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QuadPlus: Design, Modeling, and Receding Horizon-based Control of a Hyperdynamic Quadrotor

Karanjot Singh, Mohit Mehndiratta, and Mir Feroskhan*

Abstract—The maneuverability of standard quadrotors with coplanar propellers is limited by their inherent underactuation. To overcome this challenge, in-flight, active propeller tilting has been widely investigated in the literature. However, bi-axial propeller tilting that renders extensive thrust vectoring has not been explored much owing to the ensuing mechanical complexities. Therefore, this paper presents an innovative design capable of achieving independent bi-axial tilting of 100° and 180° about two perpendicular axes while keeping the mechanical complexity relatively low. The developed quadrotor, aptly named *QuadPlus*, efficiently combines actuator redundancy and propeller rotation in a compact package, compared to its state-of-the-art counterparts. The hyperdynamic *QuadPlus* with a total of 12-DoFs can control its attitude independent of the position, thus enabling effective maneuvering through narrow spaces. Moreover, a novel cascade approach comprising of a high-level nonlinear model predictive control (NMPC) algorithm is adopted to obtain the optimal actuator configuration for an underdetermined system while dealing with the physical constraints. Also, proportional-integral-derivative controllers are employed at low-level to track attitude references generated by the navigation algorithm. Finally, with the help of realistic Gazebo simulations, the efficacy of the system is demonstrated by tracking complex 3-D trajectories which replicate the motion in a constrained environment. Overall, the obtained results manifest *QuadPlus*'s capability of achieving independent position and attitude control even with multiple actuator saturation. The Authors envision that the proposed simplistic design would stimulate interest in the community for exploring the benefits offered by bi-axial propeller tilting platforms.

Index Terms—Tiltrotor, Bi-axial propeller tilting, Overactuated quadrotor, Thrust vectoring, Model Predictive Control.

I. INTRODUCTION

The last decade had witnessed substantial advancements in the embedded technology leading to widespread use of autonomous aerial robots for applications ranging from package delivery [1], infrastructure surveillance [2], transportation [3], [4], precision agriculture [5], and aerial mapping and imaging [6]. Besides, operations such as search and rescue [7] involve flights through cluttered and constrained surroundings (refer Fig. 1), thereby requiring a highly maneuverable platform. Another application where maneuverability comes in handy is aerial manipulation [8]. Therein, the forces need to be applied along specific directions, thus requiring independent position and attitude control. While the widely utilized standard quadrotors with coplanar propellers suffer from inherent underactuation, the need for maneuverability enhancement seems apparent. In this respect, a fully actuated (or overactuated) aerial robot provides the capability to counterbalance

Figure 1: (a) *QuadPlus* maneuvering through narrow space during search and rescue operation. (b) Illustration of the two axes of tilting for each propeller, (c) Application of bi-axial propeller tilting for hovering in a desired hyperplane at an arbitrary angle: γ to the horizontal plane.

the reaction forces originating from the physical interaction while maintaining a stable attitude.

Conventionally, the multirotors developed with the aim of maneuvering through tight spaces focus on shape-morphing to achieve momentary size reduction. The designs in [9], [10] reduce the span of the platform dynamically in flight to enable high-speed navigation through unstructured spaces. The foldable drone in [11] is further able to achieve different morphologies in order to adapt to various shapes of the spatial openings. However, the morphological changes lead to loss

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of controllability of one or more *degrees-of-freedom* (DoFs). Besides, they require switching between various morphology-based adaptive control strategies, which may lead to in-flight instability. On the other hand, the multi-link aerial robot in [12] achieves overactuation by aligning the propellers in different planes. The resultant aerial robot is highly maneuverable and robust against failures and external interactions. Nevertheless, the multiple mechanical linkages underlie severe design complexities with a high loading of the propellers due to increased weight.

Keeping the aforementioned challenges in mind, the contributions of this work are two folds. First is the design of an innovative quadrotor platform – *QuadPlus* – with rotationally fully-actuated (2-DoF) propellers. A balanced approach between mechanical complexity and maneuverability is targeted using independently controlled servo motors to actuate the bi-axial propeller rotation. Consequently, the proposed overactuated and hyperdynamic quadrotor is capable of thrust vectoring that renders simultaneous control of all 6-DoFs. In essence, the focus of the design is to achieve independent position and attitude control of the platform for navigating through constrained spaces rather than via shape morphing. Additionally, the tilting constraints on propeller rotation and the overactuated nature of the system renders its control a constrained optimization problem. Therefore, the second contribution of this work is the development of a novel receding horizon principle-based cascade control approach that facilitates a decoupled position and attitude control. At high-level, a nonlinear model predictive control (NMPC) algorithm performs the position and heading tracking. Essentially, it computes the optimal propeller tilt angles along with the total thrust for an underdetermined system while catering to the propeller tilting constraints. On the other hand, the low-level proportional-integral-derivative (PID) controllers are tasked with maintaining the quadrotor's attitude by commanding differential thrust from the propellers. The efficacy of the designed platform and the proposed control framework is manifested for tracking 3-D trajectories in a realistic Gazebo simulation environment. The rationale behind selecting these complicated trajectories is to mimic a constrained environment that is encountered during the search and rescue operation.

The work in this paper is presented as follows. Section II discusses the relevant research carried out on the tilt-rotor platforms. Section III provides design details of the fully decoupled bi-axial tilting mechanism of the propellers. Section IV describes the mathematical modeling of the proposed design. Section V explains the proposed cascade control framework. The simulation results pertaining to the trajectory tracking are illustrated in Section VI. Finally, the drawn conclusions and possible future works are mentioned in Section VII.

II. RELATED WORK

A. Design Methodologies

The concept of thrust vectoring in multirotor aerial robots has been explored using *fixed* or *active* propeller tilting. A detailed analysis of the hexrotor platform with *fixed* tilting angles about a single axis has been conducted in [13], [14].

Further optimization of this design is carried out in [15] by fixedly tilting the propellers about two conceivable axes of rotation. The platform is rendered fully actuated by controlling the direction of the thrust vector using the rotational speeds of the tilted propellers. The omission of additional actuators such as servo motors also makes the system reliable and robust to failure. However, due to their fixed orientation, the propellers partially oppose the forces created by each other in an attempt to direct the resultant thrust vector along the desired trajectory. Therefore, it can be optimized only for a specific trajectory following [16]. Consequently, the fully actuated hexrotor configuration achieves enhanced maneuverability by sacrificing the energy efficiency. An eight rotor configuration with four fixed main (longitudinal) and lateral rotors each is introduced in [17] which aims to decouple the attitude and position control. However, the flow interaction between the lateral and main propellers brings additional nonlinearities into the system, thereby, making the control complicated.

In order to overcome the inherent underactuation associated with a quadrotor, a design with single-axis *active* tilting of the propellers has been presented in [18]. The additional 4-DoFs connected with the propeller tilting make the system overactuated and allow full controllability over the 6-DoFs of the quadrotor. Consequently, it is possible to hover in an arbitrary orientation and also apply force in the lateral directions [19]. However, the design is inherently limited by the finite tilting angles of the actuators about one axis. It is unable to counter the gyroscopic moments generated due to propeller rotation, thereby, affecting the transient response of the system. The servo dynamics also need to be considered to improve the system response in case of sudden external disturbances emerging from interaction with the environment.

In [20], bi-axial *active* tilting of the propellers with a limited range of rotation about both axes has been utilized to improve the transient response of the quadrotor. A push-pull lever mechanism is used in conjunction with a servo motor and gear arrangement to enable lateral axis tilting. Thus, additional actuation is achieved by increasing the mechanical complexity of the design while also making it sensitive to breakdowns (gear alignment, lever arrangement). Another bi-axial tilting approach involves the propellers being placed in the center of a sizeable C-shaped channel [21]. Although the design enables extensive bi-axial rotation, it seems unwieldy, thus neutralizing the maneuverability gained due to the propeller rotation. A recent approach towards bi-axial tilting in a quadrotor focuses on addressing the geometrical constraints associated with tilting using a combination of linear servos and compound joints [22]. The design is able to overcome the constraints with regards to inward tilting of the propellers. However, in order to reduce the mechanical and control complexity, the propellers are kept co-planar even after rotation. This will affect the maneuverability of the platform as the thrust vector of all the propellers always points in the same direction.

B. Control Techniques

The additional DoFs added with the tilting of the propellers end up making the system overactuated. As such, position and

attitude can be controlled independently as shown in [18], [23]. It uses a feedback linearized controller with pseudo inverse approach for control allocation. Another control approach for dealing with large attitude changes has been described in [19]. PID controllers for the position and attitude control are used to generate pseudo control outputs (forces and moments). The determination of tilt angles is based on inverse kinematics and is used for position control only. Attitude control is performed using differential thrust generation of the propellers. These linearized approaches are shown to perform maneuvers involving on-the-spot rotations as well as 2-D trajectory tracking with the desired attitude. However, these approaches are not able to account for actuator saturation considering the limited angles of tilting.

In [24], a cascaded approach based on PID control is used for attitude control alongside position control. The outer loop of the attitude controller is used for generating reference body rates based on attitude errors. While the inner loop has moments as control outputs. In order to account for actuator saturation, the nonlinear control allocation problem is linearized by distinguishing vertical and lateral forces. Nevertheless, maximum propeller speed limits cannot be handled by this approach. Due to the nonlinear coupling between the tilt angles and angular velocities, it is also not possible to separate the slower dynamics of tilting (servo-controlled) from the faster dynamics of moment generation (thrust from the propeller rotation). Similar control approach explored in [25] lays emphasis on deriving dynamic equations and control strategy for an eight rotor configuration while ignoring the aerodynamic effects originating from a close interaction between the propellers.

Keeping in mind the nonlinear and constrained nature of the control problem, (N)MPC is a viable candidate. Over other model-based control approaches, it has been remarkably successful in control and planning of numerous robotic platforms and tasks, including: (i) ground and aerial robots in [26]–[28], (ii) aerial manipulation tasks in [29], [30], and (iii) aerial operations under uncertain conditions in [31], [32]. The underlying reason for this success is its unique ability to simultaneously handle constraints and optimize performance through recursive online optimization. Given the requirement of constraint-based optimization for the control of the hyperdynamic quadrotor, NMPC is used in this work to manage the following challenges: (i) tilt angle constraints of the servos and (ii) highly nonlinear and coupled translational and yaw dynamics.

III. DESIGN CONCEPTUALIZATION

The conception of the proposed design stemmed from the principle that two rotational DoFs are sufficient to describe the orientation of a propeller in 3-D space. The employment of separate actuators enables us to explore different practicable alignments for achieving a higher range of rotation about each axis. For this purpose, Dynamixel AX-12A dual shaft servo is used for the integration of multiple servo modules in various configurations. It also provides the capability to track its position, speed of rotation, and torque with the help

Figure 2: Components associated with each rotation: (a) Dynamixel AX-12A servo, (b) U-shaped bracket F2 attached to the servo shaft, (c) Bracket F3 used for mounting the base of the servo.

of a fully integrated controller. The U-shaped bracket F2 (depicted in Fig. 2) attached to the high-torque dual output servo shaft provides a level base for mounting the motors sequentially. Correspondingly, the bracket F3 can be attached to the servo body frame to align it along the desired axis of rotation. Therefore, the mounting brackets help explore different configurations for aligning the servo motors and keep the design compact. The detailed design description can be viewed at: <https://www.youtube.com/watch?v=he5imbx4K9k>.

A. Initial Design Abstractions

The existing designs discussed in Section II-A either have a very limited or null rotation about the axis lateral to the length of the arm (classified here as lateral axis). A higher range of motion for the bi-axial rotation of the propeller helps in improving the maneuverability through more extensive thrust vectoring. To achieve a high range of rotation about each axis, three different alignments of the servo motors are analyzed as depicted in Fig. 3. Configuration 1 is distinct from the other configurations as the top servo is responsible for the rotation about the local lateral axis. Correspondingly, the bottom servo manages the rotation about the axis aligned along the length of the arm (classified here as longitudinal axis). The top servo is mounted directly on the shaft of the bottom servo using bracket F3 while the BLDC motor driving the propeller is mounted on the top servo using bracket F2. The inward tilting of this configuration is limited by the interference of the propeller with the arm (carbon fiber tube) and has a range of about 45° (see Fig. 3a).

For both the remaining configurations, the top servo carries out the rotation about the local longitudinal axis and the bottom servo performs the rotation about the local lateral axis. The bracket F2 is used for mounting the top servo (which is secured to bracket F3) and the BLDC motor successively. The only difference between configurations 2 and 3 is the alignment of the bottom servo. It is aligned vertically for configuration 2 with the inward tilting limit of about 50° caused by the propeller interference with the arm similar

Figure 3: Illustration of different arrangements of servo motors for one arm with the causes of their respective inward tilting limits (highlighted in red): (a) Configuration 1 with approx. 45° tilt limit, (b) Configuration 2 with approx. 50° tilt limit, (c) Configuration 3 with approx. 10° tilt limit.

to configuration 1 (refer Fig. 3b). On the other hand, the horizontal alignment of the bottom servo in configuration 3 limits the inward tilting to slightly more than 10° due to the interference of the bracket F2 with the servo body frame as observed in Fig. 3c.

B. Finalized Configuration

Based on an elementary topological analysis, it is determined that the following conditions need to be satisfied for obtaining an optimal design with respect to the rotational actuation:

- The line joining the centers of rotation of both axes is aligned parallel to the axis orthogonal to the plane of propeller rotation (vertical axis in case of default orientation as shown in Fig. 4).
- The center for the generation of thrust should be closer to the CG of the whole body to reduce the length of undesired moment arms upon rotation.

The first criterion results from the fact that any offset between the centers of rotation of the servos along the longitudinal or lateral axis leads to additional torque loading

of the shaft. Therefore, configuration 1 is not considered for further analysis as it does not meet the first criterion when the propeller is aligned vertically (default propeller orientation). Nevertheless, both configurations 2 and 3 satisfy the first criterion in default orientation. Even though forces are created along longitudinal and/or lateral axes of the servo after propeller rotation, both the designs are kept compact to keep the moment arms short.

Remark 1. Dynamixel AX-12A has a stall torque of 1.5 N-m and a maximum no-load speed of 6.178 rad/s at 12 V input. As suggested by the manufacturer, a stable motion is achieved if the mean dynamic load on the servo shaft is kept at 20% or less of the stall torque [33]. Based on a dynamic performance test of the servo, peak instantaneous loads of 32.2% and 31.4% are obtained for the bottom and top servo respectively while operating at an angular speed of 1.571 rad/s. The operational speed of the servo motor is finalized based on these observations. However, the details of the testing are beyond the scope of this paper and will be discussed in future work.

Once the centers of rotation are aligned vertically, it is desired to keep the center of thrust generation (propeller origin) as close as possible to the CG of the whole quadrotor to meet the second criterion. Considering the differences in servo alignment in the remaining configurations, this requirement can be reduced to the minimization of h_b (offset between bottom servo origin and body CG) as observed in Fig. 6. Consequently, configuration 3 is finalized as the most practicable sub-optimal solution meeting the above mentioned criteria. It has minimal geometrical constraints and provides decoupled control over each axis of rotation. The range of motion achievable about both axes of rotation with this configuration is

Figure 4: Vertical alignment of the centers of rotation.

Figure 5: Illustration of the range of motion about each axis of rotation. Note that the center line denotes the default configuration with propeller aligned along the vertical axis. Also, clockwise and counter-clockwise rotations are illustrated using different color schemes.

illustrated in Fig. 5. As observed, a rotation of 100° about the lateral axis and 180° about the longitudinal axis is obtained. Due to the extensive thrust vectoring, the moment arms are kept shorter while maintaining effective controllability, thus keeping the design compact.

Since the design of *QuadPlus* is symmetric, the corresponding dimensions for each arm remain identical. The parameters are listed in Tab. I for reference and the corresponding dimensions for Arm 3 are used for illustration in Fig. 6.

Table I: Intrinsic parameters of proposed quadrotor.

Symbol	Description	Value
m	Mass of quadrotor	2.012 kg
l_a	Arm length	0.235 m
l_3	offset from o_{E_3} to o_{B_3}	0.068 m
l_h	offset from o_{P_3} to o_{B_3}	0.060 m
h_b	offset from CG to o_{E_3}	-0.007 m
I_{xx}	Mass MOI about x_b	0.037 kg-m ²
I_{yy}	Mass MOI about y_b	0.038 kg-m ²
I_{zz}	Mass MOI about z_b	0.057 kg-m ²
K_T	Force coefficient	$1.43e-5$ N-s ²
K_M	Drag-moment coefficient	$2.01e-7$ Nm-s ²

C. Comparison with existing active tilting platforms

The essential features of the *active tilting* platforms discussed in Section II-A are compared with *QuadPlus* in Fig. 7. The feature ‘*Total DoFs*’ signifies the degree of overactuation with a number greater than six (referring to the six-DoFs to be controlled) denoting the actuator redundancy. The ‘*Longitudinal rotation*’ and ‘*Lateral rotation*’ depict the range of bi-axial rotation available about the longitudinal and lateral axes respectively. The feature ‘*1/No. of arms*’ is indicative of the degree of compactness with a larger number denoting a more compact platform. And finally, ‘*Prop./mass*’ represents the

loading of each propeller used with a larger number denoting lower loading. Holocopter is a lightweight quadrotor with single-axis tilting provided by the servo mounted at the end of each arm resulting in low design complexity. However, its maneuverability is affected by the limited range of propeller rotation for thrust vectoring. *QuadPlus* has the advantage of being extremely maneuverable as it maintains a compact size and relatively low weight combined with bi-axial propeller tilting. Moreover, the extensive 3-D thrust vectoring enables energy-efficient flight, even in narrow spaces. Voliro, which is the closest comparison in terms of overactuation (total DoFs), achieves it with two additional propellers indicative of the larger size. It achieves omnidirectional maneuverability in hexrotor configuration with full 360° single-axis tilting. However, the large size of Voliro leads to a high power consumption flight. Both *QuadPlus* and Voliro can maintain almost similar propeller loading signified by the number of propellers per unit weight. In *QuadPlus*, the high range of propeller rotation about both axes removes the need for additional propellers to achieve high maneuverability.

The bi-axial tilting quadrotor developed by Junaid et al. uses a push-pull lever mechanism making it susceptible to mechanical breakdowns. It has reduced maneuverability as more weight is supported per propeller resulting in high propeller loading. Furthermore, it has a limited rotation about both axes. On the other hand, TiltDrone equates quite closely with regards to the longitudinal and lateral propeller rotation while keeping a relatively low propeller loading. The use of compound joints facilitates bi-axial tilting using two servo motors only, thus making it a lightweight platform. It provides a simple control allocation solution with all the propellers always in the same plane. However, its maneuverability is affected by the constrained tilting resulting from the design limitations. Moreover, the yaw control poses a problem as all the propellers always generate thrust in the same direction resulting in relatively fewer DoFs than *QuadPlus*. Therefore, *QuadPlus* yields the best combination of overactuation and independent propeller rotation in a compact package.

Figure 6: Illustration of reference dimensional labels.

IV. SYSTEM LEVEL MODELING

Each arm on the proposed quadrotor consists of two rotating assemblies with the bottom servo motor fixed on the base frame. The first assembly comprises of the bracket F2, and the top servo (with bracket F3) while the second assembly consists of the bracket F2 fixed to the BLDC motor and propeller. Each rotating assembly is assumed to be rigid

Figure 7: Radar chart comparing the various platforms with active propeller tilting: namely, *QuadPlus*, holocoaster [18], Voliro [24], quadrotor developed by Junaid et al. [20], Tilt-drone [22]. A larger covered area for a given platform depicts better desired characteristics.

with an associated local reference frame that is attached at the center of the servo motor. The angle of rotation about local X -axis is referred as *bank angle* ($\alpha_i, \forall i = 1, \dots, 4$) while angle of rotation about local Y -axis is referred as *elevation angle* ($\beta_i, \forall i = 1, \dots, 4$). This terminology is used for ensuing discussions. The body frame of reference $\mathcal{F}_b(\mathcal{O}_b, x_b, y_b, z_b)$ is attached at the center of the base plate of quadrotor which is moving with respect to an Earth-fixed inertial frame of reference $\mathcal{F}_I(\mathcal{O}, x, y, z)$. The corresponding frames of reference for propeller ($\mathcal{F}_{P_i}(\mathcal{O}_{P_i}, x_{P_i}, y_{P_i}, z_{P_i}), \forall i = 1, \dots, 4$), banking ($\mathcal{F}_{B_i}(\mathcal{O}_{B_i}, x_{B_i}, y_{B_i}, z_{B_i}), \forall i = 1, \dots, 4$) and elevation ($\mathcal{F}_{E_i}(\mathcal{O}_{E_i}, x_{E_i}, y_{E_i}, z_{E_i}), \forall i = 1, \dots, 4$) are defined at the respective centers of rotation as illustrated in Fig. 6. The reference frames for the arms are obtained by translation of the body frame along the respective axes as shown in Fig. 8.

Figure 8: Schematic diagram depicting the coordinate systems and reference frames.

A. Kinematic Equations

The kinematic equations for determining the position and attitude of the quadrotor in frame $\mathcal{F}_I$ are obtained by using the transformation from frame $\mathcal{F}_b$ to frame $\mathcal{F}_I$:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = {}^I\mathbf{R}_b \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = {}^I\mathbf{T}_b \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \quad (1)$$

where x, y, z and ϕ, θ, ψ are the translational position and rotational attitude, respectively, defined in frame $\mathcal{F}_I$; u, v, w and p, q, r are the translational and rotational velocities defined in frame $\mathcal{F}_b$; ${}^I\mathbf{R}_b$ is the translation transformation matrix between frames $\mathcal{F}_I$ and $\mathcal{F}_b$, whereas ${}^I\mathbf{T}_b$ maps the rotational velocity component from $\mathcal{F}_b$ to $\mathcal{F}_I$. The transformation matrices are expressed as ($c : \cos, s : \sin, t : \tan$):

$${}^I\mathbf{R}_b = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}, \quad (2)$$

$${}^I\mathbf{T}_b = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix}. \quad (3)$$

B. Dynamic Equations

The rigid-body dynamic equations are derived based on Newton-Euler formulation in the body coordinate system. The quadrotor design is symmetric with a diagonal inertia matrix. Within these equations, the quadrotor is assumed to be a point mass, wherein all the forces act at the center of gravity. In general, they are categorized as:

Force Equations

$$\dot{u} = rv - qw + g \sin\theta + \frac{1}{m} {}^bT_x, \quad (4a)$$

$$\dot{v} = pw - ru - g \sin\phi \cos\theta + \frac{1}{m} {}^bT_y, \quad (4b)$$

$$\dot{w} = qu - pv - g \cos\phi \cos\theta + \frac{1}{m} {}^bT_z. \quad (4c)$$

Moment Equations

$$\dot{p} = \left(\frac{I_{yy} - I_{zz}}{I_{xx}} \right) qr + \left(\frac{1}{I_{xx}} \right) {}^bM_x, \quad (5a)$$

$$\dot{q} = \left(\frac{I_{zz} - I_{xx}}{I_{yy}} \right) pr + \left(\frac{1}{I_{yy}} \right) {}^bM_y, \quad (5b)$$

$$\dot{r} = \left(\frac{I_{xx} - I_{yy}}{I_{zz}} \right) pq + \left(\frac{1}{I_{zz}} \right) {}^bM_z. \quad (5c)$$

where ${}^bT_x, {}^bT_y, {}^bT_z$ are the total external forces and ${}^bM_x, {}^bM_y, {}^bM_z$ are the total external moments acting in frame $\mathcal{F}_b$. In addition, I_{xx}, I_{yy} , and I_{zz} represent the moments of inertia of the entire quadrotor in frame $\mathcal{F}_b$ about axes x_b, y_b and z_b , respectively.

1) *External Forces and Moments*: The aerodynamic effects induced by the propeller rotation include thrust generation (lift) and drag-moment responsible for opposing the rotation. The thrust generated by the propeller is orthogonal to its plane of rotation while the drag-moment consists of the moment created by integrating the horizontal forces acting on the blade element. Using a combination of momentum and blade

element theory, steady state thrust and drag-moment generated by a propeller can be modeled as:

$$T_i = K_T \Omega_i^2 \quad \text{and} \quad M_i = K_M \Omega_i^2. \quad (6)$$

where T_i and M_i are the force and drag-moment generated by the i^{th} propeller with Ω_i angular velocity. In addition, K_T and K_M are the force and drag-moment coefficients, respectively, of the propeller. The thrust generated by the propeller in its own frame of reference $\mathcal{F}_{P_i}$ is written as:

$${}^{P_i}\mathbf{T} = \begin{bmatrix} 0 \\ 0 \\ T_i \end{bmatrix}, \forall i = 1, \dots, 4. \quad (7)$$

The propeller rotates with respect to the upper servo which in turn rotates with respect to the bottom servo fixed to the arm and the body. Therefore, the rotation transform from frame $\mathcal{F}_{P_i}$ frame $\mathcal{F}_b$ can be expressed as ($c : \cos$, $s : \sin$):

$$\begin{aligned} {}^b\mathbf{R}_{P_i} &= \begin{bmatrix} c\beta_i & 0 & s\beta_i \\ s\alpha_i s\beta_i & c\alpha_i & -s\alpha_i c\beta_i \\ -c\alpha_i s\beta_i & s\alpha_i & c\alpha_i c\beta_i \end{bmatrix}, \forall i = 1, 2, \\ {}^b\mathbf{R}_{P_i} &= \begin{bmatrix} c\beta_i & s\beta_i s\alpha_i & s\beta_i c\alpha_i \\ 0 & c\alpha_i & -s\alpha_i \\ -s\beta_i & c\beta_i s\alpha_i & c\beta_i c\alpha_i \end{bmatrix}, \forall i = 3, 4. \end{aligned} \quad (8)$$

As such, the thrust generated by the propeller in the frame $\mathcal{F}_b$ can be expressed as:

$${}^b\mathbf{T}_{P_i} = {}^b\mathbf{R}_{P_i} {}^{P_i}\mathbf{T}, \quad (9)$$

$$\begin{aligned} {}^b\mathbf{T}_{P_i} &= \begin{bmatrix} T_i \sin\beta_i \\ -T_i \sin\alpha_i \cos\beta_i \\ T_i \cos\alpha_i \cos\beta_i \end{bmatrix}, \forall i = 1, 2, \\ {}^b\mathbf{T}_{P_i} &= \begin{bmatrix} T_i \sin\beta_i \cos\alpha_i \\ -T_i \sin\alpha_i \\ T_i \cos\beta_i \cos\alpha_i \end{bmatrix}, \forall i = 3, 4. \end{aligned} \quad (10)$$

Similarly, considering counter-clockwise rotation for propellers 1 and 2 and clockwise rotation for propellers 3 and 4, the drag-moments due to each propeller in the frame $\mathcal{F}_b$ is obtained as:

$$\begin{aligned} {}^b\mathbf{M}_{d_i} &= \begin{bmatrix} -M_i \sin\beta_i \\ M_i \sin\alpha_i \cos\beta_i \\ -M_i \cos\alpha_i \cos\beta_i \end{bmatrix}, \forall i = 1, 2, \\ {}^b\mathbf{M}_{d_i} &= \begin{bmatrix} M_i \sin\beta_i \cos\alpha_i \\ -M_i \sin\alpha_i \\ M_i \cos\beta_i \cos\alpha_i \end{bmatrix}, \forall i = 3, 4. \end{aligned} \quad (11)$$

The displacement vector of $\mathcal{O}_{P_i}$ with respect to $\mathcal{O}_b$ is determined using the respective rotation and translation transforms and can be written as:

$$\begin{aligned} {}^b\mathbf{d}_{P_i} &= \begin{bmatrix} l_{hi} \sin\beta_i \\ -(l_3 + l_h \cos\beta_i) \sin\alpha_i + (-1)^i l_a \\ (l_3 + l_h \cos\beta_i) \cos\alpha_i + h_b \end{bmatrix}, \\ &\quad \forall i = 1, 2, \\ {}^b\mathbf{d}_{P_i} &= \begin{bmatrix} (l_3 + l_h \cos\alpha_i) \sin\beta_i + (-1)^i l_a \\ -l_{hi} \sin\alpha_i \\ (l_3 + l_h \cos\alpha_i) \cos\beta_i + h_b \end{bmatrix}, \\ &\quad \forall i = 3, 4. \end{aligned} \quad (12)$$

where l_3 , l_h and l_a are depicted in Fig. 6 and defined in Tab. I. The moment generated by thrust vectoring of each propeller in frame $\mathcal{F}_b$ can then be expressed as:

$${}^b\mathbf{M}_{T_i} = {}^b\mathbf{d}_{P_i} \times {}^b\mathbf{T}_{P_i}, \quad (13)$$

$$\begin{aligned} {}^b\mathbf{M}_{T_i} &= \begin{bmatrix} T_i \cos\beta_i \{h_b \sin\alpha_i + (-1)^i l_a \cos\alpha_i\} \\ T_i \sin\beta_i \{l_3 \cos\alpha_i + h_b\} \\ T_i \sin\beta_i \{l_3 \sin\alpha_i - (-1)^i l_a\} \end{bmatrix}, \\ &\quad \forall i = 1, 2, \\ {}^b\mathbf{M}_{T_i} &= \begin{bmatrix} T_i \sin\alpha_i \{l_3 \cos\beta_i + h_b\} \\ T_i \cos\alpha_i \{h_b \sin\beta_i + (-1)^i l_a \cos\beta_i\} \\ -T_i \sin\alpha_i \{l_3 \sin\beta_i - (-1)^i l_a\} \end{bmatrix}, \\ &\quad \forall i = 3, 4. \end{aligned} \quad (14)$$

Consequently, the total external force (${}^b\mathbf{T}_{ext}$) and total external moment (${}^b\mathbf{M}_{ext}$) acting in the frame $\mathcal{F}_b$ can finally be written as:

$${}^b\mathbf{T}_{ext} = \begin{bmatrix} {}^bT_x \\ {}^bT_y \\ {}^bT_z \end{bmatrix} = \sum_{i=1}^4 {}^b\mathbf{T}_{P_i}, \quad (15)$$

$${}^b\mathbf{M}_{ext} = \begin{bmatrix} {}^bM_x \\ {}^bM_y \\ {}^bM_z \end{bmatrix} = \sum_{i=1}^4 [{}^b\mathbf{M}_{T_i} + {}^b\mathbf{M}_{d_i}]. \quad (16)$$

For implementation purpose, the presented nonlinear model is discretized incorporating the direct multiple shooting method with a shooting grid size of 0.01s. In addition, an explicit Runge-Kutta 4th order integrator is utilized with 2-steps per shooting interval. The final full-state discretized nonlinear model of the over-actuated quadrotor can be expressed as:

$$\mathbf{x}_{k+1} = \mathbf{f}_d(\mathbf{x}_k, \mathbf{u}_k). \quad (17)$$

where the state vector $\mathbf{x} \in \mathbb{R}^{12}$ and control vector $\mathbf{u} \in \mathbb{R}^{12}$ are:

$$\mathbf{x} = [x, y, z, u, v, w, \phi, \theta, \psi, p, q, r]^T, \quad (18)$$

$$\mathbf{u} = [[\alpha_i], [\beta_i], [\Omega_i]]^T, \forall i = 1, \dots, 4. \quad (19)$$

wherein the discrete state function is denoted by $\mathbf{f}_d(\cdot, \cdot): \mathbb{R}^{12} \times \mathbb{R}^{12} \rightarrow \mathbb{R}^{12}$.

V. CONTROL FRAMEWORK

In this work, a receding horizon principle-based cascade scheme is proposed for the control of the overactuated quadrotor as summarized in a block diagram shown in Fig. 9. The designed scheme employs a high-level NMPC which utilizes the slow dynamics associated with orientation control of the propellers for controlling 4-DoFs (position and yaw) of the quadrotor. While a low-level PID controller is employed for controlling the remaining 2-DoFs (pitch and roll) using the fast dynamics of differential thrust generation. Consequently, the control output from the NMPC are servo angles (α 's and β 's) and the total thrust (bT_z), whereas the control outputs from the low-level PID controller are the desired roll and pitch moments (M_x, M_y). The displacement vectors used for calculating the moments are expressed as:

$$d_{x_i} = h_b \sin \alpha_i + (-1)^i l_a \cos \alpha_i, \forall i = 1, 2, \quad (20a)$$

$$d_{x_i} = l_3 \cos \beta_i + h_b, \forall i = 3, 4, \quad (20b)$$

$$d_{y_i} = l_3 \cos \alpha_i + h_b, \forall i = 1, 2, \quad (20c)$$

$$d_{y_i} = h_b \sin \beta_i + (-1)^i l_a \cos \beta_i, \forall i = 3, 4. \quad (20d)$$

Finally, a control allocation block is incorporated that maps the thrust and the moments commands to the rotational velocities of the rotors. It utilizes the expression given in (21), wherein $s : \sin$, $c : \cos$. It is to be noted that the above control allocation matrix is computed utilizing the external force (only along z -direction) and external moment equations in (15) and (16), respectively.

A. High-level NMPC Algorithm

(N)MPC is an advanced dynamic optimization-based feedback control strategy whose tracking performance is critically linked with the accuracy of the system model. In essence, it computes an optimal control action by optimizing the model's behavior over a finite prediction horizon window (N_c). The optimal system forecast results from an open-loop optimal control problem (OCP) whose solution for the current state and time gives an optimal sequence of control actions. Finally, the first term of the obtained control sequence is regarded as the optimal control action.

In this work, the NMPC is designed to track the trajectory commanded by the navigation algorithm. In that vein, its OCP in a least-square form is formulated as:

$$\min_{\mathbf{x}_k, \mathbf{u}_k} \frac{1}{2} \left\{ \sum_{k=0}^{N_c-1} \left(\left\| \mathbf{x}_k - \mathbf{x}_k^{\text{ref}} \right\|_{\mathbf{W}_x}^2 + \left\| \mathbf{u}_k - \mathbf{u}_k^{\text{ref}} \right\|_{\mathbf{W}_u}^2 \right) + \left\| \mathbf{x}_{N_c} - \mathbf{x}_{N_c}^{\text{ref}} \right\|_{\mathbf{W}_{N_c}}^2 \right\} \quad (22a)$$

$$\text{s.t. } \mathbf{x}_{k+1} = \mathbf{f}_d(\mathbf{x}_k, \mathbf{u}_k), \quad k \in [0, N_c - 1], \quad (22b)$$

$$\mathbf{u}_{k,\min} \leq \mathbf{u}_k \leq \mathbf{u}_{k,\max}, \quad k \in [0, N_c - 1], \quad (22c)$$

where $\mathbf{x}_k^{\text{ref}}$ and $\mathbf{u}_k^{\text{ref}}$ represent the state and control references, respectively; terminal state reference is given by $\mathbf{x}_{N_c}^{\text{ref}}$, $\mathbf{W}_x \in \mathbb{R}^{12 \times 12}$, $\mathbf{W}_u \in \mathbb{R}^{9 \times 9}$, and $\mathbf{W}_{N_c} \in \mathbb{R}^{12 \times 12}$ are the corresponding diagonal weight matrices. Furthermore, the terms $\mathbf{u}_{k,\min} \leq \mathbf{u}_{k,\max} \in \mathbb{R}^9$, specify the lower and upper bounds on

the control inputs, respectively. It is to be noted that $\mathbf{x}_0$ is taken as the current state measurement for initializing the OCP at each sampling time.

Remark 2. Note that the first and second expressions in (22a) result in a stage cost that penalizes the state and control trajectories from their specified references over the prediction horizon. While the former set the state targets, the latter is incorporated to favor control solutions that lie near predetermined nominal values rather than some arbitrary values resulting from other local minima.

Moreover, the last expression in (22a) represents the terminal penalty which is the cost incurred due to the finite prediction horizon. Essentially, this term is included in the problem formulation for stability reasons [34]. One may note that the stability and optimality proofs of NMPC are out of scope in this paper, however, the interested readers are referred to [35], [36].

For the trajectory tracking NMPC design, the state and control vectors comprise of:

$$\mathbf{x}_{\text{NMPC}} = [x, y, z, u, v, w, \phi, \theta, \psi, p, q, r]^T, \quad (23)$$

$$\mathbf{u}_{\text{NMPC}} = \left[[\alpha_i], [\beta_i], {}^bT_z \right]^T, \forall i = 1, \dots, 4. \quad (24)$$

Besides, the following state and control trajectories are given as references for the optimization problem:

$$\mathbf{x}^{\text{ref}} = \mathbf{x}_{N_c}^{\text{ref}} = [x_r, y_r, z_r, u_r, v_r, w_r, \phi_r, \theta_r, \psi_r, p_r, q_r, r_r]^T, \quad (25)$$

$$\mathbf{u}^{\text{ref}} = [-\phi_r, -\phi_r, -\phi_r, -\phi_r, -\theta_r, -\theta_r, -\theta_r, -\theta_r, mg]^T, \quad (26)$$

where x_r, y_r, z_r and u_r, v_r, w_r are the position and linear velocity references, respectively, and ϕ_r, θ_r, ψ_r and p_r, q_r, r_r are the attitude and angular rate references, respectively, that are commanded by the navigation algorithm. Besides, the term m represents the total takeoff mass (2.012 kg) of the robot and $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity. Note that, while the state reference in (25) is based on the commanded trajectory, the control reference in (26) is obtained by utilizing the dynamic equations and balancing the forces along the respective directions.

In order to comply with the actuator limits of the system, the following constraints are defined:

$$0.5mg \text{ (N)} \leq {}^bT_z \leq 1.5mg \text{ (N)}, \quad (27a)$$

$$-10 \text{ (}^\circ\text{)} \leq \alpha_1, \beta_3 \leq 60 \text{ (}^\circ\text{)}, \quad (27b)$$

$$-60 \text{ (}^\circ\text{)} \leq \alpha_2, \beta_4 \leq 10 \text{ (}^\circ\text{)}, \quad (27c)$$

$$-60 \text{ (}^\circ\text{)} \leq \alpha_3, \alpha_4, \beta_1, \beta_2 \leq 60 \text{ (}^\circ\text{)}. \quad (27d)$$

Moreover, the following weight matrices for the optimization problem are obtained utilizing the approach proposed in authors' previous work [37]:

$$\mathbf{W}_x = \text{diag}(150, 150, 150, 10, 10, 3, 0.01, 0.01, 4.3, 0.01, 0.01, 0.7),$$

$$\mathbf{W}_u = \text{diag}(30, 30, 30, 30, 30, 30, 30, 30, 0.055),$$

$$\mathbf{W}_{N_c} = 1.5\mathbf{W}_x.$$

One may note a contrast in the weight values for roll and pitch angles with that of the yaw. This is because the NMPC

$$\begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} = \begin{bmatrix} K_T c \alpha_1 c \beta_1 & K_T c \alpha_2 c \beta_2 & K_T c \alpha_3 c \beta_3 & K_T c \alpha_4 c \beta_4 \\ K_T c \beta_1 d_{x_1} - K_M s \beta_1 & K_T c \beta_2 d_{x_2} - K_M s \beta_2 & K_T s \alpha_3 d_{x_3} + K_M s \beta_3 c \alpha_3 & K_T s \alpha_4 d_{x_4} + K_M s \beta_4 c \alpha_4 \\ K_T s \beta_1 d_{y_1} + K_M s \alpha_1 c \beta_1 & K_T s \beta_2 d_{y_2} + K_M s \alpha_2 c \beta_2 & K_T c \alpha_3 d_{y_3} - K_M s \alpha_3 & K_T c \alpha_4 d_{y_4} - K_M s \alpha_4 \\ -K_M & -K_M & K_M & K_M \end{bmatrix}^{-1} \begin{bmatrix} {}^b T_z \\ M_x \\ M_y \\ 0 \end{bmatrix}. \quad (21)$$

Figure 9: Block diagram of the proposed cascade control scheme. The attitude reference vector for the low-level controller comprises of: $\mathbf{x}_{att}^{ref} = [\phi_r, \theta_r, p_r, q_r]^T$. Also, the servo angle and angular velocity vectors are respectively given as: $\alpha = [\alpha_i]^T$, $\beta = [\beta_i]^T$, $\Omega = [\Omega_i]^T$, $\forall i = 1, \dots, 4$.

does not directly manipulate the former, while they are mainly included for the feasibility of the OCP. Also, the above definition of the terminal weight matrix ($\mathbf{W}_{N_c}$) weighs it more in comparison to the stage cost weight matrix ($\mathbf{W}_x$). This is a typical way of definition for ensuring the stability of the OCP. Furthermore, the prediction horizon $N_c = 30$ is selected along with a sampling time of 0.01s for computational tractability.

Remark 3. It is to be noted that for the OCP formulation of high-level NMPC, the parameterization of the nonlinear model is also done with respect to the rotational velocities of the rotors (Ω 's). Hence, these rotor velocities are fed to high-level NMPC along with the other states, as also illustrated in Fig. 9.

1) *Solution Technique:* Direct and indirect methods are the two widely adopted approaches for solving the optimization problem of NMPC. Amongst them, the former methodology is utilized here due to its several computational advantages. Besides, the direct methods are further classified as sequential or simultaneous, wherein a variant of the simultaneous technique, i.e., direct multiple shooting method is preferred over the sequential technique (single shooting). The underlying reason is its several advantages that include flexibility in problem initialization along with algorithm parallelization, resulting in faster convergence properties especially for unstable systems [38]. The result from the direct multiple shooting method is a discretized OCP that further reduces to a sequential quadratic program (SQP) after linearization. Finally, with the help of the generalized Gauss-Newton (GGN) method, the solution to the obtained SQP is computed. Moreover, the adopted GGN method is a tailored variant of the classical Newton method which is solved with the help of a special real-time iteration (RTI) scheme proposed in [39]. In essence, the maximum number of Gauss-Newton iterations are constrained to one within the RTI scheme, whereby each new OCP is initialized with the solution from the previous iteration. This implies faster convergence which is crucial for real-time feasibility

of the NMPC over low-cost embedded processors.

As a solution platform, ACADO toolkit [40] is adopted for this work which incorporates the direct multiple shooting method and RTI approach altogether for solving the OCP in (22). The ACADO toolkit is an open-source C++ based software environment that provides a link to an exhaustive code library for optimization solvers. It also has a code generation package that creates tailored (well structured to avoid redundant computations), self-contained C-codes based on a user-specified symbolic (N)MPC problem formulation. Furthermore, its usage can be summarized as: firstly, the optimization problem in terms of system equations and constraints is defined in a C++ environment and then, the self-contained C codes are obtained utilizing its code generation package. Later, these generated C codes can be imported to any computational platform for their final execution.

B. Low-level PID Controller

The low-level controller is designed to track the roll and pitch attitude setpoints as commanded by the navigation algorithm. For that purpose, it employs a P-PID architecture (also implemented in Pixhawk) that comprises of a P controller for attitude and PID controller for angular rate. This low-level control architecture is utilized for each of the two axes (roll and pitch) separately, as their rotational dynamics is considered to be decoupled for a standard quadrotor.

The combined control vector of the low-level controller can be written as: $\mathbf{u}_{P-PID} = [M_x, M_y]^T$, where M_x and M_y are the desired moments about x_b and y_b , respectively. Besides, the incorporated low-level PID gains for roll and roll-rate tracking are given as follows:

$$P_{roll} = 6.5,$$

$$P_{roll-rate} = 0.15, I_{roll-rate} = 0.05, D_{roll-rate} = 0.003,$$

and the PID gains for pitch and pitch-rate tracking are:

$$P_{pitch} = 6.5,$$

$$P_{pitch-rate} = 0.15, I_{pitch-rate} = 0.05, D_{pitch-rate} = 0.003.$$

It is to be noted that the above gains are obtained via the trial-and-error method such that a stable attitude tracking is realized.

VI. SIMULATION RESULTS

The proposed cascade control framework is implemented for trajectory tracking application using the overactuated quadrotor. In this work, we conduct simulation-based testing of the control framework over a Gazebo simulation platform. The underlying reason to choose Gazebo is its reliable physics engine that facilitates realistic simulated dynamics. A demonstration video depicting the simulations can be found at: <https://www.youtube.com/watch?v=he5imbx4K9k>.

For this purpose, the Gazebo model of the quadrotor is prepared along with the necessary plugins to control the tilt servos. Besides, PX4 flight stack [41] provides the necessary communication channel such that the quadrotor can be controlled by sending commands over ROS (robot operating system) topics. Moreover, PX4 flight stack also accommodates a PID-based low-level controller with a similar architecture that is presented previously. Therefore, it is utilized for the desired pitch and roll control with the specified gains. It is to be noted that the yaw controller of the PX4 flight stack is purposely disabled so that the high-level NMPC can manage it.

In terms of the real-time implementation of NMPC, the ACADO generated C codes are imported and executed on a laptop having an Intel i7-4710MQ @ 2.50 GHz processor with eight cores and 16 GB RAM. Although the optimization problem of NMPC is computationally demanding, a sampling frequency of about 35-Hz is achieved owing to the RTI scheme of ACADO toolkit and our efficiently created C++ scripts.

In order to demonstrate the capabilities of the overactuated platform and cascade control framework in tracking complicated 3-D trajectories, a spatial Viviani's curve is selected as the desired trajectory. In the subsequent part, two flight cases are tested, namely Viviani's curve with static attitude and Viviani's curve with sinusoidally varying attitude. Moreover, the underlying actuator-saturation tendency within these trajectories further validates the efficacy of the proposed control framework. In essence, the trajectory has a length of 3.2m and 1.6m along x - and y -directions, respectively, along with 0.8m variation in altitude (along z -direction) and 1 m/s flight speed. The following equations are used to generate the trajectory:

$$x = 1.6 \sin \frac{t}{3.2}, y = 0.8 \sin \frac{t}{1.6}, z = 0.4(\cos \frac{t}{1.6} - 1). \quad (28)$$

A. Tracking with Static Attitude

In this test case, the quadrotor tracks a Viviani's curve-type position trajectory while maintaining a static roll and pitch attitude of 35° and 30° , respectively. The overall position tracking performance is illustrated in Figs 10a and 10b. Besides, the resulting absolute position errors along the three directions are depicted in Fig. 10d. Quantitatively, the proposed high-level NMPC resulted in the mean absolute error values of 6.67cm, 4.13cm, and 4.22cm along x -, y -, and z -directions, respectively, and the mean Euclidean error value of 9.88cm. Such low errors imply an accurate position tracking even while maintaining a static attitude.

Remark 4. In the present work, the tracking error is considered to be the deviation of the robot from the closest point on the commanded trajectory.

The attitude tracking performance is presented in Fig. 10c. As can be seen, the quadrotor satisfactorily maintains the commanded attitude within approximately $\pm 5^\circ$ for roll and pitch and $\pm 10^\circ$ for yaw. Moreover, the absolute attitude tracking errors incurred by the proposed cascade control framework are shown in Fig. 10e. Essentially, it resulted in mean absolute roll, pitch, and yaw error values of 2° , 2.5° , and 5° , respectively, and the mean Euclidean error value of about 6° .

The control outputs from high-level NMPC are depicted in Fig. 10f. As observed, all 8-DoFs associated with propeller orientation are actuated accordingly for accurate position tracking while holding a static attitude. Even though α_1 and β_3 are saturated from the beginning, the controller is able to make use of the overactuation capabilities to obtain near-optimal solutions for trajectory tracking. Correspondingly, the relatively higher yaw tracking error may be attributed to the secondary coupling effects originating from the position tracking affected by actuator saturation as both yaw and position control are performed in the high-level controller.

B. Tracking with Sinusoidal Attitude

In the second test case, the quadrotor tracks a Viviani's curve-type position trajectory along with a sinusoidally varying attitude trajectory. The peak amplitudes in pitch and roll are commanded as 25° , and in yaw as 50° . The overall position tracking performance is depicted in Figs 11a and 11b. Also, the absolute position errors along the three directions are depicted in Fig. 11d. In summary, the proposed high-level NMPC resulted in the corresponding mean absolute error values of 7.09cm, 6.69cm, and 3.84cm, and the mean Euclidean error value of 11.97cm. Unlike the previous case, the complexity of tracking simultaneous changes in commanded attitude and position result in two-pairs of tilt servos being intermittently saturated. Nevertheless, low magnitudes of the error values compared to desired commands represent a near-accurate position tracking even with sinusoidally varying attitude reference.

The attitude tracking performance can be visualized in Figs. 11c and 11e. The quadrotor accurately tracks the commanded attitude while resulting in mean absolute roll, pitch, and yaw error values of 4° , 4° , and 9° , respectively, and the mean Euclidean error of about 11° . Significant yaw and pitch tracking errors of around 40° and 20° respectively, are observed in Fig. 11e at the beginning of the second repetition of the trajectory. While the yaw tracking error occurs as the commanded yaw is in direct opposition to the direction of motion, the pitch tracking error can be attributed to the lack of differential thrust available to control both roll and pitch simultaneously along with the altitude. Even then, the controller is robust enough to stabilize the system and achieve accurate position tracking as depicted in Fig. 11b.

Remark 5. As mentioned, the attitude tracking performance gets affected by intermittent saturation of two or more tilt servos. In such a scenario, a centralized control approach that could fully exploit the system redundancy can help improve the tracking performance. Consequently, the controller may be able to engage unsaturated actuators to generate the desired residual forces and moments.

The control outputs from high-level NMPC are depicted in Fig. 11f. Similar to the previous test case, all 8-DoFs associated with propeller orientation are actuated. Even though the tilt servos α_1 , α_2 , β_3 , and β_4 with limitations in inward tilting saturate at various points in the trajectory, they successfully recover as well. This again validates the capability of the proposed high-level NMPC in safely dealing with actuator saturation and its recovery.

Figure 10: Simulation tracking results for Viviani's curve-type trajectory with constant attitude command of $\phi = 35^\circ$ and $\theta = 30^\circ$.

The single Gauss-Newton iteration per sampling instant, as adopted within the RTI scheme, would result in a suboptimal solution for NMPC. Hence, it is beneficial to check the optimality of the obtained solution. In that vein, the Karush-Kuhn-Tucker (KKT) tolerance along with the objective value is obtained and plotted in Fig. 12 for both the tests. Recall that for a linear system, these tolerance values reduce to zero. Whereas, such high values are expected for the overactuated quadrotor due to the severe nonlinearities. Besides, the objective value is high as at least two of the tilt servos are always saturated within both the test cases. Nevertheless, their low and not too drastically varying magnitudes still signify well-defined optimization problems. Finally, the average and maximum executions times within both the cases for high-level NMPC are about 23ms and 50ms, respectively. These numbers further validate the realtime feasibility of the proposed framework that too on a low-cost single-board computer.

VII. CONCLUSION

In this work, an innovative hyperdynamic quadrotor: *QuadPlus*, has been introduced to cater to the maneuverability challenges associated with search and rescue missions. The proposed design employs vertically aligned dual-shaft servo motors to independently control the 2-DoF orientation of each propeller of the quadrotor. Consequently, propeller rotations of 100° and 180° are achieved about the lateral and longitudinal

axis of the arm, respectively. In addition, to comply with the identified physical limits in the range of propeller rotation, a cascade approach has been proposed for 6-DoF control. The constraints-based optimization problem for obtaining the propeller orientation is defined using high-level NMPC and is used for position and heading control. While attitude (pitch and roll) control is performed using low-level PID controllers engaging moments generated from differential thrust.

The efficacy of the proposed control framework has been evaluated via realistic Gazebo simulations for Viviani's curve-type trajectory. Two test scenarios involving static attitude and sinusoidally varying attitude are flown. The obtained results validate the need for 2-DoF orientation control of each propeller as all the rotation angles are activated during the trajectory tracking. Besides, the results also demonstrate the effectiveness of the controller in handling the actuator saturation, followed by a successful recovery. Thus, *QuadPlus* is shown to be capable of maneuvering through constrained spaces by controlling its attitude while performing complex 3-D trajectory. As future work, a centralized control approach that could fully exploit the actuator redundancy will be evaluated for the *QuadPlus* platform.

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Figure 11: Simulation tracking results for Viviani type trajectory with sinusoidal attitude command having peak amplitude in $\phi = \theta = 30^\circ$ and $\psi = 50^\circ$.

Figure 12: KKT tolerance and objective value for high-level NMPC.

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