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2017

Ng, B. F., Palacios, R. & Graham, J. M. R. (2017). Model-based aeroelastic analysis and blade load alleviation of offshore wind turbines. *International Journal of Control*, 90(1), 15-36. <https://dx.doi.org/10.1080/00207179.2015.1068456>

<https://hdl.handle.net/10356/155384>

<https://doi.org/10.1080/00207179.2015.1068456>

This is an Accepted Manuscript of an article published by Taylor & Francis in *International Journal of Control* on 31 Jul 2015, available online:
<http://www.tandfonline.com/10.1080/00207179.2015.1068456>.

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Model-based Aeroelastic Analysis and Blade Load Alleviation of Offshore Wind Turbines

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(v4.0 released February 2014, Manuscript contains 7460 words)

Offshore wind turbines take advantage of the vast energy resource in open waters but face structural-integrity challenges specific to their operating environment that require cost-effective load alleviation solutions. This paper introduces a computational methodology for model-based two and three-dimensional design of load alleviation systems on offshore wind turbines. The aero-hydro-servoelastic model is formulated in a convenient state-space representation, coupling a multi-body composite beam description of the main structural elements with unsteady vortex-lattice aerodynamics and Morison's description of the hydrodynamics. The aerodynamics does not require empirical corrections and focuses on a control-oriented approach to the modelling. Numerical results show that through trailing-edge flaps actuated by a robust controller, more than 60% reduction in dynamic loading due to atmospheric turbulence can be achieved for the sectional model and close to 13% reduction in blade loads are obtained for the complete three-dimensional floating turbine.

Keywords: Aeroelasticity; Load control; Composite beams; Vortex methods; Floating wind turbine

1. Introduction

The development of offshore wind turbines has been gaining momentum to take advantage of the vast wind energy resource in open waters where there are less turbulent winds and more predictable power generation (Musial & Butterfield, 2006). Situated offshore, there are also less constraints in terms of turbine size, noise and visual impact. At the point of writing, offshore installations are dominated by fixed-bottom structures, such as monopile, tripod, gravity and suction bucket foundations. However, beyond a depth of approximately 50 m, these support structures are no longer commercially viable and floating concepts may be needed (Lackner & Rotea, 2011; Musial, Butterfield, & Boone, 2004).

The development of floating turbines is still in its infancy and the ones built so far are for research purposes. They are attractive in their ability to access deep waters currently out-of-bounds to its fixed-bottom counterparts, and to avoid protected offshore regions and shipping lanes (Sebastian & Lackner, 2013). Much of the technology for the floating structures are derived from oil platforms and can be classified based on their mooring systems, tanks and ballast options (Bir & Jonkman, 2007). With proven track record in oil and gas explorations, the long-term survivability of these floating structures is not of concern. The main challenge is in their adaptation to the specific operating conditions surrounding horizontal-axis wind turbines (HAWT) and in developing cost-effective floating concepts that are attractive to the wind industry (J. M. Jonkman, 2009).

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The dynamical analysis of floating wind turbines is multi-disciplinary, encompassing aerodynamics, hydrodynamics, controls and structures, which will be referred here as the aero-hydro-servoelastic (AHSE) analysis. The common approach to AHSE modelling is to include a floating/supported platform into existing aeroservoelastic codes that are proven in analysing land-based turbines (Bossanyi, 2003; J. M. Jonkman & Buhl Jr., 2005; Larsen & Hansen, 2012; Øye, 1996). This avoids code verifications for the part of turbine above sea-level. In fact, most of the recent work has used this approach and has based their floating/supported concepts on the models proposed in Passon et al. (2007) and J. M. Jonkman and Musial (2010). This led to a collaborative effort to benchmark offshore turbine codes on common test cases, leading to improvements in code accuracy. The results are particularly important to the advancement of offshore wind energy deployment.

Offshore turbines operate in relatively different conditions compared to their land-based counterparts (Bir & Jonkman, 2007). Most importantly, there is more skewed inflow due to the motion of the platform, resulting in rapid local velocity changes and greater unsteady effects that could violate the fundamental assumptions underpinning momentum-based approaches (Hansen, Sørensen, Voutsinas, Sørensen, & Madsen, 2006; Manwell, McGowan, & Rogers, 2009). Hence, more rigorous modelling techniques, such as the unsteady vortex-lattice method (UVLM), could provide insights and better predictions of the dynamic loadings (Leishman, 2002; Sebastian & Lackner, 2012). The UVLM does not rely on empirical corrections commonly found in blade-element methods and is able to capture three-dimensional effects on blades and control surfaces. The main limitation of vortex methods in wind turbine applications has been the assumption of attached flow conditions, preventing its use in operating regimes where blades are at high angles of attack or in the inboard regions of the blade. Attempts have been made by several researchers to include viscosity and flow separation (Sørensen, 1983; Voutsinas, 2006), however there are still unresolved issues related to convergence (Hansen et al., 2006). With advances in computing power, computational fluid dynamics (CFD) solving the Navier-Stokes equations are also increasingly being used (Corson, Griffith, Ashwill, & Shakib, 2012; Simms, Schreck, Hand, & Fingersh, 2001).

The modelling of floating turbines below the water surface requires a structural description of the platform (J. M. Jonkman, 2010) and a statistically representative model of the ocean wave. Together, they deliver wave-induced dynamics on all structural components of the turbine on top of the already complicated wind-induced loadings. Under such complex operating environment, floating turbines will likely necessitate load alleviation mechanisms to extend their component fatigue lives. The application of *smart rotor* concepts could provide solutions for fatigue load alleviation (Barlas & van Kuik, 2010) on these floating turbines, reducing the cost of energy and potentially relaxing component design objectives and constraints. *Smart rotors* originate from well-developed aeronautic applications (Bernhard & Chopra, 1998; Cook, Palacios, & Goulart, 2013) and incorporate actively controlled devices that are distributed along the blade. They possess the agility to rapidly respond to changes in a dynamic loading environment and include the use of trailing-edge flaps, microtabs, camber control or active twist. While *smart rotors* are rather mature in air vehicles, they are not readily transferable to wind turbine applications given the different operating conditions where loadings could vary in type and in scale. For instance, wind turbines are much larger in size and their blades are subject to greater stresses. Also, inspection and maintenance on wind turbines happen less frequently than on air vehicles due to difficulty in ground access. Individual pitch controls can also be considered for load alleviation and as demonstrated in Ng, Palacios, Graham, Kerrigan, and Hesse (2015) for the land-based version, the load reduction performance is higher than trailing-edge flaps. However, the power required for actuation is expected to be higher with increased wear and tear on pitch bearings. Individual pitch also lacks the agility and localised properties that flaps possess.

To date, most of the investigations into *smart wind turbine rotors* have been numerical with only a couple of experiments for proof-of-concept and are based on land-based turbines. These include wind-tunnel experiments (Barlas, van Wingerden, Hulskamp, van Kuik, & Bersee, 2013) and also full-scale implementation (J. C. Berg, Barone, & Yoder, 2014; J. C. Berg, Resor, Paquette,

& White, 2014; Castaignet et al., 2014) to justify the practical aspects of *smart rotors*. There are still numerous challenges to overcome, mainly in the modelling, control, implementation and finally gaining confidence and acceptance from industry. Most crucially, with the increase in size of wind turbines and the development of floating concepts for larger energy capture, high-fidelity and control-oriented AHSE modelling approaches are needed to predict turbine blade dynamics and for load alleviation simulation studies.

In this paper, we present an AHSE modelling of floating wind turbines with actively-controlled trailing-edge aerodynamic surfaces (flaps), and demonstrate their dynamic load reduction (blade flapwise root-bending moment) capabilities through robust control methods ($\mathcal{H}_\infty$). We seek to provide a control-oriented aeroelastic model without empirical data through the use of UVLM in the aerodynamics module coupled to a monolithic description of the structural dynamics. With the attached flow condition of the UVLM, the existing formulation will be more relevant in above-rated operating conditions where blades are at lower angles of attack. This is also the regime where *smart rotor* concepts are shown to be more effective in load reductions (Bergami & Gaunaa, 2012). We will approach the problem from two fronts - a simple two-dimensional sectional model of the blade for preliminary investigations and the three-dimensional model of the floating turbine for detailed blade load alleviation analysis.

In Section 2.1, structural representations of the two and three-dimensional models will first be introduced, followed by the vortex-based unsteady aerodynamics in Section 2.2 and the coupled aeroelastic description in Section 2.3. The two-dimensional aeroelastic model contains a simple plunge-pitch aerofoil coupled to the vortex-particle unsteady aerodynamics in a state-space form (Ng, Palacios, Graham, & Kerrigan, 2012). It provides a basic model to appreciate the fundamental concept and understand the versatility of the formulation, accompanied with simple closed-loop investigations. These results can be compared to component (blade) and full turbine models to assess whether two-dimensional analysis is sufficient for wind turbine active load alleviation studies. Details for numerically implementing this two-dimensional model is also included in Appendix A. The complete three-dimensional aeroelastic model is developed using similar steps as its two-dimensional counterpart but with three-dimensional sub-modules for the structures and aerodynamics. The methodology has been explored by Ng et al. (2015) for land-based turbines, coupling the vortex-lattice unsteady aerodynamics model with a multi-body composite structural description of the turbine. Here, the model will be augmented with floating hydrodynamic capabilities, accompanied with the wave/wind representations in Section 2.4 and the controller implementation in Section 2.5. The two-dimensional model will then be validated for flutter speeds before being analysed in closed-loop for dynamic load alleviation in Section 3. This will be followed by Section 4 on the numerical results of the three-dimensional model for which the conceptual NREL 5MW floating wind turbine is chosen to demonstrate the AHSE formulation. The paper will conclude with a summary of the key findings and suggestions for further work in Section 5.

2. State-space Aeroservoelastic Formulation

The state-space two and three-dimensional aeroservoelastic formulations will be presented, starting with the definitions of their structural modules in Section 2.1 and the vortex-based unsteady aerodynamics in Section 2.2. This will be followed by the description of the fluid-structure coupling to arrive at the final state-space formulation in Section 2.3.

2.1 Structural Dynamics

The two-dimensional structural model of a plunge-pitch aerofoil will be presented in Section 2.1.1 followed by the three-dimensional flexible multi-body dynamics in Section 2.1.2.

Figure 1. 2-dof aeroelastic system.

2.1.1 Two-dimensional Plunge-pitch Aerofoil

The two-dimensional structural model includes a two degrees-of-freedom (2-dof) plunge-pitch aerofoil restrained by a pair of springs (Bisplinghoff, Ashley, & Halfman, 1955), as shown in Figure 1. The spring constants, k_h and k_α , restrain the plunge and pitch motions, respectively. The model has a half chord length of b , with the distance from mid-chord to elastic axis and flap hinge denoted by ab and eb , respectively. Freestream velocity is denoted by U_∞ , pitch angle by α , plunge by h and flap deflection angle by β . The convention taken for h is positive downwards, α is positive nose up about the elastic axis and β is positive flap down about the hinge. Lift L is positive upwards and moment M_α about the elastic axis is positive when the aerofoil pitches nose up. The vertical gust in the freestream is given by w_g .

The dynamic equation-of-motion (EoM) is written in non-dimensional form as

$$\mathcal{M}\ddot{\mathbf{q}} + \mathcal{K}\mathbf{q} = \mathbf{Q}_s, \quad (1)$$

where $\mathcal{M} = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} \frac{h}{b} \\ \alpha \end{bmatrix}$, $\mathcal{K} = \begin{bmatrix} \omega_h^2 & 0 \\ 0 & \omega_\alpha^2 r_\alpha^2 \end{bmatrix}$ and $\mathbf{Q}_s = \begin{bmatrix} -\frac{L}{Mb} \\ \frac{M_\alpha}{Mb^2} \end{bmatrix}$. Details on the computation of L and M_α can be found in Appendix A. In the equations, $x_\alpha = \frac{S_\alpha}{Mb}$ is the distance between centre of gravity and elastic axis and $r_\alpha^2 = \frac{I_\alpha}{Mb^2}$ is the aerofoil's radius of gyration (both normalised by b). The terms $\omega_h = \sqrt{\frac{k_h}{M}}$ and $\omega_\alpha = \sqrt{\frac{k_\alpha}{I_\alpha}}$ represent the uncoupled natural frequencies of the plunge and pitch modes, respectively. M is the mass per unit span, S_α is the static moment of aerofoil section about the elastic axis and I_α is the aerofoil moment of inertia per unit span about the elastic axis. $\ddot{h}$ and $\ddot{\alpha}$ are the second derivatives of plunge and pitch, respectively. The flap is assumed to have a rigid link and its mass is considered to be small such that any inertial effects from flap deflection is ignored. Eq. (1) is subsequently discretised using the backward Euler method (Géradin & Rixen, 1997) such that the applied forces are computed in the t_{n+1} time-step to match the unsteady surface pressures defined by the discrete vortex-particle method.

2.1.2 Three-dimensional Multi-body Structural Dynamics

In the three-dimensional structural module, the rotor blades are modelled using composite beams described in a moving frame of reference (Géradin & Cardona, 2001; Simo & Vu-Quoc, 1988), and includes geometrically nonlinear deformation of the blades (Hesse & Palacios, 2012). The addition of the tower defines a flexible multi-body system, in which the composite beam descriptions of the rotor and tower are connected using Lagrange multipliers including a prescribed constant angular

Figure 2. (a) Structural model of the floating wind turbine. (b) Aerodynamic model of rotor. For ease of visualisation, four panels per chord is used for UVLM.

velocity.

In this study, the floating turbine platform is represented as a beam element spanning from the base of the tower to the centre of gravity (c.g.) of the platform, as shown in Figure 2. A point mass including inertia is then added to the c.g. location. As a result, the motion of this floating platform constitutes rigid-body dynamics at the base of the entire system. The complete linearised finite element EoM describing the floating turbine has the following structure

$$\begin{aligned}
 & \begin{bmatrix} \mathcal{M}_t^{SS} & \mathcal{M}_t^{SR} & 0 & 0 & -\Lambda^\top \\ \mathcal{M}_t^{RS} & \mathcal{M}_t^{RR} & 0 & 0 & 0 \\ 0 & \mathcal{M}_r^{SR} \mathbf{T}_{tr}(t) & \mathcal{M}_r^{SS} & \mathcal{M}_r^{SR} & 0 \\ 0 & \mathcal{M}_r^{RR} \mathbf{T}_{tr}(t) & \mathcal{M}_r^{RS} & \mathcal{M}_r^{RR} & \mathbf{A}_{CC}^\top(t) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \dot{\eta}_t \\ \Delta \dot{\nu}_t \\ \Delta \dot{\eta}_r \\ \Delta \dot{\nu}_r \\ \Delta \dot{\lambda} \end{bmatrix} + \begin{bmatrix} \mathcal{C}_t^{SS} & \mathcal{C}_t^{SR} & 0 & 0 & 0 \\ \mathcal{C}_t^{RS} & \mathcal{C}_t^{RR} + \mathbf{B}_f & 0 & 0 & 0 \\ 0 & \mathcal{C}_r^{SR} \mathbf{T}_{tr}(t) & \mathcal{C}_r^{SS} & \mathcal{C}_r^{SR} & 0 \\ 0 & \mathcal{C}_r^{RR} \mathbf{T}_{tr}(t) & \mathcal{C}_r^{RS} & \mathcal{C}_r^{RR} & 0 \\ \Lambda & 0 & 0 & -\mathbf{A}_{CC}(t) & 0 \end{bmatrix} \begin{bmatrix} \Delta \dot{\eta}_t \\ \Delta \dot{\nu}_t \\ \Delta \dot{\eta}_r \\ \Delta \dot{\nu}_r \\ \Delta \dot{\lambda} \end{bmatrix} \\
 & + \begin{bmatrix} \mathbf{K}_t^{SS} & 0 & 0 & 0 & 0 \\ \mathbf{K}_t^{RS} & \mathbf{C}_{ml} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{K}_r^{SS} & 0 & 0 \\ 0 & 0 & \mathbf{K}_r^{RS} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \eta_t \\ \int \Delta \nu_t dt \\ \Delta \eta_r \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{Q}_{ext} \\ \int \mathbf{Q}_{ext} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (2)
 \end{aligned}$$

where $\mathcal{M}$, $\mathcal{C}$ and $\mathbf{K}$ are the discrete mass, damping and stiffness matrices. The superscripts S and R have the structural and rigid-body definitions, respectively, and the subscripts t and r represents the tower and rotor, respectively. The matrices Λ and $\mathbf{A}_{CC}$ enforce velocity constraints between the tower top and rotor hub, with the latter further accounting for the prescribed rotor angular velocity Ω . The external forces and moments are represented by $\mathbf{Q}_{ext}$ and are used in the coupling with aerodynamics. The primary variable $\boldsymbol{\eta}$ contains all the nodal displacements and rotations, $\boldsymbol{\nu}$ contains the rigid-body motions and $\boldsymbol{\lambda}$ are the Lagrange multipliers (Ng et al., 2015).

The transformation $\mathbf{T}_{tr}(t)$ map base motions and the additional kinematics due to rotation of tower moment arm onto the rotor hub. These have not been enforced through the Lagrange mul-

tipliers and are hence included for the rotor dynamics. The entries $\mathbf{B}_f$ and $\mathbf{C}_{ml}$ are the additional damping of platform and stiffness from mooring lines, respectively. The buoyancy, hydrodynamic and mooring line forces have been lumped into $\mathbf{Q}_w$ and are included on the right hand side of Eq. (2). The continuous-time structural EoM of the floating wind turbine in Eq. (2) is subsequently discretised using the Newmark- β method to couple with the discrete-time vortex-lattice unsteady aerodynamics.

2.2 State-space Vortex Unsteady Aerodynamics

The definition of the unsteady aerodynamics in both the two and three-dimensional aeroelastic models will be based on potential flow theory. The governing equation for low-speed potential flow is the Laplace's equation, which can be solved by representing the potential flow field in terms of singularities (Green's theorem) (Katz & Plotkin, 2001; Murua, Palacios, & Graham, 2012a). This approach involves distributing elementary solutions over the body surface and its wake that satisfy the problem boundary conditions. The solution to Laplace's equation is then reduced to finding the unknown singularity distribution subject to the non-penetrating boundary condition (Neumann) specified at a set of control points (collocation points) on the body surface.

The time-domain vortex method uses vortices as singularity solutions to the Laplace's equation. In two dimensions, the aerofoil is assumed to be a flat plate and discretised using equally spaced panels, each of which contains one vortex-particle and one collocation point, as illustrated in Figure 3. In three dimensions, the vortices are located in lattices distributed across the blades and wakes shown in Figure 4. The vortex description allows for cambered shapes and direct modelling of lifting surfaces such as trailing-edge flaps. Thickness can be included through source elements but they are of second-order effects on lift and has shown to be negligible when computing loads (Bergami, Gaunaa, & Heinz, 2013). In order to enable the solution to be written in a linear state-space representation, the wake is prescribed in a helicoidal shape (Chattot, 2007) as shown in Figure 2. The chordwise length of the shed wake vortex rings from trailing-edge is $U_\infty \times \Delta t$, where the relative freestream velocity U_∞ varies linearly along the span of the blade. The time-step Δt is chosen based on the blade mid-span chordwise UVLM discretisation, given by $\Delta t = c_m / (U_{\infty,m} d)$, where d is the number of bound chordwise panels, c_m and $U_{\infty,m}$ are the local chord length and relative velocity at blade mid-span, respectively. The shed wake is then assumed to be transported downstream of the rotor by the inflow velocity and without wake expansion.

Free-wake models are known to give better predictions of aerodynamic loads over prescribed and rigid-wake models (Leishman, 2002). However, free-wake models are computationally more demanding as they require to solve the Biot-Savart law at every time step (Katz & Plotkin, 2001). A parametric study to understand the effects of wake roll-up showed that models with free and prescribed wake displayed little differences in terms of the computed unsteady aerodynamic forces (Murua et al., 2012a). Previous studies comparing both wake models reported less than 5% difference in power coefficients over a range of tip-speed-ratios (TSR) from 3 to 12 (Simoes & Graham, 1992). It should also be emphasised that the assumptions taken in this study are not intrinsic of the method (UVLM) adopted and we have only used them after direct comparison with free-wake results so as to provide a trade-off between minimum accuracy losses and maximum computational gains.

For either the two or three-dimensional problems, the equation defining the non-penetrating boundary condition and propagation of wake downstream can be written in symbolic form, as

$$\mathbf{A}_2 \mathbf{\Gamma}^{n+1} = \mathbf{A}_1 \mathbf{\Gamma}^n + \mathbf{T}_q(t) \mathbf{q}^{n+1} + \mathbf{W}_{\beta,\delta}^n, \quad (3)$$

where $\mathbf{\Gamma}$ contains all the bound and wake circulations, $\mathbf{q}$ is the vector of all structural, rigid-body dof and $\mathbf{W}_{\beta,\delta}$ is the concatenated vector of downwash due to flap deflections β and external perturbations in the flow field (gust) δ . The downwash $\mathbf{W}_{\beta,\delta}$ is approximated by its value at time-

Figure 3. Simplified vortex-particle model (illustrated with 8 aerofoil panels, 2 of which account for the flap).

Figure 4. UVLM definition of three-dimensional bound and wake panel distribution and numbering. Includes illustration of fluid-structure coupling where aerodynamic loads computed at centre of vortex ring's leading segment are mapped onto structural nodes (red arrows), and structural dof at nodes are projected onto aerodynamic collocation point as downwash (green arrows).

step t_n such that the final EoM is in explicit form. The coefficients $\mathbf{A}_1$ and $\mathbf{A}_2$ are the aerodynamic influence coefficient matrices due to the blade and wake, resolved using the Biot-Savart law (Katz & Plotkin, 2001). The coupling term $\mathbf{T}_q(t)$ is used to map structural and rigid-body dof as downwash on bound collocation points and is time-varying due to the azimuth dependence of the rotor with respect to the tower. In the two-dimensional model, this coupling term will be time-invariant.

Aerodynamic forces and moments can be computed using the unsteady Bernoulli equation (Katz & Plotkin, 2001) and is symbolically represented, as

$$\mathbf{Q}_a^{n+1} = \mathbf{F}_2 \mathbf{\Gamma}^{n+1} + \mathbf{F}_1 \mathbf{\Gamma}^n, \quad (4)$$

where $\mathbf{Q}_a$ are the computed aerodynamics forces and moments. To facilitate any future numerical implementation of the two-dimensional vortex-particle solution, the procedure and matrix definitions are included in detail in Appendix A. Note that the three-dimensional vortex-lattice essentially relies on the same steps but with three-dimensional representations of the algorithms.

Figure 5. Block diagram of fluid-structure coupling in the three-dimensional aeroelastic model and closed-loop configuration with external controllers.

2.3 Fluid-structure Coupling

In the two-dimensional model, aerodynamic loads computed at vortex points are mapped to the structural elastic axis, and in turn, structural plunge and pitch motions are mapped as downwash on the panel collocation points. In the three-dimensional model, aerodynamic loads at the centre of each bound vortex ring's leading segment are mapped onto the structural nodes. This is under the assumption that the lifting surface remains undeformed with the spanwise aerodynamic grid coinciding with the finite-element description of the beam (Murua et al., 2012a). Also, it assumes that the aerodynamic forces are approximated as isolated loads at the centre of each bound vortex ring's leading segment. Next, structural displacements, rotations and velocities of the beam are mapped onto aerodynamic collocation points as downwash (See Figure 4). Note also that in-plane motions are not modelled in the two-dimensional plunge-pitch aerofoil but they are included in the three-dimensional model.

The discretised structural dynamics for both the two and three-dimensional models can symbolically be expressed as

$$\mathbf{N}_2(t)\mathbf{q}^{n+1} + \mathbf{N}_1(t)\mathbf{q}^n = \mathbf{Q}_s^{n+1} + \mathbf{f}_w^n, \quad (5)$$

where $\mathbf{Q}_s$ contains the external forces acting on the structural nodes. Consolidating Eqs. (3), (4) and (5), we obtain in matrix form

$$\begin{bmatrix} \mathbf{A}_2 & -\mathbf{T}_q(t) \\ -\mathbf{F}_2 & \mathbf{N}_2(t) \end{bmatrix} \begin{Bmatrix} \boldsymbol{\Gamma} \\ \mathbf{q} \end{Bmatrix}^{n+1} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{F}_1 & -\mathbf{N}_1(t) \end{bmatrix} \begin{Bmatrix} \boldsymbol{\Gamma} \\ \mathbf{q} \end{Bmatrix}^n + \begin{Bmatrix} \mathbf{W}_{\beta,\delta} \\ \mathbf{0} \end{Bmatrix}^n + \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_w \end{Bmatrix}^n. \quad (6)$$

In the above equation, the first row accounts for the non-penetrating boundary condition of Eq. (3). The second row equates the discretised structural forces $\mathbf{Q}_s$ in Eq. (5) to the aerodynamic loads $\mathbf{Q}_a$ in Eq. (4). A block diagram illustrating the fluid-structure coupling of the existing aeroelastic model with feedback to external controllers is shown in Figure 5. In the case of the two-dimensional model, $\mathbf{T}_q$, $\mathbf{N}_1$ and $\mathbf{N}_2$ will be time-invariant and $\mathbf{f}_w$ will not appear.

The resulting EoM (6) can be written in a linear time-varying state-space realisation

$$\begin{aligned} \mathbf{x}^{n+1} &= \mathbf{A}(t)\mathbf{x}^n + \mathbf{B}\boldsymbol{\beta}^n + \mathbf{G} \begin{Bmatrix} \boldsymbol{\delta} \\ \mathbf{f}_w \end{Bmatrix}^n, \\ \mathbf{y}^n &= \mathbf{C}\mathbf{x}^n + \mathbf{D}\boldsymbol{\beta}^n + \mathbf{H} \begin{Bmatrix} \boldsymbol{\delta} \\ \mathbf{f}_w \end{Bmatrix}^n, \end{aligned} \quad (7)$$

where $\mathbf{A}$ is a function of the azimuth angle $\theta(t)$. If forces are defined in the output $\mathbf{y}$, they can be resolved from either the aerodynamics $\mathbf{Q}_a$ of Eq. (4) or structures $\mathbf{Q}_s$ of Eq. (5). The state vector that completely defines the three-dimensional aeroelastic system is $\mathbf{x}^\top = [\Delta\boldsymbol{\Gamma}^\top \mid \Delta\boldsymbol{\eta}_t^\top \ \Delta\dot{\boldsymbol{\eta}}_t^\top \ \Delta\boldsymbol{\eta}_r^\top \ \Delta\dot{\boldsymbol{\eta}}_r^\top \mid \Delta\boldsymbol{\nu}_t^\top \ \Delta\boldsymbol{\nu}_r^\top \ \Delta\boldsymbol{\lambda}^\top]$ containing the aerodynamic, structure and rigid-body states. For the two-dimensional model, Eq. (7) will be linear time-invariant and $\mathbf{x}^\top = [\Delta\boldsymbol{\Gamma}^\top \mid \Delta h \ \Delta\alpha \ \Delta\dot{h} \ \Delta\dot{\alpha}]$ containing the aerodynamic and structure states. The combined hydrodynamic, buoyancy and mooring line forces acting on the platform are lumped in $\mathbf{f}_w$ and only appear in the three-dimensional model.

2.4 Wave Kinematics and Inflow Turbulence

The incident-wave kinematics in irregular seas are determined using a wave spectrum combined with linear Airy wave theory (Lamb, 1997). The fundamental assumption in Airy wave theory is that the fluid is inviscid, incompressible and irrotational, allowing the use of potential flow solutions to model the motion of gravity waves on the surface of a fluid.

Airy wave theory describes the elevation e_v of the wave as a harmonic motion, given by

$$e_v = a \sin(\omega_w t - kx), \quad (8)$$

where a is the amplitude (given by half the wave height), $k = 2\pi/\lambda$ is the angular wave number, λ is the wavelength, ω_w is the wave frequency and x is the horizontal position of the wave particle (to describe the time lag) (Moe, 2007).

For irregular waves, either the Pierson-Moskowitz or JONSWAP (JOint North Sea WAVE Project) spectrum are commonly used to combine with Airy wave theory, to define a and ω_w in Eq. (8) (IEC 61400-3, 2009). Here, the Pierson-Moskowitz spectrum is adopted as it caters for fully developed seas and for wind blowing over a large area for a long time. The one-sided spectrum for the Pierson-Moskowitz spectrum is given by J. M. Jonkman (2007), as

$$S(\omega_w)_{PM} = \frac{1}{2\pi} \frac{5}{16} H_s^2 T_p \left(\frac{\omega_w T_p}{2\pi} \right)^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega_w T_p}{2\pi} \right)^{-4} \right], \quad (9)$$

where H_s is the significant wave height and T_p is the peak spectral period.

A realistic wind field in numerical simulations provides a huge benefit to predicting loads in an operating environment that is as close as possible to the actual conditions. The IEC 61400-1 (2006) is the international standard providing sets of design requirements to ensure turbines are properly engineered to withstand the operating conditions throughout a lifetime of at least 20 years. To simulate the full wind field in accordance to IEC DLC 1.2 for fatigue analysis, the open-source tool, Turbsim, developed by the US National Renewable Energy Laboratory, is used (B. J. Jonkman & Kilcher, 2012).

2.5 Controller Description

In this study, $\mathcal{H}_\infty$ controls will be considered for its robustness properties. We will discuss the control model and selection of weights in this section.

In $\mathcal{H}_\infty$ controls (Green & Limebeer, 1995; Skogestad & Postlethwaite, 2005), the objective is to minimise the $\mathcal{L}_2$ gain of the linear fractional map $F(\mathbf{P}, \mathbf{K})$

$$\|F(\mathbf{P}, \mathbf{K})\|_\infty = \max_{\delta \neq 0} \frac{\|z\|_{\mathcal{L}_2}}{\|\delta\|_{\mathcal{L}_2}}, \quad (10)$$

where $\mathbf{P}$ and $\mathbf{K}$ are the plant and controller, respectively. This signifies that the worst case response of the system to an input disturbance (maximum singular value) is minimised. Hence, in the presence of bounded uncertainties, closed-loop performance and stability are guaranteed (Zhou & Doyle, 1998).

For the two-dimensional model, Eq. (7) will be linear time-invariant and provides the state-space EoM for controller design. For the three-dimensional model, Eq. (7) is linear time-varying as the turbine dynamics are dependent on the blade's azimuth location with respect to the tower, appearing through the constrained structural EoM and aerodynamic coupling terms. However, in the unassembled rotor or rotor blade descriptions, they are linear time-invariant without the tower.

Three-dimensional aeroelastic models developed using vortex-based aerodynamics are often large even after model reduction (around a thousand states to have an accurate description of the dynamics) and are not efficient for synthesising the controller. Considering that blade loads are the focus in this study, adopting the unassembled rotor blade description provides a small, linear and representative model for controller design (Larsen, Madsen, & Thomsen, 2005; Leithead, Neilson, Dominguez, & Dutka, 2009; Plumley, Leithead, Jamieson, Bossanyi, & Graham, 2014). This unassembled rotor blade uses the same aeroelastic description as the full turbine, coupling a composite beam with the UVLM to arrive at a linear time-invariant state-space representation. It will be used to design the controller and then simulated in closed-loop with the linear time-varying complete turbine model. Nonetheless, it is worth mentioning that methods exist to design controllers directly from the complete turbine in the non-linear description, using system identification (Barlas, van der Veen, & van Kuik, 2012; Bergami & Poulsen, 2014) or different treatments of periodicity (Bottasso, Cacciola, & Riva, 2014; Geyler & Caselitz, 2008; Lu, Bowyer, & Jones, 2014; Ozdemir, Seiler, & Balas, 2011).

The linear EoM of the two-dimensional aerofoil and three-dimensional rotor or rotor blade for the controller design can be expressed as (Ng, Hesse, Palacios, Graham, & Kerrigan, 2014a)

$$\begin{aligned} \mathbf{x}_c^{n+1} &= \mathbf{A}_c \mathbf{x}_c^n + \mathbf{B}_c \boldsymbol{\beta}_c^n + \mathbf{G}_c \boldsymbol{\delta}_c^n, \\ \mathbf{y}_c^n &= \mathbf{C}_c \mathbf{x}_c^n + \mathbf{D}_c \boldsymbol{\beta}_c^n + \mathbf{H}_c \boldsymbol{\delta}_c^n, \end{aligned} \quad (11)$$

with subscript c denoting the control model.

The objective function to be minimised is given by

$$\mathbf{z} = \begin{bmatrix} \mathbf{Q}^{\frac{1}{2}} & 0 \\ 0 & \mathbf{R}^{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} \mathbf{y}_c \\ \boldsymbol{\beta}_c \end{bmatrix}, \quad (12)$$

containing the weighted measurement $\mathbf{y}_c$ and control input $\boldsymbol{\beta}_c$ from Eq. (11). The weights on measurement $\mathbf{Q}$ are increased relative to control input weight $\mathbf{R}$ until the actuator limits are encountered (D. E. Berg, Wilson, Resor, Barone, & Berg, 2009). In this study, trailing-edge flaps have been selected as the actuating mechanism for active load alleviation. Compared to pitch actuation which involve the entire blade, flaps are smaller, localised, faster in actuation and does not involve actuating the entire blade. However, if pitch controls is sought for in the modelling,

Figure 6. Block diagram of control model.

this only requires simple modification to the aerodynamics by imposing additional downwash on all the aerofoil/blade panels from pitch actuation. For both the two and three-dimensional models, the limits on flap actuation β and $\dot{\beta}$ are $\pm 10^\circ$ and $\pm 100^\circ/\text{s}$, respectively. The block diagram for the control model is shown in Figure 6.

3. Numerical Results for Sectional Model

The two-dimensional model serves as a starting point to investigate the effectiveness of active controls in reducing sectional blade loads on turbines. The model of a typical section will first be numerically verified by comparing flutter speeds against published results and subsequently demonstrated for load alleviation using actively controlled trailing-edge flaps. It also provides a simple model to investigate controller sensitivity to uncertainties resulting from relaxed spatial discretisation in the unsteady aerodynamics. In all the simulations that follow, the wind field generated by Turbsim contains a turbulent component described by the von Kármán spectrum and a wind shear with power law of exponent 0.2 (IEC 61400-1, 2006). Each simulation contains 6×600 seconds of wind field, with a characteristic hub height turbulence intensity of 17.5%.

3.1 Sectional Model Numerical Verification

The two-dimensional aeroelastic system presented in Section 2 is modelled with a converged spatial discretisation on the aerodynamics using 50 equally distributed vortex-particles on the aerofoil and a wake of 5 chord lengths. The flutter speeds are then computed and compared against another aeroelastic model with Wagner’s indicial step response for its unsteady aerodynamics, as shown in Figure 7. Also included in the plot is the flutter boundary by Murua, Palacios, and Peiró (2010) where the aerodynamics are computed using a finite-order approximation to Theodorsen’s function (Theodorsen & Garrick, 1938). All models are using the same plunge-pitch aerofoil structural description in Figure 1. As observed, there is generally a good agreement between the three aeroelastic models considered. Although not shown here, similar flutter results had also been reported by Zeiler (2000) and Bergami and Gaunaa (2010).

3.2 Dynamic Load Reduction using Controllers from Converged Models

The sectional model is now tuned to match the first flapwise and first torsional modes (0.68 Hz and 7.83 Hz) of the conceptual NREL 5MW wind turbine blade (J. M. Jonkman, Butterfield, Musial, & Scott, 2009) for dynamic load alleviation studies. The input disturbance (gust) is generated from Turbsim containing both turbulence and shear effects. The $\mathcal{H}_\infty$ controller is synthesised from the converged vortex aeroelastic model (with 50 panels per chord and 5 wake chords) and the solution is time-marched at 28.5 Hz. Using a 10% trailing-edge flap and the coefficient of lift C_L as the measurement feedback, an average 60% reduction in root-mean-square (rms) of C_L is achieved without frequency weighting. The PSD for the coefficient of lift is shown in Figure 8 where the reduction is across most frequencies. It is evident that the loads are dominated by the

Figure 7. Flutter boundaries of 2-dof aerofoil. Validating current vortex aeroelastic model with Murua et al. (2010) and a separate aeroelastic model with the unsteady aerodynamics replaced by Wagner’s indicial step response function ($a = -0.3$, $\kappa = \frac{\pi \rho b^2}{M} = 0.05$, $r_\alpha^2 = 0.25$, $b = 1.0$).

Figure 8. PSD of lift coefficient on the 2-dof aerofoil in open and closed-loop with active flap $\mathcal{H}_\infty$ controls.

low frequencies, especially the 1P (once-per-revolution). Incremental static loads are also reduced but since we are looking at incremental loads about the operating condition, this implies that targeted rotor speeds could be better maintained. The controller could also be designed to target specific frequency bands, such as the 1P. Although not attempted here in the two-dimensional case, frequency weighting will be used in the next section for the three-dimensional case. The PSD is computed using Welch’s method with eight segments of 50% overlap to smoothen the power spectrum (Welch, 1967). Note also that the maximum ΔC_L for static flap deflection of $\pm 10^\circ$ is 0.424.

3.3 Controller Synthesis from Spatially Coarse Vortex Aeroelastic Model

Given that $\mathcal{H}_\infty$ controllers have the same size as the model, control synthesis using high-order converged vortex-based aeroelastic models is computationally expensive, and will be especially so in full turbine models. At this stage, one will ask if the control model has to be that fine in order

Table 1. Performance of low order panel model for controller synthesis, reduced through relaxed spatial discretisation and balanced truncation.

Case	No. of panels in control model	Total no. of states in control model	% reduction in rms C_L in closed-loop with converged model
1	50 (Converged)	356	60.7
2	40	286	60.5
3	30	216	60.5
4	20	146	42.0
5	10	76	29.4
6	Balanced truncation on converged model	5	60.0

Figure 9. Open and closed-loop Bode plots from gust input to lift coefficient for the 2-dof model. Showing the open-loop converged model of 50 panels per chord and the closed-loop cases 1, 5 and 6 from Table 1.

to deliver good performances, noting that controllers are known to perform well in the presence of uncertainties depending on the level of model mismatch (Cook et al., 2013; Jones & Kerrigan, 2010; Lanzon & Papageorgiou, 2009; Vinnicombe, 1996). To investigate, the number of panels per chord in the vortex aeroelastic model was reduced for control synthesis, but tested in closed-loop with the converged aeroelastic model. This enables us to understand the limits to which controllers developed from spatially coarse models, which had not converged for open-loop predictions, were capable of rejecting disturbances on converged models. Also, it allows us to obtain control models of reduced number of states to speed up closed-loop numerical simulations.

As shown in Table 1, a controller developed using the vortex model with 30 panels per chord (case 3) is able to maintain the 60% reduction in rms C_L when applied in closed-loop with the converged model of 50 panels per chord. However, the maximum achievable performance deteriorates (by almost half) as we reduce the number of panels per chord on control model to 20 and then to 10. The closed-loop Bode plots for cases 1 and 5 are shown in Figure 9. In all cases, the flap deflection angles and rates were kept within $\pm 10^\circ$ and $\pm 100^\circ/\text{s}$.

As a comparison, balanced truncation (Skogestad & Postlethwaite, 2005) was also used to reduce the converged aeroelastic model (50 panels per chord) to just 5 states. When the controller is developed using this reduced model, it is capable of achieving as good performance as the controller

Figure 10. Open-loop Bode plots from gust input to lift coefficient for the 2-dof model. Showing the converged model of 50 panels per chord and the reduced order models through spatial discretisation (10 panels per chord) and balanced truncation (5 states).

designed from the converged model, as shown in case 6 of Table 1 and the closed-loop Bode plot in Figure 9.

Looking further at the open-loop Bode plots from gust input to lift coefficient in Figure 10 for the various reduced models used for control design, it is evident that the balanced truncated model of 5 states provides slightly better representation of the converged model than that of a spatially coarse model of 76 states. Even though the low frequency gains in open-loop for 10 panels per chord matches the converged model, the closed-loop performance did not perform well across all frequencies (shown previously in case 5 in Figure 9). This indicates that for vortex-based aeroelastic models, the spatial discretisation needs to be quite resolved to capture critical system dynamics.

4. Numerical Results for Floating Turbine Model

The three-dimensional AHSE framework is now used to model the conceptual floating NREL 5MW turbine, together with numerical verification, implementation studies and closed-loop load alleviation using trailing-edge flaps.

4.1 Conceptual Floating NREL 5MW Turbine

The conceptual floating turbine is based upon the works of J. M. Jonkman (2007). It adopts the NREL 5MW turbine (J. M. Jonkman et al., 2009) as a baseline model with support foundations from Statoil’s ‘Hywind’ project (J. M. Jonkman, 2010). The original NREL tower is shortened in the floating configuration to accommodate part of the floating platform above sea-level. The tower base is elevated at 10 m above sea-level and the draft is 120 m below sea-level, as shown in Figure 11. All other components such as rotor, nacelle and hub are kept as in the original NREL description.

The detailed description for the floating platform and mooring system can be found in J. M. Jonkman (2010) and J. M. Jonkman and Musial (2010). These are made publicly available through the Offshore Code Collaboration Comparison (OC3) project (Cordle & Jonkman, 2011; J. M. Jonkman & Musial, 2010). In these documentations, a hydrodynamics model based on sanitised properties of Statoil’s ‘Hywind’ floating turbine was furnished using Morison’s equation. Morison’s representation is valid for slender, vertical surface-piercing cylinders that extend to the sea floor, and accounts for relative kinematics between the fluid and substructure. It is a semi-

20 equally distributed spanwise and chordwise panels on each blade, respectively, and keeping a quarter rotor diameter of wake profile downstream, is sufficient to capture dynamics in both the blades and flaps.

The complete AHSE model of the turbine contains in excess of 30,000 states and has to be reduced for computational efficiency. The concept of model reduction is a relatively well-established subject in control literature. To effectively formulate a large aeroelastic model that is suitable for model reductions is an area that requires specific treatment. Having the EoM written in a state-space description from first principles, as demonstrated here, is the first step to facilitate model reductions.

The structural and aerodynamic models are reduced prior to coupling for increased flexibility and control over the aeroelastic sub-modules. The structural model is reduced through projection and truncation on the natural vibration modes. As for the aerodynamics, they are reduced through balanced truncation, which has shown to be effective in reducing large aeroservoelastic vortex-based models (Hesse & Palacios, 2014). The detailed process for reducing the current model had been presented in a recent work by the authors (Ng et al., 2015) and here, we will only summarise its key aspects. For the structural model, the lowest 10 tower modes and 140 rotor modes are retained. The reason for the large number of rotor modes is to capture the quasi-static torsional amplitudes. For the unsteady aerodynamics, it is linear prior to coupling and balanced truncation was applied to reduce the number of states to 350. This is determined by looking at the Hankel singular values of the balanced UVLM model. The final AHSE system contains close to 900 states and is simulated at 28.5 Hz.

Even after the model reductions, this size of the full turbine model is still considerably large for controller design. As our focus is on alleviating blade root loads and that the NREL tower was found to be relatively rigid, the control model was chosen to be one of the rotor blades described in a rotating frame of reference and written in a linear time-invariant state-space representation. Details of the single blade control model can be found in Ng et al. (2014a) and the resulting controller contains 50 states. The use of a clamped blade for controller design does neglect important couplings, particularly rigid-body motion at its root representative of tower motion. In a recent paper by Ng, Hesse, Palacios, Graham, and Kerrigan (2014b), this control model was augmented with rigid body motion at its root. The closed-loop results on the full turbine were no better than the base case controller synthesized without rigid-body motions, indicating gust disturbance dominating the response of the system and justifies the use of the clamped blade for controller design. Note that in the simulation of the full turbine, the blades are free to move about their base through simply rigid links with the hub.

The numerical verification of the proposed aeroelastic model for the land-based NREL turbine was performed in Ng et al. (2015) and Ng et al. (2014b). To verify implementation of the floating conditions, a regular Airy wave of height 6 m and period 10 s was simulated on the floating turbine. To avoid exciting high frequency dynamics, the wave was input into the system gradually using a ramp function for the first 100 s. The surge and pitch motions of the platform are shown in Figure 12. The amplitudes reported (J. M. Jonkman & Musial, 2010) (FAST/ADAMS) for heave, surge and pitch are 0.25 m, 1.50 m and 0.8° , respectively, and are very close to those attained through the current model. For the comparison, the turbine is in a stationary configuration with rigid rotor. The elastic tower fore-aft motion is shown in Figure 13 and also agrees well with literature (J. M. Jonkman & Musial, 2010), which reported an amplitude of 0.23 m. This provides the confidence that the dynamics of the free-floating platform and tower are captured through the structural model and that the wave kinematics are properly implemented.

Figure 12. Resulting platform motion of the stationary floating turbine with a regular Airy wave of height 6 m and period 10 s. To avoid exciting high frequency dynamics, a ramp function is prescribed for the first 100 s of wave input.

Figure 13. Resulting tower-top fore-aft elastic deformation of the stationary floating turbine with a regular Airy wave of height 6 m and period 10 s. To avoid exciting high frequency dynamics, a ramp function is prescribed for the first 100 s of wave input.

4.3 Closed-loop Performance with Active Trailing-edge Flaps

The inclusion of irregular wave kinematics introduces low frequency content on the loading of turbine components. This is evident through Figure 14 comparing blade flapwise root-bending moments (RBM) on the land-based and floating turbines in the presence of full wind field and irregular waves. The turbine is assumed to operate in a rated 11 m/s inflow and TSR of 7. For the design of a load alleviation system on actual blades, it is expected that different angular velocities will have to be considered to derive a family of linear controllers with gain-scheduling. Each simulation is performed on 6×600 seconds of wind field with characteristic hub height turbulence intensity of 17.5%. For each 600 seconds of simulation, the computation took approximately 30 minutes on a single processor machine.

As mentioned, the controller is designed on a clamped rotating blade and has a linear state-space description. The closed-loop configuration is shown in Figure 15 in which three independent and similar controllers, each formulated from a clamped rotating blade as in Eq. (11), are connected to the turbine. The measurement feedback is chosen to be the flapwise RBM and the control input is saturated ($\pm 10^\circ$) before being fed into the system. The flaps are chosen to occupy 10% of the local chord and located from 70% to 90% span (Ng et al., 2014a). This location is found to give a large moment arm from the blade root and the dimensions are not too large that may become heavy to actuate (Andersen, Gaunaa, Bak, & Buhl, 2006). The key operating parameters for different simulation cases and flap dimensions are shown in Table 2.

Table 3 shows the closed-loop performance of trailing-edge flaps on the floating NREL 5MW turbine, in the presence of full wind field and irregular waves. The results for its land-based coun-

Table 2. Key operating parameters for simulation and flap dimensions.

Wind (Land/Float)	Inflow	11m/s
	Tip-speed-ratio	7
	Turbulence spectrum	von Kármán
	Turbulence intensity	17.5%
	Shear	power law exponent of 0.2
Wave (Float)	Spectrum	Pierson-Moskowitz ($H_s = 6\text{m}$, $T_p = 10\text{s}$)
Flaps (Land/Float)	Maximum angle	10°
	Maximum rate	$100^\circ/\text{s}$
	Chordwise dimension	10% local chord
	Spanwise location	70% to 90% span

Figure 14. PSD of root-bending moments on one of the rotor blades in open-loop, comparing the land-based and floating turbine with irregular waves (both in the presence of full wind field).

Figure 15. Closed-loop block diagram for the three-dimensional model.

Figure 16. PSD of flapwise RBM on one of the floating wind turbine rotor blades in open and closed-loop with active load alleviation without frequency weighting. In the presence of irregular wave, wind shear and turbulence inflow.

terpart from Ng et al. (2015) are also listed as case 3 for comparison. Considering case 1 for the floating turbine without frequency weighting, we observe around 13% and 10% reduction in rms and maximum values of flapwise RBM, respectively. The reduction in damage-equivalent-load (DEL) of the RBM is similar to that of the reduction in maximum values. For the calculations of DEL for fatigue analysis (Freebury & Musial, 2000; Hayman, 2012; Hendriks & Bulder, 1995), a S-N slope of 10, which is typical for composite materials has been chosen. Here, the DEL is computed from the time-series of flapwise RBM from simulation and are without weighting of the wind speeds. As such, the average DEL reduction of around 11% here, serves as an estimate for the fatigue loads.

When compared to the land-based turbine (comparing cases 1 and 3), their percentage reduction in loads are largely similar but the absolute rms RBM value for the floating turbine is significantly higher due to the presence of low frequency wave excitations. Also, the flap activity β is higher for the floating case. The PSD for the floating turbine with irregular wave (case 1) is shown in Figure 16 without frequency weighting. The reduction in loads is across most frequencies, including those emanating from the wave kinematics and 1P wind shear effects. Incremental static loads are also reduced that could potentially be used to maintain targeted rotor speeds. Separately, specific frequency bands could be targeted by filtering the measurements prior to the feedback to controller. The results for low-pass and band-pass 0.2 Hz and 0.4 Hz are included in Table 3 (case 2). In the case of low-pass where the low frequency range of platform-induced blade loads are considered with the per-revolution loads, the resulting rms, maximum and fatigue reductions are almost similar to the unfiltered case. However, their actuator duties in terms of β are reduced by more than four times. In the case of band-pass where the focus is on the per-revolution loads (1P or 2P) in Table 3, the load reductions are smaller despite having a larger actuator duty compared to the low-pass.

For all the load alleviation cases considered here for the floating turbine with active controls, there is a very slight increase in platform motions (surge, heave and pitch) that is below 1%. Further to that, the reduction in rms and maximum values of the tower top fore-aft deflection are relatively small. These tower loads are not acted on by the active mechanisms, which had been designed to target blade loads. Nonetheless, despite using a clamped rotating blade as the control model without knowledge of any rigid-body motions from the tower, the controller has performed well in alleviating blade loads that are induced by both the wind and wave excitations.

Table 3. Closed-loop alleviation of blade loads on the floating NREL 5MW wind turbine under the action of a full wind field in combination with irregular waves. Comparing (1) active flaps alone (2) active flaps with filtered measurements (3) land-based turbine. The abbreviations are: lp (low-pass), bp (band-pass).

Case	Config	OL rms RBM (Nm)	%rd rms RBM	%rd max RBM	%rd DEL RBM	max $\dot{\beta}$ (°/s)
1	Float	1.78×10^6	13.2	10.8	11.3	99.6
2	Float (lp 0.2 Hz)	1.78×10^6	13.1	10.3	10.7	18.9
2	Float (lp 0.4 Hz)	1.78×10^6	13.2	10.6	10.8	24.5
2	Float (bp 0.2 Hz)	1.78×10^6	9.48	7.58	7.99	29.2
2	Float (bp 0.4 Hz)	1.78×10^6	4.86	4.40	4.57	37.4
3	Land	1.26×10^6	13.0	12.6	13.2	37.0

5. Conclusions

Computational modelling for active load alleviation in floating wind turbines require a multi-disciplinary approach, encompassing the unsteady aerodynamics, hydrodynamics, controls and structural dynamics in a unified framework. The challenge is to seek an enhancement to existing modelling techniques for improved capture of turbine dynamics, yet retaining numerical efficiency and control design capabilities. The solution adopted here is to resolve the aerodynamics using the unsteady vortex-lattice method to administer better characterisations of three-dimensional flows across the rotor. With a prescribed helicoidal wake, a linear state-space description of the unsteady vortex-lattice method can be formulated to allow reduced-order modelling and controller design. The state-space UVLM is easily coupled to any linearised structural dynamics and here, a multi-body description composed of composite beams has been used to represent the rotor blades and tower/platform. This unified aeroelastic framework enables a comprehensive understanding of the aeroelastic behaviour in large wind turbines and to discover the potential advantages of *smart rotor* control concepts.

Two and three-dimensional aeroelastic descriptions have been presented. The two-dimensional model provides a simplified illustration of the proposed modelling technique in a control-oriented approach. It couples a vortex-particle description with a plunge-pitch aerofoil, representative of a section through the turbine blade. Using $\mathcal{H}_\infty$ controlled trailing-edge flaps, close to 60% reduction in root-mean-square of lift coefficient was obtained. The limits to which spatially coarse models can be used for controller design was also studied, with results showing that closed-loop performances began to deteriorate as model discretisation is relaxed by more than half. In the complete three-dimensional model of the floating turbine with representative wind and wave input disturbances, around 13% reduction in blade loads (root-bending moments) can be obtained through trailing-edge flaps. The control model was based on a clamped rotating blade and was capable of acting on both the wind and wave-induced loadings.

There is still much more to be done in the understanding of wind-wave interaction and mechanisms for load alleviation in floating wind turbines, and the presented aero-hydro-elastic framework with control capabilities aims to facilitate the investigations. In particular, it would be suitable for control to incorporate output disturbance models from wind/wave excitations and making use of preview information from sensors or the knowledge of periodicity for feedforward control.

Acknowledgements

The first author would like to thank the Singapore National Research Foundation, Energy Innovation Programme Office for their funding support.

References

- Andersen, P. B., Gaunaa, M., Bak, C., & Buhl, T. (2006). Load alleviation on wind turbine blades using variable airfoil geometry. In *Proceedings of the EWEC*. Athens, Greece.
- Barlas, T. K., van der Veen, G. J., & van Kuik, G. A. M. (2012). Model predictive control for wind turbines with distributed active flaps: Incorporating inflow signals and actuator constraints. *Wind Energy*, 15(5), 757-771. (<http://dx.doi.org/10.1002/we.503>)
- Barlas, T. K., & van Kuik, G. A. M. (2010). Review of state of the art in smart rotor control research for wind turbines. *Progress in Aerospace Sciences*, 46(1), 1-27. (<http://dx.doi.org/10.1016/j.paerosci.2009.08.002>)
- Barlas, T. K., van Wingerden, W., Hulskamp, A. W., van Kuik, G. A. M., & Bersee, H. E. N. (2013). Smart dynamic rotor control using active flaps on a small-scale wind turbine: Aeroelastic modeling and comparison with wind tunnel measurements. *Wind Energy*, 16(8), 1287-1301. (<http://dx.doi.org/10.1002/we.1560>)
- Berg, D. E., Wilson, D. G., Resor, B. R., Barone, M. F., & Berg, J. C. (2009). Active aerodynamic blade load control impacts on utility-scale wind turbines. In *Proceedings of the AWEA Windpower*. Chicago, IL, USA.
- Berg, J. C., Barone, M. F., & Yoder, N. C. (2014). *SMART wind turbine rotor: Data analysis and conclusions* (Tech. Rep.). SAND2014-0712.
- Berg, J. C., Resor, B. R., Paquette, J. A., & White, J. R. (2014). *SMART wind turbine rotor: Design and field test* (Tech. Rep.). SAND2014-0681.
- Bergami, L., & Gaunaa, M. (2010). Stability investigation of an airfoil section with active flap control. *Wind Energy*, 13(2-3), 151-166. (<http://dx.doi.org/10.1002/we.354>)
- Bergami, L., & Gaunaa, M. (2012). Analysis of aeroelastic loads and their contributions to fatigue damage. In *The science of making torque from wind*. Oldenburg, Germany.
- Bergami, L., Gaunaa, M., & Heinz, J. (2013). Indicial lift response function: An empirical relation for finite-thickness airfoils, and effects on aeroelastic simulations. *Wind Energy*, 16(5), 681-693. (<http://dx.doi.org/10.1002/we.1516>)
- Bergami, L., & Poulsen, N. K. (2014). A smart rotor configuration with linear quadratic control of adaptive trailing edge flaps for active load alleviation. *Wind Energy*. (<http://dx.doi.org/10.1002/we.1716>)
- Bernhard, A. P. F., & Chopra, I. (1998). Trailing edge flap activated by a piezo-induced bending-torsion coupled beam. *Journal of the American Helicopter Society*, 44(1), 3-15. (<http://dx.doi.org/10.4050/JAHS.44.3>)
- Bir, G. S., & Jonkman, J. M. (2007). *Aeroelastic instabilities of large offshore and onshore wind turbines* (Tech. Rep.). NREL/CP-500-41804.
- Bisplinghoff, R. L., Ashley, H., & Halfman, R. L. (1955). *Aeroelasticity*. Cambridge, MA, USA: Addison-Wesley Pub. Co.
- Bossanyi, E. A. (2003). *GH Bladed theory manual* (Tech. Rep.). 282/BR/009.
- Bottasso, C. L., Cacciola, S., & Riva, R. (2014). Floquet stability analysis of wind turbines using input-output models. In *Proceedings of AIAA Scitech - 32nd ASME Wind Energy Symposium*. National Harbor, MD, USA. (<http://dx.doi.org/10.2514/6.2014-0713>)
- Castaingnet, D., Barlas, T., Buhl, T., Poulsen, N. K., Wedel-Heinen, J. J., Olesen, N. A., ... Kim, T. (2014). Full-scale test of trailing edge flaps on a Vestas V27 wind turbine: Active load reduction and system identification. *Wind Energy*, 17(4), 549-564. (<http://dx.doi.org/10.1002/we.1589>)
- Chattot, J.-J. (2007). Helicoidal vortex model for wind turbine aeroelastic simulation. *Computers and Structures*, 85(11-14), 1072-1079. (<http://dx.doi.org/10.1016/j.compstruc.2006.11.013>)
- Cook, R., Palacios, R., & Goulart, P. J. (2013). Robust gust alleviation and stabilization of very flexible aircraft. *AIAA Journal*, 51(2). (<http://dx.doi.org/10.2514/1.J051697>)

- Cordle, A., & Jonkman, J. M. (2011). State of the art in floating wind turbine design tools. In *Proceedings of the 21st International Offshore and Polar Engineering Conference*. Maui, HI, USA.
- Corson, D. A., Griffith, T., Ashwill, T., & Shakib, F. (2012). Investigating aeroelastic performance of multi-megawatt wind turbine rotors using CFD. In *Proceedings of the 53rd AIAA Structures, Structural Dynamics, and Materials Conference*. Honolulu, HI, USA. (<http://dx.doi.org/10.2514/6.2012-1827>)
- Freebury, G., & Musial, W. (2000). Determining equivalent damage loading for full-scale wind turbine blade fatigue tests. In *Proceedings of the 19th ASME Wind Energy Symposium*. Reno, NV, USA.
- Géradin, M., & Cardona, A. (2001). *Flexible multibody dynamics: A finite element approach*. Chichester, England; New York, USA: John Wiley.
- Géradin, M., & Rixen, D. (1997). *Mechanical vibrations: Theory and application to structural dynamics* (2nd ed.). Chichester, England; New York, USA: John Wiley.
- Geyler, M., & Caselitz, P. (2008). Robust multivariable pitch control design for load reduction on large wind turbines. *Journal of Solar Energy Engineering*, 130(3), 031014. (<http://dx.doi.org/10.1115/1.2931510>)
- Green, M., & Limebeer, D. J. N. (1995). *Linear robust control*. Englewood Cliffs, NJ, USA: Prentice Hall.
- Hall, K. C. (1994). Eigenanalysis of unsteady flows about airfoils, cascades, and wings. *AIAA Journal*, 32(1). (<http://dx.doi.org/10.2514/3.12309>)
- Hansen, M., Sørensen, J. N., Voutsinas, S. G., Sørensen, N., & Madsen, H. A. (2006). State of the art in wind turbine aerodynamics and aeroelasticity. *Progress in Aerospace Sciences*, 42(4), 285-330. (<http://dx.doi.org/10.1016/j.paerosci.2006.10.002>)
- Hayman, G. J. (2012). *MLife theory manual for version 1.00* (Tech. Rep.). NREL. (<http://wind.nrel.gov/designcodes/postprocessors/MLife/>)
- Hendriks, H. B., & Bulder, B. H. (1995). *Fatigue equivalent load cycle method - a general method to compare the fatigue loading of different load spectrums* (Tech. Rep.). ECN-C-95-074.
- Hesse, H., & Palacios, R. (2012). Consistent structural linearisation in flexible-body dynamics with large rigid-body motion. *Computers and Structures*, 110-111(0), 1-14. (<http://dx.doi.org/10.1016/j.compstruc.2012.05.011>)
- Hesse, H., & Palacios, R. (2014). Reduced-order aeroelastic models for the dynamics of maneuvering flexible aircraft. *AIAA Journal*, 52(8). (<http://dx.doi.org/10.2514/1.J052684>)
- IEC 61400-1. (2006). *Wind turbines - part 1: Design requirements*. (3rd ed.; Tech. Rep.). International Electrotechnical Commission.
- IEC 61400-3. (2009). *Wind turbines - part 3: Design requirements for offshore wind turbines*. (1st ed.; Tech. Rep.). International Electrotechnical Commission.
- Jones, B. L., & Kerrigan, E. C. (2010). When is the discretization of a spatially distributed system good enough for control? *Automatica*, 46(9), 1462-1468. (<http://dx.doi.org/10.1016/j.automatica.2010.06.001>)
- Jonkman, B. J., & Kilcher, L. (2012). *TurbSim user's guide: Version 1.06.00* (Tech. Rep.). NREL. (<http://wind.nrel.gov/designcodes/preprocessors/turbsim/>)
- Jonkman, J. M. (2007). *Dynamic modeling and loads analysis of an offshore floating wind turbine* (Tech. Rep.). NREL/TP-500-41958.
- Jonkman, J. M. (2009). Dynamics of offshore floating wind turbines - model development and verification. *Wind Energy*, 12, 459-492. (<http://dx.doi.org/10.1002/we.347>)
- Jonkman, J. M. (2010). *Definition of the floating system for phase IV of OC3* (Tech. Rep.). NREL/TP-500-47535.
- Jonkman, J. M., & Buhl Jr., M. L. (2005). *FAST user's guide* (Tech. Rep.). NREL/EL-500-38230. (<http://wind.nrel.gov/designcodes/simulators/fast/>)
- Jonkman, J. M., Butterfield, S., Musial, W., & Scott, G. (2009). *Definition of a 5-MW wind turbine for offshore system development* (Tech. Rep.). NREL/TP-500-38060.
- Jonkman, J. M., & Musial, W. (2010). *Offshore code comparison collaboration (OC3) for IEA task 23 offshore wind technology and deployment* (Tech. Rep.). NREL/TP-5000-48191.
- Katz, J., & Plotkin, A. (2001). *Low speed aerodynamics* (2nd ed.). Cambridge, UK; New York, USA: Cambridge University Press.
- Lackner, M. A., & Rotea, M. A. (2011). Structural control of floating wind turbines. *Mechatronics*, 21, 704-719. (<http://dx.doi.org/10.1016/j.mechatronics.2010.11.007>)

- Lamb, H. (1997). *Hydrodynamics* (6th ed.). Cambridge, UK; New York, USA: Cambridge University Press.
- Lanzon, A., & Papageorgiou, G. (2009). Distance measures for uncertain linear systems: A general theory. *Automatic Control, IEEE Trans*, 54(7), 1532-1547. (<http://dx.doi.org/10.1109/TAC.2009.2022098>)
- Larsen, T. J., & Hansen, A. M. (2012). *How 2 HAWC2, the user's manual* (Tech. Rep.). Risø-R-1597(ver. 4-3)(EN).
- Larsen, T. J., Madsen, H., & Thomsen, K. (2005). Active load reduction using individual pitch, based on local blade flow measurements. *Wind Energy*, 8(1), 67-80. (<http://dx.doi.org/10.1002/we.141>)
- Leishman, J. G. (2002). Challenges in modeling the unsteady aerodynamics of wind turbines. In *Proceedings of the 21st ASME Wind Energy Symposium*. Reno, NV, USA. (<http://dx.doi.org/10.2514/6.2002-37>)
- Leithead, W. E., Neilson, V., Dominguez, S., & Dutka, A. (2009). A novel approach to structural load control using intelligent actuators. In *Proceedings of the 17th Mediterranean Conference on Control and Automation*. Makedonia Palace, Thessaloniki, Greece. (<http://dx.doi.org/10.1109/MED.2009.5164719>)
- Lu, Q., Bowyer, R., & Jones, B. L. (2014). Analysis and design of Coleman transform-based individual pitch controllers for wind turbine load reduction. *Wind Energy*. (<http://dx.doi.org/10.1002/we.1769>)
- Manwell, J. F., McGowan, J. G., & Rogers, A. L. (2009). *Wind energy explained: Theory, design and application* (2nd ed.). Chichester, UK: Wiley.
- Moe, G. (2007). *Linear wave theory* (Tech. Rep.). IEA task 23 offshore wind technology and deployment subtask 2 Code Comparison Collaborative OC3.
- Murua, J., Palacios, R., & Graham, J. M. R. (2012a). Applications of the unsteady vortex-lattice method in aircraft aeroelasticity and flight dynamics. *Progress in Aerospace Sciences*, 55, 46-72. (<http://dx.doi.org/10.1016/j.paerosci.2012.06.001>)
- Murua, J., Palacios, R., & Graham, J. M. R. (2012b). Assessment of wake-tail interference effects on the dynamics of flexible aircraft. *AIAA Journal*, 50(7), 1575-1585. (<http://dx.doi.org/10.2514/1.J051543>)
- Murua, J., Palacios, R., & Peiró, J. (2010). Camber effects in the dynamic aeroelasticity of compliant airfoils. *Journal of Fluids and Structures*, 26(4), 527-543. (<http://dx.doi.org/10.1016/j.jfluidstructs.2010.01.009>)
- Musial, W., & Butterfield, S. (2006). *Energy from offshore wind* (Tech. Rep.). NREL/CP-500-39450.
- Musial, W., Butterfield, S., & Boone, A. (2004). *Feasibility of floating platform systems for wind turbines* (Tech. Rep.). NREL/CP-500-34874.
- Ng, B. F., Hesse, H., Palacios, R., Graham, J. M. R., & Kerrigan, E. C. (2014a). Aeroservoelastic state-space vortex lattice modeling and load alleviation of wind turbine blades. *Wind Energy*. (<http://dx.doi.org/10.1002/we.1752>)
- Ng, B. F., Hesse, H., Palacios, R., Graham, J. M. R., & Kerrigan, E. C. (2014b). Model-based aeroservoelastic design and load alleviation of large wind turbines. In *Proceedings of AIAA Scitech - 32nd ASME Wind Energy Symposium*. National Harbor, MD, USA. (<http://dx.doi.org/10.2514/6.2014-0521>)
- Ng, B. F., Palacios, R., Graham, J. M. R., & Kerrigan, E. C. (2012). Robust control synthesis for gust load alleviation from large aeroelastic models with relaxation of spatial discretisation. In *Proceedings of the EWECC*. Copenhagen, Denmark.
- Ng, B. F., Palacios, R., Graham, J. M. R., Kerrigan, E. C., & Hesse, H. (2015). Aerodynamic load control in HAWT with combined aeroelastic tailoring and trailing-edge flaps. *Wind Energy*. (<http://dx.doi.org/10.1002/we.1830>)
- Øye, S. (1996). Flex4 simulation of wind turbine dynamics. In *Proceedings of the IEA 28th Meeting of Experts on State of the Art of Aeroelastic Codes for Wind Turbine Calculations* (p. 71-76). Lyngby, Denmark.
- Ozdemir, A. A., Seiler, P. J., & Balas, G. J. (2011). Performance of disturbance augmented control design in turbulent wind conditions. *Mechatronics*, 21(4), 634-644. (<http://dx.doi.org/10.1016/j.mechatronics.2011.01.008>)
- Passon, P., Kühn, M., Butterfield, S., Jonkman, J. M., Camp, T., & Larsen, T. J. (2007). OC3 - benchmark exercise of aero-elastic offshore wind turbine codes. *Journal of Physics Conference Series: The Science of Making Torque from Wind*, 75:012071. (<http://dx.doi.org/10.1088/1742-6596/75/1/012071>)
- Plumley, C., Leithead, W., Jamieson, P., Bossanyi, E., & Graham, J. M. R. (2014). Com-

- parison of individual pitch and smart rotor control strategies for load reduction. *Journal of Physics Conference Series: The Science of Making Torque from Wind*, 524:012054. (<http://dx.doi.org/10.1088/1742-6596/524/1/012054>)
- Sebastian, T., & Lackner, M. A. (2012). Development of a free vortex wake method code for offshore floating wind turbines. *Renewable Energy*, 46, 269-275. (<http://dx.doi.org/10.1016/j.renene.2012.03.033>)
- Sebastian, T., & Lackner, M. A. (2013). Characterization of the unsteady aerodynamics of offshore floating wind turbines. *Wind Energy*, 16, 339-352. (<http://dx.doi.org/10.1002/we.545>)
- Simms, D., Schreck, S., Hand, M., & Fingersh, L. (2001). *NREL unsteady aerodynamics experiment in the NASA-Ames wind tunnel: A comparison of predictions to measurements*. (Tech. Rep.). NREL/TP-500-29494.
- Simo, J. C., & Vu-Quoc, L. (1988). On the dynamics in space of rods undergoing large motions - a geometrically exact approach. *Computer Methods in Applied Mechanics and Engineering*, 66(2), 125-161. ([http://dx.doi.org/10.1016/0045-7825\(88\)90073-4](http://dx.doi.org/10.1016/0045-7825(88)90073-4))
- Simoes, F. J., & Graham, J. M. R. (1992). Application of a free vortex wake model to a horizontal axis wind turbine. *Wind Engineering and Industrial Aerodynamics*, 39(1-3), 129-138. ([http://dx.doi.org/10.1016/0167-6105\(92\)90539-M](http://dx.doi.org/10.1016/0167-6105(92)90539-M))
- Skogestad, S., & Postlethwaite, I. (2005). *Multivariable feedback control: Analysis and design* (2nd ed.). Chichester, England, UK; Hoboken, NJ, USA: John Wiley.
- Sørensen, J. N. (1983). *Three-level viscous-inviscid interaction technique for the prediction of separated flow past rotating wing* (Unpublished doctoral dissertation). Technical University of Denmark - AFM-83-03.
- Theodorsen, T., & Garrick, I. E. (1938). *Mechanism of flutter, a theoretical and experimental investigation of the flutter problem* (Tech. Rep.). NACA 685.
- Vinnicombe, G. (1996). The robustness of feedback systems with bounded complexity controllers. *Automatic Control, IEEE Transactions*, 41(6), 795-803. (<http://dx.doi.org/10.1109/9.506232>)
- Voutsinas, S. G. (2006). Vortex methods in aeronautics: How to make things work. *International Journal of Computational Fluid Dynamics*, 20(1), 3-18. (<http://dx.doi.org/10.1080/10618560600566059>)
- Welch, P. D. (1967). The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms. *IEEE Trans. Audio Electroacoustic*, AU-15(2). (<http://dx.doi.org/10.1109/TAU.1967.1161901>)
- Zeiler, T. A. (2000). Results of Theodorsen and Garrick revisited. *Journal of Aircraft*, 37(5), 918.
- Zhao, Y. H., & Hu, H. Y. (2004). Aeroelastic analysis of a non-linear airfoil based on unsteady vortex lattice model. *Journal of Sound and Vibration*, 276(3-5), 491-510. (<http://dx.doi.org/10.1016/j.jsv.2003.08.004>)
- Zhou, K., & Doyle, J. C. (1998). *Essentials of robust control*. Englewood Cliffs, NJ, USA: Prentice Hall.

Appendix A. Numerical Implementation of Two-dimensional Vortex-particle Aeroelastic Model

The time-domain vortex-particle method uses vortices as singularity solutions to the Laplace's equation. To solve for the singularities, the non-penetrating boundary condition is imposed on collocation points to obtain the relationship (Katz & Plotkin, 2001)

$$A_{ij}\Gamma_j + w_i = 0, \quad (\text{A1})$$

where A_{ij} is the influence coefficient that gives the induced velocity normal to aerofoil surface at collocation point i due to a vortex-particle of *unit strength* at point j . The second term, w_i , is the downwash at collocation point i due to motion of the aerofoil, flap deflection and incident gust.

The influence coefficient A is given by

$$A_{ij} = \mathbf{n}_i \cdot \begin{bmatrix} u_{ij} \\ w_{ij} \end{bmatrix}, \quad (\text{A2})$$

such that

$$\begin{bmatrix} u_{ij} \\ w_{ij} \end{bmatrix} = \frac{1}{2\pi r_i^2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_{c,i} - x_{v,j} \\ z_{c,i} - z_{v,j} \end{bmatrix},$$

where u_{ij} and w_{ij} are components of velocity in the x and z directions induced at collocation point i by a unit vortex at point j . The coordinates $(x_{c,i}, z_{c,i})$ and $(x_{v,j}, z_{v,j})$ refer to the locations of the collocation point i and vortex-particle j , respectively (see Figure 3), and r_i gives the distance between them. $\mathbf{n}_i$ is a unit vector normal to the surface at collocation point i .

To model the wake vortices, we impose Kelvin's condition dictating the conservation of circulation Γ in the fluid surrounding both the aerofoil and its wake. By equating the sum of all the circulation from the aerofoil and most recent wake vortex in the current time-step to the total circulation in previous time-step, we obtain

$$\Gamma_{w,1}^{n+1} + \sum_{j=1}^N \Gamma_{a,j}^{n+1} = \sum_{j=1}^N \Gamma_{a,j}^n, \quad (\text{A3})$$

where $\Gamma_{a,j}$ is the vortex circulation at point j on the aerofoil and $\Gamma_{w,1}$ is the circulation of the most recent wake vortex shed at aerofoil trailing-edge. n is the number of time-steps that also corresponds to the number of wake panels, and N is the number of panels on the aerofoil.

Once the vortex-particle at the trailing-edge is shed into the wake, it is convected downstream such that

$$\Gamma_{w,k}^{n+1} = \Gamma_{w,k-1}^n, \quad (\text{A4})$$

where $\Gamma_{w,k}$ is the circulation of the k^{th} wake vortex-particle. The position of the wake vortex-particle is determined by both the freestream velocity and also velocities induced by all other vortices. This free wake model introduces non-linearities as the locations of the wake vortex-particles become a function of time. To obtain linear models for control methods and for computation efficiency, a prescribed wake model that neglects roll-up effects due to self-induced velocities can be implemented with minimal impact on the accuracy of the unsteady responses (Murua, Palacios, & Graham, 2012b). In this study, a planar wake has been assumed.

Putting the above equations together, we obtain a state-space expression in terms of aerofoil vorticity, $\mathbf{\Gamma}_a \in \mathbb{R}^{n_a \times 1}$, and wake vorticity, $\mathbf{\Gamma}_w \in \mathbb{R}^{n_w \times 1}$ (Hall, 1994; Zhao & Hu, 2004), written as

$$\begin{bmatrix} \mathbf{A}_a & \mathbf{A}_w \\ \mathbf{A}_{wa} & \mathbf{A}_{ww} \end{bmatrix} \begin{Bmatrix} \mathbf{\Gamma}_a \\ \mathbf{\Gamma}_w \end{Bmatrix}^{n+1} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{wa} & \mathbf{B}_{ww} \end{bmatrix} \begin{Bmatrix} \mathbf{\Gamma}_a \\ \mathbf{\Gamma}_w \end{Bmatrix}^n + \mathbf{T}_q \mathbf{q}^{n+1} + \mathbf{W}_{\beta,\delta}^n, \quad (\text{A5})$$

where $\mathbf{q} = [h \quad \alpha \quad \dot{h} \quad \dot{\alpha}]^T$, and the total number of vortex-particles on the aerofoil and wake are denoted by n_a and n_w , respectively. Note that Eqs. (A5) and (3) are equivalent. The first row in the above equation imposes the Neumann boundary condition in Eq. (A1) with $\mathbf{A}_a \in \mathbb{R}^{n_a \times n_a}$ and $\mathbf{A}_w \in \mathbb{R}^{n_a \times n_w}$. The second row containing $\mathbf{A}_{wa} \in \mathbb{R}^{n_w \times n_a}$, $\mathbf{A}_{ww} \in \mathbb{R}^{n_w \times n_w}$, $\mathbf{B}_{wa} \in \mathbb{R}^{n_w \times n_a}$ and $\mathbf{B}_{ww} \in \mathbb{R}^{n_w \times n_w}$ imposes Kelvin's condition in Eq. (A3) to dictate the conservation of circulation in the fluid surrounding the aerofoil, its wake and the convection of wake downstream. They are defined

as $\mathbf{A}_{wa} = \mathbf{B}_{wa} = \begin{bmatrix} \mathbf{1}_{1 \times n_a} \\ \mathbf{0}_{(n_w-1) \times n_a} \end{bmatrix}$, $\mathbf{A}_{ww} = \mathbf{I}_{n_w \times n_w}$, and $\mathbf{B}_{ww} = \begin{bmatrix} \mathbf{0}_{1 \times (n_w-1)} & \mathbf{0}_{1 \times 1} \\ \mathbf{I}_{(n_w-1) \times (n_w-1)} & \begin{bmatrix} \mathbf{0}_{(n_w-2) \times 1} \\ r \end{bmatrix} \end{bmatrix}$,

where $\mathbf{I}$, $\mathbf{1}$ and $\mathbf{0}$ are the identity, ones and zeros matrices, respectively, and their sizes are defined in the accompanying subscript. The relaxation factor r is included to avoid sudden changes in the downwash from the finite length of the wake vortices.

The transformation $\mathbf{T}_q \in \mathbb{R}^{(n_a+n_w) \times 4}$ in Eq. (A5) is used to map the aerofoil structural dof as downwash (velocity) onto the collocation points. The last term $\mathbf{W}_{\beta,\delta} \in \mathbb{R}^{(n_a+n_w) \times 1}$ accounts for the downwash on aerofoil collocation points due to flap deflection and incident gust, expressed as

$$\mathbf{W}_{\beta,\delta}^n = \mathbf{W}_\beta \boldsymbol{\beta}^n + \mathbf{W}_\delta \boldsymbol{\delta}^n, \quad (\text{A6})$$

where $\boldsymbol{\beta} = [\beta \ \dot{\beta}]^\top$ and $\boldsymbol{\delta} = [\delta_1 \ \dots \ \delta_{n_a}]^\top$, such that β is the flap deflection angle and $\delta = \frac{w_g}{U_\infty}$ is the induced gust angle as shown in Figure 1. Note that $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ are written in the time-steps t_n for the final EoM to be in explicit form. The matrices $\mathbf{T}_q$, $\mathbf{W}_\beta \in \mathbb{R}^{(n_a+n_w) \times 2}$ and $\mathbf{W}_\delta \in \mathbb{R}^{(n_a+n_w) \times n_a}$ have their entries along the i^{th} row and j^{th} column expressed as

$$\mathbf{T}_q = \begin{cases} 0 & \text{if } i \leq n_a \text{ and } j = 1 \\ -n_{z,i} U_\infty & \text{if } i \leq n_a \text{ and } j = 2 \\ -n_{z,i} & \text{if } i \leq n_a \text{ and } j = 3, \\ -n_{z,i} x_{c,k}^{ea} & \text{if } i \leq n_a \text{ and } j = 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{W}_\beta = \begin{cases} 0 & \text{if } i < i_h \text{ and } j = 1 \\ -n_{z,i} U_\infty & \text{if } i_h \leq i \leq n_a \text{ and } j = 1 \\ 0 & \text{if } i < i_h \text{ and } j = 2 \\ -n_{z,i} x_{c,i}^h & \text{if } i_h \leq i \leq n_a \text{ and } j = 2 \\ 0 & \text{otherwise} \end{cases},$$

$$\mathbf{W}_\delta = \begin{cases} -n_{z,i} U_\infty & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}.$$

The normal to the aerofoil surface at collocation point i is given by $n_{z,i}$ and the distance between the i^{th} collocation point and the elastic axis by $x_{c,i}^{ea}$. The collocation points covering the trailing-edge flaps starts from the hinge i_h to trailing edge n_a , and $x_{c,i}^h$ is the distance from collocation point i in the flap to the hinge axis. Note that the plunge h has no bearing on the aerodynamic downwash.

In computing the pressures and loads on the aerofoil, we make use of the two-dimensional unsteady Bernoulli equation (Katz & Plotkin, 2001). In continuous time, the pressure difference Δp_k across the lower and upper surfaces of the j^{th} vortex-particle at distance x from the leading edge is

$$\Delta p_j(t) = \rho \left[U_\infty(t) \Gamma_a(x, t) + \frac{\partial}{\partial t} \int_0^x \Gamma_a(x, t) dx \right], \quad (\text{A7})$$

where ρ is density of air. Integrating Eq. (A7) over the entire aerofoil and writing in discrete-time

state-space non-dimensional form, we obtain for the lift

$$L^{n+1} = \begin{bmatrix} \rho U_\infty^{n+1} + \frac{\rho(c-x_{v,1})}{\Delta t} \\ \rho U_\infty^{n+1} + \frac{\rho(c-x_{v,2})}{\Delta t} \\ \vdots \\ \rho U_\infty^{n+1} + \frac{\rho(c-x_{v,n_a})}{\Delta t} \end{bmatrix}^\top \begin{Bmatrix} \Gamma_{a,1} \\ \Gamma_{a,2} \\ \vdots \\ \Gamma_{a,n_a} \end{Bmatrix}^{n+1} + \begin{bmatrix} \frac{\rho(-c+x_{v,1})}{\Delta t} \\ \frac{\rho(-c+x_{v,2})}{\Delta t} \\ \vdots \\ \frac{\rho(-c+x_{v,n_a})}{\Delta t} \end{bmatrix}^\top \begin{Bmatrix} \Gamma_{a,1} \\ \Gamma_{a,2} \\ \vdots \\ \Gamma_{a,n_a} \end{Bmatrix}^n. \quad (\text{A8})$$

A similar expression can be obtained for the moment about the elastic axis

$$M_\alpha^{n+1} = - \begin{bmatrix} \rho U_\infty^{n+1}(x_{v,1} - x_{ea}) + \frac{\rho}{\Delta t} \Pi_1 \\ \rho U_\infty^{n+1}(x_{v,2} - x_{ea}) + \frac{\rho}{\Delta t} \Pi_2 \\ \vdots \\ \rho U_\infty^{n+1}(x_{v,n_a} - x_{ea}) + \frac{\rho}{\Delta t} \Pi_{n_a} \end{bmatrix}^\top \begin{Bmatrix} \Gamma_{a,1} \\ \Gamma_{a,2} \\ \vdots \\ \Gamma_{a,n_a} \end{Bmatrix}^{n+1} + \begin{bmatrix} \frac{\rho}{\Delta t} \Pi_1 \\ \frac{\rho}{\Delta t} \Pi_2 \\ \vdots \\ \frac{\rho}{\Delta t} \Pi_{n_a} \end{bmatrix}^\top \begin{Bmatrix} \Gamma_{a,1} \\ \Gamma_{a,2} \\ \vdots \\ \Gamma_{a,n_a} \end{Bmatrix}^n, \quad (\text{A9})$$

where $\Pi_j = \left(\frac{c^2}{2} - c x_{ea}\right) - \left(\frac{x_{v,j}^2}{2} - x_{v,j} x_{ea}\right)$ and $j \in [1 \dots n_a]$. x_v and x_{ea} are the locations of the vortex-particle and elastic axis measured from the leading edge, respectively, and c is the chord. The lift and moment coefficients about the elastic axis can also be written symbolically, as

$$\mathbf{Q}_a^{n+1} = \mathbf{F}_2 \mathbf{\Gamma}^{n+1} + \mathbf{F}_1 \mathbf{\Gamma}^n, \quad (4)$$

where $\mathbf{Q}_a^{n+1} = \begin{Bmatrix} L^{n+1} \\ M_\alpha^{n+1} \end{Bmatrix}$ and $\mathbf{\Gamma} = \begin{Bmatrix} \mathbf{\Gamma}_a \\ \mathbf{\Gamma}_w \end{Bmatrix}$. The coefficients $\mathbf{F}_1 \in \mathbb{R}^{2 \times (n_a + n_w)}$ and $\mathbf{F}_2 \in \mathbb{R}^{2 \times (n_a + n_w)}$ are obtained from Eqs (A8) and (A9) with additional columns of zeros multiplying $\mathbf{\Gamma}_w$.

To arrive at the two-dimensional aeroelastic model of the 2-dof aerofoil, the aerodynamics definition in Eqs. (4) and (A5) are coupled to the structural dynamics in Eq. (1) using the fluid-structure coupling in Section 2.3.