Finite element stress and failure analysis of railway track structural system

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Finite Element Stress and Failure Analysis of Railway Track Structural System

By

Liu Zhufeng

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

**A thesis submitted to Nanyang Technological University in partial fulfillment of
the requirements for the degree of Doctor of Philosophy**

2021

Statement of Originality

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Prof. Pang Hock Lye John

Authorship Attribution Statement

This thesis contains material from 4 paper published in the following peer-reviewed journal(s) / from papers accepted at conferences in which I am listed as an author.

Chapters 4 are published as Z. Liu, X. Shi, K.S. Tsang, H.J. Hoh, J.H.L. Pang. Dynamic finite element modeling and fatigue damage analysis of thermite welds. *Fatigue & Fracture of Engineering Material & Structures*. DOI: 10.1111/ffe.13091.

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Liu Zhufeng

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Abstract

Railway track comprises of continuous welded rail mounted with rail clips on sleepers integrated to a ballast or concrete slab track form system. Thermite weld is an important structural component connecting rail steel segments to form a continuous track structure and the rail clip fastening system provides flexibility and turnover resistance for running rails. The failure of rails and rail clips compromises the structural integrity in the railway track system which can lead to unplanned downtime and possible safety issues such as derailment in railway operations. The objective of this research is to study the fatigue failures of the thermite weld and rail clips subjected to train wheel-induced dynamic loadings. Under complex multiaxial loadings in service, head-to-web, web-to-foot and rail foot regions along the weld edge of thermite weld and arch regions of rail clips are susceptible to fatigue crack formation.

In this research, a fatigue assessment methodology integrates the experiment work and finite element modeling work leading to fatigue life prediction models for the thermite weld and rail clip failures. Firstly, the mechanical and fatigue properties of rail and clip steels were investigated through a series of experimental studies. On-site wireless strain measurement was also conducted to record the train wheel-induced strain spectrum at a thermite weld of a sharp curve in a metro tunnel. Secondly, a three-dimensional finite element (FE) model for railway track system including the thermite weld and rail clips

was developed by using an implicit-explicit sequence analysis to investigate the stress states at the regions of interest. The model was validated by a train roll-in test and a good consistency is observed between the simulation results and field measurements. Lastly, a time-efficient fatigue analysis methodology was proposed with the smallest enclosing circle algorithm implemented to search the critical plane orientations and calculate the maximum fatigue parameter for performing multiaxial fatigue analysis. The fatigue damage and crack initiation life of the thermite weld and rail clips were predicted according to stress-based and strain-based fatigue models.

The fatigue assessment methodology developed for the thermite weld and rail clips provides a comprehensive and rapid evaluation of the failures observed in the railway track system. The influence of change in material, geometry, loading conditions, and track structures on fatigue life can also be evaluated in a short period.

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Chapter 1. Introduction

The modern railway track is an integrated structure consisting of continuously welded rails supported by sleepers attached via base plates with clips and rest on the ballast or concrete slab that spread the train wheel-induced dynamic loadings. Thermite welds and rail clips play a key role in the railway track system since the fatigue failures of these two components can lead to unplanned system downtime and cause track reliability and safety issues such as derailment in railway operations [1-5]. Despite regular inspections and replacements of thermite welds and rail clips [6-8], a fatigue assessment method through experiments, finite element simulation and fatigue life prediction models is a necessity in understanding the failure mechanism. This fatigue assessment method [9-11] is shown in Figure 1-1.

Figure 1.1. A fatigue assessment method integrates experiments, finite element models and fatigue models

1.1. Background

Railway track is a structure consisting of continuous welded rail, rail fasteners, sleepers, and ballast or concrete slabs. Rail steel is a vital component of railway track and is directly linked to safety and rail failure concerns. The assembly and replacement of rails are usually transported to the site in the form of long segments and done by welding two adjoining rail ends. Among all the welding techniques, thermite welding which is also known as aluminothermic weld has been widely used in application due to its well-established procedure for on-site rail repair and replacement and joining with relatively low cost, fast installation, and equipment portability [12, 13]. Thermite welding is a fusion welding process that takes place in the sand mould between two rail ends. The welded joints were created by a chemical reaction between aluminium powder and iron oxide with a large amount of energy being released.

However, rail break studies show that most of the rail failures originated from fatigue cracks that were nucleated in thermite welds which are the weakest parts used for connecting rail steel segments to form a continuous track structure system [3, 14]. Thermite weld generally exhibits poor fatigue properties compared to the base rail material under train wheel cyclic loadings. Excluding failures occurred at top of rail head which was mainly caused by rolling contact fatigue [15], fatigue failures of thermite weld at the high rails as shown in Figure 1-2 have become the most serious problem on track reliability and safety especially at curve segments [4, 16].

Figure 1-2. Fatigue failure sites observed at head-to-web, web-to-foot and rail foot regions.

The rail fastening system is also one of the essential components of railway systems which provides flexibility and turnover resistance for rails. Rail clips, which are in contact with the foot of the rail and anchor the rail to the sleeper, are several orders higher replacement frequency compared with other components including thermite weld due to high installation stress and fatigue problem. The fracture of rail clips can lead to loss of gauge restraint and cause a direct impact on railway operational safety even potentially lead to train derailment. Two types of rail clips, Pandrol E-clip and fast-clip [17-19] are widely used in mass rapid transit (MRT) system due to the virtue of easy installation and removal, and good capability to maintain rail position. The

typical failure locations of these two kinds of clips are shown in Figure 1-3.

Figure 1-3. Typical fatigue failure modes: (A) E-clip at front and rear arch and, (B) fast-clip at the rear arch near the shoulder.

The fatigue crack formation at the thermite weld is due to the stress concentration effect of the weld geometry and defects and the complex stress state as the train wheel passes by. Crack initiation is the dominant fatigue failure mode of rail clips subjected to the complex stress state under train wheel-induced cyclic loading. Therefore, the material characteristics and fatigue property of the thermite weld and rail clips and the fatigue damage at the failure regions must be analyzed to have a deep understanding of the fatigue failures under service conditions.

1.2. Objective and scope

The objective of this thesis is to develop an integrated fatigue assessment methodology (Experiments, finite element models and fatigue life prediction models) for the thermite weld and rail clips under complex train wheel-induced loading conditions. The methodology focuses on the multiaxial fatigue analysis and life prediction for the thermite weld and rail clips and the inputs of the fatigue model consist of the material characteristics and fatigue properties obtained from laboratory and field experiments and multiaxial stress and strain states obtained from finite element simulations. The four major research objectives are as follows:

- 1.1 To characterize the material properties and fatigue performances for the thermite weld and rail clips using the laboratory testing techniques. This involves: (a) Tensile, hardness and fatigue tests for thermite weld steel and clip steel to obtain the fatigue life curves; (b) Microstructure analysis and chemical composition analysis for thermite weld steel; (c) Static pull-up tests for rail clips to investigate their structural deformation; (d) Structural fatigue loading test for fast-clip; and (e) Inverted four-point bend tests for thermite weld to investigate the failure at head-to-web regions.
- 1.2 To examine the strain behavior of the thermite weld at a depot and in a metro tunnel under service loadings through onsite strain measurement technologies. This

involves: (a) Strain measurement through a wired system for a static roll-in test conducted at a depot; and (b) Wireless strain measurement at a curved track in a metro tunnel.

1.3 To develop finite element models to compute the complex stress and strain tensors for the thermite weld and rail clips at the regions of interest under different loading conditions. This involves: (a) A multibody system dynamic model to estimate the wheel-rail interactions; (b) An implicit-explicit sequence approach to analyze the complex stress/strain and deformation generated from finite element models; and (c) Validation of the finite element models by comparing simulation and strain measurement results.

1.4 To develop an algorithm and computer program for the implementation of critical plane based multiaxial fatigue models to provide fatigue assessment and life predictions of the thermite weld and rail clips. This involves: (a) To develop a linear-time smallest enclosing circle algorithm using Python code for the critical plane searching process; and (b) To apply stress-based and strain-based multiaxial fatigue model for the fatigue damage calculation and life prediction.

In order to accomplish these objectives, a fatigue assessment methodology has been proposed as shown in Figure 1-4. The procedures for carrying the methodology are summarized in the following:

Firstly, obtain the fatigue life curves for smooth specimens extracted from the thermite weld and rail clips as well as the structural fatigue life curves for the thermite weld and rail clips. The material characteristics obtained through tensile, hardness tests can also be used as inputs in finite element models.

Secondly, calculate the multiaxial stress/strain histories from implicit-explicit sequence finite element analysis and validate the models by comparing the simulation results with the field strain measurement results.

Thirdly, transform the stress/strain tensor of each time step according to potential candidate planes and calculate fatigue damage using the proposed time-efficient computing algorithm as well as predict the service life of the thermite weld and rail clips with the fatigue life curves.

Finally, the effects of defects on the weld edge of thermite weld were studied especially at head-to-web regions through Neuber's and Glinka's rules.

Finite Element Stress and Failure Analysis of Railway Track Structural System

Figure 1-4. Research roadmap for failure assessment of the thermite weld and rail clips

1.3. Research gap and novelty

In general, the performance of rail tracks depends greatly upon the structural integrity of railway components. Field observations suggest that most of the service failures initiate at the thermite weld and rail clips. Failure observations show that most of the thermite weld failures originated from rail base, web-to-foot and head-to-web regions [3, 4, 20]. The typical failure locations of rail clips are at the arches. Studies have been carried out to investigate the failure mechanism of the thermite weld and rail clips, but some limitations exist and a comprehensive fatigue assessment at these regions is needed to understand the cause and risk of such fatigue failures.

Firstly, the susceptibility of the thermite weld to fatigue damage formation on the weld edge is not well researched. Thermite welds are considered the track discontinuities of the running surface because of the geometry irregularity and different mechanical properties. Thus, from previous researches, more attention was drawn on the rolling contact fatigue problems of the rail head [21, 22]. However, the failures at head-to-web, web-to-foot and rail foot regions are found to be a frequent service interruption [23] but few studies have been carried out to investigate the failure mechanism [3], particularly at head-to-web regions. The fatigue cracks initiated at head-to-web and web-to-foot regions are insidious and difficult to be detected before a final fracture occurs. Fatigue assessment and service life prediction of the thermite weld is a necessity in order to maintain the performance and structural integrity of the railway track system.

Secondly, the complex stress states and rail deformation resulting from wheel-rail contact which causes these failures should be investigated and further used for multiaxial fatigue analysis. In early studies, a beam theory analysis was used to obtain the stress and deformation state of the rail [24, 25]. Later, multibody computational methods were used by researchers to analyze wheel-rail interaction since the interaction is much more complex than theoretical assumptions [26, 27]. Recently, static [12, 28-30] and dynamic [31-34] finite element analysis was usually adopted since the stress distribution in the track and the thermite weld cannot be solved using multibody computational methods. However, few studies have been carried out to include rail clips in finite element models [35] and the effect of rail clips has not been clearly revealed. Also, in the ideal situation, the field strain measurement results should be used to compare with the simulation results to validate the models and obtain meaningful results.

Thirdly, fatigue failure of rail clips can lead to loss of gauge restraint and cause operational safety concerns. Experimental and numerical studies have been carried out to analyze the dynamic behavior of the whole fastening system [36, 37] and the fatigue life prediction of rail clips have been performed by some researchers [38-40]. However, the failure mechanism of rail clips has not been completely understood under cyclic wheel loadings and crack initiation fatigue analysis of rail clips is needed since crack nucleation and early growth consume most of the service life for high strength steel.

Lastly, fatigue damage analysis is a common technology for studying the failures of the thermite weld and rail clips due to the cyclic nature of wheel loads [39-42]. Various multiaxial fatigue models have been proposed to quantify the fatigue damage of a structure that experiences a multiaxial stress state. Multi-axial fatigue models can be classified according to the fatigue index used in the model as a scalar or in a critical plane format [43]. Critical plane fatigue criteria are developed based on experimental observations that fatigue cracks initiate on the critical plane within a material and usually applied for fatigue analysis of railway components [44-46]. However, the critical plane searching process is a time-consuming process for a huge number of points since the traditional minimum circumscribed circle method searches for all pairs of points and the triplets consisting of any instantaneous of the whole loading cycle [10, 47]. Therefore, developing a time-efficient critical plane searching algorithm is a necessity to calculate the fatigue damage of the thermite weld and rail clips despite the maximum rectangular hull method has been proposed [48-50].

This study provides a fatigue assessment methodology of the thermite weld and rail clips. The material characteristics and fatigue property are obtained through a series of material tests and structural tests. Implicit-explicit sequence finite element analysis is carried out and the models are validated through field strain measurements. A time-efficient multiaxial fatigue analysis methodology is proposed to calculate the fatigue damage. This study will provide valuable insights on the multiaxial fatigue analysis of

the thermite weld and rail clip failures subjected to dynamic loadings and shed new light on the potential future improvements on the railway track structural system.

1.4. Outline of the thesis

This thesis contains seven chapters. While the current chapter (Chapter 1) is a general introduction, Chapter 2 deals with a review of the existing literature relevant to the railway track structural system and fatigue failure investigations on the thermite weld and rail clips. It begins with a brief introduction of the railway components and their failures as well as the modeling methods. This follows by a detailed review of multiaxial fatigue models and the material characteristics. In addition, the rain-flow cycle counting approach and the notch correction for defects are also introduced. Chapter 3 presents the laboratory and field experiments of the thermite weld and the laboratory experiments of rail clips. The effect of possible surface defects on the thermite weld at head-to-web regions is examined. Chapter 4 details the stress and fatigue assessment of the thermite weld. An implicit-explicit finite element model is developed and validated with field measurement results. A linear-time critical plane searching algorithm for multiaxial fatigue analysis is proposed. Parametric studies are also performed in this chapter. Chapter 5 explores the effect of missing clips on the fatigue damage of the thermite weld and the fatigue damage for four types of railway track structural systems. Defect analysis of the thermite weld based on several methods

is also discussed. Chapter 6 performs fatigue life predictions of E-clip and fast-clip based on Fatemi-Socie strain-based fatigue criterion. The effect of installment stress and slab track on the fatigue life is also discussed. Chapter 7 summarizes and discusses the findings in the thesis as well as recommendations for future works.

Chapter 2. Literature review

In this chapter, the modeling and fatigue assessment methods of railway track structural system were reviewed. The literature review on the railway track structural systems and the common fatigue failure modes occurring in the thermite weld and rail clips were performed. The modeling methods to understand the failure mechanism of the railway system were documented. A detailed review of multiaxial fatigue models and applying critical plane fatigue criteria to explain the failure mechanism of railway system were also performed. In addition, rainflow cycle counting approach and defect models were also highlighted.

2.1. Railway Track Structural System

Railway comprises of many different sub-systems and components from various specialist suppliers, when assemble along the defined alignment and with considerations of the associated interfaces must function holistically as a system to provide safe support and guidance to the trains in service. For ballast track system, it comprises of rails, rail fastenings, sleepers, ballast, subgrade, and drainage [51]. And slab track system comprises of rails, rail fastenings, concrete sleepers, and concrete slab [52]. The typical ballast and concrete slab track system is shown in Figure 2-1:

Figure 2-1. (A) Ballast track system and (B) Concrete slab track system

2.1.1. Running Rail and Thermite Weld

The steel rail is a vital component of modern railway track and is directly linked to safety of the whole system since it undergoes the largest stress due to the direct contact with the train wheels [21, 53]. Modern rails are usually manufactured at a fixed length, sent to the construction sites, and joined end-to-end by welding process to form the continuous welded rail (CWR) to enable an even surface for the train wheels to roll on. Two rail steel grades R260 and R350HT in accordance with EN13674 are use in Singapore. R260 rails are use in straight tracks and curves of large radii ($\geq 700\text{m}$) in the mainline. R350HT rails are hardened in the manufacturing process for use in locations which are exposed to adverse operating conditions especially in curves of smaller radii ($< 700\text{m}$).

Thermite welding process is the most popular method in the field to join and repair the

rails due to its simplicity, portability, low cost and can be performed onsite after the rails have been laid in position [3, 10]. The process consists in mixing the iron oxide and the aluminium powder in the sand mould and characterized by chemical reaction to fuse the rails to form a joint through molten iron [1] as shown in Figure 2-2. The reliability of the thermite welds is difficult to assess because each one is produced with a separate casting and many variables may influence its properties such as alignment of rail ends, size of the gap between rail ends, rail cross-section variance, cleanliness of rail ends and mold, work quality of the mold assembling, amount and duration of preheating and so on [13, 54].

Figure 2-2. (A) Schematic illustration of thermite welding process [55] and (B) Thermite welding process conducted onsite

2.1.2. Rail Fastening System

A rail fastening system is the assembly of components such as rail clips, rail-pads, insulators and base plates [51, 56] primarily to secure in positions and maintain gauge of the running rails. It provides flexibility and elasticity for the rail steel to transmit heavy dynamic wheel loads of passing trains to the supporting sleeper and insulates the negative electrical current return through the running rails. Rail clips in contact with the foot of the rail can anchor the rail and providing resistance to prevent rail turnover under wheel-induced lateral forces. Both the rail pad and base plate can provide the resilience to absorb impact and vibration from passing trains. There are many different rail fastening systems available from the various specialist suppliers. Figure 2-3 illustrates two types of rail fastening systems.

Figure 2-3. (A) E-clip and (B) fast-clip rail fastening systems

2.1.3. Sleepers

Sleepers are the perpendicular elements of the railway track that can spread wheel loads to ballast or concrete slab and allow good anchors of the rail fastenings which secure running rails [56]. Sleepers are spaced regularly to ensure an even track stiffness and their shape and spacing are designed with the mainline speed and loading. Sleepers were traditionally made from wood but are now mostly made of concrete. Steel sleepers are also used in some countries due to greater resistance to wear and excellent gauge restraint despite increasing steel cost and fear of corrosion [57].

2.1.4. Ballast and Concrete Slab

Most of the modern railway tracks are a traditional ballasted type [51] typically used in elevated and at-grade track sections. Ballast is graded hard crushed rocks from the igneous origin, compacted into uniform, dense and free-draining matrix, to provide vertical and lateral supports to the track and transfers the wheel load evenly to a larger area. The ballast bed is placed under the sleepers and spread to a certain width on either end of the sleepers. Regular maintenance is usually conducted to restore the track alignment due to the slow deterioration of ballast material.

The concept of concrete slab used for the non-ballasted track is being applied especially in tunnels and for high-speed operation due to its virtues of low maintenance, low structure height, high lateral resistance, and no problems with churning of ballast

particles [52]. Concrete sleepers are encased in the concrete slab for the widely used Rheda slab track system [58].

2.2. Investigation on the Thermite Weld and Rail Clip Failures

2.2.1. Thermite Weld Failures

The thermite welds are the weakest parts used for connecting rail steel segments to form a continuous track structure system [14, 59]. The past studies showed that broken rails and welds are the major reasons for train derailments [3] and more than 60 % of rail failures are caused by welded joints failures [4] with the most frequent failures of the thermite welds [60]. The failure modes can be classified by the fractured plane and specific locations such as head, head-to-web, web-to-foot, foot tip and rail base [3] as shown in Figure 2-4.

Recent researches have shown that straight break [9, 20] or transverse failure is one of the most frequent failure modes for the thermite weld. The cracks of transverse failures initiate from the stress concentration points either at the weld centreline or edge of the weld collar on the transverse plane [4, 54] and propagate vertically or with a small angle to the transverse plane. The critical positions of transverse failures are head-to-web, web-to-foot, foot tip regions at the field side and head-to-web regions at the gauge side as shown in Figure 2-5.

Figure 2-4. (A) Definition of fractured plane and (B) Failure locations on the transverse plane

The crack initiation and propagation of transverse failures are highly dependent on the loading condition, service environment, welding process, post-heat treatment and finishing surface. Mutton et al. [4] showed that most transverse failures initiated at the centreline of the welds because of centreline shrinkage cavity defects due to insufficient preheat. Salehi et al. [9] found that the transverse failures initiate at the edge of the weld collar at head-to-web, web-to-foot and foot regions and this type of failure is more likely occur in colder months due to the longitudinal tensile stresses induced in the thermite weld. Tavakoli [3] found that the presence of surface defects at web-to-foot region can be considered as a potential reason for accelerating the transverse failure in the thermite weld.

Figure 2-5. (A) Illustration of transverse plane, (B) Critical failure locations, and (C) (D) (E) Straight break failure of the thermite weld as a result of fatigue cracking initiated at the edge of the weld collar at the foot tip [9], head-to-web and web-to-foot regions [1]

2.2.1.1. Complex Stress State of the Thermite Weld

In service, the 3D dynamic loadings resulting from wheel-rail contact during wheel passage and the bending effect resulting from the loadings can be summarized in Figure 2-6. The vertical force based on the dynamic effect of train weight will cause the in-plane bending effect of the rail under the support of sleepers. The lateral force is induced

by train curving effects including centrifugal forces and hunting effect and will cause the out-of-plane bending effect since the wheel flange is trying to push the railhead. The lateral torsional bending effect is caused by lateral displacement and twisting of the rail because the wheel-rail contact point deviates from the center of the railhead. The longitudinal force due to friction and traction effects will cause the in-plane shear stress along the longitudinal direction.

Figure 2-6. (A) 3D forces acted on the rail in service [53] and (B) Bending effects

The thermite welds are subjected to a complex stress state that comprises train wheel-induced stress, thermal stress and residual stress. The train wheel-induced stress is caused by the combination of in-plane, out-of-plane and lateral torsional bending effects. The thermal stress is due to thermal expansion and contraction from track installation and environmental temperature changes. The residual stress introduced during the rail steel manufacturing process such as head hardening or straightening and

during the thermite welding process also exerts influence on thermite weld failures [54, 61]. These three types of stresses that the rail experience during its service life is illustrated in Figure 2-7.

Figure 2-7. Complex stress state on the rail in service [53]

2.2.1.2. Material Properties of the Thermite Weld

Apart from the complex stress state, the mechanical properties of the thermite weld and the presence of surface defects also have a strong influence on the thermite weld failures. The weak material resistance of the thermite weld, resulting from the change of material characteristics such as hardness, toughness and strength, is easy for fatigue cracks to initiate and propagate [62]. The surface defects at the weld edge could be the stress concentration points that increase the fatigue damage and reduce the service life of the

thermite weld [3, 63, 64].

Hardness indicates the resistance of metal to deformation and investigation on hardness can reveal other mechanical properties such as strength and brittleness. The longitudinal hardness distributions of the thermite weld are shown in Figure 2-8 and the large hardness variation results in material discontinuities and causes fatigue crack initiation.

Figure 2-8. Longitudinal hardness distribution of thermite weld measured by (A) Saita et al. [1] and (B) Jilabi [54]

Fracture toughness is important for metal damage tolerance studies and it represents the ability to absorb energy and plastically deform without fracturing. Most of the fracture toughness tests such as mode I fracture toughness tests and fatigue crack growth tests were conducted on the parent metal [65, 66] and the test of the thermite weld metal was seldom reported. The results showed that the fracture toughness of parent metal increases with the rail grade. A recent study done by Wang et al. [67] as shown in Table 2-1 reveals that the thermite welds are more critical to fracture than the parent rail because of a much lower Charpy impact energy.

Table 2-1. Charpy impact energy of weld and base metal [67]

$T/^{\circ}\text{C}$	U71Mn rail steel		U75V rail steel		U71Mn weld	U75V weld
	U-Notch	V-Notch	U-Notch	V-Notch	U-Notch	U-Notch
20	18.29	7.51	15.84	10.00	2.87	2.55
0	15.35	4.48	11.66	6.53	2.35	2.50
-20	10.78	4.31	9.60	3.92	1.86	2.06
-40	8.09	3.66	5.55	2.94	1.82	1.91
-60	6.27	3.28	5.69	4.25	1.67	1.53

The strength property indicates ability to withstand the load without failure. Researchers have conducted tensile tests for the specimens extracted from thermite weld and parent rail to investigate their strength [67, 68] and the results are listed in Table 2-2 and Table 2-3.

The weld metal exhibits lower ultimate strength and elongation as well as limited ductility or even no measurable ductility in terms of area reduction compared with the parent material indicate its brittle behaviour. The higher yield-to-ultimate ratio also

implies the thermite weld material is more sensitive to fracture.

Table 2-2. Tensile properties of weld and parent steel [68]

Material	Ultimate tensile strength		Elongation ^(a)	Reduction in area
	ksi	MPa		
Type A				
Rail 1 ^(b)	143.0	985.8	13.0	13.6
Rail 2 ^(b)	131.5	906.5	12.0	13.3
Weld 1	116.3	801.8	2.5	N.D. ^(c)
Weld 2	106.4	733.5	2.5	N.D.
Weld 3	115.0	792.8	1.0	N.D.
Weld 4	115.5	796.2	1.0	N.D.
Weld 5	112.3	774.2	1.0	N.D.
Weld 6	112.7	776.9	1.0	N.D.
Weld 7	115.9	799.0	1.0	N.D.
Weld 8	110.4	761.1	1.0	N.D.
Weld 9	108.3	746.6	1.0	N.D.
Type B				
Weld 1x	115.3	792.8	2.7	1.8
Weld 2x	121.0	834.1	2.3	3.5
Weld 3x	121.8	839.6	2.2	2.6
Weld 4x ^(d)	123.0	847.9	3.5	2.9
Weld 5x	128.7	887.2	5.6	2.9

^(a) Based on 5. In. (127 mm) gage length

^(b) Yield strengths are 91.6 ksi (631.5 MPa) and 84.0 ksi (579.4 MPa) for rails 1 and 2, respectively.

^(c) N.D. – Not detected

^(d) Did not break in center

Table 2-3. Tensile properties of weld and base metal [67]

T / °C	U71Mn base metal					U75V base metal				
	R _{p0.2} / MPa	R _m / MPa	R _{p0.2} /R _m	δ / %	φ / %	R _{p0.2} / MPa	R _m / MPa	R _{p0.2} /R _m	δ / %	φ / %
20	500.0	932.2	0.54	13.58	19.98	601.9	1071.9	0.56	9.99	10.05
0	521.5	949.2	0.55	14.52	18.39	620.7	1086.4	0.57	7.98	7.21
-20	531.6	968.4	0.55	12.59	14.62	658.9	1101.0	0.60	7.11	7.12
-40	540.4	979.6	0.55	11.25	10.91	664.9	1057.8	0.63	4.27	4.06
-60	559.9	995.0	0.56	11.25	11.01	676.6	992.0	0.68	2.95	2.98
T / °C	U71Mn thermite weld metal					U75V thermite weld metal				
	R _{p0.2} / MPa	R _m / MPa	R _{p0.2} /R _m	δ / %	φ / %	R _{p0.2} / MPa	R _m / MPa	R _{p0.2} /R _m	δ / %	φ / %
20	675.7	996.7	0.68	3.09	4.22	695.0	945.6	0.81	1.08	2.27
0	685.2	950.5	0.72	2.03	4.17	708.4	929.6	0.76	0.95	2.66
-20	693.7	894.2	0.78	1.70	3.08	724.7	890.6	0.81	0.79	2.65
-40	701.5	860.6	0.82	1.69	2.46	739.2	849.8	0.87	0.79	2.39
-60	709.3	832.6	0.85	0.73	2.43	745.9	832.7	0.90	0.75	2.33

In addition, a four-point bend fatigue test is usually applied to evaluate the fatigue performance of the full-scale thermite weld structures and some results are shown in Figure 2-9.

(A)

(B)

Figure 2-9. Four-point bend fatigue test results [20, 69]

Ekberg et al. [69] conducted tests for rail welds with different welding technologies and hot rolled rail and the worst fatigue performance was observed for the thermite weld.

He also found that post weld treatment can improve the fatigue properties of the thermite weld. Lawrence et al. [20] found that the failures initiating from the stress concentration point at web-to-foot and rail foot regions because of insufficient heat input and oxides contamination. By optimizing the mold design and welding conditions, he concluded that the service cycles of the thermite weld will improve.

2.2.1.3. Thermite Weld Defects

Apart from the weak material resistance, various defects such as cold-laps, shrinkage cavity, inclusions and micro-pores [41, 70] can form in the thermite weld due to the casting nature of the process [3]. Therefore, the fatigue properties of the thermite weld are even worse. Several studies have been carried out to investigate the effect of defects and failure observations show that most of the thermite weld failures originated from fatigue cracks nucleated at the weld defects on the critical positions as shown in Figure 2-5 [3, 20]. Fry et al. [41] investigated the effect of several types of rail head defects on the fatigue crack initiation life under different railway operating conditions and concluded that the shape and orientation of pores have a strong influence on fatigue defects formation in thermite weld. Romano et al. [71] built a model of thermite weld with semi-elliptical surface crack at the rail base and concluded that the rail temperature is a main factor of failure. Mouallif et al. investigated the effect of inclusion defects [63] and internal defects [64] at rail base through Finite Element Analysis and concluded that the local embrittlement phenomenon of thermite weld may facilitate crack

propagation at stress concentration points of the defects. Tavakoli [3] observed that the fatigue cracks nucleated from defects at web-to-foot regions as shown in Figure 2-10. And he implemented half-penny shaped defects at web-to-foot and rail base regions and found that fatigue crack initiation life at pre-existing defects dramatically decreases and the possibility of fatigue crack initiation at web-to-foot is less than rail base.

Figure 2-10. Defects at web-to-foot regions [3]

2.2.2. Rail Clip Failures

The rail clip is an important structural component of railway track systems providing flexibility and turnover resistance for running rails. Two types of clips, E-clip and fast-clip [18, 19] are widely used in mass rapid transit systems due to ease of installation, removal, and ability to maintain rail position safely while other types of clips are usually used for heavy haul freight railroad tracks [17]. The technical terminologies and sketches of these two types of rail clips are shown in Figure 2-11. A high replacement frequency of the rail clip was observed compared with other components and the typical failure locations are shown in Figure 2-12. However, rail clip failures are seldom

reported and fewer studies have been performed to investigate the failure mechanism compared with the thermite weld. Deshimaru et al. [38] conducted a full-scale laboratory test on a track segment with a length of 8 sleeper spans and proposed a linear relationship for stress on rail clips based on wheel load and lateral force according to experiment results. Hasap et al. [35, 37] conducted fatigue tests for the E-clip under a combination of static toe load and repeated wheel load and found the fractured location is at the rear arch. They also concluded that the reduction of toe load may enhance the fatigue resistance of the E-clip. Tamagawa et al. [18] conducted tensile and hardness tests for the fast-clip steel and obtained a fatigue limit diagram with different fatigue strength through fast-clip repeat loading experiments.

Figure 2-11. Technical terminologies and sketches for (A) E-clip and (B) fast-clip

Figure 2-12. Typical failure modes: (A) E-clip at the rear arch, (B) E-clip at the front arch, and (C) fast-clip at the rear arch near the shoulder

2.3. Modeling Methods for Railway System

In addition to experimental investigation on the thermite weld and rail clips failures, several modeling methods have been developed to interpret the relationship between train wheel-induced loads and track deformation behavior. Since the early of 20th century, analytical models are applied by researchers to simplify the problems and obtain the stress and deformation state of the rail based on homogeneous situations [72]. For an idealized track-sleeper system, a beam theory analysis is usually applied that the rail is always modeled as either as a Euler–Bernoulli beam or Rayleigh-Timoshenko beam rests on the elastic foundations [73, 74]. However, the interaction between wheels and tracks is much more complex than theoretical assumptions. Multibody dynamic models are therefore used by researchers to analyze wheel-rail interaction [26, 27]. This method can simulate the dynamic effect of the vehicle and the track components considering the structural flexibility. However, the stress distribution in the track

especially at the thermite weld and rail clips cannot be solved using this method. Finite element models are then used to numerically investigate the mechanical behavior and stress condition causing the fracture and fatigue failures of the thermite weld [63, 75] and rail clips [35, 76].

2.3.1. Analytical Models

Figure 2-13. Rail bending stress as a function of the wheel position under lateral and eccentric vertical loadings [53]

The analytical models are preferably based on continuously supported track assumptions although some analytical models can handle discrete support properties [72]. The simplest model to consider the bending force on the rail is Winkler beam model which assumes that the rail is sitting on a continuous elastic foundation [73]. Orringer et al. [77] developed a model to predict rail bending stress as a function of the

wheel position under lateral and eccentric vertical loadings as shown in Figure 2-13.

It shows that although the bending in the lateral direction is possible, the dominant bending stress is along the longitudinal direction which makes the rail susceptible to mode I fatigue cracking. A more complex discrete spring-damper supporting model [25] can be used to take into account variations in track component properties and track geometric errors as shown in Figure 2-14. It also shows that the dynamic response of the track at rail joints increases as the train speed increasing.

Figure 2-14. 2D vertical train-track interaction discrete supporting model [25]

When dealing with wheel-rail contact problems, the contact pressure and the mechanical behaviors such as wear and rolling contact fatigue at the contact areas on

the rail can be estimated based on Hertz contact [78] or Non-Hertz contact theories [79].

2.3.2. Numerical Models

2.3.2.1. *Multibody System Dynamic Analysis*

Figure 2-15. Illustration of multibody dynamics simulation [81]

Multibody system dynamic models are usually used to investigate wear and fatigue damage of rails and wheels when the contact area and contact stress are required precise evaluation of the contact between the wheel and rail [80, 81]. A rigid model of a railway track is usually used to simplify the computational process but it does not always reflect actual operation conditions in wheel/track interaction [82]. Therefore, in some cases of simulation computations, track flexibility which is mainly reflected in the various stiffness and damping parameters of the track is considered to investigate the dynamic response of vehicle-track mechanical system [83]. An example of vehicle-track multibody dynamic models considering all the components in the railway systems is from Bai's work [81] as shown in Figure 2-15.

2.3.2.2. Finite Element Analysis

To investigate the stress distribution and fatigue damage on the thermite weld and rail clips, finite element analysis is usually performed because the complex geometry features cannot be embedded in analytical models and multibody dynamic models. The structural response (stress/strain) of a welded rail or rail clip under wheel loads is usually solved through finite element models with a finite length of track [3, 76]. Some finite element models established for the study of rail welds are shown in Figure 2-16.

Figure 2-16. Finite element models for the rail welds

Salehi et al. [9] developed an elastic thermo-structural finite element model with a track length of 3 meters including rail, thermite weld, 6 sleepers and elastic foundation

underneath the sleepers. A uniform contact pressure was applied on the rail through a moving load technology to simulate the wheel-induced loadings. The model was validated by strain gauge measurement results on a thermite weld specimen undergoing a three-point bend test. A higher stress and fatigue damage were observed at the under-head radius and foot tip of rail field side compared with other regions. He also found that the fatigue damage at the under-head radius increases as the lateral force increase and the ambient temperature decrease.

Tavakoli [3] developed a 3D finite element model that comprises seven main parts: axle, wheel, rail with extending beam elements, thermite weld, base plates, wood sleepers, and vertical spring elements representing the ballast and subgrade material. An elastic material model was used in his study since the research objective is to investigate the stress field at web-to-foot and foot regions at the field side which are far from the contact region. He found that the stress at the rail foot tip increases with a weld collar and a soft foundation and the stress is even higher at the web-to-foot region and rail bottom when a planar defect exists.

Lu et al. [11] developed a dynamic finite element model to investigate axle box acceleration response in the vertical direction. A static calculation is performed first under the gravity loading to obtain a steady initial state used as an input for the dynamic calculation. The model is validated by axle box acceleration response in the frequency domain. He found that fatigue damage is more significant with shorter and deeper

welded joints taken the influence of the cyclic loading condition into consideration.

Yu et al. [84] developed a dynamic finite element model to investigate the effect of weld irregularities on the rail head under the high running speed. The thermite weld was constructed in the model and was about 0.22mm higher than the rail surface. He found that the maximum value of wheel-rail vertical force at the thermite weld increases with the increase of train speed and the unevenness of the rail surface when the speed is over 250km/h.

In addition to the rail weld models, some finite element models of rail clips have also been developed by researchers to investigate their stress and fatigue behaviors as shown in Figure 2-17.

Ferreño et al. [85] built a finite element model including the clip, rail foot, base plate and the bolt to investigate the structural behavior of the SKL-1 fastening system and validated it through a series of fatigue tests. The conclusion is that the fatigue failure of the clip under normal conditions is very unlikely to occur based on the experimental and FE results.

Chen et al. [86] developed a 3D finite element model including the rail, fastening system and concrete sleepers. The pressure loadings were applied on the toes of the clips to simulate the clip lifting process and a vertical load on the top of the rail head and a lateral load pointing from the gauge side to the field side were applied to simulate

the wheel loading. They found a tensile stress concentration presented between the two inserts of the shoulder in the model which may lead to shoulder pull-out failure of the fastening system. And yielding was also observed in the clip when the ratio of lateral and vertical forces is 0.5.

Hasap et al. [37] investigate the toe load effect on the deformation and fatigue resistance of the E-clip through a linear elastic finite element model. The clips were simplified by linear springs with the stiffness of 1.1 kN/mm and static analysis was performed. Higher deformation ranges were observed on the outer clips with the contribution of the lateral load. They also found that a higher level of the toe load had no influence on the fatigue life of the clip (run-out after 5-million cycles) under normal wheel load but reduced the fatigue resistance of the clip under impact wheel load.

Hong et al. [40] developed a static finite element model for the E-clip and predicted the fatigue life with a combined solution of ABAQUS and FE-SAFE. The critical positions were the rear arch and the contact between the front arch and heel obtained from the model. They concluded that the fatigue behavior of the E-clip is highly dependent on the insertion depth and the clip deflection.

(A) Ref [85]

(B) Ref[86]

(C) Ref [37]

(D) Ref [40]

Figure 2-17. Finite element models for the rail clips

However, the approximation of wheel-induced loading in the rail clip models and static FE models of rail welds may underestimate the stress induced by the inertia effects during dynamic contact between wheel and rail track. In addition, rail clips were seldom considered in the finite element model when investigating the failure mechanism of the thermite weld and field measurement results were rarely used to calibrate the model. Therefore, a finite element model including rail clips is necessary to investigate the fatigue failures of the thermite welds and the model should be calibrated by a real-time

measurement conducted onsite. Also, the dynamic contact between wheel and rail track should be implemented in the FE model when studying the rail clip failures.

2.4. Fatigue Assessment Approach

Fatigue is a progressive and localized structural damage that is a common source of failure in railway structural systems because of the cyclic nature of wheel-induced loadings. In general, the total fatigue life N_f of a component can be divided into two phases from the fatigue damage accumulation point of view [87]:

$$N_f = N_i + N_p \quad (2-1)$$

Where N_i is the number of cycles required for crack nucleation and early crack growth, and N_p is the number of cycles for an initiated crack to propagate to the final fracture or a critical size. For high strength pearlitic rails, the crack nucleation and early growth stage accounts for the majority of total fatigue life and the crack propagation life usually analyzed by fracture mechanics is assumed to be negligible [3, 88]. Crack initiation analysis and life predictions of railway components based on multiaxial fatigue models have been carried out by many researchers from recent studies [3, 10, 46].

2.4.1. Fatigue Damage Mechanism

Fatigue damage that accumulated in materials is caused by fatigue crack initiation and growth by the cyclic loadings. A schematic illustration for a crack nucleation on the surface of a ductile metal is shown in Figure 2-18. In this example, nucleation of the slip band was caused by the cyclic shear stress or strain. The slip occurs due to the movement of dislocations caused by the cyclic shear stress and the slip bands are formed due to atoms slip on specific crystallographic planes (slip planes) [89]. The slip band will form first on the crystallographic planes of the most favorably oriented grain subjected to the applied shear stress. Only a few slip bands will form on a few favorably oriented grains when the applied stress is low while a large number of slip bands will form at a higher stress and these slip bands will form a single dominant crack under the cyclic loadings [90, 91].

Figure 2-18. Formation of slip bands [91]

In a pure metal single crystal, the slip bands are uniform on the metal surface and slip begins when shear stress on the slip plane reaches critical resolved shear stress.

Polycrystalline metals have a complex nucleation process influenced by grain boundaries, precipitates and impurities [90]. The grain boundaries can stop dislocation movement and finer grains are usually preferred due to the superior mechanical properties. The nucleation process can be described by the intrusion-extrusion model shown in Figure 2-18. Some slip bands that come out of the metal surface are called extrusions and those go into the surface are called intrusions.

A crack formed on a component surface will first initiate in the most favorable surface grain and then grows into the adjacent one as shown in Figure 2-19.

Figure 2-19. Stage I and Stage II crack growth processes [91]

The overall growth path of the crack is still along the planes of maximum shear stress even though the grain orientations of the material are random. Once the initiated crack grows large enough, it will turn to grow along the plane perpendicular to the applied tensile stress.

Carpinteri and Spagnoli [92] proposed a formula to estimate the angle δ between the normal direction of the crack initiation plane in stage one and weighted mean direction of the maximum principal stress for hard metals:

$$\delta = 45 \frac{3}{2} \left[1 - \left(\frac{t_{-1}}{f_{-1}} \right)^2 \right] \quad (2-2)$$

Where f_{-1} and t_{-1} are the reversed bending and torsional fatigue limits obtained from the fatigue tests. The angle δ is dependent on the metal material and is equal to 45° when the endurance limit ratio is equal to the brittle-ductile threshold value which is also the empirical value for the pearlite steel ($t_{-1}/f_{-1}=1/\sqrt{3}$) [93].

For stage II crack growth, fracture mechanics is frequently used to evaluate the effect of the crack geometry, operating conditions, and material properties on the fatigue life. The fracture mechanics approach is usually applied to predict the fatigue life of welded structures with the known presence of a crack [71, 94]. Figure 2-20 illustrates three crack modes. Mode I cracks are caused by the tensile loads and both Mode II and Mode III cracks can be produced under the torsional loads. Mode II cracks appear on the surface and grow along the surface under the in-plane shear loadings and Mode III cracks propagate into the surface under the out-of-plane shear loadings.

Figure 2-20. Three modes of fracture [91]

Brown and Miller [95] proposed two growth ways for an initiated shear crack as shown in Figure 2-21. Case A cracks grow along the free surface along the crack length direction and Case B cracks grow into the surface along the crack depth direction.

Figure 2-21. Two ways for crack growth [91]

2.4.2. Multiaxial Fatigue Models

Fatigue failures are common for an engineering structure subjected to a multiaxial stress field that generally results from the complex external loadings and irregular geometries. Multiaxial fatigue analysis is complicated compared with simple load cases and various models have been developed to quantify the fatigue damage because the problem is less satisfactorily solved by uniaxial methods. Several reviews and comparison of existing multiaxial fatigue models can be found from the literature [96-99]. The most fundamental part of any multiaxial fatigue model is to calculate the fatigue damage and convert it to a uniaxial state. The parameters of multiaxial fatigue models are functions used to isolate the stress and/or strain components that contribute most to the fatigue damage. Usually, the parameters can be obtained through uniaxial fatigue experiments.

Although numerous multiaxial fatigue models have been proposed in the past studies to calculate the fatigue damage and predict the fatigue life under multiaxial loading conditions, there is no universally accepted multiaxial fatigue model yet [90]. In general, the multiaxial fatigue models can be classified into three categories: stress-based, strain-based and energy-based models [100, 101]. The stress-based models are more suitable for high cycle fatigue regimes where the plastic deformation is not significant. The strain-based models, on the other hand, take into account plastic strains and are applicable for both high cycle fatigue and low cycle fatigue regimes. The energy-based models based on strain energy density generally can be expressed as the summation of

products of stress and strain tensor components.

Another way to classify multiaxial fatigue models is based on the format of the fatigue index: a scalar or a critical plane format [3, 43]. The calculated fatigue damage of the scalar format models is a scalar value averaged over all the material planes. The energy-based and equivalent stress models are two examples of the scalar format models. On the other hand, the critical plane approach is proposed to find the plane on which fatigue may germinate directly or indirectly and is useful for a subsequent crack propagation analysis.

In recent decades, critical plane format multiaxial fatigue models have been applied by many researchers to investigate the fatigue failures and predict the fatigue life of railway components subjected to cyclic wheel loadings [3, 10, 46]. In this chapter, some of the stress-based and strain-based multiaxial fatigue models with the critical plane format are reviewed to provide a better understanding of the multiaxial fatigue problems that occurred in the thermite weld and rail clips.

2.4.2.1. Critical Plane Approach Overview

Several multiaxial fatigue models have been proposed in recent decades, but there is no general agreement on the optimal model. However, the critical plane concept is considered as one of the most effective approaches in predicting the fatigue life of a structure subjected to multiaxial stress states due to their reasonable accuracy and

ability to predict the potential crack plane orientation [90, 102]. The critical plane format models postulate that only shear and normal stresses and strains on the critical plane and perpendicular to the critical plane contribute the crack initiation process. The plane corresponding to the largest fatigue damage is considered as the critical plane. According to the physical observation of Forsyth [103], the crack initiates and propagates first along a shear plane (critical plane) and then propagates in a plane perpendicular to the maximum tensile principal stress direction.

The orientation (φ , θ) of a plane passing through a material point is usually expressed in spherical coordinates as shown in Figure 2-22.

Figure 2-22. (A) Spherical coordinate system [104], (B) The critical plane Δ expressed in spherical coordinates [105]

Δ represents a material plane and n is the normal direction. θ is the angle between

vector n and Z -axis and φ is the angle between the projection of vector n on OXY plane and X -axis. The coordinates of n , l , r can be expressed in the $OXYZ$ frame:

$$n = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{bmatrix} \quad (2-3)$$

$$l = \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} = \begin{bmatrix} -\sin \varphi \\ \cos \varphi \\ 0 \end{bmatrix} \quad (2-4)$$

$$r = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} -\cos \theta \cos \varphi \\ -\cos \theta \sin \varphi \\ \sin \theta \end{bmatrix} \quad (2-5)$$

Then the traction vector T_n on the oblique material plane Δ is:

$$T_n = \vec{n} \cdot \vec{\sigma} = \begin{Bmatrix} \sigma_{xx}n_x + \sigma_{yx}n_y + \sigma_{zx}n_z \\ \sigma_{xy}n_x + \sigma_{yy}n_y + \sigma_{zy}n_z \\ \sigma_{xz}n_x + \sigma_{yz}n_y + \sigma_{zz}n_z \end{Bmatrix} \quad (2-6)$$

A shear stress τ acting on the plane Δ along any line ξ is usually called resolved shear stress. If a unit vector $\vec{m}$ is used to represent ξ , then τ is expressed as:

$$\tau = \vec{n} \cdot \vec{\sigma} \cdot \vec{m} \quad (2-7)$$

Where, χ is the angle between vector $\vec{m}$ and l -axis, and the coordinate of $\vec{m}$ in the $OXYZ$ frame is:

$$m = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} -\sin \varphi \cos \chi - \cos \theta \cos \varphi \sin \chi \\ \cos \varphi \cos \chi - \cos \theta \sin \varphi \sin \chi \\ \sin \theta \sin \chi \end{bmatrix} \quad (2-8)$$

It is clear that the resolved shear stress is a function of all the angles φ , θ , χ and time t for a loading history [105]. The resolved shear stress amplitude τ_a can be solved on

the plane Δ :

$$\tau_a(\varphi, \theta, \chi) = \frac{1}{2} \left[\max_t \tau_a(\varphi, \theta, \chi, t) - \min_t \tau_a(\varphi, \theta, \chi, t) \right] \quad (2-9)$$

Figure 2-23. (A) Illustration of the shear stress curve on the plane Δ , and (B) (C) (D) (E) (F) Five stress amplitude calculation methods

Several methods have been developed for calculating the shear stress amplitude: Longest Chord Method, Longest Projection Method, Minimum Circumscribed Circle, Minimum Circumscribed Ellipse and Maximum Rectangular Hull as shown in Figure 2-23. The Longest Chord Method is the simplest one [106] but the mean shear stress is difficult to be defined unequivocally when two or more reference chords having the

same length [107]. Grubisic and Simbürger [108, 109] proposed another simple method called the Longest Projection Method. But an obvious drawback presented in this method is that the mean shear stress could be equal to zero even with the presence of a shear stress curve with non-zero mean stresses [107]. Dang Van and Papadopoulos [47, 109] proposed the Minimum Circumscribed Circle method to overcome the problems encountered when determining the mean stress from the above-mentioned two methods. The radius of this minimum circle circumscribed all the points of a shear stress curve is τ_a . Although the phase difference in out-of-phase bending and torsion does not affect the fatigue strength for a high fatigue life component according to the conclusion drawn by Papadopoulos [110], the out-of-phase loading effect has drawn the attention of some researchers and the Minimum Circumscribed Ellipse method [111-115] was proposed to provide a more conservative prediction especially for low cycle fatigue problems. τ_a can be calculated as follows:

$$\tau_a = \sqrt{R_{max}^2 + R_{min}^2} \quad (2-10)$$

However, the computation complexity of the Minimum Circumscribed Ellipse method for searching critical plane is higher than that required by the Minimum Circumscribed Circle method. Therefore, the Maximum Rectangular Hull method was proposed by Araújo [48-50, 114] to provide a fast alternative measure of the shear stress amplitude. τ_a then can be calculated as:

$$\tau_a = \frac{1}{2} \max \sqrt{a^2 + b^2} \quad (2-11)$$

2.4.2.2. Stress-Based Criterion

For some stress-based criteria stated below, the critical plane orientation can be directly obtained when calculating the fatigue damage.

Classical and modified Dang Van Criteria

Dang Van [47] proposed his classical criterion based on the local fatigue phenomenon that fatigue behavior is governed by mesoscopic stress state. The fatigue will happen when the most unfavorably oriented grains subject to a loading beyond elastic shakedown limit even though the macroscopic behavior remains elastic. Dang Van believed that the resolved shear stress as well as the hydrostatic stress on the critical plane govern the fatigue process, and postulated in 1973 that fatigue crack may initiate on the critical plane of a particular point of material when the equivalent stress for any plane and any moment is no longer satisfied the following equation:

$$\forall t \forall n \tau_a + a_{DV} \sigma_H \leq b_{DV} \quad (2-12)$$

Where σ_H is the time dependent hydrostatic stress. Eq (2-12) can also be expressed as:

$$\max_{\vec{n}} \max_t [\tau_a + a_{DV} \sigma_H] > b_{DV} \quad (2-13)$$

a_{DV} and b_{DV} are the parameters obtained from reversed bending (f_{-1}) and torsional fatigue (t_{-1}) limit tests:

$$a_{DV} = 3 \left(\frac{t_{-1}}{f_{-1}} - \frac{1}{2} \right), b_{DV} = t_{-1} \quad (2-14)$$

A $\tau_a - \sigma_H$ diagram can be used to represent the Dang Van criterion as shown in Figure 2-24. The stress path is defined by time dependent values of $\tau_a(t)$ and $\sigma_H(t)$ and the fatigue limit is introduced by a line with a negative slope. The fatigue crack will initiate if a part of the stress path is above the criterion line.

Figure 2-24. Graphic representation of Dang Van criterion

Dang Van also proposed a simplified model by slightly modifying the original criterion [47]. At instant t , the maximum shear stress $\tau(t)$ is given by:

$$\tau(t) = \frac{\sigma_I(t) - \sigma_{III}(t)}{2} \quad (2-15)$$

Where, $\sigma_I(t)$ and $\sigma_{III}(t)$ are the largest and smallest eigenvalue of $\sigma(t)$. Then the modified Dang Van criterion can be expressed as:

$$\max_t [\tau(t) + a_{DV}\sigma_H] > b_{DV} \quad (2-16)$$

Findley and Matake Criterion

Findley proposed that the tensile normal stress acting perpendicularly to the maximum shear plane can affect the fatigue crack initiation process since it leads to the separation of crack surfaces and accelerates the movement of dislocations [116]. Matake [117] modified the critical plane searching method proposed by Findley who incorrectly considered the mean torsion effect in his criterion. The critical plane according to Matake is the plane on which the resolved shear stress amplitude peaks:

$$(\varphi, \theta): \max_{(\varphi, \theta)} \tau_a \quad (2-17)$$

The fatigue crack may initiate when the following inequality satisfies:

$$\max_t [\tau_a + k_M N_{max}] > \lambda_M \quad (2-18)$$

$$k_M = 2 \left(\frac{t_{-1}}{f_{-1}} \right) - 1, \lambda_M = t_{-1} \quad (2-19)$$

Where, N_{max} is the maximum normal stress acting on the critical plane. k_M and λ_M are the Matake parameter obtained from reversed bending (f_{-1}) and torsional fatigue (t_{-1}) limit tests, respectively.

McDiarmid criterion

The method proposed by McDiarmid for searching critical plane is same as Matake criterion and the ultimate tensile strength σ_u of the material was considered in his model [118].

$$T_a + \frac{t_{-1}}{2\sigma_u} N_{max} > t_{-1} \quad (2-20)$$

For the indirectly critical plane approach, the critical plane is searched by considering the fatigue fracture plane and the angle between these two planes. The fatigue fracture plane is defined according to normal or principal stress by different researchers.

Carpinteri and Spagnoli criterion

Carpinteri and Spagnoli [92] proposed that the critical plane orientation is correlated with maximum weighted mean principal stress direction. The mean principal stress directions are determined by averaging the instantaneous values of the principal Euler angles (ϕ , θ , Ψ) as shown in Figure 2-25.

$$\hat{\phi} = \frac{1}{W} \sum_1^N \phi(t_i) W(t_i), \hat{\theta} = \frac{1}{W} \sum_1^N \theta(t_i) W(t_i), \hat{\Psi} = \frac{1}{W} \sum_1^N \Psi(t_i) W(t_i) \quad (2-21)$$

$$W = \sum_1^N W(t_i), W(t_i) = \begin{cases} 0 & , \sigma_1(t_i) < cf_{-1} \\ \left(\frac{\sigma_1(t_i)}{f_{-1}} \right)^{-\frac{1}{m}}, & \sigma_1(t_i) \geq cf_{-1} \end{cases} \quad (2-22)$$

m is the negative slope of the S-N curve and c is safety factor with $0 < c \leq 1$.

Figure 2-25. Principal stress directions described by the Euler angles ϕ , θ , Ψ [92]

The normal vector $\vec{n}$ of the critical plane can be determined by the angle (in degree) between $\vec{n}$ and $\vec{1}$ calculated from Eq (2-2). The fatigue damage is then determined by a combination of maximum normal stress N_{max} and shear stress amplitude τ_a acting on the critical plane:

$$\sqrt{N_{max}^2 + \left(\frac{f_{-1}}{t_{-1}}\right)^2 \tau_a^2} > f_{-1} \quad (2-23)$$

Liu and Mahadevan criterion

Liu and Mahadevan [119] proposed that the fatigue fracture plane experiences the maximum normal stress amplitude. The critical plane is not an actual crack plane and the angle α between the critical plane and the fracture plane which can be obtained from fully reversed bending and torsion experiments.

$$\cos(2\alpha) = \frac{-2 + \sqrt{4 - 4\left(\frac{1}{s^2} - 3\right)(5 - \frac{1}{s^2} - 4s^2)}}{2\left(5 - \frac{1}{s^2} - 4s^2\right)}, s = \frac{t_{-1}}{f_{-1}} \quad (2-24)$$

Then the fatigue damage can be computed with the normal stress amplitude and the shear stress amplitude acting on the critical plane and assessed by a material parameter γ .

$$\sqrt{\left(\frac{\sigma_a}{f_{-1}}\right)^2 + \left(\frac{\tau_a}{t_{-1}}\right)^2} > \gamma, \gamma = [\cos^2(2\alpha) s^2 + \sin^2(2\alpha)]^{1/2} \quad (2-25)$$

2.4.2.3. Strain-Based Criterion

Fatemi-Socie criterion

The Fatemi-Socie criterion shows that the fatigue crack initiates at the plane where the shear strain amplitude maximizes. The physical basis of the Fatemi-Socie method is that the fatigue life of the material will be reduced when tensile stresses normal to the plane of the crack are present because they reduce the friction forces acting on the irregular shapes of crack surfaces [45]. The initiation process is dominated by the maximum shear strain amplitude on the critical plane. The additional cyclic hardening can result in a short life due to the out-of-phase loading effect, which is taken into account by yield strength normalized maximum normal stress perpendicular to the critical plane during the loading cycle.

The equivalent shear strain amplitude includes maximum shear strain amplitude and normalized maximum normal stress on the critical plane is calculated as:

$$\gamma_{eq} = \frac{\Delta\gamma}{2} \left(1 + k \frac{\sigma_{n,max}}{\sigma_y} \right) = f_{FS}(N_f) \quad (2-26)$$

where σ_y is the material yield stress; $\Delta\gamma$ is the shear strain range on the critical plane and $\sigma_{n,max}$ is the maximum normal stress perpendicular to the critical plane; $f_{FS}(N_f)$ is related to number of cycles to failure N_f . $f_{FS}(N_f)$ can be obtained based on Coffin-Manson uniaxial theory:

$$f_{FS}(N_f) = \frac{\tau_f'}{G} (2N_f)^{b'} + \gamma' (2N_f)^{c'} \quad (2-27)$$

where $G = E/2(1 + \nu)$ is the shear modulus; ν is the Poisson's ratio; k is the normal stress sensitivity coefficient which depends on material properties, τ_f' and b'

are shear fatigue strength coefficient and exponent; γ' and c' are shear fatigue ductility coefficient and exponent. Their values can be calculated using von Mises, Tresca and maximum principal strain criterion [120-122]. For von Mises criteria:

$$\tau'_f \approx \frac{\sigma'_f}{\sqrt{3}}, \gamma' \approx \sqrt{3}\varepsilon'_f, b' \approx b \text{ and } c' \approx c \quad (2-28)$$

For Tresca criteria:

$$\tau'_f \approx \frac{\sigma'_f}{2}, \gamma' \approx 1.5\varepsilon'_f, b' \approx b \text{ and } c' \approx c \quad (2-29)$$

And for maximum principal strain criteria:

$$\tau'_f \approx \frac{\sigma'_f}{1+\nu}, \gamma' \approx 2\varepsilon'_f, b' \approx b \text{ and } c' \approx c \quad (2-30)$$

where, σ'_f , ε'_f , b and c are fatigue strength coefficient, fatigue ductility coefficient, fatigue strength exponent and fatigue ductility exponent, respectively, which can be estimated from material tensile and hardness tests.

To obtain the normal stress sensitivity coefficient k , the shear strain amplitude $\frac{\Delta\gamma}{2}$ and the maximum normal stress $\sigma_{n,max}$ on the critical plane from Eq (2-26) under fully-reversed uniaxial case can be expressed as below [123, 124]:

$$\frac{\Delta\gamma}{2} = (1 + \nu) \frac{\sigma'_f}{E} (2N_f)^b + (1 + \nu_p) \varepsilon'_f (2N_f)^c \quad (2-31)$$

$$\sigma_{n,max} = \frac{\sigma'_f}{2} (2N_f)^b \quad (2-32)$$

Then, k can be estimated from Eq (2-26), (2-27), (2-31) and (2-32) according to the fatigue life:

$$k = \left[\frac{\frac{\tau'_f(2N_f)^{b'} + \gamma'(2N_f)^{c'}}{G}}{(1+\nu)\frac{\sigma'_f(2N_f)^b}{E} + (1+\nu_p)\varepsilon'_f(2N_f)^c} - 1 \right] \frac{2\sigma_y}{\sigma'_f(2N_f)^b} \quad (2-33)$$

where $\nu_p = 0.5$ is the plastic Poisson's ratio.

2.4.3. Multiaxial Fatigue Analysis for the thermite weld

Salehi et al. [9, 10] implemented the Dan Van criterion and the Minimum Circumscribed Circle method to investigate the fatigue damage of the thermite weld at the field side. He found that the fatigue damage maximizes at the under-head radius as shown in Figure 2-26(A). The effects of the contact pressure location, lateral force, longitudinal force, ambient temperature, and weld geometry were also investigated as shown in Figure 2-26(B)-(F). The fatigue damage at the under-head radius increases dramatically when the contact press is eccentric on the rail head, the lateral force increases, the ambient temperature is low, and the weld has a large flank angle. And the fatigue damage is less affected by the longitudinal force acting on the rail.

Figure 2-26. Multiaxial fatigue analysis of the thermite weld based on the Dang Van criterion [10]

Tavakoli [3] implemented the Findley fatigue criterion and predicted the fatigue life of the thermite weld at web-to-foot and rail foot regions. A larger fatigue damage and shorter fatigue life were obtained at the rail base corner as shown in Table 2-4. The effect of cold weathers was also incorporated into the fatigue analysis and the fatigue damage is amplified by a factor of 3 when the ambient temperature falls 100 °C below the rail neutral temperature.

Table 2-4. Fatigue damage calculated at possible fatigue crack nucleation sites [3]

	Location	f_{FIN} (MPa)	θ°	ϕ°	n_1	n_2	n_3
1	Base Corner	70	145	345	0.55	-0.15	-0.82
2	Base center	44	30	15	0.48	0.13	0.87
3	Web-base fillet	40	170	60	0.09	0.15	-0.98

2.5. Damage Tolerance Investigation

2.5.1. Rainflow Cycle Counting Approach

Apart from multiaxial fatigue analysis, rainflow cycle counting approach is usually used to convert the strains measured from complex structures to cumulative damage for life prediction. It is widely used for complex loading conditions as a damage prediction indicator [125, 126]. The sketch for rainflow cycle counting method is shown in Figure 2-27. A-B-D, D-E-G-I and I-J are half cycles and B-C-B', E-F-E' and G-H-G' are full cycles. The cycles, mean stress and stress amplitude are used for accumulated damage calculation.

Figure 2-27. Sketch for rainflow counting method.

A generalized damage accumulation law can be applied to calculate the fatigue damage for each full cycle and half cycle [127]:

$$D = \sum D_i = \sum \alpha \left(\frac{n_i}{N_{fi}} \right)^{m(\sigma)} \quad (2-34)$$

Where, $m(\sigma)$ is the exponent and equals to one when Miner's linear damage

accumulation rule is considered, α is a constant value to adjust the relationship between accumulated damage and each cycle or half cycle. $n_i = 0.5$ for half cycle and $n_i = 1$ for full cycle. N_{fi} is the predicted number of loading cycles a material experiences fatigue failure based on a uniaxial S-N curve.

2.5.2. Defect Analysis

2.5.2.1. Notch correction

With the present of the defects such as cold-laps, shrinkage cavity, inclusions, and micro-pores [2, 3], the crack initiation life prediction of the thermite weld requires detail knowledge of the local stress-strain history at the notch foot. Two notable models for considering the notch effect namely Neuber's and Glinka's rules are frequently used for estimating notch root stresses and strains [128, 129]. The first step of these two models is to relate the local stress σ and strain ε to the nominal remote stress S or strain e with the theoretical Stress Concentration Factor K_t . Within the elastic range, the local stress and strain can be calculated as:

$$\frac{\sigma}{S} = \frac{\varepsilon}{e} = K_t \quad (2-35)$$

For the notch strains and stresses after yielding occurs, Neuber and Glinka (also known as equivalent strain energy density method) hypothesizes can be expressed in the following forms:

$$SeK_t^2 = \sigma \varepsilon, \text{ for Neuber's rule} \quad (2-36)$$

$$\frac{SeK_t^2}{2} = \int \sigma d\varepsilon, \text{ for Glinka's rule} \quad (2-37)$$

When a Ramberg-Osgood-type equation is used to express the material cyclic stress-strain relationship:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'} \right)^{1/n'} \quad (2-38)$$

The general form of Neuber's and Glinka's rules can be expressed in the following forms if the nominal stress does not exceeds eighty percent of the material yield strength [130]:

$$K_t^2 \frac{S_a^2}{E} = \frac{\sigma_a^2}{E} + \sigma_a \left(\frac{\sigma_a}{K'} \right)^{1/n'}, \text{ for Neuber's rule} \quad (2-39)$$

$$K_t^2 \frac{S_a^2}{E} = \frac{\sigma_a^2}{E} + \frac{2\sigma_a}{n'+1} \left(\frac{\sigma_a}{K'} \right)^{1/n'}, \text{ for Glinka's rule} \quad (2-40)$$

Where the subscript a means stress amplitude. K' and n' are cyclic strength coefficient and cyclic strain hardening exponent, which can be obtained from tensile or hardness test results [131].

Neuber's rule assumes that the actual total strain energy density (the strain energy plus the complimentary strain energy density) at the notch tip is equal to the pseudo total

strain energy density [132, 133] while Glinka's rule states that the actual elastoplastic strain energy density is equal to the pseudo strain energy density as if a material hypothetically remains elastic [129, 134]. The graphic representation for Neuber's and Glinka's rules is shown in Figure 2-28.

Figure 2-28. Graphical representation of (A) Neuber's, (B) Glinka's rules [134]

2.5.2.2. Effect of Small Defects on Fatigue Behavior

Small defects such as inclusions and micro-pores [41, 70] can form in the thermite weld due to the casting nature of the process [3]. To evaluate the effect of small defects on the fatigue limit of materials, Murakami [135, 136] proposed a model with defect area as a geometrical parameter. The stress intensity factor (SIF) range for mode I cracks can be expressed as:

$$\Delta K_I = 0.65 \Delta \sigma_0 \sqrt{\pi \sqrt{\text{area}}}, \text{ for surface defects} \quad (2-41)$$

$$\Delta K_I = 0.5 \Delta \sigma_0 \sqrt{\pi \sqrt{\text{area}}}, \text{ for internal defects} \quad (2-42)$$

Where, $\Delta\sigma_0$ is the applied remote stress range. He also developed two formulas to express the threshold SIF range ΔK_{th} for a small crack (surface or internal) as a function of the defect size $\sqrt{area}$ and the Vickers hardness H_V :

$$\Delta K_{th} = 3.3 \times 10^{-3}(H_V + 120)(\sqrt{area})^{1/3}, \text{ for surface defects} \quad (2-43)$$

$$\Delta K_{th} = 2.77 \times 10^{-3}(H_V + 120)(\sqrt{area})^{1/3}, \text{ for internal defects} \quad (2-44)$$

The fatigue limit for defective materials with surface or internal cracks can be obtained by solving Eq (2-41) and (2-43) or Eq (2-42) and (2-44):

$$\Delta\sigma_w = 1.43(H_V + 120)/(\sqrt{area})^{1/6}, \text{ for surface defects} \quad (2-45)$$

$$\Delta\sigma_w = 1.56(H_V + 120)/(\sqrt{area})^{1/6}, \text{ for internal defects} \quad (2-46)$$

These equations are applicable for materials with hardness from 70 to 720 and the defect size $\sqrt{area}$ up to 1000 μm [137].

2.6. Summary

From literature review, the thermite welds are the weakest link in a railway steel track and the failures will cause transport downtime, costs of the replacement and even potential risk of derailment. Compared with other failures such as rolling contact fatigue on the rail head, defects occurring at the head-to-web and web-to-foot regions are more difficult to be detected from inspection. The fracture and fatigue mechanism of

transverse failures for the thermite weld has been investigated through finite element simulations and multiaxial fatigue analysis. As for the rail clips, several studies have been carried out to investigate their material and structural behaviors.

However, the fatigue behavior of the thermite weld at critical locations especially the head-to-web regions and rail clips has not received the proper amount of investigation.

And the effect of the rail clips on the dynamic behavior of the thermite weld is not well understood. In addition, fatigue assessment of the thermite weld and rail clips incorporating a fast multi-axiality analysis approach is a necessity.

Chapter 3. Experimental Study

Based on the literature review in Chapter 2, the mechanical properties and structural behavior of the thermite weld and rail clips have not been clearly revealed. The experimental study in this chapter involves the material characteristics and fatigue property of the thermite weld and rail clips as well as their structural behaviors under fatigue load conditions. Field strain measurement through the strain rosette attached at head-to-web regions of the thermite welds in a depot or a tunnel under service loading conditions is also performed.

3.1. Material Test

3.1.1. Tensile Test

Figure 3-1. (A) Experiment set-up and (B) dimensions of specimens according to ASTM A370 standard [138]

Tensile tests were conducted for in accordance with ASTM A370 standard [138]. The experiment set-up and dimension of the specimens are shown in Figure 3-1.

3.1.1.1. Tensile Test for Thermite Weld

The tensile test results for thermite weld steel and R260 parent rail steel are shown in Figure 3-2 and Table 3-1. The result for the R260 rail steel is similar to those in literature [65].

Figure 3-2. Stress-strain curves for thermite weld steel and R260 parent rail steel

Table 3-1. Tensile tests results for thermite weld steel and R260 parent rail steel

Specimen	Diameter (mm)	Elongation (%)	Area Reduction (%)	of 0.2% Offset Strength (MPa)	Yield Strength (MPa)	Ultimate Strength (MPa)	Tensile
Thermite Weld	6.18	0.6	1	720		758	
Parent Rail	4.02	10	15	534		986	

3.1.1.2. Tensile Test for Rail Clip

The tensile test results for clip steel are shown in Figure 3-3 and Table 3-2.

Figure 3-3. Stress-strain curve of tensile test specimens for clip steel

Table 3-2. Clip material tensile tests results

Sample	Diameter	Elongation	Area of	0.2% Offset Yield	Ultimate Tensile	Young's
Number	(mm)	(%)	Reduction (%)	Strength (MPa)	Strength (MPa)	Modulus (GPa)
1	3.97	14	35	1306	1464	217
2	3.97	13	37	1310	1473	216
3	3.98	12	35	1318	1474	217

The tensile properties for rail steel and clip steel will be used as inputs of finite element models in the subsequent chapters.

3.1.2. Hardness Test

The material hardness test conducted on the thermite weld and rail clip was through a

Digital Rockwell hardness tester shown in Figure 3-4. A Rockwell C diamond type indenter was used in accordance with ASTM E18 standard [139].

Figure 3-4. Digital Rockwell hardness tester

3.1.2.1. Hardness Test for Thermite Weld

The hardness test for the thermite weld steel was conducted at grip locations of the rotating bending fatigue test specimens described below and the correlation of convex cylindrical surface was considered according to British Standard ISO 6508-1 [140]. The mean and standard deviation of Rockwell C hardness test results were 28.7 HRC and 2.8 HRC, respectively.

3.1.2.2. Hardness Test for Rail Clip

The hardness test for the clip steel was conducted on the flat surface of a fast clip. The average hardness was 39.7 HRC and 1.2 HRC, respectively.

3.1.3. Uniaxial Fatigue Test

In order to obtain material stress/strain life curves, uniaxial fatigue tests were conducted for the thermite weld steel and rail clip steel.

3.1.3.1. Rotating Bending Fatigue (RBF) Test for Thermite Weld Steel

The rail head was cut from 400mm thermite weld rail as shown in Figure 3-5(A). Fifteen RBF specimens were extracted from the rail head longitudinally according to British Standard ISO 1143 [141] as shown in Figure 3-5(B). RBF tests were conducted on the RBF testing machine with a rotational speed of 3000 RPM. The endurance limit was set as 10-million cycles. The equipment set-up was shown in Figure 3-6.

Figure 3-5. (A) Thermite weld specimen and the cut rail head, and (B) Extracted RBF specimens machined according to British Standard ISO 1143 [141]

The loading conditions and test results were listed in Table 3-3 and Figure 3-7. The failure locations of 14 failed specimens were near the central position where experience

the largest stress level under the bending moment expect for No.6 specimen in the red circle. The result for No.6 specimen will be excluded for fatigue life curve fitting since this specimen failed prematurely and the failure location was at grip location which experience a much lower stress level.

Figure 3-6. Rotating bending fatigue test machine

Figure 3-7. Specimens after rotating bending fatigue tests

Table 3-3. Loading and fatigue life for thermite weld steel

Specimen	Bending Moment (N*m)	Stress Amplitude (MPa)	Cycles	Remark
1	6.32	500	45765	
2	5.69	450	70724	
3	4.74	375	301051	
4	4.76	370	363091	
5	4.45	350	1E7	Survive due to lower stress level
6	4.68	368	115684	Not count, failed at grip location
7	4.56	365	1E7	Survive due to lower stress level
8	4.68	370	3161282	
9	4.43	350	1E7	Survive due to lower stress level
10	5.09	400	134322	
11	6.21	500	117972	
12	5.66	450	248495	
13	4.74	375	336398	
14	7.04	550	28946	
15	4.60	365	4312244	

Scanning Electron Microscopy and Energy Dispersive X-Ray Spectroscopy studies

Scanning Electron Microscopy (SEM) study was performed for all 15 specimens. The fractured surface of the 14 specimens that failed within gauge marks revealed that the near-surface micro-pores initiate the fatigue failure from Figure 3-8 and Figure 3-9. It also showed that the fatigue life of the specimens was dominant by the crack initiation process since the fast fracture region accounted for most areas on the fracture surface

as shown in Figure 3-8.

Figure 3-8. Two crack initiation points found by SEM for No.1 specimen

Figure 3-9. SEM micrographs of crack initiation points for (A) No.3 specimen, (B) No.12 specimen, and (C) No.10 specimen

Different sizes, shapes and orientations of micro-pores and their distances to the surface may cause the fatigue life to vary even when a similar stress level was applied. For a higher stress level, multiple crack initiation positions can be observed at the near-

surface micro-pores and one dominant the crack growth shown in Figure 3-8. And there is only one crack origin for lower stress levels as shown in Figure 3-9. The size of the near-surface micro-pores can range from less than a hundred microns up to a few hundred microns. The distance to the surface can be closed to a few microns.

As for No.6 specimen, a shining region was presented at the cross-section of the specimen as shown in Figure 3-10(A) and main chemical composition is Al confirmed by Energy Dispersive X-Ray Spectroscopy (EDS). The powder-like structure with the main chemical composition of Al and Fe also presented as shown in Figure 3-10(C).

Figure 3-10. Fracture surface located at the grip location of No.6 specimen

3.1.3.2. Tensile Fatigue Test for Rail Clip Steel

The specimens with a continuous radius between ends were prepared in accordance with ASTM E466 standard [142] and tensile fatigue tests for rail clip steel were conducted and completed at DNV. The test set-up on the high-frequency (88.8 Hz) resonance fatigue machine and dimension of the specimens are shown in Figure 3-11.

The stress range and number of fatigue cycles are reported in Table 3-4.

Figure 3-11. (A) Fatigue experiment set up according to ASTM E466 standard [142], and (B) Dimensions for fatigue test specimens

Table 3-4. Loading and fatigue life for clip steel

Specimen	1	2	3	4	5	6	7	8
Diameter (mm)	4.10	3.97	3.97	3.94	3.93	3.94	3.92	3.92
Maximum Load (kN)	9.99	5.30	7.90	8.19	9.50	8.80	10.67	8.33
Minimum Load (kN)	0.09	0.34	0.88	0.87	1.01	0.88	1.02	0.79
Nominal Stress level (MPa)	750	401	567	600	700	650	800	625
No. of cycle	116923	5E6	5E6	5E6	70630	123656	83551	5E6
Status	Broken	Survive	Survive	Survive	Broken	Broken	Broken	Survive

The fatigue life curves will be plotted and used for fatigue assessment in the subsequent chapters.

3.2. Inverted Fatigue Test for Thermite Weld

Ekberg and Lawrence [20, 69] conducted non-inverted fatigue tests for the thermite weld and they both found that the failures initiating from the stress concentration point at web-to-foot or rail foot regions. In this research, inverted four-point bend fatigue tests were conducted to investigate the fatigue behavior of thermite weld structure at head-to-web regions.

The sketches of non-inverted and inverted four-point bend fatigue test set-up were shown in Figure 3-12. Four inverted tests for thermite weld without peening were conducted at TUV SUD and one with a higher loading was conducted at DNV. The loading frequency of four-point bend fatigue tests was 3 Hz conducted at TUV SUD and 0.3 Hz at DNV. The experiment set-ups are shown in Figure 3-13. The loading conditions and fatigue life for inverted tests were listed in Table 3-5.

Figure 3-12. Sketches for (A) Non-inverted and (B) Inverted tests

The thermite weld specimens were placed horizontally on two point supports with the weld midway between them as shown in Figure 3-13(A) and (B) and the inner span was

set to 200 mm. The outer span of the test machine was 1100 mm at TUV SUD and 1000 mm at DNV. The maximum loading cycle was set at the value of 2 million. No.4 test specimen was subjected to non-destructive crack detection and remained intact after 2 million cycles.

Figure 3-13. Inverted fatigue test set up at (A) TUV SUD and (B) DNV

Table 3-5. Loading and fatigue life for thermite weld specimens

Test	P_{max} (kN)	P_{min} (kN)	Cycles	Remark
1	550.01	50.00	701030	
2	550.01	50.00	1303159	Specimen after peening
3	500.00	50.00	293729	
4	390.00	39.00	2000000	Survive
5	900.00	90.00	46339	Test conducted at DNV

Hammer peening has been conducted for one thermite weld specimen at the weld toe regions. The specimen was also subjected to an inverted four-point bend test to

investigate the effect of defects. The specimen after peening showed a shining surface along the weld edge at head-to-web and web-to-foot regions as shown in Figure 3-14.

Figure 3-14. (B) Hammer peening process and weld toe surfaces (A) before and (C) after peening

Figure 3-15. (A) crack initiation at head-to-web region for specimen without peening, (B) crack initiation at rail head for specimen with peening

The fatigue life of the specimen after peening increased almost two times compared to the specimen without peening when the maximum applied load was 550 kN and the crack initiation point shifted from head-to-web region to rail head as shown in Figure 3-15. This could indicate that the stress concentration effect of defects at the thermite weld head-to-web region is reduced through peening process. However, more tests should be conducted for further studies to provide a more comprehensive investigation.

3.3. Static Pull-up test for E-Clip and Fast-Clip

The E-clip and fast-clip pull up experiment were conducted as shown in Figure 3-16 in order to study the deformation behaviors of the clips. The straight end of the E-clip was clamped by a steel plate fixed at its location. The steady increased vertical loading was applied at the other end of E-clip through a steel holder. Two strain rosettes were mounted at the front arch and rear arch of the E-clip shown in Figure 3-16(A). The E-clip response became non-linear when the loading was around 14 kN shown in Figure 3-16(B). The maximum load was 33.32 kN.

Two legs of the fast-clip were clamped by a fixed fixture with two shoulder housings as shown in Figure 3-16(C). The upward loading was applied at the front seat of the fast-clip through a funnel-shaped steel holder connected to a bolt. Two strain rosettes were mounted on two rear arches of the fast-clip. The fast-clip response became non-linear when the loading was around 38 kN shown in Figure 3-16(D) and the maximum

load was 51.43 kN. Both E-clip and fast-clip were under permanent deformation but unable to achieve failure.

Figure 3-16. (A) E-clip and (C) fast-clip experiment set ups and strain rosette locations, (B) E-clip and (D) fast-clip loading and deformation histories

3.4. Fast-Clip Fatigue Loading Test

The fast-clip fatigue loading experimental set-up is shown in Figure 3-17. A fixture was designed and fabricated to grip the two legs of the fast-clip. Upward cyclic loading was exerted through a metal block connected to a bolt which is allowed to swing freely to allow for opening of the fast-clip at high load. The load was controlled electronically, and the loading frequency is 2 Hz.

Figure 3-17. (A) Fast-clip fatigue loading experiment set up, and (B) Fast-clip specimens after fatigue tests

Six fatigue tests were conducted and a vertical shift of 8 kN was applied for each loading level to consider the clip opening force during the installation procedure. The R-ratio was 0.1 for all six loading levels after subtracting the vertical shift. The loading levels and life cycles were listed in Table 3-6. Based on the results, No.1 to 4 specimens failed at the inside of the rear arch. No.5 and 6 specimens did not fail after one million cycles and three million cycles, respectively.

Table 3-6. Fast-clip fatigue test results

Test	P_{min} (kN)	P_{max} (kN)	Nominal P_{min} (kN)	Nominal P_{max} (kN)	Cycles	Status
1	10.1	29.0	2.1	21.0	2,422	Failed
2	9.8	26.0	1.8	18.0	2,522	Failed
3	9.2	20.0	1.2	12.0	7,828	Failed
4	8.4	12.0	0.4	4.0	124,830	Failed
5	8.1	9.0	0.1	1.0	1E6	No Failure
6	8.275	10.75	0.275	2.75	3E6	No Failure

Six fatigue tests were conducted and a vertical shift of 8 kN was applied for each loading level to consider the clip opening force during the installation procedure. The R-ratio was 0.1 for all six loading levels after subtracting the vertical shift. The loading levels and life cycles were listed in Table 3-6. Based on the results, No.1 to 4 specimens failed at the inside of the rear arch. No.5 and 6 specimens did not fail after one million cycles and three million cycles, respectively.

3.5. Strain Measurement

Wireless strain measurements were conducted at a depot and in a metro tunnel. The strains were measured through rosettes attached at head-to-web and web-to-foot regions near the weld toes. The strain rosettes were covered with water and Electromagnetic Interference (EMI) proof tapes. The results were measured and transmitted with strain

rosettes connected with a V-Link-200 Wireless Sensor Node powered by batteries and the transmitter was put in a metal box to prevent EMI. Finally, the data was received by a LORD sensing data gateway such as the WSDA-Base-LXRS connected with a laptop which installed SensorConnect software.

3.5.1. Roll-in Test at the Depot

Figure 3-18. Strain measurement conducted on a straight track at a depot

A roll-in test was conducted on a straight track at a depot. An empty train ran on a straight track with a nearly constant velocity of 59 km/h. The wireless strain measurement test set-up is shown in Figure 3-18. The sampling rate of the signals was

128 Hz. The strain readings for the rosette attached at head-to-web regions are shown in Figure 3-19. Each peak refers to a wheelset passed by and there are 24 wheelsets for a 6-car train.

Figure 3-19. Measurement results from the roll-in test

3.5.2. Strain Measurement in the Tunnel

A real-time strain measurement was conducted on a 300 m radius curved track in a metro tunnel. The experiment set-up is shown in Figure 3-20. The strain rosettes were attached ~10mm away from the weld to parent rail interface. The transmitters, receivers and laptop were placed in the boxes and tied tightly to the pipe close to the wall in order to minimize their movements when a train passes by. A higher sampling rate of 1024 Hz was set to record the signals to capture more data points when a train passes a curved track.

Figure 3-20. Strain measurement conducted on a 300 m radius curved track in a metro tunnel

The recorded strains at head-to-web corner in the field side are shown in Figure 3-21.

51 trains were successfully monitored through the strains recorded in the laptop within the battery level. From Figure 3-21(B), the train speed can be calculated as 52 km/h.

(A)

(B)

Figure 3-21. Strain measurement in the field side at the head-to-web corner (A) for 51 trains and (B) for one train

3.6. Summary

The material characteristic and fatigue properties of thermite weld steel and rail clip steel were investigated experimentally. Compared with the parent rail metal, the yield stress of thermite weld steel increases while the ductility reduces dramatically. This brittle behavior was also confirmed by RBF tests since the fast fracture region accounted for most areas on the fracture surface. The cracks initiated at the near-surface micro-pores for all the specimens failed within gauge marks. The scatter found in RBF test data could be explained by the different sizes and distances to surface of the near-surface micro-pores observed by SEM.

Inverted four-point bend fatigue tests for the thermite weld were carried out for the first time to study the fatigue failures at head-to-web regions. Hammer peening was conducted and the specimen with peening showed a longer fatigue life ($\sim 2x$) than the specimen without peening under the same loading condition.

The clip steel exhibits a better material characteristic indicated by hardness, tensile and uniaxial fatigue tests. The static pull-up and fatigue loading tests were conducted to investigate the structural deformation behavior and fatigue properties of the clips.

Real-time strain measurements were conducted at a depot and in a metro tunnel through a wireless system. The signals were successfully recorded by the strain rosette attached at the head-to-web corner and each peak indicates a wheelset passed by the thermite

weld. The roll-in strain measurement results on the straight track will be used to validate finite element models in the next chapter. The on-site strain measurement will be used for fatigue damage accumulation analysis with the rainflow cycle counting approach in Chapter 5.

Chapter 4. Stress and Fatigue Assessment of Thermite Weld

Stress and fatigue assessment of the thermite weld is necessary to gain a comprehensive understanding of the fatigue failures at the weld edge as stated in Chapter 1 and Chapter 2. This chapter describes a fatigue damage assessment method combining multibody system dynamic method and finite element method. The wheel-track interaction was first simulated by multibody dynamic software to obtain the dynamic response and was then applied as an input of the FE models as shown in Figure 4-1. The model was validated and compared with the roll-in test. Finally, a time-efficient multi-axial fatigue assessment algorithm was developed to calculate the fatigue damage according to the stress/strain histories obtained from FE models.

Figure 4-1. A diagram of combining multibody system dynamic method and finite element method in order to perform stress and fatigue analysis.

4.1. Multibody System Dynamic Model

Multibody dynamic analysis was first performed with the commercial software

Universal Mechanism (UM) based on Bai's work [81]. The parameters of structures of vehicle and track are listed in Table 4-1. X, Y and Z indicate the longitudinal, lateral and vertical directions, respectively.

Table 4-1. Simulation parameters in UM

Parameter	Value	Parameter	Value
Car body weight, M_c (kg)	3.83×10^4	Primary suspension stiffness, K_{fx} (MN/m)	10
Bogie weight, M_t (kg)	3.0×10^3	Primary suspension stiffness, K_{fy} (MN/m)	5
Wheelset weight, M_w (kg)	1.4×10^3	Primary suspension stiffness, K_{fz} (MN/m)	0.55
Rail Weight, m_r (kg/m)	60.64	Primary suspension damping, C_{fz} (KN s/m)	6
Sleeper weight, M_s (kg)	349	Car moment of inertia, I_{cx} (kg m ²)	8.56×10^4
Equivalent ballast weight, M_b (kg)	466	Bogie moment of inertia, I_{tx} (kg m ²)	2.26×10^3
Secondary suspension stiffness, K_{cx} (MN/m)	0.15	Bogie moment of inertia, I_{ty} (kg m ²)	2.71×10^3
Secondary suspension stiffness, K_{cy} (MN/m)	0.15	Bogie moment of inertia, I_{tz} (kg m ²)	3.16×10^3
Secondary suspension stiffness, K_{cz} (MN/m)	0.4	Wheelset moment of inertia, I_{tx} (kg m ²)	915
Secondary suspension damping, C_{cy} (KN s/m)	60	Wheelset moment of inertia, I_{ty} (kg m ²)	140
Secondary suspension damping, C_{cz} (KN s/m)	80	Wheelset moment of inertia, I_{tz} (kg m ²)	915
Coefficients of equivalent spring between rail and sleeper, K_{rsy} (MN/m)	29.47	Coefficients of equivalent spring between sleeper and ballast, K_{sby} (MN/m)	70
Coefficients of equivalent damper between rail and sleeper, C_{rsy} (KN s/m)	50	Coefficients of equivalent damper between sleeper and ballast, C_{sby} (KN s/m)	60
Coefficients of equivalent spring between rail	78	Coefficients of equivalent spring for	78

and sleeper, K_{rsz} (MN/m)		ballast, K_{bz} (MN/m)	
Coefficients of equivalent damper between		Coefficients of equivalent damper or	
	50		80
rail and sleeper, C_{rsz} (KN s/m)		ballast, C_{bz} (KN s/m)	

4.2. Finite Element Model

In order to have a deep understanding about the combined effect of in-plane bending, out-of-plane bending and lateral torsional bending of the thermite weld as well as to obtain the stress tensor of stress concentration points at the weld collar under a cyclic loading for fatigue analysis, a detailed 3D dynamic finite element model with a thermite weld was modeled in the ABAQUS finite element software [143]. A half model was built due to geometrical symmetry and also to reduce the computational cost and memory.

Both implicit and explicit methods were used for analysis. The implicit analysis was used first for a static wheel-rail contact calculation to establish a stable state to apply the results as an initial state for the following explicit analysis. The implicit-explicit sequential model was chosen because there are some inherent differences between static and dynamic calculation methods, such as the inertia effect which is considered in dynamic approach but not in static approach, and an initial disturbance will interfere the calculated results at the beginning of the dynamic calculation.

4.2.1. Rail Length Effect Study

For the preliminary study, the length of rail was set as 4.9 m, 6.3 m and 7.7 m respectively as shown in Figure 4-2. Five different locations on the weld collar were selected to check the effect of rail length and the longitudinal stresses are shown in Figure 4-3. The boundary conditions were the same for these three models as follow: The two rail ends are constrained in the longitudinal direction. The bottom of the sleepers is fixed. The lateral movement of sleepers is constrained.

Figure 4-2. Three different sizes of model with same boundary condition: (A) 4.9 m, (B) 6.3 m and (C) 7.7 m

There are no significant differences among all three models. Thus, the rail length in this research was set as 4.9 m which could minimize the rail end effect while reducing calculation time. Similar length was used in other researchers' work [3, 12, 94].

(A)

(B)

(C)

(D)

(E)

(F)

Figure 4-3. (A) Sketch of 5 positions and the longitudinal stress at (B) under-head radius, (C) head-to-web corner, (D) middle web, (E) web-to-foot corner and (F) foot tip

4.2.2. Rigid Wheel Simplification Study

The wheel was set as a rigid body in this study instead of a deformable wheel since this study is interested in the regions below the under-head radius. The comparison results of rigid wheel and deformable wheel are shown in Figure 4-4.

Figure 4-4. Comparison results for (A) (B) rigid wheel, and (C) (D) deformable wheel

Even though the stresses at the wheel-rail direct contact position showed some difference for rigid and deformable wheels, the stresses value and profile did not show much difference at the positions of interest below the railhead. Therefore, the rigid body assumption for the wheel was adopted in this study.

4.2.3. Mesh Refinement Study

In order to capture the stress concentration effect at the thermite weld collar, a finer mesh was applied near the weld edge. A mesh refinement study was performed to investigate the mesh size sensitivity. Three average mesh sizes of 2 mm, 1 mm and 0.5 mm were used. The mesh details of the rail with a thermite weld were shown in Figure 4-5. All three models were submitted to High Performance Computer (HPC) to check their computational efficiency and costs.

(A)

(B)

(C)

Figure 4-5. Mesh details for (A) 2mm average mesh, (B) 1 mm average mesh and (C) 0.5 mm average mesh

Since the stresses at the rail field side are larger than the stresses at the rail gauge side due to the presence of lateral force, three locations (head-to-web, web-to-foot, and foot

tip) at the field side were selected for comparison. The comparison results of longitudinal stress of three mesh size models are shown in Figure 4-6. Little difference in stress component can be observed for 1 mm and 0.5 mm models while the stresses are much smaller for the 2 mm model. And the model with finer mesh shows more oscillations. When considering the computational cost, the 0.5 mm model takes 400 cores HPC to run 4~5 days (~ SGD 1000) to finish one simulation while it takes 32 cores HPC to run 5~6 days (~ SGD 80) for 1 mm model and 1~2 days (~ SGD 30) for 2 mm model. Therefore, the 1 mm mesh model was used in this research considering the calculation efficiency and cost. Similar mesh size was used by other researchers in dynamic finite element models [11, 32].

(A)

(B)

(C)

Figure 4-6. Longitudinal stress at (A) head-to-web corner, (B) web-to-foot and (C) foot tip in models with 2mm, 1 mm and 0.5 mm average mesh size

4.2.4. Finite Element Model Establishing

The whole FE model consisted of a wheel, a rail track with a thermite weld, base plates, rubber pads, rail clips, sleepers and ballast as shown in Figure 4-7. The UIC60E2 rail profile according to BS EN 13674 and S1002 wheel profile according to BS EN 13715 were considered in the model. The length of the rail was chosen as 4.9 m. The shape of thermite weld was constructed based on a thermite weld specimen. Eight sleepers and base plates with a total number of 12 E-clips were incorporated into the model. A 4.5 mm thin layer rubber rail-pads were constructed beneath the rail bottom and sat on the base plates. A ballast layer with 300 mm thickness was also constructed beneath the sleepers.

Figure 4-7. Finite element model for the ballast track system

Only the longitudinal movement of two rail ends was constrained because the rail is

mounted with clips on the base plates. The cross-section of sleepers was fixed in x-axis to prevent the lateral movement and the bottom of ballast was fixed. Base plates, rubber pads and clips shared the same nodes at their interfaces. General contact in ABAQUS was used between the wheel and the rail, the rail and rubber pads, the base plates and clips. The friction factor used was 0.3 which is usually adopted in the literature [144, 145]. It should be noted that even for the straight track model the contact point between wheel and rail head was not located on the center line of rail cross-section and was offset towards the inside of the rail.

The 3D cyclic loading was realized through a moving wheel. The forces acting on the wheel were obtained through the multibody system dynamic model stated above. The preliminary results revealed that the wheel-rail contact forces for all three directions do not vary much for a 4.9 m length (less than 2%), therefore, a constant 3D loading was applied on the wheel. The simulation parameters obtained from the literature [146-148] and experiments are listed in Table 4-2.

As for the mesh, 1 mm denser mesh was applied at the junction area between the plain rail and the thermite weld. The largest mesh size of about 35 mm was applied at the rail ends and a transitional mesh was considered from the ends to the middle, to reduce element numbers and computational time. C3D8R for sleepers and ballast whose geometries is easily meshed, and C3D4 for rails and base plates (including rail-pads and clips) with highly complex geometries were chosen for the finite element mesh.

The mesh of whole model consists of 923412 elements (51084 hexahedral elements of type C3D8R and 872328 tetrahedral elements of type C3D4) and 237113 nodes.

Table 4-2. Simulation parameters in finite element models

Parts	Material	Material properties				
		Density	Elastic Modulus	Poisson's	Yield Stress	Ultimate Stress
		(kg/m ³)	(MPa)	Ratio	(MPa)	(MPa)
Rail	Parent steel	7850	210000	0.3	654	926
Sleeper	Concrete	2500	38000	0.15	-	-
	Wood	800	9520	0.3	-	-
Ballast	Stone	1902	150~450	0.15	-	-
Wheel, Base plate and clip	Steel	7850	210000	0.3	-	-
Rail pad	Rubber	920	2.29	0.47	-	-

4.3. Comparison between Simulation and Experimental Results

The finite element model was validated by a train roll-in test as stated in Chapter 3. The rail was constructed on wood sleepers with E-clips constraining the rails with base plates, and ballast layer was embedded below the wood sleepers. The elastic modulus of the ballast layer was selected as 150 MPa since new-laid ballast was used. The train wheel-induced loading is 62.2 kN based on the empty car weight and the wheel-rail dynamic contact forces obtained from the UM model. Only the first wheelset of the first

bogie on the motor car was used in FE model to eliminate subsequent wheelsets' effect on the strain history. The comparison was shown in Figure 4-8 and a good consistency can be observed between simulation and field roll-in measurement results. Different boundary conditions were also investigated and the results are shown in Appendix A.

Figure 4-8 also illustrates that both vertical and longitudinal strain at the head-to-web region experienced tension deformation before the wheel passed by. The vertical strain gradually increased to tension peak 1 with a tensile strain of $\sim 40 \mu\epsilon$, then suddenly dropped to compression peak 2 with a compressive strain of $\sim 85 \mu\epsilon$ in Figure 4-8(A).

Figure 4-8. Comparison results of measured strain and simulation at (A) Vertical direction (B) Longitudinal direction

Figure 4-8 also illustrates that both vertical and longitudinal strain at the head-to-web region experienced tension deformation before the wheel passed by. The vertical strain gradually increased to tension peak 1 with a tensile strain of $\sim 40 \mu\epsilon$, then suddenly

dropped to compression peak 2 with a compressive strain of $\sim 85 \mu\epsilon$ in Figure 4-8(A).

(A)

(B)

Figure 4-9. Local deformation when the wheel rolls near the thermite weld: (A) vertical strain is plotted (B) longitudinal strain is plotted

To investigate this rapid change, the vertical local deformation of the track is plotted in Figure 4-9(A). There was a tension deformation region on the track just ahead of the wheel due to the dynamic effect of rail track materials. On one hand, the wheel was pushing off rail materials in front it, on the other hand, due to inertia effect, rail material

further from the wheel was not moving. This mechanism caused a ‘jam’ in the rail materials just ahead of the wheel and a tensile vertical strain occurs. Meanwhile, the transition area from tension to compression region was quite small, causing a sudden drop from tensile strain peak to compressive strain peak. After the wheel passed by, the compressive strain in the track decreased.

For the longitudinal strain shown in Figure 4-8(B), the tension peak 1 occurred earlier than the first vertical tension peak in Figure 4-8(A). Then, the strain gradually changed to compressive strain until it reached the compression peak 2 of $\sim 55 \mu\epsilon$ just when the wheel reached the strain gauge location. After that, the longitudinal strain gradually changed to tensile strain again and reached tension peak 3 after the wheel passed by. Different from the vertical strain in Figure 4-8(A), the longitudinal strain shows a symmetric pattern in Figure 4-8(B). There was a wave-like deformation of the rail when the wheel is rolling, and this deformation pattern had been identified by Orringer et al. and others [77, 149] as ‘reverse bending’. There was a tension deformation region ahead from wheel position and the strain gradually changed to compression when it got close to the wheel. The distance from the maximum tensile strain on the rail to the wheel position was predicted by Zerbst et al. [53] as:

$$d = \frac{\pi}{2\lambda_V} \quad (4-1)$$

Where λ_V is a parameter related with the rail geometry and the vertical foundation

stiffness k_v as:

$$\lambda_v = \sqrt[4]{\frac{k_v}{4E \cdot I_{xx}}} \quad (4-2)$$

where the vertical foundation stiffness k_v for a typically maintained wood-track is 30 MN/m² [77], $E = 210$ GPa is the elasticity modulus of the rail steel and I_{xx} is the second moment of inertia with respect to the horizontal rail axis. For the UIC60E2 rail profile in this study, $I_{xx} = 3038$ cm⁴ [53] was used. The distance d can be calculated from Eq (4-1) as ~ 1.5 m. In the roll-in test, this distance is 1 m which can be calculated using the time between peak 1 and peak 2 multiplied by the train velocity. The difference between the theoretical result and test result can be attributed to the dynamic effect of the wheel. Since Eq (4-1) is derived under static condition, the two distances between peak 1 and peak 2, peak 2 and peak 3 are the same. However, in the real situation, the moving wheel will ‘chase’ the deformation wave ahead of it, resulting in a decrease of the distances between peak 1 and peak 2, and an increase of the distances between peak 2 and peak 3.

4.4. Stress Analysis

The model was constructed based on the above-mentioned profile and material properties. A 300 m radius curved track and 100 mm cant height was used to construct

the multibody system dynamic model, a train with a full load of 64 tons and a constant velocity of 70 km/h were used to conduct the analysis. The 3D wheel-rail contact forces obtained from UM model were 131 kN for the vertical load, 53 kN for the lateral load and 2 kN for the longitudinal load. This 3D loading was then applied in Finite Element model as the input.

The stress results of the FE model are plotted in Figure 4-10, 4-11 and 4-12. Figure 4-10 shows the longitudinal stress from cross-section and field side section views when the wheel just locates at the weld central position from the side view. Figure 4-11 and 4-12 show the longitudinal stress and Tresca shear stress profiles for the elements at field side weld collar edge at the front to rolling direction.

(A)

(B)

Figure 4-10. Longitudinal stress distribution of rail for (a) cross-section view and (b) field side view

From Figure 4-10(A), a higher longitudinal stress at field side is observed due to the combination effect of in-plane bending, out-of-plane bending and torsional lateral bending under the vertical and lateral forces acting on the rail offset from the central position. An asymmetric stress profile is observed from Figure 4-10(B). The reason behind this is that the rail inertia effect was considered in the explicit FE model which makes the rail material jam at the front of rolling direction. Therefore, the longitudinal stress becomes more tensile at the rail head and foot of the field side in the opposite direction of rolling direction.

Figure 4-11. Longitudinal stress distribution at the field side according to rail height and time

Figure 4-12. Tresca shear stress distribution at field side with rail height and time

From Figure 4-11, the longitudinal stress at the rail under-head of field side gradually becomes tensile first and then peaks when the wheel just passes the welded portion,

followed by a large compression stress and gradually return to zero as the wheel moves away from the weld. The reason behind this is that the wheel tries to push the rail head material when the wheel rolling towards the weld and the adjacent rail will be stretched while the rail under direct contact position is compressed, the out-of-plane bending and torsional lateral bending effect will counter the rail in-plane bending effect when the wheel just arrives at the weld which makes the tension peak. The in-plane bending will dominant and there will be a large compression stress when the wheel just passes by the weld. Finally, the stress will gradually back to zero when the wheel is far away from the weld. As for the longitudinal stress at the rail foot of field side, the stress becomes compression firstly due to the inertia effect. There is another phenomenon that the rail foot region undergoes a longer tensile stress history than the rail under-head radius because of the rail in-plane bending effect.

From Figure 4-12, a higher shear stress state can also be observed at rail under-head and foot regions of the field side. The shear stress peaks when the wheel just arrives at the weld due to the combination effect of rail in-plane bending, out-of-plane bending and lateral torsional bending. The shear stress at the rail foot is larger than that at the rail under-head because rail foot is closer to the rail neutral axis and the bending moment is larger.

Figure 4-13. Longitudinal Stress Distribution (a) when wheel rolls to 0.105 m and (b) 0.21 m ahead from central line of the weld

Figure 4-14. Deformation pattern of rail (scaled 700 times)

Rail deformation behaviors were also shown in Figure 4-13 and 4-14. When the wheel-rail contact point is near the weld as shown in Figure 4-13(A), the maximum tensile stress occurs at the base collar edge due to the in-plane bending of the rail. High compression stress was observed at under-head fillet due to bending. However, tensile stress occurs at the under-head radius when the wheel moves a little further from the

weld as shown in Figure 4-13(B) because of the ‘reverse bending’ mentioned before.

The scaled vertical displacements as shown in Figure 4-14 also indicate the in-plane bending of rail and ‘reverse bending’.

4.5. Fatigue Analysis

After obtaining the stress histories and performing the stress analysis, multiaxial fatigue analysis with stress-based criteria was performed to calculate and compare the high cycle fatigue damage as well as the critical plane orientations. The fatigue parameters can be obtained from the rotating bending fatigue tests. The fitted stress-life curve is shown in Figure 4-15 and the relationship can be expressed as:

$$\sigma_a = \sigma'_f (2N_f)^b = 935.48 \times (2N_f)^{-0.06} \quad (4-3)$$

where σ_a is the stress amplitude, N_f is the number of cycle to failure when $R = -1$ thus $2N_f$ is the number of reversals. The coefficient of determination for the fitted stress-life curve was 0.69. The scatter found in RBF test data especially for lower stress levels could be explained by high cycle fatigue (HCF) mechanism. For HCF, crack initiation stage dominates the total fatigue life and the initiation of fatigue cracks at surface or micro-pores/cavities just beneath the surface [150-152]. The scatter band for the thermite weld material is about 5x as shown in Figure 4-15.

The reversed bending fatigue limit for 10^7 cycles is 341 MPa from Eq (4-3). The

estimated reversed torsional fatigue limit is 198 MPa when the empirical value of 0.58 is used for the ratio of reversed torsion and bending fatigue limit [93, 135, 153].

Figure 4-15. Fatigue life curve of RBF test data

In this chapter, three stress-based multiaxial fatigue criteria namely Dang Van, Matake and Liu & Mahadevan criteria were applied for fatigue analysis.

4.5.1. Fatigue Assessment with Smallest Enclosing Circle Algorithm

The original Minimum Circumscribed Circle algorithm for searching the critical plane passing through each material point of interest is time-consuming and not suitable to complex engineering problems, since it searches all pairs of points as well as the triplets

consisting of any instantaneous of the whole loading cycle to find the smallest radius which can circumscribe all the points and it has a time complexity of $O(n^3)$. The number of operations (OP) for one material point can be expressed as [10]:

$$OP = 360 \times 90 \times \left[\frac{t!}{2!(t-2)!} (t-2) + \frac{t!}{3!(t-3)!} (t-3) \right] \quad (4-4)$$

Where t is the number of time instance within a loading cycle.

Therefore, in this research, a linear time algorithm proposed by Welzl [154] was adopted and it has a time complexity of $O(n)$ which is proved by Megiddo [155]. This algorithm developed in Python code was applied in multiaxial fatigue models to reduce the computation cost. The steps are described below and shown in Figure 4-16:

Step 1: Obtain the stress histories of all the elements of interest from FE simulation results and import the stress tensors of all time instance during the loading cycle to multiaxial fatigue models as inputs.

Step 2: Decompose the stress tensors according to all planes represented by the spherical angles (φ, θ) passing through all material points of interest as well as treat the shear stress for all time instance as points according to their directions and values.

Step 3: Shuffle the points on one plane and start from two points to construct the smallest enclosing circle. Construct the smallest enclosing circle for the $i - 1$ points and add the i th point to construct the updated smallest enclosing circle.

Step 4: Iterate Step 2 and Step 3 for all planes to find the critical plane and the radius of the circle which is the maximum resolved shear stress. Finally, apply multiaxial fatigue models and report the fatigue damage as well as the critical plane orientations.

Figure 4-16. Critical plane searching method and fatigue damage calculation diagram

The fatigue damage based on three multiaxial fatigue criteria implemented with the smallest enclosing circle algorithm and calculated from the stress histories is plotted in Figure 4-17. From Figure 4-17, for a train that runs on a 300 m radius and 100 mm cant height curved track, the fatigue damage is more severe at the foot tip and exceeds the fatigue limit from all three criteria. This indicates that the fatigue crack has a large probability to initiate at rail foot tip within 4 years when about 300 trains pass a thermite weld every day.

Figure 4-17. Fatigue damage for Dang Van, Mataka and Liu & Mahadevan multiaxial fatigue criteria

Overall, the damage graphs in this study are not so smooth comparing with the fatigue damage obtained by using static analysis from the literature [9]. The reason behind this

is due to the inherent larger oscillation characteristic of meshed elements in dynamic analysis comparing with static analysis and the difficulty in well-controlling the mesh quality. Even though the fatigue damage at the head-to-web and web-to-foot regions does not exceed the reversed torsional fatigue limit, the value is still large and around 100 MPa. Due to the presence of weld defects, the fatigue cracks will have a large possibility to initiate at these two regions.

It should be noted that the fatigue damage is also large (maximum 120 MPa to 150 MPa calculated from Matake and Liu & Mahadevan criterions) at under-head radius which shows a good consistency with the literature [9]. However, there is some little difference between the damage profiles calculated by multiaxial criteria proposed by Matake and Dang Van, and the difference is slightly larger when comparing Liu & Mahadevan criterion with the previous two criteria. The reason behind this is that the dominant components and weights in these three proposed criteria are different. For example, Liu & Mahadevan criterion searches the critical plane which experiences the largest normal stress amplitude and considers the non-linear combination of the normal stress amplitude and shear stress amplitude, while Dang Van and Matake believe the shear stress is the major reason cause the crack initiation problems so that their criteria search the critical plane which experiences the maximum resolved shear stress.

Even though the criteria proposed by Dang Van and Matake have a similar concept, there are also some differences between these two criteria: Dang Van believes the

hydrostatic stress affect the crack initiation behavior while Matake considers the maximum normal stress perpendicular to the critical plane result in crack initiation, and the weight of the second component from these two criteria is different calculated from pure reversed bending and torsional loading cases.

Different locations on the thermite weld are shown in Figure 4-18 and the critical plane orientations of each elements reported from the Python codes are summarized in Table 4-3. Dang Van and Matake criterions are analyzed first since they both searches the critical plane based on maximum shear stress. The angle θ for the rail foot tip is closed to 45 degree because the longitudinal stress and shear stress have the highest values and those values are comparable. For the elements at the middle and upper rail foot, the angle θ does not vary much which is closed to 0 degree and the angle φ of the critical plane direction implies the direction of x' -axis is nearly perpendicular to the rail profile while the angle φ and θ change at the profile transition region. The angle θ and φ do not vary much for the elements located at the web-to-foot, rail web and head-to-web regions, and the angle is almost constant at middle web region (55 mm to 80 mm). The angle θ of the elements at upper head-to-web regions are larger than the elements at the rail web because the longitudinal stress at the upper head-to-web region is larger and the angle φ also implies the direction of x' -axis is nearly perpendicular to the rail profile. The angle θ is also closed to 45 degree at under-head radius because the longitudinal and shear stress have the second highest values at this region. Overall,

Dang Van criterion is more conservative at the under-head radius and foot tip regions which experience a large longitudinal tensile stress than Matake criterion, since a larger longitudinal stress contributes to a larger hydrostatic stress and the hydrostatic component in Dang Van criterion accounts for more weight than the normal stress component in Matake criterion.

Figure 4-18. Locations on the thermite weld

Table 4-3. Critical plane orientations and fatigue damage for different rail

locations (70 km/h velocity, 300 m radius and 100 mm cant height)

Rail	Dang Van			Matake			Liu & Mahadevan		
locations	φ	θ	Damage	φ	θ	Damage	φ	θ	Damage
A	11-320°	31 or 58°	152-235MPa	11-320°	31 or 58°	143-215MPa	126-147°	14-17°	179-276MPa
B	2-87°	30-34°	70-158MPa	2-87°	30-34°	66-145MPa	163-297°	11-20°	81-189MPa
C	172-209°	14-20°	64-77MPa	172-209°	14-20°	67-76MPa	114-138°	25-32°	65-80MPa
D	170-180°	11-12°	75-82MPa	170-180°	11-12°	76-81MPa	98-107°	39-45°	63-71MPa
E	130-172°	10-15°	59-75MPa	130-172°	10-15°	64-77MPa	101-124°	19-24°	55-63MPa
F	280-347°	22-29°	59-107MPa	280-347°	22-29°	64-97MPa	94-101°	14-18°	57-125MPa
G	89-326°	32 or 59°	112-123MPa	89-326°	32 or 59°	95-113MPa	53-100°	12-18°	123-146MPa

As for Liu & Mahadevan criterion, the angle θ does not change much at all the locations. The angle φ at the rail foot (below 25 mm) makes the direction of x' -axis nearly perpendicular to the rail profile but not so absolute at the upper rail foot transition regions. The angle φ implies the direction of x' -axis not perpendicular to the rail profile is due to a highly complex stress state at the upper web-to-foot transition region and changes to 114~138 degree at the lower web region. The reason for the angle θ is not zero which means the maximum normal stress amplitude direction is not aligned with the longitudinal direction (z -axis) is since the rail undergoes a complex stress state instead of uniaxial loading. The angle θ is smaller and closer to zero at the foot tip and under-head radius because the longitudinal stress is larger at these regions and has the dominant effect on the maximum normal stress amplitude direction while the angle θ

is larger at web region because the Tresca shear stress is comparable with the longitudinal stress.

4.5.2. Ballast Effect

The influence of the ballast stiffness was investigated firstly. Three values of 150 MPa for a new-laid ballast layer, 300 MPa and 450 MPa for a degradation ballast layer were selected and applied in FE models. The rest parameters used in FE models remained the same. The maximum longitudinal stress of these three cases is shown in Figure 4-19.

Figure 4-19. Maximum longitudinal stress for three ballast materials

A larger stress can be observed at the under-head radius for a degradation ballast layer while the stress at rail foot tip is larger for a new-laid ballast layer. However, the fatigue

damage is not sensitive to the ballast elastic modulus changes even though Liu & Mahadevan criterion was applied as shown in Figure 4-20. This is because the stress amplitude is used for fatigue damage calculation rather than the maximum stress and ballast stiffness does not exert a significant effect on the dynamic stress range. Therefore, the elastic modulus of the ballast layer was selected as 150 MPa for all the analysis and a new-laid ballast layer was considered in this research.

Figure 4-20. Fatigue damage according to Liu & Mahadevan criterion

A larger stress can be observed at the under-head radius for a degradation ballast layer while the stress at rail foot tip is larger for a new-laid ballast layer. However, the fatigue damage is not sensitive to the ballast elastic modulus changes even though Liu & Mahadevan criterion was applied as shown in Figure 4-20. This is because the stress amplitude is used for fatigue damage calculation rather than the maximum stress and

ballast stiffness does not exert a significant effect on the dynamic stress range. Therefore, the elastic modulus of the ballast layer was selected as 150 MPa for all the analysis and a new-laid ballast layer was considered in this research.

4.5.3. Sleeper Material Effect

The effect of the sleep materials was also investigated. The parameters used in FE model remain the same as mentioned before, except the wood properties are used to substitute the concrete sleeper material. The fatigue damage calculated from the stress history based on the three criteria are plotted in Figure 4-21. From Figure 4-21, overall a small difference of the fatigue damage (less than 10%) is obtained for the wood and concrete sleeper, but a smaller fatigue damage at the under-head radius and rail foot is found for the wood sleeper. This is because the stiffness of the wood sleeper is smaller than that of the concrete sleeper so that the in-plane bending effect of rail is less severe for the wood sleeper. It is also interesting to note that the fatigue damage for the wood sleeper calculated by Dang Van and Mataka criteria is slightly smaller at the head-to-web, web and web-to-foot regions, while the fatigue damage calculated by Liu & Mahadevan criterion is slightly larger at these regions. The fatigue damage at the web-to-foot region for all the three criteria is larger for wood sleeper because the Tresca shear stress is larger while the other components such as hydrostatic stress and maximum normal stress times a weight factor does not change much. The reason for

the fatigue damage is smaller at the web and head-to-web regions is similar that the Tresca shear stress at these regions are smaller for the wood sleeper while the other components do not increase based on Dang Van and Matake criteria. However, the fatigue damage at these regions is larger for the wood sleeper according to Liu & Mahadevan criterion. This can be attributed to the larger normal stress amplitude and shear stress at these regions which results in a higher fatigue damage value for the wood sleeper since the critical plane searching method for Liu & Mahadevan criterion is different.

Figure 4-21. Comparison results of fatigue damage based on three multiaxial fatigue criteria for wood and concrete sleeper materials.

As for the critical plane orientation, the overall directions are quite similar for both cases. However, the angle θ is a little bit larger for both Dang Van and Matake criteria

and smaller for Liu & Mahadevan criterion at the under-head radius and rail foot tip, and opposite at the head-to-web, web and web-to-foot regions. This is because the longitudinal stress is larger at the under-head radius and rail foot and smaller at the head-to-web, web and web-to-foot regions which makes the shear stress-based critical plane tilt away from the longitudinal direction (z -axis) and the normal stress-based critical plane tilt towards the longitudinal direction at the under-head radius and rail foot tip and opposite at the head-to-web, web and web-to-foot regions.

4.5.4. Train Velocity Effect

The velocity of the train is an essential factor which affect the 3D dynamic train wheel-induced loadings when a train is passing a curved track. Therefore, the fatigue assessments through different train velocities (40 km/h, 70 km/h and 100 km/h) are performed to investigate the effect of train velocity on the fatigue damage. Concrete sleepers were selected and other parameters remained the same as before. The 3D dynamic wheel-rail contact forces are 157 kN for the vertical direction, 62.8 kN for the lateral direction and 1.1 kN for the longitudinal direction according to the case of a 100 km/h train runs on a 300 m curved track, and the forces are 107 kN, 41 kN and 6 kN for a train velocity of 40 km/h. The results of fatigue damage according to different train velocities are plotted in Figure 4-22.

(A)

(B)

(C)

Figure 4-22. Fatigue damage due to train velocity effect (40 km/h, 70 km/h and 100 km/h) based on: (A) Dang Van, (B) Mataka and (C) Liu & Mahadevan criteria

From Figure 4-22, an obvious difference between the fatigue damage curves can be monitored according to different train velocities of 40 km/h, 70 km/h and 100 km/h. A larger difference of the damage is presented at the upper web to head regions when the speed increases from 40 km/h to 70 km/h, while a larger difference is obtained at the regions below the rail neutral axis and above the rail foot when the speed increases from 70 km/h to 100 km/h.

(A)

(B)

Figure 4-23. (A) Longitudinal stress and (B) Tresca shear stress for the train speed of 40 km/h, 70 km/h and 100 km/h, respectively

The reason is that the out-of-plane bending and lateral torsional bending effect cause a relative large increase of the longitudinal stress and Tresca shear stress at the under-head radius comparing with other regions when the speed increases from 40 km/h to 70

km/h, while the in-plane bending effect dominates which makes the longitudinal stress and Tresca shear stress increase dramatically at the lower web to upper foot regions when the speed increases from 70 km/h to 100 km/h (shown in Figure 4-23).

There is an interesting phenomenon should be noted that fatigue damage at the rail foot tip does not increase and even decrease based on Liu & Mahadevan criterion, this is due to the virtue of the clips periodical opening and closing effects which can release the stress at the rail foot regions to prevent a large stress and damage comparing with the fixed the rail-bottom boundary conditions which is usually adopted for simplification.

As for the critical plane orientations, the overall angle almost remains the same at the regions of interest for all the three situations. Based on the angle θ calculated by Liu & Mahadevan criterion, the angle increases at the lower web to upper foot regions (rail height from 25 mm to 50 mm) as the train velocity increases which means the critical plane for the maximum normal stress amplitude is less likely parallel to the rail section. This is because the fact that the Tresca shear stress increases faster at these regions than the longitudinal stress when the speed increases. The angle θ also increases at the head-to-web and upper web regions (from 93 mm to 125 mm) as the velocity increases. This is due to the in-plane bending effect counters the out-of-plane bending effect which makes the longitudinal tensile stress at the head-to-web and upper web regions even smaller for the 100 km/h speed situation. This makes the critical plane orientation less

perpendicular to the longitudinal direction.

The angle θ calculated by both Dang Van and Mataka criteria decreases at the lower web to upper foot regions according to the similar reason.

4.5.5. Track Curvature Effect

Two more cases with 500 m and 700 m curved track are investigated to study the track curvature effect. Concrete material is used for sleepers and other conditions remain the same. The 3D dynamic wheel-rail contact forces calculated from UM are 113 kN for the vertical direction, 42.2 kN for the lateral direction and 4.6 kN for the longitudinal direction according to the case of a 70 km/h train negotiates a 500 m curved track, and the forces are 104 kN, 33.6 kN and 4.9 kN for a 700 m curved track. In this research, two-point contact between wheel and rail is considered for all three models. The fatigue damage values calculated by the three criteria are plotted in Figure 4-24.

From Figure 4-24, a larger damage for a 300 m curved track and a smaller damage for a 700 m curved track can be obtained for all the three criteria. For the curved track of 500 m and 700 m, the values of damage increase obviously at the under-head radius, head-to-web, middle and upper web regions and have a little increment at the lower web to rail foot regions when the radius decreases. For the curved track of 300 m and 500 m, the values increase at the rail foot, upper web-to-foot, middle and upper web regions and vary little at the other regions when the radius decreases. The reason is

because the out-of-plane bending effect has a great effect when the radius varies from 700 m to 500 m which makes the rail head bend and twist to the field side, and this results in a higher longitudinal and shear stress at the regions above the neutral axis. The increasing vertical force for the radius decreasing from 500 m to 300 m increase the in-plane bending effect and the lateral-torsional buckling effect towards the gauge side which balances the out-of-plane bending effect, this results in a less increment of stress value at the regions above the neutral axis and a larger increment at the rail foot.

(A)

(B)

(C)

Figure 4-24. Fatigue damage for 300 m, 500 m and 700 m radii curved track based on: (A) Dang Van, (B) Matake and (C) Liu & Mahadevan criteria

For the critical plane direction, the value of angle θ does not vary much for the three

cases according to all the three criteria except little difference shown in some elements which is due to the numerical errors. This is because the longitudinal stress and shear stress increase simultaneously with a similar increment ratio as the radius decreases, which makes the overall critical plane orientations almost the same.

4.6. Investigations on Out-of-phase Effect and Gauge Side Damage

4.6.1. Fatigue Assessment with Maximum Rectangular Hull Method

Figure 4-25. Rotating callipers algorithm with Maximum Rectangular Hull method implemented in Dang Van criterion

Maximum Rectangular Hull method was also applied in this research to investigate the out-of-phase effect. A linear time rotating callipers algorithm [156] was implemented together with Maximum Rectangular Hull method in Dang Van criterion for the critical plane searching process. The calculation steps are shown in Figure 4-25. The comparison results of Maximum Rectangular Hull and Minimum Circumscribe Circle methods are shown in Figure 4-26.

Figure 4-26. Comparison results for Maximum Rectangular Hull and Minimum Circumscribe Circle methods

A more conservative result can be observed for Maximum Rectangular Hull method at the web regions. The reason is that the principal stress orientations at the web region rotate during the loading cycle since the other five stress components are comparable to the longitudinal stress. The fatigue damage at the web-to-foot region is also slightly larger when Maximum Rectangular Hull method is used. However, the out-of-phase

effect is not obvious at under-head radius, head-to-web and rail foot regions because of relatively larger longitudinal stresses which determine the principal stress orientations. Therefore, the predicted fatigue damage and fatigue life at these regions are less likely affected by the resolved shear stress calculation methods.

The computational time for the Minimum Circumscribe Circle method implemented with the smallest enclosing circle algorithm is slightly shorter (20%~30%) than the Maximum Rectangular Hull method. This is because the resolved shear stress amplitude is calculated for each rotation in the Maximum Rectangular Hull method while the result is just saved without calculation if the added point is inside the circle when the smallest enclosing circle algorithm is used.

4.6.2. Fatigue Damage on the Gauge Side

From the field observations, there are some failures also initiated at the head-to-web region on the gauge side. The fatigue damage calculated from Dang Van criterion on the gauge side for a train with a speed of 70 km/h runs on a 300 m radius and 100 mm cant height curved track is shown in Figure 4-27.

Surprisingly, the fatigue damage at the under-head radius and head-to-web region is larger on the gauge side rather than the field side. The reason can be explained by a larger resolved shear stress amplitude at these regions because the resolved shear stress amplitude also increases as compressive stress increasing. The maximum longitudinal

stress and resolved shear stress amplitude are shown in Figure 4-28.

Although the longitudinal stress on the gauge side is smaller and a large compressive stress occurs when the wheel just reaches the weld due to the out-of-plane bending effect, the tensile stress at the under-head radius can still reach ~ 110 MPa as shown in Figure 4-28(A) due to the reverse bending effect. Therefore, the resolved shear stress amplitude at the under-head radius and head-to-web regions is larger on the gauge side because the maximum longitudinal stress is tensile stress and a large compression stress presents during the loading cycle.

Figure 4-27. Fatigue damage on the gauge side and field side

(A)

(B)

Figure 4-28. (A) Maximum longitudinal stress and (B) resolved shear stress amplitude on both the gauge side and field side

4.7. Summary

In this chapter, stress and fatigue assessment methodology was developed. The 3D wheel-rail contact forces were obtained through multibody system dynamic models and applied in finite element models. The multiaxial stress histories of the thermite weld were obtained from finite element analysis following an implicit-explicit modelling sequence and validated by strain measurement at a depot through a wireless strain sensor system. A linear time smallest enclosing circle algorithm, implemented in three stress-based multiaxial fatigue criteria, was proposed to reduce the computation costs for the critical plane searching process.

The fatigue analysis reveals the rail foot tip is the most critical location which has a large probability to initiate a crack within 4 years. The under-head radius, head-to-web and web-to-foot regions also endure relatively large damage values. The fatigue damage analysis shows that the Liu & Mahadevan criterion gives the most conservative prediction at the under-head radius, web-to-foot and rail foot regions while Matake criterion gives the most conservative prediction at head-to-web and upper web regions.

The predicted critical plane orientations for fatigue crack initiation based on shear stress-based criteria (Dang Van and Matake) and normal stress-based criterion (Liu & Mahadevan) are different. The predicted critical plane orientations for all these three criteria are not along the longitudinal rail direction, which can explain that some rails

fractured with a small angle to the transverse plane. Parametric studies were also performed in this research. The ballast stiffness and sleeper materials do not show much influence on the fatigue behavior while the train velocity and the curved track radius exert a great influence.

The out-of-phase effect was also investigated through the Maximum Rectangular Hull method. A more conservative result can be observed for the Maximum Rectangular Hull method at the web regions while little difference presented at under-head radius, head-to-web and rail foot tip. As for the gauge side, relatively large fatigue damage also presents at the head-to-web region indicates it is a possible failure site.

Chapter 5. Constraints and Defect Studies of Thermite Weld

In this chapter, the effect of different constraints such as missing clips and different track types on the fatigue damage of the thermite weld were investigated. Defect analysis through several methods was also performed to provide a comprehensive analysis of the thermite weld failures.

5.1. Fatigue Assessment Considering Missing Clip Effect

The rail clips may not be in their original positions because of fracture failures or pop-out and this can result in a lack of constraint on the rail foot. Therefore, six models were established to investigate the effect of missing clips as shown in Figure 5-1. The material properties, boundary conditions and interactions kept the same as stated in Chapter 4.

Figure 5-1. Clip missed before and after the thermite weld

The fatigue damage was calculated according to the smallest enclosing algorithm and compared with the original model introduced in Chapter 4. The fatigue damage based on Dang Van criterion is shown in Figure 5-2 and Figure 5-3.

(A)

(B)

Figure 5-2. Fatigue damage on the field side calculated based on six missing clip FE models

(A)

(B)

Figure 5-3. Fatigue damage on the gauge side calculated based on six missing clip FE models

The fatigue damage at the foot tip on the field side slightly decreases when the clips especially the gauge side clips are lost (235 MPa for the original model, 220 MPa for the case 2 and 223 for the case 5). This is because the upward displacement of the rail

foot is less constrained and the in-plane bending effect is reduced. A larger difference can be observed at the foot tip when the gauge side clips are not at their original locations. (100 MPa for the original model, 134 MPa for the case 1 and 123 MPa). This is because of the twisting behavior when only one side is constrained under the lateral force. The asymmetrical phenomenon can be observed for the clips lost before and after the weld which is because the rail is not constrained symmetrically by the clips on both sides and a larger difference will present if the clips closer to the weld are lost. Overall, the missing clips do not exert a significant effect on the fatigue damage of the thermite weld and the fatigue life will barely change since the rail clips will be replaced during routine maintenance.

5.2. Fatigue Assessment for Four Track Structural Systems

In service, two types of track systems (ballast track and slab track) with E-clip or fast-clip are commonly used for different rail segments. Therefore, three track FE models together with the original model studied in Chapter 4 were established to investigate the effect of different track types as shown in Figure 5-4. All other conditions remain the same as described in Chapter 4. It should be noted that for there is an additional layer of the rubber pad placed between the base plate and sleeper for the slab track with the E-clip. The fatigue damage calculated with the Dang Van criterion is shown in Figure 5-5.

Figure 5-4. Four track structural systems: (A) ballast track with E-clip, (B) ballast track with fast-clip, (C) slab track with E-clip and (D) slab track with fast-clip

(B)

Figure 5-5. Fatigue damage for four track systems: (A) on the field side and (B) on the gauge side

Compared with the ballast track, the fatigue damage is larger on both the field side and gauge side when a slab track system is used. On the field side, the fatigue damage is larger for the ballast track with the fast-clips and for the slab track with the E-clips. On the gauge side, the fatigue damage for the slab track systems is larger at the under-head radius, web-to-foot and foot tip regions with the fast-clips and the difference is small at the web region. This is because the in-plane bending effect is larger for the E-clip system which causes larger stresses at the regions away from the neutral axis. Similar results can be observed for the ballast track system, but the fatigue damage is larger at the under-head radius, web-to-foot and foot tip regions with the fast-clips. Interestingly, for the same rail clips, the fatigue damage curves of the ballast track and slab track are

close to each other on the gauge side. The fatigue damage curves of the ballast and slab track systems with the fast-clips are also close. However, the difference is more significant for the ballast and slab track systems with the E-clips due to an additional layer of the rubber pad placed between the base plate and sleeper. The difference in the stiffness under the rail can result in a significant change in the in-plane bending effect.

For these four types of track structural systems, the fatigue damage at the foot tip on the field side all exceeds the fatigue limit. This implies a large probability for the crack initiating at the foot tip within 4 years. The fatigue damage is also large and close to the fatigue limit at the web-to-foot region for the slab track with E-clips. As for the gauge side, the fatigue damage at the under-head radius (~ 197 MPa) closes to the fatigue limit for the slab track with E-clips and the head-to-web region also experiences large fatigue damage. The fatigue damage for the slab track with fast-clips is also large (~ 168 MPa) at the under-head radius.

Overall, the fatigue damage for the slab track system with E-clips is more severe than the other three systems which agrees well with the field observations that more fatigue failures of the thermite weld happened in the tunnel where the slab track with E-clips is applied in recent years. The fact that the train speed previously was 70 km/h when passing a 300 m curved track in the tunnel while the train speed reduces to 50~60 km/h recently also indicates the poor performance of this track system.

5.3. Defect analysis of thermite weld

5.3.1. Crack Initiation Risk for Small Defects

Small defects either internal or surface can form in the thermite weld especially at the head-to-web and web-to-foot transition regions due to the complex geometries casting nature of the process [3]. The fatigue resistance will reduce as the defect sizes increase. The fatigue limit can be calculated from Eq (2-45) and (2-46) for different sizes of surface and internal defects. Assuming the ratio of the reversed torsional fatigue limit and bending fatigue limit t_{-1}/f_{-1} is 0.58, the torsional fatigue limit can be expressed as:

$$t_{-1} = 0.8294(H_V + 120)/(\sqrt{area})^{1/6}, \text{ for surface defects} \quad (5-1)$$

$$t_{-1} = 0.9048(H_V + 120)/(\sqrt{area})^{1/6}, \text{ for internal defects} \quad (5-2)$$

The hardness of the thermite weld at the head-to-web region is 28.7 HRC as measured in Chapter 3 and is equivalent to 291 HV. The fatigue crack initiation risk can be estimated as the ratio of calculated fatigue damage and Dang Van critical limit for each defect size. The risk equals one indicates a large probability of the occurrence of fatigue failures. The fatigue crack initiation risks at four locations (head-to-web and web-to-foot regions on both gauge and field sides) are plotted in Figure 5-6.

(A)

(B)

Figure 5-6. Fatigue Crack Initiation Risk at four transit areas with different sizes of (A) surface defect and (B) internal defect

The fatigue crack initiation risk is low when the defect is small (less than 300 μm for surface defects and less than 500 μm for internal defects) at all four regions, but the risk increases considerably with the increasing defect size. The crack initiation risk at the head-to-web regions on the gauge side is the highest because of a larger resolved shear stress amplitude presented in the previous chapter. When the defect size is larger than a certain value (300 μm for surface defects and 500 μm for internal defects), the risk is higher than one which indicates the damage tolerance limit of thermite weld at this region. The crack initiation risks at the head-to-web and web-to-foot regions on the field side are also quite high as they reach ~ 0.9 and ~ 0.8 for larger surface and internal cracks. It seems quite difficult for crack to initiate at the web-to-foot region on the gauge side as the fatigue crack initiation risk is only 0.4~0.5 even there is a 1000 μm defect.

Considering the longitudinal stress analysis result in section 4.6.2, the maximum longitudinal tensile stress at the head-to-web region on the field side, the head-to-web region on the gauge side, the web-to-foot region on the field side and the web-to-foot region on the gauge side are 190 MPa, 120 MPa, 170 MPa and 60 MPa, respectively. Combining the high fatigue crack initiation risks in the first three regions and low fatigue crack initiation risks in the last region, the fatigue fracture will very rarely occur in the web-to-foot region on the gauge side but occurs in the other three weld transit regions which agree well with the field observations as mentioned in the introduction chapter.

5.3.2. Notch Correction

Due to the presents of defects on the thermite weld, the fatigue life curve of the thermite weld structure is below the fatigue life curve of the smooth specimen extracted from the thermite weld although the stress concentration effect of the weld geometry is considered as shown in Figure 5-7.

Figure 5-7. Scatter plot of the fatigue test results from Chapter 3

It should be noted here that the mean stress effect was considered for the inverted tests and the Half-slope model was applied since it provides the best results for simple non-optimized solutions based on static material parameters [157] when calculating the strain amplitude. The Half-slope model is expressed as:

$$\sigma_{a,eq} = \frac{\sigma_a}{1 - \frac{\sigma_m}{2\sigma_u}} \quad (5-3)$$

Therefore, the stress concentration effect of defects on the thermite should be considered to correlate the fatigue life of the thermite weld material and structure. The flow chart of the notch effect correction of the defects is shown in Figure 5-8.

Figure 5-8. Flow chart of the notch effect correction for defects on the thermite weld

5.3.2.1. Fatigue Parameters Estimation for RBF Tests

The stress-life ($S-N$) and strain life ($\varepsilon-N$) methods usually used for fatigue crack initiation or nucleation life prediction can be expressed as:

$$\sigma_a = \sigma'_f (2N_f)^b, \text{ for } S - N \text{ method} \quad (5-4)$$

$$\varepsilon_a = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c, \text{ for } \varepsilon - N \text{ method} \quad (5-5)$$

where σ_a is the stress amplitude, N_f is the number of cycle to failure when $R = -1$ thus $2N_f$ is the number of reversals. Two fatigue property parameters: fatigue strength coefficient σ'_f and fatigue strength exponent b can be obtained by curve fitting of stress-controlled fatigue test results. ε'_f and c represent the fatigue ductility coefficient and exponent. A value of -0.56 was suggested for the fatigue ductility exponent c and the fatigue ductility coefficient ε'_f can be estimated according to hardness test result given by Roessle and Fatemi [158]:

$$\varepsilon'_f = \frac{1}{E} [0.32(HB)^2 - 487(HB) + 191000] \quad (5-6)$$

For a Ramberg-Osgood-type equation as described in Eq(2-38), the cyclic strength coefficient K' and cyclic strain hardening exponent n' of the material can be obtained from tensile test results. The correlations can be expressed according to the steel group for $\sigma_u/\sigma_y \leq 1.2$ [131]:

$$K' = 3.0 \times 10^{-4} (\sigma_u)^2 + 0.23(\sigma_u) + 619 \quad (5-7)$$

$$n' = -0.37 \log \left(\frac{3.0 \times 10^{-4} (\sigma_y)^2 - 0.15(\sigma_y) + 526}{3.0 \times 10^{-4} (\sigma_u)^2 + 0.23(\sigma_u) + 619} \right) \quad (5-8)$$

The thermite weld material parameters obtained from experimental results as stated in Chapter 3 for stress-life and strain-life methods are listed in Table 5-1. The fatigue life curve for the RBF tests was shown in Figure 5-9 and stress-life method fitted better with the test data because the plastic part was added on the elastic part for strain-life method. The coefficient of determination for stress-life and strain-life methods were 0.69 and 0.57, respectively. The scatter found in RBF test data could be explained by different sizes, shapes and orientations of micro-pores and their distances to the surface proved by SEM in Chapter 3.

Table 5-1. Material parameters for the thermite weld material

Yield Stress, Ultimate Stress,		Rockwell C	Fatigue strength	Fatigue strength	Fatigue	Fatigue ductility
σ_y (MPa)	σ_u (MPa)	hardness, HRC	coefficient, σ'_f (MPa)	exponent, b	ductility coefficient, ϵ'_f	exponent, c
720	758	28.7	935.48	-0.06	0.3855	-0.56

Figure 5-9. Fatigue life curve of RBF test data based on stress-life and strain-life methods

5.3.2.2. Modelling Defects on the Specimen and Stress Concentration Factor (SCF)

The effects of defect shape, dimension, and orientation on K_t were investigated through FE method. Three types of surface defects such as spheroidal (S), semi-ellipsoidal (E) and coin (C) shapes with different dimensions were modelled in this research. The 3D geometries of the specimen and these defects are shown in Figure 5-10. The spheroidal shaped defects were studied with six radii such as 25, 50, 75, 100, 150 and 200 μm . The semi-ellipsoidal shaped defects were studied with minor axis of 50 and 100 μm and the aspect ratios of 1, 2, 3, 5 and 10 for major axis. The radii and

thickness of coin shaped defects were 50, 100, 150 and 200 μm . The orientations of these defects were shown in Figure 5-10(C). For coin shaped defects, C-X and C-Z means the thickness direction align with X and Z axes. E-X, E-Y and E-Z means the direction of major radii align with X, Y and Z axes for semi-ellipsoidal shaped defects.

Figure 5-10. (A) 3D model in Abaqus, (B) 3D sketches for spheroidal, semi-ellipsoidal and coin shaped surface defects and (C) X-Z plane layout for spheroidal, semi-ellipsoidal and coin shaped surface defects

The SCF K_t and the von Mises stress clouds for some of the simulation results were shown in Figure 5-11. The maximum stresses for spheroidal and semi-ellipsoidal shape defects with the major axis along Y and Z axes (E-Y and E-Z) located near the surface with the smallest radii in the planes parallel with the bending moment direction. The maximum stresses for semi-ellipsoidal shape defects with the major axis along X axis

(E-X) shifted from the surface to the bottom. The stress concentration points for coin shape defects located at four corners.

Figure 5-11. Von Mises stress clouds: (A) spheroidal defect with a radius of 150 μm , (B)(C) coin defect with the radius and thickness of 150 μm , (D)(E)(F) semi-ellipsoidal defect with the minor axis of 100 μm and the aspect ratio is 3

The SCF K_t for spheroidal shape defects is 1.8 to 2.0 and increases with the radius. The SCF K_t for coin shape defects along X axis is 1.9 to 2.1 and for the defects along Z axis is 2.0 to 2.1. The SCF K_t for semi-ellipsoidal shape defects with the major axis along X, Y and Z axes is 2.1 ~ 2.7, 2.2 ~2.6 and 1.1 ~ 1.5, respectively. The SCF K_t increases when the dimension of minor axis for all three orientations. When the aspect

ratio increases, the SCF K_t increases for the major axis of ellipsoidal shape defects along X and Y axes while decreases for the major axis along Z axis. Overall, semi-ellipsoidal shape defects present a higher stress concentration effect.

5.3.2.3. Notch Correction and Fatigue Life for Thermite Weld Structure

In this research, Neuber's and Glinka's rules described in Chapter 2 were used to consider the notch effect for the defects. As shown in Eq (2-39) and (2-40), the actual stress amplitude increases as the SCF K_t increases when notch effect was considered according to Neuber and Glinka models which means the predicted fatigue life goes in the opposite direction to K_t . The SCF K_t with the values of 2.0~2.7 were implemented to estimate the fatigue life based on stress-life and strain-life methods. The results were shown in Figure 5-12. Overall, the strain-life method showed a better prediction than the stress-life method and the strain-life curve was above the stress-life curve for all the values of SCF K_t .

(C) $K_t=2.2$

(D) $K_t=2.3$

(E) $K_t=2.4$

(F) $K_t=2.5$

(G) $K_t=2.6$

(H) $K_t=2.7$

Figure 5-12. Fatigue life prediction used stress-life and strain-life methods

considering notch effect based on Neuber and Glinka models with different K_t

The predicted fatigue life curves with Glinka's rule shows a better curve fitting compared with Neuber's rule. Neuber's rule provides a conservative prediction compared to Glinka's rule especially for a shorter fatigue life. The reason is that Neuber's rule is a special case of Glinka's rule when the dissipation of the heat energy is neglected [159] and will overestimate the local stresses and strains when localized plastic yielding occurs at the notch tip [129, 134, 160]. The strain-life method with Glinka's rule provides the best prediction with the inverted fatigue test data when the SCF K_t is 2.6 as shown in Figure 5-12(G). The coefficients of determination are 0.56 and 0.68 for Neuber's and Glinka's rule. As for the stress-life method, SCF $K_t=2.4$ provides a better curve fitting with the coefficients of determination of 0.44 and 0.51 for Neuber's and Glinka's rule as shown in Figure 5-12(E). From Figure 5-12(B) and (C), it seems that SCF $K_t = 2.1$ and 2.2 are suitable for the thermite weld specimen with peening when stress-life and strain-life method are choosing. This could be explained by the reason that the sizes of the surface defects are reduced and the severity of the stress concentration effect at the head-to-web region is alleviated through peening.

5.3.3. Fatigue Life Prediction with Rainflow Cycle Counting Approach

Figure 5-13. Flow chart for fatigue life prediction at critical locations on the thermite weld

Combining the strain measurements and FE analysis results, the service stress spectrum using the variable amplitude strain analysis spectrum at the stress concentration site can be obtained and the fatigue life at critical locations on the thermite weld can be predicted using damage accumulation method. The fatigue life prediction process with rainflow cycle counting approach is shown in Figure 5-13.

FE model of the slab track system with E-clips was established first as described in the previous section to compare the simulation results with on-site strain measurement results at the strain rosette location. The comparison results are plotted in Figure 5-14.

Figure 5-14. FE results comparing with test results for (A) longitudinal strain and (B) vertical strain

The simulation results show the same trend exists both in longitudinal strain and vertical strain compared to test results. In Figure 5-14(A), both simulation and test results show a tension peak at ~ 1.36 s. This tensile spike can be considered as the main reason for crack propagation at the thermite weld, especially for a transverse failure. The difference in strain peak value between FE results and test results is due to irregular geometry of the test weld and the complex vehicle-track interaction and boundary conditions for a curved track cannot be accurately captured in models.

To determine the stress scale factor from strain measurement location to the weld edge, stress profiles of two elements at these two locations from FE models were compared and the scale factor for weld fracture location at head-web corner is 3.0 in this example. The rainflow matrix based on the rainflow cycle counting approach obtained from a commercial software DATS analysis is shown in Table 5-2.

Table 5-2. Damage accumulation based on rainflow cycle counting approach

Stress (Pa)		Mean	7.04E+06												
Range	After scale		1	2	3	4	5	6	7	8	9	10	Total	Expected life	n/N_i
9.12E+06	2.74E+07	1	0	0	0	0	4	2	0	0	0	1	7	3.87E+08	1.8077E-08
1.17E+07	3.50E+07	2	0	0	0	0	1	1	1	0	0	3	6	1.70E+08	3.52022E-08
1.42E+07	4.27E+07	3	0	0	0	0	0	1	1	0	0	1	3	8.83E+07	3.39813E-08
1.68E+07	5.03E+07	4	0	0	0	0	0	0	0	0	0	0	0	5.10E+07	0
1.93E+07	5.80E+07	5	0	0	1	0	0	0	0	0	0	0	1	3.18E+07	3.1422E-08
2.19E+07	6.56E+07	6	0	0	1	1	0	0	0	0	0	0	2	2.11E+07	9.4954E-08
2.44E+07	7.33E+07	7	0	2	4	0	0	0	0	0	0	0	6	1.46E+07	4.11241E-07
2.70E+07	8.09E+07	8	1	4	0	0	0	0	0	0	0	0	5	1.05E+07	4.77011E-07
2.95E+07	8.86E+07	9	2	1	0	0	0	0	0	0	0	0	3	7.76E+06	3.86643E-07
3.21E+07	9.62E+07	10	1	2	1	0	0	0	0	0	0	0	4	5.89E+06	6.79285E-07
														Total	2.16782E-06

The S-N curve expressed below is from where the crack growth life was predicted for an initial crack size of 0.025 mm to a final crack size of 5mm.

$$N = \left(\frac{S}{10383} \right)^{-3.33} \quad (5-9)$$

The total damage calculated for per train is 2.17e-6 shown in Table 5-2. Then the expected number of trains pass by is 461294, equivalent to fatigue life of ~4 years.

5.4. Summary

In this chapter, finite element and fatigue analysis were carried out to investigate the missing clip effect. Overall, the missing clips do not exert a significant effect on the fatigue damage of the thermite weld and the fatigue life will barely change since the rail clips will be replaced during routine maintenance.

The effect of track types was investigated through FE models of four track structural systems: ballast track with E-clips, ballast track with fast-clips, slab track with E-clips and slab track with fast-clips. Overall, the fatigue damage for the slab track system with E-clips is more severe than the other three systems. For these four types of track structural systems, the fatigue damage at the foot tip on the field side all exceeds the fatigue limit which implies a large probability for the crack initiating at the foot tip within 4 years. The fatigue damage at the under-head radius and head-to-web region on the gauge side closes to the fatigue limit for the slab track with E-clips and the high damage regions agree well with the field observations.

Defect analysis through several methods was also performed. The fatigue resistance will reduce as the defect sizes increase. The fatigue crack initiation risk that is defined as the ratio of calculated fatigue damage and fatigue limit for each defect size indicates a large probability of the occurrence of fatigue failures at the head-to-web and web-to-foot regions on the field side as well as the head-to-web on the gauge side. When considering the notch effect on the fatigue life curve of the thermite weld material, Glinka's rule with the strain-life approach gives the best prediction for the inverted four-

point bend fatigue test results. According to the on-site strain measurement results, the crack propagation life is ~4years calculated with the rainflow cycle counting approach.

Chapter 6. Fatigue Assessment of Rail Clips

Fatigue assessment of rail clips is also necessary to understand the failures of E-clip and fast-clip as stated in Chapter 1 and Chapter 2. In this chapter, the fatigue properties of clip material and structure were analyzed by using the strain-life method. The fatigue damage and fatigue life assessments were analysed using the Fatemi-Socie multiaxial fatigue criterion implemented with the smallest enclosing circle algorithm introduced in Chapter 4 as shown in Figure 6-1. Finally, the installation and slab track effects on the fatigue life were also investigated.

Figure 6-1. Smallest enclosing circle algorithm with Fatemi-Socie criterion

6.1. Structural Validation of Rail Clips

6.1.1. Static Pull-up Tests for Structural Validation

Static pull-up experiments were conducted in Chapter 3 to study the deformation behavior of E-clip and fast-clip. The structural behavior of the clips was also investigated through implicit FE models of E-clip and fast-clip as shown in Figure 6-2(A) and (C), respectively. A steel plate was modelled to simulate the clamped condition of E-clip leg in Figure 6-2(A). A funnel-shaped steel holder was constructed and contact with the front seat of fast-clip shown in Figure 6-2(C). The loading conditions were shown in Figure 3-16. The element type applied in both models was C3D4 tetrahedron element and the average mesh size was 1 mm. The comparison results are shown in Figure 6-2(B) and (D). Overall, the simulation results for all the channels compare well with experimental results for E-clip and fast-clip.

Figure 6-2 Finite element models of (A) E-clip and (C) fast-clip, and comparison results for (B) E-clip and (D) fast-clip

6.1.2. Strain-life Relationship for Fast-clip

6.1.2.1. Strain-life Relationship for Clip Steel

When using strain-life approach to simulate the cyclic stress-strain relationship, the total strain can be expressed as summation of elastic and plastic components [161]:

$$\varepsilon_a = \varepsilon_{ea} + \varepsilon_{pa} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (6-1)$$

In this research, two strain-life models namely the Roessle and Fatemi (R-F) model and the Muralidharan and Manson (M-M) model were investigated to assess their suitability for clip material. R-F model [158] which is based on hardness and elasticity modulus can be expressed as:

$$\varepsilon_a = \frac{4.25(HB)+225}{E} (2N_f)^{-0.09} + \frac{0.32(HB)^2-487(HB)+191000}{E} (2N_f)^{-0.56} \quad (6-2)$$

where HB is Brinell hardness of the material.

M-M model [162] based on material ultimate tensile strength and true fracture ductility is expressed as:

$$\varepsilon_a = 0.623E \left(\frac{\sigma_u}{E} \right)^{0.832} (2N_f)^{-0.09} + 0.0196\varepsilon_f^{0.155} \left(\frac{\sigma_u}{E} \right)^{-0.53} (2N_f)^{-0.56} \quad (6-3)$$

where ε_f is true fracture ductility which can be calculated according to area of reduction $\%RA$ of the specimen based on constant volume assumption:

$$\varepsilon_f = \ln \left(\frac{100}{100 - \%RA} \right) \quad (6-4)$$

The experimental results in section 3.1.3 and two strain-life models are plotted in Figure 6-3. A better fit with experimental data can be observed for the Roessle and Fatemi (R-F) model.

Figure 6-3. Strain-life curve based on Roessle and Fatemi model (R-F model) and Muralidharan and Manson model (M-M model)

It should be noted here that the mean stress effect was considered and the Half-slope model was applied as stated in Chapter 5. The Half-slope model is expressed as:

$$\sigma_{a,eq} = \frac{\sigma_a}{1 - \frac{\sigma_m}{2\sigma_u}} \quad (6-5)$$

6.1.2.2. Strain-life Relationship for Clip Structure

The geometric model of the Fast-clip fatigue test is shown in Figure 6-4. A Ramberg-Osgood-type material with isotropic hardening was applied for the Fast-clip material properties. The cyclic strength coefficient and cyclic strain hardening exponent K' and n' in Eq (2-38) can be solved in a compatibility method:

$$n' = \frac{b}{c}, K' = \frac{\sigma_f'}{(\epsilon_f')^{b/c}} \quad (6-6)$$

Figure 6-4. Geometric model of Fast-clip fatigue test

The two legs were fixed as they were clamped by a rigid fixture. One upward cycle

loading was applied on the metal block shown in Figure 6-4. The mean load and amplitude were determined according to actual experimental conditions as listed in Table 3-6. The tetrahedron element, C3D4, was used to model the fast-clip, with an average element size of 1 mm and a total number of 99,551 elements. The metal block was modelled as a rigid body. Stress and strain histories for the elements at the rear arch were extracted from simulation results for fatigue analysis. The fatigue damage for six FE models was calculated based on Fatemi-Socie criterion and three criteria (von Mises, Tresca and maximum principal strain) from Eq (2-28), (2-29) and (2-30) were used to check their suitability. The results were plotted in Figure 6-5.

From Figure 6-5, the fourth point is below the curve for all criteria and the fifth point is further below the curve because the limit was set to one million cycles and no failure occurred for the lowest loading level. The maximum principal strain criterion is the most conservative since the last point in which no failure occurred is also above the curve. Overall, von Mises criterion shows the best accuracy and will be applied for further analysis.

(A)

(B)

(C)

Figure 6-5. Estimated strain-life curves based on (A) von Mises, (B) Tresca and (C) maximum principal strain criteria

6.2. Finite Element Modelling and Fatigue Analysis for Rail Clips

6.2.1.1. Finite Element Models Establishment

Figure 6-6. FE models of railway track with fine mesh of (A) fast-clip and (B) E-clip (BC 1 and BC 2: constrain lateral and longitudinal movement)

The FE models for ballast track structures including E-clip or Fast-clip are shown in

Figure 6-6. The curved track radius and train velocity considered in this study are 300 m and 70 km/h respectively. Implicit-explicit sequence FE analysis was performed as stated in Chapter 4. General contact was assigned among wheel, rail, clips and rail-pads with a friction coefficient of 0.3. The same boundary conditions as described in Chapter 4 were applied to constrain the models: the bottom of ballast layer was fixed; the lateral movement (along x-axis) of sleepers was constrained and the longitudinal movement (along z-axis) of rail was constrained.

All the simulation parameters obtained from experimental tests and literature [147, 148] were listed in Table 6-1.

Table 6-1. Material properties used in FE models

Parts	Material properties				
	Density	Elastic Modulus	Poisson's	Yield Stress	Ultimate Stress
	(kg/m ³)	(MPa)	Ratio	(MPa)	(MPa)
Rail	7850	210000	0.3	654	926
Sleeper	2500	38000	0.15	-	-
Ballast	1902	150~450	0.15	-	-
Rail clip	7850	216700	0.3	1311.3	1470.3
Wheel and Base plate	7850	210000	0.3	-	-
Rail pad	920	2.29	0.47	-	-

The linear hexahedral element C3D8R was used to model the track structures including wheel, rail, sleepers and ballast and the tetrahedron element C3D4 was used to model

the fastening system including rail clips, rail pad and base plate. The mesh of the fast-clip track model consists of 2032628 elements (85828 hexahedral elements and 1946800 tetrahedral elements) and 484345 nodes. The mesh of the E-clip track model consists of 1761578 elements (85828 hexahedral elements and 1675750 tetrahedral elements) and 427640 nodes. The average mesh size for fast-clip and E-clip was 1 mm and coarser mesh was applied for other parts except for base plate shoulder in order to reduce computational cost.

6.2.1.2. Fatigue Analysis and Life Prediction

The Tresca stress field outputs are shown in Figure 6-7. From Figure 6-7, the Tresca stress is larger for the fast-clip and E-clip at the gauge side and maximizes when the wheel is closed to the middle sleeper because the wheel-induced lateral force pushes the rail towards the field side. The Tresca stress maximizes at the field side when the wheel near the sleepers because the rail clips at the gauge side constrain the rail and rail vibrates on the base plate. For the E-clip, the sharp bend at the rear arch is more critical than the front arch at the gauge side (The Tresca stress is about 256 MPa at the gauge side and 71 MPa at the field side) and contrary at the field side (The Tresca stress is about 127 MPa at the gauge side and 164 MPa at the field side). As for the fast-clip, the Tresca stress maximizes at the rear arch near shoulder for the gauge side (413 MPa) while maximizes at the rear arch far from the shoulder for the field side (169MPa). The potential fatigue failure locations agree well with the actual rail clip fatigue failure sites.

Figure 6-7. Tresca stress field outputs of rail clips in ballast track: (A), (B), (C) for gauge side and (D), (E), (F) for field side

The stress-strain histories for the regions of interest of rail clips were extracted and processed for multiaxial fatigue analysis based on the Fatemi-Socie fatigue criterion implemented with the time-efficient critical plane searching approach described in Figure 6-1. The maximum shear strain amplitude and the maximum normal stress perpendicular to the critical plane were obtained, and the fatigue damage and fatigue life were calculated in Table 6-2.

From Table 6-2, it can be seen that the predicted lifetime is over 100-million cycles which is equivalent to over 38 years of service life for the rear arch and the front arch of E-clip for the gauge side and at the front arch of E-clip for the field side.

Comparatively, the fast-clip at the rear arch near shoulder of the rail gauge side has the largest fatigue damage under wheel-induced dynamic loading. The predicted fatigue life for this location is about 12.6-million cycles and the equivalent service life is 4.8 years, which provides a relatively accurate prediction of fatigue life for clips in service.

Table 6-2. Predicted rail clips fatigue life

E-clip						Fast-clip					
Locations		$\Delta\gamma/2$	$\sigma_{n,max}$ (MPa)	$f_{FS}(N_f)$	N_f	Locations		$\Delta\gamma/2$	$\sigma_{n,max}$ (MPa)	$f_{FS}(N_f)$	N_f
Rear arch	Gauge side	1.43e-3	176	1.75e-3	> 1e+8	Rear arch near shoulder	Gauge side	2.23e-140	2.70e-3	1.26e+7	
	Field side	6.0e-4	50	7.0e-4	> 1e+8		Field side	7.70e-149	1.01e-3	> 1e+8	
Front arch	Gauge side	1.30e-3	154	1.58e-3	> 1e+8						
	Field side	1.34e-3	94	1.54e-3	> 1e+8						

6.3. Parametric Studies for Rail Clips' Fatigue Life

6.3.1. Rail Clip Installation Effect

During the installation process, the rail clips are normally lifted to a certain height in order to exert a clamping force on the rail foot and hold the rail to the baseplate. The installation of fast-clip involves a 2-step procedure shown in Fig 6-8(A). The first step

is to set the fast-clip into its position followed by the second step to further lock the clip in the housing. The installation of the E-clip is simpler which just uses a tool to pull the E-clip into its position shown in Figure 6-8(B).

Figure 6-8. Installation of (A) fast-clip and (B) E-clip

Therefore, the installation effect on the fatigue life of rail clips was also investigated in this research and parametric studies were performed to analyse the effect of different installation stress levels. Different installation stress levels were obtained from FE models by lifting the clip toes with different distances (1mm, 2mm, 4mm, 6mm and 8mm). The installation stress was then combined with the wheel-induced dynamic stress to calculate the fatigue damage. The fatigue damage and estimated lifetime are shown in Table 6-3 and 6-4.

Table 6-3. Fatigue damage calculated under different stress level

Clip Deformation	E-clip locations				Fast-clip	
	Rear arch		Front arch		Rear arch near shoulder	
	Gauge side	Field side	Gauge side	Field side	Gauge side	Field side
1 mm	1.95e-3	9.2e-4	1.67e-3	1.53e-3	2.88e-3	1.18e-3
2 mm	2.1e-3	1.08e-3	1.77e-3	1.56e-3	3.05e-3	1.32e-3
4 mm	2.38e-3	1.32e-3	1.98e-3	1.78e-3	3.35e-3	1.55e-3
6 mm	2.58e-3	1.45e-3	2.17e-3	1.96e-3	3.5e-3	1.66e-3
8 mm	2.69e-3	1.51e-3	2.23e-3	2.11e-3	3.58e-3	1.81e-3

Table 6-4. Fatigue life predicted under different stress level

Clip Deformation	E-clip locations				Fast-clip	
	Rear arch		Front arch		Rear arch near shoulder	
	Gauge side	Field side	Gauge side	Field side	Gauge side	Field side
1 mm	> 1e+8	> 1e+8	> 1e+8	> 1e+8	6.49e+6	> 1e+8
2 mm	> 1e+8	> 1e+8	> 1e+8	> 1e+8	3.59e+6	> 1e+8
4 mm	4.87e+7	> 1e+8	> 1e+8	> 1e+8	1.45e+6	> 1e+8
6 mm	2.03e+7	> 1e+8	> 1e+8	> 1e+8	9.45e+5	> 1e+8
8 mm	1.32e+7	> 1e+8	6.04e+7	> 1e+8	7.66e+5	> 1e+8

Overall, from Table 6-3, an obvious effect can be observed on the fatigue damage even when the fast-clip and E-clip are lifted at a small distance. From Table 6-4, the predicted fatigue life of the fast-clip at the gauge side reduces as the clip deformation increasing and is even less than a million cycles (less than a year) when lifting the fast-clip to 6 mm. The estimated lifetime is also reduced dramatically for the E-clip at the gauge side

which implies potentially failure at the rear arch and the front arch of the E-clip under cyclic train wheel-induced loading. For both E-clip and fast-clip at the field side, even though the fatigue damage increases with the installation stress levels, the fatigue life is above 100-million cycles implies a long service life.

6.3.2. Fatigue Life prediction for Slab Track System

The effect of track types on the fatigue life of rail clips was also investigated and finite element models of two slab track systems with E-clip and fast-clip were established as shown in Figure 6-9. It should be noted that an addition layer of the rail pad was placed between the base plate and the concrete sleeper. The boundary conditions and mesh were kept the same as those applied in the ballast track models except that the bottom of the sleepers was fixed for the slab track system.

Figure 6-9. FE models of slab track with fine mesh of (A) fast-clip and (B) E-clip (BC 1 and BC 2: constrain lateral and longitudinal movement)

The Tresca stress field outputs of the slab tracks are shown in Figure 6-10. From Figure 6-10, the Tresca stress is also larger for the fast-clip and E-clip at the gauge side and maximizes when the wheel is closed to the middle sleeper. For the field side, the Tresca stress also maximizes when the wheel near the sleepers. However, the maximum Tresca stress and the high-stress contour are smaller for the fast-clip at both gauge (386 MPa for the slab track and 413 MPa for the ballast track) and field sides (165 MPa for the slab track and 169 MPa for the ballast track) in a slab track. The maximum Tresca stress location at the rear arch also shifts towards the base plate shoulder as shown in Figure 6-10(C) and (F).

Compared with the ballast track, the maximum Tresca stress increases at the front arch of the E-clip at the gauge side (153 MPa for the slab track and 127 MPa for the ballast track) and decreases at the rear arch (227 MPa for the slab track and 256 MPa for the ballast track). The maximum Tresca stress location does not vary at the rear arch while slightly shifts towards the clip toe at the front arch. As for the field side, the maximum Tresca stress increases at both the front arch (234 MPa for the slab track and 164 MPa for the ballast track) and rear arch (145 MPa for the slab track and 71 MPa for the ballast track). The maximum Tresca stress location at the front arch slightly shifts downside and remains the same at the rear arch.

Figure 6-10. Tresca stress field outputs of rail clips in slab track: (A), (B), (C) for gauge side and (D), (E), (F) for field side

The calculated fatigue damage and lifetime for the slab tracks are listed in Table 6-5.

From Table 6-5, larger fatigue damage on the E-clip can be observed for the slab tracks at both the field side and the gauge side. The fatigue damage is even doubled at the field side rear arch. However, the predicted fatigue life is above 100-million cycles indicates a long service life. As for the fast-clip, the fatigue damage at the rear arch increases at the field side but decreases at the gauge side for the slab track. Even though the Tresca stress is smaller for the slab track, an increase in the shear strain amplitude offset the decrease in the maximum normal stress causes the fatigue damage to increase slightly. The predicted fatigue life for the fast-clip on a slab track at the gauge side is around 20-million cycles and the equivalent service life is 7.6 years which is longer than the estimated lifetime of the fast-clip on the ballast track.

Table 6-5. Life prediction of rail clips for slab track

E-clip						Fast-clip					
Locations		$\Delta\gamma/2$	$\sigma_{n,max}$ (MPa)	$f_{FS}(N_f)$	N_f	Locations		$\Delta\gamma/2$	$\sigma_{n,max}$ (MPa)	$f_{FS}(N_f)$	N_f
Rear arch	Gauge side	1.81e-3	128	2.05e-3	> 1e+8	Rear arch	Gauge side	1.96e-3	377	2.58e-3	2.0e+7
	Field side	1.19e-3	112	1.40e-3	> 1e+8		Field side	1.03e-3	80	1.18e-3	> 1e+8
Front arch	Gauge side	1.64e-3	156	1.93e-3	> 1e+8	Front arch	Gauge side	1.03e-3	80	1.18e-3	> 1e+8
	Field side	1.85e-3	189	2.20e-3	> 1e+8		Field side	1.03e-3	80	1.18e-3	> 1e+8

6.4. Summary

In this chapter, finite element and fatigue analysis on the fast-clips and E-clips was carried out to investigate their fatigue behavior. The Fatemi-Socie multiaxial fatigue criterion implemented with the smallest enclosing circle algorithm based on multiaxial strain and stress states was applied to calculate the fatigue damage of rail clips. The fatigue life was estimated through a strain-life relationship derived from the uniaxial fatigue tests described in Chapter 3. Compared with the Muralidharan and Manson model, the Roessle and Fatemi strain-life model agreed better with the material fatigue test results which was used for further investigation.

Overall, the fatigue analysis reveals the fatigue damage locations are at the rear arch and the front arch of the E-clip, as well as at the rear arch near the shoulder of the fast-clip for the gauge side and shows a good agreement with the actual rail clip fatigue failure sites. The rear arch of the fast-clip at the gauge side is the most critical location and the predicted fatigue life is 4.8 years for the ballast track.

The effect of the rail clip installation process and slab track on the fatigue life were also investigated. The fatigue damage increases for both fast-clip and E-clip as the clip deformation increasing. The predicted fatigue life of the fast-clip is even less than a million cycles (less than a year) when lifting the fast-clip to 6 mm. The estimated lifetime is also reduced dramatically for the E-clip at the gauge side which implies

potential failures occurred at the rear arch and the front arch of the E-clip under cyclic train wheel-induced loading. As for the slab track, larger fatigue damage on the E-clip can be observed at both the field side and the gauge side. The fatigue damage also increases at the rear arch of the fast-clip on the field side. The fatigue damage on the gauge side slightly decreases and the estimated lifetime is about 7.6 years.

Chapter 7. Summary of findings and Recommendations

7.1. Summary of finds

In this research, a fatigue assessment methodology was developed for the railway track structural systems through experiments, finite element models and fatigue analysis.

Based on the objectives and the research work carried out, the following summary of findings can be made for the thermite weld and rail clips:

1) A series of experiments were conducted to investigate the material characteristics, fatigue properties and structural behaviors of the thermite weld and rail clips because the fatigue life curves of the thermite weld and rail clip materials were not well reported.

1-a) The material characteristic and fatigue properties of thermite weld steel were investigated experimentally. Compared with the parent rail metal, the yield stress of thermite weld steel increases while the ductility reduces dramatically. The brittle behavior of the thermite weld steel was confirmed by SEM since the fast fracture region accounted for most areas on the fracture surface of the fatigue test specimens. The crack initiation points are observed at the near-surface micro-pores for all the rotating bending fatigue test specimens that failed within gauge marks. The scatter found in RBF test data could be explained by the different sizes and distances to surface of the near-surface micro-pores observed by SEM.

1-b) The fatigue life curve of the thermite weld structure was obtained through inverted four-point bend tests. Hammer peening was conducted to provide a better finishing surface at the head-to-web and web-to-foot regions and the specimen with peening showed a longer fatigue life ($\sim 2x$) than the specimen without peening under the same loading condition. Glinka's rule with the strain-life approach gives the best prediction for the inverted four-point bend fatigue test results when considering the notch effect on the fatigue life curve of the thermite weld material.

1-c) The clip steel exhibits a better material characteristic indicated by hardness, tensile and uniaxial fatigue tests. Compared with the Muralidharan and Manson model, the Roessle and Fatemi strain-life model agreed better with the material fatigue test results when obtaining the strain-life curve.

2) On-site strain measurements were carried out through a wireless system at a depot and in a tunnel, and the real-time strain measurement results were used to validate the finite element models.

2-a) Real-time strain measurements were conducted at a depot and in a metro tunnel through a wireless system. The signals were successfully recorded by the strain rosette attached at the head-to-web corner with each peak indicating a wheelset passed by the thermite weld. The roll-in strain measurement results on the straight track were used to validate the finite element model and a good agreement between the simulation results

and test results was achieved.

2-b) The on-site strain measurement results recorded by the strain rosette attached to the thermite weld in the tunnel were processed using the rainflow cycle counting approach. The crack propagation life of ~4years was predicted according to the fatigue crack growth life and damage accumulation rule.

3) A sequential method of multibody system dynamic analysis and implicit-explicit sequence finite element analysis was developed to provide a better estimation of the multiaxial stress states and in-depth analysis on the thermite weld and rail clips which were not well researched previously.

3-a) The three-dimensional wheel-rail contact forces as input in the finite element model were obtained through the multibody system dynamic models. The multiaxial stress histories of the thermite weld were then obtained from finite element analysis following an implicit-explicit modelling sequence. The model was validated by the roll-in strain measurement and revealed a phenomenon of ‘reverse bending’ of the thermite weld together with the experiment.

3-b) A larger longitudinal stress is observed at the under-head radius and foot tip of the thermite weld on the field side due to the combination effect of in-plane bending, out-of-plane bending and torsional lateral bending. A higher shear stress state can also be observed at these two regions. The Tresca shear stress cloud of the rail clips indicates

the critical positions on the arches undergo larger stress states which agrees well with the actual failures.

4) In-depth multiaxial fatigue analysis was performed for the thermite weld and rail clips. The fatigue damage reveals high fatigue crack initiation possibilities at failure locations which agrees well with the field observations. The results were published in three Tier 1 journals.

4-a) The fatigue damage at the critical locations on the thermite weld were investigated through stress-based multiaxial fatigue criteria implemented with the smallest enclosing circle algorithm proposed in this research. The fatigue analysis reveals the rail foot tip is the most critical location which has a large probability to initiate a crack within 4 years. The under-head radius, head-to-web and web-to-foot regions also endure relatively large damage values which agree well with the field observations. The fatigue damage analysis shows that the Liu & Mahadevan criterion gives the most conservative prediction at the under-head radius, web-to-foot and rail foot regions while Mataka criterion gives the most conservative prediction at head-to-web and upper web regions. Parametric studies for different operational conditions were also performed. The ballast stiffness and sleeper materials do not show much influence on the fatigue behavior while the train velocity and the curved track radius exert a great influence.

4-b) The effect of missing clips was investigated, and the fatigue analysis reveals that

the fatigue damage of the thermite weld is not affected significantly by the missing clips and the fatigue life will barely change since the rail clips will be replaced during routine maintenance. The effect of track types was also investigated through FE models of four track structural systems. Larger fatigue damage for the slab track system with E-clips was observed compared with the other three systems. The large fatigue damage at the under-head radius and head-to-web region on the gauge side for the slab track with E-clips shows a good agreement with the fatigue failure site in service. In addition, the effect of small defects was investigated. The fatigue crack initiation risk indicates a large probability of the occurrence of fatigue failures at the head-to-web and web-to-foot regions on the field side as well as the head-to-web on the gauge side.

4-c) The fatigue analysis of the rail clips was performed through Fatemi-Socie multiaxial fatigue criterion and finite element analysis which were validated by the static pull-up and fatigue loading tests. It reveals the fatigue damage locations are at the rear arch and the front arch of the E-clip as well as at the rear arch near the shoulder of the fast-clip for the gauge side and shows a good agreement with the actual rail clip fatigue failure sites. It also concludes that the rear arch of the fast-clip at the gauge side is the most critical location and the predicted fatigue life is 4.8 years.

4-d) The effect of the rail clip installation process was investigated. The fatigue damage increases for both fast-clip and E-clip as the clip deformation increasing. The predicted fatigue life of the fast-clip is less than a million cycles (less than a year) when lifting

the fast-clip to 6 mm. The estimated lifetime is also reduced dramatically for the E-clip at the gauge side which implies potential failure occurred at the rear arch and the front arch of the E-clip under cyclic train wheel-induced loading. The fatigue analysis was also performed to investigate the effect of track types and larger fatigue damage on the E-clip is observed at both the field side and the gauge side for the slab track. The fatigue damage also increases at the rear arch of the fast-clip on the field side. However, the fatigue damage on the gauge side slightly decreases compared with the ballast track and the estimated lifetime is about 7.6 years.

7.2. Conclusions

In this study, a dynamic modeling method is developed for the track structural system and fatigue assessment is conducted for the thermite weld and rail clips. The results reveal a higher crack initiation probability at head-to-web, web-to-foot and foot tip regions. The rotating bending fatigue tests show the weak material property at the head-to-web region and the inverted four-point bend fatigue tests indicate the head-to-web region is the critical failure location. And the crack will propagate under a relatively large tensile stress once the crack initiates at the head-to-web region since it is more difficult to check when the rail is in service.

The fatigue analysis also reveals the critical locations are at the rear arch and the front arch of the E-clip, as well as at the rear arch near the shoulder of the fast-clip for the

gauge side. The fatigue life of rail clips shows a good agreement with the actual failures when different installation stress levels are considered.

7.3. Recommendations for Future Work

The current research developed an integrated fatigue assessment method for the thermite weld and rail clips. Several experiments have been conducted and finite element models implemented with a time-efficient fatigue analysis methodology have been developed to understand their fatigue performance. Some further works are recommended based on the current research as below:

1. The wheel-induced loadings were obtained from multibody system dynamic models and then applied in finite element models. Although the finite element model was validated through a train roll-in test, the experimental studies of wheel-rail contact forces especially at curve segments such as attaching the accelerometer on the wheel are needed for further studies to have a better understanding of the wheel-rail interactions and perform a more accurate analysis.
2. When conducting the fatigue assessment of the thermite weld and rail clips, the geometry models of the thermite weld and rail clips were assumed to be defect-free in finite element analysis. The fatigue analysis was only performed according to the fatigue limit adjusted for small defects, notch correction and rainflow cycle

counting approach. Fatigue propagation analysis considering different defect and crack sizes using fracture mechanics is needed for further studies to have a more comprehensive understanding of their failure mechanism.

3. The specimens extracted from the head-to-web regions are only used for conducting the rotating bending fatigue test. The torsional fatigue limit used for fatigue damage assessment was estimated according to the empirical value for the pearlite steel and this may not precisely reflect the actual values. Alternative torsional fatigue tests should be conducted to provide a better evaluation of the parameters in multiaxial fatigue models.
4. When investigating the effect of defects on the thermite weld, the stress/strain histories from smooth welded rail were used for fatigue damage and risk assessment by comparing with the fatigue life curves adjusted with different stress concentration factors. However, this may not accurately reflect the actual multiaxial stress/strain states on the defects of the thermite weld. Dynamic finite element analysis of the thermite weld with different types of defects and estimating the elastoplastic stresses and strains at notches are interesting areas that worth further investigation.

Publication arising from the thesis

Published Journal Paper:

1. Z. Liu, X. Shi, K.S. Tsang, H.J. Hoh, J.H.L. Pang. Dynamic finite element modeling and fatigue damage analysis of thermite welds. *Fatigue & Fracture of Engineering Material & Structures*. DOI: 10.1111/ffe.13091.
2. Z. Liu, K.S. Tsang, Y. Liu, and J.H.L. Pang. Finite Element and Experimental Study on Multiaxial Fatigue Analysis of Rail Clip Failures. *Fatigue & Fracture of Engineering Material & Structures*. DOI: 10.1111/ffe.13310.
3. X. Shi, Y. Liu, Z. Liu, H.J. Hoh, K.S. Tsang, J.H.L. Pang. An Integrated Fatigue Assessment Approach of Rail Welds Using Dynamic 3D FE Simulation and Strain Monitoring Technique, *Engineering Failure Analysis*. DOI: 10.1016/j.engfailanal.2020.105080.

Conference Paper:

1. X. Shi, Z. Liu, K.S. Tsang, H.J. Hoh, Y. Liu, K.J. Tan, P.S. Yong, J.H.L. Pang. Wireless Strain Sensors for Dynamic Stress Analysis of Rail Steel Structural Integrity. *Advances in Condition Monitoring and Structural Health Monitoring*.

Submitted Journal Paper:

1. Z. Liu, K.S. Tsang, Y. Liu, and J.H.L. Pang. Investigation on fatigue characteristic of thermite weld through inverted four-point bend tests.

Appendix A: Investigation on Finite Element Model Validation

The comparison results for roll-in test and FE models without clips are plotted in Figure A-1.

Figure A-1. (A) Vertical and (B) longitudinal strains for tie interaction, (C) Vertical and (D) longitudinal strains for coupling interaction

References

- [1] K. Saita, K. Karimine, M. Ueda, K. Iwano, T. Yamamoto, and K. Hiroguchi, "Trends in rail welding technologies and our future approach," *Nippon steel & sumitomo metal technical report*, vol. 105, pp. 84-92, 2013.
- [2] U. Zerbst, M. Schödel, and R. Heyder, "Damage tolerance investigations on rails," *Engineering Fracture Mechanics*, vol. 76, no. 17, pp. 2637-2653, 2009.
- [3] M. Tavakoli, "Multi-Axial Fatigue Analysis of Thermite Rail Welds," 2018.
- [4] P. J. Mutton and E. Alvarez, "Failure modes in aluminothermic rail welds under high axle load conditions," *Engineering Failure Analysis*, vol. 11, no. 2, pp. 151-166, 2004.
- [5] S. Morales-Ivorra, J. I. Real, C. Hernández, and L. Montalbán, "Derailment risk and dynamics of railway vehicles in curved tracks: Analysis of the effect of failed fasteners," *Journal of Modern Transportation*, vol. 24, no. 1, pp. 38-47, 2016.
- [6] H. Shitara, Y. TERASHITA, M. TATSUMI, and Y. Fukada, "Nondestructive testing and evaluation methods for rail welds in Japan," *Quarterly Report of RTRI*, vol. 44, no. 2, pp. 53-58, 2003.
- [7] H.-M. Thomas, T. Heckel, and G. Hanspach, "Advantage of a combined ultrasonic and eddy current examination for railway inspection trains," *Insight-Non-Destructive Testing and Condition Monitoring*, vol. 49, no. 6, pp. 341-344, 2007.
- [8] M. Singh, S. Singh, J. Jaiswal, and J. Hempshall, "Autonomous rail track inspection using vision based system," in *2006 IEEE International Conference on Computational Intelligence for Homeland Security and Personal Safety*, 2006: IEEE, pp. 56-59.
- [9] I. Salehi, A. Kapoor, and P. Mutton, "Multi-axial fatigue analysis of aluminothermic rail welds under high axle load conditions," *International Journal of Fatigue*, vol. 33, no. 9, pp. 1324-1336, 2011.
- [10] I. Salehi, "Fatigue and Fracture Behaviour of Aluminothermic Rail Welds under High Axle Load Conditions," Swinburne University of Technology, 2013.
- [11] C. Lu, J. Nieto, I. Puy, J. Melendez, and J. Martínez-Esnaola, "Fatigue prediction of rail welded joints," *International Journal of Fatigue*, vol. 113, pp. 78-87, 2018.
- [12] I. Salehi, A. Kapoor, P. Mutton, and J. Alserda, "Improving the reliability of aluminothermic rail welds under high axle load conditions," *CORE 2010: Rail*,

Rejuvenation and Renaissance, p. 9, 2010.

- [13] C. Lonsdale and M. Engineer, "Thermite rail welding: history process developments, current practice and outlook for the 21st century," in *Proc. AREMA 1999 Annu. Conf.*, 1999, vol. 1895: Citeseer, p. 18.
- [14] R. Baptista, T. Santos, J. Marques, M. Guedes, and V. Infante, "Fatigue behavior and microstructural characterization of a high strength steel for welded railway rails," *International journal of fatigue*, vol. 117, pp. 1-8, 2018.
- [15] J. Ringsberg, M. Loo-Morrey, B. Josefson, A. Kapoor, and J. H. Beynon, "Prediction of fatigue crack initiation for rolling contact fatigue," *International Journal of fatigue*, vol. 22, no. 3, pp. 205-215, 2000.
- [16] Y. Li, C. Liu, N. Xu, X. Wu, W. Guo, and J. Shi, "A failure study of the railway rail serviced for heavy cargo trains," *Case Studies in Engineering Failure Analysis*, vol. 1, no. 4, pp. 243-248, 2013.
- [17] D. E. Holder, M. V. Csenge, Y. Qian, M. S. Dersch, J. R. Edwards, and B. J. Van Dyk, "Laboratory investigation of the Skl-style fastening system's lateral load performance under heavy haul freight railroad loads," *Engineering Structures*, vol. 139, pp. 71-80, 2017.
- [18] S. Tamagawa, H. Kataoka, and T. Deshimaru, "A fatigue limit diagram for plastic rail clips," *WIT Transactions on The Built Environment*, vol. 135, pp. 839-848, 2014.
- [19] L. Ling, W. Li, H. Shang, X. Xiao, Z. Wen, and X. Jin, "Experimental and numerical investigation of the effect of rail corrugation on the behaviour of rail fastenings," *Vehicle System Dynamics*, vol. 52, no. 9, pp. 1211-1231, 2014.
- [20] F. V. Lawrence, E. T. Ross, and C. P. Barkan, "Reliability of improved thermite welds," *Final report for high-speed rail IDEA project*, vol. 41, 2004.
- [21] D. Cannon, K. O. Edel, S. Grassie, and K. Sawley, "Rail defects: an overview," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 26, no. 10, pp. 865-886, 2003.
- [22] F. Sadeghi, B. Jalalahmadi, T. S. Slack, N. Raje, and N. K. Arakere, "A review of rolling contact fatigue," *Journal of tribology*, vol. 131, no. 4, 2009.
- [23] P. Mutton, "Horizontal split web fatigue failure of rail welds," in *Proc. Conf. Failure Analysis in Materials Engineering, Institute of Metals and Materials Australasia (IMMA) Melbourne*, 1994.
- [24] W. Zhai and X. Sun, "A detailed model for investigating vertical interaction

- between railway vehicle and track," *vehicle system dynamics*, vol. 23, no. S1, pp. 603-615, 1994.
- [25] W. Zhai and Z. Cai, "Dynamic interaction between a lumped mass vehicle and a discretely supported continuous rail track," *Computers & structures*, vol. 63, no. 5, pp. 987-997, 1997.
 - [26] G. Ding, Y. He, Y. Zou, R. Li, X. Ma, and S. Qin, "Multi-body dynamics-based sensitivity analysis for a railway vehicle," *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 229, no. 5, pp. 518-529, 2015.
 - [27] S. K. Sharma, "Multibody analysis of longitudinal train dynamics on the passenger ride performance due to brake application," *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics*, vol. 233, no. 2, pp. 266-279, 2019.
 - [28] S. A. Ranjha, P. J. Mutton, and A. Kapoor, "Fatigue analysis of the rail underhead radius under high axle load conditions," in *Advanced Materials Research*, 2014, vol. 891: Trans Tech Publ, pp. 1181-1187.
 - [29] S. A. Ranjhaa, A. Kulkarnia, P. Ekambarama, and P. M. A. Kapoora, "Finite element prediction of the stress state of a rail underhead radius under high axle load conditions," *Int. J. Condit. Monitor. Diagn. Eng. Manage.*, vol. 19, no. 2, pp. 11-16, 2016.
 - [30] I. Salehi, P. Mutton, and A. Kapoor, "The effect of geometric features on multi-axial fatigue behaviour of aluminothermic rail welds," *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 226, no. 4, pp. 360-370, 2012.
 - [31] J. Bian, Y. Gu, and M. H. Murray, "A dynamic wheel-rail impact analysis of railway track under wheel flat by finite element analysis," *Vehicle System Dynamics*, vol. 51, no. 6, pp. 784-797, 2013.
 - [32] Z. Yang, A. Boogaard, Z. Wei, J. Liu, R. Dollevoet, and Z. Li, "Numerical study of wheel-rail impact contact solutions at an insulated rail joint," *International Journal of Mechanical Sciences*, vol. 138, pp. 310-322, 2018.
 - [33] J. Xu, P. Wang, Y. Gao, J. Chen, and R. Chen, "Geometry evolution of rail weld irregularity and the effect on wheel-rail dynamic interaction in heavy haul railways," *Engineering Failure Analysis*, vol. 81, pp. 31-44, 2017.
 - [34] W. Cai, Z. Wen, X. Jin, and W. Zhai, "Dynamic stress analysis of rail joint with height difference defect using finite element method," *Engineering Failure*

Analysis, vol. 14, no. 8, pp. 1488-1499, 2007.

- [35] A. Hasap, P. Paitekul, N. Noraphaiphapaksa, and C. Kanchanomai, "Analysis of the fatigue performance of elastic rail clip," *Engineering Failure Analysis*, vol. 92, pp. 195-204, 2018.
- [36] X. Zhao, Z. Li, and R. Dollevoet, "Influence of the fastening modeling on the vehicle-track interaction at singular rail surface defects," *Journal of Computational and Nonlinear Dynamics*, vol. 9, no. 3, 2014.
- [37] A. Hasap, P. Paitekul, N. Noraphaiphapaksa, and C. Kanchanomai, "Influence of toe load on the fatigue resistance of elastic rail clips," *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 232, no. 4, pp. 1078-1087, 2018.
- [38] T. DESHIMARU, S. TAMAGAWA, and H. KATAOKA, "Permissible lateral force and fatigue life for rail fastening system," *Quarterly Report of RTRI*, vol. 58, no. 3, pp. 236-241, 2017.
- [39] S. Mohammadzadeh, S. Ahadi, and M. Nouri, "Stress-based fatigue reliability analysis of the rail fastening spring clip under traffic loads," *Latin American Journal of Solids and Structures*, vol. 11, no. 6, pp. 993-1011, 2014.
- [40] X. Hong, G. Xiao, W. Haoyu, L. Xing, and W. Sixing, "Fatigue damage analysis and life prediction of e-clip in railway fasteners based on ABAQUS and FE-SAFE," *Advances in Mechanical Engineering*, vol. 10, no. 3, p. 1687814018767249, 2018.
- [41] G. Fry, F. Lawrence, and A. Robinson, "A model for fatigue defect nucleation in thermite rail welds," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 19, no. 6, pp. 655-668, 1996.
- [42] B. L. Josefson and J. W. Ringsberg, "Assessment of uncertainties in life prediction of fatigue crack initiation and propagation in welded rails," *International journal of fatigue*, vol. 31, no. 8-9, pp. 1413-1421, 2009.
- [43] Y. Jiang, O. Hertel, and M. Vormwald, "An experimental evaluation of three critical plane multiaxial fatigue criteria," *International Journal of Fatigue*, vol. 29, no. 8, pp. 1490-1502, 2007.
- [44] L. Reis, B. Li, and M. De Freitas, "A multiaxial fatigue approach to Rolling Contact Fatigue in railways," *International Journal of Fatigue*, vol. 67, pp. 191-202, 2014.
- [45] M. Kiani and G. Fry, "F," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 41, no. 2, pp. 412-424, 2018.

- [46] J. W. Ringsberg, "Life prediction of rolling contact fatigue crack initiation," *International Journal of fatigue*, vol. 23, no. 7, pp. 575-586, 2001.
- [47] P. Ballard, K. D. Van, A. Deperrois, and Y. Papadopoulos, "High cycle fatigue and a finite element analysis," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 18, no. 3, pp. 397-411, 1995.
- [48] J. Araújo, A. Dantas, F. Castro, E. Mamiya, and J. Ferreira, "On the characterization of the critical plane with a simple and fast alternative measure of the shear stress amplitude in multiaxial fatigue," *International Journal of Fatigue*, vol. 33, no. 8, pp. 1092-1100, 2011.
- [49] F. Castro, J. Araújo, E. Mamiya, and N. Zouain, "Remarks on multiaxial fatigue limit criteria based on prismatic hulls and ellipsoids," *International journal of fatigue*, vol. 31, no. 11-12, pp. 1875-1881, 2009.
- [50] E. Mamiya, J. Araújo, and F. Castro, "Prismatic hull: a new measure of shear stress amplitude in multiaxial high cycle fatigue," *International Journal of Fatigue*, vol. 31, no. 7, pp. 1144-1153, 2009.
- [51] C. Esveld and C. Esveld, *Modern railway track*. MRT-productions Zaltbommel, 2001.
- [52] C. Esveld, "Recent developments in slab track," *European railway review*, vol. 9, no. 2, pp. 81-85, 2003.
- [53] U. Zerbst, R. Lundén, K.-O. Edel, and R. A. Smith, "Introduction to the damage tolerance behaviour of railway rails—a review," *Engineering fracture mechanics*, vol. 76, no. 17, pp. 2563-2601, 2009.
- [54] A. S. J. A. Z. Jilabi, *Welding of rail steels*. The University of Manchester (United Kingdom), 2015.
- [55] Y. Liu, K. S. Tsang, N. A. Subramaniam, and J. H. L. Pang, "Structural fatigue investigation of thermite welded rail joints considering weld-induced residual stress and stress relaxation by cyclic load," *Engineering Structures*, vol. 235, p. 112033, 2021.
- [56] C. F. Bonnett, *Practical railway engineering*. Imperial College Press, 2005.
- [57] J. Sadeghi and P. Barati, "Comparisons of the mechanical properties of timber, steel and concrete sleepers," *Structure and Infrastructure Engineering*, vol. 8, no. 12, pp. 1151-1159, 2012.
- [58] G. Michas, "Slab track systems for high-speed railways," ed, 2012.
- [59] Y. Liu, B. Stratman, and S. Mahadevan, "Fatigue crack initiation life prediction

- of railroad wheels," *International journal of fatigue*, vol. 28, no. 7, pp. 747-756, 2006.
- [60] Y. Terashita and M. Tatsumi, "Analysis of damaged rail weld," *Quarterly Report of RTRI*, vol. 44, no. 2, pp. 59-64, 2003.
 - [61] A. Skyttebol, B. L. Josefson, and J. W. Ringsberg, "Fatigue crack growth in a welded rail under the influence of residual stresses," *Engineering Fracture Mechanics*, vol. 72, no. 2, pp. 271-285, 2005.
 - [62] J. A. Oderio, "A metallurgical study of the detail fractures in thermite-welded railroad rails," University of Illinois at Urbana-Champaign, 1992.
 - [63] Z. Mouallif, B. Radi, and I. Mouallif, "Effect of the Inclusion Defect on the Mechanical Behavior of Thermite Welds," *Adv. Theor. Appl. Mech*, vol. 9, no. 1, pp. 11-20, 2016.
 - [64] I. Mouallif, Z. Mouallif, A. Benali, and F. Sidki, "Finite element modeling of the aluminothermic welding with internal defects and experimental analysis," in *MATEC Web of Conferences*, 2012, vol. 1: EDP Sciences, p. 00002.
 - [65] A. Motameni and A. N. ERASLAN, "Fracture and fatigue crack growth characterization of conventional and head hardened railway rail steels," 2016.
 - [66] S. Maya-Johnson, A. Ramirez, and A. Toro, "Fatigue crack growth rate of two pearlitic rail steels," *Engineering fracture mechanics*, vol. 138, pp. 63-72, 2015.
 - [67] Y.-q. Wang, H. Zhou, Y.-j. Shi, and B.-r. Feng, "Mechanical properties and fracture toughness of rail steels and thermite welds at low temperature," *International Journal of Minerals, Metallurgy, and Materials*, vol. 19, no. 5, pp. 409-420, 2012.
 - [68] J. Myers, G. Geiger, and D. R. Poirier, "Structure and Properties of Thermite Welds in Rails," *WELDING J.*, vol. 61, no. 8, pp. 258-268, 1982.
 - [69] A. Ekberg and B. Paulsson, *INNOTRACK: concluding technical report*. International Union of Railways (UIC), 2010.
 - [70] Y. Chen, F. V. Lawrence, C. P. Barkan, and J. Dantzig, "Weld defect formation in rail thermite welds," *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 220, no. 4, pp. 373-384, 2006.
 - [71] S. Romano, D. Manenti, S. Beretta, and U. Zerbst, "Semi-probabilistic method for residual lifetime of aluminothermic welded rails with foot cracks," *Theoretical and Applied Fracture Mechanics*, vol. 85, pp. 398-411, 2016.
 - [72] A. P. De Man, "A survey of dynamic railway track properties and their quality,"

Delft University of Technology, TU Delft, 2002.

- [73] S. Iwnicki, *Handbook of railway vehicle dynamics*. CRC press, 2006.
- [74] S. P. Timoshenko, "X. On the transverse vibrations of bars of uniform cross-section," *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, vol. 43, no. 253, pp. 125-131, 1922.
- [75] Z. Wen, G. Xiao, X. Xiao, X. Jin, and M. Zhu, "Dynamic vehicle-track interaction and plastic deformation of rail at rail welds," *Engineering Failure Analysis*, vol. 16, no. 4, pp. 1221-1237, 2009.
- [76] Z. Zhang, "Finite element analysis of railway track under vehicle dynamic impact and longitudinal loads," 2015.
- [77] O. Orringer, Y. Tang, J. Gordon, D. Jeong, J. Morris, and A. Perlman, "Crack propagation life of detail fractures in rails," United States. Federal Railroad Administration, 1988.
- [78] T. Farris, L. Keer, and R. Steele, "Life prediction for unstable shell growth in rails," 1990.
- [79] J. Piotrowski and W. Kik, "A simplified model of wheel/rail contact mechanics for non-Hertzian problems and its application in rail vehicle dynamic simulations," *Vehicle System Dynamics*, vol. 46, no. 1-2, pp. 27-48, 2008.
- [80] J. A. Bomidi, N. Weinzapfel, F. Sadeghi, A. Liebel, and J. Weber, "An improved approach for 3D rolling contact fatigue simulations with microstructure topology," *Tribology transactions*, vol. 56, no. 3, pp. 385-399, 2013.
- [81] L. Bai *et al.*, "An integrated approach for predicting wear and fatigue damage of wheels and rails in SMRT systems," in *2018 International Conference on Intelligent Rail Transportation (ICIRT)*, 2018: IEEE, pp. 1-5.
- [82] T. Lack and J. Gerlici, "A modified strip method to speed up the calculation of normal stress between wheel and rail," in *Applied mechanics and materials*, 2014, vol. 486: Trans Tech Publ, pp. 359-370.
- [83] J. Dižo, M. Blatnický, and S. Steišūnas, "Multibody Simulation of Rail Vehicle Running Considering Track Flexibility," in *Proceedings of the International Conference TRANSBALICA*, 2019: Springer, pp. 309-319.
- [84] M. Yu, W. Wang, and J. Liu, "A Transient Finite Element Model to Simulate the Physical Mechanisms of High-speed Wheel/rail Rolling Contact on a Rail Welding Joint," 2017.
- [85] D. Ferreño *et al.*, "Experimental and finite element fatigue assessment of the

- spring clip of the SKL-1 railway fastening system," *Engineering Structures*, vol. 188, pp. 553-563, 2019.
- [86] Z. Chen, M. Shin, S. Wei, B. Andrawes, and D. A. Kuchma, "Finite element modeling and validation of the fastening systems and concrete sleepers used in North America," *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 228, no. 6, pp. 590-602, 2014.
 - [87] A. Fatemi and L. Yang, "Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials," *International journal of fatigue*, vol. 20, no. 1, pp. 9-34, 1998.
 - [88] G. T. Fry, "A fatigue model for thermite rail welds," University of Illinois at Urbana-Champaign, 1995.
 - [89] D. Hull and D. J. Bacon, *Introduction to dislocations*. Butterworth-Heinemann, 2001.
 - [90] A. Ince, "Development of computational multiaxial fatigue modelling for notched components," 2012.
 - [91] D. Socie and G. B. Marquis, *Multiaxial fatigue*. Society of Automotive Engineers Warrendale, PA, 2000.
 - [92] A. Carpinteri and A. Spagnoli, "Multiaxial high-cycle fatigue criterion for hard metals," *International Journal of Fatigue*, vol. 23, no. 2, pp. 135-145, 2001.
 - [93] Y.-L. Lee, J. Pan, R. Hathaway, and M. Barkey, *Fatigue testing and analysis: theory and practice*. Butterworth-Heinemann, 2005.
 - [94] H. Desimone and S. Beretta, "Mechanisms of mixed mode fatigue crack propagation at rail butt-welds," *International journal of fatigue*, vol. 28, no. 5-6, pp. 635-642, 2006.
 - [95] M. W. Brown and K. Miller, "A theory for fatigue failure under multiaxial stress-strain conditions," *Proceedings of the Institution of Mechanical engineers*, vol. 187, no. 1, pp. 745-755, 1973.
 - [96] A. Karolczuk and E. Macha, "A review of critical plane orientations in multiaxial fatigue failure criteria of metallic materials," *International Journal of Fracture*, vol. 134, no. 3-4, p. 267, 2005.
 - [97] B. Li, L. Reis, and M. De Freitas, "Comparative study of multiaxial fatigue damage models for ductile structural steels and brittle materials," *International Journal of Fatigue*, vol. 31, no. 11-12, pp. 1895-1906, 2009.
 - [98] B.-R. You and S.-B. Lee, "A critical review on multiaxial fatigue assessments of

- metals," *International Journal of Fatigue*, vol. 18, no. 4, pp. 235-244, 1996.
- [99] M. Bäckström and G. Marquis, "A review of multiaxial fatigue of weldments: experimental results, design code and critical plane approaches," *Fatigue & fracture of engineering materials & structures*, vol. 24, no. 5, pp. 279-291, 2001.
- [100] Y.-Y. Wang and W.-X. Yao, "Evaluation and comparison of several multiaxial fatigue criteria," *International Journal of Fatigue*, vol. 26, no. 1, pp. 17-25, 2004.
- [101] W. Maktouf, K. Ammar, I. B. Naceur, and K. Sai, "Multiaxial high-cycle fatigue criteria and life prediction: Application to gas turbine blade," *International Journal of Fatigue*, vol. 92, pp. 25-35, 2016.
- [102] A. Fatemi and N. Shamsaei, "Multiaxial fatigue: An overview and some approximation models for life estimation," *International Journal of Fatigue*, vol. 33, no. 8, pp. 948-958, 2011.
- [103] P. Forsyth, "A two stage process of fatigue crack growth," in *Proceedings of the Crack Propagation Symposium*, 1961, vol. 1: Cranfield, pp. 76-94.
- [104] S. K. Gupta, T. Fesich, X. Schuler, V. Bhasin, K. Vaze, and E. Roos, "A critical plane based model for fatigue assessment under fixed and rotating principal direction loading," 2011.
- [105] I. V. Papadopoulos, "Long life fatigue under multiaxial loading," *International Journal of Fatigue*, vol. 23, no. 10, pp. 839-849, 2001.
- [106] J. Lemaitre and J.-L. Chaboche, *Mechanics of solid materials*. Cambridge university press, 1994.
- [107] L. Susmel, "A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems," *International Journal of Fatigue*, vol. 32, no. 11, pp. 1875-1883, 2010.
- [108] V. Grubisic and A. Simbürger, "Fatigue under combined out-of-phase multiaxial stresses," in *Fatigue Testing and Design 2, Proceedings of SEE International Conference*, 1976, vol. 5, pp. 27.1-27.8.
- [109] I. V. Papadopoulos, "Critical plane approaches in high - cycle fatigue: on the definition of the amplitude and mean value of the shear stress acting on the critical plane," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 21, no. 3, pp. 269-285, 1998.
- [110] I. V. Papadopoulos, P. Davoli, C. Gorla, M. Filippini, and A. Bernasconi, "A comparative study of multiaxial high-cycle fatigue criteria for metals," *International Journal of Fatigue*, vol. 19, no. 3, pp. 219-235, 1997.

- [111] B. Li, J. Santos, and M. D. Freitas, "A computerized procedure for long - life fatigue assessment under complex multiaxial loading," *Fatigue & fracture of engineering materials & structures*, vol. 24, no. 3, pp. 165-177, 2001.
- [112] B. Li and M. de Freitas, "A procedure for fast evaluation of high-cycle fatigue under multiaxial random loading," *Journal of Mechanical Design*, vol. 124, no. 3, pp. 558-563, 2002.
- [113] B. Li, J. Santos, and M. d. Freitas, "A unified numerical approach for multiaxial fatigue limit evaluation," 2000.
- [114] C. A. Gonçalves, J. A. Araújo, and E. N. Mamiya, "Multiaxial fatigue: a stress based criterion for hard metals," *International Journal of Fatigue*, vol. 27, no. 2, pp. 177-187, 2005.
- [115] N. Zouain, E. N. Mamiya, and F. Comes, "Using enclosing ellipsoids in multiaxial fatigue strength criteria," *European Journal of Mechanics-A/Solids*, vol. 25, no. 1, pp. 51-71, 2006.
- [116] W. N. Findley, "A theory for the effect of mean stress on fatigue of metals under combined torsion and axial load or bending," *Journal of Engineering for Industry*, vol. 81, no. 4, pp. 301-305, 1959.
- [117] I. V. Papadopoulos, P. Davoli, C. Gorla, M. Filippini, and A. Bernasconi, "A comparative study of multiaxial high-cycle fatigue criteria for metals," *International Journal of Fatigue* \$V 19\$, no. 3, pp. 219-235, 1997.
- [118] D. McDiarmid, "A general criterion for high cycle multiaxial fatigue failure," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 14, no. 4, pp. 429-453, 1991.
- [119] Y. Liu and S. Mahadevan, "Multiaxial high-cycle fatigue criterion and life prediction for metals," *International Journal of Fatigue*, vol. 27, no. 7, pp. 790-800, 2005.
- [120] K. Kim, X. Chen, C. Han, and H. Lee, "Estimation methods for fatigue properties of steels under axial and torsional loading," *International journal of fatigue*, vol. 24, no. 7, pp. 783-793, 2002.
- [121] S. Zhu, S. Foletti, and S. Beretta, "Probabilistic framework for multiaxial LCF assessment under material variability," *International Journal of Fatigue*, vol. 103, pp. 371-385, 2017.
- [122] D. McClafline and A. Fatemi, "Torsional deformation and fatigue of hardened steel including mean stress and stress gradient effects," *International Journal of Fatigue*, vol. 26, no. 7, pp. 773-784, 2004.

- [123] S.-P. Zhu, Z.-Y. Yu, J. Correia, A. De Jesus, and F. Berto, "Evaluation and comparison of critical plane criteria for multiaxial fatigue analysis of ductile and brittle materials," *International Journal of Fatigue*, vol. 112, pp. 279-288, 2018.
- [124] Y. Wang and L. Susmel, "The Modified Manson–Coffin Curve Method to estimate fatigue lifetime under complex constant and variable amplitude multiaxial fatigue loading," *International Journal of Fatigue*, vol. 83, pp. 135-149, 2016.
- [125] G. Marsh *et al.*, "Review and application of Rainflow residue processing techniques for accurate fatigue damage estimation," *International Journal of Fatigue*, vol. 82, pp. 757-765, 2016.
- [126] A. Niesłony, "Determination of fragments of multiaxial service loading strongly influencing the fatigue of machine components," *Mechanical Systems and Signal Processing*, vol. 23, no. 8, pp. 2712-2721, 2009.
- [127] K. Rege and D. G. Pavlou, "A one-parameter nonlinear fatigue damage accumulation model," *International Journal of Fatigue*, vol. 98, pp. 234-246, 2017.
- [128] K. Molski and G. Glinka, "A method of elastic-plastic stress and strain calculation at a notch root," *Materials Science and Engineering*, vol. 50, no. 1, pp. 93-100, 1981.
- [129] M. Knop, R. Jones, L. Molent, and C. Wang, "On the Glinka and Neuber methods for calculating notch tip strains under cyclic load spectra," *International journal of fatigue*, vol. 22, no. 9, pp. 743-755, 2000.
- [130] M. Kiani, J. Ishak, M. W. Keller, and S. Tipton, "Comparison between nonlinear FEA and notch strain analysis for modeling elastoplastic stress-strain response in crossbores," *International Journal of Mechanical Sciences*, vol. 118, pp. 45-54, 2016.
- [131] Z. Lopez and A. Fatemi, "A method of predicting cyclic stress–strain curve from tensile properties for steels," *Materials Science and Engineering: A*, vol. 556, pp. 540-550, 2012.
- [132] D. Kujawski and P. C. Sree, "On deviatoric interpretation of Neuber’s rule and the SWT parameter," *Theoretical and Applied Fracture Mechanics*, vol. 71, pp. 44-50, 2014.
- [133] M. Zappalorto and D. Kujawski, "Neuber’s rules and other solutions: theoretical differences, formal analogies and energy interpretations," *Theoretical and Applied Fracture Mechanics*, vol. 79, pp. 2-13, 2015.

- [134] A. Ince and D. Bang, "Deviatoric Neuber method for stress and strain analysis at notches under multiaxial loadings," *International Journal of Fatigue*, vol. 102, pp. 229-240, 2017.
- [135] Y. Murakami, *Metal fatigue: effects of small defects and nonmetallic inclusions*. Academic Press, 2019.
- [136] Y. Murakami, "Analysis of stress intensity factors of modes I, II and III for inclined surface cracks of arbitrary shape," *Engineering Fracture Mechanics*, vol. 22, no. 1, pp. 101-114, 1985.
- [137] Y. Murakami and M. Endo, "Effects of hardness and crack geometries on ΔK_{th} of small cracks emanating from small defects, The behaviour of short fatigue cracks, Edited by KJ Miller, ER de los Rios," ed: Mechanical Engineering Publications, 1986.
- [138] *ASTM A370-17, Standard Test Methods and Definitions for Mechanical Testing of Steel Products*, ASTM International, West Conshohocken, 2017.
- [139] *ASTM E18-17, Standard Test Methods for Rockwell Hardness of Metallic Materials*, ASTM International, West Conshohocken, 2017.
- [140] *ISO 6508, Metallic materials — Rockwell hardness test*, 2015.
- [141] *ISO 1143:2010, Metallic materials — Rotating bar bending fatigue testing*, 2010.
- [142] *ASTM E466-15, Standard Practice for Conducting Force Controlled Constant Amplitude Axial Fatigue Tests of Metallic Materials*, ASTM International, West Conshohocken, 2015.
- [143] Simulia, "ABAQUS 6.14 Analysis User's Manual," 2014.
- [144] M. Kiani, "Multi-Scale Fatigue Damage Life Assessment of a Railway Wheel Using a Critical-Plane Model," 2017.
- [145] A. Ekberg and E. Kabo, "Fatigue of railway wheels and rails under rolling contact and thermal loading—an overview," *Wear*, vol. 258, no. 7-8, pp. 1288-1300, 2005.
- [146] A. Ticoalu, T. Aravinthan, and W. Karunasena, "An investigation on the stiffness of timber sleepers for the design of fibre composite sleepers," in *Proceedings of the 20th Australasian Conference on the Mechanics of Structures and Materials (ACMSM 20)*, 2008: Taylor & Francis (CRC Press), pp. 865-870.
- [147] B. Indraratna, C. Rujikiatkamjorn, J. S. J. Vinod, and H. Khabbaz, "A review of ballast characteristics, geosynthetics, confining pressures and native vegetation in rail track stabilisation," 2009.

- [148] S. Kaewunruen, T. Lewandrowski, and K. Chamniprasart, "Dynamic responses of interspersed railway tracks to moving train loads," *International Journal of Structural Stability and Dynamics*, vol. 18, no. 01, p. 1850011, 2018.
- [149] N. Zong and M. Dhanasekar, "Sleeper embedded insulated rail joints for minimising the number of modes of failure," *Engineering Failure Analysis*, vol. 76, pp. 27-43, 2017.
- [150] H. Mughrabi, "On 'multi - stage' fatigue life diagrams and the relevant life - controlling mechanisms in ultrahigh - cycle fatigue," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 25, no. 8 - 9, pp. 755-764, 2002.
- [151] V. Gaur, V. Doquet, E. Persent, C. Mareau, E. Roguet, and J. Kittel, "Surface versus internal fatigue crack initiation in steel: Influence of mean stress," *International Journal of Fatigue*, vol. 82, pp. 437-448, 2016.
- [152] P. Heuler, C. Berger, and J. Motz, "Fatigue behaviour of steel castings containing near - surface defects," *Fatigue & Fracture of Engineering Materials & Structures*, vol. 16, no. 1, pp. 115-136, 1993.
- [153] M. Endo and Y. Murakami, "Effects of an artificial small defect on torsional fatigue strength of steels," *Journal of Engineering Materials and Technology*, vol. 109, no. 2, pp. 124-129, 1987.
- [154] E. Welzl, "Smallest enclosing disks (balls and ellipsoids)," in *New results and new trends in computer science*: Springer, 1991, pp. 359-370.
- [155] N. Megiddo, "Linear-time algorithms for linear programming in R^3 and related problems," *SIAM journal on computing*, vol. 12, no. 4, pp. 759-776, 1983.
- [156] G. T. Toussaint, "Solving geometric problems with the rotating calipers," in *Proc. IEEE Melecon*, 1983, vol. 83, p. A10.
- [157] J. Papuga, I. Vízková, M. Lutovinov, and M. Nesládek, "Mean stress effect in stress-life fatigue prediction re-evaluated," in *MATEC Web of Conferences*, 2018, vol. 165: EDP Sciences, p. 10018.
- [158] M. Roessle and A. Fatemi, "Strain-controlled fatigue properties of steels and some simple approximations," *International journal of fatigue*, vol. 22, no. 6, pp. 495-511, 2000.
- [159] D. Ye, S. Matsuoka, N. Suzuki, and Y. Maeda, "Further investigation of Neuber's rule and the equivalent strain energy density (ESED) method," *International journal of fatigue*, vol. 26, no. 5, pp. 447-455, 2004.
- [160] A. Moftakhar, A. Buczynski, and G. Glinka, "Calculation of elasto-plastic strains

and stresses in notches under multiaxial loading," *International journal of fracture*, vol. 70, no. 4, pp. 357-373, 1994.

[161] S. Suresh, *Fatigue of materials*. Cambridge university press, 1998.

[162] U. Muralidharan and S. Manson, "A modified universal slopes equation for estimation of fatigue characteristics of metals," 1988.