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A DIFFERENTIAL EVOLUTION APPROACH TO NOISE
REDUCTION IN BIOMEDICAL IMAGES

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**A DIFFERENTIAL EVOLUTION APPROACH TO
NOISE REDUCTION
IN BIOMEDICAL IMAGES**

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SCHOOL OF CHEMICAL AND BIOMEDICAL ENGINEERING
2009**

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A DIFFERENTIAL EVOLUTION APPROACH TO NOISE REDUCTION IN BIOMEDICAL IMAGES

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Abstract

Tomographic imaging modalities such as Magnetic Resonance Imaging (MRI), Optical Coherence Tomography (OCT) and Positron emission tomography (PET) are proving to be immensely helpful in the diagnosis of various pathologies in clinical environments. Images obtained using these modalities span the range from organ level to tissue level and represent the condensation of the information associated with the objects that are imaged. The potential of this information increases significantly, if these images are effectively processed and analyzed. However, the quality of medical images is, usually, compromised due to the undesirable noise which reduces the signal to noise ratio (SNR) of these images. The degradation of images by noise influences the diagnosis of critical diseases in clinical environments. A novel noise reduction method based on wavelets and differential evolution has been proposed in this work. The proposed method was tested on the images that were acquired from MRI, OCT and PET. The application of this method improved SNR by a considerable amount while the loss in contrast to noise ratio (CNR) was kept at minimum level. A comprehensive analysis of processed images, on the basis of different image quality indices, showed that the proposed method, effectively, reduces noise without sacrificing the image resolution.

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LIST OF ABBREVIATIONS

CNR	Contrast-To-Noise Ratio
CWT	Continuous Wavelet Transform
DE	Differential Evolution
DEWDF	Differential Evolution based Wavelet Domain Filtering
DWT	Wavelet Transform
FID	Free Induction Decay
FWHM	Full Width Half Maximum
MR	Magnetic Resonance
MRI	Magnetic Resonance Imaging
MRS	Magnetic Resonance Spectroscopy
NMR	Nuclear Magnetic Resonance
OCDR	Optical Coherence-Domain Rectometry
OCT	Optical Coherence Tomography

PET	Positron Emission Tomography
PSF	Point Spread Function
RF	Radio Frequency
ROI	Region of Interest
SNR	Signal-To-Noise Ratio
TE	Time to Echo
TR	Time to Repeat
UIQI	Universal Image Quality Index
VOR	Volume of Response
WDF	Wavelet Domain Filtering

CHAPTER 1

INTRODUCTION

Medical Imaging is the tool to visualize and analyze the structures inside the human body. Surgery and endoscopy involve invasive techniques in order to allow direct visualization of internal organs. But these are invasive techniques and have potential to cause damage or trauma to the body. Medical imaging techniques are advantageous because they allow the visualization of the inside of body in ways that are less invasive than surgery or endoscopy and, hence, they are categorized into non-invasive techniques. These imaging techniques allow us to see inside the human body using intrinsically different type of signals and they are called as imaging modalities. The signals in these modalities, usually, represent parameters of the human tissues such as reflectivity in ultrasound imaging, linear attenuated coefficient in computed tomography and hydrogen proton density in magnetic resonance imaging. These signals, hence, depend on the physical processes governing a given imaging modality. Interestingly, different imaging modalities are based upon fundamentally similar physical processes [1, 2]. Some form of energy is measured after it is passed through a region of the body, and from these measurements, mathematical estimates

are computed. The images are then produced on the basis of distribution of interactions between the energy and the body tissues. However, the specific details of applying these principles and the information content and quality of the final images are quite different. This is due to the different energy/tissue interactions that are measures of the varying biological, physical, and technological constraints imposed in obtaining these measurements practically [2]. The images produced using these different medical imaging modalities span orders of magnitude in scale, ranging from molecules to cells and cells to organ systems and the full body. Because the variety of disease processes and abnormalities affecting all regions of the human body are so numerous and different, each imaging modality possesses attributes that make it uniquely helpful in providing the desired understanding and/or discrimination of the disease or abnormality. Therefore, even after the rapid advances in imaging technology, no single image modality has, hitherto, prevailed to the complete exclusion of others.

Nevertheless, all these modalities share at least one common characteristic: although, the images acquired using each of these modalities contain vital diagnostic information their quality is often obscured by the presence of undesirable noise. The extraction of this information from the noisy images requires the application of few noise reduction algorithms before the images can be used for practical purposes such as diagnosis of diseases in a clinical environment. Moreover, the segmentation and object recognition algorithms work better when there is less noise in the images. Hence, achieving the full potential of imaging modalities is dependent on

1.1. Aim of project

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the effective processing and analysis of the images in order to faithfully extract the information. This faithful extraction of hidden information, enables understanding and insights into the structure and function of the object(s) being imaged.

1.1 Aim of project

Images acquired using different modalities can be enhanced by removing the noise from them or their visibility can be improved by increasing the contrast in the image. Image enhancement techniques such as contrast stretching and noise filters are the commonly used methods that aim to improve the quality of an image. This typically involves making certain features in the image more recognizable and prominent. Examples are amplification of edges or reduction of noise to increase the contrast between regions of an image. More specialized operations are also possible, for example, increasing the visibility of the features at a certain scale or with a certain spectral signature. Usually, noise reduction methods introduce undesired blurring effect in processed images as they fail to retain the sharp edges in images. Among all the methods for noise reduction, wavelet domain filtering stands out as one of the best methods to reduce the noise significantly while retaining the edges in the processed image. However, wavelet based method requires the selection of few parameters in order to produce optimum results. The aim of this project was to design and test a novel algorithm for the optimization and automation of wavelet domain filtering. In order to achieve this aim, the application of an evolutionary

algorithm called Different Evolution has been explored. The proposed algorithm was tested on the images acquired using three different medical imaging modalities *viz.*, Magnetic Resonance Imaging (MRI), Optical Coherence Tomography (OCT) and Positron Emission Tomography (PET).

1.2 Structure of Thesis

The Thesis follows the pattern of a research article. Chapter 1(the current chapter) introduces the thesis with the precise background information on the subject of thesis. It also puts forth the aim of this project followed by the structure of thesis(current section). Chapter 2, i.e., Literature review and Theoretical background, introduces the basic concepts pertaining to the different imaging modalities. In the same chapter, detail descriptions of Wavelets and Differential Evolution are also provided to allow the segregation of existing theory and proposed method. For the sake of better integrity of events with their order of occurrences, the literature review is presented along with the theoretical background. Chapter 3, called Materials and Method, relates the concepts discussed in previous chapter to the proposed method by providing the details of different material components and their use in the implementation of method in conjunction with existing theories. A comprehensive discussion of results, that were obtained after the application of method on images from different modalities, has been given in next chapter. Finally, the summary of the proposed method along with its application and achieved results is presented in

1.2. Structure of Thesis

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Conclusion. The papers that have been generated from this work have been enumerated in a separate list which is appended to this thesis as ‘List of Publications’ at the end just before the Bibliography. All the cited references have been provided in Bibliography.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL BACKGROUND

2.1 Wavelets in medical imaging

An important area of development in medical imaging has been the post processing of images after their reconstruction. Various methods in image processing have been proposed and they are reviewed in the following section. One of the most fundamental problems in image processing is to find a suitable representation of the data that will facilitate an analysis procedure. One way to achieve this goal is to use transformation, or decomposition of the image on a set of basis functions prior to processing in the transform domain. Transform theory has played a key role in image processing for a number of years, and it continues to be a topic of interest in theoretical as well as applied work in this field. Image transforms are used widely in many image processing fields, including image enhancement, restoration, encoding, and description [3]. Among the existing transforms, none has received so much response and enthusiasm in the medical image processing community as received

2.1. Wavelets in medical imaging

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by wavelet transforms. Wavelets provide a unifying framework for decomposing images, volumes, and time-series data into their elementary constituents across scale. Wavelet methods of analysis and representation are presently having a significant impact on the science of medical imaging and the diagnosis of disease and screening protocols. Because of a powerful underlying mathematical theory, they offer exciting opportunities for the design of new multiresolution image processing algorithms. Wavelets can be traced all the way back to Alfred Haar in 1910 [4], but since two publications in the late 1980s by S. Mallat [5] and I. Daubechies [6], wavelet has now become a major field in itself. In those papers, S. Mallat identified the important concept of multiresolution analysis which is the corner stone of modern wavelet theory, while I. Daubechies constructed the first orthogonal wavelet bases that were compactly supported. After that, a milestone paper by Grossmann and Morlet [7] is now considered as the beginning point of modern wavelet analysis. Interesting, historical anecdotes and folklore associated with wavelets can be found in the book of B. Burke Hubbard [8]. The first journal paper describing a wavelet application in medical imaging was related to noise reduction in MRI by soft- thresholding in the wavelet domain which appeared in 1991 [9]. This work, provided the earliest description of a wavelet denoising method that has become extremely popular through the impulsion of Donoho et al. A large range of wavelet applications in medical imaging is provided in [10]. Due to their multiresolution property, wavelets have achieved great success in signal denoising and, hence, wavelet denoising methods have been intensively researched over the past two decades. Pizurica et al.

2.1. Wavelets in medical imaging

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proposed a novel wavelet denoising method that uses a local statistical model for improved signal estimation and noise suppression [11]. Willett and Nowak [12] introduced a new multiscale image model, using piecewise planar basis functions, and applied their method to the reconstruction of photon-limited data (with Poisson noise). They developed penalized maximum likelihood methods for image denoising, deconvolution, and tomographic reconstruction. The approach taken by Kalifa et al. includes a nonlinear noise reduction step that is implemented by thresholding in the transformed domain; the key here was to select a transform (wavelet packet) that was optimized for the problem and data at hand [13]. Donoho and Johnstone provided an asymptotically optimal choice for a single thresholding parameter under soft thresholding and an assumption of white noise [14, 15]. Following their work, Nason [16] and Jansen, Malfait, and Bultheel [17] proposed cross validation (CV) and generalized cross validation (GCV) techniques, respectively, for thresholding parameter selection. Johnstone and Silverman [18] suggested dealing with correlated noise by grouping wavelet coefficients according to their resolution levels and then selecting thresholding parameters separately for each resolution level. Jansen and Bultheel [19] subsequently revised the GCV technique to accommodate correlated noise. Thresholding rule specification has been further examined by Antoniadis and Fan [20] and by Figueiredo and Nowak [21]. Later, empirical Bayes frameworks were proposed by Jansen and Bultheel [22] and by Johnstone and Silverman [23]. As the wavelet based thresholding depends on the number of resolution levels, Nason [24] designed a procedure for determining the number of resolution

levels at which thresholding is performed. To tackle the issues of basis selection for wavelet based image processing, several studies have been done by Candes and Donoho [25] and by Ferrando and Kolasa [26]. Although there have been many popular image processing methods for speckle reduction in OCT such as median filtering [27], homomorphic Wiener filtering [28], multi-resolution wavelet analysis [29, 30] and adaptive smoothing [31] the only post-processing method that has been reported specifically for speckle reduction in OCT is a wavelet filter that employs nonlinear thresholds [32]. The reported results indicate that when the frequency bands and thresholds are chosen properly wavelet filtering can improve the contrast-to-noise ratio of highly scattering features corrupted with speckle. However, as the power spectral densities of signal-carrying and signal degrading speckle overlap, some loss of useful information is inevitable. Hence, wavelet processing is found to be quite useful for removing the fine granular noise that pervades OCT images of dense tissue. Although, substantial improvement in image quality has been achieved through some skillful adaptation of available techniques, none of these techniques has proved to be entirely satisfactory and that leaves the scope for further improvement in this area.

2.2 Wavelet domain filtering for images

The wavelet transform is an analysis tool that expands signals onto orthogonal and semi-orthogonal bases. Since wavelets have their energy well localized in time or

Figure 2.1: Time frequency localization of wavelet transform

space, the wavelet transform is especially suitable for the analysis of non-stationary signals, such as medical images and time-activity data. The energy of a signal is partitioned through a wavelet expansion to express simultaneous time (space) and frequency local information (Figure 2.1). In wavelet domain filtering, 2-D wavelet transform of the input image is calculated to decompose the image into shifted and scaled versions of a finite time waveform. This waveform is called as mother wavelet and the result of this decomposition results into detail and approximation coefficients. The basis functions used in wavelet transforms are locally supported; they are nonzero only over part of the domain represented. Wavelet bases are bases of nested function spaces, which can be used to analyze signals at multiple scales. Best-basis selection consists of finding the mathematical function that not only completely represents the original signal, but that also concentrates the maximum amount of structured signal into the minimum number of coefficients. However, in

2.2. Wavelet domain filtering for images

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many applications, the key signal features are not well known and the optimal basis functions cannot be specified in advance. In such cases, a basis with general properties is preferable. Of a particular interest to the image processing is the fact that wavelets preserve the sharp transitions in images and depict them extremely well in wavelet expansions [33]. Due to this special treatment of edges, wavelet transforms have been very attractive in image filtering. A more comprehensive description of wavelet transforms for images can be found in ‘Digital Image Processing’ by Gonzalez and Woods [34]. In next few subsections, the concept of wavelet transform is introduced to prepare a foundation for discussion on wavelet based noise filtering. It should be noted that the description is just a brief overview of very wide area of wavelets. Readers are encouraged to refer to an excellent book on wavelets by Stéphane Mallat [35] for further details.

2.2.1 One-dimensional wavelet transform

The one-dimensional continuous wavelet transform (CWT) of a function $f(x)$ is written as:

$$W(s, \tau) = \int f(x) \psi_{s,\tau}^*(x) dx \quad (2.1)$$

where $*$ represents complex conjugation. This equation shows how a function $f(s)$ is decomposed into a set of basis functions $\psi_{s,\tau}^*(t)$, called the wavelets. The variables s and τ represent scale and translation dimensions in wavelet domain. After the desired processing in wavelet domain, function $f(x)$ can be recovered with the

2.2. Wavelet domain filtering for images

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application of inverse wavelet transform that is given by following equation:

$$f(x) = \int \int W(s, \tau) \psi_{s,\tau} d(\tau) ds \quad (2.2)$$

The wavelets are generated from a single mother wavelet $\psi(t)$ by scaling and translation. The scaling and translation is performed according to the Equation 2.3.

$$\psi_{s,\tau}(x) = \frac{1}{\sqrt{s}} \psi\left(\frac{\tau - t}{s}\right) \quad (2.3)$$

In the above equation s is scale factor and τ is the translation factor. To normalize energy across different scales, a factor of $\frac{1}{\sqrt{s}}$ is used in Equation 2.3. From Equations 2.1, 2.2 and 2.3, it can be observed that, unlike in Fourier transform, the wavelet basis functions are not specified initially in wavelet transform. Therefore, by defining a framework, the theory of wavelet transforms allows the users to deal with the general properties of the wavelets and wavelet transforms only. Because of this reason, it gives the flexibility to design wavelets according to particular application.

As wavelet transform derives most of its fundamental ideas from Fourier series, it has also inherited the series expansion property that made Fourier series so popular. Like Fourier expansion, wavelets also can be used to represent a signal function in terms of a expansion series. The wavelet series expansion is related to wavelet $\psi(x)$ and scaling function $\varphi(x)$ in accordance with following equation:

$$f(x) = \sum_k c_{j_0}(k) \varphi_{j_0,k}(x) + \sum_{j=j_0}^{\infty} \sum_k d_j(k) \psi_{j,k}(x) \quad (2.4)$$

In above equation, j_0 is an arbitrary scale. The coefficients $c_{j_0}(k)$'s and $d_j(k)$'s are called approximation or scaling coefficients and detail or wavelet coefficients.

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This is because the first sum in Equation 2.4 uses scaling functions to provide an approximation of $f(x)$ at scale j_0 for each higher scale $j \geq j_0$ in the second sum, a finer resolution function (a sum of wavelets) is added to the approximation to provide increasing detail. If the expansion functions form an orthonormal basis these coefficients are calculated from:

$$c_{j_0}(k) = \langle f(x), \varphi_{j_0,k}(x) \rangle = \int f(x) \varphi_{j_0,k}(x) dx \quad (2.5)$$

and

$$d_j(k) = \langle f(x), \psi_{j,k}(x) \rangle = \int f(x) \psi_{j,k}(x) dx \quad (2.6)$$

where $\langle \rangle$ represents inner product such that for any two functions a and b the inner product is defined as $\langle a, b \rangle$.

When discrete wavelets are used to transform a continuous signal the result is a series of wavelet coefficients, and it is referred to as the wavelet series decomposition. The signal can then be reconstructed back from its wavelet series decomposition [36]. In many practical applications the signal of interest is sampled. With a discrete signal like images, wavelet transform must be discrete too but the discrete wavelets are not time-discrete - only the translation and the scale steps are discrete. If the signal function being expanded is a sequence of numbers, the resulting coefficients are called the discrete wavelet transform (DWT) of $f(x)$. The DWT derives from the equations that were discussed above for CWT and is given by:

$$W_\varphi(j_0, k) = \frac{1}{\sqrt{M}} \sum_x f(x) \varphi_{j_0,k}(x) \quad (2.7)$$

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$$W_\psi(j, k) = \frac{1}{\sqrt{M}} \sum_x f(x) \psi_{j,k}(x) \quad (2.8)$$

for $j \geq j_0$. The discrete wavelet expansion of $f(x)$ can be represented in terms of coefficients that were given by Equations 2.7 and 2.8.

$$f(x) = \frac{1}{\sqrt{M}} \sum_k W_\varphi(j_0, k) \varphi_{j_0,k}(x) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_k W_\psi(j, k) \psi_{j,k}(x) \quad (2.9)$$

Here, $f(x)$, $\varphi_{j_0,k}(x)$ and $\psi_{j,k}(x)$ are functions of the discrete variable $x = 0, 1, 2, \dots, M-1$. Usually, j_0 assumes the value 0 and M is selected to be the power of 2 (i.e., $M = 2^j$) in order to facilitate the calculation of sum over $x = 0, 1, 2, \dots, M-1$, $j = 0, 1, 2, \dots, J-1$, and $k = 0, 1, 2, \dots, 2^j - 1$. The coefficients $W_\varphi(j_0, k)$ and $W_\psi(j, k)$ in Equations 2.7 and 2.8 correspond to the $c_{j_0}(k)$'s and $d_j(k)$'s in Equations 2.5 and 2.6.

If images are considered from a mathematical point of view, they are nothing but two-dimensional arrays of intensity values that correspond to a two-dimensional discrete signal from signal processing perspective. Those intensity or discrete signal values possess locally varying statistics that result from different combinations of abrupt features such as edges and contrasting homogeneous regions in the image. Hence, in order to analyze the two-dimensional signal, two-dimensional wavelet transform is required.

2.2.2 Two-dimensional wavelet transform

Wavelet transform in two dimension requires the scaled and translated form of two-dimensional scaling function, $\varphi(x)(y)$ and a two-dimensional wavelet $\psi(x, y)$. The

2.2. Wavelet domain filtering for images

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expressions to obtain the scaled and translated forms are shown in following equations:

$$\varphi_{j,m,n}(x, y) = 2^{j/2} \varphi(2^j x - m, 2^j y - n) \quad (2.10)$$

$$\psi_{j,m,n}^i(x, y) = 2^{j/2} \psi^i(2^j x - m, 2^j y - n) \quad (2.11)$$

where i is a superscript such that $i = H, V, D$. This subscript identifies the directional wavelets, ψ^H, ψ^V and ψ^D which measure the functional variations along the horizontal, vertical and diagonal directions respectively. The functions from Equations 2.10 and 2.11 are used to calculate discrete wavelet transform of an image, $I(x, y)$ of size $M \times N$ that, in turn, produces approximation coefficients(A) and detail coefficients(D). The expressions to calculate these coefficients are given by following equations:

$$A_\varphi(j_0, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I(x, y) \varphi_{j_0,m,n}(x, y) \quad (2.12)$$

$$D_\psi^i(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I(x, y) \psi_{j,m,n}^i(x, y) \quad (2.13)$$

Here, j_0 is an arbitrary starting scale. The coefficients, $A_\varphi(j_0, m, n)$ represent the approximation coefficients of image at scale j_0 and $D_\psi^i(j, m, n)$ represent the detail coefficients that add horizontal, vertical and diagonal details for the scales $j \geq j_0$. Usually, the approximation coefficients are calculated for $j = 0$ and detail coefficients are calculated for $j = 0, 1, 2, \dots, R$ such that $N = M = 2^R$ and $m, n = 0, 1, 2, \dots, 2^{j-1}$.

2.2.3 Thresholding

The aim of thresholding method is to discard or reduce the value of wavelet coefficients that are below a preset threshold value. This is because the higher values of detail coefficients are considered to be the representative of noise in wavelet domain. Once the thresholding is applied, the image can be reconstructed from the remaining significant coefficients. Among the existing methods of thresholding, soft and hard thresholding methods are most popular. While the soft thresholding shrinks the input wavelet coefficient by an amount equal to threshold value, the hard thresholds preserve the input coefficient, if it is greater than the threshold. However, both methods reduce the value of coefficients to zero if the the value of coefficient is lesser than or equal to the threshold value. For a given function $f(x)$, if t is the selected threshold value, following equations are used to apply hard and soft thresholding.

Hard thresholding

$$f(x) = \begin{cases} x & \text{if } |x| > t \\ 0 & \text{otherwise} \end{cases} \quad (2.14)$$

Soft thresholding

$$f(x) = \begin{cases} x - t & \text{if } |x| > t \\ 0 & \text{otherwise} \end{cases} \quad (2.15)$$

We have adopted the soft thresholding method in this study because the recent studies in wavelet domain filtering suggested that the soft thresholding produces

Figure 2.2: Thresholding methods

smoother images[37].

The effect of hard and soft thresholding on a function is depicted in Figures 2.2(a) and 2.2(b) respectively.

2.2.4 Inverse wavelet transform

The image is reconstructed once the thresholding is applied to the detail coefficients. The level of decomposition (j) and wavelet basis, that were used in the section 2.2.2, remain same while the detail coefficients obtained after thresholding are put into the following equation to reconstruct the image via inverse wavelet transform.

$$I(x, y) = \sum_m \sum_n A_\varphi(j_0, m, n) \varphi_{j_0, m, n}(x, y) + \sum_{i=H, V, D} \sum_{j=j_0}^{\infty} \sum_m \sum_n D_\psi^i(j, m, n) \psi_{j_0, m, n}^i(x, y) \quad (2.16)$$

2.3. Differential Evolution (DE)

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where $k = \frac{1}{\sqrt{MN}}$. In our research work, possibility of finding the optimal threshold, optimal resolution level and the optimal basis for wavelet denoising was explored by using differential evolution technique [38]. Differential evolution (DE) is an evolutionary algorithm which, although quite new, has the power to solve complex problems where more than two parameters need to be evaluated to optimize one or more objective functions. It is a population based method which has initialization, mutation, recombination and selection as its main components [38]. The details of our method are presented in chapter of Materials and Methods. The next Section briefly describes the basic concept and advantages of DE.

2.3 Differential Evolution (DE)

A conventional direct search method uses a strategy that generates variations of the design parameter vectors. Once a variation is generated, the new parameter vector is accepted or not. The new parameter vector is accepted in the case it reduces the objective function value. This method is usually named the greedy search. The greedy search converges fast but can be trapped by local minima. This disadvantage can be eliminated by running several vectors simultaneously. This is the main idea of differential evolution (DE) algorithm. DE is a population-based heuristic method proposed by Storn and Price [38] for global optimization over continuous spaces. It has been applied to several engineering problems in different areas [39–44]. The main difference between the genetic algorithm and DE codes the space to be

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searched as a set of D -dimensional floating-point vectors. Initially, a population of P randomly chosen vectors is created, and DE attempts to replace all the vectors of the population, with better ones (generated by specific variation operators) per iteration. The overall iterative process cycles until a suitable, user-defined stopping criterion is satisfied. DE uses a random variation that originates from the initialization of the population and the iterated random selection and recombination of candidate solutions within this population. A newly created candidate solution is accepted only if it is better than its parent. In this way, DE recombines existing solutions using difference vectors, which makes the step size and direction of search very efficient. Moreover, DE handles local optima efficiently as suggested by [45]. When DE discovers a local optimum, it generates another offspring which is located in its own vicinity to attract other members of the population. Meanwhile, all kinds of other recombinations within the population are explored that prevents the local optimum from attracting all candidate solutions. For the better understanding of our method, it will be better to define the following terms in the beginning itself:

1. Parameters: The elements whose optimal values need to be estimated.
2. Parameter vector: An array of parameters.
3. Target vector: Best vector in present generation.
4. Trial Vector: Mutated recombinant vector that challenges target vector.
5. Population: A population is a collection of parameter vectors. Here, P denotes

2.3. Differential Evolution (DE)

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the maximum number of vectors in a population.

6. Target population: The population consisting of target vectors.
7. Trial Population: Population of trial vectors
8. Generation: Generation number represents the iteration number. It keeps track of how many target populations have been evolved. The final generation holds the best parameter vectors. G denotes the generation number and G_{max} denotes the maximum generation number.
9. Objective function: The fitness function that decides whether the trial vector displaces target vector to enter into next generation or the target vector retains its position.
10. Initialization parameters: Parameters F , CR and G_{max} are called as Mutation factor, Cross-over ratio and maximum generation number respectively. They are defined later in subsequent chapters.

The detailed working of DE in correlation with our method is described in chapter on Material and Methods i.e., Chapter 3. To provide a brief overview, a flowchart of basic DE algorithm to select optimum parameter vector is shown in Figure 2.3. Because DE requires objective function to select the best parameters, we used widely accepted image quality indices in our method which ensures that images are denoised effectively. The image quality indices, that were used in this method, are discussed in next section.

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Figure 2.3: Flowchart for DE algorithm

2.4 Image quality indices

The processed images are, usually, analyzed using visual inspection because the main aim of denoising methods is to facilitate the diagnosis by physicians. However, visual inspection alone can not be used to analyze the images comprehensively because it depends on viewing conditions and thus, generates different measurement results in different viewing conditions. The use of mathematical defined image quality measures, along with visual inspection, makes the image analysis independent of viewing conditions and individual observers. The image quality indices serve two purposes: first, they are used in the proposed method to calculate objective function and second, they are used to test and compare the efficacy of image filtering methods after the image denoising. In this study, four widely accepted indices of merit were used and they are discussed further in following subsections.

2.4.1 Signal-To-Noise Ratio

Signal-To-Noise Ratio (SNR) is widely used as the key indicator for noise reduction methods and it is calculated from the following equation [46]:

$$SNR = \frac{\mu(ROI)}{\sigma(background)} \quad (2.17)$$

Here, ROI is the region of interest, $\mu(ROI)$ refers to mean pixel value of ROI and $\sigma(background)$ represents standard deviation of the pixels in the background region. The ROI and background were selected in the original image and the same location and size were used for denoised images. It should be noted that although

the background region differs in location with ROI, its size is kept equal to the size of ROI.

2.4.2 Contrast-To-Noise Ratio

Another key indicator of image quality is Contrast-To-Noise Ratio (CNR) that must not deteriorate by a significant amount even after the application of a noise filtering method. CNR can be calculated using the following equation [46]:

$$CNR = \mu(ROI) - \mu(background) \quad (2.18)$$

A significant reduction in CNR also indicates the negative impact on image resolution since CNR also depends on the resolution of the image[46].

2.4.3 Universal Image Quality Index

In 2000, Wang and Bovik proposed a new merit of index called, Universal Image Quality Index ($UIQI$), to take correlation, luminance distortion and contrast distortion into account while assessing the image quality [47]. The expression for $UIQI$ takes two images, x and y as input images and calculates the $UIQI$ value using variances, correlation and average values of pixel intensities in the input images. Moreover, this method of assessing image quality does not depend on viewing conditions and individual observer and hence, it can be used to calculate objective function. The expression to calculate $UIQI$ is as follows.

$$UIQI = \frac{4\sigma_{xy}\bar{x}\bar{y}}{(\sigma_x^2 + \sigma_y^2)((\bar{x})^2 + (\bar{y})^2)} \quad (2.19)$$

where $\bar{x}$ and $\bar{y}$ are the averages values of the images. σ_x^2 and σ_y^2 in the Equation 2.19 are the local variances of images, x and y respectively and σ_{xy} is given by following equation:

$$\sigma_{xy} = \frac{1}{N-1} \sum_{i=1}^N (\bar{x}_i - \bar{x})(\bar{y}_i - \bar{y}) \quad (2.20)$$

where N is total number of pixels in an image.

2.4.4 Full Width Half Maximum

One of the major concerns in image denoising methods has been the maintenance of image resolution. Although the existing methods reduce noise, they also tend to blur the sharp edges, thereby, reducing the image resolution. To measure the resolution loss by our method, noisy point and line phantom images have been used. The point spread function (PSF) of these images are evaluated before and after the application of proposed method. Full Width Half Maximum (FWHM) value, which is calculated as full width of PSF taken at half the maximum of profile, is generally considered as the measure of spatial resolution. PSF represents the response of system to a point source of activity. FWHM indicates the minimum distance between two objects that can be distinguished [48]. A narrow FWHM means better resolution whereas the wider FWHM represents worse resolution. Figure 2.4 gives an idea how FWHM is calculated for a Gaussian PSF.

In subsequent sections, the theoretical backgrounds of different medical imaging modalities have been discussed along with the causes of noise in acquired images.

Figure 2.4: FWHM of a Gaussian PSF

2.5 Positron Emission Tomography (PET)

Popular medical imaging technologies such as computed tomography and magnetic resonance imaging are used when non-invasive studies are required. Although these imaging techniques are important tools for providing structural details of the internal body organs, they fail to register the physiological changes inside the tissues that usually precede the structural changes. In such cases Positron Emission tomography (PET) has proven to be very useful. PET is now one of the most exciting tomographic techniques in modern medical imaging. In PET, positron emitting radiotracers are used, which makes it a functional imaging technique and places it in the family of nuclear medicine modalities [49]. As PET relies on detecting the physiological or molecular changes rather than the structural changes, it can detect an abnormality, such as tumor, at early stages. PET is superior to other imaging

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modalities in determining physiological functions because of the readily prepared radiotracers, such as oxygen, carbon, and nitrogen, are essential elements of the human body which are well incorporated into normal physiological processes.

The historical developments in positron emission tomography (PET) are covered in this section, concentrating on the major events and factors influencing development of PET and its acceptance as a clinical tool. A detailed history of PET is beyond the scope of this report and, hence, only a few important milestones are reviewed. For detailed information, [50] is a very good source. The first family of instruments was developed several decades ago by G Brownell and his group [51]. The first instrument had dual planar detectors [51] working in coincidence and providing longitudinal tomographic images. It progressed from there to rotating detectors and using transverse reconstruction algorithms to obtain transverse sections and finally to complete circular stationary detector arrangements. Earlier, only a single slice [52] was used and subsequently increased number of transverse slices were introduced [53]. Hal Anger also worked on positron imaging almost as soon as he developed the single photon Anger camera [54]. Early efforts to achieve transverse section images in nuclear medicine relied on iterative methods [55, 56], but this changed rapidly with the introduction of x-ray computed tomography and the filtered backprojection technique. As a result, PET scanners were developed at a number of academic institutions [53, 57–62]. A schematic diagram of PET scanner is shown in Figure 2.5. After the introduction of transverse section reconstruction using filtered backprojection, PET imaging advanced rapidly because of

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Figure 2.5: A schematic diagram of PET scanner

the development of FDG ($[^{18}\text{F}]2\text{-fluoro-2-deoxy-D-glucose}$)[63] at the same time, which created an opportunity to study brain function in a way that was impossible up to that point. The hardware improved at fast pace and that gave rise to the need of better reconstruction algorithms and image analysis methods. Filtered backprojection was the obvious initial choice to reconstruct PET images, since this was the standard method for Computed Tomography (CT) and Single Photon Emission Computed Tomography (SPECT). Iterative algorithms for reconstruction have the disadvantage of being more computationally intensive and the image changes both qualitatively and quantitatively as the number of iterations and the relaxation parameter(s) change. Since PET was largely, if not exclusively, used for research studies in the early years of PET, quantitative accuracy was critical. But once the computing power became sufficient to make iterative reconstruction practical, the

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image quality improved significantly using iterative algorithms which was then used for both the emission and the transmission scans. Later, the iterative algorithms became the method of choice in attenuation corrected clinical PET imaging.

2.5.1 Basic principles

PET is a medical imaging technique that employs radioactively labeled compounds to study a wide range of biological processes and physiological pathways in vivo [64]. Before the PET examination, a radionuclide is produced in cyclotron and attached or tagged to a natural body compound such as glucose. This radioactive substance is then administered to the patient. As the nuclei of radioactive substances are unstable in nature, they decay with time (according to their half lives) by emitting positrons and it is their unique decay characteristics which is responsible for the formation of PET images. The positrons, which are emitted by decaying radionuclide, are actually the antiparticle to the electrons with the same mass but with opposite electric charge. When a positron is emitted from the nucleus, it travels a short distance of around few tenths of a millimeter before undergoing an annihilation reaction with an electron in tissue. As a result of this annihilation, two gamma photons are simultaneously emitted in opposite directions. Each gamma photon, emitted from this annihilation reaction, carries energy of 511keV [65]. The detection of two, simultaneously released, gamma photons is then registered by external detectors surrounding the patient. Figure 2.6, shows the schematic diagram of positron annihilation and gamma photon ejection. If the detection is successfully

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Figure 2.6: Annihilation reaction in PET

Figure 2.7: Images of lymphoma patient acquired using PET Scanner

registered, then each gamma photon pair defines a line through the imaging volume along which the decay of the nucleus must have occurred. The collected data is reconstructed into cross-sectional images using standard computed tomography reconstruction methods such as filtered back projection [65]. The images obtained after reconstruction reflect the quantitative distribution of radionuclide in the body. Figure 2.7, shows the variants of typical PET image of a human subject with lymphoma (source: Siemens Medical Solutions).

2.5. Positron Emission Tomography (PET)

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2.5.2 Noise in PET

In PET noise there are several physical factors that affect the quantitative accuracy of PET images. Some of the most important factors include statistics of radioactive decay, attenuation and scatter [66].

Statistics of Radioactive Decay

The radioactive half-life, or $T_{1/2}$ value, is the measure most commonly used to quantify the decay rate, and refers to the time required for half of the radioactive atoms to undergo transformation. Although the half-life of an isotopic species is characteristically constant, the process of radioactive decay is fundamentally statistical. In other words, a radioisotope of essentially infinite half-life (i.e., infinitely constant radioactivity) will produce variations in individually recorded values over identical intervals of time. This variation in sampling is independent of detection method and instead derives from the intrinsic probabilistic nature of radioactive decay that follows a Poisson distribution. The spatial equivalent of this temporal variation in sampling is a mottled appearance in PET images. Thus, the statistical nature of radioactive decay itself accounts for random inconsistencies in PET images. This statistical noise is inversely related to the square root of the counts acquired and is easily reduced in images by collecting more counts. Greater instrument sensitivity and longer sampling times are the two principle ways in which PET improve counting statistics. It depends on which approach is taken as the improvements in statistical noise are generally traded for sacrifices in either temporal or spatial

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resolution.

Attenuation

A significant number of high-energy gamma (γ) photons escape detection by PET scanners as a result of their interactions with surrounding tissues. There are two major types of interactions: Compton scattering and photoelectric absorption. Together they result in only a fraction of an internal radioactive signal reaching external radiation detectors. The physical basis for these attenuation effects derives from the interaction of gamma rays with atomic electrons. In Compton scattering, a gamma photon is deflected from its original trajectory after colliding with an electron and loses a fraction of its original energy in the process. Alternatively, collision may instead result in the complete absorption of a photon's energy. In the latter instance, the imparted kinetic energy is generally sufficient to eject the electron from the atom, and thus gamma radiation is said to be ionizing. Because a photon's chances of being scattered or absorbed become greater as the distance a photon must travel increases, attenuation is said to be depth dependent. Linear attenuation is largely depth independent in PET because of coincidence detection. More simply explicated, the probability that dual photons will be detected is equal for all such pairs along a given trajectory because the combined distance traveled by each pair is identical.

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Scatter

Accurate reconstruction in PET depends on the detection of high-energy photons that travel in a straight path. However, Compton effects cause photons to deviate from their original trajectories. Since many scattered photons retain a sufficient degree of energy to escape, the detection of scattered events leads to the misinterpretation of photon trajectories and produces errors in image reconstruction. The net effect of such errors is similar to that of accidental coincidences, namely an increase in background noise that compromises image contrast. The effects of scatter can be countered with the help of energies of detected photons as the photons lose some of their energies when they are scattered. Hence, some scatter can be eliminated if the count for only those photons are considered that are recorded simultaneously and whose energies lie in a predefined window of 511keV energy. Although this provision will allow less scattered photons to be detected, there will still be some scattered photons with a high energy to fall inside the energy window. This condition is illustrated in Figure 2.8. This problem can be solved by selecting two sets of count: one set for energy window photons and another for those coincident photons that have energies in lower window. The correction is then obtained after subtraction of lower energy photons from higher energy photons. However, lower count rates for the lower energy window results in variable estimates of scatter and hence, scatter correction adds noise.

Adding to the list of already existing problems in PET, there are at least two

2.5. Positron Emission Tomography (PET)

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Figure 2.8: Provision of energy windows to counter scatter in PET

other events that reduce the signal intensity of PET signal and hence, allows noise to dominate. These two events are mentioned very briefly in the following text:

Multiple incidences

When three or more detectors are hit approximately at same time, a multiple incidence occurs. This results in the conflict and hence, these events are ignored.

Randoms

Randoms are those types of multiple events, wherein two simultaneous annihilations have exactly one photon detected. This causes the false increment of count by the two detectors for same volume of response (VOR).

2.6 Magnetic Resonance Imaging (MRI)

Magnetic resonance imaging (MRI) is one of the most promising computerized imaging modalities in present clinical environments. Not only does MRI provide capabilities to obtain images of pathology that are similar to images obtained from other modalities, it also provides additional information that is not available from any other imaging technique. The revolution in medical imaging, called MRI, originated from the work done on nuclear magnetic resonance (NMR or MR) by Bloch et al and Purcell et al in 1946 [67, 68]. Development in MR saw a rapid movement after its introduction to the scientific community and as a result it got transformed into a sophisticated technique with applications in a wide variety of disciplines that now include physics, chemistry, biology, and medicine. The effect of MR has been widespread as it proved to be an invaluable tool for molecular structure determination and investigation of molecular dynamics in solids and liquids. In last few years, applications of MR to studies of living systems have developed along two parallel and perhaps complementary directions. In one direction, MR has evolved into a spectroscopic method that provides chemical information from selected regions within a living tissue. This new method, called magnetic resonance spectroscopy (MRS), has opened up new avenues to obtain valuable metabolic data that are directly related to the state of health of the tissue and hence, it can be used to monitor tissue response to therapy. The other direction has taken MR into the realm of clinical imaging as an imaging tool to provide anatomic and pathologic information. With the de-

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velopment of pulse Fourier transform techniques in the late 1960s [69], MR forayed into diverse elds of study and a further impetus to its popularity was provided by the development of fast Fourier transform algorithms, advances in computer technology, and the advent of high-eld superconducting magnets. The introduction of new experimental concepts such as two-dimensional MR has further broadened its applications [70] to the magnetic resonance imaging (MRI) technique. Tomography is an important area in imaging science and the word tomography comes from Greek terms *tomos*=cut and *graphien*=to write. From application point of view, MRI is also a tomographic technique that is concerned with creating images of the internal (anatomical or functional) organization of an object without physically cutting it open. It produces images of internal physical and chemical characteristics of an object from externally measured MR signals. It was not until Raymond Damadian showed that nuclear magnetic times of tissues and tumors differed, did MRI generate enough interest for the detection of diseases. Later, the technique *Magnetic Resonance Zeugmatography* was introduced by Paul Lauterbur who first demonstrated MRI on small test tube in 1973, a contribution for which he was later awarded Nobel Prize [70]. It was shown by Paul Lauterbaaur that image formation using MR signals can be made possible by the spatial information encoding principles, originally named zeugmatography [70, 71].

In its present state, an MRI scanner(Figure 2.9) outputs a multidimensional data array (or image) representing the spatial distribution of some measured physical quantity. However, unlike other tomographic imaging techniques, MRI can

Figure 2.9: A schematic diagram of MRI scanner

generate two-dimensional sectional images at any orientation, three-dimensional volumetric images, or even four-dimensional images representing spatial-spectral or spatial-temporal (i.e., in cardiac imaging) distributions. Incidentally, no mechanical adjustments to the imaging machinery are involved in generating these images. Whereas, the other members of emission tomography family such as positron emission tomography (PET) and single photon computed tomography (SPECT) require injection of radioactive isotopes into the object to generate signal, the MR signals used for image formation come directly from the object itself. Because MRI operates in the radio frequency (RF) range, the imaging process does not involve the use of ionizing radiation and hence, it does not have the associated potential harmful effects. Moreover, MR Images are extremely rich in information content. The pixel values in MR image depend on a lot of intrinsic parameters, including the spin-lattice

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relaxation time (T1), the spin-spin relaxation time (T2), molecular motions (such as diffusion and perfusion), susceptibility effects, and chemical shift differences. The effects of these parameters on MR images can be suppressed or enhanced in a specific experiment by another set of operator-selectable parameters, such as repetition time (TR) and echo time (TE). Therefore, an MR image obtained from the same anatomical site can look drastically different with different data acquisition protocols. In the next few subsections, the basic physics of MRI is presented along with other important factors that affect MR images such as noise. The description in this manuscript is only a summarised form of the extensive literature [72–81] that has served as the information source for the following subsections.

2.6.1 Basic principles

MRI works on the principle of magnetic resonance. The nucleus of an atom, that contains neutrons and protons, generally exhibits a magnetic moment(μ) that depends on the number of protons and neutrons present inside. Additionally, a property called spin(short for nuclear spin angular momentum number) is also associated with the number of neutrons and protons. Because of the nature of its natural abundance in tissues (Table 2.1) that are largely made of water, the hydrogen nucleus, a proton, is used most often in MRI. Protons in hydrogen atom spin around an axis giving rise to a magnetization effect by generating a small magnetic field. The spin angular momentum (p) that can be considered as an outcome of the rotational or spinning motion of the nucleus about its own axis related to magnetic moment(μ)

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Table 2.1: NMR related properties of few nuclei

Nucleus	Gyromagnetic Ratio (MHz/T)	Natural Abundance (%)
^1H	42.58	99.98
^{13}C	10.71	1.11
^{19}F	40.05	100
^{31}P	17.23	100
^{23}Na	11.26	100

by following equation:

$$\mu = \Upsilon p \quad (2.21)$$

where Υ is the gyromagnetic ratio that is proportional to the charge-to-mass ratio of the particular nucleus. Table 2.1 provides the gyromagnetic ratios and other NMR properties for few nuclei [82].

Under the earth's weak magnetic field they precess or spin on their axes (Figure 2.10) randomly so that their magnetic fields tend to cancel out each other. However, when the protons are subjected to a powerful external magnetic field, a small fraction of these spinning protons line up either in parallel or anti parallel direction compared to the direction of the external magnetic field (B_0). When this alignment occurs the protons start spinning at a frequency called Larmor frequency, the value of which lies in the radio frequency range. The Larmor frequency (ν) depends on gyromagnetic

Figure 2.10: Precession in MRI

ratio Υ and the external magnetic field B_0 .

$$\nu = \left(\frac{\Upsilon}{2\pi}\right)B_0 \quad (2.22)$$

During the alignment process, protons are found in two slightly different energy states; E_{low} is the lower energy state where most protons are found and E_{high} is the higher energy state where anti-parallel aligned protons are found. When external radio frequency pulses are sent into the body, the protons are forced to rotate in phase with the radio frequency and they attract extra energy and jump to a higher state(excitement).The change in energy is related to larmor frequency in the following manner:

$$\Delta E = h\nu \quad (2.23)$$

where h is Planck's constant.From equations 2.23 and 2.22 we obtain Eq. 2.24 that connects change in energy to the larmor frequency and to the gyromagnetic ratio.

$$\Delta E = h\nu = h\left(\frac{\Upsilon}{2\pi}\right)B_0 \quad (2.24)$$

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A simultaneous external orthogonal magnetic field ensures that only a band of protons in the body is excited, where the radiofrequency and magnetic field strength match. When the stimulation by radio frequency pulses is turned off, the protons return to their normal state and original orientation of their magnetic (relaxation) and while doing so, their acquired energy is emitted as a weak radio-signal. Another external orthogonal magnetic field, that is applied during the emission, separates the emitted radiofrequency according to position in the body. Since the protons from different regions of the body can be made to precess at different frequencies (their magneto-resonance frequencies), the electromagnetic energy at these frequencies yields signals that are location dependent. Hence an overall analysis of radiofrequency and intensity allows generation of a cross sectional image. Because the proton density is different in various tissues of the body, MRI generates different signal intensities during the relaxation. A high signal intensity produces bright area in the final image whereas a low intensity produces dark areas. The time for relaxation is the factor controlling the outcome of the MRI procedure, gathering images with different contrast. T1 weighted images in which fat and watery substance appear dark are generated by applying short relaxation time between excitements and they are used to differentiate between white and grey matter of the brain. T2 weighted images, in which watery and diseased tissues are bright, are generated by applying longer time between excitements. These images are, generally, used for investigation of diseased parts of the brain. In most investigations, both T1- and T2- weighted images are generated.

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2.6.2 Noise in MRI

Noise obscures diagnostically important information in medical images. In MRI, noise can be attributed to two types of sources. While one type of sources can be controlled directly by means of systems settings, the other type of sources are attributable to the instrument itself and to the patient. In the instrument, noise arises from statistical fluctuations of the signal sensed by the receiver coils e.g., FID (Free Induction Decay) or echo. The dominant source in that type of source is Johnson noise, which is generally caused by the thermal agitation of electrons or ions in a conductor. Johnson noise arises predominantly from the electrolytes in the patient, but it can also arise from within the receive coil and the electronics attached to it. Moreover, the strength of magnetic field(B_0) causes more spins to participate, with a resulting increase in the size of magnetization vector [82]. This, in turn, results in the increase in signal intensity and hence less noisy image could be produced. The reasoning of increasing magnetic field may seem to be a simple and flawless solution to decrease noise but there are side-effects associated with increased magnetic field that are increased T1 times, power deposition effects, and an increased tendency toward artifact production. There are many other ways of decreasing the noise by changing hardware settings or by changing few acquisition parameters but there are always tradeoffs that have to be dealt with. In this scenario, post-acquisition image processing methods prove to be very useful as they require no changes in machine settings. Instead, the quality of final image can be efficiently controlled after the

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image has been reconstructed and saved into the repositories. From image processing literature, it has been found that in MRI, the acquired complex data is known to be corrupted by white noise having a Gaussian probability distribution (PD) [83]. After inverse Fourier transformation the real and imaginary data is still corrupted with Gaussian noise because of the orthogonality of the Fourier transform. Usually, the processing is done on magnitude and phase images, although all information is present in the real and imaginary images, since they have more physical meaning (proton density flow, etc.). However, computation of a magnitude image is a non-linear operation which changes the data distribution which in turn varies with signal-to-noise ratio (SNR). It has been shown that the data in a magnitude image changes from Gaussian distributed to Rician distributed [83, 84]. Because of the Rice distribution properties, it tends to be Rayleigh distributed when the SNR goes to zero but approaches a Gaussian distribution at high SNR [83–87].

2.7 Optical Coherence Tomography (OCT)

The ability to visualize cellular features in tissue is both clinically important for disease diagnosis and scientifically important for understanding fundamental biological processes. A pathological examination of cellular features, mainly, involves the visualization of cellular structures, including their properties and size [88]. The increasing interest in tissue engineering has prompted the need to visualize and understand the complex process of tissue development under controlled conditions

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including the need to understand how cells function, proliferate, and organize into biocompatible tissues [89]. Optical imaging techniques, such as optical coherence tomography (OCT), improve our ability to visualize these cellular features in vivo, without the need to invasively sample, process, section, and stain tissue specimens to make a diagnosis. OCT is an imaging technique that uses low coherence interferometry to image below the surface of an object. Its main applications include medical imaging, non-destructive testing and optical sensing, with many other applications still emerging [90]. OCT produces high resolution cross-sectional images of the internal microstructure of living tissue and its first applications in medicine were reported less than a decade ago [91–95]. Optical coherence-domain reflectometry (OCDR), a 1-dimensional (1-D) optical ranging technique, which was developed originally for finding faults in fiber optic cables and network components, set the ground for OCT when its ability to probe the eye and other biological tissues was recognized after its development [96–102]. OCT's optical sectioning ability, which is achieved by exploiting the short temporal coherence of a broadband light source, enables the imaging of microscopic structures in tissue at depths beyond the reach of conventional microscopes. Probing depths, exceeding 2 cm in transparent tissues including the eye and the frog embryo [103, 104] and equal to 12 mm beneath the surface of the skin and of other highly scattering tissues has already been demonstrated [94–107]. OCT is somewhat analogous to ultrasound B mode imaging, except that OCT imaging is performed by measuring the intensity of reflected or backscattered light rather than acoustical waves. Tomographic images are generated by scanning

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an optical beam across the tissue and measuring the echo time delay and intensity of light that is reflected or backscattered from internal tissue microstructures. OCT's resolution is mainly limited by the temporal coherence properties of the light source, usually a superluminescent diode, to typically 10-15 μm . Although ultrasonic imaging achieves greater probing depths but with lower resolution [108], OCT is advantageous due to its relative simplicity and lower cost of the system hardware. It is believed that multiple scattering, which becomes dominant at large depths, is the fundamental limitation preventing OCT from reaching a large probing depth in turbid media [109]. A comprehensive information on the applications of OCT is given in reviews by Fujimoto et al. [94], Izatt et al. [110], and Fercher [111].

2.7.1 Basic principles

OCT is a high resolution optical imaging technique which is used to visualize the internal microstructure in highly scattering media, e.g. biological tissue. In principle, OCT system measures the time delay and magnitude of optical echoes at different transverse positions. This system is based on a Michelson interferometer, using an oscillator mirror in reference arm and the tissue sample in the sample arm. The sample is illuminated by the measurement beam of low-coherence Michelson interferometer and the light backscattered is brought to interfere with the interferometer's reference beam. Interference will only occur if the optical path length of the reference beam coincides with the optical path length of the reference beam within the coherence length. The distance between two sample sites is then esti-

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Figure 2.11: Schematic of a simple OCT system

ated by the distance of the interference patterns corresponding to these sites using the reference mirror positions. The oscillations of the mirror provide interferometric data for different depths in the sample. A simple illustration of OCT system is depicted in Figure 2.11. Two-dimensional OCT image can be obtained by adding lateral scanning, which allows building a cross sectional image. Figure 2.12 shows a typical OCT image of a finger tip. (source: Department of dermatology of the St.-Josef-Hospital, Bochum, Germany). Figure 2.13 gives an idea how a commercial OCT looks like.

One distinct advantage of this optical approach is that the background signal can be minimized because the optical signal is nonexistent until the optical probe comes into contact with some tissue. With PET, it is not possible to control when the radioactivity is active or inactive (once the radioactive substance is administered); only the passage of time allows background signal to clear.

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Figure 2.12: A typical OCT image

Figure 2.13: A commercial OCT system

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2.7.2 Noise in OCT

Optical coherence tomography, being a noninvasive imaging method, relies on the intrinsic variation of tissue properties to differentiate tissue constituents. The spatial variation of the coherent backscatter cross section, in many applications, is the primary source of contrast in the OCT images of tissues. But the spatial and temporal coherence of the optical waves backscattered from a tissue give rise to speckle, an insidious form of noise that degrades the quality of OCT images. Speckle was observed when reflection of a laser beam from a rough surface showed a distinctive granular or mottled appearance [112]. Later, researchers realized the full significance of speckle as a fundamental property of signals and images acquired by all types of narrowband detection systems, which include radar, ultrasound, and radio astronomy. In addition to the optical properties and motion of the target object, speckle is influenced by the size and temporal coherence of the light source, multiple scattering and phase aberrations of the propagating beam, and the aperture of the detector. All of these variables contribute to the observed characteristics of speckle in optical coherence tomography of living tissue. The detailed theoretical analysis of these variables can be read from [113]. In the context of optical coherence tomography, the objective of speckle reduction is to suppress signal degrading speckle and accentuate signal-carrying speckle. The different types of speckle are distinguished by their correlation spot sizes and by the frequencies of the signals they generate during scanning [114].

CHAPTER 3

MATERIALS AND METHODS

3.1 Acquisition of PET Images

The PET images for our study were acquired using Siemens Biograph PET/CT scanner at the Department of Nuclear Medicine and PET in Singapore General Hospital (SGH), Singapore. A Jaszczak Emission Computed Tomography (ECT) phantom of cylindrical shape with a diameter of 21cm and a height of 31cm was used along with different inserts. The phantoms are shown in Figure 3.1. We used a PET radioactive tracer ^{18}F Fluoride with a radioactivity of 296MBq to facilitate sufficient emission of positrons. The duration of scan was 3 minutes/bed position and we employed 2 bed positions to cover the whole phantom. The imaging system uses a filtered back-projection algorithm combining a ramp filter for back-projection and low pass filter to mitigate high frequency noise.

3.1.1 Image Preprocessing

PET images have noise levels depending upon the length of time the emission is observed [115]. Different exposure times result in different count rates and, as a

3.1. Acquisition of PET Images

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Figure 3.1: Phantoms used for acquisition of PET images

result, the amount of noise in a PET image varies with the emission observation time. In order to simulate the effect of different count rates on amount of noise in images, we considered four different levels of noise with 1 being the lowest level and 4 being the highest. This consideration of different noise levels allows us to test the robustness of our method for images with varying degree of emission observation time. As the detection of photons in PET scanner follows Poisson distribution, we added Poisson noise to the images before the application of de-noising algorithm [37]. Thus, the test data consisted of 4 noisy images representing different levels of noise.

3.2 Acquisition of MR Images

For this study, Knee MR images of human subjects were acquired using a standard 1.5-T clinical body scanner that was equipped with a quadrature birdcage lower-extremity probe coil. The scanning of patients was done in the supine position and to prevent movement during the scan, a soft foam packing was placed around the knee. A 3-D acquisition with voxel size $0.47 \times 0.47 \times 1.0$ mm over a 120-mm-square field of view (FOV) was performed with a fat-suppressed spoiled-gradient (SPGR) sequence (58/6/30). From each volume acquisition, we acquired a total of 65 sagittal slices. The other parameters that were used for acquisition are as follows: TR = 58, TE = 6, flip=30; FOV=120 mm; matrix size 256×256 ; slice thickness=1.0 mm.

3.3 Acquisition of OCT Images

Oncidium orchid leaves, that were collected from the garden of the National Institute of Education (NIE), Singapore, were used as the object for OCT imaging. A Fourier domain OCT (FD OCT) system was used in this work to achieve high speed, high resolution imaging, similar to [116, 117]. The schematic diagram of our measurement system is shown in Figure 3.2. The light source used in the system is an INTEGRAL OCT laser (FEMTOLASERS Produktions GmbH) with a center wavelength of 800nm and an optical bandwidth of 140nm. Experimental axial resolution was 4m in air. The lateral resolution was 9 m, measured using a USAF target card (Edmond Optics V38-257). The signal-to-noise ratio of our system was

3.3. Acquisition of OCT Images

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Figure 3.2: Image acquisition system for OCT images

93 dB. Two microscope objective lenses ($NA = 0.3$) were placed in both the sample and reference arm. The placement of a objective lens in the reference arm is for mitigating dispersion effects thereby achieving a good axial resolution. The power delivered to the plant leaf was about 5 mW which together with the objective lens had irradiance that was lower than the damage threshold of plant leaves [28]. Although the laser source was a Ti:Sapphire femto-second laser, the pulse width of the laser pulses had been broadened significantly due to dispersion after passing through various optical components. Thus, the peak power was also not sufficient to damage the leaves. The principles of OCT have been described in [98]. Light reflected from the reference arm and the sample arm is combined in the interferometer and observed in the frequency domain using a spectrometer (Ocean Optics HR4000). The original optical spectrum is modulated at a frequency directly proportional to the optical pathlength between the reference mirror and the sample. The spatial reflectance profile of the tissue is obtained from an inverse Fourier transform of the optical spectrum. The scan rate was about 1 A-line/s and is limited by the acqui-

Figure 3.3: OCT image acquisition setup

sition time of the spectrometer and the signal processing time for data scaling and Fourier transforms. The photographs of our OCT image acquisition setup depicting reference arm and sample arm are shown in Figure 3.3.

3.4 DE based Wavelet Domain Filtering (DEWDF)

The image acquisition processes are followed by the removal of noise from the acquired images. In this study, wavelet domain filtering is applied in conjunction with DE for the noise filtration. All the important concepts and literature review have already been covered in Chapter 2. Here, the description of method is provided in precise steps which are followed while implementing the proposed method. The proposed method, hereafter, will be referred to as DE based Wavelet Domain Filtering (DEWDF).

3.4.1 Wavelet Domain Filtering (WDF)

From the conceptual description in Chapter 2, it can be showed that wavelet domain filtering for two-dimensional images require three major steps:

Image decomposition via 2-D wavelet transform

The discrete wavelet transform translates the image content into an approximation subband and a set of detail subbands at different orientations and resolution scales. Typically, the band pass content at each scale is divided into three orientation subbands characterized by horizontal, vertical and diagonal directions. The approximation subband consists of so-called *scaling* coefficients and the detail subband are composed of *detail* or *wavelet* coefficients. Figure 3.4 depicts an arrangement wherein the image is decomposed into approximation and detail subbands. In the figure, A represents approximation subband whereas H, V, and D represent *detail* or *wavelet* subbands in horizontal, vertical and diagonal directions respectively. The level, up to which the image is decomposed, is decided by the user at the beginning of decomposition process. Also, before the decomposition is applied, a wavelet basis has to be selected.

Thresholding

Once the image is decomposed into approximation and detail coefficients, thresholding is applied to remove undesired coefficients. It should be noted that only detail coefficients are used for thresholding since it has been observed that noise compo-

Figure 3.4: Wavelet decomposition scheme for images

nents reside in detail subbands. In the proposed method, soft thresholding is given preference over hard thresholding due to the better smoothing properties of former. The value of threshold is either selected with the help of a statistical equation [118] or it can be selected by the user.

Image reconstruction

Post thresholding, image is reconstructed back from the thresholded detail coefficients and unchanged approximation coefficients. The wavelet basis type and the decomposition level that are decided by user before the decomposition step are again used in the reconstruction process. This reconstruction process provides the

3.4. DE based Wavelet Domain Filtering (DEWDF)

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denoised image. From the above discussion, it can be inferred that wavelet domain filtering requires the selection of decomposition level, wavelet type and threshold value. In earlier studies, these three parameters have been selected manually by trial and error or some statistical expressions were used to estimate one or more parameters [24–26, 37]. Shih *et al* applied wavelet domain filtering on PET images and showed that these parameters significantly influence the noise reduction in images [37]. As opposed to their method that involved the selection of these parameters by human observer driven trial and error process we designed an automatic process to select these parameters objectively. For an efficient processing the parameters are selected using DE.

3.4.2 Parameter Selection

The DE algorithm for parameter selection can be summarized in the following steps.

Step 1: Start

Step 2: Initialize the target population for first generation

$$T_{vn,gn} = [t_{en,vn,gn}, t_{en+1,vn,gn}, \dots, t_{D,vn,gn}] \quad (3.1)$$

T is the target population and $T(vn, gn)$ refers to the vn th target vector in generation number gn . Each generation maintains a pair of populations called target and trial populations with each having P parameter vectors. A vector has three elements denoted by smaller case version of the symbol used for the vector. For example, in

3.4. DE based Wavelet Domain Filtering (DEWDF)

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Equation 3.1, t denotes an element of target vector T and its subscripts en , vn and gn refer to the element number, vector number and generation number respectively. D refers to the number of elements in each parameter vector. Since we have three parameters to select, 3 is taken as dimension number, D and the three elements of vector are threshold value, decomposition level and wavelet type.

Step 3: Use mutation to generate mutant vector population from the population of target vectors.

$$M_{vn,gn} = T_{r1,gn} + F \times (T_{r2,gn} - T_{r3,gn}) \quad (3.2)$$

Mutation, in general terms, is defined as an alteration or change. But in the context of DE, mutation can also be seen as change with a random element. Unlike other evolutionary algorithms, wherein the effects of mutation are simulated with additive increments that are randomly generated by a predefined probability distribution function (PDF), DE uses a uniform PDF not to generate increments, but to randomly sample vector differences as shown by the $(T_{r2,gn} - T_{r3,gn})$ term in Equation 3.2. Thus, the parameter vectors $T_{r1,gn}$, $T_{r2,gn}$ and $T_{r3,gn}$ are randomly selected from target population T to generate mutant vector with a condition that $r1 \neq r2 \neq r3 \neq vn$. The mutation factor, $F \in [0,1+]$ controls the rate at which the population evolves. F is usually selected from the range $(0,1)$ where 1.0 is an empirically derived upper limit. This empirical value has been derived from the fact that no function that has been successfully optimized has required the value of F greater than 1. However, the solutions can still be obtained if F is allowed to have

3.4. DE based Wavelet Domain Filtering (DEWDF)

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value greater than 1, but then the algorithm consumes more time and the solution obtained is less reliable. The case $F = 1$, however, renders a indistinguishability to distinct vector combinations, a fact evident from Equation 3.2. It must be noted that because F only affects the relative step size due to the self-adaptive distribution of vector differences, it can be kept constant during optimization without compromising DEs ability to generate steps of the required size.

Step 4: Use recombination to generate trial vector population(C) from target(T) and mutant(M) populations.

$$c_{en,vn,gn} = \begin{cases} m_{en,vn,gn} & \text{if } rand_{vn}(0,1) \leq CR \text{ or } vn = vn_{rand} \\ c_{en,vn,gn} & \text{otherwise} \end{cases} \quad (3.3)$$

DE complements the differential mutation strategy by employing *uniform crossover* or *discrete recombination* given by Equation 3.3. Recombination is used to randomly exchange or merge parameters from two or more vectors in order to create one or more trial vectors. Since it only copies information, crossover can be applied to binary, real-valued or even symbolic data. The crossover probability, $CR \in [0, 1]$, is a user defined value that controls the fraction of parameter values that are copied from the mutant. In Equation 3.3, $rand_{vn}(0,1)$ is a uniform random number generator and vn_{rand} is a randomly chosen index from mutant population to ensure that trial vector C does not duplicate $T_{vn,gn}$.

Step 5: Apply selection process to obtain the target population for next generation

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$gn + 1$.

$$T_{vn,gn+1} = \begin{cases} C_{vn,gn} & \text{if } f(C_{vn,gn}) \leq f(T_{vn,gn}) \\ T_{vn,gn} & \text{otherwise} \end{cases} \quad (3.4)$$

Selection can be applied to a population at two stages in the evolutionary process. Usually, the highest selection probability is assigned to the vectors that provide best function values, hence, making them the most likely to be chosen for mating. The methods for assigning selection probabilities involve additional assumptions about how to map objective function values to a set of probabilities. Classic DE randomly selects parent vectors without regard for their objective function values. In contrast to parent selection, survivor selection, also called replacement, chooses the next generation of vectors from the current generation of vectors and trial vectors. A selection pressure is generally applied when choosing vectors to recombine or when choosing survivors. In case of DE, first, the randomly chosen vectors are combined and then, a selection pressure is applied only when picking survivors. The reason for this selection process, is that using both parent and survivor selection can cause premature convergence to a local optimum.

The selection process involves function evaluation which relies on the calculation of objective function(OF) for two images that are obtained after the application of WDF on input image with target and trial vectors. The difference of these two OF values serve as the objective function that has to be minimized. A tournament selection then selects which vector moves to next generation. If the trial vector, $C_{vn,gn}$, has an equal or lower objective function value than that of its target vector,

3.4. DE based Wavelet Domain Filtering (DEWDF)**59**

$T_{vn,gn}$, it replaces the target vector in the next generation; otherwise, the target retains its place in the population for at least one more generation. DE integrates recombination and selection tightly by comparing each trial vector with the target vector which inherits its parameters.

Step 6: Increment generation number.

$$gn = gn + 1 \quad (3.5)$$

Step 7: Compare generation number(gn) with G_{max} (the maximum generation number set in the beginning)

$$gn = \begin{cases} G_{max} & \text{terminate and go to go to Step:8} \\ else & \text{go to Step:3} \end{cases} \quad (3.6)$$

Step 8: Best vector from final generation is saved as optimum vector. The optimum vector obtained after this step contains those values of threshold, wavelet type and decomposition level which will provide the best noise reduction in noisy images. Because the selection process used image quality indices as objective functions and only the vectors with best values of objective function were selected, the chosen optimum vector can be considered as the best possible combination for image denoising.

Step 9: Use optimum parameter vector to apply WDF on input image.

Step 10: End

3.4. DE based Wavelet Domain Filtering (DEWDF)

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Figure 3.5: Overview of proposed method

An overview of whole method is shown in Figure 3.5.

It has been already discussed in previous chapters that although wavelets have been successful in noise reduction, the wavelet based methods are still dependent on manual selection of parameters. As manual selection is always prone to bias, the proposed DE driven automatic selection of parameters, based on reliable image quality indices, provides a fast, efficient and robust solution to noise reduction in medical images. An example of successful application of DE in medical imaging is

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medical image registration method proposed by Salomon *et al* [119]. It is shown in this study that differential evolution can be considered as an appealing candidate for fast and high-quality 3-D image registrations.

A simple example of DE's main components is shown in Figure 3.6 for population of 10 vectors with $\text{foo}()$ as objective function. The initialization stage is shown in Figure 3.6(a) wherein preset parameter bounds define the domain from which the initial population vectors are chosen. Each vector is indexed with a number from 0 to NP - 1 where NP is initial population size. DE generates new points that are perturbations of existing points, but these deviations are neither reflections nor samples from a predefined probability density function. Instead of using probability density function, DE perturbs vectors with the scaled difference of two randomly selected population vectors the illustration of which is shown in Figure 3.6(b). The trial vector, u_0 , is obtained by adding the scaled, random vector difference to a third randomly selected population vector (Figure 3.6(c)). During the selection process (Figure 3.6(d), the trial vector competes against the population vector of the same index, which in this case is number 0. The winner from this selection goes to next generation and the process is continued until the termination criterion is met.

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Figure 3.6: Main steps of Differential Evolution: An example

CHAPTER 4

RESULTS AND DISCUSSION

Almost all automation methods require certain initialization parameters at the beginning and once these parameters are set, the methods continue with their designated tasks without any further tuning. Our automation works on the similar lines as it requires the setting of few parameters such as population size, P , mutation factor, F , maximum generation number, G_{max} and cross-over probability, CR to start the search and selection process. Based on the previous studies, it has been found that a population size of three times the vector dimension, D is ideal that led us to select 30 as the value of P . The selection of mutation factor F is very important as it controls the diversity in population and convergence speed. Whereas selection reduces diversity, mutation tends to increase the diversity of a population. Hence, the magnitude of F should be sufficiently high to counteract this selection pressure. In 2002, Zaharie demonstrated that there is, in fact, an effective lower limit for F . She found that when F is too small, the population can converge even if the selection pressure is absent [120]. Another study, which explored DEs performance on two of the same test functions that Zaharie used, showed that $F \leq 0.4$

was not useful [121]. In our method, F was used with values ranging from 0.5 to 0.8 based on previous studies and we found that $F = 0.7$ provided sufficiently high magnitude to increase population diversity while still keeping the convergence time low. The main role of CR is to provide the means to exploit decomposability of a decomposable objective function with an added advantage of providing extra diversity to the possible trial vector space, especially near $CR = 1$. So CR should be close to 1 in the case of parameter-dependent functions so that the performance losses associated with using a low CR are minimized. Thus, we used CR as 0.8 which also provides high cross-over events. G_{max} was selected on the empirical basis and it was observed that a value of 50 provides fast convergence while allowing the DE to search the parameter space comprehensively. The range for threshold values and decomposition levels were set as 0-20 and 1-4 respectively. Since Daubechies db/Db wavelets are the most commonly used biorthogonal wavelets in WDF, we have used ten orders of dbN/DbN wavelets where N represents the order of db/Db wavelet [122]. The algorithm was implemented in MATLAB on Windows platform. The specific results, that were obtained after the application of DEWDF on images from different modalities, are presented in following sections. Different variants of low pass filter, median filter and Wiener filter with masks of 3×3 5×5 and 7×7 were selected because these methods do not require any selection of parameters for different set of images and are independent of any human interference after the initialization.

In Chapter 3, objective function appeared as the fitness function in DEWDF

algorithm. The main advantage of using a general representation of the objective function in DEWDF algorithm instead of a pre-defined mathematical measure is that a variety of criterion for parameter selection can be used for different types of images. Moreover, this provision allows modularity in the algorithm. In next few sections, the results of the application of DEWDF on different types have been reported. Since the images were acquired from different modalities, they possess different characteristics from one another. The results are taken from different research papers that were submitted/presented by author to journals and conferences. Hence, the presentation of results vary from one modality to another.

4.1 Application of DEWDF on PET Images

DEWDF was applied to PET images with the expression $\frac{1}{SNR(image)}$ in objective function. Because SNR requires the selection of signal and background ROIs, these ROIS were also fed as input to DEWDF to allow the calculation of objective function. The aim of this experiment was to maximize the SNR of PET images.

4.1.1 Parameter Distribution

As mentioned in the Section 3.1, there were 4 sets of 50 images that were denoised with DEWDF. Each set of 50 images had a different level of noise. The noise was added in the image preprocessing step with the help of MATLAB's inbuilt *imnoise* function. Since the noise in PET is Poisson distributed, Poisson noise was added to

the acquired images. The distribution of parameters that were selected by DEWDF for noise levels 1 and 2 is shown in Figure 4.1 whereas the distribution for noise levels 3 and 4 is shown in Figure 4.2. From these figures, it can be observed that DEWDF's selection of parameters tend to vary significantly for higher levels of noise. This trend can be attributed to the fact that higher levels of noise make DEWDF more selective and aggressive in finding the optimum values. For lower levels of noise, the search landscape is more flat and hence, the variation in parameter selection is not so high. Moreover, it has been observed that the images in the center of series tend to assume higher values of threshold and lower order of Daubechies. The decomposition levels were more stable as the algorithm favored higher levels. This behaviour can be explained with the multi-resolution theory of images wherein a higher scale means less frequency and lower means higher frequency. In terms of decomposition levels, as we go down the scale number of levels increase. Due to the established fact that noise resides in higher frequency components, it is not difficult to understand that in wavelet domain, lower scales and hence, higher levels of decomposition, have more noise. This fact is corroborated by the DEWDF parameter selection i.e., thresholding at higher levels are more likely to remove noise than the lower levels. The threshold selection doesn't seem to follow particular pattern and it can be assumed that it is because the combination of parameters that work for one image and its different noisy version doesn't work for other images. Since images do not follow any pattern in series, it is obvious that parameters also would show random pattern.

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Figure 4.1: Parameter distribution for images with Noise Levels 1 and 2

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(a) Threshold Values: Noise Level 3

(b) Decomposition Levels: Noise Level 3

(c) Wavelet Types: Noise Level 3

(d) Threshold Values: Noise Level 4

(e) Decomposition Levels: Noise Level 4

(f) Wavelet Types: Noise Level 4

Figure 4.2: Parameter distribution for images with Noise Levels 3 and 4

4.1.2 Objective comparison: Image quality indices

The objective comparison on the basis of image quality indices have been shown in Table 4.1. For this comparison, different variants of median and Wiener filter were selected. The results of objective comparison based on image quality indices are tabulated in Table 4.1. It was observed that median filter was able to improve SNR by a more than 16% in comparison to other methods except DEWDF. But, looking at the figures in next two columns i.e., UIQI and CNR change, it becomes clear that median filter with mask size 7×7 brings down UIQI value to a very low level of 0.76 and reduces CNR by more than 15%. This change in two of the important image quality indices is unacceptable in clinical conditions. Comparing these figures with the those obtained after the application of DEWDF it can be observed that DEWDF performs far better when compared to other methods. It achieves the SNR improvement of more than 24% but keeps UIQI at acceptable level of 0.98. Additionally, it improves the CNR by a percentage of more than 8%. Wiener filtering is not far behind in keeping the UIQI at higher level but it makes very little difference in SNR improvement and fails to maintain CNR. The effect of these methods will be more visible in next section where the images filtered with different methods are presented.

4.1. Application of DEWDF on PET Images

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Table 4.1: Objective comparison for PET images

Methods	Input image SNR(dB)	Output image SNR(dB)	% SNR change	UIQI values	% CNR change
3×3 Median	17.22	18.45	7.12	0.81	-17.99
5×5 Median	17.22	19.20	11.50	0.77	-17.33
7×7 Median	17.22	20.09	16.64	0.76	-15.95
3×3 Wiener	17.22	17.70	2.81	0.979	-20.01
5×5 Wiener	17.22	17.93	4.09	0.975	-19.98
7×7 Wiener	17.22	18.00	4.56	0.972	-20.26
DEWDF	17.22	21.60	24.44	0.98	8.91

4.1.3 Visual inspection of denoised PET images

Because of the space constraint, only one image and its filtered versions are shown here. As mentioned earlier, four levels of noise were considered to check the robustness of method. But, for visual inspection, image number 21 with the noise level 4 was selected. This selection was made on a random basis for the images in middle of the sequence i.e., from image number 20 to 30. The selected noisy image is shown in Figure 4.3(a). Images that were processed using Median filter with masks of sizes 3×3, 5×5 and 7×7 are shown in Figures 4.3(b), 4.3(c) and 4.3(d) respectively. The resultant images after the application of Wiener filter with mask sizes 3×3, 5×5 and 7×7 are shown in Figures 4.3(e), 4.3(f) and 4.3(g) respectively. The result of

the application of DEWDF original image is shown in Figure 4.3(h) . From these figures, it can be observed that the application of some filters such as median filter, with mask 5×5 and 7×7 results in the loss of original signal itself. As shown in Figure 4.3, DEWDF outperforms other methods by a substantial difference on the basis of visual inspection.

4.2 Application of DEWDF on MR Images

A series of 64 images was acquired using the image acquisition method discussed in Section 3.2. All these images were used as inputs to the DEWDF method(Section 3.4.2).

4.2.1 Distribution of parameters

The distribution of parameters that were selected by DEWDF for all of the 64 images are shown in Figure 4.4. From Figure 4.4(a), it can be observed that the threshold values tend to assume higher values for the images that are in the middle of whole sequence. This trend can be attributed to the fact that the images in the middle of scanning sequence possess more complex information than the images at the periphery of sequence. Since those images are more likely to have a higher amount of noise than the others, higher threshold values were used by DEWDF to remove more detail coefficients. The distribution of decomposition levels in Figure 4.4(a) suggests that higher decomposition levels provide better noise reduction. This trend,

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(a) Noisy input

(b) Median 3×3

(c) Median 5×5

(d) Median 7×7

(e) Wiener 3×3

(f) Wiener 5×5

(g) Wiener 7×7

(h) DEWDF

Figure 4.3: Visual inspection of denoised PET images

in fact, follows the suggestion by many studies in past that the higher decomposition results in better denoising. The trend of wavelet types in Figure 4.4(c) suggests that higher orders of Daubechies waveletes are preferred for images in center while the lower orders, especially first order, dominate at the periphery.

4.2.2 Objective comparison: Image quality indices

The results of the image quality tests using the indices of merit discussed in Section 2.4 are presented in this section. A very popular adaptive noise reduction called Wiener filter with spatial masks of size 3×3 and 5×5 was used in our study to compare the results of DEWDF with those of Wiener filter. We have concentrated on two types of tissues in MR images: cartilage and soft tissue. In order to see the effect of our method on these two important tissue types, we selected two ROIs: one for cartilage and another for soft tissue. The background was kept same for both of these ROIs. Figures 4.5(a) and 4.5(b) show the ROIs and background region for cartilage and soft tissue in image number 32 and 48 respectively. The images 32 and 48 from the sequence of 64 were selected as test images because they represent the approximate centers of whole sequence.

We have segregated the comparative results for three indices in Tables 4.2, 4.3 and 4.4. These Tables contain the input/output values of corresponding index of merit as well as the percentage change for different tissue types and different noise reduction methods. Table 4.2 with SNR values shows that Wiener filter with mask 5×5 has better SNR values than the values that DEWDF was able to manage with

4.2. Application of DEWDF on MR Images

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(a) Threshold Values

(b) Decomposition Levels

(c) Wavelet Types

Figure 4.4: Parameter distribution for whole series of MR Images

4.2. Application of DEWDF on MR Images

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(a) Image 32

(b) Cartilage

Figure 4.5: ROIs for Different tissues in MR images

Table 4.2: Comparison of SNR values for MR images

Image	Region	Method	Input SNR (dB)	Output SNR (dB)	% SNR improvement
32	Cartilage	W3×3	25.05	32.51	29.77
		W5×5	25.05	36.19	44.45
		DEWDF	25.05	31.40	25.32
	Soft tissue	W3×3	26.83	34.78	29.60
		W5×5	26.83	38.77	44.49
		DEWDF	26.83	33.30	24.09
48	Cartilage	W3×3	25.12	34.46	37.16
		W5×5	25.12	35.88	42.82
		DEWDF	25.12	36.08	43.63
	Soft tissue	W3×3	26.01	37.20	43.02
		W5×5	26.01	39.58	52.17
		DEWDF	26.01	39.63	52.32

4.2. Application of DEWDF on MR Images

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Table 4.3: Comparison of CNR values for MR images

Image	Region	Method	Input CNR)	Output CNR	% CNR change
32	Cartilage	W3×3	65.80	65.56	-0.36
		W5×5	65.80	65.29	-0.76
		DEWDF	65.80	65.80	0.00
	Soft tissue	W3×3	85.80	85.80	0.00
		W5×5	85.80	85.81	0.01
		DEWDF	85.80	85.77	-0.03
48	Cartilage	W3×3	63.22	61.79	-2.25
		W5×5	63.22	60.80	-3.82
		DEWDF	63.22	63.07	0.23
	Soft tissue	W3×3	75.52	75.81	0.38
		W5×5	75.52	76.04	0.68
		DEWDF	75.52	75.79	0.35

image 32. But the observation of image results in next section will show that Wiener filter has been able to achieve a higher SNR improvement at the cost of visible image quality. Also, the SNR improvement was found to be better in soft tissue than in cartilage. DEWDF also maintains resolution as corroborated by the values in Table 4.3 that contains the CNR values. The change in CNR values was positive for both cartilage and soft tissue with DEWDF whereas the Wiener filter caused the reduction in contrast for both types of tissues. The same observation applies to images in Figures 4.6 and 4.7, that show no significant change in contrast for images processed with DEWDF. Lastly, we compare the images based on the index of merit that was also used as objective function in this study. As expected, the change in

4.2. Application of DEWDF on MR Images

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Table 4.4: Comparison of UIQI values for MR images

Image	Region	Method	Input UIQI)	Output UIQI	% UIQI change
32	Cartilage	W3×3	1.00	0.95	-5.36
		W5×5	1.00	0.85	-15.28
		DEWDF	1.00	1.00	0.00
	Soft tissue	W3×3	1.00	0.67	-32.65
		W5×5	1.00	0.37	-62.82
		DEWDF	1.00	0.95	-5.00
48	Cartilage	W3×3	1.00	0.91	-8.57
		W5×5	1.00	0.84	-15.77
		DEWDF	1.00	0.99	-0/91
	Soft tissue	W3×3	1.00	0.77	-23.37
		W5×5	1.00	0.67	-32.83
		DEWDF	1.00	0.95	-4.66

values of UIQI was very low for the images filtered by DEWDF. Although the values of UIQI decrease by some percentage, as indicated by Table 4.4, they never go beyond 5%. In comparison to DEWDF Wiener filter suffers from a higher loss of UIQI values which go to a percentage as high as 62.82%.

4.2.3 Visual inspection of denoised MR images

The subjective comparison of images processed with different filters is shown in Figures 4.6 and 4.7.

Even without the help of objective values of indices of merit, it can be observed that there is less or no degradation of image quality in the DEWDF processed

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Figure 4.6: Image number 32 filtered with different methods

images. The DEWDF processed images look visibly smoother and there is less blurring at the edges. This can be attributed to the fact that, due to the strong correlation of features across scales in wavelet domain, they are retained by the method whereas the noise is removed because it is less correlated across scales. Whereas Wiener filter tends to close its high-frequency passing band exclusively, our method provides feature-sensitive selectivity in passing high-frequency data at very low SNR.

Figure 4.7: Image number 48 filtered with different methods

4.3 Application of DEWDF on OCT Images

After using both SNR and UIQI separately in the calculation of objective functions ($\frac{1}{SNR}$ in PET, and UIQI difference in MRI), both of these indices were used as objective functions in application of DEWDF on OCT images. Here, DEWDF with $\frac{1}{SNR}$ is called DE-SNR and DEWDF with UIQI is called as DE-UIQI. Since SNR is used as objective function, ROI for signal and background had to be selected before the application of method.

4.3.1 Objective comparison and parameter distribution for OCT images

Based on SNR improvements, our method performs significantly better than the low pass, median and Wiener filters. DE-SNR and DE-Q in Table 4.5 represent DEWDF method using SNR and UIQI as objective functions respectively. From the Table 4.5 it can be observed that the value of UIQI decreases with the increase in size of spatial filters. This decrease in UIQI value corresponds to the high distortion and higher loss of signal. Also, the SNR improvements using other spatial filters was not at par with the improvement achieved by DEWDF. An improvement of more than 20% in SNR and negligible distortion shown by UIQI indicate that our method is more robust and efficient in reducing speckle noise from OCT images.

Effect of ROI on image denoising

An OCT image contains different areas with varying degree of significance. Some regions consist of only background area while others provide a relatively significant information. The regions with high intensity can be considered as those areas in object that reflect maximum number of photons and hence appear brighter in resulting OCT image. While the high intensity areas may not seem to get affected by speckle, the gray regions with average intensities are mostly obscured by grainy appearance of speckle. Because the amount of visible speckle was found to be varying in different regions of image, we considered 5 regions in our input image with

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Table 4.5: Objective comparison for OCT images

Method	Mask size	SNR (dB)	UIQI values	% SNR improvement
LPF	3×3	1.11	0.99	3.06
	5×5	1.11	0.99	3.07
	7×7	1.11	0.9	3.07
Median	3×3	1.12	0.96	3.96
	5×5	1.15	0.94	6.7
	7×7	1.17	0.92	9.0
Wiener	3×3	1.12	0.98	4.46
	5×5	1.15	0.96	7.01
	7×7	1.18	0.95	9.29
DE-SNR	N.A.	2.64	0.98	20.35
DE-UIQI	N.A.	2.48	0.99	13.09

different levels of intensity values. These regions, as shown in Figure 4.8, were selected to understand how the region selection influences the improvement in SNR and UIQI values. Referring to Figure 4.8, there are five ROIs with labels A, B, C, D and E. It can be observed that their selection takes into consideration the areas with highest to lowest intensity values. This choice was driven by the hypothesis that areas with more noise will tend to make the algorithm more aggressive with regards to thresholding of low scale(or high frequency) detail coefficients. The effect of selecting regions on different parameters and image metrics have been shown in Table 4.6

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Figure 4.8: ROIs for OCT image

Table 4.6: Effect of ROI on OCT images

Region	Method	% SNR improvement	UIQI values
A	DE-SNR	65.03	0.83
	DE-UIQI	25.17	0.98
B	DE-SNR	3.2	1.00
	DE-UIQI	2.61	1.00
C	DE-SNR	29.63	0.85
	DE-UIQI	10.57	0.97
D	DE-SNR	25.42	0.96
	DE-UIQI	11.85	1.00
E	DE-SNR	26.81	0.96
	DE-UIQI	17.64	0.98

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Table 4.7: Parameter distribution in OCT images

Region	Method	Threshold values	Decomposition levels	Wavelet types
A	DE-SNR	32.09	4	db8
	DE-UIQI	17.76	1	db5
B	DE-SNR	35.15	1	db10
	DE-UIQI	13.00	1	db10
C	DE-SNR	35.16	3	db9
	DE-UIQI	10.53	1	db6
D	DE-SNR	22.94	1	db2
	DE-UIQI	13.92	1	db5
E	DE-SNR	18.13	1	db1
	DE-UIQI	12.99	1	db10

The table shows that for all these ROIs a different combination of parameters was selected by our method. This variation in combination for different levels of noise proves our hypothesis that a wavelet based de-noising can be optimized and automated by selecting optimum set of parameters based on a purely objective method.

From these results we observed that the selection of SNR as objective function tend to make the thresholding of coefficients more aggressive. As shown by the SNR and UIQI values in Tables, DEWDF with SNR as objective function (DE-SNR) achieved higher values of SNR improvement than the DEWDF with UIQI difference as objective function (DE-Q). On the other hand, DE-Q achieved lesser improvement in SNR in comparison to DE-SNR. But at the same time DE-Q does not reduce values of UIQI to a large extent which in case of DE-SNR is reduced by a

higher value. This observation may be attributed to the fact that DE-SNR tries to reduce the standard deviation whereas DE-Q tends to maximize covariance between the original and processed image. Furthermore, the adaptability of our algorithm with the selection of different ROIs was visible from the values in Table 4.6. It can be observed from Tables 4.7 and 4.8 that when an ROI with less intensity area was selected, the algorithm selected a lower threshold whereas a higher threshold value was selected in case of higher intensity ROIs.

4.3.2 Visual inspection of denoised OCT images

Images in Figures 4.9 are the input image and the processed images using different filters. Figures 4.10(a) and 4.10(b) show the images processed using DEWDF with different objective functions. From Figure 4.9, it can be observed that the application of existing filters with mask larger masks such as 5×5 and 7×7 results in the loss of original signal itself.

The blurring is excessive and details are completely lost in case of median 5×5 and median 7×7 and Wiener 7×7 filters. Referring to the Figures 4.10(a) and 4.10(b), DEWDF performs denoising without blurring the significant edges. Although a small amount of smoothness has been introduced by our method too, it is still negligible in contrast to smoothness introduced by other methods.

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Figure 4.9: Visual inspection of denoised OCT images

Figure 4.10: DE with different objective functions for OCT images

4.4 Effect of DEWDF on image resolution

To study the effect on resolution loss by our method, we have used a noisy line phantom image. The point spread function (PSF) of this image was evaluated before and after the application of proposed method. Since most of PET, MR and OCR images are usually corrupted with Poisson, Gaussian and speckle noise respectively, the phantoms were also processed in order to add similar type of noise to them. Figures 4.11(a), 4.11(b), 4.11(c) and 4.11(d) show the original line phantom image, image with Poisson noise, image with Gaussian noise and image with speckle noise respectively. Images shown in Figure 4.11 were processed using DEWDF with UIQI difference as the objective function and the processed images have been shown in Figure 4.12.

Table 4.8 contains the value of FWHM for these images before and after the processing. It can be observed from the values in Table 4.8 that FWHM values which were increased by the addition of noise in original phantom image have been restored nearly to the original value after the processing. This restoration indicates

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Table 4.8: FWHM values for phantom images

Image	Input FWHM	Output FWHM
Original	5.29	N.A.
With Poisson noise	5.87	5.50
With Gaussian noise	7.43	5.90
With speckle noise	5.45	5.36

that our method reduces different types of noise without sacrificing resolution to a high extent.

A comparison between the Figures 4.12(b), 4.12(a) and 4.12(c) suggests that the bright circles in the de-noised image are more distinguishable from the background after the application of our method. This can be explained with the fact that noise structures which might have been forming a visible connection between two distinct circles would have increased the FWHM value. After these noise structures are removed or mitigated, the circles appear visibly distinct. The result is then reflected in FWHM values which get decreased to suggest that resolution has, in fact, been improved.

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Figure 4.11: Line phantom images

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(a) Poisson noise

(b) Gaussian noise

(c) Speckle noise

Figure 4.12: Line phantom images denoised with DEWDF

CONCLUSION

A novel effective technique to computerize the selection of parameters in wavelet domain filtering is introduced in this work. This technique precludes the interference of human observer while preserving the edges and removing the noise simultaneously. The parameters were selected on the basis of objective functions such as UIQI and SNR by comparing the changes in objective function value effected by the different parameters. The expression of UIQI uses correlation between different image data. This allows the comparison of the image data at given scale to the correlation of the data at that scale with those at other scales. Due to the strong correlation of features across scales in wavelet domain, they are retained by the method whereas the noise is removed because it is less correlated across scales. SNR calculation is based on the mean and standard deviation of signal and background region respectively. The use of SNR as objective function produced less noisy images but there was a slightly more reduction in UIQI when compared to the images that were denoised with UIQI as objective function in DEWDF. Unlike Wiener filter that tends to close its high-frequency passing band exclusively, our method provides feature-sensitive selectivity in passing high-frequency data at very low SNR. This feature sensitive selectivity is especially useful in reduction of noise in MR and PET images that have particularly

low SNR in various regions. In OCT images, the speckle noise dominates due to multiple scattering at large depths. Because the power spectral densities of signal-carrying and signal degrading speckle overlap this noise can be reduced only if the frequency bands are selected properly. A wavelet based thresholding method with optimum thresholding improves the signal-to-noise ratio of speckle corrupted regions. Our method not only finds a suitable threshold level, it also finds the best resolution level to apply that threshold. Hence, the selective threshold value along with the optimum resolution level significantly reduces the speckle that pervades OCT images of dense tissue. From the objective comparison, it appears that the SNR improvement by Wiener filter is at par with our method but the visual appearance of the images processed by DEWDF is superior to that of the Wiener-filtered images. In comparison to DEWDF and Wiener filter, Median filter's performance was found to be below par. Our results show that DEWDF technique can improve SNR by more than 40%, on an average, from the tested images while maintaining the UIQI change to a less than 5% of original values. The change in CNR was extremely low that suggested that the contrast and resolution were not sacrificed at the cost of improvement in SNR. This technique can be extended to allow the incorporation of more objective functions in future. Since the algorithm is modular in structure, more parameters, that influence the wavelet domain filtering, can be included for different applications. Although, the method has been tested for PET, MR and OCT images in this work, there is no limitation within the algorithm that will prohibit its use in the denoising of images from other modalities such as Ultrasound and Computed

tomography etc. The spatially selective nature of wavelet domain filtering makes DEWDF it useful in spatially dependent noise reduction. Due to the automation suggested in this work, the noise reduction process can be easily integrated into the large scale image processing workflows that would include image segmentation and object recognition as their components.

LIST OF PUBLICATIONS

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2. Vikas Gupta, Chueh Loo Poh, Elena Lim, and Chi Chiu Chan. An evolutionary algorithm to automate noise reduction in MR images, *Fifth International Conference of IEEE on Information Technology and Applications in Biomedicine*, 148-151, 2008.
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5. Vikas Gupta, Chi Chiu Chan, Chueh Loo Poh, Tzu Hao Chow, Tay Chia Meng, and Ng Beng Koon. Speckle noise reduction in OCT images using an evolutionary algorithm. *Computer Methods and Programs in Biomedicine* (Submitted)
6. Vikas Gupta, Chi Chiu Chan, Chueh Loo Poh, and Elena Lim. Computerized wavelet domain filtering for MRI with Differential Evolution. *Computers in Biology and Medicine* (Submitted)

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