensate tilts the dispersion of y linearly polarized polaritons. A non zero velocity can be easily induced in a polariton condensate by the choice of a coherent optical incident field with non zero in plane wave vector. The two key ingredients in our scheme involve an exciton polariton honeycomb lattice with zigzag edges and a non zero velocity of the x polarized polariton condensates, which then shifts the two Dirac points of the y linearly polarized polaritons in energy resulting in antichiral edge states where the modes corresponding to both the edges propagate in the same direction. Due to the spatial separation of the edge and bulk modes backscattering is suppressed even in the presence of disorder.

Chapter 5

One-way reflection-free exciton-polariton spin filtering channel

It has been recognized long ago that one of the crucial ingredients for optical circuits is the isolation of input and output [211]. Equivalently, circuits should operate in a one-way fashion, where feedback in the backward direction is suppressed. As Maxwell's equations are time reversal invariant, the realization of such a requirement is not obvious in many optical systems, and reflections must be considered carefully. With the emergence of topological photonics [93, 102], special techniques were introduced to artificially break time-reversal symmetry (e.g., helical waveguide arrays as in Floquet topological insulators [213]) or exotic material solutions employed at particular wavelengths (e.g., magnetically sensitive gyrotropic materials [91]). One of the well-known features of topological photonic lattices is the appearance of chiral edge states. As these propagate in only a certain direction and do not undergo backscattering, it has been natural to expect that such states are relevant for the robust propagation of optical signals.

Instead of breaking time-reversal symmetry directly in a photonic system, an

alternative is to couple it to a system in which time-reversal symmetry is broken. In particular, when photons are coupled to excitons in semiconductor micropillars they hybridize to form exciton-polaritons, which inherit a Zeeman splitting due to their excitonic component when placed in a magnetic field [214]. Since such micropillars can be arranged into lattices it has been possible to form topological polariton systems [183], accounting for the effect of spin-orbit coupling [94–99].

Exciton-polaritons also exhibit a significant nonlinearity, which in theory was predicted to itself induce topological behavior [97], antichiral behavior [215], non-Hermitian bi-skin effect [228], and topological solitons [196]. Separate from topological physics, nonlinearity was shown to provide low-energy polaritonic switches [3, 7, 8], reservoir networks [229–232], artificial life [233], and realize other information processing elements such as transistors [3–6], routers [216–218], and amplifiers [9, 10]. The possible use of topological polariton states at the edges of a strip to couple these elements suffers however from an obvious drawback: such states come in pairs, which propagate in opposite directions on the opposite edges of the strip [94, 96]. Consequently, there will be an unwanted counter-propagating signal at the opposite edge of the strip leading to potentially detrimental feedback (unless the strip is made very wide, which would lead to other problems in scalability).

Furthermore, it has long been expected that polaritonic devices would make use of the spin degree of freedom [221], which can take one of two spin projections on the structure growth axis (denoted $\sigma_{\pm}$). Individual spin switches [222] and gates [223] have encoded information in this spin. Unfortunately though, topological polariton states have a specific polarization that mixes the σ_{+} and σ_{-} components [94, 96], which would not allow the preservation of individual σ_{+} or σ_{-} spin polarized wavepackets.

In this chapter, we aim to develop the recent advances in topological polariton physics to introduce a channel for connecting polaritons of a given spin. We aim

Figure 5.1: (a) Schematic diagram of coupled polariton micropillars arranged in a honeycomb strip where two different sub-lattices are indicated by two colors. The considered system can act as a spin filter by allowing the polaritons with one of the spins in the system to propagate robustly from right to left while the propagation in the opposite direction is restricted. The red arrow indicates the perpendicularly applied magnetic field (H_z). (b) Potential application of the polariton spin filter (SF) as a channel that can be used to couple two polariton networks without feedback. The same can not be done using a polariton topological insulator (TI) where there exists a possibility of feedback via an opposite edge state. This figure is taken from Ref. [219].

for the channel to be reasonably thin and allow only one spin to propagate in a specific direction. To meet this aim, we consider a strip of honeycomb polariton lattice, which is composed of two interlocking sublattices, A and B (see Fig. 5.1(a)). The sites of the different sublattices are engineered with different onsite energy. Accounting for a magnetic field and spin-orbit coupling, we find that the states at different edges are split in energy and become almost completely spin polarized. Furthermore, an edge state of a given spin can be embedded in an energy range where the only existing states have opposite spin polarization. As the scattering of polaritons with disorder tends to be spin conserving, this maintains a unidirectional propagating edge state for a specific spin, which is not possible

with a regular topological polariton system (see Fig. 5.1(b)). A further key difference with existing topological polariton realizations is that the strip can be made remarkably thin (three lattice constants thick) and still maintain its unidirectional spin polarized behavior.

5.1 Scheme

We start by considering a planar semiconductor microcavity containing exciton-polaritons subjected to a resonant excitation, expressed by the driven dissipative Schrödinger equation,

$$i\hbar \frac{\partial \phi_{\sigma_{\pm}}(x, y)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m_{pol}} + V(x, y) - i\frac{\Gamma}{2} \pm \Delta_z \right] \phi_{\sigma_{\pm}}(x, y) + \Delta_T \left(i\frac{\partial}{\partial x} \pm \frac{\partial}{\partial y} \right)^2 \phi_{\sigma_{\mp}}(x, y) + F_{\sigma_{\pm}}(x, y) e^{i(k_0 x - \omega_p t)}. \quad (5.1)$$

This is the same equation as introduced in Eq. (1.84), although including the effect of an applied magnetic field perpendicular to the microcavity plane (Faraday configuration). To recall, here $\phi_{\sigma_{\pm}}(x, y)$ are the wave functions for $\sigma_{\pm}$ spin polarized polaritons; $V(x, y)$ is a potential describing a strip of honeycomb lattice having zigzag edges along the y direction; Γ is a uniform decay throughout the sample; $2\Delta_z$ and Δ_T are the Zeeman and TE-TM splittings, respectively (where the Zeeman splitting is the consequence of the applied magnetic field). $F_{\sigma_{\pm}}(x, y)$ are the circular polarization components of a resonant excitation with frequency ω_p . Each micropillar is modelled by placing a Gaussian of width σ at each site [198] and the lattice has a periodicity a along the x direction, such that,

$$V(x, y) = V(x + a, y) = -(v_o \pm \delta) \sum_{x_n, y_n} \exp \left[-\frac{(x - x_n)^2 + (y - y_n)^2}{\sigma^2} \right]. \quad (5.2)$$

Here the summation runs over all sites, $(v_o + \delta)$ is the potential depth corresponding to the A sublattices, and $(v_o - \delta)$ is the potential depth corresponding to the B sublattices. In principle, the energy detuning, δ , between the A and B sublattices can be induced by locally adjusting the thickness of the cavity or by non-resonantly pumping one of the sublattices, where the excitonic reservoir will induce a blueshift locally [224]. Alternatively, micropillars corresponding to the A and B sublattices could be engineered with different diameters (see sec. 5.3). In order to know the characteristics of the system we consider the linear band structure by setting $F_{\pm} = 0$ and $\Gamma = 0$ in Eq. (5.1) and after applying the Bloch theorem the wave function can be written as, $\phi_{\sigma\pm}(x, y) = u_{\sigma\pm}(x, y) \exp\{ikx\}$. Here $u_{\sigma\pm}(x, y)$ are the Bloch wave functions, which are periodic along the x direction and k is the wave-vector in the x direction. Upon substitution, Eq. (5.1) leads to an eigenvalue problem, which can be solved to obtain the band structure. In Figs. 5.2 (a-b) and (c-d) we plot the dispersion without and with the onsite term, respectively. For $\delta = 0$, we obtain the band structure corresponding to the polariton topological insulator (TI) (see Fig. 5.2 (a)) where the time reversal symmetry is broken by the Zeeman splitting. Here, the bulk modes are gapped and in the band gap robust edge states appear. In this case, the edge states come in counter propagating pairs, meaning that within the band gap of each propagating edge state there is another one propagating in the opposite direction. The inclusion of δ breaks the inversion symmetry and the parameters can be chosen such that the band gap at one of the Dirac points gets closed resulting in a topologically trivial system. The dispersion of the bulk modes is similar to the ones in bilayer graphene with a perpendicular bias [225]. Although, in this case the edge states are completely separated in energy (see Fig. 5.2 (c)). Note that there is no counter propagating edge mode present in the energy window of interest. We have also plotted the localization of the edge states in Fig. 5.2(e), which shows that the edge states in the spin filter (SF) regime are more localized than the standard polariton TI.

Figure 5.2: Band structure of the system under consideration of onsite energy $\delta = 0$ in (a-b) and $\delta \neq 0$ in (c-d). In (a) and (c) the states residing in bulk are shown in grey whereas blue and magenta correspond to the states located near the two edges. In (b) and (d) the same dispersions with spin contribution are plotted where red corresponds to the states having spin σ_+ and green corresponds to the states having spin σ_- . The dashed lines in (c-d) represent the energy window where the edge state is surrounded by the bulk states with opposite spins and also there is no counter propagating edge state. (e) comparison of the localization of the edge states between the polariton SF and polariton TI. This figure is taken from Ref. [219].

To emphasize the interest of the obtained states, we plot the band structures and colour code them depending upon the contribution of the spins. In the case of a polariton TI (see Fig. 5.2 (b)) we find that within the bandgap the edge states have mixed spin. On the other hand in the SF regime (Fig. 5.2 (d)) the edge states have almost pure spin and more importantly the top most edge state is surrounded by only the bulk modes with opposite spin polarization. We can then expect σ_+ spin polarized polaritons to propagate through the edge mode without backscattering, manifesting a robust SF. Considering typical parameters, we have set: $a = 3 \mu\text{m}$; $m_{pol} = 2.5 \times 10^{-5}m_e$ with m_e the free electron mass; $v_o = 16.5 \text{ meV}$; $\delta = 0.54 \text{ meV}$; $\sigma = 0.45 \mu\text{m}$; $\Delta_T = 0.15 \text{ meV}\mu\text{m}^2$; and $\Delta_Z = 0.18 \text{ meV}$. This gives an energy interval of the SF edge state of around 0.12 meV , which is well above the linewidth of modern samples. Generally, the SF regime can be found for $\varepsilon_g \simeq 2\Delta_z \geq \Delta_T$, where ε_g is the trivial bandgap induced by the broken inversion symmetry (see sec. 5.4) and Δ_T determines the group velocity of the edge state. It should be stressed that instead of the magnetic field (Zeeman splitting), the nonresonant excitation can itself break the symmetry between the two spins [226, 227]. For the rest of the chapter we will focus on the top most edge state and although here only σ_+ spin polarized polaritons are filtered, the same can be done for the σ_- spin polarized polaritons (see sec. 5.3).

Spin filtering— To validate our claim we excite the top most edge state resonantly with a continuous linearly polarized Gaussian shaped pump by setting $F_{\sigma_+} = F_{\sigma_-}$. The spin polarization degree is defined as,

$$S_z(x, y) = \frac{|\phi_{\sigma_+}(x, y)|^2 - |\phi_{\sigma_-}(x, y)|^2}{|\phi_{\sigma_+}(x, y)|^2 + |\phi_{\sigma_-}(x, y)|^2}. \quad (5.3)$$

The spatial distribution of $S_z(x, y)$ is plotted in Fig. 5.3(a) in the SF regime, which shows that although we have excited both spin states, only σ_+ spin polarized polaritons populate the top edge and propagate over $100 \mu\text{m}$ from the spot of

Figure 5.3: Spatial distribution of the spin degree defined in Eq. 5.3 for the polariton SF is shown in (a), where polaritons having both spins are pumped but only σ_+ spin polarized polaritons flow one way. The same for polariton TI is shown in (b), where polaritons with mixed spins flow counter-clock wise through the edges of the sample. (c) Feedback suppression effect where the σ_+ spin polarized polaritons can not flow from left to right, instead they are converted to σ_- spin polarized polaritons and scatter randomly into the bulk. The red arrows indicate the position of the continuous pump. Parameters: Pump width = $9 \mu\text{m}$; $\Gamma = 0.01 \text{ meV}$; $k_0 = 2.5/a$ and $\omega_p = -1.65 \text{ meV}/\hbar$ are used in (a) and (c); $k_0 = 2.35/a$ and $\omega_p = -1.98 \text{ meV}/\hbar$ are used in (b). This figure is taken from Ref. [219].

excitation. Due to the finite lifetime the intensity decreases with the increase of propagation distance. σ_- spin polarized polaritons are instead scattered randomly into the bulk. To demonstrate that there is no state of σ_+ spin polarized polaritons

to propagate in the reverse direction, we consider injecting both spins from the opposite corner of the top edge (see Fig. 5.3(c)). We observe that the σ_+ spin polarized polaritons can not propagate from left to right of the system; instead they are converted to σ_- polaritons and scatter into the bulk. This confirms that the system can act as a feedback free unidirectional spin filter. This is also similar to the recently proposed non-reciprocal transport of polaritons in an elliptical pillar chain [220]. Although in that case the spin of the polaritons are not conserved. On the other hand, in a polariton TI the polaritons with mixed spins rotate through the edges of the sample in anticlockwise direction (See Fig. 5.3(b)). Although the exact contribution of the spins in $S_z(x, y)$ can be changed slightly depending upon the state that is being excited, there is always a significant contribution from both the spins.

In all our calculations, we have taken a lifetime around 60 ps, close to that reported in [183, 224]. The SF would also work with shorter lifetime, although the propagation distance would decrease (see Fig. 5.9). Higher lifetimes such as those reported in [205] are thus the ideal.

5.2 Robustness and strip width

Fig. 5.4 (a-d) consider the propagation of a σ_+ polarized pulse (obtained by multiplying the coherent excitation with a Gaussian in time) in the case where a micropillar is removed from the top edge. We observe that near the defect the pulse spreads a little but does not backscatter and after some time the pulse passes the defect. This further proves the robustness of the edge state.

In real systems, disorder is present throughout. Fig. 5.4 (e) shows the propagation of a σ_+ polarized pulse in the presence of a continuous disorder potential, which is characterized by a root mean squared amplitude of $30\mu\text{eV}$ and a correlation length of $3\mu\text{m}$ (corresponding to typical experimental values [209]). Again,

Figure 5.4: (a-d) Propagation of a σ_+ coherent pulse in a honeycomb lattice strip with one of the sites removed. (e) Propagation of the same pulse in a honeycomb lattice strip with spatial disorder. The colour scale represents the σ_+ polarized intensity, integrated over the strip width (y -coordinate). The green curve indicates the average position of the wavepacket and the dashed lines the variance. These are defined as $x_m = \int x |\psi_{\sigma_+}|^2 dx dy / I_0$ and $x_v^2 = \int x^2 |\psi_{\sigma_+}|^2 dx dy / I_0 - x_m^2$, respectively, where $I_0 = \int |\psi_{\sigma_+}|^2 dx dy$. The parameters were the same as in Fig. 5.3(a). This figure is taken from Ref. [219].

no backscattering of the σ_+ polarized wavepacket is observed.

In addition to supporting one-way propagation, a channel for connecting information processing elements should not be too wide, so as not to limit the footprint area of future systems. In Figs. 5.5 (a-b) the band structure for a narrow strip having three unit cells along the y direction is plotted for a polariton TI and SF, respectively. Due to the lack of bulk states, the well spread edge states in the polariton TI (see Fig. 5.2(e)) vanish while comparatively more localized edge states

Figure 5.5: Band structure corresponding to a very short strip consisting of three unit cells along the y direction for TI shown in (a) and the same for SF shown in (b). The edge states in the topological gap corresponding to the TI disappear. (c) Spatial distribution of the spin degree for SF. The red arrow indicates the position of continuous pump. Parameters were kept the same as those in Fig. 5.3(a). This figure is taken from Ref. [219].

in the polariton SF remain unaffected resulting in σ_+ spin propagation as shown in Fig. 5.5 (c).

5.3 Band structure of the spin filter using micropillars of different diameters

In this section, we consider micropillars of two different diameters to construct the honeycomb lattice. In the considered system two types of micropillars have different mode localization energies due to their different diameters, which automatically gives rise to the onsite detuning, δ . In Fig. 5.6(a) a schematic diagram of such a system of micropillars arranged in a honeycomb lattice is considered, which is infinite along the x direction and finite along the y direction having the zigzag edges. The band structure is calculated the same way as done in sec. 5.1. The diameters of the micropillars are considered to be $2.4 \mu\text{m}$ and $1.9 \mu\text{m}$ [183], which represent the A and B sublattices, respectively, and the depth of the micropillars are the same and taken 16.5 meV . In order to realize the coupling between the two neighboring sites, the micropillars have an overlap between them such that the unit cell size along the x direction is $3 \mu\text{m}$. In Fig. 5.6(b) the band structure of the system is plotted without taking into account the Zeeman splitting, $2\Delta_z$, and the TE-TM splitting, Δ_T . There is a trivial bandgap, ε_g , which is due to the breaking of the inversion symmetry. Since, the edges of the lattice are composed of different types of micropillars, the edge states located at the two edges of the lattice are no longer symmetric. There is no term ($\Delta_z = 0$) to break the symmetry between the σ_+ and σ_- spin polarized polaritons and as a result the band structure is degenerate for both spins. The effect of Δ_z is shown in Fig. 5.6(c), which lifts the degeneracy between the two spins and the band structure of the σ_+ and σ_- polarized polaritons gets shifted in energy by $2\Delta_z$. Figs. 5.6(d-e) show the SF band structure taking into account both Δ_z and Δ_T terms. The parameters are chosen such that the σ_- spin polarized edge state is surrounded by the σ_+ spin polarized bulk modes and hence one can show that the σ_- spin polarized polaritons will propagate robustly along the lower edge from left to right (determined by the

group velocity of the edge state which is positive in this case) and the system will act as a SF where σ_- spin polarized polaritons will be filtered out. It should be noted that, by reversing the sign of the magnetic field (which changes Δ_z to $-\Delta_z$) one can easily change the spin polarization of the SF edge state (see Fig. 5.7).

5.4 Effect of TE-TM splitting on the band structure

In this section we obtain the band structure by solving Eqs. (5.1–5.2) corresponding to different Δ_T . In Fig. 5.8(a) the band structure of the system without Δ_z and Δ_T is plotted which shows the trivial bandgap similar to Fig. 5.6(b). Fig. 5.8(b) shows the effect of Δ_z which shifts the band structure for $\sigma_{\pm}$ spin polarized polaritons in energy. Figs. 5.8(c-d) the band structures corresponding to $\Delta_T = 0.09 \text{ meV}\mu\text{m}^2$ are plotted which shows that the SF regime can still exist even for a lower value of TE-TM splitting. However, the band structure starts to deviate from the SF regime for larger values of TE-TM splitting, $\Delta_T = 0.24 \text{ meV}\mu\text{m}^2$, as shown in 5.8(e-f). The SF edge state starts to be flat and also two edge states start to appear within the same energy interval. To conclude, there is a great degree of freedom in choosing the parameters as long as $\varepsilon_g \simeq 2\Delta_z \geq \Delta_T$.

5.5 Effect of the polariton lifetime on the propagation distance

In this section, the propagation distance for different polariton lifetime is plotted using Eqs. (5.1, 5.2, and 5.3). For polaritons with lifetime 30 ps the propagation distance is about 60 μm . As expected, the propagation distance increases with polariton lifetime and the propagation distance reaches around 100 μm for 50 ps

Figure 5.6: (a) Schematic diagram of a honeycomb lattice having zigzag edges consisting of two different types of micropillars having different diameters. Band structure of the system under consideration for $\Delta_z = \Delta_T = 0$ in (b); for $\Delta_z = 0.18$ meV and $\Delta_T = 0$ meV μm^2 in (c); and for $\Delta_z = 0.18$ meV and $\Delta_T = 0.1$ meV μm^2 in (d-e). In (b) and (d) the bulk modes are represented in grey whereas blue and magenta represent the states located at the two edges. In (c) and (e) red and green represent the σ_+ and σ_- spin polarized modes, respectively. The mass of the polaritons is $2.5 \times 10^{-5} m_e$ with m_e the free electron mass. This figure is taken from Ref. [219].

Figure 5.7: Band structure of the system shown in Fig. 5.6(a) for $\Delta_z = \Delta_T = 0$ in (a); for $\Delta_z = -0.18$ meV and $\Delta_T = 0$ meV μm^2 in (b); and for $\Delta_z = -0.18$ meV and $\Delta_T = 0.1$ meV μm^2 in (c-d). The colour codings are the same as those in Fig. 5.6. In this case the SF works for σ_+ spin polarized polaritons. This figure is taken from Ref. [219].

(see Fig. 5.9).

5.6 Discussion and Conclusion

Modern electronic devices and integrated circuits are limited by heating and time delay of communication between components. Optical devices seem promising candidates for fast interconnections and higher fidelity may reduce error correction overhead, resulting in less heating [1]. But optics needs to operate at lower

Figure 5.8: Band structure of the system under consideration for $\Delta_z = \Delta_T = 0$ in (a), for $\Delta_z = 0.18$ meV and $\Delta_T = 0$ meV μm^2 in (b), for $\Delta_z = 0.18$ meV and $\Delta_T = 0.09$ meV μm^2 in (c-d), and for $\Delta_z = 0.18$ meV and $\Delta_T = 2.4$ meV μm^2 in (e-f). The colour codings are the same as those in Fig. 5.6. This figure is taken from Ref. [219].

Figure 5.9: Spin propagation in real space for different values of Γ which corresponds to the polariton lifetime of 30 ps in (a), 40 ps in (b), 50 ps in (c), and 60 ps in (d). The red arrows indicate the position of the continuous pump. All other parameters were kept the same as those in Fig. 5.3(a). This figure is taken from Ref. [219].

threshold to be relevant, for which nonlinear optical systems are essential. Along with the nonlinearity, the spin degree freedom of exciton-polaritons make them extremely promising for nonlinear spintronics devices. The coupling between the exciton polariton condensates in lattices have been demonstrated but this coupling is always bidirectional and consequently, there will be unwanted feedback. Although the proposed spin filter is not itself a device it can be an essential component of complete future circuits or networks. We have proposed theoretically a scheme to obtain an optical spin filter based on exciton-polaritons. The naturally present spin orbit interaction along with Zeeman splitting and an onsite detuning between sublattices in a polariton honeycomb lattice gives rise to a band structure where all the edge states of the system split in energy and there exists an energy window where one of the edge states with pure spin is surrounded by bulk states with opposite spin. Since there is no state with the same spin to backscatter to, polaritons having one of the spins in the system can propagate robustly through the edge state and thus spin filtering is achieved. The absence of the pairing backward propagating edge state reduces feedback in the system. Furthermore, the edge state in the polariton SF is more localized than those in a polariton TI, which allows to operate with very narrow channels. In contrast, regular polariton TI systems would require much wider strips/lattices to support robust propagation and would cause mixing between spin polarized states.

Conclusions

This thesis focuses on the dynamics of two types of light matter strongly coupled quasi particles. The first one is the dressed electronic system where the band structure of the direct band gap QW gets renormalized in presence of intense electromagnetic field. The other one is the system of exciton-polaritons, where cavity photons hybridize with QW excitons to have part exciton and part photon nature.

In chapter 2, we considered the realization of a tunable terahertz light emitting diode from a quantum well with dressed electrons placed in a highly doped p-n junction. In the considered system the strong resonant dressing field forms dynamic Stark gaps in the valence and conduction bands and the electric field inside the p-n junction makes the QW asymmetric. It is shown that the electrons transiting through the light induced Stark gaps in the conduction band emit photons with energy directly proportional to the dressing field. This scheme is tunable, compact, and shows a fair efficiency.

Next in chapter 3, we studied the nonlinear optical properties of a semiconductor quantum well (QW) irradiated by a two-mode electromagnetic wave consisting of a strong resonant dressing field and a weak off-resonant driving field. In the considered strongly coupled electron-field system, the dressing field opens dynamic Stark gaps in the electron energy spectrum of the QW, whereas the driving field induces electron oscillations in the QW plane. Since the gapped electron spectrum restricts the amplitude of the oscillations, the emission of a frequency comb from

the QW appears. Therefore, the doubly-driven QW operates as a nonlinear optical element, which can be used, particularly, for optically controlled generation of terahertz radiation.

In chapter 4, we presented a scheme to obtain antichiral edge states in an exciton-polariton honeycomb lattice with strip geometry, where the modes corresponding to both edges propagate in the same direction. Under resonant pumping the effect of a polariton condensate with nonzero velocity in one linear polarization is predicted to tilt the dispersion of polaritons in the other, which results in an energy shift between two Dirac cones and the otherwise flat edge states become tilted. Our simulations show that due to the spatial separation from the bulk modes the edge modes are reasonably robust against scattering with disorder (even though they are topologically trivial).

In the final chapter, we considered exciton-polaritons in a strip of honeycomb lattice with zigzag edges and it is shown that the interplay among the spin-orbit coupling, Zeeman splitting, and an onsite detuning between sublattices can give rise to a band structure where all the edge states of the system split in energy. Within an energy interval, one of the spin polarized edge states resides with the gap-less bulk having opposite spin. Being surrounded by opposite spin and the absence of the backward propagating edge state ensures both reflection free and feedback suppressed one-way flow of polaritons with one particular spin in the system. The edge states in this system are more localized than those in the standard topological polariton systems and are fully spin polarized. This paves the way for feedback free spin-selective polariton channels for transferring information in polariton networks.

In the future it'll be really interesting if these theoretical proposals can be realized experimentally.

List of publications

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- S. Mandal, R. Banerjee, E. A. Ostrovskaya, and T. C. H. Liew, “Nonreciprocal transport of exciton polaritons in a non-Hermitian chain”, Phys. Rev. Lett. **125** (2020): 123902.
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