Autonomous vehicle following : a virtual trailer link approach

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AUTONOMOUS VEHICLE FOLLOWING - A VIRTUAL TRAILER LINK APPROACH

NG TECK CHEW

School of Electrical & Electronic Engineering

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Abstract

This thesis addresses the automation of the vehicle following function in an urban city environment, i.e., travelling under heavy traffic conditions or in a ‘stop-and-go’ motion. A virtual trailer link model for vehicle following has been proposed. With this perspective, the leader is represented as a tractor pulling the follower, which is modelled as a trailer, in the form of a virtual link. The optimum configuration and the length of the virtual trailer link model have been determined by taking into consideration the safe following distance as well as general car-like vehicle dynamics and constraints. In implementing the virtual trailer link model for vehicle following, sensors are required for the estimation of the relative pose and velocity of the lead vehicle in relation to the follower. However, inherent sensor noise, as well as limitations on their fields of view and resolution can affect the performance of the vehicle following function. A Bayesian formulation is thus proposed to model the process and sensor noise in the system. The key to a tractable solution for this formulation is based on the justified assumption that the pose of the follower vehicle is statistically independent of that of the leader. By estimating the poses of both vehicles, together with the uncertainties of the system, it is possible to minimize the path deviations between them. Moreover, as a result of uncertainties in the

system, the computed driving commands based on the virtual trailer link model need to be optimized. Hence, a metric is required to evaluate and optimize the driving commands for the follower vehicle. An information theoretic framework is proposed. The aim of this framework is to select an optimal control input to the follower so as to minimize the pose error between the vehicles. Under this framework, the relative information has been used as a metric to evaluate a sequence of controlling actions, which act as inputs to the follower vehicle.

Extensive simulations and experiments are carried out and detailed results are presented. This new framework for a vehicle following system has been proven to be intrinsically safe. The developed vehicle following system is compared with other published systems, showing improved path deviation during vehicle following operations.

Extended Abstract

Land transportation is an important component of the activities of modern-day living. Currently, high traffic densities, safety, air pollution and fuel costs are some of the major concerns in terms of land transportation at all levels of decision making worldwide. In order to address some of these problems, investigations into the deployment of vehicles with various degrees of computer-controlled functions (also known as Intelligent Vehicles) are being conducted. The vehicle following function, in which a follower vehicle can *autonomously* follow a lead vehicle, is one solution that is proposed to counter some of these challenges facing land transportation today. It can optimize traffic flow, regulate the inter-vehicle distance and hence enhance road safety. In addition, it can lead to the reduction of fuel consumption. The vehicle following function entails the production of a complex system with the use of computer-controlled processes. It involves the use of complex tasks such as perception, situation understanding, decision making and vehicle actuation. Several proposed solutions have yielded different levels of vehicle automation either in a stand alone manner, i.e., without external infrastructure or communication systems support, or through the use of road infrastructure and inter-vehicle communication systems. Despite these efforts, there are few stand alone solutions that automate the vehicle following

function, in particular, when the vehicles travel in a complex environment such as in city dwellings. Sensor limitations and system uncertainties linked to road complexity and a cluttered environment present major difficulties to a reliable and safe vehicle following function for urban city travelling.

This thesis addresses the automation of the vehicle following function in an urban city environment, i.e., travelling under heavy traffic conditions or in a ‘stop-and-go’ motion. A theoretical framework for vehicle following has been established based on the Bayesian estimation framework. The purpose is to achieve a reliable and safer vehicle following response, despite the physical limitations of the sensors used and despite the constraints on the vehicles. The approach is based on a stand alone system. This is achieved by analyzing the information that can be inferred from the data acquired by the onboard sensors of the follower vehicle, and by focusing on the strategy for vehicle following. This thesis studies the perception process, vehicle tracking algorithms and generation of the set points that the led vehicle is to follow. The targeted application for the proposed system is the ‘stop-and-go’ scenario where traffic flow is characterized by slow moving vehicles due to congestion.

The aim of the vehicle following function is to command the follower vehicle to trail the trajectory of the lead vehicle. In addition, an inter-vehicle safe distance is desired, which will allow the led vehicle to stop safely in the event of any unexpected action on the part of the lead vehicle. In addition, both the longitudinal and lateral path deviations between the two vehicles must be minimized in order to ensure successful vehicle following. Hence, the led vehicle

is to remain as close as possible to the trajectory of the lead vehicle, and yet keep within, what is considered to be, the safe traversable areas. In order to fulfill these important criteria, this thesis proposes and implements a virtual trailer link model for vehicle following. The concept of the virtual trailer link model was inspired by the physical tractor-trailer system used in truck towing trailers of containers on motorways and the towing of caravans by passenger vehicles. The mechanical off-hooked trailer system is shown to be capable of following close to the trajectory traced by the towing vehicle. Within this perspective, the leader is represented as a tractor pulling the follower, which is modelled as a trailer, in the form of a virtual link. The optimum configuration and the length of the virtual trailer link model have been determined by taking into consideration the safe following distance as well as general car-like vehicle dynamics and constraints.

In implementing the virtual trailer link model for vehicle following, sensors are required for the estimation of the relative pose and velocity of the lead vehicle in relation to the follower. However, inherent sensor noise, as well as limitations on their fields of view and resolution can affect the performance of the vehicle following function. A Bayesian formulation is proposed to model the process and sensor noise in the system. The key to a tractable solution for this formulation is based on the justified assumption that the pose of the follower vehicle is statistically independent of that of the leader. By estimating the poses of both vehicles, together with the uncertainties of the system, it is possible to minimize the path deviations between them.

The implementation of the vehicle following function raises certain issues, includ-

ing computer and sensor latencies. The latencies in sensing, data processing and decision making are significant for the real time implementation of the vehicle following function. That is to say, it is inevitable for a lag to exist between the actual positions of the lead and led vehicles; otherwise, the two vehicles will collide. The lag introduces the inherent latencies in the system and the required safe inter-distance between the vehicles is a necessary precondition for safe vehicle following. With system latency, the driving commands applied to the lead vehicle can always only be applied to the follower after a time lapse. Moreover, as a result of uncertainties in the system, the computed driving commands based on the virtual trailer link model need to be optimized. Hence, a metric is required to evaluate and optimize the driving commands for the follower vehicle. An information theoretical framework is proposed. The aim of this framework is to select an optimal control input to the follower so as to minimize the pose error between the vehicles. Under this framework, the relative information has been used as a metric to evaluate a sequence of controlling actions, which act as inputs to the follower vehicle.

This thesis provides a Bayesian estimation framework with a virtual trailer link model for vehicle following. Extensive simulations and experiments are carried out and detailed results are presented. This new framework for a vehicle following system has been proven to be intrinsically safe. The developed vehicle following system is compared with other published systems, showing improved path deviation during vehicle following operations.

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CHAPTER 1

Introduction

1.1 Background

Since the invention of motor powered vehicles, they have become an important part of our way of life. However, this has resulted in a steady increase in traffic density, unacceptable levels of traffic accidents and air pollution [1]. At the moment, areas with dense traffic conditions exist in every part of the world, with vehicles following one another closely while travelling at high speeds on motorways or in congested traffic in urban areas. Following a vehicle over a long period of time becomes a repetitive task for a driver. The task is made even more tiresome if it involves a continuous stop-and-go motion. These conditions can result in driver distraction and impatience that eventually induces traffic accidents at different levels of seriousness. These conditions also lead to increased fuel consumption as well as air pollution [2],[3].

Vehicles have undergone profound changes over the last few years. They have evolved over a short period of time from being thermo-mechanical systems to systems that comprise a substantial quantity of electronic components and micro-

controllers. Today, there exists a network of dedicated processors controlling most vehicle functions from the monitoring of engine performance and brake actuation to the sensing of the driver/passenger awareness state [4]. Over the years, research efforts have focused on automating the driver task in vehicle following, to various degrees of success. One of the most successful computer controlled functions in the market is Automatic Cruise Control (ACC). It has been successfully deployed in trucks and luxury cars in the US, Europe and Japan [5],[6],[7],[8],[9]. This function maintains the vehicle travelling speed as set by the driver despite changes in the road profile. The advantage of this function is that it relieves the driver of the task of pressing the 'accelerator pedal'. Automatic Gear controllers are another result of the vehicle automation function. These controllers have evolved into robotized gearboxes and are well accepted by drivers today. The successful introduction of these functions, as well as the studies on the use of exteroceptive sensors for safety and control-related tasks onboard passenger vehicles, are leading the way to the introduction of Adaptive Cruise Control and the autonomous guidance of vehicles [10]. These applications include vehicle platooning, where vehicles trail each other by forming a column. This kind of capability is of interest to the military, as it is an essential function for the deployment of robot-type vehicles as part of the Future Combat Systems (FCS) program of the US Department of Defence [11]. Automating such functions through the use of autonomous platooning vehicles allows for the removal of soldiers from harms way. The US Army has recently demonstrated the platooning capabilities of their new vehicle STRYKER [12], that travels at 40 mph in motorway like conditions. The advantage of this function is the enhancement

of the conveying of supplies and reduction in the number of casualties in standard logistical operations.

Since the early eighties, there has been much interest in the automation of several vehicle navigation functions; the vehicle following function is very much a part of these efforts [10]. Several proposed solutions have achieved different levels of vehicle automation, either in a stand alone manner or through the use of road infrastructure and inter-vehicle communications [13],[14],[15],[16]. Despite these advances, few stand alone solutions exist that automate the vehicle following function, particularly when vehicles traverse in dense urban agglomerations such as the centres of large cities.

In order to automate the vehicle following function, the lead vehicle must be detected and tracked (perception), the relative spatial distance and heading between the leader and follower vehicles must be estimated (situation understanding), the following strategy must be rationalised and defined (decision-making) and the command signals for the follower vehicle must be generated (vehicle actuation). An effective automated vehicle following system can significantly affect the traffic flow on the motorway and in the city. With the automated vehicle following system, in which inter-vehicle spacing between cars is reduced [17], the throughput of the traffic will be increased, thus easing traffic congestion. Shorter and almost constant inter-vehicle distances not only reduce the drag coefficient but also enable the optimization of the acceleration and deceleration stages of the vehicle, thus reducing overall fuel consumption [18] for high speed vehicle following applications. Furthermore, these inter-vehicle distances that depend

on the led vehicle's ability to stop safely under all conditions, have improved the safety of the vehicle following system. The use of exteroceptive onboard sensors as a part of these systems provides an opportunity to deploy other safety enhancement functions such as pedestrian detection, etc. [2]. However, sensor constraints in terms of field of view, resolution, etc. together with the complexity of urban traffic conditions present major challenges to a functional vehicle following function.

This dissertation addresses the automation of the vehicle following function in urban environments. To this end, a theoretical framework is proposed that is formulated to achieve a reliable and safe vehicle following response despite the constraints placed by the physical limitations of the sensors used and the vehicle constraints. The method is based on a stand alone solution, that is, without the use of external infrastructure or communications support. This is achieved by processing the information inferred from the onboard sensor of the follower vehicle, and by focusing on the strategy used to track the lead vehicle. For this purpose, the perception process, tracking algorithms and the generation of the set points that the led vehicle is to follow are studied. The scenarios in which the proposed application is to operate are in traffic congestions where vehicles will stop-and-go very often and hence move slowly for short periods of time.

1.2 Rationale

Since the invention of the motor vehicle, there has been a gradual modernization of its capabilities. This has resulted in the transformation of what

were essentially thermo-mechanical systems into complex mechatronic solutions where networks of microcontrollers control the vehicle capabilities. A parallel development to this is that vehicle speeds and the number of vehicles on the road have increased. This has led to the problem of traffic congestion, particularly in urban areas, and has also led to a frightening number of road accidents [19]. Traffic congestions are typical scenarios that occur as people commute from the countryside to urban agglomerations and vice versa during peak hours. Drivers spend long hours in stop-and-go situations that make them tired and hence affects their driving behavior. In addition, air pollution levels and fuel consumption have been following an upward trend because of the effect of vehicles maneuvering in stop-and-go traffic.

Schemes to reduce these increasing difficulties have been proposed and implemented over the past few years. Governments and legislative bodies such as the European Commission¹ have proposed schemes to regulate traffic and to improve road safety. Vehicle manufacturers participate actively in these schemes not only for safety reasons but also because these may come up as part of future legislation issues. Partial solutions, such as the use of Intelligent Transportation Systems (ITS), involving the use of communications technologies from vehicle to vehicle (V2V) and vehicle to infrastructure (V2I), are being considered. Another approach is to use stand alone solutions, which are known as Intelligent Vehicles Systems (IVS). These include several sensor-based computer controlled vehicle navigation functions under the name of Advanced Driving Assistance Systems

¹<http://ec.europa.eu/transport/road>

(ADAS). Examples include lane detection, lane keeping, automatic cruise control, parking assist, pre-crash breaking, and so on [20]. While most of these solutions are already deployed in series vehicles, they are limited in their use as safety devices that provide information or warning signals to drivers, and there is minimal computer actuation of the vehicle manoeuvres.

To contribute towards enhancing car safety, improving driver comfort and improving traffic flow, car manufacturers have deployed an active system, Adaptive Cruise Control (ACC), for keeping a safe distance between vehicles and improving traffic flow. These systems operate only at speeds higher than 50 km/h and on straight roads with low traffic density. These constraints have made the ACC system unsuitable for driving in urban agglomerations. In order for this function to be applicable in dense traffic conditions next to large city dwellings, there are multiple technical challenges that must be addressed. These challenges include the selection of sensors, information processing, situation awareness and safety concerns.

Congestion by commuters on approaching roads to large agglomerations can be reduced through the automation of the low speed motion of these vehicles, as identified by Bishop [21]. The trend is for computer control of the vehicle motion through the use of sensor-based systems that could be either stand alone or cooperative solutions. The former implies the use of vehicle on board sensors to detect and track the vehicle which is to be followed without the need for any infrastructure. The latter relies on information that can be embedded or transmitted from the infrastructure to the subject vehicle.

Various demonstrations have been conducted for vehicle following systems on the motorway (e.g PATH and CHAUFFEUR). There has been few examples of vehicle following systems in urban areas. CyberCars [22] has demonstrated the feasibility of this function at low speeds in urban compounds. However, the demonstration was not conducted on congested roads. Little research for vehicle following function has focused in theoretical terms for urban environments with vehicles moving in a stop-and-go manner.

There are three major benefits of having an autonomous vehicle following system that aid drivers trapped in traffic congestion in the city [23]. These are: comfort, safety and economic and environmental benefits.

- **Comfort** This is the main concern for vehicle manufacturers. Within this context, facilitating the driver tasks when in traffic jams is a very important consideration. Driver comfort will be increased significantly if the stop-and-go vehicle motion can be automated. In the case of an elderly driving population, this becomes even more significant.
- **Safety** Collisions often occur because of tail gating; thus, it is important to maintain a safe distance between vehicles. The presence of motorcycles or pedestrians between vehicles presents a hazard in this regard. Onboard vehicle sensors should detect these obstacles and the system could issue a warning or stop the vehicle.
- **Economic and Environmental** The manner in which the leader is tracked can be optimised to reduce fuel consumption. If this is extended to several

vehicles, then not only are fuel savings made but this would also result in a reduction of CO₂ emissions.

The vehicle following function for urban scenarios would contribute as a solution to several problem areas that have arisen from current traffic conditions.

1.3 The Autonomous Vehicle Following Function

The vehicle following function involves two vehicles, a leader and a follower. The follower vehicle trails the trajectory of the lead vehicle while maintaining an inter-vehicle distance that enables it to stop safely if the leader suddenly comes to a halt. Therefore, the follower vehicle is equipped with sensors in order to detect and track the leader as well as its ego-state. Proprioceptive sensors such as encoders, gyroscopes, etc are used to estimate the ego-state of the follower vehicle. Exteroceptive sensors such as scanning lasers, radars and stereovision cameras are used to estimate the state of the lead vehicle in relation to the follower vehicle. The trajectory of the lead vehicle can be inferred from the data collected by the follower vehicle onboard sensors when both vehicles are moving in the same direction. As the trajectory of the lead vehicle is unknown to the follower vehicle, the follower must perceive the whereabouts of the lead vehicle and thus make a best estimate of the pose of the leader relative to it.

The autonomous vehicle following function under computer control can be divided into three major sub-functions - namely, the detection and tracking of the lead vehicle, the generation of target points and the actuation of the follower vehicle to reach these target points. Figure 1.1 illustrates the relationships between

these sub-functions and the manner in which they constitute a closed loop control system. Each of these sub-functions is described in detail in the next few sections. The purpose is to provide an analysis of the underlying components of the autonomous vehicle following function and thus be able to look in detail, from an analytical perspective, at the scientific challenges that it represents.

Figure 1.1: Process flow of a typical automated vehicle following function. The vehicle following mission involves three major functions: Detection and tracking of the lead vehicle, vehicle following modelling and following the leader.

1.3.1 Detection and tracking of the lead vehicle

The primary capability of an autonomous vehicle function is the detection and tracking of the lead vehicle. For this purpose, on-board exteroceptive sensors are used in the lead vehicle. These can include monocular or stereo cameras, infrared cameras, millimeter wave radars and scanning laser radars (LIDAR). Their primary function is to generate sufficient data which should enable the system to detect and locate relevant objects within the sensors' field of vision. These are then classified as static or moving entities (including other vehicles) . The

next step is to identify the relevant lead vehicle in front of the follower vehicle which is then tracked. Sensor physics implies constraints on the performance of the sensors in terms of field of view, resolution, sampling rates and response bandwidth.

1.3.2 Vehicle following modelling: Target points generation

Upon detection and tracking of the lead vehicle, its pose relative to the ego-vehicle is estimated. However, in order to achieve a safe and reliable vehicle following, a minimum inter-vehicle distance is required. In addition, the resulting path deviations between the two vehicles must be minimized. The led vehicle must follow the lead vehicle closely while avoiding collision with the lead vehicle and hitting a road curb, especially if the lead vehicle stops abruptly. These two “conflicting” vehicle following conditions are the main challenges in the vehicle following system function. The model also includes the estimation of the poses of both vehicles and the generation of desired target points for the led vehicle to follow. The estimation process occurs while both vehicles are in motion. Thus, disturbances such as sensor noise, vehicle pitching effects and vehicle dynamics must be considered when modelling the system.

1.3.3 Following the lead vehicle

The purpose of the vehicle following function is to command a led vehicle to trail a lead vehicle closely. This can be considered to be fully successful if the follower vehicle exactly tracks the lead vehicle’s trajectory at a safe distance. However, this is not generally the case and errors will exist. Uncertainty and latency in

the perception process, together with vehicle manoeuvring constraints, introduce errors into the tracking and distance control process. Conventional approaches often address this problem as a control issue, and seek ways to reduce the tracking errors through the implementation of different control-based strategies which do not take into account uncertainty in the estimation processes involved. Another challenge are the metrics required to quantify the control commands for the led vehicle. This involves an evaluation of the diving commands issued to minimize the path deviations between both vehicles throughout the entire vehicle following process.

1.4 Statement of research issues

The technical feasibility of deploying vehicle following systems has been demonstrated by several research groups in the past decade. Landmark demonstrations have been conducted in Europe, USA and Japan. In Europe, such projects include CyberCars [22] led by INRIA (France) and the Program for European Traffic with Highest Efficiency and Unprecedented Safety (PROMETHEUS) [24],[25],[26], a landmark project sponsored by the European Commission. Other projects include CHAUFFEUR (in Italy) [15] and Praxitele (in France) [27], which were demonstrated in the late 1990s. In the United States, the PATH program has demonstrated a vehicle platooning system (DEMO 97) at the National Automated Highway Systems Consortium Technical Feasibility Demonstration held in San Diego as far back as in 1997 [28]. A similar demonstration was held in Japan known as DEMO2000 [29]. The methods involved the use of inter-vehicle

communications, infrastructure and dedicated lead vehicles.

Most of the examples of proposed solutions developed so far have one significant constraint, that is, they are dependent on the availability of infrastructure, communications links or customized vehicles. These are therefore not stand alone solutions. As a result, they are not flexible because of safety concerns and the need for cooperation either from other vehicles, infrastructure or the presence of a customized vehicle. Moreover, current vehicle following is achieved through longitudinal and lateral controls. Longitudinal control maintains a constant distance between the lead and follower vehicles based on system safety considerations and the response of the vehicles. Lateral control ensures that the relative yaw angle between the vehicles is reduced to zero. These control strategies rely on the detection and tracking of the lead vehicle by the onboard sensors. As a result, sensor uncertainty may produce large disturbances to the control strategies.

The research effort also targeted motorway applications for heavy vehicles. Stand alone solutions applicable to urban environments are being developed by passenger car manufacturers such as Nissan and BMW . The vehicle following function in urban environments means that inter-vehicle distances are shorter, there is a higher likelihood of the presence of obstacles between the vehicles (occlusion effects) as well as a stop-and-go motion. Further, vehicle manoeuvres are more complex; that is, turns are tighter and long straight runs are shorter. The problem in these cases is one of perception, tracking, vehicle control and safety.

Based on the above considerations, the following research issues will be addressed in this thesis for the vehicle following function that is applicable in urban environments and in the presence of a stop-and-go motion:

- **Mathematical formulation.** Extensive successful research efforts have concentrated on the design of vehicle following controllers. These efforts have provided theoretical formulations for the implementation of the control of the vehicle actuators. However, there is a lack of an analytical formulation of the vehicle following function as a complete system. This is particularly important as a successful vehicle following function relies on a combination of a perception system, tracking algorithms and control strategy [30], which need to be examined in detail.
- **Autonomous vehicle following function.** The led vehicle must be able to track and trail any lead vehicle directly in front of it, without cutting corners. Its deployment must be independent of other vehicles and road infrastructure. Hence, a stand alone autonomous solution under safety constraints is required.
- **Sensor uncertainty.** Stand alone automated vehicle following can be achieved if the led vehicle is equipped with sensors capable of tracking the lead vehicle. However, as both vehicles move and the environment is cluttered, sensor data can be noisy and contain measurement errors. Therefore, one of the fundamental requirements of a successful vehicle following function is to model the sensor uncertainty. This introduces errors to the estimation process of the tracking system and thus their effects need to be

analyzed in order to be incorporated into the overall system to enhance its overall response. Furthermore, sensor limitations such as the field of view, resolution, bandwidth and time response are some other constraints that require careful consideration.

1.5 Thesis Statement and Objectives

The premise of this research is that the vehicle following function for stop-and-go motion in urban environments can be improved through the application of a probabilistic formulation. The formulation includes a perception-tracking process, the use of a virtual trailer link attached to the lead vehicle and an information theoretic vehicle control strategy. This research proposes that by fusing these strategies into an unified framework, it is possible to achieve a safe and reliable vehicle following function.

The objectives of this research are:

- To formulate a system model for vehicle following, while taking into account the limitations on the trailing distance of the led vehicle, as a means of ensuring the tracking process and a safe inter-vehicle distance.
- To develop a mathematical formulation for the vehicle following function that allows for limitations and uncertainties in the perception process. The formulation should permit an analytical method for vehicle following implementation.
- To develop a vehicle following strategy that exploits the mathematical

formulation and vehicle following model.

- To demonstrate analytically the feasibility of the proposed framework through case-based experimentation studies.

1.6 Research methods

This thesis is an extension of work done by the author on a research-demonstration platform on autonomous unmanned ground vehicles for deployment in jungle environments [31],[32]. As part of this large scale project [33], the vehicle following function was developed based on the use of exteroceptive and proprioceptive sensors, as well as inter-vehicle communications [31]. Several challenging issues surfaced during the deployment of the experimental platform. Firstly, the use of inter-vehicle communication depended on the reliability of the communication systems and a cooperative vehicle. Next, the pose of the lead vehicle needed to be accurately estimated. However, a good position estimate is difficult to achieve in cluttered environments [34],[35]. In addition, sensor limitations and uncertainty in the estimation process meant that multiple issues were found to result in discontinuities in the vehicle following functions and path deviations. This experience has provided much insight into the physics, perception and tracking mechanisms of the vehicle following system.

Autonomous vehicle following is essentially a problem of target (lead vehicle) tracking and following. The onboard system tracks and estimates the relative pose of the lead vehicle based on the sensor information. It then sets up a safe following strategy and commands the follower vehicle to trail the trajectory of the

lead vehicle. The goal is to provide a safe and comfortable ride for the driver. The approaches taken to achieve the objective defined for this thesis are:

1. *Literature review.* To complement the experimental insight of the vehicle following problem, a literature search was conducted to understand the methods used elsewhere, as well as the challenges experienced and analyses made. Domains related to the perception process and the relative positioning between vehicles were also examined.
2. *Strategy to minimize path deviation for vehicle following.* As the follower vehicle trails a lead vehicle, it must not cut corners or collide with the leader. Thus, the safety requirement of the vehicle following function is to minimize the path deviation between the two vehicles. The design requirements for the vehicle following system were analyzed in consideration of the safety and string stability issues. Based on the analysis and literature review, it was found that it is not safe to use the current relative position of the lead vehicle as the tracking point for vehicle following. A geometrical solution was sought and a virtual trailer link model was proposed and formulated. Through a series of simulations and experimentation, the off-hooked virtual trailer link model was analyzed and has proven to be the ideal solution for the vehicle following function.
3. *Dealing with uncertainty - Bayesian formulation.* The literature and experimental experiences have identified that one major function is to perceive the leader pose from the ego vehicle. Sensor limitations, when deployed in an outdoor environment, introduce much uncertainty to this

process. Perception and uncertainty in the measurement models in this case are stochastic. Hence, a method that incorporates these issues into the overall estimation process must be found.

4. *Vehicle detection and tracking.* In order to be able to trail the leader, the follower has to extract the data from noisy and sometimes corrupted measurements. At the same time, the issue of ‘loss of measurement data’ needs to be addressed in the event that the sensor reveals no available measurement due to its pitching effects. Field data was collected for analysis. A measurement model was then formulated to find the best fit for the vehicle outline and the sensor data.
5. *A metric to quantify the vehicle following system.* From the literature review, it was learnt that it is difficult to quantify the performance of vehicle following on line. Furthermore, the majority of vehicle following controllers are based on deterministic control theory. However, the effect of controls applied to a follower vehicle may cause uncertainty because of the system latency and system disturbances. A metric is therefore required to quantify the performance of the vehicle following function, on line, taking into consideration the system latency and uncertainty.
6. *Demonstration.* To demonstrate the formulation and model of the vehicle following system, simulations and real experiments were planned. Through a series of exhaustive tests and examinations, the usability of the vehicle following function is justified.

1.7 Contributions of this thesis

The core framework and strategy proposed in this thesis are fundamentally based on well founded theoretical formulations. An overview of the primary contributions follows:

- A frequent challenge with vehicle following system involves achieving a fixed, and considered "safe", distance which the follower should maintain between the leader. Furthermore, it is widely acknowledged that optimal vehicle following requires the path traced by the leader vehicle to be exactly followed by the follower as such an approach eliminates the potential hazard of cutting corners. To jointly address these issues, a virtual trailer link model is proposed in this thesis. A virtual point is projected at a fixed distance from the rear of the leader (a virtual trailer), which guarantees the satisfaction of the safe distance criteria, while allowing for the accurate following of the leader's path.
- Path tracking applications such as vehicle following, typically involve the estimation of uncertain states (leader/follower vehicle pose), through the incorporation of uncertain measurement data (perhaps a laser sensor). Such systems are widely accepted to be well modeled through stochastic methods such as the Bayesian recursion. Such a recursion unifies sensing uncertainties of sensors and provides a formal mathematical basis which can then be solved through various filtering algorithms. Following an extensive review of the literature, such a formal stochastic formulation is seen to be lacking. Therefore, this thesis formalises the autonomous

vehicle following problem in a Bayesian framework for the first time. Recursive filters are then developed to estimate the states of interest and an observability analysis of the following system is also carried out.

- Vehicle following requires the location of the leading vehicle (or a virtual trailer) to be estimated at each time and subsequently followed. To estimate the location, the following vehicle generally uses an onboard sensor (such as laser, camera, radar etc) to outwardly sense the environment and try to obtain the leader's location. Naturally, such sensors are subject to various sources of noise, which make it a challenge to automatically interpret the sensor data and extract the vehicle location measurement. This thesis addresses this issue for laser sensors and proposes a recursive algorithm to detect the leading vehicle from data subject to numerous outliers and noise. This allows for a robust following system to be developed.
- Given accurate measurements of the leading vehicle, its path must still be accurately followed. To achieve this, a relative information minimisation approach is proposed in this thesis which examines the statistical information between the estimated poses of the leading and following vehicles. This new approach is seen to outperform standard methods of vehicle path following. Detailed case studies are also carried out to rigourously examine the feasibility and practicality of the proposed system.

1.8 Thesis Overview

The rest of this thesis is structured as follows:

Chapter 2 presents a literature review of the available vehicle following systems. Three categories of vehicle following systems are studied in the review. These three categories of vehicle are implemented on the motorways, in the urban environment and for the defense industry. Both the advantages and disadvantages of these vehicle following systems are analyzed. A brief discussion of the research gaps identified in the review is also presented. This review allows for the research challenges in slow speeds urban vehicle following to be identified.

Chapter 3 introduces the virtual trailer link model for the vehicle following function. The direct-hooked, off-hooked and generic kinematic configurations of the virtual trailer link models will be presented. The performances of each of the kinematic configurations implemented on various types of roads will be analyzed. The path deviations of each model will be compared. Based on the system design criteria and the performance of each kinematic configuration, an optimum virtual trailer link model will be designed for the vehicle following system. An experiment using real test drive data is also included. The experimental results and the implementation issues of the virtual trailer link model for vehicle following will be discussed.

Chapter 4 presents a Bayesian formulation for the vehicle following system. Based on some justifiable assumptions, a generic Bayesian formulation for vehicle following will be proposed. The observability of the vehicle following system can be analyzed based on the formulation. Furthermore, implementation issues such as sensor uncertainties will also be addressed. Some experimental results will be presented and discussed so as to provide evidence to validate the mathematical

formulation.

Chapter 5 proposes a metric to quantify the optimum driving commands for vehicle following. The metric was derived from the probabilistic perspective. The information theoretic vehicle following system will then be introduced. A detailed formulation will be provided and experimentations will be carried out as validation.

Chapter 6 concludes the thesis and proposes some recommendations for further research.

CHAPTER 2

Related Work

2.1 Introduction

Commercial demands [36] and army logistical applications [11] are the main drivers of the development of vehicle following systems. For example, in the commercial arena, the land transportation of goods containers between ports and cities is an important activity that contributes much to the economic growth of a country. Platooning of a fleet of trucks is one commercially viable solution that would help reduce the operating costs and increase the efficiency of this logistical activity [15],[16]. Truck platooning can also help reduce driver fatigue and hence improve road safety [37]. Commercially sponsored projects such as PATH [16], CHAUFFEUR [15], ARCOS [38], ARGO [39], etc are some examples of landmarks projects that have demonstrated successful implementation of the vehicle following function on motorways. Successes in motorway vehicle following implementations have motivated both the commercial and research communities to apply similar technologies to conditions of urban traffic congestion. The main objective is to reduce the burden on the driver during traffic congestions and thus

provide a more comfortable drive. Such efforts have been contributed by entities including PRAXITELE [27], the LASMEA [40] project, the ARCOS [38] project, etc. In terms of army logistical applications, robotic vehicle implementation has been of great interest to defense groups around the world [11]. The defense policies aim to reduce the risks to human life through the deployment of robotic vehicles. For example, it is dangerous for soldiers to be providing logistical supplies on the front line during times of war. Armoured vehicle platooning can help to minimise the number of soldier casualties. Some examples of such defense projects are DEMO III [41], SKYTREK [42], ULYSSES [33], etc.

This chapter summarises the research endeavours related to vehicle following systems. The purpose is first to establish a theoretical framework within which vehicle following problems can be addressed. Next, the pending research issues and constraints of such systems that are presented in the literature, and the implementations of such systems so far, will be identified and discussed.

Previous research has demonstrated the successful use of vehicle following systems particularly on the motorway, in the urban environment and in military applications. Thus, this chapter classifies vehicle following systems into these three categories. Section 2.2 reviews the research on vehicle following systems that have targeted motorway applications. Section 2.3 reviews the research on urban vehicle following systems, while Section 2.4 reviews the research carried out in the defense industry. These sections study the strategies used in the vehicle following systems. The purpose is to determine the differences in the strategies used to solve vehicle following problems, and in particular, to identify the manner

in which follower vehicle control is implemented. In other words, it is important to question how a follower vehicle is controlled while taking into account two conflicting requirements - tracking the lead vehicle as closely as possible while maintaining a sufficiently large inter-vehicle distance such that the follower vehicle can stop safely when required to. A discussion of the research issues to do with vehicle following systems is presented in Section 2.5. The chapter concludes with a summary of research gaps, in terms of issues that remain to be solved as well as a formulation of the problem as it stands and how it will be addressed in this thesis.

2.2 Vehicle Following Systems On Motorways

Motorways are dedicated roads for passenger cars, trucks and motorcycles. They are generally designed with low curvature. No pedestrians or bicycles are allowed on the motorway. This means that vehicle speeds on the motorway tend to be high ($>70\text{km/h}$). As motorways are links between two cities, commuters rely on these roads to travel between them. Traffic conditions on motorways have become quite congested, especially during peak hours. As a result of this, vehicle speeds have dropped and road efficiency has decreased. In view of these problems, vehicle following systems have been introduced to improve the capacity of motorways.

The manner in which vehicle following systems are implemented on motorways can be used to classify them into two categories. The first type of vehicle following system makes use of communications technologies to form what can be called cooperative solutions to vehicle following. The second type of vehicle following

system is based on self-sufficient solutions, that is, an autonomous system.

2.2.1 Cooperative Vehicle Following Systems

Cooperative vehicle following systems rely on communication systems for vehicle control. Either vehicle-to-infrastructure or vehicle-to-vehicle communication systems can be utilised. In the vehicle-to-infrastructure communication setup, the follower vehicle will sense a special installation on the road, such as magnetic markers, for lateral control of the vehicle. In the vehicle-to-vehicle setup, the leader will constantly transmit its vehicular status to the follower via a communication link. The follower vehicle, upon receiving the information from the leader, will perform the longitudinal and/or lateral control of the follower in order to follow the lead vehicle closely. The advantages of cooperative vehicle following systems are that no estimation of the vehicular state of the leader is required, data can be quickly processed and minimum environmental sensing by the vehicle is required. However, there is a risk of system breakdown if the communication system fails.

In the Chauffeur project [43], an electronic tow-bar system was implemented for vehicle platooning on a motorway. This electronic tow-bar system was aimed at automatically maintaining a small spacing between the two vehicles, while the follower vehicle automatically followed the trajectory traced by the lead vehicle. All the vehicles in a platoon followed the lead vehicle and the system was independent of any road infrastructure. In addition, inter-vehicle communication systems were installed on all vehicles in the platoon. Information that included the relative distance between adjacent vehicles and the tow-bar

angles were transmitted to vehicles in the platoon. The relative distance and tow-bar angles were uncovered by an infrared camera that recognized a dedicated pattern mounted on the back of the preceding vehicle. The yaw rate and speed were sensed for each vehicle in the platoon and may have been communicated via the inter-vehicle communication system. In the Chauffeur project, Pandeli [43] implemented a combination of longitudinal and lateral controllers in the follower vehicles. However, the trial results were not presented [43],[44]. Nevertheless, based on the descriptions found in the documents [43],[44], the lateral controller attempted to maintain a zero tow-bar angle between the two vehicles. Unfortunately, there is a disadvantage in maintaining a 'zero' tow-bar angle. Assuming that the controller works perfectly in maintaining a zero tow-bar angle, when the lead vehicle makes a curve turning, the controller will immediately take effect and this will result in the follower vehicle being steered into the turning mode. This may cause the follower vehicle to deviate from the trajectory of the lead vehicle.

In the PATH project [16], a demonstration was conducted in San Diego in 1997 [45] where a dedicated lane with magnetic markers embedded in the road [46] was designed for the lateral control of the vehicles. The magnetic markers were binary coded such that each marker was unique to the lane in which it was placed. During this demonstration, Tan [45] tested a vehicle following system with eight vehicles forming a platoon. Both the longitudinal and lateral controllers were implemented in the vehicles [47]. The separation distance between vehicles was 6.5m and all the vehicles operated at a speed of 27 m/s

(96 km/h). Radars were installed on the vehicles in order to determine the relative speed of each individual preceding vehicle. The signals from the magnetic markers were decoded to enable the lane following function in the vehicles. In the demonstration, a 10 cm lateral error was achieved. However, the error represents the difference between the current position of the vehicle in relation to the lane markers. The path deviation between vehicles was not presented in the documents.

Swaroop [48] has proposed a controller for a follower vehicle in an emergency lane change maneuver. Vehicle following was approached using the real-time trajectory curvature information generated and transmitted by the lead vehicle via the inter-vehicle communication system. The challenge identified by Swaroop [48] in the design of the controller was in integrating the lateral and longitudinal controllers so as to enable the follower to track the trajectory of the lead vehicle and to maintain a desirable following distance. Swaroop [48] noted that the availability of lateral deviation from the specified trajectory and the road preview information in the control system have an effect on the ride quality and tracking performance of a lateral vehicle guidance control system. The challenge that Swaroop [48] faced in implementing the vehicle following system was that the desirable trajectory to follow was not pre-specified and was not stationary. Swaroop [48] also noted that the absolute deviation from the preceding vehicle's trajectory was difficult to sense and therefore impossible to infer, especially without any form of communication between the vehicles. The problem was compounded when the follower vehicle maintained a short headway distance and

its sensors were blinded by the lead vehicle. In the design of the controller, the trajectory of the lead vehicle was estimated based on the sensor data and the information transmitted from the lead vehicle. The purpose of estimating the trajectory of the lead vehicle was to determine the desired velocity and relative position of the follower vehicle in relation to the lead vehicle. The paper [48] made several assumptions about the controller design. It was assumed that the lead vehicle had the requisite capability to sense and track the trajectory accurately. The radius of curvature and the length of the trajectory required to ensure a safe maneuver are assumed to be much larger than the length of a two vehicle platoon. In this sense, then, the two vehicles in the platoon were assumed to be traversing a road of the same curvature. Although [48] has since offered a better solution for vehicle following, which includes both lateral and longitudinal controllers, the system is very much dependent on inter-vehicle communication systems. The assumption that the two vehicles are moving along the same curvature is only valid for short headway vehicle following. It has been proven by [49] that a minimum headway for vehicle following is required; hence, the assumption made by [48] may not be valid for real system implementation.

Takehiko [50] has proposed three methods for solving vehicle following problems. The first is the point follower system, where all vehicles in the platoon individually follow the points (be it magnetic markers or information obtained from wayside communications) that represent the road. This approach is similar to road following and may not be considered to be successful as the vehicle following system does not physically follow any lead vehicle. The second method

is the longitudinal control system, where the follower vehicle uses on-board sensors to locate the position of the preceding vehicle, its own velocity, yaw rate and side slip angle. In addition to wayside vehicle communication systems for the lead vehicle, an inter-vehicle communication system was used. The third method proposed is trajectory following which aims to estimate the path of the preceding vehicle on the relative coordinates of the led vehicle. The follower records the positions of the preceding vehicle in its vehicular coordinates and transforms them in accordance with the vehicle maneuver. A group of these updated positions provide the estimated trajectory of the preceding vehicle. This paper [50] assumed that the lateral tracking error of the preceding vehicle was acquired through the inter-vehicle communication system. A trajectory follower algorithm was implemented to trail the trajectory of the leader vehicle. Certain assumptions were made during the design phase. Firstly, the follower vehicle was assumed to be able to recognize the position of the preceding vehicle. Secondly, the follower vehicle was assumed to be able to acquire its velocity, yaw rate and side slope angle data from the on board sensors. Thirdly, the information required of the preceding vehicle was assumed to be acquired through a radio transmission. Finally, the lateral tracking error of the proceeding vehicle was assumed to be acquired through the communication link. The simulation results of the trajectory following method showed that some path deviations (the exact quantity was not presented) continued to exist for the path tracking algorithm, especially when the lead vehicle was changing lanes. This path deviation filtered down all the way to the last follower vehicle in the platoon. The tracking error was significant for the last vehicle in the platoon. This result reinforces the idea

that platoon stability is directly affected by the path deviation of all the follower vehicles in the platoon. The simulation results have proven that pure longitudinal control (2nd approach in the paper [50]) for a vehicle following system will cause the follower vehicle to look for the shortest path to follow. This is undesirable when the lead vehicle is maneuvering on a curved path as the follower will tend to cut the corner when pursuing the leader.

Japanese research in the Mechanical Engineering Laboratory (MEL¹) has developed a vehicle following system called DEMO2000 [29],[51]. Communication among vehicles was implemented here. The aim of the DEMO2000 project was to test the feasibility of the inter-vehicle communication system. A radar was used to sense the relative speed of the leader in relation to the follower [29]. A constant headway of 25m was set as the safe inter-vehicle spacing during the operation and a longitudinal controller was implemented to maintain this constant spacing. No experimental results were presented in the papers.

2.2.2 Autonomous Vehicle Following Systems

A vehicle following system can be classified as autonomous if it relies only on the use of its sensors and other on board sources of information to accomplish the required tracking, actions and functions. The vehicles that fall under this classification are independent of each other and the road infrastructure. Autonomous vehicle following systems allow for fast deployment as no modification to the road infrastructure or the vehicles is required. Hence, autonomous vehicle

¹MEL has been renamed as National Institute of Advanced Industrial Science and Technology (AIST) in 2001.

following systems can co-exist with other vehicles that are not installed with a vehicle following capability.

Stefan [52],[53] has analysed the implementation of the autonomous vehicle following system using a lateral controller to interpolate the trajectory between the two vehicles. Stefan [52] demonstrated that a straight line (point to point) or a curve fitting solution will not represent the true trajectory of the lead vehicle as those paths have deviated from the lead vehicle's actual trajectory. Also, these deviations can be proportionally scaled to the distance between the lead and follower vehicles. In platooning applications, this deviation can cause the autonomous vehicle to hit an obstacle, such as a parked vehicle or a road curb on a curved road. Platoon stability would also be affected, as the path deviation from a lead vehicle filters down to all the follower vehicles in the platoon. In view of this, Stefan proposed a trajectory reconstruction model that enables the tracking of the lead vehicle. His model states that a follower will trail the path rather than the current position of the leader. A stereo pair camera is installed in the follower vehicle to estimate the relative position and orientation of the lead vehicle, and a path is reconstructed using the estimated poses of the leader and the follower. A global map is built based on the estimation results. The proposed algorithm [52],[53] uses the time history associated with the lead vehicle over a certain period of time, as shown in Figure 2.1. In this case, the position coordinates of the lead vehicle and the motion parameters of the follower vehicle were stored with the time stamped on the map. From the map, a tracking point for vehicle following in both lateral and longitudinal directions was then selected. The algorithm

Figure 2.1: Trajectory reconstruction-based vehicle following from Stefan [52]. The positions of the lead vehicle are recorded up to time t_n . A lookahead distance is imposed for safety purposes.

assumes that the lead vehicle moves at such a slow speed that it is maneuvering along a straight path within this short period of time. Some experiments were carried out on straight and clothoid paths. The speed of the vehicles was limited to 10m/s (30 km/h) during the experiments. A maximum path deviation of 70cm was observed, based on the results presented in publications [52],[53]. The solution proposed by Stefan has highlighted that the follower vehicle should trail the path of the lead vehicle instead of the momentary point on the lead vehicle as observed by the sensors. Recording the path of the leader on a map for vehicle following was shown to be feasible. As a safe following distance is required, Stefan introduced a lookahead distance as a safety separation for his follower vehicle. However, a straight path was constructed from the centre of the follower vehicle extending

to a point defined by the lookahead distance. This short section of the straight path may not reflect the actual path of the lead vehicle. Hence, a path deviation is expected, which has been observed from the experiments conducted by Stefan [52].

The PATH [16] project implemented a magnetic referencing system. The magnetic markers were buried in the centre of a dedicated lane on the motorway, as shown in Figure 2.2. All the embedded markers were spaced equally apart.

Figure 2.2: Magnetic markers used in the PATH project. The markers were embedded in the lane. A vehicle mounted with magnetometers can read the magnetic signals from the markers. This allows for the possibility of lane following.

The PATH vehicles relied on two magnetometers installed on the front and back bumpers of the vehicle for localisation of the vehicles in relation to the centre of the lane. Reading and decoding the magnetic signals allowed for the vehicle to be guided along the dedicated lane. However, Lu et. al. [54],[55],[56],[57],[58],[59] foresaw a system reliability problem - that it is possible that this lane following method for vehicle following applications would malfunction if the magnetome-

ters were faulty. As a result, Lu et.al. designed an autonomous vehicle following system using a laser scanner [55], without the aid of magnetic markers. The laser scanner used for this purpose had a field of view of 12 degrees. As the laser measurement data tends to be noisy, an extended Kalman filter [60] was formulated to estimate the relative position and lateral deviation of the leader in relation to the follower. The system was tested on curved paths, with the maximum speed of the vehicles set at 8m/s (28 km/h). A maximum path deviation of 35cm was observed in the experiments [59].

In another project under PATH, White [61] used a laser scanner that was installed at the front of the follower to estimate the relative distance and orientation of the leader. Two leader's path estimation methods for vehicle following were investigated. The first method involved constructing a straight path between the current positions of the two vehicles. In the second method, a constant curved path between the two vehicles was assumed to exist. Simulations were conducted to test the performance of these two methods. The results showed that, when using the former method, the path deviation between the two vehicles faced a problem in that the follower vehicle would cut the corner. The "cutting the corner" effect was shown to be a serious one, especially when the separation between the two vehicles was more than 10m. When using the latter method, a path deviation of less than 10cm was in fact, achieved in the simulation runs. White's observations [61] make up an important finding on the appropriate strategy to use for vehicle following, that the linear path representation of the trajectory of the lead vehicle can lead to a large path deviation in a vehicle following system.

In the ARGO project [39],[62],[63], supported by the Italian National Research Council (CNR), the vehicle following function was an extension of a road following function. Figure 2.3 shows a picture of the ARGO vehicle and a stereo camera pair mounted on the top corners of the windscreen in the vehicle. The stereo camera was used for recognition of the leader based on the symmetry characteristic in the image.

Figure 2.3: ARGO vehicle equipped with stereo cameras. The cameras are mounted at the top corners of the windscreen.

The relative distance, speed and heading of the leader were estimated from the captured image, for the purpose of vehicle localisation. The centre point of the rear of the lead vehicle, as estimated by the vision algorithm, was set as the target point for vehicle following. Broggi [39] has suggested that this target point following of the lead vehicle is insufficient for vehicle following and will result in the follower vehicle hitting the road curb, especially when the vehicle is curve following. Hence, a virtual point that is projected from the target point at a certain lookahead distance was set as the input to the vehicle controller for vehicle

following, as shown in Figure 2.4. Though the road following function was tested on a motorway, no experimental results were presented for vehicle following.

Figure 2.4: The ARGO vehicle detected the target point located at the rear of the lead vehicle. An estimated target point for vehicle following was computed based on the given lookahead distance.

In the Chauffeur Assistant project [64], a vehicle following system was designed based on the fusion of the advanced cruise control and lane keeping algorithms. A radar was used to estimate the relative distance, speed and lateral position of the leader in relation to the follower. A camera was used to detect the lane marking for estimating the position of the follower vehicle in relation to the lane. A Kalman filter was used to track the lead vehicle. Based on the output of the filter, longitudinal control was implemented for keeping a minimal and safe distance from the leader by adapting the speed of the ego vehicle automatically. Experiments were carried out at a maximum vehicular speed of 22 m/s (80km/h). The results of the experiment showed a maximum path deviation of 5m.

2.3 Vehicle Following Systems In Urban Environments

The urban environment is characterised by high traffic density and complex geometrical road and street layouts. There are traffic junctions with traffic lights controlling the flow of vehicles. Vehicles in urban environments can travel at speeds slower than 14 m/s (50 km/h). Vehicle following in the urban environment requires a less complex longitudinal controller, while lateral controllers should be emphasized if the lead vehicle is to constantly maneuver along curved paths.

In this section, the most representative systems within this vehicle following class are presented and analysed, with the emphasis placed on the perception systems used, the vehicle following strategy and the performance of each system.

Pham et al. [65],[66],[67] have implemented a vehicle following function by imitating human driving practices. In the vehicle following model [65],[66],[67], a virtual point was defined in front of the direction of motion of the follower vehicle. This virtual point was defined to be at an angle that deviated from the steering angle of the follower vehicle and at a fixed distance from the centre of the front steering wheel of the follower vehicle, as shown in Figure 2.5.

The control law marked this virtual point as a target for tracking (i.e., the virtual position of the target vehicle) during vehicle following. Pham's model [65] has proven that the virtual reference point must be located at the point of intersection of the longitudinal axis of the leader and along the extension of the direction of the steering wheels. Pham [65] has verified that this system was unable to track the trajectory of the leader, especially when the leader was maneuvering along

Figure 2.5: Virtual point Based vehicle following by Pham [65]. A virtual point P_r is defined at a distance l from the follower and at an angle k times that of the steering angle, γ , of the follower.

a curved path. To solve this problem, Pham [65] suggested that the position of the proposed virtual point should be a function of the characteristic of the desired trajectory and the instantaneous configuration of the vehicles. In other words, the virtual point should be a function of the curvature of the desired path and the velocity of the leader. For instance, when the vehicle is moving along a straight path, the virtual point should be defined at a farther distance away from the follower than when the lead vehicle is making turns. For the perception system, a laser scanner was mounted onto the follower vehicle. Three reflectors were mounted onto the rear of the lead vehicle so as to allow the relative position and orientation of the leader to the follower vehicle to be estimated. Two electric vehicles were used, and the maximum speed of the vehicles was set at 1 m/s. The

results of the experiment demonstrated vehicle following on straight and curved paths. A maximum path deviation of 0.5m was observed when the vehicles were making turns. Pham [65] has thus demonstrated with this system that vehicle following can be realized without the use of any inter-vehicle communication system. It has also been shown that the vehicle following system should not simply track the current position of the lead vehicle directly.

Franke et al, at the Daimler-Benz Research Centre [68], have implemented a truck platooning application for 7.7 ton trucks. In this system, longitudinal and lateral controllers were both implemented in the follower vehicle. A constant time gap strategy was used to control the follower vehicle in the longitudinal direction. An array of infrared lights arranged in a checked pattern was mounted at the rear of the lead vehicle. An infrared camera was used to detect this checked pattern, and the relative distance and orientation of the leader vehicle was estimated based on the captured image. A wireless communication link between vehicles was also installed, which was used to transmit the states of the leader to the follower vehicle. Before implementation, a simulation was conducted with a platoon of 4 trucks. The results of the simulation showed that the maximum path deviation (longitudinal error) was 0.5m for the first follower at a speed of 10 to 15 m/s.

Daimler Benz Research has further applied the above concept [68] to the platooning of trucks in inner cities [69],[70], under a project called UTA (Urban Traffic Assistant). Instead of the infrared camera, a stereo camera pair and a color camera located at the front of the follower vehicle were used to track the lead

vehicle, and a Kalman filter was used to estimate the relative distance, speed and acceleration of the leader in the longitudinal and lateral directions in relation to the follower. A constant time gap with a safe separation strategy was used to ensure a safe inter-vehicle distance. A tow-bar strategy was implemented to correct the lateral positional errors. The researchers at Daimler-Benz recognized that the tow-bar strategy only allows for an approximate trajectory following of the lead vehicle and tends to cut corners in narrow turns. This shortcoming was overcome by incorporating a lane detection algorithm into the system that acted as a lateral controller. The relative position of the follower vehicle in relation to the lane was computed in order to keep the vehicle on the lane. The vehicle following capability of the UTA vehicle, moving at a speed of up to 10m/s , was demonstrated in Germany. In the demonstration, the desired safe inter-vehicle distance was set at 10m and a time headway of 1 second was set in the longitudinal controller. The maximum error in the inter-vehicle separation, as compared to the measured values, was about 5m.

Parent and Daviet [71] at INRIA (France) have designed a vehicle following system for operation on urban roads. The operating speed of the vehicles was limited to 10km/h, as two slow speed electric vehicles were used. Parent [72] carried out a study with the vehicles travelling at this speed, and showed that a data acquisition rate of 10Hz (or 10 frames per second) or above for the perception system is sufficient to ensure safety. In this proposed system, an infrared reflector was mounted onto the rear of the lead vehicle. A camera and an infrared flash light were used to compute the position of the leader. This was achieved by

projecting the infrared light source onto the rear body of the leader to obtain a contrasted image. The vision algorithm then performed shape recognition in order to track the leader. The outputs of the vision algorithm were the estimated relative distance, lateral deviation, speed and acceleration of the leader. Prior to the experimentation on the road, simulation tests obtained a maximum lateral error of 60cm. The perception system proposed by Parent and Daviet [71] is unique; however, only short range infrared illumination can be used. Higher energy power is required for long range infrared illumination, but this may pose a radiation risk to the environment.

Daviet [73],[74],[75],[76] has further extended the vehicle following concept as part of the PRAXITELE project. In the PRAXITELE project [27],[77], electric cars were used as the platform for experimentation. These vehicles were intended for an operating speed of up to 60km/h for inner-city traffic. Instead of using the infrared flash as an illuminating device, as proposed in [71], the reflector mounted on the lead vehicle was replaced by three rows of Light Emitting Diode (LEDs). With this new setup, the orientation of the lead vehicle was computed as observed by the follower through a camera. Both the longitudinal and lateral vehicle controllers were implemented. The longitudinal controller was designed to regulate the relative separation distance between the two vehicles. The proposed lateral controller was aimed at aligning the steering angle of the follower parallel to the orientation of the lead vehicle. Daviet [73],[74] predicted that this proposed wheel alignment strategy for lateral following would cause the follower vehicle to cut the corner when negotiating a curved road. A lateral error

of up to 60cm was indeed obtained later when the experiments were conducted [74],[75], when the vehicles were travelling at a speed of 60km/h.

Another vehicle following system was designed in INRIA by Selim [78]. In this project [78], a visual tracking method was used to track the lead vehicle. A reference template was mounted onto the rear of the lead vehicle. The vision algorithm based on the captured images estimated the relative position and orientation of the lead vehicle in relation to the follower. These estimations were made by analysing the geometrical distortion of the reference template. A virtual reference frame was constructed based on the estimated pose of the lead vehicle. This virtual frame was obtained by translating the lead vehicle's reference frame by a specific distance. The purpose of the vehicle following controller was to track this virtual frame. An experiment was conducted with vehicle speeds of lower than 1m/s. The results showed a maximum lateral following error of about 2m when the vehicles were making turning maneuvers.

Research has also been conducted in LASMEA² on a vehicle following function [40],[79],[80]. In this design, differential RTK-GPSs (Real Time Kinematic Global Positioning Systems) were installed in both the lead and follower vehicles. During the vehicle following operation, the position (hence the path) of the leader was transmitted over to the follower via an inter-vehicle communication system. The follower vehicle maintained a constant curvilinear distance from the lead vehicle during following. Experiments were carried out on the LASMEA campus. The speed of the leader was set at 1m/s. A path following error of 3cm and 10cm was

²<http://www.lasmea.univ-bpclermont.fr>

achieved when the system was tested on a straight and curved path respectively. Martinet from LASMEA [81] has made further improvements to the system by anticipating the possible loss of a GPS signal during vehicle following in an urban environment. A monocular camera was then installed in the follower vehicle. A trajectory learning step was established in order for the system to build a reference trajectory for vehicle following. During the vehicle following stage, the sensory information (features in the scene in this case) from the database was loaded and compared to the images captured during the operation. In doing so, the relative localisation of vehicles was achieved. A path deviation of 10cm and 35cm was achieved, without the GPS information, for vehicle following along straight and curved paths respectively.

In the ARCOS [38] project, a French program, Martinez and Canudas [82],[83] have proposed a variable time headway model for vehicle following. This proposed model considered only the longitudinal control of the follower vehicle. The main purpose of this model was to ensure the safety of the vehicles during close following. An inter-vehicle communication system was used in the vehicle following experiments. The absolute position and speed of the lead vehicle were estimated and transmitted to the follower vehicle via radio-frequency modem. The vehicle following system was tested in a stop and go scenario, with the lead vehicle travelling at a speed of up to 7m/s. The proposed variable time headway model was implemented to maintain a safe vehicle following distance. A longitudinal error of 2m was reported. Another test scenario, where an emergency brake was applied to the lead vehicle, was carried out to further test the response

of the proposed model. A longitudinal error of 5m was achieved in this case, when the vehicles were travelling at a maximum speed of 20m/s before the lead vehicle made an emergency hard stop.

Before implementing a vehicle following system, Yi and Moon [84],[85] simulated the performance of vehicle following. The effect on safety by the inter-vehicle distance control and inter-vehicle time headway algorithms were both studied in detail. The studies were verified in the simulation and the speed of the vehicles was limited to 40km/h, in order to simulate low speed driving on an urban road. Yi and Gu [86],[87] carried out some real experiments to determine a practical setting for the time and distance headways for vehicle following. The driving parameters, time and distance headway of a human driver were collected using a radar in these experiments. A constant safety separation together with constant time gap policies were then modelled based on the data collected from the manual driving experiments. The results of the experiments yielded a Root-Mean-Square (RMS) inter-vehicle distance error of 1.19m.

2.4 Vehicle Following Systems In the Defense Industry

The vehicle following function has been implemented in the US Future Combat System (FCS) programme [11], Demo II and Demo III [11],[41],[88],[89],[90], as shown in Figure 2.6.

In the Demo III vehicles, Global Positioning System (GPS) receivers and a pair of modems were used. A GPS was installed on both the lead and follower vehicles, and the data augmented with the vehicle speed was recorded by a manually

Figure 2.6: Demo III vehicles.

driven lead vehicle. The recorded data was then transmitted over to the follower for spontaneous generation and adjustment of the follower vehicle's path. The follower vehicle relied on the perception system to avoid obstacles on the path taken by the leader. Sensors, including passive color CCD, high resolution stereo, stereo infrared, a laser scanner and millimeter wave radar, were installed in the Demo III vehicle. The advantage of the Demo III vehicle setup is that the execution of the vehicle following task can be carried out immediately, or days after receiving information from the leader. Furthermore, as the lead vehicle can be driven manually by a human driver, the path provided by the leader has proven to be traversable. Path planning by the follower is therefore not required. As the planning and obstacle detection tasks were minimised, the Demo III vehicle was operated in semi-autonomous mode at a maximum speed of 100km/h. An improved version of Demo III called STRYKER [12] was able to further incorporate the terrain data for the follower. No performance data is available in the literature. The main advantage of the Demo III vehicle is that minimum data processing from the perception sensors is required and the system can operate during the day as well as at night. However, its reliability depends

on the availability of the GPS data, which cannot be guaranteed in built areas or where dense tree foliage blocks satellite signals. Thus, although this is a proven solution, reliance on GPS data alone makes it unsuitable for autonomous driving in urban environments where the multi-path problem poses a major challenge [91]. Demo III vehicles also rely heavily on the communication link between the vehicles.

The United States Army has sponsored a project called BART (Binocular Autonomous Research Team) [42]. This system implemented a binocular vision system to obtain the range and heading of the lead vehicle and a neural network to control the follower vehicle. In the neural network training phase, data (vehicle speeds and inter-vehicle spacing) was collected from real driver maneuvers along a straight path, during lane changing and when making left/right turns. Two independent networks were trained, one for longitudinal (speed) control and the other for lateral (steering) control of the vehicle. The BART vehicle was aimed at developing a generic system that could be implemented on any vehicle regardless of the dynamics of the selected vehicle. Path deviation was not the main concern of this project [42]. The performance of the vehicle following system was compared to manual driving. Qualitative results presented in [42] claim that the BART vehicle performed close to the human driving behavior. Slow speed following up to 24km/h (6m/s) was tested and a 20m inter-vehicle distance was maintained throughout the following.

In another military sponsored project called ULYSSES [31],[32],[33], vehicle following was deployed in an urban jungle environment. A laser line scanner

(from SICK optics³) was used as the main vehicle tracking sensor. The maximum achievable speed of the system was 4 m/s. Because of the vehicle dynamics of the follower, which was a 12 ton Armour Personnel Carrier (APC), shown in Figure 2.7, a pitching effect was inevitable when the vehicle was deployed in an urban jungle environment. As a result, the sensor data was noisy and uncertainties in the measurement data posed a challenge to the vehicle following function. GPS coupled with inter-vehicle communication systems were introduced to resolve this issue. In this case, GPS data was used to compensate the noisy laser tracking data. A path planner [92] was also included to constrain the follower along the path of the lead vehicle. The estimated position of the leader was treated as a moving target point in the local path planning. As the ULYSSES project was at the proof of concept phase, no quantitative analysis of the performance of the vehicle following was presented.

Figure 2.7: Ulysses vehicle.

³www.sickusa.com

2.5 Research Issues in Vehicle Following Systems

The objective of the vehicle following function is for a follower vehicle to trail the path of the leader vehicle. Based on the discussions outlined in sections 2.2 to 2.4, vehicle following has been demonstrated in two classes of applications:

- **Vehicle trajectory following:** In this class of applications, the follower vehicle must follow exactly the tyre-marks of the preceding vehicle. Hence, the follower vehicle needs to also sense other objects besides the leader so as to avoid collision with the curbs. Urban vehicle following in congested conditions is one such example. In congested traffic, the vehicles are tail gating one another at a small inter-vehicle spacing. As a result, the field of view of the sensors on board the follower vehicle will inevitably be occluded by the leader due to this small spacing. With reduced field of view, insufficient environment information surrounding the ego vehicle can be acquired, thus creating the danger of collision to the vehicles. Hence, for safety reasons, the follower vehicle must trail the lead vehicle with a minimum path deviation. Furthermore, in order to avoid possible collision with a preceding vehicle, a safe distance between the lead and the follower vehicles must be maintained so that the follower can stop safely when the leader stops abruptly. Next, the follower vehicle must exactly trail the path of the lead vehicle throughout the vehicle following operation. In doing so, safe vehicle following can be achieved without the follower vehicle hitting the curbs or cutting the corners. In addition, in order to increase the efficiency of the rate of traffic flow, the follower must follow the lead vehicle

closely. When there is a small spacing between the vehicles, it becomes impossible for other vehicles to cut in between them.

- Vehicle path following: in this case, the follower vehicle emulates (replicates) the behavior of the lead vehicle. This is, in effect, for a follower to traverse on the same path, and not on the same local trajectory, as the leader vehicle. ACC [6], [7] and multiple robotic formation systems [93], [94], [95] are some of the examples of this class of vehicle following application. For this application, the follower sometimes relies on information from either on board sensors or environmental information communicated via infrastructure, to refine its path. For example, the on board sensors, such as lasers or cameras, are used to sense the road lane, curbs and other dynamic obstacles to enable the follower vehicle to replicate the behavior of the lead vehicle.

The results of experiments conducted on vehicle following systems that have been implemented as described in Sections 2.2 to 2.4 are encouraging. Many research issues have been addressed in these experiments. However, there remain some pending research issues which will be discussed in the next section. The research issues for vehicle following can be broadly classified into two major areas: vehicle following control strategies and the perception systems used in the systems.

2.5.1 Vehicle following Control Strategies

The speeds of the vehicles involved in motorway vehicle following systems are typically higher than 70 km/h [21]. As the road geometry of the motorway

includes small curvatures, most vehicle following systems have implemented a longitudinal controller in the follower vehicle for trailing the lead vehicle. The concept of the longitudinal controller has been applied to automatically control a vehicle, such as the PATH vehicles [29],[38],[64],[50],[51],[68], in the direction of motion of the vehicles. The controller maintains a safe inter-vehicle spacing between the vehicles when travelling at high speed on the motorways. This specific control law is designed based on information such as the velocity and acceleration of both vehicles and the distance between them. The vehicular information obtained from the leader's on board sensors is communicated to the follower through the wireless link between vehicles [29],[31],[48],[50],[51],[69],[96]. These can be regarded as speed controllers. The velocity profile of the vehicle is proportional to the inter-distance between the lead and follower vehicles. Constant time [68],[86],[87] and constant separation headway [29],[51] control strategies have been implemented to maintain safe following at high speed. For the constant time headway strategy [68],[86],[87], the inter-vehicle distance is proportional to the relative speed of the leader and the follower. Thus, a longer safe following distance is required when the relative speed of the vehicles is high. On the other hand, for a constant separation headway controller [29],[51], a fixed following distance, on top of a safety separation, is required. However, these strategies only address the longitudinal control of the vehicle following function and are applicable for vehicle following maneuvering along a straight path, such as a motorway. The geometry of the roads in an urban environment is much more complex. For instance, the road curvatures are higher than that of a motorway. Hence, the vehicle controller's

task is to ensure both the longitudinal as well as the lateral following of the leader by the follower.

Lateral controllers generate steering commands that minimize the lateral error between the lead and follower vehicles [38],[52],[53],[54],[64],[65],[77]. The lateral control of the follower vehicle can be implemented using inter-vehicle communication systems [29] or vehicle to infrastructure communication systems [16],[29]. In the inter-vehicle communication setups, the state of the lead vehicle, including its speed and acceleration, are transmitted to the follower vehicles. On the other hand, in the vehicle to infrastructure communication setups, the control software embedded in the road infrastructure will set the desired vehicle speed and inter-vehicle separation distance for the vehicle following systems. The relative speed and separation between vehicles can be computed based on the available information and the information obtained from the ego vehicle's onboard sensors. Another common vehicle to infrastructure communication system uses magnetic road markers embedded in the road for guiding the vehicles in the lateral directions [54],[46]. Follower vehicles are equipped with magnetometers to interpret the deviation of the vehicles from the lane. In this way, these setups are able to minimize the vehicles' dependence on environmental perception systems to ensure safe following. However, concerns around system reliability and user privacy remain. Breakdown of the communication systems and the loss of personal information that is tracked during implementation are some issues that have yet to be resolved.

Another lateral controller that is discussed in the literature relies on the lane

markers painted on the roads [64]. The images captured by the on-board cameras are used to detect the lane markers (such as painted lines). This information has been used as feedback in the vehicle controller to keep the vehicle in the lane. Although breakthrough solutions that rely on road infrastructure for vehicle lateral following have been proposed and demonstrated, some technical issues do need to be addressed. The vehicle following systems that use the lane markers for controlling the vehicle in the lane are applicable to vehicles travelling along motorways. Multiple lanes are available on the motorway and there are clear road markers (strips) that divide the lanes. However, it is common in the urban environment to have a single lane with no road markers. In such cases, the control software must detect the road dividers or curbs for lateral control of the vehicle following system. Moreover, there are traffic junctions in the urban environment with no road markers. Yellow boxes would usually be painted on the road instead. All these constraints pose research challenges for vehicle following in urban environments.

Besides implementing vehicle following using either longitudinal or lateral controllers, there have been other research endeavours that have combined these two controllers. In order for a follower to trail the path of the leader, the lead vehicle's path has been recorded in some systems [41],[88],[89],[90]. In these projects, a GPS was used for the localisation of the vehicles. The position of the lead vehicle is transmitted to the follower for vehicle path following purposes. There are some advantages to this type of vehicle following method. Firstly, the exact path of the leader is guaranteed. Secondly, low computation is required

if GPS is used. However, this method relies on the accuracy and reliability of the GPS. Data uncertainties and the canyon effect in the urban environment are some of the main concerns when using GPS as the localisation sensor for vehicle following.

Vehicle following has also been implemented by following a point associated with the leader [15],[54],[39],[77]. In this method, the centre of the rear of the lead vehicle is the point of interest to be followed. The method is similar to pure pursuit [97] and constitutes a reactive vehicle following system. The on board sensors perceive and estimate the relative position of the leader, and the follower's controller will momentarily pursue this point. As the orientation information of the leader is not considered, it is inevitable for the follower vehicle to cut a corner while curve following [39]. Virtual point following methods have been proposed to minimise this effect [39],[65], where a virtual point is computed based on the sensory information and the control model. However, the point and virtual point following methods have one major drawback; i.e., the estimated points may not exactly represent the path of the leader. Thus, some path deviation between the vehicles is to be expected.

To address the issues of path deviation and cutting of corners, [52] has implemented a path reconstruction method for vehicle following. The position and orientation of both vehicles were estimated and recorded in the system memory. During vehicle following, the recorded points were used as the reference for following. This method has solved the cutting the corner issue for vehicle following. However, the reliability of the system depends on the accuracy and

reliability of the sensors used.

Based on the analysis above, some research issues in designing a vehicle following system can be identified. It has been observed that longitudinal controllers have been successfully implemented for motorway vehicle following systems. These controllers regulate the position of the follower vehicle in the direction of motion of the vehicle. However, as road conditions are more complex in urban environments, with many sharp turnings, longitudinal controllers are unable to cope with the large curvature of the roads. On the other hand, lateral controllers that cooperate with the road infrastructure as a guide for the follower vehicles have been tested successfully on motorways. Technically, these control methodologies are meant for road following and it is assumed that the lead vehicle is maneuvering along the centre of a lane. Other types of road markers, such as yellow boxes, do exist on urban roads; however a simple lane detection algorithm may not be sufficient for the purpose of vehicle following. Hence, the lateral control algorithms for vehicle following require further investigation when the systems are implemented in urban environments. Moreover, cutting of corners by the follower is one of the major problems for vehicle following systems on a complex road. The safety of the vehicles and other road users would be compromised if this effect occurs. Longitudinal and lateral controllers are insufficient to solve this problem as the exact path of the leader is not represented in the systems.

In view of the above issues, it becomes clear that there is a need to model the exact path of the lead vehicle for an urban vehicle following system. The new

model must be able to trace the path of the leader while at the same time take into consideration the uncertainties of the sensors used.

2.5.2 Perception in Vehicle Following Systems

Perception is one of the most important factors in achieving a successful vehicle following system. Careful analysis of the appropriate sensors for this application is essential during the design stage [98]. Hence, an understanding of the characteristics of each selected sensor is of utmost importance.

Radars (Radio Detection and Ranging)

Radars have been made commercially available at relatively low cost since 1999 [99]. For example, radars from Eaton (www.eaton.com/VORAD) and Daimler Chrysler, shown in Figure 2.8, have been widely used for the cruise control of trucks.

Figure 2.8: Radar unit from (a) EATON VORAD and (b) Daimler Chrysler.

Radar controllers compute the relative information based on the time of flight principle and relative velocity using the principle of Doppler effect. Radars can be used to compute the relative speed and distance of two vehicles up to a vehicle separation distance of 100m. Two common frequency ranges used in radars are

24GHz and 77GHz . The field of view of a radar for vehicle cruise control, is typically 12 degrees at a speed resolution of 0.2km/h. Radars can be used under all weather conditions such as rain and fog and are independent of environmental illumination. Hence, they can be used around the clock. Radars constantly emit electromagnetic waves for computation of the relative information of a target. Interference issues will arise if two antennas from two similar radars are pointing at each other, resulting in inaccurate or noisy data. As the data output from the radars is in discrete form, a low computation time is required for data processing.

Radars are the most common sensors used for longitudinal vehicle following on motorways [16],[29],[41],[64],[45],[46],[47],[51],[86],[87],[88],[89],[90]. The relative speed and distance obtained from the radar mounted on a follower vehicle are used as the input to the longitudinal controllers. The controllers then regulate the desired speed and acceleration of the follower for the purpose of vehicle following. For vehicles moving at a high speed on a motorway, the relative speed during vehicle following is typically low (sometimes near zero), as both vehicles are moving at about the same speed. However, radars provide poor estimates of lateral position of the target vehicle. Hence, radars may not be suitable for low speed vehicle following systems, such as the one used for urban applications.

Laser Line Scanners

Laser line scanners have gained popularity in mobile robotic applications [100], [101]. Besides being used for vehicle following applications [33],[32],[41],[54],[55], [56],[57],[58],[59],[61],[65],[88],[89],[90],[31], laser scanners have been utilised extensively as the primary sensor for vehicle navigation and obstacle detection,

for example, in the recent DARPA⁴ urban challenge. Figure 2.9 shows three of the winning vehicles from CMU, Stanford University and Virginia Technology University. These vehicles are mounted with laser scanners for navigation purposes.

Figure 2.9: The first three teams that have completed the DARPA urban challenge 2007.

(a) Vehicle from Carnegie Mellon University, (b) Vehicle from Stanford University, and (c) Vehicle from Virginia Technology University. All these vehicles relied heavily on the laser scanner for obstacle detection and path planning.

Laser scanners emit near infrared (typical wavelength around 906nm) pulses and measure the reflected pulses. The relative distance between the object and the scanner can be obtained based on the time of flight principle. Two dimensional profile scanning has been achieved through an internal rotating prism installed in the scanner. The field of view of the 2D laser scanner can be as wide as 270 degrees. The angular resolution is 0.125 degrees; however, the scanning frequency can be as low as 10 Hz, depending on the angular resolution of the scanner. Commercial suppliers, such as IBEO⁵ and SICK⁶, have been developing laser scanners that can provide a sensing range of up to 50m. These scanners

⁴<http://www.darpa.mil/GRANDCHALLENGE/>

⁵<http://www.ibeo-as-de>

⁶<http://www.sickusa.com>

are eye safe and hence popular in outdoor mobile robotic applications. However, the accuracy of the laser scanner can be affected by dust and rain. For vehicle following applications, a 2D laser scanner provides only single line scanning, or up to 4 lines scanning (IBEO model); therefore, low computational resources are required for data processing. As only a single line of data is obtained, it is challenging to differentiate the target vehicle from the obstacles. Information such as the relative position and orientation of the lead vehicle can be estimated from the data obtained from the scanner.

Similar to Radars, laser scanners emit light waves in order to compute range information. With a laser scanner, the vehicle pitching effect constitutes another challenging research issue for vehicle tracking. Because of the pitching effect, the laser scanner may not be pointing at the target vehicle. Probabilistic estimators [54],[102],[60] are normally used to estimate the pose of the lead vehicle based on the range data. These estimators model the uncertainties of the range measurement for position estimation of the targets.

Cameras

A reliance on environmental illumination to provide information is a major reason for the general affordability of cameras [20], and drives their subsequent widespread usage in numerous intelligent transportation system (ITS) applications such as speed or traffic congestion monitoring and automated parking systems. With fast paced technological advances and resolution increases in the images acquired by cameras, target classification and discrimination based on object size or shape were made possible [15],[40],[52],[53],[64],[43]. Research

into 3D scene reconstruction also gained popularity as fusing two images allowed for range readings to be extracted as a function of the disparity [39],[62],[63]. Given images of high resolution, extensive environmental information can be extracted and estimated. However, such rich information content comes at a high computational cost and coupled with poor discrimination capabilities in the presence of obstructions or camouflage imposes performance limitations on most cameras and their processing algorithms.

Designed to sense and detect wavelengths beyond the vision spectrum range in the electromagnetic spectrum, forward looking infrared (FLIR) or thermographic cameras detect environmental infrared radiation as opposed to visible light. As such, these sensors are almost illumination independent and are operable day or night. Attractive attributes such as near or far range capabilities and robustness to smoke, haze, light camouflage, light rain and mist make FLIR cameras suitable to applications such as vehicle following [70],[41], [103]. This added robustness however, usually comes at a higher cost than normal cameras.

Active versions of FLIR cameras, known as near infrared (NIR) cameras, are also available on the market place. These sensors emit infrared radiation into the region of interest and analyze the returned signal intensities. Through such principles, detailed IR imagery is possible, even with road markings (in poorly illuminated situations) being evident in an NIR scan of a road surface. However, they are not used as extensively as FLIR cameras due to their sensitivity to the headlights of oncoming traffic, street lights and poor reflectivity at off-normal angles from smooth surfaces such as road signs. Furthermore, multiple NIR

cameras operating in unison can be susceptible to image saturation should they mutually scan each other.

2.6 Summary

This chapter has presented a literature review of research endeavours on vehicle following systems. For the purpose of analysis, vehicle following systems have been classified in this thesis based on their applications. Three applications have been identified: vehicle following systems on motorways, in the urban environment and in the defense industry. Fruitful results have been achieved for motorway (high speed) vehicle following. Many demonstrations were conducted in the late 1990s and early 2000. Landmark demonstrations such as the Demo 97 in San Diego Highway, USA and Demo 2000 in Japan are some of the success stories for motorway vehicle following.

There are two types of vehicle following implementations - autonomous (stand alone) and cooperative vehicle following systems. Cooperative vehicle following systems have some advantages over autonomous vehicle following systems. More information on the dynamics of the lead vehicle can be obtained in cooperative vehicle following systems. As for the autonomous vehicle following systems, all the information on the lead vehicle is obtained from onboard sensors. However, unlike autonomous vehicle following systems, cooperative vehicle following systems can only operate within the dedicated groups of vehicles, as they require the setting up of a communication link in the lead vehicle or infrastructure.

The lead vehicle in vehicle following systems must trace and repeat the trajectory of the lead vehicle. Dependence on inter-vehicle communications and GPS should be minimized, if not eliminated, as systems that depend on inter-vehicle communication systems face the risk of degraded performance in the event that the communication system fails. Also, systems relying on GPS may face signal reliability issues as a result of the canopy effect in the urban environment.

Longitudinal controllers allow the follower vehicle to follow the lead vehicle while keeping a separation by adapting its speed automatically. Longitudinal controllers alone offer a good vehicle following model only when the two vehicles are moving along a straight road. The system will be degraded the moment the lead vehicle begins to make a lane change or curve negotiation, thus resulting in the follower vehicle cutting the corner in pursuit of the leader. A combination of longitudinal and lateral controls has shown much improvement in vehicle following systems. However, these systems have the same problems as systems with longitudinal controllers only, when the lead vehicle is making a path change. This is because of the longitudinal component in the vehicle following algorithm. The real trajectory following for the vehicle following function problem remains unsolved. The control algorithm should minimize the path deviation during the vehicle following process and avoid the platoon stability problem if more than two vehicles are in the platoon.

Vehicle following systems for the urban environment (slow speed) have gained popularity of late. End users have shown interest in automated vehicle following systems, especially for congested traffic conditions. Also, safety and passenger

comfort are the main driving forces for a slow speed vehicle following system. Hence, market opportunities exist for slow speed vehicle following systems, especially for stop-and-go applications.

The concepts for motorway vehicle following cannot be applied directly to slow speed urban vehicle following. There are three clear differences between these two types of vehicle following. Firstly, perception systems used for motorway vehicle following may not function to standard for slow speed vehicle following systems. Second, the traffic conditions for motorways and city areas are different. Finally, the inter-vehicle separation requirement for the two scenarios are distinct.

The literature review on low speed vehicle following systems for the urban driving environment has highlighted some research gaps. A few research challenges exist in slow speed vehicle following systems. Current urban vehicle following systems either follow the current position of the leader, a virtual point projected along the axle of the leader or the estimated path recorded by the system. The experiments presented in the literature have indicated that there are path deviations when the systems operate along curved paths, thus increasing the risk of cutting the corner. A kinematic model for vehicle following is therefore required. The follower vehicle must follow the exact path of the leader while maintaining a safe distance from the leader. Nevertheless, another possible solution to reduce the risk of follower vehicle cutting the corner includes using perception systems to detect the curbs and obstacles. The follower vehicle can then use the information to refine the coarse trajectory generated by the vehicle following function. This thesis will

concentrate on the former solution.

Traffic conditions in city areas are much more complex than motorways conditions. Many other road users share the confined roads in the city. The controller in the follower vehicles must be able to detect all these other users and avoid hitting them. Also, roads in the city areas have a lower radius of curvature. The follower vehicle must be able to negotiate these roads while following the leader closely, without cutting corners. Furthermore, as the roads are not as straight as the motorway, both longitudinal and lateral vehicle following are required. Hence, constant time headway or constant separation headway vehicle following strategies may not be sufficient in this case. Next, no inter-vehicle or vehicle to infrastructure communication systems should be installed in a standalone vehicle following system. On-board sensors in the ego vehicle therefore play an important role in slow speed vehicle following. However, data uncertainties in the perception systems can degrade the performance of vehicle following systems, as noisy and corrupted data may exist in cluttered city environments.

In conclusion, this chapter has identified the major research issues for slow speed urban vehicle following systems. The following chapters will address the issues identified above, and a new framework and model for slow speed urban vehicle following will be formulated.

CHAPTER 3

The Virtual Trailer Link For Vehicle Following

3.1 Introduction

Safety is a major concern in the design of a vehicle following system. The follower vehicle must be able to stop safely and avoid collisions with vulnerable road users and other vehicles. Within this context, the follower vehicle must comply with two fundamental requirements. Firstly, it must be able to track and follow closely the path traced by the lead vehicle. This implies that the path deviation between the two vehicles must be minimized. Secondly, it must be able to stop in a controlled and safe manner, even when the lead vehicle stops abruptly.

There are several different ways of tracking the relative pose (position and orientation) of a lead vehicle in relation to the follower. One method is through the use of its proprioceptive sensors and a GPS (Global Positioning System) receiver. This information can be transmitted to the follower using an inter-vehicle communication system [13],[14]. Despite the fact that the reliability of

intercommunications systems is often an issue, this method is easy to implement although it also relies on the precision of the lead vehicle's pose estimates [104],[105]. Alternatively, the pose of the lead vehicle can be estimated using the onboard exteroceptive sensors in the follower vehicle [52],[53]. With the relative pose information of the lead vehicle, a path between the two vehicles can be constructed. On the other hand, in order to ensure vehicle safety, various safety policies for a vehicle following system have been implemented. For example, the PATH vehicles [16] use a constant time headway and CHAUFFEUR vehicles [15] use an electronic tow-bar system, as described in section 2.2.1 of Chapter 2, to ensure a safe separation distance between the lead and follower vehicles. Both longitudinal and lateral controllers have been implemented in the use of these strategies.

This chapter studies in detail the fundamental system requirements described above for a vehicle following system. The issues associated with vehicle tracking and inter-vehicle distances will be discussed, and the vehicle following function will be defined in a different and novel manner. The aim is to create an artificial inter-vehicle distance that allows for the minimization of path deviation by the follower while maintaining a safe distance between both vehicles. The proposed approach asserts that it is possible to comply with the previously defined fundamental requirements. The solution proposed in this research is the creation of a *virtual trailer link* attached to the lead vehicle. The proposed configuration also allows the follower vehicle to track the virtual trailer rather than the lead vehicle itself. It is postulated that by using the virtual trailer model that is

linked to the lead vehicle, the path deviation between the two vehicles can be minimized while a safe following distance is maintained. Furthermore, unlike conventional methods where a led vehicle follows a focus point on the lead vehicle, here, the led vehicle follows the estimated trajectory of a *virtual trailer*, which is predicted through observation of the maneuverings of the lead vehicle. Neither communication links between the two vehicles nor the installation of special road infrastructures are required for the successful implementation of this method.

The concept of the virtual trailer link model is introduced in Section 3.2. The performance of the steady and transient states of the virtual trailer link models are analyzed in Section 3.3. Taking into consideration the string stability of the vehicle following system and vehicle safety issues, the design specifications for the vehicle following system using the virtual trailer link model are discussed in Section 3.4. Experimental results are presented in Section 3.6.

3.2 The Concept: Virtual Trailer Link Model

A trailer system comprises two or more bodies connected by a rotational joint such as a kingpin hitch; i.e., a car-like tractor towing some passive trailers. The towing vehicle can perform motions that are similar to those of a car: it drives forward or backward while possibly steering left or right. Each trailer has a single axle and its wheels are all non-steerable. Each axle is hitched to the preceding trailer by means of a rigid bar that connects it to the middle axle of the vehicle in front of it. Hence, it can be seen that only axle-to-axle connections exist in a trailer configuration. In this kinematic configuration, the trailers will follow the path

that is dictated by the motion of the towing vehicle [106].

Kinematic configurations in the form of trailer links are commonly used in towing mobile platform configurations. For example, heavy trucks pull their own trailers that are linked by solid tow bars between them plus a rotational joint, as shown in Figure 3.1(a). Other examples of the use of these configurations are in trains, trams, etc. Figure 3.1(b) shows a typical passenger tram used in a theme park. These kinematic configurations have been successfully deployed for conveying passengers to various points of interest. A significant characteristic of these kinematic configurations is that they allow for tight manoeuvring and a close proximity of the trailer to the pulling vehicle, resulting in a similar trajectory that avoids collisions with road curbs.

(a)

(b)

Figure 3.1: Trailer systems deployed in the transportation sector and in theme parks. (a) A trailer system for transporting containers. (b) Tram rides are common in theme parks for transporting visitors to various point of interest within the park. (picture source: wikipedia)

In vehicle following systems, it is possible to model the lead vehicle as a towing vehicle that virtually pulls the follower vehicle. The follower vehicle can then be modelled as a trailer. There are two general trailer configurations, namely the

direct-hooked and the off-hooked kinematic configurations, as shown in Figures 3.2 and 3.7 respectively. In order to introduce the concept of the trailer system to the vehicle following system, the safety, steady state and transient state responses of the lead and led vehicle kinematic configurations must be analysed. This analysis will be conducted in the following sections.

When analysing the steady state response, the towing (lead vehicle)¹ vehicle is assumed to be moving at a constant speed with a constant angular velocity. In other words, the lead vehicle is instructed to maneuver in a circular motion. The transient state simulation test results will be presented in order to analyse the performance of the vehicle following systems. In this case, the lead vehicle will be commanded to maneuver in clothoids and sinusoidal paths. The purpose is to evaluate the path following performances of the vehicle following system with the virtual trailer kinematic configuration when the lead vehicle manoeuvres around sharp turns. This path also represents constraints typically found in urban road environments.

In order to further investigate the adaptability of the virtual trailer configuration for vehicle following, string stability and safety issues for the vehicle following system are considered and analysed. The results of this analysis will assist with the formulation of a set of design specifications for vehicle following using a virtual trailer link kinematic configuration. An optimised kinematic configuration for vehicle following is then proposed and implemented.

¹In this thesis, the lead vehicle is modelled as a towing vehicle and the follower vehicle as a trailer. Hence, the terms 'lead vehicle' will be used throughout the thesis to the 'towing vehicle'.

3.3 Performance Analysis of Virtual Trailer Link Models

This section analyses the steady and transient states of the virtual trailer link configuration. The purpose is to evaluate the amount of path deviation between the lead vehicle and the virtual trailer (follower) during various types of maneuvers, through simulation runs that model driving scenarios in several types of conditions. The vehicles are assumed to have non-holonomic constraints [107], [108]. Thus, the vehicle wheels are assumed to roll without slippage. Two kinematic configurations are analysed, namely direct-hooked and off-hooked virtual link kinematic configurations.

3.3.1 Direct-Hooked Kinematic Configuration

In a direct-hooked kinematic configuration, the lead vehicle is modelled as a two-driving wheel mobile robot pulling some trailers, as shown in Figure 3.2. The wheels of the trailer are non-steerable in this kinematic configuration. Each trailer is hooked up, as a flexible link to the mid-point of the rear wheels axle of the preceding trailer or pulling vehicle. In Figure 3.2, v and w represent the linear and angular velocities of the lead vehicle. θ_i represents the orientation of each trailer and L_i is the length of the connecting link.

The kinematic model of trailer i can be described as follows:

$$\dot{\theta}_i = \frac{1}{L_i} \sin(\theta_{i-1} - \theta_i) v_{i-1} \quad (3.1)$$

$$v_i = \cos(\theta_{i-1} - \theta_i) v_{i-1} \quad (3.2)$$

From Equations 3.1, it is clear that the angular velocity of trailer i , $\dot{\theta}_i$, is independent of the angular velocity of the lead vehicle, $\dot{\theta}_{i-1}$. This shows that

Figure 3.2: Direct-Hooked kinematic configuration. The lead vehicle is pulling two trailers in this case. The first trailer is attached to the lead vehicle at a rotational joint using a kingpin hinge and the second trailer is attached in a similar manner to the first trailer.

when the lead vehicle switches from, for instance, a straight path to a circular one, the trailer will maintain its current straight path for a short period of time before it begins to change its trajectory. This is a unique feature of the virtual trailer link kinematic model for vehicle following. The trajectories of both vehicles are the same, but at different time intervals. For example, when both vehicles are moving through a roundabout, the lead vehicle may gradually steer to exit the

roundabout, which is a straight path. During this path transition, the follower is expected to remain in the transition path rather than move along the straight path. This maneuver is desirable for the vehicle following function to achieve a minimum path deviation and is illustrated in Figure 3.3.

Figure 3.3: Initially, both vehicles are moving in a roundabout. At a certain instant in time, the lead vehicle is at position **L1** and the follower is at **F1**. When the lead vehicle gradually exits the roundabout onto a straight path at position **L2**, the follower vehicle should continue its motion in the roundabout to position **F2**. It should not cut the corner, for instance, to move to the dummy position **F3** in pursuit of the lead vehicle. Cutting the corner may result in the follower vehicle colliding with the nearby vehicle or hitting the road curb.

By setting the linear (v) and angular (ω) velocities of the lead vehicle constant, the lead vehicle will move in a circular motion. The spatial relationship between both vehicles is shown in Figure 3.4. By setting $i=1$, the relative angle between the lead vehicle and the trailer is

$$\varphi = \theta_0 - \theta_1 \quad (3.3)$$

and the angular velocity of the lead vehicle is

$$\dot{\theta}_0 = w \quad (3.4)$$

Figure 3.4: A 1-link direct-hooked trailer system configuration . R_1 and R_2 are the radii of the instantaneous centre of rotation (ICR) for the trailer and the lead vehicle. L_1 is the link length of the trailer.

From equation 3.1, with $i = 1$, $\dot{\theta}_1 = \frac{v}{L_1} \sin(\varphi)$, then

$$\begin{aligned} \dot{\varphi} &= \dot{\theta}_0 - \dot{\theta}_1 \\ &= w - \frac{v}{L_1} \sin \varphi_0 \end{aligned} \quad (3.5)$$

When $\dot{\varphi} = 0$, the rate of change of the relative angle between the lead vehicle and the trailer is zero. At this time, the trailer angle, φ , between the lead vehicle and the trailer is constant. Thus, the system is defined as being in an equilibrium state. From Equation 3.5, the system equilibrium point can be obtained as follows:

$$\begin{aligned} \varphi_0 &= \sin^{-1}\left(\frac{wL_1}{v}\right) \quad \text{where} \quad (wL_1 \leq v) \\ &= \sin^{-1}\left(\frac{L_1}{R_2}\right) \quad \text{since} \quad \left(\frac{v}{w} = R_2\right) \end{aligned} \quad (3.6)$$

where φ_0 is the steady state relative angle between the lead vehicle and the trailer.

The condition that $L_1 \leq R_2$, as indicated in Equation 3.6, sets constraints on the design of the link length for the direct-hooked kinematic configuration. Therefore, the maximum link length L_1 is determined by the minimum radius of curvature that the virtual trailer can steer to. This constraint also sets the limit on the maximum link length required for vehicle following modelled as direct-hooked kinematic configuration.

Steady-State Errors

From Figure 3.4, the relationship between R_1 and R_2 can be calculated as:

$$R_2^2 = R_1^2 + L_1^2 \quad (3.7)$$

The above equation indicates that R_2 is always greater than R_1 . This implies that the trailer will never follow exactly the path of the lead vehicle in all circumstances except when motion occurs along a straight path. The tracking error for this configuration can be defined as:

$$\varepsilon = R_2 - R_1 = R_2 - \sqrt{R_2^2 - L_1^2} \quad \text{where} \quad (R_2 > L_1) \quad (3.8)$$

The constraint, $R_2 > L_1$ in Equation 3.8 indicates again that for a given link length, L_1 , there is a minimum turning radius for the trailer.

Figure 3.5 illustrates the trend of tracking errors with respect to the turning radius of the lead vehicle. For a direct-hooked kinematic configuration with link length $L_1 < 3\text{m}$, the tracking error is large ($> 1\text{m}$) when the turning curvature

Figure 3.5: Relationship between tracking error and the lead vehicle radius of curvature (R_2) for direct-hooked kinematic configuration having different link lengths.

of the virtual trailer is small. Also, the tracking error converges to a small value ($< 0.1m$) when the radius of curvature to follow is large ($> 60m$).

Transient State Analysis - Clothoid paths

For transient state error analysis, the lead vehicle is commanded to maneuver from a straight path and gradually steer into a circular path at a constant rate, simulating a clothoid path. Table 3.1 summaries the path deviations of the direct-hooked kinematic configuration. During this maneuver, when both the lead vehicle and virtual trailer are moving along a straight path, the path deviation is zero. However, as the lead vehicle begins to gradually transit to a clothoid path with radius of curvature R_c , the path deviation increases. It should be noted that the path deviation converges to a steady state error as shown in Figure 3.5.

For all link lengths, the path deviations decrease when the radius of the path increases. This is related to the fact that the path can be regarded as a straight

Table 3.1: Direct-Hooked Trailer kinematic configuration: Transient Errors under various link lengths and radii of curvature. The values indicated in the table are the largest path deviation values during path transition (clothoid path).

$L_1(\text{m})$	Transient Error (m) when $R_c=(\text{m})$					
	10	20	30	40	50	60
3	0.47	0.25	0.15	0.12	0.08	0.07
4	0.85	0.40	0.27	0.20	0.16	0.13
5	1.25	0.62	0.42	0.32	0.25	0.21
6	2.00	0.92	0.60	0.45	0.36	0.30

path when the curvature radius of the clothoid path is large. The path deviation will eventually converge to zero if the radius of curvature, R_c , of the clothoid path is large, which can be considered as a straight path.

Transient-State Analysis - Sinusoidal Paths

In order to conduct a detailed analysis of the transient response of the direct-hooked kinematic configuration, the trajectory of the platform is considered to be represented by sinusoidal shape paths. The lead vehicle is commanded to steer on sinusoidal paths. The steering angle of the lead vehicle is defined by:

$$\gamma(i) = \gamma_{max} \times \sin(\pi \times \delta_t \times i/T) \quad (3.9)$$

where $\gamma(i)$ is the steering angle of the lead vehicle at time step i , γ_{max} denotes the maximum allowable steering angle of the lead vehicle, δ_t denotes the sampling time in seconds and T is a variable that determines the turning rate of the lead vehicle. Note that the parameter T affects the desired steering angle of the vehicle. Figures 3.6 (a) to (c) show the path deviations of the direct-hooked

kinematic configuration for various link lengths.

Figure 3.6: Path Deviation for Direct-Hooked kinematic configuration when (a) $T=30$, (b) $T=40$ and (c) $T=50$.

The path deviation is small ($< 0.1\text{ m}$) when the link length is set at 4 m . However, as the link length increases, so does the path deviation. Also, the path deviation is at its maximum when $\gamma(i)$ is at its maximum and is independent of T , the turning rate of the lead vehicle. As observed in Figure 3.6, regardless of the turning rate, T , of the lead vehicle, the maximum path deviations are the same for the same selected link length of the direct-hooked kinematic configuration. Moreover, for

any particular setting of the link length, L , the path deviation is independent of the turning rate, T , of the lead vehicle. It can therefore be concluded from these observations that there is a trade off between the selection of the virtual trailer link length and the performance of the direct-hooked kinematic configuration for vehicle following. In addition, the transient state path deviation is influenced by the turning radius of the lead vehicle.

From Equation 3.9, it can be seen that the parameter T will determine the steering angle and the turning rate of the lead vehicle. Meanwhile, it can be inferred from Equation 3.5 that the relative angle, φ , between the lead vehicle and the trailer is affected by the turning rate of the lead vehicle. Therefore, the following constraints on the maximum allowable velocity of the lead vehicle must be observed:

$$\begin{aligned} -1 &\leq \sin \varphi \leq 1 \\ -1 &\leq \frac{L_1}{v}(w - \dot{\varphi}) \leq 1 \\ \frac{-1}{(w - \dot{\varphi})} &\leq \frac{L_1}{v} \leq \frac{1}{(w - \dot{\varphi})} \end{aligned} \quad (3.10)$$

Equation 3.10 sets constraints on the selection of the link length of a direct-hooked kinematic configuration. Table 3.2 summarises the maximum allowable velocity imposed on the lead vehicle under various T . To achieve a stable vehicle following system, the maximum allowable speed imposed on the lead vehicle decreases proportionately with the turning rate of the lead vehicle. For example, consider a case where $T = 30$, which implies that the lead vehicle is negotiating

Table 3.2: Maximum allowable speed of lead vehicle under various T , for direct-hooked kinematic configuration.

T	Max. Allowable Speed (m/s) of lead vehicle when $L_1=(m)$				
	2	3	4	5	6
10	3	3	3	3	3
20	6	6	6	6	6
30	9	9	9	9	9
40	13	13	13	12	12
50	16	16	16	16	16

a turn. If the lead vehicle is moving at a speed that is slower than $9m/s$, then it is possible to model the follower vehicle as a direct-hooked virtual trailer. Thus, it is clear that Table 3.2 sets a design constraint on the vehicle following system using a direct-hooked kinematic configuration. When T is at a lower value, which signifies that the vehicle is negotiating a tight curve, the speed requirement for the lead vehicle should be reduced. As an analogy, in practice, while negotiating a tight curve, a human driver will normally reduce the vehicle speed.

The results of the above analysis have shown that the direct-hooked kinematic configuration is a potential model for representing the vehicle following system. It is possible to steer the lead vehicle from one maneuver to another without immediately affecting the maneuver of the follower - an important characteristic for vehicle following. Furthermore, the link length between the two vehicles is determined by the minimum radius of curvature of the path towards which the vehicles are to be deployed. However, there are some trade-offs when adopting

the direct-hooked kinematic configuration for vehicle following. A shorter link length ($< 3m$) allows for small path deviations to be achieved, but at the cost of a reduction in the inter vehicle separation. This may result in a collision between the two vehicles during an emergency, when the lead vehicle stops abruptly.

3.3.2 Off-Hooked Kinematic Configuration

Unlike the direct-hooked kinematic configuration, the trailers in the off-hooked kinematic configuration, as shown in Figure 3.7, are not directly attached to the centre of the previous axle but rather, are at a distance away from this point.

Figure 3.7: Representation of an off-hooked kinematic configuration.

The kinematic equations of a general kinematic configuration can be expressed as follows:

$$\dot{\theta}_i = \frac{1}{L_i} \{v_{i-1} \sin(\theta_{i-1} - \theta_i) - \dot{\theta}_{i-1} D_{i-1} \cos(\theta_{i-1} - \theta_i)\} \quad (3.11)$$

$$v_i = v_{i-1} \cos(\theta_{i-1} - \theta_i) + \dot{\theta}_{i-1} D_{i-1} \sin(\theta_{i-1} - \theta_i) \quad (3.12)$$

where L_i and D_i indicate the lengths of the front and rear connecting links respectively of trailer i . An off-hooked kinematic configuration can be obtained by setting $L_i = D_{i-1}$. It can be seen that Equations 3.12 and 3.11 are the same as Equations 3.1 and 3.2 when the length of the rear connecting link, $D_{i-1} = 0$.

Steady-State Analysis

Figure 3.8 shows the steady state configuration of a 2-link off-hooked kinematic configuration. Using kinematic and geometrical principles, this configuration can be expressed analytically as:

$$\alpha + \beta = \varphi, \quad (3.13)$$

$$R_1^2 + L_1^2 = R_0^2 + D_0^2$$

$$R_1^2 - R_0^2 = D_0^2 - L_1^2, \quad (3.14)$$

$$\begin{aligned} \tan(\alpha) &= \frac{L_1}{R_1} \\ \tan(\beta) &= \frac{D_0}{R_0}, \end{aligned} \quad (3.15)$$

and,

$$\tan(\alpha + \beta) = \frac{L_1 R_0 + D_0 R_1}{R_1 R_0 - L_1 D_0} \quad (3.16)$$

From Equation 3.16, it can be inferred that for the lead vehicle and the virtual trailer travelling on the same segment of curved road, ie, $R_1 = R_0$, the condition, $L_1 = D_0$ is required. This means that the length of the follower vehicle must be equal to the virtual link length. Note that the existence of a virtual link between vehicles in the off-hooked kinematic configuration has implicitly set up an inter-vehicle separation for vehicle following.

Figure 3.8: Off-Hooked kinematic configuration in equilibrium.

From Figure 3.8, and under the conditions that $L_1 = D_0 = L$ and $R_1 = R_0 = R$, we have $\tan(\alpha) = \tan(\beta) = \frac{L}{R}$. This implies that,

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \quad (3.17)$$

$$\tan \varphi = \frac{2RL}{R^2 - L^2} \quad (3.18)$$

where $\varphi = \alpha + \beta$.

In general, by defining the path deviation as $\varepsilon = R_1^2 - R_0^2$, then

$$\varepsilon = \begin{cases} = 0, & L_1 = D_0 \\ = |L_1^2 - D_0^2|, & \text{otherwise} \end{cases} \quad (3.19)$$

Since the lead vehicle is modelled as a car, its steering angle is constrained by, $\varphi < 90^\circ$ ². With this constraint, from Equation 3.18, $R \neq L$. This implies that the minimum radius of turning curvature, R , for the off-hooked kinematic

²The maximum allowable steering angle for a passenger car is typically $< 40^\circ$.

configuration is equal to the length of the link between the lead vehicle and the trailer. From Equation 3.19, if $L_1 = D_0$, then the path of the trailer will be identical to that of the lead vehicle, as shown in Figure 3.8. This implies that the path deviation of the off-hooked kinematic configuration will converge to zero in a steady state.

Transient State Analysis - Clothoid paths

For transient state analysis, the lead vehicle is assumed to initially be moving along a straight path and gradually merge into a circular path, representing a clothoid path. The paths of both the lead vehicle and the follower are recorded and the path deviations are computed for the purpose of analysis.

Table 3.3 summarizes the maximum path deviation results when the lead vehicle and the trailer maneuver along the clothoid paths. The path deviations for all link lengths converge to zero for all clothoid paths as represented by the parameter R_c . This is due to the fact that the vehicles are moving on a circular path immediately exiting from the clothoid path. The maximum transient error increases proportionally to the link length of the off-hooked kinematic configuration for all the clothoids paths tested. Also, for a fixed link-length, the transient error decreases as the radius, R_c , of the clothoid path increases.

Figure 3.9 shows the trend of the path deviations for the off-hooked kinematic configuration with link length $L = 3\text{m}$. It can be seen from the figure that as the lead vehicle transits from a straight path ($R_c = \infty$) to a clothoid path (R_c decreases from ∞ to a fixed value), the initial path deviation is at a maximum.

Table 3.3: Maximum transient errors for off-hooked kinematic configuration with various equal link lengths and radii of curvature

D=L (m)	Transient Error (m) when $R_c=(m)$					Error $\rightarrow 0$?
	11	13.5	18.5	27.5	55	
3	0.25	0.20	0.15	0.10	0.05	yes
4	0.45	0.35	0.30	0.18	0.10	yes
5	0.68	0.48	0.42	0.258	0.14	yes

However, as R_c decreases, the corresponding path deviation decreases as well and eventually converges to 0. This is the unique characteristic of the off-hooked kinematic configuration.

Figure 3.9: Path deviation of off-hooked kinematic configuration when the link length $L=3m$, under various transition paths (clothoid). The steady state errors eventually converge to zero.

Transient State Analysis - Sinusoidal paths

As with the off-hooked kinematic configuration, the lead vehicle will maneuver along sinusoidal paths to investigate the transient error of the system.

Figure 3.10 shows the path deviations for off-hooked kinematic configurations with various link lengths, L , when $T = 50$, with link lengths up to $5m$. The absolute path deviations are relatively small ($< 0.07m$) for all sinusoidal paths. This observation indicates that the vehicle following system with off-hooked kinematic configuration has a small path deviation when the vehicles are maneuvering on sinusoidal paths. Also, the path deviations are considered to be negligible (7 cm) compared to the width of a typical road, which is typically $5m$.

Figure 3.10: Path Deviation for Off-Hooked kinematic configuration when $T=50$.

In order to investigate the maximum operating speed and steering rate of the off-hooked kinematic configuration, simulation runs are conducted by varying the velocity of the lead vehicle when it is maneuvering along a sinusoidal path. Table 3.4 summarises the maximum allowable velocity and the corresponding steering rate under various values of T , that should be imposed on the lead vehicle for

Table 3.4: Maximum allowable velocity/steering rate of the lead vehicle under various T .

T	$\dot{\gamma}$ (deg/s)	Velocity (m/s) of lead vehicle when $L=(m)$			
		3	4	5	6
10	18	30	40	50	60
20	9	30	40	50	60
30	6	30	40	50	60
40	4.5	30	40	50	60
50	3.6	30	40	50	60

vehicle following. Overall, the maximum allowable velocity is large (*up to 60m/s*). In practice, however, a driver should not drive a vehicle at such a high speed when negotiating curve roads, ie, when T is small. Hence, theoretically, off-hooked kinematic configuration is applicable for vehicle following up to a high speed of 60 m/s (100 km/h). Furthermore, when T is small, the maximum allowable steering rate is as high as $18deg/s$. Under this high steering rate, the follower vehicle is still able to trail the lead vehicle. All these observations have lead to a conclusion that the off-hooked kinematic configuration is favorable for vehicle following modelling for motorways and urban environments.

The above evaluations have allowed for an examination of an off-hooked kinematic configuration for a vehicle following system model. With the off-hooked kinematic configuration, the steady state path deviation will converge to zero if the length of the virtual link is set to be the same as the virtual trailer. The minimum allowable turning radius of the system is constrained by the virtual link length, while the transient and sinusoidal path deviations are proportional

to the link length. Furthermore, the off-hooked kinematic configuration has been found to be suitable for both a high and low speed vehicle following system.

3.4 Design Considerations for the use of the Virtual Trailer Link Model

The performances of the direct-hooked and off-hooked kinematic configurations for vehicle following were analyzed in Sections 3.3.1 and 3.3.2 respectively. In order to apply the virtual trailer link model to vehicle following, various design considerations are required to ensure optimisation of the design parameters for the model. The following sections will evaluate and determine the optimal design parameters for the model.

3.4.1 Virtual Trailer Link Model for Vehicle Following Systems

The kinematic formulations of a general virtual trailer kinematic configuration are represented by Equations 3.11 and 3.12. The direct-hooked and off-hooked kinematic configurations can be obtained by setting $D_i = 0$ and $D_{i-1} = L_i$ respectively. In general, with a combination of settings for D_i and L_i , various kinematic configurations can be obtained. This section proposes a suitable kinematic configuration for the vehicle following model.

Optimised Kinematic Configuration for Vehicle Following Systems

The length of a passenger car, on average, is between 3 and 4m. In the virtual trailer link model, the follower vehicle will be modelled as a trailer. The length of the virtual trailer will be defined here as being of the same length as that of a

passenger car.

It has been shown in Section 3.3.2 that the steady state path deviation is minimal (a value of zero in this case) when the link length is $L_i = D_{i-1}$. Hence, other virtual trailer link models with various combinations of L_i and D_{i-1} settings will yield some steady state error.

Table 3.5 shows the maximum transient state path deviation of the general kinematic configuration with various L_i and D_{i-1} settings under clothoid paths (when $R_c = 11m$). In summary, a general virtual link model performs better than a direct-hooked kinematic configuration but does not perform better than an off-hooked kinematic configuration. Hence, the off-hooked kinematic configuration is the optimal model for a vehicle following system. By setting the virtual trailer link $D = L$ to be the same as the length of the follower vehicle, a safe and reliable vehicle following system can be obtained.

Table 3.5: Comparison of maximum path deviations for general virtual trailer system

D(m)	L(m)	Max. Path Deviation Under Clothoid Path $R_c = 11$	Error $\rightarrow 0$?
4	3	0.5	No
3	4	0.3	No
2	3	0.1	No
2	4	0.05	No

3.4.2 The Issue of String Stability

Based on the analysis in Section 3.3, the off-hooked kinematic configuration exhibited minimal path deviation when following the trajectories generated by

the lead vehicle. This kinematic configuration can be applied to the vehicle following system. The towing vehicle can be modelled as a lead vehicle. The corresponding kinematics of the virtual trailer, under the influence of the lead vehicles motion, can be modelled as the follower vehicle. Therefore, the task of the follower vehicle will only be to track the position of the virtual trailer. As there are no physical links in the virtual trailer model, the dynamics of the virtual trailer can be ignored, thus simplifying the modelling process.

The design of the virtual trailer kinematic configuration for vehicle following involves selection of the optimal configuration. This is determined by two parameters - the number of virtual trailer links and the length of each link. As discussed in Section 3.3.2, the off-hooked kinematic configuration with 2-link configuration has a zero steady state error if $D = L$, as shown in Figure 3.8. The results from Section 3.3.2 can be generalized to a case of an n -links trailer system. Figure 3.11 shows the setup of the off-hooked kinematic configuration with n links. Through analogy, it is important for the virtual trailer number 2 to be able to follow the trajectory of virtual trailer number 1 if the length of the link $D_1 = L_2$. In general, the i -th virtual trailer will be able to follow the trajectory of the $(i - 1)$ -th virtual trailer if the length of the linkages is set as $D_{i-1} = L_i$. Hence, under ideal conditions, the n -th virtual trailer should follow the trajectory of the lead vehicle.

However, sensor uncertainties, measurement errors and other external disturbances tend to affect the performance of the system. The existence of steady state errors or measurement errors in the $(i + 1)$ -th virtual trailer can filter down to

Figure 3.11: Off-hooked kinematic configuration with n links

the subsequent $(i + 2)$ -th trailer. This error propagation will affect the *string stability* [96] of the connected virtual trailer system, and thus, the ability of the vehicle following system to form a platoon. By definition, the string stability of a vehicle platoon ensures that the inter-vehicular spacing errors of all vehicles are bound uniformly in time provided the initial spacing errors of all the vehicles are bound. Theoretically, in order to maximize the string stability of a platoon, the total number of links required in the virtual trailer link model should be minimized.

Hence, it can be concluded that the optimum trailer systems should have minimum linkages set up with the length of the front and rear connection links being equal.

3.4.3 Design Consideration : Safety

Another important design constraint on a vehicle following system is the need to maintain sufficient inter-vehicle spacing. The spacing must be such that if the

lead vehicle stops instantaneously, the follower vehicle should be able to stop from its current speed without colliding with the lead vehicle, which is now stationary [109]. However, this design constraint contradicts the design constraint discussed in the previous section, which is the need for the follower vehicle to track the lead vehicle as closely as possible. Thus, a design compromise is required in order to minimize the tracking error while maintaining a safe inter-vehicle distance.

Safe distance between cars

The stopping distance of a vehicle, from the time a triggering event has occurred, is a time dependent function. It is a function of the environmental perception, vehicle decision, response, actuation and braking times. When the lead vehicle brakes suddenly, the time and hence the distance that the follower will travel before stopping can be computed as follows:

- **Perception and Decision Making Time, $\tau_{\text{perception}+\text{decision}}$:** This is the amount of time required, $\tau_{\text{perception}}$, for the perception system in the follower vehicle to sense the lead vehicle and for the algorithm to process, τ_{decision} , the sensor data so as to estimate the maneuver of the lead vehicle. This can be defined by:

$$\tau_{\text{perception}+\text{decision}} = \tau_{\text{perception}} + \tau_{\text{decision}} \quad (3.20)$$

The perception of the lead vehicle can be obtained using a Ladar, radar or stereo cameras. These sensors typically operate at a sampling period equal to $80ms$ ($\tau_{\text{perception}}$) or better. The computation time for tracking the leader vehicle based on the sensor data is taken to be $20ms$ (τ_{decision}). Therefore the

total amount of time will be:

$$\tau_{\text{perception}+\text{decision}} = 100ms. \quad (3.21)$$

- **Actuator Reaction Time, τ_{reaction} :** For a vehicle to start to brake, an actuator must be activated, which can be a part of the Electronic Stability Program (ESP) unit [110]³, an electric motor acting on the brakes, etc. These actuators have their own response time, which is typically defined as:

$$\tau_{\text{reaction}} = 100ms \quad (3.22)$$

- **Time to Brake, τ_{braking} :** The braking time for a vehicle depends mainly on its initial velocity (before braking) and the coefficient of friction between the vehicle tires and the road surface. It is the frictional force that reduces the kinetic energy of a vehicle to zero during braking. There are two types of frictional forces, namely static and kinetic. A static frictional force is when the wheels of the car continue to turn while braking. A kinetic frictional force is in operation when the wheels are locked and slide over the road surface. The braking time can be expressed as the amount of distance required for the vehicle to brake. This can be calculated using the work done by the static frictional force, which is:

$$\text{Workdone by static frictional force} = -\mu_{\text{static}}m_{\text{car}}gd = -\frac{1}{2}m_{\text{car}}v_{\text{follower}}^2 \quad (3.23)$$

where μ_{static} denotes the coefficient of static friction between the tires and the road, m_{car} is the mass of the follower vehicle, g is the gravitational force,

³ESP unit is designed to detect a difference between the driver's control inputs and the actual response of the vehicle. It then brakes the wheels or reduces the engine power to help correct the understeer or oversteer of the vehicle.

d is the minimum stopping distance of the follower vehicle and $v_{follower}$ is the speed of the follower vehicle (before braking).

Hence, the stopping distance is

$$d = \frac{v_{follower}^2}{2\mu_{static}g} \quad (3.24)$$

Minimum Braking Distance

The typical speed of a vehicle moving in congested traffic is between 20km/h ($\sim 5.55\text{m/s}$) and 40km/h ($\sim 11.11\text{m/s}$). This is defined as the speed of the lead vehicle following application.

- **Distance travelled during sensing and decision making:**

$$d_{\text{perception+decision}} = v_{follower} * \tau_{\text{perception+decision}} \quad (3.25)$$

- **Distance travelled during actuator's reaction time:**

$$d_{\text{reaction}} = v_{follower} * \tau_{\text{reaction}} \quad (3.26)$$

- **Distance travelled during vehicle braking time:**

$$d_{\text{braking}} = \frac{v_{follower}^2}{2\mu_{static}g} \quad (3.27)$$

Hence, the minimum braking distance, $d_{\text{min.brake}}$, of a follower vehicle in response to a sudden vehicle braking of the lead vehicle is

$$d_{\text{min.brake}} = d_{\text{perception+decision}} + d_{\text{reaction}} + d_{\text{braking}} \quad (3.28)$$

The minimum braking distances for various vehicle speeds are shown in Figure 3.12 for both dry and wet road surface conditions. This figure can be used in

the design of a vehicle following system. The optimal number of virtual trailers required in the modelling can then be determined based on the operating speed of the system.

Figure 3.12: Minimum braking distance for a follower travelling on dry and wet roads, at various speeds.

The analysis in the previous section has established that equating the length of the virtual trailer link with that of the follower vehicle provides a good indicator of the inter-vehicle distance. At low speeds, a single virtual trailer link is sufficient. For a typical passenger car, this will be between 3 and 4m. Table 3.6 shows the relationship between the vehicle speeds and a safe inter-vehicle distance expressed as the number of virtual trailer links. It is important to note that for a typical low vehicle speed of 20 km/h as encountered in many urban situations, the inter-vehicle distance is equal to the length of a single virtual trailer link.

The virtual trailer link model is safe to use if the total number of links is selected correctly. However, the higher the number of links, the larger will be the tracking

Table 3.6: Number of Virtual Trailers required under various operating speeds

$v_{follower}$ (km/h)	10	20	30	40	50	60
No. of trailers	1	1	2	3	3	4
safety separation (m)	4	4	12	20	20	28

error. Nevertheless, it can be argued that the faster the vehicles move, the less stringent will the accuracy be as a larger manoeuvring space is required when operating at high speeds.

3.4.4 Specifications of the Virtual Trailer Link Model for Vehicle Following

Table 3.7 summarizes the three important criteria and the selected parameters for the use of the virtual trailer kinematic configuration in a vehicle following system. Based on the analysis outlined in this chapter, it can be concluded that **a 1-link off-hooked kinematic configuration, with the length of the virtual trailer link being equal to the length of the follower vehicle, is the best model for a slow speed vehicle following system.**

Table 3.7: Design considerations for a vehicle following system

System requirements	Design specifications
Minimum path deviation	off-hooked Virtual Trailer Link Model selected
Platoon stability	Minimize number of Virtual Trailer Links
Safety considerations	At least 1 car length between vehicles

3.5 Virtual Trailer Link Model vs Leader-Follower Formation Control Strategy

The concept of virtual trailer link model for vehicle following has some similarities with respect to the leader-follower formation control strategy [94], [95], [93]. The Leader-follower formation control strategy has been used in multiple robot systems. In this work, a desired formation shape is maintained during the robotic following operation. The formation can be of any shape. Two typical formations are as shown in figure 3.13.

Figure 3.13: Leader-follower formation control of mobile robots. (a) Triangle formation control. In this case, lead vehicle 1 and followers 2 and 3 formed a triangular shape. Throughout the motion, this triangular shape is maintained. (b) Line formation. In this formation, the concept can be applied to vehicular vehicle following application. Notice that the follower may not follow exactly the trajectory of the lead vehicle and some path deviation may occurred. This configuration is similar to the direct-hooked kinematic configuration as described in section 3.3.1.

When a line formation is formed, the multiple robotic control strategy can be implemented as a vehicle following system in ITS, which is the main application of this thesis. In the leader-follower line formation control strategy, it is desired to maintain a relative position (i.e a safe inter-vehicle separation) between the lead vehicle and followers. Hence, a reactive controller is usually designed for the follower in response to the maneuvers of the lead vehicle. Furthermore, both vehicles may not be necessary travelling exactly on a same trajectory. For example, the follower may deviate away from the trajectory of the lead vehicle in an attempt to avoid collision with dynamic obstacles such as other road vehicles. In fact, vehicle following systems that have been implemented on highway such as the one in Chauffeur [15] and PATH [16] vehicles are similar to the leader-follower line formation control strategy. For example, in Chauffeur vehicle [15], a constant separation between two vehicles has been maintained and at the same time, the follower vehicle attempted to maintain a zero relative orientation between vehicles. On the other hand, the off-hooked virtual trailer link model targets the application of vehicle following system on congested traffic. Under this scenario, the follower has to trail exactly the trajectory of a lead vehicle. Due to the occlusion of perception system by the lead vehicle, the sensors onboard the follower may not be able to perceive the surrounding environment effectively. With an off-hooked virtual trailer kinematic configuration, a minimum path deviation between vehicles is guaranteed, thus the safety of the vehicles and environment are assured. Furthermore, in an off-hooked virtual trailer kinematic configuration, a constant path length between two vehicles is maintained. However, in the leader-follower line formation, a

constant separation between two vehicles is maintained. Hence, the objective of achieving minimal path deviation between two vehicles can not be attained by simply specifying a desired separation between the lead vehicle and follower while leaving the relative bearing free under leader-follower formation control strategy. By doing so, a direct hooked kinematic configuration is obtained, and as discussed in section 3.3.1, this kinematic configuration will result in the follower cutting corners during curve road negotiation.

3.6 Experiments With Real Vehicles

A theoretical analysis of the off-hooked kinematic configuration has been conducted in the previous sections. However, before implementing the virtual trailer kinematic configuration for the vehicle following system, its principle must be validated, preferably in a test environment. For this purpose, experimental trials were performed in an urban environment. The experimental setup comprised a pick-up truck equipped with multiple sensors for data collection purposes, as shown in Figure 3.14. A Global Positioning System (GPS) was mounted on the roof of the vehicle. The GPS was coupled with an Inertial Measurement Unit (IMU) [91] for better positioning (with $10m$ accuracy) of the vehicle. The poses and velocities of the vehicle during the experiment were logged at a frequency of 10Hz.

The vehicle was manually driven on a path in an urban environment. The trajectories of the vehicle included being moved along a straight road, a clothoid path and a sharp turning road. A section of the data collected is presented in

Figure 3.14: Vehicle used in the experiments. The GPS data was fused with the data from the IMU.

Figure 3.15.

Figure 3.15: A section of the trajectories of the experimental vehicle and its off-hooked kinematic configuration with $D=L=3\text{m}$. The positions of the vehicles are normalised with respect to a reference frame. The thick line (green colour) represents the trajectory of the instrumented vehicle. The thin line (blue colour) represents the trajectory of the virtual trailer. The numbers beside the lines are the times after the start of the experiment.

Figure 3.15 shows that the vehicle started at position $(-200\text{m}, -270\text{m})$ and ended at position $(-1000\text{m}, -370\text{m})$. For visualization purposes, the trajectory of the

vehicle has been represented by a thick line. The numbers beside the trajectory of the vehicle denote the time at which the vehicle was positioned after the start of the experiment. From the figure, it can be observed that the vehicle was moving along the path with a low curvature from time 0 to 300s. The vehicle then made a left turn at time 300s. It then continued in straight motion and made a left turn again at time 400s. The vehicle continued its path along another section of a low curvature until the end of the experiment.

Figure 3.16 shows the orientation of the vehicle throughout the experiment. There were some sharp changes in orientation at times 30, 90, 100, 150, 290, 330 to 390, 470 and 550 seconds. In practice, these estimated sharp changes are not realistic due to the non-holonomic constraints of the vehicle. The GPS and IMU modules contributed to the possible sources of error. These include multipath effects and satellite biases.

Figure 3.17 shows the velocity profiles of the experimental vehicle. The speed of the vehicle was maintained at about 1.8 m/s throughout the experiment except at some instances. The vehicle decelerated at times 190, 300, 400 and 430 seconds when it was negotiating turns with a high curvature.

In the experiment, the length of the off-hooked virtual trailer link and the virtual trailer were both set at 3m. The trajectory of the off-hooked virtual trailer, responding to the maneuvers of the experimental vehicle, is shown in Figure 3.15 as a thinner line. The path deviation between the experimental vehicle and the off-hooked virtual trailer is shown in Figure 3.18. As expected, the path errors were large when the vehicle was making sharp turns at times 30, 90, 100, 150,

Figure 3.16: Vehicle orientation (solid line) of the experimental vehicle. The dotted line represents the orientation of the virtual trailer. A time delay between the orientations of the lead vehicle and the follower can be observed as shown in the zoomed view. The experimental vehicle was making a sharp turn (i.e, a high rate of change in orientation) at the time instances marked with a circle in the figure.

Figure 3.17: Velocity profile of the experimental vehicle.

290, 330, 390, 470 and 550 seconds. The error distributions of the path deviations are shown in Figure 3.19. Overall, more than 90% of the path deviations were within 20cm.

Figure 3.18: Path deviation of the outdoor test.

Figure 3.19: Path error distribution.

The path deviation at time 100s was as large as 60cm. The noisy orientation data contributed to the error. As observed from Figure 3.16, at the time instance of 100s, there was a sudden change in the vehicle orientation. This sudden change caused the virtual trailer to swivel. By examining the trajectory of the

experimental vehicle at time 100s (see Figure 3.20), the orientation of the virtual trailer was indeed affected at positions (-370m,-240m) and (-394m,-234m).

Figure 3.20: Path deviation of the outdoor test between time interval 50 to 120 seconds.

Similarly, the path deviations were relative large ($> 40cm$) between time interval 300 and 400 seconds. Figures 3.21 and 3.22 show a zoomed view of the trajectories of the vehicle and its virtual trailer between time interval 300 and 400 seconds.

Figure 3.21: Path deviation of the outdoor test between time interval 250 to 310 second.

To further investigate the performance of the off-hooked kinematic configuration under real conditions, another similar experiment was set up using the same

Figure 3.22: Path deviation of the outdoor test between time interval 380 to 420 seconds.

experimental vehicle. For this experiment, higher speeds for the lead vehicle were set and the lead vehicle was commanded to accelerate and decelerate along the test path. A section of the trajectory is shown in Figure 3.23. In this experiment, the lead vehicle first made a right angle turn (section A-B in Figure 3.23) followed by another right turn (section B-C). It then continued to make two left turns (section C-D and D-E-F). The vehicle then continued along a straight path (section F-H).

The velocity profile of the lead vehicle is shown in Figure 3.24. The lead vehicle accelerated to a speed of 6m/s (section A-B in Figure 3.24). When making a right turn, the lead vehicle slowed down to a speed of 2.5 m/s. In the subsequent turns, the speed of the lead vehicle slowed to between 3 and 4 m/s. The vehicle then accelerated to a maximum speed of 12 m/s (about 43 km/h) while moving along a straight path (section F-H in figure 3.24).

Figure 3.25 shows the detailed views of the trajectories of the sections labelled in Figure 3.23. Figure 3.25(a) shows that the lead vehicle gradually accelerated to a

Figure 3.23: A section of the trajectory of the lead vehicle and its corresponding off-hooked kinematic configuration with $D=L=3\text{m}$. The positions of the vehicles are normalised with respect to a reference frame. The thick line (red colour) represents the trajectory of the instrumented vehicle. The thin line (green colour) represents the trajectory of the virtual trailer.

Figure 3.24: Velocity of the lead vehicle.

speed of 6m/s while negotiating a right turn. To determine the path deviation, the GPS information from the lead vehicle was used as a reference trajectory. Relative to this reference trajectory, the maximum path deviation along this path section was about 3m. The radius of curvature of this path section was small (tight right angle turn). As discussed in Section 3.3.2 and as shown in Figure 3.9, the expected initial path deviation is large if the off-hooked kinematic configuration is implemented on the low radius of curvature path. Hence, it can be seen from Figures 3.25 (a) to (c) that as the lead vehicle steered into the transition path (clothoid), a large initial path deviation (3m in this case) was observed and the path deviation decreased as the vehicle exited from the clothoid path. One reason for the large (3m) deviation is the discrete discontinuities in the GPS data as observed from Figures 3.25 (a) to (c). Discontinuities, of approximately 1m, in the trajectory of the lead vehicle have been observed in the GPS data. These discontinuities have also contributed to the large path deviation between the trajectories of the lead vehicle and the virtual trailer. Hence, a method to deal with these uncertainties in the data measurement is certainly required and will be addressed in the next chapter.

When the vehicle moved along a straight path (section D-E), the path deviation was small ($< 0.3m$). It can be noted in Figure 3.25(d) that the path of the lead vehicle was not straight despite the fact that it was moving on a straight road during the experiment. The errors were caused by the GPS data received. The path deviation in this case was about 0.5m. Finally, when the vehicle made a gradual left turn from point E to F, the path deviation was about 0.2m.

Figure 3.25: Zoomed in view of sections of the trajectory as shown in Figure 3.23 (a) section A-B, (b) section B-C, (c) section C-D, (d) section D-E and (e) section E-F.

3.7 Summary

This chapter has introduced the concept of the virtual trailer link with the purpose of minimizing the path deviation between the follower and lead vehicles while at the same time maintaining a safe separation distance. The validity of the method was demonstrated theoretically.

The concept of the virtual trailer link model for vehicle following was evaluated analytically in this chapter. Two kinematic configurations, direct-hooked and off-hooked, have been examined in detail. Based on the analyses, off-hooked kinematic configuration has been shown to exhibit superior characteristics for vehicle following as compared to direct-hooked kinematic configuration. By setting the virtual link length to be the same as the length of the follower, a minimal path deviation can be achieved for a vehicle maneuvering along straight, circular, clothoid and sinusoidal paths. Furthermore, the off-hooked kinematic configuration can be modelled for vehicle following both in motorways (high speed vehicle following) and the urban environment (slow speed vehicle following).

As the virtual trailer link model represents a very important component of the strategy, the proposed model was tested via experimentation. A specially equipped vehicle was used and driven to emulate various traffic scenarios, and its trajectories were recorded in real-time.

Based on the system analysis and the experimental results, it can be concluded that a 1-link off-hooked kinematic configuration, with the length of the virtual trailer link being equal to the length of the follower vehicle, is the best model for

slow speed vehicle following system. The lead vehicle is modelled as a towing vehicle pulling the follower vehicle, which is modelled as a virtual trailer.

The experiments have demonstrated a very important phenomenon, that the off-hooked kinematic configuration is sensitive to the rate of orientation change in the lead vehicle as well as the data uncertainties in the perception system. This implies that potential errors from the acquisition process of the onboard sensors can affect the performance of the model. It is therefore necessary to include countermeasures that palliate their effects and thus improve the overall performance of the vehicle following system. The above issues will be discussed in Chapter 4, which addresses the problem of the estimation of the position and orientation of the vehicles and proposes a new formulation for vehicle following.

CHAPTER 4

Bayesian Estimation Formulation For Vehicle Following

4.1 Introduction

A simplified block diagram of the vehicle following system consists of two feedback loops, as shown in Figure 4.1. The inner loop comprises a motion controller that maintains the stable traction of the vehicle. The outer loop guides the follower vehicle to follow the estimated trajectory of the lead vehicle.

The follower vehicle is assumed to have on board sensors, which are capable of estimating the orientation of the lead vehicle in relation to the follower vehicle's frame of reference. The process of implementing the vehicle following system can be sub-divided as follows:

- Localization of the follower vehicle.
- Detection and tracking of the lead vehicle.
- Following the lead vehicle.

Figure 4.1: Control block diagram for the proposed vehicle following system.

The first two components will now be individually addressed in this chapter. The third component involves the control of the follower vehicle and will be addressed in Chapter 5.

From figure 4.1, an estimate of the pose (position and orientation) of the lead vehicle is required in order to help solve the vehicle following problem. A recursive filter is a suitable solution to this problem. Due to the stochastic nature of the system and measurement models, a Bayesian formulation is used to represent the state of the system. The probability density functions (pdfs) of the states of the lead and follower vehicles are sought, as a solution to the estimation problem. A factored solution is proposed based on the assumption - that the pose of the follower vehicle is statistically independent of that of the lead vehicle when conditioned on the history of the followers' inputs and the sensor observations made by the follower vehicle. The vehicle following system is partitioned as

two separate estimation processes, one for the localization of the follower, and the other to track the lead vehicle and the pose of the virtual trailer link. The complete formulation is then tested in simulated and real outdoor environments.

This chapter provides a mathematical formulation of a generic vehicle following system based on Bayes theory. A systematic derivation of the mathematical formulation for vehicle following is outlined in Section 4.2. As the system requires an estimate of the poses of the vehicles, the observability of the system will be analyzed in Section 4.3. The system architecture for the realisation of the vehicle following system is presented in Section 4.4. A case study of the proposed formulation will be provided in Section 4.4 and the research challenges faced resulting from the case study is presented in Section 4.6. Results will be presented and discussed in Section 4.6.

4.2 Problem Formulation

The poses of the follower and lead vehicles in relation to a known reference frame, or the relative poses of the vehicles, are required to ensure the success of vehicle following systems. Mathematically, the complete vehicle following system can be formulated as a probability density function (pdf)¹:

$$P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_k) \quad (4.1)$$

where $\mathbf{x}_{F,k}$ and $\mathbf{x}_{L,k}$ are the poses of the follower and lead vehicles respectively at time k , $\mathbf{U}_k = (\mathbf{u}_0, \mathbf{u}_1, \dots, \mathbf{u}_k)$ is the history of the control inputs (for example,

¹The lowercase notation, eg $\mathbf{x}_k$ denotes the current state at time k and the uppercase notation, eg $\mathbf{X}_k$ denotes the entire history of the state up to and including time k .

the speed and steering angle commands) of the follower vehicle, and $\mathbf{Z}_k = (\mathbf{z}_0, \mathbf{z}_1, \dots, \mathbf{z}_k)$ is the history of the sensor measurement data collected up to, and including, time k . For a tractable solution to the vehicle following problem, the following assumptions are made.

- The vehicle following function is a Markov process and the current measurement $\mathbf{z}_k$ is independent of $\mathbf{Z}_{k-1}$ and $\mathbf{U}_k$, when conditioned on the pose of the follower vehicle. Hence,

$$P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_k) \propto P(\mathbf{z}_k | \mathbf{x}_{F,k}, \mathbf{x}_{L,k}) P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}) \quad (4.2)$$

- Two separate sensors may be used in the pose estimation process. For example, odometry may be used for the localization of the follower vehicle while a range sensor may be used to obtain the pose of the lead vehicle. Hence, the measurement vector $\mathbf{Z}_k$ may be expressed as two independent measurement vectors, when conditioned on the pose of the follower:

$$\mathbf{Z}_k = (\mathbf{z}_0^p, \mathbf{z}_0^r, \mathbf{z}_1^p, \mathbf{z}_1^r, \dots, \mathbf{z}_k^p, \mathbf{z}_k^r) = (\mathbf{Z}_k^p, \mathbf{Z}_k^r) \quad (4.3)$$

where $\mathbf{Z}_k^p = (\mathbf{z}_0^p, \mathbf{z}_1^p, \dots, \mathbf{z}_k^p)$ and $\mathbf{Z}_k^r = (\mathbf{z}_0^r, \mathbf{z}_1^r, \dots, \mathbf{z}_k^r)$ are the proprioceptive sensor measurement vector and range sensor measurement vector respectively, obtained up to, and including, time k . Hence,

$$\begin{aligned} & P(\mathbf{z}_k | \mathbf{x}_{F,k}, \mathbf{x}_{L,k}) P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}) \\ &= P(\mathbf{z}_k^p, \mathbf{z}_k^r | \mathbf{x}_{F,k}, \mathbf{x}_{L,k}) \times P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p, \mathbf{Z}_{k-1}^r) \\ &= P(\mathbf{z}_k^p | \mathbf{x}_{F,k}, \mathbf{x}_{L,k}) P(\mathbf{z}_k^r | \mathbf{x}_{F,k}, \mathbf{x}_{L,k}) \times P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p, \mathbf{Z}_{k-1}^r) \end{aligned} \quad (4.4)$$

- As the sensor measurement, $\mathbf{z}_k^p$ will be used for the estimation of the pose of the follower vehicle, it will not be affected by the pose of the lead vehicle.

$\mathbf{z}_k^p$ can then be assumed to be independent of $\mathbf{x}_{L,k}$, when conditioned on the current state of the follower vehicle. Hence,

$$P(\mathbf{z}_k^p | \mathbf{x}_{F,k}, \mathbf{x}_{L,k}) = P(\mathbf{z}_k^p | \mathbf{x}_{F,k}) \quad (4.5)$$

- In the vehicle following function, a control command (e.g steering angle and velocity), to be issued to the follower vehicle, must be computed based on the pose of the lead vehicle. This will affect the future pose of the follower vehicle. Thus, the state of the follower vehicle is statistically independent of the state of the leader, when conditioned on the history of the inputs to, and observations made from, the follower vehicle. Hence,

$$P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p, \mathbf{Z}_{k-1}^r) = P(\mathbf{x}_{F,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p, \mathbf{Z}_{k-1}^r) \times P(\mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^r) \quad (4.6)$$

- The history of the sensor measurement, $\mathbf{Z}^r$, will be used to estimate the pose of the lead vehicle. The pose of the follower vehicle will therefore not be affected by it, when conditioned on the history of the inputs to the follower, and the observations $\mathbf{Z}^p$ made.

$$P(\mathbf{x}_{F,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p, \mathbf{Z}_{k-1}^r) = P(\mathbf{x}_{F,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p) \quad (4.7)$$

- The pose of the lead vehicle $\mathbf{x}_{L,k}$ is independent of the history of the inputs to the follower conditioned on the history of the observations (of the leader).

$$P(\mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^r) = P(\mathbf{x}_{L,k} | \mathbf{Z}_{k-1}^r) \quad (4.8)$$

By consolidating Equations 4.1 to 4.8, the formulation for the vehicle following model can be factored as:

$$\begin{aligned}
 &P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_k) \\
 &\propto \underbrace{P(\mathbf{z}_k^p | \mathbf{x}_{F,k})P(\mathbf{x}_{F,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p)}_{\text{localization of follower}} \times \underbrace{P(\mathbf{z}_k^r | \mathbf{x}_{F,k}, \mathbf{x}_{L,k})P(\mathbf{x}_{L,k} | \mathbf{Z}_{k-1}^r)}_{\text{Tracking of lead vehicle w.r.t follower}} \quad (4.9)
 \end{aligned}$$

Thus, it can be concluded that the joint posterior for the vehicle following system can be factored into two separate estimation processes, one for the localization of the follower and the other to track the lead vehicle.

4.3 Observability Issues In A Probabilistic Vehicle Following System

Although vehicle following can be achieved using the Bayesian estimation process, the observability of a system is also an important factor for state estimation. If a system is observable, it will contain all the necessary information for the estimation with an error which is bounded [111].

Hermann and Krener [112] have come up with the concept of “*local distinguishability*” and the corresponding concept of “observability rank condition” to assist with an analysis of the observability of a nonlinear system. The formulations [112] refer to methods of observability analysis for a nonlinear system. As in most cases, the associated linearised system, of a nonlinear system, is not observable; however, it has the *local distinguishability* property as defined in [112].

Consider a nonlinear system, Σ :

$$\Sigma \begin{cases} \dot{X} = f(X, U) & X \in \mathbb{R}^n, U \in \mathbb{R}^m \\ \Lambda = g(X) & \Lambda \in \mathbb{R}^p \end{cases} \quad (4.10)$$

where $X \in \mathbb{R}^n$ is the state vector that is an element of n -dimensional manifold Σ , $U \in \mathbb{R}^m$ is the m -dimensional input vector and $\Lambda \in \mathbb{R}^p$ is the measurable output of the system Σ . The system Σ is observable, if and only if, the state can be expressed as a function, $\Phi(\cdot)$, of the observation Λ , the input U and their derivatives with respect to time:

$$X = \Phi(\Lambda, \dot{\Lambda}, \dots, \Lambda^{(k_1)}, U, \dot{U}, \dots, U^{(k_2)}) \quad (4.11)$$

The observation matrix of the system (with dimensions of $n \times n \times p$), $\mathcal{O}$, can be defined as:

$$\mathcal{O} = \begin{bmatrix} \partial g_1 & \partial L_f g_1 & \dots & \partial g_p & \dots & \partial L_f^{n-1} g_p \end{bmatrix} \quad (4.12)$$

where

- ∂g_i is the gradient vector of g_i with respect to the state X .
- $L_f g_i$ is the Lie derivative of g_i with respect to f , i.e., $L_f g_i = (\partial g_i / \partial x) \cdot f = \dot{\Lambda}$

As described in [112], a sufficient condition for the conclusion of the observability of a system can be derived by computing the rank of an observability matrix (with dimensions of $n \times n \times p$) $\mathcal{O}$. If $\text{rank}(\mathcal{O}) = n$, i.e., full rank, then the system is observable. On the other hand, as indicated in [113] and [114], it is not necessary to check for all determinants of all possible minors in $\mathcal{O}$. Rather, it is sufficient that at least one $n \times n$ minor of $\mathcal{O}$ has full rank in order to guarantee the system's observability.

4.3.1 Vehicle Following System Modelling

In this analysis of the observability of the vehicle following system, only two vehicles, one leader and one follower, will be considered. In addition, the vehicles are assumed to be moving in a 2D environment. The state of the vehicles is represented in Equation 4.13, where (x, y, θ) represents the absolute positions and orientation of the vehicle, respectively, with reference to a known global frame. The subscripts $_F$ and $_L$ represent the follower and lead vehicle respectively.

$$\mathbf{X} = \begin{bmatrix} x_F & y_F & \theta_F & x_L & y_L & \theta_L \end{bmatrix}^T \quad (4.13)$$

For the type of vehicle following under consideration, it can be assumed that no prior knowledge of the lead vehicle is available. Hence, only the input controls (V_F, ω_F) to the follower vehicle are available for system analysis, where V_F and ω_F are the translational and angular velocities of the follower vehicle respectively.

It is assumed that the follower vehicle is equipped with an on board gyroscope, and a sensor that can provide both range and bearing information of the environment as shown in Figure 4.2. The observation model of the system can be formulated as shown in Equation 4.14

Measurement model:

$$\mathbf{H} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} \sqrt{x_r^2 + y_r^2} \\ \frac{\pi}{2} - \theta_F + \tan^{-1} \frac{y_r}{x_r} \\ \frac{\pi}{2} - \theta_F \end{bmatrix} \quad (4.14)$$

where h_1 is the relative range of the two vehicles, h_2 is the relative bearing of the lead vehicle in relation to the follower vehicle as measured by the laser scanner

Figure 4.2: Measurement model for follower vehicle.

and h_3 is the absolute orientation of the follower vehicle in relation to a known reference frame.

Since only the relative range and bearing can be obtained from the sensor on board the follower vehicle, it is convenient to express the system state in terms of the relative positions, x_r and y_r , of the two vehicles. Furthermore, as no prior information regarding the state and control input of the lead vehicle is available, these parameters must be estimated. Hence, the new system model can be expressed as shown in Equation 4.15.

System model:

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, \mathbf{U}) = \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_F \\ \dot{\theta}_L \\ \dot{V}_L \end{bmatrix} = \begin{bmatrix} V_L \cos \theta_L - V_F \cos \theta_F \\ V_L \sin \theta_L - V_F \sin \theta_F \\ \omega_F \\ \omega_L \\ a_L \end{bmatrix} \quad (4.15)$$

where ω_L and ω_F are the angular velocities of the lead and follower vehicles respectively and a_L is the acceleration of the lead vehicle.

4.3.2 Observability of Vehicle Following System using Range and Bearing Sensors

The method introduced in [112] will be used to analyse the observability of the vehicle following system using only a laser scanner as the main measurement sensor. In particular, the dimension of the observability matrix, $\mathcal{O}$ and its dependency on the control inputs to the vehicle following system will be determined and analysed. This can be done by inhibiting one of the control inputs, V_F or ω_F , or both.

The gradient vector of h_i with respect to the state X is:

$$\begin{aligned} \partial h_1 &= \begin{bmatrix} \frac{x_r}{\sqrt{\Delta}} & \frac{y_r}{\sqrt{\Delta}} & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} A_0 & A_1 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (4.16)$$

$$\begin{aligned} \partial h_2 &= \begin{bmatrix} \frac{-y_r}{x_r^2 + y_r^2} & \frac{x_r}{x_r^2 + y_r^2} & -1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} B_0 & B_1 & -1 & 0 & 0 \end{bmatrix} \end{aligned} \quad (4.17)$$

$$\partial h_3 = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \end{bmatrix} \quad (4.18)$$

The first order Lie derivatives can be computed as:

$$L_{f(X,U)}\partial h_1 = \begin{bmatrix} D_0 & D_1 & D_2 & D_3 & D_4 \end{bmatrix} \quad (4.19)$$

where

$$D_0 = \frac{1}{\Delta^{\frac{3}{2}}}(x_r y_r (V_F S_F - V_L S_L) + (V_L C_L - V_F C_F)(\Delta - x_r^2))$$

$$D_1 = \frac{1}{\Delta^{\frac{3}{2}}}(x_r y_r (V_F C_F - V_L C_L) + (V_L S_L - V_F S_F)(\Delta - y_r^2))$$

$$D_2 = x_r V_F S_F / \sqrt{\Delta} - y_r V_F C_F / \sqrt{\Delta}$$

$$D_3 = -x_r V_L S_L / \sqrt{\Delta} + y_r V_L C_L / \sqrt{\Delta}$$

$$D_4 = x_r C_L / \sqrt{\Delta} + y_r S_L / \sqrt{\Delta}$$

$$\Delta = x_r^2 + y_r^2$$

$$S_L = \sin \theta_L, S_F = \sin \theta_F$$

$$C_L = \cos \theta_L, C_F = \cos \theta_F$$

$$L_{f(X,U)}\partial h_2 = \begin{bmatrix} E_0 & E_1 & E_2 & E_3 & E_4 \end{bmatrix} \quad (4.20)$$

where

$$E_0 = \frac{1}{\Delta^2}(2x_r y_r (V_L C_L - V_F C_F) + (V_L S_L - V_F S_F)(y_r^2 - x_r^2))$$

$$E_1 = \frac{1}{\Delta^2}(-2x_r y_r (V_L S_L - V_F S_F) - (V_L C_L - V_F C_F)(x_r^2 - y_r^2))$$

$$E_2 = -x_r V_F C_F / \Delta - y_r V_F S_F / \Delta$$

$$E_3 = x_r V_L C_L / \Delta + y_r V_L S_L / \Delta$$

$$E_4 = x_r S_L / \Delta - y_r C_L / \Delta$$

By combining the above elements, an observability matrix $\mathcal{O}$ can be constructed

as:

$$\mathcal{O} = \begin{bmatrix} \partial h_1 \\ \partial h_2 \\ \partial h_3 \\ L_{f(X,U)}\partial h_1 \\ L_{f(X,U)}\partial h_2 \end{bmatrix} = \begin{bmatrix} A_0 & A_1 & 0 & 0 & 0 \\ B_0 & B_1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ D_0 & D_1 & D_2 & D_3 & D_4 \\ E_0 & E_1 & E_2 & E_3 & E_4 \end{bmatrix} \quad (4.21)$$

The determinant of the observability matrix is:

$$\begin{aligned} \text{Det}(\mathcal{O}) &= (A_1 B_0 - A_0 B_1)(D_3 E_4 - D_4 E_3) \\ &= \frac{-V_L}{\Delta^{\frac{7}{2}}}((x_r C_L + y_r S_L)^2 - (y_r C_L - x_r S_L)^2) \end{aligned} \quad (4.22)$$

$\mathcal{O}$ will not have full rank if $\text{Det}(\mathcal{O}) = 0$. For this to happen,

$$V_L = 0 \quad (4.23)$$

$$\text{or,} \quad \tan \theta_L = y_r / x_r \quad (4.24)$$

From Equation 4.15, if we set $V_F = 0$, Equation 4.24 can be obtained. This implies that if the longitudinal velocity of the follower vehicle is zero, regardless of its angular velocity, there is a finite possibility of the observability condition being violated. Furthermore, given a stationary follower and a maneuvering leader, it is likely, that the leader will exit the field of view of the follower's exteroceptive sensors. In such a scenario, the rank of the observability matrix will be reduced. Given that, an intuitive conclusion can be drawn: The observability of the vehicle following system adopting a laser scanner and gyroscope is conditioned on a continuously detectable (within the field of view) leader vehicle.

4.4 Case Studies

Some case studies will be presented in this section. For demonstration purposes, the Extended Kalman Filter (EKF) will be extensively discussed, the derivation and notation for which can be found in [115]. However, the generic formulation presented above can also be applied when using other filters, such as the Unscented Kalman Filter (UKF) [116],[117] or Particle Filters [118],[119],[102].

4.4.1 Localization of the Follower Vehicle

The reference coordinate system and the pose of the follower vehicle are shown in Figure 4.3. The Ackerman model [120] is used to describe the motion model of the follower vehicle.

One simple method for localization is via dead reckoning sensors such as wheel encoders. Unfortunately, dead reckoning involves direct instrument integration, which causes unbounded errors to be accumulated over time.

A gyroscope is introduced to reduce the problem of orientation drift. A gyroscope can be mounted at the centre of the rear axle, and its position will be the reference point for the localization of the follower vehicle. The gyroscope provides information on the change in orientation of the vehicle with a reasonable angular bias. The off the shelf, optical fiber gyroscope² provides an angular bias of $1^\circ/hr$ [91]. Under the Ackerman model, the motion of the follower can now be described

²In our project, we use the KVH DSP-5000 Fiber Optic Gyro from KVH Industrial Inc. www.kvh.com.

Figure 4.3: Pose of the follower vehicle.

in terms of a non-linear state transition equation as follows:

$$\mathbf{x}_F(k+1) = \begin{bmatrix} x_F(k+1) \\ y_F(k+1) \\ \phi_F(k+1) \end{bmatrix}$$

$$= \begin{bmatrix} x_F(k) + V_F(k)\Delta T \sin \phi_F(k) \\ y_F(k) + V_F(k)\Delta T \cos \phi_F(k) \\ \phi_F(k) + \frac{V_F(k)\Delta T}{a} \tan \gamma(k) \end{bmatrix} + \mathbf{w}_F(k) \quad (4.25)$$

where x_F and y_F are the coordinates of the follower with respect to the global coordinate system and ϕ_F is its heading angle. $\mathbf{w}_F(k)$ denotes the process noise vector. The control inputs, $V_F(k)$ and $\gamma(k)$, are the longitudinal velocity and steering angle of the vehicle respectively. ΔT is the sampling time of the localization process.

As the gyroscope is used to measure the orientation of the vehicle, the observation

model is:

$$\varphi_{\text{gyro}}(k) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_F(k) \\ y_F(k) \\ \phi_F(k) \end{bmatrix} + \mu_F(k) \quad (4.26)$$

where $\mu_F(k)$ denotes the gyro sensor measurement noise. With the above system (Equation 4.25) and measurement (Equation 4.26) models, the updated pose of the follower vehicle can be obtained using filtering techniques such as the Extended Kalman Filter or Particle Filter [102]. This updated information will be used as the observation information to update the pose of the lead vehicle as described in Section 4.4.2.

4.4.2 Detection and Tracking of the Lead Vehicle

Tracking of the lead vehicle involves its detection and the estimation of its pose in relation to the follower.

Detecting the lead vehicle

Lead vehicle detection is one of the most important tasks for reliable vehicle following. Typical objects on the road include cars, trucks, buses, pedestrians and cyclists, which must not be misclassified. The current experiments use a laser scanner (SICK LMS291³) to perceive the environment. As a first approximation, the rear of the lead vehicle is assumed to be represented by a line in the 2D laser range data. Simplistically, at this stage, a line model will be used to detect the lead vehicle.

³www.sickusa.com

A data segmentation procedure must be carried out in order to identify the potential candidates for line modelling. With the segmented data, a line modelling procedure will be performed to estimate the pose of the possible leader. Simple Nearest Neighbor based data association, based on the previous leader pose and its current predicted pose, is then carried out to estimate its updated pose.

As for the line model, use of the popular Hough Transform representation is proposed. Nevertheless, other methods, such as RANSAC, split and merge algorithm, line regression algorithm [121], to name a few, can be used for line modelling. The shortest distance, $\varepsilon(k)$, from the laser scanner to the target line, and its corresponding bearing, $\rho_L(k)$ are shown in Figure 4.4. The Hough Transform representation for line extraction has been successfully implemented in many robotic applications [121]. However, some issues need to be addressed before the algorithm can be implemented. For example, there is the problem of choosing an appropriate grid size in the Hough Space. It is therefore proposed that a recursive filter (in this case the EKF) be used to address the above issues while maintaining the use of the Hough Transform parameters $\varepsilon(k)$ and $\rho_L(k)$. For a given line, $\varepsilon(k)$ and $\rho_L(k)$ are constant. With this information and the assumption of a straight line, it is possible to predict the next range value, $r(k+1)$, in terms of $r(k)$, $\rho_L(k)$ and $\theta(k)$, where $\theta(k)$ denotes the bearing of the point with

range $r(k)$ at time k . The prediction equation is:

$$\mathbf{X}_{\text{line}}(k+1) = \begin{bmatrix} r(k+1) \\ \theta(k+1) \\ \rho_L(k+1) \\ \varepsilon(k+1) \end{bmatrix} = \begin{bmatrix} \frac{r(k)\cos(\rho_L(k)-\theta(k))}{\cos(\rho_L(k)-\theta(k)-\Psi)} \\ \theta(k) + \Psi \\ \rho_L(k) \\ \varepsilon(k) \end{bmatrix} + \mathbf{w}_1(k) \quad (4.27)$$

where $\Psi = 0.5^\circ$ is the angular resolution of the laser scanner used and $\mathbf{w}_1(k)$ is the process noise vector. Since, in general, a range finder produces range values

Figure 4.4: A line model representation for detection of the lead vehicle. All measurements are made with respect to the laser scanner frame of reference.

in a sequence with respect to the scanning angle, an EKF can be formulated to recursively estimate the parameters, $\hat{\rho}_L(k|k)$ and $\hat{\varepsilon}(k|k)$, of the line using the laser scan data as the observations. The estimated values of $\rho_L(k)$, $\varepsilon(k)$ and the corresponding error covariance matrix,

$$\Sigma_{\text{line model}} = \begin{bmatrix} \sigma_{\rho_L}^2 & \sigma_{\rho_L}\sigma_{\varepsilon} \\ \sigma_{\varepsilon}\sigma_{\rho_L} & \sigma_{\varepsilon}^2 \end{bmatrix} \quad (4.28)$$

Figure 4.5: Estimation of the range (S), bearing (θ) and orientation of the leader vehicle (ρ_L).

will be used for estimating the relative pose of the lead vehicle.

Estimating the relative pose of the lead vehicle

From the line model, estimates of the range, S , to the centre point of the rear of the lead vehicle and its bearing, θ , relative to the sensor can be obtained by using the geometric properties shown in Figure 4.5. Taking the centre of the rear side of the lead vehicle as a point of reference, the bearing of the lead vehicle is simply:

$$\theta = \frac{\theta_{start} + \theta_{end}}{2.0} \quad (4.29)$$

It is assumed that the errors in determining θ_{start} and θ_{end} are not correlated. As θ_{start} and θ_{end} are measured relative to the sensor frame, the assumption is conditioned on the pose of the follower. The estimated bearing variance, in vehicle

coordinates, may be computed as:

$$\sigma_{\theta}^2 = \nabla_{\Theta} \begin{bmatrix} \sigma_{\theta_{start}}^2 & 0 \\ 0 & \sigma_{\theta_{end}}^2 \end{bmatrix} \nabla_{\Theta}^T = \frac{1}{2} \sigma_{laser, \theta}^2 \quad (4.30)$$

where

$$\nabla_{\Theta} = \begin{bmatrix} \frac{\partial \theta}{\partial \theta_{start}} & \frac{\partial \theta}{\partial \theta_{end}} \end{bmatrix} \quad (4.31)$$

$$\frac{\partial \theta}{\partial \theta_{start}} = \frac{\partial \theta}{\partial \theta_{end}} = \frac{1}{2} \quad (4.32)$$

assuming

$$\sigma_{\theta_{start}}^2 = \sigma_{\theta_{end}}^2 = \sigma_{laser, \theta}^2 \quad (4.33)$$

where $\sigma_{laser, \theta}^2$ is the variance of the angular measurement of the laser scanner.

The estimated range S , computed from the sensor to the centre of the target line is:

$$S = \frac{\varepsilon}{2} \left\{ \frac{1}{\cos(\rho_L - \theta_{start})} + \frac{1}{\cos(\rho_L - \theta_{end})} \right\} \quad (4.34)$$

The range variance is:

$$\begin{aligned} \sigma_S^2 &= \begin{bmatrix} \frac{\delta S}{\delta \varepsilon} & \frac{\delta S}{\delta \rho_L} & \frac{\delta S}{\delta \theta_s} & \frac{\delta S}{\delta \theta_e} \end{bmatrix} \begin{bmatrix} \sigma_{\varepsilon}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\rho_L}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\theta_s}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\theta_e}^2 \end{bmatrix} \begin{bmatrix} \frac{\delta S}{\delta \varepsilon} \\ \frac{\delta S}{\delta \rho_L} \\ \frac{\delta S}{\delta \theta_s} \\ \frac{\delta S}{\delta \theta_e} \end{bmatrix} \\ &= \left(\frac{\delta S}{\delta \varepsilon} \right)^2 \sigma_{\varepsilon}^2 + \left(\frac{\delta S}{\delta \rho_L} \right)^2 \sigma_{\rho_L}^2 + \left(\frac{\delta S}{\delta \theta_s} \right)^2 \sigma_{\theta_s}^2 + \left(\frac{\delta S}{\delta \theta_e} \right)^2 \sigma_{\theta_e}^2 \end{aligned} \quad (4.35)$$

where θ_s and θ_e denote θ_{start} and θ_{end} respectively.

$$\frac{\delta S}{\delta \rho_L} = \frac{\varepsilon}{2} \left[\frac{\sin(\rho_L - \theta_s)}{\cos^2(\rho_L - \theta_s)} + \frac{\sin(\rho_L - \theta_e)}{\cos^2(\rho_L - \theta_e)} \right] \quad (4.36)$$

$$\frac{\delta S}{\delta \varepsilon} = \frac{1}{2} \left[\frac{1}{\cos(\rho_L - \theta_s)} + \frac{1}{\cos(\rho_L - \theta_e)} \right] \quad (4.37)$$

$$\frac{\delta S}{\delta \theta_s} = \frac{-\varepsilon}{2} \left[\frac{\sin(\rho_L - \theta_s)}{\cos^2(\rho_L - \theta_s)} \right] \quad (4.38)$$

$$\frac{\delta S}{\delta \theta_e} = \frac{-\varepsilon}{2} \left[\frac{\sin(\rho_L - \theta_e)}{\cos^2(\rho_L - \theta_e)} \right] \quad (4.39)$$

and σ_ε^2 and $\sigma_{\rho_L}^2$ are the variances extracted from the covariance matrix $\Sigma_{line \text{ model}}$, as explained earlier. From Equations 4.36 to 4.39, there will be no solution for Equation 4.35 if $\rho_L - \theta_s = 90^\circ$ or $\rho_L - \theta_{es} = 90^\circ$. However, from Figure 4.5, this will never happen in practice.

Localizing the Lead Vehicle with the Virtual Trailer Link Model

The laser range finder data allows us to obtain the range (S) to the rear centre point, its bearing (θ) and the orientation (ρ_L) of the lead vehicle in relation to the follower, as described earlier.

In this application, the sampling time for the data acquisition of the laser information has been set at 0.1 seconds. With this sampling rate, it can reasonably be assumed that the lead vehicle is travelling at a constant velocity and a constant turning rate, thus allowing for the use of a constant velocity kinematic model.

The kinematic model for the lead vehicle and the corresponding VT link is:

$$\mathbf{X}_L(k+1) = \begin{bmatrix} x_L(k+1) \\ y_L(k+1) \\ \theta_L(k+1) \\ \dot{\theta}_L(k+1) \\ V_L(k+1) \\ \phi_L(k+1) \\ \dot{\phi}_L(k+1) \end{bmatrix} + \mathbf{w}_L(\mathbf{k})$$

$$= \begin{bmatrix} x_L(k) + V_L(k)\Delta T \cos(\theta_L(k)) \\ y_L(k) + V_L(k)\Delta T \sin(\theta_L(k)) \\ \theta_L(k) + \Delta T \dot{\theta}_L(k) \\ \dot{\theta}_L(k) \\ V_L(k) \\ \phi_L(k) + \Delta T \dot{\phi}_L(k) \\ -\frac{V_L(k) \sin(\phi_L(k))}{a} + \dot{\theta}_L(k)(1 + \cos(\phi_L(k))) \end{bmatrix} + \mathbf{w}_L(\mathbf{k}) \quad (4.40)$$

where $x_L(k+1)$, $y_L(k+1)$, $\theta_L(k+1)$, $\dot{\theta}_L(k+1)$ and $V_L(k+1)$ are the x and y positions, orientation, angular velocity and longitudinal velocity of the lead vehicle, with respect to the given reference coordinate system, respectively. ϕ_L is the trailer angle between the axis of the virtual trailer and the longitudinal axis of the lead vehicle, $\dot{\phi}_L$ is the rotational speed of the trailer and $\mathbf{w}_L(\mathbf{k})$ is the process noise vector.

As the model described above is a non-linear system, the EKF is again used to predict the states of the lead vehicle and the corresponding virtual trailer. Hence,

the Jacobian representation of the system model is:

$$\nabla f_L(k) = \begin{bmatrix} 1 & 0 & h & 0 & b & 0 & 0 \\ 0 & 1 & c & 0 & d & 0 & 0 \\ 0 & 0 & 1 & \Delta T & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & e & f & g & 0 \end{bmatrix} \quad (4.41)$$

where

$$h = -V_L(k)\Delta T \sin(\theta_L(k))$$

$$b = \Delta T \cos(\theta_L(k))$$

$$c = V_L(k)\Delta T \cos(\theta_L(k))$$

$$d = \Delta T \sin(\theta_L(k))$$

$$e = 1 + \cos(\phi_L(k))$$

$$f = \frac{-\sin(\phi_L(k))}{a}, \text{ and,}$$

$$g = -\frac{V_L(k) \cos(\phi_L(k))}{a} - \dot{\theta}_L(k) \sin(\phi_L(k))$$

With the laser scanner mounted onto the follower vehicle, the pose of the lead vehicle can be estimated, as outlined in earlier in this section. An “indirect” measurement formulation is used, such that the measurement vector used for computing the pose of the lead vehicle, from Figure 4.5, is :

$$\mathbf{Z}_L(\mathbf{k}) = \begin{bmatrix} Z_{x_L} \\ Z_{y_L} \\ Z_{\theta_L} \end{bmatrix} = \begin{bmatrix} x_F + S \cos(\theta - \phi_F) + a \cos(\frac{\pi}{2} - \phi_F) \\ y_F + S \sin(\theta - \phi_F) + a \sin(\frac{\pi}{2} - \phi_F) \\ \rho_L - \phi_F \end{bmatrix} + \mu_L(k) \quad (4.42)$$

with the observation variance :

$$\Sigma_{Z_L}(k+1) = \nabla Z_L R_{Z_L} \nabla Z_L^T \quad (4.43)$$

where $\mu_L(k)$ is the measurement noise vector and the Jacobian representation of the measurement model is

$$\nabla Z_L = \begin{bmatrix} \frac{\partial Z_L(k)}{\partial S} & \frac{\partial Z_L(k)}{\partial \theta} & \frac{\partial Z_L(k)}{\partial \rho_L} & \frac{\partial Z_L(k)}{\partial x_F} & \frac{\partial Z_L(k)}{\partial y_F} & \frac{\partial Z_L(k)}{\partial \phi_F} \end{bmatrix} \quad (4.44)$$

The 6×6 measurement noise covariance matrix, assuming the laser data to be independent (please refer to the next subsection for an explanation of this assumption) of the pose of the follower vehicle, is

$$R_{Z_L} = \begin{bmatrix} \sigma_{SS}^2 & \sigma_{\theta S}^2 & \sigma_{\rho_L S}^2 & 0 & 0 & 0 \\ \sigma_{S\theta}^2 & \sigma_{\theta\theta}^2 & \sigma_{\rho_L \theta}^2 & 0 & 0 & 0 \\ \sigma_{S\rho_L}^2 & \sigma_{\theta\rho_L}^2 & \sigma_{\rho_L \rho_L}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{X_F X_F}^2 & \sigma_{Y_F X_F}^2 & \sigma_{\phi_F X_F}^2 \\ 0 & 0 & 0 & \sigma_{X_F Y_F}^2 & \sigma_{Y_F Y_F}^2 & \sigma_{\phi_F Y_F}^2 \\ 0 & 0 & 0 & \sigma_{X_F \phi_F}^2 & \sigma_{Y_F \phi_F}^2 & \sigma_{\phi_F \phi_F}^2 \end{bmatrix} \quad (4.45)$$

A Note on Independence Assumptions

In the problem formulation section 4.2, it was argued that the vehicle following system can be partitioned into two separate estimation processes, one for the localization of the follower and the other to track the lead vehicle. The implementation in this thesis has intentionally performed the localization of the follower vehicle first, followed by the tracking of the lead vehicle. In doing so, it can be assumed that the laser data is conditionally independent of the pose of the follower vehicle, as the information on the pose of the follower vehicle has already been obtained before the pose of the lead vehicle was estimated.

4.5 System Evaluation

Based on the proposed Bayesian formulation and the system and measurement models for vehicle following systems, a case study on the implementation of the system using the Extended Kalman Filter (EKF) [60] as the main estimator will now be presented.

Prior to the analysis of the performance of the proposed vehicle following system, it is necessary to have a full understanding of the system. The case study of the proposed system uses two different sensors, a gyroscope [91] and a laser scanner together with an EKF, as a way of estimating the states of both the lead and follower vehicles. In order to ensure the successful implementation of the proposed formulation, three major issues must be addressed:

- How to associate the current sensor information to the sensor information received in the previous observation (data association problem).
- How to deal with sensor data that is corrupted by the pitching effect of the vehicle.
- How to impose kinematic constraints on the lead vehicle model.

4.5.1 Data Association Issues

The information received from the laser scanner reflects the relative pose of the lead vehicle in relation to the follower vehicle. The information received at every time instant does not have a common reference frame as the scanner is in motion together with the follower vehicle. At this early stage in this system, the generally

difficult data association problem is assumed to be solved by implementing the nearest neighbor method [60]. A coordinate transformation of the poses of the lead vehicle, in the previous and current time instance, to a common reference frame can be performed. Once the correspondence has been established, the EKF can be used to predict the next pose of the lead vehicle based on the last estimate of the pose and motion of the lead vehicle.

4.5.2 Measurement Noise and Filter Tuning

As proposed in Section 4.4.2, the computation of the pose of the lead vehicle is indirect; i.e., an EKF is used to estimate the pose of the lead vehicle by considering the centre of the rear of the lead vehicle as the measurement, rather than directly using the information obtained from the laser data. Furthermore, the data received from the scanner may have been corrupted for various reasons, such as the pitching of the sensor due to the unevenness of the road surface.

The effectiveness of the EKF depends on the accuracy of the observation noise covariance matrix. In many cases, a static, predetermined covariance matrix cannot provide a good estimate of the state of the system. The computed orientation of the lead vehicle can vary significantly if the laser data received is corrupted by the motion of the vehicle that carries the sensor. In this case, if there are outliers in the laser data, the error in estimating the relative position of the leader will be small compared to the error in estimating the orientation of the lead vehicle. The rapid change in the quality of this orientation data will result in an inaccurate computation of the filter gain and hence, in an undesirable, large correction of the estimate of the vehicle's orientation.

In this system, a simple yet effective dynamic observation noise model has been proposed to improve the effectiveness of the filtering process. This model examines the measurement innovation against the predicted orientation of the lead vehicle. If the innovation is beyond a certain threshold level, a larger observation noise covariance is used in the filter. The basic underlying concept behind this method is that if the innovation is large, which implies that there is a large discrepancy between the predicted and measured values, the prediction model can be trusted more than the measurement model, based on our “unmodelled” knowledge of the negative effects of the sensing caused by vehicle pitching.

There are several things to consider in the fine tuning of the threshold of the aforementioned innovation. The laser may scan the rear and/or the side surface of the lead vehicle. Both of these surfaces appear as a line and therefore the lead vehicle’s side could be mistaken as its rear. With a short sampling interval for the measurement (typically 0.1 seconds to acquire an entire scan in this case), it is certain that any road vehicle will not be able to turn through 90° in this time period. The threshold level on the orientation innovation is therefore set at a conservative level, to be within $\pm 10^\circ$.

4.5.3 Kinematic Constraints

Considering that the update rate of the filters is set at 0.1 seconds, the kinematic constraints of the lead vehicle system model, shown in Equation 4.40, are used. The maximum speed of the lead vehicle is also known if its specifications are known. In this case, the maximum speed of the lead vehicle has been set at

3m/s. Further, it is known that under normal driving conditions on the road, a vehicle cannot be steered beyond a certain angle. Based on these assumptions, constraints can be set on the estimated pose of the lead vehicle. As mentioned in the previous subsection, measurement noise from the sensor is inevitable. Incorrect estimates of the inter-vehicle spacing results in an incorrect estimate of the velocity of the lead vehicle. Hence, by setting a maximum value for the estimated velocity of the leader, the data association performance can be improved.

4.6 Experimental Results

Simulated and real experiments have been carried out to validate the performance of the proposed formulation. For demonstration purposes, EKF's were used as the main estimators in both experiments and repetitive simulation runs were performed. Figures 4.6 to 4.13 show the results of a representative simulation run. Comparisons of the results with those reported from other systems will be made in the real experimental section.

4.6.1 Simulation

A stand-alone software was written to control the lead vehicle. To test the robustness of the formulation, the lead vehicle was commanded to travel first along a straight path (from time 0 to 200, ie, 0 to 20s since $\Delta T = 0.1s$), followed by a clothoid path with a transition to the right (from time 200 to 300), a tight left turn (large curvature simulating a 'U-turn' path, from time 300 to 500), straight (from time 550 to 700) and finally a clothoid path with a transition to the left (from

time 700 to 820), as shown in Figures 4.6 (orientation profiles of the vehicles) and 4.7 (trajectories of the vehicles). The lead vehicle was commanded to gradually transit from the straight to the circular path, thus simulating a clothoid path on a typical urban road. During motion, the lead vehicle was accelerated to various speeds, as shown in Figure 4.11 (velocity profile of the vehicles).

In implementing the proposed formulation, another software system was used to command the follower vehicle. A simple pure pursuit control strategy was implemented to command the follower vehicle to trail the computed pose of the virtual trailer. To make sure the simulation results are comparable to the actual system, the standard deviation settings of all the sensors used were set as close to the real values as possible. The standard deviations (as obtained from the respective data sheets) of the gyroscope, laser range and laser bearing measurements were set at 1.28° , 5cm and 0.5° respectively. The relative pose of the lead vehicle, and the virtual trailer, in relation to the follower was generated at each time step (0.1s in this case). In this simulation, the data association problem, in identifying the lead vehicle, was assumed solved. Furthermore, the lengths (distance between the front and rear wheels) of both vehicles were 1.2m (which is identical to the length of the experimental vehicle). Hence, based on the system design requirements for the virtual trailer as presented in Chapter 3, its length was 1.2m .

As shown in Figure 4.6, the follower vehicle was able to align itself with the lead vehicle, albeit with a small delay. In particular, at time 200, when the lead vehicle had begun to make a transition to the right, the follower vehicle maintained its

Figure 4.6: Orientations of the leader, virtual trailer and follower. (See text for the descriptions of the maneuverings of the vehicles and the virtual trailer.)

Figure 4.7: Ground truth for the lead and follower vehicles.

straight maneuver despite a change in the maneuvering of the leader. Similarly, at time 550, the follower vehicle again maintained its straight maneuver. This is a result of the off-hooked kinematic configuration described in Chapter 3. These results are significant for the vehicle following function. With the off-hooked kinematic configuration, the follower vehicle can trail the trajectory of the lead vehicle and thus avoid hitting the road curb while the vehicles are on a transition path.

Figure 4.8 shows that the follower vehicle was able to trail behind the lead vehicle during most of the trajectory. However, there were path deviations between the two vehicles when they were negotiating curves. Nevertheless, the estimated poses of the lead vehicle and the virtual trailer were close to each other. A zoomed in view of Figure 4.8 (shown in Figure 4.9), shows that the maximum path error between the path of the lead vehicle and that of the virtual trailer was less than 20cm, which occurs when both vehicles make a 'U-turn'. This result is encouraging.

Table 4.1 shows a summary of the error distribution of the path deviation between the leader and the virtual trailer. To our knowledge, no other vehicle following system has demonstrated the capability to follow a leader in a tight 'U-turn' maneuver (the steering angle of the lead vehicle was set at 20° throughout the turn). Further, the path errors were typically less than 5cm in all cases. The path deviations between the two vehicles was small ($< 30\text{cm}$ on average) even when the lead vehicle accelerated during turning at times 200 and 650. This error is much smaller than that of the system designed by Lu [54], which reported a maximum

Figure 4.8: Simulation run. Various types of paths such as a straight line, left and right turns are included. The positions of the virtual trailer follow the path of the lead vehicle closely.

Table 4.1: Error distribution for path deviation between Leader and Virtual Trailer

Error(m)	Frequency
0 to 0.05	600
0.05 to 0.10	105
0.10 to 0.15	100
0.15 to 0.20	10
>0.20	5

path deviation of 35cm. A more comprehensive comparison will be made in the real experimental section.

Figure 4.10 shows a plot of the errors between the actual and estimated positions of the lead and follower vehicles respectively. The positional errors are less than 20cm from time 0 to 450. As the lead vehicle started to make a tight turn, the positional errors for both vehicles increased with time. The increase in positional errors was expected, as only odometry information is used for the localization of

Figure 4.9: Zoomed in view of the area enclosed by the square box in Figure 4.8. The maximum error between the trajectories of the leader and the virtual trailer is around 20cm.

both vehicles. Although the absolute positional errors were large (about 50cm), the relative positional errors between the two vehicles were small, as shown in Figure 4.7. The effect of the absolute positional errors on the vehicle following function is insignificant as the vehicle following function operates on the relative poses of the two vehicles.

Figure 4.11 shows profiles of the commanded and estimated velocities of the lead vehicle and the commanded velocity of the follower vehicle versus time. The estimated velocity of the lead vehicle was close to its actual velocity versus time. The errors in the estimate are less than $0.2m/s$. From time 0 to 200, when the lead vehicle was moving along a straight path, the estimator had accurately predicted its velocity. However, from time 200 to 600, the lead vehicle was commanded to make various turns, and the estimator was unable to predict the velocity of the

Figure 4.10: Positional errors between the ground truth and the estimated positions of the lead and follower vehicles.

lead vehicle correctly. As shown in Equation 4.40, the lead vehicle was modelled to be moving at a constant velocity and at a constant turning rate. This model has therefore been shown to be inadequate for this type of maneuver. This is the main factor in the inaccurate estimation of the leader's velocity. Other available vehicle motion models, such as the Interactive Multiple Model (IMM) filter [115], could be used to cater to various other types of maneuvers.

Figure 4.12 shows a plot of the errors in the inter-vehicle distance between the two vehicles. The errors were computed by comparing the measured inter-vehicle distances with the actual inter-vehicle distances. 95% and 90% of the errors were within $\pm 25\text{cm}$ and $\pm 20\text{cm}$ respectively.

Figure 4.13 shows a plot of the errors in the relative angle between the two vehicles. The errors were computed by comparing the measured relative angle against the actual relative angle. From time 0 to 200 and 650 to 700, the errors in

Table 4.2: Performance of system on roundabout

Radius(m)	Error(m)
10	< 0.25
20	< 0.18
30	< 0.18
40	< 0.18

relative angle were almost zero, since both vehicles were moving along a straight path. As expected, the errors increased when the vehicles were negotiating turns. As the positional errors for the follower were relatively small, as shown in Figure 4.8, we are confident that, with the proposed formulation, the follower vehicle will be able to trail the lead vehicle successfully.

Tables 4.2 and 4.3 summarize the performance of the proposed system under simulated roundabout (circular) and transition paths. The radii of a roundabout can range from 10m to 40m. For the purpose of analysis, both vehicles were commanded to move along the circular path, at various radii. The average path deviations were then computed. The average path deviation between both vehicles was small, in the range of 0.18m to 0.25m (Table 4.2).

As for the transition paths (clothoids), both vehicles were commanded to move along a straight path and gradually steer into the roundabout. The average path deviation between the vehicles was small, in the range of 0.10m to 0.30m (Table 4.3).

Figure 4.11: Commanded and estimated velocity profiles of the lead and follower vehicles.

Figure 4.12: Inter-vehicle (between leader and follower) separation errors. The errors were obtained by comparing the difference between the real (ground truth) inter-vehicle separation and the estimated inter-vehicle separation.

Table 4.3: Performance of system on Clothoid

Straight path to Radius, R_c (m)	Error(m)
10	< 0.30
20	< 0.25
30	< 0.18
40	< 0.10

Figure 4.13: Relative orientation (between leader and follower) errors. The errors were obtained by comparing the difference between the real (ground truth) relative orientation and the estimated relative orientation.

4.6.2 Real Experimentation

The follower vehicle used in this project is a RobuCar, from RoboSoft⁴, as shown in Figure 4.14. It is a four wheel drive vehicle for outdoor applications. In this experiment, the RobuCar was configured as a back wheel driving and front wheel steering vehicle. The SICK laser scanner was mounted onto the front of the RobuCar, while the gyroscope was mounted onto the middle of the rear wheel axle. On board data acquisition cards were installed to capture the laser information,

⁴<http://www.robosoft.fr/robucar.html>

Figure 4.14: Setup of the lead and follower vehicles. The lead vehicle, Club Car - CarryAll II, was manually driven around the test track in the car park. The follower vehicle was manually driven to follow the leader. The laser scanner was mounted onto the front of the follower and the gyroscope was mounted onto the centre of the rear axle of the follower.

gyroscope readings and wheel encoder speeds. The lead vehicle is a golf cart, Club Car - CarryAll II⁵. The lead vehicle was manually driven around the test track, which comprised a straight path and both left and right turns. The control issues of the follower vehicle are not considered here since the main purpose of the experiment was to evaluate the capability of the proposed formulation to track the lead vehicle, using the virtual trailer, while the follower vehicle was in motion. Hence, the follower vehicle was manually driven to trail behind the lead vehicle.

The test track was located in a carpark on campus. As there are tall buildings surrounding the test track, the GPS systems failed to function satisfactorily. Hence, the experiment was designed such that the trajectories of both vehicles

⁵<http://www.clubcar.com>

were easily obtained for comparison. For example, the vehicle was commanded to move in one maneuver, such as a single straight path or a turn only, at a time. In this way, the performance of the system could be evaluated.

Figure 4.15 shows the plot of the paths for the follower, leader and virtual trailer. As there was a time lag between the paths of the leader and virtual trailer, it is not practical to compute the path deviations between them. Hence, manual inspection of the data was carried out. The paths of the vehicles and the virtual trailer were plotted as the experiment was carried out. The path deviations between the estimated lead vehicle and the virtual trailer were observed to be small. The path deviations were small (typically $< 5cm$) when the vehicles were moving along a straight path. However, there were errors of about $15cm$ when the vehicles were negotiating a curve, at positions around (25,8) in Figure 4.15. By investigating the laser data, it was observed that the main cause of these deviations was the noisy range data. The laser points hit the side of the lead vehicle, which comprises the contour of the driver's seat and some other equipment mounted on the back of the lead vehicle, as shown in Figure 4.16. The segmentation and line extraction algorithms were unable to identify the lead vehicle, thus resulting in a false estimate of the pose of the lead vehicle. As some measurement data was lost, the filter had to sometimes rely on the prediction results. This contributed to the path deviations. Once the measurement data was restored, the filter functioned as expected and the errors in the poses of the leader and the virtual trailer were significantly reduced.

Figure 4.15: Result of the estimated vehicle and virtual trailer positions. Both the lead and follower vehicles were driven manually on a test track. The estimated paths for both the lead vehicle and the virtual trailer were close to each other. (Please refer to the text for a detailed analysis of the result)

Figure 4.16: Projection of the range data from the laser scanner onto the X-Y plane. (a) A good laser return, reflected from the lead vehicle. The laser data can be easily fitted onto a straight line model. (b) The laser data causes the straight line model to provide a false detection.

Figure 4.17: Residues in the x position and the 2-sigma upper and lower bounds. The residues were well within the bounds. The residues were noisy at around time step 500. This is due to the noisy data from the laser sensor.

Figure 4.18: Residues in the y position and the 2-sigma upper and lower bounds. The residues were well within the bounds.

Figure 4.19: Residues in the orientation of the lead vehicle and the 2-sigma upper and lower bounds. The residues were well within the bounds.

Figures 4.17 to 4.19 show the pose innovations⁶ of the lead vehicle. It is interesting to observe from Figures 4.17 and 4.18 that the positional errors of the lead vehicle fall well within 2σ bounds. The majority of the 2σ bounds for both x and y positions estimates are $< 20cm$. The positional errors became large when the lead vehicle turned. As described in the previous paragraph, the large errors were a result of the noisy observation data. As for the orientation innovation for the leader, as shown in Figure 4.19, the errors were small. Again, the larger errors occurred when the lead vehicle turned, and the observation data became unavailable.

No straightforward direct comparison of the performance of this system with previous work can be made, as the variables used to describe vehicles' poses are not always the same from one system to another. Also, the performance

⁶also know as 'residual' of the filter. It measures the difference between the predicted and the observed state of the system.

of the vehicle following system depends greatly on the actuator of the follower vehicle. However, in order to provide an indication of the achieved performance, a comparison of the maximum path deviation⁷ with the works of Lu [54], Stefan [52] and Wang [67] has been made. The path based vehicle following algorithm implemented by Stefan [52] achieved a maximum path deviation (between the two vehicles) of approximately 70 cm. The system implemented by Lu [54] achieved a maximum path deviation of 35 cm. The vehicle following system by Wang [67] achieved a maximum path deviation of 50cm. As for this case, the maximum path deviation between the lead vehicle and the virtual trailer was 15 cm. The comparison may not be exact as the control of the follower vehicle has not yet been implemented here. However, this system is already performing at least 2 times better than the others, even before the introduction of the control algorithm. We are confident that with a good vehicle controller, this system can perform on par with Lu's [54].

To further verify the performance of this system, the inter-vehicle distance between the two vehicles was manually computed and the results were compared with the estimated inter-vehicle distance. Figure 4.20 shows the plot of the two inter-vehicle distances mentioned. It was observed that the actual inter-vehicle distance was noisy and that there were 'jumps' in the data. The inter-vehicle distance computed from the filter was much smoother. Hence, if the noisy measurement data were to feed into the vehicle following controller, there

⁷for other similar work, it is the deviation between the two vehicles, while for this case, it is the deviations between the lead vehicle and the virtual trailer as a fully automated vehicle following system has not yet been implemented

Figure 4.20: Inter-vehicle distance between the lead and follower vehicles. The top figure shows the inter-vehicle distance computed manually. The lower figure shows the computed inter-vehicle distance between the two vehicles.

may be system stability issues since the follower vehicle would be commanded to accelerate at one instant and decelerate at the next, which demonstrates the advantages of the proposed estimation process. To further evaluate the capability of the proposed vehicle following formulation in tracking the leader vehicle while the follower vehicle is in motion, another experiment was conducted. The goal is to verify that the proposed formulation yields a reasonable pose estimate of the leader vehicle with respect to the follower vehicle. Hence, the experiment will concentrate on the performance of the Bayesian formulation and the control technique of the follower vehicle will only be discussed in the next chapter. The leader vehicle was manually driven around the test track as shown in Figure 4.21. Since the control issues of the follower vehicle are not considered here, the follower vehicle was manually driven to trail behind the leader vehicle. In addition, the leader vehicle was purposely driven on the outer lane (as shown in

Figure 4.21: The test track is located in the NTU campus. There are two lanes in this test track, the outer lane and the inner lane. The leader vehicle was controlled to move along the outer lane and the follower vehicle on the inner lane.

Figure 4.21) of the test track, and the follower vehicle was constrained to travel along the inner lane of the test track. The test track formed a loop. The separation between the inner lanes on the left and on the right of the track is approximately 20m and the width of each lane is approximately 4m. The total length of the test track is about 65m.

Figures 4.22 and 4.23 show the results of the experiment. The following observations were made:

- The estimated trajectory of the leader (in Figure 4.22) is smooth. The estimated trajectory for the follower vehicle, along the y-direction, does not seem to be straight. This is expected as the follower vehicle was actually manually driven and some adjustment to the steering of the follower vehicle were needed due to the dynamics, such as slippage, of the follower. Further, there is a large spacing (approximately 4 meters) between both estimated

Figure 4.22: Result of the estimated vehicle positions. The estimated paths are smooth as expected, since the vehicles were driven in different lanes on the road. There is a clear separation between the two estimated paths. (Please refer to text for details)

trajectories, when the vehicles move in the y direction. This matches our experimental setup, and approximate “ground truth” observations, as both vehicles were driven in neighbouring lanes, during their y direction of motion, the centers of which are indeed separated by approximately 4 meters. Also, the estimated separation between the the left and right inner lane of the track is about 20m. Again, this matches out experimental setup.

- More than 95% of the orientation residues of the estimator fell within the 2σ bounds (Figure 4.23).

Finally, during the experiments, we drove both of the vehicles around the test track. From Figure 4.22, it can be observed that the estimated paths for both vehicles formed a loop, which is an indication of correct filter performance, since the actual start and end locations were approximately the same for each vehicle.

Figure 4.23: Residues in x, y and θ of the leader vehicle and the corresponding 2-sigma upper and lower bounds.

4.6.3 High Speed Vehicle Following

This section evaluates the performance of the proposed following system under the practical scenario of real road vehicles operating in an urban environment at realistic velocities (5 - 72km/hr) [122], [123]. The analysis extends the experimentation from the small scale electric vehicles previously carried out, verifying the scalability and applicability of the proposed system to real engine driven road vehicles.

In this high-speed trial, the leading vehicle, a Multiple Purpose Vehicle (MPV),

was driven on a test road with a trajectory as indicated in figure 4.24, which comprises a roundabout, straight sections as well as both left and right turns. The vehicle was manually driven from point A to F, at speeds as high as 20 m/s (72 km/h)), as depicted in the velocity measurements presented in figure 4.25. Figure 4.26 shows the zoomed in view of the sections labelled from 'A' to 'F'. The follower vehicle, also a MPV, equipped with wheel encoders and a laser range scanner, was manually driven behind the lead vehicle [122], [123].

Figure 4.24: A High speed vehicle following experiment. This figure shows the result of the estimated leader vehicle and virtual trailer positions. The thick line (green) represents the estimated trajectory of the leader, with the thin blue line representing that of the virtual trailer.

Figure 4.25: A High speed vehicle following experiment. This figure gives the velocity profiles of the leader and follower vehicles. Labels 'A' to 'F' indicate the velocities of the vehicles at the corresponding labelled positions given in figure 4.24.

Given that the length of the follower is approximately 4.4m, the virtual trailer length was also set at 4.4m, according to the design requirements outlined previously in chapter 3. Localisation estimates of the follower were obtained through a fused GPS / odometry solution [122], [123]. After completion of the experiment, the following observations were notable:

- The vehicles initially accelerated to a speed of 9 m/s (32.4 km/hr) from the starting location to the position marked 'A'. Before entering the roundabout,

both vehicles decelerated to 2.5 m/s (9 km/h) and slowly accelerated again to 7.5 m/s (27 km/h) during the curved maneuver around the roundabout from 'A' to 'B'. The vehicles then drove at various speeds ranging from 2 m/s (7.2 km/h) to 14 m/s (50.4 km/h) along section B-C. The vehicles then executed a small left turn along section C-D with velocity ranging from 5m/s (18 km/h) to 7.5 m/s (27 km/h). Finally, the vehicles travelled on a gradually curving path along section E-F, recording speeds as high as 20 m/s (72 km/h).

- Figure 4.26(a) shows a zoomed-in view of section A-B. It can be observed that the path deviation between the leader and the virtual trailer is approximately 20 cm just prior to the vehicle entering the roundabout. While moving around the roundabout, the path deviation gradually increases to approximately 50 cm (at position (175,-165)). This is due to the effect of the proposed virtual trailer model. While entering the roundabout, the vehicles are actually moving on a transition path (or clothoid), thus minor path deviation is inevitable. At position (165,-188), there is a large path deviation between the leader and the estimated position of the virtual trailer. By examining the vehicle orientation shown in figure 4.27, it can be observed that the leader vehicle has actually experienced a sudden change in orientation as shown in figure 4.28. This resulted in the laser mis-interpreting the position of the leader, thus causing an incorrect pose estimate of the leader. Nevertheless, the extended Kalman Filter rectified this random fluctuation and reduced the path deviation between the leader and the virtual trailer over subsequent measurement updates.

This highlights the advantage of using a Bayesian formulation coupled with the virtual trailer kinematic configuration for vehicle following, as proposed in this thesis.

- Figures 4.26 (b) to (d) show the zoomed view of the remaining sections of the experiment. Despite a highly fluctuating vehicle velocity along these sections, the maximum path deviation from the filter output is seen to be less than 50 cm with the average path deviation being consistently maintained at approximately 30 cm.

4.7 Summary

A Bayesian formulation, together with a virtual trailer link model, aimed at producing a safe autonomous vehicle following system has been demonstrated in this chapter. With this formulation, and the dynamic modelling of the uncertainties in the sensor information, the potential for an autonomous vehicle following function has been demonstrated.

This chapter has shown that a vehicle following system can be formulated as two separate Bayesian estimation processes, one for the localization of the follower vehicle and another to track the lead vehicle. The pose of the follower vehicle can be estimated by using on board odometry and a gyroscope. The pose of the leader can be estimated with an EKF by using the information from a laser scanner, which is mounted onto the follower vehicle, together with the estimated pose of the follower vehicle, based on the assumption that the laser measurement is independent of the pose of the follower vehicle. This assumption

Figure 4.26: Zoomed-in view of vehicle trajectories. (a) Section A-B, (b) Section C-D, (c) Section D-E and (d) Section E-F.

can be made because localization of the follower vehicle has been performed prior to the estimation of the pose of the lead vehicle in the implementation of the formulation. To estimate the pose of the lead vehicle, the measurement uncertainties in the sensor measurement have to be compensated for. As both the lead and follower vehicles are constantly in motion, the data from the laser scanner can be noisy and corrupted. The data becomes corrupted due to the pitching effect of the vehicles and as a result, the performance of the EKF used in

Figure 4.27: Orientation of leader and follower.

Figure 4.28: The lead vehicle swivelled at step 370 to 400.

the estimation process can be degraded. It has been shown that the performance can be improved by implementing different error covariance matrices for the laser measurements. Alternatively, the pitching effect can be rectified by improving the perception system on board the vehicle.

As the true values of the states of both vehicles are unknown, the innovation sequences have been used as the principle means to analyze and validate the performance of this formulation. The experimental results have provided evidence that it is feasible to have two separate Bayesian estimation filters for a vehicle following system. The innovation sequences for both the position and orientation of the lead vehicle are well within 2σ confidence bounds. Furthermore, with the introduction of dynamic noise covariance matrices for compensating the laser uncertainties, the innovation sequences for the pose estimation of the lead vehicle have improved significantly.

CHAPTER 5

The Relative Information Metric For Vehicle Following

5.1 Introduction

This chapter focuses on the automatic generation of driving commands for a follower vehicle in pursuit of a lead vehicle. Information theoretic frameworks have been used extensively in mobile robotic applications. Typical applications are surveillance systems using Unmanned Aerial Vehicles (UAVs) [124] and the active exploration of an area using Unmanned Ground Vehicles (UGVs) [102]. These systems aim to maximize the knowledge, or information, gained by the robot, through optimized control actions [125],[126],[127]. The strategy is also aimed at minimizing the uncertainties of the system state through the selection of a sequence of control actions. Moreover, information theoretic frameworks have been used in the machine vision community as a tool in image association. For example, the relative information (Kullback-Leibler or K-L distance) was used as a measure for optimal feature selection such that the feature was selected by

maximizing the K-L distance between target classes [128].

In anticipation of the challenges in the vehicle following function in urban environments and the advantages of information theoretic framework in minimising system state uncertainties, a relative information based vehicle following control strategy will be formulated in this chapter. Using a mathematical formulation of the vehicle following task and problem definitions, outlined in Section 5.2, research issues such as vehicle constraints and sensor uncertainties imposed in implementation will be discussed in Section 5.3. By formulating the vehicle following system in a Bayesian representation, two probabilistic distributions describing the uncertainties of the states of the lead and the follower vehicles are obtained. The driving commands for the follower vehicle, computed based on the minimization of the relative information (K-L distance) between the two vehicles, will be presented in Section 5.4. Before issuing an order for action to the follower vehicle, a series of achievable actions can be identified. A set of expected predictions of uncertainties for the follower vehicle can be obtained, using the achievable actions as input in the predictions. By computing the relative information based on this series of expected uncertainties with respect to the uncertainty of the state of the lead vehicle, a desired action for the follower vehicle can be obtained by selecting the action that yields minimum relative information for the system (section 5.5).

The information theoretic vehicle following algorithm is tested experimentally by analyzing the performance of the follower vehicle when the lead vehicle is in various type of maneuvers. The experimental results will be presented and

discussed in detail in Section 5.6.

5.2 Definition of a Vehicle Following System: Task and problem formulation

The vehicle following function can be defined in an analytical manner as follows:

Definition 1: Conventionally, vehicle following is achieved when the follower attains the pose of the lead vehicle some instance of time later, that is,

$$\mathbf{x}_F(t) = \mathbf{x}_L(t - m), \quad m > 0 \quad \forall t > 0 \quad (5.1)$$

where t is the time and m is the amount of time that has elapsed for a follower to reach the position of, and align with, the leader. $\mathbf{x}_F(t) = [x_F(t) \ y_F(t) \ \theta_F(t)]^T$ and $\mathbf{x}_L(t) = [x_L(t) \ y_L(t) \ \theta_L(t)]^T$ are the poses of follower and leader at time t respectively.

For safe vehicle following, the following criteria must be fulfilled:

$$\| \mathbf{x}_F(t) - \mathbf{x}_L(t) \| > \mathbf{d}_m \quad (5.2)$$

where $\mathbf{d}_m > 0$ is defined as the minimum safety separation distance between the two vehicles at any given time.

Furthermore, the velocities, v_L and v_F , and steering angles, ϕ_L and ϕ_F , of the lead and follower vehicles at all times are constrained as

$$0 < v_L \leq V_{Lmax} \quad (5.3)$$

$$-\alpha_{Lmax} \leq \phi_L \leq \alpha_{Lmax} \quad (5.4)$$

$$0 < v_F \leq V_{Fmax} \quad (5.5)$$

$$-\alpha_{Fmax} \leq \phi_F \leq \alpha_{Fmax} \quad (5.6)$$

where V_{Lmax} , V_{Fmax} , α_{Lmax} and $\alpha_{Fmax} \in \mathbf{R}^+$ are the maximum achievable velocities and steering angles of the leader (indicated as subscript L) and follower (indicated as subscript F) respectively.

However, it was demonstrated in Chapter 3 that vehicle following can be achieved through the implementation of a virtual trailer link model. In this model, the lead vehicle is modelled as the towing vehicle and the follower as the trailer. As the model suggests, the lead vehicle (towing) is effectively pulling a follower (trailer) via a virtual trailer link. It has been proven that the length of the virtual trailer link must be set to be the length of the follower for an intrinsically safe vehicle following system.

Definition 2: With the virtual trailer link model, vehicle following is redefined as:

$$\|\mathbf{x}_F(t + \delta t) - \mathbf{x}_T(t)\| = 0 \quad \forall t > 0 \quad (5.7)$$

where $\mathbf{x}_T(t) = [x_T(t) \ y_T(t) \ \theta_T(t)]^T$ is the pose of the virtual trailer at time t and δt is the time increment between measurements.

As presented in Chapter 4, the motion model of the follower, in discrete time state space, can be represented as ¹:

$$\mathbf{x}_{F,k+1} = \mathbf{f}(\mathbf{X}_{L,k}, \mathbf{X}_{T,k}, \mathbf{U}_k, \omega_k) \quad (5.8)$$

¹The lowercase notation, eg $\mathbf{x}_{L,k}$ denotes the current state and the uppercase notation, eg $\mathbf{X}_{L,k}$ denotes the entire history of the state up to and including time k . The state in discrete time space is represented by subscript k , eg $\mathbf{x}_{L,k}$. Continuous time space is denoted in the form of $\mathbf{x}_L(t)$.

where $\mathbf{X}_{F,k}$, $\mathbf{X}_{L,k}$ and $\mathbf{X}_{T,k}$ are the histories of the state of the follower, leader and virtual trailer respectively. $\mathbf{U}_k$ is a vector of motion control signals input to the follower vehicle, ω_k is the motion uncertainty and $\mathbf{f}(\cdots)$ is a non-linear function representing the motion of the follower.

The sensor is used to acquire the noisy observation $\mathbf{z}_k$ of the lead vehicle taken from the follower. The sensor model, as presented in Chapter 4 and is shown here for easy reference, is described as:

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_{L,k}, \mathbf{x}_{F,k}, \nu_k) \quad (5.9)$$

where ν_k is the sensor noise and $\mathbf{h}(\cdots)$ is a non-linear function representing the sensor model.

Both the sensor and motion uncertainties will be modelled by random variables. The sequences $\{\nu_0, \nu_1, \dots, \nu_k\}$ and $\{\omega_0, \omega_1, \dots, \omega_k\}$ are assumed to be independent, zero mean, white processes with known covariances.

Furthermore, a complete vehicle following system, as presented in Chapter 4, can be formulated as a probability density function (pdf):

$$\begin{aligned} & P(\mathbf{x}_{F,k}, \mathbf{x}_{L,k} | \mathbf{U}_k, \mathbf{Z}_k) \\ & \propto \underbrace{P(\mathbf{z}_k^p | \mathbf{x}_{F,k}) P(\mathbf{x}_{F,k} | \mathbf{U}_k, \mathbf{Z}_{k-1}^p)}_{\text{localization of follower}} \times \underbrace{P(\mathbf{z}_k^r | \mathbf{x}_{F,k}, \mathbf{x}_{L,k}) P(\mathbf{x}_{L,k} | \mathbf{Z}_{k-1}^r)}_{\text{Tracking of lead vehicle w.r.t follower}} \quad (5.10) \end{aligned}$$

where $\mathbf{z}_k^p$ and $\mathbf{z}_k^r$ are the current observations made by the proprioceptive and exteroceptive sensors on board the follower vehicle. In vehicle following, the follower is able to predict its pose with the on board sensors. The follower can also

make observations of the leader and predict its relative pose. The advantage in representing the vehicle following system under a Bayesian framework is that the uncertainties in the system and sensor models are considered in the formulation.

With the Bayesian framework shown in Equation 5.10, the history of observations and control signals are recorded and the poses of both vehicles are estimated. Collating this information into an information vector, $\mathbf{I}_k$, yields the following equation,

$$\mathbf{I}_k = \{\mathbf{x}_{F,k}, \mathbf{x}_{L,k}, \mathbf{Z}_k, \mathbf{U}_{k-1}\} \quad (5.11)$$

The control (steering and speed) commands for the follower vehicle are the input to the vehicle following system. However, because of the vehicle's kinematic constraints, there are limits to the steering and heading commands which are achievable.

Definition 3: The admissible control signals at time k are the collection of all available control signals $\mathbf{A}_k$. Therefore,

$$\mathbf{A}_k = \{\mathbf{a}_0(\mathbf{I}_k), \mathbf{a}_1(\mathbf{I}_k), \dots, \mathbf{a}_{N-1}(\mathbf{I}_k)\} \quad (5.12)$$

where $\mathbf{a}_i$ is defined as a function of $\mathbf{I}_k$ at time k and N is the total number of admissible control signals.

From *definitions 2* and *3* and the constraints of Equations 5.3 to 5.6, it is possible to define the problem of vehicle following as finding an optimized control action from the admissible control signals (in *definition 3*). The condition in *definition 2* (Equation 5.7) must be fulfilled under the constraints defined in *definition 1*

(Equations 5.3 to 5.6). Hence, the problem of vehicle following can be defined as:

$$\mathbf{a}_{k+1}^* = \arg \min_{\mathbf{A}_k} \|\hat{\mathbf{x}}_{F,(k+1)} - \hat{\mathbf{x}}_{T,k}\| \quad (5.13)$$

where $\mathbf{a}_{k+1}^*$ is the optimized control action for the follower vehicle. Equation 5.13 can be viewed as an optimization problem. The aim is to search for an optimized control signal that is to be input to the controller of the follower vehicle, in the admissible command space. A metric (or objective function) can be formulated for this purpose.

5.3 Issues in Vehicle Following

For our vehicle following system, it has been shown that it is possible to estimate the poses of both the leader, and hence the pose of the virtual trailer, and follower vehicles using the on board sensors, as in Equation 5.10. However, there are two main issues that need to be considered.

Sensor Model Uncertainty

As well as the uncertainty in the relative pose estimation of the lead vehicle that must be considered by the follower when determining its next control action, the possible consequences of sensor uncertainty that might cause the vehicle following operation to fail must also be considered during implementation. All these uncertainties must be weighted by the probability that failures might actually occur.

The uncertainty analysis herein uses a first order covariance propagation technique [129]. The analysis will focus on the influence of the pose estimation of

the lead vehicle on the uncertainty of the virtual trailer pose estimate. The covariance propagation is based on the linearisation of non-linear equations describing dependencies between the variables in the measurement process by Taylor series expansion, and the computation of the proper Jacobians.

The pose of the lead vehicle can be formulated as a non-linear function $\mathbf{f}_L$:

$$\mathbf{x}_{L,k} = \mathbf{f}_L(\mathbf{x}_{F,k}, d_k, \phi_k, \phi_{L,k}) \quad (5.14)$$

where $\mathbf{x}_{F,k}$ denotes the pose of the follower vehicle at time k , d_k denotes the relative distance between the two vehicles at time k , ϕ_k denotes the relative bearing of the lead vehicle in relation to the follower and $\phi_{L,k}$ is the orientation of the lead vehicle.

It can be assumed that two independent sensors are used to observe the states of the follower and lead vehicles. The measurements for $\mathbf{x}_{F,k}$ and $(d_k, \phi_k, \phi_{L,k})$ can therefore be assumed to be statistically independent. Hence, the covariance matrix for $\mathbf{x}_{L,k}$ can be formulated as:

$$\Sigma_L = \mathbf{F}_L \Sigma_F \mathbf{F}_L^T + \mathbf{F}_M \Sigma_M \mathbf{F}_M^T \quad (5.15)$$

where $\mathbf{F}_L = \nabla_{\mathbf{x}_{F,k}} \mathbf{f}_L(\mathbf{x}_{F,k}, d_k, \phi_k, \phi_{L,k})$ is the Jacobian matrix of the nonlinear function $\mathbf{f}_L$ evaluated around the mean value $\hat{\mathbf{x}}_{F,k}$, $\mathbf{F}_M = \nabla_{(d,\phi,\phi_L)} \mathbf{f}_L(\mathbf{x}_F, d, \phi, \phi_L)$ is the Jacobian matrix of the nonlinear function $\mathbf{f}_L$ evaluated around the mean value $\hat{\mathbf{x}}_M$, $\mathbf{F}_L^T$ and $\mathbf{F}_M^T$ are the transpose matrices of $\mathbf{F}_L$ and $\mathbf{F}_M$ respectively, $\mathbf{x}_M = [d \ \phi \ \phi_L]^T$ with corresponding covariance matrix Σ_M and Σ_F is the covariance matrix of the follower pose, $\mathbf{x}_F$.

On the other hand, the pose of the virtual trailer is;

$$\mathbf{x}_T = \mathbf{f}_T(\mathbf{x}_L, L, \phi_T) \quad (5.16)$$

where $\mathbf{x}_T$ denotes the pose of the virtual trailer, ϕ_T is the trailer angle, L is the virtual trailer link length. Also, the system model for the virtual trailer angle,

$$\dot{\phi}_T(k) = \mathbf{f}(v_L(k), \phi_T(k), \dot{\phi}_T, L) \quad (5.17)$$

is a function of the leader's velocity, the virtual trailer angle, leader's orientation and the lengths of the virtual trailer, L . For an analysis of the sensor uncertainty, the lead vehicle is assumed to be moving along a straight line, in a short system sampling interval. Based on the above assumption, the virtual trailer angle can be assumed to be statistically independent of the pose of the lead vehicle. However, in real applications, the above assumption may not be valid and care has to be taken when dealing with the above.

With the above assumption, the pose uncertainty of the virtual trailer can again be computed by expanding the non-linear function f_T as a first order Taylor series around the mean, $\hat{\mathbf{x}}_T$. From the theory of uncertainty propagation [129], the covariance of $\mathbf{x}_T$, is:

$$\begin{aligned} \Sigma_T &= \nabla_{(\mathbf{x}_T)} \mathbf{f}_T(\mathbf{x}_L, \phi_T) \Sigma_L \nabla_{(\mathbf{x}_T)} \mathbf{f}_T(\mathbf{x}_L, \phi_T)^T + \nabla_{\phi_T} \mathbf{f}_T(\mathbf{x}_L, \phi_T) \nabla_{\phi_T}^T \\ &= \mathbf{F}_T \Sigma_L \mathbf{F}_T^T + \mathbf{F}_{\phi_T} \Sigma_{\phi_T} \mathbf{F}_{\phi_T}^T \end{aligned} \quad (5.18)$$

where $\mathbf{F}_T = \nabla_{\mathbf{x}_T} \mathbf{f}_T(\mathbf{x}_L, \phi_T)$ is the Jacobian matrix of the nonlinear function $\mathbf{f}_T$ evaluated around the mean value $\hat{\mathbf{x}}_T$, $\mathbf{F}_T^T$ is the transpose of $\mathbf{F}_T$, $\mathbf{F}_{\phi_T} = \nabla_{\phi_T} \mathbf{f}_T(\mathbf{x}_L, \phi_T)$ is the Jacobian matrix of the nonlinear function $\mathbf{F}_T$ evaluated around the mean value $\hat{\phi}_T$, $\mathbf{F}_{\phi_T}^T$ is the transposition of $\mathbf{F}_{\phi_T}$ and Σ_T is the covariance matrix of the $\mathbf{F}_T$.

The error covariance matrix for the virtual trailer is contributed by the sum of the errors in the follower vehicle pose estimate and the error in the sensor measurement.

Vehicle Constraints

From the on board computer, a command is sent to the follower vehicle so that it can maneuver towards the pose of the virtual trailer at time k . This is based on the estimations of the leader and virtual trailer poses relative to the follower vehicle, as shown in Figure 5.1. However, at any given time k , in practice, the pose of the follower vehicle may not allow it to attain the pose of the virtual trailer, due to a violation of the kinematic constraints.

ICR: Instantaneous Center of Rotation

Figure 5.1: Demonstration of vehicle kinematic constraints. a) At time k , the follower vehicle observes the lead vehicle and estimates the pose of the virtual trailer. A control command is generated based on the relative poses of both vehicles and the follower vehicle kinematics. b) At $k + 1$ the vehicle follower reaches the expected position. This may not match that of the virtual trailer at time k due to the kinematic constraints of the vehicle.

To minimize the effects of sensor uncertainty and vehicle kinematic constraints, the concept of relative information is used to determine the control actions for the follower vehicle. This is made possible by Equation 5.10. Two probabilistic distributions, representing the uncertainty of the poses of the vehicles, can be obtained in the recursive estimation process and then be used in the computation of relative information.

5.4 A Metric for Vehicle Following using Relative Information

As the relative information formulation will be used in this paper, a summary of the concept is included here.

Relative information (K-L distance) [130] is a metric that quantifies the “closeness” of two probability density functions. The K-L distance is defined as:

$$H(Q\|P) = \sum_{x_i} Q_{x_i} \cdot \log\left(\frac{Q_{x_i}}{P_{x_i}}\right) \quad (5.19)$$

where Q and P are the two distributions to be compared. If the distributions are similar, then the K-L distance will be close to zero. $[H(Q\|P) \geq 0$ with equality if and only if $Q = P]$.

The K-L distance is a measure of the “goodness of fit” or “closeness” of the two distributions. As compared to the information gain measure, where the change in entropy only quantifies how much of the probability distribution changes, the K-L distance represents a measure of how much the distribution has moved. For example, if P and Q are the same distributions, translated by different mean

values, the change in entropy (ie., the information gained), is zero whereas the K-L distance is not.

For the case of two Gaussian distributions [131],[132],

$$H(Q||P) = \frac{1}{2} \log \frac{|\Sigma_P|}{|\Sigma_Q|} + \frac{1}{2} Tr\{\Sigma_P^{-1}(\Sigma_Q - \Sigma_P)\} + \frac{1}{2}(\mu_Q - \mu_P)^T \Sigma_P^{-1}(\mu_Q - \mu_P) \quad (5.20)$$

where $Tr\{.\}$ denotes the trace of the matrix, (μ_P, Σ_P) and (μ_Q, Σ_Q) are the mean and covariance matrix pairs for Gaussian distributions P and Q respectively. The first term on the right hand side of Equation 5.20 represents the information gained, the second term represents mutual information and the last term is the Mahalanobis distance of the two pdfs.

Equation 5.20 has three important pieces of information embedded within the relative information.

- Mahalanobis Distance [133]: This is a popular matrix for data association in solving the target tracking problem [60]. The Mahalanobis distance has been used for validating whether a candidate measurement belongs to a target by checking if this candidate measurement falls within the σ bound of the target. In the context of the vehicle following application, the Mahalanobis distance can be used to quantify if the pose of a follower vehicle (at time $t+1$) falls within the σ bound of the estimated pose of the lead vehicle (obtained at time t).
- Information Gain: This theory has been applied extensively in the robotic exploration problem in the artificial intelligence (AI) community [102],[124],[125],[126],[127]. The theory has been used to strategically

control the robot to a desired position so as to maximally reduce the uncertainty about the environment. In the context of the vehicle following application, the information gained during vehicle following will evaluate its performance.

- **Mutual Information:** Mutual information is a measure of the information that two random variables share. It is a measure of the reduction of the entropy of one random variable given the other random variable and vice versa. Mutual information quantifies the distance between the joint distribution of the two random variables. If the two random variables are identical, the mutual information will have a magnitude of 1. In the context of vehicle following, unified mutual information is desirable. Intuitively, the aim of vehicle following is to command the follower vehicle to the pose of the lead vehicle (as at time t) at the next time step. Hence, an identical probabilistic distribution of the poses of the follower (at $t + 1$) and leader (at t) is desired.

Although the three metrics above provide certain confidence in the quality of vehicle following, individually, they are insufficient to quantify the performance of vehicle following. The Mahalanobis distance only reflects the difference between the mean values of the two distributions over the σ bound of one of the distributions, and only measures how far off the follower vehicle is within the uncertainty bound of the leader estimate. The uncertainty of the estimate in the pose of the follower is not considered in the Mahalanobis distance measure. On the other hand, both the information gain and mutual information matrices are

concerned about the uncertainties of the pose estimates, while the mean values of the poses are not considered. From Equation 5.20, if the covariance matrices of the two distributions to be compared are of the same magnitude, the K-L distance is exactly the same as the measure of the Mahalanobis distance. On the other hand, in the case of the two distributions having the same mean values, the K-L distance measures the information gained and the mutual information. Hence, the K-L distance formulation compares both the mean values and covariance matrices of the two distributions under consideration.

As stated in Equation 5.13, one solution is to strategically locate an optimized control action for the follower vehicle so as to minimize the path deviation between the vehicles. Hence, by combining the three metrics into one, which is the relative information, the optimized vehicle following system can then be achieved.

5.5 Generalized Information Theoretic Vehicle Following in a Finite Time Window

In general, the vehicle following algorithm can be formulated in a finite time horizon $[k, k+M]$, where k is the current time step and M is the finite time window size in the time horizon.

Suppose that the follower vehicle is controlled by a set of actions at each time step denoted by

$$\mathbf{U} = \{\mathbf{u}_{(k+i)}\}_{i=0,1,2,\dots,M} \quad (5.21)$$

where $\mathbf{u}_{(k+i)}$ is the vector of actions specifying the control command issued to the follower vehicle at time $k + i$. As the follower vehicle is modelled as a robotic car, the control action $u(k)$ is a vector of steering angle and speed controls.

At every time step, the follower vehicle makes observations about the lead vehicle. The observation is denoted as

$$\mathbf{Z} = \{\mathbf{z}_{k+i}\}_{i=0,1,2,\dots,M} \quad (5.22)$$

Let $\mathcal{N}_F$ and $\mathcal{N}_T$ denote the normal distribution functions representing the mean pose vectors and covariance matrices of the follower and virtual trailer respectively for all time steps $j \in [k, k + M]$ defined in the time horizon.

$$\mathcal{N}_T = \{\mathcal{N}_{T,j}\}_{j=k,k+1,\dots,k+M} \quad (5.23)$$

$$\mathcal{N}_F = \{\mathcal{N}_{F,j}\}_{j=k,k+1,\dots,K+M} \quad (5.24)$$

The terms $\mathcal{N}_{F,j}$ and $\mathcal{N}_{T,j}$ denote the distribution functions computed at time step j .

Let the admissible control command, at time step k , for the follower vehicle be denoted as

$$\mathbf{A}_k = \{\mathbf{a}_n\}_{n=0,1,2,\dots,N-1} \quad (5.25)$$

where $\mathbf{a}_n$ is the control command to be input to the follower and N is the total number of admissible commands available for the follower.

The information theoretic vehicle following problem can now be formulated as follows:

$$\mathbf{a}_{(k+1)}^* = \arg \min_{\mathbf{A}} \{C(H_j(\mathcal{N}_{F,j+1} \parallel \mathcal{N}_{T,j}))\}_{j=(k,k+1,\dots,k+M)} \quad (5.26)$$

subject to the constraints

$$\mathbf{g}(\mathbf{x}_k, \mathbf{x}_{(k+1)}, \dots, \mathbf{x}_{(k+N-1)}, \mathbf{u}_k, \mathbf{u}_{(k+1)}, \dots, \mathbf{u}_{(k+N-1)}) \leq \mathbf{g}_{th} \quad (5.27)$$

where $C(\cdot)$ is the composite scalar function representing the K-L distance, $H_j(\cdot)$ is the K-L distance computed at time j , $\mathbf{x}_k$ is the augmented state vector of both the virtual trailer and follower vehicle, $\mathbf{g}(\cdot)$ is the nonlinear constraint vector function and $\mathbf{g}_{th}$ is the constraint threshold vector. The constraints include the maximum allowable steering angle of the vehicle, safe following distance and the allowable following speed.

Equation 5.26 provides an unique decision-theoretic solution to the vehicle following problem. In general, a control command, such as velocity or steering angle, for the follower vehicle can be generated by analyzing the relative information between the two vehicles over a certain time horizon. However, the optimization of Equation 5.26 involves complex computations, which involve multiple iterations. The iteration scales in the order of $\mathcal{O}(N^{M+1})$. Hence, for implementation, the look-ahead time horizon for optimization is limited to one time step, which is also known as the greedy method [102].

5.5.1 Greedy Algorithm for Information Theoretic Vehicle Following

From Equation 5.10, under Gaussian distribution assumptions, the estimated poses of the follower and lead vehicles can be obtained using recursive filters such as the Extended Kalman Filter.

At time k , let $N_{F,k}(\hat{\mathbf{x}}_{F,k|k}, P_{F,k|k})$ and $N_{T,k}(\hat{\mathbf{x}}_{T,k|k}, P_{T,k|k})$ denote the normal distribution functions of the estimated poses of the follower and virtual trailer

respectively. $P_{F,k|k}$ and $P_{T,k|k}$ are the covariances for the follower and virtual trailer respectively. It is possible to predict the pdf of the follower vehicle, at time $k + 1$, based on the control command to be issued to the follower vehicle at time k .

Let $N_{F,k+1,\mathbf{a}_n}(\hat{\mathbf{x}}_{F,k+1|k}, P_{F,k+1|k})$ denote the predicted pdf of the follower vehicle at time $k + 1$ based on a certain admissible vehicle command, $\mathbf{a}_n$, as defined in Equation 5.25. The aim of the greedy algorithm is to determine a control command, $\mathbf{a}_{(k+1)}^*$, to be sent to the follower, that yields a minimum K-L distance between the two distributions, $N_{T,k}(\hat{\mathbf{x}}_{T,k|k}, P_{T,k|k})$ and $N_{F,k+1,\mathbf{a}_n}(\hat{\mathbf{x}}_{F,k+1|k}, P_{F,k+1|k})$. Assuming that both distributions are Gaussian, the K-L distance between them can be computed as:

$$H(P_{F,k+1|k} \| P_{T,k|k}) = \frac{1}{2} \log \frac{|P_{T,k|k}|}{|P_{F,k+1|k}|} + \frac{1}{2} \text{Tr}\{P_{T,k|k}^{-1}(P_{F,k+1|k} - P_{T,k|k})\} + \frac{1}{2}(\hat{\mathbf{x}}_{F,k+1|k} - \hat{\mathbf{x}}_{T,k|k})^T P_{T,k|k}^{-1}(\hat{\mathbf{x}}_{F,k+1|k} - \hat{\mathbf{x}}_{T,k|k}) \quad (5.28)$$

The optimization step for computing the control actions for the follower vehicle is:

$$\mathbf{a}_{(k+1)}^* = \arg \min_{\mathbf{A}_k} H(N_{F,k+1,\mathbf{a}_n}(\hat{\mathbf{x}}_{F,k+1|k}, P_{F,k+1|k}) \| N_{T,k}(\hat{\mathbf{x}}_{T,k|k}, P_{T,k|k})) \quad (5.29)$$

under the vehicle constraints as defined by Equations 5.3 to 5.6.

5.5.2 KL-Based Vehicle Following: Concept demonstration

For demonstration purposes, consider that vehicle following is carried out in 1-dimensional space. Both vehicles will be moving on a straight path and are under forward velocity control only. At time t , the follower makes an estimate of the

position of itself and the leader based on the onboard sensors. Suppose that

$N(\hat{X}_F, \Sigma_F)$ is the pdf for the follower, and,

$N(\hat{X}_L, \Sigma_L)$ is the pdf for the leader, at time t .

In order for the follower to trail the trajectory of the leader, a series of velocity commands, v_1 to v_n where n is the total number of unique velocity inputs available to the follower, can be selected to achieve this goal. However, there will be a command that will result in a minimum path deviation between these two vehicles. The optimisation involves the computation of the KL metric.

For each $v_i|_{i=1 \text{ to } n}$, the corresponding KL value is computed as follows:

$$H_i = \frac{1}{2} \left[\log \frac{\Sigma_L}{\Sigma_{F_i}} + \frac{(\Sigma_{F_i} - \Sigma_L)}{\Sigma_L} + \frac{(\hat{X}_{F_i} - \hat{X}_L)^2}{\Sigma_L} \right] \quad (5.30)$$

where $N(\hat{X}_{F_i}, \Sigma_{F_i})$ is the estimated pdf of the follower if the velocity command v_i is the input to the follower. Assume that $\Sigma_{F_i} = k_i \Sigma_L$, where $k_i|_{i=1 \text{ to } n}$ are positive real numbers. Thus, equation 5.30 can be further simplified to

$$\begin{aligned} H_i &= \frac{1}{2} \left[\log \frac{1}{k_i} + (k_i - 1) + \frac{(\hat{X}_{F_i} - \hat{X}_L)^2}{\Sigma_L} \right] \\ &= \frac{1}{2} \left[(k_i - 1) - \log k_i + \frac{(\hat{X}_{F_i} - \hat{X}_L)^2}{\Sigma_L} \right] \end{aligned} \quad (5.31)$$

Equation 5.31 can be minimised if the following two terms are minimised:

- $(\hat{X}_{F_i} - \hat{X}_L)^2$
- $(k_i - 1) - \log k_i$

It is trivial that $(\hat{X}_{F_i} - \hat{X}_L)^2$ is minimal if the estimated position of the follower at next time step, $t + 1$, is closest to the estimate of the position of the leader (at time t).

Figure 5.2 shows a plot of the 2nd term, $(k_i - 1) - \log k_i$. From the figure, it can be observed that minimal value occurred when $k_i = 1$. This result, together with the above observations, match the basic concept of the KL metric for vehicle following. Ideal vehicle following can be achieved if a control input is selected such that the pdf of the follower (at next time step, $t + 1$) is identical to that the leader (at time step t). Furthermore, the pdfs of both vehicles are not a function of the KL metric. In fact, the KL metric has anticipated and minimised the position estimate of the follower. That is, the optimise choice of input command to the follower will place the follower at a strategic position in an attempt to minimise the path deviation between both vehicles.

Figure 5.2: Plot of $(K_i + 1) - \log(K_i)$. In order to achieve value of KL, $(K_i + 1) - \log(K_i)$ has to be minimised. Observed that minimal value can be achieved at $K_i = 1$.

5.6 Implementation and Experimental Results

Figure 5.3 illustrates a simplified block diagram for the vehicle following function discussed here. There are four major modules and each of the functionalities can be described as follows:

Figure 5.3: Control block diagram for the proposed vehicle following system.

- **Perception Module (PM):** The follower vehicle is assumed to have on board sensors. In our implementation, the odometry data and the information from a gyroscope were used to localize the follower vehicle. The pose of the lead vehicle is detected using a laser scanner, camera or fusion of both.
- **Pose Estimation Module (PEM):** With the observation received from **PM**,

both the poses of the leader and follower can be obtained using Equation 5.10.

- **Virtual Trailer Module (VTM):** This module receives the estimated poses of the leader and follower from **PEM** and generates the estimated pose of the virtual trailer.
- **K-L Module (KLM):** The greedy method presented in 5.5.1 is implemented to determine the control actions for the vehicle following function. A series of possible steering commands are used as input to compute the predicted poses of the follower vehicle and virtual trailer at the next time step. The K-L distances are then computed and the control action resulting from the minimum K-L distance is then selected.

Figure 5.3 demonstrates the inter-relationship between the perception system and the vehicle actuation system (vehicle controller). In that, the perception system provides the location of the lead vehicle and the vehicle actuation system commands the follower to pursue the lead vehicle in response to the information obtained from the perception system.

Sensor data acquisition on moving platforms has always being a challenging research issue. Sensor data can be noisy and has to be compensated for. The dynamics of the moving platform and uncertainties in the sensory data have to be considered in the design as these two are coupled. For example, in vehicle following application, false perception information obtained may lead to wrong commands being computed and issued to the vehicle actuation system. As a

result, the follower vehicle may be steered into a wrong path thus causing failure in the subsequent detection of the lead vehicle as well as the stability issue in the system.

The basis of the proposed complete vehicle following control system was formulated based on the above concepts. The pose estimation module (PEM) models the states of both the lead and follower vehicles based on the odometry data (for localisation of follower) from the vehicle and sensory information (for tracking of the lead vehicle) obtained from the perception module (PM). With the pose estimates of the vehicles, the pose of the virtual trailer can then be formulated by Virtual trailer module (VTM). From the pdfs of the pose estimates of the vehicles and virtual trailer, and the kinematic models of the vehicles, optimised control actions (vehicle speed and steering control) are then computed by the K-L module (KLM). Hence, the control actions, to be fed into the vehicle controller, are functions of the sensor uncertainties (equation 5.28). Furthermore, the uncertainties in the pose estimates of the vehicles are also taken into consideration. The KL formulation actually anticipates the sensor uncertainties by optimizing the control actions for the follower. Hence, a closely coupled controller can be achieved.

The entire algorithm is summarised in Table 5.1.

The approach in this thesis is to formulate a model for vehicle following system and verify the basic theory through experiments and simulations. Some assumptions have been made in the simulation runs. That is, the rate of data acquisition and pose estimation process are fast (10 Hz). Also, as discussed in

Table 5.1: Algorithm for vehicle following function

Steps	Actions	Formulation
1	Current Pose estimates	$\hat{\mathbf{x}}_{F,k k}, \hat{\mathbf{x}}_{T,k k}$
2	Predict follower pose from achievable actions	$\hat{\mathbf{x}}_{F,k+1 k}$
3	Compute KL distance	$H_j \forall j = [1, M]$
4	Choose input and move follower	$\mathbf{a}^*(k+1)$
5	Observe pose of leader	$\mathbf{z}(k+1)$
6	Estimate new poses	$\hat{\mathbf{x}}_{F,k+1 k+1}, \hat{\mathbf{x}}_{T,k+1 k+1}$
7	Loop steps 1 to 6	-

section 4.5, the perception data is conditionally independent of the pose of the follower vehicle, as the information on the pose of the follower vehicle has already been obtained before the pose of the lead vehicle was estimated.

5.6.1 Experimental Setup

A popular simulator, the USARSim (Urban Search and Rescue Simulator), developed by researchers in CMU [134], was used as the platform for validation of the vehicle following algorithms. USARSim is a robot simulator based on the industrial game engine Unreal Engine 2004 (www.unrealtournament.com). The Unreal Engine has been deployed for the development of networked multi-player 3D games, and has solved many of the issues related to modelling, animation and rendering of the virtual environment. Furthermore, the dynamics of rigid bodies (the vehicles in this case) is transparently handled by the Karma physical engine [135]. In order to ensure that the simulation results were comparable to the actual system, the standard deviation settings of all the sensors used were set as close to the real values as possible. The standard deviations (as obtained

from the respective data sheets ²), of the gyroscope, laser range and laser bearing measurements were set to 1.28° , 5cm and 0.5° respectively.

An open test area and two vehicles were built for the simulation in this study, as shown in Figure 5.4. During the experiments, the lead vehicle was controlled by a standalone program and the actual position (ground truth) was recorded. The follower vehicle was controlled by the program with the KL algorithm.

Figure 5.4: Simulation environment for vehicle following using the Unreal Game Engine. The follower vehicle is equipped with a 2D laser scanner. The laser scanners are attached to the bumper of the vehicle.

5.6.2 Performance Analysis

To test the feasibility of the new vehicle following theory, two test scenarios are considered. In the first test, the lead vehicle was commanded to maneuver along a straight path followed by left and right turns, simulating vehicle moving on a road junction and a roundabout. The next test involved following a leader on an 'S-curve'. The purpose was to test if the proposed vehicle following method could

²In this simulation, the KVH DSP-5000 fiber Optic Gyro from KVH Industrial, Inc (www.kvh.com) and the SICK LMS290 laser scanner (www.sickusa.com) are simulated.

cope with sharp curve turnings in both directions, i.e, sharp left and right turns. The trajectory represents constraints found in typical urban road environments and attempts to challenge the controller response.

Test Scenario 1

The purpose of this experiment was to test the flexibility of the vehicle following algorithm. The lead vehicle was commanded to manoeuvre along a straight path for about 150 time steps, make a right turn for another 150 time steps and finally to make a left turn for about 300 time steps. Figures 5.5 to 5.7 show the results of the run.

Figure 5.5(a) shows the error between the two trajectories to be rather small despite the trajectory following error as estimated by the filter shown in Figure 5.5(b). The small following error shown in Figure 5.5 has suggested that the information theoretic based vehicle is robust enough to cope with various kinds of maneuvers.

From Figure 5.6, it can be seen that the orientation error, for time step up to 150, when following the leader's trajectory, is almost close to zero, as is to be expected for any vehicle following system moving along a straight path. At time step 150, the lead vehicle began to make a curved maneuver. It can be observed from Figure 5.6 that the orientation of the follower vehicle still maintained the straight moving angle for a period of time despite the curved maneuver of the leader. This is the effect of the virtual trailer link model described in Chapter 3. This is an important observation as it is undesirable for the follower vehicle

Figure 5.5: S-Path Trajectory. (a) The ground truth of vehicle trajectories. (b) Trajectories estimated by the filter.

to start making a turn when the lead vehicle has begun the curved maneuver. The result of the virtual trailer link model yields a better following performance and hence prevents the follower vehicle from hitting obstacles such as road curbs. There are, however, orientation differences when the vehicles are maneuvering along the curved path. This error is also to be expected as the two vehicles should not be aligned with the same orientation when they are travelling along a curve.

Figure 5.7 shows the performance of the information theoretic vehicle following algorithm. The inter-vehicle distance remains relatively constant throughout the run. Initially, from time step 0 to 50, the inter-vehicle distance increases over time. This is mainly caused by the initial relative position of the two vehicles. After the initialization of the vehicle following algorithm, the inter-vehicle separation increases over time until it approaches a separation distance equivalent to the total length of the virtual links (4m in this case). As the separation distance increases, the K-L distance increases. However, an

Figure 5.6: Orientations of the lead vehicle, the virtual trailer and the follower vehicle as estimated by the filter.

acceleration period exists for the follower vehicle from the start of the algorithm; hence, the K-L distance increases with respect to the inter-vehicle distance. Note that the orientation difference between the lead and follower vehicles increases from approximately 0° to 20° from time step 150 to 200. This is the virtual trailer link effect. As the lead vehicle begins to make the curved maneuver, the follower vehicle continues to move along a straight path, hence, the angular difference increases.

Test Scenario 2

As shown in Figure 5.8, the lead vehicle is commanded to manoeuvre along a straight path for a short period of time, and then to make a series of left and right turns.

Figure 5.9 shows the KL distances computed based on the estimated poses and Equation 5.28 when the lead vehicle is moving straight ahead (top figure), making

Figure 5.7: Comparison of the KL distances with respect to the inter-vehicle distance and the orientation difference.

Figure 5.8: The ground truth of both the lead and follower vehicle trajectories. Zoomed views (a) and (b) show the detailed trajectories of the vehicles at various time steps.

a left turn (middle figure) and making a right turn (bottom figure). For this experiment, the total number of admissible steering angles, $N = 40$, with an angular resolution of 1 degree. As observed in Figure 5.9, the minimum K-L distance can be obtained, and hence the optimized steering angle can be chosen.

Figure 5.9: Plot of K-L distance when the vehicles are moving in a straight line (top), turn left (middle), turn right (bottom).

Figure 5.10 shows the path deviation between the vehicles and the corresponding KL distances. The small path deviation shown in Figure 5.10 has suggested that the information theoretic based vehicle following is robust enough to cope with various kinds of maneuvers. Furthermore, from Figure 5.10, it can be seen that the trend in the path deviation plot resembles the trend in the K-L distance plot. For instance, this can be observed from the figure between time steps 400 and 800 (from Figure 5.10 and the location marked 'A' in Figure 5.8). From time steps 400 to 580 (zoomed view (a) in Figure 5.8), the leader is making a gradual left turn. As the turning rate is gradual, the orientation difference between the two vehicles will eventually decrease while the inter-vehicle separation distance between the

two vehicles will remain constant. Therefore, the KL distance decreases. From time steps 580 to 630 (zoomed view (a) in Figure 5.8), the lead vehicle is making a sharp right turn. As the turn becomes sharp, the orientation difference between the vehicles will be large and hence the K-L distance increases. From time step 630 to 800 (zoomed view (b) in Figure 5.8), the lead vehicle is making a gradual left turn. Similarly, the K-L distance decreases gradually. The above scenarios have verified that the K-L distance can be used as a metric for evaluating the performance of the vehicle following function.

Figure 5.10: Path deviation and corresponding KL distance.

To further validate the performance of the K-L metric in steering control of the following, its results are compared with the pursuit algorithm [97]. Pure pursuit algorithm has been widely used as a steering controller for autonomous vehicles [15], [16], [17]. This control strategy geometrically computes the curvature that will drive the follower vehicle from its current position to a goal position, which is the current position of the leader in this case.

Figure 5.11: Plot of optimized steering angles computed from the K-L distance metric and a pure pursuit algorithm.

Figure 5.11 shows the steering angles computed from the K-L metric using a pure pursuit algorithm [97]. It can be observed from the figure that the pure pursuit algorithm has computed the steering angles to be greater than 20 degrees when the lead vehicle is making sharp turns. These angles have exceeded the maximum allowable angle for a typical vehicle. Also, the transition of the steering angles from the left to the right turn is large for a pure pursuit algorithm. This may cause discomfort to the driver. On the other hand, the steering angles generated by the K-L metric are within the allowable steering angles of the vehicle. The transition of the steering angles generated by the K-L metric is gradual.

To further evaluate the effectiveness of the K-L metric for vehicle following, another experiment was conducted. In this experiment, the follower vehicle is commanded to follow the leader, whose trajectory is shown in Figure 5.8. However, for this experiment, the driving commands for the follower is computed

using the Mahalanobis Distance (MD in short) only. The formulation of the Mahalanobis distance is similar to Equation 5.20, by setting the first two terms in the equation to zero. Figure 5.12 shows the trajectories of the leader and follower, for the case of the follower using the MD and KL control methods. Although the follower vehicle that is controlled using the MD metric is able to closely follow the leader, the path deviation is larger compared to the trajectory of the vehicle, when controlled by the KL metric. The path deviations of both experiments are shown in Figure 5.13. The results have further reinforced the advantage of the KL metric for vehicle following. As discussed in section 5.4, the Mahalanobis Distance alone only quantifies if the pose of the follower vehicle falls within the σ bound of the estimated pose of the leader vehicle. On the other hand, the KL metric takes into consideration not only the Mahalanobis distance of the two vehicles, but in addition, the mutual information and the information gained in the process of vehicle following.

From figures 5.10 and 5.13, the maximum path deviation achieved by the KL metric is less than 35 *cm*. As discussed in Chapter 4, the path based vehicle following algorithm implemented by Stefan [52] achieved a maximum path deviation of approximately 70 *cm*. The system implemented by Lu [54] achieved a maximum path deviation of 35 *cm*. The vehicle following system by Wang [65] achieved a maximum path deviation of 50 *cm*. Thus, the proposed KL metric can perform on par with the system by Lu [54].

Overall, the K-L algorithm is able to optimize the control actions for the follower vehicle to achieve close pursuit of the lead vehicle while, at the same time,

Figure 5.12: Zoomed view (section marked with 'A' in Figure 5.8) of the trajectories of the leader and followers (commanded with the KL and MD metrics).

Figure 5.13: Path deviations of vehicle following using the KL and MD metrics.

providing a smooth input to the follower vehicle.

5.7 Summary

Autonomous vehicle following capabilities have been achieved using an information theoretic framework. It optimizes the control inputs for the follower vehicle so as to minimize the pose error between the follower and lead vehicles. Both the follower vehicle's constraints and the uncertainties in the estimation of the poses of both vehicles have been considered within the framework. Under this framework, the relative information or K-L distance has been used as a metric to evaluate a sequence of control actions, which are used as inputs to the follower vehicle. The method has been simulated and the results have shown that the information theoretic vehicle following system is robust to estimation errors and the safe separation of the vehicles has been considered. The system is robust as uncertainties in the estimation of the poses of both vehicles are considered and taken into account as part of the vehicle following function. The inter-vehicle distance is maintained as desired and thus it is possible to conclude that the follower vehicle is in a position to stop safely in case of emergencies.

CHAPTER 6

Conclusion

6.1 Introduction

This thesis has formulated a workable framework for vehicle following. A vehicle following system has been designed based on an analysis of both the research and application challenges. This thesis targeted the application of vehicle following in urban environments, particularly in a stop-and-go traffic scenario.

This chapter reviews the motivations and objectives of this thesis, the challenges addressed, the main contributions and recommendations for future research.

6.2 Challenges in vehicle following

Traffic congestion is a global issue today. It has affected mankind both economically and environmentally. Congestions on both motorways and in city areas is becoming a serious concern as there is an increasing trend of the number of motorcars on the roads. There have thus far been many initiatives that focus on solving these issues. Fruitful results have been achieved; however, they have only been successful in solving motorway congestion. Unfortunately, these solutions

cannot be applied directly to city congestion problems, as the implementation requirements and the complex city environment have made the design of vehicle following for city areas technically challenging. The road designs in city areas are significantly different from that of motorways. Motorways tend to have a higher radius of curvature compared to the roads in city areas. It is also common to have roads with sharp turns in the city area. Furthermore, there are many other road users in city areas, including pedestrians and cyclists, who share the road with motorcars, that make the design of vehicle following systems difficult. Also, the sensors used in motorway vehicle following may not be useful for city area vehicle following systems, as there are many stationary objects, such as parked vehicles, in city areas.

Safety is another point that must be considered when designing a vehicle following system. The designed system must be safe for implementation. No collisions with other vehicles or road users is to be tolerated. Furthermore, there should be no cutting of corners and hitting of road curbs when a follower trails behind a leader.

The vehicle following function deals with two contradictory functional requirements. For safety reasons, a follower must maintain a far distance away from the leader. On the other hand, to ensure efficient vehicle following, a follower must trail as close as possible to the leader. Hence, a good and effective vehicle following system must be able to strike a balance in achieving these two contradictory requirements.

The objectives of this thesis have been identified in providing a scientific solution

to the above challenges.

6.3 Thesis Achievements

The research objectives of this thesis have been accomplished. The challenges of a vehicle following system have been investigated. Trajectory following of the lead vehicle by the follower vehicle is the main focus of this research. Several techniques and methodologies of vehicle following systems, operating at both high and low speeds, have been reviewed. However, the majority of these approaches concentrate on the control issues for the follower vehicles and place little effort on overall system modelling. This thesis has shown that the overall system modelling for vehicle following is an important factor in ensuring a good performance. This is particularly important with the limitations placed on the perception systems and when the system is to be deployed in the urban environment.

A Bayesian formulation for the vehicle following system has been mathematically derived. Further, a virtual trailer link model for safe and comfortable vehicle following has also been formulated. By incorporating the virtual trailer link model into the Bayesian formulation for vehicle following, minimal path deviation between the lead and led vehicles has been achieved. For a complete vehicle following system, an information theoretic framework has been introduced. The information theoretic framework has proven to be effective in achieving optimum control of the follower in its pursuit of the leader. Both simulations and experiments have been carried to further support the theoretical portion of the

thesis.

6.4 Main Contributions

Virtual Trailer Link Model. The performance of the commercially available trailer systems have been evaluated. The off-hooked kinematic model has been identified as having the potential to closely follow the trajectory traced by the lead vehicle. By configuring the length of the virtual link and the virtual trailer to be of the same size, the virtual off-hooked kinematic configuration can be implemented in the vehicle following system. A kinematic model for the virtual trailer system has been formulated. The trajectory following capability of the virtual trailer model had been compared to a closely related concept found in the literature [52], [54], [65]. In addition, simulation tests and experiments have been carried out to further evaluate and understand the problem faced by vehicle following in a real environment. The proposed virtual off-hooked kinematic model has been shown to result in low tracking errors as compared to the direct-hooked kinematic model.

Bayesian Formulation for Vehicle Following. Autonomous vehicle following can only be achieved if the poses of both the follower and lead vehicles are continuously estimated. This can be achieved by using a Bayesian estimation technique together with a virtual trailer link model. The advantage of such a model is that the follower vehicle will trail a virtual trailer, modelled as an attachment to the lead vehicle, instead of the lead

vehicle itself, so that a safe spacing between the two vehicles is guaranteed. A generic mathematical formulation for the vehicle following system has been derived based on Bayes' theory. The advantage of this formulation is that the system uncertainties have been modelled in a probabilistic manner. The key to a tractable solution to this vehicle following problem is the justifiable assumption that the pose of the follower vehicle is statistically independent of that of the leader, when conditioned on the history of the follower vehicle's inputs and the sensor observations made by the follower vehicle.

The pose of the follower vehicle was estimated using a recursive estimator. In a separate estimator, the poses of the virtual trailer and the lead vehicle were augmented in the tracking process of the lead vehicle. The aim was to command the follower vehicle to trail the estimated pose of the virtual trailer link model, computed with an on board sensor mounted on the follower vehicle. To compensate for the uncertainties in the sensor data, a dynamic measurement noise model was implemented. Firstly, the simulation results were presented, for the case of a follower vehicle trailing a lead vehicle, using both straight and curved paths, with simulated, noise corrupted range measurements of the lead vehicle recorded from the follower vehicle. Real experiments were also carried out in a car park. The performance of the algorithm was analyzed, using both static and dynamic measurement noise models in the estimation stages of the filters.

Formulation for vehicle detection. The Hough Transform representation was

proposed for vehicle detection. A laser scanner was assumed to be the sensor used for vehicle tracking. In addition, the back of the lead vehicle was modelled as a line. The Hough Transform representation, for line extraction, has been successfully implemented in many robotic applications. However, there are issues that need to be addressed before the algorithm can be implemented. For example, choosing an appropriate grid size in the Hough Space is a problem. Hence, a recursive filter (in this case the EKF) is proposed to address the above issues while maintaining the use of the Hough Transform parameters. To identify if a laser range measurement is associated with a line, the innovation and the innovation covariance obtained in the EKF have been used to set up a validation gate. With this validation gate, any measurements that fall outside it will be classified as outliers.

Information Theoretic Vehicle Following. Vehicle following can be achieved by minimizing the relative information (Kullback-Leibler or K-L distance) between the estimated poses of the lead and follower vehicles. In order to achieve successful vehicle following, a Bayesian formulation for the system has been derived, and two probabilistic distributions, one for each vehicle's pose, can be obtained. Based on the assumption that the two pose distributions are Gaussian functions, the K-L distance of the vehicle following system can be computed from these two computed distributions. With a series of achievable actions, such as steering and velocity commands, for the follower vehicle at each pose prediction step, and by minimizing the

K-L distance, an optimized action for the follower vehicle can be obtained. The information theoretic vehicle following algorithm has been tested under a simulated environment by analyzing the performance of the follower vehicle when the lead vehicle undergoes various kinds of maneuvers. The simulated experimental results validate that the follower is able to trail the trajectories of the lead vehicle satisfactorily while maintaining a safe following distance at the same time.

6.5 Directions for future research

The first significant result of this thesis is the new vehicle following modelling method using a virtual trailer link model derived in Chapter 3. The modelling involves the selection of optimized virtual trailer link parameters for vehicle following. It has been shown that one virtual trailer link model is sufficient for safe and reliable slow speed vehicle following in congested traffic. It would be interesting to extend the virtual trailer link model to motorway vehicle following and platooning. A platoon of vehicles can be modelled as a series of virtual trailers. Both reliability and cost efficiency can be achieved with this new model. It is reliable as the virtual trailer link model has proven to be intrinsically safe if the link parameters are properly designed. It is cost effective as the system can be independent of any road infrastructure, such as road markers. However, as the number of vehicles in the platoon increases, the string stability of the platoon becomes an issue. The vehicle following model must guarantee that the propagation of the path deviation, as obtained from the first follower, to the subsequent follower, is minimized, if not completely eliminated.

Although the off-hooked kinematic configuration for vehicle following, as presented in this thesis, has been proven to be feasible, there are some research issues that need to be addressed. For example, the maximum number of virtual trailer links allowed for a reliable vehicle following that is also safe for implementation needs to be verified. This is a challenging research issue. As the separation between the two vehicle increases, the level of difficulty in the perception of the lead vehicle from the follower will increase accordingly. This is because of the limitation of the sensors. In order to improve the reliability of the perception system, fusion of the information from multiple sensors is recommended. For example, a line laser scanner provides a direct range measurement of the target vehicle, and a camera can provide rich color information about the scene. By fusing these two kinds of information using synchronized data acquisition, the probability of accurate detection and classification of the leader may be increased. Fusion of information with other sensors such as radars and infra-red cameras is yet another option.

As presented in Chapter 5, the commands for the follower vehicle in pursuing the leader can be achieved through the optimization of the KL metric. This concept has been verified using a greedy optimization method. Future improvements would be possible if the optimization process is extended to 2- or even 3-steps in the time horizon to search for the best control command for the follower. This would require the minimisation of the right hand side of Equation 5.26, when M is increased. This is possible because of the recent advancements in computing technology.

As with any autonomous system designed to work in practical outdoor scenarios, a realistic vehicle following algorithm must be robust to the random and difficult to model behaviour of other road participants such as interfering vehicles, cyclists and pedestrians. With this increased complexity, common and readily available a priori information such as a digital map may be exploited and integrated into the proposed vehicle following system to improve performance. With such an aided-following technique, the follower vehicle may enhance its prediction models and derive improved following trajectories, using the expected path to be traverse by the lead vehicle. An interactive-multi-modal (IMM) [115] approach may be adopted to switch between prediction models (constant velocity, constant turn etc.) based on the digital map data. Furthermore, environmental cues such as road junctions may be incorporated into the algorithm to predict abrupt velocity changes of the lead vehicle and improve overall vehicle following capabilities. Active exploitation of the surrounding environment also presents promise as an enhancement to the proposed method. The incorporation of active localisation and tracking methods (for example, SLAM [120] formulation) into the vehicle following algorithm would also be expected to improve the robustness of the vehicle following system.

APPENDIX A

Contextual Definitions

This thesis deals with the implementation of the vehicle following function from access roads to large conglomerations where stop-and-go conditions occur. The tight curves in villages and trajectories through smaller streets are not considered in this thesis. This section provides an overview of road types that vehicles with the presented vehicle following system should traverse. The notion of road curvature is presented and the computation of the path deviation is presented for analytical purposes.

A.0.1 Straight and Circular roads

As the name implies, a straight road is a path that has infinite curvature. Normally, no steering of the vehicle is required when it is travelling on a straight road.

A circular road can also be termed a roundabout. It is a special road junction at which traffic enters a one-way stream around a central island (Figure [A.1](#)).

Figure A.1: Typical roundabout (or circle). (source: wikipedia)

A.0.2 Clothoid - the transition path

A clothoid path is also known as a transition path (Figure A.2).

Figure A.2: Typical transition paths on motorways. (source: wikipedia)

Transition paths are commonly used to join straight and circular roads. They provide a comfortable transition between the two elements with different curvatures. A frequently used transition curve is called a clothoid or Euler spiral [136],[137]. Due to the characteristics of vehicle steering geometry, a transient

state (or distance) exists where it is possible for the front (steered) wheels of a vehicle travelling at slow speed to follow a circular path while the steering angle is changed from straight ahead to the maximum angle required to describe the turn. This is largely the reason why no transition is required for intersection turns and most curves with a design speed of below 60 km/h. The clothoid spiral was adopted for road transition design because of the simplifications that were assumed to exist, as alternative forms could not be conveniently incorporated into computer programs that came to be used for design calculations [136]. The basic properties of the clothoid can be found in [136],[137]. A clothoid path is a curve that is defined by the property that curvature (which is $(1/R_c)$) varies uniformly along the length of the spiral (Figure A.3). Therefore, the curvature at the end of the spiral is proportional to the length of the clothoid:

$$\frac{1}{R_c} \propto L_c \quad \text{or} \quad R_c L_c = \text{constant} \quad (\text{A.1})$$

where L_c is the length of the clothoid and R_c is the radius of the clothoid at the end of the transition.

Figure A.3: Transition curves with different R_c and L_c settings.

A.0.3 Computation of Path Deviation

The aim of the vehicle following function is to place the follower onto the trajectory of the leader. Hence, a method of quantifying the performance of the vehicle following system is required. For a vehicle following a circular path, the difference between the radii of the follower's path and that of the leader's path is a natural choice for computing the path deviation between the two vehicles. However, for other types of maneuvers, it is difficult to have a good performance index for path deviation. Lateral path deviation between two trajectories is commonly used as quantitative data for a performance measure of vehicle following systems. However, this computed path deviation only reflects the absolute extent of deviation between the two trajectories. A new method of computing the path deviation between two vehicles is thus proposed here. Figure A.4 shows a path traced by the lead vehicle (solid line) and the tracked path traced out by the follower vehicle (dotted line), enlarged for the purposes of illustration. Assume that the sample time is small enough such that the consecutively acquired positions of the lead vehicle can be considered to be a straight line (piecewise linear path). Two consecutive positions of the lead vehicle and the current position of the follower vehicle are used to form a virtual triangle. The area of this virtual triangle can be computed and used to represent the tracking error of the follower vehicle in relation to the lead vehicle. From Figure

A.4,

$$\begin{aligned} \text{Area of } \triangle O_{i-1}O_iT_i &= \frac{1}{2} \begin{vmatrix} T_{i,x} & O_{i-1,x} & O_{i,x} \\ T_{i,y} & O_{i-1,y} & O_{i,y} \\ 1 & 1 & 1 \end{vmatrix} \\ &= \frac{1}{2}bh \end{aligned} \quad (\text{A.2})$$

where T_i and O_i are the position vectors of the follower and leader at time i respectively as shown in Figure A.4. b is the length of the base of $\triangle O_{i-1}O_iT_i$ and h is the height of the $\triangle O_{i-1}O_iT_i$ with reference to the base, b .

Therefore,

$$h = \frac{2}{b} \times \text{Area of } \triangle O_{i-1}O_iT_i \quad (\text{A.3})$$

From Figure A.4, h is the shortest distance from the current position of the follower, at point T_i , to a line formed by joining the current and previous positions, O_i and O_{i-1} respectively, of the lead vehicle. Hence, the magnitude of the variable h is the path deviation between the two vehicles. Moreover, if $h = 0$, this implies that the three vertices are co-linear. Then, the follower vehicle is said to be aligned to the path of the leader. The sign in h indicates if the follower vehicle is on the left-hand or right-hand side of the lead vehicle. This information can be useful for steering control of the follower in its pursuit of the leader. Hence, the proposed method of computing the path deviation, for the vehicle following function, provides both the lateral path deviation and the positional information of the follower with respect to the leader.

O_i : position of leader at time i
 T_i : position of follower at time i

Figure A.4: Computation of path errors

APPENDIX B

Publications

B.1 Published Articles

1. **T. C. Ng**, M. D. Adams and J. I. Guzmán, “A Relative Information Metric for Vehicle Following Systems“ *IROS 2008*, pp 1038-1043, 2008.
2. **T. C. Ng**, M. D. Adams and J. I. Guzmán, “Bayesian Estimation of Follower and Leader Vehicle Poses with a Virtual Trailer Link Model“ *Int. Journal of Robotics Research*, Vol 27, No.1 pp 91-106, 2008
3. **T. C. Ng**, M. D. Adams and J. I. Guzmán, “An Information Theoretic Vehicle Following System“ *IEEE Intelligent Vehicles Symposium IV07*, pp 1184-1189, 2007
4. **T. C. Ng**, M.D. Adams and J. I. Guzmán, “Autonomous Vehicle-Following Systems: A Virtual Trailer Link Model”, *IROS 05*, pp 2968-2973, 2005.
5. **T. C. Ng**, J. I. Guzmán, J. Shen, Z. Gong, H. Wang and C. Chen, “Vehicle following with obstacle avoidance capabilities in natural environments“ *Proceedings. ICRA 04*. Vol 5, pp. 4283-4288, 2004
6. **T. C. Ng**, J. I. Guzmán and J. C. Tan, “Development of a 3D LADAR system for autonomous navigation“ *IEEE Conference on Robotics, Automation and*

Mechatronics, Vol 2, pp 792 - 797, 2004

7. C. Chen, H. Wang, T. C. Ng, J. I. Guzmán, J. Shen and C. W. Chan, “Target-tracking and path planning for vehicle following in jungle environment“ IEEE Control, Automation, Robotics and Computer Vision, Vol 1, pp 455 - 460 , 2004
8. J. Shen, J. I. Guzmán, T. C. Ng and B. S. Chew, “A collaborative-shared control system with safe obstacle avoidance capability“ IEEE Conference on Robotics, Automation and Mechatronics, Vol 1, pp 119 - 123, 2004

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