Anchor-free multi-level self-localization in ad-hoc networks

Song, Yang; Bajaj, Ian; Rabiee, Ramtin; Tay, Wee Peng

2021

Song, Y., Bajaj, I., Rabiee, R. & Tay, W. P. (2021). Anchor-free multi-level self-localization in ad-hoc networks. 2021 IEEE Wireless Communications and Networking Conference (WCNC). <https://dx.doi.org/10.1109/WCNC49053.2021.9417602>

<https://hdl.handle.net/10356/155063>

<https://doi.org/10.1109/WCNC49053.2021.9417602>

© 2021 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. The published version is available at: <https://doi.org/10.1109/WCNC49053.2021.9417602>.

Downloaded on 26 Apr 2023 17:09:06 SGT

Anchor-Free Multi-Level Self-Localization in Ad-hoc Networks

Yang Song¹, Ian Bajaj¹, Ramtin Rabiee² and Wee Peng Tay¹

Abstract—In this paper, we propose a multi-level localization algorithm that breaks a centralized localization problem into a cluster-level distributed localization problem, where each cluster is a centralized unit. In contrast to fully distributed localization, the cluster-level distributed scheme results in reduction in contention, communication overheads, convergence time and energy consumption because cluster heads are responsible for the *intra*-cluster positioning on behalf of the whole cluster. To generate a global map, the cluster heads communicate with their direct neighbors to carry out *inter*-cluster ranging and positioning. The proposed method is suitable for large ad-hoc networks where most agents are low-cost, low-power RF transceivers used for ranging only while some agents are integrated with microcomputers such as Raspberry Pis capable of running intra and inter-cluster localization algorithms. The proposed system can work without anchor nodes and thus it can be deployed in the environments such as urban canyon, inside multi-story buildings, airports, and underground shopping malls where access to anchors or Global Navigation Satellite System (GNSS) is limited or prohibitive. We exploit a hybrid of two well-known methods: multidimensional scaling (MDS) and extended Kalman filtering (EKF) to effectively construct local and global position maps, even in the absence of GNSS information, anchors, or a complete ranging matrix.

I. INTRODUCTION

Wireless sensor network (WSN) localization opens up a variety of location-aware applications in areas as diverse as next-generation cellular networks, vehicular networks, ad-hoc sensor networks, underwater acoustic sensor networks, as well as a number of military applications [1], [2]. The basic concept is to deploy a large number of low-cost, low-power sensor nodes (agents or anchors) that acquire and process data. In many practical environments such as enclosed areas and urban canyons where GNSS signal is not accessible and there are no pre-installed anchors, relative position determination would be a promising solution.

In contrast to fully distributed localization, the cluster-level distributed scheme results in reduction in contention, communication overheads, convergence time and energy consumption because cluster heads are responsible for the *intra*-cluster positioning on behalf of the whole cluster. To this end, we propose a multi-level localization that retains the scalability of the distributed algorithm at the cluster-level while letting each cluster be a centralized unit where a cluster head performs intra-cluster localization on behalf of the whole cluster.

Within a cluster, the cluster head coordinates with cluster members to carry out peer-to-peer (p2p) ranging which can be done through wireless technologies such as ultra wideband (UWB), LoRaWAN and Bluetooth Low Energy (BLE). Based on the local ranging matrix, the cluster head can then perform intra-cluster localization. To accommodate the ad-hoc nature of the network, the cluster head periodically broadcasts a beacon signal in order to renew the cluster members. Upon a merging request from a neighbouring cluster, two cluster heads communicate with each other to arrange inter-cluster ranging and perform inter-cluster positioning. The goal of inter-cluster positioning is to align one cluster's coordinate system to the other cluster's. Once two clusters are merged, either cluster head can broadcast "merging" request and any cluster who receives the request and wants to merge can acknowledge the request. This process repeats until all the clusters are merged into one common coordinate system. We call this whole process "cluster expansion".

The idea of anchor-free multi-level self-localization has been investigated using multidimensional scaling (MDS) method in [3]–[8]. MDS is a classic technique that seeks vectorial representations for data points given the pairwise distances between them. In case some distances are missing, MDS-MAP [3] proposes to complete the distance matrix with the shortest paths using Dijkstra's shortest path algorithm. To handle irregularly-shaped networks where the shortest path distance between two agents does not correspond well to their Euclidean distance, MDS-MAP(P) [4] computes local maps from each agent and then merges local maps into a global map. MDS-MAP(P) requires intensive computational resource for each agent. To alleviate this problem, several cluster-based MDS methods [5]–[8] are proposed to partition the network into overlapped clusters and generate local maps within each cluster and merge local maps into a global one. These cluster-based MDS methods share the same idea with our multi-level self-localization but they are mainly used to localize the static agents while ours is more capable of tracking mobile agents.

Bayesian filtering [9]–[13] has been widely adopted to track the dynamics of the mobile agents. These methods are mostly anchor-based which fuse wireless ranges together with other information such as GNSS pseudoranges [11] or measurements from on-board sensors [12], [13], via extended Kalman filtering (EKF) and/or particle filtering (PF). In the absence of anchors and GNSS signals, [14], [15] apply EKF to track mobile agents' relative locations using p2p ranges only. Our algorithmic backbone is EKF with MDS used for velocity estimation.

Our contributions are

- 1) We propose a EKF-MDS based localization algorithm to track the dynamic interactions between the clusters.

¹Y. Song, I. Bajaj and W. P. Tay are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. songy, ianbajaj, wptay@ntu.edu.sg

²R. Rabiee is with the Department of Applied Physics and Electronics, Umeå University, Sweden. ramtin.rabiee@umu.se

This research is supported by A*STAR under its RIE2020 Advanced Manufacturing and Engineering (AME) Industry Alignment Fund – Pre Positioning (IAF-PP) (Grant No. A19D6a0053).

- 2) The proposed method can work without anchors.
- 3) We provide a sufficient condition for uniquely merging two clusters.

The rest of this paper is organized as follows. In Section II, we present our problem formulation. In Section III, we propose a multi-level localization framework: intra-cluster localization, inter-cluster localization and cluster expansion, and also provide the sufficient condition for uniquely merging two clusters. The corresponding simulation and experimental results are documented, respectively, in Section IV and Section V, with our conclusions in Section VI.

Notations: We use $\mathbb{R}$ to denote the set of real numbers. The normal distribution with mean μ and variance σ^2 is denoted as $\mathcal{N}(\mu, \sigma^2)$. The uniform distribution with the lower endpoints a and upper endpoint b is denoted as $Unif(a, b)$. We use $^\top$ to represent matrix transpose. We use $\mathbf{I}_n$ to denote an identity matrix of size $n \times n$ and $\mathbf{1}_m$ to denote a column vector of size m with all entries being 1. We use $[\mathbf{A}]_{\mathcal{P}, \mathcal{Q}}$ to denote the sub-matrix of matrix $\mathbf{A}$ consisting of the entries $\mathbf{A}(i, j)$ for all $(i, j) \in \mathcal{P} \times \mathcal{Q}$.

II. PROBLEM FORMULATION

If agent i can range with agent j , agent i is a neighbour of agent j , denoted as $i \in \mathcal{B}_j$, and vice versa. Suppose there are N agents, after measuring all p2p ranges at time step k , we may form a ranging matrix $\mathbf{Z}_k \in \mathbb{R}^{N \times N}$ whose entries are

$$[\mathbf{Z}_k]_{i,j} = \begin{cases} \|\mathbf{p}_{i,k} - \mathbf{p}_{j,k}\| + n_k, & \text{if } i \in \mathcal{B}_j, \\ 0, & \text{otherwise,} \end{cases}$$

where $\mathbf{p}_{i,k}$ is the location of agent i at time step k , $\|\cdot\|$ denotes the Euclidean norm and $n_k \in \mathcal{N}(0, \sigma_n^2)$ represents independent and identically distributed (i.i.d.) additive ranging noise. The Gaussian assumption has been widely adopted under line-of-sight (LOS) scenario [16].

We propose to group agents into L disjoint clusters and designate each cluster a cluster head who is responsible for collecting the intra-cluster p2p ranges and computing the relative locations of the cluster members so that the localization complexity disperses over all clusters.

The clustering operation partitions the global ranging matrix $\mathbf{Z}_k$ into L local (or intra-cluster) ranging matrices and multiple inter-cluster ranging matrices:

$$\mathbf{Z}_k = \begin{bmatrix} [\mathbf{Z}_k]_{\mathcal{I}_1, \mathcal{I}_1} & [\mathbf{Z}_k]_{\mathcal{I}_1, \mathcal{I}_2} & \cdots & [\mathbf{Z}_k]_{\mathcal{I}_1, \mathcal{I}_L} \\ [\mathbf{Z}_k]_{\mathcal{I}_2, \mathcal{I}_1} & [\mathbf{Z}_k]_{\mathcal{I}_2, \mathcal{I}_2} & \cdots & [\mathbf{Z}_k]_{\mathcal{I}_2, \mathcal{I}_L} \\ \vdots & \vdots & \ddots & \vdots \\ [\mathbf{Z}_k]_{\mathcal{I}_L, \mathcal{I}_1} & [\mathbf{Z}_k]_{\mathcal{I}_L, \mathcal{I}_2} & \cdots & [\mathbf{Z}_k]_{\mathcal{I}_L, \mathcal{I}_L} \end{bmatrix},$$

where $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_i} \in \mathbb{R}^{N_i \times N_i}$ denote the intra-cluster ranging matrix for cluster i and $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j} \in \mathbb{R}^{N_i \times N_j}, \forall i \neq j$ denote the inter-cluster ranging matrices between cluster i and cluster j .

Our objective is to

- 1) perform intra-cluster localization for all clusters based on $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_i} \in \mathbb{R}^{N_i \times N_i}$, and

- 2) *uniquely* merge clusters based on $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j} \in \mathbb{R}^{N_i \times N_j}, i \neq j$.

Whenever the GNSS signals are accessible by some agents, it is straightforward to map the relative location to the global one.

III. MULTI-LEVEL LOCALIZATION

The system architecture is shown in Fig. 1 where the intra-cluster localization for all clusters can be carried out independently and the clusters' locations along with the inter-cluster ranges $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}, i \neq j$ will then be used for inter-cluster localization. Fig. 2 illustrates that each cluster performs intra-cluster positioning in its own coordinate system while inter-cluster positioning merges all clusters' coordinate systems into a global one. In what follows, we will show how these can be done.

A. Intra-Cluster Localization

Let $\mathbf{m}_k^{(i)} = \left[\left(\mathbf{X}_k^{(i)} \right)^\top, \left(\dot{\mathbf{X}}_k^{(i)} \right)^\top \right]^\top \in \mathbb{R}^{4N_i}$ represent the to-be-estimated states in cluster i at time k , where

$$\begin{aligned} \mathbf{X}_k^{(i)} &= \left[\left(\mathbf{p}_{1,k}^{(i)} \right)^\top, \dots, \left(\mathbf{p}_{N_i,k}^{(i)} \right)^\top \right]^\top, \\ \dot{\mathbf{X}}_k^{(i)} &= \left[\left(\dot{\mathbf{p}}_{1,k}^{(i)} \right)^\top, \dots, \left(\dot{\mathbf{p}}_{N_i,k}^{(i)} \right)^\top \right]^\top, \end{aligned}$$

where $\mathbf{p}_{n,k}^{(i)} = \left[x_{n,k}^{(i)}, y_{n,k}^{(i)} \right]^\top$ contains x and y local coordinates of agent n in cluster i , and $\dot{\mathbf{p}}_{n,k}^{(i)} = \left[\dot{x}_{n,k}^{(i)}, \dot{y}_{n,k}^{(i)} \right]^\top$ contains x and y local velocities of agent n in cluster i . In the remainder of this section, we will discuss how the intra-cluster localization can be done by utilizing two methods: EKF and MDS, and propose a framework to integrate the two.

1) *EKF Prediction:* The state $\mathbf{m}_k^{(i)}$ is evolved according to the following dynamical model:

$$\mathbf{m}_{k|k-1}^{(i)} = \mathbf{F}_k \mathbf{m}_{k-1}^{(i)} + \mathbf{G}_k \mathbf{w}_k,$$

where the transition matrix $\mathbf{F}_k$ equals to $\begin{bmatrix} \mathbf{I}_{2N_i} & \Delta_{T_s} \mathbf{I}_{2N_i} \\ \mathbf{0} & \mathbf{I}_{2N_i} \end{bmatrix}$,

and $\mathbf{G}_k$ equals to $\begin{bmatrix} \Delta_{T_s} \mathbf{I}_{2N_i} \\ \mathbf{I}_{2N_i} \end{bmatrix}$, the state noise $\mathbf{w}_k$ is zero-mean and has covariance $\mathbf{Q}_k = \sigma_w^2 \mathbf{G}_k \mathbf{G}_k^\top$ [17] and Δ_{T_s} is the sampling interval between two time steps.

2) *EKF Update:* Stacking the non-zero entries in the upper triangular part of $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_i}$ into a column vector $\mathbf{z}_k^{(i)}$. Then, we have

$$z_k^{(i)}(\ell) = h \left(\mathbf{p}_{\ell_1,k}^{(i)}, \mathbf{p}_{\ell_2,k}^{(i)} \right) + n_k = \left\| \mathbf{p}_{\ell_1,k}^{(i)} - \mathbf{p}_{\ell_2,k}^{(i)} \right\| + n_k,$$

where the index ℓ is associated with the agent pair $\{\ell_1, \ell_2\}$ in cluster i and $0 \leq \ell_1 < \ell_2 \leq N_i$. The update process proceeds

Fig. 1. The multi-level localization system.

Fig. 2. Local coordinate systems and global coordinate system.

via a standard EKF : shown below

$$\begin{aligned} \mathbf{m}_{k|k-1}^{(i)} &= \mathbf{F}_k \mathbf{m}_{k-1|k-1}^{(i)}, \\ \mathbf{P}_{k|k-1} &= \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^\top + \mathbf{Q}_k, \\ \mathbf{S}_k &= \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^\top + \sigma_n^2 \mathbf{I}, \\ \mathbf{K}_k &= \mathbf{P}_{k|k-1} \mathbf{H}_k^\top \mathbf{S}_k^{-1}, \\ \mathbf{m}_{k|k}^{(i)} &= \mathbf{m}_{k|k-1}^{(i)} + \mathbf{K}_k \left(\mathbf{z}_k^{(i)} - h \left(\mathbf{m}_{k|k-1}^{(i)} \right) \right), \\ \mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}, \end{aligned}$$

where the measurement matrix $\mathbf{H}_k \in \mathbb{R}^{M \times 4N_i}$ is defined to be the Jacobian $\frac{\partial h}{\partial \mathbf{m}} \big|_{\mathbf{m}_{k|k-1}^{(i)}}$ evaluated at $\mathbf{m}_{k|k-1}^{(i)}$:

$$\begin{aligned} [\mathbf{H}_k]_{\ell, \ell_1} &= \frac{x_{\ell_1, k} - x_{\ell_2, k}}{\|\mathbf{p}_{\ell_1, k} - \mathbf{p}_{\ell_2, k}\|}, \\ [\mathbf{H}_k]_{\ell, \ell_2} &= \frac{x_{\ell_2, k} - x_{\ell_1, k}}{\|\mathbf{p}_{\ell_1, k} - \mathbf{p}_{\ell_2, k}\|}, \\ [\mathbf{H}_k]_{\ell, \ell_1 + N_i} &= \frac{y_{\ell_1, k} - y_{\ell_2, k}}{\|\mathbf{p}_{\ell_1, k} - \mathbf{p}_{\ell_2, k}\|}, \\ [\mathbf{H}_k]_{\ell, \ell_2 + N_i} &= \frac{y_{\ell_2, k} - y_{\ell_1, k}}{\|\mathbf{p}_{\ell_1, k} - \mathbf{p}_{\ell_2, k}\|}. \end{aligned}$$

The last $2N_i$ columns of $\mathbf{H}_k$, which associated with derivatives of h w.r.t. velocities $\dot{\mathbf{X}}_k$, are zeros.

3) *MDS-Based Velocity Correction*: As foreshadowed in EKF update, the velocities are not measured directly but are derived from the agents' trajectories. The accuracy of

velocity estimation strongly depends on the assumed motion model. Here, we consider to control any misrepresentation of a dynamic motion model by estimating the velocities using MDS-MAP. We input the intra-cluster range matrix $[Z_k]_{\mathcal{I}_i, \mathcal{I}_i}$ to the MDS-MAP procedure given below

1. Fill the missing ranges in the distance matrix $[Z_k]_{\mathcal{I}_i, \mathcal{I}_i}$ with shortest paths.
2. Compute Gram matrix $[Z_k]_{\mathcal{I}_i, \mathcal{I}_i} \odot [Z_k]_{\mathcal{I}_i, \mathcal{I}_i}$, where $\odot$ denotes element-wise product.
3. Compute $\mathbf{B} = \frac{-1}{2} \mathbf{J} \mathbf{G} \mathbf{J}$, where $\mathbf{J} = \mathbf{I} - \frac{1}{N_i} \mathbf{1} \mathbf{1}^\top$.
4. Eigen-decomposition of $\mathbf{B} = \mathbf{V} \boldsymbol{\Sigma} \mathbf{V}^\top$.
5. The D -dimensional ($D = 2$ or 3) position $\mathbf{X}_{k,\text{MDS}}^{(i)} = [\boldsymbol{\Sigma}_{\mathcal{I}, \mathcal{I}}^{1/2} [\mathbf{V}]_{:, \mathcal{I}}^\top]^\top$ with index set $\mathcal{I} = \{1, \dots, D\}$ associated with D largest eigenvalues.

to get an one-step position estimate $\mathbf{X}_{k,\text{MDS}}^{(i)}$ at time instant k which (unlike EKF) is not correlated with the previous results. Since the MDS estimate $\mathbf{X}_{k,\text{MDS}}^{(i)}$ is invariant under flip, translation, and rotation, we heuristically choose three agents with the least change in velocities as compared to the previous state, to align the MDS coordinate system to the EKF coordinate system. After coordinates alignment, we use the aligned MDS positions $\mathbf{x}_{n,k,\text{MDS}}^{(i)}, \mathbf{y}_{n,k,\text{MDS}}^{(i)}$ to compute the velocity by

$$\begin{aligned} \dot{\mathbf{x}}_{n,k}^{(i)} &= \alpha \frac{\mathbf{x}_{n,k,\text{MDS}}^{(i)} - \mathbf{x}_{n,k-1,\text{MDS}}^{(i)}}{\Delta T_s} + (1 - \alpha) \dot{\mathbf{x}}_{n,k}^{(i)}, \\ \dot{\mathbf{y}}_{n,k}^{(i)} &= \alpha \frac{\mathbf{y}_{n,k,\text{MDS}}^{(i)} - \mathbf{y}_{n,k-1,\text{MDS}}^{(i)}}{\Delta T_s} + (1 - \alpha) \dot{\mathbf{y}}_{n,k}^{(i)}, \end{aligned}$$

where $n = 1, \dots, N_i$ and $\alpha \in [0, 1]$ is a trade-off parameter.

B. Inter-Cluster Localization

Given location estimates $\mathbf{X}_k^{(i)}, \mathbf{X}_k^{(j)}$ for clusters i and j and the inter-cluster ranging matrix $[Z_k]_{\mathcal{I}_i, \mathcal{I}_j}^1$, we keep the

¹One cluster head can command its members to do ranging with the members in the other cluster.

master cluster's, say cluster i 's, location $\mathbf{X}_k^{(i)}$ fixed while merging the client cluster, i.e., cluster j to it by translating and rotating cluster $\mathbf{X}_k^{(j)}$. Let cluster j 's displacement along x - and y -axis at time k be $t_{x,k}^{(j \rightarrow i)}$ and $t_{y,k}^{(j \rightarrow i)}$, respectively, and its rotation angle be $\theta_k^{j \rightarrow i}$. The interaction between two clusters is parametrized by these three variables and is substantiated by $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$.

Stacking the non-zero entries of $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$ into a column vector yields $\mathbf{z}_k$. Then, the measurement model would be

$$\begin{aligned} [\mathbf{z}_k]_\ell &= g\left(\theta_k^{j \rightarrow i}, t_{x,k}^{j \rightarrow i}, t_{y,k}^{j \rightarrow i}\right) + n_k \\ &= \left\| \hat{\mathbf{p}}_{\ell_1, k}^{(i)} - \mathbf{R}(\theta_k^{j \rightarrow i}) \hat{\mathbf{p}}_{\ell_2, k}^{(j)} - \begin{bmatrix} t_{x,k}^{j \rightarrow i} \\ t_{y,k}^{j \rightarrow i} \end{bmatrix} \right\| + n_k, \end{aligned}$$

where $\mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ denotes the rotation matrix, index $\ell = 1, \dots, M$ is associated with the agent pair $\{\ell_1, \ell_2\}$, $1 \leq \ell_1 \leq N_i$, $1 \leq \ell_2 \leq N_j$. Let the state vector be $\boldsymbol{\xi}_k = [\theta_k^{j \rightarrow i}, t_{x,k}^{j \rightarrow i}, t_{y,k}^{j \rightarrow i}]^\top$ and it evolves according to $\boldsymbol{\xi}_{k|k-1} = \boldsymbol{\xi}_{k-1} + \boldsymbol{\nu}_k$, where the process noise $\boldsymbol{\nu}_k \in \mathcal{N}(\mathbf{0}, \mathbf{Q}_k)$ with $\mathbf{Q}_k = \text{diag}(\sigma_\theta^2, \sigma_{t_x}^2, \sigma_{t_y}^2)$.

The update process proceeds via the standard EKF given in Section III-A2 where the transition matrix $\mathbf{F}_k$ becomes an identity matrix and the measurement matrix $\mathbf{H}_k \in \mathbb{R}^{M \times 3}$ is defined to be the Jacobian $\left. \frac{\partial g}{\partial \boldsymbol{\xi}} \right|_{\boldsymbol{\xi}_{k|k-1}}$ evaluated at $\boldsymbol{\xi}_{k|k-1}$:

$$\begin{aligned} [\mathbf{H}_k]_{\ell, 1} &= \frac{(x_{\ell_1, k}^{(i)} - x_{\ell_2, k}^{j \rightarrow i}) (\sin(\theta_k^{j \rightarrow i}) x_{\ell_2, k}^{(j)} + \cos(\theta_k^{j \rightarrow i}) y_{\ell_2, k}^{(j)})}{\|\mathbf{p}_{\ell_1, k}^{(i)} - \mathbf{p}_{\ell_2, k}^{j \rightarrow i}\|} \\ &\quad + \frac{(y_{\ell_1, k}^{(i)} - y_{\ell_2, k}^{j \rightarrow i}) (\sin(\theta_k^{j \rightarrow i}) y_{\ell_2, k}^{(j)} - \cos(\theta_k^{j \rightarrow i}) x_{\ell_2, k}^{(j)})}{\|\mathbf{p}_{\ell_1, k}^{(i)} - \mathbf{p}_{\ell_2, k}^{j \rightarrow i}\|}, \\ [\mathbf{H}_k]_{\ell, 2} &= \frac{-(x_{\ell_1, k}^{(i)} - x_{\ell_2, k}^{j \rightarrow i})}{\|\mathbf{p}_{\ell_1, k}^{(i)} - \mathbf{p}_{\ell_2, k}^{j \rightarrow i}\|}, \\ [\mathbf{H}_k]_{\ell, 3} &= \frac{-(y_{\ell_1, k}^{(i)} - y_{\ell_2, k}^{j \rightarrow i})}{\|\mathbf{p}_{\ell_1, k}^{(i)} - \mathbf{p}_{\ell_2, k}^{j \rightarrow i}\|}, \end{aligned}$$

where $\mathbf{p}_{\ell_2, k}^{j \rightarrow i} = [x_{\ell_2, k}^{j \rightarrow i}, y_{\ell_2, k}^{j \rightarrow i}]^\top = \mathbf{R}(\theta_k^{j \rightarrow i}) \mathbf{p}_{\ell_2, k}^{(j)} + [t_{x,k}^{j \rightarrow i}, t_{y,k}^{j \rightarrow i}]^\top$.

C. Cluster Expansion

The cluster expansion follows the process pipeline shown in Fig. 3 and it can be implemented in a distributed manner. The implementation procedure is given below.

- 1) Each cluster performs intra-cluster ranging to get $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_i}$, $\forall i$ and runs *intra-cluster* localization.
- 2) Cluster i broadcasts its cluster-location $\mathbf{X}_k^{(i)}$. If cluster i has either been merged or has global location information.
- 3) The neighbouring clusters acknowledge with their cluster IDs at random time. Then, cluster i performs inter-cluster ranging with its neighbours one-by-one to get $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$, $j \in \mathcal{N}_i$.
- 4) Cluster j , $\forall j \in \mathcal{N}_i$ runs *inter-cluster* localization algorithm introduced in Section III-B in order to merge to clusters i .

Fig. 3. Cluster expansion over time.

The cluster head may periodically broadcast a beacon message to update its cluster members. In the case that an agent has never been recruited by any cluster head for a while, it nominates itself to be a cluster head and recruit its own members.

Here we assume the merging always starts from one reference cluster, e.g., cluster A in Fig. 3. If a cluster receives "merging" requests from more than one neighbouring clusters, it simply chooses one to merge with.

D. Unique Solution for Inter-Cluster Localization

Here, we derive a sufficient condition for uniquely merging two clusters using graph theory.

Assumption 1. Each cluster contains more than two agents.

Assumption 2. Each cluster is a globally rigid graph.

Lemma 1. (Connelly [18], Jackson [19]) A graph $\mathcal{G} \in \mathbb{R}^2$ is globally rigid iff $\mathcal{G}$ is 3-connected and deleting any edge of $\mathcal{G}$ must leave a generically locally rigid graph.

Lemma 2. (Theorem 7. in [20]) Let $\mathcal{G}_1 = (\mathcal{V}_1, \mathcal{L}_1)$ and $\mathcal{G}_2 = (\mathcal{V}_2, \mathcal{L}_2)$ be two globally rigid graphs with disjoint set of vertices. There exists a bipartite graph $\mathcal{K} = (\mathcal{U}_3, \mathcal{V}_3, \mathcal{L}_3)$ ² such that $\mathcal{U}_3 \subset \mathcal{V}_1$, $\mathcal{V}_3 \subset \mathcal{V}_2$, $|\mathcal{U}_3 \cap \mathcal{V}_1| \geq 3$ and $|\mathcal{V}_3 \cap \mathcal{V}_2| \geq 3$, and $|\mathcal{L}_3| \geq 4$, then $\mathcal{G} = \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{K}$ is globally rigid.

Lemma 2 can be interpreted in terms of inter-cluster range matrix $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$: The unique solution for the inter-cluster localization exists if there exists a 3×3 submatrix in $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$ such that there are at least four non-zero entries and no empty columns/rows.

² $\mathcal{K} = (\mathcal{U}, \mathcal{V}, \mathcal{L})$ denotes a bipartite graph whose partition has two independent sets $\mathcal{U}$ and $\mathcal{V}$ and every edge in $\mathcal{L}$ connects a vertex in $\mathcal{U}$ to $\mathcal{V}$.

Fig. 4. Inter-cluster connection between five clusters.

Assumption 3. If a range measured between two agents coming from two clusters is constantly missing, it implies the distance between them is beyond the communication coverage.

This assumption says by utilizing the information that are embedded in the missing ranges we do not need as many ranges in $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$ as Lemma 2 requires to make a graph determined by $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$ globally rigid.

Lemma 3. (Theorem 4. in [21]) Let $\mathcal{G}_1 = (\mathcal{V}_1, \mathcal{L}_1)$ and $\mathcal{G}_2 = (\mathcal{V}_2, \mathcal{L}_2)$ be two rigid graphs with disjoint set of vertices. There exists a bipartite graph $\mathcal{K} = (\mathcal{U}_3, \mathcal{V}_3, \mathcal{L}_3)$ such that $|\mathcal{U}_3| = |\mathcal{U}_3 \cap \mathcal{V}_1| = 2$, $|\mathcal{V}_3| = |\mathcal{V}_3 \cap \mathcal{V}_2| = 2$ and $|\mathcal{L}_3| = 3$, then $\mathcal{G} = \mathcal{G}_1 \cup \mathcal{G}_2 \cup \mathcal{K}$ is rigid.

Proposition 1. There exists a unique solution for merging two clusters if there exists a 3×3 matrix $\mathbf{A} = \begin{bmatrix} \mathbf{B} & 0 \\ 0 & \chi \end{bmatrix}$, where $\mathbf{A}$ is a submatrix of $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$, $\mathbf{B} \in \mathbb{R}^{2 \times 2}$ has at least three non-zero entries, and $\chi \geq 0$.

Proof: Due to Assumption 2 and Lemma 3, two clusters connected via $\mathbf{B}$ is a rigid graph but is not globally rigid, and thus it may suffer flip ambiguities. In case $\chi > 0$, Lemma 2 ensures the two clusters connected via $\mathbf{A}$ is globally rigid, and thus a unique solution exists. In case $\chi = 0$, we can always replace χ by the shortest path connecting the two agents associated with χ . Due to Assumption 3, the flip ambiguity embedded in the rigid graph can thus be resolved. ■

Remark 1. If some missing ranges in $[\mathbf{Z}_k]_{\mathcal{I}_i, \mathcal{I}_j}$ are replaced with shortest paths, the EKF assigns large variance to these ranges in order to account for their inaccuracy.

IV. NUMERICAL RESULTS

Firstly, we consider a scenario where there are $L = 5$ clusters and each one contains $N_i = 10, \forall i = 1, \dots, 5$ agents. At time $k = 0$, we let $x_{n,0}^{(i)} = x_0^{(i)} + a_{n,0}^{(i)}, y_{n,0}^{(i)} = y_0^{(i)} + b_{n,0}^{(i)}$ with $x_0^{(i)}, y_0^{(i)} \sim \mathcal{N}(0, 100m^2), a_{n,0}^{(i)}, b_{n,0}^{(i)} \sim \text{Unif}(0, 10m)$ and $\dot{x}_{n,0}^{(i)}, \dot{y}_{n,0}^{(i)} \sim \text{Unif}(0.4m, 0.6m/s)$. We assume the agents in one cluster share the one acceleration which is drawn from $\text{Unif}(-0.1m/s^2, 0.1m/s^2)$. The simulated range noise follows $\mathcal{N}(0, (0.3m)^2)$ and sampling period $\Delta T_s = 0.5s$. The five clusters' inter connections are depicted in Fig. 4 which does not change over time. Cluster 1 is elected as the reference cluster. From $k = 5$ onwards, cluster 2 merges to cluster 1. From $k = 10$ onwards, cluster 3 merges to cluster 1. From $k = 15$ onwards, cluster 4 merges to cluster 2. From

$k = 20$ onwards, cluster 5 merges to cluster 3. The whole process ends at $k = 50$. Note that the simulated merging time schedule doesn't imply any time synchronization in the practical implementation. We randomly drop 20% ranges from the inter-cluster ranging matrix and 10% ranges from the intra-cluster ranging matrix and replace them by their corresponding shortest paths. The results in Fig. 5 show that the proposed method is more appropriate than the cluster-based MDS [6] to track the dynamic changes between the clusters.

Next, we consider two clusters and each contains six agents and assume their intra-cluster localizations have been done at every time instant k using the proposed algorithm in Section III-A. Let $z_{k,\max}^{(1,2)}$ denotes the maximum range in $[\mathbf{Z}_k]_{\mathcal{I}_1, \mathcal{I}_2}$. The following four inter-cluster connectivity settings are considered: 1) agents are fully connected 2) the ranges that are greater than $0.5z_{k,\max}^{(1,2)}$ are ignored, 3) the ranges that are greater than $0.3z_{k,\max}^{(1,2)}$ are ignored, 4) the ranges that are greater than $0.3z_{k,\max}^{(1,2)}$ are replaced with their corresponding shortest paths. The merging is carried out over 200 time steps and the results are shown in Fig. 6. Two things can be observed: 1) the convergence rate depends on the sparsity of the intersection graph; 2) adding virtual ranges help speed up the convergence (see purple and green curves).

V. EXPERIMENTAL RESULTS

We perform a real-time tracking of six Decawave's DW1000 UWB transceivers in a workshop of size $15m \times 10m$. Six UWB agents are split into two clusters: three agents $\{1,2,3\}$ in the master cluster and the other three agents $\{4,5,6\}$ form the client cluster. Agents 1 and 4 are the group heads. The overall positioning update rate is about 2Hz. As shown in Fig. 7, at $k = 0$ six agents are randomly initialized at the vicinity of $(0,0)$. After five seconds (at $k = 10$), two clusters' intra-cluster localizations converge and it means the relative positions between the agents within each cluster are correct but the relative position between two clusters is still evolving. After another 2.5 seconds (at $k = 15$), the inter-cluster localization converges.

VI. CONCLUSION

In this paper we have tackled multi-level localization for ad-hoc sensor networks. Global mapping was reduced to individual local cluster problems that are distributed but centralized to a cluster head responsible for agents in its vicinity. At the local cluster level, MDS-MAP based method was considered to deal with missed ranges. At the inter-cluster level, we have studied the sufficient conditions needed to uniquely merge any two clusters. As a future work, we will investigate how a cluster can be simultaneously merged to multiple clusters who share a common coordinate system.

REFERENCES

- [1] F. Gustafsson and F. Gunnarsson, "Mobile positioning using wireless networks: Possibilities and fundamental limitations based on available wireless network measurements," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 41–53, Jul. 2005.

Fig. 5. Comparison of cluster-based MDS [6] and proposed EKF-based multi-level method in terms of the global localization error, i.e., after merging all clusters into a global coordinate system, over 50 time steps.

Fig. 6. Inter-cluster localization error under four inter-cluster connectivity settings.

Fig. 7. Real-time multi-level localization results using six UWB agents at time step $k = 0, 1, 10, 15$, where the crosses denote ground truth locations.

[2] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molisch, H. V. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios: A look at positioning aspects for future sensor networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 70–84, Jul. 2005.

[3] Y. Shang, W. Ruml, Y. Zhang, and M. P. J. Fromherz, "Localization from mere connectivity," in *Proc. ACM Int. Sym. on Mobile Ad Hoc Networking and Computing*, 2003, pp. 201–212.

[4] Y. Shang, W. Ruml, Y. Zhang, and M. Fromherz, "Localization from connectivity in sensor networks," *IEEE Trans Parallel Distrib Syst.*, vol. 15, no. 11, pp. 961–974, Nov. 2004.

[5] B. Stojkoska, D. Davcev, and A. Kulakov, "Cluster-based MDS algorithm for nodes localization in wireless sensor networks with irregular topologies," in *Proc. Int. Conf. Soft Computing as Transdisciplinary Science and Technology*, 2008, p. 384–389.

[6] G. Yu and S. Wang, "A hierarchical MDS-based localization algorithm for wireless sensor networks," in *Proc. Int. Conf. Advanced Information Networking and Applications*, 2008, pp. 748–754.

[7] M. Shon, M. Jo, and H. Choo, "An interactive cluster-based MDS localization scheme for multimedia information in wireless sensor networks," *Comput. Commun.*, vol. 35, no. 15, pp. 1921 – 1929, 2012.

[8] N. Saeed and H. Nam, "Cluster based multidimensional scaling for irregular cognitive radio networks localization," *IEEE Trans. Signal Process.*, vol. 64, no. 10, pp. 2649–2659, 2016.

[9] A. Boukerche, H. A. Oliveira, E. F. Nakamura, and A. A. Loureiro, "Vehicular ad hoc networks: A new challenge for localization-based systems," *Computer Communications*, vol. 31, no. 12, pp. 2838 – 2849, 2008.

[10] N. Alam, A. Tabatabaei Balaei, and A. G. Dempster, "Relative positioning enhancement in VANETs: A tight integration approach," *IEEE Trans Intell Transp Syst.*, vol. 14, no. 1, pp. 47–55, Mar. 2013.

[11] K. Lassoued, I. Fantoni, and P. Bonnifait, "Mutual localization and positioning of vehicles sharing GNSS pseudoranges: Sequential bayesian approach and experiments," in *Proc. IEEE Int. Conf. Intelligent Transportation Systems*, Sep. 2015, pp. 1896–1901.

[12] R. Rabiee, X. Zhong, Y. Yan, and W. P. Tay, "Laif: A lane-level self-positioning scheme for vehicles in gnss-denied environments," *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 8, pp. 2944–2961, 2019.

[13] S. van der Helm, M. Coppola, K. N. McGuire, and G. C. H. E. de Croon, "On-board range-based relative localization for micro air vehicles in indoor leader-follower flight," *Autonomous Robots*, Mar. 2019.

[14] J.-L. Blanco, J. Gonzalez, and J.-A. Fernandez-Madrigal, "A pure probabilistic approach to range-only SLAM," in *Int. Conf. on Robotics and Automation*, 2008.

[15] Y. Song, M. Guan, W. P. Tay, C. L. Law, and C. Wen, "UWB/LiDAR fusion for cooperative range-only slam," in *Int. Conf. on Robotics and Automation*, 2019.

[16] I. Guvenc and C. Chong, "A survey on toa based wireless localization and nlos mitigation techniques," *IEEE Commun. Surv. Tut.*, vol. 11, no. 3, pp. 107–124, 2009.

[17] X. R. Li and V. P. Jilkov, "A survey of maneuvering target tracking: dynamic models," in *Proc. SPIE Conf. Signal and Data Processing of Small Targets*, 2000.

[18] R. Connelly, "Generic global rigidity," *Discrete & Computational Geometry*, vol. 33, no. 4, pp. 549–563, Apr. 2005.

[19] B. Jackson and T. Jordan, "Connected rigidity matroids and unique realizations of graphs," *Journal of Combinatorial Theory, Series B*, vol. 94, no. 1, pp. 1–29, May 2005.

[20] T. Eren, B. Anderson, W. Whiteley, A. Morse, and P. Belhumeur, "Merging globally rigid formations of mobile autonomous agents," in *Proc. Int. Joint Conf. Autonomous Agents and Multiagent Systems*, New York, NY, USA, Jul. 2004.

[21] R. Olfati-Saber and R. Murray, "Graph rigidity and distributed formation stabilization of multi-vehicle systems," in *Proc. IEEE Conf. Decision and Control*, Las Vegas, NV, USA, Dec. 2002.