Receding horizon-based fault-tolerant control of QuadPlus : an over-actuated quadrotor

Mehndiratta, Mohit; Singh, Karanjot; Kayacan, Erdal; Feroskhan, Mir

2021

Mehndiratta, M., Singh, K., Kayacan, E. & Feroskhan, M. (2021). Receding horizon-based fault-tolerant control of QuadPlus : an over-actuated quadrotor. 2021 IEEE 17th International Conference on Automation Science and Engineering (CASE), 853-859.
<https://dx.doi.org/10.1109/CASE49439.2021.9551527>

<https://hdl.handle.net/10356/154766>

<https://doi.org/10.1109/CASE49439.2021.9551527>

© 2021 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. The published version is available at:
<https://doi.org/10.1109/CASE49439.2021.9551527>.

Downloaded on 26 Apr 2023 21:57:27 SGT

Receding Horizon-based Fault-tolerant Control of *QuadPlus*: an Over-actuated Quadrotor

Mohit Mehndiratta¹, Karanjot Singh², Erdal Kayacan³, and Mir Feroskhan²

Abstract—Highly maneuverable, over-actuated aerial robots have gained increasing interest in various inspection applications. However, since these systems carry expensive equipment and must operate in the vicinity of humans, their fail-safe operation is paramount. In this study, we propose a centralized nonlinear model predictive control (NMPC) method to facilitate fault-tolerant control (FTC) of an over-actuated quadrotor against a propeller failure. Thanks to the novel mechanical design, the hyperdynamic quadrotor can independently command and control all 6-degrees-of-freedom (DoFs). Additionally, the underlying reconfigurability feature of the designed NMPC makes it appropriate for normal as well as faulty operations. Moreover, the centralized nature of the control framework fully exploits the actuator redundancy, thereby ensuring complete system control without losing any DoF. The efficacy of the proposed FTC framework is elaborated throughout intensive simulations utilizing a high-fidelity model over two different trajectories. From the sequential failure cases, it is shown that – even with fault detection delay up to 1s – the aerial robot satisfactorily tracks the reference trajectory.

I. INTRODUCTION

Multirotor aerial robots are being extensively utilized for infrastructure inspection such as bridges [1], buildings [2], tunnels [3], and wind turbines [4]. Since these inspection tasks (as depicted in Fig. 1) are mainly carried out in the vicinity of human operators, fault-tolerant control (FTC) of the aerial robots becomes essential to their safety. In the event of a propeller failure, the system loses actuation capacity, which may lead to instability. One of the approaches to stabilize the aerial robot post-failure is based on online parameter estimation, which can learn the residual nonlinear dynamics and subsequently update the controller [5], [6]. Another approach revolves around the conservation of angular momentum to maintain stability [7]. However, these approaches relinquish control over a degree-of-freedom (DoF) as the actuation capacity gets compromised, thus are less viable for tasks requiring precise tracking, specifically in inspection-related applications.

As an alternative strategy, an over-actuated aerial robot can handle failures while retaining control over its position and attitude [8]. To do so, its control allocation is reconfigured to exploit the input redundancy [9]. The optimum propeller tilt angles can then be computed to stabilize the aerial

Fig. 1: Water tank inspection using *QuadPlus*. An over-actuated aerial robot can handle failures during inspection operations while retaining control over its position and attitude.

robot in case of propeller failure [10]. However, the actuator saturation needs to be accounted for while reallocating the inputs [11]. Therefore, the control of an over-actuated system poses multiple challenges: (i) determination of the optimal actuator inputs for an under-determined system, (ii) physical actuator constraints, and (iii) changing dynamics due to propeller failure. Owing to its unique ability of systematic constraint handling and (online) recursive performance optimization, the nonlinear model predictive control (NMPC) method has been illustrated as a promising candidate [12]–[15]. A cascaded control approach with NMPC for fast attitude control has been shown to stabilize the aerial robot with one propeller failure in [16]. Similarly, a reconfigurable NMPC in authors' previous work [17] has demonstrated a satisfactory tracking performance while obeying the actuator constraints. However, the main limitation has been the underlying cascade scheme that does not fully exploit the actuator redundancy, thus resulting in a suboptimal solution.

In this work, we propose a centralized NMPC for the FTC application of an over-actuated quadrotor against single propeller failure. The novel quadrotor design introduces additional DoFs using bi-axial tilting of propellers. Essentially, the independently controlled actuators – yielding a high range of rotation about each axis – are used to perform the $SE(2)$ active tilting of each propeller during flight. Thanks to these additional DoFs, the quadrotor not only stabilizes but also maintains decoupled control over position and attitude, even after propeller failure. Additionally, the proposed centralized NMPC approach with reconfigurability

¹Sensible 4 Oy, Helsinki, Southern Finland, Finland. mohit.mehndiratta@sensible4.fi

²School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore. karanjot001@e.ntu.edu.sg, mir.feroskhan@ntu.edu.sg

³Artificial Intelligence in Robotics (AiR) Laboratory, Department of Electrical and Computer Engineering, Aarhus University, Aarhus C, Denmark. erdal@ece.au.dk

caters to both normal and faulty operations. It also allows the full exploitation of the actuator redundancy, thereby resulting in the optimal solution. Furthermore, to compensate for the fault-detection time from the detection method (left for future work), the designed NMPC can tackle detection delays up to 1s, which is almost 100 times the adopted sampling time and is considered reasonable for aerial applications [18].

II. PROBLEM STATEMENT

The designed platform (shown in Fig. 1) is capable of extensive thrust vectoring utilizing the decoupled bi-axial rotation of each propeller. Two independent actuators are aligned along each of the longitudinal and lateral axes for performing the rotation. To keep the design compact, the actuators are aligned sequentially in a vertical alignment which leads to a vertical shift of the center of rotation for the top actuator as illustrated in Fig. 2. The quadrotor is over-actuated with a total 12-DoFs (3-DoFs per propeller) which allows full control over its 6-DoFs (associated with position and attitude) even with multiple propeller failures. In such a scenario, the control capacity is primarily limited by the loading of the remaining propellers. Consequently, the cost of fault-tolerance is manifested in the form of an increase in mechanical complexity and weight of the quadrotor due to additional actuators.

Next, we derive the dynamic equations for developing a high-fidelity simulation model for the designed quadrotor. The rotation mechanism is assumed to be composed of rigid components; wherein the respective rotations about local X and Y -axes are represented by bank angles, α_i and elevation angles, β_i , $\forall i = 1, \dots, 4$. Also, we define the propeller frame of reference ($\mathcal{F}_{P_i}$, $\forall i = 1, \dots, 4$) at the center of the propeller, banking ($\mathcal{F}_{B_i}$, $\forall i = 1, \dots, 4$) and elevation ($\mathcal{F}_{E_i}$, $\forall i = 1, \dots, 4$) frames of references at the respective centers of rotation. The body frame of reference $\mathcal{F}_b$ is defined at the CG of the quadrotor and the inertial frame of reference $\mathcal{F}_I$ is assumed fixed to the Earth.

The position and attitude in the inertial frame $\mathcal{F}_I$ are determined using the following transformations:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \mathbf{R}_{bI} \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \mathbf{\Gamma}_{bI} \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad (1)$$

where x, y, z and ϕ, θ, ψ are the position and attitude, respectively in frame $\mathcal{F}_I$; u, v, w and p, q, r are the translational and angular velocities defined in frame $\mathcal{F}_b$; $\mathbf{R}_{bI}$ and $\mathbf{\Gamma}_{bI}$ are the transforms for mapping translational and angular velocities from frame $\mathcal{F}_b$ to $\mathcal{F}_I$. The transformation matrices can be written as ($c : \cos, s : \sin, t : \tan$):

$$\mathbf{R}_{bI} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}, \quad (2)$$

$$\mathbf{\Gamma}_{bI} = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix}. \quad (3)$$

Fig. 2: Schematic diagram depicting the reference frames.

The 6-DoF dynamic equations are expressed as:

Force Equation

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m} \mathbf{T}_b - \boldsymbol{\omega}_b \times \mathbf{V}_b - \mathbf{R}_{bI}^T \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}, \quad (4)$$

Moment Equation

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \mathbf{I}^{-1} \mathbf{M}_b - \boldsymbol{\omega}_b \times \mathbf{I} \boldsymbol{\omega}_b, \quad (5)$$

where $\mathbf{T}_b$ and $\mathbf{M}_b$ are the total external force and moment, respectively, $\boldsymbol{\omega}_b$ and $\mathbf{V}_b$ are the angular and linear velocity expressed in the body frame respectively, and $\mathbf{I}$ is the moment of inertia of the quadrotor.

The total angular velocity of the propeller with respect to the body frame expressed in the frame $\mathcal{F}_{P_i}$ is determined by accounting for the rotation about each center of rotation:

$$\begin{aligned} \boldsymbol{\omega}_{P_i b} &= \boldsymbol{\omega}_{P_i} + \mathbf{R}_{P_i E_i}^T \boldsymbol{\omega}_{E_i} + \mathbf{R}_{P_i B_i}^T (\boldsymbol{\omega}_{B_i} + \boldsymbol{\omega}_b), \forall i = 1, 2, \\ \boldsymbol{\omega}_{P_i b} &= \boldsymbol{\omega}_{P_i} + \mathbf{R}_{P_i B_i}^T \boldsymbol{\omega}_{B_i} + \mathbf{R}_{P_i b}^T (\boldsymbol{\omega}_{E_i} + \boldsymbol{\omega}_b), \forall i = 3, 4, \end{aligned} \quad (6)$$

where $\boldsymbol{\omega}_{P_i b}$ is the resultant angular velocity of the propeller with respect to the body frame, $\boldsymbol{\omega}_{P_i}$, $\boldsymbol{\omega}_{E_i}$, $\boldsymbol{\omega}_{B_i}$ are the angular velocities of the propeller, elevation, and banking mechanism, respectively; $\mathbf{R}_{P_i E_i}$ and $\mathbf{R}_{P_i B_i}$ are the rotation transforms from frame $\mathcal{F}_{P_i}$ to frames $\mathcal{F}_{E_i}$ and $\mathcal{F}_{B_i}$, respectively. The variables required for determining the total angular velocity of various propellers can be written as:

$$\boldsymbol{\omega}_{P_i} = \begin{bmatrix} 0 \\ 0 \\ \Omega_i \end{bmatrix}, \quad \boldsymbol{\omega}_b = \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad \forall i = 1, \dots, 4, \quad (7)$$

$$\boldsymbol{\omega}_{E_i} = \begin{bmatrix} 0 \\ \dot{\beta}_i \\ 0 \end{bmatrix}, \quad \boldsymbol{\omega}_{B_i} = \begin{bmatrix} \dot{\alpha}_i \\ 0 \\ 0 \end{bmatrix}, \quad \forall i = 1, \dots, 4, \quad (8)$$

$$\mathbf{R}_{P_i E_i} = \begin{bmatrix} \cos\beta_i & 0 & \sin\beta_i \\ 0 & 1 & 0 \\ -\sin\beta_i & 0 & \cos\beta_i \end{bmatrix}, \quad \forall i = 1, 2, \quad (9)$$

$$\mathbf{R}_{P_i B_i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix}, \quad \forall i = 3, 4, \quad (10)$$

$$\mathbf{R}_{P_i b} = \mathbf{R}_{P_i B_i} \mathbf{R}_{P_i E_i}, \forall i = 1, 2, \quad (11)$$

$$\mathbf{R}_{P_i b} = \mathbf{R}_{P_i E_i} \mathbf{R}_{P_i B_i}, \forall i = 3, 4. \quad (12)$$

The total angular acceleration is determined by taking the derivative of (6) and expressed as follows:

$$\begin{aligned} \dot{\boldsymbol{\omega}}_{P_i b} &= \dot{\boldsymbol{\omega}}_{P_i} + \mathbf{R}_{P_i E_i}^T \dot{\boldsymbol{\omega}}_{E_i} + \mathbf{R}_{P_i b}^T (\dot{\boldsymbol{\omega}}_{B_i} + \dot{\boldsymbol{\omega}}_b) \\ &\quad - \mathbf{R}_{P_i E_i}^T \boldsymbol{\omega}_{E_i} \times \{ \mathbf{R}_{P_i b}^T (\boldsymbol{\omega}_{B_i} + \boldsymbol{\omega}_b) \}, \forall i = 1, 2, \\ \dot{\boldsymbol{\omega}}_{P_i b} &= \dot{\boldsymbol{\omega}}_{P_i} + \mathbf{R}_{P_i B_i}^T \dot{\boldsymbol{\omega}}_{B_i} + \mathbf{R}_{P_i b}^T (\dot{\boldsymbol{\omega}}_{E_i} + \dot{\boldsymbol{\omega}}_b) \\ &\quad - \mathbf{R}_{P_i B_i}^T \boldsymbol{\omega}_{B_i} \times \{ \mathbf{R}_{P_i b}^T (\boldsymbol{\omega}_{E_i} + \boldsymbol{\omega}_b) \}, \forall i = 3, 4. \end{aligned} \quad (13)$$

The gyroscopic moment generated due to the orientation change of the propeller is computed in the non-inertial frame $\mathcal{F}_{P_i}$ using the generalized Euler's equation of motion:

$$\mathbf{M}_{GP_i} = \mathbf{I}_{P_i} \dot{\boldsymbol{\omega}}_{P_i b} + \boldsymbol{\omega}_{P_i b} \times \mathbf{I}_{P_i} \boldsymbol{\omega}_{P_i b}, \forall i = 1, \dots, 4. \quad (14)$$

Consequently, the total gyroscopic moment experienced at the CG of the quadrotor is computed by taking the reaction of the gyroscopic moments generated by the propellers in their own frames and then transforming it to the frame $\mathcal{F}_b$:

$$\mathbf{M}_{Gb} = - \sum_{i=1}^4 \mathbf{R}_{P_i b} \mathbf{M}_{GP_i} \quad (15)$$

Thrust and drag-moment coefficients (K_T and K_M , respectively) are used to compute the steady state thrust and drag-moment as follows:

$$T_i = K_T \Omega_i^2 \quad \text{and} \quad M_i = K_M \Omega_i^2 \quad (16)$$

where T_i and M_i are the thrust and drag-moment generated by the i^{th} propeller with Ω_i as the angular velocity. The drag-moment expressed in the frame $\mathcal{F}_b$ is determined as follows:

$$\mathbf{M}_{dib} = \mathbf{R}_{P_i b} [0 \quad 0 \quad M_i]^T, \forall i = 1, \dots, 4. \quad (17)$$

It is to be noted that in 17, M_i is negative for CCW rotation (propellers 3,4) and positive for CW rotation (propellers 1,2). The displacement of each propeller origin with respect to the body CG is required to determine the moment generated due to thrust generation. These displacement vectors are determined using the respective rotation and translation transforms as follows:

$$\begin{aligned} \mathbf{d}_{P_i b} &= \begin{bmatrix} l_h \sin \beta_i \\ -(l_3 + l_h \cos \beta_i) \sin \alpha_i + (-1)^i l_a \\ (l_3 + l_h \cos \beta_i) \cos \alpha_i + h_b \end{bmatrix}, \\ &\quad \forall i = 1, 2, \\ \mathbf{d}_{P_i b} &= \begin{bmatrix} (l_3 + l_h \cos \alpha_i) \sin \beta_i + (-1)^i l_a \\ -l_h \sin \alpha_i \\ (l_3 + l_h \cos \alpha_i) \cos \beta_i + h_b \end{bmatrix}, \\ &\quad \forall i = 3, 4, \end{aligned} \quad (18)$$

where l_3 , l_h , l_a and h_b are defined in Table I. The thrust moment experienced in the body frame $\mathcal{F}_b$ can then be computed as:

$$\mathbf{M}_{Tib} = \mathbf{d}_{P_i b} \times \mathbf{T}_{P_i b}, \forall i = 1, \dots, 4, \quad (19a)$$

$$\text{s.t. } \mathbf{T}_{P_i b} = \mathbf{R}_{P_i b} [0 \quad 0 \quad T_i]^T. \quad (19b)$$

TABLE I: Intrinsic parameters of proposed quadrotor.

Symbol	Description	Value
m	Mass	2.012 kg
l_a	Arm length	0.235 m
l_3	Offset from $\mathcal{F}_{E_i}$ to $\mathcal{F}_{B_i}$	0.068 m
l_h	Offset from $\mathcal{F}_{P_i}$ to $\mathcal{F}_{B_i}$	0.060 m
h_b	Offset from CG to $\mathcal{F}_{E_i}$	-0.007 m
K_T	Force coefficient	$1.43\text{e} - 5 \text{ N}\cdot\text{s}^2$
K_M	Drag-moment coefficient	$2.01\text{e} - 7 \text{ Nm}\cdot\text{s}^2$

Finally, the total external force and total external moment experienced by the quadrotor can be summed up as:

$$\mathbf{T}_b = \sum_{i=1}^4 \mathbf{T}_{P_i b}, \quad (20)$$

$$\mathbf{M}_b = \sum_{i=1}^4 [\mathbf{M}_{Tib} + \mathbf{M}_{dib}] + \mathbf{M}_{Gb}. \quad (21)$$

The final full-state discretized nonlinear model of the over-actuated quadrotor can be expressed as:

$$\mathbf{x}_{k+1} = \mathbf{f}_d(\mathbf{x}_k, \mathbf{u}_k), \quad (22)$$

$$\mathbf{y}_k = \mathbf{h}_d(\mathbf{x}_k, \mathbf{p}), \quad (23)$$

where the state vector $\mathbf{x} \in \mathbb{R}^{12}$ and control vector $\mathbf{u} \in \mathbb{R}^{12}$ are composed of:

$$\mathbf{x} = [x, y, z, u, v, w, \phi, \theta, \psi, p, q, r]^T, \quad (24)$$

$$\mathbf{u} = [[\alpha_i]_4, [\beta_i]_4, [\Omega_i]_4]^T, \forall i = 1, \dots, 4, \quad (25)$$

wherein $[\cdot]_n$ represents a vector in $\mathbb{R}^{1 \times n}$. The measurement vector $\mathbf{y} \in \mathbb{R}^{16}$ is concatenation of all states and a fault indication vector which is given by $\mathbf{p} \in \mathbb{R}^4 = [\gamma_i | i = 1, \dots, 4]^T$. In essence, the vector $\mathbf{p}$ contains fault information in terms of parameter γ_i which reflects the normal ($\gamma_i = 1$) or faulty ($\gamma_i = 0$) propeller operation. Moreover, the discrete state and measurement functions are denoted by $\mathbf{f}_d(\cdot, \cdot): \mathbb{R}^{12} \times \mathbb{R}^{12} \rightarrow \mathbb{R}^{12}$ and $\mathbf{h}_d(\cdot, \cdot): \mathbb{R}^{12+4} \rightarrow \mathbb{R}^{16}$, respectively.

III. FAULT-TOLERANT CONTROL

In this work, a centralized control approach (Fig. 3) is proposed to realize fault-tolerance by the over-actuated quadrotor. The designed NMPC tracks the position and attitude reference generated by the navigation algorithm. Accordingly, it commands the servo angles (α 's and β 's) and rotational velocities (Ω 's) of the propellers, which reflects the normal operation of the overall system. In an event of a propeller failure, the NMPC reconfigures the dynamic model online in accordance with the fault information ($\mathbf{p}$). In other words, the underlying optimal control problem (OCP) is updated such that the control commands are allocated to the remaining functional actuators.

Fig. 3: Block diagram of the proposed NMPC-based FTC scheme. Note that the dotted fault-detection block is left for future work and only depicted for enhanced readability.

A. Reconfigurable NMPC Design

The reconfigurable NMPC utilizes a least-square problem formulation in discrete-time:

$$\min_{\mathbf{x}_k, \mathbf{u}_k} \frac{1}{2} \left\{ \sum_{k=0}^{N_c-1} (\mathcal{C}_x + \mathcal{C}_u) + \mathcal{C}_{N_c} \right\} \quad (26a)$$

$$\text{s.t. } \mathbf{x}_{k+1} = \mathbf{f}_d(\mathbf{x}_k, \mathbf{u}_k, \mathbf{p}), \quad k \in [0, N_c - 1], \quad (26b)$$

$$\mathbf{x}_{k,\min} \leq \mathbf{x}_k \leq \mathbf{x}_{k,\max}, \quad k \in [0, N_c], \quad (26c)$$

$$\mathbf{u}_{k,\min} \leq \mathbf{u}_k \leq \mathbf{u}_{k,\max}, \quad k \in [0, N_c - 1]. \quad (26d)$$

In (26a), the components inside summation represent the stage cost and are expressed as:

$$\mathcal{C}_x = (\mathbf{x}_k - \mathbf{x}_k^{\text{ref}})^T \mathbf{W}_x (\mathbf{x}_k - \mathbf{x}_k^{\text{ref}}), \quad (27)$$

$$\mathcal{C}_u = (\mathbf{u}_k - \mathbf{u}_k^{\text{ref}})^T \mathbf{W}_u (\mathbf{u}_k - \mathbf{u}_k^{\text{ref}}). \quad (28)$$

They penalize the deviations of the predicted states ($\mathbf{x}_k$) and control ($\mathbf{u}_k$) trajectories from their references, $\mathbf{x}_k^{\text{ref}}$ and $\mathbf{u}_k^{\text{ref}}$, respectively. Besides, $\mathbf{W}_x \in \mathbb{R}^{12 \times 12}$ and $\mathbf{W}_u \in \mathbb{R}^{12 \times 12}$ are the corresponding state and control weight matrices which are selected to be diagonal and positive-(semi)definite. The second component in (26a) represents the terminal cost, expressed as:

$$\mathcal{C}_{N_c} = (\mathbf{x}_{N_c} - \mathbf{x}_{N_c}^{\text{ref}})^T \mathbf{W}_{N_c} (\mathbf{x}_{N_c} - \mathbf{x}_{N_c}^{\text{ref}}), \quad (29)$$

which penalizes the finite nature of the prediction horizon and is mainly added for stability reasons [19]. Also, $\mathbf{W}_{N_c} \in \mathbb{R}^{12 \times 12}$ is the corresponding positive-(semi)definite weight matrix which is also selected as diagonal. Furthermore, the terms $\mathbf{x}_{k,\min} \leq \mathbf{x}_{k,\max} \in \mathbb{R}^{12}$ and $\mathbf{u}_{k,\min} \leq \mathbf{u}_{k,\max} \in \mathbb{R}^{12}$ in (26d), specify the lower and upper bounds on the states and controls, respectively.

For the simultaneous position and attitude tracking, the NMPC is designed with the full state and control vectors given in (24). Besides, the following state trajectory is given as reference for the optimization problem:

$$\mathbf{x}^{\text{ref}} = \mathbf{x}_{N_c}^{\text{ref}} = [x_r, y_r, z_r, u_r, v_r, w_r, \phi_r, \theta_r, \psi_r, p_r, q_r, r_r]^T, \quad (30)$$

wherein $[x_r, y_r, z_r]$, $[u_r, v_r, w_r]$, $[\phi_r, \theta_r, \psi_r]$, and $[p_r, q_r, r_r]$ are the position, velocity, attitude, and angular rate references, respectively, that are commanded by the navigation algorithm. The control reference trajectory, on the other

hand, depends on the normal or faulty propeller operation, i.e., for the normal operation:

$$\mathbf{u}^{\text{ref}} = [[-\phi_r]_4, [-\theta_r]_4, [600]_4]^T, \quad (31)$$

whereas, for the faulty operation:

$$\mathbf{u}^{\text{ref}} = [[-\phi_r]_4, [-\theta_r]_4, [\gamma_i 400]_2, [850]_2]^T, \quad \forall i = 1, 2, \quad (32)$$

$$\mathbf{u}^{\text{ref}} = [[-\phi_r]_4, [-\theta_r]_4, [850]_2, [\gamma_i 400]_2]^T, \quad \forall i = 3, 4. \quad (33)$$

Note that the control reference either from (32) or from (33) is incorporated for the corresponding propeller failure.

To comply with the actuator limits, the following constraints are defined:

$$-10 \text{ (}^\circ\text{)} \leq \alpha_1, \beta_3 \leq 60 \text{ (}^\circ\text{)}, \quad (34a)$$

$$-60 \text{ (}^\circ\text{)} \leq \alpha_2, \beta_4 \leq 10 \text{ (}^\circ\text{)}, \quad (34b)$$

$$-60 \text{ (}^\circ\text{)} \leq \alpha_3, \alpha_4, \beta_1, \beta_2 \leq 60 \text{ (}^\circ\text{)}, \quad (34c)$$

$$0 \text{ (rad/s)} \leq \Omega_i \leq 1000 \text{ (rad/s)}, \quad \forall i = 1, \dots, 4. \quad (34d)$$

Moreover, the following weight matrices for the optimization problem are obtained utilizing the algorithm proposed in the authors' previous work [20]:

$$\mathbf{W}_x = \text{diag}(170, 170, 170, 3, 3, 3, 14, 14, 20, 0.2, 0.2, 0.3),$$

$$\mathbf{W}_u = \text{diag}(20, 20, 15, 15, 15, 15, 20, 20, [7 \times 10^{(-1)^{\gamma_i} 5}]_4), \quad \forall i = 1, \dots, 4,$$

$$\mathbf{W}_{N_c} = 1.5 \mathbf{W}_x.$$

Note that in the control weight matrix ($\mathbf{W}_u$), the corresponding propeller is weighted with a low or high number for normal or faulty operation, respectively. Besides, the above definition of the terminal weight matrix ($\mathbf{W}_{N_c}$) weighs it more in comparison to the stage cost weight matrix ($\mathbf{W}_x$). This is a typical way of defining them, wherein the reason being is to ensure the stability of the OCP.

For real-time feasibility, the optimization problem of NMPC is solved utilizing the direct multiple shooting method with a shooting grid size of 0.01s. The reason to adopt direct solution methodology underlies its several advantages over the indirect techniques, wherein the most prominent being the ability to easily accommodate inequality as well as equality constraints. Also, an explicit Runge-Kutta 4th order integrator is incorporated with 2-steps per shooting interval. Subsequently, the resulting discretized OCP reduces to a sequential quadratic program which is further solved via the generalized Gauss-Newton method with the help of a special real-time iteration scheme proposed in [21]. Furthermore, the prediction horizon $N_c = 30$ is selected along with a sampling time of 0.01s for computational tractability.

IV. SIMULATION RESULTS

In this section, we illustrate the tracking performance of the over-actuated quadrotor incorporating the proposed FTC framework. The simulation testing is carried out in

Matlab/Simulink environment incorporating the high-fidelity model of the quadrotor presented in Section II. In the subsequent part, we analyze the tracking performance for two reference trajectories. The first trajectory is typically utilized for inspection-related applications. The second trajectory is an 8-shaped reference that is adopted to replicate extreme operation, thereby challenging the controller. In both cases, the quadrotor is initialized with zero state and control vectors, and a fixed sampling time of 0.01s is maintained for the entire simulation. In the beginning, it gradually climbs to 1.5m height and hovers there until the trajectory starts at $t = 5s$. Moreover, while tracking the commanded attitude, a fault in Ω_3 is induced (i.e., $\gamma_3 = 0$) at $t = 15s$ for 10s. The normal operation ($\gamma_3 = 1$) resumes at $t = 25s$, followed by the fault in Ω_1 at $t = 30s$ for another 10s. Finally, the normal operation resumes at $t = 40s$ and stays until the end of the simulation. Note that in this work, since the closed-loop does not incorporate a fault-detection system, we also analyze its robustness to various fault-detection delay times. The results in this section are generated with a fault-detection time of 0.5s, which is 50 times the utilized sampling time. Moreover, the designed NMPC can handle fault-detection delays up to 1s.

A. Inspection Trajectory

In the inspection-oriented trajectory, firstly, the quadrotor is commanded to reach a position setpoint [1m,1m,1.5m] in the 3D space, followed by gradually attaining the attitude setpoint $[30^\circ, 30^\circ, 0^\circ]$. The overall tracking performance in the presence of intermittent faults can be visualized in Fig. 4. As evident from Figs. 4a, 4b, and 4c, the quadrotor successfully recovers and stays on course even after experiencing sequential propeller failures. For the fault-detection delay of 0.5s, the recorded mean absolute errors along three directions ($|e_x|$, $|e_y|$, $|e_z|$) and the mean Euclidean error are listed in Table II. Besides, the obtained maximum deviations from the trajectory at the failure instant are 13.8cm, 14.8cm, and 12.4cm, along the three directions. As anticipated, these deviations are directly dependent on the detection delay time such that they decrease by 46.4%, 56.8%, 74.2% or increase by 43.5%, 35.8%, 29.8% for detection delay as low as 0.2s or as high as 1s, respectively. Moreover, the incurred mean absolute roll, pitch, yaw errors, and the mean Euclidean error are listed in Table III. Furthermore, the corresponding maximum attitude deviations at fault instants with detection delay of 0.5s are 36.3° , 31.3° , and 11.5° , wherein they also similarly vary with detection delay times as the position deviations.

The servo angle and propeller velocity commands from the fault-tolerant NMPC are shown in Figs. 4d and 4e, respectively, wherein, the black dotted lines represent the specified constraints. Note that although the servo angles corresponding to the faulty propeller are operational, they do not have any influence on the dynamics. As evident from the figures, the control commands always stay within the constrained boundaries, even with the sudden jumps that arise at the fault instants. It can also be seen that the

TABLE II: Mean absolute errors and mean Euclidean error for position with 0.5s detection delay time. Unit [cm].

Trajectory	$ e_x $	$ e_y $	$ e_z $	Euc.
Inspection	3.5	3.0	2.0	5.9
8-shaped	11.4	12.9	6.5	20.5

TABLE III: Mean absolute errors and mean Euclidean error for attitude with 0.5s detection delay time. Unit $[\circ]$.

Trajectory	$ e_x $	$ e_y $	$ e_z $	Euc.
Inspection	2.7	3.8	0.8	5.2
8-shaped	6.7	5.9	3.1	11.2

designed NMPC optimally allocates the controls between the functional actuators even when some are already saturated, thereby adapting itself for different operating conditions. Thanks to this reconfigurability, the quadrotor continues to follow the commanded position and attitude trajectory. In other words, the NMPC optimally utilizes the remaining actuators to keep tracking the trajectory satisfactorily. All this eventually supports the real-time feasibility of the proposed control framework.

Since the gyroscopic moments are not explicitly considered within the model that underly NMPC, it is worth knowing the influence of gyroscopic moments on the overall system. Hence, they are presented along with the total moments in Fig. 4f. For a quantitative visualization, the ratios of gyroscopic moments to the total moments along the three directions are 0.55, 0.04, and 0.59. This validates the robustness of the designed NMPC in catering to the unmodeled dynamics, thereby illustrating the real-time feasibility. Furthermore, the average execution time for the fault-tolerant NMPC is 11 milliseconds. Typically, it is sufficient if the controller could run at half the sampling of the closed-loop. However, in this study, the controller almost runs at the adopted sampling time. In future work, it is envisioned to utilize an even faster solution methodology and reduce the model complexity within NMPC design to decrease the overall execution time for real-time application.

B. 8-shaped Trajectory

Next, we illustrate the controller's performance in recovering from failure in an aggressive flight. For that purpose, an 8-shaped trajectory of 1.6m width along the x-direction and 0.8m width along y-direction is commanded at almost 0.8m/s after the initial climb to 1.5m height. Then, to further challenge the control framework, an asynchronous sinusoidal attitude trajectory is commanded with the amplitude varying between $\pm 30^\circ$ for roll and pitch, and $\pm 25^\circ$ for yaw. The overall tracking performance in the presence of intermittent faults and with a fault-detection delay of 0.5s can be visualized in Fig. 5. As evident from Figs. 5a and 5b, the quadrotor continues to follow the commanded position trajectory even with propeller failure. The recorded mean absolute errors and the mean Euclidean error are listed in Table II. As anticipated, their magnitudes increase by a fair amount over

Fig. 4: Simulation tracking results for an inspection-oriented trajectory with a detection delay time of 0.5s, wherein red-dotted and cyan-dotted lines represent the failure and normal operation instants, respectively.

the error values for inspection trajectory, which is mainly due to the involved complexities. Besides, the corresponding maximum deviations from the trajectory at failure instants are 27.9cm, 30.8cm, and 25.3cm. In terms of the attitude tracking performance, depicted in Fig. 5c, the obtained errors are listed in Table III, wherein they also show a similar trend with the position errors.

The servo angle and propeller velocity commands are illustrated in Figs. 4d and 4e, respectively. While the control commands always stay bounded, they gracefully recover from saturation even with failures. In other words, the NMPC still optimally allocates the control to the remaining actuators when at least two of the servos are always saturated at the failure instants. This eventually renders the quadrotor to stay on course without any catastrophic failure, thus validating the real-time applicability of the FTC framework. Finally, the influence of gyroscopic moments in comparison to the total moments can be visualized in Fig. 5f. The ratios of gyroscopic moments to the total moments along the three directions are 0.02, 0.19, and 0.02, which again validates the robustness of the designed NMPC in handling modeling uncertainties even for this aggressive trajectory.

V. CONCLUSIONS AND FUTURE WORK

Appreciating the importance of safety within inspection-related applications of aerial robots, an FTC framework has been proposed for a novel, over-actuated quadrotor. The centralized NMPC is the core of the designed framework which can safely recover the aerial robot in the event of complete loss of one propeller. The compelling feature of the designed NMPC is the reconfigurability that renders it to be a universal controller for normal and faulty operations, thereby eradicating the need for redundancy. Moreover, its fault-tolerant performance has been illustrated via a high-fidelity simulation model over two trajectories with different complexities while accommodating fault-detection delays up to 1s. The results manifest that the quadrotor continues to individually track position and attitude satisfactorily even after losing a propeller. In future work, the focus would be to reduce the complexity of the underlying model in NMPC to expedite the solution time, followed by the incorporation of a fault-detection method.

VI. ACKNOWLEDGMENT

This research is supported by the Ministry of Education, Singapore, under its Academic Research Fund Tier 1 (RG69/20). The authors would also like to thank Civil

Fig. 5: Simulation tracking results for an 8-shaped trajectory with asynchronous sinusoidal attitude command having peak amplitude in $\phi = \theta = 30^\circ$ and $\psi = 25^\circ$. Also, red-dotted and cyan-dotted lines represent the failure and normal operation instants, respectively.

Aviation Authority of Singapore (CAAS) and the Air Traffic Management Research Institute (ATMRI), Singapore for their support.

REFERENCES

- [1] P. J. Sanchez-Cuevas, G. Heredia, and A. Ollero, "Multirotor uas for bridge inspection by contact using the ceiling effect," in *2017 International Conference on Unmanned Aircraft Systems (ICUAS)*, 2017, pp. 767–774.
- [2] J. M. Vazquez-Nicolas, E. Zamora, I. Gonzalez-Hernandez, R. Lozano, and H. Sossa, "Towards automatic inspection: crack recognition based on quadrotor uav-taken images," in *2018 International Conference on Unmanned Aircraft Systems (ICUAS)*, 2018, pp. 654–659.
- [3] B. B. Kocer, T. Tjahjowidodo, M. Pratama, and G. G. L. Seet, "Inspection-while-flying: An autonomous contact-based nondestructive test using uav-tools," *Automation in Construction*, vol. 106, p. 102895, 2019. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0926580519301451>
- [4] A. Khadka, B. Fick, A. Afshar, M. Tavakoli, and J. Baqersad, "Non-contact vibration monitoring of rotating wind turbines using a semi-autonomous uav," *Mechanical Systems and Signal Processing*, vol. 138, p. 106446, 2020. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0888327019306673>
- [5] H. A. Izadi, Y. Zhang, and B. W. Gordon, "Fault tolerant model predictive control of quad-rotor helicopters with actuator fault estimation," *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 6343 – 6348, 2011, 18th IFAC World Congress. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S1474667016446227>
- [6] I. Sadeghzadeh, A. Mehta, and Y. Zhang, "Fault/damage tolerant control of a quadrotor helicopter uav using model reference adaptive control and gain-scheduled pid," in *AIAA Guidance, Navigation, and Control Conference*, 2011, p. 6716.
- [7] M. W. Mueller and R. D'Andrea, "Stability and control of a quadcopter despite the complete loss of one, two, or three propellers," in *2014 IEEE International Conference on Robotics and Automation (ICRA)*, 2014, pp. 45–52.
- [8] G. Michieletto, M. Ryll, and A. Franchi, "Control of statically hoverable multi-rotor aerial vehicles and application to rotor-failure robustness for hexarotors," in *2017 IEEE International Conference on Robotics and Automation (ICRA)*, 2017, pp. 2747–2752.
- [9] T. Péni, B. Vanek, G. Lipták, Z. Szabó, and J. Bokor, "Nullspace-based input reconfiguration architecture for overactuated aerial vehicles," *IEEE Transactions on Control Systems Technology*, vol. 26, no. 5, pp. 1826–1833, 2018.
- [10] J. I. Giribet, R. S. Sanchez-Pena, and A. S. Ghersin, "Analysis and design of a tilted rotor hexacopter for fault tolerance," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 4, pp. 1555–1567, 2016.
- [11] J. Zhang, P. Bhardwaj, S. A. Raab, S. Saboo, and F. Holzapfel, "Control allocation framework for a tilt-rotor vertical take-off and landing transition aircraft configuration," in *2018 Applied Aerodynamics Conference*, 2018, p. 3480.
- [12] M. Mehndiratta and E. Kayacan, "A constrained instantaneous learning approach for aerial package delivery robots: onboard implementation and experimental results," *Autonomous Robots*, vol. 43, no. 8, pp. 2209–2228, Dec 2019.
- [13] M. Mehndiratta, E. Kayacan, S. Patel, E. Kayacan, and G. Chowdhary, "Learning-based fast nonlinear model predictive control for custom-made 3D printed ground and aerial robots," *Control Engineering*, 2019.

- [14] M. Mehndiratta and E. Kayacan, "Online learning-based receding horizon control of tilt-rotor tricopter: A cascade implementation," in *2018 American Control Conference (ACC)*, June 2018, pp. 1–6.
- [15] U. Eren, A. Prach, B. B. Koçer, S. V. Raković, E. Kayacan, and B. Açıkmeşe, "Model predictive control in aerospace systems: Current state and opportunities," *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 7, pp. 1541–1566, 2017. [Online]. Available: <https://doi.org/10.2514/1.G002507>
- [16] M. Kamel, K. Alexis, M. Achtelik, and R. Siegwart, "Fast nonlinear model predictive control for multicopter attitude tracking on so(3)," in *2015 IEEE Conference on Control Applications (CCA)*, 2015, pp. 1160–1166.
- [17] M. Mehndiratta and E. Kayacan, "Reconfigurable fault-tolerant nmppc for y6 coaxial tricopter with complete loss of one rotor," in *2018 IEEE Conference on Control Technology and Applications (CCTA)*, Aug 2018, pp. 774–780.
- [18] M. Saied, B. Lussier, I. Fantoni, C. Francis, H. Shraim, and G. Sanahuja, "Fault diagnosis and fault-tolerant control strategy for rotor failure in an octorotor," in *2015 IEEE International Conference on Robotics and Automation (ICRA)*, 2015, pp. 5266–5271.
- [19] T. Kraus, H. Ferreau, E. Kayacan, H. Ramon, J. D. Baerdemaeker, M. Diehl, and W. Saeys, "Moving horizon estimation and nonlinear model predictive control for autonomous agricultural vehicles," *Computers and Electronics in Agriculture*, vol. 98, pp. 25 – 33, 2013.
- [20] M. Mehndiratta, E. Camci, and E. Kayacan, "Automated tuning of nonlinear model predictive controller by reinforcement learning," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Oct 2018.
- [21] M. Diehl, H. Bock, J. P. Schlöder, R. Findeisen, Z. Nagy, and F. Allgöwer, "Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations," *Journal of Process Control*, vol. 12, no. 4, pp. 577 – 585, 2002.