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2020

Saramäki, T., Liu, Q. & Lim, Y. C. (2020). Use of common parts in masking filters for complexity reduction in FRM-based FIR filters. 2020 IEEE International Symposium on Circuits and Systems (ISCAS), 1-5. <https://dx.doi.org/10.1109/ISCAS45731.2020.9180936>

<https://hdl.handle.net/10356/147450>

<https://doi.org/10.1109/ISCAS45731.2020.9180936>

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Use of Common Parts in Masking Filters for Complexity Reduction in FRM-Based FIR Filters

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Abstract—A very efficient technique for significantly reducing the number of multipliers and adders in implementing narrow transition band linear-phase finite-impulse response (FIR) digital filters is to use the frequency-response masking (FRM) approach. This paper studies the construction of the masking filter pair in the FRM technique using a common part for further reducing the arithmetic complexity. Extensive simulations are included showing that the use of the common part always considerably decreases the arithmetic complexity at the expense of a slight increase in the overall filter order.

I. INTRODUCTION

In many digital signal processing applications, finite-impulse response (FIR) filters are preferred over their infinite-impulse response (IIR) counterparts as the former can be designed with exactly linear phase and they are free of stability problems and parasitic oscillations. Furthermore, the output noise due to the multiplication round-off errors and the sensitivity to variations in filter coefficients are usually lower for FIR filters. The major disadvantage of FIR filters is the fact that their orders are roughly inversely proportional to the transition bandwidth, which is not the case with IIR filters. This makes the implementation of narrow transition bandwidth FIR filters very costly compared to their IIR filter counterparts.

One of the most efficient techniques for synthesizing narrow transition band linear-phase FIR digital filters with a drastically reduced number of multipliers and adders compared with the conventional direct-form implementation is the frequency-response masking (FRM) approach [1]–[18]. In the FRM technique, an FIR filter is synthesized using an interconnection of several FIR subfilters as shown in Fig. 1 where L is a positive integer and N_F is the order of filter $F(z)$ which is even. The z -transform transfer function of the overall filter $F(z)$ is given by

$$H(z) = F(z^L)G_1(z) + [z^{-\frac{LN_F}{2}} - F(z^L)]G_2(z) \quad (1)$$

where $F(z^L)$ is the bandedge shaping filter obtained by replacing each delay element of the prototype bandedge shaping filter $F(z)$ by L delay elements. $G_1(z)$ and $G_2(z)$ are the transfer functions of the two masking filters. The filter orders of $G_1(z)$ and $G_2(z)$ must be both even or both odd and an appropriate number of delay elements must be cascaded to the masking filter with lower order such that the delays of the two masking filters become the same.

Fig. 1. The structure of an FIR filter synthesized using the FRM technique.

Fig. 2. The structure of an FIR filter synthesized using the FRM technique where the masking filters are constructed using a common filter part $G_3(z)$.

It has been demonstrated in several articles [3], [6], [10] that the arithmetic complexity of the FRM-based FIR filters can be reduced if the masking filters are constructed by the use of a common filter part $G_3(z)$ as shown in Fig. 2. In this case, the overall filter transfer function takes the form

$$H(z) = \left\{ F(z^L)G_1(z) + [z^{-\frac{LN_F}{2}} - F(z^L)]G_2(z) \right\} G_3(z). \quad (2)$$

The transfer functions $G_1(z)$, $G_2(z)$ and $G_3(z)$ in the structure of Fig. 2 are thus related to $G_1(z)$ and $G_2(z)$ in Fig. 1 through the substitutions of

$$G_1(z)G_3(z) \equiv G_1(z) \text{ and } G_2(z)G_3(z) \equiv G_2(z). \quad (3)$$

The reverse is not possible except for the case where $G_1(z)$ and $G_2(z)$ in Fig. 1 are separable as shown in (3). The key idea of using the common filter part to reduce the arithmetic complexity lies in the fact that the two masking filters share this common part that is implemented only once and simultaneously for both masking filters. In the original case without the common part, two masking filters are implemented in their entirety separately and independently of one another.

In this paper, a thorough study on the benefits of using a common part for the masking filters in the FRM-based FIR filters is performed. By means of several examples, it is shown that the use of the common part always reduces the overall number of multipliers at the expense of a slight increase in the overall filter order. It has been experimentally observed

that if the number of multipliers is of the main interest, then the best results are obtained by selecting either $G_1(z)$ or $G_2(z)$ in Fig. 2 to be a pure delay line.

This paper is organized as follows. An efficient joint optimization approach to designing FRM-based FIR filters with structures shown in Figs. 1 and 2 is first introduced in Section II. Section III illustrates then, by means of an example, the usefulness of the common part in constructing the two masking filters in comparison with the design with separate masking filters. Section IV presents a detailed comparison of FRM-based FIR filters with and without using a common part for the masking filters, which is obtained by extensive simulations. The results show that in different cases (different L), using the common part results in a considerable reduction in the number of multipliers. Finally, in Section V some concluding remarks are drawn.

II. JOINT OPTIMIZATION OF SUBFILTERS

In this section, we present an efficient optimization method that jointly optimizes all the subfilters in the FRM filters shown in Figs. 1 and 2.

The zero-phase frequency response of the overall filter in Fig. 2 can be written as

$$H(\omega) = \{F(L\omega)G_1(\omega) + [1 - F(L\omega)]G_2(\omega)\}G_3(\omega), \quad (4)$$

where

$$F(\omega) = f\left(\frac{N_F}{2}\right) + \sum_{n=1}^{\frac{N_F}{2}} 2f\left(\frac{N_F}{2} - n\right) \cos(\omega n)$$

and for $x = 1, 2, 3$,

$$G_x(\omega) = \begin{cases} g_x(n) \left(\frac{N_x}{2}\right) + \sum_{n=1}^{\frac{N_x}{2}} 2g_x\left(\frac{N_x}{2} - n\right) \cos(\omega n), & \text{for even } N_x \\ \sum_{n=1}^{\frac{N_x+1}{2}} 2g_x\left(\frac{N_x+1}{2} - n\right) \cos\left(\omega\left(n - \frac{1}{2}\right)\right), & \text{for odd } N_x \end{cases}.$$

In the above equations, $F(\omega)$ and $G_x(\omega)$ are the zero-phase frequency responses of $F(z)$ and $G_x(z)$, respectively. The impulse-response coefficients of $F(z)$ and $G_x(z)$ are denoted as $f(n)$ and $g_x(n)$, respectively. N_F is the order of $F(z)$ and N_x is the order of $G_x(z)$.

Alternatively, by denoting the coefficient vector of $F(z)$ as

$$\mathbf{a}_f = \left[f\left(\frac{N_F}{2}\right), 2f\left(\frac{N_F}{2} - 1\right), \dots, 2f(0) \right]^T$$

and the coefficient vectors of $G_x(z)$ as

$$\mathbf{a}_x = \begin{cases} \left[g_x\left(\frac{N_x}{2}\right), 2g_x\left(\frac{N_x}{2} - 1\right), \dots, 2g_x(0) \right]^T, & \text{for even } N_x \\ \left[2g_x\left(\frac{N_x-1}{2}\right), 2g_x\left(\frac{N_x-3}{2}\right), \dots, 2g_x(0) \right]^T, & \text{for odd } N_x \end{cases}$$

Fig. 3. Magnitude responses for the FRM-based FIR filters without (red line) and with (blue line) a common part. (a). Overall response. (b). Passband details.

the overall zero-phase frequency response can be expressed as

$$H(\omega, \mathbf{x}) = \{F(L\omega, \mathbf{a}_f)G_1(\omega, \mathbf{a}_1) + [1 - F(L\omega, \mathbf{a}_f)]G_2(\omega, \mathbf{a}_2)\}G_3(\omega, \mathbf{a}_3) \quad (5)$$

where $\mathbf{x} = [\mathbf{a}_f \ \mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3]^T$.

The optimization problem for designing the FRM filter in Fig. 2 can then be formulated as

$$\underset{\mathbf{x}, \delta}{\text{minimize}} \quad \delta \quad (6a)$$

$$\text{subject to} \quad |H(\omega, \mathbf{x}) - 1| \leq \delta, \quad \forall \omega \in [0, \omega_p] \quad (6b)$$

$$W_s(\omega) |H(\omega, \mathbf{x})| \leq \delta, \quad \forall \omega \in [\omega_s, \pi] \quad (6c)$$

where $W_s(\omega)$ is the weight function for the stopband.

Note that when the masking filters are separately implemented (see Fig. 1), we have $N_3 = 0$. Thus, problem (6) can also be used to design the FRM filter in Fig. 1. A local optimum (or possibly a global optimum) of problem (6) can be obtained using sequential convex programming [19] with proper selected initial solutions.

III. MOTIVATION FOR THE USE OF A COMMON PART TO CONSTRUCT THE MASKING FILTERS

To illustrate the benefits of using a common part in constructing the masking filters, consider the criteria in [1], that is, the passband and stopband edge angles are at $\omega_p = 0.6\pi$ and $\omega_s = 0.61\pi$, the minimum stopband attenuation is 40 dB, and the maximum passband peak-to-peak ripple is 0.2 dB.

A. FRM-Based FIR Filters with Separate Masking Filters

If the masking filters are separately implemented as shown Fig. 1, then for $L = 14$, the number of multipliers in the overall filter is minimized by $N_F = 28$, $N_1 = 43$, and $N_2 = 25$. The resulting number of multipliers is 50 and the overall filter order is 435. For the direct-form FIR filter implementation, the corresponding figures are 191 and 381, respectively. Hence, the arithmetic complexity of the present FRM-based FIR filter design is 26.2% of that of the direct-form FIR implementation at the expense of a 14.2% increase in the filter order. The magnitude response of the FRM-based filter is shown by the red line in Fig. 3.

Fig. 4. Construction of the first overall masking filter $G_1(z)G_3(z)$ (black line) with the aid of the common part $G_3(z)$ (blue line) and the “correction” term $G_1(z)$ (red line).

B. FRM-Based FIR Filters with a Common Part for Two Masking Filters

If in the implementation, where the masking filters share a common part (see Fig. 2), it is desired that the resulting order of the second making filter $[G_2(z)G_3(z)]$ remains the same, then the number of overall multipliers is minimized by selecting $N_F = 28$, $N_1 = 28$, $N_2 = 0$, and $N_3 = 25$. The blue line in Fig. 3 shows the magnitude response of this design. For this design, the number of multipliers decreases from 50 to 43 when compared with the filter with separately designed masking filters. The price to be paid is a slight increase in the overall filter order (from 435 to 445). It is worth emphasizing the fact that if the masking filters sharing the common part are separately implemented, i.e., implemented as $G_1(z)G_3(z)$ and $G_2(z)G_3(z)$, then their orders are $28 + 25 = 53$ and 25 and the overall number of multipliers increases to 55.

Fig. 4 illustrates how the first overall masking filter $G_1(z)G_3(z)$ is constructed from the common part $G_3(z)$, which is the second overall masking filter, and the “correction” term $G_1(z)$. It can be seen that most of the zeros on the unit circle are provided by $G_3(z)$ (blue line), whereas $G_1(z)$ (red line) concentrates on making the passband of the overall masking filter (black line) wider in addition to generating some more zeros on the unit circle. It is worth emphasizing that in the practical implementation it is required that $G_2(z) = z^{-N_1/2} = z^{-14}$ in Fig. 2 in order for making the delays through both branches the same. The overall second masking filter is thus $z^{-14}G_3(z)$. Note that the additional delay is half the order of $G_1(z)$.

C. Comparisons

Figs. 5 and 6 compare the impulse responses and zero plots of the overall masking filters such that the responses for the separate design (design with a common part) are on the left (right). Similar comparisons between the bandedge shaping filters are shown in Fig. 7. Fig 8, in turn, compares the magnitude responses of the first and second masking filters and those of the bandedge shaping and overall filters. Responses for the FRM filter with separate masking filters (masking filters sharing a common part) are shown by red (blue). Overall, it can be seen from Figs. 5-8 that the FRM

Fig. 5. Impulse responses and zero plots for the first overall masking filters. (a) FRM-based FIR filter without a common part. (b) FRM-based FIR filter with a common part.

Fig. 6. Impulse responses and zero plots for the second overall masking filters. (a) FRM-based FIR filter without a common part. (b) FRM-based FIR filter with a common part.

filters with and without a common part for the masking filters behave quite similarly. There are two major differences. First, the overall first masking filters are $G_1(z)$ and $G_1(z)G_3(z)$ for the designs without and with a common part, respectively. From Fig. 5, it is seen that their impulse responses around the centers of the responses are very similar. This explains the similarities between their magnitudes responses shown in Fig. 8(a). Second, the overall second masking filters are $G_2(z)$ and $z^{-14}G_3(z)$. As seen from Fig. 6, when omitting the additional delays, their characteristics are quite the same.

D. Minimum Complexity Design

Selecting $G_2(z)$ to be a pure delay is only one option. If $G_2(z)$ consists of a transfer function of order 2 (4) [8] {16} and a proper number of delays, then the given criteria are met by selecting the order of $G_3(z)$ to be $25 - 2 = 23$ ($25 - 4 = 21$) [$25 - 8 = 17$] { $25 - 16 = 9$ } and that of $G_1(z)$ to be $28 + 2 = 30$ ($28 + 4 = 32$) [$28 + 8 - 2 = 34$] { $36 = 28 + 16 - 8$ }. In all the cases, the order of $G_2(z)G_3(z)$ remains on purpose to be 25, whereas the order of $G_1(z)$ is the minimum even integer required to meet the criteria. The order of $G_1(z)G_3(z)$ remains the same in the first two cases, whereas in the third (fourth)

Fig. 7. Impulse responses and zero plots for the bandedge shaping filters. (a) FRM-based FIR filter without a common part. (b) FRM-based FIR filter with a common part.

Fig. 8. Magnitude responses for the FRM-based FIR filters without (red line) and with (blue line) a common part. (a) First overall masking filters. (b) Second overall masking filters. (c) Bandedge shaping filters. (d) Overall filters.

case, it is reduced by 2 (8) from 53 to 51 (45). The resulting number of multipliers is 44 (45) [47] {48} in comparison with 43 and the overall filter order is 445 (445) [443] {437} in comparison with 445.

For the overall filter with separate masking filters, the number of multipliers and the order are, according to the previous discussion, 50 and 435, respectively. Hence, the above four FRM-based FIR filters are in between the filter without a common part for the masking filters and the filter with a common part in the special case where $G_2(z)$ is a pure delay line. In general, if the target is to minimize the number of multipliers, then the best design for all specifications considered so far has been obtained by selecting either $G_1(z)$ or $G_2(z)$ to be simply a delay line, i.e., $N_1 = 0$ or $N_2 = 0$.

IV. SIMULATION RESULTS

Table I provides various pairwise comparisons between FRM-based FIR filters with and without using a common part for constructing two masking filters. The criteria to be met are those specified in the previous section. As special cases, the minimum-order direct-form FIR filter is included

TABLE I
COMPARISON BETWEEN FRM-BASED FIR FILTERS SYNTHESIZED WITH AND WITHOUT A COMMON PART FOR THE MASKING FILTERS

L	N_F	N_1	N_2	N_3	Order	Number of Multipliers	Arithmetic Complexity	Expense
1	381	0	0	0	381	191	100%	0%
9	44	40	32	0	436	61	31.9%	14.4%
4	94	10	12	0	388	61	31.9%	1.8%
4	94	0	10	7	393	59	30.9%	3.1%
9	44	29	15	0	425	46	24.1%	11.5%
9	44	18	0	17	431	41	21.5%	13.1%
9	42	30	18	0	408	48	25.1%	7.1%
9	42	18	0	22	418	44	23.0%	9.7%
9	40	35	21	0	395	50	26.2%	3.7%
9	40	16	0	32	408	47	24.6%	7.1%
14	28	43	25	0	435	50	26.2%	14.2%
14	28	24	0	28	444	43	22.5%	16.5%

in the first row and the FRM filter obtained by the original synthesis scheme proposed in [1] in the second row. In addition to supplying the number of multipliers and the filter orders for each comparison filter pair, the number of multipliers is compared with the minimum-order direct FIR filter design by means of the arithmetic complexity. The expense, in turn, indicates the increase in the overall filter order.

As seen in Table I, the best FRM-based FIR filter with a common part for $L = 14$ is achieved from the filter of Section III-B by decreasing N_1 from 28 to 24 and by increasing N_3 from 25 to 28. The overall number of multipliers remains the same (43), but the overall filter order decreases by one (from 445 to 444). It should be noted that the same number of multipliers and the same order is also obtained by $N_1 = 20$ and $N_3 = 32$. This design is slightly worse since the stopband attenuation is lower (40.2 dB compared with 40.8 dB).

The filters in the pair for $L = 9$ and $N_F = 44$ are the best ones (with and without the common part) in terms of the number of multipliers. The filter pairs for $L = 9$ and $N_F = 42$ and $L = 9$ and $N_F = 40$ have been inserted in order to study how much the order of the bandedge shaping filter can be lowered. It is seen that this decrease results in filters with lowered filter orders at the expense of more multipliers. In this respect, the filters for $L = 4$ increase the order of the minimum-order direct FIR filter only slightly while they still provide a considerable reduction in the number of multipliers. Overall, it is observed from Table I that the use of the common part reduces in each case the number of multipliers at the expense of a very small increase in the overall filter order.

V. CONCLUSION

A thorough investigation on the use of a common part for the masking filters in the FRM-based FIR filters has been performed in this paper. Simulation results have demonstrated that the use of the common part can decrease the arithmetic complexity significantly. The price to pay is a slight increase in the overall filter order. Future work is devoted to studying to which extent the use of common parts can be utilized in the multistage FRM approach.

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