Reliability assessment of damaged ductile RC frame structures against progressive collapse in close-in detonation conditions

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**RELIABILITY ASSESSMENT OF DAMAGED DUCTILE RC
FRAME STRUCTURES AGAINST PROGRESSIVE
COLLAPSE IN CLOSE-IN DETONATION CONDITIONS**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2009

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AGAINST PROGRESSIVE COLLAPSE IN CLOSE-IN DETONATION CONDITIONS

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To my family

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SUMMARY

A new approach is developed based upon the alternative load path (ALP) concept to assess the reliability of multi-storey ductile frame structures subjected to one column failure under close-in detonation conditions. This approach is based on a combination of the Monte Carlo Simulation method and an iterative algorithm developed for calculating structural performance function. The damaged ductile frame structure is simulated with a lumped plasticity model, while the singularity of the structural stiffness matrix is adopted as a reasonable failure criterion for structures under static loading conditions. A performance function is established based on the virtual work principle, critical collapse mechanism criterion and a construction system. An iterative procedure is developed to solve the performance function where the interaction between the axial force and the strength of the end springs is considered in the calculation of the minimum internal virtual work. A flowchart based on a combination of the developed procedure and the Monte Carlo Simulation method (MCS) is presented for the reliability assessment of damaged structures under static loading conditions.

In order to consider the dynamic effects induced by the sudden loss of a column in a close-in detonation, the method for the reliability assessment of the damaged ductile frame against progressive collapse under static loading conditions is further developed. The safety of the frame structure is evaluated based on two aspects: (1) the flexural response and (2) the shear response of the structure. Two performance functions are formed for structural flexural response to consider the structural collapse due to the lack of strength or deformation capacity of the structural weakest collapse mechanism. A third performance function is constructed to consider shear response. Since any of the above three performance functions less than zero will lead to structure progressive collapse, a global performance function is calculated by taking the minimum of these functions. The effectiveness of the developed method is

demonstrated by assessing the reliability of a four-storey reinforced concrete (RC) frame structure and the results exhibit the significance of the dynamic effects on structural reliability against progressive collapse.

Three groups of parametric studies are conducted to investigate the effects of different parameters on structural reliability against progressive collapse. In the first group of studies, the parameters considered include the coefficient of variations (CoVs) of the strength and the elastic modulus of concrete and reinforcing bars, the CoVs of the cross sectional effective depth and width of structural members and the CoVs of the dead and live load (DLL & LL). The second group of studies investigates parameters including the reinforcement area, the mean value of structural members' cross sectional effective depth and the mean value of concrete strength. The third group of studies focuses on the influence of the number of structural storeys. The results obtained from the parametric study are discussed from two perspectives: (1) the effect of the parameters on the failure mode of critical zones and (2) the effect of the parameters on structural reliability measured by the occurrence probability of structural collapse events.

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LIST OF SYMBOLS

A_s and A_s'	Longitudinal reinforcement area
$A_{v,k}$	Shear reinforcement area at the k^{th} section
A_r	Reinforcement area of the beams of the reference structure
A_a	Reinforcement area of the beams adopted in parametric studies
A_{r4}	Reinforcement area of the beams in the first four storeys of the newly built five-storey frame structure
A_{a5}	Reinforcement area of the beams in the fifth storey of the newly built five-storey frame structure
ASF	Average sensitivity factor
AVR	Average variation ratio
a	Acceleration of structural global response
b	Width of structural member
c and c'	Concrete covers
CoV_{sg}	Generally accepted value of the CoV of yield strength of reinforcing bars
CoV_{dg}	Generally accepted value of the CoV of effective depth of structural members
CoV_{DLg}	Generally accepted value of the CoV of Dead Load
CoV_{LLg}	Generally accepted value of the CoV of Live Load
CoV_{cg}	Generally accepted value of the CoV of compressive strength of concrete
CoV_{ecg}	Generally accepted value of the CoV of elastic modulus coefficient of concrete
CoV_{wg}	Generally accepted value of the CoV of width of structural members
CoV_{sa}	Modified value of the CoV of yield strength of reinforcing bars

CoV_{da}	Modified value of the CoV of effective depth of structural members
CoV_{DLa}	Modified value of the CoV of Dead Load
CoV_{LLa}	Modified value of the CoV of Live Load
CoV_{ca}	Modified value of the CoV of compressive strength of concrete
CoV_{eca}	Modified value of the CoV of elastic modulus coefficient of concrete
CoV_{wa}	Modified value of the CoV of width of structural members
CoV_{DL}	Coefficient of variation of dead load
CoV_{LL}	Coefficient of variation of live load
CoV_{fc}	Coefficient of variation of concrete compressive strength
CoV_{fy}	Coefficient of variation of steel yielding strength
d	Effective depth of structural members
d_k	Effective depth of the k^{th} section
DL	Dead load
DL_k	Characteristic value of dead load
DL_m	Mean value of dead load
$D_{[I,II,III],i}^{[b1,b2,c1,c2]}$	Director coefficients of the ESPHs on Beam-1, Beam-2, Column-1 and Column-2 within Type-I ~ Type-III critical zones in the i^{th} storey
E_c	Concrete elastic modulus
E_f	Structural failure event
f_c	Concrete compressive strength
f_{ck}	Characteristic compressive strength of concrete
f_{cm}	Mean value of compressive strength of concrete
f_y	Yielding strength of reinforcing steel

f_{yk}	Characteristic yielding strength of reinforcing steel
f_{ym}	Mean value of yielding strength of reinforcing steel
F_{bc}	Axial forces on indirectly connected columns and structural beams
F_l^i	Vertical force transferred from Part-b
F_r^i	Vertical force transferred from Part-c
$F_{c1}^{I \sim III, i}$	Axial forces on Column-1s in Type-I, II and III critical zones respectively, in the i^{th} storey
$F_{c2}^{I \sim III, i}$	Axial forces on Column-2s in Type-I, II and III critical zones respectively, in the i^{th} storey
$F_{c2}^{I \sim III, i-1}$	Axial forces on Column-2s in Type-I, II and III critical zones respectively, in the $(i-1)^{th}$ storey
F_E	External force on the structure
F_I	Inertial force
F_R	Recovering force due to structural deformation
$f_{vy, k}$	Yield strength of the shear reinforcement at the k^{th} section
h	Height of structural member
K_s	Structural stiffness matrix
l	Total length of the damaged span
l_l	Length of the left damaged span
l_r	Length of the right damaged span
LL	Live load
LL_k	Characteristic value of live load
LL_m	Mean value of live load
$L1$	Storey of the failed column
$L2$	Roof storey
M_u	Strength of end springs in structural lumped plasticity model

M_{+u}	Positive strength of end springs
M_{-u}	Negative strength of end springs
$M_{u,b1}^{I-III}$	Strength of end springs on Beam-1s in Type-I, II and III critical zones, respectively
$M_{+u,b1}^{I-III}$	Positive strength of end springs on Beam-1s in Type-I, II and III critical zones, respectively
$M_{-u,b1}^{I-III}$	Negative strength of end springs on Beam-1s in Type-I, II and III critical zones, respectively
$M_{+u,b1}^{I-III,i}$	Positive strength of end springs on Beam-1s in Type-I, II and III critical zones, respectively, in the i^{th} storey
$M_{-u,b1}^{I-III,i}$	Negative strength of end springs on Beam-1s in Type-I, II and III critical zones, respectively, in the i^{th} storey
$M_{u,b2}^{I-III}$	Strength of end springs on Beam-2s in Type-I, II and III critical zones, respectively
$M_{+u,b2}^{I-III}$	Positive strength of end springs on Beam-2s in Type-I, II and III critical zones, respectively
$M_{-u,b2}^{I-III}$	Negative strength of end springs on Beam-2s in Type-I, II and III critical zones, respectively
$M_{+u,b2}^{I-III,i}$	Positive strength of end springs on Beam-2s in Type-I, II and III critical zones, respectively, in the i^{th} storey
$M_{-u,b2}^{I-III,i}$	Negative strength of end springs on Beam-2s in Type-I, II and III critical zones, respectively, in the i^{th} storey
$M_{u,c1}^{I-III}$	Strength of end springs on Column-1s in Type-I, II and III critical zones, respectively
$M_{+u,c1}^{I-III}$	Positive strength of end springs on Column-1s in Type-I, II and III critical zones, respectively
$M_{-u,c1}^{I-III}$	Negative strength of end springs on Column-1s in Type-I, II and III critical zones, respectively
$M_{+u,c1}^{I-III,i}$	Positive strength of end springs on Column-1s in Type-I, II and III critical zones, respectively, in the i^{th} storey
$M_{-u,c1}^{I-III,i}$	Negative strength of end springs on Column-1s in Type-I, II and III critical zones, respectively, in the i^{th} storey

$M_{u,c2}^{I \sim III}$	Strength of the end spring on Column-2s in Type-I, II and III critical zones, respectively
$M_{+u,c2}^{I \sim III}$	Positive strength of end springs on Column-2s in Type-I, II and III critical zones, respectively
$M_{-u,c2}^{I \sim III}$	Negative strength of end springs on Column-2 in Type-I, II and III critical zones, respectively
$M_{+u,c2}^{I \sim III,i}$	Positive strength of end springs on Column-2s in Type-I, II and III critical zones, respectively, in the i^{th} storey
$M_{-u,c2}^{I \sim III,i}$	Negative strength of end springs on Column-2s in Type-I, II and III critical zones, respectively, in the i^{th} storey
M_{ur}	Flexural strength of structural members
M_{+ur}	Positive flexural strength of structural members
M_{-ur}	Negative flexural strength of structural members
$M_{+ur,bc}$	Positive flexural strength of indirectly connected columns and structural beams
$M_{-ur,bc}$	Negative flexural strength of indirectly connected columns and structural beams
$M_{+ur,b1}^{II,i}$	Positive strength of end sections of Beam-1s in Type-II critical zones in the i^{th} storey
$M_{+ur,b2}^{II,i}$	Positive strength of end sections of Beam-2s in Type-II critical zones in the i^{th} storey
$M_{l,i}^{leftend}$	The moment at the left end section of the beam in the left damaged span and the i^{th} storey
$M_{l,i}^{rightend}$	The moment at the right end section of the beam in the left damaged span and the i^{th} storey
$M_{r,i}^{leftend}$	The moment at the left end section of the beam in the right damaged span and the i^{th} storey
$M_{r,i}^{rightend}$	The moment at the right end section of the beam in the right damaged span and the i^{th} storey
$M_{l,i}(x)$	The moment distribution of the beam in the left damaged span and the i^{th} storey
$M_{r,i}(x)$	The moment distribution of the beam in the right damaged span and the i^{th} storey

$m_{i,j}$	Lumped mass concentrated on the j^{th} node of the i^{th} storey
$m_{i,j}^{bl}$	Half of the total masse of the beam at the left side of the j^{th} node of the i^{th} storey
$m_{i,j}^{br}$	Half of the total masse of the beam at the right side of the j^{th} node of the i^{th} storey
$m_{i,j}^{cu}$	Half of the total masse of the column at the upper side of the j^{th} node of the i^{th} storey
$m_{i,j}^{cl}$	Half of the total masse of the column at the lower side of the j^{th} node of the i^{th} storey
$m_{i,k}$	Lumped mass concentrated at the node on the damaged axis and the i^{th} storey
$m_{L1,k}$	Lumped mass concentrated at the node on the damaged axis and the storey of the failed column
N_s	Number of structural samples used in MCS
nc	Number of the considered cross sections.
np	Number of the RPHs
P_f	Occurrence probability of structural failure event
P_v	Concentrated force vector on the nodes above the failed column
$P_{v,i}$	The component of P_v in the i^{th} storey, ($i = L1 \cdots L2$)
$P_{v,L1}$	The component of P_v in the $L1$ storey
Q_j	Plastic rotation of the j^{th} ESPH
$Q_{[I,II,III],i}^{[b1,b2,c1,c2]}$	Plastic rotations of the ESPHs at the end sections of Beam-1, Beam-2, Column-1 and Column-2 within Type-I ~ Type-III critical zones in the i^{th} storey
$Q_{[I,II,III],i}^{[b1,b2,c1,c2],o}$	Plastic rotations obtained from the structural analysis with the model depicted in Table 4.2 and Figure 4.9 for corresponding ESPHs at the end sections of Beam-1, Beam-2, Column-1 and Column-2 within Type-I ~ Type-III critical zones in the i^{th} storey
q_i	Service loadings in the damaged span and the i^{th} storey
$q_{L,i}$	Service loading in the left damaged span and the i^{th} storey

$q_{r,i}$	Service loading in the right damaged span and the i^{th} storey
R_m	Strength of the equivalent SDOF system
s_k	Spacing of the shear reinforcement at k^{th} the cross section
$V_{r,k}$	Shear strength of the k^{th} section
$V_{c,k}$	Shear resistance provided by concrete at the k^{th} section
$V_{s,k}$	Shear resistance provided by the shear reinforcement at the k^{th} section
$V_{f,k}$	Shear force at the k^{th} section
v	Velocity of structural global response
$V_{b1}^{II,i}$	Shear force at the end section of Beam-1 in Type-II critical zone and the i^{th} storey
$V_{b2}^{II,i}$	Shear force at the end section of Beam-2 in Type-II critical zone and the i^{th} storey
δW_{int}	Internal virtual work done by the damaged structure
$\min(\delta W_{int})$	Minimum internal virtual work done by the structural weakest collapse mechanism
$\delta W_{int,j}$	Internal virtual work done by the j^{th} critical zone
$\min(\delta W_{int,j})$	Minimum internal virtual work done by the j^{th} critical zone
$\delta W_{int}^{I \sim III}$	Internal virtual work done by Type-I, II and III critical zones, respectively
$\min(\delta W_{int}^{I \sim III})$	Minimum internal virtual work done by Type-I, II and III critical zones, respectively
$\delta W_{int}^{I,1 \sim 3}$	Internal virtual work done by Type-I critical zone in Sub-spaces 1, 2 and 3, respectively
$\min(\delta W_{int}^{I,1 \sim 3})$	Minimum internal virtual work done by Type-I critical zone in Sub-spaces 1, 2 and 3, respectively
$\delta W_{int}^{II,1 \sim 4}$	Internal virtual work done by Type-II critical zone in Sub-spaces 1, 2, 3 and 4, respectively
$\min(\delta W_{int}^{II,1 \sim 4})$	Minimum internal virtual work done by Type-II critical zone in Sub-spaces 1, 2, 3 and 4, respectively

$\delta W_{\text{int}}^{III,1\sim3}$	Internal virtual work done by Type-III critical zone in Sub-spaces 1, 2 and 3, respectively
$\min(\delta W_{\text{int}}^{III,1\sim3})$	Minimum internal virtual work done by Type-III critical zone in Sub-spaces 1, 2 and 3, respectively
$\min(\delta W_{\text{int},i}^I)$	Minimum internal virtual work done by Type-I critical zone in the i^{th} storey
$\min(\delta W_{\text{int},i}^{II})$	Minimum internal virtual work done by Type-II critical zone in the i^{th} storey
$\min(\delta W_{\text{int},i}^{III})$	Minimum internal virtual work done by Type- III critical zone in the i^{th} storey
$\min(\delta W_{\text{int},a})$	Minimum internal virtual work done by Type-I, II and III critical zones in Part-a
$\min(\delta W_{\text{int},a11})$	Minimum internal virtual work done by Type-I critical zone in Part-a11
δW_{ext}	External virtual work done by service loadings
$\delta W_{\text{ext}}^{PV}$	External virtual work done by virtual concentrated force vector P_v
δW_{ext}^t	Total external virtual work done by service loadings and P_v
$\delta W_{\text{ext},i}$	External virtual work done by service loadings in the i^{th} storey
$\delta W_{\text{ext},a}$	External virtual work in Part-a
$\delta W_{\text{ext},a11}$	External virtual work in Part-a11
$W_{\text{int},u}$	Energy dissipation capacity of RWCM.
$W_{\text{ext},u}$	External work done by the service loadings in damaged spans at the deformation level of y_u
$W_{\text{ext},u}^{l,i}$	External work by the service loadings in the left damaged span and the i^{th} storey at the deformation level of y_u
$W_{\text{ext},u}^{r,i}$	External work by the service loadings in the right damaged span and the i^{th} storey at the deformation level of y_u
δW_E	Virtual work by F_E
δW_I	Virtual work by F_I
δW_R	Virtual work by F_R

$\delta W_{\text{int},i}^{I \sim III, \text{real}}$	Internal virtual work provided by the RPHs within Type-I ~ III critical zones in the i^{th} storey
$x_{l,i}$	Position of the positive RPH on beam in the left damaged span and the i^{th} storey
$x_{r,i}$	Position of the positive RPH on beam in the right damaged span and the i^{th} storey
y_e	Yield displacement of the equivalent SDOF system
$y_{e,j}$	The displacement which satisfies $\theta_j(y_{e,j}) = 0$
y_m	Maximum displacement of the damaged structure
y_u	Displacement capacity of the equivalent SDOF system
z_1	The first performance function of the damaged structure
z_2	The second performance function of the damaged structure
z_3	The third performance function of the damaged structure
z	Performance function of the damaged structure
z_C	Performance function of the construction system
α	Rotation of the node in critical zone
α_k	Angle between the shear reinforcement and the member's axis at the k^{th} cross section
$\delta \theta_i^{I \sim III}$	Virtual plastic rotations of the RPHs within Type-I ~ Type-III critical zones in the i^{th} storey under $\delta \Delta = 1$
θ_j	Plastic rotations of the j^{th} RPH
$\theta_{[b1,b2,c1,c2]}^{[I,II,III],i}$	Plastic rotations of the RPHs on Beam-1, Beam-2, Column-1 and Column-2 within Type-I ~ Type-III critical zones in the i^{th} storey
$\theta_{u,j}$	Rotational capacity of the j^{th} RPH
$\gamma_{b1}^{I \sim III}$	Virtual rotation of the ESPHs on Beam-1 in Type-I, II and III critical zones, respectively
$\gamma_{b2}^{I \sim III}$	Virtual rotation of the ESPHs on Beam-2 in Type-I, II and III critical zones, respectively

γ_{c1}^{I-III}	Virtual rotation of the ESPHs on Column-1 in Type-I, II and III critical zones, respectively
γ_{c2}^{I-III}	Virtual rotation of the ESPHs on Column-2 in Type-I, II and III critical zone, respectively
ψ_{LL}	Combination factor for live load
Δ	Displacement at the upper node of the failed column
$\Delta_{l,i}^u(x)$	Displacement distribution of the beam in the left damaged span and the i^{th} storey at the displacement level of y_u
$\Delta_{l,i}^{u,1}(x)$	The first component of $\Delta_{l,i}^u(x)$
$\Delta_{l,i}^{u,2}(x)$	The section component of $\Delta_{l,i}^u(x)$
$\Delta_{l,i}^{u,3}(x)$	The third component of $\Delta_{l,i}^u(x)$
$\Delta_{r,i}^u(x)$	Displacement distribution of the beam in the right damaged span and the i^{th} storey at the displacement level of y_u
$\Delta_{r,i}^{u,1}(x)$	The first component of $\Delta_{r,i}^u(x)$
$\Delta_{r,i}^{u,2}(x)$	The section component of $\Delta_{r,i}^u(x)$
$\Delta_{r,i}^{u,3}(x)$	The third component of $\Delta_{r,i}^u(x)$
$\delta\Delta$	Virtual displacement at the upper node of the failed column
r_{cs}	Parameter modification ratio of CoV_{sa}
r_{cd}	Parameter modification ratio of CoV_{da}
r_{cdl}	Parameter modification ratio of CoV_{DLa}
r_{cll}	Parameter modification ratio of CoV_{LLa}
r_{cc}	Parameter modification ratio of CoV_{ca}
r_{cec}	Parameter modification ratio of CoV_{eca}
r_{cw}	Parameter modification ratio of CoV_{wa}
r_{ma}	Parameter modification ratio of A_a
r_{md}	Parameter modification ratio of μ_{da}

r_{mc}	Parameter modification ratio of μ_{ca}
r_{sma}	Parameter modification ratio of A_{a5}
μ_{dr}	Mean values of cross sectional effective depths of the beams of the reference structure
μ_{da}	Mean values of cross sectional effective depths of the beams adopted in parametric studies
μ_{cr}	Mean values of compressive strength of concrete of the reference structure
μ_{ca}	Mean values of compressive strength of concrete adopted in parametric studies

CHAPTER ONE

INTRODUCTION

1.1 Problem Statements

Structural progressive collapse in a close-in detonation is the total or partial collapse of the building initiated by the localised failure of the blast loaded structural members. Considerations of how to prevent progressive collapse have been widely discussed since the collapse of the Ronan Point in the UK in 1968. A number of papers focusing on this topic were published throughout the 1970s. However, concerns over progressive collapse waned in the 1980s, mainly due to the relative peacefulness of the world and the lack of significant collapses. It is only after several recent structural collapse events resulting from terrorist attacks (such as the collapse of the Murrah Federal Building in Oklahoma City in 1995 and that of the World Trade Center Towers in New York City in 2001) that the interests in the research of structural resistance against progressive collapse have been regenerated.

The whole process of the collapse can be divided into three phases: (1) the blast occurrence phase, (2) the structural local response phase and (3) the structural global response phase. Accordingly, different methods can be employed to mitigate the possibility of structural progressive collapse in blast events. These methods include the event control method, the specific load resistance method, the alternative load path method and the indirect design method. The basic concept of the alternative load path method is that structures should be able to sustain local damages induced by blast pressures or other causes through providing an alternative load path to redistribute the loads previously carried by the damaged members. Since this method excludes the necessity for considering the uncertainty of the blast or other abnormal events in structural design and assessment, it has been widely adopted by current building codes [A1, A2, B10, D1, E4, G1, I1, N3] and research

activities [A6, B1, B11, G2, H1, K1, K2, K3, M1, M2, M3, M4, M16, P1]. However, it should be pointed out that current implementations of the alternative load path method are generally in a deterministic manner where all the parameters are assumed to be well-defined and deterministic. In reality, the random variations of the quantities including the applied loads, strength and stiffness of all construction materials and structural geometries are inevitable and have a significant influence on structural safety [K4, M5, R1]. Considering this fact, structural reliability assessment based on the probability theory should be the most sensible way for evaluating structural safety.

There are two types of approaches for evaluating the reliability of a damaged structure system against progressive collapse. The first type is based on a combination of a nonlinear finite element structural model and a probabilistic method such as the Monte Carlo simulation (MCS) or the first order reliability method (FORM) [V1, V2, V3]. The second type relies on the identification of all structural major collapse mechanisms and is particularly suitable for ductile frame structures. Although these two types of approaches made great improvement in structural reliability assessment, the huge amount of computational work limits their applications and further studies are necessary.

1.2 Research Objectives and Scopes

The main objective of this thesis is to develop a practical approach for the reliability assessment of damaged ductile reinforced concrete (RC) frame structural systems against progressive collapse in close-in detonations. The close-in blast loading is localized on a small part of the structure and is generally characterized by its extremely high magnitude and very short duration. Therefore, the initial local damage in this condition is modelled by a sudden removal of one structural column. The research is conducted with the following focuses:

- To develop a new approach for assessing the reliability of damaged ductile frame structures against progressive collapse under static service loading conditions.

- To further develop the above approach so that the dynamic effects induced by the sudden loss of one structural column in a close-in blast event are considered in the reliability assessment.
- To study the effects of different parameters on structural reliability against progressive collapse after the initial failure of a certain structural column.

1.3 Thesis Organization

The introduction of the problems, the objectives and the scopes of this study are defined in this chapter.

Chapter 2 carries out a literature review of previous research works on structural progressive collapse, which includes an overview of three structural progressive collapse events, the methods applicable for preventing structural progressive collapse, the relevant provisions in current design codes, the existing research on the structural resistance against progressive collapse and the reliability assessment of structural systems with local damages.

Chapter 3 focuses on the formulation of a performance function for damaged ductile frame structures against progressive collapse under static service loadings. The structure is modeled with the lumped plasticity model where springs are applied at the ends of structural members to simulate the nonlinear behaviour of the plastic hinges which may occur during the response of the structural system under external loadings. Based on this structural model, a performance function is established based on the virtual work principle and the critical collapse mechanism criterion. The dynamic effects induced by the sudden loss of a structural column are not considered in this chapter for simplicity purposes but they will be addressed in Chapter 5.

Chapter 4 presents an iterative procedure to solve the performance function developed in Chapter 3. The interaction between the strength of plastic hinges and

structural members' axial force is taken into account in the iterative calculation. In addition, the positions of the real plastic hinges, which may not be located on the end sections of beams, are considered by using reduced strength for corresponding springs. A flowchart based on a combination of the iterative procedure and Monte Carlo Simulation method is provided for reliability assessment of damaged ductile frame structures against progressive collapse under static service loadings.

Chapter 5 further develops the approach provided in Chapters 3 and 4. Structural collapses due to the lack of strength or deformation capacity of the structural weakest collapse mechanism or due to the lack of shear strength are studied in this chapter. Correspondingly, three performance functions are deduced. A global performance function is then established by taking the minimum of these three functions. The dynamic effects by the sudden loss of a structural column are considered in the deduction of the performance functions from three aspects: applying the service loadings as rectangular dynamic forces, utilizing a virtual concentrated force vector to simulate the inertial forces after the formation of structural weakest collapse mechanism, and considering the maximum dynamic response in the second performance function.

Chapter 6 carries out extensive parametric studies to investigate the effects of different parameters on structural reliability against progressive collapse after the failure of a structural column. The studies are divided into three groups with different objectives. The parameters considered in the first group of studies include the coefficient of variations (CoVs) of the strength and the elastic modulus of concrete and reinforcing bars, the CoVs of the cross sectional effective depth and width of structural members and the CoVs of the dead and live load (DLL & LL). The second group of studies investigates parameters including the reinforcement area, the mean value of structural member cross sectional effective depth and the mean value of concrete strength. The effects of the number of structural storeys are studied in the third group of parametric studies. The results obtained from the parametric studies are discussed from two perspectives: the effect of the parameters

on the failure mode of critical zones and the effect of the parameters on structural reliability measured by the occurrence probability of structural collapse events.

Chapter 7 concludes the work of the present study and recommends future work.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction and Background

With the global increase in terrorist activities, public or commercial buildings have become increasingly likely targets of explosive threats at close distances. Blast loadings, acting on buildings in close-in detonations, are extraordinary dynamic loads which are characterized by their great intensities and short durations [B2, B3, B4, B5, F1, H2, O1, T1]. If the structure is not sufficiently robust and resilient, progressive collapse will be triggered by the high blast loading. In spite of the low probability of such occurrences, progressive collapse of a structure can result in significant loss of lives and thus has become one of the major concerns of current research works.

This chapter presents a summary of the previous research works on structural progressive collapse. An overview of three recent progressive collapse events is provided in Section 2.2. Subsequently, the available methods to reduce the risk of structural progressive collapse under close-in blast conditions are discussed in the third section. Section 2.4 covers the most popular design standards and codes related to structural progressive collapse. A brief survey on the analytical models and related studies about the behaviour of damaged structures after the removal of one of their primary members is given in Section 2.5. The sixth section reviews the existing probabilistic theories and the research related to the reliability assessment of a structural system with local damages against progressive collapse. Finally, the key points in this chapter are summarized in the last section.

2.2 Review of Progressive Collapse Events and Lessons

2.2.1 Ronan Point Building Failure

The first event that evokes civil engineers' concentration on structural progressive collapse is the failure of the 23-storey Ronan Point Apartment building. In May 1968, a small gas explosion occurred in the southeast corner on the 18th storey of this building. The blast pressure blew out the supporting precast walls in this storey, which in turn caused the collapse of the upper four floors [G3]. The collapse progressed towards the ground due to the impact of the failed upper floors on the floors below. The result of the partial progressive collapse of Ronan Point is shown in Figure 2.1.

Figure 2.1 Progressive Collapse of Ronan Point [G3]

2.2.2 Oklahoma City Incident

Concerns over progressive collapse waned during the 1980s, mainly due to the relative peacefulness of the world and the lack of significant collapses. However, renewed interest in structural resistance against this type of failure was regenerated by the bombing of the Alfred Murrah federal building. In 1995, the Alfred P. Murrah federal office building in Oklahoma City was heavily damaged by a terrorist bomb blast, which was estimated to contain about 2174 kg of ANFO, equivalent to about 1800 kg TNT charge and located at a distance about 3.05 m from building.

Major structural damage and building collapse occurred to the Murrah building in this bombing incident. The damage distribution and its failure mechanism had been widely studied in [C1, H3, M8, S2]. In summary, the damage observed was categorized into two parts: direct air-blast damage and collateral effects caused by progressive collapse. As a direct result of the bombing, the Murrah Building suffered important structural member damage including the failure of three intermediate critical columns supporting a third-floor girder on the north side. Some floor slabs in the proximity of the bomb were also directly demolished. Since three principal columns G16, G20 and G24 were destroyed by the blast forces, whether the remaining structures could sustain the dead and live loadings was further studied by Sozen [S2]. It was found that with the loss of these three intermediate principal columns, the transfer girder supporting the upper portion of the building failed which in turn led to the progressive collapse of a major part of this building. The structure after progressive collapse is shown in Fig. 2.2.

Fig. 2.2 Progressive Collapse of Murrah Building [S2]

2.2.3 WTC Progressive Collapse

The collapse of the World Trade Center occurred after the September 11 attacks. Each of the two towers of the World Trade Center (WTC) was hit by an airliner that had been hijacked by Al Qaeda operatives. The south tower collapsed at 9:59 a.m., less than an hour after being hit, and the north tower followed at 10:28 a.m.

The collapse of the World Trade Center on September 11, 2001, led to demands by the public to amend current building codes and provide protection against progressive collapse caused by extreme events. Following September 11, the literature on progressive collapse mitigation has expanded significantly. Important issues examined by investigators include events leading to progressive collapse, assessment of loads, analysis methods, and design philosophy [B12, F3, H7, N4, U1, R3].

2.2.4 Structural Progressive Collapse in Close-in Detonation

Blast loading was a type of abnormal loads which had not been included in the building codes until then. It was not sufficient to assume that the structures designed for normal conditions could withstand the close-in blast impulse or other abnormal loads [R2]. Although certain forms of construction were more susceptible to progressive collapse than others, it had been pointed out that this type of collapse can occur in all types of construction [T2]. In order to reduce the possibility of structural failure in this manner, researches were carried out concentrated on blast loadings [B6, N1], structural member's responses subjected to blast loadings and the local damages induced thereby [A3, B2, M6, T1, T3], dynamic response of structure systems after the sudden loss of a primary structural member [B1, G2, G4, H1, K1, M1, M2, M3, M4], design methods against progressive collapse [E1, F2, M7] and structural risk analysis under abnormal loadings [S1, E3]. In addition, worldwide modification of design codes had been made to accommodate the necessity for structures to possess adequate resistance against progressive collapse.

A review of the blast incidents and related researches demonstrates that the whole process of the structural progressive collapse in a close-in detonation can be divided into three phases: the blast occurrence phase, the structural local response phase and the structural global response phase. In the first phase, a wavefront with high overpressure is produced as a result of the detonation. The intensity of the wavefront is determined by the charge weight and standoff distance of the explosion event. As the wavefront propagates outward and impacts on structural members close to the detonation, the process enters the second phase, where significant local response is induced to structure. The blast loaded structural members including the columns, beams and walls may be severely damaged or even completely destroyed by the blast impulse, which in turn leads to the release of the internal forces carried by these members. Following the local damage of structural members, global dynamic structural response occurs in the third phase and additional damages are induced to other members because of the load redistribution. The degree of the additional damages depends on the continuity, integrity and robustness of the

remaining structure. Progressive collapse is the extreme case of the spread of the local failure from member to member horizontally or vertically through the initially undamaged portions of the structure and resulting eventually in the collapse of the entire building or a disproportionately large part of it. The investigation of previous incidents shows that the most devastating loss of lives and social property occurs during the third phase i.e. the global response phase when the structural is subjected to collapse.

2.3 Methods against Structural Progressive Collapse

In order to mitigate the possibility of structural progressive collapse in a blast event, different methods with respect to above three phases can be employed, namely, the event control method, the specific load resistance method and the alternative load path method. A brief description of these methods is presented herein.

2.3.1 Event Control Method

The event control method used in diminishing the risk of structural progressive collapse under blast conditions is to prevent the occurrence of the detonation event or to reduce the intensity of the blast pressure acting on structural members by limiting the charge weight and standoff distance, *etc.* The use of a barrier to disable the access of the terrorist bomb car to structural buildings is a typical example of this method. It needs to be pointed out that the event control method does not increase structural continuity, integrity and robustness. In addition, this method is regarded as beyond the scope of work of engineers [L1]. Thus, other methods should be considered in structural design.

2.3.2 Specific Load Resistance Method

Protective design is a typical implementation of the specific local resistance design method for structures to prevent progressive collapse under detonation conditions. It requires structural members to be able to sustain the blast loadings without causing

unacceptable local damages. A successful application of this method relies on three key points i.e. the explosion event to be rationally pre-defined; the blast loading to be reasonably modelled and the corresponding structural member's response to be accurately evaluated. Although, many documents focusing on the later two points were published [A3, B2, B3, N1, M6, T1, T3], which had greatly improved the application of this method in structural design against progressive collapse in blast condition, a major drawback exists that the blast event should be known previously and that such a knowledge is difficult to ascertain. In addition, this method could be uneconomical considering the significant uncertainties about the blast event.

2.3.3 Alternative Load Path Method

The alternative load path method concerns the structural global behaviour after certain primary structural members are destroyed by blast loadings. The basic concept of this method is that structures should be able to sustain the local damages induced by blast loadings (or other causes) through providing an alternative load path to redistribute the loads previously carried by the damaged members.

In the implementation of this method, one or more of primary members (generally columns) are assumed to have failed and thus to be removed from the structural model. The remaining structure is then analyzed. The dynamic effect due to the sudden loss of the member by the blast loading will play an important role in determining the structural response and thus should be carefully considered [K1, P1]. The analysis continues until there are no other member failures or the structure remains capable of supporting its loads despite the loss of various structural components. The main advantage of the alternative load path method is that its results are independent of the blast event and thus is a favourable way for reducing the risk of structural progressive collapse in such events.

2.3.4 Indirect Design Method

The specific load resistance method and the alternative load path method are generally classified as the direct design method since these two methods calculate structural resistance explicitly. There is also another method called the indirect design method, which considers structural resistance against progressive collapse by specifying the minimum requirements on structural continuity, integrity, strength and ductility. Because this method is simple and believed to be able to improve overall structural response, it has been integrated into most building codes and specifications. However, some researchers have criticized the method as it gives no calculations as to how a structure behaves should one of its critical members fail.

2.4 Existing Design Guidelines Related to Progressive Collapse

Prevention of progressive collapse has garnered increasing interest since the failure of the Ronan Point Apartment, the Murrah Building and WTC towers. Many guidelines and provisions in prevention of progressive collapse have been well addressed, which include ASCE 7-02 [A1], the Interagency Security Committee (ISC) security criteria [I1], the DoD (Department of Defense) guidance [D1], and GSA (General Services Administration) guidance [G1], British Standards [B10] and Eurocode [E4]. All of these design guides make significant efforts to define procedures to prevent progressive collapse and are briefly discussed below.

2.4.1 ASCE 7-02 Guidance

The standard ASCE 7-02 [A1] “Minimum Design Loads for Buildings and Other Structures” gives an explicit requirement on structural integrity for preventing progressive collapse, which states *“Buildings and other structures shall be designed to sustain local damaged with the structural system as a whole remaining stable and not being damaged to an extent disproportionate to the original local damaged. This shall be achieved through an arrangement of the structural members that provides stability to the entire structural system by transferring loads from any locally damaged region to adjacent regions capable of resisting those loads without*

collapse.” Clearly, this statement focuses on the alternative load path method in structural design and can be accomplished by providing sufficient continuity, redundancy and ductility for the structure.

No specific design criterion is given in ASCE 7-02. However, three methods, including the indirect design method, the alternative load path method and the specific load resistance method are discussed in its commentary. In addition, different load combinations are recommended according to the method employed.

2.4.2 ISC Security Criteria

In “ISC (The Interagency Security Committee) security criteria for New Federal Office Buildings and Major Modernization Projects” [I1], the alternative load path concept has also been suggested for the design of structures in resisting progressive collapse with local damages. It is described that “*Designs that facilitate or are vulnerable to progressive collapse must be avoided. At a minimum, all new facilities shall be designed for the loss of a column for one floor above grade at the building perimeter without progressive collapse.*” The design is to be carried out based on a combination of the dead load plus a realistic estimate of the live load. Although either static or dynamic analysis for structure design against progressive collapse is allowed in the criteria, it gives no information on how to perform such analysis. In addition, the ISC document makes a reference to the ASCE 7-1995 [A2] standard for details.

2.4.3 DoD Guidance

The main objective of the document “DoD (Department of Defense) Interim Antiterrorism/Force Protection Construction Standards – Progressive Collapse Design Guidance” [D1] is to provide guidelines for diminishing the risk of structural collapse from terrorist attacks for DoD facilities. It gives a more explicit procedure for designing structures to resist progressive collapse with alternative load path and requires performing a series of analyses of a structure subjected to the

removal of a primary load-carrying member at different positions according to the possible locations of the blast event. Static linear-elastic models or nonlinear models are allowed to track the structural responses after its initial damage.

2.4.4 GSA Guidance

The most detailed approach for the analyses of structural progressive collapse was developed by the GSA (General Services Administration) guidance in the document “Progressive Collapse Analysis and Design Guidelines for New Federal Office Building and Major Modernization Projects” in order to reduce and/or assess of the potential for progressive collapse in new or existing buildings [G1]. Similar to the procedure in DoD guidance, it entails a series of analyses of the structure damaged with a primary member removed, one at a time, due to the potential variation of the event location. Different types of structural models are permitted such as the linear-elastic or nonlinear and the two-dimensional or three-dimensional models. According to the analysis techniques employed, various load factors are recommended. For example, twice of the gravity load (Dead + 25% Live) is applied to the linear static analysis in order to consider the dynamic effect of the sudden loss of the member. On the other hand, only the un-factored gravity loads are suggested for nonlinear dynamic analysis.

2.4.5 British Standard

After Ronan Point collapse, British Standards stated explicit design provisions against progressive collapse. The general tying of various structural elements to provide continuity and redundancy is emphasized in the standards. Ties enhance the resistance of wall panels to being blown away in an explosion and the ability of a structure to bridge over a lost support. In designing for this possibility, various structural elements are considered lost one at a time [B10]. In addition, structural elements deemed vital to a building stability shall be designed as key elements, which are able to withstand accidental loads (i.e, a pressure of 34kPa) [B10].

2.4.6 Eurocode

Eurocode provides general design guideline to avoid progressive collapse such as a selection of a good structural layout. It also recommends tying the building together and defines values for ties forces. This code is adopted by many European countries that may also supplement it with national standards [E4].

2.5 Progressive Collapse Analysis of Damaged Structures

A better understanding of the behaviour of damaged structures with one or more primary members failed is crucial for a successful application of the alternative load path method in design. Therefore, analysis of the damaged structural response receives substantial attention. Analytical methods for this purpose vary greatly from the simple linear static procedure to the complex nonlinear dynamic analysis [M1]. A brief review of the most popular analytical methods and related studies on structural progressive collapse is given in this section.

2.5.1 Linear Static Analysis

Linear static analysis of the progressive collapse for a damaged structure is widely used because of its simplicity. During the analysis, one or more load carrying members are assumed to fail and removed from the structural model. The remaining structure is then analyzed with the service loadings statically applied on it while the responses of all members are checked against acceptable criteria. In the case of any additional damage being identified on structural members, the model should be modified correspondingly and re-analyzed from its initially unloaded condition. The procedure continues until no further members fails or the remaining structure is capable of supporting the loads despite the loss of the bearing members. Based on the results of linear static analysis, static load redistribution for the remaining structure is obtained.

Columns are important in transmitting vertical and lateral loads to the foundations and are the major load-carrying members of building frames. Due to the blast event, the critical columns may be removed from the frame. As a result, the forces carried previously by the failed columns have to be transferred to adjacent members through another load path. Haluk, *et al* [H1], through an extensive of linear static analyses, investigated the redistribution paths of the forces resulting from a column failure and identified the basic structure defense mechanisms in the damaged building frame. Baldridge and Humay [B7] studied the significance of structural continuity and ductility in resisting progressive collapse when one structural column was assumed to fail initially. Grierson *et al* [G2] developed a simplified linear static method for simulating structural response during progressive collapse. The results from the developed linear static method exhibit good match with those obtained through the nonlinear static analysis [G5].

2.5.2 Nonlinear Static Analysis

The nonlinear static analysis of the behaviour of a damaged structure is an implementation of the pushover analysis with the loads applied in vertical direction. The basic differences between the linear and nonlinear static analysis are that the considerations for structural stiffness degradation due to the plastic response and the application of the service loadings are in a step by step manner in the structural nonlinear static analysis. The advantages and disadvantages of the nonlinear static method for structural progressive collapse simulation are summarized by Marjanishvili [M1]. Although this method is relatively simple as compared to the nonlinear dynamic analysis, its results tend to be overly conservative.

2.5.3 Dynamic Analysis

The dynamic effect caused by the sudden failure of structural members can be significant on structural response. There are a numbers of papers focusing on this topic. In the early of 1980s, McConnel and Kelly [M3] made a study of the progressive collapse of warehouse racking initiated by collision with a moving fork-

lift truck. By following the collapse with a time-step integration method, it was concluded that for this structural type, the dynamic effects were significant in the early stages of collapse. Subsequently, Pretlove *et al* [P1] discussed the importance of the dynamic effects of the sudden failure of a structural member on the progressive failure of a simple uniaxial tension structure and of a tension spoke wheel. The research demonstrated that a static analysis can be unconservative in predicting the potential of progressive collapse for a structure damaged by the sudden removal of a primary member. More recently, Morris [M4], Malla and Nalluri [M9] studied the dynamic response of truss-type structures after the failure of a structural member.

As for frame structures, Kaewkulchai and Williamson [K1, K2, K3] developed a beam element formulation and solution procedure for dynamic progressive collapse analysis of damaged planar frames. The study addressed the significance of dynamic load redistribution following the failure of one or more elements. Through the examples studied, it was pointed out that a static analysis could not provide a conservative estimate of the collapse potentiality of damaged frame structures.

For concrete buildings, Hakuno and Meguro [H4] used a method called the Extended Distinct-Element Method (EDEM) to model collapse of the structure under seismic or other dynamic loadings. The EDEM is a modification of the distinct-element method (DEM) originally proposed by Cundall and Strack [C2]. The element model consists of a rigid element connecting to two springs (horizontal and vertical direction), which are used to represent material properties. Hakuno and Meguro extended DEM to analyze the fracture of concrete structure, where additional mortar springs called pore springs were introduced between elements in both direction to model fracture of cement in concrete. The collapse simulation of concrete frames by horizontal seismic oscillation was performed using EDEM. To reduce the computational time, only a few elements for the cross section of the frame members were utilized sacrificing solution accuracy.

Buscemi and Marjanishvili [B1] developed a single-degree-of-freedom (SDOF) model for determining the potential of a structure to progressive collapse using dynamic analysis. A nonlinear response chart for progressive collapse analysis was developed to investigate the potential for progressive collapse by evaluating the maximum ductility and corresponding rotations of critical structural load-bearing members.

In addition, the dynamic response of structures during progressive collapse is also studied using the commercial software such as ABAQUS, LS-DYNA, or AUTODYNE. Li, *et al* [L2] studied the progressive collapse of a high rise concrete structure subjected to column failure through a two steps of analysis. The initial condition of the building and the structural dynamic response after column failure were simulated with these two steps, respectively. The results of the simulation demonstrated the stress redistribution among the remaining members. Luccioni, *et al* [L3] analyzed the collapse of a reinforced concrete building under blast loads with AUTODYNE. The whole process from the detonation of the explosive charge to the complete demolition, including the propagation of the blast wave and its interaction with the structure was reproduced. The analysis consumed approximately 310 hours in a system with a Pentium IV processor and 500MHz RIMM memory.

2.6 Reliability Assessment of Damaged Structures

The studies carried out by previous researches for preventing structural progressive collapse based on the alternative load path method are greatly helpful for engineers in structural design and assessment. However, most of them addressed this issue in a deterministic way where all parameters were assumed to be well-defined and deterministic. In reality, the random variations of the quantities including the applied loads, strength and stiffness of all construction materials and structural geometries are inevitable and have a significant influence on structural safety [K4, M5, R1, V1, V2]. Considering this fact, structural reliability assessment based on probability theory should be the most sensible way for evaluating structural safety.

The response of the remaining structure, after the initial damage, may be accompanied by significant cracking and plastic deformation. As a result, non-linear behaviour has to be considered and the current approach to reliability provision of the analyzed structure based on partial safety factors is no longer applicable [E2]. A direct application of the probabilistic approach for structural reliability assessment becomes necessary. In this section, the existing reliability theory including the first order reliability method (FORM) and the second order reliability method (SORM) is reviewed as well as the current researches related to the reliability assessment of damaged reinforced concrete structures.

2.6.1 Review of Reliability Theory

The structural statistical reliability theory deals with the determination of the occurrence probability P_f of the structure failure event based on the scatter of the applied loads and structural resistance properties [B8, B9, H5, H6 L4, R1, T4]. The failure event of the structure is described by a performance function z :

$$z = z(X), \quad X = (X_1, X_2, \dots, X_n) \quad (2.1)$$

where $X_1, X_2, \dots, X_n$ denote the basic random variables; X is named the random space of the concerned structure. By definition, $z(X) < 0$ indicates the occurrence of the structural failure event, whereas $z(X) > 0$ implies that the structure will be safe. In addition, the equation $z(X) = 0$ defines the limit state surface. Consequently, the probability of structural failure event can be expressed by the following function:

$$P_f = P_f(z < 0) \quad (2.2)$$

Monte Carlo Simulation (MCS) is a reliable tool for numerical integration of Equation (2.2) and can be utilized when dealing with a large number of random

variable or complicated performance function. It involves sampling at random to simulate artificially a large number of experiments and to observe the result. The equation for calculating P_f with MCS is given as:

$$P_f \approx \frac{N(z(\hat{X}) \leq 0)}{N_s} \quad (2.3)$$

where $\hat{X}$ denotes the sample value of the random variables X ; $N(z(\hat{X}) \leq 0)$ indicate the number of the observations for which $z(\hat{X}) \leq 0$; N_s is the total sample number. Obviously, the total number N_s of samples required is related to the accuracy for P_f .

2.6.2 Reliability Assessment of Damaged Structures

Generally, there are two types of approaches for the reliability assessment of a damaged structure system against progressive collapse. The first type is based on a combination of a nonlinear finite element structural model and a probabilistic method such as the Monte Carlo simulation (MCS). Nonlinear finite element analysis of the structural response is carried out to evaluate the structural performance function, while MCS is used to determine the reliability of the structure. After the failure criteria for a structure system is properly defined, the approaches of this type can be theoretically implemented to all kinds of structural reliability assessment problems. However, at the present stage, there is a major obstacle that heavily limits its applications in civil engineering especially in the reliability assessment of a damaged structure against progressive collapse. That is a huge amount of computational effort to be needed. Since significant cracking and plastic deformation is involved, the numerical simulation of the damaged structure against progressive collapse will be a time consuming task due to its extensive nonlinear response. When combined with MCS, thousands of such numerical simulations are required for assessing the reliability of the damaged structure with a

member removed at just one location. This process has to be repeated when the structure is initially damaged at other locations.

The second type of approaches for structural system reliability assessment requires the identification of the collapse mechanisms of the structure as the first step. It can be done through the ‘event-tree’ method [M5]. Based on the identified collapse mechanisms, the occurrence probability of structural failure event and the system reliability index can be evaluated by MCS or empirical formulae. In addition, some algorithms have been provided to predict the bounds of the system’s reliability.

Apparently, the second type of approaches relies heavily on the identification of structural collapse mechanisms and will generally give an unconservative estimation if any major mechanism is missed. However, the number of structural collapse mechanisms is greatly dependent on the degree of indeterminacy of the concerned structure. For a civil building with lots of members and in turn a high degree of indeterminacy, exhaustive enumeration of all possible mechanisms becomes a challenging task. Although there are some techniques such as the ‘truncated enumeration’ method [M10, M11] applicable for finding major structural collapse mechanisms, it is still a difficult and time consuming task and requires further research.

The plastic analysis method [C6, R4] can also be used for structural collapse mechanism determination, which has its roots in UK in the 1950s for steel and concrete structures and in Denmark for slabs. Its popularity migrated to the United States by the later 1950’s, but was soon subsumed in the early 1960’s by the advent of elastic computer analysis of structures. According to this method, the strength of a ductile elastic-perfect-plastic frame structure is determined by its collapse mechanism. After the formation of this mechanism, the bending moment distribution remains unchanged and further deformation focus on certain critical locations (plastic hinges).

The strength of the structure or the collapse mechanism induced by an external load equal to structural strength lies between the results obtained from the lower bound and upper bound theorems. The lower bound theorem states that a load calculated based on the equilibrium condition and an assumed bending moment distribution which nowhere exceeds corresponding the plastic moment (bending strength of the cross section) is less than or equal to the strength of the structure [C6, R4]. Such a moment distribution is referred to be statically admissible. The upper bound theorem states that a load computed on the basis of an assumed collapse mechanism will be greater than or at best equal to the strength of the structure. The deformation shape represented by the collapse mechanism is said to be kinematically admissible. If both theorems yield the same results, the calculated load is equal to structural strength (uniqueness theorem) while the corresponding mechanism is the real collapse mechanism of the structure under this load. The plastic analysis method is of great value in defining the failure mechanism for simple structures such as continuous beams *etc.* However, for the reliability assessment of a complex multi-storey frame structure, the huge number of possible collapse mechanisms still presents difficulties and more research should be done.

2.7 Summary

With the increasing likeliness of close-in detonations, improving structural resistance against progressive collapse in close-in detonation has become increasingly important in structural design. Investigation of the structural collapse events shows that the whole collapse process can be divided into three phases and the most devastating casualties will occur in the third phase if the structure is not suitably designed for sustaining local damages induced by blast loadings.

Different design methods can be employed accordingly to the three phases in progressive collapse events. By discussing the implementation of these methods, the alternative load path approach is regarded as a favorable procedure since it is event independent. The ability of the alternative load path method for explicit consideration of the structural global response is another major advantage of this

approach. As a result, it is recommended by many of current design guidelines. Some of these guidelines give out detailed procedure to carry out structural progressive collapse analysis based on alternative load path method.

Several analysis methods are available for studying structural response after the sudden failure of structure members. These methods will be carried out at different computational costs and provide different accuracy. The linear static and nonlinear static methods are simple and are allowed by many current design codes. Some researchers used these two methods in their studies. However, it is difficult to make a reasonable consideration for the dynamic effect triggered by the sudden failure of structural member. The load factor of 2 given in design standards to be used in the static analyses will produce overly conservative results. The dynamic effect proves to be substantially important in structural design and assessment against progressive collapse through a large number of studies. Therefore, the complex nonlinear dynamic analysis method which gives the most realistic simulation for structural response is favoured by most researchers. However, this method can be very time consuming.

A direct application of the probabilistic approach for structural reliability assessment against progressive collapse is necessary considering the fact that the inevitable variations of the quantities including the applied loads, strength and stiffness of all construction materials and structural geometries have a significant influence on structural safety against progressive collapse. There are generally two types of approaches for structural reliability evaluation for damaged structures with one or more primary members failed. The first type of approaches for structural reliability assessment, which is based on a combination of a nonlinear finite element model and a probabilistic method such as MCS, requires huge amount of computational effort. The second type of structural reliability assessment approaches relies heavily on the identification of all major structural collapse mechanisms. If any of these mechanisms is missed, an un-conservative estimation will be obtained. However, the exhaustive enumeration of all major structural

collapse mechanisms keeps challenging and presents an important limitation to the application of this type of approaches.

CHAPTER THREE

RELIABILITY ASSESSMENT OF DAMAGED DUCTILE RC FRAME STRUCTURAL SYSTEM AGAINST PROGRESSIVE COLLAPSE UNDER STATIC LOADING CONDITIONS

— PART I. PERFORMANCE FUNCTION

3.1 Introduction

Preventing structural progressive collapse in an event of close-in explosions has become an increasingly important concern in civil engineering. For this purpose, the alternative load path method (ALPM) has been widely accepted by many researchers. According to ALPM, a new approach is developed in this chapter to assess the reliability of ductile reinforced concrete (RC) frame structures subjected to one column failure in close-in explosion events. The approach is based on a combination of the Monte Carlo Simulation (MCS) method and an iterative algorithm for calculating the structural performance function.

This chapter concentrates on the construction of the performance function of damaged structures under static service loadings. The lumped plasticity model is used to simulate the damaged frame structure, while the singularity of the structural stiffness matrix is adopted as a reasonable criterion for structural global collapse. Three types of critical zones are identified and an optimum model is presented to determine the possible failure modes of each zone. Finally, a performance function in terms of the minimum internal virtual work minus the external virtual work done by service loadings is established based on the virtual work principle, critical collapse mechanism criterion and a construction system. The solution of this function will be addressed in Chapter 4.

3.2 Structural Modelling and Failure Criterion

Kaewkulchai and Williamson [K1, K2, K3] has demonstrated the good performance of the lumped plasticity model in the analysis of structural progressive collapse for ductile frame structures. With this model, the structure system is treated as an assembly of beam and column members which are represented by the beam-column elements jointed at different nodes. Additional springs are applied at both ends of the beam-column elements to simulate the nonlinear behavior of the plastic hinges that may occur during the response of the structural system under external loadings. The middle part of each element between its end springs is regarded to be elastic. The structure model is plotted in Figure 3.1.

(a). The structural system

Figure 3.1 The Lumped Plasticity Model

(b). The beam-column element

(c). The resistance function of the end spring

Figure 3.1 (Continued)

In Figure 3.1(c), M_u indicates the strength of the end spring. In the event of a close-in explosion, the corresponding element of the column subjected to failure needs to be removed from the intact structural model, and the loadings carried by this element have to be transferred to others. As a result, a chain of load redistribution occurs within the remaining structure, which in turn induces additional damage to other elements. If the structure is not sufficiently robust, it will lose its balance entirely and result in a progressive collapse. In order to measure the occurrence possibility of structural collapse in such situation, a criterion has to be defined so that it is able to properly reflect the global failure of the system. Under

static loading conditions, the singularity of the structural stiffness matrix is considered to satisfy this requirement, and is expressed mathematically as:

$$Det(K_s) = 0 \quad (3.1)$$

In Equation 3.1, K_s denotes the stiffness matrix of the damaged frame structure; $Det(\bullet)$ is a function of the stiffness matrix calculating its determinant. When the value of this function is zero, the singularity of the structural stiffness matrix is reached.

For a ductile frame simulated with the lumped plasticity model, Equation (3.1) also indicates the formation of a ductile collapse mechanism in the damaged structure with sufficient plastic hinges occurring on different structural elements. Depending on whether the initially failed column is located on an internal axis or on an external axis, there are generally two types of structural progressive collapses with their collapse mechanisms schematically plotted in Figure 3.2. However, if the damaged frame structure subjected to the failure of an external column is extended as demonstrated in Figure 3.2(c) with the strength of the elements and the service loadings within the extended part being zeros, the second type of structural progressive collapse can be treated as a special case of that in Figure 3.2(a). Therefore, an approach will be developed in the following sections to assess the reliability of the damaged frame with an initial failure of an internal column and is applicable for both the cases above. For the convenience of later discussion, some definitions need to be given herein:

- 1) The critical zones as shown in Figure 3.2 indicate the zones within which the plastic hinges would form and possibly contribute to the structural collapse mechanism. It does not mean that the plastic hinges cannot form in other regions but the hinges in other regions will not be part of the mechanism. It can be observed that there are totally three types of critical zones involved in the progressive collapse of the damaged frame structure, namely: Type-I, Type-II or Type-III critical zone. The superscript of *I*, *II* or

III in any symbol appearing in the later sections will be used to indicate them.

- 2) In each critical zone, the springs representing the possible plastic hinges during progressive collapse are all located at the ends of the structural elements as shown in Figure 3.2(d) with their strengths initially taken as the flexural strengths of the relative members. However, it may induce an unconservative error in structural reliability assessment as the real plastic hinges on structural beams are not necessarily formed at beams' end sections. This error can be corrected by employing the reduced strengths for the corresponding springs, which will be discussed in Chapter 4. In order to distinguish between the real plastic hinge and the plastic hinge represented by the yielded end spring, they are named RPH and ESPH, respectively.
- 3) The strength of the real structural members and that of the end springs are denoted by M_{ur} and M_u , respectively. If the bottom fabric (for beam) or left fabric (for column) of a structural member is in tension, then the related strength is generally called the positive strength in this study and will be indicated by a symbol + before the subscript *ur* or *u* such as M_{+ur} or M_{+u} . On the other hand, M_{-ur} and M_{-u} are used to represent the negative strength for the real structural members and the end springs.
- 4) The plastic hinges corresponding to M_{+ur} , M_{-ur} , M_{+u} and M_{-u} are identified as positive RPH, negative RPH, positive ESPH and negative ESPH, respectively.
- 5) From Figure 3.2(a) to 3.2(c), it can be seen that different numbers of structural beams and columns exist in different critical zones. In order to obtain a uniform identification for the members in each critical zone, the beam on the left side of the zone is named Beam-1 while the beam on the right is named Beam-2 as depicted in Figure 3.2(d). Similarly, the columns

on the lower and upper sides are named Column-1 and Column-2, respectively. These members will be specified by the subscripts $b1$, $b2$, $c1$ and $c2$ if applicable.

- 6) The directly connected columns are defined to be the columns that are directly connected to the critical zones, while the indirectly connected columns represent all the columns other than the directly connected columns.
- 7) The damaged span indicates the total span between Type-I and Type-III critical zones, which can be further divided into the left damaged span between Type-I and Type-II critical zones and the right damaged span between Type-II and Type-III critical zones. The left and right damaged spans will be denoted by the subscripts of l and r , respectively.

(a). Collapse mechanism for the damaged frame with an internal column initially failed

Figure 3.2 Collapse Mechanisms of the Damage Frame

(b). Collapse mechanism of the damaged frame with an external column initially failed

(c). Collapse mechanism of the extended damaged frame with an external column initially failed

Figure 3.2 (Continued)

(d). Details of the critical zone

Figure 3.2 (Continued)

As discussed previously, the global failure of the damaged ductile frame simulated with the lumped plasticity model is characterized by the formation of a ductile collapse mechanism where sufficient ESPHs occur within the critical zones. It is therefore reasonable to assume that the virtual deformation corresponding to a given virtual displacement $\delta\Delta$ after the formation of the collapse mechanism is contributed only by the virtual rotations of the ESPHs of the mechanism. Consequently, by selecting the occurrence possibility of structural failure event P_f as a rational measurement, the structural reliability can be evaluated based on the virtual work principle and is expressed as:

$$P_f = P(z < 0) \quad (3.2a)$$

where

$$z = \delta W_{\text{int}} - \delta W_{\text{ext}} \quad (3.2b)$$

Equation (3.2b) is the structural performance function, where δW_{int} and δW_{ext} indicate the internal virtual work done by the ESPHs of the collapse mechanism and

the external virtual work done by the service loadings respectively. Apparently, if δW_{int} is less than δW_{ext} ($z < 0$), the structure will lose its balance entirely and collapse. On the contrary, when δW_{int} is greater than δW_{ext} ($z > 0$), there exists a stable state for the structural system sustaining the service loadings. In addition, $z = 0$ indicates that the structure reaches its ultimate state with a collapse mechanism appearing in it.

In order to solve Equation (3.2a) for a multi-storey ductile frame structure, the Monte Carlo Simulation method (MCS), which proves to be a reliable tool for the reliability assessment of complicated systems, can be adopted. The procedure is described as follows:

- 1) Generate structural samples based on the given distribution properties of the considered random parameters;
- 2) Determine the collapse mechanism for each sample of the damaged frame structure;
- 3) Calculate the internal and the external virtual works under $\delta \Delta$ according to the mechanism obtained in Step 2, and check the value of the performance function;
- 4) Repeat Steps 2 to 3 for different samples of the damaged frame structure, and count the number of the structural failure event ($z < 0$);
- 5) Calculate P_f through dividing the number of failure events by the total number of structural samples.

Based on this procedure, it can be seen that during the solution of P_f , of key importance are the determination of structural collapse mechanism and the

calculation of the performance function. The approach dealing with these two problems is addressed in this study.

3.3 External Virtual Work

Since the virtual deformation is contributed only by the virtual rotation of the ESPHs of the collapse mechanism, the virtual vertical displacement of the damaged structure under $\delta\Delta$ will be step-linearly distributed along the damaged span. The virtual work $\delta W_{ext,i}$ done by the service loadings $q_i(x)$ in the i^{th} storey can be obtained with:

$$\delta W_{ext,i} = \int_0^l q_i(x) \times \delta\Delta(x) dx \quad (3.3)$$

where l represents the length of the damaged span and $\delta\Delta(x)$ denotes the virtual displacement of $q_i(x)$ along the damaged span, given as:

$$\delta\Delta(x) = \delta\Delta \times x / l_l \quad (0 \leq x \leq l_l) \quad (3.4a)$$

$$\delta\Delta(x) = \delta\Delta \times (1 - \frac{x - l_l}{l_r}) \quad (l_l \leq x \leq l) \quad (3.4b)$$

If $q_i(x)$ is uniformly distributed within each span, substituting $\delta\Delta(x)$ into Equation (3.3) produces:

$$\delta W_{ext,i} = \frac{1}{2} q_{l,i} \times \delta\Delta \times l_l + \frac{1}{2} q_{r,i} \times \delta\Delta \times l_r \quad (3.5)$$

where $q_{l,i}$ and $q_{r,i}$ indicate the magnitude of the uniformly applied service loadings on the left and right damaged spans, respectively; l_l and l_r are the corresponding

span lengths. The external virtual work δW_{ext} in Equation (3.2b) should be the sum of $\delta W_{ext,i}$ in all the storeys from the storey of the failed column ($L1$) to the roof storey ($L2$), thus

$$\delta W_{ext} = \sum_{i=L1}^{L2} \delta W_{ext,i} \quad (3.6)$$

3.4 Internal Virtual Work

In general, a large number of collapse mechanisms exist according to the locations where the ESPHs occur during the progressive collapse of the damaged frame. For example, if all ESPHs appear on the ends of the beams, a beam collapse mechanism is formed as shown in Figure 3.3(a). On the other hand, when some ESPHs appear on beams and others on columns, the mechanism will be a mixed collapse mechanism as plotted in Figure 3.3(b).

(a) The beam collapse mechanism

Figure 3.3 Beam and Mixed Collapse Mechanisms of the Damage Frame

(b) The mixed collapse mechanism

Figure 3.3 (Continued)

The calculation of the internal virtual work δW_{int} in Equation (3.2b) is directly dependent on the mechanism selected to represent the structural failure event. Since all of the mechanisms may induce structural failure, they have to be considered simultaneously in the reliability assessment of the damaged structure. The traditional approach for solving such a problem entails identifying all of the possible mechanisms, which proves to be a complicated task due to the large number of possibility. In order to deal with this problem, the critical collapse mechanism criterion is adopted to identify the weakest collapse mechanism and the corresponding internal virtual work is calculated.

3.4.1 Critical Collapse Mechanism Criterion

The critical collapse mechanism criterion states that the damaged frame structure will fail if its weakest collapse mechanism, which produces the minimum value of the performance function under a given virtual $\delta \Delta$, cannot guarantee the balance of the structure. Mathematically, it can be expressed as:

$$z = \min(\delta W_{\text{int}} - \delta W_{\text{ext}}) = \min(\delta W_{\text{int}}) - \delta W_{\text{ext}} \quad (3.7a)$$

with

$$z < 0 \Rightarrow E_f \text{ and } E_f \Rightarrow z < 0 \quad (3.7b)$$

Equation (3.7a) is the modified form of the structural performance function and E_f indicates the structural failure event. In addition, $\min(\delta W_{\text{int}})$ in Equation (3.7a) denotes the minimum internal virtual work of the damaged structure under $\delta \Delta$, which is done by the ESPHs of the structural weakest collapse mechanism. This criterion is apparently reasonable for structural reliability assessment as the structure will be safe only when all of the possible collapse mechanisms prove to be able to provide adequate strength to keep structural stable. As a result, Equation (3.7) presents a sufficient and necessary condition for checking the occurrence of structure failure event. Moreover, this criterion also indicates that if the structure is staying in its ultimate state with $z = 0$, the corresponding mechanism appearing in the structure must be the weakest collapse mechanism.

3.4.2 Minimum Internal Virtual Work

The internal virtual work δW_{int} is determined by the virtual rotations of the ESPHs within the critical zones. Therefore, δW_{int} can be obtained by:

$$\delta W_{\text{int}} = \sum_{j=1}^c \delta W_{\text{int},j} \quad (3.8)$$

where $\delta W_{\text{int},j}$ indicates the internal virtual work done by the ESPHs within the j^{th} critical zone; c represents the number of critical zones involved. Minimizing δW_{int} in Equation (3.8) produces:

$$\min(\delta W_{\text{int}}) = \sum_{j=1}^c \left(\min(\delta W_{\text{int},j}) \right) \quad (3.9)$$

Based on Equations (3.9), it can be seen that the calculation of the minimum internal virtual work $\min(\delta W_{\text{int}})$ of the damaged structure has been transformed into a series of problems of finding the minimum internal virtual work produced by each critical zone $\min(\delta W_{\text{int},j})$. The solution for these problems will be deduced with an optimum model addressed in the following section.

3.5 Possible Failure Modes of Critical Zones

3.5.1 Deformation of Critical Zones

The internal virtual work produced by a critical zone ($\delta W_{\text{int},j}$) is determined by the virtual rotations of related ESPHs. However, it is currently unknown where the ESPHs will form during the structural progressive collapse. In order to cover all possibility in the deduction of the formula for $\min(\delta W_{\text{int},j})$, it is preliminarily assumed that ESPHs are formed on all the end springs of a critical zone and an arbitrary rotation α is introduced to its node to determine the ESPHs' virtual rotations. For the three types of critical zones demonstrated in Figure 3.2, each of their deformation versus α and $\delta\Delta$ is plotted in Figure 3.4.

Figure 3.4 Deformation of Different Types of Critical Zones

(b). Type-II critical zone

(c). Type-III critical zone

Figure 3.4 (Continued)

Figure 3.4(a) plots the deformation of Type-I critical zone on the left axis adjacent to the initially failed internal column. The axis of Beam-2 will experience a rigid rotation of $\delta\Delta/l_l$ since the virtual deformations under $\delta\Delta$ are concentrated on ESPHs within critical zones. Given an arbitrary rotation α of the node with the positive direction being counter-clockwise, the virtual rotations of the ESPHs are functions of $\delta\Delta/l_l$ and α , which are expressed as:

$$\gamma_{c1}^I = \gamma_{c2}^I = |\alpha| \quad (3.10a)$$

$$\gamma_{b1}^I = |\alpha| \quad (3.10b)$$

$$\gamma_{b2}^I = |\alpha + \delta\Delta / l_l| \quad (3.10c)$$

where γ_{c1}^I , γ_{c2}^I , γ_{b1}^I and γ_{b2}^I denote the virtual rotations of the ESPHs on Column-1, Column-2, Beam-1 and Beam-2, respectively, in this type of critical zone.

The deformation of Type-II critical zone on the axis of the failed internal column under given $\delta\Delta$, is shown in Figure 3.4(b). The rigid rotations of beam axes on the left and right sides of the zone are $\delta\Delta / l_l$ and $\delta\Delta / l_r$, respectively. The relationships between the virtual rotations of the ESPHs and $\delta\Delta / l_l$, $\delta\Delta / l_r$ and α are written in Equation (3.11):

$$\gamma_{c2}^{II} = \gamma_{c2}^{II} = |\alpha| \quad (3.11a)$$

$$\gamma_{b1}^{II} = |\alpha + \delta\Delta / l_l| \quad (3.11b)$$

$$\gamma_{b2}^{II} = |\alpha - \delta\Delta / l_r| \quad (3.11c)$$

Figure 3.4(c) demonstrates the deformation of Type-III critical zone on the right axis adjacent to the failed internal column. The virtual rotations of the ESPHs with a given $\delta\Delta$ and α are:

$$\gamma_{c1}^{III} = \gamma_{c2}^{III} = |\alpha| \quad (3.12a)$$

$$\gamma_{b1}^{III} = |\alpha - \delta\Delta / l_r| \quad (3.12b)$$

$$\gamma_{b2}^{III} = |\alpha| \quad (3.12c)$$

The internal virtual work by each type of the critical zone is the sum of the products of the virtual rotations and the strength of corresponding end springs and thus can be obtained as:

For Type-I critical zone:

$$\delta W_{\text{int}}^I = M_{u,b1}^I |\alpha| + M_{u,b2}^I |\alpha + \delta\Delta / l_l| + (M_{u,c1}^I + M_{u,c2}^I) |\alpha| \quad (3.13)$$

For Type-II critical zone:

$$\delta W_{\text{int}}^{II} = M_{u,b1}^{II} |\alpha + \delta\Delta / l_l| + M_{u,b2}^{II} |\alpha - \delta\Delta / l_r| + (M_{u,c1}^{II} + M_{u,c2}^{II}) |\alpha| \quad (3.14)$$

For Type-III critical zone:

$$\delta W_{\text{int}}^{III} = M_{u,b1}^{III} |\alpha - \delta\Delta / l_r| + M_{u,b2}^{III} |\alpha| + (M_{u,c1}^{III} + M_{u,c2}^{III}) |\alpha| \quad (3.15)$$

In Equations (3.13)~(3.15), δW_{int}^I , $\delta W_{\text{int}}^{II}$ and $\delta W_{\text{int}}^{III}$ indicate the internal virtual work produced by a single critical zone of the Type-I, Type-II, and Type-III respectively; $M_{u,b1}^{I-III}$, $M_{u,b2}^{I-III}$, $M_{u,c1}^{I-III}$ and $M_{u,c2}^{I-III}$ represent the strength of the end springs on Beam-1, Beam-2, Column-1 and Column-2 in different types of critical zones. It can be observed that with the given $\delta\Delta$, the amount of internal virtual work provided by a certain critical zone will vary with α . In order to obtain the minimum internal virtual work by each critical zone, an unconstrained nonlinear optimum model is formulated as:

$$\begin{cases} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int},j} = f(\alpha) \end{cases} \quad (3.16)$$

where $\delta W_{\text{int},j} = f(\alpha)$ is the objective function of α as expressed in Equations (3.13) ~ (3.15) with respect to the type of the critical zones. Since any critical zone in the damaged ductile frame structure concerned in this study will belong to one of the three types as plotted in Figure 3.2, the close-form solution of Equation (3.16) will be deduced for each type.

3.5.2 Type-I Critical Zone

By substituting Equation (3.13) into Equation (3.16), the unconstrained optimum model for the Type-I critical zone becomes:

$$\begin{cases} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^I = M_{u,b1}^I |\alpha| + M_{u,b2}^I |\alpha + \delta\Delta/l_l| + (M_{u,c1}^I + M_{u,c2}^I) |\alpha| \end{cases} \quad (3.17)$$

The continuous variation of α from $-\infty$ to $+\infty$ is defined as the one-dimensional optimum α space while the points $\alpha + \delta\Delta/l_l = 0$ and $\alpha = 0$ as the characteristic points. The whole α space is divided into 3 sub-spaces by these two characteristic points as plotted in Figure 3.5(a). Consequentially, the unconstrained nonlinear optimum model in Equation (3.17) can be transformed into 3 constrained linear optimum models corresponding to these sub-spaces.

In Sub-space 1, it is reformed as:

$$\begin{cases} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{I,1} = -(M_{-u,b1}^I + M_{+u,b2}^I + M_{+u,c1}^I + M_{-u,c2}^I) \alpha \\ & \quad - M_{+u,b2}^I \delta\Delta/l_l \\ \text{subject to} & \alpha \leq -\delta\Delta/l_l \end{cases} \quad (3.18)$$

It is well understood that for a constrained linear optimum problem to be solved, the solution will be located on its boundaries [A4], which are $\alpha = -\infty$ and $\alpha = -\delta\Delta/l_l$ for the current circumstance. Considering that the objective function $\delta W_{\text{int}}^{I,1}$ in Equation (3.18) is a monotonically decreasing function of α , the minimum value of $\delta W_{\text{int}}^{I,1}$ is obviously to be on $\alpha = -\delta\Delta/l_l$ with:

$$\min(\delta W_{\text{int}}^{I,1}) = (M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I)\delta\Delta/l_l \quad (3.19)$$

The relationship between $\delta W_{\text{int}}^{I,1}$ and α is represented by the radial AB in Figure 3.5(a) while the shape of Type-I critical zone in Sub-space 1 is plotted in Figure 3.5(b).

In Sub-space 2 where $-\delta\Delta/l_l \leq \alpha \leq 0$, the optimum model of Equation (3.17) becomes:

$$\begin{cases} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{I,2} = (-M_{-u,b1}^I + M_{-u,b2}^I - M_{+u,c1}^I - M_{-u,c2}^I)\alpha \\ & + M_{-u,b2}^I\delta\Delta/l_l \\ \text{subject to} & -\delta\Delta/l_l \leq \alpha \leq 0 \end{cases} \quad (3.20)$$

Two boundary points $\alpha = -\delta\Delta/l_l$ and $\alpha = 0$ exist. Whichever leads to the least value of the objective function $\delta W_{\text{int}}^{I,2}$ will be the location of the solution. Therefore, by substituting $\alpha = -\delta\Delta/l_l$ and $\alpha = 0$ into the objective function in Equation (3.20) respectively, and taking the minimum value, it can be obtained that:

$$\min(\delta W_{\text{int}}^{I,2}) = \min[(M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I)\delta\Delta/l_l, M_{-u,b2}^I\delta\Delta/l_l] \quad (3.21)$$

The curve of $\delta W_{\text{int}}^{I,2}$ versus α is plotted by either the line BC or BC' in Figure 3.5(a) depending on the relationship between $M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I$ and $M_{-u,b2}^I$. The shape of the critical zone in this sub-space is shown in Figure 3.5(c).

In Sub-space 3 where $\alpha \geq 0$, Equation (3.17) is transformed into:

$$\left\{ \begin{array}{ll} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{I,3} = (M_{+u,b1}^I + M_{-u,b2}^I + M_{-u,c1}^I + M_{+u,c2}^I)\alpha \\ & + M_{-u,b2}^I \delta \Delta / l_l \\ \text{subject to} & \alpha \geq 0 \end{array} \right. \quad (3.22)$$

It is different from the model presented in Equation (3.18), being that the objective function $\delta W_{\text{int}}^{I,3}$ here increases monotonically. Accordingly, the minimized value of $\delta W_{\text{int}}^{I,3}$ will lie on the left boundary of $\alpha = 0$, expressed as:

$$\min(\delta W_{\text{int}}^{I,3}) = M_{-u,b2}^I \delta \Delta / l_l \quad (3.23)$$

The objective function $\delta W_{\text{int}}^{I,3}$ in this space is demonstrated through CD or $C'D'$ in Figure 3.5(a) with the shape of Type-I critical zone shown in Figure 3.5(d).

Since it is necessary to find the minimum internal virtual work over the whole α space as formulated by Equation (3.17), combining Equations (3.19), (3.21) and (3.23) produces:

$$\begin{aligned} \min(\delta W_{\text{int}}^I) &= \min[\min(\delta W_{\text{int}}^{I,1}), \min(\delta W_{\text{int}}^{I,2}), \min(\delta W_{\text{int}}^{I,3})] \\ &= \min[(M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I) \delta \Delta / l_l, M_{-u,b2}^I \delta \Delta / l_l] \end{aligned} \quad (3.24)$$

Equation (3.24) can be explained as follows. When the Type-I critical zone is concerned in the progressive collapse of the damaged frame structure with an

internal column initially destroyed by blast loadings, given a displacement $\delta\Delta$, there are only two possible ways for this type of critical zone to deform in providing the minimum internal virtual work. If the sum of the strength of the end springs on Beam-1, Column-1 and Column-2 is less than the strength of the spring on Beam-2 ($M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I < M_{-u,b2}^I$), the solution of Equation (3.17) will be located on the point B in Figure 3.5(a) with $\min(\delta W_{\text{int}}^I) = (M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I)\delta\Delta/l_l$ and $\alpha = -\delta\Delta/l_l$. Correspondingly, three ESPHs at the ends of Beam-1, Column-1 and Column-2, respectively, will occur in generating the structural weakest collapse mechanism. In addition, the virtual deformation and internal virtual work will concentrate on the three ESPHs. Otherwise, if ($M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I > M_{-u,b2}^I$), point C in Figure 3.5(a) will be the solution with $\min(\delta W_{\text{int}}^I) = M_{-u,b2}^I\delta\Delta/l_l$ and $\alpha = 0$. In this case, only one ESPH at the end of Beam-2 will appear to produce the weakest mechanism, while the virtual deformation and internal virtual work will focus on this ESPH. These two possible ways of deformation are called the possible failure modes for Type-I critical zone and are plotted in Figures 3.5(e) and 3.5(f).

Figure 3.5 Possible Failure Modes of Type-I critical zone

Figure 3.5 (Continued)

3.5.3 Type-II Critical Zone

Above discussion presents the method to deduce the formula to calculate the minimum internal virtual work provided by Type-I critical zones under $\delta\Delta$. This method is also applicable to other types of critical zones. The unconstrained optimum model for minimum internal virtual work is formulated for Type-II critical zone by substituting Equation (3.14) into Equation (3.16) as:

$$\begin{cases} \text{require} & \alpha \\ \text{minimize} & \mathcal{W}_{\text{int}}^{\text{II}} = M_{u,b1}^{\text{II}} |\alpha + \delta\Delta / l_l| + M_{u,b2}^{\text{II}} |\alpha - \delta\Delta / l_r| \\ & + (M_{u,c1}^{\text{II}} + M_{u,c2}^{\text{II}}) |\alpha| \end{cases} \quad (3.25)$$

The whole α space will be divided into four sub-spaces by its characteristic points $\alpha + \delta\Delta/l_l = 0$, $\alpha = 0$ and $\alpha - \delta\Delta/l_r = 0$ as shown in Figure 3.6(a). Accordingly, Equation (3.25) can be divided into four constrained linear optimum models to calculate the minimum internal virtual work in each sub-space, which are expressed as follows:

In Sub-space 1

$$\left\{ \begin{array}{ll} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{II,1} = -(M_{-u,b1}^{II} + M_{+u,b2}^{II} + M_{+u,c1}^{II} + M_{-u,c2}^{II})\alpha \\ & - (M_{-u,b1}^{II}\delta\Delta/l_l - M_{+u,b2}^{II}\delta\Delta/l_r) \\ \text{subject to} & \alpha \leq -\delta\Delta/l_l \end{array} \right. \quad (3.26)$$

In Sub-space 2

$$\left\{ \begin{array}{ll} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{II,2} = (M_{+u,b1}^{II} - M_{+u,b2}^{II} - M_{+u,c1}^{II} - M_{-u,c2}^{II})\alpha \\ & + (M_{+u,b1}^{II}\delta\Delta/l_l + M_{+u,b2}^{II}\delta\Delta/l_r) \\ \text{subject to} & -\delta\Delta/l_l \leq \alpha \leq 0 \end{array} \right. \quad (3.27)$$

In Sub-space 3

$$\left\{ \begin{array}{ll} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{II,3} = (M_{+u,b1}^{II} - M_{+u,b2}^{II} + M_{-u,c1}^{II} + M_{+u,c2}^{II})\alpha \\ & + (M_{+u,b1}^{II}\delta\Delta/l_l + M_{+u,b2}^{II}\delta\Delta/l_r) \\ \text{subject to} & 0 \leq \alpha \leq \delta\Delta/l_r \end{array} \right. \quad (3.28)$$

In Sub-space 4

$$\left\{ \begin{array}{ll} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{II,4} = (M_{+u,b1}^{II} + M_{-u,b2}^{II} + M_{-u,c1}^{II} + M_{+u,c2}^{II})\alpha \\ & + (M_{+u,b1}^{II} \delta \Delta / l_l - M_{-u,b2}^{II} \delta \Delta / l_r) \\ \text{subject to} & \alpha \geq \delta \Delta / l_r \end{array} \right. \quad (3.29)$$

The curves of the objective functions $\delta W_{\text{int}}^{II,1} \sim \delta W_{\text{int}}^{II,4}$ versus α in the corresponding sub-space are plotted in Figure 3.6(a), while the shapes of the critical zone in different sub-spaces are shown in Figure 3.6(b) ~ 3.6(e). Solving Equations (3.26) ~ (3.29) and combining their results, the minimum virtual work provided by Type-II critical zone over the whole α space is:

$$\begin{aligned} \min(\delta W_{\text{int}}^{II}) &= \min[\min(\delta W_{\text{int}}^{II,1}), \min(\delta W_{\text{int}}^{II,2}), \min(\delta W_{\text{int}}^{II,3}), \min(\delta W_{\text{int}}^{II,4})] \\ &= \min[(M_{+u,b2}^{II} + M_{+u,c1}^{II} + M_{-u,c2}^{II})\delta \Delta / l_l + M_{+u,b2}^{II} \delta \Delta / l_r, \\ &\quad M_{+u,b1}^{II} \delta \Delta / l_l + M_{+u,b2}^{II} \delta \Delta / l_r, \\ &\quad M_{+u,b1}^{II} \delta \Delta / l_l + (M_{+u,b1}^{II} + M_{-u,c1}^{II} + M_{+u,c2}^{II})\delta \Delta / l_r] \end{aligned} \quad (3.30)$$

From Equation (3.30), it can be concluded that there are three possible failure modes for Type-II critical zone in structural progressive collapse providing the minimum internal virtual work under $\delta \Delta$. Depending on the relationships of the strength of the end springs, these failure modes of the zone and the related ESPHs are shown in Figures 3.6 (f) ~ 3.6(h).

(a). Four sub-spaces

(b). Type-II critical zone in Sub-space 1

(c). Type-II critical zone in Sub-space 2

(d). Type-II critical zone in Sub-space 3

(e). Type-II critical zone in Sub-space 4

Figure 3.6 Possible Failure Modes of Type-II Critical Zone

(f). The first possible failure mode and the related ESPHs
($\alpha = -\delta\Delta / l_r$)

(g). The second possible failure mode and the related ESPHs
($\alpha = 0$)

(h). The third possible failure mode and the related ESPHs
($\alpha = \delta\Delta / l_r$)

Figure 3.6 (Continued)

3.5.4 Type-III Critical Zone

The unconstrained optimum model for minimum internal virtual work by Type-III critical zone is written as:

$$\begin{cases} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{\text{III}} = M_{u,b1}^{\text{III}} |\alpha - \delta\Delta / l_r| + M_{u,b1}^{\text{III}} |\alpha| + (M_{u,c1}^{\text{III}} + M_{u,c2}^{\text{III}}) |\alpha| \end{cases} \quad (3.31)$$

There are two characteristic points $\alpha = 0$ and $\alpha - \delta\Delta / l_r = 0$ dividing the whole α space into three sub-spaces as demonstrated in Figure 3.7(a). In each sub-space, the optimum model of Equation (3.31) becomes:

In Sub-space 1

$$\begin{cases} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{\text{III},1} = -(M_{-u,b1}^{\text{III}} + M_{+u,b2}^{\text{III}} + M_{+u,c1}^{\text{III}} + M_{-u,c2}^{\text{III}}) \alpha \\ & + M_{-u,b1}^{\text{III}} \delta\Delta / l_r \\ \text{subject to} & \alpha \leq 0 \end{cases} \quad (3.32)$$

In Sub-space 2

$$\left\{ \begin{array}{ll} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{\text{III},2} = \left(-M_{-u,b1}^{\text{III}} + M_{-u,b2}^{\text{III}} + M_{-u,c1}^{\text{III}} + M_{+u,c2}^{\text{III}} \right) \alpha \\ & + M_{-u,b1}^{\text{III}} \delta \Delta / l_r \\ \text{subject to} & 0 \leq \alpha \leq \delta \Delta / l_r \end{array} \right. \quad (3.33)$$

In Sub-space 3

$$\left\{ \begin{array}{ll} \text{require} & \alpha \\ \text{minimize} & \delta W_{\text{int}}^{\text{III},3} = \left(M_{+u,b1}^{\text{III}} + M_{-u,b2}^{\text{III}} + M_{-u,c1}^{\text{III}} + M_{+u,c2}^{\text{III}} \right) \alpha \\ & + M_{+u,b1}^{\text{III}} \delta \Delta / l_r \\ \text{subject to} & \alpha \geq \delta \Delta / l_r \end{array} \right. \quad (3.34)$$

The shapes of the critical zone in these sub-spaces are plotted in Figures 3.7(b) ~ 3.7(d). By solving Equations (3.32) ~ (3.34), the minimum internal virtual work by this critical zone under $\delta \Delta$ is achieved as:

$$\begin{aligned} \min(\delta W_{\text{int}}^{\text{III}}) &= \min \left[\min(\delta W_{\text{int}}^{\text{III},1}), \min(\delta W_{\text{int}}^{\text{III},2}), \min(\delta W_{\text{int}}^{\text{III},3}) \right] \\ &= \min \left[M_{-u,b1}^{\text{III}} \delta \Delta / l_r, \left(M_{-u,b2}^{\text{III}} + M_{-u,c1}^{\text{III}} + M_{+u,c2}^{\text{III}} \right) \delta \Delta / l_r \right] \end{aligned} \quad (3.35)$$

Therefore, there exist two possible failure modes for Type-III critical zone, which are exhibited in Figures 3.7(e) ~ 3.7(f).

(a). Three sub-spaces

(b). Type-III critical zone in Sub-space 1

(c). Type-III critical zone in Sub-space 2

(d). Type-III critical zone in Sub-space 3

(e). The first possible failure mode and the related ESPH
($\alpha = 0$)

(f). The second possible failure mode and the related ESPHs
($\alpha = \delta\Delta / l_r$)

Figure 3.7 Possible Failure Modes of Type-III Critical Zone

3.5.5 Verification

In order to verify above optimum model, a simple example is discussed here where a Type-I critical zone is loaded by a constant service loadings q and an increasing shear force V at the right end. The strength of related members is demonstrated in Figure 3.8(a) where $M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I > M_{-u,b2}^I$. With the continuous increasing of V , a failure mechanism will finally form in this zone. According to Section 3.5.2, this failure mechanism can be characterized by the formation of a plastic hinge at the left end of the member Beam-2 as plotted in Figure 3.5(f). Therefore, the true limit load of V , based on the upper bound theorem, should be less than or at most equal to $(M_{-u,b2}^I - 0.5 \times q \times l_{beam-2}^2) / l_{beam-2}$.

The moment diagram of the critical zone based on above failure mechanism and the inequality $M_{-u,b1}^I + M_{+u,c1}^I + M_{-u,c2}^I > M_{-u,b2}^I$ is plotted in Figure 3.8(b). It can be seen that there is no location in this diagram where the bending moment exceeds its corresponding strength. According to the lower bound theorem [C6, R4], $(M_{-u,b2}^I - 0.5 \times q \times l_{beam-2}^2) / l_{beam-2}$ should be the lower bound of V as well. Thus, $(M_{-u,b2}^I - 0.5 \times q \times l_{beam-2}^2) / l_{beam-2}$ is the true limit of V . The above discussion proves that Section 3.5.2 generates the correct failure mechanism.

Figure 3.8 Example

3.6 The Performance Function

From above discussion, the minimum internal virtual work of the damaged frame structure can be obtained by substituting Equations (3.24), (3.30), and (3.35) into Equation (3.9), and further the performance function against progressive collapse based on the virtual work principle and the critical collapse mechanism criterion can be rewritten as:

$$\begin{aligned}
 z &= \min(\delta W_{\text{int}}) - \delta W_{\text{ext}} \\
 &= \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^I) + \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^{II}) + \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^{III}) - \sum_{i=L1}^{L2} \delta W_{\text{ext},i} \\
 &= \sum_{i=L1}^{L2} \min \left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} + M_{-u,c2}^{I,i} \right) \delta \Delta / l_l, M_{-u,b2}^{I,i} \delta \Delta / l_l \right] \\
 &\quad + \sum_{i=L1}^{L2} \min \left[\left(M_{+u,b2}^{II,i} + M_{+u,c1}^{II,i} + M_{-u,c2}^{II,i} \right) \delta \Delta / l_l + M_{+u,b2}^{II,i} \delta \Delta / l_r, \right. \\
 &\quad \left. M_{+u,b1}^{II,i} \delta \Delta / l_l + M_{+u,b2}^{II,i} \delta \Delta / l_r, \right. \\
 &\quad \left. M_{+u,b1}^{II,i} \delta \Delta / l_l + \left(M_{+u,b1}^{II,i} + M_{-u,c1}^{II,i} + M_{+u,c2}^{II,i} \right) \delta \Delta / l_r \right] \\
 &\quad + \sum_{i=L1}^{L2} \min \left[M_{-u,b1}^{III,i} \delta \Delta / l_r, \left(M_{-u,b2}^{III,i} + M_{-u,c1}^{III,i} + M_{+u,c2}^{III,i} \right) \delta \Delta / l_r \right] \\
 &\quad - \sum_{i=L1}^{L2} \frac{1}{2} q_{l,i} \times \delta \Delta \times l_l - \sum_{i=L1}^{L2} \frac{1}{2} q_{r,i} \times \delta \Delta \times l_r
 \end{aligned} \tag{3.36}$$

In Equation (3.36), i denotes the i^{th} storey varying from $L1$ (the storey of the failed column) to $L2$ (the roof storey). Obviously, if the strengths of all involved end springs are available, the calculation of the performance function will be straightforward by substituting these values into Equation (3.36). However, the problem lies in the fact that the strength of the end springs, in a reasonable structural model, shall be determined closely related to the axial force on corresponding members. While the axial force on different members will experience different degrees of variation after the initial failure of the column, it becomes difficult for the direct prediction of the strength of the end springs and further the solution of the Equation (3.36) based on the damaged frame structural

model in Figure 3.2(a). To deal with this problem, a new structural system, namely, a construction system, is created as shown in Figure 3.9.

Figure 3.9 A Construction System

The only difference of the construction system in Figure 3.9 from the real system in Figure 3.2(a) lies in that an additional virtual concentrated force vector $P_v = \{p_{v,i}\}$ is applied to the damaged frame at the nodes above the failed column, making the structure staying in its ultimate state. Based on the critical collapse mechanism criterion as discussed in Section 3.4.1, it is known that the weakest collapse mechanism must appear in the construction system. Given a $\delta\Delta$, the total external virtual work δW_{ext}^t , which now includes two parts: the external virtual work δW_{ext} by service loadings and the additional external virtual work δW_{ext}^{pv} by P_v , should be equal to the minimum internal virtual work $\min(\delta W_{int})$ of the construction system:

$$\delta W_{ext}^t = \delta W_{ext} + \delta W_{ext}^{pv} \quad (3.37a)$$

$$\begin{aligned} z_C &= \min(\delta W_{\text{int}}) - \delta W_{\text{ext}}^t \\ &= \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^I) + \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^{II}) + \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^{III}) - \delta W_{\text{ext}}^t = 0 \end{aligned} \quad (3.37b)$$

In Equation (3.37b), z_C denotes the performance function of the construction system, which should be equal to zero as this system is in its ultimate state. Substituting Equation (3.37a) into (3.37b) obtains:

$$\begin{aligned} z &= \min(\delta W_{\text{int}}) - \delta W_{\text{ext}} \\ &= \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^I) + \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^{II}) + \sum_{i=L1}^{L2} \min(\delta W_{\text{int},i}^{III}) - \sum_{i=L1}^{L2} \delta W_{\text{ext}} \\ &= \delta W_{\text{ext}}^{pv} = \sum_{i=L1}^{L2} P_{v,i} * \delta \Delta \end{aligned} \quad (3.38)$$

It needs to be pointed out that although Equation (3.38) is established based on the construction system in Figure 3.9, the solution of this equation can still reflect the safety status of the real damaged frame structure plotted in Figure 3.2(a). For example, if the structure is designed robustly enough to sustain the initial column failure without causing progressive collapse under static service loadings, then a downward P_v is needed for the damaged frame to reach its ultimate state and thus the value of z in Equation (3.38) will be positive. Otherwise, P_v has to be an upward supporting force vector with z being negative. Therefore, Equation (3.38) solved based on the construction system can still be treated as the performance function for the real structure system and will be adopted in structural reliability assessment.

3.7 Summary

This chapter concentrates on the construction of the performance function for the reliability assessment of damaged ductile RC frame structures against progressive collapse under static loading conditions. The lumped plasticity model is selected to simulate the structure, which proves good performance by previous studies. Based

on the virtual work principle and the critical collapse mechanism criterion, a performance function (3.7) of the damaged frame structure is established in terms of the minimum internal virtual work minus the external virtual work done by service loadings. The equation for the external virtual work is developed directly by calculating the sum of the definite integrals of the functions $q_i(x) \times \delta\Delta(x)$ along the damaged span. In order to compute the minimum internal virtual work by the structural weakest collapse mechanism, three types of critical zones are identified, while the relationship of the internal virtual work by each type versus the virtual displacement $\delta\Delta$ and the node rotation α is deduced. An optimum model for evaluating the minimum internal virtual work by critical zones is provided and the solution of this optimum model discloses the possible failure modes for each critical zone. By substituting the formulae for the external virtual work and the minimum internal virtual work of all critical zones, the performance function of Equation (3.7) is rewritten as Equation (3.36). However, intensive inspection shows that direct solution of the rewritten performance function is difficult. To solve this problem, a construction system is established by applying a virtual concentrated force vector $P_v = \{p_{v,i}\}$ on the damaged frame. It is discussed that based on this construction system, the solution of the performance function of Equation (3.38) in terms of the minimum internal virtual work minus the external virtual work by service loadings is still applicable for the reliability assessment of the real damaged structure. An iterative procedure to calculate this function will be discussed in Chapter 4.

CHAPTER FOUR

RELIABILITY ASSESSMENT OF DAMAGED DUCTILE RC FRAME STRUCTURAL SYSTEM AGAINST PROGRESSIVE COLLAPSE UNDER STATIC LOADING CONDITIONS

— PART II. RELIABILITY ASSESSMENT

4.1 Introduction

A performance function for the reliability assessment of the damaged frame structure against progressive collapse under static loading conditions has been presented in Equation (3.36) according to the virtual work principle and the critical collapse mechanism criterion. However, the solution of the performance function of Equation (3.36) is not straightforward due to the dependency of the strength of the end springs on the axial force in corresponding structural members. To deal with this, a construction system is created, where a virtual concentrated force vector $P_v = \{p_{v,i}\}$ is applied to the damaged frame at the nodes above the failed column making the structure staying in its ultimate state. The discussion in Section 3.6 shows that based on this construction system, the function in terms of the minimum internal virtual work minus the external virtual work done by service loadings, as given in Equation (3.38), can still be adopted as the performance function for the reliability assessment of the real damaged structure.

An iterative procedure is developed in this chapter to solve the performance function of Equation (3.38). The minimum internal virtual work by critical zones is computed in such a manner that the interaction between the axial force and the strength of the end springs on directly connected columns is taken into account explicitly. In order to consider the positions of the real plastic hinges (RPHs) that

are not located at the end sections of structural beams, equations according to the moment diagrams are established to calculate the reduced strength for corresponding end springs. A numerical example is adopted to illustrate the effectiveness of utilizing the calculated reduced strength to obtain the solution of the performance function. In addition, the axial force on the indirectly connected columns and beams as well as the strength of the related end springs are evaluated iteratively with the calculation of structural weakest collapse mechanism. Based on a combination of the developed procedure and the Monte Carlo Simulation method, a flowchart is presented to assess the reliability of the damaged ductile frame structure against progressive collapse.

Finally, implementation of this flowchart into a four-storey RC frame structure subjected to two different cases of column failure is demonstrated, where the uncertainties in material properties, applied loadings and structural geometries are considered. The results are presented in this chapter including the occurrence probability of structural collapse event, the minimum internal virtual work, the external virtual work and the structural performance function. In addition, the failure mode of each critical zone, which occurs in structural weakest collapse mechanism, is also recorded for all structural samples used in the Monte Carlo Simulation. The numerical results show that the loss of an external column will present a more critical case for this particular frame than the loss of an internal column, and that the ESPHs in generating the structural weakest collapse mechanism are predominantly located on beams in damaged spans.

4.2 The Iterative Procedure

Before developing the procedure for solving the performance function of Equation (3.38), structural members of the damaged frame are divided into three groups: directly connected columns, indirectly connected columns and structural beams. The directly connected columns are defined to be the columns that are directly connected to the critical zones, while the indirectly connected columns represent all the columns other than the directly connected columns.

It had been demonstrated by Haluk *et al* [H1] that significant change of member's axial force would occur primarily in the directly connected columns during the progressive collapse of the damaged frame. In contrast, the variation of the axial force in other members was much slighter and thus could be regarded to be less important. Accordingly, the solution of the performance function of Equation (3.38) will be developed in an iterative manner as given below:

- 1) Calculate the initial axial forces on indirectly connected columns and structural beams (F_{bc}) through structural analysis of the intact frame under service loadings, and determine the flexural strengths of these members ($M_{\pm ur, bc}$) based on the corresponding $P - M$ curves [N2, M12];
- 2) Fix F_{bc} and $M_{\pm ur, bc}$, and compute the minimum internal virtual work $\min(\delta W_{int, i}^I)$, $\min(\delta W_{int, i}^{II})$, $\min(\delta W_{int, i}^{III})$, $\min(\delta W_{int})$ as well as the structural weakest collapse mechanism;
- 3) Update F_{bc} according to the obtained structural weakest collapse mechanism and computed $M_{\pm ur, bc}$ according to the renewed F_{bc} ;
- 4) Repeat Steps 2 and 3 until the convergences for F_{bc} and $M_{\pm ur, bc}$ are achieved.

Note that since the change of F_{bc} from its initial value is slight during the progressive collapse of the damaged structure, convergence will be easily attained after several cycles of the above iterative calculation.

4.3 Minimum Internal Virtual Work

In order to calculate the minimum internal virtual work (Step 2 in above iterative procedure), the i^{th} storey of the construction system is divided into three parts (Part-

a, Part-b and Part-c) as shown in Figure 4.1(a), while the working state of Part-a is plotted in Figure 4.1(b).

(a). The isolated three parts

(b). Working state of Part-a

Figure 4.1 Isolating Three Parts of the Construction System in the i^{th} Storey

(c). Further separating Part-a in $M_{+u,b2}^{II,i} > M_{+u,b1}^{II,i}$

(d). Further separating Part-a in $M_{+u,b2}^{II,i} < M_{+u,b1}^{II,i}$

Figure 4.1 (Continued)

In Figure 4.1(b), $F_{c1}^{I,i}$, $F_{c1}^{II,i}$, $F_{c1}^{III,i}$, $F_{c2}^{I,i}$, $F_{c2}^{II,i}$, $F_{c2}^{III,i}$ denote the axial forces on directly connected columns; $M_{-u,b1}^{I,i}$, $M_{-u,b2}^{I,i}$, $M_{+u,c1}^{I,i}$, $M_{-u,c2}^{I,i}$, $M_{+u,b1}^{II,i}$, $M_{+u,b2}^{II,i}$, $M_{\pm u,c1}^{II,i}$, $M_{\pm u,c2}^{II,i}$, $M_{-u,b1}^{III,i}$, $M_{-u,b2}^{III,i}$, $M_{+u,c2}^{III,i}$, $M_{-u,c1}^{III,i}$ represent the strength of different end springs. The symbols F_l^i and F_r^i indicate the vertical forces transferred from Part-b and Part-c respectively, while $q_{l,i}$, $q_{r,i}$ and $p_{v,i}$ are the two service loadings and the virtual concentrated force on the i^{th} storey.

Among all the above parameters, the values of $q_{l,i}$, $q_{r,i}$ are given for each structural sample; F_l^i , F_r^i can be determined from the vertical balance of Part-b and Part-c since the axial forces on indirectly connected columns are fixed before the calculation of the minimum internal virtual work; $p_{v,i}$ has to be calculated with the method addressed in Section 4.5; $M_{-u,b1}^{I,i}$, $M_{-u,b2}^{I,i}$, $M_{+u,b1}^{II,i}$, $M_{+u,b2}^{II,i}$, $M_{-u,b1}^{III,i}$ and $M_{-u,b2}^{III,i}$ are initially taken as the corresponding structural beams' flexural strengths which are also fixed in each cycle of the calculation as addressed in Section 4.2. Note that if necessary, the reduced strength for the end springs on beams will be evaluated according to Section 4.4 and assigned to the related parameters of $M_{+u,b1}^{II,i}$ and/or $M_{+u,b2}^{II,i}$. In addition, $M_{+u,c1}^{I,i}$, $M_{\pm u,c1}^{II,i}$, $M_{-u,c1}^{III,i}$, $M_{-u,c2}^{I,i}$, $M_{\pm u,c2}^{II,i}$ and $M_{+u,c2}^{III,i}$ are functions of $F_{c1}^{I,i}$, $F_{c1}^{II,i}$, $F_{c1}^{III,i}$, $F_{c2}^{I,i}$, $F_{c2}^{II,i}$ and $F_{c2}^{III,i}$ respectively, where $F_{c2}^{I,i}$, $F_{c2}^{II,i}$ and $F_{c2}^{III,i}$ can be obtained from the solution of the upper storeys. Therefore, it can be seen that for the computation of the minimum internal virtual work $\min(\delta W_{int,i}^I)$, $\min(\delta W_{int,i}^{II})$ and $\min(\delta W_{int,i}^{III})$ by the three critical zones as shown in Figure 4.1(b), only $F_{c1}^{I,i}$, $F_{c1}^{II,i}$ and $F_{c1}^{III,i}$ remain here to be solved. For this purpose, three equations are established as follows.

4.3.1 The first and Second Equations

The first equation is built up based on the virtual work principle. Since the construction system shown in Figure 4.1(a) is in its ultimate state, given a $\delta\Delta$, the

minimum internal virtual work within the isolated Part-a $\delta W_{\text{int},a}$ will be the sum of those produced by the involved three critical zones, expressed as:

$$\begin{aligned}
 \min(\delta W_{\text{int},a}) &= \min(\delta W_{\text{int},i}^I) + \min(\delta W_{\text{int},i}^{II}) + \min(\delta W_{\text{int},i}^{III}) \\
 &= \min\left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i}(F_{c1}^{I,i}) + M_{-u,c2}^{I,i}\right)\delta\Delta/l_l, M_{-u,b2}^{I,i}\delta\Delta/l_l\right] \\
 &\quad + \min\left[\left(M_{+u,b2}^{II,i} + M_{+u,c1}^{II,i}(F_{c1}^{II,i}) + M_{-u,c2}^{II,i}\right)\delta\Delta/l_l + M_{+u,b2}^{II,i}\delta\Delta/l_r, \right. \\
 &\quad \left. M_{+u,b1}^{II,i}\delta\Delta/l_l + M_{+u,b2}^{II,i}\delta\Delta/l_r, \right. \\
 &\quad \left. M_{+u,b1}^{II,i}\delta\Delta/l_l + \left(M_{+u,b1}^{II,i} + M_{-u,c1}^{II,i}(F_{c1}^{II,i}) + M_{+u,c2}^{II,i}\right)\delta\Delta/l_r\right] \\
 &\quad + \min\left[M_{-u,b1}^{III,i}\delta\Delta/l_r, \left(M_{-u,b2}^{III,i} + M_{-u,c1}^{III,i}(F_{c1}^{III,i}) + M_{+u,c2}^{III,i}\right)\delta\Delta/l_r\right]
 \end{aligned} \tag{4.1}$$

The external virtual work $\delta W_{\text{ext},a}$ in this part is calculated by:

$$\delta W_{\text{ext},a} = \frac{1}{2}q_{l,i} \times \delta\Delta \times l_l + \frac{1}{2}q_{r,i} \times \delta\Delta \times l_r + F_{c2}^{II,i} \times \delta\Delta + p_{v,i} \times \delta\Delta - F_{c1}^{II,i} \times \delta\Delta \tag{4.2}$$

Equating Equation (4.1) with Equation (4.2) produces the first equation:

$$\begin{aligned}
 &\min\left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i}(F_{c1}^{I,i}) + M_{-u,c2}^{I,i}\right)/l_l, M_{-u,b2}^{I,i}/l_l\right] \\
 &+ \min\left[\left(M_{+u,b2}^{II,i} + M_{+u,c1}^{II,i}(F_{c1}^{II,i}) + M_{-u,c2}^{II,i}\right)/l_l + M_{+u,b2}^{II,i}/l_r, \right. \\
 &\quad \left. M_{+u,b1}^{II,i}/l_l + M_{+u,b2}^{II,i}/l_r, \right. \\
 &\quad \left. M_{+u,b1}^{II,i}/l_l + \left(M_{+u,b1}^{II,i} + M_{-u,c1}^{II,i}(F_{c1}^{II,i}) + M_{+u,c2}^{II,i}\right)/l_r\right] \\
 &+ \min\left[M_{-u,b1}^{III,i}/l_r, \left(M_{-u,b2}^{III,i} + M_{-u,c1}^{III,i}(F_{c1}^{III,i}) + M_{+u,c2}^{III,i}\right)/l_r\right] \\
 &= \frac{1}{2}q_{l,i} \times l_l + \frac{1}{2}q_{r,i} \times l_r + F_{c2}^{II,i} + p_{v,i} - F_{c1}^{II,i}
 \end{aligned} \tag{4.3}$$

where $M_{+u,c1}^{I,i}(F_{c1}^{I,i})$, $M_{\pm u,c1}^{II,i}(F_{c1}^{II,i})$ and $M_{-u,c1}^{III,i}(F_{c1}^{III,i})$ represent the functions of $M_{+u,c1}^{I,i}$, $M_{\pm u,c1}^{II,i}$, $M_{-u,c1}^{III,i}$ versus $F_{c1}^{I,i}$, $F_{c1}^{II,i}$, $F_{c1}^{III,i}$, respectively.

The second equation can be built up from the vertical balance of Part-a, and is written as:

$$F_{c1}^{I,i} + F_{c1}^{II,i} + F_{c1}^{III,i} = F_{c2}^{I,i} + F_{c2}^{II,i} + F_{c2}^{III,i} + F_l^i + F_r^i + q_{l,i} \times l_l + q_{r,i} \times l_r + p_{v,i} \quad (4.4)$$

4.3.2 Behaviour of the Type-II Critical Zone

In order to construct the third equation, the behaviour of the Type-II critical zone will be inspected. The analysis carried out in Section 3.5.3 shows that for this type of critical zone in structural progressive collapse, there are three possible failure modes exhibited in Figures 3.6(f) ~ 3.6(h), and its minimum internal virtual work can be calculated with Equation (3.30). According to the relationship between $M_{+u,b1}^{II}$ and $M_{+u,b2}^{II}$ in Equation (3.30), three cases i.e. $M_{+u,b1}^{II} < M_{+u,b2}^{II}$, $M_{+u,b1}^{II} > M_{+u,b2}^{II}$ and $M_{+u,b1}^{II} = M_{+u,b2}^{II}$ are investigated as follows.

For the first case where $M_{+u,b1}^{II} < M_{+u,b2}^{II}$, it can be deduced that:

$$(M_{+u,b2}^{II} + M_{+u,c1}^{II} + M_{-u,c2}^{II})\delta\Delta/l_l + M_{+u,b2}^{II}\delta\Delta/l_r > M_{+u,b1}^{II}\delta\Delta/l_l + M_{+u,b2}^{II}\delta\Delta/l_r \quad (4.5)$$

As a result, Equation (3.30) is simplified by removing the first term from the $\min[\bullet]$ function on the right side as:

$$\min(\delta W_{int}^{II}) = \min \left[M_{+u,b1}^{II}\delta\Delta/l_l + M_{+u,b2}^{II}\delta\Delta/l_r, \right. \\ \left. M_{+u,b1}^{II}\delta\Delta/l_l + (M_{+u,b1}^{II} + M_{-u,c1}^{II} + M_{+u,c2}^{II})\delta\Delta/l_r \right] \quad (4.6)$$

Correspondingly, the two possible failure modes demonstrated by Figures 3.6(g) ~ 3.6(h) remain. It can be seen that in any of these two possible failure modes, an ESPH with its strength $M_{+u,b1}^{II}$ occurs at the end section of Beam-1 (the beam on the left side of the critical zone as shown in Figure 3.2(d)).

For the second case where $M_{+u,b1}^{II} > M_{+u,b2}^{II}$, the following equation can be obtained:

$$M_{+u,b1}^{II} \delta\Delta / l_l + M_{+u,b2}^{II} \delta\Delta / l_r < M_{+u,b1}^{II} \delta\Delta / l_l + (M_{+u,b1}^{II} + M_{-u,c1}^{II} + M_{+u,c2}^{II}) \delta\Delta / l_r \quad (4.7)$$

Consequently, the third term in the $\min[\bullet]$ function on the right of Equation (3.30) is deleted, and the minimum internal virtual work by Type-II critical zones is calculated by:

$$\min(\delta W_{\text{int}}^{II}) = \min \left[(M_{+u,b2}^{II} + M_{+u,c1}^{II} + M_{-u,c2}^{II}) \delta\Delta / l_l + M_{+u,b2}^{II} \delta\Delta / l_r, \right. \\ \left. M_{+u,b1}^{II} \delta\Delta / l_l + M_{+u,b2}^{II} \delta\Delta / l_r \right] \quad (4.8)$$

In this situation, the two possible failure modes shown by Figures 3.6(f) ~ 3.6(g) remain, while in both of these two possible modes an ESPH corresponding to $M_{+u,b2}^{II}$ can be observed at the end of Beam-2 (the beam on right side of the critical zone).

For the third case where $M_{+u,b1}^{II} = M_{+u,b2}^{II}$, the minimum term in Equation (3.30) is

$M_{+u,b1}^{II} \delta\Delta / l_l + M_{+u,b2}^{II} \delta\Delta / l_r$, therefore

$$\min(\delta W_{\text{int}}^{II}) = M_{+u,b1}^{II} \delta\Delta / l_l + M_{+u,b2}^{II} \delta\Delta / l_r \quad (4.9)$$

The only possible failure mode, when $M_{+u,b1}^{II} = M_{+u,b2}^{II}$, is plotted in Figure 3.6(g) where two ESPHs appear at the end of both Beam-1 and Beam-2, respectively.

Combining the above investigations, it can be concluded that for a Type-II critical zone in the construction system, an ESPH corresponding to the smaller strength between $M_{+u,b1}^{II}$ and $M_{+u,b2}^{II}$ will appear at the end of relative beam, or two ESPHs at the end of both Beam-1 and Beam-2 when $M_{+u,b1}^{II} = M_{+u,b2}^{II}$.

4.3.3 The Third Equation

Based on the above conclusion drawn in Section 4.3.2, the third equation that is needed to calculate $F_{c1}^{I,i}$, $F_{c1}^{II,i}$, $F_{c1}^{III,i}$ in Part-a will be created with respect to different cases including $M_{+u,b1}^{II,i} < M_{+u,b2}^{II,i}$, $M_{+u,b1}^{II,i} > M_{+u,b2}^{II,i}$ and $M_{+u,b1}^{II,i} = M_{+u,b2}^{II,i}$.

In Figure 4.1(b), if $M_{+u,b1}^{II,i} < M_{+u,b2}^{II,i}$, an ESPH will appear at the end of Beam-1 in the Type-II critical zone in Part-a. As a result, Part-a can be further separated into two parts (Part-a11 and Part-a12) as shown in Figure 4.1(c), where $V_{b1}^{II,i}$ is the shear force on the section. It is obvious that a Type-I critical zone is involved in Part-a11 and the minimum internal virtual work $\min(\delta W_{\text{int},a11})$ within Part-a11 can be calculated by:

$$\min(\delta W_{\text{int},a11}) = \min\left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} (F_{c1}^{I,i}) + M_{-u,c2}^{I,i}\right) \delta \Delta / l_l, M_{-u,b2}^{I,i} \delta \Delta / l_l\right] \quad (4.10)$$

The corresponding external virtual work $\min(\delta W_{\text{ext},a11})$ is:

$$\delta W_{\text{ext},a11} = \frac{1}{2} q_{l,i} \times \delta \Delta \times l_l - V_{b1}^{II,i} \times \delta \Delta - M_{+u,b1}^{II,i} \times \delta \Delta / l_l \quad (4.11)$$

Equating Equations (4.10) with Equation (4.11) produces:

$$\min\left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} (F_{c1}^{I,i}) + M_{-u,c2}^{I,i}\right), M_{-u,b2}^{I,i}\right] + M_{+u,b1}^{II,i} = \frac{1}{2} q_{l,i} \times l_l^2 - V_{b1}^{II,i} \times l_l \quad (4.12)$$

In order to satisfy the vertical balance of Part-a11, the shear force $V_{b1}^{II,i}$ can be determined by:

$$V_{b1}^{II,i} = F_{c2}^{I,i} + F_l^i + q_{l,i} \times l_l - F_{c1}^{I,i} \quad (4.13)$$

Substituting Equation (4.13) into Equation (4.12), the third equation is finally established as:

$$\begin{aligned} & \min \left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} (F_{c1}^{I,i}) + M_{-u,c2}^{I,i} \right), M_{-u,b2}^{I,i} \right] + M_{+u,b1}^{II,i} \\ & = \left(F_{c1}^{I,i} - F_{c2}^{I,i} - F_l^i - \frac{1}{2} q_{l,i} \times l_l \right) \times l_l \end{aligned} \quad (4.14)$$

The same method described above can be utilized to construct the third equation when $M_{+u,b1}^{II,i} > M_{+u,b2}^{II,i}$. Since an ESPH will occur at the end of Beam-2 in the Type-II critical zone, Part-a can be divided into Part-a21 and Part-a22 as shown in Figure 4.1(d). Based on the virtual work principle and the vertical balance of Part-a22, the third equation is written as:

$$\begin{aligned} & \min \left[M_{-u,b1}^{III,i}, \left(M_{-u,b2}^{III,i} + M_{-u,c1}^{III,i} (F_{c1}^{III,i}) + M_{+u,c2}^{III,i} \right) \right] + M_{+u,b2}^{II,i} \\ & = \left(F_{c1}^{III,i} - F_{c2}^{III,i} - F_r^i - \frac{1}{2} q_{r,i} \times l_r \right) \times l_r \end{aligned} \quad (4.15)$$

As for $M_{+u,b1}^{II,i} = M_{+u,b2}^{II,i}$, either of Equation (4.14) and Equation (4.15) can be adopted as the third equation, since two ESPHs will occur at the end of both Beam-1 and Beam-2 in the Type-II critical zone in Part-a to form the structural weakest collapse mechanism.

4.3.4 Solution of the Three Equations

According to Equations (4.3), (4.4) and (4.14) established for $M_{+u,b1}^{II,i} \leq M_{+u,b2}^{II,i}$ or Equations (4.3), (4.4) and (4.15) for $M_{+u,b1}^{II,i} \geq M_{+u,b2}^{II,i}$, the procedure for calculating the axial force of $F_{c1}^{I,i}$, $F_{c1}^{II,i}$ and $F_{c1}^{III,i}$ in Figure 4.1(b) is straightforward and addressed herein.

For $M_{+u,b1}^{II,i} \leq M_{+u,b2}^{II,i}$, inspection of Equations (4.3), (4.4) and (4.14) reveals there is only one variable $F_{c1}^{I,i}$ existing in Equation (4.14). Therefore, the value of $F_{c1}^{I,i}$ can be obtained directly by solving this equation using the method demonstrated in Figure 4.2.

Figure 4.2 Solution of the Axial Force $F_{c1}^{I,i}$

Let $M_i^I(F_{c1}^{I,i}) = \min \left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i}(F_{c1}^{I,i}) + M_{-u,c2}^{I,i} \right), M_{-u,b2}^{I,i} \right]$, the relationships of $M_{+u,c1}^{I,i}(F_{c1}^{I,i})$, $\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i}(F_{c1}^{I,i}) + M_{-u,c2}^{I,i} \right)$, M_i^I and $M_i^I + M_{+u,b1}^{II,i}$ versus $F_{c1}^{I,i}$ can be schematically depicted by the curves in Figure 4.2, where Curve-1 indicates the interaction between the axial force $F_{c1}^{I,i}$ and the strength of the spring $M_{+u,c1}^{I,i}$ that is fixed at the end section of Column-1 in the Type-I critical zone. The $P-M$ [N2, M12] curve of this column shall be used for Curve-1. For simplicity Curve-I is approximately represented by a multi-linear shape i.e. Line-I in Figure 4.2. Translating Line-I upward by $M_{-u,b1}^{I,i} + M_{-u,c2}^{I,i}$ yields Line-II, which reflects the relationship between $M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i}(F_{c1}^{I,i}) + M_{-u,c2}^{I,i}$ and $F_{c1}^{I,i}$. In order to achieve the function $M_i^I(F_{c1}^{I,i})$ defined above, Line-III is obtained by comparing Line-II with a

horizontal dash line $M = M_{-u,b2}^{I,i}$ and taking the smaller value of them. The translation of Line-III by $M_{+u,b1}^{II,i}$ leads to Line-IV of $M_i^I(F_{c1}^{I,i}) + M_{+u,b1}^{II,i}$. Finally, the solution of $F_{c1}^{I,i}$ from Equation 4.14 is acquired by finding the intersection point between Line-IV and the line expressed as $M = \left(F_{c1}^{I,i} - F_{c2}^{I,i} - F_l^i - \frac{1}{2} q_{l,i} \times l_l \right) \times l_l$.

With $F_{c1}^{I,i}$ obtained, the values of $F_{c1}^{II,i}$ and $F_{c1}^{III,i}$ can be calculated through Equations (4.3) and (4.4). Note that Inequality (4.5) exists in the case of $M_{+u,b1}^{II,i} \leq M_{+u,b2}^{II,i}$, the term $(M_{+u,b2}^{II,i} + M_{+u,c1}^{II,i} + M_{-u,c2}^{II,i}) \delta \Delta / l_l + M_{+u,b2}^{II,i} \delta \Delta / l_r$ can be removed from the second $\min[\bullet]$ function on the left side of Equation (4.3). Consequently, substituting Equations (4.4) and (4.14) into (4.3) yields:

$$\begin{aligned} & \min \left[M_{+u,b2}^{II,i}, \left(M_{+u,b1}^{II,i} + M_{-u,c1}^{II,i} (F_{c1}^{II,i}) + M_{+u,c2}^{II,i} \right) \right] \\ & + \min \left[M_{-u,b1}^{III,i}, \left(M_{-u,b2}^{III,i} + M_{-u,c1}^{III,i} (A - F_{c1}^{II,i}) + M_{+u,c2}^{III,i} \right) \right] \\ & = \left(q_{l,i} \times l_l + \frac{1}{2} q_{r,i} \times l_r + F_{c2}^{I,i} + F_{c2}^{II,i} + F_l^i + p_{v,i} - F_{c1}^{I,i} - F_{c1}^{II,i} \right) \times l_r \end{aligned} \quad (4.16a)$$

where

$$A = F_{c2}^{I,i} + F_{c2}^{II,i} + F_{c2}^{III,i} + F_l^i + F_r^i + q_{l,i} \times l_l + q_{r,i} \times l_r + p_{v,i} - F_{c1}^{I,i} \quad (4.16b)$$

Only one variable $F_{c1}^{II,i}$ remains in Equation (4.16a), therefore its value can be determined by solving the equation using the same method as described in Figure 4.2. Eventually, $F_{c1}^{III,i}$ is calculated according to Equation (4.4).

As for the case where $M_{+u,b1}^{II,i} \geq M_{+u,b2}^{II,i}$, a similar procedure can be used for calculating the axial forces $F_{c1}^{I,i}$, $F_{c1}^{II,i}$ and $F_{c1}^{III,i}$ with Equations (4.3), (4.4) and (4.15). Firstly, $F_{c1}^{III,i}$ is directly solved through Equation (4.15). Then by substituting Equations (4.4) and (4.15) into Equation (4.3) and deleting the term

$M_{+u,b1}^{II,i} \delta \Delta / l_l + (M_{+u,b1}^{II,i} + M_{-u,c1}^{II,i} + M_{+u,c2}^{II,i}) \delta \Delta / l_r$ from the second $\min[\bullet]$ function on the left side of Equation (4.3), $F_{c1}^{II,i}$ is computed from the following expression:

$$\begin{aligned} & \min \left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} (A - F_{c1}^{II,i}) + M_{-u,c2}^{I,i} \right), M_{-u,b2}^{I,i} \right] \\ & + \min \left[\left(M_{+u,b2}^{II,i} + M_{+u,c1}^{II,i} (F_{c1}^{II,i}) + M_{-u,c2}^{II,i} \right), M_{+u,b1}^{II,i} \right] \\ & = \left(\frac{1}{2} q_{l,i} \times l_l + q_r^i \times l_r + F_{c2}^{II,i} + F_{c2}^{III,i} + F_r^i + p_{v,i} - F_{c1}^{II,i} - F_{c1}^{III,i} \right) \times l_l \end{aligned} \quad (4.17a)$$

where

$$A = F_{c2}^{I,i} + F_{c2}^{II,i} + F_{c2}^{III,i} + F_l^i + F_r^i + q_{l,i} \times l_l + q_{r,i} \times l_r + p_{v,i} - F_{c1}^{III,i} \quad (4.17b)$$

Equation (4.4) is still applicable for the calculation of $F_{c1}^{I,i}$.

At the present stage, the axial forces $F_{c1}^{I,i}$, $F_{c1}^{II,i}$ and $F_{c1}^{III,i}$ in Column-1s of the three critical zones in Part-a are all established. As a result, the strengths of related end springs $M_{+u,c1}^{I,i}$, $M_{\pm u,c1}^{II,i}$ and $M_{-u,c1}^{III,i}$ in Figure 4.1(b) can be determined through the $P-M$ curves of the corresponding columns. Subsequently, the minimum internal virtual works produced by the critical zones in this storey including $\min(\delta W_{int,i}^I)$, $\min(\delta W_{int,i}^{II})$ and $\min(\delta W_{int,i}^{III})$ can be derived with Equations (3.24), (3.30) and (3.35), respectively.

The axial forces in Column-2s of the three critical zones in the $(i-1)^{th}$ storey i.e. $F_{c2}^{I,i-1}$, $F_{c2}^{II,i-1}$ and $F_{c2}^{III,i-1}$ should be equal to $F_{c1}^{I,i}$, $F_{c1}^{II,i}$ and $F_{c1}^{III,i}$ respectively. In addition, the strength of the corresponding end springs on these columns ($M_{-u,c2}^{I,i-1}$, $M_{\pm u,c2}^{II,i-1}$ and $M_{+u,c2}^{III,i-1}$) can be evaluated from these columns' $P-M$ curves. Therefore, the process for calculating the minimum internal virtual works by each critical zone in the $(i-1)^{th}$ storey will continue. As this process goes all the way from the roof storey ($L2$) to the storey of the initially failed column ($L1$), the

minimum internal virtual work by every critical zone and that by the weakest collapse mechanism of the construction system are acquired.

4.4 Positions of Real Plastic Hinges

The structural lumped plasticity model adopted in this study for the calculation of the external virtual work and the minimum internal virtual work is built up in such a manner that the springs representing the possible plastic hinges are all fixed at the end sections of structural members. In addition, the strength of the end springs is initially taken as the flexural strength of the related members. It should be pointed out that although the computational work needed in structural reliability assessment is significantly reduced with this model, errors could be induced in identifying the positive RPHs on beams in Type-II critical zones since the maximum positive moments along the beams in the damaged span do not necessarily appear at the end sections as shown in Figure 4.3. These errors will finally cause the performance function and the reliability of the damaged frame to be overestimated. Thus, in order to achieve a more reasonable assessment for the damaged structure against progressive collapse, the positions of the positive RPHs on beams in Type-II critical zones have to be considered. One practical approach for this purpose is to employ the reduced strength for the corresponding end springs. The equations to compute the reduced strength are introduced in this section.

Figure 4.3 Moment Distribution along the Damaged Span of the Construction System

4.4.1 Positions of Real Plastic Hinges on Beams in Left Damaged Span

The moment distribution along the beams in the damaged span of the construction system is schematically plotted in Figure 4.3. In order to demonstrate the approach for considering the positions of the positive RPHs on beams in Type-II critical zones, the beam on the i^{th} storey in the left damaged span as circled in Figure 4.3 is selected, and a positive RPH is assumed to have formed on this member. Since the construction system in Figure 4.1 is in its ultimate state, the working state and moment diagram of the selected beam can be schematically plotted in Figures 4.4(a) ~ 4.4(b). If the maximum positive moment, which reaches the beam's positive flexural strength $M_{+ur,bl}^{II,i}$, occurs at some location denoted by $x_{l,i}$ within the span, then the positive RPH will appear there and apparently the moment at the right end section can only attain a value less than $M_{+ur,bl}^{II,i}$ as shown in Figure 4.4(b). In order to calculate the value of $x_{l,i}$ in Figure 4.4(b), the beam is divided into two parts at $x_{l,i}$ with the left part depicted in Figure 4.4(c).

(a). Working state

(b). Moment distribution

(c). Left part of the beam

Figure 4.4 Position of the Positive RPH on the Beam in the Left Damaged Span

Two equations are established based on the virtual work principle and the vertical balance of the left part of the beam in Figure 4.4(c), expressed respectively as:

$$\begin{aligned} & \min \left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} (F_{c1}^{I,i}) + M_{-u,c2}^{I,i} \right) / x_{l,i}, M_{-u,b2}^{I,i} / x_{l,i} \right] + M_{+ur,b1}^{II,i} / x_{l,i} \\ & = \frac{1}{2} \times q_{l,i} \times x_{l,i} \end{aligned} \quad (4.18a)$$

and

$$F_{c1}^{I,i} = F_{c2}^{I,i} + F_l^i + q_{l,i} \times x_{l,i} \quad (4.18b)$$

Substituting Equation (4.18b) into Equation (4.18a) produces:

$$\begin{aligned} & \min \left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} (F_{c2}^{I,i} + F_l^i + q_{l,i} \times x_{l,i}) + M_{-u,c2}^{I,i} \right) / x_{l,i}, M_{-u,b2}^{I,i} / x_{l,i} \right] + M_{+ur,b1}^{II,i} \\ & = \frac{1}{2 \times q_{l,i}} (q_{l,i} \times x_{l,i})^2 \end{aligned} \quad (4.19)$$

By solving Equation (4.19) with the method described in Section 4.3.4, $x_{l,i}$ is acquired. Obviously, if $x_{l,i}$ is beyond the span of the beam ($x_{l,i} \geq l_l$), the positive RPH will occur at the right end section, and thus the strength of the corresponding end spring $M_{+u,b1}^{II,i}$ can be simply taken as the flexural strength of this member $M_{+ur,b1}^{II,i}$ with no error induced in the structural reliability assessment. However, if $x_{l,i} \leq l_l$, the positive RPH should be located at the calculated position of $x_{l,i}$. In this case, the moment at the right end section of the beam cannot reach $M_{+ur,b1}^{II,i}$. Therefore, reduced strength must be employed for the corresponding end spring $M_{+u,b1}^{II,i}$ in the structural model. The calculation of the reduced strength can be carried out according to the moment diagram of the member shown in Figure 4.4(b) for two objectives.

- 1) To ensure that the maximum positive moment of the select beam will not exceed its positive flexural strength $M_{+ur,b1}^{II,i}$;
- 2) To guarantee that the accuracy of the performance function calculated based on the construction system in Figure 4.1(a) will not be affected by the fixation of the springs at end sections.

Based on above discussion, the reduced strength for the corresponding end spring $M_{+u,b1}^{II,i}$ on Beam-1 in Type-II critical zone can be obtained by:

$$M_{+u,b1}^{II,i} = M_{+ur,b1}^{II,i} - \frac{1}{2} q_{l,i} \times (l_l - x_{l,i})^2 \quad (4.20)$$

4.4.2 Positions of Real Plastic Hinges on Beams in Right Damaged Span

As for the beam within the right damaged span, the position of its positive RPH can be taken in to account by same approach as that addressed in Section 4.4.1 but with a different set of parameters shown in Figures 4.5(a) ~ 4.5(c).

(a). Working state

Figure 4.5 Position of the Positive RPH on the Beam in the Right Damaged Span

(b). Moment distribution

(c). Right part of the beam

Figure 4.5 (Continued)

The equations for calculating $x_{r,i}$ and the reduced strength of the corresponding end spring $M_{+u,b2}^{II,i}$ on Beam-2 in Type-II critical zone are given respectively as:

$$M_{+ur,b2}^{II,i} + \min \left[M_{-u,b1}^{III,i}, \left(M_{-u,b2}^{III,i} + M_{-u,c1}^{III,i} \left(F_{c2}^{III,i} + F_r^i + q_{r,i} \times x_{r,i} \right) + M_{+u,c2}^{III,i} \right) \right] \quad (4.21)$$

$$= \frac{1}{2 \times q_{r,i}} \left(q_{r,i} \times x_{r,i} \right)^2$$

$$M_{+u,b2}^{II,i} = M_{+ur,b2}^{II,i} - \frac{1}{2} q_{r,i} \times (l_r - x_{r,i})^2 \quad (4.22)$$

4.4.3 Numerical Example

In order to demonstrate the effectiveness of utilizing the reduced strength to account for the position of the positive RPH, a simple example is used here of a beam which collapses under the uniformly distributed service loading q of 50 KN/m. The positive strength M_{+ur} and negative strength M_{-ur} of this beam are assumed to be 225 KNm and 400 KNm, respectively, while the boundary condition is plotted in Figure 4.6. A construction system is created by applying a virtual concentrated force P_v at the left end section making the beam staying in its ultimate state. In addition, two springs are introduced on the construction system to simulate the behaviour of the plastic hinges. Based on this construction system and according to the locations and the strengths of the springs, three different models for calculating the external virtual work δW_{ext} done by q , the internal virtual work δW_{int} and the performance function z in terms of δW_{int} minus δW_{ext} will be inspected.

Figure 4.6 A Example Beam

The first model shown in Table 4.1 reflects the real collapse mechanism of the construction beam system. In this model, a spring named Spring-1 is located at the

position of the positive RPH with $x = 5000$ mm, where x is determined with Equation (4.21) by taking $M_{+ur,b2}^{II,i} = 225$ KN.m, $M_{-u,b1}^{III,i} = 400$ KN.m, $q_{r,i} = 50$ KN/m, $l_r = 6000$ mm and the other parameters equal to infinity. The strength of Spring-1 is denoted by M_1 and is taken as the positive strength of the beam M_{+ur} as so to represent the contribution of the positive RPH. In order to simulate the negative RPH, another spring namely Spring-2 with its strength M_2 equal to M_{-ur} is used and fixed at the right end section. Given $\delta\Delta = 1000$ mm, the virtual displacement distribution $\delta\Delta(x)$ of the first model is plotted in Table 4.1. The virtual rotation Q_{exa} of the plastic hinges represented by the yielded springs (ESPHs) under $\delta\Delta$ is $1/5$.

In the second model, Spring-1 is fixed at the left end section of the beam while its strength M_1 is still taken as M_{+ur} . This model is to investigate the error that will be induced by fixing the ESPH at the end section of the beam without considering the position effect of the positive RPH. The only difference between the third and the second model is that a reduced strength of 200 KNm is employed for M_1 in the third model to consider the position of the positive RPH which is not at the end section. The value of the reduced strength is calculated with Equation (4.22) by taking $M_{+ur,b2}^{II,i} = 225$ KN.m, $q_{r,i} = 50$ KN/m, $l_r = 6000$ mm and $x_{r,i} = 5000$ mm. The virtual displacement of the second and third model under $\delta\Delta = 1000$ mm is linearly distributed as plotted in Table 4.1, while the rotation Q_{exa} of the ESPHs is $1/6$.

For the above three models, the external virtual work δW_{ext} done by the service loading q , the internal virtual work δW_{int} , the performance function z as well as the virtual concentrated force P_v are evaluated with $\delta W_{ext} = \int_0^l q \times \delta\Delta(x) dx$, $\delta W_{int} = (M_1 + M_2) \times Q_{exa}$, $z = \delta W_{int} - \delta W_{ext}$ and $P_v = z$, respectively. The results are listed in Table 4.1.

Table 4.1 Effectiveness of the Reduced Strength in Considering the Position of RPHs

	The shape of the collapsed beam under $\delta\Delta = 1$	M_1 (KNm)	M_2 (KNm)	Position (spring-1)	δW_{int} (KNm)	δW_{ext} (KNm)	z (KNm)	P_v (KN)
The first method		M_{+ur} (225)	M_{-ur} (400)	x (5000)	125	175	-50	50
	Moment Diagram						
The second method		M_{+ur} (225)	M_{-ur} (400)	l (6000)	104	150	-45.8	45.8
	Moment Diagram						
The third method		M_{+u} (200)	M_{-ur} (400)	l (6000)	100	150	-50	50
	Moment Diagram						

From Table 4.1, it can be seen that the moment diagram obtained from the first model nowhere exceeds the beam's capacity while the bending moment on the two springs reaches yield to form a collapse mechanism. Based on the upper bound and lower bound theorems, this observation demonstrates that the failure mode of the first model (or the location of the two plastic hinges modelled by related springs) presents the true situation.

Comparing the results obtained from the first and the second model listed in Table 4.1 shows that by fixing the springs at the beam's end section, the performance function z in the second model is overestimated by about 8.4% due to the error induced in identifying the positive RPH with the spring. However, this error is compensated by using the reduced strength for Spring-1 in the third model, where an accurate performance function is attained. In addition, it also can be seen that the moment diagrams obtained from the first and the third model matches very well.

Based on above discussion, it can be concluded that the utilization of the reduced strength for corresponding springs is a proper approach to consider the position of the positive RPHs on beams in structural assessment.

4.4.4 Flowchart for Minimum Internal Virtual Work

Through combining Section 4.3 and Section 4.4, the flowchart for calculating the minimum internal virtual work $\min(\delta W_{\text{int},i}^{I-III})$ and $\min(\delta W_{\text{int}})$ is given in Figure 4.7.

Figure 4.7 Flowchart for the Minimum Internal Virtual Work

4.5 Determination of the Virtual Concentrated Force Vector P_v

As mentioned in Chapter 3, a construction system is established by applying a virtual concentrated force vector $P_v = \{p_{v,i}\}$ ($i = L1 \cdots L2$) on the damaged frame to make the structure stay in its ultimate state as shown in Figure 3.9. Therefore, under $\delta\Delta$, the external virtual work by P_v denoted as δW_{ext}^{pv} should be equal to the value of the performance function which is constructed in terms of the minimum internal virtual work $\min(\delta W_{int})$ minus the external virtual work δW_{ext} done by service loadings, expressed as:

$$\delta W_{ext}^{pv} = \delta\Delta \times p_{v,L1} \sum_{i=L1}^{L2} p_{v,i} / p_{v,L1} = \min(\delta W_{int}) - \delta W_{ext} \quad (4.23)$$

If the ratio of $p_{v,i} / p_{v,L1}$ is predefined, the virtual force on i^{th} storey p_i can be calculated by:

$$p_{v,i} = p_{v,L1} \times p_{v,i} / p_{v,L1} \quad (4.24)$$

where $p_{v,L1}$ can be evaluated with Equation (4.23), written as:

$$p_{v,L1} = \left(\min(\delta W_{int}) - \delta W_{ext} \right) / \left(\delta\Delta \times \sum_{i=L1}^{L2} p_{v,i} / p_{v,L1} \right) \quad (4.25)$$

According to Equation (4.24) and (4.25), it can be seen that the magnitude of the concentrated virtual force $\{p_{v,i}\}$ is determined by the minimum internal virtual work $\min(\delta W_{int})$. On the other hand, Equations (4.3) and (4.4) indicate that the solutions of the axial forces on directly connected columns $F_{c1}^{I,i}$, $F_{c1}^{II,i}$, $F_{c1}^{III,i}$ and further the minimum internal virtual work $\min(\delta W_{int})$ of the construction system

are effected by the magnitude of $\{p_{v,i}\}$. Thus, an iterative procedure is required to calculate $\{p_{v,i}\}$. The flowchart of the iterative procedure is shown in Figure 4.8.

* ϵ_1 : A small positive value

Figure 4.8 Flowchart for the Virtual Concentrated Force Vector

The progressive collapse of the damaged frame with a column initially failed in a close-in detonation is a dynamic process. During this process, the inertia forces induced by the acceleration of the structural mass will influence structural response. By properly defining the ratio $p_{v,i}/p_{v,L1}$, the concentrated virtual force vector P_v applied on the construction system can take the inertia force into account in the computation of the structural weakest collapse mechanism. The specific meaning for P_v and a reasonable model for $p_{v,i}/p_{v,L1}$ will be addressed in details in Chapter 5.

4.6 Axial Forces on Indirectly Connected Columns and Beams

4.6.1 Modification of Structural Model

Sections 4.3, 4.4 and 4.5 describe the approach to calculate the performance function z of the damaged frame structure based on the construction system. Within those sections, the axial forces F_{bc} on the indirectly connected columns and structural beams are fixed. In order to obtain a more accurate solution of the performance function, F_{bc} should be updated according to the acquired weakest collapse mechanism. For this purpose, structural analysis of the construction system with a modified model needs to be carried out. The modification of the structure model is executed by three steps as described below.

- 1) Employ the hinge connections to replace the ESPHs corresponding to the failure mode of the critical zones calculated with flowchart (Figure 4.8);
- 2) Apply moment loadings on both sides of the above hinge connections. The amplitudes of the moment loadings should be equal to the strengths of the related ESPHs. After the first and second steps, the three types of critical zones in the modified structural model are plotted in Table 4.2 according to their different failure modes.

- 3) Introduce an additional vertical support at the upper node of the failed column as shown in Figure 4.9. This step is to avoid the singularity of the structural stiffness matrix that is induced by the first and second steps. It should be pointed out that the reaction force exerted on the structure by this additional vertical support is zero, and thus the axial forces on indirectly connected columns and structural beams (F_{bc}) will not be affected by the third step of modification.

With the above three steps, the modified model of the construction system is plotted in Figure 4.9.

Table 4.2 Models of Critical Zones in Different Failure Modes

	Original critical zone	The first failure mode	The second failure mode	The third failure mode
Type-I
Type-II
Type-III

Figure 4.9 The Modified Model of the Construction System

4.6.2 Flowcharts for Performance Function and Structural Reliability

The solution of the axial forces F_{bc} on the indirectly connected columns and structural beams will interact with the determination of the structural weakest collapse mechanism. As a result, an iterative procedure is established. According to the weakest collapse mechanism and the corresponding minimum internal virtual work finally obtained after the convergence of the F_{bc} , the performance function is computed with Equation (3.38). The flowchart for the computation of the performance function is plotted in Figure 4.10.

The structural reliability assessment of the damaged frame structure against progressive collapse is carried out based on a combination of the flowchart (Figure 4.10) and Monte Carlo Simulation method. The steps of the combination are given in Section 3.2, while the flowchart is demonstrated in Figure 4.11.

* ϵ_2 : A small positive value

Figure 4.10 Flowchart for the Calculation of the Performance Function

Figure 4.11 Flowchart for Reliability Assessment of the Damaged Structure

4.7 Numerical Example

The occurrence probability of progressive collapse for a symmetric three-bay four-storey RC frame structure shown in Figure 4.12(a) is evaluated based on the flowchart plotted in Figure 4.11. The structure is subjected to uniformly distributed service loadings including a dead load DL and a live load LL with their characteristic value to be $DL_k = 27 \text{ kN/m}$ and $LL_k = 30 \text{ kN/m}$. The characteristic compressive strength of concrete f_{ck} is set to be 30 MPa while the characteristic yielding strength of the reinforcing hot-rolled steel f_{yk} is equal to 400 MPa . The dimension and reinforcement of the structural members are presented in Figure 4.12(a) with the detailed data listed in Table 4.3. The beams within the same span and the columns on the same axis have identical cross sectional dimensions and reinforcement. Two types of initial column failure due to different close-in

detonation in the first level are checked and the corresponding damaged frame structures are plotted in Figures 4.12(b)~ 4.12(c).

(a). The intact frame structure

(b). Case 1: external column failure

(c). Case 2: internal column failure

Figure 4.12 A Four-storey RC Frame Structure

Table 4.3 Dimension and Reinforcement for Different Cross Sections

Cross Sections		h (mm)	b (mm)	c' (mm)	c (mm)	A_s' (mm ²)	A_s (mm ²)
1-1	500	500	30	30	2462	2462
	2-2	500	500	30	30	2826	2826
	3-3	600	300	30	30	1661	1245
	4-4	600	300	30	30	1963	1471
	5-5	600	300	30	30	1963	1471

4.7.1 Basic Random Variables

According to the analysis conducted by Val *et al* [V1, V2, V3], the compressive strength of concrete f_c , the yield strength of reinforcing steel f_y and the effective depth of structural members d have been identified as parameters which have a major influence on structural reliability. Along with the service loadings DL and LL , they are defined as the basic random variables and are considered in the example.

According to [C3], the mean value of the compressive strength of concrete f_{cm} can be estimated by:

$$f_{cm} = f_{ck} + 8 \quad (4.26)$$

Considering that f_{ck} is defined as a 0.05-fractile of f_c and assuming that f_c is normally distributed, its coefficient of variation (CoV) may be calculated by:

$$CoV_{f_c} = 8 / [1.64 \times (f_{ck} + 8)] \quad (4.27)$$

As for f_y , its CoV (CoV_{fy}) is taken to be equal to 0.08 [V2, M13] while its mean value f_{ym} can be obtained as:

$$f_{ym} = f_{yk} / (1 - 1.64 \times CoV_{fy}) \quad (4.28)$$

Different types of distribution were proposed for f_y including the normal, lognormal and beta distribution [M13]. In this study, it is taken as a normal random variable.

The dead load DL is also considered to be normal random variable while the live load LL is represented by a type 1 extreme value distribution. The CoVs for these two parameters (CoV_{DL} , CoV_{LL}) are 0.05 and 0.40 respectively and their mean (DL_m , LL_m) are determined by [B12, C4, I2, V2]:

$$DL_m = 1.05 \times DL_k \quad (4.29)$$

and

$$LL_m = \psi_{LL} LL_k / (1 + 1.305 \times CoV_{LL}) \quad (4.30)$$

where ψ_{LL} , taken as 0.6 in this example, denotes the frequent value of the live load used for an design situation associated with the column failure in detonation conditions.

According to [M14], the effective depths d of different structural members are assumed to be normally distributed with mean values equal to their nominal values and CoVs being 0.05.

In addition to the five basic random variables mentioned above, the uncertainties of two other parameters including concrete elastic modulus E_c and structural members' width b are also accounted for in the example. Considering the high

degree of correlation between E_c and f_c , the former can be estimated by [C3, M15, V2]:

$$E_c = \alpha_E (0.1 f_c)^{1/3} \quad (4.31)$$

where α_E namely elastic modulus coefficient of concrete herein is assumed to be a independent random variable. Based on previous research, the statistic properties of all random variables considered are summarized in Table 4.4.

Table 4.4 Distributions of Random Variables

<i>Parameters</i>	<i>d</i>	<i>b</i>	<i>f_c</i>	<i>α</i>	<i>f_y</i>	<i>DL</i>	<i>LL</i>
Distribution type	Normal	Normal	Normal	Normal	Normal	Normal	Type 1 extreme value
Mean value	Nominal value *	Nominal value **	38	2.15E+4	460	28.35	11.8
CoV	0.05	0.03	0.13	0.15	0.08	0.05	0.4

* $d = h - c$ or $d = h - c'$; h and c are listed in Table 4.3.

** b is listed in Table 4.3.

4.7.2 Results and Discussion

Assuming that each of the involved random variable is fully correlated within a structural member but independent between different members, there are a total of 164 random variables for the example structure. By setting the ratio $p_{v,i}/p_{v,L1}$ in Equations (4.24) and (4.25) equal to 1, the occurrence probabilities of the damaged frame collapse events P_f are calculated based on the flowchart of Figure 4.11. In order to ensure that the convergent result is attained, different numbers N_s of the structural samples are adopted in the MCSs. The curves of P_f versus N_s are plotted in Figure 4.13.

Figure 4.13 Collapse Probability versus the Number of Structural Samples

Figure 4.13 shows that P_f converge quickly with N_s increasing from 2000 to 80000. When 80000 structural samples are used in the MCS, the resulted P_f are 0.22883 and 0.018288 for the two initial column failure cases, respectively. It is obvious that the loss of an external column is more critical for this particular frame structure than the loss of an internal column. It can be explained by the following reasons. Firstly, more members are involved in generating the structural weakest collapse mechanism after the loss of an internal column, which will not only increase the mean value of $\min(\delta W_{\text{int}})$ but reduce its CoV. Secondly, the CoV of δW_{ext} is reduced in the case of an internal column failure due to the fact that the service loadings on more structural members are included in the calculation of δW_{ext} . Finally, the beams in the middle span of the structure are more strongly reinforced than those in the side span. It will be discussed in the following part of this section that the strength of beams is primarily important for this frame against progressive collapse since the ESPHs of structural weakest collapse mechanism are most likely to be located on beams.

In addition to P_f , the distributions of $\min(\delta W_{\text{int}})$, δW_{ext} and z are plotted in Figure 4.14.

Figure 4.14 Distributions of $\min(\delta W_{\text{int}})$, δW_{ext} and z

Along with the histograms of $\min(\delta W_{\text{int}})$, δW_{ext} and z obtained from the MCS with 80000 structural samples, the normal probability density functions according to the mean values and the CoVs of the related parameter are also exhibited in Figure 4.14. It can be observed that the distributions of $\min(\delta W_{\text{int}})$ have a good match with the corresponding normal probability density curves in both cases since all the random parameters including f_c , f_y and d , which are important to the flexural strength of structural members and thus important to $\min(\delta W_{\text{int}})$, are assumed to be normally distributed. However, apparent deviations between the histograms of δW_{ext} and the normal density curves can be observed in Figures 4.14(c) and 4.14(d) due to the type 1 extreme value distribution for the live load LL .

The critical zones' failure modes which occur in structural weakest collapse mechanism are recorded for each structural sample. In addition, the total numbers of such occurrence for the 80000 structural samples used in the MCS are counted and are listed in Table 4.5 for both initial column loss cases.

Table 4.5 Occurrence Numbers of Different Failure Modes of Critical Zones

	Critical zone	Occurrence numbers of critical zones' failure modes (The first mode / The second mode / The third mode*)			
		Storey 1	Storey 2	Storey 3	Storey 4
Case 1	Type-II	0 / 79968 / 32	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -*	80000 / 0 / -*	80000 / 0 / -	80000 / 0 / -*
Case 2	Type-I	0 / 80000 / -*	0 / 80000 / -*	0 / 80000 / -*	6834 / 73166 / -*
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -*	80000 / 0 / -*	80000 / 0 / -*	80000 / 0 / -*

*: The third failure mode does not exist for the denoted critical zone

It can be seen from Table 4.5 that the Type-I, Type-II and Type-III critical zones of all storeys will fail according to the 2nd, 2nd and 1st modes, respectively, for a great

part of the 80000 samples. This observation indicates that the ESPHs of the structural weakest collapse mechanisms are predominantly located on the beams in the damaged span according to Figures 3.5(f), 3.6(g) and 3.7(e). Only in a few samples, the Type-II critical zones in the first and the fourth storeys (for Case 1) or the Type-I critical zone in the fourth storey (for second case) will fail in the mode with an ESPH appearing on related column. It is due to the fact that there is only one structural column connected with one beam for the above three critical zones. Although the column is intentionally designed to be stronger than the beam, there will still be some probability for the flexural strength of the beam to be greater than that of the connected column within the 80000 samples considering the uncertainties of f_c , f_y and d . This possibility further leads to ESPH appearing on column in the few samples. Nevertheless, it can be concluded based on Table 4.5 that the most effective way to improve the safety of this frame structure against progressive collapse should be increasing the strength of structural beams.

4.8 Summary

The procedure to solve the performance function of Equation (3.38) based on the construction system is discussed for assessing the reliability of a multi-storey RC frame structure subject to the column failure against progressive collapse. The interactive effects between the strength of the end springs and the axial forces on related members are considered in the calculation of the minimum internal work. In addition, reduced strengths are computed to account for the positions of positive EPHs that are not located on the beams' end sections. Modification of structural model is carried out based on the weakest collapse mechanism, while the axial forces on indirectly connected columns and beams are iteratively undated according to the revised model. The flowcharts for calculating the minimum internal work, the virtual concentrated force vector and the reliability of damaged structure are presented.

Implementations of the procedure developed are demonstrated for a four-storey RC frame structure under two different situations of column failure. The uncertainties in

material properties, applied loadings and structural geometries are accounted for. The occurrence numbers of the critical zones' failure modes are counted and the collapse probabilities for the damaged structure are computed. Numerical results show that the failure probability for this particular frame structure subjected to the loss of an external column is much higher than that subjected to the loss of an internal column. In addition, the ESPHs of the weakest collapse mechanism for the damaged frames are predominantly located on the beams in the damaged span for both column loss cases. Chapter 3 and Chapter 4 present the approach for the reliability assessment of damaged RC frame structural system against progressive collapse under static loading conditions. The dynamic effects induced by the sudden removal of the column in a close-in explosion on the system reliability against progressive collapse will be discussed in Chapter 5.

CHAPTER FIVE

RELIABILITY ASSESSMENT OF DAMAGED DUCTILE RC FRAME STRUCTURAL SYSTEM AGAINST PROGRESSIVE COLLAPSE UNDER DYNAMIC LOADING CONDITIONS

— PART I. PROCEDURE

5.1 Introduction

The method presented in Chapters 3 and 4 assesses the reliability of the damaged frame structure against the progressive collapse, where the failed column is removed from the structural model and the service loadings including the dead and live loads are statically applied to the damaged structure. This method is applicable for the situation where the failure of the column is assumed to be at a very low speed and thus little dynamic response would be evoked to the structure. However, in the case of a close-in detonation, this assumption is no longer valid since the destruction of the column by blast pressure occurs within a significantly short period of time, generally in the order of several milliseconds. As a result, the structural response will considerably be influenced by the dynamic effects induced by the sudden loss of the column.

In order to consider the dynamic effects induced by the sudden loss of a column in a close-in detonation, the method presented in Chapters 3 and 4 is further developed in this chapter for the reliability assessment of the damaged ductile frame against progressive collapse under dynamic loading conditions. The safety of the frame structure is evaluated based on two aspects: the flexural response and the shear response of the structure. As for structural flexural response, two performance functions are formed to consider the structural collapse due to the lack of strength or deformation capacity for structural weakest collapse mechanism. In the calculation

of the second function, the damaged structure is simplified into a single-degree-of-freedom (SDOF) system to dissipate external work while the resistance function of the SDOF system is determined by the strength and rotational capacity of the plastic hinges occurring in structural collapse. In addition, a third performance function is constructed to consider shear response, where the shear force on different structural members is obtained from structural weakest collapse mechanism. Finally, a global performance function of the damaged structure is taken to be the minimum of the above three functions.

Implementations of the developed method are demonstrated to assess the reliability of the same four-storey reinforced concrete (RC) frame structure adopted in Chapter 4 and the result shows a significant increase on the occurrence probability of structural collapse event (P_f) due to the dynamic effects of the sudden loss of a structure column.

5.2 Structural Model and Assumptions

5.2.1 Structure Model

The lumped plasticity model presented in Chapter 3 will once again be utilized here to represent the force-displacement relationship of the structure. Since the dynamic response of the structure has to be considered, the inertial force produced by structural mass and acceleration shall be included in the equilibrium equation (called the equation of motion).

There are two well-known mass matrices which can be used to represent the continuous distribution of the mass over the elements' length in structural dynamic response analysis: the consistent mass matrix and the lumped mass matrix. The consistent mass matrix depends on the shape function of the element that is used to generate the stiffness matrix. The lumped mass matrix is generated by concentrating the continuous mass of each element on its ends, generally half of the total element mass on each end. Although it is difficult to assert that which mass matrix is better

for general application, Cook *et al* [C5] pointed out that the consistent mass matrix usually yields more accurate results in eigenvalue problems while the lumped mass matrix is simpler and more suitable for inelastic dynamic response analysis of frame structures. The lumped mass matrix has been widely employed by finite element analysis software to simulate the structural inelastic dynamic responses. With the lumped mass matrix, the structural model is schematically plotted in Figure 5.1.

Figure 5.1 Structural Model for Dynamic Analysis

In Figure 5.1, $m_{i,j}$ indicates the lumped mass concentrated on the j^{th} node of the i^{th} storey. It consists of four parts: halves of the total masses of the left and the right connected beams ($m_{i,j}^{bl}$ and $m_{i,j}^{br}$) and halves of the total masses of the upper and lower connected columns ($m_{i,j}^{cu}$ and $m_{i,j}^{cl}$). Therefore, $m_{i,j}$ is expressed as:

$$m_{i,j} = \frac{1}{2} (m_{i,j}^{bl} + m_{i,j}^{br} + m_{i,j}^{cu} + m_{i,j}^{cl}) \quad (5.1)$$

5.2.2 Simplifications and Assumptions

The remaining frame structure will experience the dynamic response when subjected to the sudden loss of a structural column in the event of a close-in detonation. To develop the method for reliability assessment of the damaged structure against progressive collapse considering the dynamic effects, the following assumptions are made:

- 1) The dynamic response of the damaged structure induced by the initial column loss can be calculated through three steps: removing the failed column from the structural model, applying the service loadings in the spans other than the damaged spans statically, and applying the service loadings in the damaged spans as rectangular dynamic forces. The treatment of the service loading as rectangular dynamic forces has been substantiated by Kaewkulchai [K2], who pointed out that the time histories of the structural response analyzed by simplifying the service loadings to be rectangular dynamic forces were quite close to (with slight conservative error) the real time histories of the structural response after the column failure.
- 2) The dynamic response of the damaged structure can be divided into two categories: the global structural dynamic response (GSDR) and the local dynamic vibrations of the structural elements (LDVOE) such as the axial vibrations of structural columns. The GSDR, whose natural period is generally in the order of seconds, is characterized by the large vertical deformation of the damaged span in a global manner. Excessive GSDR will directly lead to progressive collapse of the damaged structure. The LDVOE induced by the changes of elements' internal force, is usually of small amplitude and has a significantly higher frequency than the GSDR. It will be quickly damped out by structural damping. Therefore, the LDVOE is neglected in the following reliability assessment.

- 3) There are a number of documents focused on structural damping and a 5% of critical damping is generally accepted for concrete structure under seismic condition. However, damping effect on global structural response is ignored in this study considering the following three points:
- Damping effect for a damaged structure against progressive collapse may not be as significant as it is under seismic condition, since the dynamic force here is rectangular instead of cyclical loading.
 - In structural progressive collapse, large plastic deformation is included and the structural stiffness keeps changing with the increase of plastic deformation. Thus, 5% critical damping may not be applicable and more experimental study is needed for determining a reasonable value of it.
 - Structural damping can be directly incorporated into a dynamic finite element analysis simply by employing a damping matrix. However, it becomes very difficult to consider the damping effect in structural reliability assessment since the structural nonlinear dynamic finite element analysis is impractical for this purpose as discussed in Chapter 2. Therefore, damping effect is ignored to simplify the problem, which will lead to a conservative result.
- 4) Two kinds of structural failure are needed to be considered here: the excessive flexural responses of the damaged frame and the excessive shear responses. For the flexural responses, since the scope of this study is on the reliability assessment of the ductile reinforced concrete (RC) frame structure such as those seismically designed, the weakest collapse mechanism consisting of a number of plastic hinges with sufficient rotational capacity will control the progressive collapse event. The damaged structure can be simplified into be an equivalent SDOF system as shown in Figure 5.2. As the rectangular dynamic service loadings acting on damaged spans, the system will respond from its initial state (State “O” ($\Delta = 0$)) in Figure

5.2(a)), through State “A” ($\Delta = y_e$) plotted in Figure 5.2(b) where the weakest collapse mechanism occurs, till the maximum response State “B” ($\Delta = y_m$) in Figure 5.2(c) is reached. It is obvious that if State “B” is within the deformation capacity of the weakest collapse mechanism (State “C” where $\Delta = y_u$ as shown in Figure 5.2(d)), the structure will be safe from progressive collapse induced by excessive flexural response. The resistance function of the equivalent SDOF system is plotted in Figure 5.2(d). In order to reasonably represent the energy dissipation capacity of the damaged structure with the area under the curve OAC, the resistance function of the equivalent SDOF system can be defined as follows:

- Its strength R_m is equal to the energy dissipated by the weakest collapse mechanism giving unit deformation ($\delta\Delta = 1$).
- The yield displacement y_e is the displacement corresponding to the initiation of the final plastic hinge of the weakest mechanism and R_m will be kept constant from y_e to the displacement capacity y_u of the system.
- y_u can be taken as the displacement corresponding to the first exceedance of the rotational deformation capacity of the plastic hinges of the mechanism. Further response will induce a radical drop down to the resistance of the weakest mechanism and the SDOF system.
- The resistance function of the equivalent SDOF system is indicated by the lines connecting the three points “O”, “A” and “C” as shown in Figure 5.2 (d).

In fact, before the formation of the weakest mechanism, the plasticity of the damaged structure will gradually develop from its initial state of “O” to “A”.

From this point of view, the above way in constructing the resistance function of the SDOF system is conservative.

- 5) In order to consider the shear response in the reliability assessment of the damaged structure, it is assumed that brittle shear failure of the damaged frame will occur prematurely if the shear strength of any structural member is exceeded by the shear force during the dynamic response of the structure. The shear strength of a structural member is determined by its dimensions, the strength of the construction materials and the reinforcement *etc* [M12]. The shear force depends on the magnitude of structural dynamic response and thus an accurate evaluation of the shear force entails a detailed nonlinear dynamic finite element analysis which makes the reliability assessment of the damaged structure impractical. In order to simplify the computation of the shear force, it is also assumed that the weakest collapse mechanism has occurred to the damaged frame structure and thus the maximum shear force on each member can be evaluated based on the weakest mechanism. This assumption may lead to a conservative result for the reliability assessment when the frame is designed sufficiently strong to be able to sustain more than approximately twice of the static service loadings on the damaged spans. However, consider the fact that for ductile frames, the structural collapse event is primarily induced by the lack of strength or deformation capacity of structural weakest collapse mechanism, the above conservative error will be negligible.
- 6) The initial state of the damaged structure plotted in Figure 5.2 (a) is important in its reliability assessment against progressive collapse. In order to determine the damaged structure's initial state, a structural analysis needs to be carried out previously and some important parameters (such as the location of the node above the removed column and the bending moments of the beams in the damaged spans *etc*) be recorded. The following discussing is based on this initial state.

Initial state of the damaged structure model

Initial state of the equivalent SDOF system

(a) State "O"

Figure 5.2 Simplification of the Damaged Structure into An Equivalent SDOF System

Initiation of the weakest collapse mechanism

Yield of the equivalent SDOF system

(b) State “A”

Figure 5.2 (Continued)

Maximum response of the damaged structure

Maximum response of the equivalent SDOF system

(c) State “B”

Figure 5.2 (Continued)

(d) Resistance function of the equivalent SDOF system

Figure 5.2 (Continued)

5.3 Performance Functions

5.3.1 Construction of Performance Functions

Three performance functions with respect to structural flexural and shear responses are constructed for the reliability assessment. As the basic requirement for the damaged structure to be safe from progressive collapse, its weakest collapse mechanism should be able to provide sufficient strength to support the service loadings. Accordingly, the first performance function is built up based on the virtual work principle as given in Chapter 3:

$$z_1 = \min(\delta W_{\text{int}}) - \delta W_{\text{ext}} \quad (5.2a)$$

The second condition which the damaged frame shall satisfy in preventing progressive collapse is that the deformation capacity of the weakest mechanism y_u needs to be adequate to sustain the maximum dynamic response y_m under the rectangular dynamic force as shown in Figure 5.2. If expressed in terms of energy, it indicates that the energy dissipation capacity of the mechanism ($W_{\text{int},u}$) shall be

greater than the external works ($W_{ext,u}$) done by the service loadings on the damaged spans at the deformation level of y_u . Therefore the second performance function is written as:

$$z_2 = W_{int,u} - W_{ext,u} \quad (5.2b)$$

Obviously, when $z_2 > 0$, the response of the structure will rebound before y_u and the structure will be safe against the collapse which is induced by excessive large deformation. On the other hand, if $z_2 < 0$, y_m will exceed y_u and structural collapse occurs.

In addition, the structure should not be subjected to the brittle shear failure. According to the discussion in Section 5.2, the third performance function is expressed by:

$$z_3 = \min(\{V_{r,k}\} - \{V_{f,k}\}) \quad k = 1 \cdots nc \quad (5.2c)$$

where $\{V_{r,k}\}$ indicates the accumulation of the shear strength of the end sections of all structural members; $\{V_{f,k}\}$ is the accumulation of the shear force calculated based on structural weakest mechanism; nc denotes the number of the considered cross sections. In addition, $\min(\bullet)$ calculates the minimum component of $\{V_{r,k}\} - \{V_{f,k}\}$.

Since any of the above three performance functions less than zero can lead to structure collapse, they should be considered simultaneously in the reliability assessment. Consequently, a global performance function z for the damaged frame structure is employed, which takes the minimum value among z_1 , z_2 and z_3 :

$$z = \min(z_1, z_2, z_3) \quad (5.2d)$$

It can be seen that all of the three performance functions (z_1 , z_2 and z_3) which are involved in the calculation of z depend on structural weakest collapse mechanism. Therefore, the identification of this mechanism under the dynamic conditions is particularly important for structural reliability assessment.

5.3.2 Structural Weakest Collapse Mechanism and Solution of the First Performance Function

With the column removed from the structural model and the service loadings instantaneously applied on the damaged spans, the dynamic response will be evoked to the damaged structure from its initial state plotted in Figure 5.2(a). The equation of motion can be written as:

$$F_I + F_R = F_E \quad (5.3a)$$

where F_I , F_R and F_E denote the inertial force, the restoring force and the external force, respectively. Equation (5.3a) indicates that the structure will keep in balance if an additional force equal to F_I but in the opposite direction is applied on the structure. Thus, given a virtual deformation, Equation (5.3a) can be expressed based on the virtual work principle in another form as:

$$\delta W_I + \delta W_R = \delta W_E \quad (5.3b)$$

where δW_I , δW_R and δW_E represent the virtual works by F_I , F_R and F_E under the given virtual deformation respectively.

As the response of the damaged structure exceeds State “A” in Figure 5.2(b) ($\Delta \geq y_e$), structural weakest collapse mechanism is formed. Further structural deformation will be concentrated on the rotation of the plastic hinges of the mechanism and the dynamic response of the structure can be schematically plotted as in Figure 5.3.

Figure 5.3 Dynamic Response of the Damaged Structure when $\Delta \geq y_e$

In Figure 5.3, v and a are the velocity and the acceleration of the global response of the structure at the axes k of the failed column. Within this structural response stage where Δ changes from y_e to y_m , the virtual works δW_R and δW_E in Equation (5.3b) can be the internal virtual works by the plastic hinges of the weakest mechanism and the external virtual works by the service loadings within the damaged spans under $\delta\Delta$, which are denoted by $\min(\delta W_{\text{int}})$ and δW_{ext} respectively. In addition, the value of δW_I can be calculated as:

$$\delta W_I = \delta\Delta \times \sum_{i=L1}^{L2} (m_{i,k} a) \quad (5.4a)$$

or

$$\delta W_I = \delta\Delta \times m_{L1,k} \times a \sum_{i=L1}^{L2} (m_{i,k} / m_{L1,k}) \quad (5.4b)$$

where $m_{i,k}$ indicates the lumped mass concentrated at the node on Axis k and the i^{th} storey. Substituting Equation (5.4b) into Equation (5.3b) and replacing δW_R and δW_E by $\min(\delta W_{int})$ and δW_{ext} produces:

$$\delta \Delta \times m_{L1,k} \times (-a) \sum_{i=L1}^{L2} (m_{i,k} / m_{L1,k}) = \min(\delta W_{int}) - \delta W_{ext} \quad (5.5)$$

Comparing Equation (5.5) with Equations (3.38) and (4.23), it can be seen that by taking $p_{v,i} / p_{v,L1}$ in Equation (4.23) to be equal to $m_{i,k} / m_{L1,k}$ and replacing $p_{v,L1}$ with $m_{L1,k} \times (-a)$, Equations (3.38) and (4.23) for calculating the weakest mechanism of the damaged structure under static loading conditions are exactly the same as Equation (5.5). Therefore, the approach discussed in Chapter 4 for solving Equation (3.38) or (4.23) is also applicable for Equation (5.5) to determine the structural weakest collapse mechanism under dynamic conditions and to calculate the corresponding $\min(\delta W_{int})$, δW_{ext} as well as the first performance function z_1 in Equation (5.2a).

5.3.3 Solution of the Second Performance Function

The second performance function shown in Equation (5.2b) is constructed based on the relationship between the energy dissipation capacity of the weakest collapse mechanism $W_{int,u}$ and the external work $W_{ext,u}$ by the service loadings on the damaged span at the response level of y_u . This performance function is presented to assess whether the displacement capacity of the weakest mechanism y_u can accommodate the maximum dynamic response y_m . As discussed previously, by ignoring the local dynamic vibration of different structural element (LDVOE), the flexural response of the damaged structure can be simplified into an equivalent SDOF system with the area under its resistance function curve shown in Figure

5.2(d) to conservatively represent $W_{int,u}$. Accordingly, Equation (5.2b) can be rewritten as:

$$z_2 = W_{int,u} - W_{ext,u} = R_m y_u - \frac{1}{2} R_m y_e - W_{ext,u} \quad (5.6)$$

where R_m , y_e and y_u denote the strength, the yielding displacement and the displacement capacity of the SDOF system, respectively.

It is obvious that R_m , y_e and y_u depend on the weakest collapse mechanism of the damaged structure. However, the weakest collapse mechanism determined with the approach discussed in Chapters 3 and 4 is idealized by fixing all of the plastic hinges at the end sections of structural members and using the reduced strength $M_{+u,b1}^{II,i}$ or $M_{+u,b2}^{II,i}$ for the end springs on beams within Type-II critical zones to compensate the error that may be induced. Although it can lead to the correct solution of the first performance function, the idealized weakest mechanism does not exactly reflect the real weakest collapse mechanism of the damaged frame structure. In order to distinguish the idealized weakest mechanism and the real weakest collapse mechanism, they are denoted by IWCM and RWCM, respectively. In addition, the plastic hinge represented by the yielded spring at the end section of a structural member is called ESPH while the real plastic hinge in structural RWCM is named RPH. In fact, the RWCM can be easily deduced from the IWCM by moving the ESPHs on beams within Type-II critical zones (if applicable) to the locations of the corresponding RPHs (i.e. the $x_{l,i}$ or $x_{r,i}$ calculated in Chapter 4) and applying the flexural strengths of real structural members $M_{+ur,b1}^{II,i}$ or $M_{+ur,b2}^{II,i}$ to these hinges.

To obtain a more reasonable evaluation of the second performance function z_2 , the four parameters R_m , y_e , y_u and $W_{ext,u}$ in Equation (5.6) should be calculated based on the RWCM. The procedures to find them are discussed in the following sections.

5.3.3.1 Strength of the SDOF System - R_m

The strength R_m of the SDOF system is equal to the energy dissipated by the RPHs of the RWCM under unit displacement $\delta\Delta = 1$. It is written as:

$$R_m = \sum_{i=L1}^{L2} \delta W_{\text{int},i}^{I,\text{real}} + \sum_{i=L1}^{L2} \delta W_{\text{int},i}^{II,\text{real}} + \sum_{i=L1}^{L2} \delta W_{\text{int},i}^{III,\text{real}} \quad (5.7)$$

where $\delta W_{\text{int},i}^{I \sim III,\text{real}}$ is the internal energy provided by the RPHs within Type-I ~ Type-III critical zones on the i^{th} storey. To solve Equation (5.7), the deformation of the damaged structure and the circled part in the i^{th} storey under $\delta\Delta = 1$ as well as the corresponding rotations of the RPHs within different types of critical zone are plotted in Figure 5.4 and Figure 5.5, respectively.

(a) The damaged structure

Figure 5.4 Deformation of the Damaged Structure
and the Circled i^{th} Storey under $\delta\Delta = 1$

(b) The circled i^{th} storey (Type-II critical zone in the first failure mode)

(c) The circled i^{th} storey (Type-II critical zone in the second failure mode)

Figure 5.4 (Continued)

(d) The circled i^{th} storey (Type-II critical zone in the third failure mode)

Figure 5.4 (Continued)

Figure 5.5 Rotations of the RPHs under $\delta\Delta = 1$

(c) RPHs within Type-II
critical zone in the 1st failure mode

(d) RPHs within Type-II
critical zone in the 2nd failure mode

(e) RPHs within Type-II critical zone in the 3rd failure mode

(f) RPHs within Type-III
critical zone in the 1st failure mode

(g) RPHs within Type-III
critical zone in the 2nd failure mode

Figure 5.5 (Continued)

In Figures 5.4 and 5.5, $\delta\theta_i^I$ and $\delta\theta_i^{III}$ are the rotations of the RPHs in Type-I and Type-III critical zones on the i^{th} storey under $\delta\Delta = 1$, respectively. Considering that further deformation will be concentrated on the RPHs after the formation of the RWCM, the values of $\delta\theta_i^I$ and $\delta\theta_i^{III}$ can be determined through $x_{l,i}$, $x_{r,i}$ and the failure mode of the Type-II critical zone as shown in Figures 5.4(b)~(d). The formulae for these two variables are listed in Table 5.1.

Table 5.1 Rotations of RPHs in Type-I and Type-III Critical Zones

$(\delta\theta_i^I \text{ and } \delta\theta_i^{III})$			
	Failure mode of Type-II critical zone		
	First mode	Second mode	Third mode
$\delta\theta_i^I$	$1/l_l$	$1/x_{l,i}$	$(l_l + l_r - x_{l,i})/(l_r \times x_{l,i})$
$\delta\theta_i^{III}$	$(l_l + l_r - x_{r,i})/(l_l \times x_{r,i})$	$1/x_{r,i}$	$1/l_r$

In addition, the rotations of the RPHs in Type-II critical zone are different combinations of $\delta\theta_i^I$ and $\delta\theta_i^{III}$ with respect to the failure mode of this type of critical zone, which are demonstrated in Figures 5.5(c) ~ (e). Based on $\delta\theta_i^I$ and $\delta\theta_i^{III}$, the value of $\delta W_{int,i}^{I-III,real}$ can be calculated with the following equations.

For Type-I critical zone:

$$\delta W_{int,i}^{I,real} = \min \left[\left(M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} + M_{-u,c2}^{I,i} \right), M_{-u,b2}^{I,i} \right] \times \delta\theta_i^I \quad (5.8)$$

For Type-II critical zone in the first failure mode:

$$\delta W_{int,i}^{II,real} = \left(M_{+u,c1}^{II,i} + M_{-u,c2}^{II,i} \right) \times \delta\theta_i^I + M_{+ur,b2}^{II,i} \times (\delta\theta_i^I + \delta\theta_i^{III}) \quad (5.9a)$$

For Type-II critical zone in the second failure mode:

$$\delta W_{\text{int},i}^{II,real} = M_{+ur,b1}^{II,i} \times \delta \theta_i^I + M_{+ur,b2}^{II,i} \times \delta \theta_i^{III} \quad (5.9b)$$

For Type-II critical zone in the third failure mode:

$$\delta W_{\text{int},i}^{II,real} = M_{+ur,b1}^{II,i} \times (\delta \theta_i^I + \delta \theta_i^{III}) + (M_{-u,c1}^{II,i} + M_{+u,c2}^{II,i}) \times \delta \theta_i^{III} \quad (5.9c)$$

For Type-III critical zone:

$$\delta W_{\text{int},i}^{III,real} = \min \left[M_{-u,b1}^{III,i}, \left(M_{-u,b2}^{III,i} + M_{-u,c1}^{III,i} + M_{+u,c2}^{III,i} \right) \right] \times \delta \theta_i^{III} \quad (5.10)$$

Substituting Equations (5.8) ~ (5.10) into Equation (5.7), the strength of the equivalent SDOF system R_m is obtained.

5.3.3.2 Yield Displacement of the SDOF System - y_e

The yield displacement y_e of the SDOF system is taken as the displacement corresponding to the initiation of the final RPH of the RWCM. If the plastic rotations of the RPHs is denoted with a vector $\theta = \{\theta_j\}$ where $j = 1 \cdots np$ (np is the number of the RPHs), θ_j will be a function of the structural global response Δ and can be symbolized as $\theta_j(\Delta)$. Thus, y_e can be mathematically expressed with the following equation:

$$\min \{\theta_j(y_e)\} = \varepsilon \quad j = 1 \cdots np \quad \varepsilon \rightarrow 0 \quad (5.11)$$

where ε indicates an arbitrarily small positive value. Obviously, the solution of the Equation (5.11) depends on the function $\theta_j(\Delta)$. In order to obtain $\theta_j(\Delta)$, the

relationship between the plastic rotations of the ESPHs represented by $Q = \{Q_j\}$ and the structural global response Δ should firstly be established as $Q = \{Q_j(\Delta)\}$. The function of $Q_j(\Delta)$ can be obtained based on the structural model described in Figure 4.9 and Table 4.2. Given a large vertical displacement Δ to the additional support in Figure 4.9, the corresponding $Q_j(\Delta)$ is listed in Table 5.2.

Table 5.2 Rotations of the ESPHs $Q_j(\Delta)$ and Sectional Moments at Both Ends of Beams in the Damaged Span ($M_{l,i}^{leftend}$, $M_{l,i}^{rightend}$, $M_{r,i}^{leftend}$ and $M_{r,i}^{rightend}$)

Rotations of the ESPHs in Type-I critical zone on the i^{th} storey		
	The first failure mode	The second failure mode	The third Failure mode
ESPH on Beam-1	$D_{b1}^{I,i} = -1$ $Q_{b1}^{I,i} = D_{b1}^{I,i} \times Q_{b1,o}^{I,i} + \frac{\Delta}{l_i}$		
ESPH on Beam-2		$D_{b2}^{I,i} = -1$ $Q_{b2}^{I,i} = D_{b2}^{I,i} \times Q_{b2,o}^{I,i} + \frac{\Delta}{l_i}$	
ESPH on Column-1			$D_{c1}^{I,i} = 1$ $Q_{c1}^{I,i} = D_{c1}^{I,i} \times Q_{c1,o}^{I,i} + \frac{\Delta}{l_i}$
ESPH on Column-2			$D_{c2}^{I,i} = -1$ $Q_{c2}^{I,i} = D_{c2}^{I,i} \times Q_{c2,o}^{I,i} + \frac{\Delta}{l_i}$
$M_{l,i}^{leftend}$	$-M_{-u,b1}^{I,i} - M_{+u,c1}^{I,i} - M_{-u,c2}^{I,i}$		$-M_{-u,b2}^{I,i}$

Table 5.2 (Continued)

Rotation of the ESPHs in Type-II critical zone on the i^{th} storey		
	The first failure mode	The second failure mode	The third failure mode
<div data-bbox="268 1048 475 1077">ESPH on Beam-1</div> <div data-bbox="268 1223 475 1252">ESPH on Beam-2</div> <div data-bbox="255 1384 488 1413">ESPH on Column-1</div> <div data-bbox="255 1545 488 1574">ESPH on Column-2</div>
		$D_{b1}^{II,i} = 1$ $Q_{b1}^{II,i} = D_{b1}^{II,i} \times Q_{b1,o}^{II,i} + \frac{\Delta}{l_l}$	$D_{b1}^{II,i} = 1$ $Q_{b1}^{II,i} = D_{b1}^{II,i} \times Q_{b1,o}^{II,i} + \frac{\Delta}{l_l} + \frac{\Delta}{l_r}$
	$D_{b2}^{II,i} = 1$ $Q_{b2}^{II,i} = D_{b2}^{II,i} \times Q_{b2,o}^{II,i} + \frac{\Delta}{l_l} + \frac{\Delta}{l_r}$	$D_{b2}^{II,i} = 1$ $Q_{b2}^{II,i} = D_{b2}^{II,i} \times Q_{b2,o}^{II,i} + \frac{\Delta}{l_r}$	
	$D_{c1}^{II,i} = 1$ $Q_{c1}^{II,i} = D_{c1}^{II,i} \times Q_{c1,o}^{II,i} + \frac{\Delta}{l_l}$		$D_{c1}^{II,i} = -1$ $Q_{c1}^{II,i} = D_{c1}^{II,i} \times Q_{c1,o}^{II,i} + \frac{\Delta}{l_r}$
<div data-bbox="320 1657 421 1709">$M_{l,i}^{rightend}$</div> <div data-bbox="325 1749 416 1800">$M_{r,i}^{leftend}$</div>	$D_{c2}^{II,i} = -1$ $Q_{c2}^{II,i} = D_{c2}^{II,i} \times Q_{c2,o}^{II,i} + \frac{\Delta}{l_l}$		$D_{c2}^{II,i} = 1$ $Q_{c2}^{II,i} = D_{c2}^{II,i} \times Q_{c2,o}^{II,i} + \frac{\Delta}{l_r}$
	$M_{+u,b2}^{II,i} + M_{+u,c1}^{II,i} + M_{-u,c2}^{II,i}$ $M_{+u,b2}^{II,i}$	$M_{+u,b1}^{II,i}$ $M_{+u,b2}^{II,i}$	$M_{+u,b1}^{II,i}$ $M_{+u,b1}^{II,i} + M_{-u,c1}^{II,i} + M_{+u,c2}^{II,i}$

Table 5.2 (Continued)

Rotation of the ESPHs in Type-III critical zone on the i^{th} storey		
	The first failure mode	The second failure mode	The third failure mode
ESPH on Beam-1	$D_{b1}^{III,i} = -1$ $Q_{b1}^{III,i} = D_{b1}^{III,i} \times Q_{b1,o}^{III,i} + \frac{\Delta}{l_r}$
ESPH on Beam-2		$D_{b2}^{III,i} = -1$ $Q_{b2}^{III,i} = D_{b2}^{III,i} \times Q_{b2,o}^{III,i} + \frac{\Delta}{l_r}$	
ESPH on Column-1		$D_{c1}^{III,i} = -1$ $Q_{c1}^{III,i} = D_{c1}^{III,i} \times Q_{c1,o}^{III,i} + \frac{\Delta}{l_r}$	
ESPH on Column-2		$D_{c2}^{III,i} = 1$ $Q_{c2}^{III,i} = D_{c2}^{III,i} \times Q_{c2,o}^{III,i} + \frac{\Delta}{l_r}$	
$M_{r,i}^{rightend}$	$-M_{-u,b1}^{III,i}$	$-M_{-u,b2}^{III,i} - M_{-u,c1}^{III,i} - M_{+u,c2}^{III,i}$	

In Table 5.2, $Q_{[b1,b2,c1,c2]}^{[I,II,III],i}$ are the plastic rotations of the ESPHs at the end sections of Beam-1, Beam-2, Column-1 and Column-2 within Type-I ~ Type-III critical zone on the i^{th} storey; $Q_{[b1,b2,c1,c2],o}^{[I,II,III],i}$ are the plastic rotations of corresponding ESPHs

obtained from the structural analysis with the model depicted in Table 4.2, Figure 4.9 and moving the additional vertical support in Figure 4.9 to its initial position. The direction of $Q_{[b1,b2,c1,c2],o}^{[I,II,III],i}$ should follow the definition in Chapter 3. In addition, director coefficients $D_{[b1,b2,c1,c2]}^{[I,II,III],i}$ are introduced in Table 5.2 to indicate the direction of the ESPHs of the IWCM. There are only two available values for $D_{[b1,b2,c1,c2]}^{[I,II,III],i}$: -1 for the negative ESPH and 1 for the positive ESPH. Based on Table 5.2, it can be observed that Q_j will increase linearly with the increase of Δ after the formation of the IWCM.

To derive the relationship between $\theta_j(\Delta)$ and $Q_j(\Delta)$, an example of the circled part on the i^{th} storey of the damaged structure shown in Figure 5.4(a) is discussed. Suppose that the first failure mode occurs to the three critical zones included in the selected part, the ESPHs of the IWCM, the RPHs of the RWCM and their corresponding plastic rotations under Δ are plotted in Figure 5.6.

Figure 5.6 Rotation of the ESPHs and RPHs on the Beams in the i^{th} Storey

In Figure 5.6, $x_{r,i}$ denotes the location of the RPH on the beam within Type-II critical zone. It can be seen that the plastic rotations of the RPHs are equal to those of the ESPHs except that $\theta_{b2}^{II,i} > Q_{b2}^{II,i}$ and $\theta_{b1}^{III,i} > Q_{b1}^{III,i}$. The relationships between $\theta_{b2}^{II,i}$, $\theta_{b1}^{III,i}$ and $Q_{b2}^{II,i}$, $Q_{b1}^{III,i}$ are deduced based on Figure 5.6, expressed respectively as:

$$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times \left(1 + \frac{l_r - x_{r,i}}{x_{r,i}} \right) \quad (5.12a)$$

$$\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times \frac{l_r - x_{r,i}}{x_{r,i}} \quad (5.12b)$$

Substituting $Q_{b2}^{II,i}(\Delta)$ and $Q_{b1}^{III,i}(\Delta)$ listed in Table 5.2 into Equations (5.12a) and (5.12b) produces:

$$\theta_{b2}^{II,i} = D_{b2}^{II,i} \times Q_{b2,o}^{II,i} \times \left(1 + \frac{l_r - x_{r,i}}{x_{r,i}} \right) + \Delta \times \left(\frac{1}{l_l} + \frac{1}{l_r} \right) \times \left(1 + \frac{l_r - x_{r,i}}{x_{r,i}} \right) \quad (5.13a)$$

$$\theta_{b1}^{III,i} = D_{b2}^{II,i} \times Q_{b2,o}^{II,i} \times \left(\frac{l_r - x_{r,i}}{x_{r,i}} \right) + D_{b1}^{III,i} \times Q_{b1,o}^{III,i} + \Delta \times \frac{l_r - x_{r,i}}{x_{r,i}} \times \left(\frac{1}{l_l} + \frac{1}{l_r} \right) + \frac{\Delta}{l_r} \quad (5.13b)$$

Obviously, $\theta_{b2}^{II,i}(\Delta)$ and $\theta_{b1}^{III,i}(\Delta)$ are linearly increasing functions of Δ . For the rotations of other RPHs ($\theta_{b1}^{I,i}$, $\theta_{c1}^{I,i}$, $\theta_{c2}^{I,i}$, $\theta_{c1}^{II,i}$, $\theta_{c2}^{II,i}$), they are equal to the rotations of the corresponding ESPHs ($Q_{b1}^{I,i}$, $Q_{c1}^{I,i}$, $Q_{c2}^{I,i}$, $Q_{c1}^{II,i}$, $Q_{c2}^{II,i}$) as shown in Figure 5.6 and thus are linearly related to Δ as well. The same method addressed above can be employed to deduce the relationship between θ_j and Q_j , when other failure modes are identified for the three critical zones. The results are listed in Table 5.3.

Table 5.3 Relationships between Rotations of RPHs and ESPHs *

Type-I critical zone	Type-II critical zone	Type-III critical zone	The relationship between θ_j and Q_j
The 1 st mode	The 1 st mode	The 1 st mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times \left[1 + \left(l_r - x_{r,i} \right) / x_{r,i} \right]$ $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$
The 1 st mode	The 1 st mode	The 2 nd mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times \left[1 + \left(l_r - x_{r,i} \right) / x_{r,i} \right]$ $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$ $\theta_{c1}^{III,i} = Q_{c1}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$ $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$
The 1 st mode	The 2 nd mode	The 1 st mode	$\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times \left(l_l - x_{l,i} \right) / x_{l,i}$ $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times \left(l_l - x_{l,i} \right) / x_{l,i}$ $\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times \left(l_l - x_{l,i} \right) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times \left[1 + \left(l_l - x_{l,i} \right) / x_{l,i} \right]$ $\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times \left[1 + \left(l_r - x_{r,i} \right) / x_{r,i} \right]$ $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$
The 1 st mode	The 2 nd mode	The 2 nd mode	$\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times \left(l_l - x_{l,i} \right) / x_{l,i}$ $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times \left(l_l - x_{l,i} \right) / x_{l,i}$ $\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times \left(l_l - x_{l,i} \right) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times \left[1 + \left(l_l - x_{l,i} \right) / x_{l,i} \right]$ $\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times \left[1 + \left(l_r - x_{r,i} \right) / x_{r,i} \right]$ $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$ $\theta_{c1}^{III,i} = Q_{c1}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$ $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{b2}^{II,i} \times \left(l_r - x_{r,i} \right) / x_{r,i}$

Table 5.3 (Continued)

Type-I critical zone	Type-II critical zone	Type-III critical zone	The relationship between θ_j and Q_j
The 1 st mode	The 3 rd mode	The 1 st mode	$\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$
The 1 st mode	The 3 rd mode	The 2 nd mode	$\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$
The 2 nd mode	The 1 st mode	The 1 st mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$ $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
The 2 nd mode	The 1 st mode	The 2 nd mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$ $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c1}^{III,i} = Q_{c1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
The 2 nd mode	The 2 nd mode	The 1 st mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$ $\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$ $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$

Table 5.3 (Continued)

Type-I critical zone	Type-II critical zone	Type-III critical zone	The relationship between θ_j and Q_j
The 2 nd mode	The 2 nd mode	The 2 nd mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$ $\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$ $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c1}^{III,i} = Q_{c1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
The 2 nd mode	The 3 rd mode	The 1 st mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$
The 2 nd mode	The 3 rd mode	The 2 nd mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$

*: The rotation of the RPHs (θ_j) that is not given in Table 5.3 is equal to the rotation of the ESPHs (Q_j).

By substituting $Q_j(\Delta)$ given in Table 5.2 into the equations listed in Table 5.3, the function of $\theta_j(\Delta)$ for all RPHs is obtained with respect to the failure mode of related critical zones. Based on the $\theta_j(\Delta)$ obtained, the calculation of y_e with Equation (5.11) becomes simple. Firstly, compute the function $\theta_j(\Delta)$ with $\Delta = \Delta_1$ and $\Delta = \Delta_2$ respectively, where Δ_1 , Δ_2 are two arbitrarily given values. Then, determine the displacement $y_{e,j}$ to satisfy $\theta_j(y_{e,j}) = 0$. Since $\theta_j(\Delta)$ is a linearly increasing function, $y_{e,j}$ can be solved with:

$$y_{e,j} = \Delta_1 - \frac{\theta_j(\Delta_1)}{\theta_j(\Delta_2) - \theta_j(\Delta_1)} \times (\Delta_2 - \Delta_1) \quad (5.14)$$

Repeat above calculations for all RPHs ($j = 1 \cdots np$) to gain a vector $\{y_{e,j}\}$. Finally, y_e is acquired by finding the maximum element in $\{y_{e,j}\}$, that is $y_e = \max\{y_{e,j}\}$.

5.3.3.3 Displacement Capacity of the SDOF System - y_u

The displacement capacity y_u of the SDOF system is defined to be the displacement corresponding to the first exceedance of the rotational capacity of the RPHs of the RWCM. The main consideration for such a definition is that the strength of the RWCM will quickly drop as the structural global response Δ surpasses y_u . Thus, it is assumed that the structural collapse begins when Δ reaches y_u . However, it should be pointed out that the progressive collapse of the damaged frame structure is a complicated process. The collapse of the structure may spread over all the structural storeys from the first floor to the roof, or stop somewhere leaving several storeys intact. Therefore, an accurate mathematical definition for structural progressive collapse is almost impossible. $\Delta = y_u$ can only indicate the initiation of such a disaster. Nevertheless, it is adopted as a criterion here for assessing the possibility of structural progressive collapse since it is simple and conservative. According to the definition, y_u is mathematically expressed as:

$$\min\{\theta_{u,j} - \theta_j(y_u)\} = \varepsilon \quad j = 1 \cdots np \quad \varepsilon \rightarrow 0 \quad (5.15)$$

where $\theta_{u,j}$ is the rotational capacity of the j^{th} RPH [D2]. Based on the function $\theta_j(\Delta)$ obtained above, Equation (5.15) can be solved with the same method as that for Equation (5.11).

5.3.3.4 External Work $W_{ext,u}$

The external work $W_{ext,u}$ by the service loadings on the damaged spans at the response level of y_u should be the sum of those on the i^{th} structural storey (i ranging from L_1 to L_2) as:

$$\begin{aligned} W_{ext,u} &= \sum_{i=L_1}^{L_2} (W_{ext,u}^{l,i} + W_{ext,u}^{r,i}) \\ &= \sum_{i=L_1}^{L_2} \left[\int_0^{l_l} q_{l,i}(x) \Delta_{l,i}^u(x) dx + \int_0^{l_r} q_{r,i}(x) \Delta_{r,i}^u(x) dx \right] \end{aligned} \quad (5.16)$$

where $W_{ext,u}^{l,i}$ and $W_{ext,u}^{r,i}$ are the external work by the service loadings on the left and right damaged spans, respectively; l_l and l_r are the corresponding span lengths; $q_{l,i}(x)$ and $q_{r,i}(x)$ denote the distribution of the service loadings; $\Delta_{l,i}^u(x)$ and $\Delta_{r,i}^u(x)$ represent the displacement functions of the beams in the damaged spans when the global response (Δ) of the RWCM reaches y_u . Equation (5.16) demonstrates that $W_{ext,u}$ is dependent on $\Delta_{l,i}^u(x)$ and $\Delta_{r,i}^u(x)$ when l_l , l_r , $q_{l,i}(x)$ and $q_{r,i}(x)$ are available.

Actually, the displacement function of any beam in either the left or the right damaged span ($\Delta_{l,i}^u(x)$ or $\Delta_{r,i}^u(x)$) is a combination of three items, expressed as:

$$\Delta_{l,i}^u(x) = \Delta_{l,i}^{u,1}(x) + \Delta_{l,i}^{u,2}(x) + \Delta_{l,i}^{u,3}(x) \quad (5.17a)$$

$$\Delta_{r,i}^u(x) = \Delta_{r,i}^{u,1}(x) + \Delta_{r,i}^{u,2}(x) + \Delta_{r,i}^{u,3}(x) \quad (5.17b)$$

The shapes of $\Delta_{l,i}^{u,1}(x)$, $\Delta_{l,i}^{u,2}(x)$ and $\Delta_{l,i}^{u,3}(x)$ as well as those of $\Delta_{r,i}^{u,1}(x)$, $\Delta_{r,i}^{u,2}(x)$ and $\Delta_{r,i}^{u,3}(x)$ are plotted in Figure 5.7.

(a) The first item of $\Delta_{l,i}^u(x)$

(b) The second item of $\Delta_{l,i}^u(x)$

(c) The third item of $\Delta_{l,i}^u(x)$

Figure 5.7 Three Items of the Displacement Functions
for the Beams in the Damaged Span

(d) The first item of $\Delta_{r,i}^u(x)$

(e) The second item of $\Delta_{r,i}^u(x)$

(f) The third item of $\Delta_{r,i}^u(x)$

Figure 5.7 (Continued)

The first item of the displacement functions $\Delta_{l,i}^{u,1}(x)$ and $\Delta_{r,i}^{u,1}(x)$ in Figures 5.7(a) and (d) indicates the shape of the beams in the left and the right damaged span due to the elastic deformation under their end section moments $(M_{l,i}^{leftend} - M_{l,i,o}^{leftend}, M_{l,i}^{rightend} - M_{l,i,o}^{rightend})$, $(M_{r,i}^{leftend} - M_{r,i,o}^{leftend}, M_{r,i}^{rightend} - M_{r,i,o}^{rightend})$ and the service loadings $q_{l,i}(x)$, $q_{r,i}(x)$. If denoting the moment distributions of the beams with $M_{l,i}(x)$ and $M_{r,i}(x)$, then $\Delta_{l,i}^{u,1}(x)$ and $\Delta_{r,i}^{u,1}(x)$ can be obtained respectively with:

$$\Delta_{l,i}^{u,1}(x) = \int \left[\int \frac{-M_{l,i}(x)}{EI} dx \right] dx \quad \text{with} \quad \Delta_{l,i}^{u,1}(0) = 0 \quad \text{and} \quad \Delta_{l,i}^{u,1}(l_l) = 0 \quad (5.18a)$$

$$\Delta_{r,i}^{u,1}(x) = \int \left[\int \frac{-M_{r,i}(x)}{EI} dx \right] dx \quad \text{with} \quad \Delta_{r,i}^{u,1}(0) = 0 \quad \text{and} \quad \Delta_{r,i}^{u,1}(l_r) = 0 \quad (5.18b)$$

The second item of the displacement functions $\Delta_{l,i}^{u,2}(x)$ and $\Delta_{r,i}^{u,2}(x)$ in Figures 6.7(b) and (e) shows the rigid rotation of the beams due to the global response of $\Delta = y_u$, which can be written respectively as:

$$\Delta_{l,i}^{u,2}(x) = y_u \times \frac{x}{l_l} \quad (5.19a)$$

$$\Delta_{r,i}^{u,2}(x) = y_u \times \left(1 - \frac{x}{l_r} \right) \quad (5.19b)$$

The third item of $\Delta_{l,i}^{u,3}(x)$ and $\Delta_{r,i}^{u,3}(x)$ in Figures 6.7(c) and (f) is associated with $\theta_{b1}^{II,i}(y_u)$ and $\theta_{b2}^{II,i}(y_u)$, which are the plastic rotations of the RPHs on Beam-1 and Beam-2 in Type-II critical zone at the response level y_u . Therefore, it can be deduced as:

$$\left\{ \begin{array}{l} \Delta_{l,i}^{u,3}(x) = \frac{x \times (l_l - x_{l,i}) \times \theta_{b1}^{II,i}(y_u)}{l_l} \quad (\text{for } x = 0 \rightarrow x_{l,i}) \\ \Delta_{l,i}^{u,3}(x) = \frac{x_{l,i} \times (l_l - x_{l,i}) \times \theta_{b1}^{II,i}(y_u)}{l_l} - \frac{(x - x_{l,i}) \times x_{l,i} \times \theta_{b1}^{II,i}(y_u)}{l_l} \quad (\text{for } x = x_{l,i} \rightarrow l_l) \end{array} \right. \quad (5.20a)$$

$$\left\{ \begin{array}{l} \Delta_{r,i}^{u,3}(x) = \frac{x \times x_{r,i} \times \theta_{b2}^{II,i}(y_u)}{l_r} \quad (\text{for } x = 0 \rightarrow l_r - x_{r,i}) \\ \Delta_{r,i}^{u,3}(x) = (l_r - x_{r,i}) \times \theta_{b2}^{II,i}(y_u) - \frac{x \times (l_r - x_{r,i}) \times \theta_{b2}^{II,i}(y_u)}{l_r} \quad (\text{for } x = x_{r,i} \rightarrow l_r) \end{array} \right. \quad (5.20b)$$

In Equation (5.20), $x_{l,i}$ and $x_{r,i}$ represent the location of the RPHs along the structural beam. $\theta_{b1}^{II,i}(y_u)$ and $\theta_{b2}^{II,i}(y_u)$ can be calculated by substituting $\Delta = y_u$ into the functions listed in Tables 5.2 and 5.3

Substituting Equations (5.17) ~ (5.20) into Equation (5.16), the external work $W_{ext,u}$ corresponding to y_u can be obtained. For the frame structure under uniformly distributed service loadings within each span, the external work on the beams in the left and the right damaged span ($W_{ext,u}^{l,i}$ and $W_{ext,u}^{r,i}$) can be calculated by:

$$W_{ext,u}^{l,i} = q_{l,i} \times \left[\frac{q_{l,i} \times l_l^5}{120EI} + \frac{(M_{l,i}^{leftend} + M_{l,i}^{rightend} - M_{l,i,o}^{leftend} - M_{l,i,o}^{rightend}) \times l_l^3}{24EI} + \frac{y_u \times l_l}{2} + \frac{\theta_{b1}^{II,i}(y_u) \times x_{l,i} \times (l_l - x_{l,i})}{2} \right] \quad (5.21a)$$

$$W_{ext,u}^{r,i} = q_{r,i} \times \left[\frac{q_{r,i} \times l_r^5}{120EI} + \frac{(M_{r,i}^{leftend} + M_{r,i}^{rightend} - M_{r,i,o}^{leftend} - M_{r,i,o}^{rightend}) \times l_r^3}{24EI} + \frac{y_u \times l_r}{2} + \frac{\theta_{b2}^{II,i}(y_u) \times x_{r,i} \times (l_r - x_{r,i})}{2} \right] \quad (5.21b)$$

where $M_{l,i}^{leftend}$, $M_{l,i}^{rightend}$, $M_{r,i}^{leftend}$ and $M_{r,i}^{rightend}$ indicate the end section moments of the beams in the left and right damaged spans respectively, which are dependent on the failure modes of the critical zones as listed in Table 5.2; $M_{l,i,o}^{leftend}$, $M_{l,i,o}^{rightend}$, $M_{r,i,o}^{leftend}$ and $M_{r,i,o}^{rightend}$ are their initial values which can be obtained from the analysis of the initial state of the damaged structure shown in Figure 5.2(a).

After these four parameters (R_m , y_e , y_u and $W_{ext,u}$) are determined, the second performance function z_2 will be calculated by substituting their values into in Equation (5.6).

5.3.4 Solution of the Third Performance Function

In order to consider the structural collapse due to brittle shear failure in the reliability assessment, the third performance function (5.2c) needs to be evaluated. It can be seen that the result of this function depends on the shear strength of structural members' cross sections and the shear force after the formation of the structural weakest collapse mechanism. The “Unified Facilities Criteria (UFC) – Design of Buildings to Resist Progressive Collapse” [D2] recommended the shear strength to be calculated based on the design code ACI 318-02 [A5]. Thus, $V_{r,k}$ in Equation (5.2c) can be written as:

$$V_{r,k} = V_{c,k} + V_{s,k} \quad (5.22)$$

In Equation (5.22), $V_{r,k}$ is the shear strength of the cross section k ; $V_{c,k}$ denotes the shear resistance provided by concrete which is determined by the dimensions of the

cross section, the compressive strength of the concrete material, the shear span-to-depth ratio and the axial force on the member *etc.* In addition, $V_{s,k}$ represents the contribution of the shear reinforcement and can be calculated by:

$$V_{s,k} = \frac{A_{v,k} f_{vy,k} (\sin \alpha_k + \cos \alpha_k) d_k}{s_k} \quad (5.23)$$

where $A_{v,k}$ and $f_{vy,k}$ indicate the area and the yield strength of the shear reinforcement at the cross section k ; d_k is the effective depth equal to the distance from the extreme compression fiber to the centroid of the longitudinal tension bars; s_k is the spacing of the shear reinforcement; α_k is the angle between the shear reinforcement and the member's axis.

Since it is assumed that the weakest collapse mechanism has occurred to the damaged structure, the shear force $V_{f,k}$ in Equation (5.2c) can be obtained from the structural analysis based on the structural model depicted in Figure 4.9 and Table 4.2. Two points need to be pointed out here that:

- 1) The calculation of $V_{f,k}$ does not induce any additional computation work for the reliability assessment since the needed structural analysis has already been carried out in the calculation of the IWCM.
- 2) Although the structural model given in Figures 4.9 and 4.2 is based on the IWCM instead of the RWCM, it does not result in any additional error in $V_{f,k}$. This is because the reduced strength of the ESPH on Beam-2 in Type-II critical zone is computed based on the member's real moment diagram which further leads the moment and shear force distributions of the damaged structure with IWCM to be the same as those of the damaged structure with RWCM.

5.4 Flowchart for Reliability Assessment

Based on the discussion in Section 5.3, the flowchart for the calculation of the performance functions z_1 , z_2 , z_3 and z of the damaged ductile frame structure against progressive collapse under dynamic loading conditions is given in Figure 5.8.

Figure 5.8 Flowchart for the Calculation of the Performance Functions z_1 , z_2 , z_3 and z

When Monte-Carlo Simulation (MCS) method is employed, the flowchart for reliability assessment of the damaged flexural frame structure against progressive collapse under dynamic loading conditions is given in Figure 5.9.

Figure 5.9 Flowchart for Reliability Assessment of the Damaged Structure under Dynamic Loading Conditions

5.5 Numerical Example

The same four-storey three-bay frame structure in Chapter 4 is selected to illustrate the implementation of the approach addressed in this chapter to assess the reliability of the damaged frame structure against progressive collapse accounting for the dynamic effects of the sudden loss of a column and the structural brittle shear failure. Closed stirrups with the area $A_{v,k} = 200 \text{ mm}^2$ are used in all structure members as the shear reinforcement. The stirrups are laid at a spacing of $s_k = 200 \text{ mm}$, while their characteristic yield strength is taken to be 300 MPa .

According to Equation (4.28), the mean value of strength of the shear reinforcement is 345 MPa. As for other data necessary for the structural reliability assessment, they can be found in Figure 4.12 and Tables 4.3 and 4.4.

Two cases of initial column failure (external-column and internal-column) in close-in blast conditions are considered as shown in Figure (4.12). The structure is assumed to require a High Level of Protection, thus based on “Unified Facilities Criteria (UFC) – Design of Buildings to Resist Progressive Collapse” [D2], the rotational capacities of the real plastic hinges $\theta_{u,j}$ are taken to be 4 degree. With Monte Carlo Simulation (MCS), the probabilities of the four performance functions (z_1 , z_2 , z_3 and z) less than zero, $P_f(z_1 < 0)$, $P_f(z_2 < 0)$, $P_f(z_3 < 0)$ and $P_f(z < 0)$, are shown in Figure 5.10 with respect to different numbers of structural samples N_s by Curves 1 ~ 4, respectively.

(a) Case 1 (external column failure initially) (b) Case 2 (internal column failure initially)

Figure 5.10 $P_f(z_1 < 0)$, $P_f(z_2 < 0)$, $P_f(z_3 < 0)$ and $P_f(z < 0)$
versus the Number of Structural Samples

In Figures 5.10(a) and 5.10(b), Curves 1 ~ 3 demonstrate the probability of structural failure by three individual reasons: the lack of strength of the weakest

collapse mechanism to resist the external service loadings ($z_1 < 0$); the lack of deformation capacity of the weakest collapse mechanism to sustain the maximum structural dynamic response ($z_2 < 0$) and the lack of the shear strength of structural members ($z_3 < 0$). On the other hand, Curve 4 reflects the reliability of the damaged frame structure against progressive collapse in a comprehensive manner, where the global performance function z takes the above these three performance functions (z_1 , z_2 and z_3) into account by finding their minimum. Based on Figure 5.10, several observations can be made as follows.

- 1) Curve 3 in Figures 5.10(a) and (b) is located near the horizontal axis with $P_f(z_3 < 0) \approx 0$. By comparing Curve 3 with Curves 1 and 2, it can be obtained that the structural collapse due to the lack of shear strength of structural members is negligible in the reliability assessment of the adopted example of the ductile frame structure damaged with one column removed.
- 2) Curve 2 is significantly higher than Curve 1, which indicates a significant impact of the dynamic effect induced by the sudden loss of a structural column on the reliability of the damaged frame structure against progressive collapse.

For instance, if the dynamic effect is ignored, z_2 will be deleted from the function $z = \min(z_1, z_2, z_3)$, which in turn leads Curve 2 to be deleted from Figure 5.10. As a result, Curve 4 will nearly coincide with Curve 1 considering that $P_f(z_3 < 0) \approx 0$. In this case, the reliability $P_f(z < 0)$ calculated through MCS with 80000 structural samples will approximately be 0.22883 (the right end point of Curve-1 in Figure 5.10(a)) for the damaged frame structure with an external column initially failed. On the other hand, when the dynamic response of the frame is taken into account after the sudden failure of the external column, this value rises up to 0.3771 (the right end point of Curve-4 in Figure 5.10(a)).

- 3) Curve 2 is quite close to Curve 4 as demonstrated in Figures 5.10(a) and (b). As a result it can be concluded that the second performance function plays a dominant role in the structural reliability assessment.

In addition to the results of $P_f(z_1 < 0)$, $P_f(z_2 < 0)$, $P_f(z_3 < 0)$ and $P_f(z < 0)$ discussed above, the distributions of the performance functions (z_1 , z_2 , z_3 and z) are plotted in Figure 5.11. Along with the histograms of z_1 , z_2 , z_3 and z obtained from the MCS with 80000 samples, the normal probability density functions according to the mean values and the coefficients of variation (CoVs) of these parameters are also exhibited in Figure 5.11. Comparing the graphs in Figures 5.11, it can be observed that the CoVs of z_2 is the largest among z_1 , z_2 and z_3 , which are 3.7072 and 0.68133 respectively for Case 1 (external column failure initially) and Case 2 (internal column failure initially). It is reasonable since the dynamic flexural response of the damaged structure controls the structural collapse events. Moreover, the CoVs of z i.e. 8.4028 and 0.69069 respectively for Case 1 and 2, are higher than those of z_2 . It is also seen that the distribution of z has a significant difference from the normal probability density function.

(a). Distribution of z_1 in Case 1

(e). Distribution of z_1 in Case 2

Figure 5.11 Distributions of the Performance Functions (z_1 , z_2 , z_3 and z)

(b). Distribution of z_2 in Case 1

(f). Distribution of z_2 in Case 2

(c). Distribution of z_3 in Case 1

(g). Distribution of z_3 in Case 2

(d). Distribution of z in Case 1

(h). Distribution of z in Case 2

Figure 5.11 (Continued)

5.6 Summary

The method given in Chapters 3 and 4 is further developed in this chapter to perform a reliability assessment of the damaged frame structure accounting for the dynamic effects of the sudden loss of a column. Three different conditions which can lead to structural progressive collapse are considered here: the lack of strength for the weakest collapse mechanism, the lack of deformation capacity for the weakest collapse mechanism, and the lack of the shear strength of structural members.

Two performance functions associated with structural flexural response are formed in the structural reliability assessment. The first function considers the relationship between the strength of the weakest collapse mechanism and the external service loadings in term of internal and external virtual works under $\delta\Delta = 1$. The discussion in Section 5.3.2 reveals that the method given in Chapters 3 and 4 is still valid for solving this function.

The second function determines whether the damaged structural will be subjected to progressive collapse due to excessive dynamic response by comparing the energy dissipation capacity of the structure and the external works at the deformation level of y_u . In order to calculate the second function, the damaged structure is simplified into a single-degree-of-freedom (SDOF) system to dissipate the external work. The parameters including the strength R_m , the yield displacement y_e and the displacement capacity y_u of the SDOF system are determined through the weakest collapse mechanism of the damaged structure and the resistance and rotational capacity of related plastic hinges.

A third performance function is constructed to consider the possibility of the structural collapse due to the lack of shear strength of structural members. The shear strength of members is calculated according to current standards. Assuming that the idealized weakest collapse mechanism has occurred to the damaged frame, the shear

force on structural members is obtained from the structural analysis with the model given in Table 4.2 and Figure 4.9.

Since any of above three performance functions less than zero will lead to structural collapse, they should be considered simultaneously in the reliability assessment. Consequently a global performance function z of the damaged structure is employed, which takes the minimum value of these functions. The reliability of the damaged structure is assessed based on a combination of the Monte Carlo Simulation method and the calculation of this global performance function. The flowchart is given in Figures 5.8 and 5.9.

Implementations of the developed method are demonstrated to assess the reliability of the same example of the four-storey reinforced concrete (RC) frame structure as utilized in Chapter 4 against progressive collapse. The results show that the dynamic effect induced by the sudden loss of structural column plays a significant role during structural progressive collapse while the second performance function proves to be of utmost importance for the reliability assessment of the adopted ductile frame against progressive collapse.

CHAPTER SIX

RELIABILITY ASSESSMENT OF DAMAGED DUCTILE RC FRAME STRUCTURE AGAINST PROGRESSIVE COLLAPSE UNDER DYNAMIC LOADING CONDITIONS

— PART II. PARAMETRIC STUDY

6.1 Introduction

This chapter focuses on the effects of different parameters on the structural reliability against progressive collapse after the initial failure of a certain structural column. Extensive parametric studies are carried out based on the methods discussed in previous chapters. The studies are performed using the four-storey ductile frame structure, which is adopted in Chapters 4 and 5, and is named as the reference structure herein. The dimensions, reinforcement areas, external loadings and the corresponding random properties of the reference structure are given in Sections 4.7 and 5.5. Two cases of initial column failure as plotted in Fig. 4.11(b) and (c) are considered in this chapter. The method for the parametric studies and the considered parameters are addressed in the following section.

6.2 Three Groups of Parametric Studies

There are three interesting topics about the safety of damaged ductile frame structures against progressive collapse. They are the effects of the random properties of different structural parameters on structural reliability, the most efficient way for civil engineers to improve the structural safety and the effects of the storey numbers. Based on these topics, the parametric studies are divided into three sub-groups.

6.2.1 Group I of Parametric Studies

The structural behaviours under external loadings will inevitably exhibit random characteristics due to the random properties of structural parameters and external loadings. As a result, the most scientific way to define structural safety is to use structural reliability measured with the probability of the occurrence of a structural failure event i.e. $P_f(z < 0)$. However, it is expected that different parameters with various random properties will exert different effects on structural reliability. A better understanding of the relations between the random properties of the structural parameters and $P_f(z < 0)$ would be helpful for structural design and assessment.

Generally, the degree of the randomness of a structural parameter can be measured by its coefficient of variation (CoV). The smaller the CoV, the more concentrated the distribution of the parameter is around its mean value. For a parameter with a CoV=0, it becomes a deterministic quantity. There are lots of documents, which adopt statistical studies to determine the COVs of structural parameters [C3, C4, M13, M14, M15, V1, V2, V3]. Their results, summarized in Table 4.4 have been employed in previous chapters. In this chapter, an important part of the effort made is to study the significance of the COVs of the structural parameters on the reliability of damaged ductile frame structures against progressive collapse. As such, the CoVs of different structural parameters are modified independently and the corresponding $P_f(z < 0)$ of the damaged four-storey frame structure is computed after removing the initially failed column. The considered parameters in this group of studies include CoVs of the strength and the elastic modulus of concrete and reinforcing bars, CoVs of the cross sectional effective depth and width of structural members and CoVs of the external loads including the dead load (DL) and the live load (LL), which are listed in Table 6.1.

Table 6.1 Group I of Parameters

(a). CoV of Strength of Reinforcing Bar						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	CS-E1	CS-E2	CS-E3	CS-I1	CS-I2	CS-I3
CoV _{sg}	0.08	0.08	0.08	0.08	0.08	0.08
CoV _{sa}	0.0	0.04	0.08	0.0	0.04	0.08
$r_{cs} = \text{CoV}_{sa} / \text{CoV}_{sg}$	0.0	0.5	1.0	0.0	0.5	1.0
(b). CoV of Structural Members' Cross Sectional Effective Depth						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	CD-E1	CD-E2	CD-E3	CD-I1	CD-I2	CD-I3
CoV _{dg}	0.05	0.05	0.05	0.05	0.05	0.05
CoV _{da}	0.0	0.025	0.05	0.0	0.025	0.05
$r_{cd} = \text{CoV}_{da} / \text{CoV}_{dg}$	0.0	0.5	1.0	0.0	0.5	1.0
(c). CoV of Dead Load (DL)						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	CDL-E1	CDL-E2	CDL-E3	CDL-I1	CDL-I2	CDL-I3
CoV _{DLg}	0.05	0.05	0.05	0.05	0.05	0.05
CoV _{DLa}	0.0	0.025	0.05	0.0	0.025	0.05
$r_{cdl} = \text{CoV}_{DLa} / \text{CoV}_{DLg}$	0.0	0.5	1.0	0.0	0.5	1.0
(d). CoV of Live Load (LL)						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	CLL-E1	CLL-E2	CEC-E3	CLL-I1	CLL-I2	CLL-I3
CoV _{LLg}	0.4	0.4	0.4	0.4	0.4	0.4
CoV _{LLa}	0.0	0.2	0.4	0.0	0.2	0.4
$r_{cll} = \text{CoV}_{LLa} / \text{CoV}_{LLg}$	0.0	0.5	1.0	0.0	0.5	1.0
(e). CoV of Concrete Strength						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	CC-E1	CC-E2	CC-E3	CC-I1	CC-I2	CC-I3
CoV _{cg}	0.13	0.13	0.13	0.13	0.13	0.13
CoV _{ca}	0.0	0.065	0.13	0.0	0.065	0.13
$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0
(f). CoV of Elastic Modulus Coefficient of Concrete						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	CEC-E1	CEC-E2	CEC-E3	CEC-I1	CEC-I2	CEC-I3
CoV _{ecg}	0.15	0.15	0.15	0.15	0.15	0.15
CoV _{eca}	0.0	0.075	0.15	0.0	0.075	0.15
$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0
(g). CoV of Structural Members' Cross Sectional Width						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	CW-E1	CW-E2	CW-E3	CW-I1	CW-I2	CW-I3
CoV _{wg}	0.03	0.03	0.03	0.03	0.03	0.03
CoV _{wa}	0.0	0.015	0.03	0.0	0.015	0.03
$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0

In Table 6.1, the Studied Case (SC) represents the case for which the reliability of the damaged four-storey frame structure is evaluated. CoV_{sg}, CoV_{dg}, CoV_{DLg},

CoV_{LLg} , CoV_{cg} , CoV_{ecg} and CoV_{wg} demonstrate the generally accepted values of the CoVs of related parameters extracted from previous statistic researches. They are listed in the table only for reference. CoV_{sa} , CoV_{da} , CoV_{DLa} , CoV_{LLa} , CoV_{ca} , CoV_{eca} and CoV_{wa} denote the modified values, which are adopted in the parametric studies herein. r_{cs} , r_{cd} , r_{cdl} , r_{cll} , r_{cc} , r_{cec} and r_{cw} are referred as parameter modification ratio (PMR), which calculate the ratio between the modified CoVs used in analyses and their generally accepted values. The PMR indicates the degree of the modification.

6.2.2 Group II of Parametric Studies

The randomness of the structural parameters considered in Group I from the parametric studies conducted are inevitable and are currently not within the limits of control of engineering. It is impractical to improve the structural safety against progressive collapse by limiting their CoVs. Therefore, it becomes pertinent to determine an efficient design procedure to increase the structural reliability of a structure posterior to failure of a column. For this purpose, the effects of the reinforcement area, the mean value of structural members' cross sectional effective depth and the mean value of concrete strength are explored. The related parameters based on the reference frame structure are modified and studied independently. These parameters are listed in Table 6.2.

In Table 6.2, A_r and μ_{dr} denote the reinforcement areas and mean values of cross sectional effective depths of the reference structural beams as given in Table 4.3, while A_a and μ_{da} indicate their modified values adopted in Group II of the parametric studies respectively. The example in Chapter 4 shows that the plastic hinges of the weakest collapse mechanisms for the damaged reference frame structures in Figures 4.11(b) and 4.11(c) are predominantly located on the beams. The modifications of these two parameters are performed only on structural beams. In addition, μ_{cr} and μ_{ca} denote the mean value of concrete strength in the reference structure and its modified values employed in the related cases.

Table 6.2 Group II of Parameters

(a). Reinforcement Areas of Structural Beams						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	MA-E1	MA-E2	MA-E3	MA-I1	MA-I2	MA-I3
A_r	Table 4.3	Table 4.3	Table 4.3	Table 4.3	Table 4.3	Table 4.3
A_a	$0.95 \times A_r$	$1.0 \times A_r$	$1.05 \times A_r$	$0.95 \times A_r$	$1.0 \times A_r$	$1.05 \times A_r$
$r_{ma} = A_a / A_r$	0.95	1.0	1.05	0.95	1.0	1.05
(b). Mean Values of Structural Beams' Cross Sectional Effective Depths						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	MD-E1	MD-E2	MD-E3	MD-I1	MD-I2	MD-I3
μ_{dr}	Table 4.3	Table 4.3	Table 4.3	Table 4.3	Table 4.3	Table 4.3
μ_{da}	$0.95 \times \mu_{dr}$	$1.0 \times \mu_{dr}$	$1.05 \times \mu_{dr}$	$0.95 \times \mu_{dr}$	$1.0 \times \mu_{dr}$	$1.05 \times \mu_{dr}$
$r_{md} = \mu_{da} / \mu_{dr}$	0.95	1.0	1.05	0.95	1.0	1.05
(c). Mean Value of Concrete Strength						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied Case	MC-E1	MC-E2	MC-E3	MC-I1	MC-I2	MC-I3
μ_{cr}	38	38	38	38	38	38
μ_{ca}	36.1	38	39.9	36.1	38	39.9
$r_{mc} = \mu_{ca} / \mu_{cr}$	0.95	1.0	1.05	0.95	1.0	1.05

6.2.3 Group III of Parametric Studies

The third group of the parametric studies focuses on the effect of the number of structural storeys on resistance against progressive collapse. A five-storey frame structure is built up with its first four storeys to be exactly the same as the reference structure. However, the reinforcement areas of the beams in the fifth storey will be varied from 0.9 to 1.1 times of those of the corresponding beams' sections in the lower storeys. The rationale behind the variation is to accommodate for the fact that frame structures are rarely designed in a manner where all structural members in different storeys are identically reinforced. The studied cases are tabulated in Table 6.3.

Table 6.3 Group III of Parameter

Reinforcement Areas of Structural Beams in the Fifth Storey						
Initially failed column	External column-Fig. 6.1(b)			Internal column-Fig. 6.1(c)		
Studied case	SA-E1	SA-E2	SA-E3	SA-I1	SA-I2	SA-I3
A_{r4}	Table 4.3	Table 4.3	Table 4.3	Table 4.3	Table 4.3	Table 4.3
A_{a5}	$0.9 \times A_{r4}$	$1.0 \times A_{r4}$	$1.1 \times A_{r4}$	$0.9 \times A_{r4}$	$1.0 \times A_{r4}$	$1.1 \times A_{r4}$
$r_{5ma} = A_{a5} / A_{r4}$	0.9	1.0	1.1	0.9	1.0	1.1

In Table 6.3, A_{r4} represents the reinforcement areas of the first four storeys of the newly built frame structure, the numerical value of which can be found in Table 4.3. In addition, A_{a5} represents the beam's reinforcement areas in the fifth storey. The intact five-storey frame and the two cases of damaged frame with different column failure are plotted in Fig. 6.1.

(a). The intact frame structure

(b). Case 1: External column failure

(c). Case 2: Internal column failure

Figure 6.1 A Five-storey Frame Structure

6.3 Results and Discussions

The reliability of the damaged structure from the parametric studies listed in Tables 6.1 ~ 6.3 is assessed with the Monte Carlo Simulation (MCS) method as developed from the previous chapters. In each assessment, 80000 samples are applied to achieve a convergent simulation. The effects of the parameters considered are investigated based on the results generated by the MCS method and analyzed from two aspects. Namely these are their effects on the critical zones' failure mode which occurs in the weakest collapse mechanism of the structure and their effects on structural reliability against progressive collapse.

For the first aspect, the failure modes of the critical zones which occur in the weakest collapse mechanism are recorded for all samples. According to these records, the occurrence numbers of different failure modes are counted and enumerated in Tables 6.4 ~ 6.7, Tables 6.9 ~ 6.12, Tables 6.14 ~ 6.19, Tables 6.21 ~ 6.26 and Tables 6.28 ~ 6.29 for each critical zone.

As for the second aspect, Tables 6.8, 6.13, 6.20, 6.27 and 6.30 list the calculated values of $P_f(z < 0)$, where z is the performance function as defined in Equation (5.2d). In order to quantify the effect of the parameter on structural reliability explicitly, two factors, namely, the average sensitivity factor (ASF) and the average variation ratio (AVR) of $P_f(z < 0)$ are evaluated as follows.

$$ASF = \frac{\left| P_f(z < 0, PMR = x_2) - P_f(z < 0, PMR = x_1) \right|}{x_2 - x_1} \quad (6.1a)$$

$$AVR = \frac{P_f(z < 0, PMR = x_2) / P_f(z < 0, PMR = x_1) - 1}{x_2 - x_1} \quad (6.1b)$$

In Equations (6.1), $x_1 = 0, 0.95$ and 0.9 , respectively, in Group I, II and III parametric studies, while $x_2 = 1, 1.05$ and 1.1 correspondingly. $P_f(z < 0, PMR = x)$ denotes $P_f(z < 0)$ under the related value of PMR. From Equation (6.1), it is clear that the larger the ASF or the greater the distance between AVR and zero, the greater the significance of the effect of the parameter to the structural reliability against progressive collapse. In addition to these two factors, the curves of $P_f(z < 0)$ with respect to PMR are plotted in Figures 6.2 ~ 6.12 to show the effect of the parameters as well.

6.3.1 Group I of Parametric Studies

The results of Group I parametric studies are discussed for CoV_{sa} and CoV_{da} , CoV_{DLa} and CoV_{LLa} , CoV_{ca} , CoV_{eca} and CoV_{wa} respectively according to the above two aspects of the effects of the parameter on the damaged frame structure against progressive collapse.

For CoV_{sa} and CoV_{da} , the occurrence numbers of different failure modes of each critical zone recorded in the MCSs with different values of CoV_{sa} and CoV_{da} are presented in Tables 6.4 to 6.7. By comparison, it can be seen that CoV_{sa} and CoV_{da} show to have important influence on the occurrence of the failure modes for two critical zones. They are, the Type-II critical zone in the 1st storey of the damaged frame structure with the initial failure of an external column and the Type-I critical zone in the 4th storey of the damaged frame with the initial failure of an internal column. For example, in the parametric study of CoV_{sa} , the occurrence number for the first failure mode of the Type-I critical zone in the 4th storey increases significantly from 735 to 2141 to 6737 as shown in Table 6.5 for the cases of CS-I1, CS-I2 and CS-I3 with CoV_{sa} increasing from 0 to 0.04 to 0.08, while the number of the second failure mode of the same critical zone decreases from 79265 to 77859 to 73263.

It can be explained as follows. The above two critical zones are constructed by one column connecting with one beam. Since the frame structure is designed in such a manner with the column's strength to some extent higher than the beam's strength, the probability of the plastic hinge forming on the beam shall be greater than that on the column when the weakest collapse mechanism is produced. It is demonstrated by the obviously larger occurrence numbers of the second failure mode in these two critical zones. However, the flexural strength of the structural members is considerably dependent on the strength of the reinforcing bars and their cross sectional effective depth. With the CoV_{sa} or CoV_{da} enlarged, the CoV of the flexural strength of the members will be amplified as well. It leads to a higher occurrence possibility of the column plastic hinge corresponding to the increased occurrence numbers of the failure modes other than the second mode in these zones to generate the collapse mechanism. For the convenience of later discussion, the critical zone constructed by one beam connecting with one column is named two member critical zone (TM critical zone).

The effects of CoV_{sa} and CoV_{da} on structural reliability, the $P_f(z < 0)$ obtained from the MCS method for related studied cases, as well as the corresponding ASFs and AVRs calculated with Equation (6.1) are given in Table 6.8. It can be seen that for the damaged ductile frame structure with initial external or internal column failure, the ASF and AVR resulted from the studies of CoV_{da} are 0.0259 and 0.0737 or 0.0220 and 0.4239 respectively. This indicates that the $P_f(z < 0)$ will be increased either by a significant magnitude of up to 0.0259 or by a significant ratio of up to 42.39% as the PMR rises from 0 to 1 (CoV_{da} from 0 to 0.05). An interesting point, which should be noted here, is that although the ASF (0.0220) calculated in case of the structure damaged with initial internal column failure is smaller than the ASF (0.0259) in the case when the structure damaged with external column failure. The AVR (0.4239) in the former structural damage case is obviously larger than the AVR (0.0737) in the later case. This is due to the far smaller values of $P_f(z < 0)$ in CD-I1, CD-I2 and CD-I3 as compared with those in CD-E1, CD-E2 and CD-E3. Therefore, if only ASF is employed in the study of the parameter's effect on

structural reliability, some important information may be ignored. As for the effect of CoV_{sa} , Table 6.8 shows that the values of ASF and AVR obtained in the parametric studies of CoV_{sa} are close to those obtained in the studies of CoV_{da} , which in turn demonstrates the similar influence of CoV_{sa} .

Besides Table 6.8, Figures 6.2 and 6.3 plot the curves of $P_f(z < 0)$ versus PMRs which are denoted by r_{cs} and r_{cd} in the studies of CoV_{sa} and CoV_{da} , respectively. The quick increases of the curves with r_{cs} and r_{cd} also exhibit the significance of CoV_{sa} and CoV_{da} on $P_f(z < 0)$.

Table 6.4 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of CoV_{sa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CS-E1	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CS-E2	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CS-E3	Type-II	0 / 79967 / 33	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.5 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of CoV_{sa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CS-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	735 / 79265 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CS-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	2141 / 77859 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CS-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6737 / 73263 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.6 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of CoV_{da})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CD-E1	Type-II	0 / 79996 / 4	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CD-E2	Type-II	0 / 79986 / 14	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CD-E3	Type-II	0 / 79968 / 32	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.7 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of CoV_{da})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CD-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	3788 / 76212 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CD-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	4561 / 75439 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CD-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6834 / 73166 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.8 $P_f(z < 0)$ in the Parametric Studies of CoV_{sa} and CoV_{da}

(a). CoV of Strength of Reinforcing Bar						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	CS-E1	CS-E2	CS-E3	CS-I1	CS-I2	CS-I3
$r_{cs} = \text{CoV}_{sa} / \text{CoV}_{sg}$	0.0	0.5	1.0	0.0	0.5	1.0
$P_f(z < 0)$	0.3486	0.3565	0.3740	0.0486	0.0545	0.0736
ASF	0.0254			0.0250		
AVR	0.0729			0.5144		
(b). CoV of Structural Members' Cross Sectional Effective Depth						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	CD-E1	CD-E2	CD-E3	CD-I1	CD-I2	CD-I3
$r_{cd} = \text{CoV}_{da} / \text{CoV}_{dg}$	0.0	0.5	1.0	0.0	0.5	1.0
$P_f(z < 0)$	0.3512	0.3575	0.3771	0.0519	0.0583	0.0739
ASF	0.0259			0.0220		
AVR	0.0737			0.4239

Figure 6.2 Effects of CoV_{sa} on $P_f(z < 0)$

Figure 6.3 Effects of CoV_{da} on $P_f(z < 0)$

For CoV_{DLa} and CoV_{LLa} , the occurrence numbers obtained from MCS for different failure mode of each critical zone are tabulated in Tables 6.9 to 6.12. It can be seen that although CoV_{DLa} and CoV_{LLa} have been remarkably modified from 0 to 0.05 and from 0 to 0.4, respectively, their effects on critical zones' failure modes in producing the structural collapse mechanism are almost negligible. Only slight variations on the occurrence numbers of the failure modes for the TM critical zones can be observed. For example, Table 6.10 shows that the occurrence numbers of the first failure mode of the Type-I critical zone on the 4th storey of the damaged frame are 6712, 6849 and 6741 for the cases CDL-I1, CDL-I2 and CDL-I3, respectively, with the CoV_{DLa} being 0, 0.025 and 0.05. This variation could be due to the random sampling in the MCS, and is ignorable as compared to those observed in the parametric studies of CoV_{sa} and CoV_{da} . The minor influence of CoV_{DLa} and CoV_{LLa} on this aspect is understandable since the failure mode of the critical zones composing the structural collapse mechanism is predominately determined by the relative flexural strength of the connected members as illustrated in the Equations (3.24), (3.30) and (3.35).

As for the effects of CoV_{DLa} and CoV_{LLa} on structural reliability, Table 6.13 lists the values of $P_f (z < 0)$ calculated from the MCS for the related cases included in the parametric studies of CoV_{DLa} and CoV_{LLa} , as well as the corresponding ASFs and AVRs. In addition, Figures 6.4 ~ 6.5 plot the curves of $P_f (z < 0)$ with respect to r_{cdl} or r_{ccl} , respectively. Three points can be observed:

- 1) CoV_{LLa} plays a substantially important role in structural reliability. As the PMR varies from 0 to 1 (CoV_{LLa} from 0 to 0.4), the value of $P_f (z < 0)$ rises from 0.3281 to 0.3743 or from 0.0136 to 0.0734 for the damaged frame structure with external or internal column failure. Correspondingly, the ASFs are evaluated to be 0.0462 and 0.0598 for the two column failure cases, while the AVRs are 0.1408 and 4.3971. It indicates a major increase

of up to 0.0598 or by 439.7% in $P_f(z < 0)$ for the damaged structure in different column failure cases.

- 2) The ASF obtained in the studies of CoV_{DLa} with the structure damaged by either external or internal column failure is 0.0061 or 0.0053, respectively. The magnitudes are rather small as compared to the ASFs evaluated in the study of CoV_{LLa} . In addition, the AVR (0.0165) obtained in case of the external column failure is also unimportant. However, the AVR (0.0774) calculated while the frame structure is assumed to be damaged with internal column failure still shows the significant influence of CoV_{DLa} on structural reliability. It indicates that a considerable increase of 7.74% can be expected for $P_f(z < 0)$ with CoV_{DLa} varying from 0 to 0.05.
- 3) Comparing the ASFs and AVRs in both studies of CoV_{DLa} and CoV_{LLa} demonstrates that the influence of CoV_{LLa} on structural reliability against progressive collapse is by far more prominent for the four-storey frame structure. This phenomenon is also shown by the much steeper slopes of the curves of $P_f(z < 0)$ in Figures 6.5 than those in Figure 6.4. It is because of the large difference between the generally accepted values of CoV_{DLg} (0.05) and CoV_{LLg} (0.4). Although the same degree of modification for these two parameters (r_{cdl} and $r_{\text{cll}} = 0$ to 0.5 to 1.0) is adopted in the studies, the absolute variations of CoV_{LLa} ($0 \rightarrow 0.4$) are much greater than that of the CoV_{DLa} ($0 \rightarrow 0.05$).

Table 6.9 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of CoV_{DLa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CDL-E1	Type-II	0 / 79955 / 45	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CDL-E2	Type-II	0 / 79962 / 38	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CDL-E3	Type-II	0 / 79950 / 50	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.10 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of CoV_{DLa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CDL-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6712 / 73288 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CDL-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6849 / 73151 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CDL-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6741 / 73259 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.11 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of CoV_{LLa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CLL-E1	Type-II	0 / 79961 / 39	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CLL-E2	Type-II	0 / 79957 / 43	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CLL-E3	Type-II	0 / 79957 / 43	0 / 80000 / 0	0 / 80000 / 0	0 / 79997 / 3
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.12 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of CoV_{LLa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CLL-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6859 / 73141 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CLL-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6859 / 73141 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CLL-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6792 / 73208 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.13 $P_f(z < 0)$ in the Parametric Studies of CoV_{DLa} and CoV_{LLa}

(c). CoV of Dead Load (DL)						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	CDL-E1	CDL-E2	CDL-E3	CDL-I1	CDL-I2	CDL-I3
$r_{cdl} = \text{CoV}_{\text{DLa}} / \text{CoV}_{\text{DLg}}$	0.0	0.5	1.0	0.0	0.5	1.0
$P_f(z < 0)$	0.3687	0.3720	0.3748	0.0685	0.0696	0.0738
ASF	0.0061			0.0053		
AVR	0.0165			0.0774		

(d). CoV of Live Load (LL)						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	CLL-E1	CLL-E2	CEC-E3	CLL-I1	CLL-I2	CLL-I3
$r_{cdl} = \text{CoV}_{\text{DLa}} / \text{CoV}_{\text{DLg}}$	0.0	0.5	1.0	0.0	0.5	1.0
$P_f(z < 0)$	0.3281	0.3470	0.3743	0.0136	0.0281	0.0734
ASF	0.0462			0.0598		
AVR	0.1408			4.3971

(a) Damaged Frame Structure with External Column Failure

(b) Damaged Frame Structure with Internal Column Failure

Figure 6.4 Effects of CoV_{DLa} on $P_f(z < 0)$

Figure 6.5 Effects of CoV_{LLa} on $P_f(z < 0)$

For CoV_{ca} , CoV_{eca} and CoV_{wa} , the occurrence numbers of the failure modes are tabulated in Tables 6.14 to 6.19 for all critical zones from the cases studied. In addition, the values of $P_f(z < 0)$ as well as the calculated ASFs and AVRs are listed in Table 6.20. Based on these results, it is clear that CoV_{ca} , CoV_{eca} and CoV_{wa} do not have a remarkable influence on both the failure modes of critical zones generating the structural collapse mechanism and the structural reliability against progressive collapse. Only the TM critical zones experience slight variations on the occurrence numbers of the failure modes. The maximum magnitudes of ASF and AVR obtained in the studies of these three parameters are 0.0022 and 0.0166 respectively, as listed in Table 6.20. It indicates that the increase of $P_f(z < 0)$, as PRM varies from 0 to 1, is not greater than 0.0022 or 1.66% for the four-storey frame structure damaged with initial failure of different columns. The insignificance of CoV_{ca} , CoV_{eca} and CoV_{wa} on structural reliability against progressive collapse is also reflected by the curves of $P_f(z < 0)$ versus PMR (r_{cc} , r_{cec} or r_{cw}), which appear nearly horizontal lines as plotted in Figures 6.6 ~ 6.8.

Table 6.14 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of CoV_{ca})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CC-E1	Type-II	0 / 79953 / 47	0 / 80000 / 0	0 / 80000 / 0	0 / 79997 / 3
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CC-E2	Type-II	0 / 79962 / 38	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CC-E3	Type-II	0 / 79958 / 42	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.15 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of CoV_{ca})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CC-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6747 / 73253 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CC-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6866 / 73134 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CC-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6863 / 73137 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.16 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of CoV_{eca})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CEC-E1	Type-II	0 / 79958 / 42	0 / 80000 / 0	0 / 80000 / 0	0 / 79997 / 3
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CEC-E2	Type-II	0 / 79951 / 49	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CEC-E3	Type-II	0 / 79962 / 38	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.17 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of CoV_{eca})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CEC-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6935 / 73065 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CEC-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6765 / 73235 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CEC-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6788 / 73212 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.18 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of CoV_{wa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CW-E1	Type-II	0 / 79965 / 35	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CW-E2	Type-II	0 / 79958 / 42	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CW-E3	Type-II	0 / 79963 / 37	0 / 80000 / 0	0 / 80000 / 0	0 / 79996 / 4
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.19 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of CoV_{wa})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
CW-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6920 / 73080 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CW-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6879 / 73121 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
CW-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6738 / 73262 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.20 $P_f(z < 0)$ in the Parametric Studies of CoV_{ca} , CoV_{eca} and CoV_{wa}

(e). CoV of Concrete Strength						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	CC-E1	CC-E2	CC-E3	CC-I1	CC-I2	CC-I3
$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0
$P_f(z < 0)$	0.3738	0.3739	0.3746	0.0728	0.0730	0.0739
ASF	0.0008			0.0011		
AVR	0.0021			0.0151		
(f). CoV of Elastic Modulus Coefficient of Concrete						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	CEC-E1	CEC-E2	CEC-E3	CEC-I1	CEC-I2	CEC-I3
$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0
$P_f(z < 0)$	0.3734	0.3740	0.3756	0.0724	0.0729	0.0736
ASF	0.0022			0.0012		
AVR	0.0059			0.0166		
(g). CoV of Structural Members' Cross Sectional Width						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	CW-E1	CW-E2	CW-E3	CW-I1	CW-I2	CW-I3
$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0
$P_f(z < 0)$	0.3748	0.3751	0.3758	0.0729	0.0730	0.0735
ASF	0.0010			0.0006		
AVR	0.0027			0.0082

(a) Damaged Frame Structure with External Column Failure

(b) Damaged Frame Structure with Internal Column Failure

Figure 6.6 Effects of CoV_{ca} on $P_f(z < 0)$

Figure 6.7 Effects of CoV_{eca} on $P_f(z < 0)$

Figure 6.8 Effects of CoV_{wa} on $P_f(z < 0)$

6.3.2 Group II of Parametric Studies

The objective of Group II parametric study is to find the most efficient design method to improve structural reliability against progressive collapse. The studied parameters and cases are listed in Table 6.2. The results of this group of studies obtained from MCS will be demonstrated and discussed here.

Tables 6.21 ~ 6.26 present the occurrence numbers of all critical zones' failure modes recorded from MCS with 8×10^4 samples for all the cases given in Table 6.2. It can be seen that by modifying the parameters of A_a (reinforcement area in beams) and μ_{da} (mean value of beam's effective depth) with r_{ma} and r_{md} from 0.95 to 1.00 to 1.05, the occurrence numbers of different failure modes for the TM critical zones vary obviously. This phenomenon is due to the determining position of A_a and μ_{da} on flexural strength of structural members. For example, in the cases of MA-I1, MA-I2 and MA-I3, the flexural strength of the beam in the Type-I critical zone on the 4th storey will be evidently improved as A_a rises from $0.95 A_r$ to $1.0 A_r$ to $1.05 A_r$. Consequently, the probability of a plastic hinge forming on the connected column becomes much higher in producing the collapse mechanism, which further leads to the occurrence numbers of the first failure mode increase apparently from 3230 to 6836 to 12166. In contrast, Tables 6.25 and 6.26 exhibit the limited influence of μ_{ca} (mean value of concrete strength) on the failure modes of critical zones.

For the case MA-I3, the higher possibility of plastic hinge forming in column will increase the likelihood of a sway mode of structural failure. However, considering the low potential that a denotation will occur together with an event which can induce significant horizontal force (such as seismic), this structural failure mode is ignored in this study.

As for the effects of A_a , μ_{da} and μ_{ca} on structural reliability, the values of $P_f(z < 0)$ obtained from MCS with 8×10^4 samples as well as the ASFs and AVRs are given in Table 6.27. It is clear that A_a and μ_{da} can exert similarly drastic impact on $P_f(z < 0)$. In case of the four-storey frame structure damaged with external column failure, the corresponding ASFs are 4.422 and 5.114 respectively, indicating the decreases of more than 0.4 in the magnitude of $P_f(z < 0)$ with only about 10% increase of A_a or μ_{da} (from $0.95 A_r$ to $1.05 A_r$ or from $0.95 \mu_{dr}$ to $1.05 \mu_{dr}$). While in case of the structure damaged with internal column failure, the AVRs are -9.507 and -9.748 respectively, which demonstrate that $P_f(z < 0)$ is reduced by a ratio of more than 90% from MA-I1 to MA-I3 or from MD-I1 to MD-I3.

On the other hand, the benefit on the structural reliability against progressive collapse by increasing μ_{ca} is limited as compared to that by increasing A_a or μ_{da} . It can be seen that only in case of the frame structure damaged with internal column failure, a considerable decrease ratio of 9.02% ($= 10\% \times \text{AVR}$) in $P_f(z < 0)$ is attained with an increase in μ_{ca} from $0.95 \mu_{cr}$ to $1.05 \mu_{cr}$.

Table 6.21 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of A_a)

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
MA-E1	Type-II	0 / 79991 / 9	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MA-E2	Type-II	0 / 79953 / 47	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MA-E3	Type-II	0 / 79848 / 152	0 / 80000 / 0	0 / 80000 / 0	0 / 79990 / 10
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.22 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of A_d)

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
MA-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	3230 / 76770 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MA-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6836 / 73164 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MA-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	12166 / 67834 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.23 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of μ_{da})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
MD-E1	Type-II	0 / 79989 / 11	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MD-E2	Type-II	0 / 79958 / 42	0 / 80000 / 0	0 / 80000 / 0	0 / 79997 / 3
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MD-E3	Type-II	0 / 79855 / 145	0 / 80000 / 0	0 / 80000 / 0	0 / 79994 / 6
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.24 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of μ_{da})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
MD-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	2944 / 77056 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MD-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6937 / 73063 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MD-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	13045 / 66955 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.25 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Parametric Study of μ_{ca})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
MC-E1	Type-II	0 / 79962 / 38	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MC-E2	Type-II	0 / 79955 / 45	0 / 80000 / 0	0 / 80000 / 0	0 / 79999 / 1
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MC-E3	Type-II	0 / 79962 / 38	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.26 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Parametric Study of μ_{ca})

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)			
		Storey 1	Storey 2	Storey 3	Storey 4
MC-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6677 / 73323 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MC-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6708 / 73292 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
MC-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	7056 / 72944 / -
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.27 $P_f(z < 0)$ for Group II of Parametric Studies

(a). Reinforcement Areas of Structural Beams						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	MA-E1	MA-E2	MA-E3	MA-I1	MA-I2	MA-I3
$r_{ma} = A_a / A_r$	0.95	1.0	1.05	0.95	1.0	1.05
$P_f (z < 0)$	0.6266	0.3746	0.1844	0.2739	0.0739	0.0135
ASF	4.422			2.604		
AVR	-7.057			-9.507		
(b). Mean Values of Structural Beams' Cross Sectional Effective Depths						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	MD-E1	MD-E2	MD-E3	MD-I1	MD-I2	MD-I3
$r_{md} = \mu_{da} / \mu_{dr}$	0.95	1.0	1.05	0.95	1.0	1.05
$P_f (z < 0)$	0.6705	0.3738	0.1591	0.3413	0.0736	0.0086
ASF	5.114			3.327		
AVR	-7.627			-9.748		
(c). Mean Value of Concrete Strength						
Initially failed column	External column-Fig. 4.11(b)			Internal column-Fig. 4.11(c)		
Studied case	MC-E1	MC-E2	MC-E3	MC-I1	MC-I2	MC-I3
$r_{mc} = \mu_{ca} / \mu_{cr}$	0.95	1.0	1.05	0.95	1.0	1.05
$P_f (z < 0)$	0.3816	0.3756	0.3723	0.0765	0.0735	0.0696
ASF	0.093			0.069		
AVR	-0.2437			-0.902

(a) Damaged Frame Structure with External Column Failure

(b) Damaged Frame Structure with Internal Column Failure

Figure 6.9 Effects of A_a on $P_f(z < 0)$

Figure 6.10 Effects of μ_{da} on $P_f(z < 0)$

Figure 6.11 Effects of μ_{ca} on $P_f(z < 0)$

6.3.3 Group III of Parametric Studies

The purpose of Group III parametric study is to investigate the effect of the number of structural storeys on its reliability against progressive collapse. The results from the MCS for the involved cases listed in Table 6.3 are given as follows.

The occurrence numbers of different failure mode of the critical zones are recorded in Tables 6.28 and 6.29. It can be seen that the significant variations of the failure modes' occurrence numbers only appear for the TM critical zone in the roof storey of the damaged five-storey frame structure (i.e. the Type-II critical zone in the 5th storey when the frame is damaged with external column failure or the Type-I critical zone in the 5th storey when the frame is damaged with internal column failure). This variation is due to the increase of the reinforcement area in the beams of the 5th storey with r_{5mp} being 0.9, 1.0 and 1.1 for the related cases.

In addition, the results of $P_f(z < 0)$ for different studied cases involved in Group III parametric study are enumerated in Table 6.30. The comparison of $P_f(z < 0)$ between the cases MA-E2 and SA-E2 or between the cases MA-I2 and SA-I2 reveals that if the additional 5th storey is designed as robust as the lower four storeys (PMR=1), the increase of the storey number of the ductile frame structure from four to five has obvious positive benefits on structural reliability against progressive collapse. The value of $P_f(z < 0)$ decreases by about 16.3% from 0.3746 (MA-E2) to 0.3134 (SA-E2) in the case of external column failure or by about 25.3% from 0.0739 (MA-I2) to 0.0552 (SA-I2) in the case of internal column failure. This positive effect of the presence of the additional 5th storey can be explained from two points: 1) it facilitates the development of Vierendeel (frame) action in the frame; 2) it also helps reducing the CoV of $W_{int,u}$ and $W_{ext,u}$.

However, the parametric studies in this group also demonstrated that if the members in the additional 5th storey are not designed as strong as those in the other four

storeys (as in SA-E1 or SA-I1), the global structural reliability will apparently be diminished by this weaker storey. For example, in SA-E1 where the beams' reinforcement area in the 5th storey is reduced by only 10% from that in SA-E2, the occurrence probability of the structural collapse event $P_f(z < 0)$ resulted by the MCS dramatically rose from 0.3134 to 0.4140. Comparing the values of $P_f(z < 0)$ obtained among the cases of SA-E1, SA-E2 and MA-E2 or among the cases of SA-I1, SA-I2 and MA-I2, which are 0.4140, 0.3134 and 0.3746 or 0.1106, 0.0552 and 0.0739 respectively, it can be seen that the increase of structural storeys produce obvious positive effects on structural reliability against progressive collapse. However, if the members of the increased storey are not as robust as those of other storeys, the negative influence may counteract with the positive effects, resulting in an even lower structural reliability for the damaged frame as compared to that of the structure without the increased storey. On the other hand, if the members of the increased storey are more robustly designed than other storeys, the reliability of the damaged structure would be considerably improved as shown by SA-E3 and SA-I3 with the corresponding $P_f(z < 0)$ reduced to 0.2317 and 0.0256, respectively.

The ASFs and AVRs calculated with the damaged five-storey frame structure with the reinforcement areas in its roof storey beams increasing from $0.9A_{r4}$ to $1.1A_{r4}$ are given in Table 6.30. The considerable absolute magnitude of these two factors (0.9115 and 2.2017 in the case of external column failure or 0.4250 and 3.8427 in the case of internal column failure) do illustrate the important influence of the robustness of the increased 5th storey on structural global reliability.

Table 6.28 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with External Column Failure in Group III Parametric Study)

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)				
		Storey 1	Storey 2	Storey 3	Storey 4	Storey 5
SA-E1	Type-II	0 / 79958 / 42	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
SA-E2	Type-II	0 / 79951 / 49	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 79998 / 2
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
SA-E3	Type-II	0 / 79962 / 38	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 79953 / 47
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.29 Occurrence Numbers of Different Failure Modes of Critical Zones
(Damaged Structure with Internal Column Failure in Group III Parametric Study)

Studied case	Critical zone	Occurrence numbers of failure modes (The first mode / The second mode / The third mode)				
		Storey 1	Storey 2	Storey 3	Storey 4	Storey 5
SA-I1	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	1344/78656/-
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
SA-I2	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	6878/73122/-
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -
SA-I3	Type-I	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	0 / 80000 / -	19430/60570/-
	Type-II	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0	0 / 80000 / 0
	Type-III	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -	80000 / 0 / -

Table 6.30 $P_f(z < 0)$ for Group III of Parametric Studies

Reinforcement Areas of Structural Beams in the Fifth Storey						
Initially failed column	External column-Fig. 6.1(b)			Internal column-Fig. 6.1(c)		
Studied case	SA-E1	SA-E2	SA-E3	SA-I1	SA-I2	SA-I3
$r_{5ma} = A_{a5} / A_{r4}$	0.9	1.0	1.1	0.9	1.0	1.1
$P_f(z < 0)$	0.4140	0.3134	0.2317	0.1106	0.0552	0.0256
ASF	0.9115			0.4250		
AVR	-2.2017			-3.8427

Figure 6.12 Effects of A_{a5} on $P_f(z < 0)$

6.4 Summary

Three groups of parametric studies are carried out in this chapter investigating the effects of different parameters on structural reliability against progressive collapse. The results of the parametric studies are discussed from two perspectives: the effects of the related parameter on the failure mode of critical zones and on the

structural reliability measured by $P_f (z < 0)$. Based on the studies, several points can be summarized as follows.

The CoVs of the reinforcing bar's strength and the structural members' effective depth have important influences on both the failure modes of the TM critical zones in generating the structural weakest collapse mechanism and the structural reliability. The ASF and AVR calculated based on the parametric studies of CoV_{sa} and CoV_{da} are given in Table 6.8 with their maximum values being 0.0259 and 0.5144 respectively. The CoVs of the external loadings (DL and LL) are significant only for $P_f (z < 0)$. The minor effect of CoV_{DLa} and CoV_{LLa} on the failure mode of the critical zone is understandable since the failure mode of the critical zone occurring in the structural weakest collapse mechanism is determined by the relative flexural strength of the connected members as shown in Equations (3.24), (3.30) and (3.35). As for the CoVs of the strength and the elastic modulus of concrete, the width of the structural members, their effect on the damaged ductile frame structures against progressive collapse is almost ignorable. According to the Group I parametric studies, if the time consuming structural reliability assessment is not available for civil engineers and only the deterministic design method is applicable, conservative factors shall be considered at least for the structural parameters including the strength of the reinforcing bars, effective depth of structural members, the DL and the LL, in the design of the frame structure so as to obtain a reasonable level of confidence about its structural safety after a certain column has initially failed.

The results of the Group II parametric studies show that the most efficient method to improve the structural reliability against progressive collapse is to increase the structural member's reinforcement area A_a or its effective depth μ_{da} . For the adopted four-storey frame structure in both external and internal column initial failure cases, the values of ASF and AVR listed in Table 6.27 indicate the major decrease of $P_f (z < 0)$ as the PRM increases from 0.95 to 1.0. The structural reliability can also be improved by the increase of concrete strength. However,

compared to A_a and μ_{da} , the same percentage increase in concrete strength μ_{ca} will lead to a far less significant drop in $P_f(z < 0)$ as denoted by the corresponding smaller absolute values of ASF and AVR.

For the adopted ductile frame structures, Group III of the parametric studies demonstrates that an increase in the number of structural storeys from 4 to 5 results in a decrease of $P_f(z < 0)$, when the members in the 5th storey are identically reinforced as those in the first four storeys. In addition, the results of the Group III parametric studies also show the effect of the robustness of a single storey on structural reliability against progressive collapse. The occurrence probability of structural collapse event $P_f(z < 0)$ increases remarkably with the decrease of A_{r5} from $1.05A_{r4}$ to $1.0A_{r4}$ to $0.9A_{r4}$. The corresponding ASFs and AVRs are given in Table 6.30.

CHAPTER SEVEN

CONCLUSIONS AND FUTURE WORK

The alternative load path method (ALPM) proves a reasonable approach for the structural design against progressive collapse in close-in detonation events. The idea behind this method is that the remaining structure after the failure of the primary structural members should be able to sustain service loadings without further collapse disproportional to the initial local damage. Based on the ALPM, the reliability of damaged structures against progressive collapse is investigated in this study, where the initial local damage under the close-in detonation condition is modelled by the sudden removal of one structural column. The summary and conclusions of the study are given in the following.

7.1 Summary and Conclusions

By modelling the structure with the lumped plasticity model and adopting the singularity of structural stiffness matrix as a rational criterion for structural global failure, a performance function of Equation (3.7) is established in Chapter 3 for damaged ductile frames against progressive collapse under static loading conditions. This performance function is based on the virtual work principle and the critical collapse mechanism criterion. The critical collapse mechanism states that the structure will fail if the structural weakest collapse mechanism producing the minimum value of structural performance function can not provide sufficient resistance for the structure. Thus, the utilization of the critical collapse mechanism criterion combines all possible structural collapse mechanisms into one performance function and can greatly reduce the computational work for structural reliability assessment.

The performance function of Equation (3.7) is expressed in terms of the minimum internal virtual work $\min(\delta W_{\text{int}})$ done by the structural weakest collapse mechanism minus the external virtual work δW_{ext} done by static service loadings. Moreover, $\min(\delta W_{\text{int}})$ is the sum of the minimum internal virtual work $\min(\delta W_{\text{int},j})$ provided by each critical zone. In order to obtain $\min(\delta W_{\text{int},j})$, three types of critical zones are identified and the formulae for the minimum internal virtual work $\min(\delta W_{\text{int}}^{I\sim III})$ by each type of critical zone are deduced with an linear optimum model. The solution of this linear optimum model shows that the strength of related ESPHs determines $\min(\delta W_{\text{int}}^{I\sim III})$ as well as the critical zone's failure mode which will occur during structural progressive collapse.

Based on the formulae for $\min(\delta W_{\text{int}}^{I\sim III})$, the performance function of Equation (3.7) is rewritten as Equation (3.36). However, due to the dependency of the strength of the end springs on the axial force of structural members, it is found that the performance function of Equation (3.36) can not be calculated directly. To deal with this issue, a construction system is established by applying an additional virtual concentrated force vector to structural model. Based on this construction system, further discussion shows that the function of Equation (3.38) in terms of the minimum internal virtual work $\min(\delta W_{\text{int}})$ minus the external virtual work δW_{ext} by static service loadings is still applicable for reliability assessment of the real damaged structure.

An iterative procedure for solving the function of Equation (3.38) is developed in Chapter 4. In each cycle of the calculation, three equations are deduced to determine the axial force on directly connect columns in the i^{th} storey. The calculation starts from the roof storey to the storey of the failed column and thus the minimum internal virtual works including $\min(\delta W_{\text{int},i}^I)$, $\min(\delta W_{\text{int},i}^{II})$, $\min(\delta W_{\text{int},i}^{III})$

and $\min(\delta W_{\text{int}})$ are obtained provided that the virtual concentrated force vector and the axial forces on indirectly connected columns and beams are available.

The positions of the real plastic hinges (RPHs) on the beams in Type-II critical zones are taken into consideration by using the reduced strength for corresponding end springs. The equations for the position of RPHs and the reduced strength of the end springs are deduced based on real moment diagram. Therefore, the maximum beam moments, the moment distributions and the result of the function of Equation (3.38) derived based on the construction system will keep unaffected by fixing all springs at end sections of structural members. A simple example justifies the utilization of the reduced strength for end springs to consider the position of RPHs. Iterative procedures are further presented for calculating the virtual concentrated force vector and the axial forces on indirectly connected columns and structural beams based on the given ratio of $p_{v,i}/p_{v,L1}$ and a modified damaged structural model, respectively.

Based on a combination of the Monte Carlo Simulation method and the calculation of structural performance function of Equation (3.38), a flowchart for reliability assessment of the damaged structure against progressive collapse under static loading conditions is presented. According to this flowchart, the reliability of a four-storey ductile frame structure under two cases of column failure (external column failure and internal column failure) is studied. Results show that the plastic hinges composing the structural weakest collapse mechanism are predominately located on structural beams in the damaged span. Therefore, reinforcing the beams should be the most effective method to improve the damaged structural resistance against collapse. The results also show the good match of the minimum internal virtual work distribution with a normal probability density function. However, due to the fact that the live loads follow an extreme value type I distribution, apparent deviations between the histograms of δW_{ext} and the normal density curves can be observed.

Dynamic effects due to the sudden loss of a structural column are significant for the response of the damaged structure. The reliability assessment method presented in Chapters 3 and 4 is further developed in Chapter 5 to take the dynamic effects into account. Three performance functions for structural collapse due to the lack of strength and deformation capacity of structural weakest collapse mechanism and the lack of shear strength are constructed.

The first function considers structural collapse due to the lack of strength of the weakest mechanism. The discussion about structural dynamic response after the formation of the weakest mechanism shows that the first performance function for the damaged structure under dynamic loading conditions could be exactly the same as that for the structure under static loading conditions by reasonably defining $p_{v,i}/p_{v,L1}$. In this case, the virtual concentrated force vector will reflect the influence of the inertial force. Accordingly, the iterative procedure for structure under static conditions is still valid.

The second function considers structural collapse due to the lack of deformation capacity of the weakest mechanism and is created based on the internal work and the external work at a displacement level of y_u . In the calculation of this function, the structure is simplified into a single degree of freedom system. Equations for determining the resistance function of the SDOF system are obtained based on the strength and rotational capacity of real plastic hinges (RPHs).

The third function determines structural collapse in shear. For simplicity, it is assumed that the weakest collapse mechanism has occurred to the damaged frame structure and thus the maximum shear force on each member can be evaluated based on the weakest mechanism. This assumption is conservative for ductile frames which are strong enough to sustain more than approximately twice of the service loadings in the damaged span. However, considering the dominant position of structural flexural response in the reliability assessment of ductile frames, this conservative error should be negligible.

Since any of the above three performance function less than zero will lead to structural progressive collapse, a global performance function is calculated by taking the minimum of these performance functions. While combined with MCS, the reliability of damaged structures against progressive collapse under dynamic loading conditions is evaluated. Example results show that the dynamic effects by the sudden loss of a column are significant for structural reliability. It is also shown that the second performance function plays a predominant role in the reliability assessment.

Three groups of parametric studies are carried out to investigate the effects of different parameters on structural reliability. Based on the first group of studies, it is found that the CoVs of both the reinforcing bar's strength and the structural members' effective depth have important influences on the failure modes of the TM critical zones in generating the structural weakest collapse mechanism as well as the structural reliability. However, the CoVs of the external loadings (DL and LL) are significant only for $P_f(z < 0)$.

The second group of studies reveals that increasing structural member's reinforcement area A_a or their effective depth μ_{da} should be the most efficient method to improve the structural reliability against progressive collapse. Structural reliability can also be improved by the increase of concrete strength. However, as compared to A_a and μ_{da} , the same percentage increase in concrete strength μ_{ca} will lead to a far less significant drop in $P_f(z < 0)$.

Increasing the number of storeys could bring about positive effects for the damaged ductile frame structure against progressive collapse provided that the increased storey is designed as strong as the other storeys. However, the third group of studies also shows that when the increased storey is weaker than the other, a remarkable negative effect is made on structural reliability.

7.2 Future work

This study aims to develop a practical approach for reliability assessment of ductile frame structures subjected to one column failure in close-in blast conditions. Although, it presents some improvement in the concerned topic, modification and further development of the method are needed. Valuable future work is given as follows.

Present studies are concentrated only on the reliability assessment for two-dimensional ductile frames. In reality, most of frame buildings are three-dimensional structures. Further development of this study to the reliability assessment of three-dimensional frames is worthy of consideration.

Experimental studies about the cracking pattern and the flexural strength of structural beams during building progressive collapse in a close-in blast event is also valuable for future research. Due to the large positive moment on structural beams in damaged spans around Type-II critical zones, the concrete cracking become significant. The bond between steel and concrete may be damaged by cracking and the positive moment resistance of the beam may not be fully utilized.

Future experimental studies could also be performed about the position of positive plastic hinges on beams in damaged spans. Although equations for calculating this parameter are deduced mathematically, experimental verifications are needed.

Since structural progressive collapses triggered by close-in blast events are focused upon in this study, the collapse of the structure considered will propagate in vertical direction. Studies about structural progressive collapse propagating in lateral deformation are also of importance. For example, collapse of seismically designed frame structures may occur due to a small local failure under design seismic loadings. The local failure can be induced by human error in design or by bad workmanship in construction *etc.* In this case, it is possible for the local failure to

propagate in lateral direction under seismic loadings, which finally results in progressive collapse of the structure.

In this study, the initial local damage is modelled by a sudden removal of a structural column. It could be reasonable for structure in a close-in blast event. However, the above approach for considering the initial local failure may not be applicable for the structure under other types of abnormal events such as the distant blast incidents where more than one of the structural columns could be destroyed by the blast loadings. Therefore, the reliability of structures subjected to other types of initial failure is also worthy of further investigations.

REFERENCES

- [A1] ASCE 7-02, (2002), Minimum Design Loads for Buildings and other Structures, American Society of Civil Engineers, New York.
- [A2] ASCE 7-95, (1995), Minimum Design Loads for Buildings and Other Structures, American Society of Civil Engineers, New York.
- [A3] Allgood J. R. and Swihart G. R., (1970), “Design of Flexural Members for Static and Blast Loadings”, ACI Monograph Series, No. 5.
- [A4] Arora J. S., (2004), Introduction to Optimum Design, Academic Press.
- [A5] ACI, (2002), Building Code Requirements for Structural Concrete and Commentary (ACI 318-02), American Concrete Institute, Farmington Hills, Michigan.
- [A6] Allen, D. and W.R. Schriever, (1973), “Progressive collapse, abnormal loads and building codes,” in Structural Failures: Modes, Causes, Responsibilities, ASCE, New York, NY, pp. 21 – 48.
- [B1] Buscemi N. and Marjanishvili S., (2005), “SDOF Model for Progressive Collapse Analysis”, Proceedings of the 2005 Structures Congress and the 2005 Forensic Engineering Symposium, ASCE.
- [B2] Biggs J. M., (1964), Introduction to Structural Dynamics, McGraw-Hill, New York.
- [B3] Baker W. E., Cox. P. A., Westine P. S., Kulesz J. J. and Strehlow R. A., (1983), An Explosion Hazard and Evaluation: Fundamental Studies in Engineering, Elsevier Scientific Publishing Company.

- [B4] Brode H. L., (1955), "Numerical Solution of Spherical Blast Waves", Journal of Applied Physics, No. 6
- [B5] Bangash M. Y. H., (1997), Impact and Explosion: Analysis and Design, Blackwell Scientific Publication, Oxford.
- [B6] Baker W. E., (1973), Explosions in Air, University of Texas Press, Austin, Texas.
- [B7] Baldridge S. M. and Humay F. K., (2003), "Preventing Progressive Collapse in Concrete Buildings", Concrete International, Vol. 25 (11), pp: 73 – 79.
- [B8] Bjerager P., (1989), "Plastic Systems Reliability by LP and FORM", Computers and Structures, Vol. 31, pp: 187-196.
- [B9] Breitung K., (1984), "Asymptotic Approximations for Multi-normal Integrals", Journal of Engineering Mechanics, ASCE, Vol. 110 (3), pp: 357-366.
- [B10] BS 8110, (1997), Structural Use of Concrete, Part 1: Code of Practise for Design and Construction, British Standards Institute.
- [B11] Breen, J.E. and Siess, C.P. (1979), "Progressive collapse – symposium summary." ACI Journal. Vol.76(9), pp997 – 1004.
- [B12] BRE Centre for Fire Safety Engineering, University of Edinburgh (2006). "Dalmarnock Full-Scale Experiments 25 & 26 July 2006". BRE Centre for Fire Safety Engineering, University of Edinburgh.

- [B12] Bartlett, F.M., and MacGregor, J.G. 1996, 'Statistical analysis of the compressive strength of concrete in structures', ACI Materials. Journal, Vol. 93(2).
- [C1] Corley W. G., Mlakar P. F., Sozen M. A. and Thornton C. H., (1998), "The Oklahoma City Bombing: Summary and Recommendation for Multi-hazard Mitigation", Journal of Performance of Constructed Facilities, ASCE, Vol. 12 (3), pp: 100-112.
- [C2] Cundall P. A. and Strack O. D. L., (1979), "Discrete Numerical Model for Granular Assemblies", Geotechnique, London, Vol. 29 (1), pp: 47-65.
- [C3] Comite Euro-International du Beton, (1993), "CEB-FIP Model Code 1990", CEB Bulletin d'Information No. 213/213, Lausanne, Switzerland.
- [C4] Comite Euro-International du Beton, (1976), "Common Unified Rules for Different Types of Construction and Materials", CEB Bulletin d'Information No. 116-E, Paris, France.
- [C5] Cook R. D., Malkus D. S. and Plesha M. E., (1989), Concepts and Applications of Finite Element Analysis, 3rd Edition, John Wiley & Sons, New York.
- [C6] Chakrabarty J. , (2006), Theory of plasticity, Butterworth-Heinemann.
- [D1] DoD, (2001), DoD Interim Antiterrorism/Force Protection Construction Standards – Progressive Collapse Design Guidance, Department of Defense, Washington, DC.

- [D2] DoD, (2005), Unified Facilities Criteria - Design of Buildings to Resist Progressive Collapse, Department of Defense, Washington, DC.
- [E1] Ellingwood B. and Leyendecker E. V., (1978), "Approaches for Design against Progressive Collapse", Journal of Structure Division, ASCE, Vol. 104 (3), pp: 413 - 423.
- [E2] Eibl J., (1991), "Safety Considerations for Nonlinear Analysis", Proceedings of IABSE Colloquium on Structural Concrete, International Association for Bridge and Structural Engineering, Stuttgart, Germany, pp: 337-342.
- [E3] Ellingwood B. (2006), "Mitigating Risk from Abnormal Loads and Progressive Collapse", Journal of Performance Construction Facility, Vol.20(4), pp. 315-323.
- [E4] Eurocode 2 (1996), Design of Concrete Structures, Comite Europeen de Normalisation.
- [F1] Forbes D. F., (1999), "Blast Loading on Petrochemical Buildings", Journal of Energy Engineering, ASCE, Vol. 125 (3), pp: 94-102.
- [F2] Fintel M. and Schultz D. M., (1976), "Philosophy for Structural Integrity of Large Panel Buildings," Journal of the Prestressed Concrete Institute, Vol.21, No.3, pp: 46-69.
- [F3] Field, Andy (2004). "A Look Inside a Radical New Theory of the WTC Collapse". Fire/Rescue News. Retrieved on 2006-07-28.
- [G1] GSA, (2000), Progressive Collapse Analysis and Design Guidelines for New Federal Office Buildings and Major Modernization Projects, General Services Administration, Washington, DC.

- [G2] Grierson D. E., Safi M., Xu L. and Liu Y. X., (2005), “Simplified Methods for Progressive Collapse Analysis of Buildings”, Proceedings of the 2005 Structures Congress and the 2005 Forensic Engineering Symposium, ASCE.
- [G3] Griffiths H., Pugsley A. G., and Saunders O., (1968), Report of the Inquiry into the Collapse of Flats at Ronan Point, Canning Town, Her Majesty’s Stationary Office, London, England.
- [G4] Gross J. L., and McGuire W., (1983), “Progressive Collapse Resistant Design”, Journal of Structural Engineering, ASCE, Vol. 109 (1), pp: 1-15.
- [G5] Grierson D. E., Xu L. and Liu Y. X., (2005), “Progressive Failure Analysis of Buildings Subjected to Abnormal Loading”, Journal of Computer Aided Civil and Infrastructure Engineering, Vol.20 (3).
- [H1] Haluk S., Ergin C. and Sinan A., (1994), “Resistance Mechanisms in RC Building Frames Subjected to Column Failure”, Journal of Structural Engineering, Vol. 120 (3), pp: 765-782.
- [H2] Hyde D. W., (1991), ConWep: Weapons Effects Calculation Program Based on Technique Manual, TM5-855-1, United States Army Engineer Waterways Experiment Station.
- [H3] Hinman E. E. and Hammond D. J., (1997), Lessons from the Oklahoma City Bombing: Defensive Design Techniques, Published by ASCE press. New York.
- [H4] Hakuno M. and Megrro K. (1993) “Simulation of Concrete-frame Collapse due to Dynamic Loading”, Journal of Engineering Mechanics, Vol.119 (9): 1709-1723.

- [H5] Hasofer A. M. and Lind N. C., (1974), "Exact and Invariant Second-moment Code Format", Journal of the Engineering Mechanics Division, ASCE, Vol. 100 (1), pp: 111-121.
- [H6] Hohenbichler M., Gollwitzer S., Kruse W. and Rackwitz R. (1987), "New Light on First- and Second-order Reliability Method", Structural Safety, Vol. 4, pp:267-284.
- [H7] Hamburger, Ronald, et al. (2006), "World Trade Center Building Performance Study", Federal Emergency Management Agency.
- [I1] ISC, (2001), ISC Security Design Criteria for New Federal Office Buildings and Major Modernization Projects, Interagency Security Committee. Washington, DC.
- [I2] Israel M., Ellingwood B. and Corotis R. (1987), 'Reliability-based code formulations for reinforced concrete. buildings', J. Struct. Division, ASCE, 113 (10).
- [K1] Kaewkulchai G. and Williamson E. B., (2003), "Dynamic Behaviour of Planar Frames during Progressive Collapse", Proceedings of the 16th ASCE Engineering Mechanics Conference, ASCE.
- [K2] Kaewkulchai G., (2003), Dynamic Progressive Collapse of Frame Structures, Ph.D. Dissertation, University of Texas at Austin.
- [K3] Kaewkulchai G. and Williamson E. B., (2004), "Beam Element Formulation and Solution Procedure for Dynamic Progressive Collapse Analysis", Computers and Structures, Vol. 82, pp: 639-651.
- [K4] Kottegoda N. T. and Rosso R., (1997), Statistics, Probability, and Reliability for Civil and Environmental Engineers, McGraw-Hill.

- [L1] Leyendecker E. V., and Ellingwood B. R., (1977), “Design Methods for Reducing the Risk of Progressive Collapse in Buildings,” NBS Building Science Series 98, National Bureau of Standards, Washington, DC.

- [L2] Li B., Huang Z. W. and Pan T. C., (2003), “ The Local and Global Structural Responses of a Tall Building Subjected to Close-in Detonations”, Conference on Design and Analysis of Protective Structures against Impact/Impulsive/Shock Loads (DAPSIL), Japan, pp: 388-401.

- [L3] Luccioni B. M., Ambrosini R. D. and Danesi R. F., (2004), “Analysis of Building Collapse under Blast Loads”, Engineering Structures, Vol. 26 (1), pp: 63-71.

- [L4] Liu P. L. and Kiureghian A. D., (1991), “Optimization Algorithms for Structural Reliability”, Structural Safety, Vol. 9, pp: 161-177.

- [M1] Marjanishvili S. M. (2004), “Progressive Analysis Procedure for Progressive Collapse”, Journal of Performance of Constructed Facilities, Vol. 18 (2), pp: 79-85.

- [M2] Malla R. B. and Aalluri B., (2000), “Dynamic Nonlinear Member Failure Propagation in Truss Structures”, Structure Engineering and Mechanics, Vol.9 (2), pp: 111-126.

- [M3] McConnel R. E. and Kelly S. J., (1983), “Structural Aspects of the Progressive Collapse of Warehouse Racking”, Structural Engineer, 61A, pp: 343-347.

- [M4] Morris N. F., (1993), "Effect of Member Snap on Space Truss Collapse", *Journal of Engineering Mechanics*, Vol.119 (4), pp: 870-886.
- [M5] Melchers R. E., (1999), *Structural Reliability Analysis and Prediction*, 2nd Edition, John Wiley & Sons.
- [M6] May G. C. and Smith P. D., (1995), *Blast Effect on Building: Design of Buildings to Optimize Resistance to Blast Loading*, Thomas Telford.
- [M7] McGuire W., (1974), "Prevention of Progressive Collapse," *Proceedings of the Regional Conference on Tall Buildings*, Asian Institute of Technology, Bangkok, Thailand.
- [M8] Mlakar P. F., Corley W. G. and Sozen M. A., et al. (1998), "The Oklahoma City Bombing: Analysis of Blast Damage to the Murrah Building", *Journal of Performance of Constructed Facilities*, ASCE, Vol. 12 (3), pp: 113-119.
- [M9] Malla R. B. and Nalluri B., (1995), "Dynamic Effects of Member Failure on Response of Truss-type Space Structures," *Journal of Spacecraft and Rockets*, Vol. 32(3), pp: 545-551.
- [M10] Melchers R. E. and Tang L. K., (1985), "Failure Modes in Complex Stochastic Systems", *Proceedings of the 4th International Conference On Structural Safety and Reliability*, Vol. 1, Kobe, Japan, pp: 97-106.
- [M11] Melchers R. E. and Tang L. K., (1984), "Dominant Failure Modes in Stochastic Structural Systems", *Structural Safety*, Vol. 2.
- [M12] MacGregor J. G. and Wight J. K., (2004), *Reinforced Concrete: Mechanics and Design*, Prentice Hall.

- [M13] MacGregor J. G., Mirza S. A. and Ellingwood B., (1983), “Statistical Analysis of Resistance of Reinforced and Pre-stressed Concrete Members”, ACI Journal, Vol. (80), pp: 167-176.
- [M14] Mirza S. A. and MacGregor J. G., (1979), “Variations in Dimensions of Reinforced Concrete Members”, Journal of the Structural Division, ASCE, Vol.(105), pp: 751-766.
- [M15] Mirza S. A., Hatzinikolas M. and MacGregor J. G., (1979), “Statistical Description of Strength of Concrete”, Journal of Structural Division, ASCE, Vol. (105), pp: 1021-1037.
- [M16] Mainstone, R.J. (1974), The Hazards of Explosion, Impact and Other Random Loadings on Tall Buildings - B.R.E. Current Paper No. CP 64/74.
- [N1] Newmark N. M., (1972), “External Blast”, Proceedings of International Conference on the Planning and Design of Tall Building, Lehigh University, Vol. Ib, pp: 661-676.
- [N2] Nawy E. G., (2002), Reinforced Concrete – A Fundamental Approach, Prentice Hall International Series in Civil Engineering and Engineering Mechanics.
- [N3] NISTIR 7396, (2006), Best Practices for Reducing the Potential for Progressive Collapse in Buildings.
- [N4] National Institute of Standards and Technology, (2005), Final Reports of the Federal Building and Fire Investigation of the World Trade Center Disaster.

- [O1] Olatidoye O., Sarathy S., and Jones G. et al. (1999), A Representative Survey of Blast Loading Models and Damage Assessment Methods for Buildings Subjected to Explosive Blasts, CEWES MSRC/PET TR/98-36, Nichols Research Corporation.

- [P1] Pretlove A. J., Ramsden M. and Atkins A. G., (1991), "Dynamic Effects in Progressive Failure of Structures", International Journal of Impact Engineering, Vol. 11 (4), pp: 539-546.

- [R1] Rachwitz R. and Fiessler B., (1978), "Structural Reliability under Combined Random Load Sequences", Computers and Structures, Vol. 9, pp: 489-494.

- [R2] Rodin J., (1969) "Statistical Design Against Progressive Collapse", The Consulting Engineer, London, England.

- [R3] Robertson, Leslie E. (2002). "Reflections on the World Trade Center". The Bridge Volume 32, Number 1. National Academy of Engineering.

- [R4] Roger L. Brockenbrough, Frederick S. Merritt, (1999), Structural Steel Designer's Handbook, Third version, McGraw-Hill Professional.

- [S1] Somes N. F., (1973), "Abnormal Loadings on Buildings and Progressive Collapse", Center for Building Technology, National Bureau of Standards, Washington, DC.

- [S2] Sozen M. A., Thornton C. H., Corley W. G. and Mlakar P. F., (1998), "The Oklahoma City Bombing: Structural and Mechanisms of the Murrah Building", Journal of Performance of Constructed Facilities, ASCE, Vol. 12 (3), pp: 120-136.

- [T1] TM 5-855-1, (1986), Fundamental of Protective Design for Conventional Weapons, United States Department of the Army, Washington, DC.
- [T2] Taylor D. A., (1975), "Progressive Collapse", Canadian Journal of Civil Engineering, Vol. 2(4).
- [T3] TM5-1300, (1990), Structures to Resist the Effects of Accidental Explosions, Volume IV, Reinforced Concrete Design, United States Department of the Army, Washington, DC.
- [T4] Turk G., Ramirez M. R. and Corotis R., (1994), "Structural Reliability Analysis of Nonlinear Systems", Proceedings of the 6th International Conference on Structural Safety and Reliability, pp: 1345-1352, Balkema, Rotterdam.
- [U1] Usmani, A.S., Y.C. Chung, J.L. Torero (2003). "How did the World Trade Center Collapse: A New Theory", Fire Safety Journal, Vo.38(6).
- [V1] Val D., Bljugar F. and Yankelevsky D., (1996), "Optimization Problem Solution in Reliability Analysis of RC Structures", Computers and Structures, Vol. 60 (3), pp: 351-355.
- [V2] Val D., Bljugar F. and Yankelevsky D., (1997), "Reliability Evaluation in Nonlinear Analysis of Reinforced Concrete Structures", Structural Safety, Vol. 19 (2), pp: 203-217.
- [V3] Val D., Bljugar F. and Yankelevsky D., (1997), "Reliability Assessment of Damaged RC Framed Structures", Journal of Structural Engineering, ASCE, Vol. 123 (7), pp: 889-895.