Robust congestion management and service restoration for active distribution networks

Shen, Feifan

2021

Shen, F. (2021). Robust congestion management and service restoration for active distribution networks. Doctoral thesis, Nanyang Technological University, Singapore.
<https://hdl.handle.net/10356/153882>

<https://hdl.handle.net/10356/153882>

<https://doi.org/10.32657/10356/153882>

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**ROBUST CONGESTION MANAGEMENT AND
SERVICE RESTORATION FOR ACTIVE
DISTRIBUTION NETWORKS**

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ROBUST CONGESTION MANAGEMENT AND SERVICE RESTORATION FOR ACTIVE DISTRIBUTION NETWORKS

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2021

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from 11 papers published in the following peer-reviewed journals or conferences where I was the first and/or corresponding author.

Chapter 1 is published as Feifan Shen, Qiuwei Wu*, and Yusheng Xue, “Review of Service Restoration for Distribution Networks,” *Journal of Modern of Power Systems and Clean Energy*, **8**, 1-14 (2020). DOI: 10.35833/MPCE.2018.000782.

The contributions of the co-authors are as follows:

- A/Prof. Qiuwei Wu provided the research direction and a lot of suggestions on the paper organizaion.
- I wrote the draft of the manuscript, and the manuscript was revised by A/Prof. Qiuwei Wu and Prof. Yusheng Xue.

Chapter 1 is also published as Feifan Shen, Qiuwei Wu*, Shaojun Huang, Juan Camilo López, Canbing Li, and Bin Zhou, “Review of Service Restoration Methods in Distribution Networks,” 2018 *IEEE PES Innovative Smart Grid Technologies Conference Europe* (2018). DOI: 10.1109/ISGTEurope.2018.8571821.

The contributions of the co-authors are as follows:

- A/Prof. Qiuwei Wu provided the initial research idea.
- I co-designed this study with Dr. Shaojun Huang and discussed the paper organization and content with Dr. Juan Camilo López.

- I wrote the draft of the manuscript, and the manuscript was revised by Prof. Canbing Li and A/Prof. Bin Zhou.

Chapter 2 is published as Feifan Shen, Shaojun Huang, Qiuwei Wu*, Sami Repo, Yan Xu, and Jacob Østergard. Comprehensive Congestion Management for Distribution Networks Based on Dynamic Tariff, Reconfiguration and Re-profiling Product. *IEEE Trans. Smart Grid*, **10**, 4795-4805 (2019). DOI: 10.1109/TSG.2018.2868755.

The contributions of the co-authors are as follows:

- A/Prof. Qiuwei Wu provided the research direction, and I co-designed this study with A/Prof. Qiuwei Wu and Dr. Shaojun Huang.
- I implemented codes on GAMS and carried out simulations, and simulation results were analyzed by me together with A/Prof. Qiuwei Wu and A/Prof. Xu Yan. A/Prof. Qiuwei Wu and A/Prof. Xu Yan also provided guidance on manuscript writing.
- I wrote the draft of the manuscript, and the manuscript was revised by A/Prof. Qiuwei Wu, A/Prof. Xu Yan, Prof. Sami Repo and Jacob Østergard.

Chapter 2 is also published as Feifan Shen, Qiuwei Wu*, and Shaojun Huang. Day-ahead Congestion Management Scheme for Distribution Networks with Dynamic Tariff and Re-profiling Products. *International Journal of Smart Grid and Clean Energy*, **9**, 91-101 (2020). DOI: <https://doi.org/10.12720/sgce.9.1.91-101>.

The contributions of the co-authors are as follows:

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- I prepared the draft of the manuscript, and the manuscript was revised by A/Prof. Qiuwei Wu.

Chapter 3 is published as Feifan Shen, QiuweiWu*, Xiaolong Jin, Menglin Zhang, SaeedTei-mourzadeh, Osman Bulent Tor, “Coordination of dynamic tariff and scheduled reprofiling product for day-ahead congestion management of distribution networks,” *International Journal of Electrical Power and Energy System*, **135**, 107672 (2022). DOI: <https://doi.org/10.1016/j.ijepes.2021.107612>.

The contributions of the co-authors are as follows:

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- Dr. Saeed Teimourzadeh and Dr. Osman Bulent Tor provided suggestions on the framework design.
- I carried out the simulations, and the simulation results were analyzed by me together with Dr. Xiaolong Jin and Dr. Menglin Zhang.
- I prepared the draft of the manuscript, and the manuscript was revised by A/Prof. Qiuwei Wu.

Chapter 4 is published as Feifan Shen, QiuweiWu*, “Robust dynamic tariff method for day-ahead congestion management of distribution networks,” *International Journal of Electrical Power and Energy System*, **134**, 107366 (2022). DOI: <https://doi.org/10.1016/j.ijepes.2021.107366>.

The contributions of the co-authors are as follows:

- I designed the study and conducted simulations.
- The simulation results were analyzed by me together with A/Prof. Qiuwei Wu.
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Chapter 5 is published as Feifan Shen, Qiuwei Wu*, Shaojun Huang, Xinyu Chen, and Yan Xu, “Two-Tier Demand Response with Flexible Demand Swap and Transactive Control for Real-Time Congestion Management in Distribution Networks,” *International Journal of Electrical Power and Energy System*, **114**, **105399** (2020). DOI: <https://doi.org/10.1016/j.ijepes.2019.105399>.

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Chapter 6 is published as Feifan Shen, Qiuwei Wu*, Yan Xu, Fangxing Li, Fei Teng, and Goran Strbac, “Hierarchical Service Restoration Scheme for Active Distribution Networks Based on Alternating Direction of Multipliers (ADMM),” *International Journal of Electrical Power and Energy System*, **118**, 105809 (2020). DOI: <https://doi.org/10.1016/j.ijepes.2019.105809>.

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- I prepared the draft of the manuscript, and the manuscript was revised by A/Prof. Qiuwei Wu, A/Prof. Yan Xu, Prof. Fangxing Li and Prof. Goran Strbac.

Chapter 7 is published as Feifan Shen, Juan C. Lopez, Qiuwei Wu*, Marcos J. Rider, Tianguang Lu, and Nikos D. Hatziargyriou, “Distributed Self-healing Scheme for Unbalanced Electrical Distribution Systems Based on Alternating Direction Method of Multipliers (ADMM),” *IEEE Trans. Power System*, **35**, 2190-2199 (2020). DOI: [10.1109/TPWRS.2019.2958090](https://doi.org/10.1109/TPWRS.2019.2958090).

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- The manuscript was revised by Dr. Juan C. Lopez, A/Prof. Qiuwei Wu, Prof. Marcos J. Rider, and Prof. Nikos D. Hatziargyriou.

Chapter 7 is also published as Feifan Shen, Qiuwei Wu, Juan Camilo López, and Marcos J Rider, “Distributed Service Restoration of Active Electrical Distribution Systems using ADMM,” *10th Conference on Innovative Smart Grid Technologies*, 2019. DOI: 10.1109/ISGT.2019.8791626.

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- The manuscript was revised by A/Prof. Qiuwei Wu and Prof. Marcos J. Rider.

Chapter 8 is also published as Feifan Shen, Qiuwei Wu*, Jin Zhao, Wei Wei, Nikos D. Hatziargyrious, and Feng Liu, “Distributed Risk-limiting Load Restoration in Unbalanced Distribution Systems with Networked Microgrids,” *IEEE Trans. Smart Grid*, **11**, 4574-4586 (2020). DOI: 10.1109/TSG.2018.2803141.

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Acknowledgements

First and foremost, I would like to express my sincere gratitude to my PhD supervisors Associated Prof. Xu Yan (Nanyang Technological University), Associate Prof. Qiuwei Wu (Technical University of Denmark) and Associate Prof. Arne Hejde Nielsen (Technical University of Denmark) for giving me the opportunity to pursue a PhD degree. I would like to thank them for their patient guidance, continuous support, and inspiration during the three years of studies. I have been fortunate to work with these passionate and excellent researchers.

I particularly thank my supervisor Xu Yan for hosting me during my stay at NTU and for embracing me into his research group.

Last but not least, I am deeply grateful to my family for their love, encouragement, continuous support, and understanding of my decision.

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Summary

As a promising solution to reduce greenhouse gas emission and alleviate energy shortage concerns, it is expected that the distributed energy resources (DERs) will be largely deployed in modern power systems. The deployment of DERs accelerates the transformation of the passive distribution network into the active distribution network (ADN) and could offer various benefits to the operation of the distribution network, such as the infrastructure upgrade deferral, and power quality and operational reliability improvement. However, the uncoordinated DER operation has adverse impacts and poses great challenges to the operation of the ADN. In order to accommodate the large-scale DERs while ensuring secure and efficient management of ADNs, it is important to leverage the benefits of DERs while mitigating the adverse impacts.

Congestion could occur in the ADN due to the uncoordinated DER operation. Electrical vehicles (EVs) and heat pumps (HPs) as two popular DERs at the demand side can shift power consumption in response to electricity prices, which may cause congestion at those hours with low electricity prices. One focus of this thesis is to study market-based congestion management methods for ADNs.

Integrating different types of congestion management methods is believed to be a promising way to achieve a more efficient congestion management solution. In this context, this thesis proposes a comprehensive congestion management scheme to integrate the dynamic tariff (DT) (price-based

method), network reconfiguration, and re-profiling product (incentive-based method) to alleviate day-ahead congestion. Moreover, a relaxed reconfiguration-based DT optimization model is formulated, which can ensure a feasible DT solution and set a threshold to DTs. Despite the proposed integration of the DT and incentive-based method improves the congestion management effectiveness, the integration is implemented in a simple way through independent optimizations of the DT and re-profiling product. In addition, the integration may be limited due to potential conflicts with the current distribution system operator (DSO) regulations. Therefore, this thesis further proposes a coordination scheme of the DT method and incentive-based method, in which the DSO is in a profit-neutral position through the integrated optimization of the two methods.

Additionally, the DT method may be ineffective because the energy plans made at the aggregator side may not respect network constraints due to forecast errors. Therefore, this thesis proposes a robust DT method to deal with the uncertainty, which ensures that the network constraints are satisfied even under the worst uncertainty realization scenario. As important as day-ahead congestion management, real-time congestion should be addressed as well. This thesis proposes a two-tier real-time congestion management method based on the flexible demand swap and transactive control. Real-time congestion can be alleviated efficiently by the flexible demand swap while considering the end-user's rebound conditions and its willingness to change day-ahead schedules through a transactive control-based interaction mechanism.

In the case of contingency, after fault location and isolation, service restoration is carried out to rapidly restore as many outage areas as possible through network reconfiguration and microgrid (MG) formation and through dispatching DERs. In the existing literature, most of the service restoration models are solved by centralized algorithms based on centralized infrastructure, which is faced with a few drawbacks. Firstly, it may suffer from a heavy computation burden with the increasing size and complexity of the ADN. Secondly, the centralized infrastructure is costly and has a single-point failure risk. Thirdly, it is challenging for the centralized infrastructure to adapt to system changes, e.g., the integration of DERs. Finally, it compromises privacy information protection. Therefore, another focus of the thesis is to design service restoration methods that can overcome the above-mentioned drawbacks.

A hierarchical service restoration method is proposed based on the alternating direction method of multipliers (ADMM), in which the service restoration problem is solved in a hierarchical manner to reduce the computation burden for large-scale ADNs. Nevertheless, it still needs a central controller to send out control commands in a centralized manner. Therefore, this thesis further proposes a distributed service restoration method based on the ADMM, in which the service restoration problem is decomposed and solved in a distributed manner. The distributed method is more resilient to controller failures and can provide privacy information protection. However, these proposed methods are mainly based on the power fed by the upstream transmission network and post fault reconfiguration without considering the uncertainties of DERs and load consumption. Therefore, the thesis proposes a

distributed risk-limiting service restoration method for the ADN with networked MGs, in which each MG individually uses its local DERs to restore loads without violating risk-limiting constraints of the whole network.

In summary, several market-based congestion management methods and service restoration methods are proposed for the secure and efficient operation and management of ADNs.

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List of Abbreviations

ADN	active distribution network
AMI	advanced metering infrastructure
AGG	aggregator
B-CRP	bi-directional conditional re-profiling product
BRP	balance responsible party
CRP	conditional re-profiling product
CLPU	cold load pick up
CVaR	conditional value-at-risk
DA	distribution automation
DAS	distribution automation system
DER	distributed energy resources
DG	distributed generation
DLMP	distributional locational marginal price
DPT	dynamic power tariff
DP	dynamic programming
DS	dynamic subsidy
DSO	distribution system operator
DT	dynamic tariff
EV	electric vehicle
ESS	energy storage system
FDS	flexible demand swap
FLECH	flexibility clearing house
FT	flexibility table
FRR	frequency response rate
GA	genetic algorithm
HP	heat pump
ICT	information, communication technology
IGDT	information gap decision theory
KKT	Karush-Kuhn-Tucker
LFM	local flexibility market
LMP	locational marginal price
LSP	line shadow price
LV	low voltage
MAS	multi-agent system
MG	microgrid
MINLP	mixed integer nonlinear programming
MILP	mixed integer linear programming
MISOCP	mixed integer second-order cone programming
MIP	mixed integer programming
MT	wind turbine

MV	medium voltage
NLP	non-linear programming
OPF	optimal power flow
PDF	probability distribution function
PSA	parallel simulated annealing
PSO	particle swarm algorithm
PV	photovoltaic
PTDF	power transfer distribution factor
RE	relative error
SRP	scheduled re-profiling product
TS	tube search
TN	transmission network
VR	voltage regulator
WP	wind power
WPS	wind power system
WT	wind turbine

List of Symbols

Sets:

$\mathcal{N}^T$	set of day-ahead planning period
$\bar{\mathcal{N}}^T$	subset of $\mathcal{N}^T$ for market agent
$\mathcal{N}^a$	set of aggregators
$\mathcal{N}_d^{(1)}, \mathcal{N}_d^{(2)}$	set of load points in $S^{(1)}$ and set of load points in $S^{(2)}$
$\mathcal{N}^{sw}$	set of available swaps
$\mathcal{N}^{sw,*}$	set of selected swaps
$\mathcal{N}^c$	set of customers who participate in the transactive market
$\mathcal{N}, \mathcal{N}^S$	set of nodes and set of main source nodes
$\mathcal{N}^L, \mathcal{N}^M$	sets of loads and MTs
$\mathcal{N}^W, \mathcal{N}^P$	set of WTs and PVs
$\mathcal{N}^E, \mathcal{N}^{cap}$	set of ESS and capacitors
$\mathcal{N}^{ev}, \mathcal{N}^{hp}$	set of EVs and HPs
$\mathcal{N}_i^{ev}, \mathcal{N}_i^{hp}$	set of EVs and HPs having contracts with i -th aggregator
$\mathcal{C}_{s,d,t}$	set of customers belonging to d -th side of s -th swap in period t
$\mathcal{L}, \mathcal{L}^T$	set of branches and switches, and set of tie-switches
$\mathcal{L}^B, \mathcal{L}^S$	set of branches and set of switches
$\mathcal{L}^N, \mathcal{L}^{VR}$	set of branches without VRs, and set of branches with VRs
$\mathcal{L}^F$	set of switches connected to the faulted zone
$\mathcal{L}^c$	set of congestion lines
$\mathcal{T}$	set of restoration time steps
$\mathcal{Z}$	set of zones
$\mathcal{S}$	set of restoration steps, i.e., $s_1, s_2, \dots, s_{\max}$
$\mathcal{S}^{sw}$	set of sides of a swap $\{d_1, d_2\}$, d_1 represents $S^{(1)}$, d_2 represents $S^{(2)}$
$\mathcal{B}_z$	set of nodes belonging to zone $z \in \mathcal{Z}$
$\mathcal{H}$	set of discrete segments
$\mathcal{J}$	set of scenarios used in CVaR
Φ	set of three phases $\{a, b, c\}$

Parameters:

A_{i,t,t_-}	coefficient matrix describing the relation between the power consumption and temperature change of the household
$B_{i,t}$	price sensitivity coefficient matrix
C	reduced incidence matrix

D_t	power transfer distribution factor (PTDF) of the network in the day-ahead planning period t
E_i	customer to load bus mapping matrix
G	reduced incidence matrix
H_t^v	voltage incident matrix in the day-head planning period t
$\bar{H}_{ij}$	the upper bound of fictitious active power flow from nodes i to j
$\bar{I}_{ij}^{\text{sqr}}$	maximum square current magnitude of the circuit ij
$K_{i,t}^{a,\text{max/min}}$	maximum/minimum household inside temperature limit
$K_{i,0}^a$	initial household inside temperature
$K_{i,t}^{\text{des}}$	the desired household inside temperature of i -th customer in period t
K_i^D	fictitious active demand at node i
M	a very big number used in the big-M method
$P_t^{(i,g)}$	the number of offered flexibility service products of the i -th aggregator at node g in the day-ahead planning period t
$\bar{P}_i^G, \bar{Q}_i^G$	maximum active and reactive DG generations at node i
P_i^D, Q_i^D	active and reactive demands at node i
$\bar{P}_i^{\text{MT,max/min}}$	maximum/minimum limit of MT active power generation
$\bar{P}_i^{\text{MT,max/min}}$	MT ramp-up/ramp-down limit
$\bar{Q}_i^{\text{MT,max/min}}$	maximum/minimum limit of MT reactive power generation
$\bar{Q}_i^{\text{PV,max/min}}$	maximum/minimum limit of PV reactive power generation
$\bar{Q}_i^{\text{WT,max/min}}$	maximum/minimum limit of WT reactive power generation
R, X	diagonal resistance and reactance matrix
R_{ij}, X_{ij}	resistance and reactance of the circuit between nodes i and j
$\mathbf{R}_{ij}, \mathbf{X}_{ij}$	three phase resistance and reactance matrix of the circuit between nodes i and j
$\tilde{\mathbf{R}}_{ij}, \tilde{\mathbf{X}}_{ij}$	three phase equivalent resistance and reactance matrix of the circuit between nodes i and j
$\bar{V}^{\text{sqr}}, \underline{V}^{\text{sqr}}$	maximum and minimum limits of the squared voltage magnitude
$V_t^{\text{b,sqr}}$	squared nominal voltage magnitude at the substation node
$b_{i,s,d,t}$	mapping matrix. $b_{i,s,d,t} = 1$ if customer i belongs to the d -th side of s -th swap in period t ; otherwise, $b_{i,s,d,t} = 0$
$a_{i,t}$	the i -th EV availability in period t
$a_{i,t}^a$	the accurate i -th EV availability in period t

$a_{i,t}^{\text{dw}}$	the maximum downward EV availability forecast error
$a_{i,t}^{\text{uw}}$	the maximum upward EV availability forecast error
$\bar{a}_{ij,t}^{\text{min}}$	minimum limit of the VR voltage ratio
c_t	base spot price in period t
c^{p}	the profit coefficient of the aggregator
c_i^{G}	cost of generation at node i
c_{ij}^{ls}	cost of line losses of the circuit between nodes i and j
c_{ij}^{s}	cost of a switching action of the switch between nodes i and j
c_i^{r}	cost of load shedding at node i
c_z^{U}	cost of de-energizing zone z : $c_i^{\text{r}} P_i^{\text{D}}$, $i \in \mathcal{B}(z)$
$\bar{c}_i^{\text{P}}$	load priority coefficients
$\bar{c}_i^{\text{M}}$	energy production cost of MT at node i
$\bar{c}_{ij}^{\text{VR}}$	operation cost of the VR between nodes i and j
$d_{i,t}$	i -th EV discharging power due to driving in period t
$d_{i,t}^{\text{a}}$	accurate EV discharging power
$d_{i,t}^{\text{dw}}$	maximum downward forecast error of EV discharging power
$d_{i,t}^{\text{uw}}$	maximum upward forecast error of EV discharging power
e_i^{min}	lower limit of the SOC level of i -th EV
e_i^{max}	upper limit of the SOC level of i -th EV
$e_{i,0}$	initial SOC level of i -th EV
f_t^{max}	line loading limit
f_l	l -th congestion line's overloading value
f^{r}	the remaining capacity of uncongested lines
l_t^{loss}	cost coefficient of the line losses
m_i	number of customers of i -th aggregator
n_v	total vertexes of the graph
$\bar{n}_{ij}^{\text{tap,ini}}$	VR initial tap position
$p_{i,t}^{\text{min}}$	lower charging power limit of i -th EV in the planning period t
$p_{i,t}^{\text{max}}$	upper charging power limit of i -th EV in the planning period t
$\tilde{p}_{i,t}^{\text{min}}$	lower power consumption limit of i -th HP in the planning period t
$\tilde{p}_{i,t}^{\text{max}}$	upper power consumption limit of i -th HP in the planning period t

p_t^c	conventional active demand at each node in the planning period t
p_t^{fl}	price of the flexibility service product in the planning period t
p_t^g	power injection of DGs
$p_{t_1}^d$	the amount of a power consumption reduction at the congestion hour t_1 in each swap
$p_{j^*}^c$	conventional active demand at node j^*
$p_{i,t}^{\text{da}}$	the day-ahead scheduled power consumption of i -th customer in period t
$p_{s,t}^{\text{r,max}}$	the maximum allowable amount of energy rebound of s -th swap in period t
q_t^c	reactive power demand at each node in the planning period t
$q_{j^*}^c$	conventional reactive demand at node j^*
w_t^l, w_t^v	penalty coefficients
π_t^{fl}	volume of the SRP
$\delta_{i,t}^{\text{max}}$	the allowable maximum modification of the power consumption target of i -th customer in period t
λ^{max}	the maximum clearing price of the transactive market
$\theta_{i,t}$	the inverse of the response curve's slope of i -th customer in period t
t_1	the time period in which a power consumption reduction is required
$\hat{z}_i$	the index of the zone to which node i belongs
w_{ij}^{ini}	initial status of the switch between nodes i and j
n_l	the number of branches
$\varepsilon_1, \varepsilon_2$	specified thresholds for primal and dual residuals
ρ	the penalty parameter of the ADMM-based algorithm
μ, τ	parameters used for tuning the penalty parameter ρ
$\bar{z}$	the maximum value of variable z
β_i^{CLPU}	cold load percentage
λ_i^{CLPU}	percentage of additional power absorbed due to CLPU
η_i^c, η_i^d	ESS charging and discharging efficiency coefficients

Continuous variables:

$E_{i,t}^r$	received revenue of i -th customer in period t
$\bar{E}_{i,t}^{\text{ES}}$	SOC level of the ESS at node i at time step t

$\vec{E}_{t,t}^{\text{ES,ini}}$	initial ESS SOC level
$\vec{E}_{t,t}^{\text{ES,max/min}}$	maximum/minimum limit of the ESS SOC level
F_t^p / F_t^q	active/reactive power flow
$\vec{P}_{i,t}^{\text{DER}}$	DER (WT, PV, ESS) active power generation at node i at time step t
$P_{ij,s}, Q_{ij,s}$	active and reactive power flow from nodes i to j at step s
$P_{i,s}^G, Q_{i,s}^G$	active and reactive DG generations at node i at step s
$\vec{P}_{i,t}^{\text{M}}$	MT generation at node i at time step t
$\vec{P}_{i,t}^{\text{D,F}}$	expected active demand
$\vec{P}_{i,t}^{\text{CLPU}}$	additional active power absorbed due to CLPU phenomenon
$\vec{P}_{i,t}^{\text{MT/PV/WT}}$	MT/PV/WT active power generation at node i at time step t
$\vec{P}_{i,t}^{\text{PV/WT,F}}$	forecasted PV/WT active generation
$\vec{P}_{i,t}^{\text{ES}}$	net active power generation of the ESS at node i at time step t
$\vec{P}_{i,t}^{\text{ES,D/C}}$	ESS discharging/charging power
$\vec{Q}_{i,t}^{\text{D,F}}$	expected reactive demand
$\vec{Q}_{i,t}^{\text{MT/PV/WT}}$	MT/PV/WT reactive power generation at node i at time step t
$\vec{Q}_{i,t}^{\text{ES}}$	ESS reactive power generation at node i at time step t
$\vec{Q}_i^{\text{E,max/min}}$	maximum/minimum limits of the ESS reactive power generation
$\vec{Q}_{i,t}^{\text{cap}}$	capacity of the capacitor at node i at time step t
$\vec{Q}_{i,t}^{\text{DER}}$	DER (WT, PV, ESS) reactive power generation at node i at time step t
K_t	fictious active power flow
$K_{i,s}^G$	fictious DG active power generation at node i at step s
$I_{ij,s}^{\text{sqr}}$	squared current magnitude from nodes i to j at step s
$V_{i,s}^{\text{sqr}}$	squared voltage magnitude at node i at step s
V_t^{sqr}	squared voltage magnitude in period t
ΔV_t^{sqr}	voltage magnitude drops of lines in period t
$x, \vec{x}$	$\vec{x}$ is the three-phase counterpart of x
$\vec{a}_{ij,t}$	VR voltage ratio
$\Delta a_{ij,t}$	change rate of the VR voltage ratio
$\pi_{e,t}^e, \pi_{h,t}^h$	the response elastic coefficients of e -th EV and h -th HP at hour t
$f_{e,t}^e, f_{h,t}^h$	the amounts of flexibility provided by e -th EV and h -th EV at hour t

$p_{i,t}$	charging power of i -th EV in the planning period t
$\tilde{p}_{i,t}$	power consumption of i -th HP in the planning period t
$p_{i,t}^*$	final charging power of i -th EV in the planning period t after the incentive-based method
$\tilde{p}_{i,t}^*$	final power consumption of i -th HP in the planning period t after the incentive-based method
$\bar{p}_t^{\text{gen}}$	power injection of DGs after providing flexibility
$\bar{p}_t^{\text{load}}$	active power demand at each node after providing flexibility
$p_{s,t}^r$	the amount of energy rebound in period t in s -th swap
$p_{i,t}^{\text{tar}}$	the power consumption target of i -th customer in period t
$p_{i,t}^a$	the actual power consumption of i -th customer in period t
$p_{e,t}^{\text{e,init}}$	EV charging power of e -th EV in period t
$p_{h,t}^{\text{h,init}}$	HP power consumption of h -th HP in period t
$p_{e,t}^e$	final EV charging power of e -th EV in period t in the incentive-based method
$p_{h,t}^h$	final HP power consumption of h -th HP in period t in the incentive-based method
$r_{s,d,t}$	The total difference between the actual and day-ahead scheduled power consumption of those customers who belong to d -th side of s -th swap in period t .
$r_{i,s}$	percentage of load shedding at node i at step s
r_t^{inc}	incentive (flexibility price) at hour t in the incentive-based method
$s_{t,jk}$	auxiliary variable of line jk
t_2	t_2 is the time period in which the energy rebound occurs
α_t^l	auxiliary variables of line loading constraints
α_t^v	auxiliary variables of voltage magnitude constraints
$\delta_{i,t}$	the modification of the power consumption target of i -th customer in period t
$\lambda_{s,d,t}^{\text{cl}}$	the clearing price for d -th side of s -th swap in period t
$p_{t_2}^r$	set of day-ahead planning period
x^{ld}	the total overloading amount during the planning horizon
x^{wc}	the total overloading amount during the planning horizon in the worst-case scenario

$z^+, z^-, \Delta z$	auxiliary variables used to linearize z^2
β_{ij}	the auxiliary variable associated with the switch between nodes i and j
Cost_s	the total payment of s -th swap
Pr_s	set of day-ahead planning period
$\theta_{e,t}^e, \theta_{h,t}^h$	auxiliary variables associated with providing flexibility

Binary variables:

$w_{ij,s}$	status of the switch ij (the switch between nodes i and j) at step s , where $w_{ij} = 1$ if the switch is closed at step s ; otherwise, $w_{ij} = 0$.
$x_{t,jk}$	status of sectionalizing switch jk in the planning period t , $x_{t,jk} = 1$ if the switch jk is closed; otherwise, $x_{t,jk} = 0$
$x_{z,s}$	status of zone z at step s , where $x_{z,s} = 1$ if the zone is energized at step s ; otherwise, $x_{z,s} = 0$
$x_{s,j}^{\text{sw}}$	binary variable representing whether the j -th load point in s -th swap is selected
$x_{jk,t}^{\text{ld}}$	overloading status of line jk in period t , $x_{jk,t}^{\text{ld}} = 1$ if line loading on line jk exceed its line capacity; otherwise, $x_{jk,t}^{\text{ld}} = 0$
$y_{s,j}^{\text{sw}}$	binary variable representing whether the j -th load point in s -th swap is selected
$y_{ij,s}^u, y_{ij,s}^d$	auxiliary binary variables associated with switching
z_t^{pur}	purchase status of the flexibility service product
$z_{i,t}^1, z_{i,t}^2$	binary variables indicating the direction of power consumption modi- fication of i -th customer in period t
$z_{i,t}$	binary variable indicating whether the power consumption modifica- tion reaches the maximum allowable value
$z_{e,t}^e, z_{h,t}^h$	binary variables associated with the customer's response.

Integer variables:

$\vec{s}_{ij,t}^{\text{VR}}$	number of tap operations of the VR
$\vec{n}_{ij,t}^{\text{tap}}$	integer variable representing the tap position of the tap-changer

Other symbols:

$\mathbf{1}$	a vector with all ones
$\{v\}_i$	i -th element of vector v

$ \ast $	cardinality pf sets, e.g., $ \mathcal{N} , \mathcal{N}^s , \mathcal{H} $
$\langle \vec{x}, \vec{y} \rangle$	inner product of the two vectors
$\vec{x} \odot \vec{y}$	element-wise product of the two vectors

CHAPTER 1.

INTRODUCTION

Section 1.3 of this chapter is mainly based on the published articles 1 and 2¹.

1.1 Background

In the next 25 years, it is expected that the worlds' total energy consumption will increase by one-third and the pressure on energy shortage is increasing rapidly [1]. In this regard, the Danish government has set up an ambitious energy strategy to realize independence from coal, oil, and gas by 2050 [1]. The strategy presents a wide breadth of energy policy initiatives to reduce the use of fossil fuel and increase the share of renewable energy. As driven by the energy shortage and environment issues and a range of energy policies, the distributed energy resources (DERs), such as photovoltaics (PVs), small wind power systems (WPSs), electric vehicles (EVs), and heat pumps (HPs), will be massively deployed in power systems in the future [2], [3].

For instance, in 2019, the PV installation exceeds 100 GW for the third time in a row [4]. The cumulative capacities of the top 10 countries in 2019 are shown in Fig. 1-1 [4]. China leads the PV market with a cumulative capacity of 204.7 GW, followed by the USA (75.9GW) and Japan (63GW).

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¹ Section 1.3 of this chapter is based on the published articles 1 and 2: 1) **Feifan Shen**, Qiuwei Wu*, and Yusheng Xue, "Review of Service Restoration for Distribution Networks," *Journal of Modern of Power Systems and Clean Energy*, vol. 8, no. 1, pp. 1-14, Jan. 2020; 2) **Feifan Shen**, Qiuwei Wu, Shaojun Huang, Juan Camilo López, Canbing Li, and Bin Zhou, "Review of Service Restoration Methods in Distribution Networks," *2018 IEEE PES Innovative Smart Grid Technologies Conference Europe* (ISGT-Europe), 2018.

Fig. 1-1 Cumulative capacities of top 10 countries in 2019

The global EV sale market has also experienced rapid growth in recent years, as shown in Fig. 1-2. According to the Global EV Outlook 2020, the EV sale reached 2.1 million globally in 2019 and boosts the stock to 7.2 million EVs [5]. The EV units of the top three countries/regions are shown in Fig. 1-3. China is the world's largest EV sale market with 2.3 million EVs in active use, followed by the European Union (1.2 million) and USA (1.1 million) [6].

Fig. 1-2 Global EV stock during 2015-2019

Fig. 1-3 EV units of top three countries/regions in 2019

The large-scale integration of DERs accelerates the paradigm shift of the distribution network from a passive network to a more flexible and active network [7]. The active distribution network (ADN) can take advantage of the information, communication technologies (ICT) to proactively coordinate the operation and control of DERs, enabling a secure and efficient solution to the management of the distribution network.

The impacts of DERs on the operation of ADNs have been widely discussed [2]. The integration of DREs provides both challenges and benefits to the planning and operation of ADNs. Regarding the operational challenges, the DER operation complicates the power flow pattern by introducing bi-directional power flow and incurs varying operation conditions along with network constraints violations. For example, the non-coordinated EV charging could cause congestion issues [8], including the feeder overloading problem (loading exceeds the feeder physical limit) and voltage deviation problem (bus voltage deviation exceeds the deviation limit, typically ± 0.1 p.u.). Regarding the operational benefits, the strategical DER operation could reduce line losses, improve power quality, and enhance the resilience of the ADN. Especially after a contingency event, the DERs could actively contribute to the recovery of the ADN by acting as support sources to provide electricity supply [9].

To achieve the secure and efficient operation of ADNs, the PhD project mainly focuses on the congestion issue of the ADN caused by the non-coordinated operation of DERs at the demand-side, i.e., flexible demands EVs and HPs, and on ser-

vice restoration of ADNs after the contingency. The following two subsection presents literature reviews on congestion management and service restoration of ADs, followed by contributions of the thesis.

1.2 Literature Review on Congestion Management of ADNs

The high penetration of DERs poses many challenges to the secure operation of ADNs. One major challenge is feeder congestion caused by the non-coordinated operation of DERs at the demand side, such as simultaneous EV charging and HP power consumption. A huge number of congestion management methods have been developed to alleviate congestion of ADNs. They can be categorized into three types: 1) network reinforcement; 2) direct congestion management; and 3) indirect or market-based congestion management.

Network reinforcement is to expand or harden the existing feeders to improve their current carrying capabilities [10]. It requires an additional capital investment of power components, such as lines and transformers. In direct congestion management methods, the distribution system operator (DSO) directly controls the existing power system components to alleviate congestion, e.g., through network reconfiguration by operating sectionalizing and tie switches [11], and/or through direct voltage control by adjusting on-load tap changers (OLTCs) [12], and/or through direct active/reactive power control [13]. The indirect methods, namely market-based methods, are based on the existing or newly designed markets, such as the day-ahead/real-time energy market and flexibility market. They aim to use price or incentive signals to motivate customers to change their power consumption patterns in order to alleviate congestion.

The market-based congestion management methods as the focus of the thesis are reviewed in subsections 1.2.1 and 1.2.2 according to the used time frame and signal type (price and incentive).

1.2.1 Day-Ahead Market-based Methods

1.2.1.1 Price-Based Methods

The distribution locational marginal price (DLMP) extended from the transmission-level locational marginal price (LMP) concept was developed in [14]-[18] to alleviate distribution-level congestion. The DLMP-based method could motivate DGs to produce more power locally to reduce energy delivery from the remote areas at congestion hours, which could alleviate congestion and at the same time

increase DGs' revenues. The DLMP is calculated through the energy market clearing problem, which is similar to the LMP calculation. However, how the distribution-level energy market clearing is coordinated with the transmission-level energy market clearing has not been addressed.

The dynamic tariff (DT) method [19]-[23], dynamic power tariff (DPT) method [24], dynamic subsidy (DS) method [25], and line shadow price (LSP) method [26], [27] have been developed to alleviate congestion of ADNs through rescheduling flexible demands such as EVs and HPs. Unlike the DLMP-based method, the DT, DPT, DS, and LSP methods are not based on the market clearing problem and can be seamlessly integrated into the existing day-ahead energy market, e.g., the Nordic spot market. These methods have a common feature that the final electricity prices, i.e., spot prices plus DTs or spot prices minus DSs, are higher at congestion hours than those at un-congestion hours. Consequently, customers will shift power consumption at congestion hours to un-congestion hours in order to minimize energy costs, thereby alleviating congestion.

The DT method as the main focus of the thesis is a decentralized congestion management method, where aggregators make energy plans independently based on spot prices and DTs published by the DSO. Since the DTs contain network constraint information, the energy plans made by the aggregator respect network constraints as the DSO expects. A DT model was formulated in [19] to alleviate congestion induced by EVs. The DTs are obtained by solving a social welfare optimization model considering EVs as flexible demands. However, due to the linear formulation of the DT model, there might be a multiple-response issue that the energy plans made by the aggregators are different from the ones the DSO expects, so congestion may not be alleviated. To resolve this issue, the price sensitivity coefficient is used in [20] in order to formulate a quadratic programming (QP)-based DT model, which ensures a unique response solution of the aggregators to the DTs.

The DSO charges tariffs without considering power consumption levels in the DT method whereas it charges tariffs based on consuming power in the DPT method in [24]. In the DPT method, the customers consuming more power will pay higher tariffs than those consuming lower power, which can reflect real contributions of customers to the distribution investment, operation and maintenance costs. Instead of imposing tariffs on customers, a DS method was proposed in [25], in which the DSO pays subsidies to customers at un-congestion hours, which consequently leads to higher final prices (spot prices minus subsidies) at congestion hours. Therefore, the DS plays the same role as the DT in terms of shifting power consumption. The

LSP method proposed in [26], [27] works similarly to the DT method. However, the tariffs are calculated with an iterative procedure between the DSO and aggregators based on the dual decomposition.

In the above-mentioned DT methods, there is an assumption that the DSO and aggregators use the same energy requirement parameters, such as EV driving parameters. However, the DSO's forecast errors cannot be eliminated, which may result in a failure of the DT method. Therefore, it is important to deal with the uncertainty in the DT framework. In [22], an uncertainty management scheme for the DT method was developed, in which a chance-constrained DT model is formulated. By solving the model with an iterative procedure, a certain security margin of the line capacity is reserved to ensure that the congestion probability is lower than a certain level with a specified confidence level. However, the probability distributions of forecast errors need to be known beforehand. In [23], a distributed optimization-based DT method was proposed to relieve the DSO's forecast burden. In the proposed method, the DSO does not need to forecast energy requirements of the aggregators and the DTs are obtained through an iterative procedure between the DSO and aggregators. However, the high communication burden induced by the iterative procedure may compromise the effectiveness of the proposed method.

The coordination of the price-based method and the direct control method could be beneficial. In [21], an optimal reconfiguration-based DT model was proposed, in which network reconfiguration is incorporated into the DT model to alleviate congestion and line loss reduction. In [27], network reconfiguration is served as a backup congestion management method if the LSP method is failed.

1.2.1.2 Incentive-Based Methods

In incentive-based methods, the aggregators or customers are rewarded for rescheduling power consumption.

A monetary incentive-based method was proposed in [28] to coordinate flexible demands to alleviate day-ahead congestion. The power consumption rescheduling is based on spot prices and additional incentives. Since the proper incentive is chosen from a given set of incentives using an iterative process between the DSO and customers, the incentive may not be optimal.

Another type of incentive-based method is to establish a flexibility market, such as the flexibility clearing house (FLECH) [29] and local flexibility market (LFM) [30]-[36]. In these methods, the DSO buys flexibility service products provided by

the aggregators or customers to alleviate congestion. The FLECH is a service-oriented platform aiming to trade flexibility service products, such as Flexibility Service of Overload Planned (FSOP), and Flexibility Service of Overload Reserve (FSOR). In addition, three types of advance demand products are defined in [37], including Scheduled Re-Profiling (SRP) product, Conditional Re-Profiling (CRP) product, and Bi-Directional Conditional Re-Profiling (B-CRP) product.

The LFM in [30] is based on bilateral flexibility contracts with multiple contracts between each pair of agents. The contract trading process is carried out with an iterative process between the DSO and aggregators and between the aggregator and customers. The LFMs in [31]-[36] are pool-based markets where flexibility sellers offer flexibility service products in the market, and the market operator clears the market to satisfy the flexibility buyers' flexibility requirements based on different market clearing models.

A day-ahead LFM market and an intraday LFM market were proposed in [32] for day-ahead and intraday congestion management, in which market participants consist of the DSO, market operator, balance responsible party (BRP), and aggregator. The DSO buys flexibility service products in the day-ahead LFM to alleviate congestion, whereas the BRP competes with the DSO in the intraday LFM for buying flexibility service products to minimize imbalance costs. The day-ahead and intraday market clearing problems are formulated as mixed integer linear programming (MILP) models and solved by the market operator with the social welfare maximization. A similar LFM was proposed in [33] for day-ahead congestion management, in which micro-grids (MGs) provide flexibility service products and the DSO is regarded as the market operator. The MG agent aggregates flexibility from the DERs, and the market clearing problem is formulated as a non-linear programming (NLP) model and solved by the DSO with the minimum procurement cost.

The above-mentioned LFMs are cleared before the day-ahead energy market clearing, so the LFM clearing solution is used to adjust original energy schedules to alleviate congestion. In contrast, the LFMs in [34]-[36] operates after the day-ahead energy market clearing if the accepted energy schedules in the energy market result in congestion. The market clearing problems in [34]-[36] are formulated as NLP models and solved by the DSO to minimize flexibility procurement costs. In these LFMs, the imbalance cost induced by rescheduling energy schedules after the day-ahead energy market clearing has not been addressed.

A hierarchy congestion management scheme was proposed in [38] to integrate the SRPs and network reconfiguration in the tertiary control level for day-ahead congestion. The tertiary controller considers network reconfiguration as primary means to alleviate congestion. If network reconfiguration cannot resolve all congestion, the DSO will buy SRPs and CRPs to resolve the remaining congestion.

1.2.2 Real-time Market-Based Methods

Real-time congestion due to forecast errors and unexpected power component failures should be addressed as well. Real-time congestion management is usually implemented 5-60 minutes prior to the operation time [39]. The existing real-time market-based congestion management methods are limited, and most of them are incentive-based methods.

In [39], a flexible demand swap (FDS)-based method was developed to use flexible demand swap to alleviate real-time congestion. The flexible demand swap represents spatial and temporal power consumption/generation change, which can address both the rebound effect and imbalance issue. An agent-based real-time congestion management scheme was introduced in [40], in which DERs are represented by agents to send flexibility service bids to the DSO, and then the DSO selects flexibility bids with the minimized procurement cost to alleviate congestion. In addition, direct load control is considered as a backup option for real-time congestion management.

1.3 Literature Review on Service Restoration of ADNs

The Advanced Metering Infrastructure (AMI) and Distribution Automation (DA) provide ADNs self-healing capability. Self-healing represents the ability of the network, after a fault, to automatically and intelligently restore itself to the best possible status with minimum human intervention based on a set of equipment, algorithms and communication technologies [41]. A successful self-healing scheme depends on the complete implementation of fault location, fault isolation, and service restoration. As an essential aspect and complicated task of self-healing, service restoration has drawn considerable attention. Thus, the Ph.D. program focuses on service restoration of self-healing for ADNs.

An ADN shown in Fig. 1-4 is used to illustrate service restoration procedures through network reconfiguration or islanded MGs. The network has six load zones and a DG unit with the black-start capability. Suppose that a permanent fault occurs in zone 1, then switches S1 and S2 are opened to isolate zone 1. Finally, the DSO

reconfigures the network topology to restore the outage zone 2 by closing tie-switch 1 (TS1), as shown in Fig. 1-5. During network reconfiguration, operational constraints must be respected, such as power balance constraints, voltage and current magnitudes constraints and radial topology constraints.

Suppose that a permanent fault occurs in zone 5. After the fault, the DG unit should be disconnected immediately to avoid adverse impacts according to the current operational policies [41]. To isolate the faulty zone 5, switches S5 and S6 are opened. In such a case, restoration paths of the utility power supply are unavailable. Therefore, the DG unit is restarted and dispatched to supply outage loads locally and form an MG, as shown in Fig. 1-5. Due to operational constraints of the DG unit, outage loads are restored sequentially while maintaining generation-load balance.

Fig. 1-4 Initial topology of the ADN

Fig. 1-5 Illustration of the service restoration procedure

The service restoration methods that require communication systems, as the main topic of the thesis, can be categorized into three types, including 1) centralized method: each local agent communicates with a central controller that performs computations based on overall system information; 2) distributed method: each agent communicates with its neighbors to perform computations and there is not a central controller; and 3) hierarchical method: agents communicate in a hierarchical structure to perform computations, and local agents communicate with a central controller. The existing service restoration methods are organized in Table 1-1 according to the communication architecture and used techniques including expert systems, heuristic algorithms, meta-heuristic algorithms, graph theory, mathematical programming, and multi-agent systems (MASs).

Table 1-1 Category of the existing service restoration methods requiring communication systems

Communication architecture	Mainly used techniques
Centralized	expert systems, heuristic algorithms, meta-heuristic algorithms, graph theory, mathematical programming
Distributed/hierarchical	mathematical programming, MASs

1.3.1 Centralized Methods

Centralized methods are based on the distribution automation system (DAS) consisting of a master station, substations, DA devices, and a communication system. By collecting global information, the service restoration scheme can be optimized.

However, centralized methods are based on centralized infrastructure and need to transmit a large amount of filed data and process computations using a powerful control center with expensive computational capacity. Hence, centralized methods may suffer from the heavy computation burden and the single point failure risk, i.e., a failure of the central controller would cause a failure of service restoration. Moreover, they also compromise the privacy information protection and it is challenging for the centralized infrastructure to adapt to system changes.

1.3.1.1 Expert System

An expert system has three principal parts [42]: knowledge base, database, and inference engine, as shown in Fig. 1-6. The knowledge base consists of the domain knowledge used for problem-solving. The domain knowledge is translated into rules, e.g., IF (condition)-THEN (action) rules. The database includes a set of facts used to match rules in the knowledge base. The inference engine links the facts with rules and conducts the reasoning process to obtain a solution.

Fig. 1-6 Structure of the expert system

The knowledge bases developed in [43] and [44] are based on IF-THEN rules acquired from the expert knowledge of power engineers. With the knowledge bases, the expert systems can restore the system with zone restoration or group (multi-zones) restoration. In [45], an expert system was developed based on the colored petri net inference model that has parallel-like inference characteristic, which helps find restoration plans in case of multiple contingencies. An object-oriented programming (OOP) technique-based expert system was developed in [46], which organizes power system component data in a hierarchical way and uses the OOP technique to represent rules. Hence, the inference ability of the expert system is enhanced.

The expert system-based method can obtain feasible restoration solutions quickly, whereas it is difficult to construct a large knowledge base and is costly to maintain a large-scale expert system. In addition, the optimality of solutions cannot be guaranteed.

1.3.1.2 Heuristic Algorithm

In heuristic algorithms, the heuristics are transformed into heuristic algorithms to guide the solution searching.

A heuristic service restoration procedure was translated into a heuristic algorithm in [47], in which the de-energized loads are firstly restored by supporting feeders and then restored by corresponding supporting laterals in order to restore the maximum loads with a minimized number of switching operations. A heuristic algorithm based on the power flow calculation was developed in [48]. Firstly, all available switches regarded as modifiable fictitious current sources are closed to create a meshed network. Then, an optimal power flow study is conducted to find the switch carrying the least current that will be opened to eliminate a network loop. Repeat this process until there is no network loop. The switch is opened in such a fashion so that the disturbance in the power flow pattern induced by opening switches is minimized. Finally, a radial network with acceptable operation conditions can be obtained.

In [49] and [50], multi-tier heuristic algorithms were developed. The heuristic procedure is that the switches and supporting feeders in tier 1, which are directly related to out-of-service portions of the network, are firstly used to try to restore all de-energized loads; if it fails, the outer switches and feeders related to tier 1 are used to restore remaining de-energized loads. Moreover, load curtailment is considered in the heuristic procedure.

The decision tree concept was used in [51]-[53] to develop heuristic algorithms. These algorithms construct the solution space as a decision tree and use different search techniques to search the tree. Starting from the root node, each of its successor nodes is assigned with a value (either 0 or 1) representing the status of a switch, i.e., the switch is closed if the value is 1; otherwise, the switch is open. As moving down on the tree, all nodes are assigned with binary values. Finally, any path between the root node and a leaf node of the tree represents a combination of switching operations, namely a restoration solution. Thus, all restoration solutions can be obtained through the exhaustive search of the tree using different search techniques

and evaluated according to specific requirements. In [51], the depth-first search technique was used to search for desired restoration solutions.

In [52] and [53], a node of the tree represents a configuration of the network. The transition from a node to one of its successor nodes represents a pair of switch operations, e.g., open a sectionalizing switch and close a tie-switch. Through searching the tree from the root node (initial post-fault configuration) to a leaf node (targeted configuration), a combination of switching operations can be found to reach the target configuration. The breadth-first search technique that can obtain the shortest path from the root node to the leaf node was used in [52] to search for the restoration solution with the minimum number of switching operations. The A* search technique was used in [53] to search for the optimal solution.

The fuzzy set theory was used in [54] and [55] to deal with the imprecise expression of heuristic rules and the uncertainty. The heuristic algorithm in [47] is improved in [54] by introducing a fuzzy cause-effect network to deal with the imprecise expression of heuristic rules. In [55], the uncertainty of loads was modeled by fuzzy set variables and the heuristic procedure in [47] was used to find restoration solutions.

The advantage of heuristic algorithms is that feasible solutions can be obtained quickly. However, they require problem-dependent knowledge, i.e., heuristics, to search for solutions.

1.3.1.3 Graph Theory

A distribution network shown in Fig. 1-4 can be represented as a graph. The vertices represent source nodes and load zones, and edges represent switches. Therefore, service restoration is to find a spanning tree that represents a radial configuration of the network with a desirable operation condition. Different graph theory-based methods were proposed in [56]-[60] to find spanning trees.

In [56], according to the fundamental loop concept, a spanning tree can be obtained by removing one edge from each fundamental loop. Then, all radial configurations are obtained and can be evaluated through the power flow study. In [57], energized nodes connecting de-energized zones are treated as source nodes as well. Then, the modified Prim's algorithm was used to find spanning trees, i.e., a forest, which denotes that de-energized zones are supplied by multiple sources. In [58], an "interested tree" concept representing a radial network configuration where all loads are supplied by substations was proposed. An algorithm was developed to extract

all “interested trees” from the generated spanning trees to achieve complete service restoration.

Graph theory-based service restoration methods through the MG formation were proposed in [59] and [60]. A fundamental loop-based algorithm was developed in [59] to form multiple MGs to implement service restoration while considering source nodes and DG nodes as root nodes. A cut set theory-based spanning tree search algorithm was proposed in [60], in which each MG is modelled as a virtual feeder and the DERs are modeled as root nodes. According to the cyclic interchange operation concept in graph theory, a spanning tree is generated by operating a pair of switches. Then, a radial network configuration incorporating MGs can be found for service restoration.

In the graph theory-based methods, although the optimal solution can be obtained through an exhaustive search, the above methods may be costly and unattractive due to a large number of spanning trees for large-scale ADNs.

1.3.1.4 Mathematical Programming

The service restoration problem can be formulated as the MINLP model [61], MILP model [62], [63], and mixed integer second order cone programming (MISOCP) model [64]. Then, the commercial solvers, such as CPLEX [65], can be used to solve the service restoration models to obtain service restoration solutions.

Two MINLP models were formulated in [61] for service restoration of unbalanced ADNs based on the branch flow model and bus injection model, respectively. The formulated MINLP model is a non-convex combinatorial problem, which is hard to solve. Hence, the linearization and convex relaxation techniques have been applied.

The authors in [62] and [63] proposed a two-stage strategy to solve the MINLP model. At the first stage, the piecewise linearization technique is used to linearize non-linear constraints of the MINLP model and obtain an MILP model. Then, using the obtained binary values at the first stage, which represents a configuration of the network, an NLP model is solved at the second stage to obtain continuous variables representing an operation condition of the network. In addition, the optimal switching sequence during service restoration is considered in [63].

The convex relaxation technique was used in [64] to transform the MINLP model into a MISOCP model by relaxing the non-linear constraint as a second-order cone

constraint. Compared with the MINLP model, the MISOCP model can be solved efficiently using commercial solvers based on the convex optimization techniques, such as branch and bound (B&B) [66].

Service restoration using MGs have been studied in [67]-[70]. A comprehensive strategy considering the normal operation mode and the self-healing mode was developed in [67]. In the normal operation mode, the DGs are dispatched to minimize energy costs of the network. In the self-healing mode, an MINLP model is formulated to optimally sectionalize the network into several MGs, in each of which self-adequacy is maintained. In [68], an MILP model was formulated to maximize the pick-up amount of critical loads by controlling remotely controlled switches and DG units while satisfying the self-adequacy and operation constraints of the MG formation. However, the formulated model is a single-step model without considering restoration steps. In [69] and [70], two sequential service restoration strategies were proposed to determine the optimal switching sequence to form MGs to restore outage areas for balanced and unbalanced networks, respectively.

The uncertainties of DG units and load consumption during service restoration have been investigated in [67] and [71]-[75]. In [67], the uncertainties of DG units and load consumptions are characterized by scenarios, and the rolling-horizon technique is used to make restoration decisions based on the latest information under uncertain environment. The authors in [71] proposed a two-stage robust restoration model to consider the uncertainties of DG units and load consumption. The robust model is to obtain a restoration solution that is optimal to the worst-case scenario and is feasible to all scenarios. The authors in [72] formulated an information gap decision theory-based robust restoration model. For a given uncertain set of DG outputs and load consumption, the solution can guarantee that the restored load amount would not fall below a specified threshold. In [73], the uncertainty of load consumption was modeled by fuzzy variables and a fuzzy MIP model was developed to perform risk management for service restoration. In [74], the uncertainty of the renewable energy generation was modeled using a data-driven chance-constrained method. Accordingly, a chance-constrained service restoration strategy was developed to generate the restoration sequence for MGs after blackouts. In [75], a risk-limiting service restoration strategy was proposed for MGs using the risk-limiting dispatch method. The stochastic WT and PV power outputs were modeled by probability density functions (PDFs), and a Gaussian mixture model is used to represent a joint PDF of WT and PV power outputs.

The dynamic programming technique was applied to decompose a decision-making process into a sequence of decision steps over time. In [76], the energized sequence of feeders is represented as a sequence of states, and an enhanced dynamic programming method is used to select the optimal sequence of states, i.e. the optimal energized sequence. In [77], the bender decomposition technique was used to decompose the service restoration problem into a master problem determining the configuration and a slave problem minimizing line losses. Then, the master and slaver problems are solved iteratively to derive the restoration solution.

The advantage of the mathematical programming-based methods is the ability to formulate and solve complex instances of the service restoration problem. In such a case, the global optimal or near-optimal restoration solution can be obtained. However, with the increasing size and complexity of the network, it may have a heavy computation burden for solving the mathematical programming models.

1.3.1.5 Meta-Heuristic Algorithms

The service restoration problem can be solved by meta-heuristic algorithms, which use natural intelligence to explore the search space. Meta-heuristic algorithms have similar procedures with different searching and encoding strategies. Meta-heuristic algorithms include the genetic algorithm (GA) [78], [79], tube search (TS) algorithm [80], particle swarm optimization (PSO) algorithm [81], parallel-simulated annealing (PSA) algorithm [82], and etc.

In [82], four meta-heuristic algorithms (GA, TS, reactive TS, and PSA) are described, and their solutions are compared in terms of the average calculation time and solution quality. More advanced meta-heuristic algorithms have been developed in [83] and [84] for service restoration. A non-dominated sorting genetic algorithm-II was adopted in [83], which can deal with the multi-objective issue without using weighting coefficients. In the above-mentioned algorithms, the switching sequence is not considered. Hence, the authors in [84] proposed a multi-objective evolutionary algorithm based on the node-depth encoding method, which can provide the switching sequence solution.

The fuzzy set theory has been utilized to optimize solutions obtained through meta-heuristic algorithms. In [85], an interactive fuzzy satisfying method based on the fuzzy set theory and GA was proposed to deal with imprecise nature in multi-objectives of the problem.

The meta-heuristic algorithms do not require problem-dependent heuristics and can be applied for different types of problems; however, they cannot guarantee the optimality of solutions.

1.3.1.6 Hybrid Methods

Different techniques have been combined to have an optimized service restoration solution. In [86], the expert system is firstly used to divide the whole network into several sub-networks, and then service restoration of each sub-network is formulated as an MILP model. In [87], the heuristic algorithm is firstly used to find the possible restoration plans that require one switching operation. If all these restoration plans are failed, the restoration problem is formulated as a MIP model and solved to obtain restoration solutions. A two-phase strategy was proposed in [88], in which the target configuration is obtained through the GA algorithm in the first phase, and the optimal switching sequence is obtained using the dynamic programming technique in the second phase.

1.3.2 Distributed and Hierarchical Methods

Distributed and hierarchical methods use distributed intelligence to solve service restoration problems. In these methods, the MAS and mathematical programming are used.

1.3.2.1 Multi-Agent Systems

In the MAS, the distribution network is divided into multiple sub-networks, each of which is controlled by a local agent. Each agent can perceive and react changes in its environment autonomously and coordinate to solve problems according to specified heuristic rules. The MAS-based service restoration methods are based on different types of agents and communication architecture. In the distributed MAS, agents communicate with neighbors without central negotiation agents, whereas central negotiation agents are needed to coordinate agents in the hierarchical MAS.

The hierarchical MASs were developed in [89]-[91]. In the MAS in [89], the bus agent and facilitator agent were used. The bus agent is to restore loads directly connected to its bus and the facilitator agent as the negotiation agent coordinates all bus agents. A two-layer MAS was developed in [90], in which a feeder is divided into several zones, each of which has a zone agent detecting status of the zone and communicating with its feeder agent. The feeder agent as a negotiate agent coordinates affiliated zone agents through communicating with adjacent feeder agents. A similar two-layer MAS was designed in [91] with the zone agent

being replaced by the load agent. Moreover, an additional regulator agent was added in the upper layer to detect and adjust the nodal voltage.

The distributed MASs were designed in [92]-[95]. In [92], switch agents, load agents, and generator agents communicate with neighbor agents to make decisions. In [93], the zone agent has four possible states, including faulty zone agent, down zone agent, zone tie agent, and healthy zone agent. When a fault occurs, each zone agent determines its state based on the fault location and communicates with each other to make decisions.

The MASs were designed in [94] and [95] for service restoration through the MG formation. In [94], the MAS is based on the “team” concept representing a group of segments bounded by switches. The switch and distributed energy storage agents within a team can communicate with each other and a team can communicate with other teams next to it. The teams of agents negotiate to determine the network configuration and control distributed energy storage systems to form MGs. In [95], load agents, EV aggregator agents, DG agents, and switch agents are coordinated to determine the network configuration incorporating MGs and power outputs provided by EVs and DGs. Based on the MAS in [95], the uncertainties of loads and DERs are considered in [96] by generating scenarios using Monte Carlo simulations.

The MAS-based service restoration methods can achieve distributed implementation and obtain feasible solutions quickly; however, the agent-based decision-making uses heuristic rules, which cannot ensure the optimality of solutions.

1.3.2.2 Mathematical Programming

The distributed algorithm has been used to solve the service restoration problem in a distributed manner. An alternating direction method of multipliers (ADMM)-based distributed service restoration strategy was proposed in [97], in which the service restoration problem is formulated as a MISOCP model that is decomposed by the ADMM and solved by local agents at each node in parallel. However, the strategy is designed for black-start service restoration and does not consider network configuration.

1.3.2.3 Hybrid Methods

In [98], the mathematical programming model is combined with the MAS. In the MAS, feeder agents negotiate with each other to obtain a reduced network model of the portion of the network involved in service restoration. Then, based on the

reduced network model, a MISOCP service restoration model was formulated and solved to obtain restoration solutions.

1.4 Motivations and Contributions of the Thesis

The motivations and contributions of the thesis are briefly described as follows. The detailed descriptions can be found in each chapter.

Chapter 2: As previously mentioned, integrating different types of congestion management methods is believed to be a promising way to achieve a better congestion management solution. However, the integration of the price-based method and incentive-based method has not been studied in the existing literature. In this context, the contribution of this chapter is to propose a comprehensive congestion management scheme to integrate the DT (price-based method), network reconfiguration, and re-profiling product (incentive-based method) to alleviate day-ahead congestion. In the proposed scheme, the DT method integrated with network reconfiguration is used to alleviate day-ahead congestion firstly. Then, re-profiling products traded in the LFM are used to resolve the remaining congestion. Moreover, a relaxed reconfiguration-based DT optimization model is formulated, which can ensure a feasible DT solution when the original DT method is infeasible and can set a threshold to DTs.

Chapter 3: Despite the integration of the DT and re-profiling product is studied in chapter 2, it is implemented in a simple way that the congestion is distributed between the two methods by setting appropriate coefficients according to the maximum limit of DTs. The DT and re-profiling product are optimized independently. In order to optimally use the synergy between the two methods, an integrated optimization of the two methods needs to be studied. In addition, the proposed integration in chapter 2 may be limited due to potential conflicts with the current DSO regulations. From the implementation point of view, the DSO's revenues by collecting tariffs should be restricted and the DSO's costs for purchasing flexibility service products should be recovered in an appropriate manner. The contribution of this chapter is to propose a coordination scheme of the DT method and incentive-based method, in which the DSO is in a profit-neutral position through the integrated optimization of the two methods. In the proposed coordination scheme, the DSO does not have congestion management costs or revenues.

Chapter 4: The uncertainty exists in the DT framework because the DSO needs to forecast energy requirements at the aggregator side. Due to the inherently inevitable forecast error, the energy plans made at the aggregator side may not respect

network constraints, which makes the DT method ineffective. Therefore, the contribution of this chapter is to propose a robust DT method to deal with the uncertainty. The proposed robust DT method ensures that the network constraints are satisfied even in the worst uncertainty realization scenario.

Chapter 5: Real-time congestion caused by the forecast error or power system component's failure should be addressed as well. Since end-users have already made energy schedules in the day-ahead energy market, two important aspects should be considered in real-time congestion management, including the rebound effect and end-user's willingness to change day-ahead schedules. Therefore, the contribution of this chapter is to propose a two-tier real-time congestion management scheme based on the flexible demand swap and transactive control. Real-time congestion can be alleviated efficiently by the flexible demand swap while considering the end-user's energy rebound conditions and willingness to change day-ahead schedules through a transactive control-based interaction mechanism.

Chapter 6: In order to consider complicated instances of service restoration, such as the multi-objective and uncertainty, and to obtain an optimized service restoration plan, the service restoration problem is formulated as an optimization model that is usually solved in a centralized manner in the existing literature. The centralized method may suffer from a heavy computation burden as the size and complexity of the ADN increase. Therefore, the contribution of this chapter is to propose a hierarchical service restoration method based on the ADMM to reduce the computation complexity of service restoration. In the proposed method, each load zone as a radial portion of the network has a controller solving its own sub-problem with local network data and information from the central controller. The central controller solves a global coordination problem with information collected from each zone controller.

Chapter 7: Although the proposed hierarchical service restoration method can reduce the computation burden, it still needs a central controller to send out control commands in a centralized manner. The centralized infrastructure is costly and suffers from the single-point failure risk. Moreover, it is challenging for the centralized infrastructure to adapt to system changes, and it compromises privacy information protection. Therefore, the contribution of this chapter is to propose a distributed service restoration method based on the ADMM. In the distributed method, the service restoration problem is distributed among the load zones, each of which has a local controller that exchanges information with neighbor controllers.

Chapter 8: The proposed hierarchical and distributed service restoration methods are mainly based on power fed by the upstream transmission network and network reconfiguration. However, they may be ineffective in case of the unavailability of power from the upstream transmission network. The MG can be used to deal with this challenge by utilizing its local DERs to restore loads. In addition, the uncertainty of the DERs brings risks to secure service restoration. Therefore, the contribution of this chapter is to develop a distributed risk-limiting service restoration method for the ADN with networked MGs. In the proposed method, the risk limitation is addressed by the conditional value-at-risk index (CVaR). Each MG can make risk-limiting service restoration decisions individually without violating the risk-limiting constraints of the whole network.

1.5 Structure of the Thesis

The thesis is organized as follows,

Chapter 2 presents a comprehensive day-ahead congestion management method for ADNs based on the DT, network reconfiguration, and re-profiling product. Chapter 3 investigates the coordination of the DT method and incentive-based method to alleviate day-ahead congestion of ADNs. Chapter 4 presents a robust DT method to deal with the uncertainty in the DT framework. Chapter 5 presents a two-tier real-time congestion management method for ADNs based on the flexible demand swap and transactive control. Chapter 6 proposes a hierarchical service restoration method for ADNs based on the ADMM. Chapter 7 proposes a distributed service restoration method for ADNs based on the ADMM. Chapter 8 presents a distributed risk-limiting service restoration method for the ADN with MGs based on the ADMM and CVaR. Conclusions and future work are provided in chapter 9.

CHAPTER 2.

COMPREHENSIVE DAY-AHEAD CONGESTION MANAGEMENT BASED ON THE DYNAMIC TARIFF, NETWORK RECONFIGURATION, AND RE-PROFILING PRODUCT

This chapter presents a comprehensive day-ahead congestion management scheme based on the dynamic tariff (DT), network reconfiguration, and re-profiling product to effectively alleviate day-ahead congestion of ADNs. This chapter is based on the published articles 3 and 4 with changes to coherently fit into the framework of this thesis².

2.1 Introduction

Although the existing market-based methods are effective in congestion management, there are issues when only one method is implemented. The key feature of the DT method is that the optimal energy plans made at the DSO side can be realized at the aggregator side through the DTs. Thus, the effectiveness of the DT method depends on the DT calculation. However, the DT optimization model at the DSO side may be infeasible when severe congestion occurs, resulting in a failure of the DT calculation. In addition, even if the DT calculation is feasible, customers may face very high DTs under the severe congestion situation. Therefore, in order

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² This chapter is based on published articles 3 and 4: 1) **Feifan Shen**, Shaojun Huang, Qiuwei Wu*, Sami Repo, Yan Xu, and Jacob Østergard, "Comprehensive Congestion Management for Distribution Networks Based on Dynamic Tariff, Reconfiguration and Re-profiling Product," *IEEE Trans. Smart Grid*, vol. 10, no. 5, pp. 4795-4805, Sep. 2019; and 2) **Feifan Shen**, Qiuwei Wu*, and Shaojun Huang, "Day-ahead Congestion Management Scheme for Distribution Networks with Dynamic Tariff and Re-profiling Products," *International Journal of Smart Grid and Clean Energy*, vol. 9, no. 1, pp. 91-101, Jan. 2020.

to resolve the above-mentioned issues, a relaxed reconfiguration-based DT optimization model is proposed in this chapter, which can ensure a feasible DT solution when the original DT optimization model is feasible and set a threshold to DTs.

As mentioned in chapter 1, integrating different congestion management methods is a promising way to achieve a better solution for congestion management. In recent studies, the direct congestion management method is combined with either the price-based method or incentive-based method. A reconfiguration-based DT model was proposed in [21] to incorporate network reconfiguration into the DT method. The DTs contains both network constraint information and line loss information, which helps alleviate congestion and reduce line losses. In [27], network reconfiguration was considered as a backup option if the line shadow price (LSP) method is failed. A hierarchical congestion management scheme was developed in [38], in which network reconfiguration is combined with re-profiling products (SRPs and CRPs) in the tertiary control layer to alleviate congestion. In addition, direct active power control was used in [13] as a backup tool for the market-based method in which customers adjust their initial load profiles in response to dynamic price signals sent by the aggregators.

However, integrating the price-based method (DT) and incentive-based method (re-profiling product) for congestion management has not been studied. There are a few possible advantages for the integration, including 1) the DSO does not necessarily need to impose very high DTs to alleviate congestion because it can purchase flexibility products to alleviate part of congestion; and 2) the DSO and aggregators can share the responsibility of congestion management because they pay congestion management costs together. Therefore, a comprehensive day-ahead congestion management scheme based on the DT, network reconfiguration, and re-profiling product is proposed in this chapter.

The contributions of this chapter are summarized as follows: 1) propose a relaxed reconfiguration-based DT model that can ensure a feasible solution in case the original unrelaxed DT model is infeasible and can limit DTs to be below a preset threshold; 2) consider both the line loss reduction and voltage magnitude constraints in the proposed DT model; and 3) propose a comprehensive scheme to combine the price-based method, direct control method, and incentive-based method to effectively alleviate day-ahead congestion.

This chapter is organized as follows. Section 2.2 describes concepts of the DT, network reconfiguration, and re-profiling product, and illustrates the framework of

the comprehensive scheme. Section 2.3 presents model formulations and the algorithm flowchart of the comprehensive scheme. In section 2.4, case studies are given and discussed, followed by a summary in section 2.5.

2.2 Comprehensive Congestion Management Scheme

This section firstly describes the DT, network reconfiguration, and re-profiling product for alleviating congestion. Then, a comprehensive congestion management scheme is illustrated.

2.2.1 Dynamic Tariff Method

The DT method as the focus of this thesis is a decentralized market-based congestion management method. According to [20], the basic procedure of the DT method to alleviate day-ahead congestion is shown in Fig. 2-1.

Fig. 2-1 Basic procedure of the DT method to alleviate day-ahead congestion

Firstly, the DSO and aggregators obtain necessary data. The DSO obtains the grid model and forecasted spot prices. Moreover, the DSO obtains flexible demand data from the aggregators or by its own predictions. Secondly, the DSO solves the DSO optimization model considering network constraints to obtain DTs ($E_i^T \rho_i$) that are sent to the aggregators. The DSO optimization model is formulated in (2.1)-(2.7) considering EVs and HPs as flexible demands. Thirdly, after receiving DTs, the aggregators solve their own optimization models based on the forecasted spot prices and DTs. The aggregator optimization model is formulated in (2.8)-(2.12). Finally, the aggregators submit the energy bids to the day-ahead energy market.

By comparing the KKT conditions of the DSO optimization model with the ones of the aggregator optimization model, it can be proven in Appendix A that the solution obtained at the aggregator side is the same as the one at the DSO side. Therefore, the energy plans made by the aggregators respect network constraints.

As discussed above, the DT method is carried out before day-ahead energy market clearing. In the DT method, the DSO is the only responsible actor for congestion management.

DSO optimization model:

$$\min \sum_{t \in \mathcal{N}^T} \left[\sum_{i \in \mathcal{N}^a} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_t \mathbf{1})^T p_{i,t} + \tilde{p}_{i,t}^T B_{i,t} \tilde{p}_{i,t} + (c_t \mathbf{1})^T \tilde{p}_{i,t} \right) \right], \quad (2.1)$$

$$GF_t^p = p_t^c + \sum_{i \in \mathcal{N}_a} E_i(p_{i,t} + \tilde{p}_{i,t}); \forall t \in \mathcal{N}^T \quad (\rho_t), \quad (2.2)$$

$$F_t^p \leq f_t^{\max}; \forall t \in \mathcal{N}^T \quad (\gamma_t), \quad (2.3)$$

$$e_{i,t}^{\min} \leq \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\mu_{i,t}^-, \mu_{i,t}^+), \quad (2.4)$$

$$K_{i,t}^{a,\min} \leq \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a \leq K_{i,t}^{a,\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\tilde{\mu}_{i,t}^-, \tilde{\mu}_{i,t}^+), \quad (2.5)$$

$$p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t} p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\sigma_{i,t}^-, \sigma_{i,t}^+), \quad (2.6)$$

$$\tilde{p}_{i,t}^{\min} \leq \tilde{p}_{i,t} \leq \tilde{p}_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\tilde{\sigma}_{i,t}^-, \tilde{\sigma}_{i,t}^+), \quad (2.7)$$

Objective function (2.1) is to minimize energy costs of EVs and HPs, where $\mathcal{N}^T$ is the set of planning periods, $\mathcal{N}^a$ is the set of aggregators, $p_{i,t} \in R^{m_i \times 1}$ is EV charging power, $\tilde{p}_{i,t} \in R^{m_i \times 1}$ is HP power consumption, c_t is the forecasted spot price, and $B_{i,t}$ is the price sensitivity coefficient matrix. Constraint (2.2) represents the active power balance equation, where p_t^c is the active demand, E_i is the customer to load bus mapping matrix, G is the load bus to line incidence matrix, and F_t^p represents line power flow. Constraint (2.3) is the line loading constraint, where $f_t^{\max}$ is the

line capacity. Constraints (2.4) is the EV SOC level constraint, where $d_{i,t}$ is EV power consumption due to driving, $e_{i,0}$ is the initial SOC level, and $e_{i,t}^{\min}$ and $e_{i,t}^{\max}$ are the minimum and maximum limits of the SOC level. Constraint (2.5) is the household inside temperature constraint, where $A_{i,t,-}$ is the power to temperature matrix, $K_{i,0}^a$ is initial household inside temperature, and $K_{i,t}^{a,\min}$ and $K_{i,t}^{a,\max}$ are the minimum and maximum limits of household inside temperature. Constraint (2.6) is the EV charging power constraint, where $a_{i,t}$ is the EV availability, and $p_{i,t}^{\min}$ and $p_{i,t}^{\max}$ are the minimum and maximum limits of EV charging power. Constraint (2.7) is the HP power consumption constraint, where $\tilde{p}_{i,t}^{\min}$ and $\tilde{p}_{i,t}^{\max}$ are the minimum and maximum limits of HP power consumption. The dual variables of each constraint are provided in its right-side bracket.

Aggregator optimization model:

$$\begin{aligned} \min \quad & \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_t \mathbf{1} + E_i^T \rho_t)^T p_{i,t} \right) \\ & + \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left(\tilde{p}_{i,t}^T B_{i,t} \tilde{p}_{i,t} + (c_t \mathbf{1} + E_i^T \rho_t)^T \tilde{p}_{i,t} \right), \end{aligned} \quad (2.8)$$

$$e_{i,t}^{\min} \leq \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\mu_{i,t}^-, \mu_{i,t}^+), \quad (2.9)$$

$$K_{i,t}^{a,\min} \leq \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a \leq K_{i,t}^{a,\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\tilde{\mu}_{i,t}^-, \tilde{\mu}_{i,t}^+), \quad (2.10)$$

$$p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t} p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\sigma_{i,t}^-, \sigma_{i,t}^+), \quad (2.11)$$

$$\tilde{p}_{i,t}^{\min} \leq \tilde{p}_{i,t} \leq \tilde{p}_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\tilde{\sigma}_{i,t}^-, \tilde{\sigma}_{i,t}^+), \quad (2.12)$$

2.2.2 Network Reconfiguration

Network reconfiguration is a direct congestion management method implemented by the DSO in different time frames, such as day-ahead and real-time time frames. Network reconfiguration is to change network topology to realize certain objectives, such as line loss reduction and congestion mitigation. By opening normally closed

sectionalizing switches and closing normally open tie-switches, power consumption can be shifted among feeders to alleviate congestion.

2.2.3 Re-Profiling Product

The DSO can procure flexibility service products in the day-ahead flexibility market to alleviate congestion. The day-ahead flexibility market is a flexibility trading platform. Different from the DT method, the aggregators gather flexibility from flexible demands and trade them in the flexibility market. Two types of flexibility service products (SRPs/CRPs), which will be used in this chapter, were defined in [37]. The SRP means that the aggregator has the obligation to produce or consume assigned power during an assigned period while CRP means that the aggregator must have the capacity to change assigned power at a designated period.

2.2.4 Framework of the Comprehensive Scheme

In this subsection, the framework of the comprehensive scheme is described. As mentioned previously, the above-mentioned three methods are implemented in the day-ahead time frame. The DT method is carried out to formulate the energy bids before the energy bidding process while the re-profiling products are used after the energy schedules are formulated. Thus, it is feasible to combine the DT method and re-profiling product procurement in sequence. Since a combined optimization of flexible demands and network topology can achieve an optimized solution for congestion management before the energy bidding process, network reconfiguration is incorporated into the DT framework. Based on the above analyses, the proposed comprehensive scheme consists of two steps: the DT and network reconfiguration are integrated to mitigate congestion before the energy bidding process at the first step, then the re-profiling products are traded to mitigate remaining congestion after formulating energy schedules at the second step.

There are three actors in the comprehensive scheme, including the DSO, aggregator, and market operator. The roles of these actors in the comprehensive scheme are described as follows,

- DSO: the DSO operates the distribution network in order to deliver electricity to customers in a cost-effective and secure way. In order to alleviate congestion using the comprehensive scheme, the DSO calculates DTs at the first step and procures flexibility service products at the second step.
- Aggregator: the aggregators act as mediators among the DSO, customer, and market operator. Although the aggregator has different roles in the DT method

and flexibility market, a generic aggregator concept was proposed in [37] to combine them. The generic aggregator has the following roles: 1) submit energy bids and purchase energy in the day-ahead energy market on behalf of flexible demands; 2) mobilize flexible demands and create optimal energy schedules for them; 3) gather flexibility from flexible demands and bid flexibility service products in the flexibility market; 4) manage flexible demands to provide flexibility services. In the scheme, each customer is assigned to an aggregator by a contract and can choose to be involved in either the DT method or the flexibility market. It is also assumed that each aggregator has contracts with a small portion of customers at each load bus so that the aggregators do not have market power.

- **Market operator:** there are two market operators in the proposed comprehensive scheme, including the day-ahead energy market operator and flexibility market operator. The day-ahead energy market operator clears the day-ahead energy market to formulate day-ahead energy schedules. The flexibility market operator runs the flexibility market as an independent entity and clears the flexibility market to obtain flexibility trading results to help the DSO mitigate day-ahead congestion.

The framework of the comprehensive scheme is illustrated in Fig. 2-2. All functionalities of the three actors (the DSO, aggregator and market operator) are illustrated in separate blocks. The procedures of using the comprehensive scheme to alleviate congestion in the day-ahead time frame are described as follows.

Firstly, the DSO obtains all necessary data. The DSO determines the optimal network topology and DTs. After receiving DTs from the DSO, the aggregators manage those flexible demands that are willing to respond to the DTs based on the final prices and submit the energy bids to the day-ahead energy market. At the same time, the aggregators gather flexibility from the flexible demands that are willing to provide flexibility, and then offer flexibility service bids to the day-ahead flexibility market. After the energy bidding process, the DSO validates if the proposed energy schedules lead to congestion. If congestion does not exist, the day-ahead energy market and day-ahead flexibility market are closed. Otherwise, the DSO will request the market agent to purchase flexibility service products in the flexibility market to alleviate congestion. After the flexibility market clears, the aggregators will be informed about the accepted flexibility products. As such, a final energy schedule is determined.

Fig. 2-2 Framework of the comprehensive scheme

2.3 Model Formulations of the Comprehensive Scheme

This subsection presents model formulations and the algorithm flowchart of the comprehensive scheme. It is assumed that EVs, HPs, and onsite DGs provide flexibility, and a part of EVs and HPs respond to DTs while the rest of them and onsite DGs provide flexibility.

2.3.1 Relaxed Reconfiguration-Based DT Model at the First Step

A relaxed reconfiguration-based DT model that can resolve the infeasible issue and limit the maximum DT is proposed.

2.3.1.1 Relaxed Reconfiguration-Based DSO Optimization Model

The relaxed reconfiguration-based DSO optimization is used to determine the optimal topology schedule and DTs. Compared with the DSO optimizations in [20], line loss reduction and voltage limits are included, and the auxiliary variables are introduced to relax the line loading and voltage magnitude constraints. These auxiliary variables ensure that the relaxed reconfiguration-based DSO optimization always has a feasible solution. Moreover, the penalty terms associated with the auxiliary variables in the objective function enable the DSO to decide how much the line loading and voltage magnitudes constraints should be relaxed. The relaxed reconfiguration-based DSO optimization is formulated as an MILP model, as below,

$$\min \sum_{i \in \mathcal{N}^T} \left[\sum_{i \in \mathcal{N}^a} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_t \mathbf{1})^T p_{i,t} + \tilde{p}_{i,t}^T B_{i,t} \tilde{p}_{i,t} + (c_t \mathbf{1})^T \tilde{p}_{i,t} \right), \quad (2.13) \right. \\ \left. \left(w_t^l \mathbf{1} \right)^T \alpha_t^l + \left(w_t^v \mathbf{1} \right)^T \alpha_t^v + l_t^{\text{loss}} \left(F_t^p \right)^T R F_t^p \right]$$

$$G F_t^p = p_t^c - p_t^g + \sum_{i \in \mathcal{N}_a} E_i \left(p_{i,t} + \tilde{p}_{i,t} \right); \forall t \in \mathcal{N}^T, \quad (2.14)$$

$$G F_t^q = q_t^c; \forall t \in \mathcal{N}^T, \quad (2.15)$$

$$\left| F_t^p \right| \leq f_t^{\max} + \alpha_t^l; \forall t \in \mathcal{N}^T \left(\pi_t^-, \pi_t^+ \right), \quad (2.16)$$

$$\left| \left\{ F_t^p \right\}_{jk} \right| \leq M x_{t,jk}; \forall jk \in \mathcal{L}^s, t \in \mathcal{N}^T, \quad (2.17)$$

$$\left| \left\{ F_t^q \right\}_{jk} \right| \leq M x_{t,jk}; \forall jk \in \mathcal{L}^s, t \in \mathcal{N}^T, \quad (2.18)$$

$$\left\{ V_t^{\text{sqr}} \right\}_j - \left\{ V_t^{\text{sqr}} \right\}_k = 2 \left(\left\{ R F_t^p \right\}_{jk} + \left\{ X F_t^q \right\}_{jk} \right); \forall jk \in \mathcal{L}^B, t \in \mathcal{N}^T, \quad (2.19)$$

$$\left\{ V_t^{\text{sqr}} \right\}_j - \left\{ V_t^{\text{sqr}} \right\}_k = 2 \left(\left\{ R F_t^p \right\}_{jk} + \left\{ X F_t^q \right\}_{jk} \right) + s_{t,jk}; \forall jk \in \mathcal{L}^s, t \in \mathcal{N}^T, \quad (2.20)$$

$$\left| s_{t,jk} \right| \leq M \left(1 - x_{t,jk} \right), \forall jk \in \mathcal{L}^s, t \in \mathcal{N}^T, \quad (2.21)$$

$$\underline{V}^{\text{sqr}} - \alpha_t^v \leq V_t^{\text{sqr}} \leq \bar{V}^{\text{sqr}} + \alpha_t^v; \forall t \in \mathcal{N}^T \left(\xi_t^-, \xi_t^+ \right), \quad (2.22)$$

$$e_{i,t}^{\min} \leq \sum_{t_- \leq t} \left(p_{i,t_-} - d_{i,t_-} \right) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \left(\mu_{i,t}^-, \mu_{i,t}^+ \right), \quad (2.23)$$

$$K_{i,t}^{a,\min} \leq \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a \leq K_{i,t}^{a,\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \left(\tilde{\mu}_{i,t}^-, \tilde{\mu}_{i,t}^+ \right), \quad (2.24)$$

$$p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t} p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \left(\sigma_{i,t}^-, \sigma_{i,t}^+ \right), \quad (2.25)$$

$$\tilde{p}_{i,t}^{\min} \leq \tilde{p}_{i,t} \leq \tilde{p}_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \left(\tilde{\sigma}_{i,t}^-, \tilde{\sigma}_{i,t}^+ \right), \quad (2.26)$$

$$\alpha_t^l \geq 0; \forall t \in \mathcal{N}^T \left(\beta_t^l \right), \quad (2.27)$$

$$\alpha_t^v \geq 0; \forall t \in \mathcal{N}^T \left(\beta_t^v \right), \quad (2.28)$$

$$GK_t = 1; \forall t \in \mathcal{N}^T, \quad (2.29)$$

$$\left| \{K_t\}_{jk} \right| \leq Mx_{t,jk}; \forall jk \in \mathcal{L}^S, t \in \mathcal{N}^T, \quad (2.30)$$

$$\sum_{jk \in \mathcal{N}_s} x_{t,jk} = n_v - 1; \forall t \in \mathcal{N}^T, \quad (2.31)$$

The objective function in (2.13) has three parts: the first four terms represent energy costs of EVs and HPs; the fifth and sixth terms represent penalty costs of constraint relaxation, where w_t^l and w_t^v are penalty coefficients, and α_t^l and α_t^v are auxiliary variables of the line loading constraint and voltage magnitude constraint, respectively; and the seventh term is the line loss cost, where l_t^{loss} is the line loss cost coefficient, F_t^p is active power flow, and R is the line resistance matrix. Constraints (2.14) and (2.15) represent active and reactive power flow balance equations, respectively, where F_t^q is reactive power flow, q_t^c is the reactive demand, and p_t^g is the active power generation. Constraint (2.16) is the relaxed line loading constraint. Constraints (2.17) and (2.18) force active and reactive power to be zero if the corresponding switch is open, where $\mathcal{L}^S$ is the set of lines with sectionalizing switches, M is a very big number, and $x_{t,jk}$ represents the status of the switch ij , $x_{t,jk} = 1$ if the switch ij is closed; otherwise, $x_{t,jk} = 0$. Constraint (2.19) represents voltage drops for those lines without switches, where $\mathcal{L}^B$ is the set of lines without switches, and $R+jX$ is the line impedance. Constraints (2.20) and (2.21) represent that voltage drop equations are canceled for the lines whose switches are open, where $s_{t,jk}$ is the auxiliary variable. Constraint (2.22) is the relaxed voltage magnitude constraint. Constraint (2.23) is the EV SOC level constraint. Constraint (2.24) is the household inside temperature constraint. Constraints (2.25) and (2.26) are the EV charging power constraint and HP power consumption constraint, respectively. Constraints (2.27) and (2.28) are non-negativity constraints of auxiliary variables. Constraint (2.29) forces all load points being connected by supplying a unit fictitious active power to each load point from the substation, where K_t is fictitious active power

flow. Constraint (2.30) forces fictitious active power flow to be zero if the corresponding switch is open. Constraints (2.14), (2.15) and (2.29)-(2.31) can ensure a connected radial topology of the network. Constraint (2.31) ensures that the number of edges of the resulting graph (after reconfiguration) is the number of vertexes n_v minus one. The dual variables of constraints are given in the brackets.

The DTs are calculated by solving the relaxed reconfiguration-based DSO optimization model and sent to the aggregators. The dual variables of an MILP problem can be derived based on the assumption that very small changes at the right sides of constraints will not change the discrete variables [21]. Suppose that the optimal solution $(p_{i,t}^*, \tilde{p}_{i,t}^*, x_{i,jk}^*)$ is obtained, the dual variables can be calculated from the KKT conditions of the relaxed DSO optimization with discrete variables being fixed as $x_{i,jk}^*$. In order to clearly explain the superior feature of the relaxed optimization model, the relaxed optimization model with fixed discrete variables $(x_{i,jk}^*)$ is transformed into an equivalent model. After obtaining the solution of discrete variables $(x_{i,jk}^*)$, the network model is known in each optimization period. Thus, the power flow equations (2.14)-(2.15) and (2.17)-(2.18) are transformed into (2.32)-(2.33) by using the power transfer distribution factor (PTDF) D_t . Equations (2.19)-(2.21) are transformed into (2.34)-(2.35) by using the voltage incident matrix H_t^v (a matrix describing the relation between nodal voltage and voltage drop ΔV_t^{sqf} of each line), where $V_t^{\text{b,sqf}}$ is the squared voltage magnitude at the substation node. The equivalent model is formulated as follows,

$$\min (2.13)$$

$$\text{subject to: (2.16), (2.22)-(2.28)}$$

$$F_t^p = D_t \left(p_t^c - p_t^g + \sum_{i \in \mathcal{N}^a} E_i (p_{i,t} + \tilde{p}_{i,t}) \right); \forall t \in \mathcal{N}^T (\lambda_t), \quad (2.32)$$

$$F_t^q = D_t q_t^c; \forall t \in \mathcal{N}^T, \quad (2.33)$$

$$\Delta V_t^{\text{sqf}} = 2(RF_t^p + XF_t^q); \forall t \in \mathcal{N}^T (\omega_t), \quad (2.34)$$

$$V_t^{\text{sqf}} = V_t^{\text{b,sqf}} - H_t^v \Delta V_t^{\text{sqf}}; \forall t \in \mathcal{N}^T (\gamma_t), \quad (2.35)$$

The dual variables can be obtained from the KKT conditions of the above equivalent model. The corresponding KKT conditions are as follows,

$$B_{i,t} p_{i,t} + c_t \mathbf{1} + E_i^T D_i^T \lambda_t + \sum_{t \leq t_+} (\mu_{i,t_+}^+ - \mu_{i,t_+}^-) + (\sigma_{i,t}^+ - \sigma_{i,t}^-) = 0, \quad (2.36)$$

$$B_{i,t} \tilde{p}_{i,t} + c_t \mathbf{1} + E_i^T D_i^T \lambda_t + \sum_{t \leq t_+} A_{i,t_+,t}^T (\tilde{\mu}_{i,t_+}^+ - \tilde{\mu}_{i,t_+}^-) + (\tilde{\sigma}_{i,t}^+ - \tilde{\sigma}_{i,t}^-) = 0, \quad (2.37)$$

$$l_t^{\text{loss}} R F_t^{\text{p}} - \lambda_t + \pi_t^+ - \pi_t^- + 2R^T \omega_t = 0, \quad (2.38)$$

$$\gamma_t + \xi_t^+ - \xi_t^- = 0, \quad (2.39)$$

$$-\omega_t + (H_t^v)^T \gamma_t = 0, \quad (2.40)$$

$$(w_t^l \mathbf{1}) - \pi_t^+ - \pi_t^- - \beta_t^l = 0, \quad (2.41)$$

$$(w_t^v \mathbf{1}) - \xi_t^+ - \xi_t^- - \beta_t^v = 0, \quad (2.42)$$

After calculating dual variables, the derived DTs, defined by $D_i^T \lambda_t$, will be sent to the aggregators. Then, each aggregator makes its own energy plans using the following optimization problem.

2.3.1.2 Aggregator Optimization

For aggregator i ,

$$\begin{aligned} \min \quad & \sum_{t \in \mathcal{N}^T} \left[\frac{1}{2} p_{i,t}^T B_{i,t} p_{i,t} + (c_t \mathbf{1} + E_i^T D_i^T \lambda_t)^T p_{i,t} \right. \\ & \left. + \frac{1}{2} \tilde{p}_{i,t}^T B_{i,t} \tilde{p}_{i,t} + (c_t \mathbf{1} + E_i^T D_i^T \lambda_t)^T \tilde{p}_{i,t} \right], \end{aligned} \quad (2.43)$$

subject to: (2.23)-(2.26).

The objective is to minimize the energy costs subject to constraints (2.23)-(2.26). Comparing the KKT conditions of the aggregator optimization with (2.36)-(2.42),

it is concluded that the optimal solutions $(p_{i,t}^*, \tilde{p}_{i,t}^*)$ at the DSO side can be obtained at the aggregator side by independent optimizations.

2.3.1.3 Existence of Upper Limit of DTs

In the KKT conditions (2.36)-(2.42), the Lagrange multipliers, $\pi_t^-, \pi_t^+, \xi_t^-, \xi_t^+, \mu_{i,t}^-, \mu_{i,t}^+, \tilde{\mu}_{i,t}^-, \tilde{\mu}_{i,t}^+, \sigma_t^+, \sigma_t^-, \tilde{\sigma}_t^+, \tilde{\sigma}_t^-, \beta_t^l, \beta_t^v$, all the terms of matrix D_t, H_t^v, R , cost coefficient l_t^{loss} , and penalty coefficients w_t^l, w_t^v are all non-negative. According to (2.38)-(2.40), the derived DTs satisfy the following equation:

$$D_t^T \lambda_t = D_t^T l_t^{\text{loss}} R F_t^p + D_t^T (\pi_t^+ - \pi_t^-) + D_t^T \left(2R^T (H_t^v)^T (\xi_t^- - \xi_t^+) \right), \quad (2.44)$$

It is obvious that the DTs are determined by three parts that are associated with the line loss, line loading constraint, and voltage magnitude constraint, respectively. If the line loading and voltage magnitude constraints do not bind, the corresponding part in (2.44) is zero. The existence of the upper limit of DTs is described as follows.

According to (2.44), the following inequality is satisfied,

$$|\lambda_t| \leq l_t^{\text{loss}} R |F_t^p| + (\pi_t^+ + \pi_t^-) + 2R^T (H_t^v)^T (\xi_t^- + \xi_t^+), \quad (2.45)$$

According to (2.41), the following inequality is obtained,

$$\pi_t^+ + \pi_t^- \leq (w_t^l \mathbf{1}), \quad (2.46)$$

According to (2.42), the following inequality is obtained,

$$\xi_t^- + \xi_t^+ \leq (w_t^v \mathbf{1}), \quad (2.47)$$

In addition, the EV charging power constraint (2.25) and HP power consumption (2.26) ensure that line loading has the maximum limit $F_t^{\text{p,lim}}$. Therefore, the following inequality is satisfied,

$$|F_t^p| \leq F_t^{\text{p,lim}}, \quad (2.48)$$

According to (2.45)-(2.48), the DTs satisfy the following inequality,

$$\left| D_t^T \lambda_t \right| \leq D_t^T l_t^{\text{loss}} R F_t^{\text{p,lim}} + D_t^T \left(w_t^l \mathbf{1} \right) + 2 D_t^T R^T \left(H_t^v \right)^T \left(w_t^v \mathbf{1} \right), \quad (2.49)$$

Thus, the DTs have an upper bound that is determined by parameters D_t , R , H_t^v , $F_t^{\text{p,lim}}$, l_t^{loss} , and penalty coefficients w_t^l and w_t^v . Before solving the relaxed reconfiguration-based DSO optimization, the DSO knows all possible solutions of network reconfiguration, which means that the DSO knows all possible D_t and H_t^v in (2.49). In addition, the maximum loading limit $F_t^{\text{p,lim}}$, matrix R , and coefficient l_t^{loss} are known parameters for the DSO. Based on the above analyses, the DSO can preset the maximum limit of DTs by choosing appropriate penalty coefficients. The DSO can set penalty coefficients by investigating the acceptable maximum DTs of customers. In addition, according to (2.46)-(2.48), it can be concluded that each part of DTs can be regulated separately.

2.3.2 Market Agent Model at the Second Step

If congestion cannot be completely alleviated at the first step, flexibility service products (SRPs/CRPs) are used to alleviate the remaining congestion. Since the focus is on the day-ahead time frame, SRPs are used only. As assumed in [37], the SRP is characterized by time of service, location (node) of service, type of service, volume (p_t^{fl}), and price (π_t^{fl}). Regarding the flexibility bidding and pricing, assumptions are made according to the discussions in [99]. The SRP bid is in a stepwise form in the day-ahead flexibility market, which is similar to the generation bidding in the day-ahead energy market.

After receiving a request for purchasing flexibility to alleviate congestion, the market agent helps the DSO procure flexibility service products with minimum costs. The optimization model is formulated as follows,

$$\min \sum_{t \in \bar{\mathcal{N}}^T} \sum_{i \in \bar{\mathcal{N}}^a} \sum_{g \in \bar{\mathcal{N}}} \sum_{f \in P_t^{(i,g)}} \left\{ \pi_t^{\text{fl}} \right\}_{i,g,f} \left\{ p_t^{\text{fl}} \right\}_{i,g,f} \left\{ z_t^{\text{pur}} \right\}_{i,g,f}, \quad (2.50)$$

$$F_t^{\text{p}} = D_t \left(\bar{p}_t^{\text{load}} - \bar{p}_t^{\text{gen}} \right); \forall t \in \bar{\mathcal{N}}^T, \quad (2.51)$$

$$F_t^{\text{q}} = D_t \left(q_t^{\text{c}} \right); \forall t \in \bar{\mathcal{N}}^T, \quad (2.52)$$

$$\left| F_t^{\text{p}} \right| \leq f_t^{\text{max}}; \forall t \in \bar{\mathcal{N}}^T, \quad (2.53)$$

$$\Delta V_t^{\text{sqr}} = 2(RF_t^{\text{p}} + XF_t^{\text{q}}); \forall t \in \bar{\mathcal{N}}^{\text{T}}, \quad (2.54)$$

$$V_t^{\text{sqr}} = V_t^{\text{b,sqr}} - H_t \Delta V_t^{\text{sqr}}, \forall t \in \bar{\mathcal{N}}^{\text{T}}, \quad (2.55)$$

$$\underline{V}^{\text{sqr}} \leq V_t^{\text{sqr}} \leq \bar{V}^{\text{sqr}}, \forall t \in \bar{\mathcal{N}}^{\text{T}}, \quad (2.56)$$

$$\{z_t^{\text{pur}}\}_{i,g,f} \in \{0,1\}, \forall i \in \mathcal{N}^{\text{a}}, g \in \mathcal{N}, f \in P_t^{(i,g)}, \quad (2.57)$$

$$\sum_{f \in P_t^{(i,g)}} \{z_t^{\text{pur}}\}_f \leq 1, \forall i \in \mathcal{N}^{\text{a}}, g \in \mathcal{N}, \quad (2.58)$$

The objective function (2.50) minimizes the procurement cost of flexibility service products, where $\bar{\mathcal{N}}^{\text{T}}$ is the set of planning periods for market agent, $P_t^{(i,g)}$ is the set of offered SRPs provided by i -th aggregator at node g in period t , π_t^{fi} is the volume of flexibility provided in the SRP, p_t^{fi} is the price of the SRP, and z_t^{pur} represents the purchase status of the SRP, z_t^{pur} is one if the corresponding SRP is purchased; otherwise, z_t^{pur} is zero. Constraints (2.51) and (2.52) are power balance equations, where $\bar{p}_t^{\text{gen}}$ and $\bar{p}_t^{\text{load}}$ are, respectively, the active generation and active demand after providing flexibility. Constraint (2.53) represents the line loading constraint. Constraints (2.54) and (2.55) calculate nodal voltage magnitudes. Constraint (2.56) is the voltage magnitude constraint. The binary nature of z_t^{pur} is defined in (2.57). Constraint (2.58) represents that the number of purchased SRPs of each aggregator at each node in each period should be one at most.

2.3.3 Algorithm Flowchart of the Comprehensive Scheme

The algorithm of the comprehensive scheme includes the reconfiguration-based DT (R-DT) algorithm and market agent algorithm. The algorithm flowchart of the comprehensive algorithm is illustrated in Fig. 2-3.

Fig. 2-3 The algorithm flowchart of the comprehensive scheme.

The procedure of the algorithm is described as follows. At first, the algorithm reads input data. Then, it runs the R-DT algorithm. The R-DT algorithm first reads necessary data, such as predicted spot prices and topology information, and then solves the relaxed reconfiguration-based DSO optimization. Finally, the optimal topology schedule and DTs are obtained. With these DTs, it solves the aggregator optimization model and formulates the energy schedules. After this, the power flow study is conducted to check if the formulated energy schedules result in congestion. If congestion does not exist, the algorithm terminates; otherwise, a request signal is sent to the market agent. The market agent algorithm reads SRP bid data, optimal topology schedule data and other necessary data. Then, the market agent optimization model is solved to obtain SRP trading results. If congestion is completely resolved after purchasing SRPs, the selected SRP bids and proposed energy schedules are combined to create the final energy schedules; otherwise, output an alarm signal.

2.4 Case Studies

In order to validate the effectiveness of the proposed comprehensive scheme for congestion management of ADNs, three case studies were conducted using the Danish driving pattern [101] and the modified Bus 4 distribution network of the Roy Billinton Test System (RBTS) [102].

2.4.1 Grid Data and Simulation Parameters

Fig. 2-4 shows the modified single line diagram of the Bus 4 distribution network. Line segments of feeders are labeled as L1-L53, e.g., L2, L4, L6, L8, L9, L11, L12, and L13 are the transformers connecting the load points LP₁-LP₇ and DG1. In addition, the equivalent graphic diagram of the network is shown in Fig. 2-5, where the dotted lines indicate that the corresponding switches are open.

The detailed data of load points, key parameters of EVs and HPs, and forecast spot prices can be found in Appendix B. It is assumed that each residential customer owns one EV and one HP and has contracts with two aggregators (AGGs). In the three case studies, AGG1 has contracts with 80 customers per load point, DG5 and DG6 while AGG2 has contracts with the rest 120 customers per load point, DG1-DG4. Moreover, 80 percent of EVs and HPs at each load point react to DTs, and the rest EVs, HPs and all DGs provide SRPs. The lower voltage magnitude limit is set as 0.952 p.u. in order to reserve a small security margin compared to the physical limit (0.94 p.u.) to consider approximated errors of voltage magnitudes. DG parameters, line loading limits, and line impedance are listed in Table 2-1. The simulations were carried out using the Gurobi solver [103].

Fig. 2-4 The modified Bus 4 distribution system of the RBTS

Fig. 2-5 Equivalent graphic diagram

Table 2-1 DG parameters, line loading limits, and line impedance

parameters	value
Active power capacity of DG1	500kW
Active power capacity of DG2, 3, 4, 5, 6	300kW
Line loading limit of L1: Case1 and 2/Case3	8000/ 7300kW
Line loading limit of L14: Case1 and 2/Case3	8300/ 8100kW
Line loading limit of L28: Case1 and 2/Case3	8900/ 9500kW
Line loading limit of L41: Case1 and 2/Case3	8000/ 7900kW
Resistance of the lines	0.26 ohm/km
Reactance of the lines	0.037 ohm/km

2.4.2 Case Study Results

In Case 1, the advantage of integrating network reconfiguration with the DT method is verified. Case 2 demonstrates that the relaxed reconfiguration-based DT model can limit the maximum DT. Case 3 demonstrates that the relaxed reconfiguration-based DT model can ensure a feasible solution in case the original unrelaxed DT model is infeasible.

2.4.2.1 Case1

In this case, the DT model without considering network reconfiguration does not have a feasible solution for congestion management. After solving the unrelaxed reconfiguration-based DSO optimization model (removing auxiliary variables α_t^l and α_t^v from the relaxed reconfiguration-based DSO optimization model), the resulting line loadings on L1, L14, L28 and L41 at critical hours are shown in Fig. 2-6 (d_ev and d_hp are EVs and HPs that react to DTs, p_loads are EVs and HPs that provide SRPs, e_loads are equivalent conventional loads). It is shown that all line loadings do not exceed limits. It is shown in Fig. 2-7 that network topology changes frequently in order to achieve a feasible solution for congestion management. Therefore, it is concluded that the integration of the DT and network reconfiguration can achieve a better solution for congestion management. In addition, the DTs are listed in Table 2-2. Since line loading on L14 reaches the limit at t_{18} and t_{19} , high DTs (see case1 in Table 2-2) appear at LP₁₆. Moreover, since the line loss is considered in the objective function, DTs are not zero even if there is no congestion, e.g., LP₂₉-LP₃₇ at t_{19} .

After receiving DTs from the DSO, the aggregators solve their own optimization models based on the DTs and spot prices. As shown in Fig. 2-8, the resulting line loadings at the aggregator side are the same as the ones at the DSO side, which demonstrates that the decentralized congestion management manner is achieved through the DTs. It is shown in Fig. 2-9 that the voltage magnitude of the critical bus LP₄₀ is above the minimum limit, and the maximum approximation error is around 1%.

Fig. 2-6 Line loadings of the unrelaxed reconfiguration-based DSO optimization in case1.

Fig. 2-7 Optimal topology schedule of the network in the planning horizon.

Fig. 2-8 Line loadings of the aggregator optimization with DTs in case1.

Table 2-2 A Part of DTs in three cases

LP	Case1		Case2		Case3	
time	t_{18}	t_{19}	t_{18}	t_{19}	t_{18}	t_{19}
16	2.5758	6.3251	0.9251	2.5013	0.0012	0.0011
19	2.5765	6.3259	0.9259	2.5022	0.0019	0.0017
21	2.5773	6.3266	0.9266	2.5031	0.0026	0.0023
22	2.5773	6.3267	0.9267	2.5031	0.0027	0.0023
24	2.5777	6.3271	0.9270	2.5037	0.0030	0.0026
26	2.5777	6.3273	0.9270	2.5040	0.0030	0.0026
27	2.5777	6.3273	0.9270	2.5040	0.0030	0.0026
29	7.4928	0.0012	2.5014	0.0012	0.4192	0.0014
31	7.4938	0.0020	2.5025	0.0020	0.4202	0.0023
32	7.4937	0.0019	2.5024	0.0020	0.4202	0.0023
34	7.4944	0.0024	2.5031	0.0025	0.4209	0.0030
35	7.4945	0.0025	2.5032	0.0025	0.4210	0.0030
37	7.4948	0.0026	2.5035	0.0027	0.4213	0.0033
39	7.4948	6.3274	2.5036	2.5042	0.4214	0.0033
40	7.4949	6.3274	2.5036	2.5044	0.4214	0.0033
43	5.3677	0.0011	2.5012	2.5011	3.5012	3.5016
46	5.3686	0.0018	2.5020	2.5018	3.5019	3.5018
55	5.3694	0.0024	2.5027	2.5025	3.5033	3.5030

Fig. 2-9 Voltage profile of LP40 in case 1

2.4.2.2 Case2

In case 1, congestion can be alleviated by the reconfiguration-based DT method. However, as shown in Table 2-2 (case1), the maximum DT is 7.4949 DKK/kWh at bus LP₄₀ at t_{18} , which denotes that customers face very high DTs. In order to resolve this issue, the relaxed reconfiguration-based DT model is solved. The maximum limit of DTs is set as 3 DKK/kWh in this case. Given that the voltage magnitudes are not violated, the third part of the DT in (2.44) is zero. Therefore, there is no need to relax voltage magnitude constraints and the penalty coefficient w_t^v is set as a very big number, e.g., $1e10$. Since the line loss contributes a very small portion of DTs (the reason is that the upper bound of the first part of DTs ($D_{it}^{\text{loss}} RF_t^{\text{p,lim}}$) is very small), the high DTs are caused by the line loading constraint and should be restricted by w_t^l . Thus, we set w_t^l to be 2.5 to restrict the maximum limit of the second part of DTs.

After solving the relaxed reconfiguration-based DT model, it is shown in Fig. 2-10 that line loadings on L14, L18 and L41 exceed the limits at t_{18} and t_{19} . The overloading amounts are listed in Table 2-3. As shown in Table 2-2 (case2), the maximum DT (2.5044 DKK/kWh) is restricted.

Fig. 2-10 Line loadings of the relaxed reconfiguration-based DSO optimization in case 2.

Table 2-3 Overloading amounts after the first step

unit (kW)		congestion lines		after first step	
time				t_{18}	t_{19}
Case2	L1			-	-
	L14			-	299.30
	L28			301.30	-
	L41			185.22	68.10
Case3	L1			425.74	238.97
	L14			-	-
	L28			-	-
	L41			134.47	58.96

Then, the market agent algorithm is implemented to alleviate remaining congestion. Several assumptions are made in the study: 1) the aggregators gather flexibility from EVs, HPs and DGs, and sent SRPs bids to the day-ahead flexibility markets; 2) two types of SRPs are offered, namely power consumption decrease (Type I) and generation increase (Type II). For example, Table 2-4 lists the SRP bids at t_{18} . The market agent model is solved to mitigate unsolved congestion on L28 and L41

at t_{18} with minimized SRP procurement costs. The selected SRP bids are highlighted in bold in Table 2-4. After purchasing SRPs, line loadings on L28 and L41 are below the limits as shown in Fig. 2-11. Therefore, the comprehensive scheme can successfully alleviate congestion without incurring very high DTs.

Table 2-4 Offered SRP bids at t_{18} in Case 2

LP	type	AGG 1				AGG 2			
		volume (kW)		price (DKK/kWh)		volume (kW)		price (DKK/kWh)	
29	I	20	30	0.44	0.49	30	40	0.45	0.48
31	I	12	20	0.35	0.42	18	30	0.37	0.44
32	I	20	30	0.43	0.46	30	40	0.44	0.49
34	I	16	28	0.38	0.44	24	36	0.40	0.46
35	I	20	28	0.42	0.49	30	42	0.44	0.51
37	I	12	22	0.37	0.43	18	33	0.39	0.45
42	II	-	-	-	-	60	80	0.79	0.88
43	I	15	24	0.37	0.41	18	24	0.39	0.41
45	II	60	70	0.77	0.85	-	-	-	-
46	I	12	16	0.37	0.39	17	36	0.37	0.48
48	I	15	28	0.37	0.41	19	36	0.39	0.48
49	I	13	25	0.35	0.41	19	24	0.40	0.43
51	II	40	50	0.57	0.65	-	-	-	-
52	I	12	21	0.35	0.42	24	38	0.40	0.47
55	I	11	25	0.34	0.43	17	20	0.39	0.41

Fig. 2-11 Line loadings before and after applying market agent algorithm in case 2.

2.4.2.3 Case 3

In this case, the unrelaxed reconfiguration-based DT model is infeasible when more severe congestion appears. However, the relaxed DT model has a feasible DT solution and the proposed comprehensive scheme can effectively alleviate such severe congestion.

Under the severe congestion situation, it is acceptable to set a higher limit for DTs. In this case, the maximum limit of DTs is set as 4 DKK/kWh, and the penalty coefficient w_t^l and w_t^y are set as 3.5 and 10, respectively. After solving the relaxed reconfiguration-based DT optimization model, it is shown in Fig. 2-12 that congestion is not completely alleviated on L1 and L41 at t_{18} and t_{19} . As shown in Table 2-2 (case3), the maximum DT (3.5033 DKK/kWh) is restricted. Due to unsolved congestion, SRPs are used. The selected SRP bids at t_{18} are highlighted in bold in Table 2-5. Finally, line loadings on L1 and L41 are below the loading limits, as shown in Fig. 2-13. In addition, it is shown in Fig. 2-14 that the voltage profile of the critical bus LP₄₀ is above the minimum limit. It is concluded that the relaxed DT model can ensure a feasible DT solution and the proposed comprehensive scheme can alleviate congestion efficiently.

Fig. 2-12 Line loadings of the relaxed reconfiguration-based DSO optimization in case 3.

Fig. 2-13 Line loadings before and after applying market agent algorithm in case 3.

Table 2-5 Offered scheduled re-profiling products at t_{18} in case 3

LP	type	AGG 1				AGG 2			
		volume (kW)		price (DKK/kWh)		volume (kW)		price (DKK/kWh)	
1	I	20	30	0.41	0.49	35	40	0.48	0.54
2	I	15	20	0.38	0.42	18	30	0.37	0.44
3	I	25	30	0.42	0.46	28	40	0.42	0.53
4	I	23	30	0.39	0.45	24	36	0.40	0.46
6	I	25	35	0.44	0.49	32	42	0.45	0.51
8	II	-	-	-	-	80	100	0.82	0.91
42	II	-	-	-	-	60	80	0.79	0.88
43	I	18	24	0.39	0.41	18	24	0.39	0.41
45	II	50	70	0.67	0.89	-	-	-	-
46	I	12	16	0.37	0.39	19	36	0.38	0.48
48	I	10	28	0.35	0.44	19	37	0.39	0.53
49	I	15	25	0.39	0.41	22	24	0.40	0.41
51	II	40	50	0.57	0.65	-	-	-	-
52	I	12	21	0.36	0.42	27	38	0.42	0.47
55	I	11	25	0.34	0.43	18	20	0.39	0.41

Fig. 2-14 Voltage profile of LP₄₀ in case 3.

2.5 Summary

In this chapter, a comprehensive congestion management scheme is proposed to combine the DT, network reconfiguration, and re-profiling product to alleviate day-ahead congestion. In the comprehensive scheme, network reconfiguration is integrated with the DT method to alleviate congestion at the first step. Then, re-profiling products are used at the second step to alleviate unsolved congestion. Particularly, a relaxed reconfiguration-based DT optimization model is formulated to resolve the possible infeasible issue and regulate DTs. The numerical results show that the proposed comprehensive scheme can utilize flexibility from EVs, HPs and DGs to effectively alleviate day-ahead congestion and can ensure that DTs are below a preset threshold.

CHAPTER 3.

COORDINATION OF THE DYNAMIC TARIFF AND INCENTIVE-BASED METHODS FOR DAY-AHEAD CONGESTION MANAGEMENT

This chapter presents a coordination scheme of the DT and incentive-based methods to alleviate day-ahead congestion of the ADN while ensuring that the DSO does not have congestion management costs or revenues. This chapter is based on the published article 5 with changes to coherently fit into the framework of this thesis³.

3.1 Introduction

In chapter 2, the integration of the DT method and incentive-based method is investigated. An important question for the integration is to determine how congestion is distributed between the two methods. In the proposed comprehensive scheme, by setting proper penalty coefficients in the relaxed reconfiguration-based DT model, the maximum limits for DTs are set, which determines the percentage of congestion alleviated by the DT method. Then, the remaining congestion is alleviated by using re-profiling products. Therefore, the integration is based on independent optimizations of the DT and re-profiling product. In order to optimally use the synergy between the two methods, an integrated optimization needs to be studied.

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³ This chapter is based on published articles 5: **Feifan Shen**, Qiuwei Wu*, Xiaolong Jin, Menglin Zhang, Saeed Tei-mourzadeh, Osman Bulent Tor, "Coordination of dynamic tariff and scheduled reprofiling product for day-ahead congestion management of distribution networks," *International Journal of Electrical Power and Energy System*, vol. 135, pp. 107672, Feb. 2022.

In the integration of the DT method and incentive-based method, the DSO gains revenues in the DT method by collecting tariffs whereas it has costs for purchasing flexibility service products in the incentive-based method. In the proposed comprehensive scheme, whether the DSO has revenues or costs in the integration depends on the congestion situation. From the implementation point of view, there might be conflicts with the current DSO's regulations. For example, according to the Danish Energy Regulation Authority (DERA) regulations, the DSO's revenue should be restricted by a revenue cap [104]. Moreover, the DSO's costs for purchasing flexibility service products need to be recovered in an appropriate manner [105]. To avoid the potential conflicts, a straightforward way is to enable the DSO to be in a profit-neutral position in the integration to alleviate congestion. This means that the DSO does not have revenues or costs in terms of congestion management.

Considering the above-mentioned two aspects, a coordination scheme of the DT method and incentive-based method is proposed, in which the DSO is in a profit-neutral position through an integrated optimization of the DT and incentive-based methods. Firstly, the coordination problem is formulated as an MINLP model. Then, the non-linear constraints of the MINLP model are linearized, and the resulting model is solved by a PSO-based solution strategy. The contributions of this chapter are summarized as follows: 1) an MINLP model is formulated for the incentive-based method to determine re-profiling amounts and incentives; 2) a coordination scheme of the DT and incentive-based methods is proposed; and 3) a PSO-based solution strategy is proposed to solve the coordination model.

The rest of this chapter is organized as follows. Section 3.2 provides model formulations to calculate DTs and determine re-profiling amounts and incentives. Section 3.3 illustrates the framework of the coordination scheme and presents the coordination model. Case study results are provided and discussed in section 3.4, followed by a summary in section 3.5.

3.2 Model formulations

In this section, the model formulations of the DT and incentive-based methods are presented.

3.2.1 DT Model

In the DT method, the DTs are price signals that ensure the energy plans made at the aggregator side being the same as the ones at the DSO side. In this chapter, EVs

and HPs are considered as flexible demands to respond to DTs. The standard optimization model at the DSO side used to calculate DTs is formulated in (2.1)-(2.7) in section 2.2.1, chapter 2. In the DT model, the DTs are calculated using the Lagrange multipliers (ρ_t) obtained after solving the DT model. Since the DTs need to be used to represent congestion management costs in an explicit expression in the coordination model, a two-level DSO optimization model that calculates DTs without using the Lagrange multipliers is formulated as below,

Upper level:

$$\sum_{t \in \mathcal{N}^T} \rho_t, \quad (3.1)$$

$$GF_l^p = p_t^c + \sum_{i \in \mathcal{N}^a} E_i (p_{i,t} + \tilde{p}_{i,t}); \forall t \in \mathcal{N}^T, \quad (3.2)$$

$$F_t^p \leq f_t^{\max}; \forall t \in \mathcal{N}^T, \quad (3.3)$$

$$0 \leq \rho_t \leq \rho^{\max}; \forall t \in \mathcal{N}^T, \quad (3.4)$$

Lower level:

$$p_{i,t}, \tilde{p}_{i,t} \in \left\{ \begin{array}{l} \min \quad \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_t 1 + E_i^T \rho_t)^T \tilde{p}_{i,t} \right) \\ \quad + \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left(\tilde{p}_{i,t}^T B_{i,t} \tilde{p}_{i,t} + (c_t 1 + E_i^T \rho_t)^T \tilde{p}_{i,t} \right) \\ s.t. \quad e_{i,t}^{\min} \leq \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} \leq e_{i,t}^{\max}; \\ \quad \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\mu_{i,t}^-, \mu_{i,t}^+) \\ \quad K_{i,t}^{a,\min} \leq \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a \leq K_{i,t}^{a,\max}; \\ \quad \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\tilde{\mu}_{i,t}^-, \tilde{\mu}_{i,t}^+) \\ \quad p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t} p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\sigma_{i,t}^-, \sigma_{i,t}^+) \\ \quad \tilde{p}_{i,t}^{\min} \leq \tilde{p}_{i,t} \leq \tilde{p}_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \quad (\tilde{\sigma}_{i,t}^-, \tilde{\sigma}_{i,t}^+) \end{array} \right. , \quad (3.5)$$

The lower-level optimization model in (3.5) is to minimize energy costs of flexible demands based on spot prices and DTs, subject to the EV driving constraints and

HP thermal constraints (see section 2.2.1 in chapter 2 for detailed descriptions). In the upper-level optimization model, the objective function (3.1) is to minimize DTs. Constraint (3.2) is the active power balance equation. Constraint (3.3) is the line loading constraint. The DTs are constrained in constraint (3.4). The upper-level optimization model is to minimize DTs while ensuring the line loading constraints being satisfied with the solution $(p_{i,t}, \tilde{p}_{i,t})$ obtained from the lower-level model. Therefore, the two-level optimization model is to find such a set of minimum DTs, based on which the energy plans obtained by solving the lower-level optimization model, i.e., the aggregator model, satisfy line loading constraints.

3.2.2 Incentive-Based Model

In the incentive-based method, the DSO pays for flexibility to alleviate congestion, and the aggregator and customers are rewarded for providing flexibility. The incentive-based model is formulated as below,

$$\begin{aligned} \min \quad & \sum_{t \in \mathcal{N}^T} \left(\sum_{e \in \mathcal{N}^{\text{ev}}} (f_{e,t}^e)^2 / \pi_{e,t}^e + \sum_{h \in \mathcal{N}^{\text{hp}}} (f_{h,t}^h)^2 / \pi_{h,t}^h \right) \\ & \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left[(p_{i,t}^*)^T B_{i,t} p_{i,t}^* + (c_t + E_t^T \rho_t) p_{i,t}^* \right] , \\ & + \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left[(\tilde{p}_{i,t}^*)^T B_{i,t} \tilde{p}_{i,t}^* + (c_t + E_t^T \rho_t) \tilde{p}_{i,t}^* \right] \end{aligned} \quad (3.6)$$

$$\begin{cases} f_{e,t}^e = z_{e,t}^e \pi_{e,t} r_t^{\text{inc}}, \forall e \in \mathcal{N}^{\text{ev}}, t \in \mathcal{N}^T \\ f_{h,t}^h = z_{h,t}^h \pi_{h,t} r_t^{\text{inc}}, \forall h \in \mathcal{N}^{\text{hp}}, t \in \mathcal{N}^T \end{cases} \quad (3.7)$$

$$\begin{cases} p_{e,t}^e = p_{e,t}^{\text{e,init}} - f_{e,t}^e + \theta_{e,t}^e, \forall t \in \mathcal{N}^T \\ p_{h,t}^h = p_{h,t}^{\text{h,init}} - f_{h,t}^h + \theta_{h,t}^h, \forall t \in \mathcal{N}^T \end{cases} \quad (3.8)$$

$$\begin{cases} -M(1 - z_{e,t}^e) \leq \theta_{e,t}^e \leq M(1 - z_{e,t}^e), \forall e \in \mathcal{N}^{\text{ev}}, t \in \mathcal{N}^T \\ -M(1 - z_{h,t}^h) \leq \theta_{h,t}^h \leq M(1 - z_{h,t}^h), \forall h \in \mathcal{N}^{\text{hp}}, t \in \mathcal{N}^T \end{cases} \quad (3.9)$$

$$\begin{cases} p_{i,t}^* = \sum_{e \in \mathcal{N}_i^{\text{ev}}} p_{e,t}^e, \\ \tilde{p}_{i,t}^* = \sum_{h \in \mathcal{N}_i^{\text{hp}}} p_{h,t}^h, \end{cases} ; \forall i \in \mathcal{N}^a, \quad (3.10)$$

$$e_{i,t}^{\min} \leq \sum_{t_- \leq t} (p_{i,t_-}^* - d_{i,t_-}) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T, \quad (3.11)$$

$$K_{i,t}^{a,\min} \leq \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-}^* + K_{i,0}^a \leq K_{i,t}^{a,\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T, \quad (3.12)$$

$$p_{i,t}^{\min} \leq p_{i,t}^* \leq a_{i,t} p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T, \quad (3.13)$$

$$\tilde{p}_{i,t}^{\min} \leq \tilde{p}_{i,t}^* \leq \tilde{p}_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T, \quad (3.14)$$

$$GF_t^p = p_t^c + \sum_{i \in \mathcal{N}_a} E_i (p_{i,t}^* + \tilde{p}_{i,t}^*); \forall t \in \mathcal{N}^T \quad (\rho_t), \quad (3.15)$$

$$F_t^p \leq f_t^{\max}; \forall t \in \mathcal{N}^T, \quad (3.16)$$

The objective function (3.6) has three terms: the first term is to minimize the flexibility payment, where $\mathcal{N}^{\text{ev}}$ and $\mathcal{N}^{\text{hp}}$ are the sets of EVs and HPs, $f_{e,t}^e$ and $f_{h,t}^h$ are the amounts of flexibility provided by e -th EV and h -th EV at hour t , $\pi_{e,t}^e$ and $\pi_{h,t}^h$ are the response elastic coefficients of e -th EV and h -th HP, respectively; the last two terms are to minimize energy costs of flexible demands based on the final electricity prices (spot prices plus DTs), where $p_{i,t}^*$ and $\tilde{p}_{i,t}^*$ are the EV charging power and HP power consumption, respectively, after implementing the price-based method. Constraint (3.7) models the customer's linear response to the incentive, where r_t^{inc} is the incentive (flexibility price). It is assumed that each customer can choose whether to respond to the incentive, the customer responds if the corresponding binary variable $z_{e,t}^e$ or $z_{h,t}^h$ equals to one. The amounts of flexibility provided by EVs and HPs are calculated in (3.8) and (3.9), respectively. If the customer responds to the incentive, the amount of flexibility equals to initial power consumption ($p_{e,t}^{\text{e,init}}$, $p_{h,t}^{\text{h,init}}$) minus final power consumption ($p_{e,t}^e$, $p_{h,t}^h$), where $\theta_{e,t}^e$ and $\theta_{h,t}^h$ are the auxiliary variables. The aggregated EV charging power and HP power consumption are obtained in (3.10). The optimization model also subjects to EV driving constraints (3.11) and (3.13), household thermal constraints (3.12) and (3.14), power balance constraint (3.15), and line loading constraint (3.16).

3.3 Coordination Scheme of the DT and Incentive-Based Methods

In this section, the coordination model is formulated firstly. Then, the framework of the coordination scheme is illustrated.

3.3.1 Coordination Model

The coordination model ensures that the DSO does not have congestion management costs or revenues after implementing the DT and incentive-based methods. The coordination model is formulated as an MINLP model, as below,

$$\begin{aligned}
 \min \quad & w_1^d \left\| \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{N}^T} \rho_{n,t} \left[\sum_{i \in \mathcal{N}^a} E_{i,n} (p_{i,t}^* + \tilde{p}_{i,t}^*) + p_{n,t}^c \right] \right. \\
 & \left. - \sum_{t \in \mathcal{N}^T} \left(\sum_{e \in \mathcal{N}^{ev}} (f_{e,t}^e)^2 / \pi_{e,t}^e + \sum_{h \in \mathcal{N}^{hp}} (f_{h,t}^h)^2 / \pi_{h,t}^h \right) \right\| \\
 & + w_2^{DT} \left(\sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{N}^T} \rho_{n,t} \right) \\
 & + w_3^e \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left[p_{i,t}^{*,T} B_{i,t} p_{i,t}^* + \left(c_t + \sum_{n \in \mathcal{N}} E_{i,n} \rho_{n,t} \right) p_{i,t}^* \right] \\
 & + w_3^e \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left[\tilde{p}_{i,t}^{*,T} B_{i,t} \tilde{p}_{i,t}^* + \left(c_t + \sum_{n \in \mathcal{N}} E_{i,n} \rho_{n,t} \right) \tilde{p}_{i,t}^* \right]
 \end{aligned} \tag{3.17}$$

(3.4), (3.5) and (3.7)-(3.16)

The objective function (3.17) consists of four terms: the first term minimizes the difference between the DSO's revenues in the DT method and DSO's incentive payments in the incentive-based method; the second term minimizes DTs; and the last two terms minimize energy costs of flexible demands after the DT and incentive-based methods. Since the weight coefficient (w_1^d) of the first term is much larger than the ones of other terms (w_2^{DT} , w_3^e), the theoretical value of the first term is zero, which ensures that the DSO does not have congestion management revenues or costs.

The coordination model subjects to constraints (3.4) and (3.5) of the DT method and constraints (3.7)-(3.16) of the incentive-based method. It is noted that constraint (3.8) links the DT method and incentive-based method.

3.3.2 Framework of the Coordination Scheme

In the coordination scheme, the DT method and incentive-based method are carried out sequentially in the day-ahead time frame, as illustrated in Fig. 3-1. The procedures of the coordination scheme to alleviate day-ahead congestion are described as follows,

- 1) The DSO and aggregators obtain necessary data, such as spot prices, flexible demand energy requirements and customers' response elastic coefficients.
- 2) The DSO solves the coordination model to calculate DTs in the DT method and incentives in the incentive-based method. Then, the DSO sends DTs and incentives to the aggregators.
- 3) The DT method is carried out at the first step. Based on the DTs and spot prices, the aggregator solves the optimization model, i.e., the lower-level optimization model in (3.5), to make energy plans.
- 4) Then, the incentive-based method is implemented afterward. Based on the incentives and response elastic coefficients, each aggregator schedules contractual customers to provide flexibility.
- 5) After implementing the DT method and incentive-based method, the final day-ahead energy schedules are formulated. Then, the DSO will check if the final energy schedules lead to congestion.
- 6) Finally, the financial settlement is carried out.

Fig. 3-1 Framework of the coordination scheme

3.4 Model Reformulation and Solution Strategy

To make the coordination model computational tractable, non-linear constraints (3.5) and (3.7) of the MINLP model are firstly linearized. Although the MINLP

model with linear constraints can be further transformed into an MILP model by linearizing the objective function, it is still hard to solve the resulting MILP model with commercial solvers due to a large number of binary variables. Therefore, a PSO-based solution strategy is proposed to solve the MINLP model with linear constraints.

3.4.1 Model Reformulation

3.4.1.1 Transformation of the Lower-Level Optimization Model in (3.5)

In the lower-level optimization model, variables ρ_t are obtained from the upper-level optimization model and are regarded as parameters in the lower-level optimization model. Therefore, the lower-level optimization model in (3.5) is a convex model and can be replaced by its KKT conditions, as below,

$$B_{i,t} p_{i,t} + c_i \mathbf{1} + E_i^T \rho_t + \sum_{t \leq t_+} (\mu_{i,t_+}^+ - \mu_{i,t_+}^-) + (\sigma_{i,t}^+ - \sigma_{i,t}^-) = 0, \quad (3.18)$$

$$B_{i,t} \tilde{p}_{i,t} + c_i \mathbf{1} + E_i^T \rho_t + \sum_{t \leq t_+} A_{i,t_+,t}^T (\tilde{\mu}_{i,t_+}^+ - \tilde{\mu}_{i,t_+}^-) + (\tilde{\sigma}_{i,t}^+ - \tilde{\sigma}_{i,t}^-) = 0, \quad (3.19)$$

$$e_{i,t}^{\min} - \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) - e_{i,0} \leq 0 \perp \mu_{i,t}^- \geq 0, \quad (3.20)$$

$$\sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} - e_{i,t}^{\max} \leq 0 \perp \mu_{i,t}^+ \geq 0, \quad (3.21)$$

$$K_{i,t}^{a,\min} - \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} - K_{i,0}^a \leq 0 \perp \tilde{\mu}_{i,t}^- \geq 0, \quad (3.22)$$

$$\sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a - K_{i,t}^{a,\max} \leq 0 \perp \tilde{\mu}_{i,t}^+ \geq 0, \quad (3.23)$$

$$p_{i,t}^{\min} - p_{i,t} \leq 0 \perp \sigma_{i,t}^- \geq 0, \quad (3.24)$$

$$p_{i,t} - a_{i,t} p_{i,t}^{\max} \leq 0 \perp \sigma_{i,t}^+ \geq 0, \quad (3.25)$$

$$\tilde{p}_{i,t}^{\min} - \tilde{p}_{i,t} \leq 0 \perp \tilde{\sigma}_{i,t}^- \leq 0, \quad (3.26)$$

$$\tilde{p}_{i,t} - \tilde{p}_{i,t}^{\max} \leq 0 \perp \tilde{\sigma}_{i,t}^+ \geq 0, \quad (3.27)$$

However, constraints (3.18)-(3.27) are non-linear due to product terms. Therefore, constraints (3.18)-(3.27) are linearized by introducing the big-M method and can be replaced by linear inequality constraints in (3.28)-(3.35), as below,

$$\begin{cases} 0 \leq \mu_{i,t}^- \leq Mb_{i,t}^-; \\ 0 \leq -e_{i,t}^{\min} + \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} \leq M(1 - b_{i,t}^-); \end{cases} \quad (3.28)$$

$$\begin{cases} 0 \leq \mu_{i,t}^+ \leq Mb_{i,t}^+; \\ 0 \leq -\sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) - e_{i,0} + e_{i,t}^{\max} \leq M(1 - b_{i,t}^+); \end{cases} \quad (3.29)$$

$$\begin{cases} 0 \leq \tilde{\mu}_{i,t}^- \leq M\tilde{b}_{i,t}^-; \\ 0 \leq -K_{i,t}^{a,\min} + \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a \leq M(1 - \tilde{b}_{i,t}^-); \end{cases} \quad (3.30)$$

$$\begin{cases} 0 \leq \tilde{\mu}_{i,t}^+ \leq M\tilde{b}_{i,t}^+; \\ 0 \leq -\sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} - K_{i,0}^a + K_{i,t}^{a,\max} \leq M(1 - \tilde{b}_{i,t}^+); \end{cases} \quad (3.31)$$

$$\begin{cases} 0 \leq \sigma_{i,t}^- \leq Mh_{i,t}^-; \\ 0 \leq -p_{i,t}^{\min} + p_{i,t} \leq M(1 - h_{i,t}^-); \end{cases} \quad (3.32)$$

$$\begin{cases} 0 \leq \sigma_{i,t}^+ \leq Mh_{i,t}^+; \\ 0 \leq -p_{i,t} + a_{i,t} p_{i,t}^{\max} \leq M(1 - h_{i,t}^+); \end{cases} \quad (3.33)$$

$$\begin{cases} 0 \leq \tilde{\sigma}_{i,t}^- \leq M\tilde{h}_{i,t}^-; \\ 0 \leq -\tilde{p}_{i,t}^{\min} + \tilde{p}_{i,t} \leq M(1 - \tilde{h}_{i,t}^-); \end{cases} \quad (3.34)$$

$$\begin{cases} 0 \leq \tilde{\sigma}_{i,t}^+ \leq M\tilde{h}_{i,t}^+; \\ 0 \leq -\tilde{p}_{i,t} + \tilde{p}_{i,t}^{\max} \leq M(1 - \tilde{h}_{i,t}^+); \end{cases} \quad (3.35)$$

where $b_{i,t}^-$, $b_{i,t}^+$, $\tilde{b}_{i,t}^-$, $\tilde{b}_{i,t}^+$, $h_{i,t}^-$, $h_{i,t}^+$, $\tilde{h}_{i,t}^-$, and $\tilde{h}_{i,t}^+$ are auxiliary variables used in the big-M method for linearization.

3.4.1.2 Linearization of Non-Linear Constraint (3.7)

Constraint (3.7) is non-linear due to the product of a binary variable and a continuous variable. Similarly, constraint (3.7) can be linearized in (3.36) and (3.37) by introducing the big-M method, as below,

$$\begin{cases} 0 \leq f_{e,t}^e \leq Mz_{e,t}^e; \\ \pi_{e,t}r - M(1 - z_{e,t}^e) \leq f_{e,t}^e \leq \pi_{e,t}r; \end{cases}, \quad (3.36)$$

$$\begin{cases} 0 \leq f_{h,t}^h \leq Mz_{h,t}^h; \\ \pi_{h,t}r - M(1 - z_{h,t}^h) \leq f_{h,t}^h \leq \pi_{h,t}r; \end{cases}, \quad (3.37)$$

Finally, the original MINLP model is reformulated as an MINLP model with linear constraints, as below,

$$\text{Min (3.17)}$$

$$\text{s.t. (3.4), and (3.8)-(3.16), (3.28)-(3.37),}$$

It should be mentioned that although the above MINLP model can be linearized into an MILP model by linearizing the objective function (3.17), it is still hard to solve the resulting MILP model due to the large number of binary variables. Therefore, a PSO-based solution strategy is presented to solve the above MINLP model.

3.4.2 Solution Strategy

The algorithm flowchart of the PSO-based solution strategy is shown in Fig. 3-2. In the algorithm, a particle represents a DT solution. Firstly, for each particle, the linearized DT model is solved to obtain EV charging power and HP power consumption, which are regarded as parameters in the incentive-based model. Then, the linearized incentive-based model is solved to obtain final EV charging power and HP power consumption and incentive payments. Finally, the particle is evaluated by the fitness function defined in the objective function in (3.17). The particle is better if it has a smaller objective value. After evaluating all particles in a population, the population is updated using (3.38), and all updated particles are evaluated. Repeat the update and evaluation process until the termination conditions are

satisfied. The detailed descriptions of particle update can be found in [106]. It should be mentioned that, in order to ensure a feasible solution for the incentive-based model for different particles, the line capacity constraint (3.16) is relaxed and an additional relaxation penalty term is included in the objective function (3.6). Accordingly, the relaxation penalty term with a very big weight coefficient is included in the fitness function in (3.17) to exclude those DT solutions with which congestion cannot be alleviated.

$$\begin{cases} \mathbf{v}_i^k = w\mathbf{v}_i^{k-1} + c_1u_1(\mathbf{p}_i^{k-1} - \mathbf{x}_i^{k-1}) + c_2u_2(\mathbf{g}_i^{k-1} - \mathbf{x}_i^{k-1}); \\ \mathbf{x}_i^k = \mathbf{x}_i^{k-1} + \mathbf{v}_i^k; \end{cases} \forall i \in P, \quad (3.38)$$

where P is the set of particles; w , c_1 , u_1 , c_2 , and u_2 are population update parameters, $\mathbf{v}_i^k$ represents the update velocity of i -th particle at k -th iteration, $\mathbf{x}_i^k$ represents the position of i -th particle at k -th iteration, i.e., values of DTs, $\mathbf{p}_i^{k-1}$ represents the best position of i -th particle until $(k-1)$ -th iteration, $\mathbf{g}_i^{k-1}$ represents the best position of the population until $(k-1)$ -th iteration.

Fig. 3-2 Algorithm flowchart of the PSO-based solution strategy

3.5 Case Studies

3.5.1 Grid Data and Simulation Parameters

The effectiveness of the proposed coordination scheme for day-ahead congestion management of ADNs was demonstrated using the Bus 4 distribution network of the Roy Billinton Test System (RBTS) [106], as shown in Fig. 3-3. Line segments of feeder 1 are labeled as L1-L12 and load points of feeder 1 are labeled as LP1-LP7. The data of load points is provided in Table 3-1. Line loading limits of L2 and L7 are 1200 kW and 3000 kW, respectively. It is assumed that each customer at LP₁, LP₄ and LP₅ owns an EV and a HP and has contract with two aggregators (AGG). AGG 1 has contracts with 80 customers per load point and AGG 2 has contracts with 120 customers per load point. The key parameters of EVs and spot prices can be found in Appendix B. The weight coefficient w_1^d , w_2^{DT} , w_3^e are 10, 1000, and 1, respectively. In the PSO-based algorithm, a population has 5 particles

and 100 generations are used for particle update. Coefficient w is set as 1, and c_1 , u_1 , c_2 , and u_2 are set as 2. The simulations were carried out on GAMS with CPLEX solver [65].

Fig. 3-3 The diagram of the Bus 4 distribution network

Table 3-1 Load point data

load points (LPs)	customer type	peak conv. load [kW]	number of customers per LP
LP1-4,	residential	886.9	100
LP5,	residential	813.7	100
LP6-7, LP16-17,	commercial	415.0	10

3.5.2 Congestion Management Effectiveness

Before congestion management, it is shown in Fig. 3-4 that congestion occurs on L2 at t_{24} and on L7 at t_{18} . The overloading amounts are listed in Table 3-2. In order to alleviate congestion, the coordination scheme is carried out. The line loadings on L2 and L7 after implementing the DT method are shown in Fig. 3-5. As shown in Table 3-2, congestion on L2 and L7 is not completely alleviated. Moreover, new congestion occurs on L7 at t_{17} because the aggregators shift power consumption from t_{18} to t_{17} in order to alleviate congestion at t_{18} . The obtained DTs are listed in Table 3-3. Then, the incentive-based method is implemented to alleviate the unsolved congestion. The line loadings on L2 and L7 after the incentive-based method are shown Fig. 3-6, which demonstrates that the coordination scheme can effectively alleviate congestion. The incentives are 0.82 DKK, 13.73 DKK and 1.41 DKK at t_{17} , t_{18} , and t_{24} , respectively.

The DT method and proposed integration method in chapter 2 are carried out to compared with the coordination scheme. Both the DT method and integration method can alleviate congestion effectively. The obtained DTs in the DT method and integration method are listed in Table 3-4. In the DT method, there are high DTs at t_{18} , whereas the maximum DT is restricted in the integration method (the preset maximum DT limit is 1 DKK/kW). It can be seen that the integration method has higher DTs compared with the coordination scheme. This is because the integrated method would use DTs to alleviate congestion to the maximum extent without considering the coordination with the incentive-based method.

Fig. 3-4 Line loadings on L2 and L7 before congestion management

Fig. 3-5 Line loadings on L2 and L7 after implementing the DT method

Fig. 3-6 Line loadings on L2 and L7 after implementing the incentive-based method

Table 3-2 The overloading amounts of L2 and L7

unit [kW]	congestion lines	t_{17}	t_{18}	t_{24}
no management	L2	-	-	70.88
	L7	-	104.50	-
after DT method	L2	-	-	1.30
	L7	0.40	80.81	-

Table 3-3 DTs in the coordination scheme (unit: DKK)

load points	t_{17}	t_{18}	t_{24}
LP ₁	-	-	0.06956
LP ₄	0.02397	0.45000	-
LP ₅	0.02397	0.45000	-

Table 3-4 DTs in the DT method and integration method (unit: DKK)

	load points	t_{17}	t_{18}	t_{24}
DT method	LP ₁	-	-	0.06956
	LP ₄	0.40239	1.32729	-
	LP ₅	0.40239	1.32729	-
Integration method	LP ₁	-	-	0.06956
	LP ₄	0.26519	1.00000	-
	LP ₅	0.26519	1.00000	-

3.5.3 Financial Settlement

The DSO's revenues or costs in the DT method, integration method and proposed coordination method are listed in Table 3-5. The DSO in the DT method has the largest revenues by collecting tariffs. In the integration method, congestion is distributed between the DT and incentive-based methods. Therefore, the DSO's net revenue is smaller than the one in the DT method. In the coordination method, the difference between the DSO's revenue and cost is 0.012 DKK, which is very small and can be ignored. Therefore, the DSO is in a neutral profit position in the coordination scheme.

Table 3-5 The DSO's revenues or costs in the three methods

unit [DKK]	revenues in DT	costs in incentive-based method	net revenues
DT method	3857.387	-	3857.387
integration method	2842.451	1130.844	1711.607
coordination method	1113.268	1113.280	0.012

3.5.4 Performance of the PSO-based solution strategy

The PSO solution search process is shown in Fig. 3-7 and Fig. 3-8. It can be seen that the PSO-based algorithm can gradually obtain better solutions during the

search process. With the final DT solution, the difference between the DSO’s revenue and cost is very small (0.012DKK), which demonstrates that the PSO-based solution strategy can obtain a high-quality solution.

Fig. 3-7 Objection function value of the best solution during the solution search process

Fig. 3-8 The difference between the DSO’s revenue and cost obtained with the best solution during the solution search process

3.6 Summary

In this chapter, a coordination scheme of the DT method and incentive-based method was proposed to alleviate day-ahead congestion. In the coordination scheme, the DT method is used to alleviate part of congestion at the first step. Then, the remaining congestion is alleviated by the incentive-based method at the second step. The coordination problem is formulated as an MINLP model. Then, the model is linearized by applying linearization techniques and solved by the proposed PSO-based solution strategy. The numerical results demonstrate that the coordination scheme can efficiently resolve day-ahead congestion while ensuring that the DSO

is in a profit-neutral position and does not have congestion managements costs or revenues.

CHAPTER 4.

ROBUST DYNAMIC TARIFF METHOD FOR DAY-AHEAD CONGESTION MANAGEMENT OF ACTIVE DISTRIBUTION NETWORKS

This chapter presents a robust DT method to alleviate day-ahead congestion of ADNs, which can deal with the uncertainty in the DT framework. This chapter is based on the published article 6 with changes to coherently fit into the framework of this thesis⁴.

4.1 Introduction

The deterministic market-based methods have been used in chapters 2 and 3 to perform day-ahead congestion management of distribution networks. However, the uncertainty of DERs has significant impacts on the congestion management effectiveness and should be considered. The transmission-level congestion management methods considering the uncertainty such as uncertain renewable energy and loads have been widely studied. In [108], a distributionally robust model considering dynamic line ratings was developed for transmission line congestion management. The proposed method uses dynamic ratings on critical lines to keep the risk of overloading below a preset level considering thermal rating forecast errors. In [109], a stochastic congestion management method was developed to consider the power system component's uncertainty, such as the outage of generation units and transmission lines. In [110], EVs act as small storage systems to alleviate congestion,

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⁴ This chapter is based on published articles 6: Feifan Shen, QiuweiWu*, "Robust dynamic tariff method for day-ahead congestion management of distribution networks," *International Journal of Electrical Power and Energy System*, vol. 134, pp. 107366, Jan. 2022.

and the congestion management problem is formulated considering the uncertainties of wind power, solar power, EVs, and loads. The uncertainty is characterized by scenarios derived from the PDFs of uncertain variables.

The congestion management methods for ADNs considering the uncertainty have been proposed in [111]-[113]. In [111], an information gap decision theory (IGDT)-based congestion management method considering uncertain wind power was proposed. An IGDT-based robust model and an IGDT-based opportunistic model are formulated to make risk-averse and risk-seeking congestion management decisions, respectively. In [112], a robust optimization model was developed to optimize the ESS operation to alleviate congestion considering the uncertainties of ESS SOC level, flexible demands, and renewable energy. In [113], the day-ahead congestion management problem for distribution networks is formulated as a bi-level optimization model. Then the robust optimization theory is introduced to deal with the uncertainties of DG units and market prices.

This chapter aims to alleviate day-ahead congestion for ADNs using the DT method. In the DT method [19], [20], [22], [23], the DSO uses price signals instead of direct control commands to influence power consumption behavior to carry out congestion management in a decentralized manner. However, the uncertainty exists in the DT framework due to its decentralized nature. In the DT method, the DSO needs to predict energy requirements at the aggregator side such that the DSO and aggregators use the same energy requirement parameters in the optimization models such as the EV availability. However, the prediction is not perfect and would lead to a certain level of uncertainty in the DT method. The studies on the uncertainty in the DT framework are limited. In [22], a chance-constrained DT method was developed, in which the robustness guarantees that the congestion probability is under a certain level with a given confidence level. However, the PDFs of forecast errors must be known previously, which is not practical and may hinder the application of the congestion management method. In [23], the DTs are calculated using an iterative procedure between the DSO and aggregators, so the DSO does not need to predict energy requirement data at the aggregator side. However, the DSO and aggregators may have heavy communication burdens due to the iterative procedure, which may compromise the congestion management effectiveness. To overcome the above-mentioned issues, a robust DT method is proposed in this chapter. In the robust DT method, a three-level robust DT model is formulated to obtain a robust DT solution, based on which the energy plans made at the aggregator side satisfy the line loading limits even under the worst uncertainty realization scenario. The contributions of this chapter are summarized as follows: 1) a three-level robust DT

optimization model is formulated; and 2) an iterative procedure to solve the proposed three-level robust model is developed.

The rest of this chapter is organized as follows. Section 4.2 presents the model formulation of the deterministic DT method and describes the uncertainty in the DT framework. Section 4.3 presents the robust DT model formulation and solution strategy. Case study results are discussed in section 4.4, followed by a summary in section 4.5.

4.2 Model Formulations and Uncertainty of the DT Method

This section firstly formulates the deterministic DT model and then describes the uncertainty in the DT framework.

4.2.1 Deterministic DT Model

As mentioned previously in chapter 3.2.1, the DSO can solve a two-level optimization model to calculate DTs without calculating Lagrange multipliers. In this chapter, EVs are considered as flexible demands only. The two-level DSO optimization model in chapter 3 is adapted as below,

Upper level:

$$\min \sum_{t \in \mathcal{N}^T} \rho_t, \quad (4.1)$$

$$GF_t^p = p_t^c + \sum_{i \in \mathcal{N}^a} E_i p_{i,t}; \forall t \in \mathcal{N}^T, \quad (4.2)$$

$$x_{jk,t}^{\text{ld}} = \begin{cases} 1, & \{F_t^p\}_{jk} \geq \{f_t^{\max}\}_{jk}; \\ 0, & \{F_t^p\}_{jk} < \{f_t^{\max}\}_{jk}; \end{cases} \forall jk \in \mathcal{L}, t \in \mathcal{N}^T, \quad (4.3)$$

$$x^{\text{ld}} = \sum_{jk \in \mathcal{L}} \sum_{t \in \mathcal{N}^T} x_{jk,t} \left(\{F_t^p\}_{jk} - \{f_t^{\max}\}_{jk} \right), \quad (4.4)$$

$$x^{\text{ld}} \leq 0; , \quad (4.5)$$

$$\rho_t \geq 0; \forall t \in \mathcal{N}^T, \quad (4.6)$$

Lower level:

$$p_{i,t}, \tilde{p}_{i,t} \in \begin{cases} \min & \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_i \mathbf{1} + E_i^T \rho_t)^T p_{i,t} \right) \\ s.t. & e_{i,t}^{\min} \leq \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T, \\ & p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t} p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \end{cases} \quad (4.7)$$

The lower-level optimization model in (4.7) is to minimize the energy costs of EVs based on spot prices and DTs, subject to EV driving constraints. The objective function (4.1) of the upper-level optimization model minimizes DTs. Constraint (4.2) represents the active power balance equation. The overloading status ($x_{jk,t}^{\text{ld}}$) of each line at each hour is determined in constraint (4.3), $x_{jk,t}^{\text{ld}}=1$ if the line loading on the line jk exceeds its line capacity; otherwise $x_{jk,t}^{\text{ld}}=0$. The total overloading amount (x^{ld}) of all lines during the planning horizon is calculated in (4.4). Constraint (4.5) represents that no congestion occurs during the planning horizon. Constraint (4.6) is the non-negativity constraint of DTs. The upper-level optimization model is to minimize DTs which ensures the line loading limits being respected with the solution obtained from the lower-level model.

After the DSO solves the two-level optimization model and obtains DTs, the aggregator solves the aggregator optimization model, i.e., the lower-level optimization model in (4.7), to make energy plans.

4.2.2 Uncertainty in the DT framework

In the deterministic DT method, the DSO and aggregators use the same parameters, including the price sensitivity coefficient, spot prices, mapping coefficients, and EV driving parameters. A few consumptions are made: 1) the price sensitivity coefficient is used to resolve the multiple response issue, so the DSO will share it with the aggregators; 2) this chapter focuses on the forecast error of EV driving behavior, so the customer to bus load mapping coefficients and EV charging power limits should be reported to the DSO; 3) since the DT method is based on the price signal (spot price plus DTs), the DSO and aggregators are suggested to use the same forecasted spot prices, which can be realized by choosing the same third-party to provide forecast values; and 4) since this chapter investigates the discrepancy between the energy plans made at the DSO side and actual energy plans made at

the aggregator side due to the DSO's forecast error, the EV driving parameters used by the aggregators are assumed to be accurate.

In the two-level DSO optimization model, the uncertain parameters are EV power consumption $d_{i,t}$ and EV charging availability $a_{i,t}$. There are two ways to model uncertain parameters. One is to model uncertain parameters by scenarios derived from the PDFs of uncertain variables. The other one is to model uncertain parameters by polyhedral uncertainty bounds. The uncertain bounds are used to model uncertain parameters in this chapter since it can ensure the solution is feasible for every uncertainty realization. The uncertain parameters $d_{i,t}$ and $a_{i,t}$ are modeled by polyhedral bounds in (4.8),

$$\begin{cases} d_{i,t} \in [d_{i,t}^a - d_{i,t}^{dw}, d_{i,t}^a + d_{i,t}^{uw}]; \\ a_{i,t} \in [a_{i,t}^a - a_{i,t}^{dw}, a_{i,t}^a + a_{i,t}^{uw}]; \end{cases}, \quad (4.8)$$

where $d_{i,t}^a$ and $a_{i,t}^a$ are the accurate EV power consumption and EV availability, respectively, $d_{i,t}^{dw}$ and $d_{i,t}^{uw}$ are the maximum downward and upward forecast errors from the accurate EV power consumption, respectively, and $a_{i,t}^{dw}$ and $a_{i,t}^{uw}$ are the maximum downward and upward forecast errors from the accurate EV availability, respectively.

4.3 Robust DT Model and Solution Strategy

This section firstly formulates the robust DT model and then presents a solution strategy to solve the robust DT model.

4.3.1 Robust DT Model

A three-level robust DSO optimization model is formulated to obtain a robust DT solution. The robustness means that the robust DT solution ensures the line loading constraints being respected in the worst-case scenario. The three-level optimization model is formulated as below,

Upper level:

$$\min \sum_{t \in \mathcal{N}^T} \rho_t, \quad (4.9)$$

$$\rho_t \geq 0; \forall t \in \mathcal{N}^T, \quad (4.10)$$

$$x^{\text{wc}} \leq 0; , \quad (4.11)$$

Middle level:

$$x^{\text{wc}} = \max \sum_{jk \in \mathcal{L}} \sum_{t \in \mathcal{N}^T} x_{jk,t}^{\text{ld}} \left(\{F_t^{\text{p}}\}_{jk} - \{f_t^{\text{max}}\}_{jk} \right), \quad (4.12)$$

$$x_{jk,t}^{\text{ld}} = \begin{cases} 1, & \{F_t^{\text{p}}\}_{jk} \geq \{f_t^{\text{max}}\}_{jk}; \\ 0, & \{F_t^{\text{p}}\}_{jk} < \{f_t^{\text{max}}\}_{jk}; \end{cases} \quad \forall jk \in \mathcal{L}, t \in \mathcal{N}^T, \quad (4.13)$$

$$GF_t^{\text{p}} = p_t^{\text{c}} + \sum_{i \in \mathcal{N}^{\text{a}}} E_i p_{i,t}; \forall t \in \mathcal{N}^T, \quad (4.14)$$

$$\begin{cases} d_{i,t} \in [d_{i,t}^{\text{a}} - d_{i,t}^{\text{dw}}, d_{i,t}^{\text{a}} + d_{i,t}^{\text{uw}}]; \\ a_{i,t} \in [a_{i,t}^{\text{a}} - a_{i,t}^{\text{dw}}, a_{i,t}^{\text{a}} + a_{i,t}^{\text{uw}}]; \end{cases}, \quad (4.15)$$

Lower level:

$$p_{i,t} \in \begin{cases} \min \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^{\text{a}}} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_t 1 + E_i^T \rho_t) p_{i,t} \right) \\ s.t. \quad e_{i,t}^{\min} \leq \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^{\text{a}}, t \in \mathcal{N}^T, \\ p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t} p_{i,t}^{\max}; \forall i \in \mathcal{N}^{\text{a}}, t \in \mathcal{N}^T \end{cases} \quad (4.16)$$

The lower-level optimization model in (4.16) is to minimize the energy costs of EVs, subject to EV charging constraints. The middle-level optimization model is to find the worst-case scenario in the predefined uncertain bounds. The worst-case scenario is the scenario having the largest overloading amount (x^{wc}). The upper-level optimization model is to minimize DTs. With the three-level structure, the three-level optimization model is to find such a set of minimum DTs which ensures the energy plans made at the aggregator side satisfy line loading constraints even in the worst-case scenario.

After the DSO solves the three-level optimization model and obtains a robust DT solution, the aggregator solves the following aggregator optimization model to make energy plans.

For aggregator i ,

$$\begin{cases} \min & \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_i \mathbf{1} + E_i^T \rho_t) p_{i,t} \right) \\ \text{s.t.} & e_{i,t}^{\min} \leq \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}^a) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T, \\ & p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t}^a p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \end{cases} \quad (4.17)$$

4.3.2 Solution Strategy for the Robust DT Model

The three-level DSO optimization model in (4.9)-(4.16) is a non-convex model that is hard to solve. To efficiently solve the model, linearization techniques are applied firstly to obtain an MILP model. Then, an iterative procedure is used to solve the MILP model.

4.3.2.1 Transformation of the Lower-Level Optimization Model

As mentioned previously in chapter 3, the lower-level optimization model can be replaced by its KKT conditions. Then, the KKT conditions are further linearized in (4.18)-(4.22) by using the big-M method.

$$B_{i,t} p_{i,t} + c_i \mathbf{1} + E_i^T \rho_t + \sum_{t \leq t_+} (\mu_{i,t_+}^+ - \mu_{i,t_+}^-) + (\sigma_{i,t}^+ - \sigma_{i,t}^-) = 0, \quad (4.18)$$

$$\begin{cases} 0 \leq \mu_{i,t}^- \leq M b_{i,t}^-; \\ 0 \leq -e_{i,t}^{\min} + \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}^a) + e_{i,0} \leq M(1 - b_{i,t}^-); \end{cases} \quad (4.19)$$

$$\begin{cases} 0 \leq \mu_{i,t}^+ \leq M b_{i,t}^+; \\ 0 \leq -\sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}^a) - e_{i,0} + e_{i,t}^{\max} \leq M(1 - b_{i,t}^+); \end{cases} \quad (4.20)$$

$$\begin{cases} 0 \leq \sigma_{i,t}^- \leq M h_{i,t}^-; \\ 0 \leq -p_{i,t}^{\min} + p_{i,t} \leq M(1 - h_{i,t}^-); \end{cases} \quad (4.21)$$

$$\begin{cases} 0 \leq \sigma_{i,t}^+ \leq Mh_{i,t}^+; \\ 0 \leq -p_{i,t} + a_{i,t}p_{i,t}^{\max} \leq M(1-h_{i,t}^+); \end{cases}, \quad (4.22)$$

4.3.2.2 Linearization of the Objective Function (4.12)

The objective function (4.12) is non-linear due to the product of a binary variable and a continuous variable. By introducing the big-M method, the objective function (4.12) can be linearized in (4.23) and (4.24), where $z_{jk,t}^{\text{ld}}$ is the auxiliary variable used for linearization.

$$x^{\text{wc}} = \max \sum_{jk \in \mathcal{L}} \sum_{t \in \mathcal{N}^T} \left(z_{jk,t}^{\text{ld}} - x_{jk,t}^{\text{ld}} \{f_t^{\max}\}_{jk} \right), \quad (4.23)$$

$$\begin{cases} -Mx_{jk,t}^{\text{ld}} \leq z_{jk,t}^{\text{ld}} \leq Mx_{jk,t}^{\text{ld}}; \\ \{F_t^{\text{p}}\}_{jk} - M(1-x_{jk,t}^{\text{ld}}) \leq z_{jk,t}^{\text{ld}} \leq \{F_t^{\text{p}}\}_{jk} + M(1-x_{jk,t}^{\text{ld}}); \end{cases} \forall jk \in \mathcal{L}, t \in \mathcal{N}^T, \quad (4.24)$$

Finally, the three-level DSO optimization model is reformulated as a two-level MILP model as below,

Upper level: min (4.9),

Subject to: (4.10), (4.11),

Middle level: max (4.23),

Subject to: (4.13)-(4.15), (4.18)-(4.22), (4.24)

4.3.2.3 Solution Strategy for the Two-Level MILP Model

An iterative procedure is used to solve the two-level MILP model. In the model, the DTs are decision variables in the upper-level model and are regarded as parameters in the middle-level model. In the iterative procedure, the DTs are considered as control parameters in the middle-level model. The DTs are updated iteratively by solving the standard DSO optimization model in (4.25) (one-level optimization model) with line loading limits $f_t^{\max,k}$ taking different values at different iteration step k . The iterative procedure is described in detail as below,

- 1) The DSO and aggregators obtain necessary data, such as the price sensitivity coefficient and forecasted spot prices.
- 2) The DSO solves the standard DSO optimization model in (4.25) with initial line loading limits to calculate DTs.
- 3) With the DTs obtained at step 2), the DSO solves the middle-level model in the two-level MILP model to calculate the overloading amount (x^{wc}) in the worst-case scenario.
- 4) Check if constraint (4.11) is satisfied. If no congestion occurs, then publish the DTs as the final DT solution, otherwise, go to step 5)
- 5) Push down the line loading limits in the standard DSO optimization model with a small step, e.g., a step with 50 kW. Then, go back to step 2) to solve the standard DSO optimization model with the updated line loading limits. It is noted that the line loading limits are fixed in the two-level MILP model.

Finally, after the iterative procedure, the aggregator solves the aggregator model based on the robust DT solution to obtain the actual energy plans.

Standard DSO optimization:

$$\left\{ \begin{array}{l} \min \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^a} \left(p_{i,t}^T B_{i,t} p_{i,t} + (c_t 1)^T p_{i,t} \right) \\ s.t. \quad GF_t^p = p_t^c + \sum_{i \in \mathcal{N}^a} E_i p_{i,t}; \forall t \in \mathcal{N}^T \quad (\rho_t) \\ F_t^p \leq f_t^{\max,k}; \forall t \in \mathcal{N}^T \\ e_{i,t}^{\min} \leq \sum_{t_- \leq t} \left(p_{i,t_-} - d_{i,t_-}^a \right) + e_{i,0} \leq e_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \\ p_{i,t}^{\min} \leq p_{i,t} \leq a_{i,t}^a p_{i,t}^{\max}; \forall i \in \mathcal{N}^a, t \in \mathcal{N}^T \end{array} \right. , \quad (4.25)$$

4.4 Case Studies

In order to validate the effectiveness of the proposed robust DT method for congestion management of distribution networks, case studies were conducted on the Bus 4 distribution system of the Roy Billinton Test System (RBTS) [102], as shown in Fig. 4-1. Line segments of feeder 1 are labeled as L1-L12 and load points of feeder 1 are labeled as LP1-LP7. It is assumed that each customer owns an EV and

has a contract with one of the two aggregators (AGG1-AGG2). AGG 1 has contracts with 80 customers per load point, and AGG 2 has contracts with 120 customers per load point. The detailed data of load points, key parameters of EVs, and forecasted spot prices can be found in Appendix B. The line loading limits of L2 and L7 are 1050 kW and 3100 kW, respectively. In the uncertain bounds, the maximum upward and downward forecast errors from the accurate EV power consumption is 0.2 p.u., and the maximum upward and downward forecast errors from the accurate EV availability is 0.1 p.u.. The simulations were carried out on GAMS with the Gurobi solver.

Without congestion management, congestion occurs on L2 and L7 at t_{24} , as shown in Fig. 4-2. Then, the deterministic DT method and robust DT method are carried out to perform congestion management.

Fig. 4-1 Bus 4 distribution network of the RBTS

Fig. 4-2 Line loadings on L1 and L7 without congestion management

4.4.1 Deterministic DT Method

In the deterministic DT method, after the DSO solves the DSO optimization model with forecasted EV driving parameters, the line loadings on L2 and L7 at the DSO side are shown in Fig. 4-3 and DTs are listed in Table 4-1. It is shown that congestions on L2 and L7 are alleviated. As proven in Appendix A, the energy plans at the aggregator side should be the same as the ones at the DSO side if the DSO and aggregators use the same EV driving parameters. Therefore, the congestion is alleviated at the aggregator side if the forecast is perfect. However, this might not be the case in practice due to the inevitable DSO’s forecast error. Based on the DTs and spot prices, the aggregator solves the aggregator optimization model with accurate EV driving parameters to obtain actual energy plans, as shown in Fig. 4-4. It is shown that congestion still occurs on L2 and L7 at t_{24} because the accurate EV power consumption demands are larger than the DSO’ forecasted values and thus EVs need to charge more power at t_{24} .

Table 4-1 DTs obtained with the deterministic method and robust method

DTs (DKK)	t_{23}	t_{24}				
load points	LP ₁	LP ₁	LP ₄	LP ₅	LP ₆	LP ₇
deterministic DTs	-	0.0699	0.0695	0.0695	0.0695	0.0695
robust DTs	0.00678	0.0768	0.0697	0.0697	0.0697	0.0697

Fig. 4-3 Line loadings at the DSO side after the deterministic method

Fig. 4-4 Line loadings at the aggregator side after the deterministic method

4.4.2 Robust DT Method

In order to deal with the DSO’s forecast error, the robust DT method is carried out. After the DSO solves the robust model, the DTs are listed in Table 4-1. Moreover,

the congestion lines and overloading amount under the worst-case scenario during the iterative procedure are shown in Table 4-2. After 6 iterations, the robust DT solution is obtained. It is shown that the robust DTs are larger than the deterministic ones at t_{24} , and DTs are required at t_{23} in the robust method. This is because larger DTs are required to shift more power consumption at t_{23} and t_{24} to off-peak hours to deal with forecast errors. Moreover, the overloading amount under the worst-case scenario is reduced gradually during the iterative procedure.

Table 4-2 Congestion lines and overloading amount under the worst-case scenario

iterations	1	2	3	4	5	6
congestion lines	L1, L7	L1, L7	L7	L7	L7	-
overloading amount [kW]	345.36	295.36	129.99	80.00	30.00	0

Based on the spot prices and robust DTs, the aggregator solves the aggregator optimization to obtain actual energy plans using the accurate EV driving parameters. The line loadings on L2 and L7 at the aggregator side are shown in Fig. 4-5. It is shown that congestion is completely alleviated based on the robust DT solution eve if there is a discrepancy between the forecasted and accurate parameters.

Fig. 4-5 Line loadings at the aggregator side in the robust DT method

4.5 Summary

In the DT framework, the uncertainty exists because the DSO needs to forecast the energy requirements at the aggregator side, and forecast errors are inherently inevitable. In such a case, the aggregator's energy plans may not be the same as the ones at the DSO side and thus congestion cannot be alleviated as the DSO expects. To deal with the forecast error, a robust DT method is proposed, in which a three-level robust optimization model is formulated and solved to obtain a robust DT solution with an iterative procedure. The robustness in the DT method means that the line loading constraints are satisfied even under the worst-case scenario. Based on the robust DT solution, the aggregator's energy plans respect the line loading constraints even if there is a discrepancy between the DSO's forecasted EV driving parameters and accurate parameters used by the aggregators.

CHAPTER 5.

TWO-TIER REAL-TIME CONGESTION MANAGEMENT BASED ON FLEXIBLE DEMAND SWAP AND TRANSACTIVE CONTROL

This chapter presents a two-tier real-time congestion management method based on the flexible demand swap and transactive control to deal with real-time congestion of ADNs. This chapter is based on the published article 7 with minor changes to fit into the framework of this thesis⁵.

5.1 Introduction

In addition to day-ahead congestion, the forecast error and unexpected power system component's failure would lead to a discrepancy from the day-ahead schedules and cause real-time congestion. Therefore, it is also important to implement congestion management in real-time operation, i.e., 5-60 minutes prior to operation time [39]. Currently, the studies on real-time congestion management methods at the distribution level are limited. In [39], a flexible demand swap (FDS) concept was proposed and used to alleviate real-time congestion. The FDS represents the spatial and temporal power consumption or generation change, which can address both the rebound effect and imbalance issue. An agent-based real-time congestion management scheme was introduced in [40], in which DERs are represented by agents to send flexibility service bids to the DSO, and then the DSO selects flexi-

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⁵ This chapter is based on the published article 7: **Feifan Shen**, Qiuwei Wu*, Shaojun Huang, Xinyu Chen, and Yan Xu, "Two-Tier Demand Response with Flexible Demand Swap and Transactive Control for Real-Time Congestion Management in Distribution Networks," *International Journal of Electrical Power and Energy System*, vol. 114, pp. 105399, Jan. 2020.

bility service bids with the minimized cost to alleviate real-time congestion. In addition, direct active power control is considered as a back-up option for maker-based congestion management.

Two important aspects should be considered for real-time congestion management since the customers have already made day-ahead energy schedules. The first one is to satisfy customer's specific requirements on energy rebound conditions, i.e., the rebound period and rebound amount (rebound percentage). If specific requirements on the rebound percentage cannot be met, certain operational constraints will be violated, e.g., a rebound percentage of 100% for HPs may result in household temperature exceeding the upper limit because the required rebound percentage is actually lower than 100% [114], [115]. The second one is to consider the customer's willingness to provide flexibility in order to reduce discomfort to them. For example, conservative customers may be not willing to provide flexibility.

However, the above-mentioned two aspects have not been considered in the existing real-time congestion management methods. In [39], [40], specific requirements on energy rebound conditions are not considered. For example in [39], an energy rebound percentage of 100% was forced to occur in a specified period, so the customer cannot determine its own energy rebound conditions. In addition, none of them has considered the customer's willingness to provide flexibility. To resolve the abovementioned issues, a two-tier real-time congestion management method based on the flexible demand swap and transactive control is proposed in this chapter. The benefits of the proposed method are summarized as follows: 1) the proposed method considers the customer's specific requirements on energy rebound conditions by allowing the customers to rebound energy in multiple periods and determine rebound percentages; and 2) the proposed method considers the customer's willingness to provide flexibility.

The rest of this chapter is organized as follows. Section 5.2 introduces concepts of the FDS and transactive control. Section 5.3 presents the framework of the proposed two-tier real-time congestion management. The model formulations and algorithm of the proposed method are provided in section 5.4. Case study results are discussed in section 5.5, followed by a summary in section 5.6.

5.2 Concepts of the Flexible Demand Swap and Transactive Control

In this section, concepts of the flexible demand swap and transactive control are described in detail.

5.2.1 Flexible Demand Swap

The FDS can be used to alleviate real-time congestion in distribution networks without causing system imbalance and forming new peaks [39]. A swap has two sides. One side of a swap ($S^{(1)}$) is defined as,

$$S^{(1)} : \{LP_j | j \in N_d^{(1)}\}, (t_1, -p_{t_1}^d), (t_2, +p_{t_2}^r), \quad (5.1)$$

where the set $\{LP_j | j \in N_d^{(1)}\}$ represents a joint of load points providing flexibility, notations “-” and “+” represent the power consumption decrease and increase, respectively, t_1 is the time period (when and how long) in which a power consumption reduction is required, $-p_{t_1}^d$ represents a power consumption reduction with the amount of $p_{t_1}^d$ at the specified load points in period t_1 , t_2 is the period in which energy rebound occurs, and $+p_{t_2}^r$ means a power consumption increase with the amount of $p_{t_2}^r$ in the period t_2 at the specified load points, which allows customers to rebound energy.

The other side of the swap ($S^{(2)}$) is defined as,

$$S^{(2)} : \{LP_j | j \in N_d^{(2)}\}, (t_1, +p_{t_1}^d), (t_2, -p_{t_2}^r), \quad (5.2)$$

This side of the swap represents the opposite flexibility that is used to maintain the system balance. For example, in the period t_1 , a consumption increase with the amount of $p_{t_1}^d$ is required at load points $\{LP_j | j \in N_d^{(2)}\}$ to compensate the consumption reduction at load points $\{LP_j | j \in N_d^{(1)}\}$ stipulated in $S^{(1)}$. Similarly, the swap ensures the system balance in the period t_2 .

The Bus 4 distribution network of Roy Billinton Test System (RBTS) shown in Fig. 5-1 is used to illustrate how the FDS is used to alleviate congestion. Line segments of feeder 1 are labelled as L1-L12 and the first line segment of feeder 2 is labelled as L14. Load points are labelled as LP₁₋₇, LP₁₁₋₁₆, LP₁₈₋₂₅ and LP₃₂₋₃₈. It is assumed that congestion occurs on L3 (overloading by p^d) at time 18:00 and lasts for 1 hour, and that there are enough capacities on L1 and L2 to implement opposite flexibility. Moreover, suppose that there is enough capacity on L3 during 19:00-20:00 to rebound energy with the amount of p^r . Then, a complete swap is formulated as,

$$\begin{cases} S^{(1)} : \{LP_{2-5}\}, (18:00, 1h, -p^d), (19:00, 1h, +p^r) \\ S^{(2)} : \{LP_1\}, (18:00, 1h, +p^d), (19:00, 1h, -p^r) \end{cases}, \quad (5.3)$$

As stipulated in the swap in (5.3), customers at LP_{2-5} reduce power consumption at 18:00 and rebound energy at 19:00, so congestion at 18:00 is alleviated without causing new congestion. At the same time, customers at LP_1 provide opposite power consumption changes to maintain the system balance.

Fig. 5-1 Single line diagram of the distribution network

5.2.2 Concept of transactive control

Transactive control is a distributed control strategy that utilizes economic signals for delivering the desire to change operation status of responsive loads [116]. In the proposed method, each aggregator has contracts with a number of customers. Based on the transactive control concept, the aggregator constructs a transactive market with its contractual customers, in which the customers submit their power consumption targets and response curves in the specified period and they are rewarded for modifications of power consumption targets. The response curve is illustrated in Fig. 5-2.

Fig. 5-2 Response curve of a customer

The response curve models the customer's power consumption modification ($\delta_{i,t}$) in response to a clearing price (λ_t^{cl}). The coefficient γ is the slope of the response curve, denoting the customer's willingness to change its power consumption target. A smaller γ means a larger power consumption modification with the same price, which means that the customer is more willing to change its power consumption target. After receiving response curves and power consumption targets, the aggregator conducts an optimization to clear the market and broadcasts the clearing price to customers. With the clearing price and response curve, each customer determines its power consumption modification using (5.4),

$$\delta_{i,t} = \begin{cases} \theta_{i,t} \lambda_t^{\text{cl}}, & \lambda_t^{\text{cl}} \leq \delta_{i,t}^{\text{max}} / \theta_{i,t} \\ \delta_{i,t}^{\text{max}}, & \lambda_t^{\text{cl}} > \delta_{i,t}^{\text{max}} / \theta_{i,t} \end{cases}, \quad (5.4)$$

where $\theta_{i,t}$ is the inverse of the response curve's slope. Accordingly, the customer's actual power consumption is equal to the power consumption target plus or minus the power consumption modification as below,

$$p_{i,t}^{\text{a}} = p_{i,t}^{\text{tar}} + (z_{i,t}^1 - z_{i,t}^2) \delta_{i,t}, \quad \forall i \in \mathcal{N}^{\text{c}}, t \in \mathcal{N}^{\text{T}}, \quad (5.5)$$

where $p_{i,t}^{\text{tar}}$ is the power consumption target of i -th customer in period t , $p_{i,t}^{\text{a}}$ is the actual power consumption of i -th customer in period t , $z_{i,t}^1$ and $z_{i,t}^2$ are binary variables representing the direction of the power consumption modification. The customer decreases its power consumption if $z_{i,t}^1 = 0$ and $z_{i,t}^2 = 1$. Then, the received revenue ($E_{i,t}^{\text{r}}$) of each customer can be calculated using (5.6),

$$E_{i,t}^r = \lambda_t^{\text{cl}} \delta_{i,t}, \quad (5.6)$$

In the transactive control concept, the customer can change its willingness to change the power consumption target in each period. If a customer wants to be more conservative to provide flexibility, it can increase its response curve' slope. Consequently, the modification of the power consumption target and received revenue are reduced.

An example is used to illustrate how the transactive market operates. In $S^{(1)}$ in (5.3), a power consumption reduction with the total amount of p^d is required to alleviate congestion on L3 at 18:00. It is assumed that the customers located at LP_2 are willing to participate in the transactive market. Firstly, the aggregator constructs a transactive market with its contractual customers, and the customers located at LP_2 submit their response curves and power consumption targets at time 18:00 to the market. Then, the aggregator clears the market in order to have a power consumption reduction with the amount of p^d and informs customers about the clearing price. Based on the clearing price and response curves, each customer determines its actual power consumption. As a result, a power consumption reduction with the amount of p^d is realized.

5.3 Framework of the Real-Time Congestion Management

In order to consider customers' specific requirements on energy rebound conditions, a new market operation mechanism is developed for the FDS market. The new market operation mechanism allows the aggregators to determine swap bids considering customers' requirements. Then, the aggregators submit swap bids to the swap market, and the DSO selects swaps with the minimized procurement cost. This is different from the swap market in [39], where the DSO determines swaps directly without considering customer's requirements. Moreover, the transactive control concept is used to develop an interactive mechanism between the aggregator and customers in order to consider the customer's willingness to provide flexibility. Based on the FDS, swap market, and transactive control, a real-time congestion management method with a two-tier framework is proposed, as shown in Fig. 5-3.

In the upper tier, the DSO constructs a swap market with the aggregators. Firstly, according to congestion information, the DSO publishes Flexibility Tables (FTs) to the swap market. The FT consists of specific requirements on swaps. Secondly, the aggregators respond to FTs by submitting swap bids to the swap market. Thirdly,

the DSO evaluates all the submitted swap bids and chooses the best swap bid. In the lower tier, the aggregator constructs a transactive market with its contractual customers after receiving FTs. In the market, an interactive process is carried out to determine the swap bids.

Fig. 5-3 Structure of the two-tier framework

The procedures of using the proposed method to alleviate real-time congestion are illustrated in Fig. 5-4 and has the following five steps:

- 1) Prepare FTs: the DSO identifies network congestion 15 minutes in advance so that there is enough time to set up swap market and transactive markets. Then, the DSO formulates FTs according to congestion information.
- 2) Publish FTs: the DSO constructs a swap market and publishes FTs to the market.
- 3) Aggregators respond to FTs: in order to respond to FTs, each aggregator constructs a transactive market with its contractual customers and determines swap bids through an interactive process. Then, the aggregators close transactive markets and submit their swap bids to the swap market.

4) Evaluation and selection: the DSO evaluates all the swap bids and chooses the best swap bids according to requirements. Then, the DSO closes the swap market and informs the aggregators about selected swap bids.

5) Swap Implementation and financial settlement: the aggregators implement the selected swaps at the operation time. After the implementation, the DSO carries out financial settlement for the selected swaps.

Fig. 5-4 The procedures of using the proposed method to alleviate real-time congestion

5.3.1 Swap Market for the DSO and Aggregators

The DSO issues the FTs in the swap market. The FT is defined as,

$$\begin{cases} S^{(1)} : \{LP_j | j \in N_d^{(1)}\}, (t_1, -p_{t_1}^d), (t_2, +, \leq p_{t_2}^{r, \max}), \dots, (t_i, +, \leq p_{t_i}^{r, \max}) \\ S^{(2)} : \{LP_j | j \in N_d^{(2)}\}, (t_1, +p_{t_1}^d), (t_2, -, \leq p_{t_2}^{r, \max}), \dots, (t_i, -, \leq p_{t_i}^{r, \max}) \end{cases}, \quad (5.7)$$

The FT lists the requirements on swaps for alleviating real-time congestion. For example, the FT determines the required amount of a power consumption reduction and the required amount of a power consumption increase in congestion periods (t_1), and the candidate load points providing flexibility. Unlike a swap, the FT stipulates the maximum allowable energy rebound amounts ($p_{t_2}^{r, \max}, \dots, p_{t_i}^{r, \max}$) in multiple periods instead of a fixed amount in a specified period, e.g., $+p_{t_2}^r$ in time period t_2 in (5.1), which allows the customers to rebound energy according to specific requirements. In addition, the FT guarantees that a power consumption increase must be compensated by a power consumption reduction at the same time, e.g., in period t_2 , a power consumption increase ($+p_{t_2}^r$) in $S^{(1)}$ should be accompanied with a power consumption decrease ($-p_{t_2}^r$) in $S^{(2)}$.

5.3.2 Transactive Market for the Aggregator and Customers

In order to provide swap bids, each aggregator constructs a transactive market with its contractual customers. In the transactive market, an interactive process is carried out, as shown in Fig. 5-5.

Fig. 5-5 The interactive process between the aggregator and customers

In the transactive market, the aggregator first sends the request of providing flexibility to the customers located at the candidate load points given in the FTs. Each customer can choose to accept or reject the request. For example in (5.7), after customers submit power consumption targets and response curves of period t_1 to the transactive market, the aggregator clears the market in order to change customer's power consumption targets to provide a power consumption reduction with the amount of $p_{t_1}^d$ in $S^{(1)}$ and a power consumption increase with the same amount in $S^{(2)}$ in period t_1 . Then, the aggregator informs customers about the clearing price,

based on which the customers determine actual power consumption in period t_1 . After this, the customers submit power consumption targets and response curves to the transactive market for period t_2 . Then, the aggregator clears the market to modify customers' power consumption targets so that the energy rebound amount does not exceed the maximum allowable amount ($p_{t_2}^{r,\max}$) given in the FT, and the system balance is maintained. The market clearing process repeats for the following periods until all customers do not need to rebound energy, e.g., the customer's household inside temperature is restored to a satisfactory level. In the interactive process, once a customer does not need to rebound energy, it will no longer participate in the transactive market.

Finally, the energy rebound amount in each period is determined through the interactive process, and a swap is formulated as,

$$\left\{ \begin{array}{l} S^{(1)} : \{LP_j \mid j \in N_d^{(1)}\}, (t_1, -p_{t_1}^d), (t_2, +p_{t_2}^r), \dots, (t_i, +p_{t_i}^r) \\ S^{(2)} : \{LP_j \mid j \in N_d^{(2)}\}, (t_1, +p_{t_1}^d), (t_2, -p_{t_2}^r), \dots, (t_i, -p_{t_i}^r) \end{array} \right\}, \quad (5.8)$$

In (5.8), the energy rebound amount in each period ($p_{t_i}^r$) respects the limits given in the FT. Then, the aggregator puts a price tag to each swap according to the payments to customers and expected profit for providing the swap. Finally, a complete swap bid is formulated as,

$$\text{Swap bid:} \left\{ \begin{array}{l} \text{Price: price(DKK)} \\ \text{Pattern:} \left\{ \begin{array}{l} S^{(1)} : \{LP_j \mid j \in N_d^{(1)}\}, (t_1, -p_{t_1}^d), (t_2, +p_{t_2}^r), \dots, (t_i, +p_{t_i}^r) \\ S^{(2)} : \{LP_j \mid j \in N_d^{(2)}\}, (t_1, +p_{t_1}^d), (t_2, -p_{t_2}^r), \dots, (t_i, -p_{t_i}^r) \end{array} \right\} \end{array} \right\}, \quad (5.9)$$

5.4 Model Formulations and Algorithm

The model formulations of the FTs and swap bid are described in this section. For the clarity of swap implementation, the amount of power consumption reduction in each congestion period in each swap is standardized to be a fixed number p^d , e.g., 100kW, and the duration of each time period is one hour.

5.4.1 Formulation of FTs

As described in section 5.3.1, the $S^{(1)}$ of the FT stipulates the candidate load points to provide flexibility and the maximum allowable energy rebound amount in each

period. The set of candidate load points can be obtained by solving the following MILP model,

Optimization 1 (OPT.1)

$$\min \sum_{s \in \mathcal{N}^{\text{sw}}, j \in \mathcal{N}^{\text{d}}} x_{s,j}^{\text{sw}}, \quad (5.10)$$

$$\sum_{s \in \mathcal{N}^{\text{sw}}, j \in \mathcal{N}^{\text{d}}} p^{\text{d}} T_{l,j} x_{s,j}^{\text{sw}} \geq f_l, \quad \forall l \in \mathcal{L}^{\text{c}}, \quad (5.11)$$

$$\sum_{j \in \mathcal{N}^{\text{d}}} x_{s,j}^{\text{sw}} \leq 1, \quad \forall s \in \mathcal{N}^{\text{sw}}, \quad (5.12)$$

$$x_{s,j}^{\text{sw}} \in \{0,1\}, \quad \forall s \in \mathcal{N}^{\text{sw}}, j \in \mathcal{N}^{\text{d}}, \quad (5.13)$$

where $\mathcal{N}^{\text{d}}$ is the set of load points, $\mathcal{N}^{\text{sw}}$ is the set of available swaps, $\mathcal{L}^{\text{c}}$ is the set of congestion lines, l denotes the l -th congestion line, f_l is the l -th congestion line's overloading amount, $x_{s,j}^{\text{sw}}$ is the binary variable representing whether the j -th load point in the s -th swap is selected, and $T_{l,j}$ is the cell at l -th row, j -th column of the power transfer distribution factor (PTDF) matrix.

The objective function (5.10) is to minimize the number of swaps needed for congestion management. For example, if the DSO gives a set $\mathcal{N}^{\text{sw}} = \{s_1, s_2, s_3, \dots, s_{10}\}$ and the objective value of the above model (OPT.1) is 2, which means that only two swaps are needed to alleviate congestion. Constraint (5.11) ensures that congestion on each congestion line is alleviated. Constraint (5.12) means that each swap includes one load point at most. Constraint (5.13) defines the binary nature of the variable $x_{s,j}^{\text{sw}}$. After solving OPT.1, the minimum number of needed swaps, e.g., S_1 and S_2 , and one set of candidate load points to provide flexibility are obtained as,

$$\begin{cases} S_1^{(1)} = \{\text{LP}_{j_1}, t_1, -p^{\text{d}}\} \\ S_2^{(1)} = \{\text{LP}_{j_2}, t_1, -p^{\text{d}}\} \end{cases}, \quad (5.14)$$

After obtaining the above solution, add a new constraint ($x_{s,j_1}^+ + x_{s,j_2}^- \leq 1$) to OPT.1 to exclude the solution. The constraint means that the already obtained load points (LP_{j_1}, LP_{j_2}) cannot occur in the new solution. Then, solve OPT.1 again and a new solution can be found. Repeat this process until no solution can be found, or the objective value increases, or the total number of solutions reaches a predefined number. Finally, all the candidate load points can be grouped in the following form,

$$\begin{cases} S_1^{(1)} = \{LP_{j_1}, LP_{j_3}, \dots, LP_{j_{n1}}, t_1, -p^d\} \\ S_2^{(1)} = \{LP_{j_2}, LP_{j_4}, \dots, LP_{j_{n2}}, t_1, -p^d\} \end{cases}, \quad (5.15)$$

Then, the DSO assigns the maximum allowable energy rebound amount to $S_1^{(1)}$ and $S_2^{(1)}$. The DSO can forecast remaining capacities of those critical lines associated with energy rebound and then allocate the remaining capacities to each swap. For example, if two swaps (S_1 and S_2) are used to alleviate congestion on the same line, half of the remaining capacity of the line in each period is allocated to each swap. So far, the $S_1^{(1)}$ and $S_2^{(1)}$ of two FTs are formulated as,

$$\begin{cases} S_1^{(1)} = \{LP_{j_1}, LP_{j_3}, \dots, LP_{j_{n1}}, (t_1, -p^d), (t_2, +, \leq p_{1,t_2}^{r,\max}), \dots, (t_i, +, \leq p_{1,t_i}^{\max})\} \\ S_2^{(1)} = \{LP_{j_2}, LP_{j_4}, \dots, LP_{j_{n2}}, (t_1, -p^d), (t_2, +, \leq p_{2,t_2}^{r,\max}), \dots, (t_i, +, \leq p_{2,t_i}^{\max})\} \end{cases}, \quad (5.16)$$

The other side of a swap $S^{(2)}$ is to find all candidate load points to provide opposite flexibility in period t_1 to guarantee system balance. According to the number of selected swaps in $S^{(1)}$, the following MILP problem (OPT.2) with a dummy objective function is used to generate $S^{(2)}$,

OPT.2

$$\min 0, \quad (5.17)$$

$$\sum_{s \in \mathcal{N}^{sw,*}, j \in \mathcal{N}^d} p^d T_j y_{s,j}^{sw} \leq f^r, \quad (5.18)$$

$$\sum_{j \in \mathcal{N}^d} y_{s,j}^{sw} \leq 1, \quad \forall s \in \mathcal{N}^{sw,*}, \quad (5.19)$$

$$V_0(1 - \frac{1}{V_0^2} \sum_{j \in \mathcal{N}^d} ((p_j^c + p^d \sum_{s \in \mathcal{N}^{sw,*}} y_{s,j}^{sw}) R_{j,j'}^s + q_j^c X_{j,j'}^s)) \geq V^{\min}, \quad \forall j \in \mathcal{N}^d, \quad (5.20)$$

$$y_{s,j}^{sw} \in \{0,1\}, \quad \forall s \in \mathcal{N}^{sw,*}, j \in \mathcal{N}^d, \quad (5.21)$$

where $\mathcal{N}^{sw,*}$ is the set of selected swaps in $S^{(1)}$, $y_{s,j}^{sw}$ is the binary variable representing whether the j -th load point in s -th swap is selected, f^r is the remaining capacity of uncongested lines, V_0 is the voltage magnitude at the substation node; $V^{\min}$ is the lower limit of voltage magnitude, p_j^c is the conventional active load at node j , q_j^c is the conventional reactive load at node j , T_j is the j -th column of the PTDF matrix, and matrix R^s and X^s are real and imagine parts of the inverse matrix of the partial nodal admittance Y_{LL} , which is a submatrix of the admittance matrix Y :

$$Y = \begin{bmatrix} Y_{00} & Y_{0L} \\ Y_{L0} & Y_{LL} \end{bmatrix}$$

Constraint (5.18) ensures that congestion does not occur after providing the opposite flexibility. Constraint (5.19) ensures that each swap includes one load point at most. Constraint (5.20) calculates nodal voltages using an approximation method in [117] and ensures that voltage limits are not violated. Similarly, after repeating solving OPT.2 and excluding the already obtained solution, all candidate load points can be found and the $S^{(2)}$ of FTs is formed as,

$$\begin{cases} S_1^{(2)} = \{LP_j \mid j \in N_{1,d}^{(2)}, (t_1, +p^d)\} \\ S_2^{(2)} = \{LP_j \mid j \in N_{2,d}^{(2)}, (t_1, +p^d)\} \end{cases}, \quad (5.22)$$

5.4.2 Swap Bids Formulations

The aggregator interacts with its contractual customers to formulate swap bids according to the FTs. In the study, HPs as flexible demands provide flexibility. It is assumed that the customer implements the day-ahead energy schedule if it does not participate in the transactive market.

5.4.2.1 Customer's Power Consumption Target

For each period, each customer determines a power consumption target to maintain the desired household inside temperature. The power consumption target of i -th customer in period t is calculated using the following equation,

$$K_{i,t}^{\text{des}} = K_{i,t_0} + \sum_{t_0 < t^* < t} A_{i,t^*} p_{i,t^*}^{\text{a}} + A_{i,t} p_{i,t}^{\text{tar}}, \quad \forall t \in \mathcal{N}^{\text{T}}, i \in \mathcal{N}^{\text{c}}, \quad (5.23)$$

where $\mathcal{N}^{\text{c}}$ is the set of customers participating in the transactive market, $K_{i,t}^{\text{des}}$ is the desired household inside temperature of i -th customer in period t , K_{i,t_0} is the initial household inside temperature of i -th customer one hour before providing flexibility, and $A_{i,t}$ is the power to temperature matrix of i -th customer in period t .

5.4.2.2 Swap Formulation

After receiving customers' response curves and power consumption targets, each aggregator conducts the following optimization model (OPT.3) to clear the transactive market for each period and determines swaps.

The optimization model for period t is formulated as below,

OPT.3

$$\min \sum_{s \in \mathcal{N}^{\text{sw},*}} \sum_{i \in \mathcal{N}^{\text{c}}} \sum_{d \in \mathcal{S}^{\text{sw}}} b_{i,s,d,t} \theta_{i,t} \left(\lambda_{s,d,t}^{\text{cl}} \right)^2, \quad \forall t \in \mathcal{N}^{\text{T}}, \quad (5.24)$$

$$\sum_{i \in \mathcal{N}^{\text{c}}} b_{i,s,d,t} (p_{i,t}^{\text{a}} - p_{i,t}^{\text{da}}) = r_{s,d,t}, \quad \forall s \in \mathcal{N}^{\text{sw},*}, d \in \mathcal{S}^{\text{sw}}, t \in \mathcal{N}^{\text{T}}, \quad (5.25)$$

$$r_{s,d,t} = -p^{\text{d}}, \quad d = d_1, t = t_1, \forall s \in \mathcal{N}^{\text{sw},*}, \quad (5.26)$$

$$r_{s,d,t} \leq p_{s,t}^{\text{r,max}}, \quad d = d_1, \forall t > t_1, s \in \mathcal{N}^{\text{sw},*}, \quad (5.27)$$

$$\sum_d r_{s,d,t} = 0, \quad \forall t \geq t_1, s \in \mathcal{N}^{\text{sw},*}, \quad (5.28)$$

$$p_{i,t}^{\text{a}} = p_{i,t}^{\text{tar}} + (z_{i,t}^1 - z_{i,t}^2) \delta_{i,t}, \quad \forall i \in \mathcal{N}^{\text{c}}, t \in \mathcal{N}^{\text{T}}, \quad (5.29)$$

$$\delta_{i,t} = \theta_{i,t} \lambda_{s,d,t}^{\text{cl}} (1 - z_{i,t}) + \delta_{i,t}^{\text{max}} z_{i,t}, \quad \forall i \in \mathcal{C}_{s,d,t}, s \in \mathcal{N}^{\text{sw},*}, d \in \mathcal{S}^{\text{sw}}, t \in \mathcal{N}^{\text{T}}, \quad (5.30)$$

$$\delta_{i,t}^{\max} z_{i,t} \leq \theta_{i,t} \lambda_{s,d,t}^{cl} \leq \delta_{i,t}^{\max} (1 - z_{i,t}) + \theta_{i,t} \lambda_{i,t}^{\max} z_{i,t}, \quad (5.31)$$

$$\forall i \in C_{s,d,t}, s \in \mathcal{N}^{\text{sw},*}, d \in \mathcal{S}^{\text{sw}}, t \in \mathcal{N}^T,$$

$$z_{i,t}^1, z_{i,t}^2, z_{i,t} \in \{0,1\}, \quad (5.32)$$

$$\begin{cases} z_{i,t}^1 = z_{i-1,t}^1, \forall i > 1, i \in C_{s,d,t}, s \in \mathcal{N}^{\text{sw},*}, d \in \mathcal{S}^{\text{sw}}, t \in \mathcal{N}^T \\ z_{i,t}^2 = z_{i-1,t}^2, \forall i > 1, i \in C_{s,d,t}, s \in \mathcal{N}^{\text{sw},*}, d \in \mathcal{S}^{\text{sw}}, t \in \mathcal{N}^T \end{cases} \quad (5.33)$$

The objective function (5.24) is to minimize the total payment to customers in period t , where $\mathcal{S}^{\text{sw}}$ is the set of sides of a swap, d_1 represent $\mathcal{S}^{(1)}$ and d_2 represents $\mathcal{S}^{(2)}$; $b_{i,s,d,t}$ is the mapping coefficient, $b_{i,s,d,t} = 1$ if customer i belongs to the d -th side of s -th swap in period t ; otherwise, $b_{i,s,d,t} = 0$, and $\lambda_{s,d,t}^{cl}$ is the clearing price for d -th side of s -th swap in period t . Constraint (5.25) calculates the total difference between the actual and day-ahead scheduled power consumption, where $p_{i,t}^{\text{da}}$ is the day-ahead scheduled power consumption of i -th customer in period t , $p_{i,t}^{\text{a}}$ is the actual power consumption of i -th customer in period t , and $r_{s,d,t}$ is the total difference between the actual and day-ahead scheduled power consumption of those customers who belong to d -th side of s -th swap in period t . Constraint (5.26) represents that a power consumption reduction is required in each swap in the congestion period t_1 . Constraint (5.27) ensures that the energy rebound amount in each period and each swap does not exceed the limit given in FTs, where $p_{s,t}^{\text{r,max}}$ is the maximum allowable energy rebound amount of s -th swap in period t . Constraint (5.28) ensures that a power consumption increase is compensated by a power consumption decrease in each period and each swap. Constraints (5.29)-(5.32) model the customer's response in the transactive market, where $z_{i,t}$ is the binary variable indicating whether the power consumption modification reach the maximum allowable value. Constraint (5.33) ensures that all the customers belonging to the same side of a swap should have the same modification direction in each period.

5.4.2.3 Swap Bid Price

After solving the above model (OPT.3), the energy rebound amount in each swap and each period ($p_{s,t}^{\text{r}}$) is obtained, so the swaps are determined. Then, the aggregator determines the price for each swap using the following equation.

$$\begin{cases} \text{Cost}_s = \sum_{t \in \mathcal{N}^T} \sum_{i \in \mathcal{N}^c} \sum_{d \in \mathcal{S}^{\text{sw}}} b_{i,s,d,t} \theta_{i,t} \left(\lambda_{s,d,t}^{\text{cl}} \right)^2, \quad \forall s \in \mathcal{N}^{\text{sw},*}, \\ \text{Pr}_s = c^{\text{p}} \text{Cost}_s, \quad \forall s \in \mathcal{N}^{\text{sw},*} \end{cases}, \quad (5.34)$$

where Cost_s is the total payment of s -th swap to customers, c^{p} is the profit coefficient determined by the aggregator, and Pr_s is the price of s -th swap bid.

5.4.3 Algorithm of the Proposed Method

The algorithm of implementing the proposed method is illustrated in Table 5-1.

Table 5-1 Algorithm of implementing the proposed method

Algorithm1: Determine FTs
Input: congested period, congested lines and each congested line's loading value exceeding limit, topology of the network Output: $\mathcal{S}^{(1)}$ of the FTs 1. Solve OPT.1
Input: the number of selected swaps, the remaining capacity of uncongested lines, topology of the network Output: $\mathcal{S}^{(2)}$ of the FTs 2. Solve OPT.2
Algorithm 2: Determine swap bids
Repeat: for each period t , Initial $t=t_1$ Input: initial household temperature one hour before providing flexibilities, desired temperature, actual HP power consumption of period $t-1$, Output: HP power consumption target 3. Solve (5.23)   Input: HP power consumption target, response curve, limit of the amount of energy rebound, $b_{i,s,d,t}$: mapping matrix Output: the amount of energy rebound, clearing price 4. Solve OPT.3   Input: clearing price Output: actual HP power consumption 5. Solve (5.4) and (5.5)   Update: $t=t+1$, update $b_{i,s,d,t}$ Until: all the elements of $b_{i,s,d,t}$ are zero
Input: clearing price, response curve, $b_{i,s,d,t}$: mapping matrix Output: Pr_s : Swap bid price 6. Solve (5.34)

5.5 Case Studies

5.5.1 Grid and Simulation Parameters

Case studies were carried out on the distribution network in Fig. 5-1 to demonstrate the effectiveness and benefits of the proposed real-time congestion management method. The detailed data of load points can be found in Appendix B. The key HP parameters, line segment parameters and simulation parameters are listed in Table 5-2. It is assumed that each customer owns one HP, and there are two aggregators (AGG) in the study. AGG 1 has contracts with 80 customers per load point and AGG 2 has contracts with 120 customers per load point. Customers can be categorized into four types (Type1-Type4) according to the response rate θ . It is assumed that customers are willing to rebound energy as soon as possible. The day-ahead scheduled line loadings on L2, L3, and L13 without considering forecast errors are shown in Fig. 5-6.

Table 5-2 Key simulation parameters

parameters	value
COP of HP	2.3
Desired temperature	23 °C
Satisfying temperature range	22.7 -23.3 °C
Line loading limit of L2	1600kW
Line loading limit of L3	7200kW
Line loading limit of L13	9000kW
Resistance/reactance	0.26/0.027 omh/km
Response rate of Type1/2/3/4	0.5/0.4/0.3/0.2 kW/DKK
Lower limit of voltage	0.948 p.u.
Profit coefficient of agg1/agg2	1.3/1.5

Fig. 5-6 Day-ahead scheduled line loadings on L2, L3, and L13

5.5.2 Real-Time Congestion Management Effectiveness

In this section, the effectiveness of the proposed method to alleviate real-time congestion is demonstrated. As approaching 18:00, it is assumed that, due to the day-head forecast errors of the convention load consumption, the DSO predicts that there will be overloading of 100kW on L2 and 80kW on L3 at 18:00 and the overloading will last for 1 hour. The forecasted line loadings on L2, L3 and L13 are shown in Fig. 5-7. In order to alleviate congestion on L2 and L3, the DSO solves OPT.1 and OPT.2 and forms two FTs as listed in Table 5-3.

Fig. 5-7 Forecasted line loadings of L2, L3 and L13 at real-time operation

Table 5-3 Flexibility tables

unit (kW)	side	load point	18:00, 1h	19:00, 1h	20:00, 1h	21:00, 1h
FT1	$S_1^{(1)}$	LP ₁	-100	+, ≤6	+, ≤11	+, ≤197
	$S_1^{(2)}$	LP _{11-16, 18-25, 32-38}	+100	-, ≤6	-, ≤11	-, ≤197
FT2	$S_2^{(1)}$	LP ₂₋₅	-100	+, ≤20	+, ≤178	+, ≤947
	$S_2^{(2)}$	LP _{11-16, 18-25, 32-38}	+100	-, ≤20	-, ≤178	-, ≤947

For example, FT1 requires that a swap bid should provide a power consumption reduction with the amount of 100kW at LP₁ at 18:00 and at the same time provide opposite flexibility at LP₁₁₋₁₆, LP₁₈₋₂₅ and LP₃₂₋₃₈. Moreover, the FT1 stipulates the limits on the energy rebound amounts in the following periods and requires that swap bids should ensure the system balance. After receiving the FTs, the aggregators interact with their contractual customers to formulate swap bids and each aggregator provides two swap bids as listed in Table 5-4. In the swap market, the DSO chooses the swap bid 1 (S_1) provided by AGG2 and chooses the swap bid 2 (S_2) provided by AGG 1 because they have lower prices. Then, AGG 1 implements

S_2 and AGG 2 implements S_1 at the operation time. For example, at 18:00, AGG 2 will reduce HP power consumption by 100 kW at LP_1 to resolve congestion and will increase HP power consumption by 100kW at LP_{11-12} to ensure the system balance. Then, AGG 2 will increase HP power consumption at LP_1 by 6kW at 19:00 and by 11kW at 20:00, which allows the customer at LP_1 to restore household inside temperature. Moreover, the HP power consumption at LP_{11-12} is reduced at 19:00 and 20:00 to maintain the system balance. It is shown in Fig. 5-8 that congestion on L2 and L3 is resolved at 18:00 and energy rebound does not cause new congestion following 18:00 after implementing the selected swaps. Therefore, the proposed method can resolve real-time congestion efficiently.

Table 5-4 Swap bids provided by the aggregators

unit (kW)	swap	price (DKK)	side	load point	18:00, 1h	19:00, 1h	20:00, 1h
AGG.1	S_1	792	$S_1^{(1)}$	LP_1	-100	+6	+11
			$S_1^{(2)}$	LP_{11-12}	+100	-6	-11
	S_2	375	$S_2^{(1)}$	LP_{2-4}	-100	+20	+0.86
			$S_2^{(2)}$	LP_{13-15}	+100	-20	-0.86
AGG.2	S_1	527	$S_1^{(1)}$	LP_1	-100	+6	+11
			$S_1^{(2)}$	LP_{11-12}	+100	-6	-11
	S_2	565	$S_2^{(1)}$	LP_2	-100	+20	+2.13
			$S_2^{(2)}$	LP_{13-14}	+100	-20	-2.13

Fig. 5-8 Resulting line loadings after implementing selected swaps

The existing FDS-based method in [39] was carried out to compare with the proposed method. It is assumed that the same aggregators implement swaps and the same load points provide flexibility in the FDS method. Therefore, AGG 1 and AGG 2 implement the following two swaps, S_1^{FDS} and S_2^{FDS} , respectively, during the operation time to alleviate congestion in the FDS method.

$$S_1^{\text{FDS}} : \begin{cases} S^{(1)} : \{LP_{2-4}\}, (18:00, 1h, -100kW), (20:00, 1h, +100kW) \\ S^{(2)} : \{LP_{13-15}\}, (18:00, 1h, +100kW), (20:00, 1h, -100kW) \end{cases}, \quad (5.35)$$

$$S_2^{\text{FDS}} : \begin{cases} S^{(1)} : \{LP_1\}, (18:00, 1h, -100kW), (21:00, 1h, +100kW) \\ S^{(2)} : \{LP_{11-12}\}, (18:00, 1h, +100kW), (21:00, 1h, -100kW) \end{cases}, \quad (5.36)$$

After implementing the swaps, the resulting line loading on L2, L3 and L13 in the FDS method is shown in Fig. 5-9. It is shown that congestion is alleviated without causing new congestion. For example, AGG 2 in both methods reduces power consumption by 100kW at LP_1 at time 18:00 to resolve congestion and increases power

consumption at LP_{11-12} by 100kW at time 18:00 to ensure the system balance. However, AGG 2 in the FDS method only allows customers to rebound energy at 21:00 while customers in the proposed method can choose to rebound energy at any period after 18:00 as long as the total energy rebound amount does not exceed the limit given in FTs, e.g., 19:00 and 20:00 shown in Table 5-4.

Fig. 5-9 Resulting line loadings on L2, L3 and L13 in the FDS method

5.5.3 Customers' Requirements on Energy Rebound

The benefit of the proposed method that customer's specific requirements on energy rebound can be met is demonstrated. In the FDS method, the aggregator divides the amount of the required power consumption modification (decrease/increase) into equal units and allocates each unit to each customer. Table 5-5 lists the energy rebound amounts and percentages of the customer at LP_1 in both methods. The household inside temperatures of the customer in both methods are shown in Fig. 5-10. As shown in Table 5-5, the customer in the proposed method determines the actual energy rebound amount in each period, i.e., 0.0212 kW at 19:00 and 0.0541 kW at 20:00, and the rebound percentage (15.81%). However, AGG 2 in

the FDS method forces the customer to rebound energy with a rebound percentage of 100% (0.833 kW) at 21:00, which neglects the customer’s actual requirements on the rebound percentage. As a result, the customer’s temperature at 21:00 is higher than the satisfactory level, as shown in Fig. 5-10, because the required rebound percentage is lower than 100%. Similarly, the household temperature and power consumption of a customer located at LP₁₁ in both methods are shown in Fig. 5-11 and Fig. 5-12. As shown in Fig. 5-11, the customer’s temperature is restored to the satisfactory level at 20:00 in both methods. Therefore, the customer should not change its day-ahead scheduled HP power consumption after 20:00. However, in the FDS method, since energy rebound occurs at LP₁ at 21:00, the customer is forced to reduce its power consumption to provide opposite flexibility to ensure system balance at 21:00, as shown in Fig. 5-12, which results in lower temperature than the satisfactory level at 21:00, as shown in Fig. 5-11. In summary, the proposed two-tier method can consider customer’s specific requirements on energy rebound.

Table 5-5 The energy rebound amounts and percentages of the customer in two methods

unit (kW)	time				rebound percentage
	18:00, 1h	19:00, 1h	20:00, 1h	21:00, 1h	
Two-tier method	-0.476 ⁽¹⁾	0.0212	0.0541	0	15.81%
FDS method	-0.833 ⁽¹⁾	0	0	0.833	100%

⁽¹⁾ Notation “-” represents a reduction of HP power consumption

Fig. 5-10 Household temperatures of a customer located at LP₁ in two methods

Fig. 5-11 Household temperatures of a customer located at LP_{11} in two methods

Fig. 5-12 HP power consumption of a customer located at LP_{11} in two methods

5.5.4 Customer's Willingness to Provide Flexibility

This section demonstrates that the proposed method can consider customers' willingness to provide flexibility. The HP power consumption and household inside temperatures of Type 1 and Type 4 customers located at LP_1 are shown in Fig. 5-13 and Fig. 5-14, respectively. As shown in Fig. 5-13, since the Type 1 customer (non-conservative customer) has a larger response rate than the Type 4 customer (conservative customer), the Type 4 customer has a smaller modification of its power consumption target at 18:00, 19:00, and 20:00. Accordingly, Type 4 customer's household inside temperature is restored faster than the Type 1 customer, as shown in Fig. 5-14. To ensure system balance, the customers located at LP_{11-12} should provide opposite flexibility. The HP power consumption and household inside temperatures of Type 1 and Type 4 customers located at LP_{11} are shown in Fig. 5-15 and Fig. 5-16, respectively. Likewise, the conservative (Type 4) customer has less modification of its power consumption target and can restore its household temperature to the satisfactory level faster. Therefore, the customers' willingness to provide flexibility is considered in the proposed two-tier method. Moreover, customers can adjust their response curves to change their willingness.

Fig. 5-13 HP power consumption of Type1 and Type 4 customers

Fig. 5-14 Household inside temperatures of Type 1 and Type 4 customers

Fig. 5-15 HP power consumption of Type 1 and Type 4 customers

Fig. 5-16 Household inside temperatures of Type 1 and Type 4 customers

5.5.5 Customer's Revenues

The customers' revenues for providing flexibility are listed in Table 5-6. It is shown that the most non-conservative customer (Type 1) receives the maximum revenue since it provides more flexibility. In contrast, the most conservative customer (Type 4) receives the least revenue.

Table 5-6 The revenue of each type of customer and the total revenue of all customers

customers	type1	type2	type3	type4	total
DKK	3.223	2.578	1.934	1.289	405.617

5.6 Summary

A two-tier real-time congestion management method of distribution networks based on the flexible demand swap and transactive control is proposed in this chapter. With the FDS and swap market, congestion can be alleviated efficiently in the real-time time frame. In the swap market with the proposed operation mechanism, the customers can rebound energy in multiple periods and determine rebound percentages by themselves. Moreover, a transactive control-based interactive mechanism between the aggregators and customers is developed to consider the customer's willingness to provide flexibility. The case study results show that the proposed two-tier method can resolve real-time congestion efficiently while considering customers' specific requirements on energy rebound and willingness to provide flexibility.

CHAPTER 6.

HIERARCHICAL SERVICE RESTORATION FOR ACTIVE DISTRIBUTION NETWORKS BASED ON ALTERNATING DIRECTION METHOD OF MULTIPLIERS

This chapter presents a hierarchical service restoration scheme for ADNs based on the alternating direction method of multipliers (ADMM) in order to reduce the computation burden of service restoration for large-scale ADNs. This chapter is based on the published article 8 with changes to fit into the framework of the thesis⁶.

6.1 Introduction

The Advanced Metering Infrastructure (AMI) and Distribution Automation (DA) provide distribution networks self-healing capability. The application of self-healing enables the network, after a fault, to automatically and intelligently restore itself to the best possible status with minimum human intervention based on a set of devices, algorithms, and information and communication technologies (ICT). A self-healing scheme consists of fault location, fault isolation, and service restoration. After fault location and isolation, service restoration plans are used to restore outage areas.

As mentioned in chapter 1, most of the service restoration methods in the literature are centralized methods, which are based on the distribution automation system (DAS) consisting of a master station, sub-stations, DA devices, and a communication system. Among the centralized methods, expert systems, meta-heuristics, graph theory, and mathematical programming have been used to solve the service

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⁶ This chapter is based on the published article 8: **Feifan Shen**, Qiuwei Wu*, Yan Xu, Fangxing Li, Fei Teng, and Goran Strbac, "Hierarchical Service Restoration Scheme for Active Distribution Networks Based on Alternating Direction of Multipliers (ADMM)," *International Journal of Electrical Power and Energy System*, vol. 118, pp. 105809, Jun. 2020.

restoration problem. The main advantage of the mathematical programming-based centralized methods is the ability to formulate and solve complex instances of the service restoration problem, considering the multi-objective optimization and stochasticity. However, the computational burden of the service restoration problem grows significantly with the increasing size and complexity of the distribution network because service restoration is a combinatorial non-linear optimization problem. Thus, the centralized methods may have high computational burdens for large-scale ADNs. To resolve this issue, an ADMM-based hierarchical service restoration scheme is proposed to obtain the optimal service restoration plan in a hierarchical manner and to reduce computation complexity. As a distributed optimization technique, the ADMM has been widely used to distribute and solve convex optimization problems [118], such as the convexified optimal power flow problem [119], [120], optimal power-gas flow problem [121], and voltage control problem [122].

Firstly, the optimal service restoration problem considering the switching sequence, DG units, and controllable loads is formulated as an MINLP model. Then, by applying linearization techniques, the MINLP model is transformed into an MILP model that is decomposed and solved using the proposed ADMM-based algorithm in a hierarchical manner. The contributions of this chapter are summarized as follows: 1) propose an MILP model formulation for service restoration considering the switching sequence, DG units and controllable loads; and 2) develop an ADMM-based hierarchical solution method to solve service restoration problem and reduce computational complexity.

The structure of this chapter is organized as follows. The framework of the proposed hierarchical service restoration scheme is presented in section 6.2. Section 6.3 provides the model formulation of the optimal service restoration problem. Section 6.4 presents the ADMM-based algorithm to solve the service restoration model. Case study results are discussed in Section 6.5, followed by a summary in section 6.6.

6.2 Framework of the Proposed Hierarchical Service Restoration Scheme

The service restoration problem is solved using a hierarchical framework shown in Fig. 6-1. The proposed framework is based on independent local controllers and a central controller with limited information exchanged among them. As shown in Fig. 6-1, a zone that is the interconnected portion of the network separated by switches has a local controller. Based on the ADMM-based algorithm, each zone

controller solves its own sub-problem with local network data and information from the central controller to optimize variables associated with the zone, e.g., the amount of load shedding. The central controller solves a global problem with information collected from each zone to coordinate the status of switches and zones, i.e., opening or closing switches and energizing or de-energizing the zone.

Fig. 6-1 Framework of the proposed hierarchical service restoration scheme.

6.3 Model Formulations for the Optimal Service Restoration Problem

6.3.1 Optimal Service Restoration Model

Consider the distribution network as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ denotes the set of nodes and $\mathcal{L}$ denotes the set of branches and switches. In the network, every node and branch belong to a unique zone $z \in \mathcal{Z}$. The optimal service restoration problem is formulated as an MINLP model in (6.1)-(6.19).

6.3.1.1 Objective Function

$$\begin{aligned} \min \quad & \sum_{z \in \mathcal{Z}} \sum_{s \in \mathcal{S}} c_z^U (1 - x_{z,s}) + \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{S}} c_i^G P_{i,s}^G + \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{S}} c_i^r r_{i,s} P_i^D \\ & + \sum_{ij \in \mathcal{L}} \sum_{s \in \mathcal{S}} c_{ij}^{lss} I_{ij,s}^{lss} R_{ij} + \sum_{ij \in \mathcal{L}} \sum_{s \in \mathcal{S}} c_{ij}^s (y_{ij,s}^u + y_{ij,s}^d) \end{aligned} \quad (6.1)$$

The objective function in (6.1) minimizes five terms: the first term is to minimize the number of de-energized zones, where $\mathcal{Z}$ is the set of zones, $\mathcal{S}$ is the set of restoration steps, $x_{z,s}$ is the status of z -th zone at step s , and c_z^U is the cost to de-

energize a zone; the second term is to minimize the energy production cost, where $\mathcal{N}$ is the set of nodes, c_i^G is the energy cost of the DG unit at node i , and $P_{i,s}^G$ is the DG active power generation at node i at step s ; the third term is to minimize the load shedding cost, where c_i^r is the load shedding cost per unit at node i , P_i^D is the active demand at node i , and $r_{i,s}$ is the load shedding percentage at node i at step s ; the fourth term is to minimize line loss cost, where $\mathcal{L}$ is the set of lines, c_{ij}^{loss} is the line loss cost per unit at line ij , $I_{ij,s}^{\text{sqr}}$ is the square current magnitude at line ij from nodes i to j , and R_{ij} is the resistance at line ij ; the fifth term is to minimize the number of switching operations, where $\mathcal{L}^S$ is the set of switches, c_{ij}^s is the switching operation cost of the switch ij , and $y_{ij,s}^u$ and $y_{ij,s}^d$ are auxiliary binary variables associated with switching operations.

6.3.1.2 Operational Constraints

- Power flow equations

$$P_{i,s}^G + \sum_{ji \in \mathcal{L}} P_{ji,s} - \sum_{ij \in \mathcal{L}} (P_{ij,s} + I_{ij,s}^{\text{sqr}} R_{ij}) = P_{i,s}^D (x_{z,s} - r_{i,s}); \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}, \quad (6.2)$$

$$Q_{i,s}^G + \sum_{ji \in \mathcal{L}} Q_{ji,s} - \sum_{ij \in \mathcal{L}} (Q_{ij,s} + I_{ij,s}^{\text{sqr}} X_{ij}) = Q_{i,s}^D (x_{z,s} - r_{i,s}); \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}, \quad (6.3)$$

$$V_{j,s}^{\text{sqr}} I_{ij,s}^{\text{sqr}} = (P_{ij,s})^2 + (Q_{ij,s})^2; \forall ji \in \mathcal{L}, s \in \mathcal{S}, \quad (6.4)$$

$$V_{i,s}^{\text{sqr}} - V_{j,s}^{\text{sqr}} = 2(P_{ij,s} R_{ij} + Q_{ij,s} X_{ij}) + I_{ij,s}^{\text{sqr}} \left[(R_{ij})^2 + (X_{ij})^2 \right]; \forall ij \in \mathcal{L}^B, s \in \mathcal{S}, \quad (6.5)$$

Constraint (6.2) is the active power balance equation at each node at each restoration step, where $\mathcal{B}(z)$ is the set of nodes within z -th zone, and $P_{ij,s}$ is the active power flow on line ij from nodes i to j at step s . Constraint (6.3) is the reactive power balance equation at each node at each step, where $Q_{ij,s}$ is the reactive power flow on line ij from nodes i to j at step s , and $Q_{i,s}^G$ is the DG reactive power generation at node i at step s . Current magnitudes of all branches and switches at each step are calculated in (6.4), where $V_{j,s}^{\text{sqr}}$ is the square voltage magnitude at node j at

step s . The voltage drop at each branch is calculated in (6.5), where $\mathcal{L}^B$ is the set of branches.

- Voltage magnitude constraint

$$-\bar{V}^{\text{sqr}}(1 - w_{ij}) \leq V_{i,s}^{\text{sqr}} - V_{j,s}^{\text{sqr}} \leq \bar{V}^{\text{sqr}}(1 - w_{ij}); \forall ij \in \mathcal{L}^S, s \in \mathcal{S}, \quad (6.6)$$

$$x_{z,s} \underline{V}^{\text{sqr}} \leq V_{i,s}^{\text{sqr}} \leq x_{z,s} \bar{V}^{\text{sqr}}; \quad \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}, \quad (6.7)$$

Constraint (6.6) ensures that if a switch is closed, voltage magnitudes at both ends of the switch are identical, where w_{ij} is the status of switch ij , $w_{ij} = 1$ if the switch is closed; otherwise, $w_{ij} = 0$. Constraint (6.8) limits voltage magnitudes at all nodes at each step: if a given zone is de-energized, voltage magnitudes at all nodes within the zone are forced to be zero.

- Load shedding constraint

$$0 \leq r_{i,s} \leq x_{z,s}; \quad \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}, \quad (6.8)$$

Constraint (6.8) limits the load shedding percentage; if a zone is de-energized, the load shedding percentage at each node within the zone is forced to be zero.

- Current magnitude constraint

$$\begin{cases} I_{ij,s}^{\text{sqr}} \leq x_{z,s} \bar{I}_{ij}^{\text{sqr}}; & \forall z \in \mathcal{Z}, ij \in \mathcal{L}^B, s \in \mathcal{S} \\ I_{ij,s}^{\text{sqr}} \leq w_{ij,s} \bar{I}_{ij}^{\text{sqr}}; & \forall ij \in \mathcal{L}^S, s \in \mathcal{S} \end{cases}, \quad (6.9)$$

Constraint (6.9) limits the square current magnitudes of branches and switches at each step.

- DG generation constraint

$$\begin{cases} 0 \leq P_{i,s}^G \leq \bar{P}_i^G x_{\hat{z}_i,s}; & \forall i \in \mathcal{N}, s \in \mathcal{S} \\ 0 \leq Q_{i,s}^G \leq \bar{Q}_i^G x_{\hat{z}_i,s}; & \forall i \in \mathcal{N}, s \in \mathcal{S} \end{cases}, \quad (6.10)$$

Constraint (6.10) limits the maximum DG active and reactive generations at each node at each step.

- Switching operation constraint

$$w_{ij,s} - w_{ij,s-1} = y_{ij,s}^u - y_{ij,s}^d; \forall ij \in \mathcal{L}^s, s \in \mathcal{S} \mid w_{ij,s_1-1} = w_i^{j,ini}, \quad (6.11)$$

$$\sum_{ij \in \mathcal{L}^s} (y_{ij,s}^u + y_{ij,s}^d) \leq 1; \forall s \in \mathcal{S}, \quad (6.12)$$

Constraint (6.11) models the switching action. Constraint (6.12) ensures at most one switching operation at each step.

- Zone status constraint

$$|x_{\hat{z}_i,s} - x_{\hat{z}_j,s}| \leq 1 - w_{ij,s}; \forall ij \in \mathcal{L}^s, s \in \mathcal{S}, \quad (6.13)$$

Constraint (6.13) represents that connected zones must have the same status, where $\hat{z}_i$ is the index of the zone to which node i belongs.

- Radial topology constraint

$$\sum_{ij \in \mathcal{L}^s} w_{ij,s} x_{\hat{z}_i,s} = \sum_{z \in \mathcal{Z}} x_{z,s}; \forall s \in \mathcal{S}, \quad (6.14)$$

$$K_{i,s}^G + \sum_{ji \in \mathcal{L}} K_{ji,s} - \sum_{ij \in \mathcal{L}} K_{ij,s} = K_i^D x_{z,s}; \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}, \quad (6.15)$$

$$\begin{cases} K_{ij,s} \leq x_{z,s} \bar{K}_{ij}; & \forall z \in \mathcal{Z}, ij \in \mathcal{L}^B, s \in \mathcal{S} \\ K_{ij,s} \leq w_{ij,s} \bar{K}_{ij}; & \forall ij \in \mathcal{L}^s, s \in \mathcal{S} \end{cases}, \quad (6.16)$$

$$K_{i,s}^G = 0; \forall i \in \mathcal{N} \setminus \mathcal{N}^S, s \in \mathcal{S}, \quad (6.17)$$

Constraints (6.14)-(6.17) ensure a radial topology of the energized portion of the network at each step [123], where $\mathcal{N}^S$ is the set of substation source nodes, $K_{ij,s}$ is the fictitious active power flow from nodes i to j , $K_{i,s}^G$ is the fictitious active power

generation at node i at step s , K_i^D is the fictitious active demand at node i , and $\bar{K}_{ij}$ is the upper limit on $K_{ij,s}$. Constraints (6.15)–(6.17) force energized zones being connected to the main grid by supplying fictitious active demands of each energized zone from substations.

- Service restoration process constraint

$$|w_{ij,s+1} - w_{ij,s}| \leq 1 - w_{i^*j^*,s}^*; \forall i^*j^* \in \mathcal{L}^F, (i,j) \in \mathcal{L}^S \setminus \mathcal{L}^F, s \in \mathcal{S}, \quad (6.18)$$

Constraint (6.18) represents that those switches that are not connected to the faulty zone can operate only after the switches connected to the faulty are opened, denoting that the service restoration process starts after the faulty zone isolation.

- Binary variables

$$x_{z,s}, w_{i,s}^j, y_{i,s}^{j,u}, y_{i,s}^{j,d} \in \{0,1\}; \quad \forall z \in \mathcal{Z}, (i,j) \in \mathcal{L}^S, s \in \mathcal{S}, \quad (6.19)$$

The binary decision variables are defined in (6.19).

6.3.2 Linearization of the MINLP Model

The optimal service restoration problem formulated as an MINLP model is hard to solve. To efficiently obtain service restoration plans, linearization techniques are used to linearize non-linear constraints (6.4) and (6.14), which results in an MILP model.

6.3.2.1 Linearization of Constraint (6.4)

Firstly, the product of continuous variables $V_{j,s}^{\text{sqr}} I_{ij,s}^{\text{sqr}}$ in (6.4) is approximated in (6.20) by fixing $V_{j,s}^{\text{sqr}}$ as the square nominal value $(V^{\text{nom}})^2$. The approximation error is low because voltage magnitudes are constrained in a small range. Secondly, the square terms in (6.4) are approximated using the piece-wise linearization technique [62] as shown in (6.21)–(6.27).

$$V_{j,s}^{\text{sqr}} I_{ij,s}^{\text{sqr}} \approx (V^{\text{nom}})^2 I_{ij,s}^{\text{sqr}}; \quad (6.20)$$

$$(P_{ij,s})^2 \approx L(P_{ij,s}), (Q_{ij,s})^2 \approx L(Q_{ij,s}); \quad (6.21)$$

where function $L(z)$ is defined as follows, where $\mathcal{H}$ is the set of linearization segments.

$$L(z) = \sum_{\lambda=1}^{|\mathcal{H}|} \phi_{\lambda} \Delta z_{\lambda}, \quad (6.22)$$

$$\sum_{\lambda=1}^{|\mathcal{H}|} \Delta z_{\lambda} = z^+ + z^-, \quad (6.23)$$

$$z = z^+ - z^-, \quad (6.24)$$

$$0 \leq \Delta z_{\lambda} \leq \frac{\bar{z}}{|\mathcal{H}|}; \forall \lambda \in \mathcal{H}, \quad (6.25)$$

$$\phi_{\lambda} = (2\lambda - 1) \frac{\bar{z}}{|\mathcal{H}|}; \forall \lambda \in \mathcal{H}, \quad (6.26)$$

$$z^+, z^- \geq 0, \quad (6.27)$$

Then, the linearization approximation of (6.4) is obtained as,

$$\left(V^{\text{nom}}\right)^2 I_{ij,s}^{\text{sqr}} = L(P_{ij,s}) + L(Q_{ij,s}); \forall ij \in \mathcal{L}, s \in \mathcal{S}, \quad (6.28)$$

6.3.2.2 Linearization of Constraint (6.14)

The product of binary variables $w_{ij,s} x_{z_i,s}$ in (6.14) is replaced by a continuous auxiliary variable $\tau_{ij,s}$ as shown in (6.29).

$$\begin{cases} \sum_{ij \in \mathcal{L}^S} \tau_{ij,s} = \sum_{z \in \mathcal{Z}} x_{z,s}; \quad \forall s \in \mathcal{S} \\ w_{ij,s} + x_{z_i,s} - 1 \leq \tau_{ij,s}; \quad \forall ij \in \mathcal{L}^S, s \in \mathcal{S}, \\ 0 \leq \tau_{ij,s} \leq w_{ij,s}; \quad \forall ij \in \mathcal{L}^S, s \in \mathcal{S} \\ 0 \leq \tau_{ij,s} \leq x_{z_i,s}; \quad \forall ij \in \mathcal{L}^S, s \in \mathcal{S} \end{cases}, \quad (6.29)$$

Finally, the MILP model is formulated as (6.1)-(6.3), (6.5)-(6.13), (6.15)-(6.19) and (6.28), (6.29).

6.4 ADMM-Based Algorithm for the Service Restoration Model

Although the ADMM is originally designed to solve convex problems, some variations of ADMM have been proposed to solve non-convex instances, such as MILP models [124], [125] and cardinality constrained problems [126]. The practical evidence in these studies suggests that the ADMM is also an effective algorithm to obtain high-quality solutions for non-convex problems [124].

6.4.1 Reformulation of the MILP Model for the ADMM

For node $i \in \mathcal{N}$ at step $s \in \mathcal{S}$, define sets $\mathcal{P}_{i,s}$, $\mathcal{I}_{i,s}$, $\mathcal{V}_{i,s}$ and $\mathcal{W}_{i,s}$ as below,

$$\begin{cases} \mathcal{P}_{ij,s} := \{P_{ij,s}, H_{ij,s}, Q_{ij,s}\}; \forall ij \in \mathcal{L} \\ \mathcal{V}_{i,s} := \{V_{i,s}^{\text{sqr}}\}; \forall i \in \mathcal{N} \\ \mathcal{I}_{i,s} := \left\{ \left\{ I_{ij}^{\text{sqr}} \right\}_{ij \in \mathcal{L}}, r_i, P_i^G, Q_i^G \right\}; \forall i \in \mathcal{N} \\ \mathcal{W}_{ij,s} := \{w_{ij,s}, y_{ij}^d, y_{ij}^u, \tau_{ij,s}\}; \forall ij \in \mathcal{L}^s \end{cases}$$

The sets $\mathcal{P}_{ij,s}$, $\mathcal{I}_{i,s}$, $\mathcal{V}_{i,s}$ and $\mathcal{W}_{ij,s}$ consist of variables with the subscript of i or ij , which are considered as affiliated variables of node i . Since each node belongs to a unique zone, the variables with the subscript i or ij are exclusively optimized by the corresponding zone controller $\hat{z}_i$. For clarity, the MILP model is presented in a compact form.

The objective function in (6.1) is separable for each zone and is rewritten as,

$$\min f(x_{z,s}, \mathcal{I}_{i,s}, \mathcal{W}_{ij,s}) = \sum_{z \in \mathcal{Z}} \sum_{i \in \mathcal{B}(z)} \sum_{s \in \mathcal{S}} G_{z,i,s} \left[x_{z,s}, \mathcal{I}_{i,s}, \left\{ \mathcal{W}_{ij,s} \right\}_{\forall ij \in \mathcal{L}^s} \right]^T, \quad (6.30)$$

Constraints (6.2), (6.3), (6.5), (6.6) and (6.15) can be rewritten as,

$$\begin{aligned} A_{i,s} \left[\left\{ \mathcal{P}_{ij,s}, \mathcal{V}_{i,s}, \mathcal{V}_{j,s} \right\}_{ij \in \mathcal{L}}, \left\{ \mathcal{P}_{ji,s} \right\}_{ji \in \mathcal{L}}, \mathcal{I}_{i,s}, \left\{ \mathcal{W}_{ij,s} \right\}_{\forall ij \in \mathcal{L}^s}, x_{z,s} \right]^T &\leq a_{i,s}; \\ \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S} \end{aligned} \quad (6.31)$$

where $A_{i,s}$ and $a_{i,s}$ are the corresponding coefficient matrixes. It is obvious that adjacent nodes i and j are coupled through $\mathcal{P}_{ij,s}, \mathcal{P}_{ji,s}, \mathcal{V}_{i,s}, \mathcal{V}_{j,s}$ in (6.31), where $ij \in \mathcal{L}$, which results in a coupling between adjacent zones $\hat{z}_i$ and $\hat{z}_j$.

Constraints (6.7)-(6.11), (6.16)-(6.18) and (6.28) are separable for each zone and are rewritten as,

$$B_{i,s} \left[\left\{ \mathcal{P}_{ij,s} \right\}_{ij \in \mathcal{L}}, \mathcal{I}_{i,s}, \mathcal{V}_{i,s}, \left\{ \mathcal{W}_{ij,s} \right\}_{\forall ij \in \mathcal{L}^S}, x_{z,s} \right]^T \leq b_{i,s}; \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}, \quad (6.32)$$

where $B_{i,s}$ and $b_{i,s}$ are the corresponding coefficient matrixes.

Constraints (6.12), (6.13), and (6.29) can be rewritten as,

$$C \left[\left\{ \mathcal{W}_{ij,s} \right\}_{i \in \mathcal{L}, s \in \mathcal{S}}, \left\{ x_{z,s} \right\}_{z \in \mathcal{Z}, s \in \mathcal{S}} \right]^T \leq c, \quad (6.33)$$

where C and c are the corresponding coefficient matrixes.

Then, the MILP model is expressed by the compact form in (6.30)-(6.33). To use the ADMM to decompose the MILP model, a set of auxiliary variables ($\tilde{\mathcal{P}}_{ij,s}, \hat{\mathcal{P}}_{ij,s}, \tilde{\mathcal{V}}_{i,s}, \hat{\mathcal{V}}_{ij,s}, \tilde{\mathcal{W}}_{ij,s}, \tilde{x}_{z,s}$) and equality constraints are introduced as shown in (6.38). By replacing the original variables in (6.30)-(6.32) with the auxiliary variables, the MILP model is rewritten as,

$$\min f(\tilde{x}_{z,s}, \mathcal{I}_{i,s}, \tilde{\mathcal{W}}_{ij,s}), \quad (6.34)$$

$$A_{i,s} \left[\left\{ \tilde{\mathcal{P}}_{ij,s}, \tilde{\mathcal{V}}_{i,s}, \hat{\mathcal{V}}_{ij,s} \right\}_{ij \in \mathcal{L}}, \left\{ \hat{\mathcal{P}}_{ij,s} \right\}_{ji \in \mathcal{L}}, \mathcal{I}_{i,s}, \left\{ \tilde{\mathcal{W}}_{ij,s} \right\}_{\forall ij \in \mathcal{L}^S}, \tilde{x}_{z,s} \right]^T \leq a_{i,s}; \quad (6.35)$$

$$\forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}$$

$$B_{i,s} \left[\left\{ \tilde{\mathcal{P}}_{ij,s} \right\}_{ij \in \mathcal{L}}, \mathcal{I}_{i,s}, \tilde{\mathcal{V}}_{i,s}, \left\{ \tilde{\mathcal{W}}_{ij,s} \right\}_{\forall ij \in \mathcal{L}^S}, \tilde{x}_{z,s} \right] \leq b_{i,s}; \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), s \in \mathcal{S}, \quad (6.36)$$

$$C \left[\left\{ \mathcal{W}_{ij,s} \right\}_{\forall ij \in \mathcal{L}^S, s \in \mathcal{S}}, \left\{ x_{z,s} \right\}_{z \in \mathcal{Z}, s \in \mathcal{S}} \right]^T \leq c; \quad (6.37)$$

$$\begin{cases} \mathcal{P}_{ij,s} = \tilde{\mathcal{P}}_{ij,s}; & \forall ij \in \mathcal{L}, s \in \mathcal{S} \\ \mathcal{P}_{ji,s} = \hat{\mathcal{P}}_{ij,s}; & \forall ji \in \mathcal{L}, s \in \mathcal{S} \\ \mathcal{V}_{i,s} = \tilde{\mathcal{V}}_{i,s}, \mathcal{V}_{j,s} = \hat{\mathcal{V}}_{ij,s}; & \forall i \in \mathcal{N}, ij \in \mathcal{L}, s \in \mathcal{S}, \\ \mathcal{W}_{ij,s} = \tilde{\mathcal{W}}_{ij,s}; & \forall ij \in \mathcal{L}^S, s \in \mathcal{S} \\ x_{z,s} = \tilde{x}_{z,s}; & \forall z \in \mathcal{Z}, s \in \mathcal{S} \\ \tilde{\mathcal{P}}_{ij,s}, \hat{\mathcal{P}}_{ij,s}, \tilde{\mathcal{W}}_{ij,s}, \tilde{x}_{z,s} \in \mathcal{R} \end{cases}, \quad (6.38)$$

Note that, those variables with the subscript of i or ij exclusively belong to node i . Therefore, (6.35) and (6.36) can be decomposed among each zone.

6.4.2 ADMM-Based Algorithm

6.4.2.1 Augmented Lagrangian

For brevity, define sets $\mathcal{X}_1$, $\mathcal{X}_2$ and Λ as below,

$$\begin{cases} \mathcal{X}_1 := \left\{ \tilde{\mathcal{P}}_{ij,s}, \hat{\mathcal{P}}_{ij,s}, \mathcal{I}_{i,s}, \tilde{\mathcal{V}}_{i,s}, \hat{\mathcal{V}}_{ij,s}, \tilde{\mathcal{W}}_{ij,s}, \tilde{x}_{z,s} \mid (6.35), (6.36) \right\}; \\ \mathcal{X}_2 := \left\{ \mathcal{P}_{ij,s}, \mathcal{W}_{ij,s}, \mathcal{V}_{i,s}, \mathcal{V}_{j,s}, x_{z,s} \mid (6.37) \right\}; \\ \Lambda := \left\{ \mathcal{A}_{ij,s}, \mathcal{B}_{ij,s}, \mathcal{C}_{i,s}, \mathcal{D}_{ij,s}, \mathcal{F}_{ij,s}, \mathcal{G}_{z,s} \right\}; \end{cases}$$

where $\mathcal{X}_1$ and $\mathcal{X}_2$ are the sets of primal variables and Λ is the set of dual variables. Then, by adding equality constraint (6.38) into the objective function (6.34) through the dual variables, the augmented Lagrangian of the MILP problem is formulated as,

$$\begin{aligned} & \min_{\mathcal{X}_1, \mathcal{X}_2, \Lambda} f(\tilde{x}_{z,s}, \mathcal{I}_{i,s}, \tilde{\mathcal{W}}_{ij,s}) + \\ & + \frac{\rho}{2} \left\{ \sum_{ij \in \mathcal{L}, s \in \mathcal{S}} \left(\|\mathcal{P}_{ij,s} - \tilde{\mathcal{P}}_{ij,s} + \mathcal{A}_{ij,s}\|^2 - \|\mathcal{A}_{ij,s}\|^2 \right) + \sum_{ji \in \mathcal{L}, s \in \mathcal{S}} \left(\|\mathcal{P}_{ji,s} - \hat{\mathcal{P}}_{ij,s} + \mathcal{B}_{ij,s}\|^2 - \|\mathcal{B}_{ij,s}\|^2 \right) \right. \\ & + \sum_{ij \in \mathcal{L}, s \in \mathcal{S}} \left(\|\mathcal{V}_{j,s} - \hat{\mathcal{V}}_{ij,s} + \mathcal{C}_{ij,s}\|^2 - \|\mathcal{C}_{ij,s}\|^2 \right) + \sum_{i \in \mathcal{N}, s \in \mathcal{S}} \left(\|\mathcal{V}_{i,s} - \tilde{\mathcal{V}}_{i,s} + \mathcal{D}_{i,s}\|^2 - \|\mathcal{D}_{i,s}\|^2 \right) \\ & \left. + \sum_{ij \in \mathcal{L}^S, s \in \mathcal{S}} \left(\|\mathcal{W}_{ij,s} - \tilde{\mathcal{W}}_{ij,s} + \mathcal{F}_{ij,s}\|^2 - \|\mathcal{F}_{ij,s}\|^2 \right) + \sum_{z \in \mathcal{Z}, s \in \mathcal{S}} \left(\|x_{z,s} - \tilde{x}_{z,s} + \mathcal{G}_{z,s}\|^2 - \|\mathcal{G}_{z,s}\|^2 \right) \right\} \end{aligned}$$

Then, the ADMM-based algorithm is used to solve the augmented Lagrangian in an iterative manner.

6.4.2.2 Sub-Problem I

Sub-problem I is a QP problem given by (6.39). At the k -th iteration, variables in $\mathcal{X}_1$ are optimized, and other variables are fixed as the values obtained at the $(k-1)$ -th iteration, i.e., $\mathcal{X}_2^{k-1}$ and Λ^{k-1} .

$$\begin{aligned} \min_{\mathcal{X}_1} f & \left(\tilde{x}_{z,s}, \mathcal{I}_{i,s}, \tilde{\mathcal{W}}_{ij,s} \right) \\ & + \frac{\rho}{2} \left\{ \sum_{ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ij,s}^{k-1} - \tilde{\mathcal{P}}_{ij,s} + \mathcal{A}_{ij,s}^{k-1} \right\|^2 + \sum_{ji \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ji,s}^{k-1} - \hat{\mathcal{P}}_{ij,s} + \mathcal{B}_{ij,s}^{k-1} \right\|^2 \right. \\ & + \sum_{ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{V}_{j,s}^{k-1} - \hat{\mathcal{V}}_{ij,s} + \mathcal{C}_{ij,s}^{k-1} \right\|^2 + \sum_{i \in \mathcal{N}, s \in \mathcal{S}} \left\| \mathcal{V}_{i,s}^{k-1} - \tilde{\mathcal{V}}_{i,s} + \mathcal{D}_{i,s}^{k-1} \right\|^2 \\ & \left. + \sum_{ij \in \mathcal{L}^S, s \in \mathcal{S}} \left\| \mathcal{W}_{ij,s}^{k-1} - \tilde{\mathcal{W}}_{ij,s} + \mathcal{F}_{ij,s}^{k-1} \right\|^2 + \sum_{z \in \mathcal{Z}, s \in \mathcal{S}} \left\| x_{z,s}^{k-1} - \tilde{x}_{z,s} + \mathcal{G}_{z,s}^{k-1} \right\|^2 \right\} \end{aligned} \quad (6.39)$$

The objective function in (6.39) and constraints (6.35) and (6.36) are separable for each zone. Thus, variables in $\mathcal{X}_1$ can be optimized in parallel for each zone. In sub-problem I, each zone controller solves its sub-problem locally.

6.4.2.3 Sub-Problem II

Sub-problem II is a MIQP problem shown in (6.40). At the k -th iteration, variables in $\mathcal{X}_2$ are optimized with obtained $\mathcal{X}_1^{k-1}$ and Λ^{k-1} .

$$\begin{aligned} \min_{\mathcal{X}_2} \frac{\rho}{2} & \left\{ \sum_{ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ij,s} - \tilde{\mathcal{P}}_{ij,s}^k + \mathcal{A}_{ij,s}^{k-1} \right\|^2 + \sum_{ji \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ji,s} - \hat{\mathcal{P}}_{ij,s}^k + \mathcal{B}_{ij,s}^{k-1} \right\|^2 \right. \\ & + \sum_{ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{V}_{j,s} - \hat{\mathcal{V}}_{ij,s}^k + \mathcal{C}_{ij,s}^{k-1} \right\|^2 + \sum_{i \in \mathcal{N}, s \in \mathcal{S}} \left\| \mathcal{V}_{i,s} - \tilde{\mathcal{V}}_{i,s}^k + \mathcal{D}_{i,s}^{k-1} \right\|^2 \\ & \left. + \sum_{ij \in \mathcal{L}^S, s \in \mathcal{S}} \left\| \mathcal{W}_{ij,s} - \tilde{\mathcal{W}}_{ij,s}^k + \mathcal{F}_{ij,s}^{k-1} \right\|^2 + \sum_{z \in \mathcal{Z}, s \in \mathcal{S}} \left\| x_{z,s} - \tilde{x}_{z,s}^k + \mathcal{G}_{z,s}^{k-1} \right\|^2 \right\} \end{aligned} \quad (6.40)$$

In (6.40), $\mathcal{W}_{ij,s}$ and $x_{z,s}$ are coupled in $\mathcal{X}_2$, as shown in (6.37). Therefore, variables in $\mathcal{W}_{ij,s}$ and $x_{z,s}$ are optimized in a centralized way by the central controller. It is

noted that variables in $\mathcal{P}_{ij,s}$, and $\mathcal{V}_{i,s}$ are free variables and the associated terms in the objective function are separable for each zone. Therefore, these variables can be optimized in parallel by local zone controllers.

6.4.2.4 Dual Variable Update and Stopping Criterion

After solving sub-problems I and II, dual variables are updated using (6.41) at the k -th iteration. In the ADMM-based algorithm, primal and dual variables are updated iteratively. The iteration process stops till aggregated primal residuals r^k and dual residuals d^k are lower than specified thresholds ε_1 and ε_2 , as shown in (6.42)-(6.44).

$$\begin{cases} \mathcal{A}_{ij,s}^k = \mathcal{P}_{ij,s}^k - \tilde{\mathcal{P}}_{ij,s}^k + \mathcal{A}_{ij,s}^{k-1}; & \forall ij \in \mathcal{L}, s \in \mathcal{S} \\ \mathcal{B}_{ji,s}^k = \mathcal{P}_{ji,s}^k - \hat{\mathcal{P}}_{ji,s}^k + \mathcal{B}_{ji,s}^{k-1}; & \forall ji \in \mathcal{L}, s \in \mathcal{S} \\ \mathcal{C}_{ij,s}^k = \mathcal{V}_{j,s}^k - \hat{\mathcal{V}}_{ij,s}^k + \mathcal{C}_{ij,s}^{k-1}; & \forall ij \in \mathcal{L}, s \in \mathcal{S} \\ \mathcal{D}_{i,s}^k = \mathcal{V}_{i,s}^k - \tilde{\mathcal{V}}_{i,s}^k + \mathcal{D}_{i,s}^{k-1}; & \forall i \in \mathcal{N}, s \in \mathcal{S} \\ \mathcal{F}_{ij,s}^k = \mathcal{W}_{ij,s}^k - \tilde{\mathcal{W}}_{ij,s}^k + \mathcal{F}_{ij,s}^{k-1}; & \forall ij \in \mathcal{L}, s \in \mathcal{S} \\ \mathcal{G}_{z,s}^k = \mathcal{X}_{z,s}^k - \tilde{\mathcal{X}}_{z,s}^k + \mathcal{G}_{z,s}^{k-1}; & \forall z \in \mathcal{Z}, s \in \mathcal{S} \end{cases}, \quad (6.41)$$

$$\begin{aligned} r_z^k = & \left\{ \sum_{i \in \mathcal{B}(z), ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ij,s}^k - \tilde{\mathcal{P}}_{ij,s}^k \right\| + \sum_{i \in \mathcal{B}(z), ji \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ji,s}^k - \hat{\mathcal{P}}_{ji,s}^k \right\| \right. \\ & \sum_{i \in \mathcal{B}(z), ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{V}_{j,s}^k - \hat{\mathcal{V}}_{ij,s}^k \right\| + \sum_{i \in \mathcal{B}(z), s \in \mathcal{S}} \left\| \mathcal{V}_{i,s}^k - \tilde{\mathcal{V}}_{i,s}^k \right\|^2 \\ & \left. \sum_{i \in \mathcal{B}(z), ij \in \mathcal{L}^S, s \in \mathcal{S}} \left\| \mathcal{W}_{ij,s}^k - \tilde{\mathcal{W}}_{ij,s}^k \right\|^2 \right\} + \sum_{s \in \mathcal{S}} \left\| \mathcal{X}_{z,s}^k - \tilde{\mathcal{X}}_{z,s}^k \right\|^2; \forall z \in \mathcal{Z} \end{aligned}, \quad (6.42)$$

$$\begin{aligned} d_z^k = & \left\{ \sum_{i \in \mathcal{B}(z), ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ij,s}^k - \mathcal{P}_{ij,s}^{k-1} \right\| + \sum_{i \in \mathcal{B}(z), ji \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{P}_{ji,s}^k - \mathcal{P}_{ji,s}^{k-1} \right\| \right. \\ & + \sum_{i \in \mathcal{B}(z), ij \in \mathcal{L}, s \in \mathcal{S}} \left\| \mathcal{V}_{j,s}^k - \mathcal{V}_{j,s}^{k-1} \right\| + \sum_{i \in \mathcal{B}(z), s \in \mathcal{S}} \left\| \mathcal{V}_{i,s}^k - \mathcal{V}_{i,s}^{k-1} \right\|^2 \\ & \left. + \sum_{i \in \mathcal{B}(z), ij \in \mathcal{L}^S, s \in \mathcal{S}} \left\| \mathcal{W}_{ij,s}^k - \mathcal{W}_{ij,s}^{k-1} \right\|^2 \right\} + \sum_{s \in \mathcal{S}} \left\| \mathcal{X}_{z,s}^k - \mathcal{X}_{z,s}^{k-1} \right\|^2; \forall z \in \mathcal{Z} \end{aligned}, \quad (6.43)$$

$$\begin{cases} r^k = \sum_{z \in \mathcal{Z}} r_z^k \leq \varepsilon_1; \\ d^k = \sum_{z \in \mathcal{Z}} d_z^k \leq \varepsilon_2; \end{cases} \quad (6.44)$$

6.4.2.5 Dynamic Penalty Parameter Tuning

A dynamic penalty parameter tuning technique in [118] is used to update the penalty parameter. The penalty parameter ρ_z for each zone is updated according to primal and dual residuals of each zone, as shown in (6.45). The penalty parameter is updated in such a fashion in order to balance convergence of primal and dual residuals.

$$\rho_z^{k+1} = \begin{cases} \rho_z^k / (1 + \tau); & \text{if } \|r_z^k\| \leq \mu \|d_z^k\| \\ (1 + \tau) \rho_z^k; & \text{if } \|d_z^k\| \leq \mu \|r_z^k\|, \\ \rho_z^k; & \text{otherwise} \end{cases} \quad (6.45)$$

6.5 Information Exchanges in the Hierarchal Framework

6.5.1.1 Information Exchanges in Sub-Problem I

In sub-problem I, zone controllers collect $\mathcal{X}_2^*$, i.e., $x_{z,s}^*$, $\mathcal{W}_{ij,s}^*$, $\mathcal{V}_{i,s}^*$ and $\mathcal{P}_{ij,s}^*$, from the central controller and solve local sub-problems in parallel to optimize $\mathcal{X}_1$, as shown in Fig. 6-2. After solving the sub-problem I, in order to optimize $\mathcal{X}_2$ in sub-problem II, the central controller collects a part of $\mathcal{X}_1^*$, i.e., $\tilde{x}_{z,s}^*$, $\tilde{\mathcal{W}}_{ij,s}^*$, $\hat{\mathcal{V}}_{ij,s}^*$ and $\hat{\mathcal{P}}_{ij,s}^*$, from zone controllers.

6.5.1.2 Information Exchanges in Sub-Problem II

In sub-problem II, the central controller solves a global problem and zone controllers solve local sub-problems to optimize $\mathcal{X}_2$. As shown in Fig. 6-3, and zone controllers collect $\hat{\mathcal{P}}_{ij,s}^*$ and $\hat{\mathcal{V}}_{ij,s}^*$ from the central controller. After solving sub-problem II, in order to optimize $\mathcal{X}_1$ in the sub-problem I, the central controller collects $\mathcal{V}_{i,s}^*$ and $\mathcal{P}_{ij,s}^*$ from local controllers.

The information exchanges between zone controllers and the central controller are limited because only the information associated with zone status and boundary nodes is exchanged.

Fig. 6-2 Information exchanges in sub-problem I.

Fig. 6-3 Information exchanges in sub-problem II.

6.6 Case Studies

Three case studies were conducted on the 44-node test system, the modified IEEE 123-node system and the Brazil distribution system to validate the effectiveness of

the proposed hierarchical service restoration scheme. In the proposed ADMM-based algorithm, parameters c_i^G is 0.1 m.u. (monetary unit)/MW for DG units and 0.2 m.u./MW for main sources, c_{ij}^{lss} is 0.1 m.u./MW, c_i^r is 5 m.u./MW, and c_{ij}^s is 0.1 m.u.. The threshold ε_1 is set as 0.001 in cases 1 and 2 and 0.01 in case 3, and ε_2 is set as 0.01 in all cases. The number of discrete segments $|\mathcal{H}|$ and the maximum number of switching operations $s_{\max}$ are set as 30 and 8, respectively. The lower limit of voltage magnitudes is set as 0.95 p.u. Simulations were carried out using the GAMS with the CPLEX solver.

6.6.1 Case1: 44-node Test System

The 44-node test system in Fig. 6-4 consists of three feeders and nine zones with two DG units connected at zone 3 (DG1) and zone 9 (DG2). DG1 and DG2 operate with the unity power factor and their capacities are 100 kW. Closed switches and open switches are represented by black and white colored boxes, respectively. The detailed data of loads and circuits can be found in Appendix B. The following three scenarios were considered to validate the effectiveness of the proposed hierarchical service restoration scheme. The service restoration results of the three scenarios are summarized in Table 6-1, including the minimum voltage magnitude during service restoration, switching sequence, objective function value, and number of iterations of the ADMM-based algorithm.

- Base scenario: a permanent fault occurs in zone 1 with DG operation and load shedding
- Scenario 1: compared with the base scenario, DG1 is disconnected.
- Scenario 2: compared with the base scenario, load shedding is not considered.

Fig. 6-4 Initial topology of the 44-node test system

Table 6-1 Service restoration results in case 1

scenarios		base	no. 1	no. 2
switching sequence	s_1	s1-1 $\uparrow^1$	s1-1 $\uparrow$	s1-1 $\uparrow$
	s_2	4-5 $\uparrow$	4-5 $\uparrow$	4-5 $\uparrow$
	s_3	8-22 $\downarrow^1$	11-25 $\downarrow$	7-9 $\uparrow$
	s_4	-	7-9 $\uparrow$	8-22 $\downarrow$
	s_5	-	8-22 $\downarrow$	11-25 $\downarrow$
$V^{\min 2}$		0.965	0.950	0.959
O.F. ³		11.799	13.444	14.988
Num. ⁴		898	1771	1117

$\uparrow^1$ and $\downarrow^1$: Symbols “ $\uparrow$ ” and “ $\downarrow$ ” indicate the opening and closure of a switch, respectively; $V^{\min 2}$: minimal voltage magnitude [p.u.]; O.F.³: objective function value [m.u.]; Num.⁴: number of iterations of the ADMM-based algorithm.

6.6.1.1 Base Scenario

As shown in Table 6-1, the switch s1-1 (the switch between source 1 and node 1) and switch 4-5 (the switch between nodes 4 and 5) are opened to isolate the faulty zone 1 at steps s_1 and s_2 , respectively. Then, the switch 8-22 is closed to resupply

outage zones 2 and 3 at step s_3 . In this scenario, operational constraints are not violated, and two DG units output maximum power after service restoration because they have lower energy costs than the main sources.

6.6.1.2 Scenario 1

As shown in Table 6-1, outage zones 2 and 3 are supplied by closing the switch 11-25 at step s_3 . In order to ensure that voltage magnitudes are maintained within limits at this step, the system sheds 23.50 kW and 4.70 kVar loads at node 5, and 23.5 kW and 4.70 kVar loads at node 8. In this scenario, the switch 11-25 is closed at step s_3 instead of the switch 8-22 because of the limited current capacity of the switch 8-22. Since DG1 does not operate, more energy must be provided by main source 2 to supply loads in zone 3. In such a case, closing the switch 8-22 results in congestion and has higher load shedding costs. Finally, outage zones 2 and 3 are supplied separately by opening the switch 7-9 and closing the switch 8-22. During service restoration, the minimal voltage magnitude is 0.95 p.u. at nodes 5 and 6 at step s_3 . Compared with the base scenario, this scenario requires more switching operations and has higher total costs. It is obvious that DG units can improve the restoration ability of the distribution network.

6.6.1.3 Scenario 2

As shown in Table 6-1, different from scenario 1, the switch 7-9 is opened firstly to separate outages zones 2 and 3 before restoring them. Then, outage zone 2 is restored at step s_4 by closing the switch 8-22 and zone 3 is restored at step s_5 by closing the switch 11-25. In this scenario, operational constraints are not violated. Compared with scenario 1, this scenario has higher total costs because zones 2 and 3 can be restored only when they can be restored completely. Therefore, the appropriate load shedding can facilitate service restoration.

Fig. 6-5 Initial topology of the modified IEEE 123-node system

6.6.2 Case 2: IEEE 123-Node System

The modified IEEE 123-node system in Fig. 6-5 is used to further demonstrate the proposed hierarchical service restoration scheme. The system has two feeders and seven zones with three DG units connected at zones 3 (DG1), 5 (DG2) and 7 (DG3). DG units operate with the unity power factor and their capacities are 100 kW, 15 kW and 80 kW, respectively. Detailed parameters of circuits and loads are available online [127].

In this case, all possible faulty zones were tested. The service restoration results are summarized in Table 6-2. It is shown the proposed hierarchical scheme can restore the system while maintaining the faulty zone isolated in all tests. Moreover, the proposed scheme guarantees that operational constraints are not violated, and the energized portion of the system has a radial topology at each step.

As shown in Table 6-2, if the fault occurs in those zones locating at the end of feeders, i.e., zone 3, zone 4 or zone 7, only one switch operates during the service restoration process because the isolation of these zones will not affect healthy zones. When the fault occurs in other zones, i.e., zones 1, 2, 5 and 6, additional switching operations are required to restore outage zones. In addition, the numbers of iterations of the ADMM-based algorithm are listed in Table 6-2.

Table 6-2 Service restoration results in Case 2

		faulty zone						
		1	2	3	4	5	6	7
sw ¹	s ₁	s1-149↑	13-18↑	18-135↑	13-152↑	s2-300↑	97-197↑	72-76↑
	s ₂	13-18↑	18-135↑	-	-	97-197↑	72-76↑	-
	s ₃	13-152↑	151-300↓	-	-	54-94↓	54-94↓	-
	s ₄	151-300↓	-	-	-	72-76↑	-	-
	s ₅	39-66 ↓	-	-	-	60-160↓	-	-
V ^{min}		0.971	0.975	0.975	0.975	0.950	0.958	0.975
Num.		615	607	445	243	723	445	362

sw¹: Switching sequence

6.6.3 Case 3: Brazil 948-Node Distribution System

The modified Brazil 948-node distribution system shown in Fig. 6-6 is used to demonstrate the scalability of the proposed hierarchical scheme. The system consists of 948 nodes, 9 DGs, 85 loads zones and 95 switches. The detailed parameters of transmission lines and loads are available online [128]. The capacities of DGs units in zones 51, 55, 40, 47, 85, 83 are 50 kW and capacities of other DGs are 20 kW.

Suppose that a permanent fault occurs in zone 48 at feeder 2. After implementing the proposed ADMM-based algorithm, the hierarchical scheme isolates the faulty zone at the first three restoration steps, i.e., the switch between zones 25 and 48 is opened at s_1 to isolate the faulty zone from the upstream substation. The switches between zones 48 and 50 and between zones 48 and 49 are opened at s_2 and s_3 , respectively, to separate the faulty zone from the downstream outage zones 49-55. Then, at step s_4 , outage zones 49, 51-55 are supplied by closing the switch between zones 32 and 49. Finally, feeder 1 supplies the outage zone 50 by closing the tie-switch between zones 50 and 85 at step s_6 . The reconfigured system has a radial topology with the minimum voltage magnitude of 0.975 p.u. at node 925 in zone 38. Moreover, no load shedding is needed, and all DG units use their maximum capacities. In this case, the proposed ADMM-based algorithm needs 1523 iterations to converge.

Fig. 6-6 Initial topology of the modified Brazil 948-node distribution system

6.6.4 Sensitivity Analysis to the Penalty Parameter

Case 1 is used to conduct the sensitivity analysis by implementing the proposed ADMM-based algorithm with different penalty parameters between 50 and 400. Table 6-3 shows the numbers of iterations in three scenarios of case 1 with and without the penalty parameter tuning technique. Within a certain parameter range, the algorithm has better convergence performance as ρ^0 increases. However, it may have slow convergence if ρ^0 is too large, e.g., $\rho^0 = 400$. Moreover, Table 6-3 shows that the algorithm with the penalty parameter tuning technique has smaller average numbers of iterations. In most of the scenarios, the convergence process can be accelerated by using the varying penalty parameter. Fig. 6-7 shows the convergence processes of primal and dual residuals with the penalty parameter tuning technique ($\rho^0 = 200$), which demonstrates the feasibility and effectiveness of the proposed ADMM-based algorithm.

Table 6-3 Numbers of iterations in three scenarios in case 1 without and with the penalty parameter tuning technique

num.	without ρ tuning technique				with ρ tuning technique			
ρ^0	scen. 1	scen. 2	scen. 3	average	scen. 1	scen. 2	scen. 3	average
50	1520	1798	1187	1501.7	1168	1571	1552	1430.3
100	1127	1771	1208	1368.7	1060	1067	1325	1150.7
200	806	1406	1186	1132.6	808	1165	1204	1059.0
300	991	1840	1334	1388.3	1100	886	1356	1114.0
400	930	1947	1295	1424.0	792	983	1487	1087.3

Fig. 6-7 Convergence of the ADMM-based algorithm with the penalty tuning technique ($\rho^0 = 200$)

6.6.5 Discussions on Solution Quality and Computation Efficiency

Comparisons between the hierarchical solution and centralized solution in case 1 are given in Table 6-4. The centralized solution is obtained by solving the MILP model using the CPLEX solver with a maximum optimality gap of 1%. Table 6-4 lists objective function values and active/reactive power generations in the two schemes. It is shown that active and reactive power generations in the two schemes are almost the same and the maximum relative error of objective function values is lower than 0.1%, which demonstrates that the proposed hierarchical scheme can reproduce the centralized solution.

Table 6-5 summarizes the computation complexity of the centralized and hierarchical schemes in case 3. Compared with the centralized scheme, the average number of continuous variables of each controller in each calculation is reduced by 99.4% in the hierarchical scheme. Moreover, zone controllers optimize continuous variables only and the central controller optimizes binary variables only. Therefore, the computational complexity of the service restoration problem is reduced substantially by using the proposed hierarchical scheme.

The computation time in all cases with the centralized and hierarchical schemes is compared in Table 6-6. The hierarchical scheme can obtain solutions within 120 s in all cases. As for the 44-node and 123-node systems, the centralized scheme needs

less computation time than the hierarchical scheme. This is because the centralized algorithm can solve small-scale problems efficiently whereas the ADMM-based algorithm needs iterations to obtain solutions. However, for the large-scale Brazil 948-node system, the computation time of the ADMM-based algorithm is 115 s that is 96.15% lower than the one with the centralized algorithm. The centralized algorithm suffers from a heavy computation burden because the computation complexity grows exponentially as the number of binary variables increases. In contrast, the ADMM-based algorithm can decompose the large-scale problem into smaller-scale problems and solve them efficiently.

In summary, the hierarchical scheme can obtain optimal solutions with less computation complexity. For large-scale systems, the computation time decreases substantially by using the proposed hierarchical scheme.

Table 6-4 Centralized and hierarchical solutions in case 1

scenario	values	centralized scheme	hierarchical scheme
base	O.F. ¹	11.799	11.799
	APSN 2 ²	0.715	0.715
	APSN 3	0.612	0.612
	RPSN 2 ³	0.175	0.175
	RPSN 3	0.152	0.152
no. 1	O.F.	13.444	13.455
	APSN 2	0.820	0.820
	APSN 3	0.612	0.612
	RPSN 2	0.180	0.180
	RPSN 3	0.152	0.152
no. 2	O.F.	14.988	14.988
	APSN 2	0.820	0.820
	APSN 3	0.612	0.612
	RPSN 2	0.180	0.180
	RPSN 3	0.152	0.152

¹ O.F.: Objective function value

² APSN i : Active power source at source node i (MW)

³ RPSN i : Reactive power source at source node i (MVar)

Table 6-5 Computational complexity comparisons between two schemes in case 3

number of variables	centralized scheme	hierarchical scheme	
	central controller	zone controller	central controller
continuous variables	481216	2860	0
integer variables	2476	0	2476

Table 6-6 Computation time of two schemes in three cases

computation time (s)	44-node system			123-node system							real Brazil system
	no.1	no.2	no.3	no.1	no.2	no.3	no.4	no.5	no.6	no.7	
centralized	2.3	5.5	4.8	8.9	9.6	8.4	8.9	11.2	10.4	10.2	2993.3
hierarchical	22.1	41.9	54.2	48.0	44.9	29.6	17.6	73.8	34.0	21.8	115.1

6.7 Summary

This chapter proposes a hierarchical service restoration scheme for ADNs based on the ADMM. The optimal service restoration problem is firstly formulated as an MINLP model. Then, the MINLP model is transformed into an MILP model using linearization techniques, which is decomposed and solved by the ADMM-based algorithm. Through decomposition, the service restoration problem is solved based on a hierarchical framework, in which each zone controller solves its local sub-problem with information from the central controller and the central controller solves a global coordination problem with information from each zone controller. The numerical results demonstrate that the proposed hierarchical service restoration scheme can restore the network while ensuring that operation constraints are not violated. Compared with the centralized scheme, the proposed hierarchical scheme can obtain the optimal service restoration plan with less computational complexity.

CHAPTER 7.

DISTRIBUTED SERVICE RESTORATION FOR ACTIVE DISTRIBUTION NETWORKS BASED ON ALTERNATING DIRECTION METHOD OF MULTIPLIERS

This chapter presents a distributed service restoration scheme for ADNs based on the ADMM, which can obtain the optimal service restoration plan in a distributed manner. This chapter is mainly based on the published articles 9 and 10⁷.

7.1 Introduction

In chapter 6, a hierarchical service restoration scheme is presented. Although it can reduce the computation complexity compared with the centralized scheme, it still needs a central controller to collect data from local controllers and send out control commands in a centralized manner. As mentioned previously, there are a few drawbacks with the centralized method. The centralized communication infrastructure may suffer from the single-point failure risk. If the central controller is ineffective, the service restoration process is failed. In addition, it compromises the privacy information protection because service restoration involves different independent entities, each of which has its own objectives and may be not willing to share its own privacy information. Moreover, it is challenging for the centralized infrastructure to adapt to system changes, e.g., the integration of DREs [97].

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⁷ This chapter is based on the published articles 9 and 10: 1) **Feifan Shen**, Juan C. Lopez, Qiuwei Wu*, Marcos J. Rider, Tianguang Lu, and Nikos D. Hatziargyriou, "Distributed Self-healing Scheme for Unbalanced Electrical Distribution Systems Based on Alternating Direction Method of Multipliers (ADMM)," *IEEE Trans. Power System*, vol. 35, no. 3, pp. 2190-2199, May 2020; 2) **Feifan Shen**, Qiuwei Wu, Juan Camilo López, and Marcos J Rider, "Distributed Service Restoration of Active Electrical Distribution Systems using ADMM," *10th Conference on Innovative Smart Grid Technologies*, 2019.

On the other hand, in the existing distributed methods, the multi-agent system (MAS)-based methods use simple heuristic rules that disregard the mathematical programming aspect of service restoration. Thus, the optimality of solutions cannot be guaranteed with the MAS-based methods. An ADMM-based distributed service restoration strategy was developed in [97] to solve the service restoration problem in distributed manner. However, the strategy is designed for black start restoration and network reconfiguration as an important option to facilitate service restoration is not considered in the model formulation.

To resolve the above issues, a distributed service restoration scheme for the unbalanced ADN is proposed in this chapter based on the ADMM, which can obtain the optimal service restoration plan in a distributed manner. Compared with the distributed method in [97], network reconfiguration is considered in the proposed scheme, in which the optimal service restoration problem is formulated as a MIP model considering radial topology constraints. To decompose the MIP model and solve it in a distributed manner, a set of radial topology constraints with a decomposable structure is introduced, and the binary variables are handled by projection functions in the ADMM-based algorithm.

Firstly, the optimal service restoration problem is formulated as an MINLP model, including decomposable radiality constraints. Then, the MINLP model is transformed into an MISOCP model using the convex relaxation. Finally, the MISOCP model is decomposed and solved using the proposed ADMM-based algorithm. Through the decomposition, the service restoration problem is distributed among the zones of the network, where each zone is a radial portion of the network separated by the remotely controlled switches. Each zone has a service restoration controller that exchanges information with neighbor controllers.

The contributions of this chapter are summarized as follows: 1) propose an MISOCP model for the optimal service restoration problem of the unbalanced ADN, considering dispatchable DG units, controllable loads, and switches; 2) develop an ADMM-based solution method to solve the MISOCP model in a distributed manner.

The rest of this chapter is organized as follows. Section 7.2 presents the optimal service restoration model for the unbalanced ADN. Section 7.3 describes the proposed ADMM-based algorithm to solve the service restoration problem in a distributed manner. Section 7.4 discusses the case study results, followed by a summary in section 7.5.

7.2 Optimal Service Restoration Model for Unbalanced ADNs

The optimal service restoration problem of the unbalanced ADN can be represented by an MINLP model in (7.1)-(7.12). It is noted the variables (x_i, x_{ij}) and parameters (a_i, a_{ij}) defined in chapter 6 have been extended in this chapter to represent three phase variables $(\vec{x}_i, \vec{x}_{ij})$ and three phase parameters $(\vec{a}_i, \vec{a}_{ij})$. Set Φ represents three phases $\{a, b, c\}$. $\vec{x}_i(\phi)$ and $\vec{x}_{ij}(\phi)$ denotes the ϕ -th element of the three-phase variables. The inner and element-wise products of two vectors (three-phase variables) are represented by $\langle \vec{x}, \vec{y} \rangle$ and $\vec{x} \odot \vec{y}$, respectively.

7.2.1 Objective Function

The objective function in (7.1) consists of five terms: the first term minimizes the number of de-energized zones; the second term minimizes the energy production costs; the third term minimizes the number of switching operations; the fourth term minimizes the load shedding cost, where T is the operation duration of the restored system; the last term minimizes the line loss cost.

$$\min \left\{ \sum_{z \in \mathcal{Z}} c_z^U (1 - x_z) + \sum_{i \in \mathcal{N}} T \langle \vec{c}_i^G, \vec{P}_i^G \rangle + \sum_{ij \in \mathcal{L}^S} c_{ij}^S s_{ij} + \sum_{i \in \mathcal{N}} T \langle \vec{c}_i^r, \vec{r}_i \odot \vec{P}_i^D \rangle + \sum_{ij \in \mathcal{L}} T \langle \vec{c}_{ij}^{\text{loss}}, \vec{I}_{ij}^{\text{sqr}} \odot \vec{R}_{ij} \rangle \right\}, \quad (7.1)$$

7.2.2 Constraints

Each constraint of the MINLP model is defined for each zone $z \in \mathcal{Z}$, as follows,

$$\vec{P}_i^G + \sum_{ji \in \mathcal{L}} \vec{P}_{ji} - \sum_{ij \in \mathcal{L}} (\vec{P}_{ij} + \mathbf{R}_{ij} \vec{I}_{ij}^{\text{sqr}}) = \vec{P}_i^D \odot (x_z - \vec{r}_i); \forall i \in \mathcal{B}(z), \quad (7.2)$$

$$\vec{Q}_i^G + \sum_{ji \in \mathcal{L}} \vec{Q}_{ji} - \sum_{ij \in \mathcal{L}} (\vec{Q}_{ij} + \mathbf{X}_{ij} \vec{I}_{ij}^{\text{sqr}}) = \vec{Q}_i^D \odot (x_z - \vec{r}_i); \forall i \in \mathcal{B}(z), \quad (7.3)$$

$$\vec{V}_j^{\text{sqr}} \odot \vec{I}_{ij}^{\text{sqr}} = (\vec{P}_{ij})^{\odot 2} + (\vec{Q}_{ij})^{\odot 2}; \forall i \in \mathcal{B}(z), ij \in \mathcal{L}, \quad (7.4)$$

$$\vec{V}_i^{\text{sqr}} - \vec{V}_j^{\text{sqr}} = 2(\tilde{\mathbf{R}}_{ij} \vec{P}_{ij} + \tilde{\mathbf{X}}_{ij} \vec{Q}_{ij}) + |\mathbf{Z}_{ij}|^{\odot 2} \vec{I}_{ij}^{\text{sqr}}; \forall i \in \mathcal{B}(z), ij \in \mathcal{L}^B, \quad (7.5)$$

$$-\bar{V}^{\text{sqr}}(1-w_{ij}) \leq \bar{V}_i^{\text{sqr}} - \bar{V}_j^{\text{sqr}} \leq \bar{V}^{\text{sqr}}(1-w_{ij}); \forall i \in \mathcal{B}(z), ij \in \mathcal{L}^s, \quad (7.6)$$

$$x_z \underline{V}^{\text{sqr}} \leq \bar{V}_i^{\text{sqr}}(\phi) \leq x_z \bar{V}^{\text{sqr}}; \forall i \in \mathcal{B}(z), \phi \in \Phi, \quad (7.7)$$

$$0 \leq r_i(\phi) \leq x_z; \forall i \in \mathcal{B}(z), \phi \in \Phi, \quad (7.8)$$

$$\begin{cases} \bar{I}_{ij}^{\text{sqr}}(\phi) \leq x_z \bar{I}_{ij}^{\text{sqr}}; & \forall i \in \mathcal{B}(z), ij \in \mathcal{L}^B, \phi \in \Phi \\ \bar{I}_{ij}^{\text{sqr}}(\phi) \leq w_{ij} \bar{I}_{ij}^{\text{sqr}}; & \forall i \in \mathcal{B}(z), ij \in \mathcal{L}^s, \phi \in \Phi \end{cases}, \quad (7.9)$$

$$\begin{cases} 0 \leq \bar{P}_i^G(\phi) \leq \bar{P}_i^G x_{z_i}; & \forall i \in \mathcal{B}(z), \phi \in \Phi \\ 0 \leq \bar{Q}_i^G(\phi) \leq \bar{Q}_i^G x_{z_i}; & \forall i \in \mathcal{B}(z), \phi \in \Phi \end{cases}, \quad (7.10)$$

$$\begin{cases} s_{ij} \geq w_{ij} - w_{ij}^{\text{ini}}; \\ s_{ij} \geq w_{ij} - w_{ij}^{\text{ini}}; \end{cases} \forall i \in \mathcal{B}(z), ij \in \mathcal{L}^s, \quad (7.11)$$

$$w_{ij}, x_z \in \{0, 1\}, \quad (7.12)$$

Constraints (7.2) and (7.3) represent unbalanced three-phase active and reactive power equations at each node, respectively [97], where $\mathbf{Z}_{ij} = \mathbf{R}_{ij} + j\mathbf{X}_{ij}$ is the three-phase line impedance matrix and $\mathbf{Z}_{ij} \in \mathbb{C}$. Current magnitudes of all branches and switches at each phase are calculated in constraint (7.4). The voltage drop of each branch at each phase is calculated in (7.5), where $\tilde{\mathbf{R}}_{ij} + j\tilde{\mathbf{X}}_{ij}$ is the equivalent three-phase line impedance matrix. The derivation procedure of the equivalent matrix can be found in [97]. Constraint (7.6) guarantees that if a given switch is closed (i.e., $w_{ij}=1$), the voltage magnitudes at two ends of the switch are identical; otherwise, they can vary freely within their limits. Constraint (7.7) limits the voltage magnitudes: if a given zone is de-energized, voltage magnitudes are forced to zero. Constraint (7.8) limits the load shedding percentage at each node and each phase: if a given zone is de-energized (i.e., $x_z = 0$), the load shedding percentages at all nodes in the zone are zero (i.e., $r_i(\phi) = 0$). Constraint (7.9) limits the current magnitudes of all branches and switches at each phase: if a given zone is de-energized, current magnitudes of branches within the zone are forced to be zero, and if a given switch is open, the current magnitude of the switch is forced to be zero. Constraint (7.10) limits the maximum DG active and reactive power generations at each node

and phase. The switching operation is modeled in (7.11). Finally, the binary nature of x_z and w_{ij} is defined in (7.12).

After restoration, radial topology of the energized portion of the ADN is guaranteed by power balance constraints (7.2) and (7.3) and spanning tree constraints (7.13) and (7.14) [129]. Compared with the conventional radiality constraints used in chapter 6 [123], constraints (7.13) and (7.14) are defined for each zone, where β_{ij} is the auxiliary variable associated with the switch between nodes i and j .

$$\begin{cases} \beta_{ij} + \beta_{ji} = w_{ij}; \forall ij \in \mathcal{L}^s \\ \sum_{ji \in \mathcal{L}^s} \beta_{ij} + \sum_{ij \in \mathcal{L}^s} \beta_{ji} = x_z; \forall i \in \mathcal{B}(z), z \in \mathcal{Z} \end{cases} \quad (7.13)$$

$$\begin{cases} \beta_{ij} = 0; \forall ij \in \mathcal{L}^s, i \in \mathcal{N}^s \\ \beta_{ij}, \beta_{ji} \in [0, 1]; \forall ij \in \mathcal{L}^s \end{cases} \quad (7.14)$$

Due to expression (7.4), the optimal service restoration problem is a non-convex combinatorial problem, which is hard to solve. Thus, a relaxation technique is used to transform the MINLP problem into an MISOCP problem, which can be efficiently solved using several commercial solvers based on convex optimization techniques. A convex relaxation is applied by converting (7.4) into the second-order cone inequality in (7.15), which results in an MISOCP problem, given by objective function (7.1), subject to constraints (7.2), (7.3) (7.5)-(7.15). It is checked that the equality constraint (7.4) is satisfied for all the tests in section 7.4.

As shown in the proposed MISOCP model, each constraint is defined for each zone, resulting in a decomposable structure that can be solved by the ADMM in a distributed manner, as explained in the following section.

$$\vec{V}_j^{\text{sqr}} \odot \vec{I}_{ij}^{\text{sqr}} \geq \left(\vec{P}_{ij} \right)^{\odot 2} + \left(\vec{Q}_{ij} \right)^{\odot 2}; \forall i \in \mathcal{B}(z), ij \in \mathcal{L}, \quad (7.15)$$

7.3 ADMM-based Algorithm for the Service Restoration Problem

In this section, the ADMM algorithm is adapted to solve the optimal service restoration problem in a distributed manner. Firstly, the MISOCP problem is reformulated for the implementation of the ADMM algorithm. Then, the augmented Lagrangian of the problem is introduced. Finally, the ADMM-based algorithm is presented.

7.3.1 Reformulation of the MISOCP Model

As discussed in chapter 6, section 6.4.1, the MISOCP model can be written in a compact form in a similar manner. For clarity, define sets $\mathcal{P}_{ij}$, $\mathcal{V}_i$, and $\mathcal{I}_i$ for node $i \in \mathcal{N}$ as below,

$$\begin{cases} \mathcal{P}_{ij} := \{\vec{P}_{ij}, \vec{Q}_{ij}\}; \\ \mathcal{V}_i := \{\vec{V}_i^{\text{sqf}}\}; \\ \mathcal{I}_i := \left\{ \left\{ \vec{I}_{ij}^{\text{sqf}} \right\}_{ij \in \mathcal{L}}, \vec{r}_i, \vec{P}_i^G, \vec{Q}_i^G, \left\{ s_{ij} \right\}_{ij \in \mathcal{L}^s} \right\}; \end{cases}$$

The variables defined in sets $\mathcal{P}_{ij}$, $\mathcal{V}_i$, and $\mathcal{I}_i$ have the subscript of i or ij and are considered as affiliated variables of node i . Since each node belongs to a unique zone, these variables are exclusively optimized by the corresponding zone controller $\hat{z}_i$. The MISOCP model is rewritten in a compact form as below,

$$\min \sum_{z \in \mathcal{Z}} \sum_{i \in \mathcal{B}(z)} G_{z,i} [x_z, \mathcal{I}_i]^T, \quad (7.16)$$

$$A_i \left[\left\{ \mathcal{P}_{ij}, \mathcal{V}_i, \mathcal{V}_j \right\}_{ij \in \mathcal{L}}, \left\{ \mathcal{P}_{ji} \right\}_{ji \in \mathcal{L}}, \mathcal{I}_i, x_z \right]^T \leq a_i; \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), \quad (7.17)$$

$$B_i \left[\mathcal{P}_{ij}, \mathcal{I}_i, \mathcal{V}_i, x_z \right]^T \leq b_i; \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), \quad (7.18)$$

(7.12)-(7.14)

The objective function (7.1) is written in a compact form in (7.16), which is separable for each zone. Constraint (7.17) is the compact form of coupled constraints (7.2), (7.3), (7.5), (7.6) and (7.15), through which variables of boundary nodes of adjacent zones are coupled. Constraint (7.18) is the compact form of independent constraints of each zone, i.e., (7.7)-(7.11). In order to use the ADMM algorithm to distribute the MISOCP model, a set of auxiliary variables and equality constraints are introduced, as shown in (7.24). By replacing the original variables with the auxiliary variables in (7.13)-(7.14) and (7.16)-(7.18), the MISOCP model is rewritten as,

$$\min f(\tilde{x}_z, \mathcal{I}_i) = \sum_{z \in \mathcal{Z}} \sum_{i \in \mathcal{B}(z)} G_{z,i}(\tilde{x}_z, \mathcal{I}_i), \quad (7.19)$$

$$A_i \left[\left\{ \tilde{\mathcal{P}}_{ij} \right\}_{ij \in \mathcal{L}}, \left\{ \hat{\mathcal{P}}_{ij} \right\}_{ji \in \mathcal{L}}, \tilde{\mathcal{V}}_i, \hat{\mathcal{V}}_j, \mathcal{I}_i, \tilde{x}_z \right]^T \leq a_i; \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), \quad (7.20)$$

$$B_i \left[\tilde{\mathcal{P}}_{ij}, \tilde{\mathcal{V}}_i, \mathcal{I}_i, \tilde{x}_z \right]^T \leq b_i; \forall z \in \mathcal{Z}, i \in \mathcal{B}(z), \quad (7.21)$$

$$\begin{cases} \tilde{\beta}_{ij} + \hat{\beta}_{ij} = \tilde{w}_{ij}; \forall ij \in \mathcal{L}^s \\ \sum_{ji \in \mathcal{L}^s} \tilde{\beta}_{ij} + \sum_{ij \in \mathcal{L}^s} \tilde{\beta}_{ij} = \tilde{x}_z; \forall i \in \mathcal{B}(z), z \in \mathcal{Z} \end{cases}, \quad (7.22)$$

$$\begin{cases} \tilde{\beta}_{ij} = 0; \forall ij \in \mathcal{L}^s, i \in \mathcal{N}^s \\ \tilde{\beta}_{ij}, \hat{\beta}_{ij} \in [0, 1]; \forall ij \in \mathcal{L}^s \end{cases}, \quad (7.23)$$

$$\begin{cases} \mathcal{P}_{ij} = \tilde{\mathcal{P}}_{ij}, \mathcal{V}_i = \tilde{\mathcal{V}}_i, \mathcal{V}_j = \hat{\mathcal{V}}_j; \quad \forall ij \in \mathcal{L} \\ \mathcal{P}_{ji} = \hat{\mathcal{P}}_{ij}; \quad \forall ji \in \mathcal{L} \\ x_z = \tilde{x}_z; \quad \forall z \in \mathcal{Z} \\ \beta_{ij} = \tilde{\beta}_{ij}, \beta_{ji} = \hat{\beta}_{ij}, w_{ij} = \tilde{w}_{ij}; \quad \forall ij \in \mathcal{L}^s \\ \beta_{ij} = \tilde{\beta}_{ij}; \quad \forall ji \in \mathcal{L}^s \\ \tilde{x}_z, \tilde{w}_{ij} \in [0, 1] \end{cases}, \quad (7.24)$$

It is noted that the variables with the subscript of i or ij exclusively belong to node i . Therefore, constraints (7.20)-(7.23) can be decomposed among each zone after the model reformulation.

7.3.2 Augmented Lagrangian

The augmented Lagrangian is formulated by adding equality constraints in (7.24) into the objective function of the MISOCP model through dual variables Λ defined in (7.26), as follows,

$$\begin{aligned}
\min f(\tilde{x}_{z,s}, \mathcal{I}_{i,s}) &+ \frac{\rho}{2} \left\{ \sum_{z \in \mathcal{Z}} (\|x_z - \tilde{x}_z + \varpi_z\|^2 - \|\varpi_z\|^2) \right. \\
&+ \sum_{ij \in \mathcal{L}^S} (\|w_{ij} - \tilde{w}_{ij} + \zeta_{ij}\|^2 - \|\zeta_{ij}\|^2) + \sum_{ij \in \mathcal{L}} (\|\mathcal{P}_{ij} - \tilde{\mathcal{P}}_{ij} + \mathcal{A}_{ij}\|^2 - \|\mathcal{A}_{ij}\|^2) \\
&+ \sum_{ji \in \mathcal{L}} (\|\mathcal{P}_{ji} - \hat{\mathcal{P}}_{ij} + \mathcal{B}_{ij}\|^2 - \|\mathcal{B}_{ij}\|^2) + \sum_{ij \in \mathcal{L}} (\|\mathcal{V}_j - \hat{\mathcal{V}}_{ij} + \mathcal{C}_{ij}\|^2 - \|\mathcal{C}_{ij}\|^2) \quad , \quad (7.25) \\
&+ \sum_{i \in \mathcal{N}} (\|\mathcal{V}_i - \tilde{\mathcal{V}}_i + \mathcal{D}_i\|^2 - \|\mathcal{D}_i\|^2) + \sum_{ij \in \mathcal{L}^S} (\|\beta_{ij} - \tilde{\beta}_{ij} + \tau_{ij}\|^2 - \|\tau_{ij}\|^2) \\
&\left. \sum_{ij \in \mathcal{L}^S} (\|\beta_{ji} - \hat{\beta}_{ij} + \nu_{ij}\|^2 - \|\nu_{ij}\|^2) + \sum_{ji \in \mathcal{L}^S} (\|\beta_{ij} - \tilde{\beta}_{ij} + \psi_{ij}\|^2 - \|\psi_{ij}\|^2) \right\}
\end{aligned}$$

Subject to: (7.20)-(7.23).

It is noted that binary variables are only included in the objective function of the augmented Lagrangian and all constraints of the Lagrangian function can be decomposed among each zone. The augmented Lagrangian is optimized over two groups of primal variables $\mathbf{X}$ and $\mathbf{Y}$ and one group of dual variables $\mathbf{\Lambda}$, as defined in (7.26). For ease of presentation, objective function (7.25) is expressed in a compact form by $g(\mathbf{X}, \mathbf{Y}, \mathbf{\Lambda})$.

$$\begin{cases} \mathbf{X} = [\tilde{\mathcal{P}}_{ij}, \hat{\mathcal{P}}_{ij}, \tilde{\mathcal{V}}_i, \hat{\mathcal{V}}_{ij}, \tilde{\beta}_{ij}, \hat{\beta}_{ij}, \tilde{x}_z, \tilde{w}_{ij}, \mathcal{I}_i] \\ \mathbf{Y} = [\mathcal{P}_{ij}, \mathcal{V}_i, \beta_{ij}, x_z, w_{ij}] \\ \mathbf{\Lambda} = [\mathcal{A}_{ij}, \mathcal{B}_{ij}, \mathcal{C}_{ij}, \mathcal{D}_i, \tau_{ij}, \psi_{ij}, \nu_{ij}, \varpi_{ij}, \zeta_{ij}] \end{cases} \quad , \quad (7.26)$$

7.3.3 ADMM-Based Algorithm

In the ADMM-based algorithm, optimization over primal variables is separated into two sub-problems (sub-problems I and II). Primal variables $\mathbf{X}$ and $\mathbf{Y}$ and dual variables $\mathbf{\Lambda}$ are updated in an iterative process.

7.3.3.1 Sub-Problem I

At the k -th iteration, sub-problem I is given by (7.27), subject to constraints of the augmented Lagrangian, in which primal variables $\mathbf{X}^k$ are optimized, and primal variables $\mathbf{Y}$ and dual variables $\mathbf{\Lambda}$ are considered as parameters (i.e., $\mathbf{Y}^{*,k-1}$ and $\mathbf{\Lambda}^{*,k-1}$) obtained at the $(k-1)$ -th iteration.

Sub-problem I:

$$g(\mathbf{X}^k, \mathbf{Y}^{*,k-1}, \mathbf{\Lambda}^{*,k-1}), \quad (7.27)$$

Subject to: (7.20)-(7.23).

7.3.3.2 Sub-Problem II

After solving sub-problem I, sub-problem II is solved at k -th iteration to optimize primal variables $\mathbf{Y}^k$ with the obtained $\mathbf{X}^{*,k}$ and $\mathbf{\Lambda}^{*,k-1}$.

Sub-problem II:

$$g(\mathbf{X}^{*,k}, \mathbf{Y}^k, \mathbf{\Lambda}^{*,k-1}), \quad (7.28)$$

Note that sub-problem II is a non-constrained MIQP problem due to the binary nature of x_z and w_{ij} . Instead of solving the MIQP problem, the binary terms in the objective function associated with x_z and w_{ij} are excluded and the problem is solved as a QP problem. Then, the binary variables are obtained using the following projection function,

$$\begin{cases} x_z^k = \Pi_x(\tilde{x}_z^{*,k} + \varpi_z^{*,k-1}); \\ w_{ij}^k = \Pi_w(\tilde{w}_{ij}^{*,k} + \zeta_{ij}^{*,k-1}); \end{cases} \quad (7.29)$$

Here, Π denotes the projection onto x and w . Since x_z and w_{ij} are binary variables, Π rounds each entry to its nearest binary value. Since the sub-problem II is a non-constrained problem and the binary variables are not coupled with other decision variables, using the projection function and solving the MIQP problem directly are equivalent in terms of obtaining solutions of binary variables. It is worth noting that sub-problem II can be easily calculated for each zone in a distributed manner.

7.3.3.3 Dual Variable Update

Among zones, the exchanged primal variables are defined in $\mathbf{X}^{\text{ex}}$ in (7.30) and $\mathbf{Y}$ in (7.26). Once the exchanged variables are updated by solving the two sub-problems at the k -th iteration, dual variables can be updated using (7.31).

$$\mathbf{X}^{\text{ex}} = [\tilde{\mathcal{P}}_{ij}, \hat{\mathcal{P}}_{ij}, \tilde{\mathcal{V}}_i, \hat{\mathcal{V}}_{ij}, \tilde{\beta}_{ij}, \hat{\beta}_{ij}, \tilde{x}_z, \tilde{w}_{ij}], \quad (7.30)$$

$$\Lambda^{*,k} = \Lambda^{*,k-1} + (\mathbf{X}^{\text{ex},*,k} - \mathbf{Y}^{*,k}), \quad (7.31)$$

7.3.3.4 Stopping Criterion

The primal variables and dual variables are updated sequentially until the aggregated residual between exchanged variables is lower than a specified tolerance ε , as shown in (7.32).

$$\|\mathbf{X}^{\text{ex},*} - \mathbf{Y}^*\| \leq \varepsilon, \quad (7.32)$$

7.4 Case Studies and Results

To validate the efficiency of the proposed distributed service restoration scheme, two case studies were conducted with an unbalanced 44-node test system and the modified IEEE 123-node system. The proposed ADMM-based algorithm was implemented on GAMS with the CPLEX solver. In the ADMM-based algorithm, the weight coefficients are determined based on the priority of each objective. Restoring power to outage zones is the main goal of service restoration and has the top priority. The minimization of the number of switching operations is to reduce restoration time and has the second priority. The line losses reduction has the least priority. Therefore, the cost of de-energizing a zone c_z^U is set as 50 m.u. (monetary units), the cost of load shedding $\vec{c}_i^r$ is set as 10 m.u./MWh, the cost of a switching action c_{ij}^s is set as 5 m.u., and the cost of line losses $\vec{c}_{ij}^{\text{loss}}$ is set as 0.1 m.u./MWh. The generation cost coefficients of the main sources and DG units are approximately based on their generation costs. The cost coefficient $\vec{c}_i^G$ is set as 0.2 m.u./MWh for each main source and set as 0.1 m.u./MWh for each DG unit. The tolerance ε is 10^{-3} , and the adjustable parameter ρ is 350 in Case 1 and 200 in Case 2. The operational time of the restored system T is assumed to be 1 hour. The lower limit of the voltage magnitude is set as 0.95 p.u. in both cases.

Fig. 7-1 unbalanced 44-node test system

7.4.1 Case 1: Unbalanced 44-Node Test System

In this case, an unbalanced 44-node test system shown in Fig. 7-1 is used to verify the effectiveness of the proposed ADMM-based algorithm. The system consists of three separated feeders with nine zones and two DG units connected at zone 3 (DG 1) and zone 9 (DG 2), respectively. Open switches are represented as white-colored boxes, and closed switches are represented as black-colored boxes. Detailed data of loads and branches can be found in Appendix B. DG units operate with unity power factors and their capacities for each phase are 100 kW. Four scenarios were conducted in this case as below,

- Scenario 1: a fault occurs at the load zone adjacent to the source node, and DG operation is considered.
- Scenario 2: compared with scenario 1, the operation of DG1 is not considered.
- Scenario 3: a fault occurs at the source node or upstream to the source node, and DG operation is considered.
- Scenario 4: multiple faults at middle load zones are considered with DG operation.

The restoration results of the four scenarios are summarized in Table 7-1.

Table 7-1 Service restoration results in case 1

scenarios	no.1	no.2	no.3	no.4
faulty zone	zone1	zone1	source node	zones 2, 5
opening switches	r1-1, 4-5	r1-1, 4-5	r1-1	4-5, 7-9, 17-19, 20-23
closed switches	8-22	11-25	3-15	11-25, 26-39
line losses [kW]	53.31	77.46	46.34	58.05
$V^{\min}$ [p.u.]	$V_{13}(a) = 0.965$	$V_8(a) = 0.950$	$V_{41}(a) = 0.966$	$V_{13}(a) = 0.950$
$I^{\max}$ [A]	$I_{2,14}(a) = 92.4$	$I_{2,14}(a) = 99.9$	$I_{2,14}(a) = 104.6$	$I_{3,30}(a) = 112.7$
load shedding (P) [kW]	/	48.27	/	61.48
load shedding (Q) [kW]	/	9.65	/	12.30
loads served (P) [kW]	4800	4751.73	4800	4738.52
loads served (Q) [kW]	960	950.35	960	947.50

7.4.1.1 Scenario 1

After executing the proposed ADMM-based algorithm, the restored system isolates the faulty zone by opening the switch s1-1 (the switch between source node 1 and node 1) and switch 4-5 (the switch between nodes 4 and 5). Then, the switch 8-22 is closed to supply zones 2 and 3. The restored system has 53.32 kW of line losses and the minimum voltage magnitude is 0.965 p.u. at node 13 at phase a . Current magnitudes are within their operational limits and both DG units use their maximum power capacities because of lower energy production costs.

7.4.1.2 Scenario 2

Assume the same fault occurs at zone 1 while the operation of DG 1 is not considered. After executing the proposed ADMM-based algorithm, zones 2 and 3 are supplied by closing the switch 11-25 instead of the switch 8-22. Since DG 1 is disconnected, more energy must be transmitted from the main sources to supply zone 3. Due to the limited capacity of the switch 8-22, closing this switch results in a larger amount of load shedding and a higher total cost than closing the switch 11-25. The restored system sheds 24.147 kW and 4.829 kVar loads at node 5 at phase a , and 24.120 kW and 4.824 kVar loads at node 8 at phase a to ensure that all operational

constraints are not violated. The minimum voltage magnitude is 0.950 p.u. at node 6 at phase a .

7.4.1.3 Scenario 3

In this scenario, suppose that a fault occurs at source node 1 or upstream to source node 1. In such a case, the utility power from source node 1 is not available. After executing the proposed ADMM-based algorithm, the source node 1 is isolated by opening the switch s_{1-1} . Then, the switch 3-15 is closed to supply outage zones 2 and 3. The restored system has 46.52 kW of line losses. All operational constraints are satisfied, and two DG units operate at the maximum capacities.

7.4.1.4 Scenario 4

In this scenario, multiple faults are simulated. Suppose that two faults occur at zones 2 and 5, respectively. Then, the switch 4-5 and switch 7-9 are opened to isolate the faulty zone 2. The switch 17-19 and switch 20-23 are opened to isolate the faulty zone 5. Then, the switch 11-25 and switch 26-39 are closed to supply outages zones 3 and 6. To respect voltage magnitude constraints, the system sheds 61.48 kW and 12.30 kVar loads at node 12 at phase a . The minimum voltage magnitude is 0.950 p.u. at node 13 at phase a . In addition, both DG units use their maximum power capacities.

7.4.2 Case 2: Unbalanced IEEE 123-Node System

In this case, the effectiveness of the proposed ADMM-based algorithm under switch maintenance and communication loss is validated on the modified IEEE 123-node test system shown in Fig. 7-2. The unbalanced system consists of two feeders and six DG units that operate with unity power factors. Detailed load and branch data can be found in [127]. The service restoration results of the four scenarios in case 2 are summarized in Table 7-2.

Fig. 7-2 Modified IEEE 123-node system

7.4.2.1 Scenario 1

In this scenario, a fault occurs at zone 2 and the DG unit at node 250 is disconnected. The switch 13-18 and switch 18-135 are opened to isolate the faulty zone. Then, the switch 39-66 is closed to supply zone 3. The restored system has 32.98 kW of line losses and the minimum voltage is 0.971 p.u. at node 51 at phase *a*. All operation constraints are satisfied in this scenario, and DG units operate at their maximum power capacities.

7.4.2.2 Scenario 2: Switching Maintenance

In this scenario, the same fault at zone 2 is simulated while the switch 39-66 is under maintenance. In such a case, the switch 151-300 is closed to supply zone 3. The restored test has 26.32 kW of line losses and the minimum voltage magnitude is 0.977 p.u. at node 94, phase *a*. All operational constraints are respected in this scenario.

7.4.2.3 Scenario 3: Communication Loss of the Zone Downstream to the Fault

Under certain situations, such as the failure of communication devices, the communication loss could happen. If all communication within a zone is lost, the service restoration scheme should guarantee that the zone without communication does not participate in the restoration process and remains its initial status prior to service restoration. It is assumed that a fault occurs at zone 1 and its downstream zone 2 loses communication. Thus, zone 2 should remain de-energized and its controller does not participate in the restoration process. After implementing the proposed ADMM-based algorithm, zones 3 and 4 are supplied by closing the switch 151-300 and switch 39-66. Moreover, all operational constraints are not violated.

Table 7-2 Service restoration results in case 2

scenarios	no.1	no.2	no.3	no.4
faulty zone	zone 2	zone 2	zone 1	zone 1
opening switches	13-18, 18-135	13-18, 18-135	13-18, 13-152, 18-135	13-18, 13-152
closed switches	39-66	151-300	151-300, 39-66	151-300, 39-66
line losses [kW]	32.98	26.32	23.58	26.82
$V^{\min}$	$V_{51}(a) = 0.971$	$V_{94}(a) = 0.977$	$V_{52}(a) = 0.975$	$V_{52}(a) = 0.962$
$I^{\max}$	$I_{r2,300}(a) =$ 118.4	$I_{r2,300}(a) =$ 143.3	$I_{r2,300}(a) =$ 193.0	$I_{r3,30}(a) = 212.0$
load shedding (P) [kW]	/	/	/	62.43
load shedding (Q) [kW]	/	/	/	31.22
loads served (P) [kW]	3490	3490	3490	3427.57
loads served (Q) [kW]	1920	1920	1920	1888.78

7.4.2.4 Scenario 4: Communication Loss of the Zone Not Downstream to the Fault

In this scenario, assume the same fault at zone 1, and zone 7 loses communication. In such a case, zone 7 should remain energized and its controller does not participate in the restoration process. All variables associated with the zone, e.g., power flow at the switch 72-76, are set as the values obtained in the normal operation.

After executing the ADMM-based algorithm, zones 2, 3 and 4 are supplied, and the restored system sheds loads, e.g., sheds 18.378 kW and 9.189 kVar loads at load 70 at phase *a*, to ensure that variables associated with zone 7 remain unchanged.

7.4.3 Performance of the ADMM-based Algorithm

7.4.3.1 Convergence Performance

The numbers of iterations in both cases are listed in Table 7-3. For example, in case 1, the convergence processes of the aggregated residuals in four scenarios are shown in Fig. 7-3. It is shown that the algorithm converges to the specified tolerance after a reasonable number of iterations. As shown in Fig. 7-4, the binary variables associated with the switch 8-22 (switch a) and switch 11-25 (switch b) converge before 20 iterations in scenarios 1 and 2, which demonstrates the effectiveness of the projection function to deal with binary variables.

Table 7-3 Numbers of iterations in both cases

num. iterations	scenario 1	scenario 2	scenario 3	scenario 4
case1	791	735	581	749
case2	691	558	493	662

Fig. 7-3 Convergence processes of the aggregated residuals in case 1

Fig. 7-4 Convergence processes of binary variables associated with the switch 8-22 and switch 11-25 in scenarios 1 and 2, case 1

7.4.3.2 Sensitivity Analysis

The impacts of the penalty parameter ρ and tolerance ε on the convergence process and accuracy of the ADMM-based algorithm are studied in case 1. The numbers of iterations in case 1 with different values of ρ are listed in Table 7-4. It is shown that an improper value of ρ may cause slow convergence, e.g., $\rho = 50$ in scenarios 2 and 4. Therefore, it is important to choose an appropriate value for ρ . In case 1, ρ is set as 350 because it has the least average number of iterations for all scenarios.

Table 7-4 Numbers of iterations with parameters ρ in Case 1

ρ	scenario 1	scenario 2	scenario 3	scenario 4	average
50	1332	2385	879	2440	1759.00
100	995	1089	813	1051	987.00
200	885	824	787	774	817.50
300	735	750	760	910	788.75
350	791	735	581	749	714.00
400	787	1090	582	746	801.25
500	744	1100	695	914	863.25
600	654	1290	543	1086	983.25

The impacts of the convergence tolerance ε on the solution accuracy and number of iterations are shown in Table 7-5. The accuracy of the obtained solution is evaluated by the relative error (RE) using (7.33), where $\mathbf{s}$ is the vector of nodal power injections obtained with the ADMM-based algorithm, and $\mathbf{s}^*$ is the vector of nodal

power injections obtained with the centralized algorithm. The centralized solution is obtained by solving the formulated MISOCP problem in section 7.2 using the CPLEX solver with a zero-optimality gap. As shown in Table 7-5, the solution accuracy is improved as the convergence tolerance decreases. In the meantime, the algorithm needs more iterations to converge. Compared with the solution using $\varepsilon = 1 \times 10^{-2}$, the average RE obtained with $\varepsilon = 1 \times 10^{-3}$ decreases by 281 times and the average number of iterations increases by 2.41 times. Although the average RE can be further reduced by 3.9 times when $\varepsilon = 1 \times 10^{-4}$, the average number of iterations increases by 2.9 times. Therefore, the algorithm with $\varepsilon = 1 \times 10^{-3}$ has a good trade-off between the solution accuracy and converge performance.

$$RE = \frac{\|s - s^*\|}{\|s^*\|}, \quad (7.33)$$

Table 7-5 Solution accuracy and number of iterations with different convergence tolerances in case 1

convergence tolerance	scenarios	1×10^{-2}	1×10^{-3}	1×10^{-4}
relative error	scenario 1	1.41×10^{-1}	5.45×10^{-4}	4.40×10^{-4}
	scenario 2	2.19×10^{-1}	7.67×10^{-3}	1.60×10^{-3}
	scenario 3	4.91×10^{-3}	6.21×10^{-4}	3.44×10^{-4}
	scenario 4	5.71	1.26×10^{-2}	3.10×10^{-3}
	average	1.52	5.40×10^{-3}	1.40×10^{-3}
number of iterations	scenario 1	221	791	1620
	scenario 2	368	735	1989
	scenario 3	324	581	1176
	scenario 4	269	749	3509
	average	295.5	714.0	2073.5

7.4.3.3 Solution Comparisons between Centralized Method and Distributed Method

The RE of the objective function value obtained with the distributed method is shown in Table 7-6. It is shown that the maximum RE is lower than 1.7×10^{-4} . Moreover, as shown in Table 7-5, the RE of the distributed solution is lower than 1.3×10^{-2} . Therefore, the proposed distributed method can reproduce the centralized solution.

Table 7-6 Relative errors (REs) of the objective function values in both cases

scenarios	case1	case2
scenario 1	1.10×10^{-6}	3.80×10^{-6}
scenario 2	5.75×10^{-6}	2.49×10^{-6}
scenario 3	3.80×10^{-7}	1.03×10^{-6}
scenario 4	2.84×10^{-5}	1.69×10^{-4}

7.4.3.4 Computation Efficiency

Regarding the optimization problem, the centralized method solves a MISOCP problem whereas the distributed method solves smaller scale QP problems and applies projection functions. Table 7-7 summarizes the numbers of variables of optimization problems in the two methods for both cases, scenario 1. Compared with the centralized method, the average numbers of continuous variables of each controller in each calculation are reduced by 87.19% in case 1, and by 87.92% in case 2. Moreover, there are no integer variables involved in the optimization in the distributed method. Thus, the controller in the distributed method solves smaller scale and simpler optimization problems.

The computation time of the distributed algorithm at each iteration is the time duration of the most time-consuming controller. The average computation time per iteration in each scenario in both cases is shown in Table 7-8. Note that the average computation time per iteration in case 2 is larger than the one in case 1 because each controller solves a larger problem in case 2. The total computation time of the distributed algorithm and centralized algorithm is summarized in Table 7-9. It is shown that the proposed distributed method can obtain solutions within 15 seconds in both cases. Although the distributed method needs more computation time than the centralized method, it does not need a powerful central controller and it can protect privacy information, e.g., a DG unit is an independent entity participating in service restoration without sharing its privacy information to other agents.

Table 7-7 Comparison of optimization problems in both cases, scenario 1

number of variables		centralized method	distributed method
case 1	continuous	1046	134
	integer	62	0
case 2	continuous	3666	443
	integer	39	0

Table 7-8 Average computation time per iteration in both cases

seconds [s]	case1	case2
scenario 1	0.0105	0.0211
scenario 2	0.0106	0.0178
scenario 3	0.0112	0.0166
scenario 4	0.0135	0.0166

Table 7-9 Computation time of centralized and distributed methods

seconds [s]	case1		case2	
	centralized	distributed	centralized	distributed
scenario 1	1.52	8.34	2.34	14.56
scenario 2	0.87	7.81	1.79	9.95
scenario 3	1.00	6.53	2.52	8.18
scenario 4	0.935	10.08	2.95	11.02

7.5 Summary

In this chapter, the service restoration problem of the unbalanced ADN is formulated as an MINLP model, including radiality constraints with a decomposable structure. Then, the MINLP model is transformed into an MISOCP model, which is decomposed and solved using an ADMM-based algorithm that handles binary decision variables by projection functions. With the proposed decomposition, the service restoration problem is distributed among the zones of the ADN, where each zone has a service restoration controller that exchanges information with neighbor controllers. The effectiveness of the proposed distributed service restoration scheme was demonstrated on the unbalanced 44-node system and the modified IEEE 123-node system. In all cases, the proposed distributed method can restore the system effectively. Moreover, it can reproduce the optimal service restoration plan obtained with the centralized method and can ensure privacy information protection.

CHAPTER 8.

DISTRIBUTED RISK-LIMITING SERVICE RESTORATION FOR ACTIVE DISTRIBUTION NETWORKS WITH NETWORKED MICROGRIDS

This chapter presents a distributed risk-limiting service restoration scheme for the ADN with MGs to deal with the uncertainties of DERs and load consumption. This chapter is based on the published paper 11 with minor changes to fit into the framework of the thesis⁸.

8.1 Introduction

Conventional service restoration methods for ADNs are mainly based on the power fed by the upstream transmission networks (TNs) and, if needed, post-fault system reconfiguration. However, conventional service restoration methods may be ineffective in case of the unavailability of power from the upstream TNs due to damages in upstream TNs. Recently, utilizing MGs to deal with this challenge has been studied. A MG is a small-scale distribution network consisting of loads and different types of DERs at the low voltage (LV) or medium voltage (MV) level [131]. MGs can be used for service restoration in two ways: 1) MGs are considered as emergency sources to restore loads [132], [133]; 2) the MG utilizes its local DERs to restore its own loads [68]-[70], [67], [74], or networked MGs are formed to restore loads together [75]. This chapter focuses on the networked MGs to restore loads.

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⁸ This chapter is mainly based on the published paper 11: **Feifan Shen**, Qiuwei Wu*, Jin Zhao, Wei Wei, Nikos D. Hatziaargyrious, and Feng Liu, "Distributed Risk-limiting Load Restoration in Unbalanced Distribution Systems with Networked Microgrids," *IEEE Trans. Smart Grid*, vol.11, no.6, pp. 4574-4586, May 2020.

In [68], the formation of multiple MGs in the distribution network was proposed to restore loads. The restoration problem in [68] was formulated as a single time-step model without considering the restoration sequence. In [69] and [70], sequential service restoration strategies were proposed to provide a series of control actions in MGs to coordinate DERs and switches to restore loads sequentially. In the above studies, the uncertainties of load consumption and DER generations, such as WTs and PVs, were not considered. However, the uncertainty brings risks to the secure operation of the MG and has significant impacts on service restoration. A rolling-horizon optimization-based stochastic service restoration strategy was proposed in [67] to sectionalize the outage portion of the distribution network into multiple MGs. In [74], a chance-constrained service restoration strategy was proposed, which generates the restoration sequence for MGs after blackouts. In [75], a risk-limiting service restoration strategy was proposed for MGs using the risk-limiting dispatch method.

The above-mentioned service restoration methods utilizing MGs are based on centralized optimization algorithms. There are some challenges with these methods. Firstly, a central controller needs to collect and process all the data and send control commands, thus suffering from the single-point failure risk and a heavy computation burden for large-scale networks. Secondly, since MGs may belong to different owners, they may be not willing to share privacy information during restoration [134], e.g., schedules of DERs. Therefore, it is important to study distributed service restoration in networked MGs.

In addition, the conditional value-at-risk (CVaR) is used to model distributed risk limitation. The CVaR is a widely used risk-limiting index to deal with the uncertainty, such as in optimal bidding of generation units [135], reactive power planning [136], unit commitment [137], and transmission-level service restoration [138], [139]. The CVaR has the sub-additivity feature, which makes it easier to realize the distributed risk limitation. In the CVaR-based formulation, the risk-limiting constraint can be directly decomposed among sub-systems such that each sub-system can make risk-limiting decisions individually without violating the global risk-limiting constraint.

This chapter proposes a distributed risk-limiting service restoration scheme for the unbalanced ADN with networked MGs based on the CVaR and ADMM. In this method, the CVaR is used as a risk-limiting index, based on which the optimal risk-limiting service restoration problem is formulated as an MINLP model considering discrete variables and uncertainties of DERs and loads. Then, the MINLP model is

transformed into an MILP model by applying linearization techniques. With the ADMM and sub-additivity of the CVaR, the MILP model can be solved in a distributed manner among MGs by iteratively solving small-scale QP sub-problems and applying projection functions to deal with discrete variables. Consequently, distributed risk limitation is realized among MGs, each of which can make service restoration decisions individually without violating risk-limiting constraints of the whole system.

The contributions of this chapter are summarized as follows: 1) develop a distributed risk-limiting service restoration scheme for the unbalanced ADN with networked MGs; 2) formulate MINLP and MILP models for the CVaR-based optimal risk-limiting service restoration problem; and 3) decompose the risk-limiting service restoration scheme by introducing the distributed risk limitation using the sub-additivity of the CVaR.

The rest of the chapter is organized as follows. Section 8.2 formulates the MINLP model of the optimal risk-limiting service restoration problem of the ADN with networked MGs. Section 8.3 describes the MILP model formulation and decomposition of the model. Section 8.4 presents the ADMM-based distributed solution method for the MILP model. Case studies are provided in section 8.5, followed by a summary in section 8.6.

8.2 Optimal Risk-Limiting Service Restoration Problem

This section first introduces the service restoration scenario under consideration. Then, the MINLP model of the optimal risk-limiting service restoration problem is formulated.

8.2.1 Service Restoration Scenario

An unbalanced ADN with three MGs shown in Fig. 8-1 is used to illustrate the service restoration scenario under consideration. The three MGs are connected by tie-lines and each MG consists of different types of DERs, i.e., micro-turbines (MTs), wind turbines (WTs), photovoltaics (PVs), and energy storage systems (ESSs). In the study, MTs and ESSs are dispatchable DERs and used to regulate frequency and voltage during the restoration process. The frequency and voltage regulation capabilities of the MT and ESS depend on their capacities. Moreover, the frequency regulation capability of the ESS is limited by its state of charge (SOC) level. The WTs and PVs are non-dispatchable DERs and their active power outputs

are assumed to be the forecast values. Therefore, they do not participate in frequency regulation. Reactive power outputs of the WT and PV can be controlled to regulate voltage.

Suppose that the substation (s/s) is damaged due to extreme events and thus power from the upstream TN is not available for service restoration. During the outage period, networked MGs control DERs and conventional devices, i.e., voltage regulators (VRs) and capacitor banks, to pick up loads in a sequential procedure. During restoration, the frequency variation after a load pick-up is critical and requires special attention [140]. The risk of the frequency security violation due to uncertainties is limited by the CVaR-based method in this study. It is assumed that there are sufficient resources to regulate voltage, such as VRs, capacitor banks and DERs, so the risk of voltage violation is not considered. Since this study focuses on risk management of the frequency security, the uncertainties of DER generation and loads that significantly affect the active power balance of the system are considered. It is assumed that accurate system parameters are used, e.g., line impedance.

Fig. 8-1 The unbalanced ADN with MGs

8.2.2 Optimal Risk-Limiting Service Restoration Model

The optimal risk-limiting service restoration problem is formulated as an MINLP model in (8.1)-(8.25). It is noted the variables ($\bar{x}_i$, $\bar{x}_{ij}$) and parameters ($\bar{a}_i$, $\bar{a}_{ij}$) defined in chapter 7 have been extended as $\bar{x}_{i,t}$, $\bar{x}_{ij,t}$, $\bar{a}_{i,t}$, and $\bar{a}_{ij,t}$ in this chapter to include the time step index. The variables and parameters already provided in

chapters 6 and 7 will not defined in this chapter. The node set $\mathcal{N}$ consists of $\mathcal{N}^L$, $\mathcal{N}^M$, $\mathcal{N}^W$, $\mathcal{N}^P$, $\mathcal{N}^E$, and $\mathcal{N}^{\text{cap}}$, representing sets of loads, MTs, WT, PVs, ESSs and capacitor banks, respectively. Sets $\mathcal{L}^N$, $\mathcal{L}^{\text{VR}}$, and $\mathcal{L}^T$ denote sets of lines without VRs, lines with VRs and tie-lines connecting MGs, respectively. Set $\mathcal{T}$ is the set of restoration time steps.

- Objective function

The objective function (8.1) consists of four terms: the first term maximizes the load pick-up amount considering priorities of loads, where $\vec{c}_i^P$ is the priority coefficient, and Δt is the duration of a time step; the second term minimizes line losses; the third term minimizes MT generation costs, where $\vec{c}_i^M$ is the MT energy production cost, and $\vec{P}_{i,t}^M$ is the MT active power generation; the last term minimizes operation costs of VRs, where $\vec{s}_{ij,t}^{\text{VR}}$ is the number of tap operations of the VR, and $\vec{c}_{ij}^{\text{VR}}$ is the cost coefficient. It is worth noting that minimizing the VR operation costs will not deteriorate load pick-up performance since the coefficients associated with the load pick-up amount are much larger than the ones of the VR operation costs.

$$\begin{aligned} \max \quad & \sum_{i \in \mathcal{N}^L} \sum_{t \in \mathcal{T}} \langle \vec{c}_i^P, \vec{P}_{i,t}^D \rangle \Delta t - \sum_{ij \in \mathcal{L}} \sum_{t \in \mathcal{T}} \langle \vec{c}_{ij}^{\text{loss}}, \vec{I}_{ij,t}^{\text{sq}} \odot \vec{R}_{ij} \rangle \Delta t \\ & - \sum_{i \in \mathcal{N}^M} \sum_{t \in \mathcal{T}} \langle \vec{c}_i^M, \vec{P}_{i,t}^M \rangle \Delta t - \sum_{ij \in \mathcal{L}^{\text{VR}}} \sum_{t \in \mathcal{T}} \langle \vec{c}_{ij}^{\text{VR}}, \vec{s}_{ij,t}^{\text{VR}} \rangle, \end{aligned} \quad (8.1)$$

- Load constraints

The loads restored at node i at time step t ($\vec{P}_{i,t}^D + j\vec{Q}_{i,t}^D$) are modeled as,

$$\begin{cases} \vec{P}_{i,t}^D = x_{i,t}^L \vec{P}_{i,t}^{\text{D},F}; \quad \forall i \in \mathcal{N}^L, t \in \mathcal{T} \\ \vec{Q}_{i,t}^D = x_{i,t}^L \vec{Q}_{i,t}^{\text{D},F}; \quad \forall i \in \mathcal{N}^L, t \in \mathcal{T} \end{cases}, \quad (8.2)$$

where $x_{i,t}^L$ is the binary variable representing load pick-up status, $x_{i,t}^L = 1$ if the load is restored; otherwise, $x_{i,t}^L = 0$, $\vec{P}_{i,t}^{\text{D},F} + j\vec{Q}_{i,t}^{\text{D},F}$ represents the expected amount of loads.

The cold load pick-up phenomenon (CLPU) is modeled by (8.3) [97], where $\vec{P}_{i,t}^{\text{CLPU}}$

denotes additional active power absorbed due to the CLPU, which is included in the right-hand side of (8.2).

$$\bar{P}_{i,t}^{\text{CLPU}} = \beta_i \lambda_i (x_{i,t}^{\text{L}} - x_{i,t-1}^{\text{L}}) \bar{P}_{i,t}^{\text{D,F}}; \quad \forall i \in \mathcal{N}^{\text{L}}, t \in \mathcal{T}, \quad (8.3)$$

where β_i^{CLPU} is the cold load percentage and λ_i^{CLPU} is the percentage of additional power absorbed. Note that adjustable loads can be modeled by defining $x_{i,t}^{\text{L}}$ as the continuous variable.

- Voltage constraints

The squared voltage magnitude should be within the maximum and minimum limits as,

$$\underline{V}^{\text{sqr}} \leq \bar{V}_{i,t}^{\text{sqr}}(\phi) \leq \bar{V}^{\text{sqr}}; \quad \forall i \in \mathcal{N}, t \in \mathcal{T}, \phi \in \psi, \quad (8.4)$$

- Line current capacity constraints

The squared current magnitude should be lower than the maximum limit as,

$$\bar{I}_{ij,t}^{\text{sqr}}(\phi) \leq \bar{I}_{ij}^{\text{sqr}}; \quad \forall ij \in \mathcal{L}, t \in \mathcal{T}, \phi \in \psi, \quad (8.5)$$

- MT operational constraints

The MT power generation ($\bar{P}_{i,t}^{\text{MT}} + j \bar{Q}_{i,t}^{\text{MT}}$) is constrained by the maximum and minimum limits ($\bar{P}_i^{\text{MT,max}}, \bar{Q}_i^{\text{MT,max}}, \bar{P}_i^{\text{MT,min}}, \bar{Q}_i^{\text{MT,min}}$) in (8.6). Constraint (8.7) is the ramping rate constraint, where $\bar{P}_i^{\text{MT,ru}}$ and $\bar{P}_i^{\text{MT,rd}}$ are the ramp-up and ramp-down limits, respectively.

$$\begin{cases} \bar{P}_i^{\text{MT,min}} \leq \bar{P}_{i,t}^{\text{MT}} \leq \bar{P}_i^{\text{MT,max}}; \\ \bar{Q}_i^{\text{MT,min}} \leq \bar{Q}_{i,t}^{\text{MT}} \leq \bar{Q}_i^{\text{MT,max}}; \end{cases} \quad \forall i \in \mathcal{N}^{\text{M}}, t \in \mathcal{T}, \quad (8.6)$$

$$\begin{cases} \bar{P}_i^{\text{MT,rd}} \leq \bar{P}_{i,t}^{\text{MT}} - \bar{P}_{i,t-1}^{\text{MT}} \leq \bar{P}_i^{\text{MT,ru}}; \quad \forall i \in \mathcal{N}^{\text{M}}, t \in \mathcal{T} \big|_{t \neq t_1}, \\ \bar{P}_{i,t}^{\text{MT}} \leq \bar{P}_i^{\text{MT,ru}}; \quad \forall i \in \mathcal{N}^{\text{M}}, t = t_1 \end{cases} \quad (8.7)$$

- WT and PV operation constraints

The PV and WT active power generations ($\vec{P}_{i,t}^{\text{PV}}$, $\vec{P}_{i,t}^{\text{WT}}$) are equal to their generation forecasts ($\vec{P}_{i,t}^{\text{PV,F}}$, $\vec{P}_{i,t}^{\text{WT,F}}$) in (8.8). The PV and WT reactive power generations ($\vec{Q}_{i,t}^{\text{PV}}$, $\vec{Q}_{i,t}^{\text{WT}}$) are constrained by the maximum and minimum limits ($\vec{Q}_i^{\text{PV,max}}$, $\vec{Q}_i^{\text{PV,min}}$, $\vec{Q}_{i,t}^{\text{WT,max}}$, $\vec{Q}_{i,t}^{\text{WT,min}}$) in (8.9).

$$\begin{cases} \vec{P}_{i,t}^{\text{PV}} = \vec{P}_{i,t}^{\text{PV,F}}; \quad \forall i \in \mathcal{N}^{\text{P}}, t \in \mathcal{T} \\ \vec{P}_{i,t}^{\text{WT}} = \vec{P}_{i,t}^{\text{WT,F}}; \quad \forall i \in \mathcal{N}^{\text{W}}, t \in \mathcal{T} \end{cases}, \quad (8.8)$$

$$\begin{cases} \vec{Q}_i^{\text{PV,min}} \leq \vec{Q}_{i,t}^{\text{PV}} \leq \vec{Q}_i^{\text{PV,max}}; \quad \forall i \in \mathcal{N}^{\text{P}}, t \in \mathcal{T} \\ \vec{Q}_{i,t}^{\text{WT,min}} \leq \vec{Q}_{i,t}^{\text{WT}} \leq \vec{Q}_{i,t}^{\text{WT,max}}; \quad \forall i \in \mathcal{N}^{\text{W}}, t \in \mathcal{T} \end{cases}, \quad (8.9)$$

- Energy storage system operational constraints

The ESSs are modeled based on the per-phase basis [141] in (8.10)-(8.15). Constraint (8.10) calculates the net active power generation ($\vec{P}_{i,t}^{\text{ES}}$) of the ESS, which is the difference between its discharging power ($\vec{P}_{i,t}^{\text{ES,D}}$) and charging power ($\vec{P}_{i,t}^{\text{ES,C}}$).

$$\vec{P}_{i,t}^{\text{ES}} = \vec{P}_{i,t}^{\text{ES,D}} - \vec{P}_{i,t}^{\text{ES,C}}; \forall i \in \mathcal{N}^{\text{E}}, t \in \mathcal{T}, \quad (8.10)$$

Constraint (8.11) ensures that the ESS can only operate in one of the three modes, i.e., charging, discharging or idle mode.

$$\vec{x}_{i,t}^{\text{ES,C}}(\phi) + \vec{x}_{i,t}^{\text{ES,D}}(\phi) \leq 1; \forall i \in \mathcal{N}^{\text{E}}, t \in \mathcal{T}, \phi \in \psi, \quad (8.11)$$

where $\vec{x}_{i,t}^{\text{ES,C}}$ and $\vec{x}_{i,t}^{\text{ES,D}}$ are binary variables, $\vec{x}_{i,t}^{\text{ES,C}}=1$ if the ESS operates in the charging mode, $\vec{x}_{i,t}^{\text{ES,D}}=1$ if the ESS operates in the discharging mode, and both of them are zero if the ESS operates in the idle mode.

Constraints (12) and (13) are active and reactive power generation constraints of the ESS.

$$\begin{cases} \vec{x}_{i,t}^{\text{ES,C}} \odot \vec{P}_i^{\text{EC,min}} \leq \vec{P}_{i,t}^{\text{ES,C}} \leq \vec{x}_{i,t}^{\text{ES,C}} \odot \vec{P}_i^{\text{EC,max}}; \forall i \in \mathcal{N}^{\text{E}}, t \in \mathcal{T} \\ \vec{x}_{i,t}^{\text{ES,D}} \odot \vec{P}_i^{\text{ED,min}} \leq \vec{P}_{i,t}^{\text{ES,D}} \leq \vec{x}_{i,t}^{\text{ES,D}} \odot \vec{P}_i^{\text{ED,max}}; \forall i \in \mathcal{N}^{\text{E}}, t \in \mathcal{T} \end{cases}, \quad (8.12)$$

$$\vec{x}_{i,t}^{\text{ES}} \odot \vec{Q}_i^{\text{E,min}} \leq \vec{Q}_{i,t}^{\text{ES}} \leq \vec{x}_{i,t}^{\text{ES}} \odot \vec{Q}_i^{\text{E,max}}; \forall i \in \mathcal{N}^{\text{E}}, t \in \mathcal{T}, \quad (8.13)$$

where $\vec{P}_i^{\text{EC,max}}$ and $\vec{P}_i^{\text{EC,min}}$ are the maximum and minimum limits of active charging power, $\vec{P}_i^{\text{ED,max}}$ and $\vec{P}_i^{\text{ED,min}}$ are the maximum and minimum limits of active discharging power, $\vec{Q}_{i,t}^{\text{ES}}$ is the reactive power output of the ESS, and $\vec{Q}_i^{\text{E,max}}$ and $\vec{Q}_i^{\text{E,min}}$ are the maximum and minimum limits of reactive power.

Constraint (8.14) represents the energy balance relation of the ESS, where $\vec{E}_{i,t}^{\text{ES}}$ is the state of charge (SOC) level of the ESS, $\vec{E}_{i,t}^{\text{ES,ini}}$ is the initial SOC, η_i^{C} and η_i^{D} are charging and discharging efficiency coefficients, respectively.

$$\begin{cases} \vec{E}_{i,t}^{\text{ES}} = \vec{E}_{i,t-1}^{\text{ES}} + \eta_i^{\text{C}} \vec{P}_{i,t}^{\text{ES,C}} \Delta t - \frac{1}{\eta_i^{\text{D}}} \vec{P}_{i,t}^{\text{ES,D}} \Delta t; \forall i \in \mathcal{N}^{\text{E}}, t \in \mathcal{T} \big|_{t \neq t_1}; \\ \vec{E}_{i,t}^{\text{ES}} = \vec{E}_{i,t}^{\text{ES,ini}}; \forall i \in \mathcal{N}^{\text{E}}, t = t_1 \end{cases}, \quad (8.14)$$

Constraint (8.15) stipulates that the SOC level should be within the maximum and minimum limits ($\vec{E}_{i,t}^{\text{ES,max}}$, $\vec{E}_{i,t}^{\text{ES,min}}$), respectively.

$$\vec{E}_{i,t}^{\text{ES,min}} \leq \vec{E}_{i,t}^{\text{ES}} \leq \vec{E}_{i,t}^{\text{ES,max}}; \forall i \in \mathcal{N}^{\text{E}}, t \in \mathcal{T}, \quad (8.15)$$

- Capacitor bank constraints

The shunt capacitor bank is modeled in (8.16), where $\vec{Q}_{i,t}^{\text{cap}}$ is the capacitor capacity and $x_{i,t}^{\text{cp}}$ is the binary variable representing the switch on/off status. The capacitor switches on if $x_{i,t}^{\text{cp}} = 1$; otherwise, the capacitor switches off.

$$\vec{Q}_{i,t}^{\text{cap}} = x_{i,t}^{\text{cp}} \vec{Q}_i^{\text{C}}; \forall i \in \mathcal{N}^{\text{cap}}, t \in \mathcal{T}, \quad (8.16)$$

- Three-phase unbalanced power flow constraints

$$\vec{P}_{i,t}^{\text{DER}} + \sum_{j \in \mathcal{L}} \vec{P}_{ji,t} - \sum_{ij \in \mathcal{L}} (\vec{P}_{ij,t} + \mathbf{R}_{ij} \vec{I}_{ij,t}^{\text{sqr}}) = \vec{P}_{i,t}^{\text{D}}; \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad (8.17)$$

$$\vec{Q}_{i,t}^{\text{DER}} + \vec{Q}_{i,t}^{\text{cap}} + \sum_{ji \in \mathcal{L}} \vec{Q}_{ji,t} - \sum_{ij \in \mathcal{L}} (\vec{Q}_{ij,t} + \mathbf{X}_{ij} \vec{I}_{ij,t}^{\text{sqr}}) = \vec{Q}_{i,t}^{\text{D}}; \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad (8.18)$$

$$\vec{V}_{i,t}^{\text{sqr}} - \vec{V}_{j,t}^{\text{sqr}} = 2 \left(\tilde{\mathbf{R}}_{ij} \vec{P}_{ij,t} + \tilde{\mathbf{X}}_{ij} \vec{Q}_{ij,t} \right) + \left| \mathbf{Z}_{ij} \right|^{\odot 2} \vec{I}_{ij,t}^{\text{sqr}}; \forall ij \in \mathcal{L}^{\text{N}}, t \in \mathcal{T}, \quad (8.19)$$

$$\vec{V}_{j,t}^{\text{sqr}} \odot \vec{I}_{ij,t}^{\text{sqr}} = \left(\vec{P}_{ij,t} \right)^{\odot 2} + \left(\vec{Q}_{ij,t} \right)^{\odot 2}; \forall ij \in \mathcal{L}, t \in \mathcal{T}, \quad (8.20)$$

Constraints (8.17) and (8.18) represent active and reactive power balance at node i at time step t , respectively, where $\vec{P}_{i,t}^{\text{DER}} + j \vec{Q}_{i,t}^{\text{DER}}$ represents the total power generation of DERs. Constraint (8.19) represents the voltage drop of line ij at time step t . Constraint (8.20) defines the relation between the current magnitude and voltage magnitude.

- Voltage regulator constraints

The VRs can be modeled by ideal three-phase transformers [97]. A line ij with a VR can be divided into a line ij^* containing a tap-changer, which is modeled in (8.21), and a line j^*j having the same impedance as the line ij .

$$\vec{V}_{i,t}^{\text{sqr}} = \vec{a}_{ij,t}^{\odot 2} \odot \vec{V}_{j^*,t}^{\text{sqr}}; \forall ij \in \mathcal{L}^{\text{VR}}, t \in \mathcal{T}, \quad (8.21)$$

$$\vec{a}_{ij,t} = \vec{a}_{ij,t}^{\min} + \Delta a_{ij,t} \vec{n}_{ij,t}^{\text{tap}}; \forall ij \in \mathcal{L}^{\text{VR}}, t \in \mathcal{T}, \quad (8.22)$$

$$\vec{n}_{ij,t}^{\text{tap}}(\phi) \in \{0, 1, \dots, n^{\max}\}, \forall \phi \in \psi, \quad (8.23)$$

where $\vec{a}_{ij,t}$ represents the voltage ratio, $\vec{n}_{ij,t}^{\text{tap}}$ is an integer variable representing the tap position of the tap-changer, $\Delta a_{ij,t}$ represents the change rate of the voltage ratio, and $\vec{a}_{ij,t}^{\min}$ is the lower limit of the voltage ratio. The number of tap operations is modeled in (8.24), where $\vec{n}_{ij,t}^{\text{tap,ini}}$ is the initial tap position.

$$\begin{cases} \vec{s}_{ij,t}^{\text{VR}} \geq \vec{n}_{ij,t}^{\text{tap}} - \vec{n}_{ij,t-1}^{\text{tap}}; \quad \forall ij \in \mathcal{L}^{\text{VR}}, t \in \mathcal{T} \Big|_{t \neq t_1} \\ \vec{s}_{ij,t}^{\text{VR}} \geq \vec{n}_{ij,t-1}^{\text{tap}} - \vec{n}_{ij,t}^{\text{tap}}; \quad \forall ij \in \mathcal{L}^{\text{VR}}, t \in \mathcal{T} \Big|_{t \neq t_1} \\ \vec{s}_{ij,t}^{\text{VR}} \geq \vec{n}_{ij,t}^{\text{tap}} - \vec{n}_{ij}^{\text{tap,ini}}; \quad \forall ij \in \mathcal{L}^{\text{VR}}, t = t_1 \\ \vec{s}_{ij,t}^{\text{VR}} \geq \vec{n}_{ij}^{\text{tap,ini}} - \vec{n}_{ij,t}^{\text{tap}}; \quad \forall ij \in \mathcal{L}^{\text{VR}}, t = t_1 \end{cases}, \quad (8.24)$$

- Load pick-up sequencing constraints

Constraint (23) represents that the load cannot be tripped once it has been restored.

$$x_{i,t}^{\text{L}} \geq x_{i,t-1}^{\text{L}}; \forall i \in \mathcal{N}^{\text{L}}, t \in \mathcal{T}, \quad (8.25)$$

- Risk-limiting frequency response rate constraints

The frequency response rate (FRR) is used to measure the frequency deviation after a sudden load pick-up, which can be modeled by a simplified inequality in (24) [74]. The FRR constraint denotes that the equivalent incremental load pick-up amount in each time step should be smaller than the maximum allowable amount that is proportional to the total capacity of all MTs and those ESSs with discharging modes.

$$\begin{aligned} & \sum_{i \in \mathcal{N}^{\text{L}}} \sum_{\phi \in \mathcal{V}} \left(\Delta \vec{P}_{i,t}^{\text{D}}(\phi) - \Delta \vec{P}_{i,t}^{\text{PV}}(\phi) - \Delta \vec{P}_{i,t}^{\text{WT}}(\phi) \right) \\ & \leq \gamma \left(\sum_{\phi \in \mathcal{V}} \sum_{i \in \mathcal{N}^{\text{M}}} \vec{P}_i^{\text{MT,max}}(\phi) + \sum_{i \in \mathcal{N}^{\text{E}}} \left\langle \vec{x}_{i,t}^{\text{ES,D}}, \vec{P}_i^{\text{ED,max}} \right\rangle \right); \forall t \in \mathcal{T}, \end{aligned} \quad (8.26)$$

where

$$\begin{cases} \Delta \vec{P}_{i,t}^{\text{D}} = \vec{P}_{i,t}^{\text{D}} - \vec{P}_{i,t-1}^{\text{D}}; & \forall i \in \mathcal{N}^{\text{L}}, t \in \mathcal{T} \\ \Delta \vec{P}_{i,t}^{\text{PV}} = \vec{P}_{i,t}^{\text{PV,F}} - \vec{P}_{i,t-1}^{\text{PV,F}}; & \forall i \in \mathcal{N}^{\text{P}}, t \in \mathcal{T} \\ \Delta \vec{P}_{i,t}^{\text{WT}} = \vec{P}_{i,t}^{\text{WT,F}} - \vec{P}_{i,t-1}^{\text{WT,F}}; & \forall i \in \mathcal{N}^{\text{W}}, t \in \mathcal{T} \end{cases}$$

For simplicity, the compact form of (8.26) is,

$$R(\mathbf{x}^{\text{L}}, \mathbf{y}) \leq R_b; \mathbf{x}^{\text{L}} = [x_{i,t}^{\text{L}}]; \mathbf{y} = [\vec{P}_{i,t}^{\text{D,F}}, \vec{P}_{i,t}^{\text{WT,F}}, \vec{P}_{i,t}^{\text{PV,F}}]$$

where function $R(\mathbf{x}^L, \mathbf{y})$ calculates the incremental load pick-up amount and R_b is the maximum allowable amount. In practice, since forecast errors are inevitable, the forecasts in $\mathbf{y}$ are uncertain variables $\mathbf{y}^u$. Therefore, the risk of the FRR constraint violation caused by the uncertainties should be limited. Based on the CVaR, the risk-limiting FRR constraint is formulated as,

$$\text{CVaR}\left(R(\mathbf{x}^L, \mathbf{y}^u)\right) \leq R_b; \mathbf{y}^u = \left[\tilde{P}_{i,t}^{\text{D},u}, \tilde{P}_{i,t}^{\text{WT},u}, \tilde{P}_{i,t}^{\text{PV},u}\right], \quad (8.27)$$

where $\tilde{P}_{i,t}^{(\cdot),u}$ represents uncertain active loads and active power generation of WTs/PVs. Constraint (8.27) represents that the CVaR value of the uncertain incremental load pick-up amount $R(\mathbf{x}^L, \mathbf{y}^u)$ should be smaller than R_b . By definition, the value-at-risk value is the least value ξ such that $R(\cdot)$ does not exceed the value ξ with a specified probability level α , whereas the CVaR value is the conditional expected value of $R(\cdot)$ above the value ξ . The CVaR value of $R(\mathbf{x}^L, \mathbf{y}^u)$ with the probability level α is calculated as,

$$\text{CVaR}\left(R(\mathbf{x}^L, \mathbf{y}^u)\right) = (1-\alpha)^{-1} \int_{R(\mathbf{x}^L, \mathbf{y}^u) \geq \xi} R(\mathbf{x}^L, \mathbf{y}^u) p(\mathbf{y}^u) d\mathbf{y}^u, \quad (8.28)$$

where $p(\mathbf{y}^u)$ denotes the probability distribution of $\mathbf{y}^u$.

Due to non-linear constraints (8.20), (8.21), and (8.27), and integer variables, the optimal risk-limiting service restoration model is an MINLP model.

8.2.3 Discussions on the Risk-Limiting Service Restoration Model

The purpose of this chapter is to develop a distributed decision-making method for service restoration under uncertain conditions. The decision-making process is to decide the optimal restoration operations such as load pick-up sequence based on the steady-state model. The dynamic stability of the MG during restoration is a critical issue and needs to be considered during the decision-making process. To capture dynamics in the steady-state decision-making model, a prevailing way is to incorporate equivalent dynamic constraints. Therefore, the FRR constraint is incorporated into the steady-state model to measure the largest frequency deviation during the transient process and ensures the frequency stability of the obtained restoration operations. The detailed analysis of frequency and voltage dynamics requires dynamic simulations and is generally studied separately to verify the restoration operations obtained in the decision-making process. Therefore, detailed dynamic modelling is out of scope of this chapter.

8.3 MILP Model Formulation and Decomposition

To make the optimal risk-limiting service restoration problem computationally tractable, the MINLP model is transformed into the MILP model by applying linearization techniques. Then, the decomposition of the MILP model among MGs is presented.

8.3.1 MILP Model Formulation

8.3.1.1 Linearization of (8.20)

As discussed in chapter 6, section 6.3.2.1, the product of variables $(\vec{V}_{j,t}^{\text{sqr}} \odot \vec{I}_{ij,t}^{\text{sqr}})$ in (8.20) can be approximated by fixing $\vec{V}_{j,t}^{\text{sqr}}$ as the squared nominal value $(V^{\text{nom}})^2$ and using the piecewise linearization technique. The linearization approximation of (8.20) is obtained as,

$$(V^{\text{nom}})^2 \vec{I}_{ij,t}^{\text{sqr}} = L(\vec{P}_{ij,t}) + L(\vec{Q}_{ij,t}); \forall ij \in \mathcal{L}, t \in \mathcal{T}, \quad (8.29)$$

where the function L is defined in (6.22)-(6.27) in chapter 6, section 6.3.2.1.

8.3.1.2 Linearization of (8.21)

The binary expansion scheme [142] is used to linearize (8.21). The integer variable $\vec{n}_{ij,t}^{\text{tap}}$ is represented by a binary equation in (8.30), where $\mathcal{H}$ is the set of binary representation and $\vec{\lambda}_{ij,t}^h$ is the binary variable.

$$\vec{n}_{ij,t}^{\text{tap}} = \sum_{h=1}^{|\mathcal{H}|} 2^{h-1} \vec{\lambda}_{ij,t}^h, \quad (8.30)$$

By introducing three auxiliary variables $\vec{m}_{ij,t}$, $\vec{e}_{ij,t}^h$ and $\vec{d}_{ij,t}^h$, and using the big-M method, the constraint (8.21) is linearized in (8.31), as below,

$$\begin{cases} \vec{V}_{i,t}^{\text{sqr}} = a_{ij,t}^{\min} \vec{m}_{ij,t} + \Delta a_{ij,t} \sum_{h=1}^H 2^{h-1} \vec{d}_{ij,t}^h \\ \vec{m}_{ij,t} = \vec{a}_{ij,t} \vec{V}_{j^*,t}^{\text{sqr}} = a_{ij,t}^{\min} \vec{V}_{j^*,t}^{\text{sqr}} + \Delta a_{ij,t} \sum_{h=1}^H 2^{h-1} \vec{e}_{ij,t}^h \\ 0 \leq \vec{e}_{ij,t}^h \leq M \vec{\lambda}_{ij,t}^h, 0 \leq \vec{d}_{ij,t}^h \leq M \vec{\lambda}_{ij,t}^h \\ 0 \leq \vec{V}_{j^*,t}^{\text{sqr}} - \vec{e}_{ij,t}^h \leq M(1 - \vec{\lambda}_{ij,t}^h), 0 \leq \vec{m}_{ij,t} - \vec{d}_{ij,t}^h \leq M(1 - \vec{\lambda}_{ij,t}^h) \end{cases}, \quad (8.31)$$

8.3.1.3 Linearization of (8.26)

According to [143], the CVaR value can be approximately calculated using (8.32) by discretizing uncertain variables $\mathbf{y}^u$ into a collection of scenarios $\mathbf{y}_j^u$, where $\mathcal{J}$ is the set of scenarios, π_j is the probability of scenario $\mathbf{y}_j^u$, and $[a]^+ = \max\{a, 0\}$.

$$\text{CVaR}\left(R(\mathbf{x}, \mathbf{y}^u)\right) = \xi + (1 - \alpha)^{-1} \sum_{j=1}^{|\mathcal{J}|} \pi_j \left[R(\mathbf{x}, \mathbf{y}_j^u) - \xi \right]^+, \quad (8.32)$$

Then, the non-linear constraint (8.27) can be replaced by linear equations in (8.33) by introducing the auxiliary variable v_j .

$$\begin{cases} \text{CVaR}\left(R(\mathbf{x}, \mathbf{y}^u)\right) = \xi + (1 - \alpha)^{-1} \sum_{j=1}^{|\mathcal{J}|} \pi_j v_j \leq R_b; \\ v_j \geq R(\mathbf{x}, \mathbf{y}_j^u) - \xi, \quad v_j \geq 0; \quad j \in \mathcal{J} \end{cases}, \quad (8.33)$$

Finally, the MILP model is formulated by replacing non-linear constraints (8.20), (8.21) and (8.27) with linear constraints (8.29), (8.31) and (8.33).

8.3.2 Decomposition of the MILP model

For clarity of presentation, the time step subscript t is omitted. As we did in chapters 6 and 7, define a set $\mathcal{K}_i := \{a_i, a_{ij}\}$, which consists of all variables with the subscript of i and ij and represents exclusively affiliated variables of node i . According to the structure of networked MGs, the MILP model is rewritten in a compact form and decomposed as below,

$$\min \sum_{\text{MG}_m \in \mathcal{M}} f_{\text{MG}_m}(\mathbf{x}_{\text{MG}_m}^b, \mathbf{x}_{\text{MG}_m}^c), \quad (8.34)$$

$$g_{ij}(\vec{V}_i^{\text{sqr}}, \vec{V}_j^{\text{sqr}}, \vec{I}_{ij}^{\text{sqr}}, \vec{P}_{ij}, \vec{Q}_{ij}) = 0; \forall ij \in \mathcal{L}^T, \quad (8.35)$$

$$g_i(\vec{P}_{ji}, \vec{Q}_{ji}, \vec{P}_{ij}, \vec{Q}_{ij}, \vec{I}_{ij}^{\text{sqr}}) = 0; \forall i \in \mathcal{B}_m, j \in \mathcal{B}_n, \text{MG}_{m/n} \in \mathcal{M}, \quad (8.36)$$

$$g_{\text{MG}_m}(\mathbf{x}_{\text{MG}_m}^b, \mathbf{x}_{\text{MG}_m}^c, \vec{V}_i^{\text{sqr}}, \vec{P}_{ij}, \vec{Q}_{ij}) \leq 0; \forall ij \in \mathcal{L}^T, \text{MG}_m \in \mathcal{M}, \quad (8.37)$$

$$\text{CVaR}\left(\sum_{\text{MG}_m \in \mathcal{M}} R_{\text{MG}_m}\right) \leq R_b, \quad (8.38)$$

The objective function in (8.34) is the compact form of (8.1), where $\text{MG}_m \in \mathcal{M}$ represents m -th MG, f_{MG_m} is the objective function of MG_m , $\mathbf{x}_{\text{MG}_m}^b$ and $\mathbf{x}_{\text{MG}_m}^c$ represent binary and continuous variables of all nodes in MG_m , respectively. Constraints (8.35) and (8.36) represent voltage drop constraints of tie lines $ij \in \mathcal{L}^T$ and power balance constraints of the boundary node $i \in \mathcal{B}_m$ in MG_m . As shown in (8.35) and (8.36), variables of boundary node $i \in \mathcal{B}_m$ in MG_m , i.e., $\vec{V}_i^{\text{sqr}}, \vec{P}_{ij}, \vec{Q}_{ij}$ are coupled with variables of its adjacent boundary node $i \in \mathcal{B}_n$ in MG_n , i.e., $\vec{V}_j^{\text{sqr}}, \vec{P}_{ji}, \vec{Q}_{ji}$. Constraint (8.37) represents independent constraints of MG_m with exclusive variables. Constraint (8.38) originates from the risk-limiting FRR constraint (8.26) which involves all MGs' situations, where uncertain variable R_{MG_m} represents the incremental load pickup amount in MG_m .

To implement the distributed method, each MG should solve its own problem with exclusive variables. However, due to the coupled constraints (8.35), (8.36) and (8.38), the MILP model in (8.34)-(8.38) is not intrinsically separable among MGs. Therefore, an ADMM based solution method is introduced to solve the MILP model in a distributed manner.

8.4 ADMM-based Solution Method

This subsection presents an ADMM-based solution method to solve the MILP model in a distributed manner among MGs.

8.4.1 Decoupling of Coupled Constraints

Before solving the MILP model, the coupled constraints are decoupled by introducing auxiliary variables. For simplicity, the symbol of the three-phase variable $\vec{x}$ is simplified as x .

8.4.1.1 Decoupling of Power Flow Constraints (8.35) and (8.36)

To decouple (8.35) and (8.36), a set of auxiliary variables and a set of equality constraints are introduced as,

$$\begin{cases} P_{ji} = \hat{P}_{ij}, Q_{ji} = \hat{Q}_{ij}; \forall ji \in \mathcal{L}^T \\ P_{ij} = \tilde{P}_{ij}, Q_{ij} = \tilde{Q}_{ij}, V_i^{\text{sqr}} = \tilde{V}_i^{\text{sqr}}, V_j^{\text{sqr}} = \hat{V}_j^{\text{sqr}}; \forall ij \in \mathcal{L}^T \end{cases}, \quad (8.39)$$

By substituting auxiliary variables in (8.35)-(8.37), these constraints are transformed into (8.40), which can be fully decoupled among MGs, each of which optimizes its affiliated variables.

$$\begin{cases} g_{ij}(\tilde{V}_i^{\text{sqr}}, \hat{V}_j^{\text{sqr}}, I_{ij}^{\text{sqr}}, \tilde{P}_{ij}, \tilde{Q}_{ij}) = 0; & \forall (i, j) \in \mathcal{L}^T \\ g_i(\hat{P}_{ij}, \hat{Q}_{ij}, \tilde{P}_{ij}, \tilde{Q}_{ij}, I_{ij}^{\text{sqr}}) = 0; & \forall i \in \mathcal{B}_m, j \in \mathcal{B}_n, \text{MG}_{m/n} \in \mathcal{M}, \\ g_{\text{MG}_m}(\mathbf{x}_{\text{MG}_m}^b, \mathbf{x}_{\text{MG}_m}^c, \tilde{V}_i^{i, \text{sqr}}, \tilde{P}_{ij}, \tilde{Q}_{ij}) \leq 0; \forall ij \in \mathcal{L}^T, \text{MG}_m \in \mathcal{M} \end{cases} \quad (8.40)$$

8.4.1.2 Decoupling of the Risk-Limiting FRR Constraint (8.38)

The sub-additivity of the CVaR is shown in (8.41), which means that the CVaR value of the overall network is smaller than the sum of CVaR values of all MGs. Therefore, if the sum of CVaR values of all MGs is smaller than the CVaR bound R_b , as shown in (8.42), the risk-limiting constraint (8.38) is satisfied. Then, in order to have the distributed risk limitation, the distributed constraint (8.43) is introduced, where each MG limits its own CVaR value. If the distributed constraint (8.43) is satisfied, constraint (8.42) and coupled constraint (8.38) are satisfied. Therefore, the solution obtained with the model using the distributed constraint (8.43) respects the coupled risk-limiting constraint (8.38).

$$\text{CVaR}\left(\sum_{\text{MG}_m \in \mathcal{M}} R_{\text{MG}_m}\right) \leq \sum_{\text{MG}_m \in \mathcal{M}} \text{CVaR}(R_{\text{MG}_m}), \quad (8.41)$$

$$\text{CVaR}\left(\sum_{\text{MG}_m \in \mathcal{M}} R_{\text{MG}_m}\right) \leq \sum_{\text{MG}_m \in \mathcal{M}} \text{CVaR}(R_{\text{MG}_m}) \leq R_b, \quad (8.42)$$

$$\begin{cases} \text{CVaR}(R_{\text{MG}_m}) \leq R_{b,\text{MG}_m}; \text{MG}_m \in \mathcal{M} \\ R_{b,\text{MG}_m} = \left(\sum_{\phi \in \mathcal{V}} \sum_{i \in \mathcal{V}_{\text{MG}_m}^{\text{M}}} \bar{P}_i^{\text{MT,max}}(\phi) + \sum_{i \in \mathcal{V}_{\text{MG}_m}^{\text{E}}} \langle \vec{x}_{i,t}^{\text{ES,D}}, \bar{P}_i^{\text{ED,max}} \rangle \right), \\ \sum_{\text{MG}_m} R_{b,\text{MG}_m} = R_b; \end{cases} \quad (8.43)$$

where R_{b,MG_m} is the maximum allowable incremental load pick-up amount of MG_m .

8.4.1.3 Binary Variables Processing

In the MILP model, all binary variables $\mathbf{x}_{\text{MG}_m}^b$ are redefined as continuous variables, and an equality constraint (8.44) and binary variables $\tilde{\mathbf{x}}_{\text{MG}_m}^b$ are introduced as,

$$\mathbf{x}_{\text{MG}_m}^b = \tilde{\mathbf{x}}_{\text{MG}_m}^b, \mathbf{x}_{\text{MG}_m}^b \in [0,1], \tilde{\mathbf{x}}_{\text{MG}_m}^b \in \{0,1\}; \forall \text{MG}_m \in \mathcal{M}, \quad (8.44)$$

8.4.2 Augmented Lagrangian

By adding equality constraints (8.39) and (8.44) into the objective function (8.34) through dual variables Λ defined in (8.45) and the penalty parameter ρ , the augmented Lagrangian of the MILP model is formulated, given by (8.46), subject to (8.40) and (8.42). The augmented Lagrangian is optimized over primal variables $(\mathbf{X}, \mathbf{Y})$ defined in (8.47) and dual variables Λ .

$$\Lambda = \left\{ \chi_{ij}, \eta_{ij}, \nu_{ij}, \varsigma_{ij}, \kappa_i, \sigma_{ij}, \mathbf{o}_{\text{MG}_m} \right\};, \quad (8.45)$$

$$\begin{aligned}
 \min \sum_{\text{MG}_m \in \mathcal{M}} f_{\text{MG}_m}(\mathbf{x}_{\text{MG}_m}^b, \mathbf{A}_{\text{MG}_m}) &+ \frac{\rho}{2} \left[\sum_{ji \in \mathcal{L}^T} \left(\|P_{ji} - \tilde{P}_{ij} + \chi_{ij}\|^2 \right. \right. \\
 &+ \|\chi_{ij}\|^2 + \|\mathcal{Q}_{ji} - \tilde{\mathcal{Q}}_{ij} + \eta_{ij}\|^2 - \|\eta_{ij}\|^2 \Big) + \sum_{ij \in \mathcal{L}^T} \left(\|P_{ij} - \hat{P}_{ij} + \mathbf{v}_{ij}\|^2 \right. \\
 &- \|\mathbf{v}_{ij}\|^2 + \|\mathcal{Q}_{ij} - \hat{\mathcal{Q}}_{ij} + \varsigma_{ij}\|^2 - \|\varsigma_{ij}\|^2 + \|V_i^{\text{sqr}} - \tilde{V}_i^{i,\text{sqr}} + \kappa_i\|^2 \\
 &- \|\kappa_i\|^2 + \|V_j^{\text{sqr}} - \hat{V}_j^{j,\text{sqr}} + \sigma_{ij}\|^2 - \|\sigma_{ij}\|^2 \Big) + \sum_{\text{MG}_m \in \mathcal{M}} \left(\|\mathbf{x}_{\text{MG}_m}^b \right. \\
 &- \tilde{\mathbf{x}}_{\text{MG}_m}^b + \mathbf{o}_{\text{MG}_m} \Big\|^2 - \|\mathbf{o}_{\text{MG}_m}\|^2 \Big); \tag{8.46}
 \end{aligned}$$

subject to: (8.40) and (8.43).

$$\begin{cases} \mathbf{X} = \{ \tilde{V}_i^{\text{sqr}}, \hat{V}_{ij}^{\text{sqr}}, \tilde{P}_{ij}, \tilde{\mathcal{Q}}_{ij}, \hat{P}_{ij}, \hat{\mathcal{Q}}_{ij}, \mathbf{x}_{\text{MG}_m}^b, \mathbf{x}_{\text{MG}_m}^c \}; \\ \mathbf{Y} = \{ V_i, P_{ij}, \mathcal{Q}_{ij}, \tilde{\mathbf{x}}_{\text{MG}_m}^b \}; \end{cases} \tag{8.47}$$

8.4.3 ADMM-Based algorithm

The augmented Lagrangian is solved by the ADMM-based algorithm in an iterative procedure. For the ease of presentation, the objective function (8.46) of the augmented Lagrangian is represented by a compact form $g(\mathbf{X}, \mathbf{Y}, \Lambda)$.

8.4.3.1 Sub-Problem I

At the k -th iteration, primal variables $\mathbf{X}^k$ are optimized in sub-problem I, given by (8.48), subject to (8.40) and (8.43), with parameters $\mathbf{Y}^{*,k-1}$ and $\Lambda^{*,k-1}$ obtained at the $(k-1)$ -th iteration.

Sub-problem I:

$$g(\mathbf{X}^k, \mathbf{Y}^{*,k-1}, \Lambda^{*,k-1}), \tag{8.48}$$

subject to: (8.40) and (8.43)

Sub-problem I is a QP problem whose objective function and constraints can be fully decoupled among MGs. Therefore, sub-problem I can be solved in a distributed manner among MGs.

8.4.3.2 Sub-Problem II

The primal variables $\mathbf{Y}^k$ are optimized in sub-problem II, given by (8.49), with parameters $\mathbf{X}^{*,k-1}$ and $\Lambda^{*,k-1}$.

Sub-problem II:

$$g(\mathbf{X}^{*,k-1}, \mathbf{Y}^k, \Lambda^{*,k-1}), \quad (8.49)$$

Sub-problem II is a non-constrained MIQP problem. Instead of solving the MIQP problem to optimize binary variables, the binary variables are optimized using the projection function in (8.50), where Π_x denotes the projection function, which rounds each entry to its nearest binary value. Therefore, the last term in (8.46) associated with binary variables is excluded in sub-problem II and primal variables $\mathbf{Y}^k$ are optimized by solving a QP problem, which can be fully decoupled among MGs.

$$\tilde{\mathbf{x}}_{\text{MG}_m}^{\text{b},k} = \Pi_x(\mathbf{x}_{\text{MG}_m}^{\text{b},*,k-1} + \mathbf{o}_{\text{MG}_m}^{*,k-1});, \quad (8.50)$$

8.4.3.3 Update of Dual Variables

After optimizing primal variables, dual variables are updated in (8.51) using the primal variables $\mathbf{Y}$ and exchanged variables $\mathbf{X}^{\text{ex}}$ defined in (8.52),

$$\Lambda^k = \Lambda^{*,k-1} + (\mathbf{X}^{\text{ex},*,k-1} - \mathbf{Y}^{*,k});, \quad (8.51)$$

$$\mathbf{X}^{\text{ex}} = \{\tilde{V}_i^{\text{sqr}}, \hat{V}_{ij}^{\text{sqr}}, \tilde{P}_{ij}, \tilde{Q}_{ij}, \hat{P}_{ij}, \hat{Q}_{ij}, \mathbf{x}_{\text{MG}_m}^{\text{b}}\};, \quad (8.52)$$

8.4.3.4 Stop Criteria

The primal variables and dual variables are updated iteratively till primal and dual residuals are lower than specified thresholds, shown in (8.53).

$$\|\mathbf{X}^{\text{ex},*,k} - \mathbf{Y}^{*,k}\|^2 \leq \sigma_1, \|\mathbf{X}^{\text{ex},*,k} - \mathbf{X}^{\text{ex},*,k-1}\|^2 \leq \sigma_2, \quad (8.53)$$

8.5 Case Studies and Results

The modified IEEE 123-node system with three MGs in Fig. 8-1 is used to demonstrate the effectiveness of the proposed distributed risk-limiting load restoration

method. The detailed parameters of the system can be found in [127]. The DERs under consideration are categorized in Table 8-1 with respect to their black start and dispatchable capabilities. The MT at node 55 provides the reference voltage for the system [140]. The restoration horizon has 6 time steps and each step has a duration of 15 minutes. The lower and upper limits of the voltage magnitude are 0.95 p.u. and 1.05 p.u., respectively. The probability level α is 0.9. In the ADMM-based algorithm, convergence thresholds σ_1 and σ_2 are set as $10^{-4}\sqrt{|\mathcal{N}|}$ [97]. The costs of the MT generation, line loss and VR operation at a time are set as 0.1 m.u./kWh, 0.1 m.u./kWh, and 0.01 m.u., respectively. To ensure that minimizing the operation costs will not compromise the maximization of the load pick-up amount, the priority coefficients of the critical and non-critical loads are set as 1.5 m.u./kWh and 1.0 m.u./kWh, respectively.

When applying the CVaR method to the case with multiple random variables, i.e., WT and PV active power outputs and load forecast errors, the correlations among random variables should be considered when generating scenarios. In the study, the forecast error scenarios considering correlations are obtained using the copula function-based scenario generation method [144], [145]. 1000 scenarios are generated and reduced to 20 scenarios by using the simultaneous backward reduction method [146]. The proposed algorithm is implemented using GAMS with the CPLEX solver.

Table 8-1 Category of distributed energy resources (DERs)

DERs		locations (node no.)	black-start capability
dispatchable	MTs	8, 55, 28, 40, 105, 76	55
	ESSs	23, 79	-
non-dispatchable	WTs	152, 61, 48, 300, 87	-
	PVs	53, 30, 50, 108, 450	-

8.5.1 Convergence Analysis

The convergence analysis is conducted by implementing the algorithm with different values of ρ between 15 and 500. Table 8-2 shows that the algorithm can reach convergence within 700 iterations for different values of ρ . Moreover, the algorithm can converge to the optimal solution of the Augmented Lagrangian with these values of ρ , which shows a good convergence ability of the algorithm. In addition, the convergence performance of the algorithm highly depends on the selection of ρ . For instance, an improper value $\rho = 500$ would cause slow convergence. Within the certain range, the algorithm with $\rho = 30$ has the best convergence performance. The convergence process of the ADMM-based algorithm with $\rho = 30$ is shown in Fig.

8-2. It can be seen that the primal and dual residuals are lower than thresholds after 182 iterations, which demonstrates the feasibility and convergence of the proposed algorithm.

Table 8-2 Numbers of iterations with different values of ρ

ρ	15	20	30	40	50	100	150
number of iterations	237	187	182	166	257	500	508
ρ	200	250	300	350	400	450	500
number of iterations	619	604	599	655	671	651	617

Fig. 8-2 Convergence process of the proposed algorithm ($\rho = 30$)

8.5.2 Risk-Limiting Load Restoration Performance

8.5.2.1 Restoration Procedure

Three methods are conducted to demonstrate the effectiveness of the proposed distributed risk-limiting scheme. The three methods are: 1) the optimal service restoration without considering risk limitation (OLR); 2) the centralized risk-limiting service restoration (CRLR); 3) the proposed distributed risk-limiting service restoration (DRLR).

The accumulated percentages of the load pick-up amount (load pick-up results), risk indexes R_{index} and CVaR values with the three methods are listed in Table 8-3. The load pick-up results have been validated by conducting the power flow analysis based on the expected forecast values. 1000 scenarios randomly generated by the Monte Carlo simulation are used to check the risk index. The risk index at each time step denotes the percentage of those scenarios in which the FRR constraints are violated among the 1000 scenarios.

As shown in Table 8-3, all loads can be restored during the restoration horizon in three methods. Although the CRLR and DRLR methods pick up loads slowly and

have lower objective values compared with the OLR method, the risk indexes are reduced substantially. For example, the CRLR and DRLR methods pick up fewer loads at step t_1 in order to reduce the risk of the FRR constraint violation. Moreover, the DRLR method picks up fewest loads and consequently has the smallest CVaR value and risk index. In addition, Table 8-3 shows that the CVaR values in the DRLR method are lower than its bounds (R_b) at each step. This means that the risk-limiting constraints of the whole network are satisfied if the risk-limiting constraints of each MG are satisfied. The above results demonstrate that the proposed DRLR method can restore loads efficiently with a smaller risk index and distributed risk-limiting ability.

Table 8-3 Load pick up results of three methods

index	method	time steps					obj. value
		t_1	t_2	t_3	t_4	t_5	
load pick-up	OLR	32.94%	56.0%	77.4%	100%	100%	306.40
	CRLR	29.0%	54.4%	73.9%	93.2%	100%	297.43
	DRLR	27.8%	50.2%	70.2%	88.8%	100%	288.16
R_{index}	OLR	32.7%	49.4%	47.8%	47.6%	100%	/
	CRLR	3.6%	6.2%	20.3%	6.4%	0	/
	DRLR	0.4%	0.7%	5.5%	0.8%	0	/
CVaR	OLR	66.1	68.2	65.1	69.2	-12.3	6.0
	CRLR	62.3	62.2	61.8	61.8	13.6	0.7
	DRLR	59.6	57.2	58.2	56.5	32.6	-1.9
R_b	CRLR	62.4	62.4	62.4	62.4	60.0	61.2
	DRLR	62.4	62.4	62.4	61.2	61.2	60.0

Furthermore, the following three scenarios are conducted to study the impacts of the VR operation and CLPU phenomenon on service restoration.

- Base scenario: the VR operation and CLPU phenomenon are considered in this scenario.
- Scenario 1: compared with the base scenario, the VR operation is not considered.
- Scenario 2: compared with the base scenario, the CLPU phenomenon is not considered.

Load pick-up results in the three scenarios are listed in Table 8-4. Compared with the base scenario, loads are picked up slowly in scenario 1. Only 95.6% of loads

are restored at the end of the restoration horizon even if additional four restoration steps are included, i.e., a horizon with 10 restoration steps. This is because the loads at node 66 remain de-energized to avoid the voltage magnitude violation at node 64 at phase *b*. Thus, using VRs can facilitate load restoration. In scenario 2, the restoration procedure picks up loads more rapidly and has a larger objective value. It means that the CLPU phenomenon defers the restoration procedure because additional power is absorbed due to the CLPU.

Table 8-4 Load pick up results in three scenarios

	time steps						obj. value
	t_1	t_2	t_3	t_4	t_5	t_6	
base	27.8%	50.2%	70.2%	88.8%	100%	100%	288.16
sce1	27.8%	50.2%	70.2%	88.5%	95.6%	95.6%	283.29
sce2	35.9%	55.7%	75.8%	91.1%	100%	100%	302.29

8.5.2.2 Flexible Risk Management

The proposed DRLR method keeps the flexible risk management ability of the CVaR method. Table 8-5 shows load pick-up results with different probability levels α . It is shown that a larger probability level leads to a more conservative restoration solution with a slower load pick-up procedure and a smaller risk index. Therefore, flexible risk management can be realized by adjusting the probability level.

Table 8-5 Load pick-up results with different probability levels

α	time steps					obj. value	$R_{\text{index in total}}$
	t_1	t_2	t_3	t_4	t_5		
0.70	28.1%	51.7%	72.2%	91.7%	100%	291.87	16.6%
0.80	28.1%	50.5%	71.8%	90.7%	100%	290.33	12.9%
0.95	28.3%	49.0%	69.6%	87.9%	100%	287.00	5.8%

8.5.3 Computation Efficiency of the DRLR method

The computation time of the DRLR method is 355.3 seconds, which is lower than 455.2 seconds of the CRLR method. As far as the communication delay of the distributed method is concerned, it is assumed that the synchronous setting of the ADMM algorithm is used, which means that all sub-problems need to be solved before starting the next iteration no matter how much time the communication delay is [147]. Therefore, the communication delay would not affect the effectiveness of the proposed algorithm, but it would increase the total execution time (computation time plus the total communication delay). Since the system of networked

MGs is not big and suppose that there are direct fiber optical links between neighboring controllers, the communication delay is small and assumed to be 1 second.

Regarding the model size, the CRLR method solves an MILP problem with 93195 continuous variables and 648 binary variables, whereas the DRLR method solves smaller-scale QP problems, each of which has in average 16335 continuous variables in each calculation. The binary variables are dealt with by the projection function and not included in the optimization model. Therefore, the DRLR method outperforms the CRLR method in terms of computation efficiency.

8.5.4 Faults on Lines, Controllers or Communication Links

Three fault conditions are simulated as below,

- Scenario 3: a fault occurs on the line between nodes 35 and 135 within MG₂.
- Scenario 4: restoration controller of MG₂ has a fault and does not participate in the restoration process.
- Scenario 5: faults occur on communication links between restoration controllers.

8.5.4.1 Scenario 3

In this scenario, the switch 18-135 (the switch between nodes 18 and 135) is opened to isolate the fault and nodes 35-51, 135 and 151 should remain de-energized during restoration. The load pick-up result in Table 8-6 shows that the remaining 79.5% of loads can be fully restored after 5 restoration steps.

8.5.4.2 Scenario 4

If one controller fails, the remaining controllers would still work. In this scenario, the tie-switch 13-18 is opened and the loads in MG₂ remain de-energized. The load pick-up result in Table 8-6 shows that all loads in MG₁ and MG₃ (68.8% of loads) can be restored during the restoration horizon. However, the centralized method has the single-point failure risk. If the central controller fails, the restoration process cannot start.

8.5.4.3 Scenario 5

In this scenario, all MGs are disconnected by opening tie-switches and each controller makes restoration decisions based on local information. The load pick-up

result is shown in Table 8-6. Since MGs are disconnected, there are no power exchanges between MGs to support the restoration process. Therefore, it has the slower restoration procedure when all MGs are disconnected.

Table 8-6 Restoration results with three fault conditions

	time steps						obj. value
	t_1	t_2	t_3	t_4	t_5	t_6	
sce3	27.9%	44.3%	56.4%	68.8%	79.5%	79.5%	214.3
sce4	19.1%	33.5%	45.2%	58.2%	68.8%	68.8%	178.2
sce5	27.8%	50.7%	69.6%	88.2%	100%	100%	286.7

8.5.5 Impact of the Number of Reduced Scenarios on Solutions

Table 8-7 shows the load pick-up results with different numbers of reduced scenarios, i.e., 4, 20 and 50 scenarios. It is shown that it has a more conservative solution with a lower risk index as the number of reduced scenarios increases. This is expected because the uncertainty can be captured more accurately with more scenarios considered in the model. However, the number of scenarios affects the computation complexity of the problem and thus affects convergence performance of the algorithm. In the study, 20 scenarios are chosen because it has a good tradeoff between the computation complexity and uncertainty modeling accuracy.

Table 8-7 Load pick-up results with different reduced scenarios ($\rho=30$)

#	time steps					computation time	R_{index} in total
	t_1	t_2	t_3	t_4	t_5		
4	27.4%	49.0%	68.8%	88.8%	100%	254.8s	15.3%
20	27.8%	50.2%	70.2%	88.8%	100%	428.7s	7.4%
50	27.1%	48.7%	68.2%	87.1%	100%	581.7s	2.8%

#: the number of reduced scenarios

Table 8-8 Computation complexity of the problem with different restoration horizons

number of restoration steps		6	8	10
variables	binary	648	864	1080
	continuous	93195	124602	155729
constraints		10716	14669	18342
computation time (s)		455.3	633.9	1333.1

8.5.6 Impact of the Restoration Horizon on Computation Complexity

The computation complexity of the problem is summarized in Table 8-8 with different numbers of restoration steps, i.e., 6, 8 and 10 steps. The numbers of variables and constraints of the problem and computation time are listed. It is shown that the

problem considering a longer restoration horizon has more variables and constraints and needs more computation time. Since all loads can be restored after 5 restoration steps, as shown in Table 8-3, the restoration horizon with 6 restoration steps is chosen.

8.5.7 Analysis of Linearization of the MINLP Model

To deal with the voltage linearization error after applying linearization techniques, the minimum limit of the voltage magnitude is set as 0.95 p.u. in order to reserve a small security margin (0.02 p.u.) compared with the assumed physical limit 0.93 p.u.. The voltage linearization errors during the restoration horizon at phase *c* are shown in Fig. 3. The maximum error of three phases is 0.018 p.u. that is smaller than the preset security margin. Therefore, the voltage security of the solution obtained with the MILP model is guaranteed.

Fig. 8-3 The voltage linearization errors at phase *c*

The centralized load pick-up solutions of the MINLP model and MILP model are shown in Table 8-9. It can be seen that two solutions have similar restoration procedures and the relative difference of the objective value is 0.19%, which demonstrates that the MILP model can obtain a high-quality solution. Moreover, the distributed load pick-up solution is a feasible load pick-up solution of the original MINLP model. The relative difference of the MINLP objective function value between the distributed load pick-up solution and MINLP solution is 3.2%.

Table 8-9 Centralized load pick-up solutions of the MINLP and MILP problems

problems	time steps					obj. value
	t_1	t_2	t_3	t_4	t_5	
MINLP	31.2%	52.4%	73.7%	93.5%	100%	298.00
MILP	29.0%	54.4%	73.9%	93.2%	100%	297.43

8.6 Summary

This chapter proposes a distributed risk-limiting service restoration method for the unbalanced ADN with networked MGs based on the CVaR and ADMM. By using the CVaR as the risk-limiting index, the optimal risk-limiting service restoration problem is formulated as an MINLP model that is transformed into an MILP model by applying linearization techniques. With the sub-additivity of the CVaR and ADMM, the MILP model is decoupled and solved in a distributed manner, realizing the distributed risk limitation among MGs. The numerical results demonstrate that the proposed method can restore loads efficiently with a smaller risk index and can realize distributed and flexible risk management. Moreover, the proposed distributed method is more robust against different fault conditions compared with the centralized method.

CHAPTER 9.

CONCLUSION AND RECOMMENDATIONS

9.1 Conclusion

In this thesis, several market-based congestion management methods and service restoration methods were proposed in order to overcome the shortcomings of the existing methods and achieve the more efficient and secure operation and management of ADNs.

9.1.1 Congestion Management

The integration of different types of congestion management methods has been investigated. Firstly, a comprehensive congestion management scheme was proposed to combine the DT (price-based method), network reconfiguration (direct control method), and re-profiling product (incentive-based method) to alleviate day-ahead congestion. In the comprehensive scheme, the DT method integrated with network reconfiguration is used to alleviate congestion at the first step. Then, re-profiling products are traded at the second step to alleviate unsolved congestion. In the comprehensive scheme, a relaxed configuration-based DT optimization model was formulated, which considers both voltage magnitude limits and line loss reduction and resolves the infeasible issue of the original DT method. Moreover, the maximum DT is restricted by setting penalty coefficients, which ensures that customers would not face very high DTs. However, the integration of the DT and re-profiling product is implemented in a simple way by independent optimizations of the DT and incentive-based methods. In order to optimally utilize the synergy between the two methods and avoid potential conflicts with the current DSO regulations, this thesis further proposed a coordination scheme of the DT method and incentive-based method, in which the DSO is in a profit-neutral position through the integrated optimization of the two methods. The coordination problem is formulated as an MINLP model, which is then linearized and solved by a proposed PSO-based solution strategy.

In the DT method, there is an assumption that the DSO and aggregators use the same energy requirement parameters, which might not be the case in practice due to the DSO's forecast error. To overcome this issue, a robust DT method was proposed, in which a three-level robust optimization model is formulated and solved to obtain the robust DT solution with an iterative procedure. Based on the robust DT solution, the aggregator's energy plans respect the line loading constraints even if there is a discrepancy between the DSO's forecasted parameters and aggregators' accurate parameters.

In addition to day-ahead congestion, it is also important to deal with real-time congestion. Therefore, a two-tier real-time congestion management scheme based on the flexible demand swap and transactive control was proposed. In the lower tier, a transactive control-based interaction mechanism is developed to formulate flexible demand swap bids while considering the customer's specific energy rebound conditions and willingness to provide flexibility. In the upper tier, the flexible demand swap bids are traded in the swap market to alleviate congestion according to the DSO's requirements on congestion management.

9.1.2 Service Restoration

Most of the existing service restoration optimization models are solved by centralized algorithms based on centralized infrastructure, so they are faced with a few drawbacks regarding the computation complexity, single-point failure risk, difficulty in adapting to system changes, and privacy information protection. To reduce the computation complexity, a hierarchical service restoration scheme was proposed, in which the optimal service restoration problem is firstly formulated as a MINLP model. Then, by applying linearization techniques, the MINLP model is transformed into an MILP model that is decomposed and solved by the ADMM-based algorithm. Through decomposition, the service restoration problem is solved based on a hierarchical framework, in which each zone controller solves its local sub-problem and the central controller solves a global coordination problem. By using the proposed hierarchical service restoration scheme, the computational complexity of service restoration can be reduced substantially as compared with the centralized scheme.

However, such a hierarchical scheme is still based on centralized infrastructure. Therefore, we proposed a distributed service restoration scheme based on the ADMM for the unbalanced ADN. The service restoration problem is firstly formulated as a MINLP model, including radiality constraints with a decomposable structure. Then, the MINLP model is transformed into a MISOCP model through

convex relaxation, which is decomposed and solved using an ADMM-based algorithm that handles binary decision variables by projection functions. Through the decomposition, the service restoration problem is distributed among the zones of the ADN. The proposed distributed service restoration scheme does not have the single-point failure risk and respects privacy information protection.

In order to implement service restoration in case of the unavailability of power from the upstream TN and to consider the uncertainty of DERs, a distributed risk-limiting service restoration scheme was proposed for the unbalanced ADN with networked MGs based on the CVaR and ADMM. The CVaR-based optimal risk-limiting service restoration problem is firstly formulated as an MINLP model that is then transformed into an MILP model by applying linearization techniques. By utilizing the sub-additivity feature of the CVaR and ADMM, the MILP model is decoupled and solved in a distributed manner, which results in the distributed risk limitation among MGs. Each MG can make risk-limiting service restoration decisions individually without violating the risk-limiting constraint of the whole network.

9.2 Recommendations for Future Work

Several suggestions on future work includes:

- The DT method in this thesis is based on the DC OPF model to calculate DTs, the accuracy of the DC model for distribution networks is therefore a concern. Moreover, it is important to model the unbalanced characteristic of the distribution network. The future work will study a more practical and accurate OPF model for the DT method.
- The DT method in this thesis uses flexible demands, i.e., EVs and HPs, to alleviate day-ahead congestion. Congestion could also be alleviated by adjusting DERs that produce power locally, such as ESSs. The future work will study the integration of different types of DERs in the DT method to alleviate congestion more efficiently.
- Further organize different market-based congestion management methods to form a congestion management system to alleviate congestion in different time frames more thoroughly.
- In service restoration, network reconfiguration and the MG formation are two means to restore outage loads and are usually implemented separately.

The combined optimization of network reconfiguration and the MG formation could realize more efficient service restoration and will be investigated in the future.

Author's Publications

The publications in relation to the PhD study include 10 journal papers and 2 conference papers and are listed:

1. **Feifan Shen**, Qiuwei Wu*, and Yusheng Xue, "Review of Service Restoration for Distribution Networks," *Journal of Modern of Power Systems and Clean Energy*, vol. 8, no. 1, pp. 1-14, Jan. 2020.
2. **Feifan Shen**, Qiuwei Wu, Shaojun Huang, Juan Camilo López, Canbing Li, and Bin Zhou, "Review of Service Restoration Methods in Distribution Networks," 2018 *IEEE PES Innovative Smart Grid Technologies Conference Europe* (ISGT-Europe), 2018.
3. **Feifan Shen**, Shaojun Huang, Qiuwei Wu*, Sami Repo, Yan Xu, and Jacob Østergard, "Comprehensive Congestion Management for Distribution Networks Based on Dynamic Tariff, Reconfiguration and Re-profiling Product," *IEEE Trans. Smart Grid*, vol. 10, no. 5, pp. 4795-4805, Sep. 2019.
4. **Feifan Shen**, Qiuwei Wu*, and Shaojun Huang, "Day-ahead Congestion Management Scheme for Distribution Networks with Dynamic Tariff and Re-profiling Products," *International Journal of Smart Grid and Clean Energy*, vol. 9, no. 1, pp. 91-101, Jan. 2020.
5. **Feifan Shen**, Qiuwei Wu*, Xiaolong Jin, Menglin Zhang, Saeed Teimourzadeh, Osman Bulent Tor, "Coordination of dynamic tariff and scheduled reprofiling product for day-ahead congestion management of distribution networks," *International Journal of Electrical Power and Energy System*, vol. 135, pp. 107672, Feb. 2022.
6. **Feifan Shen**, Qiuwei Wu*, "Robust dynamic tariff method for day-ahead congestion management of distribution networks," *International Journal of Electrical Power and Energy System*, vol. 134, pp. 107366, Jan. 2022.
7. **Feifan Shen**, Qiuwei Wu*, Shaojun Huang, Xinyu Chen, and Yan Xu, "Two-Tier Demand Response with Flexible Demand Swap and Transac-

- tive Control for Real-Time Congestion Management in Distribution Networks,” *International Journal of Electrical Power and Energy System*, vol. 114, pp. 105399, Jan. 2020.
8. **Feifan Shen**, Qiuwei Wu*, Yan Xu, Fangxing Li, Fei Teng, and Goran Strbac, “Hierarchical Service Restoration Scheme for Active Distribution Networks Based on Alternating Direction of Multipliers (ADMM),” *International Journal of Electrical Power and Energy System*, vol. 118, pp. 105809, Jun. 2020.
  9. **Feifan Shen**, Juan C. Lopez, Qiuwei Wu*, Marcos J. Rider, Tianguang Lu, and Nikos D. Hatziargyriou, “Distributed Self-healing Scheme for Unbalanced Electrical Distribution Systems Based on Alternating Direction Method of Multipliers (ADMM),” *IEEE Trans. Power System*, vol. 35, no. 3, pp. 2190-2199, May 2020.
  10. **Feifan Shen**, Qiuwei Wu, Juan Camilo López, and Marcos J Rider, “Distributed Service Restoration of Active Electrical Distribution Systems using ADMM,” *10th Conference on Innovative Smart Grid Technologies*, 2019.
  11. **Feifan Shen**, Qiuwei Wu*, Jin Zhao, Wei Wei, Nikos D. Hatziargyriou, and Feng Liu, “Distributed Risk-limiting Load Restoration in Unbalanced Distribution Systems with Networked Microgrids,” *IEEE Trans. Smart Grid*, vol.11, no.6, pp. 4574-4586, May 2020.
  12. **Feifan Shen**, Qiuwei Wu*, Xiaolong Jin, Bin Zhou, Canbing Li, and Yan Xu, “ADMM-based Market Clearing and Optimal Flexibility Bidding of Distribution-level Flexibility Market for Day-Ahead Congestion Management of Distribution Networks,” *International Journal of Electrical Power and Energy Systems*, vol. 123, pp. 106266, Dec. 2020.

The publications 1-11 are incorporated in the thesis, while the rest of them are cited as references wherever relevant. For the sake of consistency, certain notations in this thesis differ from the notations used in the publications.

APPENDIX A

SOLUTION EQUIVALENCE PROOF OF THE DSO OPTIMIZATION AND AGGREGATOR OPTIMIZATION IN THE DYNAMIC TARIFF METHOD

The solution equivalence between the DSO optimization model and aggregator optimization model in the DT method is proven in this section.

A.1 The KKT Conditions of the DSO Optimization Model

The KKT conditions of the DSO optimization model in (2.1)-(2.7) are formulated in (A1)-(A12), as below,

$$B_{i,t} p_{i,t} + c_t \mathbf{1} + E_i^T \rho_t + \sum_{t \leq t_+} (\mu_{i,t_+}^+ - \mu_{i,t_+}^-) + (\sigma_{i,t}^+ - \sigma_{i,t}^-) = 0, \quad (\text{A1})$$

$$B_{i,t} \tilde{p}_{i,t} + c_t \mathbf{1} + E_i^T \rho_t + \sum_{t \leq t_+} A_{i,t_+,t}^T (\tilde{\mu}_{i,t_+}^+ - \tilde{\mu}_{i,t_+}^-) + (\tilde{\sigma}_{i,t}^+ - \tilde{\sigma}_{i,t}^-) = 0, \quad (\text{A2})$$

$$G^T \rho_t + \gamma_t = 0, \quad (\text{A3})$$

$$(F_t^p - f_t^{\max}) \perp \gamma_t = 0, \quad (\text{A4})$$

$$e_{i,t}^{\min} - \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) - e_{i,0} \leq 0 \perp \mu_{i,t}^- \geq 0, \quad (\text{A5})$$

$$\sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} - e_{i,t}^{\max} \leq 0 \perp \mu_{i,t}^+ \geq 0, \quad (\text{A6})$$

$$K_{i,t}^{a,\min} - \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} - K_{i,0}^a \leq 0 \perp \tilde{\mu}_{i,t}^- \geq 0, \quad (\text{A7})$$

$$\sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a - K_{i,t}^{a,\max} \leq 0 \perp \tilde{\mu}_{i,t}^+ \geq 0, \quad (\text{A8})$$

$$p_{i,t}^{\min} - p_{i,t} \leq 0 \perp \sigma_{i,t}^- \geq 0, \quad (\text{A9})$$

$$p_{i,t} - a_{i,t} p_{i,t}^{\max} \leq 0 \perp \sigma_{i,t}^+ \geq 0, \quad (\text{A10})$$

$$\tilde{p}_{i,t}^{\min} - \tilde{p}_{i,t} \leq 0 \perp \tilde{\sigma}_{i,t}^- \leq 0, \quad (\text{A11})$$

$$\tilde{p}_{i,t} - \tilde{p}_{i,t}^{\max} \leq 0 \perp \tilde{\sigma}_{i,t}^+ \geq 0, \quad (\text{A12})$$

A.2 The KKT Conditions of the Aggregator Optimization Model

The KKT conditions of the aggregator optimization model in (2.8)-(2.12) are formulated in (A13)-(A22), as below,

$$B_{i,t} p_{i,t} + c_t \mathbf{1} + E_i^T \rho_t + \sum_{t \leq t_+} (\mu_{i,t_+}^+ - \mu_{i,t_+}^-) + (\sigma_{i,t}^+ - \sigma_{i,t}^-) = 0, \quad (\text{A13})$$

$$B_{i,t} \tilde{p}_{i,t} + c_t \mathbf{1} + E_i^T \rho_t + \sum_{t \leq t_+} A_{i,t,t_+}^T (\tilde{\mu}_{i,t_+}^+ - \tilde{\mu}_{i,t_+}^-) + (\tilde{\sigma}_{i,t}^+ - \tilde{\sigma}_{i,t}^-) = 0, \quad (\text{A14})$$

$$e_{i,t}^{\min} - \sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) - e_{i,0} \leq 0 \perp \mu_{i,t}^- \geq 0, \quad (\text{A15})$$

$$\sum_{t_- \leq t} (p_{i,t_-} - d_{i,t_-}) + e_{i,0} - e_{i,t}^{\max} \leq 0 \perp \mu_{i,t}^+ \geq 0, \quad (\text{A16})$$

$$K_{i,t}^{a,\min} - \sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} - K_{i,0}^a \leq 0 \perp \tilde{\mu}_{i,t}^- \geq 0, \quad (\text{A17})$$

$$\sum_{t_- \leq t} A_{i,t,t_-} \tilde{p}_{i,t_-} + K_{i,0}^a - K_{i,t}^{a,\max} \leq 0 \perp \tilde{\mu}_{i,t}^+ \geq 0, \quad (\text{A18})$$

$$p_{i,t}^{\min} - p_{i,t} \leq 0 \perp \sigma_{i,t}^- \geq 0, \quad (\text{A19})$$

$$p_{i,t} - a_{i,t} p_{i,t}^{\max} \leq 0 \perp \sigma_{i,t}^+ \geq 0, \quad (\text{A20})$$

$$\tilde{p}_{i,t}^{\min} - \tilde{p}_{i,t} \leq 0 \perp \tilde{\sigma}_{i,t}^- \leq 0, \quad (\text{A21})$$

$$\tilde{p}_{i,t} - \tilde{p}_{i,t}^{\max} \leq 0 \perp \tilde{\sigma}_{i,t}^+ \geq 0, \quad (\text{A22})$$

A.3 Solution Equivalence Proof

Suppose that $s^* := p_{i,t}^*, \tilde{p}_{i,t}^*, \rho_t^*, \gamma_t^*, \mu_{i,t}^{+,*}, \mu_{i,t}^{-,*}, \tilde{\mu}_{i,t}^{+,*}, \tilde{\mu}_{i,t}^{-,*}, \sigma_{i,t}^{+,*}, \sigma_{i,t}^{-,*}, \tilde{\sigma}_{i,t}^{+,*}, \tilde{\sigma}_{i,t}^{-,*}$ is a solution of the DSO optimization model and the solution also satisfies the KKT conditions (A1)-(A12). By comparing the KKT conditions, the solution also satisfies the KKT conditions (A13)-(A22) and constraints of the aggregator optimization model. Therefore, s^* is also a solution of the aggregator optimization model. It is concluded that the DSO solution must be the aggregator solution.

On the other hand, it should be proven that the aggregator solution must be the DSO solution. Suppose a solution s^{**} is the aggregator solution but not the DSO solution. According to the previous conclusion, the solution s^* is the DSO solution and the aggregator solution as well. Due to convexity of the aggregator optimization model, the aggregator has a unique solution. Therefore, $s^* = s^{**}$ and s^{**} is also the DSO solution, which contradicts the assumption that the solution s^{**} is the aggregator solution but not the DSO solution. Based on these discussions, the aggregator solution must be the DSO solution. Therefore, the solution equivalence between the DSO optimization model and aggregator optimization model is proven.

APPENDIX B

SIMULATION PARAMETERS

B.1 Grid Data

The case studies in chapters 2-5 are conducted on the Bus 4 distribution network of the Roy Billinton Test System (RBTS) [102]. Its basic structure is shown in Fig. B-1. In chapters 2 and 4, some specific modifications have been made to fit into the case study design.

The customer type at each load point and the peak conventional load per load point are listed in Table B-1. The 24-hour conventional load profile for each load point can be obtained by multiplying the peak load of each load point by the load coefficient listed in Table B-6. It is assumed that each residential customer has one EV and/or HP.

Fig. B-1 Single line diagram of the distribution network

Table B-1 Load points data

load points (LP)	customer type	peak conv. load per point (kW)	number of customers per point
LP _{1-4,11-13,18-21,32-35}	residential	886.9	200
LP _{5,14,15,22,23, 36, 37}	residential	813.7	200
LP _{6,7,16,17,24,25,38}	commercial	671.4	10

The 44-node distribution network [62] shown in Fig. B-2 is used in case studies in chapters 6 and 7. For the balanced case in chapter 6, the node data is listed in Table B-2 (Nodes not included in the table are interconnection nodes without loads). For simplicity, all branch parameters are considered equal, with $R_{ij} = 0.9 \Omega$, $X_{ij} = 0.9 \Omega$, $\bar{I}_{ij}^{sqr} = 2000A$. For the unbalanced case in chapter 7, the load at phase b at each node is 80% of the load at phase a , and load at phase c at each node is 60% of the load at phase a . The phase to phase impedance is considered as zero.

Fig. B-2 Single diagram of the 44-node distribution network

Table B-2 Node data

Node [<i>i</i>]	P_i [kW]	Q_i [kVar]
2	100	20
5	100	20
8	100	20
10	100	20
12	100	20
14	100	20
20	100	20
24	100	20
27	100	20
30	100	20
35	200	40
40	200	40
41	200	40

B.2 EV Parameters.

The key parameters of EVs are listed in Table B-3. The EV availability is obtained from the driving pattern study in [101]. The data of 50 EVs is used to represent a large EV fleet in case studies and is listed in Table B-4. The EV is available if it can charge power.

Table B-3 Key parameters of EVs

parameter	value
EV battery size	25 kWh
Peak charging power	11 kW (3 phase)
Energy consumption per km	150 Wh/km
Minimum SOC	20%
Maximum SOC	85%

B.3 Diagonal Matrix B

In the case studies, diagonal matrix B (price sensitive matrix) has same diagonal elements, which are equal to 0.0001.

Table B-4 EV availability of a sample of 50 EVs

	e0001	e0002	e0003	e0004	e0005	e0006	e0007	e0008	e0009	e0010	e0011	e0012	e0013	e0014	e0015	e0016	e0017	e0018	e0019
t_1	1	1	0.16667	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1
t_2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1
t_3	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1
t_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
t_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
t_6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
t_7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
t_8	1	1	1	1	1	1	1	1	1	0.35	0.76667	1	0.016667	1	0.85	1	1	1	0.51667
t_9	1	1	1	1	1	1	1	1	0.016667	0	0	1	0	1	0	1	1	1	0
t_{10}	0.51667	0.1	1	1	1	1	1	1	0.016667	0	0	1	0	1	0	1	0.016667	0.21667	0
t_{11}	0	0	1	1	1	1	1	1	0	0	0	1	0	0.15	0	1	0.083333	1	0
t_{12}	0	1	1	1	1	1	1	1	0	0	0	1	0	1	0	1	1	1	0
t_{13}	0	1	1	1	1	0.51667	1	1	0	0	0	1	0	1	0	1	1	1	0
t_{14}	0	1	1	1	1	0.25	1	1	0	0	0	1	0	1	0	1	0.016667	1	0
t_{15}	0	1	1	1	0.6	1	1	1	0	0	0	1	0	1	0	1	0	1	0
t_{16}	0	1	1	1	1	1	0.18333	1	0	0.26667	0	1	0.083333	1	0	1	0.66667	1	0
t_{17}	0	1	0.016667	1	1	1	0	1	0.18333	0.91667	0	1	1	1	0.6	1	1	1	0
t_{18}	0	1	0.55	1	1	1	0	1	0.91667	0.51667	0.91667	0.76667	1	1	1	1	1	1	0
t_{19}	0	1	1	1	0.51667	1	0.8	1	1	0	0.35	0	1	1	1	1	1	1	0
t_{20}	0	1	1	1	0	0.6	1	1	1	0	0.45	0	1	1	1	1	1	1	0.68333
t_{21}	0	1	1	1	0	0	1	1	1	0	1	0	1	1	1	1	1	1	1
t_{22}	0.58333	1	1	1	0	0	1	1	1	0	1	0	1	1	1	1	1	1	1
t_{23}	1	1	1	1	0	0	1	1	1	0	1	0.91667	1	1	1	1	1	1	1
t_{24}	1	1	1	1	0	0	0.6	1	1	0.41667	1	0.8	1	1	1	1	1	1	1

[illegible]

	e0037	e0038	e0039	e0040	e0041	e0042	e0043	e0044	e0045	e0046	e0047	e0048	e0049	e0050
t_1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
t_2	1	1	1	1	1	1	1	1	1	0	1	1	1	1
t_3	1	1	1	1	1	1	1	1	1	0	1	1	1	1
t_4	1	0.016667	1	1	1	1	1	1	1	1	1	1	1	1
t_5	1	0	1	1	1	1	1	1	1	1	1	1	1	1
t_6	1	0	1	1	1	1	1	1	1	1	1	1	1	1
t_7	1	0	1	1	1	1	1	1	1	1	1	0.35	1	1
t_8	0.68333	0.75	1	1	1	1	0.51667	1	1	0.93333	0.016667	0	0.93333	1
t_9	0	1	1	1	1	1	0	1	0.016667	0.66667	0	0	0	1
t_{10}	0	1	1	1	1	1	0	1	0	1	0	0	0	1
t_{11}	0	1	0.016667	1	1	1	0	1	0	1	0	0	0	0.6
t_{12}	0	1	0	1	1	1	0	1	0	1	0	0	0	0.58333
t_{13}	0	1	0.86667	1	0.51667	1	0	1	0	1	0	0.25	0	1
t_{14}	0	1	1	1	0	1	0	1	0	1	0	1	0	1
t_{15}	0	1	1	1	0	1	0	1	0	0.51667	0	1	0	1
t_{16}	0	1	1	1	0	1	0	0.26667	0	1	0	0.51667	0	1
t_{17}	0.61667	1	1	1	0.66667	1	0	0.55	0.91667	1	0.083333	0.63333	0	1
t_{18}	1	1	1	1	1	1	0.91667	1	1	1	1	0.26667	0	1
t_{19}	1	1	1	1	1	1	1	1	1	1	1	0	0	1
t_{20}	1	1	1	1	1	1	1	1	1	1	1	0	1	1
t_{21}	1	1	1	1	1	1	1	1	1	1	0.76667	0	1	1
t_{22}	1	1	1	1	1	1	1	1	1	1	0.66667	0	1	1
t_{23}	1	1	1	1	1	1	1	1	1	1	1	0	1	1
t_{24}	1	1	1	1	1	1	1	1	1	1	1	0.66667	1	1

B.4 HP and House Thermal Parameters

The key parameters of HPs are listed in Table B-5. The house thermal parameters can be found in [20] and [148].

Table B-5 Key parameters of HPs

parameter	value
COP of HP	2.3
Initial Inside Air Temp.	20 °C
Initial Structure Temp.	15 °C
Min Temp. of the House	20 °C
Max Temp. of the House	24 °C

B.5 Forecasted Day-Ahead Spot Prices, Outside Temperature and Conventional Load Coefficients

The predicted day-ahead spot price (c_t), outside temperature, and conventional load coefficients are shown in Table B-6.

Table B-6 Forecasted day-ahead spot prices, outside temperature, and conventional load coefficient

time	outside Temp. (°C)	day-ahead spot price (DKK/kWh)	conv. load coefficient
t_1	1.0000	0.37109	0.43491
t_2	1.9799	0.38109	0.42124
t_3	1.9942	0.35177	0.41575
t_4	1.9449	0.34177	0.42715
t_5	3.0281	0.30115	0.44823
t_6	3.0027	0.31115	0.54529
t_7	3.0019	0.32018	0.64725
t_8	3.0258	0.33018	0.62588
t_9	3.7540	0.34309	0.60073
t_{10}	3.9839	0.30309	0.56767
t_{11}	4.0278	0.35185	0.54251
t_{12}	4.6447	0.36185	0.54043
t_{13}	5.0854	0.37168	0.53420
t_{14}	3.9327	0.41612	0.54061
t_{15}	4.5145	0.35155	0.55981
t_{16}	5.0699	0.38838	0.69040
t_{17}	3.8971	0.35125	0.93859
t_{18}	4.0556	0.33125	1.00000
t_{19}	2.5406	0.30465	0.92232
t_{20}	2.0209	0.31465	0.84300
t_{21}	2.0014	0.30212	0.75081
t_{22}	1.9993	0.32212	0.64897
t_{23}	1.9902	0.29414	0.52515
t_{24}	1.8782	0.22414	0.43786

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