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Key Points:

- A new length scale is proposed for describing the size of the large-scale flow structure associated with local scour
- Different types of clear-water scour can be compared in a consistent approach based on the new length scale
- The scour depth normalized by the new length scale varies in a fashion that is comparable to the bedload function

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Unified Criterion for Clear-Water Local Scour Induced by Junction Flows and Wall Jets

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Abstract Local scour in a sediment bed varies depending on the type, size and shape of hydraulic structures, and the properties of approach flow and sediment particles. Published studies are hitherto confined primarily to empirical experiments based on particular types of hydraulic structure and there is almost no comparison of scour amongst different types of hydraulic structures. This study aims to provide an attempt to unify different types of clear-water scour based on a new length scale, which is proposed to characterize the size of the large-scale flow structure that governs scouring processes. The new length scale serves as a generalized hydraulic radius, which is applicable for all types of clear-water scour related to junction flows at bridge piers and abutments, and wall jets including culverts. The analysis indicates that when normalized with the new length scale, the equilibrium scour depths can be described with a unified function of the densimetric Froude number, regardless of different scour types. In addition, this study also shows that the proposed function varies with the scour efficiency, in a fashion resembling the bedload function in the range of low to high transport regimes. This finding, which is founded on solid physical grounds, reveals that different kinds of local scour phenomena can be interpreted in unison rather than based on the individual type of hydraulic structures.

1. Introduction

Sediment erosion and deposition are the two basic processes that shape rivers and coasts. The processes become complex in the presence of human interventions, such as bridge construction, pipeline installation, and various offshore developments, which may cause intensive local scour (Graf & Altinakar, 2017). Around manmade structures, the formation of local scour holes may endanger their foundations, and sometimes lead to catastrophic structural failure (Hong et al., 2012). There have been substantial research results over the past eight decades, which were not only discussed in archival publications but also summarized in published reports and design manuals (Adduce & Rocca, 2006; Adduce & Sciortino, 2006; Breusers & Raudkivi, 1991; Ettema et al., 2011; Fael et al., 2006; Hoffmans & Verheij, 1997; Kirkil & Constantinescu, 2010; Melville & Coleman, 2000; Radice et al., 2009; Sheppard et al., 2011; Sumer & Fredsøe, 2002). Significant efforts are also made recently (Cheng & Wei, 2019; Cheng, Wei, et al., 2020; Coscarella et al., 2020; Guan et al., 2016; Manes & Brocchini, 2015; Si et al., 2018; 2019; Wei et al., 2020). Given the intricate scouring mechanisms that are as yet not fully understood, past studies usually treated different types of scour independently under simplified laboratory conditions. For example, local scour at a bridge pier has seldom been analyzed together with that induced by a wall jet as they occur in different flow scenarios. One exception is that bridge abutment scour has been compared to bridge pier scour, because an abutment may be approximated to be one-half of a wide pier (Melville, 1997). Notwithstanding, there is almost no common ground for comparing scour of different origins, such as, pier, abutment, culvert, and wall jet.

This paper aims to evaluate the equilibrium clear-water scour induced by junction flows, at bridge piers and abutments, as well as wall jets and culverts in a consistent approach. Key to resolving the problem is the development of a general length scale based on the size of the large-scale flow structure that governs the scouring processes, which is presented in Section 2. In Section 3, we conduct a scaling analysis of the scour depth using the proposed length scale. Section 4 shows that the proposed functional relation agrees well with empirical results for the case of pier scour. With an analysis of experimental data published in the literature, Section 5 demonstrates that the scour depth varies similarly across various cases associated with junction flows and wall jets, and the normalized scour depth can be expressed as a unified function of the densimetric Froude number. In the analysis, we also consider the effects of flow depth, viscosity, sediment

Figure 1. Schematic of junction flows at (a) bridge pier and (b) abutment, and (c) wall jet.

nonuniformity and grain size. In Section 6, the observed variation in the normalized scour depth is further shown to be comparable to that in bedload sediment transport from the low to high transport regimes. The present study is limited to three-dimensional cases, and thus the scour induced by two-dimensional wall jets is excluded.

2. Universal Length Scale

Choosing appropriate characteristic length and velocity scales is essential for performing scaling analyses of sediment transport and local scour problems (Raudkivi, 1998; Yalin, 1977). In comparison to the velocity scale, which is usually represented by the approach flow velocity, the length scale of flows around structures like a pier is not straightforward as the flow is not only confined by the channel cross section but also disturbed by the structure. Several length parameters are influential in scour-related processes. For example, pier scour is subject to pier width b , flow depth h , and channel width B . The effects of h and B may be ignored for deep flows occurring in a wide channel ($h \gg b$, $B \gg b$). For this simplest case, the scour depth at a bridge pier would only be associated with the pier width. This could be the reason why in most pier-scour studies, the scour depth is often scaled with the pier width (Chiew, 1984; Chiew & Lim, 2000; Ettema, 1976; Melville, 1997; Oliveto & Hager, 2002; Raudkivi & Ettema, 1983; Sheppard & Miller, 2006). For scour induced by three-dimensional wall jets, the outlet size (e.g., pipe parameter) can be simply taken as the length parameter for scaling the scour depth (Chiew & Lim, 1996; Rajaratnam & Berry, 1977). To compare different types of scour, it is imperative to know under what conditions (including the length scale) the scour depth at a pier, for example, would be equivalent to that induced by a wall jet for the same incoming flow velocity and bed sediment.

No matter what factors cause the local scour, the scour depth is always indicative of the energy dissipation of turbulent flow. For a given incoming flow velocity, the energy of the flow depends on the extent of the flow. This is because the kinetic energy of a bulk flow, which is proportional to mV^2 (with m denoting the mass and V the average flow velocity), increases with increasing mass or extent of the flow. Moreover, the extent of flow would be represented by the large-scale flow structure, which contains most of the turbulent energy. This will be further discussed for the different scour cases.

2.1. Case I: Junction Flow at a Pier

For the junction flow at a bridge pier (see Figure 1a), it is the obstruction induced by the pier that dictates the intensity of the flow driving the scouring action. The wider the pier, the larger the obstruction, and thus more energy is needed to initiate and sustain scour. It is noted that the pier obstruction is also affected by the flow depth, particularly in shallow flows. Cheng and Wei (2019) argued that the effective size of the obstruction can physically be linked to the large-scale vortex system that forms in front of the pier. They reported that the size of the obstruction related to a pier, R_p , can be generally defined as

$$R_p = \frac{bh}{b + 2h} \text{ or } \frac{1}{R_p} = \frac{1}{0.5b} + \frac{1}{h} \quad (1)$$

Figure 2. Variation of pier hydraulic radius. It approaches one-half of the pier width for narrow piers ($b \ll h$) and the flow depth for wide piers ($b \gg h$).

where b is the pier width, h is the flow depth and subscript p denotes the pier. Equation 1 indicates that $R_p \rightarrow 0.5b$ for relatively deep flows or narrow piers ($h \gg b$), and $R_p \rightarrow h$ for relatively shallow flows or wide piers ($h \ll b$).

Cheng and Wei (2019) further showed that R_p essentially quantifies the size of the horseshoe vortex system in front of a pier (Shen et al., 1969), which can be measured as the distance from the separation point of the approach boundary layer to the nose of the pier. Some published R_p -values are compared in Figure 2, in which the data points denote the laboratory measurements of the vortex size reported by Kothiyari et al. (1992) and Sumer and Fredsøe (2002). Figure 2 reveals that R_p approaches $0.5b$ for narrow piers ($b \ll h$) and h for wide piers ($b \gg h$).

This behavior is similar to the conventional hydraulic radius of a rectangular channel, R_h , defined as

$$R_h = \frac{Bh}{B + 2h} \text{ or } \frac{1}{R_h} = \frac{1}{0.5B} + \frac{1}{h} \quad (2)$$

Equation 2 shows that R_h varies between two extreme values, that is $0.5B$ for narrow channels and h for wide channels. It is also noted that R_h describes the dimension of the large-scale flow structure that is confined by the cross section of open channel flows (Cheng & Wei, 2019; Gioia & Bombardelli, 2001). Given the similarity between R_p and R_h , R_p is referred to as the pier hydraulic radius. Since R_p characterizes the size of the large-scale vortex system, it is deemed to be the governing length parameter that dictates pier-scour.

2.2. Case II: Junction Flow at an Abutment

In the presence of a pier, the approach flow separates into two streams when it arrives at the pier, of which each is diverted downstream only by one-half of the pier. With this consideration, an abutment may be approximated as one-half of a wide pier. Therefore, to apply Equation 1 to an abutment (see Figure 1b), it can be modified as

$$R_a = \frac{Lh}{L + h} \text{ or } \frac{1}{R_a} = \frac{1}{L} + \frac{1}{h} \quad (3)$$

where subscript a denotes the abutment, and L is the abutment length in the direction perpendicular to the main flow. Similar to R_p , this abutment hydraulic radius is the governing length parameter that dictates abutment-scour.

2.3. Case III: Wall Jet Through a Culvert Outlet

A culvert drains water in full flowing or nonfull flowing states, with the former resembling a three-dimensional wall jet. In comparison with local scour induced by junction flows at a pier or abutment, jet scour does not involve a solid structure that induces a fluid-structure interaction. However, the size of the large-scale flow structure associated with a jet through a circular pipe appears much simpler to determine because the flow is confined by the cross section at the outlet of the pipe without the three-dimensional flow separation associated with junction flows. Thus, its relevant length scale R_j may be quantified using the conventional circular pipe hydraulic radius, that is one quarter of the pipe diameter D ,

$$R_j = \frac{D}{4} \quad (4)$$

where subscript j denotes the jet.

The above approach can be further generalized for noncircular wall jets and nonfull flowing culverts, in which the length scale is calculated as the ratio of the cross-sectional area to the wetted perimeter at the outlet. It should be mentioned that in this study, only the horizontal culvert and wall jet are considered, and the tail water effect is ignored.

3. Scaling Analysis

Regardless of the types of scour, the equilibrium scour depth d_{se} under clear-water conditions essentially depends on how the sediment bed responds to the energy of the flow field. If the approach flow is described using the universal length scale $R (= R_p, R_a \text{ or } R_j)$ and the approach velocity U , for a sediment bed consisting of noncohesive uniform grains, d_{se} is functionally related to parameters as follows:

$$d_{se} = f(R, U, d_{50}, \rho, \rho_s, g, \nu) \quad (5)$$

where d_{50} is the median particle diameter of sediment, ρ is the fluid density, ρ_s is the particle density, g is the gravitational acceleration and ν is the fluid viscosity.

In Equation 5, R is used to replace certain geometric dimensions, for example, b and h for a pier. The basic idea here is to use R , a combination of b and h , to characterize the domain of the turbulent flow approaching the pier. This is different from the consideration in usual dimensional analysis that involves both b and h . However, to employ such a combined parameter is not completely new. A well-known example is related to the conventional analysis of flow resistance in an open channel. For example, the friction factor f is computed as $f = 8gR_h S / U^2$, where S is the energy slope. Here, the computation is conducted based only on the combined parameter, $R_h [= Bh / (B + 2h)]$ for a rectangular cross-section. This implies that it is not necessary to apply B and h individually for quantifying the bulk flow properties.

The present study aims to simplify the analysis with a similar idea. For example, for a bridge pier, we attempt to directly use a single length scale R_p rather than two length parameters, b and h . Being different from b and h , which are geometric dimensions of a pier, R_p serves as a measurement of the dimension of the flow structure, that is the vortex formed in front of a pier. In other words, as a single length scale, R_p is a superior choice when compared with b and h in the description of the scour process. This could be further illustrated by considering the extreme case of $h \gg b$, for which the effect of h would become insignificant on the scour depth. However, R_p is still justifiable as it reduces to $0.5b$ when $h \gg b$. Similarly, for the case of $h \ll b$, the effect of b can be ignored, while R_p reduces to h . This means that R_p can be considered as a generalized length parameter. As a result, the application of the single combined parameter R_p (rather than both h and b) would simplify relevant analyses.

Applying the Buckingham Pi theorem to Equation 5 by choosing R , U and ρ as repeating variables yields

$$\frac{d_{se}}{R} = f\left(\frac{d_{50}}{R}, \frac{\rho_s}{\rho}, \frac{U}{\sqrt{gR}}, \frac{UR}{\nu}\right) \quad (6)$$

Dividing $\frac{U}{\sqrt{gR}}$ by $\sqrt{\frac{d_{50}}{R}}$ and rearranging the dimensionless parameters in Equation 6 yields

$$\frac{d_{se}}{R} = f\left(\frac{d_{50}}{R}, \frac{\rho_s}{\rho}, \frac{U}{\sqrt{gd_{50}}}, \frac{UR}{\nu}\right) \quad (7)$$

Next, Equation 7 is simplified by considering the gravitational effect on submerged particles, that is replacing g with a modified gravitational acceleration, $g' = \frac{\rho_s - \rho}{\rho} g$. As a result, $\frac{\rho_s}{\rho}$ is dropped from Equation 7.

It should be noted that this replacement cannot be derived based on the Buckingham Pi theorem, but it is a useful manipulation, as explained in the following.

The subject of sediment transport or local scour concerns two phases, sediment particles and fluid. The effective density of a particle submerged in fluid is the difference of particle density ρ_s and fluid density ρ ,

that is $\rho_s - \rho$. The relative effective density is equal to $\frac{\rho_s - \rho}{\rho}$. The multiplication of $\frac{\rho_s - \rho}{\rho}$ and the gravitational acceleration g , that is g' , delineates the relative submerged weight of a particle per unit volume. This combination has been widely applied to consider the submerged weight effect in the formation of important parameters. For example, to describe the submerged weight effect on the settling velocity w , the drag can be normalized as the drag coefficient $C_D = \frac{4}{3} \frac{g' d_{50}}{w^2}$. Other successful applications of g' in studies of sediment transport include the bed shear stress τ_b being normalized as $\theta = \frac{\tau_b}{\rho g' d_{50}}$ (Shields number), and the volumetric sediment transport rate q_s being normalized as $\phi = \frac{q_s}{\sqrt{g' d_{50}^3}}$ (Einstein number).

With the above consideration, Equation 7 is simplified as

$$\frac{d_{se}}{R} = f\left(\frac{d_{50}}{R}, F_d, \frac{UR}{\nu}\right) \quad (8)$$

where F_d is the densimetric Froude number defined as

$$F_d = \frac{U}{\sqrt{g' d_{50}}} \quad (9)$$

The densimetric Froude number describes the relative fluid force in comparison to the submerged particle weight.

Equation 8 shows that the normalized scour depth depends on the densimetric Froude number, the relative particle size and the Reynolds number defined based on R . If both viscous and particle size effects are assumed negligible, the functional relationship given in Equation 8 can be further simplified as

$$\frac{d_{se}}{R} = f(F_d) \quad (10)$$

In the subsequent data analysis, the viscous effect will be evaluated in detail by checking if the approach flow is in the transitional or fully rough turbulence stage, for which a procedure is developed for the computation of the friction factor with the data used. As shown later in the analysis, the viscous effect on the scour depth appears to be insignificant. In addition, for the case of relatively large particle size, a particle size correction will be performed based on previous studies of pier scour, with the details provided in Section 5.

In a typical scouring phenomenon, turbulence may play an important role in sediment-flow interactions in different stages of the scouring processes (Coscarella et al., 2020; Manes & Brocchini, 2015). However, in the proposed equation, Equation 10, the turbulent components of the flow are not explicitly considered. This is because the bulk properties of the approach flow could better embody the large-scale flow structure that dominates the scouring processes and eventually controls the equilibrium scour depth. Therefore, the present study focuses on the bulk properties (e.g., R and U) of the approach flow rather than turbulent flow characteristics observed during the scouring process or within the scour hole.

4. Comparisons With Empirical Functions for the Case of Pier Scour

In the following, it is shown that Equation 10 is comparable to some simple functions commonly used in previous studies of pier scour. First, by ignoring the viscous effect, $\sqrt{g' d}$ is proportional to the settling velocity w , that is $\sqrt{g' d} \sim w$ (Cheng, 1997, 2008a). Second, one may also take the critical depth-averaged flow velocity for bed sediment entrainment U_c to be proportional to w , that is $U_c \sim w$ (Yang, 1973). Therefore, $\sqrt{g' d} \sim U_c$. This means that $F_d \sim U / U_c$, and thus Equation 10 can be rewritten as

$$\frac{d_{se}}{R} = f\left(\frac{U}{U_c}\right) \quad (11)$$

For narrow piers ($b \ll h$), one may take $R \approx 0.5b$ based on Equation 1. Therefore, Equation 11 shows that d_{se}/b depends on U/U_c , that is

$$\frac{d_{se}}{b} = f\left(\frac{U}{U_c}\right) \quad (12)$$

This dependence has been confirmed by previous experimental studies (Chiew & Melville, 1987; Raudkivi & Ettema, 1983). In comparison to Equation 12, Equation 10 may be applied generally without the need to consider the pier-narrowness effect, as was customarily done in published studies.

Furthermore, Equation 10 shows that when F_d approaches a constant, d_{se}/R would also become a constant. As discussed above, $F_d \sim U / U_c$. Therefore, when the approach flow velocity upstream a pier is equal to the critical velocity for incipient sediment motion, that is $U = U_c$, F_d can be considered as a constant. This yields

$$\frac{d_{se}}{R} = c \quad (13)$$

where c is a constant. Applying Equation 13 for the case of pier scour and using Equation 1 to express R , one gets

$$\frac{d_{se}}{b} = \frac{ch / b}{1 + 2h / b} \quad (14)$$

The clear-water scour at a pier under the condition of $U \approx U_c$ has been investigated by Melville and Coleman (2000) who expressed the upper limit of the maximum scour depth for circular piers as follows:

$$\frac{d_{se}}{b} = \begin{cases} 2.4 & \text{for } h / b \geq 1.43 \\ 2\sqrt{h / b} & \text{for } 0.2 < h / b < 1.43 \\ 4.5h / b & \text{for } h / b \leq 0.2 \end{cases} \quad (15)$$

Both Equations 14 and 15 are compared with published experimental data (Basak et al., 1975; Melville & Coleman, 2000) in Figure 3. It shows that Equation 14 with $c = 4.6$ represents well the trendline of the experimental data, while Equation 14 with $c = 5.6$ is comparable to the empirical formula, Equation 15, for describing the upper bound of the data.

In addition, a result similar to Equation 13 has also been obtained recently for wave-induced pipeline scour by Cheng et al. (2020). They show that if the current velocity is smaller than the maximum orbital velocity of water particles at the bed, the scour depth normalized by the size of vortex street, which is similar to d_{se} / R , can be approximated as a constant for both wave-alone and wave-plus-current conditions.

5. Comparisons With Experimental Data for All Scour Cases

For comparison purposes, three large databases that were previously published (summarized in Table 1) are used to validate Equation 10 for different types of clear-water scour at bridge pier, abutment, and culvert (including circular wall jet, noncircular wall jet, and nonfull flowing culvert). For local scour at a bridge pier or abutment, the data used for analysis are those reported by Ettema (1980) and Molinas (2003). Ettema (1980) study of clear-water pier scour is considered classical, which comprehensively addresses the effect of factors including pier size, flow depth, particle size and gradation. Molinas' (2003) experimental work appears more systematic than others, in which three flume experiments were employed to collect a large amount of data for both uniform and nonuniform sediment conditions. For culvert scour, Tan et al. (2020) recently compiled 588 sets of data from extensive published literature, which mainly covers three cases, that

Figure 3. Normalized scour depth remains unchanged for clear-water pier scour under the condition of $U \approx U_c$.

is circular wall jet, noncircular wall jet and nonfull flowing culvert. In addition, the local scour data for relatively large-scale circular piers (Mueller & Wagner, 2005; Sheppard et al., 2004) are also included for analysis. To reduce uncertainties in scour depth measurements, all the datasets with scour depth less than 5 mm are excluded. For the abutment scour (Molinas, 2003), the datasets with the relatively wide width of 0.46 m are also removed from the database, as the effect of the longitudinal dimension of an abutment is not considered in the present analysis.

To further prepare for the analysis, Equation 10 needs to be corrected to account for the sidewall and sediment gradation effects. In addition, both viscous and grain size effects are also considered.

5.1. Sidewall Effects

Given the limited width of a typical laboratory flume, experimental data are usually influenced by the sidewall of the flume. This requires sidewall corrections to be made for the flow depth and approach flow velocity. By noting that the conventional hydraulic radius in open-channel flows physically represents an effective flow depth that accounts for the sidewall effects, Equations 1 and 3 can be modified by replacing the flow depth h with the hydraulic radius R_h , which yields

Table 1
Summary of Datasets Used for the Present Analysis

Reference	Scour type	No. of datasets	$d_{50}(mm)$	σ_g	F_{d84}	$\frac{d_{se}}{R}$	$\frac{R}{d_{50}}$	$\frac{b}{d_{50}}$	$\frac{B}{b}$	$\frac{B}{L}$
Ettema (1980)	Pier	97	0.24–7.8	1.1–1.3	1.56–4.11	0.62–7.83	1.8–416.8	3.7–1,000	6.3–52.3	/
Molinas (2003)	Pier	184	0.55–16.7	1.2–3.7	0.48–4.52	0.10–5.37	3.4–131.1	10.5–392.7	8.6–31.58	/
	Abutment	214	0.78–1.80	1.2–3.9	0.85–2.90	0.14–4.87	9.3–55.1	/	/	6.0–24.4
Sheppard et al. (2004)	Pier (large-scale)	14	0.22–2.9	1.21–1.51	2.47–5.75	2.75–6.06	42.7–1,652.7	138.5–4,136.4	6.7–55.5	/
Mueller and Wagner (2005)	Pier (large-scale)	8	7.51	6.9	1.85–2.51	0.98–2.40	109.3–122.2	319.6	/	/
Tan et al. (2020)	Circular wall jet	302	0.24–204.2	1.0–4.09	1.68–73.06	1.38–76.4	0.37–25.4	/	/	/
	Noncircular wall jet	74	0.11–2.46	1.13–1.5	1.72–18.91	1.65–16.0	2.7–70.5	/	/	/
	Nonfull flowing culvert	171	0.49–5.2	1.13–3.83	0.99–6.68	1.27–9.83	3.4–54.9	/	/	/

Figure 4. Plan view of approach and side flow velocities for a pier.

to be considered based on U_1 and U_2 rather than U . It should be mentioned that such an approach has been successfully applied in studies of open channel flows with vegetation simulated using cylindrical rods. For example, Cheng and Nguyen (2011) showed that a general drag coefficient function can be established provided that both the vegetation-related drag coefficient and Reynolds number are redefined based on the average velocity at the side of cylindrical rods.

5.2. Sediment Gradation Effects

In practice, a natural sediment bed often comprises nonuniformly distributed particles, for which the gradation effect needs to be considered. For sediment beds comprising nonuniform particles, the representative particle size is not necessarily the median diameter d_{50} . Instead, it could be related to both d_{50} and the geometric standard deviation or gradation coefficient σ_g , which is defined as $\sqrt{d_{84} / d_{16}}$, with d_{84} and d_{16} denoting the 84th and 16th percentiles of the particle size distribution, respectively. For the case that the particle size distribution can be approximated as lognormal, the representative particle diameter d_r of a sediment bed can be expressed as (Cheng, 2016)

$$d_r = d_{50} \sigma_g^n \quad (18)$$

where n is an exponent. For a given particle size distribution, the d_r -related percentile p_r varies with n as follows:

$$p_r = 50 \left[\operatorname{erf} \left(\frac{n}{\sqrt{2}} \right) + 1 \right] \quad (19)$$

When $n = 1$, $p_r \approx 84$ and $d_r \approx d_{84}$. The choice of $n = 1$ is verified using the clear-water pier scour data of Molinas (2003), for which a series of correlation analyses was performed between d_{se} / R_v and F_d that is defined based on different representative particle sizes. The result plotted in Figure 5 clearly shows that the squared coefficient of correlation peaks when $n = 1$ or $d_r \approx d_{84}$. This implies that the representative particle size

Figure 5. Variation of squared correlation coefficient with the power index n in Equation 18. The representative particle diameter is d_{84} for $n = 1$.

Figure 6. Nikuradse-style friction factor diagram for two-dimensional open channel flow.

can be taken to be d_{84} . It should be mentioned that the same approximation has also been implemented in the previous studies of bed resistance (Cheng, 2017; Ferguson, 2007; Whiting & Dietrich, 1990).

As a result, the densimetric Froude number given in Equation 10 is revised as

$$F_{d84} = \frac{U}{\sqrt{g'd_{84}}} \quad (20)$$

5.3. Viscous Effect of Approach Flows

The viscous effect of the approach flows can be negligible for scour depth prediction, as shown here. To assess viscous effects, a modified Nikuradse friction factor diagram for two-dimensional open channel flows is used, which can be expressed in the following empirical function (Cheng, 2008b):

$$\frac{1}{f} = \left(\frac{Re_h}{24} \right)^\alpha \left(1.8 \log \frac{Re_h}{2.1} \right)^{2(1-\alpha)\beta} \left(2 \log \frac{11.8h}{k_s} \right)^{2(1-\alpha)(1-\beta)} \quad (21)$$

where f is the friction factor, $Re_h = Uh / \nu$, k_s is the roughness height,

$$\alpha = \frac{1}{1 + \left(\frac{Re_h}{850} \right)^9} \quad (22)$$

and

$$\beta = \frac{1}{1 + \left(\frac{Re_h}{160h / k_s} \right)^2} \quad (23)$$

Equation 21 is plotted in Figure 6, which is superimposed with a division curve given by

$$f_c = \left(\frac{200h}{k_s Re_h} \right)^2 \quad (24)$$

where f_c is the critical friction factor for the division of fully rough bed and transitional bed. Equation 24 can be rewritten as $\frac{u_* k_s}{\nu} \approx 70$, which was previously presented by Rouse in 1943 (Brown, 2002). If $f > f_c$, the channel bed can be considered fully rough so that the viscous effect can be ignored. In Equation 21, the flow depth is used for two-dimensional flows, and it may be replaced with the hydraulic radius for three-dimensional flows.

For the experimental data used in the present study, the friction factor f can be computed using Equation 21 with the measurements of U , h , and d_{50} , as the shear velocity was not provided in the relevant studies. In the computation, k_s is taken to be $2d_{50}$ and the kinematic viscosity ν is taken to be $10^{-6} \text{ m}^2/\text{s}$. Then, Equation 24 is used to calculate f_c . By applying the criterion of $f > f_c$, the data can be separated into two groups, one for fully rough bed condition and the other for transitional bed condition. The computed result shows that the exclusion of the data for the transitional bed condition does not necessarily improve the data correlation. For example, for the pier scour data provided by Molinas (2003), the correlation coefficient varies from 0.74 to 0.75, if only the cases of fully rough bed (34 out of 184) are considered. However, the influence becomes negative for the abutment scour data provided by Molinas (2003), for which the correlation coefficient

Figure 7. Variation of grain size correction factor with the ratio of pier width to grain diameter.

decreases from 0.73 to 0.70, if only the cases of fully rough bed (167 out of 214) are considered. Similar inconclusive assessment results are also obtained for the cases of jet and culvert scour.

The above results imply that the viscous effect is negligible for scour depth prediction at least for the data used in the present study. This could be explained by noting the large-scale turbulence. For example, for the upstream flow approaching a pier, the flow could be transitional, for which the viscosity would play a role in the bed shear. However, such a viscous effect would not be important for local scour around a pier, as the pier-induced turbulent eddies (rather than the regular boundary shear) would dominate flow-sediment interactions locally. This is why there is almost no involvement of the viscosity factor in typical local scour analyses in the literature. In the analysis of scale effect in pier scour experiments, Ettema et al. (1998) stated that the pier Reynolds number is not a significant parameter if the flow around a pier is fully turbulent.

5.4. Grain Size Effects

The grain size effect on local scour may be considerable for laboratory modeling studies. Raudkivi and Ettema (1983) reported that for clear-water pier scour, the equilibrium scour depth d_{se} normalized by the pier width is unaffected by the particle diameter provided that $b/d_{50} > 20 \sim 25$. They explained that for particles greater than the size of the groove excavated at the base by the downflow in front of the pier, they would dissipate some energy of the downflow and thus impede the erosion process. Chiew (1984) showed that for live bed scour, d_{se}/b increases almost linearly with increasing pier to sediment size ratio for $b/d_{50} < 50$, after which d_{se}/b becomes independent of b/d_{50} . However, an opposing result was reported by Lee and Sturm (2009) based on a more comprehensive analysis of both laboratory and field data. Lee and Sturm (2009) showed that a relatively large grain size would increase the scour depth for b/d_{50} in the range of 10–200, and the grain size effect is negligible only for $b/d_{50} > 200$.

In Figure 7, two series of data are plotted, one presented by Lee and Sturm (2009) for the Froude number of the approach flow < 0.4 , and the other by Chiew (1984). In the figure, $C_{d50} = \frac{d_{se}|_{b \gg d_{50}}}{d_{se}}$ is defined as the ratio of the scour depth without the grain size effect to that with the effect. According to Lee and Sturm (2009), $d_{se}|_{b \gg d_{50}} \approx 1.3b$ for $b/d_{50} > 200$. From Figure 7, it is interesting to observe that the grain size effects reported by the two studies differ clearly from each other for $b/d_{50} = 10 - 200$.

Figure 8. Relationship between normalized scour depth and densimetric Froude number. For $F_{d84} > 3$, the relationship becomes linear. (a) log-log plot; (b) semi-log plot.

Based on the data presented largely by Lee and Sturm (2009) for $b/d_{50} = 10 - 5000$ and the Froude number of the approach flow < 0.4 , the grain size correction factor is proposed here as

$$C_{d50} = \frac{8.5}{b/d_{50}} + 1 - \exp\left(-0.01 \frac{b}{d_{50}}\right) \quad (25)$$

Figure 7 shows that Equation 25 generally represents the data from Lee and Sturm (2009) and also the trendline of the data from Chiew (1984) for small values of b/d_{50} . For the present study, Equation 25 is used to correct the grain size effect on the pier scour.

5.5. Data Processing and Comparison

The following procedure is applied to the data analysis and subsequent comparison:

- (1) Calculate the densimetric Froude number using Equation 20, where the average flow velocity U is replaced with $UB/(B - b)$ for pier scour, and $UB/(B - L)$ for abutment scour;
- (2) Calculate R for the three cases of local scour, using Equation 16 for bridge piers, Equation 17 for abutments, and Equation 4 for culverts (or wall jets);
- (3) Use Equation 25 to calculate C_{d50} and then correct scour depth as $C_{d50}d_{se}$; and
- (4) Plot d_{se}/R against F_{d84} .

Figure 8 presents the relationship between d_{se}/R and F_{d84} for the four cases, that is local scour at piers, abutments, culverts and wall jets. Moreover, superimposed in Figure 8 is the best-fit curve given as follows:

$$\frac{d_{se}}{R} = 1.3F_{d84} \exp\left(-\frac{7}{F_{d84}^5}\right) \quad (26)$$

Due to the large size of the database, there is significant overlapping of the data points in Figure 8. For clarity, all series of data are re-plotted in Figure 9, with four panels, each assigned to an individual case, namely circular and noncircular wall jets, nonfull flowing culverts, piers, and abutments. In particular, in Figure 9c, the categories of data series concerning regular pier scour (Ettema, 1980; Molinas, 2003) and large-scale pier scour (Sheppard et al., 2004; Mueller & Wagner, 2005) are plotted together. It shows that the two categories of data generally agree with each other, in spite of the scattering data points.

From Figures 8 and 9, it can be seen that for the high densimetric Froude number ($F_{d84} > 3$), the relationship is linear, that is $d_{se}/R \approx 1.3F_{d84}$, which applies largely to the cases of culvert and wall jet. For $F_{d84} < 3$, significant reductions in d_{se}/R occur, which applies to the cases of piers and abutments. In spite of the different variations, all the data seem to consistently follow the common trend line as described by Equation 26. This result implies that the generalized hydraulic radius proposed in this study is acceptable to correlate all the data despite the different scour types.

Figure 9. Variation of normalized scour depth with densimetric Froude number for four cases: (a) circular and noncircular wall jets; (b) nonfull flowing culvert; (c) bridge pier; and (d) abutment. The linear relationship exists for $F_{d84} > 3$.

6. Comparison of D_{se}/R With Einstein Number

To better decipher the different variations in d_{se}/R , one may make an analogy between local scour and bed-load transport. This is because the relationship between d_{se}/R and F_d , as shown in Figures 8a, is strikingly similar to that between the dimensionless bedload transport rate (or Einstein number ϕ) and the dimensionless shear stress (or Shields number θ). To the best of our knowledge, there is no such comparisons reported in the literature. Although a scour hole results from sediment transport that occurs locally, it seems to evolve in its own fashion, of which the mechanism is much more complex than the description presented in

Figure 10. Variation of bedload transport rate with Shields number.

gets $q_s \sim u_s \delta$ without considering small variations in the bed porosity. In addition, if the viscous effect is ignored, the particle settling velocity, $w = 1.15\sqrt{g'd_{50}}$ (Cheng, 1997). Substituting these results into Equation 27 yields

$$\phi \sim \frac{u_s \delta}{w d_{50}} \quad (28)$$

Furthermore, by assuming that $u_s \sim w$, Equation 28 can be simplified as

$$\phi \sim \frac{\delta}{d_{50}} \quad (29)$$

Although the assumption of $u_s \sim w$ is crude, it is acceptable for scaling analyses. This is explained in the following steps. First, the particle velocity is proportional to the shear velocity u_* (Cheng & Emadzadeh, 2014; Fernandez Luque & van Beek, 1976). Based on their experimental data, Fernandez Luque and van Beek 1976 obtained

$$u_s = 11.5(u_* - 0.7u_{*c}) \quad (30)$$

where u_{*c} is the critical shear velocity for incipient sediment motion. If u_{*c} is relatively small, Equation 30 gives $u_s \sim u_*$. Next, if the viscous effect is ignored, the critical condition for initial particle suspension can be expressed as $w > 2u_*$ (Cheng & Chiew, 1999). This implies that for bedload transport, $u_{*c} < u_* < 0.5w$. On the other hand, assuming that the critical Shields number, $\theta_c = u_{*c}^2 / (g'd_{50}) = 0.06$, $u_{*c} = 0.25\sqrt{g'd_{50}} = 0.22w$ by applying $\sqrt{g'd_{50}} = 0.87w$ (Cheng, 1997). Therefore, $0.22w < u_* < 0.5w$. This means that u_* is in the same order as w for bedload transport. With the above considerations, it is reasonable to apply the assumption of $u_s \sim w$ for the analysis.

Equation 29 indicates that the dimensionless bedload transport simply represents the ratio of the two length parameters, that is δ / d_{50} , the bedload thickness relative to the particle size. In this sense, the bedload layer

common bedload studies. At present, it is still unclear how to evaluate the variation in scour depth with bulk flow conditions in terms of bedload transport rate. As a preliminary attempt, with the proposed length scale R , this section aims to establish such a connection by re-interpreting the dimensionless bedload transport rate or Einstein number.

In terms of ϕ and θ , Figure 10 plots five classical series of bedload transport rate data reported by Gilbert (1914), Meyer-Peter and Müller (1948), Paintal (1971), Taylor and Vanoni (1972), and Wilson (1966). By comparing Figures 8 and 10, the following observations can be made.

First, both F_d and θ are comparable for the description of the flow intensity in relation to the submerged weight of particles. Figure 8 shows that d_{se}/R increases with increasing F_d , and the increase is rapid for relatively small F_d . Similar variations also can be observed in the relationship of $\phi - \theta$ shown in Figure 10, which depicts how the variation of ϕ with θ is significant for low θ .

Second, similar to d_{se}/R , ϕ can be considered as a length indicator of bedload transport rate, which is scaled with the eddy size in the near-bed flow. For uniform sediments, the dimensionless bedload transport rate ϕ is defined as

$$\phi = \frac{q_s}{\sqrt{g'd_{50}}d_{50}} \quad (27)$$

where q_s is the volumetric bedload transport rate per unit width. If the average particle velocity is u_s , and the bedload layer thickness is δ , one

thickness can be understood as a length indicator of the transport capacity in comparison to the eddy size for a given flow condition, because d_{50} represents the eddy size in the near-bed flow that plays a more significant role than others in the consideration of bed particle motion (Gioia & Bombardelli, 2001).

The above length ratio related to the bedload transport is similar to d_{se}/R . For simplicity, considering the case of local scour at a narrow pier in a wide channel, the flow depth and sidewall effects are negligible, which yields $R \approx 0.5b$. Therefore, $d_{se}/R \sim d_{se}/b$. Here, the equilibrium scour depth d_{se} is the result of the local scour capacity for a given flow condition, and the pier size b characterizes the size of the vortex system that governs the local scour. Consequently, it can be deduced that the physical meaning of the normalized scour depth d_{se}/R is similar to that of the dimensionless bedload transport rate ϕ .

Third, similar to the bedload transport rate that is related to the pickup probability of bed particles, the scour depth varies depending on scour efficiency. It is noted that the variation in the bedload transport rate is closely related to bedload intermittency. Shih and Diplas (2018) reported that by excluding intermittency, the bedload transport rate can be simply expressed as

$$\phi = c_1 \theta^{1.5} \quad (31)$$

where c_1 is a constant. The intermittency is closely related to turbulence, which significantly affects the bed load transport in the intermediate to weak transport regimes. It reflects the efficiency of the flow in transporting sediment particles. This efficiency would approach zero for very weak sediment transport and be close to unity for very high transport rates. To incorporate the effect of intermittency, Cheng, Lu, and Wei (2020) theoretically derived a bedload formula that is applicable for all weak to high transport regimes,

$$\phi = 13p\theta^{1.5} \quad (32)$$

where p is the pickup probability that can be formulated by considering the turbulent effect on a bed particle (Cheng & Chiew, 1998). With reference to Equation 32, Equation 26 can be revised to be

$$\frac{d_{se}}{R} = 1.3e_s F_{d84} \quad (33)$$

where e_s is referred to as the scour efficiency. Comparing Equation 26 and Equation 33 yields

$$e_s = \exp\left(-\frac{7}{F_{d84}^5}\right) \quad (34)$$

Based on Equation 34 and also the experimental data presented in Figure 8, it is postulated that the scour efficiency approaches unity for the cases of wall jet and culvert scour. This could be understood by noting that the large-scale flow, once released from the culvert outlet, will impact the sediment bed directly and uninterrupted. In contrast, the scour efficiency would be reduced in junction flows for the cases of pier and abutment, as observed in Figure 9, where the relevant data patently deviates from the linear relation. The reduction in the efficiency could be closely associated with the intermittency that is inherent in the large-scale vortex system around piers and abutments, for which the oscillations associated with the vortex system were discussed by Kirkil and Constantinescu (2010), Koken and Constantinescu (2008), and Koken and Constantinescu (2011).

Figure 8 also shows that at $F_{d84} \approx 3$, the variation of d_{se}/R with increasing F_{d84} switches from a nonlinear to linear trend, with the scour efficiency gradually increasing toward unity. Similarly, the transition of bedload transport occurs at $\theta \approx 0.1$, when the variation of ϕ with $\theta^{1.5}$ changes from nonlinear to linear, with the bedload intermittency gradually dying out (see Figure 10). The two turning points are interestingly comparable. For a rough bed, u_* can be related to U as follows:

$$\frac{U}{u_*} = 2.5 \ln\left(\frac{h}{k_s}\right) + 6 \quad (35)$$

where k_s is the roughness height. For $h / k_s = 10 - 1000$, $u_* = (0.043 - 0.085)U$. With this result and assuming that $d_{84} / d_{50} \approx 1.3$ for uniform sediments, the turning point of $\theta \approx 0.1$ in Figure 10 can be rewritten as $F_{d84} = 3.3-6.4$, which is slightly greater than $F_{d84} \approx 3$, as observed in Figure 8. This resemblance further reflects the distinct similarity between local scour and bedload transport.

7. Conclusions

In past studies, the equilibrium clear-water scour depth was related to flow and sediment properties, usually on empirical grounds. In the present study, the scour depth is normalized with a new length scale, which is similar to the conventional hydraulic radius, but applicable for all scour cases of bridge piers, abutments, wall jets and culverts. The results obtained in the present study are summarized as follows:

- (1) The new length scale is useful for quantifying the size of the large-scale flow structure, which depends on the flow obstruction caused by piers or abutments, or the boundary confinement of the flow outlet associated with culverts or wall jets.
- (2) The normalized scour depth can be described using a unified function of the densimetric Froude number for all the different types of scour considered.
- (3) The function appears to be linear for the cases of wall jet and culvert scour, but nonlinear for the cases of pier and abutment scour.
- (4) The trend of the unified function can be further understood by making analogy with the bedload transport formula. Such a comparison has not been reported in the literature, to the best of our knowledge. The analysis shows that the scour efficiency approaches unity for the cases of wall jet and culvert, but significantly reduces for the case of junction flows at piers and abutments.

The present analysis focuses on the scaling analysis for the different scour cases, but has several limitations. The robustness of the proposed approach would be definitely improved by validating it with data in a wider range. The grain density in the data used in this study is almost constant, and thus the result may not be applicable for scour cases with light sediment materials. The effects of the shape of piers and abutments are also not considered in the analysis. How the structure shape and orientation modify the proposed hydraulic radius is worthy of further efforts.

Notation

B	channel width;
b	pier width;
c	constant;
c_1	coefficient;
C_{d50}	correction factor for grain size effect;
C_D	drag coefficient;
D	pipe diameter;
d_{16}	16th percentiles of the particle size distribution;
d_{50}	median particle diameter;
d_{84}	84th percentiles of the particle size distribution;
d_r	representative particle diameter;
d_{se}	equilibrium scour depth;
e_s	scour efficiency;
F_d	densimetric Froude number;
F_{d84}	d_{84} based densimetric Froude number;
f	friction factor;
f_c	critical friction factor for the division of fully rough bed and transitional bed;
g	gravitational acceleration;
g'	$(\rho_s/\rho - 1)g$;
h	flow depth;
k_s	roughness height;

L	abutment length;
m	mass;
n	exponent;
p	pickup probability;
p_r	d_r -related percentile;
q_s	volumetric bedload transport rate;
R	dimension of large-scale vortex system;
R_a	dimension of large-scale vortex system related to an abutment;
R_h	hydraulic radius;
Re_h	Reynolds number based on flow depth;
R_j	dimension of large-scale vortex system related to a wall jet;
R_p	dimension of large-scale vortex system related to a pier;
S	energy slope;
U	approach depth-averaged flow velocity;
U_1	flow velocity at a side position of a pier;
U_c	critical depth-averaged flow velocity for incipient sediment motion;
u_*	shear velocity;
u_{*c}	critical shear velocity;
u_s	average particle velocity;
V	average flow velocity;
w	particle settling velocity;
α	exponent;
β	exponent;
δ	bedload layer thickness;
θ	dimensionless bed shear stress or Shields number;
θ_c	critical Shields number;
μ	dynamic fluid viscosity;
ν	kinematic fluid viscosity;
ρ	fluid density;
ρ_s	particle density;
σ_g	gradation coefficient;
τ_b	bed shear stress; and
ϕ	dimensionless sediment transport rate or Einstein number.

Data Availability Statement

All the data used in this study are available from Mendeley Data repository (<https://doi.org/10.17632/8gf3fwgznp.1>).

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References

- Adduce, C., & Rocca, M. L. (2006). Local scouring due to turbulent water jets downstream of a trapezoidal drop: Laboratory experiments and stability analysis. *Water Resources Research*, 42, 67–80. <https://doi.org/10.1029/2005wr004139>
- Adduce, C., & Sciortino, G. (2006). Scour due to a horizontal turbulent jet: Numerical and experimental investigation. *Journal of Hydraulic Research*, 44(5), 663–673. <https://doi.org/10.1080/00221686.2006.9521715>
- Basak, V., Basamisli, Y., & Ergun, O. (1975). *Maximum equilibrium scour depth around linear-axis square cross-section pier groups (In Turkish)*. Ankara, Turkey. Devlet su Isteri Genel Mudurlugu.
- Breusers, H. N. C., & Raudkivi, A. J. (1991). *Scouring*, IAHR, A. A. Balkema Rotterdam. Taylor & Francis.
- Brown, G. O. (2002). The history of the Darcy-Weisbach equation for pipe flow resistance. In *Environmental and water Resources history sessions at ASCE civil engineering conference and exposition 2002*. Washington, D.C., United States. American Society of Civil Engineers. [https://doi.org/10.1061/40650\(2003\)4](https://doi.org/10.1061/40650(2003)4)
- Cheng, N.-S. (1997). Simplified settling velocity formula for sediment particle. *Journal of Hydraulic Engineering*, 123(2), 149–152. [https://doi.org/10.1061/\(asce\)0733-9429\(1997\)123:2\(149\)](https://doi.org/10.1061/(asce)0733-9429(1997)123:2(149))
- Cheng, N.-S. (2008a). Comparison of settling-velocity-based formulas for threshold of sediment motion. *Journal of Hydraulic Engineering*, 134(8), 1136–1141. [https://doi.org/10.1061/\(asce\)0733-9429\(2008\)134:8\(1136\)](https://doi.org/10.1061/(asce)0733-9429(2008)134:8(1136))
- Cheng, N.-S. (2008b). Formulas for friction factor in transitional regimes. *Journal of Hydraulic Engineering*, 134(9), 1357–1362. [https://doi.org/10.1061/\(asce\)0733-9429\(2008\)134:9\(1357\)](https://doi.org/10.1061/(asce)0733-9429(2008)134:9(1357))

- Cheng, N.-S. (2016). Representative grain size and equivalent roughness height of a sediment bed. *Journal of Hydraulic Engineering*, 142(1). 06015016. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0001069](https://doi.org/10.1061/(asce)hy.1943-7900.0001069)
- Cheng, N.-S. (2017). Simple modification of Manning-Strickler formula for large-scale roughness. *Journal of Hydraulic Engineering*, 143(9). 04017031. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0001345](https://doi.org/10.1061/(asce)hy.1943-7900.0001345)
- Cheng, N.-S., & Chiew, Y.-M. (1998). Pickup probability for sediment entrainment. *Journal of Hydraulic Engineering*, 124(2), 232–235. [https://doi.org/10.1061/\(asce\)0733-9429\(1998\)124:2\(232\)](https://doi.org/10.1061/(asce)0733-9429(1998)124:2(232))
- Cheng, N.-S., & Chiew, Y.-M. (1999). Analysis of initiation of sediment suspension from bed load. *Journal of Hydraulic Engineering*, 125(8), 855–861. [https://doi.org/10.1061/\(asce\)0733-9429\(1999\)125:8\(855\)](https://doi.org/10.1061/(asce)0733-9429(1999)125:8(855))
- Cheng, N.-S., & Emadzadeh, A. (2014). Average velocity of solitary coarse grain in flows over smooth and rough beds. *Journal of Hydraulic Engineering*, 140(6). 04014015. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0000875](https://doi.org/10.1061/(asce)hy.1943-7900.0000875)
- Cheng, N.-S., Lu, Y. S., & Wei, M. X. (2020). Simple formulation of bedload sediment transport rate based on a novel definition of pickup probability. *Journal of Hydraulic Engineering*, 146(9). 06020012. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0001797](https://doi.org/10.1061/(asce)hy.1943-7900.0001797)
- Cheng, N.-S., & Nguyen, H. T. (2011). Hydraulic radius for evaluating resistance induced by simulated emergent vegetation in open-channel flows. *Journal of Hydraulic Engineering*, 137(9), 995–1004. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0000377](https://doi.org/10.1061/(asce)hy.1943-7900.0000377)
- Cheng, N.-S., & Wei, M. (2019). Scaling of scour depth at bridge pier based on characteristic dimension of large-scale vortex. *Water*, 11(12), 2458. <https://doi.org/10.3390/w11122458>
- Cheng, N.-S., Wei, M., Xu, P., & Mao, R. (2020). Length scale for evaluating wave-induced pipeline scour. *Ocean Engineering*, 218, 108153. <https://doi.org/10.1016/j.oceaneng.2020.108153>
- Chiew, Y. M. (1984). *Local scour at bridge piers*. Rep. No. 355. Auckland, New Zealand. School of Engineering, The University of Auckland.
- Chiew, Y. M., & Lim, F.-H. (2000). Failure behavior of riprap layer at bridge piers under live-bed conditions. *Journal of Hydraulic Engineering*, 126(1), 43–55. [https://doi.org/10.1061/\(asce\)0733-9429\(2000\)126:1\(43\)](https://doi.org/10.1061/(asce)0733-9429(2000)126:1(43))
- Chiew, Y. M., & Lim, S.-Y. (1996). Local scour by a deeply submerged horizontal circular jet. *Journal of Hydraulic Engineering*, 122(9), 529–532. [https://doi.org/10.1061/\(asce\)0733-9429\(1996\)122:9\(529\)](https://doi.org/10.1061/(asce)0733-9429(1996)122:9(529))
- Chiew, Y. M., & Melville, B. W. (1987). Local scour around bridge piers. *Journal of Hydraulic Research*, 25(1), 15–26. <https://doi.org/10.1080/00221688709499285>
- Coscarella, F., Gaudio, R., & Manes, C. (2020). Near-bed eddy scales and clear-water local scouring around vertical cylinders. *Journal of Hydraulic Research*, 58(6), 968–981. <https://doi.org/10.1080/00221686.2019.1698668>
- Ettema, R. (1976). *Influence of bed material gradation on local scour*. Rep. No. 124. Auckland, New Zealand. School of Engineering, The University of Auckland.
- Ettema, R. (1980). *Scour at bridge piers*. Rep. No. 216. Auckland, New Zealand. School of Engineering, University of Auckland.
- Ettema, R., Constantinescu, G., & Melville, B. W. (2011). *Evaluation of bridge scour research: Pier scour processes and predictions*. Washington, DC: The National Academies Press.
- Ettema, R., Melville, B. W., & Barkdoll, B. (1998). Scale effect in pier-scour experiments. *Journal of Hydraulic Engineering*, 124(6), 639–642. [https://doi.org/10.1061/\(asce\)0733-9429\(1998\)124:6\(639\)](https://doi.org/10.1061/(asce)0733-9429(1998)124:6(639))
- Fael, C. M. S., Simarro-Grande, G., Martín-Vide, J.-P., & Cardoso, A. H. (2006). Local scour at vertical-wall abutments under clear-water flow conditions. *Water Resources Research*, 42, W10408. <https://doi.org/10.1029/2005wr004443>
- Ferguson, R. I. (2007). Flow resistance equations for gravel- and boulder-bed streams. *Water Resources Research*, 43, W05427. <https://doi.org/10.1029/2006wr005422>
- Fernandez Luque, R., & van Beek, R. (1976). Erosion and transport of bed-load sediment. *Journal of Hydraulic Research*, 14(2), 127–144. <https://doi.org/10.1080/00221687609499677>
- Gilbert, G. K. (1914). *The transportation of debris by running water*. Reston, VA: U.S. Geological Survey. <https://doi.org/10.3133/pp86>
- Gioia, G., & Bombardelli, F. A. (2001). Scaling and similarity in rough channel flows. *Physical Review Letters*, 88(1), 014501. <https://doi.org/10.1103/physrevlett.88.014501>
- Graf, W. H., & Altinakar, M. S. (2017). *Local scour, paper presented at river system analysis and management*. Springer.
- Guan, D., Melville, B., & Friedrich, H. (2016). Local scour at submerged weirs in sand-bed channels. *Journal of Hydraulic Research*, 54(2), 1–13. <https://doi.org/10.1080/00221686.2015.1132275>
- Hoffmans, G. J. C. M., & Verheij, H. J. (1997). *Scour manual*. The Netherlands: Balkema Publishing Co.
- Hong, J.-H., Chiew, Y.-M., Lu, J.-Y., Lai, J.-S., & Lin, Y.-B. (2012). Houfeng bridge failure in taiwan. *Journal of Hydraulic Engineering*, 138(2), 186–198. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0000430](https://doi.org/10.1061/(asce)hy.1943-7900.0000430)
- Kirkil, G., & Constantinescu, G. (2010). Flow and turbulence structure around an in-stream rectangular cylinder with scour hole. *Water Resources Research*, 46, W11549. <https://doi.org/10.1029/2010wr009336>
- Koken, M., & Constantinescu, G. (2008). An investigation of the flow and scour mechanisms around isolated spur dikes in a shallow open channel: 1. Conditions corresponding to the initiation of the erosion and deposition process. *Water Resources Research*, 44, W08406. <https://doi.org/10.1029/2007wr006489>
- Koken, M., & Constantinescu, G. (2011). Flow and turbulence structure around a spur dike in a channel with a large scour hole. *Water Resources Research*, 47, W12511. <https://doi.org/10.1029/2011wr010710>
- Kothiyari, U. C., Garde, R. C. J., & Ranga Raju, K. G. (1992). Temporal variation of scour around circular bridge piers. *Journal of Hydraulic Engineering*, 118(8), 1091–1106. [https://doi.org/10.1061/\(asce\)0733-9429\(1992\)118:8\(1091\)](https://doi.org/10.1061/(asce)0733-9429(1992)118:8(1091))
- Lee, S. O., & Sturm, T. W. (2009). Effect of sediment size scaling on physical modeling of bridge pier scour. *Journal of Hydraulic Engineering*, 135(10), 793–802. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0000091](https://doi.org/10.1061/(asce)hy.1943-7900.0000091)
- Manes, C., & Brocchini, M. (2015). Local scour around structures and the phenomenology of turbulence. *Journal of Fluid Mechanics*, 779, 309–324. <https://doi.org/10.1017/jfm.2015.389>
- Melville, B. W. (1997). Pier and Abutment Scour: Integrated Approach. *Journal of Hydraulic Engineering*, 123(2), 125–136. [https://doi.org/10.1061/\(asce\)0733-9429\(1997\)123:2\(125\)](https://doi.org/10.1061/(asce)0733-9429(1997)123:2(125))
- Melville, B. W., & Coleman, S. E. (2000). *Bridge scour*. Colorado: Water Resources Publication.
- Meyer-Peter, E., & Müller, R. (1948). Formulas for bed-load transport. In *Proc., 2nd meeting of international association for hydraulic research*. Stockholm, Sweden: IAHR.
- Molinas, A. (2003). *Bridge scour in nonuniform sediment mixtures and in cohesive materials: Synthesis report*, Laboratory Final Report Rep. FHWA-RD-03-083. McLean, Virginia: U. S. Department of Transportation, Federal Highway Administration.
- Mueller, D. S., & Wagner, C. R. (2005). *Field observations and evaluations of streambed scour at bridges, Final Report Rep. FHWA-RD-03-052*. McLean, VA: U.S. Department of Transportation, Federal Highway Administration.

- Oliveto, G., & Hager, W. H. (2002). Temporal evolution of clear-water pier and abutment scour. *Journal of Hydraulic Engineering*, 128(9), 811–820. [https://doi.org/10.1061/\(asce\)0733-9429\(2002\)128:9\(811\)](https://doi.org/10.1061/(asce)0733-9429(2002)128:9(811))
- Paintal, A. S. (1971). Concept of critical shear stress in loose boundary open channels. *Journal of Hydraulic Research*, 9(1), 91–113. <https://doi.org/10.1080/00221687109500339>
- Radice, A., Porta, G., & Franzetti, S. (2009). Analysis of the time-averaged properties of sediment motion in a local scour process. *Water Resources Research*, 45, W03401. <https://doi.org/10.1029/2007wr006754>
- Rajaratnam, N., & Berry, B. (1977). Erosion by circular turbulent wall jets. *Journal of Hydraulic Research*, 15(3), 277–289. <https://doi.org/10.1080/00221687709499648>
- Raudkivi, A. J. (1998). *Loose boundary hydraulics*. London: CRC Press.
- Raudkivi, A. J., & Ettema, R. (1983). Clear-water scour at cylindrical piers. *Journal of Hydraulic Engineering*, 109(3), 338–350. [https://doi.org/10.1061/\(asce\)0733-9429\(1983\)109:3\(338\)](https://doi.org/10.1061/(asce)0733-9429(1983)109:3(338))
- Shen, H. W., Schneider, V. R., & Karaki, S. (1969). Local scour around bridge piers. *Journal of the Hydraulics Division*, 95(6), 1919–1940. <https://doi.org/10.1061/jyceaj.0002197>
- Sheppard, D. M., Demir, H., & Melville, B. W. (2011). *Scour at wide piers and long skewed piers*. Washington, DC: The National Academies Press.
- Sheppard, D. M., & Miller, W. (2006). Live-bed local pier scour experiments. *Journal of Hydraulic Engineering*, 132(7), 635–642. [https://doi.org/10.1061/\(asce\)0733-9429\(2006\)132:7\(635\)](https://doi.org/10.1061/(asce)0733-9429(2006)132:7(635))
- Sheppard, D. M., Odeh, M., & Glasser, T. (2004). Large scale clear-water local pier scour experiments. *Journal of Hydraulic Engineering*, 130(10), 957–963. [https://doi.org/10.1061/\(asce\)0733-9429\(2004\)130:10\(957\)](https://doi.org/10.1061/(asce)0733-9429(2004)130:10(957))
- Shih, W., & Diplas, P. (2018). A unified approach to bed load transport description over a wide range of flow conditions via the use of conditional data treatment. *Water Resources Research*, 54, 3490–3509. <https://doi.org/10.1029/2017wr022373>
- Si, J.-H., Lim, S.-Y., & Wang, X.-K. (2018). Flow structures in evolving scour holes caused by a plunging jet downstream of a weir. *Journal of Hydraulic Engineering*, 144(6), 04018018. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0001452](https://doi.org/10.1061/(asce)hy.1943-7900.0001452)
- Si, J.-H., Lim, S.-Y., & Wang, X.-K. (2019). Jet-flipping in scour hole downstream of unsubmerged weir with apron. *Journal of Hydraulic Engineering*, 145(10), 04019035. [https://doi.org/10.1061/\(asce\)hy.1943-7900.0001634](https://doi.org/10.1061/(asce)hy.1943-7900.0001634)
- Sumer, B. M., & Fredsøe, J. (2002). *The mechanics of scour in the marine environment*. Hackensack, NJ: World Scientific Publishing Company. <https://doi.org/10.1142/4942>
- Tan, S. M., Lim, S. Y., Wei, M. X., & Cheng, N.-S. (2020). Application of densimetric Froude number for evaluating the maximum culvert scour depth. *Journal of Irrigation and Drainage Engineering*, 146(8), 04020020. [https://doi.org/10.1061/\(asce\)ir.1943-4774.0001487](https://doi.org/10.1061/(asce)ir.1943-4774.0001487)
- Taylor, B. D., & Vanoni, V. A. (1972). Temperature effects in low-transport, flat-bed flows. *Journal of the Hydraulics Division*, 98(8), 1427–1445. <https://doi.org/10.1061/jyceaj.0003375>
- Wei, M., Chiew, Y.-M., & Cheng, N.-S. (2020). Recent advances in understanding propeller jet flow and its impact on scour. *Physics of Fluids*, 32(10), 101303. <https://doi.org/10.1063/5.0023266>
- Whiting, P. J., & Dietrich, W. E. (1990). Boundary shear stress and roughness over mobile alluvial beds. *Journal of Hydraulic Engineering*, 116(12), 1495–1511. [https://doi.org/10.1061/\(asce\)0733-9429\(1990\)116:12\(1495\)](https://doi.org/10.1061/(asce)0733-9429(1990)116:12(1495))
- Wilson, K. C. (1966). Bed-load transport at high shear stress. *Journal of the Hydraulics Division*, 92(6), 49–59. <https://doi.org/10.1061/jyceaj.0001562>
- Yalin, M. S. (1977). *Mechanics of sediment transport*. Oxford, New York: Pergamon Press. <https://doi.org/10.1061/9780872620834.124>
- Yang, C. T. (1973). Incipient motion and sediment transport. *Journal of the Hydraulics Division*, 99(10), 1679–1704. <https://doi.org/10.1061/jyceaj.0003766>