Robust control of power converter via uncertainty and disturbance estimator

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Robust Control of Power Converter Via Uncertainty and Disturbance Estimator

Li Ziqi

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Master of Engineering

2021

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

18-01-2021

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Hung Dinh Nguyen

Authorship Attribution Statement

This thesis contains material from one paper accepted at conference in which I am listed as an author.

Chapter 4 is published as Ziqi Li, Xin Zhang, "Enhancing Inverters Output Voltage Quality Via Uncertainty and Disturbance Estimator-Based Controller with a Multi-Band-Stop Filter," 9th IEEE International Conference on Power Electronics, Drives and Energy Systems (PEDES) 2020.

The contributions of the co-authors are as follows:

- Assistant Professor Zhang Xin provided the initial project direction.
- I prepared the manuscript drafts. The manuscript was revised by Assistant Professor Zhang Xin.
- I co-designed the study with Assistant Professor Zhang Xin and performed all the laboratory work at the School of Electrical and Electronic Engineering. I also analyzed the data.
- All experiments were conducted by me.
- I participated and presented in the 9th IEEE International Conference on PEDES 2020, organized at Malaviya National Institute of Technology Jaipur, Rajasthan, India from 16th to 19th December 2020.

18-01-2021

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Summary

The uncertainty and disturbance estimator (UDE)-based robust control method has been extensively studied and employed in many uncertain systems to realize disturbance rejection and address the instability issues. Among those uncertain systems, inverters play an important part in energy conversion applications. Existing works in the literature mainly focus on reducing the total harmonic distortion (THD) of inverter's output voltage under nonlinear loads, which could enhance the quality of the inverter's output voltage. However, such works cannot assure the robustness in the presence of fundamental frequency fluctuations. In this thesis, based on the UDE-based robust control structure, dual-loop control framework is employed to decouple the reference tracking and disturbance rejection of the inverter in the frequency domain. A multi-band-stop filter is designed to minimize the total harmonic distortion (THD) of inverter's output voltage by modifying the magnitude of output impedance only around the harmonic frequencies directly. In addition, increasing the bandwidth of each band-stop filter could guarantee the robust performance to fundamental frequency fluctuations, thus enhancing the quality of inverter's output voltage as well. Besides, the design with multi-time-delayed filter to further reduce output voltage THD was introduced, including the principle and overall control framework. Finally, the performance of the proposed control design is verified with the experimental results. Also, the experiment compared with UDE-based controller equipped with single-time-delayed filter and multi-time-delayed filter has been conducted.

Index Terms—uncertainty and disturbance estimator (UDE), total harmonic distortion (THD), multi-band-stop filter, voltage quality.

Acronyms

UDE	Uncertainty and Disturbance Estimator
THD	Total Harmonic Distortion
SMC	Sliding-mode Control
ADRC	Active Disturbance Rejection Control
EID	Equivalent Input Disturbance Control
DOBC	Disturbance-observer-based Controller
MPC	Model Predictive Control
LUD	Lumped Uncertainty and Disturbance
PWM	Pulse Width Modulation
DSP	Digital Signal Processing
PM	Phase Margin
GM	Gain Margin
DR	Diode Rectifier
HIL	Hardware-In-the-loop

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Chapter 1 Introduction

1.1 Motivation

With the unprecedentedly rapid development of technology worldwide, power systems have become more flexible and innovative than before. Many systems have unknown dynamics, modeling errors, various sorts of disturbances, and noise problems. In order to wrestle structured or unstructured uncertainties and reject external disturbances in these uncertain systems, a variety of robust control techniques have been put forward, for example, adaptive robust control [1]-[3], sliding-mode control (SMC) [4], extended-state-observer-based control [5]-[7], robust nonlinear predictive controller [8], [9], active disturbance rejection control (ADRC) [10], [11], equivalent input disturbance (EID) control [12], disturbance-observer-based controller (DOBC) [13]-[17]. Nevertheless, the above-mentioned control methods all have their constraints and rather complex to be implemented in practice.

Sustainable energy has been applied to various situations such as distributed generation, hybrid electric transportation, smart grids, etc. Among these applications, inverters play a rather important part in energy transformation systems. Many research works have been studied on control problems regarding inverters [18]. Among these control problems, reducing the total harmonic distortion (THD) of inverter's output voltage under nonlinear loads is one of the most common challenges. The existing of harmonic components in output voltage is mainly attributed to the nonlinear loads. When current and voltage of the loads are not linear dependent, the non-sinusoidal current will lead to harmonic components in output voltage.

In order to minimize the output voltage THD of the inverter under nonlinear loads, deadbeat controller [19], hysteresis controller [20], sliding-mode controller [4], Lyapunov function-based control [21], [22] and model predictive control (MPC) [23] have been utilized to minimize the inverter's output voltage THD. Recently, output

impedance-based approaches have become popular since its advantages on load sharing between inverters working in parallel [19], [25]. Another control approach to minimize voltage THD is to directly modify the magnitude of output impedance of the inverter. Therefore, the issue of reducing THD of output voltage can be converted to the problem of minimizing the output impedance magnitude of the inverter, which could enhance the output voltage quality of the inverter [19], [26]. Taking nonlinear loads into consideration, only the values of output impedance at harmonic frequencies are of significance, particularly odd multiples of base frequency in single-phase systems. As for three-phase systems, the harmonic components are mainly $6n \pm 1$ multiples of fundamental frequency. The output impedance values at the rest of bandwidth are irrelevant for output voltage THD reduction. The methods to minimize voltage THD via modifying the output impedance of the inverter can be classified into two main types, the first type possesses single-degree-of-freedom structure, such as multiresonant and repetitive control methods [27]-[30]. The above-mentioned control methods do not decouple the reference tracking and disturbance rejection loops due to the limited control bandwidth. However, the coupled two loops have mutual effects with each other. The second type owns two-degree-of-freedom feature, for example, disturbance-observer-based controller (DOBC) [13]-[17] and uncertainty and disturbance estimator (UDE)-based controller [31]-[35]. They employ two-degree-of-freedom structure to decouple the reference tracking and disturbance rejection of inverters in the frequency domain, which could fulfill asymptotic reference tracking and disturbance rejection in the meanwhile [36]. The DOBC is similar with UDE-based control that the disturbance is extracted from the system dynamics and then estimated by a low-pass filter. The DOBC estimates and cancels the lumped uncertainty and disturbance (LUD) to nominalize the plant, thus making the tracking controller to modify the tracking response of the nominal plant [37]. Nevertheless, the DOBC requires the nominal plant model and needs to take the inverse of the plant model, thus making it complex to be implemented in practice.

The core idea of UDE-based controller is that any engineering signal could be

approximated in the frequency domain via an appropriate filter. Hence, UDE-based robust control method stands out among previous methods due to the following three advantages: (1) UDE-based control can fast estimate the lumped uncertain term, which is a function of control inputs, states and disturbances, and it only requires the frequency information of the disturbances without knowing the magnitude bounds; (2) Since Laplace transform can be utilized in designing some time-varying systems, UDE-based control can be applied to a wider type of systems, such as non-linear systems, time-varying systems and time-delayed systems; (3) It does not need a mathematical model, which can be regarded as a filter problem. In conclusion, UDE-based control is easily to be implemented and reveals good robust performance simultaneously. In addition, UDE-based control approach has been extensively applied to the power systems and many other applications, for instance, the wind turbine [38], robot systems [39], aircraft industry [40], active magnetic bearing [41] and heat conducting systems [42]. UDE-based control has been found rather useful in the above systems and industries.

1.2 Objectives

In order to eliminate the effects to the inverter's output voltage brought by the nonlinear loads and simultaneously guarantee the robustness in the presence of base frequency fluctuations, UDE-based control approach is utilized to directly minimize the magnitude of output impedance at harmonic frequencies. Dual-loop control framework is employed to decouple the reference tracking and disturbance rejection of inverters in the frequency domain, thus directly shaping the magnitude of output impedance [43], [44]. Moreover, dual-loop control structure typically reveals good robust performance [45].

In this thesis, based on the UDE control theory, the target is to enhance the output voltage quality of the single-phase inverter feeding nonlinear loads via reducing the THD of output voltage. The single-phase inverter is an LC-filter based inverter. Hence, inductor current is regarded as the inner loop variable compensated by a proportional controller. As for the outer loop, it adopts an UDE-based compensator to fulfill output impedance reduction and guarantee robust performance to base frequency fluctuations. Due to the

two-degree-of-freedom structure of UDE-based control method, dual-loop control framework is utilized to realize decoupling of tracking and disturbance rejection of the inverter. It is distinct that the UDE-based voltage controller could directly modify the magnitude of output impedance via a suitable filter design without knowing the output current value, which could diminish the THD of output voltage. Instead of commonly used low-pass filter which owns constraints due to the limited control bandwidth, a multi-band-stop filter structure is employed to directly minimize the magnitude of output impedance at harmonic frequencies, thus diminishing the output voltage THD as well [46]. In single-phase systems, harmonic components are mainly odd multiples of base frequency with 50 Hz. In addition, increasing the bandwidth of each band-stop filter could assure the robustness to base frequency fluctuations, thus enhancing the quality of inverter output voltage as well. In the presence of fundamental frequency fluctuations up to ± 1 Hz which are likely to occur, the output waveforms of the single-phase inverter have few changes with nonlinear loads. It means that the existence of fundamental frequency fluctuations up to ± 1 Hz has little influence on the system output waveforms and output voltage THD, thus assuring the robust performance. Therefore, the proposed multi-band-stop filter with UDE-based controller could reduce the output voltage THD and guarantee good robustness against fundamental frequency fluctuations in the meanwhile. Also, comparison experiment of output voltage THD with single-time-delayed filter and multi-time-delayed filter has been conducted.

1.3 Major contribution of the Thesis

Due to the two-degree-of-freedom feature of UDE-based control method, the UDE-filter is employed to directly minimize the magnitude of output impedance around harmonic frequencies, thus reducing voltage THD. Then the quality of inverter's output voltage is enhanced with THD minimization. In addition, since the fundamental frequency fluctuations are likely to occur in practice, increasing the bandwidth of each band-stop filter could assure the robust performance against base frequency fluctuations.

1.4 Organization

This thesis can be divided into seven parts in general. Chapter 1 is the introduction of the thesis, including the background of various control methods in uncertain systems, classification of previous techniques to output voltage THD minimization of the inverter. Also, the objectives and major contribution of the thesis are discussed. Chapter 2 illustrates the literature review of existing works, and their drawbacks are demonstrated respectively. In Chapter 3, the design of UDE-based robust control structure and the advantages are introduced in detail. Chapter 4 presents the proposed control framework, including the inner loop controller, outer voltage controller and the construction of UDE-based voltage controller. Chapter 5 introduces the control principle of UDE-based controller with multi-time-delayed filter. In Chapter 6, experimental verification of the proposed multi-band-stop structure and comparison with single-time-delayed filter and multi-time-delayed filter to inverter's output voltage quality is demonstrated respectively. Conclusion of the thesis and recommendations for future work are summarized in Chapter 7.

Chapter 2 Literature Review

2.1 Problem description

Disturbances and uncertainties extensively exist in all industrial systems and have bad influence on performance and stability of the control systems [47]-[49]. Hence, addressing uncertainties and rejecting external disturbances have become an important objective in control system design. When the uncertainty or disturbance is measurable, the measured information can be feedforwarded to compensate or eliminate the effects of the disturbance before it affects the whole system. Nevertheless, in common situations, the external disturbance cannot be measured directly, or it is rather tough to be measured. Therefore, several other control techniques have been put forward. For example, adaptive control [1]-[3] and sliding-mode control (SMC) [4]. Another type of methods directly extracts the uncertainty and disturbance information from the system dynamics and input value. Then estimating the disturbance from known variables to compensate or attenuate the influence of external disturbances to the system. For example, active disturbance rejection control (ADRC) [10], [11], equivalent input disturbance (EID) control [12] and disturbance-observer-based control (DOBC) [13]-[17]. The main control approaches to wrestle structured or unstructured uncertainties and reject external disturbances in uncertain systems are introduced as follows.

2.2 Existing research works

2.2.1 Adaptive control and sliding-mode control

Adaptive control method is capable of addressing the parametric uncertainties through modifying the control parameters [1]-[3]. Besides, it can solve adaptive neural network control problems for nonparametric uncertainties by identifying it based on the common

function approximation theory, such as unmodeled dynamics in neural network systems [50], [51]. Assuming a nominal model of the plant and selecting a suitable controller, then the closed-loop performance and stability of the system could be guaranteed with external disturbances. As for sliding-mode control (SMC), it has been investigated to ensure the robustness of sliding motion on the sliding surface when model uncertainties and external disturbances exist [4]. The uncertainty is wrestled by the high-gain feedback for the purpose of assuring the accessibility of the sliding mode.

2.2.2 Active disturbance rejection control, equivalent input disturbance control and disturbance-observer-based control

Another class of methods mainly depends on the extraction of the uncertainty and disturbance information from the system dynamics and input signal directly. The active disturbance rejection control (ADRC) treats the total disturbance including both the uncertainty and disturbance as an additional state [10], [11]. It estimates disturbances via an extended state observer [52]. Nevertheless, the boundedness of the derivative of the disturbance is required in the ADRC design. As for UDE-based control, it does not need the assumption on the disturbances and only needs the frequency information of the disturbances for the filter design. Equivalent input disturbance (EID) control approach interprets the uncertainty and disturbance as being applied at the input and then estimates it [12]. However, the EID approach requires that the disturbance be applied to the input channel, whereas UDE-based robust control does not have this constraint.

The disturbance-observer-based control (DOBC) estimates the uncertainty and disturbance by employing the inverse of the nominal plant model and a Q-filter [13]-[17]. The DOBC could be applied to stable, unstable, minimum phase and nonminimum phase systems with different bandwidth constraints. The DOBC is similar with UDE-based control that the disturbance is extracted from the system dynamics and then estimated by a low-pass filter, and both of them own the two-degree-of-freedom structure. Then the tracking and disturbance rejection are decoupled in the frequency

domain. Nevertheless, the DOBC requires the nominal plant model and involves taking the inverse of the plant model in frequency domain, whereas the UDE-based control does not.

A simple linear DOBC system is presented in Fig. 2.1. As seen from the block diagram, $G(s)$ represents the real physical plant, $G_n(s)$ denotes the nominal model to design the controller, $Q(s)$ is a stable filter, c is the feedback controller output, y_r is the reference signal, $\hat{y}$ is the measured output, n is the measurement noise, d is the external disturbance, d_l is the lumped uncertainty and disturbance (LUD), $\hat{d}_l$ is the estimation value of lumped uncertainty and disturbance (LUD). It is apparent that when the disturbance d and uncertainty are absent and the nominal plant $G_n(s)$ is equal to the real physical system, the inner loop with the disturbance estimation and compensation will not work. Hence, the baseline controller $C(s)$ could be designed based on the tracking performance and system stability, whereas the inner loop is designed to reject disturbance. In order to satisfy the two conflicting requirements, the feedback loop and the inner disturbance attenuation loop are designed separately. Moreover, the nominal performance will not be sacrificed to guarantee good robust performance, since the inner loop will not work when the uncertainty is absent, thus making it a classic feedback loop with the controller $C(s)$. When the uncertainty exists, the inner loop will be activated to reject disturbances and suppress uncertainties.

Figure 2.1. Block diagram of a linear DOBC system.

Chapter 3 Design of UDE-based robust control structure

3.1 Control framework

Setting an uncertain linear time-invariant system, described as follows:

$$\frac{dx(t)}{dt} = (A + \Delta A)x(t) + (B + \Delta B)u(t) + d(t) \quad (1)$$

where $x(t)$ is the system state, $u(t)$ is the control input, A denotes the known system matrix, B denotes the control matrix, ΔA denotes the unknown model uncertainties and $d(t)$ denotes the external disturbances of the system. The lumped uncertainty term is given by:

$$u_d(t) = \Delta A \cdot x(t) + \Delta B u(t) + d(t) \quad (2)$$

Combining (1) and (2), lumped uncertainty term u_d can be expressed as:

$$u_d(t) = \frac{dx(t)}{dt} - Ax(t) - Bu(t) \quad (3)$$

In order to eliminate the effects of external disturbances, an UDE is designed to estimate the unknown term u_d , which can be estimated by an UDE as following:

$$\hat{u}_d(t) = \left[\frac{dx(t)}{dt} - Ax(t) - Bu(t) \right] * g(t) \quad (4)$$

where “*” refers to the conventional convolutional operator and $g(t)$ is the impulse response of a frequency-selective filter with unity gain and zero phase shift, which covers the bandwidth of lumped uncertainty term u_d . The target of the controller is to make the system output accurately tracks the reference signal r when the uncertainties and disturbances exist. Then a reference model is introduced, presented as below:

$$\frac{dx_m(t)}{dt} = A_m x_m(t) + B_m u_m(t) \quad (5)$$

with $x_m(t)$ and $u_m(t)$ denoting the reference model state and reference model input

respectively, A_m and B_m denoting the system matrix and input matrix of the reference model respectively. It is apparent that the reference model matrix A_m should be stable to realize reference tracking performance. The state error between the reference system and the nominal plant is expressed as:

$$e(t) = x_m(t) - x(t) \quad (6)$$

The desired dynamics of the state error are presented as follows:

$$\frac{de(t)}{dt} = (A_m + K)e(t) \quad (7)$$

with K denoting the feedback gain. Substituting (1) and (5) into (7), the controller $u(t)$ is deduced as:

$$Bu(t) = (A_m - A)x(t) + B_m u_m(t) - Ke(t) - u_d(t) \quad (8)$$

Then replacing u_d with $\hat{u}_d$ in (8), the UDE-based control law in time domain is described as:

$$u(t) = B^+ [(A_m - A)x(t) + B_m u_m(t) - Ke(t) - \hat{u}_d(t)] \quad (9)$$

where $B^+ = (B^T B)^{-1} B^T$ refers to the pseudo-inverse of control matrix B since matrix B is not a square matrix and cannot take the inverse matrix of it directly. Taking Laplace transform of (9), then substituting (4) into (9):

$$\begin{aligned} U(s) &= B^+ \left\{ (A_m - A)X(s) + B_m U_m(s) - KE(s) - [sX(s) - AX(s) - BU(s)G(s)] \right\} \end{aligned} \quad (10)$$

with $G(s)$ denoting the Laplace transform of $g(t)$. Simplifying (10) and taking the inverse Laplace transform, the UDE-based control principle in time domain described by filter transfer function $G(s)$ is represented as:

$$\begin{aligned} u(t) &= B^+ \left\{ -Ax(t) + L^{-1} \left[\frac{1}{1-G(s)} \right] * [A_m x(t) + B_m u_m(t) - Ke(t)] - L^{-1} \left[\frac{sG(s)}{1-G(s)} \right] * x(t) \right\} \end{aligned} \quad (11)$$

where $L^{-1}[\cdot]$ refers to the inverse Laplace transform operator.

From the general control principle in (11), the UDE-based robust control framework is illustrated in Fig. 3.1. As shown in block diagram, the whole control framework consists of three loops. The first loop is reference model control loop, in which the internal model theory is employed for the controller design. Then the target of controller is to fulfill the asymptotic tracking of the reference signal. As for the second loop, it is the tracking error feedback control loop. The purpose of this feedback loop is to make the practical system state tracking the reference model state by driving the state error to zero. In addition, the third loop is the feedforward control loop for the uncertainties compensation and elimination, in which the UDE is utilized to estimate the uncertainties and disturbances. The design of feedback control loop and feedforward control loop are decoupled in the frequency domain, otherwise the coupled two loops will affect each other. This is the so-called two-degree-of-freedom characteristic, which could fulfill asymptotic reference tracking and external disturbance rejection in the meanwhile [32].

Figure 3.1. Block diagram of UDE-based robust control framework.

3.2 Design of the reference system and UDE-filter

In order to fulfill the two-degree-of-freedom structure, the asymptotic reference tracking and disturbance rejection should be realized simultaneously. The internal model principle is adopted for the design of reference model controller $N_r(s)/M_r(s)$ and the

UDE-filter $G(s)$.

The reference signal r is given before, the external system model is demonstrated as follows:

$$\begin{cases} \dot{\omega}_r(t) = S_r \omega(t) \\ r(t) = P_r \omega(t) \end{cases} \quad (12)$$

where ω_r is the external system state, the reference signal r determines the matrices S_r and P_r . Then the value of $M_r(s)$ should be chosen as:

$$M_r(s) = |sI - S_r| \quad (13)$$

The value of $N_r(s)$ is a polynomial function to make the reference system controller $N_r(s)/M_r(s)$ stable and fulfill good tracking performance as well. As for UDE-filter $G(s)$, the main requirement is that the bandwidth of the filter should cover the spectrum of lumped uncertainty and disturbance u_d . Adopting the internal model principle to design the UDE-filter, which can be described as follows:

$$G(s) = 1 - \frac{M_d(s)}{N_d(s)} \quad (14)$$

where $M_d(s)$ is determined by the external system model of the lumped uncertainty term u_d , which is described as:

$$M_d(s) = lcm(|sI - S_d|, |sI - S_d|) \quad (15)$$

where ‘ lcm ’ refers to the least common multiple operator. $N_d(s)$ is a suitable polynomial function of s , which contributes to the stability of UDE-filter $G(s)$. Besides, the value of $M_d(\infty)/N_d(\infty)$ is finite. Consequently, UDE-filter $G(s)$ is designed as a common low-pass filter, represented as follows:

$$G(s) = 1 - \frac{s}{s+a} = \frac{a}{s+a} \quad (16)$$

with ‘ a ’ expressing the positive constant. The design of filter $G(s)$ should consider the bandwidth conditions of both the reference signal r and lumped uncertainty and

disturbances term u_d .

Therefore, compared with other disturbance-observer-based control (DOBC) which needs the magnitude information of the disturbance, the design of UDE-filter only requires the frequency or the bandwidth information of the disturbances. Besides, the frequency information of a signal is easy to be available in actual situation. The UDE-based controller is model-free and easy to be implemented in practice which formulates the disturbance estimation as an explicit filter issue.

3.3 Stability analysis

Then the stability of the closed-loop system is analyzed. From (7), $A_m + K$ is required to be Hurwitz and the stability condition of the uncertain system proposed in [36] is described as follows:

$$(I - BB^+) [(A_m - A)x(t) + B_m u_m(t) - u_d(t) - Ke(t)] = 0 \quad (17)$$

Nevertheless, the above constraint is rather complicated to judge the stability of the system due to the existence of several variables, such as $x(t)$, $u_m(t)$, $u_d(t)$ and $e(t)$. Hence, it is shown that choosing an appropriate A_m and K to assure the closed-loop stability is rather complicated, particularly in high-order systems with uncertainties and disturbances.

Substituting (9) and (4) into (1), the closed-loop dynamics of the system state is deduced as:

$$\begin{aligned} \frac{dx(t)}{dt} = & [A + BB^+(A_m - A)]x(t) + BB^+[B_m u_m(t) - Ke(t)] \\ & + BB^+ \bar{u}_d(t) + (I - BB^+)u_d(t) \end{aligned} \quad (18)$$

where $\bar{u}_d(t) = u_d(t) - \hat{u}_d(t)$ is the estimation error of lumped uncertainty term.

Combing (5) and (18), the closed-loop dynamics of the tracking error is deduced as follows:

$$\begin{aligned} \frac{de(t)}{dt} = & A_m x_m(t) + B_m u_m(t) - [A + BB^+(A_m - A)]x(t) \\ & - BB^+[B_m u_m(t) - Ke(t) - \hat{u}_d(t)] - u_d(t) \end{aligned} \quad (19)$$

Defining the closed-loop system matrix A_c as:

$$A_c = A_m + K - (I - BB^+)(A_m + K - A) \quad (20)$$

Then the tracking error can be simplified as follows:

$$\frac{de(t)}{dt} = A_c e(t) + (I - BB^+)(A_m + K - A) \quad (21)$$

Substituting (20) into (18), the closed-loop dynamics of system state can be described as:

$$\begin{aligned} \frac{dx(t)}{dt} = & A_c x(t) + BB^+[B_m u_m(t) - Kx_m(t) + \bar{u}_d(t)] \\ & + (I - BB^+)u_d(t) \end{aligned} \quad (22)$$

According to (4), and taking Laplace transform of $\bar{u}_d(t)$:

$$\bar{U}_d(s) = [\Delta A \cdot X(s) + D(s)][1 - G(s)] \quad (23)$$

Then taking Laplace transform of (22) and substituting (23) into it, the closed-loop system dynamics is represented as:

$$\begin{aligned} & \{I - (sI - A_c)^{-1}[I - BB^+G(s)]\Delta A\}X(s) \\ & = (sI - A_c)^{-1}\{BB^+[B_m U_m(s) - KX_m(s)] + [I - BB^+G(s)]D(s)\} \end{aligned} \quad (24)$$

From (24), the equivalent block diagram of the closed-loop system is shown in Fig. 3.2, in which $\hat{U}(s)$ denotes the equivalent input and $\hat{D}(s)$ denotes the equivalent external disturbance. According to the small-gain theory, the stability conditions of the closed-loop system with the UDE-based controller are summarized as following: (1) A_m is Hurwitz; (2) A_c is Hurwitz; (3) The 2-norm of the uncertainty matrix ΔA is smaller than a known positive number γ . UDE-filter $G(s)$ should satisfy the condition below:

$$\|(sI - A_c)^{-1}[I - BB^+G(s)]\|_\infty < \frac{1}{\gamma} \quad (25)$$

Figure 3.2. Equivalent block diagram of the closed-loop system.

3.4 Comparison with other robust controllers

As has been introduced in Chapter 2.2, The disturbance-observer-based control (DOBC) is similar with UDE-based control that the disturbance is extracted from the system dynamics and then estimated by a low-pass filter, and both of them own the two-degree-of-freedom structure. Nevertheless, the DOBC requires the nominal plant model and involves taking the inverse of the plant model in frequency domain, whereas the UDE-based control does not. In Chapter 2.2, Fig. 2.1 presents a simple linear DOBC system and its basic principle was introduced. The active disturbance rejection control (ADRC) treats the total disturbance including both the uncertainty and disturbance as an additional state. It estimates uncertainties and disturbances via an extended state observer. Nevertheless, the boundedness of the derivative of the disturbance is required in the ADRC design. As for UDE-based control, it does not need the assumption on the disturbances and only needs the frequency information of the disturbances for the filter design.

In order to demonstrate the advantage of UDE-based robust controller, the comparative experiment with DOBC and ADRC was conducted. The experimental validation was conducted on an uncertain servo system. UDE-filter was selected as a third-order low-pass filter. The desired reference trajectory is $r(t) = 30 \sin(2\pi t)$. The external disturbance was added at $t = 4s$, and the system uncertainty added at $t = 8s$. The internal-model-

embedded DOBC was considered for comparison. As for the ADRC, it estimates uncertainties and disturbances via an extended state observer, and a third-order extended state observer was employed for comparison. Fig. 3.3 shows the lumped uncertainty estimation and controller output of three controllers respectively. The lumped uncertainty estimation and controller output of UDE-based control are on the left, and the middle figures are the results of DOBC. And the results of ADRC are on the right. As seen from Fig. 3.3 (a), compared with UDE-based controller, the estimation waveforms of DOBC and ADRC are both unstable with oscillations. Also, as shown in Fig. 3.3 (b), the DOBC and ADRC both have serious high-frequency spikes in the control signal compared with UDE-based controller. Therefore, experimental results have demonstrated good overall performance of UDE-based controller, with comparison to the DOBC and ADRC.

Figure 3.3. Comparison of UDE (left) with DOBC (middle) and ADRC (right). (a) lumped uncertainty estimation. (b) controller output.

Chapter 4 Proposed UDE-based controller with multi-band-stop filter

4.1 Proposed system

A representative single-phase inverter which is made up of a dc source v_{dc} , an inverter leg and an LC filter with a nonlinear load, as presented in Fig. 4.1. The main reason why numerous harmonic components exist in output voltage is the presence of nonlinear loads. Since the current and the voltage of the loads are not linear dependent, the output current will become non-sinusoidal, which leads to the appearance of numerous harmonic components in output voltage. Therefore, the problem of harmonic components mainly owes to the nonlinear loads of the inverter.

Seen from Fig. 4.1, the controller u is transferred to a pulse width modulation (PWM) signal to control the inverter leg. When neglecting both the equivalent series resistances of the inductor and the capacitor, the single-phase inverter system could be described by:

$$\begin{cases} u_o(t) = u(t - T_d) v_{dc} \\ L \frac{di_L(t)}{dt} = u_o(t) - v_o(t) \\ C \frac{dv_o(t)}{dt} = i_L(t) - i_o(t) \end{cases} \quad (26)$$

where T_d expresses the overall sampling and switching delay.

Figure 4.1. Representative single-phase inverter with LC-filter.

4.1.1 Pulse width modulation (PWM) sampling

Asymmetric PWM is employed to fulfill the overall delay reduction [53]. Fig. 4.2 demonstrates the switching cycle timing chart, in which the duty ratio is updated twice in every PWM cycle. Seen from the chart, the carrier is a triangular wave. When the cycle starts, this is the first PWM update time which means the on time. When the carrier wave reaches the peak value, the second PWM update acts, thus referring to the off time. The total delay is composed of PWM transportation time delay T_{PWM} and digital signal processing (DSP) computational time T_c . For the proposed structure, the maximum DSP computational time was determined by the experiments, which contributes to the total sampling and switching delay $T_d=45\mu s$.

Figure 4.2. Switching cycle timing chart.

4.1.2 Inner loop control

In the single-phase inverter system, inductor current can be regarded as the inner loop parameter. And a proportional controller compensates the inductor current. From Fig. 4.1, the inductor current can be represented as:

$$\frac{di_L(t)}{dt} = L^{-1} [u(t - T_d) v_{dc} - v_o(t)] \quad (27)$$

The control input signal can be described as:

$$u(t) = \frac{[u'(t) + v_o(t)]}{v_{dc}} \quad (28)$$

Substituting (28) into (27), the inductor current can be deduced as follows:

$$\frac{di_L(t)}{dt} = L^{-1} \left\{ \frac{v_{dc}}{v_{dc}(t-T_d)} [u'(t-T_d) + v_o(t-T_d)] - v_o(t) \right\} \quad (29)$$

Since the bandwidths of v_{dc} and v_o are both much smaller than $1/T_d$, the inductor current can be linearized, which is approximately represented as follows:

$$\frac{di_L(t)}{dt} \approx L^{-1} u'(t-T_d) \quad (30)$$

Since the modified plant is disturbance-free, a proportional controller which is applied to compensate the current loop is described as follows:

$$u'(t) = K_{PI} [i_L^*(t) - i_L(t)] \quad (31)$$

where i_L^* expresses the inductor current reference signal. Current loop gain $L_I(s)$ and current loop closed-loop transfer function $G_I(s)$ in frequency domain are presented as follows respectively:

$$\begin{cases} L_I(s) = \frac{K_{PI} L^{-1}}{s} e^{-T_d s} \\ G_I(s) = \frac{L_I(s)}{1 + L_I(s)} = \frac{K_{PI} L^{-1}}{s e^{T_d s} + K_{PI} L^{-1}} \end{cases} \quad (32)$$

When $K_{PI}=59$, current loop gain possesses phase margin (PM) with 45° and gain margin (GM) with 6 dB with the bandwidth of 2762 Hz, which could guarantee the stability of the inverter. The Bode plot of the current loop gain is shown in Fig. 4.3. As shown in the Bode plot, when the magnitude of current loop gain is zero, the phase margin is obtained as 45° . When the phase of current loop gain is -180° , the gain margin is obtained as 6 dB.

Figure 4.3. Bode plot of current loop gain.

4.1.3 Outer loop control

The current loop closed-loop transfer function $G_I(s)$ can be regarded as an actuator to the outer voltage loop. Rearranging (26), output voltage dynamics can be deduced as follows:

$$\frac{dv_o(t)}{dt} = C^{-1} [i_L(t) - i_o(t)] = (C_n + \Delta C)^{-1} [i_L^*(t) + \Delta i_L(t) - i_o(t)] \quad (33)$$

where C_n and ΔC refer to the nominal and uncertain values of capacitor. i_L^* and Δi_L denote inductor current reference signal and tracking error. i_o^d denotes the gross lumped uncertainty and disturbance output current. Then the output voltage can be written as:

$$\frac{dv_o(t)}{dt} = C_n^{-1} [i_L^*(t) - i_o^d(t)] \quad (34)$$

Combing (33) and (34), the gross lumped uncertainty and disturbance output current is described as:

$$i_o^d(t) = C^{-1} \Delta C i_L^*(t) + C^{-1} C_n [\Delta i_L(t) - i_o(t)] \quad (35)$$

Setting a linear time-invariant stable reference model, with v_o^* expressing the output voltage reference value, v_R denoting the reference state. ω_R expresses the tracking bandwidth parameter and it is a positive value. The reference model is expressed as:

$$\frac{dv_R(t)}{dt} = -\omega_R v_R(t) + \omega_R v_O^*(t) \quad (36)$$

The tracking error is the difference value between the reference model and the actual output voltage value, shown as below:

$$e(t) = v_R(t) - v_O(t) \quad (37)$$

The goal of the voltage controller is to make the value of state error $e(t)$ converge to zero, which is expected to realize the stable dynamics as follows:

$$\frac{de(t)}{dt} = -\omega_R e(t) \quad (38)$$

Combing (33)-(38), the inductor current reference signal can be deduced as:

$$i_L^*(t) = C_n \cdot \omega_R [v_O^*(t) - v_O(t)] + i_O^d(t) \quad (39)$$

Setting voltage proportional parameter $K_{PV} = C_n \cdot \omega_R$. Since the value of lumped uncertainty and disturbance output current i_O^d is unknown, the inductor current reference value cannot be utilized to control the system. Then the UDE-based method is introduced to estimate the unknown parameter. i_O^d can be substituted by the following known parameter. Simplifying (34), i_O^d is described as following:

$$i_O^d(t) = i_L^*(t) - C_n \frac{dv_O(t)}{dt} \quad (40)$$

A UDE-based theory is designed for the estimation of the unknown parameter i_O^d . It is estimated by an UDE with its filter principle, expressed as follows:

$$\hat{i}_O^d(t) = i_O^d(t) * g(t) = \left[i_L^*(t) - C_n \frac{dv_O(t)}{dt} \right] * g(t) \quad (41)$$

where $g(t)$ refers to the impulse response of a frequency-selective filter $G(s)$ and ‘*’ denotes the convolutional operative symbol.

Substituting (41) into (39), $\hat{i}_O^d$ is a replacement of the unknown parameter i_O^d . Then the inductor current reference value which is regarded as the control principle in time domain is represented as follows:

$$i_L^*(t) = i_{L0}^*(t) + \hat{i}_O^d(t) = K_{PV} \left[v_O^*(t) - v_O(t) \right] + \left[i_L^*(t) - C_n \frac{dv_O(t)}{dt} \right]^* g(t) \quad (42)$$

According to the control principle (42) and voltage controller equation (28) and (31), the desired control framework is demonstrated in Fig. 4.4. The block diagram is made up of three parts in general. The inner current loop, the outer voltage loop and the control object which is the single-phase inverter with LC-based filter.

Figure 4.4. Proposed control framework.

Then taking Laplace transform of (42), the general control principle in frequency domain is deduced as:

$$I_L^*(s) = C_n \left[\frac{\omega_R}{1-G(s)} V_o^*(s) - \frac{\omega_R + sG(s)}{1-G(s)} V_o(s) \right] \quad (43)$$

Setting two parameters $F_a(s)$ and $F_b(s)$ to simplify the inductor current reference value in frequency domain, which are shown as follows:

$$\begin{cases} F_a(s) = \frac{\omega_R}{1-G(s)} \\ F_b(s) = \frac{\omega_R + sG(s)}{1-G(s)} \end{cases} \quad (44)$$

Substituting (44) into (43), the control principle in frequency domain can be obtained as:

$$I_L^*(s) = C_n [F_a(s)V_o^*(s) - F_b(s)V_o(s)] \quad (45)$$

Fig. 4.5 shows the complete voltage loop block diagram. Current loop closed-loop transfer function $G_I(s)$ can be regarded as the actuator to the outer voltage loop which is utilized to activate the outer loop. Then the voltage loop gain $L_V(s)$ can be expressed as:

$$L_V(s) = \frac{\omega_R + sG(s)}{s[1 - G(s)]} G_I(s) \quad (46)$$

Figure 4.5. Equivalent voltage loop block diagram.

Substituting (45) into (33), (34) and simplifying it, the voltage loop closed-loop dynamics can be described as:

$$V_o(s) = \frac{F_a(s)G_I(s)}{s + F_b(s)G_I(s)} V_o^*(s) - \frac{C_n^{-1}}{s + F_b(s)G_I(s)} I_o^d(s) \quad (47)$$

with $T_V(s) = \frac{F_a(s)G_I(s)}{s + F_b(s)G_I(s)}$, $Z_o(s) = \frac{C_n^{-1}}{s + F_b(s)G_I(s)}$. Simplifying (47), the closed-

loop dynamics can be expressed as:

$$V_o(s) = T_V(s)V_o^*(s) - Z_o(s)I_o^d(s) \quad (48)$$

Therefore, the single-phase inverter with the UDE-based control theory can be regarded as a circuit diagram composed of a voltage source $T_V(s)V_o^*(s)$ and an output impedance $Z_o(s)$, which is presented in Fig. 4.6. In the equivalent circuit diagram, $V_o(s)$ is regarded as the output voltage and the lumped uncertainty and disturbance output current $I_o^d(s)$ is regarded as the output current. It is distinct that the actual output is close to the reference signal when the value of $T_V(s)$ is close to one, which could satisfy a good tracking

capability. Besides, in view of the external disturbance elimination, the lumped uncertainty and disturbance output current should be close to zero and the value of output impedance $Z_o(s)$ should be close to zero.

Figure 4.6. Equivalent circuit diagram of the single-phase inverter.

4.2 Proposed UDE-based controller with multi-band-stop filter

4.2.1 Total harmonic distortion (THD)

It is well known that the quality of output voltage is mainly judged by the total harmonic distortion (THD) of inverter's output voltage. In order to enhance the inverter's output voltage quality, output voltage THD should be cut down. Firstly, the total uncertainty and disturbance current and output voltage reference signal can be given by:

$$\begin{cases} i_o^d(t) = \sum_{n=1}^{\infty} I_n \sin(n\omega_0 t + \phi_n) \\ v_o^*(t) = V_M^* \sin \omega_0 t \end{cases} \quad (49)$$

The output voltage is composed of the fundamental component v_{O1} and harmonic components v_{OH} . Substituting (49) into (48), output voltage can be deduced as follows:

$$\begin{aligned} v_o(t) &= V_M^* \sin \omega_0 t - \sum_{n=1}^{\infty} I_n |Z_o(n\omega_0)| \times \sin[n\omega_0 t + \phi_n + \arg Z_o(n\omega_0)] \\ &= v_{O1}(t) + v_{OH}(t) \end{aligned} \quad (50)$$

When $n=1$, the fundamental component of output voltage v_{O1} is described as:

$$\begin{aligned} v_{O1}(t) &= V_M^* \sin \omega_0 t - I_1 |Z_o(\omega_0)| \times \sin[n\omega_0 t + \phi_1 + \arg Z_o(\omega_0)] \\ &= V_1 \sin(\omega_0 t + \theta) \end{aligned} \quad (51)$$

The difference value of (50) and (51) is the harmonic components of the output voltage v_{OH} , expressed as:

$$v_{OH}(t) = -\sum_{n=2}^{\infty} I_n |Z_o(jn\omega_0)| \times \sin[n\omega_0 t + \phi_n + \arg Z_o(jn\omega_0)] \quad (52)$$

The output voltage's fundamental component and harmonic component are orthogonal and decoupled. The quality of the voltage is mainly judged by the total harmonic distortion (THD). The definition of THD is the square root of the ratio of total harmonic components' square and fundamental component's square.

$$THD_v = \frac{\sqrt{\sum_{n=2}^{\infty} [I_n |Z_o(jn\omega_0)|]^2}}{V_1} \quad (53)$$

From (53), the value of THD is mainly determined by the magnitude of output impedance at harmonic frequencies. As for single-phase systems, the most harmonic components come from odd multiples of base frequency 50 Hz. However, in three-phase systems, the harmonic components mainly consist of $6n \pm 1$ multiples of base frequency. Therefore, minimizing voltage THD is equivalent to the reduction of magnitude of output impedance at odd multiples of fundamental frequency. The minimization of voltage THD means that the output voltage's quality of the single-phase inverter is enhanced. However, as shown in (51), when voltage THD is minimized, output voltage is still not equal to the voltage reference signal since there is a voltage drop on the output impedance at fundamental frequency. Hence, the target is to cut down the magnitude of output impedance with the order of the highest load harmonics, thus diminishing the voltage THD. It is distinct that only the magnitudes of output impedance at harmonic frequencies need to be reduced, while the magnitudes at the other range of frequencies are not important since they possess the least harmonic components and have little effect to output voltage THD reduction.

4.2.2 Reference tracking

When current loop closed-loop transfer function $G(s)=1$, rearranging (47), closed-loop

voltage dynamics can be deduced as:

$$V_o(s) = \frac{\omega_R}{s + \omega_R} V_o^*(s) - \frac{C_n^{-1}}{s + \omega_R} [1 - G(s)] I_o^d(s) \quad (54)$$

with $T_v(s) = \frac{\omega_R}{s + \omega_R}$, $Z_o(s) = \frac{C_n^{-1}}{s + \omega_R} [1 - G(s)]$.

It is apparent that the output impedance consists of two frequency-selective filters cascaded in series, which can be expressed as follows:

$$\begin{cases} Z_{o1}(s) = \frac{C_n^{-1}}{s + \omega_R} \\ Z_{o2}(s) = 1 - G(s) \end{cases} \quad (55)$$

The magnitude of two parts of output impedance should be diminished respectively. As for $Z_{o1}(s)$, when $1/C_n > \omega_R$, the magnitude of $Z_{o1}(s)$ is larger than 0 dB in the range of frequencies which are smaller than $\omega = \sqrt{C_n^{-2} - \omega_R^2}$. As seen from (55), increasing tracking bandwidth ω_R could minimize the magnitude of $Z_{o1}(s)$. As for the second filter $Z_{o2}(s)$, the magnitude at harmonic frequencies should be minimize to zero to fulfill THD reduction. Nonetheless, in view of the stability margins, the control bandwidth is limited. Choosing a suitable UDE-filter $G(s)$ could satisfy the output impedance reduction and guarantee the stability margins as well. As has been mentioned in current loop control, the minimum stability margin is set to be PM of 45° and GM of 6 dB, respectively. The minimum value of tracking bandwidth ω_R is set to be $10 \cdot \omega_0$, which could fulfill good reference tracking and assure the stability margins requirement simultaneously.

4.2.3 Common UDE-filter

The most common utilized UDE-based controller filter is the low-pass Butterworth filter. Nonetheless, to realize external disturbance rejection, increasing filter order and increasing cut off frequency are employed in [32]. There are several contradictions when taking the limited control bandwidth into consideration. Table 1 summarizes the transfer function and cut off frequency parameters of various low-pass Butterworth filters with

different orders. ω_R denotes the tracking bandwidth and ω_F refers to the cut off frequency of the filters.

Table 1. Parameters of different low-pass Butterworth filter

order	$\omega_R/2\pi$	$\omega_F/2\pi$	$G(s)$
0	1196	0	0
1	500	664	$\frac{\omega_F}{s + \omega_F}$
20	500	530	$\frac{\omega_F^2}{s^2 + 1.41\omega_F \cdot s + \omega_F^2}$
21	500	393	$\frac{1.41\omega_F \cdot s + \omega_F^2}{s^2 + 1.41\omega_F \cdot s + \omega_F^2}$
32	500	279	$\frac{2\omega_F \cdot s^2 + 2\omega_F^2 \cdot s + \omega_F^3}{s^3 + 2\omega_F \cdot s^2 + 2\omega_F^2 \cdot s + \omega_F^3}$

The filter cut off frequency is set according to the minimum stability margin which is set to be PM of 45° and GM of 6 dB, respectively. From Table 1, the cut off frequency of $G_{20}(s)$ which is 530 Hz is larger than the cut off frequency $G_{21}(s)$ which is 393 Hz. Nevertheless, for disturbance rejection ability, filter $G_{21}(s)$ is better than the filter $G_{20}(s)$. Moreover, when the order of the filter becomes larger, the cut off frequency becomes smaller. Increasing the order of low-pass Butterworth filter could diminish its cut off frequency. Since the magnitude of output impedance at fundamental frequency with larger order is smaller than that with smaller order, the filters with larger order possess better disturbance rejection capability. Increasing the filters order could assure better disturbance rejection ability in the range of low frequencies, especially at fundamental frequency. As the range of higher frequencies, for instance, from 100 Hz to 2000 Hz, the magnitude of output impedance is larger than zero, which leads to the bad disturbance rejection ability in the range of higher frequencies. Hence, increasing the order of low-pass Butterworth filter could decrease the cut off frequency, while could

not guarantee the magnitude of output impedance reduction in the range of medium frequencies. The voltage THD deduction could not be realized as well.

Therefore, the common utilized low-pass Butterworth filter owns the disadvantage of limited control bandwidth, thus making it hard to minimize the magnitude of output impedance at the range of higher frequencies. Then the output voltage THD could not be cut down. Compared with the low-pass filter with limited control bandwidth problems, the proposed multi-band-stop filter could directly diminish the output impedance around all the harmonic frequencies, particularly odd multiples of base frequency with 50 Hz in single-phase inverter.

4.2.4 Design of UDE-based controller with multi-band-stop filter

As has been mentioned, output impedance consists of two frequency-selective filters cascaded in series. $Z_{O2}(s)=1-G(s)$ mainly modify the output impedance. Then selecting an appropriate output impedance $Z_{O2}(s)$ to design the UDE-filter $G(s)$, the output impedance could be directly reduced only at the harmonic frequencies. In the single-phase inverter, the harmonic components are mainly odd multiples of fundamental frequency. Hence, a connection of several cascaded band-stop filters is applied to constitute $Z_{O2}(s)$, and the n th harmonic filter stop band is from $n \cdot \omega_0 \cdot l$ to $n \cdot \omega_0 / l$. The value of l is smaller than one. Output impedance $Z_{O2}(s)$ is expressed as follows:

$$Z_{O2}(s) = \prod_{n=1}^N H_n(s) \quad (56)$$

Fig. 4.7 demonstrates the magnitude of the ideal output impedance $Z_{O2}(s)$. In order to assure the robustness to fundamental frequency fluctuations, the stop band bandwidth of each band-stop filter should be increased as much as possible. It means that the value of l should be minimized. However, due to the minimum stability margin which is set to be PM of 45° and GM of 6 dB respectively, the bandwidth of each filter cannot be increased unlimitedly. Therefore, maximum stop band bandwidth is divided by n band-stop filters cascaded with each other.

Figure 4.7. Magnitude of output impedance $Z_{O2}(s)$.

Then elliptic band-stop filters have been chosen to make up output impedance $Z_{O2}(s)$ since they could acquire a transition width with the smallest order [54]. Hence, $Z_{O2}(s)$ is designed to be six cascaded second-order filters ($n=1,3,5,7,9,11$), with pass-band and stop-band ripples of 0.35 and 60dB respectively. In view of the minimum stability margin with PM of 45° and GM of 6 dB respectively, the smallest value of l is 0.9. Fig. 4.8 and Fig. 4.9 show the Bode plot of $Z_{O1}(s)$ and $Z_{O2}(s)$ respectively. As seen from the Bode plot, the magnitude of $Z_{O1}(s)$ is always 20 dB approximately at the harmonic frequencies. Since the base frequency is 50 Hz, the harmonic components are mainly odd multiples of base frequency 50 Hz which are 50, 150, 250, 350, 450, 550 Hz respectively. At these harmonic frequencies, the output impedance is always resistive with zero phase shift.

Figure 4.8. Bode plot of $Z_{O1}(s)$.

Figure 4.9. Bode plot of $Z_{O2}(s)$.

The UDE-filter $G(s)=1- Z_{O2}(s)$ could be deduced by the output impedance $Z_{O2}(s)$, and the Bode plot of UDE-filter $G(s)$ is shown in Fig. 4.10. The magnitude of UDE-filter owns unity gain and zero phase shift at the first six odd multiples of base frequency with 50 Hz. Fig. 4.11 presents the voltage loop gain $L_V(s)$. As shown in the Bode plot, the gain margin of 6 dB is the stability margin constraints and not the phase margin of 45° .

Figure 4.10. Bode plot of designed UDE-filter $G(s)$.

Figure 4.11. Bode plot of voltage loop gain.

Chapter 5 UDE-based controller with multi-time-delayed filter

5.1 Design of multi-time-delayed filter

Setting a stable uncertain linear single-input single-output system with disturbances:

$$y(s) = P(s)u(s) = [P_n(s) + \Delta P(s)][u_n(s) + f(s)] \quad (57)$$

where $y(s)$ denotes the system output, $P_n(s)$ and $\Delta P(s)$ are nominal and uncertain components of plant P respectively, $u(s)$ denotes the system input, $u_n(s)$ is the control input and $f(s)$ represents the external disturbance. Introducing the lumped uncertainty and disturbance term $u_d(s)$, (57) could be simplified as:

$$y(s) = P(s)u(s) = P_n(s)[u_n(s) + u_d(s)] \quad (58)$$

Since $u_d(s)$ owns odd symmetry, it only contains odd harmonics which could be described by:

$$u_d(t) = \sum_{n=1, \text{odd}}^{\infty} D_n \sin(n\omega_0 t + \theta_n) \quad (59)$$

with D_n denoting the amplitude and θ_n denoting the phase of the n th disturbance harmonic. Taking the two-degree-of-freedom feature into consideration, reference tracking and disturbance rejection should be satisfied in the meanwhile. Then the control input $u_n(s)$ is set to be a split control parameter, shown as follows:

$$u_n(t) = u_{nt}(t) - u_{nd}(t) \quad (60)$$

with $u_{nt}(t)$ representing tracking controller's output value, $u_{nd}(t)$ denoting uncertainty and disturbance estimator (UDE) estimation value, respectively. Selecting a suitable UDE estimation value, the value of $u_{nd}(t)$ is equal to the value of lumped uncertainty and disturbance term $u_d(t)$. Then (58) could be rewritten as below:

$$y(s) = P_n(s)u_{nt}(s) \quad (61)$$

The tracking controller and UDE-based controller should be decoupled with limited control bandwidth constraints and minimum stability margins [55], [56]. Then rearranging (58), the lumped uncertainty and disturbance term $u_d(s)$ could be given by:

$$u_d(s) = P_n^{-1}(s)y(s) - u_{nt}(s) \quad (62)$$

The lumped uncertainty term is approximated by an UDE via a linear filter $G_f(s)$. The filter possesses unity gain and zero phase shift at odd multiples of base frequency. $u_d(s)$ is estimated by UDE-based control method, shown as following:

$$u_{nd}(s) = u_d(s)G_f(s) = \{P_n^{-1}(s)y(s) - [u_{nt}(s) - u_{nd}(s)]\}G_f(s) \quad (63)$$

Then simplifying (63), the estimation value $u_{nd}(s)$ could be expressed as:

$$u_{nd}(s) = \frac{G_f(s)}{1 - G_f(s)} [P_n^{-1}(s)y(s) - u_{nt}(s)] \quad (64)$$

Then substituting (63) into (58), system output $y(s)$ could be represented as:

$$y(s) = P_n(s) \{u_{nt}(s) + u_d(s)[1 - G_f(s)]\} \quad (65)$$

It is apparent that when linear filter $G_f(s)$ owns unity gain and zero phase shift at odd multiples of base frequency, the value of $1 - G_f(s)$ is equal to zero and the lumped uncertainty and disturbance term $u_d(s)$ can be entirely eliminated. Since $u_d(t)$ in (59) only contains odd harmonics, it could be described as:

$$u_d(t) = -u_d \left(t - \frac{T_0}{2} \right) \quad (66)$$

where $T_0 = 2\pi/\omega_0$, taking Laplace of (66), lumped uncertainty term $u_d(s)$ in the frequency domain could be given by:

$$u_d(s) = -u_d(s)e^{-\frac{T_0}{2}s} \quad (67)$$

Since $u_d(t)$ in (66) possesses infinite bandwidth, $u_d(t)$ cannot be applied to be estimated directly. Hence, a low-pass filter $Q(s)$ is introduced to limit the bandwidth of $u_d(s)$ in (67). Then UDE estimation value $u_{nd}(s)$ is described as:

$$u_{nd}(s) = u_d(s) \left[-Q(s) e^{-\left(\frac{T_0}{2} - \Delta T\right)s} \right] \quad (68)$$

where ΔT denotes the delay of low-pass filter $Q(s)$ at base frequency ω_0 . $G_{f1}(s)$ is regarded as a single-time-delayed filter, which could be given by:

$$G_{f1}(s) = -Q(s) e^{-\left(\frac{T_0}{2} - \Delta T\right)s} \quad (69)$$

In the range of pass band of the low-pass filter $Q(s)$, $H_{f1}(s)$ is equal to $1 - G_{f1}(s)$, which could be deduced as:

$$H_{f1}(s) = 1 - G_{f1}(s) = 1 + e^{-\frac{T_0}{2}s} \quad (70)$$

The magnitude of $H_{f1}(s)$ could be given by:

$$\left| H_{f1}(j\omega) \right| = \left[2 + 2 \cos \left(\frac{T_0}{2} \omega \right) \right]^{\frac{1}{2}} \quad (71)$$

Consequently, the magnitude of $H_{f1}(s)$ is always zero when n is an odd number, and is always two when n is an even number, which could be described as follows:

$$\left| H_{f1}(jn\omega_0) \right| = \begin{cases} 0, & \text{odd } n \\ 2, & \text{even } n \end{cases} \quad (72)$$

The Bode plot of magnitude of $H_{f1}(s)$ is shown in Figure 5.1. The normalized frequency is ω/ω_0 . It is evident that the magnitude of $H_{f1}(s)$ is close to zero at odd multiples of base frequency, and at even multiples of base frequency, the magnitude is close to two.

Figure 5.1. Bode plot of $|H_f(s)|$.

For the purpose of enhancing the robust performance against fundamental frequency fluctuations [57], [58], (66) could be derived to a common form as below:

$$u_d(t) = (-1)^m u_d(t - mT_0/2) \quad (73)$$

with $m=1,2,3\dots$. Then taking Laplace transform of (73), the lumped uncertainty term in the frequency domain could be deduced as follows:

$$u_d(s) = (-1)^m u_d(s) e^{-m\frac{T_0}{2}s} \quad (74)$$

Introducing a parameter k_m , shown as below:

$$\sum_{m=1}^M k_m = 1 \quad (75)$$

Then (74) can be described as:

$$u_d(s) = \sum_{m=1}^M k_m (-1)^m u_d(s) e^{-m\frac{T_0}{2}s} \quad (76)$$

Similarly, since $u_d(s)$ in (76) owns infinite bandwidth, it cannot be employed to UDE-based controller directly. Introducing the low-pass filter $Q(s)$, the lumped uncertainty term is estimated by an UDE:

$$u_{nd}(s) = u_d(s) Q(s) \sum_{m=1}^M k_m (-1)^m e^{-\left(m\frac{T_0}{2} - \Delta T\right)s} \quad (77)$$

Similarly, $G_{fM}(s)$ could be regarded as a multi-time-delayed filter, which is given by:

$$G_{fM}(s) = Q(s) \sum_{m=1}^M k_m (-1)^m e^{-\left(m\frac{T_0}{2} - \Delta T\right)s} \quad (78)$$

According to [59], the parameter k_m is chosen to decrease the influence on multi-time-delayed filter $G_{fM}(s)$ in the presence of fundamental frequency fluctuations, thus enhancing the robust performance around odd multiples of base frequency. Therefore, the derivative of $G_{fM}(s)$ from one to (M-1) order at odd multiples of fundamental frequency should be set to zero, given by the following (M-1) equations:

$$\left. \frac{d^l G_{fM}(s)}{ds^l} \right|_{s=jn\omega_{0,n} \text{ odd}} = 0, l = 1, 2, \dots, M-1 \quad (79)$$

Substituting (79) into (78), the condition of parameter k_m could be derived as:

$$\sum_{m=1}^M m^l k_m = 0, l = 1, 2, \dots, M-1 \quad (80)$$

Combing (75) and (80), the parameter values of k_m from M=1 to M= 5 are listed in Table 2. When considering low-pass filter $Q(s)$ as an ideal condition, which means it is equal to one, within the pass band of $Q(s)$, $H_{fM}(s)$ is equal to $1 - G_{fM}(s)$, which could be deduced as:

$$H_{fM}(s) = 1 - G_{fM}(s) = 1 - \sum_{m=1}^M k_m (-1)^m e^{-m\frac{T_0}{2}s} \quad (81)$$

Table 2. Parameter values of k_m

M	k_1	k_2	k_3	k_4	k_5
1	1	—	—	—	—
2	2	-1	—	—	—
3	3	-3	1	—	—
4	4	-6	4	-1	—
5	5	-10	10	-5	1

The magnitude of $H_{fM}(s)$ can be deduced as below:

$$H_{fM}(j\omega) = \left[2 + 2 \cos\left(\frac{T_0}{2} \omega\right) \right]^{\frac{1}{2}M} \quad (82)$$

It is obvious that the magnitude of $H_{fM}(s)$ is always zero when n is an odd number and $H_{fM}(s)$ is 2^M when n is an even number, which can be expressed as:

$$|H_{fM}(jn\omega_0)| = \begin{cases} 0, & \text{odd } n \\ 2^M, & \text{even } n \end{cases} \quad (83)$$

The Bode plot of the magnitude of $H_{fM}(s)$ when order $M=1,2,3,4$ is presented in Figure 5.2, and the normalized frequency is ω/ω_0 . As seen from the Bode plot, increasing the delay order M would increase the magnitude of $H_{fM}(s)$.

Figure 5.2. Bode plot of $|H_{fM}(s)|$ with $M=1,2,3,4$.

In addition, in order to verify the fact that increasing delay order could enhance the robust performance against base frequency fluctuations. The ratio of magnitude of $H_{fM}(s)$ and $H_{fM+1}(s)$ can be given by:

$$|R_{H1}(j\omega)| = \frac{|H_{fM}(j\omega)|}{|H_{fM+1}(j\omega)|} = |H_{f1}(j\omega)|^{-1} = \left[2 + 2 \cos\left(\frac{T_0}{2} \omega\right) \right]^{-\frac{1}{2}} \quad (84)$$

When the normalized frequency ω/ω_0 is close to $n\pm 1/3$, the magnitude of ratio is close to one which means that $H_{fM}(s)$ is equal to $H_{fM+1}(s)$. At these frequencies, the robust performance would not be enhanced. At frequencies around n multiples of fundamental

frequency, the robustness could be enhanced against small frequency fluctuations, as desired.

5.2 Overall control framework

The system output signal $y(t)$ should track the reference value $y^*(t)$, which is described as below:

$$y^*(t) = R \sin \omega_0 t \quad (85)$$

In order to make the tracking error converge to zero, a proportional tracking controller $C_t(s)$ is employed to fulfill good tracking performance. The output of tracking controller could be represented as:

$$u_{nt}(s) = C_t(s) [y^*(s) - y(s)] \quad (86)$$

From (60), combining (86) and (77), the control input $u_n(t)$ is the difference value between $u_{nt}(t)$ and $u_{nd}(t)$, which could be deduced as:

$$u_n(s) = \frac{C_t(s)}{1 - G_{fM}(s)} y^*(s) - \frac{C_t(s) + G_{fM}(s) P_n^{-1}(s)}{1 - G_{fM}(s)} y(s) \quad (87)$$

Then introducing an actuator $T_a(s)$ with limited bandwidth, the system input signal $u(s)$ and output value $y(s)$ could be described as:

$$\begin{cases} u(s) = u_n(s) T_a(s) + u_d(s) \\ y(s) = \frac{P_n(s) C_t(s)}{1 + P_n(s) C_t(s)} y^*(s) + \frac{[1 - G_{fM}(s)] P_n(s)}{[1 + P_n(s) C_t(s)] T_a(s)} u_d(s) \end{cases} \quad (88)$$

From (88), the overall control framework is shown in Figure 5.3. A compromise between reference tracking and disturbance rejection would exist since the limited control bandwidth.

Figure 5.3. Overall control framework.

5.3 Application to single-phase inverter

In Chapter 4, a single-phase inverter with LC filter has been introduced. Similarly, the inductor current is treated as the inner loop parameter compensated by a proportional controller. As for the outer voltage loop, UDE-based controller is employed to compensate the effects. The current loop closed-loop transfer function $G_I(s)$ is equivalent to T_a in (88). $G_I(s)$ can be regarded as the actuator to the outer voltage loop, as noted in Chapter 4. Then the output voltage dynamics can replace (58) and (88), given by:

$$v_o(s) = P_n(s) [u_n(s)T_a(s) + u_d(s)] = \frac{1}{C_n s} [i_L^*(s)G_I(s) + i_o^d(s)] \quad (89)$$

It is apparent that $y(s)$ is equivalent to $v_o(s)$, $P_n(s)$ is equivalent to $1/C_n s$, $u_n(s)$ is equivalent to $i_L^*(s)$ and $u_d(s)$ is equivalent to $i_o^d(s)$. From (48) in Chapter 4, the output voltage of the inverter can be deduced as:

$$v_o(s) = T_v(s)v_o^*(s) + Z_o(s)i_o^d(s) \quad (90)$$

The multi-time-delayed filter $G_{fM}(s)$ is designed according to (81), which could make the output impedance to zero. Then the output voltage THD can be reduced accordingly. The inductor current reference value $i_L^*(s)$ can be given by:

$$i_L^*(s) = i_{L0}^*(s) - \hat{i}_o^d(s) = u_{nt}(s) - u_{nd}(s) \quad (91)$$

where $i_{Lo}^*(s)$ is the output of the tracking controller which is equivalent to $u_{nt}(s)$, given by:

$$i_{Lo}^*(s) = K_{PV} [v_o^*(s) - v_o(s)] \quad (92)$$

It is obvious that (92) is equivalent to (86), and tracking controller C_t is equivalent to K_{PV} . According to [60], the low-pass filter $Q(s)$ is chosen to be a third-order low-pass Butterworth filter to construct the multi-time-delayed filter $G_{fM}(s)$ in (78).

Chapter 6 Experimental Results

In order to verify the feasibility and reliability of the proposed control method, the model of the single-phase half-bridge inverter with the proposed multi-band-stop filter was built. The parameter values of the single-phase inverter system are summarized in Table 3. Firstly, the load is set to be a linear load with $33\ \Omega$ which could be regarded as a baseline compared with nonlinear load. Then the linear load is substituted by a diode rectifier (DR) with heavy RC load which consists of a capacitor and a resistor. Fig. 6.1 presents the inverter system with nonlinear load. The parameter values of the nonlinear load are listed in Table 4.

OPAL-RT platform is utilized to execute the whole model in an analog control system, and the oscilloscope was employed to capture the waveforms of output voltage and load current of the single-phase inverter. The Hardware-In-the-loop (HIL) simulation results could justify the performance of the proposed control structure, which could realize the THD reduction of inverter's output voltage and guarantee the robustness capability in the presence of base frequency fluctuations simultaneously. Therefore, the quality of inverter's output voltage could be enhanced. Fig. 6.2 shows the whole HIL experimental simulation.

Table 3. Inverter system parameter values

Parameter	Value	Units
Switching frequency, T_s^{-1}	15	kHz
Filter inductance, L	3.4	<i>mH</i>
Filter capacitance, C	30	μF
Base frequency, ω_0	100π	rad/s
DC voltage, v_{dc}	400	V

Figure 6.1. Single-phase inverter with nonlinear load.

Table 4. Nonlinear load parameter values

Parameter	Value	Units
Load resistance, R_L	50	Ω
Load capacitance, C_L	570	μF

Figure 6.2. Hardware-In-the-loop (HIL) simulation.

6.1 Operation under fundamental frequency

In the first experiment, the single-phase inverter is under nominal fundamental frequency of 50 Hz. The magnitude of output voltage reference signal V_o^* was set to be 155V. Single-phase inverter with proposed multi-band-stop filter with linear loads and nonlinear loads under steady-state operation and transients were experimented respectively. In the case of linear loads, Fig. 6.3(a) presents the waveforms of voltage reference signal, output voltage and load current under steady-state operation with linear loads. Fig. 6.3(b) and (c) demonstrate the full-load to no-load and no-load to full-load transitions with linear loads, respectively. As for nonlinear loads situation, Fig. 6.4(a), (b), (c) demonstrate the output waveforms under steady-state, full-load to no-load and

no-load to full-load transitions respectively. As seen from the output waveforms under both steady-state and transients condition, output voltage waveforms well track the reference signal in both steady state and transients. The system owns good performance with both linear and nonlinear loads, especially good steady state performance.

(a)

(b)

(c)

Figure 6.3. Output voltage and current with linear loads, base frequency. (a) steady-state operation. (b) full-load to no-load transition. (c) no-load to full-load transition.

(a)

(b)

(c)

Figure 6.4. Output voltage and current with nonlinear loads, base frequency. (a) steady-state operation. (b) full-load to no-load transition. (c) no-load to full-load transition.

Fig. 6.5 and Fig. 6.6 present the THDs of output voltage with linear and nonlinear loads respectively. In the case of linear loads, THD=1.02% could be regarded as a reference value compared with THD with nonlinear loads. Seen from the spectra of output voltage under nonlinear loads, it is shown that the voltage harmonics of first six odd multiples up to 11th are almost inexistent, as expected. Corresponding THD of output voltage was attained as 2.14%, compared with THD=8.7% with the employment of first order low-pass Butterworth filter and THD=15.1% with the employment of third order low-pass Butterworth filter as the UDE-filter under similar operating parameters. The value of

THD with proposed multi-band-stop filter is much smaller than those of classical low-pass filters. The experimental results validate the feasibility of the proposed control structure, which could minimize the THD of single-phase inverter's output voltage. Therefore, the inverter's output voltage quality is enhanced with the THD reduction.

Figure 6.5. Output voltage spectra with linear loads.

Figure 6.6. Output voltage spectra with nonlinear loads.

6.2 Operation under fundamental frequency fluctuations

In the second experiment, since the base frequency fluctuations around ± 1 Hz are likely to occur in practical application, the single-phase inverter system under steady-state operation with nonlinear loads under ± 1 Hz fundamental frequency fluctuations was conducted to justify the robust performance of the proposed multi-band-stop filter structure. Fig. 6.7 demonstrates the output voltage and load current waveforms under the frequency of 49, 49.5, 50.5, 51Hz, respectively. As seen from the waveforms, the output voltage and load current have few changes under four different frequencies around

base frequency. The experimental results of output voltage THD under fundamental frequency fluctuations up to ± 1 Hz are presented in Fig. 6.8. It is obvious that the value of voltage THD is still low in the presence of base frequency fluctuations up to ± 1 Hz. Even at the frequency of 49 Hz and 51 Hz, the maximum THD is still not very large, approximately 2.8%. Consequently, the proposed UDE-controller design with a multi-band-stop filter could assure the robustness against fundamental frequency fluctuations, thus enhancing the output voltage quality of the single-phase inverter as well.

(a)

(b)

(c)

(d)

Figure 6.7. Steady-state operation under ± 1 Hz fundamental frequency fluctuations. (a) 49 Hz. (b) 49.5 Hz. (c) 50.5 Hz. (d) 51Hz.

Figure 6.8. Output voltage THD under fundamental frequency fluctuations.

6.3 Comparison with single-time-delayed filter and multi-time-delayed filter

UDE-based controller with single-time-delayed filter and multi-time-delayed filter were conducted in the same inverter system to further reduce the output voltage THD under same operating parameters. Including the inverter systems parameters and nonlinear load parameters. The delay order M is set to be three. Figure 6.9 presents the output voltage normalized spectra with multi-time-delayed filter and multi-band-stop filter, respectively. As seen from the output voltage spectra, the harmonic components with multi-time-delayed filter is smaller than those with multi-band-stop filter at most of the harmonic frequencies. The output voltage THD is attained as 1.82% with time-delayed-filter compared with 2.14% with multi-band-stop filter. Therefore, UDE-based controller with multi-time-delayed filter could further reduce the output voltage THD,

thus better enhancing the inverter's output voltage quality.

Figure 6.9. Comparison of Output voltage spectra.

Comparison experiment has been conducted in the presence of fundamental frequency fluctuations up to ± 1 Hz to verify the robust ability with three different filters. Figure 6.10 demonstrates the output voltage THD under base frequency fluctuations up to ± 1 Hz with three types of filters, respectively. As seen from the figure, output voltage THD with multi-time-delayed filter is smaller than that equipped with multi-band-stop filter and is much smaller than that with single-time-delayed filter. Besides, THD can remain low in a rather large range of frequencies from 49.5 Hz to 50.5 Hz with multi-time-delayed filter, compared with multi-band-stop filter. The output voltage THD can remain low only in a small range of frequencies around the base frequency 50 Hz. Therefore, multi-time-delayed filter owns better robust performance compared with the UDE-controller with multi-band-stop filter.

Figure 6.10. Comparison of output voltage THD under fundamental frequency fluctuations.

Chapter 7 Conclusion and Recommendations

7.1 Conclusion

Due to the two-degree-of-freedom structure, UDE-based robust control method is employed to many uncertain systems to wrestle structured or unstructured uncertainties and reject external disturbances. Compared with other robust control methods such as active disturbance rejection control (ADRC), equivalent-input-disturbance (EID) control approach and disturbance-observer-based control (DOBC), UDE-based control method stands out due to its explicit principle and good robust performance. It only requires the bandwidth value of the disturbances rather than the magnitude information, and it could be regarded as a filter problem which does not need an accurate mathematical model.

Sustainable energy has been applied to various situations such as distributed generation, hybrid electric transportation, smart grids, etc. Among these applications, inverters play a rather important part in energy transformation systems. Many research works have been studied on control problems regarding inverters. Among the control problems, reducing the total harmonic distortion (THD) of inverter's output voltage under nonlinear loads is one of the most common troubles. The issue of reducing THD of output voltage can be converted to the problem of minimizing the output impedance magnitude of the inverters.

In this thesis, the object is to fulfill output voltage quality enhancement of the single-phase half-bridge inverter with an LC filter. Inductor current treats as the inner loop variable compensated by a proportional controller. As for the outer loop, it adopts an UDE-based compensator to fulfill output impedance reduction around harmonic frequencies and guarantee robust performance to base frequency fluctuations. Due to the two-degree-of-freedom characteristic of UDE-based control approach, dual-loop control framework is utilized to realize decoupling of reference tracking and disturbance rejection of the inverter in the frequency domain. It is distinct that the UDE-based

voltage controller could directly modify the magnitude of output impedance via a suitable filter design without knowing the value of output current. Instead of commonly utilized low-pass filter which possesses constraints due to the limited control bandwidth, a multi-band-stop filter structure is employed to directly minimize the magnitude of output impedance around harmonic frequencies, thus diminishing the output voltage THD. In single-phase systems, harmonic components are mainly odd multiples of base frequency 50 Hz. In addition, increasing the bandwidth of each band-stop filter could assure the robustness against base frequency fluctuations which are likely to occur in practical applications, thus also enhancing the quality of inverter's output voltage. Therefore, the proposed multi-band-stop filter with UDE-based controller could not only reduce the output voltage THD, but also guarantee good robustness in the presence of fundamental frequency fluctuations. In addition, in order to further reduce the output voltage THD and improve the robustness, the design with multi-time-delayed filter was introduced, including the principle and overall control framework.

Finally, experiments under linear loads and nonlinear loads were conducted respectively. Besides, the values of output voltage THD under fundamental frequency fluctuations of ± 1 Hz were depicted. Also, comparison experiments of inverter's output voltage with other types of filters were conducted. Consequently, experimental results could well verify the performance of the proposed control design.

7.2 Recommendations for future research

In the single-phase inverter with an LC-filter feeding a nonlinear load, uncertainty and disturbance estimator (UDE)-based controller with a multi-band-stop filter can realize output voltage THD minimization and guarantee good robust performance to the base frequency fluctuations in the meanwhile. Hence, the inverter's output voltage quality is enhanced with the THD reduction. Compared with common employed first order low-pass Butterworth filter and third order low-pass filter, output voltage THD is minimized significantly from 8.7% and 15.1% to 2.14% with the multi-band-stop filter. And THD is reduced to 1.82% with the employment of multi-time-delayed filter. Nevertheless, it

takes approximate one cycle for the single-phase inverter system to settle down with both linear and nonlinear loads. The transient response performance needs to be improved further since both the multi-time-delayed filter and multi-band-stop filter possess a rather long convergence period. The settling time is sacrificed to realize good performance under steady-state operation.

For the purpose of increasing the transient response speed in single-phase inverter and realize output voltage THD minimization simultaneously, UDE-based controller should be improved to satisfy the inverter's voltage quality enhancement. For future work, it is suggested that another modified type of UDE-filter could be applied to better diminish the output voltage THD of the inverter, as well as increase the transient response speed.

Furthermore, this thesis only studied the single-phase inverter with an LC filter. However, there are various type of filters in inverters, such as LCL-filter which is extensively adopted nowadays. Therefore, more methods on enhancing voltage quality of LCL-filter based single-phase inverter via the UDE-based controller are suggested to be studied in the future.

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Author's Publications

Conference publication:

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