Energy absorption characteristics of sandwich structures subjected to low-velocity impact

Foo, Choon Chiang

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**ENERGY ABSORPTION CHARACTERISTICS OF SANDWICH
STRUCTURES SUBJECTED TO LOW-VELOCITY IMPACT**

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SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

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Abstract

This dissertation presents experimental, numerical and analytical investigations of sandwich plates subjected to quasi-static loading and low-velocity impact. The objectives of this research are to predict the low-velocity impact response and damage in a sandwich structure, and to characterise the energy absorbed by the structure. Aluminium sandwich plates and composite sandwich plates made of Nomex honeycombs and carbon/epoxy skins were investigated under both static indentation and low-velocity impact loadings using a hemispherical indenter/impactor. Emphasis was placed on the damage characteristics and the energy absorption capabilities of these structures. Based on the least-squares method, a single equation that links absorbed energy with the impact energy and the damage initiation threshold energy was derived for composite sandwich plates. It was found that the proportion of impact energy absorbed by the composite plate was inversely related to the damage initiation energy, but directly related to the relative loss of the plate's transverse stiffness after damage. This energy equation is useful for further studies on damage resistance and tolerance.

A three-dimensional FE model was also developed to simulate the indentation and impact tests. In contrast to the equivalent continuum core normally used by other investigators, the cellular honeycomb core was discretely modelled with shell elements so that it was geometrically more accurate. A progressive damage model was also included to predict damage initiation and progression in the laminated skins. Comparison of numerical results with test results demonstrated the ability of the model to capture the impact characteristics. Core damage was identified to be one of the damage mechanisms at initial damage. Parametric studies also showed that denser cores resulted in greater peak loads and smaller damage profiles in the impacted structure. However, the energy absorbed during impact was independent of the core density.

Finally, an analytical model was proposed to predict the impact response of a sandwich structure beyond the onset of damage. Closed-form solutions were derived for three parameters that described the plate's structural behaviour, namely, the plate's elastic structural stiffness, the critical load at the onset of damage, and the reduced stiffness after damage. The critical load was found to be theoretically predictable by accounting for the elastic energies absorbed by the sandwich plate up to core failure. These parameters were then included in a modified energy-balance model coupled with the law of conservation of momentum to predict transient load and deflection histories for the plate subjected to impact. This impact model is an efficient design tool which can complement detailed FE simulations.

Chapter 1

Introduction

1.1 Background

Sandwich structures consist of two stiff and thin skins separated by a lightweight core (Figure 1.1). Typically, the skins are made of aluminium or fibre-reinforced composites, while the cores are foams or honeycombs, with a wide range of materials and properties for each type. The facesheets carry in-plane and bending forces, while the core keeps the facesheets apart and carries transverse loads. This results in an efficient lightweight structure with high specific bending strengths and stiffnesses. Due to their structural efficiency, sandwich structures are widely used in lightweight constructions especially in aerospace industries. In aircraft structures, sandwich materials are found in secondary structures such as spoilers, floor panels, nacelles, and fairings.

Figure 1.1: Typical sandwich construction, from [1]

In the service life of a sandwich component in aircrafts, impacts are expected to arise from a variety of causes. Typical in-service impacts include debris propelled at high velocities from the runway during aircraft takeoffs and landings, or even collisions by

birds. Others include tools dropping on the structure during the manufacturing process or during maintenance. In sandwich structures, impacts create internal damage that easily escapes visual detection. Reduction of structural stiffness and strength can occur, and subsequently, propagate under further loading [2]. This relatively poor resistance to localised impact has become a concern for both manufacturers and end-users who need to locate damages for repair. Moreover, widespread application of sandwich composites in aerospace industries, particularly in aircraft primary structures, has thus been inhibited due to the lack of understanding of low-velocity impact damage and its effect on structural performance. Due to these concerns, the behaviour of sandwich structures under impact has received increasing attention.

One common approach to analyse the effect of impact on sandwich structures is to separately address impact damage resistance and impact damage tolerance. Impact damage resistance deals with the response and damage caused by impact. Due to the complicated nature of impact damage, a general approach for predicting damage initiation and propagation is still absent. The analysis for laminated composite plates subjected to transverse loads has always been complicated by factors such as the variation of property through the thickness, geometric and material nonlinearity, and transverse shear effect. In sandwich structures, the core further compounds the analysis; relatively few analytical solutions have been proposed for sandwich structures because of the complex interaction between the composite facesheet and the core during deformation. Furthermore most existing impact models, such as the spring-mass and energy-balance models [2], assume elastic behaviour, and they cease to be valid after the onset of damage.

A great deal of research work to address impact damage resistance comprises experimental studies quantifying damage. However, due to the lack of sophisticated equipment to monitor real-time internal damage, such studies are mostly limited to damage characteristics at the final state and residual strength measurements. Although testing can yield data for a particular plate and load, it is not generally feasible to implement testing for a wide range of variables because material test programmes are very expensive for industry.

Driven by the need to reduce the reliance on testing, researchers and designers have turned their attention to the finite element method (FEM). FEM can be applied to plates and shells of various shapes, sizes and compositions, which are subjected to different loadings and supports. The versatility of the technique, combined with the substantial time and cost savings that can be achieved, makes FEM a cost-effective approach to design sandwich structures.

In order to simulate real-world behaviour, analysts who have previously adopted a mostly linear approach to simulation are developing models that incorporate non-linearities to account for realistic stresses and deformations. In honeycomb sandwich structures [3–5], the multi-cellular core is usually replaced with an equivalent continuum model and analysed in terms of its effective properties. A continuum model may seem a convenient way to represent the real core geometrically, but errors exist when it is used to model damage [6, 7]. One reason is that it may be difficult to simulate the exact damage progression due to the discontinuous surfaces of the honeycomb core in contact with the facesheets. This limitation can be overcome by modelling the honeycomb core explicitly using discrete shell elements to obtain more realistic distributions of stresses and strains [7].

1.2 Objectives

The main objectives of this research were to:

1. develop a three-dimensional FE model and an analytical model to predict the low-velocity impact response and damage of a composite sandwich plate.
2. characterise the energy absorbed by a composite sandwich plate subjected to low-velocity impact.

1.3 Scope

To justify the need for current research, a literature review was carried out. This review focused on the work on impact damage resistance and previous analytical methods used to predict deformation and damage in sandwich structures. A combination of experimental investigation, analytical development, and finite element modelling was then used to achieve the research objectives.

1.3.1 Experimental investigation

Quasi-static indentation and low-velocity impact tests were conducted on aluminium sandwich plates and composite sandwich plates which were made of Nomex honeycombs and carbon/epoxy skins. The purposes of the experimental investigation were three-fold: (1) to determine if the impact events were quasi-statically equivalent; (2) to establish a connection between the impact response and the energy absorbed by the sandwich specimens; and (3) to validate the finite element and analytical models.

1.3.2 Analytical modelling

An analytical model was proposed to extend the validity of the energy-balance model beyond the elastic regime. In this model, three parameters were derived: the elastic stiffness, the critical load at the onset of damage, and the stiffness after damage. The critical load at the onset of damage was derived by considering the elastic energy absorbed by the plate up to core failure. Closed-form analytical solutions were also provided for the elastic stiffness and the reduced stiffness after damage. The three parameters were subsequently included in a modified energy-balance model which was coupled with the law of conservation of momentum to predict the load and deflection histories of the sandwich plate subjected to impact.

1.3.3 Finite Element modelling

A FE sandwich model that comprised a cellular core was developed to analyse the response of both aluminium and composite sandwich plates subjected to indentation and impact. In the case of the composite sandwich plate, a progressive damage model was also included to predict matrix failure, delamination, and fibre failure for the laminated skins. Additionally, the FE model was used to characterise the impact damage resistance and energy absorption properties of the sandwich structures.

1.4 Outline

The overall work is organised in the subsequent chapters as follows. Relevant work in the literature is reviewed in Chapter 2. The experimental investigation is presented in Chapter 3, where the test specimens, experimental set-ups and test procedures are described. The analytical model to predict the low-velocity impact response of the sandwich plate is then detailed in Chapter 4, which is also where the elastic and reduced stiffnesses of the plate, as well as the critical load at damage initiation, are theoretically derived. Next, the FE sandwich model is described and several modelling considerations are addressed in Chapter 5. Results from tests, along with those from the analytical and FE models, are presented in Chapter 6, and the implications of these results are also discussed. Finally, in Chapter 7, conclusions are drawn from the current investigation and recommendations are proposed for further work.

Chapter 2

Literature Review

2.1 Introduction

Sandwich structures with laminated facings are widely used in many applications because of the well-known advantages of this type of construction, particularly their high specific bending stiffness. Despite this, the main weakness of such structures has always been the poor rigidity in the transverse direction. Consequently, impact is a key issue in the design of sandwich structures and may be the limiting design issue in many cases.

Efforts aimed at investigating the impact of sandwich structures can be broadly classified into three areas: impact dynamics, damage resistance and damage tolerance. Impact dynamics analysis is carried out to predict the structure's impact response using a mathematical model that accounts for the motions of the projectile and target, as well as the projectile-target interaction. Damage resistance is the measure of the ability of a structure to resist damage due to a foreign object impact, whereas damage tolerance is the ability of the structure to perform satisfactorily with the presence of impact damage. Most investigations can involve one or a combination of these three aspects of impact. The focus of this study is on impact dynamics and impact damage resistance.

This review begins with the work on the impact damage resistance of composite sandwich structures. The use of drop-weight impact tests and static indentation tests to simulate low-velocity impact damage are first elaborated. The types and sequence of damage which occur in impacted or indented sandwich panels, as well as the parameters affecting damage initiation and growth, are then described. Next, existing analytical

models that are used to predict the impact response and damage are reviewed. Because of the difficulties encountered in modelling deformation and damage analytically, impact analysis of sandwich structures has also relied heavily on finite element analysis. There is a growing interest in progressive damage modelling for sandwich structures, and several issues relevant to this topic are examined. In particular, the choice of failure criteria and material property degradation models are discussed, along with various modelling approaches for honeycomb sandwich structures. Finally, to conclude, several needs that serve as motivation for the current study are identified.

The objective of this chapter is to summarise the essential features of other work which relates to this study. Due to the vast amount of publications available in the literature, it is necessary to limit the scope of this review to the articles which have provided direction to this research and support a particular claim of this dissertation. Although the emphasis here is on composite honeycomb sandwich structures subjected to low-velocity impacts, previous work on monolithic laminates is also included to supplement and substantiate various ideas and theories.

2.2 Impact Damage Resistance

2.2.1 Impact testing

Research work to address impact damage resistance of composite sandwich structures is dominated by experimental studies quantifying damage [2, 8–16]. In such studies, impact tests are first conducted based on a set of parameters, and various damage assessment methods are used to identify the damage mechanisms sustained after the impact event. Subsequently a relationship is established between the critical parameters and the damage mechanisms. For instance, one conventional way often used to identify damage initiation is to establish the relationship between impact energy and damage area after a large number of coupon impact tests [2, 17, 18]. In most conventional low-velocity impact tests, a projectile of large mass drops onto the target with low velocity, a scenario which

is typical of tools dropping on aircraft structures during maintenance or manufacturing. Instrumented drop-weight tests are very popular because they are simple to perform and easily repeatable [19]. The principal measurement in these tests is the load-time history, which can be integrated to produce the displacement and absorbed energy of the impacted plate. Figure 2.1 gives an example of the load and absorbed energy histories measured in such tests.

Low-velocity impacts often cause damage in sandwich structures that is difficult to detect by a simple visual inspection: damage in opaque laminates such as carbon/epoxy composites are almost indiscernible and indentations can be inconspicuous as compared to the entire sandwich panel. Thus, various experimental techniques have been developed to assess and quantify the impact damage. As described in [2], some of these techniques, such as photomicroscopy, are destructive, while others like ultrasonic imaging, X-ray scanning, and thermography are non-destructive. However both types of methods are laborious and time-intensive, and they can only determine the state of the damage at the end of the impact event. Moreover in some cases, the effectiveness of these methods has been questioned [2, 18, 20, 21].

Alternatively, investigators have used the recorded load history curves to characterise elastic behaviour, failure initiation and failure propagation for the structure in terms of impact force and energy [9, 16, 18, 19, 22–25]. When the impact energy of the projectile is high enough to exceed the damage threshold of the plate, the load-time history increases up to a critical value before a sudden drop occurs at the onset of damage (Fig. 2.1). For rate-insensitive materials, previous studies have shown that this critical load is independent of impact energy [18, 25–28]. Consequently, the damage initiation threshold load of a plate can be easily revealed using its impact response derived from a single test. This approach of determining damage initiation eliminates the need to examine impacted specimens and is significantly faster and cheaper [18].

In the literature, there is evidence to suggest that low-velocity impact tests strongly correlate with static indentation tests for carbon-epoxy laminates [22, 26, 29–31] and

Figure 2.1: Impact load and absorbed energy histories of 10 mm thick glass-fibre reinforced laminates, from [18].

Nomex honeycomb-cored sandwich panels with thin carbon-epoxy facesheets [9, 13, 15, 24], in terms of load-displacement responses and damage characteristics. On the other hand, several studies have also indicated that the quasi-static equivalence for low-velocity tests is limited; for instance, slight differences in values of damage initiation loads have been attributed to inertial resistance for thick facesheets [9], harmonic oscillations due to the dynamic event [26], and material strain-rate effects [32]. Given that there are many variables involved in all these studies, such as the boundary conditions and specimen size, a conclusive generalisation cannot be drawn [29]. Nevertheless, these findings seem to suggest that, at least within a small margin, the static indentation and low-velocity impact test data may be superimposed over one another.

Because of this strong correlation between the quasi-static and dynamic test data, many researchers have advocated the use of a quasi-static test to model low-velocity impact events [8,24,29,33,34]. The benefits of conducting a preliminary static indentation test prior to an impact test are well-documented in [19,24,29], amongst which is the ability to detect damage initiation and propagation more easily in static tests due to the absence of harmonic oscillations. Furthermore, the quasi-static load-displacement curves can be easily translated into load-energy plots [24], which would be useful to predict the impact

load generated at various impact energies.

2.2.2 Failure modes in damage

Extensive experimental studies have been carried out to investigate the process of damage initiation and propagation [9, 10, 13, 24, 34–36]. For low-velocity impact or indentation loading on sandwich structures which does not result in perforation, the damage is typically confined to the top facing, the core-facing interface and the core, with little damage to the bottom facing. The damage in the impacted facesheet is similar to that observed in monolithic composites, although it is more localised in the contact area underneath the projectile due to the core [37]. It is generally accepted that the primary damage in laminated skins comprise of matrix cracking, delamination, and fibre breakage [2, 14, 38]. Due to the low fracture toughness of the matrices used in modern composite materials, matrix cracks initiate first under tensile stress due to stress concentrations at the fibre-matrix interface [29, 32, 39] at relatively low energy levels. For thick laminates, cracks appear in the first layer due to the high localised contact stresses, whereas for thin laminates, bending stresses in the lowest layer initiate the cracks (Fig. 2.2). As the energy increases, these cracks propagate until they reach the interface of a neighbouring ply with a different fiber orientation. Subsequently, matrix cracks coalesce at these interfaces with high interlaminar stresses, and delamination initiates. As the impact velocity or indentation force increases even further, fiber damage starts to occur; a permanent dent or crack visible on the surface of the panel generally indicates significant facesheet damage [15, 40].

Core materials are expected to substantially affect the damage initiation characteristics of sandwich panels because they generally have lower mechanical properties than skins due to their lower density [13]. In honeycomb cores, the first sign of core damage includes buckling and crushing underneath the projectile [2, 15, 41]. At higher impact energies, cracks may start to develop in brittle honeycomb cores, such as phenolic glass, whereas in more ductile honeycombs like Nomex and aluminium, cell walls may split along the

Figure 2.2: Damage patterns in (a) thick and (b) thin composite laminates, from [2].

thickness direction [17]. In some instances, debonding at the core-facing interface has been documented for sandwich panels with aluminium honeycombs [41].

Figure 2.3: Debonding at cell-wall interfaces, from [42].

Core crushing is a complicated mechanical phenomenon characterised by the appearance of various folds and failures in the cellular structure. Extensive experimental studies have been carried out to investigate the behaviour and deformation mechanisms of honeycombs under quasi-static compression [2,42–46] and dynamic loading conditions [45,47]. Such studies mostly centre on the uniaxial compression behaviour of aluminium and Nomex honeycombs, which are frequently used as cores in sandwich structures. More recently, the crushing of aluminium honeycombs has been studied extensively by Mohr

and Doyoyo [48–53]. They [49, 53] found that elastic buckling occurs before the initial peak load, and likened it to the plastic collapse model of thin plates by von Karman et al. (cited in [49]). According to them, elastic buckling changes the homogenous membrane stress state in an initially flat cell wall, resulting in stress concentrations along the wall boundaries. Subsequently, plastic collapse occurs near the loaded edge of the honeycombs due to yielding along these boundaries. On the other hand, Nomex honeycombs appear to have a slightly complicated crushing behaviour. Although the folding phenomenon seems similar to that observed in aluminium honeycombs, the fold angles are sharper due to the different plasticity of the materials [44]. Cracking of the resin layers on the cell walls, localised tearing at the vertical edges of these walls, as well as debonding at the cell-wall interfaces (Fig. 2.3) have also been reported [42, 44].

2.2.3 Parameters affecting impact damage

Knowledge garnered from the vast amount of experimental studies has led to an understanding of which parameters affect the initiation and propagation of impact damage. In general, parameters that affect the overall stiffness of the structure have a significant effect on the impact response and damage resistance of the structure [2, 14]. The target stiffness affects the magnitude of the maximum impact load, which in turn influences the nature and extent of damage induced during impact.

These governing parameters may include the material properties of the skins and core, the structural configuration, and the projectile characteristics [2]. In composite sandwich structures, the material properties of the fibres and matrix in the facesheet have a significant effect on impact resistance. For instance, using a high failure strain of the fibres as well as toughened matrices improves the impact resistance of monolithic laminates [2, 14]. Core properties also influence the impact damage resistance of sandwich panels. Denser cores were found to retain greater stiffness after damage initiation [24], increase the damage initiation load [13, 54] and the peak impact load [36], as well as reduce the amount of damage in the top skin and core [36]. Likewise, a thicker skin

also increases the damage initiation threshold load in sandwich composites [9, 13, 24, 54]. Similarly, the size and the shape of the projectile are important parameters. Under indentation, the stiffness increases with indenter size [34] (Fig. 2.4), while a flat-end indenter results in higher stiffness and threshold loads, as compared to a hemispherical indenter [13, 54].

Figure 2.4: Indentation responses of sandwich panels under indentors of different diameters, from [34].

The typical parametric studies provide useful insights which could be used to develop improved materials systems and to design impact-resistant structures. However, most experimental studies usually considered a single configuration and examined the effect of some arbitrary parameter [2]. Several investigators have pointed out that there is a lack of an universal governing parameter for impact damage [2, 55, 56]. Christoforou [55] wrote that this results in unnecessary and cost-ineffective test programmes that produce repetitive and, in some instances, seemingly contradictory data. In addition, such parametric investigations usually rely on a single parameter to assess the relative performance of test

specimens. Damage maps that plot damage area [2,17,18,56] or even dent depths [2,6,57] against impact energy are commonly presented. Likewise, peak impact force is widely accepted as a damage metric for low-velocity impacts, given its linear relationship with damage beyond a threshold value [18,23,28,35,55,58,59], as shown in Fig. 2.5.

Figure 2.5: Damage size plotted against peak impact force for two laminates of different thicknesses [59].

Figure 2.6: Delamination areas as a function of two mass/velocity combinations for identical impact energies on thick composite laminates, from [35].

However, several concerns have been raised regarding the overall applicability of such

Figure 2.7: Peak impact load plotted against impact energy for carbon/epoxy laminates, from [22]. Beyond the elastic regime, reported experimental values were lower than theoretical ones which were predicted using a simple spring-mass (SM) model prediction.

damage maps plotted against either impact energy or peak force. Previous studies have indicated that damage in composite laminates impacted by a large mass projectile depends on the transverse stiffness of the plate [28, 59], which means that the damage-energy maps are only valid for the particular configurations reported. On the other hand, damage inflicted by a smaller mass impactor may vary with mass and velocity even at the same impact energy [35, 59]; an impactor with smaller mass may cause more severe damage at the same impact energy (Fig. 2.6). In fact Zhou [55] argued that the only advantage of using impact energy as a parameter is its convenience of being readily defined at the start of the impact test. Furthermore, because the value of the energy threshold for damage initiation is usually fairly small compared to the impact energy, data scatter which is inevitable with composite materials makes it difficult to locate the damage threshold using impact energy as a parameter [18, 59]. Similarly, some investigators have observed that the peak load remained relatively constant despite the increase in damage size for composite

laminates [26, 29–31] (Fig. 2.7). In that case, the peak load cannot be used independently to assess the impact performance of a structure.

Given that there are disadvantages in using either impact energy or peak force on its own to correlate impact response with damage, investigators have turned their attention to the use of other parameters or a combination of parameters. For instance, normalised threshold loads [23, 25, 55] or normalised impact durations [31] have been proposed to predict residual properties of damaged structures. Recently, the case of using absorbed energy plots to assess damage [16, 31, 60, 61] has also been made, which is based on the fact that absorbed energy directly relates to the amount of damage sustained by the specimen [21]. All these emerging studies suggest that there is a desire to establish a better, or more efficient, method to interpret test data which could be used to assess the relative impact performance of composite structures.

2.3 Analytical Modelling of Impact Response and Damage

Through the vast amount of experimental data and observations collected over the years, the basic damage mechanisms of low-velocity impact damage appears to be well understood. However, relatively fewer analytical solutions have been proposed for sandwich structures because of the complex interaction between the composite facesheet and core during deformation and failure. To simplify the problem, the impact response is usually decoupled into local and global responses [32, 62–64]. In this context, the global response refers to the dynamic structural response of the plate, whereas the local response refers to the indentation caused by the impactor. Subsequently each response is analysed separately, while ignoring the interaction between the two.

2.3.1 Impact models

According to Abrate [65], models to obtain the global response of the plate can be classified into three main types: complete models, energy-balance models, and spring-mass models. In a complete model, the dynamic response of the structure is modelled completely using an appropriate structural theory; several conventional beam, plate, and shell theories used to analyse composite structures are briefly presented in [2]. Although complete models may be suitable for a wider range of plates, they are onerous to solve when large deflections, transverse shear effects, and more complicated architectures have to be considered [56]. Hence such complex models are less appealing to designers.

Besides complete models, the spring-mass models [32, 63, 66–69] and energy-balance models [8, 69, 70] are two other mathematical models used extensively to analyse impact dynamics. In the spring-mass model (Fig. 2.8), a combination of bending, shear, membrane and contact springs may be used to represent the effective structural stiffness of the system. The elastic response may then be solved from the dynamics equations of the model. Under elastic conditions, the load history is a bell-shaped curve where the peak load is calculated by equating the impact energy and the strain energy of the spring system [26].

Figure 2.8: A spring-mass model, from [2].

On the other hand, the energy-balance model assumes quasi-static behaviour and uses the principle of conservation of total energy in the impactor-plate system. Consequently, the kinetic energy of the impactor is equated to the sum of the energies due to contact, bending, shear, and membrane deformations. The energy-balance model assumes that the projectile becomes stationary when the structure reaches its maximum deflection, and the initial kinetic energy is used to deform the structure. One benefit of the model is that energy partitioning is possible; deformation energies can be quantified and identified separately [8, 71]. However, unlike the spring-mass model, this model only yields the maximum impact force but not the load-time history [2, 8].

Because of their simplicity and efficiency, the spring-mass and energy-balance models have been popular with many investigators and they have been used with some success to predict the impact force for composite laminates [58, 59, 69], aluminium sandwich plates [8], as well as composite sandwich structures [66, 70, 72]. However, one serious drawback of these elastic models is that they cease to be valid after the onset of damage and thus they are unable to model damage initiation and propagation. Feraboli [22, 26] published load-energy plots for carbon-epoxy laminates subjected to low-velocity impacts, and used these plots to show that the spring-mass model overestimates the peak impact load for the plates after the onset of damage (Figure 2.7). Similarly, the energy-balance model is inaccurate when damage initiates at higher impact energies [8, 71]. Modified spring-mass models have been proposed to account for damage [22, 66, 68], but most are heavily based on empirical correlation [22, 66]; hence they introduce additional unknowns that have to be determined experimentally.

2.3.2 Contact laws

In the local analysis, most analytical methods attempt to solve for the load-indentation response and to predict damage initiation. Because the local indentation is often of the same order or greater than the overall displacement of the plate, the inclusion of the local indentation in the impact model becomes important [2, 62]. The load is related to the

indentation by a simple relationship, which is known as the contact law. The Hertzian contact law was first used by Timoshenko in 1913 to study the impact of a beam by a steel sphere (cited in [2]), which is expressed as

$$P = k\alpha^n \quad (2.1)$$

where P is the contact force, α is the indentation, k is the contact stiffness and n is a constant ($\frac{3}{2}$ in this case). Since then, this approach has been used extensively for the impact analysis of composite materials [2, 69, 73], with the contact stiffness, k , defined as

$$k = \frac{4}{3} \frac{R^{1/2}}{\frac{1-\nu_r^2}{E_r} + \frac{1}{E_P}} \quad (2.2)$$

where R is the radius of the impactor, ν_r and E_r are the Poisson's ratio and the Young's modulus of the impactor respectively, and E_P is the Young's modulus of the laminated plate in the thickness direction. Here, the material is assumed to remain linearly elastic throughout the impact, and material damage sustained by the plate during impact is ignored.

However the Hertzian law cannot be used for sandwich structures because the indentation of sandwich structures is largely influenced by the behaviour of the core [2, 63, 74]. For most practical cases of sandwich structures, the core is much softer than the facesheets in the out-of-plane direction, and consequently, core deformation becomes dominant. Moreover, the Hertzian law does not account for the thickness of the core relative to those of the facesheets, the difference in moduli between the facesheets and the core, as well as the bonding between the facesheet and the core [74].

Consequently, the formulation of an accurate contact law for sandwich structures has received a good deal of attention. Although the contact law can be determined experimentally [34, 75], this method is not ideal because the parameters are material-dependent and a new indentation test is required for each plate configuration. For instance,

Figure 2.9: Three possible regimes of modelling local indentation response for sandwich plate [32]: (a) plate resting on elastic foundation for very small indentation prior to core crushing; (b) plate on rigid-plastic foundation for larger indentation which is smaller than the skin thickness; (c) membrane on rigid-plastic foundation for indentation larger than the skin thickness.

Hazizan and Cantwell [8] conducted static indentation tests on aluminium sandwich beams and fitted the load-indentation data to the contact law, in Eq. 2.1. The average value of n was found to be approximately 1.2 (Fig. 2.10). On the other hand, Lee et al. (cited in [2]) reported that the value of n for a foam-core sandwich plate with graphite-epoxy facings was 0.8.

Therefore, analytical methods have been proposed to derive the contact law using the elasticity theory [74, 76]. Other analytical methods to determine the local indentation response include modelling the top facesheet resting on a foundation (Fig. 2.9): the foundation is assumed to be either elastic for small indentation prior to core failure [2, 77–80], or rigid-plastic with membrane theory and core yielding further incorporated for large deformation [32, 70, 81, 82]. For the elastic foundation approach, some of these analyses are based on the classical Winkler foundation model [2, 77, 83–85], where the core is modelled as a continuously distributed set of independent springs, while others apply a two-parameter foundation model to account for the shearing interaction between the loaded facesheet and the core [78–80]. Abrate [2] suggests that the initial local stiffness

Figure 2.10: The variation of parameter n with crosshead displacement rate for aluminium sandwich beams under indentation, from [8].

for sandwich plates with quasi-isotropic laminated facesheets can be estimated as

$$K_{loc} = 8 \sqrt{Dk_c} \quad (2.3)$$

where D is the bending rigidity of the laminate and k_c is the elastic modulus of the foundation. This result is derived from considering an infinite isotropic plate resting on an elastic foundation subjected to a concentrated load [86], and ignores the interaction between the local bending and the plate boundary.

2.3.3 Damage prediction

Apart from the load-indentation response, several researchers have attempted to predict the damage initiation threshold load, which is indicated by a sudden drop in the load-history plots with a subsequent reduction in the plate's transverse stiffness. In composite laminates, the initial damage has been attributed to the onset of large unstable delamination growth [22, 26, 28, 59, 68, 87, 88] or the initiation of internal shear matrix cracks [89]; thus critical loads for these failure modes had been derived accordingly.

On the other hand, it appears that delamination and matrix cracking do not have any significant effect on the stiffnesses for composite sandwich structures [28, 90]. Instead, numerous experimental studies have associated core damage with the damage initiation threshold load for honeycomb sandwich plates [9, 13, 15, 24, 54, 90].

Figure 2.11: Free-body diagram at instant of top skin shear failure under impact by cylindrical projectile, from [91].

To model core damage under local indentation, Olsson [62] proposed an indentation model that consisted of an elastic facesheet resting on an elastic foundation. By matching the stress in the core with its compressive yield strength and assuming a foundation stiffness based on isotropic cores, he derived the critical load for core crushing as

$$\begin{aligned}
 P_{cr} &= 8\sigma_u \sqrt{D_f h_c / (1.38 E_{3c})} & \text{for } h_c \leq h_c^* \\
 P_{cr} &= 3\sqrt{3}\sigma_u \left(2D_f / E_{3c}\right)^{2/3} & \text{for } h_c > h_c^* \\
 \text{where } h_c^* &= \frac{32}{27} \left(\frac{16D_f}{E_{3c}}\right)^{1/3}
 \end{aligned} \tag{2.4}$$

Figure 2.12: Core shear failure, from [91].

where σ_u is the compressive strength of the core, D_f is the effective bending stiffness of the top skin, h_c is the core thickness, and E_{3c} is the out-of-plane stiffness for the core. Although impressive, Eq. 2.4 does not account for the influence of the indenter, which has been shown to affect the response of sandwich structures under indentation [34,54,92,93]. Core yielding for foam-cored sandwich structures has also been considered by others [83, 84, 94]. The common feature of these approaches is that they consider the honeycomb as a homogeneous material. Others have also employed static contact laws to model damage in honeycomb cores by representing the core either as a continuum model [75] or as a grid of nonlinear springs [44, 95], although no explicit expression of the critical load for core damage was given. On the other hand, critical loads for other failure modes such as shear (Fig. 2.11) and tensile fracture of top facesheet, core shear failure (Fig. 2.12), and tensile failure of the bottom facesheet had been considered and derived separately by Fatt and her co-workers [81, 91]. Similarly, they attempted to predict the failure load for facesheet cracking using the Maximum Stress and Tsai-Hill criteria [81], but the predicted failure loads were underestimated.

2.4 Progressive Damage Analysis

Most analytical methods do not account for the fact that the damage is a progressive phenomenon. Because the mechanisms for damage propagation and accumulation are so complicated, analytical models are often impractical and, maybe, infeasible for progressive damage modelling. Instead, the finite element method is probably the most suitable tool.

Progressive damage modelling for composites has been the focus of extensive research in recent years [96]. Although damage may initiate in the structure in the early stages of loading, the ultimate failure of a composite structure rarely occurs at the load corresponding to the initial damage. Instead, the structure would only start to lose its structural integrity as damage accumulates and propagates to large and critical regions of the structure under increasing load. A typical progressive damage model comprises the following steps [96–100]: stress analysis, failure analysis and material property degradation. Stress analysis is first performed to determine the three-dimensional (3D) stress state in the lamina. Based on the material strengths of the lamina, failure criteria are then used to detect failure initiation in the lamina. Subsequently upon failure detection, material degradation models are applied to propagate the failure and establish new degraded material properties for the failed lamina.

2.4.1 Failure criteria

Initial failure in the lamina of the composite can be predicted by applying an appropriate failure criterion. The purpose of a failure criterion is to determine the strength and the mode of failure of a lamina in a state of combined stress. An extensive amount of work has been expended over the past few decades in searching for the elusive “best” failure theory, as indicated by the wide variety of empirical approaches proposed [101, 102]. Although significant progress has been made, the failure analysis of composite laminates appears far from being mature, as evident from the World-Wide Failure Exercise conducted in 1998, whereby several of the leading failure theories were compared against experimental

tests [102]. There seems no criterion that is universally accepted as adequate under general loading conditions.

Lamina failure criteria are often classified into three groups [103, 104]: the limit criteria, the interactive criteria, and the mode-based criteria. The limit criteria, such as the maximum stress criterion, may be simple to apply but they do not consider the interaction among the stresses. The interactive criteria, which assume that all stress components simultaneously contribute to the failure of the composite, are usually expressed in the form of a single equation and give a global indication of failure. Tsai-Hill and Tsai-Wu failure criteria, which can be found in most textbooks on composite materials, belong to this category. Although the simplicity in using a single equation to predict failure in a lamina is attractive, all the failure mechanisms must be included simultaneously for any loading [103]. More importantly, these interactive criteria are not useful for modelling damage propagation because they do not reveal the failure mode. On the other hand, the mode-based failure criteria distinguish individual failure modes in the lamina.

A seminal work for mode-based failure criteria was first proposed by Hashin and Rotem [105] for unidirectional fibre composites, which was later modified by Hashin [106]. Hashin considered two failure modes that are influenced by the fibre and the matrix separately, and developed a failure criterion for each mode. Furthermore, he recognised that a composite typically has different ultimate strengths in tension and compression, and thus the fibre and matrix failure criteria have tensile and compressive sub-forms. The significance of Hashin's works [105, 106] is to introduce an approach to define failure criteria for composites based on failure modes, which consequently led to the development of several failure criteria of similar forms by many other researchers such as Yamada and Sun [107], Chang and his co-workers [108, 109], and Christensen [110].

Because each mode-based failure criteria does not consider all failure modes that can possibly occur in fibrous composites, the accuracy of the failure criterion depends on whether the assumed failure mechanism actually happens for a particular loading. Despite

that limitation, mode-based failure criteria have been used extensively [3, 40, 111–114] to predict impact damage, notably with some success, due to their relative ease of implementation in FE codes. Moreover, as compared to the limit and interactive failure criteria, they are more useful in a progressive failure analysis because they identify the various failure modes and the order in which they occur so that subsequent material degradation modelling can follow.

2.4.2 Failure models

A common approach to model damage in composites is the material property degradation method. Essentially, a set of material degradation rules is used to modify the material properties of the failed element once failure is predicted in a lamina using a failure criterion. By doing so, it is assumed that the damage within an element has an effect on the elastic properties of that element only, and that the damaged material can be substituted with an equivalent material that has degraded properties. The main purpose of these degradation rules is to inhibit the ply from sustaining further load, depending on the failure modes. The severity of damage may be characterised by damage evolution laws which are typically determined through continuum damage mechanics [109, 115, 116]. However it is not easy to determine the degraded properties of the lamina, which may be complicated by factors such as the crack density in the laminate [109, 115]. Thus, various degradation rules that are mainly empirical have been proposed.

Most of the material degradation models are based on the stiffness reduction approach, where the stiffnesses of the lamina are either reduced instantly or gradually to zero. Gradual unloading models include the constant degradation method [97], the exponential degradation method [117], and the Weibull distribution [108]. For the instantaneous unloading case, the constitutive properties of failed elements are reduced to zero or a small fraction of the original value upon failure detection in order to circumvent numerical ill-conditioning. This small fraction of the original value is often arbitrary; in terms of percentage, it can be as high as 50% for carbon reinforced plastics [118], or 0.1% as

suggested in [96]. In addition, different degradation factors can also be applied separately for each failure mode in order to represent the material more realistically [98, 119].

Although models to predict matrix and fibre failure are well developed, modelling methods for delamination are more complicated and are still an active research problem [96, 120, 121]. One approach to model delamination growth is to use fracture mechanics but such methods require the existence of initial cracks in the FE model [96, 122, 123]. Unlike in isotropic solids, it is usually not possible to clearly define and identify cracks in composites [124]. Another alternative is to include special interface elements between plies where delaminations are likely to occur [38, 123, 125–127]. This approach predicts delamination initiation using a stress-based initiation criteria, such as the quadratic interaction of the interlaminar stresses [128], and then applies the fracture mechanics method to simulate crack propagation. However, a very fine mesh at the crack tip is often required to overcome convergence problems due to numerical instability caused by the softening material model associated with the energy release [96, 123]. Consequently, the mesh requirements for accurate delamination modelling may be prohibitive computationally. On the other hand, several researchers have also used stress-based methods to predict both delamination initiation and growth due to their relative ease of implementation in FE codes [28, 100, 112, 117, 129]. In some instances, excellent agreement between experimental and predicted results has been reported even when delamination is not modelled, which suggests that delamination has minor effect on the plate's stiffness and impact response [40, 97, 130].

2.4.3 General Finite Element modelling of sandwich structures

Although a fair amount of success has been achieved in predicting low-velocity impact damage in carbon monolithic panels [28, 39, 117, 129, 131], less work has been carried out on composite sandwich panels consisting of composite facesheets supporting Nomex honeycomb cores. In FE analysis, computational expenses for honeycomb sandwich models increase rapidly as the number of cells in the core increases. Therefore, in order

to attain efficiency, the honeycomb core is usually meshed with 3D solid elements to represent an equivalent continuum model [3–5, 7, 132] (Fig. 2.13), while 2D plate or shell finite elements are used for the skins [3, 7]. However, since the contact load distribution in the laminate is inherently a 3D problem, the 2D elements may prove inadequate when transverse stresses are required for failure analysis. By considering the honeycomb as a homogeneous material, the response is then described in terms of macroscopic stresses and strains. Instead of considering the real cellular structure, the sandwich core is analysed in terms of its effective properties, which have to be determined by mechanical testing or analytical approximation. Various analytical techniques have been proposed to predict the effective continuum properties of the core in terms of its geometric and material characteristics [43, 46, 133–136].

Figure 2.13: Sandwich continuum finite element model used to predict impact damage, from [5].

To model damage in the continuum core, several approaches have been proposed. Some researchers have applied an elliptic yield criterion which accounts for the transverse normal and shear stresses to predict the onset of core plasticity [4, 90]. Similarly, the core has also been idealised as an isotropic, elastic-perfectly plastic material in the transverse direction [3, 5, 99]. Atkay et al. [5] proposed an element elimination technique which removes finite elements upon reaching a threshold stress or strain value. However, as they

pointed out [137], such an approach cannot model a realistic impact response since failed elements also contribute to the damage resistance even after failure. Horrigan et al. [114] proposed an isotropic continuum damage model, but this modelling was limited to small deformations since the plastic stress flow did not represent the real damage in the core, as pointed out by Castanie et al. [95].

Although a continuum core meshed with solid elements may seem a convenient way to represent the honeycomb core geometrically, errors have been attributed to the continuum model when it is used to model damage [7, 114]. One reason is that it may be difficult to simulate the exact damage progression using solid elements due to the discontinuous surfaces of the honeycomb core in contact with the facesheets. The onset of damage initiation and propagation in the honeycomb core may be highly sensitive to the local damage distribution along the cells. Moreover, damage is assumed to progress at an even rate throughout the continuum model, whereas damage in the test specimens may occur at a distance of approximately cellular width apart [114]. Thus, the local stress field and damage distribution may not be accurately represented in the core, especially in the impact damage region.

The limitation of the continuum model can be overcome by adopting a micromechanical approach, where the actual discrete hexagonal microstructure of the honeycomb is taken into consideration. Here, each cellular cell is modelled explicitly with shell elements, such that the final model is an accurate and detailed representation of the real geometry. Previous investigations have assumed this modelling approach. Nguyen et al. [138] accurately simulated the impact response for aluminium honeycomb sandwich plates, and found that the structural response and impact damage resistance of these materials are sensitive to the core geometry. Atkay et al. [7] also modelled the transverse crushing behaviour of aluminium and Nomex honeycomb cores and reported good correlation with test data. Similarly, Mohr and Doyoyo [52, 53] analysed the deformation mechanisms of aluminium honeycombs under large deformation, and found that the constitutive behaviour of these honeycombs during plastic collapse is largely controlled

by folding systems. All these studies also demonstrate that a micromechanical model is useful to study the effect of core geometry on the structural response.

2.5 Concluding Remarks

Based on the literature reviewed, several needs have been identified as motivation for the current work. First, a great deal of experimental work has resulted in considerable understanding of the behavior of composite sandwich structures subjected to low-velocity impact. For instance, the damage modes in sandwich panels under localised loading are fairly well documented. Although observations from these studies have led to a greater understanding of parameters that affect damage initiation and propagation, questions are still being asked of the current methods used to assess the impact performance of composite structures. Previously, most researchers have used a single damage metric, which is either the impact energy or the peak impact force, to characterise the impact damage resistance of composite structures, which has resulted in large but expensive databases. An improved technique to correlate the impact response with damage is desirable, so that test results among structures of various configurations or materials can be better compared.

Second, accurate prediction of impact response and damage of composite sandwich structures has remained a tremendous challenge due to the complex failure mechanisms of the facesheets and core. A number of mathematical models to predict the impact response of composite structures have been proposed in the literature. Most of these models are valid for elastic impact events, but the impact response of the structure after the onset of damage is not well dealt with. Furthermore, there are very few attempts to theoretically quantify the damage initiation threshold load for composite sandwich panels under indentation.

Lastly, there are still no widely established methods to predict the structural performance of composite sandwich structures subject to low-velocity impact. Due to its versatility, FE analysis is often used to study the failure of these structures and to identify

critical parameters for design purposes. In FE modelling of honeycomb sandwich structures, the core is often meshed with solid elements to represent an equivalent continuum model, which does not model damage realistically. On the other hand, the micromechanical approach, in which the cell-walls are explicitly modelled using shell elements, has been used successfully for aluminium honeycombs [7, 52, 53, 138]. However it has not been presented extensively in the literature so far, particularly for Nomex honeycombs supporting carbon/epoxy skins.

Chapter 3

Experimental Investigation

This chapter presents the experimental investigation carried out in this study. The primary objectives of the experimental work were to guide the development of the analytical and FE models, and subsequently, to validate these models with test data. Thick square aluminium and composite sandwich plates of various configurations were subjected to a transverse load at their centre by a hemispherical indenter (impactor); both quasi-static indentation and low-velocity impact tests were performed and compared. In addition, flatwise compression tests were conducted on bare Nomex honeycombs to characterise their out-of-plane compressive behaviour. The test specimens are first introduced, and the test systems are later described.

3.1 Specimen Description

The specimens for the static indentation and low-velocity drop weight tests were aluminium sandwich plates and composite sandwich plates that were made of carbon/epoxy facesheets and Nomex honeycomb cores.

3.1.1 Aluminium sandwich plates

The aluminium sandwich plates comprised of aluminium alloy 3003-H19 foil for the honeycomb core and aluminium alloy 1100-H14 for the facesheets. Each plate measured 100 mm $\times$ 100 mm, with a core thickness of 20 mm and a thickness of 0.75 mm for each top and bottom facesheet. The density of the aluminium honeycomb was 72.0 kg/m³, and

the cell size was 6.35 mm. In the impact tests, the aluminium sandwich panels were struck by an impactor with a mass of 2.65 kg at various impact velocities. The drop height of the impactor was adjusted from 0.033 m to 0.50 m to achieve a range of impact energies that varied from 0.85 J to 13.0 J. Initially, to ensure the experimental procedure was repeatable, each impact test was performed on four different specimens, while the quasi-static indentation test was performed on three different specimens. However, because good repeatability was observed, the number of specimens for repeatability assurance was reduced during the later stages of the testing.

3.1.2 Composite sandwich panels

Prepared in-house, the composite sandwich panels consisted of commercially available Fibredux® 913C-HTA carbon fibre-epoxy composites bonded to HexWeb® A1 Nomex honeycomb core with 2 layers of Redux 335K adhesive films. First, each lamina was cut to size 100 mm by 100 mm and hand-laid, and the laminated facings were then cured at temperature up to 120°C according to the curing cycle shown in Figure 3.1. Later, the cured laminates were assembled with the honeycomb core and adhesive films, and subsequently cured again. The end product was a complete sandwich plate ready for structural tests (Fig. 3.2). Table 3.1 shows the list of specimen types used in the static indentation and low-velocity impact tests, with various core thickness, cell size and laminates orientation.

Figure 3.1: The curing cycle for composite laminates and sandwich plates.

Figure 3.2: A typical composite sandwich specimen.

Table 3.1: List of composite sandwich specimens used in the static indentation and low-velocity impact tests.

Specimen Identity	Skin stacking sequence	Skin thickness (mm)	Core cell size (mm)	Core height (mm)	Loading rate (m/s)
C1/3/15	[0/90/0/90/0] _s ¹	1.25	3	15	QS ² , 1.18, 1.65
C1/6/15	[0/90/0/90/0] _s	1.25	6	15	QS; 1.65
C1/13/15	[0/90/0/90/0] _s	1.25	13	15	QS; 1.18, 1.65, 1.98, 2.34, 2.75
C1/3/20	[0/90/0/90/0] _s	1.25	3	20	QS; 1.65
C1/6/20	[0/90/0/90/0] _s	1.25	6	20	QS; 1.65
C1/13/20	[0/90/0/90/0] _s	1.25	13	20	QS; 1.65
C1/3/25	[0/90/0/90/0] _s	1.25	3	25	QS; 1.65
C1/6/25	[0/90/0/90/0] _s	1.25	6	25	QS; 1.65
C1/13/25	[0/90/0/90/0] _s	1.25	13	25	QS; 1.65
C2/3/15	[+45/-45/0/90/0] _s	1.25	3	15	QS; 1.18
C2/13/15	[+45/-45/0/90/0] _s	1.25	13	15	QS; 1.18, 1.65
C3/3/15	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	3	15	QS
C3/6/15	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	6	15	QS
C3/13/15	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	13	15	QS
C3/3/20	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	3	20	QS
C3/6/20	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	6	20	QS
C3/13/20	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	13	20	QS
C3/3/25	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	3	25	QS
C3/6/25	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	6	25	QS
C3/13/25	[+45/0 ₂ /90/0 ₂ /-45] _s	1.75	13	25	QS

¹ The subscript *s* denotes symmetry about the midplane of the laminate.² ‘QS’ refers to quasi-static indentation test.

The material properties of the laminate provided by the manufacturers are listed in Table 3.2. The aerospace grade Nomex honeycombs are made of Dupont's Nomex aramid-fibre paper in an expansion process, where sheets of Nomex paper are first stacked and adhesively bonded together. After expanded to form an array of hexagonal cells, this block of paper sheets is then dipped in phenolic resin several times to produce a low-density honeycomb core with high strength and very good fire resistance [139, 140]. The geometry of the hexagonal honeycomb core is illustrated in Figure 3.3. The cell-walls oriented in the ribbon direction (2-direction) of the core are referred to as the ribbon walls, and the remaining cell-walls in the core, which are inclined at an angle, are the free walls. Table 3.3 lists the properties of the honeycombs, all of which have a nominal core density of 64 kg/m^3 .

Table 3.2: Material properties for Fibredux® 913C-HTA carbon-epoxy laminates.

Property	Value
Longitudinal stiffness, E_{11} (GPa)	150
Transverse stiffness, E_{22} (GPa)	9.5
Out-of-plane stiffness, E_{33} (GPa)	9.5
Poisson's ratios, ν_{12} and ν_{13}	0.263
Poisson's ratio, ν_{23}	0.458
Shear moduli, G_{12} and G_{13} (GPa)	5.43
Shear modulus, G_{23} (GPa)	3.26
Longitudinal tensile strength, X_t (MPa)	1900
Longitudinal compressive strength, X_c (MPa)	1550
Transverse tensile strength, Y_t (MPa)	65.5
Transverse compressive strength, Y_c (MPa) ¹	140
Interlaminar shear strength, S (MPa)	101.2
Out-of-plane tensile strength, Z_t (MPa)	65.5
Density, ρ (kg/m^3)	1100

¹ Estimated from Ref. [141]

Figure 3.3: Co-ordinates and geometrical parameters of hexagonal honeycomb structure.

Table 3.3: Dimensions and material properties for HexWeb[®] A1 Nomex honeycomb cores with nominal core density of 64 kg/m³.

Honeycomb Core	c (mm)	h_c (mm)	l (mm)	h (mm)	t (mm)	E_{33c} (MPa)	G_{23c} (MPa)	G_{13c} (MPa)
3-15	3	15	1.7	1.7	0.11	201	63	35
3-20	3	20	2.0	2.0	0.15	181	63	35
3-25	3	25	2.0	2.0	0.14	181	63	35
6-15	6	15	4.0	4.0	0.19	150	55	33
6-20	6	20	4.0	4.0	0.17	169	55	33
6-25	6	25	4.0	4.0	0.17	173	55	33
13-15	13	15	7.5	7.5	0.60	139	55	32
13-20	13	20	8.0	6.0	0.41	209	55	32
13-25	13	25	8.0	6.0	0.36	200	55	32

3.2 Quasi-static Indentation Test

The objectives of conducting a quasi-static test prior to an impact test are two-fold. First, in a static test, the displacement of the indenter can be better controlled, resulting in more accurate measurements of transverse force. Consequently, damage initiation and propagation in the specimen is easier to detect. In addition, the effect of time-dependent dynamic processes, which may complicate the analysis in the impact test, is eliminated in a static test. For instance, the strain-rate effects of the constituent materials and the support conditions may influence the behaviour of the sandwich panel under impact. Moreover, inertial oscillations induced in the impact event may result in a recorded load-time curve that is difficult to interpret [19]. Therefore, a preliminary static test can provide a baseline for the impact analysis. Second, despite the similarities between quasi-static indentation and drop-weight impact tests in terms of load-displacement responses and damage characteristics for honeycomb sandwich panels [8, 24, 34], the quasi-static assumption for low-velocity impact has also been shown to be limited [9, 29]. Therefore, by comparing the response of the sandwich plate in both tests, the limit beyond which the quasi-static assumption ceases to be valid can be defined.

The static indentation tests were conducted using the Instron 5500R test system (Fig. 3.4), where a 13.1 mm diameter hemispherical steel hardened indenter was loaded statically onto the specimen under displacement control at a constant cross-head speed of 0.5 mm/min. A 10 kN load cell which was located between the indenter and the crosshead measured the contact force, while a displacement transducer (LVDT) mounted under the specimen recorded the displacement of the centre of the bottom facesheet. After each test, the compressive load versus indenter displacement data were collected via a digital data acquisition system.

Figure 3.5 shows the sandwich plate in the restraint fixture. The square specimen, with its centre located directly underneath the indenter, was positioned between the top and bottom clamp plates which had circular openings of diameter 76.4 mm. The clamp plates were then bolted in place to secure the sandwich plate. One benefit of such a configuration

Figure 3.4: Schematic of Instron 5500R static indentation test system.

Figure 3.5: Schematic set-up of the clamped sandwich plate in the static and impact tests.

is the polar symmetry resulting from the circular opening. A spherical indenter (impactor) was chosen in this work because this shape is the most widely discussed in the literature. The absence of sharp edges on the indenter also reduces the chances of discontinuities in the deformation profile on the indented facesheet, which may further complicate any analysis. In addition, the indenter used in the static tests and the impactor used in the impact tests had the same diameter for consistency, so that comparisons could be made later.

3.3 Low-velocity Impact Test

The use of instrumented impact test devices in assessing the dynamic response of composite materials has long been popular among researchers due to the relative ease and speed with which a large number of impact tests can be performed. In these tests, the principal measurement is the load-time history. Consequently, investigators are able to characterise elastic behaviour, failure initiation, and failure propagation for the composite structure in terms of impact force and energy [9, 19, 22]. Although other procedures, such as C-scan, radiography, and photomicrography, can be used to assess the extent of damage in the structure, these techniques are generally time-consuming and labour-intensive. Since damage degrades the stiffness of the plate, and subsequently affects the load response, the load-time and load-deflection histories can provide some insight in the mechanisms of damage. Thus one objective was to correlate the observed load-time history to the damage sustained by the plate.

3.3.1 Drop-weight test set-up

The low-velocity drop-weight impact tests were carried out using the the Instron Dynatup 8250 impact testing machine (Fig. 3.6). Its principal features are: (1) a stiff, guided, near-free-falling mass; (2) a force transducer mounted in the falling impactor (tup), which has a capacity of 15.56 kN; (3) a hemispherical 13.1 mm diameter steel tup tip; (4) a velocity

Figure 3.6: Schematic of Dynatup 8250 drop weight impact test system.

detector to measure the tup velocity just prior to impact, and to trigger data collection; (5) a set of pneumatic clamp plates to hold the specimen in place; (6) a rebound brake assembly to prevent multiple impacts; and (7) a digital data acquisition system.

The impactor mass, which was 2.65 kg in all tests, was first raised to a certain height and then released. The free-falling impactor would fall along two smooth guided columns, and through the centre hole of the clamp plate of diameter 76.4 mm to strike the specimen. The support conditions for the specimen were similar to those used in the static test (Fig. 3.5). The pneumatic clamp plates prevented any movement of the specimen, without causing any buckling of the honeycomb core prior to impact. After the first impact, the rebound brake was activated to support the crosshead, and thus the impactor was only allowed to strike the specimen once. Impact force was measured discretely over time with the force transducer during impact, and the acceleration was calculated by the impact

force divided by the impactor mass. Subsequently, the velocity and the displacement of the impactor were derived by integration. These data were later used to generate graphical plots for the impact load and the displacement of the impactor as functions of time.

3.3.2 Energy calculations

In the impact test, impact energy which is defined as the incident kinetic energy of the impactor is a parameter widely used to characterise the event. Generally, different impact energies are obtained by varying the mass of the impactor, M_{imp} , and the height from which it is dropped. Energy losses which occur during the free-fall of the impactor, due to sliding friction along the guide rails and air resistance, etc., are accounted for by measuring the velocity of the impactor just prior to impact, $V_{imp}(0)$. Impact energy, E_{imp} , is thus calculated as

$$E_{imp} = \frac{M_{imp}}{2} V_{imp}(0)^2 \quad (3.1)$$

The net portion of the impact energy transferred to the plate at any given point in time is calculated by the kinetic energy lost by the impactor. Consequently, the impact energy is completely transferred to the plate at the point of the impactor's maximum displacement. A fraction of the impact energy, E_{el} , is stored in the plate as elastic strain energy and restituted back to the impactor during rebound. The remaining energy, E_{abs} , is absorbed by the system. Thus,

$$E_{imp} = E_{el} + E_{abs} \quad (3.2)$$

The absorbed energy consists of two parts. First, energy is dissipated from the plate due to damage progression, E_{dam} , which manifests in the form of failure modes. Second, energy is also dissipated in non-conservative phenomena, E_{nc} , such as vibration, heat, friction, and sound, etc. Thus,

$$E_{abs} = E_{dam} + E_{nc} \quad (3.3)$$

The terms, E_{el} and E_{abs} , can be evaluated by comparing the loading and unloading load-displacement curves of any instrumented test [27, 30, 34], where E_{abs} is the area enclosed

between the loading and unloading portions of the curve and E_{el} is the area under the unloading portion. However, E_{nc} and E_{dam} cannot be easily separated, and E_{nc} is usually assumed to be small as compared to E_{dam} . Frictional losses in the contact area between the impactor and the plate during impact are negligible since the specimens in this study did not suffer perforation under low-velocity impact, as also noted by [16]. In addition, the energy dissipated due to vibration can be ignored for low-velocity impacts which are quasi-statically equivalent, according to other references [9, 19, 142]. This quasi-static assumption will be confirmed later.

3.4 Flatwise Compression Test on Bare Honeycombs

The flatwise compression tests served two purposes in this study. First, the honeycomb properties commonly provided by the manufacturers are limited and are insufficient for designers to model the material of the honeycomb structures in commercial FE codes [136, 143]. One simple way of overcoming this limitation is to perform flatwise compression tests on bare Nomex honeycomb cores, so that the test data could be used to calibrate the material model. Second, previous studies have indicated that the onset of damage in sandwich panels subjected to quasi-static indentation and low-velocity impact is due to core failure [9, 15, 24, 90]. Based on this finding, an analytical method was developed to predict the critical load at failure initiation by considering the elastic energy absorbed by the plate up to the onset of core damage. Accordingly, the energy absorbed by the core under compression up to the onset of damage was measured in the compression test.

Figure 3.7 shows three different core sizes (9, 33, and 60 cells) which were considered for each core configuration (Table 3.3). With three samples for each core size tested, there were 81 test specimens altogether. Each specimen was carefully cut out of large honeycomb plates to ensure that the cells were complete with intact peripheral walls.

Subsequently, the flatwise compression tests on these honeycombs were carried out using the Instron 5500R test machine (Figure 3.8). The specimens were crushed at a

Figure 3.7: Three different sizes of honeycomb cores 13-15.

Figure 3.8: Flatwise compressive tests on bare honeycombs.

slow displacement rate of 0.5 mm/min with a 150 kN load cell; at low loads, a 5 kN load cell was used to ensure more accurate measurements. Flat metal plates were used to crush large specimens that exceeded the boundaries of the loading area (Figure 3.8(b)). The crosshead displacement and force data were recorded to produce compressive load-deformation curves.

Figure 3.9 shows a typical load-displacement curve obtained from the flatwise compression tests. Initially, the compressive load increased linearly due to the elastic bending of the thin cell walls until a critical load, P^{Pk} , was reached, and core failure is assumed to initiate at this point. After the peak load, a sharp drop to a plateau load was observed. The area under the load-displacement curve up to the peak load is the energy absorbed by the core at initial failure, U^{Pk} . The elastic strain energy absorbed by the core per unit area at initial failure is defined as

$$U^* = \frac{U^{Pk}}{A_{core}} \quad (3.4)$$

Figure 3.9: A load-displacement curve for compressive test on 3-15 bare Nomex honeycombs with 60 cells.

where A_{core} is the effective crushing area of the core. This elastic strain energy U^* is a material property of the honeycomb core (Table 3.4), and the core is deemed to have failed once the energy absorbed by the core (per unit area) exceeds U^* .

Table 3.4: Elastic strain energies per unit area of honeycomb cores under flatwise compression (mean $\pm$ standard deviation).

Honeycomb Configuration	Elastic strain energy per unit area, U^* (kJ/m ²)
3-15	0.992 ± 0.142
3-20	1.095 ± 0.163
3-25	0.968 ± 0.095
6-15	1.045 ± 0.079
6-20	0.992 ± 0.099
6-25	1.306 ± 0.113
13-15	0.976 ± 0.071
13-20	0.948 ± 0.162
13-25	1.115 ± 0.151

3.5 Summary

Quasi-static indentation and low-velocity drop weight impact tests were performed on aluminium and composite sandwich plates. Beside the sandwich plates, bare honeycombs were also loaded in static flatwise compression tests. The results from all these tests would be used to validate the analytical and FE models presented in the following chapters.

Chapter 4

Analytical Formulation

4.1 Introduction

One objective in understanding the behaviour of sandwich structures under impact is to develop an analytical model to predict the impact force history and the overall response of the structure to that impact. A typical load-time history and load-displacement plot for a composite sandwich subjected to a low-velocity impact test is shown in Fig. 4.1. Initially, the impact force increases in a sinusoidal-like manner with time and linearly with the displacement of the impactor (with slope K_0), as observed in Refs. [22, 26, 31, 59]. As the force increases up to a critical value (P_1), there is a sudden load drop indicating the onset of damage. This is followed by a subsequent increase to the maximum load at a lower stiffness (K_{dam}), before rebounding occurs.

Previous studies [8, 22, 26, 31, 59, 71] have shown that elastic impact models, such as the spring-mass and energy-balance models, tend to overestimate the peak impact load for the plates after the onset of damage (Fig. 4.1), since these models do not account for damage initiation and propagation. Thus the impact response of the structure after the onset of damage is not well dealt with.

Here, the energy-balance model [2, 65, 69, 144] is modified to extend its validity beyond the elastic regime. The following sections describe the methods used to derive the elastic stiffness (K_0), the critical load at damage initiation (P_1), and the reduced transverse stiffness of the plate after damage (K_{dam}). These three parameters are then incorporated into the modified energy-balance model to predict the low-velocity impact response of

sandwich plates. Under low-velocity impact, the sandwich plate essentially deforms under quasi-static loading [8, 24, 29, 33]. For this reason, the static indentation problem of a clamped circular composite sandwich plate centrally loaded by a rigid hemispherical indenter is first considered in this analysis.

Figure 4.1: Schematic of a typical load-displacement and load-time plot for a composite structure under impact.

4.2 Elastic Stiffness of Sandwich Plate

An important first step in the impact analysis of sandwich panels is to determine the elastic response of the sandwich plate prior to damage. One approach is to decouple the local and global responses and ignore any interaction between the two, so that these stiffnesses can be determined separately. This approach which is used to simplify the problem is common in the literature [32, 62–64, 70]. The local indentation problem of a rigidly supported sandwich plate is first solved by employing a simple indentation model in which the indented facesheet rests on an elastic core foundation. Subsequently, the

classical plate theory is used to derive the bending and shear stiffnesses, which constitute the global stiffness for the sandwich plate.

4.2.1 Local indentation

A rigidly supported sandwich plate undergoes only local deformation, α , which consists of both top facesheet indentation and core deformation. Although the use of the Hertzian contact law to analyse the local deformation for isotropic homogenous linear elastic bodies is well-established, the Hertzian contact law may not be appropriate for sandwich plates because the core is more flexible than the facesheets in the transverse direction. Consequently, the indentation of sandwich plates is dominated by the deformation of the core [2].

Figure 4.2: Local indentation model.

The principle of minimum total potential energy, which provides considerable ease in solving problems involving orthotropic plates, is used here to derive the elastic local stiffness, K_{loc} , of a clamped circular sandwich panel indented by a spherical indenter at its middle (Fig. 4.2). The top facesheet is modelled as a plate resting on the core subjected to a concentrated load, with membrane stretching of the facesheet neglected for very small indentation. Accordingly the facesheet is only bonded to the core discretely, thus the shear resistance between the laminate and the core is ignored. In addition, the facesheet is assumed to remain undamaged. Likewise, the core assumes an elastic Winkler foundation model. This approach is similar to those adopted by previous studies who have

successfully modelled the elastic response of sandwich panels under local indentation assuming an elastic core behaviour [62, 77–82]. The elastic modulus of the foundation k_c , which has the dimensions of force per unit surface area of plate per unit deflection, is related to the modulus of the core in the out-of-plane direction E_{33c} and the thickness of the core h_c by [2, 77, 83, 84]

$$k_c = \frac{E_{33c}}{h_c} \quad (4.1)$$

Figure 4.3: Indentation of top facesheet by spherical indenter.

The localised indentation area is assumed to be clamped at its boundary [70], with the profile of the local indentation represented by

$$\alpha(r) = \alpha_0 \left(1 - \frac{r^2}{a^2}\right)^2 \quad (4.2)$$

where α_0 and a are the transverse deflection and the radius of region of local indentation on the top facesheet, respectively (See Fig. 4.3). This indentation profile approximates the lowest mode of vibration for a clamped circular plate [145], where the shape of the indentation profile is symmetrical about its centre. In addition, the boundary conditions $\alpha = 0$ and $d\alpha/dr = 0$ at $r = a$, as well as the symmetrical condition, $d\alpha/dr = 0$ at $r = 0$ are satisfied.

According to Abrate [2], laminated facesheets with more than 6 plies and a symmetric lay-up can be modelled as orthotropic plates. Thus, under axisymmetrical bending, the strain energy of the elastic circular clamped facesheet can be expressed as (see

Appendix A)

$$U_1 = \frac{32\pi D_f \alpha_0^2}{3a^2} \quad (4.3)$$

where an equivalent bending stiffness of the orthotropic facesheet, D_f , is given by

$$D_f = \frac{1}{8} [3D_{11} + 2(D_{12} + 2D_{66}) + 3D_{22}] \quad (4.4)$$

in which D_{ij} are terms from the laminate bending stiffness matrix. The strain energy due to the deformation of the elastic foundation is given as [146]

$$U_2 = \int_0^{2\pi} \int_0^a \frac{1}{2} k_c \alpha^2 r dr d\theta = \frac{\pi k_c a^2 \alpha_0^2}{10} \quad (4.5)$$

The work done by the contact load P is then

$$W = P(\alpha)_{r=0} = P\alpha_0 \quad (4.6)$$

Therefore, the total potential energy is

$$\Pi = U_1 + U_2 - W \quad (4.7)$$

Applying the minimising condition, $\partial\Pi/\partial a$, it yields

$$a = \sqrt[4]{\frac{320D_f}{3k_c}} \quad (4.8)$$

Again, applying $\partial\Pi/\partial\alpha_0$ and substituting Eq. 4.8, the localised load-indentation relationship is derived as

$$P = \left(\pi \sqrt{\frac{256}{15} D_f k_c} \right) \alpha_0 \quad (4.9)$$

In other words, the initial local stiffness of a rigidly supported circular sandwich plate with orthotropic facesheets is

$$K_{loc} = 12.98 \sqrt{D_f k_c} \quad (4.10)$$

The local stiffness is directly proportional to the foundation modulus of the core and the bending stiffness of the top facesheet. As observed by many investigators [2, 13, 147], the initial load varies linearly with indentation prior to core failure, so Eq. 4.10 is applicable. This result is also in contrast with Eq. 2.3 on page 21, which is derived for an infinitely large isotropic plate.

4.2.2 Global deflection

When the sandwich plate is clamped around its edges, it experiences both local and global deformation. Global deformation, w , refers to the bending and shearing of the entire sandwich plate. In reality, there exists an interaction between the local and global deformations: as the core crushes during local deformation, its height reduces and the global bending and shearing stiffness of the sandwich plate becomes smaller [32, 62]. However, given that the impact damage is small and localised around the impactor for low-velocity impacts, this influence on global plate behaviour may be neglected. In addition, as the core is much thicker than the facesheets, the membrane stretching of the facesheets is assumed to be negligible. The membrane stiffness of the core is also negligible [70]. Thus, the load sustained by the plate is related to this global deflection [69] by

$$P = K_{glo} w_0 \quad (4.11)$$

where $K_{glo} = K_b K_s / (K_b + K_s)$ is the effective global stiffness due to bending stiffness, K_b , and shear stiffness, K_s .

The derivation of the bending stiffness and shear stiffness hereafter follows closely to that adopted by Zhou and Stronge [70]. The effective shear stiffness of the sandwich plate, K_s , is derived by dividing the central load, which is assumed to act over a contact area with radius R_c in the form of a uniformly distributed pressure, by the shear deflection

at the centre of the plate to give [70]

$$K_s = \frac{4\pi G_c (h_f + h_c)^2}{h_c (1 + 2 \ln(R_p/R_c))} \quad (4.12)$$

where $G_c = (G_{13} + G_{23})/2$ is the average out-of-plane shear moduli of the core; h_c is the thickness of the core; R_p and R_c are the outer and contact radii of the plate, respectively. An initial value of R_c is required to obtain K_s , where $R_c = \sqrt{2\alpha_0 R_i - \alpha_0^2}$ by geometrical relation in which R_i is the radius of the indenter [2]. Further numerical calculations showed that the effective shear stiffness is insensitive to this assumed contact radius, as also observed in [70]. Hence it may be justifiable to substitute the mean value for the function, $\ln(R_p/R_c)$, for the range of α_0 from 0 to R_{ind} ; the mean value for the function was calculated to be 2.07.

The governing differential equation for bending of an orthotropic plate is expressed as

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} = q \quad (4.13)$$

where D_{ij} are the bending stiffnesses of the orthotropic plate and w is the central deflection due to the loading per unit area, q . For a plate that is clamped around its edge and subjected to a uniformly distributed transverse loading, the expressions for the deflection of both isotropic and orthotropic plates can have the same form, provided an equivalent bending stiffness is used for the orthotropic plate [148, 149]. This equivalent bending stiffness is identical to that expressed in Eq. 4.4 (see also Appendix A). Consequently, using the classical plate theory, the effective bending stiffness of the sandwich plate, K_b , can be derived as [69, 86, 146]

$$K_b = \frac{16\pi D'_{sw}}{R_p^2} \quad (4.14)$$

where D'_{sw} is the equivalent bending stiffness of the clamped sandwich plate [86, 148]. Using Eq. 4.4, D'_{sw} can be expressed as,

$$D'_{sw} = \frac{1}{8} [3D_{11,sw} + 2(D_{12,sw} + 2D_{66,sw}) + 3D_{22,sw}] \quad (4.15)$$

where the $D_{ij,sw}$ terms are quantities from the $[D]$ stiffness matrix of the sandwich panel. Appendix B presents explicit expressions for the bending stiffnesses of sandwich panels with thin stiff facesheets on a soft core. Equation 4.14 is different from the one presented by Zhou and Stronge [70], which had considered the bending stiffness of an isotropic facesheet.

Consequently, the elastic stiffness of the sandwich panel, K_0 , is then,

$$\frac{1}{K_0} = \frac{1}{K_{loc}} + \frac{1}{K_{glo}} \quad (4.16)$$

4.3 Onset of Damage

At the onset of damage (P_1 in Fig. 4.1), there is a substantial drop in the structural stiffness, and core failure has been identified to occur at that point [9, 15, 24, 90]. In addition, several researchers have presented evidence showing that there is a specific value of impact force and energy at the onset of damage, which is also independent of the impact energy [26, 29, 30, 59].

Previously, the theoretical load at the onset of core yielding in foam-cored sandwich structures has been investigated in Refs. [83, 84, 150], where the core is usually modelled as a homogenous material. In honeycomb sandwich structures, Zheng et al. [75] proposed a failure criterion based on the compressive yield strength of the core to model damage. Similarly, Castanie and colleagues [44, 95] represented the honeycomb core as a grid of nonlinear springs and used a crushing law which was empirically determined from flatwise compression tests on bare honeycombs to model damage. In these cases, no explicit expression for the critical load for core damage was provided. Olsson [62, 82] presented an explicit expression for the critical load for core crushing by using small deflection theory for a plate resting on an elastic foundation. However he did not account for the influence of the indenter, which has been shown to affect the local indentation of sandwich structures [34, 54, 92, 93]. Several analytical studies [62, 64, 78] have also attempted to predict the size of core crush area by adopting the elastic core foundation

model.

Here, a method is proposed to predict the load at failure initiation (P_1) by considering the elastic energy absorbed by the plate up to the onset of damage. Under indentation loading, the core in the sandwich plate is subjected to both shear and out-of-plane compressive stresses [151]. However Aminanda et al. [44, 95] examined the behaviour of Nomex honeycomb core under compression, and found that the compression load is supported mainly by these vertical edges in the hexagonal cell. Neglecting the influence of shear loading, they then modelled the honeycomb core as a grid of nonlinear springs located exactly at the vertical edges, and accurately predicted the indentation of sandwich structures with metallic skins and honeycomb cores. Moreover, they [95] found that the shearing of the cell walls becomes important only for sharp indentors on thin facesheets; however, the influence of the shear force is negligible for spherical indentors. These results suggest that, for the current aim of predicting the load at initial failure, the behaviour of the core under indentation can be modelled by considering its compression behaviour, while neglecting the influence of shear loading.

First, the strain energy absorbed by the core at initial failure under local indentation is determined. As described earlier on page 44, the elastic strain energy U^* , which is a material property of the honeycomb core derived from the flatwise compression test, is a measure of the energy absorbed by the core per unit area at initial failure. In other words, this elastic strain energy is a parameter that indicates the onset of core failure. Consequently, under local indentation, the energy absorbed by the core up to initial damage, U_{core}^{th} , is estimated by multiplying the planar core damage area by U^* . The core damage region which is assumed to be circular is characterized by an average diameter $2R_{cr}$. It is also assumed that this radius R_{cr} is a function of the indenter's radius,

$$R_{cr} = \beta R_{ind} \quad (4.17)$$

where β is a constant uniquely dependent on the materials of the top skin and core under concern. Accordingly, the energy absorbed by the core up to initial damage under local

indentation U_{core}^{th} is determined as

$$U_{core}^{th} = \pi R_{cr}^2 \times U^* = \pi \beta^2 R_{ind}^2 U^* \quad (4.18)$$

Here, the energy to deform the surrounding cell-walls that have not yet failed is assumed to be negligible. This is reasonable since the initial damage is expected to be highly localised in the vicinity of the indenter.

Next, the elastic strain energy of the top facesheet at the onset of damage, U_{tfs}^{th} , is calculated. The top facesheet is assumed to be elastic at the onset of core damage. Additionally, the interaction between the core and the top facesheet, as well as the deformation of the bottom facesheet, is ignored. The elastic strain energy of the top facesheet under local indentation (Fig. 4.3) comprises the bending and membrane stretching energies (U_b and U_m). For an undamaged facesheet, its elastic bending energy U_b is expressed in Eq. 4.3. Similarly for a clamped circular plate, the membrane stretching energy U_m can be determined as [70]

$$U_m = 2\pi \int_0^a \left(\frac{N_r \varepsilon_r}{2} + \frac{N_\theta \varepsilon_\theta}{2} \right) r dr = \frac{\pi E_f h_f}{1 - \nu_f^2} \int_0^a (\varepsilon_r^2 + \varepsilon_\theta^2 + 2\nu_f \varepsilon_r \varepsilon_\theta) r dr \quad (4.19)$$

where radial strain, $\varepsilon_r = \frac{du}{dr} + \frac{1}{2}(\frac{d\alpha}{dr})^2$, and circumferential strain, $\varepsilon_\theta = \frac{u}{r}$. For $\nu_f = 0.3$, Eq. (4.19) can be simplified to

$$U_m = 2.59\pi D_f \left(\frac{\alpha_0^4}{a^2 h_f^2} \right) \quad (4.20)$$

Accordingly, the local indentation α_0^{th} and the radius of the local indentation area a^{th} are required to determine the elastic energies U_b^{th} and U_m^{th} at the onset of initial damage. Note that the radius of the local deformation zone a , as presented in Eq. 4.8, is valid only for the elastic indentation before permanent core indentation and damage. Therefore it is reasonable to estimate the radius of the local indentation zone on the facesheet at the onset of damage, a^{th} , using Eq. 4.8. Although the indentation is assumed to be circular here, in

reality, the indentation is elliptical in an orthotropic plate because the elastic constants of the plate are direction-dependent. However, according to Olsson [152], this effect is small. In his paper, he referred to a numerical solution by Greszczuk and Chao (cited in [152]) which showed that the axis ratio of the ellipse was only 1.07 for $E_{1f}/E_{2f} = 14.3$.

Next, to calculate the local indentation α_0^{th} , assume that the load varies linearly with the local indentation prior to initial damage [10, 13, 70], such that $P = K_{loc}\alpha_0$. As such, the energy due to local indentation in the contact region can be expressed as

$$U_c = \int_0^{\alpha_0} P d\alpha_0 = \int_0^{\alpha_0} K_{loc}\alpha_0 d\alpha_0 = \frac{1}{2}K_{loc}\alpha_0^2 = \frac{P^2}{2K_{loc}} \quad (4.21)$$

This energy is then equated to the energies sustained by the core and the top facesheet under local indentation,

$$U_c = U_{core} + (U_b + U_m)_{fs} \quad (4.22)$$

Substituting Eqs. 4.3, 4.18, 4.20, and 4.21 into Eq. 4.22 yields

$$\frac{1}{2}K_{loc}\alpha_0^2 = \pi\beta^2 R_{ind}^2 U^* + \frac{32\pi D_f \alpha_0^2}{3a^2} \left(1 + 0.244 \frac{\alpha_0^2}{h_f^2} \right) \quad (4.23)$$

By solving Eq. 4.23, four roots would be obtained, of which the smallest positive root is the indentation at the onset of failure α_0^{th} . Subsequently, the local indentation energy U_c^{th} can be determined. Note that, if $\alpha_0^{th} < h_f$, the strain energy for the top facesheet is mainly due to bending [146]. In that case, Eq. 4.23 reduces to a simple quadratic equation if the contribution of the membrane stretching energy is ignored.

Similarly, again assuming that the load varies linearly with the global deformation (w_0) prior to initial damage [10, 13, 70], i.e., $P = K_{glo}w_0$, the energy due to global deformation in the form of bending and shear deformations is

$$U_{bs} = \int_0^{w_0} P dw_0 = \int_0^{w_0} K_{glo}w_0 dw_0 = \frac{1}{2}K_{glo}w_0^2 = \frac{P^2}{2K_{glo}} \quad (4.24)$$

The global deflection of the sandwich plate w_0^{th} can be found by equating Eqs. 4.9

and 4.11,

$$K_{bs}w_0^{th} = K_{loc}\alpha_0^{th} \quad (4.25)$$

Therefore, the threshold energy at the onset of damage, U^{th} , is the sum of U_c and U_{bs} up to that instant,

$$U^{th} = U_c^{th} + U_{bs}^{th} \quad (4.26)$$

Previous studies have shown that there is a critical impact force, P_1 at the onset of damage [22, 26, 29, 30, 59]. In order to relate U^{th} to the damage initiation load P_1 , the energies used to deform the plate can be expressed as a function of the load [10, 13, 70]. Accordingly, the total work done on the plate by the indenter can be written as

$$U_{Total} = U_c + U_{bs} = \frac{P^2}{2} \left(\frac{1}{K_{loc}} + \frac{1}{K_{glo}} \right) = \frac{P^2}{2K_0} \quad (4.27)$$

The above equation implies that the load sustained by the plate at any instant depends on the instantaneous work done on the plate and its elastic structural stiffness. Therefore P_1 is,

$$P_1 = \sqrt{2K_0U^{th}} \quad (4.28)$$

This result is analogous to the peak impact load P_{peak} derived in References [2, 22, 26], which use an elastic spring-mass model to show that,

$$P_{peak} = \sqrt{2K_0E_{imp}} \quad (4.29)$$

where E_{imp} is the impact energy and K_0 is the structural stiffness prior to damage. However Eq. 4.29 ceases to work beyond the elastic regime. Accordingly, there exists a threshold impact energy, U^{th} , at which damage starts to occur. If the impact energy is lower than the threshold energy, no damage will occur and the peak load can be predicted by Eq. 4.29. Conversely, if the impact energy exceeds this threshold value, damage will initiate.

4.4 Stiffness after Damage

Experimental studies [9, 13, 26, 27, 30] find that the onset of damage is indicated by the sudden decrease in the stiffness of the plate, which is represented by the slope of the load-displacement curve, from the initial stiffness K_0 to the reduced stiffness after damage K_{dam} (see Fig. 4.1). Therefore, it is essential to account for the progressive change in the plate's stiffness to accurately represent the entire impact event when damage has initiated. In the literature, it is well-accepted that most of the energy absorbed by the plate during impact is dissipated in the form of damage modes, and the extent of damage is reflected in the reduction of the plate's stiffness. Feraboli and Kedward [31] found that the reduced stiffnesses for impacted carbon/epoxy laminates were related to the damage initiation energy and the impact energy. Lifshitz et al. [27] also reported that the relative loss in energy was related to the decrease in stiffness for carbon fibre reinforced plastic (CFRP) beams subjected to low-velocity impacts by the relation

$$\frac{K_{dam}}{K_0} = 1 - \frac{E_{abs}}{E_{imp}} \quad (4.30)$$

where E_{imp} is the impact energy, and E_{abs} is the energy absorbed mainly in damage mechanisms and dissipated in non-conservative phenomena (Eq. 3.3). Using Eq. 3.2, the above equation can also be expressed as

$$\frac{K_{dam}}{K_0} = \frac{E_{el}}{E_{imp}} \quad (4.31)$$

where E_{el} is the elastic strain energy. Simply put, the fraction of the recoverable energy in the impact event gives a measure of the plate's reduced stiffness with respect to its initial stiffness.

In an elastic impact event, where no damage occurs, the entire impact energy is recoverable. Accordingly, the peak impact load sustained by the plate (P_{max}^{el} in Fig. 4.1) can be predicted in Eq. 4.29. However, after the onset of damage, some of the impact energy is dissipated in the form of damage modes, such as matrix failure, delamination,

fibre breakage, and core damage. In other words, the initiation and propagation of damage reduces the amount of energy available to do work on the plate [22]. According to Feraboli [22, 26], the effective impact energy then reduces to E_{imp}^{eff} ,

$$E_{imp}^{eff} = E_{imp} - E_{abs} \quad (4.32)$$

Consequently, the peak load sustained by the damaged plate would be less than that sustained in an elastic impact. Following Eq. 4.29, the peak load sustained by the damaged plate is then [26]

$$P_{max}^{dam} = \sqrt{2 \cdot K_0 \cdot E_{imp}^{eff}} = \sqrt{2 \cdot K_0 \cdot (E_{imp} - E_{abs})} \quad (4.33)$$

Using Equations 4.29 and 4.33, the ratio of the reduced stiffness of the plate to its initial stiffness can then be expressed as

$$\frac{K_{dam}}{K_0} = \frac{E_{imp} - E_{abs}}{E_{imp}} = \left(\frac{P_{max}^{dam}}{P_{max}^{el}} \right)^2 \quad (4.34)$$

It is assumed that the energy dissipated due to damage during unloading is negligible; in other words, only the energy dissipated before reaching the peak load is considered.

4.5 Impact Model

Previously, the simple energy-balance model has been used by other investigators to predict the elastic impact force, without producing the load history, for a wide range of structures which include composite laminates [58, 59, 69], aluminium sandwich plates [8], and composite sandwich structures [66, 70, 72]. In this analysis, the energy-balance model is modified to derive the load and deflection histories for the sandwich plate under impact. The maximum impact load and maximum plate deflection are assumed to occur when the velocity of the impactor becomes zero. By the law of conservation of energy, assuming no other energy losses, the total work done by the impact load (Eq. 4.27) on the plate at

Figure 4.4: Calculation procedure for analytical impact model: (a) flowchart; and (b) approximation of integral in Eq. 4.36 using trapezoidal rule in representative load-time plot.

any instant t is equal to the change in kinetic energy of the impactor at that instant,

$$U_c + U_{bs} = \frac{P^2}{2K_0} = \frac{1}{2}M_{imp} (V_{imp}^2 - V(t)^2) \quad (4.35)$$

where M_{imp} denotes the mass of the impactor, V_{imp} is the impact velocity and $V(t)$ refers to the velocity of the impactor at time t . The impact load is also a function of time, i.e., $P = P(t)$. By the law of conservation of impulse-momentum,

$$M_{imp} (V_{imp} - V(t)) = \int_0^t P dt \quad (4.36)$$

The load and velocity histories are then solved using Eqs. 4.35 and 4.36. Subsequently, the deflection of the impactor is obtained by integrating the velocity history. Once the load reached the critical load P_1 , the elastic stiffness K_0 is degraded to the reduced stiffness K_{dam} to account for damage. The integral on the right-hand side of Eq. 4.36 is approximated by the area under the load-time curve using the trapezoidal rule. No unloading is considered. Figure 4.4 illustrates the calculation procedure for the impact model.

4.6 Summary

In this chapter, an analytical model is developed to characterise the elastic response, impact damage initiation, and residual stiffness degradation of idealised composite sandwich panels under quasi-static indentation, in order to predict the low-velocity impact response of such structures. Closed-form solutions are provided to theoretically predict the elastic stiffness, the load at the onset of damage, and the damaged stiffness.

The elastic local stiffness is first derived using an energy method which assumes small deflection and elastic core response, while the global stiffness is determined using the classical plate theory. Next, the load at the onset of initial damage is predicted by accounting for the elastic energies absorbed by the core and the top facesheet up to

initial failure in the core. Because absorbed energy is a direct indication of the damage state accumulated in the structure, the subsequent degradation in the plate's stiffness is related to the relative loss in impact energy. Finally, the laws of conservation of energy and momentum are coupled in a modified energy-balance model (Eqs. 4.35 and 4.36) to predict the low-velocity impact response of sandwich plates using the stiffnesses and the load at initial failure derived from the quasi-static analysis.

Closed-form analytical solutions that can accurately predict deformations and damage reduce the reliance on empirical correlation and would be useful for design purposes. In addition, an accurate analytical model provides fast benchmark solutions that serve as a guide for more detailed and time-consuming finite element analysis, the accuracy of which depends heavily on user-defined material behaviour. Nevertheless, robust closed-form analytical models may be difficult to obtain in some cases, given the wide range of potential material systems and impact parameters involved in any study. For this reason, other modelling techniques, such as FEA, may prove more fruitful and useful.

Chapter 5

Finite Element Modelling

5.1 Introduction

A three-dimensional finite element model of the honeycomb sandwich plate was developed in the commercial finite element software, ABAQUS v6.6, to investigate the response of the structure subjected to a transverse load applied at its centre. Both quasi-static indentation and low-velocity impact problems were solved using the explicit solution method in ABAQUS/Explicit. In this chapter, the modelling approach and considerations in setting up the FE model are first elaborated. Two sandwich plates of different materials are considered: an aluminium sandwich plate and a composite sandwich plate composed of carbon/epoxy facesheets and Nomex honeycomb core. Finally, a progressive damage model that is used to predict damage initiation and progression in the laminated skins is described.

5.2 Modelling Approach and Considerations

ABAQUS/Explicit was employed to model quasi-static indentation loading and low-velocity impact of honeycomb sandwich panels. In this section, the modelling of the honeycomb sandwich plate is first described. Next, some considerations in using the explicit dynamic procedure to solve the quasi-static problem are addressed. Subsequently, a mesh convergence study is also presented.

5.2.1 Modelling of honeycomb sandwich plate

In the FE analysis, a high level of modelling realism is desired so that experimental and numerical results can be compared. By making simplified but reasonable assumptions, the motivation is to replicate the static indentation test and drop weight impact test in an artificial and simplified domain, such that they resemble the actual experiments as closely as possible, without incurring high computational expenses. The following points are relevant for both static and impact analyses, unless stated otherwise.

Honeycomb core

Figure 5.1: A unit honeycomb cell

A unit cell of the hexagonal honeycomb as depicted in Figure 5.1 was first modelled, where the cell size and core height, as well as the lengths and thicknesses of the free walls and ribbon walls, were specified as input variables. The numerical models had the same dimensions as the specimens used in the experimental study. The unit cell was then replicated in the 1-direction and 2-direction to produce many individual cells, and these cells were merged together to assemble the honeycomb core, where the axis of cells was oriented perpendicular to the facesheets. The core was then meshed with 4-node

linear, reduced-integration shell elements (S4R). Because the strength of the adhesive bond at the ribbon wall is finite in a real honeycomb, intercellular delamination between adjacent cell walls may occur. By merging individual cells to assemble the core, a single layer of shell elements was used to represent the ribbon wall. Therefore, the adhesive bonding between each honeycomb cell was assumed to be perfect, and the intercellular delamination between neighbouring cells was neglected. Cell wall fracture, which was unlikely for the range of impact energies considered in this study, was also not included in the model.

Here, each cell was explicitly modelled with shell elements to give an accurate and detailed representation of the actual hexagonal honeycomb structure. This modelling approach has been adopted in previous FE studies for aluminium honeycombs [7, 52, 53, 138] and Nomex honeycombs [7]. Given the discontinuous surfaces of the honeycomb core in contact with the facesheets, such an approach is believed to be able to simulate the stress field and damage progression along the cells more accurately.

Although geometrically correct, the cellular core model becomes infeasible when the structure is large or contains a large number of elements, due to the exorbitant computational expenses required, as also noted in [7]. This was the problem faced in modelling the 3 mm cell-sized core which had a greater number of cells due to its smaller cells. Moreover, given that the deformation and damage were expected to be highly localised in the vicinity of the projectile, an entire core meshed with shell elements was unnecessary and inefficient. Therefore, to circumvent this problem, the modelling approach for the 3 mm cell-sized core was modified. Instead of entirely meshed with shell elements, the 3 mm cell-sized core consisted of two regions coupled together (see Figure 5.2): 1) a central region of 72 cells meshed with shell elements; and 2) an equivalent homogenous core meshed with continuum 8-node linear, reduced-integration solid elements (C38DR) that surrounded the central region. Previously, several researchers have used solid elements to model the honeycomb core with some success [3–5, 132], although others have reported difficulties to model impact damage

realistically using solid elements [7,114,137]. Thus this combined cellular and continuum core model was expected to result in faster solution by reducing the number of elements in the core, while still allowing realistic stress distributions to be obtained in the critical region.

Figure 5.2: Two FE models for honeycomb core: (a) discrete cellular core model entirely meshed with shell elements, and (b) combined core model where shell elements in the central region are surrounded by solid elements.

Facesheets

In the aluminium sandwich plate, shell elements were used to model the aluminium facesheets of 0.75 mm thickness for convenience and computational savings. However, in the composite sandwich plate, the facesheets were meshed with 8-node linear reduced-integration solid elements (C3D8R), with each ply in the laminate represented by a solid element in the through-thickness direction. Solid elements were used because out-of-plane stresses through the facesheet's thickness were required to predict interlaminar failure. However due to the poor aspect ratio of these solid elements, a reasonably fine mesh in the vicinity of the indenter was needed to achieve convergence. The use of one element per ply also inadvertently led to high aspect ratios in the elements located away from the centre of the plate, where the mesh was less refined. Highly distorted elements are overly stiff because they are less able to represent more complicated field variations as undistorted elements [153]. Nevertheless, because linear elements in ABAQUS adopt the reduced-integration scheme [154], the inherent flexibility of these linear elements

would offset the effect of high aspect ratios [119]. According to Cook et al. [153], reduced integration has a softening effect because some polynomials of the lower-order rule disappear at the integration points and hence make no contribution to strain energy.

Skin-core interface

Visual inspection of the impacted specimens did not reveal any significant debond damage at the skin-core interface. Because epoxy adhesives are generally stronger than the core itself, debonding is seldom a problem for perfect bonds [46]. Consequently, the adhesive bonding between the facesheet and the core was assumed to be perfect, and a surface-based tie constraint was adopted at the facesheet-core interfaces. Although the use of compatible meshes with common nodes at the skin-core interface was an option, the tied constraint provided a convenient alternative to tie meshes of highly dissimilar refinement together. Accordingly, each node of the honeycomb core at the interface was constrained to have the same translational and rotational motion as the node on the facesheet to which it was “tied”. The default adjust of the slave nodes (ADJUST=YES) was used in the tie formulation. In this case, the slave nodes on the core were automatically repositioned by ABAQUS in the initial configuration to resolve gaps such that the surfaces were just touching [154]. For facesheets meshed with solid elements, only the displacement degrees of freedom were constrained. Consequently, tied surfaces initially in contact were prevented from penetrating, separating, or sliding relative to one another.

Rigid impactor

To reduce CPU time, the 13.1 mm diameter steel spherical impactor (indenter) was modelled as a rigid body. The terms ‘impactor’ and ‘indenter’ are used interchangeably here. In ABAQUS, a rigid body can be modelled either as an analytical rigid surface or a discrete rigid body meshed with elements. However, it was found that a rigid surface resulted in significant penetration. Hence the impactor was modelled as a discrete body by using 4-node linear tetrahedron continuum elements (C3D4) and applying a rigid body

constraint. The rigid body reference node which governed the motion of the impactor was located at the bottom-most position of the impactor, and the impactor was constrained to move only in the out-of-plane direction (3–direction) of the plate. In this case, the position of the rigid body reference node was not important since no rotations were applied to the body. The impactor had a Young’s modulus of 200 GPa, with a Poisson’s ratio of 0.3. In all impact simulations, the impactor had a density of $2.25 \times 10^6 \text{ kg/m}^3$ to reflect its mass of 2.65 kg. In addition, an initial velocity v_0 was assigned to the impactor at its reference node to simulate a free-falling mass under gravity. All simulations commenced with the impactor situated just 0.1 mm above the sandwich plate to reduce the runtime. On the other hand, in the static analysis, a downward displacement load in the 3–direction was prescribed on the indenter’s reference node, but gravity and the initial velocity were not assigned.

Contact interaction

The general contact algorithm in ABAQUS/Explicit was used to simulate the contact between the rigid impactor and the central region of the top skin, as well as other contact interactions that could potentially happen between the remaining regions of the model. One such instance was the localised crushing of the cellular walls in the core. Moreover, because the laminated skins were meshed with solid elements that might fail, the contact domain had to include both exterior and interior faces of the solid elements that could potentially come into contact with the impactor. In ABAQUS, this general contact algorithm enforces contact constraints using a penalty contact method. The penalty stiffness that relates the contact force to the penetration distance is chosen automatically by ABAQUS/Explicit so that the effect on the computational expenses is minimal, while ensuring that the penetration is not significant [154]. The mesh on the top facesheet had to be refined adequately enough to interact with the rigid impactor, such that the impactor did not penetrate the facesheet. Post-processing of results to check the penetration in the contact region was carried out in all models.

Boundary conditions

The support fixture in the experiments facilitated as circular clamped boundary conditions. As such, the boundary conditions of the area outside of the 76.4 mm diameter hole on both facesheets were prescribed to be fixed, i.e. the six translational and rotational degrees of freedom were set to zero. Friction between the clamp plates and the facesheets was also ignored. Figure 5.3(a) shows the FE square plate model of actual size and geometry. The sandwich plate was also modelled as a circular plate of diameter 76.4 mm, clamped at its top and bottom circumferential edges (Fig. 5.3(b)).

The $100 \times 100 \text{ mm}^2$ square model was compared with an idealized circular model of diameter 76.4 mm to study the effect of membrane reaction of the facesheets on the impact response of the aluminium sandwich plate under clamped conditions. The load histories for the two plates impacted at 7 J were almost identical (Fig. 5.3(c)). Because the number of elements used for the circular plate was less than that for the square one, the runtime for the circular plate was approximately 75% of that for the square one in this case. The computational savings became much more significant for the composite sandwich model which included solid elements to model the skins. Subsequently, the circular plate model which was computationally more efficient was used for the composite sandwich plates.

5.2.2 Quasi-static analysis with ABAQUS/Explicit

Two FE solution techniques are generally used in commercial FE software packages: the implicit and explicit solution methods [153]. In the implicit method, a set of nonlinear dynamic equilibrium equations is solved simultaneously and iteratively until a convergence criterion is satisfied for each increment. The iterative approach employed may have difficulty to achieve convergence in analyses with highly nonlinear material models or complicated contact conditions [154–156]. Even if convergence is possible, such analyses are expensive due to the large number of iterations required. In recent years, the use of the explicit method in nonlinear finite element analysis has become

Figure 5.3: Two FE models of clamped aluminium sandwich plate with impactor: (a) a square plate of actual size and geometry, and (b) an idealised circular plate. Load-time plots for both models subjected to 7 J impact are compared in (c).

increasingly prevalent, notably in the areas of composite impact analysis [39, 127] and metal forming simulation [155, 156].

Explicit solution method

An outline of the explicit procedure is given as follows. By explicitly integrating the equations of motion through time, the kinematic conditions at one increment are calculated based on the conditions at the previous increment. At the beginning of each

increment, ABAQUS/Explicit first solves for dynamic equilibrium:

$$\underline{M}\ddot{\underline{u}} = \underline{P} - \underline{I} \quad (5.1)$$

where $\underline{M}$ is the lumped mass matrix, $\ddot{\underline{u}}$ are the nodal accelerations, $\underline{P}$ is the vector of externally applied forces, and $\underline{I}$ is the vector of internal element forces. The accelerations are then calculated as

$$\ddot{\underline{u}}^{(i)} = \underline{M}^{-1} \cdot (\underline{P} - \underline{I})^{(i)} \quad (5.2)$$

where the superscripts refer to the time increment. Because a lumped mass matrix is used, the acceleration of any node is determined completely by its mass and the net force acting on it, eliminating the need for simultaneous equations. Using the central difference rule, the accelerations are integrated through time to first solve for the velocities at the middle of the current increment, and subsequently, the displacements at the end of the increment:

$$\dot{\underline{u}}^{(i+\frac{1}{2})} = \dot{\underline{u}}^{(i-\frac{1}{2})} + \frac{\Delta t^{(i+1)} + \Delta t^{(i)}}{2} \ddot{\underline{u}}^{(i)} \quad (5.3)$$

$$\underline{u}^{(i+1)} = \underline{u}^{(i)} + \Delta t^{(i+1)} \dot{\underline{u}}^{(i+\frac{1}{2})} \quad (5.4)$$

Because the explicit procedure assumes accelerations remain constant during an increment, small time increments are necessary to maintain accuracy of solution. As a result, analyses typically require many thousands of increments. Nevertheless, with no simultaneous equations involved, each time increment is computationally inexpensive to solve. Moreover, convergence problems are not encountered because there is no iteration required.

Originally developed to solve dynamic events in which inertial effects are significant, the explicit procedure has also proved to be a viable alternative in solving highly nonlinear quasi-static problems with complicated contact conditions [154–156]. However, it is impractical to perform the static analysis in its natural time scale since the large number of small time increments would result in lengthy run-time. Thus it is desired to speed up the

process artificially, whilst ensuring that inertial effects remain small. For rate-insensitive materials, one option is to increase the loading rate, which is inversely related to the load duration. In a quasi-static analysis, the response of the structure is usually dominated by its lowest mode [153, 154]. As a guideline for selecting optimal loading rates, Kutt et al. [155] recommended that the duration of the load step be at least five times greater than the fundamental natural period of the model, while step times that are 10-50 times greater have also been suggested by ABAQUS [154]. In addition, several studies have indicated that inertial effects in the model are negligible when the ratio of kinetic energy to the total internal energy is below 5% [155–157].

Accordingly, the following steps were carried out to solve the static indentation problem using the explicit procedure. First, a frequency extraction analysis was carried out to determine the fundamental frequency for the sandwich model. Explicit quasi-static analyses were then carried out with increasing step times that were at least 10 times larger than the fundamental period, in order to converge on a quasi-static solution. Finally, kinetic and internal energies results were checked during post-processing to ensure that dynamic effects were insignificant.

As an illustration, consider the quasi-static indentation of the aluminium sandwich plate. The fundamental period (T) of the plate was first found to be 0.23 ms, and quasi-static analyses were then carried out with three different step-times (2.3 ms, 4.6 ms and 6.9 ms). Figure 5.4 shows the load-displacement plots of the plate indented to a maximum displacement of 6.0 mm. The results for the three cases were within approximately 5% of each other, and the load-displacement history became less oscillatory with increasing step-time. The kinetic energy history for the plate in the second attempt (step-time of 4.6 ms) was almost negligible as compared to its internal energy history throughout the step (Fig. 5.5), which indicated a quasi-static solution. Thus the results showed that a step time that was 20 times greater than T was adequate for convergence in this case. Similar steps were carried out to analyse composite sandwich plates; for these plates, experience indicated that a step time that was 10 times greater than the corresponding T was able to

achieve converged results.

Figure 5.4: Load-displacement plots for explicit quasi-static analyses of aluminium sandwich plates with three different step times.

Figure 5.5: Kinetic and internal energy histories plotted against displacement for the case of $t_{step} = 20T$.

5.2.3 Mesh convergence

Mesh convergence is another issue that is paramount to the accuracy of the FE solution. Coarse meshes can give inaccurate results, while computational expenses are higher for finer meshes. As the mesh density is increased, the numerical solution will converge toward an unique value. In order to achieve a compromise between efficiency and accuracy, a mesh convergence study was carried out to ensure the mesh refinement in the sandwich model was sufficiently fine enough.

To illustrate, four mesh refinements for the aluminium sandwich plate subjected to an impact energy of 7.0 J were created, and the results of each of the four mesh densities are compared and tabulated in Table 5.1. The influence of the mesh density on the displacement of the mid-point of the top facesheet, and on the peak von Mises stress at the mid-point of bottom facesheet, was considered. The coarsest mesh, mesh A, predicted less accurate results for the displacement at the top facesheet, but the results were close for the remaining meshes. Hence for displacements, the results had converged for mesh B. However, the peak stress on the bottom facesheet converged much more slowly than the displacements. Because stresses and strains are calculated from the displacement gradients, a much finer mesh is required to predict accurate displacement gradients than is needed to calculate accurate displacements. As the difference in stress values for meshes C and D was less than 2%, mesh D was deemed to have converged. Figure 5.6 illustrates the load histories of the FE models with meshes C and D. The difference in the results is not large. Similar mesh convergence studies were carried out for other models.

Table 5.1: Results of mesh convergence study.

Mesh	No. of elements	Displacement of mid-point of top facesheet (mm)	Stress of mid-point of bottom facesheet (MPa)
A	11847	-4.28	93.4
B	15875	-4.43	85.6
C	17667	-4.44	70.8
D	23755	-4.46	69.5

Figure 5.6: Load-time plots of meshes C and D.

5.3 Material Model for Aluminium Sandwich Plate

The FE model was first validated by modelling an aluminium honeycomb sandwich panel primarily because aluminium is isotropic and its material properties are well-established. The facesheets and the core for the aluminium sandwich plates were defined in ABAQUS/Explicit as isotropic, bilinear, plastic materials, the properties of which are presented in Table 5.2. The symbols ρ , σ_y , σ_u and ν denote density, yield strength, tensile strength and Poisson's ratio respectively. The bilinear model assumed a tangent modulus, E_t , which was $0.7 \times$ the Young's modulus, E with linear strain-hardening. This is cited to be typical for aluminium alloys in [138]. Studies have indicated that aluminium alloys are essentially rate-insensitive [158], and thus rate dependency was not considered.

Table 5.2: Material properties for aluminium panels

Property	Facesheets (1100-H14 aluminium alloy)	Core (3003-H19 foil aluminium alloy)
ρ (kg/m^3)	2700	2700
E (GPa)	70.0	70.0
σ_y (MPa)	117	220
σ_u (MPa)	124	250
ν	0.33	0.33

5.4 Material Model for Composite Sandwich Plate

Unlike for metals, where extensive information on the dynamic material properties at high strain rates are available in the literature, there are no universally accepted material laws for composite materials in dynamic simulations. As remarked by Atkay et al. [5], the dynamic failure behaviour of composites is difficult and complicated to model, compounded by several reasons: the wide range of fibres and matrices available, the possibility of interaction between fibre-dominated or matrix-dominated failure modes, and the strain-rate dependency of material properties.

In this section, the material models for the Nomex honeycomb core and the carbon-epoxy laminates are presented. According to several studies [14, 22, 159], modern carbon/epoxy systems show no sensitivity to strain-rate effects, so strain-rate dependency was not considered. For Nomex honeycombs, the strain-rate effect has also been investigated experimentally [45, 47]. Goldsmith et al. [47] carried out flatwise compression tests under high loading rates (10–40 m/s) and found that the increase in dynamic crush strength of Nomex honeycombs is only about 10% higher than the static value. Similarly, Heimbs et al. [45] reported a 10% increase in the crush strength for stabilised compression of Nomex specimens at a strain-rate of 150 s^{-1} . Given that the loading rates in the current impact tests were relatively low, the strain-rate effects for the Nomex honeycombs were also not included for simplicity.

5.4.1 Nomex honeycomb core

For the cellular core, the cell-wall solid was modelled as an isotropic, elastic perfectly-plastic material under compression, which is a reasonable assumption for the observed behavior of typical honeycombs [7, 46, 82, 99]. Measured nonlinear compressive stress-strain data under uniaxial loading, as described in Chapter 3, were used to curve fit the idealised form of the material model. In doing so, the out-of-plane loading (in 3-direction) was assumed to have little effect on failure in the in-plane directions, and that the in-plane failures of the core were inconsequential to the failure behaviour of the core [43]. This assumption is justified by the fact that the elastic stiffnesses in the in-plane direction are typically one to two orders of magnitudes lower than those in the out-of-plane direction [45, 139]. In addition, the constitutive response for honeycombs under tension, which is different from that under compression [46], was not considered since out-of-plane tensile loads are rarely applied to a honeycomb in most practical applications [52].

On the other hand, the continuum core which surrounded the cellular core in 3 mm cell-sized core was modelled as an orthotropic elastic solid, defined by the elastic stiffnesses in the 1, 2, and 3-directions. The out-of-plane properties were again determined directly from the flatwise compression test data. However, the in-plane properties are seldom tested and values are usually not provided by the manufacturers as they are extremely low [139]. Heimbs et al. [45] carried out in-plane crushing tests on Nomex honeycombs and reported that the in-plane stiffnesses, expressed in terms of factors, are 140 and 250 times smaller for the 2-direction and the 1-direction, respectively, as compared to the out-of-plane value. Sometimes, the factors could also be in the range of 1000 [139]. Nevertheless, initial FE simulations indicated that the in-plane stiffnesses had a negligible effect on the load-deformation response of the cores subjected to out-of-plane crushing, and thus a factor of 100 was assumed.

In order to qualify the material model for the core, a separate FE investigation was conducted to simulate the out-of-plane compression behaviour of bare honeycombs. As an

illustration, Fig. 5.7 compares the experimental and predicted load-displacement response for 3 mm and 13 mm cell-sized Nomex honeycombs with a core height of 15 mm. For the 13 mm cell-sized honeycombs, after an initial nonlinear region, possibly due to the establishing of contact evenly across the specimen, the test specimen exhibited a linear behavior before failure occurred in the form of a load drop, to about half the maximum value. The numerical model shows a reasonable agreement with the experimental crushing response, in terms of the linear slope, the maximum load, as well as the crushing load after failure. In the FE model, yielding was observed at the top edges of the honeycomb cells at the peak compressive load, which indicated the initiation of damage. As core crushing propagated, progressive plastic folding of the cell-walls was observed (Fig. 5.8(a)).

Figure 5.7: Load-displacement curves of (a) 13-15 honeycomb core of 9 cells, and (b) 3-15 honeycomb core of 33 cells under flatwise compression; numerical curves slightly displaced to facilitate comparison.

Figure 5.8: Deformation plots of 13-15 Nomex honeycombs under flatwise compression at approximately 30% compressive strain from (a) FE analysis, and (b) experiment.

5.4.2 Progressive damage model for composite laminate

A progressive damage model, which integrated stress analysis, failure analysis and material property degradation, was implemented in ABAQUS/Explicit using an user subroutine (VUMAT) to predict failure in the composite laminates. The flowchart in Fig. 5.9 depicts the sequence of events in the analysis. At every time step, element strain increments at each Gauss integration point (material point) were passed into the subroutine. Assuming a linear elastic orthotropic behaviour, trial stresses were calculated and then tested for failure using the specified failure criteria. If failure occurred, material properties were degraded, and stresses were recalculated with the modified material properties. Subsequently, the material properties were stored as state variables and returned to the solver, along with the stresses, at the end of each time increment. Finally, the analysis proceeded on to the next time step, and the whole procedure was repeated.

Figure 5.9: Flowchart that depicts the sequence of events each time the user-defined subroutine for progressive damage model is called.

Linear elastic stress analysis

Stress analysis was first performed by assuming a three-dimensional (3D) linear elastic orthotropic behaviour for the composite laminates. Trial stresses were calculated based on the stress-strain relationship:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & 0 & 0 & 0 \\ & D_{2222} & D_{2233} & 0 & 0 & 0 \\ & & D_{3333} & 0 & 0 & 0 \\ & & & D_{1212} & 0 & 0 \\ & symm & & & D_{1313} & 0 \\ & & & & & D_{2323} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \quad (5.5)$$

A local orthogonal coordinate system is defined in which the 1- and 2-axes are parallel and transverse to the fibres, respectively, and the 3-axis coincides with the through-thickness direction. The terms of the stiffness matrix, D_{ijkl} , are related to the 9 engineering constants associated with the material's principal directions by

$$D_{1111} = E_1(1 - \nu_{23}\nu_{32})\Upsilon$$

$$D_{2222} = E_2(1 - \nu_{13}\nu_{31})\Upsilon$$

$$D_{3333} = E_3(1 - \nu_{12}\nu_{21})\Upsilon$$

$$D_{1122} = E_1(\nu_{21} + \nu_{31}\nu_{23})\Upsilon$$

$$D_{1133} = E_1(\nu_{31} + \nu_{21}\nu_{32})\Upsilon$$

$$D_{2233} = E_2(\nu_{32} + \nu_{12}\nu_{31})\Upsilon$$

$$\Upsilon = \frac{1}{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13}}$$

Failure analysis

After the trial stresses were computed, a set of failure criteria was used to detect failure. There are three main forms of failure in the composite laminate: fiber failure, matrix

failure, and delamination. The stress-based 3D Hashin criteria [106] were used to predict the initiation of four in-plane failure modes in the matrix and fibre under both tension and compression. The Hashin criteria were chosen because they could distinguish individual failure modes and the order in which they occur. In addition the criteria have been relatively successful when used to predict failure for carbon/epoxy laminates in previous studies (see for example [40, 117]).

For the case of fibre tensile failure, the interaction of the shear stresses was found to result in conservative predictions, as also observed in [100]. Moreover, according to [101], the physical basis of the contribution of the shear stresses to the failure of the fibres under tension is not exactly clear. Thus the contribution of the shear stresses to the failure of fibres was ignored, and the criterion for fibre tensile fracture simplified to the maximum stress criterion (Eq. 5.6). Accordingly, the Hashin criteria in the three-dimensional form are given as:

Fibre tensile fracture, $\sigma_{11} \geq 0$:

$$\sigma_{11} = X_t \quad (5.6)$$

Fibre compressive fracture, $\sigma_{11} < 0$:

$$\sigma_{11} = X_c \quad (5.7)$$

Matrix tensile or shear cracking failure, $(\sigma_{22} + \sigma_{33}) \geq 0$:

$$\frac{(\sigma_{22} + \sigma_{33})^2}{Y_t^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2 - \sigma_{22}\sigma_{33}}{S^2} = 1 \quad (5.8)$$

Matrix compressive failure, $(\sigma_{22} + \sigma_{33}) < 0$:

$$\frac{1}{Y_c} \left[\left(\frac{Y_c}{2S} \right)^2 - 1 \right] (\sigma_{22} + \sigma_{33}) + \frac{(\sigma_{22} + \sigma_{33})^2}{4S^2} + \frac{\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2 - \sigma_{22}\sigma_{33}}{S^2} = 1 \quad (5.9)$$

where the σ_{ij} terms are components of the stress tensor. The notation of the above quantities refer to the same local layer coordinate system described earlier. The quantities in the denominators are the strengths in the corresponding directions.

Some researchers have used a stress-based criterion to model delamination [100, 112,

117, 129] due to its relative ease of implementation in explicit FE codes. Hou et al. [112] argued that compressive stress constrains crack opening, and thus delamination should not be allowed to occur while an element is under compression in the through-thickness direction. Previous experimental studies have also found that there is no delamination in a narrow band immediately adjacent to the impact point [129], which suggests that the stress field in that area does not reach a level to initiate delamination. Thus, only delamination under tensile σ_{33} was considered, and it was assumed that interlaminar delamination initiated under the following criterion:

$$\text{Delamination, } \sigma_{33} \geq 0 : \left(\frac{\sigma_{33}}{Z_t} \right)^2 + \frac{\sigma_{23}^2 + \sigma_{13}^2}{S^2} = 1 \quad (5.10)$$

Material property degradation

Once failure was predicted in an element, damage was simulated by degrading the material properties of the failed elements. Material degradation was carried out by assuming that damage within an element had an effect on the elastic properties of that element only. The aim was to inhibit the element's load-carrying capabilities according to the mode of failure.

When matrix failure was detected, it was assumed that the matrix could not carry any additional load, and E_{22} of the failed element was reduced to zero. For fibre failure, the material point was flagged for deletion. Consequently, all the elastic properties were reduced to zero, and zero stresses and strain increments were passed into the subroutine for the deleted material point. When delamination was detected, the material was assumed to lose its ability to carry the load in the out-of-plane direction, and thus E_{33} , G_{13} and G_{23} were reduced to zero. Table 5.3 summarises the degradation rules. In order to avoid numerical instability during solution, the degraded material properties were set to a small value such that the effective stiffness was zero while still retaining a 'numerical' stiffness [96]. In addition, due to restrictions on the engineering properties of composites [160], the Poisson's ratios were degraded for compatibility.

Table 5.3: Material property degradation rules

Mode of failure	Material property degradation rules
Matrix tensile cracking	$E_{22} = \nu_{12} = 0$
Matrix compressive failure	$E_{22} = \nu_{12} = 0$
Fibre tensile failure	Material point deleted (all stiffnesses and stresses set to 0)
Fibre compressive failure	Material point deleted (all stiffnesses and stresses set to 0)
Delamination under tension	$E_{33} = G_{13} = G_{23} = \nu_{13} = \nu_{23} = 0$

Implementation in ABAQUS/Explicit

Any material properties that were required in the user subroutine (VUMAT) to define the failure model had to be directly specified in the ABAQUS input file using certain keywords. Examples of the input files are provided in Appendix C. Such properties include the engineering constants for the laminate which were declared as solution-dependent state variables. Space for these state variables at each material point were first allocated using the *DEPVAR keyword. The material properties were then initialised using the *INITIAL CONDITIONS keyword. On the other hand, the constant laminate strengths were defined using the *USER MATERIAL keyword. In addition, an index and a flag for each failure mode were declared as state variables. All failure indices and flags were initialised to zero. Subsequently during analysis, when a failure index exceeded unity, the corresponding failure flag would be set to a value of one to indicate failure initiation. The exception to this is the element deletion flag for fibre failure. In ABAQUS, the element deletion flag must be initialised to a value of one to indicate that the material point is active; upon fibre failure, the flag is then set to zero for the material point to be deleted [154].

In the VUMAT subroutine, the algorithm for the damage model was implemented as follows:

Do the following for all material points:

Read in all state variables at the beginning of the increment.

Read in laminate strengths.

Read in strain increments $\Delta\epsilon$.

Compute the stiffness matrix $\mathbf{D}$.

Calculate elastic trial stresses, $\sigma_{\text{new}}^{\text{trial}}$:

$$\sigma_{\text{new}}^{\text{trial}} = \sigma_{\text{old}} + \mathbf{D}\Delta\epsilon$$

Using trial stresses, calculate failure index for fibre failure.

If index > 1 , then flag material point for deletion.

End if.

Similarly, calculate failure index for matrix failure.

If index > 1 and $(\sigma_{22} + \sigma_{33} > 0)$, then

Flag material point for matrix tensile failure.

Else if index > 1 and $(\sigma_{22} + \sigma_{33} < 0)$, then

Flag material point for matrix compressive failure.

End if.

Also calculate failure index for delamination under tension.

If index > 1 , then

Flag material point for delamination failure under tension.

If material point is flagged for matrix failure or delamination, then

Reassign degraded material properties.

Recalculate new stresses using new material properties.

End if.

Update new stress state and new state variables at end of increment.

End Do.

5.5 Summary

A three-dimensional FE model for the simulation of impact and quasi-static indentation of sandwich structures has been presented. In contrast to the equivalent continuum core normally used [3–5], the cellular honeycomb core was discretely modelled with shell

elements so that it is geometrically more accurate. The aim was to obtain more realistic distributions of stresses and strains. The modelling method would be validated first through the analysis of aluminium sandwich specimens, whose material properties are well-established.

Subsequently, a progressive failure analysis for composite sandwich structures was also developed. Failure criteria and material degradation rules were incorporated into a progressive damage model for the composite laminate, which was implemented as an user subroutine in ABAQUS/Explicit. Although the adopted failure criteria were based on the work of other researchers, the current combination of failure criteria and degradation laws proposed for the laminated skins, coupled with the geometrically correct honeycomb core, presents a new investigation in the numerical analysis of composite sandwich structures. A FE sandwich model capable of predicting failure would facilitate design processes in engineering structures, and minimise the amount of prototyping and testing required in the development of impact-resistant sandwich structures.

Chapter 6

Results and Discussions

All experimental and numerical results are presented in this chapter, and they are divided into three sections. In the first section, experimental results which consist of both aluminium and composite sandwich panels subjected to quasi-static indentation and low-velocity impact loadings are presented. Next, numerical results from the finite element models which were described in Chapter 5 are validated with experimental results. Finally, results from the analytical impact model that was developed to predict the impact response of the sandwich plate are presented. Where appropriate, analytical and FE predictions are compared with the corresponding experimental data.

6.1 Static Indentation and Low-velocity Impact Tests

Experimental results for aluminium sandwich plates are first presented and discussed. Both quasi-static indentation and dynamic impact tests are considered and compared. These are followed by the results obtained for composite sandwich plates. A new technique is then introduced to characterise the impact response of the composite sandwich plates and derive an empirical equation based on the least-squares method. Essentially, this equation quantifies the energy absorbed by the sandwich composite by correlating the impact energy and the damage initiation threshold energy of the plate.

6.1.1 Aluminium sandwich panels

Static indentation tests

Quasi-static indentation tests were conducted on three aluminium sandwich panels, and the load-displacement curves for these samples are presented in Fig. 6.1. Good repeatability was observed in terms of the load-displacement response. Initially the slope of the curve was steep, and then it decreased when the load reached around 500 N. Subsequently, the load rose almost steadily up to the ultimate failure load (P_f), where a sudden load drop occurred. The area under the load-deflection curve up to P_f was calculated to give the static energy for failure (E_s). The results for the three specimens were $P_f = 2.83 \pm 0.09$ kN and $E_s = 7.64 \pm 0.44$ J (mean $\pm$ standard deviation).

Figure 6.1: Load-displacement curves for aluminium panels subjected to static indentation.

In a separate diagnostic test, an identical aluminium sandwich panel was indented up to various loads at three different positions on the top skin. Subsequently, the indented facesheet was removed from the specimen using a pair of pliers to reveal the underlying damage in the core, as shown in Fig. 6.2. When the transverse point load was first applied

up to 500 N, there was hardly any visible indentation on the skin, suggesting that plastic deformation had not yet occurred. For the loading up to 1750 N, the indentation was more obvious but there was no fracture. However, when the specimen was loaded to P_f , a circumferential crack which measured roughly half the circumference of the indenter was present on the indented facesheet. Other tiny cracks also appeared to propagate from the initial circumferential crack. This strongly suggests that the indented plate failed by the fracture of the top skin. Because the core thickness was greater than the indenter's radius, penetration of the top skin would have occurred before the applied force could act on the lower skin, and thus no damage was observed on the bottom facesheet. In the core, the cell walls had crushed to a maximum distance of about 2 mm directly beneath the point where fracture had occurred. No significant damage to the core was observed for the other two loadings.

Figure 6.2: Damage sustained in (a) indented facesheet and (b) core of aluminium sandwich plate under various quasi-static loads.

Impact tests on aluminium sandwich panels

Next, load-time histories and load-deflection plots for four samples of aluminium sandwich panels impacted at 7.0 J by the 2.65 kg impactor are shown in Fig. 6.3. Very little scatter is observed in these curves, and this consistency again emphasises the overall reliability of the present experimental procedure. Therefore, only one set of test results

is presented for each sandwich plate in subsequent graphs. The load-deflection curves in Fig. 6.3 exhibited a steep initial stiffness immediately upon impact up to 0.5 kN, where the stiffness reduced and remained relatively constant up to the maximum load of 2.6 kN, before unloading occurred.

Figure 6.3: Load-time and load-displacement plots for impact on aluminium sandwich plates at 7 J.

Identical aluminium sandwich specimens were also impacted at four other impact energies of 0.85, 2.0, 10.0, and 13.0 J. The load-deflection curves for the five impact cases are shown in Fig. 6.4, with the quasi-static curve also included for comparison (from Fig. 6.1). For a structure subjected to a quasi-static loading, a load-deflection curve depicts its equilibrium states before, during and after damage [9]. By comparing the load-displacement curves of the static and impact events, the influence of the loading rates can be evaluated. Accordingly, any difference between a quasi-static load-displacement curve and that of a impact process can be attributed to the inertial effect present in the dynamic impact process.

In Fig. 6.4, the loading paths of all impact events follow the static curve closely,

Figure 6.4: Comparison of the load-deflection curves of impact tests at increasing energy levels with the quasi-static indentation curve for aluminium sandwich plates.

implying that the low-velocity impact is quasi-statically equivalent. For impact energies less than E_s , unloading occurred at the respective peak loads. On the other hand, for impact energies greater than E_s (10.0 and 13.0 J), the peak impact loads exceeded the static failure load P_f and reached 3 kN approximately. Subsequently, there was a load drop at the peak load before unloading occurred at a lower load. Figure 6.5 presents the photographs of the damage on the impacted facesheets taken after testing. Dents generally increased in size with increasing impact energy. When the impact energy was 7 J and below, there was no crack or tear in the facesheet, unlike for the cases of 10 and 13 J. Comparing the damage sustained on the top skins in Figs. 6.2(a) and 6.5, the damage before and after ultimate failure appears to be similar in both indentation and impact tests.

Essentially, the quasi-static nature of low-velocity impact events allows the results from static indentation tests to be directly used for impact tests, provided that E_{imp} is lower than E_s . Figure 6.6 shows the quasi-static load-energy curve for the aluminium

Figure 6.5: Damage observed on top facesheets for four impact energies: (a) 2, (b) 7, (c) 10, and (d) 13 J.

Figure 6.6: Quasi-static load-energy curve and superimposed peak load vs. impact energy test data for aluminium sandwich panels.

sandwich panel, in which the energy on the x-axis is directly obtained by integrating the load-deflection curve. Also included in Fig. 6.6 are the peak load vs. impact energy data points for the five impact cases. For impact energies lower than E_s , the data points agreed very well with the static curve. As pointed out in Ref. [19], the quasi-static energy curve is useful because it allows the extrapolation of peak impact loads that are sustained by the plate at various impact energies below E_s . As such, a single quasi-static load-energy curve could produce the equivalent impact load-energy data, which would otherwise be obtained from a large number of impact tests.

Figure 6.7: Ratio of absorbed energy (E_{abs}) to static energy for failure (E_s) plotted against impact energy (E_{imp}) for aluminium sandwich panels.

Next, the absorbed energy E_{abs} is normalised by the static energy at failure E_s and plotted against impact energy E_{imp} , as shown in Fig. 6.7. The energy ratio (E_{abs}/E_s) increases linearly with increasing impact energy; and by using the linear least-squares regression method based on the five impact data points, $E_{abs}/E_s = 0.128E_{imp}$. The energy ratio is unity at an impact energy of 7.8 J approximately. When the energy ratio is greater

than 1, fracture and tearing were evident on the impacted facesheets (Fig.6.5(c) and (d)). As highlighted earlier, this damage mode closely resembles the one observed in the static test which was loaded beyond the ultimate load (Fig. 6.2). Conversely, when the ratio is less than 1, the only observed damage on the impact facesheet is a dent (Fig. 6.5(a) and (b)).

The preceding observations suggest that the amount of absorbed energy could serve as an indicator to the extent of damage sustained in a low-velocity impact, as demonstrated by previous studies [9, 30, 34]. This result is expected because the strain energy released during damage progression can be viewed as a major source of energy loss. Plastic deformation in the aluminium sandwich plate appears to constitute a majority of this absorbed energy (Fig. 6.5). Given that it is almost impossible to precisely measure impact damage in the core and the skin using current damage assessment methods, the use of absorbed energy as a damage measure is a significant alternative. By presenting the test data using the normalised energy ratio in Fig. 6.7, one could easily estimate the impact energy at which top skin fracture is likely to initiate. Alternatively, the energy absorbed by the plate at a particular E_{imp} could also be determined using the same curve in Fig. 6.7, provided E_s is known.

6.1.2 Composite sandwich plates

Typical bending behaviour of clamped sandwich panels

The typical load-displacement curve for clamped composite sandwich panels loaded up to ultimate failure under quasi-static loading is shown in Figure 6.8. There were three deformation stages that could be characterised by three critical loads: threshold load for initial damage (P_1), load for secondary damage (P_2), and load at ultimate failure (P_f). The stiffnesses of the plate were represented by the slopes of the load-displacement curve. Initially, the structure deformed in a linear elastic manner until the initial damage threshold load was reached. At this point, the slope of the curve suddenly decreased which indicated the onset of initial damage, as also observed in [13, 24, 34, 54]. As the

load increased, a number of minor load drops occurred without significantly affecting the stiffness of the structure. In this second deformation stage, the slope between the threshold and secondary damage loads was almost linear, although there was a noticeable steepening of the slope towards the secondary damage load. Membrane stretching of the top skin was likely to be significant due to the small specimen size with a clamped boundary. The load drop after P_2 was typically between 100–500 N.

Figure 6.8: Typical load-displacement curve of composite sandwich plate C1/13/15 loaded by hemispherical indenter.

Subsequently, the final deformation stage was characterised by a number of smaller load drops which occurred more frequently as the panel approached ultimate failure. Such behaviour was due likely to the crushing of honeycomb cells in conjunction with significant damage propagation in the top skin. In addition, the slope after P_2 was more erratic, and sometimes nonlinear. For simplicity, the post-secondary damage region was linearised to determine the slope. Ultimate failure was characterised by a sudden major load drop, which represented an abrupt loss of load-bearing capability for the plate. In some cases, this load drop occurred at a load lower than the secondary damage load. Nevertheless, the load drop always coincided with an audible crack observed in the tests,

which suggests the fracture of the top skin, as also reported in other tests [15, 34, 40]. Given that the loaded top skin contributed largely to the damage resistance of the structure, the failure of the top skin was regarded as the ultimate failure in this study. No debonding between the top skin and core was detected for the specimens, and all bottom skins remained intact after ultimate failure.

The critical loads and the slopes of the load-displacement curves for the composite sandwich plates in the static indentation tests are summarised in Tables 6.1 and 6.2. The critical loads were generally very repeatable (Table 6.1). The initial damage threshold loads for all the plates were between 26% and 40% of the respective ultimate loads. For sandwich plates with thicker skins (plates with C3 skins), the critical loads were higher, which is consistent with earlier findings reporting that an increase in skin thickness increases the initial damage threshold load [9, 13, 54]. One explanation is the greater flexural rigidity and local contact stiffness associated for such plates with thicker facesheets. The thicker skin absorbs more energy in bending and distributes the contact force over a greater area. Consequently, the core becomes more shielded from the contact stresses, and the onset of damage in the core happens at a greater load. Conversely for sandwich plates with thin facesheets, the contact force is transferred directly to the core over a relatively smaller area. As a result, core damage is expected to be more severe and this may also explain why the loss of stiffness after P_1 was larger for the plates with thinner skins (Table 6.2).

Table 6.1: Summary of critical load and displacement values for composite sandwich panels under quasi-static indentation.

Specimen	Initial damage		Secondary damage		Top skin fracture	
	Load (kN)	Displ. (mm)	Load (kN)	Displ. (mm)	Load (kN)	Displ. (mm)
C1/3/15-1	1.01	1.00	3.06	4.72	3.86	8.26
C1/3/15-2	1.15	1.06	3.13	4.18	3.30	6.00
C1/3/15-3	1.03	1.06	2.94	4.25	4.01	7.14
Average $\pm$ SD	1.06 $\pm$ 0.08	1.04 $\pm$ 0.03	3.04 $\pm$ 0.10	4.38 $\pm$ 0.29	3.72 $\pm$ 0.37	7.13 $\pm$ 1.13
C1/6/15-1	0.98	0.92	3.05	4.20	3.11	5.94
C1/6/15-2	0.92	0.90	3.03	4.00	3.58	7.17
C1/6/15-3	1.04	0.93	3.07	4.05	3.08	5.12
Average $\pm$ SD	0.98 $\pm$ 0.06	0.92 $\pm$ 0.02	3.05 $\pm$ 0.02	4.08 $\pm$ 0.10	3.26 $\pm$ 0.28	6.08 $\pm$ 1.03
C1/13/15-1	0.98	0.84	3.16	4.26	3.18	5.94
C1/13/15-2	1.06	1.01	3.43	4.33	3.73	7.23
C1/13/15-3	1.09	1.15	3.61	4.82	3.56	7.04
Average $\pm$ SD	1.04 $\pm$ 0.06	1.00 $\pm$ 0.15	3.40 $\pm$ 0.23	4.47 $\pm$ 0.30	3.49 $\pm$ 0.28	6.74 $\pm$ 0.70
C1/3/20-1	1.01	0.87	2.90	3.63	3.07	5.15
C1/3/20-2	1.01	0.93	3.26	4.23	3.22	5.49
C1/3/20-3	1.00	1.08	2.90	3.66	3.16	5.27
Average $\pm$ SD	1.01 $\pm$ 0.01	0.96 $\pm$ 0.11	3.02 $\pm$ 0.21	3.84 $\pm$ 0.34	3.15 $\pm$ 0.07	5.30 $\pm$ 0.18
C1/6/20-1	0.95	0.96	2.74	3.40	3.58	6.69
C1/6/20-2	1.08	1.05	3.20	4.60	3.66	7.54
C1/6/20-3	0.98	0.98	3.07	4.17	2.64	4.43
Average $\pm$ SD	1.00 $\pm$ 0.07	1.00 $\pm$ 0.05	3.01 $\pm$ 0.24	4.06 $\pm$ 0.60	3.29 $\pm$ 0.57	6.22 $\pm$ 1.61
C1/13/20-1	1.00	0.88	3.30	4.09	3.52	5.70
C1/13/20-2	1.04	0.89	3.31	4.56	3.44	6.09
C1/13/20-3	0.97	0.92	2.90	3.92	3.00	5.58
Average $\pm$ SD	1.00 $\pm$ 0.04	0.90 $\pm$ 0.02	3.17 $\pm$ 0.23	4.19 $\pm$ 0.33	3.32 $\pm$ 0.28	5.79 $\pm$ 0.26
C1/3/25-1	1.02	0.84	3.16	3.80	3.45	5.74
C1/3/25-2	1.02	0.86	3.15	3.89	2.84	4.67
C1/3/25-3	1.05	0.88	3.23	3.94	3.70	5.82
Average $\pm$ SD	1.03 $\pm$ 0.01	0.86 $\pm$ 0.02	3.18 $\pm$ 0.04	3.88 $\pm$ 0.07	3.33 $\pm$ 0.45	5.41 $\pm$ 0.64
C1/6/25-1	0.97	0.92	3.16	3.85	3.45	5.52
C1/6/25-2	1.08	0.91	3.31	3.99	3.74	5.87
C1/6/25-3	0.95	0.90	2.96	3.69	3.48	6.24
Average $\pm$ SD	1.00 $\pm$ 0.07	0.91 $\pm$ 0.01	3.14 $\pm$ 0.17	3.84 $\pm$ 0.15	3.56 $\pm$ 0.15	5.88 $\pm$ 0.36
C1/13/25-1	1.02	0.84	3.23	3.81	3.70	5.80
C1/13/25-2	1.08	0.90	3.56	4.10	3.64	6.45
C1/13/25-3	1.15	0.89	3.30	4.30	3.27	6.14
Average $\pm$ SD	1.08 $\pm$ 0.06	0.88 $\pm$ 0.03	3.36 $\pm$ 0.17	4.07 $\pm$ 0.25	3.54 $\pm$ 0.23	6.13 $\pm$ 0.33
C2/3/15-1	0.95	0.87	3.30	3.55	3.40	3.87
C2/3/15-2	0.97	0.83	3.35	3.60	3.44	3.98
Average $\pm$ SD	0.96 $\pm$ 0.01	0.85 $\pm$ 0.02	3.32 $\pm$ 0.03	3.57 $\pm$ 0.04	3.42 $\pm$ 0.03	3.92 $\pm$ 0.07
C2/13/15-1	1.04	0.95	3.58	4.01	3.40	4.23
C2/13/15-2	1.14	1.03	-	-	3.43	3.89
Average $\pm$ SD	1.09 $\pm$ 0.07	0.99 $\pm$ 0.05	3.58	4.01	3.41 $\pm$ 0.03	4.06 $\pm$ 0.24

Continued on next page

Table 6.1 – Continued.

Specimen	Initial damage		Secondary damage		Top skin fracture	
	Load (kN)	Displ. (mm)	Load (kN)	Displ. (mm)	Load (kN)	Displ. (mm)
C3/3/15-1	1.51	1.04	4.52	4.13	4.87	5.03
C3/3/15-2	1.51	1.10	4.04	4.05	3.96	5.10
C3/3/15-3	1.50	0.90	4.37	3.70	4.73	5.29
Average $\pm$ SD	1.51 $\pm$ 0.01	1.01 $\pm$ 0.10	4.31 $\pm$ 0.24	3.96 $\pm$ 0.23	4.52 $\pm$ 0.49	5.14 $\pm$ 0.13
C3/6/15-1	1.59	1.11	3.67	3.45	3.94	5.23
C3/6/15-2	1.45	0.87	3.98	3.71	4.78	5.46
C3/6/15-3	1.55	1.13	3.67	3.24	5.30	5.41
Average $\pm$ SD	1.53 $\pm$ 0.07	1.03 $\pm$ 0.14	3.77 $\pm$ 0.18	3.46 $\pm$ 0.24	4.67 $\pm$ 0.69	5.37 $\pm$ 0.12
C3/13/15-1	1.55	0.88	4.05	4.17	4.66	5.12
C3/13/15-2	1.38	0.78	3.79	3.69	4.37	4.89
C3/13/15-3	1.45	0.80	3.89	3.59	4.50	4.87
Average $\pm$ SD	1.46 $\pm$ 0.09	0.82 $\pm$ 0.06	3.91 $\pm$ 0.13	3.82 $\pm$ 0.31	4.51 $\pm$ 0.15	4.96 $\pm$ 0.14
C3/3/20-1	1.48	0.96	4.58	3.85	4.43	4.01
C3/3/20-2	1.62	1.02	4.14	3.46	4.21	4.87
C3/3/20-3	1.25	0.90	4.32	3.95	3.80	4.23
Average $\pm$ SD	1.45 $\pm$ 0.19	0.96 $\pm$ 0.06	4.35 $\pm$ 0.22	3.75 $\pm$ 0.26	4.15 $\pm$ 0.32	4.37 $\pm$ 0.45
C3/6/20-1	1.50	1.33	4.13	3.83	4.85	5.11
C3/6/20-2	1.31	0.97	3.90	3.67	4.95	6.07
Average $\pm$ SD	1.41 $\pm$ 0.13	1.15 $\pm$ 0.26	4.01 $\pm$ 0.16	3.75 $\pm$ 0.11	4.90 $\pm$ 0.07	5.59 $\pm$ 0.68
C3/13/20-1	1.33	0.74	3.77	3.25	4.86	5.27
C3/13/20-2	1.42	0.86	4.48	3.57	4.42	4.17
C3/13/20-3	1.44	0.81	4.09	3.53	4.28	4.78
Average $\pm$ SD	1.39 $\pm$ 0.06	0.80 $\pm$ 0.06	4.12 $\pm$ 0.36	3.45 $\pm$ 0.18	4.52 $\pm$ 0.30	4.74 $\pm$ 0.55
C3/3/25-1	1.45	0.88	4.23	3.50	4.37	4.37
C3/3/25-2	1.41	0.88	4.38	3.70	4.77	4.94
C3/3/25-3	1.47	0.90	4.39	3.91	4.37	5.26
Average $\pm$ SD	1.44 $\pm$ 0.03	0.88 $\pm$ 0.01	4.33 $\pm$ 0.09	3.70 $\pm$ 0.20	4.50 $\pm$ 0.23	4.85 $\pm$ 0.45
C3/6/25-1	1.72	0.99	4.16	3.37	4.97	5.94
C3/6/25-2	1.47	0.82	-	-	4.72	4.11
C3/6/25-3	1.45	0.81	4.17	3.63	4.63	4.71
Average $\pm$ SD	1.55 $\pm$ 0.15	0.87 $\pm$ 0.10	4.16 $\pm$ 0.01	3.50 $\pm$ 0.19	4.77 $\pm$ 0.18	4.92 $\pm$ 0.93
C3/13/25-1	1.46	1.10	4.24	3.59	4.17	4.62
C3/13/25-2	1.64	0.88	4.57	3.70	4.62	4.64
C3/13/25-3	1.47	1.05	4.51	3.96	3.92	5.19
Average $\pm$ SD	1.52 $\pm$ 0.10	1.01 $\pm$ 0.11	4.44 $\pm$ 0.18	3.75 $\pm$ 0.19	4.24 $\pm$ 0.35	4.82 $\pm$ 0.32

Similarly, panels with thicker skins exhibited greater stiffnesses in the linear elastic and secondary damage regions (Table 6.2). Panels with thicker cores were also stiffer under bending in the linear elastic region, which could be attributed to the increased transverse shear resistance offered by the thicker cores. However, after the onset of initial damage, the stiffnesses in the secondary damage regions were relatively similar

for plates with the same skins. For plates with C1 and C2 skins (same thickness), the stiffness losses after initial damage averaged approximately 50% and 40%, respectively. This difference may be attributed to the stacking sequence of the skins. According to studies cited in [14], composites having $+/- 45^\circ$ surface plies protect the underlying 0° load-bearing plies from damage and hence are more damage-resistant than $(0^\circ, 90^\circ)$, $(0^\circ, +/- 45^\circ)$ and $(0^\circ, 90^\circ, +/- 45^\circ)$ laminates. On the other hand, the stacking sequence does not seem to influence the critical loads for plates with C1 and C2 skins (Table 6.1). This is similar to previous results that found that the stacking sequence had no measurable effect on the energy required for damage initiation for composite laminates [2].

Table 6.2: Summary of stiffness of load-displacement curves for composite sandwich panels under quasi-static indentation.

Specimen	Linear elastic region (kN/mm)	Secondary damage region (kN/mm)	Stiffness loss from initial to secondary region	Post-secondary damage region (kN/mm)
C1/3/15-1	1.29	0.609		0.308
C1/3/15-2	1.30	0.705		0.266
C1/3/15-3	1.26	0.716		0.399
Average $\pm$ SD	1.28 $\pm$ 0.02	0.677 $\pm$ 0.059	47.3%	0.324 $\pm$ 0.068
C1/6/15-1	1.40	0.663		0.351
C1/6/15-2	1.36	0.703		0.344
C1/6/15-3	1.42	0.673		0.444
Average $\pm$ SD	1.40 $\pm$ 0.03	0.680 $\pm$ 0.020	51.3%	0.379 $\pm$ 0.056
C1/13/15-1	1.39	0.636		0.291
C1/13/15-2	1.43	0.717		0.224
C1/13/15-3	1.28	0.676		0.335
Average $\pm$ SD	1.37 $\pm$ 0.08	0.676 $\pm$ 0.040	50.5%	0.284 $\pm$ 0.056
C1/3/20-1	1.46	0.722		0.268
C1/3/20-2	1.30	0.736		0.449
C1/3/20-3	1.41	0.741		0.474
Average $\pm$ SD	1.39 $\pm$ 0.08	0.733 $\pm$ 0.010	47.3%	0.397 $\pm$ 0.112
C1/6/20-1	1.52	0.724		0.434
C1/6/20-2	1.37	0.710		0.382
C1/6/20-3	1.47	0.668		-
Average $\pm$ SD	1.45 $\pm$ 0.08	0.701 $\pm$ 0.029	51.8%	0.408 $\pm$ 0.037
C1/13/20-1	1.48	0.682		0.408
C1/13/20-2	1.40	0.620		0.474
C1/13/20-3	1.44	0.683		0.461
Average $\pm$ SD	1.44 $\pm$ 0.04	0.662 $\pm$ 0.036	54.1%	0.447 $\pm$ 0.035
C1/3/25-1	1.48	0.774		0.516
C1/3/25-2	1.53	0.713		0.329
C1/3/25-3	1.47	0.739		0.479
Average $\pm$ SD	1.49 $\pm$ 0.03	0.742 $\pm$ 0.031	50.3%	0.441 $\pm$ 0.099
C1/6/25-1	1.57	0.729		0.535
C1/6/25-2	1.50	0.746		0.458
C1/6/25-3	1.50	0.743		0.470
Average $\pm$ SD	1.53 $\pm$ 0.04	0.739 $\pm$ 0.009	51.5%	0.488 $\pm$ 0.041
C1/13/25-1	1.40	0.751		0.541
C1/13/25-2	1.50	0.754		0.290
C1/13/25-3	1.54	0.616		0.433
Average $\pm$ SD	1.48 $\pm$ 0.07	0.707 $\pm$ 0.079	52.2%	0.421 $\pm$ 0.126
C2/3/15-1	1.43	0.882		-
C2/3/15-2	1.37	0.864		-
Average $\pm$ SD	1.40 $\pm$ 0.04	0.873 $\pm$ 0.013	37.6%	-
C2/13/15-1	1.29	0.761		-
C2/13/15-2	1.30	0.774		-
Average $\pm$ SD	1.30 $\pm$ 0.01	0.767 $\pm$ 0.009	40.7%	-

Continued on next page

Table 6.2 – Continued.

Specimen	Linear elastic region (kN/mm)	Secondary damage region (kN/mm)	Stiffness loss from initial to secondary region	Post-secondary damage region (kN/mm)
C3/3/15-1	1.66	1.09		0.528
C3/3/15-2	1.57	1.07		0.512
C3/3/15-3	1.91	1.08		0.650
Average $\pm$ SD	1.71 $\pm$ 0.18	1.08 $\pm$ 0.01	37.0%	0.563 $\pm$ 0.075
C3/6/15-1	1.77	0.889		0.565
C3/6/15-2	1.90	0.973		0.547
C3/6/15-3	1.59	1.09		0.841
Average $\pm$ SD	1.76 $\pm$ 0.16	0.985 $\pm$ 0.103	43.9%	0.651 $\pm$ 0.165
C3/13/15-1	1.86	0.809		0.774
C3/13/15-2	1.92	0.874		0.502
C3/13/15-3	1.86	0.889		0.520
Average $\pm$ SD	1.88 $\pm$ 0.04	0.858 $\pm$ 0.042	54.4%	0.599 $\pm$ 0.152
C3/3/20-1	1.88	1.09		-
C3/3/20-2	1.74	1.09		0.053
C3/3/20-3	1.77	1.05		-
Average $\pm$ SD	1.80 $\pm$ 0.07	1.08 $\pm$ 0.02	40.1%	0.053
C3/6/20-1	1.83	1.10		0.835
C3/6/20-2	1.98	0.973		0.526
Average $\pm$ SD	1.90 $\pm$ 0.10	1.03 $\pm$ 0.07	45.9%	0.719 $\pm$ 0.168
C3/13/20-1	1.91	1.15		0.585
C3/13/20-2	1.88	0.976		-
C3/13/20-3	2.02	1.08		0.344
Average $\pm$ SD	1.94 $\pm$ 0.07	1.07 $\pm$ 0.09	45.0%	0.465 $\pm$ 0.171
C3/3/25-1	2.04	1.11		0.479
C3/3/25-2	1.77	1.09		0.785
C3/3/25-3	1.94	1.10		0.212
Average $\pm$ SD	1.91 $\pm$ 0.13	1.10 $\pm$ 0.01	42.6%	0.492 $\pm$ 0.287
C3/6/25-1	2.07	1.08		0.451
C3/6/25-2	2.12	1.05		-
C3/6/25-3	2.03	1.02		0.571
Average $\pm$ SD	2.07 $\pm$ 0.04	1.05 $\pm$ 0.03	49.1%	0.511 $\pm$ 0.085
C3/13/25-1	1.54	1.15		0.443
C3/13/25-2	2.04	1.10		0.400
C3/13/25-3	1.85	1.04		-
Average $\pm$ SD	1.81 $\pm$ 0.25	1.10 $\pm$ 0.06	39.4%	0.305 $\pm$ 0.203

Low-velocity impact response

First, the typical load-displacement curves for composite sandwich plate C1/13/15 struck at various impact energies of ranging from 1.8 to 10.0 J are presented in Figure 6.9. The onset of initial damage was clearly reflected in the load-displacement plot, in the form

of a minor load drop and a subsequent slope decrease which represented a reduction in the stiffness of the plate. This initial threshold load was independent of impact energy and was similar to the quasi-static value (Table 6.1). In addition, the elastic stiffness and the stiffness after damage, represented by the slopes of the curve, were approximately constant with increasing impact energies. Hence inertial stiffening was not very significant.

After the initiation of damage, the load increased up to a maximum before unloading occurred (1.8–5.2 J). During unloading, the load decreased with decreasing displacement, as the impactor rebounded toward its original position. The enclosed area under the load-displacement curve represents the portion of impact energy absorbed by the plate, E_{abs} , which is non-recoverable. The increasing hysteresis area indicates the increasing absorbed energy at higher impact energies. On the other hand, in the cases of impact velocities 7.3 and 10.0 J, unloading did not occur at the maximum load. Instead, there was a major load drop at the peak load, before unloading occurred at maximum displacement. For these two cases, visible radial cracks were detected on the impacted facesheets (Fig. 6.10), whereas small depressions that were barely noticeable were found on the top skins of the other plates impacted at lower energies. Hence the major load drop was very likely due to the fracture of the top skin.

Next, the impact load-energy curves for the sandwich plates impacted at 3.6 J are compared with the corresponding quasi-static curves in Fig. 6.11. The curves for all plates almost coincide, which highlights the quasi-static nature of the low-velocity impact events. Thus it is reasonable to predict the damage initiation load, which is indicated by the load drop in the curves, by considering a quasi-static test.

Figure 6.9: Load-displacement curves of C1/13/15 at various impact energies.

Figure 6.10: Top facing of Plate C1/13/15 impacted at 7.3J.

Figure 6.11: Load-energy curves for composite sandwich plates. Solid lines are for impacts at 3.6 J whereas dotted lines refer to quasi-static results.

6.1.3 Energy profiling curves

In the literature, it has been shown that the energy absorbed by a test specimen increases quadratically with impact energy [26, 31, 60, 61], while the critical energy for damage initiation is rate independent [22, 26]. This absorbed energy has also been used to directly relate the amount of damage in the structure measured by conventional damage assessment methods [21]. Liu [61] showed that graphs of absorbed energy vs. impact energy could be used to determine the penetration and perforation thresholds for glass/epoxy composite laminates. However, such energy plots depend strongly on many parameters, such as the impactor size, the thickness of the laminate, and the stacking sequences of the skins, as indicated in [61].

As an illustration, Fig. 6.12 shows the energy ratio E_{abs}/E_{imp} plotted against the impact energy E_{imp} for 69 sandwich composites of various core thicknesses and cell sizes with two different laminate orientations, impacted at various velocities (Table 3.1). The graph indicates that absorbed energy increases with impact energy as damage in the specimen becomes more severe, which is a common finding as mentioned earlier [26, 31, 60, 61]. Apart from that, however, there is no other discernible trend with different values of E_{abs} recorded for different types of plates even at E_{imp} which are almost similar. This is expected because the impact characteristics differ for each plate configuration. The response of a structure depends on its geometry and material, as well as the velocity of the impactor. These parameters strongly influence the entire outcome of the impact behaviour. Obviously, if absorbed energy vs. impact energy plots were to be used to characterise the impact response for these plates, as suggested by Liu [61], many such graphs would be needed since each configuration requires its own plot.

Therefore, a better way of presenting the impact test data is introduced. For the same data in Fig. 6.12, the absorbed energy and the impact energy are first normalised by the critical energy for damage initiation (U_1), and then the normalised ratios are plotted against each other in Fig. 6.13. An equal-energy line is also included for comparison. The data points in the figure can be separated into two groups based on the observed damage

Figure 6.12: Normalised absorbed energies of composite sandwich specimens of various configurations impacted at a range of energies.

modes on the impacted facesheets. In the first group, a cluster of data points is located either on or close to the equal-energy line. The relative closeness between these points and the line indicate that the absorbed energy is almost or entirely equal to the impact energy, which suggests that penetration of the impactor into the plate had already occurred. Two other observations were common for this group of specimens: a major load drop at the maximum load in the load-displacement curve (Fig. 6.9) as well as visible cracking of the top skin (Fig. 6.10).

In the other region, where the energy ratios are farther below the equal-energy line, this group of specimens suffered barely visible impact damage (BVID) in the form of permanent indentation on the impacted facesheet, without skin fracture. Although very difficult to detect visually, this type of damage is particularly insidious and dangerous because it could propagate under further loading and significantly weaken the structure causing early failure. A clearer understanding of this subject would aid in the design of safe and durable composite sandwich structures. The focus of this current investigation is

Figure 6.13: Normalised absorbed energies plotted against normalised impact energies of composite sandwich specimens subjected to low-velocity impact.

on the BVID region.

First, the data points in the BVID region are re-plotted again in Fig. 6.14 after inverting the x-axis in Fig. 6.13. The power regression curve in Fig 6.14 is the best-fitted curve based on the least-squares method, and the equation of the curve is given as:

$$\frac{E_{abs}}{U_1} = 0.3494 \left(\frac{U_1}{E_{imp}} \right)^{-1.2042} \quad (6.1)$$

with a R-squared value of 0.986. Next, by multiplying both sides by U_1/E_{imp} , Eq. 6.1 is rearranged to give:

$$\frac{E_{abs}}{E_{imp}} = 0.3494 \left(\frac{U_1}{E_{imp}} \right)^{-0.2042} \quad (6.2)$$

Equation 6.2 relates the absorbed energy to the critical energy for damage initiation and the impact energy (E_{imp}) for the specimens with BVID. Equation 6.2 is only valid for $0.048 < U_1/E_{imp} < 0.244$ based on available data; these limits could be extended to a wider range under more testing. The significance of this empirical energy equation, which sets

Figure 6.14: Normalised energies of 65 composite sandwich specimens which suffered barely visible impact damage.

it apart from those proposed in the literature (see, for example, Refs. [31, 61, 161]), is the inclusion of U_1 which is a characteristic of the sandwich plate. More importantly, U_1 defines the onset of damage and characterises the impact damage resistance of the structure.

A series of curves is plotted within the limits specified earlier for U_1/E_{imp} by varying either E_{imp} or U_1 in Eq. 6.2. Figure 6.15(b) shows that a plate with a lower U_1 absorbs more energy at a given impact energy level. Such curves would be useful for design; for instance, if the maximum allowable E_{abs}/E_{imp} is 0.6 for an impact energy of 4 J, then U_1 should be greater than 0.3 J (Fig. 6.15(b)). Furthermore, recall that E_{abs}/E_{imp} is directly related to the damaged stiffness of the plate after impact (K_{dam}), as stated in Eq. 4.30 (results that substantiate Eq. 4.30 would be presented later on page 143). Hence, Eq. 6.2 directly allows one to infer the residual stiffness that will be sustained by a damaged specimen at any impact energy level by simply knowing the value of the damage initiation threshold impact energy, without the need to conduct further experiments.

Figure 6.15: Series of curves derived from Equation 6.2 by varying (a) impact energy; and (b) damage initiation threshold energy.

It is also important to emphasise the significance of data presentation using the non-dimensional energy ratios. A single characteristic energy curve (Fig. 6.14) that can represent the impact response of various structures with different thicknesses, core heights and cell size ratio is useful. This also means that the number of impact tests required to produce this energy curve can be reduced. With the empirical relationship, the energy absorbed by the structure at a given impact energy can be forecasted simply by determining U_1 , which is rate-independent. As such, U_1 can be determined from a quasi-static analysis, which is advantageous because damage detection is easier in static tests, and so, the damage initiation energy is easier to be quantified [19].

6.2 Results from the Finite Element Analysis

In this section, the numerical results from the finite element models are compared with the experimental results for both aluminium and composite sandwich plates. A good correlation with test data would provide the confidence to use the FE models to investigate the impact response of these sandwich plates. Using the FE model, the damage characteristics of the composite plates, as well as the energy absorption characteristics of the skins and the core, are investigated further. Lastly, results from a series of parametric studies conducted to determine the effect of various geometric parameters on the damage resistance, as well as the energy absorption capabilities, of the sandwich plate, are presented.

6.2.1 Validation of FE model with experimental results

Aluminium sandwich panel

The FE model is first validated with experimental test results for aluminium sandwich plates. A good correlation between the test data and numerical predictions is achieved for the impact analysis. The load-time and load-displacement curves for the 7.0 J impact case agree well in terms of peak load and overall profile, as shown in Fig 6.16. The shorter impact duration for the FE case was probably due to the stiffer FE model, although an excellent prediction of the residual dent depth was achieved. Figure 6.17 illustrates the predicted impact damage on the top facesheet at the end of the impact event for the 7.0 J impact case, along with the test result. The contour plot for the equivalent plastic strain, which is used to represent the inelastic deformation in the facesheet, is shown. The size of the predicted damage area agrees well with the experimental result, which are both circular in shape.

Figure 6.18 compares the predicted and experimental maximum deflections of the impacted facesheet and peak impact loads for a range of impact energies from 0.85 J to 13.0 J. The two sets of results show a good agreement, with the largest error of 15%

recorded for the predicted peak load when the impact energy was 0.85 J. Again, the damage area on the top facesheet for both experimental and FE studies compare well for the same range of impact energies (Table 6.3). All these results demonstrate the capability of the numerical model to predict the impact event and to represent the damage on the impacted facesheet adequately.

Figure 6.16: Response curves for aluminium sandwich panel impacted at 7 J.

Figure 6.17: Predicted impact damage on top facesheet for (a) numerical simulation compared with (b) experimental result for an impact energy of 7 J.

Figure 6.18: Comparison of experimental and numerical results for (a) maximum deflection and (b) peak load over a range of impact energies.

Table 6.3: Diameter of damage area (mm) on top facesheet over a range of impact energies.

Impact Energy (J)	Experiment	Numerical
2.0	15.0	18.8
7.0	18.5	22.0
10.0	21.0	24.4
13.0	23.0	25.6

Composite sandwich panel

Next, a series of composite sandwich plates with various combinations of skins and cores were considered for the impact simulations. These plates were impacted at a range of impact energies from 1.8 J to 5.2 J. Figure 6.19 compares the predicted threshold and maximum loads (P_1 and P_{max}) with the test results for these plates. The predicted damage initiation threshold energies (U_1) as well as absorbed energies (E_{abs}) are presented in Fig. 6.20. The finite element predictions are generally satisfactory with a maximum

error of +18% recorded for E_{abs} in the case of Plate C2/3/15 impacted at 1.8 J. Apart from that overestimated result, all absorbed energies were under-predicted within -18%. The correlation between predicted results and test data provides credibility to the FE model and justifies the use of the chosen failure criteria for the range of impact responses considered in this study.

However, the FE model was unable to simulate the unloading at higher impact energies, particularly for cases where top skin fracture was evident in experiments. For instance, in the case of 7 J, the predicted absorbed energy could not be determined because the impactor failed to rebound after the major load drop at the peak load and remained wedged inside the plate. Consequently, the simulation did not complete. Similar difficulties were encountered when the model was used to simulate the static indentation beyond the maximum load. In the current degradation scheme, elements that have failed by fibre failure are removed. However some elements may not reach a stress state that is high enough to initiate fibre failure. Consequently these remaining elements experience large distortions as the impactor penetrates into the top skin. As a result, the element length in the load direction becomes excessively small, and this has an effect of reducing the time step significantly in explicit code simulations. As such, solution becomes practically impossible, and the simulation terminates eventually. An improved numerical model to better model skin fracture for impact simulations at higher impact energies, while retaining good correlation with test results, remains a challenging problem.

Computational cost

Table 6.4 shows the CPU cost of the quasi-static and impact simulations considered in this analysis. Depending on the sandwich models and analysis times, the runtimes of these simulations ranged between 1 hrs 10 mins and 31 hrs 30 mins on a personal computer with a Intel® Pentium® 4 3.0 Ghz processor. The CPU costs for the aluminium sandwich models were relatively lower compared to the composite sandwich models. In particular, simulations for the composite sandwich models with 3 mm-sized cores were the most

Figure 6.19: Comparison of predicted and experimental damage initiation threshold loads ($\circ$) and maximum loads ($\square$) for composite sandwich plates under low-velocity impacts.

Figure 6.20: Comparison of predicted and experimental damage initiation threshold energies ($\circ$) and absorbed energies ($\square$) for composite sandwich plates under low-velocity impacts.

computationally expensive, with runtimes that exceeded one day. This was attributed to the large number of elements (>40000) in these models.

Table 6.4: Analysis cases and CPU costs for square aluminium sandwich models (ALSW) and circular composite sandwich models (CSW) in ABAQUS/Explicit.

Analysis Type ¹	No. of Elements	Analysis Time (ms)	CPU Time(s) ²
ALSW (QS)	21959	4.60	5220
ALSW (0.85 J)	23755	4.50	4205
ALSW (2.0 J)	23755	4.50	5340
ALSW (7.0 J)	23755	4.50	6219
ALSW (10.0 J)	23755	5.00	6556
ALSW (13.0 J)	23755	5.00	6771
CSW C1/3/15 (QS)	41971	2.50	42018
CSW C2/3/15 (QS)	41971	4.00	54080
CSW C3/3/15 (QS)	50053	3.00	69660
CSW C1/13/15 (QS)	16852	2.50	10875
CSW C2/13/15 (QS)	16821	2.00	17527
CSW C3/13/15 (QS)	22617	2.00	19208
CSW C1/3/15 (1.8 J)	42792	5.80	107460
CSW C1/3/15 (3.6 J)	42792	6.00	111600
CSW C1/13/15 (1.8 J)	16852	6.25	42308
CSW C1/13/15 (3.6 J)	16852	6.60	44215
CSW C1/13/15 (1.8 J)	16852	6.90	46805
CSW C2/3/15 (1.8 J)	42422	5.30	113430
CSW C2/13/15 (1.8 J)	19571	6.00	46205
CSW C2/13/15 (3.6 J)	19571	6.00	46800
CSW C1/13/20 (3.6 J)	20191	6.00	29450

¹ Terms in brackets refer to impact energy while ‘QS’ refers to quasi-static indentation.

² 3600 s = 1 hr; 86400 s = 24 hrs.

Figure 6.21: Comparison of predicted and experimental load-displacement curves for sandwich plates C1/3/15, C2/3/15, C3/3/15 under quasi-static indentation.

Figure 6.22: Comparison of predicted and experimental load-displacement curves for sandwich plates C1/13/15, C2/13/15, C3/13/15 under quasi-static indentation.

6.2.2 Damage characteristics of composite sandwich plates under quasi-static indentation

The load-displacement plots of six sandwich panels with various cores and skins are presented in Figures 6.21 and 6.22. The predicted load-displacement behaviour using the FE model generally agrees well with the experimental result in terms of overall trend up to maximum load. In order to further understand the nature of damage in the composite sandwich plate, the onset of failure modes were identified and located using the FE progressive failure model. The failure modes were based on the set of failure criteria described in Section 5.4.2.

The general sequence of damage could be described as follows. In the initial stage of the loading, matrix failure in the form of matrix crushing and matrix cracking was common, which is expected given that the matrix is weak in the transverse directions. Matrix crushing always initiated in the top ply of the laminate, which was induced by the high contact stresses. On the other hand, matrix cracking initiated in the bottom ply where the tensile stresses were the highest since bending dominated in the early stage of loading, and then propagated towards the inner plies. Subsequently, following matrix failure, a small delamination would initiate in the top ply around the indenter as a result of the contact stresses, which is similar to the observation reported in [112, 162]. In addition, delamination on this proximal side of the plate extended very little under further loading. However, in a separate diagnostic experimental test for Plate C1/13/15, where loading was terminated once the initial threshold load was exceeded, no delamination was detected on the top surface of the specimen.

Although matrix failure and some small delamination at the top and middle plies developed at an early stage of the loading, the loss in stiffnesses for the damaged elements was not serious enough to affect the overall stiffness of the structure, with no effect on the load-displacement curve observed up to that point. This observation is in accordance with previous findings reported in [23, 28, 88, 90, 162]. Davies and Zhang [28] suggested that, although delamination may split the laminate into separate components, these components

still have to rotate together at the ends of the delamination under transverse loading and thus the stiffness of the plate is not significantly reduced. In addition, matrix crushing was not expected to adversely affect the response because it was always limited to a localised region directly underneath the indenter, which was similar to the finding reported in [112].

As the load increased, the remaining undamaged stiffer elements had to carry a greater portion of the additional load. In addition, elements which had sustained matrix failure could not carry any more load in the matrix, which led to an increase in stress level in the fibre direction. Subsequently, core damage then initiated in the honeycomb cell that was located closest to the indenter, at the vertical edges and cell-walls almost simultaneously. At this instant, delamination quickly advanced toward the inner plies near the mid-plane, where the maximum shear stresses were found [62, 112]. This was followed quickly by fibre compressive failure in the top ply underneath the indenter. This abrupt loss in stiffnesses, combined with the removal of elements due to fibre failure, contributed to the subsequent decrease of the plate's stiffness at the damage initiation threshold load (P_1). This result is consistent with earlier experimental findings that reported a simultaneous occurrence of core crush and delamination at the onset of initial failure for sandwich plates in bending [13, 54].

After the onset of initial failure, fibre breakage also occurred at the bottom ply of the top skin and propagated in the inner plies. Delamination was mostly found in the mid-plane of the laminate, and because compressive σ_{33} inhibited delamination from propagating, no delamination was found in a narrow region directly adjacent to the indent point, which are in accordance with reported experimental findings [129]. In addition, matrix cracks propagated extensively near the bottom plies and also at the top surface where the facesheet experienced further stretching due to the relatively soft core. Core damage which was initially concentrated in a zone about the size of the indenter also spread to other surrounding cells, with plastic folding observed in the cell-walls near the top edge of the core. Around the maximum load, extensive fibre failure had occurred through the depth in the localised region underneath the indenter, indicating imminent

skin fracture. Figure 6.23 illustrates the damage progression in the sandwich plate up to maximum load.

Figure 6.23: Sketches of the cross-section of the FE sandwich model during static loading up to maximum load.

The load transfer between the core and the indented skin may be understood better by examining in detail the interaction between the indenter and the honeycomb cells underneath. This is particularly important for honeycomb cores given that the finite cell size and the discontinuous surfaces of the core with the facesheet may both affect the initiation and propagation of core damage. Visualisation of core damage was possible here using the FE model because the honeycomb core was modelled discretely with shell

elements. Figure 6.24 depicts the damage in two cores with cell sizes of 3 mm and 13 mm at loads near the damage initiation loads (P_1) under indentation. The honeycombs deformed plastically in a progressive manner, where damage initiated directly underneath the indent point and then propagated outward along the top edge of the core. This observation indicates that the stress and strain fields in the core were non-uniform, with the largest stress and strain magnitude located underneath the load point. As such, existing FE models that assume a homogeneous continuum model for honeycomb cores [3,6,114] would not be able to represent damage in the core realistically.

Figure 6.24: Predicted core damage from FE model at loads near P_1 for indentation of Plates (a) C1/13/15, and (b) C1/3/15.

In the core with the 3 mm-sized cells, the contact load was resisted by a number of cell-walls, given that the cells were packed more closely together. As a result, a few cells were crushed at the onset of damage, and the planar damaged area in the core measured slightly less than the indenter size ($\varnothing 13.1$ mm) at a load near P_1 . In contrast, damage in the other core occurred almost simultaneously at the 6 vertical edges and cellular walls in one cell,

which was about the indenter size (Fig. 6.24(a)). In the experiments, the respective total displacements at the initial damage were about 1.0 mm (Table 6.1). Following Zhou et al. [13], the contact area at the skin-core interface could be estimated by assuming that the direct action of the indenter sheared through the top skins at an angle of 45° (Fig. 6.25). Accordingly, these contact areas are approximately 9.5 mm in diameter for the C1/13/15 and C1/3/15 plates, which compare reasonably with the FE results in Fig. 6.24. Moreover, the FE results also infer that it may be possible to predict P_1 by determining the onset of core failure.

Figure 6.25: Illustration of interaction between hemispherical indenter and the sandwich plate assuming direct shear through [13].

6.2.3 Energy partitioning plots

Unlike most previous experimental investigations, where the main focus was limited to the overall load response and the final damage state in the sandwich plate, the individual energy absorption capability of the skins and core was investigated further using the FE model. The absorbed energy of each part was obtained from the internal energy history in ABAQUS.

Figure 6.26(a) first shows the internal energy profiles, plotted against the indenter's

displacement, for each individual part in plate C1/13/15 (U_{part}) under quasi-static indentation. The internal energy of the top skin was higher than that for the core throughout the entire loading, while the contribution by the bottom skin was almost negligible. Because the plates considered were relatively thick, the bending of the lower face was insignificant under indentation.

Arguably, Fig. 6.26(a) may not be very useful on its own. However by normalising U_{part} by the total internal energy for the plate (U_{Total}), further insights into the deformation of the plate could be obtained, as shown in Fig. 6.26(b). At the onset of initial damage (point A), the percentage of total internal energy for the core suddenly dropped 12% while that of the top skin increased 13%. This is highly indicative of damage initiation in the core. Shortly after, there was another drop for the top skin at point B, which coincided with extensive fibre failure in plies 6–10 of the top skin. As delamination and fibre damage propagated further in the top skin, the bending stiffness weakened and membrane stretching became more significant, as also indicated by the slope increase in the load-displacement curve toward the ultimate failure load in Fig. 6.22. Due to the stiffening of the top skin, $U_{Top\ skin}/U_{Total}$ gradually increased beyond point C. In addition, the gradual decrease in the internal energy contributed by the core may be attributed to the lower load required to sustain core crushing after the onset of core failure.

Apart from using the internal energy plots to correlate damage initiation and propagation in the sandwich plate, kinetic and internal energy plots could also be used to understand the energy transfer occurring in the plates during impact. Figure 6.27 shows the kinetic and internal energy histories for the aluminium sandwich plate at an impact energy of 7.0 J. The internal energy comprises the energy dissipated by plastic deformation and the recoverable strain energy. The total energy for the whole system, which is the summation of the kinetic energy and internal energy, is constant throughout the impact event.

When the impactor strikes the plate, a contact pressure arises in the small contact area between the two bodies, which results in local deformation and subsequently, indentation

Figure 6.26: Internal energy plots for the core and the skins in C1/13/15 under quasi-static indentation.

in the contact area [163]. Due to this contact pressure, a resultant force acts equally in opposite directions on both colliding bodies. This impact force which initially increases with indentation reduces the speed at which the impactor approaches the plate. As a result, the impactor slows down and loses kinetic energy. This continues until it reaches a point where the work done by the impact force is able to bring the impactor to a halt. At this point of maximum displacement, the kinetic energy of the impactor becomes zero. Simultaneously, the equal but opposite impact force acting on the plate does work and increases the internal energy of the plate until it reaches a maximum. As shown in Fig. 6.27, the internal energy for the top facesheet is higher than that for the core, and it accounts for 54% of the total internal energy for the plate. However, the internal energy of the bottom facesheet is almost negligible, which is due to the highly localised impact damage in the upper facesheet and core near the impact point.

Upon unloading, the stored elastic strain energy is released, and this energy release

Figure 6.27: Kinetic and internal energy plots for aluminium sandwich panel subjected to 7 J impact.

generates the force to accelerate the impactor as it rebounds from the plate. The kinetic energy of the impactor at the instant it separates from the plate is 0.26 J, which equates to the release of strain energy for the sandwich plate. Therefore, approximately 96% of the impact energy has been absorbed by the plate. Given that the internal energy comprises the energy dissipated due to plasticity and the recoverable strain energy, this implies that a major portion of the plate's internal energy has been dissipated due to plasticity.

Next, the absorbed energy normalised by the impact energy (E_{abs}/E_{imp}) is plotted against the impact energy for the FE models of composite sandwich plates C1/13/15 and C2/13/15 in Figures 6.28 and 6.29, respectively. Included are also the test data and the empirical energy curve (Eq. 6.2) for comparison. The empirical energy curves for both plates were obtained by substituting the static test values of U_1 into Eq. 6.2; the mean values of U_1 were 0.466 J and 0.508 J for Plates C1/13/15 and C2/13/15, respectively.

Figure 6.28: Normalised absorbed energy plots for Plate C1/13/15 impacted between 1.8 J to 5.2 J.

Figure 6.29: Normalised absorbed energy plots for Plate C2/13/15 impacted between 1.8 J to 5.2 J.

Figure 6.30: Comparison of energies absorbed by the top skins and cores of Plates C1/13/15 and C2/13/15 impacted between 1.8 J to 5.2 J.

In both plates, the predicted absorbed energy increased with impact energy, which are in accordance with experimental results, with negligible contribution from the bottom skins. The rate of increase in energy absorbed was rather small, which is due likely to the localised nature of impact damage. Figure 6.29 indicates that approximately 50% of the impact energy of the projectile was absorbed by both the top skin and the core for C2/13/15 over the range of impact energies investigated here. This result suggests that the distribution of the impact energy throughout the plate's components does not vary with impact conditions. In contrast, the energy absorbed by the core increases with increasing impact energy for C1/13/15, while the top skin absorbs relatively the same amount of energy (Fig. 6.28).

The energy absorption profiles for the top skin and the core for the two plates are then compared against each other in Fig. 6.30. The top skin absorbed approximately the same amount of energy for the two plates over the range of impact energies here, whereas the core in C1/13/15 absorbed more energy. Due to its higher U_1 , Plate C2/13/15 is

expected to absorb less energy as compared to Plate C1/13/15, as also inferred by Eq. 6.2. For this range of impact energies, the damage in the top skin was composed mainly of matrix failure and delamination, prior to extensive fibre failure which would culminate in catastrophic skin fracture. Because of the low fracture toughness in the matrix, the energy dissipated by these failure modes in the top skins was relatively low. As such the remaining contribution of absorbed energy, which increases with E_{imp} , would have to come from the core.

6.2.4 Parametric studies

Figure 6.31: The basic honeycomb element [139].

A series of parametric studies was conducted using the FE sandwich model to identify the pertinent parameters of the sandwich plate that would affect its impact damage resistance and energy absorption capabilities. First, the effect of various geometric parameters, such as foil thickness and cell size, on the impact damage resistance of the core and top facesheet for aluminium sandwich plates was investigated. Assuming that each core cell of the model is a perfect regular hexagonal cell unit of Fig. 6.31, the

honeycomb density (HD) can be derived as [139],

$$HD = \frac{2(b+l)t\rho}{(b+l\cos\theta)(2l\sin\theta)} = \frac{1.54t\rho}{b} \text{ kg/m}^3 \quad (6.3)$$

where ρ is the density of the cell-wall material. The factor 1.54 is for cores where the thickness of ribbon walls is twice that of the free wall. Likewise, it can be easily shown that the factor is 1.15 for cores where all cell-walls have an uniform thickness (see also Ref. [46]). Based on Eq. 6.3, the density of the aluminium honeycomb core was increased by adjusting the cell wall thickness t , and the node width b (Table 6.5).

Table 6.5: List of aluminium sandwich plates with increasing honeycomb core densities.

Plate	Cell wall thickness, t (mm)	Node width, b (mm)	Core density, HD (kg/m ³)
AP1	0.0635	7.51	35.2
AP2	0.0635	4.62	57.2
AP3	0.0508	3.67	57.6
AP4	0.0635	3.67	72.0
AP5	0.0762	3.67	86.4

In this parametric study, the aluminium sandwich plates with a constant core height of 20 mm were subjected to a 7.0 J impact, and the resulting damage to the impacted facesheet and core are compared in Figs. 6.32 and 6.33. The contour plots for the equivalent plastic strain at the end of the impact event are presented. This strain is a scalar variable that is used to represent the inelastic deformation in the facesheet and core. The damage profile on the impacted facesheets were circular in shape, while the damaged areas in the honeycomb core were localised, and concentrated mainly in the vicinity of the impact point and in the upper half of the core. As shown in both figures, the size of the damaged areas decreased with increasing core density, implying that a denser core enhances the damage resistance of the structure. Because the core density is proportional to the ratio $t\rho/b$ (Eq. 6.3), a cell-wall with higher density and greater thickness will result in a more damage-resistant core. Smaller cell sizes will also improve the resistance of the core to impact damage.

Figure 6.32: Predicted damage areas on impacted facesheet for core densities of (a) 35 kg/m^3 , (b) 57 kg/m^3 , (c) 72 kg/m^3 and (d) 86 kg/m^3 .

Figure 6.33: Predicted damage areas in the honeycomb core at mid-section for core densities of (a) 35 kg/m^3 , (b) 57 kg/m^3 , (c) 72 kg/m^3 and (d) 86 kg/m^3 .

Figure 6.34 also illustrates the variation of the absorbed energies, which are nor-

malised against impact energies, and the peak impact loads for the range of honeycomb core densities listed in Table 6.5. The amount of energy absorbed is almost identical for all core densities, which could be expected because core crushing occurs in a small region relative to the size of the whole plate. As such, the influence of the core density on the energy absorbed due to this localised damage is almost negligible as compared to, say, global crushing of the entire plate. On the other hand, denser cores experience higher peak impact loads (Fig. 6.34(b)). One explanation could be that the number of cells packed within the core increases as the core density increases. Consequently, the impact load would be resisted by more cell walls within the same impact zone, and the plate becomes stiffer.

Figure 6.34: Variation of (a) percentage of absorbed energy with respect to impact energy, and (b) peak load, over a range of core densities.

Figures 6.35 and 6.36 show the damage profiles for two sandwich plates AP2 and AP3. Despite having a core density which is almost equivalent, the damage profiles for both plates in the impacted facesheet and core are unexpectedly different. Between the

two plates, the core in plate AP3 has a smaller foil thickness (Table 6.5). Given that plate AP3 also has a smaller cell size, more honeycomb cells are packed in its core, and so the space across each separate cell-wall is closer. Consequently, the stress sustained by the impacted facesheet on plate AP3 would be transferred onto this greater number of cell-walls. This might result in a smaller yielded region on the facesheet. However, due to the smaller foil thickness in plate AP3, the cell-walls are more susceptible to crushing, which explains the larger damaged region in the core for plate AP3.

Figure 6.35: Comparison of top facesheet damage areas for plates (a) AP3 and (b) AP2, whose core densities are approximately 57 kg/m^3 .

Figure 6.36: Comparison of damage areas in honeycomb core for plates (a) AP3 and (b) AP2, whose core densities are approximately 57 kg/m^3 .

Next, the effect of core density on the energy absorption capability of composite sandwich plates was investigated. A composite sandwich plate (C2/13/15) with skin configuration $[+45/-45/0/90/0]_s$ and a constant core height of 15 mm and cell size of 13 mm impacted at 1.8 J was considered. With the cell-wall material assumed to remain

constant, the cell-wall thicknesses were varied to give four different core densities ranging from 33–87 kg/m³, listed in Table 6.6.

Table 6.6: List of composite sandwich plates with increasing honeycomb core densities.

Plate	Thickness of freewall, (mm)	Thickness of ribbon wall (mm)	Core density (kg/m ³)
CP1	0.3	0.3	32.6
CP2	0.3	0.6	44.0
CP3	0.6	0.6	65.3
CP4	0.6	1.2	87.3

Figure 6.37 shows the absorbed energy ratio for the four composite sandwich plates. The energy absorbed by the plate decreased very slightly with increasing relative density until the relative density became greater than 0.1, where the core absorbed less energy than the top skin. Since the energy absorbed by the top skin remained relatively constant for all four plates, this decrease in absorbed energy is attributed to the denser core. Figure 6.38 shows the smaller damaged region in the core with increasing core density.

Next, the initiation loads for various failure modes in the plates (Table 6.6) are plotted against the relative density of the core in Figure 6.39. The damage initiation load P_1 which marks the onset of damage in the plate was inferred from the load response; it was represented by a slight load-drop and a subsequent decrease in the slope of the load-displacement curve. The initiation loads for matrix failure, fibre failure and delamination in the top skin were based on the failure criteria listed in Section 5.4.2, while core yielding was used to represent core damage. As the relative density of the core increased, the maximum impact load (P_{max}) increased, while P_1 remained relatively constant. Matrix failure and delamination in the top skin initiated at low loads, although no effect was observed on the load-displacement response at those loads.

Figure 6.39 shows that the load at the onset of core damage increases with the relative density. For plates CP1 and CP2, core damage occurred at 71% and 77% of P_1 , respectively, possibly due to the thinner walls. As pointed out earlier in Section 4.3, some studies have identified core failure to occur at the onset of damage [9,15,24,90]. The

Figure 6.37: Absorbed energy ratio plotted against relative density for composite sandwich plates CP1 to CP4 (Table 6.6).

Figure 6.38: Core damage at the end of the impact event for (a) CP1, (b) CP2, (c) CP3, and (d) CP4.

Figure 6.39: Initiation loads (P_i) for various failure modes in composite sandwich plates of various core densities. Subscript ‘i’ represents ‘CORE’, ‘MCH’, ‘MCK’, ‘DELAM’, and ‘FF’, which refer to core failure, matrix crushing, matrix cracking, delamination, and fibre failure, respectively.

present result indicates that, for cores with very low densities, associating the onset of core damage with P_1 may lead to conservative estimates. However, even though core damage initiated at loads slightly lower than P_1 for plates CP1 and CP2, further propagation of core damage up to P_1 was very limited, judging from the very slight increase in dissipated energy in the core, which was 0.08 J and 0.06 J for CP1 and CP2, respectively. In other words, the energy absorbed by the core as the load increases up to P_1 is only slightly greater than the energy it absorbs at the initiation of core damage.

On the other hand, the initiation of fibre failure closely coincided with the incidence of P_1 in all 4 cases. In the FE model, fibre failure always initiated in the top ply on the impacted facesheet, directly underneath the indenter. Although no fibre fracture was observed on the test specimens, it must be noted that fibre fracture may be very difficult to detect via visual inspection, given that impacted specimens have to be de-plyed and

examined in detail using a scanning electron microscope [28]. This finding also agrees well with some other studies which reported that the damage initiation load (P_1) also corresponds to the initiation of fibre fracture in the top skin, which could appear as micro-cracking on the impacted surface and as tensile failure on the back surface of the top skin [34, 88, 93].

6.3 Analytical Model

This section presents the results of the analytical model developed in Chapter 4 which is used to predict the impact force history of sandwich structures subjected to low-velocity impact. Three parameters were first determined: the elastic stiffness K_0 , the damage initiation threshold load P_1 , and the stiffness after damage K_{dam} . The threshold load, along with the elastic and damaged stiffnesses, was then used in the modified energy-balance model to predict the impact response of the composite sandwich plate. For the aluminium sandwich plate, the stiffnesses were determined numerically from an quasi-static explicit FE analysis, and then incorporated into the energy-balance model to obtain the impact response.

6.3.1 Elastic structural stiffness

Table 6.7 shows the theoretical stiffnesses, with the elastic stiffness K_0 compared against the experimental values for 20 configurations of composite sandwich plates (Table 3.1) loaded under quasi-static indentation. The predicted results are in good agreement with the experiments. Increasing the skin thickness increases the flexural rigidity of the panels as well as the local contact resistance. To illustrate, compare the plates with C1 and C3 skins ($h_f = 1.25$ and 1.75 mm, respectively). For the same core configuration, the plates with the thicker skins have a greater elastic stiffness, which is mainly due to the large increase in local stiffness K_{loc} . In fact, for an increase of 40% in skin thickness, K_{loc} increases about 66%, whereas the global stiffness K_{glo} increases merely 6%. The global stiffness also increases with core thickness, which is expected since the flexural stiffness (K_b) of a panel is proportional to the term $h_f(h_c + h_f)^2$ according to sandwich theory [77,164]. Conversely, K_{loc} decreases with core thickness according to the definition of the foundation modulus (Eq. 4.1).

At this stage, it is appropriate to comment on the elastic foundation approach assumed in this work. First, the shearing interaction between the loaded facesheet and the core material is not accounted for by modelling the core as a Winkler foundation.

Table 6.7: Comparison of predicted and mean experimental values for elastic stiffnesses (kN/mm) for composite sandwich plates loaded by indentation.

Sandwich Configuration	Theoretical			Experimental K_0
	K_{loc}	K_{glo}	K_0	
C1/3/15	4.83	2.10	1.46	1.28 ± 0.02
C1/6/15	4.17	1.88	1.30	1.40 ± 0.03
C1/13/15	4.01	1.86	1.27	1.37 ± 0.08
C1/3/20	3.97	2.69	1.60	1.39 ± 0.09
C1/6/20	3.84	2.42	1.48	1.45 ± 0.08
C1/13/20	4.26	2.39	1.53	1.44 ± 0.04
C1/3/25	3.54	3.29	1.71	1.49 ± 0.03
C1/6/25	3.47	2.96	1.60	1.53 ± 0.04
C1/13/25	3.73	2.92	1.64	1.48 ± 0.07
C2/3/15	4.83	2.10	1.46	1.40 ± 0.04
C2/13/15	4.00	1.86	1.27	1.30 ± 0.01
C3/3/15	8.00	2.23	1.74	1.71 ± 0.18
C3/6/15	6.91	2.00	1.55	1.76 ± 0.16
C3/13/15	6.64	1.98	1.53	1.91 ± 0.04
C3/3/20	6.57	2.82	1.98	1.80 ± 0.07
C3/6/20	6.35	2.54	1.81	1.90 ± 0.10
C3/13/20	7.06	2.51	1.85	1.94 ± 0.08
C3/3/25	5.87	3.42	2.16	1.91 ± 0.14
C3/6/25	5.74	3.07	2.00	2.07 ± 0.04
C3/13/25	6.19	3.04	2.04	1.81 ± 0.25

Second, such a core is in a uniaxial stress state with stresses and displacements related through the elastic foundation modulus k_c , which depends on the geometry of the core. One problem with this approach is that the stresses and displacements cannot be simultaneously matched to the three-dimensional solution for thick cores, as pointed out in Refs. [62, 165]. Although various definitions of the foundation modulus have been used over the years by different researchers [62, 78, 79, 165], a very simple definition which is also popular with other investigators [2, 77, 83–85] is used here. Third, some may also question the applicability of the elastic foundation approach for honeycomb cores [78]. Because honeycomb cores are discrete in nature, they do not support the facesheets of a sandwich panel continuously but discretely along the edges of the honeycomb cells, unlike polymeric foams for instance. Thomsen [78] commented that when the dimensions of the cells are small compared to the skin thickness, the elastic foundation formulation is expected to yield reasonable results. On the other hand, he argued that if the cell size of

the honeycomb is comparable to, or larger than, the skin thickness, the loaded face may tend to act like plates within the boundaries of each cell. Nevertheless, bearing in mind that the primary objective for this part of the work is to obtain the elastic stiffness K_0 , the current approach is deemed reasonable.

6.3.2 Damage initiation threshold load

Next, the damage initiation threshold load P_1 of the composite sandwich plate is predicted based on the assumption that core damage occurs at the onset of damage for the plate. This is also in accordance with the finite element results presented in the previous section which indicate that core damage does occur either at or very near to P_1 .

In order to determine the energy absorbed by the core under local indentation up to initial damage, the size of the planar damage area in the core, as measured by the radius R_{cr} , has to be known. R_{cr} was earlier assumed to be a function of the indenter's radius R_{ind} , where $R_{cr} = \beta R_{ind}$ (Eq. 4.17). One simple method to obtain R_{cr} is to estimate the contact area at the skin-core interface by examining the interaction between the indenter and the honeycomb cells underneath (Fig 6.25), as described earlier on page 122. Under bending, the clamped sandwich plate underwent both global deformation and local indentation. Subsequently, the honeycomb cells in the vicinity of the indent point rotated inward, while the top skin wrapped around the advancing indenter (Fig. 6.40), as pointed out in [13]. As a result, a larger contact area of the core was exposed to crushing by the indenter. Conversely, if the sandwich plate was rigidly supported, the core crushing zone would be smaller since only local indentation of the indenter into the top facesheet would occur.

Accordingly, given that the total displacements at initial damage were approximately 1.0 mm (Table 6.1), R_{cr} for the plates with skin thicknesses of 1.25 mm and 1.75 mm were calculated to be about 4.75 mm and 5.25 mm ($\beta \approx 0.73$ and 0.80), respectively. The predicted damage initiation loads compare well with experimental values for the composite sandwich plates subjected to static indentation, as shown in Table 6.8. Previously, Turk and Fatt [81] assumed an effective radius of $0.4R_{ind}$ in a similar attempt to calculate

Figure 6.40: Illustration of honeycomb cells undergoing crushing as the plate bends under advancing hemispherical indenter.

the energy dissipated in crushing the honeycomb under a hemispherical indenter for a composite sandwich plate consisting of graphite/epoxy laminates and Nomex honeycomb. The constant β appears to be unique for various types of sandwich plates which comprise different cores and laminates.

The current approach decouples the local and global responses of the sandwich plate, while ignoring any interaction between the two, which is a classical method often used by investigators to study the elastic response of sandwich panels subjected to a point load [32, 64, 78, 82]. One main advantage of this approach is that the energy due to local indentation U_c can be segregated from the energy due to global deformation U_{bs} . Essentially, core crushing under local indentation is one of the main concerns at initial damage whereas plate bending is usually elastic. Moreover the elastic strain energy in a large sandwich structure is invariably higher than that in a typical test specimen. To illustrate, Figure 6.41 shows the contribution of U_c and U_{bs} at the onset of damage as a function of the panel size. With increasing plate radius, the global deformation increases as the flexural rigidity decreases. Consequently, U_{bs} and U_1 increase. On the other hand,

Table 6.8: Comparison of predicted and experimental values (mean $\pm$ standard deviation) for damage initiation threshold loads for composite sandwich plates loaded by indentation.

Sandwich Configuration	Damage initiation loads P_1 (kN)		Pred. P_1 /Expt. P_1
	Predicted	Experimental	
C1/3/15	1.17	1.06 ± 0.08	1.10
C1/6/15	1.12	0.98 ± 0.06	1.14
C1/13/15	1.06	1.04 ± 0.06	1.01
C1/3/20	1.12	1.01 ± 0.01	1.11
C1/6/20	1.04	1.00 ± 0.07	1.04
C1/13/20	1.08	1.00 ± 0.04	1.07
C1/3/25	0.99	1.03 ± 0.01	0.96
C1/6/25	1.14	1.00 ± 0.07	1.14
C1/13/25	1.09	1.08 ± 0.06	1.01
C2/3/15	1.17	0.96 ± 0.01	1.22
C2/13/15	1.06	1.09 ± 0.07	0.97
C3/3/15	1.66	1.51 ± 0.01	1.10
C3/6/15	1.58	1.53 ± 0.07	1.04
C3/13/15	1.42	1.46 ± 0.09	0.97
C3/3/20	1.58	1.45 ± 0.19	1.09
C3/6/20	1.48	1.41 ± 0.13	1.05
C3/13/20	1.52	1.39 ± 0.06	1.09
C3/3/25	1.41	1.44 ± 0.03	0.97
C3/6/25	1.61	1.55 ± 0.15	1.04
C3/13/25	1.55	1.52 ± 0.10	1.01

U_c should not be affected because the local response of the plate does not depend on the plate radius. This suggests that the damage initiation energy for a larger plate is merely the sum of the constant U_c and the additional U_{bs} . In other words, a plate with a larger radius will store more energy in elastic modes of deformation. This implies that at a given energy level, damage will therefore be less in specimens of larger diameters, as also observed in other tests performed on laminates [166, 167].

At the onset of damage, the damage initiation energy U_1 consists of the energy due to bending and shear deformations U_{bs} , and the energy due to localised indentation U_c . By assuming that the load varies linearly with deformations at that instant, a dimensionless

Figure 6.41: Normalised energies due to local indentation and global deformation at the onset of damage as a function of radius for Plate C1/13/15.

parameter η , which is the ratio of the local indentation to the global deformation at the plate centre, can be defined as

$$\eta = \frac{\alpha_0}{w_0} = \frac{K_{glo}}{K_{loc}} \quad (6.4)$$

Furthermore, by using Eqs. 4.21 and 4.24, it can be shown that U_c and U_{bs} are related to U_1 by η ,

$$\frac{U_c}{U_1} = \frac{\eta}{1 + \eta} \quad \text{and} \quad \frac{U_{bs}}{U_1} = \frac{1}{1 + \eta} \quad (6.5)$$

Figure 6.42 illustrates the contribution of U_c and U_{bs} to U_1 as a function of η . At $\eta=1$, the local and global deformations are of the same magnitude. If η is small, the bending and shear strain energies account for a significant portion of U_1 and the global deformation dominates the overall response of the panel. Conversely, a large η ($\eta > 1$) indicates that the global deformation becomes negligible in comparison with localised indentation.

Figure 6.42: Normalised energies due to local indentation and global deformation at the onset of damage as a function of the stiffness ratio.

6.3.3 Reduced stiffness after damage

Beyond the damage initiation load, the specimen is damaged and this damage is reflected in the reduction of the plate's transverse stiffness from K_0 to K_{dam} . Eleven sandwich configurations of various cores and skins were impacted at various E_{imp} (Table 3.1), and the ratio of the plate's stiffness after and before impact (K_{dam}/K_0) is plotted as a function of the ratio of the energy absorbed E_{abs} to the impact energy E_{imp} in Fig. 6.43. The reduced stiffness K_{dam} was determined as the final slope of the load-displacement curve just prior to unloading. The experimental points are shown in Fig. 6.43 to be approximated by a straight line,

$$\frac{K_{dam}}{K_0} = 1 - \frac{E_{abs}}{E_{imp}} \quad (6.6)$$

Note that the ratio K_{dam}/K_0 for the sandwich plates which suffered barely visible impact damage (BVID) in this study lies mainly in the range of 0.4–0.6. The above equation implies that the plate's reduced stiffness is a function of the recoverable energy at the

end of the impact event, and is identical to the one identified by Lifshitz et al. [27] for CFRP beams. Consequently, the residual stiffness of the damaged plate could be easily determined since the absorbed energy E_{abs} is often readily available (or easily calculated) from an impact test [22,26]. Also recall the empirical energy equation (Eq. 6.2) presented earlier on page 108 for composite sandwich plates which had sustained BVID. Based on that energy equation, the energy absorbed could be predicted by knowing the damage initiation energy (U_1) and the impact energy. Subsequently, the reduced stiffness of the plate may be found using Eq. 6.6.

Figure 6.43: Relative reduction in the stiffness of the plate as a function of relative loss in impact energy.

Another equation that could be used to predict the reduced stiffness of the plate is Eq. 4.34 (page 60). Based on Eq. 4.34, the predicted K_{dam}/K_0 are compared against the test data in Table 6.9, with the predictions generally lower than the experimental values. One possible reason is that the actual peak load (P_{max}^{dam}) sustained by a damaged plate is always likely to be underestimated in Eq. 4.33, which assumes that all the energy is dissipated in damage before the peak load is reached. However some of the absorbed energy may be dissipated after the peak load and during unloading [22]. In addition, Eq. 4.34 is invalid when top skin fracture occurs, as seen in the cases of Plate C1/13/15

impacted at 7.26 and 10.0 J. Energy dissipated in fibre breakage after the peak load and during unloading is expected to be significant, which explains the discrepancies in the results.

Table 6.9: Predicted ratio of damaged stiffness to elastic stiffness for composite sandwich plates impacted at various energies using Equation 4.34 compared against experimental values.

Sandwich Configuration	Nominal E_{imp} (J)	K_{Dam}/K_0	
		Predicted (Eq. 4.34)	Experiment
C1/3/15	1.84	0.510	0.517
C1/3/15	3.83	0.412	0.532
C1/6/15	3.83	0.432	0.485
C1/13/15	1.84	0.525	0.480
C1/13/15	3.83	0.423	0.531
C1/13/15	5.04	0.336	0.380
C1/13/15	7.26	0.321	0.186
C1/13/15	10.0	0.199	0
C1/3/20	3.83	0.376	0.390
C1/6/20	3.83	0.388	0.431
C1/13/20	3.83	0.422	0.521
C1/3/25	3.83	0.440	0.441
C1/6/25	3.83	0.378	0.471
C1/13/25	3.83	0.443	0.508
C2/3/15	1.84	0.563	0.567
C2/13/15	1.84	0.532	0.554

One advantage of using Eq. 4.34 is that the elastic peak load P_{max}^{el} could be easily calculated if one knows the plate's initial structural stiffness and the impact energy (Eq. 4.29). Apart from being directly available in an impact test, the peak load sustained by the damaged plate P_{max}^{dam} may also be predicted theoretically. Fatt and Park [91] derived closed-form solutions for several failure modes of composite sandwich plates, which include tensile and shear fracture of the top skin, core shear failure, and tensile failure of the bottom skin. Because radial cracks in the impacted facesheet were observed in the test specimens that suffered from visible damage (Fig 6.10), tensile fracture of the top skin appears to be the failure mode that coincided with the major load drop at the peak

load in the load-time history. According to [91], the failure load due to tensile fracture of the top skin is

$$P_f = dA_{11}\varepsilon_{cr}\sqrt{2\varepsilon_{cr}} + 2\pi qR_e^2 \quad (6.7)$$

where d is the total crack length, A_{11} is the laminate extensional stiffness, ε_{cr} is the tensile strain for facesheet fracture, q is the crushing strength of the core, and R_e is the effective radius of the impactor ($R_e = 0.4 \times R_{imp}$). As an example, consider the sandwich plate C1/13/15 impacted at 5 J by the 13.1 mm hemispherical impactor. The other variables in Eq. 6.7 are: $A_{11} = 1.178 \times 10^8$ N/m, $\varepsilon_{cr}=0.012$, $q=1$ MPa, and an estimated total radial crack length $d=10$ mm. Accordingly, the predicted P_{max} is 2.19 kN, which compares reasonably with the measured experimental value of 2.26 kN.

6.3.4 Impact response

Finally, the impact model described in Section 4.5 was used to derive the response of the composite sandwich plates subjected to low-velocity impact. For each plate, the elastic stiffness K_0 , the damage initiation load P_1 and the stiffness after damage K_{dam} were predicted as detailed in the preceding sections. Figures 6.44–6.47 show the experimental and predicted loadtime histories and load-deflection histories for several composite sandwich plates subjected to low-velocity impacts. In the load-time curves, the load initially increased linearly up to the critical load P_1 and then dropped suddenly. Subsequently the load increased to a maximum at a reduced stiffness. The predicted results are comparable with test data, in terms of the critical and peak loads, as well as the overall behaviour. This approach demonstrates the capability of the modified energy-balance model to reproduce the low-velocity impact response of a sandwich plate by using just three parameters (K_0 , P_1 , and K_{dam}).

Unloading was not considered for the impacts in Figures 6.44–6.47 because the contact law for the unloading phase must be determined empirically for these cases. Beyond the onset of damage, the loading and unloading phases are significantly different due to the substantial amount of energy dissipated in damage [2]. On the other hand,

Figure 6.44: Load-time and load-deflection histories for Plates C1/13/15 and C2/13/15 under 1.8 J impact. Solid lines are representative experimental curves whereas dotted lines refer to numerical predictions.

the modified energy-balance model is capable of predicting the entire response for purely elastic impacts. Figure 6.48 shows the response of Plate C2/13/15 impacted at 0.334 J, before the damage threshold is reached. The analytical result is compared with the FE prediction because it was not possible to obtain a purely elastic response even at the lowest drop height in the current experimental setup. Both results are comparable, with the most notable difference in the solution runtime; the FE analysis took a few hours on a home PC with a Pentium® 4 Processor while a few minutes was only required for the analytical solution. Due to the elastic impact, loading and unloading followed the same paths with no hysteresis loop (Fig 6.48); no energy was dissipated due to damage. The load-time history also showed a half-sine wave which is representative of elastic impacts [22].

The analytical impact model was also used to predict the low-velocity impact response of the aluminium sandwich plate. Using an explicit FE quasi-static analysis, the local stiffness K_{loc} and global stiffness K_{glo} were determined by decoupling the local and

Figure 6.45: Load-time and load-deflection histories for Plates C1/3/15, C1/6/15, and C1/13/15 under 3.6 J impact. Solid lines are representative experimental curves whereas dotted lines refer to numerical predictions.

Figure 6.46: Load-time and load-deflection histories for Plates C1/3/20, C1/6/20, and C1/13/20 under 3.6 J impact. Solid lines are representative experimental curves whereas dotted lines refer to numerical predictions.

Figure 6.47: Load-time and load-deflection histories for Plates C1/3/25, C1/6/25, and C1/13/25 under 3.6 J impact. Solid lines are representative experimental curves whereas dotted lines refer to numerical predictions.

Figure 6.48: Simulated linear elastic impact response of Plate C2/13/15 prior to damage.

Figure 6.49: Load vs. global deflection and load vs. local indentation curves of aluminium sandwich panel indented to 2 mm in a FE explicit quasi-static analysis.

global responses of the clamped aluminium plate under indentation (Fig.6.49). The global stiffness remained relatively constant ($P = K_{glow_0}$), with the behavior of the aluminium sandwich panel dominated by local indentation. Moreover, contrary to the linear contact law observed for composite sandwich plates, the contact load P for the aluminium sandwich plate is related to the local indentation α_0 by the power law,

$$P = K_{loc}\alpha_0^n \quad (6.8)$$

where n is some constant. Consequently, the energy due to local indentation is then

$$U_c = \int_0^{\alpha_0} P d\alpha_0 = \frac{P^{(1+\frac{1}{n})}}{(n+1) K_{loc}^{\frac{1}{n}}} \quad (6.9)$$

Figure 6.50: Predicted and experimental response curves for aluminium sandwich plates impacted at 2.0 J and 7.0 J.

Figure 6.50 shows the load-time and load-deflection histories for the aluminium

sandwich plate for two impact energies of 2.0 and 7.0 J; results are compared with both test data and FE simulation results. The comparisons indicate that a good agreement existed between the experimental and predicted results, in terms of peak load and overall profile. The analytical model was able to predict the impact response reasonably well up to the point of maximum load. In addition, the stiffness of the plate, as indicated by the slope of the load-deflection curve, was well represented by both numerical and analytical models.

The good correlation between predicted results and test data for both composite and aluminium sandwich plates implies that the quasi-static assumption adopted for the energy-balance model is valid here. For a low-velocity impact to be considered quasi-static, the suggested upper limit for velocity ranges from 10 to 100 m/s, and a range of 10 to 20 m/s has been suggested for typical composite materials [67]. Likewise, an impactor-plate mass ratio of 2 is sufficient to ensure quasi-static impact response; however, an impactor that has a mass at least 10 times greater than its target is recommended [19].

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

This research aimed to predict the low-velocity impact response and damage of sandwich structures, and to characterise the energy absorbed by these structures. The following conclusions are made.

7.1.1 Energy absorption characteristics of sandwich plates under low-velocity impact

Based on the least-squares method, an empirical equation that links the absorbed energy to the impact energy was derived for composite sandwich plates. This single equation could predict the absorbed energies for sandwich plates of various skin and core configurations, because it incorporates a crucial parameter that is missing in previous studies — the damage initiation threshold energy. One inference of this equation is that absorbed energy is inversely related to the damage initiation energy. Since sandwich plates with thicker facesheets have higher damage initiation energies, as shown in the static tests, the absorbed energy for these plates under impact would be lower. In addition, the proportion of impact energy absorbed by the plates was related to the relative loss of the plate's transverse stiffness after damage. This energy equation is thus useful for future studies on damage resistance and tolerance in composite sandwich structures.

The use of absorbed energy as a damage measure was further demonstrated in aluminium sandwich plates. Test results revealed that the ratio of absorbed energy

normalised by the static energy for failure reflected the extent of damage in these plates. Fracture and tearing were evident on the impacted facesheet when this energy ratio exceeded unity. Therefore the results suggest that it is possible to predict failure for these plates without relying on post-impact inspection of the specimen.

7.1.2 FE model to predict response and damage due to localised loading

A three-dimensional FE sandwich model that comprised a cellular core, modelled with shell elements, was developed and validated in this research. The FE model was first validated with test data for aluminium sandwich specimens impacted at a range of energies from 0.85 to 13.0 J. The impact load-time history, peak loads and maximum deflections, and specimen damage areas were predicted reliably. More specifically, the largest error of 15% was reported for the peak load when the plate was impacted at 0.85 J.

Subsequently, the FE analysis was extended to composite sandwich plates, where a progressive damage model based on the Hashin failure criteria and a delamination criterion was included to predict the damage mechanisms and failure in the laminated skins. The FE model could model the behaviour of six composite sandwich plates under indentation; in particular, the stiffnesses and the critical load at the onset of damage were well represented, as shown by the load-displacement curves. For impact simulation, nine composite sandwich plates of various combinations of skins and cores were impacted at a range of impact energies from 1.8 to 5.2 J. The peak impact loads, the absorbed energies, as well as the damage initiation threshold loads and energies, were predicted within 18% of the experimental results.

The FE model was also used to determine the effect of various geometric parameters, such as cell-wall thickness and cell size, on the damage resistance of the structure. Results showed that plates with denser cores experienced smaller damage profiles and greater peak loads, indicating that a denser core enhances the damage resistance of the structure. On the other hand, the core density had little effect on the energy absorbed during impact.

The thickness and density of the cell-wall, as well as the cell size of the honeycomb, are important design parameters that directly affect the core density. Such information would facilitate the design of a more efficient impact-resistant structure, particularly in the preliminary design stage before testing and prototyping.

7.1.3 Structural behaviour

Damage characteristics of clamped composite sandwich panels under indentation were investigated using the FE sandwich model. Although matrix failure occurred very early in the top skin, it did not affect the behaviour of these plates. At the onset of initial damage, which was marked by a slope change in the load history, the plates exhibited a simultaneous occurrence of core failure, onset of delamination in the top skin, and fibre fracture in a small region contacting the indenter. The findings suggest that the damage initiation load for these plates could be associated with the onset of core failure. Further loading resulted in continued core crush, propagation of delamination and fibre fracture, which culminated in top-skin failure.

In addition, damage initiation and propagation of these plates were characterised using load-displacement curves. The critical and ultimate loads as well as the stiffnesses in the linear elastic and secondary damage regions all increased significantly when the skin thickness was increased. The loss of stiffness after the initial damage load was also smaller for plates with thicker skins. One reason is the greater flexural rigidity and local contact stiffness associated for such plates. Panels with thicker cores also had higher initial stiffnesses due to the greater flexural rigidity and transverse shear resistance.

An analytical model was also proposed to predict the critical load at the onset of damage. Due to the quasi-static nature of the low-velocity impacts of sandwich plates as indicated by test results, the critical load P_1 could be derived from a static test. The critical load was theoretically predicted by accounting for the elastic energy absorbed by the plate up to the point of core failure. At that load, the corresponding damage initiation energy U_1 consists of the energy due to bending and shear deformations U_{bs} and the

energy due to localised indentation U_c . These energies depend on the local and global stiffness. When the local stiffness is large compared to the global stiffness, the global deformation dominates the overall response of the plate and U_c can be ignored.

Impact test results also showed that the relative loss of the plate's transverse stiffness after damage (K_{dam}/K_0) was related to the relative loss in impact energy (E_{abs}/E_{imp}). Alternatively, the reduced stiffness after damage (K_{dam}) could also be estimated using the respective peak loads sustained in an actual impact and in an elastic impact, provided no penetration of the top skin has occurred. These expressions can be used to evaluate the residual stiffness of the damaged structure quickly, and eliminate the need to conduct further testing.

7.1.4 Analytical Model to Predict Impact Response

The modified energy-balance model was shown to be capable of predicting the low-velocity impact response of a composite sandwich plate by using just three parameters (K_0 , P_1 , and K_{dam}) to account for elastic response, damage initiation, and propagation. Alternatively, the stiffnesses could be derived from a quasi-static FE analysis, as illustrated in the case of aluminium sandwich plates. The load-time history and load-deflection history up to maximum load were well predicted in terms of peak load and overall behaviour for all plates tested. This impact model is an extension of the original energy-balance model, which is largely limited to elastic impacts and does not produce load histories. Because it is fast and efficient, this model is suitable for initial design of sandwich panels susceptible to impact, and can complement detailed FE simulations.

7.2 Recommendations for Future Work

The current work raises several issues which require further investigation. The following recommendations are proposed.

7.2.1 Validity of the empirical energy equation for other plates

The empirical energy equation was limited to plates composed of the same materials that had sustained barely visible damage prior to projectile penetration. Future research should be carried out to check whether the equation is valid for other material systems. The present findings will be further established if other researchers can achieve the same results.

7.2.2 FE model to simulate impacts at higher energies

One limitation of the FE model was its inability to simulate the unloading of the impactor at higher impact energies, particularly for cases where top skin fracture was evident in experiments. It was believed that the simulations did not complete due to large distortions of the elements. An improved material degradation procedure may circumvent this problem.

In addition, fracture of the cell-walls in the core may occur at higher impact energies. Consequently, this would require a different approach in terms of modelling. In particular, the in-plane deformations of the core are expected to influence its out-of-plane behaviour. Strain-rate effects, which had been neglected in this study, would also become important.

7.2.3 Local indentation damage model

The analytical model proposed to predict the onset of damage was limited to local indentation damage. Because the global deflection of the sandwich panel is dominated by shearing, this motivates further studies to investigate core shear rupture damage, which is another concern.

In addition, the energy absorbed by the core under local indentation up to initial damage was quantified based on empirical observations. It is expected that this energy would depend strongly on the diameter of the indenter, as well as the material properties of the constituent skins and cores. Future research should be carried out to examine this relationship.

Appendices

Appendix A

Strain energy of a circular clamped orthotropic plate

For a rectangular orthotropic plate, the strain energy due to bending is given as [168]

$$U = \frac{1}{2} \int \int \left[D_{11} \left(\frac{\partial^2 \alpha}{\partial x^2} \right)^2 + 2D_{12} \left(\frac{\partial^2 \alpha}{\partial x^2} \right) \left(\frac{\partial^2 \alpha}{\partial y^2} \right) + D_{22} \left(\frac{\partial^2 \alpha}{\partial y^2} \right)^2 + 4D_{66} \left(\frac{\partial^2 \alpha}{\partial x \partial y} \right)^2 \right] dx dy \quad (\text{A.1})$$

where D_{ij} are the bending rigidities of the laminate, and α is the transverse deflection of the plate. In the case of axisymmetrical bending for a circular plate, where the applied load and boundary conditions are independent of the angle θ , the deflection α of the plate is a function of the radial position r only. Hence in Eq. A.1, the following terms can be expressed in polar coordinates

$$\begin{aligned} \frac{\partial^2 \alpha}{\partial x^2} &= \frac{d^2 \alpha}{dr^2} \cos^2 \theta + \frac{d\alpha}{dr} \frac{\sin^2 \theta}{r} \\ \frac{\partial^2 \alpha}{\partial y^2} &= \frac{d^2 \alpha}{dr^2} \sin^2 \theta + \frac{d\alpha}{dr} \frac{\cos^2 \theta}{r} \\ \frac{\partial^2 \alpha}{\partial x \partial y} &= \frac{d^2 \alpha}{dr^2} \sin \theta \cos \theta - \frac{d\alpha}{dr} \frac{\sin \theta \cos \theta}{r} \\ dx dy &= r dr d\theta \end{aligned} \quad (\text{A.2})$$

Assume that the deflection of a circular clamped plate is approximated as [145]

$$\alpha(r) = \alpha_0 \left(1 - \frac{r^2}{a^2}\right)^2 \quad (\text{A.3})$$

where α_0 and a are the transverse deflection and the radius of the plate, respectively. Consequently, the strain energy due to bending of an orthotropic circular plate is then derived as

$$U_b = \frac{4\pi (3D_{11} + 2(D_{12} + 2D_{66}) + 3D_{22}) \alpha_0^2}{3a^2} \quad (\text{A.4})$$

For an isotropic circular plate, the bending strain energy is

$$\begin{aligned} U_b &= \frac{D_f}{2} \int_0^{2\pi} \int_0^a \left[\left(\frac{\partial^2 \alpha}{\partial r^2} \right)^2 + \frac{1}{r^2} \left(\frac{\partial \alpha}{\partial r} \right)^2 + \frac{2\nu}{r} \frac{\partial \alpha}{\partial r} \frac{\partial^2 \alpha}{\partial r^2} \right] r \, dr \, d\theta \\ &= \frac{32\pi D_f \alpha_0^2}{3a^2} \end{aligned} \quad (\text{A.5})$$

where the bending stiffness of the facesheet is $D_f = E_f h_f^3 / [12(1 - \nu_f^2)]$; E_f , h_f , and ν_f refer to the Young's modulus, thickness and Poisson's ratio of the facesheet, respectively. This implies that the expressions for the bending deflection of isotropic and orthotropic plates can have exactly the same form if Eq. A.4 is re-written as

$$U_b = \frac{32\pi D' \alpha_0^2}{3a^2} \quad (\text{A.6})$$

in which

$$D' = \frac{1}{8} [3D_{11} + 2(D_{12} + 2D_{66}) + 3D_{22}] \quad (\text{A.7})$$

Appendix B

Bending stiffness components for sandwich plate

In the sandwich plate, each facesheet and the core — regardless of the core height (h_c) — can be considered as the three laminae that compose the overall sandwich laminate [77]. In this way, the composite laminate theory can be used to analyse the sandwich structure. Consequently, the bending stiffness quantities for a sandwich plate with two identical thin stiff orthotropic facesheets on a soft core are [62]

$$D_{ij} = \frac{h_f(h_c + h_f)^2}{2(1 - \nu_{12f}\nu_{21f})} \begin{bmatrix} E_{1f} & \nu_{12f}E_{2f} & 0 \\ \nu_{12f}E_{2f} & E_{2f} & 0 \\ 0 & 0 & G_{12f}(1 - \nu_{12f}\nu_{21f}) \end{bmatrix} \quad (\text{B.1})$$

where the subscripts f and c refer to the properties of facesheet and core, respectively. The following is also assumed [77]: the core is weak compared to the facesheet; the core does not contribute significantly to the stiffness quantities; and the thickness of the sandwich plate remains constant.

Appendix C

ABAQUS Input files

Examples of the input files for the simulation of quasi-static indentation and impact of aluminium and composite sandwich models in ABAQUS/Explicit are provided here. Because of the large models, a complete input file would consist of a huge amount of data which define the nodes, elements, and etc. Hence for readability, only parts of the input files are included here.

C.1 Quasi-static Indentation of Aluminium Sandwich Plate

```
*Heading
  Square aluminium sandwich plate under quasi-static indentation
** PARTS
**
*Part , name=Impactor
*End Part
**
*Part , name=Core
*End Part
**
*Part , name=facesheet-shell
*End Part
**
** ASSEMBLY
**
*Assembly , name=Assembly
**
*Instance , name=Core-1, part=Core
*Element , type=S4R
..
*End Instance
**
*Instance , name=facesheet-shell-top , part=facesheet-shell
*Element , type=S4R
..
*End Instance
```

```

**
*Instance , name=facesheet-shell-bot , part=facesheet-shell
*Element , type=S4R
..
*End Instance
**
*Instance , name=Impactor-1, part=Impactor
*Element , type=C3D4
..
*End Instance
**
*Nset , nset=hc-corebot , instance=Core-1
..
*Elset , elset=hc-corebot , instance=Core-1
..
*Nset , nset=core , instance=Core-1, generate
..
*Elset , elset=core , instance=Core-1, generate
..
*Nset , nset=hc-coretop , instance=Core-1
..
*Elset , elset=hc-coretop , instance=Core-1
..
*Nset , nset=CLAMPED, instance=facesheet-shell-top
..
*Nset , nset=CLAMPED, instance=facesheet-shell-bot
..
*Elset , elset=CLAMPED, instance=facesheet-shell-top
..
*Elset , elset=CLAMPED, instance=facesheet-shell-bot
..
*Nset , nset=BotFS_N , instance=facesheet-shell-bot
..
*Nset , nset=PLATEN, instance=facesheet-shell-top
..
*Nset , nset=ImpN , instance=Impactor-1
..
*Elset , elset=Imp , instance=Impactor-1
..
*Surface , type=ELEMENT, name=impsurf
..
*Surface , type=NODE, name=hc-corebot_CNS_ , internal
..
*Surface , type=NODE, name=hc-coretop_CNS_ , internal
..
**
*** Defines the tie constraints between skins and core
** Constraint: BFS-core
*Tie , name=BFS-core , adjust=yes
hc-corebot_CNS_ , BFS-top
** Constraint: RigidImp
*Rigid Body, ref node=ImpN, elset=Imp
** Constraint: TFS-core
*Tie , name=TFS-core , adjust=yes
hc-coretop_CNS_ , TFS-bot
*End Assembly

```

```

**
*** Defines a smooth step amplitude curve for prescribed displacement
    of indenter
*Amplitude, name=move, definition=SMOOTH STEP
0., 0., 0.0001, 1.
*Amplitude, name=indent, definition=SMOOTH STEP
0., 0., 0.0046, 1.
** MATERIALS
**
*Material, name="AL 1100-H14 BL"
*Density
2700.,
*Elastic
6.9e+10, 0.33
*Plastic
1.17e+08, 0.
1.192e+08, 7.87e-06
1.222e+08, 2.595e-05
1.242e+08, 3.8e-05
*Material, name=Al3003-H19-foil-BL
*Density
2700.,
*Elastic
6.9e+10, 0.33
*Plastic
2.207e+08, 0.
2.308e+08, 4.464e-05
2.409e+08, 0.000104
2.509e+08, 0.0001632
*Material, name=Steel.BIGIMP
*Density
2.251e+06,
*Elastic
2e+11, 0.3
**
** INTERACTION PROPERTIES
**
*Surface Interaction, name=nofric
**
** BOUNDARY CONDITIONS
*** Clamp plate at its surfaces on skins
** Name: ClampedEdges Type: Symmetry/Antisymmetry/Encastre
*Boundary
CLAMPED, ENCASTRE
** Name: U3Imp Type: Displacement/Rotation
*** Constrain the impactor to move in Z-dir only
*Boundary
ImpN, 1, 1
ImpN, 2, 2
ImpN, 4, 4
ImpN, 5, 5
ImpN, 6, 6
**
** INTERACTIONS
**

```

```

** Interaction: Defines contact between all surfaces using general
   contact algorithm
*Contact, op=NEW
*Contact Inclusions, all exterior
*Contact property assignment
  , , nofric
** -----
**
** STEP: Move downward
*** In this step, the impactor moves downward 0.1 mm
*Step, name=Move
Move 0.1mm
*Dynamic, Explicit
  , 0.0001
*Bulk Viscosity
0.06, 1.2
**
** BOUNDARY CONDITIONS
**
** Name: U3Imp Type: Displacement/Rotation
*Boundary, amplitude=move
ImpN, 1, 1
ImpN, 2, 2
ImpN, 3, 3, -0.0001
ImpN, 4, 4
ImpN, 5, 5
ImpN, 6, 6
**
** OUTPUT REQUESTS
**
*Restart, write, number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, number intervals=5
*Node Output
RF, U, V
*Element Output
LE, PE, PEEQ, S
*Contact Output
CSTRESS,
**
** HISTORY OUTPUT: Energy
**
*Output, history, variable=PRESELECT, time interval=5e-06
**
** HISTORY OUTPUT: U3_botfsN
**
*Output, history, time interval=5e-06
*Node Output, nset=BotFS_N
U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output, nset=CLAMPED
RF3,

```

```

**
** HISTORY OUTPUT: U3_IMP
**
**Node Output , nset=ImpN
U3,
**
** HISTORY OUTPUT: U3_PlateN
**
**Node Output , nset=PLATEN
U3,
**End Step
** -----
**
** STEP: Indent
**
*** In this step , the indenter indents downward a further 5.5 mm
**Step , name=indent
indent 5.5mm downward for 4.6ms
**Dynamic , Explicit
, 0.0046
**Bulk Viscosity
0.06, 1.2
**
** BOUNDARY CONDITIONS
**
** Name: U3Imp Type: Displacement/Rotation
**Boundary , amplitude=indent
_PickedSet890 , 1, 1
_PickedSet890 , 2, 2
_PickedSet890 , 3, 3, -0.0055
_PickedSet890 , 4, 4
_PickedSet890 , 5, 5
_PickedSet890 , 6, 6
**
** OUTPUT REQUESTS
**
**Restart , write , number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
**Output , field , number intervals=100
**Node Output
RF, U, V
**Element Output
LE, PE, PEEQ, S
**Contact Output
CSTRESS,
**
** HISTORY OUTPUT: Energy
**
**Output , history , variable=PRESELECT
**
** HISTORY OUTPUT: U3_botfsN
**
**Output , history
**Node Output , nset=BotFS_N

```

```

U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output , nset=CLAMPED
RF3,
**
** HISTORY OUTPUT: U3_IMP
**
*Node Output , nset=ImpN
U3,
**
** HISTORY OUTPUT: U3_PlateN
**
*Node Output , nset=PLATEN
U3,
*End Step

```

C.2 Impact of Aluminium Sandwich Plate

```

*Heading
  Square aluminium sandwich plate impacted at 2.3 m/s
** PARTS
**
*Part , name=Impactor
*End Part
**
*Part , name=Core
*End Part
**
*Part , name=facesheet-shell
*End Part
**
** ASSEMBLY
**
*Assembly , name=Assembly
**
*Instance , name=Core-1, part=Core
*Element , type=S4R
..
*End Instance
**
*Instance , name=facesheet-shell-top , part=facesheet-shell
*Element , type=S4R
..
*End Instance
**
*Instance , name=facesheet-shell-bot , part=facesheet-shell
*Element , type=S4R
..
*End Instance
**
*Instance , name=Impactor-1, part=Impactor
*Element , type=C3D4

```

```

..
*End Instance
**
*Nset, nset=hc-corebot, instance=Core-1
..
*Elset, elset=hc-corebot, instance=Core-1
..
*Nset, nset=core, instance=Core-1, generate
..
*Elset, elset=core, instance=Core-1, generate
..
*Nset, nset=hc-coretop, instance=Core-1
..
*Elset, elset=hc-coretop, instance=Core-1
..
*Nset, nset=CLAMPED, instance=facesheet-shell-top
..
*Nset, nset=CLAMPED, instance=facesheet-shell-bot
..
*Elset, elset=CLAMPED, instance=facesheet-shell-top
..
*Elset, elset=CLAMPED, instance=facesheet-shell-bot
..
*Nset, nset=BotFS_N, instance=facesheet-shell-bot
..
*Nset, nset=PLATEN, instance=facesheet-shell-top
..
*Nset, nset=ImpN, instance=Impactor-1
..
*Elset, elset=Imp, instance=Impactor-1
..
*Surface, type=ELEMENT, name=impsurf
..
*Surface, type=NODE, name=hc-corebot_CNS_, internal
..
*Surface, type=NODE, name=hc-coretop_CNS_, internal
..
**
*** Defines the tie constraints between skins and core
** Constraint: BFS-core
*Tie, name=BFS-core, adjust=yes
hc-corebot_CNS_, BFS-top
** Constraint: RigidImp
*Rigid Body, ref node=ImpN, elset=Imp
** Constraint: TFS-core
*Tie, name=TFS-core, adjust=yes
hc-coretop_CNS_, TFS-bot
*End Assembly
**
** MATERIALS
**
*Material, name="AL 1100-H14 BL"
*Density
2700.,
*Elastic
6.9e+10, 0.33

```



```

*Plastic
  1.17e+08,      0.
  1.192e+08,    7.87e-06
  1.222e+08,    2.595e-05
  1.242e+08,    3.8e-05
*Material , name=Al3003-H19-foil-BL
*Density
2700.,
*Elastic
  6.9e+10, 0.33
*Plastic
  2.207e+08,      0.
  2.308e+08,    4.464e-05
  2.409e+08,    0.000104
  2.509e+08,    0.0001632
*Material , name=Steel_BIGIMP
*Density
  2.251e+06,
*Elastic
  2e+11, 0.3
**
** INTERACTION PROPERTIES
**
*Surface Interaction , name=nofric
**
** BOUNDARY CONDITIONS
*** Clamp plate at its surfaces on skins
** Name: ClampedEdges Type: Symmetry/Antisymmetry/Encastre
*Boundary
CLAMPED, ENCASTRE
** Name: U3Imp Type: Displacement/Rotation
*** Constrain the impactor to move in Z-dir only
*Boundary
ImpN, 1, 1
ImpN, 2, 2
ImpN, 4, 4
ImpN, 5, 5
ImpN, 6, 6
**
** FIELDS
*** Defines initial velocity of impactor
** Name: initVel Type: Velocity
*Initial Conditions , type=VELOCITY
ImpN, 1, 0.
ImpN, 2, 0.
ImpN, 3, -2.3
** INTERACTIONS
**
** Interaction: Defines contact between all surfaces using general
      contact algorithm
*Contact , op=NEW
*Contact Inclusions , all exterior
*Contact property assignment
  , , nofric
** -----
**

```

```

** STEP: Drop
*** In this step, the impactor drops 0.1 mm under gravity
*Step, name=Drop
drop0.1mm
*Dynamic, Explicit
, 4.3474e-05
*Bulk Viscosity
0.06, 1.2
**
** LOADS
**
** Name: gravity    Type: Gravity
*Dload
Imp, GRAV, 9.81, 0., 0., -1.
**
** OUTPUT REQUESTS
**
*Restart, write, number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, number intervals=5
*Node Output
RF, U, V
*Element Output
LE, PE, PEEQ, S
*Contact Output
CSTRESS,
**
** HISTORY OUTPUT: Energy
**
*Output, history, variable=PRESELECT, time interval=8.6948e-06
**
** HISTORY OUTPUT: U3_botfsN
**
*Output, history, time interval=8.6948e-06
*Node Output, nset=BotFS_N
U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output, nset=CLAMPED
RF3,
**
** HISTORY OUTPUT: U3_IMP
**
*Node Output, nset=ImpN
U3,
**
** HISTORY OUTPUT: U3_PlateN
**
*Node Output, nset=PLATEN
U3,
*End Step
** -----
**

```

```

** STEP: impact
**
*Step, name=impact
impact for 4.2 ms
*Dynamic, Explicit
, 0.0042
*Bulk Viscosity
0.06, 1.2
**
**
** OUTPUT REQUESTS
**
*Restart, write, number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, number intervals=100
*Node Output
RF, U, V
*Element Output
LE, PE, PEEQ, S
*Contact Output
CSTRESS,
**
** HISTORY OUTPUT: Energy
**
*Output, history, variable=PRESELECT
**
** HISTORY OUTPUT: U3_botfsN
**
*Output, history
*Node Output, nset=BotFS_N
U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output, nset=CLAMPED
RF3,
**
** HISTORY OUTPUT: U3_IMP
**
*Node Output, nset=ImpN
U3,
**
** HISTORY OUTPUT: U3_PlateN
**
*Node Output, nset=PLATEN
U3,
*End Step

```

C.3 Quasi-static Indentation of Composite Sandwich Plate

```

*Heading
Quasi-static indentation of composite sandwich model C2/13/15
*Part, name="Indentor"
*End Part
**
*Part, name="Core"
*End Part
**
*Part, name="Solid skin"
*End Part
**
** ASSEMBLY
**
*Assembly, name=Assembly
*Instance, name="Indentor-1", part="Indentor"
*Element, type=C3D4
..
..
*End Instance
**
*Instance, name="Core-1", part="Core"
*Element, type=S4R
..
..
*End Instance
**
*Instance, name="Solid skin-1", part="Solid skin"
*Element, type=C3D8R
..
..
*End Instance
**
*Instance, name="Solid skin-2", part="Solid skin"
*Element, type=C3D8R
..
..
*End Instance
**
*** Defines the node sets for boundary and loading conditions, and
    output requests
*Nset, nset=IndentorN, instance="Indentor-1"
..
*Elset, elset=Indentor, instance="Indentor-1"
..
*Nset, nset=TFS, instance="Solid skin-1", generate
..
*Elset, elset=TFS, instance="Solid skin-1", generate
..
*Nset, nset=core, instance="Core-1", generate
..
*Elset, elset=core, instance="Core-1", generate
..

```

```

*Nset, nset=hc-coretop, instance="Core-1"
..
*Elset, elset=hc-coretop, instance="Core-1"
..
*Nset, nset=hc-corebot, instance="Core-1"
..
*Elset, elset=hc-corebot, instance="Core-1"
..
*Nset, nset=contactregion, instance="Solid skin-1"
..
*Elset, elset=contactregion, instance="Solid skin-1", generate
..
*Nset, nset=TFSN, instance="Solid skin-1"
..
*Nset, nset=skins, instance="Solid skin-1", generate
..
*Nset, nset=skins, instance="Solid skin-2", generate
..
*Elset, elset=skins, instance="Solid skin-1", generate
..
*Elset, elset=skins, instance="Solid skin-2", generate
..
*Nset, nset=CLAMPED, instance="Solid skin-1"
..
*Nset, nset=CLAMPED, instance="Solid skin-2"
..
*Elset, elset=CLAMPED, instance="Solid skin-1"
..
*Elset, elset=CLAMPED, instance="Solid skin-2"
..
*Nset, nset=BFS, instance="Solid skin-2", generate
..
*Elset, elset=BFS, instance="Solid skin-2", generate
..
*Nset, nset=BFSN, instance="Solid skin-2"
..
*Surface, type=ELEMENT, name=impsurf
..
*Surface, type=ELEMENT, name=TFS-bot
..
*Surface, type=ELEMENT, name=BFS-top
..
*Surface, type=NODE, name=contactregion_CNS_, internal
..
*Surface, type=NODE, name=hc-corebot_CNS_, internal
..
*Surface, type=NODE, name=hc-coretop_CNS_, internal
..
**
*** Defines an element-based surface (surferode)
*** which consists of both exterior and interior faces
*** of elements in a region (contactregion)
*** in the top skin adjacent to impactor.
*surface, name=extCR, type=element
contactregion
*surface, name=intCR, type=element

```

```

contactregion, interior
*surface, name=surferode, combine=union
extCR, intCR
**
*** Defines the tie constraints between skins and core
** Constraint: BFS-core
*Tie, name=BFS-core, adjust=yes, type=NODE TO SURFACE
hc-corebot_CNS_, BFS-top
** Constraint: RigidImp
*Rigid Body, ref node=IndentorN, elset=Indentor
** Constraint: TFS-core
*Tie, name=TFS-core, adjust=yes, type=NODE TO SURFACE
hc-coretop_CNS_, TFS-bot
*End Assembly
**
** MATERIALS
**
*Material, name=Lamina
*Density
1100.,
*** Declare the state variables for use in VUMAT.
*** The 16th flag is set to the deletion flag for fibre failure.
*Depvar, delete=16
21,
*** Input material strengths for lamina
*User Material, constants=6
1.9e+09, -1.55e+09, 6.55e+07, -1.4e+08, 1.012e+08, 6.55e+07
*Material, name=Steel_BIGIMP_2650g
*Density
2.252e+06,
*Elastic
2e+11, 0.3
*Material, name="nomex core"
*Density
710.,
*Elastic
2e+09, 0.4
*Plastic
3e+07, 0.
**
** INTERACTION PROPERTIES
**
*Surface Interaction, name=nofric
*Friction
0.,
**
** BOUNDARY CONDITIONS
*** Clamps the sandwich plate around its circumferential edges on the
skins
** Name: ClampedEdges Type: Symmetry/Antisymmetry/Encastre
*Boundary
CLAMPED, ENCASTRE
**
*** Initialise state variables for VUMAT, where
*** State variable 1 = E1
*** State variable 2 = E2

```

```

*** State variable 3 = E3
*** State variable 4 = v12
*** State variable 5 = v13
*** State variable 6 = v23
*** State variable 7 = v21
*** State variable 8 = v31
*** State variable 9 = v32
*** State variable 10 = G12
*** State variable 11 = G13
*** State variable 12 = G23
*** State variable 13 to 21 = Indices and flags for failure modes
*** State variable 16 = Element deletion flag for fibre failure
    initialised to 1
*Initial Conditions, type=Solution
skins, 150e9, 9.5e9, 9.5e9, 0.263, 0.263, 0.458, 0.0167
0.0167, 0.458, 5.43e9, 5.43e9, 3.26e9, 0, 0, 0
1,0,0,0,0,0
***
**
** INTERACTIONS
**
*** Interaction: Defines contact using general contact algorithm
*** Surface erosion is also modelled.
*** Once an element in top skin fails by fibre failure (deleted),
*** its faces are removed from contact domain,
*** and any interior faces that have been exposed are activated.
*Contact
*Contact inclusions
,
surferode,
, surferode
*Contact Controls Assignment, Nodal erosion=NO
*Contact property assignment
, , nofric
**
*** Defines a smooth step amplitude curve for prescribed displacement
    of indenter
*Amplitude, name=Amp-1, definition=SMOOTH STEP
0., 0., 0.0001, 1.
*Amplitude, name=Amp-2, definition=SMOOTH STEP
0., 0., 0.002, 1.
** -----
**
** STEP: Drop
*** In this step, the indenter displaces downward 0.1 mm
*Step, name=Drop
Move downward 0.1mm in 0.1 ms
*Dynamic, Explicit
, 0.0001
*Bulk Viscosity
0.06, 1.2
**
** BOUNDARY CONDITIONS
**
** Name: U3Imp Type: Displacement/Rotation
*Boundary, amplitude=Amp-1

```

```

IndentorN, 1, 1
IndentorN, 2, 2
IndentorN, 3, 3, -0.0001
IndentorN, 4, 4
IndentorN, 5, 5
IndentorN, 6, 6
**
*** OUTPUT REQUESTS : Defines the output required.
**
*Restart, write, number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, number interval=5
*Node Output
RF, U, V
*Element Output, directions=YES
LE, PE, PEEQ, S
*Contact Output
CSTRESS,
**
** FIELD OUTPUT: status_sdv
**
*Element Output, elset=skins, directions=YES
SDV, STATUS
**
** HISTORY OUTPUT: U3BFS
**
*Node Output, nset=BFSN
U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output, nset=CLAMPED
RF3,
**
** HISTORY OUTPUT: U3_INDENTOR
**
*Node Output, nset=IndentorN
U3
**
** HISTORY OUTPUT: U3TFSN
**
*Node Output, nset=TFSN
U3,
**
** HISTORY OUTPUT: Energy
**
*Output, history, variable=PRESELECT, time interval=5e-06
*End Step
** -----
**
** STEP: indent
*** In this step, the indenter displaces downward a further 4 mm
*Step, name=indent
indent 4mm

```



```

*Dynamic, Explicit
, 0.002
*Bulk Viscosity
0.06, 1.2
**
** BOUNDARY CONDITIONS
**
** Name: U3Imp Type: Displacement/Rotation
*Boundary, amplitude=Amp-2
IndentorN, 1, 1
IndentorN, 2, 2
IndentorN, 3, 3, -0.004
IndentorN, 4, 4
IndentorN, 5, 5
IndentorN, 6, 6
**
** OUTPUT REQUESTS - Defines the output required.
**
*Restart, write, number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, number interval=50
*Node Output
RF, U, V
*Element Output, directions=YES
LE, PE, PEEQ, S
*Contact Output
CSTRESS,
**
** FIELD OUTPUT: status_sdv
**
*Element Output, elset=skins, directions=YES
SDV, STATUS
**
** HISTORY OUTPUT: U3BFS
**
*Node Output, nset=BFSN
U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output, nset=CLAMPED
RF3,
**
** HISTORY OUTPUT: U3_INDENTOR
**
*Node Output, nset=IndentorN
U3
**
** HISTORY OUTPUT: U3TFSN
**
*Node Output, nset=TFSN
U3,
**
** HISTORY OUTPUT: Energy

```

```

**
*Output, history, variable=PRESELECT
*End Step

```

C.4 Impact of Composite Sandwich Plate

```

*Heading
Composite sandwich model C2/13/15 impacted at 1.8 J
*Part, name="Impactor"
*End Part
**
*Part, name="Core"
*End Part
**
*Part, name="Solid skin"
*End Part
**
** ASSEMBLY
**
*Assembly, name=Assembly
*Instance, name="Impactor-1", part="Impactor"
*Element, type=C3D4
..
..
*End Instance
**
*Instance, name="Core-1", part="Core"
*Element, type=S4R
..
..
*End Instance
**
*Instance, name="Solid skin-1", part="Solid skin"
*Element, type=C3D8R
..
..
*End Instance
**
*Instance, name="Solid skin-2", part="Solid skin"
*Element, type=C3D8R
..
..
*End Instance
**
*** Defines the node sets which are used to define boundary and loading
    conditions
*Nset, nset=ImpN, instance="Impactor-1"
..
*Elset, elset=Imp, instance="Impactor-1"
..
*Nset, nset=TFS, instance="Solid skin-1", generate
..
*Elset, elset=TFS, instance="Solid skin-1", generate
..

```

```

*Nset, nset=core, instance="Core-1", generate
..
*Elset, elset=core, instance="Core-1", generate
..
*Nset, nset=hc-coretop, instance="Core-1"
..
*Elset, elset=hc-coretop, instance="Core-1"
..
*Nset, nset=hc-corebot, instance="Core-1"
..
*Elset, elset=hc-corebot, instance="Core-1"
..
*Nset, nset=contactregion, instance="Solid skin-1"
..
*Elset, elset=contactregion, instance="Solid skin-1", generate
..
*Nset, nset=TFSN, instance="Solid skin-1"
..
*Nset, nset=skins, instance="Solid skin-1", generate
..
*Nset, nset=skins, instance="Solid skin-2", generate
..
*Elset, elset=skins, instance="Solid skin-1", generate
..
*Elset, elset=skins, instance="Solid skin-2", generate
..
*Nset, nset=CLAMPED, instance="Solid skin-1"
..
*Nset, nset=CLAMPED, instance="Solid skin-2"
..
*Elset, elset=CLAMPED, instance="Solid skin-1"
..
*Elset, elset=CLAMPED, instance="Solid skin-2"
..
*Nset, nset=BFS, instance="Solid skin-2", generate
..
*Elset, elset=BFS, instance="Solid skin-2", generate
..
*Nset, nset=BFSN, instance="Solid skin-2"
..
*Surface, type=ELEMENT, name=impsurf
..
*Surface, type=ELEMENT, name=TFS-bot
..
*Surface, type=ELEMENT, name=BFS-top
..
*Surface, type=NODE, name=contactregion_CNS_, internal
..
*Surface, type=NODE, name=hc-corebot_CNS_, internal
..
*Surface, type=NODE, name=hc-coretop_CNS_, internal
..
**
*** Defines an element-based surface (surferode)
*** which consists of both exterior and interior faces
*** of elements in a region (contactregion)

```

```

*** in the top skin adjacent to impactor.
*surface, name=extCR, type=element
contactregion
*surface, name=intCR, type=element
contactregion, interior
*surface, name=surferode, combine=union
extCR, intCR
**
*** Defines the tie constraints between skins and core
** Constraint: BFS-core
*Tie, name=BFS-core, adjust=yes, type=NODE TO SURFACE
hc-corebot_CNS_, BFS-top
** Constraint: RigidImp
*Rigid Body, ref node=ImpN, elset=Imp
** Constraint: TFS-core
*Tie, name=TFS-core, adjust=yes, type=NODE TO SURFACE
hc-coretop_CNS_, TFS-bot
*End Assembly
**
** MATERIALS
**
*Material, name=Lamina
*Density
1100.,
*** Declare the state variables for use in VUMAT.
*** The 16th flag is set to the deletion flag for fibre failure.
*Depvar, delete=16
21,
*** Input material strengths for lamina
*User Material, constants=6
1.9e+09, -1.55e+09, 6.55e+07, -1.4e+08, 1.012e+08, 6.55e+07
*Material, name=Steel.BIGIMP_2650g
*Density
2.252e+06,
*Elastic
2e+11, 0.3
*Material, name="nomex core"
*Density
710.,
*Elastic
2e+09, 0.4
*Plastic
3e+07,0.
**
** INTERACTION PROPERTIES
**
*Surface Interaction, name=nofric
*Friction
0.,
**
** BOUNDARY CONDITIONS
*** Clamps the sandwich plate around its circumferential edges on the
skins
** Name: ClampedEdges Type: Symmetry/Antisymmetry/Encastre
*Boundary
CLAMPED, ENCASTRE

```

```

** Name: U3Imp Type: Displacement/Rotation
*** Constrain the impactor to move in Z-dir only
*Boundary
ImpN, 1, 1
ImpN, 2, 2
ImpN, 4, 4
ImpN, 5, 5
ImpN, 6, 6
**
**
** PREDEFINED FIELDS
*** Defines initial velocity of impactor
** Name: initVel Type: Velocity
*Initial Conditions, type=VELOCITY
ImpN, 1, 0.
ImpN, 2, 0.
ImpN, 3, -1.18
**
**
*** Initialise state variables for VUMAT, where
*** State variable 1 = E1
*** State variable 2 = E2
*** State variable 3 = E3
*** State variable 4 = v12
*** State variable 5 = v13
*** State variable 6 = v23
*** State variable 7 = v21
*** State variable 8 = v31
*** State variable 9 = v32
*** State variable 10 = G12
*** State variable 11 = G13
*** State variable 12 = G23
*** State variable 13 to 21 = Indices and flags for failure modes
*** State variable 16 = Element deletion flag for fibre failure
    initialised to 1
*Initial Conditions, type=Solution
skins, 150e9, 9.5e9, 9.5e9, 0.263, 0.263, 0.458, 0.0167
0.0167, 0.458, 5.43e9, 5.43e9, 3.26e9, 0, 0, 0
1,0,0,0,0,0
***
**
** INTERACTIONS
**
*** Interaction: Defines contact using general contact algorithm
*** Surface erosion is also modelled.
*** Once an element in top skin fails by fibre failure (deleted),
*** its faces are removed from contact domain,
*** and any interior faces that have been exposed are activated.
*Contact
*Contact inclusions
,
surferode,
, surferode
*Contact Controls Assignment, Nodal erosion=NO
*Contact property assignment
, , nofric

```

```

**
** -----
**
** STEP: Drop
*** In this step, the impactor drops 0.1 mm under gravity
*Step, name=Drop
drop 0.1 mm with velocity of 1.18m/s
*Dynamic, Explicit
, 8.47159e-05
*Bulk Viscosity
0.06, 1.2
**
** LOADS
**
** Name: gravity    Type: Gravity
*Dload
Imp, GRAV, 9.81, 0., 0., -1.
**
*** OUTPUT REQUESTS : Defines the output required.
**
*Restart, write, number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, number interval=5
*Node Output
RF, U, V
*Element Output, directions=YES
LE, PE, PEEQ, S
*Contact Output
CSTRESS,
**
** FIELD OUTPUT: status_sdv
**
*Element Output, elset=skins, directions=YES
SDV, STATUS
**
** HISTORY OUTPUT: U3BFS
**
*Node Output, nset=BFSN
U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output, nset=CLAMPED
RF3,
**
** HISTORY OUTPUT: U3_V3_IMP
**
*Node Output, nset=ImpN
U3, V3
**
** HISTORY OUTPUT: U3TFSN
**
*Node Output, nset=TFSN
U3,

```

```

**
** HISTORY OUTPUT: Energy
**
*Output, history, variable=PRESELECT, time interval=4.2358e-06
*End Step
** -----
**
** STEP: impact
*Step, name=impact
impact
*Dynamic, Explicit
, 0.006
*Bulk Viscosity
0.06, 1.2
**
** OUTPUT REQUESTS - Defines the output required.
**
*Restart, write, number interval=1, time marks=NO
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, number interval=50
*Node Output
RF, U, V
*Element Output, directions=YES
LE, PE, PEEQ, S
*Contact Output
CSTRESS,
**
** FIELD OUTPUT: status_sdv
**
*Element Output, elset=skins, directions=YES
SDV, STATUS
**
** HISTORY OUTPUT: U3BFS
**
*Node Output, nset=BFSN
U3,
**
** HISTORY OUTPUT: RF3
**
*Node Output, nset=CLAMPED
RF3,
**
** HISTORY OUTPUT: U3_V3_IMP
**
*Node Output, nset=ImpN
U3, V3
**
** HISTORY OUTPUT: U3TFSN
**
*Node Output, nset=TFSN
U3,
**
** HISTORY OUTPUT: Energy
**

```

```
*Output , history , variable=PRESELECT  
*End Step
```


List of Publications

International Peer-reviewed Journals

1. C. C. Foo, G. B. Chai and L. K. Seah. Quasi-static and low-velocity impact failure of aluminium honeycomb sandwich panels. *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, 220:53–66, 2006.
2. C. C. Foo, G. B. Chai and L. K. Seah. Mechanical properties of Nomex material and Nomex honeycomb structure. *Composite Structures*, 80:588–594, 2007.
3. C. C. Foo, L. K. Seah and G. B. Chai. Low-velocity impact failure of aluminium honeycomb sandwich panels. *Composite Structures*, 85:20–28, 2008.
4. C. C. Foo, G. B. Chai and L. K. Seah. A model to predict low-velocity impact response and damage in sandwich composites. *Composites Science and Technology*, 68:1348–1356, 2008.

International Conferences

1. C. C. Foo, L. K. Seah, G. B. Chai, and Y. C. Peh. Mechanical properties of aramid paper honeycomb. In *Proceedings of Twelve International Conference on Composites/Nano-Engineering (ICCE-12)*, edited by D. Hui, Tenerife, Spain, 1-6 August 2005.
2. C. C. Foo, L. K. Seah, and G. B. Chai. Low-velocity impact failure of aluminium honeycomb sandwich panels. In *Proceedings of Fifth International Conference on Mechanics and Materials in Design*, edited by J. F. Silva Gomes and S. A. Meguid, Porto, Portugal, 24-26 July 2006.

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