Thermal and flow characteristics under oscillating acoustic field in a channel

Harikumar, Govind

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**THERMAL AND FLOW CHARACTERISTICS UNDER
OSCILLATING ACOUSTIC FIELD IN A CHANNEL**

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Interdisciplinary Graduate School
Energy Research Institute @ NTU (ERI@N)

2021

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OSCILLATING ACOUSTIC FIELD IN A CHANNEL**

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**Interdisciplinary Graduate School
Energy Research Institute @ NTU (ERI@N)**

A thesis submitted to the Nanyang Technological University in partial fulfillment of
the requirement for the degree of
Doctor of Philosophy

2021

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Govind Harikumar

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Authorship Attribution Statement

This thesis contains material from 1 paper accepted at conferences and 1 paper published in a peer-reviewed journal in which I am listed as an author.

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The contributions of the co-authors are as follows:

- Prof. Duan and Dr. Dubey suggested initial project direction and guided the work.
- Dr. Wang helped with designing the study and gave technical advice.
- Mr. Ho assisted with pressure and temperature data collection.
- I conducted the study, analyzed the data, and wrote the manuscript. The manuscript was revised by Prof. Duan.

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The contributions of the co-authors are as follows:

- Prof. Duan was involved in conceptualization, design of study, interpretation of data as well review and editing of manuscript.
- Dr. Shen was involved in conceptualization, design of study, analysis and interpretation of data and co-wrote the original draft of the manuscript with me.
- I was involved in conceptualization, design of study, analysis and interpretation of data and co-wrote the original draft of the manuscript.
- Dr. Wang helped in conceptualization and design of the study.

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Abstract

Thermal and fluid flow processes undergone by a gas in an oscillating acoustic field can convert sources of heat energy into acoustic power or use acoustic power to generate or pump heat from low to high temperatures. Thermoacoustic systems utilize this process for heating, cooling or power generation. They are environmentally friendly devices that use inert gases and have no or minimal moving parts, thereby offering high reliability and a long life span. The stack is the key component in thermoacoustic systems where energy conversion takes place. However, the underlying flow and heat transfer mechanisms of the oscillating fields in the internal channels of stacks is not well understood and therefore warrant further exploration.

In this work, two different thermoacoustic systems were investigated. First, the standing wave system with a straight duct was designed and constructed. For this system, porous steel wool which was characterized based on the ratio of hydraulic radius to the thermal penetration depth was investigated and found to generate considerably high temperature difference which illustrates the thermoacoustic effect and the viability of using porous media as an alternative to regular geometries of stacks in standing wave systems. Additionally, to investigate the flow field behind channels of the stack, time-resolved particle image velocimetry measurements (PIV) were conducted and the phase averaged results revealed two pairs of counter-

rotating dominant and residual vortices. Non-linear vortex shedding phenomenon was also observed which may affect the heat transfer between the fluid and stack.

The second system investigated was the looped thermoacoustic system, where a part of the straight duct is replaced by a looped tube enclosing the stack. The phase averaged results of the time resolved PIV measurements showed the evolution of dominant and residual vortices in the flow field in which the dominant vortices remained attached to the ends of the stack and the corresponding residual vortices propagate with the mean flow. Here, due to the complex geometry of the loop, proper orthogonal decomposition was applied to identify the small-scale fluctuations from the prominent flow structures. Temporal coefficients revealed that the vortical flow structures varied from cycle to cycle.

Finally, a transient computational fluid dynamic model was developed to observe the velocity, temperature and flow fields within channels of the stack. The simulation results showed good agreement with the experimental measurements at several frequencies and generated similar flow structures. A third pair of counter-rotating inner vortices formed during suction and a residual vortex layer were observed within the channel. Velocity profiles inside the channel showed the existence of peaks near the walls and a reduction of velocity at the channel core which was most prominent at resonance frequency. The temperature profile was also found to have analogous peaks and dips which were dependent on the velocity profile and the direction of heated gas flow. The distribution of peaks or dips in the channel varied with the viscous and thermal penetration depths which decreased with increasing mean pressure. These variations could affect the heat transfer between the stack and fluid where the average heat flux in a cycle was found to increase with Reynolds number for different working fluids.

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List of Symbols

Abbreviations

AHX	Ambient Heat Exchanger
CFD	Computational Fluid Dynamics
CHX	Cold Heat Exchanger
COP	Coefficient of performance
DAQ	Data Acquisition
DR	Drive ratio
DV	Dominant vortices
FFT	Fast Fourier Transform
HHX	Hot Heat Exchanger
IV	Inner vortices
PIV	Particle Image Velocimetry
POD	Proper Orthogonal Decomposition
RANS	Reynolds Averaged Navier-Stokes equations
RV	Residual vortices

SST	Shear Stress Transport
SVD	Singular Value Decomposition
UDF	User Defined Function

Greek Symbols

δ	Plate thickness
δ_k	Thermal penetration depth
δ_v	Viscous penetration depth
δ_{ij}	Kroneckar delta
ϵ	Effect of stack material
γ	Ratio of specific heat
κ	Thermal conductivity
κ_s	Thermal conductivity of stack
λ	Wavelength
λ_2	Lambda-2 criterion for vortex identification
μ	Viscosity
ν	Kinematic viscosity
Ω	Vorticity
ω	Angular frequency
Φ	Modes matrix
Π	Perimeter of wire

Ψ	Snapshot coefficient matrix
ρ	Density
ρ_m	Mean density
ρ_s	density of solid
Σ	Singular value matrix
σ_{ij}	Stress tensor (N/m ²)
T	Temperature
τ	Time step
T_1	Complex temperature
T_m	Mean temperature
$\varkappa$	Turbulence kinetic energy
φ	scalar quantity

Roman Symbols

A	Cross-sectional area
a	Speed of sound in medium
A_g	Cross sectional area of gas
A_s	Area of stack
A_c	cross-sectional area of wire
A_{gs}	gas-solid contact area
a_i	Temporal coefficient of mode i

C_p	Isobaric specific heat
D	Center to center distance between plates
d	Plate separation
E	Acoustic energy
E	Total energy (J/Kg)
f	frequency
f_k	Thermal function
f_v	Viscous function
H	Total energy flux
K	Kinetic energy
k	Wave number
KC	Keulegan-Carpenter number
L	Length of the resonator
l_0	Half-plate thickness
L_s	Stack length
m_s	mass of solid
P	Input power
p	Pressure
p_1	Complex pressure amplitude
p_m	Mean pressure

Pr	Prandtl number
q	Heat flux
r_h	hydraulic radius
Re	Reynolds number
S	Control area
S_h	Heat source (J/Kg)
T	Time period of oscillating flow
t	Time
T_1	Cold end temperature
T_2	Hot end temepature
u	x component of velocity
U_1	Complex volume velocity
u_M	mean amplitude of axial velocity
v	y component of velocity
V_g	Volume of gas
V_{total}	Total volume of stack
Wo	Womersly number
X	Snapshot matrix
y_0	Half-plate spacing

Special Symbols

$'$	Fluctuating component
$\overline{}$	Phase-average
i	$\sqrt{-1}$
$Im[]$	Imaginary part of complex number
$Re[]$	Real part of complex number

Chapter 1

Introduction

1.1 Background

The coupling of thermodynamic and fluid dynamic aspects of a gas in an oscillating acoustic field or a sound wave leads to the inter-conversion between sound and heat energy. This phenomenon dealing with creation or amplification of an acoustic wave created by a temperature difference or vice-versa for the conversion between acoustic and thermal energy is called the thermoacoustic effect. Some of the earlier observations of the thermoacoustic effect were seen in the case of glass blowing such as the Sondhauss tube [1] oscillations and in the cryogenic phenomenon called Taconis oscillations [2]. It was Rott [3, 4] who first developed the mathematical framework for the gas oscillations in a channel in the presence of a temperature gradient, in large part to explain the Taconis oscillations. Many years later, several devices started to emerge making use of this phenomenon for refrigeration such as the device by Høfler [5, 6] and separately in the development of pulse tube cryo-coolers [7–9]. Stirling cycle based machines were identified by

Ceperley [10, 11] to have time phasing similar to a traveling wave in their regenerator which lead to the development of a new class of thermoacoustic devices [12, 13]. Design methodologies were subsequently developed largely based on the works of Swift with his group at the Los Alamos laboratory [14–16] for constructing thermoacoustic engines and refrigerators.

A typical thermoacoustic device consists of a stack or regenerator surrounded by heat exchangers on either side called the ‘thermoacoustic core’ which is placed inside a long duct containing the oscillating gas called the resonator. Depending on the direction of energy conversion, the thermoacoustic device may either be a prime-mover or a refrigerator/heat pump. In a prime-mover also known as a heat engine, a temperature difference between the stack ends is utilized to produce acoustic work in the form of pressure waves. These pressure waves can in turn be used to generate electric power by means of acoustic-electric converters [17]. Meanwhile in a refrigerator/heat-pump, acoustic energy in the form of pressure oscillations is utilized to create a temperature difference across the stack. The distinction between the refrigerator and heat pump depends on the desired effect, wherein a refrigerator is used for cooling while a heat-pump is used for heating or upgrading waste heat.

1.2 Motivation

The exponential growth of energy demands of the world population and the need to move towards more environmentally friendly sources of energy, to mitigate the damaging consequences of climate change have surfaced the need for alternate means of energy conversion. Thermoacoustic systems have several advantages over conventional energy conversion technologies in this regard [18]. The systems are environmentally friendly with no harmful emissions due to the inert nature of the working fluids like helium and air.

Additionally, these working fluids generally do not undergo phase change under the normal conditions and therefore can accommodate a wide range of temperatures. The systems, having no or minimal number moving parts at high temperature zones which improves their reliability, provide easy maintenance and potentially longer life spans. They also do not use any exotic materials and the system is highly scalable. Although they are generally simple to construct, the design process is tedious due to the complex nature of the oscillating acoustic field and its interaction with the internal structures of the thermoacoustic system. This leads to non-linear phenomenon that can cause unexpected deviations from the established design guidelines. Therefore studying these non-linear phenomenon and their mechanisms through experiments or simulations can further understand thermoacoustic systems and help improve this clean and promising technology.

1.3 Objective

The main objective of this work is to further improve the understanding of thermoacoustic phenomenon by characterizing the oscillating flow field in and around the vicinity of the stack and identify the evolution of non-linearity in the flow using a combination of experimental measurements, data analysis and computational modeling. Although several experimental studies have confirmed the presence of non-linear phenomenon like vortex generation in standing wave thermoacoustic systems with a straight resonator, seldom attention has been given to the looped thermoacoustic system which is investigated predominantly in this work. This will prove beneficial to understanding the limitations of design methodologies currently used, which are mainly based on the linear theory of thermoacoustics and using steady flow correlations. The evolution of vortical structures and non-periodicity of small-scale fluctuations have been studied using experimental

data and the velocity and temperature fields in the vicinity and channels of the stack were investigated using computational modeling. Knowledge of the flow structures and their behavior can be helpful to identify ways to improve heat and mass transfer during design stages and CFD models which can incorporate these non-linear phenomena can be used as a convenient tool to investigate thermoacoustic systems. Further details regarding the scope of the work is detailed in the next section.

1.4 Scope

Experimental and numerical investigation of fluid flow and thermal processes around thermoacoustic stacks will help further understand the features of the oscillating acoustic fields in the context of thermoacoustics. Temperature measurements using a steel wool stack can demonstrate the viability of using low-cost and easily accessible porous materials as thermoacoustic stacks for energy conversion. Additionally, conventional thermoacoustic systems are designed based on the linear theory of thermoacoustics, however non-linear phenomenon like vortex generation, acoustic streaming and turbulence are known to affect the performance of thermoacoustic systems [14]. Experimental particle image velocimetry (PIV) measurements combined with statistical analysis can be used as effective tools to visualize flow field. A baseline for the PIV measurements is established using the standing wave thermoacoustic system and compared with similar work in literature. Furthermore, for complicated geometries in thermoacoustic systems like looped tubes, proper orthogonal decomposition (POD) analysis can be used breakdown the oscillating flow field into the dominant flow and other small scale vortical structures. Numerical modeling using computational fluid dynamics (CFD) software which can be validated with the experimental results can help further expand the study by making it pos-

sible to measure thermal and flow parameters that would have too costly and time-consuming to investigate experimentally. The combination of experimental measurements of the oscillating acoustic field near the stack and CFD modeling of the entire thermoacoustic system will provide novel insights regarding the features of the oscillating flow in looped thermoacoustic systems. The scope of this work is listed below.

- (i) To design and construct a standing wave thermoacoustic system with a straight resonator to study the thermoacoustic effect
- (ii) To investigate the effectiveness of using steel wool as the stack for thermoacoustic energy conversion
- (iii) To measure using PIV technique and analyze the flow structures behind the stack in the standing wave thermoacoustic system
- (iv) To design and construct a looped thermoacoustic system with the stack enclosed inside a feedback loop and attached to a straight resonator
- (v) To measure using PIV technique and analyze the flow structures behind the stack in the looped thermoacoustic system
- (vi) To use POD analysis to breakdown the oscillating flow in looped thermoacoustic system into periodic dominant and small scale unsteady flow fluctuations
- (vii) To develop a CFD model based on the experimental setup, in commercial software ANSYS FLUENT, validate the model and further expand the study of the thermal and flow fields in the looped thermoacoustic system

1.5 Outline of thesis

- Chapter 2 provides information about the theoretical framework for understanding the fundamentals of the thermoacoustic effect. It also gives the necessary background on the studies of oscillating flows in the context of thermoacoustic systems and identifies possible areas that need further investigation. Finally, details are provided regarding the numerical modeling of thermoacoustic systems with reference to previous studies.
- Chapter 3 includes the thermal study of the steel wool stack and flow visualization study of the parallel plate stack in a standing wave thermoacoustic system. Statistical analysis is used to identify the characteristics of the flow field.
- Chapter 4 includes the flow visualization study in a looped thermoacoustic system, with the addition of POD analysis for the more complex system. The flow structure evolution and the fluctuations of small-scale vortical structures are studied.
- Chapter 5 includes numerical modeling based on the looped thermoacoustic system to visualize the flow and temperature fields inside the stack with varying parameters.
- Chapter 6 summarizes the overall conclusions from the different studies in the thesis and offers insight into potential future works

Chapter 2

Literature Review

2.1 An overview of thermoacoustics

Thermoacoustic processes involve the creation or amplification of an acoustic wave due to a temperature difference or the generation of a temperature difference caused by the propagation of an acoustic wave. The traversal of an acoustic wave in a medium, results in the displacement of the medium and causes pressure, density and temperature variations associated with it. These fluctuations are too minute to be observed in everyday situations whereas the acoustic waves associated with common thermoacoustic phenomenon have extreme magnitudes [19]. The presence of a porous medium (called stacks or regenerators) with much higher heat capacity than that of the medium through which the acoustic wave propagates, results in magnification of the thermoacoustic effect. Based on the dominant type of acoustic wave present, the thermoacoustic systems can be broadly classified as standing wave or traveling wave systems [18]. In a standing wave thermoacoustic system, the pressure and velocity are out of phase by around 90°

and the fluid undergoes a thermodynamic cycle closely resembling the Brayton cycle. Standing wave thermoacoustic systems require imperfect thermal contact between the gas and the stack and therefore have inherently less efficiency. However, by virtue of their extreme simplicity, many devices are still devices have been developed based on the standing wave thermoacoustic system. [5, 20, 21]. When the traveling wave dominates in the system, the pressure and velocity are in phase in the regenerator and the gas undergoes a thermodynamic cycle similar to the Stirling cycle [22–24]. The traveling wave phasing is achieved by positioning the regenerator inside a feedback loop [10]. Even though the traveling wave systems have inherently higher efficiencies, the non-zero time averaged mass flow phenomena like Gedeon acoustic streaming [25, 26] can be observed inside the feedback loop which can deteriorate its performance due to the unwanted convective heat transfer from hot region to cold region.

2.1.1 Thermoacoustic devices and their applications

Figure 2.1: Miniature thermoacoustic refrigerator interfaced with a micro-circuit [27].

Thermoacoustic devices can be used for the recovery of low grade heat [28] from different sources. These devices work on inert working fluids like helium or air and have minimal environmental impact. They have also very few or no moving parts improving their reliability in the long run. They

Figure 2.2: Miniature thermoacoustic prime mover interfaced with a micro-circuit [27].

are relatively much more compact than other heat recovery systems and have been implemented in various scales and domains. Heat driven thermoacoustic engines can be used to generate acoustic waves which can be converted into electrical power [29–31]. Wang et al. [23,32] had developed a thermoacoustic Stirling generator which can make use of the LNG cold energy and gas turbine exhaust energy capable of producing up to 3 KW mechanical power and 1 kW electric power at 295 °C.. Spoelstra et al. [19] had developed a thermoacoustic engine driven by low grade heat source of 110 °C with three regenerator stages.

Miniaturization has become a recent field of interest in thermoacoustics specifically because it shows great potential for cooling and waste heat management [34] in micro-electronics. Typically miniaturization with reduction in the length results in increase of frequency and for compatible sizes required for integration into micro-circuits, the resonant frequency reaches the kilo-Hertz range. The different approaches to thermal management is discussed in [34]. Symko et al. [27] designed and developed miniature thermoacoustic systems which can be interfaced with electronic circuits. Fig.2.1 shows an acoustic cooler driven by a piezo-electric driver connected to a power transistor for cooling. The length of the resonator was around 4 cm, with air as the working fluid. They have obtained a temperature difference of 11 K and were able to achieve a cooling power of 1 W with a

Figure 2.3: Thermoacoustically driven thermoacoustic cooler [33].

COP of 2-3 for the acoustic cooler. Alternatively, a thermoacoustic engine or prime mover can also be used to convert heat directly to sound of high frequency as shown in Fig.2.2. The device had a resonator of length 2 cm and produces sound in the kilohertz range. They have obtained appreciable efficiencies of 10 % of the Carnot efficiency for the prime mover with a heat input of 2 W and a temperature difference of 40 K with high power densities. Sahoo et al. [35] designed a thermoacoustic energy converter which can be integrated into vehicle exhausts. Abdoulla et al. [36] designed low cost a laboratory demonstrator for a thermoacoustic micro-generator running on the waste heat from cooking activities.

Besides power generation, thermoacoustic engines can be used to drive heat pumps which can upgrade waste heat from various sources into more usable process heat [19, 37]. Thermoacoustic refrigerators which make use of acoustic energy to produce cooling can be also be powered by waste heat driven thermoacoustic engines [38, 39]. Fig.2.3 shows a thermoacoustic refrigerator driven by a combination of a standing wave and traveling wave thermoacoustic engines developed by Kang et al. [33]. The use of multiple stages in thermoacoustic engines such as the novel four stage travelling wave engine by de Blok [40] shown in Fig.2.4 helps to lower the onset temperature difference of thermoacoustic oscillations enabling us to utilize even lower quality heat sources for energy conversion.

Figure 2.4: Novel multi-stage thermoacoustic system designed by de Blok [40].

In addition to using waste heat sources, thermoacoustic systems can also make use of renewable sources of energy like solar energy [41–43]. Fig. 2.5 shows a thermoacoustically driven thermoacoustic refrigerator powered by solar energy making use of Fresnel lenses [41]. Another known application for these thermoacoustic devices is in the separation of gas-mixtures using acoustic power and boundary layer processes [44, 45, 45]. Overall, thermoacoustic systems have promising potential to be clean and reliable energy

conversion technologies. However, there is still much progress to be made in the efforts to improve their efficiency. Therefore, it is of great interest to delve deeper into the working principles of these thermoacoustic devices to improve designs and overall efficiency in order to make them commercially viable. The underlying theory of the thermoacoustic effect in these devices is discussed in the next section.

Figure 2.5: Thermoacoustic refrigerator driven by solar energy developed by Adeff and Hoffer [41].

2.1.2 Thermodynamic cycle

The working principle of a thermoacoustic system is explained here in terms of the thermodynamic cycle undergone by a gas parcel in a standing wave acoustic field. The thermoacoustic core where the energy conversion takes place is termed as the ‘stack’. The stack is a porous medium whose hydraulic radius, r_h defined as the ratio of gas volume to the gas-solid contact area [46], is comparable to the thermal penetration depth, δ_k . The thermal penetration depth is defined as the distance through which, heat can diffuse

Figure 2.6: Schematic representation of the basic components in a standing wave engine.

into the fluid surrounding the stack in half cycle of oscillations. [46].

$$\delta_k = \sqrt{\frac{2\kappa}{\omega\rho_m C_p}} \quad (2.1)$$

where κ is the thermal conductivity of the gas, ρ_m is the mean density, C_p is the isobaric specific heat capacity and ω is the angular frequency of oscillations.

Similarly the viscous penetration depth, δ_v is the distance within which viscous effects are prominent in the fluid surrounding the stack.

$$\delta_v = \sqrt{\frac{2\mu}{\omega\rho_m}} \quad (2.2)$$

Where μ is the viscosity of the working fluid.

In a standing wave field, the pressure lags behind the velocity by a phase of 90° . Also, an imperfect thermal contact is required by design. As shown in Fig.2.6, a simple standing wave engine consists of a hot heat exchanger (HHX), the stack and an ambient heat exchanger (AHX) enclosed

Figure 2.7: Thermodynamic cycle in standing wave systems. Adapted and redrawn from [46].

inside a resonator. The right end is kept open at ambient air temperature. When heat is applied and a sufficient temperature gradient is established across the stack, spontaneous oscillations begin to occur [14]. The intensity of these oscillations increase when the amount of heat is increased and finally begins to approach a steady state. The heat flows from the hot end heat exchanger to the ambient end heat exchanger and acoustic power flows through the stack. The thermodynamic cycle undergone by a parcel of gas in pressure-volume coordinates is shown in Fig.2.7. Initially in step 1, the gas is compressed and simultaneously moved leftward by the wave in a nearly adiabatic process. Now in step 2, heat is transferred from the solid walls to the gas parcel and it expands. This continues into step 3 where the gas parcel expands into a larger volume while simultaneously being moved rightward by the wave. Finally, in step 4, heat is transferred from the gas to the solid walls and thereby contracts bringing it back to the beginning of the cycle. Since the thermal expansion takes place at a higher pressure than thermal contraction, net work is done by the gas parcel on its surroundings. The

combined effect of the work done by all such individual gas parcels in the stack contribute to the production of acoustic power, adding energy in every cycle and self-sustaining the oscillations. In essence, the gas absorbs heat Q_h from the HHX at a higher temperature and rejects heat Q_a at the AHX at a lower temperature thereby producing acoustic power in the stack. For steps 1 and 3, the gas should be isolated from the solid which brings about the need for imperfect thermal contact. This is guaranteed by the distance between the gas and nearest solid surface being of the order of the thermal boundary layer of the gas. But this causes relatively higher resistance to the heat transfer process, making standing wave engine inherently less efficient although being simple in design. If the magnitude of temperature gradient is not sufficient, heat transfer process would take place in the reverse direction. The gas will reject heat to the solid at high temperature and absorb heat at lower temperatures leading to the generation of refrigeration cycle seen in heat pumps or refrigerators. Therefore there must be a critical temperature gradient which determines if the working fluid in the thermoacoustic system undergoes a heat-engine cycle or refrigeration cycle. This critical temperature gradient is defined as [14]

$$\nabla T_{crit} = \frac{\omega A |p_1|}{\rho_m C_p |U_1|} \quad (2.3)$$

where A is the cross-sectional area, p_1 and U_1 are the oscillating pressure and volume velocities respectively.

In a standing wave refrigerator or heat pump, each individual parcel of gas within the channels of the stack, transfers heat along its length up through the temperature gradient by using the acoustic power and achieves the required thermoacoustic heat pumping effect. As shown in Fig.2.8 (a) if the left heat exchanger is kept at ambient temperature levels, then the

Figure 2.8: Schematic representation of the basic components in a standing wave (a) refrigerator and (b) heat pump.

system absorbs cooling power Q_c from an external load essentially a refrigerator. Whereas as shown in Fig.2.8 (b), if the right end is the one kept at ambient conditions, thermal power Q_h is released at the other end.

2.1.3 Linear thermoacoustics

The original thermoacoustic theory was pioneered by Rott [3, 4]. He had assumed small, steady state, mono-frequency sinusoidal oscillations of variable quantities like pressure, velocity and temperature. The waves are assumed to propagate in the x -direction such that the length scales obey $\delta_k < \lambda$ and $\delta_v < \lambda$. Later in 1988, Swift [15, 47] reworked the theory for low

amplitudes by assuming sinusoidal time dependence for all quantities which can be expressed as

$$p = p_m + Re[p_1(x)e^{i\omega t}] \quad (2.4)$$

where p is the pressure, subscript m refers to the mean value, subscript 1 denotes a complex function of x , ω is the angular velocity and t is the time such that $e^{i\omega t}$ reflects the sinusoidal time dependence.

Similarly the velocity and temperature fluctuations can be expressed as

$$U = Re[U_1(x)e^{i\omega t}] \quad (2.5)$$

$$u = Re[u_1(x, y, z)e^{i\omega t}] \quad (2.6)$$

$$T = T_m(x) + Re[T_1(x, y, z)e^{i\omega t}] \quad (2.7)$$

Here U is the volumetric velocity and u is the acoustic velocity. T is the temperature field and T_m denotes the mean temperature. The density field ρ is also similar to T .

The geometry considered to illustrate the thermoacoustic effect is shown in Fig.2.9. It consists of a parallel plate stack of thickness $2l$ and spacing $2y_0$. The plates are assumed to be stationary and rigid. Using the fluctuating quantities described above, we can obtain the expressions for continuity, momentum and energy equations as given below,

Figure 2.9: The thermoacoustic core.

$$\frac{dU_1}{dx} = -\frac{i\omega A}{\rho_m a^2} \left[1 + \frac{(\gamma - 1)f_k}{1 + \epsilon} \right] p_1 + \frac{f_k - f_v}{(1 - f_v)(1 - Pr)(1 + \epsilon)} \frac{U_1}{T_m} \frac{dT_m}{dx} \quad (2.8)$$

$$\frac{dp_1}{dx} = -\frac{i\omega \rho_m}{1 - f_v} \frac{U_1}{A} \quad (2.9)$$

$$\frac{dT_m}{dx} = \frac{\dot{H} - \frac{1}{2} Re \left[p_1 \tilde{U}_1 \left(1 - \frac{f_k - f_v}{(1 + \epsilon)(1 + Pr)(1 - f_v)} \right) \right]}{\frac{\rho_m C_p |U_1|^2}{2A\omega(1 - Pr)|1 - f_v|^2} Im \left[f_v + \frac{(f_k - f_v)(1 + \epsilon f_v / f_k)}{(1 + \epsilon)(1 + Pr)} \right] - (A\kappa + A_s \kappa_s)} \quad (2.10)$$

where p_1 and U_1 are the complex pressure and volume velocity, a is the speed of sound, ρ_m and T_m are the mean density and temperature, $\dot{H}$ is the total power flow or total energy flux, C_p , A , γ , Pr & κ are the isobaric specific heat, ratio of specific heats, Area, Prandtl number and Thermal conductivity respectively of the working fluid and A_s & κ_s the Area and Thermal conductivity of the stack material respectively. ϵ represents the effect of specific heat and thermal conductivity of the stack material on the heat transfer between the working fluid and solid. It is assumed to be zero for an ideal

solid with infinite specific heat and thermal conductivity [14]. f_k and f_v are the thermal and viscous functions. f_v and f_k depends on the geometry of the channel under and has been calculated for various geometries as illustrated in [14].

Equations 2.8-2.10 represent five variables namely, $Re[p_1]$, $Im[p_1]$, $Re[U_1]$, $Im[U_1]$ and T_m which represent the physics of the thermoacoustic effect. This set of five coupled equations (the first two being complex) can be used for analysis of thermoacoustic systems.

But this linear model can further be simplified by assuming the working fluid to be an ideal gas and having pure standing wave phasing. We also include some additional approximations namely, the ‘boundary layer approximation’ and ‘short stack approximation’. The boundary layer approximation assumes that the half plate spacing, $y_0 \gg \delta_k, \delta_v$. This means that the boundary layer limit values of f_k and f_v can be used. The ‘short stack approximation’ [14] assumes the length of the stack to be considerably smaller than the wavelength such that pressure and velocity inside the stack can be considered independent of x . The stack material is assumed to be an ideal solid ($\epsilon = 0$). Also the temperature difference along the stack should be much lower than the average temperature within the stack. The pressure and velocity for this standing wave system can be represented as,

$$p_1 = p_0 \cos kx, \quad (2.11)$$

$$u_1 = \left(1 + \frac{l}{y_0}\right) \left(\frac{p_0}{\rho_m a}\right) \sin kx \quad (2.12)$$

We can derive expressions for the total energy flux and acoustic power flowing through the stack [14]. The final expressions are as shown

below,

$$\dot{H} = \frac{A}{4} \frac{\delta_k}{r_h} \frac{|p_1| |U_1|}{A(1+Pr)\Lambda} \left[\Gamma \frac{1 + \sqrt{Pr} + Pr}{1 + \sqrt{Pr}} - \left(1 + \sqrt{Pr} - \frac{\delta_v}{y_0} \right) \right] - (A\kappa + A_s \kappa_s) \nabla T \quad (2.13)$$

$$\Delta \dot{E} = \frac{A}{4} \frac{L_s}{r_h} \left[\frac{(\gamma - 1) |p_1|^2 \delta_k \omega}{\gamma p_m} \left(\frac{\Gamma}{(1 + \sqrt{Pr})\Lambda} - 1 \right) - \frac{\rho_m |U_1|^2 \delta_v \omega}{A^2 \Lambda} \right] \quad (2.14)$$

where,

L_s is the stack length,

$$\Gamma = \frac{\frac{dT_m}{dx}}{\nabla T_{crit}}, \quad (2.15)$$

$$\nabla T_{crit} = \frac{\omega |p_1|}{\rho_m C_p |U_1|}, \quad (2.16)$$

$$\Lambda = 1 - \frac{\delta_v}{r_h} + \frac{\delta_v^2}{2r_h^2} \quad (2.17)$$

In case of the resonator, energy is dissipated due to viscous shear and thermal relaxation [14] and the expression for the acoustic energy loss per unit surface area for zero temperature gradient along the axis is given by

$$\frac{d\dot{E}}{dS} = -\frac{1}{4}\rho_m\left(\frac{U_1}{A}\right)^2\delta_v\omega - \frac{1}{4}\frac{(|p_1|)^2}{\gamma p_m}(\gamma - 1)\delta_k\omega \quad (2.18)$$

Using the linear theory, it is possible to model and predict the performance of various stacks and regenerators which constitute a thermoacoustic device. The Los Alamos team [48] also developed a program code for simulating and designing thermoacoustic engines and refrigerators with simple geometries which is based on the model of linear thermoacoustics called Design Environment for Low-Amplitude Thermoacoustic Energy Conversion (DeltaEC) [49]. The code makes use of ‘Rott’s linear approximations’ to solve the one-dimensional wave equations. It can be used to design dimensions of the key components in thermoacoustic devices and is used to develop practical designs based for thermoacoustic engines and refrigerators with simple geometries. It can also be used as a comparison tool for benchmarking data obtained experimentally or by numerical analysis [24, 31, 39].

2.1.4 Limitations of linear thermoacoustics

Linear thermoacoustics described in section 2.1.3 is generally applicable only for steady-state low amplitude operations of thermoacoustic devices and therefore has its limitations. It fails to explain the behavior of these devices in more realistic conditions. Oscillating flow even though well understood in the case of simple geometries, is still unfamiliar through tortuous porous media geometries. Turbulence and its associated losses will also become significant at higher pressure amplitudes particularly in resonator components [14]. The entrance length (when the velocity profile becomes fully developed) for oscillating flows will have viscous and thermal effects which are not well understood. It is assumed that for a stack, the entrance length will be typically of the order of displacement amplitude of the gas [14].

At the interface between thermoacoustic components, continuity of complex variables like pressure, volume velocity and temperature assumed in Rott's theory may not always hold valid. These discontinuities are also of the order of the gas displacement amplitude and cause dissipation of acoustic power and entropy generation in the device. Harmonics which have negligible effect at lower amplitudes and are usually associated with fourth order power terms will also become significant as we move to design devices for higher amplitudes. Besides these, two other effects relating to the oscillating flow taking place in thermoacoustic components are vortex shedding and streaming. Vortex formation and streaming in addition to causing dissipation of acoustic power may also play a role in the heat transfer mechanism of the stack. Streaming [50] is responsible for convective heat transfer losses in thermoacoustic cores and at high pressure amplitudes, the losses can be significant. Complex flow patterns resulting from the generation of vorticity in the flow can either enhance or degrade the heat transfer process. These effects which become prominent at higher amplitudes can be better understood using non-linear thermoacoustics which began its development in the 1990s [18]. Further discussion regarding the flow phenomenon associated with the interaction of the oscillating acoustic field with the components inside the thermoacoustic system are given in the next section

2.2 Flow measurements of oscillating acoustic fields in stacks

The interaction of oscillating flow fields with internal structures in thermoacoustic devices lead to non-linear effects like vortex shedding and streaming in the vicinity of the stack or regenerator ends. An illustration of the vortex generation and shedding phenomenon is shown in Fig.2.10. The flow fields in these cases cannot be accurately described by the existing

Figure 2.10: Generation of vortical structures at the ends of stack plates reported by Shi et al. [51].

linear theory of thermoacoustics developed by Swift exclusively for low amplitude oscillations. At very high pressure amplitudes or drive ratios (ratio of pressure amplitude to mean pressure) strong vortex generation and shedding can be observed similar to the von Karman vortex streets formed in steady flows past bluff bodies [52] but made even more complicated here due to the impingement of vortices formed in the first half-cycle on the newly formed second-half cycle vortices during flow reversal [53]. These flow patterns and the turbulence phenomenon play a critical role in the heat transfer mechanism between the solid in the stack and fluid.

To overcome these limitations and to understand the behavior of the oscillatory flow in the neighborhood of the thermoacoustic core (i.e. the stack), Particle Image Velocimetry (PIV) technique can be used [54, 55]. In PIV, the flow is visualized by illuminating micron sized tracer particles which are injected into the flow field using a droplet or aerosol generator with a dual pulse laser system. The velocity field can be estimated by observing the motion of these tracer particles in the flow. This is done by taking pairs of

images by a high speed camera within a very small time interval in between them and using cross-correlation to calculate the relative displacement of the particles and thereby the flow field. This technique is generally a non-intrusive one and has negligible effect on the flow field. Several studies have used PIV measurements to visualize the flow field in thermoacoustic systems. Blanc-Benon et al. [56] first visualized the instantaneous flow fields in thermoacoustic stacks for low Mach-numbers and found they are in agreement with computational results. Berson et al. [57] demonstrated that counter rotating vortices are generated at the edges of the plates. These vortices are symmetrical and attached to the plates at lower drive ratios or pressure amplitudes but detach at higher pressure amplitudes. Mao et al. [58,59] observed that the most characteristic vortices are seen in the ejection stage of the flow and that these symmetrical attached vortices are either elongated or concentrated near the edges of the plates. When the amplitude of oscillations increase, either vortex breakup or alternating vortex shedding pattern will be seen for elongated and concentrated vortices respectively. Shi et al. [60] made use of non-dimensional parameters like Reynolds number (Re), Womersley number (Wo) and Keulegan-Carpenter number (KC) for describing the evolution of vortices in the oscillating flow past the parallel plate stack. They observed nine unique evolution paths for the vortices involving a different sequence of eight typical wake patterns. Aben et al. [61] used the ratio of Strouhal number (St) based on the driving flow frequency and the Reynolds number (Re) based on plate separation to categorize the number of vortices generated at the plate edges and also showed the influence of porosity and plate shape on the vortex pattern. Both, Aben et al. [61] and Babaei et al. [62] had measured and confirmed the presence of secondary streaming velocities in standing wave systems which might affect the performance of the stacks. Ramadan et al. [63] further showed that these streaming patterns cause deviations from theoretical predictions in the mean velocity field

at high amplitudes. In another study, Shi et al. [64] made use of the St calculated based on the primary vortex shedding frequency obtained from Fast Fourier Transform (FFT) analysis of a hot-wire signal and Re to characterize the vortex shedding process and suggested that the vortex shedding frequency during ejection phases behaved similarly to that of steady flows. Zhang et al. [65] included hot and cold heat exchangers in addition to the parallel plate stack in their setup and studied the influence of factors like driving power, plate thickness, plate spacing and shape of the plate ends on the ejection vortices. They also suggested that displacement amplitude of suction vortices would play a key role in the heat transfer between the stack plates and the heat exchangers. Pan et al. [66, 67] observed the shift in the velocity distributions at different tilt angles of the resonator.

Besides the typical straight-tube geometry systems, thermoacoustic systems with complicated tube geometries are also investigated in literature. Yazaki et al. [12] tested a thermoacoustic engine in a looped tube and found that the engine in a looped tube amplified the traveling wave power, resulting to significantly smaller onset temperature ratios than those for the engine in a resonator. Backhaus and Swift [13, 68] developed a laboratory prototype of a hybrid thermoacoustic engine, a combination of a resonator tube and a looped tube at the end, and achieved a remarkable efficiency of 0.30, comparable to that of the common internal combustion engine and piston-driven Stirling engines. Ueda et al. [69] investigated the acoustic field in the engine similar to Backhaus and Swift's prototype, focusing on the phase between the velocity and the pressure. They found that the negative phase lead of the velocity relative to the pressure of about -20° resulted in the high efficiency of such systems. Later, a second regenerator was installed at a precise position to act as a heat pump in the looped tube and a low temperature of $-25^\circ C$ could be achieved [70]. Yu et al. [71] found that there are two oscillation modes in these systems and the high frequency mode can

transition into the low frequency mode. The regenerator hydraulic radius, resonator length, and mean pressure influenced these two oscillation modes and thus the characteristics of the thermoacoustic system [72]. Biwa et al. [73] used multiple regenerators to decrease the critical temperature ratio and proposed a quality factor to predict whether the thermoacoustic engine really starts up or not.

Despite the prevalence of such looped geometries in thermoacoustic systems and their promising potential, few studies have focused their attention on the evolution of flow structures and non-linear phenomenon like vortex generation and shedding in these looped geometries. As known from previous studies of the flow structure around the stack in the straight resonator [58–60,64], it is understood that the flow behavior strongly influences the performance of thermoacoustic systems. Therefore, a similar approach to examine the flow evolution around the stack for looped thermoacoustic systems will prove beneficial in expanding existing theoretical knowledge regarding such systems. In this study a baseline for investigating the oscillating flow is established using the standing wave thermoacoustic system with a straight resonator. PIV measurements of velocity components and vorticity are used to identify relevant non-linear phenomenon in the oscillating flow field. With this understanding, the approach is expanded to include the looped thermoacoustic system where the stack is enclosed within a feedback tube. From literature, it is understood that this geometry is more prevalent in highly efficient thermoacoustic systems however, the flow structure has been investigated rarely. Therefore, PIV technique can be used to visualize the flow field in the looped thermoacoustic system and help understand any peculiar features of the oscillating flow related to such geometries.

All these experimental investigations are extremely useful in understanding the flow field in thermoacoustic systems, but they are also limited

in their scope because of restrictions in the observations regions as well as cost and time constraints. Therefore, to compliment the experimental investigation, numerical modeling can be capable tool for providing additional insight in other regions of the flow field. A discussion on the numerical modeling of thermoacoustic systems is mentioned in the next section.

2.3 Numerical modeling of thermal and flow fields in stacks

Figure 2.11: Computational domain used by Cao et al. shown by dashed lines (top) and the expanded view (bottom) of the single plate in resonator. Adapted and redrawn from [74].

As discussed previously, the limited applicability of the linear thermoacoustic theory to low pressure amplitudes and the experimental studies of the stack confirming the presence of non-linear flow phenomenon that affect performance suggest the need for a more accurate model for thermo-

coustic flow processes. Several studies were conducted over the years which were progressively improved from single plate studies to finally simulating the flow in the entire thermoacoustic system. Details regarding these earlier models will introduce some interesting features regarding the modeling of thermoacoustic flow.

An early simplistic model of a thermoacoustic stack plate was considered by Cao et al. [74] consisting of a single plate and a resonance tube with an oscillating flow boundary condition. The computational domain is denoted in dashed lines with an enlarged view of the stack plate at the bottom as shown in Fig.2.11. The continuity, momentum and energy equations were solved for a viscous and compressible fluid. The ‘SOLA-ICE’ algorithm [75] was used to solve the governing equations numerically. It was found that the energy flux density in the vertical direction operated in a length scale proportional to the gas displacement amplitude.

Worlikar et al. developed a two-dimensional unsteady model for low mach number flows in the region between two stack plates. Heat exchanger plates were included in later models in the form of isothermal plates in perfect thermal contact with the stack plates. They had found good agreement with the recommendations of Swift [47] concerning simplified heat exchanger scaling. However their results showed that the theoretical calculations underestimated the work required and overestimated the cooling load which may due to huge thermal loss. [76–78]. This model was expanded upon by Besnoin et al. [79] to study the effects of heat exchanger length and position on the cooling load. They also observed wave-like motion the gaps of the plates which enhanced the cooling load.

Ishikawa and Mee used a two dimensional Navier-Stokes solver to study energy transport near thermoacoustic cores [80]. They found that both

‘short’ and ‘long’ plates showed heat pumping effect when they were spaced at distances greater than the thermal penetration depth. They also found that the dissipation of energy increased quadratically with mach number and reported vortical motion outside the plates. Marx et al. [81–83] investigated non-linearities in the temperature field and established a critical stack length of four times the gas displacement where these non-linearities become dominant. It was found that these thermal harmonics degraded the performance of stacks at high drive ratios and when placed close to the velocity anti-nodes. A computational method was developed by Piccolo [84,85] for estimating the heat transfer coefficient between the stack and fluid and finding optimized lengths for heat exchangers. He also further investigated the performance characteristics of parallel fin heat exchangers.

Zoontjens et al. [86] departed from the numerical approach of dealing with a single plate to a thermoacoustic couple using commercial CFD code ANSYS FLUENT [87]. Oscillating pressure boundary conditions were used to simulate the thermoacoustic process and the heat flux and flow field distributions at the ends of the plate were studied. It revealed that the plate thickness had great influence on vortex generation and the distribution of heat flux.

Figure 2.12: Computational domain of entire thermoacoustic engine used by Zink et al. [88].

All these efforts on the numerical modeling of the thermoacoustic stacks eventually lead to the simulation of entire thermoacoustic systems. Nijeholt et al. [89] developed a CFD model for a traveling wave thermoacoustic engine and demonstrated the prevalence of non-linearities at high

amplitudes. Zink et al. [88, 90] explored the effects of resonator curvature for the whole thermoacoustic engine with a temperature gradient boundary condition applied using ANSYS FLUENT. They later demonstrated thermoacoustic cooling effect in a refrigerator model with the addition of a second stack in the system. A three-dimensional model for the stack was developed by El-Rahman et al. [91] which showed the effectiveness of square pores in heat transfer compared to parallel plate structure in stacks.

CFD models have also been used recently to visualize flow structures and temperature fields near the stacks of thermoacoustic systems. Yu et al. [92] developed the CFD model for standing wave engine with an operating frequency of 300 Hz in FLUENT. They had measured the onset temperatures at different mean pressures and also observed the flow field and vortex generation at the stack ends. Jaworski et al. [93] used large eddy simulation scheme to investigate entrance effects at the inlet of the stack plates. Both the flow field and temperature fields were calculated by Tasnim et al. [94] using commercial CFD code and showed the development of temperature difference in the stack. Low amplitude oscillating flow was investigated using CFD by Mohd Saat et al. [95] and identified the distortion of velocity field by temperature gradients. Yu et al. [96] investigated both experimentally and using CFD the temperature fields inside the parallel plates and found that time-dependent temperature gradients in the transverse direction enhanced the heat transfer between the fluid and solid.

Comparisons of the models in all these studies with experimental results showed good agreement. This justifies the use of CFD as an excellent tool for the investigation of flow and thermal behavior of thermoacoustic systems. Furthermore they provide detailed insight into the finer details which might not be available through experimental measurements alone. However, similar to the experimental cases, the CFD models here only investigate

the velocity and temperature fields in thermoacoustic systems with typical straight resonators. Here, in this work the more complex looped thermoacoustic system is also investigated experimentally using PIV measurements and analysis. The PIV data of the looped thermoacoustic system could then be used to validate a CFD model which can effectively investigate such systems. This CFD model will be able to incorporate the non-linear flow effects by making use of the Reynolds Averaged Navier Stokes equations and appropriate turbulence models and would potentially reveal more characteristics of the flow field than possible using traditional modeling approaches based on the linear theory of thermoacoustics and the steady state assumptions used in designing such thermoacoustic systems. Therefore, it will be interesting to investigate the oscillating flow in looped geometry and understand the flow features which may play a key role in heat transfer.

2.4 Summary

A critical review of the various aspects of thermoacoustic systems including the applications and the theoretical framework behind the thermoacoustic energy conversion process was discussed. The crucial role of the stack in thermoacoustic systems was also explained. Although the linear thermoacoustic theory has been used extensively to design thermoacoustic systems, the limitations of the theory in the high pressure amplitude regimes have been made obvious through numerous experimental and numerical studies of thermoacoustic systems. They revealed that non-linear phenomenon are widely prevalent in thermoacoustic systems even at relatively low amplitudes. Vortex generation and shedding in particular has been observed in many of the cases and this could play a significant role in transferring heat flux across the stack and between heat exchangers in practical systems. Furthermore, the flow and thermal behavior of looped geometries

in thermoacoustic systems have rarely been investigated using experimental or numerical methods. It is in this context that two different experimental test setups, the first one being a standing wave system with the stack in a straight resonator and the second with the stack enclosed in a looped tube will be designed and constructed to study these phenomenon in detail. This study will help to identify potential flow and thermal characteristics of the thermoacoustic phenomenon which would otherwise be neglected in traditional modeling approaches but could play a significant role in either enhancing or degrading the heat transfer processes in these systems. Following this, the next chapter deals with the experimental measurements in the first system.

Chapter 3

Experimental investigation of oscillating flow in a standing wave thermoacoustic system^{*}

3.1 Background

In this chapter, a standing wave thermoacoustic system characterized by a straight-tube resonator which was designed and constructed in-house is used to study the thermal and flow field characteristics of the oscillating flow in the system. The standing wave thermoacoustic system driven by a loudspeaker with a stack inside the resonator is the most basic form of a thermoacoustic heat pump and can demonstrate the generation of temperature difference at its core. Two experimental tests were conducted on this system and analyzed in this chapter. First off, low cost and easily

^{*}Some materials in this chapter has been published as ‘Govind Harikumar, Ker Hin Ho, Kai Wang, Swapnil Dubey, Fei Duan, “Thermoacoustic energy conversion in a square duct”, Energy Procedia 2018’

available porous materials like steel wool can generate temperature difference by utilizing acoustic energy from the loudspeaker. If the effective pore size or hydraulic radius is comparable to the thermal penetration depth (δ_k), then porous materials can also be used as the stack for standing wave type thermoacoustic systems. The first experiment demonstrates the use of steel wool as the stack inside the thermoacoustic system to generate temperature difference. In the second part, a parallel plate aluminium stack is introduced in the standing wave thermoacoustic system and is used to establish the viability of using PIV technique to visualize the flow structure evolution in the internal structures of thermoacoustics systems and identify the key flow features which will help support further investigations on more complex systems.

3.2 Experiments

3.2.1 Experimental setup

Figure 3.1: Schematic of standing wave thermoacoustic system with steel wool regenerator.

The schematic of the experimental setup is shown in Fig.3.1. The

standing wave thermoacoustic system consists of a 1.02 m long straight resonator made of acrylic plates of thickness 8 mm glued together to form the duct. The resonator is connected to a loudspeaker (Visaton TW 250 XS-8 Ohm) mounted inside a wooden box at one end and is closed at the other end. The straight duct is divided into three sections (380 mm, 260 mm and 380 mm respectively) connected by flanges and having an internal square cross-section of 100 mm $\times$ 100 mm. The loudspeaker is driven by a power amplifier (QSC RMX 850a) and the power input is monitored using a digital power meter (GW-INSTEK GPM-8213). To accomodate the change in cross-section from the loudspeaker to the square duct, a conical transition of length 150 mm and made of aluminium is used. The working fluid is air with a mean pressure of 1 bar and a mean temperature of 293.15 K. The oscillating flow is induced in the system using a function generator (RIGOL DG4062) which outputs sine wave signals of the required frequency to the loudspeaker via the amplifier. A photograph of the assembled standing wave thermoacoustic system can be found in appendix A in Fig.A.1 and the accompanying steel wool stack in Fig.A.2

In this study, initially steel wool is placed as uniformly as possible in the middle section of the resonator (260 mm length) by arranging cut pieces of steel wool wires with orientation along the direction of propagation and locking them in position using metal gaskets screwed in at the ends of the section. This functions as a regenerator in the system. The aim here is to identify whether steel wool is viable as a low-cost thermoacoustic stack which can generate significant temperature difference. This is advantageous since random porous materials like steel wool require no complicated fabrication techniques to manufacture and is readily available. Furthermore, it can demonstrate the thermoacoustic heat pumping effect in action in the experimental test rig which will serve as a basis for future studies. A pressure sensor P1 (HUBA-511) is mounted at the closed end (which is the pressure

anti-node) to measure the maximum value of pressure fluctuations in the system. Two thin wire type-K thermocouples are mounted on either ends of the regenerator using thermal paste to measure the temperatures. The thermocouples are calibrated by using a thermal bath available in the laboratory and the pressure sensor was factory calibrated (by ISOLAB (SINGAPORE) Pte Ltd) with voltage for accuracy. Data acquisition cards NI-6211 and NI-9214 (DAQ-1 and DAQ-2) are used to collect the pressure and temperature data respectively.

3.2.2 Temperature and pressure measurement procedure

The system is driven at resonance frequency (determined as later explained in the results section) of 56 Hz and the power input is varied from 5 W to 30 W by adjusting the gain on the amplifier. Measurements are taken after around 90 seconds in each case so that the temperature differences become stable. A set of five readings are taken for each set to ensure repeatability. The estimated uncertainty for pressure and temperature measurements under 4 % and 1 % respectively for the set of measurements. Details of the uncertainty measurement are given appendix C. The hydraulic radius of steel wool in the stack is varied by taking different mass of steel wool inside the same middle section of the resonator. The cases investigated are shown in table 3.1. The hydraulic radius is estimated for each case as derived from its basic definition which is the ratio of volume of gas to the gas-solid contact area ($r_h = \frac{V_g}{A_{gs}}$). Assuming an uniform size for the wires, we can determine the cross-sectional area (A_c) and the perimeter (Π) by measuring the individual wire diameter. Measurements from Leica DCM8 microscope revealed an average wire diameter of 40 μm . For a given mass (m_s) and density (ρ_s) of solid, the gas-solid contact area is given by,

$$A_{gs} = \frac{\Pi}{A_c} \cdot \frac{m_s}{\rho_s} \quad (3.1)$$

The volume of gas can be estimated by subtracting the volume of the solid from the total volume of the stack section (V_{total}). Therefore the hydraulic radius for the sample can be calculated using,

$$r_h = \frac{V_{total} - \frac{m_s}{\rho_s}}{\frac{\Pi}{A_c} \cdot \frac{m_s}{\rho_s}} \quad (3.2)$$

At the resonance frequency of 56 Hz (discussed later in results) the thermal penetration depth (δ_k) for air is approximately 0.35 mm. The ratio of r_h/δ_k is one of the characteristic features of stacks which influences the thermoacoustic energy conversion process [14]. Table 3.1 shows the specifications of each case tested in this study.

Table 3.1: Specifications of steel wool stack.

Case	Mass (g)	$r_h(mm)$	r_h/δ_k
M1	74.6	2.74	7.8
M2	103.1	1.98	5.7
M3	154.4	1.32	3.8
M4	184.2	1.10	3.1
M5	213.7	0.95	2.7

3.2.3 Flow visualization setup

The schematic of the experimental setup for the flow visualization study of parallel plate stack is shown in Fig.3.2. The standing wave thermoacoustic system described in section 3.2.1 is used in this study. Instead of steel wool, a stack of parallel plates made of aluminium is placed at a distance of 450 mm from the starting point of the resonator since in this flow visualization study, the aim is to identify the flow features prevalent in

Figure 3.2: Schematic of PIV measurement setup.

the vicinity of the channels commonly seen in the parallel plate stacks and heat exchangers of thermoacoustic systems. The stack is 100 mm long in the direction of flow propagation and has the same width as the resonator duct. Each plate has a thickness of $\delta=2$ mm and a separation of $d=5$ mm and are held together using four vertical rods at the corner. The distance between the plate center lines is $D=7$ mm and the stack is locked in position using screws on the top side of the duct. Similar to the previous experiment, the maximum pressure amplitude was monitored using pressure sensor P1 connected to the DAQ. The oscillating flow is driven by the loudspeaker using the sine wave signal from the function generator and powered by the amplifier. Flow measurements are made using a time-resolved Particle Image Velocimetry (PIV) system. To visualize the flow field, the resonator is seeded with olive oil particles with a mean diameter of $1\ \mu\text{m}$ from a custom-made aerosol generator. The oil tracer particles are illuminated by a laser sheet generated by a Nd-YAG dual pulse laser (Photonics Industries, DM20-527DH) and divergent optics. Pairs of images were acquired using a high

-speed camera (Phantom V711) with a resolution of 1280×800 pixels. The lens has a field of view of about $24 \text{ mm} \times 15 \text{ mm}$. The region of interest in the flow field is the area in between two plates at the middle. The firing of the dual pulse laser and the camera are triggered in-sync using the synchronizer linked to the function generator. A photograph of the various equipment used can be found in appendix A Fig.A.4

3.2.4 PIV measurement procedure

Depending on the acoustic frequency of the flow, the sample rate for the image capture was varied between 2250 Hz and 2800 Hz. The time interval between the two images in a pair captured by the high speed camera is adjusted by changing the timing between the two laser pulses and was varied between 40-100 μs depending on the particle displacement in the images. The images were analyzed using cross-correlation algorithm in PIVLab code [97–99]. Multiple passes of interrogation windows with an overlap of 50 % and minimum size 32×32 pixels were used on the raw images. The additional settings for PIVLab is given in appendix B Fig.B.1. The uncertainty in the velocity components measurements from pixels in PIV is estimated to be under 0.4 %. Velocity components u and v and the vorticity Ω can be calculated in post-processing.

Several cases of oscillating flow were measured by changing the driving power (5 W to 30 W) and the flow frequency (45 Hz, 56 Hz, 90 Hz and 120 Hz). The parameters for each flow case are given in table 3.2. Re is the Reynolds number based on the phase-averaged mean velocity amplitude $\overline{u_M}$ at $x=17 \text{ mm}$ (obtained later from PIV measurements in the region in front of the stack between the plates) in the far field ($x > D$) and $y \in [-\frac{D}{2}, \frac{D}{2}]$ calculated by

Table 3.2: Flow cases measured in standing wave thermoacoustic system.

Frequency (Hz)	Power (W)	Pressure amplitude (Pa)	Re	KC
56	5	1531.8	262	2.8
56	10	2104.5	369	3.9
56	15	2508.8	436	4.6
56	20	2808.9	506	5.4
56	25	3083.5	558	5.9
56	30	3345.4	621	6.6
45	25	2609	337	4.5
90	25	1468.5	348	2.3
112	25	1974.7	552	2.9

$$Re = \frac{\overline{u_M} d}{\nu} \quad (3.3)$$

Where ν is the kinematic viscosity

KC is the Kuelegan-Carpenter number such that,

$$KC = \frac{\overline{u_M}}{\omega d} \quad (3.4)$$

Where $\omega = 2\pi f$ is the angular frequency. About 5000 pairs of images are captured for each case and the phase averaged results are used to visualize the flow.

3.3 Results and discussion

3.3.1 Resonance frequency

Initially, the resonance frequency of the standing wave thermoacoustic system is determined by measuring the pressure amplitude at the pressure anti-node (P1) against frequencies in the range of 40-100 Hz with

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Figure 3.3: Determination of resonance frequency for standing wave thermoacoustic system.

a constant power input from the loudspeaker (5 W). Fig.3.3 shows that the system undergoes resonance at a frequency of 56 Hz as evident from the peak in the pressure amplitude at the same input power. This is slightly lower than the theoretical prediction of a quarter wavelength resonator which is $f = a/4L \approx 84$ Hz . This deviation is because the standing wave system is driven by a loudspeaker and that affects the frequency of the entire system. We can also see that the pressure amplitude again starts to increase from 90 Hz to 112 Hz here indicating the presence of a possible second resonance peak after 112 Hz. The thermoacoustic system will perform its best at the resonant frequency. Further measurements with steel wool are conducted at this resonant frequency with varying input power from the loudspeaker in the range of 5-30 W.

3.3.2 Thermal characteristics of steel wool stack

Fig.3.4 (a) shows the effect of input power on the maximum pressure amplitude in the thermoacoustic system for the different cases. Naturally, increasing the driving power of the loudspeaker induces stronger pres-

Figure 3.4: Variation of (a) Maximum pressure amplitude and (b) Temperature difference with input power for different mass of steel wool.

sure fluctuations however, the 'damping effect' of steel wool with increasing mass is evident. Consequently case M1 with the lowest mass has the strongest pressure fluctuations for all input powers and case M5 has the weakest respectively. This is directly relatable to the results of temperature difference shown Fig.3.4(b). The temperature difference is highest for M1 with a maximum value of 16.1°C at 30 W input power. While the temperature difference increases with input power for cases M1, M2 and M3, it is fairly stable after $P=10$ W for cases M4 and M5. Their temperatures differences reach maximum values of only 3.9 °C and 3.1 °C at 20 W and 25 W respectively. This may be due to large conduction losses from the hot side to the cold side in these configurations due to their tightly packed nature.

Fig.3.5(a)-(e) show the trends of hot and cold end temperatures with input power of the cases M1-M5 respectively. It is worth noting that for all cases except M1, the temperatures of both hot and cold ends increase due to an increase in total energy in the system. The hot end reaches a very high value of 41.5 °C. However, the cold end temperature for M1 remains relatively stable at around 25-26 °C because of stronger thermoacoustic heat pumping effect described in chapter 2 section 2.1.2. For cases M2 and M3,

Figure 3.5: Variation of T1 (cold end) and T2 (hot end) with input power for different mass of steel wool.

even though the hot ends reach high temperatures of 38.1 °C and 37.1 °C respectively, the proportionate rise of the cold end temperatures to 27.9 °C and 30.6 °C brings down the temperature difference to 10.1 °C and 6.5 °C respectively.

Generally for parallel plate geometry stacks, a r_h/δ_k values of 1 to

2 is recommended [14, 100, 101]. However in the case of steel wool material it is found that much higher ratios lead to better performance. Admittedly, the random nature of such porous materials like steel wool make it difficult to accurately quantify geometrical parameters in a similar manner to more precise geometries like parallel plate configurations. Nonetheless, the results support the use of such porous materials as stacks for standing wave thermoacoustic systems with the added advantage being easy availability, much lower costs and no complicated fabrication techniques necessary.

3.3.3 Flow evolution in parallel plate stack

Figure 3.6: Phase averaged axial velocity $\bar{u}$ at $x=1.5$ mm and $y=0$. The temporal step between the scatters is $0.02T$. The solid lines represent the local linear fitting and tagged with local slopes $\partial\bar{u}/\partial(t/T)$.

To identify the typical flow characteristics seen behind the stack in the standing wave thermoacoustic system, in this section the results with case $f=56$ Hz (which is the resonant frequency determined in section 3.3.1) and $P=25$ W are discussed in detail. The flow is divided into 50 phases or instances with $t/T=0$ defined on the basis of phase averaged axial velocity component $\bar{u}$ at $x=1.5$ mm $y=0$ as shown in figure 3.6. Here T is the

time period of the oscillating flow and t is the time. For convenience in this discussion, the parts of the flow with a positive $\bar{u}$ is defined as the 'ejection' stage when flow direction is out of the channel and that with a negative $\bar{u}$ is defined as the suction stage when the flow direction is into the channel. Both ejection and suction stages have periods of acceleration and deceleration as seen from the slopes indicated by solid blue lines. In the accelerating period of the ejection stage the slope is nearly constant with a value of $\partial\bar{u}/\partial(t/T) \cong 33.7 \text{ m/s}^2$. This is followed by inflections which are evidence of strong vorticity fields in the plate vicinity which will be discussed subsequently. The two peaks indicate the rotation of vortices affecting the local velocity. During the deceleration stage, a steep slope in the ejection stage is followed by a more gradual slope in the suction stage. This indicates that the suction stage ($\approx 53\%T$) is slightly longer than the ejection stage ($\approx 47\%T$) in this case. The maximum velocity in the ejection stage is $\bar{u} = 4.0 \text{ m/s}$ and in the suction stage is $\bar{u} = -4.4 \text{ m/s}$ respectively.

Figure 3.7: Phase averaged vorticity contours at $t/T=0$ to $t/T=0.9$ in the case of $f=56 \text{ Hz}$ and $P=25 \text{ W}$ with superimposed contour lines at $\lambda_2=-10^4$ with cross symbols showing the position of local minimum of λ_2 .

Fig.3.7 shows the phase averaged results of vorticity field at ten selected time instants from $t/T=0$ to $t/T=0.9$. To identify the vortex regions and distinguish them from shear layers, the λ_2 criterion is used [102]. Con-

Figure 3.8: Phase averaged velocity vector maps at $t/T=0$ to $t/T=0.9$ in the case of $f=56$ Hz and $P=25$ W.

four lines of $\lambda_2 < -10^4$ are superimposed on the color coded vorticity contours and the local minimum values of λ_2 are marked by a cross symbol which represents the vortex cores. The rectangular grey blocks on the y axis represent the ends of the plates. Additionally, Fig.3.8 represents the corresponding velocity vector map (colored by magnitude) at the same time instants.

At the beginning of the cycle at $t=0$, the velocity magnitudes are relatively low especially near the plate ends and is just about to switch directions. The velocity magnitude aligned with the channel center is clearly larger because of transverse area contraction when entering the channel. At $t=0.1T$ the flow is clearly in the ejection stage and the flow expands into the open area from the channels. The difference in velocity magnitudes between the channel or stack outlet and the remaining parts of the flow generates vorticity. A pair of counter-rotating vortices (with opposite signs of vorticity) are formed attached to the end of the plates. These vortices which are formed by the rolling up of shear layers due to sudden expansion of the transverse area of the channel are referred to as dominant vortices. Accompanying them, are weaker vorticity concentrations of opposite sign and that are formed during the previous cycle and exiting from the channel are called

residual vortices. Details regarding the formation of residual vortices from within the plate channels are discussed later in detail in chapter 5 when the entire flow field surrounding the stack plates is visualized using the CFD model. At $t/T=0.2T$ both dominant and residual vortices are seen emerging from the plates and are denoted as ‘DV’ and ‘RV’ respectively in Fig.3.7. The dominant vortices then elongate and grow as the flow progresses. From $t=0.4T$ to $t=0.6T$ we can see a pattern of alternate vorticity concentrations forming in the flow. As clearly seen in $t=0.4T$, this alternating ‘vortex street’ is a result of vortex shedding phenomenon from elongated dominant vortices and the weaker residual vortices remaining in the flow field moving downstream with the mean flow. The vortex breakup could be induced as a result of the pressure gradient and general instability due to high vorticity concentrations when the flow accelerates [51,103].

When the flow enters the suction stage from $t=0.5 T$, the strength of the vorticity concentrations start to decrease. The detached vortices start reversing their direction of propagation and the dominant vortices attached to the ends of the stack start to shrink. At $t=0.6T$ the dominant vortices that were attached to the stack in the first half of the cycle ($t=0T$ to $t=0.4T$) have almost completely disappeared, although the detached remnants from the previous cycle still remain in the flow along with the residual vortices. Again, due to the increase in velocity near the stack due to reduction in transverse area, shear layers of the opposite sign start to form and generate dominant vortices of the opposite sign attached to the stack plates as seen from $t=0.7T$. It is also notable from the velocity map that the mean velocity in these parts of the suction stage and higher than their ejection counterparts (comparing $t=0.3T$ and $t=0.8T$). The remnant vortices from the ejection stage then merge with the newly formed shear layer as evident in $t=0.9T$. This completes one full cycle and the same pattern repeats.

3.3.3.A Effects of frequency on flow behavior

Figure 3.9: Phase-averaged vorticity contours with superimposed contour lines of $\lambda_2 < -10^4$ and vortex cores for (a) $f=45$ Hz (b) $f=56$ Hz (c) $f=90$ Hz and (d) $f=112$ Hz, at instants from $t/T = 0$ to $t/T = 0.8$, respectively.

In this section, we examine the flow evolution in cases driven by the different loudspeaker frequency but with the same input power. Fig.3.9 shows the phase-averaged vorticity contours with superimposed contour lines of $\lambda_2 < -10^4$ from $t/T=0$ to $t/T=0.8$ to identify the vortices and their cores. Looking at five similar time instances from $t=0.2$ to $t=0.4T$ during ejection, we observe that the dominant vortex shedding is predominantly visible only at $f=56$ Hz. In all other cases, the vortex street is formed from the detached residual vortices which is evident from their opposite signs with respect to the attached dominant vortices. The residual vortices blown out from the channel during ejection and when the flow reverses they dissipate in strength

and merge back with newly formed dominant vortices. This suggests that the vorticity concentration is largely dependent on the amplitude of pressure fluctuations (see table 3.2). Consequently, the adverse pressure gradient in the oscillating flow is a key factor in the breakup of the dominant vortices attached to the stack plates.

Figure 3.10: Phase-averaged kinetic energy K in the control area ($x \in [0.25, 18]mm$ and $y \in [-\frac{D}{2}, \frac{D}{2}]$) for $f=45$ Hz, $f=56$ Hz, $f=90$ Hz, and $f=112$ Hz, respectively.

The kinetic energy of the flow plays a key role in determining the efficiency of thermoacoustic energy conversion process [104]. This is because, the kinetic energy is dissipated as heat within the viscous penetration depth of the stack during each cycle [47]. For comparing the different flow frequencies, the phase-averaged kinetic energy, $K = (\sum(u^2 + v^2)\Delta S)/S$ (S is the control area), is used to show the magnitude of the induced kinetic energy in the cycle, shown as Fig.3.10. The control area for comparison is $x \in [0.25, 18]mm$ and $y \in [-\frac{D}{2}, \frac{D}{2}]$, which can cover the dominant area of fluctuations in the flow field. The variation of kinetic energy for each case is characterized by two peaks, one during ejection and another during suction stages. During flow reversal, the kinetic energy is low as the base flow ve-

locity also reaches minimum and changes direction. The kinetic energy for $f=90$ Hz is the minimum owing to its extremely low pressure amplitude as seen in table 3.2. However, $f=112$ Hz, while in-fact being the second harmonic has a relatively lower pressure amplitude due to harmonic suppression since the entire resonator cross-section is not independent of x . Despite its low pressure amplitude, $f=112$ Hz has considerable kinetic energy because of its relatively higher frequency value. This means that both frequency and pressure amplitude play a key role in determining the induced kinetic energy. It is also worth noting that the peak value of kinetic is higher in the suction stage than in the ejection stage. This is because the mean velocity of the base flow is higher during suction than ejection which can be seen in Fig.3.8 by comparing $t/T=0.8$ and $t/T=0.3$. This maybe a result of the position of the stack in the resonator.

3.3.3.B Effect of driving power on flow behavior

In this section, we compare the flow field at different power input levels for the resonant frequency $f=56$ Hz. Fig.3.11 shows the phase averaged vorticity contours with superimposed contour lines of $\lambda_2 < -10^4$ with cross symbols indicating the local minimum λ_2 representing vortex cores. As observed in the case of frequency variations, here also there seems to be a threshold pressure amplitude above which the dominant vortices begin to shed. Since the loudspeaker power input directly increases the the pressure amplitude, we can see stronger vorticity concentrations as we move from $P=5$ W to $P=30$ W. The cases $P=5$ W and 10 W, have the dominant vortex remaining attached to the plate ends during ejection stage. Here again, the residual vortices are the major components of the vortex street because of the opposite sign of vorticity concentrations. It is noted that the size of the dominant vortex during ejection also enlarges with increase in power for these cases. This can be seen by observing $t/T=0.2$ to $t/T=0.4$. For $P=$

Figure 3.11: Phase-averaged vorticity contours with superimposed contour lines of $\lambda_2 < -10^4$ and vortex cores for $f=56\text{ Hz}$ and (a) $P=5\text{ W}$, (b) $P=10\text{ W}$, (c) $P=15\text{ W}$, (d) $P=20\text{ W}$, and (e) $P=30\text{ W}$, at instants from $t/T = 0$ to $t/T = 0.8$, respectively.

15 W, it seems that the dominant vortices adjacent to the plate center-line grow very large in size and essentially break off the smaller two dominant vortices. For $P=20\text{ W}$ and $P=30\text{ W}$, the dominant vortex also breaks up during ejection. This is confirmed by the presence of same sign vorticity concentrations as well as a reduction in the length of the dominant vortices as seen from $t/T=0.2$ to $t/T=0.4$.

Figure 3.12: Phase-averaged kinetic energy K in the control area ($x \in [0.25, 18]$ mm and $y \in [-\frac{D}{2}, \frac{D}{2}]$) for $f=56$ Hz and P from 5 W to 30 W respectively.

Comparing the average kinetic energies for the different power inputs in Fig.3.12 it is obvious that an increase in power input leads to an overall higher kinetic energy output. As in the previous case, the kinetic energy is almost zero during flow reversal. Interestingly, the kinetic energy peaks of the suction stage are not always higher than that in the ejection stage. This means that the mean flow has a higher magnitude in the suction stage only for $P=15$ W, 20 W and 25 W. It is speculated that at the lower power inputs, the pressure gradients might not be strong enough to counteract any geometrically induced adverse pressure gradients in the suction stage. However, this does not explain the lower kinetic energy of the suction stage at $P=30$ W, granted the difference is relatively smaller here. This may be explained by noting that at $P=30$ W, the vorticity concentrations are very large and there is also dominant vortex shedding. These form a part of the vortex street and during flow reversal travel back to the stack plates and cause significant flow retardation because of retaining relatively higher amounts of vorticity.

3.4 Summary

A thermal study of the standing wave thermoacoustic system was conducted using porous steel wool stack to generate temperature difference similar to a heat pump. It is clear that the ratio $r_h\delta_k$ has a significant impact on the thermoacoustic energy transfer process. From the tested mass samples, it was found that a r_h/δ_k value of 7.8 had the highest temperature difference of 16.1 °C. This shows that porous materials like steel wool can be good candidate to use as stacks in thermoacoustic systems with high heat capacity and low conduction losses at optimum r_h/δ_k . In comparison to other stack geometries like, parallel plate stacks, spiral stacks and pin array stacks [105], these porous materials can be easily obtained for a very low cost and require no complicated fabrication techniques . Therefore porous material based thermoacoustic systems can be a viable option for cost-effective cooling or heating. The generated temperature differences could be larger and sustained with added heat exchangers to absorb and reject heat in future investigations.

Secondly, in this study, the flow structure evolution behind the stack of a standing wave thermoacoustic system was observed and analyzed. The one common feature of all cases investigated with varying frequency and varying power is the symmetry of the flow structures with respect to the center-line between the plates. This is expected because Re and KC values are in the order of $O(10^2)$ and $O(10^0)$, respectively [51,61,106]. The pressure amplitude is found to be a key threshold criterion which determines whether there is shedding of dominant vortices attached to the stack plates or not. Vortex shedding and the traversal of detached as well as residual vortices following the mean flow are characteristic features for all cases studied. The evolution of kinetic energy is dependent on both frequency and pressure fluctuations. At lower pressure amplitudes, geometrically induced gradients

may also affect the peaks in the suction and ejection stage. Very strong vortices induced during ejection stage of the flow as seen in the case of $P=30$ W could indeed affect the kinetic energy variation in the suction flow.

Overall this study establishes a baseline for investigating the flow around the stack using PIV in a straight-tube and will help expand the study to more complex geometries. It was found that the vortex structures generated could indeed play a role in the heat and mass transfer by convection between the stack and hot and cold heat exchangers in practical systems. The length and geometry of the heat exchangers could be designed in such a way to make use of the extended and traveling vortices. It would also help to design the systems with specific limits of pressure amplitude and ranges of frequency in mind to avoid unnecessary vortex shedding and very high peaks in kinetic energy which may dissipate heat and generate losses. Therefore, the flow behavior can either enhance or degrade the thermoacoustic phenomenon and its understanding is essential particularly at high pressure amplitudes of operation. Since looped geometries are also seen in many practical thermoacoustic devices, in the following chapter, we will explore the vortex generation phenomenon in a looped thermoacoustic system where the stack is contained inside loop attached to the resonator.

Chapter 4

Experimental investigation of oscillating flow in a looped thermoacoustic system^{*}

4.1 Background

Building on the experimental and statistical techniques and the knowledge we gained from studying the standing wave thermoacoustic system from the previous chapter, here we introduce a more complicated looped thermoacoustic system which was designed and fabricated in-house. Here, a part of the straight-tube resonator from the standing wave system is replaced with a closed loop and the stack is enclosed within it.

The flow features in the stacks of looped thermoacoustic systems have been rarely studied even though similar configurations are seen in a

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variety of thermoacoustic devices showing good potential as mentioned in chapter 2 section 2.2. Similar to the flow measurement using PIV technique in the previous chapter, here we investigate the oscillating flow behavior behind the stack in a closed loop. This will help better understand the underlying flow features which will play a significant role to improve design choices for looped thermoacoustic systems. It is also worth mentioning that a thermal study was not conducted here because the system was designed with visualizing the flow morphology around the stack in mind. The parallel plate stack configuration is again used here since the aim is to characterize the flow behavior in the channels using PIV which is relevant to stack and heat exchanger designs in thermoacoustic systems. A random porous material like steel wool would be difficult to characterize even though it may generate better temperature differences. Time-resolved PIV technique is used to capture the development of phase-averaged oscillating flow fields. Additionally, in this chapter, due to the more complex nature of the looped thermoacoustic system, proper orthogonal decomposition (POD) analysis is applied to identify non-periodic flow features which may impact the heat and mass transfer mechanisms. Modal reduction of the oscillating flow will help distinguish these smaller scale fluctuations from the dominant oscillating flow. The spatio-temporal relationships between mode 1 (dominant flow structure) and higher modes (shear layers and vortices) are investigated.

4.2 Experiments

4.2.1 Experimental setup

The schematic of the experimental setup is shown in Fig.4.1. The looped thermoacoustic system is a modification of the standing wave thermoacoustic system in chapter 3. The straight resonator duct of length 0.38

Figure 4.1: Schematic of the experimental setup.

m is kept intact and attached to the loudspeaker on one end. However, the opposite end is now attached to square looped tube with a total length of 1.6 m. The internal cross-section throughout the tubes kept the same (100×100 mm). The parallel plate aluminum stack is used here with 14 plates of length 100 mm and thickness 2 mm and the center line distance $D=7$ mm. The working fluid is air at atmospheric pressure and at a mean temperature of 293.15 K. The function generator coupled with the amplifier and loudspeaker is used to drive the oscillating flow. Here, five pressure sensors are mounted at different locations on the duct to measure the pressure amplitude and the data is collected using the DAQ. A photograph of the looped thermoacoustic system is shown in appendix A Fig.A.3

The time-resolved PIV system used in chapter 3 is again used here to visualize the flow field. Micron sized oil tracer particles are injected into the flow using the aerosol generator and illuminated by the laser sheet. The high speed camera captures the particle images at resolution of 1280×800 pixels in a field of view of $24 \text{ mm} \times 15 \text{ mm}$. The synchronizer connected to the second channel of the function generator triggers both the laser firing and image capture from the high speed camera.

4.2.2 Experimental procedure

Table 4.1: Flow cases measured in looped thermoacoustic system.

Frequency (Hz)	Power (W)	Pressure amplitude (Pa)	Re	KC
45	5	1181.3	107	1.4
45	10	1648.9	152	2.0
45	15	2045.2	182	2.4
45	20	2303	211	2.8
45	25	2499	232	3.1
45	30	2666.3	259	3.4
65	25	1386.8	168	1.5
90	25	883.1	146	1.0
120	25	1519.6	315	1.6

In this study , several cases of oscillating flow are measured with various frequencies, 45 Hz, 65 Hz, 90 Hz and 120 Hz and driving powers ranging from 5 W to 30 W. The parameters for each case are given table 4.1. Re and KC values are calculated based on the mean velocity at $x=17$ mm in the far field ($x>D$) and $y \in [-\frac{D}{2}, \frac{D}{2}]$ (the same as chapter 3 section 3.2.4).

Based on the flow frequency, the sampling rate is varied between 2250 Hz and 3250 Hz. The time interval between two images in a pair is adjusted from 40-100 μm depending on particle displacement. PIVLab [97–99] with cross-correlation algorithm is again used to analyze the image pairs and calculate the flow field. Multiple interrogation window sizes up to a minimum of 32×32 pixels and 50% overlap are used. 5000 pairs of images are analyzed to obtain the time resolved and phase-averaged flow patterns. The uncertainty in the value of pressure amplitude from the pressure sensors is under 4 % and that of velocity measurement from PIV is under 0.4 %.

In addition to statistical analysis carried out in the previous chapter, here proper orthogonal decomposition (POD) is applied to post-process

the PIV results. POD method can extract physical modes from the main flow based on energy optimization and decompose the flow into a combination of coherent flow structures [107]. The POD algorithm is a useful tool in fluid flow analysis. [108–111]

The temporal snapshots from the time-resolved PIV can be arranged into a matrix of the form,

$$X = [x(t_1) \ x(t_2) \ \dots \ x(t_m)]. \quad (4.1)$$

Here, $x(t_i)$ is the reshaped column vector of the original snapshot matrix $u(t_i)$ and m is the number of snapshots.

The core idea behind POD is to find the eigenvectors ϕ_i (modes) that can represent the original data in an optimal way and the corresponding eigenvalues λ_i (energy) from

$$K\phi_i = \lambda_i\phi_i, \quad (4.2)$$

where K is given by

$$K = XX^T. \quad (4.3)$$

By using Singular Value Decomposition (SVD),

$$X = \Phi\Sigma\Psi^T, \quad (4.4)$$

Where the left and right singular vectors Φ and Ψ , and diagonal matrix Σ of the singular values ($\sigma_1, \sigma_2, \dots, \sigma_m$) can be solved. Φ represents the modes matrix $[\phi_1 \ \phi_2 \ \dots \ \phi_m]$, Ψ contains the snapshots coefficients $[\psi_1 \ \psi_2 \ \dots \ \psi_m]$, and the singular values are related to the eigenvalues such that

$\sigma_i^2 = \lambda_i$. Thus, for a complex flow field, the original snapshot at t_i can be reconstructed as the sum of a time-averaged flow field $\bar{u}$ and a combination of modes with their corresponding temporal coefficients

$$u(t_i) = \bar{u} + \sum_{i=1}^m \sigma_i \phi_i \psi_i^T. \quad (4.5)$$

4.3 Results and discussion

4.3.1 Resonant frequency

Similar to the standing wave thermoacoustic system, here also the resonance frequencies of the system are determined by plotting the maximum pressure amplitude at P4 against the frequencies in the range of 35-150 Hz. From Fig.4.2(a), we can find two peaks with the highest magnitude of pressure amplitude for the same input power ($P=10$ W) which shows that 45 Hz and 120 Hz are the first and second resonance frequencies respectively.

Figure 4.2: (a) Pressure amplitude at different frequencies measured at point 4; (b) the pressures at different locations in the looped thermoacoustic system ($f = 45$ Hz and $P = 25$ W).

Additionally, the phase-averaged pressure data at the five locations in the looped thermoacoustic system for the case $f=45$ Hz and $P=25$ compared with the input signal from the function generator is shown in

Fig.4.2(b). The pressure signals have around 90° phase lag with the input signal. This may be due to the natural resonance characteristics of the loudspeaker system. The maximum value of pressure is marked by a solid dot on each curve and it is obvious that there is phase lag between the different locations. It is observed that this lag is a function of the distance from the loudspeaker.

4.3.2 Flow structure evolution

Figure 4.3: The phase-averaged velocity component $\bar{u}$ at the location $x = 1.5 \text{ mm}$ and $y = 0$. The temporal step between scatters is $0.02 T$. The solid lines present the local linear fitting, tagged with local slopes $\partial\bar{u}/\partial(t/T)$.

To identify the typical flow pattern characteristics in the looped thermoacoustic system, the case with $f=45 \text{ Hz}$ (the first resonance frequency) and $P = 25 \text{ W}$ is discussed here in detail. For this study, $t = 0$ is defined as the zero phase angle of the temporal coefficient of the POD mode 1 (discussed in POD analysis later). In Fig.4.3, the phase-averaged velocity component $\bar{u}$ at the location $x = 1.5 \text{ mm}$ and $y = 0$ is plotted. The

velocity magnitude varies from $\bar{u}_{min} = -2.2 \text{ m/s}$ to $\bar{u}_{max} = 2.0 \text{ m/s}$ periodically. As per previous conventions, the period with positive $\bar{u}$ is defined as the ejection stage, and that with negative $\bar{u}$ is defined as the suction stage. Inflections are again observed in the ejection stage of vortical rotation near the stack ends. This can affect the local velocity in the region leading to two peaks as seen in the Fig.4.3. The bottom of the velocity curve is flat due to the reduced area of the transverse section. As highlighted by the solid blue lines, the slope of the velocity curve midway during acceleration is almost constant with a value $\partial\bar{u}/\partial(t/T) \approx 12.7 \text{ m/s}^2$. However, during deceleration, the slope has a sudden drop between $t/T = 0.52 \sim 0.56$, from $\partial\bar{u}/\partial(t/T) \approx -8.2 \text{ m/s}^2$ to $\partial\bar{u}/\partial(t/T) \approx -17.9 \text{ m/s}^2$. Since the slopes during the ejection stage is smaller than that during the suction stage, the duration of the ejection stage ($\sim 58\% T$) is longer than that of the suction stage ($\sim 46\% T$).

Fig.4.4 shows the phase-averaged flow field at ten phases or instances, from $t = 0$ to $t = 0.9T$. As in chapter 3 section 3.3, vortices are distinguished from shear layers using λ_2 criterion with superimposed contour lines of $\lambda_2 < -10^4$ and the position of local λ_2 minimum marked with cross symbols indicating vortex cores. . The gray blocks on the y -axis indicate end of the stack plates. The corresponding velocity vector maps are shown directly below the vorticity contours. Vectors are colored to highlight their magnitude.

In Fig.4.4(a), at $t = 0$, the overall velocity magnitude is very small. Parts of flow are still going into the channel due to hysteresis from the previous cycle. At $t = 0.1 T$, the flow switches direction and enters the ejection stage. The overall velocity magnitude is still quite small while the velocity near the outlet of the channel is larger. Thus, shear layers are formed and vorticity is generated. At $t = 0.2 T$, pairs of counter-rotating vortices

Figure 4.4: Phase-averaged vorticity contours (top) and velocity vector maps (bottom) at (a): $0 \sim 0.4 T$ and (b): $0.5 \sim 0.9 T$ in the case $f = 45$ Hz and $P = 25$ W. Superimposed contour lines represent the level of $\lambda_2 = -10^4$ and cross symbols show the position of local minimum λ_2 .

form at the end of plates. These are the dominant vortices identified in the previous study which are formed due to rolling up of shear layers as seen in the velocity vector map. The vortices with weaker vorticity concentrations are the corresponding residual vortices which are formed in the previous suction stage and ejected from with the plate channels. From $t = 0.2 T$ to $t = 0.4 T$, the dominant vortices elongate while they keep attaching to the end of the plates. Meanwhile, more residual vortices of opposite vorticities exit the channel. Here, we observe only the residual vortices moving downstream due to entrainment by the mean flow. The dominant vortices remain attached to the stack plates throughout the entire ejection stage and there is no vortex breakup unlike the previous study.

In Fig.4.4(b), as a result of the deceleration of the base flow, the size of dominant vortices shrink slightly at $t = 0.5 T$ and the transport of residual vortices slow down. At $t = 0.6 T$, near the stack, the strength of dominant vortices reduce dramatically due to the loss of vorticity feeding from the shear flow. Remnants of residual vortices are still observable away from the stack but they become much weaker and their flow is reversed. From $t = 0.6 T$ to $t = 0.9 T$, dominant vortices generated in the previous cycle which were attached to the plates ($t = 0 \sim 0.4 T$) have fully dissipated. As a result of the reduction of the transverse section area caused by the stack, the velocity magnitude of the flow. Newly formed shear layers in the reversed flow generate dominant vortices with opposite signs as that of the previous cycle. The retreating weaker residual vortices then merge with this new shear layer. From the velocity vector map, it is also known that the mean velocity magnitude in the far field ($x > D$) is larger during the suction stage than that during the ejection stage. This completes one full cycle and the flow structure repeats.

4.3.3 Effect of flow frequency

Figure 4.5: Phase-averaged vorticity contours with superimposed contour lines of $\lambda_2 < -10^4$ and vortex cores for (a) $f = 45 \text{ Hz}$, (b) $f = 65 \text{ Hz}$, (c) $f = 90 \text{ Hz}$, and (d) $f = 120 \text{ Hz}$, at instants from $t/T = 0$ to $t/T = 0.8$, respectively.

In this section, the flow frequency is varied between $f=45 \text{ Hz}$, $f=65 \text{ Hz}$, $f=90 \text{ Hz}$ and $f=120 \text{ Hz}$ keeping a constant power input from the loud-speaker at $P=25 \text{ W}$. More details for the cases are given in table 4.1. Fig.4.5 shows a comparison of the vorticity contours with varying frequency from phases $t/T = 0$ to $t/T = 0.8$. Contour lines of $\lambda_2 < -10^4$ are superimposed to identify vortices and corresponding vortex cores (minimum λ_2) are marked by a cross. All cases have similar flow patterns in this study. During the ejection stage, shear flow generated at the outlet of the stack rolls up into dominant vortices and remain attached to the plates throughout the

cycle. Meanwhile, the residual vortices that are left behind follow the base flow. During the suction stage, dominant vortices are pulled inward and the residual vortices become weaker and also traverse back near the stack. The vortices generated during the ejection stage dissipate locally but the suction induced shear flow also generates a smaller vortices near the stack entrance. This can be observed from $t/T=0.6$ to $t/T=0.8$.

With changing acoustic frequency, the vorticity concentrations differ significantly and decreases in the order $f=45$ Hz, $f=120$ Hz, $f=65$ Hz and $f=90$ Hz. This is expected because 45 Hz and 120 Hz are the first and second resonance frequencies respectively with much higher pressure amplitudes than the rest. As seen in Fig.4.5(a), $f=45$ Hz has the strongest and largest vorticity concentrations. It is also noted that the residual vortices are transported much further into the flow field than the rest of the cases. Since the entire resonator cross-section is not independent of x , harmonic suppression leads to resonance at non-integral multiples of 45 Hz. Consequently, the second harmonic frequency $f = 90$ Hz has the lowest amplitude and therefore the weakest and smallest dominant vortices as clearly seen in Fig.4.5 (c). The residual vortices stay very close to the stack.

Fig.4.6 shows the variation of phase averaged kinetic energy K defined in the previous chapter in the same control area S which is $x \in [0.25, 18]mm$ and $y \in [-\frac{D}{2}, \frac{D}{2}]$ and covers the dominant area of fluctuations. The induced kinetic energy will be dissipated as heat from the stacks viscous boundary layers. Similar to the previous study, the kinetic energy curve shows two peaks, one during ejection stage and one during suction stage, with the latter being higher due to larger velocity of base flow during suction as evident from Fig.4.4 by comparing $t/T=0.8$ and $t/T=0.3$. During transition between the two stages, the kinetic energy is very low, due to the low velocity of the base flow (see in Fig.4.4). Interestingly, case $f = 120$ Hz

Figure 4.6: Phase-averaged kinetic energy K in the control area ($x \in [0.25, 18]mm$ and $y \in [-\frac{D}{2}, \frac{D}{2}]$) for $f = 45 \text{ Hz}$, $f = 65 \text{ Hz}$, $f = 90 \text{ Hz}$, and $f = 120 \text{ Hz}$, respectively.

has a much higher averaged kinetic energy during the entire cycle than the remaining cases. This may be because, the kinetic energy is also heavily dependent on the speed of the fluctuations or in other words the frequency in addition to the pressure amplitude. Since $f=120 \text{ Hz}$ is the second resonance frequency, it has considerably large pressure amplitudes and is also a high frequency flow. Therefore it has the highest kinetic energy values compared to the rest. As expected, the average kinetic energy for $f = 90 \text{ Hz}$ is always the minimum. For each individual evolution curve, the averaged kinetic energy is higher during the suction stage than that during the ejection stage. This is because that the mean velocity of the base flow is larger during the suction stage for all cases of flow frequency.

Figure 4.7: Phase-averaged vorticity contours with superimposed contour lines of $\lambda_2 < -10^4$ and vortex cores for $f = 45 \text{ Hz}$ and (a) $P = 5 \text{ W}$, (b) $P = 10 \text{ W}$, (c) $P = 15 \text{ W}$, (d) $P = 20 \text{ W}$, and (e) $P = 30 \text{ W}$, at instants from $t/T = 0$ to $t/T = 0.8$, respectively.

4.3.4 Effects of driving power

In this section, the flow evolution is compared at the resonance frequency $f=45$ Hz with the input driving power of the loudspeaker varying from $5W$ to $30W$. Fig.4.7 shows the comparison of the vorticity contours with varying driving power from instances $t/T = 0$ to $t/T = 0.8$. Contour lines of $\lambda_2 < -10^4$ are superimposed to identify vortices and corresponding vortex cores (minimum λ_2) are marked by a cross. As expected, increasing the input power leads to larger periodic fluctuations of the flow field. Therefore successively larger and stronger vortices are observed with increasing power. The dominant vortices remain attached to the plate for all power inputs indicating that the threshold pressure amplitude for vortex instability and breakup has not been reached unlike the case for the standing wave system. In the case of residual vortices beside their increase in size and vorticity magnitude, their displacement across the flow field also increases. As a result, more number of residual vortices are observed. As observed for all cases in both standing wave and looped thermoacoustic system, more power means that the higher vorticity concentrations can sustain longer in the flow field and even during flow reversal. This helps the generation of more residual vortices in the next cycle.

From the kinetic energy evolution curves in Fig.4.8, it is obvious that the peak values of phase average kinetic energy also increases with the input power. However the increment among the higher power cases $P \geq 20W$ is less than jump between the lower power cases. During transition between the ejection stage and the suction stage, $t/T \in [0, 0.1]$, $t/T \in [0.4, 0.6]$ and $t/T \in [0.9, 1]$, the values of K are almost the same in cases with $P \geq 20W$. Since the velocity of the base flow is very low during these periods the major contribution of kinetic energy should come from the vortical structures. This implies that the overall strength of vorticities may be at the same level during

Figure 4.8: Phase-averaged kinetic energy K in the control area ($x \in [0.25, 18]mm$ and $y \in [-\frac{D}{2}, \frac{D}{2}]$) for $f = 45 Hz$ and P from $5W$ to $30W$, respectively.

these periods for cases with $P \geq 20W$. Therefore, the difference in peak values among the cases should be primarily because of differences in base flow rather than the vortices. Accordingly, the cases with $P < 20W$ should have comparable contributions to the kinetic energy from both the base flow and the local vortical structures during the entire cycle. As discussed in chapter 3 section 3.3, both the pressure fluctuations and geometrically induced pressure gradients can influence the mean velocities of the base flow. For the case of the looped thermoacoustic system, it is observed that the mean velocity of the suction flow is always greater than the ejection flow. This means that the peak kinetic energy is always higher during suction stage for this system in the range of driving powers tested.

4.3.5 POD analysis

Due to the complex geometry of the looped thermoacoustic system proper orthogonal decomposition (POD) is used in this study to identify the most energetic modes of the fluctuating flow behind the stack. Since POD analysis is sensitive to the number of snapshots [110], a series of analysis with various number of snapshots are compared initially to obtain the criterion for a converged result. Based on the eigenvalues of the modes, the number of snapshots $N=2500$ is necessary so that the eigenvalue variation of mode 1 is less than 1% when an additional 500 snapshots are included in the analysis. Therefore, the number of snapshots used in this study with $N=5000$ will be sufficient for a reliable POD result.

Figure 4.9: The energy distribution of modes (solid markers) and the corresponding cumulative summation (open markers) for cases with the same power $P=25$ W.

First off, the energy fraction of POD modes are calculated to understand the kinetic energy distribution among modes. For each data set, the POD analysis can obtain 5000 modes. In Fig4.9, the energy distribution of the first 500 modes and the corresponding cumulative summation are plotted for cases with the same power $P=25$ W at various frequencies. Here, the energy fraction is defined as $\lambda_i / \sum_{n=1}^N \lambda_n$ for the i -th mode and the

cumulative summation is $\sum_{m=1}^i \lambda_m / \sum_{n=1}^N \lambda_n$ at mode i . Above the break in the vertical axis, the scale is linear while the break, the scale is in common logarithm. Remarkably, the first mode in all cases have an energy contribution which more than 95% of the total energy. Then the energy fraction decreases rapidly with the increase of mode index. It is worth pointing out that the energy distributions for the different cases are slightly different. For case 45 Hz, the energy fraction of mode 1 is 95.9% while it exceeds 99% in the other three frequencies. Accordingly, higher order modes in case 45 Hz contribute more kinetic energy. This implies that fluctuations represented by higher order modes are more significant in case 45 Hz. Although mode 2 in case 45 Hz only has 1.34% of the total kinetic energy, it is still larger than those of subsequent modes. The energy fractions of mode 3 to mode 5 are in the same level indicating that they may represent the small scale fluctuations of the same basic structures (discussed subsequently). This phenomenon is observed in other cases as well.

Figure 4.10: The energy distribution of modes (solid markers) and the corresponding cumulative summation (open markers) for cases at the same frequency $f=45$ Hz. The inset graph is the energy fractions of mode 1 versus the input power.

Fig.4.10 shows the comparison of energy distributions among cases with different input power at $f=45$ Hz. As magnified in the inset graph, the energy fraction of mode 1 is non-monotonic with the input power. The

minimum energy fraction of mode 1 appears in case $P=20\text{ W}$, around 95.6%. Case $P = 5\text{ W}$ has the maximum energy fraction of mode 1, almost 98.6%. This non-monotonic tendency implies that the flow fluctuations of higher order modes are enhanced by increasing the input power up-to $P=20\text{ W}$. Once the input power is over $P=20\text{ W}$, the higher order oscillations are suppressed slightly. Therefore the cumulative summation of case $P = 20\text{ W}$ is always the smallest, while that of case $P = 5\text{ W}$ is the largest. It is also observed that the energy fraction of mode 3 is very close to that of mode 2 in case $P=5\text{ W}$ which may indicate that they are the fluctuations of same flow structure. For the remaining cases, mode 3 is most likely the same flow structure fluctuations as the subsequent modes.

Figure 4.11: (a-e) vorticity contour and (f-j) velocity vector map of mode 1 to mode 5, respectively. ($f=45\text{ Hz}$ and $P=25\text{ W}$).

Based on the energy distribution of the POD modes, the corresponding fluctuations of the flow structures for the first 5 modes in the case $f=45\text{ Hz}$ and $P=25\text{ W}$ are shown in Fig.4.11. For this comparison, the vorticity and velocity magnitudes are normalized by their maximum values in each mode. According to velocity vector map, it is known that mode 1 represents the periodic oscillation of the base flow. The pairs of vorticity concentrations at the end of plates show the ejection/suction induced shear

flow due to change in available transverse section area. This makes sense because the shear flow is coherent to the base flow. This result is also consistent with observations from Fig.4.4, Fig.4.5 and Fig.4.7 that show that vorticities of the shear flow alternate their sign from the ejection stage to the suction stage. From the vorticity contour and the velocity vector map, it is obvious that mode 2 shows the magnitude fluctuations of vortical flow structures. So mode 2 indeed corresponds to the formation and dissipation of the dominant and residual vortices. For mode 3, the velocity vector map shows two areas with relatively large scales of velocity fluctuations behind plates. Looking at the vorticity contour, these two structures seem to represent the oscillation of dominant vortices in the y direction. The vorticity contour in the remaining area shows fluctuations of vortical flow structures, indicating the propagation of residual vortices. Mode 4 and mode 5 also consist mainly of two fluctuations similar to mode 3 namely, the vertical oscillation of dominant vortices and the fluctuation of residual vortices. However, there are also displacements between vorticity concentrations, and their signs are opposite. From these three vector maps, areas with significant velocity oscillations are also shifted and velocity vectors are in opposite directions. They are typical modes showing the propagation of flow structures [110].

In addition to the spatial features, the temporal features of these modes can also be obtained from POD analysis since the original PIV data is time-resolved in this study. Fig.4.12(a) shows the temporal coefficient of mode 1 versus the time. The evolution of the coefficient follows the pattern of a sine wave, with the same cycle duration T as the input signal of the system. Therefore, the zero phase angle of the coefficient of mode 1 is used as the initial time $t/T=0$. As shown in Fig.4.11(f), the base flow structure of mode 1 is into the channel. Therefore, the period of negative coefficient of mode 1 is the ejection stage and the period of positive coefficient is the suction stage. It is also noted that the peak magnitude of the coefficient

Figure 4.12: (a) Temporal coefficient of mode 1, and (b-e) Relationship of temporal coefficients between mode 2 to mode 5 with mode 1. ($f=45$ Hz and $P=25$ W).

changes slightly from one cycle to another. This cycle-cycle variation could be the result of remnant flow structures persisting from the end of one cycle to the beginning of the next cycle and affects the base flow evolution.

To further understand the relationships between mode 1 (the dominant flow) and the higher modes, temporal coefficients of mode 2 to mode 5 are plotted against the temporal coefficient of mode 1 as seen in Fig.s4.12(b-e). The black dot scatters are 1000 raw data points, and the red line with directional arrows represent the phase-averaged evolution. Specifically, the instants at $t/T = 0, 0.25, 0.5$ and 0.75 are highlighted with orange dots and marked by arrows. In Fig.4.12(b), all the scatters are highly concentrated around the phase-averaged curve, indicating relatively stable temporal-relationship between mode 1 and mode 2. The evolution loop is in the counter clockwise direction, showing that they are the same cycle. Around $t/T=0.75$, a_2 is close to zero and its variation is very small, indicating the disappearance of vortices. The minimum a_2 appears at $t/T \approx 0.44$, showing the maximum strength of vortices. These observations are consistent with the phase-averaged results shown in Fig.5.4. It is also noted that the evolution curve for some cycles may tie into a knot near $t/T=0.75$. This implies that fluctuations of the suction induced shear flow and the reversal of residual vortices is slightly varying between cycles, which can be a result of the unsteady interaction between these flow structures.

The phase-averaged relationship between mode 1 and mode 3 is in the shape of the number ‘8’ as seen in Fig.4.12(c). It is in counter clockwise in the left loop and clockwise in the right loop. Therefore, the dominant frequency of mode 3 maybe twice that of mode 1. In Fig.4.12(d), the phase-averaged relationship between mode 1 and mode 4 is also in the pattern of ‘8’, horizontally symmetric to that of mode 3. As known from Fig.4.11, the velocity fluctuations for the dominant vortices are in opposite directions for

mode 3 and mode 4. Therefore, the symmetric temporal coefficients during the ejection stage in mode 3 and mode 4 represent the same fluctuation i.e the vertical oscillation of the dominant vortices. Whereas in the suction stage, due the dissipation of vortices, the magnitudes of coefficients for mode 3 and mode 4 are relatively small. It is also noted that the variation of the scatters in cycles are quite large in Fig.4.12(c) and (d), implying that the coefficient is time-dependent. In other words, the strength of these vortical fluctuations vary from cycle to cycle. The pattern of phase-averaged relationship between mode 1 and mode 5 has two knots. Therefore, the dominant frequency of mode 3 could be triple that of mode 1. From $t/T \approx 0.8$ to $t/T \approx 0.2$, scatters of the coefficients of mode 5 are concentrated around zero. The magnitude changes significantly in the remaining period, showing that they indicate the propagation of residual vortices.

Figure 4.13: Spectra of temporal coefficients for $f=45$ Hz and $P=25$ W.

The temporal coefficients can be further analyzed to reveal the frequency features of the modes. Fast Fourier transform (FFT) is applied here and the resulting spectra is shown in Fig.4.13. Mode 1 has a peak frequency of 45 Hz which is the same as that of the base flow or input signal. Mode 2 also has a dominant frequency of 45 Hz however, there is also a peak at the second harmonic 90 Hz. However, mode 3 and mode 4 have peaks at 90 Hz and even more smaller higher harmonic peaks. For mode 5 the dominant frequency is 135 Hz with a significant peak also at 90 Hz. From the evolution of temporal coefficients mentioned in Fig.4.12

we know that knots may sometimes appear and disappear due to varying magnitudes. For example, the knots near $t/T = 0.75$ in mode 2 and mode 5, and the knots near $t/T = 0$ in mode 3 and mode 4 are alternating between cycles. Consequently, along with the dominant frequencies in the particular mode (which may be double or triple the fundamental frequency), some lower frequency peaks may also be observed due to the disappearance of knots.

4.3.5.A Comparisons between POD results

Figure 4.14: (a-c) Vorticity contour and velocity vector map of mode 1, mode 2, and mode 3, respectively; relationships of coefficients for (d) mode 2 and (e) mode 3; (f) spectra of temporal coefficients. (45Hz and $P = 5\text{ W}$).

The POD results for the case $f=45\text{ Hz}$ and $P=5\text{W}$ and $f=90\text{ hz}$ and $P=25\text{ W}$ are introduced here for comparison. Since mode 1 to mode 3 are representative of the flow structure fluctuations as seen from the previous case, Fig.s4.14(a)-(c) shows only these 3 modes. Mode 1 as before represents the periodic fluctuations in the base flow and induced shear flow near the stack. Mode 2 and mode 3 represent the fluctuations of dominant and residual vortices. Looking at Fig.s4.14(d) and (e), we observe that even though the phase-averaged patterns form a single loop, occasional knots can still be observed near $t/T=0.75$ for mode 3. As a result, the dominant frequency peaks are 45 Hz for all modes, but mode 3 also has harmonic

peaks. The displaced positions of the scatters between cycles signify temporal variations for these kind of fluctuations. Additionally, the strength of the residual vortices are much lower compared to the dominant vortices and their propagation distance is limited. Hence, these modes already include the fluctuations resulting from vorticity propagation.

Figure 4.15: (a-e) Vorticity contours and velocity vector map for modes 1 to 5, respectively; (f-i) relationships between coefficients for mode 2 to mode 5; (j) spectra of temporal coefficients. ($f=90$ Hz and $P=5$ W).

Fig.4.15 shows the POD results for the case $f=90$ Hz and $P=25$ W. As established in earlier cases, mode 1 is the periodic fluctuation of base flow and induced shear flow while mode 2 represents the fluctuations of the dominant and residual vortices. Interestingly, here the dominant vortices are also weak and therefore comparable in size to the residual vortices as seen in Fig.4.5(c). Hence their propagation can be captured by mode 3 and 5, although their kinetic energy fractions are still very low ($< 0.1\%$). Vertical fluctuations of the flow structure can be seen in mode 3 which

could be a result of the loop geometry. The relationships between temporal coefficients indicate that the fluctuation of the dominant vortices occurs during $t/T=0.75$ to $t/T=0.25$ and the fluctuation of the residual vortices is significant between $t/T=0.25$ and $t/T=0.75$, respectively. The frequency spectra also reveal that mode 1 and 2 have the same frequency as the input signal and the higher harmonics represent fluctuations of residual vortices.

4.3.6 Turbulence levels

The statistical analysis of the PIV data could characterize the flow evolution inside the looped thermoacoustic system, as shown in Figs.4.4, 4.5 and 4.7. However, the phase-averaging of velocity and vorticity might have erased some un-steady features of the flow field. Subsequent POD analysis of the oscillating flow revealed the prevalence of coherent small scale vortical structures in the flow field. Therefore the quantity $\varkappa$ defined as $\varkappa = \overline{u'^2 + v'^2}$ is introduced to quantify the velocity variations. Here, $\overline{u'^2}$ is the spatial-averaged squares of variations of velocity component u in a snapshot, and $\overline{\overline{u'^2}}$ is the corresponding phase-averaged result. Fig.4.16 shows the phase-and spatial-averaged squares of velocity variations in the control area $x \in [0.25, 18]mm$ and $y \in [-\frac{D}{2}, \frac{D}{2}]$. This $\varkappa$ value can be used to approximate the turbulence kinetic energy [58]. It is obvious that the $\varkappa$ value becomes significant when the oscillation of the flow field becomes stronger. For instance, the $\varkappa$ value is close to zero in the case $f=65$ Hz and case $f = 90$ Hz, while it is over 0.01 in the case $f=45$ Hz with the same power. For individual cases, the $\varkappa$ value also increases greatly during the period from $t/T = 0.2$ to $t/T = 0.8$, when vortical flow structures exist in the flow field. It also observed that $\varkappa$ value is not monotonic with the input power (see $P=30$ W). This may because the kinetic energy contribution from mode 1 decreases up to $P=20$ W and then increases as seen in the inset graph in

Fig.4.10 which indicates that higher order oscillations are suppressed slightly after a certain limit. The dominant flow overwhelmingly stands out leading to slightly lower $\varkappa$ value. All these features shown in Fig.4.16 confirm that the flow field is not strictly repeatable.

Figure 4.16: The phase- and spatial-averaged squares of velocity variations in the control area $x \in [0.25, 18]mm$ and $y \in [-\frac{D}{2}, \frac{D}{2}]$.

4.4 Summary

The flow behavior near the stack in a looped thermoacoustic engine was investigated using time-resolved PIV technique. In addition to statistical analysis, the POD method was also employed to understand the flow physics. All cases in this study exhibited a symmetric behavior with respect to the center line between plates with Re and KC values of the order of $O(10^2)$ and $O(10^0)$. However the distinguishing feature here is the complete absence of dominant vortex breakup seen in the standing wave thermoacoustic system as well as several similar studies in literature. [58, 60, 61] in all the cases tested. The vortex street is formed due to the propagation

of the residual vortices while dominant vortices may elongate but remain attached to the stack plates during ejection. The vorticity concentrations are the strongest at resonance frequencies and increase with input power. However, the turbulence level is not monotonic with the input power. POD analysis decomposed the oscillating flow into the dominant base flow and fluctuations of small-scale vortical vortices. The spatio-temporal relationship between the flow structure fluctuations revealed cycle to cycle variability in the oscillating flow. Furthermore, the increments in phase-averaged turbulence kinetic energy suggests that these unsteady variations are indeed significant. These small scale coherent structures could affect the local mass and heat transfer. Modal analysis was used to find the spatio-temporal relationships between the higher modes relative to mode 1 or the base flow. The results reveal a non-periodic nature to these modal flow features indicating the drawback of traditional design methodologies especially for high amplitude operating conditions, when these effects may be enhanced. The non-periodicity of these flow fluctuations as well vortex formation and their propagation could be considered when designing key components in thermoacoustic systems like the stack and heat exchangers. A direct measurement of hit on performance is not possible here due to lack of heat exchangers in this experimental test rig, however the key flow features identified here would still be even more relevant in thermoacoustic prime movers and heat pumps. Therefore, such variations of the flow field should be taken into consideration when designing thermoacoustic systems.

The combination of PIV measurements, statistical analysis and POD could successfully study the flow structures in the looped thermoacoustic system. Now to further examine the looped thermoacoustic system, a numerical investigation using ANSYS FLUENT will be done in the next chapter. This will be used to study the thermal and flow field inside the channels of the stack which were previously not accessible.

Chapter 5

Numerical investigation of thermal and flow fields in a looped thermoacoustic system

5.1 Background

Computational methods offer a robust solution to study operating conditions beyond the scope of the experimental setup available. Chapter 2 section 2.3 helps establish numerical methods including modeling using commercial CFD code like ANSYS FLUENT as a viable tool to simulate oscillating flows in simple thermoacoustic systems. As discussed, several studies have explored successfully the velocity fields, temperature fields and flow structures in thermoacoustic systems with a straight resonator. These were shown to reveal additional characteristics of the oscillating flow inside the channels of the stack. Here in this study, expanding on the experimental results in chapter 4, the more complex looped thermoacoustic which has

been given rare attention is modeled using CFD in ANSYS FLUENT for numerical investigation of velocity, temperature and flow fields. Furthermore, the model will be validated against experimental results and will be used to study the new regions and parameters previously not possible due to practical limitations of the experimental study. This can help justify the use of the transient CFD model as a proper investigative tool to study the looped thermoacoustic system which includes non-linear effects and with the added advantage of minimal time and cost restrictions compared to testing several configurations experimentally.

5.2 Computational model

Figure 5.1: Computational domain (red dotted lines) in the looped thermoacoustic system with enlarged view of mesh near stack.

The computational model is developed based on the experimental setup of the looped thermoacoustic system described in Chapter 4. A 2-dimensional model covering the height and length of the resonator from the location of the first pressure sensor P1 to the beginning of the loop (0.34 m) and the entire length of the loop (1.6 m) is modeled in commercial CFD software ANSYS FLUENT 19.5. The computational domain with the boundaries and the fine mesh generated are shown in 5.1. The parallel plate

stack is modeled using 14 rectangular slabs of length 100 mm and breadth 2 mm with a spacing of 5 mm and located 0.546 m from P1 similar to the experiment. The distance between the plate center lines is $D=7$ mm.

The computational model is designed to simulate the vortex generation at the ends of the parallel plates in the presence of oscillatory flow past the stack assembly. Periodic boundary conditions are not used due to the looped geometry and the unique position of the stack assembly. In order to accurately capture the non-linear flow phenomenon near the vicinity of the stack, a very dense mesh is used on either side of the stack plates covering a length of length of around $100\delta_v$, the viscous penetration depth [112] and covers the region of interest. The remaining portion of the computational model uses a coarser mesh as consists of only the flow in the empty resonator.

The working fluid for the system is air governed by the ideal gas equation to accommodate changes in density. A pressure inlet is introduced at the location of P1 which simulates the oscillating driving pressure from the loudspeaker in the experiment at the same location. The sinusoidal pressure amplitude is supplied by means of a User Defined Function (UDF) written for ANSYS FLUENT and corresponds to the pressure amplitudes measured by the pressure sensor P1 in the experiment. The walls of the resonator are made of acrylic and adiabatic boundary conditions are applied. The stack is made of aluminium and thermally coupled boundary conditions with the solid and fluid.

The turbulent flow is modeled using Reynolds Averaged Navier-Stokes (RANS) equations. In the RANS model, the instantaneous variables in the Navier-Stokes equations are replaced by the ensemble average and the corresponding fluctuating components. The velocity component is represented as

$$u_i = \overline{u_i} + u'_i \quad (5.1)$$

Where $i=1,2,3$ are the components in index notation.

For scalar quantities (such as pressure) denoted by φ ,

$$\varphi = \overline{\varphi} + \varphi' \quad (5.2)$$

Now, substituting equations 5.1-5.2 in the continuity, and momentum equations of the standard Navier-stokes equations, we get the RANS equations in Cartesian tensor form as shown below.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (5.3)$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right] + \frac{\partial}{\partial x_j} (-\rho \overline{u'_i u'_j}) \quad (5.4)$$

Where ρ is the density, p is the pressure, μ is the effective viscosity and δ_{ij} is the Kronecker delta. The new term introduced here $-\rho \overline{u'_i u'_j}$, are the Reynolds stresses which are further solved in ANSYS FLUENT using Shear Stress Transport (SST) $k-\omega$ [113] model to close equation 5.4. The SST- $k-\omega$ model is used to simulate turbulence phenomenon here since it was found to represent the flow structure most accurately and is also confirmed by previous studies of similar flows [114].

The energy equation for turbulent heat transport is modeled as follows.

$$\frac{\partial(\rho E)}{\partial t} + \frac{\partial[u_i(\rho E + p)]}{\partial x_i} = \frac{\partial}{\partial x_j} \left[\kappa \frac{\partial T}{\partial x_j} + u_i \cdot (\sigma_{ij}) \right] + S_h \quad (5.5)$$

Where E is the total energy, T is the temperature, κ is the effective thermal conductivity, S_h is any user added heat source and τ_{ij} is the stress tensor defined by

$$\sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \delta_{ij} \frac{\partial u_k}{\partial x_k} \quad (5.6)$$

Gravity is also incorporated in the model with a value of $g=-9.8$ m/s². Pressure-based solver is used to solve the transient simulation with the PISO scheme applied for pressure-velocity coupling. PRESTO! is used for spatial discretization of pressure while second order upwind discretizations are applied for density, momentum, turbulent kinetic energy and energy. Convergence criteria of 10^{-4} is used for continuity and momentum equations while 10^{-6} is applied for the energy equation. More details regarding the choice of user settings in FLUENT can be found in [87]. A maximum of 50 iterations per time step are allowed for solution to converge. Solutions are allowed to run for each case, until the velocity and pressure amplitudes reach a steady oscillatory state. The details of the time-step and mesh density are detailed in the next section.

5.3 Results and Discussion

5.3.1 Grid and time-step dependency

Four cases of mesh grids were tested for grid independency of the solution. The mesh was successively refined in the vicinity of the stack plate

Figure 5.2: Mesh grid sizes for (a) Mesh 1 (b) Mesh 2 (c) Mesh 3 and (d) Mesh 4 zoomed inside the channels of the parallel plate stack. (meshing of plate omitted here).

Table 5.1: Time step values compared for study.

Case	Time step duration
τ_1	$\frac{1}{50f}$
τ_2	$\frac{1}{100f}$
τ_3	$\frac{1}{200f}$

assembly in the region of interest. This would be sufficient as the relevant flow phenomenon in this study are confined to that region. The number of elements in each mesh grid and their relative structures are shown in Fig.5.2. For grid independency, the axial velocity profile with respect to the distance from the edge of the stack plate to the middle of the channel located 25 mm from the middle of the plates and at the ejection ($t/T = 0.2$) and suction time ($t/T = 0.8$) for all four mesh densities are compared.

For solving this transient model proper time steps need to be selected to achieve good convergence. The time-step is selected based on $\tau = \frac{1}{N.f}$, where f is the frequency of oscillating flow and N is a suitable number by which a single cycle of the flow is divided so as to get converged solutions in the minimum number of iterations while at the same time keeping the total simulation run time reasonable. As given in table 5.1, $N=50,100$ and 200 were tested for time-step independence by compar-

ing the time-variation of axial velocity at a distance of 18 mm from ends of the stack for one full cycle. The results for the grid independency and time-step independency study are shown in 5.3 and 5.4 respectively. Mesh 3 with 329094 elements and $N=100$ was found to give a grid and time-step independent solution with reasonable computational time and accuracy as shown.

Figure 5.3: Grid independence study for velocity profile within stack channel.

Figure 5.4: Time-step independence study for axial velocity variations in one cycle.

5.3.2 Model validation

For the purposes of initial validation, comparisons are made to the experimental test case of the looped thermoacoustic system operating at a frequency of 45 Hz and an input power of 25 W supplied to the loudspeaker.

The corresponding drive ratio (ratio of pressure amplitude to mean pressure) at P1 is 1.48 % at a mean pressure of 1 bar and mean temperature of 293.15 K. The same input conditions of drive ratio, mean pressure and mean temperature are applied to the CFD model as well. The simulation is run for 2500 time steps which correspond to 25 flow cycles when the velocity and pressure amplitudes are stable. The selection initial time $t/T=0$ for this comparison is the same as experimental results discussed in chapter 4.

Figure 5.5: Pressure amplitude at location P5 for model and experiment.

Figure 5.6: Velocity amplitude at location 18 mm away from stack plates for model and experiment.

A good agreement between experiment (EXP) and simulation (CFD) results is obtained as will be described here. Fig.5.5 shows the variations of pressure amplitude for the simulation and experiment in one cycle at the location of P5 reading in the experiment. For both cases, the pressure starts initially much lower than the mean value resulting in a large negative

pressure amplitude and later peaks nearly midway through the cycle. The difference between the mean amplitudes of pressure oscillations between the simulation and experimental measurements is 4.29 %. Additionally, Fig.5.6 shows the variation of axial velocity in the simulation for one full cycle at 18 mm from the plate ends into the far field compared with the phase averaged results from the PIV measurements in the experiments. Here also, the simulation results agree well the experimental results with a 4.2 % difference between mean amplitudes of axial velocity. Although the curves have nearly identical peak velocity (≈ 3.3 % difference) during the second half cycle, there are some disparities in the peak velocity amplitude during the positive half of the cycle (≈ 21.2 % difference) and consequently some deviations from the experimental values which may be due to three-dimensional nature of the physical setup. However, a key consideration for this comparison is the nature of the flow structures developed during the acoustic cycle. Due to the significance of flow structures near the stack vicinity on the heat transfer in thermoacoustic systems, we can also help establish the validity of the model by comparing the temporal evolution of these structures from the CFD simulation with the experimental results as shown in Fig.5.7. The red dotted rectangular area signifies the region of interest in common with the observation region in the experiment. The pattern of the flow structures are indeed very similar for both cases. We can clearly see the evolution of a pair of dominant vortices (DV) which are attached to the ends of the stack plates and formed by rolling up of the shear layer. The residual vortices (RV) are the weaker vorticity concentrations that are remnant structures formed in the suction stage of the preceding cycle and ejected from within the channel. These are also seen propagating with the main flow at various time instances in both the simulation and experiment results. Fig.5.7(c) compares the span-wise averaged vorticity magnitude normalized with the mean vorticity (Ω/Ω_m) and we can observe similar trends in the vorticity profile

along the axial direction of the flow in the region of interest. There are some disparities in the locations of the secondary peaks in vorticity magnitudes particularly at $t/T=0.6$ and $t/T=0.8$ but these may be due to the turbulent nature of the flow and cycle to cycle variations of the flow structures which are inevitable as concluded from the proper orthogonal decomposition (POD) analyses conducted from the experimental data in the chapter 4. Nonetheless, the general similarity of the flow patterns and the vorticity distributions along with the similar pressure and velocity oscillations validate the model for the hybrid thermoacoustic system. Therefore, the CFD model can be used to further investigate the flow and temperature field for the stack which will be discussed in the subsequent sections.

Figure 5.7: Comparison of vorticity contours with superimposed contour lines of $\lambda_2 < 10^{-3}$ and vortex cores between (a) CFD (b) Experiment and (c) normalized span-average vorticity for $y \in [-D, D]$ for both at instants $t/T=0$ to $t/T=0.8$.

5.3.3 Flow inside the stack channel

In this section, we discuss the flow features, velocity and temperature profiles found inside the channels of the parallel plate stack for oscillat-

ing flow at resonance frequency of the looped thermoacoustic system $f=45$ Hz and reference power input of $P=25$ W at a drive ratio at P1 of 1.48 %. Investigation of this region was previously hindered due to the limitations of the PIV setup. With the CFD model we can observe this region as well which will help provide a more complete picture of the flow features in the looped thermoacoustic system.

Figure 5.8: Vorticity contours showing the evolution of dominant, residual and inner vortices at instants $t/T=0.4$ to $t/T=0.6$ marked as DV, RV and IV respectively.

Fig.5.8 shows the three types of vortical structures forming near the end of the parallel plate stack. As discussed in chapter 3 & 4, the strong dominant vortices (DV) formed by the rolling up of shear layers remain attached to the end of the plates and the weaker residual vortices (RV) left behind from the previous suction stage move downstream. The residual vortices are in fact emerging from within the channel as seen in Figs 5.7 & 5.8. These are remnants of the vortices generated in the suction stage that traverse inside the channel and then get blown out during the next ejection stage. However, just at the very end of the plates, at $t/T=0.4$ we can observe a third pair of previously undetected vortices emerging called hereafter as 'inner vortices' (IV). The inner vortices are formed when the pair of vortices sucked back into the channel during flow reversal induce counter-rotating vortical structures near the plate surface. The inner vortices then grow in size from $t/t=0.4$ to $t/T=0.5$ and finally merge with the shear layer

on the surface inside the channel at instant $t/T=0.6$. During this same time interval, the dominant vortices which previously had been attached to the plate ends now seem to jump over the merged shear layer and gradually decrease in intensity inside the channel and then later disappear.

Figure 5.9: Temporal evolution of vorticity contours around the entire length of the stack plate from $t/T=0$ to $t/T=0.9$ for $f=45 \text{ Hz}$ and $P=25 \text{ W}$.

Fig.5.9 shows the temporal evolution of the vorticity contours over the entire length of the parallel plates (7 & 8 from the top) for ten phases in the cycle. Here $t/T=0$ represents the same initial phase as in the ex-

Figure 5.10: Axial mean velocity at $x=-68\text{mm}$ (left end), $x=0$ (middle) and $x=68\text{ mm}$ (right end).

periment. The axial velocity of the base flow spatially averaged along the width of the channel ($y \in [-\frac{d}{2}, \frac{d}{2}]$) at $x=-50\text{ mm}$ (left end of the plates), $x=0$ (middle) and $x=50\text{ mm}$ (right end) during one cycle is shown in Fig. 5.10 for reference. The slightly higher maximum velocity magnitudes at the left and right ends may be due to the presence of strong vorticity concentrations. At $t/T=0.1$, we can start to see the emergence of dominant vortices forming from the shear layers on the plate surface. Expanding on the experimental observations with a longer region of interest, we can see that a pair of residual vortices observed are an extension of a weaker second vorticity pair left behind inside the channel and generated from the previous cycle as explained above, extending throughout the channel. This residual vorticity layer can be clearly seen at $t/T=0.2$ during ejection and successively becomes weaker as the flow progresses leaving behind only small vortical structures as seen in $t/T=0.7$ and finally disappears at $t/T=0.8$ during suction. Looking at the left end of the stack plates, we can see similar dominant vortices developing attached to the plates and surrounded by residual vortices weaker in magnitude. However, the shape and size of the vortices are different and are not symmetric. This is expected because the left end of the plates are near the bend in the feedback loop and will cause distortions in the vortical structures as shown.

Figure 5.11: Temporal evolution of temperature contours around the entire length of the stack plate from $t/T=0$ to $t/T=0.9$ for $f=45 \text{ Hz}$ and $P=25 \text{ W}$.

Fig.5.11 presents the corresponding temperature contours at ten phases in the cycle. We observe that the gas near the left end of the stack is always relatively cooler compared to the right end. This is because of the net thermoacoustic heat pumping effect. Additionally, the temperature oscillations described by thermoacoustic theory [4] can be clearly seen as the cycle progresses. The maximum temperatures are observed towards the middle ($t/T=0.5$) of the cycle when pressure amplitude is also approaching

maximum value. The temperature contours show high temperature concentrations (HTC) surrounding the edges of the stack plates in the right end (hot end) which are clearly visible at $t/T=0.2$ and 0.3 . Alternatively, low temperature concentrations (LTC) are observed at the edges of stack plates in the left end of the stack (cold end), as clearly seen during $t/T=0.7$ and 0.8 . This phenomenon will be explained subsequently based on the velocity and temperature profiles inside the channel of stack plates.

Figure 5.12: Axial velocity profile in the channel between two plates at $x=25$ mm from the center of the plate to the right from $t/T=0$ to $t/T=0.9$ for $f=45$ Hz and $P=25$ W.

Fig.5.12 presents the velocity profile in the channel between the plates at 25 mm from the center for ten phases in the cycle. A characteristic feature of the oscillating flow seen here is the annular effect in the velocity profiles [115,116] wherein the maximum velocity occurs near to the wall rather than at the channel center. These velocity peaks are particularly obvious and closest to the wall at $t/T=0$ and $t/T=0.5$ during beginning and flow reversal stages of the cycle respectively and highlighted with red dotted circles in Fig. 5.12. The distance of the peaks from the wall depends on the

viscous penetration depth of the gas ($\delta_v = \sqrt{\frac{2\mu}{\omega\rho_m}}$). The residual shear layer and vortical structures inside the channel may also retard the velocity away from the walls near the center and accelerate the velocity near the edges.

Figure 5.13: Temperature profile in the channel between two plates at $x=25$ mm from the center of the plate to the right from $t/T=0$ to $t/T=0.9$ for $f=45$ Hz and $P=25$ W.

Analogous to the velocity profile, the temperature profile also exhibits the characteristic ‘annular effect’ of oscillating flows with the temperature peaks clearly visible at $t/T=0.2$ and $t/T=0.3$ and temperatures dips clearly visible at $t/T=0.7$ and $t/T=0.8$ as seen in Fig.5.13 and highlighted with red dotted circles. The distance of the peaks or dips from the wall depends on the thermal penetration depth of the gas ($\delta_k = \sqrt{\frac{2\kappa}{\omega\rho_m C_p}}$). Similar phenomenon have also been reported in literature for thermoacoustic systems incorporating heat exchangers [96]. The temperature peaks occur when the flow is from left to right and layers of hot gas within the thermal boundary layer flow into the right side rising the temperature relative to the core. This can be seen comparing $t/T=0.2, 0.3$ between Figs.5.11 and 5.12 in the ejection stage of the cycle. Similarly, when the flow is from right

to left, the hotter gas propagates in the reverse direction and heats up the central core leading to a dip near the walls as observed in Figs.5.13 and 5.12 at $t/T=0.7, 0.8$ during the suction stage. This annular effect is responsible for the high/low temperature concentrations at edges of the plates observed in the temperature contours shown in Fig.5.11. The velocity profile clearly influences the motion of the hot gas in the thermal boundary layer and thereby also the temperature profile. Therefore, this characteristic feature of the temperature distribution may play a key role in either improving or degrading the heat transfer between the stack and working fluid by creating temperature gradients.

5.3.4 Effect of acoustic frequency

Table 5.2: Cases of frequency studied in hybrid thermoacoustic system.

f (Hz)	Power (W)	DR at P1 %	Re	KC	δ_v (mm)	δ_k (mm)
45	25	1.48	225	3.0	0.33	0.39
65	25	0.25	190	1.8	0.23	0.28
90	25	0.41	143	1.0	0.15	0.17
120	25	1.92	341	1.7	0.10	0.12

In this section, the acoustic frequency is varied between $f=45$ Hz, 65 Hz, 90 Hz and 120 Hz applying the corresponding drive ratios with the same input power ($P=25$ W) for all frequencies. To replicate the conditions of the experiment in chapter 4 for further analysis, the drive ratio which generates the same input power for each frequency is calculated from the measured values of pressure amplitude at pressure-inlet P1 and the mean pressure at that condition. This drive ratio is applied to the pressure inlet P1 of the CFD model. The parameters for the various cases are given in table 5.2. Re and KC values are based on the mean amplitude of axial velocity at $x=18$ mm in the far field ($x \gg D$). The operating pressure is 1 bar and the mean temperature is 293.15 K for all cases. For this study, in

order to compare the same time instances at different frequencies, the non-dimensional flow time had to be redefined such that the initial time $t/T=0$ is when the pressure amplitude is at its maximum value.

Figure 5.14: Comparison of (a) pressure amplitude and (b) axial velocity with frequency between experimental measurements and CFD results.

Fig. 5.14(a) & (b) shows the comparison of pressure amplitude at P5 and the axial velocity at 18 mm from the stack ends, of the simulation results with the experimental measurements performed on the hybrid thermoacoustic system with different frequencies ($f=45$ Hz has been omitted here for clarity since it has already been discussed in section 5.3.2). We can see a good agreement between both results for all cases with the exception of slight deviation in pressure amplitude for $f=65$ Hz. This may be due to external factors like variability of loudspeaker performance when the measurement was performed on the experimental system. The order of maximum pressure amplitude and axial velocity also follow that in the experiment with close values for $f=65$ Hz and $f=120$ Hz and $f=90$ Hz having the lowest. It is also noted here that 45 Hz has the highest peak pressure amplitude and axial velocity by comparing Fig.5.14 with Figs 5.5-5.6, because of it being the resonance frequency of the system. Overall, the similar trends in pressure amplitude and velocity further strengthen the validity of the CFD model for the system. The dependence of the profile of velocity

amplitude on acoustic frequency will be discussed below.

Figure 5.15: Velocity profile in the channel between plates at a distance of 25 mm from the center at instances (a) $t/T=0.2$ (b) $t/T=0.4$ (c) $t/T=0.6$ and (d) $t/T=0.8$ at different frequency.

Fig.5.15 shows the comparison of velocity profiles inside the channel at 25 mm from the stack center at $t/T=0.2, 0.4, 0.6$ and 0.8 for the acoustic frequencies investigated. At all instances, 120 Hz has the highest velocity amplitude while 90 Hz has the lowest. This is because 120 Hz being the second resonant frequency has considerably strong pressure oscillations as indicated before and also rapid acceleration owing to its very high frequency. These two factors contribute to the overall high magnitude of velocity oscillations for 120 Hz. As explained in chapter 4 harmonic suppression for integral multiples of the first resonant frequency leads to much weaker low

pressure oscillations for 90 Hz and as a result has the lowest velocity amplitude as well. The difference is more subtle when comparing the velocity oscillations of 45 Hz and 65 Hz because of their close frequencies and pressure amplitudes. As a result, during the ejection stages at $t/T=0.2$ and $t/T=0.4$ when the velocity magnitudes are generally smaller, the differences between them are very small whereas the differences are magnified during the faster suction stages at $t/T=0.6$ and $t/T=0.8$. The characteristic velocity peaks (annular effect) near the wall are also present here with an increasing frequency shifting the peak further towards the wall. This is because an increase in frequency leads to smaller viscous penetration depths (δ_v). Therefore the size of the in-viscid region increases with frequency.

Fig.5.16 shows the temperature profiles in the same cross-section and time instances as before. The annular effect for temperature in the form of temperature peaks and dips are also prevalent here particularly at $t/T=0.2$ and $t/T=0.8$. With a change in acoustic frequency both the thermal penetration depth (δ_k) and the gas displacement are affected. δ_k decreases whereas the gas displacement is also dependent on the velocity. At $t/T=0.2$, 45 Hz exhibits the highest temperature peaks due to the annular effect but surprisingly the core temperatures for both 45 Hz and 65 Hz are lower than 90 Hz and 120 Hz. This may be due to the hysteresis effect of the previous acoustic cycle bringing down the core temperature of the comparatively slower frequencies. At $t/T=0.4$ and $t/T=0.6$, temperature peaks/dips are not particularly significant because of the higher overall temperatures. Finally at $t/T=0.8$, we can observe temperature dips for all frequencies. Here the pressure amplitude greatly influences the oscillations of core temperature with 90 Hz showing the smallest change and 45 Hz showing the largest.

Figure 5.16: Temperature profile in the channel between plates at a distance of 25 mm from the center at instances (a) $t/T=0.2$ (b) $t/T=0.4$ (c) $t/T=0.6$ and (d) $t/T=0.8$ at different frequency.

Table 5.3: Cases of mean pressure studied in hybrid thermoacoustic system.

p_m (bar)	f (Hz)	DR at P1 %	Re	KC	δ_v (mm)	δ_k (mm)
1	45	1.48	225	3.0	0.33	0.39
2	45	1.48	452	3.0	0.23	0.28
5	45	1.48	1129	3.0	0.15	0.17
10	45	1.48	2261	3.0	0.10	0.12

5.3.5 Effect of mean pressure

In this section, the mean pressure of the working fluid (air) was varied between $p_m=1$ bar, 2 bar, 5 bar and 10 bar keeping the same drive ratio (DR=1.48 %) at the pressure inlet P1 (the value is the same as that of

Figure 5.17: Variation of (a) pressure amplitude and (b) axial velocity with mean pressure.

the experimental case with $f=45$ Hz and $P=25$ W). The parameters for the different cases are given in table 5.3. Fig.5.17 (a) and (b) shows the variations of pressure amplitude at P5 and axial velocity amplitude at 18 mm from the stack ends respectively. Since the drive ratio is constant, the pressure amplitude increases with increasing mean pressure, whereas the axial velocity remains largely identical for all cases. This is because the velocity directly scales with pressure amplitude but inversely scales with the mean density [14], thereby maintaining a constant value when the mean pressure increases leading to an increase in both density and pressure amplitude.

Fig 5.18 shows the comparison of velocity profiles inside the channel at 25 mm from the stack center at $t/T=0.2, 0.4, 0.6$ and 0.8 for the mean pressures investigated. The overall trends in velocity profiles remain identical for all the mean pressures. However, a more closer inspection of the velocity profile reveals that an increase in mean pressure causes the velocity peaks caused by the annular effect to shift closer towards the stack walls with an increase in mean pressure. This is due to the reduction in viscous penetration depth (δ_v) with increasing mean pressure/density. As a result of this shifting velocity peaks, during the positive flow directions at $t/T=0.2$

Figure 5.18: Velocity profile in the channel between plates at a distance of 25 mm from the center of the plate at instances (a) $t/T=0.2$ (b) $t/T=0.4$ (c) $t/T=0.6$ and (d) $t/T=0.8$ at different mean pressures.

and $t/T=0.4$, the velocity amplitude in the central in-viscid region at 1 bar tends to be the largest whereas in the negative flow direction the velocity at $t/T=0.6$, it becomes the lowest. However at $t/T=0.8$, due to the relatively close peaks, the trend is again reversed.

The temperature profiles along the same cross-section and instances for different mean pressures are shown in Fig.5.19. Observing the temperature peaks at $t/T=0.2$, we can see that 1 bar mean pressure exhibits the most drastic inflections in temperature moving from the center of the channel to the plate surface. This is because of the larger thermal penetration depth (δ_k) at lower mean pressure. Similarly at $t/T=0.8$, the width of the

Figure 5.19: Temperature profile in the channel between plates at a distance of 25 mm from the center of the plate at instances (a) $t/T=0.2$ (b) $t/T=0.4$ (c) $t/T=0.6$ and (d) $t/T=0.8$ at different mean pressures.

higher temperature central region is smaller for 1 bar due to the larger thermal boundary layer. Neither peaks nor dips are obviously distinguishable for $t/T=0.4$ and $t/T=0.6$ on account of overall higher temperatures at these instances.

The flow patterns for all operating pressures are identical with the key differentiation being the reduction of size of the shear layer attached to the plates with increasing mean pressure as seen in Fig.5.20. this due to difference in viscous penetration depth explained earlier.

Figure 5.20: Vorticity contour around the plates at $t/T=0.4$ at different mean pressures.

5.3.6 Effect of working fluid

Table 5.4: Cases of working fluids studied in hybrid thermoacoustic system.

Gas	f (Hz)	DR at P1 %	Re	KC	δ_v (mm)	δ_k (mm)
Air	45	1.48	355	3.0	0.33	0.39
He	45	1.48	14	1.5	0.92	1.12
Ar	45	1.48	237	2.9	0.31	0.38
CO ₂	45	1.48	840	6.0	0.24	0.27

Figure 5.21: Variation of (a) pressure amplitude and (b) axial velocity with the working fluid.

The nature of the working fluid used plays a critical role in the thermoacoustic energy conversion phenomenon. In this section, different

working fluids commonly used in thermoacoustic systems such as Helium (He), Argon (Ar) and Carbon-dioxide (CO_2) are investigated and compared to air used in the original experimental study. The mean pressure is kept is 1 bar and the frequency f is 45 Hz. The same drive ratio (1.48 %) is applied for all fluid cases at the pressure inlet P1 corresponding to $P=25$ W in the experiment. The parameters of the different cases are given in table 5.4. Fig. 5.21 (a) and (b) presents the maximum pressure amplitude at P5 and the axial velocity variations at $x=18$ mm in one cycle for the different working fluids. The magnitudes of the pressure amplitude and velocity oscillations are determined by the interplay of a number of factors such as fluid density, speed of sound in the fluid and Prandtl number [14]. The results here indicate that even with the same pressure amplitude supplied at the inlet P1, the maximum pressure amplitude and consequently also the maximum axial velocity varies according to the working fluid. This directly follows the KC values for the different fluid cases with the highest for CO_2 , lowest for He and close values for air and Ar.

Fig.5.22 shows the comparison of velocity profiles inside the channel at 25 mm from the stack center at $t/T=0.2, 0.4, 0.6$ and 0.8 for the different working fluids. He, which has the largest δ_v value exhibits the conventional parabolic velocity profile in all phases since the annular effect is not seen due to large thickness of viscous boundary layer. Air and Ar with very close δ_v values show very similar velocity profiles with relatively smaller peaks near the plate surface. CO_2 with the smallest δ_v distance shows drastic peaks during ejection stages with a retardation of central core velocity resulting in an S-like profile as seen before in Fig. 5.18 (b). The higher velocities during suction stage seem to counteract this velocity reduction in the core as seen during $t/T=0.6$ and 0.8 .

Fig. 5.23 presents the corresponding temperature profile inside the

Figure 5.22: Velocity profile in the channel between plates at a distance of 25 mm from the center at instances (a) $t/T=0.2$ (b) $t/T=0.4$ (c) $t/T=0.6$ and (d) $t/T=0.8$ for different working fluids.

channel at 25 mm from the stack center at $t/T=0.2, 0.4, 0.6$ and 0.8 for the different working fluids. At $t/T=0.2$, two of the characteristic peaks of temperature are observed near the plate edges for all fluids. The distance of the peak from the edges changes with δ_k . At $t/T=0.4$ and 0.6 , the higher mean temperatures obscure the annular effect leading to a more uniform temperature distribution away from the boundary layer for air, Ar and CO_2 while the larger δ_k creates a parabolic distribution for He. The lower velocities at different instances for He may also lead to slower movement of hot gas across the channel during suction limiting the annular effect. When the flow reaches $t/T=0.8$, the temperature dips are seen due to movement of hot

Figure 5.23: Temperature profile in the channel between plates at a distance of 25 mm from the center at instances (a) $t/T=0.2$ (b) $t/T=0.4$ (c) $t/T=0.6$ and (d) $t/T=0.8$ for different working fluids.

gas towards the right during suction stage.

5.3.7 Discussions on heat transfer

The influence of the annular effect seen in both velocity and temperature profiles is largely dependent on the thickness of the viscous and thermal boundary layers compared to the channel spacing. This can be an important consideration when designing heat exchangers to be used in conjunction with the stack ends. The velocity peaks and the residual shear layer can reduce the core velocity. The velocity profile can also influence the movement of hot gas in the thermal boundary layer during ejection and

suction stages of the flow. Additionally, the temperature annular effect and the movement of the working fluid can create temperature gradients which can affect the heat transfer between the solid surface and the working fluid. More details regarding the heat transfer can be investigated by considering net heat flux generated over a cycle in the walls of the stack for different cases of working fluids and mean pressures.

Figure 5.24: Average heat flux over the length of a plate wall against $\log_{10}(Re)$ at difference mean pressures for different working fluids.

The magnitude of line average heat flux ($q = \kappa \frac{dT}{dy}_{wall}$, where κ is the thermal conductivity of the gas and $\frac{dT}{dy}_{wall}$ is the transverse temperature gradient at the wall interface) over the entire length of the wall for a single stack plate in one acoustic cycle is estimated and plotted against the Reynolds number ($(\log_{10}(Re))$) for different working fluids air, He, Ar and CO₂ as shown in Fig.5.24. For each gas the Reynolds number is calculated at mean pressures 1 bar, 2 bar, 5 bar and 10 bar. When the mean pressure increases for a gas, even though the mean velocity amplitude u_M remains the same, the density of the fluid ρ increases. Therefore the Reynolds number also goes up. The averaged heat flux in one cycle is from the fluid to the

solid and increases with an increase in mean pressure because the thermal penetration depth reduces creating a larger temperature gradient and facilitating stronger heat transfer between the stack wall and the much denser fluid. Among the working fluids tested, CO₂ generates the highest heat flux while He generates the lowest at $f=45$ Hz. This is largely because the speed of sound in the medium is different for each fluid and affects the resonance phenomenon. Since the speed of sound for Helium is much higher, the resonance frequency range would also be higher resulting in generally low heat flux at this frequency. Meanwhile CO₂ has much higher heat capacity than the rest and also has a speed of sound comparable to Air and Ar, resulting in the highest heat flux in this frequency. It should be noted that designing the system with optimal stack plate spacing and other geometric dimensions will vastly improve the heat fluxes for the different cases. Nonetheless, features of the oscillating flow like the vortical structures and annular effect should be considered to generate high heat flux across the stack and thereby good performance.

5.4 Summary

A transient CFD model was developed based on the SST- $k - \omega$ turbulence model to study the flow structures, temperature and velocity profiles in the channel of the stack for the looped thermoacoustic system. The numerical results showed good agreement with the experimental results of pressure amplitude and axial velocity over one cycle measured at several frequencies and generated identical flow structures. The simulation results revealed the formation of previously undetected inner vortices during flow reversal which later merge with the shear layer on the plate surface. The vorticity contours also showed a residual vorticity layer along the length of the channel which emerge out of the channel to become the residual vortices

observed in the experiment. The presence of counter-rotating pairs of dominant, residual and inner vortices can influence the axial velocity profiles by accelerating the flow near the stack walls and decelerating it at the center. This effect of velocity peaks near the wall was also observed by examining the velocity profile in the channel and is similar to the annular effect reported in oscillating flows. The temperature profile also shows peaks and dips at various instances due to movement of working gas within the channel. The flow history was shown to affect the temperature distribution at the beginning of the cycle particularly for lower frequencies. The distance of the viscous and thermal boundary layers from the stack wall largely influences the position of these velocity peaks and temperature peaks or dips respectively. They are more prominent at resonance frequencies and at the same frequency, increasing the mean pressure shifts them closer to the wall. Therefore the annular effect shapes the velocity and temperature distributions and can affect heat transfer between stack and working fluid. The average heat flux over one flow cycle was shown to increase with Reynolds number for different working fluids. Future investigations may provide further insight on whether the annular effect can be utilized to enhance heat transfer especially in thermoacoustic engines and refrigerators with hot and cold heat exchangers. All these nuances in velocity and temperature distributions and heat flux, along with the evolution of flow structures provide more insight into the nature of the oscillating flow in the looped thermoacoustic system. The CFD model developed in the study could successfully investigate the oscillating flow field in the looped thermoacoustic system and overcome the limitations of linear thermoacoustics which is commonly used for designing these devices.

Chapter 6

Conclusion and Future work

6.1 Conclusions

In this work the thermal and flow characteristics of the oscillating acoustic field were investigated in and around the vicinity of the thermoacoustic stack using a combination of experimental, statistical and numerical methodologies. The major findings from the different studies are listed below.

- (i) A standing wave thermoacoustic system with a straight resonator was designed and fabricated in house to demonstrate the thermoacoustic heat pumping effect. Usually, the stack takes the form of precise geometries that may require fairly accurate fabrication techniques such as parallel plate stack, pin array stack, honeycomb stack etc. Here, as a side study, steel wool which is a low cost and easily available porous material with a random geometry has been used as the stack and was able to generate significant temperature difference between the ends. Therefore, the study shows that a porous material based

thermoacoustic stacks could be viable options to reduce the cost of thermoacoustic systems.

- (ii) Non-linear effects have been a cause of concern for the design of thermoacoustic devices especially at significant pressure amplitudes. This can cause a decrease in efficiency as well as inaccurate results when using conventional thermoacoustic design methodologies that rely on the linear theory. With the use of PIV flow visualization and measurement, the flow field around the stack region was investigated first in the standing wave thermoacoustic system with a straight resonator to establish a baseline with previous studies in literature. In this study, prominently, vortex formation at the ends of the parallel plates could be observed in the oscillating flow. Statistical analysis was used to obtain the phase averaged velocity and vorticity fields. Counter rotating vortices were shown to influence the local velocity at the entrance region of the stack. The kinetic energy was found to increase with input power and highest at harmonic frequencies. Symmetric vortices attached to the plate ends called dominant vortices were found which would break up due to instability at higher pressure amplitudes. The characteristic vortex street also consisted of residual vortices generated between the plate edges during the preceding suction stage of the cycle. These vortical structures could indeed carry heat to and from the stack to adjacent hot and cold heat exchangers in real thermoacoustic systems. This mechanism of heat transport by the vortical structures could either lead to unwanted loss of heat loss or even enhance the thermoacoustic heat pumping effect [14]. Therefore careful consideration of the presence of vortical structures is indeed necessary when designing the stack and heat exchangers assembly which forms the thermoacoustic core in order to operate them at high power and pressure amplitudes.

- (iii) The flow behavior and evolution in the looped thermoacoustic system was investigated using time-resolved PIV. The two typical vortex structures-the dominant and residual vortices were observed here as well. However, the characteristic vortex street for the looped systems consisted of only the residual vortices. No vortex shedding was observed even at comparable pressure amplitudes relative to the standing wave system. The kinetic energy increased monotonically with input power and was highest at resonance frequencies. As mentioned previously, the vortical structures can carry kinetic energy and dissipate it as heat which can function as a heat transfer mechanism between the stack, heat exchangers and the working gas. Vortex structures and their evolution during oscillation can be taken into consideration when modeling real thermoacoustic systems. Proper orthogonal decomposition method was also used here owing to the more complex geometry, to better understand the flow physics. POD analysis decomposed the flow into a periodic base flow and small scale coherent vortical structures. Modal analysis revealed the spatio-temporal relationship between them and the base flow. The increase in phase averaged turbulent kinetic energy implies that these flow structures are in fact unsteady between cycles. Therefore the non-periodic nature of the flow field variations should also be taken into consideration when modeling the heat transfer in such systems especially at high pressure amplitudes.
- (iv) A transient CFD model of the looped thermoacoustic system was developed in ANSYS FLUENT. The CFD model was validated with experimental results at several frequencies and were used to visualize the thermal and flow fields within the channels of the stack which was previously not possible due to limitations of the experimental setup. The results of vorticity contours revealed the origin of residual vor-

tices seen in the flow field to be formed from a remnant vorticity layer withdrawn into the channels during the suction stage and later exiting the channels during ejection. It also showed the presence of a third pair of inner vortices which merge with the shear layer. An annular effect in the velocity and temperature profiles between the channels of the stack could be observed where the velocity exhibited peaks near the wall while the temperature showed peaks or dips depending on the movement of the fluid within the channel. The variations of velocity and temperature profiles are also influenced by the mean pressure, frequency and the nature of working fluid due to changing thickness of the viscous and thermal boundary layers. Hysteresis effects from previous acoustic cycle can also affect the velocity and temperature changes within the channels of the stack. It was also observed that the heat flux between the fluid and stack increases with Reynolds number for different working fluid tested. The CFD model can effectively serve as a design tool to work with non-linear effects such as vortex formation and turbulence which can help accurately predict velocity and temperature profiles and heat flux generated at various operating conditions which may not be possible to test experimentally because of cost and time constraints. The interaction of vortex structures from the stack with heat exchanger modules could be explored in future models for a more complete understanding of the entire thermoacoustic system.

The results from all these studies suggest the importance of vortical structures, their ability to transport and dissipate heat in the form kinetic energy and the non-periodic nature of small-scale flow fluctuations with particular importance to the more geometrically complex and relevant looped thermoacoustic system. The modeling of mass and heat transfer between the stack and adjacent heat exchangers could benefit from the proper use and

understanding of these flow phenomena. They could be re-enforced to the enhance the heat transfer or minimized when there is significant heat loss. CFD simulations which have been validated are therefore essential during the design stages of thermoacoustic systems especially when aiming for high power and pressure amplitudes.

6.2 Future work recommendations

With the aim of further advancing research on oscillating flows and thermoacoustics some recommendations for future studies are described here.

- (i) Stereoscopic PIV systems that are an extension of the time resolved PIV system used in this work that includes two cameras that can capture all three components of the velocity field could be beneficial to identify any three-dimensional effects of the vortical structures near the stacks. Furthermore, POD analysis could be applied for the standing wave thermoacoustic system as well for comparison with the looped thermoacoustic system. In future studies experimental test setups or CFD models with different stack geometries like honeycombs or pin-array stacks and even higher pressure amplitudes where the unsteady flow behavior would heavily influence the performance of the system could be studied. Heat exchangers could be added to create significant temperature gradients across the stack. The CFD models could also be validated with temperature data from faster and more accurate measurement techniques such as Planar Laser Induced Fluorescence (PLIF).
- (ii) Developing a three-dimensional CFD model based on both the standing wave and looped thermoacoustic systems could be prove as a useful

tool to explore any unforeseen flow field morphology. But the main hindrance in this case would be being able to run the model in reasonable amounts of time and with minimum computational costs. Therefore efforts would need to be made in optimizing the user settings. This would particularly be useful to examine the turbulence at high Reynolds numbers.

- (iii) The thermoacoustic systems explored in this work were not designed with the aim of using as practical energy recovery technology but rather examining the underlying flow physics. In future efforts, similar investigations could be conducted on more application oriented systems like thermoacoustic systems that can utilize waste heat for cooling heating or power generation. These would include all components including stacks with spacing or pore size optimized for efficiency and also hot and cold heat exchangers to absorb and reject heat. In this more practical model which would be built up from our existing setup, thermodynamic cycle analysis would prove extremely useful. The performance measured in terms of COP or overall efficiency of the system and the flow of heat flux in the stack and the heat exchangers could be calculated using experimental measurements and analyzed in detail with comparisons to linear thermoacoustic model and CFD results.

Publications

Conference Papers

G. Harikumar, K. H. Ho, K. Wang, S. Dubey, and F. Duan. Thermoacoustic energy conversion in a square duct. *Energy Procedia* 158, 1811–1816 (2019). DOI: 10.1016/j.egypro.2019.01.425.

Journal Papers

L. Shen, G. Harikumar, K. Wang, and F. Duan. Flow visualization in a hybrid thermoacoustic system. *Experimental Thermal and Fluid Science* 125, 110374 (2021). DOI: 0.1016/j.expthermflusci.2021.110374.

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Appendix A

Photographs of experimental setup and equipment

The actual photographs of the standing wave thermoacoustic system and the looped thermoacoustic system designed and analyzed in this work are presented here along with the steel wool stack and the various equipment used for the different studies.

Figure A.1: Photograph of standing wave thermoacoustic system.

Figure A.2: Photograph of steel wool regenerator.

Figure A.3: Photograph of looped thermoacoustic system.

Figure A.4: Photograph of equipment used namely, (a) Dual pulse Nd-YAG laser (b) Synchronizer (c) Function generator (d) Aerosol generator (e) NI-6211 DAQ (f) NI-9214 DAQ (g) Power amplifier and (h) Loudspeaker.

Appendix B

PIVLab user settings

The user settings applied in PIVLab program [97–99] for the processing of the frames captured by the high speed camera is shown here. The program uses cross-correlation to calculate the velocity vectors with further options available for post-processing including calculating the vorticity.

PIV settings (CTRL+S)

Help:

Suggest settings

PIV algorithm

☒ FFT window deformation
 ☐ DCC (deprecated)

Pass 1

Interrogation area	Step
128	32
	= 75%

Pass 2...4

Interrogation area	Step
☒ Pass 2	
64	32
☒ Pass 3	
32	16
☐ Pass 4	
32	16

Sub-pixel estimator

Gauss 2x3-point

☐ Disable auto-correlation

Correlation quality

Normal (recommended)

Analyze current frame

Image pre-processing (CTRL+I)

☒ Enable CLAHE

Window size [px] 10

☒ Enable highpass

Kernel size [px] 5

☐ Enable intensity capping

☐ Wiener2 denoise filter

Window size [px] 3

☒ Auto contrast stretch

minimum: 0.047058

maximum: 0.67058

Preview current frame

Background subtraction GUI

Figure B.1: User settings for analysis using PIVLab.

Appendix C

Uncertainty Estimation

The value of experimental uncertainty for measured parameter Z is calculated by statistical analysis from a set of observations as given by

$$U = k \cdot \frac{\sigma}{\sqrt{n}} \quad (\text{C.1})$$

Here n is the number of sample observations, k is the coverage factor for 95 % confidence interval from the t-distribution table and σ is the standard deviation defined by

$$\sigma = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^n (Z - \bar{Z})^2} \quad (\text{C.2})$$

Where $\bar{Z} = \frac{1}{n} \sum_{i=1}^n (Z)$ is the mean of n repeated observations.