Radiation of open-ended waveguides through a multi-layer superstrate

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**RADIATION OF OPEN-ENDED WAVEGUIDES
THROUGH A MULTI-LAYER SUPERSTRATE**

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Summary

In this thesis, the radiation of open-ended waveguides radiating into a half free space through a layered superstrate is analyzed.

Firstly, surface waves propagating in a multilayer medium backed by a conductive ground plane are studied. The fields' expressions for *TM* and *TE* modes are formulated in a recursive form, respectively. Transcendental equations are built up to find the propagation constant k_y so that the fields in all layers can be obtained. Numerical results are provided to illustrate the surface wave propagating in one-layer and two-layer media, respectively. Parametric studies are carried out to show the relationship between the layered dielectric medium's parameters and the surface wave's propagation constant.

Radiation from infinitely flanged open-ended waveguides is then studied to verify the efficiency of a numerical method. Firstly, the formulation for a general open-ended waveguide is provided to obtain the reflection coefficient matrix at the opening. The field equation containing double surface integrations on the aperture is obtained. These surface integrals are then converted to one integral in the spectral domain through the Sommerfeld identity. Secondly, an efficient method for calculating the reflection coefficient matrix of open-ended coaxial lines is proposed. The generalized pencil of functions method (GPOF) is employed to fit the Sommerfeld integrands with expo-

nential series so that the integrals can be analytically solved in closed form. Thirdly, the accelerating technique is applied to calculate the reflection coefficient matrix of open-ended rectangular waveguides. In addition to the GPOF method, Gaussian quadrature is also employed to accelerate the computation. Numerical results for the reflection coefficient of an open-ended WR-90 waveguide are provided. It is shown that the proposed method is very efficient in calculating the reflection coefficient matrix of open-ended waveguides.

A full-wave analysis of an open-ended waveguide radiating through multi-layer superstrate is also provided. The spectral dyadic Green's function for a multi-layer medium is formulated through cascaded matrices, each of which corresponds to one layer of the superstrate independently. The reflection coefficient matrix is derived by enforcing the field matching condition at the opening of the waveguide. The GPOF method and Gaussian quadrature are employed to improve the computational efficiency after poles in the integrand are removed. The accuracy, efficiency and convergence of this numerical method are examined. The principle of high directivity of the open-ended rectangular waveguide radiating through the two-layer superstrate is investigated and a simple transmission line model is also proposed to explain the gain-enhancement effect of the superstrate. Radiation from an open-ended coaxial line through a two-layer superstrate is also analyzed, which show the expedience of our method in solving omnidirectional problem.

Experimental studies of open-ended waveguides radiating through a two-layer superstrate is finally carried out. Experimental results show that a single cavity-backed aperture antenna can achieve a high-gain of 16.7dBi

through a two-layer superstrate. The effects of the superstrate and the ground plane of finite sizes are then studied. Moreover, this gain enhancement effect of the two-layer superstrate is also proven to be valid for array configurations.

Chapter 1

Introduction

1.1 Motivation

Maxwell's equations are the foundation of modern electromagnetic theory, which were introduced by J.C. Maxwell [1] in 1873. After this landmark achievement, the first pair of antennas in the world was invented in 1886 by H. Hertz. Nowadays, various antennas have been invented and are being widely used in all of the world.

High gain antennas are one of the most favored types by many radio applications because they have many advantages. For example, a high signal to noise ratio (SNR) is usually desired by the receiver in a wireless communication system. Since it is almost impossible to suppress the noise from the environment, we can only manage to increase the received signal power. According to the Friis transmission equation [2], the received signal power is in proportion to the gains of both transmitting and receiving antennas for a given transmitting power. Therefore, using antennas of high gain can lead to an increment of SNR. Moreover, using high gain antennas in wireless communication systems can reduce the interference between radio links

and the power consumption of mobile terminals. Another advantage of high directive antenna is its high resolution, which is very useful in improving the positioning accuracy of radar and other positioning systems. Since the directivity is equal to the number of targets uniformly distributed in the free space that the antenna can distinguish [3], the higher the antenna's directivity is, the smaller is the area in which the target can be located. This property is very useful for RAdio Detection And Ranging (RADAR) systems in searching the target objects such as aircrafts or ships. It is noted that the value of an antenna's directivity is equal to its gain if there is no dissipative loss [4].

As the antenna design techniques blossom since the Hertz's dipole and loop came around, many methods for gain-enhancement have been proposed. Beyond the traditional technique such as using large physical aperture size and in-phase antenna array, some antennas use parasitic elements to achieve a high gain such as Yagi antenna [5], which usually consists of a driving element, a reflective element and a few directive elements. Mutual coupling between elements is utilized to generate a pencil-like end-fire radiation pattern. Recently, metamaterials have also been proposed to improve the antennas' gain [6–9].

It has been reported in [10] that a multi-layer superstrate can enhance the antenna's gain as long as the layered medium is properly designed. The technique of gain-enhancement through a layered medium has many advantages over other methods.

1. It is simple to implement and of low cost since no complicated structure or special material is required;
2. It can be applied to various types of antennas such as printed dipoles

[10], patch antennas [16, 18] or aperture antennas [19–21], which extends the range of its applications;

3. As the adopted superstrate is usually thin and flat, it is quite useful in applications that have strict surface requirement, such as the antennas used on aircrafts or satellites.

Because of these advantages, this gain-enhancement technique has received intensive attention. The effect of the layered superstrate on printed antennas has been studied in [10–14]. In these early works, full-wave analysis was provided initially and then a simplified transmission line model was proposed to explain the principle of the gain-enhancement effect of the layered superstrate. Recent works on this topic were also carried out in experiments [18] or analysis technique [16]. The gain-enhancement effect of the superstrate on aperture antenna was briefly introduced in [19, 20] and a cavity-backed slot antenna radiating through a multi-layer superstrate is analyzed in [21].

In addition to the gain-enhancement effect of the layered superstrate, to find a numerical technique that can efficiently evaluate the Sommerfeld integrals is also a point of interest in the analysis of radiation from an open-ended waveguide. The integration is usually time-consuming and the result is hard to converge because the integrand is slowly decaying and oscillating. Therefore, to improve the computation efficiency is within the scope of my work. When the opening of the waveguide is covered by a superstrate, the Sommerfeld integrand will contain one or more poles, which correspond to different modes of surface waves. These poles may affect the evaluation accuracy if they are not treated properly. Poles in the Sommerfeld integrand can be removed by the residue theorem [22]. Consequently, my work begins with the study of surface waves propagating in a multi-layer superstrate.

1.2 Objectives

In responding to the issues mentioned in Section 1.1, this thesis aims at developing an efficient method for analyzing the open-ended waveguides radiating through a layered superstrate and investigating the gain-enhancement principle of the layered medium on aperture antennas. Base on these main objectives, following tasks have to be completed:

1. Since the propagation constants of surface waves will be used in the analysis of open-ended waveguide radiating through a layered superstrate, the propagation of surface waves along a multilayer medium is to be studied. The field equations in each layer and transcendental equations are to be formulated to solve the propagation constants of all modes that can propagate along the superstrate. Each propagation constant corresponds to the position of a pole in the Sommerfeld integrand studied in the later chapters. Numerical results of one-layer models and two-layer models are examined to show the influences of each layer's thickness and permittivity on the propagation constants.
2. An efficient numerical method is introduced to evaluate the Sommerfeld integrals, which is firstly applied to solve the reflection coefficient matrix of open-ended waveguides radiating into a half space. Approximations to the integrand will be made by the generalized pencil of functions (GPOF) method and Gaussian quadrature so that the evaluation of the integral can be accelerated. An open-ended coaxial line and an open-ended rectangular waveguide are considered to demonstrate the efficiency and accuracy of the proposed method.
3. The analysis of an open-ended rectangular waveguide radiating through

a two-layer superstrate is rigorously performed to demonstrate its high directivity. The GPOF method and Gaussian quadrature are employed to efficiently evaluate the Sommerfeld integrals, which is more sophisticated due to the complicated Green's function of the two-layer superstrate. The accuracy, efficiency and convergence of this numerical method are verified. Besides the numerical technique for evaluating the Sommerfeld integrals, the principle of the high directivity of an open-ended waveguide radiating through a two-layer superstrate is also analyzed. An open-ended coaxial line radiating through the two-layer superstrate will be given as well to show the application of my method in solving omnidirectional problem.

4. Experimental studies are carried out to verify the gain-enhancement effect of a two-layer superstrate on a cavity-backed slot antenna, which exhibits a better return loss performance than open-ended waveguides. Since the sizes of the superstrate and the ground plane in my theoretical models are considered to be infinitely large in the horizontal direction, which is actually infeasible, the relationship between the antenna's gain and the horizontal sizes of the antenna, which include the superstrate size and the ground plane size, are investigated to show how large they need to be in practice. The gain-enhancement effect of the superstrate on a two-slot array and a four-slot spiral array of circular polarization are also demonstrated.

1.3 Major Contributions of the Thesis

The major contributions of my work presented in this thesis are listed as follows:

1. An efficient method is developed to calculate the Sommerfeld integrals used for evaluating the reflection coefficient of open-ended waveguides. The integrands, which are functions of the radial wave number k_ρ , often have branch point [49], denoted by k_1 in this thesis. Therefore, it is not smooth along the integral path from 0 to $+\infty$ and approximation to the whole integrand is not easy to be made along the whole path. Therefore, I proposed two solutions to this problem. The first is for the case when the denominator of the Green's function takes the form $\sqrt{k_1^2 - k_\rho^2}$. In this case, the curve of integrand function excluding the denominator $\sqrt{k_1^2 - k_\rho^2}$ is fitted by exponential series through the GPOF method and the result of integration is a linear combination of a Struve function and modified Bessel functions. The second is for the case when the integrand is not infinite at the branch point, such as in a layered medium. The integral path is firstly divided by the branch point so that the integrals in the two segments can be calculated separately. From 0 to the branch point, the integral is evaluated by Gaussian quadrature. The damped and oscillating tails of the integrands after the branch point are approximated by exponential series through the GPOF method, the result of which is represented by the coefficients of the series. If the model is omnidirectional, such as a coaxial line, the integration in azimuth direction can be avoided by proper manipulations; otherwise the Gaussian quadrature is employed to evaluate the integral in this direction. It will be shown that

the proposed numerical method is efficient and accurate in evaluating Sommerfeld integrals.

2. The principle of high directive radiation through a two-layer superstrate is studied. The influences of the superstrate and the excitation are considered, respectively. A superstrate is found to have significant focusing effect in the vertical direction when

- the dielectric constant of the first layer is 1;
- the thickness of the first layer is a multiple of half free space wavelength;
- the dielectric constant of the second layer, denoted by ϵ_r in this thesis, is very high;
- the thickness of the second layer is $(n/2 + 1/4)\lambda_2$, where n is any non-negative integer and λ_2 is the wavelength in the second layer.

It is also found that the directivity is approximately proportional to the value of ϵ_r when above conditions are met. The spectrum of the excitation is also required to have non-zero value in the position that corresponds to the expected direction of high directivity.

3. A simple model is proposed to illustrate the high-gain conditions of a two-layer superstrate in the view of transmission line theory. In this model, the opening of the waveguide is represented by a voltage source. Applying the transmission line theory, it is found that the voltage amplification factor is equal to the square root of the relative permittivity of the second layer when the permittivity of the first layer is 1. This model provides a very simple way to estimate the gain-enhancement effect of the superstrate.

4. Experimental studies are carried out to verify the high-gain effect of the superstrate on cavity-backed slot antennas, which have much better return loss performance than open-ended waveguides. The measured results show that a two-layer superstrate that satisfies the gain-enhancement conditions can greatly improve the gain of both slot element and slot arrays, which therefore verify the proposed principles of high directive radiation. Moreover, the parametric study of the horizontal sizes of the superstrate and ground plane is provided to show how the antenna gain varies with the two sizes.

1.4 Organization of the Thesis

The rest of this thesis is organized as follows. A literature review is presented in Chapter 2. Previous works on high directive radiation, analysis of open-ended waveguides and numerical techniques for evaluating Sommerfeld integrals are briefly reviewed.

In Chapter 3, surface waves propagating in a layered medium are studied. The recursive matrix equations of the electric and magnetic fields in a multi-layer medium are provided for TM modes and TE modes, respectively. Transcendental equations are formulated according to the boundary conditions so that the propagation constant can be solved. Numerical results of the normalized propagation constant of TM_0 mode and TE_1 mode are provided. For each mode, firstly, their propagation in a one-layer medium is considered and the influences of the relative thickness and the relative permittivity of the substrate are discussed. Next, the surface wave in a two-layer medium is studied. Moreover, higher-order surface wave modes in a two-layer medium are also discussed.

An efficient method introduced in Chapter 4 for full-wave analysis of open-ended waveguides radiating into a half space. Firstly, the field equation of an arbitrary waveguide is formulated to calculate the reflection coefficient matrix in the spectral domain. In order to efficiently evaluate the Sommerfeld integrals, the GPOF method and Gaussian quadrature are employed. An open-ended coaxial line and a rectangular waveguide are considered, respectively, which therefore demonstrates the high efficiency and accuracy of the method in analyzing the radiation of open-ended waveguides into a half space.

In Chapter 5, an efficient full-wave analysis of an open-ended rectangular waveguide radiating through a two-layer superstrate is provided. Firstly, the formulation of the spectral dyadic Green's function in a multi-layer superstrate is given. Then the expression of the spectral dyadic Green's function for a two-layer superstrate is given explicitly. After that, the reflection coefficient matrix is obtained by evaluating the Sommerfeld integrals efficiently through the GPOF method and Gaussian quadrature after removal of the poles in the integrand. The accuracy, efficiency and convergency performance of the numerical method are demonstrated. After that the principles of high directivity is also studied by considering the Green's function and the spectrum of the excitation. In addition, a simplified transmission line model is proposed to estimate the gain-enhancement effect of the superstrate. To be complete, an open-ended coaxial line radiating through the two-layer superstrate is analyzed to show the expedience of the proposed method in solving the omnidirectional problem.

Chapter 6 presents experimental studies of cavity-backed slot antennas radiating through a two-layer superstrate. Experimental results of a single-

slot antenna is provided to demonstrate the high-gain effect of the two-layer superstrate. Parametric study is also performed to show the effects of the horizontal sizes of the superstrate and the ground plane on the single-slot antenna gain. Experimental results of a two-slot array and a spirally arranged four-slot array of circular polarization are also presented to show the effect of the two-layer superstrate on slot arrays.

Conclusions of the thesis and recommended future work are presented in Chapter 7.

Chapter 2

Literature Review

This chapter consists of three sections. The review of popular gain-enhancement techniques is provided in Section 2.1. Previous works on radiation from open-ended waveguides are reviewed in Section 2.2. Several numerical techniques for evaluating Sommerfeld integral are reviewed in Section 2.3.

2.1 Gain-enhancement Techniques

Study on high gain antennas dates back to decades ago [23, 24] since they find wide use in many communication and radar systems. Many antenna design techniques have been proposed to improve the antenna's gain in the past decades [25–30]. In this section, the principle of high-directive radiation is briefly discussed and some of the commonly used gain enhancement techniques are then introduced.

2.1.1 Principle of High Directive Radiation

To generate a high directive radiation beam, a uniform or almost-uniform field distribution over a large aperture is required. This principle can be

easily understood with the following equation,

$$D = 4\pi \frac{A_e}{\lambda^2} \quad (2.1)$$

where D and A_e are the directivity and effective aperture of the antenna, respectively. λ is the wavelength. The aperture efficiency is defined as $\Upsilon_{ap} = A_e/A_p$. Therefore a large physical aperture A_p together with a high aperture efficiency Υ_{ap} , which is usually represented by the uniformity of field distribution over the aperture, can result in an antenna of high gain. Antennas with uniform field distribution along the aperture have found many implementations such as horn antennas, which usually have a very large physical aperture. A tapered guiding structure is employed to ensure the uniformity of the fields when the cross section is enlarged. The uniformity of the fields at the opening of the horn contributes to a high aperture efficiency and in turn results in a high gain. Beyond the guided wave along a tapered waveguide, other techniques can also be adopted to generate uniform fields. For parabolic reflector antennas and lenz antennas, which are usually excited by a point source, curved surface and dielectric lenz are used, respectively, to compensate for the phase difference of a spherical wave arriving at the aperture plane. The principles of these three types of high gain antennas are illustrated in Fig. 2.1.

Employing antenna arrays consisting of identical elements is an even more direct way since it aims to generate uniform fields at the aperture through many conformal antenna elements, which may have a shorter front-to-back dimension than the aforementioned types but the cost is more complexity in design and a new defect - mutual coupling, which could decrease the radiating efficiency and in turn affect the antenna's gain.

(a) Horn antenna

(b) Parabolic reflector antenna

(c) Lenz antenna

Figure 2.1 Typical high-gain antennas.

2.1.2 Parasitic Elements

However, mutual coupling does not play negative role against efficient radiation in all cases. Parasitic effect can also be utilized to enhance the antenna's gain. Yagi-Uda antenna [5] is a special antenna that can generate high directive radiation through parasitic elements. A typical Yagi-Uda antenna consists of a driving dipole as the excitation, one reflector which is inductive and one or more directors that are capacitive. Besides dipole antennas, the gain-enhancement effect of parasitic elements can also be applied to other kinds of antennas. For example, a parasitic slot array was introduced in [31]. A stacked square patch antenna and a stacked circular patch antenna were presented in [32] and [33], respectively. It was reported that all of these proposed antennas achieved high gains through mutual coupling. It is also noticed that the bandwidth of such antennas is usually very narrow.

2.1.3 Metamaterial

Metamaterial is defined as *macroscopic composites having a man-made, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses to specific excitation* [34]. The properties of this material are determined more by its structure rather than its composition. In the past decade, various metamaterials were proposed such as double negative (DNG) materials [36]; photonic band gap (PBG) structured materials [37, 55]; and high-impedance surfaces [38]. Metamaterials are also used in antenna design to improve the performance. A metamaterial superstrate composed of stacked S-shaped split-ring resonators was demonstrated to exhibit gain-enhancement effect in [6]. A superstrate with circular patch being cut periodically was proposed

in [7] to demonstrate that it can enhance the antenna's gain. Magnetic photonic crystal operating in frozen mode was reported to be able to improve the antenna gain in [8, 9]. Gain-enhancement by adoption of degenerate band-edge crystal was introduced in [39, 40]. PBG superstrates composed of periodic air-filled and metallic blocks inside a host dielectric substrate were proposed in [41] and [42], respectively. A study on high directive electromagnetic band gap (EBG) resonator antenna using frequency selective surface (FSS) was provided in [43, 44]. Metamaterial electromagnetic insulators are introduced in [45] to suppress mutual coupling between closely arranged array elements, which therefore results in a superdirective array. A high-gain subwavelength resonant cavity antenna was proposed in [46], which consists of a metamaterial ground plane, a superimposed metallodielectric electromagnetic band gap (MEBG) array and a single dipole placed between them as the excitation. Although a metamaterial can effectively enhance the antenna's gain as has been shown in these works, it often adopts a complicated pattern or structure, which therefore increases the cost of design and fabrication. In this thesis, the gain-enhancement effect of a simple structure - layered superstrate - will be studied.

2.1.4 Mutli-layer Superstrate

The technique of gain-enhancement through a multi-layer superstrate was proposed in [10]. It was shown that when the parameters of the layered medium meet the resonant conditions, which were illustrated by a transmission line model [10], the antenna gain can be significantly improved.

Waves propagating in layered media have been intensively studied in many works [47–50]. The printed antennas radiating through a multi-layered

superstrate were analyzed in [11,12]. the issues on impedance characteristics and loss through surface wave are investigated in detail [17]. Further analytical studies on the gain-enhancement effect of the multi-layer superstrate were carried out in [13–16]. Experimental studies based on the resonating conditions were also provided in [51–54] to demonstrate the gain-enhancement effect. Besides printed antennas, which are usually embedded in one layer of substrate or on the interface between two layers, aperture antennas, which are opened on the ground plane and radiating through a layered medium, have been studied by many researchers as well, such as in [19–21]. Fig. 2.2 depicts the models of two types of sources radiating through a two-layer medium. In this thesis, I focus on the radiation from an open-ended waveguide through a layered medium.

Most of the previous works assumed that the excitation is very simple, which could help to understand the effect of the layered medium. However, a practical feeding element is not necessarily as simple as a point source or a certain mode. Therefore, to provide a more accurate analysis of open-ended waveguides radiating through a layered medium, the reflection coefficient matrix should be solved to exactly describe the source antenna before calculating the far field pattern, which is one of the contributions of this thesis.

2.2 Radiation from Open-ended Waveguides

An open-ended waveguide usually refers to a waveguide abruptly terminated in a plane perpendicular to the longitudinal direction and waves from the aperture can radiate into a half space [56]. Open-ended waveguides can be classified according to the shapes of their cross-section and some examples are given in Fig. 2.3. Besides the cross-section shape, an open-ended wave-

(a) Printed circuit antenna

(b) Aperture antenna

Figure 2.2 Radiation through a two-layer medium.

Figure 2.3 Examples of open-ended waveguides.

uide can be either flanged or without flange. The open-ended waveguides with an infinite flange are considered in this thesis because of the fact that the diffraction by the flange edge can be easily estimated using the geometry theory of diffraction (GTD). Further discussions on the effect of the flange can be found in [57–59]. Open-ended waveguides have many advantages such as simplicity, high power capacity and natural integration with the feeding part, thus, they have found many applications, such as antenna elements in phased arrays [60], non-destructive measurements [61,62] and fatigue detection [63], etc. Therefore, they have received considerable attentions and many works have been published to analyze the radiation from open-ended waveguides. The analysis process usually consists of two steps: the first is to find the field within the waveguide, especially the field at the aperture; the second is to calculate the radiated fields in the open space according to the results obtained in the first step. The second step is usually not very difficult when the field along the aperture is known [3]. Therefore, most researchers made their efforts in investigating the first step.

2.2.1 Radiating into a Half Space

The simplest model is an open-ended waveguide radiating directly into a homogeneous half space. Among previous works, some researchers were interested in characteristics of a particular model. For an example, the radiation from an open-ended waveguide into a lossy half free space was studied in [65], in which an aperture admittance model was proposed based on the properties of driving point admittance of passive and stable one-port networks; the mutual coupling between two open-ended coaxial lines was discussed in [66]. Some others focused on the techniques for deriving the scattering matrix (reflection coefficient matrix) $\mathbf{\Gamma}$ at the aperture. Many methods have been proposed during the past decades. The correlation matrix method [67] is based on the principle of power conservation through the aperture; the transverse operator method [68] offers a simple and systematic formulation of the radiation problem of open-ended waveguide by field expansion in a series of normal transverse electric (TE) and transverse magnetic (TM) modes. The extrapolation method was proposed in [69], in which the convergence problem in a lossless model is avoided by extrapolating from the results of lossy models; the Kobayashi potential method proposed in [70] converged faster than the method of moments (MoM). However, this method is only applicable to simple shapes, such as circular and rectangular plates and apertures. In this thesis, the matrix equation, which is used to solve the reflection coefficient matrix, is derived by the hybrid mode matching and method of moments [71], which has been proven effective in analyzing the radiation from a waveguide. The transverse electric field E_t along the aperture was converted to surface magnetic current through equivalent principle [64], which was illustrated in Fig. 2.4. The magnetic field in the half

Figure 2.4 The equivalent principle applied to an aperture.

space is then associated with the surface magnetic current through the half space Green's function. The scattering matrix can be obtained by applying the field matching along the aperture and the orthogonality of the mode functions.

2.2.2 Radiating through a Planar Medium

When the aperture of a waveguide is covered by a planar medium, the analysis becomes more difficult since the Green's function is more complicated. The simplest case, in which open-ended waveguides radiating through a one-layer superstrate were studied by many researchers such as W. F. Croswell [73], C. P. Wu [74] and G. Panariello [75], etc.. The admittance of an open-ended waveguide radiating into a half free space through a multi-layer medium was studied in [72], in which the half free space is approximated by a very large waveguide. The multi-layer effects on cavity-backed slot antennas were studied in [21] by using a hybrid finite element method (FEM) and method of moments, in which the fields in multi-layer superstrate is solved by MoM while in the cavity they are solved by FEM. Full-wave analysis of a coaxial line radiating through a multi-layer medium was presented in [76] in order to measure the permittivity and permeability of materials. In [77], the method of characteristic mode was used to calculate the reflection from an open-ended waveguide terminated by a multi-layer medium numeri-

cally. It is noticed that the scattering at the opening of the waveguides were intensively studied, but few discussions were made on the gain-enhancement effect of the layered medium. In this thesis, the principle of the high directive radiation is investigated jointly with the reflection coefficient matrix derived at the waveguide's opening.

2.3 Evaluation of Sommerfeld Integral

In this thesis, the radiation of open-ended waveguides through a layered medium is to be analyzed. Therefore, evaluating the Sommerfeld integral is unavoidable. On one side, the Sommerfeld integrals usually cannot be worked out analytically due to the complexity of the integrand, which consists of the Green's function and the function of the source. On the other side, evaluation of the integrals numerically is time consuming since the integrand is oscillating and slowly decaying so that result is hard to converge. In order to improve the integration efficiency, many evaluation techniques have been proposed. The numerical techniques for Sommerfeld integration usually have two steps:

1. To determine the integration path, which could be modified from the original one [78]. This modification contributes to two benefits:
 - to avoid the poles and branch cuts;
 - to accelerate the convergence of the integration;
2. To replace the integrand with some other functions by transformation and approximation, the integral of which is more expedient to evaluate.

Usually these two aspects are adopted unitedly. Some numerical techniques for evaluating Sommerfeld integral are reviewed in this section, which is by

no means a complete list of all previous works on this topic.

Gauss-Laguerre quadrature and Romberg-Shanks composite method were proposed in [79] to be jointly used in calculating Sommerfeld integral. In this algorithm, when the horizontal distance between the observed point and the source is greater than their altitude distance, the integral is calculated through Gauss-Laguerre quadrature and the accuracy is determined by the approximation of the integrand by a polynomial; when horizontal distance is shorter than the vertical distance, Romberg-Shanks method is adopted, which segments the integral path by nulls of the Bessel function and calculates the integral of each interval, which leads to a series with its limits being the the result of the integration.

An automatic Romemberg check Newton-Cotes quadrature was used to calculate the Sommerfeld integral for a printed dipole in [80], in which the integration was carried out in a very large area, as shown in Fig. 2.5 (M was proposed to be 10^4). This square area was then divided into four parts marked by I, II, III and IV, respectively. Part I was a circular narrow strip, in which the integrand was singular and the integral was calculated using the residue theorem. The integrals in the rest three parts were evaluated using the self-check Newton-Cotes quadrature since they were all finite integrals without poles. The integration in the strip without pattern between area I, III and IV was ignored because of its narrow width δ .

In [81], the method of steepest descent path was employed in computation of the integral. The original integral path was firstly transformed to the steepest descent path through

$$\xi = \pm \left[\pi/2 + j \ln(t^2 + j + |t|\sqrt{t^2 + 2j}) \right] + \theta_2$$

Figure 2.5 Integral area division in [80].

After that, the Sommerfeld integral took the form of

$$\propto \int_{-\infty}^{+\infty} Q(t) e^{-\beta t^2} dt \quad (2.2)$$

When $Q(t)$ was approximated by the first term of its Taylor's series, the integration was analytically solved. This algorithm assumed that no poles or branch cut was intersected by the path.

In [82], the Sommerfeld integral path was transformed from the real axis to a curve enclosing the branch cut, due to which the poles were disposed separately. Since Hankel function with large argument can be approximated by

$$H_0^{(2)}(z) \approx \sqrt{\frac{2}{\pi z}} e^{-j(z - \frac{\pi}{4})}$$

and after some substitution, the integrand became a product of an algebraic function and an exponential function, which was then solved by second-order asymptotic evaluation. However, it is seen that the approximation performance of both the methods in [81] and [82] is not very good when the

argument of the integrand is small.

The Chebyshev decomposition method was proposed in [83]. The integral path was divided into two parts: one is finite with poles and the other is semi-infinite without poles. The integrand in the first part can be well fitted with Chebyshev polynomial after the poles were removed; but in the second part, two methods were proposed: in the first method, the semi-infinite integral was mapped into a finite interval using the hypertrigonal function and then fit the integrand with Chebyshev polynomial, which was not so accurate to imitate the fluctuation of the integrand after squeezing of the integral interval; the second method separated the integrand into a real part and an imaginary part, then two large enough but finite intervals were chosen for the two integrals, respectively. Within these two intervals, the integrands were fitted with Chebyshev polynomials. After some manipulations, the Sommerfeld integral was finally evaluated by a summation of products of Bessel functions.

In [84], the longitudinal current density in the Sommerfeld integral was represented by the triangular subdomain basis function with edge condition. Therefore, the integral with respect to the transverse wave number k_y was represented in terms of the square of the spherical Legendre function of the first kind, which was then approximated by a summation of natural logarithmic functions. Since the integral with respect to k_x could be analytically worked out, it resulted in very simple polynomials. The integral path $[-2L, 2L]$ with respect to the χ , which was introduced when converting the spatial Green's function into spectral domain, was divided into three segments. The integrals within the two parts without singular points were worked out numerically while in the rest part with poles the integral was

evaluated in a closed form. This method indicated that if the base functions of the source were carefully chosen, the integration could be greatly facilitated.

A leading-order approximations method was introduced in [85] to greatly accelerate the computation of Sommerfeld integrals. This method first adopted the steepest descent path and leading-order approximations were then employed to accelerate the integration process. In [86], the leading-order approximation was adopted jointly with the Galerkin method to efficiently solve the Sommerfeld integral for an arbitrarily shaped and oriented wire antenna. It was shown that this method is efficient in solving the radiation from wire antennas.

Complex image method was proposed in [87], in which the quasi-static component and the surface wave component in the integrand, which was a function of k_ρ , were firstly removed. The remainder except the Hankel function was approximated by several damped exponential series of k_z through the Prony's method [88]. Therefore, the integral was analytically evaluated using the Sommerfeld Identity. A similar algorithm was proposed in [89,90], in which the Prony's method in [87] was replaced by the generalized pencil of functions method, which was more robust and less noise sensitive. It has been noticed that the computation efficiency can be greatly improved if those complicated integrands are approximated by some other functions which can be easily integrated. Especially when the integrands are exponential functions, the results can be represented directly by the coefficients of the exponential functions. Therefore, the problem of numerical integration has been converted to the problem of coefficients estimation.

Among the techniques for estimation, Prony's method is very popular for estimating the exponential coefficients. An alternative method is the pencil of functions (POF) method [91, 92], which was used to modeling an impulse response [93]. Matrix pencil method was presented as a variation of POF in [95], which was proven to be very efficient in estimating parameters of exponential sinusoids [96] and computing Sommerfeld integral tails [98]. The generalized pencil of functions method was proposed in [94], in which the eigenvalues of the matrix pencil are found by solving the equation $Y_1^+ Y_2 \mathbf{p}_i = z_i \mathbf{p}_i$, where Y_1 and Y_2 are the information matrices, the superscript $+$ denotes the pseudo-inverse. $\mathbf{p}_i$ is the generalized eigenvector and z_i is the generalized eigenvalue. It was reported that the generalized pencil of functions method has better performance than Prony's method with respect to computation and noise sensitivity.

Compared with approximation methods using other functions, curve fitting using exponential functions with complex coefficients is more wise because: 1) they have similar features with the Sommerfeld integrand: decaying and oscillating; 2) the fitting is not restricted within a finite range; 3) no complicated transform is pre-required; 4) the integration result can directly come up with the coefficients of exponential functions. Therefore, this technique will be adopted in my work.

Chapter 3

Surface Wave in a Layered Medium

3.1 Introduction

Surface wave is also known as Zenneck wave, which was initially studied by Uller [99] in 1903 and later by Zenneck [100] in 1907. This special kind of wave can be excited in and propagate along the surfaces of many kinds of structures, such as a dielectric rod or slab, a corrugated plane or a dielectric-coated wire, etc. This thesis is to analyze the open-ended waveguide radiating through a multilayer superstrate, in which surface waves appear unavoidably. It will be shown later that locating poles, which represent the propagation constants of different surface wave modes, is an important step in solving the whole problem. In this chapter, the propagation of the surface waves in a multilayer planar medium backed by a ground plane is to be studied.

A general multilayer model is shown in Fig. 3.1. There are totally N dielectric layers stacked one by one, which are indexed by $i (i = 1, \dots, N)$

Figure 3.1 Model of multi-layered medium.

from bottom to top. The thickness of layer i and the position of the interface between layer i and layer $i + 1$ are represented by h_i and d_i , respectively. Obviously, I have

$$d_i = \sum_{k=1}^i h_k$$

The permittivity and permeability of layer i are ϵ_i and μ_i , respectively. The half space on layer N is indexed by $i = N + 1$ for convenience. It should be mentioned that this semi-infinite layer is unnecessary to be a half free space since the propagation of the surface wave is only affected by the ratios of the permittivities and permeabilities between layers, as will be shown later. Below the first layer is a perfectly conductive ground plane. All layers are infinitely large horizontally (in the x-y plane). From the view of spectral analysis, any wave can be represented by a superposition of waves propagating in all directions. Therefore, the surface wave propagating along the positive y-axis with $k_x = 0$ is considered without losing the generality. Moreover, if a surface wave is propagating along the y axis, its propagation

constant k_y should be greater than k_{N+1} , which results in the exponential attenuation of the field intensities along the $+z$ axis in the $(N+1)$ th layer. For convenience, the wavelength in layer i is denoted by λ_i .

3.2 Formulation

In this chapter, the field equations [101] are established for two types of surface wave: TM and TE modes, respectively. Since the assumption has been made that the surface waves propagate along the y -axis. After this definition, the field of TM modes can be described by H_x , E_y and E_z while the H_x is orthogonal to the propagating direction of the wave; for TE modes, the field components consist of E_x , H_y and H_z while E_x is orthogonal to the propagating direction of the wave. Since the components that are used to describe the fields of the two classes of modes are different, the field equations are formulated separately and given in Appendix A.

3.2.1 Fields of TM Modes

In the layered medium, the three field components in the i th layer for TM modes are given by H_{x_i} , E_{y_i} and E_{z_i} , among which H_{x_i} and E_{y_i} can be derived by Eq. (3.1).

$$\begin{bmatrix} H_{x_i} \\ E_{y_i} \end{bmatrix} = \begin{cases} \mathbf{P}_1 \mathbf{U}_1 \mathbf{P}_1^{-1} \begin{bmatrix} 2A \\ 0 \end{bmatrix} & i = 1 \\ \mathbf{P}_i \mathbf{U}_i \mathbf{P}_i^{-1} \begin{bmatrix} H_{x_{i-1}}|_{z=d_{i-1}} \\ E_{y_{i-1}}|_{z=d_{i-1}} \end{bmatrix} & i = 2, 3, \dots, N+1 \end{cases} \quad (3.1)$$

with

$$A = C e^{-j k_y y} \quad (3.2)$$

$$\mathbf{P}_i = \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_{z_i}}{\omega \epsilon_i} \end{bmatrix} \quad (3.3)$$

$$\mathbf{U}_i = \begin{bmatrix} \cos k_{z_i}(z - d_{i-1}) & j \sin k_{z_i}(z - d_{i-1}) \\ j \sin k_{z_i}(z - d_{i-1}) & \cos k_{z_i}(z - d_{i-1}) \end{bmatrix} \quad (3.4)$$

where ω is the angular frequency, C is the amplitude constant, k_y is the propagation constant and k_{z_i} is the wave number in the i th layer with respect to the $+z$ direction. k_{z_i} can be derived by k_y through

$$k_{z_i}^2 + k_y^2 = k_i^2 \quad (3.5)$$

where $k_i = \omega \sqrt{\mu_i \epsilon_i}$. It is found that Eq. (3.1) is a set of recursive matrix equation, in which the fields in layer i are calculated through the fields in layer $i - 1$ except for the first layer, which can be obtained independently. The detailed derivation process of Eq. (3.1) is provided in Appendix A. The field component E_{z_i} is given by

$$E_{z_i} = \frac{k_y}{\omega \epsilon_i} H_{x_i}. \quad (3.6)$$

with $k_y/(\omega \epsilon_i)$ being the mode impedance of TM modes in the i th layer.

Since the fields in layer $N + 1$ can be derived through Eq. (3.1) and the propagation constant k_y of the surface wave is contained implicitly in the matrix equations, if the relationship between horizontal electric and

magnetic fields at $z = d_N$ is found, k_y can be obtained. It is noted that the propagation of a surface wave requires that the intensities of the fields are attenuated exponentially in the $(N + 1)$ th layer as z approaching $+\infty$, which indicates that the factor containing z within the expressions of the horizontal fields in the $(N + 1)$ th layer should take the form of $e^{-jk_{z_{N+1}}z}$ with $-jk_{z_{N+1}}$ being a negative real number.

In the $(N + 1)$ th layer, the fields are given as follows,

$$\begin{bmatrix} H_{x_{N+1}} \\ E_{y_{N+1}} \end{bmatrix} = \begin{bmatrix} \cos k_{z_{N+1}}(z - d_N)H_{x_N}|_{z=d_N} + j \sin k_{z_N}(z - d_N)\frac{\omega\epsilon_{N+1}}{k_{z_{N+1}}}E_{y_N}|_{z=d_N} \\ j \sin k_{z_{N+1}}(z - d_N)\frac{k_{z_{N+1}}}{\omega\epsilon_{N+1}}H_{x_N}|_{z=d_N} + \cos k_{z_N}(z - d_N)E_{y_N}|_{z=d_N} \end{bmatrix} \quad (3.7)$$

In order to let $H_{x_{N+1}}$ and $E_{y_{N+1}}$ in Eq. (3.7) attenuate along the $+z$ axis, the following condition should be satisfied,

$$H_{x_N}|_{z=d_N} + \frac{\omega\epsilon_{N+1}}{k_{z_{N+1}}}E_{y_N}|_{z=d_N} = 0 \quad (3.8)$$

which is a transcendental equation that can be used to calculate k_y . It is noticed that Eq. (3.8) may have more than one real solution greater than k_{N+1} . Each solution is the k_y of one TM mode. The largest one corresponds to the TM_0 mode surface wave; the second largest solution, if exists, is the k_y of the TM_1 mode, and so on.

When $N = 1$, Eq. (3.8) becomes

$$1 + j\frac{k_{z_1}\epsilon_{r_2}}{k_{z_2}\epsilon_{r_1}}\tan k_{z_1}h_1 = 0 \quad (3.9)$$

where ϵ_{r_i} is the relative permittivity of layer i . Considering Eq. (3.5), k_y

and k_{z_i} can be represented in terms of k_i and θ_i as

$$\begin{cases} k_y = k_i \sec \theta_i \\ k_{z_i} = -jk_i \tan \theta_i \end{cases} \quad i = 1, 2, \dots, N+1 \quad (3.10)$$

where θ_i can be either real or imaginary. Substituting Eq. (3.10) into Eq. (3.9), I have

$$1 + j \frac{\tan \theta_1}{\tan \theta_2} \sqrt{\frac{\epsilon_{r2} \mu_{r1}}{\epsilon_{r1} \mu_{r2}}} \tan \left(-j2\pi \bar{h}_1 \sqrt{\frac{\epsilon_{r1} \mu_{r1}}{\epsilon_{r2} \mu_{r2}}} \tan \theta_1 \right) = 0 \quad (3.11)$$

where

$$\theta_i = \text{arcsec} \left(\bar{k}_y \sqrt{\frac{\epsilon_{r2} \mu_{r2}}{\epsilon_{r_i} \mu_{r_i}}} \right) \quad i = 1, 2 \quad (3.12)$$

and $\bar{h}_1 = h_1/\lambda_2$ is the relative thickness of the substrate with λ_2 being the wavelength in the second layer, $\bar{k}_y = k_y/k_2$ is the normalized propagation constant. It is found that the normalized transmission constant $\bar{k}_y$ that satisfies Eq. (3.11), if exists, is determined by $\bar{h}_1$, $\epsilon_{r1}/\epsilon_{r2}$ and μ_{r1}/μ_{r2} , all of which are relative quantities, which explains why the second layer are not necessary to be a half free space.

When $N = 2$, Eq. (3.8) becomes

$$1 - \frac{\epsilon_{r2} k_{z1}}{\epsilon_{r1} k_{z2}} \mathbb{T}_1 \mathbb{T}_2 + j \left(\frac{\epsilon_{r3} k_{z2}}{\epsilon_{r2} k_{z3}} \mathbb{T}_2 + \frac{\epsilon_{r3} k_{z1}}{\epsilon_{r1} k_{z3}} \mathbb{T}_1 \right) = 0 \quad (3.13)$$

where $\mathbb{T}_i = \tan k_{z_i} h_i$ ($i = 1, 2$). Similar to the case of $N = 1$, Eq. (3.13) can

be rewritten as

$$1 - \frac{\tan \theta_1}{\tan \theta_2} \sqrt{\frac{\epsilon_{r2}\mu_{r1}}{\epsilon_{r1}\mu_{r2}}} \mathbb{T}_1^* \mathbb{T}_2^* + j \left(\frac{\tan \theta_2}{\tan \theta_3} \sqrt{\frac{\epsilon_{r3}\mu_{r2}}{\epsilon_{r2}\mu_{r3}}} \mathbb{T}_2^* + \frac{\tan \theta_1}{\tan \theta_3} \sqrt{\frac{\epsilon_{r3}\mu_{r1}}{\epsilon_{r1}\mu_{r3}}} \mathbb{T}_1^* \right) = 0 \quad (3.14)$$

where

$$\mathbb{T}_i^* = \tan \left(-j2\pi \bar{h}_i \sqrt{\frac{\epsilon_{r_i}\mu_{r_i}}{\epsilon_{r3}\mu_3}} \tan \theta_i \right) \quad i = 1, 2$$

and

$$\begin{cases} \theta_i = \operatorname{arccsc} \bar{k}_y \sqrt{\frac{\epsilon_{r3}\mu_{r3}}{\epsilon_{r_i}\mu_{r_i}}} & i = 1, 2, 3 \\ \bar{h}_i = h_i/\lambda_3 & i = 1, 2 \\ \bar{k}_y = k_y/k_3 \end{cases}$$

λ_3 is the wavelength in the free space. It is seen in Eq. (3.14) that the normalized propagation constant $\bar{k}_y$, if exists, is determined by $\epsilon_{r1}/\epsilon_{r3}$, $\epsilon_{r2}/\epsilon_{r3}$, μ_{r1}/μ_{r2} , μ_{r2}/μ_{r3} , $\bar{h}_1$ and $\bar{h}_2$. It is also noticed that all of them are relative quantities.

3.2.2 Fields of TE Modes

Similar to the TM modes, the field components of TE modes consist of E_{x_i} , H_{y_i} and H_{z_i} in the i th layer. The recursive field expressions of the TE modes are given by Eq. (3.15), in which only E_{x_i} and H_{y_i} are involved.

$$\begin{bmatrix} E_{x_i} \\ H_{y_i} \end{bmatrix} = \begin{cases} \mathbf{Q}_1 \mathbf{U}_1 \mathbf{Q}_1^{-1} \begin{bmatrix} 0 \\ 2A \end{bmatrix} & i = 1 \\ \mathbf{Q}_i \mathbf{U}_i \mathbf{Q}_i^{-1} \begin{bmatrix} E_{x_{i-1}}|_{z=d_{i-1}} \\ H_{y_{i-1}}|_{z=d_{i-1}} \end{bmatrix} & i = 2, 3, \dots, N+1 \end{cases} \quad (3.15)$$

where

$$\mathbf{Q}_i = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{k_{z_i}}{\omega\mu_i} \end{bmatrix}$$

and $\mathbf{U}_i$ is given by Eq. (3.4). The third component H_{z_i} is given by

$$H_{z_i} = -\frac{k_y}{\omega\mu_i} E_{x_i} \quad (3.16)$$

with $\frac{k_y}{\omega\mu_i}$ being the mode impedance of the TE modes in the i th layer.

For TE modes, the horizontal fields in the half free space are

$$\begin{bmatrix} E_{x_{N+1}} \\ H_{y_{N+1}} \end{bmatrix} = \begin{bmatrix} \cos k_{z_i}(z - d_{i-1})E_{x_N}|_{z=d_N} - j \sin k_{z_i}(z - d_{i-1})\frac{\omega\mu_i}{k_{z_i}}H_{y_N}|_{z=d_N} \\ -j \sin k_{z_i}(z - d_{i-1})\frac{k_{z_i}}{\omega\mu_i}E_{x_N}|_{z=d_N} + \cos k_{z_i}(z - d_{i-1})H_{y_N}|_{z=d_N} \end{bmatrix} \quad (3.17)$$

and the corresponding transcendental equation of k_y is given by

$$E_{x_N}|_{z=d_N} - \frac{\omega\mu_{N+1}}{k_{z_{N+1}}}H_{y_N}|_{z=d_N} = 0 \quad (3.18)$$

It is noticed that Eq. (3.18) could have more than one solution. Similar to the case of TM modes, the mode that corresponds to the largest k_y is

assigned as TE_1 mode; the mode that corresponds to the second largest k_y is TE_2 mode, and so on.

When $N = 1$, Eq. (3.18) becomes

$$j \frac{k_{z2} \mu_{r1}}{k_{z1} \mu_{r2}} \tan k_{z1} h_1 + 1 = 0 \quad (3.19)$$

Substituting Eq. (3.10) into Eq. (3.19), I have

$$j \frac{\tan \theta_2}{\tan \theta_1} \sqrt{\frac{\epsilon_{r2} \mu_{r1}}{\epsilon_{r1} \mu_{r2}}} \tan \left(-j 2\pi \bar{h}_1 \sqrt{\frac{\epsilon_{r1} \mu_{r1}}{\epsilon_{r2} \mu_{r2}}} \tan \theta_1 \right) + 1 = 0 \quad (3.20)$$

It is found in Eq. (3.20) that the normalized propagation constant $\bar{k}_y$ is also determined by the same relative quantities as those for TM modes when $N = 1$.

When $N = 2$, Eq. (3.18) becomes

$$1 - \frac{k_{z2} \mu_{r1}}{k_{z1} \mu_{r2}} \mathbb{T}_1 \mathbb{T}_2 + j \left(\frac{k_{z3} \mu_{r1}}{k_{z1} \mu_{r3}} \mathbb{T}_1 + \frac{k_{z3} \mu_{r2}}{k_{z2} \mu_{r3}} \mathbb{T}_2 \right) = 0 \quad (3.21)$$

Substituting Eq. (3.10) to Eq. (3.21), the following equation is derived,

$$1 - \frac{\tan \theta_2}{\tan \theta_1} \sqrt{\frac{\epsilon_{r2} \mu_{r1}}{\epsilon_{r1} \mu_{r2}}} \mathbb{T}_1^* \mathbb{T}_2^* + j \left(\frac{\tan \theta_3}{\tan \theta_1} \sqrt{\frac{\epsilon_{r3} \mu_{r1}}{\epsilon_{r1} \mu_{r3}}} \mathbb{T}_1^* + \frac{\tan \theta_3}{\tan \theta_2} \sqrt{\frac{\epsilon_{r3} \mu_{r2}}{\epsilon_{r2} \mu_{r3}}} \mathbb{T}_2^* \right) = 0 \quad (3.22)$$

Therefore, it can be concluded that the normalized propagation constant $\bar{k}_y$ of TE modes is also determined by six relative quantities: $\bar{h}_1$, $\bar{h}_2$, $\epsilon_{r1}/\epsilon_{r3}$, $\epsilon_{r2}/\epsilon_{r3}$, μ_{r1}/μ_{r3} and μ_{r2}/μ_{r3} , which are the same as those for TM modes when $N = 2$.

Figure 3.2 Model of one-layer medium backed by a ground plane.

3.3 Results and Discussions

It has been shown that the normalized propagation constant $\bar{k}_y$ for TM modes and TE modes can be derived by solving Eq. (3.8) and Eq. (3.18), respectively. For a surface wave to propagate along the interface of the layered medium, k_y should be real and $k_{z_{N+1}}$ be imaginary, which requires $\bar{k}_y > 1$. I also notice that both of the two transcendental equations could have more than one solutions greater than 1. For conciseness, I assume that $\epsilon_{r_{N+1}} = 1$ and $\mu_{r_i} = 1$ for any i , which indicates that the $(N + 1)$ th layer is a half free space.

3.3.1 TM_0 Mode

One-layer Medium

At the beginning, let's consider the simplest case where $N = 1$. The model is shown in Fig. 3.2, in which there is only one layer of flat dielectric slab backed by a ground plane. The thickness and relative permittivity of the substrate are h_1 and ϵ_{r_1} , respectively. Above the substrate is the half free space. The TM_0 surface wave is to propagate along the y axis, while the field intensity is attenuated along the $+z$ axis in the half free space.

Figure 3.3 Normalized propagation constant of TM_0 mode with respect to relative thickness of the substrate.

Fig. 3.3 shows the influences of $\bar{h}_1$ and ϵ_{r1} on the normalized propagation constant $\bar{k}_y$ of TM_0 mode. The three curves are for different ϵ_{r1} values. It is seen $\bar{k}_y$ becomes greater as $\bar{h}_1$ increases. Given a $\bar{h}_1$, a larger ϵ_{r1} contributes to a greater $\bar{k}_y$. Therefore, $\bar{k}_y$ is an increasing function of both $\bar{h}_1$ and $\bar{h}_2$. When $\bar{h}_1 \rightarrow 0$, $\bar{k}_y \rightarrow 1$ for all curves, which means the decay of the fields' intensities along the $+z$ axis in the half free space slows down. Therefore it can be concluded that as long as the substrate is thin enough, the surface wave in the half free space approximates to a plane wave propagating along the $+y$ axis. When $\bar{h}_1$ is very large, $\bar{k}_y$ approaches the limit of $\sqrt{\epsilon_{r1}}$, which means the fields vary slowly along the $+z$ axis in the substrate, while decay very fast in the half free space. Therefore the surface wave regresses to a plane wave propagating along the y axis in the substrate.

Figure 3.4 The model of a two-layer medium backed by a ground plane.

Two-layer Medium

A two-layer model ($N = 2$) is shown in Fig. 3.4. The relative permittivities and thicknesses of the first and second layer are ϵ_{r1} , ϵ_{r2} , h_1 and h_2 , respectively. Quantities in the half free space are indexed by 3. There are two type of configurations with respect to permittivities of the two layers. One is $\epsilon_{r1} < \epsilon_{r2}$ while the other is $\epsilon_{r1} > \epsilon_{r2}$. In this part, the higher one is assigned with 10 and the lower one is 2.

Fig. 3.5 shows the normalized propagation constant of TM_0 mode with respect to the relative thicknesses of the two layers when $\epsilon_{r1} = 2$ and $\epsilon_{r2} = 10$. Each curve in the figure represents the relationship between $\bar{k}_y$ and $\bar{h}_2$ for a given $\bar{h}_1$. The following information can be found in this figure:

1. If $\lambda_3 \rightarrow +\infty$, which indicates the frequency becomes 0, it is found that $\bar{h}_1 \rightarrow 0$ and $\bar{h}_2 \rightarrow 0$ and the normalized propagation constant $\bar{k}_y \rightarrow 1$. Therefore the cutoff frequency is 0. It is noticed that for h_1 and h_2 of any finite values, this result always exists, which leads to the conclusion that the cutoff frequency of TM_0 mode is 0, which is independent of the thickness of any layer.
2. $\bar{k}_y$ is an increasing function of $\bar{h}_2$.

3. All curves share a common upper bound, which is $\bar{k}_y = \sqrt{\epsilon_{r2}}$. The lower bound is determined by $\bar{h}_1$ when $\bar{h}_2 = 0$, in which case the two-layer model reduces to one-layer.
4. There is a point passed by all curves. The position of this point can be derived as follows. Eq. (3.14) can be rewritten to be

$$1 + j \frac{\tan \theta_2}{\tan \theta_3} \sqrt{\frac{1}{\epsilon_{r2}}} \mathbb{T}_2^* + j \tan \theta_1 \mathbb{T}_1^* \left(j \frac{\mathbb{T}_2^*}{\tan \theta_2} \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} + \frac{1}{\tan \theta_3} \sqrt{\frac{1}{\epsilon_{r1}}} \right) = 0 \quad (3.23)$$

If $\bar{k}_y = \sqrt{\epsilon_{r1}}$, I have $\tan \theta_1 = 0$ and the term $\bar{h}_1$ in $\mathbb{T}_1^*$ is eliminated from Eq. (3.23)

$$1 + j \frac{\tan \phi_2}{\tan \phi_3} \sqrt{\frac{1}{\epsilon_{r2}}} \tan \left(-j 2\pi \bar{h}_2 \sqrt{\epsilon_{r2}} \tan \phi_2 \right) = 0 \quad (3.24)$$

where $\phi_2 = \text{arcsec} \sqrt{\epsilon_{r1}/\epsilon_{r2}}$ and $\phi_3 = \text{arcsec} \sqrt{\epsilon_{r1}}$. Solving Eq. (3.24), I have

$$\bar{h}_2 = \frac{j}{2\pi \tan \phi_2} \sqrt{\frac{1}{\epsilon_{r2}}} \arctan \left(j \frac{\tan \phi_3}{\tan \phi_2} \sqrt{\epsilon_{r2}} \right) \quad (3.25)$$

Since Eq. (3.24) is an equation with respect to only $\bar{h}_2$, which indicates that whatever $\bar{h}_1$ is, the point $\left(\frac{j}{2\pi \tan \phi_2} \sqrt{\frac{1}{\epsilon_{r2}}} \arctan \left(j \frac{\tan \phi_3}{\tan \phi_2} \sqrt{\epsilon_{r2}} \right), \sqrt{\epsilon_{r1}} \right)$ on the $\bar{h}_2 - \bar{k}_y$ plane is always passed by the curves. Moreover, it is found that $k_{z1} = 0$ when $\bar{k}_y = \sqrt{\epsilon_{r1}}$. Therefore, the fields in the first layer only vary in the propagation direction since $k_x = k_{z1} = 0$ and $k_y = k_1$.

5. When $\bar{h}_2 < j \arctan \left(j \sqrt{\epsilon_{r2}} \tan \phi_3 / \tan \phi_2 \right) / (2\pi \tan \phi_2 \sqrt{\epsilon_{r2}})$, $\bar{k}_y < \sqrt{\epsilon_{r1}}$. The fields in the first layer vary sinusoidally along the z axis. More-

Figure 3.5 Normalized propagation constant of TM_0 mode with respect to the relative thicknesses ($\epsilon_{r1} = 2$, $\epsilon_{r2} = 10$).

over, a larger $\bar{h}_1$ contributes to a greater $\bar{k}_y$.

6. When $\bar{h}_2 > j \arctan(j\sqrt{\epsilon_{r2}} \tan \phi_3 / \tan \phi_2) / (2\pi \tan \phi_2 \sqrt{\epsilon_{r2}})$, it is found that $\bar{k}_y > \sqrt{\epsilon_{r1}}$. The fields in the first layer vary exponentially along the z axis. On the contrary, a larger $\bar{h}_1$ makes $\bar{k}_y$ smaller. As $\bar{h}_2$ becomes larger, all curves are converged and asymptotic to $\bar{k}_y = \sqrt{\epsilon_{r2}}$, which indicates that the influence of both $\bar{h}_1$ and $\bar{h}_2$ is weakened. Therefore, increasing the relative thickness of either layer further cannot affect the normalized propagation constant significantly.

Fig. 3.6 shows the normalized propagation constant of TM_0 mode with respect to the relative thicknesses of the two layers when $\epsilon_{r1} = 10$ and $\epsilon_{r2} = 2$. In this figure, it can be seen

1. When $\lambda_3 \rightarrow +\infty$, I find that $\bar{h}_1 \rightarrow 0$, $\bar{h}_2 \rightarrow 0$ and $\bar{k}_y \rightarrow 1$. Therefore the cutoff frequency is 0.

Figure 3.6 Normalized propagation constant of TM_0 mode with respect to the relative thicknesses of the two layers ($\epsilon_{r1} = 10$, $\epsilon_{r2} = 2$).

2. $\bar{k}_y$ is an increasing function of both $\bar{h}_1$ and $\bar{h}_2$. When $\bar{h}_1 \rightarrow +\infty$, The curves approach to an upper bound $\bar{k}_y = \sqrt{\epsilon_{r1}}$. When $\bar{h}_2$ increases, $\bar{k}_y$ has also a limit located between $\sqrt{\epsilon_{r1}}$ and $\sqrt{\epsilon_{r2}}$. This limit is determined by $\bar{h}_1$ through Eq. (3.14). The lower bounds of the curves are determined by $\bar{h}_2$ when $\bar{h}_1 = 0$, which is equivalent to a one-layer model.
3. When $\epsilon_{r1} > \epsilon_{r2}$, neither a flat curve nor a point that is passed by all the curves can be found in the figure. Therefore, $\bar{h}_1$ and $\bar{h}_2$ can always affect the propagation constant of the surface wave more or less.

3.3.2 TE_1 Mode

One-layer Medium

Fig. 3.7 shows the influences of $\bar{h}_1$ and ϵ_{r_1} on the normalized propagation constant $\bar{k}_y$ of TE_1 mode when $N = 1$. The condition for a surface wave to propagate is $\bar{k}_y > 1$. Substituting this condition into Eq. (3.20), I have

$$\bar{h}_1 > \frac{1}{4} \sqrt{\frac{1}{\epsilon_{r_1} - 1}} \quad (3.26)$$

which is in accord to the result given in [102]. This threshold is the minimum relative thickness that supports the propagation of TE_1 mode surface wave. The cutoff frequency can be obtained by replacing λ_2 with c/f_c in Eq. (3.26)

$$f_{TE_1} = \frac{c}{4h_1} \sqrt{\frac{1}{\epsilon_{r_1} - 1}} \quad (3.27)$$

where c is the speed of light in free space. It is found in Eq. (3.26) that if either h_1 or ϵ_{r_1} is large enough a TE_1 mode surface wave can always propagate since $f_{TE_1} \rightarrow 0$. As shown in the figure, when ϵ_{r_1} is assigned with 2, 5 and 10, the smallest $\bar{h}_1$ that can carry a TE_1 surface wave equals to 1/4, 1/8 and 1/12, respectively. When $\bar{h}_1$ is slightly larger than the minimum relative thickness, $\bar{k}_y \approx 1$, which indicates the fields is attenuated very slowly along the $+z$ axis in the half free space and the propagation of the surface wave is similar to that of a plane wave. As $\bar{h}_1$ increases, $\bar{k}_y$ approaches to $\sqrt{\epsilon_{r_1}}$ and the fields varies slowly in the first layer and decay very fast in the half free space along the z direction; therefore, it can be approximated by a plane wave propagating in the slab.

Figure 3.7 Normalized propagation constant of TE_1 mode with respect to relative thickness of the substrate.

Two-layer Medium

Fig. 3.8 shows the normalized propagation constant of TE_1 mode with respect to the relative thicknesses of the two layers when $\epsilon_{r1} = 2$ and $\epsilon_{r2} = 10$. In this figure, it is found that

1. If either $\bar{h}_1 = 0$ or $\bar{h}_2 = 0$, the two-layer model reduces to the one layer case and the cutoff condition is given by Eq. (3.26). When both $\bar{h}_1$ and $\bar{h}_2$ are nonzero, the cutoff condition can be obtained by solving Eq. (3.22) numerically. It is found that the condition for a TE_1 mode surface wave to propagate in the two-layer medium is jointly determined by $\bar{h}_1$ and $\bar{h}_2$. Therefore, it is a curve on the $\bar{h}_1 = \bar{h}_2$ plane that separates the propagation area from the cutoff area, which is shown in Fig. 3.9. If the point $(\bar{h}_1, \bar{h}_2)$ is located in the propagation area, the TE_1 surface wave can propagate; it cannot if the point is in

Figure 3.8 Normalized propagation constant of TE_1 mode with respect to the relative thicknesses of the two layers ($\epsilon_{r1} = 2$, $\epsilon_{r2} = 10$).

the cutoff area. It is also noticed that the two-ends of the boundary curve are $(0, 1/4)$ and $(1/12, 0)$, which correspond to the one layer case for $\epsilon_{r1} = 2$ and $\epsilon_{r1} = 10$, respectively.

2. It can also be seen in Fig. 3.8 that $\bar{k}_y$ is an increasing function of both $\bar{h}_1$ and $\bar{h}_2$. As $\bar{h}_2$ increases to $+\infty$, the upper bound of all curves is found to be $\sqrt{\epsilon_{r2}} = \sqrt{10}$. The lower bound is determined by the value of $\bar{h}_1$. When $\bar{h}_1 < 1/(4\sqrt{\epsilon_{r1}-1})$, the lower bound is 1; when $\bar{h}_1 > 1/(4\sqrt{\epsilon_{r1}-1})$, the lower bound is reached when $\bar{h}_2 = 0$ and the $\bar{k}_y$ can be found in Fig. 3.7 for $\epsilon_{r1} = 2$.

Fig. 3.10 shows the normalized propagation constant of TE_1 mode with respect to the relative thicknesses of the two layers when $\epsilon_{r1} = 10$ and $\epsilon_{r2} = 2$. In this figure, it is found that

1. $\bar{k}_y$ is an increasing function of both $\bar{h}_1$ and $\bar{h}_2$.

Figure 3.9 Cutoff boundary of TE_1 mode with respect to the relative thicknesses of the two layers ($\epsilon_{r1} = 2$, $\epsilon_{r2} = 10$).

2. The upper bound of $\bar{k}_y$ for any $\bar{h}_2$ is $\sqrt{\epsilon_{r1}} = \sqrt{10}$ when $\bar{h}_1$ is approaching $+\infty$. Especially when $\bar{h}_1 > 0.1$, all curves that represent different $\bar{h}_2$ converge, which indicates that the influence of the parameter $\bar{h}_2$ diminishes.
3. Similar to the previous case where $\epsilon_{r2} > \epsilon_{r1}$, the cutoff condition here is also jointly determined by the values of $\bar{h}_1$ and $\bar{h}_2$. A concave curve can be drawn on the $\bar{h}_1 - \bar{h}_2$ plane to separate the cutoff area and the propagation area, which is shown in Fig. 3.11. Comparing the convex curve in Fig. 3.9 with this curve, we can found that the positions of the two ends of the curves are exchanged and the cutoff area in the later figure is larger. For a two-layer model as shown in Fig. 3.4, a TE_1 mode surface wave is possible to propagate when the second layer is of a higher permittivity; while it cannot propagate when the stacking

Figure 3.10 Normalized propagation constant of TE_1 mode with respect to the relative thicknesses of the two layers ($\epsilon_{r1} = 10$, $\epsilon_{r2} = 2$).

order of two layers are exchanged. The inversion of this argument is untenable.

3.3.3 Higher-order Modes

In this subsection, higher-order surface wave modes in a two-layer medium are studied with respect to the relative thickness of the first layer $\bar{h}_1$. The dielectric constants of layers one and two are 1 and 10.2, respectively; the relative thicknesses of the two layers has the relationship of $\bar{h}_1 = 2\bar{h}_2\sqrt{10.2}$. Parameters of the model are so chosen that the results can be used later in Section 5.4.

Figs. 3.12 and 3.13 compare the normalized propagation constant of higher-order TM and TE modes with respect to $\bar{h}_1$. It is seen that a higher-order mode has a higher cutoff frequency and smaller propagation constant

Figure 3.11 Cutoff boundary of TE_1 mode with respect to the relative thicknesses of the two layers ($\epsilon_{r1} = 10$, $\epsilon_{r2} = 2$).

for a given $\bar{h}_1$. In these two figures, I also notice that when $\bar{h}_1 = 0.5$, only TM_0 and TE_1 modes can propagate and the normalized propagation constant are 1.048 and 2.0068, respectively.

3.4 Conclusions

The propagation of surface waves in a layered medium has been analyzed in this chapter. First, matrix equations for the fields of TM modes and TE modes are given, respectively. It is found that the horizontal electric and magnetic fields in the i th layer can be expressed by the same fields components in the $(i - 1)$ th layer except for the fields in the first layer, which is given explicitly. In order to derive the propagation constant k_y , transcendental equations are formulated for TM modes and TE modes, respectively. The normalized propagation constant $\bar{k}_y$ is determined by $\bar{h}_i$, $\epsilon_{r_i}/\epsilon_{r_{N+1}}$ and

Figure 3.12 Cutoff boundary of TE_1 mode with respect to the relative thicknesses of the two layers ($\epsilon_{r1} = 1$, $\epsilon_{r2} = 10.2$).

Figure 3.13 Cutoff boundary of TE_1 mode with respect to the relative thicknesses of the two layers ($\epsilon_{r1} = 1$, $\epsilon_{r2} = 10.2$).

$\mu_{r_i}/\mu_{r_{N+1}}$ for $i = 1, \dots, N$. Therefore, the normalized propagation constant $\bar{k}_y$ will be the same if these relative values are unchanged. The sufficient and necessary condition for a surface wave to propagate is $\bar{k}_y > 1$.

Numerical results are provided to show the effects of $\bar{h}_i$, $\epsilon_{r_i}/\epsilon_{r_{N+1}}$ on the normalized propagation constant $\bar{k}_y$ of both TM_0 mode and TE_1 mode when $N = 1, 2$, respectively, while the relative permeabilities of all layers are assumed to be 1. The following conclusions can be drawn after analyzing the results:

1. The cutoff frequency of TM_0 mode in an arbitrary one-layer or two-layer medium is 0 as long as the permittivity of the $(N + 1)$ th layer is not the highest. This requirement for the permittivity of the $(N + 1)$ th layer ensures that the field intensities are always attenuated along the $+z$ axis.
2. As long as the relative thickness of the layer of the highest permittivity is large enough, $\bar{k}_y$ always approaches the square root of the highest relative permittivity.
3. When $N = 1$, $\bar{k}_y$ is an increasing function of $\bar{h}_1$. When $N = 2$, $\bar{k}_y$ is also an increasing function of $\bar{h}_1$ and $\bar{h}_2$ except for the case when $\epsilon_{r_1} < \epsilon_{r_2}$ and $\bar{h}_2 > j \arctan(j\sqrt{\epsilon_{r_2}} \tan \phi_3 / \tan \phi_2) / (2\pi \tan \phi_2 \sqrt{\epsilon_{r_2}})$, in which $\bar{k}_y$ is a decreasing function of $\bar{h}_1$, but it is still an increasing function of $\bar{h}_2$.
4. In a two-layer model, if the permittivity of each layer is given, the cutoff condition for TE_1 mode is jointly determined by $\bar{h}_1$ and $\bar{h}_2$. When $\epsilon_{r_1} < \epsilon_{r_2}$, the boundary on the $\bar{h}_1 - \bar{h}_2$ plane that separates the propagation area and cutoff area is a convex curve; while it becomes

a concave one when $\epsilon_{r_1} > \epsilon_{r_2}$. The two end points of the boundary curve are located on the $\bar{h}_1$ and $\bar{h}_2$ axes at $1/(4\sqrt{\epsilon_{r_1}})$ and $1/(4\sqrt{\epsilon_{r_2}})$, respectively.

Chapter 4

Open-ended Waveguides

Radiating into a Half Free Space

4.1 Introduction

Open-ended waveguides are widely used in many applications, such as aeronautics, large phased array systems, thermography, and non-destructive measurement, etc. Therefore, they have received extensive attention for decades [56, 62]. Various methods were proposed to analyze the radiation from open-ended waveguides [67–70]. In this chapter, a method is proposed to analyze the radiation from open-ended waveguides to validate its efficiency, which will later be applied to a more complicated model in Chapter 5. The matrix equation for the reflection coefficient matrix at the opening is established using the mode-matching method, which contains double surface integrals. To facilitate the calculation, a two-step approach is proposed: (1) The double surface integral on the aperture is converted into one integral in

Figure 4.1 Side-view of an open-ended waveguide radiating into a half free space.

the spectral domain using the Sommerfeld identity and the Graf's addition theorem; (2) The spectral integral, which contains poles, is then converted to the summation of a short series using the generalized pencil of functions method (GPOF) and the Gaussian quadrature. Through these two steps, the calculation is then proven to be greatly facilitated and highly efficient.

Fig. 4.1 shows the side-view of an open-ended waveguide of arbitrary cross-section. The waveguide is terminated at the opening on an infinitely large flange (or ground plane). The proposed method [104] is to calculate the reflection coefficient matrix at the opening and then the radiation pattern of the open-ended waveguide through the equivalence principle.

4.2 General Formulation

4.2.1 Reflection Coefficient Matrix

In a waveguide of arbitrary cross section, the transverse electric and magnetic fields can be expressed as:

$$\begin{cases} \vec{E}_t(\vec{r})|_{z<0} = \sum_n \left(A_n^+ e^{-j\beta_n z} + A_n^- e^{j\beta_n z} \right) \vec{e}_n(\vec{\rho}) \\ \vec{H}_t(\vec{r})|_{z<0} = \sum_n \left(A_n^+ e^{-j\beta_n z} - A_n^- e^{j\beta_n z} \right) \frac{1}{Z_n} \hat{z} \times \vec{e}_n(\vec{\rho}) \end{cases} \quad (4.1)$$

where $\vec{r}$ is the vector representing the position of an observed point, n is the mode index, A_n^+ and A_n^- represent the magnitudes of the incident and reflected wave of the n th mode, respectively, β_n is the phase constant along the $+z$ direction. Z_n is the mode impedance of the n th mode in the waveguide. $\vec{e}_n(\vec{\rho})$ is the modal function of the n th mode, which depends on the materials of the waveguide and the shape of its cross section. $\vec{\rho}$ is the projection of $\vec{r}$ on the ground plane.

At the opening of the waveguide, where $z = 0^-$, Eq. (4.1) becomes

$$\begin{cases} \vec{E}_t(\vec{r})|_{z=0^-} = \vec{E}_t(\vec{\rho}) = \sum_n \left(A_n^+ + A_n^- \right) \vec{e}_n(\vec{\rho}) \\ \vec{H}_t(\vec{r})|_{z=0^-} = \vec{H}_t(\vec{\rho}) = \sum_n \left(A_n^+ - A_n^- \right) \frac{1}{Z_n} \hat{z} \times \vec{e}_n(\vec{\rho}) \end{cases} \quad (4.2)$$

Now considering the fields in the half free space, I have

$$\vec{H}_t(\vec{r})|_{z>0} = -j\omega \vec{F}(\vec{r}) - \nabla \Phi_m(\vec{r}) \quad (4.3)$$

where $\vec{F}$ is the electric vector potential and Φ_m is the magnetic scalar po-

tential, which are given by

$$\begin{cases} \vec{F}(\vec{r}) = \epsilon \iint_{S_a} \vec{M}_s(\vec{r}') G(\vec{r}|\vec{r}') dS' \\ \Phi_m(\vec{r}) = \frac{j}{\omega\mu\epsilon} \nabla \cdot \vec{F}(\vec{r}) \end{cases} \quad (4.4)$$

and $G(\vec{r}|\vec{r}')$ is the scalar Green's function for the half free space,

$$G(\vec{r}|\vec{r}') = \frac{e^{-jk|\vec{r}-\vec{r}'|}}{2\pi|\vec{r}-\vec{r}'|}. \quad (4.5)$$

Since the equivalent magnetic current on the aperture is given by

$$\vec{M}_s(\vec{\rho}) = \vec{E}_t(\vec{\rho}) \times \hat{z} = \sum_n (A_n^+ + A_n^-) \vec{e}_n(\vec{\rho}) \times \hat{z}. \quad (4.6)$$

the transverse magnetic field $\vec{H}_t(\vec{r})$ in the half free space can be obtained according to Eq. (4.3)-Eq. (4.6),

$$\begin{aligned} \vec{H}_t(\vec{r}) = \sum_n (A_n^+ + A_n^-) \left\{ j\omega\epsilon \iint_{S_a} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{2\pi|\vec{r}-\vec{r}'|} \hat{z} \times \vec{e}_n(\vec{\rho}') dS' \right. \\ \left. + \frac{j}{\omega\mu} \nabla \iint_{S_a} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{2\pi|\vec{r}-\vec{r}'|} \nabla' \cdot \hat{z} \times \vec{e}_n(\vec{\rho}') dS' \right\} \end{aligned} \quad (4.7)$$

Considering the continuity of the transverse magnetic field at the opening of the waveguide, which is $\vec{H}_t|_{z=0^-} = \vec{H}_t|_{z=0^+}$, if both sides of Eq. (4.7) are multiplied with $\hat{z} \times \vec{e}_m(\vec{r})$, and then integrate on the aperture S_a , the

following equation can be obtained:

$$\begin{aligned}
 & \sum_n (A_n^+ - A_n^-) \frac{1}{Z_n} \iint_{S_a} \hat{z} \times \vec{e}_n(\vec{\rho}) \cdot \hat{z} \times \vec{e}_m(\vec{\rho}) dS \\
 &= \frac{j}{\omega\mu} \sum_n (A_n^+ + A_n^-) \left\{ k^2 \iint_{S_a} \iint_{S_a} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{2\pi|\vec{r}-\vec{r}'|} \hat{z} \times \vec{e}_n(\vec{\rho}') \hat{z} \times \vec{e}_m(\vec{\rho}) dS' dS \right. \\
 & \quad \left. + \iint_{S_a} \iint_{S_a} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{2\pi|\vec{r}-\vec{r}'|} [\nabla' \cdot \hat{z} \times \vec{e}_n(\vec{\rho}')] [\nabla \cdot \hat{z} \times \vec{e}_m(\vec{\rho})] dS' dS \right\} \quad (4.8)
 \end{aligned}$$

which can also be expressed by a matrix equation.

$$\mathbf{V}(\mathbf{A}^+ - \mathbf{A}^-) = \mathbf{U}(\mathbf{A}^+ + \mathbf{A}^-) \quad (4.9)$$

where $\mathbf{A}^+ = [A_1^+, A_2^+, \dots, A_n^+ \dots]^T$ and $\mathbf{A}^- = [A_1^-, A_2^-, \dots, A_n^-, \dots]^T$ are two column vectors representing the magnitudes of incident and reflected wave, respectively. The superscript T is the transpose operator. $\mathbf{V} = \{v_{mn}\}$ and $\mathbf{U} = \{u_{mn}\}$ are matrices of the same dimension. Elements in $\mathbf{V}$ and $\mathbf{U}$ are respectively given by

$$v_{mn} = \frac{1}{Z_n} \iint_{S_a} \hat{z} \times \vec{e}_n(\vec{\rho}) \cdot \hat{z} \times \vec{e}_m(\vec{\rho}) dS = \frac{1}{Z_n} \delta_{mn} \quad (4.10)$$

and

$$u_{mn} = u_{mn}^I - u_{mn}^{II} \quad (4.11)$$

where

$$u_{mn}^I = \frac{jk^2}{2\pi\omega\mu} \iint_{S_a} \iint_{S_a} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \hat{z} \times \vec{e}_n(\vec{\rho}') \hat{z} \times \vec{e}_m(\vec{\rho}) dS' dS \quad (4.12)$$

$$u_{mn}^{\text{II}} = \frac{-j}{2\pi\omega\mu} \iint_{S_a} \iint_{S_a} \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \left[\nabla' \cdot \hat{z} \times \vec{e}_n(\vec{\rho}') \right] \left[\nabla \cdot \hat{z} \times \vec{e}_m(\vec{\rho}) \right] dS' dS \quad (4.13)$$

Since the Sommerfeld identity is given by

$$\frac{e^{-jkR}}{R} = -j \int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho d) e^{-jk_z|z|} \quad (4.14)$$

where $R^2 = |\vec{r} - \vec{r}'|^2 = d^2 + z^2$. When $z = 0$, I have $R = d = |\vec{\rho} - \vec{\rho}'|$ and

$$\frac{e^{-jkd}}{d} = -j \int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho d). \quad (4.15)$$

Substituting Eq. (4.15) into Eq. (4.12) and Eq. (4.13), respectively, I obtain

$$u_{mn}^{\text{I}} = \frac{k^2}{2\pi\omega\mu} \iint_{S_a} \iint_{S_a} \int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho |\vec{\rho} - \vec{\rho}'|) \hat{z} \times \vec{e}_n(\vec{\rho}') \cdot \hat{z} \times \vec{e}_m(\vec{\rho}) dS' dS \quad (4.16)$$

and

$$u_{mn}^{\text{II}} = \frac{1}{2\pi\omega\mu} \iint_{S_a} \iint_{S_a} \int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho |\vec{\rho} - \vec{\rho}'|) \left[\nabla' \cdot \hat{z} \times \vec{e}_n(\vec{\rho}') \right] \left[\nabla \cdot \hat{z} \times \vec{e}_m(\vec{\rho}) \right] dS' dS \quad (4.17)$$

Using the Graf's addition theorem [103],

$$J_0(k_\rho |\vec{\rho} - \vec{\rho}'|) = \sum_{s=-\infty}^{\infty} J_s(k_\rho \rho) J_s(k_\rho \rho') \cos s(\phi - \phi') \quad (4.18)$$

where ϕ and ϕ' are the angles of vectors $\vec{\rho}$ and $\vec{\rho}'$, respectively. I can then

reach the following results.

$$u_{mn}^I = \frac{k^2}{2\pi\omega\mu} \int_0^\infty dk_\rho \frac{k_\rho}{k_{zs=-\infty}} \sum_{s=-\infty}^\infty \iint_{S_a} \iint_{S_a} J_s(k_\rho\rho) J_s(k_\rho\rho') \cos s(\phi - \phi') \hat{z} \times \vec{e}_n(\vec{\rho}') \cdot \hat{z} \times \vec{e}_m(\vec{\rho}) dS' dS \quad (4.19)$$

and

$$u_{mn}^II = \frac{1}{2\pi\omega\mu} \int_0^\infty dk_\rho \frac{k_\rho}{k_z} \sum_{s=-\infty}^\infty \iint_{S_a} \iint_{S_a} J_s(k_\rho\rho) J_s(k_\rho\rho') \cos s(\phi - \phi') [\nabla' \cdot \hat{z} \times \vec{e}_n(\vec{\rho}')] [\nabla \cdot \hat{z} \times \vec{e}_m(\vec{\rho})] dS' dS \quad (4.20)$$

If the functions of the all modes in the waveguide are known, the two double integrals over the opening surface S_a can be worked out. Finally, u_{mn} can be calculated numerically through the one-dimensional integral in the spectral domain. Since the two double integrals are replaced by one-dimensional integral, the computation can be significantly facilitated.

From Eq. (4.9), the relationship between $\mathbf{A}^+$ and $\mathbf{A}^-$ can be shown as follows

$$\mathbf{A}^- = (\mathbf{U} + \mathbf{V})^{-1} (\mathbf{V} - \mathbf{U}) \mathbf{A}^+ \quad (4.21)$$

and the reflection coefficient matrix $\mathbf{\Gamma}$ is defined as

$$\mathbf{\Gamma} = (\mathbf{U} + \mathbf{V})^{-1} (\mathbf{V} - \mathbf{U}) \quad (4.22)$$

Consequently, with $\mathbf{A}^-$ being known for a given $\mathbf{A}^+$, the radiation pattern can be calculated. The validity of this method is to be verified in the next section by considering an open-ended coaxial line and an open-ended rect-

angular waveguide.

4.2.2 Calculating the Radiation Pattern

It is noticed that the radiation pattern can be derived directly through Eq. (4.7), but evaluating the complicated integral for all spatial point represented by $\vec{r}$ is not straightforward. Since in a half free space, the far-field radiation pattern can be derived from the spectrum of the field distribution [105], the acquisition process can be greatly simplified and is introduced in this subsection.

The formulas for calculating the far-field radiation pattern of aperture antenna are given by [106],

$$\begin{cases} \vec{E}_r \approx 0 \\ \vec{E}_\theta \approx -\frac{jke^{-jkr}}{4\pi r}(L_\phi + \eta N_\theta) \\ \vec{E}_\phi \approx +\frac{jke^{-jkr}}{4\pi r}(L_\theta - \eta N_\phi) \end{cases} \quad (4.23)$$

where

$$\begin{aligned} N_\theta &= \iint_S [J_x \cos \theta \cos \phi + J_y \cos \theta \sin \phi - J_z \sin \theta] e^{jkr' \cos \psi} ds' \\ N_\phi &= \iint_S [-J_x \sin \phi + J_y \cos \phi] e^{jkr' \cos \psi} ds' \\ L_\theta &= \iint_S [M_x \cos \theta \cos \phi + M_y \cos \theta \sin \phi - M_z \sin \theta] e^{jkr' \cos \psi} ds' \\ L_\phi &= \iint_S [-M_x \sin \phi + J_y \cos \phi] e^{jkr' \cos \psi} ds' \end{aligned}$$

and the electric and magnetic currents $\mathbf{J}_s$ and $\mathbf{M}_s$ have the following equiv-

alent relationship with the transverse fields

$$\begin{cases} \vec{M}_s = -\hat{z} \times \vec{E}_t|_{z=0} \\ \vec{J}_s = \hat{z} \times \vec{H}_t|_{z=0} \end{cases}$$

In Eq. (4.23), r is the distance from the point of observation to the origin point; k is the free space wavenumber and $\eta = 120\pi\Omega$.

Considering the definition of double Fourier transform [107]

$$\begin{aligned} \tilde{E}_t(k_x, k_y) &= \iint_S \vec{E}_t(x, y) e^{-jk_x x} e^{-jk_y y} dx dy \\ \tilde{H}_t(k_x, k_y) &= \iint_S \vec{H}_t(x, y) e^{-jk_x x} e^{-jk_y y} dx dy \end{aligned}$$

and the coordinate system transformation

$$e^{jkr' \cos \psi} ds' = e^{jk_x x'} e^{jk_y y'} dx dy$$

the nonzero components in Eq. (4.23) become

$$\begin{aligned} \vec{E}_\theta &= \frac{jke^{-jkr}}{4\pi r} (\cos \phi \tilde{E}_x^- + \sin \phi \tilde{E}_y^- + \eta \cos \theta \cos \phi \tilde{H}_y^- - \eta \cos \theta \sin \phi \tilde{H}_x^-)|_{z=0} \\ \vec{E}_\phi &= \frac{jke^{-jkr}}{4\pi r} (-\cos \theta \sin \phi \tilde{E}_x^- + \cos \theta \cos \phi \tilde{E}_y^- - \eta \sin \phi \tilde{H}_y^- - \eta \cos \phi \tilde{H}_x^-)|_{z=0} \end{aligned} \quad (4.24)$$

where

$$\begin{cases} k_x = k \sin \theta \cos \phi \\ k_y = k \sin \theta \sin \phi \\ k_z = k \cos \theta \end{cases} \quad (4.25)$$

and

$$\begin{cases} \tilde{E}_x^-(k_x, k_y) = \tilde{E}_x(-k_x, -k_y) \\ \tilde{E}_y^-(k_x, k_y) = \tilde{E}_y(-k_x, -k_y) \\ \tilde{H}_x^-(k_x, k_y) = \tilde{H}_x(-k_x, -k_y) \\ \tilde{H}_y^-(k_x, k_y) = \tilde{H}_y(-k_x, -k_y) \end{cases}$$

Moreover, it is not difficult to derive the following matrix equation from Maxwell's equations,

$$\begin{bmatrix} \tilde{H}_y \\ -\tilde{H}_x \end{bmatrix} = \frac{1}{\omega\mu k_z} \begin{bmatrix} k^2 - k_y^2 & k_x k_y \\ k_x k_y & k^2 - k_x^2 \end{bmatrix} \begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \end{bmatrix} \quad (4.26)$$

Substituting Eq. (4.25) in Eq. (4.26), I have

$$\begin{bmatrix} \tilde{H}_{y3}^- \\ -\tilde{H}_{x3}^- \end{bmatrix} = \frac{1}{\eta \cos \theta} \begin{bmatrix} 1 - \sin^2 \theta \sin^2 \phi & \sin^2 \theta \cos \phi \sin \phi \\ \sin^2 \theta \cos \phi \sin \phi & 1 - \sin^2 \theta \cos^2 \phi \end{bmatrix} \begin{bmatrix} \tilde{E}_{x3}^- \\ \tilde{E}_{y3}^- \end{bmatrix} \quad (4.27)$$

Substituting Eq. (4.27) into Eq. (4.24), the formula for calculating far-field radiation pattern in term of spectrum of the transverse electric field in the half free space is found to be

$$\begin{bmatrix} \vec{E}_\theta \\ \vec{E}_\phi \end{bmatrix} = \frac{jke^{-jkr}}{2\pi r} \begin{bmatrix} \cos \phi & \sin \phi \\ -\cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \begin{bmatrix} \tilde{E}_x^- \\ \tilde{E}_y^- \end{bmatrix} \Big|_{z=0} \quad (4.28)$$

Since the transverse electric field vanishes except at the waveguide opening, the spectrum of the transverse electric field at $z = 0$ is equal to $\tilde{\mathbf{E}}_w|_{z=0}$, which is a superposition of the spectra of all waveguide modes at the opening. According to Eq. (4.21) and Eq. (4.22), when $\mathbf{A}^+$ and $\mathbf{\Gamma}$ are known, $\mathbf{A}^-$ can

Figure 4.2 3D-view of an open-ended coaxial line radiating into a half space.

be calculated by Eq. (4.21). Therefore, $\tilde{\mathbf{E}}_w|_{z=0}$ is known and the radiation pattern can be obtained using Eq. (4.28).

4.3 Open-ended Coaxial Line

4.3.1 Formulation

Fig. 4.2 shows the 3-dimensional view of an open-ended coaxial line [108], which is opened on an infinitely large and perfectly conductive flange. The inner and outer radii are a and b respectively. Usually the incident wave is the dominant TEM mode, then the reflected wave only consists of TEM and TM_{0n} modes with the symmetry of the open-ended coaxial line model. The opening area between $\rho = a$ and $\rho = b$ is labeled by S_a .

In the coaxial line, the mode functions in the cylindrical coordinate sys-

tem are given by [109]

$$\vec{e}_n(\vec{\rho}) = \begin{cases} \frac{1}{\sqrt{2\pi \ln b/a}} \frac{\hat{\rho}}{\rho} & \text{TEM}(n=0) \\ N_{0n}^e \frac{\chi_{0n}}{a} Z'_0\left(\frac{\chi_{0n}}{a}\rho\right) \hat{\rho} & \text{TM}(n=1,2,\dots) \end{cases} \quad (4.29)$$

where N_{0n}^e is the normalized coefficient, $\hat{\rho}$ is the unit vector in the radial direction.

$$Z'_0\left(\frac{\chi_{0n}}{a}\rho\right) = -J_1\left(\frac{\chi_{0n}}{a}\rho\right) Y_0(\chi_{0n}) + Y_1\left(\frac{\chi_{0n}}{a}\rho\right) J_0(\chi_{0n})$$

χ_{0n} is the n th nonvanishing root of the Bessel-Neumann combination $Z_0(b\chi_{0n}/a)$ and J_n and Y_n are Bessel functions of the first and second kinds, respectively.

Since $\vec{e}_n$ in Eq. (4.29) contains only component in the $\hat{\rho}$ direction and it does not vary with ϕ , I have

$$\nabla \cdot \hat{z} \times \vec{e}_n(\vec{\rho}) = 0 \quad (n = 0, 1, 2, \dots) \quad (4.30)$$

for TEM and TM_{0n} ($n = 1, 2, \dots$) modes. Therefore, $u_{mn}^{II} = 0$ and $u_{mn} = u_{mn}^I$.

It is noticed that the mode functions $\vec{e}_m$ and $\vec{e}_n$ are independent of ϕ and ϕ' , respectively, Eq. (4.19) becomes

$$u_{mn}^I = \frac{k^2}{2\pi\omega\mu} \int_0^\infty dk_\rho \frac{k_\rho}{k_z} \sum_{s=-\infty}^\infty \int_a^b J_s(k_\rho\rho) e_m(\rho) \rho d\rho \int_a^b J_s(k_\rho\rho') e_n(\rho') \rho' d\rho' \int_0^{2\pi} \int_0^{2\pi} \cos s(\phi - \phi') \cos(\phi - \phi') d\phi d\phi' \quad (4.31)$$

The double integral of ϕ and ϕ' in Eq. (4.31) is nonzero only when $s = \pm 1$,

therefore, I obtain

$$u_{mn} = \frac{2\pi k^2}{\omega\mu} \int_0^\infty dk_\rho \frac{k_\rho}{k_z} X_n(k_\rho) X_m(k_\rho) \quad (4.32)$$

where $X_i(k_\rho)$ is defined as

$$X_i(k_\rho) = \int_a^b J_1(k_\rho \rho) e_i(\rho) \rho d\rho \quad (4.33)$$

and the scalar function $e_n(\rho)$ is the magnitude of the vector function $\vec{e}_n(\rho)$. Substituting Eq. (4.29) into Eq. (4.33), one can readily see that $X_n(k_\rho)$ can be analytically worked out.

$$X_i(\rho) = \begin{cases} \frac{1}{\sqrt{2\pi \ln b/a}} \frac{J_0(k_\rho a) - J_0(k_\rho b)}{k_\rho} & TEM \\ \frac{2N_{0n}^e k_\rho}{\pi [(\chi_{0n}/a)^2 - k_\rho^2]} \left[J_0(k_\rho a) - \frac{J_0(\chi_{0n})}{J_0(\chi_{0n} b/a)} J_0(k_\rho b) \right] & TM_{0n} \end{cases} \quad (4.34)$$

$X_m(k_\rho) X_n(k_\rho)$ in Eq. (4.32) is then sampled and those sample points are used by the generalized pencil of functions method, which is described in Appendix B, to estimate the coefficient of an exponential series and the function product has now been approximated by

$$X_m(k_\rho) X_n(k_\rho) \approx \sum_{q=1}^Q A_{mn,q} e^{-B_{mn,q} k_\rho} \quad (4.35)$$

where $A_{mn,q}$ and $B_{mn,q}$ are complex coefficients. The integrals in Eq. (4.32) can therefore be expressed explicitly by using the known definite integral

formulas [110],

$$\begin{aligned}
 u_{mn} &= \frac{2\pi k^2}{\omega\mu} \sum_q A_{mn,q} \left[\int_0^k \frac{k_\rho e^{-B_{mn,q}k_\rho}}{\sqrt{k^2 - k_\rho^2}} dk_\rho + j \int_k^\infty \frac{k_\rho e^{-B_{mn,q}k_\rho}}{\sqrt{k_\rho^2 - k^2}} dk_\rho \right] \\
 &= \frac{2\pi k^2}{\omega\mu} \sum_q A_{mn,q} \left\{ \frac{\pi k}{2} [L_1(B_{mn,q}k) - I_1(B_{mn,q}k)] \right. \\
 &\quad \left. + k + jkK_1(B_{mn,q}k) \right\}
 \end{aligned} \tag{4.36}$$

where $L_1(\cdot)$ is the modified Struve function, $I_1(\cdot)$ and $K_1(\cdot)$ are modified Bessel functions of imaginary argument. Therefore, each element in matrix $\mathbf{U}$ is expressed analytically in closed-form and the reflection coefficient matrix can finally be calculated efficiently using Eq. (4.10), Eq. (4.22) and Eq. (4.36).

4.3.2 Results and Discussions

In this section, an open-ended coaxial line identical to [62] is considered to verify the validity of my method. The inner and outer radii of the coaxial line are 1.4364mm and 4.725mm, respectively. The coaxial line and the half open space are assumed to be filled with a lossless dielectric material of relative permittivity of 2.05. The metallic flange is assumed to be infinitely large and perfectly conducting. For this example, $N = 3$ is sufficient to obtain convergent results.

Fig. 4.3 to 4.5 show the curve fitting performance using the GPOF method when calculating u_{00} , u_{01} and u_{11} at 1 GHz, respectively. For uniformity, 64 sampling points are employed and the number of the exponential terms $Q = 27$ in the computation of all elements in matrix $\mathbf{U}$. It is seen from Figs 4.3, 4.4 and 4.5 that the exponential series can closely fit of the

Figure 4.3 Curve fitting results for u_{00} .

Figure 4.4 Curve fitting results for u_{01} .

Figure 4.5 Curve fitting results for u_{11} .

original curves though the latter varies a lot. Therefore, the good curve fitting performance using the generalized pencil of functions method has been demonstrated.

Fig. 4.6 shows the reflection coefficient of the open-ended coaxial line for the TEM mode. It is seen that the results obtained using my method are in very good agreement with that obtained by Ansoft's High Frequency Structure Simulator (HFSS). It should be mentioned here that it took 4 seconds to compute the reflection coefficient at one frequency while HFSS used 130 seconds. The comparison is carried out using a computer equipped with Intel Core2 6300 CPU and 1G memory. Our calculated radiation pattern is shown in Fig. 4.7, which agrees very well with that by HFSS perfectly.

(a) Magnitude

(b) Phase

Figure 4.6 Reflection coefficient of an open-ended coaxial line radiating into the half space.

Figure 4.7 Radiation pattern of an open-ended coaxial line ($f = 1\text{GHz}$).

4.4 Open-ended Rectangular Waveguide

4.4.1 Formulation

In this section, the efficient method proposed in Section 4.3 is modified to calculate the reflection coefficient matrix of a rectangular waveguide [111]. The 3D-view of an open-ended rectangular waveguide with infinite flange is shown in Fig.4.8. The length and width of the waveguide cross section are a and b , respectively. It is assumed that the incident wave is of TE_{10} mode and radiates into the half space through the opening. Since geometrical discontinuity exists at the interface between the waveguide and the half space, reflection could come out at the opening of the waveguide. The relationship between the incident wave and the reflected wave is represented by the reflection coefficient matrix $\mathbf{\Gamma}$, which is defined in Eq. (4.22).

Since a wave propagating in a rectangular waveguide can be identified

Figure 4.8 Geometry of a rectangular waveguide radiating into a half free space

as TE_{mn} or TM_{mn} mode, each valid combination of (m, n) for either TE modes or TM modes should be mapped into a unique index $l (l = 1, \dots, N)$ in $\mathbf{A}^+$ and $\mathbf{A}^-$, which therefore corresponds to a column or row of Γ . The mapping rules are as follows:

1. For a given frequency and dimension of waveguide's cross section, the cutoff wavenumbers $k_{c,mn} = \sqrt{(m\pi/a)^2 + (n\pi/b)^2}$ of all modes in the rectangular waveguide are sorted out incrementally, which means the mode with smaller $k_{c,mn}$ is mapped to smaller l ;
2. The mode with smaller m will be mapped to smaller l if two modes have the same $k_{c,mn}$;
3. For a given (m, n) , TE_{mn} mode will be mapped to smaller l if both TE_{mn} and TM_{mn} exist.

Through this mapping process, all the modes originally indexed by (m, n) as well as mode type are uniquely re-indexed by l .

The transverse electric field at the opening of the rectangular waveguide,

where $z = 0$, can be represented as:

$$\vec{E}_t(\vec{r})|_{z=0} = \sum_{l=0}^N (A_l^+ + A_l^-) \vec{e}_l(\vec{\rho}) \quad (4.37)$$

where N is the number of considered modes, A_l^+ and A_l^- represent the amplitude coefficients of the incident and reflected waves of the l th mode, respectively, $\vec{r}$ is the position of a field point in the vector form and $\vec{\rho}$ is its projection on $x - y$ plane,

$$\vec{e}_l(\vec{\rho}) = \begin{cases} N_{mn} \left[\hat{x} \cdot \frac{n\pi}{b} \mathbb{X} - \hat{y} \cdot \frac{m\pi}{a} \mathbb{Y} \right] & \text{TE mode} \\ N_{mn} \left[\hat{x} \cdot \frac{m\pi}{a} \mathbb{X} + \hat{y} \cdot \frac{n\pi}{b} \mathbb{Y} \right] & \text{TM mode} \end{cases} \quad (4.38)$$

where

$$\begin{cases} \mathbb{X} = \cos \frac{m\pi(x+a/2)}{a} \sin \frac{n\pi(y+b/2)}{b} \\ \mathbb{Y} = \sin \frac{m\pi(x+a/2)}{a} \cos \frac{n\pi(y+b/2)}{b} \end{cases} \quad -\frac{a}{2} < x < \frac{a}{2} \text{ and } -\frac{b}{2} < y < \frac{b}{2}$$

N_{mn} is the normalization coefficient, which is given by

$$N_{mn} = \sqrt{\frac{(2 - \delta_{m0})(2 - \delta_{n0})}{ab}} \frac{1}{k_{cl}}$$

k_{cl} is the cutoff wavenumber of the l th mode, the function δ_{mn} is defined as

$$\delta_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

It can be derived from Formula 9.1.21 in [103] that

$$J_s(k_\rho \rho) e^{js\phi} = \frac{j^s}{2\pi} \int_0^{2\pi} e^{js\theta} e^{-jk_\rho(x \cos \theta + y \sin \theta)} d\theta \quad (4.39)$$

Using Eq. (4.39), one can get

$$\begin{aligned} & \sum_{s=-\infty}^{+\infty} J_s(k_\rho \rho) J_s(k_\rho \rho') \cos s(\phi - \phi') \\ &= \frac{1}{2\pi} \int_0^{2\pi} e^{-jk_\rho(x \cos \theta + y \sin \theta)} e^{jk_\rho(x' \cos \theta + y' \sin \theta)} d\theta \end{aligned} \quad (4.40)$$

The detailed derivation processes of Eq. (4.39) and Eq. (4.40) are given in Appendix C. Substituting Eq. (4.38) and Eq. (4.40) into Eq. (4.19) and Eq. (4.20), respectively and after some manipulations I obtain the following equations,

$$u_{ll'} = \frac{k^2 a^2 b^2 N_l N_{l'}}{(2\pi)^2 \omega \mu} \int_0^\infty \int_0^{2\pi} \frac{k_\rho}{k_z} (\Phi_1 \Psi_1 + \Phi_2 \Psi_2 - \Phi_3 \Psi_3)_{ll'} dk_\rho d\theta \quad (4.41)$$

where

$$\begin{cases} \Psi_1 = \mathbb{A}(m, k_x a) \mathbb{B}(n, k_y b) \mathbb{A}(m', -k_x a) \mathbb{B}(n', -k_y b) \\ \Psi_2 = \mathbb{B}(m, k_x a) \mathbb{A}(n, k_y b) \mathbb{B}(m', -k_x a) \mathbb{A}(n', -k_y b) \\ \Psi_3 = \mathbb{B}(m, k_x a) \mathbb{B}(n, k_y b) \mathbb{B}(m', -k_x a) \mathbb{B}(n', -k_y b) \end{cases}$$

and

$$\begin{cases} \mathbb{A}(m, \alpha) = \frac{1}{2j} \left[j^m \text{sinc} \frac{\alpha - m\pi}{2\pi} - j^{-m} \text{sinc} \frac{\alpha + m\pi}{2\pi} \right] \\ \mathbb{B}(m, \alpha) = \frac{1}{2} \left[j^m \text{sinc} \frac{\alpha - m\pi}{2\pi} + j^{-m} \text{sinc} \frac{\alpha + m\pi}{2\pi} \right] \end{cases} \quad (4.42)$$

The parameters Φ_1 , Φ_2 and Φ_3 are listed in Table 4.1, where $TE - TM$ refers to the situation when the l th mode is a TE mode and the l' th mode is a TM mode.

Due to the symmetry of the integrand with respect to θ , the integration

TABLE 4.1 Table of parameters Φ_1 , Φ_2 and Φ_3

	Φ_1	Φ_2	Φ_3
$TE - TE$	$\frac{\pi^2 m m'}{a^2}$	$\frac{\pi^2 n n'}{b^2}$	$\frac{k_{cl}^2 k_{cl'}^2}{k^2}$
$TM - TM$	$\frac{\pi^2 n n'}{b^2}$	$\frac{\pi^2 m m'}{a^2}$	0
$TE - TM$	$\frac{\pi^2 m n'}{ab}$	$-\frac{\pi^2 n m'}{ab}$	0
$TM - TE$	$\frac{\pi^2 n m'}{ab}$	$-\frac{\pi^2 m n'}{ab}$	0

range in Eq. (4.41) to $(0, \pi/2)$ can be reduced and $u_{ll'}$ becomes

$$u_{ll'} = \frac{k^2 a^2 b^2 N_l N_{l'}}{\pi^2 \omega \mu} \int_0^{\pi/2} F_{ll'}(\theta) d\theta \quad (4.43)$$

where

$$F_{ll'}(\theta) = \int_0^{+\infty} \frac{k_\rho}{k_z} \Theta_{ll'}(\theta, k_\rho) dk_\rho \quad (4.44)$$

and

$$\Theta_{ll'}(\theta, k_\rho) = (\Phi_1 \Psi_1 + \Phi_2 \Psi_2 - \Phi_3 \Psi_3)_{ll'} \quad (4.45)$$

For a given θ , applying the GPOF method to approximate the function $\Theta_{ll'}$ with an exponential series, I have

$$\Theta_{ll'}(\theta, k_\rho) \approx \sum_{q=1}^Q C_{q,ll'} e^{D_{q,ll'} k_\rho} \quad (4.46)$$

where $C_{q,ll'}$ and $D_{q,ll'}$ are complex coefficients. Since $\Theta_{ll'}$ is decaying as k_ρ approaches $+\infty$, the real parts of all $D_{q,ll'}$ must be negative so that the exponential series can follow the decreasing. Then, substituting Eq. (4.46)

into Eq. (4.44), the following equation is obtained,

$$u_{ll'} \approx \frac{a^2 b^2 N_l N_{l'}}{\pi^2 \omega \mu} \int_0^{\frac{\pi}{2}} \int_0^{+\infty} \frac{k_\rho}{k_z} \sum_{q=1}^Q C_{q,ll'} e^{D_{q,ll'} k_\rho} dk_\rho d\theta \quad (4.47)$$

The integration of k_ρ in Eq. (4.47) can be worked out analytically, which is similar to Eq. (4.36) except that the result here is a function of θ .

$$\begin{aligned} & \int_0^{+\infty} \frac{k_\rho}{k_z} \sum_{q=1}^Q C_{q,ll'} e^{D_{q,ll'} k_\rho} dk_\rho \\ &= \sum_{q=1}^Q C_{q,ll'} \left\{ \frac{\pi k}{2} [\mathbf{L}_1(k D_{q,ll'}) - I_1(k D_{q,ll'})] + k + j k K_1(k D_{q,ll'}) \right\} \\ &= \hat{F}_{ll'}(\theta) \end{aligned} \quad (4.48)$$

where $\mathbf{L}_1(\cdot)$ is the modified Struve function of order 1, $I_1(\cdot)$ and $K_1(\cdot)$ are the modified Bessel functions. It is noted that the pole ($k_z = 0$) in the integral of Eq. (4.44) is also eliminated analytically through Eq. (4.48). The integral of θ can be obtained using Gaussian Quadrature.

$$\int_0^{\frac{\pi}{2}} \hat{F}_{ll'}(\theta) d\theta \approx \frac{\pi}{4} \sum_{t=1}^T w_t \hat{F}_{ll'}(\theta_t) \quad (4.49)$$

Finally, the $\mathbf{U}$ matrix elements can be evaluated using the following short series.

$$u_{ll'} \approx \frac{a^2 b^2 N_l N_{l'}}{\pi^2 \omega \mu} \frac{\pi}{4} \sum_{t=1}^T w_t \hat{F}_{ll'}(\theta_t) \quad (4.50)$$

The calculation of $u_{ll'}$ has now been converted from the double surface integrals with poles to the summation of a short series and the computation efficiency is greatly improved. After $\mathbf{U}$ is obtained, the reflection coefficient matrix $\mathbf{\Gamma}$ can be derived through Eq. (4.22).

4.4.2 Results and Discussions

An open-ended WR-90 waveguide model with infinite flange is analyzed in this section to demonstrate the efficiency of my method. The dimensions a and b of the waveguide are $22.86mm$ and $10.16mm$, respectively. The number of modes considered in the rectangular waveguide is set to be $N = 30$, which is large enough to obtain convergent results. The number of exponential terms is chosen to be $Q = 20$, a choice after a trade-off between computation time and accuracy.

In order to demonstrate the accuracy of the GPOF method, $\Theta_{11}(\theta, k_\rho)$ is plotted against its approximation by the summation of the exponential series in Fig.4.9-4.13. It is seen that Θ_{11} and its corresponding exponential series agree very well with each other at the abscissas (θ_{1-5}) of the 5-point Gaussian quadrature, which are listed in Table 4.2. Fig.4.14 shows the curves of $F_{11}(\theta)$ and the values of $\hat{F}(\theta_i)$ on these abscissas. The very good agreement between $F_{11}(\theta)$ and $\hat{F}_{11}(\theta_i)$ indicates that the results obtained by Eq. (4.50) are very good approximations of Eq. (4.43).

The reflection coefficient results obtained with my proposed method are compared with those obtained by HFSS. Fig.4.15 shows the magnitude and phase of the reflection coefficient for the dominant TE_{10} mode. In this figure, it can be seen that the proposed method is very accurate in calculating the reflection coefficient of an open-ended waveguide. Furthermore, a comparison of CPU time is also carried out using a computer with Intel Core2 6300 CPU and 1G memory. Due to the accelerating technique and removal of poles, the computation time of the proposed method is 5 seconds, while the time spent by using HFSS is 150 seconds. It can therefore be concluded that the method proposed in this chapter is very efficient in calculating the

Figure 4.9 Curve fitting results of Θ_{11} at $\theta = \theta_1$.

Figure 4.10 Curve fitting results of Θ_{11} at $\theta = \theta_2$.

 Figure 4.11 Curve fitting results of Θ_{11} at $\theta = \theta_3$.

 Figure 4.12 Curve fitting results of Θ_{11} at $\theta = \theta_4$.

 Figure 4.13 Curve fitting results of Θ_{11} at $\theta = \theta_5$.

Figure 4.14 Comparison of values on abscissas of the 5-point Gaussian quadrature

TABLE 4.2 Table of θ values adopted for 5-point Gaussian quadrature.

θ_1	4.2219°
θ_2	20.768°
θ_3	45.000°
θ_4	69.231°
θ_5	85.778°

reflection coefficients of open-ended rectangular waveguides.

Fig. 4.16 shows the radiation pattern of the open-ended rectangular waveguide at 10GHz, where 6 modes are considered. The directivity of the open-ended WR-90 waveguide is 6.5dBi.

4.5 Conclusions

Radiation from open-ended waveguides has been studied in this chapter. Field equations have been obtained using the mode-matching method. Sommerfeld identity has then been employed to convert the multiple integrals on the opening of the waveguides into the integral in the spectral domain. For an open-ended coaxial line, the Sommerfeld integrand could be approximated by a summation of a short series using the GPOF method; for an open-ended rectangular waveguide, Gaussian quadrature has also been proposed to evaluate the integral in the azimuth direction. The results have shown that my method is very accurate and efficient in analyzing the radiation from open-ended coaxial lines and open-ended rectangular waveguides into half space.

(a) Magnitude

(b) Phase

Figure 4.15 Reflection coefficient of the TE_{10} mode in the open-ended rectangular waveguide

Figure 4.16 Radiation pattern of an open-ended rectangular waveguide ($f = 10\text{GHz}$).

Chapter 5

Open-ended Waveguides

Radiating through a Two-layer Superstrate

5.1 Introduction

In this chapter, a full-wave analysis of the radiation from open-ended waveguides through a two-layer superstrate is presented [112]. Firstly, the spectral dyadic Green's function in a multilayer superstrate is formulated so that it can be derived through cascaded matrices, each of which represents the effect of one layer of the superstrate independently. Therefore, it can be modified conveniently for arbitrary superstrate configurations. Next, the reflection coefficient matrix $\mathbf{\Gamma}$ at the end of the waveguide can be derived by solving Sommerfeld integrals, in which the generalized pencil of functions (GPOF) method [94] and Gaussian quadrature are employed, which were adopted in Chapter 4 to accelerate the numerical computation of the integrals. The accuracy and efficiency of the proposed method are validated by an analysis of

a rectangular waveguide radiating through the two-layer superstrate. The convergence behavior of this efficient method is also investigated. During the analysis, it is found that the directivity of the open-ended rectangular waveguide can be greatly improved if a specific superstrate is adopted, therefore, both the Green's function of the superstrate and the excitation are studied to uncover the principle of high directive radiation of this model. A coaxial line model is also considered to demonstrate the proposed analysis method in solving omnidirectional problems.

The rest of this chapter is organized as follows. In Section 5.2, the spectral dyadic Green's function for a multilayer superstrate is formulated and the explicit expression for a two-layer superstrate is given. In Section 5.3, the two-layer superstrate excited by a rectangular waveguide is analyzed and the Green's function and the spectral functions of the transverse fields in the waveguide are utilized together to solve the reflection coefficient matrix through Sommerfeld integrals, which is jointly evaluated by the GPOF method and Gaussian quadrature. Validation of the proposed analysis method is presented in Section 5.4, in which the reflection coefficient and the radiation pattern are examined. The accuracy, efficiency and convergence of the numerical evaluation technique are also demonstrated in this section. A study on the principle of high directivity is provided in Section 5.5, in which the effects of the superstrate and the excitation are investigated, respectively. After that, the radiation from an open-ended coaxial line through the two-layer superstrate is analyzed in Section 5.6 to demonstrate the proposed method applied to omnidirectional situation. Conclusions are given in Section 5.7.

Figure 5.1 Side view of an open-ended waveguide radiating into the half space through a multi-layer superstrate.

5.2 Spectral Dyadic Green's Function

5.2.1 Multilayer Superstrate

The model of an open-ended waveguide radiating through a multilayer superstrate is illustrated in Fig. 5.1. The superstrate consists of N dielectric layers which are infinitely large in horizontal direction. The permittivity, permeability and thickness of the i th layer are ϵ_i , μ_i and h_i , respectively. The interface between the i th layer and the $(i+1)$ th layer is at $z = d_i$. Above the superstrate is the half free space which is indexed with $N+1$ for convenience. A waveguide opening on an infinitely large ground plane is mounted at the bottom of the superstrate. In both the waveguide and superstrate, the fields are superpositions of incident wave's fields (propagating in $+z$ direction) and reflected wave's (propagating in $-z$ direction), while in the half free space, no wave travels downwards. In the N -layer superstrate, the spectra

of transverse electric and magnetic fields in the i th layer can be given by

$$\begin{bmatrix} \tilde{\mathbf{E}}_i \\ \tilde{\mathbf{H}}_i \end{bmatrix} = j \begin{bmatrix} k_{z_i} \mathbf{I} & k_{z_i} \mathbf{I} \\ \mathbf{B}_i & -\mathbf{B}_i \\ \omega\mu_i & -\omega\mu_i \end{bmatrix} \begin{bmatrix} \mathbf{I}e^{-jk_{z_i}(z-d_{i-1})} & 0 \\ 0 & \mathbf{I}e^{jk_{z_i}(z-d_{i-1})} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{I}}_i \\ \tilde{\mathbf{R}}_i \end{bmatrix} \quad (5.1)$$

where

$$\begin{aligned} \tilde{\mathbf{E}}_i &= \begin{bmatrix} \tilde{E}_{x_i} \\ \tilde{E}_{y_i} \end{bmatrix}, \tilde{\mathbf{H}}_i = \begin{bmatrix} \tilde{H}_{y_i} \\ -\tilde{H}_{x_i} \end{bmatrix}, \tilde{\mathbf{I}}_i = \begin{bmatrix} \tilde{I}_{x_i} \\ \tilde{I}_{y_i} \end{bmatrix}, \tilde{\mathbf{R}}_i = \begin{bmatrix} \tilde{R}_{x_i} \\ \tilde{R}_{y_i} \end{bmatrix}, \\ \mathbf{B}_i &= \begin{bmatrix} k_i^2 - k_y^2 & k_x k_y \\ k_x k_y & k_i^2 - k_x^2 \end{bmatrix} \text{ and } \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$\tilde{I}_{x_i}, \tilde{I}_{y_i}$ are the spectral functions of the x and y components of the incident wave, respectively. $\tilde{R}_{x_i}$ and $\tilde{R}_{y_i}$ are the corresponding functions for the reflected wave.

Considering the continuity of transverse electric and magnetic fields at $z = d_i$ ($i = 1, 2, \dots, N$) and $\tilde{\mathbf{R}}_{N+1} = 0$, the fields' relationship between the first layer and the half free space is given by

$$\begin{bmatrix} \tilde{\mathbf{E}}_1|_{z=0} \\ \tilde{\mathbf{H}}_1|_{z=0} \end{bmatrix} = j \mathbf{T}_1 \mathbf{T}_2 \cdots \mathbf{T}_{N-1} \mathbf{T}_N \begin{bmatrix} k_{z_{N+1}} \mathbf{I} \\ \mathbf{B}_{N+1} \\ \omega\mu_{N+1} \end{bmatrix} \tilde{\mathbf{I}}_{N+1} \quad (5.2)$$

where

$$\mathbf{T}_i = \begin{bmatrix} \mathbf{I}\mathbb{C}_i & j\mathbf{B}_i^{-1}\omega\mu_i k_{z_i} \mathbb{S}_i \\ j\mathbf{B}_i \mathbb{S}_i / (\omega\mu_i k_{z_i}) & \mathbf{I}\mathbb{C}_i \end{bmatrix}$$

$$\mathbb{C}_i = \cos(k_{z_i} h_i) \text{ and } \mathbb{S}_i = \sin(k_{z_i} h_i) \quad (5.3)$$

It is seen that the effect of the i th layer on Eq. (5.2) is represented by the matrix $\mathbf{T}_i$. The benefit brought by this concise expression is that if a layer is changed or added or removed, only the corresponding $\mathbf{T}_i$ is required to be modified or inserted or deleted from Eq. (5.2) with other parts unchanged. This feature of independence is very useful since no sophisticated modification of the formulation is needed when the superstrate configuration varies from one to another. Letting

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{bmatrix} = \prod_{i=1}^N \mathbf{T}_i \quad (5.4)$$

The expression of $\widetilde{\mathbf{H}}_1|_{z=0}$ in terms of $\widetilde{\mathbf{E}}_1|_{z=0}$ is found,

$$\widetilde{\mathbf{H}}_1|_{z=0} = \mathbf{G}\widetilde{\mathbf{E}}_1|_{z=0} \quad (5.5)$$

where $\mathbf{G}$ is the spectral dyadic Green's function given by

$$\mathbf{G} = \mathbf{C}\mathbf{D}^{-1} \quad (5.6)$$

$$\begin{cases} \mathbf{C} = \left[k_{z_{N+1}} \mathbf{W}_{21} + \mathbf{W}_{22} \frac{\mathbf{B}_{N+1}}{\omega \mu_{N+1}} \right] \\ \mathbf{D} = \left[k_{z_{N+1}} \mathbf{W}_{11} + \mathbf{W}_{12} \frac{\mathbf{B}_{N+1}}{\omega \mu_{N+1}} \right] \end{cases} \quad (5.7)$$

It is seen that this expression of $\mathbf{G}$ in terms of $\mathbf{W}$ is very concise compared with those in [114, 115].

5.2.2 Two-layer Superstrate

The following assumptions are made for this model for simplicity:

1. $\mu_1 = \mu_2 = \mu_3 = \mu$ and μ is the permeability of free space;
2. $\epsilon_{r_3} = \epsilon_{r_1} = 1, \epsilon_{r_2} = \epsilon_r \gg 1$;

With the assumptions made above, I have $k_3 = k_1, k_{z_3} = k_{z_1}, \mathbf{B}_3 = \mathbf{B}_1$. The matrices $\mathbf{C}$ and $\mathbf{D}$ in Eq. (5.7) can also be explicitly shown as

$$\begin{cases} \mathbf{C} = \frac{1}{\omega\mu} \left[\mathbf{B}_1 \mathbb{C}_2 \mathbb{L}_1 + j \mathbf{B}_2 \mathbb{C}_1 \mathbb{S}_2 \frac{k_{z_1}}{k_{z_2}} - \mathbf{B}_1 \mathbf{B}_2^{-1} \mathbf{B}_1 \mathbb{S}_1 \mathbb{S}_2 \frac{k_{z_2}}{k_{z_1}} \right] \\ \mathbf{D} = \mathbb{L}_1 \mathbb{C}_2 k_{z_1} - \mathbf{B}_1^{-1} \mathbf{B}_2 \mathbb{S}_1 \mathbb{S}_2 \frac{k_{z_1}^2}{k_{z_2}} + j \mathbf{B}_2^{-1} \mathbf{B}_1 \mathbb{C}_1 \mathbb{S}_2 k_{z_2} \end{cases} \quad (5.8)$$

where $\mathbb{L}_i = \mathbb{C}_i + j \mathbb{S}_i$.

Let's define the 2×2 matrix $\mathbf{G}$ as follows,

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \frac{1}{\mathbb{D}} \begin{bmatrix} g'_{11} & g'_{12} \\ g'_{21} & g'_{22} \end{bmatrix} \quad (5.9)$$

Substituting Eq. (5.8) into Eq. (5.6) and comparing with Eq. (5.9), all elements of $\mathbf{G}$ are explicitly shown as

$$\begin{aligned} \mathbb{D} = & k_{z_1} \mathbb{J}_2^2 + \mathbb{K}_2^2 \left[\mathbb{K}_1^2 - \frac{\mathbb{J}_1^2}{\epsilon_r^2} \right] \epsilon_r k_{z_1} - \mathbb{J}_2 \mathbb{F}_2 \left(\mathbb{K}_1 - j \frac{\mathbb{J}_1}{\epsilon_r} \right) (\epsilon_r k_{z_1}^2 + k_{z_2}^2) \\ & - j \mathbb{J}_1 \mathbb{F}_1 \mathbb{F}_2^2 \left(\frac{k_{z_2}^4 + \epsilon_r^2 k_{z_1}^4}{\epsilon_r} \right) \end{aligned} \quad (5.10)$$

$$g'_{11} = \sum_{s=1}^6 \mathbb{X}_s \quad (5.11)$$

$$g'_{12} = k_x k_y \sum_{s=1}^6 \mathbb{Y}_s \quad (5.12)$$

where

$$\left\{ \begin{array}{l} \mathbb{X}_1 = \mathbb{J}_2^2(k_1^2 - k_y^2) \\ \mathbb{X}_2 = \frac{\mathbb{J}_2\mathbb{F}_1\mathbb{F}_2}{\epsilon_r} \left\{ k_y^2 [(\epsilon_r^2 - 1)k_1^2 + \epsilon_r(\epsilon_r + 1)k_{z_1}^2] - k_1^2 [(\epsilon_r^2 - 1)k_1^2 + (\epsilon_r + 1)k_{z_1}^2] \right\} \\ \mathbb{X}_3 = j\mathbb{J}_1\mathbb{J}_2\mathbb{F}_2 \left(1 + \frac{1}{\epsilon_r} \right) k_{z_1} (\epsilon_r k_1^2 - k_y^2) \\ \mathbb{X}_4 = \frac{\mathbb{F}_1^2\mathbb{F}_2^2}{\epsilon_r} \left\{ k_1^2 k_{z_2}^4 - k_y^2 [(\epsilon_r - 1)^2 k_1^4 + (\epsilon_r^2 - 1)k_1^2 k_{z_1}^2 + \epsilon_r^2 k_{z_1}^4] \right\} \\ \mathbb{X}_5 = -\frac{\mathbb{J}_1^2\mathbb{F}_2^2}{\epsilon_r} \left\{ \epsilon_r^2 k_1^2 k_{z_1}^2 + k_y^2 [(\epsilon_r - 1)^2 k_1^2 - k_{z_1}^2] \right\} \\ \mathbb{X}_6 = -j\mathbb{J}_1\mathbb{K}_1\mathbb{K}_2^2 (k_1^2 - k_y^2) \left(\epsilon_r + \frac{1}{\epsilon_r} \right) \end{array} \right.$$

$$\left\{ \begin{array}{l} \mathbb{Y}_1 = \mathbb{J}_2^2 \\ \mathbb{Y}_2 = -\frac{\mathbb{J}_2\mathbb{F}_1\mathbb{F}_2}{\epsilon_r} [(\epsilon_r^2 - 1)k_1^2 + \epsilon_r(\epsilon_r + 1)k_{z_1}^2] \\ \mathbb{Y}_3 = j\mathbb{J}_1\mathbb{J}_2\mathbb{F}_2 \left(1 + \frac{1}{\epsilon_r} \right) k_{z_1} \\ \mathbb{Y}_4 = \frac{\mathbb{F}_1^2\mathbb{F}_2^2}{\epsilon_r} [(\epsilon_r - 1)^2 k_1^4 + (\epsilon_r^2 - 1)k_1^2 k_{z_1}^2 + \epsilon_r^2 k_{z_1}^4] \\ \mathbb{Y}_5 = -\frac{\mathbb{J}_1^2\mathbb{F}_2^2}{\epsilon_r} [k_{z_2}^2 - \epsilon_r(\epsilon_r - 1)k_1^2] \\ \mathbb{Y}_6 = -j\mathbb{J}_1\mathbb{K}_1\mathbb{K}_2^2 \left(\epsilon_r + \frac{1}{\epsilon_r} \right) \end{array} \right.$$

and

$$\left\{ \begin{array}{l} \mathbb{J}_i = \mathbb{L}_i^{-1}\mathbb{C}_i \\ \mathbb{K}_i = \mathbb{L}_i^{-1}\mathbb{S}_i \\ \mathbb{F}_i = \mathbb{K}_i/k_{z_i} \end{array} \quad (i = 1, 2) \right. \quad (5.13)$$

It is not difficult to find that g'_{22} can be derived from g'_{11} by exchanging k_x and k_y and $g'_{12} = g'_{21}$.

Figure 5.2 3-D view of an open-ended rectangular waveguide radiating through a two-layer superstrate.

5.3 Excitation by a Rectangular Waveguide

5.3.1 Reflection Coefficient Matrix

In this section, an open-ended rectangular waveguide radiating through a two-layer superstrate is considered. The geometry of the model is shown in Fig. 5.2. The superstrate considered here follows the assumption made in subsection 5.2.2. The length and width of the waveguide aperture are denoted by a and b , respectively. Waves are expected to travel from the rectangular waveguide and radiate into the half free space through the two-layer superstrate.

The spatial expressions of the transverse electric fields of a particular mode in a rectangular waveguide are given as follows,

$$\left\{ \begin{array}{l} \vec{\mathbf{e}}_{mn}^h = N_{mn} \begin{bmatrix} \frac{n\pi}{b} S_x \\ -\frac{m\pi}{a} S_y \end{bmatrix} \quad \text{TE mode} \\ \vec{\mathbf{e}}_{mn}^e = N_{mn} \begin{bmatrix} \frac{m\pi}{a} S_x \\ \frac{n\pi}{b} S_y \end{bmatrix} \quad \text{TM mode} \end{array} \right. \quad (5.14)$$

where N_{mn} is the normalized magnitude, a and b are the sizes of the rectangular waveguide in $\hat{x}$ and $\hat{y}$ directions, respectively,

$$S_x = \begin{cases} \cos \frac{m\pi(x + \frac{a}{2})}{a} \sin \frac{n\pi(y + \frac{b}{2})}{b} & -\frac{a}{2} < x < \frac{a}{2}, -\frac{b}{2} < y < \frac{b}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$S_y = \begin{cases} \sin \frac{m\pi(x + \frac{a}{2})}{a} \cos \frac{n\pi(y + \frac{b}{2})}{b} & -\frac{a}{2} < x < \frac{a}{2}, -\frac{b}{2} < y < \frac{b}{2} \\ 0 & \text{elsewhere} \end{cases}$$

It is seen that all TE or TM modes can be numbered with an integer pair (m, n) . In order to simplify the notation, each mode is represented with only one integer l ($l = 1, 2, \dots, L$). With one indexing number, all modes considered in the waveguide are rearranged and the electric and magnetic fields in the waveguide are given by

$$\begin{cases} \vec{\mathbf{E}}_w = \sum_{l=1}^L (A_l^+ e^{-j\beta_l z} + A_l^- e^{j\beta_l z}) \vec{\mathbf{e}}_l \\ \vec{\mathbf{H}}_w = \sum_{l=1}^L (A_l^+ e^{-j\beta_l z} - A_l^- e^{j\beta_l z}) \vec{\mathbf{e}}_l Y_l \end{cases} \quad (5.15)$$

where A_l^+ and A_l^- are the magnitudes of mode l propagating in $+\hat{z}$ and $-\hat{z}$ directions, respectively. Y_l is the mode admittance of the l th mode. At the opening of the waveguide ($z = 0$), Eq. (5.15) can be expressed by matrix equations

$$\begin{cases} \vec{\mathbf{E}}_w|_{z=0^-} = \vec{\mathbf{e}} (\mathbf{A}^+ + \mathbf{A}^-) \\ \vec{\mathbf{H}}_w|_{z=0^-} = \vec{\mathbf{e}} \mathbf{Y} (\mathbf{A}^+ - \mathbf{A}^-) \end{cases} \quad (5.16)$$

where

$$\vec{\mathbf{e}} = (\vec{\mathbf{e}}_1, \vec{\mathbf{e}}_2, \dots, \vec{\mathbf{e}}_L) = \begin{pmatrix} \vec{e}_{x_1} & \vec{e}_{x_2} & \cdots & \vec{e}_{x_L} \\ \vec{e}_{y_1} & \vec{e}_{y_2} & \cdots & \vec{e}_{y_L} \end{pmatrix} \quad (5.17)$$

$$\mathbf{A}^+ = (A_1^+, A_2^+, \dots, A_L^+)^T$$

$$\mathbf{A}^- = (A_1^-, A_2^-, \dots, A_L^-)^T$$

$$\mathbf{Y} = \begin{bmatrix} Y_1 & & & \\ & Y_2 & & \\ & & \ddots & \\ & & & Y_L \end{bmatrix} \quad (5.18)$$

Taking double Fourier transform on both sides of Eq. (5.16), I have

$$\tilde{\mathbf{E}}_w|_{z=0^-} = \tilde{\mathbf{e}} (\mathbf{A}^+ + \mathbf{A}^-) \quad (5.19)$$

$$\tilde{\mathbf{H}}_w|_{z=0^-} = \tilde{\mathbf{e}} \mathbf{Y} (\mathbf{A}^+ - \mathbf{A}^-) \quad (5.20)$$

where $\tilde{\mathbf{e}} = (\tilde{\mathbf{e}}_1, \tilde{\mathbf{e}}_2, \dots, \tilde{\mathbf{e}}_L)$ is the double Fourier transform of $\vec{\mathbf{e}}$ and

$$\tilde{\mathbf{e}}_l = \begin{cases} \tilde{\mathbf{e}}_{mn}^h = \pi N_{mn} \begin{bmatrix} an\mathbb{B}(m, k_x a)\mathbb{A}(n, k_y b) \\ -bm\pi\mathbb{A}(m, k_x a)\mathbb{B}(n, k_y b) \end{bmatrix} \\ \tilde{\mathbf{e}}_{mn}^e = \pi N_{mn} \begin{bmatrix} bm\mathbb{B}(m, k_x a)\mathbb{A}(n, k_y b) \\ an\pi\mathbb{A}(m, k_x a)\mathbb{B}(n, k_y b) \end{bmatrix} \end{cases} \quad (5.21)$$

where the functions $\mathbb{A}(m, \alpha)$ and $\mathbb{B}(m, \alpha)$ have been defined in Eq. (4.42).

Considering the continuity of the transverse fields along the waveguide

opening, substituting Eq. (5.19) into Eq. (5.5), and taking double inverse Fourier transform on both sides of the obtained equation, I have

$$\vec{\mathbf{H}}_1|_{z=0} = \frac{1}{(2\pi)^2\omega\mu_0} \left(\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{G} \tilde{\mathbf{e}} e^{jk_x x} e^{jk_y y} dk_x dk_y \right) (\mathbf{A}^+ + \mathbf{A}^-) \quad (5.22)$$

Left-multiplying both sides of Eq. (5.22) by $\vec{\mathbf{e}}^T$ and integrating on the waveguide opening S and considering Eq. (5.20) over the aperture, the following equation is derived,

$$\begin{aligned} & \left(\iint_S \vec{\mathbf{e}}^T \vec{\mathbf{e}} dS \right) \mathbf{Y}(\mathbf{A}^+ - \mathbf{A}^-) \\ &= \frac{1}{(2\pi)^2\omega\mu_0} \left[\iint_S \vec{\mathbf{e}}^T \left(\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathbf{G} \tilde{\mathbf{e}} e^{jk_x x} e^{jk_y y} dk_x dk_y \right) dS \right] (\mathbf{A}^+ + \mathbf{A}^-) \end{aligned} \quad (5.23)$$

Since the term $\iint_S \vec{\mathbf{e}}^T \vec{\mathbf{e}} dS$ results in a unit matrix, it can be removed from the left-hand side of Eq. (5.23). Then, exchanging the order of the integrations on the right-hand side of Eq. (5.23) and the following matrix equation is obtained,

$$\mathbf{Y}(\mathbf{A}^+ - \mathbf{A}^-) = \mathbf{U}(\mathbf{A}^+ + \mathbf{A}^-) \quad (5.24)$$

where

$$\mathbf{U} = \frac{1}{(2\pi)^2\omega\mu_0} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\tilde{\mathbf{e}}^-)^T \mathbf{G} \tilde{\mathbf{e}} dk_x dk_y \quad (5.25)$$

and

$$\tilde{\mathbf{e}}^- = \iint_S \vec{\mathbf{e}} e^{jk_x x} e^{jk_y y} dx dy \quad (5.26)$$

Therefore, the reflection coefficient matrix $\mathbf{\Gamma}$ is of the form

$$\mathbf{\Gamma} = (\mathbf{Y} + \mathbf{U})^{-1}(\mathbf{Y} - \mathbf{U}) \quad (5.27)$$

Since $\mathbf{Y}$ has been well defined in Eq. (5.18), the next step is to calculate the matrix $\mathbf{U}$. According to Eq. (5.25), the element in the l th row and l' th column of $\mathbf{U}$ is

$$u_{ll'} = \frac{1}{(2\pi)^2 \omega \mu_0} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\tilde{\mathbf{e}}_l^-)^T \mathbf{G} \tilde{\mathbf{e}}_{l'} dk_x dk_y \quad (5.28)$$

where

$$\tilde{\mathbf{e}}_l^- = (-1)^{m+n+1} \tilde{\mathbf{e}}_l$$

As shown in Section 5.2, the dyadic Green's function $\mathbf{G}$ is symmetrical, it is not difficult to find that $u_{ll'} = u_{l'l}$ as long as m and n have the same parity with m' and n' , respectively, which indicates only the elements in the lower triangular or the upper triangular part of $\mathbf{U}$ are required to be evaluated. Substituting Eq. (5.9) and Eq. (5.21) into Eq. (5.28), $u_{ll'}$ becomes

$$u_{ll'} = \frac{(-1)^{m+n+1}}{(2\pi)^2 \omega \mu_0} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} Z dk_x dk_y \quad (5.29)$$

where

$$Z = N_{mn} N_{m'n'} \pi^2 (q_1 g_{11} \xi_{BA} \xi'_{BA} + q_2 g_{12} \xi_{AB} \xi'_{BA} + q_3 g_{12} \xi_{BA} \xi'_{AB} + q_4 g_{22} \xi_{AB} \xi'_{AB}) \quad (5.30)$$

The expressions of q_s ($s = 1, 2, 3, 4$) are listed in Table 5.1. and functions

TABLE 5.1 Expressions of q_1 , q_2 and q_3 for different cases of mutual coupling between modes.

	q_1	q_2	q_3	q_4
TE-TE	a^2nn'	$-abmn'$	$-abnm'$	b^2mm'
TE-TM	$abnm'$	$-b^2mm'$	a^2nn'	$-abmn'$
TM-TE	$abmn'$	a^2nn'	$-b^2mm'$	$-abnm'$
TM-TM	b^2mm'	$abnm'$	$abmn'$	a^2mn'

ξ_{AB} , ξ_{BA} , ξ'_{AB} and ξ'_{BA} are defined as

$$\begin{cases} \xi_{AB} = \mathbb{A}(m, k_x a) \mathbb{B}(n, k_y b) \\ \xi_{BA} = \mathbb{B}(m, k_x a) \mathbb{A}(n, k_y b) \\ \xi'_{AB} = \mathbb{A}(m', k_x a) \mathbb{B}(n', k_y b) \\ \xi'_{BA} = \mathbb{B}(m', k_x a) \mathbb{A}(n', k_y b) \end{cases}$$

Z is found to be a function of k_x and k_y in Eq. (5.30). The parity of the integrand Z with respect to k_x and k_y is determined by the combination of m, m', n, n' , which is illustrated in Table. 5.2. The result of integration in Eq. (5.29) is 0 if either $m + m'$ or $n + n'$ is odd. This indicates that the l th mode and l' th mode can not be coupled when either $m + m'$ or $n + n'$ is odd. Therefore, if a mode can not be coupled from the incident mode, this mode can be removed from matrix $\mathbf{U}$, which decreases the dimension of $\mathbf{U}$ without affecting the accuracy. Moreover, when both $m + m'$ and $n + n'$ are even, Z is even with respect to both k_x and k_y and the integration domain can be reduced to only one quadrant, as shown in Eq. (5.31). Furthermore, it can be concluded from Eq. (5.30) that the integral domain can always

TABLE 5.2 The parity of Z with respect to k_x and k_y .

	$m + m'$ is odd	$m + m'$ is even
the parity of Z with respect to k_x	odd	even
	$n + n'$ is odd	$n + n'$ is even
the parity of Z with respect to k_y	odd	even

be reduced to one quadrant as long as the spectra of both the l th and l' th mode in the waveguide are symmetric or antisymmetric with respect to k_x and k_y .

$$u_{ll'} = \begin{cases} \frac{1}{\pi^2 \omega \mu_0} \int_0^{+\infty} \int_0^{+\infty} Z(k_x, k_y) dk_x dk_y & m + m' \text{ and } n + n' \text{ are even} \\ 0 & \text{otherwise} \end{cases} \quad (5.31)$$

After some observations on the spectral dyadic Green's function, it is found that $\mathbf{G}$ has poles on the (k_x, k_y) plane, which corresponds to surface waves in the superstrate. The positions of the poles depend on the value of $k_x^2 + k_y^2$, therefore it is more convenient to extract the poles from the integrand in a polar coordinates system. Using the equations $k_x = k_\rho \cos \psi$, $k_y = k_\rho \sin \psi$, Eq. (5.31) is transformed from Cartesian coordinate to cylindrical coordinate system.

$$u_{ll'} = \begin{cases} \frac{1}{\pi^2 \omega \mu_0} \int_0^{\pi/2} \int_0^{+\infty} Q(k_\rho, \psi) dk_\rho d\psi & m + m' \text{ and } n + n' \text{ are even} \\ 0 & \text{otherwise} \end{cases} \quad (5.32)$$

where $Q(k_\rho, \psi) = k_\rho Z(k_\rho \cos \psi, k_\rho \sin \psi)$. Now the positions of the poles are only related to k_ρ and integration of the poles can be obtained by the theorem

of residue. Usually the number and positions of poles on the integration path of k_ρ can be obtained by solving a transcendental equation numerically, which was provided in Chapter 3. If there are J poles $(k_{p_1}, k_{p_2}, \dots, k_{p_J})$, for a given $\psi(\psi = \psi_t)$, the integration of k_ρ in Eq. (5.32) can be decomposed as

$$\int_0^{+\infty} Q(k_\rho, \psi_t) dk_\rho = \int_0^{+\infty} \left(Q^*(k_\rho, \psi_t) + \sum_{s=1}^J \frac{2k_{p_s} R_s(\psi_t)}{k_\rho^2 - k_{p_s}^2} \right) dk_\rho \quad (5.33)$$

where $Q^*(k_\rho, \psi_t) = Q(k_\rho, \psi_t) - \sum_{s=1}^J \frac{2k_{p_s} R_s(\psi_t)}{k_\rho^2 - k_{p_s}^2}$ is a function of k_ρ without poles, $R_s(\psi_t)$ is residues of $Q(k_\rho, \psi)$ at (k_{p_s}, ψ_t) . Therefore, when both $m+m'$ and $n+n'$ are even, the expression of $u_{ll'}$ in Eq. (5.32) becomes

$$u_{ll'} = \frac{1}{\pi^2 \omega \mu_0} \int_0^{\pi/2} \left(\int_0^{+\infty} Q^*(k_\rho, \psi_t) dk_\rho - j\pi \sum_{s=1}^J R_s(\psi_t) \right) d\psi \quad (5.34)$$

When the integration in Eq. (5.34) is numerically solved, the reflection coefficient matrix $\mathbf{\Gamma}$ is then obtained through Eq. (5.27).

5.3.2 Efficient Evaluation Technique

It is noticed that calculating the integration in Eq. (5.34) directly by summing up a lot of sample points may consume a lot of time since the expression of $Q^*(k_\rho, \psi)$ is very complicated. Therefore, an accelerating technique is proposed in this part to improve the calculation efficiency. Firstly, it is noticed that the wavenumber k_1 corresponds to the half free space since $k_1 = k_3$. As has been pointed out in [49], $k_\rho = k_1$ is a branch point and the integrand $Q^*(k_\rho, \psi_t)$ with respect to k_ρ is not smooth there. The integration interval of k_ρ is segmented into $(0, k_1)$ and $(k_1, +\infty)$ so that the curve is smooth within each range and less sample points are required later. Secondly, the integrals in the two intervals are calculated separately. Gaussian quadrature

are adopted to evaluate the integral in $(0, k_1)$.

$$\int_0^{k_1} Q^*(k_\rho, \psi_t) dk_\rho = \frac{k_1}{2} \sum_{s=1}^{K_1} w_s Q^*(k_{\rho_s}, \psi_t) \quad (5.35)$$

where K_1 is the number of abscissas with respect to k_ρ from 0 to k_1 , w_s and k_{ρ_s} are the weights and abscissas, respectively. The generalized pencil of functions (GPOF) method [94] is employed to fit the integrand $Q^*(k_\rho, \psi_t)$ in $(k_1, +\infty)$ with a fitting function $Q_{ap}^*(k_\rho, \psi_t)$, which is the summation of a series of exponential functions through some sample points.

$$Q^*(k_\rho, \psi_t) \approx Q_{ap}^*(k_\rho, \psi_t) = \sum_{s=1}^{M_e} c_s e^{d_s k_\rho}$$

Therefore, the integration of k_ρ in $(k_1, +\infty)$ is converted to a simple summation as shown in Eq. (5.36).

$$\int_{k_1}^{+\infty} Q^*(k_\rho, \psi_t) dk_\rho = \int_{k_1}^{+\infty} \sum_{s=1}^{M_e} c_s e^{d_s k_\rho} dk_\rho = - \sum_{s=1}^{M_e} \frac{c_s}{d_s} e^{d_s k_1} \quad (5.36)$$

where M_e is the number of terms in the exponential series, c_s and d_s are the resultant coefficients obtained with GPOF, which are dependent on ψ_t implicitly. Since $Q^*(k_\rho, \psi)$ diminishes as k_ρ approaches to $+\infty$, the real part of d_s is always negative for $s = 1, 2, \dots, M_e$.

It is also found that the integration in Eq. (5.34) with respect to ψ can also be calculated by Gaussian quadrature. Therefore, the double integral of u_W has now been converted to a summation as

$$u_W = \begin{cases} \frac{1}{4\pi\omega\mu_0} \sum_{t=1}^{K_2} w_t F(\psi_t) & \text{both } m+m' \text{ and } n+n' \text{ are even} \\ 0 & \text{otherwise} \end{cases} \quad (5.37)$$

TABLE 5.3 Values of parameters.

Name of parameter	Description	Value
L	number of considered modes	11
K_1	number of abscissas with respect to k_ρ	9
K_2	number of abscissas with respect to ψ	9
M_s	number of sample points for GPOF	300
M_e	number of exponential terms	7

where K_2 is the number of abscissas with respect to ψ and ψ_t represents the abscissas in interval $(0, \pi/2)$ and

$$F(\psi_t) = \left[\frac{k_1}{2} \sum_{s=1}^{K_1} w_s Q^*(k_{\rho_s}, \psi) - \sum_{s=1}^{M_e} \frac{c_s}{d_s} e^{d_s k_1} - j\pi \sum_{s=1}^J R_s(\psi) \right]_{\psi=\psi_t} \quad (5.38)$$

It will be shown later that the computation efficiency has been greatly improved after using this accelerating technique.

5.4 Validation

The radiation of an open-ended rectangular waveguide through a two-layer superstrate as depicted in Fig. 5.2 is analyzed to demonstrate the proposed method. The model of the rectangular waveguide is WR-90, which is opened on an infinitely large ground plane. The cross-section dimension is 22.86mm by 10.16mm. The thicknesses of the first and second layers of the superstrate are 14.993mm and 2.3473mm, respectively. The relative permittivity of the second layer is $\epsilon_r = 10.2$. The feeding (incident) wave from $-\infty$ along the waveguide is assumed to be TE_{10} mode. The values of parameters are set as list in Table. 5.3. The abscissas of 9-point Gaussian quadrature with respect to k_ρ and ψ are derived from the abscissas y_i given in [-1,1] through

TABLE 5.4 Parameters for 9-point Gaussian quadrature.

index of point (i)	y_i	weight (w_i)	k_{ρ_i}/k_1	ψ_i/π
1	-0.9682	0.0813	0.0159	0.0080
2	-0.8360	0.1806	0.0820	0.0410
3	-0.6133	0.2606	0.1933	0.0967
4	-0.3243	0.3123	0.3379	0.1689
5	0	0.3302	0.5000	0.2500
6	0.3243	0.3123	0.6621	0.3311
7	0.6133	0.2606	0.8067	0.4033
8	0.8360	0.1806	0.9180	0.4590
9	0.9682	0.0813	0.9841	0.4920

Eq. (5.39) and Eq. (5.40), respectively.

$$k_{\rho_s} = \frac{y_s + 1}{2} k_1 \quad s = 1, 2, \dots, 9 \quad (5.39)$$

$$\psi_t = \frac{(y_t + 1)\pi}{4} \quad t = 1, 2, \dots, 9 \quad (5.40)$$

The results are listed in Table. 5.3.

5.4.1 Evaluation of the Integral

In this subsection, the results obtained by numerically evaluating the integrals of $u_{ll'}$ by the GPOF method and Gaussian quadrature are studied. Since the matrix $\mathbf{U}$ is found to be symmetrical, the integral required to be numerically evaluated is $(L+1)L/2$. The operating frequency is 10GHz. For conciseness, only the fitting results in u_{11} ($l = l' = 1$) are shown here.

Fitting the Integrand Using GPOF

The curve fitting results obtained by the GPOF method are examined. The approximating functions Q_{ap}^* are compared with the original integrand Q^*

 Figure 5.3 Curve fitting results at ψ_1 ($f=10\text{GHz}$).

for all ψ_t ($t = 1, 2, \dots, 9$) in Fig. 5.3-5.11. It is seen in these figures that the curves Q_{ap}^* are in very good agreement with Q^* for all ψ_t ($t = 1, 2, \dots, 9$) though the integrand with respect to k_ρ varies a lot when ψ changes. Therefore, these curve fitting results demonstrate the effectiveness of the GPOF method in approximating the Sommerfeld integrand.

Results of Gaussian Quadrature

Gaussian quadrature is employed to evaluate the integral of k_ρ in $(0, k_1)$ and the integral of ψ from 0 to $\pi/2$. The numerical results of evaluating the integral of k_ρ by Gaussian quadrature are given in Table. 5.5. The reference values are obtained by decreasing the sampling interval until the variation of the summation results does not exceed 1%. At last, the number of the sample point employed to calculate the reference value is 1047, which is much

 Figure 5.4 Curve fitting results at ψ_2 ($f=10\text{GHz}$).

 Figure 5.5 Curve fitting results at ψ_3 ($f=10\text{GHz}$).

 Figure 5.6 Curve fitting results at ψ_4 ($f=10\text{GHz}$).

 Figure 5.7 Curve fitting results at ψ_5 ($f=10\text{GHz}$).

 Figure 5.8 Curve fitting results at ψ_6 ($f=10\text{GHz}$).

 Figure 5.9 Curve fitting results at ψ_7 ($f=10\text{GHz}$).

 Figure 5.10 Curve fitting results at ψ_8 ($f=10\text{GHz}$).

 Figure 5.11 Curve fitting results at ψ_9 ($f=10\text{GHz}$).

TABLE 5.5 Comparison of integration results with respect to k_ρ in $(0, k_1)$.

ψ	reference values	results by Gaussian quadrature	error(%)
ψ_1	$0.6597 + 0.8177i$	$0.6611 + 0.8170i$	0.1490
ψ_2	$0.6654 + 0.8217i$	$0.6666 + 0.8212i$	0.1230
ψ_3	$0.6928 + 0.8409i$	$0.6928 + 0.8407i$	0.0184
ψ_4	$0.7626 + 0.8881i$	$0.7599 + 0.8891i$	0.2460
ψ_5	$0.8850 + 0.9665i$	$0.8769 + 0.9698i$	0.6674
ψ_6	$1.0346 + 1.0572i$	$1.0193 + 1.0638i$	1.1265
ψ_7	$1.1543 + 1.1271i$	$1.1330 + 1.1366i$	1.4456
ψ_8	$1.2115 + 1.1598i$	$1.1872 + 1.1707i$	1.5880
ψ_9	$1.2243 + 1.1671i$	$1.1994 + 1.1783i$	1.6142

greater than the 9 points used by Gaussian quadrature. It is also noticed that the relative error is below 2% which is a good tradeoff between the efficiency and accuracy. When Gaussian quadrature is applied to evaluating the integral with respect to ψ from 0 to $\pi/2$, the numerical result is $(2.043 + 2.614j) \times 10^{-3}$. If it is compared with the result by direct summation, in which 90 sample points are used and the value is $(2.043 + 2.617) \times 10^{-3}$, the relative error less than 0.1%. Therefore, it is seen that Gaussian quadrature shows very good performance in evaluating the integral of ψ .

5.4.2 Reflection Coefficient

When the matrix $\mathbf{U}$ is obtained, the reflection coefficient matrix $\mathbf{\Gamma}$ can be derived through Eq. (5.27). Fig. 5.12 presents the reflection coefficient results with respect to frequencies obtained using this method, which are compared with those by Ansoft's HFSS. It is seen in the figure that the results obtained by the two methods agree very well with each other, which therefore verifies the accuracy of my method in calculating the reflection coefficient of an open-ended waveguide radiating through the two-layer superstrate.

(a) Magnitude

(b) Phase

Figure 5.12 Calculated results for the reflection coefficient of an open-ended WR-90 waveguide with a two-layer superstrate ($L = 9$, $K_1 = 9$, $K_2 = 9$, $M_s = 300$, $M_e = 7$).

TABLE 5.6 Comparison of normalized computation time.

Error tolerance	HFSS	Proposed method
0.02	136	3.7s
0.002	328	9.6s

5.4.3 Computational Efficiency

In order to show the efficiency of my method, The computation time consumed by the proposed method and HFSS are compared. The computer is equipped with Intel Core2 6300 CPU and 1G memory. the CPU time of the full-wave analysis is presented and the computation time results are given in Table 5.6. The employed computer is equipped with Intel Core2 6300 CPU and 1G memory. It is seen that the computation time of my method is much less than that spent by HFSS. More time can be saved by using my method when higher accuracy is expected.

5.4.4 Convergence Study

The convergence behaviors of the proposed method with respect to the following parameters are studied at 10GHz in terms of the reflection coefficient.

1. L : the number of modes considered in the waveguide,
2. K_1 : the number of abscissas in calculating the integral of k_ρ from 0 to k_1 ,
3. K_2 : the number of abscissas in calculating the integral of ψ from 0 to $\pi/2$,
4. M_s : the number of sample points employed by GPOF to fit the integration tail,
5. M_e : the number of exponential terms generated by GPOF.

TABLE 5.7 Reflection coefficient with respect to the mode number L
 $(K_1 = 9, K_2 = 9, M_s = 300, M_e = 7)$.

L	Reflection coefficient
3	-0.3165-0.4342j
7	-0.3177-0.4296j
11	-0.3184-0.4286j
15	-0.3189-0.4284j
19	-0.3189-0.4284j

TABLE 5.8 Reflection coefficient with respect to the number of abscissas
 K_1 ($L = 19, K_2 = 9, M_s = 300, M_e = 7$).

K_1	Reflection coefficient
7	-0.3266-0.4252j
9	-0.3189-0.4284j
11	-0.3191-0.4285j

Table 5.7 presents the convergence results with respect to the number of modes considered in the waveguide while $K_1 = 9$, $K_2 = 9$, $M_s = 300$, $M_e = 7$. It is seen in this table that when 3 modes are considered the error is about 0.006. Variation of the reflection coefficient diminishes as the number of considered modes increases to 15, which indicates that convergent results can be obtained with a relative small mode number.

Table 5.8 examines the convergence of my method with respect to the number of abscissas adopted by the Gaussian quadrature in calculating the integral of k_ρ from 0 to k_1 while $L = 19$, $K_2 = 9$, $M_s = 300$, $M_e = 7$. It is seen that the change of the reflection coefficient result is very small when the number of abscissas is greater than 9, which therefore indicates that a convergent result of the integration can be obtained with 9 points Gaussian quadrature.

Table 5.9 shows the convergence behavior of my method with respect to the number of abscissas adopted by the Gaussian quadrature in calculating the integral of ψ from 0 to $\pi/2$ while $L = 19$, $K_1 = 9$, $M_s = 300$, $M_e = 7$. It

TABLE 5.9 Reflection coefficient with respect to the number of abscissas K_2 ($L = 19, K_1 = 9, M_s = 300, M_e = 7$).

K_2	Reflection coefficient
7	-0.3192-0.4290j
9	-0.3189-0.4284j
11	-0.3189-0.4284j

TABLE 5.10 Reflection coefficient with respect to the number of sample points M_s ($L = 19, K_1 = 9, K_2 = 9, M_e = 7$).

M_s	Reflection coefficient
50	-0.3224-0.4290j
100	-0.3184-0.4312j
150	-0.3189-0.4281j
300	-0.3189-0.4284j

is found that the reflection coefficient result is unchanged from $K_2 = 9$. It can therefore be concluded that the convergence performance is quite good when $K_2 \geq 9$.

The convergence behavior of my method with respect to the sample points adopted by GPOF is shown in Table 5.10 while $L = 19, K_1 = 9, K_2 = 9, M_e = 7$. It is noted that as more sample points of the integrand are used, the variation of the reflection coefficient becomes smaller, which leads to convergent results. As the number of required sample points is related to the shape of the fitted curve, it is shown in our study that 300 sample points are enough to accurately fit all the integrand tails in matrix $\mathbf{U}$ by GPOF.

TABLE 5.11 Reflection coefficient with respect to the number of exponential terms M_e ($L = 19, K_1 = 9, K_2 = 9, M_s = 300$).

M_e	Reflection coefficient
5	-0.3233-0.4280j
7	-0.3189-0.4284j
9	-0.3188-0.4295j

Table 5.11 shows the convergence behavior of my method with respect to the number of exponential terms while $L = 19$, $K_1 = 9$, $K_2 = 9$, $M_s = 300$. It is seen that when M_e increases from 5 to 7, the relative variation of the reflection coefficient is about 0.8%. When M_e increases from 7 to 9, the relative variation decreases to 0.2%. It is found that 7 exponential terms can provide quite accurate result in our calculation.

5.5 Radiation Pattern and Directivity

5.5.1 Calculating the Radiation Pattern

According to Eq. (5.24) and Eq. (5.27), when $\mathbf{A}^+$ and $\mathbf{\Gamma}$ are known, $\mathbf{A}^-$ can be derived by

$$\mathbf{A}^- = \mathbf{\Gamma} \mathbf{A}^+$$

The spectrum of the transverse electric field at the opening $\tilde{\mathbf{E}}_w|_{z=0}$ can be obtained through Eq. (5.16). Substituting Eq. (5.2) into Eq. (5.1) when $N = 2$, the spectra of the electric and magnetic fields in the half free space (layer 3) are derived.

$$\begin{cases} \tilde{\mathbf{E}}_3 = k_{z_1} \mathbf{D}^{-1} e^{-jk_{z_1}(z-d_2)} \tilde{\mathbf{E}}_w|_{z=0} \\ \tilde{\mathbf{H}}_3 = \frac{1}{\omega\mu} \mathbf{B}_1 \mathbf{D}^{-1} e^{-jk_{z_1}(z-d_2)} \tilde{\mathbf{E}}_w|_{z=0} \end{cases} \quad (5.41)$$

Similar to the formulation in Subsection 4.2.2, the far field radiation pattern of aperture antenna can be calculated through Eq. 12-10a to 12-10f in [106]. However, in this case, the electric surface current (J_s) and magnetic surface current (M_s) are both distributed across the top surface the

superstrate rather than the waveguide opening. The relationship between the transverse fields and the surface currents on the $z = d_2$ plane is denoted by

$$\begin{cases} \vec{M}_s = -\hat{z} \times \vec{E}_t|_{z=d_2} \\ \vec{J}_s = \hat{z} \times \vec{H}_t|_{z=d_2} \end{cases}$$

the far-field electric components $\vec{E}_\theta$ and $\vec{E}_\phi$ are given as follows.

$$\begin{aligned} \vec{E}_\theta &= \frac{jk_1 e^{-jk_1 r}}{4\pi r} \left[\sin \phi \tilde{E}_{y_3}^- + \cos \phi \tilde{E}_{x_3}^- - \eta \left(-\cos \theta \cos \phi \widetilde{H}_{y_3}^- + \cos \theta \sin \phi \widetilde{H}_{x_3}^- \right) \right] \\ \vec{E}_\phi &= \frac{jk_1 e^{-jk_1 r}}{4\pi r} \left[\cos \theta \cos \phi \tilde{E}_{y_3}^- - \cos \theta \sin \phi \tilde{E}_{x_3}^- - \eta \left(\sin \phi \widetilde{H}_{y_3}^- + \cos \phi \widetilde{H}_{x_3}^- \right) \right] \end{aligned} \quad (5.42)$$

where

$$\begin{cases} k_x = k_1 \sin \theta \cos \phi \\ k_y = k_1 \sin \theta \sin \phi \\ k_{z_1} = k_1 \cos \theta \end{cases} \quad (5.43)$$

and

$$\begin{cases} \tilde{E}_{x_3}^-(k_x, k_y) = \tilde{E}_{x_3}^-(-k_x, -k_y) \\ \tilde{E}_{y_3}^-(k_x, k_y) = \tilde{E}_{y_3}^-(-k_x, -k_y) \\ \widetilde{H}_{x_3}^-(k_x, k_y) = \widetilde{H}_{x_3}^-(-k_x, -k_y) \\ \widetilde{H}_{y_3}^-(k_x, k_y) = \widetilde{H}_{y_3}^-(-k_x, -k_y) \end{cases}$$

r is the distance from the point of observation to the origin point and $\eta = 120\pi\Omega$. The variable z is set to be d_2 so that the exponential term $e^{-k_{z_1}(z-d_2)}$ can be removed. It is seen that the radiation pattern can be derived directly

from the spectra of the fields in the half free space. Moreover, it is noticed that

$$\widetilde{\mathbf{H}}_3 = \frac{1}{\omega \mu k_{z_1}} \mathbf{B}_3 \widetilde{\mathbf{E}}_3 \quad (5.44)$$

Replacing k_x , k_y , k_{z_1} in Eq. (5.44) by Eq. (5.43), I have

$$\begin{bmatrix} \widetilde{H}_{y_3}^- \\ -\widetilde{H}_{x_3}^- \end{bmatrix} = \frac{1}{\eta \cos \theta} \begin{bmatrix} 1 - \sin^2 \theta \sin^2 \phi & \sin^2 \theta \cos \phi \sin \phi \\ \sin^2 \theta \cos \phi \sin \phi & 1 - \sin^2 \theta \cos^2 \phi \end{bmatrix} \begin{bmatrix} \widetilde{E}_{x_3}^- \\ \widetilde{E}_{y_3}^- \end{bmatrix} \quad (5.45)$$

Substituting Eq. (5.45) into Eq. (5.42), the following equations are arrived at,

$$\begin{bmatrix} \vec{E}_\theta \\ \vec{E}_\phi \end{bmatrix} = \frac{jk_1 e^{-jk_1 r}}{2\pi r} \begin{bmatrix} \cos \phi & \sin \phi \\ -\cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \begin{bmatrix} \widetilde{E}_{x_3}^- \\ \widetilde{E}_{y_3}^- \end{bmatrix} \quad (5.46)$$

which therefore indicates that the radiation pattern can be derived by the spectrum of the transverse electric field along the aperture independently. Since the relationship between $\widetilde{\mathbf{H}}_3$ and $\widetilde{\mathbf{E}}_3$ is linear as shown in Eq. (5.44), the radiation pattern can also be calculated using $\widetilde{\mathbf{H}}_3$.

When the radiation pattern is known, the directivity of the aperture antenna can be calculated according to its definition,

$$\text{directivity} = \frac{4\pi P(\theta, \phi)}{\int_0^{2\pi} \int_0^{\pi/2} P(\theta', \phi') \sin \theta' d\theta' d\phi'} \quad (5.47)$$

where

$$P(\theta, \phi) = (|\vec{E}_\theta|^2 + |\vec{E}_\phi|^2)/\eta \quad (5.48)$$

Substituting Eq. (5.46) into Eq. (5.48) and suppress the term of r , I have

$$\begin{aligned}
 P = \frac{1}{\eta} \Big\{ & \cos^2 \theta \left[|\tilde{E}_{x_3}^-|^2 \sin^2 \phi + |\tilde{E}_{y_3}^-|^2 \cos^2 \phi - \sin \phi \cos \phi \left(\tilde{E}_{x_3}^{-*} \tilde{E}_{y_3}^- + \tilde{E}_{x_3}^- \tilde{E}_{y_3}^{-*} \right) \right] \\
 & + |\tilde{E}_{x_3}^-|^2 \cos^2 \phi + |\tilde{E}_{y_3}^-|^2 \sin^2 \phi + \sin \phi \cos \phi \left(\tilde{E}_{x_3}^{-*} \tilde{E}_{y_3}^- + \tilde{E}_{x_3}^- \tilde{E}_{y_3}^{-*} \right) \Big\}
 \end{aligned}
 \tag{5.49}$$

From Eq. (5.47) and Eq. (5.49), it is seen that to generate high-directive radiation in an expected direction $(\theta_{max}, \phi_{max})$, the function P is required to achieve the maximum value when $\theta = \theta_{max}$ and $\phi = \phi_{max}$, which increases the numerator in Eq. (5.47), and decrease rapidly as (θ, ϕ) are far from $(\theta_{max}, \phi_{max})$, which decreases the denominator in Eq. (5.47). This is the principle of high directivity in view of spectrum analysis. If the spectrum of the excitation $\tilde{E}_w|_{z=0}$ can not satisfy this requirement, a certain structure, such as the two-layered superstrate proposed in Section 5.4, is introduced to transform it into a new spectrum, which is $\tilde{E}_3$ in our two-layer superstrate model, through a specific Green's function so that the requirement for high directivity can be satisfied.

It is noted that the far field radiation pattern considered in this thesis does not include surface waves. Therefore, the surface waves do not contribute to the calculation of directivity. When the antenna gain is considered, the power brought away by the surface waves should be taken into account. The evaluation of the loss caused by surface waves have been discussed in [17].

Fig. 5.13 presents the comparison of radiation pattern results at $10GHz$ obtained by Eq. (5.46) and those by Ansoft Designer. It is seen in the figure that the results obtained by both methods are in very good agreement,

which proves the correctness of the formulation as well as the accuracy of my method in calculating the radiation pattern. When the radiation pattern is known, the directivity is found to be 16dBi, which is much higher than that of a open-ended WR-90 waveguide without the superstrate given in Subsection 4.4.2.

5.5.2 Effects of the Superstrate

It has been shown that the radiation pattern is directly related to spectra of the fields in the half free space, which are determined by both the spectral Green's function and the spectrum of the excitation. In this subsection, the influence of the Green's function in first and the effect of the source will be discussed in the next subsection.

Let's consider a point source located at the origin with time harmonic electric field in the $\hat{y}$ direction, which is shown in Fig. 5.14. The spatial function of the point source is given by

$$\vec{E}_p = \delta(x)\delta(y)\hat{y}$$

with the time harmonic factor $e^{j\omega t}$ suppressed. Therefore, its spectrum is

$$\tilde{E}_p = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta(x)\delta(y)\hat{y}e^{-jk_x x}e^{-jk_y y}dxdy = \hat{y} \quad (5.50)$$

 (a) $\phi = 0^\circ$

 (b) $\phi = 90^\circ$

Figure 5.13 Radiation pattern of an open-ended WR-90 waveguide with a two-layer superstrate ($L = 9$, $K_1 = 9$, $K_2 = 9$, $M_s = 300$, $M_e = 7$, $f = 10GHz$).

Figure 5.14 Model of the two-layer superstrate excited by a point source.

Substituting Eq. (5.50) into Eq. (5.41), I have

$$\begin{aligned}\tilde{E}_{x_3} &= \frac{k_{z_1} \mathbb{S}_2}{k_{z_2} \mathbb{D}^*} \left(\mathbb{S}_1 + j \frac{\mathbb{C}_1}{\epsilon_r} \right) k_x k_y (1 - \epsilon_r) \\ \tilde{E}_{y_3} &= \frac{k_{z_1}}{\mathbb{D}^*} \left\{ k_{z_1} \mathbb{L}_1 \mathbb{C}_2 - \frac{\mathbb{S}_2}{k_{z_2}} \left[k_1^2 (j \mathbb{C}_1 - \mathbb{S}_1 \epsilon_r) + k_x^2 \left(\mathbb{S}_1 \epsilon_r + \frac{\mathbb{C}_1}{\epsilon_r} \right) - j k_y^2 \mathbb{L}_1 \right] \right\}\end{aligned}\quad (5.51)$$

with

$$\begin{aligned}\mathbb{D}^* &= k_{z_1}^2 \mathbb{L}_1^2 \mathbb{C}_2^2 + \mathbb{S}_2^2 \left(\mathbb{S}_1^2 - \frac{\mathbb{C}_1^2}{\epsilon_r^2} \right) \epsilon_r k_{z_1}^2 - k_{z_1} \mathbb{L}_1 \mathbb{C}_2 \frac{\mathbb{S}_2}{k_{z_2}} \left(\mathbb{S}_1 - j \frac{\mathbb{C}_1}{\epsilon_r} \right) (\epsilon_r k_{z_1}^2 + k_{z_2}^2) \\ &\quad - j \frac{\mathbb{C}_1 \mathbb{S}_1 \mathbb{S}_2^2}{k_{z_2}^2 \epsilon_r} [(k_{z_2}^4 + \epsilon_r^2 k_{z_1}^4)]\end{aligned}$$

$\mathbb{D}^*$ can also be derived by

$$\mathbb{D}^* = k_{z_1} \mathbb{L}_1^{-2} \mathbb{L}_2^{-2} \mathbb{D}$$

where $\mathbb{D}$ has been defined in Eq. (5.10). $\mathbb{C}_i$, $\mathbb{S}_i$ and $\mathbb{L}_i$ ($i = 1, 2$) have been defined in Section 5.2.

Substituting Eq. (5.51) into Eq. (5.46), the radiation pattern is obtained in terms of E_θ and E_ϕ . The power density P , given by Eq. (5.48), is a

Figure 5.15 Radiation pattern of a y-polarized point source ($h_1 = 0$, $h_2 = 0$).

function of θ and ϕ . Therefore, the patterns of power density are examined with respect to the superstrate's parameters: h_1 , h_2 and ϵ_r , respectively.

- **h_1 and h_2**

In order to show the influence of the superstrate on the radiation pattern of a point source, h_1 and h_2 are normalized to λ_1 and λ_2 , respectively, while ϵ_r remains a constant 10.2. λ_1 and λ_2 are the wavelengths in the first and second layer of the superstrate, respectively. The calculated results are shown in Fig. 5.16-5.22

Firstly, a special case where $h_2 = 0$ is studied, which is equivalent to the point source radiating directly into a half free space. The radiation pattern is given in Fig. 5.15 and the directivity is 3 (about 4.77dBi), which are references for analyzing the effect of the two-layer superstrate.

 Figure 5.16 Radiation pattern of a point source ($h_1 = 0$, $\epsilon_r = 10.2$).

 (a) $h_2 = \lambda_2/8$

 (b) $h_2 = \lambda_2/4$

 (c) $h_2 = 3\lambda_2/8$

 (d) $h_2 = \lambda_2/2$

 (e) $h_2 = 5\lambda_2/8$

 (f) $h_2 = 3\lambda_2/4$

 Figure 5.17 Radiation pattern of a point source ($h_1 = \lambda_1/4$, $\epsilon_r = 10.2$).

 Figure 5.18 Radiation pattern of a point source ($h_1 = \lambda_1/2$, $\epsilon_r = 10.2$).

 Figure 5.19 Radiation pattern of a point source ($h_1 = 3\lambda_1/4$, $\epsilon_r = 10.2$).

 Figure 5.20 Radiation pattern of a point source ($h_1 = \lambda_1$, $\epsilon_r = 10.2$).

Figure 5.21 Radiation pattern of a point source ($h_1 = 5\lambda_1/4$, $\epsilon_r = 10.2$).

Figure 5.22 Radiation pattern of a point source ($h_1 = 3\lambda_1/2$, $\epsilon_r = 10.2$).

Fig. 5.16 shows the radiation patterns for different h_2 when $h_1 = 0$, in which the two-layer superstrate regresses to a one-layer case with its thickness and relative permittivity being h_2 and ϵ_r , respectively. It is seen the focus effect of the one-layer superstrate is very weak although it is a little more directive than the pattern in Fig. 5.15.

Fig. 5.17 shows the radiation patterns for different h_2 when $h_1/\lambda_1 = 1/4$. It is seen that most of the radiation patterns are bifurcated except when $h_2/\lambda_2 = 1/2$. It is noticed that the lobes are very slim though the patterns split, which indicates intensive radiation in the directions where θ is between 60° and 90° and $\phi = 90^\circ$ and 270° . Therefore, such configurations except for $h_2/\lambda_2 = 1/2$ could be candidates if this kind of pattern is desired.

Fig. 5.18 shows the radiation patterns for different h_2 when $h_1/\lambda_1 = 1/2$. It is seen in Fig. 5.18(a) the beam is very slim for $\phi = 0^\circ$ while in the plane $\phi = 90^\circ$ there are two big side lobes. When $h_2/\lambda_2 = 1/4$, the two side lobes are suppressed and a very high directivity is generated (33.197 or 15.2dBi). Compared with that of the reference case, the directivity has been increased by 10.4dB. When h_2/λ_2 increases to $3/8$, the pattern splits in both $\phi = 0^\circ$ and $\phi = 90^\circ$ planes. When $h_2/\lambda_2 = 1/2$, the directivity becomes very low. The patterns in (e) and (f) are similar to those in (a) and (b), respectively.

Fig. 5.19 shows the radiation patterns for different h_2 when $h_1/\lambda_1 = 3/4$. The patterns are all bifurcated except for $h_2/\lambda_2 = 1/2$, but they are different from those in Fig. 5.17 because 1) the patterns in this case are much slimmer; 2) the pointed directions in term of θ are located between 30° and 60° ; 3) the pencil-like slanted lobes are in both $\phi = 0^\circ$ and $\phi = 90^\circ$ planes.

Fig. 5.20 shows the radiation patterns for different h_2 when $h_1/\lambda_1 = 1$. It is found that high-directivity can be obtained when $h_2/\lambda_2 = 1/4$ or $3/4$

TABLE 5.12 Directivities of a point source radiating through a two-layer medium.

		h_1/λ_1						
		0	1/4	1/2	3/4	1	5/4	3/2
$\frac{h_2}{\lambda_2}$	1/8	3.998	8.034	10.643	16.497	15.555	15.826	18.218
	1/4	3.856	9.957	33.197	23.316	31.802	26.509	28.628
	3/8	3.752	8.374	13.347	17.641	14.962	17.749	16.045
	1/2	3.158	3.604	3.412	3.786	4.092	4.453	4.792
	5/8	4.271	7.552	10.145	13.892	13.757	15.694	13.931
	3/4	4.991	10.026	33.775	29.006	31.263	30.139	30.992

but the side lobe level is very high. It is also found the side lobes are all pointing to the direction of $\theta \approx 60^\circ$.

Fig. 5.21 shows the radiation patterns for different h_2 when $h_1/\lambda_1 = 5/4$. It is seen that all radiation patterns have four pencil-like branches except for $h_2/\lambda_2 = 1/2$. The bigger two of them point to $30^\circ < \theta < 60^\circ$ and the others point to $60^\circ < \theta < 90^\circ$. Also this beam splitting exists in both $\phi = 0^\circ$ and $\phi = 90^\circ$.

Fig. 5.22 shows the radiation patterns for different h_2 when $h_1/\lambda_1 = 3/2$. The patterns in this figure resemble those in Fig. 5.20 except that 1) the beam is slimmer; 2) the number side lobes in $\phi = 0^\circ$ doubles, which are pointing to $\theta \approx 50^\circ$ and $\theta \approx 70^\circ$.

The directivity results are given in Table 5.12. It is seen that directivities less than 5 are located in the first column and the fourth row, which represents $h_1/\lambda_1 = 0$ and $h_2/\lambda_2 = 1/2$, respectively. When $h_1/\lambda_1 \geq 1/2$, the directivity does not change drastically as h_1/λ_1 varies, which therefore results in two high-directive ridges at $h_2/\lambda_2 = 1/4$ and $h_2/\lambda_2 = 3/4$ while the peaks are located at $h_1/\lambda_1 = 1/2$ and $h_1/\lambda_1 = 3/2$.

TABLE 5.13 Directivity with respect to ϵ_r ($h_1/\lambda_1 = 1/2$, $h_2/\lambda_2 = 1/4$).

ϵ_r	directivity	directivity/ ϵ_r
2	6.8	3.4
5	16.3	3.3
10	32.5	3.3
20	65.7	3.3
50	166.7	3.3
100	335.8	3.4

• ϵ_r

The relative permittivity of the second layer of the superstrate ϵ_r is another important parameters besides the thicknesses of the two layers in our model. In order to show the effect of ϵ_r on the high directive radiation of the point source, let's set $h_1/\lambda_1 = 1/2$ and $h_2/\lambda_2 = 1/4$. The calculated radiation patterns for different ϵ_r are shown in Fig. 5.23. It is seen that as ϵ_r grows larger, the beamwidth becomes narrower and the side lobe level in the $\phi = 90^\circ$ plane decreases. The directivity results with respect to different values of ϵ_r are given in Table 5.13. In accordance with the radiation patterns, the directivity values are proportional to the relative permittivity and the ratios are provided in the third column of the table. It is seen that the directivity values (not in dBi) can be approximated by the values of ϵ_r multiplied by 3.3, which is close to the directivity (3dBi) of a point source radiating directly into a half free space. In other words, the directivity of the point source can be enlarged approximately by a factor of ϵ_r .

5.5.3 Effects of the Excitation

Since the radiation pattern is jointly determined by the Green's function and the excitation, the effects of the source on the radiation through the two-layered is studied in this subsection.

Figure 5.23 Radiation pattern of a point source ($h_1/\lambda_1 = 1/2$, $h_2/\lambda_2 = 1/4$).

TABLE 5.14 Comparison of directivities of TE_{10}/TE_{30} mode with/without the superstrate.

	TE_{10}	TE_{30}
with superstrate	38.66	47.67
no superstrate	4.27	7.04
improvement in multiple	9.05	6.77

The results of the simplest excitation - a point source - have been given in subsection 5.5.2. It is seen that the directivity can be improved greatly in some cases and can barely in some others. Therefore, it is very important to find the right superstrate configurations for a certain source. Now a question comes up, whether a layered superstrate, which can improve the directivity of a certain source, is able to improve those of any other excitations. Before answering this question, the radiation patterns of TE_{10} mode and TE_{30} mode in a WR-90 open-ended waveguide are examined, respectively. The parameters of the superstrate, if applied, are $h_1 = \lambda_1/2$, $h_2 = \lambda_2/4$ and $\epsilon_r = 10.2$.

The comparison of radiation patterns generated by excitation of TE_{10} mode and TE_{30} mode are given in Fig. 5.24 and 5.25, respectively. The directivity results are given in Table 5.14. It is seen that when the superstrate is employed, both radiation patterns of TE_{10} and TE_{31} modes are slimed down and the directivities are greatly improved. As to the absolute directivity, TE_{30} mode is higher than TE_{10} mode in both cases, while for the extent of improvement, the two-layer superstrate is more effective on TE_{10} mode than TE_{30} mode.

If attention is paid to the spectra of TE_{10} and TE_{30} as well as the previously discussed point source, it can be found that they are all non-zero at $\theta = 0$. The spectra of TE_{10} and TE_{30} in a WR-90 rectangular waveguide at 10GHz are depicted in Fig. 5.26.

(a) without the superstrate

(b) with the superstrate

 Figure 5.24 Comparison of radiation patterns generated by TE_{10} mode ($f=10\text{GHz}$).

(a) without the superstrate

(b) with the superstrate

Figure 5.25 Comparison of radiation patterns generated by TE_{30} mode ($f=10\text{GHz}$).

(a) TE_{10} mode

(b) TE_{30} mode

Figure 5.26 Spectrum of TE_{10} and TE_{30} modes in a WR-90 waveguide ($f=10\text{GHz}$).

The spectrum of TE_{12} mode at 10GHz is shown in Fig. 5.27. It is seen that both the $\hat{x}$ component and $\hat{y}$ component are zero at $k_x = k_y = 0$, which is equivalent to $\theta = 0$.

The resultant radiation patterns generated by the TE_{12} mode distributed at the aperture of an open-ended WR-90 waveguide with and without the aforementioned two-layer superstrate ($h_1 = \lambda_1/2$, $h_2 = \lambda_2/4$ and $\epsilon_r = 10.2$) are given in Fig. 5.28. It is seen both of the patterns do not show an intensive radiation but a null in the direction of $\theta = 0$. For the case without the superstrate, it is easy to understand the existence of the null since the spectrum of TE_{12} mode is null at that point, as has been shown in Fig. 5.27 and the process by Eq. (5.49) does not change the value at this point. Therefore, the radiation pattern in Fig. 5.28(a) is null when $\theta = 0$. As for the case with the superstrate, it is noticed that both the $\hat{x}$ and $\hat{y}$ components of the spectrum are null at $k_\rho = 0$, while the linear transformation by Eq. (5.41) cannot remove the null point from the spectrum since it is equivalent to the superposition of two re-weighted quantities, which are null at the same point. So no matter what weight number is used, the null point always stays. Following the process without superstrate, the null point is transferred to the radiation pattern. Therefore, the reason why the directivity cannot be improved by the two-layer superstrate has been demonstrated for the excitation of TE_{12} mode. A generalized conclusion followed by the reasoning states: If the spectrum of the excitation is null at a certain point, no matter what a planar, layered, dielectric superstrate is adopted, the resultant radiation pattern is null in the corresponding direction.

(a) $\hat{y}$ component

(b) $\hat{x}$ component

Figure 5.27 Spectrum of TE_{12} mode in a WR-90 waveguide ($f=10\text{GHz}$).

(a) without superstrate

(b) with superstrate

 Figure 5.28 Radiation pattern of TE_{12} mode in a WR-90 waveguide ($f=10\text{GHz}$).

Figure 5.29 Geometry of a cavity-backed slot antenna radiating through a two-layer superstrate.

5.5.4 Transmission Line Model

It is found that the antenna directivity can be obtained using Eq. (5.47), but it is not straightforward since a two-dimensional integration is required to calculate the total radiated power. In this subsection, a very simple transmission line model is established to estimate the directivity of open-ended waveguide radiating through a multilayer superstrate and obtain insight into the principle of gain-enhancement.

Fig. 5.29 shows the geometry of a two-layer superstrate fed by a cavity-backed slot. A rectangular slot is cut on an infinitely large ground plane and it is backed with a rectangular cavity. A coaxial probe is used to excite the cavity. The fields in the cavity are then coupled to the slot and radiate into a half free-space through a two-layer superstrate. Right on the ground plane is layer 1, whose height and dielectric constant are h_1 and 1, respectively. Dielectric layer 2 of thickness h_1 and dielectric constant ϵ_r is placed right on layer 1. In this section, both the superstrate and ground plane are assumed

Figure 5.30 Equivalent circuit model.

to be infinitely large. The topmost part of the model is a half free space extending to infinity horizontally and upwards.

According to Huygens' Principle [116], any point on the slot aperture can be considered as the source for secondary radiation, then a point source on the slot surface is taken into account as an example. Consider a radiated wave leaving the superstrate-covered slot antenna at an angle of θ , which is the angle between the radiating direction of interest and the vertical direction. As shown in Fig. 5.29, the angles away from the slot normal in layer 1 and layer 2 can be θ_1 and θ_2 , respectively. They can be readily obtained by Snell's law.

$$\begin{cases} \theta_1 = \theta \\ \theta_2 = \arcsin(\sin \theta / \sqrt{\epsilon_r}) \end{cases} \quad (5.52)$$

For this considered wave radiated from the slot surface, an equivalent circuit model can be obtained, as shown in Fig. 5.30. Z_0 is the characteristic

impedance of the transmission line representing the free space, which is a constant ($120\pi\Omega$). Z_1 and Z_2 are the characteristic impedances for layer 1 and layer 2, respectively, and they have the form of

$$Z_1 = Z_0 \text{ and } Z_2 = Z_0/\sqrt{\epsilon_r} \quad (5.53)$$

The lengths of the transmission lines representing the paths over which the wave travels through layer 1 and layer 2 are l_1 and l_2 , respectively,

$$l_1 = h_1/\cos\theta_1 \text{ and } l_2 = h_2/\cos\theta_2 \quad (5.54)$$

For convenience, let's define

$$\begin{cases} \gamma_1 = 2\pi l_1/\lambda_1 = 2\pi l_1 f/c_0 \\ \gamma_2 = 2\pi l_2/\lambda_2 = 2\pi l_2 f\sqrt{\epsilon_r}/c_0 \end{cases} \quad (5.55)$$

for the phase delays the wave undergoes layer 1 and layer 2, respectively. Here λ_1 and λ_2 are wave lengths in layer 1 and layer 2, respectively, c_0 is the speed of light in vacuum and f is the operating frequency. According to Schelkunoff's equivalence theorem, an equivalent magnetic current can be placed over the slot surface with the slot aperture replaced with a perfectly conducting surface. Therefore, a voltage source, representing the equivalent magnetic current, is placed on the bottom of layer 1 in the model shown in Fig. 5.30.

As reported in [10], if a high gain is expected in the θ direction, the following conditions must be satisfied.

$$\gamma_1 = \pi, \quad \gamma_2 = \pi/2 \text{ and } \epsilon_r \gg 1 \quad (5.56)$$

Therefore, when a high gain is desired in the upward direction ($\theta = 0$), the thicknesses of layer 1 and layer 2 should be half and one quarter guided wavelength, respectively and layer 2 is of high dielectric constant. The above gain-enhancement condition can be clearly explained using the equivalent circuit model in Fig. 5.30. Since $\gamma_1 = \pi$, the voltage at point A is

$$V_A = V_A^+ + V_A^- = -V_s \quad (5.57)$$

where V_A^+ and V_A^- are the voltages of the waves traveling upward and downward at interface A, respectively, as shown in Fig. 5.30. The voltage at interface B can then be obtained as

$$V_B = V_B^+ + V_B^- = V_A^+ e^{-j\gamma_2} + V_A^- e^{j\gamma_2} \quad (5.58)$$

This voltage V_B is the output voltage that will travel along the semi-infinite transmission line of characteristic impedance Z_0 . Using the relationship between the incident voltage V_B^+ and the reflected voltage V_B^-

$$\Gamma_B = \frac{V_B^-}{V_B^+} = \frac{Z_0 - Z_{r_2}}{Z_0 + Z_{r_2}} \quad (5.59)$$

and Eq. (5.56), one can easily get

$$\left| \frac{V_B}{V_S} \right| = \frac{Z_0}{Z_{r_2}} = \sqrt{\epsilon_r} \quad (5.60)$$

Therefore Eq. (5.60) shows that for a given voltage source, the output voltage is proportional to $\sqrt{\epsilon_r}$, an amplification factor of the source voltage. This indicates that an intensive radiation will occur in the direction with respect to θ where Eq. (5.56) is satisfied. Since the “antenna gain” is defined

Figure 5.31 3-D view of an open-ended coaxial line radiating through a two-layer superstrate.

as the ratio of the maximum radiation intensity to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically, a high-gain slot antenna can thus be obtained if conditions in Eq. (5.56) are satisfied in the direction of $\theta = 0$.

5.6 Excitation by a Coaxial Line

5.6.1 Formulation

In this section, the proposed numerical method is applied to an open-ended coaxial line to show its efficiency in solving omnidirectional problems. The model considered is shown in Fig. 5.31, which is similar to that in Fig. 5.2 except for the cross-section of the waveguide. The inner and outer radii of the coaxial line are denoted by a and b , respectively.

The mode functions of electric field at the opening of the coaxial line have been given in Eq. (4.29). Therefore, the x component of $\vec{e}_n(\vec{\rho})$ is $e_n(\rho) \cos \phi$

and its double fourier transform $\mathcal{F}_x(k_x, k_y)$ is expressed as

$$\begin{aligned}
 \mathcal{F}_{x_n}(k_x, k_y) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e(\rho) \cos \phi e^{-jk_x x} e^{-jk_y y} dx dy \\
 &= \int_0^{+\infty} \int_0^{+2\pi} e_n(\rho) \cos \phi e^{-jk_\rho \rho \cos(\phi-\psi)} d\phi \rho d\rho \\
 &= \int_0^{+\infty} e_n(\rho) \rho \int_0^{+2\pi} \cos \phi e^{-jk_\rho \rho \cos(\phi-\psi)} d\phi d\rho
 \end{aligned} \tag{5.61}$$

Since

$$\int_0^{+2\pi} \cos \phi e^{-jk_\rho \rho \cos(\phi-\psi)} d\phi = -j2\pi J_1(k_\rho \rho) \cos \psi \tag{5.62}$$

where ψ is the angular quantity in the spectral domain and can be derived from $k_\rho e^{j\psi} = k_x + jk_y$. Substituting Eq. (5.62) into Eq. (5.61), the spectrum of the x component becomes

$$\mathcal{F}_{x_n}(k_\rho, \psi) = -j2\pi \cos \psi \int_0^{+\infty} e_n(\rho) J_1(k_\rho \rho) \rho d\rho \tag{5.63}$$

For a coaxial line whose inner and outer radii are a and b , respectively, the transverse electric components $e_n(\rho)$ is zero when $\rho < a$ or $\rho > b$. Therefore, I have

$$\mathcal{F}_{x_n}(k_\rho, \psi) = -j2\pi \cos \psi \int_a^b e_n(\rho) J_1(k_\rho \rho) \rho d\rho \tag{5.64}$$

Substituting Eq. (4.29) into Eq. (5.64), the spectra of the x components

of TEM mode and TM_{0n} modes can be obtained. For TEM mode,

$$\begin{aligned}\mathcal{F}_{x_0}(k_\rho, \psi) &= -\frac{j2\pi \cos \psi}{\sqrt{2\pi \ln b/a}} \int_a^b J_1(k_\rho \rho) d\rho \\ &= -\frac{j2\pi \cos \psi}{k_\rho \sqrt{2\pi \ln b/a}} [J_0(k_\rho a) - J_0(k_\rho b)]\end{aligned}\quad (5.65)$$

For TM_{0n} modes,

$$\begin{aligned}\mathcal{F}_{x_n}(k_\rho, \psi) &= -j2\pi \cos \psi N_{0n}^e \int_a^b \frac{\chi_{0n}}{a} Z' \left(\frac{\chi_{0n}}{a} \rho \right) J_1(k_\rho \rho) \rho d\rho \\ &= j2\pi \cos \psi N_{0n}^e \int_a^b \frac{\chi_{0n}}{a} \left[J_1 \left(\frac{\chi_{0n}}{a} \rho \right) Y_0(\chi_{0n}) - Y_1 \left(\frac{\chi_{0n}}{a} \rho \right) J_0(\chi_{0n}) \right] J_1(k_\rho \rho) \rho d\rho \\ &= \frac{j4N_{0n}^e k_\rho \cos \psi}{(\chi_{0n}/a)^2 - k_\rho^2} \left[J_0(k_\rho a) - \frac{J_0(\chi_{0n})}{J_0(\chi_{0n}b/a)} J_0(k_\rho b) \right]\end{aligned}\quad (5.66)$$

Similarly, for the y component, I have

$$\mathcal{F}_{y_0}(k_\rho, \psi) = -\frac{j2\pi \sin \psi}{k_\rho \sqrt{2\pi \ln b/a}} [J_0(k_\rho a) - J_0(k_\rho b)] \quad (5.67)$$

for TEM mode and

$$\mathcal{F}_{y_n}(k_\rho, \psi) = \frac{j4N_{0n}^e k_\rho \sin \psi}{(\chi_{0n}/a)^2 - k_\rho^2} \left[J_0(k_\rho a) - \frac{J_0(\chi_{0n})}{J_0(\chi_{0n}b/a)} J_0(k_\rho b) \right] \quad (5.68)$$

for TM_{0n} modes.

Now substituting Eq. (5.65)-Eq. (5.68) into Eq. (5.28) and after some manipulations, the matrix $\mathbf{U}$'s element becomes

$$u_{nn'} = \frac{1}{2\pi\omega\mu} \int_0^{+\infty} \frac{1}{\mathbb{D}} \mathcal{E}_n(k_\rho) \mathcal{E}_{n'}(k_\rho) \mathcal{A}(k_\rho) k_\rho dk_\rho \quad (5.69)$$

where

$$\mathcal{E}_n(k_\rho) = \begin{cases} -\frac{j2\pi}{k_\rho^2 \sqrt{2\pi \ln b/a}} [J_0(k_\rho a) - J_0(k_\rho b)] \\ \frac{j4N_{0n}^e}{(\chi_{0n}/a)^2 - k_\rho^2} \left[J_0(k_\rho a) - \frac{J_0(\chi_{0n})}{J_0(\chi_{0n}b/a)} J_0(k_\rho b) \right] \end{cases} \quad (5.70)$$

$$\begin{aligned} \mathcal{A}(k_\rho) = & k_\rho^2 \left\{ -\mathbb{J}_2^2 k_1^2 + \frac{\mathbb{J}_2 \mathbb{F}_1 \mathbb{F}_2}{\epsilon_r} k_1^2 [(\epsilon_r^2 - 1)k_1^2 + (\epsilon_r + 1)k_{z_1}^2] - \frac{\mathbb{F}_1^2 \mathbb{F}_2^2}{\epsilon_r} k_1^2 k_{z_2}^4 \right. \\ & \left. - j\mathbb{J}_1 \mathbb{J}_2 \mathbb{F}_2 (\epsilon_r + 1) k_1^2 k_{z_1}^2 + \mathbb{J}_1^2 \mathbb{F}_2^2 \epsilon_r k_1^2 k_{z_1}^2 + j\mathbb{J}_1 \mathbb{K}_1 \mathbb{K}_2^2 k_1^2 \left(\epsilon_r + \frac{1}{\epsilon_r} \right) \right\} \end{aligned} \quad (5.71)$$

and $\mathbb{D}$ is given in Eq. (5.10). Due to the nulls of $\mathbb{D}$, the integrand in Eq. (5.69) has poles along the integration path, which represent the modes of surface waves. Therefore, the theorem of residue is employed to remove the poles, the process of which has been illustrated in Section 5.3. After that, the integrand becomes smooth piecewise so that the GPOF method and Gaussian quadrature can be applied to evaluate the integral.

5.6.2 Results and Discussions

Now let's consider an open-ended coaxial line mode with $a = 3.5mm$ and $b = 7mm$. The superstrate configuration is the same as the one used in Section 5.4, where $h_1 = 14.993mm$, $h_2 = 2.3473mm$ and $\epsilon_r = 10.2$. In our approximation, 5 modes are considered, which are TEM and TM_{0n} modes ($i = 1, 2, 3, 4$). 100 sample points are adopted by the GPOF method, which results in a series of 30 exponential functions. It is found that the average computation time consumed by this method is 1/5 of that by direct summing up the integrand.

The reflection coefficient results with respect to frequency are shown in Fig. 5.32. It is seen in the figure that the results obtained by my method agree with those by Ansoft's HFSS, which therefore verifies the accuracy of my method in calculating the reflection coefficient of an open-ended coaxial line radiating through the two-layer superstrate.

The calculated radiation pattern is shown in Fig. 5.33. Since the dominant TEM mode is omnidirectional, which therefore results in a null point at $k_\rho = 0$, the radiation pattern also has a null in the direction of $\theta = 0$. This is in agreement with the conclusion drawn in subsection 5.5.3.

5.7 Conclusions

An efficient full-wave analysis of open-ended waveguides radiating through a two-layer superstrate has been presented in this chapter. The field equations have been set up in form of cascaded matrices, through which the spectral dyadic Green's function for the multi-layer superstrate has been derived. The GPOF method and Gaussian quadrature have been proposed to evaluate the Sommerfeld integral, which therefore showed that the computation efficiency has been greatly improved. After the reflection coefficient matrix was obtained, the radiation pattern and the directivity have been calculated. An open-ended WR-90 waveguide with a specific two-layer superstrate has been studied to verify the efficiency of my method. Numerical results have shown that the proposed method is very accurate and efficient. The convergence behaviors of my method and the influence of the superstrate's dimensions on the open-ended waveguide's directivity have also been examined. The effect of the two-layer superstrate on the radiation pattern and directivity of the open-ended waveguide has been analyzed and the influences of

(a) Magnitude.

(b) Phase.

Figure 5.32 3-D view of an open-ended rectangular waveguide radiating through a two-layer superstrate.

Figure 5.33 Radiation pattern of an open-ended coaxial line through a two-layer superstrate.

superstrate and the excitation have been discussed, respectively. The analysis of an open-ended coaxial line radiating through a two-layer superstrate has been provided to show the effectiveness of our method in analyzing an omnidirectional model.

Chapter 6

Experimental Studies

6.1 Introduction

In Chapter 5, the radiation of open-ended waveguides through a two-layer superstrate has been studied by theoretical analysis. It has been found that a very high directivity can be obtained if the two-layer is of a specific configuration, which is very useful in high-gain applications. But there are two intrinsic issues in the theoretical model: (1) The ground and superstrate can not be infinitely large, then, what are the sufficient sizes? (2) The matching at the opening of the waveguide is usually not very good, which results in a high return loss and is unfavorable in antenna design. For the first issue, a parametric study on the size of the superstrate and ground plane is carried out; for the latter one, a probe-fed cavity-backed slot is employed, since a proper cavity may acts as an impedance matching network, the return loss performance can be improved.

Cavity-backed slot (CBS) antennas are extensively used in satellite communications [117] and airborne phased arrays [118] because of their unique features: (i) they can provide a unidirectional radiation [119]; (ii) mutual

coupling between elements is relatively low, which is desirable in the design of phased arrays [118]; and (iii) they can be readily flush mounted to flying objects' surface [120]. CBS antennas can either be fed by a coaxial probe [121], microstrip line [122], or co-planar waveguide (CPW) [123].

Contributions are made in the following aspects [124]. Firstly, The measured gain of a single slot antenna can achieve 16.7dBi, which is 10.7dB higher than that of a usual slot antenna. Secondly, the parametric study on the size of the superstrate and ground plane are presented, which answers the first question prompted at the beginning of this section. Thirdly, a two-slot array radiating through the two-layer superstrate is studied, which shows the gain is 1.7dB higher than that of a single slot. Finally, the radiation of a rationally arranged four-slot array through the two-layer superstrate is investigated. Both the high-gain effect of superstrate and its excellent circular polarization performance are demonstrated. All the simulations in this chapter are carried out using Ansoft's HFSS except for those mentioned otherwise.

The rest of the chapter is organized as follows. Three high-gain slot antennas backed cavities, including a single-aperture, a two-slot linear array and a four-slot spiral array of circular polarization, are fabricated and examined in Sections 6.2, 6.3 and 6.4, respectively. Both simulated and measured results are provided for these three antennas to show the high gain effect of the proposed two-layer superstrate. Conclusions are given in Section 6.5.

(a) three-dimensional view

(b) cross-sectional view

Figure 6.1 Geometry of a single-aperture antenna covered with a two-layer superstrate (view is not to scale).

6.2 Single Aperture Antenna

6.2.1 Antenna Geometry

A single-aperture antenna operating at 9.5GHz is introduced in this section. The geometry of the antenna is shown in Fig. 6.1. A rectangular slot is cut at the center of a square ground plane. The size of the ground plane is $200mm \times 200mm$, which is electrically large and the diffraction from its edges can be minimized. The dimension of the slot is $25mm \times 5mm$. On the bottom side of the ground plane, the slot is enclosed by a rectangular cavity. The depth of the cavity is $18mm$ and dimension of its cross section is the same as the slot. This cavity is fed by a probe, which is extended from a Sub-Miniature A (SMA) connector. It is found that a good impedance match is achieved when the feeding position is placed at the center of the wide edge and $8mm$ below the slot while the length of the probe projected into the cavity is $4.5mm$. The radius of the probe is $0.635mm$, which is also the radius of the inner conductor of a SMA connector. On the top side of the ground plane, the slot is covered by a square two-layer superstrate. The side length of the superstrate is $80mm$. The size of the superstrate is not as large as the ground plane because it is found that such a size is large enough to generate a uniform field distribution on the top surface of the superstrate that leads to the expected high gain. As pointed out in Chapter 5, a high gain can be obtained if ϵ_r , the dielectric constant of layer 2, is very large. Therefore, RT/Duroid 6010LM, whose dielectric constant is 10.2 [125], is selected as the material of layer 2. Layer 1 of the superstrate is a styrofoam slab and its dielectric constant is 1.03 [126], which is very close to that of free space. The thicknesses of the two layers are $2.54mm$

TABLE 6.1 Parameters of the single-aperture antenna covered with a two-layer superstrate.

Parameter	Dimension(mm)
Side length of the square ground plane (g)	200
Side length of the square superstrate (s)	80
Slot length (l)	25
Slot width (w)	5
Thickness of the Duroid 6010 layer (t)	2.54
Thickness of the Styrofoam layer (h)	16
Cavity depth (d)	18
Depth of feeding position (u)	8
Probe length (p)	4.5

and $16mm$, respectively, so that the conditions represented by Eq. (5.56) can be satisfied for $\theta = 0$. It is noticed that these two thicknesses are not exactly in accordance with Eq. (5.56) because the superstrate is not infinitely large. Small adjustment is then required to maximize the antenna gain. The parameters of the single-aperture antenna covered by a two-layer superstrate are listed in Table 6.1.

6.2.2 Results and Discussions

Fig. 6.2 provides the simulated and measured return loss results of the single-aperture antenna. It is seen that there are two neighboring resonant points: one at about 9.5GHz and the other near 10GHz. The -10dB return loss bandwidth of this single-aperture antenna ranges from 9.3GHz to 10.5GHz, totally 1.2GHz or 12% of the center frequency. The return loss performance of this cavity-backed slot antenna is much better than that of an open-ended waveguide as shown in Fig. 5.12. The abrupt termination of the waveguide leads to impedance mismatching at the interface between the waveguide and the open area, which can deteriorate the return loss performance of a simple open-ended waveguide. However, for a CBS antenna, a good impedance

Figure 6.2 Simulated and measured return loss results of the single-aperture antenna.

matching at the coaxial port can be realized if the cavity and the probe are carefully designed and our antenna is a good example. Simulated results for a larger superstrate ($s = 120mm, 160mm$) are also provided to show the influence of the superstrate size on the return loss performance. It is seen that as the superstrate size is enlarged to $120mm$, the return loss increases slightly while the $1.2GHz$ bandwidth does not shrink. The curves for $s = 120mm$ and $s = 160mm$ are almost identical, which indicates that the return loss results converge as the size is larger than $120mm$.

Fig. 6.3 shows the simulated electric field distribution on the upper face of the superstrate. It is seen that the field is quite uniform in direction and phase, which will reasonably result in a high gain in the upward direction, as expected.

Fig. 6.4 and Fig. 6.5 show the simulated and measured radiation pat-

Figure 6.3 Simulated electric field distribution on the upper face of the superstrate for a single-aperture antenna ($f = 9.5GHz$).

terns of the single-aperture antenna in E-plane and H-plane, respectively. In both figures, the measured and simulated results are in very good agreement. The side lobe level is lower than -15dB. The measured gain of the single-aperture antenna is 16.7dBi. This is much higher than that of the single-aperture antenna without the superstrate, which is around 6dBi. The effective aperture size obtained using Eq. (2.1) is $3716mm^2$ and the aperture efficiency is 0.58. This gain-enhancement performance is actually significant since the uniform field distribution on such an large area is realized by only a two-layer superstrate of a little more than half a wavelength in thickness. Fig. 6.6 presents the variation of the single-aperture antenna's gain with respect to frequency. It is seen that the maximum measured gain is achieved at 9.5GHz and the bandwidth gain product is about 28GHz. As the operating frequency shifts away from 9.5GHz, the conditions represented by Eq. (5.56) are not satisfied, which results in the decrease of the antenna gain.

Figure 6.4 Simulated and measured E-plane radiation pattern (scaled in dBi) of the single-aperture antenna ($f = 9.5GHz$, $\phi = 0^\circ$).

Figure 6.5 Simulated and measured H-plane radiation pattern (scaled in dBi) of the single-aperture antenna ($f = 9.5GHz$, $\phi = 90^\circ$).

Figure 6.6 Measured single-aperture antenna gain with respect to the operating frequency.

6.2.3 Parametric Study

A parametric study is carried out in this subsection to show the effect of the horizontal dimensions of the superstrate and ground plane on the single-aperture antenna's gain. It should be mentioned here that the length of the probe may required to be adjusted for impedance matching. Unspecified dimensions of the slot antenna are can be found in Table 6.1.

Fig. 6.7 shows the variation of the gain vs. the ground plane size of the single-aperture antenna. It is found that when the ground plane size is less than 2.5 wavelengths, the slot antenna's gain is considerably affected by its size and the antenna gain increases rapidly with the increase of the ground plane size. However, when the size of the ground plane is larger than 2.5 wavelengths, the antenna gain does not increase monotonically any longer and converges gradually to the gain obtained in the case where both s and g approach to the $+\infty$.

The relationship between the antenna gain and the side length (s) of

Figure 6.7 Variation of the single-aperture antenna's gain with respect to the size of the ground plane.

the square superstrate is illustrated in Fig. 6.8. The antenna gain increases rapidly with the enlargement of superstrate size until s reaches around 3 wavelengths. Further enlargement of the superstrate size will result in slight oscillation of the antenna gain, which eventually converges to the gain of the antenna with its superstrate being infinitely large.

6.3 Two-Slot Array

6.3.1 Array Configuration

A two-slot array is described in this subsection to examine the gain-enhancement effect of the same two-layer superstrate on the slot array configuration. The geometry of the two-slot array is depicted in Fig. 6.9. All parameters in this figure are the same as those of the single-aperture, which are listed in Table 6.1. A new parameter here is $e = 15mm$, which represents the distance

Figure 6.8 Variation of the single-aperture antenna's gain with respect to the side length of the square two-layer superstrate.

between the two radiating slots from edge to edge as shown in the figure.

Feeding the two slots from the same direction is more commonly used in practical applications, but this will make it very hard to secure the cable to the SMA connector mounted between the two-cavities of our experimental model. The solution to this problem is to arrange the two SMA connectors symmetrically as shown in Fig. 6.9. When this design is applied, both SMA connectors can be easily secured, but Port 1 and Port 2 should have a phase difference of π , which results in counteraction of the fields generated by Slot 1 and Slot 2 in the direction of $\theta = 0$. As has been demonstrated in Subsection 5.5.3, this counteraction will result in a null in the upward direction. Therefore, a compensation for this phase difference is required, which is realized by a delay line of half a wavelength connecting to Port 2.

Figure 6.9 Configuration of the two-slot array.

6.3.2 Results and Discussions

Fig. 6.10 provides the simulated and measured S-parameters at the SMA ports of the two-slot array. The return loss at Port 1 and the mutual coupling between the two ports are plotted in this figure. The return loss at Port 2 is omitted because of the symmetrical structure. The measured -10dB return loss bandwidth of one slot element is from 9.3GHz to 10.5GHz, which is slightly narrower than the simulated one. The difference between the simulated and measured curves beyond 10GHz is due to the fact that there are slight dimensional differences due to fabrication tolerance between the measured model and the simulated one. The mutual coupling between the two elements is less than -13dB, as shown in Fig. 6.10.

Fig. 6.11 shows the simulated and measured results of E-plane and H-plane radiation patterns of the two-slot array. The side lobe level is lower than that of the single-aperture antenna. The measured gain of this two-slot array is 18.4dBi, which is 1.7dB rather than 3dB higher than that of a single element. It is noticed that both the two slot elements are off-centered

Figure 6.10 Simulated and measured S-parameters of the two-slot array.

from the center of the square superstrate and the slots are now closer to the edge of the superstrate, which is similar to the situation with a smaller superstrate covering the each slot elements. The gain of each element in the two-slot array is now lower than that of the single-aperture antenna. The lower gain of two elements in the array configuration accounts for the gain increase by 1.7dB for the two-slot array, instead of 3dB. From the measured gain results, it can be concluded that the two-layer superstrate can greatly enhance the gain of the two-slot array.

6.4 Four-Slot Array of Circular-polarization

6.4.1 Array Configuration

After the study of a slot element and a two-slot array, a four-slot array of circular polarization is considered in this subsection. The geometry of the

(a) $\phi = 0^\circ$

(b) $\phi = 90^\circ$

Figure 6.11 plane radiation pattern (scaled in dBi) of the two-slot array ($f = 9.5GHz$).

array antenna is shown in Fig. 6.12. It is seen that the four slots are arranged in a rotationally symmetrical manner. All parameters of the four-slot array are the same as those listed in Table 6.1 except m and n , which are used to locate the position of each slot. In this slot array, $m = 5mm$, $n = 7.5mm$. The label $R - R$ in Fig. 6.12(a) marks the reference plane where the cross-sectional view in Fig. 6.12(b) is located. The photo of the fabricated antenna array is given in Fig. 6.13, where the two-layer superstrate was processed to be transparent so that the four slots covered by it can be seen.

6.4.2 Results and Discussions

The return loss of one slot element in the four-slot array configuration is depicted in Fig. 6.14. The measured -10dB return loss bandwidth shows the band is from about 9.2GHz to 11.3GHz, which is slightly wider than both the single-aperture configuration and the two-slot array. This band extension is due to the mutual coupling between elements.

Fig. 6.15 presents the mutual coupling between slot elements in the four-slot array configuration. Only S_{12} and S_{13} are provided because of symmetry of the array. It is seen that the coupling to element 1 from element 3 is much higher than from element 2, which indicates that mutual coupling between parallel elements with the same polarization is more significant. The coupling between elements 1 and 3 is close to -10dB at 9.5GHz which is due to the small separation between them.

Since the four slots are arranged spirally, they are fed with equal amplitude and 90° phase difference in clockwise direction so as to produce circularly polarized fields in the broadside direction. This function is realized by a cascaded Wilkinson power divider and the phase difference is generated by

(a) Top view

(a) Corss-section view on the reference plane (R-R).

Figure 6.12 Geometry of the four-slot array.

Figure 6.13 Photo of the four-slot array with the superstrate disposed to be semitransparent.

Figure 6.14 Measured and simulated return loss results of one element in the four-slot array.

Figure 6.15 Measured and simulated mutual coupling between elements of the four-slot array.

four microstrip delay lines of different length. A photo of the feeding circuit is shown in Fig. 6.16.

Fig. 6.17 shows the simulated and measured radiation pattern of the four-slot array. It is seen that the two curves agree with each other very well and the measured gain is 18.4dBi, the same as that of the two-slot array. Without the superstrate, the gain of the slot array is only 9dBi. Therefore, a 9.4dBi gain-enhancement effect of the superstrate is validated by this slot array antenna. The 3dB beamwidth is about 20° . When $\theta = \pm 20^\circ$, the power density is 10dB less the peak in $\theta = 0^\circ$. The side lobe is below -15dB due to the close element spacing.

Fig. 6.18 shows the axial ratio result of the four-slot array antenna with respect to the elevation angle θ . It is seen that the measured result agree with the simulated one very well in the range of $-20^\circ < \theta < 20^\circ$, in which the

Figure 6.16 Photo of the feeding circuit.

Figure 6.17 Measured and simulated radiation pattern (scaled in dBi) of the four-slot array ($f = 9.5GHz$, $\phi = 0^\circ$).

Figure 6.18 Axial ratio of the four-slot array with respect to θ at 9.5GHz ($\phi = 90^\circ$).

axial ratio is lower than 3dB. If the slim radiation pattern of the radiation pattern is considered, it can be found that a good circularly polarized field can be produced within the spatial coverage of the main lobe. Therefore this antenna is a very good candidate for circularly polarized high-gain antenna. When $|\theta| > 20^\circ$, the agreement of the two curves becomes worse. This is because of the lower power density at this range and signal is so weak that it is easy to be overwhelmed by noise.

Fig. 6.19 shows the axial ratio result of the four-slot array antenna with respect to frequency. The two curves are obtained by measurement and simulation using HFSS, respectively. It is noticed that they are all below 3dB from 9GHz to 10.2GHz though the measured axial ratio is not as low as the simulated one. The reason for the difference is the usage of the delay lines, which is designed for the center frequency 9.5GHz and the delay effect

Figure 6.19 Axial ratio of the four-slot array with respect to frequency at $\theta = 0^\circ$ ($\phi = 90^\circ$).

of which is frequency dependent. As the operating frequency moves away from 9.5GHz, the phase difference between the four element is no longer 90° , therefore, the axial ratio increases on both end of the measured curve. While in HFSS, it very easy to set the phase difference between port to be exactly 90° over any frequency band, therefore, the simulated result given by HFSS is much better than the measured one.

6.5 Conclusions

The gain-enhancement effect of a two-layer superstrate on cavity-backed slot antenna and arrays has been studied in this chapter. It has been shown that if the thickness and dielectric constant of each layer of the superstrate are properly chosen, an aperture antenna can achieve a very high gain. Simulated and experimental results have shown that this two-layer superstrate

can enhance the gain of the single-aperture slot antenna to 16.7dBi, which is much higher than that of a conventional slot antenna. In our experiment, a square superstrate of $80mm \times 80mm$ in size has been proven to be large enough to show obvious gain-enhancement effect and the -10dB return loss bandwidth of our high-gain cavity-backed slot antenna has been found to be about 12%.

The gain-enhancement effect of the two-layer superstrate on a two-slot array or a four-slot array of circular polarization has also been studied. The antenna gain results for both arrays are 18.4dBi, which has verified that the proposed two-layer superstrate is very effective in gain enhancement for slot arrays. Moreover, our four-slot spiral array has very good circular polarization performance since its axial ratio is lower than 3dB within the range of half power beamwidth.

Chapter 7

Conclusion and Recommendations

7.1 Conclusion

The work presented in this thesis focuses on the analysis of open-ended waveguides radiating through a layered medium. After a brief literature review on relevant works, surface waves propagating in a layered medium has been studied in Chapter 3. Since the surface wave exists in the form of certain modes in a layered medium and these modes can be classified as either *TM* modes or *TE* modes according to the directions of their electric and magnetic components, field equations have been formulated for these two types of modes, respectively. The derived field equations have been given in a recursive form, in which the electric and magnetic fields in the i th layer are expressed in terms of the fields in the $(i - 1)$ th layer. An exception is the fields in the first layer, which is given explicitly. Since I do not consider the situation where the excitation is placed in the half free space, therefore the term with $e^{jk_{zN+1}z}$ is removed. This condition has been utilized to build tran-

scendental equations for TM modes and TE modes, respectively, through which the propagation constant can be evaluated. After some observations on the transcendental equations, I have found that it is more convenient to describe the surface waves propagating in the layered medium by relative quantities such as normalized propagation constant, relative thickness and relative permittivity of each layer, which have been defined in Chapter 3. By solving the transcendental equations numerically, the parametric studies of surface waves in one- and two-layer media have been provided, respectively. This work not only shows how the layered medium affects the propagation of a surface wave, but also helps to find the number of poles and locate the positions of poles in the Sommerfeld integrand, which will be used in later work.

A numerical method has been proposed in Chapter 4 to analyze open-ended waveguide radiating into a half space. In order to obtain the reflection coefficient matrix at the opening of the waveguide of arbitrary cross-section, field equations has been formulated by field matching along the opening, which has then been converted into the spectral domain through the Sommerfeld identity. Further discussion has been carried out on an open-ended coaxial line, in which the GPOF method has been employed to approximate the part of the integrand without the denominator k_z . Therefore the spectral integrations have been analytically worked out. Since only some sample points are needed to determine the coefficients of the exponential series, the computational efficiency has been greatly improved. Next I have combined the generalized pencil of matrix method with Gaussian quadrature to solve the reflection coefficient matrix of an open-ended rectangular waveguide, the spectrum of which is no longer omnidirectional. The GPOF method

has been adopted to calculate the spectral integrals in the radial direction and Gaussian quadrature has been employed to facilitate the integration in the azimuth direction. It has been found that the proposed method is very efficient in solving the radiation problem of open-ended waveguides.

After applying the proposed method to the analysis of open-ended waveguides radiating into a half free space, the method is used to analyze the radiation of an open-ended rectangular waveguide through a layered superstrate. First, the spectral dyadic Green's function in layered medium is derived, which is also very convenient to be extended to arbitrary superstrate configurations. Particularly, the spectral dyadic Green's function of a specific two-layer superstrate has been given explicitly. With the spectra of modes in the waveguide being known, an integration equation has been formulated through field matching along the opening of the waveguide to solve the reflection coefficient matrix. The method proposed in Chapter 4 has now been slightly modified to numerically evaluating the Sommerfeld integrals. Different from the situation when an open-ended waveguide radiating directly into a half free space, the Sommerfeld integrand has poles, which correspond to surface waves in the layered superstrate, should be removed by the residue theorem before applying the GPOF method and Gaussian quadrature. An example of WR-90 has been given to demonstrate the good performance of the method in terms of accuracy, efficiency and convergence. The principle of high directivity has also been studied and the requirement for the Green's function of the superstrate and the spectrum of the excitation has been given. Besides the rectangular waveguide, the excitation by an open-ended coaxial line has also been analyzed, in which only the numerical evaluation of a radial integral is required due to the omnidirectional

integrand.

Experimental studies have been carried out to verify the gain-enhancement effect of two-layer superstrate studied in Chapter 5 on cavity-backed slot antennas. Both the experimental and simulated results have shown that the gain of a single slot antenna can be greatly improved by the superstrate if the high gain conditions are met. The influences of the superstrate's and ground's sizes on the antenna's gain have also been studied. After that, a two-slot array and a four-slot spiral array radiating through the superstrate have been analyzed, which has shown that the two-layer superstrate can also greatly improve the gain of antenna arrays.

7.2 Recommendations for Further Research

Although many efforts have been made on the study of open-ended waveguide radiating through a layered superstrate as has been shown in this thesis, there is further work on this topic that deserves to be studied. Some of them are recommended as follows:

1. Theoretical formulation and experimental results of a rectangular waveguide radiating through a two-layer superstrate have been provided in this thesis, other frequently used waveguides such as circular waveguides or elliptical waveguides also deserves consideration. Since the excitation is changed, different features of the radiation could be exhibited.
2. As shown in this thesis, the GPOF method has been adopted to accelerate the calculation of the spectral integrals. Beyond the validity of this method, how to optimize the parameters of the GPOF method in

numerical evaluation the Sommerfeld integral is of great value since it can maximize the computational efficiency. Besides the GPOF, there are other algorithms that can realize the approximation such as the Prony's method. A comparison between these methods is also advised.

3. Quantitative evaluation of the dissipative loss in surface wave is another interesting point. This work will contribute to design of high-gain antennas when the dissipation caused by surface waves is a significant factor to be considered.
4. Although some experimental results of slot arrays have been given in the thesis, a theoretical analysis on the mutual coupling between antenna elements under a multi-layer superstrate would extend the understanding of the antenna model.

List of Author's Publications

Journal Papers

1. W. Tan, L.C. Lee and Z. Shen, "A circularly polarized microstrip-fed T-slot antenna array on a conducting cylinder," *IEEE Antennas Wireless Propagat. Letter*, vol. 5, no. 1, pp. 507-510, Dec. 2006.
2. W. Tan and Z. Shen, "Efficient analysis of open-ended coaxial line using Sommerfeld identity and matrix pencil method," *IEEE Microwave Wireless Components Letters*, vol. 18, no. 1, pp. 7-9, Jan. 2008.
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4. W. Tan, Z. Shen and Z. Shao, "Radiation of high-gain cavity-backed slot antennas through a two-Layer superstrate," *IEEE Antennas Propagat. Magazine*, vol. 50, no. 3, pp.78-87, Jun. 2008.
5. W. Tan and Z. Shen, "Efficient full-wave analysis of an open-ended waveguide radiating through a multilayer superstrate," Accepted by *Radio Science*, 2008.

Conference Papers

1. W. Tan, Z. Shen, Z. Shao, and M. Fujise, "A gain-enhanced microstrip-fed cavity-backed slot antenna," *Proc. Asia-Pacific Microwave Conference*, vol. 2, pp. 4-7, Dec. 2005.
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3. Y. Liu, Z. Shen, B. Zheng and W. Tan, "A dual-band beam-switched slot array for GSM 900/1800MHz," *Proc. Asia-Pacific Microwave Conference*, Dec. pp. 2070 - 2073, 2006.
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APPENDIX A

Surface Wave Equations in a Layered Medium

In this appendix, the equations for the transverse fields in a multilayer medium are to be derived for TM and TE mode surface waves, respectively.

First, Maxwell's equations are given as follows

$$\begin{cases} \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \\ \nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \end{cases} \quad (\text{A-1})$$

In an ideal dielectric material, electric current $\vec{J} = 0$. Therefore, I have the following two sets of independent equations

$$\begin{cases} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \epsilon \frac{\partial E_x}{\partial t} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \epsilon \frac{\partial E_y}{\partial t} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \epsilon \frac{\partial E_z}{\partial t} \end{cases} \quad (\text{A-2})$$

$$\begin{cases} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial H_z}{\partial t} \end{cases} \quad (\text{A-3})$$

Since I have assumed that the surface wave is propagating along the y axis, which means $\frac{\partial}{\partial x} = 0$. Therefore, from Eq. (A-2) and Eq. (A-3), the following two sets of equations are obtained, which are for TM and TE modes, respectively.

$$\text{TM Modes} \begin{cases} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \\ \frac{\partial H_x}{\partial z} = j\omega\epsilon E_y \\ -\frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \end{cases} \quad (\text{A-4})$$

$$\text{TE modes} \begin{cases} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x \\ \frac{\partial E_x}{\partial z} = -j\omega\mu H_y \\ -\frac{\partial E_x}{\partial y} = -j\omega\mu H_z \end{cases} \quad (\text{A-5})$$

From Eq. (A-4), I have

$$\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} + \omega^2 \mu \epsilon H_x = 0 \quad (\text{A-6})$$

Considering the unidirectional propagation along the positive y axis, H_x is

$$H_x = A \left(e^{-jk_z z} - R e^{jk_z z} \right) \quad (\text{A-7})$$

where $A = C e^{-jk_y y}$, C is the magnitude constant, R is the relative magni-

tude, the time harmonic $e^{j\omega t}$ is suppressed. Substituting Eq. (A-7) into Eq. (A-4), E_z and E_y are derived as follows,

$$\begin{cases} E_y = -A \frac{k_z}{\omega\epsilon} (e^{-jk_z z} + R e^{jk_z z}) \\ E_z = A \frac{k_y}{\omega\epsilon} (e^{-jk_z z} - R e^{jk_z z}) \end{cases} \quad (\text{A-8})$$

R can be obtained by applying the boundary conditions at all interfaces between layers. Therefore, the expressions of the fields in the i th layer in terms of $(i-1)$ th layer are found,

$$\begin{cases} H_{x_i} = \frac{1}{2} \left(H_{x_{i-1}}|_{z=d_{i-1}} - \frac{\omega\epsilon_i}{k_{z_i}} E_{y_{i-1}}|_{z=d_{i-1}} \right) e^{-jk_{z_i}(z-d_{i-1})} \\ \quad + \frac{1}{2} \left(H_{x_{i-1}}|_{z=d_{i-1}} + \frac{\omega\epsilon_i}{k_{z_i}} E_{y_{i-1}}|_{z=d_{i-1}} \right) e^{jk_{z_i}(z-d_{i-1})} \\ \quad = \cos k_{z_i}(z-d_{i-1}) H_{x_{i-1}}|_{z=d_{i-1}} + \frac{j\omega\epsilon_i}{k_{z_i}} \sin k_{z_i}(z-d_{i-1}) E_{y_{i-1}}|_{z=d_{i-1}} \\ E_{y_i} = -\frac{1}{2} \left(\frac{k_{z_i}}{\omega\epsilon_i} H_{x_{i-1}}|_{z=d_{i-1}} - E_{y_{i-1}}|_{z=d_{i-1}} \right) e^{-jk_{z_i}(z-d_{i-1})} \\ \quad + \frac{1}{2} \left(\frac{k_{z_i}}{\omega\epsilon_i} H_{x_{i-1}}|_{z=d_{i-1}} + E_{y_{i-1}}|_{z=d_{i-1}} \right) e^{jk_{z_i}(z-d_{i-1})} \\ \quad = \frac{j k_{z_i}}{\omega\epsilon_i} \sin k_{z_i}(z-d_{i-1}) H_{x_{i-1}}|_{z=d_{i-1}} + \cos k_{z_i}(z-d_{i-1}) E_{y_{i-1}}|_{z=d_{i-1}} \end{cases} \quad (\text{A-9})$$

Eq. (A-9) can be replaced by a matrix equation

$$\begin{bmatrix} H_{x_i} \\ E_{y_i} \end{bmatrix} = \mathbf{P}_i \mathbf{U}_i \mathbf{P}_i^{-1} \begin{bmatrix} H_{x_{i-1}}|_{z=d_{i-1}} \\ E_{y_{i-1}}|_{z=d_{i-1}} \end{bmatrix} \quad (\text{A-10})$$

where

$$\mathbf{P}_i = \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_{z_i}}{\omega\epsilon_i} \end{bmatrix} \quad (\text{A-11})$$

$$\mathbf{U}_i = \begin{bmatrix} \cos k_{z_i}(z - d_{i-1}) & j \sin k_{z_i}(z - d_{i-1}) \\ j \sin k_{z_i}(z - d_{i-1}) & \cos k_{z_i}(z - d_{i-1}) \end{bmatrix} \quad (\text{A-12})$$

In summary, the fields of TM mode in the layered medium are given as

$$\begin{bmatrix} H_{x_i} \\ E_{y_i} \end{bmatrix} = \begin{cases} \mathbf{P}_1 \mathbf{U}_1 \mathbf{P}_1^{-1} \begin{bmatrix} 2A \\ 0 \end{bmatrix} & i = 1 \\ \mathbf{P}_i \mathbf{U}_i \mathbf{P}_i^{-1} \begin{bmatrix} H_{x_{i-1}}|_{z=d_{i-1}} \\ E_{y_{i-1}}|_{z=d_{i-1}} \end{bmatrix} & i = 2, 3, \dots, N+1 \end{cases} \quad (\text{A-13})$$

Similarly, from Eq. (A-5), I have the following equations for TE modes

$$\begin{bmatrix} E_{x_i} \\ H_{y_i} \end{bmatrix} = \begin{cases} \mathbf{Q}_1 \mathbf{U}_1 \mathbf{Q}_1^{-1} \begin{bmatrix} 0 \\ 2A \end{bmatrix} & i = 1 \\ \mathbf{Q}_i \mathbf{U}_i \mathbf{Q}_i^{-1} \begin{bmatrix} E_{x_{i-1}}|_{z=d_{i-1}} \\ H_{y_{i-1}}|_{z=d_{i-1}} \end{bmatrix} & i = 2, 3, \dots, N+1 \end{cases} \quad (\text{A-14})$$

where

$$\mathbf{Q}_i = \begin{bmatrix} 1 & 0 \\ 0 & \frac{k_{z_i}}{\omega\mu_i} \end{bmatrix} \quad (\text{A-15})$$

APPENDIX B

Generalized Pencil of Functions Method

The generalized pencil of function (GPOF) method, which is introduced in [94], is to be briefly introduced in this appendix. Using this method, the complex spectrum of the target function are extracted, which are represented by exponential series.

$$y(k) \approx \sum_{m=1}^M b_m e^{s_m k}$$

where $\mathbf{y} = y(k)$ ($k = 0, 1, \dots, N - 1$) are the given sample sequence to be fitted, $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_M)$ and $\mathbf{s} = (\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_M)$ are the coefficients to be solved. N is the amount of sample points to be employed. M is the expected number of exponential series.

The adopted GPOF algorithm is illustrated as follows:

First, in order to extract the complex spectrum, two matrices $\mathbf{Y}_1$ and

$\mathbf{Y}_2$ are defined, whose elements are samples of the to-be fitted curve,

$$\mathbf{Y}_1 = \begin{bmatrix} y_0 & y_1 & \cdots & y_{N-M-1} \\ y_1 & y_2 & \cdots & y_{N-M} \\ \vdots & \vdots & \ddots & \vdots \\ y_{M-1} & y_M & \cdots & y_{N-2} \end{bmatrix}$$

and

$$\mathbf{Y}_2 = \begin{bmatrix} y_1 & y_2 & \cdots & y_{N-M} \\ y_2 & y_3 & \cdots & y_{N-M+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_M & y_{M+1} & \cdots & y_{N-1} \end{bmatrix}$$

The singular value decomposition (SVD) [128] of $\mathbf{Y}_1$ is given by

$$\mathbf{Y}_1 = \mathbf{U}\mathbf{D}\mathbf{V}^T$$

where $\mathbf{U}$ is a $M \times M$ matrix, $\mathbf{V}$ is a $(N - M) \times (N - M)$ matrix and $\mathbf{D}$ is a $M \times (N - M)$ diagonal matrix. The superscript H denotes the conjugate transpose of a matrix. Let's define another matrix $\mathbf{Z}$ as

$$\mathbf{Z} = \mathbf{D}^{-1}\mathbf{U}^H\mathbf{Y}_2\mathbf{V}'$$

where $\mathbf{V}'$ is a matrix that consists of the first M columns of $\mathbf{V}$. It is therefore found that $\mathbf{Z}$ is a $M \times M$ square matrix. Its eigenvalues are solved to be $\mathbf{d} = (d_1, d_2, \cdots, d_M)$, which are also used to calculate $\mathbf{s}$ through $\mathbf{s} = \ln \mathbf{d}$.

The coefficients b_m are obtained using the least square method.

$$\mathbf{b} = [b_1, b_2, \dots, b_M]^T = \mathbf{G}^{-1} \mathbf{y}$$

where

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{s_1} & e^{s_2} & \dots & e^{s_M} \\ \vdots & \vdots & \ddots & \vdots \\ e^{s_1(N-1)} & e^{s_2(N-1)} & \dots & e^{s_M(N-1)} \end{bmatrix}$$

and

$$\mathbf{y} = [y_1, y_2, \dots, y_{N-1}]^T$$

The superscripts -1 and T denote the pseudo-inverse of a rectangular matrix and transpose operation, respectively. Since both $\mathbf{s}$ and $\mathbf{b}$ are obtained, the original function $y(k)$ is now represented by

$$y(k) \approx \sum_{m=1}^M b_m e^{s_m k}.$$

APPENDIX C

Derivation of Eq. (4.39) and Eq. (4.40)

The detailed derivation process of Eq. (4.39) and Eq. (4.40) is to be provided in this appendix.

From [103], it is found that

$$\begin{aligned} J_s(k_\rho \rho) &= \frac{j^{-s}}{\pi} \int_0^\pi e^{jk_\rho \rho \cos \theta} \cos s\theta d\theta \\ &= \frac{j^{-s}}{2\pi} \left(\int_0^\pi e^{jk_\rho \rho \cos \theta} e^{js\theta} d\theta + \int_0^\pi e^{jk_\rho \rho \cos \theta} e^{-js\theta} d\theta \right) \\ &= \frac{j^{-s}}{2\pi} \left(\int_0^\pi e^{jk_\rho \rho \cos \theta} e^{js\theta} d\theta + \int_{-\pi}^0 e^{jk_\rho \rho \cos \theta} e^{js\theta} d\theta \right) \\ &= \frac{j^{-s}}{2\pi} \int_{-\pi}^\pi e^{jk_\rho \rho \cos \theta} e^{js\theta} d\theta \end{aligned} \tag{C-1}$$

It is noticed that the integrand is a periodic function with its period being 2π since s is an integer. Substituting θ in Eq. (C-1) by $\theta' - \phi - \pi$ and the

following equation is obtained,

$$\begin{aligned}
 J_s(k_\rho \rho) &= \frac{j^{-s}}{2\pi} \int_0^{2\pi} e^{jk_\rho \rho \cos(\theta' - \phi - \pi)} e^{js(\theta' - \phi - \pi)} d\theta' \\
 &= \frac{j^s}{2\pi} e^{-js\phi} \int_0^{2\pi} e^{-jk_\rho \rho (\cos \theta' \cos \phi + \sin \theta' \sin \phi)} e^{js\theta'} d\theta' \\
 &= \frac{j^s}{2\pi} e^{-js\phi} \int_0^{2\pi} e^{-jk_\rho (x \cos \theta' + y \sin \theta')} e^{js\theta'} d\theta' \tag{C-2}
 \end{aligned}$$

where

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

Multiplying both sides of Eq. (C-2) with $e^{js\phi}$, it becomes,

$$J_s(k_\rho \rho) e^{js\phi} = \frac{j^s}{2\pi} \int_0^{2\pi} e^{-jk_\rho (x \cos \theta + y \sin \theta)} e^{js\theta} d\theta \tag{C-3}$$

Now we consider the summation in Eq. (4.40). After applying Eq. (C-3)

to it, I have

$$\begin{aligned}
 & \sum_{s=-\infty}^{+\infty} J_s(k_\rho \rho) J_s(k_\rho \rho') \cos s(\phi - \phi') \\
 &= \sum_{s=-\infty}^{+\infty} \frac{1}{2} \left[J_s(k_\rho \rho) e^{js\phi} J_s(k_\rho \rho') e^{-js\phi'} + J_s(k_\rho \rho) e^{-js\phi} J_s(k_\rho \rho') e^{js\phi'} \right] \\
 &= \sum_{s=-\infty}^{+\infty} \frac{(-1)^s}{8\pi^2} \left[\int_0^{2\pi} e^{js\theta} e^{-jk_\rho(x \cos \theta + y \sin \theta)} d\theta \int_0^{2\pi} e^{-js\theta'} e^{-jk_\rho(x' \cos \theta' + y' \sin \theta')} d\theta' \right. \\
 &\quad \left. + \int_0^{2\pi} e^{-js\theta} e^{-jk_\rho(x \cos \theta + y \sin \theta)} d\theta \int_0^{2\pi} e^{js\theta'} e^{-jk_\rho(x' \cos \theta' + y' \sin \theta')} d\theta' \right] \\
 &= \frac{1}{8\pi^2} \left[\int_0^{2\pi} e^{-jk_\rho(x \cos \theta + y \sin \theta)} \int_0^{2\pi} e^{-jk_\rho(x' \cos \theta' + y' \sin \theta')} \sum_{s=-\infty}^{+\infty} e^{js(\theta - \theta' + \pi)} d\theta' d\theta \right. \\
 &\quad \left. + \int_0^{2\pi} e^{-jk_\rho(x \cos \theta + y \sin \theta)} \int_0^{2\pi} e^{-jk_\rho(x' \cos \theta' + y' \sin \theta')} \sum_{s=-\infty}^{+\infty} e^{-js(\theta - \theta' + \pi)} d\theta' d\theta \right] \\
 &= \frac{1}{8\pi^2} \int_0^{2\pi} e^{-jk_\rho(x \cos \theta + y \sin \theta)} \int_0^{2\pi} e^{-jk_\rho(x' \cos \theta' + y' \sin \theta')} \\
 &\quad \left[\sum_{s=-\infty}^{+\infty} e^{js(\theta - \theta' + \pi)} + e^{-js(\theta - \theta' + \pi)} \right] d\theta' d\theta \tag{C-4}
 \end{aligned}$$

where

$$x' = \rho' \cos \phi'$$

$$y' = \rho' \sin \phi'$$

Since [127]

$$\sum_{n=-\infty}^{\infty} \delta(x - 2n\pi) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} e^{-jsx},$$

Eq. (C-4) becomes

$$\begin{aligned}
& \sum_{s=-\infty}^{+\infty} J_s(k_\rho \rho) J_s(k_\rho \rho') \cos s(\phi - \phi') \\
&= \frac{1}{2\pi} \int_0^{2\pi} e^{-jk_\rho(x \cos \theta + y \sin \theta)} \int_0^{2\pi} e^{-jk_\rho(x' \cos \theta' + y' \sin \theta')} \sum_{n=-\infty}^{+\infty} \delta(\theta - \theta' + \pi + 2n\pi) d\theta' d\theta \\
&= \frac{1}{2\pi} \int_0^{2\pi} e^{-jk_\rho(x \cos \theta + y \sin \theta)} e^{jk_\rho(x' \cos \theta + y' \sin \theta)} d\theta
\end{aligned}$$

which is Eq. (4.40).