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Impact response and energy absorption of functionally graded foam under temperature gradient environment

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Abstract

As an effective strategy to enhance the energy absorption and impact resistance capacities of foam materials, density-gradient design has been widely proposed as a viable solution. However, the influence of thermal environment on the impact response and energy absorption of functionally graded foam still remains unclear. In the present paper, an analytical model based on the double shock wave theory is presented to examine the impact performance of density-graded foam rods under temperature gradient environments. A density-graded foam with one end supported and the other end struck by a block of mass is presented to evaluate their impact performances. Both the positive and negative density-gradient models with power-law profiles along the axial direction of the foam rod are considered. The proposed theoretical representation is validated against several previous models, including a finite element (FE) simulation. It is demonstrated that the temperature gradient may lead to transformations in the deformation pattern for the positive and negative density-graded foams. Moreover, the temperature gradient can enlarge the impact resistance property in most cases and can reduce the energy absorption capacity of density-graded foams.

Keywords: impact response; functionally graded foam; thermal environment; energy absorption; impact resistance.

1. Introduction

Due to their superior mechanical, thermal and acoustic properties, foam materials have

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garnered significant attention in the field of transport, aerospace, and civil engineering. Particularly, their lightweight and outstanding energy absorption capacities make them attractive candidates for protecting the structure/human from damage/injury under impact and crash loadings [1-8]. There are many theoretical, numerical and experimental analyses carried out to understand their dynamic behaviors. Theoretically, Reid and Peng [9] initially proposed the shock wave theory in the framework of the rigid, perfectly-plastic, locking (R-PP-L) idealization to reveal the dynamic crushing process of foam rod being struck by a rigid projectile. Subsequently, several complex constitutive relationships including the elastic, perfectly-plastic, locking (E-PP-L) model [10], the elastic, perfectly-plastic, hardening (E-PP-H) model [11], as well as the rigid, plastic-hardening (R-PH) model [12] and the rigid, softening-hardening (R-SH) model [13] were proposed as improvements to the shock wave theory. These studies all illustrate that the shock wave theory can capture the impact crushing response of foam materials effectively. Numerically, finite element (FE) analyses based on the periodic Voronoi model [14-16] and hollow spheres model [17] were carried out to explain the dynamic crushing behavior of foam materials. In addition, various dynamic compression and impact experiments on foam materials were also conducted to quantify their impact responses and energy absorption characteristics [18-21].

Notably, most of the abovementioned studies are focused on traditional foam materials with uniform densities. To improve the energy absorption and impact resistance capacities of foam materials, the concept of functionally graded design is introduced. Functionally graded material is a class of materials whose physical properties vary smoothly from one surface to another, which is a concept proposed by material scientists in the early 1980s for ultrahigh temperature protection in space vehicles [22]. The material can overcome delamination and stress concentration in traditionally layered materials. Employing the FE simulation study, Den et al. [23] analyzed the energy absorption and dynamic response of closed-cell metal foams with graded porosity distributions via the Voronoi model. Similar numerical models were also established by Yang et al. [24] and Liang et al. [25]. Besides, Zhou et al. [26] studied the crushing process of foam materials subjected to an impact loading by using the hollow sphere model. Furthermore, Cui et al. [27] designed a helmet with functionally graded foam core to obtain an optimization choice for impact resistance. Other crashworthiness systems such as metallic tubes and sandwich structures filled with graded foam core were also designed and numerically studied [28-31]. By means of

experiments, the quasi-static and dynamic behaviors of graded foam materials are also widely studied. For instance, He et al. [32] adopted the quasi-static compression experiment to inspect the crushing behavior of graded foam materials. Xia et al. [33] set up a blast test program to investigate the effectiveness of density-graded foams on preventing the blast loadings. Hangai et al. [34] employed the drop weight impact test to reveal the dynamic characteristics of functionally graded aluminum foams.

Apart from the numerical and experimental studies, theoretical investigations were also carried out by many academic researchers. In the framework of the shock wave theory, Liu et al. [35] studied the dynamic process of a density-graded foam rod impinging onto the rigid wall with an initial velocity. Shen et al. [36, 37] analyzed the impact response of a graded foam rod struck by a rigid mass as well as a mass-attached graded foam rod impinging onto a rigid wall based on the same theory. In their studies, an interesting phenomenon, i.e., the double shock wave mode was first observed and analyzed theoretically, and this double shock wave phenomenon is also validated by comparison with FE simulations. Similar phenomenon was also detected and reported by Karagiozova and Alves [38]. Based on this double shock wave theory, the impact behaviors of more complex graded foam with nonlinear continuous density-gradient [39], with middle-high and middle-low density-gradient [40], as well as with varying cross-section [41] were also investigated. In addition, the dynamic behavior of graded foam subject to underwater blast loading is also evaluated by Chen et al. [42, 43] by employing the fluid-structure interaction method.

However, most above reports on the foam materials are limited to room temperature environment, and the influences of elevated temperature on their dynamic behaviors are often neglected. As a commonly-used material in the impact resistance system, they may also be subjected to high temperatures, such as the protection system in space vehicles to prevent damages caused by the impact from space debris. Under such conditions, the foam materials would need to withstand extreme temperatures and large temperature gradients. Hence, understanding the influences of temperature on the impact behaviors of foam materials becomes critical and needs to be addressed. In literature, there are a few academic researches focusing on the quasi-static crushing of foam materials under high temperature environment. For example, Linul et al. [44] conducted a quasi-static compression experiment to estimate the material properties of closed-cell aluminum-alloy foam materials under different temperatures. Similarly, the effects of high

temperature on the quasi-static compressive behaviors of open-cell foams and carbon nanotube-reinforced aluminum foams were also investigated experimentally [45, 46]. From their studies, it was found that elevated temperatures have large influence over the plateau stress and Young's modulus of foam materials. Moreover, the effect of elevated temperatures on the indentation [47] and three-point bending [48] of aluminum foam core sandwich structures was also analyzed. Few researchers also devoted to examining the influence of external temperature environment on the impact behavior of foam materials. Ling et al. [49] experimentally studied the repeated impact behaviors of foam sandwich plates under low temperatures. Li and Lu [50] presented a theoretical model to reveal the dynamic crushing behavior of cellular foams under initial velocity impact in temperature gradient environments.

According to the literature summarized above, despite the preliminary studies, the underlying deformation mechanism of foam materials subject to impact loadings in thermal environments is still not thoroughly understood. In particular, the effects of temperature gradient on the impact performances of graded foam materials are still missing. In the present paper, a theoretical model on the basis of the double shock wave theory is introduced to describe the dynamic behaviors of density-graded foam materials with temperature gradient. A stationary graded foam rod subject to an impact by a mass block is taken into consideration. The density of the foam rod is distributed along the axial direction with a power law profile, and both negative and positive density-gradient models are analyzed. The proposed theoretical model is validated by comparing with previous studies and further verified with FE simulations. The influences of temperature gradient and density gradient on the deformation mechanism, energy absorption, and impact resistance capacities of foam materials are investigated in detail. It is expected that the present work can provide guidelines to the application of graded foam materials in the impact protection system under thermal environment.

2. Theoretical modeling

2.1. Problem formulation

Consider a stationary graded foam rod of initial length L_0 in the thermal environment with initial temperatures T_p and T_d at the striking and supported ends ($T_p \geq T_d$), respectively, as depicted in Fig.1. Besides, it is assumed that temperature varies linearly along the axial direction

of the graded foam rod under the striker-rod impact scenario, i.e., a stationary graded foam rod with one (distal) end supported, while the other (proximal) end struck by a mass block M_p with an initial velocity V_0 . Let A_0 denote the cross-sectional area of the graded foam rod, which remains constant during the whole crushing process. The initial densities at the striking and supported ends of the graded foam rod are represented by ρ_{i0} and ρ_{s0} , respectively. It is assumed that the initial density along the axial direction of the graded foam rod satisfies the following form

$$\rho_0(X) = \rho_{i0} + (\rho_{s0} - \rho_{i0}) \left(\frac{X}{L_0} \right)^K, \quad (1)$$

where K stands for a power-law index, which controls the density distribution of the foam rod, and $K = 1$ represents the linear gradient distribution. X denotes an arbitrary position in the axial direction of the foam rod. The density gradient of the graded foam rod can be expressed as

$$g = \frac{\rho_{s0}}{\rho_{i0}}. \quad (2)$$

Following this definition, the uniform, positive and negative density distributions can be, respectively, denoted by $g = 1$, $g > 1$, and $g < 1$. In the present study, a simple case neglecting the temperature rise due to the plastic deformation of foam materials under impact loading is adopted. To account for the heat generated during the plastic crushing process, the reader may refer to Refs.[51, 52].

From Gibson and Ashby's analysis [53], the densification strain ε_d and quasi-static plateau stress σ_p of low-density foam material while neglecting the thermal effect can be denoted as functions of its relative density, given by

$$\varepsilon_d(X) = 1 - B_1 \frac{\rho_0(X)}{\rho_b}, \quad (3)$$

$$\sigma_p(X) = B_2 \left(\frac{\rho_0(X)}{\rho_s} \right)^{3/2} \sigma_b, \quad (4)$$

where $\rho_0(X)$ stands for the density of the functionally graded foam rod at any position X , ρ_b and σ_b are, respectively, the density and yield stress of the base material from which the foam material is made. B_1 and B_2 are material constants determined by experiments. By taking into account the temperature gradient, the quasi-static plateau stress of the foam rod at any arbitrary position can be re-expressed as [50]

$$\sigma_p(X) = B_4(T_d) \left(\frac{\rho_0(X)}{\rho_b} \right)^{3/2} \sigma_b + B_3(T_p - T_d) \left(1 - \frac{X}{L_0} \right), \quad (5)$$

where B_3 is a coefficient reflecting the influence of temperature gradient on the quasi-static plateau stress of the foam rod; $B_4(T_d)$ is a material parameter related to the temperature in the supported end, which can be expressed as $B_4(T_d) = B_2(1 - \alpha_m T_d / T_m)$ and T_m denotes the melting temperature of the base material from which the foam material is made, and α_m denotes a material constant [54]. It should be noted that the maximum external temperature considered in this paper should not be larger than the value of T_m .

Fig.1. Sketch of the impact of a mass block on the graded foam rod with temperature gradient.

2.2. Negative density-graded foam

The R-PP-L idealization and one-dimensional shock wave theory are employed to set up the governing equations of the graded foam rod struck by a mass block under the temperature gradient. According to the R-PP-L material idealization, the elastic wave speed is assumed to be infinite, which means that the elastic wave can reach the ends of the graded foam rod immediately once the foam is impinged. Thus, the stress just ahead of the shock wave front is always equal to the quasi-static plateau stress at the location X_0 . As presented in Fig.2, when a compressive shock wave front propagates from a material location X_0 to $X_0 + dX$ during a small time interval dt , a discontinuous jump across the shock wave front can be found for the particle velocity V , compressive strain ε , and compressive stress σ . The Lagrangian wave speed of the shock wave front can be calculated as $C = dX/dt$. The displacement continuity condition and the momentum conservation condition over the shock wave front of the graded foam rod can be separately expressed as [11]

$$[V] = C[\varepsilon], \quad (6)$$

$$[\sigma] = \rho(X_0)C[V], \quad (7)$$

where the symbol $[\]$ stands for a discontinuous jump of physical quantities $\{\sigma, \varepsilon, V\}$ over the shock wave front, which can be denoted as

$$[\Phi] = \Phi^- - \Phi^+, \quad (8)$$

where Φ^- and Φ^+ represent the values of the physical quantities behind and ahead of the shock wave front, respectively.

Fig.2. Schematic of a discontinuous shock wave propagating along the foam rod.

In the studies presented by Shen et al.[36, 37], it is revealed that shock wave fronts form both at the striking and supported ends when a mass block is launched to the negative foam rod, i.e., the double shock wave model can be observed. Additionally, when a negative density-graded foam rod is struck by a mass block, the impinging layer crushes immediately and the mass block will attach to this end with a common velocity V_1 . Meanwhile, a shock wave will appear at the weaker end of the foam rod and it travels towards the proximal end with a velocity C_2 , as depicted in Fig.3. Let V_2 denote the particle velocity of the un-deformed region. In addition, denote X_1 and X_2 the Lagrangian locations of the forward and backward shock wave fronts, respectively. The velocities of these two shock wave fronts C_1 and C_2 can be separately formulated as

$$C_1 = \frac{dX_1}{dt}, \quad C_2 = -\frac{dX_2}{dt}. \quad (9)$$

On the basis of the R-PP-L material model, the material density across the forward shock wave front changes from $\rho_0(X_1)$ to ρ_b , while it jumps from $\rho_0(X_2)$ to ρ_b at the backward shock wave front. During the crushing process, the mass density, the particle velocity, the compressive stress and strain ahead of the forward shock wave front can be given as

$$\rho_1^+ = \rho_0(X_1), \quad V_1^+ = V_2, \quad \sigma_1^+ = \sigma_p(X_1), \quad \varepsilon_1^+ = 0. \quad (10)$$

Whilst behind this shock wave front, the corresponding physical quantities are

$$\rho_1^- = \rho_b, \quad V_1^- = V_1, \quad \sigma_1^- = \sigma_d(X_1), \quad \varepsilon_1^- = \varepsilon_d(X_1). \quad (11)$$

These quantities ahead of the backward shock wave front satisfy

$$\rho_2^+ = \rho_0(X_2), \quad V_2^+ = V_2, \quad \sigma_2^+ = \sigma_p(X_2), \quad \varepsilon_2^+ = 0, \quad (12)$$

while behind this shock wave front, one has

$$\rho_2^- = \rho_b, \quad V_2^- = 0, \quad \sigma_2^- = \sigma_d(X_2), \quad \varepsilon_2^- = \varepsilon_d(X_2). \quad (13)$$

Inserting Eqs.(10) and (11) into Eq.(6) leads to

$$V_1 - V_2 = C_1 \varepsilon_d(X_1). \quad (14)$$

Substituting Eqs.(10) and (11) into Eq.(7) yields

$$\sigma_d(X_1) - \sigma_p(X_1) = C_1 \rho(X_1) (V_1 - V_2). \quad (15)$$

Similarly, combining Eqs. (10)-(11) with Eqs.(6) and (7), one can obtain

$$V_2 = C_2 \varepsilon_d(X_1), \quad (16)$$

$$\sigma_d(X_2) - \sigma_p(X_2) = C_2 \rho(X_2) V_2. \quad (17)$$

Eliminating C_1 from Eqs. (14) and (15) leads to

$$\sigma_d(X_1) = \sigma_p(X_1) + \frac{\rho(X_1)}{\varepsilon_d(X_1)} (V_1 - V_2)^2, \quad (18)$$

Furthermore, eliminating C_2 from Eqs. (16) and (17) results in

$$\sigma_d(X_2) = \sigma_p(X_2) + \frac{\rho(X_2)}{\varepsilon_d(X_2)} V_2^2. \quad (19)$$

Here, Eqs.(18) and (19) reveal the relationship between the dynamic stress and quasi-static plateau stress of the negative graded foam rod. One can conclude that the stress enhancement phenomenon appears both in the forward and backward shock wave fronts. However, the above two equations are based on the R-PP-L material idealization that neglects the dynamic constitutive relation for the foam rod. Investigations on the dynamic constitutive relation of foam material subjected to impact loadings have been carried out by several researchers, and more information can be found in Refs.[20, 55, 56].

Fig.3. The double shock wave model for a negative density-graded foam rod subjected to mass impact: (a) initial stage, (b) intermediate stage.

As the densified region behind the backward shock wave front is in equilibrium, the dynamic stress $\sigma_d(X_2)$ in the above equation is also equal to the reaction stress σ_s at the supported end [37]. With the aid of momentum conservation, the dynamic stress at the striking end σ_i can be calculated as follows

$$\sigma_i = \frac{M_p}{A_0} \frac{dV_1}{dt}, \quad (20)$$

Substituting Eq.(9) into Eqs.(14) and (16), one has

$$\frac{dX_1}{dt} = \frac{V_1 - V_2}{\varepsilon_d(X_1)}, \quad (21)$$

$$\frac{dX_2}{dt} = -\frac{V_2}{\varepsilon_d(X_2)}. \quad (22)$$

The momentum conservation with respect to the forward densified region and the attached mass block M_p can be expressed as

$$(m_1 + M_p)dV_1 = -\sigma_d(X_1)A_0dt, \quad (23)$$

where $m_1 = \int_0^{X_1} \rho_0(X)dXA_0$ denotes the total mass of the forward densified region. For the un-deformed region of the graded foam rod, the momentum conservation yields

$$m_2dV_2 = [\sigma_p(X_1) - \sigma_p(X_2)]A_0dt, \quad (24)$$

where $m_2 = \int_{X_1}^{X_2} \rho_0(X)dXA_0$ denotes the total mass of the un-deformed region of the graded foam rod.

By inserting Eq.(18) into Eqs.(23) and (24), the above two equations can be re-written as

$$\frac{dV_1}{dt} = -\frac{\sigma_p(X_1) + \frac{\rho(X_1)}{\varepsilon_d(X_1)}(V_1 - V_2)^2}{(\int_0^{X_1} \rho_0(X) dX + M_p/A_0)}, \quad (25)$$

$$\frac{dV_2}{dt} = \frac{\sigma_p(X_1) - \sigma_p(X_2)}{\int_{X_1}^{X_2} \rho_0(X) dX}. \quad (26)$$

Here, Eqs.(21), (22), (25), and (26) gives the governing differential equations of the functionally graded foam rod with temperature gradient struck by a rigid mass block. The graded foam rod is un-deformed and stays stationary at the instant $t = 0^-$. After impact at $t = 0^+$, the impinging layer obtains a velocity V_1 and travels towards the distal end simultaneously. From Eq.(26), one can detect that the monotonicity of V_2 is determined by the sign of $\sigma_p(X_1) - \sigma_p(X_2)$. Particularly, for positive graded foam rod, one has $dV_2/dt < 0$ due to $\sigma_p(X_1) - \sigma_p(X_2) < 0$. However, V_2 is equal to zero at the beginning and has to increase from impact, which means only a single shock wave front appears at the striking end, with speed corresponding to V_1 . In fact, the crushing process of the positive graded foam rod is similar to that of a uniform foam rod and there exist two deformation patterns. The first deformation pattern happens when the functionally graded foam rod is fully crushed while the velocity satisfies $V_1 \neq 0$ (Positive model I). The dynamic process after being wholly crushed is not calculated further in the present paper. If the impinging process terminates as $V_1 = 0$, Positive model II takes place, and the foam rod may not be completely crushed at this instance.

For the negative graded foam rod subjected to mass impact, one can observe the shock wave front both at the striking and supported ends because $\sigma_p(X_1) - \sigma_p(X_2) > 0$. Under this situation, three deformation patterns may exist, i.e., in Negative model I, the decrement of V_1 is accompanied by the increment of V_2 , and the functionally graded foam rod is wholly crushed before the two velocities reach the same value. If the negative graded foam rod is not wholly crushed at the instant when V_1 and V_2 achieve the same value, the velocities will decrease at the same rate subsequently. At this stage, the backward densified region gets increasingly enlarged, while the forward densified region remains a constant. If the functionally graded foam rod has $V_1 = V_2 \neq 0$ when it is fully crushed, this is represented as Negative model II. On the other hand, if the rod is not wholly crushed at the moment when $V_1 = V_2 = 0$, this is represented as Negative model

III.

For convenience, the following non-dimensional parameters are introduced

$$\tau = \frac{t}{L_0 / \sqrt{\sigma_{ave} / \rho_{ave}}}, \quad \alpha = \frac{M_p}{A_0 \int_0^{L_0} \rho_0(X) dX} = \frac{M_p}{A_0 L_0 \rho_{ave}},$$

$$x_i = \frac{X_i}{L_0}, \quad v_i = V_i / \sqrt{\sigma_{ave} / \rho_{ave}}, \quad (27)$$

where $i=0, 1, 2$, τ denotes the non-dimensional time, α represents the mass ratio of the rigid mass block to the graded foam rod, x_i and v_i stand for the non-dimensional lengths and velocities, respectively, ρ_{ave} denotes the average density of the functionally graded foam rod and σ_{ave} represents the corresponding plateau stress without considering the temperature gradient effect. By using Eqs.(1) and (2), the average density ρ_{ave} can be calculated as

$$\rho_{ave} = \frac{\int_0^{L_0} \rho(X) dX}{L_0} = \frac{\rho_{s0} + K \rho_{i0}}{K+1} = \frac{(g+K) \rho_{i0}}{K+1}. \quad (28)$$

With the help of Eqs.(27) and (28), the normalization of Eqs.(21), (22), (25), and (26) yields

$$\frac{dx_1}{d\tau} = \frac{1}{1 - \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K}} (v_1 - v_2), \quad (29)$$

$$\frac{dx_2}{d\tau} = - \frac{v_2}{1 - \frac{[1 + (g-1)(x_2)^K](K+1)}{g+K}} (1 - \varepsilon_{dave}), \quad (30)$$

$$\frac{dv_1}{d\tau} = - \frac{(1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} \right]^{3/2} + \beta \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d)(1 - x_1)}{\left\{ \int_0^{x_1} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi + \alpha \right\} \left\{ \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} \right\}} (v_1 - v_2)^2$$

$$- \frac{\left\{ 1 - \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} (1 - \varepsilon_{dave}) \right\} \left\{ \int_0^{x_1} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi + \alpha \right\}}{\left\{ \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} \right\}} (v_1 - v_2)^2 \quad (31)$$

$$\frac{dv_2}{d\tau} = \frac{(1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} \right]^{3/2} + \beta \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d)(1 - x_1)}{\int_{x_1}^{x_2} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi} - \frac{(1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x_2)^K](K+1)}{g+K} \right]^{3/2} + \beta \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d)(1 - x_2)}{\int_{x_1}^{x_2} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi}, \quad (32)$$

where ε_{dave} is the average densification strain, which is corresponding to the average density ρ_{ave} of the graded foam rod; $\beta = \frac{B_3 B_1^{3/2}}{B_2 \sigma_b}$ denotes a material parameter that reflects the effect of temperature gradient. The derivation of above equations is given in Appendix A. The initial conditions at $\tau = 0$ are

$$v_1(0) = v_0, v_2(0) = 0, x_1(0) = 0, x_2(0) = 1. \quad (33)$$

With the help of above initial conditions, the governing equations presented in Eqs.(29)-(32) can be solved by employing the Runge-Kutta method. After v_1 and v_2 approach the same values, they will reduce together. The initial conditions for this stage are

$$\tau = \tau^*, v_1 = v_2 = v^*, x_1 = x_1^*, x_2 = x_2^*. \quad (34)$$

The location of the deformation front for the forward wave x_1 remains a constant during this stage, i.e., $\frac{dx_1}{d\tau} = 0$. The momentum conservation over the backward shock wave front can be calculated as

$$\frac{dV_1}{dt} = - \frac{\sigma_p(X_2)}{(\int_0^{x_2} \rho_0(X) dX + M_p / A_0)}. \quad (35)$$

By using Eqs.(27) and (28), the normalization of the above equation can be expressed as

$$\frac{dv^*}{d\tau} = - \frac{(1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x_2)^K](K+1)}{g+K} \right]^{3/2} + \beta \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d)(1 - x_2)}{\left\{ \int_0^{x_2} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi + \alpha \right\}}. \quad (36)$$

The dynamic response at $\tau \geq \tau^*$ can be obtained by using the governing equations

Eqs.(29),(30) and (36), as well as initial conditions Eq.(34).

2.3. Positive density-graded foam

As analyzed above, only a single shock wave front can be observed in the functionally graded foam rod when the density gradient is positive, and this shock wave would propagate from the striking end to the supported end, as presented in Fig.4. As a matter of fact, the single shock wave model can be treated as a specialized form of the double shock wave model. The dimensionless governing equations of this model can be obtained by inserting $v_2 \equiv 0$ and $x_2 \equiv 1$ into Eqs.(29)-(32). Similar to the negative density-graded foam model presented in the above section, the impact response of a positive density-graded foam rod subjected to an impact by a rigid mass block with consideration of the temperature gradient effect can be calculated numerically.

Fig.4. The single shock wave model for a positive density-graded foam rod subjected to mass impact: (a) initial stage, (b) intermediate stage.

2.4. Energy absorption capacity

During quasi-static compression, the total energy absorbed in the crushing process for the foam rod with arbitrary density-graded distributions and temperature gradients can be calculated as[36, 39]

$$E_q = \int_0^{L_0} A_0 \sigma_p(X) \varepsilon(X) dX . \quad (37)$$

For a uniform foam rod whose mass density equals to the average density of the density-graded foam ρ_{ave} , and neglecting the effect of the temperature gradient, the above equation can be simplified as[36, 39]

$$E_{qave} = A_0 \sigma_{ave} \varepsilon_{dave} , \quad (38)$$

where σ_{ave} and ε_{dave} are, respectively, the quasi-static plateau stress and densification strain of the uniform foam rod without considering the temperature gradient effect.

It is of practical interest to consider the special case that the input kinetic energy of the rigid mass block is just exhausted when the functionally graded foam rod is wholly crushed. The critical velocity of the rigid mass block is significant to assess the energy absorption capacity of the foam rod with different density-graded distributions and temperature gradients. Here, this critical velocity is called as the densification velocity V_d [36, 40], and the corresponding critical input energy is named as the densification energy, which can be given as

$$E_d = \frac{1}{2} M_p V_d^2. \quad (39)$$

One can introduce an energy ratio ξ to describe the energy absorption capacity of the density-graded foam rod, which satisfies

$$\xi = \frac{E_d}{E_{qave}} = \frac{1}{2\varepsilon_{dave}} \alpha v_d^2, \quad (40)$$

where v_d denotes the dimensionless densification velocity of the functionally graded foam rod.

3. Validation of theoretical model

To the best of the authors' knowledge, there are currently no studies on graded foam under temperature environment suitable for validation of the combined effect of functional grading and temperature. In this section, two examples are presented for the validation of the theoretical model involving separate effects. In the first example, a linearly negative or positive graded foam rod subjected to a mass impact under room temperature is considered, i.e., $T_p = T_d = 25^\circ\text{C}$ are employed in the present theoretical model and the power law index is set as $K = 1$. The density gradients are, respectively, specified as $g = 2$ and $g = 0.5$ for the positive and negative gradient cases, which correspond to $\Theta_\rho = 0.667$ and -0.667 in Shen et al.'s study [36], respectively. The mass ratio and non-dimensional initial velocity of the mass block are specified as $\alpha = 0.5$ and $v_0 = 2$ ($V_0 \approx 153\text{m/s}$), respectively. The velocities of the graded foam rod predicted by the present theoretical model and Shen et al.'s model [36] are compared in Fig.5 both for the positive and negative gradient cases. It is demonstrated that the change trends for these two models are identical both for positive and negative density-graded foam rods under mass impact, indicating the present model can reduce to Shen et al.'s model [36] when the temperature gradient effect is

neglected.

In the second comparison example, a uniform foam rod under temperature gradient and impingement by a mass block is taken into consideration. In this theoretical model, the density gradient is set as $g = 1$, and the temperatures in the impinging and support ends are fixed as $T_p = 125^\circ\text{C}$ and $T_d = 25^\circ\text{C}$, respectively. The mass ratio $\alpha = 0.5$ and initial velocity $v_0 = 2$ ($V_0 \approx 153\text{m/s}$) of the mass block are adopted. As analyzed above, only a shock front exists for this situation, and the velocity and deformation history of the foam rod are calculated, which gives a comparison with Li et al.'s study[50], as presented in Fig.6. Good agreement can also be observed between the present model and Li et al.'s study[50] without considering the density gradient of the foam rod, providing another validation for the present model.

Fig.5. Time-history of velocity for functionally graded foam rod struck by rigid mass block under room temperature: (a) positive graded foam rod, (b) negative graded foam rod.

Fig.6. Dynamic response of a uniform foam rod struck by mass block under temperature gradient.

4. Finite element modeling

In this section, the dynamic response of a density-graded foam specimen in the temperature gradient environment and subjected to mass impact is numerically investigated by using the FE simulation via ABAQUS/Explicit code. A cylindrical foam rod of length 102 mm and radius 1 mm is taken into account. Due to the symmetry of this problem, one-quarter of the foam specimen is adopted and the symmetry boundary condition is employed in the symmetry planes. One end of the density-graded foam rod is supported and the other end is struck by a mass block. This mass block is simulated by a moving rigid plane with a total mass $\alpha = 1$ and an initial velocity $v_0 = 0.8$ ($V_0 \approx 63\text{m/s}$). The surface to surface contact law with no friction is utilized to define the contact behavior between the rigid plane and the density-graded foam rod surface. The foam rod is meshed with the reduced integration, eight nodes linear brick, hourglass controlled solid elements (C3D8R). Sufficiently large number of portions (51 equal-size layers with length 2 mm each) is divided to simulate the graded foam materials with continuously varying densities. Each divided portion is treated as a uniform material, and the gradient can be reached by assigning different densities for each layer. For each small equivalent piece, 40 elements are divided in the length direction, while 97 elements are divided in the cross-section direction. The approximate element size of the present FE model is $e \approx 0.1\text{mm}$, and the detailed FE model is presented in Fig.7. In addition, mesh sensitivity study is carried out by comparing the present model with another rough model (element size $e \approx 0.2\text{mm}$). As shown in Table 1, the final total length and the impact duration are almost unchanged through mesh refinement, indicating that the current element size is sufficient for convergence.

Fig.7. FE model of graded foam rod: (a) whole mode, (b) elements in each portion, (c) elements in the cross-section.

Table 1 Mesh sensitivity study of the present FE model.

Model	negative ($g = 0.5$)		positive ($g = 2$)	
	final total length	impact duration	final total length	impact duration
Theoretical model	0.6639	1.2776	0.5595	1.1576
Present FE model ($e \approx 0.1\text{mm}$)	0.6706	1.2213	0.5822	1.0426

Rough FE model ($e \approx 0.2\text{mm}$)	0.6709	1.2208	0.5827	1.0421
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The crushable foam model with isotropic hardening and zero plastic Poisson's ratio is employed to simulate the crushing behavior of the density-graded foam rod. The base material of the foam rod is selected as aluminum with mass density $\rho_d = 2700 \text{ kg/m}^3$, Young's modulus $E_d = 70 \text{ GPa}$, and yield stress $\sigma_d = 170 \text{ MPa}$. Two different density profiles with identical weight, i.e., positive density gradient $g = 2$, and negative density gradient $g = 0.5$ are taken into consideration. In this numerical study, the power-law index is set as $K = 1$ and the average relative density is fixed as 0.15 for all cases, i.e., the initial densities at the striking and supported ends are, respectively, set as $\rho_{i0} = 0.2\rho_d$ and $\rho_{s0} = 0.1\rho_d$ for the negative foam rod, while they are specified as $\rho_{i0} = 0.1\rho_d$ and $\rho_{s0} = 0.2\rho_d$ for the positive foam rod. The graded foam rod placed in the thermal environment with a temperature gradient changing linearly along the axial direction is considered. The material properties of foam specimen obeying the R-PP-L idealization model are applied. The densification strain and the quasi-static plateau stress of the foam rod can be obtained through Eqs.(3)-(5) where the material parameters are given as $B_1 = 1.4$ [53], $B_2 = 0.2488$ [40], $B_3 = -3.88/T_m$ [50], $\beta_m = 1$, and $T_m = 660^\circ\text{C}$ [54]. From Refs. [39, 54, 57], Young's modulus of the foam specimen by considering the temperature gradient effect can be determined by using the relationship $E = B_5 \left(1 - \beta_m \frac{T}{T_m}\right) E_d \left(\frac{\rho_0}{\rho_d}\right)^2$, in which $B_5 = 1$ [39] and $\beta_m = 1$ [54] are adopted in this numerical analysis. A large tangent modulus $E_t = 0.1E_d$ is employed as the compressive strain is larger than the densification strain and the elastic Poisson's ratio is fixed at $\nu_e = 0.3$.

The distributions of equivalent plastic strain for the negative and positive graded foam rod struck by a mass block are plotted in Figs.8 and 9, respectively. Here, the temperature in the striking and supported ends are specified as $T_p = 125^\circ\text{C}$ and $T_d = 25^\circ\text{C}$, respectively. It is clear that two shock wave fronts appear both in the striking and supported ends when the density gradient is negative, while only one shock wave front is observed in the positive density-graded foam rod. Particularly, from Fig.8, it can be found that the forward deformed region increases remarkably at the initial impinging stage, while it remains a constant when v_1 and v_2 reach the same value ($\tau \approx 0.42$). The shock wave front in the supported end forms immediately when the negative foam rod is struck by the mass block and travels leftward then, and the backward

deformed region always increases during the whole impinging process.

Fig.8. Distribution of equivalent plastic strain (PEEQ) for negative graded foam rod struck by a mass block from the left.

Fig.9. Distribution of equivalent plastic strain (PEEQ) for positive graded foam rod struck by a mass block from the left.

The influence of temperature gradient on the time history of velocity of the negative graded foam rod can be easily observed from Fig.10(a). In this figure, the theoretical and FE simulation results for the dynamic response of negative density graded foam rod with and without temperature gradients are compared. In the FE model, the velocity of an arbitrary node in the striking end is selected to represent the velocity in the forward densified region v_1 , while a node almost in the middle of the un-deformed region is selected to calculate the velocity in the un-deformed region v_2 . It is found that the velocity in the forward densified region v_1 decreases significantly after the negative-graded foam rod is struck by a mass block, while the velocity in the un-deformed region v_2 increases slightly. The process continues until v_1 and v_2 reach the same value, which they will then decrease at the same rate to zero. When the temperature gradient is taken into account, v_1 increases slightly, and v_2 decreases by a significant amount, leading to the

delay for v_1 and v_2 to reach a common value. Besides, the duration of the impact becomes longer when the temperature gradient is taken into consideration.

The time history of the velocity for the positive graded foam rod with and without considering the temperature gradient effect is compared in Fig.10(b) for both the theoretical modeling and FE simulation. From Fig.9, it is observed that only the region close to the striking end is crushed after impact, and the velocity of the densified region decreases until the impact process terminates (see in Fig.10(b)). Comparing with the impact under room temperature ($T_p = T_d = 25^\circ\text{C}$), the temperature gradient can enhance the impact velocity and the duration of the impact process.

Fig.10. Time history of velocity for (a) negative, and (b) positive graded foam rods under mass impact.

The comparison of dynamic stresses at the supported end predicted by the theoretical and FE simulation is depicted in Fig.11 for both negative and positive graded foam rods when struck by a mass block. As presented in Fig.11(a), it is demonstrated that the stress at the supported end of the negative graded foam rod undergoes a step increase at first and subsequently remains constant. Under external temperature gradient, maximum stress is observed to be reduced at the supported end. Fig.11(b) depicts the stress at the supported end for the foam rod with positive density gradient, indicating a similar trend where the stress at the supported end decreases when the temperature gradient effect is taken into account. The FE simulation results gave similar predictions to the theoretical model.

The dynamic stresses at the striking end for the negative and positive graded foams struck by a mass block are presented in Figs.12(a) and (b), respectively. One can find that the stress

decreases significantly for both the negative and positive density gradient cases. In particular, a second reduction can be observed for the negative graded foam at the moment when $v_1 = v_2$. The temperature gradient reduces the stress at the striking end for both the negative and positive graded foams. Furthermore, good agreement between theoretical and FE simulation results can be observed.

Fig.11. Dynamic stress at the supported end for (a) negative, and (b) positive graded foam rods.

Fig.12. Dynamic stress at the striking end for (a) negative, and (b) positive graded foam rods.

5. Results and discussion

In this section, detailed investigation into the impact response and energy absorption of graded foam rod under temperature gradient environment is presented. Similar to the material parameters used in Section 3, $\beta_m = 1$, $T_m = 660^\circ\text{C}$, $B_1 = 1.4$, $B_2 = 0.2488$, $B_3 = -3.88/T_m$ are adopted. In addition, the average densification strain is fixed at $\varepsilon_{dave} = 0.8$ for the following numerical studies unless otherwise specified.

5.1. Density distribution

In this subsection, the density distribution along the axial direction of the graded foam rod is

plotted in Fig.13. The density ratio of negative foam rod decreases along the axial direction, while an opposite trend can be found for the positive foam rod. On the other hand, the density ratio along the axial direction is unchanged for the uniform foam rod. The density distribution profile is controlled by the value of power-law index K , i.e., $K = 1$ represents the linear density graded distribution, while $K = 2$, and 3 are nonlinear distributions. Although the density distribution profile is distinct for different power-law indexes, the areas under these curves are identical, indicating they have the same total masses, which is useful for the following comparing studies.

Fig.13. Density distribution along the axial direction of the graded foam rod.

5.2. Negative density-graded foam

The time history of the velocity, deformation, stresses in the striking and supported ends for a negative graded foam rod struck by a mass block with mass ratio $\alpha = 0.5$ and initial velocity $v_0 = 2.1$ ($V_0 = 160\text{m/s}$) is plotted in Fig.14. Here, a negative graded foam rod with density gradient $g = 0.5$ and power-law index $K = 1$ is taken into account. It is shown that Negative model III occurs in the room temperature environment ($T_p = 25^\circ\text{C}$), i.e., the decrease of v_1 is accompanied with the increase of v_2 until they reach the same value, after which, they reduce to zero at the same rate, as presented in Fig.14(a). By increasing the temperature gradient (the temperature in the supported end is fixed), a slight increment can be observed for v_1 , while v_2 decreases prominently, leading to a delay for v_1 and v_2 achieve a common value. From Fig.14(b), the normalized location of the deformation front for the forward wave x_1 first increases, which then remains constant when v_1

and v_2 reach the same value, while the distance travelled by the backward shock wave front towards the mass block x_2 always increases during the whole impact process. It can also be observed that the forward region gets more deformed, whereas the backward region becomes less deformed as the temperature gradient increases. Moreover, by increasing the temperature gradient, the length of the un-deformed region gets smaller, and Negative model III turns to Negative model II. From Figs.14(c) and (d), it is clear that the dynamic stress in the striking end σ_i / σ_{ave} decreases initially before reaching a constant value, while the dynamic stress in the supported end σ_s / σ_{ave} for all temperature gradients increases initially and then decreases. The time history of σ_i / σ_{ave} is insensitive to the temperature gradient, but the maximum stress at the initial instance decreases slightly with increasing temperature gradient. The temperature gradient also reduces the dynamic stress in the supported end σ_s / σ_{ave} .

Fig.14. Impact response of a negative graded foam rod struck by mass block with $\alpha = 0.5$ and $v_0 = 2.1$: (a) velocity, (b) displacement, (c) σ_i / σ_{ave} , (d) σ_s / σ_{ave} .

Another numerical example for a negative graded foam rod struck by a mass block is presented in Fig.15. Here, parameters $g = 0.5$, $K = 1$, $\alpha = 1$ and $v_0 = 1.5$ ($V_0 \approx 115\text{m/s}$) are adopted. A similar change trend caused by the temperature gradient can also be observed, i.e., by increasing

the temperature gradient, the velocity v_1 and length x_1 both increase, accompanied by a decrement in velocity v_2 . Besides this, the foam rod is wholly crushed much more quickly as temperature gradient is increased. The temperature gradient has little influence on the change trend of the dynamic stress in the striking end σ_i / σ_{ave} , while playing a significant role on the dynamic stress in the supported end σ_s / σ_{ave} .

Fig.15. Impact response of a negative graded foam rod struck by mass block with $\alpha = 1$ and $v_0 = 1.5$: (a) velocity, (b) displacement, (c) σ_i / σ_{ave} , (d) σ_s / σ_{ave} .

The maximum stress in the striking and supported ends as a function of the temperature gradient for a negative graded foam rod under the impact loading is shown in Fig.16. The density gradient of the foam rod is chosen to be $g = 0.5$, with mass ratio and initial velocity of the mass block separately set as $\alpha = 0.5$ and $v_0 = 2.1$ ($V_0 \approx 160\text{m/s}$). It is shown that the maximum stress in the striking and supported ends both decrease by raising the temperature gradient. For a specified temperature gradient, the maximum stress in the striking end decreases with the increase of the power-law index, while an opposite trend can be observed for the maximum stress in the supported end.

Fig.16. The maximum stress in the (a) striking and (b) supported ends for the negative graded foam rod struck by mass block.

The influences of temperature gradient and power-law index on the densification velocity and energy absorption of the negative graded foam rod are shown in Fig.17. A negative graded foam rod with $g = 0.5$ struck by a mass block with $\alpha = 0.5$ is considered. It is observed that as the temperature gradient is increased, the densification velocity v_d and the energy ratio ξ both reduce. When the temperature gradient is fixed, an increment can be observed for the densification velocity v_d and the energy ratio ξ as the power-law index increases.

Fig.17. The (a) densification velocity and (b) energy absorption for the negative graded foam rod struck by a mass block with $\alpha = 0.5$.

5.3. Positive density-graded foam

The dynamic response of a positive foam rod struck by a mass block is shown in Fig.18 for different temperature gradients. In this numerical example, a positive foam rod with density gradient $g = 2$ and power-law index $K = 1$ is considered, and the mass ratio and initial velocity for the mass block are set as $\alpha = 0.5$ and $v_0 = 2.1$ ($V_0 \approx 160\text{m/s}$), respectively. As observed, the velocity v_1 and the distance travelled by the forward shock wave front x_1 both get larger with the

increase in the temperature gradient. Additionally, the impact duration increases with increasing temperature gradient. As the foam rod is not wholly crushed, and increasing the temperature gradient would result in an increment of the deformed region (shown in Fig.18(b)), which would cost more time (i.e., the impact duration increases). The dynamic stress in the striking and supported ends show almost the same trend regardless of the temperature gradient, i.e., the stress in the striking end always decreases, while the stress in the supported end always increases during the whole crushing process. The temperature gradient will lead to a reduction of the maximum stress in the striking end, while an opposite trend can be found for the maximum stress in the supported end.

Fig.18. Impact response of a positive graded foam rod struck by mass block with $\alpha = 0.5$ and $v_0 = 2.1$: (a) velocity, (b) displacement, (c) σ_i / σ_{ave} , (d) σ_s / σ_{ave} .

Fig. 19 provides another case study for a positive foam rod struck by a mass block with mass ratio $\alpha = 1$ and initial velocity $v_0 = 1.5$ ($V_0 \approx 115\text{m/s}$). In this figure, a model transformation phenomenon can be observed, i.e., Positive model II in the room temperature would turn to Positive model I when the temperature gradient is considered, which means the foam rod will be

wholly crushed before v_1 decreases to zero as the temperature gradient is applied. One can also observe that the impact duration reduces with increase in the temperature gradient (shown in Fig.19(a)) and as a result, the foam rod would be wholly crushed much faster (the impact response after wholly crushed is not considered in the present study). The maximum stress in the striking end σ_i / σ_{ave} can be found at the initial instance of impact and decreases as the temperature gradient increases. On the other hand, the maximum stress in the supported end σ_s / σ_{ave} can be observed at the moment when the impact process is terminated and its value increases slightly as the temperature gradient is raised.

Fig.19. Impact response of a positive graded foam rod struck by mass block with $\alpha = 1$ and $v_0 = 1.5$: (a) velocity, (b) displacement, (c) σ_i / σ_{ave} , (d) σ_s / σ_{ave} .

The maximum stress versus the temperature gradient for the positive graded rod with different power-law indexes is illustrated in Fig.20. The density gradient $g = 2$, mass ratio of the mass block $\alpha = 0.5$ and initial velocity $v_0 = 2.1$ ($V_0 \approx 160\text{m/s}$) are used in this numerical analysis. It is observed that the temperature gradient can reduce the maximum stress at the striking end, while having an opposite effect on the maximum stress in the supported end. When the temperature

gradient is fixed, the maximum stress in the striking and supported ends both gets larger as the value of the power-law index is increased.

Fig.20. The maximum stress in the (a) striking and (b) supported ends for the positive graded foam rod.

The non-dimensional densification velocity v_d and the energy ratio ξ of the positive graded foam rod against the temperature gradient are presented in Fig.21. The density gradient is set as $g = 2$, and the mass ratio of the mass block is fixed as $\alpha = 0.5$ in this analysis. It is demonstrated that both the densification velocity v_d and energy absorption capacity decrease as the temperature gradient is increased. The influence of the power-law index on the densification velocity v_d and the energy ratio ξ is insignificant.

Fig.21. The (a) densification velocity and (b) energy absorption for the positive graded foam rod struck by a mass block with $\alpha = 0.5$.

5.4. Comparison for different density gradient models

The maximum dynamic stresses against the temperature gradient in the striking and supported ends are compared in Fig.22 for three different density-gradient models. Here, a foam

rod struck by a mass block with $\alpha = 0.5$ and $v_0 = 2.1$ ($V_0 \approx 160\text{m/s}$) is taken into consideration. The density gradients are set as $g = 0.5, 1$, and 2 for the negative, uniform and positive density-graded foam rods, respectively. It is clear that the positive density-gradient model can provide the smallest dynamic stress in the striking end, while the stress in the supported end for the negative density-graded foam is the smallest. The maximum stress in the striking end reduces with increased temperature gradient for all density gradient models, while the maximum stress in the supported end decreases for the negative foam rod and increases for the uniform and positive graded foam rods.

Fig.22. Comparison of the maximum stress in the (a) striking and (b) supported ends for different foam rod models.

A comparison of the densification velocity and energy absorption for different foam rod models is exhibited in Fig.23. Here, $g = 0.5, 1$, and 2 are used to represent the negative, uniform and positive graded foam rods, respectively, and the mass ratio $\alpha = 0.5$ for the mass block is adopted in the analyses. As the temperature gradient increases, a reduction in the densification velocity v_d and the energy ratio ξ is observed. The densification velocity and energy absorption capacity of the positive density-graded foam rod are much larger than those of the uniform and negative density-graded models. Moreover, the negative density-gradient distribution reduces the absorption capacity of foam materials under temperature gradient environment.

Fig.23. Comparison of the (a) densification velocity and (b) energy absorption for different foam rod models.

6. Conclusions

In this paper, the dynamic response and energy absorption capacity of a functionally graded foam rod exposed to temperature gradient environment and struck by a mass block is investigated theoretically and numerically. Based on the double shock wave theory and the R-PP-L idealization, the governing equations are developed and the validation of the present theoretical model is performed by comparing with previous models and FE simulations. The influence of temperature gradient on the dynamic response of deformation and velocity, as well as the dynamic stress and energy absorption characteristics of graded foam rod are analyzed in detail. From the present study, some concluding remarks are drawn:

1. There are two shock wave fronts in the negative density-graded foam rod, while only one shock wave front is found in the positive density-graded foam rod. The temperature gradient can enlarge the deformed regions both for the negative and positive density-graded foam models. Additionally, the transformation of the deformation pattern can be observed for all density-graded models as the temperature gradient is applied.
2. Based on maximum stress, the positive density-gradient foam is helpful in protecting the object in the striking end, and the temperature gradient can enhance its protective ability. While the negative density-graded foam has the best impact resistance capacity for the object in the support end, and its capacity enlarges with the raise of the temperature gradient.
3. The positive density-gradient foam model can achieve the highest energy absorption under temperature gradient environment, followed by uniform and negative density-gradient models,

respectively. For all density-graded models, the energy absorption capacity reduces as the temperature gradient increases.

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Appendix A

The ratio of the density at any position X can be calculated by dividing Eq.(1) with Eq.(28), i.e.,

$$\frac{\rho_0(X)}{\rho_{ave}} = \frac{[1 + (g-1)(x)^K](K+1)}{g+K} \quad (\text{A.1})$$

By inserting Eq.(A.1) into Eq.(3), the non-dimensional densification strain at any position X can be obtained as follows

$$\varepsilon_d(X) = 1 - B_1 \frac{\rho_0(X)}{\rho_b} = 1 - B_1 \frac{\rho_0(X)}{\rho_{ave}} \frac{\rho_{ave}}{\rho_b} = 1 - \frac{[1 + (g-1)(x)^K](K+1)}{g+K} (1 - \varepsilon_{dave}) \quad (\text{A.2})$$

where ε_{dave} is the densification strain corresponding to the average density ρ_{ave} of the graded foam rod. By combining Eqs.(4) and (5), the non-dimensional plateau stress can be derived as

$$\begin{aligned} \frac{\sigma_p(X)}{\sigma_{ave}} &= \frac{B_2(1 - \alpha_m T_d / T_m) \left(\frac{\rho_0(X)}{\rho_b} \right)^{3/2} \sigma_b + B_3(T_p - T_d) \left(1 - \frac{X}{L_0} \right)}{B_2 \left(\frac{\rho_{ave}}{\rho_b} \right)^{3/2} \sigma_b} \\ &= (1 - \alpha_m T_d / T_m) \left(\frac{\rho_0(X)}{\rho_{ave}} \right)^{3/2} + \frac{B_3}{B_2 \sigma_b} \left(\frac{\rho_{ave}}{\rho_b} \right)^{-3/2} (T_p - T_d) (1 - x) \\ &= (1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x)^K](K+1)}{g+K} \right]^{3/2} + \frac{B_3 B_1^{3/2}}{B_2 \sigma_b} \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d) (1 - x) \end{aligned} \quad (\text{A.3})$$

The non-dimensional stress caused by the inertial effect expressed in Eq.(18) can be given as

$$\begin{aligned} \frac{\frac{\rho(X_1)}{\varepsilon_d(X_1)} (V_1 - V_2)^2}{\sigma_{ave}} &= \frac{1}{\varepsilon_d(X_1)} \frac{\rho(X_1)}{\rho_{ave}} \frac{\rho_{ave}}{\sigma_{ave}} (v_1 - v_2)^2 \frac{\sigma_{ave}}{\rho_{ave}} = \frac{1}{\varepsilon_d(X_1)} \frac{\rho(X_1)}{\rho_{ave}} (v_1 - v_2)^2 \\ &= \frac{g+K}{1 - \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} (1 - \varepsilon_{dave})} (v_1 - v_2)^2 \end{aligned} \quad (\text{A.4})$$

With the help of the above equations, the normalization of Eqs.(21), (22), (25), and (26) can be

finally calculated as

$$\frac{dx_1}{d\tau} = \frac{1}{1 - \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K}} (v_1 - v_2) \quad (\text{A.5})$$

$$\frac{dx_2}{d\tau} = - \frac{v_2}{1 - \frac{[1 + (g-1)(x_2)^K](K+1)}{g+K}} (1 - \varepsilon_{dave}) \quad (\text{A.6})$$

$$\begin{aligned} \frac{dv_1}{d\tau} = & - \frac{(1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} \right]^{3/2} + \beta \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d)(1 - x_1)}{\left\{ \int_0^{x_1} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi + \alpha \right\}} \\ & - \frac{\left\{ \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} \right\}}{\left\{ 1 - \frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} (1 - \varepsilon_{dave}) \right\} \left\{ \int_0^{x_1} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi + \alpha \right\}} (v_1 - v_2)^2 \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \frac{dv_2}{d\tau} = & - \frac{(1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x_1)^K](K+1)}{g+K} \right]^{3/2} + \beta \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d)(1 - x_1)}{\int_{x_1}^{x_2} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi} \\ & - \frac{(1 - \alpha_m T_d / T_m) \left[\frac{[1 + (g-1)(x_2)^K](K+1)}{g+K} \right]^{3/2} + \beta \left(\frac{1}{1 - \varepsilon_{dave}} \right)^{3/2} (T_p - T_d)(1 - x_2)}{\int_{x_1}^{x_2} \left[\frac{[1 + (g-1)(\xi)^K](K+1)}{g+K} \right] d\xi} \end{aligned} \quad (\text{A.8})$$

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