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A Frictional Contact-Pattern-Based Model for Inserting a Flexible Shaft into Curved Channels

Jiajun Liu, Lin Cao*, Muneaki Miyasaka, and Soo Jay Phee

Abstract— Flexible endoscopy and catheterization typically involve inserting a flexible shaft into a curved channel. Understanding the mechanics involved in the insertion process is crucial for the structural design, actuation, sensing, control, and navigation of these flexible medical tools. However, the ever-changing contacts and friction between the insertion shaft and the pathway make the mechanics complicated. Existing analytical models simplify the problem by neglecting the friction and assuming specific boundary conditions that are valid only in a few specific instances. In the meantime, FEM models have trade-offs between computation speed, accuracy, and stability. This paper presents an efficient theoretical framework to model the insertion process with friction, promoting fast and accurate computation of the mechanics involved. The inserting shaft is segmented based on the evolving contacts; system equations are formulated with friction-included force equilibrium and boundary conditions. The model is verified through experiments; channels with different shapes/curvatures were considered. The root-mean-square errors between the model and measured insertion forces are less than 0.055N (average percentage error less than 9.62%). This model will enhance the fundamental understanding of the insertion process's mechanics and benefit the engineering (design, actuation, and control) and medical practices of related medical tools (e.g., endoscopic instruments and catheters).

Index Terms—insertion, contact force, friction, quasi-static, modeling.

I. INTRODUCTION

In the medical field, using flexible medical instruments such as flexible endoscopes and catheters to access the human body has been a widely adopted approach for minimally invasive interventions. It will continue to be a strong trend for new technology development, e.g., the Monarch robotic catheter system (Auris Health Inc., US) and the ION robotic catheter system (Intuitive Surgical Inc, US), which have recently been approved for lung diagnosis. Maneuvering a flexible instrument or shaft into a curved pathway involves complex, evolving contacts and friction between the shaft and pathway and the large bending deformation of the flexible shaft. Thus, there is a general need to understand fundamentally, model, and simulate the interaction between the flexible shaft and the tortuous pathway during the insertion process. In this paper, "shaft" is a generalized term for flexible instruments such as catheters, endoscopes, or guidewires; the "channel" is for related anatomy or tool lumens such as blood vessels, respiratory tract, or the tool channels of an endoscope.

Modeling such a process is a fundamental scientific problem

that could also benefit both medical and engineering practices. For instance, these models could be used to construct a virtual simulator for training [1]–[3], to conduct pre-operation planning [4], and to help design the instruments [5], [6]. Virtual training allows the specialist to learn/practice/create new techniques without risking patients' lives and exposure to X-rays for catheterization and thus is safe, cost-effective, and efficient compared to physical trials using human cadavers and animals. The specialist could practice and plan the procedures on the model before the surgical operation. The instrument's performance could be evaluated conveniently using the model, and the iterative design could be performed without building the prototypes and conducting physical tests. These models could be applied for robotic catheter systems to calculate the instruments' states for haptic feedback or enhanced control [7]–[9].

The Finite Element Method (FEM) has been used to model the insertion of a flexible shaft into a curved channel. FEM is a versatile method that can simulate the non-linear interactions between shaft and channel with high-fidelity. The shaft is discretized into a series of finite beam elements for the insertion problem. External loads on the element nodes are calculated using the contact models based on the penalty method. FEM [10]–[12] have been developed to model the navigation of flexible needles in the tissue; studies [13], [14] modeled the interactions between flexible instruments and the blood vessel or endoscopic tool channel considering the nonlinearity of the material. Normal contact force and respective frictions could be calculated based on the plastic deformation of the channel [9], [13]–[15]. These forces are then used as the external loads on the nodes. FEM is a powerful tool for high fidelity modeling; however, the external loads on the element's body are ignored or approximately converted to the nodes. The accuracy of the simulation is heavily dependent on the resolution of the meshing. Thus, it results in a trade-off between accuracy and efficiency (computation time).

Insertion force has been theoretically formulated based on the gradient of the elastic energy under ideal assumptions (zero friction, zero clearance, and non-deformable channel) [16], [17].

With non-zero clearance, the shaft contacts the channel in different manners. Studies [18]–[22] simulated the quasi-static deformation of the flexible shaft in a confined space with loads and constraints defined on the shaft's ends. The shaft is divided

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into different elements based on how it contacts the channel. The shaft is modeled based on the contact locations and types (contact pattern), i.e., contact-pattern-based (CPB) models. Chen *et al.* [18] modeled the shaft deformation inside the circular channel with external loads defined at shaft ends. They also summarized the transformation of contact patterns when external loads change. Liu *et al.* [20] studied the deformation of the flexible blade inside the circular channel considering the static, rolling, and sliding friction at contacts. The rolling friction was observed when a sine wave signal was used as the input motion at one end while the other end was constrained. Denoël *et al.* [23] proposed a methodology to model the insertion process in a frictionless channel with the distal tip trajectory defined. In [19], a CPB model with the reference frame defined on the channel's centerline was proposed, simplifying the numerical solving process and achieving the same simulation results in [18]. Furthermore, the CPB method could search the prescribed contact patterns [23] or transform the existing contact pattern based on defined rules [19]. Huynen *et al.* [21] applied the CPB model to frictionless three-dimensional confined deformations and proposed the formulation for a flexible rod attached to a smooth surface [24]. In most studies, the shooting method and collocation method were applied to solve the system equations with boundary conditions. Liakou *et al.* [25] proposed an optimization method to solve the system equations for confined deformation problems and demonstrated the efficiency of simulating the shaft deformation inside a straight tube.

Most CPB models assume zero friction between the shaft and channel and define the distal tip's trajectory, which is not realistic for inserting a shaft into a hollow channel. Moreover, the channel is assumed to be rigid. The contact forces along the shaft are calculated from the deformation of the shafts in these CPB models. In contrast, the FEM model evenly meshes the shaft and uses the penalty method to calculate the contact forces at the nodes only. The shaft is modeled from one end to another using the chain algorithm [26] for both FEM [9], [14], and CPB methods [21], [27]. FEM methods simulate the insertion process with high fidelity and heavy computation load. In contrast, CPB models have better calculation efficiency than FEM [23], but are limited by the ideal assumptions and difficult to verify experimentally.

This study aims to develop a comprehensive model for the insertion process with realistic assumptions, allowing fast computation of the mechanics involved. A novel CPB method is proposed to model the flexible shaft's quasi-static insertion process into a curved channel. The main contributions are a) the proposed CPB model can generate the contacts, contact forces, internal forces, deformations, shapes of the shaft during the insertion under quasi-static motion; b) the friction and clearance are considered in this model while the tip is constrained by the interactions with the channel, which is more realistic and results in a well-determined system; c) various experiments and simulations are conducted to illustrate the accuracy and calculation speed of the model.

The remainder of the paper is organized as follows: Section II describes the methodology of this model. Section III

introduces the experiments and results for the verification of the proposed method. Section IV discusses the limitations and advantages of the proposed model. Finally, section V concludes the paper.

II. METHODOLOGY

The contact-pattern-based model formulates the shaft's deformation inside the channel based on their interactions (contacts' forces and contact pattern), which results in various constraints. These contacts' forces and contact patterns can be optimized to satisfy these constraints. A transformation process is employed to handle the changes of contact patterns during the insertion. First, we need to introduce the description of the shaft and channel.

A. Problem Description

Fig. 1 shows a channel with two straight ends and a middle arc section of radius R & span-angle Θ . The channel's wall is assumed to be rigid (non-deformable). A flexible shaft or rod with bending stiffness EI is inserted from the proximal end of the channel. The clearance, which is the difference between the channel inner diameter (ID) and the shaft outer diameter (OD), is σ . The shape of the channel is known. Thus, inclined angle θ and the coordinates of the centerline are known.

Figure 1. A deformed flexible shaft in that channel which has a constant-radius arc and straight ends.

In this paper, the shaft is treated as a curve-like elastic body, and its inner diameter is ignored. For a given point on the shaft at length $s \in [0, L]$, the inclined angles α is a function of s . The coordinate of this point could be expressed as $(x(s), y(s)) = (\int_0^s \cos \alpha(s) ds - D, \int_0^s \sin \alpha(s) ds)$. The shaft is inserted from the channel's proximal end with coordinates $(-D, 0)$ and zero inclined angle $\alpha_0 = 0$. The insertion speed is assumed to be constant along the x-axis.

It is assumed that Coulomb friction with the coefficient of friction (CoF) μ develops at all contacts. The shaft's mass is assumed to be negligible, leading to the negligible shaft's inertia. Thus, the quasi-static model is suitable for this problem. Moreover, the shaft is assumed to be inextensible and uniform in stiffness and density. The channel and shaft are horizontal; the mass/gravity of the shaft and the resulted friction force are negligible.

We also assume that the normal contact force N is perpendicular to the tangent direction of the channel and pointing to the channel's centerline. The friction force f is along the tangent direction but opposite the insertion direction.

B. Segmentation of the Shaft and System Equations

The shaft is segmented into several elements based on how it contacts the channel. As depicted in Fig 2, the discrete contact element (DCE) only touches the channel at one or both ends. In contrast, the continuous contact element (CCE) touches the channel's wall along its body. The sequence of DCE/CCE is the global contact pattern. The end nodes of the elements are indexed from 0 (proximal tip) to n (distal tip) for n elements.

Figure 2. Various contact types and contact elements. Macro images (a,b,d,e,f) were taken by a 5mp f/2.4 camera. (a) The discrete contact on the outer wall; (b) The discrete contact on the inner wall; (c) a DCE element with both ends on the inner wall; (d) the macro picture of left contact in (c); (e) the macro picture of right contact in (c); (f) a CCE element on the outer wall.

The shaft's internal forces at length s along the axial direction is $F_a(s)$ and the one along the radial direction is $F_r(s)$. The second subscript might be used as $F_{a,m} = F_a(s = L_m)$ to describe the axial forces at node $m \in [0, n]$. The superscripts '+' or '-' indicate the point approaches the node m as $F_{a,m}^+ = \lim_{s \rightarrow L_m^+} F_a(s)$ & $F_{a,m}^- = \lim_{s \rightarrow L_m^-} F_a(s)$. In the rest of this paper, the independent variable s is omitted for a clear presentation.

1) Nodes

There are three types of nodes, as shown in Fig.3. At the proximal end (node 0 in Fig. 3a), there are the insertion force F_{ins} , contact normal force N_0 and contact friction f_0 . And the internal forces ($F_{a,0}^+$, $F_{r,0}^+$) are balanced with external forces (N_0, f_0, F_{ins}) as:

$$\begin{aligned} F_{ins} - F_{a,0}^+ + f_0 &= 0 \\ -F_{r,0}^+ + N_0 &= 0 \end{aligned} \quad (1)$$

Fig. 3b describes force diagrams for middle nodes ($m \in [1, n-1]$). The force equilibriums are formulated by:

$$\begin{aligned} F_{r,m}^+ &= F_{r,m}^- + N_m \\ F_{a,m}^+ &= F_{a,m}^- + f_m \end{aligned} \quad (2)$$

In addition, these middle nodes are shared by adjacent elements and the shaft is tangential with the wall at these nodes, which leads to $\alpha_m = \lim_{s \rightarrow L_m^+} \alpha(s) = \lim_{s \rightarrow L_m^-} \alpha(s)$ and $\kappa_m = \alpha'_m = \lim_{s \rightarrow L_m^+} \alpha'(s) = \lim_{s \rightarrow L_m^-} \alpha'(s)$.

The force diagram for distal node n is shown in Fig.3c, the inclined angle at the shaft and the channel are α_n and θ_n respectively. The force equilibriums at node n are:

$$\begin{aligned} F_{r,n}^+ \cos(\theta_n - \alpha_n) - F_{a,n}^+ \sin(\theta_n - \alpha_n) + N_n &= 0 \\ F_{r,n}^+ \sin(\theta_n - \alpha_n) + F_{a,n}^+ \cos(\theta_n - \alpha_n) + f_n &= 0 \end{aligned} \quad (3)$$

The Coulomb friction model is applied to these nodes with CoF, μ . The direction of friction is defined along the opposite shaft's axial direction:

$$f_m = -\mu |N_m| \quad (4)$$

Figure 3. The force diagrams of nodes. (a) the proximal node, $m = 0$ (b) middle nodes $m \in [1, n - 1]$, (c) the distal node, $m = n$

2) Discrete Contact Element (DCE)

As shown in Fig. 4, the internal forces F_a in axial direction and F_r in the radial direction at any point $s \in [L_{m-1}, L_m]$ of the m^{th} element can be written as Eq. (5) using inclined angle α based on force equilibriums.

$$\begin{aligned} F_a &= F_{a,m-1}^+ \cos(\alpha - \alpha_{m-1}) + F_{r,m-1}^+ \sin(\alpha - \alpha_{m-1}) \\ F_r &= -F_{a,m-1}^+ \sin(\alpha - \alpha_{m-1}) + F_{r,m-1}^+ \cos(\alpha - \alpha_{m-1}) \end{aligned} \quad (5)$$

Figure 4. The force diagrams for the DCE (left) and its infinitesimal segment (right)

A linear relationship between bending moment M and curvature α' is used:

$$EI\alpha' = M \quad (6)$$

where E is Young's modulus, I is the second moment of inertia, and EI is the bending stiffness.

Then the bending moment balance for an infinitesimal could be formulated as:

$$-M' + F_r = 0 \quad (7)$$

Substituting Eq. (6) and Eq. (7) into Eq. (5) yields below governing ordinary differential equation (ODE) for the element.

$$\begin{aligned} -EI\alpha'' - F_{a,m-1}^+ \sin(\alpha - \alpha_{m-1}) \\ + F_{r,m-1}^+ \cos(\alpha - \alpha_{m-1}) &= 0 \end{aligned} \quad (8)$$

Initial boundary conditions ($\alpha_{m-1}, \alpha'_{m-1}$) and coefficients ($F_{a,m-1}^+, F_{r,m-1}^+$) are needed to integrate this ODE over the element length $L_m - L_{m-1}$.

3) Continuous Contact Elements (CCE)

The shape of continuous contact elements is identical to the channel wall's shape. The distributed load along the element is defined as $p(s)$. For an infinitesimal section shown in Fig. 5, the force and moment equilibrium equations are as below:

$$\begin{aligned} -\mu|p| - F_a' + F_r\alpha' &= 0 \\ -p + F_r' + F_a\alpha' &= 0 \end{aligned} \quad (9)$$

Eq. (7) is also valid for CCE, leading to the solution of radial force F_r . Substituting Eq. (6)(7) into Eq. (9) results in the governing equation for continuous contact arc:

$$-\mu[EI\alpha'''] - \mu[F_a\alpha'] - F_a' + EI\alpha''\alpha' = 0 \quad (10)$$

Since the element's shape is identical to the channel wall's shape, the inclined angle α , curvature α' , and higher-order differential terms of the inclined angle are considered known. Then, Eq. (10) is an ODE regarding the internal axial force, and it can be integrated over the element length $L_m - L_{m-1}$ with the initial boundary condition $F_{a,m-1}^-$. Moreover, for constant-radius shapes ($\alpha''' = \theta''' = 0, \alpha'' = \theta'' = 0$ and $\alpha' = 1/R$), Eq. (10) is simplified to $\mu|F_a\alpha'| = F_a'$ which is analytically solvable as $F_a = F_{a,m-1}^- e^{-\mu(s-L_{m-1})/R}$ [28].

Figure 5. The force diagrams for the CCE (left) and its infinitesimal segment (right)

4) The Chain Algorithm

Eq. (8) requires $F_{a,m-1}^+, F_{r,m-1}^+, N_{m-1}, \alpha_{m-1}, \alpha'_{m-1}$ as inputs, and Eq. (10) requires $F_{a,m-1}^+$ as input to integrate over the element length $L_m - L_{m-1}$. After the integration, the inclined angle & curvature can be obtained for DCE, and the axial force along the element could be obtained for CCE. The internal forces at the distal end $F_{a,m}^-$ & $F_{r,m}^-$ could be calculated using Eq. (5) or Eq. (9).

The inclined angle, curvature, and internal forces at m^{th} node, which is obtained by integrating governing equation of m^{th} element, are used as the inputs for the next element. With a guessed normal force N_m , the calculation could propagate to the proximal end. It is a typical chain algorithm [26]. The proximal node information (F_{ins}, α'_0, N_0 for DCE; F_{ins} for CCE) are required to initiate the chain algorithm.

These governing ODEs Eq. (8) & Eq. (10) generate the shaft shape based on the inputs provided. However, the inputs vary with different contact patterns. Therefore, the shaft's shape should satisfy various constraints which are more than or equal to the inputs to avoid an underdetermined system.

C. Constraints and Objective Function

Geometrically, the shaft should be contained by the channel as the assumed sequence of DCE/CCE. Meanwhile, the balances of forces and force moments need to be satisfied. They result in the constraints listed below:

(a) the distance from nodes to channel's wall d_m is zero

$$d_m = R_m - R + \frac{\sigma}{2} \cdot \text{sign}(R - R_m) = 0 \quad (11)$$

$$R_m = \sqrt{(x_m)^2 + \left(y_m - R - \frac{\sigma}{2}\right)^2}$$

where, $x_m = \int_0^{L_m} \cos \alpha(s) ds - D$ & $y_m = \int_0^{L_m} \sin \alpha(s) ds$ are the coordinates of m^{th} node on the shaft;

(b) the shaft's and channel's inclined angles at nodes are the same,

$$\theta_m - \alpha_m = 0, m \in [1, n-1] \quad (12)$$

(c) the curvature at the proximal node of CCE and respective channel wall's curvature, if m^{th} element is a CCE

$$\alpha'_{m-1} - \theta'_{m-1} = 0 \quad (13)$$

(d) the calculated external forces at the shaft's distal tip

should satisfy the Coulomb friction model,

$$f_n + \mu|N_n| = 0 \quad (14)$$

(e) the distal tip has zero curvature since no external bending moment applied,

$$\alpha'_n = 0 \quad (15)$$

Table I lists the unknown inputs and constraints for DCE/CCE at different locations of the shaft. Therefore, it can be concluded that the number of constraints is always equal to the number of unknowns for our proposed model.

Meanwhile, the shaft should follow the minimum total potential energy principle, leading to a non-negative objective function regarding the elastic energy of the shaft: $P_e = \min\left(\frac{1}{2} \int_0^L (\alpha'(s))^2 ds\right)$.

Type/location of element	Number of inputs	Number of constraints	Difference
One discrete contact	3 (F_{ins}, N_0, α'_0)	3 (Eq. 11,14,15)	0
One continuous contact	1 (F_{ins})	1 (Eq.14)	0
Discrete	first 4 ($l_1, F_{ins}, N_0, \alpha'_0$)	2 (Eq. 11,12)	2
	middle 2 (l_{m+1}, N_m)	2 (Eq. 11,12)	0
	last 1 (N_{n-1})	3 (Eq. 11,14,15)	-2
Continuous	first 2 (l_1, F_{ins})	0	2
	middle 1 (l_{m+1})	1 (Eq. 13)	0
	last 0	2 (Eq. 14,15)	-2

D. Optimizing for a given insertion length

The system is optimized for the unknown inputs with respect to the zero-value constraints and non-negative objective function at a given insertion length. We choose the non-linear least-square solver (LSQNONLIN) from MATLAB for easy implementation and a good balance between efficiency and accuracy [27]. Moreover, the shooting method [18] and the collocation method [21] could be applied to solve the system equations regarding the constraints. We also convert the inputs to their dimensionless form (with asterisks) using: $(l^*, x^*, y^*) = \frac{(l, x, y)}{R}$, $(F^*, N^*, f^*) = \frac{(F, N, f)}{EI/R^2}$. The fix-step Runge Kutta 4th order (RK4) method is used to integrate the governing ODEs. The step size for RK4 is 10^{-4} .

For the LSQNONLIN solver, the tolerances (the function tolerance, optimality tolerance, and step tolerance) are configured as 10^{-7} , 10^{-7} , and 10^{-7} ; the maximum number of iterations and the maximum number of function counts are configured to 20 & 100 respectively as the stop criteria. The algorithm for the solver is set to 'trust-region-reflective'. The limits for contact forces are set to $[10^{-6}, 30]$. The limit for the m^{th} contact location is set to $[\min(L_{m-1} + \beta_1\sigma, L_m - \beta_2\sigma), \min(L_m + \beta_2\sigma, L_{m+1} - \beta_3\sigma)]$, which is bounded by shaft length within $[D, L]$. The coefficients β_1, β_2 , & $\beta_3 \in [0.5, 32]$ have default value 4. If the upper or lower limits are reached, the coefficients are doubled or halved, respectively. All constraints with a coefficient 10^3 are used as objective functions of LSQNONLIN. The configuration of the workstation are: processor: Inter ® Core i5-7500; RAM: 16G; MATLAB ® R2018b.

E. Transformation of The Contact Pattern

At the beginning of the insertion process, the shaft is aligned

to the channel's outer wall. After the shaft enters the arc section, a single DCE is formed and treated as a cantilever beam. When the shaft touches the inner wall, a DCE-DCE contact pattern is formed, and it converts to CCE-DCE-DCE (CDD) immediately. In this process, the forces in the system are trivial, and the contacts of this CDD could be approximated by a straight line, which connects the start point, tangential with the inner wall and touching the outer wall [23]. Therefore, the CDD is used as the initial contact pattern. As the insertion length increases, the shaft develops various contacts with the channel. The contact pattern transforms as the insertion continues. The external loads at the shaft's tips cause the interaction between the channel and shaft, described by various global contact patterns [18], [19]. The global contact patterns were prescribed and searched in these studies, which is not optimal for the complex channels. Thus, we adopt the local transformation and focus on DCE and CCE only.

Figure 6. The illustration of three local contact pattern transformations.

Fig. 6 illustrates three local transformations. In transformation type **a**, the shaft is deformed to approach the channel's wall (refer to Fig. 2 b to Fig. 2 c). The appearance of the contact can be determined by checking the distance from the shaft to the channel's wall:

$$d_{distance}(s) = \left(R_{dis} - R + \frac{\sigma}{2} \cdot \text{sign}(R - R_{dis}) \right) \cdot \text{sign}(R - R_{dis}) + \varepsilon_d \quad (16)$$

where $R_{dis} = \sqrt{(x(s))^2 + (y(s) - R - \frac{\sigma}{2})^2}$ and $\varepsilon_d = \frac{\sigma}{100}$ is the tolerance. If a point on the shaft had a negative distance to the channel's wall, it is a contact point. Local transformation type **a** describes the case that extra contacts appear within a DCE, a node is added to this DCE and breaks it into two DCE.

As the insertion continues, the insertion force increases. Many extra contacts may develop near an existing contact node to form CCE, which is the local transformation type **b** (referring to Fig. 2c to Fig. 2d). The section from the node to extra contacts is considered as CCE first (Fig. 6b). If the CCE's distributed normal force: $p = +F_r' + F_a\alpha'$ from Eq. (10) does not satisfy that $p \cdot \text{sign}(R_{dis} - R) > 0$, the CCE could not form, and DCE should be applied as local transformation type **c** (referring to Fig. 2a to Fig. 2b).

These local transformations could be reversed in certain conditions. If the contact force ($N_m < \frac{EI}{R^2}/100$, for reversed Fig. 6a) or element length is near zero ($L_m - L_{m-1} < \sigma/100$, for reversed Fig. 6b&6c), the contact node is removed, and adjacent elements are merged.

The previous insertion step's contact pattern is adopted for a new insertion length. Then shaft can be optimized for various

inputs (unknowns) listed in Table I. The optimized results and respective shaft's shape may satisfy the constraints generated but violate the assumed contact pattern. The violations include having extra contacts, having near-zero normal force, and near-zero element length. Thus, the contact pattern transformations are essential to modify the assumed contact pattern. After the transformation, the optimization is performed again, and the optimized shape will be checked. If multiple locations satisfy the transformation condition simultaneously, the location with a larger curvature is considered first.

F. Special scenarios

There are three special scenarios during the insertion process, in which we simplify the DCE / CCE discussed.

Scenario 1: The shaft's first element is CCE with the channel's straight entry. The curvature for this element is zero, resulting in zero derivatives of axial force F_a' , the radial forces F_r and distributed normal force p . The insertion force F_{ins} transmits to the distal node without frictional loss. The F_{ins} is the sole input that initiates the chain algorithm.

Scenario 2: The shaft's last element is CCE with the channel's straight exit. Since there is no external bending moment on the distal end, the curvature is zero. It leads to zero value for the derivative of axial force F_a' , the radial forces F_r and distributed normal force $p(s)$. If a non-zero $F_{a,n-1}^+$ exists, with zero derivatives of axial force F_a' , the radial forces F_r and distributed normal force p , $F_{a,n}^- = F_{a,n-1}^+$ also exists and there are no external forces to balance it as $f = -\int \mu p ds = 0$. Then, the force balance cannot be achieved on the last element. Thus, $F_{a,n-1}^+$ should be zero. Then, the Eq.(14) and (15) hold for the last element's proximal node.

Scenario 3: There are discontinuities in curvature on CCE. The contact nodes are assigned to the points of curvature discontinuity. The Eq. (14) is applied to these points of discontinuity, leading to the solution of the normal contact forces N_m on these points. The only input is the axial force at the proximal node. Thus, it can be treated as one continuous contact.

G. Overview of the model

Fig. 7 presents the overview of the proposed model. There are three major steps:

a) Optimization: The shaft is segmented into a chain of elements based on the assumed contact pattern. Then, the shaft's governing ODEs can be formulated, and respective constraints/objective functions are generated. Finally, the LSQNONLIN solver solves the governing ODEs and the respective constraints/objective function with previous optimized results as initial guesses. If the LSQNONLIN solver did not reach the defined function tolerance at the end of optimization, the solving process would be repeated ten times at most.

b) Transformation: The transformation function checks the optimized results. The assumed contact pattern would be transformed, and step *a*) will be repeated until the optimized results match the assumed contact pattern.

c) *Insertion*: The insertion length increases. Then, a)-b) are repeated till the target insertion length is reached.

Figure 7. The flow chart of the model. The blue zone (optimize) is the core that forms and solves the system equations. The yellow zone (transformation) is the inner loop that transforms the contact pattern when necessary. The red zone (insert) is the outer loop that increases the insertion length.

Referring to Fig. 1, the shaft is aligned with the channel's outer wall, a CCE will be developed as the first element, and it falls into *Scenario 1*. The insertion force F_{ins} will be the sole input, and normal forces and element length will be affected by the insertion force. Thus, we could validate the proposed model by comparing the insertion force from experiments and simulation. Moreover, the channel has a straight end, which indicates that *Scenario 2* happens at the end of the insertion.

III. EXPERIMENTS AND RESULTS

A. Experimental Setup

A testbed was developed to measure the proximal insertion force. The channels with different bending angles, curvature, a single bend, or two bends were considered.

Fig. 8a shows the experimental setup, which consists of the motor block and the channel platform. The motor block inserts a flexible shaft into the hollow channel fixed on the platform. The hollow channel is set to different shapes by the channel restraints made from the acrylic board. Moreover, the channel restraints can be used as the channel directly, as shown in Fig. 8b. The motor block is mounted parallel to the x-axis of the platform. The distance between the motor block and the entry of the arc section is 100 mm.

Figure 8. (a) The platform's top view (b) The channel restraints with a protractor and clear acrylic cover.

Figure 9. (a) sectioned top view of motor block (b) the side view of the motor block and the force diagram.

Fig.9a shows the top view of the sectioned motor block. The motor (FAULHABER 2342S024C) and a passive encoder (SCANCON 2RM3600-D, 3600 pulse/rev) with the roller (MISUMI, ROGAN25-8-15, diameter 25 mm) or a pair of motors were used to insert the shaft.

The shaft is aligned to the channel's outer wall, and it is fed through the stabilizing tube (ID 1.5 mm, stainless steel, inside the motor block). The stabilizing tube is fixed in position by two pairs of bearings with grooves. This design reduces the shaft's vibration during the insertion. Fig. 9b shows the side view of the motor block. The slider (SSEBWZ16-110,

MISUMI) carries the motor block sandwiched by two loadcells (FUTEK LSB200, capacity 4.48 N for loadcell A and 8.96 N for loadcell B, accuracy 0.009 N). The readings from loadcell A and B were labeled as F_A & F_B respectively. The loadcells were preloaded to 1~3 N to ensure continuous non-zero measurement. The insertion force was calculated by $F_{ins} = F_A - F_B + f_{slider}$. The friction force of the slider f_{slider} was neglected, and the insertion force was calculated by $F_{ins} = F_A - F_B$.

The super-elastic nitinol rod was used as the flexible shaft. The bending stiffness EI of the nitinol rods was measured through the cantilever bending test. Two tubes (PTFE tube: inner diameter 2mm, CoF 0.06; FEP tube: ID 2mm & 3mm, CoF 0.09) and the acrylic constraints (CoF 0.10) were used as the hollow channel. The friction coefficients were measured by pulling the flexible shaft made from nitinol through a pair of grips made from the channel's material. The friction coefficient equals to half of the average pull force divided by the average of normal forces.

Figure 10. The various channel shapes are presented. The parameters of the "C" shape are listed in TABLE II. The "S" shape has a radius of 150mm. The centerline of varying curvature shape ("V" shape) is defined by a quadratic Bezier curve having the control points $(0, \sigma/2)$, $(0.14, \sigma/2)$ & $(0.15, 0.13 + \sigma/2)$

TABLE II

THE PARAMETERS OF C SHAPE

	Channel ID (mm)	Shaft OD (mm)	EI (Nm ²)	CoF	Radius (mm)	Span angle (degree)
Case A	3.3	1.3	0.010	0.10	100	180
Case 1	2	1.3	0.010	0.06	120	180
Case 2	2	1.3	0.010	0.06	120	90
Case 3	2	1.3	0.010	0.06	150	180
Case 4	2	1.3	0.010	0.09	120	180
Case 5	3	1.3	0.010	0.09	120	180
Case 6	2	1.0	0.003	0.06	120	180

As shown in Fig.10, three types of shapes were tested. 1) The channel consists of only one constant-radius arc with a straight start and end ("C" shape). The parameters related to the shaft & channel are listed in TABLE II. 2) The "S" shape channel had a positive 90-degree arc following by a negative 90-degree arc, and both arcs had a radius of 150 mm. 3) The "V" shape channel had a varying curvature in the arc section. The centerline of this variable curvature arc was constructed by a quadratic Bezier

curve with a straight start and end. The minimal radius of the shape was 0.125 m. For "S" shape and "V" shape, the other parameters were: shaft OD 1.3 mm, channel ID 2.0 mm, and friction coefficient 0.06.

First, we wanted to validate the "C" shape channel's contact pattern transformation (Case A). A protractor was fixed to the inner restraint to measure the inclined angle during insertion. The inclined angle of contacts and the insertion forces are compared with the proposed model's simulation. Then, experiments were performed on various "C" shapes (Case 1~6), "S" shape, and "V" shape channels. The insertion forces are compared with the proposed model. The experimental results are presented and analyzed in this section.

B. Validation of contact pattern on Case A

The contacts' inclined angles were measured using the protractor attached to the acrylic restraint. The insertion speed was 3.43 mm/s. The contact node inclined angles were measured every five seconds (round marker) and compared with the simulation result in Fig. 11.

Figure 11. The contact node inclined angles from the experiment (exp) and the proposed model (sim) for Case A.

The contact patterns observed during the insertion are depicted in Fig. 12 and illustrated in the supplementary video. Because the nodes 0 & 1 were within the channel's straight part, their inclined angles were always zero and not presented in Fig. 11.

The appearance of discrete contact (local transformation type **a**) happened at **b** & **d**; the development of DCE (local transformation type **c**) happened at **c**; the development of CCE (local transformation type **b**) happened at **e**. After the shaft's tip extending out of the channel's arc section at **f**, the CCE's length was reduced (**f-g**, between nodes 3 and 4). Node 7 extended along the channel's straight part (the inclined angles of node 7 and node 6 were overlapped in Fig 11 between **f** & **g**). After **g**, there was no change on all the contact node inclined angles. These local transformations and the other contact's inclined angles match the proposed model's results with an average error of 3-degree and a maximum error of 15-degree at node 4.

Node 5 moved from 3-degree to 165-degree on the channel's inner wall, while the tip (node 6) moved from 0 to 180 degrees on the channel's outer wall. Thus, the distal element swept through the most arc section (3-degree to 165-degree). In other words, the channel shape affected at least the distal element, which would be reflected in the insertion force. So, for example,

the tip extended out of the arc section at f , and the insertion force started to drop.

Figure 12. The contact pattern transformation is shown. The subfigure is matched with the insertion lengths labels in Fig. 11. The global contact pattern is presented using D & C as the abbreviation of DCE and CCE, respectively. For example, CDD is the abbreviation form of global contact pattern CCE, DCE, DCE. (a) before entering the arc section of the channel; (b) CDD; (c) CDDD; (d) CDDDD; (e) CDDCDD; (f) after exiting the arc section of the channel (g) CDDCDDC.

Figure 13. (a) The insertion forces from the experiment and the simulation were compared. The alphabet labels for insertion length are in line with the one in Fig. 11. (b) The gradients of the insertion forces from the experiment and the proposed model were compared.

At the same time, the insertion forces were measured and compared with the proposed model in Fig. 13a. The root-mean-square (RMS) error was 0.055N, while the average percentage error was 4.5%. The gradient of the insertion force is presented in Fig 13b. A low-pass filter of cutoff frequency 0.1Hz was applied to the experimental data. The a - g were marked at the same insertion length as Fig 11. The CCE at point e appeared

earlier than the model, as shown in Fig 13b and Fig 11. Large changes in gradients were identified from the force plots at b, c, d, e, g, f , when the contact patterns transform or the channel shape changed, while the insertion force was smooth at points a . In this case, we can conclude that some of the contact pattern changes and shape changes could be observed from the insertion force.

C. Insertion into "C" Shapes (Case 1~5)

In these tests, PTFE and FEP tubes were fixed inside the channel restraints as the channel. They have a CoF of 0.06 and 0.09 within the reported CoF 0.04 ~ 0.2 (between tissue and medical instruments [29], [30]). It was hard to observe the contacts through the tubes. Thus, we only measured the insertion forces and evaluated the proposed model based on them.

Figure 14. The experimental insertion forces were compared with the simulation result for various C shape channels.

TABLE III
THE ERRORS OF C SHAPE CASES

	Case A	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
RMS error (N)	0.055	0.040	0.043	0.048	0.046	0.052	0.020
Average % error	4.5%	3.12 %	3.49 %	3.48 %	2.80 %	5.52 %	6.53%

The experimental and simulation results and relative errors are presented in Fig. 14 and TABLE III. In general, the proposed model simulated the insertion forces with an RMS error less than 0.055 N. Moreover, Case 1, 3~6 had the same global contact pattern transformation as Case A. In Case 2, the shaft extended out of the channel before forming CCE at the middle.

D. Insertion into "S" Shape & "V" Shape

As depicted in Fig. 15, the measured insertion force was similar to the proposed CPB method in the "S" shape channel with an RMS error of 0.044N and an average percentage error of 3.98%. At the first 90-degree (before point a), the "S" shape insertion force was nearly the same as the "C" shape (Case 3) insertion force with an RMS difference of 0.038N. Fig. 16 shows the simulated and measured insertion force for the "V" shape channel. The experimental insertion force matched the proposed method's simulation with a 0.037 N RMS error and 9.62% average percentage error.

Figure 15. The experimental insertion force of the "S" shape channel was compared with the proposed model.

Figure 16. The experimental insertion force of the "V" shape channel was compared with the proposed model. The maximum curvature of the channel was 9.4 m^{-1} .

IV. DISCUSSION

There are good agreements in the contact patterns and the insert forces from the results presented. Then, we would like to discuss some features, advantages, and limitations of the proposed method.

The proposed model simulated the interaction between the shaft and channel in contact patterns without additional sensory inputs. The simulated transformation of contact patterns matched with experiments, and the local transformations **a**, **b** & **c** could be observed. There was a reduction of CCE's length in Fig.12 f-g, but it did not shrink to zero. The reverse processes of local transformations **a**, **b** & **c** did not appear in the experiments, but they should be easily observed in the extraction process. Moreover, this method could be applied to the extraction process, which is part of future work.

Some features in the insertion force profile could be explained using the proposed model. In Fig 15 and the supplementary video, the insertion force started to drop after point **a** and formed a valley between **b** and **c**. In the simulation, the shaft's tip traveled from one side to the other for the S shape channel after the first 90-degree arc section. The insertion force dropped at **a-b** because the following -90-degree channel provided space for the shaft to recover slightly towards its neutral shape. Then, the insertion force was nearly unchanging

at **b-c**, at which the proposed model showed that the contacts inside the first 90-degree were stable when the tip traveled from one side to the other side. At **c**, the tip touched the other wall, and the insertion force started to rise. Then, the shaft was deformed to the other side to pass through the channel resulting in a rise after point **c**. The insertion forces from the experiment and simulation were very close for the "S" and "V" shape channels, showing the proposed method's ability and potential to handle complex shapes (multiple arc sections and varying curvature).

Figure 17. The simulation results of various step sizes (0.1 mm, 1 mm, and 10 mm) for C shape case 2 are shown. The radius R is 0.12 m. The inset depicts the details between insertion length 0.2m to 0.22m.

Referring to Fig. 17, the proposed method took 1 minute, 5 minutes, and 27 minutes to simulate the insertion on "C" shape case 2 for step sizes 10 mm, 1 mm, and 0.1 mm, respectively. While FEM model that we constructed based on [9] took 31 mins for 12 elements and 153 mins for 20 elements to simulate the insertion on the same case. The proposed model is stable over various step sizes and yield the same numerical values. Moreover, the adaptive step-size scheme based on optimization parameters could be applied to improve efficiency. The proposed method is developed from frictionless CPB method [18], [19], [21], [23], considering the Coulomb friction under quasi-static motion [31] and relaxing the requirements on trajectory or position at distal tip [20]–[22], [24], [25], [27]. We have illustrated that this insertion problem is well-defined. The number of constraints is sufficient to ensure the convergences on various step-sizes.

The theoretical model proposed in [17] assumed that the channel is frictionless and has zero clearance. The insertion force modeled is $F_{ins} = EI\kappa^2/2 = EI/2R^2$, which had a dimensionless value $F_{ins} R^2/EI = 1/2$. By setting the CoF to zero and gradually reducing the clearance from 1.0 mm to 0.1mm, the simulation results were presented in Fig. 18 with the theoretical values. As the clearance reduced from 1.0mm to 0.1mm, the dimensionless insertion force $F_{ins} R^2/EI$ from the

proposed method converged towards the theoretical value.

Figure 18. The comparison of simulations and theoretical model in the frictionless channel (radius 150 mm and span angle 90-degree). For the theoretical model, the clearance was zero. For simulation, the clearances were 1.0, 0.7, 0.4, and 0.1 mm

The super-elastic nitinol rods used approximately meet some of the ideal assumptions. The nitinol wire weighed 7.5g, which suits the negligible weight assumption. The measured bending deflection of the 1.3mm diameter nitinol wire at the middle point was 11 mm, which is 1.24% of its 887 mm length. Although the deformation is small, it may result in errors like CCE's early appearance in Fig. 11, point *e*. Moreover, the motor block could clamp and insert without slip at a maximum insertion force of 2 N, limiting the selection of channels' materials and design. The testbed was designed to measure the insertion force because it is the sole energy input to this planar system and triggers the changes in the normal forces and the elements' length. Thus, we focused on the insertion force for horizontal planar cases as explained at the end of Section II G. In addition, *Scenario 1* does not always hold for three-dimensional channels. Thus, other loads at the proximal end (torque, moment, and normal forces) must be included in three-dimensional cases.

V. CONCLUSION

In this work, a quasi-static contact-pattern-based framework was proposed to model the insertion of a flexible shaft into a curved channel considering both friction and clearance. By segmenting the shaft based on the contacts between the shaft and channel, the model efficiently finds the evolving contact pattern, external forces, and the shape of the insertion shaft while satisfying various constraints. In addition, the shaft's behavior during the insertion could be modeled efficiently and accurately compared with the experimental results.

The advantage of the proposed model over existing mathematical models is that friction is realistically considered, and no pre-defined trajectory or constraint at the tip of the shaft. Besides, this model could solve the insertion process at various step-sizes, depending on accuracy and computation speed requirements. Finally, as demonstrated, this model enables fundamental understanding and analysis of the insertion problem by generating detailed information of the insertion process, e.g., the evolving deformation, stress, contact locations, and contact forces of the curved shaft lumen.

From the application point of view, we believe this model could benefit medical and engineering practices such as pre-operative planning, intervention simulator for training, design, and control of flexible tools such as manual/robotic catheters and flexible endoscopes [32]–[35]. The proposed model could obtain the contact forces and location information to avoid potential damage to the tissue during insertion. In addition, the contacts' information could be fused with other sensory inputs (e.g., position and orientations from magnetic tracking sensors) to improve the maneuverability of instruments.

Several limitations exist in this proposed model. (1) The channel is assumed rigid, as many other models did, while a channel made of biological tissue may have viscoelasticity; (2) The Coulomb friction model is suitable for the sliding friction in this work. However, manual insertion is a dynamic process requiring insertion, rotation, and extraction. More sophisticated friction models [9] could be explored to handle static friction and the transition from static to sliding friction. (3) Shear and torsion effects are neglected; the shear effects in the targeted scenario are minor, while the torsion effect may be significant when the inserting shaft (e.g., a catheter) is expected to transmit torques and axial rotation. (4) Only two-dimensional cases are considered. The presented examples and experiments are planar scenarios; the same methodology can be extended to three-dimensional cases. Future studies in these areas will be pursued.

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