Essays on blood supply management

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ESSAYS ON BLOOD SUPPLY MANAGEMENT

JUAN RAMON LIBORO SENG
NANYANG BUSINESS SCHOOL

2021

Essays on Blood Supply Management

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Nanyang Business School

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2021

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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JUAN RAMON LIBORO SENG

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Geoffrey Bryan Ang Chua

Authorship Attribution Statement

This thesis contains material from papers accepted in the following conferences in which I am listed as an author.

I presented Chapter 3 in the 2019 *MSOM Conference* held in Singapore on June 30-July 2, 2019 under the title “Blood Inventory Management with Stochastic Supply and Cool-Down”. I also presented it in the 2020 *INFORMS Annual Meeting* held virtually from November 8-11, 2020 under the title “Inventory Models with Regenerative Capacity”

I presented Chapter 5 in the 2020 *Analytics for X Conference* held virtually from June 30-July 2, 2021 under the title “Ethically Complex Blood Supply Interventions During Pandemics”.

The contributions of the co-authors are as follows:

- Dr. Chua provided the initial project direction and edited the draft for submission
- I did the analysis while having constant discussion on mathematical techniques and correctness with Dr. Chua. I also wrote the paper, and Dr. Chua reviewed and provided comments on the contents.

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JUAN RAMON LIBORO SENG

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Summary

There is a global shortage of blood with countries signaling the need to improve collection in order to address blood requirements. This dissertation focuses on three essays related to blood supply and blood donor management with the aim of addressing this global shortage. We do this by providing insights and tools into better blood inventory management, donor recruitment and retention, and disaster-related policy for blood banks.

One of the defining features of blood donation is a fixed rest time between donations for donors. This effectively restricts the ability of willing donors to donate more often than they want to. From a blood bank's perspective, this also reduces the number of times they can ask a donor to donate. We call this process *regeneration* with blood donors being an example of *regenerative capacity* while noting that this changes a blood bank's optimal inventory policy. In Chapter 3, *regenerative capacity* is analyzed, advancing not only blood inventory management but also generalizes capacitated inventory systems. The chapter considers both backlogging and lost-sales with an initial assumption that no donors drop-off from the donor pool once they become a part of it. We prove the optimality of a threshold policy in both cases. We further characterize a rationing and donor-build up policy in the lost-sales setting if few donors are available because of a resource-conserving and capacity building mind-set from blood banks. This is then extended to a more realistic scenario where donors are allowed to drop-off. In this case, blood banks can also build up their inventory in anticipation of future drop-offs. We then provide a numerical study that looks at the effect of reducing regeneration on the cost incurred. We find that because blood is considered a *valuable* resource, reducing regeneration can significantly reduce cost. Lastly, we prove the optimality

of threshold policies when there is perishability with a lifetime of one period, non-identical but independent demand, and joint recruitment of donors and ordering of blood.

The blood shortage problem becomes even more apparent when there are disasters. In the two part essay in Chapters 4 and 5, we examine blood supply interventions and the dynamics of blood supply during disasters. For Chapter 4, we construct an analytical model which looks at the equivalence of three types of interventions in a steady-state system, namely interventions related to: (1) recruitment and retention, (2) donation frequency, and (3) reducing regeneration time. We also present a methodology for comparing the efficiency of each of the three interventions in reaching a desired target donation rate. Next, we compare the steady-state results with that of the more realistic non-steady state scenario through an iterative calculation procedure and find that the donations fluctuate heavily during and after disasters of varying lengths. Despite this, however, the equivalences between the interventions continue to hold. This volatility can be overcome by assuming that blood banks have the capability to control donation rates and by implementing a dynamic donation rate strategy. Next in Chapter 5, we gain further insights from the analytical model by constructing a stochastic inventory model with regenerative capacity, followed by a discrete-event simulation study. The results show that more mature blood banks have more strategies available at their disposal when it comes to trying to maintain fill-rates. We also observe that the dynamic donation rate strategy results in lower fill-rates during disasters when the fill-rate is low despite being able to meet donation targets, and that multiple waves with short lead times exacerbate this effect. Lastly, we introduce three ethically complex interventions: (1) demand deferral, (2) rationing of blood, and (3) tapping non-traditional donor pools and show that these can be implemented from both the analytical and simulation models.

Finally, in Chapter 6, we address the trade-off between recruitment and retention of blood donors. As blood banks try to increase donation by using marketing campaigns to target groups of donors, budget constraints leave them with a choice of either recruiting new donors or retaining old ones. We use a multi-period network flow model and an equivalent mixed integer linear program to solve the problem given four sample countries. We find that if the objective function is to maximize number of donations, blood banks would focus on retaining those who have already

donated rather than getting new donors. However, this leaves the blood bank vulnerable to having an older donor pool. We then introduce a hybrid objective function that balances maximizing new donors with maximizing the number of donations. A numerical study confirms that the hybrid objective increases new donors significantly while reducing the number of donations by a relatively smaller amount. The final analysis compares the strategies of short-term and long-term planning horizons, such that long-term planning horizons focus on recruitment of new donors at the start and retention of more regular donors towards the end.ail

Chapter 1

Thesis Contribution

This thesis contributes to research and literature in inventory theory and blood supply management. In Chapter 3, we advance inventory theory by introducing a generalization of capacity called *regenerative capacity*, which has not yet been studied analytically in previous literature. It opens up a new line of research in inventory theory considering there are numerous resources that regenerate after utilization (e.g. blood banking, forestry, fisheries). We quantify through a numerical study the impact of changing regeneration time on cost, which enables us to influence policy-making when they conduct cost-benefit analysis of determining proper regeneration times of donors. The results are extended to important blood banking features such as perishability, seasonality, and donor drop-offs.

Chapters 4 and 5 provides a method for blood banks to choose which interventions to use during disasters. Chapter 4 provides the analytical basis, with further support that it can work even in volatile, non-steady state situations. A new dynamic donation rate strategy is also introduced, which can be utilized by blood banks who have control on donation rates. Then in chapter 5, we also provide a simulation tool that can be utilized when planning for pandemics and disasters. Blood banks can easily change the input into the model and use the results to inform policy makers of which interventions (ethically complex or otherwise) may be used to improve fill-rates. The introduction of ethically complex interventions and how it can improve fill-rates also provides a new perspective in the debate of whether or not the interventions should be implemented. The model also allows for easy evaluation and implementation of interventions. Since literature in blood

supply management uses DES extensively, the model we created is flexible enough to incorporate these interventions and be evaluated when pandemics or disasters hit.

Lastly, in Chapter 6, we provide a stylized model for recruitment and retention of donors. The constant problem of allocation of resources for recruitment and retention that blood banks face can be addressed and solved to optimality by the model that we created. Blood banks would be able to use it in their planning processes and can easily be adjusted to take into account any marketing constraints. It also provides insights into the difference in optimal strategies involved with varying planning horizons, as well as alternative measures that incorporate both new donors and number of donations. The model starts the foundation of using network theory in recruitment and marketing literature.

Chapter 2

Introduction

Blood and blood products are an important resource in any country's health system. These are required for use in surgical procedures, maintenance in general medicine, treatment of blood diseases, and handling of medical emergencies (Singapore Red Cross, 2018). Whole blood and its components are extracted from donors and travel through a Blood Supply Chain (BSC) before being transfused to patients. This journey from extraction to transfusion is complex, and from an operations perspective, three important features are present: (1) Outdating or Perishability of Blood, (2) Cross-matching or Patient-Blood Compatibility, and (3) Demand and Supply Seasonality and Stochasticity.

Various studies have been done on the management of BSCs taking into consideration these three features (see for example Beliën and Forcé, 2012 and Pirabán et al., 2019), but the most pressing need comes from being able to manage blood supply and donors. Majority of blood is still obtained through voluntary donations from the public, and there has been a significant decline in donations globally in recent years in conjunction with rising demand. Singapore for example projects that demand for blood will rise by 3-5% year on year – double by 2030 – but is faced with a 13% reduction in its donor pool for individuals aged 16-25 (Poon, 2017). The US also experienced a steady decline in donor turnout from 3.8 million in 2009 to 2.7 million in 2017, which dangerously closed the gap between supply and demand (NBCDFW, 2018). Our own interviews with senior management of blood collection groups also revealed that there is a focus on being able to manage (i.e. recruit, retain, and incentivize) donors to donate in order to meet blood requirements.

Further, in a study by Roberts et al. (2019), they estimated that the global need-to-supply ratio of blood is 1.12, and that 119 out of 195 countries studied do not have sufficient blood. A closer look reveals that low and middle-income countries bear the burden of even lower donation rates than their high-income counterparts with high-income countries collecting 40% of total blood donations globally while only accounting for 16% of the population (WHO, 2020). Even more concerning is the fact that this disparity is aggravated during disasters like pandemics where infrastructure and resource differences result in wide gaps in blood collection rates.

These highlight the importance of blood supply management research during periods of peace and disasters across countries of different demographic and socioeconomic statuses. In this dissertation, we discuss three studies that focus on ways to improve blood inventory and donor management given these scenarios. In the process, we are able to provide insights into blood management which can hopefully influence public policy and save more lives. A general theme throughout this dissertation is the treatment of blood and donors as inventory units. While this is a simplifying assumption, it is a necessary (but not restrictive) one in order to be able to use certain tools of operations management such as stochastic dynamic programming, discrete-event simulation, and network flow models.

In Chapter 3, we focus on *donor regeneration*, or the rest needed after donating before being allowed to donate again. Analyzing this feature of blood donation gives blood banks the knowledge on the effect of regeneration on their inventory policies, which they can now adjust accordingly based on the optimal policies obtained in this chapter. This chapter also introduces the inventory theme to the dissertation by thinking of donors as production capacity (more specifically, *regenerative capacity*) and the blood that's extracted as inventory. We provide a stochastic dynamic programming model and obtain the optimal blood extraction policy given the number of donors available and regenerating. The regeneration process deviates from typical capacitated inventory models because donors can be re-utilized, and blood can be extracted again in a future period. We obtain an optimal threshold policy for both the backlogging and lost-sales cases assuming there are no donor drop-offs and there is *iid* demand. This optimal threshold is further characterized by a rationing policy in the lost-sales setting. A numerical study looks at the impact of reducing regeneration in blood donors and how the myopic policy can be a good policy substitute. We then extend the model to a more realistic assumption where

there are donor drop-offs, thus incorporating loss of donors in the model. The resulting extension shows that when there are drop-offs, blood banks would then build up more inventory beyond the threshold provided in the no drop-off case. Lastly, we examine the case where blood outdates if not used immediately, and the case where there is independent but not identically distributed demand to take into consideration perishability and seasonality, respectively.

Next, in Chapters 4 and 5, we look at blood donor management during disasters and the interventions that blood banks have at their disposal. We examine the effect of standard and ethically complex interventions such as recruitment and retention, developing a culture of donation, reducing regeneration, rationing, deferral of demand, and using non-traditional donors on fill-rates during disasters. The analysis is first done by constructing an analytical model in Chapter 4 to come up with an equivalence among the interventions. That is, for example, what equivalent increase in donors should we have in order to get the same effect of reducing regeneration time by the same amount. A simple methodology to compare which intervention is more effective is illustrated in this chapter by introducing an efficiency parameter. Then, the volatility resulting from implementation of these interventions is analyzed by coming up with an iterative calculation procedure and a dynamic donation rate strategy is suggested. Next, an inventory theory-based Discrete-Event Simulation (DES) model in Chapter 5 is constructed to see the effectiveness of this strategy in addressing demand. We do this by comparing the dynamic donation rate strategy and a fixed donation rate strategy for two sample countries and obtaining the resulting fill-rates. We also extend this analysis to multiple disaster waves.

Lastly, in Chapter 6, we focus on the problem of optimal recruitment and retention of blood donors. Blood banks use marketing and recruiting campaigns to be able to either recruit new donors or retain old ones. However, a trade-off occurs in deciding which group to target given a limited budget. We use a novel approach in solving the donor recruitment problem by converting it to a network flow model and solving it through a Mixed Integer Linear Programming (MILP) with the objective of maximizing donations. This is solved for four sample countries where we examine the effect of different strategies on the resulting total donors and donations. We then propose a hybrid objective that incorporates both the number of new donors and donations, and show that the hybrid objective can result in only a slightly lower

number of donations for an equivalent trade-off of a high number of new donors. Lastly, we compare the optimal strategies when the planning horizon changes from the short-term to the long-term. The resulting model from this chapter gives a recruitment and retention optimization tool for blood banks to use when deciding on who to target and when to target them.

The rest of the dissertation proceeds as follows: in Chapter 3, we discuss donor regeneration and regenerative capacity. Chapters 4 and 5 look at blood donor management and interventions during disasters. Chapter 6 then examines the optimal recruitment and retention of blood donors, and finally Chapter 7 concludes the dissertation and provides on-going and areas for future research.

Chapter 3

Inventory Control Models with Regenerative Capacity

3.1 Introduction

In this chapter, we introduce a new class of stochastic, inventory problems where capacity regenerates. We consider donors as production capacity and blood as inventory and note that we use the terms donors and capacity, and blood and inventory interchangeably. In each period, capacity that is utilized starts regenerating and can be used again (or finishes regeneration) after a certain amount of time. On the other hand, unutilized capacity is carried over into the next period as additional available capacity. Effectively, the capacities in future periods are dependent on decisions in the current and previous periods. There is a trade-off between a decision to order now and incur holding cost but have more available capacity in the future, or defer ordering for lesser holding cost but have lower available capacity overall in the rest of the planning horizon. We call this *regenerative capacity*, by virtue of the regeneration that happens after the utilization of capacity.

This novel concept is a generalization of capacity in inventory control models, and as mentioned is most apparent in blood banking where blood donors who donate today need to rest for eight to twelve weeks before becoming eligible for donation again, whereas those who haven't donated today can still do so tomorrow. Interestingly, this can also be seen in other industries such as the dairy industry's harvesting of milk from cows. They need to take into account the rest that the

animals need for them to remain productive. In aviation, pilots need to rest for a certain amount of time before being allowed to fly a plane again. With the recent call from governments to be more critical of the use of natural resources, regulations on the logging and fishing industry necessitate that certain sections of forest and ocean have to be given time to recover after utilization. The applications of regenerative capacity are numerous, but it can broadly be summarized to cover settings where capacity is a resource that needs to recover before being available for utilization again. This concept becomes more critical now as it comes with increasing regulatory pressure on industries to be more sustainable and mindful of the effects of their actions on stakeholders (e.g. health of donors and pilots, recovery of natural resources, etc.). With increasing regulation though, is more complexity on the management of these resources. Now, not only should demand and supply be matched optimally to reduce costs, but decisions should also be made so as to be able to build a pipeline of future capacity. Without regenerative capacity (i.e. independent capacity), this is not a problem because capacity is known from the start of the planning horizon. This is not the case however, when we generalize it to include regeneration.

Blood supply management also usually focuses on perishability of blood and blood products. We refer to the statistic mentioned in the Introduction of this dissertation which says that there is very high need-to-supply ratio of blood. This means that perishability of blood, though important, must be considered in the context of relatively few donors compared to the demand. This chapter then focuses on the upstream stages of blood supply chains at the blood extraction side.

The structure of the optimal policy for this class of problems is still unknown and will be shown for the backlogging and lost-sales case in this chapter. We also extend the results to the perishable case with lifetime of one period, the case where there is independent but non-identical demand, and where recruitment and donor extraction decisions are jointly made. The rest of the chapter is organized as follows: the remainder of this section discusses preliminaries on the process of regeneration, which is required for a more complete discussion of related literature, as well as the related literature. Section 3.2 introduces the models and provides the results, Section 3.3 focuses on the impact of regenerative capacity on cost through a numerical study, and Section 3.4 discusses some extensions.

3.1.1 Preliminaries

We begin our discussion on regenerative capacity (even before the related literature – for better comprehension) by first introducing *incoming capacity*. The incoming capacity u_s is a known capacity that gets added into the system at a period s from an external source. These are the blood donors who are recruited and included in the donor pool in each period. All the u_s is contained in the capacity vector $\mathbf{u} = (u_1, u_2, \dots, u_T)$, such that $\sum_{i=1}^s u_i$ is the total incoming capacity (e.g. total number of donors in the donor pool) in period s .

Let a_s be the order in period s . We define the regeneration time k as the number of periods an a_s equal amount of capacity in inventory ordered in s becomes unavailable, before it becomes available again as capacity after $k+1$ periods. For example, say we have a capacity vector $\mathbf{u} = (5, 6, 4, 3)$ with $k = 2$. In the first period, the capacity is equal to the incoming capacity $u_1 = 5$ and so if we order $a_1 = 4$ units in period 1, period 2's capacity will be $6 + (5 - 4) = 7$ while period 4 will get an additional $a_1 = 4$ units of capacity as well. This indicates that the capacity equal to inventory ordered in $s = 1$ regenerates in period $s = 2$ and 3, and becomes available in $s = 4$. At the same time, the unutilized capacity $5 - 4 = 1$ gets carried over into period $s = 2$, increasing its capacity. Suppose instead $a_1 = 3$ units. This means that the capacity in period 2 becomes $6 + (5 - 3) = 8$ while period 4 will get an additional $a_1 = 3$ units. This is clearly different from the capacities previously when $a_1 = 4$. This simple example demonstrates how decisions in past periods will now have a larger impact on the state space of future periods, because ordering (non-ordering) would expand (restrict) the possible future states of both capacity and ending inventory.

From here, we can have a better understanding of the related literature and how special cases of regenerative capacity have been analyzed.

3.1.2 Related Literature

3.1.2.1 Inventory Theory

This chapter contributes to the literature on capacitated inventory systems which have been studied extensively. We briefly summarize the most relevant ones to this

chapter. For example, Federgruen and Zheng (1986, 1986b) look at the optimal policies for capacitated systems using the average and discounted cost criterion. Roughly, the optimal policy is to raise inventory to be as close as possible to a threshold given the limits on ordering (i.e. capacity). However, in majority of capacitated inventory literature, capacity is known with certainty throughout the planning horizon, which as was discussed in Section 3.1.1 is not the case for regenerative capacity.

There are random capacity models like those studied by Wang and Gerchak (1996) and more recently Feng and Shanthikumar (2018), which retain the optimality of a threshold policy, but are then dependent on other states like on-hand inventory. More closely related to regenerative capacity would be the study by Jaksic et al. (2006) where future capacity is revealed when progressing from period to period. The difference with regenerative capacity however is that the realized capacities are not influenced by prior decisions. Rather, they are drawn from a known probability distribution, and unused capacity is lost. The optimal policy remains to be a state-dependent order-up-to threshold.

Regenerative Capacity is also related to remanufacturing and reverse logistics. In remanufacturing, inventory that has been used to satisfy demand may return, be refurbished, and used to satisfy another unit of demand in the future. Some examples of these studies are the ones by Zhou and Yu (2011) and Zhou et al. (2011) who were able to find the optimal policies for multiple inventory classes and for joint pricing and remanufacturing decisions. However, in remanufacturing, the arrival of cores (i.e. returned items which in this case represent the production limit of remanufacturing) are random similar to that of stochastic capacity, whereas the regeneration time is fixed. Remanufacturing is also a separate process from manufacturing, such that the sharing of fixed capacity of a manufacturing facility is needed (Gong and Chao, 2013).

A special case of regenerative capacity, and closest to this chapter, is the study by Benjaafar et al. (2017) where they likened capacity to a *scarce resource*. Capacity in this case can be thought of as a resource that needs to be rationed in order to satisfy demand. In their model, the utilized capacity does not regenerate in a future period. Rather, there is a total capacity known at the start that they can utilize throughout the planning horizon. The unutilized capacity is then carried over in the next period. What is interesting is that we can say that their capacity

does regenerate, but the regeneration period goes beyond the planning horizon, effectively removing it from the analysis. We can also say that the incoming capacity is zero for all periods except for the first period, making the first period's incoming capacity be the total capacity that needs to be rationed. This is precisely the model that Benjaafar et al. (2017) considered. They use the setting of production limits/quotas in fisheries, logging, and carbon emissions and other harmful pollutants as the motivation to their model, and it supports our generalization further, indicating that problems with production limits fall under the larger class of problems with regenerative capacity. In fact, Benjaafar et al.'s (2017) model (although they maximize revenue) is a special case of the model discussed in this paper.

Similarly, Gong and Zhou (2013) looked at optimal production planning in the context of emissions trading. In their study, capacity is the allowance level that a firm can either sell or buy more of. They find that the optimal policy is defined by two-thresholds – the buy-up-to and sell-down-to threshold which guides how much allowance they should have in each period. Unutilized capacity can be carried over into the next period, or it can be sold to other firms. However, regeneration is also not observed in this context because what was utilized cannot be used again in a future period.

We make use of L^h -convexity in order to analyze regenerative capacity. These have been widely used in inventory literature because it is able to provide insights into the structure of optimal thresholds especially in multi-state problems. For example, Zipkin (2008) proved L^h -convexity in lost-sales models with lead time, while the same was proven by Huh and Janakiraman (2010) for serial inventory systems. Interestingly, regenerative capacity does have some similarities with inventory problems with lead time or serial systems in that in these systems, inventory progresses through *levels* (i.e. time before arrival for lead time, echelons for serial systems) before it can get to its final destination as inventory. There are some major differences, however, between these systems and that of regenerative capacity. The biggest one is that in regenerative capacity, capacity is what goes through regeneration stages before it can be reusable, in contrast to inventory in both lead time and serial systems. For serial systems where inventory that can be transferred to a lower echelon is dictated by capacity in the form of inventory in the higher

echelon, another important difference is that this transfer is a decision. In regenerative capacity, capacity automatically goes through the regeneration process once an ordering decision has been made. Regenerative capacity systems also exhibit a cyclical usage of inventory/capacity, while serial systems and those with lead-time are linear.

We note that existing models cannot handle the problem of regenerative capacity. In this sense, regenerative capacity is a novel concept that is yet to be studied. At the same time, capacitated inventory systems, more complex inventory systems in existing literature (Benjaafar et al., 2017), and the motivational examples mentioned can be expressed as cases of regenerative capacity. More concretely, these are summarized in Table 3.1. This paper then advances inventory theory to encompass a more complex class of problems while providing some additional insight in existing capacitated inventory models.

TABLE 3.1: Inventory Systems and Corresponding k Values

	Dynamic Capacity	Benjaafar et al., 2017	Fixed Capacity	Donor Cool-Down
k	0	$T - 1$	0	$0 < k < T - 1$
u_t	<i>unrestricted</i>	$u_1 > 0, u_t = 0 \forall t > 1$	$u_1 > 0, u_t = 0 \forall t > 1$	≥ 0

3.1.2.2 Blood Inventory Management

This chapter also contributes to blood inventory management research. Blood inventory management has been studied extensively in literature, with Piraban et al. (2019) providing the latest review of blood supply chain papers. Majority of the research done in this area focuses on evaluating the performance of inventory policies in terms of shortage and outdates while explicitly considering either perishability or blood type compatibility. Civelek et al. (2015) for example consider an inventory model that explicitly includes a mismatch cost when demand for younger blood is satisfied by an older unit. Sarhangian et al. (2018) evaluated an age-based threshold policy that balances transfusion of younger blood while trading off a small increase in outdates. Chen et al. (2019) and Osorio et al. (2017) both study the decision of collecting blood inventory either through whole-blood donations or apheresis donations with the idea that platelets obtained from apheresis donations outdate faster. Abouee-Mehrizi et al. (2019) studied the optimal allocation policy for multiple blood types considering perishability. These

papers consider perishability but do not include regeneration of donors. As such, the results can only apply in cases where the number of donations exceeds the amount of demand. This is because an implicit assumption made is that there are enough donors at any given point in time to donate, which is most likely not the case in majority of blood banking systems and hospitals. Obtaining the optimal blood inventory policy with regeneration is then an important stepping stone linking perishable blood inventory management with donor rest. This chapter provides the initial results for regeneration of donors, and some preliminary results taking into consideration regeneration with perishability.

3.2 Stochastic Blood Inventory Problem with Regenerative Capacity

3.2.1 Set-up and Notations

We first introduce the notations that will be used throughout the paper. We also let $(x)^+ = \max(0, x)$.

TABLE 3.2: Model Notations

States	Description
x_s	Starting inventory level at period s
$\mathbf{x}_s$	vector of length k , $\mathbf{x}_s = (x_{s,1}, x_{s,2}, \dots, x_{s,k})$
$q_{s,j}$	Capacity regenerating in $k + 1 - j$ periods: $q_{s,j} = a_{s-j}$
$\mathbf{q}_s$	Regeneration Schedule, vector of length k , $\mathbf{q}_s = (q_{s,1}, q_{s,2}, \dots, q_{s,k})$
Parameters	
u_s	Incoming capacity in period s
$\underline{u}_s$	Realized capacity in period s
D	<i>iid</i> demand with cdf F and pdf f
c, h, p	ordering, holding, and backlog cost per unit
$\mathbf{e}$	the all ones vector such that $\mathbf{e} = (1, 1, 1, \dots)$
Decisions	
a_s	Order quantity in period s
y_s	Inventory at period s after ordering but before demand: $x_s + a_s = y_s$

Consider a periodic inventory system over a finite planning horizon $s = 1, \dots, T$ with the objective of minimizing total expected cost. Demand is *iid* and there is no lead time. There are no fixed ordering costs, and the relevant costs are unit ordering, holding, and lost-sales or backlogging costs. In a blood bank, these costs represent the equivalent recruitment/processing cost (c), storage cost (h), and health/quality of life/expedited transfer costs (p). In each period, there is incoming capacity of u_s new donors. Capacity regenerates, that is, donors who donate return after a regeneration time k , $0 \leq k < \infty$, $k \in \mathbb{Z}^+$ and donor who do not donate are carried over to the next period. Similar to Benjaafar et al. (2017), we assume that both demand and capacity are continuous and that there is a one-to-one correspondence between donors and the amount of blood donated and demanded. We recognize that blood donors are discrete units, but using the continuous case allows for easier illustration of the proofs. The theorems of discrete convexity (including $L^\natural$ -convexity) which are used extensively in this chapter still apply in the continuous case (see Zipkin, 2008 for details). Further, the reverse can be used in numerical calculations when $L^\natural$ convex functions are restricted to integer solutions since continuous $L^\natural$ convex functions when restricted to an integer domain remain $L^\natural$ convex.

The sequence of events are as follows: at the beginning of each period s , the on-hand inventory as well as $\mathbf{q}_s$ are observed. An order $0 \leq a_s \leq u_s$ is then placed and arrives instantaneously during the same period. The states are updated accordingly depending on a_s , demand arrives, and costs are calculated.

We consider the case where unsatisfied demand is lost, although the basic structural result can apply to the backlogging case. From the perspective of blood-banking, backlogging represents an external blood bank that supplies blood to hospitals. In this case, unsatisfied demand can usually be addressed in a later period because hospitals order to maintain their own inventories. It does not necessarily mean that they are already unable to satisfy blood requirements. On the otherhand, the lost-sales model represents a hospital blood-bank that gathers and uses its own blood inventory. The demand it cannot satisfy from its own blood collection efforts must be obtained from other sources, usually an external blood-bank or other hospitals.

3.2.2 Regenerative Capacity

We briefly discussed the dynamics of the regeneration process in Section 3.1.1 and discuss the formal definition of regeneration in this section. Regeneration is a property of capacity that allows utilized capacity to come back and be utilized again after a set amount of time k . Let $\underline{u}_s$ be the capacity that gets actualized in period s . It is the physical manifestation of capacity that restricts the increase of inventory to an upper bound and we call this *realized capacity* such that

$$\underline{u}_s = u_s + \underline{u}_{s-1} - a_{s-1} + a_{s-k-1} \quad (3.1)$$

$$\begin{aligned} &= u_s + \sum_{i=1}^{s-1} (u_i - a_i) + \sum_{i=1}^{s-k-1} a_i \\ &= \sum_{i=1}^s u_i - \sum_{i=s-k}^{s-1} a_i \end{aligned} \quad (3.2)$$

From (3.1), realized capacity is a sum of three components: (1) the incoming capacity (u_s), (2) the unutilized capacity in period $s-1$ ($\underline{u}_{s-1} - a_{s-1}$), and (3) the order from period $s-k-1$ (a_{s-k-1}). This captures the three important properties of regeneration; namely, that new capacity can enter the system, that unused capacity is carried over to the next period, and that capacity regenerates after $k+1$ periods. Upon simplifying, we get the result in (3.2) which shows that the realized capacity in s is dependent on the ordering decisions made in periods $s-k$ to $s-1$. This is intuitive because if regeneration time has a length of k , then the past k periods' orders are unavailable since they are still in the regeneration process. Figure 3.1 shows the dynamics of realized capacity and how utilized and unutilized capacity progresses in a planning horizon as described in (3.1).

FIGURE 3.1: Dynamics of Realized, Utilized, and Unutilized Capacity

FIGURE 3.2: Regeneration Schedule Starting Period s

Let $\mathbf{q}_s = (q_{s,1}, q_{s,2}, \dots, q_{s,k})$ where $q_{s,i}$ represents the capacity that will regenerate in $k+1-i$ periods (i.e. it will regenerate in period $s+k+1-i$). If $s-i \leq 0$, then $q_{s,i} = 0$. We call $\mathbf{q}_s$ the *regeneration schedule* because it contains information on when and how many donors will regenerate. We note that $q_{s,i} = a_{s-i}$ meaning that the i -th component of $\mathbf{q}_s$ is the order quantity in period $s-i$. This is consistent with the definition of the regeneration schedule because the order quantity a_{s-i} in period $s-i$ will regenerate in period $s+k+1-i$. Figure 3.2 shows an illustration of the regeneration schedule starting with an order in period $s-k$. We point out the change in realized capacity from period to period depending on the regeneration schedule. Observe that the reduction in the sum of incoming capacity by an order in s , a_s gets removed from the computation of realized capacity in period $s+k+1$.

We discuss some of the special cases found in Table 3.1. The first would be when $k = 0$ with non-decreasing capacity (including fixed capacity inventory systems). Since there is instantaneous regeneration, we know with certainty through (3.2) that any future realized capacity will be the sum of all previous incoming capacities. This is because utilized capacity in the current period will regenerate in the next period and what is not utilized will be carried over in the next period as well. In this case, the realized capacity is known with complete certainty throughout the planning horizon, and the capacities are independent of any decisions made.

Another special case is when $k = T - 1$ with non-decreasing capacity. Here, utilized capacity does not regenerate during the planning horizon, but unutilized capacity still gets carried over into the next period. In this case, while realized capacity is not known per period and is dependent on ordering and non-ordering decisions, we do know the total capacity throughout the planning horizon.

3.2.3 Dynamic Programming Formulation

Regenerative capacity inventory problems can be formulated as a stochastic dynamic program with the objective of minimizing expected cost. We begin our analysis by introducing the dynamic programming formulation for the lost-sales case:

$$\begin{aligned}
 V_s(x_s, \mathbf{q}_s) &= \min_{0 \leq a_s \leq \sum_{i=1}^s u_i - \sum_{i=1}^k q_{s,i}} \left\{ ca_s + G(x_s + a_s) + \right. \\
 &\quad \left. \alpha \mathbb{E} V_{s+1}[(x_s + a_s - D)^+, (a_s, q_{s,1}, q_{s,2}, \dots, q_{s,k-1})] \right\} \\
 &= \min_{0 \leq a_s \leq \sum_{i=1}^s u_i - \sum_{i=1}^k q_{s,i}} \left\{ ca_s + G(x_s + a_s) \right. \\
 &\quad \left. + \alpha \mathbb{E} V_{s+1}[(x_s + a_s - D)^+, \mathbf{q}_{s+1}] \right\} \tag{3.3}
 \end{aligned}$$

and the backlogging case:

$$\begin{aligned}
 V_s^B(x_s, \mathbf{q}_s) &= \min_{0 \leq a_s \leq \sum_{i=1}^s u_i - \sum_{i=1}^k q_{s,i}} \left\{ ca_s + G^B(x_s + a_s) + \right. \\
 &\quad \left. \alpha \mathbb{E} V_{s+1}[x_s + a_s - D, (a_s, q_{s,1}, q_{s,2}, \dots, q_{s,k-1})] \right\} \\
 &= \min_{0 \leq a_s \leq \sum_{i=1}^s u_i - \sum_{i=1}^k q_{s,i}} \left\{ ca_s + G^B(x_s + a_s) \right. \\
 &\quad \left. + \alpha \mathbb{E} V_{s+1}[x_s + a_s - D, \mathbf{q}_{s+1}] \right\} \tag{3.4}
 \end{aligned}$$

where ca_s is the ordering cost, $G(y) = h\mathbb{E}(y - D)^+ + p\mathbb{E}(D - y)^+$ is the lost-sales and holding cost function, $G^B(y) = h\mathbb{E}(y - D)^+ + b\mathbb{E}(D - y)^+$ is the backlogging and holding cost function, and $V_{T+1} = 0$. We assume that demand is *iid*. The difference between (3.3) and (3.4) is the typical formulation for lost-sales and backlog in the transition from period s to $s+1$. The second equality in both cases above is because of the definition of $\mathbf{q}_{s+1}$ such that the transition of $\mathbf{q}_s$ to $\mathbf{q}_{s+1}$ captures capacity regeneration. Going into $s+1$, the final component of $\mathbf{q}_s$ gets removed, indicating

that $q_{s,k}$ capacity units have regenerated and are ready to be utilized in $s+1$. The remaining components move one position further down the vector with $q_{s+1,i} = q_{s,i-1}$ for $i = 2, \dots, k$. This represents the progression of the regeneration schedule by one period (as seen in Figure 3.2). Lastly, a_s becomes the first component of $\mathbf{q}_{s+1}$ to represent the inclusion of utilized capacity in s in the regeneration schedule of the next period. The capacity constraint is obtained from (3.2).

In what follows, we first provide the optimal policy for the lost-sales case when $u_s \geq 0$ for $s = 1, \dots, T$. This assumption means that there is a non-decreasing donor pool throughout the planning horizon. We then provide some results when removing the restriction on incoming capacity and allowing it to be negative, as well as examining the case where there is perishable inventory. We use the superscript B when referring to functions, variables, and parameters that are for the backlogging model. We look specifically at the lost-sales case while noting that some results are applicable to the backlogging case as well. When it is applicable, we will explicitly say that it is so. Otherwise, the results are only for the lost-sales setting.

3.2.4 Non-negative Incoming Capacity, $u_i \geq 0$

Recall our initial assumption that there is full information on u_s for all s . To obtain the optimal policy, we will do a transformation of the DP formulation in (3.3). This involves redefining the state variables which are necessary to show $L^{\natural}$ -convexity – a generalization of convexity on a lattice. Let $A \subseteq \mathbb{R}^n$ be a polyhedron that forms a lattice. By definition, a function $f : A \mapsto \mathbb{R}$ is called $L^{\natural}$ -convex if $\psi(\mathbf{a}, \xi) = f(\mathbf{a} - \xi \mathbf{e})$, $\xi \leq 0$, is submodular on $\{(\mathbf{a}, \xi) | \mathbf{a} - \xi \mathbf{e} \in A\}$. This is such that $L^{\natural}$ -convexity also implies both submodularity and convexity in each of the state vector components. $L^{\natural}$ -convexity sees many applications in inventory theory (see Zipkin, 2008; Murota, 2003; Huh and Janakiraman, 2010; Gong and Chao, 2013) and we show that it is also a very useful property when dealing with regenerative capacity. Define:

$$x_{s,i} = x_s - \sum_{j=1}^i q_{s,j} + \sum_{j=1}^{s+k-i} u_j \quad (3.5)$$

for $i = 0, 1, 2, \dots, k$ such that

$$x_{s,0} = x_s + \sum_{j=1}^{s+k} u_j \quad (3.6)$$

This state transformation, $x_{s,i}$, represents what we call the *maximum available resource* in period $s+k-i$ which is the maximum amount of demand that can be satisfied until $s+k+i$. The maximum available resource also shows that resources remain within the system but take on two types: (1) usable, demand-fulfilling resources (i.e. inventory x_s and available capacity $\sum_{j=1}^{s+k-i} u_j - \sum_{j=1}^i q_{s,j}$), and (2) unusable, regenerating resources (i.e. regeneration vector components $q_{s,j}$) which would later be converted to the first type.

This is one of the key points in our analysis of regenerative capacity – that we look at capacity as a resource that is yet to be converted to usable inventory. This is in contrast to past analysis of capacitated systems which look at capacity as an upper bound of production or of storage. An alternative understanding of regenerative capacity problems is that it is a problem that deals with managing resources in its entirety including those that are regenerating.

For example, in the case of blood donors, the maximum available resource consists of the blood already collected and in inventory, the donors who are already part of the donor pool, and future incoming donors. Here, blood that is not yet donated by donors is a resource that can be converted to blood inventory. This means that as of period s , we can project the maximum amount of blood that the system can have which is equal to what is in inventory as usable blood, and what is still with current and future donors as unextracted blood.

Next, let $\mathbf{x}_s = (x_{s,1}, x_{s,2}, \dots, x_{s,k})$. With the definition in (3.6), $\mathbf{x}_1 = (x_{s,0}, x_{s,0}, \dots, x_{s,0})$. We highlight that

$$\begin{aligned} \mathbf{x}_s &= (x_{s,1}, x_{s,2}, \dots, x_{s,k}) \\ &= (x_{s,0} - q_{s,1}, x_{s,0} - q_{s,1} - q_{s,2}, \dots, x_{s,0} - q_{s,1} - q_{s,2} - \dots - q_{s,k}) \end{aligned} \quad (3.7)$$

is a vector in which component i is the sum of components 1 to $i+1$ of $(x_s, -\mathbf{q}_s)$ and the constant $\sum_{i=1}^{s+k-i} u_i$. This means that we can change (3.3) into:

$$\begin{aligned} \bar{V}_s(x_{s,0}, \mathbf{x}_s) &= \min_{y_s \in \mathcal{A}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) \\ &\quad - \min(y_s, D)\mathbf{e}] \} \\ &= \min_{y_s \in \mathcal{A}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} \bar{V}_{s+1}[x_{s+1,0}, \mathbf{x}_{s+1}] \} \end{aligned} \quad (3.8)$$

where $\bar{G}(y_s) = (c - p)y_s + (h + p - \alpha c)\mathbb{E}(y_s - D)^+ + \mu p$, $\mu = \mathbb{E}[D]$, $\bar{V}_{T+1} = 0$, and

$$\mathcal{A}_s = \{y : x_{s,0} - \sum_{i=1}^{s+k} u_i \leq y \leq x_{s,k}\} \quad (3.9)$$

The lower bound of $\mathcal{A}_s$ is the on-hand inventory, while the upper bound is the maximum available resource (i.e. on-hand inventory and unconverted, non-regenerating, and arrived resources). Note that $y_s - \min(y_s, D) = (y_s - D)^+$. This transformation can also be done in the backlogging case by changing the state-transition to subtract only D instead of $\min(y_s, D)$. Figure 3.3 illustrates that the state transition in the transformed DP preserves the regeneration schedule. First, the difference between $y_s + \sum_{i=1}^{s+k} u_i$ and $x_{s,0}$ is $a_s + u_{s+k+1}$ which maintains the definition of $\mathbf{x}_{s+1}$. Removing $x_{s,k}$ through the transition means that the only instance of a_{s-k} appearing in any of the components get removed from the vector resulting in the regeneration of a_{s-k} . Moving each component one step further down the vector advances the regeneration schedule by one period. Lastly, subtracting the same quantity to each component $\min(y_s, D)$ maintains the regeneration schedule. We conclude that the original DP in (3.3) and the DP in (3.8) are equivalent because one is just a transformation of the other. We can therefore analyze regenerative capacity problems by looking at the projected resource pool rather than the on-hand inventory. The implication is that in managing regenerative capacity inventory systems, it is not enough to only consider the on-hand inventory, but we must include both the incoming capacity and the regenerating capacity as sources of this inventory.

In natural resource management (i.e. forestry and fisheries), regenerative capacity says that there must be an awareness of the total amount of resources available as well as those that are still recovering (i.e. regenerating) from logging and fishing activities. This indicates the importance of regenerative capacity in sustainability and environmental efforts. This is also true from a blood bank's perspective, such that both the blood inventory on-hand and the number of donors have to be taken into consideration when making inventory decisions. This is in contrast to the status quo where blood banks primarily look only at the total inventory on-hand while not considering regenerating donors.

If we let $\bar{H}_s(y_s, x_{s,0}, \mathbf{x}_s) = \bar{G}(y_s) + \alpha \mathbb{E} \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - D]$, then we can also write $\bar{V}_s(x_{s,0}, \mathbf{x}_s) = \min_{y_s \in \mathcal{A}_s} \bar{H}_s(y_s, x_{s,0}, \mathbf{x}_s)$. We can do a

FIGURE 3.3: Regeneration Schedule Starting Period s with state transformation $\mathbf{x}_s$

similar transformation for the backlogging case resulting in functions $\bar{H}_s^B(y_s, x_{s,0}, \mathbf{x}_s)$ and $\bar{V}_s^B(x_{s,0}, \mathbf{x}_s)$.

Lemma 3.1. $\bar{H}_s(y_s, x_{s,0}, \mathbf{x}_s)$ and $\bar{H}_s^B(y_s, x_{s,0}, \mathbf{x}_s)$ are $L^{\natural}$ -convex in $(y_s, x_{s,0}, \mathbf{x}_s)$ and $\bar{V}_s(x_{s,0}, \mathbf{x}_s)$ and $\bar{V}_s^B(x_{s,0}, \mathbf{x}_s)$ are $L^{\natural}$ -convex in $(x_{s,0}, \mathbf{x}_s)$.

Since $\bar{V}_s$ is $L^{\natural}$ -convex, then the optimal policy is a threshold policy that is dependent on $(x_{s,0}, \mathbf{x}_s)$ and equivalently $(x_s, \mathbf{q}_s)$ if we translate the states into those of the original DP. Let $y_s^L(x_{s,0}, \mathbf{x}_s) = \arg \min_{y_s \in \mathcal{A}_s} \bar{H}_s(y_s, x_{s,0}, \mathbf{x}_s)$ and the unconstrained minimizer $y_s^*(x_{s,0}, \mathbf{x}_s) = \arg \min_{y_s} \bar{H}_s(y_s, x_{s,0}, \mathbf{x}_s)$. We then have the following theorem:

Theorem 3.2. (a) The optimal ordering policy in the backlogging and lost-sales cases is a state-dependent threshold policy defined by $y_s^*(x_{s,0}, \mathbf{x}_s)$ such that

$$y_s^L(x_{s,0}, \mathbf{x}_s) = \begin{cases} x_{s,0} - \sum_{i=1}^{s+k} u_i & \text{if } x_{s,0} - \sum_{i=1}^{s+k} u_i > y_s^*(x_{s,0}, \mathbf{x}_s) \\ y_s^*(x_{s,0}, \mathbf{x}_s) & \text{if } x_{s,0} - \sum_{i=1}^{s+k} u_i \leq y_s^*(x_{s,0}, \mathbf{x}_s) \leq x_{s,k} \\ x_{s,k} u_i & \text{if } x_{s,k} < y_s^*(x_{s,0}, \mathbf{x}_s) \end{cases}$$

(b) $y^*(x_{s,0}, \mathbf{x}_s)$ is non-decreasing in $(x_{s,0}, \mathbf{x}_s)$ and

$$y^*((x_{s,0}, \mathbf{x}_s) + \omega \mathbf{e}) \leq y^*(x_{s,0}, \mathbf{x}_s) + \omega$$

(c) when $\mathbf{x}_s$ is transformed to the original $\mathbf{q}_s$ states, we have the following series of inequalities

$$-1 \leq \frac{\partial y_s^*(x_s, \mathbf{q}_s)}{\partial q_{s,1}} \leq \frac{\partial y_s^*(x_s, \mathbf{q}_s)}{\partial q_{s,2}} \leq \dots \leq \frac{\partial y_s^*(x_s, \mathbf{q}_s)}{\partial q_{s,k}} \leq 0 \leq \frac{\partial y_s^*(x_s, \mathbf{q}_s)}{\partial x_s} \leq 1$$

(d) if $\mathbf{u}$ changes and $u_i \geq 0$, then

$$0 \leq \frac{\partial y_s^*(\mathbf{u})}{\partial u_i} \leq 1$$

for $i = s, s+1, \dots, T$

Theorem 3.2 (b), (c), and (d) all point to the bounded sensitivity of the ordering threshold with respect to the regenerating capacity and the incoming capacity. (b) and (d) together say that an increase in new donors would increase the order quantity, indicating less conservatism in managing donors if there is a large donor pool. (c) and (d) together say that if there are more donors in the regeneration schedule, then the order quantity of blood decreases as a way to conserve available donors while waiting for the regeneration process to finish. This conservatism is higher if the time left before finishing regeneration for each donor is longer. The more donors that are regenerating, the less available donors there are, which leads to a *resource-conserving mindset* in managing blood inventory.

This threshold policy is similar to that of the results obtained by Abouee-Mehrizi et al. (2019) in their study of blood allocation with multiple priority classes and perishability. The model does not have regeneration nor capacity, but the threshold in their case depends on the amount of on-hand blood inventory at each remaining shelf life. We can infer from their results and Theorem 3.2 that the pipeline of resources in the upstream (i.e. donors) and downstream (i.e. blood) portions of a blood supply chain influence the optimal order quantity of blood. A good avenue for further study would therefore be to link the bounded and monotone sensitivity results for both perishability and regeneration to see which has a larger impact on cost.

Figures 3.4a and 3.4b illustrate the results of Theorem 3.2 (a), (b), and (c). Notice that the threshold is dependent on $(x_{s,0}, \mathbf{x}_s)$ and that it is increasing in $x_{s,0}$ and decreasing with $q_{s,i}$. Further, the threshold is more sensitive to $q_{s,1}$ than to $q_{s,2}$. Figures 3.5a and 3.5b on the other hand illustrate the results in Theorem 3.2 (d), which are the order-up-to levels for different values of k at varying u_1 and u_2 respectively.

FIGURE 3.4: Optimal Order Up-to Level y_s^L for varying $x_{1,0}$ and $q_{1,i}$ (Demand is Uniform on $[1, 10]$, $T = 5$, $\alpha = 1$, $p = 9$, $c = 1$, $h = 3$, $k = 2$, $u_1 = 10$)

FIGURE 3.5: $y_s^L(x_{s,0}, \mathbf{x}_s)$ Comparison of different k for varying u_i . (Demand is Uniform on $[1, 10]$, $T = 5$, $\alpha = 1$, $p = 10$, $c = 3$, $h = 1$)

We next examine the effect of changing the regeneration schedule as well as the length of the regeneration time on total expected cost. The results are summarized in the following Proposition.

Proposition 1. (a) The following inequalities are true for the cost function $\bar{V}_s$

$$c - p \leq \sum_{i=0}^k \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,i}}, \quad (3.10)$$

$$c - p \leq \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,0}}, \quad (3.11)$$

and

$$c - p \leq \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,i}} \leq 0 \quad (3.12)$$

for $i = 1, 2, \dots, k$.

(b) $\bar{V}_s$ is non-decreasing in k .

Proposition 1 (a) says that an increase in the on-hand inventory has a maximum cost reduction of $c - p$, while having more capacity coming off regeneration will only be beneficial to the system. Figure 3.6 graphically shows property (b) of Proposition 1. The longer the regeneration time, the more difficult it will be to satisfy demand because capacity will be in an unusable state for a longer period, and so costs will be higher. However, this can be off-set by an increase in incoming capacity such that with high enough incoming capacity, a system with long regeneration times can have the same cost as that of a system with instantaneous regeneration (i.e. $k = 0$).

FIGURE 3.6: Cost Comparison of different k for varying u_1 . (Demand is Uniform on $[1, 10]$, $T = 5$, $\alpha = 1$, $p = 10$, $c = 3$, $h = 1$)

Before we proceed with a more in-depth analysis of the optimal policy for the general lost-sales case, we first look at the special case of Benjaafar et al. (2017)

but extend it such that $u_i \geq 0$. In their study, $u_1 > 0$ and $u_i = 0$ for $i = 2, \dots, T$. This means that the total capacity that can be utilized is available in period 1 and there will be no additional incoming capacity in future periods. Extending this to $u_i \geq 0$ expands the context of Benjaafar et al.'s work to cover the case where allowed carbon utilization is increased progressively in tranches rather than a lump-sum amount at the beginning. In their work, the problem is analyzed through a relaxation that removes the on-hand inventory as a state in the function which enables ordering below the on-hand inventory level. The relaxation is valid because the on-hand inventory component does not transition to become the component that defines the upper bound of the action set in period $s + k + 1$. That is, $k = T - 1$ and therefore utilized capacity will not regenerate within the planning horizon. This relaxation is what will be utilized in the proof for Theorem 3.3, where we similarly relax the lower bound of the action set $\mathcal{A}_s$ to 0, such that the new relaxed action set is $\hat{\mathcal{A}}_s = \{y : 0 \leq y \leq x_{s,k}\}$. Note that k can be greater than or equal to T in this case, but it can be reduced to $k = T - 1$ since both will have the same feature of utilized capacity not regenerating within the planning horizon. We then have the following result on the optimal threshold:

Theorem 3.3. *Let $y^L = F^{-1}\left(\frac{p-c}{h+p-ac}\right)$ be the myopic order quantity. Given a lost-sales inventory problem with planning horizon T , regeneration period $k = T - 1$, and incoming capacity $u_i \geq 0$ for $i = 1, \dots, T$, the optimal threshold has the following properties for $s = 1, \dots, T$:*

(a) *if $x_{s,k-i} \geq (i + 1)y^L$ for $i = 0, \dots, T - s$, then $y_s^L(x_{s,0}, \mathbf{x}_s) = y_s^*(x_{s,0}, \mathbf{x}_s) = y^L$.*

(b) *else,*

$$\frac{x_{s,k}}{n} \leq y_s^*(x_{s,0}, \mathbf{x}_s) \leq y^L$$

where

$$n = \begin{cases} \bar{t} - s + 1 & \text{if } \bar{t} \exists \\ k - s + 2 & \text{otherwise} \end{cases}$$

and

$$\bar{t} = \max\{t \geq s \mid x_{s,s-t+k} < (t - s + 1)y^L \text{ and } x_{s,s-t+k-1} \geq (t - s + 2)y^L\}$$

Theorem 3.3 (a) shows that if the maximum available resources are high enough in all periods starting from s , then the optimal policy is the myopic policy. (b) on the otherhand says that if the maximum available resources is not enough to merit the use of the myopic policy in each period, then we must ration resources. The amount that will be rationed is dependent on the maximum available resources until the period ($\bar{t}$) that will be enough to use the myopic policy for the rest of the planning horizon. We can see that the myopic policy level becomes an important anchor in describing the strategies that are employed by firms when managing regenerative capacity systems. In this case, there is an emphasis on the influence of resource quantity on how complex its management would be. Having more of a resource results in a simpler, straightforward myopic policy while less of a resource requires some form of conservation and rationing below the myopic policy.

Going back to the general regenerative capacity case, $x_{s,0}$ cannot be dropped when $k < T - 1$ because the sequence of states is necessary to capture regeneration. This means that the relaxation done for Theorem 3.3 cannot be used on its own. To address this, we create a revised version of the general problem as follows:

$$\begin{aligned} \hat{V}_s(x_{s,0}, \mathbf{x}_s) = & \min_{y_s \in \hat{\mathcal{A}}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} \hat{V}_{s+1} [(\max(y_s, x_{s,0} - \sum_{i=1}^{s+k} u_i) + \sum_{i=1}^{s+k+1} u_i, \\ & x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(\max(y_s, x_{s,0} - \sum_{i=1}^{s+k} u_i), D) \mathbf{e}] \} \end{aligned} \quad (3.13)$$

with the revised versions of the functions and thresholds denoted with over-hat notation. We change the state transition such that the first component in $\hat{V}_{s+1}$ becomes $\max(y_s, x_{s,0} - \sum_{i=1}^{s+k} u_i) + \sum_{i=1}^{s+k+1} u_i$ instead of $y_s + \sum_{i=1}^{s+k+1} u_i$. The term that gets subtracted to each component also changes from $\min(y_s, D)$ to $\min(\max(y_s, x_{s,0} - \sum_{i=1}^{s+k} u_i), D)$. This change in the state transition solves the problem of dropping on-hand inventory. If y_s is less than or equal to the on-hand inventory, then $\max(y_s, x_{s,0} - \sum_{i=1}^{s+k} u_i) = x_{s,0} - \sum_{i=1}^{s+k} u_i$ and no capacity was utilized. This state transition preserves the regeneration schedule even with the relaxation of the order level's lower bound.

The relaxed version is also L^1 -convex as summarized by the lemma below. This is then followed by a theorem on additional properties of the optimal threshold for the relaxed problem for general k .

Lemma 3.4. *The functions associated with the relaxed problem: $\hat{H}_s(y_s, x_{s,0}, \mathbf{x}_s)$ is L^1 -convex in $(y_s, x_{s,0}, \mathbf{x}_s)$ and $\hat{V}_s(x_{s,0}, \mathbf{x}_s)$ is L^1 -convex in $(x_{s,0}, \mathbf{x}_s)$.*

Theorem 3.5. *The optimal threshold $y_s^*(x_{s,0}, \mathbf{x}_s) = \hat{y}_s^*(x_{s,0}, \mathbf{x}_s)$ for periods $s = 1, \dots, T$ in the regenerative capacity problem with lost-sales, regeneration time k , and incoming capacity $u_i \geq 0$ for $i = 1, \dots, T$. It also has the following property:*

$$\min(\bar{y}^1, \bar{y}^2) \leq y_s^*(x_{s,0}, \mathbf{x}_s) \leq \bar{y}^3$$

where $\bar{y}^1 = \Phi^{-1}\left(\frac{(1-\alpha)(p-c)}{(1-\alpha)(h+p-\alpha c)+\alpha^2(p-c)}\right)$, $\bar{y}^2 = \Phi^{-1}\left(\frac{p-c}{2(h+p-\alpha c)}\right)$, and $\bar{y}^3 = \Phi^{-1}\left(\frac{(1+\alpha)(p-c)}{h+p-\alpha c}\right)$

Theorem 3.5 says that the optimal threshold is lower-bounded, indicating that there is still rationing of resources that happens even with regeneration within a planning horizon. However, the threshold is no longer dependent on the number of periods remaining in the planning horizon. There is also an upperbound indicating that there is merit to building up a pipeline of resources in the future (this case will be seen further when we remove the restrictions on u_s). A simple corollary to this is similar to that of Theorem 3.3 (a), which is that if there is enough resources to use the myopic policy in each period, then the myopic policy is optimal. Figure 3.7 shows examples of the bounds on the optimal order up to level for different α values. We also specify the areas where ordering is done for more available resources in the future and where rationing takes place. Notice that the interval between the lower bound and upper bound gets smaller as α decreases. If $\alpha = 0$, as in typical capacitated inventory models, the problem reduces to a series of newsvendor models which is also confirmed by the convergence of the bounds to y^L .

3.3 Impact of Regenerative Capacity

In this section, we discuss the impact of regenerative capacity on cost through a numerical study. From Proposition 1 (b), we know that longer regeneration periods result in higher cost. This implicitly says that regulations that increase regeneration time for donors result in higher cost to blood banks assuming the increase allows for full/normal utilization of resources. Blood banks would have less donors during a planning horizon because of the additional time needed for them to regenerate which leads to higher shortage costs. Given this realization (and assumption),

FIGURE 3.7: Sample Bounds and Optimal Order Up-To Level y_s^L given x_1 for various α values (Demand is Uniform on $[1, 10]$, $T = 5, p = 9, c = 6, h = 3, u_1 = 5$)

a cost-minimizing blood bank benefits from lower regeneration and would prefer it to be as low as possible (i.e. $k = 0$) so that it would have the maximum donors to satisfy demand and avoid shortage. However, reducing regeneration time clearly has negative externalities such as risks related to iron deficiency and anemia. Conversely, increasing regeneration can make donors recover properly, protecting their health. The goal of the numerical study is to examine the extent of the decrease in cost if regeneration is decreased to $\tilde{k} < k$, for example if regulations are relaxed in order to increase blood supply. Though the externalities are not captured in the model, the relative cost of lower regeneration compared to increased regeneration serves as a trade-off benchmark for blood banks and policy makers when deciding on what regeneration period should be followed.

We define $\frac{h}{p}$ and $\frac{c}{p}$ as the holding-shortage and ordering-shortage cost ratios. The analysis focuses on the magnitude of the cost reduction in relation to these cost ratios.

Table 3.3 shows the financial trade-off of reducing regeneration from $k = 3$ to $\tilde{k} = 0, 1$, and 2 for varying cost ratios. We include the case where there is incoming capacity in the middle of the planning horizon, $u_{\bar{T}} > 0$ where $\bar{T} = \frac{T-1}{2} + 1$ to show the effect of incoming capacity. We show representative results for $T = 15$ and Uniform demand on $[1, 8]$.

TABLE 3.3: Percent Cost Decrease from $k = 3$ to $\check{k}$ for $T = 15$, Demand is Uniform on $[1, 8]$

p	$(\frac{c}{p}, c)$	$(\frac{h}{p}, h)$	$u_{\bar{T}} = 0$									$u_{\bar{T}} = 0.3$								
			$\check{k} = 0$	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
			$u_1 = 0.8y^L$			$1.2y^L$			$1.5y^L$			$0.8y^L$			$1.2y^L$			$1.5y^L$		
6	(0.17, 1)	(0.17, 1)	60	27	8	56	42	14	50	45	20	58	32	8	52	42	16	46	42	21
		(0.5, 3)	45	20	6	40	30	11	33	30	14	45	22	6	37	30	12	30	28	14
		(0.83, 5)	38	19	6	32	24	9	29	25	11	36	20	6	30	24	10	26	23	11
	(0.5, 3)	(0.17, 1)	29	14	4	24	19	7	19	17	8	27	15	4	21	17	7	16	15	8
		(0.5, 3)	21	10	3	17	13	5	15	13	5	20	11	3	16	12	5	13	12	5
		(0.83, 5)	17	8	3	16	10	3	12	10	5	17	9	3	15	10	3	10	9	4
	(0.83, 5)	(0.17, 1)	7	3	1	5	4	1	4	4	2	6	3	1	5	4	1	4	3	2
		(0.5, 3)	5	2	1	4	2	1	3	3	1	5	2	1	4	2	1	3	3	1
		(0.83, 5)	3	1	0	4	2	1	3	2	1	3	1	0	4	2	1	3	2	1
9	(0.11, 1)	(0.11, 1)	71	37	11	67	55	21	59	56	33	69	43	13	63	54	19	52	51	31
		(0.33, 3)	54	22	6	54	38	13	48	41	18	55	25	7	52	38	14	45	40	18
		(0.56, 5)	48	21	6	46	30	10	43	33	12	48	23	7	44	31	11	40	32	13
	(0.33, 3)	(0.11, 1)	46	21	6	42	31	10	36	32	13	44	24	6	38	30	11	32	30	14
		(0.33, 3)	36	15	4	32	23	8	26	23	11	36	17	5	30	23	9	24	22	11
		(0.56, 5)	31	15	5	27	19	7	24	20	8	30	16	5	25	19	7	22	19	8
	(0.56, 5)	(0.11, 1)	27	12	4	23	17	6	18	17	8	25	14	4	20	16	6	15	14	7
		(0.33, 3)	20	9	3	17	12	4	15	12	5	20	10	3	16	12	4	14	12	5
		(0.56, 5)	17	7	2	17	9	3	12	10	4	16	8	2	15	9	3	11	10	4
12	(0.08, 1)	(0.08, 1)	77	40	11	74	60	22	69	66	40	75	46	14	70	59	20	62	60	37
		(0.25, 3)	64	30	9	60	46	17	54	49	23	62	35	10	56	46	19	49	45	23
		(0.42, 5)	54	22	7	52	38	13	45	40	18	55	25	7	50	38	14	42	39	18
	(0.25, 3)	(0.08, 1)	57	30	9	51	41	15	43	41	24	55	34	10	47	40	14	35	34	20
		(0.25, 3)	47	23	7	41	32	12	35	32	15	45	26	7	37	31	13	30	29	15
		(0.42, 5)	40	17	5	36	27	10	29	26	12	40	19	5	34	26	10	27	25	12
	(0.42, 5)	(0.08, 1)	40	21	6	34	27	10	27	26	16	38	24	7	30	26	10	21	21	13
		(0.25, 3)	32	13	4	29	21	7	23	21	9	32	14	4	27	20	7	21	19	9
		(0.42, 5)	28	13	4	25	17	6	22	18	7	27	14	4	23	17	6	20	17	7

We can see from Table 3.3 that reducing regeneration can significantly reduce the blood bank's cost. However, this is especially the case when the holding-shortage and the ordering-shortage ratios are small (e.g. higher p). For example, when $\frac{c}{p} = \frac{h}{p} = 0.08$, the average cost decrease is 50%, with the highest decrease of 77% at $\check{k} = 0$, $u_1 = 0.8y^L$, and $u_{\bar{T}} = 0$. If $\frac{c}{p}$ and $\frac{h}{p}$ are high, then the cost decrease resulting from decreasing k is also low. For example, in the case of $\frac{c}{p} = 0.83$, $\frac{h}{p} = 0.83$, the cost decrease ranges from 0 – 4%. Figure 3.8 plots the reduction in cost for varying holding-shortage and ordering-shortage ratios. We can also infer from the results that a higher u_1 and $u_{\bar{T}}$ leads to a lower cost decrease from k to $\check{k}$ by comparing the cost decrease of each $\check{k}$ at every value of u_1 and $u_{\bar{T}}$. This is because a higher incoming capacity would allow the blood bank to better be able to ration across the planning horizon.

Both the results mean that the impact of regeneration on cost depends on how rare and valuable the resource is. That is, if it is imperative that demand should be satisfied (i.e. higher p , more valuable) or if the amount of resources is smaller

FIGURE 3.8: Cost Reduction from $k = 3$ to $\check{k} = 0$ for Varying Cost Ratios, $T = 5$, Demand is Uniform on $[1, 8]$, $u_1 = 8$, $p = 20$

(i.e lower u_i , rarer) then reducing regeneration has a larger reduction in cost. In the case of blood banking, p is high because of the detrimental impact on human health by not being able to satisfy demand. It is also the case that u_i is relatively low given the shortage in supply.

Lastly, we look at the case of using the myopic policy and how it compares with the optimal policy. We use the myopic policy because it is the optimal policy when the number of donors is large enough. At the same time, the myopic policy is easy to implement. Based on the results in Table 3.4, the average difference between the myopic policy and the optimal policy is 1.7% across all scenarios examined, with a maximum difference with the optimal policy of 7.6% (when $p = 6, c = 1, h = 5$).

However, we also highlight that the performance of the myopic policy again depends on the holding-shortage and ordering-shortage ratios. In Figure 3.9, the myopic policy does not perform well in cases where (1) $\frac{c}{p}$ is low and $\frac{h}{p}$ is high, (2) $\frac{c}{p}$ is low and $\frac{h}{p}$ is moderate. Conversely, in cases where the ordering cost is high, the myopic policy performs very well with less than 1% difference with the optimal.

FIGURE 3.9: Percent Cost Difference between Myopic and Optimal Policy for Varying Cost Ratios, $k = 2$, $T = 15$, Demand is Uniform on $[1, 8]$, $u_1 = 8$, $p = 20$

3.4 Extensions

3.4.1 No restrictions on u_s

In the previous sections, we looked at the case where incoming capacity $u_s \geq 0$ for all s . This is the case when total capacity is non-decreasing. This means that cumulative capacity $\sum_{i=1}^s u_i$ as can be seen in $\mathcal{A}_s$ gets larger as time passes. However, there are many instances where there is a reduction in capacity sometime during the planning horizon. For example, blood banks face problems of donor drop-offs when eligible donors don't donate anymore because of eligibility requirements or personal choices. Another example would be when sudden restrictions on previously allowable forested or fishing areas are implemented, or even when massive deforestation or a decline in fish stocks occur. Overall, we need to take into account this decline because the gathering of resources can be restricted in the future or it can be depleted or destroyed such that even though they regenerate, they can eventually be unusable.

To model this, we relax the restriction and allow for $u_s < 0$ to indicate that capacity gets removed from the system. This would still be called "incoming capacity" although the value could now be negative. We again have to adjust the DP formulation to incorporate this relaxation. The problem with the current formulation

TABLE 3.4: Percent Cost Difference of Between the Myopic and Optimal Policy
($T = 15$, Demand is Uniform on $[1, 8]$)

p	$(\frac{c}{p}, c)$	$(\frac{h}{p}, h)$	$u_{\bar{T}} = 0$									$u_{\bar{T}} = 0.3$								
			$k = 1$			2			3			1			2			3		
			$u_1 = 0.8y^L$			$1.2y^L$			$1.5y^L$			$0.8y^L$			$1.2y^L$			$1.5y^L$		
6	(0.17, 1)	(0.17, 1)	0.1	1.0	0.7	2.1	2.4	1.8	1.6	1.6	0.4	1.4	1.8	0.6	1.7	2.1	0.0	1.2	1.4	0.0
		(0.5, 3)	1.4	2.4	1.8	3.3	6.9	5.2	0.9	5.5	4.7	2.7	3.5	1.4	2.9	6.8	4.4	0.6	5.1	4.3
		(0.83, 5)	3.4	4.2	3.2	3.7	7.6	5.8	1.9	6.8	5.4	5.5	6.1	2.7	3.1	7.3	4.6	1.4	6.4	4.4
	(0.5, 3)	(0.17, 1)	0.5	1.1	0.8	0.9	2.3	1.8	0.2	1.7	1.5	1.2	1.9	0.6	0.7	2.0	1.2	0.2	1.3	0.5
		(0.5, 3)	1.7	2.3	1.7	1.7	3.8	3.0	0.8	3.3	2.8	2.7	3.2	1.4	1.4	3.6	2.4	0.6	3.0	2.2
		(0.83, 5)	1.7	2.2	1.7	3.4	3.9	3.0	0.7	3.3	2.8	2.7	3.2	1.2	2.7	3.8	2.5	0.3	2.9	2.4
	(0.83, 5)	(0.17, 1)	0.5	0.7	0.5	0.5	1.1	0.9	0.2	0.9	0.8	0.7	1.0	0.4	0.4	1.0	0.7	0.2	0.8	0.6
		(0.5, 3)	0.4	0.4	0.3	1.0	1.2	1.0	0.5	0.9	0.8	0.4	0.4	0.3	1.0	1.2	1.0	0.5	0.9	0.8
		(0.83, 5)	0.0	0.0	0.0	0.8	0.7	0.5	0.0	0.7	0.6	0.0	0.0	0.0	0.8	0.7	0.5	0.0	0.7	0.6
9	(0.11, 1)	(0.11, 1)	0.1	0.7	0.4	1.8	1.3	0.0	0.0	0.5	0.0	1.2	1.1	0.0	0.7	0.0	0.0	0.0	0.1	0.0
		(0.33, 3)	0.4	1.5	1.1	2.5	4.4	3.1	1.7	3.5	2.1	1.2	2.2	0.8	2.5	4.3	2.5	1.6	3.2	2.0
		(0.56, 5)	1.6	2.7	2.0	2.3	5.0	3.7	1.8	4.5	3.4	3.0	4.0	1.7	2.4	4.8	2.9	1.7	4.2	2.7
	(0.33, 3)	(0.11, 1)	0.1	0.6	0.4	1.3	1.1	0.9	1.3	0.7	0.7	0.7	0.9	0.3	1.1	1.0	0.5	1.1	0.0	0.0
		(0.33, 3)	0.7	1.5	1.1	1.8	4.0	3.1	0.4	3.0	2.7	1.3	2.1	0.9	1.6	3.8	2.5	0.2	2.7	2.1
		(0.56, 5)	1.7	2.6	2.0	2.0	4.5	3.5	1.0	4.0	3.2	3.0	3.7	1.6	1.7	4.3	2.7	0.7	3.7	2.6
	(0.56, 5)	(0.11, 1)	0.2	0.6	0.5	0.6	1.3	0.9	0.3	0.8	0.5	0.7	1.1	0.4	0.4	1.1	0.2	0.1	0.7	0.2
		(0.33, 3)	0.9	1.4	1.1	1.0	2.4	1.9	0.6	2.0	1.7	1.5	2.0	0.9	0.9	2.2	1.5	0.4	1.9	1.3
		(0.56, 5)	0.9	1.4	1.1	2.0	2.5	1.9	0.4	2.1	1.8	1.5	2.0	0.8	1.7	2.4	1.6	0.2	1.8	1.3
12	(0.08, 1)	(0.08, 1)	0.1	0.3	0.2	2.6	0.0	0.0	0.8	0.0	0.0	1.0	0.4	0.0	1.5	0.0	0.0	0.2	0.0	0.0
		(0.25, 3)	0.7	1.9	1.3	2.7	4.7	3.0	0.9	3.5	1.5	2.7	3.3	1.0	2.0	4.2	0.9	0.4	3.1	0.9
		(0.42, 5)	0.9	2.0	1.5	3.1	5.9	4.3	1.1	4.7	3.9	1.7	2.9	1.1	2.9	5.8	3.6	0.7	4.5	3.1
	(0.25, 3)	(0.08, 1)	0.1	0.4	0.2	1.1	0.5	0.5	0.0	0.0	0.0	0.7	0.6	0.0	0.5	0.0	0.0	0.0	0.1	0.0
		(0.25, 3)	0.8	1.8	1.3	1.8	4.3	3.2	0.5	3.3	2.6	2.3	3.2	1.0	1.4	3.9	2.1	0.3	2.7	0.9
		(0.42, 5)	1.0	1.9	1.5	2.5	5.3	4.1	0.6	4.1	3.8	1.9	2.8	1.1	2.1	5.2	3.4	0.3	3.8	3.2
	(0.42, 5)	(0.08, 1)	0.1	0.5	0.3	0.6	0.8	0.8	0.0	0.5	0.0	0.6	0.8	0.0	0.2	0.7	0.0	0.0	0.2	0.0
		(0.25, 3)	0.4	1.0	0.8	1.2	2.7	2.1	0.5	2.0	1.8	0.8	1.5	0.6	1.1	2.6	1.7	0.4	1.8	1.3
		(0.42, 5)	1.1	1.9	1.4	1.3	3.2	2.5	0.8	2.8	2.3	2.0	2.7	1.2	1.2	3.0	1.9	0.6	2.6	1.8

is that by allowing u_s to be negative, the upper bound of $\mathcal{A}_s$ can be less than its lower bound. More concretely, there may be instances wherein having a negative incoming capacity in period s decreases the maximum order up to level (i.e. the upper bound) to a value that is less than the on-hand inventory (i.e. the lower bound).

In order to address this, we make an assumption that if there is no capacity that can get removed from the system once the negative incoming capacity arrives, then capacity gets removed during the period that it becomes available. This availability comes from either regeneration or from positive incoming capacity at a later period. We model this by introducing $\ddot{V}_s$ which is the function with no restrictions on u_i ,

$$\ddot{V}_s(x_{s,0}, \mathbf{x}_s) = \min_{y_s \in \ddot{\mathcal{A}}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} \ddot{V}_{s+1}((y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(y_s, D)\mathbf{e}) \} \quad (3.14)$$

where $\ddot{\mathcal{A}}_s = \{y : x_{s,0} - \sum_{i=1}^{s+k+1} u_i \leq y \leq \max(x_{s,0} - \sum_{i=1}^{s+k+1} u_i, x_{s,k} - \sum_{i=s+1}^{s+k+1} u_i)\}$. For example, in blood banking, suppose in period s we have $u_s < 0$ meaning donors are expected to drop-off. However, suppose as well that there are not

enough non-regenerating donors that can drop-off in the same period (i.e. $x_{s,0} - \sum_{i=1}^{s+k+1} u_i > x_{s,k} - \sum_{i=s+1}^{s+k+1} u_i$). Then from $\ddot{\mathcal{A}}_s$, there will be no available blood donors in period s and the upper bound will be the on-hand inventory. This means that donors that can drop-off in period s will drop-off, but the total number of drop-offs will still not be equal to u_s . Eventually, this will be corrected in some period $s' > s$ where we may have $x_{s',0} - \sum_{i=1}^{s'+k+1} u_i < x_{s',k} - \sum_{i=s'+1}^{s'+k+1} u_i$. At s' , there are enough additional available donors coming from either regeneration or new donors (i.e. positive incoming capacity in period $s+1, \dots, s'$) such that the remaining drop-offs from negative incoming capacity u_s gets removed. With this set-up, the regeneration schedule gets maintained while allowing drop-offs to be incorporated.

Unfortunately, the function is no longer L^h -convex. But numerical studies show that there is monotonicity in the cost function such that cost is non-increasing in $\mathbf{x}_s$. Furthermore, we observe that we lose the L^h -convexity property for more negative values of u_i because of a breakpoint in the graph that changes the trajectory of the function. on both sides of this breakpoint, the function resembles two continuous and convex functions. Figure 3.10 shows an example of this observation. We plot the total expected cost for a range of u_2 values against $q_{1,1}$, which is the original state corresponding to $x_{1,1}$. For $k = 1$, $q_{1,1}$ is the number of donors that are going to regenerate in $s = 2$. It can also be seen that with high drop-offs (i.e. $u_2 = -4, -5$) and a low number of regenerating donors (i.e. $q_{1,1} = 0, 1, 2$), costs are lower than some of the cases where drop-offs are lower. This is in contrast to Proposition 1 of the no drop-off case which says that higher incoming capacity should lead to lower costs.

Furthermore in Figure 3.11, a larger number of donor drop-offs in future periods from s (i.e. $u_i < 0$) leads to higher order quantities beyond the myopic policy level. The intuition is that since we have knowledge that donors will drop-off from the system in a future period, then we would increase our ordering so that we would have enough blood inventory once the drop-off occurs. This behavior is in anticipation of the future drop-offs that will occur. This *anticipatory ordering mindset* decreases as the drop-offs decrease (i.e. u_i becomes larger) until we eventually reach the point where we start rationing again when incoming new donors becomes positive. We note that the decrease in the order-up-to-level does not have

FIGURE 3.10: Total Expected Cost for varying $q_{1,1}$ (Demand is Uniform on $[1, 10]$, $T = 5$, $\alpha = 1$, $p = 10$, $c = 3$, $h = 1$, $u_1 = 12$, $k = 1$. Note: $y^L = 10$)

the same bounded sensitivity property as in Theorem 3.2 (d), as can be seen at the interval $u_2 = [-4, -3]$.

Figure 3.12 shows that unlike in the case where $u_i \geq 0$ (in Figure 3.6), there is a breakpoint in the decrease in cost in the unrestricted case. The breakpoint occurs at the same u_2 interval $[-4, -3]$. This further supports the observation that some threshold to anticipatory ordering exists. Beyond which, we start to ration capacity. Both figure 3.10 and 3.12 show that the cost of switching from anticipatory ordering to rationing is high. This indicates that a change in strategy brought about by a change in donor behavior can bring in additional costs.

FIGURE 3.11: $y_s^L(x_{s,0}, \mathbf{x}_s)$ for varying unrestricted u_2 (Demand is Uniform on $[1, 10]$, $T = 5$, $\alpha = 1$, $p = 10$, $c = 3$, $h = 1$, $u_1 = 12$, $k = 1$. Note: $y^L = 10$)

FIGURE 3.12: Total Expected Cost for varying unrestricted u_2 (Demand is Uniform on $[1, 10]$, $T = 5$, $\alpha = 1$, $p = 10$, $c = 3$, $h = 1$, $u_1 = 12$, $k = 1$)

3.4.2 Alternative Formulation: Donor Drop-offs

The formulation below extends the result of L^1 -convexity to the case where there are donor drop-offs. In the previous section, we assumed that u_s can be negative, which poses some problems in terms of updating the regeneration schedule. An alternative formulation would be to think of drop-offs as a multiplication of a fraction to the incoming capacity.

Let $1 - r_s$ be the percent reduction in period s of donors who are part of the system from period 1 to $s + k$. Let $\bar{u}_s$ be the total incoming capacity net of drop-offs until period $s + k$. We then have $\bar{u}_s$ as:

$$\bar{u}_s = \sum_{i=1}^{s+k} (u_i \prod_{j=i-k}^s r_j) \quad (3.15)$$

The state transformation then becomes:

$$x_{s,i} = x_s + \sum_{j=1}^{s+k-i} (u_j \prod_{l=j-k}^s r_l) - \sum_{j=1}^i (q_{s,j} \prod_{l=1}^j r_{s-l}) \quad (3.16)$$

$$\begin{aligned} \bar{V}_s(x_{s,0}, \mathbf{x}_s, \bar{u}_s) &= \min_{y_s \in \mathcal{A}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} \bar{V}_{s+1}[r_{s+1} (y_s + \bar{u}_s + u_{s+k+1}, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) \\ &\quad + ((1 - r_{s+1})y_s - \min(y_s, D)) \mathbf{e}, r_{s+1}(\bar{u}_s + u_{s+k+1})] \} \\ &= \min_{y_s \in \mathcal{A}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} \bar{V}_{s+1}[x_{s+1,0}, \mathbf{x}_{s+1}, \bar{u}_{s+1}] \} \end{aligned} \quad (3.17)$$

where $\mathcal{A}_s = \{y : x_{s,0} - \bar{u}_s \leq y \leq x_{s,k}\}$. Notice that if there are no donor drop-offs, then $\bar{u}_s = \sum_{i=1}^{s+k} u_i$ and this formulation reverts to the original formulation.

(3.17) is $L^\natural$ -convex by seeing that the multiplication of r_s to the state vector still results in a state that can be found in the feasible states of the polyhedron.

Next, we look at extensions that are applicable in the lost-sales setting.

3.4.3 Perishable Inventory

We go back to the assumption that $u_s \geq 0$. This next extension is the case where inventory is perishable with lifetime $m = 1$. That is, on-hand inventory outdates in the same period that it is obtained if it is not used to satisfy demand. This extension is important because blood and blood products are perishable, and so analyzing this extension expands the coverage of the optimal policy. The functions associated with the perishable case are denoted by a *tilde*. The challenging part of modelling perishable inventory for regenerative capacity problems is being able

to maintain the regeneration schedule with the outdating of inventory in the state transition. However, for $m = 1$, the change in (3.8) would be to subtract y_s in the state transition of the regeneration schedule instead of $\min(y_s, D)$. Such that,

$$\tilde{V}_s(x_{s,0}, \mathbf{x}_s) = \min_{y_s \in \mathcal{A}_s} \{ \tilde{G}(y_s) + \alpha \tilde{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - y_s \mathbf{e}] \} \quad (3.18)$$

where $\tilde{G}(y_s) = (c-p)y_s + (\theta + p - \alpha c)(y_s - D)^+ + \mu p$, and θ is the per unit outdating cost. The change to subtract only y_s is because unused inventory gets outdated in the same period. With this change in the DP formulation, all of the results of the non-perishable case hold for the perishable case with $m = 1$ except that we replace h with θ .

3.4.4 Donor Recruitment and Retention/Joint Capacity and Inventory Optimization

Another extension would be the case where there is joint optimization of capacity and inventory. In this extension, we assume that $k = 1$ and $m = 1$. In each period, we must decide given the total number of donors from the previous period $\tilde{u}$ and the number of donors regenerating q_s how much capacity to recruit and retain aside from deciding the order quantity of blood. This additional decision is captured by $\bar{u}_s$, where $\bar{u}_s \geq q_{s,1} = q_s$. The condition is because we assume that capacity that is regenerating cannot be dropped. This also implies that donors can be dropped if they are available (i.e. $\bar{u}_s < \tilde{u}_s$). An additional cost c_1^r is incurred for every unit of capacity that is recruited in a period, c_2^r is incurred for every unit of capacity that is retained from the previous period (including capacity on-hand in the current period), and c^m (i.e. maintenance cost) is incurred for each unit that is regenerating. With this, we have the dynamic programming formulation:

$$\tilde{V}_s^u(\tilde{u}_s, q_s) = \min_{q_s \leq \bar{u}_s} \left\{ c_1^r(\bar{u}_s - \tilde{u}_s)^+ + c_2^r \bar{u}_s + c^m q_s + \min_{0 \leq y_s \leq \bar{u}_s - q_s} \left\{ \tilde{G}(y_s) + \alpha \tilde{V}_{s+1}^u(\bar{u}_s, y_s) \right\} \right\} \quad (3.19)$$

If we let $\bar{z}_s = \bar{u}_s - q_s$ and $\tilde{z}_s = \tilde{u}_s - q_s$, then (3.19) can be changed to:

$$\begin{aligned}\tilde{V}_s^u(\tilde{z}_s, q_s) &= \min_{0 \leq \bar{z}_s} \left\{ c_1^r(\bar{z}_s - \tilde{z}_s)^+ + c_2^r \bar{z}_s + (c_2^r + c^m)q_s + \min_{0 \leq y_s \leq \bar{z}_s} \left\{ \tilde{G}(y_s) + \alpha \tilde{V}_{s+1}^u(\bar{z}_s - y_s, y_s) \right\} \right\} \\ &= (c_2^r + c^m)q_s + \min_{0 \leq \bar{z}_s} \left\{ c_1^r(\bar{z}_s - \tilde{z}_s)^+ + c_2^r \bar{z}_s + \min_{0 \leq y_s \leq \bar{z}_s} \left\{ \tilde{G}(y_s) + \alpha \tilde{V}_{s+1}^u(\bar{z}_s - y_s, y_s) \right\} \right\}\end{aligned}\quad (3.20)$$

Notice that q_s no longer becomes part of the optimization problem. We can further simplify the function by highlighting that q_{s+1} in period $s+1$ is actually y_s in period s . This means that we can move $(c_2^r + c^m)q_{s+1}$ into period s such that

$$\tilde{V}_s^t(\tilde{z}_s) = \min_{0 \leq \bar{z}_s} \left\{ c_1^r(\bar{z}_s - \tilde{z}_s)^+ + c_2^r \bar{z}_s + \min_{0 \leq y_s \leq \bar{z}_s} \left\{ \tilde{G}^t(y_s) + \alpha \tilde{V}_{s+1}^t(\bar{z}_s - y_s) \right\} \right\} \quad (3.21)$$

where $\tilde{G}^t(y) = (\alpha(c_2^r + c^m) + c - p)y_s + (\theta + p - \alpha c)\mathbb{E}(y_s - D)^+ + \mu p$, $\mu = \mathbb{E}[D]$ and if

$$\tilde{H}_s^t(z, y) = \tilde{G}^t(y) + \alpha \tilde{V}_{s+1}^t(z - y) \quad (3.22)$$

and

$$\tilde{v}_s^t(\bar{z}_s, \tilde{z}_s) = c_1^r(\bar{z}_s - \tilde{z}_s)^+ + c_2^r \bar{z}_s + \min_{0 \leq y_s \leq \bar{z}_s} \left\{ \tilde{G}^t(y_s) + \alpha \tilde{V}_{s+1}^t(\bar{z}_s - y_s) \right\} \quad (3.23)$$

then we can rewrite (3.21) as

$$\begin{aligned}\tilde{V}_s^t(\tilde{z}_s) &= \min_{0 \leq \bar{z}_s} \left\{ c_1^r(\bar{z}_s - \tilde{z}_s)^+ + c_2^r \bar{z}_s + \min_{0 \leq y_s \leq \bar{z}_s} \left\{ \tilde{H}_s^t(\bar{z}_s, y_s) \right\} \right\} \\ &= \min_{0 \leq \bar{z}_s} \left\{ \tilde{v}_s^t(\bar{z}_s, \tilde{z}_s) \right\}\end{aligned}\quad (3.24)$$

This transformation reduces the complexity of the problem by reducing the state from a vector to a scalar, and the constraints such that each one has at most two variables. This facilitates proof of convexity as summarized in the following Lemma:

Lemma 3.6. $\tilde{H}_s^t(\bar{z}_s, y_s)$ and $\tilde{v}_s^t(\bar{z}_s, \tilde{z}_s)$ are jointly convex in $(\bar{z}_s, y_s)$ and $(\bar{z}_s, \tilde{z}_s)$ respectively, while $\tilde{V}_s^t(\tilde{z}_s)$ is convex in $\tilde{z}_s$.

This can be proven through standard backward induction along with the fact that the recruitment, retention, and maintenance costs are linear. This convexity result directly implies a threshold policy for the optimal policy:

Theorem 3.7. *The optimal inventory ordering policy y_s^* is a state-dependent threshold policy that is dependent on $\bar{z}_s$ and the optimal capacity recruitment policy $\bar{z}_s^*$ is a state-dependent threshold policy that is dependent on $\tilde{z}_s$.*

The proof follows directly from the convexity of the cost functions in Lemma 3.6. What theorem 3.7 says is that recruitment (resp., dropping) of donors into(resp., out of) the donor pool depends on the initial number of non-regenerating donors, and that the order quantity of blood is dependent on the number of donors that are available for that period.

3.4.5 Non-Stationary but Independent Demand

Note that with deterministic and positive incoming capacity, Lemma 3.1, Theorem 3.2, and Proposition 1 still apply if the demand is non-stationary but independent (i.e. D_s with cdf F_s and pdf f_s). However, the other results on the properties of the threshold do not. This indicates that the optimal policy considering seasonality of blood demand would be the same.

3.5 Conclusion

In this chapter, we introduced a novel feature found in blood donors called *regeneration* and *regenerative capacity*. This is a generalization of existing capacitated inventory systems defined by two parameters: the regeneration time k and the incoming capacity vector $\mathbf{u}$. When there is regenerative capacity, donors who have donated become available after $k + 1$ periods and donors who do not donate are carried over in the next period. We performed a state transformation that changes the states to one which considers the total projected resource. From here, we show that when there are no donor drop-offs, the optimal policy is a state-dependent threshold policy by proving L^1 -convexity and that the threshold is dependent on the total projected resource and the regenerating capacity. This threshold is more sensitive to donors that are further down the regeneration schedule. The optimal policy further has the property that if donors are not enough for the planning horizon, then either rationing or donor build-up will occur. This alludes to both a

resource-conserving, and capacity building mindset for a blood bank that's trying to satisfy demand using a small number of donors.

We believe that this study can be extended further by scholars. One of the main assumptions that we have with donor regeneration is that donor arrivals are deterministic. Future studies can focus on stochastic incoming capacity to consider the case where firms do not know with certainty how many additional units of capacity will arrive, and the case where there is advanced capacity information (ACI). In this case, incoming capacity in period s will only be known a set number periods before instead of at $s = 1$. The multi-item case may also be considered, each with different regeneration times (e.g. whole blood and apheresis donation) constituting shared regenerative capacity. Firms have the added decision of which type of product should be produced from the same resource pool while noting that the processes involved in making each product has different regeneration times. Generalizing the extension for any m other than one would also fully link regenerative capacity with existing blood inventory literature that focuses heavily on perishable inventory. Lastly, it would be interesting to see the effect of quality considerations if capacity is allowed to regenerate faster, along with pricing schemes that are affected by this quality consideration.

Chapter 4

Part 1: Blood Supply Interventions During Pandemics and Disasters

Chapters 4 and 5 are two parts of one essay, entitled *Blood Supply Interventions During Pandemics and Disasters*. We split the essay into two to highlight the methodologies and models used in each part. The first part deals with the analytical model, while the second focuses on a simulation model and a numerical study.

4.1 Introduction

The core problem that blood banks are trying to solve is matching a limited, uncertain supply of blood with uncertain demand in an efficient and ethical way. Efficiency can be achieved using tools from operations management, with extensive literature examining blood supply and blood bank operations during periods of peace. However, managing the dynamics of supply and demand becomes more challenging during pandemics and disasters. In Zimrin and Hess' (2007) paper on the impact of pandemics on blood supply, they say that there "*will be an overwhelming utilization and depletion of hospital resources*", including blood because "*mobile blood drives will be lost*" and donors "*might be less willing to sit in a waiting room full of strangers*" (Zimrin and Hess, 2007). These excerpts highlight that blood is a human product and its supply is tied to the behavior of humans such that

disruptions and population characteristics sway the supply of blood in an unpredictable way. Nonetheless, blood requirements remain constant or even increase during pandemics, putting a spotlight on the importance of blood management strategies.

The COVID-19 pandemic gives us a concrete example. It is well documented that blood donor turnout during COVID-19 varies greatly across different countries. For some, donors recognize the need to help others, hence the usual donor turnout after calls for blood during disasters is seen. In others, this increase is not seen because the pandemic itself suppresses altruism due to fear or it reduces the accessibility of blood donation facilities due to lockdown and social-distancing measures (Kam, 2020; de Angelis, 2020).

The stochastic nature of blood donation gets highlighted further during disaster and pandemic scenarios. The reality, however, is that even though blood donation is stochastic, blood banks can still influence donor arrivals through *policy interventions* or simply *interventions*. This includes activities like donor recruitment drives, or donor awareness campaigns. By implementing these interventions, donors are directly (e.g. recruited in person, saw an ad online) or indirectly (e.g. culture of donation increases in the community) influenced to donate. However, little has been studied on the efficiency and equivalence of the different interventions at a blood bank's disposal. That is, when will it be "better" to do one intervention over another? For example to recruit more donors versus to make people donate more often. Blood banks have limited man-power and monetary resources and knowing when each intervention is more efficient is critical. This is especially true during disaster situations, and in lower-income countries where the need-to-supply ratio is higher and proper blood policy quite literally saves lives. Further, the reaction of donors to disaster scenarios (i.e. to either increase or decrease donation) also affects post-disaster donations. The effect of interventions is relatively unknown in this case. Lastly, the donors' reaction to calls for blood can lead to fluctuations in supply that can lead to over or under-collection of blood. How interventions can be used to mitigate these effects is not yet understood.

In this two-part study, we answer these questions by comparing three disaster interventions, namely, interventions related to (1) recruitment and retention, (2) donation frequency, and (3) regeneration time and analyze its efficiency and equivalence to each other in augmenting blood supply during disasters. We will also

analyze the post-disaster effects of implementing each of the interventions, and what happens when there are multiple disaster waves. In chapter 5 (Part 2), we introduce three ethically complex interventions: (4) deferral of demand, (5) rationing of blood, and (6) tapping non-traditional donor pools, and show that it can be implemented through the models discussed in both parts.

In this chapter (Part 1), we conduct a comparison among the interventions through a stylized donation model that determines the equivalent increase in blood donation. We determine when it is more efficient to use one over the other through the use of an efficiency parameter. Next, we then analyze the non-steady state dynamic that characterizes disaster and post-disaster periods, and see how the interventions fare in this case. This includes analyzing the length of time it takes for the system to go back into the steady state after changes in donation rates occur during and after disasters. An iterative calculation procedure is introduced that gives a good approximation of this volatility. Then, a dynamic donation rate strategy is discussed as a possible alternative to avoid volatility in blood donation. In the next chapter (Part 2), we create a stochastic inventory control model and present a numerical study through a discrete-event simulation (DES) model to gain more insights on this strategy and its effects when there are multiple disaster waves. We also include a discussion on ethically complex interventions that can be implemented through the *DES* model. Overall, these next two chapters contribute to the blood banking and blood supply chain literature by providing a simple process for comparing the efficiency of blood supply interventions during and post-disasters. It also provides an analysis of how interventions can be used to facilitate better blood banking strategies.

The rest of Chapter 4 proceeds as follows: we finish this section with a review of related literature. Next, we present the stylized donation model in Section 4.2 and compare its efficiency to each other in Section 4.3. Lastly, we present the Disaster and Post-Disaster Effects of blood donation and the interventions in 4.4. This is then immediately followed by Part 2 in Chapter 5.

4.1.1 Related Literature

4.1.1.1 Blood Supply Management

Blood is considered a critical medical resource that is used in treatments, procedures, and emergencies. However, it is also a complex resource because of its scarcity, perishability, and compatibility and donor regeneration rules. Given its importance, OM scholars have studied its supply chain extensively, with Prastacos (1984) providing the first comprehensive assessment of the decisions made in blood supply chains. More recently, Beliën and Forcé (2012) and Pirabán et al. (2019) provided an overview of work on blood supply operations.

A large section of blood management literature focuses on the creation of inventory policies that minimize shortage and outdates. Though not exclusive to blood, Nahmias and Pierskalla (1973) were one of the first to develop inventory policies for perishable items. Following their work, the proposed policies increased in complexity from double cross-matching (Dumas and Rabinowitz, 1977), expedited and regular orders (Zhou et al, 2011), quantity and time threshold policies (Lowalekar and Ravichandran, 2017), and age-based threshold policies (Abbasi and Hosseini-fard, 2014; Sarhangian et al. 2018). More recently, the optimal inventory and issuance policy for multiple blood types was obtained by Abouee-Mehrizi et al. (2019) and the optimal policy for collection and inventory control of platelets was studied by Chen et al. (2019). Paul et al. (2018) studied socially optimal contracting between regional blood banks and hospitals in order to manage inventory in multiple sites.

Perhaps the closest to Chapter 4 is the work done by Hosseinifard et al. (2019), where they examine the volatility of blood donation during and after disasters. They provided the recommendation that targeting newer donors should be done in order to compensate for the drop in repeat donors in the post-disaster period. However, the paper assumed that the increase in donations is a multiple of the steady state number of donations, which could lead to problems when it comes to regeneration (i.e. more donors donated than there are actual available, non-regenerating donors). We address this problem by changing this assumption to one where changes in the donation rate are the reason why donations increase. This assumption also means that we can analyze lowered donation during disasters – something that Hosseinifard et al. (2019) did not cover. Next, we also provide

actionable interventions that are tied to the known parameters of the model, which makes it easier for blood banks to translate and use. To the best of our knowledge, this is the first study that explicitly looks at different types of blood banking interventions.

With this analysis, we still need to see whether the interventions and strategies are able to address blood demand. However, the complexity of blood banking systems usually do not allow for a closed-form solution in majority of cases. This makes the usage of simulation in chapter 5 an attractive and widely used methodology when studying blood banking systems even for the studies stated above. The commonly used simulation method is *Discrete-Event Simulation* (DES). For example, Katsaliaki and Brailsford (2007) used DES to simulate the entire supply chain from donor to patient and included stochastic arrivals for donors, blood type compatibility, and varying shelf lives. This approach has also been used by Rytila and Spens (2006), and Alfonso et al. (2012) with the purpose of creating hospital tools that can be used to evaluate blood replenishment policies. Baesler et al. (2014) used DES along with a Pareto-Frontier analysis to find the trade-off between reducing shortage and outdates. Fontain et al (2010) and Blake et al. (2013) used DES to assess the impact of a reduction of shelf-life on shortages. Osorio et al. (2017) created an integrated DES-optimization model for production planning of RBCs and platelets and showed that using their method improves shortage, outdates, and required donors. Haijema et al. (2007) used a combination of MDP and simulation to come up with a double-level order-up-to policy which keeps track of young and total blood inventory, leading to reduced costs associated with the production of platelets. Lastly, Duan and Liao (2013, 2014) used simulation-optimization methodologies to assess replenishment policies, including the effect of shortening shelf lives.

In this chapter and the next, we use a combination of an analytical formulation and DES. This is because of the complexity of modelling blood banks during disasters, whereas the studies listed above occur during normal times.

4.1.1.2 Disaster Management and Disruptions in Blood Banks

These next two chapters also contribute to the literature on disaster management in blood banks. There is a critical nature to blood especially during disasters because

of its immediate need in saving lives. Despite this, there are very few studies that deal with blood bank disaster management. Most of the studies look at the problem from a network perspective and are concerned with transporting and positioning blood and blood facilities (Gathreh Samani et al., 2018; Sha and Huang, 2012; Jabbardazeh et al., 2014; Fahimnia et al., 2015; Ma et al., 2019). This assumes that donors would be willing to go to these sites to donate during and after the disaster, and are only concerned about having the facility in their proximity. However, this may not be true in cases where there may be very low donor turnout. Haijema et al. (2009) is partly able to address this low donor turnout by using dynamic programming and inventory theory in reducing shortages in blood banks during disruptions. However, they looked at disruptions like holiday breaks rather than disasters. These studies also focus on short mass-casualty events (e.g. tsunamis, terrorist attacks) or short disruptions (e.g. Christmas and Easter holidays) rather than sustained disasters like pandemics, which this chapter covers.

To the best of our knowledge, there is no operations management literature covering blood banking during long, sustained disasters. Rather, literature comes mostly from medical scholars. The two closest to operations management would be that of Kamp et al. (2010) and An et al. (2011). Kamp et al. (2010) used an SIR (Susceptible-Infected-Recovered) model to simulate blood banking during pandemics while considering the demographic composition of the population of donors. They found that hospitals should come up with a prioritization scheme for blood in anticipation of the drop in supply. An et al. (2011), on the other hand, created a hospital simulation tool which considers different types of donors (no donations, regular donations, sporadic donations), disasters, and demand deferrals. In their study, they looked at one-wave influenza pandemics and its impact on supply for one blood bank. In contrast, we can analyze multiple wave disasters, different types of interventions, and non-stationary demand and donation rates while presenting a more thorough mathematical formulation.

The medical literature usually focuses on blood banking best practices and supply expectations. For example, Zimrin and Hess (2007) discussed the importance of blood supply management during pandemics and suggested better recruitment of donors and triage schemes to cushion the impact on supply. Stanworth et al. (2020) discussed blood bank and hospital best practices such as availability of personnel and the use of proper inventory management techniques as integral to COVID-19

response. Rouka (2020) wrote that specialists should prepare blood supply simulations not just for COVID-19 but also for future pandemics, and that these should incorporate social distancing measures and equipment management. While these studies provide insights based on observation, they do not provide quantitative or analytical basis for their recommendations. This chapter complements this literature by providing rigorous analytical support as well as additional operational insights.

4.2 Model Setup

We only consider repeat donors in our model, and focus on the number of donations obtained from this group of donors. As such, when we refer to donors or donor pool, we only refer to repeat donors. Focusing on repeat donors allows us to isolate the effect of interventions to those who are more likely to be influenced by blood bank's interventions. As Hosseinifard et al. (2019) pointed out, while first time donors may donate during disasters, it is unlikely that they will donate after that.

In order to gain more insight into the effectiveness of interventions during disasters, we first formulate the average donation without the distinction of whether or not this occurs during a disaster. Then, in section 4.4, we will look at the distinction between different phases of a disaster timeline.

Suppose we have a donor pool of N donors who individually have a probability of donation $p \in [0, 1]$. Once a donor donates, they need to regenerate for k periods before being available again. Although each individual donor is an independent Bernoulli trial when it comes to donating, the fact that there is regeneration implies that the expected number of donations is lower compared to when there is no regeneration. On any given period, the expected value of donations, $\mathbb{E}[A]$, is given by:

$$\mathbb{E}[A] = \frac{Np}{1 + kp} \quad (4.1)$$

This value was obtained by Hosseinifard et al. (2019) through a Markov Chain approach and we refer the reader to their paper for the details of the calculation. It is clear that (4.1) depends on three key parameters: N , p , and k . Therefore, to reach this target, blood banks change N , p , and k through *interventions* and are

called as such as these represent the effects of policy decisions (N and p) and the actual policy decisions themselves (k).

Focusing only on repeat donors means that N indicates how well the blood bank is able to make first time donors become repeat donors. This process of increasing N is more commonly known as activities that promote *donor recruitment and retention*. Next, although p can be argued to be an innate property of donors (i.e. how altruistic they are), it is also a result of a blood bank's effort to foster a culture of donation within the community. Blood banks are then able to influence how often repeat donors will donate by implementing interventions that change *donation frequency*. Lastly, the value of k is a direct result of blood donation policy on the appropriate *regeneration time* of donors, which is based on donor health and safety. In most countries, this value ranges from eight to twelve weeks for whole blood donations (i.e. 56 to 84 days). We refer to the interventions related to N , p , and k as the donor recruitment and retention ($\bar{N}$), donation frequency ($\bar{p}$), and regeneration time ($\bar{k}$) interventions, respectively. With abuse of notation, $\bar{N}$, $\bar{p}$, and $\bar{k}$ simply refer to the interventions and does not change with the value of N , p , and k .

4.2.1 Equivalence of Changing N , p , and k

Implementing interventions related to the three parameters obviously changes (4.1). However, depending on the context of the blood bank, changing one parameter over another might be easier to do. For example, it might be more feasible to increase repeat donors rather than to reduce the regeneration time because a community is vulnerable to haemoglobin related health issues. We therefore analyze the equivalence among the parameters in terms of how they change (4.1). That is, a change in one parameter is equivalent to a change in the other. In this way, if an intervention that affects one parameter is difficult to implement during disasters, then implementing interventions that affect other parameters might give the same effect but is easier to implement.

We conduct pair-wise comparisons on N , p , and k to obtain the equivalence of how these parameters affect the average number of donations. We assume when changing N that we add (resp. subtract) n new (resp. current) repeat donors. p can be changed to $p' > p$ ($p' < p$ if p is reduced) by adding (resp. subtracting) q ,

$0 \leq q \leq 1 - p$ (resp. $-p \leq q \leq 0$) such that $p' = p + q$, and k can be changed by multiplying a factor l , $0 \leq l$ where if $l > 1$, then the regeneration time increases.

Let $A(i)$, $i = \bar{N}, \bar{p}, \bar{k}$ be the arrivals when intervention i is used. We have the expected arrivals given each intervention:

$$\mathbb{E}[A(\bar{N})|n] = \frac{(N + n)p}{1 + kp} \quad (4.2)$$

$$\mathbb{E}[A(\bar{k})|l] = \frac{Np}{1 + klp} \quad (4.3)$$

$$\mathbb{E}[A(\bar{p})|q] = \frac{N(p + q)}{1 + k(p + q)} \quad (4.4)$$

4.2.1.1 Comparing $\bar{N}$ and $\bar{p}$

We begin by comparing $\mathbb{E}[A(\bar{N})|n]$ and $\mathbb{E}[A(\bar{p})|q]$. For easier calculation and interpretation later on, we will use $q = p' - p$ such that $\mathbb{E}[A(\bar{p})|q] = \frac{Np'}{1 + kp'}$. Let $n^*(p, p')$ be such that $\mathbb{E}[A(\bar{N})|n^*(p, p')] = \mathbb{E}[A(\bar{p})|q]$. $n^*(p, p')$ is the value n in intervention $\bar{N}$ that would give an equal change to arrivals when q is used in intervention $\bar{p}$.

In general, given parameters b_1 and b_2 corresponding to interventions i_1 and i_2 , respectively, $b_1^*(b_2)$ is the value of b_1 in intervention i_1 that would give an equal change to arrivals when b_2 is used in intervention i_2 .

Lemma 4.1. (a) $n^*(p', p)$ is given by the following formula:

$$\frac{N(p' - p)}{p(1 + kp')} = n^*(p, p') \quad (4.5)$$

It is easy to calculate for $q^*(n)$ (i.e. the reverse equivalence), but the analysis will be the same and will not add much to the study. As such, even when comparing the other interventions, we will choose to analyze only one equivalence.

From (4.5), we can divide both sides by N and get an intervention that is not in absolute donor number, but rather a percentage of the total population of repeat donors. This removes any population size nuances that might arise when comparing $\bar{N}$ and $\bar{p}$. We let $\tilde{n}^*(p, p')$ be the corresponding percent change in the population

to match a change in p such that

$$\frac{p' - p}{p(1 + kp')} = \frac{n^*(p, p')}{N} = \tilde{n}^*(p, p') \quad (4.6)$$

Figure 4.1 graphs $\tilde{n}^*(p, p')$ for an increase of $p' - p$ given various p . First, $\tilde{n}^*(p, p')$ is increasing in $p' - p$ and p' , but there is a decreasing marginal increase. Second, it also shows that there is a diminishing marginal decrease of the value of $\tilde{n}^*(p, p')$ in p . These results can be confirmed by taking the second derivative of $\tilde{n}^*(p, p')$.

Then, we can say that if during a disaster, a blood bank needs to increase donations through $\bar{N}$, less donors are needed to be converted to regular donors if the initial donation rate is higher. More mature blood banking systems which are characterized by a well-developed culture of donation then already have a distinct advantage when trying to address disasters.

FIGURE 4.1: Graph of $\tilde{n}^*(p, p')$ for an increase of $p' - p$ given various starting p . ($k = 84$ days)

In cases where there is a drop in donation rates (i.e. $q < 0$ and therefore $p' < p$), then we can also find the equivalent increase $\tilde{n}^{**}(p, p')$ that would retain the original $\mathbb{E}[A]$.

Lemma 4.2. $\tilde{n}^{**}(p, p')$ is given by the following formula:

$$\frac{p - p'}{p'(1 + kp)} = \tilde{n}^{**}(p, p') \quad (4.7)$$

Note that $\tilde{n}^{**}(p, p') = \tilde{n}^*(p', p)$. This is because when p decreases to p' and we try to get the equivalent percent increase in N , we can think of it as having to increase the probability of donation from p' to p . This is then obtained by calculating $\tilde{n}^*(p', p)$.

As with the other comparisons, this logic of thinking of p' as the starting point of the analysis when p decreases also applies, and so we will not illustrate it further for the others.

4.2.1.2 Comparing $\bar{p}$ and $\bar{k}$

We next compare the $\bar{p}$ and $\bar{k}$ interventions. Recall that when implementing the $\bar{k}$ intervention, blood banks choose a factor l that reduces $\bar{k}$ to a fraction lk . Let $l^*(p, p', k)$ be the equivalent value of l in the $\bar{k}$ intervention if p is changed to p' in the $\bar{p}$ intervention.

Lemma 4.3. $l^*(p, p', k)$ is given by the following formula:

$$l^*(p, p', k) = \frac{1}{kp'} - \frac{1}{kp} + 1 \quad (4.8)$$

Lemma 4.3 says that $l^*(p, p', k)$ is decreasing in p' and $p' - p$, and increasing in p and k . This again shows that more mature blood banking systems with higher donation rates don't have to implement extreme reductions in k in order to get additional donations. On the other-hand, less mature blood banks need to reduce k significantly in order to get the equivalent donations. Figure 4.2 illustrates this point.

4.2.1.3 Comparing $\bar{N}$ and $\bar{k}$

Next, we compare the $\bar{N}$ and $\bar{k}$ interventions. Let $n^*(k, p, l)$ be the equivalent change in $\bar{N}$ when l is used in $\bar{k}$. Just like in the previous equivalences for the other interventions, we have the following proposition:

FIGURE 4.2: Graph of $l^*(k, p, p')$ for an increase of $p' - p$ from various starting p ($k = 56$ days)

Lemma 4.4. $n^*(k, p, l)$ is given by the following formula:

$$\frac{Nkp(1-l)}{1+lkp} = n^*(k, p, l) \quad (4.9)$$

We again let $\tilde{n}^*(k, p, l) = \frac{n^*(k, p, l)}{N}$ in order to obtain the percentage change in $\bar{N}$. $\tilde{n}^*(k, p, l)$ is decreasing in p , meaning more mature blood banks with high donation rates don't need to increase the number of donors as much to match a decrease in k .

FIGURE 4.3: Graph of $\tilde{n}^*(k, p, l)$ for $1 - l$ given p and k

4.3 Efficiency of Interventions

From section 4.2.1, we determined the equivalence among $\bar{N}$, $\bar{p}$, and $\bar{k}$. In this section, we look at when each intervention should be used by introducing and comparing efficiency measures.

Take for example $\bar{N}$ and $\bar{p}$: if we want a fair comparison between the two, we have to factor in how difficult (or easy) it is to change the equivalent $p' - p$ and $\tilde{n}^*(p, p')$ values. For example, from a blood bank's perspective, a 1 point increase in p may not be as difficult as increasing $\tilde{n}^*(p, p')$ by the same amount (here, we refer to 1 point as equivalent to 0.01). To facilitate this comparison, we introduce an effort parameter $E_{\bar{p}}$ and $E_{\bar{N}}$, such that the total effort required to implement the interventions are $E_{\bar{p}}(p' - p)$ and $E_{\bar{N}}\tilde{n}^*(p, p')$. We chose a linear effort multiplier for simplicity, but an increasing concave function would yield similar results. We then define an intervention as more *efficient* if the total effort required to execute the same increase in donation is lower than that of another intervention. This is mathematically illustrated through the following definition:

Definition 4.1. Given intervention i_1 and i_2 with its respective parameters b_1 and b_2 , intervention i_1 is more efficient than i_2 , denoted $i_1 \succ^{\text{eff}} i_2$ if:

$$E_{i_1}b_1 < E_{i_2}b_2^*(b_1) \quad (4.10)$$

We can further simplify this definition by scaling the effort parameters. We do this by dividing both sides of (4.10) by E_{i_1} and defining $\epsilon_{i_1, i_2} = \frac{E_{i_1}}{E_{i_2}}$. For the $\bar{p}$ and $\bar{N}$ interventions, we therefore have the following proposition:

Proposition 2. $\bar{p} \succ^{\text{eff}} \bar{N}$ if $p' < \frac{\epsilon_{\bar{N}, \bar{p}} - p}{pk}$

Proposition 2 says that intervention $\bar{p}$ is more efficient compared to $\bar{N}$ depending on a threshold that is increasing in the effort ratio, decreasing in p , and decreasing in k . Figure 4.4 illustrates this for various $\epsilon_{\bar{N}, \bar{p}}$. The shaded regions are where $\bar{p} \succ^{\text{eff}} \bar{N}$. The $\epsilon_{\bar{N}, \bar{p}}$ labels indicate the additional shaded region contributed by raising the effort parameter to $\epsilon_{\bar{N}, \bar{p}}$. For example, when $\epsilon_{\bar{N}, \bar{p}} = 1$, $\bar{p} \succ^{\text{eff}} \bar{N}$ in both the cyan and green regions, and when $\epsilon_{\bar{N}, \bar{p}} = 2$, $\bar{p} \succ^{\text{eff}} \bar{N}$ in the cyan, green, and magenta

regions. We note that as $\epsilon_{\bar{N}, \bar{p}}$ increases, $\bar{p}$ becomes more efficient than $\bar{N}$ for more combinations of p and p' .

FIGURE 4.4: Combinations of p and p' where $\bar{p} \succ^{\text{eff}} \bar{N}$

Aside from scaling the effort needed to do each of the interventions, we have to further make the actions done in $\bar{k}$ and $\bar{p}$ comparable. Currently, the fraction l cannot be directly compared to $p' - p$ because l represents the fraction that k gets reduced to. This poses a problem because l will be high (e.g. 0.9, 0.95) when the degree of usage of $\bar{k}$ is low, and vice versa. However, choosing l is equivalent to choosing $1 - l$, which can be interpreted as the fraction that k gets reduced by. We can then compare a 1% increase in the reduction $1 - l$ with a 1 point increase in p .

Proposition 3. $\bar{p} \succ^{\text{eff}} \bar{k}$ if $p' < \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$

Proposition 3 says that $\bar{p}$ is more efficient compared to $\bar{k}$ depending on a threshold that is increasing the effort ratio, decreasing in p , and decreasing in k , just like that of the efficiency comparison between $\bar{p}$ and $\bar{N}$. In Figure 4.2, we again see that donation frequency related activities are mostly more effective if reducing regeneration requires a significantly higher effort. If we think of effort as the action required to implement the intervention, then in practice it is more difficult to increase donation frequency than to reduce regeneration (i.e. $\epsilon_{\bar{k}, \bar{p}} < 1$) because the latter only involves allowing donors to donate earlier. This means that $\bar{p}$ is more efficient only for populations with low donation rates. But if effort also involves the health risk obtained of reducing k , then $\epsilon_{\bar{k}, \bar{p}}$ can be greater than 1 and $\bar{p}$ becomes a more efficient option for more combinations of p and p' .

An assumption that we make when using $\bar{k}$ is that $l \in [0, 1]$ when reducing k . However, there are physiological (and ethical) limitations that only allow for l to

FIGURE 4.5: Combinations of p and p' where $\bar{p} \succ^{\text{eff}} \bar{k}$

be up to a certain value $l^{\max}$, or equivalently a reduction of $1 - l^{\max}$. Beyond which, it becomes dangerous and/or impossible to use $\bar{k}$. What this means that is that there will be cases wherein even though $\bar{p} \prec^{\text{eff}} \bar{k}$, $1 - l^{\max}$ prevents $\bar{k}$ from being used, which by default makes $\bar{p}$ the only available intervention. Once we take into account this limitation, then the region where $\bar{p}$ becomes the preferred intervention becomes larger, even for high values of p . Figure 4.6 shows the region for $\bar{p}$ becomes the preferred intervention because of the limitation on reducing $k = 84$.

 FIGURE 4.6: Regions where either $\bar{p} \succ^{\text{eff}} \bar{k}$ or $\bar{p}$ is the only feasible intervention ($k = 84$ days, $1 - l^{\max} = \frac{1}{3}$, $\epsilon_{\bar{k},\bar{p}} = 0.5$)

Lastly, we present the proposition that determines when $\bar{k}$ is more efficient than $\bar{N}$:

Proposition 4. $\bar{k} \succ^{\text{eff}} \bar{N}$ if $1 + \frac{1}{kp} - \epsilon_{\bar{N},\bar{k}} < 1 - l$

This efficiency is determined by a threshold on $1 - l$ that is decreasing in k, p , and $\epsilon_{\bar{N}, \bar{k}}$. It is again intuitive that if $\epsilon_{\bar{N}, \bar{k}}$ is higher, then it is more difficult to implement $\bar{N}$ and the blood bank would prefer $\bar{k}$.

4.3.1 When do we use $\bar{N}$, $\bar{p}$, and $\bar{k}$?

In the previous section, we obtained pair-wise equivalences among the three interventions and determined when each intervention is more efficient than the other. In this section, we combine the results from the pair-wise comparisons to determine when each intervention is the most efficient compared to the other two.

For ease of analysis, the baseline intervention would be that of increasing the donation frequency from p to p' . We will determine which intervention is the most efficient by comparing the effort required to match an increase of p to p' . We can technically choose any parameter, but using combinations of p and p' is a more intuitive comparison.

Propositions 2 and 3 already indicate the regions where $\bar{p} \succ^{\text{eff}} \bar{N}$ and $\bar{p} \succ^{\text{eff}} \bar{k}$, respectively. The only remaining comparison is between $\bar{N}$ and $\bar{k}$. However, the comparison is not through the equivalence of $\bar{N}$ and $\bar{k}$ which we did in the previous section. Rather, we compare $\epsilon_{\bar{k}, \bar{n}}(1 - l^*(p, p', k))$ and $\tilde{n}^*(p, p')$, the respective equivalence values that $\bar{N}$ and $\bar{k}$ have with $\bar{p}$. We formalize this in the following proposition.

Proposition 5. $\bar{k} \succ^{\text{eff}} \bar{N}$ if $p' > \frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})}$

We then have the efficiency results for all three interventions as follows

Theorem 4.5. *If there are no restrictions on $\bar{k}$ (i.e. $1 - l^{\max} = 1$), then*

- (a) $\bar{p} \succ^{\text{eff}} \bar{N}$ and $\bar{p} \succ^{\text{eff}} \bar{k}$ if $p' < \frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp}$ and $p' < \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$
- (b) $\bar{N} \succ^{\text{eff}} \bar{p}$ and $\bar{N} \succ^{\text{eff}} \bar{k}$ if $p' > \frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp}$ and $p' < \frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})}$
- (c) $\bar{k} \succ^{\text{eff}} \bar{p}$ and $\bar{k} \succ^{\text{eff}} \bar{N}$ if $p' > \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$ and $p' > \frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})}$

In the proof for Theorem 4.5, we are able to show that this is an exhaustive list of inequalities, and that other combinations of inequalities would not be possible since the intersection is a null set.

Since $\epsilon_{\bar{k}, \bar{p}} = \epsilon_{\bar{N}, \bar{p}} \epsilon_{\bar{k}, \bar{N}}$, blood banks only need to define two of the three effort parameters to fully characterize the inequalities. Figure 4.7 shows an example of the regions where combinations of p' and p would make an intervention more efficient than the others.

$\bar{p}$ is usually the more efficient intervention when the donation rate is low. $\bar{k}$ is interesting because it is more likely the most efficient intervention, but the cap on the reduction means that it can only be used up to a certain degree, and therefore $\bar{N}$ becomes the more efficient intervention by default.

FIGURE 4.7: Combinations of p and p' where specific interventions would be the most efficient ($k = 84$ days, $1 - l^{\max} = \frac{1}{3}$, $\epsilon_{\bar{N}, \bar{p}} = 1.5$, $\epsilon_{\bar{k}, \bar{N}} = 0.95$)

Another observation in Figure 4.7 and Theorem 4.5 is that $\frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})}$ defines whether or not $\bar{N}$ will be the most efficient intervention for some $p < 1 - l^{\max}$. This can be found in the region R in the figure where the intervention shifts from $\bar{N}$ to $\bar{k}$ before being limited by $1 - l^{\max}$. This region, if it exists, can be defined as

$$R = \left\{ (p, p') : p' > p; p' < 1 - l^{\max}; p' > \frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp}; p' > \frac{\epsilon_{\bar{k}, \bar{p}}}{kp} \right\} \quad (4.11)$$

Naturally, there will be an $\epsilon_{\bar{k}, \bar{N}}$ value that makes $\bar{k}$ or $\bar{N}$ the most efficient intervention for the entire region R . We obtain these in the following proposition:

Proposition 6. If R exists, then

- (a) The $\bar{N}$ intervention would be the most efficient intervention in R if:

$$\epsilon_{\bar{k}, \bar{N}} \leq \frac{k(1 - l^{\max})}{1 + k(1 - l^{\max})} \quad (4.12)$$

(b) Assuming $1 - l^{\max} > \rho$, the $\bar{k}$ intervention would be the most efficient intervention in R if:

$$\epsilon_{\bar{k}, \bar{N}} \leq \frac{\rho k}{1 + \rho k} \quad (4.13)$$

where $\rho = \min(p_{\bar{N}, \bar{p}}^*, p_{\bar{k}, \bar{p}}^*)$, $p_{\bar{k}, \bar{p}}^* = \sqrt{\frac{\epsilon_{\bar{k}, \bar{p}}}{k}}$, and $p_{\bar{N}, \bar{p}}^*$ is the $p > 0$ that solves $\epsilon_{\bar{N}, \bar{p}} = p(kp + 1)$

We have now determined the equivalence of each of the interventions with each other, and have introduced a way to measure its efficiency. This is important for blood banks especially during disasters since it determines what intervention should be pursued given a baseline and target donation rate. All these previous results are the means to achieve the targets that are set. In the next section, we focus on blood donation during and after disasters, and the effects of the interventions.

4.4 Disaster and Post-Disaster Donations

We analyze a horizon of length T periods, with period $s = 1, \dots, T$. There are three sequential phases: the pre-disaster (PR), disaster (DIS), and post-disaster (PO) phases (in Hosseinifard et al. (2019), these are referred to as Phase I, II, and III). The DIS phase occurs from $s = s_{start}^{DIS}$ to $s = s_{end}^{DIS}$, while the PR and PO phases occur from $s = 1$ to $s_{start}^{DIS} - 1$ and $s = s_{end}^{DIS} + 1$ to T , respectively. The disaster length L , is then equal to $L = s_{end}^{DIS} - s_{start}^{DIS} + 1$.

In the PR phase, the probability of donation is denoted as $p^{PR} \in [0, 1]$. Once in the DIS phase, we assume that p^{PR} changes to a level p^{DIS} which can be higher or lower than p^{PR} . This change in donation probability is seen in all countries that are hit by disasters and pandemics and reflects an increase or decrease in the altruistic tendencies of donors when faced with emergencies (Glynn et al. 2003; Kam, 2020; de Angelis, 2020; Ji and Zheng, 2020).

Next, blood banks have a target average daily donation during disasters, $\mathbb{E}[A^{TAR}] = \frac{Np^{TAR}}{1 + kp^{TAR}}$, where p^{TAR} is the equivalent minimum probability of donation that would

reach the target daily donations. Lastly, we assume that $p^{PO} = p^{PR}$. When considering the interventions, we can have $p^{DIS} = p^{TAR}$ and find the equivalent parameters for each of the interventions that would reach p^{TAR} (i.e. using the previous section's notation, $p = p^{PR}$ and $p' = p^{TAR}$).

We are interested in looking at how L , p^{DIS} , and the interventions discussed in section 4.2 affect $\mathbb{E}[A]$ during the *DIS* and *PO* phases. In the previous sections, we implicitly assumed that $\mathbb{E}[A]$ in any given phase is already in the steady state. However, this is not immediately the case once the donation rates change or the interventions are implemented (resp. stopped) during the *DIS* (resp. *PO*) phase. For example, Figure 4.8a shows that a change in donation rates during the *DIS* phase leads to an expected number of donations that deviates from the steady state. There would be an increase in donation during the first few days, with an eventual decrease below the steady state, and if the *DIS* phase continues indefinitely, a wave like pattern occurs which eventually settles on the steady state. We refer to this non-steady state expected number of arrivals in period s as $\mathbb{E}[\hat{A}_s]$.

In order to be able to calculate $\mathbb{E}[\hat{A}_s]$, we introduce an iterative procedure that depends only on knowing the past number of donors and donation rates. More concretely, the number of donors and donation rates in period $s - 1$ and $s - k - 1$ given the current period s . This allows blood banks to better predict the effect of disasters and interventions on the number of donations.

FIGURE 4.8: Simulated Sample Graph of Progression of $\mathbb{E}[\hat{A}_s]$ Through the Different Phases

(A) Comparison of Steady State and Non-Steady State Expected Arrivals

(B) Comparison of Interventions in Non-Steady State

4.4.1 Iterative Calculation of Expected Arrivals

Assume that the system is already in the steady state during the PR phase. The expected number of donors in each state: Available (V) and Regenerating for i

periods (R_i) , $i = 1, \dots, k$, is $\frac{N}{1+kp}$ and $\frac{Np}{1+kp}$, respectively. Let $\hat{p}_s$ be the donation rate during period s , while $\hat{N}_s$ is the expected number of available donors.

Without loss of generality, assume $s_{start}^{DIS} > k$. Then the following iterative equation can be used to calculate the expected number of available donors in period s :

$$\hat{N}_s = \hat{N}_{s-1}(1 - \hat{p}_{s-1}) + \hat{N}_{s-k-1}\hat{p}_{s-k-1} \quad (4.14)$$

and the expected number of arrivals in period s is:

$$\mathbb{E}[\hat{A}_s] = \hat{N}_s \hat{p}_s \quad (4.15)$$

This is used to recursively calculate for $\hat{N}_s$ and $\mathbb{E}[\hat{A}_s]$, which allows us to analyze the dynamics of a blood bank even though it is not in the steady state.

We obtain $\mathbb{E}[\hat{A}_s]$ through this iterative procedure and compare it with the expected arrivals obtained in a simulation across varying p^{PR} , p^{DIS} , L , and N . The results show that the iterative procedure is a good approximation of the expected arrivals with the percent average difference between the simulation and the iterative procedure being less than 0.5%. We show sample simulation results in Table 4.1.

TABLE 4.1: Average Percent Difference between Simulation and Iterative Calculation Procedure ($p = 0.0338$, $N = 1,000,000$, $k = 84$)

		<i>DIS</i>			<i>PO</i>		
	$p^{DIS} - p^{PR}$	0.1	0.2	0.3	0.1	0.2	0.3
<i>L</i>	10	0.18%	0.11%	0.11%	0.07%	0.08%	0.08%
	50	0.17%	0.18%	0.18%	0.07%	0.07%	0.06%
	100	0.13%	0.14%	0.14%	0.07%	0.09%	0.09%
	200	0.12%	0.13%	0.15%	0.08%	0.11%	0.10%

This procedure then provides a way for blood banks to calculate the expected arrivals even in a non-steady state setting.

4.4.1.1 Iterative Calculation of Expected Arrivals with Interventions

We extend the iterative calculation procedure to the case when interventions are used. First, the $\bar{p}$ intervention follows the same dynamic as the change from p^{PR} to p^{DIS} , such that $p^{DIS} = p'$ and $p^{PR} = p$.

Next, If we implement the $\bar{k}$ intervention, then at $s = s_{start}^{DIS}$ the donors at states R_i , $i = 1, \dots, (1-l)k$, will all be part of A such that

$$\hat{N}_{s_{start}^{DIS}} = \frac{N}{1+kp} + \frac{(1-l)kNp}{1+kp} \quad (4.16)$$

Additionally, during the *DIS* phase, the second term of (4.14) changes such that the subscript becomes $s-lk-1$ as a result of the reduction in regeneration time:

$$\hat{N}_s = \hat{N}_{s-1}(1 - \hat{p}_{s-1}) + \hat{N}_{s-lk-1}\hat{p}_{s-lk-1} \quad (4.17)$$

Since we assume that interventions are not used anymore after the *DIS* phase, then the regeneration time will return to a length of k periods. During the first $(1-l)k$ periods of the *PO* phase, no regeneration will occur because the regeneration time is increased, which leads to

$$\hat{N}_s = \hat{N}_{s-1}(1 - \hat{p}_{s-1}) \quad (4.18)$$

After, the initial lk periods, the calculation of $\hat{N}_s$ reverts back to (4.14).

Next, if $\bar{N}$ is implemented, the most straightforward approach is to just increase $\hat{N}_{s_{start}^{DIS}}$ by $\hat{n}N$. However, there will be issues in the *PO* phase once the interventions are stopped because it will be difficult to distinguish which donors were a result of the intervention. To address this, we can instead keep track of the additional donors given by $\bar{N}$ with a new variable $\hat{M}_s$ such that

$$\hat{M}_{s_{start}^{DIS}} = \hat{n}N \quad (4.19)$$

The transition into the next periods during the *DIS* phase is then

$$\hat{M}_s = \begin{cases} \hat{M}_{s-1}(1 - \hat{p}_{s-1}) & \text{if } s - s_{start}^{DIS} < k + 1 \\ \hat{M}_{s-1}(1 - \hat{p}_{s-1}) + \hat{M}_{s-k-1}\hat{p}_{s-k-1} & \text{otherwise} \end{cases} \quad (4.20)$$

and

$$\mathbb{E}[\hat{A}_s] = (\hat{N}_s + \hat{M}_s)\hat{p}_s \quad (4.21)$$

In the *PO* phase, we simply revert back to (4.15).

Would the comparisons between interventions then have no bearing if the *DIS* phase only goes to the steady state after a long period of time? Fortunately, we can see in Figure 4.8b that regardless of whether the system is in the steady state or not, the equivalence among the interventions holds in the *DIS* phase.

However, the difference comes during the *PO* phase for the $\bar{p}$ and $\bar{k}$ interventions when compared to $\bar{N}$. The expected number of donations begins at a lower level for $\bar{p}$ and $\bar{k}$, but because the new donors are entirely separate from the original donor pool in $\bar{N}$, removing the interventions would just result in the expected number of donations reverting to the *PR* phase.

4.4.2 Occurrence of the Steady State

Next, we conduct a numerical experiment to demonstrate the difference between using the steady state formulation and the non-steady state formulation for calculating the expected number of donations in both phases. We want to know the amount of time it takes for the blood bank to reach the steady state in the *DIS* and *PO* phases. To do this, we provide a definition for the steady state.

Definition 4.2. Given a phase $P = DIS, PO$, and assuming P continues indefinitely, the steady state period $\tilde{s}^P(\delta)$ is defined as:

$$\tilde{s}^P(\delta) = \arg \min_s \left\{ s : \left| \frac{\mathbb{E}[A^P] - \mathbb{E}[\hat{A}_t]}{\mathbb{E}[A^P]} \right| \leq \delta \quad \forall t = s, s+1, \dots, s+k+1 \right\} \quad (4.22)$$

Definition 4.2 says that a system enters the steady state in period $\tilde{s}^P(\delta)$ in phase P if the percent difference of the expected arrivals with the steady state expected arrivals is less than δ for the next $k+1$ periods.

Figure 4.9a shows that it takes a significant amount of time to get into the steady state during the *DIS* phase, especially as the difference between p^{PR} and p^{DIS} gets larger. Blood banks must then be aware that a change in donation rate due to the *DIS* phase will not immediately result in the predicted steady state donation quantities. The length of the *DIS* phase would also determine whether or not the blood bank would reach the steady state. Reaching the steady state may be possible for longer disasters like pandemics, but the change in donation rate must

be sustained throughout the duration, which is a challenge in itself due to donor fatigue. It is also generally much faster to reach the steady state when decreasing the probability of donation rather than increasing it, as can be seen in Figure 4.9.

FIGURE 4.9: $\tilde{s}^P(\delta)$, $P = PO, DIS$ at each p^{DIS} for a given p^{PR} ($\delta = 0.05, k = 84$)

(A) *DIS* Phase, $p^{DIS} > p^{PR}$, log-scale

(B) *PO* Phase, $p^{DIS} > p^{PR}$, linear scale

(C) *DIS* Phase, $p^{DIS} < p^{PR}$, linear scale

(D) *PO* Phase, $p^{DIS} < p^{PR}$, log-scale

4.4.2.1 Dynamic Donation Rate Strategy

This volatility becomes a problem for blood banks since it means that either there will be too much or too little blood during and after disasters. It then begs the question of whether or not blood banks can prevent these swings in average donation rate. That is, is there a way for the interventions to be implemented such that the average donation rate will go into the steady state immediately during the *DIS* and *PO* phases. This is so that any excess or shortage of blood is avoided.

We assume that the blood bank can dynamically change the donation rate on a per period basis through the various interventions. This may be done through constant communication by the blood bank with donors, for example when blood banks display real-time blood stock information on its website. In the dynamic donation rate strategy, we equate the expected number of donations $\mathbb{E}[\hat{A}_s] = \hat{N}_s \hat{p}_s$, with the target quantity $\mathbb{E}[A^{TAR}]$ during the DIS phase or $\mathbb{E}[A^{PO}]$ during the PO phase, and get the following:

$$\hat{p}_s = \frac{\mathbb{E}[A^{TAR}]}{\hat{N}_s} \text{ or } \frac{\mathbb{E}[A^{PO}]}{\hat{N}_s} \quad (4.23)$$

Since we are able to recursively calculate for $\hat{N}_s$, then obtaining $\hat{p}$ will also be straightforward. A comparison of this dynamic donation rate compared to using p^{TAR} can be found in Figure 4.10. Clearly, $\mathbb{E}[A^{TAR}]$ and $\mathbb{E}[A^{PO}]$ can immediately be achieved when allowing for a gradual change in the donation rate. Essentially, with control over donation rates, the blood banks are able to create an artificial steady state that matches the steady state donation targets.

We emphasize the assumption that blood banks can directly control the donation rates on a daily basis. For less mature blood banks who do not have the resources, manpower, and culture of donation, this would be difficult to do. It is only when a blood bank becomes mature and have the resources to use its interventions, would this strategy be viable.

FIGURE 4.10: Comparison of $\mathbb{E}[\hat{A}_s]$ with dynamic and non-dynamic donation rates ($k = 84$ days, $N = 100, 312$, $p^{PR} = 0.03$, $p^{TAR} = 0.045$)

4.5 Preliminary Conclusion of Chapter 4

In this chapter, we provide an analytical model for blood banks to use in comparing three types of interventions. The equivalence between $\bar{N}$, $\bar{k}$, and $\bar{p}$ are obtained, and efficiency parameters are introduced to determine when each would be the best option to use in increasing donation rates. This provides blood banks with a more analytical approach to determining which intervention should be utilized in their context. This analysis was done with the assumption of steady state, which we point out to be problematic because of the volatility brought by disasters and the sudden change of donation rates. We introduce an iterative calculation procedure for obtaining the number of donations even with the volatility, and show that it is a good approximation to simulated arrivals. Lastly, we introduce a dynamic donation rate strategy that allows blood banks to immediately achieve the steady state in the *DIS* and *PO* phases by using interventions to enact per period changes to donation rates.

This concludes the analytical portion of the study. In the next chapter, we continue the essay by introducing a simulation model and comparing the results to that of the analytical model found in this chapter.

Chapter 5

Part 2: Blood Supply Interventions During Pandemics and Disasters

In this second part of *Blood Supply Interventions During Pandemics and Disasters*, we strengthen the results of the analytical model in Chapter 4 by creating a simulation model that incorporates more realistic properties of blood banks and disasters. This model is a stochastic inventory theory-based DES model that allows for full flexibility in choosing system parameters such as demand, disaster length and intensity, and number and type of donors.

We include demand and inventory processes in the DES model to fully see how the interventions can be used to meet fill-rate targets. Chapter 4's analytical model is looked at from an expected value perspective, which may result in nuances of whether or not it can really satisfy demand. Should blood banks then be more conservative or aggressive in implementing these interventions when demand is involved?

This insight is then extended to multiple disaster waves and other ethically complex interventions. Beyond this analysis, the constructed model can be implemented as a tool that can further aid blood banks in strategic and tactical policy decisions. Additionally, the model is rigorously defined based on inventory theory. In doing so, we provide OM scholars a baseline model that can be used for more complex analysis.

This chapter proceeds as follows: In Section 5.1, we focus on building the stochastic inventory control and discrete event simulation model. In particular, Section 5.1.1 discusses the segmentation of pandemics into phases, Section 5.1.2 describes the demand process, Section 5.1.3 looks at the donation process, and Section 5.1.4 describes the transition process. Section 5.2 is a numerical study on changing donation targets during disasters as well as implementation of ethically complex interventions. Section 5.3 presents some managerial insights, and lastly, Section 5.4 summarizes and concludes Chapters 4 and 5.

Throughout this chapter, we use the following notational convention. Variables in upper-case denote random variables (e.g. A) and the corresponding realizations are in lower-case (e.g. a). First-position subscripts denote time progression (e.g. a_s), whereas superscripts denote classifications (e.g. a^t). Bold type faced letters denote vectors (e.g. $\mathbf{A}_s$) and regular type faced letters with second-position subscripts denote components of the vector (e.g. $A_{s,i}$). We also let $(a)^+ = \max(a, 0)$.

5.1 Inventory Control and Discrete-Event Simulation Model

We present the model by first giving a brief overview and then discussing the details in the rest of the section. We first model the dynamics of the system including donor and demand arrivals, transitions, and pandemic progression. Then, we use the resulting simulation to check how the interventions discussed in Chapter 4 should be implemented in a non-steady state setting and when the *DIS* and *PO* phases vary.

As blood banks typically manage multiple types of blood inventory independently, it suffices to consider a blood bank that manages a single type of perishable blood inventory with lifetime m periods over a finite planning horizon T . Each period $s = 1, 2, \dots, T$ in the planning horizon represent any appropriate time interval (e.g. days, weeks, months). Without any interventions, the blood bank tries to satisfy stochastic demand D_s by accepting a stochastic number of donations from two types of donors: not yet donated (f) and regular (r) donors. Each type has n_s^f and n_s^r individuals in period s , respectively. Donors can exit the system, change their type from f to r , or enter the system as f at given rates. Once donors

have donated, they need to regenerate for k periods before being allowed to donate again in accordance with health and safety protocols set by the blood bank. The regeneration period is the minimum time between successive donations in order for a donor to recover lost blood and to stabilize haemoglobin and iron levels.

5.1.1 Phases and Segments

Just like in chapter 4, we simulate the blood bank over a finite planning horizon T and divide it into three mutually exclusive sets of periods called phases. These phases are the pre-disaster (PR), disaster (DIS), and post-disaster (PO) phases. When there are multiple disaster waves, we include a subscript for DIS_w and PO_w to denote the w -th wave. These represent the progression of disasters; PR is the status quo prior, DIS_w is the w -th wave of a disaster, and PO_w is the recovery period or the intermediate time between waves (for multiple waves)

Each disaster wave DIS_w of length L_w is further divided into three segments: the ramp-up (R_w), the plateau (PL_w) segment which is the peak, and the decline (DEC_w). The mutually exclusive sets of periods associated with each phase and segment for a sample of one and two wave disasters can be found in Table 5.1a and 5.1b, and a graphical illustration of the division can be found in Figure 5.1.

TABLE 5.1: Division of Planning Horizon into Segments and Periods

Phase	Periods (1 Wave)	Periods(2 Waves)
PR	$[1, s_{\text{start}}^{DIS_1} - 1]$	$[1, s_{\text{start}}^{DIS_1} - 1]$
DIS_1	$[s_{\text{start}}^{DIS_1}, s_{\text{end}}^{PAN_1}]$	$[s_{\text{start}}^{DIS_1}, s_{\text{end}}^{DIS_1}]$
PO_1	$[s_{\text{end}}^{DIS_1} + 1, T]$	$[s_{\text{end}}^{DIS_1} + 1, s_{\text{start}}^{DIS_2} - 1]$
DIS_2	-	$[s_{\text{start}}^{DIS_2}, s_{\text{end}}^{PAN_2}]$
PO_2	-	$[s_{\text{end}}^{DIS_2} + 1, T]$

(A) Periods Associated with each Pandemic Wave

Segment	Periods
R_w	$[s_{\text{start}}^{PAN_w}, s_{\text{end}}^{R_w}]$
PL_w	$[s_{\text{end}}^{R_w} + 1, s_{\text{start}}^{DEC_w} - 1]$
DEC_w	$[s_{\text{start}}^{DEC_w}, s_{\text{end}}^{DIS_w}]$

(B) Periods Associated with each segment in DIS_w , $w = 1, 2$

5.1.2 Blood Demand

The first important distinction among the phases is the demand rate for blood. In the model, Demand D_s is assumed to be Poisson with mean μ_s for every period s . This assumption can be easily changed as the simulation model allows for any general distribution. The model also allows for the mean demand or distribution parameter to change during each of the phases. For example, a Poisson distribution's mean demand can increase based on the number of infected individuals during a pandemic disaster if the infection can be treated with blood transfusion. On the other-hand, mean demand may remain constant if blood transfusion is not needed in treatment. This latter scenario is what has been documented with COVID-19 (Stanworth et al., 2020; Fan et al., 2020). A third scenario, where demand decreases can also be implemented through deferral of demand.

FIGURE 5.1: Sample Progression of μ_s through the Phases and Segments

Although mean demand varies throughout the year for blood banks, it is assumed during the PR phase that $\mu_s = \mu^{PR}$ is a constant. In the case of having no change in demand during the disaster, this extends to the DIS_i and PO_i phases. If demand increases due to a disaster, this assumption allows us to isolate the effect of the increase. The change in demand for blood may also occur gradually over a period of time during a pandemic or disaster. We capture this by allowing μ_s (or the relevant parameter in the case of a different assumption on demand distribution) to change depending on the phase and period.

In the model, we assume that the demand changes during R_w until the start of PL_w . At which point, the plateau or the maximum (minimum in the case of

a decrease in demand) mean demand in each wave $(1 + \gamma^{PL_w})\mu^{PR}$, is reached. $\gamma^{PL_w} \geq 0$ represents the maximum percent increase of the mean demand from PR to PAN_w . Eventually, after the plateau (PL_w) at the start of DEC_w , the mean demand decreases until the end of PAN_w . The decrease in mean demand for the waves stop at the start of PO_w at an intermediate level of demand $(1 + \gamma^{PO_w})\mu^{PR}$ where $\gamma^{PO_w} \geq 0$. γ^{PO_w} is the parameter that indicates whether or not a disaster is fully under control. A positive value shows that the mean demand is greater than μ^{PR} which means that infections are still present even after the decline. $\gamma^{PO_w} = 0$ on the other hand means that the mean demand goes back to pre-disaster levels. We assume for the final wave W that $\gamma^{PO_w} = 0$, to indicate the end of the disaster. Note that it is possible for γ^{PO_w} to be zero even if w is not the last wave, when infections can virtually disappear only to have a resurgence of cases after a few months. The assumption on the rates of change can be easily changed depending on the blood bank's assessment during a particular disaster. For example, the increase can be exponential and the decrease can be linear. An example of the values of the mean demand per period with exponential increase and linear decrease for one and two-wave disasters are shown in Table 5.2 while a graph of its progression through the phases and segments is shown in Figure 5.1. $\hat{p}_s^t$ will be discussed in Section 5.1.3.2. When demand does not change because of a disaster, $\gamma^{PO_w} = \gamma^{PL_w} = 0$. We also assume that unsatisfied demand is lost and that it is satisfied through a FIFO policy, although this can easily be changed to be a backlogging model and one where an age-based threshold is implemented.

 TABLE 5.2: Table of μ_s and $\hat{p}_s^t$ Values per Phase and Segment

Phase/Segment		1 Wave		2 Waves	
		μ_s μ^{PR}	$\hat{p}_s^t$ $p^{t,PR}$	μ_s μ^{PR}	$\hat{p}_s^t$ $p^{t,PR}$
DIS_1	PR				
	R_1	$(1 + \gamma^{PL_1})^{\alpha_1} \mu^{PR}$	$(1 + \alpha_1 \lambda) p^{t,PR}$	$(1 + \gamma^{PL_1})^{\alpha_1} \mu^{PR}$	$(1 + \alpha_1 \lambda) p^{t,PR}$
	PL_1	$(1 + \gamma^{PL_1}) \mu^{PR}$	$(1 + \lambda) p^{t,PR}$	$(1 + \gamma^{PL_1}) \mu^{PR}$	$(1 + \lambda) p^{t,PR}$
	DEC_1	$(1 + \gamma^{PL_1}(1 - \beta_1)) \mu^{PR}$	$(1 + \lambda(1 - \beta_1)) p^{t,PR}$	$((1 + \gamma^{PL_1})(1 - \beta_1) + (1 + \gamma^{PO_1})\beta_1) \mu^{PR}$	$(1 + \lambda(1 - \beta_1)) p^{t,PR}$
	PO_1	μ^{PR}	$p^{t,PR}$	$(1 + \gamma^{PO_1}) \mu^{PR}$	$p^{t,PR}$
DIS_2	R_2	-	-	$(1 + \gamma^{PO_1}) \left(\frac{1 + \gamma^{PL_2}}{1 + \gamma^{PO_1}} \right)^{\alpha_2} \mu^{PR}$	$(1 + \alpha_2 \lambda) p^{t,PR}$
	PL_2	-	-	$(1 + \gamma^{PL_2}) \mu^{PR}$	$(1 + \lambda) p^{t,PR}$
	DEC_2	-	-	$(1 + \gamma^{PL_2}(1 - \beta_2)) \mu^{PR}$	$(1 + \lambda(1 - \beta_2)) p^{t,PR}$
	PO_2	-	-	$\mu^{PR,b}$	$p^{t,PR}$

$$\text{Let } \alpha_w = \frac{s - s_{\text{start}}^{PAN_w} + 1}{s_{\text{end}}^{RW} - s_{\text{start}}^{PAN_w} + 1} \text{ and } \beta_w = \frac{s - s_{\text{start}}^{DEC_w} + 1}{s_{\text{end}}^{PAN_w} - s_{\text{start}}^{DEC_w} + 1}.$$

5.1.3 Donations and Inventory

The blood bank manages blood inventory with lifetime m periods (i.e. maximum shelf life or MSL is m). At the start of each period s , the blood bank has $x_{s,j}$ units with remaining lifetimes $m-j$ (or of age j), $j = 1, \dots, m-1$ captured by the vector $\mathbf{x}_s = (x_{s,1}, x_{s,2}, \dots, x_{s,m-1})$. Let $x_s^{\text{total}} = \sum_{j=1}^{m-1} x_{s,j}$ be the total on-hand inventory in period s which is replenished through the donor pool. We consider two donor categories: regular donors (r), and those who have never donated but are eligible (f). It is typical for blood banks to categorize donors based on their frequency (and probability) of donation, and the two categories mentioned are the most important to blood banks (Veldhuizen et al., 2013). The number of donors of each type t in period s is represented by n_s^t , $t = f, r$. Further, we define $n_s^{\text{total}} = n_s^f + n_s^r$ as the total donor pool in period s .

5.1.3.1 VNRBD

Donation rates vary for each donor type with r donating at a higher probability than f . We model donor arrivals as random variables A_s^t , $t = r, f$ with total arrivals $A_s^r + A_s^f$ in period s . We assume that A_s^t follows a binomial distribution with probability of donation $\hat{p}_s^t$ and $\hat{n}_s^t$ independent trials such that $\hat{p}_s^r > \hat{p}_s^f$ and for $t = r$,

$$\hat{n}_s^r = n_s^r - \sum_{j=1}^k q_{s,j}^r \quad (5.1)$$

n_s^r refers to the number of available, non-regenerating donors as formulated in Chapter 3. The difference is that in that chapter, $q_{s,j}^r$ is a decision variable, and there are no stochastic arrivals. In this formulation, $q_{s,j}^r = a_{s-j}^r$ which is the realization of A_{s-j}^r . The vector $\mathbf{q}_s^r = (q_{s,1}^r, q_{s,2}^r, \dots, q_{s,k}^r)$ then captures the past k arrivals from period s . There is no need to keep track of the regeneration process for n donors since they have not yet donated. Thus, for n donors, $\hat{n}_s^f = n_s^f$.

During the *DIS* phase, when the $\bar{N}$ intervention is used, the same variables $\hat{n}_s^{pd}$, $\mathbf{q}_s^{pd}$ and dynamics are used for the additional donors.

5.1.3.2 Propensity to Donate

As in chapter 4, the donation probability $\hat{p}_s^t$, for $t = f, r$ changes depending on the phase and segment of the planning horizon starting from a base probability of $p^{t,PR}$. We call this maximum increase or decrease in probability of donation as the donor pool's *Propensity to Donate* and is represented by $(1 + \lambda^w)p^{t,PR}$, $\lambda^w \geq -1$. λ^w can be thought of as a measure of additional willingness or barriers to donation for donors during wave w such that if $\lambda^w < 0$, then donors are more hesitant to donate due to the disaster and will do so with lower probability compared to the status quo. Conversely, if $\lambda^w > 0$, then donors become more altruistic and will donate with higher probability. The distinction of propensities to donate for each wave shows how donors can have different behavior depending on each wave. The increase (decrease, resp.), plateau, and decrease (increase, resp.) of $\hat{p}_s^t$ happens at same segments as that of demand and depends on the blood bank's assessment of the disaster. An example of the values for a linear increase and decrease for $\hat{p}_s^t$ can be found in Table 5.2.

Alternatively, the dynamic donation rates in section 4.4.2.1 can also be implemented easily in the simulation model.

5.1.4 Sequence of Events and State Transitions

The model starts with states $(\mathbf{x}_s, \mathbf{q}_s^r, \mathbf{q}_s^{pd}, \mathbf{n}_s)$ in every period s , where $\mathbf{n}_s = (n_s^f, n_s^r, n_s^{pd})$ is the vector of all donor types. The sequence of events are as follows:

- Donors arrive based on the available and eligible donors. Those who arrive beyond the deferral level are turned away (deferred).
- Demand D_s arrives and is satisfied to the fullest extent by on-hand inventory plus donor arrivals based on a FIFO issuing rule. Unsatisfied demand is lost.
- Donors transition into, out of, and within the system, and the regeneration and outdating processes occur.

The admission policy and processes described throughout this section allow us to simplify the dynamic programming formulation in the simulation to one that does

not require any periodic decision variables. That is, the blood bank's key decisions can either be predetermined (e.g. issuing rule, m, k) or determined at the onset of disasters (e.g. which intervention to deploy). Nonetheless, we still have to formulate the state transitions from s to $s + 1$ as follows:

$$(\mathbf{x}_{s+1}, \mathbf{q}_{s+1}^r, \mathbf{q}_{s+1}^{pd}, \mathbf{n}_{s+1}) = \left(I_s(\mathbf{x}_s, A_s^f, A_s^r, A_s^{pd}, D_s), G_s^r(\mathbf{q}_s^r, A_s^r, A_s^f), G_s^{pd}(\mathbf{q}_s^{pd}, A_s^{pd}, A_s^f), \left[N_s^f(\mathbf{n}_s, A_s^f), N_s^r(n_s^r, A_s^f), N_s^{pd}(n_s^{pd}, A_s^f) \right] \right) \quad (5.2)$$

where $I_s, N_s^f, N_s^r, N_s^{pd}, G_s^r$ and G_s^{pd} are the transition functions for (I)nventory (I_s), do(N)ors (N_s^t for donor type $t = f, r, pd$) and re(G)eneration (G_s^t for donor type $t = r, pd$).

5.1.4.1 Inventory Transition

I_s follows the standard transition of perishable inventory (see, for example, Haijema, 2014) except that the new first component of the vector in period s includes the realization of the stochastic arrivals a_s^t such that

$$I_s(\mathbf{x}_s, a_s^f, a_s^r, a_s^{pd}, d_s) = \left((a_s^f + a_s^r + a_s^{pd} - d_s - \sum_{j=1}^{m-1} x_{s,j})^+ \right)^+, \quad (5.3)$$

$$(x_{s,1} - d_s - \sum_{j=2}^{m-1} x_{s,j})^+, \dots, (x_{s,m-2} - d_s - x_{s,m-1})^+ \right)^+$$

where d_s is a realization of $D_s \sim G(\Phi)$ and G is a general distribution with parameter set Φ . When $s \in PR, PO_w$, $a_s^{pd} = 0$ since the interventions are not used.

5.1.4.2 Donor Transition

In the *PR* and *PO* phases, the donor transition functions are as follows:

$$N_s^f(\mathbf{n}_s, a_s^f) = (1 - \delta^{ex})(n_s^f - a_s^f) + \delta^{en} n_s \quad (5.4)$$

$$N_s^r(n_s^r, a_s^f) = (1 - \delta^{ex})(n_s^r + \delta^{tr} a_s^f) \quad (5.5)$$

$$N_s^{pd}(n_s^{pd}, a_s^f) = 0 \quad (5.6)$$

$$\tilde{A}_s^f \sim \text{Binomial}(\hat{n}_s^f, \hat{p}_s^f) \quad (5.7)$$

$$\tilde{A}_s^r \sim \text{Binomial}(\hat{n}_s^r, \hat{p}_s^r) \quad (5.8)$$

$$\tilde{A}_s^{pd} \sim \text{Binomial}(0, \hat{p}_s^{pd}) \quad (5.9)$$

Once A_s^f and A_s^r donors arrive according to the stochastic arrival in (5.7) and (5.8), a fixed fraction of the realized a_s^n donors δ^{tr} transitions to become r donors. The assumption is that if an n donor does not become r , then that donor will exit the system, but is immediately replaced by a new f donor. Within each donor type, a fixed fraction δ^{ex} also exit the system and a fixed fraction δ^{en} of new eligible donors equal to the total population enter the system as f . This models the entering and exiting of donors in the eligible donor pool based on their eligibility status. For example, donors who get older or sick exit the system while those who are of donating age enter. We make a similar assumption as An et al. (2011) that $\delta^{ex} = \delta^{en}$ to ensure a closed system over time. Once we enter the *DIS_w* phases, $N_s^{pd}(n_s^{pd}, a_s^f) = u_s^{pd}$. We assume that because these donors are targeted by interventions that they will not exit until the end of the *DIS* phase.

5.1.4.3 Regeneration Transition

Lastly, we present the baseline donor regeneration transition functions for $s \in PR$.

$$G_s^r(\mathbf{q}_s^r, a_s^r, a_s^f) = (1 - \delta^{ex})(a_s^r + \delta^{tr} a_s^f, q_{s,1}^r, \dots, q_{s,k-1}^r) \quad (5.10)$$

The function removes the final component of the vector and moves the rest one position further down the vector. Next, the sum of new arrivals q_s^r and donors who transition from n who become r after donating, $\delta^{tr} a_s^f$, become the first component of (5.10). Finally, the vectors are multiplied by $(1 - \delta^{ex})$ to account for the exit of donors which we assume to be distributed evenly across donors with varying

regeneration periods remaining. When reducing regeneration time is implemented as an intervention, the last $(1 - l)k$ components are removed during the PAN_w phases. Once the waves finish, then $(1 - l)k$ number of zeros are added to the end of the vector. The formulation can be found in Appendix C.1 and these transition functions complete the model.

5.2 Simulating Disasters and Interventions

The model described can be implemented as a simulation tool and allows blood banks to conduct what-if scenario analyses to aid policy decisions. For example, a blood bank can test the effect of varying the lifetime of blood through m . Another use-case would be for determining issuing rules other than FIFO or selecting donor deferral levels.

In our case, we use the tool to get more insights from the analytical model in chapter 4. We do this by simulating the extent at which interventions should be used for varying increases in demand γ^{PL} , disaster length L , and multiple disaster waves W .

The main performance measure that we will be looking at is Fill-Rate. For each period s , the fill-rate FR_s^P in phase P is:

$$FR_s^P = \min(1, \frac{x_s^{\text{total}} + \sum_{t=\{r,f,pd\}} a_s^t}{d_s}) \times 100\% \quad (5.11)$$

Fill-Rate is a measure of how much of the current period's demand is satisfied by on-hand inventory plus arrivals. We believe that this is an appropriate performance measure because blood donation requires immediate availability. Demand that cannot be satisfied by on-hand blood inventory needs to be satisfied one way or another in order to save a patient's life or improve a patient's health. Donors must get blood or else there may be severe, life-ending consequences. Fill-rate measures how well a blood bank is able to avert these repercussions.

We use available data for two countries: Norway and the UK, obtained from the WHO's *Global Status Report on Blood Safety (2016)* to obtain country-level data on blood collection by donation type, donation frequency, and number of donors. Table 5.3 summarizes this information.

TABLE 5.3: Country Information

	Norway ($u_1^r = 92,226, p^{PR} = 0.04$)			UK ($u_1^r = 100,312, p^{PR} = 0.03$)		
FR^{PR}	80	90	100	80	90	100
μ^{PR}	1,057	940	846	1,069	950	855

(A) Mean Demand during the PR phase to achieve Fill-Rate

	Norway		UK	
u_1^r	92,226		100,312	
p^{PR}	0.04		0.03	
γ^{PL}	0.1	0.2	0.1	0.2
p^{TAR} Fixed Donation Strategy	0.066	0.146	0.044	0.073

(B) Country-Related Parameters

Since we already know the equivalence of each of the interventions for $\bar{p}$, $\bar{k}$, and $\bar{N}$, we focus only on $\bar{p}$ and note that the results of Chapter 4 would determine which intervention should be used.

In all the following studies, we initially set $\mu^{PR} = \frac{\mathbb{E}[A^{PR}]}{FR^{PR}}$ so that in steady state during the pre-disaster phase, the fill-rate is FR^{PR} . Then, in the DIS phase, with an increase in demand of γ^{PL} , comparisons are made in relation to if the different interventions and strategies can lead to $FR^{DIS} = FR^{PR}$. That is, can the interventions make the fill-rate in the disaster phase the same as the fill-rate of the pre-disaster phase. The simulations have a burn-in period of 1300 periods, and the results are averaged over 600 iterations for each set of parameters.

5.2.1 Impact of Dynamic Donation Rates on Fill-Rate

In section 4.4.2.1, we determined that dynamic donation rates are less than or equal to the target donation rates of the steady state. With the inclusion of demand, we want to check whether dynamic donation rates would be able to aid in meeting fill-rate targets during the DIS and PO phases. We conduct a numerical study for various L , γ^{PL} , and FR^{PR} for both the dynamic donation rate and p^{TAR} when it is calculated through the steady state formula. We call these two as the dynamic donation rate strategy and the fixed donation rate strategy, respectively. The results of which can be found in Table 5.4.

What we can see is that even though the dynamic donation rate strategy is able to match the increase in mean demand, it is only when fill-rate is high that the fill-rate

TABLE 5.4: Fill-Rates Using Dynamic Donation Rate (Fixed Donation Rate)

	FR^{PR}	Norway						UK					
		80		90		100		80		90		100	
		0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
λ	5	79 (100)	79 (100)	89 (100)	89 (100)	100 (100)	100 (100)	80 (100)	79 (100)	90 (100)	89 (100)	100 (100)	100 (100)
	10	79 (100)	78 (100)	89 (100)	87 (100)	100 (100)	100 (100)	79 (100)	78 (100)	89 (100)	88 (100)	100 (100)	100 (100)
	50	76 (86)	72 (91)	85 (97)	81 (100)	100 (100)	100 (100)	76 (87)	73 (92)	86 (97)	82 (100)	100 (100)	100 (100)
	100	75 (82)	70 (84)	84 (92)	79 (92)	100 (100)	100 (100)	75 (82)	71 (84)	85 (92)	80 (93)	100 (100)	100 (100)
	200	75 (81)	70 (83)	84 (91)	78 (92)	100 (100)	95 (100)	75 (81)	71 (83)	84 (91)	79 (92)	100 (100)	97 (100)

(A) Average Fill-Rate during the *DIS* phase

	FR^{PR}	Norway						UK					
		80		90		100		80		90		100	
		0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
λ	5	79 (77)	79 (78)	89 (88)	88 (90)	100 (100)	100 (100)	80 (78)	79 (79)	89 (89)	89 (90)	100 (100)	100 (100)
	10	79 (74)	77 (75)	88 (87)	87 (88)	100 (100)	100 (100)	79 (75)	78 (76)	89 (88)	88 (89)	100 (100)	100 (100)
	50	77 (67)	75 (57)	87 (75)	84 (65)	100 (100)	100 (100)	77 (68)	74 (56)	87 (76)	84 (69)	100 (100)	100 (100)
	100	79 (74)	79 (70)	89 (84)	88 (84)	100 (100)	100 (100)	79 (75)	78 (70)	89 (85)	88 (82)	100 (100)	100 (100)
	200	79 (73)	78 (59)	89 (82)	88 (76)	100 (100)	98 (100)	79 (74)	77 (67)	88 (83)	87 (77)	100 (100)	97 (100)

(B) Average Fill-Rate during the first $\frac{k}{2}$ periods of the *PO* phase

values in the *DIS* phase are maintained. If the fill-rate is low, and the disaster length is high, then $FR^{DIS} < FR^{PR}$. For example, when $L = 200$, and $\gamma^{PL} = 0.2$ for Norway, the dynamic donation rate strategy would lead to $FR^{DIS} = 70\%$ compared to $FR^{PR} = 80\%$. Using the fixed donation rate strategy on the other hand leads to $FR^{DIS} > FR^{PR}$.

Conversely, during the *PO* phase, using the dynamic donation rate strategy leads to a fill-rate that is closer to FR^{PR} compared to using the fixed donation rate strategy. This is already expected from the results of section 4.4.2.1.

It is worth noting that when $FR^{PR} = 100$, using either options would maintain the fill-rate during both the *DIS* and *PO* phases. This further alludes to the options that are available to more mature blood banks who usually have high fill-rates. They can easily use either the dynamic donation rate or fixed donation rate strategies and any would be able to meet their targets. Compare this with less mature blood banks who are not able to meet demand (i.e. low FR), who have to choose whether they want to increase donations during the *DIS* phase and lose out on donations in the *PO* phase, or vice versa.

Next, we conduct analysis for two wave disasters, but this time vary the length of PO_1 , while maintaining L_1 and L_2 . Table 5.5 shows the results. The insight remains that if the fill-rate is higher, then both strategies are able to meet fill-rate targets, though there is some decrease when $FR^{PR} = 100$ for the dynamic donation strategy if the time between disaster waves is small. This highlights the impact of

multiple disaster waves, such that even mature blood banks are affected heavily if they occur in succession.

TABLE 5.5: Fill-Rates Using Dynamic Donation Rate (Fixed Donation Rate) for Two Waves ($L_1 = L_2 = 100$)

	FR^{PR}	Norway						UK					
		80		90		100		80		90		100	
		0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
PO_1 Length	Baseline One Wave	75 (82)	70 (84)	84 (92)	79 (92)	100 (100)	100 (100)	75 (82)	71 (84)	85 (92)	80 (93)	100 (100)	100 (100)
	5	74 (81)	71 (83)	84 (91)	80 (93)	100 (100)	93 (100)	74 (81)	71 (82)	84 (91)	80 (92)	100 (100)	96 (100)
	10	74 (82)	72 (83)	84 (92)	82 (94)	100 (100)	94 (100)	74 (81)	71 (83)	84 (92)	81 (93)	100 (100)	98 (100)
	50	73 (82)	70 (84)	85 (92)	84 (93)	100 (100)	100 (100)	73 (82)	71 (84)	85 (92)	85 (93)	100 (100)	100 (100)
	100	73 (82)	70 (83)	83 (92)	86 (94)	100 (100)	100 (100)	74 (81)	69 (84)	84 (92)	86 (93)	100 (100)	100 (100)
	200	73 (82)	70 (84)	83 (92)	93 (92)	100 (100)	100 (100)	74 (82)	69 (84)	83 (92)	93 (93)	100 (100)	100 (100)

(A) Average Fill-Rate during the *DIS* phase

	FR^{PR}	Norway						UK					
		80		90		100		80		90		100	
		0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
PO_1 Length	Baseline One Wave	79 (74)	79 (70)	89 (84)	88 (84)	100 (100)	100 (100)	79 (75)	78 (70)	89 (85)	88 (82)	100 (100)	100 (100)
	5	78 (73)	74 (56)	87 (82)	83 (72)	100 (100)	93 (100)	78 (74)	76 (67)	88 (84)	85 (76)	100 (100)	95 (100)
	10	77 (73)	72 (57)	86 (83)	80 (69)	100 (100)	92 (100)	78 (75)	75 (67)	87 (84)	84 (76)	100 (100)	93 (100)
	50	73 (76)	68 (77)	82 (86)	76 (92)	100 (100)	86 (100)	74 (76)	68 (72)	84 (86)	77 (84)	100 (100)	99 (100)
	100	77 (74)	73 (64)	87 (83)	82 (77)	100 (100)	100 (100)	77 (75)	75 (69)	87 (84)	85 (81)	100 (100)	100 (100)
	200	77 (74)	70 (67)	86 (84)	78 (81)	100 (100)	100 (100)	77 (75)	74 (70)	87 (85)	84 (82)	100 (100)	100 (100)

(B) Average Fill-Rate during the first $\frac{k}{2}$ periods of the *PO* phase

5.2.2 Ethically Complex Interventions

There may be cases where standard interventions such as those discussed in the previous section may not be effective in increasing the average donation rate. This may be due to the lack of responsiveness of the donor pool to the interventions, especially during disasters. Nonetheless, blood banks still need to increase donations, which compels us to consider ethically complex interventions as viable alternatives. We highlight that VNRBD should still be the focus of blood banks and that ethically complex interventions should be thought of as a last resort. There is merit however in at least being informed on the effectiveness of these types of interventions where standard strategies are exhausted. After all, inaction knowing that there are options available is also ethically complex.

We consider three ethically complex interventions that can be examined using the simulation: (1) deferring demand, (2) rationing blood, and (3) tapping non-traditional donors. We also summarize the ethical considerations for each intervention in Table 5.6. Though we already analyzed reducing regeneration, we include it in the table for completeness.

TABLE 5.6: Summary of Ethical Considerations per Stakeholder

Intervention	Donors	Patients	Culture/Systems
Reducing Regeneration	-health side-effects		
Rationing Blood		-health side-effects	
Non-Traditional Donors	-can aggravate current health issues	-possible low quality blood	
Deferring Demand		-elective surgeries can become serious	-overwhelmed blood banks after pandemic

5.2.2.1 Deferring Demand

Deferring demand refers to the intervention of rescheduling or postponing elective surgeries and treatments to after the disaster or pandemic (An et al., 2011). It is sometimes necessary to do this postponement in order to free up bed-space and resources for those affected. This leads to lower utilization of blood during the *DIS* phase because there will be fewer patients. As a result, demand deferral is the most used intervention during emergencies with an estimate of 28 million surgeries world-wide deferred during COVID-19 (Ljungqvist et al., 2020). However, it is projected that the backlog of surgeries would need a median of 45 weeks to clear up after (COVIDSurg Collaborative, 2020). This may eventually result in a deteriorating quality of life for deferred patients, even though the surgeries may initially be elective. We emphasize that deferral as an intervention is done mostly to free up bed-space an increase in patients. However, the resulting deferral directly impacts the utilization of blood which merits its inclusion as an ethically complex intervention for blood supply.

We conduct a search for the required percent reduction in demand, called the *deferral rate*, given an increase of γ^{PL} during the *DIS* phase for various FR and L , found in Table 5.7. For higher FR , the percent demand deferral is much lower than that of the increase in demand, as can be seen with $FR^{TAR} = 100$. For demand increases of $\gamma^{PL} = 0.1$, the maximum percent demand deferral is 3 and 1% for Norway and the UK, respectively. This is the same when $\gamma^{PL} = 0.2$, where the maximum demand deferral is 11 and 10%.

When FR is lower ($FR^{PR} = 80$) and the demand increase is low ($\gamma^{PL} = 0.1$), then the deferral rate more closely matches the demand increase. Lastly, for a shorter L , the deferral rate is lower because of the non-steady state dynamic.

TABLE 5.7: Deferral Rate Needed to Meet FR^{TAR} ($p^{PR} = p^{DIS}$)

DIS Length	$FR^{PR} = FR^{TAR}$	Norway						UK					
		80		90		100		80		90		100	
	γ^{PL}	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
	5	0.09	0.17	0.10	0.17	0.0	0.0	0.9	0.17	0.09	0.17	0.0	0.0
	10	0.10	0.17	0.10	0.17	0.0	0.0	0.09	0.17	0.09	0.17	0.0	0.0
	50	0.10	0.17	0.10	0.17	0.0	0.0	0.09	0.17	0.09	0.17	0.0	0.0
DIS Length	100	0.10	0.17	0.10	0.17	0.0	0.04	0.10	0.17	0.09	0.17	0.0	0.02
	200	0.10	0.17	0.10	0.17	0.03	0.11	0.10	0.17	0.10	0.17	0.01	0.10

5.2.2.2 Rationing of Blood

Blood banks can also implement a reduction in the amount of blood transfused to patients (Zimrin and Hess, 2007; Stanworth et al., 2020). This intervention rations blood to reduce demand, but may put patients at risk. For some health conditions such as thalassemia, a sudden reduction in regular transfusions can lead to pain and fatigue (CDC, 2020). In some studies, however, it has been determined that blood transfusion can be reduced during treatment of leukemia and other illnesses without increasing the mortality or morbidity rate (Zimrin and Hess, 2007). Thus, rationing of blood has been discussed as an intervention that could be used during disaster scenarios.

Rationing can be modelled similarly to the donor deferral intervention where there is a deferral rate that gets multiplied to the demand, and so the results of the simulation for deferral for the *DIS* phase would also apply.

5.2.2.3 Tapping Non-Traditional Donors

Another ethically complex intervention would be to relax eligibility requirements for donors. Note that the total number of eligible donors are a direct result of the rules dictated by a blood bank. This includes age requirements, health backgrounds, and even travel history as a means to filter those who are allowed to donate. By allowing previously ineligible donors (e.g. older donors, MSM (men having sex with men), tattooed individuals) to donate, the donor pool increases and could lead to more donations. This intervention is considered ethically complex since there is a possibility that it could lead to medical complications for donors and lower blood quality for patients (Germain 2016; Hoad et al., 2019; Davison et al. 2019).

In this case, all the analysis pertaining to $\bar{N}$ intervention would apply since there is additional donors being added into the system.

5.3 Managerial and Policy Implications of Chapters 4 and 5

One of the main policy implications from this study is the importance and advantage of being a mature blood bank with a strong culture of donation when facing disasters. In Chapter 4, we see this when the extent at which interventions are implemented become smaller as p^{PR} increases. In this chapter, this can be seen with how in simulations, a higher FR means having the option to implement either a dynamic donation rate strategy or a fixed donation rate without having to worry of drops in fill-rate in the *DIS* and *PO* phases. Even when it comes to ethically complex interventions like demand deferral, the extent of implementation as can be seen from the deferral rates is much lower in mature blood banks.

In order to prepare for the *DIS* phase, blood banks must then focus its efforts in increasing the culture of donation (i.e. increase p^{PR}) even in the *PR* phase. Chapter 4's equivalence results in Figure 4.7 allude to this. While the interventions were analyzed with disasters in mind, it is equally applicable in the *PR* phase such that if the culture of donation is not well developed (p is low), then increasing p (focusing on changing the culture of donation) is the best way to increase blood donation. This sets up the blood bank for better blood management not only in the *PR* phase but also in the *DIS* phase.

At the same time, focusing on becoming a more mature blood bank means having more control over the donation rates of its population. If that is the case, then blood banks now have the option to implement a dynamic donation rate strategy that allows for immediate achievement of the target number of donations in the *DIS* phase, which when coupled with higher fill-rates also solves the volatility problem in the *PO* phase.

Another policy implication is the impact of volatility during and after disasters. When implementing interventions, blood banks must consider the initial increase (resp. decrease) in the *DIS* (resp. *PO*) phases when planning. The reaction of the

population to disasters affects how long the system goes back to a more predictable, steady state. The dynamic donation strategy mentioned can help address this by controlling the donation rates to reach an artificial steady state that is equivalent to the steady state targets.

We also introduced the different interventions $\bar{p}$, $\bar{k}$, and $\bar{N}$, and found that each becomes the preferred intervention in different contexts during disasters. $\bar{p}$ is preferred for less mature blood banks, $\bar{k}$ is preferred for blood banks with relatively high donation rates, and $\bar{N}$ is preferred for mature blood banks. Along with this, we introduced the ethically complex interventions that can be implemented with theoretically the same results as that of the standard interventions. However, there has to be caution on the part of the blood bank when thinking about implementing these alternatives (summarized in Table 5.6). For example, rationing blood is effective because donors received less transfusion quantities than what is normally recommended by health professionals. Policy makers then have to be aware of these health implications and get knowledge on the specific illnesses and corresponding therapy and procedures that their patients need so that safety is maintained.

Deferring demand also increases FR during the pandemic, but it sacrifices a blood bank's resources in the future. Blood banks can use demand deferral if they are sure that their systems will be able to prepare and support this demand in the future and if the patients are able to wait for a long time before being entertained. Otherwise, the blood bank is merely postponing the drop in supply while exposing patients to the possibility of worsening health conditions.

When tapping non-traditional donor pools, policy makers have to consider the overall health of the population group that they are going to allow to donate. Caution has to be exercised when allowing an entire group which was previously unable to give blood to donate. Small, incremental eligibility rule relaxations should be implemented as opposed to a sweeping eligibility requirement change. For example, only those above 65 years old who meet a certain health criteria on haemoglobin levels will be allowed to donate. In some cases, outdated eligibility criteria that have been disproven (such as those for MSM and tattooed individuals (Germain et al. 2014; Germain 2016; Hoad et al. 2019)) can be removed to have a larger donor pool without having any issues.

5.4 Conclusion of Chapters 4 and 5

These last two chapters are able to provide blood banks a way to analyze and compare blood supply interventions during disasters. A comparison among three main categories of interventions: (1) recruitment and retention, (2) donation frequency, and (3) regeneration time shows that donation frequency type interventions are preferred by blood banks with low donation rates, while recruitment and retention and regeneration time are preferred by those with higher donation rates. The comparisons also show to what extent each of the available interventions should be implemented when blood banks are faced with disasters. Further, we introduce an effort factor for the interventions which allows blood banks to see which ones are more efficient in their context.

In implementing these interventions, the sudden change in donation rates leads to volatility in both the *DIS* and *PO* phases. An iterative calculation procedure which gives a good approximation of the expected number of donations can be used as a way to analyze this non-steady state scenario. Depending on whether the donation rates increase or decrease, these periods of volatility could last for long periods of time. To address this, we introduced a dynamic donation rate strategy that is obtained by recursively calculating the donation rate that matches the target level of donation.

We validated these results by constructing a simulation model that incorporates demand and inventory processes in blood banking and conducting a numerical study. We find that although the dynamic donation rate strategy is able to reach the target donation quantity, our simulation model shows that when fill-rate is low, this strategy leads to lower fill-rates during the *DIS* phase. Conversely, using the steady-state formula leads to higher than intended fill-rates in the *DIS* phase and lower fill-rates in the *PO* phase. This problem is not encountered by mature blood banks with high fill-rates which emphasizes the need for maturing blood banks to transition into mature ones. However, when there are multiple disaster waves that are not spaced far apart from each other, we find that even mature blood banks have difficulty meeting targets.

Lastly, three ethically complex interventions were also introduced. Deferring demand can help increase fill-rates, and the reduction required to reach the target fill-rates also favors mature blood banks with initially high fill-rates. Since rationing

of blood is modelled similarly to demand deferral, the extent of its implementation would also be lower in mature blood banks. Tapping non-traditional donor pools can also be implemented as a special case of the $\bar{N}$ intervention.

There are many possible avenues for future research. For chapter 4, one would be in constructing an analytical model that includes first time donors in order to see the impact of interventions for this group. Next, data can be gathered on the effort parameters for different countries as a case study to validate whether these countries are doing the more efficient interventions. In the current chapter, multiple blood types can be incorporated in the simulation, as well as checking for whether changing the issuing rules from FIFO to other more complex policies would affect the dynamic donation strategy.

Chapter 6

A Network Optimization Approach To Recruitment and Retention of Blood Donors

6.1 Introduction

Blood donation comes from willing and able blood donors. Currently, there is still no widely used and viable substitute for blood such that the most sustainable and important source still comes from donors. Donors also cannot be forced to donate in the prevailing standard of VNRBD (Voluntary Non-Remunerated Blood Donation) blood banking systems. That is, they have to donate out of their own free will without coercion. However, this does not mean that blood banks would just sit back and wait for donors to arrive at their facilities. In fact, blood banks can still influence these donors to donate through marketing and recruitment campaigns (James et al, 1996). These can range from simple SMS or e-mail messages, to more complex influencer and blood ambassador messages and even radio or television segments.

As in a commercial organization, blood banks need to focus their marketing effort to a target market so as to maximize the effectiveness of a marketing campaign. Blood banks then need to decide which donor groups to target and when they should be targeted. This requires an understanding of the demographics of a blood bank's donor pool. At the same time, these donors have varying donation rates

depending on the number of donations that they made in the past (i.e. the number of donations in their donor career). This number ranges from zero donations made as in new/prospective donors, to the hundreds for more experienced, regular donors. These donors with varying donor careers pose trade-offs for a blood bank. Usually, those who are experienced, regular donors are much older, who while they may give donations in the short-term, will also have a shorter run-time of donations in the future. On the other hand, new donors are more likely to have lower donation rates but longer donor careers ahead of them (Schreiber et al., 2003). With a limited budget, blood banks have to decide whether they should recruit new donors or retain regular ones while deciding which donor demographic should be targeted. This is a practical problem that was alluded to by the Singapore Red Cross in a published in the *Straits Times* which said that there is a decline in younger donors, and so there is a reliance on older ones (Poon, 2017). Similarly, over half of the donors in the UK are aged 45 and older (NHS Blood and Transplant, 2018), and 45% of donors in the US are 50 and older (Abbott, 2019).

In this chapter, we aim to address this problem by formulating a model that determines the optimal recruitment and retention targeting action that blood banks should take. We also want to see the effect of this optimal policy on the total number of donors to be able to quantify the trade-off of recruitment and retention. We utilize a network flow optimization approach and find that because of the ease with which it can be solved through commercially available solvers, that it can be an effective way to solve the blood bank's problem.

This chapter proceeds as follows: In the rest of this section, we discuss the related literature. In Section 6.2, we discuss the donor network formulation. Section 6.3 proceeds to discuss the equivalent MILP model. Lastly, Section 6.4 discusses the case study.

6.1.1 Related Literature

This chapter contributes to network flow literature. A single commodity network flow model (SCNF) is a type of network optimization problem where a single item (or commodity) is optimized to move from a source to a sink node given an objective and constraints on edges. The objective of maximizing the total amount of the commodity to flow from a source to a sink node is called the max-flow problem

(Ford and Fulkerson, 1962). This has been extended to a more general network model called the multi-commodity network flow model (MCNF), which has been extensively studied in literature (see for example: Wang, 2018 for a comprehensive literature review). Studies related to SCNF and MCNF fall broadly into two categories: algorithmic and applications based. Algorithmic studies focus on efficient solution methods like algorithms and heuristics to solve network flow problems. On the otherhand, applications-based literature use MCNF to solve problems by formulating them as networks. For example, it has been used in telecommunications (Lin and Yee, 1992; Shepherd and Zhang, 2001), Logistics (Hane et al. 1995; Bellmore et al. 1971), and Production Schedulling (D'amours et al. 1996). This chapter also focuses on an application of SCNF which considers donors as the unit that flows through the edges between nodes. However, we consider a multi-period setting where the objective is to maximize the sum of flows across a planning horizon.

Blood supply chains have also been modelled through networks. For example, Masoumi et al. (2017) used a network approach to determine the effect of mergers and acquisitions of blood banks on the cost across the blood supply chain. Nagurney and Dutta (2019) used a novel game theoretic network model that showed evidence of higher service quality of blood banks when competition is present. These blood bank applications focus more on blood bank behavior and positioning rather than recruitment and retention of donors.

Lastly, this chapter also contributes to donor recruitment and retention literature. Majority of blood recruitment and retention literature focuses on statistical studies to isolate the likelihood of future donation. Godin et al (2005, 2007) for example conducted surveys and used linear regression to say that the intention to donate is the biggest determinant of repeat donation. They also noted that the anticipated regret, and demographic factors like being male, having a higher education level, higher age, and a smaller distance from a blood drive makes a person likelier to donate. Bosnes et al. (2005) on the other hand used a logistic regression model to predict the probability of repeat donation. In their study, they found that personal contact as opposed to written invitation is more effective in inducing donations. Schreiber et al. (2005) noted that first time donors who returned within 12 months of their first donation were more likely to become regular donors, emphasizing the importance of retaining them within their first year. Lastly, Notari et al. (2009)

analyzed the effect of age on donor return patterns in first time blood donors in the United States and suggested that adverse events especially in younger donors should be prevented in order to increase retention. While these examples illustrate the knowledge of blood banks and scholars on determinants of future donation, they are taken in isolation from the planning process done by blood banks when deciding on marketing campaigns. That is, these studies still do not say definitively who to target given the number of each type of donor in a blood bank's donor pool. This chapter aims to address that gap.

6.2 Donor Network Formulation

We use a network optimization approach and solve the model using a linear programming formulation. We consider a fluid model where the transition rates are deterministic. These transition rates can be estimated prior to optimization using blood donor data if available. We recognize that there may be some pitfalls in sequential estimation and optimization, especially in the case of targeting donors.. This is because of the influence of targeting on donor behavior which then translates to a change in parameters (see for example Loke et al., 2021). However, this simplifying methodology allows us to gain some insight into who should be targeted by blood banks given its current donor demographics.

Suppose we have m donor categories denoted by $i = 1, 2, \dots, m$ and within each category, there are l subcategories $j = 1, 2, \dots, l$. For the purposes of this chapter, the categories refer to the division on frequency of donation, and the subcategories refer to the age of the donor. Without loss of generality, we assume that the categories refer to the absolute number of donations by each donor found in that category. For example, donors in $i = 3$ have made three donations in their donor career (further explanation for $i = m$ and how it includes donors with more than m donations will be discussed later). This division of donor categories and subcategories is a typical segmentation that blood banks use in order to be able to focus their marketing and recruitment campaigns. For example, blood banks would differentiate between one-time donors (i.e. those who donated once) and regular donors (i.e. those who donated more than once). These are further split into different age groups (e.g. below 20 years old, 20-24, 25-30, etc.) or even gender and well-being among other distinct parameters. In our case, we focus on

age as the subcategory although the model can be easily extended to cover other categorizations.

Each category and subcategory pair (i, j) represents a node in a digraph, and we will refer to (i, j) as the state that a donor is in. We consider a finite, periodic, time horizon of T such that in each period $s = 1, 2, \dots, T$, the number of donors who are in state (i, j) is $c_s^{i,j}$. Directed arcs exist from state (i, j) to state $(i, j + 1)$, $(i + 1, j + 1)$, (i, j) , and a drop-off state DO such that donors flow through the arcs into the corresponding states according to a fixed rate from period s to $s + 1$. The arcs represent different events as follows:

- $(i, j) \rightarrow (i, j + 1)$: aging of donors without additional donation in the previous period
- $(i, j) \rightarrow (i + 1, j)$: additional donation without aging in the previous period
- $(i, j) \rightarrow (i + 1, j + 1)$ aging of donors with donation done in the previous period
- $(i, j) \rightarrow (i, j)$ no aging and no donation in the previous period
- $(i, j) \rightarrow DO$ donor drops-off and becomes ineligible for donation (e.g. illness, death, age limits)

Notice that some states would have only a subset of the arcs because the direct successor states would not exist. This is true for $i = m$ and $j = l$, such that for $i = m$, only $(i, j) \rightarrow (i, j + 1)$, $(i, j) \rightarrow (i, j)$ and $(i, j) \rightarrow DO$ are applicable. This does not mean however that donors in state $i = m$ would not be donating anymore. Rather, the absence of the arc represents the fact that if more donations are made by individuals in the state, then they would still fall under that category. We see this with how blood banks group donors who have made a significant number of donations even though the range of donation quantities may be large (e.g. donors with 20 donations are grouped together with those who have made 50 donations). This also implies that donors who have these number of donations already have similar characteristics and willingness to donate such that there would be no benefit from separating the group. Similarly, for $j = l$, only $(i, j) \rightarrow (i + 1, j)$, $(i, j) \rightarrow (i, j)$, and $(i, j) \rightarrow DO$ are applicable. In this case, an aging donor would

go to the DO state and the arcs corresponding to aging would be included in the rates of $(i, j) \rightarrow DO$.

Additionally, a state $i = 0, j = 1, \dots, l$ exists with a directed arc from $(0, j) \rightarrow (1, j)$. This represents the arrival of new donors who have not yet donated in the past. the corresponding arrival into a subcategory j refers to the age group that the new donor belongs to. This gives us a network $G(N, A)$ such that the set of nodes $N = \{(i, j) : i = 0, 1, \dots, m; j = 1, 2, \dots, l\} \cup DO$ and the set of arcs A as described for each node earlier. Figure (6.1) gives an example of this network with $m = 3$ and $l = 3$.

FIGURE 6.1: Sample Donor Network with $m = 3$ and $l = 3$.

6.2.1 Transition Rates

6.2.1.1 Drop-off Rates

We first discuss the drop-off rate $q^{i,j}$, $i = 1, \dots, m$ and $j = 1, \dots, l$ and assume that the drop-offs occur at the start of every period before any donation happens. $q^{i,j}$ represents the transition rate from a state (i, j) to DO . We also assume that

the drop-off rates are independent of any targeting or marketing effort that blood banks do. What this implies is that the other transition rates apply to only $\bar{c}_s^{i,j} = c_s^{i,j}(1 - q^{i,j})$ donors of each state, because a fraction $q^{i,j}$ have already dropped-off.

If data is available, obtaining an estimate of $q^{i,j}$ can be done through logistic regression, such that

$$q^{i,j} = q(\mathbf{X}_s^d | d \in (i, j)) = \frac{\exp(\alpha^0 + \alpha \mathbf{X}_s^d)}{1 + \exp(\alpha^0 + \alpha \mathbf{X}_s^d)} \quad (6.1)$$

where $\mathbf{X}_s^d$ are donor specific covariates of all donor d that are in state (i, j) at the beginning of the planning horizon. Otherwise, $q^{i,j}$ can be extrapolated from other sources like life expectancy and even disease prevalence rates.

Recall our assumption that donors dropping-off means that they cannot come back to donate. This is equivalent to assuming that the drop-offs are because of reasons that end the donor career including reaching age requirement limits. As a result, we need to make some adjustments for the drop-off rate of $j = l$ because this is the subcategory aging means the donor would automatically drop-off. We make an assumption that aging happens according to a uniform distribution with range equal to the four times the age range L^l of the subcategory. This is because we assume that a period is equal to one regeneration period (i.e. 12 weeks = 3 months = 1/4 of a year). Therefore, the dropoff rate due to aging $q_{\text{aging}}^{i,l} = \frac{1}{4L^l}$ for state (i, l) , $i = 1, \dots, m$, and the overall drop-off rate is $q^{i,l} = q_{\text{aging}}^{i,l} + q(\mathbf{X}_s^d | d \in (i, j))$ for all i

6.2.1.2 Donation and Aging Rates

Next, we discuss the aging and donation rates for each of the states and refer to an arc that goes from $(i, j) \rightarrow (i', j')$ for some applicable state (i', j') as $(i, j)(i', j')$ for compactness. As mentioned earlier, donors in state (i, j) transition to state (i', j') according to deterministic rates. We denote these rates by a fraction $p_t^{(i,j)(i',j')}$ for $i = 1, \dots, m$ and $j = 1, \dots, l$ such that $0 \leq p_t^{(i,j)(i',j')} < 1$ for $t = NI, I$. A state can be targeted by marketing campaigns in period s and this determines which rate is used in the transition. This is given by a binary decision variable $\gamma_s^{i,j}$. If (i, j) is targeted, then $\gamma_s^{i,j} = 1$ and this changes the transition rates to $p_I^{(i,j)(i',j')}$. Else, $\gamma_s^{i,j} = 0$ and the transition rates are $p_{NI}^{(i,j)(i',j')}$. It is necessary that

$p_I^{(i,j)(i',j')} > p_{NI}^{(i,j)(i',j')}$ for all $i = 1, \dots, m$, $j = 1, \dots, l$, $i' = i + 1$, $j' = 1, \dots, l$. Otherwise, $\gamma_s^{i,j} = 0$ for all i, j and it would not make sense to target donor groups. Note that while targeting donors increases the rate of donation (i.e. $p_t^{(i,j)(i+1,j)}$ and $p_t^{(i,j)(i+1,j+1)}$) and not the rate of aging or drop-off, the rates being deterministic fractions (which should add up to one) means that $p_t^{(i,j)(i,j+1)}$, $p_t^{(i,j)(i,j)}$ would also change from the targeting. This condition is given by (6.2) below.

$$p_t^{(i,j)(i,j)} + p_t^{(i,j)(i+1,j)} + p_t^{(i,j)(i,j+1)} + p_t^{(i,j)(i+1,j+1)} = 1 \quad (6.2)$$

Similar to the drop-off rates, we can estimate the probability of donation if data is available through logistic regression as follows:

$$p(\mathbf{X}_s^d | d \in (i, j)) = \frac{\exp(\beta^0 + \beta \mathbf{X}_s^d)}{1 + \exp(\beta^0 + \beta \mathbf{X}_s^d)} \quad (6.3)$$

and noting that $p(\mathbf{X}_s^d | d \in (i, j)) = p_t^{(i,j)(i+1,j)} + p_t^{(i,j)(i+1,j+1)}$. That is, it is a sum of the transition rates for both donating and aging or not aging. $p^{\text{aging}} = p_t^{(i,j)(i,j+1)} + p_t^{(i,j)(i+1,j+1)}$ is obtained by using the same assumption as in donor drop-offs that the distribution of ages per category is uniform. Lastly, $p_t^{(i,j)(i,j)}$ can be obtained by taking the first estimate and the uniform distribution assumption to solve:

$$p_t^{(i,j)(i,j)} = (L^{i,j} - 1) \left(\frac{1 - p(\mathbf{X}_s^d | d \in (i, j))}{L^{i,j}} \right) \quad (6.4)$$

We can then solve a system of four equations and four unknowns to get the estimates of the transitions.

Recall that we assumed that a donation means a transition from category i to $i + 1$. However, in the case of when $i = m$, we assume that an additional donation would result in the donor remaining in the category (note though that it can move subcategories). This means that the transition rate $p_t^{(m,j)(m,j)}$ also includes the rate at which donors donate while still remaining in the same subcategory in the next period (i.e. $p_t^{(i,j)(i+1,j)}$ in other states). We distinguish the two rates because we will need the distinction in computing for the objective function. We let $p_{t,d}^{(m,j)(m,j)}$ and $p_{t,nd}^{(m,j)(m,j)}$ be the transition rates of a donating (d) and non-donating (nd) donor in category m such that $p_t^{(m,j)(m,j)} = p_{t,d}^{(m,j)(m,j)} + p_{t,nd}^{(m,j)(m,j)}$ for all j . $p_{t,d}^{(m,j)(m,j)}$ is computed as a result of estimating $p(\mathbf{X}_s^d | d \in (m, j)) = p_{t,d}^{(m,j)(m,j)} + p_t^{(i,j)(i+1,j+1)}$ and $p_{t,nd}^{(m,j)(m,j)}$ is obtained from solving the system of linear equations.

TABLE 6.1: Linear Programming Model Notation

Indices	Description
i	donor category from $0, 1, \dots, m$ (based on number of donations)
j	donor subcategory $1, \dots, l$ (based on donor age)
s	period from $1, \dots, T$
Parameters	
$p_t^{(i,j)(i',j')}$	transition rate from state (i, j) to (i', j') for $t = NI, I$
$C_t^{0,j}$	arrival of new donors for $j = 1, \dots, l, t = NI, I$
$c_1^{i,j}$	initial number of donors per state (i, j)
$q^{i,j}$	drop-off rate for state (i, j)
M	large, positive, constant number
B	maximum number of campaigns that can be used to target donors
Decision Variables	
$\gamma_s^{i,j}$	binary variable, 1 if state (i, j) during period s is targeted, 0 otherwise
$c_s^{i,j}$	number of donors in state (i, j) , in period $s = 2, \dots, T$

Lastly, the blood bank can also target state $(0, j)$ if it wants to recruit new donors. These donors arrive according to deterministic values $C_t^{0,j}$, $t = I, NI$, depending on whether the subcategory is targeted such that $C_I^{0,j} > C_{NI}^{0,j}$.

6.3 Mixed Integer Linear Programming Formulation

From the network formulation, we translate the problem into a linear program to facilitate an easier solution process using standard LP software.

6.3.1 Objective Function

The objective is to maximize the total number of donations received during the entire planning horizon. This is translated to an objective function which takes the product of the number of donors in each state and the transition rates: $\bar{c}_s^{i,j} p_t^{(i,j)(i+1,y)}$. We also include the number of deterministic arrivals from new donors. This is then

summed up across all periods $s = 1, 2, \dots, T$.

$$\begin{aligned}
\text{Max } Z = & \sum_{s=1}^T \left(\sum_{i=1}^{m-1} \sum_{j=1}^{l-1} \bar{c}_s^{i,j} \gamma_s^{i,j} \left(p_I^{(i,j)(i+1,j)} + p_I^{(i,j)(i+1,j+1)} \right) \right. \\
& + \sum_{i=1}^{m-1} \bar{c}_s^{i,l} \gamma_s^{i,l} p_I^{(i,l)(i+1,l)} + \sum_{j=1}^l \bar{c}_s^{m,j} \gamma_s^{m,j} p_{I,d}^{(m,j)(m,j)} + \\
& \sum_{i=1}^{m-1} \sum_{j=1}^{l-1} \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) \left(p_{NI}^{(i,j)(i+1,j)} + p_{NI}^{(i,j)(i+1,j+1)} \right) \\
& + \sum_{i=1}^{m-1} \bar{c}_s^{i,l} (1 - \gamma_s^{i,l}) p_{NI}^{(i,l)(i+1,l)} + \sum_{j=1}^l \bar{c}_s^{m,j} (1 - \gamma_s^{m,j}) p_{NI,d}^{(m,j)(m,j)} \\
& \left. + \sum_{j=1}^l \left(C_I^{0,j} \gamma_s^{0,j} + C_{NI}^{0,j} (1 - \gamma_s^{0,j}) \right) \right) \quad (6.5)
\end{aligned}$$

Note that the expression $\bar{c}_s^{i,j} \gamma_s^{i,j}$ is a product of a continuous and binary variable. Standard linearization techniques can be done to remedy this.

6.3.2 Decision Variables

The decision variables are the binary variable $\gamma_s^{i,j}$ and the donor number in each state $c_s^{i,j}$. $c_s^{i,j}$ is a decision variable that is used only to preserve the flow of donors from period to period, and would not be a main decision done by the blood banks. This can be seen by equality constraints that will be introduced in the next section. On the other hand, $\gamma_s^{i,j}$ is a direct decision by the blood bank which targets donor categories and subcategories.

6.3.3 Constraints

The constraints can be divided into two main categories: (1) Donor Flow Constraints and (2) Targeting/Budget Constraints. Donor flow constraints ensure the preservation of the number of donors in each state (i, j) for all time periods s . Because of the standard nature of these constraints, we relegate its illustration in Appendix D.2.

A more important set of constraints would be targeting/budget constraints that either restrict or require the targeting of donors. This last set of constraints is what

blood banks are faced with when they are deciding whether to recruit or retain. We highlight that the targeting strategy modelled through $\gamma_s^{i,j}$ creates a trade-off for blood banks only if there are restrictions in place. If there are no restrictions, then the optimal decisions would be for $\gamma_s^{i,j} = 1$ for all i, j, s . We illustrate some typical constraints that blood banks face when targeting donors as follows.

6.3.3.1 Budget Constraints

Blood banks typically face constraints on the number of campaigns that they can run in recruiting or retaining donors. This is usually because of monetary or budget limits that permit only a certain number to be implemented in a fiscal year. We can also think of this as a limit on donor exposure such that the more donors are exposed to recruitment efforts, the less effective future recruitment efforts will be. A limit on how many blood banks can implement would prevent this marketing saturation. The budget constraints can either be for total planning horizon as in (6.6) or for each period as in (6.7).

$$\sum_{s=1}^T \sum_{i=1}^m \sum_{j=1}^l \gamma_s^{i,j} \leq B \quad (6.6)$$

$$\sum_{i=1}^m \sum_{j=1}^l \gamma_s^{i,j} \leq B_s \quad \forall s \quad (6.7)$$

6.3.4 Target Requirement Constraints

There can also be constraints on requiring that a specific donor category or sub-category is targeted. This is usually the case when blood banks want to build their donor base and mandate that new donors be targeted.

$$\sum_{j=1}^l \gamma_s^{i,j} \geq 1 \quad \forall s, \text{ for a specific } i \quad (6.8)$$

if the target is a category, and

$$\sum_{i=1}^m \gamma_s^{i,j} \geq 1 \quad \forall s, \text{ for a specific } j \quad (6.9)$$

if the target is a sub-category. We clarify that the target requirement constraints don't mean that other nodes can't be targeted. It just ensures that specific the specific i or j determined by the blood bank will be.

6.3.4.1 Target Market Constraints

We can also implement constraints that restrict blood banks to only target one age group. This is so that marketing efforts can gain the maximum effect as campaigns are usually targeted towards specific age groups.

$$\sum_{\forall i} \gamma_s^{i,j} = 0 \quad \forall j \neq \text{specified } j' \quad (6.10)$$

We can see that modeling the donor recruitment and retention problem as a network model with binary decision variables on targeting makes it easier to include the typical campaign and marketing constraints that blood banks face.

6.4 Model Implementation and Numerical Results

6.4.1 Data Set Description

Ideally, transactional data on blood donation should be used in order to estimate the transition and drop-off rates. However, we used proxy data from WHO's *Global Status Report on Blood Safety (2016)* in order to inform the parameters of the model. We obtained number the number of regular donations, number of first-time donations, and age-specific donation rates per 1000 people on four countries: Zimbabwe, South Africa, Singapore, and Australia. These countries were selected because these four have age-specific donation rates listed in the WHO report, and it was done so to illustrate the difference in donation rates among mature and maturing countries that do not use family replacement donation (i.e. all donations come from VNRBD). We scale the blood donation data to account for 100% coverage of blood donation in each country. Since we needed drop-off rates, we used publicly available data to obtain age-specific death-rates and assumed that these

would constitute the drop-off. Lastly, we obtained population data on the different age-groups.

We follow the age-group classification used by the WHO, namely 16 to 17, 18 to 24, 25 to 44, 45 to 64, and 65 to 70. These age groups are referred to subcategory $j = 1, 2, \dots, 5$, respectively. The data set that was used and the computations to arrive at the parameters can be found in Appendix D.1. From the data, we scaled the initial number of donors to a total of 1000 to serve as the baseline number for each country.

6.4.2 Results and Analysis

We implemented the model in python 3.0 and solved it using GUROBI optimization software.

The targeting decisions using parameters obtained for each country can be found in Table 6.2. It can be seen that the optimal targeting decisions are different for each country in that they target different subcategories. However, the general observation is that optimal targeting tends to focus on increasing the donation of those who have donated once (i.e. target category 1). This is primarily because of the large number of one-time donors in each country. Given that there is a limit on the number of campaigns that can be run, a campaign that targets a lot of donors at once is a more efficient use of resources. At the same time, literature shows that if one-time donors donate for a second time, then they are more likely to be regular donors who donate for the rest of their lives assuming there is no cause for drop-off (Yu et al., 2007). If the blood-bank instead decides to require that they target new donors in each period, each would then focus on recruiting donors from the subcategories that would have the highest rate of donation. This can be a problem in the case of countries like Australia where the highest rates of donation occur in older donors. That is, there is a risk for targeting older donors because they have a shorter donor career ahead of them compared to younger donors.

Table 6.3 compares the objective values and the total number of donors at the end of the planning horizon for the optimal, target new donors, and no campaign budget scenarios. For a detailed breakdown of the total number of donors under each scenario and country, refer to Figure D.1 in Appendix D.3. The first observation is

TABLE 6.2: Optimal Donor Targeting with $T = 8$ and $B = 8$ (period s , category i , sub-category j)

Zimbabwe		South Africa		Singapore		Australia	
Optimal	Target New Donors	Optimal	Target New Donors	Optimal	Target New Donors	Optimal	Target New Donors
(1,0,1)	(1,0,1)	(1,2,3)	(1,0,3)	(1,0,3)	(1,0,3)	(1,0,4)	(1,0,4)
(1,1,1)	(2,0,1)	(4,1,3)	(2,0,3)	(2,1,3)	(2,0,3)	(1,1,4)	(2,0,4)
(2,1,1)	(3,0,1)	(5,1,3)	(3,0,3)	(3,1,3)	(3,0,3)	(2,0,4)	(3,0,4)
(3,1,2)	(4,0,1)	(6,1,3)	(4,0,3)	(4,1,3)	(4,0,3)	(2,1,4)	(4,0,4)
(4,1,2)	(5,0,1)	(7,1,3)	(5,0,3)	(5,1,3)	(5,0,3)	(3,1,4)	(5,0,4)
(5,1,2)	(6,0,1)	(7,1,4)	(6,0,3)	(6,1,3)	(6,0,3)	(4,1,4)	(6,0,4)
(6,1,2)	(7,0,1)	(8,1,3)	(7,0,4)	(7,1,3)	(7,0,3)	(5,1,4)	(7,0,4)
(7,1,2)	(8,0,2)	(8,2,3)	(8,0,2)	(8,1,3)	(8,0,5)	(5,2,4)	(8,0,2)

TABLE 6.3: Objective and Donor Values for Each Country Under Different Scenarios (% difference vs Optimal)

	Objective Value			Total Number of Donors		
	Optimal	Target New Donors	No Budget	Optimal	Target New Donors	No Budget
Zimbabwe	886	553 (-38%)	515 (-42%)	1,669	2,076 (+24%)	1,603 (-4%)
South Africa	978	854 (-13%)	843 (-14%)	1,215	1,318 (+8%)	1,214 (0%)
Singapore	1,059	855 (-19%)	820 (-23%)	1,437	1,627 (+13%)	1,405 (-2%)
Australia	1,441	1,339 (-7%)	1,271 (-12%)	1,280	1,400 (+9%)	1,238 (-3%)

that implementing campaigns can significantly increase the donations that blood banks receive. For example, in Zimbabwe and Singapore where having no budget means 42% and 23% lower donations, respectively, compared to optimal. This highlights the importance of marketing campaigns in increasing the blood supply of a country. Next, if we compare the trade-off between deciding whether to target new donors or use the optimal targeting decisions, using the optimal would result in higher donations but lower number of donors. Conversely, targeting new donors would result in a higher number of donors but a lower number of donations. This becomes an issue because blood banks would want to solve both of these problems of having more donors in the pipeline (i.e. focus on new, younger donors) while still having a large number of donations (i.e. focus on optimal actions).

6.4.2.1 Hybrid Objective

To address this, we introduce a hybrid objective function that incorporates both an increase in new donors and the number of donations in a planning horizon. The idea behind this new objective function is to assign weights W and $1 - W$, $0 \leq W \leq 1$ on each separate objective term (new donors and number of donations). This allows blood banks to choose the level at which they want to prioritize the two competing objectives. We clarify that new donors in this context refers to new arrivals into sub-category $j = 1, 2$. These represent the younger age groups and

would theoretically have longer donor careers if they stay on to become regular donors.

Let Z_1 be the expression found in (6.6), and $Z_2 = \sum_{s=1}^T \sum_{j \in \mathcal{D}} (C_I^{0,j} \gamma_s^{0,j} + C_{NI}^{0,j} (1 - \gamma_s^{0,j}))$. We have the hybrid objective function:

$$\text{Max } \bar{Z} = (1 - W)Z_1 + WZ_2 \quad (6.11)$$

where $\mathcal{D}$ is the set of all donor subcategories j that want to be tracked and increased by blood banks. For our purposes, $\mathcal{D} = \{j : j = 1, 2\}$. Figure 6.2 shows the percent difference of the objective function and the number of donors if a hybrid objective is used vs that of the original objective function. The increase in the number of new donors by using the hybrid objective can be significant. For example in the case of Australia (Fig. 6.2d), the first occurrence of a trade-off happens with a 44% increase in new donors for a 9% reduction in number of donations at $W = 0.85$. Similarly, for South Africa (at $W = 0.65$, 6.2b) and Singapore (at $W = 0.7$, 6.2c), the trade-off is 22% and 19% increase in new donors, respectively for a 4% reduction in total donations.

For Zimbabwe in Fig. 6.2a, the increase in number of new donors begins at a low $W = 0.1$ because of the high rate of arrival of new donors. This also leads to lower number of total donations for higher values of W since the objective now prioritizes allocating campaigns to recruiting donors rather than retaining donors and getting more donations.

The hybrid objective becomes a plausible substitute in the network optimization problem because it allows for an increase in new donors which blood banks need, while at the same time maintaining high donation quantities. Blood banks can adjust the weights to their needs and context and come up with a targeting policy that balances recruitment and retention.

6.4.3 Varying Time Horizons

We next examine the case of planning for different time horizons to see if there are differences among the optimal policies when T varies. The results of this study would suggest insights for short-term versus long-term planning of which types of donors should be targeted. We solve the model to optimality for $T = 4, 12, 24, 60$

FIGURE 6.2: Percent Difference of Number of Donations and Number of New Donors for Hybrid Objective vs Z_1 ($T = 8, B = 8$)

(A) Hybrid Objective Results for Zimbabwe

(B) Hybrid Objective Results for South Africa

(C) Hybrid Objective Results for Singapore

(D) Hybrid Objective Results for Australia

which corresponds to 1, 3, 6, and 15 years, respectively (recall one period is 3 months or 1 regeneration period). We assume that $B = 4$ for every four periods, that is, the blood bank can target at most four groups every year.

The optimal actions for each country can be found in Appendix D.3 in Tables D.5, D.6, D.7, and D.8. We compare the number of states in each category for a planning horizon to the same period range and states in the other planning horizons. For example, when $T = 4$, we obtain the number of states that are targeted in each category and compare that to the states that are targeted in the first four periods of $T = 12, 24$, and 60. We repeat this for $T = 12$, and compare it to the first twelve periods of $T = 24$ and 60, as well as comparing $T = 24$ with that of the first 24 periods of $T = 60$. The differences and trends can provide insight into what the targeting strategy should be depending on the length of the planning horizon.

FIGURE 6.3: Number of States Targeted in each Category and Period Range at Different T

Zimbabwe		T			
Category	Period Range	4	12	24	60
0	1 to 4	4	3	4	3
	1 to 12		9	11	11
	1 to 24			16	16
	1 to 60				27
1	1 to 4	0	1	0	1
	1 to 12		3	0	1
	1 to 24			8	8
	1 to 60				32
2	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				1
3	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0
4	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0
5	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0

South Africa		T			
Category	Period Range	4	12	24	60
0	1 to 4	2	4	4	4
	1 to 12		6	9	9
	1 to 24			9	15
	1 to 60				23
1	1 to 4	2	0	0	0
	1 to 12		6	3	3
	1 to 24			12	9
	1 to 60				34
2	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			1	0
	1 to 60				1
3	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0
4	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0
5	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			2	0
	1 to 60				2

Australia		T			
Category	Period Range	4	12	24	60
0	1 to 4	3	4	4	4
	1 to 12		8	12	12
	1 to 24			18	24
	1 to 60				30
1	1 to 4	1	0	0	0
	1 to 12		4	0	0
	1 to 24			4	0
	1 to 60				10
2	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				5
3	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				1
4	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0
5	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			2	0
	1 to 60				14

Singapore		T			
Category	Period Range	4	12	24	60
0	1 to 4	2	4	4	4
	1 to 12		4	9	10
	1 to 24			14	18
	1 to 60				31
1	1 to 4	2	0	0	0
	1 to 12		7	3	2
	1 to 24			10	6
	1 to 60				23
2	1 to 4	0	0	0	0
	1 to 12		1	0	0
	1 to 24			0	0
	1 to 60				2
3	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0
4	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				0
5	1 to 4	0	0	0	0
	1 to 12		0	0	0
	1 to 24			0	0
	1 to 60				4

We summarize the number of states targeted per category within each subset of the planning horizon in Figure 6.3. The results state that when there is a shorter planning horizon, the blood banks' optimal action would be to target a combination of new donors and those who donated once. As the planning horizon get larger, the blood banks' optimal policy shifts to one where it should target new donors at the beginning, and then target existing, more regular donors later on. We can see this by looking at the increasing number of 0 category states that are targeted from period ranges 1 to 12 and 1 to 24 as T increases, as well as the existence of targeting of category 2, 3, and 5 states when $T = 60$. This implies that the marketing strategies involved in recruitment and retention change depending on whether it is from a short-term or long-term perspective. Short-term planning will involve more retention of one-time donors, while long-term planning is focused on the build-up of new donors for the future who in turn would be targeted and retained as regular donors by then.

6.5 Conclusion of Chapter 6

In this chapter, we focused on the donor recruitment and retention problem given budget constraints. This involves deciding on which donor group to target with marketing campaigns and when to target them in a planning horizon. We used a multi-period network flow model to illustrate the problem and solved the equivalent MILP to find the optimal recruitment and retention policy given different sets

of country parameters. Based on the four representative countries, maximizing the number of donations leads countries to focus on retaining donors rather than recruiting new ones. As a result, the number of new donors is low under optimal targeting (in short-term planning scenarios). A hybrid objective which incorporates weights of both number of new donors and total number of donations addresses this concern. The results indicate that using the hybrid objective increases the number of new donors significantly compared to a low reduction in the total number of donations relative to the optimal. We also examined the optimal strategies in short-term and long-term planning scenarios which showed that long-term planning focuses on recruitment of new donors and eventually targeting of those who become more regular donors in the future. This chapter gives blood banks a tool and methodology that they can use in planning their donor recruitment and marketing campaigns. The hybrid objective also allows blood banks to change the weights easily depending on their priorities.

Further studies can focus on using transactional blood donation data. This can be used to estimate improvement in blood donation rates after targeting, which could further support the results of the study. In line with this, another possible extension would be to use data to allow for a stochastic change in the donation rates and new donor arrivals which would incorporate the effect of both inducing and changing demographics of each state. This changes the methodology from sequential estimation-optimization to a simultaneous one.

Chapter 7

Conclusion and Summary of Main Findings

Throughout the dissertation, we examined how to improve blood inventory management and fill-rate in both peace time and disasters, and characterized optimal policies for inventory and recruitment of blood donors.

In chapter 3, we introduced *regeneration* and *regenerative capacity* to describe how donors need to rest before being available to donate again. This is a generalization of capacitated inventory systems defined by two parameters: the regeneration time k and the incoming capacity vector $\mathbf{u}$. By conducting a novel state transformation, we are able to show that the optimal policy is a state-dependent threshold policy. This threshold is more sensitive to donors that are further down the regeneration schedule and has the property that if donors are not enough for the planning horizon, then either rationing or donor build-up will occur.

We showed that reducing regeneration time significantly decreases cost especially when the shortage cost is high and the incoming capacity is lower. This highlights how regulation on donor regeneration can greatly influence cost. The myopic policy is also an effective policy to use for blood banks with low ordering costs since it doesn't penalize firms from simply utilizing capacity.

We also extend the model to the case where donors are allowed to drop-off or incoming capacity is unrestricted (u_s is unr), where there is perishability of one

period ($m = 1$), where there is joint capacity and inventory optimization (for $k = 1$ and $m = 1$), and the case where there is non-stationary demand (D_s).

In Chapter 4 and 5, we focus on blood supply interventions. First, we constructed a stylized donation model that considers the steady state number of donations from repeat donors. Three interventions related to recruitment and retention, donation frequency, and reducing regeneration time were then compared and analyzed to get the equivalence among the three. We find that blood banks with lower donation rates would prefer to use donation frequency related interventions, while those with higher donation rates prefer recruitment and retention or reducing regeneration time interventions. Implementation of these interventions would lead to highly volatile donation quantities during the *DIS* and *PO* phases. However, blood banks with a good control of donation rates can implement interventions that follow a dynamic donation rate strategy. This results in being able to immediately meet donation targets and effectively removing the volatility of donation.

Then, we constructed a *DES* model based on stochastic inventory theory in order to incorporate demand and blood inventory processes in the analysis. We find through a numerical study that the dynamic donation rate strategy fails to achieve fill-rate targets if the initial fill-rate in the *DIS* phase is low, but it gives high fill-rate during the *PO* phase. Conversely, a fixed donation rate strategy gives high fill-rate in the *DIS* phase and low fill-rate in the *PO* phase. In all this, being a mature blood bank results in having high fill-rates in both phases regardless of the strategy used. It also means no being affected as much by multiple disaster waves, although short intervals between waves can still lead to lower fill-rates.

The simulation model used in Chapter 4 was essential in obtaining the insights, and can be used in similar analyses by blood banks in scenario planning. The necessity of this type of tool cannot be over-emphasized as it allows blood banks to prepare for disruptions in their operations.

Lastly, in Chapter 6, we looked at the donor recruitment and retention problem subject to budget constraints. A multi-period network flow model as used to illustrate the problem and the equivalent MILP was solved. Based on four countries, maximizing the number of donations means retaining old donors rather than recruiting new ones. As a result, the number of new donors is low under optimal targeting (in short-term planning scenarios). A hybrid objective which incorporates

weights of both number of new donors and total number of donations addresses this concern. The results indicate that the hybrid objective leads to an increase in the number of new donors and a low reduction in the total number of donations relative to the optimal. Optimal strategies in short-term and long-term planning scenarios also showed that long-term planning focuses on recruitment.

7.1 Summary

Overall, the dissertation provides (1) a new way to manage blood inventory through Chapter 3, (2) insights into the effect of disasters on blood donation and the usage of blood supply interventions through Chapters 4 5, and (3) a tool, through Chapter 6 that allows blood banks to plan their recruitment and retention strategy while incorporating both new donors and total donation. As a final note, we recall the pressing need for blood and highlight a previously made point that there is still no viable and sustainable substitute for it. It is no wonder that donation is aptly called the *gift of life*, as every additional donation that comes to address a global supply shortage literally saves lives. It is our goal from these studies that blood banks and countries are able to see that while blood may be in short supply, there are areas for improvement and consideration that could extend this gift to more people.

Appendix A

Proofs for Chapter 3

Throughout this appendix, with abuse of notation, let

$$\sum \frac{\partial w_s}{\partial x} = \frac{\partial w_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,0}} + \frac{\partial w_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,1}} + \dots + \frac{\partial w_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,k}} \quad (\text{A.1})$$

for some function $w_s(x_{s,0}, \mathbf{x}_s, \mathbf{v})$ where $\mathbf{v}$ is the vector of states other than $(x_{s,0}, \mathbf{x}_s)$.

A.1 Proof of Lemma 3.1, Lemma 3.4, and $L^\natural$ -convexity of the perishability problem with $m = 1$

The proof is based from Benjaafar et al. (2017) but includes corrections to errors in their proof. We also explicitly prove the bound of the derivative of the value function which can only be done by simultaneously showing $L^\natural$ -convexity. This was not done so in Benjaafar et al. (2017).

To prove that $\bar{V}_s(x_{s,0}, \mathbf{x}_s, u)$ is $L^\natural$ -convex, we first consider a separate dynamic programming problem:

$$g_s(x_{s,0}, \mathbf{x}_s) = \min_{y_s \in \mathcal{A}_s} \{(c - p)y_s + \mathbb{E}z_s(y_s, x_{s,0}, \mathbf{x}_s, D)\} \quad (\text{A.2})$$

where

$$\begin{aligned} z_s(y_s, x_{s,0}, \mathbf{x}_s, d) &= \min_{0 \leq l \leq \min(d, y_s)} \{ (h + p - \alpha c)(y_s - l)^+ + p(l - y_s)^+ \\ &\quad + \alpha g_{s+1} [y_s + \sum_{i=i}^{s+k+1} u_i - l, (x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - l\mathbf{e}] \} \end{aligned} \quad (\text{A.3})$$

and

$$\begin{aligned} Z_s(y_s, x_{s,0}, \mathbf{x}_s, d, l) &= (h + p - \alpha c)(y_s - l)^+ + p(l - y_s)^+ \\ &\quad + \alpha g_{s+1} [(y_s + \sum_{i=i}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - l\mathbf{e}] \end{aligned} \quad (\text{A.4})$$

such that

$$z_s(y_s, x_{s,0}, \mathbf{x}_s, d) = \min_{0 \leq l \leq \min(d, y_s)} Z_s(y_s, x_{s,0}, \mathbf{x}_s, d, l)$$

We want to show that

$$c - p \leq \sum \frac{\partial g_s}{\partial x} \quad (\text{A.5})$$

Simultaneously, we will also be able to show the $L^{\natural}$ -convexity of $\bar{V}^L$.

If $s = T$, then

$$\frac{\partial}{\partial l} Z_T(y, x_{T,0}, \mathbf{x}_T, d, l) = \begin{cases} \alpha c - h - p & \text{if } y \geq l \\ p & \text{if } y < l \end{cases} \quad (\text{A.6})$$

since $\sum \frac{\partial g_{T+1}}{\partial x} = 0$. When $\alpha c < h + p$, $\frac{\partial}{\partial l} Z_T(y, x_{T,0}, \mathbf{x}_T, d, l) < 0$ for $0 \leq l \leq \min(d, y)$. This implies that

$$\begin{aligned} g_T(x_T, \mathbf{x}_T) &= \min_{y_T \in \mathcal{A}_s} \mathbb{E} Z_s(y_T, x_{T,0}, \mathbf{x}_T, D, \min(y_T, D)) \\ &= \min_{y_T \in \mathcal{A}_s} \bar{G}(y_T) \\ &= \bar{V}_T(x_{T,0}, \mathbf{x}_T) \end{aligned} \quad (\text{A.7})$$

Where the second equality is because $(y - d)^+ = y - \min(y, d)$. $g_T(x_{T,0}, \mathbf{x}_T)$ is trivially $L^{\natural}$ -convex, meaning $\bar{V}_T(x_{T,0}, \mathbf{x}_T)$ is also $L^{\natural}$ -convex. Clearly,

$$c - p \leq c - p + (h + p - \alpha c) \int_0^{y_T} f(D) dD = \sum \frac{\partial g_T}{\partial x}$$

Next assume for period $s + 1$ that $c - p \leq \sum \frac{\partial g_{s+1}}{\partial x}$. We have

$$\frac{\partial}{\partial l} Z_s(y_s, x_{s,0}, \mathbf{x}_s, d, l) = \alpha c - h - p - \alpha \sum \frac{\partial g_{s+1}}{\partial x} < 0$$

when $y > l$ and

$$\begin{aligned} g_s(x_{s,0}, \mathbf{x}_s) &= \min_{y_s \in \mathcal{A}_s} \mathbb{E} Z_s(y_s, x_{s,0}, \mathbf{x}_s, D, \min(y_s, D)) \\ &= \min_{y_s \in \mathcal{A}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} g_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(y_s, D)\mathbf{e}] \} \\ &= \bar{V}_s(x_{s,0}, \mathbf{x}_s) \end{aligned} \tag{A.8}$$

which shows that the dynamic program examined here is the same as (3.8). Proving $L^\natural$ -convexity of $g_s(x_{s,0}, \mathbf{x}_s)$ proves the $L^\natural$ -convexity of $\bar{H}_s(y_s, x_{s,0}, \mathbf{x}_s)$ and $\bar{V}_s(x_{s,0}, \mathbf{x}_s, u)$. It is easy to see that f is $L^\natural$ -convex. Next, we define $\psi(x_{s,0}, \mathbf{x}_s, \xi) = g((x_{s,0}, \mathbf{x}_s) - \xi\mathbf{e})$ and prove that $\psi(x_{s,0}, \mathbf{x}_s, \xi)$ is submodular in $(x_{s,0}, \mathbf{x}_s, \xi)$. Proving this proves $L^\natural$ -convexity. We have,

$$\psi(x_{s,0}, \mathbf{x}_s, \xi) = \min_{y_s \in \mathcal{L}_s} \{ (c - p)y_s + \mathbb{E} z_s(y_s, x_{s,0} - \xi, \mathbf{x}_s - \xi\mathbf{e}, D) \}$$

where

$$\mathcal{L}_s = \{ y : x_{s,0} - \xi - \sum_{i=1}^{s+k} u_i \leq y \leq x_{s,k} - \xi \} \tag{A.9}$$

Let $\check{y}_s = y_s + \xi$. This means

$$\psi(x_{s,0}, \mathbf{x}_s, \xi) = \min_{\check{y}_s \in \check{\mathcal{L}}_s} (c - p)(\check{y}_s - \xi) + \mathbb{E} z_s(\check{y}_s - \xi, x_{s,0} - \xi, \mathbf{x}_s - \xi\mathbf{e}, D)$$

where

$$\check{\mathcal{L}}_s = \{ \check{y} : x_{s,0} - \xi - \sum_{i=1}^{s+k} u_i \leq \check{y} - \xi \leq x_{s,k} - \xi \} \tag{A.10}$$

Since each inequality involves exactly two variables (u_i is a constant and ξ gets cancelled) and they are of opposite signs, then $\check{\mathcal{L}}$ is a lattice by Example 2.2.7 (b) of Topkis (1998). Therefore, $\psi(x_{s,0}, \mathbf{x}_s, \xi)$ is submodular in $(x_{s,0}, \mathbf{x}_s, \xi)$ (Theorem 2.7.6, Topkis (1998)) and $g_s(x_{s,0}, \mathbf{x}_s)$ is $L^\natural$ -convex. Note that we can reach the same conclusion by utilizing Proposition 4 (e) in Chen (2017). The last step for this part

of the proof is to show that $c - p \leq \sum \frac{\partial g_s}{\partial x}$. Assume that $c - p \leq \sum \frac{\partial g_{s+1}}{\partial x}$. This leads to:

$$\begin{aligned}
\sum \frac{\partial g_s}{\partial x} &= \frac{\partial g_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,0}} + \frac{\partial g_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,1}} + \dots + \frac{\partial g_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,k}} \\
&= \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,0}} + \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,1}} + \dots + \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,k}} \\
&= \bar{G}'(y_s) + \alpha \frac{\partial}{\partial y_s} \int_0^\infty \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - \min(y_s, D)\mathbf{e}]f(D)dD \\
&\quad + \alpha \sum_{i=1}^{k-1} \frac{\partial}{\partial x_i} \int_0^\infty \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - \min(y_s, D)\mathbf{e}]f(D)dD \\
&= \bar{G}'(y_s) + \alpha \frac{\partial}{\partial y_s} \int_0^{y_s} \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}]f(D)dD \\
&\quad + \alpha \frac{\partial}{\partial y_s} \int_{y_s}^\infty \bar{V}_{s+1}[\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y_s\mathbf{e}]f(D)dD \\
&\quad + \alpha \sum_{i=1}^{k-1} \frac{\partial}{\partial x_i} \int_0^{y_s} \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}]f(D)dD \quad (\text{A.11}) \\
&\quad + \alpha \sum_{i=1}^{k-1} \frac{\partial}{\partial x_i} \int_{y_s}^\infty \bar{V}_{s+1}[\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y_s\mathbf{e}]f(D)dD \\
&= \bar{G}'(y_s) + \alpha \frac{\partial}{\partial y_s} \int_0^{y_s} \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}]f(D)dD \\
&\quad + \alpha \sum_{i=1}^{k-1} \frac{\partial}{\partial x_i} \int_0^{y_s} \bar{V}_{s+1}[(y_s + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}]f(D)dD \\
&= c - p + (h + p - \alpha c) \int_0^{y_s} f(D)dD + \alpha \sum \frac{\partial g_{s+1}}{\partial x} \int_0^{y_s} f(D)dD \\
&\geq c - p + (h + p - \alpha c) \int_0^{y_s} f(D)dD + \alpha(c - p) \int_0^{y_s} f(D)dD \\
&= c - p + (h + (1 - \alpha)p) \int_0^{y_s} f(D)dD \\
&\geq c - p
\end{aligned}$$

Where the first inequality is because of the inductive assumption on $\sum \frac{\partial g_{s+1}}{\partial x}$ $\square$

To prove the L^1 -convexity of the relaxed problem $\hat{V}$, we change the constraint $0 \leq l \leq \min(d, y_s)$ to $0 \leq l \leq \min(\max(y_s, x_{s,0} - \sum_{i=1}^{s+k+1} u_i), d)$ and use the

same approach to proving $L^{\natural}$ -convexity of g in the original problem to show the $L^{\natural}$ -convexity of $\hat{f}$. For brevity, we will just show that the new constraint can be broken down into two constraint sets that contains a maximum of two variables of opposite signs, noting that a transformation similar to the one in the original problem would just cancel out ξ .

$$\{l : 0 \leq l \leq \min(\max(y_s, x_{s,0} - \sum_{i=1}^{s+k+1} u_i), d)\} = \begin{cases} \{l : 0 \leq l \leq d, l \leq y_s\} & , y_s \geq x_{s,0} - \sum_{i=1}^{s+k+1} u_i \\ \{l : 0 \leq l \leq d, l \leq x_{s,0} - \sum_{i=1}^{s+k+1} u_i\} & , y_s < x_{s,0} - \sum_{i=1}^{s+k+1} u_i \end{cases}$$

For the backlogging case, it is easy to show $L^{\natural}$ -convexity because the transition from period s to $s+1$ involves subtracting D to every component of the new state vector instead of $\min(y_s, D)$ in the lost-sales case. Standard induction arguments, along with the fact that $\Psi(\mathbf{x}, \xi) = \psi(\mathbf{x} - \xi \mathbf{e})$ is $L^{\natural}$ -convex for some $L^{\natural}$ -convex function ψ , and usage of Proposition 4 (e) of Chen to show $L^{\natural}$ -convexity of the constraint set on $\bar{V}_s^B$ proves $L^{\natural}$ -convexity of both $\bar{V}_s^B$ and $\bar{H}_s^B$.

For the perishability case, if $m = 1$, then instead of subtracting l in the transition of g_{s+1} , we subtract y_s . The proof for $L^{\natural}$ -convexity still applies.

A.2 Proof of Theorem 3.2

(a) and (b): The optimal policy follows directly from the convexity of $\bar{V}_s$ and $y((x_{s,0}, \mathbf{x}_s) + \omega \mathbf{e}) \leq y(x_{s,0}, \mathbf{x}_s) + \omega$ and the non-decreasing property follows from Lemma 3 of Zipkin (2008).

(d): $\frac{\partial y_s^*(\mathbf{u})}{\partial u_i} \leq 1$ follows directly from (b) and since $x_{s,i} = x_{s,0} - \sum_{j=1}^i q_{s,j} + \sum_{j=1}^{s+k-i} u_j$. The lower-bound is because y_s^* is nondecreasing in $(x_{s,0}, \mathbf{x}_s)$.

(c): The proof is drawn largely from the proof of Lemma 2 and Theorem 2 in Pang et al. (2012) as follows: For ease of exposition, we let $\mathbf{x} = (x_{s,0}, \mathbf{x}_s)$, $\check{\mathbf{q}} = (x_s, \check{\mathbf{q}}_s)$, and $\mathbf{q} = (x_s, \mathbf{q}_s)$. Where $\check{\mathbf{q}}_s = -\mathbf{q}_s$. We also refer to the components of the vector in the order $i = 0, 1, \dots, k$. Lastly, let $q_s(\mathbf{x})$ be the optimal order quantity for period s , $\check{q}_s(\mathbf{x}) = -q_s(\mathbf{x})$.

Note that $\check{q}_s(\mathbf{x}) = x_{s,0} - \sum_{i=1}^{s+k} u_i - y_s(\mathbf{x})$, which is non-increasing in $\mathbf{x}$.

$$\begin{aligned} \check{q}_s(\mathbf{x} + \omega \mathbf{e}_k) &= x_{s,0} + \omega - \sum_{i=1}^{s+k} u_i - y_s(\mathbf{x} + \omega \mathbf{e}_k) \\ &\leq x_{s,0} + \omega - \sum_{i=1}^{s+k} u_i - y_s(\mathbf{x}) \\ &= \check{q}_s(\mathbf{x}) + \omega \end{aligned}$$

where the inequality is because $y_s(\mathbf{x})$ is non-decreasing in $\mathbf{x}$. We also have

$$\begin{aligned} \check{q}_s(\mathbf{x} + \omega \mathbf{e}) &= x_{s,0} + \omega - \sum_{i=1}^{s+k} u_i - y_s(\mathbf{x} + \omega \mathbf{e}) \\ &\geq x_{s,0} - \sum_{i=1}^{s+k} u_i - y_s(\mathbf{x}) \\ &= \check{q}_s(\mathbf{x}) \end{aligned}$$

Where the inequality is because of (b). Taking the above results and since $\check{q}_s(\mathbf{x})$ is non-increasing in $\mathbf{x}$ (to get the second inequality), we have:

$$\check{q}_s(\mathbf{x}) \leq \check{q}_s(\mathbf{x} + \omega \mathbf{e}) \leq \check{q}_s(\mathbf{x} + \omega \mathbf{e}_k) \leq \check{q}_s(\mathbf{x}) + \omega$$

which Pang et al.'s Lemma 2 multiplied by -1 . This directly implies

$$y_s(\mathbf{x}) + \omega \geq y_s(\mathbf{x} + \omega \mathbf{e}) \geq y_s(\mathbf{x} + \omega \mathbf{e}_k) \geq y_s(\mathbf{x})$$

From here, note that $y_s(\mathbf{x}) = y_s(\mathbf{q}) = y_s(\check{\mathbf{q}})$. That is, the threshold $y(\mathbf{x})$ is equivalently dependent on $\mathbf{q}$ and $\check{\mathbf{q}}$ because it is dependent on $\mathbf{x}$.

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_l) &= y_s(\mathbf{x} + \sum_{i=l}^k \omega \mathbf{e}_i) \\ &\leq y_s(\mathbf{x} + \sum_{i=l-1}^k \omega \mathbf{e}_i) \\ &= y_s(\check{\mathbf{q}} + \omega \mathbf{e}_{l-1}) \forall l = 2, 3, \dots, k \end{aligned}$$

$$\Rightarrow y_s(\check{\mathbf{q}} + \omega \mathbf{e}_l) - y(\check{\mathbf{q}}) \leq y_s(\check{\mathbf{q}} + \omega \mathbf{e}_{l-1}) - y(\check{\mathbf{q}}) \forall l = 2, 3, \dots, k$$

Note that $\check{\mathbf{q}} + \omega \mathbf{e}_l = -(\mathbf{q} - \omega \mathbf{e}_l)$. Therefore,

$$y_s(\mathbf{q} + \omega \mathbf{e}_l) - y(\mathbf{q}) \geq y_s(\mathbf{q} + \omega \mathbf{e}_{l-1}) - y(\mathbf{q}) \forall l = 2, 3, \dots, k$$

Next,

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_1) &= y_s(\mathbf{x} + \sum_{i=1}^k \omega \mathbf{e}_i) \\ &\leq y_s(\mathbf{x}) + \omega \\ &= y_s(\mathbf{x}) + \omega \end{aligned}$$

which implies,

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_1) - y_s(\check{\mathbf{q}}) &\leq \omega \\ \iff y_s(\mathbf{q} + \omega \mathbf{e}_1) - y_s(\mathbf{q}) &\geq -\omega \end{aligned}$$

Then,

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_k) &= y_s(\mathbf{x} + \omega \mathbf{e}_k) \\ &\geq y_s(\mathbf{x}) \\ &= y_s(\check{\mathbf{q}}) \end{aligned}$$

which again implies

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_k) - y_s(\check{\mathbf{q}}) &\geq 0 \\ \iff y_s(\mathbf{q} + \omega \mathbf{e}_k) - y_s(\mathbf{q}) &\leq 0 \end{aligned}$$

We also have

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_0) &= y_s(\mathbf{x} + \sum_{i=0}^k \omega \mathbf{e}_i) \\ &= y_s(\mathbf{x} + \omega \mathbf{e}) \\ &\geq y_s(\mathbf{x}) \\ &= y_s(\check{\mathbf{q}}) \end{aligned}$$

and

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_0) - y_s(\check{\mathbf{q}}) &\geq 0 \\ \iff y_s(\mathbf{q} + \omega \mathbf{e}_0) - y_s(\mathbf{q}) &\geq 0 \end{aligned}$$

note that the sign does not change because the first component $i = 0$ in vectors $\check{\mathbf{q}}$ and $\mathbf{q}$ are the same. Lastly,

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_0) &= y_s(\mathbf{x} + \sum_{i=0}^k \omega \mathbf{e}_i) \\ &= y_s(\mathbf{x} + \omega \mathbf{e}) \\ &\leq y_s(\mathbf{x}) + \omega \\ &= y_s(\check{\mathbf{q}}) + \omega \end{aligned}$$

and

$$\begin{aligned} y_s(\check{\mathbf{q}} + \omega \mathbf{e}_0) - y_s(\check{\mathbf{q}}) &\leq \omega \\ \iff y_s(\mathbf{q} + \omega \mathbf{e}_0) - y_s(\mathbf{q}) &\leq \omega \end{aligned}$$

Combining all these results, we have:

$$-\omega \leq y_s(\mathbf{q} + \omega \mathbf{e}_1) - y_s(\mathbf{q}) \leq y_s(\mathbf{q} + \omega \mathbf{e}_2) - y_s(\mathbf{q}) \leq \dots \leq y_s(\mathbf{q} + \omega \mathbf{e}_k) - y_s(\mathbf{q}) \leq 0 \leq y_s(\mathbf{q} + \omega \mathbf{e}_0) - y_s(\mathbf{q}) \leq \omega$$

Which if smooth translates to

$$-1 \leq \frac{\partial y(x_s, \mathbf{q}_s)}{\partial q_{s-1}} \leq \frac{\partial y(x_s, \mathbf{q}_s)}{\partial q_{s-2}} \leq \dots \leq \frac{\partial y(x_s, \mathbf{q}_s)}{\partial q_{s-k}} \leq 0 \leq \frac{\partial y(x_s, \mathbf{q}_s)}{\partial x_s} \leq 1$$

□

A.3 Proof of Proposition 1

(a): The proof for $c - p \leq \sum_{i=0}^k \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,i}}$ is shown in the proof for Lemma 3.1. $c - p \leq \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,0}}$ and $c - p \leq \frac{\partial \bar{V}_s(x_{s,0}, \mathbf{x}_s)}{\partial x_{s,i}} \leq 0$ for $i = 1, 2, \dots, k$ are proven in the same way.

(c): Increasing k restricts the usage of capacity, which results in non-decreasing cost.

A.4 Proof of Theorem 3.3

We show the relaxed version of the problem for $k = T - 1$:

$$\hat{V}_s(x_{s,0}, \mathbf{x}_s) = \min_{y_s \in \mathcal{A}_s} \{ \bar{G}(y_s) + \alpha \mathbb{E} \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(y_s, D) \mathbf{e} \right) \}$$

Note that this is different from the relaxed problem introduced in (3.13) in that we just allow for ordering below the on-hand inventory and that there is no need to adjust for the term subtracted to each component of the state. This is allowed for $k = T - 1$ since there will be no point in the planning horizon where the first component $x_{s,0}$ will be the upper bound of $\hat{A}_t$ for some $t > s$. In other words, the regeneration schedule will not be affected. We also note that in the relaxed problem, $c - p \leq \sum \frac{\partial \hat{V}_s}{\partial x} \leq 0$ following the proof from Lemma 3.1. The derivative is less than or equal to zero because of the relaxation of the constraint. We define $\hat{y}_s^*$ as the unconstrained minimizer of $\hat{H}_s$ and $\hat{y}_s^L$ is the optimal ending inventory (and the decision variable). We remove the dependence on the states for brevity and where it does not cause confusion.

(a) We prove this by induction. For the base-case at period T , if $x_{T,k} \geq y^L$, then $\hat{y}_T^L = \hat{y}_T^* = y^L$ because $\frac{\partial \hat{H}(y_T)}{\partial y_T} \Big|_{y_T=y^L} = \frac{\partial \bar{G}(y_T)}{\partial y_T} \Big|_{y_T=y^L} = 0$. Assume that (a) holds for period $s + 1$. Next, suppose the following inequalities are true:

$$\begin{aligned} x_{s,k} &\geq y^L \\ x_{s,k-1} &\geq 2y^L \\ &\vdots \\ x_{s,1} &\geq ky^L \\ x_{s,0} &\geq (k+1)y^L \end{aligned} \tag{A.12}$$

note that we include in the proof all possible inequalities that can be formed in the planning horizon, even though some of these inequalities will not be applicable by the way the conditions of (a) are defined. Nonetheless, the proof still holds with this inclusion. If $y_s^L = y_s^* = y^L$, then the state transition into period $s + 1$ would mean subtracting $\min(y^L, D)$ to all components of the state vector for any realization of D such that in

the next period, we have the following inequalities:

$$\begin{aligned}
x_{s,k-1} - \min(y^L, D) &\geq 2y^L - \min(y^L, D) &\geq y^L \\
x_{s,k-2} - \min(y^L, D) &\geq 3y^L - \min(y^L, D) &\geq 2y^L \\
&\vdots \\
x_{s,1} - \min(y^L, D) &\geq ky^L - \min(y^L, D) &\geq (k-1)y^L \\
x_{s,0} - \min(y^L, D) &\geq (k+1)y^L - \min(y^L, D) &\geq ky^L
\end{aligned}$$

Implying that $\hat{y}_{s+1}^L = \hat{y}_{s+1}^* = y^L$ by the inductive assumption. This also means that the corresponding sets of inequalities needed to follow the conditions in (a) will be true if we repeat the process until $s = T$. Lastly, it suffices to show that $\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s=y^L} = 0$. But we know if (a) is true for $s+1$, then $\frac{\partial \hat{H}_{s+1}}{\partial y_{s+1}} \Big|_{y_{s+1}=y^L} = 0$ and that $\frac{\partial \bar{G}^L(y_s)}{\partial y_s} \Big|_{y_s=y^L} = 0$. This confirms $\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s=y^L} = 0$ and proves $\hat{y}_s^L = \hat{y}_s^* = y^L$. $\square$

(b) We begin by noting that

$$\begin{aligned}
\frac{\partial \hat{H}_s}{\partial y} &= c - p + (h + p - \alpha c) \int_0^y f(D) dD + \alpha \frac{\partial}{\partial y} \int_0^y \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - D \mathbf{e} \right) \\
&\quad + \alpha \frac{\partial}{\partial y} \int_y^\infty \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y \mathbf{e} \right) f(D) dD \\
&= c - p + (h + p - \alpha c) \int_0^y f(D) dD + \alpha \frac{\partial}{\partial y} \int_y^\infty \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y \mathbf{e} \right) \\
&\leq c - p + (h + p - \alpha c) \int_0^y f(D) dD - \alpha \int_y^\infty (c - p) f(D) dD \\
&= (1 - \alpha)(c - p) + (h + (1 - \alpha)p) \int_0^y f(D) dD
\end{aligned}$$

The second equality is because we relaxed the constraint to allow for ordering below the on-hand inventory and the fact that $y + \sum_{i=1}^T u_i$ will not be part of a future constraint in the planning horizon. The first inequality follows from $c - p \leq \sum \frac{\partial \hat{V}_s}{\partial x} \leq 0$. Because of the convexity of $\frac{\partial \hat{H}_s}{\partial y}$, we can obtain the optimal y of the last line (we put emphasis on the last line and not the function itself) by equating it to 0 and solving for y to get $\dot{y} = F^{-1}(\frac{(1-\alpha)(p-c)}{h+(1-\alpha)p})$. Since $\frac{p-c}{h+p-\alpha c} \geq \frac{(1-\alpha)(p-c)}{h+(1-\alpha)p}$ then $\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s=y^L} \geq 0$ which means $\hat{y}^* \leq y^L$. This also means that

$$\hat{y}_s^L = \begin{cases} x_{s,k} & \text{if } \dot{y}_s \geq x_{s,k} \\ y_s^* & \text{if } \dot{y}_s < x_{s,k} \end{cases} \quad (\text{A.13})$$

whatever that $\hat{y}_s^*$ might be. If $\dot{y}_s < x_{s,k}$, then $\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s = \hat{y}_s^L} = 0$ which results to:

$$\begin{aligned} 0 &= \bar{G}'(\hat{y}_s^L) - \alpha(1 - F(\hat{y}_s^L)) \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - \hat{y}_s^L \mathbf{e} \right) \\ \iff \frac{\bar{G}'(\hat{y}_s^L)}{\alpha(1 - F(\hat{y}_s^L))} &= \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - \hat{y}_s^L \mathbf{e} \right) \end{aligned}$$

As in Benjaafar et al. (2017), we then have

$$\begin{aligned} \sum \frac{\partial}{\partial x} \hat{V}_s(x_{s,0}, \mathbf{x}_s) &= \bar{G}'(\hat{y}_s^L) + \alpha \int_0^y \sum \frac{\partial}{\partial x} \hat{V}_{s+1}(\hat{y}_s^L + \sum_{i=1}^{s+k+1} u_i - D, (x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}) f(D) dD \\ &\geq \bar{G}'(\hat{y}_s^L) + \alpha \int_0^y \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - \hat{y}_s^L \mathbf{e} \right) f(D) dD \\ &= \bar{G}'(\hat{y}_s^L) + \frac{F(\hat{y}_s^L)}{1 - F(\hat{y}_s^L)} \bar{G}'(\hat{y}_s^L) \\ &= \frac{\bar{G}'(\hat{y}_s^L)}{1 - F(\hat{y}_s^L)} \end{aligned}$$

where the inequality is true because of the convexity of $\hat{V}_{s+1}$. We will next analyze

$\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s = \frac{x_{s,k}}{n}}$ by considering two cases:

CASE 1: $x_{s,k-1} - \frac{x_{s,k}}{n} > \dot{y}$. We have:

$$\begin{aligned} \frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s = \frac{x_{s,k}}{n}} &= \bar{G}'\left(\frac{x_{s,k}}{n}\right) \\ &\quad - \alpha(1 - F(\frac{x_{s,k}}{n})) \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, \mathbf{x} - \frac{x_{s,k}}{n} \mathbf{e} \right) \\ &\leq \bar{G}'\left(\frac{x_{s,k}}{n}\right) - \frac{\alpha(1 - F(\frac{x_{s,k}}{n})) \bar{G}'(\hat{y}_{s+1}^L)}{1 - F(\hat{y}_{s+1}^L)} \\ &= (1 - F(\frac{x_{s,k}}{n})) \left(\frac{\bar{G}'(\frac{x_{s,k}}{n})}{(1 - F(\frac{x_{s,k}}{n}))} - \frac{\alpha \bar{G}'(\hat{y}_{s+1}^L)}{1 - F(\hat{y}_{s+1}^L)} \right) \end{aligned} \tag{A.14}$$

It can be easily shown that $\frac{\bar{G}'(y)}{1 - F(y)}$ is non-positive and non-decreasing for $y \in [0, y^L]$. It thus suffices to show that $\frac{x_{s,k}}{n} \leq y_{s+1}^L$ to prove that $\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s = \frac{x_{s,k}}{n}} \leq 0$ for case 1.

Note that in period s , the conditions of (b) state that there exists some i in $i = 0, \dots, k - s + 1$ for which $x_{s,k-i} < (i + 1)y^L$. If we follow the proof of (a), then we are subtracting a quantity that is $\leq y^L$ to the inequalities. If $x_{s,k-i} \geq (i + 1)y^L$, we know from (a) that this inequality will be preserved if we subtract a quantity that is $\leq y^L$. However, there

are instances wherein subtracting a quantity $\leq y^L$ to $x_{s,k-i} < (i+1)y^L$ reverses the inequality. This implies that in period $s+1$, n will change to n' such that $n' \leq n-1$. This is also true when no inequalities are reversed as a result of the state transition. We then have:

$$\frac{x_{s,k}}{n} \stackrel{?}{\leq} \frac{x_{s,k-1} - \frac{x_{s,k}}{n}}{n'} \quad (\text{A.15})$$

where we subtract $\frac{x_{s,k}}{n}$ on the RHS to indicate that this is the maximum amount that will be subtracted from the states during the transition from s to $s+1$ if $y_s = \frac{x_{s,k}}{n}$ is chosen as the ending inventory level. (A.15) is then equivalent to:

$$(n'+1)(x_{s,k}) \leq n(x_{s,k-1}) \quad (\text{A.16})$$

because $x_{s,k} \leq x_{s,k-1}$. Therefore, $\left. \frac{\partial \hat{H}_s}{\partial y_s} \right|_{y_s = \frac{x_{s,k}}{n}} \leq 0$.

CASE 2: $x_{s,k-1} - \frac{x_{s,k}}{n} \leq \dot{y}$

$$\begin{aligned} \left. \frac{\partial \hat{H}_s}{\partial y_s} \right|_{y_s = \frac{x_{s,k}}{n}} &= \bar{G}'\left(\frac{x_{s,k}}{n}\right) \\ &\quad - \alpha(1 - F(\frac{x_{s,k}}{n})) \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left(\sum_{i=1}^{s+k+1} u_i, \mathbf{x} - \frac{x_{s,k}}{n} \mathbf{e} \right) \\ &\leq \bar{G}'\left(\frac{x_{s,k}}{n}\right) - \alpha(1 - F(\frac{x_{s,k}}{n}))(c - p) \\ &\leq 0 \end{aligned} \quad (\text{A.17})$$

where the last inequality is because (A.15) is also true in Case 2, the assumption that $x_{s,k-1} - \frac{x_{s,k}}{n} \leq \dot{y}$, and the definition of $\dot{y}$. The proof for $\hat{y}_s^* = y_s^*$ is similar to the proof of Theorem 4 in Benjaafar et al. (2017).

A.5 Proof of Theorem 3.5

Let $y_s^0(y, x_{s,0}) = \max(y, x_{s,0} - \sum_{i=1}^{s+k+1} u_i)$. When writing $y_s^0(y, x_{s,0})$ without the function parameters, we assume that $y_s^0 = \max(\hat{y}_s^L, x_{s,0} - \sum_{i=1}^{s+k+1} u_i)$. We have $\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s = \hat{y}_s^L} = 0$ such that

$$\begin{aligned} 0 &= \bar{G}'(\hat{y}_s^L) - \alpha \int_{y_s^0}^{\infty} \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left[\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y_s^0 \mathbf{e} \right] f(D) dD + \\ &\quad \alpha \int_0^{y_s^0} \frac{\partial}{\partial y} \hat{V}_{s+1} \left[(y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D \mathbf{e} \right] f(D) dD \end{aligned} \quad (\text{A.18})$$

$$\begin{aligned} \iff &\quad \alpha \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left[\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y_s^0 \mathbf{e} \right] - \bar{G}(\hat{y}_s^L) \\ &= \alpha \int_0^{y_s^0} \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left[\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y_s^0 \mathbf{e} \right] f(D) dD \\ &\quad + \alpha \int_0^{y_s^0} \frac{\partial}{\partial y} \hat{V}_{s+1} \left[(y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D \mathbf{e} \right] f(D) dD \end{aligned} \quad (\text{A.19})$$

Since $\sum \frac{\partial}{\partial x} \hat{V}_{s+1} \leq 0$, then

$$\begin{aligned} -\bar{G}'(\hat{y}_s^L) &\geq \alpha \int_0^{y_s^0} \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left[\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y_s^0 \mathbf{e} \right] f(D) dD \\ &\quad + \alpha \int_0^{y_s^0} \frac{\partial}{\partial y} \hat{V}_{s+1} \left[(y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D \mathbf{e} \right] f(D) dD \end{aligned} \quad (\text{A.20})$$

Let

$$\begin{aligned} \Delta &= \alpha \int_0^{y_s^0} \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left[\sum_{i=1}^{s+k+1} u_i, (x_{s,0}, \dots, x_{s,k-1}) - y_s^0 \mathbf{e} \right] F(D) dD + \\ &\quad \alpha \int_0^{y_s^0} \frac{\partial}{\partial y} \hat{V}_{s+1} \left[(y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D \mathbf{e} \right] f(D) dD \end{aligned}$$

and let $\bar{y}^1 = F^{-1}(\frac{(1-\alpha)(p-c)}{(1-\alpha)(h+p-\alpha c) + \alpha^2(p-c)})$ and $\bar{y}^2 = F^{-1}(\frac{p-c}{2(h+p-\alpha c)})$. We first prove the lower bound:

CASE 1: $\bar{G}'(\hat{y}_s^L) \geq 0 \geq -\bar{G}'(\hat{y}_s^L) \geq \Delta$

$$\begin{aligned}
\sum \frac{\partial}{\partial x} \hat{V}_s(x_{s,0}, \mathbf{x}_s) &= \bar{G}'(y_s^L) + \alpha \int_0^{y_s^0} \sum \frac{\partial}{\partial x} \hat{V}_{s+1}((y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}) f(D) dD \\
&\quad + \alpha \int_0^{y_s^0} \frac{\partial}{\partial y} \hat{V}_{s+1}[(y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}] f(D) dD \\
&\geq \bar{G}'(y_s^L) + \alpha(c-p) \int_0^{y_s^0} f(D) dD
\end{aligned} \tag{A.21}$$

Where the inequality is from (A.12). We then have

$$\begin{aligned}
\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s=\bar{y}^1} &= \bar{G}'(\bar{y}^1) - \alpha(1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \sum \frac{\partial}{\partial x} \hat{V}_{s+1}[\sum_{i=1}^{s+k+1} u_i, \mathbf{x} - y_s^0(\bar{y}^1, x_{s,0})\mathbf{e}] f(D) dD + \\
&\quad \alpha \frac{\partial}{\partial y_s} \int_0^{y_s^0(\bar{y}^1, x_{s,0})} \hat{V}_{s+1}[(y_s^0(\bar{y}^1, x_{s,0}) + \sum_{i=1}^{s+k+1} u_i, \mathbf{x}) - D\mathbf{e}] f(D) dD \\
&\leq \bar{G}'(\bar{y}^1) - \alpha(1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \left(\bar{G}'(\hat{y}_{s+1}^L) \right. \\
&\quad \left. + \alpha(c-p) \int_0^{y_{s+1}^0(\hat{y}_{s+1}^L, \sum_{i=1}^{s+k+1} u_i)} f(D) dD \right) \\
&\leq (1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \left(\bar{G}'(\bar{y}^1) - \alpha \left(\bar{G}'(\hat{y}_{s+1}^L) \right. \right. \\
&\quad \left. \left. + \alpha(c-p) \int_0^{y_{s+1}^0(\hat{y}_{s+1}^L, \sum_{i=1}^{s+k+1} u_i)} f(D) dD \right) \right) \\
&\leq (1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \left((1 - \alpha) \left((c-p) + (h+p-\alpha c) \int_0^{\bar{y}^1} f(D) dD \right) - \right. \\
&\quad \left. \alpha^2(c-p) \int_0^{\bar{y}^1} f(D) dD \right) \\
&= (1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \left((1 - \alpha)(c-p) + \left((1 - \alpha)(h+p-\alpha c) \right. \right. \\
&\quad \left. \left. + \alpha^2(p-c) \right) \int_0^{\bar{y}^1} f(D) dD \right) \\
&\leq 0
\end{aligned} \tag{A.22}$$

where the second inequality is because $\bar{G}'(\bar{y}^1) < 0$, and the third is because of the inductive assumption that $\hat{y}_{s+1}^L \geq \bar{y}^1$ and $\bar{G}'(\hat{y}_{s+1}^L) \geq 0$. The last inequality is true from the definition of $\bar{y}^1$.

CASE 2: $-\bar{G}'(\hat{y}_s^L) > 0 > \bar{G}'(\hat{y}_s^L) \geq \Delta$

The inequality in (A.21) is also true in Case 2. This leads to a similar inequality as in (A.22):

$$\begin{aligned}
\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s=\bar{y}^1} &\leq (1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \left(\bar{G}'(\bar{y}^1) - \alpha \left(\bar{G}'(\hat{y}_{s+1}^L) + \alpha(c-p) \int_0^{y_{s+1}^0(\hat{y}_{s+1}^L, \sum_{i=1}^{s+k+1} u_i)} f(D) dD \right) \right) \\
&\leq (1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \left(\bar{G}'(\bar{y}^1) - \alpha \left(\bar{G}'(\bar{y}^2) + \alpha(c-p) \int_0^{\bar{y}^1} f(D) dD \right) \right) \\
&= (1 - F(y_s^0(\bar{y}^1, x_{s,0}))) \left((1 - \alpha)(c-p) + \left((1 - \alpha)(h + p - \alpha c) \right. \right. \\
&\quad \left. \left. + \alpha^2(p-c) \right) \int_0^{\bar{y}^1} f(D) dD \right) \\
&\leq 0
\end{aligned} \tag{A.23}$$

where the second inequality is because of the inductive assumption $\hat{y}_{s+1}^L \geq \bar{y}^1$. The final inequality is because of the definition of $\bar{y}^1$.

CASE 3: $-\bar{G}'(\hat{y}_s^L) > 0 \geq \Delta > \bar{G}'(\hat{y}_s^L)$ We first have:

$$\begin{aligned}
\sum \frac{\partial}{\partial x} \hat{V}_s(x_{s,0}, \mathbf{x}_s) &= \bar{G}'(\hat{y}_s^L) + \alpha \int_0^{y_s^0} \sum \frac{\partial}{\partial x} \hat{V}_{s+1}[(y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}] f(D) dD \\
&\quad + \alpha \int_0^{y_s^0} \frac{\partial}{\partial y} \hat{V}_{s+1}[(y_s^0 + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, \dots, x_{s,k-1}) - D\mathbf{e}] f(D) dD \\
&\geq 2\bar{G}'(\hat{y}_s^L)
\end{aligned} \tag{A.24}$$

Which solving for $\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s=\bar{y}^2}$ results into:

$$\begin{aligned}
\frac{\partial \hat{H}_s}{\partial y_s} \Big|_{y_s=\bar{y}^2} &= \bar{G}'(\bar{y}^2) - \alpha(1 - F(y_s^0(\bar{y}^2, x_{s,0}))) \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left[\sum_{i=1}^{s+k+1} u_i, \mathbf{x} - y_s^0(\bar{y}^2, x_{s,0})\mathbf{e} \right] f(D) dD + \\
&\quad \alpha \frac{\partial}{\partial y_s} \int_0^{y_s^0(\bar{y}^2, x_{s,0})} \hat{V}_{s+1}[(y_s^0(\bar{y}^2, x_{s,0}) + \sum_{i=1}^{s+k+1} u_i, \mathbf{x}) - D\mathbf{e}] f(D) dD \\
&\leq \bar{G}'(\bar{y}^2) - \alpha(1 - F(y_s^0(\bar{y}^2, x_{s,0}))) (2\bar{G}'(\hat{y}_{s+1}^L)) \\
&\leq \bar{G}'(\bar{y}^2) - 2\bar{G}'(\hat{y}_{s+1}^L) \\
&\leq 0
\end{aligned} \tag{A.25}$$

Where the first inequality is because of (A.24), the second inequality is because of the assumption that $\bar{G}'(\hat{y}_{s+1}^L) < 0$, and the final inequality is because of the definition of $\bar{y}^2$ and the inductive assumption on $\bar{y}^2$.

Given the results of $\bar{y}^1$ in Case 1 and 2 and $\bar{y}^2$ in Case 3, we are then sure that $\hat{y}_s^* \geq \min(\bar{y}^1, \bar{y}^2)$.

Next, we prove the upper bound.

$$\begin{aligned}
\left. \frac{\partial \hat{H}_s}{\partial y_s} \right|_{y_s = \bar{y}^4} &= \bar{G}'(\bar{y}^4) - \alpha(1 - F(y_s^0(\bar{y}^4, x_{s,0}))) \sum \frac{\partial}{\partial x} \hat{V}_{s+1} \left[\sum_{i=1}^{s+k+1} u_i, \mathbf{x} - y_s^0(\bar{y}^4, x_{s,0}) \right] + \\
&\quad \alpha \frac{\partial}{\partial y_s} \int_0^{y_s^0(\bar{y}^4, x_{s,0})} \hat{V}_{s+1} \left[(y_s^0(\bar{y}^4, x_{s,0}) + \sum_{i=1}^{s+k+1} u_i, \mathbf{x}) - D \mathbf{e} \right] f(D) dD \\
&\geq \bar{G}'(\bar{y}^4) + \alpha(c - p) \int_0^{y_s^0(\bar{y}^4, x_{s,0})} f(D) dD \\
&\geq \bar{G}'(\bar{y}^4) + \alpha(c - p) \\
&= 0
\end{aligned} \tag{A.26}$$

Where the first inequality is because of (A.20) and the final equality is because of the definition of $\bar{y}^4$.

The final part of the proof is to show that the thresholds for the relaxed problem are the same as the thresholds of the original problem. Given a starting state $(x_{s,0}, \mathbf{x}_s)$ in period s , if $y_s^* \geq x_{s,0}$, then

$$\begin{aligned}
\hat{H}_s(\hat{y}_s^*, x_{s,0}, \mathbf{x}_s) &= \bar{G}(\hat{y}_s^*) + \alpha \mathbb{E} \hat{V}_{s+1} \left[(\max(\hat{y}_s^*, x_{s,0} - \sum_{i=1}^{s+k+1} u_i) + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) \right. \\
&\quad \left. - \min(\max(\hat{y}_s^*, x_{s,0} - \sum_{i=1}^{s+k+1} u_i), D) \mathbf{e} \right] \\
&= \bar{G}(\hat{y}_s^*) + \alpha \mathbb{E} \hat{V}_{s+1} \left[(\hat{y}_s^* + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(\hat{y}_s^*, D) \mathbf{e} \right]
\end{aligned}$$

and if $\hat{y}_s^* < x_{s,0}$, then

$$\hat{H}_s(\hat{y}_s^*, x_{s,0}, \mathbf{x}_s) = \bar{G}(x_{s,0}) + \alpha \mathbb{E} \hat{V}_{s+1} \left[(x_{s,0}, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(x_{s,0} - \sum_{i=1}^{s+k+1} u_i, D) \mathbf{e} \right]$$

Notice that in both cases, if $\hat{y}_s^*$ is the threshold for the original problem $\bar{V}_{s+1}$ and $\bar{H}_{s+1}$, then if $\hat{y}_s^* \geq x_{s,0}$,

$$\bar{H}_s(\hat{y}_s^*, x_{s,0}, \mathbf{x}_s) = \bar{G}(\hat{y}_s^*) + \alpha \mathbb{E} \bar{V}_{s+1} \left[(\hat{y}_s^* + \sum_{i=1}^{s+k+1} u_i, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(\hat{y}_s^*, D) \mathbf{e} \right]$$

and if $\hat{y}_s^* < x_{s,0}$

$$\bar{H}_s(\hat{y}_s^*, x_{s,0}, \mathbf{x}_s) = \bar{G}(x_{s,0}) + \alpha \mathbb{E} \bar{V}_{s+1}[(x_{s,0}, x_{s,0}, x_{s,1}, \dots, x_{s,k-1}) - \min(x_{s,0} - \sum_{i=1}^{s+k+1} u_i, D) \mathbf{e}]$$

which has exactly the same states as the relaxed problem going into $s + 1$. If the states are the same, and because of the convexity of $\hat{V}$, then $\hat{y}_s^* = y_s^*$.

Appendix B

Appendix for Chapter 4

B.0.1 Proof of Lemma 4.1

We equate $\mathbb{E}[A(\bar{N})|n]$ and $\mathbb{E}[A(\bar{p})|q]$ to each other and solve for n . Recall that $\mathbb{E}[A(\bar{p})|q] = \mathbb{E}[A(\bar{p})|p'] = \frac{Np'}{1+kp'}$.

$$\begin{aligned}\frac{(N+n)p}{1+kp} &= \frac{Np'}{1+kp'} \\ (N+n)(1+kp')p &= Np'(1+kp) \\ np(1+kp') &= Np' + Nkpp' - Np - Nkpp' \\ n &= \frac{N(p' - p)}{p(1+kp')} \end{aligned} \tag{B.1}$$

B.0.2 Proof of Lemma 4.2

We use the same proof as that of Proposition 4.1 except that we switch p and p'

B.0.3 Proof of Lemma 4.3

We again equate $\mathbb{E}[A(\bar{k})|l]$ and $\mathbb{E}[A(\bar{p})|q]$ to each other and solve for l .

$$\begin{aligned}
\frac{Np'}{1+kp'} &= \frac{Np}{1+kp} \\
Np'(1+kp) &= Np + Nkpp' \\
Np' + Nkpp'l &= Np + Nkpp' \\
l &= \frac{p-p' + kpp'}{kpp'} \\
l &= \frac{1}{kp'} - \frac{1}{kp} + 1
\end{aligned} \tag{B.2}$$

B.0.4 Proof of Lemma 4.4

Equate $\mathbb{E}[A(\bar{N})|n]$ to $\mathbb{E}[\bar{A}(k)|l]$ to each other and solve for n .

$$\begin{aligned}
\frac{(N+n)p}{1+kp} &= \frac{Np}{1+kp} \\
(N+n)p(1+kp) &= Np(1+kp) \\
Np(1+kp) + np(1+kp) &= Np(1+kp) \\
np(1+kp) &= Np + Nkp^2 - Np - Nkp^2 \\
n &= \frac{Nkp(1-l)}{1+kp}
\end{aligned} \tag{B.3}$$

B.0.5 Proof of Propositions 2, 3, 4, and 5

We use Definition 4.1 to obtain the inequalities in the porpositions. First, we have for Proposition 2,

$$\begin{aligned}
p' - p &< \epsilon_{\bar{N}, \bar{p}} \tilde{n}^*(p, p') \\
p' - p &< \frac{\epsilon_{\bar{N}, \bar{p}}(p' - p)}{p(1+kp')} \\
p + kpp' &< \epsilon_{\bar{N}, \bar{p}} \\
p' &< \frac{\epsilon_{\bar{N}, \bar{p}} - p}{pk}
\end{aligned}$$

For proposition 3, we have

$$\begin{aligned}
p' - p &< \epsilon_{\bar{k}, \bar{p}}(1 - l^*(p, p', k)) \\
p' - p &< \epsilon_{\bar{k}, \bar{p}}\left(\frac{1}{kp} - \frac{1}{kp'}\right) \\
kpp'(p' - p) &< \epsilon_{\bar{k}, \bar{p}}(p' - p) \\
p' &< \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}
\end{aligned} \tag{B.4}$$

For proposition 4,

$$\begin{aligned}
1 - l &< \epsilon_{\bar{N}, \bar{k}} \tilde{n}^*(k, p, l) \\
1 - l &< \frac{\epsilon_{\bar{N}, \bar{k}} kp(1 - l)}{1 + lkp} \\
1 &< \frac{\epsilon_{\bar{N}, \bar{k}} kp}{1 + lkp} \\
1 + lkp &< \epsilon_{\bar{N}, \bar{k}} kp \\
\frac{1}{kp} + l &< \epsilon_{\bar{N}, \bar{k}} \\
1 - l &> 1 + \frac{1}{kp} - \epsilon_{\bar{N}, \bar{k}}
\end{aligned} \tag{B.5}$$

Lastly, for proposition 5,

$$\begin{aligned}
\epsilon_{\bar{k}, \bar{N}}(1 - l^*(p, p', k)) &< \tilde{n}^*(p, p') \\
\epsilon_{\bar{k}, \bar{N}}\left(\frac{1}{kp} - \frac{1}{kp'}\right) &< \frac{p' - p}{p(1 + kp')} \\
\frac{\epsilon_{\bar{k}, \bar{N}}(p' - p)}{kpp'} &< \frac{p' - p}{p(1 + kp')} \\
\epsilon_{\bar{k}, \bar{N}}(1 + kp') &< kp' \\
\epsilon_{\bar{k}, \bar{N}} + \epsilon_{\bar{k}, \bar{N}} kp' &< kp' \\
\epsilon_{\bar{k}, \bar{N}} &< p'(k - \epsilon_{\bar{k}, \bar{N}}) \\
\frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})} &< p'
\end{aligned} \tag{B.6}$$

B.0.6 Proof of Theorem 4.5

The proof of is straight forward and uses the intersection of the inequalities obtained from Propositions 2, 3, and 5. However, we also need to show that it is not possible to have p' such that (I) $p' < \frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp}$, $p' > \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$ and $p' < \frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})}$ or (II) $p' > \frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp}$, $p' < \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$ and $p' > \frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})}$. Notice that if these inequalities are true, then there would be a circular sequencing of the efficiency of the interventions (which is a contradiction). Since (II) just reverses the inequalities of (I), we show the proof for (I) and note that the same steps would be used for (II).

To prove (I), we only need to show that for a given p , (I-A) if $\frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp} > \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$, then $\frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})} < \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$ and (I-B) if $\frac{\epsilon_{\bar{k}, \bar{p}}}{kp} < \frac{\epsilon_{\bar{k}, \bar{N}}}{k(1 - \epsilon_{\bar{k}, \bar{N}})}$, then $\frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp} < \frac{\epsilon_{\bar{k}, \bar{p}}}{kp}$. Showing (I-A) and (I-B) means that the intersection of the inequalities is a null set. We assume that $0 < \epsilon_{\bar{k}, \bar{N}} < 1$. Otherwise, these are automatically true.

Case I-A:

$$\begin{aligned}
 \frac{\epsilon_{\bar{k}, \bar{N}}}{kp(1 - \epsilon_{\bar{k}, \bar{N}})} & \stackrel{?}{<} \frac{\epsilon_{\bar{k}, \bar{p}}}{kp} \\
 \frac{\epsilon_{\bar{k}, \bar{N}}}{1 - \epsilon_{\bar{k}, \bar{N}}} & \stackrel{?}{<} \epsilon_{\bar{k}, \bar{p}} \\
 & \Leftrightarrow \epsilon_{\bar{k}, \bar{N}} \stackrel{?}{<} \epsilon_{\bar{k}, \bar{p}} - \epsilon_{\bar{k}, \bar{p}} \epsilon_{\bar{k}, \bar{N}} \\
 \Leftrightarrow \epsilon_{\bar{k}, \bar{N}}(1 + \epsilon_{\bar{k}, \bar{p}}) & \stackrel{?}{<} \epsilon_{\bar{N}, \bar{p}} \epsilon_{\bar{k}, \bar{N}} \\
 \Leftrightarrow 1 + \epsilon_{\bar{k}, \bar{p}} & \stackrel{?}{<} \epsilon_{\bar{N}, \bar{p}} \\
 \Leftrightarrow \frac{\epsilon_{\bar{k}, \bar{p}}}{kp} & \stackrel{?}{<} \frac{\epsilon_{\bar{N}, \bar{p}} - 1}{kp} \\
 & < \frac{\epsilon_{\bar{N}, \bar{p}} - p}{kp}
 \end{aligned}$$

The first includes $p < 0$ in the denominator of the RHS which makes it a stronger result that encompasses the case of no p . The second line results from multiplying kp to both sides. The third line is a simple multiplication, followed by grouping like terms in the fourth line, along with noting that $\epsilon_{\bar{k}, \bar{p}} = \epsilon_{\bar{N}, \bar{p}} \epsilon_{\bar{k}, \bar{N}}$. The fourth and fifth lines are also simple arithmetic, and the last line is because $p < 1$. Because of the initial condition, the first line is therefore true. Case I-A is proven.

Case I-B:

$$\begin{aligned}
& \frac{\epsilon_{\bar{N},\bar{p}} - p}{kp} < \frac{\epsilon_{\bar{k},\bar{p}}}{kp} \\
& \Leftrightarrow \epsilon_{\bar{N},\bar{p}} - \epsilon_{\bar{k},\bar{p}} < p \\
& \Leftrightarrow \frac{\epsilon_{\bar{k},\bar{p}}}{\epsilon_{\bar{k},\bar{N}}} - \epsilon_{\bar{k},\bar{p}} < p \\
& \Leftrightarrow \frac{\epsilon_{\bar{k},\bar{p}}k(1 - \epsilon_{\bar{k},\bar{N}})}{\epsilon_{\bar{k},\bar{N}}} < kp \\
& \Leftrightarrow \frac{k(1 - \epsilon_{\bar{k},\bar{N}})}{\epsilon_{\bar{k},\bar{N}}} < \frac{kp}{\epsilon_{\bar{k},\bar{p}}} \\
& \Leftrightarrow \frac{\epsilon_{\bar{k},\bar{N}}}{k(1 - \epsilon_{\bar{k},\bar{N}})} > \frac{\epsilon_{\bar{k},\bar{p}}}{kp}
\end{aligned} \tag{B.7}$$

The second line is simple arithmetic, while the third is because $\epsilon_{\bar{N},\bar{p}} = \frac{\epsilon_{\bar{k},\bar{p}}}{\epsilon_{\bar{k},\bar{N}}}$. The fourth line multiplies k to both sides, the fifth divides both sides by $\epsilon_{\bar{k},\bar{p}}$, and the last line takes the inverse. Case I-B is proven.

B.0.7 Proof of Proposition 6

(a) To prove (a), we solve for $\epsilon_{\bar{k},\bar{N}}$ in $\frac{\epsilon_{\bar{k},\bar{N}}}{k(1-\epsilon_{\bar{k},\bar{N}})} < 1 - l^{\max}$ since if the threshold is higher than $1 - l^{\max}$, then the $\bar{k}$ intervention cannot be used.

$$\begin{aligned}
& \frac{\epsilon_{\bar{k},\bar{N}}}{k(1 - \epsilon_{\bar{k},\bar{N}})} < 1 - l^{\max} \\
& \Leftrightarrow \epsilon_{\bar{k},\bar{N}} < k(1 - l^{\max}) - k\epsilon_{\bar{k},\bar{N}} + kl^{\max}\epsilon_{\bar{k},\bar{N}} \\
& \Leftrightarrow \epsilon_{\bar{k},\bar{N}}(1 + k - kl^{\max}) < k(1 - l^{\max}) \\
& \epsilon_{\bar{k},\bar{N}} < \frac{k(1 - l^{\max})}{1 + k(1 - l^{\max})}
\end{aligned} \tag{B.8}$$

where each step can be done through algebra

(b) To prove (b), we note that if we are already in the region R , Theorem 4.5 says that $\bar{k}$ would be the most efficient intervention if $p' > \frac{\epsilon_{\bar{k},\bar{N}}}{k(1-\epsilon_{\bar{k},\bar{N}})}$. Since $\frac{\epsilon_{\bar{k},\bar{N}}}{k(1-\epsilon_{\bar{k},\bar{N}})}$ is a horizontal line, then we must find the lowest point of R and make sure that $\frac{\epsilon_{\bar{k},\bar{N}}}{k(1-\epsilon_{\bar{k},\bar{N}})}$ passes through it.

The lowest point of R occurs at either the intersection of $p' = p$ and $p' = \frac{\epsilon_{\bar{N},\bar{p}}^{-p}}{kp}$, which we call $p_{\bar{N},\bar{p}}^*$ or the intersection of $p' = p$ and $p' = \frac{\epsilon_{\bar{k},\bar{p}}}{kp}$, which we call $p_{\bar{k},\bar{p}}^*$.

First, to solve for $p_{\bar{N},\bar{p}}^*$, we equate $\frac{\epsilon_{\bar{N},\bar{p}}^{-p}}{kp} = p$ and solve for p . This results in $\epsilon_{\bar{N},\bar{p}} = p(kp + 1)$ which can be solved using the quadratic formula.

Next, to solve for $p_{\bar{k},\bar{p}}^*$, we again equate $\frac{\epsilon_{\bar{k},\bar{p}}}{kp} = p$ and solve for p . We then have $p_{\bar{k},\bar{p}}^* = \sqrt{\frac{\epsilon_{\bar{k},\bar{p}}}{k}}$.

Define $\rho = \min(p_{\bar{N},\bar{p}}^*, p_{\bar{k},\bar{p}}^*)$. We can then solve for $\epsilon_{\bar{k},\bar{N}}$ in the inequality $\rho > \frac{\epsilon_{\bar{k},\bar{N}}}{k(1-\epsilon_{\bar{k},\bar{N}})}$ which results in $\epsilon_{\bar{k},\bar{N}} \leq \frac{\rho k}{1+\rho k}$.

Appendix C

Appendix for Chapter 5

C.1 Regeneration Transition Functions During PAN_w

We first introduce two functions: $A(\mathbf{v}, a)$ and $S(\mathbf{v}, a)$ where $\mathbf{v} = (v_1, v_2, \dots, v_n) \in \mathbf{R}^n$ and $a \in \mathbf{Z}^+$, such that:

$$A(\mathbf{v}, a) = \mathbf{w}, \mathbf{w} \in \mathbf{R}^{n+a} \quad (\text{C.1})$$

where $w_i = v_i$ for $i = 1, \dots, n$ and $w_i = 0$ for $i = n+1, \dots, n+a$. And

$$S(\mathbf{v}, a) = \mathbf{y}, \mathbf{y} \in \mathbf{R}^{n-a} \quad (\text{C.2})$$

where $y_i = v_i$ for $i = 1, \dots, n-a$. $A(\mathbf{v}, a)$ adds a zero components to the end of the vector $\mathbf{v}$ while $S(\mathbf{v}, a)$ removes the last a components of the vector $\mathbf{v}$.

With this, the regeneration transition functions for $w = 1, 2$ are

if $s = s_{start}^{PAN_w}$ then

$$G_s^r(\mathbf{q}_s^r, a_s^r, a_s^f) = S((1 - \delta^{ex})(a_s^r + \delta^r a_s^f, q_{s,1}^r, \dots, q_{s,k-1}^r), \rho^k) \quad (\text{C.3})$$

$$G_s^{pd}(\mathbf{q}_s^{pd}, a_s^{pd}, a_s^f) = S((1 - \delta^{ex})(a_s^{pd} + (1 - \delta^r) a_s^f, q_{s,1}^{pd}, \dots, q_{s,k-1}^{pd}), \rho^k) \quad (\text{C.4})$$

if $s = s_{end}^{PAN_w}$ then

$$G_s^r(\mathbf{q}_s^r, a_s^r, a_s^f) = A((1 - \delta^{ex})(a_s^r + \delta^r a_s^f, q_{s,1}^r, \dots, q_{s,k-\rho^k-1}^r), \rho^k) \quad (C.5)$$

$$G_s^{pd}(\mathbf{q}_s^{pd}, a_s^{pd}, a_s^f) = A((1 - \delta^{ex})(a_s^{pd} + (1 - \delta^r)a_s^f, q_{s,1}^{pd}, \dots, q_{s,k-\rho^k-1}^{pd}), \rho^k) \quad (C.6)$$

if $s_{start}^{PAN_w} < 1 < s_{end}^{PAN_w}$ then

$$G_s^r(\mathbf{q}_s^r, a_s^r, a_s^f) = (1 - \delta^{ex})(a_s^r + \delta^r a_s^f, q_{s,1}^r, \dots, q_{s,k-\rho^k-1}^r) \quad (C.7)$$

$$G_s^{pd}(\mathbf{q}_s^{pd}, a_s^{pd}, a_s^f) = (1 - \delta^{ex})(a_s^{pd} + (1 - \delta^r)a_s^f, q_{s,1}^{pd}, \dots, q_{s,k-\rho^k-1}^{pd}) \quad (C.8)$$

Appendix D

Appendix for Chapter 6

D.1 Data-Set and Parameters

This section in the appendix describes the data-set that was used in Chapter 6

D.1.1 Drop-off Rates

The Drop-off rates (if data is available, $q(\mathbf{X}_s^d | d \in (i, j))$) were estimated by multiplying the percent of deaths per category to the death rate and then dividing by 1000. We assumed that the drop-off rate would be the same for subcategories that fall under the same bucket of percent of death. For example, drop-off rate for ages 16 to 17 and 18 to 24 would be the same because they all fall under the same bucket of Ages 5 to 25 in Table D.1.

$$(\text{Drop-off Rate})^{i,j} = \frac{(\text{Percent of Deaths})^{i,j}}{1000} \quad (\text{D.1})$$

TABLE D.1: Death Rate and Percentage of Deaths per Country

Country	Death Rate (per 1,000)	% deaths Under 5	Ages 5 to 25	Ages 25 to 65	Age 65 and over
Zimbabwe	8.4	25	12	42	21
South Africa	10.1	19	10	49	22
Singapore	4.9	0	1	24	75
Australia	6.7	1	1	15	83

D.1.2 Donation Rates and Obtaining the Donor Distribution

We obtained an estimate of the number of donors per state (i, j) as follows. First, we get the total number of donations by multiplying the donation rate per 1000 found in Table D.2 and the Population of state (i, j) found in Table D.3.

$$(Total\ Donations)^{i,j} = \frac{(Population)^{i,j} * (Donation\ Rate)^{i,j}}{1000} \quad (D.2)$$

Next, we multiply the Resulting Total Donations by the percent breakdown of First-Time Donations and Regular Donations in Table D.4.

$$(First-Time\ Donations)^{i,j} = (First-Time\ Donation\ Rate)^{i,j} * (Total\ Donations)^{i,j} \quad (D.3)$$

$$(Regular\ Donations)^{i,j} = (Regular\ Donation\ Rate)^{i,j} * (Total\ Donations)^{i,j} \quad (D.4)$$

The total number of first-time donations is equal to the total number of first time-donors (i.e. Category $i = 1$ donors). We then obtain new Donation Rates by subtracting the number of first-time donors from the population and recalculating the donation rates per 1000 population per age group.

$$(Donation\ Rates\ Less\ First\ Time\ Donors)^{i,j} = \frac{1000 * (Regular\ Donations)^{i,j}}{(Population)^{i,j} - (First-Time\ Donations)^{i,j}} \quad (D.5)$$

From here, we assume that $4c_1^{2,j} = 3c_1^{3,j} = 2c_1^{4,j} = c_1^{5,j}$ for all j to indicate that the total number of donors who donate regularly becomes less and less the larger the number of donations. This assumption is purely for convenience and can be changed without changing the insights. We can then easily solve for the total number of donors per category i by using the assumption that a donor in each category i has i donations. this also implies that we initially assume that donors in category i all have 5 donations. However, we reiterate that future donations by donors in this category imply that they remain in $i = 5$ assuming they do not drop-off. The resulting number of donors are then scaled to a total of 1000.

$C_I^{i,j}$ and $C_{NI}^{i,j}$ are computed as follows:

$$C_I^{i,j} = \frac{(Scaled\ First-Time\ Donations)^{i,j}}{4} \quad (D.6)$$

$$C_{NI}^{i,j} = \frac{C_I^{i,j}}{2} \quad (\text{D.7})$$

We used the donation rates per population subcategory as a proxy for probability of donation in category 1 (if data is available, $p(\mathbf{X}_s^d | d \in (i, j))$). Then we assumed that this probability of donation increases by 0.05 for each increase in i . Additionally, we assumed that if a state (i, j) is targeted, then the probability of donation increases by 0.1.

TABLE D.2: Donation Rates per 1000 People, per Age Group

Country	16 to 17	18 to 24	25 to 44	45 to 64	65 to 70
Zimbabwe	62	7	1	2	10
South Africa	40	24	21	29	20
Singapore	18	50	38	15	0
Australia	42	70	60	95	97

TABLE D.3: Population Data

Country	16 to 17	18 to 24	25 to 44	45 to 64	65 to 70
Zimbabwe	1,594,436	1,350,444	3,686,190	1,403,634	176,757
South Africa	4,656,000	4,870,000	19,290,000	9,214,000	1,276,000
Singapore	358,717	407,567	1,972,683	1,997,117	382,333
Australia	1,421,612	1,566,792	6,513,401	5,858,755	1,188,989

TABLE D.4: Break-down of donations between First-Time and Regular

Country	% First Time Donation	% Regular Donation
Zimbabwe	37%	63%
South Africa	13%	87%
Singapore	22%	78%
Australia	19%	81%

D.2 Donor Flow Constraints

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} + C_I^{0,j} \gamma_s^{0,j} + C_{NI}^{0,j} (1 - \gamma_s^{0,j}) - \bar{c}_s^{i,j} \gamma_s^{i,j} \left(p_I^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_I^{(i,j)(i+1,b)} \right) \quad (D.8)$$

$$- \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) \left(p_{NI}^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_{NI}^{(i,j)(i+1,b)} \right),$$

for $i = 1, j = 1$

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} + C_I^{0,j} \gamma_s^{0,j} + C_{NI}^{0,j} (1 - \gamma_s^{0,j}) - \bar{c}_s^{i,j} \gamma_s^{i,j} \left(p_I^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_I^{(i,j)(i+1,b)} \right) \quad (D.9)$$

$$- \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) \left(p_{NI}^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_{NI}^{(i,j)(i+1,b)} \right)$$

$$+ \bar{c}_s^{i,j-1} \gamma_s^{i,j-1} p_I^{(i,j-1)(i,j)} + \bar{c}_s^{i,j-1} (1 - \gamma_s^{i,j-1}) p_{NI}^{(i,j-1)(i,j)},$$

for $i = 1, j = 2, \dots, l-1$

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} + C_I^{0,j} \gamma_s^{0,j} + C_{NI}^{0,j} (1 - \gamma_s^{0,j}) + \bar{c}_s^{i,j-1} \gamma_s^{i,j-1} p_I^{(i,j-1)(i,j)} + \bar{c}_s^{i,j-1} (1 - \gamma_s^{i,j-1}) p_{NI}^{(i,j-1)(i,j)} \quad (D.10)$$

$$- \bar{c}_s^{i,j} \gamma_s^{i,j} p_I^{(i,j)(i+1,j)} - \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) p_{NI}^{(i,j)(i+1,j)},$$

for $i = 1, j = l$

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} - \bar{c}_s^{i,j} \gamma_s^{i,j} \left(p_I^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_I^{(i,j)(i+1,b)} \right) - \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) \left(p_{NI}^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_{NI}^{(i,j)(i+1,b)} \right) \quad (D.11)$$

$$+ \bar{c}_s^{i-1,j} \gamma_s^{i-1,j} p_I^{(i-1,j)(i,j)} + \bar{c}_s^{i-1,j} (1 - \gamma_s^{i-1,j}) p_{NI}^{(i-1,j)(i,j)},$$

for $i = 2, \dots, m-1, j = 1$

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} + \bar{c}_s^{i-1,j} \gamma_s^{i-1,j} p_I^{(i-1,j)(i,j)} + \bar{c}_s^{i-1,j} (1 - \gamma_s^{i-1,j}) p_{NI}^{(i-1,j)(i,j)} \quad (D.12)$$

$$- \bar{c}_s^{i,j} \gamma_s^{i,j} p_I^{(i,j)(i,j+1)} - \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) p_{NI}^{(i,j)(i,j+1)},$$

for $i = m, j = 1$

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} - \bar{c}_s^{i,j} \gamma_s^{i,j} \left(p_I^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_I^{(i,j)(i+1,b)} \right) - \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) \left(p_{NI}^{(i,j)(i,j+1)} + \sum_{b=j}^{j+1} p_{NI}^{(i,j)(i+1,b)} \right) \quad (D.13)$$

$$+ \bar{c}_s^{i,j-1} \gamma_s^{i,j-1} p_I^{(i,j-1)(i,j)} + \sum_{b=j-1}^j \bar{c}_s^{i-1,b} \gamma_s^{i-1,b} p_I^{(i-1,b)(i,j)}$$

$$+ \bar{c}_s^{i,j-1} (1 - \gamma_s^{i,j-1}) p_{NI}^{(i,j-1)(i,j)} + \sum_{b=j-1}^j \bar{c}_s^{i-1,b} (1 - \gamma_s^{i-1,b}) p_{NI}^{(i-1,b)(i,j)},$$

for $i = 2, \dots, m-1, j = 2, \dots, l-1$

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} + \bar{c}_s^{i,j-1} \gamma_s^{i,j-1} p_I^{(i,j-1)(i,j)} + \sum_{b=j-1}^j \bar{c}_s^{i-1,b} \gamma_s^{i-1,b} p_I^{(i-1,b)(i,j)} + \bar{c}_s^{i,j-1} (1 - \gamma_s^{i,j-1}) p_{NI}^{(i,j-1)(i,j)} \quad (D.14)$$

$$+ \sum_{b=j-1}^j \bar{c}_s^{i-1,b} (1 - \gamma_s^{i-1,b}) p_{NI}^{(i-1,b)(i,j)} - \bar{c}_s^{i,j} \gamma_s^{i,j} p_I^{(i,j)(i,j+1)} - \bar{c}_s^{i,j} (1 - \gamma_s^{i,j}) p_{NI}^{(i,j)(i,j+1)},$$

for $i = m, j = 2, \dots, l-1$

$$c_{s+1}^{i,j} = \bar{c}_s^{i,j} + \bar{c}_s^{i,j-1} \gamma_s^{i,j-1} p_I^{(i,j-1)(i,j)} + \sum_{b=j-1}^j \bar{c}_s^{i-1,b} \gamma_s^{i-1,b} p_I^{(i-1,b)(i,j)} + \bar{c}_s^{i,j-1} (1 - \gamma_s^{i,j-1}) p_{NI}^{(i,j-1)(i,j)} \quad (D.15)$$

$$+ \sum_{b=j-1}^j \bar{c}_s^{i-1,b} (1 - \gamma_s^{i-1,b}) p_{NI}^{(i-1,b)(i,j)},$$

for $i = m, j = l$

D.3 Scaled Donor Distribution and Optimal Policy for Varying Horizon Length T

TABLE D.5: Optimal Policy Comparison for Singapore at Different T

T								
4	12	24		60				
(1,0,3)	(1,0,3)	(1,0,3)	(13,0,3)	(1,0,3)	(13,0,3)	(25,0,3)	(37,0,3)	(49,0,3)
(2,0,3)	(2,0,3)	(2,0,3)	(13,1,3)	(2,0,3)	(13,1,3)	(25,1,3)	(37,1,3)	(49,1,3)
(3,1,3)	(3,0,3)	(3,0,3)	(14,1,3)	(3,0,3)	(15,0,3)	(26,0,3)	(37,2,4)	(50,0,3)
(4,1,3)	(4,0,3)	(4,0,3)	(14,1,4)	(4,0,3)	(16,0,3)	(26,1,4)	(39,1,3)	(52,5,4)
	(5,1,3)	(5,0,3)	(17,0,3)	(5,0,3)	(17,1,3)	(29,0,3)	(41,0,3)	(53,0,3)
	(6,1,3)	(6,0,3)	(17,1,3)	(6,0,3)	(18,0,3)	(29,1,3)	(41,1,4)	(53,1,3)
	(7,1,3)	(7,0,3)	(18,0,3)	(7,0,3)	(19,0,3)	(30,0,3)	(42,1,3)	(55,5,4)
	(8,1,3)	(8,1,3)	(19,1,4)	(8,1,3)	(20,0,3)	(30,1,3)	(42,1,4)	(56,1,3)
	(9,2,3)	(9,1,3)	(21,0,3)	(9,0,3)	(21,0,3)	(33,0,3)	(45,0,3)	(57,1,3)
	(10,1,4)	(10,0,3)	(21,1,4)	(9,1,3)	(21,1,3)	(33,1,3)	(46,1,3)	(58,1,3)
	(11,1,4)	(11,0,3)	(22,0,3)	(11,0,3)	(22,0,3)	(34,0,3)	(46,5,4)	(59,2,3)
	(12,1,4)	(11,1,3)	(22,1,3)	(12,0,3)	(22,1,3)	(35,1,4)	(47,0,3)	(60,5,4)

TABLE D.6: Optimal Policy Comparison for Australia at Different T

T								
4	12	24		60				
(1,0,4)	(1,0,4)	(1,0,4)	(13,0,4)	(1,0,3)	(13,0,4)	(25,0,3)	(37,1,4)	(49,5,4)
(2,0,4)	(2,0,4)	(2,0,4)	(13,1,4)	(1,0,4)	(14,0,4)	(25,0,4)	(37,5,4)	(51,5,4)
(3,0,4)	(3,0,4)	(3,0,4)	(14,0,4)	(2,0,3)	(15,0,4)	(26,0,4)	(39,1,4)	(52,1,4)
(4,1,4)	(4,0,4)	(4,0,4)	(15,0,4)	(3,0,3)	(16,0,4)	(28,0,4)	(40,2,4)	(52,5,4)
	(5,0,4)	(5,0,4)	(17,0,4)	(5,0,3)	(17,0,4)	(29,0,4)	(41,2,4)	(53,1,4)
	(6,0,4)	(6,0,4)	(17,1,4)	(5,0,4)	(18,0,4)	(29,1,4)	(42,3,4)	(53,2,4)
	(7,0,4)	(7,0,4)	(18,0,4)	(6,0,3)	(19,0,4)	(30,0,4)	(42,5,4)	(53,5,4)
	(8,0,4)	(8,0,4)	(18,1,4)	(6,0,4)	(20,0,4)	(32,1,4)	(43,5,4)	(55,5,4)
	(9,1,4)	(9,0,4)	(21,0,4)	(9,0,3)	(21,0,4)	(33,1,4)	(45,1,4)	(57,1,4)
	(10,1,4)	(10,0,4)	(21,1,4)	(9,0,4)	(22,0,4)	(34,1,4)	(45,5,4)	(57,5,4)
	(11,1,4)	(11,0,4)	(22,5,4)	(10,0,4)	(23,0,4)	(34,2,4)	(46,5,4)	(58,5,4)
	(12,1,4)	(12,0,4)	(23,5,4)	(11,0,4)	(24,0,4)	(36,2,4)	(48,5,4)	(59,5,4)

TABLE D.7: Optimal Policy Comparison for Zimbabwe at Different T

T								
4	12	24		60				
(1,0,1)	(1,0,1)	(1,1,1)	(13,0,1)	(1,1,1)	(13,0,1)	(25,0,1)	(37,0,1)	(49,0,1)
(2,0,1)	(1,1,1)	(2,0,1)	(13,1,2)	(2,0,1)	(13,1,2)	(25,1,3)	(37,1,3)	(49,1,4)
(3,0,1)	(3,0,1)	(3,0,1)	(13,1,3)	(3,0,1)	(14,0,1)	(26,0,1)	(38,0,1)	(49,2,3)
(4,0,1)	(4,0,1)	(4,0,1)	(14,0,1)	(4,0,1)	(15,0,1)	(26,1,3)	(38,1,3)	(50,1,3)
	(5,0,1)	(5,0,1)	(17,0,1)	(5,0,1)	(17,0,1)	(29,0,1)	(41,0,1)	(53,0,1)
	(6,0,1)	(6,0,1)	(17,1,3)	(6,0,1)	(17,1,2)	(29,1,3)	(41,1,4)	(53,1,3)
	(7,0,1)	(7,0,1)	(18,0,1)	(7,0,1)	(17,1,3)	(30,0,1)	(42,1,3)	(53,1,4)
	(8,0,1)	(8,0,1)	(18,1,3)	(8,0,1)	(19,1,3)	(30,1,3)	(44,1,3)	(54,1,3)
	(9,0,1)	(9,0,1)	(21,1,3)	(9,0,1)	(21,0,1)	(33,0,1)	(45,1,3)	(57,0,1)
	(9,1,2)	(9,1,2)	(22,1,3)	(10,0,1)	(21,1,3)	(33,1,3)	(46,1,3)	(57,1,3)
	(10,0,1)	(10,0,1)	(23,1,3)	(11,0,1)	(22,1,3)	(34,1,3)	(47,1,3)	(58,1,3)
	(10,1,2)	(11,0,1)	(24,1,3)	(12,0,1)	(24,1,3)	(35,1,3)	(48,1,3)	(60,1,3)

TABLE D.8: Optimal Policy Comparison for South Africa at Different T

T								
4	12	24		60				
(1,0,3)	(1,0,3)	(1,0,3)	(13,1,3)	(1,0,3)	(13,0,3)	(25,0,3)	(37,1,3)	(49,1,3)
(2,0,3)	(2,0,3)	(2,0,3)	(13,2,3)	(2,0,3)	(14,0,3)	(25,1,3)	(37,1,4)	(49,5,4)
(3,1,3)	(3,0,3)	(3,0,3)	(16,1,3)	(3,0,3)	(15,1,3)	(26,1,3)	(38,0,3)	(52,1,3)
(4,1,3)	(4,0,3)	(4,0,3)	(16,1,4)	(4,0,3)	(16,0,3)	(27,1,3)	(39,0,3)	(52,5,4)
	(5,0,3)	(5,0,3)	(17,1,3)	(5,0,3)	(17,0,3)	(29,0,3)	(41,0,3)	(53,0,3)
	(6,0,3)	(6,0,3)	(18,1,3)	(6,0,3)	(17,1,3)	(29,1,4)	(41,1,3)	(53,1,4)
	(6,1,3)	(6,1,3)	(18,1,4)	(7,0,3)	(18,1,3)	(30,1,3)	(43,1,3)	(54,1,3)
	(7,1,3)	(7,1,3)	(20,1,3)	(8,1,3)	(20,0,3)	(30,1,4)	(44,1,4)	(55,1,3)
	(9,1,3)	(9,0,3)	(22,1,3)	(9,1,3)	(21,0,3)	(33,1,3)	(45,0,3)	(57,1,3)
	(10,1,3)	(9,1,3)	(23,5,4)	(10,0,3)	(21,1,3)	(34,0,3)	(45,1,3)	(57,2,3)
	(11,1,3)	(10,1,3)	(24,1,3)	(11,0,3)	(22,1,3)	(34,1,4)	(46,1,3)	(58,1,3)
	(12,1,3)	(11,1,3)	(24,5,4)	(12,1,3)	(23,1,3)	(36,1,3)	(46,1,4)	(58,1,4)

FIGURE D.1: Scaled Donor Distribution/Heat Map per country: Baseline, Optimal ($B = 8, T = 8$), Target New Donors($B = 8, T = 8$), and No Budget($B = 0, T = 8$)

(A) Donor Distribution Results for Zimbabwe

(B) Donor Distribution Results for South Africa

(C) Donor Distribution Results for Singapore

(D) Donor Distribution Results for Australia

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