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**DYNAMICS OF QUANTUM GAS IN
NON-ABELIAN GAUGE FIELD**

MEHEDI HASAN

**SCHOOL OF PHYSICAL AND MATHEMATICAL
SCIENCES**

2021

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2021

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MEHEDI HASAN

Abstract

Experimental observation of *Zitterbewegung* is considered a holy grail of the modern physics since its debut in 1930 by Schrödinger. In this thesis we describe the first experimental observation of this effect in a two-dimensional ultracold atomic wave packet. In addition to that, we have also revealed the anisotropic nature of *Zitterbewegung*, in the presence of non-Abelian Gauge field.

To observe this effect, we have performed laser cooling in two major steps — 461 nm magneto-optical trap (MOT) and 689 nm MOT — and then loaded the atoms into an optical dipole trap. After performing optical pumping to one of the Zeeman states of the hyperfine ground-state, while atoms are inside the dipole trap, we have cooled down the strontium-87 atoms via evaporative cooling and the gas enters the quantum regime at a $\frac{T}{T_F} 0.21(4)$, with Temperature T and Fermi temperature T_F . As the temperature of the gas (40-50 nK) is far below the recoil temperature (230 nK) of the atoms, we have performed experiments with the wave packet that reveals the envisaged *Zitterbewegung* effect.

In order to demonstrate the anisotropy of the *Zitterbewegung*, we introduced a kick to the cold atomic wave packet so that the wave packet explores the momentum-dependence of the energy eigenstates of the dressed Hamiltonian. We reveal the complete anisotropic nature of the *Zitterbewegung* amplitude and the relation of the oscillation frequency with the energy-gap between the two bands. We speculate that this could be a new way to map the band-diagram of a multi-band system. All the results are analyzed and explained with analytic calculations coupled with numerical analysis.

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Chapter 1

Introduction

Dirac equation is considered as one of the building blocks of modern physics [4]. Most of the modern theories of particle and field, one way or another, are connected to this theory [5]. A consistent theory of quantum mechanics and special theory of relativity has provided us with a more transparent understanding of the atomic structure, and many more subsequent developments. This equation has not only guided us to our modern understanding, but also hinted us towards many new phenomena that were probably not envisioned by the Dirac himself.

One such modern example, in the context of condensed matter physics, is the field of topological matter and specially the emergence of quantum spin Hall effect. After the initial proposal of quantum spin Hall effect in graphene by Kane and Mele [6], building on the work of D. Haldane [7], an important observation was made by A. Bernevig and colleagues [8, 9]. The key ingredient for the experimental realization of quantum spin Hall effect is the understanding that — a band inversion inside a quantum well emulates a Dirac particle with negative mass, and then quantum spin Hall effect emerges out of this system in a natural way [10, 11]. The stage was already set by the work of Volkov and colleagues who had already pointed out such a band-inverted system, with PbSnTe/HgCdTe quantum well [12]. Soon after the theory paper of Bernevig and colleagues, quantum spin Hall effect was experimentally realized in a HgTe/(Hg,Cd)Te quantum well system [13]. After this experimental breakthrough, the field of topological matter has been at the forefront of the research in condensed matter [14–31], photonics [32–46], the cold atom community [47–60, 60–65], and many more related fields [39, 40, 66–69].

Another example of far-reaching effect of the Dirac equation, which is central to this thesis, is the *Zitterbewegung* of a particle that was predicted by Schrödinger, three years after the original paper of Dirac [70]. The *Zitterbewegung* or the trembling motion of electron inside atoms is responsible for the Darwin term in the fine structure Hamiltonian of hydrogen atom [71]. As was pointed out by Schrödinger that the order of magnitude of the *Zitterbewegung* is 10^{21} Hertz, therefore the direct observation remain elusive till today due to the limitation of the time-resolved experiments even in the state of the art time-resolved experiments [72–76].

With the wake of quantum simulation in the atomic physics, ultracold atoms or ions can simulate the same class of Hamiltonian as is found in nature, yet in a much controlled environment [77, 78]. The key idea is that, all the observables gets rescaled in the renormalized Hamiltonian. With this notion in mind, the *Zitterbewegung* has been studied theoretically and experimentally in the context of cold atoms [79–83]. The first experimental observation, via quantum simulation, was made in the group of C.F. Roos with trapped ion where they have simulated the one-dimensional Dirac equation and *Zitterbewegung* motion [79]. This effect was also observed with cold neutral atoms in the group of I. Spielman, for a one-dimensional artificial Abelian gauge field [81].

A key ingredient for the latter observation, was the presence of artificial gauge field induced by light-matter coupling. The role of artificial gauge field for neutral atoms is to emulate magnetic field for charged particles [55, 84–86]. The artificial magnetic field leading to the Lorentz force for neutral atoms, can be accomplished at least by two manners: one way is to rotate the quantum gas [87–95], and the other way is to induce geometric potentials via light-matter coupling [52, 96–104]. Note that one of the main motivation to emulate the Lorentz force for neutral particle was to induce a large magnetic flux per lattice plaquet so that the fractional quantum Hall regime becomes accessible in cold atom experiments [60–63, 105–107]. The latter can be obtain only with geometric potential [84]. Moreover with artificial gauge fields, the field have grown to see many spectacular phenomena with single particle physics from Hofstadter butterfly [53, 55], topological pumping [48–52, 57], dynamical gauge field [108–114], to name a few.

Broadly speaking, the artificial gauge field can be categorized by two main classes: Abelian and non-Abelian gauge field. The former gauge field is the one where the different components of the gauge field commute among themselves, and in

the latter case the components of the gauge field do not commute among themselves [84, 104, 115, 116]. The non-commutation of the components give rise to many interesting features to the system, namely non-Abelian Aharonov-Bohm effect, peculiar expansion dynamics in real-space, geometric quantum gate, and *Zitterbewegung* [83, 104, 117, 118], to name a few. There has been some recent development on the realization of two dimensional spin-orbit interaction system in bulk and in lattice [119–123].

One of the main motivations to employ quantum gas for quantum simulation is due to the availability of the extreme control over the system. Among different options for atoms in periodic table, the group-II atoms are particularly attractive due to the presence of a narrow transition $^1S_0 - ^3P_1$. For strontium which is the choice of atom for our setup has a linewidth of $2\pi \cdot 7.4$ kHz [1]. This narrow line is exploited in the magneto-optical trap (MOT) to reach few microkelvin temperature at the end of MOT (see Chapter-3 for detail). Another advantage of strontium is the extremely small Landé g -factor of order 10^{-4} , and this provides another advantage that the Zeeman levels of the ground state are less susceptible to ambient magnetic field [1].

The goal of this thesis is to experimentally realize the two ideas together, non-Abelian gauge field and *Zitterbewegung*, in the context of cold atom. There are two major contributions in this thesis:

1. We provide a complete description of the necessary condition for the appearance of *Zitterbewegung* and the relation among the components of gauge field, for SU(2) Hamiltonian class, as described in Chapter-2.
2. We experimentally reveal, for the first time, to the best of our knowledge, the anisotropic nature of *Zitterbewegung* in a two-dimensional non-Abelian gauge field, as described in Chapter-4.

The first contribution is a theoretical one while the second one is an experimental contribution. To accomplish the second one, we have laser-cooled the fermionic strontium-87 species and then load the atoms into an optical dipole trap to perform evaporative cooling. Afterwards, we turn on our gauge field with lasers to induce the non-Abelian gauge field. As the atoms spend time in gauge field, they undergo *Zitterbewegung* motion for an appropriately created initial internal state and momentum state.

Outline of the Thesis

The thesis is organized as follows:

- In Chapter-2, we provide the necessary theoretical background for the experimental results described in the chapters that follow. Here we provide a complete analysis on the appearance of *Zitterbewegung* in a SU(2) Hamiltonian class. We explicitly mention the relation among different components of gauge field and the scalar field, in the context of *Zitterbewegung*.
- In Chapter-3, we provide the description of our experimental set-up. We explain the cooling of 600 °C hot gas of strontium to a cold cloud of temperature $\approx 3 \mu\text{K}$. We describe the laser systems that was implemented during the period of my PhD. We briefly explain the cooling and trapping on the broad $2\pi \cdot 32 \text{ MHz}$ line (the 461 nm transition between 1S_0 and 1P_1 states). Afterwards, we describe the cooling and trapping on the intercombination line (the 689 nm transition between 1S_0 and 3P_1 states) with linewidth $2\pi \cdot 7.5 \text{ kHz}$. Here we capture more than 15 million atoms at a temperature $\approx 3 \mu\text{K}$, in this red magneto-optical trap(MOT). Afterwards we explain the loading and transferring atoms from the red MOT to the crossed-optical-dipole trap. The theory of evaporation, along with different scaling laws are described. Afterwards, the implemented evaporation sequence is described in full detail. At the end of evaporation sequence, we end up with a degenerate Fermi gas with temperature $T = 30 \text{ nK}$ and $\frac{T}{T_F} = 0.21(4)$ where T_F is the Fermi temperature. We detail the characterization of the degenerate gas and compare it with a thermal gas with $\frac{T}{T_F} = 3.1$. The signature of degeneracy — the over-shooting of occupation from the Maxwell-Boltzmann distribution reveals the quantum nature of the gas.
- In Chapter-4, we present the main experimental results on the observation of anisotropic *Zitterbewegung* in the presence of non-Abelian gauge field. We start the chapter with the initial state preparation which is of paramount importance for the experiments. After preparing the initial state, we study the dynamics of the wave packet for different initial kick with particular momentum. We reveal that the *Zitterbewegung* has both radial- and angular-dependence on the kick-momentum. We delineate few other situations where

the magnitude of kick-momentum varies at fixed angles. In those situations we reveal the linear dependence of the oscillation frequency and the momentum-independence of the damping-time for of the velocity oscillation.

- In Chapter-5, we summarize the thesis emphasizing the main results and hinting towards a possible extension to $SU(3)$ system that is work-in-progress in our lab.

Chapter 2

Theory: Artificial Gauge field and *Zitterbewegung*

2.1 Introduction

In this chapter, we detail the necessary theory to interpret the experimental results in Chapter-4. Section-2.2 outlines the emergence of gauge field in electromagnetism from a historical perspective followed by a discussion on the effect of gauge field in quantum mechanics. Afterwards in subsection-2.2.1, we mention the experimental realization of Abelian gauge field with few notable points on spin-orbit interaction in a one-dimensional system. Then we provide a detailed discussion on non-Abelian gauge field, specially the scheme that we realize experimentally in Chapter-4.

In Section-2.3, we provide the derivation of *Zitterbewegung* in the context of relativistic electron wave-packet and comment on the relation between the frequency of oscillation and Compton frequency. In subsection-2.3.2 we embark on a rather ambitious journey to classify all the possible cases (Abelian and non-Abelian) in a SU(2) Hamiltonian class, in the context of *Zitterbewegung*. Here we derive the necessary conditions for the emergence of *Zitterbewegung* and then comment on each equation. We elaborate our analytic results on non-Abelian gauge field and compare them with the numerical Monte-Carlo calculation. Finally we summarize the results of this chapter.

2.2 Gauge field: In Electromagnetism and in Neutral Atom

July 21st of the year 1820 marks the beginning of a new era of physics. On this day, world came to know about the discovery of Oersted: A magnetic needle is deflected due to the electrical current flow in a nearby circuit. This phenomena that put the electric and magnetic field on the same footing, is the beginning of a new way of thinking about field. Several other contemporaries of Oersted, namely Ampère, Faraday, Neumann, Weber and many more, were thinking about this and each of them had their own mathematical description of this phenomena. However some of descriptions differ from the another by a total differential of a scalar function; although it eventually produces a zero contribution to the total force when integrated over the entire contour [124, 125]. Later it was understood that these kind of functions do not affect the physics although the appearance of these functions provides computational ease. This was later dubbed as *gauge invariance*. These functions, named gauge field, were considered as mathematical convenience rather than some physical quantity. There are two main classes of such functions (often called potential): 1. vector gauge field $\mathbf{A}$ and scalar gauge field Φ . In classical electromagnetism, they are related to the electric and magnetic field via

$$\begin{aligned}\mathbf{E} &= -\nabla\Phi - \frac{\partial}{\partial t}\mathbf{A}, \\ \mathbf{B} &= \nabla \times \mathbf{A},\end{aligned}\tag{2.1}$$

where we notice that the transformation $\mathbf{A} \mapsto \mathbf{A} + \nabla f$ will not change the magnetic field $\mathbf{B}$ for any arbitrary scalar function f . And to invoke the invariance of the electric field $\mathbf{E}$, the scalar field Φ has to undergo the simultaneous transformation $\Phi \mapsto \Phi - \frac{\partial}{\partial t}f$. Note that f can depend on spatial and temporal coordinates and that's why this kind of freedom to choose the function f arbitrarily, is called local gauge invariance [124].

Similar quantities pop up in the description of wavefunctions in quantum mechanics [126, 127]. It was found that the physical quantities remain invariant under the local transformation of the wave function $\psi(\mathbf{r}, t)$:

$$\psi(\mathbf{r}, t) \mapsto \psi(\mathbf{r}, t) \cdot \exp\left[-i\frac{q}{\hbar}f(\mathbf{r}, t)\right],\tag{2.2}$$

where q is the charge of the particle. Note that the above transformation is equivalent to the minimal coupling substitution in Schrödinger equation,

$$p_j \mapsto p_j + qA_j, \quad (2.3)$$

where A_j is the j -th component of $\mathbf{A}$. Eqs. 2.3 and 2.2 provide us with two important messages:

- Electric and magnetic field can interact with charged particle only.
- Wave function may carry the signature of the gauge field and it can be a physical observable when two or more wave functions interfere due to the accumulation of different amount of phase by two states under the influence of same field.

The second point was realized by Aharonov and Bohm in their 1959 paper [128]. After the experimental observation of the geometric phase, in the next year by Chambers [129], physicists start to take gauge field as something more ‘real’ than just a mathematical convenience. That’s why whenever a field interact with a particle, the appearance of gauge field has become inevitable.

The reader may have noticed the omission of the word ‘charged’ in the previous sentence. This is due to the fact that atomic physicists have come up with a new way to emulate the Lorentz force, with neutral atoms [84, 85, 98, 100, 115, 130]. The generation of magnetic field by this artificial manner is done mainly via two means: 1. physically rotating the atomic cloud by leveraging the Coriolis effect, 2. inducing geometric potential by coupling with the internal degrees of freedom of the neutral atoms. While the former is an interesting approach, we will elaborate more on the latter approach since this is the method we have adopted to realize the artificial gauge field with strontium-87 atom in our experiments.

2.2.1 A Short Tour to Artificial Gauge Field

To generate artificial magnetic field, the atomic internal degrees of freedom are coupled to the external degrees of freedom of the atom, and to the geometry of the external electromagnetic field [84, 130]. The generated vector gauge field $\mathbf{A}$, of the artificial magnetic field $\mathbf{B}$, can be in one of the two broad categories:

Abelian Gauge Field: Here the components A_j of the gauge field commute among themselves. This can be accomplished when A_j s are scalars, or when A_j are matrices and commute among themselves.

Non-Abelian Gauge Field: Here the components are matrix-valued and they do not commute with each other.

2.2.1.1 Realization of Abelian Gauge Field

A typical way to realize the Abelian gauge field is to employ two counter-propagating laser beams (L_1 and L_2) as shown in Fig. 2.1. The two laser frequencies are far off-detuned from the excited state manifold $|e\rangle$ and there is a two-photon detuning of amount δ between the two laser frequencies. The blue arrow represented by L_1 can address the transition $|\uparrow\rangle \rightarrow |e\rangle$, and L_2 (shown by the orange arrow) addresses the $|\downarrow\rangle \rightarrow |e\rangle$ transition. The Rabi frequency of L_1 and L_2 is Ω . When the atom (shown by the green disk) absorbs a photon from L_1 and emits a photon to L_2 , the atom goes from $|\uparrow\rangle$ to state $|\downarrow\rangle$ and acquires a momentum $2\hbar k_L$ along $+x$ -axis, where k_L is the magnitude of the wave vector of $L_{1,2}$.

FIGURE 2.1: A typical way to realize the Abelian gauge field via Raman scheme. Here L_1 and L_2 each addresses the transition $|\uparrow\rangle \rightarrow |e\rangle$ and $|\downarrow\rangle \rightarrow |e\rangle$, respectively. The two-photon detuning is δ while both lasers $L_{1,2}$ are far off-detuned from the excited-state $|e\rangle$.

After some manipulation and disregarding the common energy shift the Hamiltonian of this system, in momentum-space, reads [100]:

$$H = \frac{\hbar^2 k_x^2}{2M} \otimes \mathbb{1} + \left(\frac{\Omega}{2} \sigma_3 + \frac{\delta}{2} \sigma_2 \right) + \frac{\hbar^2 k_L}{M} k_x \sigma_2, \quad (2.4)$$

FIGURE 2.2: Dispersion relation of a one-dimensional spin-orbit coupled system given by the Hamiltonian in Eq. 2.4. The left panel describes two shifted parabolas when the Raman coupling Ω and two-photon-detuning δ are zero. In the middle panel, a non zero Raman coupling opens the gap at zero momentum since this term behaves like a Zeeman term in the Hamiltonian given by Eq. 2.4. The right panel shows the effect of non-zero two-photon detuning δ . The dispersion relation becomes tilted as δ acts as a constant shift in momentum in Eq. 2.4.

where σ_j s are the Pauli matrices, M is the mass of the atom, k_x is the wave number associated to the momentum $p_x = \hbar k_x$, and $\hbar$ is the reduced Planck's constant. The Rabi frequency Ω plays the role of a constant magnetic field as it is proportional to σ_3 while this is the quantization axis, reminiscent of linear Zeeman effect. And the two-photon detuning δ plays the role of constant momentum shift as it is multiplied by the same Pauli matrix as the spin-orbit interaction term (the last term in Eq. 2.4). The dispersion relation of the Hamiltonian in Eq. 2.4 reads

$$E_{\pm} = \frac{\hbar^2 k_x^2}{2M} \pm \sqrt{\left(\frac{\Omega}{2}\right)^2 + \left(\frac{\delta}{2} + \frac{\hbar^2 k_L}{M} k_x\right)^2}, \quad (2.5)$$

where the $+$ ($-$) sign represents the upper(lower) branch of the dispersion relation.

The effects of the parameters Ω and δ on the dispersion relation in Eq. 2.5 are shown in Figs. 2.2. When both Ω and δ are zero, the dispersion relation is just two shifted parabolas centered around $\pm k_L$, as shown in the left panel of Fig. 2.2. The term corresponding to Ω behaves like a constant magnetic field in momentum-space, since it is proportional to σ_3 while this is the quantization axis. That's why for $\Omega \neq 0$, the gap at zero momentum opens, as shown in the middle panel of Fig. 2.2. Here $E_R = \frac{\hbar^2 k_L^2}{2M}$ is the recoil energy for the wave vector k_L . The effect of non-zero δ is shown in the right panel of Fig. 2.2. It essentially tilts and shifts the dispersion relation. One way to understand this effect is to look at the dispersion relation in

Eq. 2.5 where we see that having a non-zero δ behaves like adding (or subtracting) momentum k_x .

In this configuration, spin-orbit coupling term, i.e., the last term in Eq. 2.4, and the term in the parenthesis in Eq. 2.4, do not commute. As will be shown later that this non-commutation gives rise to one-dimensional *Zitterbewegung* in this class of system. The generated gauge field creates a non-zero magnetic field that cannot be gauged away [100]. The appearance of magnetic field in a similar system was demonstrated by the Spielman group, via the presence of vortices in a Bose-Einstein condensate [98]. The gauge field here is Abelian since there is only one component of the gauge field (proportional to p_x or k_x) and the other component is zero. Therefore the two components of the gauge field commute trivially. In this thesis, we are focused on non-Abelian gauge field, therefore we do not elaborate more on Abelian gauge field.

2.2.1.2 Non-Abelian Gauge Field

One way to realize the non-Abelian gauge field, with cold atom, is to exploit the tripod scheme, proposed in a seminal paper in Ref. [115]. This scheme is particularly suited for alkaline-earth atoms due to the presence of nuclear spin with extremely small Landé g -factor that leads to almost degenerate ground state (even at large magnetic field) with large nuclear spin. Strontium-87 is a particularly good choice for this scheme due to the presence of large nuclear spin $I_N = 9/2$ and a Landé g -factor of $-1.3 \cdot 10^{-4}$. Therefore the ground-state is almost insensitive to magnetic field. Instead of describing the non-Abelian gauge field in general, here in the following we give a focused description of the scheme that we have implemented in our experiments. A detailed discussion on different schemes can be found in Ref. [84].

In Fig. 2.3, we present the scheme that we have followed to realize the non-Abelian gauge field in our experiments. Two counter-propagating laser beams with opposite helicity $\sigma_{\pm}$ along x -axis, and one laser beam with π polarization (propagating along y -axis), act on the atomic wave packet (schematically shown by the blue circle), as shown in Fig. 2.3(a). The resonantly-addressed transition by each laser beam is shown in Fig. 2.3(b). The direction of the magnetic field is along x -axis. In the dressed atom picture, there are two degenerate dark states $|D_{1,2}\rangle$, as shown in

Fig. 2.3(c). The dark state manifold is separated from the two bright-states $|B_{1,2}\rangle$ by energy $\hbar\Omega$ where $\Omega = \sqrt{\Omega_+^2 + \Omega_\pi^2 + \Omega_-^2}$. The Rabi frequencies of the laser beams are $\Omega_{\pm,\pi}$ where the subscripts label the polarization of each beam. When Ω is large, then one can invoke adiabaticity and atoms prepared in the dark-state manifold will remain in this manifold [84, 115].

FIGURE 2.3: **Schematic of tripod laser configurations and level-diagrams.** (a) The three laser beams with polarization $\sigma_{\pm}, \pi$, and two counter-propagating (along the x axis) lasers have opposite helicity, while the laser with π polarization is orthogonal to the two aforementioned laser beams. (b) Each of the three laser beam in (a) addresses one transition that connect the $F = 9/2$ of $1S_0$ and $F = 9/2$ of $3P_1$ manifold. The corresponding m_F states of the ground-or excited-states (enumerated by the subscripts e, g) are labelled. (c) The state diagram in the dressed state picture. The two degenerate dark-states have zero projection along the excited-state $|7/2\rangle_e$, therefore immune to spontaneous emission. The two bright-states are separated from the dark states by a frequency $\Omega = \sqrt{\Omega_+^2 + \Omega_\pi^2 + \Omega_-^2}$ where each Rabi frequency is labelled by the corresponding polarization of the beam.

The expression of the $|D_1\rangle$ and $|D_2\rangle$ reads [115]

$$\begin{aligned}
 |D_1\rangle &= e^{iS_{31}} \sin(\varphi) |5/2\rangle_g - e^{iS_{32}} \cos(\varphi) |7/2\rangle_g, \\
 |D_2\rangle &= e^{iS_{31}} \cos(\theta) \cos(\varphi) |5/2\rangle_g + e^{iS_{32}} \cos(\theta) \sin(\varphi) |7/2\rangle_g - \sin(\theta) |9/2\rangle_g,
 \end{aligned}
 \tag{2.6}$$

where θ and ϕ are related to the Rabi Frequencies $\Omega_{\pm,\pi}$ in the following manner

$$\begin{aligned}\Omega_+ &= \Omega \sin(\theta) \cos(\varphi), \\ \Omega_\pi &= \Omega \sin(\theta) \sin(\varphi), \\ \Omega_- &= \Omega \cos(\theta),\end{aligned}\tag{2.7}$$

$$\Omega = \sqrt{|\Omega_+|^2 + |\Omega_\pi|^2 + |\Omega_-|^2},$$

and the phases S_{ij} read as follows

$$S_{31} = -2\kappa x, \quad S_{32} = -\kappa(x + y),\tag{2.8}$$

where κ is the wave vector of the three laser beams in Fig. 2.3. Note that the dark-states have no contribution from the excited states, as shown in Eq. 2.6.

Once we have the expressions of the dark-states $|D_{1,2}\rangle$ at our disposal, we can evaluate the matrix elements of vector gauge field $\mathbf{A}$ and scalar gauge field Φ via [115] :

$$\begin{aligned}A_{n,m} &= i\hbar \langle D_n | \nabla D_m \rangle, \\ \Phi_{n,m} &= \frac{\hbar^2}{2M} \left[\langle \nabla D_n | \nabla D_m \rangle + \sum_{k=1}^2 \langle D_n | \nabla D_k \rangle \langle D_k | \nabla D_m \rangle \right].\end{aligned}\tag{2.9}$$

For equal magnitude of the Rabi frequencies (as is the case in our experiments), i.e., $|\Omega_{\pm,\pi}| = \Omega_0$, the angles read

$$\varphi = \frac{\pi}{4}, \quad \theta = \arccos\left(\frac{1}{\sqrt{3}}\right).\tag{2.10}$$

Using the values of θ and ϕ from Eq. 2.10, and the expressions of $|D_{1,2}\rangle$, we find the components $A_{x,y}$ of the vector gauge field and the scalar gauge field Φ as

$$A_x = \frac{\hbar\kappa}{2} \begin{bmatrix} 3 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 1 \end{bmatrix}, \quad A_y = \frac{\hbar\kappa}{2} \begin{bmatrix} 1 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{3} \end{bmatrix}, \quad \Phi = \frac{\hbar^2\kappa^2}{2M} \begin{bmatrix} \frac{1}{3} & \frac{1}{3\sqrt{3}} \\ \frac{1}{3\sqrt{3}} & \frac{5}{9} \end{bmatrix}.\tag{2.11}$$

In the dark state manifold the Hamiltonian for the Schrödinger equation reads

$$H = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2M} \otimes \mathbb{1} - \frac{\hbar\kappa}{M} \left(\hat{p}_x \otimes \hat{A}_x + \hat{p}_y \otimes \hat{A}_y \right) + \underbrace{\frac{\hat{A}_x^2 + \hat{A}_y^2}{2M} + \frac{\hbar^2\kappa^2}{2M}}_{\tilde{\Delta}} \Phi, \quad (2.12)$$

FIGURE 2.4: Dispersion relation in Eq. 2.13 for non-Abelian gauge field in the Hamiltonian in Eq. 2.12. (a), (c), and (e) show the dispersion relation, as a function of p_y , for fixed momentum $p_x = 0$, $p_x = 4\hbar\kappa$, and $p_x = -4\hbar\kappa$, respectively. (b), (d), and (f) show the dispersion relation, as a function of p_x , for fixed momentum $p_y = 0$, $p_y = 4\hbar\kappa$, and $p_y = -4\hbar\kappa$, respectively. Note the asymmetric nature of the dispersion relations as they will be manifested in the experiments described in Chapter-4.

The term labelled by $\tilde{\Delta}$ will be cancelled for our experiments (as described in Chapter-4). Note that any term in the Hamiltonian that is proportional to the same Pauli matrix as on the spin-orbit interaction term will behave like a shift in momentum. Therefore we cancel this term $\tilde{\Delta}$ in our experiment since this term itself can induce *Zitterbewegung* for atomic wave packet (by acting as a momentum-kick)

and then the interpretation of the results become more challenging. Therefore to distinguish the effects of non-Abelian gauge field, namely the spin-orbit interaction terms, we have cancelled $\tilde{\Delta}$ and the only terms that remain are the vector gauge field along with the kinetic energy term in the Hamiltonian in Eq. 2.12. The experimental realization of the cancellation is described in Chapter-4.

To understand the system better we find the dispersion relation of the Hamiltonian in Eq. 2.12 with $\tilde{\Delta} = 0$,

$$E_{\pm} = \frac{p_x^2 + p_y^2}{2M} - \frac{\hbar\kappa(3p_x + p_y)}{3M} \pm \frac{\hbar\kappa}{3M} \sqrt{3p_x^2 + p_y^2}, \quad (2.13)$$

where the $+$ ($-$) represents the upper(lower) branch of the dispersion relation.

FIGURE 2.5: Dispersion relation of upper (blue)- and lower (red)-branches of Eq. 2.13 for non-Abelian gauge field in the Hamiltonian in Eq. 2.12. Here E_R is the recoil energy and corresponds to a frequency of 4.8 kHz. The path traced by the two branches are shown in the inset. The radius of the circle in the inset is $P_0 = 4\sqrt{2}\hbar\kappa$. Here the point $M_{\pi/4}$ makes an angle $\pi/4$ with the x -axis. The frequency-difference between the two branches are indeed the the frequency of *Zitterbewegung* (see Fig. 4.17 of Chapter 4, for the experimental result with the same parameters of the experiment).

Note that the dispersion relation do not remain same as we exchange p_x and p_y . This gives us a hint of the inherent anisotropy in the system we are considering. The dispersion relation is shown in Figs. 2.4 for several values of p_x and p_y . Note the degeneracy at $\mathbf{p} = 0$ in Figs. 2.4(a),(d) since this is responsible for non-zero Berry phase [83]. The anisotropy of the dispersion relation is seen even at large

momentum $p_{x,y} = 4\hbar\kappa$. This anisotropy will be manifested in the *Zitterbewegung* of the wave packet, as will be shown in Chapter-4. We show the dispersion relation on the full circle of radius $P_0 = 4\sqrt{2}\hbar\kappa$, and at an angle 45° in Fig. 2.5. Here the anisotropy of the dispersion relation is even more manifested.

Before moving on to describe the eigenstates of the dressed Hamiltonian, we note that the energy difference between the upper- and lower-branch reads

$$\Delta E = \frac{2\hbar\kappa}{3M} \sqrt{3p_x^2 + p_y^2}. \quad (2.14)$$

A typical value of ΔE is order of tens of kilohertz for $p_{x,y} \sim \hbar\kappa$, for strontium-87 with $\kappa = \frac{2\pi}{\lambda}$ and $\lambda = 689$ nm. As will be shown later in this chapter analytically, and in Chapter-4 through experiments that the frequency of *Zitterbewegung* is exactly the energy difference between the two branches. Since this time-scale is easily accessible in experiments, this suggests that the *Zitterbewegung* effect (if there is any) can be observed in this system.

FIGURE 2.6: The energy eigenstates for the Hamiltonian in Eq. 2.12 in momentum-space. The eigenstates of the upper-branch is shown in (a) where a branch-cut is present at an angle 45° with the $+x$ -axis. In (b), the eigenstates of the lower-branch is shown, and here the branch-cut is at an angle 225° with the $+x$ -axis. The effect of these two branch-cuts — the suppression of *Zitterbewegung* — will be shown in our experiments described in Chapter-4.

One of the most interesting fact about non-Abelian Gauge field is the energy eigenstates in momentum space. The eigenstates are momentum-dependent, as shown in Figs. 2.6. The left and right panel show the eigenstates of the upper- and lower-branch for the Hamiltonian in Eq. 2.12, respectively. Here the x and y components of the arrows at each momentum point represents the spin-up and spin-down components, respectively. When $\tilde{\Delta} = 0$, there is no σ_2 in Eq. 2.12, therefore the components of the eigenstates can be all real. The momentum dependence of the *Zitterbewegung* motion that will be shown experimentally in Chapter-4, is attributed to this momentum-dependence of the eigenstates. Note the presence of two branch-cuts at 45° and 225° — the eigenstates flip sign across each branch-cut. The π phase shift of the eigenstates is deeply rooted to the degeneracy at zero momentum, therefore they acquire a π Berry phase as the eigenstates move around the origin [83]. The effect of the branch-cuts are manifested in our experiments via the suppression of *Zitterbewegung* for momentum-kick along those specific angles. At any point on the branch-cut, the off-diagonal term in the Hamiltonian becomes zero, namely the two dressed bands becomes isolates. Since the *Zitterbewegung* arises due to the interference between the upper- and lower-branch, when the off-diagonal terms-becomes zero, the exchange of population between these two branches is suppressed, and subsequently the *Zitterbewegung* vanishes.

Before discussing *Zitterbewegung* in more detail, we briefly mention few proposals on the realization of non-Abelian gauge field, and few other scheme that has been experimentally realized. One of the early proposal for the realization of non-Abelian gauge field in a lattice was in Ref. [131] where authors have proposed to use state-dependent hopping to couple the internal states of atoms. There have been some other proposal to realize non-Abelian gauge field in optical lattices using external drive and different lattice configurations [59, 132, 133].

Another approach to realize the non-Abelian gauge field without the need for lattice has been suggested in Ref. [134]. We have adapted this approach because of its simplicity and the tripod scheme that the authors have proposed required three almost degenerate ground states and in strontium we have ten degenerate ground states. Therefore this scheme is particularly suited for us. Recently there has been a demonstration of the realization non-Abelian gauge field, without the need for lattice, in Ref. [135]. Here the authors have taken two states from each of the two hyperfine ground-states of rubidium and couple them via microwave. This way

the authors create two sub-spaces each containing two degenerate levels, and subsequently connect these two sub-spaces to simulate Yang monopole. This similar approach has been employed by the same group to measure the Wilson loop in a non-Abelian gauge field in Ref. [116].

2.3 *Zitterbewegung* from Ground up: Electron Wave Packet to Ultracold Atomic Wave Packet

The notion of *Zitterbewegung* — the trembling motion of a wave packet — is that the position as well as the velocity operators, in the Heisenberg picture, oscillates at some characteristic frequency, for a wave packet. We will now derive the *Zitterbewegung* effect for two different cases, first for a relativistic free particle and later perform the similar derivation for an ultracold atomic wave packet, to draw the parallel between the two cases.

2.3.1 *Zitterbewegung* for Dirac Wave Packet

The starting point is obviously the Dirac equation [4]

$$\left[c\hat{\alpha} \cdot \hat{p} + \hat{\beta}mc^2 \right] \psi = i\hbar \frac{\partial}{\partial t} \psi, \quad (2.15)$$

with Hamiltonian $\hat{H} = c\hat{\alpha} \cdot \hat{p} + \hat{\beta}mc^2$,

where c is the speed of light in vacuum, m is the rest mass of the particle, $\hat{p}$ is the momentum operator; the $\hat{\alpha}$ and $\hat{\beta}$ matrices are related to the γ matrices as follows

$$\alpha^i = \gamma^0 \gamma^i, \beta = \gamma^0, \quad (2.16)$$

where the γ matrices are related to the Pauli matrices σ^i (for $i = \{0, 1, 2, 3\}$) via

$$\gamma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & -\sigma^0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (2.17)$$

for $i = \{1, 2, 3\}$. The Heisenberg representation of the operators provides an algebraic convenience for the derivation of *Zitterbewegung* effect. Using the Heisenberg

equation of motion, the time-evolution of the position operator reads

$$\begin{aligned} i\hbar\dot{\hat{x}}^j &= [\hat{x}^j, \hat{H}] , \\ &= i\hbar c \hat{\alpha}^j, \end{aligned} \tag{2.18}$$

where we have used the following relation for an arbitrary function of operators F ,

$$[x^j, F] = i\hbar \frac{\partial}{\partial p^j} F. \tag{2.19}$$

Note that the above equation can be derived via a Taylor expansion of the operator function $\hat{F}(\hat{x}^j, \hat{p}^j)$ and then using $[x^m, p^n] = i\hbar \delta_{m,n}$. The author apologizes to the reader for the abuse of covariant- and contravariant-forms, as they do not affect the contents of this thesis.

Firstly, the expression of velocity operator in Eq. 2.18 is a bit counter-intuitive since different components of the velocity operators (i.e., $\hat{x}^j$ s) are non-commuting, namely $[\alpha^j, \alpha^k] \neq 0$. This would suggest that, in an experiment, one cannot measure these quantities simultaneously. In fact there is no problem with Eq. 2.18, and we just need to find the proper expression keeping in mind that the velocity operator is not a constant of motion for the particle — it does not commute with the Hamiltonian. Moreover, the energy eigenstates are not eigenstates of the velocity operators.

Secondly, since the eigenvalues of α_j s are ± 1 , it would seem like that a measurement of the velocity for a particle with mass m would yield $\pm c$. However classical velocity of a massive particle with mass m cannot be c , as it would be the case if the velocity eigenstates were energy eigenstates. This truly remarkable feature of this seemingly simple equation in Eq. 2.18 was pointed out by Gregory Breit in his 1928 paper where he associated the $\hat{\alpha}$ matrices with the physical quantity velocity, in parallel with the Pauli matrices that describe the spin of an electron [136].

The above two points suggest that to find a proper description of the dynamic variable velocity $\hat{v}$, we need to look at its time evolution, namely

$$i\hbar\dot{\hat{v}}^j = [\hat{v}^j, H]. \tag{2.20}$$

If one focuses on the 2-dimensional case where p_z is zero in the Hamiltonian in Eq. 2.15, the matrices follow the following differentials equations:

$$\begin{aligned}\dot{\alpha}_1 &= \frac{2ic}{\hbar} [p_1 - \alpha_1 \alpha_2 p_2 - \alpha_1 \beta mc], \\ \dot{\alpha}_2 &= \frac{2ic}{\hbar} [p_2 - \alpha_1 \alpha_2 p_1 - \alpha_2 \beta mc], \\ \dot{\beta} &= -\frac{2ic}{\hbar} \beta (\alpha_1 p_1 + \alpha_2 p_2).\end{aligned}\tag{2.21}$$

The coupled set of differential equations for the operators is non-linear, therefore we do not embark on a general solution. However, to demonstrate the phenomena, like many textbooks [126, 137], we can look at the trajectory of the wave packet for momentum $\mathbf{p} = \mathbf{0}$. Then the velocity operator takes the following form

$$\begin{aligned}\hat{\mathbf{v}} &= c\hat{\boldsymbol{\alpha}} \left[\cos(\omega t) - i\hat{\beta} \sin(\omega t) \right], \\ \text{with, } \omega &= 2\frac{mc^2}{\hbar},\end{aligned}\tag{2.22}$$

where we see that the dynamic quantity velocity oscillates at a frequency twice the Compton frequency $\omega_c = \frac{mc^2}{\hbar}$. This frequency ω is in fact nothing but the frequency difference between and positive (particle) and negative (hole) energy branches of the particle. As we will see later that the *Zitterbewegung* frequency for an atomic wave packet, in artificial gauge field, is exactly the energy difference between the upper- and lower-branch of the dispersion relation. For completeness, we write the expression of the position operator in the following

$$\hat{\mathbf{x}} = \hat{\mathbf{x}}_0 + \frac{\hbar}{2mc} \hat{\boldsymbol{\alpha}} \left[\sin(\omega t) + i\hat{\beta} \cos(\omega t) \right],\tag{2.23}$$

where $\hat{\mathbf{x}}_0$ is the position operator at $t = 0$. This motion of the center-of-mass of an electron was dubbed *Zitterbewegung* by Schrödinger [70].

To get an estimation of the oscillation frequency we plug in the rest mass of electron m and the period of oscillation turns out to be $4.02 \cdot 10^{-21}$ second — extremely small to be observed directly on a state-of-the-art time-resolved experiments [72, 73, 75, 76].

However, as was shown in page-17 in subsection- 2.2.1.2 that the two branches of the dispersion relation for atoms in artificial gauge field are separated by tens of

kilohertz. Therefore atomic wave packet seems like a viable ground to observe the *Zitterbewegung* effect. This effect was successfully observed in a trapped ion system, to simulate one-dimensional Dirac equation [79], and also in the Spielman group with neutral atoms, for Abelian gauge field, in an one-dimensional system [81].

It will be shown in Chapter-4, in the presence of non-Abelian gauge field, the system becomes much more rich in phenomenology, namely the anisotropic nature of *Zitterbewegung* with full contrast can be revealed and the frequency of oscillation is controlled by exploring the entire plane of the two-dimensional momentum-space. To understand those phenomena in the experiments, we discuss the theoretical framework in the following subsections.

2.3.2 *Zitterbewegung* for an Ultracold Atomic Wave packet: Abelian and Non-Abelian Gauge Field

We consider an atomic wave packet, in the presence of a gauge field given by the vector components $\hat{A}_{x,y}$ where the subscript represents the corresponding component, and a scalar field $\hat{\Phi}$. Note that here we consider a system with pseudo-spin $\frac{1}{2}$ while the components of the gauge field along with the scalar part are 2×2 matrices. For a particle with mass M , the Hamiltonian of the system is given by

$$\begin{aligned} H &= \frac{\hat{p}_x^2 + \hat{p}_y^2}{2M} \otimes \mathbb{1} - \alpha \left[\hat{p}_x \otimes \hat{A}_x + \hat{p}_y \otimes \hat{A}_y \right] - \beta' \hat{\Phi} + \gamma \left[\hat{A}_x^2 + \hat{A}_y^2 \right], \\ &= \frac{\hat{p}_x^2 + \hat{p}_y^2}{2M} \otimes \mathbb{1} - \alpha \left[\hat{p}_x \otimes \hat{A}_x + \hat{p}_y \otimes \hat{A}_y \right] - \beta \hat{\Phi}', \end{aligned} \quad (2.24)$$

and the gauge field are given by

$$\begin{aligned} \hat{A}_x &= c_0 \sigma_0 + c_1 \sigma_1 + c_2 \sigma_2 + c_3 \sigma_3, \\ \hat{A}_y &= d_0 \sigma_0 + d_1 \sigma_1 + d_2 \sigma_2 + d_3 \sigma_3, \\ \hat{\Phi}' &= f_0 \sigma_0 + f_1 \sigma_1 + f_2 \sigma_2 + f_3 \sigma_3, \end{aligned} \quad (2.25)$$

where α and β are scalar constants and each represents the strength of the term associated to it, σ_j s are Pauli matrices, $\{c_i, d_j, f_k\}$ are real scalars, and $\hat{p}_{x,y}$ are the momentum operators. Note that we have absorbed the terms related to β' and γ into a single parameter β with $\hat{\Phi}'$.

To see the presence (or absence) of *Zitterbewegung* in the above Hamiltonian class (for various values of the parameters $\{c_i, d_j, f_k\}$), we analyze it in the Heisenberg picture and then the role of momentum becomes an initial momentum-kick to the atomic wave packet. Moreover, the momentum operators commute with the Hamiltonian in Eq. 2.24, therefore each component is a constant of motion for the atom. We write the Heisenberg equation of motion for the Pauli matrices as follows:

$$\begin{aligned}\dot{\sigma}_1 &= -\frac{2}{\hbar}\alpha p_x [c_2\sigma_3 - c_3\sigma_2] - \frac{2}{\hbar}\alpha p_y [d_2\sigma_3 - d_3\sigma_2] + \frac{2}{\hbar}\beta [f_2\sigma_3 - f_3\sigma_2], \\ \dot{\sigma}_2 &= -\frac{2}{\hbar}\alpha p_x [c_3\sigma_1 - c_1\sigma_3] - \frac{2}{\hbar}\alpha p_y [d_3\sigma_1 - d_1\sigma_3] + \frac{2}{\hbar}\beta [f_3\sigma_1 - f_1\sigma_3], \\ \dot{\sigma}_3 &= -\frac{2}{\hbar}\alpha p_x [c_1\sigma_2 - c_2\sigma_1] - \frac{2}{\hbar}\alpha p_y [d_1\sigma_2 - d_2\sigma_1] + \frac{2}{\hbar}\beta [f_1\sigma_2 - f_2\sigma_1].\end{aligned}\tag{2.26}$$

We consider the general initial condition for the atoms:

$$|\Psi(t=0)\rangle = \begin{bmatrix} \cos(\theta) \\ e^{i\varphi} \sin(\theta) \end{bmatrix},\tag{2.27}$$

where θ and φ are the spherical angles on Bloch sphere (and they have nothing to do with the same symbols in Eq. 2.7). Now the initial condition for the dynamic variables $\sigma_{1,2,3}$ read as following:

$$\begin{aligned}\sigma_1(t=0) &= \sin(2\theta) \cos(\varphi), \\ \sigma_2(t=0) &= \sin(2\theta) \sin(\varphi), \\ \sigma_3(t=0) &= \cos(2\theta).\end{aligned}\tag{2.28}$$

We solve the coupled differential equations in Eq. 2.26 with the initial condition from Eq. 2.28. With the expressions of the time-dependent $\sigma_{1,2,3}$ at our disposal, we can derive the expressions for the components of velocity via the gauge covariant form:

$$v_j = \frac{1}{M} (p_j - M\alpha A_j).\tag{2.29}$$

In the following three cases we describe different ways to realize *Zitterbewegung* in a cold atom experiment:

At zero momentum($\mathbf{p} = 0$) kick: Here we will see that for an arbitrary gauge field (Abelian or Non-Abelian), *Zitterbewegung* appears only when *at least* one of the f_j s are non-zero, i.e., the contributions from scalar fields (those that are not multiplied by $p_{x,y}$) . Therefore the presence of the scalar field is necessity for the emergence of *Zitterbewegung*, as described in page-24.

Abelian gauge field at non-zero momentum kick: In this case, we will see that *Zitterbewegung* appears however, the amplitude of the velocity oscillation is momentum-independent, as described in page-26.

Non-Abelian gauge field at non-zero momentum kick: In this case, we will see that *Zitterbewegung* appears while the amplitude of the velocity oscillation can be controlled by magnitude and the direction of momentum-kick, as will be shown in page-30.

2.3.2.1 Abelian and Non-Abelian at Zero Momentum Kick

In this case, we set $p_{x,y} = 0$, in Eq. 2.26 with the atom in pseudo-spin ‘down’ at $t = 0$, namely $\theta = \pi$. For a general initial condition, the equation becomes too involved and it is easy to loose sight of the main message that we would like to convey. This particular choice of initial condition retains all the necessary physics while the equations are tractable. The x -component of the velocity reads

$$\begin{aligned}
 v_x &= v_{x0} + v_{x1} \cos(\omega t) + v_{x2} \sin(\omega t), \\
 v_{x0} &= \frac{1}{f_1^2 + f_2^2 + f_3^2} \cdot [c_0 (f_1^2 + f_2^2 + f_3^2) - f_3 (c_1 f_1 + c_2 f_2 + c_3 f_3)], \\
 v_{x1} &= \frac{1}{f_1^2 + f_2^2 + f_3^2} \cdot [f_3 (c_1 f_1 + c_2 f_2) - c_3 (f_1^2 + f_2^2)], \\
 v_{x2} &= \frac{c_2 f_1 - c_1 f_2}{\sqrt{f_1^2 + f_2^2 + f_3^2}}, \\
 \omega &= 2 \frac{\beta}{\hbar} \sqrt{f_1^2 + f_2^2 + f_3^2}.
 \end{aligned} \tag{2.30}$$

And the y -component of velocity read

$$\begin{aligned}
v_y &= v_{y0} + v_{y1} \cos(\omega t) + v_{y2} \sin(\omega t), \\
v_{y0} &= \frac{1}{f_1^2 + f_2^2 + f_3^2} \cdot [d_0 (f_1^2 + f_2^2 + f_3^2) - f_3 (d_1 f_1 + d_2 f_2 + d_3 f_3)], \\
v_{y1} &= \frac{1}{f_1^2 + f_2^2 + f_3^2} \cdot [f_3 (d_1 f_1 + d_2 f_2) - d_3 (f_1^2 + f_2^2)], \\
v_{y2} &= \frac{d_2 f_1 - d_1 f_2}{\sqrt{f_1^2 + f_2^2 + f_3^2}}, \\
\omega &= 2 \frac{\beta}{\hbar} \sqrt{f_1^2 + f_2^2 + f_3^2}.
\end{aligned} \tag{2.31}$$

Note that in the above expressions we have subtracted the contribution of the ballistic motion in Eq. 2.29 to highlight the contributions of the gauge field. The things that we immediately see from Eqs. 2.30 and 2.31 are the following:

- There can be oscillation of velocity, along both axes.
- The frequency of oscillation ω along both axes is same and it is non-zero when *at least* one of the coefficients f_j s of the Pauli matrices is non-zero.
- The amplitude of the motion can only be adjusted by the parameters $\{c_i, d_j, f_k\}$.

Note that the expressions in Eqs. 2.30 and 2.31 are applicable for an atomic wave packet at temperature $T = 0$. As we introduce finite temperature, the wave packet will have finite extension in the momentum-space. Therefore despite having no momentum kick, namely the average momentum of the wave packet is zero, the other momentum-components participates in the dynamics and they may give rise to an overall oscillation. The observable effect of finite temperature, on the dynamics, will be a damped sinusoid-like motion instead of a pure sinusoid-like motion [82, 83].

2.3.2.2 Abelian Gauge Field at Non-zero Momentum Kick

We consider the same general gauge field $A_{x,y}$ as in the previous case, namely

$$A_x = c_0\sigma_0 + c_1\sigma_1 + c_2\sigma_2 + c_3\sigma_3, \quad (2.32)$$

$$A_y = d_0\sigma_0 + d_1\sigma_1 + d_2\sigma_2 + d_3\sigma_3.$$

For an Abelian gauge field, the components A_x and A_y commutes, i.e.,

$$[A_x, A_y] = 0. \quad (2.33)$$

In order to satisfy Eq. 2.33 for the gauge field components in Eq. 2.32, we need to fulfil the following condition

$$\frac{c_1}{d_1} = \frac{c_2}{d_2} = \frac{c_3}{d_3}, \quad (2.34)$$

The condition in Eq. 2.34 have a nice geometric interpretation if we treat (c_1, c_2, c_3) and (d_1, d_2, d_3) as two points in three-dimensional Cartesian coordinate — the gauge field is Abelian, when the line joining this two points goes through the origin.

With this understanding at our disposal we parameterize the coefficient of the Pauli matrices as follows

$$\frac{c_1}{d_1} = \frac{c_2}{d_2} = \frac{c_3}{d_3} = \zeta, \quad (2.35)$$

and disregarding the terms proportional to β by setting $\beta = 0$ (for the time being) in Eq. 2.26, and taking θ and φ as arbitrary for the initial condition in Eq. 2.28, we solve for the dynamic variables $\sigma_{1,2,3}$ in Eq. 2.26. Afterwards we use Eq. 2.29 to find the components of velocity as

$$v_x = c_0 + d_3\zeta \cos(2\theta) + \zeta \sin(2\theta) [d_1 \cos(\varphi) + d_2 \sin(\varphi)], \quad (2.36)$$

$$v_y = d_0 + d_3 \cos(2\theta) + \sin(2\theta) [d_1 \cos(\varphi) + d_2 \sin(\varphi)].$$

Note that in the above expressions we have subtracted the contribution of the ballistic motion in Eq. 2.29 to highlight the contributions of the gauge field. The expressions of velocity in Eq. 2.36 tells us something interesting — in the absence of the scalar term (since $\beta = 0$) in the gauge field, there is no *Zitterbewegung* motion of the particle for an Abelian gauge field.

Now we can examine the system by letting $\beta \neq 0$, i.e., the scalar term to be non-zero and then two situations arises

1. The whole scalar term Φ' in Eq. 2.24, may commute with one of the components of the gauge field, i.e., $A_{x,y}$. Note that the components of the gauge field are already commuting among themselves. In this situation the *Zitterbewegung* vanishes — the velocity do not oscillate in time.
2. If the whole scalar term Φ' do not commute with at least one of the components of the gauge field (i.e., $A_{x,y}$), then the *Zitterbewegung* persists. In this case, the frequency as well as the amplitudes become dependent on the initial momentum, i.e., the amplitudes and the frequency are anisotropic in the momentum-space.
3. In the pathological case, i.e., when one of the components (e.g., $\hat{A}_y$) of the gauge field $\hat{A}_{x,y}$ is proportional to the identity matrix σ_0 , the velocity corresponding to that axis (i.e., v_y) exhibits no *Zitterbewegung* motion while the velocity along the other axis (i.e., v_x) shows *Zitterbewegung* motion if that component of the gauge field (i.e., $\hat{A}_x$) is non-commuting with Φ' .

The main message we would like to convey through the discussion above is that — even for an Abelian gauge field, non-commutation is a necessary ingredient for the presence of *Zitterbewegung* motion in an atomic wave packet.

The expressions of the frequency and amplitudes are too long to present here and therefore we opted for quoting the results in words only. And before going to the discussion of non-Abelian gauge field, here in the following we will make some remarks on the condition for the vanishing commutation of two 2×2 matrices that will help to understand the facts about Abelian gauge field better.

As we mentioned before that two 2×2 matrices $\hat{A}$ and $\hat{B}$ commute when the line joining the points $A \equiv (a_1, a_2, a_3)$ and $B \equiv (b_1, b_2, b_3)$ goes through the center, where $\hat{A}$ and $\hat{B}$ are represented as follows

$$\hat{A} = a_0\hat{\sigma}_0 + a_1\hat{\sigma}_1 + a_2\hat{\sigma}_2 + a_3\hat{\sigma}_3, \quad (2.37)$$

$$\hat{B} = b_0\hat{\sigma}_0 + b_1\hat{\sigma}_1 + b_2\hat{\sigma}_2 + b_3\hat{\sigma}_3.$$

We can stretch the above statement a little further by claiming the following

If two matrices of dimension 2×2 commute and one of the two commuting matrices commute with a third matrix of same dimension, then all three matrices commute among themselves, when neither of the three matrices are proportional to the identity matrix $\hat{\sigma}_0$.

Note that any matrix $\hat{P} = p_0 \hat{\sigma}_0$ that is proportional to identity matrix $\hat{\sigma}_0$ sits at the origin of the Cartesian coordinate as it represents the point $P \equiv (0, 0, 0)$. The claim above can be understood as follows — if neither of the points A , B , and P are at the origin, and $[\hat{A}, \hat{B}] = 0$, then the line AB must go through the origin. Now if $\hat{A}$ commutes with $\hat{P}$, then the line AP also goes through the origin. Therefore the line BP also goes through the origin, namely $[\hat{B}, \hat{P}] = 0$. We have exploited this property of commutation to arrive at the conclusions regarding the presence of *Zitterbewegung* in Abelian gauge field.

2.3.2.3 Non-Abelian Gauge Field at Non-Zero Momentum

In the general case of non-Abelian gauge field, i.e., $[A_x, A_y] \neq 0$, and $\mathbf{p} \neq 0$, the *Zitterbewegung* persists. As was mentioned in page-30, that the terms in $\tilde{\Delta}$ in Eq. 2.12 will act like an effective shift in momentum. Therefore the analysis with non-zero momentum and $\tilde{\Delta} = \hat{0}$ is the same as having $\tilde{\Delta} \neq 0$, except the renormalization of the momentum $p_{x,y}$. In the following, we consider the case when $\mathbf{p} \neq 0$ and $\tilde{\Delta} = 0$.

Since this case is central to the contents of this thesis, we will be more explicit about the expressions of the gauge field as well as to the physical realization of the corresponding gauge field. The realization of the system is shown in Figs. 4.1. We purposefully repeat part of the discussion so that this part of the calculation is self-contained.

The four-level system shown in Fig. 4.1(b) is dressed by the three laser beams — two counter-propagating laser beams with opposite helicity and one π polarized beam orthogonal to these two. The dressed states, shown in Fig. 4.1(c) has two dark states $|D_{1,2}\rangle$ that are separated by the two other bright-states $|B_{1,2}\rangle$ by a frequency Ω .

FIGURE 2.7: **Schematic of tripod laser configurations and level-diagrams.** (a) The three laser beams with polarization $\sigma_{\pm}, \pi$; the two counter-propagating (along the x axis) lasers have opposite helicity, while the laser with π polarization is orthogonal to the two aforementioned laser beams. (b) Each of the three laser beams in (a) addresses one transition that connect the $F = 9/2$ of $1S_0$ and $F = 9/2$ of $3P_1$ manifold. The corresponding m_F states of the ground- or excited-states (enumerated by the subscripts e, g) are labelled. (c) The state diagram in the dressed state picture. The two degenerate dark-states have zero projection along the excited-state $|7/2\rangle_e$, therefore immune to spontaneous emission. The two bright-states are separated from the dark states by a frequency $\Omega = \sqrt{\Omega_+^2 + \Omega_\pi^2 + \Omega_-^2}$ where each Rabi frequency is labelled by the corresponding polarization of the beam.

For equal magnitude of the Rabi frequencies $\Omega_{\pm, \pi}$, we write the Hamiltonian in the basis of the dark-states, and the Hamiltonian looks the same in Eq. 2.24 except for the fact that we explicitly write down the forms of α , β , and γ .

$$H = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2M} \otimes \mathbb{1} - \frac{\hbar\kappa}{M} \left(\hat{p}_x \otimes \hat{A}_x + \hat{p}_y \otimes \hat{A}_y \right) + \underbrace{\frac{\hat{A}_x^2 + \hat{A}_y^2}{2M} + \frac{\hbar^2\kappa^2}{2M}}_{\hat{\Delta}} \Phi, \quad (2.38)$$

where the components of the gauge field $A_{x,y}$ and the scalar field read

$$A_x = \frac{1}{2} \begin{bmatrix} 3 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 1 \end{bmatrix}, \quad A_y = \frac{1}{2} \begin{bmatrix} 1 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{3} \end{bmatrix}, \quad \Phi = \begin{bmatrix} \frac{1}{3} & \frac{1}{3\sqrt{3}} \\ \frac{1}{3\sqrt{3}} & \frac{5}{9} \end{bmatrix}. \quad (2.39)$$

To simplify the notation, we write $\hat{A}_{x,y}$ in terms of Pauli matrices $\hat{\sigma}_{0,1,2,3}$

$$\hat{A}_x = (c_0\hat{\sigma}_0 + c_1\hat{\sigma}_1 + c_3\hat{\sigma}_3), \quad (2.40)$$

$$\hat{A}_y = (d_0\hat{\sigma}_0 + d_1\hat{\sigma}_1 + d_3\hat{\sigma}_3),$$

where the values of c_i and d_j are

$$\begin{aligned} c_0 &= 1, & d_0 &= \frac{1}{3}, \\ c_1 &= \frac{1}{2\sqrt{3}}, & d_1 &= -\frac{1}{2\sqrt{3}}, \\ c_3 &= \frac{1}{2}, & d_3 &= \frac{1}{6}. \end{aligned} \tag{2.41}$$

Since the matrix elements of $A_{x,y}$ are all real, we *a priori* set the coefficients of σ_2 to be zero.

To distinguish the effect of non-Abelian nature of the gauge field, we cancel the term labelled by $\tilde{\Delta}$ in Eq. 2.38. Therefore all the f_j s in Eq. 2.26 are zero and we have chosen that the atom is initially at $|D_2\rangle$ (i.e., $\theta = \pi$ in Eq. 2.28). Then the expressions of the $\sigma_{1,2,3}$ as the dynamic variables read

$$\begin{aligned} \sigma_1(t) &= \frac{\sqrt{3}}{4} \frac{(p_y - p_x)(3p_x + p_y)}{(3p_x^2 + p_y^2)} [1 - \cos(\Omega \cdot t)], \\ \sigma_2(t) &= \frac{\sqrt{3}}{2} \frac{(p_y - p_x)}{\sqrt{3p_x^2 + p_y^2}} \sin(\Omega \cdot t), \\ \sigma_3(t) &= -\frac{(3p_x + p_y)^2}{4(3p_x^2 + p_y^2)} - \frac{3(p_x - p_y)^2}{4(3p_x^2 + p_y^2)} \cdot \cos(\Omega \cdot t), \\ \text{with } \Omega &= \frac{2\kappa}{3m} \sqrt{3p_x^2 + p_y^2}. \end{aligned} \tag{2.42}$$

Using the expressions of $\sigma_{1,2,3}$ of and using Eq. 2.29, we write down the expressions of the velocity

$$\begin{aligned} \tilde{v}_x(p_x, p_y; t) &= v_{x1} + v_{x0} \cos(\omega \cdot t), \\ \tilde{v}_y(p_x, p_y; t) &= v_{y1} + v_{y0} \cos(\omega \cdot t), \end{aligned} \tag{2.43}$$

$$\text{with } \omega = \frac{2\kappa}{3M} \sqrt{3p_x^2 + p_y^2},$$

where

$$\begin{aligned} v_{x1} &= \frac{\hbar\kappa}{M} \cdot \frac{p_x p_y - 3(p_x)^2 - 2(p_y)^2}{2[3(p_x)^2 + (p_y)^2]}, & v_{x0} &= \frac{\hbar\kappa}{M} \cdot \frac{p_y^2 - p_x p_y}{2[3(p_x)^2 + (p_y)^2]}, \\ v_{y1} &= \frac{\hbar\kappa}{M} \cdot \frac{3p_x p_y - 6(p_x)^2 - (p_y)^2}{6[3(p_x)^2 + (p_y)^2]}, & v_{y0} &= \frac{\hbar\kappa}{M} \cdot \frac{3p_x p_y - 3p_x^2}{6[3(p_x)^2 + (p_y)^2]}. \end{aligned} \tag{2.44}$$

Note that we have added ‘tilde’ in Eqs. 2.43 to emphasize the fact that this expression is valid at temperature $T = 0$. In the above expressions we have subtracted the contribution of the ballistic motion in Eq. 2.29 to highlight the contributions of the gauge field.

One way to incorporate the effects of temperature would be integrate the expressions of the velocity-components above with the appropriate momentum distribution of the atomic wave packet, namely

$$\mathbf{v}_{\text{th.}} = \int d\mathbf{p} \cdot \mathbf{v}_{T=0} \cdot f(\mathbf{p}, T), \quad (2.45)$$

where $f(\mathbf{p})$ is the momentum distribution at temperature T , and $\mathbf{v}_{T=0}$ is the velocity at $T = 0$, namely the expressions in Eq. 2.30 and 2.31.

As will be shown later in Chapter-4 that in our experiment we are dealing with a Fermi gas, therefore one should take $f(\mathbf{p})$ as the momentum distribution

$$f_{\text{FD}}(p) = \frac{1}{2\pi m k_B T} \frac{\text{Li}_2(-\zeta \mathbb{E})}{\text{Li}_3(-\zeta)},$$

$$\text{with } \mathbb{E} = \exp \left[-\frac{(p_x^2 + p_y^2)}{2m k_B T} \right], \quad \text{Li}_3(-\zeta) = -\frac{1}{6} \left(\frac{T}{T_F} \right)^3, \quad (2.46)$$

$$T_F = (6N)^{1/3} \frac{\hbar \bar{\omega}}{k_B}, \quad \bar{\omega} = \sqrt[3]{\omega_x \omega_y \omega_z},$$

where $\omega_{x,y,z}$ are the oscillation frequencies of the trap (e.g., optical dipole trap or radio-frequency trap) along the axes corresponding to the subscripts, ζ is the fugacity, N is the number of atoms, T is the temperature, and T_F is the Fermi temperature; $\text{Li}_n(x)$ is the n -order polylogarithmic function [138].

However performing integrations involving Polylogarithmic functions multiplied by algebraic and trigonometric functions is difficult and we could not find any closed form for these integrals. As an approximation, we have used the 2-dimensional Maxwell-Boltzmann distribution as the thermal distribution

$$f_{\text{MB}}(\mathbf{p}) = \frac{1}{2m k_B T} \exp \left(-\frac{\mathbf{p}^2}{2m k_B T} \right), \quad (2.47)$$

and interestingly this distribution explains our experimental results described in Chapter-4 with good accuracy. Now adding the effect of the initial kick $\mathbf{p}^0$ on the

cloud in Eq. 2.45, we perform the thermal averaging

$$\begin{aligned}
 v(\mathbf{p}^0, t)_{\text{th.}} &= \int d\mathbf{u} \cdot \left(\mathbf{v}_0(\mathbf{p}^0 + \mathbf{u}) + \mathbf{v}_1(\mathbf{p}^0 + \mathbf{u}) \cos[\omega(\mathbf{p}^0 + \mathbf{u}) \cdot t] \right) f(\mathbf{u}); \quad \mathbf{u} = \mathbf{p} - \mathbf{p}^0, \\
 &\approx \mathbf{v}_0(\mathbf{p}^0) + \mathbf{v}_1(\mathbf{p}^0) \int d\mathbf{u} \cdot \cos \left[\left(\omega(\mathbf{p}^0) + \mathbf{u} \cdot \left[\frac{\partial}{\partial \mathbf{u}} \omega(\mathbf{p}^0 + \mathbf{u}) \right]_{\mathbf{u}=\mathbf{0}} \right) \cdot t \right] f(\mathbf{u}), \\
 &\text{with } \left[\frac{\partial}{\partial \mathbf{u}} \omega(\mathbf{p}^0 + \mathbf{u}) \right]_{\mathbf{u}=\mathbf{0}} = \frac{2\kappa}{3M} \cdot \frac{1}{\sqrt{2 + \cos(2\Theta)}} \begin{pmatrix} 3 \cos(\Theta) \\ \sin(\Theta) \end{pmatrix}; \\
 &\text{and } p_x^0 = P_0 \cos(\Theta), p_y^0 = P_0 \sin(\Theta), \\
 &= \mathbf{v}_0(\mathbf{p}^0) + \mathbf{v}_1(\mathbf{p}^0) \text{Re} [\exp[i \cdot \omega(\mathbf{p}^0) \cdot t] I_0], \\
 &\text{with } I_0 = \int du_0 d\varphi \cdot u_0 \cdot \exp[i \cdot u_0 \cdot (\alpha \cos(\varphi) + \beta \sin(\varphi)) \cdot t] \cdot f(u_0), \\
 &\alpha = \frac{2\kappa}{3M} \cdot \frac{3 \cos(\Theta)}{\sqrt{2 + \cos(2\Theta)}}, \quad \beta = \frac{2\kappa}{3M} \cdot \frac{\sin(\Theta)}{\sqrt{2 + \cos(2\Theta)}}.
 \end{aligned} \tag{2.48}$$

Note that in the second line of Eq. 2.48, we have approximated $\omega(\mathbf{p}^0 + \mathbf{u})$ for small magnitude of $\mathbf{u}$ compared to the initial momentum-kick $\mathbf{p}^0$.

Now we can rewrite the expression of I_0 in Eq. 2.48 as follows

$$\begin{aligned}
 I_0 &= \int du_0 d\varphi \cdot u_0 \cdot \exp[i \cdot u_0 \cdot (\alpha \cos(\varphi) + \beta \sin(\varphi)) \cdot t] \cdot f(u_0), \\
 &= \frac{1}{\gamma} \int du_0 \cdot u_0 \cdot J_0 \left(u_0 t \sqrt{\alpha^2 + \beta^2} \right) \cdot \exp \left(-\frac{u_0^2}{\gamma} \right), \quad \text{with } \gamma = 2mk_{\text{B}}T, \\
 &= \exp \left(-t^2 \frac{\gamma}{4} (\alpha^2 + \beta^2) \right),
 \end{aligned} \tag{2.49}$$

where J_0 is the zeroth order Bessel function. Now using Eqs. 2.48 and 2.49,

$$\mathbf{v}(\mathbf{p}^0, t)_{\text{th.}} = \mathbf{v}_0(\mathbf{p}^0) + \mathbf{v}_1(\mathbf{p}^0) \cdot \cos[\omega(\mathbf{p}^0) \cdot t] \cdot \exp \left[-\left(\frac{t}{\tau} \right)^2 \right], \tag{2.50}$$

with

$$\mathbf{v}_0(\mathbf{p}^0) = \frac{\hbar\kappa}{m} \begin{bmatrix} \frac{\sin(2\Theta) - \cos(2\Theta) - 5}{4(2 + \cos(2\Theta))} \\ \frac{3\sin(2\Theta) - 5\cos(2\Theta) - 7}{12(2 + \cos(2\Theta))} \end{bmatrix}, \mathbf{v}_1(\mathbf{p}^0) = \frac{\hbar\kappa}{m\sqrt{2}} \begin{bmatrix} \frac{\sin(\Theta) \sin(\Theta - \frac{\pi}{4})}{(2 + \cos(2\Theta))} \\ \frac{\cos(\Theta) \sin(\frac{\pi}{4} - \Theta)}{(2 + \cos(2\Theta))} \end{bmatrix}, \quad (2.51)$$

$$\omega(\mathbf{p}^0) = \frac{2\kappa}{3M} P_0 \sqrt{2 + \cos(2\Theta)},$$

$$\tau = \frac{3}{\kappa} \sqrt{\frac{M}{2k_B T}} \sqrt{\frac{2 + \cos(2\Theta)}{5 + 4\cos(2\Theta)}},$$

where ω is the frequency of oscillation and τ is the damping time.

By looking at Eqs. 2.51, we can make the following experimentally verifiable predictions

1. At a fixed angle Θ , the frequency of oscillation is linear in pushing momentum P_0 .
2. For fixed value of P_0 , i.e., on a circle, the frequency of oscillation has a period of π .
3. The damping time is independent of the absolute value P_0 .
4. The amplitudes of velocity, for both components, are independent of the absolute value of pushing momentum P_0 , for large values of P_0 .
5. At $\Theta = \pi/4$ & $5\pi/4$, the amplitudes of the oscillation vanish for both components. However, as we will see later that due to finite momentum extension of the wave packet, each component exhibit tiny oscillation for pushing along $\Theta = \pi/4$ & $5\pi/4$. Note that this small magnitude is not clearly seen in our experiment, however we extract almost zero magnitude of the amplitudes at those angles, as described in Chapter-4.
6. For pushing along y -axis, i.e., for $\Theta = \pi/2$, the oscillating component is v_x . Likewise for $\Theta = 0$, only the v_y is oscillates. This is an extreme form of anisotropy.

Now we perform some numerical test of Eqs. 2.50 and 2.44, using Monte-Carlo simulation method.

2.3.3 Numerical Calculation for Non-Abelian Gauge Field

The main motivation to perform the numerical test is that due to finite temperature, the wave packet has finite extension in momentum-space, and in the derivation of Eq. 2.51 we have performed some approximation. Therefore we test the analytic expressions with numerical calculation via Monte-Carlo analysis to understand the limitations of the expressions in Eq. 2.51.

When we provide an initial kick $\mathbf{p}^0$ to the cloud, the center of the wave packet is just shifted to the point $\mathbf{p}^0$ with finite momentum extension, as shown in Fig. 2.8, where P_0 and Θ specifies the vector $\mathbf{p}^0$ in momentum-space.

FIGURE 2.8: Schematic representation of the kick provided to the atomic wave packet. Due to finite temperature of the wave packet, shown via blue disk, it has a finite extension in momentum-space. Here the mean momentum $\mathbf{p}^0$ has a magnitude of P_0 at an angle Θ with the $+x$ -axis.

To incorporate the effect of temperature, in the expressions of velocity given by Eq. 2.43, we evaluate this expression for a number of momentum points that are randomly chosen from a Gaussian distribution (given by Eq. 2.47), centered around $\mathbf{p}^0$, and the size of the Gaussian is dictated by the temperature. Then we take the numerical-average of the those at each point. In the following we compare this approach and the approximated expressions in Eq. 2.50 that is supposed to be more accurate for large values of P_0 .

2.3.3.1 Effect of Temperature

The effect of temperature T is shown in Fig. 2.9, for $\Theta = \frac{3\pi}{4}$ and $P_0 = 4$. We have chosen this value since both the components (v_x on the left panel and v_y on the right panel) of velocity have comparable amplitude of oscillation.

FIGURE 2.9: Effect of wave packet temperature on the observability of the *Zitterbewegung*. The left and right panel shows the velocity along x - and y -axis, respectively. The unit of the velocity is the recoil velocity of an atom $v_{\text{rec.}} = \frac{\hbar\kappa}{M}$. For almost zero temperature $T = 1$ pK, the velocity oscillation persists for many cycles while for temperature of order 30-60 nK, the velocity gets more damped for an increasing temperature.

Here we see that for $T = 1$ pK, the velocity is almost undamped, and for a typical values of the temperature $T = 30$, 45, and 60 nK in our experiment (as described in Chapter-4), the damping is increasingly pronounced.

2.3.3.2 Velocity at Different Magnitude of Momentum Kick

FIGURE 2.10: Comparison of the Monte-Carlo(MC) numerical calculation and the approximate formula in Eq. 2.51 for $P_0 = 1\hbar\kappa$ at an angle $\Theta = 3\pi/4$ or 135° . The left and right panel show the velocity along x - and y -axis, respectively. For a near-zero temperature $T = 1$ pK, the effect of damping is negligible, and then the Monte-Carlo calculation and the analytic expression are almost perfectly matched. As temperature increases, the limitations of Eq. 2.51 start to show up.

In this case, we keep the angle fixed and demonstrate the effect of magnitude of pushing momentum P_0 . In Fig. 2.10, we show both components of the velocity (v_x on the left panel and v_y on the right panel) for $P_0 = 1$. In Fig. 2.11, we show both components of the velocity (v_x on the left panel and v_y on the right panel) for $P_0 = 4$.

FIGURE 2.11: Comparison of the Monte-Carlo(MC) numerical calculation and the approximate formula in Eq. 2.51 for $P_0 = 4\hbar\kappa$ at an angle $\Theta = 3\pi/4$ or 135° . The left and right panel show the velocity along x - and y -axis, respectively. Due to the large magnitude of the kick $P_0 = 4\hbar\kappa$, the frequency of oscillation is higher, for the same Θ , as compared to Figs. 2.10. Note the almost perfect match between the Monte-Carlo(MC) and the approximate formula in Eq. 2.51, even at temperature $T = 45$ nK.

The main difference between Figs. 2.10 and 2.11, is the oscillation frequency which can be understood from 2.51. as it suggests that the oscillation frequency ω goes linearly with the pushing momentum, and damping τ should be the same for both cases as it is independent of P_0 .

2.3.3.3 Velocity for the Special Angle of Kick

As was mentioned in page-33, at the end of subsection 2.3.2, there are few special angles of pushing that suppress the oscillation along one axis ($\Theta = 0^\circ, 90^\circ, 180^\circ$, and 270°) or both axes ($\Theta = 45^\circ$ and 225°). Therefore we pay special attention to those angles. Note that the qualitative behaviour of the velocity at any angle Θ is same at the angle $(\Theta + \pi)$, for same P_0 . Therefore in the following we show the behavior of the velocity (both components v_x and v_y) for the angles $0^\circ, 45^\circ, 90^\circ$ for a pushing magnitude $P_0 = 1$ and $4\hbar\kappa$, at a fixed temperature $T = 45$ nK.

FIGURE 2.12: Comparison of the Monte-Carlo(MC) numerical calculation and the approximate formula in Eq. 2.51 for different magnitude of the kick P_0 at an angle $\Theta = 0$, for fixed temperature $T = 45$ nK. The left and right panel shows the velocity along x - and y -axis, respectively. In the left panel, the prediction of Eq. 2.51 is that $v_x(t) = 0$ for $\Theta = 0$. However due to finite momentum extension at $T = 45$, there is a small oscillation induced due to the neighboring momentum states around the mean momentum. The dynamics along y -axis is well captured both by Monte-Carlo calculation and the expression in Eq. 2.51.

As can be seen from Eq. 2.51, that the amplitude of v_x vanishes for *Zitterbewegung* motion. However due to finite momentum-extension of the wave packet, v_x exhibits small oscillation, as can be seen from Fig. 2.12. At small momentum P_0 , for the same temperature, the contributions from momentum other than the central momentum, is more pronounced since the dispersion relation is more flat for small momentum as was shown in subsection 2.2.1.2.

FIGURE 2.13: Comparison of the Monte-Carlo(MC) numerical calculation and the approximate formula in Eq. 2.51 for different magnitude of the kick P_0 at an angle $\Theta = \pi/4$, for fixed temperature $T = 45$ nK. The left and right panel shows the velocity along x - and y -axis, respectively. Although Eq. 2.51 suggests that the amplitudes of both v_x and v_y are zero at $\Theta = \pi/4$ or 45° , the finite temperature of the wave packet leads to non-zero amplitudes of oscillation. Note that the effect of finite temperature is more pronounced at small momentum of kick P_0 .

The angle $\Theta = \pi/4$ is particularly interesting because of the presence of the branch-cut at this angle, as shown in Figs. 2.6. Here in Fig. 2.13, we see that we have small oscillation along both axis for small momentum and this is captured by the Monte-Carlo simulation, while the approximate formula in Eq. 2.51 fail to capture the full dynamics since it is derived for large value of P_0 .

FIGURE 2.14: Comparison of the Monte-Carlo(MC) numerical calculation and the approximate formula in Eq. 2.51 for different magnitude of the kick P_0 at an angle $\Theta = 0$, for fixed temperature $T = 45$ nK. The left and right panel shows the velocity along x - and y -axis, respectively. Although Eq. 2.51 suggests that $v_y(t) = 0$ for $\Theta = \pi/2$, we see a small amplitude that is not captured by the approximate equation at small momentum while the full dynamics for $v_x(t)$ (in the left panel) is captured by Eq. 2.51.

For pushing along $\Theta = \pi/2$ is almost same $\Theta = 0$ except for the fact that here the role of v_x and v_y is flipped, as can be seen from Figs. 2.12. This rich phenomenology is experimentally realized in Chapter-4 where we scan both P_0 and Θ and reveal the complete dependence of *Zitterbewegung* on these parameters. The exciting of all the results, is the observation of anisotropic *Zitterbewegung* — the amplitude of oscillation depends strongly on the angle of pushing for a fixed value pushing magnitude P_0 .

2.3.4 Effects of Degeneracy on the Velocity Dynamics

Here we make a comparison between the Fermi-Dirac distribution $f_{\text{FD}}(\mathbf{p})$ in Eq. 2.46 and the Maxwell-Boltzmann distribution $f_{\text{MB}}(\mathbf{p})$ in Eq. 2.47, in the context of thermal averaging given by Eq. 2.45. In Fig. 2.15, we present the velocity oscillation for two different momentum-kicks, 1. at $\mathbf{P}_0 = 4\hbar\kappa$, $\Theta = 0.75\pi$, 2. $P_0 = \hbar\kappa$, $\Theta = 0.5\pi$. To demonstrate the role of the degeneracy, we compare the velocity over the time $t = 0$ to $200 \mu\text{s}$ for different values of degeneracy. Here we pay special attention to $T/T_F = 0.2$ since this is the region of degeneracy of our gas, as shown in Chapter-3.

FIGURE 2.15: (a) and (b) show the two components of velocity for $P_0 = 4\hbar\kappa$ and $\Theta = 0.75\pi$ for velocity averaging via Eq. 2.45 with Maxwell-Boltzmann (MB) and Fermi-Dirac(FD) distribution at a fixed temperature 45 nK, and different values of degeneracy T/T_F . (c) and (d) show the two components of velocity for $P_0 = 4\hbar\kappa$ and $\Theta = 0.5\pi$ for velocity averaging via Eq. 2.45 with Maxwell-Boltzmann (MB) and Fermi-Dirac(FD) distribution at a fixed temperature 45 nK, and different values of degeneracy T/T_F .

As shown in Figs. 2.15, the first visible effect of degeneracy is the pronounced damping. This smaller damping-time at highly degenerate gas can be understood from the fact that Fermi-Dirac distribution accommodates less number of particle at small momentum than a Maxwell-Boltzmann distribution at the same temperature, due to the Pauli exclusion principle. However to keep the distribution normalized, this ‘missing’ particles are accommodated at the higher momentum region. Therefore the momentum distribution of a deeply degenerate gas (at a temperature T) with a Fermi-Dirac distribution behaves ‘hotter’ than a classical gas (at the same temperature T) with Maxwell-Boltzmann distribution. This flattening of the distribution can be mathematically understood from the presence of $\text{Li}_3(-\zeta)$ in the denominator of Eq. 2.46. For small $\frac{T}{T_F}$, the fugacity ζ is large and this leads to a large value of $\text{Li}_3(-\zeta)$. For large values of T/T_F , the FD and MB almost coincide, as shown in Figs. 2.15(b) and (d). Here we have performed the

integral numerically and the integration is extremely slow since at for every point in time we need to perform a two-dimensional integral.

2.4 Summary and Outlook

In this chapter we have discussed the relevant theory of *Zitterbewegung* and non-Abelian artificial gauge field to get the ground for the experimental results described in Chapter-4. A complete analysis on the emergence of *Zitterbewegung*, in a SU(2) Hamiltonian class is provided along with the detailed calculation on non-Abelian gauge field. The expressions of velocity is derived for finite temperature via velocity averaging. The analytic results are compared with the numerical Monte-Carlo calculation with comments. At the end of the chapter we emphasized the effect of degeneracy compared to a classical gas described by Maxwell-Boltzmann distribution. The conclusions from this chapter will be demonstrated experimentally in Chapter-4.

Chapter 3

Experimental Set-up, Laser Cooling, and Evaporative Cooling

3.1 Introduction

In this Chapter, we briefly describe our experimental set-up and mention few key features of the laser cooling in 461 nm magneto-optical trap(MOT) and in 689 nm MOT. After this we further cool using evaporative cooling to achieve a sub-recoil temperature. The latter is of key importance to observe the *Zitterbewegung* effect (see Chapter 4 for more detail).

In section-3.2, we provide a description of the strontium atomic species and the energy diagram. In section-3.3, we briefly describe our experimental set-up which is detailed in Ref. [1]. In section-3.4, we describe the blue and red MOT very briefly since they have already been detailed in Refs. [1, 118]. After this, we give a detail characterization of the optical dipole trap (ODT) in our experiment in Section-3.5. Then we perform the optical pumping inside the ODT and it is detailed in Section-3.6. Afterwards, the evaporative cooling is characterized in Section-3.7 and the generation of degenerate Fermi gas (DFG) with $T/T_F = 0.21(4)$ is described, with details on the fitting of Thomas-Fermi distribution of a DGF, is described in Section-3.8.

3.2 Strontium-87 Atom: The Key player

We perform the experiments on a cold cloud of strontium-87 (see Chapter-4 for the experimental results). This alkaline-earth atom has some interesting properties that provides some advantage to cool this species, via laser cooling, down to micro-Kelvin or even sub-micro-Kelvin temperature [1, 139–145]. The key to this ultra-low temperature in a magneto-optical trap(MOT) is the presence of the narrow intercombination line transition $^1S_0 \rightarrow ^3P_1$, with linewidth $2\pi \cdot 7.5$ kHz [1, 146–148]. While the presence of nuclear spin in the fermionic strontium poses technical difficulty for efficient cooling during the MOT cycle, yet this nuclear spin has been central for the quantum simulation of $SU(N)$ symmetric system and optical clock [149–152].

Sr Isotope	Natural Abundance	Statistics	Nuclear Spin
84	0.56 %	Bosonic	0
86	9.86 %	Bosonic	0
87	7.00 %	Fermionic	9/2
88	82.58 %	Bosonic	0

TABLE 3.1: The description of most abundant isotopes of strontium. Taken from NIST database [3]

In Table-3.1, we provide the description of the stable isotopes of strontium. Despite the relatively low abundance of fermionic strontium-87, it has been possible to cool and trap a reasonable number of atoms (~ 15 million) at a temperature $3 \mu\text{K}$, at the end of the MOT cycle. We exploit the nuclear spin of the fermionic species in our tripod scheme described in the previous chapter. This high value of nuclear spin provides possibility to simulating higher spin systems.

FIGURE 3.1: The energy level diagram of strontium-87. Taken from a paper of our group [1]. With kind permission of The European Physical Journal (EPJ).

The level-diagram of strontium-87 is shown in Fig. 3.1. The transition at 461 nm and at 689 nm are used for MOT while 707 nm and 679 nm are used for repumping during the blue MOT cycle, as described in Section-3.4. In the following we describe the experimental set-up that has enabled us to perform the cooling and the subsequent experiments in artificial non-Abelian gauge field.

3.3 Description of the Experimental Set-up

The detail of our experimental set-up is discussed in Ref. [1]. In Fig. 3.2, we schematically show the experimental setup on the science table.

FIGURE 3.2: The schematic of the experimental set-up on the science table. Taken from a paper of our group [1]. With kind permission of The European Physical Journal (EPJ).

We briefly describe each part of the set-up here:

Oven: Hot vapor of strontium at temperature 600 °C come out of the nozzles of the oven. There are 900 microtubes in a 1 cm × 1 cm area and each 1 cm long microtube has inner diameter of 200 μm and an outer diameter of 300 μm. The nozzle is kept at 50 °C higher than the oven to avoid any clog.

Two-dimensional Molasses Cooling: As the atoms come out of the nozzle, the transverse velocity is reduced with a pair of molasses beam addressing the $^1S_0 \rightarrow ^1P_1$ transition with wavelength 461 nm. This helps to collimate the atomic beam.

Zeeman Slower: In order to reduce the velocity along the longitudinal direction, we use a 32 cm long Zeeman slower. The optical power for the blue beam in the Zeeman slower is 170 mW with a detuning of -408 MHz with respect to the blue transition.

Two-dimensional MOT: When the atoms leave the Zeeman slower, they enter into a two-dimensional MOT. The purpose of this MOT is cool the atoms in the transverse direction while focusing and guiding the atomic beam along the zero magnetic field line. The MOT axis is kept at an angle 30° with the Zeeman slower axis. This helps to isolate the light of Zeeman slower from the three-dimension MOT, and at the same time the unwanted atomic species that are not cooled cannot enter the science chamber [1]. The differential tube between the 2D-MOT and 3D-MOT helps to isolate the science chamber and maintain an ultra high vacuum there. Two pairs of atomic beams (one with σ_+ and the other with σ_- polarization) have a total optical power of 120 mW (with beam diameter ~ 1 cm) and is detuned by -28 MHz from the blue transition. The field gradient is 5 G/cm.

Three-dimensional MOT: In the 3D-MOT chamber we perform the blue MOT and the red MOT (both are explained later in this chapter).

3.3.1 The Blue Laser System

Our blue laser system is described in Fig. 3.3. This is commercial TOPTICA SHG-PRO laser. Here an initial 922 nm infrared light is frequency doubled via second harmonic generation to 461 nm. The total output power out of the box is 600 mW. This power is distributed to three different paths, as shown in Fig. 3.3. And each path is controlled by an acousto-optic modulator(AOM) that serves the purpose of switch as well controls the power and frequency of the corresponding beam.

Part of the beam goes to the 2D MOT where we perform transverse cooling of the atomic beam after it passes through the Zeeman slower, as was mentioned in the previous section. The other two paths are for 2D molasses cooling and the 3D MOT. We use 2 mW of infrared 922 nm light to inject a diode laser and subsequently a tapered amplified(TA) to generate 1 W of 922 nm IR at the output of TA. After coupling to fiber, this light is used to inject a commercial TOPTICA SHG PRO to generate 170 mW of 461 nm blue light.

FIGURE 3.3: The blue laser system in our experiment. Taken from a paper of our group [1]. With kind permission of The European Physical Journal (EPJ).

Note that we lock the blue laser with the $|m_J = 0\rangle$ Zeeman state of the 1S_0 ground-state to the $|m_J = -1\rangle$ state of the 1P_1 manifold, of strontium-88 with a dedicated atomic reference oven. Changing the magnetic field in the reference cell, we can switch from one atomic species to another, as described in Ref. [1].

3.3.2 Repumper Laser

Since the $^1S_0 \rightarrow ^1P_1$ transition is not closed as shown in Fig. 3.1, one out of every twenty thousand atoms from the excited-state decay via the channel $^1P_1 \rightarrow ^1D_2$, shown by the red dashed line in Fig. 3.1 [153]. Part of those atoms that decay to 1D_2 state, finally arrive at the 3P_2 (lifetime is 20 seconds at ambient temperature [145, 154]) and not further cooled by the MOT. To improve the lifetime of the blue MOT, we employ two repumper lasers at 707 nm (to address $^3P_2 \rightarrow ^3S_1$ transition), so that atoms go back to the 1S_0 state to further participate in the MOT cycle. Some atoms decay to the 3P_0 state from the 1S_0 . For those atoms, we use a 679 nm (to address $^3P_0 \rightarrow ^3S_1$ transition).

FIGURE 3.4: The repumping scheme during the blue MOT. Taken from a paper of our group [1]. With kind permission of The European Physical Journal (EPJ).

In our experiments, we have two dedicated TOPTICA DL PRO lasers operating at 707 nm and 679 nm, respectively. These lasers are frequency-locked with a wavemeter (HighFinesse WS/7 with accuracy 40 MHz). Due to the presence of hyperfine structure in Sr-87, we use an electro-optic modulator (with modulation frequency 540 MHz) to address several hyperfine transitions between the aforementioned transitions of the repumper lasers, as shown in Fig. 3.4. Since 80% of the total population in 3P_2 is in $|F = 13/2\rangle$ and $|F = 11/2\rangle$, we particularly address those transitions for the 707 nm repumper [1, 145].

3.3.3 The Red Laser System

FIGURE 3.5: The red laser system in our experiment. Taken from a paper of our group [1]. With kind permission of The European Physical Journal (EPJ).

The red laser system, shown in Fig. 3.5, addresses the transitions for the intercombination line. An external-cavity diode laser is locked to an ultra-low expansion cavity of length 10 cm and finesse 3000. The laser is locked to the cavity via Pound-Drever-Hall technique [1]. The cavity drift is 10 Hz/s and in order to compensate the drift we use saturated fluorescence spectroscopy on $|m_J = 0\rangle \rightarrow |m_J = 0\rangle$ transition from the 1S_0 to 3P_1 manifold of ^{88}Sr . A magnetic field of 10 G is applied to get rid of the contribution from other m_J states of the excited-state, during

the fluorescence spectroscopy. This frequency compensation is done with AOM1 in Fig. 3.5.

As the master laser is locked, we use this laser to inject other slave lasers, e.g., to address $|F = 9/2\rangle \rightarrow |F = 9/2\rangle$ and $|F = 9/2\rangle \rightarrow |F = 11/2\rangle$ transitions, as shown in Fig. 3.5. The 1.2 GHz EOM is placed to take into account the hyperfine shift of the $|F = 9/2\rangle \rightarrow |F = 11/2\rangle$ transition with respect to the Sr-88 transition. The output laser beam of each slave laser goes through an AOM and then fiber-coupled to send the beam at the science table.

3.4 The Laser Cooling Mechanism for Strontium-87

The cooling is performed in two separate MOTs (blue MOT and red MOT), we overlap the red and blue beams to maximize the overlap between the two MOT positions and to increase the transfer efficiency. Here we describe the operation of the two MOTs briefly.

FIGURE 3.6: The timing diagram for the blue MOT and red MOT.

3.4.1 The Blue MOT

As described in Section-3.3, after partially cooling in the 2D molasses, Zeeman slower, and in 2D MOT atoms enter the science chamber. Here three pairs (for cooling along all three spatial axes) of laser beam along with the magnetic field trap and cool the atoms. Each pair of beams consists of two counter-propagating laser beams of opposite helicity $\sigma_{\pm}$. The quadrupole magnetic field in the MOT is provided by a pair of coil in the anti-Helmholtz configuration with a magnetic field gradient 43 G/cm for the blue MOT.

The loading of the blue mot lasts for ~ 4.5 seconds and during this time, the power in three beams are 82 mW with each beam having a size of 1 cm. Then we reduce the power of the blue beams and start the low-power blue MOT cycle, as described in Fig. 3.6. This low-power blue MOT helps to cool the atoms further at a final temperature around 2 mK with ~ 40 million atoms [155]. After this stage, we start the red MOT.

3.4.2 The Red MOT

The red MOT consists of two stages. Since the temperature of the cloud after the blue MOT is still much larger than the linewidth of the intercombination line, we spectrally broaden both the red MOT beam (to address $|F = 9/2\rangle \rightarrow |F = 11/2\rangle$) and the stir beam (to address the $|F = 9/2\rangle \rightarrow |F = 9/2\rangle$). Note that the purpose of the stir beam is to keep the atoms from getting out of resonance locally due to large difference in the Landé g -factor of the ground state and the excited state $|F = 11/2\rangle$ of the 3P_1 manifold. The magnetic field gradient and the power of each beam during the MOT cycle are mentioned in Fig. 3.6.

After the broadband cycle, we start the single-band cycle of the red MOT. We lower the power of the MOT beams and increase the magnetic field gradients to compress the cloud further so that loading to the optical dipole trap is more efficient. The final temperature of the MOT is 3 μ K with more than 15 million atoms.

Although temperature of the MOT is quite cold, we need to cool the atoms further to reach a temperature below the recoil temperature 230 nK. To accomplish this

goal, we perform evaporative cooling after loading the atoms from MOT to an optical dipole trap (ODT). The loading to the optical dipole trap and the subsequent evaporation is detailed in the following sections.

3.5 Optical Dipole Trap

A classical dipole placed in a static electric field gets attracted to the region with largest electric field when the polarizability is positive. Akin to this phenomena, a neutral atom that acquires a dipole moment under the influence of a static field, seeks high-field region [130]. The potential energy $U = -\frac{1}{2}\alpha|\mathbf{E}|^2$ is negative for an atom in its ground state, thus the atom is attracted to high-intensity region.

FIGURE 3.7: A two-level system in a red-detuned light. The initial ground- and excited-states ($|e, g\rangle$ in the left panel) get dressed and the shift in opposite directions ($|1, 2\rangle$ in the right panel).

Optical dipole trap (ODT) exploits the simple principle. As shown in Fig. 3.7, a two-level system with states $|e, g\rangle$ under the influence of a far-detuned laser, shown by the red arrow. The force on the atoms is state-dependent – the ground state of the dressed state $|1\rangle$ energy is lowered. The trap depth for a two-level atoms reads

$$U(\mathbf{r}) = -\frac{3\pi c^2}{2\omega_A} \left(\frac{\Gamma}{\omega_A - \omega_L} + \frac{\Gamma}{\omega_A + \omega_L} \right) I(\mathbf{r}), \quad (3.1)$$

where ω_L and ω_A are the frequency of atomic transition and the laser frequency, respectively; I is the intensity of the laser at the position of the atom, c is the speed of light in vacuum, and Γ is the linewidth of the excited state.

For a simplified two-level atom, under the influence of a laser with electric field $\mathbf{E}$ and ignoring the contribution of the counter-rotating term in Eq. 3.1, the dressed-energies of an atom read [156]:

$$\Delta E = \pm \frac{|\langle e | \mu | g \rangle|^2}{\omega_L - \omega_A} |\mathbf{E}(\mathbf{r})|^2 = \pm \frac{3\pi c^2 \Gamma}{2\omega_L^3 \delta} I(\mathbf{r}), \quad (3.2)$$

where $\delta = \omega_L - \omega_A$ is the detuning. The upper and lower signs are for ground- and excited-states, respectively. From Eq. 3.2, it is evident that for red-detuned laser, namely $\omega_L < \omega_A$, as pictorially represented in Fig. 3.7. The atom in ground-state is drawn towards the high-intensity region (region with larger I) of the laser – the laser profile with intensity gradient behaves like a trapping potential.

The fundamental spatial mode profile a laser beam is described by a Gaussian and under the paraxial-wave approximation, for Gaussian beam that propagates long z direction, the intensity profile has the functional form [156],

$$I(\mathbf{r}) = \frac{2P_0}{\pi w_0^2} \left[1 + \left(\frac{z}{z_R} \right)^2 \right]^{-2} \exp \left[-\frac{2(x^2 + y^2)}{1 + \left(\frac{z}{z_R} \right)^2} \right], \text{ where} \quad (3.3)$$

P_0 : total power of the beam,
 w_0 : beam waist,
 $z_R = \frac{\pi w_0^2}{\lambda_L}$: Rayleigh length for laser wavelength λ_L .

As we can see from Eq. 3.3, Gaussian beam has the highest intensity at the origin $\mathbf{r} = \mathbf{0}$. Thus this beams behave like a trapping potential for atoms at the ground state, as atoms tend to accumulate near this high-intensity region. We use two such beams and overlap them with the MOT to load the atoms into the crossed-ODT shown in Fig. 3.8. The trap depth on the horizontal-plane ($z = 0$) is shown here.

As the laser is far detuned, the absorption of any photon is suppressed, the force on an atom for a far-detuned laser, i.e., $|\omega_L - \omega_A| \ll \omega_A$ is conservative.

The beam in Eq. 3.3 behaves parabolically near the origin and using Eq. 3.2 one can extract the longitudinal frequency $\omega_{\parallel}$ and transverse frequency $\omega_{\perp}$ as

$$\omega_{\perp} = 4 \frac{U_0}{M w_0^2}, \quad \omega_{\parallel} = 2 \frac{U_0}{M z_R^2}, \quad (3.4)$$

where U_0 is the trap-depth at the origin and M is the mass of the atom. Each beam is assumed to be radially symmetric, as is the case in our experiment. Note that

although a Gaussian beam is parabolic near the center, the beam is highly non-parabolic as one moves away from the center. This is a good approximation so long as the trap-depth is high, therefore the atoms experience *only* the near-parabolic part of the trap. However this harmonic approximation breaks down near the end of evaporation when the trap-depth is lowered to a much smaller value than the initial trap-depth, and atoms start to explore the *anharmonic* part of the trap.

FIGURE 3.8: Schematic of the horizontal cross-dipole of our experiment and the calculated potential profile at $z = 0$.

There is one more detail on the vertical profile of the dipole trap. In our setup, the dipole trap is crossed on the horizontal plane and gravity is along the vertical direction z (z is positive downward). Therefore, in addition to the force from the dipole trap, the atoms will also be influenced by the gravity and the total potential reads

$$U_{\text{total}} = U_0 - Mgz, \quad (3.5)$$

where U_0 is the trap depth entirely due to the dipole trap, g is the gravitation acceleration, and z is the location of the atom with mass M . We show this variation along the vertical direction in Fig. 3.9(a). Here the potential difference between the two extrema is the effective trap-depth for the atoms. Eventually the trap-depth in our system is estimated to be $\approx 72 \mu\text{K}$, taking the effect of gravity into account while this quantity is $\approx 85 \mu\text{K}$ without gravity.

FIGURE 3.9: (a) Calculated total trap-depth profile along the vertical direction in the presence of gravity and the cross-dipole-trap. The Orange and red dots represents the two extrema of the potential profile and the potential difference between this two point is the effective trap-depth of the atoms. (b) the shift of minimum and maximum trap-depth points as a function of laser power with same parameters of our experiment, namely, wavelength $\lambda = 1064 \text{ nm}$, waist $w_0 = 65 \mu\text{m}$. The red and orange lines are the trajectories of the red and orange dots in the left figures, respectively. The dashed-vertical line shows the minimum power necessary to hold a single strontium(87) atom in our dipole trap to counter-balance the gravity. Here $P_{\text{Gravity}} \approx 680 \text{ mW}$.

We would like to also point out that the minimum of the potential profile is shifted vertically as one changes the total power. This suggests that if one adiabatically ramps-down the power of the lasers, the center of the cloud will shift vertically and this shift will be most dramatic during the last part of the evaporation when the power is low. We note on passing that indeed we have observed this trend in our evaporation ramp when we lower the trap-depth significantly. However we are not going to elaborate much on this simple yet interesting detail.

3.5.1 Dipole Trap Setup and Loading Atoms into Dipole Trap

FIGURE 3.10: The schematic representation of the dipole trap setup.

In our experiment, we overlapped two Gaussian beams with wavelength 1064 nm, on the horizontal plane. The relative angle between the propagation directions of the beam is about 110° . Each of the two beam is derived from a MOPA-Mephisto 1064 nm fiber laser, and their powers are controlled independently with acousto-optic modulators (AOMs). The diffracted beam from the AOMs are fiber-coupled via polarization maintaining fiber and transferred to the science chamber where they are focused to a waist of $65 \mu\text{m}$. The power of each beam is locked via two independent PID units (Koheron PI200) and the maximal output power of the two beams at the output of the fiber read 4.8 W and 4.3 W, respectively.

FIGURE 3.11: Number of atoms loaded into the cross-dipolar trap and the temperature of the gas, as a function of the maximum trap-depth. As we can see here, the temperature and number of atoms increases almost linearly as the trap-depth is increased. The solid lines are linear fits to extract the slopes, as reported inside the main figure. The mentioned trap-depth is for the ground state, and is estimated theoretically with Eq. 3.2.

We turn on the dipole trap beams during the blue-MOT cooling. This early turn-on leads to a loading of approximately 2.5×10^6 atoms in the ODT at a temperature around $12 \mu\text{K}$. This temperature is higher than the temperature ($3 \mu\text{K}$) during the last stage of MOT. This inherent increase of temperature while loading into an optical dipole trap is shown as a function of trap-depth in the Fig. 3.11. As the ratio $\frac{N}{T}$ remains constant the ratio T/T_F with Fermi temperature T_F , increases at a rate $\sim N^{1/3}$, as the trap-depth is increased for more number of atoms. The reason for such an increased temperature could be attributed to the inhibition of MOT cooling due to the light-shift induced by the off-resonant light [157, 158].

As evident from Fig. 3.11, the number of atoms loaded into the trap, in principle, could still be improved by increasing the trap-depth. If we had opted for reducing the beam waist for a quadratic gain in the trap-depth, as shown in Eq. 3.3, the mode volume would also be reduced at a cubic rate. Thus it does not seem a feasible choice. However, if we had more power at the end of the fibers, this would increase the number of atoms loaded into the trap. Therefore, we have always put effort to couple as much light possible into the fiber.

We characterize the ODT first by measuring the lifetime of atoms inside the ODT. Theoretically the evolution of number of atoms $N(t)$ inside ODT is modeled via [156]:

$$\dot{N}(t) = -\alpha N(t) - \beta' N(t)^2 - \gamma \int d\mathbf{r} \cdot n^3(\mathbf{r}), \quad (3.6)$$

where $n(\mathbf{r})$ is the density profile and α , β' , and γ are different loss coefficient as described below:

- single-particle loss coefficient, α : although the vacuum pressure inside the science chamber is $\sim 10^{-10}$ mbar. There are still some residual gas remaining in the chamber and those gases are almost at room temperature. Any collision with those hot gases is sufficient to knock an atom out of the ODT. This is the dominant mechanism of atom loss in our dipole trap [156, 159].
- two-body loss coefficient, β' : binary collisions between atoms in the trap can release energy and increase the kinetic energy of the atoms. This energy can be sufficient for an atom to escape the trap. Two-body loss process is dominant when the atoms are not in their absolute ground-state [158–160]. The prime in the beta represents the atom number loss compared to the coefficient of density loss β . This process is supposed to be absent in our experiment since the atoms are in almost degenerate hyperfine ground-states, and collision does not change the nuclear spin [161].
- three-body loss coefficient, γ : in a collision of three atoms in the trap, two of them can form a molecule and the third atoms gain the remaining kinetic energy of the atoms [162, 163]. If this energy is sufficiently high, the atom can escape the trap. This process happens when the density of atoms is extremely high ($\sim 10^{15}$ atoms/cm³) [164]. For the density of atoms in our experiment ($\sim 10^{13}$ atoms/cm³) this process is completely negligible.

We recognize that the expression in Eq. 3.6 is Ricatti equation when we set γ to zero. Therefore we readily solve for $N(t)$ as

$$N(t) = N_0 \exp[-\alpha t] \cdot \frac{\alpha}{(\alpha + N_0 \beta') - N_0 \beta' \cdot \exp[-\alpha t]}, \quad (3.7)$$

where N_0 is the number of atoms at $t = 0$.

FIGURE 3.12: The measurement of the lifetime inside the dipole-trap. The number of atoms as a function of holding time inside the ODT is fitted with an exponential to extract the lifetime 43.9 seconds. The linear trend in the log-lin plot suggests that the dominant atom loss mechanism in our experiment is the background-gas collision.

The expression in Eq. 3.7 is a modified exponential. However, the time-evolution of number of atoms in ODT, as shown in Fig. 3.12, the log-lin plot is linear. Therefore the contribution of two-body loss is negligible.

However, there is one subtlety regarding the fitting. Even when the two-body loss is negligible, if one tries to fit the experimental data with Eq. 3.7, to extract the lifetime and the two-body loss coefficient, the fit may converge to a lifetime that is over-estimated while the two-body coefficient β' is non-zero. Therefore Eq. 3.7 should only be employed only when the log-lin plot of number of atoms versus holding time is non-linear. For our data in Fig. 3.12, we extract a lifetime of 43.9 seconds while fitting the expression in Eq. 3.7 by setting an *a priori* value $\beta' = 0$.

3.6 Optical Pumping

The idea of optical pumping is to ‘pump’ atoms out of one state to another, via optical means [165]. To illustrate the basic notion, we refer to Fig. 3.13. Here we consider a simple situation where atoms are optically excited from $|m_F = -1/2\rangle_g$ to $|m_F = 1/2\rangle_e$ via the laser light shown schematically in red, where subscripts e and g of the kets refer to excited- and ground-state manifolds, respectively. The polarization and frequency is chosen so that the laser can only mediate the $|m_F = -1/2\rangle_g \rightarrow |m_F = 1/2\rangle_e$ transition.

FIGURE 3.13: Schematic representation of optical pumping. Before optical pumping, atoms occupy both the hyperfine ground-states $|m_F = \pm 1/2\rangle_g$. In the presence of an appropriate laser beam with right polarization, shown by the ‘red arrow’, the atoms only get excited via the transition $|m_F = -1/2\rangle_g \rightarrow |m_F = 1/2\rangle_e$. Subscripts e and g of the kets refer to excited and ground-state manifolds, respectively. The atoms that relax back to the $|m_F = 1/2\rangle_g$ of the ground state keeps accumulating while population in $|m_F = -1/2\rangle_g$ is continuously depleted due to the presence of the aforementioned laser light that only mediates the transition $|m_F = -1/2\rangle_g \rightarrow |m_F = 1/2\rangle_e$.

This way, the atoms in $|m_F = 1/2\rangle_g$ are not perturbed by this light. As the atoms get excited to $|m_F = 1/2\rangle_e$, and due to the finite lifetime of the excited-state, the atoms will relax back to both $|m_F = \pm 1/2\rangle_g$, according to the branching ratio. Although the atoms in $|m_F = 1/2\rangle_g$ will be undisturbed, the atoms that go back to $|m_F = -1/2\rangle_g$ will be excited again due to the continuous presence of the laser light. Therefore while $|m_F = 1/2\rangle_g$ will be depleted, the number of atoms will

continually get accumulated in $|m_F = 1/2\rangle_g$. Thus the atoms in $|m_F = -1/2\rangle_g$ are optically pumped to $|m_F = 1/2\rangle_g$.

FIGURE 3.14: Optical Pumping of $|m_F = 1/2 - 7/2\rangle \rightarrow |m_F = 9/2\rangle$, while the Zeeman states $|m_F = (-9/2) - (-1/2)\rangle$ are intact.

As shown in Fig. 3.14, for the purpose of our gauge field experiment, we need to polarize the atoms to $|F = 9/2, m_F = 9/2\rangle_g$ of 1S_0 ground state while $|F = 9/2, m_F = 5/2\rangle_g$ and $|F = 9/2, m_F = 7/2\rangle_g$ must be empty. Therefore it is sufficient to pump atoms from $|F = 9/2, m_F = 5/2\rangle_g$ and $|F = 9/2, m_F = 7/2\rangle_g$ to $|F = 9/2, m_F = 9/2\rangle_g$. However, we pump all the atoms from $|F = 9/2, m_F = 1/2\rangle_g$ to $|F = 9/2, m_F = 7/2\rangle_g$, into the $|F = 9/2, m_F = 9/2\rangle_g$ state, as shown in Fig. 3.14. The reason we emptied $|F = 9/2, m_F = 1/2\rangle_g$ and $|F = 9/2, m_F = 3/2\rangle_g$ is two-fold:

- Firstly, as we will use the $\{|m_F = 5/2\rangle_g, |m_F = 7/2\rangle_g, |m_F = 9/2\rangle_g\}$ manifold for our gauge field experiment, and the transitions of these state with $|F = 9/2, m_F = 7/2\rangle_e$ will be resonantly addressed. Therefore having no population in $|F = 9/2, m_F = 3/2\rangle_g$ and $|F = 9/2, m_F = 1/2\rangle_g$ will allow us to directly connect $m+F=9/2$ to a dark state sensitive to our gauge field (see Chapter-4).
- Secondly, we will perform evaporative cooling after the optical pumping. Therefore it would be favourable to have more atoms in $|F = 9/2, m_F = 9/2\rangle_g$ before the onset of evaporation, so as to increase the overall signal of our gauge field experiment. Moreover atoms in $|m_F = -9/2\rangle_g$ to $|m_F = 3/2\rangle_g$

are essentially spectators and atoms in $|F = 9/2, m_F = 9/2\rangle_g$ gets redistributed among $\{|m_F = 5/2\rangle_g, |m_F = 7/2\rangle_g, |m_F = 9/2\rangle_g\}$ under the action of gauge field lasers. We also note that the atoms with negative m_F are not perturbed as they will help to thermalize the $|m_F = 9/2\rangle_g$ states during the evaporation via s -wave collision.

However, to perform the optical pumping, one needs to know the spectral positions of each m_F states in the presence of a magnetic field that sets the quantization axis. Towards that end, we apply a reasonable large magnetic field of 23.5 Gauss so that the level spacing between neighboring m_F states are ≈ 2 MHz which is larger than the Doppler broadening of the gas (~ 50 kHz). As shown in Fig. 3.14, we address each m_F states independently, namely we first optically pump atoms from $|F = 9/2, m_F = 1/2\rangle_g$ to $|F = 9/2, m_F = 3/2\rangle_g$ and then $|F = 9/2, m_F = 3/2\rangle_g$ to $|F = 9/2, m_F = 5/2\rangle_g$ and all the way to $|F = 9/2, m_F = 9/2\rangle_g$. Before engaging into the optical pumping we perform precise spectroscopy to determine the detunings from the zero-field position of each m_F levels of interests, in the excited-state manifold $|F = 9/2\rangle_e$.

3.6.1 Atom-loss Spectroscopy of $|F = 9/2\rangle \rightarrow |F = 9/2\rangle$ and Optical Pumping to $|F = 9/2, m_F = 9/2\rangle$ State

FIGURE 3.15: Illustration of the idea of atom-loss spectroscopy: at a non-zero bias field, the Zeeman sub-levels of $|F = 9/2\rangle_e$ are separated and here we show only one of those levels, namely $|m_F = 9/2\rangle_e$. When $|F = 9/2, m_F = 7/2\rangle_g \rightarrow |F = 9/2, m_F = 9/2\rangle_e$ transition is addressed on resonance, $|F = 9/2, m_F = 7/2\rangle_g$ population gets depleted while the off-resonance excitation does not deplete the population. Afterwards, this $|F = 9/2, m_F = 7/2\rangle_g$ is probed with a σ_+ light on the $|F = 9/2, m_F = 7/2\rangle_g \rightarrow |F = 11/2, m_F = 9/2\rangle_e$ transition. At the resonance of $|F = 9/2, m_F = 7/2\rangle_g \rightarrow |F = 9/2, m_F = 9/2\rangle_e$, the loss of population is maximum, i.e., remaining population is minimum.

We have chosen $|F = 9/2\rangle_g \rightarrow |F = 9/2\rangle_e$ hyperfine transition to perform the optical pumping. The reason to choose $|F = 9/2\rangle_g \rightarrow |F = 9/2\rangle_e$ transition for optical pumping is a matter of convenience, namely we can use the same lasers as our gauge field lasers while we take advantage of the high purity of polarization of these laser beams ($> 99.5\%$). In particular we use a σ_+ light to address the transitions between $|F = 9/2\rangle_g \rightarrow |F = 9/2\rangle_e$ manifolds, as shown by the solid ‘red’ lines in Fig. 3.14.

In principle, one needs a probe laser to perform the spectroscopy on $|F = 9/2\rangle_g \rightarrow |F = 9/2\rangle_e$ transitions. However, we did so without using another probe (in addition to our existing $|F = 9/2\rangle_g \rightarrow |F = 11/2\rangle_e$ probe laser beam), by performing the so called atom-loss spectroscopy and the idea is illustrated in Fig. 3.15.

FIGURE 3.16: Atom-loss spectroscopy on $|F = 9/2\rangle_g \rightarrow |F = 9/2\rangle_e$: the blue dots are the experimental data where we scan the laser frequency, at a bias field of 23.5 Gauss and when the atoms are inside the dipole trap. The maximum loss of atoms correspond to the resonant frequency of a particular transition. The m_F states associated to the transitions are shown while the population represents the ground-state populations. We fit the experimental data with a Gaussian of negative amplitude to extract the resonant frequencies. The magnetic shift of the ground state is negligible due to small nuclear Landé g -factor of -1.3×10^{-4} .

The idea can be summarized as follows where we illustrate the idea for the spectroscopy of $|F = 9/2, m_F = 7/2\rangle_g \rightarrow |F = 9/2, m_F = 9/2\rangle_e$ transition:

1. before optical pumping, all the hyperfine ground-states of 1S_0 are almost equally occupied. Then we shine a σ_+ light with a fixed frequency to address $|F = 9/2, m_F = 7/2\rangle_g \rightarrow |F = 9/2, m_F = 9/2\rangle_e$ transition. Therefore atoms from $|F = 9/2, m_F = 7/2\rangle_g$ will be *depleted* via excitation to the excited-state, and will subsequently relax back to ground-state, according to the Clebsch-Gordan coefficients of each transition.

2. afterwards, we resonantly probe the population of $|F = 9/2, m_F = 7/2\rangle_g$ using $|F = 9/2\rangle_g \rightarrow |F = 11/2\rangle_e$ probe. The resonance frequency of this transition is already known from a previous spectroscopy on $|F = 9/2, m_F = 7/2\rangle_g \rightarrow |F = 9/2, m_F = 9/2\rangle_e$.
3. As we scan the laser frequency in step-1, *maximum depletion* of atoms from $|F = 9/2, m_F = 7/2\rangle_g$ state will happen when the laser in step-1 is at resonance, as shown in Fig. 3.16.

The aforementioned approach is executed by shining $4 \mu\text{W}$ power (beam waist is 0.1 cm) for the depletion of atoms from ground-state and the pulse duration is $100 \mu\text{s}$. As we would like to perform the optical pumping when the atoms are held in the optical dipole trap(ODT), the resonant frequency from the spectroscopy will also provide the information on the differential light-shift between the ground- and excited-states. This is shown in Fig. 3.16, where we perform the aforementioned atom-loss spectroscopy either in the presence of dipole trap or in the absence of the ODT. In the latter case when we perform the atom-loss spectroscopy, we turn-off the ODT beams and wait $\sim 65 \mu\text{s}$ to make sure a complete extinction of the beams, then we send the pulse for atom-loss. This way we measure the differential light-shift and the resonance frequencies of $|F = 9/2\rangle_e$ manifold.

In Fig. 3.16, we present the spectroscopy of two transition, namely $|F = 9/2, m_F = 5/2\rangle_g \rightarrow |F = 9/2, m_F = 7/2\rangle_e$ and $|F = 9/2, m_F = 7/2\rangle_g \rightarrow |F = 9/2, m_F = 9/2\rangle_e$. As can be seen here, the transitions are blue-shifted in the presence of the dipole trap and the shift is $\sim 310 \text{ kHz}$. The level-separation between the two neighboring Zeeman states also provides us a means to a precise calibration of the magnetic field, as will be illustrated in the following.

At the bias field of 23.5 Gauss , we expect a level separation of 1.993 MHz between $|F = 9/2, m_F = 7/2\rangle_g$ and $|F = 9/2, m_F = 9/2\rangle_g$, from linear Zeeman effect with Landé g -factor $g_F = 2/33$. However, we found this separation to be 2.284 MHz from the data shown in Fig. 3.16. To understand the source of this apparent 291 kHz , we calculate level-spacing taking into account the quadratic Zeeman shift. Towards that end, one needs to diagonalize the following hyperfine Hamiltonian

$$H = H_Z + A\vec{I} \cdot \vec{J} + Q \frac{\frac{3}{2}\vec{I} \cdot \vec{J} - I(I+1)(J+1)}{2IJ(2I-1)(2J-1)}, \text{ with} \quad (3.8)$$

$$H_Z = (g_s S_z + g_l L_z - g_I I_z) \mu_0 B_z,$$

where $g_s \approx 2$ and $g_l = 1$ are the g -factors of spin and orbital angular momentum, respectively. And S_z , L_z , I_z are the z components of spin, orbital, and nuclear spin, respectively. The g -factor of nuclear spin reads $g_I = \frac{\mu_I(1-\sigma_d)}{\mu_0 I}$, where $\mu_I = -1.0924\mu_N$ (for Sr(87) [166]) is nuclear magnetic moment, $\sigma_d = 0.00354$ (for Sr(87) [167]) is diamagnetic correction, μ_N is nuclear magneton, $\mu_0 = \frac{\mu_B}{h}$ is the Bohr magneton, and h is Planck's constant.

Using the experimental value of magnetic dipole moment of 3P_1 subspace $A = -260$ MHz and ignoring the contribution from electric quadrupole $Q = -35$ MHz [168, 169], we diagonalize the Hamiltonian in Eq. 3.8. The Breit-Rabi like diagram for 3P_1 is shown in Fig. 3.17(a). In Fig. 3.17(b), the field-dependence of $|F = 9/2\rangle$ (at zero-field) manifold is shown up to 23.5 Gauss. As shown in Fig. 3.17(b), the bending of the energy levels is pronounced around 23.5 Gauss. Quantitatively, we extract the level-separation between $|F = 9/2, m_F = 7/2\rangle_e$ and $|F = 9/2, m_F = 9/2\rangle_e$ at 23.5 Gauss as 2.277 MHz which is reasonably close to the experimental value 2.284 MHz shown in Fig. 3.16.

FIGURE 3.17: The magnetic field dependence of the 3P_1 manifold: (a) the three hyperfine sub-spaces of $F = 7/2$, $9/2$, and $11/2$. (b) The magnetic field-dependence of the $F = 9/2$ manifold with respect to zero-field value up to the bias field 23.5 Gauss.

As we have the resonance position of the $|F = 9/2\rangle$ space at our disposal, we are all set for the optical pumping. Towards that goal, we broaden the laser frequency by 0.5 MHz across the resonance frequency of $|F = 9/2, m_F = 1/2\rangle_g \rightarrow |F =$

$9/2, m_F = 3/2\rangle_e$ transition, to pump atoms out of $|F = 9/2, m_F = 1/2\rangle_g$ state, to $|F = 9/2, m_F = 3/2\rangle_g$ state. The total power is $60 \mu\text{W}$ and intensity per needle of the broadened spectrum is $\approx 40I_s$, where $I_s = 3 \mu\text{W}/\text{cm}^2$ is the saturation intensity of the intercombination line. The laser pulse duration is 7 ms. We repeat the same process to optically pump from $|m_F = 3/2\rangle_g$ to $|m_F = 5/2\rangle_g$ with the same laser parameters.

However, to optically pump from $|m_F = 5/2\rangle_g$ we use 2.6 times more laser power since the Clebsch-Gordan coefficient is relatively weak. This way we empty $\{|m_F = 1/2\rangle_g, |m_F = 3/2\rangle_g, |m_F = 5/2\rangle_g, |m_F = 7/2\rangle_g\}$ states and transfer those atoms to $|m_F = 9/2\rangle_g$. At the end of optical pumping, we have 4.5 times increment of $|m_F = 9/2\rangle_g$ population and the residual population in $\{|m_F = 1/2\rangle_g, |m_F = 3/2\rangle_g, |m_F = 5/2\rangle_g, |m_F = 7/2\rangle_g\}$ states is less than 4% in total. Note that the populations in negative m_F states of the ground-state remains intact. We exploit these atoms to thermalize the atoms in $|m_F = 9/2\rangle_g$ state, during the evaporation, via s -wave collision. This is the topic of the next section.

3.7 Evaporative Cooling

As the optical dipole trap (ODT) is prepared and the atoms are loaded from the magneto-optical trap to the ODT, and atoms are partially polarized to $|F = 9/2, m_F = 9/2\rangle_g$, everything is ready for further cooling. The idea of evaporative cooling is illustrated in Fig. 3.18. When two atoms, with energy E_1 and E_2 collide elastically, they exchange their kinetic energy and the new energies of the atoms are E_3 and E_4 , respectively. The energies are subjected to the constraint: $E_1 + E_2 = E_3 + E_4$. After Collision, if the kinetic energy of one of the atoms is higher than the trap-depth U_0 , that atom will leave the trap. If one keeps lowering the trap-depth, this way the high-energy atoms are driven away from the trap and the atoms with small kinetic energy remain in the trap [170–172].

The evaporation involves mainly four parameters –

1. Trap depth $U(t)$ at time t ,

2. Number of atoms $N(t)$,
3. Collision rate $\gamma(t)$,
4. Temperature $T(t)$.

The behavior of the last three parameters can be identified using trivial scaling-law and they subsequently suggest a scheme to lower the trap-depth, as will be delineated in the following.

FIGURE 3.18: The main principle of evaporative cooling where atoms with initial energies $E_{1,2}$ go to energy states $E_{3,4}$, via elastic collision. When $E_4 > U_0$, this atom leaves the trap leaving behind an atom with smaller kinetic energy $E_3 < E_{1,2}$. This way the gas gets colder while the high-energy atoms are driven away from the trap via continuous lowering of the trap-depth.

For a trap having the zero energy at the bottom of the trap, the average energy loss-rate $\dot{E}$ due to loss of atoms reads

$$\dot{E} = \dot{N} (U + \alpha k_B T) + \frac{\dot{U}}{U} \cdot \frac{E}{2}, \quad (3.9)$$

where $\alpha k_B T$ is the average thermal energy of the escaping atoms and for s -wave collision $\alpha = (\eta - 5)/(\eta - 4)$ [173, 174], and k_B is Boltzmann constant. Treating the gas classically, the total energy reads $E = 3NkT$, and substituting this expression into Eq. 3.9, we find the relation between the number of atoms and the trap depth, at any specific time –

$$\frac{N}{N_i} = \left(\frac{U}{U_i} \right)^{3/2(\eta'-3)}, \quad (3.10)$$

where $\eta' = \eta + \alpha$, and N_i is the number of atoms in ODT for an initial trap-depth of U_i . As can be seen from the above equation that the number of atoms decreases from its initial value N_i inside the ODT, as we ramp down the trap-depth from its initial value U_i .

The relation between temperature and number of atoms has the following scaling law –

$$\frac{T}{T_i} = \left(\frac{N}{N_i} \right)^{2(\eta'-3)}, \quad (3.11)$$

where N_i and T_i are the initial number of atoms and temperature, respectively.

Next we evaluate the scaling-law of another important parameter of evaporation – the collision rate, as a function of trap-depth and N . As we drive away the high energy tail of the Boltzmann distribution, the remaining atoms need to collide among themselves to thermalize and reach an equilibrium temperature before further evaporation. The collision rate dictates how fast the remaining atoms will reach an equilibrium temperature and it depends on the trap-depth via the following relation

$$\frac{\gamma}{\gamma_i} = \left(\frac{U}{U_i} \right)^{\eta'/2(\eta'-3)}. \quad (3.12)$$

Eq. 3.12 implies that as the trap is lowered, the collision-rate decreases and the thermalization takes longer and longer. This is responsible for the slowing-down of the evaporation. Moreover, this suggests that to keep the atoms thermalized, as the ramp-down of the trapping potential progresses, one should decrease the rate at which the trap is lowered.

To find an optimum ramp for the trap-depth few things are crucial –

- The trap-depth $U(t)$ at any instant has to be larger than the temperature by a constant factor η . Usually η is picked to be 10.
- Collision rate $\gamma(t)$ has to be decent throughout the evaporation trajectory so that thermalization is guaranteed.
- The ramp has to be fast enough to prevent loss of atoms due to collision with background gas. However, too fast ramp will cause loss of atoms without any

gain in phase-space density.

Taking all the above-mentioned facts, one can analytically model an optimum ramp by starting with the s -wave Boltzmann equation

$$\dot{N}(t) = -2(\eta - 4) \exp(-\eta) \gamma(t) N(t), \quad (3.13)$$

and then using Eqs. 3.10 and 3.12, one arrives

$$U(t) = U(t=0) \left[1 + \frac{t}{\tau}\right]^{-2(\eta'-3)/\eta'}, \quad (3.14)$$

$$\text{where } \frac{1}{\tau} = \frac{2}{3}\eta'(\eta - 4) \exp(-\eta) \gamma(t=0).$$

Although the above ramp satisfy all the requirements for a ‘good’ evaporation, there are few other approaches that have been implemented to reach degeneracy in an optical dipole trap. Before presenting a detailed result of our evaporation, in a non-exhaustive list, we briefly survey here few approaches:

- In the realization of degenerate Fermi gas of Sr(87) [143, 175], a temperature $T/T_F \approx 0.26$ was achieved via evaporative cooling and the ramping of the power for the ODT laser followed

$$P(t) = P_0 / (1 + t/\tau)^\beta + P_{\text{offset}}, \quad (3.15)$$

where the term P_{offset} is included to counter-balance the gravity for some pre-defined values of τ and β .

- One can also keep one of the two lasers of the ODT intact, and reduce the power of another laser to reduce the trap-depth. The power of the laser can be decreased via exponential-law $U(t) = U(t=0) \exp(-t/\Gamma)$ with a pre-defined value of Γ .
- Sometimes the evaporation is performed in two steps – 1. an initial fast ramp-down of the trap to take advantage of the large value of collision rate, 2. a slower ramp with at the later stage of evaporation to prevent too much loss of atoms and to ensure thermalization [176].

FIGURE 3.19: The evaporation trajectory in our experiment: initially we hold the atoms in the dipole trap for three seconds. Then we ramp-down the power of both lasers using the AOMs. After 2.2 seconds, we hold the power of one laser while continuing the ramp-down of the other laser. During ‘Stage-II’, trap depth is reduced in all three directions (3-D Ev.) while during ‘Stage-III’, the evaporation happens mainly along two dimensions (2-D Ev.).

We have taken a hybrid approach for evaporation. As shown in Fig. 3.19, during our evaporation trajectory, we first hold the atoms for three seconds in the dipole trap, to capture more atoms inside the central region of the trap, this we indicate as ‘State-I’ in Fig. 3.19. Afterwards, we start the lowering of the trap-depth exponentially in time, by ramping-down both the beams power. During this time the trap-depth is lowered in all three spatial dimensions, and we mark this part of the evaporation as ‘Stage-II’ where evaporation is essentially three-dimensional. After 2.2 seconds of exponential ramp-down of power, we stop the ramp-down of one of the beams while we continue the ramp-down of the other beam. This is the stage that we call ‘Stage-III’ of the evaporation where trap-depth is lowered

mainly along two spatial dimensions. Note that the rates of ramping-down the power of the beams, during ‘Stage-II’, are slightly different. At the end of the full evaporation, we reach a temperature of ~ 30 nK with ~ 37 thousands with $T/T_F = 0.21(4)$, for a Fermi temperature $T = 143$ nK, as will be shown later.

The evolution of temperature and number of atoms during the evaporation is shown in Fig. 3.20. Here it is evident that the decrease of number of atoms is not too dramatic during the first few seconds of evaporation. However, temperature seems to show be flattened near the end of the evaporation where the gas is already in a deeply-degenerate regime with $\frac{T}{T_F} = 0.21(4)$.

FIGURE 3.20: The temperature and number of atoms during the evaporation trajectory.

To quantitatively evaluate the performance of the evaporation, one needs to know the rate of cooling as a function of the number of atoms that remain in the trap. Towards this end, we find the slope $\frac{\partial \text{Log} N}{\partial \text{Log} T}$ and compare it with the dimensionality of the evaporation at each stage of the evaporation, as shown in Fig. 3.21.

FIGURE 3.21: Left panel: the evolution of number of atoms as atoms cool down during the evaporation. The slope $\frac{\partial \text{Log} N}{\partial \text{Log} T} < D$ is a quantitative measure of the performance of the evaporation [2], where D is the dimensionality of the evaporation. The solid lines are linear fits of the experimental data shown in blue dots. From the extracted slope, it is evident that the performance of the evaporation is reasonably good. Right panel: the phase space density $\frac{T}{T_F}$ steadily decreases during the initial stage of evaporation and eventually flattens near the end when Pauli blockade come into play as the gas enters into a deeply degenerate regime.

As shown in the left panel of Fig. 3.21, during ‘Stage-II’ the slope is 0.48 which is much smaller than the dimensionality $D = 3$ of evaporation. Therefore the gain in phase space is quite fast during this ramp. The last point is evident from the right panel of Fig. 3.21 where we see a steady increase of phase space density, namely a lowered $\frac{T}{T_F}$. During the ‘Stage-III’ of evaporation gain in phase space starts to flatten as the gas enters into deeply degenerate regime. However, the efficiency of evaporation is still reasonable as $\frac{\partial \text{Log} N}{\partial \text{Log} T} = 1.1 < D = 2$ during this stage. The next section is dedicated to the characterization of the degenerate gas.

3.8 Characterization of Fermi Gas

At the end of the evaporation, we have thirty-seven thousands atoms at a temperature 30 nK. This gas has a very high phase space density, namely $\frac{T}{T_F} = 0.21(4)$. We fit the distribution of the optical density of the gas after 13 ms time-of-flight. A key feature of a degenerate Fermi gas is that, Gaussian distribution will over-estimate the population at small momentum, i.e., at the center of the cloud. Due to Pauli exclusion principle, a degenerate Fermi gas cannot have the same occupation as

a classical inside the Fermi sea. However, the distribution near the tail will look alike for a degenerate Fermi gas and a classical gas. The proper distribution that fits a Fermi gas is the Thomas-Fermi distribution (see Appendix-A for a detail of the fitting).

FIGURE 3.22: A thermal gas of atoms at $\frac{T}{T_F} = 3.1$: the data after 13 ms of time-of-flight fits well with both the Gaussian and Thomas-Fermi distribution. Here the gas is essentially a classical gas, therefore both Thomas-Fermi and Gaussian distribution are good fits.

We start with a thermal gas. And as an example, we take a gas near the beginning of evaporation where the phase space density is small $\frac{T}{T_F} = 3.1$. Here Both the Thomas-Fermi and Gaussian distribution fits the occupation of the population, as shown in Fig. 3.22.

However, for a degenerate Fermi gas, a Gaussian distribution over-estimates the occupation near the center of the cloud while a Thomas-Fermi distribution fits the whole distribution quite well, as shown in Fig. 3.23 (see Appendix-A for detail on the fitting).

FIGURE 3.23: Degenerate Fermi gas at $\frac{T}{T_F} = 0.21(4)$: the density plot shows the optical density after 13 ms of time-of-flight. The data are then summed along x - and y -axes and shown along this two axes. As shown here, Thomas-Fermi distribution fits well with the experimental data while the Gaussian fit overestimates near the center of the cloud.

Although there have been a plethora of studies to improve our understanding of Fermi gas in the last two decades, and it is still a fascinating field, we do not embark on playing with our Fermi gas mainly due to two reasons: firstly, the interaction energy in Sr(87) is rather moderate (s -wave scattering length is 97 Bohr-radius), therefore many interaction-induced phenomena might remain elusive at this scale of interaction. However, we must admit that despite the absence of magnetic Feshbach resonance, and with a moderate scattering length, the effects of interaction is readily observable in Sr(87), as reported in Ref. [176]. Secondly, our primary goal was to reduce the temperature of the cloud below the recoil temperature 230 nK, so that we can explore the effects of gauge field on atoms while temperature-induced decoherence remain negligible for a reasonable amount of time in the experiment. Therefore, we go on and use this gas to expose to

artificial gauge field and explore few interesting phenomena, as will be discussed in the next chapter.

3.9 Summary

In this chapter, we have provided a brief description of our experimental set-up and then the laser cooling of the atoms are mentioned. We gave few experimentally relevant parameters for the blue and red MOTs. Afterwards, we described the detail of the optical dipole trap and the optical pumping. We detail the full evaporation trajectory and report on the observation of degenerate Fermi gas with $\frac{T}{T_F} = 0.21(4)$. Importantly, the temperature is well below the recoil temperature which is an important point for the Gauge field experiment.

Chapter 4

Dynamics in Non-Abelian Gauge Field: *Zitterbewegung*

4.1 Introduction

This chapter is dedicated for the experiments we have performed, to observe the *Zitterbewegung* and characterize its anisotropy in an atomic wave packet in non-Abelian gauge field. The main idea of this experiment is to measure the velocity and look at the dynamics to recast it as *Zitterbewegung* effect of the position coordinate since these two quantities – the velocity and the position – oscillate at the same frequency, as was described in Chapter-2.

We detail the state preparation and explain the role of adiabaticity in the state preparation in Section-1.2. Afterwards in Section-1.3, we present our results on the observation of anisotropic *Zitterbewegung*. In this section we detail two types of experiments where we show the linear dependence of oscillation frequency of *Zitterbewegung* as a function of the kick-magnitude, and in the second type of experiments we reveal the angular dependence of the amplitude for the *Zitterbewegung*.

4.2 Initial State Preparation

FIGURE 4.1: **Schematic of tripod laser configurations and level-diagrams.** (a) The three laser beams with polarization $\sigma_{\pm}$, π , and two counter-propagating (along the x axis) lasers have opposite helicity, while the laser with π polarization is orthogonal to the two aforementioned laser beams. (b) Each of the three laser beam in (a) addresses one transition that connect the $F = 9/2$ of $1S_0$ and $F = 9/2$ of $3P_1$ manifold. The corresponding m_F states of the ground-or excited-states (enumerated by the subscripts e, g) are labelled. (c) The state diagram in the dressed state picture. The two degenerate dark-states have zero projection along the excited-state $|7/2\rangle_e$, therefore immune to spontaneous emission. The two bright-states are separated from the dark states by a frequency $\Omega = \sqrt{\Omega_+^2 + \Omega_\pi^2 + \Omega_-^2}$ where each Rabi frequency is labelled by the corresponding polarization of the beam.

As shown in Fig. 4.1(a), the key ingredients of performing this experiment are two counter-propagating, along x -axis, lasers with opposite helicity — σ_+ and σ_- polarizations — and another laser with π -polarization propagating along y -axis. These lasers address three transitions that connects three ground-states $|5/2\rangle_g$, $|7/2\rangle_g$, $|9/2\rangle_g$ of $1S_0$ manifold to the excited state $|F = 9/2, m_F = 7/2\rangle_e$ of the $3P_1$ manifold, as shown in Fig. 4.1(b).

As explained in Chapter-2, in the dressed-state picture shown in Fig. 4.1(c), this laser configuration has two degenerate dark-states $|D_{1,2}\rangle$ (with no contribution from the excited state $|7/2\rangle_e$) and two bright-states $|B_{1,2}\rangle$ (with contribution from excited-state $|7/2\rangle_e$). One of the two dark-states, namely $|D_2\rangle$, has zero projection on the ground-state Zeeman level $|9/2\rangle_g$, while $|D_1\rangle$ has non-zero projection along $|9/2\rangle_g$, as was shown in Chapter-2. Therefore $|D_2\rangle$ can be adiabatically connected to $|9/2\rangle_g$. We use this idea to prepare a definite state to commence our experiments, as will be described later in this chapter.

After the preparation of the initial state, we introduce a kick to the cloud and observe the dynamics of various quantities, namely the population of the bare atomic states $\{|5/2\rangle_g, |7/2\rangle_g, |9/2\rangle_g\}$, the x - and y -component of the velocity that we recast to the position coordinate — as described in Chapter-2 — as a mean to the observation of *Zitterbewegung*.

As explained in the previous chapter, the Zeeman state $|9/2\rangle_g$ of 1S_0 manifold is adiabatically connected to dark-state $|D_2\rangle$. Therefore, one way to prepare $|D_2\rangle$ would be to empty $|5/2\rangle_g$, and $|7/2\rangle_g$, while pumping atoms to $|9/2\rangle_g$ and then adiabatically turn-on the laser that addresses the transition $|9/2\rangle_g \rightarrow |7/2\rangle_e$, namely the σ_- beam with Rabi frequency Ω_- in Fig. 4.1(a), while we turn on σ_+ and π beams abruptly. To accomplish this goal, we need *three* ingredients, namely:

1. Atoms in Zeeman ground-states $|9/2\rangle_g$,
2. Proper purity of polarizations of the gauge field beams,
3. Well-calibrated magnetic field to address the transitions with lasers.

Each of the three points are elaborated in the following subsections.

4.2.1 Optical Pumping to $|9/2\rangle_g$

We have performed optical pumping (as described in Chapter-4) where atoms from the positive Zeeman states of the ground states are optically pumped to the Zeeman state $|m_F = 9/2\rangle_g$. The populations in the ground states, after optical pumping, are shown schematically in Fig. 4.2. Note that we need to make sure that the Zeeman states $|5/2, 7/2\rangle_g$ are empty, in order to prepare $|D_2\rangle$.

FIGURE 4.2: Schematic representation of the population in ground-state 1S_0 manifold, after optical pumping. Here the crucial thing is that $|5/2\rangle_g$ and $|7/2\rangle_g$ are empty. We emptied $|1/2\rangle_g$ and $|3/2\rangle_g$ and pumped them to $|9/2\rangle_g$, in order to increase the number of atoms in $|9/2\rangle_g$ and to suppress the off-resonant excitations.

4.2.2 Polarization of Tripod Lasers

For proper addressing of the transitions, we have taken great care to prepare the polarizations of the three gauge field beams, namely the σ_+ , σ_- , and π polarizations. The three gauge field beams are derived from the same slave laser and later distributed, as shown in the schematic of the optical set-up Fig. 4.3. The measured phase coherence-time of these three beams is order of milliseconds, as detailed in Ref. [118].

Here we adjust the quarter wave-plate of each of the σ_+ and σ_- beam to have $> 99.5\%$ accuracy of polarizations. In order to calibrate the polarization we use a rotating motor-mount polarizing beam-splitter (PBS). We record the time-trace of the beam-intensity after passing through this PBS, that is placed after the quarter-wave plate. We extract the purity of the polarization from the second harmonics of the rotating frequency of the motor [118].

We control the ratio of powers via non-polarizing beam-splitter and subsequently with an acousto-optic modulator (AOM) for each beam. The ignition and extinction of the lasers are controlled via the respective AOM of each beam. Note that the π and σ_- lasers are detuned from the σ_+ laser by -12 kHz and -24 kHz, respectively. This detuning takes care of the Zeeman-shift for each of the ground states, due to the bias magnetic field of ≈ 67 Gauss. And this frequency is determined via a careful calibration of the magnetic field which will be described in the following subsection.

FIGURE 4.3: **Schematic of the tripod laser system.** The three beams for the tripod are derived from the same laser. The power in each beam is controlled via the non-polarizing beam-splitters (NPBSs) and the individual acousto-optic modulator (AOM). Polarizations are controlled via independent quarter-wave plates for the $\sigma_{\pm}$ beams, and the π polarization is maintained via Glan polarizer. The beams are aligned with respect to the bias coil. Here ‘M’ represents a mirror.

4.2.3 Calibration of the Bias Magnetic Field

To calibrate the bias magnetic field (≈ 67 Gauss), and to find the resonance frequencies, we perform the atom-loss spectroscopy, using the same principle described in Chapter-4. We locate the resonances for the following three transitions:

- $|F = 9/2, m_F = 1/2\rangle_g \rightarrow |F = 9/2, m_F = 3/2\rangle_e$,
- $|F = 9/2, m_F = 3/2\rangle_g \rightarrow |F = 9/2, m_F = 5/2\rangle_e$,
- $|F = 9/2, m_F = 5/2\rangle_g \rightarrow |F = 9/2, m_F = 7/2\rangle_e$.

The resonance locations are shown in Fig. 4.4. Here we use σ_+ light to loose atoms from the ground-states and then probe that particular Zeeman state via a probe that addresses $|F = 9/2\rangle_g \rightarrow |F = 11/2\rangle_e$ transitions, with previously known frequencies (via spectroscopy, described in Chapter-3).

FIGURE 4.4: **Atom-loss spectroscopy for magnetic field characterization.** To locate the resonance with respect to zero-field location, and to characterize the bias field, three spectroscopy on $|F = 9/2\rangle_g \rightarrow |F = 9/2\rangle_g$ transition are performed. The corresponding m_F states of the ground- and excited-states are labelled by g and e , respectively.

In Fig. 4.4, we readily see that the two neighboring Zeeman m_F -states are not equally separated (frequency differences are 6.66 MHz and 7.41 MHz, respectively), this prohibits us to extract the magnetic field strength via the usual weak-field Zeeman splitting formula. The spectroscopy provides us two pieces of information:

1. The magnetic field strength at the level of atoms.
2. The appropriate laser frequencies to address the transitions of the tripod configuration, specially the $|5/2\rangle_g \rightarrow |7/2\rangle_e$ transition. Afterwards, we infer the frequencies of the other two tripod lasers frequencies from the extracted magnetic field.

To calibrate the magnetic field, we go back to the hyperfine Hamiltonian (as described in Chapter-4) in the 3P_1 manifold and diagonalize the following Hamiltonian:

$$H = H_Z + A\vec{I} \cdot \vec{J} + Q \frac{\frac{3}{2}\vec{I} \cdot \vec{J} - I(J+1)(I+1)}{2IJ(2I-1)(2J-1)}, \text{ with} \quad (4.1)$$

$$H_Z = (g_s S_z + g_l L_z - g_I I_z) \mu_0 B_z,$$

where $g_s \approx 2$ and $g_l = 1$ are the g -factors of spin and orbital angular momentum, respectively. And S_z , L_z , I_z are the z components of spin, orbital, and nuclear spin, respectively. The g -factor of nuclear spin reads $g_I = \frac{\mu_I(1-\sigma_d)}{\mu_0 I}$, where $\mu_I = -1.0924\mu_N$ (for Sr(87) [166]) is the nuclear magnetic moment, $\sigma_d = 0.00354$ (for Sr(87) [167]) is diamagnetic correction, μ_N is nuclear magneton, $\mu_0 = \frac{\mu_B}{h}$ is the Bohr magneton, and h is Planck's constant.

The magnetic field-dependence of the $|F = 9/2\rangle$ manifold is shown in Fig. 4.5. Afterwards, we compare the level-separation between neighboring Zeeman levels in Fig. 4.4, by treating it as an optimization problem for the unknown parameter 'Magnetic Field' B_0 . My minimizing the residue, defined as

$$\text{Residue}(B_0) = \sum_{j=1}^3 (f_j^{\text{Exp.}} - f_j^{\text{Num.}}(B_0))^2, \quad (4.2)$$

where f_j are the experimental and theoretical values of the resonance frequencies, taking the shift of ground-state into account at B_0 . The extracted value of the bias field is 67.44 Gauss. To account for the shift of the ground-states, we use

formula for linear Zeeman effect with Landé g -factor $-1.3 \cdot 10^{-4}$. The extracted magnetic field value is crucial, since it lets us infer the shifts of the ground-state Zeeman sub-levels which are of paramount importance, since any mismatch in this frequency will be reflected as a two-photon detuning in the gauge field experiments.

FIGURE 4.5: **Magnetic field-dependence of the $|F = 9/2\rangle$ subspace of 3P_1 manifold.** The location of the three resonance position of the $|7/2, 5/2, 3/2\rangle$ are shown by the three red-dots. The inset shows the residue while fitting the experimental values extracted from Fig. 4.4 treating the bias field as a free parameter. The field is found to be 67.44 Gauss which is close to the expected value 67 Gauss.

4.2.4 Cancellation of the Scalar Terms in the Gauge-field Hamiltonian

Once we have, 1. atoms only in $|9/2\rangle_g$ (and empty $|5/2\rangle_g$, and $|7/2\rangle_g$), 2. accurate polarizations of the gauge field lasers, 3. a calibrated magnetic field and the proper frequencies to address the three transitions, we are ready to turn-on the gauge field lasers for the commencement of the experiment. However, there is one little step left – namely, to cancel the scalar term in the Hamiltonian so that the effects that we envisage to observe — the *Zitterbewegung*, are entirely due to the presence of gauge field, as explained in Chapter-2. Towards that end, we briefly review

the description of the system, for the sake of continuity of this chapter, that was described in Chapter-2.

The Hamiltonian of the four-level system with detuning, in the rotating-wave approximation reads.

$$H_{\text{RWA}} = -\frac{\hbar}{2} \begin{pmatrix} -2\Delta_+ & 0 & 0 & \Omega_+ \\ 0 & -2\Delta_\pi & 0 & \Omega_\pi \\ 0 & 0 & -2\Delta_- & \Omega_- \\ \Omega_+^* & \Omega_\pi^* & \Omega_-^* & 0 \end{pmatrix}, \quad (4.3)$$

where $\Delta_{\pm,\pi}$ refer to the detuning of the corresponding lasers with Rabi frequency $\Omega_{\pm,\pi}$, respectively. Note that the Rabi frequencies have magnitude of order 250 kHz while the detunings are order of the recoil frequencies $\omega_{\text{rec.}} = 2\pi \cdot 4.7$ kHz, of the intercombination line. Therefore we can take the detunings as perturbation to the original Hamiltonian, i.e., without the detunings.

We write the Hamiltonian in the following form,

$$H_{\text{RWA}} = H_0 + H_1,$$

$$H_0 = -\frac{\hbar}{2} \begin{pmatrix} 0 & 0 & 0 & \Omega_+ \\ 0 & 0 & 0 & \Omega_\pi \\ 0 & 0 & 0 & \Omega_- \\ \Omega_+^* & \Omega_\pi^* & \Omega_-^* & 0 \end{pmatrix}, H_1 = -\frac{\hbar}{2} \begin{pmatrix} -2\Delta_+ & 0 & 0 & 0 \\ 0 & -2\Delta_\pi & 0 & 0 \\ 0 & 0 & -2\Delta_- & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (4.4)$$

where H_1 is considered as a perturbation to the Hamiltonian H_0 .

Evaluating the matrix elements $\langle D_i | H_1 | D_j \rangle$ of H_1 in the dark-state subspace given by

$$|D_1\rangle = \frac{1}{\sqrt{2}} \left(|5/2\rangle_g - |7/2\rangle_g \right), \quad (4.5)$$

$$|D_2\rangle = \frac{1}{\sqrt{6}} \left(|5/2\rangle_g + |7/2\rangle_g + 2|9/2\rangle_g \right),$$

we evaluate the matrix form of the operator H_1 in the dark-state manifold as follows

$$\tilde{H}_1 = \frac{\hbar}{6\sqrt{3}} \begin{bmatrix} 3\sqrt{3}(\Delta_+ + \Delta_\pi) & 3(\Delta_+ - \Delta_\pi) \\ 3(\Delta_+ - \Delta_\pi) & \sqrt{3}(\Delta_+ + \Delta_\pi + 4\Delta_-) \end{bmatrix}. \quad (4.6)$$

Note that in the above expressions of $|D_{1,2}\rangle$ (and in the following calculations), we have used the fact that the all three Rabi frequencies in our experiments are of equal magnitude.

The Hamiltonian of the atom, in the dark-state manifold without the contributions from detunings (i.e., $H_1 = 0$), reads

$$H = \frac{\hat{P}^2}{2M} \otimes \mathbb{1} - \frac{\hbar\kappa}{M} \left(\hat{p}_x \otimes \hat{A}_x + \hat{p}_y \otimes \hat{A}_y \right) + \underbrace{\frac{A_x^2 + A_y^2}{2M}}_{\tilde{\Delta}} + \Phi, \quad (4.7)$$

where the $A_{x,y}$ are the components of the vector gauge field and Φ is the scalar gauge field. The explicit forms of $A_{x,y}$ and Φ read

$$A_x = \frac{\hbar\kappa}{2} \begin{bmatrix} 3 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 1 \end{bmatrix}, \quad A_y = \frac{\hbar\kappa}{2} \begin{bmatrix} 1 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{3} \end{bmatrix}, \quad \Phi = \frac{\hbar^2\kappa^2}{2M} \begin{bmatrix} \frac{1}{3} & \frac{1}{3\sqrt{3}} \\ \frac{1}{3\sqrt{3}} & \frac{5}{9} \end{bmatrix}. \quad (4.8)$$

In order to cancel the contributions of terms given by

$$\tilde{\Delta} = \frac{1}{2M} (A_x^2 + A_y^2) + \Phi = \frac{\hbar^2\kappa^2}{2M} \frac{1}{\sqrt{3}} \begin{pmatrix} 3\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix}, \quad (4.9)$$

in Eq. 4.7, we leverage the matrix elements in Eq. 4.6, namely we adjust $\Delta_{\pm,\pi}$ to cancel the contributions of $\tilde{\Delta}$. Experimentally we have picked the values as follows

$$\begin{aligned} \Delta_+ &= -1 \cdot \omega_{\text{rec.}}, \\ \Delta_\pi &= +1 \cdot \omega_{\text{rec.}}, \\ \Delta_- &= +3 \cdot \omega_{\text{rec.}}, \end{aligned} \quad (4.10)$$

where $\omega_{\text{rec.}}$ is the recoil frequency of the intercombination line, namely $2\pi \cdot 4.7$ kHz. Note that while picking the values of $\Delta_{\pm,\pi}$, we have disregarded the terms proportional to the identity matrix as they contribute to a global phase in any dynamics.

After the cancellation of the scalar term in gauge field, we complete all three requirements mentioned in the beginning of Sec. 4.2. Now we are ready to prepare the initial state which is half the hurdle of the experiment. Note that we prepare

dark-state-‘2’ $|D_2\rangle$ for no particular reasons, except for the fact that this state is adiabatically connected to $|9/2\rangle_g$ state. This strategic advantage and the mechanism of adiabaticity to prepare $|D_2\rangle$, is elaborated in the following subsection.

4.2.5 Adiabaticity and Preparation of $|D_2\rangle$

FIGURE 4.6: **Laser ignition sequence and the adiabatic preparation of $|D_2\rangle$.** (a) Level-diagram of the relevant atomic states and the initial occupation of $|9/2\rangle_g$ state by the atoms (shown by orange ellipse). (b) Same as (a), however in the rotating-wave picture while all the transitions are addressed on resonance. (c) The dressed state picture when σ_+ and π beams are ‘on’ abruptly and σ_- is still ‘off’. Note the ‘intermediate’ dark- and bright-state labelled by superscript ‘0’, while the atoms are still occupying the $|9/2\rangle_g$ state. (d) The level diagram when all three beams have reached the steady-state power. the atoms were adiabatically transferred from $|9/2\rangle_g$ atomic state to the dressed state $|D_2\rangle$. (e) The laser ignition sequence starts when the σ_+ and π beams are abruptly turned-on at $t = 0$, and after a time $t_0 = 1 \mu\text{s}$ the σ_- beam power is adiabatically ramped up (for a time $t_1 = 10 \mu\text{s}$) to the steady-state value that corresponds to Rabi frequency Ω_0 , and this value is same for all three beams. As we ramp the power of σ_- beam linearly, the Rabi frequency increase via square-root, as shown by the dashed orange line. See the text below for a detail of all the steps involved.

Since the two dark-states are degenerate in the absence of any detuning term in the Hamiltonian in Eq. 4.3, any linear superposition is also a dark-state. However, to make a quantitative comparison with theory and the experiments, we need to prepare a definite dark-state. Towards that end, we leverage the fact that for one choice of dark-states basis we can have the following dark states:

$$\begin{aligned} |D_1\rangle &= \frac{1}{\sqrt{2}} \left(|5/2\rangle_g - |7/2\rangle_g \right), \\ |D_2\rangle &= \frac{1}{\sqrt{6}} \left(|5/2\rangle_g + |7/2\rangle_g + 2|9/2\rangle_g \right), \end{aligned} \quad (4.11)$$

where $|D_1\rangle$ and $|D_2\rangle$ are orthogonal.

One key feature of this choice of basis for the dark-states is that the $|9/2\rangle_g$ state is only present in $|D_2\rangle$. Therefore we can invoke the adiabaticity, namely *the invariance of the manifold-ordering during an adiabatic process*, in the subspace of the instantaneous dressed-states. The ignition sequence of the laser beams is described in Fig. 4.6(e). Here in the following we walk through different stages of the process described in Fig. 4.6(e):

Figs. 4.6(a),(b): Here we describe the lasers systems (a) and its representation in the rotating-wave-approximation (b). The ‘orange ellipse’ represents the atomic occupation (here all the atoms are initially prepared in $|9/2\rangle_g$). Note that in this representation — for resonant excitation (which is the case in our experiments) — the four atomic states have zero energy. Schematically they are equivalent to the case when all the Rabi frequencies are zero.

Fig. 4.6(c): At time $t = 0$, we *abruptly* turn-on the two lasers that address σ_+ - and π -transitions. Since this is a non-adiabatic process, the dressed states during the time $t =]0, t_0[$ have two bright-state $|B_{1,2}^{(0)}\rangle$ and one dark state $|D_1^{(0)}\rangle$. The bright-states are separated from the dark-states by a frequency $\Omega^{(0)} = \sqrt{\Omega_+^2 + \Omega_\pi^2}$. Note that neither of the bright-states or dark-states, have any contribution from the state $|9/2\rangle_g$ since this state is still isolated from the rest of the atomic system, namely the Rabi frequency $\Omega_- = 0$. This statement becomes more apparent when we look at the Hamiltonian during

the time $t =]0, t_0[$:

$$H_{\text{RWA}}^{(0)} = \begin{bmatrix} |5/2\rangle_g & |7/2\rangle_g & |9/2\rangle_g & |7/2\rangle_e \\ 0 & 0 & 0 & \Omega_+ \\ 0 & 0 & 0 & \Omega_\pi \\ 0 & 0 & 0 & 0 \\ \Omega_+ & \Omega_\pi & 0 & 0 \end{bmatrix}. \quad (4.12)$$

For time $t < t_0$, the atoms are still in $|9/2\rangle_g$. In our experiment we have $t_0 = 1 \mu\text{s}$, for the laser-intensities to stabilize.

Fig. 4.6(d): For $t =]0, t_0[$, as we enumerate the manifolds with increasing energies from the bottom in Fig. 4.6(c), the atoms are occupying the manifold with index ‘2’. For time $t = [t_0, t_0 + t_1]$, we *adiabatically* ramp the power of the laser that addresses the transition $|9/2\rangle_g \rightarrow |7/2\rangle_e$. At the end of adiabatic ramp namely at time $t > (t_0 + t_1)$, the adiabaticity guarantees that the atoms will remain in the manifold with index ‘2’. Moreover, since the projection of $|9/2\rangle_g$ onto $|D_1\rangle$ is *zero*, as shown in Eq. 4.11, there will be *zero* occupation in state $|D_1\rangle$, at the end of adiabatic ramp. This way we transfer atoms from atomic state $|9/2\rangle_g$ to the dressed state $|D_2\rangle$. In our experiments, we have used $t_1 = 10 \mu\text{s}$ and have made sure that the adiabaticity hold by scanning t_1 and looking at the state that we prepare after the adiabatic ramp.

As all three lasers are at steady-state, namely for time $t > (t_0 + t_1)$, the final Hamiltonian reads

$$H_{\text{RWA}}^{(0)} = \begin{bmatrix} |5/2\rangle_g & |7/2\rangle_g & |9/2\rangle_g & |7/2\rangle_e \\ 0 & 0 & 0 & \Omega_+ \\ 0 & 0 & 0 & \Omega_\pi \\ 0 & 0 & 0 & \Omega_- \\ \Omega_+ & \Omega_\pi & \Omega_- & 0 \end{bmatrix}, \quad (4.13)$$

and the two dark states read

$$\begin{aligned} |D_1\rangle &= \frac{1}{\sqrt{2}} \left(|5/2\rangle_g - |7/2\rangle_g \right), \\ |D_2\rangle &= \frac{1}{\sqrt{6}} \left(|5/2\rangle_g + |7/2\rangle_g + 2|9/2\rangle_g \right). \end{aligned} \quad (4.14)$$

Since we envisage to prepare $|D_2\rangle$, we expect the populations of the three atomic states $\{|5/2\rangle_g, |7/2\rangle_g, |9/2\rangle_g\}$ as follows:

$$\begin{aligned} P_{5/2} &= \frac{1}{6}, \\ P_{7/2} &= \frac{1}{6}, \\ P_{9/2} &= \frac{4}{6}. \end{aligned} \tag{4.15}$$

This is exactly what we prepare, as shown in Fig. 4.7, where we show the populations of the three Zeeman states, from the theory given by Eq. 4.15 and the measured values. Note that there are two undetermined phases for the complete characterization of the initial state $|D_2\rangle$. After preparing the initial state, we have reversed the laser ignition sequence described in Fig. 4.6 to check that all the atoms go back to $|9/2\rangle_g$. Indeed we have found that more than 97% of atoms go back to $|9/2\rangle_g$. This gives us further confidence that we have indeed prepared $|D_2\rangle$.

FIGURE 4.7: **Population of the initial state $|D_2\rangle$.** The Atomic population for the three Zeeman states of the ground state manifold 1S_0 with $F = 9/2$ are shown. The experimentally extracted values are compared against the theoretically-expected values from Eq. 4.15.

We have used a fluorescence imaging technique to measure the population of each Zeeman states. In Fig. 4.8(a), we show the experimental data where the fluorescence count is shown, after the preparation of the initial state. This image is taken after an abrupt switch-off of all the tripod beams follows by a 9 ms of time-of-flight.

We fit the data with three Gaussian functions as shown in Fig. 4.8(b). In the images, the $|5/2\rangle_g$ state is separated from the $|9/2\rangle_g$ state by $-2v_{\text{rec.}}$ along the x -axis. This is due to the fact that as we adiabatically ramp-up the power of laser corresponding to σ_- transition, atoms will absorb one photon from this laser (propagating along $-x$ -axis), and subsequently emit another photon into the direction of σ_+ (propagating along $+x$ -axis) laser, via stimulated emission, to go to the $|5/2\rangle_g$ state. Therefore the atoms in $|5/2\rangle_g$ will be in the state $|5/2_g; p_x = -2p_{\text{rec.}}, p_y = 0\rangle$ after abrupt switch off of the beams, where, where the second quantum number is to enumerate the momentum state of the particle [177, 178].

Likewise, the atoms in $|7/2\rangle_g$ will absorb one photon from σ_- laser (propagating along $-x$ -axis) and emit one photon into the π -beam (propagating along $+y$ -axis), via stimulated emission. Therefore these atoms will be in the state $|7/2_g; p_x = -p_{\text{rec.}}, p_y = -p_{\text{rec.}}\rangle$, as shown in Figs. 4.8(a) and (b).

FIGURE 4.8: **Imaging and fitting the initial state $|D_2\rangle$ in velocity-space.** (a) The fluorescence image of the initial state $|D_2\rangle$ after a 9 ms time-of-flight. The three ground-states are labelled by the corresponding m_F values, at the location in the velocity-space. At $\mathbf{v} = \mathbf{0}$, the $|9/2\rangle_g$ sits with the atoms in negative m_F states in the $|F = 9/2\rangle_g$ manifold, as explained in the text. (b) Fit of the experimental data in (a) with three simultaneous Gaussian functions. Here the centers, sizes, and the height of each Gaussian were treated as free parameters. (c) Same fit as (b) except for the fact that we artificially removed half of the total population from the Gaussian centered at $\mathbf{v} = \mathbf{0}$. Note that all other atomic population gets renormalized accordingly.

Note that, we have 50% of the total atoms in the negative Zeeman states $|j\rangle_g$ with $j = -9/2, -7/2, \dots, -1/2$. These atoms are spectator during the whole experiment since all the three gauge field lasers are far detuned ($> (20 - 30)$ MHz) from the these transitions. And those atoms just sit at the zero-momentum state with the $|9/2\rangle_g$ states that do not absorb any photon during the state preparation.

In Fig. 4.8(c), we numerically subtracted 50% of the total population, i.e., population of the negative Zeeman ground-states. Note that the fluorescence is performed on the $^1S_0 \rightarrow ^1P_1$ transition that has a line width of 32 MHz and due to the presence hyperfine states $|F = 7/2\rangle$, $|F = 9/2\rangle$, and $|F = 11/2\rangle$ in the excited-states, we cannot perform spin-dependent fluorescence on this line. However, we can resolve the momentum states from the fluorescence image. We have performed independent spin-sensitive shadow imaging via $^1S_0 \rightarrow ^3P_1$ transition on $|F = 9/2\rangle_g \rightarrow |F = 11/2\rangle$, to confirm the population shown in Fig. 4.7. The spin-resolved population and the momentum-resolved populations agree very well within our detection sensitivity. Since the signal-to-noise ratio in the fluorescence is much stronger and provides equivalent information as the shadow imaging, we opted for the fluorescence imaging for faster data-taking.

This brings us to a situation where we can play with different configurations of atoms inside the non-Abelian gauge field. In the following section, we describe the dynamics of atoms inside the gauge field when we introduce kick to atoms. One key result is the observation of anisotropic *Zitterbewegung*, in the presence of non-Abelian gauge field.

4.3 Dynamics in Gauge Field

As we prepare the initial state as the dark-state-‘2’, namely $|D_2\rangle$, we perform a set of experiments starting with this states. The main idea of those experiments is the following: We push the atoms along different directions on the xy -plane with magnitude P_0 and at an angle Θ with x -axis, as shown in Fig. 4.9(a). The mechanism that we adopt in our experiment, to realize the kick on atoms, is shown in Fig. 4.9(b).

FIGURE 4.9: **Schematic of kick and spin-texture of the upper-branch of the dressed state.** (a) The atomic cloud initially located at the center $\mathbf{p} = \mathbf{0}$ shown by the disk is displaced in momentum-space to a polar coordinate (P_0, Θ) . (b) The mechanism to introduce a kick $P_0 = Mv_0$ to an atom with mass M , at an angle Θ with the x -axis. See the text below for a detail account of the mechanism. (c) The momentum-dependent eigenvalues of the upper-branch of Hamiltonian in Eq. 4.17. Note the presence of branch-cut at $\Theta = \pi/4$.

If the atom (shown by ‘light blue disk’ in Fig. 4.9(b)) had a velocity v_0 at an angle Θ with the x -axis, the frequency of the three lasers, shown by ‘green arrows’, transform as following:

$$\begin{aligned}\omega_+ &\rightarrow \omega_+ - \kappa \cdot v_0 \cos(\Theta), \\ \omega_\pi &\rightarrow \omega_\pi - \kappa \cdot v_0 \sin(\Theta), \\ \omega_- &\rightarrow \omega_- + \kappa \cdot v_0 \cos(\Theta),\end{aligned}\tag{4.16}$$

where κ is the common wave vector of the three beams, and the frequency ω of each beam is labelled by the subscript of the corresponding polarization. Therefore, to

mimic the kick $P_0 = Mv_0$ at and angle Θ to a static atom with mass M , we just need to read-off the detunings from the expression in Eq. 4.16, to shift the center of the wave packet at point (P_0, Θ) , in momentum-space.

The Hamiltonian after cancellation of the scalar terms reads

$$H = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2M} \otimes \mathbb{1} - \frac{\hbar\kappa}{M} \left(\hat{p}_x \otimes \hat{A}_x + \hat{p}_y \otimes \hat{A}_y \right), \quad (4.17)$$

where κ is the wave vector of the gauge field lasers, M is the mass of the atom, and $\hat{p}_{x,y}$ are the momentum operators. The components of the gauge field $A_{x,y}$ are given in Eq. 4.8. A key feature of this set of $A_{x,y}$ is that they are non-commuting, namely $[A_x, A_y] \neq 0$. This non-commutation is manifested in the spin texture (in the basis of the Hamiltonian, namely the dark-state basis) of the dressed eigenstate of Hamiltonian in Eq. 4.17, as shown in Fig. 4.9(c).

There are two important points to note on the spin-texture shown in Fig. 4.9(c):

- The eigenstates are *momentum-dependent*, in stark contrast with two dimensional Abelian cases where the momentum dependence can be factored-out.
- The presence of a branch-cut, shown by the yellow line, at an angle $\pi/4$ with x -axis. This branch-cut appears from the square-root-type singularity of the eigenvalues, as described in Chapter-2. Due to the presence of this branch-cut at $\Theta = \pi/4$, we will see shortly that pushing atoms along this direction induce no oscillation.

In the following subsection we explain how we extract the velocity component of the atom inside the gauge field and it's rationale.

4.3.1 Velocity from Population

In our experiment, we measure the population of the atomic states at three different momentum, as describes in the previous section. Afterwards, we can extract the velocity-components of the atoms via two *equivalent* approaches:

1. As we have the three populations of the atomic states at our disposal, we can recast them onto the dark-state manifold to find the components $c_{1,2}$ of

$|D_{1,2}\rangle$, respectively [104, 134]. Afterwards, we can write the state of an atom at any moment by

$$|\Psi(t)\rangle = c_1(t)|D_1\rangle + c_2(t)|D_2\rangle. \quad (4.18)$$

Then the j -th components of the velocity of the atom can then be extracted via the gauge-covariant form of the velocity operator

$$\hat{v}_j = \frac{1}{M} (\hat{p}_j - \hat{A}_j). \quad (4.19)$$

2. The second approach is more intuitive and here we find the components of the velocity as follows

$$v_x = v_{\text{rec.}}(-2P_{5/2} - P_{7/2}), \quad (4.20)$$

$$v_y = v_{\text{rec.}}(-P_{7/2}).$$

Note that the ballistic velocity of the atoms are subtracted in the above equation. Therefore the contribution of the gauge field will only be manifested here.

An intuitive way to understand Eq. 4.20 is that any atom in $|5/2\rangle_g$, in total, must have *absorbed* one photon from the $-x$ -propagating laser beam (addressing $|9/2\rangle_g \xrightarrow{\text{Absorption}} |7/2\rangle_e$ transition) and *emitted* one photon along $+x$ -propagating laser beam (addressing $|7/2\rangle_e \xrightarrow[\text{Emission}]{\text{Stimulated}} |5/2\rangle_g$ transition). Note that the effect of absorbing any other photons to go to any other state ($|9/2\rangle_g$ or $|7/2\rangle_g$) and then coming back to $|5/2\rangle_g$, will induce no overall momentum since they get cancelled. Moreover, reversing the aforementioned process, i.e., going to $|5/2\rangle_g$ from $|9/2\rangle_g$, will put the atom in $|9/2_g; p_x = 0, p_y = 0\rangle$ state. By the same argument, any atom at the atomic state $|7/2\rangle_g$ will have the full state $|7/2_g; p_x = -1, p_y = -1\rangle$, where the momentum quantum numbers are in the unit of recoil momentum $\hbar\kappa$. This way we extracted the velocity components $v_{x,y}$ for different configurations by measuring the population of the atomic states at particular momentum states, using Eq. 4.20.

Note that this approach is less dependent on the model, since the first approach requires the knowledge of gauge field components which is dependent on the ratio of Rabi frequencies of the three beams, as discussed in Chapter-2. This requires

repeated measurements of the Rabi frequencies, since the components of the gauge fields depend on the ratio of Rabi frequencies, as described in Chapter-2. For all the presented results in the following, we have extracted the velocities using Eq. 4.20 and for few cases we have checked that Eq. 4.19 gives the same mean values for the components of velocity.

FIGURE 4.10: **Population and the corresponding velocity for different pushing momentum and angles.** (a)-(c) the population $P - j$ of the three atomic ground-states (labelled by the subscripts $j = 5/2, 7/2$, and $9/2$). (d)-(f) The components of velocity extracted from the population of the corresponding column. The magnitude of pushing P_0 is in unit of recoil momentum, and the direction is given by Θ . The dots are the extracted values of from experiments, and the solid lined are the numerical calculation using Heisenberg equation of motion (see Chapter-2 for more detail on the numerical calculation). Note that there is no fitting parameters for the solid lines.

In Figs. 4.10, we show the time-evolution of the atomic population, and the extracted velocity components, for three situations. In Figs. 4.10(a), we show the time-evolution of the three atomic states $\{|5/2\rangle_g, |7/2\rangle_g, |9/2\rangle_g\}$, when we introduce no pushing to the atoms. The components of the velocity are shown in Figs. 4.10(d). Note that the velocity components shows no oscillation in stark contrast with the two other cases shown in Figs. 4.10(b), (e) and Figs. 4.10(c), (f) where we pushed the atoms with momentum P_0 in unit of recoil momentum $\hbar\kappa$ along the direction Θ . Note that the velocity and the population, both exhibit a damped-oscillation. The origin of this damped oscillation is attributed to the

finite temperature of the atoms. The dots represents the experimentally measured quantities while the solid lines are numerical calculation. See Chapter-2 for the detail on the numerical calculation.

As the theory and experiment emerges in a reasonable degree, this gives us confidence that we have good control over the system and we explore few more cases systematically to understand the underlying physics. However, before that we quote some of the results from Chapter-2 in the following for the purpose of being self-contained. As described in Chapter-2, the components of the velocity at temperature $T = 0$ reads

$$\begin{aligned}\tilde{v}_x(p_x^0, p_y^0; t) &= v_{x1} + v_{x0} \cos(\omega \cdot t), \\ \tilde{v}_y(p_x^0, p_y^0; t) &= v_{y1} + v_{y0} \cos(\omega \cdot t), \\ \text{with } \omega &= \frac{2\kappa}{3M} \sqrt{3(p_x^0)^2 + (p_y^0)^2},\end{aligned}\tag{4.21}$$

where $p_{x,y}^0$ are the components of pushing momentum, M is the mass of the atom, κ is the wave vector of the gauge field laser, and $\hbar$ is Planck's constant; the amplitudes and the offset of each component read:

$$\begin{aligned}v_{x1} &= \frac{\hbar\kappa}{M} \cdot \frac{p_x^0 p_y^0 - 3(p_x^0)^2 - 2(p_y^0)^2}{2[3(p_x^0)^2 + (p_y^0)^2]}, v_{x0} = \frac{\hbar\kappa}{M} \cdot \frac{p_y^2 - p_x^0 p_y^0}{2[3(p_x^0)^2 + (p_y^0)^2]}, \\ v_{y1} &= \frac{\hbar\kappa}{M} \cdot \frac{3p_x^0 p_y^0 - 6(p_x^0)^2 - (p_y^0)^2}{6[3(p_x^0)^2 + (p_y^0)^2]}, v_{y0} = \frac{\hbar\kappa}{M} \cdot \frac{3p_x^0 p_y^0 - 3p_x^2}{6[3(p_x^0)^2 + (p_y^0)^2]},\end{aligned}\tag{4.22}$$

Note that the oscillation frequency $\omega(p_x^0, p_y^0)$ is proportional to the energy difference between the two-branches of the dressed states of the Hamiltonian in Eq. 4.17. See Chapter-2 for more detail.

As we introduce temperature T to the system, for any particular pushing, not just a single momentum takes part in the oscillation. Rather all the momentum – dictated by the extent of the wave packet in momentum-space – take part in the oscillation. And each of the momentum have different frequencies of oscillation given by the expression of $\omega(p_x^0, p_y^0)$ in Eq. 4.21. This finite momentum-broadening results in

the decay of the oscillation. The velocity components, at finite temperature T read

$$\begin{aligned}\tilde{v}_x(p_x^0, p_y^0; t) &\approx v_{x1} + v_{x0} \cos(\omega \cdot t) \cdot \exp\left[-\left(\frac{t}{\tau}\right)^2\right], \\ \tilde{v}_y(p_x^0, p_y^0; t) &\approx v_{y1} + v_{y0} \cos(\omega \cdot t) \cdot \exp\left[-\left(\frac{t}{\tau}\right)^2\right],\end{aligned}\quad (4.23)$$

$$\text{with } \omega = \frac{2\kappa}{3M} \sqrt{3(p_x^0)^2 + (p_y^0)^2}, \quad \tau = \frac{3}{\kappa} \sqrt{\frac{M}{2k_B T}} \sqrt{\frac{2 + \cos(2\arctan(p_y^0/p_x^0))}{5 + 4 \cos(2\arctan(p_y^0/p_x^0))}}.$$

Note that the $\approx$ symbol emphasizes here that the above expression is derived for the case when the magnitude of pushing P_0 is large compared to the recoil momentum $\hbar\kappa$.

The above expressions of velocity becomes much more compact as we go to polar coordinate via the substitution:

$$\begin{aligned}p_x^0 &\rightarrow P_0 \cos(\Theta), \\ p_y^0 &\rightarrow P_0 \sin(\Theta),\end{aligned}\quad (4.24)$$

where P_0 is the magnitude of pushing momentum. Then the expression of the velocity-components at finite temperature read

$$\begin{aligned}v_x(P_0, \Theta; t) &\approx v_{x1} + v_{x0} \cdot \cos(\omega \cdot t) \cdot \exp\left[-\left(\frac{t}{\tau}\right)^2\right], \\ v_y(P_0, \Theta; t) &\approx v_{y1} + v_{y0} \cdot \cos(\omega \cdot t) \cdot \exp\left[-\left(\frac{t}{\tau}\right)^2\right],\end{aligned}\quad (4.25)$$

$$\omega = \frac{2\kappa}{3m} P_0 \sqrt{(2 + \cos(2\Theta))}, \quad \tau = \frac{3}{\kappa} \sqrt{\frac{M}{2k_B T}} \sqrt{\frac{2 + \cos(2\Theta)}{5 + 4 \cos(2\Theta)}},$$

where the offset and the amplitude of each component is as follows:

$$\begin{aligned}v_{x1} &= \frac{\hbar\kappa}{m} \cdot \frac{\sin(2\Theta) - \cos(2\Theta) - 5}{4(2 + \cos(2\Theta))}, \quad v_{x0} = \frac{\hbar\kappa}{m} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sin(\Theta) \sin(\Theta - \frac{\pi}{4})}{(2 + \cos(2\Theta))}; \\ v_{y1} &= \frac{\hbar\kappa}{m} \cdot \frac{3 \sin(2\Theta) - 5 \cos(2\Theta) - 7}{12(2 + \cos(2\Theta))}, \quad v_{y0} = \frac{\hbar\kappa}{m} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\cos(\Theta) \sin(\frac{\pi}{4} - \Theta)}{(2 + \cos(2\Theta))}.\end{aligned}\quad (4.26)$$

By looking at Eqs. 4.25 and 4.26, we can make the following experimentally verifiable predictions

1. From Eq. 4.25: At a fixed angle Θ , the frequency of oscillation is linear in pushing momentum P_0 .
2. From Eq. 4.25: For fixed value of P_0 , i.e., on a circle, the frequency of oscillation has a period of π .
3. From Eq. 4.25: The damping time is independent of the absolute value P_0 .
4. From Eq. 4.26: The amplitudes of velocity, for both components, are independent of the absolute value of pushing momentum P_0 .
5. From Eq. 4.26: At $\Theta = \pi/4$ & $5\pi/4$, the amplitudes of the oscillation vanish for both components. However, as we will see later that due to finite momentum extension of the wave packet, each component exhibit tiny oscillation for pushing along $\Theta = \pi/4$ & $5\pi/4$.
6. From Eq. 4.26: For pushing along $y(x)$ -axis, i.e., for $\Theta = \pi/2$, the oscillating component is v_x . Likewise for $\Theta = 0$, only the v_y is oscillates. This is an extreme form of anisotropy. Therefore, the amplitude of each component and subsequently the absolute value of the velocity is extremely anisotropic, as will be shown in the following when we scan the angle Θ for a fixed value of P_0 .

4.3.2 Dynamics in Gauge Field: Pushing Along Lines

To explore the system more closely, we start with the situation where we push along a line $\Theta = \text{constant}$, and scan the value of P_0 and then extract three main quantity from the velocity oscillation: The frequency, amplitudes of oscillation, and the damping time. Note that although we presented our results on the achievement of degeneracy at temperature 30 nK, we have performed all our experiments for a gas at temperature around 45 nK. This is done to make sure that the temperature is reproducible since it requires more scrutiny for the alignment of the optical dipole trap beams, to maintain such a low temperature.

4.3.2.1 Pushing Along $\Theta = 0.50\pi$

FIGURE 4.11: The frequency of velocity oscillation for the v_x (a) and v_y (b) components, for different pushing momentum at a fixed angle $\Theta = \pi/2$. The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.25 and numerical calculation (see Chapter-2 for detail), respectively.

In this situation, since Θ is constant, we expect that the frequency of oscillation is linear with P_0 from Eq. 4.25. This is exactly what we see in experiment, as shown in Figs. 4.11. Here we see that the oscillation frequency of both components, vary linearly with the pushing momentum. The dots represent the extracted value from the experimental data via fitting with a function of the form in Eq. 4.25, the approximation is from Eq. 4.25 (see Chapter-2 for the derivation), and the numerical calculation performed via Monte-Carlo simulation with Eq. 4.21, for finite momentum extension of the wave packet at temperature $T = 45$ nK. See Chapter-2 for more detail on the numerical calculation.

FIGURE 4.12: The amplitudes of velocity oscillation for the v_x (a), v_y (b) components, and the absolute value of velocity (c), for different pushing momentum at a fixed angle $\Theta = \pi/2$. The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.26 and numerical calculation (see Chapter-2 for detail), respectively. Note that the amplitude of v_y is much small compared to v_x . The non-zero value of amplitude, at small momentum, is due to momentum broadening of the wave packet at finite temperature.

As mentioned in the previous subsection, due to finite momentum extension, the y -component exhibits small oscillation and due to this small amplitude of oscillation, shown in Fig. 4.12(b), the extracted frequency deviates from the line. Note that this is due to the limitation of the fitting process – it cannot extract the frequency well when the amplitudes are extremely small. As we can see in Figs. 4.12, the amplitudes have momentum-dependence for small P_0 and later it approaches the analytic expressions in Eq. 4.26 – it shows momentum-independence.

Although the damping-time is expected to be momentum-independent for large value of momentum, a tiny variation for small momentum is observed from the numerical calculations, as shown in Figs. 4.13. This discrepancy of having a systematically smaller damping-time at experiment as compared to the prediction from Eq. 4.26 can be due to the degenerate nature of the gas, as hinted in Chapter-2. Note that the error bars in Fig. 4.13(b) is quite large for large momentum. This is due to the small value of oscillation amplitudes, as shown in Fig. 4.12(b). Note that, even the numerical calculation shows wiggle for large momentum since the amplitude of oscillation along y -axis is vanishingly small for large P_0 , as shown in Eq. 4.26.

FIGURE 4.13: The damping time of velocity oscillation for the v_x (a), v_y (b) components, for different pushing momentum at a fixed angle $\Theta = \pi/2$. The dot represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.25 and numerical calculation (see Chapter-2 for detail), respectively. Since the amplitude of v_y is much small compared to v_x (see Figs. 4.12), therefore the error bars gets broader for small amplitudes of oscillation. Here the temperature for the wave packet is $T = 45$ nK.

In the following, we mention one more case for $\Theta = \text{constant}$, when both the components are oscillating with comparable amplitudes.

4.3.2.2 Pushing Along $\Theta = 0.75\pi$

FIGURE 4.14: The frequency of velocity oscillation for the v_x (a) and v_y (b) components, for different pushing momentum at a fixed angle $\Theta = 0.75\pi$, namely at 135° . The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.25 and numerical calculation (see Chapter-2 for detail), respectively.

In this case the extracted values of frequencies get aligned with the numerical calculation and the analytic form in Eq. 4.25, as shown in Figs. 4.15.

FIGURE 4.15: The amplitudes of velocity oscillation for the v_x (a), v_y (b) components, and the absolute value of velocity (c), for different pushing momentum at a fixed angle $\Theta = 0.75\pi$, namely at 135° . The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.26 and numerical calculation (see Chapter-2 for detail), respectively.

Amplitude of oscillation for each component is comparable and relatively large. Therefore the extracted values of amplitudes can be more fairly compared to the

theory prediction and turn out to be in agreement with the numerical calculation and the analytic expressions in Eq. 4.26, as shown in Figs. 4.15. The small discrepancy at large momentum could be due to the presence of non-adiabatic mechanisms.

FIGURE 4.16: The damping time of velocity oscillation for the v_x (a), v_y (b) components, for different pushing momentum at a fixed angle $\Theta = 0.75\pi$, namely at 135° . The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.25 and numerical calculation (see Chapter-2 for detail), respectively. Here the temperature for the wave packet is $T = 45$ nK.

Similar to the case with amplitudes, the extracted values of damping time is in fair agreement with the numerical calculation and the analytic expression in Eq. 4.25, as shown in Figs. 4.16. The momentum-independence is manifested from the experimental values shown in Figs. 4.16.

4.3.3 Dynamics in Gauge Field: Pushing on a Circle

We present the most interesting case now. Here we keep the pushing magnitude fixed to $P_0 = 4\sqrt{2}\hbar\kappa$ and scan the angle Θ from 0° to 360° . The results are presented in the following:

FIGURE 4.17: The frequency of velocity oscillation for the v_x (a) and v_y (b) components, for different pushing angles at a fixed Momentum $P_0 = 4\sqrt{2}$. The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.25 and numerical calculation (see Chapter-2 for detail), respectively. Note that due to the small amplitudes of the velocity components at some special angles, the fitting function returns a large error bars (see the text immediately below), and we have limited the axes for better contrast of the data of interests.

As shown in the expression of frequency in Eq. 4.25, the dependence of Θ goes like $\sim \sqrt{2 + \cos(2\Theta)}$ for the frequency. This variation is quantitatively captured in our experiment, as shown in Figs. 4.17. Moreover, the π -periodicity of the frequency is apparent from the plots of each component.

There are few special angles where the fitting is challenging when either of the two components of velocity becomes small, or both of them are extremely small. Since these points tell us more about the physics of the system than the other points, we took data specially at those points, at the expense of esthetics of the plots. In the following we enumerate those special points, and it will help us understand the large error bars at those points.

$\Theta = 0^\circ$ & 180° : Refer to Eq. 4.26, the x -component is expected to have zero amplitude for oscillation. However due to finite momentum extension, we see an overall small oscillation in x -component and the subsequent extracted values have large error bars.

$\Theta = 45^\circ$ & 225° : Refer to Eq. 4.26, both components have zero amplitudes for oscillation. However, finite momentum extension of the wave packet results

in small oscillation of both components, and the resulting fit has a large uncertainty of the extracted values.

$\Theta = 90^\circ$ & 270° : This is similar to the case with $\Theta = 0^\circ$ & 180° except for the fact that here the y -components is expected to have zero amplitude of oscillation from the estimation of Eq. 4.26.

FIGURE 4.18: The damping time of velocity oscillation for the v_x (a), v_y (b) components, for different pushing angles at a fixed momentum $P_0 = 4\sqrt{2}$. The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.25 and numerical calculation (see Chapter-2 for detail), respectively. Here the temperature for the wave packet is $T = 45$ nK.

The damping time shown in Figs. 4.18 are in reasonable agreement with the equation of τ in Eq. 4.25. The few points with large error bars are one of those points where at least one of the components or both have small amplitudes for oscillation, as shown in Figs. 4.19. Comparing Figs. 4.17 and Figs. 4.18, the damping time is minimum when the oscillation frequency is highest, and vice versa. Intuitively, we can interpret this as follows: When the oscillation frequency is large, then each mode corresponding to each momentum centered around the point (P_0, Θ) oscillate rapidly and pretty soon they become out of sync, and the overall oscillation dies away. The opposite happens for the case of large damping time.

FIGURE 4.19: The absolute value of amplitudes of velocity oscillation for the v_x (a), v_y (b) components, and the absolute value of velocity (c), for different pushing angles at a fixed momentum $P_0 = 4\sqrt{2}$. The dots represent the extracted values from the experimental data. The orange and green solid lines are the approximation given by Eq. 4.26 and numerical calculation (see Chapter-2 for detail), respectively. Note that the small magnitude of amplitudes for v_y component results in a large error bar for the fit to converge.

The most important result of this thesis is the above plot in Fig. 4.19 where we show that the amplitude of the velocity oscillation has the anisotropy with full contrast, namely it can be suppressed in few cases. This anisotropic *Zitterbewegung* has never been reported in literature, to the best of our knowledge. We see from Fig. 4.19(c), that the magnitude of the oscillation is almost entirely suppressed at $\Theta = 45^\circ$ & 225° . Remarkably, those are the angles where the branch-cuts appear for the upper- and lower-branch of the dressed-state Hamiltonian in Eq. 4.17.

This anisotropic *Zitterbewegung* is attributed to the momentum dependence of the eigenstates of the Hamiltonian in Eq. 4.17, which is a manifestation of spin-orbit interaction. This coupling of internal (i.e., atomic states) and external (i.e., momentum-states) degrees-of-freedom, namely the spin-orbit interaction induces an ether-like vacuum for atoms – in the presence of gauge field – and subsequently the Galilean invariance breaks down [179].

4.4 Summary and Outlook

In this chapter, we have presented our experiment on the observation of anisotropic *Zitterbewegung* in a non-Abelian gauge field. We have discussed the preparation of initial state and then discussed the detail on data analysis with few quoted

results from Chapter-2. The oscillation frequency of the *Zitterbewegung* is shown to the energy difference of the two branches of the dressed-Hamiltonian. We have also shown that the amplitude of velocity oscillation remain independent of the magnitude of the kick-momentum so long as the direction remain fixed. Most importantly we have revealed that the anisotropic nature of the *Zitterbewegung*. The main take-home message from this chapter is: *The presence of non-Abelian gauge field leads to an anisotropic and controllable Zitterbewegung oscillation of atomic wave packet.* All the presented results are compared with the analytic expressions and numerical calculations.

The relation between the oscillation frequency of velocity and the energy-gap between can be two bands can be new way to extract the dispersion relation of a two-band system or just to probe the energy-gap, similar to Ref. [180]. An interesting direction would be to extend this system with pseudo-spin- $\frac{1}{2}$ to higher spin models, and studying *Zitterbewegung* for a multi-band system could be interesting since it may give rise to several frequencies of oscillation when different bands are separated by different amount of energies.

Chapter 5

Summary and Outlook

In this thesis, we have presented our work on the observation of anisotropic *Zitterbewegung* in a non-Abelian gauge field with ultracold atoms. Each of the experimental results was understood with the aid of detailed analytic calculation coupled with numerical analysis. However before reaching to the point where we can observe this phenomena, there was a long way of preparing the system, cooling the gas down to the right temperature regime, and in parallel we had to have a firm grasp of the underlying theory to interpret the results. In the following we comment on these steps and suggest possible ways of improvement.

We describe the laser cooling and trapping of strontium-87, in our experiment. As there are several groups in the world performing experiments with strontium, mostly for the purpose of metrology and quantum simulation [143, 153, 181–186], our group has one of the highest flux of atoms in our apparatus [1]. With only laser-cooling we get down to $\approx 3 \mu\text{K}$ with more than 15 million atoms. As we load these atoms into a crossed-dipole trap, we see that the temperature of the atoms increases linearly with the trap-depth of the dipole trap, as shown in Fig. 4.5. Although this linear behavior was reported before, in the context of dipole trap, the mechanism of heating is not completely understood [157]. In future, it might be worth probing into this phenomena.

Afterwards we perform optical pumping on loaded atoms inside the dipole trap, to pump all the atoms of positive m_F state to $|m_F = 9/2\rangle$ state in the $|F = 9/2\rangle$ manifold of 1S_0 ground state. Note that we have kept the atoms in the negative m_F state intact for strategic advantage — for the thermalization via collision during

the evaporation. During the process of optical pumping, we have leveraged the atoms-loss spectroscopy technique that has provided us with the information on the differential light-shift due to the dipole trap. The optical pumping efficiency has been recorded as more than 95%.

Then we perform evaporation on this partially polarized gas. The evaporation was described in full detail throughout the whole trajectory. We perform the evaporation where we ramp-down the power of two beams at slightly different rates, and there are distinct regime for evaporation: 1. When we ramp-down the power of both laser beams, 2. One beam is kept at a fixed power and the other beam power keep decreasing. We show that in both regimes, there is gain in phase space via scaling law. Afterwards we characterized the gas and fit it with Thomas-Fermi distribution to reveal the degeneracy $\frac{T}{T_F} = 0.21(4)$. As this fitting scheme is quite standard and reliable [138], there is an alternative way of revealing the degeneracy via probing the phase-space noise [176, 187, 188]. Since this latter method requires taking hundreds of images and a meticulous calibration of the imaging noise, we have opted for the more tradition way of fitting, namely the Thomas-Fermi fitting. However it would be interesting to compare these two approaches, and see the onset of the suppressed fluctuation of density for the latter method.

As we have the cold gas of temperature (≈ 45 nK) that is significantly below the recoil temperature 230 nK, we went on to perform the experiments on the dynamics of this aforementioned gas in a non-Abelian gauge field. Before we could reveal the anisotropic *Zitterbewegung* of the atomic wave packet, we carefully prepared the initial state in the dark-state manifold and then introduced a momentum-kick to the cloud. The role of this momentum-kick is emphasized from a theoretical perspective in Chapter-2 where we have performed a complete theoretical analysis on the appearance of *Zitterbewegung* for all possible scenario in a SU(2) Hamiltonian class, in non-relativistic Schrödinger equation. For the observation of *Zitterbewegung*, we have performed mainly two types of experiments.

The first type of experiments are the ones where we have kept the angle of kick-momentum fixed and varied the magnitude of the kick-momentum. Here we have observed that the frequency of oscillation goes linearly with the kick-momentum while the amplitude of velocity oscillation remain constant for a fixed angle. The slope of the frequency versus momentum-kick magnitude depends on the angle at which we give the kick to the wave packet. Since the frequency of oscillation,

for particular momentum-kick, is proportional to the energy difference between the two dressed bands of the Hamiltonian, this could be a new way to map the dispersion relation of a two-band Hamiltonian, in momentum-space. A careful theoretical analysis is required to extend this method for multi-band system as several frequencies may appear when different bands cease to be equidistant in energy, at a particular momentum.

In the second type of experiments, we fix the magnitude of kick-momentum and vary the angle at which we provide the kick. Here we reveal that for two particular angles of kick-momentum, the *Zitterbewegung* vanishes, as shown in Fig. 5.19. These two angles can be traced back from the spin-texture of the eigenstate of the Hamiltonian that hosts branch-cuts at those angles, one for each of the eigenstates. The anisotropy of the amplitude is manifested over the whole 360° . To the best of our knowledge, this is the first report on this kind of anisotropy in the context of *Zitterbewegung*.

One major limitation of our experiment is that we cannot put the atoms in arbitrarily large momentum state. This is due to the fact that — a large average momentum would require a large detuning for the gauge field lasers. As soon as the detunings becomes comparable to the Rabi frequencies ($\sim 2\pi \cdot 200$ kHz in our experiments) of the gauge field laser beams, the atoms start to leave the dark state manifold and couple to the bright states of the dressed Hamiltonian. Therefore, we had to make sure that the adiabaticity condition is maintained — the atoms in the dark-states do not leave the dark-state manifold. One possible solution is to increase the Rabi frequencies of the gauge field lasers. However that would cause an increased off-resonant coupling to the states that lie outside the tripod atomic levels. This off-resonant coupling can be suppressed by increasing the magnetic field, however we are operating at the maximum possible bias magnetic field (67 Gauss) that our current source can generate via the bias coils. Having a current source of higher current would solve this problem. However, one should keep an eye on the quadratic Zeeman effect as the level-spacing between different Zeeman levels tend to increase slowly at high magnetic field (~ 100 Gauss), for the $|F = 9/2\rangle$ manifold of the 3P_1 state.

There is one particular thing of interest that we have not been able to explore during the extent of the PhD, namely the effects of the degeneracy at zero momentum. Based on our theoretical calculation, the winding number is non-zero

(see Appendix-B). This suggests the presence of a Zak phase of π as one perform a complete revolution around the zero momentum [83, 189]. This experiment we could not perform for two reasons: 1. the direct digital synthesizers (DDSs) are not suited to have time-resolution. Therefore we could not change the detunings as a function of time so that the wave packer performs a loop in momentum-space. Although we could overcome the previous limitation by employing electro-optic modulators (EOMs) to perform the loop in momentum-space, however the available phase limits the rate at which the loop can be performed. 2. One needs to full-fill the adiabaticity condition during the loop. Then we are faced with the competition between the energy gap of the two dressed bands (this demands us to go as slow as possible), and the finite damping-time of the population (this suggests to go as fast as possible). We could not find a suitable parameter region so that we can full-fill the two conditions simultaneously to perform a complete loop. One solution is to lower the temperature gas even further and this will increase the damping time (the damping-time goes like inverse-square-root of the temperature).

One could perform similar experiment with other group-II atoms (e.g., Ytterbium) due to the presence of almost degenerate ground-states. The presence of nuclear spin will prevents a large fluctuation of the ground energy levels at ambient magnetic field, and it has the similar internal energy state as strontium. Moreover, it is possible to control the ambient magnetic field at a level of few tens of μT [135], and then one could use the Zeeman sub-levels of the two hyperfine ground-states, of group-I atoms, to induce Raman-like transition (instead of microwave [135]) to have a two-dimensional spin-orbit coupled system with non-Abelian gauge field where the momentum terms of the Hamiltonian are comparable to the recoil momentum of the Raman transition. Moreover, one could envision to perform similar experiments with molecule. The presence of dipolar interaction between molecules will make the system rich in phenomenology. It would be interesting to see how the center of mass oscillates for a molecule and the average momentum gets coupled with other motional degrees of freedom [190].

Although we have presented our experiment for a $\text{SU}(N = 2)$ system, this can be extended to other systems with $N > 2$. There the frequency of the *Zitterbewegung* which is the difference between different bands, can have multiple frequency-components due to the presence of several bands. The oscillation frequency that provides information on the difference between the bands can be a powerful tool to

map the band-diagram of a multi-band system [191]. Moreover, selective excitation among particular bands, for particular choices of initial states, can be exploited to probe the energy-difference between those particular set of bands or any of the underlying symmetries of the system.

In contrast to SU(2) system where the *Zitterbewegung* can only vanish on a line (as demonstrated in this thesis), in a SU(3) system (which is currently being pursued experimentally in our lab), the *Zitterbewegung* can vanish at discrete points for one choice of gauge field, and may vanish on a line for other choices of gauge field, in the two-dimensional momentum-space. In a SU(3) system, there are eight Gell-Mann matrices, and the possible gauge field structure is richer compared to the three Pauli matrices for SU(2) systems [192]. Moreover, this kind of multi-frequency *Zitterbewegung* has no analogue in the existing literature, and may have connection with neutrino oscillation [193].

Appendix A

Detail on the Fitting of Thomas-Fermi Distribution

For a Fermi gas, the relevant parameters to be extracted from an absorption image, are the following –

- Number of atoms N ,
- The ratio $\frac{T}{T_F}$.

We proceed via the following three steps –

- **Step-1:** From the absorption image taken during the time-of-flight, we fit them with a classical Maxwell-Boltzmann distribution. This provides us with two parameters, namely 1. number of atoms N , 2. temperature of the cloud T . This step is common for all the gases, i.e., thermal gas and even for deeply degenerate gases. The number of atoms extracted from this step is reasonably correct, however the temperature from this step has to be reevaluated.
- **Step-2:** The high momentum wings of the cloud is fitted with a Gaussian distribution, while leaving ‘zero-weight’ to the central portion (e.g. $2\frac{\sigma_x+\sigma_y}{2}$) of the cloud, during fitting. The rationale behind this step is the following: the information about the temperature, for a degenerate gas, is available above the Fermi momentum, contrary to the atoms that reside deep into the

Fermi sea. The calculated temperature is a good estimation of the temperature T . After this step, a reasonable estimation could be performed on the degeneracy parameter T/T_F , with Fermi temperature $T_F = \frac{\hbar\omega}{k_B} (6N)^{1/3}$, for the trap oscillation frequency $\omega = (\omega_x\omega_y\omega_z)^{1/3}$. The measurement of ω at each point in the evaporation-trajectory is cumbersome. To avoid measuring trap frequency throughout the evaporation trajectory, we measure the power with good precision and then interpolate the trapping frequency during the evaporation trajectory, at different trap-depth.

- **Step-3:** Having a good estimation of degeneracy parameter T/T_F and the number of atoms N at our disposal, we proceed to the fitting with a Thomas-Fermi distribution

$$\text{Optical Denisity}(x, y) = \text{OD}_{\text{peak}} \frac{\text{Li}_2(-\zeta \exp(-x^2/2\sigma_{\text{TF},x}^2 - y^2/2\sigma_{\text{TF},y}^2))}{\text{Li}_2(-\zeta)}, \quad (\text{A.1})$$

where OD_{peak} represents peak optical depth and $\sigma_{\text{TF},j} = \sqrt{\frac{k_B T}{m\omega_j^2} (1 + \omega^2 t^2)}$, for a TOF absorption image of time t .

Note that the fugacity ζ , dispersion $\sigma_{\text{TF},x}$ and y are the fitting parameters in Eq. A.1. Extracted ζ is compared with the fugacity from Step-1 and 2, via $\text{Li}_3(-\zeta) = -\frac{1}{6(T/T_F)^3}$

We perform the optimization of fitting by finding the global optimum points for given intervals, of the parameters. The optimization method is quite robust. Please note, the fitting for degenerate Fermi gas should be quite precise due to the fact that the transition from classical gas to quantum gas is smooth, in stark contrast with the Bose-Einstein condensate that has a sharp transition and the bi-modal distribution can even be detected by naked eyes.

A.1 Fitting a Degenerate Fermi Gas

We show the performance of the fitting scheme for cloud of different temperature and without any noise added to the some phantom images.

We employ the following rules to extract the temperature:

1. **Gaussian:** The image is fit with a gaussian and the temperature reads $T = \frac{\sigma_x^2 m \omega^2}{k_B}$
2. **Truncated:** The center of the data is truncated for $4\sigma_x$ and then the temperature is extracted in a similar manner of Gaussian fit.
3. **Fugacity:** The temperature is extracted from fugacity ζ via

$$T = \sqrt[3]{\frac{T_F^3}{\text{Li}_3(-\zeta)}} \quad (\text{A.2})$$

4. **Logpolynomial:** This is the same as the $T = \frac{m \omega^2 \sigma_x^2}{k_B}$, and this is the dispersion in the fit of Thomas Fermi function.

FIGURE A.1: The extracted temperature using different schemes.

As shown in Fig. A.1, different fitting approaches yield different level of accuracy to extract the ratio T/T_F . We make the following remarks on the fitting:

- For high temperature ($T/T_F \geq 1$), Gaussian fit provides a good answer, this is because of the well-known fact that the gas is classical, at this range of temperature.

- For low temperature ($T/T_F < 0.5, 1$), Thomas-Fermi fit is the only one that provides a good answer and this is due to the obvious fact that the gas is dictated by Fermi-Dirac distribution, at this range of temperature.
- Interestingly, Thomas-Fermi approximation gives good result even at *high* temperature. This has the following reason: the fugacity of the gas is related to the temperature via

$$\text{Li}_3(-\zeta) = -\frac{1}{6(T/T_F)^3}, \quad (\text{A.3})$$

and at *high* T/T_F , fugacity ζ is quite *small*.

Moreover, from the expansion of $\text{Li}_n(-\zeta) = -\zeta + \frac{\zeta^2}{2^n} - \frac{\zeta^3}{3^n} + O(\zeta^4)$, we can deduce that, for *small* ζ , $\text{Li}_2(-\zeta) \approx -\zeta$.

Moreover, the optical density in Thomas-Fermi approximation reads

$$\text{Optical Density}(x, y) = \text{OD}_{\text{peak}} \frac{\text{Li}_2(-\zeta \exp(-x^2/2\sigma_{\text{TF},x}^2 - y^2/2\sigma_{\text{TF},y}^2))}{\text{Li}_2(-\zeta)}. \quad (\text{A.4})$$

If we expand the second-order logpolynomial functions in Eq. A.4, around the small parameter, then the fugacity terms in the numerator and denominator cancel. This finally recovers the Gaussian distribution at *high* temperature.

Appendix B

Winding Number

The relevant part of the Hamiltonian is

$$H = \tilde{m}_1 \sigma_1 + \tilde{m}_3 \sigma_3, \quad (\text{B.1})$$

where $\sigma_{1,3}$ are the Pauli matrices and momentum-space magnetic field components $\tilde{m}_{1,3}$ read [189]

$$\tilde{m}_1 = c_1 p_x + d_1 p_y, \quad \tilde{m}_3 = c_3 p_x + d_3 p_y,$$

$$c_1 = \frac{1}{2\sqrt{3}}, \quad d_1 = -c_1, \quad (\text{B.2})$$

$$c_3 = \frac{1}{2}, \quad d_3 = \frac{1}{6},$$

$$p_x = P_0 \cos(\theta), \quad p_y = P_0 \sin(\theta)$$

where we have parameterized the momentum $p_{x,y}$ in the polar coordinate with radial component P_0 and the polar angle θ . We would like to calculate the winding number of the Hamiltonian in Eq. B.1. Towards that end, we form the vector

$$\tilde{\mathbf{M}} = \begin{bmatrix} \tilde{m}_1 \\ \tilde{m}_3 \end{bmatrix}. \quad (\text{B.3})$$

and normalize it as follows

$$\mathbf{M} = \frac{\tilde{\mathbf{M}}}{\|\tilde{\mathbf{M}}\|} = \begin{bmatrix} m_1 \\ m_3 \end{bmatrix}, \quad (\text{B.4})$$

where

$$m_1 = \sqrt{\frac{3}{2}} \frac{\sin\left(\frac{\pi}{4} - \theta\right)}{\sqrt{2 + \cos(2\theta)}}, \quad m_2 = \frac{1}{2} \frac{(3 \cos(\theta) + \sin(\theta))}{\sqrt{2 + \cos(2\theta)}}. \quad (\text{B.5})$$

We also evaluate the quantity $\dot{\mathbf{M}}(\theta)$ as follows

$$\dot{\mathbf{M}}(\theta) = \begin{bmatrix} \dot{m}_1 \\ \dot{m}_3 \end{bmatrix}, \quad (\text{B.6})$$

$$\text{with } \dot{m}_1 = -\frac{\sqrt{3}}{2} \frac{(3 \cos(\theta) + \sin(\theta))}{[2 + \cos(2\theta)]^{3/2}}, \quad \dot{m}_3 = \frac{3}{\sqrt{2}} \frac{\sin\left(\frac{\pi}{4} - \theta\right)}{[2 + \cos(2\theta)]^{3/2}}.$$

Now we have all the ingredients to evaluate the winding number w as follows

$$w = \frac{1}{2\pi} \int_0^{2\pi} d\theta \cdot \left[\mathbf{M} \times \dot{\mathbf{M}} \right]_{\perp}, \quad (\text{B.7})$$

where $\left[\mathbf{M} \times \dot{\mathbf{M}} \right]_{\perp}$ is the out-of-plane component of the vector $\mathbf{M} \times \dot{\mathbf{M}}$ that reads

$$\begin{aligned} \left(\mathbf{M} \times \dot{\mathbf{M}} \right)_1 &= 0, \\ \left(\mathbf{M} \times \dot{\mathbf{M}} \right)_2 &= 0, \\ \left(\mathbf{M} \times \dot{\mathbf{M}} \right)_3 &= \frac{\sqrt{3}}{2 + \cos(2\theta)}. \end{aligned} \quad (\text{B.8})$$

Note that we have considered the circle on the plane of first- and second-component. After evaluating the integral in Eq. B.7, we find

$$w = +1. \quad (\text{B.9})$$

Since the winding number is a non-zero integer, we expect non-zero Zak phase when encircling around the zero momentum in the parameter-space [189].

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