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A Piecewise-Affine Decision Rule based Stochastic AC Optimal Power Flow Approach

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Abstract—This paper proposes a piecewise-affine decision rule based stochastic optimal power flow approach for reserve dispatch of generators under uncertain demand and renewable energy source generation. The ambiguity set comprises a predefined set of scenarios to capture the plausible uncertain realizations. The objective is to minimize the expected generation cost under those scenarios. A piecewise-affine decision rule is established with auxiliary control parameters to compute the optimal participation of generators depending on the amounts of reserve requirements. Such a reserve dispatch rule would be less conservative compared to the traditional affine decision rule. The results illustrate that the proposed piecewise-affine generation redispatch rule improves the optimality of operation via efficient resource utilization for reserve support.

Index Terms—AC optimal power flow, automatic generation control, piecewise-affine decision rule, stochastic optimization

I. INTRODUCTION

Optimal power flow (OPF) is a mathematical tool used by the power system operators to seek economic operation. It is a challenging task to maintain the economy and reliability of future power systems under the uncertainties attributed to increased penetration of renewable energy sources (RESs). Therefore, the optimization tools must be able to determine suitable countermeasures to mitigate the impacts of these uncertainties on system security while maintaining the economy of operations. Among the different classes of OPF problems that handle uncertainties, the stochastic or scenario-based OPF (S-OPF) formulations consider a predetermined pool of scenarios (point forecasts) of uncertain power injections [1] based on the probability distribution functions [2], [3]. Therein, the number of scenarios in the pool is a trade-off between the accuracy of the representation of uncertainties and the computational burden of the S-OPF problem. As such, the selection of scenarios is critical for the validity and the tractability of the S-OPF solution. Traditionally, the Monte Carlo based approaches are used to generate scenarios. Recently, the Latin Hypercube sampling method has been widely used due to its improved accuracy over the Monte Carlo method [4]. Further, scenario reduction methods have been proposed in the literature to represent the average impact of a large number of scenarios with reduced dimensionality to improve the tractability of the S-OPF problem [5].

In addition to the generation capability limits, the S-OPF problem accounts for the reserve capability limits of the online generators. The solution of the S-OPF problem usually include the reference power dispatch set points for generators which are capable of providing sufficient amount of reserves [3], [6], [7]. In general, the real-time power mismatch is associated with a power frequency variation. In the S-OPF literature, affine decision (AD) rules have been proposed for reserve dispatch, considering the proportional participation of online generators to mitigate the power mismatch caused by the uncertain sources in real-time operation [2], [3], [6], [7]. This is referred to as automatic generation control in which the generator power dispatch is controlled depending on the frequency variations. Many of the S-OPF approaches which derive AD rules for reserve dispatch consider decoupled network approximation for the solution tractability [1], [6]. However, the accuracy of the solution might be insufficient for some of the systems [8]. Hence, there is a research gap in the literature for an adaptable S-OPF approach (considering nonlinear network constraints) for determining a decision rule for reserve dispatch of online generators.

In general, there is a higher probability of occurring lower amounts of power mismatches compared to higher amounts of power mismatches. The generation/reserve capabilities and cost functions are different for the units in operation. As such, the low-cost units must be utilized for the highly probable lower amounts of power mismatches, wherein the expensive units can be operated only to satisfy higher amounts of power mismatches due to the generation/reserve capacity limitations of the low-cost units. Thus, the utilization of all the reserve units (as per the AD making proposed in [7]) in the constant proportion for any amount of power mismatch would result in utilizing expensive units at times when it could have been satisfied only using economic units. Such a redispatch rule might result in suboptimal utilization of generation resources leading to inefficient operation of power systems.

Therefore, this paper proposes a piecewise-affine decision (PW-AD) rule for reserve dispatch of online units based on the power mismatch (which is generally reflected via the amount of deviation in *power frequency* in real-time operation). Accordingly, the proposed PW-AD rule-based S-OPF approach aims at dispatching reserve units following their capability restrictions, nonlinear network constraints, and the cost of generation, thereby improving the efficiency of operation compared to the traditional AD rule proposed in [7].

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II. OPTIMAL POWER FLOW FORMULATION

Let $\mathcal{G}$, $\mathcal{F}$, $\mathcal{N}$, and $\mathcal{L}$ be the sets of generators, RESs, buses, and lines of the power system respectively. Let $\delta \in \mathbb{R}^{|\mathcal{N}|}$ and $V \in \mathbb{R}^{|\mathcal{N}|}$ be the vectors of bus voltage angles and magnitudes with $\Theta = [\delta^T, V^T]^T$ respectively. $(\cdot)^T$ is the transpose of vector/matrix $(\cdot)$. Let $P_G \in \mathbb{R}^{|\mathcal{G}|}$ and $Q_G \in \mathbb{R}^{|\mathcal{G}|}$ be the active and reactive power generation vectors of generators respectively. In general, the tuple (Θ, P_G, Q_G) comprises the decision variables pertaining to the OPF problem. Further, let $P_F \in \mathbb{R}^{|\mathcal{F}|}$ and $Q_F \in \mathbb{R}^{|\mathcal{F}|}$ be the active and reactive power generation vectors of RESs respectively. Finally, $P_D \in \mathbb{R}^{|\mathcal{N}|}$ and $Q_D \in \mathbb{R}^{|\mathcal{N}|}$ be the vectors of active and reactive power demand of the system. Equations (1a) and (1b) constitute the active and reactive power extractions at bus $n \in \mathcal{N}$ respectively which follow the *Kirchhoff's current and voltage laws*. Subscripts of the variables denote bus indexes; $\delta_{n,i} = \delta_n - \delta_i$. $Y_{n,i}^G$ and $Y_{n,i}^B$ are the real and imaginary parts of element (n, i) of the bus admittance matrix respectively.

$$\Phi_{P,n}(\Theta^\omega) = V_n^\omega \sum_{i=1}^{|\mathcal{N}|} V_i^\omega \left[Y_{n,i}^G \cos(\delta_{n,i}^\omega) + Y_{n,i}^B \sin(\delta_{n,i}^\omega) \right] \quad (1a)$$

$$\Phi_{Q,n}(\Theta^\omega) = V_n^\omega \sum_{i=1}^{|\mathcal{N}|} V_i^\omega \left[Y_{n,i}^G \sin(\delta_{n,i}^\omega) - Y_{n,i}^B \cos(\delta_{n,i}^\omega) \right] \quad (1b)$$

Equations (2) formulate the apparent power flow $\hat{\Phi}_{X,l}(\Theta^\omega)$ through line $l \in \mathcal{L}$ from bus $i \in \mathcal{N}$ to bus $j \in \mathcal{N}$. Therein, $P_{X,l}^{i,j}$ and $Q_{X,l}^{i,j}$ are the active and reactive power flows respectively through line $l \in \mathcal{L}$ from bus i to bus j ($i \neq j$).

$$P_{X,l}^{i,j} = -(V_i^\omega)^2 Y_{i,j}^G + V_i^\omega V_j^\omega \left(Y_{i,j}^G \cos(\delta_{i,j}^\omega) + Y_{i,j}^B \sin(\delta_{i,j}^\omega) \right) \quad (2a)$$

$$Q_{X,l}^{i,j} = (V_i^\omega)^2 Y_{i,j}^B + V_i^\omega V_j^\omega \left(Y_{i,j}^G \sin(\delta_{i,j}^\omega) - Y_{i,j}^B \cos(\delta_{i,j}^\omega) \right) \quad (2b)$$

$$\hat{\Phi}_{X,l}(\Theta^\omega) = \sqrt{(P_{X,l}^{i,j})^2 + (Q_{X,l}^{i,j})^2} \quad (2c)$$

Generation capability and network constraints of the OPF problem under scenario ω can be formulated as follows.

$$\Pi^\omega = \left\{ (\Theta^\omega, P_G^\omega, Q_G^\omega) \in \mathbb{R}^{2|\mathcal{N}|} \times \mathbb{R}^{|\mathcal{G}|} \times \mathbb{R}^{|\mathcal{G}|} : (1), (2) \right\} \quad (3a)$$

$$\Gamma_G^T P_G^\omega - \Gamma_D^T P_D^\omega + \Gamma_F^T P_F^\omega - \Phi_P(\Theta^\omega) = 0 \quad (3b)$$

$$\Gamma_G^T Q_G^\omega - \Gamma_D^T Q_D^\omega + \Gamma_F^T Q_F^\omega - \Phi_Q(\Theta^\omega) = 0 \quad (3c)$$

$$\hat{\Phi}_X(\Theta^\omega) \leq \bar{S}_X, \quad \check{\Phi}_X(\Theta^\omega) \leq \bar{S}_X \quad (3d)$$

$$V \leq V^\omega \leq \bar{V}, \quad P_G \leq P_G^\omega \leq \bar{P}_G, \quad Q_G \leq Q_G^\omega \leq \bar{Q}_G \quad (3e)$$

where Π^ω is the *feasibility space* of OPF constraints under scenario ω . Equations (3b) and (3c) are the nodal active and reactive power balance equations respectively where $\Gamma_G \in \{0, 1\}^{|\mathcal{G}| \times |\mathcal{N}|}$, $\Gamma_D \in \{0, 1\}^{|\mathcal{N}| \times |\mathcal{N}|}$, and $\Gamma_F \in \{0, 1\}^{|\mathcal{F}| \times |\mathcal{N}|}$ are the connecting matrices which map generators, demand, and RESs to the buses respectively. Constraints (3d) limit the apparent power flow through lines within their transmission capacities $\bar{S}_X$. The voltage magnitude limits for the buses and

the active and reactive power generation capability limits are enforced in (3e), where upper and lower bounds are denoted by the $(\bar{\cdot})$ and $(\underline{\cdot})$ respectively.

III. UNCERTAINTY CHARACTERIZATION

In general, uncertain power injections $\mathcal{E}$ are assumed with polyhedral uncertainty sets as defined in (4) with lower $\underline{\mathcal{E}} \leq 0$ and upper $\bar{\mathcal{E}} \geq 0$ confidence bounds [9].

$$\Omega = \left\{ \mathcal{E} = [\mathcal{E}_D^T, -\mathcal{E}_F^T]^T \in \mathbb{R}^{|\mathcal{N}|+|\mathcal{F}|} : \underline{\mathcal{E}} \leq \mathcal{E} \leq \bar{\mathcal{E}} \right\} \quad (4)$$

where $\mathcal{E}_D$ and $\mathcal{E}_F$ are the forecast errors of the load demand and RES generations respectively.

There are several tools which forecast plausible realizations of power demand and RES power output uncertainties using probability distributions and the correlations of uncertain variables. Let Υ be the set of stochastic scenarios (point forecasts) such that $\Upsilon \subset \Omega$. Normally, the higher the number of scenarios $|\Upsilon|$, the robustness of the S-OPF solution will be increased; however, the resulting S-OPF problem would be more challenging to solve.

Let $\omega = 0$ be the base scenario and $\mathcal{W} = \{0, 1, \dots, |\Upsilon|\}$. Further, uncertain injections are assumed with *constant* power factors such that the ratios ϕ_D and ϕ_F are defined by Q_D^0/P_D^0 and Q_F^0/P_F^0 respectively. $\mathcal{D}(x)$ constructs a diagonal matrix with entries of x . Then,

$$P_D^\omega = P_D^0 + \mathcal{E}_D^\omega, \quad P_F^\omega = P_F^0 + \mathcal{E}_F^\omega; \quad \forall \omega \in \mathcal{W} \setminus \{0\} \quad (5a)$$

$$Q_D^\omega = \mathcal{D}(\phi_D) P_D^\omega, \quad Q_F^\omega = \mathcal{D}(\phi_F) P_F^\omega; \quad \forall \omega \in \mathcal{W} \setminus \{0\} \quad (5b)$$

IV. STOCHASTIC OPTIMIZATION AND CORRECTIVE GENERATOR CONTROL

A. Affine Decision Rule for Generation Control

The operating point (Θ^0, P_G^0, Q_G^0) is feasible under the base scenario. The active and reactive power of generation must be adjusted under the presence of uncertainties to maintain the power balance and a feasible voltage profile while satisfying the reserve capability and network constraints. Traditionally, automatic generation control is realized via AD policies which define the generation redispatch in proportion to the power mismatch. As such, the total power mismatch, i.e., $\mathbf{1}^T \mathcal{E}^\omega$ under scenario ω (where $\mathbf{1}$ is a vector of ones with an appropriate dimension), is divided among available generators based on their individual (nonnegative) *participation factors*.

$$\mathcal{C}(\mathcal{E}^\omega) = \left\{ \alpha_g \in [0, 1]; \quad \forall g \in \mathcal{G} : \sum_{g \in \mathcal{G}} \alpha_g = 1 \right\} \quad (6a)$$

$$P_{G,g}^\omega = (\mathbf{1}^T \mathcal{E}^\omega) \alpha_g + P_{G,g}^0; \quad \forall g \in \mathcal{G}^{\mathcal{PV}} \quad (6b)$$

$$-\bar{r}_G \leq P_G^\omega - P_G^0 \leq \bar{r}_G \quad (6c)$$

$$V_n^\omega - V_n^0 = 0; \quad \forall n \in \mathcal{N}^{\delta V} \cup \mathcal{N}^{\mathcal{PV}} \quad (6d)$$

where α_g is the proportional participation factor of generator $g \in \mathcal{G}$. Equation (6a) enforces their summation to be 1 as the optimal participation of all the generators must balance the total power mismatch. Although forecast errors are the main source of power mismatch, they would further result in

an active power loss variation (usually smaller in quantity) due to the changes in power flow. This is a priori unknown quantity. The generators at the reference bus (slack bus) are usually assumed to mitigate the variations in the total power loss in addition to their contributions for correcting the power mismatch caused by the forecast errors [7]. Hence, the adjusted power dispatch values based on the proportional participation are strictly enforced only for the set of generators at PV-buses, i.e., $\mathcal{G}_{PV}$ in (6b). In (6c), the active power redispatch is restricted below the ramping/reserve capability restrictions, i.e., $\bar{r}_G \in \mathbb{R}^{|\mathcal{G}|}$ of generators. Moreover, the generator bus voltages (at the slack bus, i.e., $\mathcal{N}_{\delta V}$ and PV-buses, i.e., $\mathcal{N}_{PV}$) are maintained constant under the uncertain realizations which is enforced in (6d).

B. Piecewise-Affine Decision Rule

The conventional AD rule given in (6a) and (6b) approximates the generation redispatch proportional to the total power mismatch, i.e., $\mathbf{1}^T \mathcal{E}^\omega$ for scenario ω . However, due to different generation capabilities, and the nonlinearities in cost functions and network constraints, the optimum decision rule for generation redispatch must be *nonlinear*. However, a nonlinear decision rule makes the S-OPF problem computationally intractable. Therefore, a PW-AD rule is proposed in this paper, which has the potential to approximate the nonlinearity to a certain extent leading to enhanced economy of operations.

In general, there is a higher probability that uncertain power demand or RES power output realizations would vary around the mean values/base scenario. Hence, smaller amounts of reserve requirements are more frequent than higher amounts of reserve requirements (which is usually associated with the worst scenario). In the proposed PW-AD rule, generator participation factors are determined based on the total amount of power imbalance (deficit/surplus) caused by the uncertainties. The AD rule results in partial utilization of economic sources for lesser reserve requirements which are more frequent. In contrast, the utilization of economic sources can be enhanced with the PW-AD rule for more frequent lesser reserve requirements, while expensive sources are only employed to satisfy higher reserve requirements due to the reserve capability restrictions of economic sources. To this end, the uncertainty segments are defined as follows.

Let $\mathcal{X} = [\mathcal{X}_1, \dots, \mathcal{X}_{|\mathcal{M}|}]^T \in [0, 1]^{|\mathcal{M}|}$ be a *predefined* set of parameters such that $0 < \mathcal{X}_1 \leq \dots \leq \mathcal{X}_{|\mathcal{M}|} < 1$. Let $\hat{\xi}^\omega = [\hat{\xi}_1^\omega, \dots, \hat{\xi}_{|\mathcal{M}|}^\omega]^T \in \mathbb{R}_+^{|\mathcal{M}|}$ be the set of auxiliary uncertain variables associated with scenario $\mathcal{E}^\omega$ such that

$$\hat{\xi}_m^\omega = \max \left\{ \mathbf{1}^T \mathcal{E}^\omega - \mathcal{X}_m (\mathbf{1}^T \bar{\mathcal{E}}), 0 \right\}; \forall m \in \mathcal{M} \quad (7)$$

Similarly, let $\check{\xi}^\omega = [\check{\xi}_1^\omega, \dots, \check{\xi}_{|\mathcal{M}|}^\omega]^T \in \mathbb{R}_-^{|\mathcal{M}|}$ be the set of auxiliary uncertain variables associated with scenario $\mathcal{E}^\omega$ such that

$$\check{\xi}_m^\omega = \min \left\{ \mathbf{1}^T \mathcal{E}^\omega - \mathcal{X}_m (\mathbf{1}^T \underline{\mathcal{E}}), 0 \right\}; \forall m \in \mathcal{M} \quad (8)$$

As such, the total plausible power imbalance is divided into $2|\mathcal{M}| + 1$ segments. In the following, the PW-AD rule $\tilde{\mathcal{C}}$ with

auxiliary control parameters, i.e., β_g^m of generator $g \in \mathcal{G}$ are defined for each uncertainty segment $m \in \mathcal{M}$ and optimized for redispatching generators against the uncertainties.

$$\tilde{\mathcal{C}}(\mathcal{E}^\omega) = \left\{ \hat{\beta}_g^m, \check{\beta}_g^m \in [-1, 1]; \forall g \in \mathcal{G}, \forall m \in \mathcal{M} \right. \quad (9a)$$

$$\hat{\xi}_m^\omega \in \mathbb{R}_+, \check{\xi}_m^\omega \in \mathbb{R}_-, (7), (8); \forall m \in \mathcal{M} \quad (9b)$$

$$(6a), (6c), (6d) \quad (9c)$$

$$\sum_{g \in \mathcal{G}} \left[\hat{\beta}_g^m + \check{\beta}_g^m \right] = 0; \forall m \in \mathcal{M} \quad (9d)$$

$$P_{G,g}^\omega = \sum_{m \in \mathcal{M}} \hat{\beta}_g^m \hat{\xi}_m^\omega + \sum_{m \in \mathcal{M}} \check{\beta}_g^m \check{\xi}_m^\omega + (\mathbf{1}^T \mathcal{E}^\omega) \alpha_g + P_{G,g}^0; \forall g \in \mathcal{G}^{PV} \quad (9e)$$

$$\alpha_g + \sum_{m'=1}^m \hat{\beta}_g^{m'} \geq 0, \forall g \in \mathcal{G}, \forall m \in \mathcal{M} \quad (9f)$$

$$\alpha_g + \sum_{m'=1}^m \check{\beta}_g^{m'} \geq 0; \forall g \in \mathcal{G}, \forall m \in \mathcal{M} \quad (9g)$$

Equation (9d) along with (6a) satisfies the power balance or the complete sharing of the power mismatch among the operating units/generators. The power dispatch of generators are determined based on the PW-AD rule given in (9e). Therein, the participation factor of unit g at each uncertainty segment m is determined by α_g and $\hat{\beta}_g^m / \check{\beta}_g^m$. Hence, the PW-AD rule improves the operational flexibility via enabling the generators to contribute with different participation factors depending on the reserve requirements. Constraints (9f) and (9g) ensures the nonnegative participation of generators for sharing of power mismatch within each uncertainty segment.

C. Adaptable Stochastic OPF Formulation

S-OPF problem determines the optimal base scenario solution and the corrective control rule which allows sufficient redispatch measures against the possible uncertain realizations. To this end, adaptable S-OPF problem can be modeled as follows. Let $\mathcal{A}_G = [\alpha_g] \in [0, 1]^{|\mathcal{G}|}$, $\hat{\mathcal{B}}_G = [\hat{\beta}_g^m] \in [-1, 1]^{|\mathcal{G}| \times |\mathcal{M}|}$, and $\check{\mathcal{B}}_G = [\check{\beta}_g^m] \in [-1, 1]^{|\mathcal{G}| \times |\mathcal{M}|}$.

$$\min_{\substack{\Theta^\omega, P_G^\omega, Q_G^\omega \\ \mathcal{A}_G, \hat{\mathcal{B}}_G, \check{\mathcal{B}}_G}} \sum_{\omega \in \mathcal{W}} \mu_\omega C_G(P_G^\omega) \quad (10a)$$

$$\text{s. t. } (\Theta^\omega, P_G^\omega, Q_G^\omega) \in \Pi^\omega, \mathcal{E}^\omega \in \Upsilon; \forall \omega \in \mathcal{W} \quad (10b)$$

$$(\Theta^\omega, P_G^\omega, Q_G^\omega, \mathcal{A}_G, \hat{\mathcal{B}}_G, \check{\mathcal{B}}_G) \in \tilde{\mathcal{C}}(\mathcal{E}^\omega); \forall \omega \in \mathcal{W} \quad (10c)$$

The expected total generation cost is given in (10a) where μ_ω is the probability of uncertain realization $\mathcal{E}^\omega \in \Upsilon$ for all $\omega \in \mathcal{W}$ with $\sum_{\omega \in \mathcal{W}} \mu_\omega = 1$. OPF constraints in (3) under the point forecasts (including the base scenario) are combined in (10b). Therefore, the optimal solution $(\Theta^{0*}, P_G^{0*}, Q_G^{0*}, \mathcal{A}_G^*, \hat{\mathcal{B}}_G^*, \check{\mathcal{B}}_G^*)$ of (10) is *feasible* (thus, *robust*) against the considered set of uncertain realizations $\mathcal{E} \in \Upsilon$.

V. NUMERICAL RESULTS

In the numerical analysis, the performance of the proposed PW-AD rule $\tilde{\mathcal{C}}$ defined in (9) is compared with the traditional

AD rule $\mathcal{C}$ given in (6) based on the IEEE 24-bus system in [10]. IEEE 24-bus system comprises 33 generators and 17 loads. To support the case study, five wind power plants of 100 MW nominal active power generation with 0.6 power factor each were connected at bus 11, 12, 17, 21, and 22. The quadratic cost coefficients of generators were taken as 1% of their linear cost coefficients. In case studies, symmetrical forecast errors of $[-10\%, 10\%]$ and $[-40\%, 40\%]$ are considered for the nominal load demand and RES generation respectively.

S-OPF problem (10) was programmed in MATLAB and solved using the inbuilt interior-point solver in MATPOWER package [10]. All the simulations were performed on a PC with an Intel®Core i7-4770U four-core CPU processor running at 3.40 GHz with 8 GB of RAM.

A. Scenario Generation and Reduction

In this work, the uncertain injections are assumed with polyhedral uncertainty sets and the scenarios are generated assuming a uniform distribution within the confidence bounds. As such, 1,000 scenarios were randomly generated to reflect the realizations of uncertain injection. To improve the solution tractability, a pool of only 20 scenarios were selected in addition to the base scenario (i.e., 21 in total) following the procedure mentioned in Algorithm 2 of [5, Appendix]. Therein, the two scenarios with highest and lowest reserve requirements were included in the scenario pool to maintain the robustness of the generator set-points, i.e., the base scenario power dispatch, and the decision rule.

B. Performance Evaluation of the PW-AD Rule

For the 21 scenarios considered in the pool, the maximum per unit reserve requirement $\max_{\omega \in \mathcal{W}} \mathbf{1}^T \mathcal{E} \omega / (\mathbf{1}^T \mathcal{E})$ is 0.4811 and the minimum per unit reserve requirement $\min_{\omega \in \mathcal{W}} \mathbf{1}^T \mathcal{E} \omega / (\mathbf{1}^T \mathcal{E})$ is 0.4481. $\mathcal{X} = [0.1, 0.2, 0.3, 0.4]^T$ is taken arbitrarily. This provides 9 uncertainty segments to compute the optimal PW-AD rule. Computation time for AD rule and PW-AD rule based S-OPF problems were 36.11 s and 38.01 s respectively. Hence, computational performance is almost similar for both problems. Total operating cost resulted with PW-AD rule $\$9.1854 \times 10^4/\text{h}$ was 3.36% lower than that given by the AD rule $\$9.5043 \times 10^4/\text{h}$. Out of the 33 generators in the test system, the AD rule utilizes only 13 generators for reserve dispatch, whereas the PW-AD rule utilizes 22 generators at different levels of reserve requirements. Fig. 1 illustrates the generation redispatch variation of generators 12, 21, 24 and 33. It can be observed that the participation factors allocated by the PW-AD rule are varying with the amount of power imbalance. For instance, the participations/contributions from cheaper generators 21 and 24 are significant within the 5th (middle one) of the 9 uncertainty segments, while the generators 12 and 33 (which are more expensive) are utilized only in other uncertainty segments. As the 5th segment encloses the *mean*, the PW-AD rule utilizes economic sources to mitigate the most frequently occurring uncertainties which results in a lesser total operating cost. As such, the reduction in the total

operating cost of PW-AD rule based operation is due to the efficient utilization of resources for reserve dispatch.

Fig. 1. Reserve dispatch variation of generators 12, 21, 24 and 33 under the two decision rules.

Fig. 2 illustrates that the variation of generator voltage magnitudes for the two decision rules are almost similar.

Fig. 2. Generator voltage magnitude variation under the two decision rules.

To validate and compare the robustness of the AD rule and PW-AD rule based S-OPF solutions, out-of-sample analyses were considered. Therein, a set of scenarios comprising 1,000 uncertain realizations within the polyhedral confidence bounds were randomly and independently generated following the Latin Hypercube sampling method in [4]. Therein, a power flow simulation was executed via MATPOWER package [10] under each scenario. Voltage magnitudes of generator buses were fixed at the respected values computed in the base scenario solution of S-OPF problem for the two decision rules. Similarly, the active power dispatch of the generators apart from the slack generator were fixed using the corresponding decision rule. Then, the state variables of the system were computed for every scenario and their practicality was examined according to satisfaction of the physical/security limits. If a power flow solution reports a violation of any such limits, the S-OPF solution, i.e., the base scenario power dispatch, the generator voltage magnitudes, and the decision rule, is deemed to be infeasible for the corresponding scenario.

In power flow simulations, it was found that the S-OPF solutions based on both the decision rules are *feasible* for 99.5% of the scenarios used in the out-of-sample analyses. This is reasonable as the S-OPF problems are not aimed in fully robust solutions. In Fig. 3, the two histograms show

Fig. 3. Histogram of the total operating cost under PW-AD and AD rules considering feasible uncertain scenarios (out of 1,000 scenarios) reported in the out-of-sample analyses. Bin width of the histogram is 10^3 .

the number of feasible scenarios with respect to the total operating cost of power flow solutions for both the redispatch decision rules. The *expected* total operating cost (considering feasible scenarios only) of the power flow solutions for the PW-AD rule and the AD rule are 0.82% ($\$9.110 \times 10^4/h$) and 0.97% ($\$9.412 \times 10^4/h$) lower than the respective total operating costs of the S-OPF solutions. Thus, it can be observed that the proposed PW-AD rule improves the efficiency of operation with sufficient robustness in comparison to the AD rule.

C. Impact of the Number of Uncertainty Segments

In this section, the impact of different selections of uncertainty segments for the performance of the PW-AD rule based S-OPF solutions is examined under the same pool of scenarios Υ . Hence, the previous PW-AD rule based S-OPF solution obtained with 9 uncertainty segments (say Scheme 1) is compared against that obtained with $\mathcal{X} = [0.1, 0.3]^T$ which provides 5 uncertainty segments (say Scheme 2). The total operating cost of the solution obtained with Scheme 2 was $\$9.308 \times 10^4/h$, which is 1.33% suboptimal (higher) than that with Scheme 1. However, it is still 2.07% economical than the AD rule based S-OPF solution. Similar to the solution of Scheme 1, the solution of Scheme 2 also utilizes the same generators for reserve dispatch. Fig. 4 compares the reserve dispatch variation of generators 12, 21, 24 and 33 under the two schemes. The computation time of Scheme 2 was 38.11 s and the S-OPF solution of Scheme 2 was reported to be 99.6% robust in the out-of-sample analysis. Therefore, it can be observed that increasing the number of uncertainty segments in the PW-AD rule will reduce the total operating cost without significantly affecting the computational tractability and the robustness of the S-OPF solution.

VI. CONCLUSIONS

In this paper, the authors proposed a piecewise-affine decision rule for reserve dispatch of generators of the power systems based on stochastic optimal power flow problem. Stochastic scenarios of RES generations and load demands were generated considering the original polyhedral constraints and a scenario reduction method was applied to reduce the

Fig. 4. Reserve dispatch variation of generators 12, 21, 24 and 33 under Scheme 1 and Scheme 2.

computational burden on the S-OPF problem. The results illustrated the performance of the proposed PW-AD rule compared to the traditional affine decision rule for reserve dispatch. Therein, the proposed PW-AD rule efficiently utilizes the generation resources to satisfy the reserve requirements without compensating the robustness or the computational tractability of the S-OPF problem. It was further observed that more number of uncertainty segments will allow more flexibility for online generators to support reserve requirements. In the future, the proposed PW-AD rule can be extended for robust optimization based OPF frameworks and the nonlinear network-constrained unit commitment frameworks.

REFERENCES

- [1] Á. Lorca and X. A. Sun, "The adaptive robust multi-period alternating current optimal power flow problem," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1993–2003, Mar. 2018.
- [2] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski, "Adjustable robust solutions of uncertain linear programs," *Mathematical Programming*, vol. 99, no. 2, pp. 351–376, Mar. 2004.
- [3] A. Venzke and S. Chatzivasileiadis, "Convex relaxations of probabilistic AC optimal power flow for interconnected AC and HVDC grids," *IEEE Transactions on Power Systems*, vol. 34, no. 4, pp. 2706–2718, Jul. 2019.
- [4] P. Jirutitijaroen and C. Singh, "Comparison of simulation methods for power system reliability indexes and their distributions," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 486–493, May 2008.
- [5] J. M. Morales, S. Pineda, A. J. Conejo, and M. Carrion, "Scenario reduction for futures market trading in electricity markets," *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 878–888, May 2009.
- [6] R. A. Jabr, "Adjustable robust OPF with renewable energy sources," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4742–4751, Nov. 2013.
- [7] L. Roald and G. Andersson, "Chance-constrained AC optimal power flow: Reformulations and efficient algorithms," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2906–2918, May 2018.
- [8] A. Castillo, P. Lipka, J.-P. Watson, S. S. Oren, and R. P. O'Neill, "A successive linear programming approach to solving the IV-ACOPF," *IEEE Transactions on Power System*, vol. 31, no. 4, pp. 2752–2763, 2016.
- [9] P. Pinson and G. Kariniotakis, "Conditional prediction intervals of wind power generation," *IEEE Transactions on Power Systems*, vol. 25, no. 4, pp. 1845–1856, Nov. 2010.
- [10] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, Feb. 2011.