Fatigue crack growth studies on welded joints with multiple weld toe cracks

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SINGAPORE

**FATIGUE CRACK GROWTH STUDIES ON WELDED
JOINTS WITH MULTIPLE WELD TOE CRACKS**

TSANG KIN SHUN

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

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JOINTS WITH MULTIPLE WELD TOE CRACKS**

TSANG KIN SHUN

Tsang Kin Shun 2021

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

2021

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Tsang Kin Shun

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

15 June 2021

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Date

Pang Hock Lye John

Authorship Attribution Statement

This thesis contains material from 3 paper(s) published in the following peer-reviewed journal(s) in which I am listed as an author.

Chapter 3 is published as J. H. L. Pang, K. S. Tsang, and H. J. Hoh, “3D stress intensity factors for weld toe semi-elliptical surface cracks using XFEM,” *Marine Structures*, vol. 48, pp. 1–14, Jul. 2016.

The contributions of the co-authors are as follows:

- Prof John Pang Hock Lye conceptualized the initial project direction.
- I obtained the SIFs from the Finite Element Modelling and conducted the analysis.
- I prepared the manuscript drafts that were revised by Dr. Hoh.

Chapter 3 also contains material from H. J. Hoh, J. H. L. Pang, and K. S. Tsang, “Stress intensity factors for fatigue analysis of weld toe cracks in a girth-welded pipe,” *International Journal of Fatigue*, vol. 87, pp. 279–287, Jun. 2016.

The contributions of the co-authors are as follows:

- Prof John Pang Hock Lye conceptualized the initial project direction.
- I conducted the analysis.
- I revised the manuscript drafts that were prepared by Dr. Hoh.

Chapter 4 is published as J. H. L. Pang, H. J. Hoh, K. S. Tsang, J. Low, S. C. Kong, and W. G. Yuan, “Fatigue crack propagation analysis for multiple weld toe cracks in cut-out fatigue test specimens from a girth welded pipe,” *International Journal of Fatigue*, vol. 94, no. Part 1, pp. 158–165, Jan. 2017.

The contributions of the co-authors are as follows:

- Prof John Pang Hock Lye conceptualized the initial project direction.
- I wrote the program to incorporate the multiple fatigue crack algorithm and conducted the analysis.

- I was involved in the specimen preparation and conduct of the Stress-life curve tests at DNV GL Laboratory, Singapore with the help of Mr. Low, Mr. Kong and Mr. Yuan.
- I revised the manuscript drafts that were prepared by Dr. Hoh.

Chapter 6 is published as Y. Liu, K. S. Tsang, H. J. Hoh, X. Shi, and J. H. L. Pang, "Structural fatigue investigation of transverse surface crack growth in rail steels and thermite welds subjected to in-plane and out-of-plane loading," Engineering Structures, vol. 204, p. 110076, 2020.

The contributions of the co-authors are as follows:

- Prof John Pang Hock Lye conceptualized the initial project direction and edited the manuscript drafts.
- I conducted the analysis together with Dr. Liu and revised the manuscript drafts that were prepared by Dr. Liu, together with Dr. Hoh and Dr. Shi.
- I developed the fatigue crack growth and life prediction algorithm.

14 June 2021

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Abstract

This PhD study examines fatigue crack growth analysis and testing case studies of multiple surface cracks for assessing fatigue cracks propagation life of welded joints with surface cracks at weld toe regions. Fatigue analysis was performed based on fracture mechanics concepts using the crack propagation driving force parameter of stress intensity factor (SIF). SIFs were obtained for plain plate and welded joint specimens with semi-elliptical cracks. XFEM was employed for improving the model mesh generation and insertion of crack front sub model, allowing for an approximation of cracks without the need for the mesh to follow the crack as in conventional crack modelling techniques, significantly simplifying the modelling of cracks. The importance of partitioning of the model and orthogonality of the crack in XFEM was demonstrated, and the SIFs obtained by XFEM were compared to benchmark solutions. The M_k factor solutions were used in a fatigue life prediction model to predict the growth of fatigue cracks at weld toe regions and compared with experimental fatigue test data. A multiple crack recharacterization method incorporating SIFs and M_k factor solutions was developed as a MATLAB program for fatigue crack propagation.

S-N curve tests and FE modelling were conducted for welded pipe cut-out specimens, cruciform joints and rail welded specimens. For welded pipe cut-out specimens, an improved fatigue life analyses for multiple cracks was developed by considering the crack coalescence stage and fatigue crack closure. The prediction achieved a better fit to the tested fatigue life S-N curve results. An algorithm was developed for multiple fatigue crack propagation and demonstrated for a large number of initial (a_i) starting weld toe surface

cracks (12 to 22). The analysis provided details of the fatigue growth history of each crack, their sequence of coalescence and the evolving crack shape development (a/c vs a/t) to a dominant single, final (a_f) crack before fracture.

For cruciform joints, both tension loading and four-point bending tests were carried out to understand weld toe failures. XFEM was used to model a semi-elliptical surface crack to obtain SIFs and M_k factor solutions, which showed good agreement with established solutions. A procedure for fatigue crack growth and life prediction for cruciform joint was developed similar to that used for the welded pipe cut-out specimens, and the prediction was close to the tested fatigue life for tension loading. The algorithm of the multiple fatigue crack propagation for cruciform joints demonstrated the predicted fracture surface with 13 initial cracks presented the most detailed and best approximation of the experimental fracture surface, as compared to 5 or 1 crack.

For rail specimens, inverted four-point bend tests were conducted to determine the fatigue strength of rail welded specimens. The results showed that the fatigue life of a specimen in the inverted fatigue test was about 3X that of the conventional test. From the fracture surfaces, it could be seen that the crack initiated from the head-to-web transition area along the edge. SIF solutions of rail specimens were developed based on 3D XFEM Model, and M_k factor solutions were obtained. A more powerful scheme to account for crack propagation based on SIF known as the coupled root mean square (RMS) method proposed by Cruse and Besuner was introduced and implemented. The fatigue life prediction provided a good fit to the experimental fatigue test data,

and the predicted fracture surfaces were shown to closely resemble the actual fracture surfaces.

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List of Symbols

θ	Weld angle
a	Crack depth (mm)
a_i	Initial crack depth (mm)
a_f	Final crack depth (mm)
b	Plate half width (mm)
c	Crack half width (mm)
h	Plate half length (mm)
h_{att}	Attachment height (mm)
t	Attachment thickness (mm)
K	Stress Intensity Factor
L	Attachment footprint width
M_k	Weld toe magnification factor
T	Main plate thickness (mm)
N	Fatigue life cycle count
C	Material constant exponent in the crack propagation equation
m	Material constant exponent in the crack propagation equation
ΔK_{th}	Threshold Stress Intensity Factor
E	Modulus of Elasticity (GPa)

Chapter 1 Introduction

1.1 Background and Motivation

Fatigue crack growth is one of the most important causes affecting the long term structural integrity of engineering structures subjected to cyclic loading, such as in offshore and marine structures. This is particularly important in welds, which are susceptible to fatigue damage initiation when cyclically loaded [1]. In order to perform a safe and reliable assessment of the fatigue life based on damage-tolerant design approaches, fatigue crack growth predictions must be included in the assessment.

In welded joints, the geometry in weld toe region is a common fatigue crack initiation site along the weld toe line. In the case of lower quality weld processes, defects such as lack-of-fusion, porosity or slag intrusions may be present. This combination of weld toe stress concentration and defects provide ready sites for crack initiation of multiple weld toe surface cracks, which propagate and coalesce to form combined surface cracks and propagate until a critical crack size to failure.

Fatigue cracks can initiate either at the weld toe [2–4] or the weld root [5–9], at locations with high stress concentration for partial penetration fillet weld specimens. The sharp transition in geometry in the weld toe acts as a stress raiser for the formation of weld toe cracks, although weld root cracks may also initiate from the ends of non-welded root faces [2]. The type of crack that occurs depends on many factors, including the throat to plate thickness ratio, type of loading, residual stresses and weld defects at the weld toe and weld root (Figure 1-1).

Figure 1-1 Illustration of possible crack locations in a cruciform joint [2]

The application of fracture mechanics to weld fatigue can be roughly divided into two approaches [10]. The first approach is to use classical *damage tolerance* analysis to account for large cracks and defects, a so-called long crack in the range of millimeters. These large cracks are above the detection limits of the non-destructive testing method used in quality control and hence can be detected after welding or in service. This kind of flaws can be minimized with proper manufacturing and control and is well understood. The second approach, commonly known as the *safe life* philosophy, accounts for the small cracks and defects in total life and strength of the welded part. Instead of removing all defects that invariably exists in welded structures, a ‘fitness for purpose’ design philosophy must be adopted to define methods of assessing the significance of flaws, particularly in the context of fatigue [11].

The field of linear elastic fracture mechanics and fatigue crack propagation is applied to determine the remaining lifetime starting from an initial weld defect. The application of fracture mechanics in fatigue crack propagation analysis usually requires an initial crack, that is a relevant weld toe defect and it neglects the fatigue crack initiation part as a pre-existing crack is expected to propagate till fracture. When the fatigue initiation stage is short compared to the overall fatigue crack propagation

life, fracture mechanics can be applied to predict the remaining fatigue life [12].

Having this in mind, the following issues still have to be solved:

- In order to perform fatigue crack growth predictions, stress intensity factors (SIFs) must be calculated or obtained through numerical methods
- Multiple crack propagation at stress levels above the fatigue limit has to be taken into account
- A fatigue life prediction algorithm is needed to perform the actual fatigue crack growth predictions

Stress Intensity Factors

As it is well known, the main problem concerning the characterization of cracks and their evolution under repeated loading in the context of Linear Elastic Fracture Mechanics is the determination of the SIF at each point on the crack front [13]. Empirical SIFs equations for simple geometries are readily available in various standards [14]. For more complex geometries, different approaches have been proposed in the literature to determine the SIF: analytical methods [15–18], the weight function method [19–24], the body force method [25], empirical relationships [26–28], and numerical methods, such as the finite element method (FEM), the line spring FEM approach [29], the stiffness derivative method [30], and the virtual crack closure technique [31]. More recently, other methods have also been applied, such as the dual boundary element method [32] and the meshless methods [33].

For welded components with complex geometrical shapes, numerical finite element modeling needs to be carried out for accurate SIFs. More recently, the eXtended Finite Element Method (XFEM) was developed to allow for a significantly simplified modelling of cracks by enabling local enrichment of approximation spaces using the

concept of partition of unity [34]. While other methods of numerical simulation have been developed, the traditional finite element method remains the most common method in this area of research.

Multiple crack propagation

Surface cracks can propagate under repeated loading. During crack growth, a critical crack size can be reached, which leads to a catastrophic failure of the flawed structural components. Investigations have been made to study the interaction of static adjacent cracks [13,35–42]. The investigations above agree that interaction of adjacent cracks exist, but the degree of interaction depend on factors such as crack dimensions, aspect ratio and separation distance. The SIFs obtained from these investigations also cannot be used for fatigue growth analysis. In the majority of cases for various welded joints, the currently used weld toe magnification factors do not adequately describe the behavior of weld toe cracks.

A typical feature of weldment fatigue is multiple crack initiation along the weld toe [43]. Cracks are nucleated and grow at different positions e.g. along the weld toe [44,45]. When these cracks grow to a certain size, they coalesce to form larger cracks. It is clear, that this pattern will have an impact on the lifetime of the component, because of which it must be taken into account [43].

Fatigue Life Prediction Algorithm

Taking into account the relationship between the fatigue crack growth rate $\frac{da}{dN}$ and the cyclic stress intensity ΔK using the Paris-Erdogan Law [46], a life prediction algorithm is needed to perform the actual fatigue crack growth predictions. Currently, most algorithms available in standards or commercially only considers large cracks. These algorithms do not take into account the smaller, multiple cracks that initiate in

areas of high stress such as at weld toes. The fatigue life of these smaller, multiple cracks takes up a major part of the total fatigue life of the structure, and only merge to form large cracks in a later phase.

1.2 Objectives of Current Research

The objectives of the research work include the following:

1. To develop a multiple fatigue crack growth algorithm for multiple surface cracks.

This involves Study No. 1: XFEM calibration studies for SIF solutions and development of multiple surface crack algorithm, which aims to (a) Explore the use of XFEM to obtain SIF solutions for plain plate and M_k factor solutions for fillet welded T-butt joint plate with semi-elliptical crack (b) Calibrate XFEM model by comparison with benchmark cases and established solutions, as well as a mesh refinement study (c) Develop an algorithm for multiple crack recharacterization method (d) Incorporate SIF solutions and multiple crack recharacterization method into a MATLAB program to conduct fatigue life prediction with multiple cracks.

2. To study the fatigue crack growth behavior for welded pipe cut-out specimens with multiple weld toe cracks. This involves Study No. 2: Fatigue investigations of welded pipe cut-out specimens, which aims to (a) Conduct fatigue $S-N$ curve tests for welded pipe cut-out specimens under four-point bending (b) Obtain new SIFs using 3D XFEM model for semi-elliptical surface crack in welded pipe cut-out specimens (c) Improve the fatigue life prediction of welded pipe cut-out specimens with multiple weld toe cracks by considering crack coalescence and fatigue crack closure (d) Compare fatigue life prediction with experimental fatigue

- crack growth data (e) Develop multiple fatigue crack growth algorithm to study details of the fatigue growth of each weld toe crack and their coalescence history
3. To investigate the *S-N* fatigue curve behavior of cruciform joint specimens with weld toe cracks subjected to tension and bending loading. This involves Study No. 3: Fatigue study on cruciform welded joint specimens, which aims to (a) Understand weld toe failures of cruciform joint specimens by fatigue *S-N* curve tests for cruciform joint specimens to (b) Study the modelling of a semi-elliptical surface crack in cruciform joint to obtain new SIFs and M_k factor solutions (c) Develop procedure for fatigue crack growth and life prediction for cruciform joint (d) Study the effect of number of initial cracks used in the multiple fatigue crack growth algorithm and compare with experimental fatigue crack growth data.
 4. To study the fatigue behavior of thermite welded rail specimens using *S-N* fatigue curve tests and fatigue life prediction methods. This involves Study No. 4: Fatigue study on thermite welded four-point bending fatigue inverted specimen test and fatigue life prediction, which aims to (a) Understand fatigue failure of head-to-web weld toe cracks by conducting inverted four-point bending *S-N* curve tests of thermite welded rail specimens (b) Investigate the effect of in-plane and out-of-plane stresses on SIFs of thermite welded rail specimens using 3D XFEM (c) Improve fatigue life prediction method using coupled root mean square method to account for crack shape development (d) Compare fatigue life prediction with experimental fatigue *S-N* curve crack growth data (e) Develop multiple fatigue crack growth algorithm specifically for use in thermite welded rail specimens with weld toe crack at head-to-web area.

1.3 Scope of Current Research Work

The framework of the multiple fatigue crack growth algorithm is shown in Figure 1-2.

Figure 1-2 Framework for multiple fatigue crack growth algorithm

Based on the framework, fatigue analysis is performed based on fracture mechanics concepts with crack propagation driving force parameter of stress intensity factor (SIF). Numerical finite element modeling needs to be carried out for accurate SIFs, especially for complex geometries. The new SIFs will be compared with benchmark cases and established solutions. XFEM will be used for numerical finite element modeling.

The SIFs obtained through FEM for each welded geometry will be integrated into a crack growth model for use in fatigue life prediction. The fatigue life prediction will be compared with experimental data of different geometries to prove its effectiveness and accuracy. S-N Curve Tests will be conducted for welded pipe cut-out specimens, cruciform joints and rail specimens. With a range of materials and weld geometries, material properties and behavior of loading conditions are also needed as inputs into the prediction. The fatigue life prediction will be compared with these experimental data.

A multiple crack recharacterization method will be developed to determine the best method of crack coalescence for surface cracks. The separation distance between cracks will be considered, as well as the effect of multiple cracks. The fracture surfaces will be examined and the starting locations of multiple initial cracks will be used as inputs into the algorithm. These data and algorithm will then be incorporated into a MATLAB program, where a predicted fracture surface can be generated and compared with the experimental fracture surface.

1.4 Organization of the Thesis Structure

This thesis is partitioned into seven chapters. Chapter 1 gives a general introduction to the study, while Chapter 2 deals with a review of the existing literature, covering the fatigue of welded joints and fundamental of fatigue mechanisms and life prediction. It begins with a brief look at significant factors affecting fatigue of welded joints and a review of methods of improving the fatigue strength of welded joints. Next, the fundamentals of fracture mechanics-based fatigue life prediction is examined, starting from short cracks to large cracks. In addition, a review on the interaction of multiple surface cracks is given, ending with a look at typical structural components with surface cracks.

Chapter 3 presents Study No. 1: XFEM calibration studies for SIF solutions and development of multiple surface crack algorithm. It starts out with a parametric study using 3D XFEM to obtain new SIFs and M_k factor solutions, based on a plain plate and fillet-welded T-butt joint plate, which are also compared with benchmark cases and established solutions. A novel algorithm for multiple crack propagation is demonstrated and fatigue life prediction is conducted using an algorithm programmed in MATLAB.

Chapter 4 looks into the fatigue crack growth behavior for welded pipe cut-out specimens with multiple weld toe cracks with Study No. 2: Fatigue investigations of welded pipe cut-out specimens. Fatigue S - N curve tests are conducted for welded pipe cut-out specimens and new SIFs are obtained using 3D XFEM. An improved fatigue life prediction algorithm is used to in the prediction of fatigue life and a multiple fatigue crack growth algorithm provides details of the fatigue growth of each weld toe crack and their coalescence history.

Chapter 5 investigates the $S-N$ fatigue curve behavior of cruciform joint specimens with weld toe cracks subjected to tension and bending loading, through Study No. 3: Fatigue study on cruciform welded joint specimens. Tension and bending loading fatigue $S-N$ curve tests are conducted to study weld toe failures of cruciform joint specimens, and the modelling of a semi-elliptical surface crack in cruciform joint to obtain new SIFs and M_k factor solutions will be studied. The multiple fatigue crack growth algorithm is applied, and the results are compared with experimental fatigue crack growth data.

Chapter 6 presents the study of the fatigue behavior of thermite welded rail specimens using $S-N$ fatigue curve tests and fatigue life prediction methods, involving Study No. 4: Fatigue study on thermite welded four-point bending fatigue inverted specimen test and fatigue life prediction. The results of the inverted four-point bending $S-N$ curve tests of thermite welded rail specimens are shown, which demonstrates fatigue failure of head-to-web weld toe cracks. The SIFs solutions of thermite welded rail specimens using 3D XFEM are presented and these SIFs are incorporated into a fatigue life prediction algorithm. A multiple fatigue crack growth algorithm specifically for use in thermite welded rail specimens with weld toe crack at head-to-web area is also demonstrated.

Chapter 7 provides the overall conclusion of this research and recommendations for future works.

Chapter 2 Literature Review

This chapter presents a review of the existing literature, covering the fatigue of welded joints and fundamental of fatigue mechanisms and life prediction. It begins with a brief look at significant factors affecting fatigue of welded joints and a review of methods of improving the fatigue strength of welded joints. Next, the fundamentals of fracture mechanics-based fatigue life prediction are examined, starting from short cracks to large cracks. In addition, a review on the interaction of multiple surface cracks is given, ending with a look at typical structural components with surface cracks.

2.1 Fatigue of welded joints

2.1.1 Introduction

Fatigue failure accounts for up to 90% of failure in welded structures and joints, and the fatigue strength of welded components are often much lower than parent metals. For example, Figure 2-1 shows the fatigue strength of high tensile structural C-Mn steel, for the plain steel plate, notched plate, and plate with fillet welded attachment. As can be seen, the fatigue endurance limit for the plain steel plate and notched plate is at 300 MPa and 200 MPa respectively, but much lower for the plate with fillet welded attachment at about 30 MPa.

In the industry today, many different codes and practices exist for welded structures to minimize fatigue failures. However, many problems and failures are still out of the scope of these codes and practices. A good understanding of the fatigue behavior of welded joints is crucial in understanding the cause of their failure.

Figure 2-1 Comparison between fatigue strengths of plain steel plate, notched plate, and plate with fillet welded attachment. Steel to BS 4360, Grade 50B [47]

2.1.2 Significant Factors Affecting Fatigue of Welded Joints

Stress concentrations and weld defects

Apart from inherent flaws at weld toe, other imperfections, such as welding defects and misalignment, can occur in welded joints and may influence their mechanical integrity [43,48]. Welding flaws include porosity, slag inclusions, lack of fusion and incomplete weld root penetration. Such flaws provide additional stress concentrations which, if not more severe than those already arising from the basic weld geometry, will produce alternative sites for fatigue crack initiation and could therefore lead to a reduction in fatigue life, especially in relatively high fatigue strength joints. ISO 5817 [49] provides a list of up to 26 different types of weld imperfections, which can be summarized in Table 2-1.

Table 2-1 Weld imperfections according to ISO 5817 [49]

Defects	Examples	Importance
Cracks and crack-like imperfections	Hot and cold cracks, lack of fusion, incomplete penetration	Must be avoided or immediately subject to fracture mechanics analysis
Material imperfections which act as crack initiation sites	Slag inclusions	Paramount importance for fatigue strength and fatigue life analyses

Geometric discontinuities which increase the local stresses	Misalignment such as angular distortion or undercuts	Lower importance but may affect crack initiation and propagation
Minor imperfections	Porosity in low stress sections of the component	Little or no effect on fracture or fatigue life

Some authors [50–55] have pointed out that ISO 5817 is not meant for assessment of welds for fitness for service, but is instead meant to improve the overall quality of welding. For the purposes of crack initiation and fatigue life assessment, BS 7910 [14] can provide a better guide to assessing if flaws in metallic structures (which includes welded joints, are acceptable.

Material properties

The difference between the fatigue behavior of welded and unwelded materials is the differing significance of the crack initiation process. Although the fatigue strength of unwelded components normally increase with the material tensile strength, this is not the case for welded joints in which fatigue failure is determined mainly by crack propagation from crack-like flaws. Thus, any beneficial effect of tensile strength on fatigue must relate to the crack initiation process. Weld metal and heat affected zone have minor influence on fatigue strength because the rate of crack growth is relatively insensitive to microstructure.

2.1.3 Typical Structural Components with Surface Cracks

Plain Plates

Surface cracks in plain plates have been well researched [27,56–61] (Figure 2-2 [62]), as many types of structural components can be simplified to plate elements. The solution by Newman and Raju is the most commonly used, providing SIF solutions for surface cracks with different sizes and shapes in plain plates [26,58,59].

Figure 2-2 Proportion of research of crack growth in different structures

Pipes

Pipes are commonly used in oil and natural gas production and transportation, and semi-elliptical surface cracks initiate and propagate at critical areas, such as the hang-off zone, where there are combined tensile and bending loads, the bends, where there are high magnitude bending moments, and internally, due to the internal pressure in the pipe [62]. Figure 2-3 shows an example of a girth weld in a pipe, also called a circumferential weld.

Figure 2-3 Pipe containing girth weld

Surface cracks in pipes have been investigated in several works. Surface crack growth and SIF evaluation methods have been reviewed by Scott and Thorpe [63], Parks [64],

Pang [65], Brighenti and Carpinteri [13] and Branco et al. [66]. Experimental studies have also been done [67–70] on surface cracks in pipes to calibrate numerical and analytical methods. Other work that has been done can be found in [71–77].

Cruciform joint

Cruciform fillet welded joint is a common weld geometry found in many welded structures. Depending on where the crack propagates from, these joints can be classified into load-carrying or non-load-carrying [5]. Various researchers have done work on evaluating the fatigue and fracture strength of cracks in cruciform fillet welded joint.

Using Paris' Law [46], Frank and Fisher [78] developed two-dimensional SIF solutions by considering the non-penetrating region at the weld root as an initial crack. Based on this, Guha [79] proposed a new fracture mechanics method to predict the fatigue life of welded cruciform joints. Xu [80] carried out load-carrying welds joints analysis using two-dimensional boundary element method, while Lie [5] proposed new SIF solutions for weld toe and weld root cracks.

Rail steel

Rail steels are susceptible to fatigue failures resulting in rail breaks that can lead to unplanned downtime and safety concerns in railway operations [81–84]. Rail defects are classified by the plane of the crack surface and its specific location at rail head, head-to-web, web-to-base, base bottom or corner regions [85]. The transverse surface crack at a thermite weld cross-section is shown in Figure 2-4, where cracks arise from different stress concentration sites and propagate in the transverse plane leading to final brittle fracture [86]. The majority of past railway service transverse rail failures occurs in the head-to-web (28%), web-to-base (30%) and base (including base corner,

30%) [87,88], indicating that fatigue crack propagation analysis and life prediction of such failures are of great interest in the railway industry.

Figure 2-4 Description of crack locations on rail profile

Previously, Li et al. [89] analyzed transverse rail cracks in rails for heavy haul trains and summarized characteristics of transverse failures based on metallurgical and microstructural studies. Ochi and McEvily [90] reported mechanical tests using specimens extracted from the head of the rail to evaluate fracture toughness of rail steels. Dang Van et al. [91] developed a comprehensive simulation approach to model the initiation and propagation of transverse cracks induced by rolling-sliding contact load. Trollé et al. [92] studied and modelled rolling contact fatigue cracks in rail head using XFEM. Zhao et al. [93] investigated the SIFs of head cracks in 3D using finite element modelling (FEM) with ANSYS/LS-DYNA. These researches focused on and provided reasonable explanations for thermite weld failures at the base and head.

However, limited investigation was done on full scale testing of thermite weld failures at the head-to-web transition zone. Lawrence et al. [94] investigated the reliability of improved thermite welds where conventional (non-inverted) S-N curve were conducted. In these tests, the failures at the base of the rail as in a conventional (non-inverted) test, the location of the highest stress will be at the foot of the rail. In order to obtain meaningful fatigue test results for failures at the rail head and head-to-web

transition zone, fatigue tests of the inverted rail have to be carried out such that the location of highest stress will be at the rail head.

2.1.4 Improving the Fatigue Strength of Welded Joints

Potential failure sites of welded joints

The potential failure sites at which fatigue cracks may develop depend upon the geometry of the joint detail. In load-carrying fillet welds, fatigue cracks may initiate at the weld root and propagate through the weld metal until throat failure occurs. In continuous welds parallel to the direction of applied stress, fatigue cracks would be expected to initiate at weld defect locations, stop/start positions, or weld surface ripples. The most common failure site is the weld toe. Transverse butt welded joints (without a backing bar), attachments on the surface of a stressed member, e.g. stiffeners, and load-carrying T butt and fillet welds (if the throat size is sufficiently large) all develop fatigue cracks at the weld toe. This occurs because of the existence

of small intrusions at the weld toe which act as pre-existing cracks and start to propagate almost as soon as cyclic loading commences. In addition, the change of section occurring at the joint leads to a stress concentration at the weld toe. The growth of fatigue cracks in as-welded structures is also influenced by the presence of large tensile residual stresses as a result of the welding process.

Techniques for improving the fatigue strength of welded joints

The fatigue strength of many common welded joints are low compared with those of unwelded components, and this low fatigue strength may be the limiting factor in the design of many steel structures. It is often desirable, therefore, to increase the fatigue life of a particular joint detail, and there are four fundamental methods by which this can be achieved [22]:

- Reduction of gross loading;
- Reduction of local stress by improved structural design;
- Use of a joint detail with a higher fatigue strength;
- Local treatment of the joint detail.

In many instances, and in particular when a weld repair is being considered, it is often impracticable to implement any of the first three of these methods, so local treatment may be the only method for this purpose. Many local treatments have been developed to improve fatigue strengths. All the techniques rely to a greater or lesser extent on removing the intrusions or defects, reducing the stress concentration due to joint and weld geometry, or modifying the residual stress distribution. Two broad groups of such techniques can be identified:

- Modification of the weld toe profile. Its main objectives are the removal of the intrusions at the weld toe and the achievement of a smooth transition between

weld metal and parent plate. This will introduce a significant crack initiation phase with a resulting increase in fatigue life, and a fatigue limit may occur at stresses which are much higher than for as-welded joints.

- Modification of the residual stress distribution. It will create the compressive residual stresses in the regions of interest, where fatigue cracks are likely to initiate.

2.2 Fundamentals of Fatigue Mechanisms and Life Prediction

2.2.1 Cyclic Plastic Deformation of Material

The fatigue process is conditioned by cyclic plastic deformation. Without repeated plastic deformation there would not be fatigue. For example, the plastic strain amplitude at the fatigue limit (with cycles of the order of 10^7) is in the order of 10^{-5} for all materials. This plastic strain, applied only once, does not cause any substantial changes in materials, but the multiple repetition of this very small plastic deformation leads to cumulative damage and hence fatigue failure. It is the plastic deformation, not elastic deformation, that causes irreversible changes in the material structure.

Therefore, the cyclic plastic deformation will be considered further here in the fatigue process.

Fatigue failure is usually caused by the initiation of small surface cracks and their propagation until a critical crack size failure occurs. Small surface cracks are of particular interest because the initial crack growth rate is relative small during the initial phase of crack initiation, and hence it takes up a significant portion of the fatigue life of the part.

Figure 2-6 Fatigue life of a structure

The fatigue life of a structure under repeated loading can be divided into the crack initiation life and the crack propagation life (Figure 2-6) [95].

Generally, the initiation life of a fatigue crack can be explained as the initiation of many micro cracks and their growth in the micro-size range. It refers to micro cracks which is difficult to be detected by visual inspection or non-destructive testing techniques.

The propagation life can be explained as the merging of these micro cracks into longer macro cracks which are detectable and their propagation until final failure. The extent that these surface cracks propagate tends to depend on various factors, such as the crack geometry, number of cracks and residual stress in the material.

2.2.2 Fatigue Crack Growth Process

Fatigue Crack Initiation

Fatigue cracks nucleate from the surface because at the surface of cyclically loaded metal there is a higher stress than in the interior and therefore also a higher plastic strain due to stress concentration. The macroscopic stress concentration for many components in engineering applications is due to either notches or to bending and twisting, both of which lead to stress gradients with the highest stress on the surface. Even in uniformly stressed parts, a small degree of eccentricity in the axial load is

practically unavoidable, again leading to certain bending or twisting moments, and consequently to stress concentration in the surface layer. On the other hand, there are also microscopic stress concentrators. The stress level at the surface is sensitive to surface topography. Even very fine grinding produces grooves with depth of the order of tenths of a micron, which can increase the stress very locally by tens of percents and the resulting fatigue life may change several times. Second phase particles, such as inclusions and precipitates, having elastic properties different from the matrix, generally also serve as stress concentrators [96].

At these stress concentration locations, microcracks are usually nucleated and the number of microcracks nucleated at the surface is high. But only after the largest cracks, or the coalescence of several microcracks, reach a certain size, which is typically several tenths of a millimeter for low-strength materials and several hundredths of a millimeter for high strength materials, some continue to propagate or coalesce during further cycling till complete failure.

There are three types of nucleation site, to which local plastic strain concentration at or near the surface is common, namely fatigue slip bands, grain boundaries and surface inclusions. The initial appearance of fatigue slip bands typically coincides with the end of the macroscopic hardening/softening. A high number of microcracks usually form during the first 20 to 40% of the total fatigue life. The whole process of nucleation takes place after saturation of the bulk mechanical properties is reached. Experiments show that the removal of the surface layer prior to the stage of crack propagation considerably increases fatigue life, and polishing (for example, grinding) removes not only the microcracks, but also, if any, extrusions and intrusions. Surface layers and the stress raisers are the principal sources of fatigue damage in the prenucleation stage. Fatigue damage at this stage is to be related primarily to the

sharpness and density of the surface notches and, to a lesser extent, to the state of dislocation structure surrounding them.

In weldments, the factors such as microstructure (base metal, HAZ and weld metal) and the related yield strength and tensile strength, residual stresses and loading conditions strongly affect fatigue crack initiation life.

Fatigue Crack Propagation

The nucleation stage ends in the formation of surface microcracks. These microcracks lie along the active slip planes, i.e. along slip planes in which the shear stress has maximum values. By uniaxial loading, the maximum shear stress lies in planes oriented at 45° to the direction of applied stress. With increasing length, the growing cracks leave the original near the 45° oriented slip planes and to propagate perpendicular to the stress axis. This transition of the crack plane is also the transition from Stage I to Stage II. In Stage II of fatigue crack propagation, only one crack usually propagates, all others stop well within Stage I.

Stage I fatigue crack propagation (crystallographic) along the slip bands is controlled by the shear component of the applied stress. Non-crystallographic propagation in Stage II is in turn controlled by the normal component. Cracks nucleated at grain boundaries can propagate either along the grain boundaries or across the grains. Analogously, cracks nucleated in slip bands within the grains can propagate either across the grains (transcrystalline growth) or along the grain boundaries (intercrystalline growth). In engineering materials, the cracks very often propagate by combination of transcrystalline and intercrystalline growth.

Experimental measurements of the long crack propagation rates can be depicted schematically on a $\log(da/dN)$ versus $\log(\Delta K)$ plot in Figure 2-7. The relation in

regime B for medium crack propagation rates is a straight line which corresponds to Paris' law. In regime A, for lower propagation rates and small ΔK values, the curve approaches the threshold value ΔK_{th} asymptotically. For high ΔK values, the crack growth rate increases more quickly, but in many situations, regime C may be considered to be of minor practical importance since fatigue crack growth rates are so high and the remaining crack propagation life is low. However, it can be important where large in-service loads occur infrequently, as is the case for certain operational transients in pressure vessels. There is a large effect of microstructure on crack growth in the near-threshold region and in the region of transition to final fracture.

Figure 2-7 Schematic of fatigue crack propagation

Crack propagation life prediction can be made using the SIF range (ΔK) and crack propagation rate ($\frac{da}{dN}$) through a Paris law [46]:

$$\frac{da}{dN} = C(\Delta K)^m \quad (1)$$

where N is the cycle count, and C and m are material constants.

2.2.3 Small Cracks and Crack Closure

Small cracks

The fatigue behavior of small cracks is often very different from that of larger cracks, which is also called the "anomalous" behavior of small cracks. There is not a precise definition of what constitutes a "small" crack, but many consider cracks less than 1 mm long to be small. There are different size scales below which the growth rates of a fatigue crack may be found to exhibit a crack size-dependence.

Small cracks can be classified into microstructurally small cracks (1 to 50 μm), mechanically small cracks (10 to 1000 μm), physically small cracks (500 to 1000 μm) and chemically small crack [97]. However, in welded joints, inherent crack-like flaws are often of the order of physically small crack size. These crack-like flaws are usually present at stress concentration locations and exhibit small crack growth behavior.

Crack closure

The phenomenon of fatigue crack closure has received considerable attention as a means of explaining a number of fatigue phenomena such as the existence of crack propagation threshold, the influence of overloading cycles, load ratio and environment on crack propagation. The discovery of crack closure made clear that crack growth rates are not only influenced by the conditions ahead of the crack tip, but also by the nature of crack face contact behind the crack tip. Crack closure implies material contact between the upper and lower surface of a fatigue crack with load transmission through the contact area under tensile loading. The main mechanisms of fatigue crack growth retardation include plasticity-induced closure, roughness-induced closure, oxide-induced closure (including oxides, liquids, corrosion products) and phase

transformation-induced crack closure. For welded joints, the plasticity-induced closure is a dominant mechanism.

2.2.4 Magnification (M_k) factors and Plastic Zone Correction

When a welded attachment is added, the ratio of the SIF of a plate with welded attachment as compared to the SIF of a plain plate is defined as the magnification (M_k) factor [98,99].

$$M_k = \frac{K_{\text{(welded plate)}}}{K_{\text{(plain plate)}}} \quad (2)$$

The M_k factor quantifies the change in SIFs as a result of the presence of the weld which is then used in conjunction with existing solutions for plain plates [100].

Therefore, the M_k factors at the crack depth and crack surface can be described by

$$M_{k,a} = \frac{K_{w,a}}{K_a} \quad (3)$$

$$M_{k,c} = \frac{K_{w,c}}{K_c} \quad (4)$$

where K_a and K_c refers to the SIF at the crack depth and crack surface of a weld toe crack on a plain plate and $K_{w,a}$ and $K_{w,c}$ refers to the SIF at the crack depth and crack surface of a weld toe crack.

M_k factors are generally influenced by the size of the welded attachment, as well as the size and depth of the weld toe crack. Through the use of M_k factors, the SIFs of a weld toe crack can be predicted. Bowness and Lee [100] presented solutions for estimating M_k factors for single semi-elliptical cracks in fillet welded T-butt joints.

The M_k factors are used in the BS7910:2013 [14] and API 579-1/FFS-1 2007 [101] codes to evaluate the reliability of the structures under fatigue loading. The M_k factor solutions are used to predict the growth of fatigue cracks using a model reported earlier by Pang and Gray [102].

Incorporating the M_k factors, the SIF at the crack depth ΔK_a , and the crack surface ΔK_c , is given by:

$$\Delta K_a = M_{k,a} F \Delta \sigma \sqrt{\pi \frac{a}{Q}} \quad (5)$$

$$\Delta K_c = M_{k,c} F \Delta \sigma \sqrt{\pi \frac{a}{Q}} \quad (6)$$

As the SIFs are functions of crack depth a , the SIFs and M_k factors are able to account for the growth in crack size during the process of crack propagation.

To compute the effective crack length with plastic zone correction, plane stress and plane strain conditions are assumed at the crack surface and depth respectively. The following plastic zone corrections can be added to the crack depths and crack half widths respectively at each iteration:

$$Pz_a = \frac{1}{6\pi} \left(\frac{\Delta K_a}{\sigma_y} \right)^2 \quad (7)$$

$$Pz_c = \frac{1}{2\pi} \left(\frac{\Delta K_c}{\sigma_y} \right)^2 \quad (8)$$

The effective SIF range was corrected with a threshold SIF, ΔK_{th} . The thresholds at the crack surface, $\Delta K_{th,c}$, and crack depth, $\Delta K_{th,a}$, will both be maintained at 63 N/mm^{3/2}. The thresholds and M_k factor solutions can be combined with the Paris Law equation:

$$\int_0^{N_f} dN = \int_{a_i}^{a_f} \frac{da}{C (M_{k,a} \Delta K_a - \Delta K_{th,a})^m} \quad (9)$$

$$\int_0^{N_f} dN = \int_{a_i}^{a_f} \frac{da}{C (M_{k,c} \Delta K_c - \Delta K_{th,c})^m} \quad (10)$$

2.2.5 Interaction of Multiple Surface Cracks

Multiple small surface cracks are often found in welded components subjected to fatigue loads. These cracks often propagate from as many as tens to hundreds of small weld toe cracks along the weld toe line. Eventually, these small surface cracks coalesce to form a single crack causing final fracture.

Investigations have been made to study the interaction of static adjacent cracks [13].

Most studies have been devoted to coplanar propagation [66]. For these investigations, the SIFs are estimated using various methods, such as using FEM [35,36], body force method [37–39] and finite element alternating [40–42]. It is inevitable that there will be changes in the SIFs when an adjacent crack is approaching, and hence the term ‘interaction’ was introduced. The investigations above agree that interaction of adjacent cracks exist, but the degree of interaction depend on factors such as crack dimensions, aspect ratio and separation distance.

For fatigue assessment of multiple surface cracks, there are commonly two approaches Figure 2-8 [5]. According to ASME XI [103], BS PD6493 [104] and BS 7910:2013 [14], adjacent cracks should be recharacterized if certain geometrical conditions are met. The standards recommend that a single rectangular crack with length $2c_1 + 2c_2 + s$ and depth $\max(a_1, a_2)$ be used. This method of crack recharacterization is considered to be overly conservative as it does not take into account the stages of interaction and coalescence. In practical applications, this method is considered to severely underestimate fatigue lives.

Figure 2-8 Re-characterization rules in (a) BS 7910:2013 [14] and (b) NIIT method

A second method, known as the ‘No Interaction and Immediate Transition (NIIT)’ method, assumes that the adjacent cracks have no influence on each other before they meet, and immediately combine to form one crack when they meet. The region and cycles where the coalescence occurs is neglected. Kishimoto et al. [105] and Soboyejo et al. [106] evaluated the interaction of two coplanar surface cracks in plates subjected to tension and bending. It was concluded that the cracks grew almost independently before contact. On contact, the SIFs increased to maximum values, causing rapid transition and coalescence into a single larger crack. Lin and Smith [107,108] modelled multiple semicircular cracks in plates under tension and bending. They concluded that at initial stage, the cracks grow independently until they contact, and then quickly coalesce to form a single smooth crack with no concave points. Kamaya [36,109,110] also concluded that interacting coplanar cracks can be evaluated with an equivalent semi-elliptical crack with the same area. These results support the use of the NIIT method. Other experimental investigations into the NIIT have been made [111–115], indicating that the NIIT method is significantly less conservative than the first method.

2.3 Fatigue Crack Growth Models and Software

2.3.1 Fatigue Crack Growth Models

Two-degree-of-freedom models refer to models that are developed assuming a pre-defined crack shape. Newman and Raju [59] were the pioneers in this area,

developing models for surface cracks in plates subjected to tension and bending. The authors assumed the cracks to follow a semi-elliptical shape in the case of surface cracks. Using two key points, the deepest point and the surface point of the crack, finite element models were used to determine the stress intensity factors. Other authors built on this method of predicting crack growth in plates with surface cracks in tension and bending, or a combination of both [116–119]. For tubular joints subjected to tension and bending, Kam [120] developed a method to predict the crack aspect ratio. Carpinteri [75,121–123] has also done other work in investigating different geometries and loading. Notably, a three-parameter model was used [124,125], comprising major axis, minor axis, and the centre of the ellipse.

In the area of fatigue crack growth in pipes, Brighenti [76] computed SIFs of an external surface flaw in a round pipe subjected to an internal pressure. Iranpour and Taheri [126] simulated the crack growth using elliptical crack shapes and three-dimensional FE models to obtain the SIFs.

To account for multiple degree-of-freedom in crack growth, Smith and Cooper [127] pioneered a model based on various geometries subjected to different kinds of loading. Lin and Smith [128] concluded that two-degree-of-freedom models were more conservative, at about thirty percent lower, as compared to numerical simulations.

2.3.2 Fatigue Crack Growth Software

Many software solutions have been developed to model fatigue crack growth problems, but most of the solutions were developed by research groups and not commercially available [66]. These software can be broadly categorized into two approaches: numerical based and analytical based.

Numerical based software, more commonly known as Finite Element Method (FEM) modelling, aims to predict crack paths and geometries in two- or three-dimensional geometries. Some examples of numerical based software include FRANC3D [129], ADAPCRACK [130], ZENCRACK, CRACKTRACER [131,132], LYNX [133] and PROCRAK [134,135]. A common strategy in these numerical based software is to use the sub-modelling technique, where a global model is created, and a sub-model is used to refine the mesh near the crack region.

Analytical based software conduct fatigue crack growth predictions based on libraries of closed-form SIF solutions for cracks in different geometries and configurations. This approach is generally more time efficient because numerical models do not need to be generated. However, these software are limited by the size of their libraries, and do not take into account shape changes during crack growth and other non-linear effects. Some examples of analytical based software include NASGRO [136,137], NASCRAC [138], ESACRACK, AFGROW and VIDA.

2.3.3 Abaqus eXtended Finite Element Method

The eXtended Finite Element Method (XFEM) is a finite element method that enables local enrichment of approximation spaces using the concept of partition of unity [34], allowing for significantly simplified modelling of cracks. XFEM allows for an approximation of cracks without the need for the mesh to follow the crack as in conventional crack modelling techniques. Through the use of XFEM, weld toe cracks can be modeled in ABAQUS more easily than conventional methods.

The XFEM method is used in the approximation of discontinuities and singularities, such as displacements near cracks, through the use of enrichment functions. These enrichment functions consist of the asymptotic crack tip functions that account for the

singularity at the crack tip and a discontinuous function that represents the gap between the crack surfaces [139]. The displacement approximation function is given by [139]

$$\begin{aligned} \mathbf{u}_{\text{XFEM}}(\mathbf{x}) = & \sum_{i \in S} N_i(\mathbf{x}) \mathbf{u}_i + \sum_{i \in S_h} N_i(\mathbf{x}) H(\mathbf{x}) \mathbf{a}_i \\ & + \sum_{i \in S_c} N_i(\mathbf{x}) \sum_{a=1}^4 \varphi_a(\mathbf{x}) \mathbf{b}_i^a \end{aligned} \quad (11)$$

where $N_i(\mathbf{x})$ are standard shape functions, S is the set of nodes in the finite element space, S_h is the set of nodes enriched with the discontinuous function, S_c is the set of nodes with tip enrichment functions, $\mathbf{u}_i$ is the nodal displacement vector, $\varphi_a(\mathbf{x})$ represents the asymptotic crack tip function, $\mathbf{a}_i$ and $\mathbf{b}_i^a$ are the nodal enriched degree of freedom vector and $H(\mathbf{x})$ is the discontinuous jump function.

For a general crack, $H(\mathbf{x})$ is as follows [139]

$$H(\mathbf{x}) = \begin{cases} 1, & \text{if } (\mathbf{x} - \mathbf{x}^*) \cdot \mathbf{n} > 0 \\ -1, & \text{otherwise} \end{cases} \quad (12)$$

where $\mathbf{n}$ is the outward normal vector, $\mathbf{x}^*$ is a point on the crack close to $\mathbf{x}$.

For an isotropic material, the asymptotic crack tip function is

$$\varphi_a(\mathbf{x}) = \left[\sqrt{r_c} \sin \frac{\theta}{2}, \sqrt{r_c} \cos \frac{\theta}{2}, \sqrt{r_c} \sin \frac{\theta}{2} \sin \theta, \sqrt{r_c} \cos \frac{\theta}{2} \sin \theta \right] \quad (13)$$

where r and θ are local polar coordinates defined at the crack tip.

Finite element modelling was conducted in ABAQUS, using the eXtended Finite Element Method (XFEM) for crack modelling and direct-cyclic loading to simulate fatigue loading.

At the present moment, XFEM is unable to account for the interaction of multiple cracks as each crack defined in XFEM will have to be in a separate zone. While XFEM have been used to evaluate the SIF of surface cracks in steel plates, the applications of XFEM on offshore metallic pipes and other welded structures are still very limited in research [62].

Chapter 3 Study No. 1: XFEM Calibration Studies for SIF Solutions and Development of Multiple Surface Crack Algorithm

This chapter presents the development of the multiple fatigue crack growth algorithm, starting out with a parametric study using 3D XFEM to obtain new SIFs and M_k factors, based on a plain plate and fillet-welded T-butt joint plate, which are also compared with current solutions and fatigue data. Fatigue life prediction is conducted using an algorithm programmed in MATLAB to perform the fatigue crack growth and a novel algorithm for multiple crack propagation is demonstrated.

As mentioned previously, the Paris-Erdogan Law [46] takes into account the relationship between the fatigue crack growth rate $\frac{da}{dN}$ and the cyclic stress intensity ΔK . However, a life prediction algorithm is still needed to perform the actual fatigue crack growth predictions. This chapter focuses on the development of this algorithm.

3.1 Development of XFEM Modelling Methodology

A parametric study was carried out using 3D XFEM, with finite element models based on a plain plate and fillet welded T-butt joint plate with semi-elliptical crack as shown in Figure 3-1.

Figure 3-1 a) Plain plate b) Fillet welded T-butt joint plate [140]

Using symmetry, a 3D half model was generated using XFEM. The mesh was designed such that the mesh near the crack was heavily refined and focused, but

coarser in other areas (Figure 3-2). The area nearest to the crack was partitioned according to the shape of the crack and the mesh in this partition as fine. In the areas further away, the mesh was designed to be coarser progressively. This would allow for faster computational time, without sacrificing the accuracy of the results.

Figure 3-2 Cutaway of fine mesh model [140]

When using XFEM, the partitioning of the mesh was designed to follow the shape of the crack front (Figure 3-2) in order to obtain more robust and accurate results.

Figure 3-3 More partitioning for smaller cracks a) a/T of 0.5 b) a/T of 0.05 c) a/T of 0.005 [140]

In order to model smaller cracks of up to an a/T ratio of 0.005, more partitions had to be added to ease the transition from a fine mesh to a coarse mesh (Figure 3-3).

Without these transitional partitions, there would be too many elements in the whole model, resulting in long computational time. For example, for an a/T ratio of 0.5 (Figure 3-3a), these transitional partitions did not need to be added because the size of the mesh along the crack is relatively large and it could transit to a coarse mesh smoothly. For an a/T ratio of 0.05 (Figure 3-3b) and 0.005 (Figure 3-3c), transitional partitions would have to be added to smoothen the transition of the mesh from fine to coarse. Without these transitional partitions, the results would be inaccurate and large deviations from previous work would occur.

The models were subjected to a pure tension load on one end, and boundary conditions to prevent rigid body motion were applied to the other. Symmetry boundary conditions were also applied on the symmetry plane. 10 contours were requested from the ABAQUS output.

Mesh refinement study and methods of obtaining SIF outputs

To determine the fineness of the mesh required, a mesh refinement study was carried out. The number of elements along the crack front (Figure 3-2) was varied (30, 50 and 80 elements) and its effect on SIFs at the crack ends and crack depths was investigated. Along the crack front, 20 elements surround the crack in all directions as 10 contours are requested from the output. The sizes of these elements were adjusted accordingly to prevent any huge distortion in the elements.

Figure 3-4 SIF against Contour No. at angle of 10.5° [140]

Two methods of obtaining SIFs at the crack ends and crack depths were investigated. In the first method, the SIFs are evaluated using the contour integrals around the crack [139]. An example of the ABAQUS output is shown for an arbitrary location along the crack front (Figure 3-4) and compared with the result of Newman and Raju [26]. The SIFs obtained for the first two contours show large deviations from the results of Newman and Raju [26], stabilizing only from the 4th contour onwards. Therefore, the average of the SIFs from the 4th to 10th contour will be used to obtain reliable results. The second method to obtain SIFs is to use the method of displacement extrapolation as recommended by Kristiansen and Fu [141]. In this method, a node path was created along the opening of the crack (Figure 3-5). This node path stretched from the crack end to 9 nodes thereafter. The SIF values of each node was then plotted and then extrapolated to find the SIF value for the crack.

Figure 3-5 Inserting a node path (in red).

Figure 3-6 Illustration of crack end SIF solution

Figure 3-6 shows the method of extrapolation to find the SIF value. The three points corresponding to the SIF value of consecutive nodes with the R-squared value closest to 1 were used as points for linearizing.

The mesh refinement study will be carried out using the plain plate model (Figure 3-8a) with a crack depth ratio, a/T , of 0.005 and a crack aspect ratio, a/c , of 0.5.

Figure 3-7 Mesh refinement study comparing contour integral and displacement extrapolation methods [140]

The mesh refinement study shows that the SIFs at the crack depth (Figure 3-7a) and crack ends (Figure 3-7b) obtained from the ABAQUS output using the contour integrals will not be accurate especially at the crack end. This is possibly a result of the presence of two stress singularities [142]. On the other hand, the results obtained using displacement extrapolation were more accurate even when only 50 elements along the crack front were used. Therefore, the method of displacement extrapolation to obtain SIFs will be used.

The SIFs will be normalized with the applied stress, S_t , and crack depth, a , as

$$Y_a = \frac{K_a}{S_t \sqrt{\pi a}} \quad (14)$$

$$Y_c = \frac{K_c}{S_t \sqrt{\pi a}} \quad (15)$$

where K_a and K_c are SIFs with physical dimensions $[\text{MPa} \sqrt{\text{m}}]$. In this report, all SIFs are treated as non-dimensional unless specified otherwise.

3.2 Benchmarks

3.2.1 Plain Plate with Semi-Elliptical Crack

The surface crack SIF by Newman and Raju [26] is

$$K_I = S_t \sqrt{\pi \frac{a}{Q}} F\left(\frac{a}{T}, \frac{a}{c}, \frac{c}{b}, \theta\right) \quad (16)$$

where K_I is the mode I SIF, S_t is the remote uniform-tension stress, while a , T , b , c , and θ are specified in Figure 3-8.

Figure 3-8 Plain plate

The Stress Intensity Factor (SIF) solutions can be determined from finite element analysis [143].

Q can be approximated to be

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65} \quad \left(\frac{a}{c}\right) \leq 1 \quad (17)$$

The function F is defined as

$$F = \left[M_1 + M_2 \left(\frac{a}{T}\right)^2 + M_3 \left(\frac{a}{T}\right)^4 \right] f_\theta g f_w \quad (18)$$

where

$$M_1 = 1.13 - 0.09 \left(\frac{a}{c}\right) \quad (19)$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + \left(\frac{a}{c}\right)} \quad (20)$$

$$M_3 = 0.5 - \frac{1.0}{0.65 + \left(\frac{a}{c}\right)} + 14 \left(1.0 - \frac{a}{c}\right)^{24} \quad (21)$$

$$g = 1 + \left[0.1 + 0.35 \left(\frac{a}{T}\right)^2\right] (1 - \sin \phi)^2 \quad (22)$$

$$f_\phi = \left[\left(\frac{a}{c}\right)^2 \cos^2 \phi + \sin^2 \phi\right]^{\frac{1}{4}} \quad (23)$$

$$f_w = \left[\sec\left(\frac{\pi c}{2b} \sqrt{\frac{a}{T}}\right)\right]^{\frac{1}{2}} \quad (24)$$

In equations 22 and 23, ϕ refers to an angular parameter that is used to determine the position of the point along the crack front.

The new SIF solutions were obtained and compared with established solutions [26].

The investigation was carried out and the results are summarized in Table 3-1 and

Table 3-2. The deviation is calculated by

$$\text{Deviation (\%)} = \frac{Y_{\text{XFEM}} - Y_{\text{Newman and Raju}}}{Y_{\text{Newman and Raju}}} \times 100\% \quad (25)$$

Table 3-1 Comparison of Y_a results to Newman and Raju [20]

a/c	a/T	Crack depth, Y_a (XFEM)	Crack depth, Y_a (Newman and Raju)	Deviation (%)
1	0.5	0.70198	0.69257	1.36
1	0.1	0.67664	0.66384	1.93
1	0.05	0.67323	0.66286	1.56
0.5	0.5	1.02143	1.04076	-1.86
0.5	0.1	0.91978	0.90206	1.96
0.5	0.05	0.88521	0.89748	-1.37
0.25	0.5	1.39623	1.40381	-0.54
0.25	0.1	1.04968	1.04713	0.24
0.25	0.05	1.03867	1.03676	0.18

Table 3-2 Comparison of Y_c results to Newman and Raju [20]

a/c	a/T	Crack end, Y_c (XFEM)	Crack end, Y_c (Newman and Raju)	Deviation (%)
1	0.5	0.80578	0.82243	-2.02
1	0.1	0.73905	0.73254	0.89
1	0.05	0.73980	0.72973	1.38
0.5	0.5	0.86502	0.87391	-1.02
0.5	0.1	0.69724	0.70387	-0.94
0.5	0.05	0.68757	0.69863	-1.58
0.25	0.5	0.80571	0.83351	-3.34
0.25	0.1	0.58643	0.57776	1.50
0.25	0.05	0.58238	0.57067	2.05

Compared to the solutions by Newman and Raju [26], most of the SIFs obtained by XFEM are close to that obtained by the solutions. The maximum deviation observed is 3.34%. Hence, the techniques used in partitioning and mesh generation can be deemed to be accurate.

3.2.2 Fillet-Welded T-butt Joint Plate With Semi-Elliptical Crack [140]

The plain plate investigations of the previous section were repeated with attachments of different sizes added to the model (Figure 3-8b) with the same parameters in Figure 3-8a. The weld toe M_k factors were obtained and compared with established solutions by Bowness & Lee [100], given in Appendix A. The investigation was carried out and the results are summarized in Table 3-3. The deviation is calculated by

$$\text{Deviation (\%)} = \frac{Y_{\text{XFEM}} - Y_{\text{Bowness \& Lee}}}{Y_{\text{Bowness \& Lee}}} \times 100\% \quad (26)$$

Table 3-3: M_k factors for fillet-welded T-butt joint plate compared to Bowness & Lee [100]

L/T	a/c	a/T	Crack depth, $M_{k,a}$ (XFEM)	Crack depth, $M_{k,a}$ (Bowness and Lee)	Deviation (%)	Crack end, $M_{k,c}$ (XFEM)	Crack end, $M_{k,c}$ (Bowness and Lee)	Deviation (%)
0.5	1	0.5	0.9517	0.9845	-3.33	1.2795	1.3066	-2.07
0.5	1	0.1	0.9716	0.9931	-2.16	1.6098	1.6851	-4.47
0.5	1	0.05	1.0631	1.0942	-2.84	1.9373	2.0090	-3.56
0.5	1	0.01	1.5542	1.6071	-3.30	3.1768	3.2045	-0.87
0.5	1	0.0075	1.6993	1.7369	-2.17	3.5256	3.4959	0.85
0.5	1	0.005	1.9466	1.9472	-0.03	3.8557	3.9278	-1.83
0.5	0.5	0.5	0.9499	0.9633	-1.39	1.3787	1.3695	0.67
0.5	0.5	0.1	0.9669	0.9939	-2.72	1.7292	1.7860	-3.18
0.5	0.5	0.05	1.1014	1.0944	0.64	2.0762	2.1152	-1.84
0.5	0.5	0.01	1.5457	1.6013	-3.48	3.2864	3.3561	-2.08
0.5	0.5	0.0075	1.7014	1.7303	-1.67	3.6415	3.6377	0.10
0.5	0.5	0.005	1.9485	1.9396	0.46	4.0271	4.0472	-0.50
0.5	0.25	0.5	0.9236	0.9359	-1.31	1.3842	1.4354	-3.57
0.5	0.25	0.1	0.9826	1.0019	-1.93	1.9752	2.0262	-2.51
0.5	0.25	0.05	1.0769	1.1062	-2.65	2.3940	2.4182	-1.00
0.5	0.25	0.01	1.6109	1.6177	-0.42	3.9356	3.8459	2.33
0.5	0.25	0.0075	1.7876	1.7471	2.32	4.1378	4.1633	-0.61
0.5	0.25	0.005	1.9874	1.9569	1.56	4.6757	4.6263	1.07
1.25	1	0.5	0.9142	0.9285	-1.54	1.3211	1.3579	-2.71
1.25	1	0.1	1.0402	1.0727	-3.02	1.8940	1.9609	-3.41
1.25	1	0.05	1.1940	1.2346	-3.29	2.3194	2.3950	-3.16
1.25	1	0.01	1.8636	1.9343	-3.66	3.8679	3.9389	-1.80
1.25	1	0.0075	2.0430	2.1113	-3.24	4.2850	4.3150	-0.70
1.25	1	0.005	2.3471	2.3989	-2.16	4.9851	4.8755	2.25

1.25	0.5	0.5	0.9073	0.9073	0.00	1.3681	1.4382	-4.88
1.25	0.5	0.1	1.0403	1.0735	-3.09	2.0217	2.0770	-2.66
1.25	0.5	0.05	1.2441	1.2348	0.75	2.5061	2.5184	-0.49
1.25	0.5	0.01	1.9414	1.9285	0.67	4.0549	4.1246	-1.69
1.25	0.5	0.0075	2.0348	2.1047	-3.32	4.4878	4.4904	-0.06
1.25	0.5	0.005	2.3406	2.3913	-2.12	4.8715	5.0257	-3.07
1.25	0.25	0.5	0.8942	0.8798	1.64	1.4400	1.5330	-6.07
1.25	0.25	0.1	1.0507	1.0815	-2.85	2.2658	2.3571	-3.87
1.25	0.25	0.05	1.2188	1.2467	-2.23	2.8062	2.8745	-2.38
1.25	0.25	0.01	1.8871	1.9448	-2.97	4.5511	4.7225	-3.63
1.25	0.25	0.0075	2.0813	2.1215	-1.90	5.0287	5.1365	-2.10
1.25	0.25	0.005	2.3837	2.4086	-1.03	5.8565	5.7445	1.95
2.0	1	0.5	0.9066	0.8879	2.11	1.3130	1.3361	-1.72
2.0	1	0.1	1.0565	1.0815	-2.31	1.9415	1.9968	-2.77
2.0	1	0.05	1.2173	1.2561	-3.09	2.4025	2.4574	-2.23
2.0	1	0.01	1.9304	1.9745	-2.23	4.0383	4.0897	-1.26
2.0	1	0.0075	2.1173	2.1548	-1.74	4.4627	4.4884	-0.57
2.0	1	0.005	2.4264	2.4473	-0.86	5.1719	5.0843	1.72
2.0	0.5	0.5	0.8752	0.8667	0.98	1.3766	1.4126	-2.55
2.0	0.5	0.1	1.0602	1.0823	-2.04	2.0694	2.1102	-1.93
2.0	0.5	0.05	1.2838	1.2564	2.19	2.6001	2.5789	0.82
2.0	0.5	0.01	1.9154	1.9687	-2.71	4.2333	4.2766	-1.01
2.0	0.5	0.0075	2.1213	2.1481	-1.25	4.6762	4.6648	0.25
2.0	0.5	0.005	2.4265	2.4398	-0.54	5.0546	5.2346	-3.44
2.0	0.25	0.5	0.8528	0.8392	1.62	1.4680	1.5077	-2.64
2.0	0.25	0.1	1.0738	1.0903	-1.51	2.3617	2.3904	-1.20
2.0	0.25	0.05	1.2485	1.2682	-1.55	2.9590	2.9379	0.72
2.0	0.25	0.01	1.9484	1.9850	-1.85	4.7778	4.8903	-2.30
2.0	0.25	0.0075	2.0898	2.1650	-3.47	5.2736	5.3297	-1.05
2.0	0.25	0.005	2.3824	2.4571	-3.04	6.1101	5.9771	2.22

2.75	1	0.5	0.8899	0.8745	1.77	1.3677	1.3477	1.48
2.75	1	0.1	1.0752	1.0859	-0.99	2.0294	2.0114	0.89
2.75	1	0.05	1.2492	1.2676	-1.45	2.4931	2.4723	0.84
2.75	1	0.01	2.0011	2.0133	-0.61	4.2307	4.1091	2.96
2.75	1	0.0075	2.1973	2.2011	-0.17	4.6765	4.5088	3.72
2.75	1	0.005	2.5226	2.5061	0.66	5.0654	5.1059	-0.79
2.75	0.5	0.5	0.8736	0.8533	2.38	1.3873	1.4145	-1.93
2.75	0.5	0.1	1.0847	1.0868	-0.19	2.1629	2.1195	2.05
2.75	0.5	0.05	1.3164	1.2679	3.83	2.6307	2.5884	1.64
2.75	0.5	0.01	1.9853	2.0075	-1.10	4.3284	4.2856	1.00
2.75	0.5	0.0075	2.2001	2.1945	0.26	4.7857	4.6731	2.41
2.75	0.5	0.005	2.5204	2.4985	0.88	5.2987	5.2416	1.09
2.75	0.25	0.5	0.8454	0.8258	2.38	1.5053	1.4990	0.41
2.75	0.25	0.1	1.1017	1.0948	0.63	2.3512	2.3951	-1.83
2.75	0.25	0.05	1.2904	1.2797	0.83	2.9205	2.9435	-0.78
2.75	0.25	0.01	2.0062	2.0239	-0.87	4.9624	4.8913	1.45
2.75	0.25	0.0075	2.1629	2.2113	-2.19	5.4347	5.3287	1.99
2.75	0.25	0.005	2.4719	2.5159	-1.75	6.0619	5.9724	1.50

It is observed from Figure 3-9 that the deviation histograms show a good normal distribution of the percentage deviation with a maximum deviation of -6.07%, indicating that the proposed M_k factors show good agreement with established solutions by Bowness & Lee [100].

Effect of crack aspect ratio

The parametric study using 3D XFEM shows that as the crack size decreases, the M_k factor increases. This is illustrated with a plot of the M_k factors for an attachment footprint ratio, L/T , of 2.75 (Figure 3-10).

Figure 3-10 a) Plot of $M_{k,a}$ factors vs. a/T for L/T of 2.75 b) Plot of $M_{k,c}$ factors vs. a/T for L/T of 2.75 [140]

Referring to Figure 3-10, it can be seen that as the a/T ratio decreases, the $M_{k,a}$ and $M_{k,c}$ factors increase. The $M_{k,a}$ factors were similar across the different a/c ratios of 0.25, 0.5 and 1, ranging from 0.84 to 2.47. In comparison, the $M_{k,c}$ factors (Figure 3-10b) were much higher than the $M_{k,a}$ factors. Among the $M_{k,c}$ factors, the $M_{k,c}$ factors were the highest for the a/c ratio of 0.25, and lowest for the a/c ratio of 1.0. The $M_{k,c}$ factors range from 1.38 to 6.06. The smaller crack sizes are more severely affected by the weld attachment compared to the larger cracks, as shown by the high M_k factors observed.

Effect of L/T ratio

Comparing across different L/T ratios, the results show that M_k factors are higher for larger L/T ratios. This is illustrated with a plot of M_k factors for different L/T ratios for an a/c ratio of 0.25 (Figure 3-11).

Figure 3-11 a) Plot of $M_{k,a}$ factors vs. a/T for different L/T b) Plot of $M_{k,c}$ factors vs. a/T for different L/T [140]

Referring to Figure 3-11, it can be seen that the $M_{k,a}$ and $M_{k,c}$ factors are higher for larger L/T ratios. This shows that the stress concentration is higher when the attachment footprint of the welded attachment is larger.

With the results obtained in the above section, the M_k factors obtained though XFEM can be curve fitted into solutions, as a function of a/T ratios, expressed in the form of a power law (Eqn. 27).

In order to simplify the M_k factor solutions, the results are separated into three categories – upper bound, mean and lower bound. For each L/T ratio, the highest M_k factors, obtained from the a/c of 0.25, are placed in the upper bound category. The lowest M_k factors, obtained from the a/c of 1.0, are placed in the lower bound category. The average of the two is placed in the mean. Therefore, three equations are curve-fitted for each L/T ratio. The upper bound M_k factor solution will give the highest M_k factors, while the lower bound M_k factor solution will give the lowest M_k factors.

The M_k factor solutions follow the format of the following equation:

$$M_k = x \left(\frac{a}{T} \right)^y + z \quad 0.005 \leq \frac{a}{T} \leq 0.5 \quad (27)$$

where the coefficients x , y and z can be found in Table 3-4.

Table 3-4 Coefficients x , y and z for M_k factor solutions [140]

L/T		$M_{k,c}$			$M_{k,a}$		
		x	y	z	x	y	z
0.5	Upper Bound	0.019	-1.010	1.611	0.017	-0.814	0.879
	Mean	0.023	-0.943	1.519	0.019	-0.774	0.880
	Lower Bound	0.038	-0.814	1.376	0.024	-0.716	0.872
1.25	Upper Bound	0.074	-0.769	1.743	0.083	-0.559	0.791
	Mean	0.104	-0.691	1.509	0.089	-0.544	0.767
	Lower Bound	0.177	-0.579	1.201	0.096	-0.528	0.742
2.0	Upper Bound	0.116	-0.698	1.685	0.074	-0.584	0.817
	Mean	0.137	-0.652	1.471	0.087	-0.556	0.769
	Lower Bound	0.174	-0.592	1.230	0.102	-0.528	0.717
2.75	Upper Bound	0.086	-0.757	1.779	0.087	-0.566	0.795
	Mean	0.109	-0.697	1.584	0.096	-0.549	0.758
	Lower Bound	0.158	-0.614	1.342	0.106	-0.531	0.718

The coefficients of determination, R^2 , of all the above curved fitted solutions were more than 0.99. This indicates that the regression line has over 99% correlation to the M_k data obtained and is a close approximation of the real data points.

Figure 3-12 illustrates the upper bound, mean and lower bound categories of the M_k factor solution for the L/T ratio of 2.75 in graphical form.

Figure 3-12 Illustrations of a) $M_{k,a}$ factor solutions b) $M_{k,c}$ factor solutions for L/T of 2.75 [140]

In order to obtain the most conservative predicted fatigue life, the upper bound M_k factor solution should be used in the fatigue life prediction model. This will result in the shortest predicted fatigue life. On the other hand, the lower bound M_k factor solution will give the longest predicted fatigue life. Through the use of these practical to use M_k factor solutions, a quick prediction can be made regarding the fatigue life of a part without sacrificing accuracy and reliability.

3.3 Fatigue Life Prediction

An algorithm is programmed in MATLAB to perform the crack growth and fatigue life prediction. The Paris law constants used are $C = 5.21 \times 10^{-13}$ and $m = 3$ recommended by the BS 7910 [14]. The applied stress range is 120 MPa with stress ratio of $R = 0$, while the thickness of the plate is 50 mm. Simulation is performed for

an initial crack depth of 0.5 mm, which corresponds to an a/T ratio of 0.01 for a single crack. The cracks are allowed to propagate until a depth of 25 mm, which corresponds to an a/T ratio of 0.5. The initial widths of the cracks tested are 2.0 mm, 1.0 mm and 0.5 mm, corresponding to a/c ratios of 0.25, 0.5 and 1.0 respectively (Figure 3-14).

Figure 3-13 Comparison of predicted fatigue life to experimental data [140]

The M_k factor solutions was compared with HSE fatigue data on 16 mm thick tubular joints [98] (Figure 3-13). The Paris law constants used were $C = 1.8318 \times 10^{-13}$ and $m = 3$ [98]. The predicted life (for single surface crack) showed good agreement with the data and was conservative by 10% on average.

The algorithm is also important in the parametric studies of factors affecting fatigue life, such as the initial crack size and aspect ratio.

Figure 3-14 Illustration of single crack growth algorithm for a) a/c of 0.25 b) a/c of 0.5 c) a/c of 1.0 [140]

Figure 3-15 Changes in a/c (single crack) for a) plain plate b) plate with attachment (L/T of 2.75) [140]

The change in crack aspect ratio, a/c , for single cracks as the cracks grow from the initial a/c ratios of 1.0, 0.5 and 0.25 is illustrated in Figure 3-15. As the cracks grow, the cracks in the plain plate tend towards a semi-elliptical shape with an a/c of 0.848, while the cracks in the plates with welded attachment tend towards a semi-elliptical shape with an a/c of 0.342, for an L/T of 2.75. Therefore, smaller aspect ratios can be expected for cracks at the weld toe region than surface cracks on plain plates.

Figure 3-16 Fatigue life prediction (single crack) for a) plain plate b) plate with attachment (L/T of 2.75) [140]

From Table 3-5, it can be seen that the predicted life of a weld toe crack is severely reduced with the addition of a welded attachment. This is further illustrated in Figure

3-16b, where the number of cycles for a crack in a plate with welded attachment (L/T of 2.75) to propagate from an a/T ratio of 0.01 to 0.5 is shown for single cracks, compared to the plain plate in Figure 3-16a. The huge reduction in the predicted life, about one order of magnitude, due to the stress concentration at the weld is demonstrated. For example, for an a/c of 1.0, the predicted fatigue life was reduced from 8.587 million to 0.337 million cycles. For an a/c of 0.25, the predicted fatigue life was reduced from 2.195 million to 0.319 million cycles.

3.4 Multiple Crack Recharacterization Method

Figure 3-17 Algorithm for multiple crack propagation [140]

The multiple crack recharacterization method is as shown in Figure 3-17. The input requires information on the crack, material and fatigue parameters. Crack data would include the crack initiation sites, initial and final depth sizes, and crack aspect ratios. Material properties required are the yield stress and ultimate tensile stress. Fatigue parameters, such as C and m from the Paris law related to the fatigue propagation law are input in this stage as well. In every iteration, the crack depth is propagated by 0.0001 of the current crack depth.

The model is extended to the multiple surface cracks commonly observed at the weld toe, where each crack is treated as independent, in line with the NIIT method. The multiple cracks will coalesce as they propagate, until a single dominant crack emerges and fracture occurs (Figure 3-18). In this method of crack recharacterization, the new crack depth is taken to be the depth of the larger crack (to provide a more conservative analysis), while the new crack length is equal to the sum of the lengths of the coalescing cracks. The location of the new crack along the width, which is the midpoint of the old cracks, remains unchanged. The amount of which the crack grows in the depth-wise and length-wise direction is determined by the SIF with plastic zone correction as mentioned in the previous section.

Figure 3-18 Re-characterization model for coalescing surface cracks [140]

Fatigue life is predicted for plates with and without attachments with 2 cracks and 4 cracks initially for the same starting a/c and a/T ratios as the single cracks. The separation distance between the cracks is shown in Figure 3-19 in terms of crack width, c .

Figure 3-19 Separation distance between cracks for a) 2 cracks b) 4 cracks [140]

The upper bound M_k factor solutions are used for all simulations to obtain the most conservative estimate. 36 simulations are performed for plates with attachment with L/T ratios. In addition, 9 simulations are also performed on plain plates for comparison, for a total of 45 cases. The results are summarized in Table 3-5. For the sequence of crack coalescence, the two cracks in the middle coalesced initially, as they were the closest in proximity to each other. The coalesced crack subsequently merged with the crack on the far left, being the next closest in proximity, and finally with the crack on the far right, being the furthest.

The change in crack aspect ratio, a/c , for multiple cracks as the cracks grow from an initial a/c ratio of 0.25 is illustrated in Figure 3-20. When two cracks coalesce, there is a sharp decrease in a/c ratio, resulting in a merged crack with lower a/c at similar crack depth a/T compared to the single crack case. For the case of two cracks, one

sharp drop of the a/c ratio was observed, with two later incidents of crack coalescence observed for the four crack case.

Table 3-5 Fatigue life prediction results [140]

L/T	Initial a/c	Predicted fatigue life (millions)		
		1 crack	2 cracks	4 cracks
0.5	1.0	0.635	0.594	0.544
	0.50	0.615	0.565	0.523
	0.25	0.585	0.540	0.510
1.25	1.0	0.386	0.366	0.341
	0.50	0.377	0.350	0.326
	0.25	0.359	0.334	0.314
2.0	1.0	0.358	0.341	0.320
	0.50	0.351	0.328	0.306
	0.25	0.338	0.314	0.294
2.75	1.0	0.337	0.324	0.303
	0.50	0.330	0.311	0.290
	0.25	0.319	0.297	0.278
Plain	1.0	6.587	6.063	5.964
	0.50	3.638	3.389	3.316
	0.25	2.195	2.062	2.025

Figure 3-20 Changes in a/c (multiple cracks) for a) plain plate b) plate with attachment (L/T of 2.75) [140]

The model was further extended to predict fatigue life for up to 100 cracks.

Simulations were performed for stress ranges of 225, 250, 275, 300 and 325 MPa to

generate the fatigue S - N curve. The separation distance between the cracks was kept constant at $0.5c$, where c refers to the crack width. The deviation is calculated by

$$\text{Deviation (\%)} = \frac{N_{1 \text{ crack}} - N_{n \text{ crack}}}{N_{1 \text{ crack}}} \times 100\% \quad (28)$$

where N refers to the predicted fatigue life, and n refers to the number of cracks.

Table 3-6 Fatigue life prediction results for many cracks

Stress (MPa)	$N_{1 \text{ crack}}$	$N_{2 \text{ crack}}$	Deviation	$N_{10 \text{ crack}}$	Deviation	$N_{100 \text{ crack}}$	Deviation
225	44.095	42.651	3.27%	36.396	17.46%	24.698	43.99%
250	30.788	29.774	3.29%	25.388	17.54%	17.114	44.41%
275	22.217	21.482	3.31%	18.304	17.61%	12.247	44.88%
300	16.463	15.912	3.35%	13.538	17.77%	8.989	45.40%
325	12.468	12.048	3.37%	10.237	17.89%	6.741	45.93%

A total of 20 simulations were performed for multiple cracks and the results are summarized in Table 3-6. All initial cracks were propagated at the same time from the start. Using these values, the S - N curve can be generated (Figure 3-21). The results show that as the number of cracks increases, the predicted fatigue life decreases for the same stress range, causing a shift in the S - N curve to the left. The largest deviation that was observed was 56.93% for 100 cracks at 325 MPa. This signifies that the use of the multiple crack algorithm has a significant impact on the predicted factor life, as compared to only using a single crack fatigue life prediction algorithm.

Figure 3-21 S-N curve predicted by multiple crack fatigue life algorithm [144]

The change in crack aspect ratio, a/c , for multiple cracks as the cracks grow from an initial a/T ratio of 0.01 for the stress range of 250 MPa is also illustrated in Figure 3-22. When cracks coalesce, there is a sharp decrease in a/c ratio, resulting in a merged crack with lower a/c . This is due to the crack recharacterization method used in this fatigue life prediction model as described earlier and especially apparent where there are an increasing number of cracks (Figure 3-23).

Figure 3-22 Crack shape (a/t vs a/c) development [144]

(a) 1 crack

Figure 3-23 Illustration of Crack shape (a/t vs a/c) development

As the multiple crack recharacterization method is based on Paris Law, the prediction for fatigue crack growth analysis is valid only when the cracks have initiated, and the fatigue propagation of short cracks is not considered. Due to the small scale yielding of the rail steels and thermite weld, the behavior of short crack fatigue is also considered to be negligible. The focus of the current work is on semi-elliptical surface cracks in the weld toe region and does not consider sub-surface or internal cracks.

Another assumption is that due to the small size of the cracks, the cracks are assumed to propagate on the same 2D plane, and future research could consider a propagation algorithm for a 3D case.

Chapter 4 Study No. 2: Fatigue Investigations of Welded Pipe Cut-Out Specimens

This chapter presents the fatigue investigations of welded pipe cut-out specimens, starting with a look into the specimens used for the tests and their material properties. S-N Curve tests of welded pipe cut-out specimens under four-point bending was conducted and new SIF solutions of welded pipe cut-out specimens were developed based on 3D XFEM Model. An improved fatigue life analysis was developed which provided good fit to the tested fatigue life S-N curve results. An algorithm was developed for multiple fatigue crack propagation and the analysis provided details of the fatigue growth history of each crack, their sequence of coalescence and the evolving crack shape development

4.1 Materials and Specimens

Welded X65 offshore pipes (outer diameter 32" and 1" thickness) were cut into fatigue coupons ($t = 25.4$ mm, $b = 100$ mm) (Figure 4-1). The large width b specified allows multiple fatigue crack growth and coalescence. The actual fatigue specimen is shown in Figure 4-2.

Figure 4-1 Cutting of fatigue coupons from girth welded pipe

Figure 4-2 Welded pipe cutout specimen [145]

For the X65 steel, yield strength and ultimate tensile strength of the base material were obtained via relation to hardness using strength-hardness statistical correlation in API X65 steel [146] ($\sigma_Y = 550$ and UTS = 633 MPa). The detailed as-welded weld profile is shown in Figure 4-3. No cracks were identified prior to testing.

Figure 4-3 Profile of the girth weld

4.2 S-N Curve Tests Under Four-Point Bending

The four-point bending ($L = 240$ mm, $l = 120$ mm) is set up as shown in Figure 4-4. The specimen is oriented such that tensile stresses occur at pipe outer surface with weld cap.

Figure 4-4 Schematic of four-point bending fatigue test setup [145]

Figure 4-5 shows the actual four-point bending fatigue test setup. No grinding was applied to the specimen before testing.

Figure 4-5 Four-point bending fatigue test setup [145]

Figure 4-6 Stress-life curve for X65 steel, load ratio $R = 0.1$ [145]

The stress-life curve is shown compared to literature [147] in Figure 4-6, at load ratio of R of 0.1. No cracks were identified prior to testing.

In an attempt to view the crack shape, an additional striation marking test following the method of Schijve [148] was adopted. However, using a overload stress ratio of 1.4, no fatigue cracks were observed even after a significant number of cycles, which could be attributed to crack closure [149].

4.3 SIF Solutions of Welded Pipe Cut-Out Specimens

4.3.1 FEM Modelling Methodology

The SIFs of plain plates obtained by Newman and Raju [26] are used in the analysis, while the M_k factors were obtained by finite element analyses applied to a single crack in a cut-out plate.

The cut-out plate (Figure 4-7) has width b , wall thickness t , crack depth a and crack width c . The crack was located at the outer weld toe, with the geometry further described by dimensions of the circumferential weld: external width w_e , internal width w_i , external radius r_e and internal radius r_i .

Figure 4-7 Welded pipe cut-out with circumferential semi-elliptical surface crack

New SIF values were obtained for a cut-out plate with an external weld toe crack under tension or bending, where a/t was varied for a/c values of 0.25, 0.5 and 1.0. The weld dimensions are given in Figure 4-8.

Figure 4-8 Dimensions of the girth weld [150]

Since the study was concerned with the effects of the weld geometry, the material was taken to be homogeneous, using properties assumed in the plain pipe case ($E = 210 \text{ GPa}$, $\nu = 0.3$). Figure 4-9 shows meshing for large (Figure 4-9a-c) and small cracks (Figure 4-9d-f) for various crack aspect ratios. The loads and boundary conditions are applied as described above. Close-up views of the deformed weld toe cracks of $\frac{a}{t} = 0.5$ (large crack) and $\frac{a}{t} = 0.05$ (small crack) are shown in Figure 4-10a and Figure 4-10b, respectively.

Figure 4-9 Meshing of welded pipe for various ratios of a/t and a/c

Figure 4-10 Deformed weld toe cracks of $a/c = 0.25$ for (a) $a/t = 0.5$ and (b) $a/t = 0.05$

Post-processing requires handling of the J -integral, which refers to the path-independent contour integral around a crack used to determine the strain energy release rate. In three-dimensions the J -integral is given by

$$J = \lim_{\Gamma \rightarrow 0} \int_A \lambda(s) \mathbf{n} \cdot \left(W \mathbf{I} - \boldsymbol{\sigma} \cdot \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \right) \cdot \mathbf{q} dA \quad (29)$$

where $\lambda(s)$ is the virtual crack advance (of local direction $\mathbf{q}$) at point s along the crack front; dA is a surface element with outward normal $\mathbf{n}$; W is the elastic strain energy; $\boldsymbol{\sigma}$,

$\mathbf{u}, \mathbf{x}$ represent the surface traction, displacement and coordinate directions, respectively.

By nature of the approximation used in the finite element formulation, the J -integral evaluations in different contours may differ. Hence, the average J -integral value of the second to fifth contours was used. For linear elastic material response, the SIFs can be derived from the J -integral averages, assuming plane strain conditions throughout the crack front, using the relation

$$K_I = \sqrt{\frac{JE}{1 - \nu^2}} \quad (30)$$

where J refers to the average J -integral values, E is the Young's modulus and ν is the Poisson's ratio.

4.3.2 SIF and M_k Factor solutions

This section shows the SIF solutions and weld toe magnification factors (M_k factors) for the pipe geometry considered, as summarized in Table 4-1.

Table 4-1 SIF solutions and weld toe M_k factors

	$a/c = 1$	$a/c = 0.5$	$a/c = 0.25$	$a/c = 1$	$a/c = 0.5$	$a/c = 0.25$
a/T	K_a			K_c		
0.5	0.2109	0.3819	0.5939	0.7181	0.7173	0.7131
0.1	0.6077	0.7781	0.9354	0.9807	0.9609	0.8801
0.05	0.7219	0.9219	1.1009	1.2518	1.1320	1.0499
0.01	1.0674	1.4173	1.6800	2.0737	1.7784	1.6582
0.0075	1.1779	1.5559	1.7280	2.2331	1.8553	1.7470
0.005	1.2335	1.7756	2.0363	2.6187	2.1774	2.1041
a/T	$M_{k,a}$			$M_{k,c}$		
0.5	0.9416	0.9432	0.9852	1.0367	1.0162	0.9930
0.1	1.0214	0.9781	0.9950	1.3194	1.4336	1.4502
0.05	1.1275	1.0962	1.1052	1.6697	1.6593	1.7306
0.01	1.5718	1.5598	1.6275	2.7252	2.4495	2.8569
0.0075	1.7212	1.7554	1.6800	2.9330	2.7726	3.0205
0.005	1.8032	1.9588	1.9401	3.5009	3.2744	3.6523

The graphical plots for M_k factors for the welded pipe under tension at the deepest point $M_{k,a}$ and at the surface $M_{k,c}$ are shown in Figure 4-11 and Figure 4-12, respectively. Higher values of the M_k factors are found in the surface point as compared to the deepest point of the crack.

Figure 4-11 Weld toe magnification factors for $M_{k,a}$ [145]

Figure 4-12 Weld toe magnification factors for $M_{k,c}$ [145]

The M_k factors at the crack surface $M_{k,c}$ and at the crack depth $M_{k,a}$ of the upper bound are given by

$$M_{k,a} = 0.03901(a/t)^{-0.6289} + 0.8914$$

$$M_{k,c} = 0.262(a/t)^{-0.4547} + 0.6922$$
(31)

The solutions were curve fitted using MATLAB. The resultant curves for the $M_{k,a}$ and $M_{k,c}$ values had an R-square value of 0.9947 and 0.9975 respectively.

4.4 Fatigue Crack Growth and Life Prediction

For crack growth, an effective SIF range, ΔK_{eff} is commonly used in place of ΔK [14] to account for effects such as crack closure or threshold SIF:

$$\frac{da}{dN} = C(\Delta K_{\text{eff}})^m \quad (32)$$

Several different forms of ΔK_{eff} have been suggested. A threshold SIF (K_{th}) was introduced to explain non-propagation at lower stress levels, leading to ΔK_{eff} (with weld toe magnification M_k factors to account for welded geometry) in the form of

$$\Delta K_{\text{eff}} = M_k \Delta K - K_{\text{th}} \quad (33)$$

The K_{th} given in the BS 7910 code [14] takes the form

$$K_{\text{th}} = \begin{cases} 63 & \text{for } R \geq 0.5 \\ 170 - 214 R & \text{for } 0 \leq R < 0.5 \end{cases} \quad (34)$$

given in units of $\text{N/mm}^{3/2}$, where the load ratio is $R = \sigma_{\text{min}}/\sigma_{\text{max}}$.

Another widely accepted concept is the fatigue crack closure [151,152], which was developed to explain how stresses below a crack opening stress, σ_{op} of the fatigue cyclic load can be insufficient to propagate the crack. Some works showed that the large-crack threshold could be attributed to an increase in crack-closure behavior as load is reduced [153]. The effective stress intensity factor is related to the crack closure parameter U by

$$\Delta K_{\text{eff}} = U \cdot M_k \Delta K \quad (35)$$

where U is defined by Elber [151] as

$$U = \frac{\sigma_{\max} - \sigma_{\text{op}}}{\sigma_{\max} - \sigma_{\min}} \quad (36)$$

Codrington and Kotousov [154] employed the distributed dislocation technique to determine a crack closure model for plates of finite thickness, relating U to load ratio R by

$$U = A + B \cdot R + C \cdot R^2 \quad (37)$$

where the coefficients are

$$\begin{aligned} A &= 0.446 + 0.266 \cdot e^{-0.41\eta} \\ B &= 0.373 + 0.354 \cdot e^{-0.235\eta} \\ C &= 0.2 - 0.667 \cdot e^{-0.515\eta} \end{aligned} \quad (38)$$

for which the constraint factor η is

$$\eta = \frac{\Delta K}{(1 - R)\sigma_{\text{flow}}\sqrt{h}} \quad (39)$$

varying between plane stress ($\eta \rightarrow \infty$) and plane strain ($\eta \rightarrow 0$) for different values of half-thickness h , which refers to the thickness of the plate on either side of the crack.

The flow stress, σ_{flow} is taken to be an average of yield and tensile strength.

The following fatigue law constants were used: design values of $C = 5.21 \times 10^{-13}$ and $m = 3$ as given in the BS 7910 standard [14]. In comparison, for mean curve, values of $C = 1.5 \times 10^{-13}$ for $R = 0.1$ [155], and $m = 3$ were used. For the crack closure parameter U , at the crack surface, h in equation (39) is assumed to be 3 mm based on the weld dimensions. At the crack depth, U varies as the crack grows because η is a function of ΔK , which is given by $\Delta \sigma \sqrt{\pi a}$, while h is 50 mm. Values of the closure parameter U used are shown in Figure 4-13.

Figure 4-13 Range of crack closure parameter U in analysis [145]

Figure 4-14 Stress-life curves for X65 steel at load ratio $R = 0.1$ [145]

Based on the experiments (Curve 1 in Figure 4-14), at the highest load range 450 MPa, (82 % of yield stress), failure occurred at 130 thousand cycles, and at the lowest tested load range 225 MPa, no failure could be determined up to two million load cycles.

Curves 2 to 4 in Figure 4-14 show the predicted crack propagation life for the crack distributions observed of initial crack depth $a = 0.1$ mm. Curve 2, which uses the BS 7910 design curve predicts a fatigue life of about 10 % of the actual life. Curve 3, which uses mean curve data ($C = 1.5 \times 10^{-13}$ from [155]) with the threshold SIF

predicts higher life than Curve 2, but only half the actual life is reflected. When crack closure and equivalent area crack coalescence are considered with the mean curve constants, the prediction becomes closer to the tested fatigue life compared to the threshold SIF approach (Curve 4). Considering the equivalent area during crack coalescence increases predicted fatigue life by 9% at higher loads but becomes less significant (3%) at lower loads.

4.5 Multiple Crack Fatigue Life Prediction Model

Figure 4-15 Algorithm for multiple crack propagation [145]

The algorithm of the multiple fatigue crack propagation is shown in Figure 4-15. The input requires information on the crack, material and fatigue parameters. Crack data would include the crack initiation sites, initial and final depth sizes, and crack aspect ratios. Material properties required are the yield stress and ultimate tensile stress. Fatigue parameters, such as C and m from the Paris law related to the fatigue propagation law are input in this stage as well.

In the algorithm (Figure 4-15), the cracks are first checked against the final crack depth criteria. If the deepest crack depth does not exceed the final crack depth, the crack is propagated based on the number of cycles stepwise. All adjacent cracks are then checked for overlaps. If no overlap is detected, the final crack depth is checked again. When an overlap is detected, the new coalesced crack is represented by a crack with an area equivalent to the sum of the two original cracks [109,110], by constraining the new width as a sum of the two crack widths. This algorithm enables the coalescence stage of multiple fatigue crack growth to be considered. This new crack will replace the larger of the two original cracks, while the smaller crack is removed. After all overlapping cracks have coalesced, the final crack depth criterion is re-evaluated. If the criterion is met, the program outputs data and ends.

In this analysis of multiple fatigue crack propagation, all cracks are assumed to initiate at the same time. The interaction between multiple cracks [41,156] is not considered, leading to a non-conservative prediction.

Information on the number and position of the cracks was obtained by analyses of the fracture surfaces. A range of 12 to 22 cracks were observed by identifying the ratchet marks, and the locations of each crack initiation site were recorded in Table 4-2.

Traditionally, typical weld toe defects have been known to be 0.15 to 0.4 mm deep [157]. Assuming semi-elliptical cracks, conservative values of initial crack depth ($a =$

0.1 mm) and aspect ratio ($\frac{a}{c} = 0.25$) were used. In the analysis, no residual stresses were assumed.

Table 4-2 Crack initiation sites identified from surface of fractured samples [145]

Stress range $\Delta\sigma$ / MPa	Number of cracks	Final crack depth / mm	Distance of crack initiation sites from edge / mm				
292.5	12	18.6	13.9	17.6	20.3	24.1	29.9
			36.9	47.6	57.2	63.1	68.2
			71.7	75.7			
320	20	17.5	11.1	18.9	25.5	28.9	32.9
			35.8	38.4	41.1	43.7	48.2
			52.1	53.9	55	57.6	61.6
			69.5	77.4	80.3	84.5	89.2
337.5	14	16.25	9	16.3	21.5	27.4	33.2
			39.1	45.4	49.5	51.9	56.5
			61.4	68.8	78.5	84.8	
382.5	22	19.4	13.2	18.8	24	26.4	28.8
			32.2	36.5	40.9	44.7	49
			54.8	59.6	62.7	66.6	70.7
			75.7	80.3	83.9	87.7	90.4
			92.5	94.5			
450	18	13.3	9.5	16.2	25.3	32.5	34.3
			36.9	42.3	47.7	51.8	56.4
			60.6	64.7	67.5	68.8	70.9
			77.3	85.8	93.3		

Table 4-3 Comparison of fracture surfaces [145]

Stress range $\Delta\sigma$ / MPa	Predicted surface	Experimental fracture surface
(a) 292.5
(b) 320
(c) 337.5
(d) 382.5

The predicted fracture surfaces are also shown to match the actual fracture surfaces (Table 4-3). The asymmetry in the fracture surface is captured correctly in most cases, and the inclusion of corner cracks (or cracks close to the corner) would improve the predictions.

In Table 4-3, each crack is identified by a unique colour. When crack coalescence occurs, the coalesced crack will assume the colour of the dominant crack. Thus, as the cracks propagate deeper only the colour of the dominant crack would remain. The crack colours are also used in Figure 4-16 such that each line represents the change in crack aspect ratio over depth for a particular crack. As coalescence occurs, two lines representing two different cracks would merge into a single line, assuming the colour of the dominant crack. Hence the growth and coalescence of the cracks can be traced.

Figure 4-16 Crack aspect ratio (a/c) against crack depth (a/t) for multiple fatigue crack propagation [145]

The coalescence of cracks shown in Figure 4-16 is complicated and can be interpreted by considering Figure 4-17 and Table 4-4, which considers the 12-crack coalescence sequence under the $\Delta\sigma = 292.5 \text{ MPa}$ load (Figure 4-16a and Table 4-3a). Figure 4-17a zooms in on the crack aspect ratio vs crack depth ratio growth, Figure 4-17b shows the number associated with each crack, while Table 4-4 shows the predicted crack propagation step-by-step.

Under this load, the cracks initiate at $\frac{a}{c} = 0.25$ and the aspect ratio increases with crack depth ratio $\frac{a}{t}$ (Table 4-4a). One pair of cracks (C2 and C3) coalesce at $\frac{a}{t}$ of 0.026 (point A in Table 4-4b), with C2 assumed dominant since both are equal.

Subsequently, C10 and C11 merge (point B), with C10 assumed dominant.

Crack C2 then merges with C1 and C4 (C2 dominant, point C in Table 4-4d), while C10 merges with C12 (C10 dominant, point D in Table 4-4e). C10 subsequently merges with C9 (point E), and then with C8 (point F), with C10 emerging dominant. C2 then merges with C5 (point G) followed by C6 (point H in Table 4-4i), C2 prevailing.

Figure 4-17 Coalescence of multiple cracks in fatigue for $\Delta\sigma = 292.5$ MPa [145]

Table 4-4 Details multiple crack propagation for $\Delta\sigma = 292.5$ MPa [145]

Max $\frac{a}{t}$	Crack aspect ratio vs crack aspect ratio	Predicted fracture surface
(a) 0.025		
(b) 0.035		
(c) 0.0485		
(d) 0.0495		
(e) 0.075

At this depth of $\frac{a}{t} = 0.2$, three cracks remain: C2 (coalesced), C10 (coalesced) and C7 (which has been growing independently so far). C10 will then dominate C7 (point I in Table 4-4j), and prevail over C2 (point J). The single coalesced C10 crack will then grow to from $\frac{a}{c}$ of 0.2 to 0.35 and fall back to 0.3 before fracture, as seen in Figure 4-17a. This occurs because the crack closure parameter U increases with crack width as well, making ΔK_{eff} more significant at the crack width than at crack depth.

Chapter 5 Study No. 3: Fatigue Study on Cruciform Welded Joint Specimens

This chapter presents the fatigue study on cruciform welded joint specimens, starting with tension loading and four-point bending tests carried out to examine weld toe failures. XFEM was used to model a semi-elliptical surface crack to obtain new SIFs and new M_k factor solutions, which showed good agreement with established solutions. With fatigue crack growth and life prediction incorporated into the algorithm of the multiple fatigue crack propagation, detailed predicted fracture surfaces were generated which showed good agreement with the experimental fracture surface.

5.1 Materials and Specimens

A section of a cruciform joint is shown in Figure 5-1, where fatigue test pieces were extracted from. The actual fatigue specimen is shown in Figure 5-2. The overall dimensions of the specimens are as shown in Figure 5-3.

The base metal is a high strength carbon steel while the weld metal is an austenitic stainless-steel filler. The fillet welds are partial penetration welds with approximately 9mm weld root penetration as observed from the specimen.

Figure 5-1 Extraction of fatigue test pieces

Figure 5-2 A longitudinal section of a cruciform joint

Figure 5-3 Dimensions of cruciform welded specimens

Table 5-1 shows the weight percentage of the chemical compositions of parent metal, an armor steel, analyzed with an EDX (energy dispersive X-ray).

The carbon equivalent of the parent metal was calculated to be 1.1438%.

Table 5-2 gives the mechanical properties of the materials.

Table 5-1 Chemical compositions of the parent metal

C	Si	P	S	Cr	Mn	Ni	Mo
0.652	0.226	0.000	0.139	0.839	1.941	1.620	0.034

The carbon equivalent of the parent metal was calculated to be 1.1438%.

Table 5-2 Mechanical properties of the materials

	σ_u (MPa)	σ_y (MPa)	E (GPa)	Poisson's ratio, ν
Base metal	1600	1300	207	0.3
Weld metal	634	467	207	0.3

The elastic modulus or Young's modulus, E, the ultimate tensile strength or tensile strength, σ_u , yielding stress σ_y or offset yield strength $\sigma_{0.2}$, the percent elongation, and the percent reduction in area of the steel were measured under tensile stress test. The test results are given in Table 5-3. The total strains at σ_u and $\sigma_{0.2}$ are 3.5524% and 0.8818% respectively.

Table 5-3 Measured mechanical properties of the armoured steel

E (Gpa)	σ_u (MPa)	$\sigma_{0.2}$ (MPa)	Elongation (%)	Reduction of area (%)
210.4	1714.8	1425.0	18.0	40.39

5.2 S-N Curve Tests Under Axial Load

The S-N curve data are required for the studies on fatigue life prediction and improving fatigue strength of the welded joints. The fatigue tests were conducted with frequency from 10 to 15 Hz and the stress ratio $R = 0.1$ for all the tests. The test machine used was the Shimadzu Servopulser, a hydraulically operated testing machine with a load cell of 50kN (Figure 5-4).

Figure 5-4 Shimadzu Servopulser with Servo Controller 4830

Figure 5-5 Experimental Setup and mounting

The steel cruciform joint specimens were mounted on test jigs connected to the Shimadzu Servopulser (Figure 5-5). The plates have a cross-sectional area of 450mm^2 .

The fatigue test results of as-welded joints with weld toe and weld root failure mode are given in Table 5-4 and Table 5-5 respectively, while the S-N curves are shown in Figure 5-6 and Figure 5-7 respectively.

Table 5-4 Fatigue test results of as-welded specimens under axial load (Weld toe failures)

Stress range (MPa)	Cycles to failure	Failure mode	Specimen No.
90	4,907,000	Weld toe	32
100	3,651,672	Weld toe	15
120	1,296,010	Weld toe	4
150	1,707,477	Weld toe	29
150	1,340,000	Weld toe	17

Figure 5-6 S-N curves of the as-welded joints under axial loading (Weld toe failures)

Table 5-5 Fatigue test results of as-welded specimens under axial load (Weld root failures)

Stress range (MPa)	Cycles to failure	Failure mode	Specimen No.
120	1,774,659	Weld root	9
120	456,000	Weld root	14
200	118,179	Weld root	22
250	33,868	Weld root	38

Figure 5-7 S-N curves of the as-welded joints under axial loading (Weld root failures)

From the fatigue test results and S-N curves, it can be observed that for as-welded joints under axial loading, weld root failure tends to occur at higher loading over 120 MPa. On the other hand, the weld toe failure tends to take place at lower loading below 150 MPa.

The fracture surface of one as-welded specimen is given in Figure 5-8 and a picture of a specimen which failed at the weld root is shown in Figure 5-9. Figure 5-10 [158] explains the crack profile at the weld toe for the welded cruciform joint from research done by Ghafoori-Ahangar and Verreman, which is similar to the fracture surface in Figure 5-8.

Figure 5-8 Fracture surface of as-welded specimen

Figure 5-9 Failure location at weld root

Figure 5-10 Crack profile at the weld toe [158]

During the course of axial load testing, another joint specimen was tested under axial loading of 200 MPa and failed at the weld root with 45,776 cycles. As noted previously, failures of cruciform joints may occur at the weld root or weld toe (Figure 5-11) [159,160]. This is dependent on several influencing factors, including magnitude of the applied stress [161,162], stress amplitude [162,163] and weld size [8,162].

Figure 5-11 Different fatigue failure mode of cruciform joints [158]

The failure at the weld root could also be due to partial penetration, where there may be other flaws inside the welding part which may provide crack initiation locations, especially at the weld root. Under such circumstances, specimens may fracture at weld root and not at the weld toe under axial loading, regardless of weld toe grinding.

In order to avoid further failures of cruciform joints at the weld root, four-point bending tests were carried out, as the stress at the surface of specimen near the weld toe is largest and the stress at neutral plane near the weld root is low. The specimens are more likely to fail at the weld toe, and such bending tests can better facilitate investigation of weld toe failures.

5.3 S-N Curve Tests Under Four-Point Bending

The experimental set-up is illustrated in Figure 5-12. The results of cruciform joint under four-point bending loading are shown in Table 5-6, and S-N curves are indicated in Figure 5-13. The stress ratio is 0.1 ($R = 0.1$) for all the tests.

Figure 5-12 Four-point bending test setup

Table 5-6 Fatigue test results of welded joints loaded in four-point bending

Stress range (MPa)	Cycles to failure	Failure mode	Specimen No.
400	98,855	Weld toe	21
300	474,517	Weld toe	11
250	1,044,200	Weld toe	37
225	10,000,000	No failure	6
200	10,000,000	No failure	28

Figure 5-13 S-N curves of the welded joints loaded in four-point bending

From Figure 5-13, it can be found that the fatigue strengths of the cruciform joints under bending loading are much higher than under tension loading. At the lower stresses with longer lives, the gaps between the two kinds of loading are larger than those at the higher stresses with shorter lives.

5.4 SIF Solutions of Cruciform Joints Based On 3D XFEM Model

5.4.1 FEM Modelling Methodology

The cruciform joints have the dimensions as shown in Figure 5-14a. After assembling the model, partitioning is done to minimize distortion of elements during meshing, thereby improving computation time and the accuracy of results. As shown in Figure 5-14b, partitions were made by following the geometry of the crack front. The model was also partitioned into several parts.

Figure 5-14 a) Cruciform joint dimensions used for modelling b) Boundary conditions and loading applied to partitioned model

A loading force is applied to the model equivalent to the axial force of the machine pulling the specimen as shown in Figure 5-14b. A fixed boundary condition was set at

the bottom of the model. This is done so that this region can be constrained to remain fixed during the simulation.

In order to specify the target mesh density of a particular region, seeds are placed along the edges of the region. These seeds act as markers for ABAQUS to generate the mesh accordingly to what is desired. The seeds were positioned with a bias towards the crack edge.

The area near the crack were assigned a higher density of mesh while non-critical areas can have lower density of mesh with larger sized elements. By creating a balance of more elements at the critical areas and less elements at non-critical areas, computation time is improved without sacrificing quality of the simulation results. The meshed model is shown in Figure 5-15, which also shows the higher mesh density applied to the position of the crack at the weld toe.

Figure 5-15 Meshed model showing higher density mesh near crack

A XFEM crack was created through the interaction module of ABAQUS and assigned to the model. ϕ function of the level set method for the signed distance function (PHILSM) was requested as the field output request to enable the crack growth to be viewed in the visualization module.

Figure 5-16 Crack opening for $a = 0.5$ and $a/c = 1$

Figure 5-16 depicts a close-up view of the simulated crack opening for crack length of $a = 0.5$ with aspect ratio of $a/c = 1$. SIF values were then obtained from the data file of the simulated job. A total of 20 contours integrals were used to evaluate the SIF values. The SIFs from the first three contours were ignored, and the average of the 4th to 20th contours were used in evaluating the average SIF values at each crack location.

5.4.2 SIF and M_k Factor solutions

This section shows the SIF solutions and weld toe magnification factors (M_k factors) for the cruciform joint geometry considered, as summarized in Table 5-7.

Table 5-7 SIF solutions and weld toe M_k factors

	$a/c = 1$	$a/c = 0.5$	$a/c = 0.25$	$a/c = 1$	$a/c = 0.5$	$a/c = 0.25$
a/T	K_a			K_c		
0.5	0.5919	0.9065	1.5173	0.9455	1.0190	1.4500

0.3	0.6526	0.8717	1.1383	0.9837	0.9412	0.9197
0.1	0.8683	1.2694	1.4268	1.2747	1.1607	1.0312
0.05	1.1209	1.4174	1.6804	1.4129	1.3198	1.2262
0.025	1.3394	1.6889	2.0215	1.8578	1.8339	1.5540
a/T	$M_{k,a}$			$M_{k,c}$		
0.5	0.8442	0.8276	0.8443	1.1356	1.1079	1.3589
0.3	0.9657	0.9075	0.9376	1.2865	1.2247	1.3389
0.1	1.3079	1.4067	1.3605	1.7399	1.6484	1.7820
0.05	1.6910	1.5792	1.6205	1.9362	1.8890	2.1483
0.025	2.0213	1.8842	1.9546	2.5483	2.6299	2.7315

The graphical plots for M_k factors for the cruciform joint under tension at the deepest point $M_{k,a}$ and at the surface $M_{k,c}$ are shown in Figure 5-17. Higher values of the M_k factors are found in the surface point as compared to the deepest point of the crack.

Figure 5-17 Weld toe magnification factors $M_{k,a}$

Figure 5-18 Weld toe magnification factors $M_{k,c}$

The weld toe M_k factors were compared with established solutions by Zhao [164]. The M_k equations for the crack deepest point and the crack ends in the load-carrying cruciform welded joints under tension are given in Appendix B. The investigation was carried out and the results are summarized in Table 5-8. The deviation is calculated by

$$\text{Deviation (\%)} = \frac{Y_{\text{XFEM}} - Y_{\text{Zhao}}}{Y_{\text{Zhao}}} \times 100\% \quad (40)$$

Figure 5-19 Deviation histograms

It is observed from Figure 5-19 that the deviation histograms show a good normal distribution of the percentage deviation, indicating that the proposed M_k factors show good agreement with established solutions by Zhao [164].

Table 5-8 Comparison of $M_{k,a}$ results to Zhao

a/c	a/T	Crack depth, $M_{k,a}$ (XFEM)	Crack depth, $M_{k,a}$ (Zhao)	Deviation (%)	Crack end, $M_{k,c}$ (XFEM)	Crack end, $M_{k,c}$ (Zhao)	Deviation (%)
1	0.5	0.8442	0.7561	11.7%	1.1356	1.1761	-3.4%
1	0.3	0.9657	0.9108	6.0%	1.2865	1.3145	-2.1%
1	0.1	1.3079	1.4011	-6.7%	1.7399	1.6119	7.9%
1	0.05	1.6910	1.7172	-1.5%	1.9362	1.9735	-1.9%
1	0.025	2.0213	NA	NA	2.5483	NA	NA
0.5	0.5	0.8276	0.7561	9.5%	1.1079	1.1975	-7.5%
0.5	0.3	0.9075	0.9074	0.0%	1.2247	1.3476	-9.1%
0.5	0.1	1.4067	1.3659	3.0%	1.6484	1.6534	-0.3%
0.5	0.05	1.5792	1.6643	-5.1%	1.8890	2.0085	-5.9%
0.5	0.025	1.8842	NA	NA	2.6299	NA	NA
0.25	0.5	0.8443	0.7862	7.4%	1.3589	1.2796	6.2%
0.25	0.3	0.9376	0.9371	0.1%	1.3389	1.4246	-6.0%
0.25	0.1	1.3605	1.3771	-1.2%	1.7820	1.7237	3.4%
0.25	0.05	1.6205	1.6643	-2.6%	2.1483	2.0658	4.0%
0.25	0.025	1.9546	NA	NA	2.7315	NA	NA

The M_k factors at the crack surface $M_{km,c}$ and at the crack depth $M_{km,a}$ of the upper bound for a cruciform joint subjected to tension are curve fitted into solutions which can be expressed in the following form:

$$\begin{aligned} M_{km,a} &= 1.571(a/t)^{-0.1723} - 0.9413 \\ M_{km,c} &= 0.1693(a/t)^{-0.623} + 1.05 \end{aligned} \quad (41)$$

The solutions were curve fitted using MATLAB. The resultant curves for the $M_{km,a}$ and $M_{km,c}$ values had an R-square value of 0.9989 and 0.9944 respectively.

The M_k factors at the crack surface $M_{kb,c}$ and at the crack depth $M_{kb,a}$ of the upper bound for a cruciform joint subjected to bending are also curve fitted into solutions which can be expressed in the following form:

$$\begin{aligned} M_{kb,a} &= 0.06783(a/t)^{-0.674} + 0.7248 \\ M_{kb,c} &= 0.03155(a/t)^{-0.8868} + 1.032 \end{aligned} \quad (42)$$

The solutions were curve fitted using MATLAB. The resultant curves for the $M_{kb,a}$ and $M_{kb,c}$ values had an R-square value of 0.9975 and 0.9924 respectively.

5.5 Fatigue Crack Growth and Life Prediction

The procedure for fatigue crack growth and life prediction is similar to that of Section 4.4. An effective SIF range, ΔK_{eff} was similarly used in place of ΔK [14] to account for effects such as crack closure or threshold SIF. A threshold SIF (K_{th}) was also used with accompanying M_k factors to account for welded geometry. The following fatigue law constants were used: design values of $C = 5.21 \times 10^{-13}$ and $m = 3$ as given in BS 7910 [14]. A threshold SIF (K_{th}) of 63 was used, as well as an initial crack depth a of 0.5 mm and initial crack aspect ratio of 1.0.

Figure 5-20 Stress-life curves for cruciform joint

The fatigue life prediction for both cases of tension and bending loading are plotted in graphical form, as seen in Figure 5-20. As can be seen, the fatigue life prediction was very close to the tested fatigue life for tension loading. The prediction for bending loading was lower than that of the experimental result, providing a more conservative prediction.

Development of crack aspect ratio

Figure 5-21 Changes in a/c (single crack) for a) Tension loading b) Bending loading

Graphs of crack aspect ratio (a/c) against crack depth ratio (a/t) were plotted to study the development of the crack shapes for different kinds of loading in Figure 5-21. The initial crack aspect ratio (a/c) that were studied were 1.0, 0.5 and 0.25 with an applied tension or bending loading of 100 MPa. As the cracks grew, the cracks tended towards a semi-elliptical shape with an a/c of 0.501 in tension loading, while the cracks under bending loading tended towards a semi-elliptical shape with an a/c of 0.295. Therefore, smaller aspect ratios can be expected for cracks under bending loading than tension loading.

Effect of initial crack aspect ratio (a/c) on predicted fatigue life

Figure 5-22 Fatigue life prediction (single crack) for a) Tension loading b) Bending loading

From Figure 5-22, it can be seen that the predicted fatigue life is heavily influenced by the initial crack aspect ratio. In the case of tension loading, using an initial crack aspect ratio of 0.5 and 1.0 predicted a 26.6% and 50.3% increase in fatigue life respectively as compared to using an initial crack aspect ratio of 0.25. For bending loading, the percentages are 24.5% and 48.0% for initial crack aspect ratios of 0.5 and 1.0 respectively.

5.6 Multiple Crack Fatigue Life Prediction Model

The algorithm of the multiple fatigue crack propagation is shown in Figure 5-23, similar to the one previous used. The input requires information on the crack, material and fatigue parameters. Crack data would include the crack initiation sites, initial and final depth sizes, and crack aspect ratios. Material properties required are the yield stress and ultimate tensile stress. Fatigue parameters, such as C and m from the Paris law related to the fatigue propagation law are input in this stage as well.

Figure 5-23 Algorithm for multiple crack propagation

In the algorithm (Figure 5-23), the cracks are first checked against the final crack depth criteria. If the deepest crack depth does not exceed the final crack depth, the crack is propagated based on the number of cycles stepwise. All adjacent cracks are then checked for overlaps. If no overlap is detected, the final crack depth is checked again. When an overlap is detected, the new coalesced crack is represented by a crack with an area equivalent to the sum of the two original cracks [109,110], by constraining the new width as a sum of the two crack widths. This new crack will replace the larger of the two original cracks, while the smaller crack is removed. After all overlapping cracks have coalesced, the final crack depth criterion is re-evaluated. If the criterion is met, the program outputs data and ends. All cracks are assumed to initiate at the same time. The interaction between multiple cracks [41,156] is not considered.

Information on the number and position of the cracks was obtained by analysis of the fracture surfaces. Cracks were identified by observing the ratchet marks, and the locations of each crack initiation site are recorded in Table 5-9. Assuming semi-elliptical cracks, conservative values of initial crack depth ($a_i = 0.5$ mm), final crack depth ($a_f = 5.0$ mm) and initial crack aspect ratio ($a_i/c_i = 0.5$) were used. In the analysis, no residual stresses were assumed.

Table 5-9 Crack initiation sites identified from surface of fractured samples

Stress range $\Delta\sigma$/ MPa	Number of cracks	Distance of crack initiation sites from left edge (mm)				
90.0	13	5.5	8.9	10.9	12.5	16.6
		20.9	22.4	24.9	29.1	33.5
		35.8	37.1	40.0

Figure 5-24 Comparison of fracture surfaces a) Experimental fracture surface b) Predicted surface c) Fracture surface with overlay of predicted surface

From Figure 5-24, it can be observed that the predicted fracture surfaces are shown to match the actual fracture surfaces. The final crack shape from the fracture surface is captured adequately, and the locations where the cracks merge match up to the locations of the ratchet marks on the crack surface.

Effect of number of initial cracks used on the predicted fatigue life

A case study was conducted to determine the effect of number of initial cracks used on the predicted fatigue life and crack shape development. A total of 3 cases were studied – 13 initial cracks, 5 initial crack and 1 single initial crack. All other parameters were kept constant for this case study.

Table 5-10 Crack initiation sites for case study

Number of cracks	Distance of crack initiation sites from left edge (mm)				
	5.5	8.9	10.9	12.5	16.6
13	20.9	22.4	24.9	29.1	33.5
	35.8	37.1	40.0		
5	7.2	15.1	24.1	30.2	38
1	22.5				

The predicted fracture surfaces can be seen from Figure 5-25.

Figure 5-25 Comparison of fracture surfaces a) 13 initial cracks b) 5 initial cracks c) 1 initial crack

From the predicted fracture surfaces in Figure 5-25, it can be seen that the fracture surface with 13 initial cracks presented the most detailed and best approximation of the experimental fracture surface. Conversely, the predicted fracture surface of 1 initial crack is unable to fully capture the final crack shape from the fracture surface

and represents an oversimplification of the fatigue life prediction. The predicted fracture surface of 5 initial cracks provided an in between representation, being able to give a reasonable approximation of the experimental fracture surface with less details.

The predicted fatigue life of the 3 different cases can be found in Table 5-11, while the crack depth (a/T) vs cycles plot is given in Figure 5-26. The deviation is calculated by

$$\text{Deviation (\%)} = \frac{N_{1 \text{ crack}} - N_{n \text{ crack}}}{N_{1 \text{ crack}}} \times 100\% \quad (43)$$

where N refers to the predicted fatigue life, and n refers to the number of cracks.

Table 5-11 Effect of number of initial cracks used on predicted fatigue life

	Predicted fatigue life (cycles)	Deviation (%)
1 initial crack	1,323,215	
5 initial cracks	1,096,807	17.1
13 initial cracks	886,262	33.0

Figure 5-26 Plot of crack depth (a/T) vs cycles

From the predicted fatigue life in Figure 5-26, it can be seen that the predicted fatigue life using 1 initial crack was the longest. The use of 13 initial cracks provided the

most conservative fatigue life (33.0% shorter), followed by the predicted fatigue life of 5 initial cracks (17.1% shorter). This phenomenon happens because of the recharacterization of the multiple cracks into a single crack, where there is a sudden change in the crack shape and overall size. It is worth noting that this happens mostly towards the end of the fatigue life where large cracks are forming and being recharacterized, and the crack growth rate are largely the same when the cracks are small.

This study showed that the number of initial cracks used had an effect on the predicted fatigue life. More initial cracks provided a more conservative fatigue life, while giving a more detailed approximation of the experimental fracture surface. The predicted crack shape development is shown in Figure 5-27.

Figure 5-27 Crack shape development of a) 13 initial cracks b) 5 initial cracks c) 1 initial crack

Chapter 6 Study No. 4: Fatigue Study on Thermite Welded Four-Point Bending Fatigue Inverted Specimen Test and Fatigue Life Prediction

This chapter presents a fatigue study on thermite welded four-point bending fatigue inverted specimen test and fatigue life prediction. Inverted four-point bend tests were conducted to determine the fatigue strength of rail welded specimens, with failure locations at the head and head-to-web transition zone. Using a 3D XFEM Model, new SIF and M_k factor solutions of rail specimens were developed. The fatigue life prediction incorporated a coupled root mean square (RMS) method to provide a good fit to the experimental fatigue test data, and the predicted fracture surfaces were shown to closely resemble the actual fracture surfaces.

6.1 Materials and Specimens

The chemical composition and mechanical properties of rail steels (R260 and R350HT) are standardized and can be obtained from manufacturers, while the chemical composition and mechanical properties of thermite weld metal are shown in Table 6-1 below.

Table 6-1 Mechanical properties of Thermite weld

Hardness (BHW)	0.2% offset yield strength (MPa)	Ultimate tensile strength (MPa)	Elongation (%)	Reduction area (%)
290 ~ 325	724 ($R_{p0.2}$)	758	1	2

6.2 S-N Curve Tests for Inverted four-point bend test of rail specimen

Inverted four-point bend test of rail specimen tests of as-welded specimens and a needle peened specimen (R350HT) were conducted to determine the fatigue strength of welded specimens. EN 14730-1:2017 Annex J [165] was adopted as the test

procedure. The welded rail sample was placed horizontally inverted (with rail foot on top and rail head at bottom) on two point supports, $L = 1100$ mm apart with the weld midway between them. The inner span, W , was set to 200 mm (refer to Figure 6-1). 2 steel rods with diameter 80mm was used as loading points and support points. A load with sinusoidal waveform was applied to the sample at the frequency of 3 Hz between the maximum load and the minimum load until the test piece broke. The R ratio used for the test was 0.1. The experimental set up is shown in Figure 6-2.

Figure 6-1 Schematic of experimental set up

Figure 6-2 Experimental set up

The test results are shown below in Table 6-2, and S-N curves are indicated in Figure 6-3.

Table 6-2 Results of Fatigue Test

Stress Range (MPa)	Life (Cycles)	Specimen condition	Failure location	Specimen No.
461.10	46,339	As-welded	Weld-toe at web-to-head	3-R
320.21	701,030	As-welded	Weld-toe at web-to-head	1
288.18	293,729	As-welded	Weld-toe at web-to-head	2
224.78	2,000,000	As-welded	No failure	3
320.21	1,303,159	Peened	Parent rail initiating from the head	4

Note: R indicates that the specimen used was from previous “No failure” test

Figure 6-3 S-N curves of inverted four-point bend test of rail specimen

The test results are plotted together with previous research by Lawrence et al. [94], where conventional (non-inverted) S-N curve were conducted. From the results, it could be seen that the fatigue life of a specimen in the inverted fatigue test was about 3X that of the conventional test. The fatigue life of the peened specimen is also almost 2X of that of the as-welded specimen. This suggests that the peened specimen is more resilient to fatigue loading and needle peening has beneficial effects on the fatigue strength of welds.

The fracture surfaces of the as-welded specimens are shown in Figure 6-4, and the fracture surface of the peened specimen is shown in Figure 6-5. The orange arrows indicate probable crack initiation sites.

(a)

(b)

Figure 6-4 Fracture surfaces of as-welded specimens a) Specimen 1 b) Specimen 2 c) Specimen 3-R

From the fracture surfaces in Figure 6-4, it can be seen that the crack initiated from the head-to-web transition area along the edge as pointed out by the arrows. This was to be expected as the weld is the weakest point of the specimen undergoing 4-point bending load due to the stress concentration effects at the weld toe.

Figure 6-5 Fracture surfaces of peened specimen

For the peened specimen (Figure 6-5), the crack initiated from the head of the rail instead, which was the bottom of the 4-point bend. Closer investigation of the fracture surface revealed that the plane where the crack initiated was far away from the weld, indicating that the crack initiated from the parent rail.

6.3 SIF Solutions Based On 3D XFEM Model

6.3.1 FEM Modelling Methodology

The SIFs for different surface crack locations and sizes were derived from FEM of rail steel and thermite weld segments shown in Figure 6-6 and Figure 6-7.

(a) In-plane loading

(b) Out-of-plane loading

Figure 6-6 Loading conditions used in FEM [166]

The in-plane loading was modeled as vertical bending moment (M_x) and uniformly distributed tensile stress (N_z) as shown in Figure 6-6a while the out-of-plane loading was modeled as lateral bending (M_y) shown in Figure 6-6b.

Figure 6-7 FEMs for five different crack locations [166]

Due to the dynamic contact interaction between the wheel and rail, rail and welded joints are subjected to a wave-like deformation [167]. Both the rail head region and base region are subjected to cyclic in-plane tension and compression stress causing fatigue loading. In these models, a vertical bending was applied for the rail base cracks while a reverse bending was applied for the head cracks.

The global model of a rail section with length of 200 mm and its detailed mesh are shown in Figure 6-7. The cracks were modeled starting from 0.1 mm, propagating to final fracture crack size. The final crack sizes were set to be 25 mm for cracks in the

head (Figure 6-7b), base bottom (Figure 6-7d), base corner of field side (Figure 6-7e) and 20 mm for cracks in the head-to-web (Figure 6-7a), web-to-base (Figure 6-7c).

The service loading condition considered will result in a combination of mode I crack opening and sliding due to shear stress (modes II and III) [168]. However, rail transverse cracks propagates mainly in the transverse plane normal to the loading direction, therefore mode I failures has the most significant impact in the development of these transverse cracks. The mode I SIF can be simplified as

$$K_I = Y\sigma\sqrt{\pi a} = Y\left(\frac{a}{t}, \varphi\right)\sigma\sqrt{\pi a} \quad (44)$$

where σ is the applied remote stress, Y is the shape factor, a is the crack size and t is equal to width of rail web (16.5 mm). Following Newman and Raju's [59] classical empirical solutions, the shape factor Y was defined as a function of crack size, a and angle, φ . The angle φ starts from the x -axis following an anti-clockwise direction as shown in Figure 6-7a. The width of the web, t , was included with expression of a/t to normalize the crack size, a . To facilitate the analysis in the fitting process, the angle was normalized as:

$$\phi = (\varphi - \varphi_{\min})/(\varphi_{\max} - \varphi_{\min}) \quad (45)$$

To study the effect of the thermite weld segment on the SIF solution, a three-dimensional profile of a thermite welded joint from service was obtained using a portable laser scanner Handyscan 307 and the thermite weld geometry was built on top of the model for rail steel. Similar in-plane and out-of-plane loading, and boundary condition were used as in the rail steel models. Two different distances of the surface crack from the weld toe: 0.1 mm and 0.5 mm were modeled and calculated. The effect of weld to the SIF, M_k , is expressed as:

$$M_k = \frac{K_{(\text{welded rail})}}{K_{(\text{rail steel})}} \quad (46)$$

Uncoupled 2 Degree of Freedom and Coupled Root Mean Square Methods

In this case study, two different schemes to account for crack propagation based on SIF were investigated. The first scheme is the uncoupled 2-DOF scheme of which the two points were treated as separate point and disregarded any other intermediate points along the crack front. The crack will propagate to a new shape with an increment of da and dc in separate direction after each loading cycle as shown in Figure 6-8.

Figure 6-8 Superposition of crack growth dimension [166]

The second scheme, proposed by Cruse and Besuner [169], is known as the coupled root mean square (RMS) method. This method takes into account the effective increased average area in both deepest and surface direction. The RMS SIF expression in deepest direction is as:

$$K_{RMS,a}^2 = \frac{1}{\Delta S_a} \iint_{\Delta S_a} K^2(\phi) dS_a = \frac{2}{\pi} \int_0^\pi K^2(\phi) \sin^2 \phi d\phi \quad (47)$$

while the RMS SIF expression in surface direction is as:

$$K_{RMS,c}^2 = \frac{1}{\Delta S_c} \iint_{\Delta S_c} K^2(\phi) dS_c = \frac{2}{\pi} \int_0^\pi K^2(\phi) \cos^2 \phi d\phi \quad (48)$$

where da and dc represent the crack increment in deepest and surface direction respectively, ϕ is the angle from surface following anti-clockwise direction, ΔS_a and ΔS_c refer to the equivalent area increment after each load cycle using RMS scheme. Although only SIFs in two points were calculated, this coupled method accounts for

the SIFs all along the crack front indirectly to crack growth along depth (a) and width (c) direction. The SIF solution at the crack front in terms of crack angle ϕ , $K(\phi)$ and RMS scheme was then incorporated into the fatigue crack growth algorithm to predict the crack shape evolution and fatigue propagation life will be developed.

Crack Shape Change and Increment

In order to account for cracks with different shapes, a correction factor considering different aspect ratio a/c was applied to the semi-circular solution. Previously, Merkle [170] has summarized that the SIF solution for semi-elliptic surface cracks with any aspect ratio a/c can be expressed as:

$$K = \frac{Y\sqrt{\pi a}}{\Phi} \left[\sin^2 \phi + \left(\frac{a}{c} \right)^2 \cos^2 \phi \right]^{\frac{1}{4}} f_w \quad (49)$$

where a is the crack depth, c is the crack width, Y and θ correspond to shape factor for either rail steel or welded rail and crack angle respectively, f_w is the finite-width correction and is given as:

$$f_w = \sec \left(\frac{\pi c}{2b} \sqrt{\frac{a}{t}} \right) \quad (50)$$

Value of Φ is given as

$$\Phi = \int_0^{\frac{\pi}{2}} \left[1 - \left(1 - \frac{a^2}{c^2} \right) \sin^2 \phi \right]^{\frac{1}{2}} d\phi \quad (51)$$

For the increment of these transverse cracks during fatigue crack propagation, Corn [171] found that small semi-circular surface cracks under bending and tensile loading condition tend to retain their shape during the early fatigue crack growth stages.

However, as the surface crack grows the crack shape is expected to change according to the SIF driving force parameters. The results showed that the SIF factor at the crack front is larger in width direction than in the depth direction and it will cause the crack shape change from semi-circular to semi-elliptical shape due to the different

constraint conditions. Hence Wu [172] suggested a correction to account for the variation of the C constant with crack front angle:

$$C(\phi) = \frac{C_a}{[1+0.1 \times (1-\sin\phi)^2]} \quad (52)$$

where C_a corresponds to C constant of the crack front in the depth direction and ϕ denotes the parametric angle of crack front. The corrected Paris' Law for each point on the crack front was applied as:

$$\frac{da(\phi)}{dN} = C(\phi)[\Delta K(\phi)]^m \quad (53)$$

6.3.2 SIF and M_k Factor solutions

SIF solutions

Three loading types, vertical bending M_x , lateral bending M_y , and uniformly distributed tensile stress N_z , were applied in the FEM for cracks in each location. The geometry factor Y was then calculated using Equation (54):

$$Y = \frac{K_{(FEM)}}{\sigma\sqrt{\pi a}} \quad (54)$$

where $K_{(FEM)}$ refers to the SIF value calculated from FEM result.

A nonlinear regression method was used to obtain suitable equations for the Y factor curves. It was accomplished using iterative algorithms to estimate parameters for the equations. An equation with the following form was proposed to fit these curves:

$$Y = \left[p \left(\frac{a}{t} \right)^n \right] \left\{ 1 + \left[0.1 + c_1 \left(\frac{a}{t} \right)^2 + c_2 \left(\frac{a}{t} \right)^3 \right] [1 - \sin(\phi + \theta)]^2 \right\} \quad (55)$$

where $p \left(\frac{a}{t} \right)^n$ was applied to consider the local stress concentration raise by the geometric changes at the weld toe region, $\left[0.1 + c_1 \left(\frac{a}{t} \right)^2 + c_2 \left(\frac{a}{t} \right)^3 \right]$ is the polynomial

term to describe the trend of Y in terms of crack size a ; $[1 - \sin(\phi + \theta)]^2$ is the trigonometric term to describe the trend of Y in terms of normalized crack angle ϕ .

The trend of geometry factors Y was observed to be different for crack size smaller and larger than 1 mm. This could be due to the effect of the surrounding geometry conditions on the stress field when the crack size increases to more than 1 mm.

Therefore, curve fitting of Equation (55) to the geometry factors Y obtained from the FEM result was conducted and the value of each parameter is shown in Table 6-3. A value of R^2 , the coefficient of determination, of more than 0.9 of all curve fitted result was obtained, showing good representation of the proposed equation fitted to FEM results.

Table 6-3 Y factor parameters from FEM analysis results

		Crack Location									
		Head Crack		Head to web crack		Web to base crack		Bottom crack		Base corner crack

		$a \leq 1\text{mm}$	$a > 1\text{mm}$	$a \leq 1\text{mm}$	$a > 1\text{mm}$	$a \leq 1\text{mm}$	$a > 1\text{mm}$	$a \leq 1\text{mm}$	$a > 1\text{mm}$	$a \leq 1\text{mm}$	$a > 1\text{mm}$
Vertical bending M_x	p	.627	.647	.694	.620	.643	.607	.702	.622	.623	.935
	n	-.010	.012	.022	.067	-.002	.080	.020	-.033	-.010	.185
	c_1	109.101	.383	-9.628	-.149	-40.028	-.112	62.854	.266	210.278	1.107
	c_2	-	-.096	198.20	.159	761.947	.111	-	-.053	-	-.728
	θ	-.004	.118	-.051	-1.909	.075	2.240	.004	-	-.079	-.171
Lateral bending M_y	p	.636	.349	.693	.281	.610	.292	-	-	.641	.870
	n	-.008	-.402	.027	-.553	-.018	-.500	-	-	-.008	.101
	c_1	94.783	1.049	52.387	1.196	137.842	1.230	-	-	196.383	1.146
	c_2	-	-.336	-	-.258	-	-.456	-	-	-	-.748
	θ	.119	1.341	-.169	-1.082	.137	1.184	-	-	-.100	-.322
	p	.622	.720	.697	.686	.646	.689	.692	.711	.618	.906

Tension N_z	n	-.005	.107	.017	.060	-.006	.074	.024	.085	-.007	.162
	c_1	102.740	.068	11.706	-.332	-18.361	-.240	44.328	-.094	195.961	.248
	c_2	-	.076	-	.340	445.720	.201	-	.121	-	-.124
	θ	-.002	.214	-.045	-.435	.054	.554	.002	-	-.082	-.477

M_k Factor solutions

The weld M_k factor for transverse weld toe cracks located at head-to-web transition region with crack depth from 0.1 mm to 10 mm is shown in Figure 6-9. Two different distance of the surface crack from the weld toe: 0.1 mm and 0.5 mm were investigated. It was found that the M_k factor decreases as the crack size increases and it converge to 1.0 when the crack size reaches 10 mm in depth. It was noted that the weld magnification factor M_k is independent of crack front direction and loading conditions. Comparing across different locations of weld toe cracks, the M_k factors are higher when the crack is closer to the weld toe due to the stress concentration effects of the weld toe region. Based on these results, the M_k factors calculated from FEM results were curve-fitted as:

$$M_k = p_k \left(\frac{a}{t} \right)^{n_k} \text{ when } 0.1 \text{ mm} \leq a \leq 10 \text{ mm} \quad (56)$$

where the value of coefficients p_k and n_k were found to be 0.8908 and -0.2563 for surface cracks located at 0.1 mm distance from the weld toe region. For surface crack located 0.5 mm away from the weld toe, the coefficients p_k and n_k were 0.9644 and -0.123 respectively. The determination of the curve fitting, R^2 is more than 0.98, which indicated good fitting between the calculated result and fitted regression line.

(a) Global FE model and detailed mesh of surface crack at thermite welded joint

Crack located 0.1 mm from weld toe Crack located 0.5 mm from weld toe
(b) Results of M_k factor for head-to-web weld toe cracks

Figure 6-9 FEM and results for M_k factor [166]

6.4 Fatigue Crack Growth and Life Prediction

The SIF range (ΔK) and Paris' Law crack propagation rate (da/dN) data were numerically integrated in a fatigue crack growth life prediction algorithm developed in the MATLAB program, with Figure 6-10 showing the algorithm used. The SIFs determined through FEM analysis were used as the crack propagation driving force parameter [173–175]. The results of crack life prediction and crack shape evolution were also presented using the function of plotting in MATLAB.

The prediction for fatigue crack growth analysis is valid only when the cracks have initiated, and the fatigue propagation of short cracks is not considered. Due to the small scale yielding of the rail steels and thermite weld, the behavior of short crack fatigue is also considered to be negligible. Although residual stress has an effect on

fatigue crack initiation and early life propagation, the residual stress distribution will change once crack size increases [86]. The influence of the residual stress change on fatigue crack propagation needs a more comprehensive and detailed investigation, which is not within the scope of the current thesis.

Figure 6-10 Fatigue growth algorithm

Based on the solutions of geometry factor Y and weld magnification factor M_k at crack front, a fatigue crack growth and life algorithm was developed. For C and m constants, the values of thermite weld material were obtained from fatigue test results

while the values of rail steel material were from the reported results of Motameni [176].

Fatigue life prediction

With the fatigue crack growth and life prediction algorithm developed, the fatigue crack growth and life prediction analysis were conducted for a head-to-web crack subjected to vertical bending stress M_x . The following fatigue law constants were used: $C = 3.45 \times 10^{-16}$ and $m = 3.91$. An initial crack depth a of 0.15 mm and initial crack aspect ratio of 1.0 were used.

Figure 6-11 Stress-life curves for Inverted four-point bend test of rail specimen

The fatigue life prediction for inverted four-point bend test of rail specimen are plotted in graphical form, as seen in Figure 6-11. As can be seen, the fatigue life prediction provided a good fit to the experimental fatigue test data.

Effect of initial crack depth (a) on predicted fatigue life

A case study was conducted to determine the effect of initial crack depth (a_i) used on the predicted fatigue life. A total of 3 cases were studied – initial cracks depth (a_i) of 0.1 mm, 0.15 mm and 0.5 mm. All other parameters were kept constant for this case

study. The predicted fatigue life of the 3 different cases can be found in Table 6-4.

The deviation is calculated by

$$\text{Deviation (\%)} = \frac{N_{a_i = 0.15 \text{ mm}} - N_{a_i}}{N_{a_i = 0.15 \text{ mm}}} \times 100\% \quad (57)$$

where N refers to the predicted fatigue life.

Table 6-4 Effect of number of initial cracks used on predicted fatigue life

	Predicted fatigue life (cycles)	Deviation (%)
Initial $a_i = 0.1 \text{ mm}$	2,893,616	24.5
Initial $a_i = 0.15 \text{ mm}$	2,323,684	
Initial $a_i = 0.5 \text{ mm}$	1,228,239	-47.1

Figure 6-12 Fatigue life prediction (single crack) for different initial crack depths

From Table 6-4 and Figure 6-12, it can be seen that the predicted fatigue life is heavily influenced by the initial crack depth. Based on a load of 200 MPa, using an initial crack depth of 0.1 and 0.5 predicted a 24.5% increase and 47.1% decrease in fatigue life as compared to using an initial crack aspect ratio of 0.15 respectively.

Crack shape evolution under different loads

In the next case study, two types of loading conditions for in-plane load and out-of-plane loading were considered separately to investigate the effect of loading type on the crack shape during the fatigue crack propagation. Two methods of uncoupled 2

DOF method and coupled RMS method were applied and compared for transverse cracks initiating from head-to-web transition region. The results of aspect ratio a/c in terms of crack size during crack propagation and the crack propagation path by using two different schemes were shown in Figure 6-13a. In Figure 6-13b, each blue circle represented the crack shape with a fixed increment of crack front growth in the depth direction. Therefore, the number of the blue crack shape contours like beach marks and the spacing between each contour or beach mark described the fatigue life cycles, the crack growth and crack shape development.

(b) crack propagation and shape development

Figure 6-13 Crack growth and crack shape development [166]

The results in Figure 6-13a showed that the crack shape for transverse surface cracks tend to keep to a constant crack aspect ratio for in-plane loading conditions. For in-plane vertical bending M_x and tension N_z , the crack aspect ratio tended towards 0.8~0.9 despite starting from different initial crack sizes and shapes. However, for out-of-plane bending loading, the crack aspect ratio varies differently from the in-plane loading case. The crack aspect ratio decreases significantly until the final crack size is reached at rail break. The out-of-plane bending loading causes the fatigue crack to grow faster at the surface width directions than the depth direction, leading to a final crack shape that is semi-elliptical with a low crack aspect ratio a/c of 0.2~0.3. Furthermore, the faster crack growth behavior led to a shorter predicted fatigue life for surface cracks subjected to out-of-plane bending loading and is more significant compared to the in-plane loading cases.

6.5 Multiple Crack Fatigue Life Prediction Model

The algorithm of the multiple fatigue crack propagation is shown in Figure 6-14, similar to the one previous used. The input requires information on the crack, material and fatigue parameters. Crack data would include the crack initiation sites, initial and final depth sizes, and crack aspect ratios. Material properties required are the yield stress and ultimate tensile stress. Fatigue parameters, such as C and m from the Paris

law related to the fatigue propagation law are input in this stage as well. The crack coalescence algorithm is also similar to the one previously used.

Figure 6-14 Algorithm for multiple crack propagation

Information on the number and position of the cracks was obtained by analysis of the fracture surfaces. Cracks were identified by observing the ratchet marks, and the locations of each crack initiation site are recorded in Table 6-5. Assuming semi-elliptical cracks, conservative values of initial crack depth ($a_i = 0.15$ mm), final crack depth ($a_f = 25.0$ mm) and initial crack aspect ratio ($a_i/c_i = 1.0$) were used, same as in the fatigue life prediction. In the analysis, no residual stresses were assumed. The

locations of the crack initiation sites were defined according to the x' and y' axis as shown in Figure 6-15 for head-to-web cracks.

Figure 6-15 Axis for definition of crack initiation sites for head-to-web cracks

Table 6-5 Crack initiation sites identified from surface of fractured samples

Specimen number	Number of cracks	Distance of crack initiation sites from $x' = 0$ (mm)				
1	5	3.0	8.5	12.5	14.5	17.5
2	4	0.5	2.0	3.5	9.5	
3-R	4	0.5	1.5	3.0	7.5

(a)

Figure 6-16 Comparison of experimental (left) and predicted (right) fracture surfaces a) Specimen 1 b) Specimen 2 c) Specimen 3-R

From Figure 6-16, it can be observed that the predicted fracture surfaces are shown closely resemble the actual fracture surfaces. The final crack shape from the fracture surface is captured adequately, and the locations where the cracks merge match up to the locations of the ratchet marks on the crack surface. The plot of initial crack depth (a) against predicted fatigue life is shown in Figure 6-17a, while the predicted crack shape development of each specimen is shown in Figure 6-17b, Figure 6-17c and Figure 6-17d.

Figure 6-17 a) initial crack depth (a) against predicted fatigue life; Crack shape development of b) Specimen 1 c) Specimen 2 d) Specimen 3-R

6.6 XFEM Automated Crack Growth

In the above research study, only a static load was used to simulate the crack growth to obtain the SIFs. A case study was conducted to utilize the XFEM automated crack growth method in ABAQUS to attempt to understand and visualize the way cracks grow and develops.

In order to use XFEM automated crack growth, the initiation and propagation constants have to be defined. For crack initiation, the relative fracture energy release rate ΔG determines the onset of the fatigue crack growth. The fatigue crack growth initiation criterion is defined as

$$f = \frac{N}{c_1 \Delta G^{c_2}} \quad (58)$$

where c_1 and c_2 are material constants and N is the total number of cycles. The crack does not grow unless the above equation is satisfied and when the maximum fracture energy release rate, G_{max} , is greater than $G_{threshold}$.

For crack propagation, rate of the crack growth per cycle is governed by the Paris law

$$\frac{da}{dN} = c_3 \Delta G^{c_4} \quad (59)$$

where c_3 and c_4 are material constants.

The constants used are as shown in Table 6-6.

Table 6-6 Material constants

c_1	c_2	c_3	c_4
0.5	-0.1	3.807e-7	3.034

Modelling methodology

Figure 6-18 Location of crack in model

A 3D crack of 0.2mm radius is inserted into the model in the location as shown. The crack is defined to be an XFEM crack and allowed to grow.

The fracture criterion is set to Maximum Tangential Stress (MTS) criterion, which uses the tensile failure mode assumption. The mesh is designed such that the mesh

near the crack is heavily refined and focused, but coarser in other areas (Figure 6-18). The mesh in this region is assigned Quad (Structured). In the areas further away, the mesh is designed to be coarser progressively. The mesh in this region is assigned Quad (Free). This will allow for faster computational time, without sacrificing the accuracy of the results.

Figure 6-19 Boundary condition and bending moment

As shown in Figure 6-19, a fixed Boundary condition is applied on one side of the model, and a bending moment is applied on the other. The bending moment applied is equivalent to the experimental fatigue test conducted for specimen 1.

The preliminary results of XFEM automated crack growth are shown in Figure 6-20.

Figure 6-20 Preliminary results of XFEM automated crack growth

From the preliminary results, the advantage of XFEM to automatically determine the crack shape and crack front and grow the crack without any remeshing was demonstrated. The crack can be seen to roughly follow the semi-elliptical partitioning of the model and mesh, and the fracture surface was similar to that of Specimen 1 (Figure 6-4a).

However, the results also showed that the XFEM crack reached a critical crack size at 45,050 cycles. This was very much lower than the experimental fatigue life of 701,030 cycles for Specimen 1 (6.4%). This result was much too low for XFEM automated crack growth to be considered accurate, hence in this thesis, XFEM was only used to model cracks and to obtain SIFs, of which it was able to do so reasonably well.

Chapter 7 Conclusion & Future Work

7.1 Conclusion

In this PhD study, a new algorithm for fatigue crack propagation analysis is demonstrated using new XFEM capabilities to obtain SIFs. The multiple weld toe crack growth analysis models were verified with S-N fatigue tests of various welded geometries. Based on the objectives and the research work carried out, the following conclusions can be made:

Conclusions for Study No. 1: XFEM calibration studies for SIF solutions and development of multiple surface crack algorithm

1a) A multiple fatigue crack growth algorithm for surface cracks was developed. Stress analysis of a crack at the weld toe region of a metal plate was conducted with the use of ABAQUS XFEM. The importance of partitioning of the model and orthogonality of the crack in XFEM was demonstrated. It was found that SIFs from the 4th to 10th contour could be used to obtain reliable results. A mesh refinement study was also carried out, and it was determined that the use of 50 elements along the crack front produced accurate results. The SIFs obtained by XFEM were close to Newman and Raju [26], with a maximum deviation of 3.34%.

1b) The SIFs for a fillet-welded T-butt joint plate were also determined, and the M_k factors due to the weld were calculated. the results were compared to established solutions by Bowness & Lee [100]. The deviation histograms show a good normal distribution of the percentage deviation with a maximum deviation of -6.07%, indicating that the proposed M_k factors show good agreement with established solutions.

1c) The effect of crack aspect and L/T ratio were considered. The M_k factor solutions were then separated into three categories – upper bound, mean and lower bound, with a coefficients of determination, R^2 , of more than 0.99 for all the curved fitted solutions indicating a very good fit. It was established that in order to obtain the most conservative predicted fatigue life, the upper bound M_k factor solution should be used in the fatigue life prediction model.

1d) The M_k factor solutions were used in a fatigue life prediction model to predict the growth of fatigue cracks. The fatigue life model was compared HSE tubular joint fatigue life data and showed good agreement. The model was extended to the multiple surface cracks commonly observed at the weld toe, where each crack is treated as independent, in line with the NIIT method. The results show that as the number of cracks increases, the predicted fatigue life decreases for the same stress range, causing a shift in the $S-N$ curve to the left.

Conclusions for Study No. 2: Fatigue investigations of welded pipe cut-out specimens

2a) S-N Curve Tests of welded pipe cut-out specimens under four-point bending was conducted. The stress-life curve is compared to literature [147] and the results were similar.

2b) SIF solutions of welded pipe cut-out specimens were developed based on 3D XFEM Model, and M_k factor solutions were obtained. The solutions were curve fitted and the coefficients of determination, R^2 , were 0.9947 and 0.9975 for the crack depth and surface solutions respectively indicating a very good fit.

2c) An improved fatigue life analyses for multiple cracks was developed by considering the crack coalescence stage and fatigue crack closure. The algorithm

which used the BS 7910 design curve only predicted a fatigue life of about 10 % of the actual life. The use of mean curve data ($C = 1.5 \times 10^{-13}$ from [155]) with the threshold SIF predicts higher life than the BS 7910 design curve with only half the actual life is reflected. When crack closure and equivalent area crack coalescence are considered with the mean curve constants, the prediction becomes closer to the tested fatigue life compared to the threshold SIF approach. Considering the equivalent area during crack coalescence increases predicted fatigue life by 9% at higher loads but becomes less significant (3%) at lower loads.

2d) An algorithm of the multiple fatigue crack propagation was demonstrated. A large number of starting weld toe cracks (12 to 22) were modelled in the fatigue life prediction of the fatigue tested pipes. The analysis provided details of the fatigue growth of each crack and its coalescence history. The approach can be extended to applications in salt-water or for welds with variable amplitude loading.

Conclusions for Study No. 3: Fatigue study on cruciform welded joint specimens

3a) S-N Curve Tests of cruciform joint specimens was conducted. It was found that for the as-welded joints under tension loading, weld root failure tends to occur at higher loading over 120 MPa. On the other hand, the weld toe failure tends to take place at lower loading below 150 MPa. In order to avoid further failures of cruciform joints at the weld root, four-point bending tests were carried out, and it was found that the fatigue strengths of the cruciform joints under bending loading are much higher than under tension loading.

3b) A semi-elliptical surface crack was modelled in cruciform joint specimens to obtain SIFs. Similar partitioning was done to minimize distortion of elements during meshing. The area near the crack were assigned a higher density of mesh while non-

critical areas can have lower density of mesh with larger sized elements. By creating a balance of more elements at the critical areas and less elements at non-critical areas, computation time was improved without sacrificing quality of the results. The SIFs from the first three contours were ignored, and the average of the 4th to 20th contours were used in evaluating the average SIF values at each crack location. SIF solutions of cruciform joint specimens were developed and M_k factor solutions were obtained. The results were compared with established solutions by Zhao [164] and showed good agreement.

3c) A procedure for fatigue crack growth and life prediction for cruciform joint was developed similar to that used for the welded pipe cut-out specimens. An effective SIF range, ΔK_{eff} was similarly used in place of ΔK [14] to account for effects such as crack closure or threshold SIF. The fatigue life prediction was very close to the tested fatigue life for tension loading. The prediction for bending loading is lower than that of the experimental result, providing a more conservative prediction. The development of crack aspect ratio was investigated, and it was found that as the cracks grew, the cracks tended towards a semi-elliptical shape with an a/c of 0.501 in tension loading, while the cracks under bending loading tended towards a semi-elliptical shape with an a/c of 0.295. The effect of initial crack aspect ratio (a/c) on predicted fatigue life was also studied and it was found that fatigue life is heavily influenced by the initial crack aspect ratio. In the case of tension loading, using an initial crack aspect ratio of 0.5 and 1.0 predicted a 26.6% and 50.3% increase in fatigue life respectively as compared to using an initial crack aspect ratio of 0.25. For bending loading, the percentages are 24.5% and 48.0% for initial crack aspect ratios of 0.5 and 1.0 respectively.

3d) An algorithm of the multiple fatigue crack propagation for cruciform joints was demonstrated, similar to the one previous used. Information on the number and position of the cracks was obtained by analysis of the fracture surfaces. The predicted fracture surfaces were shown to match the actual fracture surfaces. The final crack shape from the fracture surface was captured adequately, and the locations where the cracks merge match up to the locations of the ratchet marks on the crack surface. The effect of number of initial cracks used on the predicted fatigue life was investigated, and it was found that the fracture surface with 13 initial cracks presented the most detailed and best approximation of the experimental fracture surface. The predicted fracture surface of 1 initial crack is unable to fully capture the final crack shape from the fracture surface and represents an oversimplification of the fatigue life prediction. It was also concluded that the predicted fatigue life using 1 initial crack was the longest, while the use of 13 initial cracks provided the most conservative fatigue life (33.0% shorter). This study showed that the number of initial cracks used did have an effect on the predicted fatigue life.

Conclusions for Study No. 4: Fatigue study on thermite welded four-point bending fatigue inverted specimen test and fatigue life prediction

4a) S-N curve tests for inverted four-point bending of rail specimen were conducted to determine the fatigue strength of rail welded specimens. The welded rail sample was placed horizontally inverted (with rail foot on top and rail head at bottom) on two point supports. The results showed that the fatigue life of a specimen in the inverted fatigue test was about 3X that of the conventional test. From the fracture surfaces, it could be seen that the crack initiated from the head-to-web transition area along the edge.

4b) SIF solutions of rail specimens were developed based on 3D XFEM Model, and M_k factor solutions were obtained. In-plane and out-of-plane stresses were investigated to observe their contribution to fatigue propagation of transverse cracks in rail and thermite weld sections. The in-plane loading was modeled as vertical bending moment (M_x) and uniformly distributed tensile stress (N_z) while the out-of-plane loading was modeled as lateral bending (M_y). A more powerful scheme to account for crack propagation based on SIF known as the coupled root mean square (RMS) method proposed by Cruse and Besuner [169] was introduced. The SIF solution at the crack front in terms of crack angle ϕ , $K(\phi)$ and RMS scheme was then incorporated into the fatigue crack growth algorithm to predict the crack shape evolution.

4c) The SIF and M_k factor solutions were numerically integrated in a fatigue crack growth life prediction algorithm developed in the MATLAB program. The fatigue life prediction provided a good fit to the experimental fatigue test data. A case study was conducted to investigate crack shape evolution under different loads. Two methods, including uncoupled 2 DOF method and coupled RMS method, were applied and compared for transverse cracks initiating from head-to-web transition region. The results showed that the crack shape for transverse surface cracks tend to keep to a constant crack aspect ratio for in-plane loading conditions. Also, for in-plane vertical bending M_x and tension N_z , the crack aspect ratio tended towards 0.8~0.9 despite starting from different initial crack sizes and shapes.

4d) An algorithm of the multiple fatigue crack propagation for rail specimens was demonstrated, similar to the one previous used. The locations of the crack initiation sites were defined according a rotated axis x' and y' for head-to-web cracks. It can be

observed that the predicted fracture surfaces closely resemble the actual fracture surfaces.

7.2 Recommendations for Future Research

In the current work, the development of a multiple fatigue crack growth algorithm was introduced. Based on the objectives and the research work carried out, some further works can be recommended as follows:

1. In the current work, residual stresses were not accounted for. Although the initial residual stress from the manufacturing or welding process has an effect on fatigue crack initiation and early life propagation, the residual stress distribution will change once crack size increases. For future research, residual stress effects due to the welding process can to be taken into account.
2. In the current work, the prediction for fatigue crack growth analysis is valid only when the cracks have initiated, and the fatigue propagation of short cracks is not considered. Due to the small scale yielding of the rail steels and thermite weld, the behavior of short crack fatigue is also considered to be negligible. The fatigue crack initiation becomes more important with the presence of welding defects for the accuracy of the total fatigue life prediction.
3. In addition to obtaining SIF solutions from the XFEM simulation, ABAQUS can also be used to provide visualization on how the crack grows and develops over time. In the current research study, only a static load was used to simulate the crack growth to obtain the SIFs, and only preliminary was done to examine XFEM automated crack growth. Future work can look into using cyclic loads to simulate fatigue crack growth and also extend the simulation to more complex structures.

Appendix A M_k Equations for Fillet-Welded T-Butt Joint

The M_k equations by Bowness & Lee [100] for the crack deepest point and the crack ends in the fillet-welded T-butt joint under tension and bending are given in the following:

At the crack deepest point under tension:

$$M_{k,a} = f1 + f2 + f3$$

$$f1 = 0.43358(a/t)^{(g1+(g2(a/t))^{g3})} + 0.93163 \exp((a/t)^{-0.050966})$$

$$f2 = -0.21521(1 - (a/t))^{176.4199} + 2.8141(a/t)^{(-0.1074(a/t))}$$

$$f3 = 0.33994(a/t)^{g5} + 1.9493(a/t)^{0.23003} + (g6(a/t)^2 + g7(a/t) + g8)$$

$$g1 = -1.0343(a/c)^2 - 0.15657(a/c) + 1.3409$$

$$g2 = 1.3218(a/c)^{-0.61153}$$

$$g3 = -0.87238(a/c) + 1.2788$$

$$g4 = -0.46190(a/c)^3 + 0.6709(a/c)^2 - 0.37571(a/c) + 4.6511$$

$$g5 = -0.015647(L/t)^3 + 0.09089(L/t)^2 - 0.17180(L/t) - 0.24587$$

$$g6 = -0.20136(L/t)^2 + 0.93311(L/t) - 0.41496$$

$$g7 = 0.20188(L/t)^2 - 0.97857(L/t) + 0.068225$$

$$g8 = -0.027338(L/t)^2 + 0.12551(L/t) - 11.218$$

At the crack end under tension:

$$M_{kc} = f1 \times f2 \times f3$$

$$f1 = g1(a/t)^{(g2(c/a)^2 + g3(c/a) + g4)} + g5(1 - (a/t))^{(g6(c/a)^2 + g7(c/a) + g8)} + g9$$

$$f2 = (-0.28639(a/c)^2 + 0.35411(a/c) + 1.643)(a/t)^{g9} + 0.27449(1 - (a/t))^{g10}$$

$$f3 = g11(a/t)^{0.75429} + g12 \exp((a/t)^{g13})$$

$$g1 = 0.0078157(c/a)^2 - 0.070664(c/a) + 1.8508$$

$$g2 = -0.000054546(L/t)^2 + 0.00013651(L/t) - 0.00047844$$

$$g3 = 0.00049192(L/t)^2 - 0.0013595(L/t) + 0.011400$$

$$g4 = 0.0071654(L/t)^2 - 0.033399(L/t) - 0.25064$$

$$g5 = -0.018640(c/a)^2 + 0.24311(c/a) - 1.7644$$

$$g6 = -0.0016713(L/t)^2 + 0.0090620(L/t) - 0.016479$$

$$g7 = -0.0031615(L/t)^2 - 0.010944(L/t) + 0.13967$$

$$g8 = -0.045206(L/t)^3 + 0.32380(L/t)^2 - 0.68935(L/t) + 1.4954$$

$$g9 = -0.25473(a/c)^2 + 0.40928(a/c) + 0.0021892$$

$$g10 = 37.423(a/c)^2 - 15.741(a/c) + 64.903$$

$$g11 = -0.10553(L/t)^3 + 0.59894(L/t)^2 - 1.0942(L/t) - 1.2650$$

$$g12 = 0.043891(L/t)^3 - 0.24898(L/t)^2 + 0.44732(L/t) + 0.60136$$

$$g13 = -0.011411(a/c)^2 + 0.004369(a/c) + 0.51732$$

Appendix B M_k Equations for Cruciform Joints

Figure B-1 Schematic of load-carrying cruciform welded joints with a weld toe crack [164]

The M_k equations by Zhao [164] for the crack deepest point and the crack ends in the load-carrying cruciform welded joints under tension are given in the following:

At the crack deepest point under tension (membrane) loading:

$$M_{km,a} = \left[f_{1t} \left(\frac{a}{T}, \frac{a}{c} \right) + f_{2t} \left(\frac{a}{T}, \frac{L}{W} \right) \right] f_{3t} \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T} \right)$$

$$f_{1t} \left(\frac{a}{T}, \frac{a}{c} \right) = -4.187443 \left(\frac{a}{T} \right)^{C_1} + C_2 \left[1 - \left(\frac{a}{T} \right) \right]^{C_3}$$

$$C_1 = -0.006699 \left(\frac{a}{c} \right)^2 + 0.060199 \left(\frac{a}{c} \right) + 0.132981$$

$$C_2 = 0.243472 \left(\frac{a}{c} \right)^2 - 0.631431 \left(\frac{a}{c} \right) + 4.502722$$

$$C_3 = 0.021592 \left(\frac{a}{c} \right)^2 - 0.082626 \left(\frac{a}{c} \right) - 0.047647$$

$$f_{2t} \left(\frac{a}{T}, \frac{L}{W} \right) = C_4 \left[1 - \left(\frac{a}{T} \right) \right]^{C_5} + 0.971259 \left(\frac{L}{W} \right)^{C_6}$$

$$C_4 = -2.275102 \left(\frac{L}{W} \right)^3 + 5.099321 \left(\frac{L}{W} \right)^2 - 3.768011 \left(\frac{L}{W} \right) - 0.236142$$

$$C_5 = -0.006393 \left(\frac{L}{W}\right)^{-1.709022} + 0.048061$$

$$C_6 = -3.907304 \left(\frac{L}{W}\right)^3 + 6.919132 \left(\frac{L}{W}\right)^2 - 4.633721 \left(\frac{L}{W}\right) + 0.873854$$

$$f_{3t} \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T}\right) = \left[C_7 \left(\frac{h}{T}\right)^2 + C_8 \left(\frac{h}{T}\right) + C_9 \right] \left(\frac{a}{T}\right)^{-0.005681} \\ - 0.633002 \left[1 - \left(\frac{a}{T}\right) \right]^{C_{10} \left(\frac{h}{T}\right)^2 + C_{11} \left(\frac{h}{T}\right) + C_{12}}$$

$$C_7 = -3.935803 \left(\frac{L}{W}\right)^3 + 4.905402 \left(\frac{L}{W}\right)^2 - 0.692841 \left(\frac{L}{W}\right) + 0.024166$$

$$C_8 = 10.565033 \left(\frac{L}{W}\right)^3 - 12.984031 \left(\frac{L}{W}\right)^2 + 1.674406 \left(\frac{L}{W}\right) - 0.048156$$

$$C_9 = -3.912511 \left(\frac{L}{W}\right)^3 + 4.803722 \left(\frac{L}{W}\right)^2 - 0.603372 \left(\frac{L}{W}\right) + 1.633214$$

$$C_{10} = 3.927331 \left(\frac{L}{W}\right)^2 - 4.092911 \left(\frac{L}{W}\right) + 0.401612$$

$$C_{11} = -4.910425 \left(\frac{L}{W}\right)^2 + 9.111931 \left(\frac{L}{W}\right) - 0.973012$$

$$C_{12} = 1.327904 \left(\frac{L}{W}\right)^2 - 3.213625 \left(\frac{L}{W}\right) + 0.352334$$

At the crack ends under tension (membrane) loading:

$$M_{\text{km,c}} = g_{1t} \left(\frac{a}{T}, \frac{a}{c}\right) g_{2t} \left(\frac{a}{T}, \frac{L}{W}\right) g_{3t} \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T}\right)$$

$$g_{1t} \left(\frac{a}{T}, \frac{a}{c}\right) = 0.006645 \left(\frac{a}{T}\right)^{D_1} + D_2 \left[1 - \left(\frac{a}{T}\right) \right]^{D_3}$$

$$D_1 = 0.025504 \left(\frac{a}{c}\right)^{-0.427822} - 1.413056$$

$$D_2 = 0.092443 \left(\frac{a}{c}\right)^{-0.533921} + 1.105046$$

$$D_3 = 0.259243 \exp \left[-0.267633 \left(\frac{a}{c}\right) \right] - 0.431201 \exp \left[-8.547022 \left(\frac{a}{c}\right) \right]$$

$$g_{2t} \left(\frac{a}{T}, \frac{L}{W}\right) = D_4 \left(\frac{a}{T}\right)^{0.037085} + D_5 \left[1 - \left(\frac{a}{T}\right)\right]^{2.193111}$$

$$D_4 = -2.929901 \left(\frac{L}{W}\right)^3 + 3.708066 \left(\frac{L}{W}\right)^2 - 0.146582 \left(\frac{L}{W}\right) + 0.981334$$

$$D_5 = -3.042231 \left(\frac{L}{W}\right)^3 + 4.955614 \left(\frac{L}{W}\right)^2 - 1.001302 \left(\frac{L}{W}\right) + 0.159388$$

$$g_{3t} \left(\frac{a}{T}, \frac{h}{T}\right) = D_6 \left(\frac{a}{T}\right)^{D_7} + D_8 \exp \left[\left(\frac{a}{T}\right)^{D_9} \right] + \left[D_{10} \left(\frac{L}{W}\right) + D_{11} \right] \left(\frac{a}{T}\right) \\ + \left[D_{12} \left(\frac{L}{W}\right)^3 + D_{13} \left(\frac{L}{W}\right)^2 + D_{14} \left(\frac{L}{W}\right) + D_{15} \right]$$

$$D_6 = 0.108712 \left(\frac{h}{T}\right)^3 - 0.325564 \left(\frac{h}{T}\right)^2 + 0.330611 \left(\frac{h}{T}\right) + 0.023104$$

$$D_7 = 2.427003 \exp \left[-5.451022 \left(\frac{h}{T}\right) \right] + 1.832011 \exp \left[0.023192 \left(\frac{h}{T}\right) \right]$$

$$D_8 = 0.596244 \left(\frac{h}{T}\right)^2 - 1.505243 \left(\frac{h}{T}\right) + 0.461443$$

$$D_9 = 0.002399 \left(\frac{h}{T}\right)^{-4.387023} + 0.015433$$

$$D_{10} = -0.782511 \left(\frac{h}{T}\right)^3 + 1.669321 \left(\frac{h}{T}\right)^2 - 0.731284 \left(\frac{h}{T}\right) + 0.062355$$

$$D_{11} = -0.327223 \left(\frac{h}{T}\right)^3 + 1.070706 \left(\frac{h}{T}\right)^2 - 1.183403 \left(\frac{h}{T}\right) + 0.336634$$

$$D_{12} = -4.583221 \left(\frac{h}{T}\right)^2 + 10.604032 \left(\frac{h}{T}\right) - 3.838501$$

$$D_{13} = -3.001621 \left(\frac{h}{T}\right)^3 + 13.304051 \left(\frac{h}{T}\right)^2 - 18.651014 \left(\frac{h}{T}\right) + 5.972304$$

$$D_{14} = 0.522173 \left(\frac{h}{T}\right)^3 - 2.046621 \left(\frac{h}{T}\right)^2 + 2.073301 \left(\frac{h}{T}\right) - 0.566133$$

$$D_{15} = -1.654421 \left(\frac{h}{T}\right)^2 + 4.203302 \left(\frac{h}{T}\right) - 0.340781$$

At the crack deepest point under bending loading:

$$M_{kb,a} = \left[f_{1b} \left(\frac{a}{T}, \frac{a}{c} \right) + f_{2b} \left(\frac{a}{T}, \frac{L}{W} \right) \right] f_{3b} \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T} \right)$$

$$f_{1b} \left(\frac{a}{T}, \frac{a}{c} \right) = A_1 \left(\frac{a}{T} \right)^{-0.534642} + A_2 \left[1 - \left(\frac{a}{T} \right) \right]^{0.076073}$$

$$A_1 = -0.032828 \left(\frac{a}{c} \right)^2 + 0.024593 \left(\frac{a}{c} \right) + 0.107091$$

$$A_2 = -0.303072 \left(\frac{a}{c} \right)^3 + 0.664011 \left(\frac{a}{c} \right)^2 - 0.368904 \left(\frac{a}{c} \right) + 0.717644$$

$$f_{2b} \left(\frac{a}{T}, \frac{L}{W} \right) = 0.252931 \left[1 - \left(\frac{a}{T} \right) \right]^{A_3} - 0.717241 \left(\frac{a}{T} \right)^{A_4}$$

$$A_3 = -3.093204 \left(\frac{L}{W} \right)^3 + 2.942711 \left(\frac{L}{W} \right)^2 - 0.500892 \left(\frac{L}{W} \right) - 1.149303$$

$$A_4 = -0.643833 \left(\frac{L}{W} \right)^3 + 0.856092 \left(\frac{L}{W} \right)^2 - 0.087147 \left(\frac{L}{W} \right) + 0.425022$$

$$f_{3b} \left(\frac{a}{T}, \frac{L}{W}, \frac{h}{T} \right) = \left[-0.036012 \left(\frac{h}{T} \right)^2 + 0.098204 \left(\frac{h}{T} \right) + A_5 \right] \left(\frac{a}{T} \right)^{-0.153241}$$

$$- 0.695552 \left[1 - \left(\frac{a}{T} \right) \right] \left[2.800142 \left(\frac{h}{T} \right)^2 - 7.301054 \left(\frac{h}{T} \right) + A_6 \right]$$

$$A_5 = 0.156252 \left(\frac{L}{W} \right)^2 + 0.077504 \left(\frac{L}{W} \right) + 0.803141$$

$$A_6 = -10.001022 \left(\frac{L}{W} \right) + 14.631043$$

At the crack ends under bending loading:

$$M_{kb,c} = g_{1b} \left(\frac{a}{T}, \frac{a}{c} \right) g_{2b} \left(\frac{a}{T}, \frac{L}{W} \right) g_{3b} \left(\frac{L}{W}, \frac{h}{T} \right)$$

$$B_1 = -0.002825 \left(\frac{c}{a} \right)^2 + 0.045302 \left(\frac{c}{a} \right) + 1.080021$$

$$B_2 = -0.002758 \left(\frac{c}{a} \right)^2 + 0.015421 \left(\frac{c}{a} \right) + 0.137102$$

$$g_{2b} \left(\frac{a}{T}, \frac{L}{W} \right) = B_3 \left(\frac{a}{T} \right)^{0.117099} + B_4 \left[1 - \left(\frac{a}{T} \right) \right]^{4.186371}$$

$$B_3 = -1.074002 \left(\frac{L}{W} \right)^3 + 1.148403 \left(\frac{L}{W} \right)^2 - 0.088664 \left(\frac{L}{W} \right) + 1.053501$$

$$B_4 = 0.183564 \left(\frac{L}{W} \right)^2 - 0.016366 \left(\frac{L}{W} \right) + 0.319844$$

$$g_{3b} \left(\frac{L}{W}, \frac{h}{T} \right) = B_5 \left(\frac{L}{W} \right)^2 + B_6 \left(\frac{L}{W} \right) + B_7$$

$$B_5 = -0.036422 \left(\frac{h}{T} \right)^{-1.469011} + 0.117502$$

$$B_6 = 0.028944 \left(\frac{h}{T} \right)^{-2.871042} - 0.286312$$

$$B_7 = -0.067203 \left(\frac{h}{T} \right)^2 + 0.116903 \left(\frac{h}{T} \right) + 0.961721$$

Appendix C MATLAB Code for Fatigue Propagation Life Prediction and Multiple Fatigue Crack Growth Algorithm

```

function [output1] = SEC (input1)
clc
clear all
format long

global thickness width applied_stress yield_stress CA_const M_const Tha_const
Thc_const UA_const UC_const output final_crack bending cracks mpdc10 LT
geometry applied_stressx applied_stressy applied_stressz

geometry='tubu'; % 'tubu', 'pipe', 'cruciform', 'rail_HW'

if strcmp(geometry,'tubu')
    % applied_stress=input;
    applied_stress=250; % 225-325

    % location
    cracks_number=100;
    cracks = (linspace(250/2-0.75*(cracks_number-1),250/2+0.75*(cracks_number-
1),cracks_number));

    initial_ac=1; % a/c (ratio) (expand into matrix if necessary)
    final_crack = 25; % final crack depth (length)
    cracks(:,2) = 0.5; % X65 use .001 % initial crack depth (length) (expand into matrix
if necessary)
    LT=1.25;
    thickness =50; % original=50
    width=250;
    bending=0; % 1 or 0
    yield_stress =383;
    CA_const =5.21e-13;
    M_const =3;
    % Rratio = 0.1;
    % Th = 170-240*Rratio;
    Tha_const =63;
    Thc_const=63;
end

if strcmp(geometry,'pipe')
    % applied_stress=input;
    applied_stress=450; % 292.5, 320, 337.5, 382.5, 450

    % location
    if applied_stress == 450
        cracks =
[9.536;16.237;25.258;32.474;34.278;36.856;42.268;47.680;51.804;56.443;60.567;64.

```

```

691;67.526;68.814;70.876;77.320;85.825;93.299]; %position of crack along width
(length)
    final_crack      = 13.30275229; %final crack depth (length)
    elseif applied_stress == 382.5
        cracks =
[13.221;18.750;24.038;26.442;28.846;32.212;36.538;40.865;44.712;49.038;54.808;5
9.615;62.740;66.587;70.673;75.721;80.288;83.894;87.740;90.385;92.548;94.471];
%position of crack along width (length)
    final_crack      = 19.44444444; %final crack depth (length)
    elseif applied_stress == 337.5
        cracks =
[8.967;16.304;21.467;27.446;33.152;39.130;45.380;49.457;51.902;56.522;61.413;68.
750;78.533;84.783]; %position of crack along width (length)
    final_crack      = 16.25; %final crack depth (length)
    elseif applied_stress == 320
        cracks =
[11.053;18.947;25.526;28.947;32.895;35.789;38.421;41.053;43.684;48.158;52.105;5
3.947;55.000;57.632;61.579;69.474;77.368;80.263;84.474;89.211]; %position of
crack along width (length)
    final_crack      = 17.5; %final crack depth (length)
    elseif applied_stress == 292.5
        cracks =
[13.904;17.647;20.321;24.064;29.947;36.898;47.594;57.219;63.102;68.182;71.658;7
5.668]; %position of crack along width (length)
    final_crack      = 18.61111111; %final crack depth (length)
    elseif applied_stress == 225
        cracks =
[13.904;17.647;20.321;24.064;29.947;36.898;47.594;57.219;63.102;68.182;71.658;7
5.668]; %position of crack along width (length)
    final_crack      = 18.61111111; %final crack depth (length)
    else
        cracks = [1335.2/2]; %position of crack along width (length)
        final_crack   = 16; %final crack depth (length)
    end
    initial_ac=1; %a/c (ratio) (expand into matrix if necessary)
    cracks(:,2) = 0.25; %X65 use .001 %initial crack depth (length) (expand into matrix
if necessary)

thickness=16;
width=1335.2;
% thickness =25.4;
% width=100;
bending=1; %1 or 0
yield_stress =550;
UT_stress = 633;
Sflow = (yield_stress + UT_stress)/2;
CA_const =1.8318e-13;
M_const =3.0;
Rratio = 0;
%Th = 170-240*Rratio;

```

```

Th = 0;
Tha_const = Th;
Thc_const = Th;
Smax = applied_stress/(1-Rratio);
alpha = 1.9;
A0 = (0.825-0.34*alpha+0.05*alpha^2)*(cos(pi*Smax/(2*Sflow)))^(1/alpha);
A1 = (0.415-0.071*alpha)*Smax/Sflow;
A3 = 2*A0+A1-1;
A2 = 1-A0-A1-A3;
S0 = Smax*(A0+A1*Rratio+A2*Rratio^2+A3*Rratio^3);
end

```

```

if strcmp(geometry,'cruciform')
%   applied_stress=input1;
    applied_stress = 100; % S_t or b
%   cracks = [5.5;8.9;10.9;12.5;16.6;20.9;22.4;24.9;29.1;33.5;35.8;37.1;40];
% position of crack along width (length)
%   cracks = [7.2; 15.1; 24.1; 30.2; 38]; % position of crack along width (length)
    cracks = [22.5]; % position of crack along width (length)
    cracks(:,2) = 0.5; % initial a
    final_crack = 5; % final a
    initial_ac = 0.5; % a/c (ratio) (expand into matrix if necessary)
    thickness = 10;
    width = 45;
    bending = 0; % 1 or 0
    yield_stress = 383;
    CA_const = 5.21e-13; % C constant
    M_const = 3; % m constant
    Tha_const = 63;
    Thc_const = 63;
end

```

```

if strcmp(geometry,'rail_HW')
%   crack_size = [0.1 0.2 0.5]; % initial crack size mm [0.1 0.2 0.5]
    % applied_stress=input;
    applied_stressx = 320.21; % 100,200,300 320.21, 288.18, 461.10
    applied_stressy = 0;
    applied_stressz = 0;
    % location
    thickness = 30; % unit mm
    width = 200;
    cracks_number = 1;
%   cracks = [0.1; 5; 10]; % zeros(length(crack_size),1);
    cracks = [ 8.5; 12.5; 14.5; 17.5 ]; % 320.21
%   cracks = [0.5; 2; 3.5; 9.5]; % 288.18
%   cracks = [0.5; 1.5; 3; 7.5]; % 461.10
%   cracks(:,1) = (linspace(width/2-5*3*(cracks_number-1),width/2+5*3*(cracks_number-1),cracks_number));
    final_crack = 25.0;
    initial_ac = 1.0; % a/c (ratio) (expand into matrix if necessary)

```

```

    cracks(:,2) = 0.15; % initial a
    yield_stress = 1175; % (MPa)880 for R260, 1175 for R350HT, 758 for weld
    CA_const = 3.45e-16 ; % da/dn=mm/cycle, deltaK=Mpa(mm)-0.5, 1.14e-16 For
    R260, 3.45e-16 for R350HT, 3.28e-24 for weld dadn mm/cycle delta K MPa mm^0.5
    M_const = 3.91 ; % 4.1 for R260, 3.91 for R350HT, 7.22 for weld
end

UA_const = 1;
UC_const = 1;
z = 1;
cracks(:,3) = cracks(:,2)./initial_ac; % crack_width
cracks(:,4) = 0; % cycle count
cracks(:,1) = sortrows(cracks(:,1)); % sort

while max(cracks(:,2,z)) <= final_crack
    % propagate crack
    notzero = find(cracks(:,1,z));
    for x = notzero(1,:)
        if strcmp(geometry, 'rail_HW')
            [da, dc, dna] = growcrackrail_HW(cracks(x,2,z), cracks(x,3,z));
        else
            [da, dc, dna] = growcrack(cracks(x,3,z), cracks(x,2,z));
        end
        cracks(x, :, z+1) = [cracks(x,1,z) cracks(x,2,z)+da cracks(x,3,z)+dc
        cracks(x,4,z)+dna];
    end

    % find cracks to merge
    notzero1 = find(cracks(:,1,z+1));
    for h = 1: numel(notzero1)-1
        x1 = notzero1(1,h); % x=x
        x2 = notzero1(1,h+1); % x2=x+1
        if cracks(x2,1,z+1)-cracks(x1,1,z+1)-cracks(x2,3,z+1)-cracks(x1,3,z+1) < 0
            % crack overlap condition
            area = cracks(x2,3,z+1).*cracks(x2,2,z+1)-cracks(x1,3,z+1).*cracks(x1,2,z+1);
            if area > 0
                cracks(x2,1,z+1) = cracks(x2,3,z+1)+cracks(x1,1,z+1); % location
            else
                temp = x2;
                x2 = x1;
                x1 = temp;
                cracks(x2,1,z+1) = cracks(x1,3,z+1)+cracks(x2,1,z+1); % location
            end
            %
            cracks(x+1,2,z+1) = max(cracks(x:x+1,2,z+1)); % depth

            cracks(x2,2,z+1) = (((cracks(x1,2,z+1)*cracks(x1,3,z+1)))+(cracks(x2,2,z+1)*cracks(x2,
            3,z+1)))/(cracks(x1,3,z+1)+cracks(x2,3,z+1));
            cracks(x2,3,z+1) = cracks(x1,3,z+1)+cracks(x2,3,z+1); % width
            cracks(x2,4,z+1) = max(cracks(x1,4,z+1), cracks(x2,4,z+1)); % cycle count
            cracks(x1, :, z+1) = 0;
        end
    end
end

```

```

%      cracks(:,z+1) = sortrows(cracks(:,z+1)); %sort
    end
    end
    z=z+1;
end

output=cracks(:,1:70:numel(cracks(1,1,:)));
%To extract a/c vs a/t
% output1=reshape(output(:,2,:)/thickness,[],1); %at
% output1=reshape(output(:,2,:)/output(:,3,:),[],1); %ac

%To extract a/t vs cycle
% output1=reshape(output(:,2,:)/thickness,[],1); %at 1 crack
% output1=reshape(output(:,4,:),[],1); %cycle 1 crack
% output1=reshape(max(output(:,2,:)),[],1)/thickness; %at many cracks
% output1=reshape(max(output(:,4,:)),[],1); %cycle many cracks

%To extract cycles
output1=max(cracks(:,4,end));

mpdc10 = distinguishable_colors(numel(cracks(:,1,1))); % 10x3 color list
plotcrack(5); %plot graphs (spacing)
end

function [da,dc,dna] = growcrackrail_HW(crack_depth,crack_width)
global thickness width applied_stress yield_stress angel CA_const M_const
Tha_const Thc_const UA_const UC_const bending LT applied_stressx
applied_stressy applied_stressz
    at = crack_depth/16.5;
    ac = crack_depth/crack_width;
    theta = 0.4: 0.1: 3.5;
    ac = crack_depth/crack_width;
    z = ((sin
(theta)).^2+(ac).^0.5.*(cos(theta)).^2).^0.25./(sqrt(1+1.464.*(ac)^1.65))/0.6371;
%paramters for Y factor
if crack_depth > 1 %for crack size large than 1
    px=0.620;
    nx=0.067;
    ax=-0.149;
    bx=0.159;
    cx=-1.909;
    py=0.281;
    ny=-0.553;
    ay=1.196;
    by=-0.258;
    cy=-1.082;
    pz=0.686;
    nz=0.060;
    az=-0.332;
    bz=0.340;

```

```

        cz=-0.435;
    else %for crack size smaller than 1
        px=0.694;
        nx=0.022;
        ax=-9.628;
        bx=198.206;
        cx=-0.051;
        py=0.693;
        ny=0.027;
        ay=52.387;
        by=-676.227;
        cy=-0.169;
        pz=0.697;
        nz=0.017;
        az=11.706;
        bz=-106.053;
        cz=-0.045;
    end
    fx = 1.0 + (0.1+ax*at^2+bx*at^3).*((1-sin(theta+cx)).^2);
    qx = px*(at)^nx;
    fy = 1.0 + (0.1+ay*at^2+by*at^3).*((1-sin(theta+cy)).^2);
    qy = py*(at)^ny;
    fz = 1.0 + (0.1+az*at^2+bz*at^3).*((1-sin(theta+cz)).^2);
    qz = pz*(at)^nz;

    % Mk factor, only applicable for crack size smaller than 10 mm
    if at < 0.6061
        hx = fx.*qx.*(0.9644*(at)^(-0.123)); %0.8757for 0.1mm 0.4379 for 0.5mm
        hy = fy.*qy.*(0.9644*(at)^(-0.123));
        hz = fz.*qz.*(0.9644*(at)^(-0.123));
    else
        hx = fx.*qx;
        hy = fy.*qy;
        hz = fz.*qz;
    end
    %   hx = fx.*qx;
    %   hy = fy.*qy;
    %   hz = fz.*qz;

    %correction for different shape

    SIFx = applied_stressx*(sqrt(pi*crack_depth)).*hx.*z; % K_a
    SIFy = applied_stressy*(sqrt(pi*crack_depth)).*hy.*z; % K_a
    %   SIFCy =
    ((18.8+crack_depth)/(18.8+crack_width))*applied_stressy*(sqrt(pi*crack_depth)).*hc
    y;
    SIFz = applied_stressz*(sqrt(pi*crack_depth)).*hz.*z; % K_a

    RMSax = (SIFx).^2.*(sin(theta-0.4)).^2; % value before integral
    RMScx = (SIFx).^2.*(cos(theta-0.4)).^2;

```

```

RMSay = (SIFy).^2.*(sin(theta-0.4)).^2; % value before integral
RMScy = (SIFy).^2.*(cos(theta-0.4)).^2;
RMSaz = (SIFz).^2.*(sin(theta-0.4)).^2; % value before integral
RMScz = (SIFz).^2.*(cos(theta-0.4)).^2;

QRMSax = trapz(RMSax*0.1);
QRMScx = trapz(RMScx*0.1);
QRMSay = trapz(RMSay*0.1);
QRMScy = trapz(RMScy*0.1);
QRMSaz = trapz(RMSaz*0.1);
QRMScz = trapz(RMScz*0.1);

ESIFAWx = sqrt(QRMSax/pi*2); %delta_K_a
ESIFCWx = sqrt(QRMScx/pi*2);
ESIFAWy = sqrt(QRMSay/pi*2); %delta_K_a
ESIFCWy = sqrt(QRMScy/pi*2);
ESIFAWz = sqrt(QRMSaz/pi*2); %delta_K_a
ESIFCWz = sqrt(QRMScz/pi*2);
%stress intensity factor a & c
ESIFAW = ESIFAWx+ESIFAWy+ESIFAWz; %delta_K_a
ESIFCW = ESIFCWx+ESIFCWy+ESIFCWz; %delta_K_c

% Crack Propagation calculation
da = 0.0001*crack_depth;
dna = da/(CA_const.*((ESIFAW)^M_const));
dc = dna*(CA_const).*(ESIFCW.^M_const); %dc =
dna*24.0*(CA_const*0.9^M_const).*(ESIFCW.^M_const);
end

function [da,dc,dna] = growcrack(crack_width,crack_depth)
global thickness width applied_stress yield_stress CA_const M_const Tha_const
Thc_const UA_const UC_const bending LT geometry
at = crack_depth/thickness;
ac = crack_depth/crack_width;
cw = crack_width/width;
cb = 2*cw;

z1 = 1.13-0.09*ac;
z2 = (-0.54+(0.89/(0.2+ac)))*at^2;
z3 = (0.5-(1/(0.65+ac)))+(14*(1-ac)^24))*at^4;
gc = 1.1 + (0.35*at^2);
fc = sqrt(ac);
ha = 1;
hc = 1;
if bending >0
    hc=1-0.34*at-0.11*ac*at;
    g1=-1.22-0.12*ac;
    g2=0.55-1.05*ac^0.75+0.47*ac^1.5;
    ha=1+g1*at+g2*at^2;
end

```

```

if cb <= 0.5

    fw = sqrt(1/(cos(pi*(cw)*sqrt(at))));

else

    fw = sqrt(1/(cos(pi*(0.25)*sqrt(at))));
end

q = sqrt(1+(1.464*(ac^1.65)));

% doing SIFA without plastic correction

SIFA = applied_stress*ha*(sqrt(pi*crack_depth))*(z1 + z2 + z3)*(fw/q);

% doing SIFC without plastic corection

SIFC = applied_stress*hc*sqrt(pi*crack_depth)*(z1 + z2 + z3)*fw*gc*(fc/q);

% Newman and Raju
% Plastic correction factor
PZA = 0.0530516*(SIFA/yield_stress)^2;
PZC = 0.1591549*(SIFC/yield_stress)^2;
amp = crack_depth + PZA;
cmp = crack_width + PZC;
atp = amp/thickness;
acp = amp/cmp;
cwp = cmp/width;
cbp = 2*cwp;

z1p = 1.13-0.09*acp;
z2p = (-0.54+(0.89/(0.2+acp)))*(atp)^2;
z3p = (0.5-(1/(0.65+acp)) + 14*(1-acp)^24)*(atp)^4;
gcp = 1.1 + 0.35*(atp)^2;
fcp = sqrt(acp);

if cbp < 0.5

    fwp = sqrt(1/(cos(pi*(cwp)*sqrt(atp))));

else

    fwp = sqrt(1/(cos(pi*(0.25)*sqrt(atp))));
end

qp = sqrt(1+(1.464*(acp)^1.65));

% doing SIFA with plastic correction

SIFAP = applied_stress*ha*sqrt(pi*amp)*(z1p + z2p + z3p)*fwp/qp;

% doing SIFC with plastic correction

```



```

        SIFCP = applied_stress*hc*sqrt(pi*amp)*(z1p + z2p + z3p)*fwp*gcp*fcf/qp
    ;

    if strcmp(geometry,'tubu')
        Mkc1 = [1 0.01949*at^(-1.01)+1.611 0.07387*at^(-0.7691)+1.743 0.1156*at^(-
0.6979)+1.685 0.0862*at^(-0.7571)+1.779];
        Mka1 = [1 0.01736*at^(-0.8143)+0.879 0.08266*at^(-0.5592)+0.791 0.07398*at^(-
0.5839)+0.8171 0.08703*at^(-0.5657)+0.7953];
        LT1 = [0 0.5 1.25 2 2.75];
        Mkc=interp1(LT1,Mkc1,LT);
        Mka=interp1(LT1,Mka1,LT);
    end

    if strcmp(geometry,'pipe')
        Mkc = 0.262*at^(-0.4547)-0.6922;
        Mka = 0.03901*at^(-0.6289)+ 0.8914;
    end

    if strcmp(geometry,'cruciform')
        Mkc=0.1693*at^-0.623+1.05;
        Mka=1.571*at^-0.1723-0.9413;
        if bending >0
            Mkc=0.03155*(at)^(-0.8868)+1.032;
            Mka=0.06783*(at)^(-0.674)+0.7248;
        end
    end

    ESIFAW = UA_const*(Mka*SIFAP-Tha_const);
    ESIFCW = UC_const*(Mkc*SIFCP-Thc_const);

% Crack Propagation Simulation */
da      = 0.0001*crack_depth;
dna     = da/(CA_const*(ESIFAW^M_const));
dc      = dna*CA_const*(ESIFCW^M_const);

end

function plotcrack(spacing)
global output thickness width final_crack cracks applied_stress geometry s
figure('units','normalized','outerposition',[0 0 1 1],'Color',[1.0 1.0 1.0]);

%plot a/t vs fatigue life
subplot(2,2,1);
if numel(output(:,2,1))~=1
    plot (reshape(max(output(:,4,:)),[],1),reshape(max(output(:,2,:)),[],1)/thickness,'b');
else
    plot (reshape(output(:,4,:),[],1),reshape(output(:,2,:),[],1)/thickness,'b');
end

```

```

xlabel('Number of cycles'); % x-axis label
ylabel('a/t'); % y-axis label
legend(sprintf('Predicted life = %.f cycles',
max(cracks(:,4,end))), 'Location', 'northwest');

%plot a/c vs a/t
subplot(2,2,2)
cd=reshape(output(:,2,:),numel(output(:,1,1)),[]);
cw=reshape(output(:,3,:),numel(output(:,1,1)),[]);
for x = 1:numel(cd(:,1))
    plot(cd(x,:)/thickness,cd(x,:)/cw(x,:), 'Color', colour(x)); hold on;
end
xlabel('a/t'); % x-axis label
ylabel('a/c'); % y-axis label
axis([0,final_crack/thickness,0,1]);

%plot crack growth
subplot(2,2,[3,4])
for z = 1:spacing:numel(output(1,1,:))
    for x = 1:numel(output(:,1,1))
        if output(x,1,z)~=0
            plotEllipse(output(x,3,z),output(x,2,z),[output(x,1,z),0], colour(x)); hold on;
        end
    end
end
if strcmp(geometry,'rail_HW')
    % axis([0,width,0,thickness]);
    box off;
    set(gca,'Visible','off');
else
    axis([0,width,0,thickness]);
end
grid off;
xlabel('Width'); % x-axis label
ylabel('Thickness'); % y-axis label

% title(sprintf('Stress range = %.1f MPa', applied_stress))
end

function [output2] = colour(input)
    global mpdc10
    output2 = mpdc10(input,:);
end

function plotEllipse(a,b,C,colour)
    global geometry s
    % range to plot over
    % -----
    N = 10;
    if strcmp(geometry,'rail_HW')

```

```

        theta = 0:1/N:1.5*pi+1/N;
    else
        theta = 0:1/N:pi+1/N;
    end

    % Parametric equation of the ellipse
    %-----
    state(1,:) = a*cos(theta);
    state(2,:) = b*sin(theta);

    % Coordinate transform (since your ellipse is axis aligned)
    %-----
    X = state;
    X(1,:) = X(1,:) + C(1);
    X(2,:) = X(2,:) + C(2);

    % Plot
    %-----
    sss = plot(X(1,:),X(2:,:), 'Color', colour);

    if strcmp(geometry, 'rail_HW')
        direction = [0 0 1];
        rotate(sss, direction, 19.98310652)
    end

    hold on;
    % plot(C(1),C(2),'r*');
    axis equal;
    grid;

end

```

Appendix D Publications Published During This PhD Research

Journal Papers

1. Liu Y, Tsang KS, Hoh HJ, Shi X, Pang JHL, “Structural fatigue investigation of transverse surface crack growth in rail steels and thermite welds subjected to in-plane and out-of-plane loading”, *Engineering Structures*, 201 (2020), 110076.
2. Pang JHL, Hoh HJ, Tsang KS, Low J, Kong SC and Yuan WG, “Fatigue crack propagation analysis for multiple weld toe cracks in cut-out fatigue test specimens from a girth welded pipe”, *International Journal of Fatigue*, 94 (2017) 158-165.
3. Pang JHL, Tsang KS and Hoh HJ, “3D stress intensity factors for weld toe semi-elliptical surface cracks using XFEM”, *Marine Structures*, 48 (2016) 1-14.
4. Hoh HJ, Pang JHL and Tsang KS, “Stress intensity factors for fatigue analysis of weld toe cracks in a girth-welded pipe”, *International Journal of Fatigue*, 87 (2016) 279-287.

Conference Papers

1. Hoh HJ, Pang JHL and Tsang KS, “Failure Assessment Diagram (FAD) Analysis of fatigue test results for X65 welded joints”, *Fatigue 2018, 12th International Fatigue Congress*, Poitiers, France (2018).
2. Tsang KS, Pang JHL and Hoh HJ, “Influence of weld toe radii on fatigue life prediction”, *Fatigue 2018, 12th International Fatigue Congress*, Poitiers, France (2018).
3. Hoh HJ, Pang JHL and Tsang KS, “Fatigue Modelling of Semi-elliptical Surface Cracks in Welded Pipe Geometries”, *OMAE 2016, 35th International Conference on Ocean, Offshore & Arctic Engineering*, Busan, South Korea (2016) OMAE2016-54683.
4. Tsang KS, Pang JHL and Hoh HJ, “Fatigue Algorithm for Modeling Many Weld Toe Cracks Propagating at Welded Joints”, *OMAE 2016, 35th International Conference on Ocean, Offshore & Arctic Engineering*, Busan, South Korea (2016) OMAE2016-54735.

Technical disclosure

1. Pang JHL, Tsang KS, “Fatigue Crack Growth Analysis (FCGA) Software” NTU:
TD/315/17

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