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2009

Wang, Y. (2009). Robust repetitive control with application to PWM DC/AC inverters.
Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/19008>

<https://doi.org/10.32657/10356/19008>

Robust Repetitive Control with Application to PWM DC/AC Inverters

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A thesis submitted to the Nanyang Technological University

in fulfillment of the requirements for the degree of

Doctor of Philosophy

2009

Acknowledgments

My foremost and sincere appreciation goes to my research supervisor, Dr. Wang Danwei. I would never finish this work without his excellent guidance, encouragement, and constant support. His extensive knowledge, advice, insights, and suggestions not only carried me on through difficult times, but also helped me to shape my research skills. It is a true honor to work together with him.

The research and study in Nanyang Technological University was enjoyable and rewarding. I would like to take this opportunity to thank my colleagues: Dr. Zhang Bin, Dr. Ye Yongqiang and Dr. Zhou Keliang. It was a great and valuable experience to work with you as a team. There were many inspirational and fruitful discussions and cooperations among us, from which this work has been greatly benefitted. Those days we discussed on the way to dinner and we did experiments over ten hours together will not be forgettable.

Many thanks to my friends in Intelligent Robotics Laboratory: Mr. Liu Shuyong, Mr. Tang Yu, Dr. Jia Zhen, Dr. Wang Xiao, Dr. Sun Yowchen, Mrs. Zhu Xiaolin. Your friendship and kindness make my graduate study life rich and colorful.

I am deeply indebted to my parents and families for their love, care, support, and understanding throughout my research. Finally, I would like to thank my dearest Chunmei. Her love and encouragement is a great drive to my research these years.

Notation and Symbols

$\mathbb{R}$ and $\mathbb{C}$	fields of real and complex numbers
$\mathbb{C}_-$ and $\overline{\mathbb{C}}_-$	open and closed left-half plane
$\mathbb{C}_+$ and $\overline{\mathbb{C}}_+$	open and closed right-half plane
$j\mathbb{R}$	imaginary axis
$\in$	belong to
$\subset$	subset
$\cup$	union
$\cap$	intersection
$\square$	end of proof
$\mathcal{L}_2(j\mathbb{R})$	square integrable functions on $\mathbb{C}_0$ including at ∞
$\mathcal{H}_2$	the set of $\mathcal{L}_2(j\mathbb{R})$ functions analytic in $Re(s) > 0$
$\mathcal{RH}_2$	the real rational stable subspace of $\mathcal{H}_2$
$\mathcal{L}_\infty(j\mathbb{R})$	functions bounded on $Re(s) = 0$ including at ∞
$\mathcal{H}_\infty$	the set of $\mathcal{L}_\infty(j\mathbb{R})$ functions analytic in $Re(s) > 0$
$\mathcal{RH}_\infty$	the real rational stable subspace of $\mathcal{H}_\infty$
I_n	$n \times n$ identity matrix
A^*	complex conjugate transpose of A
$\det(A)$	determinant of A
$\bar{\sigma}(A)$	the largest singular values of A
$\underline{\sigma}(A)$	the smallest singular values of A

$\mathcal{F}_l(M, Q)$	lower Linear Fractional Transformation (LFT)
$\mathcal{F}_u(M, Q)$	upper LFT

Table of Contents

Acknowledgments	i
Notation and Symbols	i
Abbreviations	viii
Symbols	viii
List of Figures	viii
List of Tables	x
Summary	xii
1 Introduction	1
1.1 Background	1
1.1.1 What is repetitive control?	1
1.1.2 A brief history	2
1.2 Literature Review	4
1.2.1 Continuous-time Domain Research	4

1.2.2	Discrete-time Domain Research	5
1.2.3	Sampled-data Research	6
1.2.4	Repetitive Control with Non-periodic Disturbances	7
1.2.5	Repetitive Control with Adaptations	7
1.2.6	Repetitive Control and Iterative Learning Control	9
1.2.7	Other Topics in Repetitive Control	9
1.2.8	Applications of Repetitive Control	10
1.3	Motivations	10
1.4	Objective	11
1.5	Major contributions of this thesis	12
1.6	Organization of the thesis	13
2	Preliminaries	15
2.1	Robustness and Uncertainties	15
2.2	Linear Fractional Transform	18
2.3	Structured Singular Value Theory	20
2.3.1	Structured Singular Value	20
2.3.2	Overview of μ Synthesis	22
3	Modelling and Robust Repetitive Control of PWM DC/AC In-	
	verters	24
3.1	Introduction	25
3.2	Plant Modelling	26

3.2.1	Linear Load PWM Inverter Modelling	26
3.2.2	Uncertainty Modelling	29
3.2.3	Performance Weighting Selection	31
3.3	Robust Repetitive Controller Design	32
3.4	Experimental Results	36
3.4.1	Controller Design	37
3.4.2	Experimental Results	40
3.4.3	Discussions	45
3.5	Conclusions	48
4	A Three-Step Design Method for Performance Improvement of Robust Repetitive Control with Application to PWM DC/AC Inverters	49
4.1	Three Step Design for Performance Improvement of Robust Repetitive Control	50
4.1.1	Step 1: K' design	50
4.1.2	Step 2: Q filter design	51
4.1.3	Step 3: K design	53
4.2	Application to PWM DC/AC Inverter	55
4.2.1	Simulation Results	56
4.2.2	Experiment Results	58
4.3	Conclusion	58
5	Robust Repetitive Control with Linear Phase Lead	61

5.1	From Frequency Domain ILC to Repetitive Control	62
5.1.1	Frequency Domain ILC	62
5.1.2	Repetitive Control	63
5.2	Repetitive Control Motivated by ILC Linear Phase Lead Design . .	65
5.3	Robust Stable Repetitive Control	66
5.4	Robust Performance Repetitive Control	70
5.5	Experiment Results	73
5.6	Conclusion	75
6	Fractional Delay Based Robust Repetitive Control with Application to PWM DC/AC Inverter	78
6.1	Introduction	79
6.2	Period Sensitivity Analysis	80
6.3	Fractional Delay based Repetitive Control	81
6.3.1	Problem Formulation	82
6.3.2	Lagrange Interpolator Method	83
6.3.3	Least Square Method	84
6.4	Application to CVCF PWM DC/AC Inverters	86
6.5	Conclusion	93
7	Conclusion and Recommendations	94
7.1	Conclusions	94
7.2	Recommendations for further research	95

7.2.1	RRLC based on LMI	95
7.2.2	Robust repetitive learning control for MIMO systems	96
7.2.3	Adaptive repetitive control with application to PWM inverters	99
Author's Publications		100
Bibliography		102

List of Figures

1.1	Repetitive Control System	2
2.1	M - Δ Loop for Stability Analysis	16
2.2	Output Multiplicative Perturbed Systems	17
2.3	The Diagram of LFTs	19
2.4	General Framework of System	19
2.5	Robust Performance vs Robust Stability	22
3.1	CVCF PWM DC/AC inverter and PWM input waveform	27
3.2	LFT representation of parametric uncertainty	30
3.3	LFT representation of plant with uncertainties	31
3.4	Block Diagram of Robust Repetitive Control System	32
3.5	LFT Form for Repetitive Control System	33
3.6	LFT Form for Robust Repetitive Control Design	34
3.7	PWM inverter nominal model with structured uncertainties	37
3.8	Unstructured uncertainty and performance weighting functions	39

3.9 RRC System Closed-loop Sensitivity and Performance Weighting Functions	41
3.10 Experiment Setup	42
3.11 Feedback Controlled Steady State Responses with Different Loads .	43
3.12 RRC Controlled Steady State Responses with Different Loads . . .	44
3.13 RRC Tracking Error Histories with Different Loads	46
3.14 RRC Controlled Response Under Load Changes	47
4.1 K' Design in TSRRC	51
4.2 Q design in TSRRC	52
4.3 Q filters and performance weighing function	55
4.4 Sensitivity functions comparison	56
4.5 Steady state simulation results with rectifier load	57
4.6 Simulation Results: Transient Errors Comparison	57
4.7 Simulation Results: RMS Errors Comparison	58
4.8 Steady state response comparison with rectifier load	59
4.9 Tracking error histories comparison with rectifier load	60
5.1 Configuration of ILC	62
5.2 Add-On Repetitive Control System Block Diagram	64
5.3 Plug-in Repetitive Control System Block Diagram	64
5.4 Robust Stable Repetitive Control System	67
5.5 LFT Form for Robust Stable Repetitive Control System	68

5.6	Robust Performance Repetitive Control System	71
5.7	Robust Stable Repetitive Control with Linear Phase Lead	73
5.8	Sensitivity Function of Repetitive System Comparison	74
5.9	Steady state response comparison with rectifier load	75
5.10	Tracking error histories comparison with rectifier load	76
6.1	Block Diagram of Repetitive Control System	80
6.2	Q_{FD}^{LI} and Q_{FD}^{LS} Comparison	88
6.3	Sensitivity Function Comparison	89
6.4	Sensitivity Function Comparison	89
6.5	Conventional RC Steady State Response with Rectifier Load	90
6.6	Least Square FD-RC Steady State Response with Rectifier Load	90
6.7	Lagrange Interpolation FD-RC Steady State Response with Rectifier Load	91
6.8	Conventional RC controlled tracking error history	91
6.9	LS RC controlled tracking error history	92
6.10	LI RC controlled tracking error history	92
7.1	Dual-stage Servo Control System Structure	98
7.2	Block Diagram of SIDO Repetitive Controller	98
7.3	Block Diagram of Adaptive Repetitive Control with ANF approach	99

List of Tables

3.1	Nominal Plant Parameters and Uncertain Bounds for Design	36
3.2	Parameters for Experiment and Simulation	36
3.3	Performance Comparisons of Two RRC Controllers	45

Summary

Repetitive control has been successfully used in various applications due to its ability of tracking or rejecting exogenous periodic signals with low costs. On the other hand, Iterative Learning Control (ILC) is a closely related research area and has been studied over 20 years. The objective of this thesis is to provide some design methods of combining ILC and robust repetitive control design methods to increase periodic disturbance rejection performance as well as guarantee worst case performance of system.

In this thesis, a sequential and iterative design methodology for robust repetitive control system is presented. It can improve performance of control system by tracking or rejecting signals with higher frequency components. Based on this framework, frequency domain ILC synthesis methods can be applied in the design sequence. This thesis devotes itself to methodologies of phase lead compensation in robust repetitive learning control.

In conventional repetitive control system, the period of exogenous signals must be the integer number of sampling points. However, it is not always satisfied in real applications. In this thesis, a systematic approach for non-integer delay repetitive control system with fixed sampling rate is proposed. The proposed fractional delay based repetitive control scheme employs the design techniques in digital signal processing theory and two different implementation structures are presented.

All proposed control schemes are applied to the control of constant-voltage constant-frequency (CVCF) pulse-width modulated (PWM) DC/AC converter systems. In

this thesis, the modelling of PWM converters is also discussed. To overcome the inherent drawback of the conventional linear load model, the load changes are modelled as plant uncertainties by Linear Fractional Transformation (LFT), where linear load differences are treated as structured uncertainties, and the nonlinear loads are considered as unstructured uncertainties. It will result in less conservative results for robust controller synthesis and could have better performance. The experimental results demonstrate the effectiveness of proposed robust repetitive learning control schemes.

Chapter 1

Introduction

Many control problems encountered in practice include those which perform the same task repeatedly. The exogenous signals in these cases are periodic, such as mechanical system with rotational motion, alternate current power system, etc. The conventional approach is just to design a classical feedback controller to meet the general performance specifications. However, we could get better tracking or rejection performance by utilizing the specific characteristic of periodic signals. Repetitive control is such an effective solution for this problem.

1.1 Background

1.1.1 What is repetitive control?

Repetitive Control is based on the Internal Model Principle (IMP) [23]. In repetitive control systems, a *periodic signal generator* $1/(1 - e^{-\tau s})$ is included in a stable feedback loop as the internal model. The repetitive control system generates an infinitely large feedback gain at periodic signal's fundamental frequency and its harmonics. Hence the periodic exogenous signals can be tracked and/or rejected

asymptotically with known period, if the closed loop system is stable.

Consider the feedback control system as shown in Figure 1.1. We assume the plant to be controlled is linear time-invariant. In Figure 1.1, P is the transfer function of the plant, C is the transfer function of compensator, M is the transfer function of the internal model, or is also called the *periodic signal generator*. Also in the same figure, r is the reference signal, d is the disturbance signal and e is the error signal which should be made small in some sense. r and/or d contains the periodic components. The periodic signal generator and the compensator together is called *Repetitive Controller* and such a feedback system is called a *Repetitive Control System*.

Figure 1.1: Repetitive Control System

1.1.2 A brief history

The concept of repetitive control was first introduced by Inoue et al. [47, 48] in the early 1980s and was successfully applied to the control of proton synchrotron magnet power supply [48]. Since then it has been extensively studied by researchers. However, it is not trivial to stabilize the repetitive system since it contains a positive feedback module as the periodic signal generator. Much research has been carried out to stabilize the repetitive control systems. In [35], the stability analysis of the infinite dimensional repetitive control system is presented. It is proved that the stability could be achieved only when the plants were proper but not strictly proper. This is because of the controller parameterization and the unrealistic demand of perfect tracking of periodic signals of high frequency.

To overcome this limitation, a low pass filter $Q(s)$ was introduced to filter out the high frequency components [35]. The robustness is achieved at the sacrifice of tracking performance at high frequency. The design of the low pass filter $Q(s)$ reflects the trade off between the closed loop system periodic disturbance rejection performance and system stability robustness.

Although the first repetitive controllers were developed in continuous time domain, they were implemented digitally. To reduce the error caused by the controller discretization, much research focused on the direct design of repetitive control in discrete time domain. Tomizuka et al. proposed a “prototype” repetitive controller based on the Zero Phase Error Tracking Control (ZPETC) for stable open loop plant. This method features a filter structure of similar order to the plant and a pure delay for efficient real time implementation. Based the same compensator structure, a zero phase low pass filter $Q(z)$ is further introduced in the internal model to account for robust stability [130, 129, 55].

To study the robust stability and performance of repetitive control systems, H_∞ optimal control and structured singular values have also been used. In [97], a two step method to design H_∞ optimal repetitive controller using infinite dimensional H_∞ optimal control theory is proposed. In [69], Guveac proposed a robust stability and performance analysis structure using structured singular value for continuous time repetitive control system, where the lower and upper bound of structured singular values of the system are approximated by substituting the delay $e^{-\tau s}$ of the internal model with 1 and -1 respectively. However, this approximation is not satisfied until the delay is large enough. Therefore, this structure is not suitable for synthesis. Based on the previous analysis results, a Robust Repetitive Control (RRC) scheme [74, 72, 75] based on structured singular value to satisfy overall system robustness performance. In this RRC scheme, a fictitious uncertainty is introduced to replace the long delay chain in the internal model of repetitive system. In [73], a two parameter scheme is then proposed to achieve better performance. With the same RRC structure, a modified structure of $Q(z)$ filter was proposed

to enhance performance at periodic disturbance's low frequency harmonics with higher order repetitive controllers [57, 58]. In [59], the feed-forward control, repetitive control and feedback control are combined together for the robust performance control. In continuous domain, H_∞ design method for MIMO repetitive control systems is proposed in [137].

The robust repetitive control is the basis of this thesis and will be discussed in detail during the following chapters.

1.2 Literature Review

1.2.1 Continuous-time Domain Research

The concept of repetitive control was first introduced in the continuous domain and much research were reported during the past years.

Since the design of the low pass filter $Q(s)$ reflects the trade off between the closed loop system periodic disturbance rejection performance and system stability robustness, some design methods were reported on Q filter design. In [67, 65], it is reported that the trade-offs between robustness and transient response for continuous time SISO LTI repetitive control systems. The transient was measured using various induced norms and the robustness was quantified by the gap metric. In this evaluation the design tools used were H_1 loop shaping and closed loop pole placement. Moon et al. In [88], a graphical $Q(s)$ filter design method is proposed based on frequency domain analysis of linear interval systems. In [97, 98], the design of $Q(s)$ filter with respect to the nominal repetitive control system is optimized to improve the nominal performance of system. In [16], the repetitive controller design for linear systems with time-varying norm-bounded uncertainties is formulated into the optimization problem with an LMI constraint of the free parameter.

Repetitive control was also applied into control of nonlinear systems. In [26, 81], the repetitive control of the “beam and ball” nonlinear systems is considered by analyzing the effectiveness of a regulator incorporating harmonic oscillators with frequencies multiple of the fundamental frequency of the periodic signals to be rejected and/or tracked. In [119], a Lyapunov-based adaptive repetitive control was presented for a class of nonlinearly parametrized systems. In [141], the existence of the solution and learning convergence property of repetitive control for nonlinear dynamical systems with nonparametric uncertainties are studied. A further extension of the RLC to cascade systems is also explored. In [120], the nonlinear internal model principle control for nonlinear systems with nonlinear disturbance dynamics are presented.

1.2.2 Discrete-time Domain Research

In discrete-time domain design, the delay $e^{-\tau s}$ in repetitive controller is replaced by z^{-N} and then the system is finite dimensional. Compared with continuous-time domain approaches, analysis and synthesis of repetitive control systems become easier. The first discrete-time domain repetitive controller was reported in [85]. In [123], a “prototype” repetitive controller was introduced based on zero phase error tracking control scheme. In [130, 129], Tsao and Tomizuka further introduced a zero phase low pass filter $Q(z)$ in the internal model to account for robust stability. A relationship between $Q(z)$ and the unmodelled dynamics was given, which provides a sufficient condition for robust stability. This structure can be applied to both minimum and non minimum phase stable plants. In [62], an optimal structure for ZPETC repetitive control system is proposed. In [30], the $Q(z)$ in the prototype repetitive control structure is substituted by $S(z)/(S(z) + R(z))$ for frequency shaping of the sensitivity function. $S(z)$ and $R(z)$ are designed to reject both the repeatable and non-repeatable disturbances.

In [103], a digital repetitive plug-in controller is developed for odd-harmonic discrete-

time periodic references and disturbances. The controller had a lower data memory occupation by taking advantage of the particular characteristics of the signals to track or attenuate. A similar structure was also developed by [15, 22]. However, the tracking performance may be sacrificed. To combine the advantages of both odd-harmonic and conventional structure, a dual mode repetitive control (DMRC) [150] was introduced. Without additional complexity and loss of tracking accuracy, DMRC can achieve faster error convergence rate than conventional RCs without performance degradation.

Multi-rate repetitive control design method are also studied by researchers [110, 107]. In [66], the nonlinear control for robust rejection of periodic disturbances was studied. Command shaping and multi-rate repetitive control were combined to improve the control performance of flexible manipulators [102]. In [24], two multi-rate repetitive controllers were proposed.

1.2.3 Sampled-data Research

One possible problem for discrete-time domain repetitive control approaches is inter-sample behavior. In discrete-time repetitive controller design, the tracking performance between sampling points is sacrificed, i.e., the inter-sample behavior is not considered in discrete-time domain design. In [36], the inter-sample ripple of repetitive control systems were studied. In [24], a multi-rate repetitive control controllers are proposed to handle the inter-sample behavior. To handle inter-sample behavior, modern sampled-data control theory was used to design repetitive control systems. In [64], Langari and Francis introduced induced power norm of steady-state tracking error and was minimized for the worst periodic input of unit power. The fast-sampling-fast-holding (FSFH) technique was used to approximate the sampled-data repetitive controller. In [49], a periodically time-varying compensator was introduced during the design of sampled-data repetitive controller. This yields extra freedom in designing this class of systems. This freedom can be effec-

tively used to increase robustness in the design. In [37], sampled-data H_∞ control theory was applied into repetitive controller design and showed that this approach significantly attenuated the ripples without sacrificing the tracking performance.

1.2.4 Repetitive Control with Non-periodic Disturbances

When the exogenous signals are not periodic, the repetitive controller may also be applied. [82] developed a repetitive control scheme with reference signals not necessarily time periodic functions. A more general case is the periodic disturbances or reference signals are existent and synchronized with a rate varying process variable. One efficient way is to design a repetitive controller in angle domain [135, 136, 121, 90]. In the angle domain, the system's time invariant dynamics become time varying and dependent on the angular speed. Therefore, the problem of repetitive control for time varying systems arises. In [135, 136], model reference control was applied to compensate for the linear time varying system, rendering a linear time invariant input/output system. Asymptotic output tracking/regulation performance is achieved by employing repetitive control loop to compensate for the linear time invariant model. In [90], the same technique was introduced to eliminate position-dependent disturbances in a constant-speed-rotation system.

1.2.5 Repetitive Control with Adaptations

Adaptive techniques have been widely used in repetitive control systems. When the period of disturbances is unknown or time-varying, recursive frequency estimation techniques can be used. In [127], a sampled data recursive scheme for identifying the period of a periodic signal with a resolution finer than the sampling interval are proposed. In addition, discrete-time self-tuning repetitive controllers, which adapt both the periodic signal period and sampling interval, are proposed based on the period identification scheme. In [9], an adaptive repetitive control, which

deals with the non-integer samples per period due to the fixed sampling rate, were presented. The proposed adaptive repetitive control consists of two portions, the repetitive controller and nominal controller, where the former uses a fictitious sampler operating at a variable sample rate maintained at multiple times of the signal frequency, while the latter uses a fixed sampling rate. In [19], an recursive period identification method are proposed for the time varying character of the period to deal with time varying periodic disturbances. In [40, 39], a repetitive control based on models of isolated frequencies rather than the commonly used delay model are presented. The relationship between different modeled frequencies is fixed by the model structure, and the fundamental frequency is obtained by gradient descent. To show the feasibility of this approach, it was used to reduce vibrations on a lever that were caused by a motor with imbalance in its rotation. In [44], an adaptive control algorithm for rejecting narrow-band noise was proposed. This algorithm was developed under the framework of robust stabilization and the internal model principle. The internal model which contained the noise dynamic was updated using the estimated disturbance signal. The parameter adaptation was constrained to meet the condition of robust stabilization when the parameters were converged.

Besides the recursive period estimation, adaptive controller can also be applied into the repetitive control systems. In [45], the problem of designing an adaptive tracking controller for repetitive signals are studied. In [131], an adaptive repetitive control scheme are proposed and applied to the control of a Pulse-Width-Modulated (PWM) inverter used in a high-performance AC power supply. The proposed adaptive repetitive controller consists of a voltage regulator using state feedback control, a repetitive controller with tuning parameters and an adaptive controller with a recursive least-squares estimator (LSE).

1.2.6 Repetitive Control and Iterative Learning Control

Iterative learning control (ILC) has close relationship with repetitive control. Longman [111, 42, 43, 79, 78] has shown that they are the same in many aspects and they are different mostly from the motivations and applications. In [104], both ILC and repetitive control design method for multi-variable systems based on internal model principle are studied. In [41], four different repetitive control schemes by experiments are studied. In [80], the tradeoffs in learning/repetitive controllers design are showed. In [112, 113], the stability boundary and frequency response stability condition for ILC and repetitive control are compared. In [99, 93], a zero-phase IIR low-pass filter are introduced for repetitive controllers, and in [41], an IIR zero-phase cutoff are implemented as a batch update.

1.2.7 Other Topics in Repetitive Control

Many other techniques are also used to polish repetitive system performance from different aspects. In [142], the repetitive system design method from the viewpoint of pole placement is proposed. All poles of the closed-loop system can be assigned and thus faster error convergence can be guaranteed. In [10], discrete wavelet transforms are applied to reduce the memory requirements of repetitive controllers. Only the highest-level approximation coefficients and a few detail coefficients in the wavelet transform have to be saved and hence the error signal is compressed. The compressed error signal is in turn reconstructed to compensate the control system. In [77], the sliding mode control characteristics are incorporated into repetitive learning control and applied into the robot control. In [34], the repetitive control for linear, periodic, time-varying systems are developed by using gain scheduling control. In [119], the discrete-time variable-structure repetitive control scheme is introduced. A time-domain sufficient conditions for the robust stability are presented. In [143], the performance of plug-in repetitive control system is analyzed. It gives an upper bound for the square integral of the tracking

error over one time period of periodic input signals based on Fourier analysis. In [114], a repetitive control scheme, which is not sensitive to disturbance period uncertainty, is proposed. In [70], simple analog circuits are applied to implement a repetitive control scheme.

1.2.8 Applications of Repetitive Control

Throughout the development of repetitive control, many possible fields of applications have been discovered since then. These applications include AC motors [116], industrial robots [5, 106, 124, 50, 132, 118], metal cutting [128, 84, 59, 125], steel casting process [83], satellite control [6], spark ignition engines [95], and PWM inverter and rectifier [149, 147, 100, 101, 21]. Repetitive control was firstly applied in Hard Disk Drive (HDD) servo control systems by Tomizuka to eliminate the Repetitive Run Out (RRO) in the tracking errors and achieved good efficiency [14], which attracted more researches [105, 32, 29, 74, 8, 7, 94, 13, 96]. Similarly, it has also been used to audio Compact Disk (CD) and optical drive servo systems [65, 88, 17, 18, 68, 11]. Repetitive control is applied to the control of six-degree of freedom (6-DOF) active hexapod based on the Stewart platform configuration [25, 20].

1.3 Motivations

Repetitive control system introduces infinitely large feedback gain at periodic signal's fundamental frequency and its harmonics, so that perfect performance can be achieved on the periodic signal as long as the closed loop system is stable. However, it is not trivial to stabilize the repetitive system since it contains a positive feedback module as the periodic signal generator. It becomes more difficult when system uncertainties exist. On the other hand, the repetitive controller will amplify some frequency components and thus may degrade system performance.

In this thesis, the research focuses on the robust stability and robust performance analysis and synthesis of repetitive control systems.

On the other hand, Iterative Learning Control (ILC) has close relationship with repetitive control. Longman [42, 79] has shown that they are the same in many aspects and they are different mostly from the motivations and applications. ILC and repetitive control have been researched under different contexts and many different approaches have been developed independently. In this thesis, the ILC design methods are utilized to improve disturbance rejection performance for repetitive system with preserving the overall system robustness.

Furthermore, the repetitive controller requires to know exactly the period of disturbance. When there are small perturbations in the disturbance period, the performance of repetitive control will degrade very fast. In this thesis, the adaptive filtering technique is used to estimate the period of disturbance online.

1.4 Objective

The objective of this thesis is to design robust repetitive controller with improved performance as well as applying the proposed design method to PWM inverters. Although the stability of repetitive control problem has been established [35, 123]. However, the robust stability analysis of repetitive control system has not been studied. In this thesis, the robust stability analysis will be established by using structured singular value theory. In [75], robust repetitive control based on structured singular value has shown successful design to satisfy overall system robust performance.

1.5 Major contributions of this thesis

The main contributions of this thesis are in the analysis and synthesis of robust repetitive control using ILC design technique. The contributions are listed as follows:

- Robust stability and performance analysis of repetitive control by using structured singular value theory.

In the repetitive control system, the Q filter in the internal model is introduced to filter out high frequency components to achieve robustness of the system. The bandwidth of Q filter and system stability and performance is therefore a tradeoff. In this thesis, the robust stability and performance evaluation is put into the framework of structured singular value theory and then this tradeoff is obvious in this framework.

- Iterative and sequential design for robust repetitive learning control system.

In the thesis, we proposed a three-step design method for performance improvement of robust repetitive control systems. The proposed design method generates a repetitive controller and feedback controller sequentially and iteratively. This proposed method enhances the performance of a repetitive control system by extending the bandwidth of low-pass filter Q to reject more high frequency harmonic disturbances as well as to guarantee the overall system robustness performance.

- Robust repetitive controller design with ILC design techniques.

Based on the three-step robust repetitive control scheme, the ILC design methods can be easily applied. The linear phase lead compensation is applied to the robust repetitive control, which can extend the bandwidth of the Q filter and help to admit to larger learning gain. Experimental results show the effectiveness of proposed method.

- Development of a fractional-delay based repetitive control scheme to handle non-integer period case.

In a repetitive control system, the period of the exogenous signals must be an integer number of sampling points. However, it can not be always satisfied in real applications. In this thesis, a systematic approach for non-integer delay repetitive control system with fixed sampling rate is proposed. The proposed fractional delay based repetitive control scheme employs the design techniques in digital signal processing theory and two different implementation structures are presented. One is easy to design and the other has optimized performance.

- Application of the proposed algorithms to PWM DC/AC inverters.

In this thesis, the proposed robust repetitive learning controllers were applied into constant-voltage constant-frequency (CVCF) pulse-width modulated (PWM) DC/AC Inverter systems. To overcome the inherent drawback of linear load model, the load changes are modelled as plant uncertainties by Linear Fractional Transformation (LFT): linear load differences are treated as structured uncertainties, and the nonlinear loads are considered as unstructured uncertainties.

1.6 Organization of the thesis

In this thesis, the analysis and synthesis of robust repetitive learning control with applications to CVCF PWM Inverters are discussed. The details of the each chapter are as follows:

Chapter 1 is the introduction of thesis, which gives basic introduction and brief history of repetitive control system, a survey of previous researches as well as the motivation and organization of this thesis.

Chapter 2 introduces some basic concepts and theory of robust optimal control. Structured Singular Value (SSV) or μ theory is the focus of this chapter.

Chapter 3 presents the modelling and robust repetitive control of constant-voltage constant-frequency (CVCF) pulse-width modulated (PWM) DC/AC Inverter systems.

Chapter 4 presents a three-step robust design method to improve performance of robust repetitive control systems. The proposed design method generates a repetitive controller and feedback controller sequentially and iteratively. This proposed method enhances the performance of a repetitive control system by extending the bandwidth of low-pass filter Q to reject more high frequency harmonic disturbances as well as to guarantee the overall system robustness performance.

Chapter 5 discusses the relationship between repetitive control and iterative learning control and then proposes robust repetitive learning control (RRLC) scheme.

Chapter 6 presents a systematic approach for non-integer delay repetitive control system with fixed sampling rate.

Chapter 2

Preliminaries

In this chapter, some basic concepts and theory of robust optimal control are introduced. Structured Singular Value (SSV) or μ theory is the focus of this chapter, since it is a powerful tool in the design and analysis of our proposed robust repetitive control scheme. The detailed discussion about robust optimal control can be found in [146].

2.1 Robustness and Uncertainties

Robust control laws are to maintain system response and error signals within pre-specified tolerances in the presence of uncertainty in the system model. Uncertainty may take many forms but among the most significant are noise/disturbance signals and the transfer function modelling errors. Another source of uncertainty is unmodelled nonlinear distortion. Uncertainty in any form is no doubt a major issue in control system designs.

In general, we are forced to use not just a single parameterized model but model sets that allow for plant dynamics that are not represented in the model structure. One such example is a model with uncertainties that are within a bound in frequency

domain. The relation to the nominal plant can be either additive form:

$$P_{\Delta}(s) = P(s) + W_1(s)\Delta(s)W_2(s), \quad \bar{\sigma}[\Delta(j\omega)] < 1, \quad \forall \omega \geq 0, \quad (2.1)$$

or multiplicative form:

$$P_{\Delta}(s) = (1 + W_1(s)\Delta(s)W_2(s))P(s). \quad (2.2)$$

where $W_1(s)$ and $W_2(s)$ are weighting functions, which are stable transfer matrices that characterize the spatial and frequency structure of the uncertainty, confines the matrix $P_{\Delta}(s)$ to the neighborhood of the nominal plant $P(s)$.

The essence for the robust stability criteria derived in the sequel is the so-called *Small Gain Theorem* [146]. Consider the interconnected system shown in Figure 2.1 with $M(s)$ a stable $p \times q$ transfer matrix.

Theorem 1 (Small Gain Theorem) *Suppose $M \in \mathcal{RH}_{\infty}$ and let $\gamma > 0$. Then the interconnected system shown in Figure 2.1 is well-posed and internally stable for all $\Delta(s) \in \mathcal{RH}_{\infty}$ with*

- (a) $\|\Delta\|_{\infty} \leq 1/\gamma$ if and only if $\|M(s)\|_{\infty} < \gamma$;
- (b) $\|\Delta\|_{\infty} < 1/\gamma$ if and only if $\|M(s)\|_{\infty} \leq \gamma$;

Figure 2.1: M - Δ Loop for Stability Analysis

Based on *Small Gain Theorem*, the system robust stability of $M - \Delta$ loop under multiplicative unstructured uncertainty has been shown. Similar results can be derived for the case of additive uncertainties. We assume that the system model is described by the following set of multiplicative perturbations:

$$\Pi = (1 + W_1 \Delta W_2)P \quad (2.3)$$

with $W_1, W_2, \Delta \in \mathcal{RH}_\infty$. Consider the feedback system shown in Figure 2.2.

Figure 2.2: Output Multiplicative Perturbed Systems

Theorem 2 Let $\Pi = \{P + W_1 \Delta W_2 : \Delta \in \mathcal{RH}_\infty\}$ and let K be a stabilizing controller for the nominal plant P . Then the closed-loop system is well-posed and internally stable for all $\|\Delta\|_\infty < 1$ if and only if $\|W_2 T_o W_1\|_\infty \leq 1$, where $T_o = I - (I + PK)^{-1} = I - S_o$

Let $T_{e\tilde{d}}$ denote the transfer matrix between $\tilde{d}$ and e . For system shown in Figure 2.2, we have

$$T_{e\tilde{d}} = W_e S_o (I + W_1 \Delta W_2 T_o)^{-1} W_d \quad (2.4)$$

and the robust performance is satisfied iff

$$\|W_2 T_o W_1\|_\infty \leq 1 \quad (2.5)$$

and

$$\|T_{e\tilde{d}}\|_\infty \leq 1, \quad \forall \Delta \in \mathcal{RH}_\infty, \quad \|\Delta\|_\infty < 1. \quad (2.6)$$

The exact analysis for this robust performance problem is not trivial and will be given by μ analysis.

2.2 Linear Fractional Transform

Linear Fractional Transformation (LFT) is a convenient tool to formulate control problems to a standard framework and thus can be treated using the same technique. The basic role of LFT is modelling uncertainty, that is “pulling out the Δ s” from the nominal systems.

Definition 1 *Let M be a complex matrix partitioned as*

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \in \mathbb{C}^{(p_1+p_2) \times (q_1+q_2)},$$

and let $\Delta_l \in \mathbb{C}^{q_2 \times p_2}$ and $\Delta_u \in \mathbb{C}^{q_1 \times p_1}$ be two complex matrices. Then we can formally define a lower LFT with respect to Δ_l as the map

$$\mathcal{F}_l(M, \bullet) : \quad \mathbb{C}^{q_2 \times p_2} \mapsto \mathbb{C}^{q_1 \times p_1}$$

with

$$\mathcal{F}_l(M, \Delta_l) = M_{11} + M_{12}\Delta_l(I - M_{22}\Delta_l)^{-1}M_{21} \quad (2.7)$$

provided that the inverse $(I - M_{22}\Delta_l)^{-1}$ exists.

There is similar definition with $\mathcal{F}_u(M, \Delta_u)$. The motivation for the terminologies of *lower* and *upper* LFTs can be shown from the following diagram representations of $\mathcal{F}_l(M, \Delta_l)$ and $\mathcal{F}_u(M, \Delta_u)$, where P is the augmented plant, Δ is the uncertainty of system and K represents the controller to be designed.

Figure 2.3: The Diagram of LFTs

Figure 2.4: General Framework of System

By LFT any interconnected system may be rearranged to fit the general framework, which is shown in Figure 2.4. Various modelling assumptions will be considered and the impact of these assumptions on analysis and synthesis methods will be explored in this general framework. Where K is controller, P is nominal system and Δ is the system perturbation. The closed-loop system is an LFT on the perturbation and the controller given by:

$$z = \mathcal{F}_u(\mathcal{F}_l(P, K), \Delta)w = \mathcal{F}_l(\mathcal{F}_u(P, \Delta), K) \quad (2.8)$$

2.3 Structured Singular Value Theory

2.3.1 Structured Singular Value

Although small gain theory can be used to derive robust stability tests under various assumptions of model uncertainties, it is rather conservative. The stability and performance analysis for systems with structured uncertainty require a new matrix function called *Structured Singular Value* which is denoted by μ . In the definition of $\mu(M)$, there is an underlying structure Δ (a prescribed set of block diagonal matrices) on which everything in the sequel depends. We define $\Delta \subset \mathbb{C}^{n \times n}$ as

$$\Delta = \{\text{diag}[\delta_1 I_{r_1}, \dots, \delta_s I_{r_s}, \Delta_1, \dots, \Delta_F] : \delta_i \in \mathbb{C}, \Delta_j \in \mathbb{C}^{m_j \times m_j}\}. \quad (2.9)$$

Definition 2 For $M \in \mathbb{C}^{n \times n}$, $\mu_\Delta(M)$ is defined as

$$\mu_\Delta(M) := \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \Delta, \det(I - M\Delta) = 0\}} \quad (2.10)$$

unless no $\Delta \in \Delta$ makes $I - M\Delta$ singular, in which case $\mu_\Delta(M) := 0$.

The most well-known use of μ as a robustness analysis tool is in the frequency domain. Suppose $G(s)$ is a stable, real-rational, multi-input multi-output transfer function of a linear system. For clarity, assume G has q_1 inputs and p_1 outputs. Let Δ be a block structure, as in Equation (2.9).

Let $\mathcal{M}(\Delta)$ denote the set of all block diagonal and stable rational transfer functions that have blocks structures such as Δ .

$$\mathcal{M}(\Delta) := \{\Delta(\bullet) \in \mathcal{RH}_\infty : \Delta(s_o) \in \Delta \text{ for all } s_o \in \bar{\mathbb{C}}_+\}$$

Theorem 3 *Let $\beta > 0$. The loop shown in Figure 2.1 is well-posed and internally stable for all $\Delta(\bullet) \in \mathcal{M}(\Delta)$ with $\|\Delta\|_\infty < \frac{1}{\beta}$ if and only if*

$$\sup_{\omega \in \mathbb{R}} \mu_\Delta(G(j\omega)) \leq \beta \quad (2.11)$$

Often, stability is not the only property of a closed-loop system that must be robust to perturbations. In most cases, long before the onset of instability, the closed-loop performance will degrade to the point of unacceptability. Such a test will indicate the worst-case level of performance degradation associated with a given level of perturbations.

Assume G_p is a stable, real-rational, proper transfer function with $q_1 + q_2$ inputs and $p_1 + p_2$ outputs. Partition G_p in the obvious manner

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},$$

so that G_{11} has q_1 inputs and p_1 outputs, and so on. Let $\Delta \subset \mathbb{C}^{q_1 \times p_1}$ be a block structure, as in Equation (2.9). Define an augmented block structure:

$$\Delta_P := \left\{ \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_f \end{bmatrix} : \Delta \in \Delta, \Delta_f \in \mathbb{C}^{q_2 \times p_2} \right\}$$

Then the system robust performance can be addressed as follows:

Theorem 4 *Let $\beta > 0$. For all $\Delta(s) \in \mathcal{M}(\Delta)$ with $\|\Delta\|_\infty < \frac{1}{\beta}$, the loop is well-posed, internally stable, and $\|\mathcal{F}_u(G_p, \Delta)\|_\infty \leq \beta$ if and only if*

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_P}(G_p(j\omega)) \leq \beta \quad (2.12)$$

This theorem says that a robust performance problem is equivalent to robust stability problem with augmented uncertainty Δ , as shown in Figure 2.5.

Figure 2.5: Robust Performance vs Robust Stability

2.3.2 Overview of μ Synthesis

For the analysis results, we see that each case eventually leads to the evaluation of

$$\|M\|_{\alpha} \quad \alpha = 2, \infty, \text{ or } \mu \quad (2.13)$$

for some transfer matrix M . Thus when the controller is put back into the problem, it involves only a simple linear fractional transformation with

$$\mathcal{F}_l(G, K) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}$$

Then leads to the synthesis problem

$$\min_K \|\mathcal{F}_l(G, K)\|_{\alpha} \quad \alpha = 2, \infty, \text{ or } \mu \quad (2.14)$$

which is subject to the internal stability of the nominal model. The cases of $\alpha = 2$ and ∞ are standard H_2 and H_{∞} problems. The synthesis for the $\alpha = \mu$ case is not yet fully solved. μ may be obtained by scaling and applying $\|\cdot\|$ (for $F \leq 3$ and $S = 0$); a reasonable approach is to “solve”

$$\min_K \inf_{D, D^{-1} \in \mathcal{H}_{\infty}} \|D\mathcal{F}_l(G, K)D^{-1}\|_{\infty} \quad (2.15)$$

by iteratively solving for K and D . This is so-called *D-K iteration*. The stable and minimum phase scaling matrix $D(s)$ is chosen such that $D(s)\Delta(s) = \Delta(s)D(s)$.

For a fixed scaling transfer matrix D , $\min_K \|D\mathcal{F}_l(G, K)D^{-1}\|_\infty$ is standard H_∞ optimization problem. For a given stabilizing controller K , $\inf_{D, D^{-1} \in \mathcal{H}_\infty} \|D\mathcal{F}_l(G, K)D^{-1}\|_\infty$ is a standard convex optimization problem. Although the joint optimization of D and K is not convex and the global convergence is not guaranteed, many designs have shown that this approach works very well.

Chapter 3

Modelling and Robust Repetitive Control of PWM DC/AC Inverters

In this chapter, modelling and robust repetitive controller of constant-voltage constant-frequency (CVCF) pulse-width modulated (PWM) DC/AC inverter systems are studied. To overcome the inherent drawback of linear load model, the load changes are modelled as plant uncertainties by Linear Fractional Transformation (LFT): linear load differences are treated as structured uncertainties, and the nonlinear loads are considered as unstructured uncertainties. The robust repetitive controlled inverter systems will have the robustness performance as well as periodic disturbance rejection performance in the presence of various loads. Experimental results verified the effectiveness of the modelling and control scheme.

3.1 Introduction

Constant-voltage-constant-frequency pulse-width modulated (CVCF PWM) DC/AC inverters [27, 33, 48, 53, 52, 54] are widely used in various power supplies. A good power supply should have the nominal constant output under disturbances and uncertainties, good dynamic response to disturbances, and remain stable under all operating conditions. Output voltage total harmonics distortion (THD) is one important index to evaluate the performance of inverters. High THD may lead to communication interference, excessive heating in capacitors and transformers etc. Nonlinear loads, such as rectifier loads, causing periodic distortion, are major sources of THD in AC power systems.

The repetitive control scheme [33, 149, 148, 131, 101] has been employed to improve tracking accuracy of PWM inverters and thus significantly reduce THD. The comparisons among different repetitive approaches can be found in [100].

However, when a repetitive controller is plugged into the existing feedback system directly, the repetitive controller will unavoidably amplify some non-periodic disturbances and thus may degrade system performance. It is necessary to take the impact of repetitive control module into consideration during the design of the feedback controller. Moreover, the prevailing feedback controllers [53], such as deadbeat controllers, one-sampling-ahead-preview (OSAP) controllers, etc., are mainly developed on the basis of PWM inverters with nominal linear load (resistor load). However the inverter should operate under various loads, conventional feedback controllers can not always achieve the desired performance (especially under nonlinear rectifier loads), and even worsen the robustness of the overall repetitive control system.

To reduce the performance degradation caused from uncertainties and disturbances, the state-of-the-art H_∞ control theory provides engineers with a powerful systematic approach to develop robust feedback controllers. Furthermore, based

on H_∞ control theory, Li and Tsao proposed Robust Repetitive Control (RRC) scheme to properly integrate the repetitive control and the feedback control to improve overall system robustness performance, and applied RRC into non-circular machining [75] and Hard Disk Drivers [74]. Weiss et. al. [137] proposed a repetitive control scheme of MIMO systems using H_∞ design and applied into DC/AC inverters in Micro-grids [138]. It initiated the applications of H_∞ repetitive control in PWM inverters with some preliminary and promising simulation results.

In this chapter, a robust H_∞ repetitive controller is proposed for CVCF PWM inverters. In the proposed robust repetitive control scheme, the plug-in repetitive control module is regarded as one component of the augmented plant during the design of robust feedback controller for PWM. Furthermore, load variations are treated as plant uncertainties by Linear Fractional Transformation (LFT), wherein, linear load differences are modelled as structured uncertainties, and the nonlinear loads are considered as unstructured uncertainties. Experiment results verified the validity of the proposed scheme.

3.2 Plant Modelling

3.2.1 Linear Load PWM Inverter Modelling

The dynamics of the CVCF PWM DC/AC inverters (as shown in Figure 3.1(a)) can be described as follows :

$$\begin{bmatrix} \dot{v}_c \\ \ddot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L_n C_n} & -\frac{1}{C_n R_n} \end{bmatrix} \begin{bmatrix} v_c \\ \dot{v}_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_n C_n} \end{bmatrix} v_{in} \quad (3.1)$$

where v_c is the output voltage; i_o is the output current; E is the DC bus voltage; L_n , C_n and R_n are the nominal values of the inductor, capacitor and load, respectively; As shown in Figure 3.1(b), the control input v_{in} is a PWM voltage pulse of

Figure 3.1: CVCF PWM DC/AC inverter and PWM input waveform

magnitude E (or $-E$) with width ΔT centered in the sampling interval T . For a linear system $\dot{x} = Ax + Bu$, its sampled-data equation can be expressed as

$$x(k+1) = e^{AT}x(k) + \int_0^T e^{A(T-\tau)}Bu(\tau)d\tau$$

Therefore, a sampled-data form for Equation (3.1) can be derived as follows:

$$\begin{aligned} x(k+1) &= A_dx(k) + B_du(k) \\ y(k) &= C_dx(k) + D_du(k) \end{aligned} \quad (3.2)$$

where $x(k) = [v_c(k) \ \dot{v}_c(k)]^T$, $u(k) = v_{in}(k)$, and

$$\begin{aligned} A_d &= \begin{bmatrix} 1 - \frac{T^2}{2L_nC_n} & T - \frac{T^2}{2C_nR_n} \\ -\frac{T}{L_nC_n} + \frac{T^2}{2L_nC_n^2R_n} & 1 - \frac{T}{C_nR_n} - \frac{T^2}{2L_nC_n} + \frac{T^2}{2C_n^2R_n^2} \end{bmatrix} \\ B_d &= \begin{bmatrix} \frac{T^2}{2L_nC_n} \\ \frac{T}{L_nC_n} - \frac{T^2}{2L_nC_n^2R_n} \end{bmatrix} \\ C_d &= [1 \ 0] \\ D_d &= 0 \end{aligned}$$

The corresponding plant transfer function is

$$P_0 = \left[\begin{array}{c|c} A_d & B_d \\ \hline C_d & D_d \end{array} \right] \quad (3.3)$$

where we have used the compact notation in robust optimal control theory [146], i.e., $P_0(z) = C_d(zI - A_d)^{-1}B_d + D_d$. The controller design will be based on the model shown in Equation (3.3) in the discrete time domain. The objective of the controller is to force the tracking error between $y(k)$ and its sinusoidal reference $y_d(k)$ under various loads.

3.2.2 Uncertainty Modelling

There are always differences, which are called uncertainties, between the model and actual plant. For CVCF PWM inverter system, the uncertainties are categorized into two classes: plant parameter uncertainties and load uncertainties. Since the PWM inverter system is modelled with linear load, different linear loads can be treated as parametric uncertainties. Nonlinear loads have different structures and are regarded as unmodelled dynamics of plant. By Linear Fractional Transformation (LFT) [146], parametric uncertainties and unmodelled dynamics are modelled as “structured” and “unstructured” uncertainties, respectively. The detail information of LFT can be found in Chapter 2.

Parametric uncertainties modelling

The parametric uncertainties are modelled as structured uncertainties of plant. The parameter variation can always be bounded within a range. For our specific system, linear load $R \in [R_{rate}, +\infty)$, where R_{rate} is the rated linear load and $R = +\infty$ represents no load. By LFT we can pull out the parametric uncertainties from the nominal plant, which is shown in Figure 3.2. It can be represented as

$$P_s(z) = \mathcal{F}_u(P_0(z), \Delta_s(z)) \quad (3.4)$$

Where $P_0(z)$ is related to the nominal plant in Equation (3.3). The detailed expression of $P_0(z)$ can be derived by MATLAB robust control Toolbox and is omitted here. $\Delta_s(z)$ is the parametric or structured uncertainty with $\Delta_s(z)$ is stable and the norm of $\Delta_s(z)$ is less than or equal to 1, i.e., $\Delta_s(z) \in \mathcal{RH}_\infty$ and $\|\Delta_s(z)\|_\infty \leq 1$.

Figure 3.2: LFT representation of parametric uncertainty

Nonlinear loads modelling

Because the nonlinear loads' different structures, they can not be modelled as parametric uncertainties. Here the nonlinear loads are regarded as unmodelled dynamics of plant and modelled as unstructured uncertainties. The unstructured uncertainty of plant can be represented as multiplicative form:

$$P_{\Delta}(z) = (1 + W_r(z)\Delta_r(z))P_s(z) \quad (3.5)$$

where $\Delta_r(z) \in \mathcal{RH}_{\infty}$ with $\|\Delta_r(z)\|_{\infty} \leq 1$. $W_r(z)$ is weighting function, which is stable transfer matrix that characterizes the spatial and frequency structure of the uncertainty, confines the matrix $P_{\Delta}(z)$ to the neighborhood of the nominal plant $P_s(z)$. For our PWM inverter system, nonlinear loads can always be bounded as equivalent resistors within the range $[R_l, R_u]$, where R_l and R_u are equivalent resistors' lower and upper bounds.

Combining both structured and unstructured uncertainties, we have completely modelled the deviations caused by different loads. Let

$$\Delta_u = \begin{bmatrix} \Delta_s & 0 \\ 0 & \Delta_r \end{bmatrix}$$

System with uncertainties can be reconfigured as the LFT form shown in Figure 3.3:

$$P_{\Delta} = \mathcal{F}_u(P_n, \Delta_u) \quad (3.6)$$

Figure 3.3: LFT representation of plant with uncertainties

where

$$P_n = \left[\begin{array}{cc|c} P_{011} & P_{012} & P_{012} \\ 0 & 0 & W_r \\ \hline P_{021} & P_{022} & P_{022} \end{array} \right] = \left[\begin{array}{c|c} P_{n11} & P_{n12} \\ \hline P_{n21} & P_{n22} \end{array} \right] \quad (3.7)$$

3.2.3 Performance Weighting Selection

Control system performance objectives can usually be specified in terms of requirements on the sensitivity functions $S_{\Delta} := 1/(1 + P_{\Delta}K)$. We use a rational transfer function W_f , which is performance weighting function, to shape the foregoing sensitivity function, i.e.,

$$\|W_f(e^{j\omega})S_{\Delta}(e^{j\omega})\|_{\infty} \leq 1 \quad (3.8)$$

Equation (5.17) indicates that if the sensitivity function lies below the inverse of the weighting function W_f in the bode diagram, the robustness performance is guaranteed. Therefore, the selection of W_f is the key to the robust controller design.

3.3 Robust Repetitive Controller Design

Figure 3.4 shows the design structure for repetitive control system, where P_n is the nominal plant model, whose definition can be found in Equation (3.7), $\Delta_u, \Delta_f, W_f \in \mathcal{RH}_\infty$ with $\|\Delta_u\|_\infty \leq 1$ and $\|\Delta_f\|_\infty \leq 1$. W_f is performance weighting function. The repetitive module G_{rep} , shown in Equation (3.9), is plugged into the system.

$$G_{rep}(z) = \frac{k_g \Phi(z) Q(z) z^{-N+m}}{1 - Q(z) z^{-N}} \quad (3.9)$$

Where k_g is the learning gain, N is the period of disturbance and m is the plant delay. Assume $N \gg m$. $Q(z)$ is a unity gain low-pass filter and is introduced here to address robust stability of repetitive control system [123]. $\Phi(z)$ is the compensator.

Figure 3.4: Block Diagram of Robust Repetitive Control System

To guarantee overall system performance, feedback controller needs to be designed in the presence of repetitive controller. Li and Tsao [75] proposed a Robust Repetitive Control (RRC) scheme based on structured singular value to satisfy overall system robustness performance. In this scheme, a fictitious uncertainty is introduced to replace the big delay in the internal module in order to reduce the order of feedback controller which is synthesized by μ theory [146].

From Figure 3.4, the repetitive control system can be converted into the LFT form in Figure 3.5.

Figure 3.5: LFT Form for Repetitive Control System

Define an augmented block structure:

$$\Delta_p := \begin{bmatrix} \Delta_f & 0 \\ 0 & \Delta_u \end{bmatrix}$$

and let

$$P_{aug} = \begin{bmatrix} W_f & W_f P_{n21} & -W_f P_{n22} \\ 0 & P_{n11} & -P_{n12} \\ (1 + G_{rep}) & (1 + G_{rep}) P_{n21} & -(1 + G_{rep}) P_{n22} \end{bmatrix} \quad (3.10)$$

The definition of P_n can be found in Equation (3.7). Then the closed-loop system in Figure 3.5 can be expressed as:

$$G_{aug}(z) = \mathcal{F}_l(P_{aug}(z), K(z)) \quad (3.11)$$

By μ -synthesis [146], we may get H_∞ controller to guarantee overall system robustness performance. However, according to H_∞ optimal control theory, the order of

controller is equal to the highest order of augmented plant P_{aug} . Observe the last row of Equation (3.10), we may find the highest order will definitely be larger than N , which is the period of periodic disturbance. It means that the order of controller K is usually large enough to make the real-time implementation impossible.

There are several techniques to reduce the order of controller: The first one is to simplify the repetitive controller: let $\Phi = 1$, $m = 1$; The second technique is to use balanced realization [146] to reduce the order of K after synthesis.

The most important technique is introducing a fictitious uncertainty Δ_k to replace the big delay z^{-N+m} in the design. Of course this generalization will bring conservatism to RRC system. Suppose that $\Delta_k \in \mathcal{RH}_\infty$ and $\|\Delta_k\|_\infty \leq 1$, it is obvious that z^{-N+m} is an admissible element of Δ_k , i.e., $z^{-N+m} \in \Delta_k$. Then the repetitive control system in Figure 3.4 can be converted into the LFT form shown in Figure 3.6.

Figure 3.6: LFT Form for Robust Repetitive Control Design

Define an augmented block structure:

$$\Delta_{RRC} := \begin{bmatrix} \Delta_f & 0 & 0 \\ 0 & \Delta_u & 0 \\ 0 & 0 & \Delta_k \end{bmatrix}$$

and let

$$P_{RRC} = \begin{bmatrix} W_f & W_f P_{n21} & 0 & -W_f P_{n22} \\ 0 & P_{n11} & 0 & -P_{n12} \\ Q & Q P_{n21} & Q & -Q P_{n22} \\ I & P_{n21} & I & -P_{n22} \end{bmatrix} \quad (3.12)$$

Compare it with Equation 3.10, we may find that the order of augmented plant has been dramatically reduced. Then the closed-loop system in Figure 3.6 can be equivalently expressed as:

$$G_{RRC}(z) = \mathcal{F}_l(P_{RRC}(z), K(z)) \quad (3.13)$$

The robustness performance of repetitive control system in Figure 3.6 can be described by the following theorem [75]:

Theorem 5 *For all $\Delta_u \in \mathcal{RH}_\infty$ with $\|\Delta_u\|_\infty \leq 1$, the loop in Figure 3.6 is robust stable and the robust performance specified by Equation (5.17) is achieved, if*

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RRC}}(G_{RRC}(e^{j\omega})) < 1 \quad (3.14)$$

In the above theorem, the robustness performance is described by the Structured Singular Value or μ theory. The variable μ is the index of system robustness. The definition of μ and introduction of μ theory can be found in [146].

In summary, the synthesis procedure of RRC controller for PWM inverter system is as follows:

- (1) Model the inverter system with nominal linear load, load changes are modelled as uncertainties of plant by LFT;

- (2) Choose the performance weighting function $W_f(e^{j\omega})$ by specific performance requirements;
- (3) Substitute z^{-N+m} in the repetitive module G_{rep} with the fictitious uncertainty Δ_k ;
- (4) Design controller K such that the system of Figure 3.6 by μ -synthesis.

Then, the RRC controlled inverter systems will have the robustness performance as well as periodic disturbance rejection performance in the presence of various loads.

3.4 Experimental Results

Table 3.1 shows the nominal plant parameter values which will be used in controller synthesis. “Uncertain Bound” columns give two parameter uncertainty bounds. Two controllers will be designed according to these two bounds. The comparisons of the two cases will show the tradeoff between system robustness and performance. Table 3.2 shows the actual parameters used in experiments.

Parameter	Nominal Value	Uncertain Bound 1	Uncertain Bound 2
L_n	$20mH$	$[10mH, 25mH]$	$[10mH, 30mH]$
C_n	$45\mu F$	$[40\mu F, 50\mu F]$	$[30\mu F, 60\mu F]$
R_n	18Ω	$[18\Omega, +\infty\Omega]$	$[8\Omega, +\infty\Omega]$

Table 3.1: Nominal Plant Parameters and Uncertain Bounds for Design

Parameter	Value	Parameter	Value
L	$20mH$	L_r	$20mH$
C	$45\mu F$	C_r	$45\mu F$
R	22Ω	R_r	22Ω
E	$70V$	T_s	$0.1mS$
y_d	$50V, 50Hz$	m	1

Table 3.2: Parameters for Experiment and Simulation

Figure 3.7: PWM inverter nominal model with structured uncertainties

3.4.1 Controller Design

Figure 3.7 shows the bode diagram of nominal plant model with various linear resistor loads as well as parameter uncertainties. Since the magnitude responses of inverters with nonlinear loads are similar to inverters with linear loads, we define a notch-filter-like weighting function W_r , which is shown in Equation (3.15).

H_∞ optimal control deals with the issue of worst-case controller design for linear plants subject to unknown disturbances and plant uncertainties. The worst-case performance is bounded by performance weighting function W_f . Two performance weighting functions are defined for the controllers with different uncertain bounds. Performance weighting function associated with Uncertain Bounds 1, which is defined as W_{f1} , is designed with 600Hz cutoff frequency and 30dB low frequency disturbance suppression ratio. Performance weighting function associated with Uncertain Bounds 2, which is defined as W_{f2} , is designed with 500Hz cutoff frequency and 20dB low frequency disturbance suppression ratio. It is obvious that

W_{f1} addresses more disturbances rejection ratio and wider bandwidth, hence the resulting closed-loop system will have better performance, such as THD, convergence time, etc.

Equation (3.16) and (3.17) are implementations of W_{f1} and W_{f2} , respectively. Figure 3.8 shows the chosen unstructured uncertainties weighting and performance weighting functions.

$$W_r(z) = \frac{0.1099z^2 - 0.1943z + 0.08616}{z^2 - 1.943z + 0.9605} \quad (3.15)$$

$$W_{f1}(z) = \frac{1.579z^2 - 0.1863z + 0.005495}{z^2 - 1.764z + 0.7775} \quad (3.16)$$

$$W_{f2}(z) = \frac{0.4697z^2 - 0.5292z + 0.1491}{z^2 - 1.811z + 0.8197} \quad (3.17)$$

Following the previous design flow of RRC, the robust H_∞ feedback controllers can be obtained. With balanced realization [146], the reduced-order controllers are shown in Equation (3.18) and (3.19), where K_1 is the controller with W_{f1} and Uncertain Bound 1 and K_2 is the controller with W_{f2} and Uncertain Bound 2.

$$K_1(z) = \frac{11.22z^4 - 13.65z^3 - 7.617z^2 + 13.72z - 3.526}{z^4 + 0.2173z^3 - 1.762z^2 - 0.1461z + 0.7455} \times 10^{-4} \quad (3.18)$$

$$K_2(z) = \frac{2.303z^4 - 3.426z^3 - 1.054z^2 + 3.434z - 1.24}{z^4 + 0.1851z^3 - 1.756z^2 - 0.163z + 0.7783} \times 10^{-4} \quad (3.19)$$

Figure 3.9 shows the closed-loop sensitivity functions of RRC controlled inverter system with K_1 and K_2 . The sensitivity functions with and without repetitive controller always lie below the inverse of performance weighting function $1/W_f$. According to Equation (5.17), it means that overall system nominal performance is satisfied. Furthermore, the peak μ value of the closed-loop system with K_1 and K_2 are 0.948 and 0.978, respectively. According to Theorem 5, the overall system

Figure 3.8: Unstructured uncertainty and performance weighting functions

robustness performances are also satisfied by both K_1 and K_2 . Note that in Figure 3.9, the additional deep notches, which is introduced by repetitive controller, in blue line address extra suppression to periodic disturbances and hence improve the tracking performance of system.

3.4.2 Experimental Results

Experiment setup has been built for the inverter system shown in Figure 3.10. Facilities include DSPACE (DS1102) DSP development toolkits, H-bridge IGBT switches inverter and oscilloscope.

Figure 3.11 shows the steady state responses of the output voltage $V_c(t)$, load current $i_o(t)$ of the robust feedback controlled inverter system with different loads. Figure 3.11(a) and 3.11(b) show that the THDs under resistor load are 4.2% and 4.8% with K_1 and K_2 respectively. Figure 3.11(c) and 3.11(d) show that the THDs with no load are 2.5% and 2.16% with K_1 and K_2 respectively. Figure 3.11(e) and 3.11(f) show that the THDs with rectifier load are 10.6% and 12.1% with K_1 and K_2 respectively. The robust controller can stabilize the system, but it can not always ensure good tracking performance with various loads.

Figure 3.12 shows the steady state responses of the output voltage $V_c(t)$, load current $i_o(t)$ of the RRC controlled inverter system with different loads. With resistor load, the THDs have been reduced to 2.5% and 2.16% with K_1 and K_2 respectively. The THDs with no loads are 0.45% and 0.42% controlled by K_1 and K_2 , respectively. For rectifier load, the THDs have been reduced to 0.87% and 1.42% with K_1 and K_2 respectively. The results show that RRC controllers K_1 and K_2 can achieve good tracking performance under various loads.

Figure 3.13 shows the transient responses of RRC controlled systems with various loads. With linear loads, the peak errors are reduced from $\pm 4V$ and $\pm 5.5V$ to $\pm 0.5V$ and $\pm 0.5V$ by K_1 and K_2 within 0.15s and 0.1s, respectively. The peak

(a) Sensitivity Function with K_1 (b) Sensitivity Function with K_2

Figure 3.9: RRC System Closed-loop Sensitivity and Performance Weighting Functions

Figure 3.10: Experiment Setup

(a) K_1 with Linear Load (b) K_2 with Linear Load (c) K_1 with No Load (d) K_2 with No Load (e) K_1 with Rectifier Load (f) K_2 with Rectifier Load

Figure 3.11: Feedback Controlled Steady State Responses with Different Loads

(a) K_1 with Linear Load (b) K_2 with Linear Load (c) K_1 with No Load (d) K_2 with No Load (e) K_1 with Rectifier Load (f) K_2 with Rectifier Load

Figure 3.12: RRC Controlled Steady State Responses with Different Loads

errors with no load are reduced from $\pm 4V$ to $\pm 0.5V$ by both K_1 and K_2 within $0.1s$. For rectifier loads, the peak errors are reduced from $\pm 8V$ and $\pm 13V$ to $\pm 0.5V$ and $\pm 1.5V$ by K_1 and K_2 within $0.15s$ and $0.2s$, respectively.

Figure 3.14 shows the transient responses to step load changes with K_2 . The responses of K_1 are similar and omitted here. When system is switched between linear resistor load and no load, the transient state lasts only 4 cycles. Even switched between rectifier load and no load, system returns to steady state within 6 cycles. The transient tracking errors vary within $\pm 4V$.

	Linear Load		No Load		Rectifier Load	
	K_1	K_2	K_1	K_2	K_1	K_2
THD	0.47%	0.42%	0.45%	0.42%	0.87%	1.42%
Peak Error	$\pm 0.5V$	$\pm 0.5V$	$\pm 0.5V$	$\pm 0.5V$	$\pm 0.5V$	$\pm 1.5V$
Convergence Time	0.15s	0.1s	0.1s	0.1s	0.15s	0.2s

Table 3.3: Performance Comparisons of Two RRC Controllers

3.4.3 Discussions

Following the design steps of RRC scheme in Section III, two controllers K_1 and K_2 with different parameter uncertain bounds are developed and applied into the experiments. Both K_1 and K_2 can stabilize the inverter system and achieve satisfactory tracking accuracies and dynamic responses. The theoretic analysis and experimental results demonstrate the effectiveness of RRC scheme.

Table 3.3 shows the performance comparisons of two RRC controllers with different loads. It shows that K_1 and K_2 behave similarly under linear load and no load; whereas, K_1 shows better tracking accuracy and faster convergence rate under rectifier load. This is because that the worse-case performance of RRC system is guaranteed by performance index W_f . W_{f1} indicates -30dB disturbances rejection ratio and 600Hz bandwidth, which is superior to W_{f1} with -20dB disturbances rejection ratio and 500Hz bandwidth. Hence, the resulting RRC controller K_1 has

(a) K_1 with Linear Load (b) K_2 with Linear Load (c) K_1 with No Load (d) K_2 with No Load (e) K_1 with Rectifier Load (f) K_2 with Rectifier Load

Figure 3.13: RRC Tracking Error Histories with Different Loads

(a) $R = +\infty \longrightarrow R = R_n$ (b) $R = R_n \longrightarrow R = +\infty$ (c) $R = +\infty \longrightarrow \text{rectifier load}$ (d) $\text{rectifier load} \longrightarrow R = +\infty$

Figure 3.14: RRC Controlled Response Under Load Changes

better disturbances rejection capability than K_1 . On the other hand, K_2 has better robustness since it tolerates larger parameter uncertain bounds. Furthermore, it is almost impossible to design a RRC controller with W_{f1} and Bound 2 for our specific system. Theoretic analysis and experimental results reveal the tradeoff between robustness and performance for RRC system, that is, better robustness means worse performance and vice versa.

Using H_∞ design method, the RRC scheme guarantees the worse-case performance with respect to parameter uncertainties and load variations. However, H_∞ design often results in high order controller. In the proposed RRC scheme, several techniques, such as introducing fictitious uncertainty, simplifying the repetitive controller and balanced realization, have been successfully employed to reduce the order of the controller.

3.5 Conclusions

In this chapter, a robust repetitive controller design method is applied to CVCF PWM inverter systems. Inverters with uncertainties and different loads are modelled via LFT to formulate to the standard framework. The inverter systems controlled by robust repetitive controller have the robustness performance as well as periodic disturbance rejection performance. To overcome the difficulties in high order controller by conventional design techniques, a fictitious uncertainty is introduced to replace a known function in plug-in repetitive module. Experimental results demonstrate the robustness and performance of the proposed method with various load conditions.

Chapter 4

A Three-Step Design Method for Performance Improvement of Robust Repetitive Control with Application to PWM DC/AC Inverters

This chapter presents a three-step robust design method to improve performance of robust repetitive control systems. The proposed design method generates a repetitive controller and feedback controller sequentially and iteratively. This proposed method enhances the performance of a repetitive control system by extending the bandwidth of low-pass filter Q to reject more high frequency harmonic disturbances as well as to guarantee the overall system robustness performance. In this method, an interim feedback controller K' is designed to enable the Q design in repetitive module. Then, Q filter is designed based on K' to satisfy robustness stability of system. Finally the feedback controller K' is redesigned as K to guarantee the overall system robustness performance. Application of the proposed method to

PWM DC/AC inverter systems is studied in details to illustrate the design procedure and performance. Experiment results demonstrate the effectiveness of the proposed approach.

4.1 Three Step Design for Performance Improvement of Robust Repetitive Control

In this chapter, we propose a design method to generate a repetitive controller and feedback controller sequentially and iteratively. This proposed method enhances the performance of a repetitive control system by extending the bandwidth of low-pass filter Q to reject more high frequency harmonic disturbances as well as to guarantee the overall system robust performance. The proposed method is referred as Three Step Repetitive Control (TSRRC) scheme.

In TSRRC, an interim feedback controller K' is firstly designed based on W_r and W_f . Then, Q filter is designed with the existence of K' to satisfy robustness stability of system. Finally the feedback controller K' is redesigned as K to guarantee the overall system robustness performance. Actually, such iterative design procedure may not limit to three steps and it could continue on the iterations.

4.1.1 Step 1: K' design

A conventional robust optimal feedback controller is designed by μ -synthesis [146] in this step. Figure 4.1 shows a system before the repetitive control module is plugged-in.

The following theorem gives the analysis result of robustness performance [146]:

Theorem 6 *For all $\Delta_r \in \mathcal{RH}_\infty$ with $\|\Delta_r\|_\infty \leq 1$, the loop in Figure 4.1 is well-*

 Figure 4.1: K' Design in TSRRC

posed, internally stable, and $\|\mathcal{F}_u(G_P, \Delta_r)\|_\infty < 1$ if and only if

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_P}(G_P(e^{j\omega})) < 1 \quad (4.1)$$

with

$$G_P = \begin{bmatrix} -T'W_r & -K'S'W_r \\ PS'W_f & S'W_f \end{bmatrix} \quad (4.2)$$

where $S' = 1/(1 + PK')$, $T' = 1 - S'$ and

$$\Delta_P = \begin{bmatrix} \Delta_r & 0 \\ 0 & \Delta_f \end{bmatrix}$$

4.1.2 Step 2: Q filter design

With a repetitive module plugged in, the system has the structure in Figure 4.2. The following theorem gives a convenient Q design method to guarantee robustness stability of repetitive control system.

Theorem 7 Consider the repetitive control system in Figure 4.2. K' is an $\mathcal{H}_\infty$ stabilizing controller to make Equation (4.1) hold. If

$$|Q| < |W_f| \quad (4.3)$$

Figure 4.2: Q design in TSRRC

then the repetitive control system is robustly stable.

Proof 1 Consider the repetitive control system shown in Figure 4.2, define

$$\Delta_s = \begin{bmatrix} \Delta_r & 0 \\ 0 & z^{-N+L} \end{bmatrix}$$

and

$$G_s = \begin{bmatrix} -T'W_r & -K'SW_r \\ PS'Q & S'Q \end{bmatrix} \quad (4.4)$$

The repetitive control system is robustly stable if and only if

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_s}(G_s(e^{j\omega})) < 1 \quad (4.5)$$

On the other hand, from Theorem 6,

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_P}(G_P(e^{j\omega})) < 1 \quad (4.6)$$

Note that $\Delta_s \subset \Delta_P$. If $|Q| < |W_f|$,

$$\begin{bmatrix} I & 0 \\ 0 & W_f^{-1}Q \end{bmatrix} G_P = G_s \implies \|G_s\|_\infty \leq \|G_P\|_\infty$$

Hence,

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_S}(G_s(e^{j\omega})) < \sup_{\omega \in \mathbb{R}} \mu_{\Delta_P}(G_P(e^{j\omega})) < 1 \quad (4.7)$$

This completes the proof.

Remark 1 Theorem 7 can only guarantee the robustness stability of system with additional repetitive control module. System robustness performance can be satisfied only by modifying the feedback controller K' in the third step.

Remark 2 Since only the performance weighting function W_f is used in step 2, one can directly start from step 2 in real applications, without designing K' .

4.1.3 Step 3: K design

The purpose of this step is to address the robustness performance of system. By theorem 5, the redesigned feedback controller K will replace K' in the system in Figure 4.2. Since the original feedback controller K' has been replaced by the K , we need guarantee that this new K can still make the Theorem 6 hold. The following theorem states that this replacement will not cause any instability of repetitive control system.

Theorem 8 Suppose an H_∞ controller K can guarantee the robustness performance of the system in Figure 4.2, i.e.,

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{TSRRC}}(G_{TSRRC}(e^{j\omega})) < 1$$

then the same controller can also guarantee robustness performance of system in Figure 4.1, i.e.,

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_P}(G_P(e^{j\omega})) < 1$$

Where

$$\Delta_{TSRRC} := \begin{bmatrix} \Delta_r & 0 & 0 \\ 0 & \Delta_k & 0 \\ 0 & 0 & \Delta_f \end{bmatrix} \quad G_{TSRRC} = \begin{bmatrix} -TW_r & -KSW_r & -KSW_r \\ PQS & QS & QS \\ PSW_f & -TW_f & SW_f \end{bmatrix}$$

Proof 2 Define

$$\tilde{G}_{TSRRC} = \left[\begin{array}{c|cc} QS & PQS & QS \\ \hline -KSW_r & -TW_r & -KSW_r \\ -TW_f & PSW_f & SW_f \end{array} \right] = \left[\begin{array}{c|c} \tilde{G}_{TSRRC11} & \tilde{G}_{TSRRC12} \\ \hline \tilde{G}_{TSRRC21} & \tilde{G}_{TSRRC22} \end{array} \right] \quad (4.8)$$

If K can guarantee the robustness performance of the system in Figure 4.2, then the following inequality holds,

$$\|\mathcal{F}_u(\tilde{G}_{TSRRC}, \Delta_k)\|_\infty < 1$$

So we have

$$\|G_P\|_\infty = \|\mathcal{F}_u(\tilde{G}_{TSRRC}, 0)\|_\infty = \|\tilde{G}_{TSRRC22}\|_\infty < 1$$

That is

$$\|\mathcal{F}_u(G_P, \Delta_r)\|_\infty < 1$$

From Theorem 6, the robustness performance of system in Figure 4.1 is guaranteed by controller K .

Hence, the design procedure of TSRRC is as follows:

- (1) Design an H_∞ feedback controller K' based on W_r and W_f . If there is no solution, choose different performance index W_f ;

- (2) Choose appropriate Q filter by $|Q| < |W_f|$;
- (3) Redesign the H_∞ feedback controller K .

Remark 3 *The feasibility of controller K' in step 1 can not always guarantee the existence of controller K in step 3. This is because when we design K , there exists additional uncertainty Δ_k introduced by repetitive control module.*

4.2 Application to PWM DC/AC Inverter

The controller design follows the procedures described. Figure 4.3 shows the design result of step 2 in TSRRC method. Compared with the Q filter of RRC, Q filter has extended the pass-band from 600Hz to 1400Hz with sharper cutoff. Note that the bandwidth of Q filter is limited by performance weighting function W_f in this scheme.

Figure 4.3: Q filters and performance weighing function

Figure 4.4 shows the resulting sensitivity function of both TSRRC and RRC approaches. The line with square marks are from the proposed TSRRC method,

which show deeper notches at the fundamental frequencies and harmonics up to 1400Hz. It represents better nominal periodic disturbance rejection performance from TSRRC scheme. Figure 4.4 also shows that both of these two lines lie under the performance weighting function $1/W_f$ and hence the nominal performance of overall system has been guaranteed. The resulting $\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{TSRRC}}(G_{TSRRC}(e^{j\omega})) = 0.997 < 1$ and according to Theorem 5 the robustness performance is achieved.

Figure 4.4: Sensitivity functions comparison

4.2.1 Simulation Results

In the simulations, the repetitive control module is plugged into the system at 0.1s. Figure 4.5 shows the steady state error with TSRRC and RRC methods. Figure 4.6 shows the transient error histories. With only robust feedback controller, the peak of error is 6V; with RRC scheme, the error peak is reduced to 2V and with TSRRC method, it can be suppressed to less than 1V. The Root-Mean-

Figure 4.5: Steady state simulation results with rectifier load

Figure 4.6: Simulation Results: Transient Errors Comparison

Square (RMS) errors of two schemes are shown in Figure 4.7. The RMS error can be suppressed $24dB$ by RRC and over $32dB$ by TSRRC. The simulation results indicate that TSRRC scheme can reduce the tracking error more efficiently.

Figure 4.7: Simulation Results: RMS Errors Comparison

4.2.2 Experiment Results

Figures 4.8(a) and 4.8(b) show the steady state error with only feedback controller. The THD is 14.83%. Figures 5.9(a) and 4.8(e) show the steady state output voltage and load current waveforms comparison with rectifier load. Less waveform distortions can be found in TSRRC controlled voltage. Figures 5.9(b) and 4.8(f) show the harmonics spectrums of output voltages. The THD with RRC is 2.22% while with TSRRC is 0.90%. It is clear that TSRRC controlled voltage contains less harmonics. The transient response of tracking error convergence with RRC and TSRRC are shown in Figure 5.10. The peak of error has been reduced to 3V with RRC and to 2V with RRC. It is clear that the proposed TSRRC method shows better tracking performance over RRC scheme.

4.3 Conclusion

In this chapter, we have shown that the newly proposed three-step robust repetitive controller design method can extend the cutoff frequency of the Q filter, and hence,

(a) Feedback Controlled Output Voltage and Load Current

(b) Feedback Controlled Output Voltage Harmonics Spectrum

(c) RRC Controlled Output Voltage and Load Current

(d) RRC Controlled Output Voltage Harmonics Spectrum

(e) TSRRC Controlled Output Voltage and Load Current

(f) TSRRC Controlled Output Voltage Harmonics Spectrum

Figure 4.8: Steady state response comparison with rectifier load

Figure 4.9: Tracking error histories comparison with rectifier load

suppress more periodic disturbance harmonics to achieve better performance. In the design, both robust stability and performance of system have been appropriately taken into account and specifications can be met in the procedure. Among the three steps in the proposed method, two steps are the design and redesign of feedback controller and this iteration ensures the robustness and performance of the closed-loop system with the plug-in repetitive controller. The application to the PWM DC/AC inverters illustrates the design procedure of the proposed method in details. Simulations and experiments of PWM inverter system are provided to demonstrate the effectiveness of the proposed method.

Chapter 5

Robust Repetitive Control with Linear Phase Lead

In this chapter, we will first discuss the relationship between repetitive control and Iterative Learning Control (ILC) and the understanding of this relationship makes ILC laws available for repetitive controller design. This repetitive control system is then put into the robust optimal control framework. System robust stable and performance conditions are explicitly expressed by structured singular value conditions. All these discussions form the basis of our proposed robust repetitive control with linear phase lead(RRCLPL). The design problem of RRCLPL will be introduced in the next chapter.

5.1 From Frequency Domain ILC to Repetitive Control

5.1.1 Frequency Domain ILC

The objective of ILC is to make the system output tracking a desired trajectory perfectly. The schematic diagram of a typical ILC system can be illustrated in Figure 5.1, in which the off-line ILC calculation is shown in dash line. $y_d(t)$ is the desired output trajectory of the plant, $u_j(t)$ and $y_j(t)$ are the input signal and practical trajectory of the j -th trial, respectively. $u_j(t)$ and $y_j(t)$ are saved in memory for ILC update. The output of ILC $u_{j+1}(t)$ is saved in memory for the next trial. Consider a single input single output (SISO) discrete linear time

Figure 5.1: Configuration of ILC

invariant (LTI) system as follows:

$$\begin{cases} x_j(k+1) = Ax_j(k) + Bu_j(k) + w_j(k) \\ y_j(k) = Cx_j(k) + v_j(k) \end{cases} \quad (5.1)$$

where the subscript j indicates the operation trial, $x(k) \in R^n$ the state vector, $y(k) \in R^p$ the output vector and $u(k) \in R^r$ the input vector, w and v the state disturbance and output disturbance respectively, which are repeated every cycle.

Taking the z transform of (5.1) for the j -th trial

$$Y_j(z) = G(z)U_j(z) + \Upsilon(z) \quad (5.2)$$

where $G(z) = C(zI - A)^{-1}B$ and $\Upsilon(z) \triangleq C(zI - A)^{-1}zx(0) + C(zI - A)^{-1}W(z) + V(z)$

Consider ILC scheme with the update law of

$$U_j(z) = Q(z) [U_{j-1}(z) + K_g z^L \Phi(z) E_{j-1}(z)] \quad (5.3)$$

with $Q(z)$ and z^L being low-pass filter and linear phase lead compensation, respectively. K_g is the learning gain and $\Phi(z)$ is the learning compensator. Then the frequency domain standard stability criterion for ILC ensuring monotonic decay of error is as follows [79]:

$$\|Q(z)[1 - K_g z^L \Phi(z) G(z)]\|_\infty < 1 \quad (5.4)$$

5.1.2 Repetitive Control

For repetitive control system with period of N , the learning law (5.3) should be updated online. Then the update law has the form of:

$$U(z) = Q(z) [z^{-N}U(z) + K_g z^L \Phi(z) z^{-N} E(z)] \quad (5.5)$$

The transfer function of repetitive module is as follows:

$$\frac{U(z)}{E(z)} = \frac{K_g z^{-N+L} \Phi(z)}{1 - Q(z) z^{-N}} \quad (5.6)$$

There are two repetitive control structures: add-on structure and plug-in structure which are shown in Figure 5.2 and Figure 5.3, respectively. It has been proved [111]

that these two structures are mathematically equivalent. In this thesis, the plug-in structure is used and we have the stability condition of repetitive control system as follows:

Figure 5.2: Add-On Repetitive Control System Block Diagram

Figure 5.3: Plug-in Repetitive Control System Block Diagram

Theorem 9 Consider the repetitive control system in Figure 5.3. If

(1) the roots of $1 + K(z)P(z) = 0$ are inside the unit circle;

(2) $\|Q(z)(1 - K_g z^L \Phi(z)T(z))\|_\infty < 1$,

$$\text{for all } z = e^{j\omega}, \omega \in [0, \pi] \quad (5.7)$$

with $T = KP/(1 + KP)$.

then the repetitive control system is stable.

It is clear that the stability condition (5.7) in Theorem 9 will become milder as $Q(z)$ approaches zero, which implies that a $Q(z)$ filter with lower cutoff frequency

will enhance system stability. However, lower cutoff frequency deteriorates periodic disturbances rejection performance and the repetitive control module become useless when $Q=0$.

Remark 4 *The design of low-pass filter $Q(z)$ represents the trade-off between periodic disturbance rejection performance and system robust stability.*

5.2 Repetitive Control Motivated by ILC Linear Phase Lead Design

The basic idea for repetitive control scheme is to design the repetitive control with frequency domain ILC approaches. Linear phase lead ILC [79, 133] has been proved to be a simple and efficient scheme and it will be used into our proposed repetitive control scheme. Consider the repetitive control system in Figure 5.3, we need to design the learning gain K_g , linear phase lead z^L and low-pass filter $Q(z)$.

For the simplicity of repetitive module, the learning compensator $\Phi(z)$ is set to be 1 during the design. Suppose closed-loop complementary sensitivity function $T(z)$ has frequency characteristics $T(e^{j\omega}) = M_T(\omega)e^{j\theta_T(\omega)}$ with $M_T(\omega)$ being its magnitude and $\theta_T(\omega)$ being its phase. Assume $Q(z) = 1$, then the repetitive control system stability condition (5.4) leads to

$$K_g M_T(\omega) < 2 \cos(\omega L + \theta_T(\omega)) \quad (5.8)$$

By realizing $K_g > 0$ and $M_T(\omega) > 0$, Equation (5.8) leads to [134]

$$-\frac{\pi}{2} < \omega L + \theta_T(\omega) < \frac{\pi}{2} \quad (5.9)$$

For stable closed-loop system, the phase response is within $[-\pi, 0]$ and the linear phase lead compensator z^L provides linear phase shift which is 0 rad at $\omega = 0$ and

$L\pi$ rad at Nyquist frequency. Hence, the phase condition (5.9) can be satisfied by choosing appropriate lead step L . Zero-phase low-pass filter $Q(z)$ cuts off the frequency components that violate the phase condition (5.9). Hence the cutoff frequency ω_c of $Q(z)$ can be determined as follows, where ϵ provides a margin for system robustness.

$$\omega_c = \max_{\omega \in \mathbb{R}^+} \{|\omega L + \theta_T(\omega)| \leq \frac{\pi}{2} - \epsilon\} \quad (5.10)$$

If phase condition (5.9) is satisfied, condition (5.8) can always be satisfied by choosing appropriate learning gain K_g :

$$0 < K_g < \frac{2 \cos(\omega L + \theta_T(\omega))}{M_T(\omega)} \quad (5.11)$$

Hence the three parameters L , $Q(z)$ and K_g can be designed by Equation (5.9), (5.10) and (5.11), respectively.

Remark 5 *The enhancement of repetitive control system performance lies in two ways: on the one hand, it compensates the phase lag of closed-loop feedback system and can be regarded as approximate zero phase error tracking compensation [123] with low costs; on the other hand, it admits wider Q filter bandwidth, which helps to reject more periodic disturbance harmonics.*

Due to the existence of plant uncertainties, it is not easy to design the repetitive controller. In this case, a robust repetitive control scheme will be further proposed.

5.3 Robust Stable Repetitive Control

The robust stable repetitive control system is shown in Figure 5.4.

Figure 5.4: Robust Stable Repetitive Control System

Define

$$\begin{aligned}
 G_{RSRLC} &= \left[\begin{array}{c|c} -TW_r & -KS W_r \\ \hline K_g P Q S & Q(z^{-L} - K_g T) \end{array} \right] \\
 &= \left[\begin{array}{c|c} G_{RSRLC11} & G_{RSRLC12} \\ \hline G_{RSRLC21} & G_{RSRLC22} \end{array} \right], \tag{5.12}
 \end{aligned}$$

and

$$\Delta_{RSRLC} = \left[\begin{array}{cc} \Delta_r & 0 \\ 0 & z^{-N+L} \end{array} \right] \tag{5.13}$$

where

$$S = \frac{1}{1 + KS}, \quad T = \frac{KS}{1 + KS}$$

Then the robust stability of repetitive control system can be described by the following theorem.

Theorem 10 *The repetitive control system in Figure 5.4 is robustly stable for all $\Delta_r \in \mathcal{RH}_\infty$ with $\|\Delta_r\|_\infty \leq 1$, if*

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RSRLC}}(G_{RSRLC}(e^{j\omega})) < 1, \tag{5.14}$$

Proof 3 Since $z^{-N+L} \in \mathcal{RH}_\infty$ and $\|z^{-N+L}\|_\infty = 1$, we have $\Delta_{RSRLC} \in \mathcal{RH}_\infty$ and $\|\Delta_{RSRLC}\|_\infty = 1$. The linear fractional transformation (LFT) form [146] of robust stable repetitive control system in Figure 5.4 can be illustrated in Figure 5.5

Figure 5.5: LFT Form for Robust Stable Repetitive Control System

where

$$P_{RSRLC} = \begin{bmatrix} 0 & 0 & -W_r \\ K_g P Q & z^{-L} Q & -K_g P Q \\ P & I & I \end{bmatrix} \quad (5.15)$$

By the definition of LFT, we have

$$G_{RSRLC} = \mathcal{F}_l(P_{RSRLC}, K)$$

Then according to [146], the theorem is proved.

Theorem 11 Consider the repetitive control system in Figure 5.4. If Theorem 10 holds, Theorem 9 will also be satisfied.

Proof 4 According to the main loop theorem (Theorem 10.6 in [146]),

$$\begin{aligned} \mu_{\Delta_{RRCs}}(G_{RRCs}(e^{j\omega})) < 1 & \Leftrightarrow \\ \left\{ \begin{array}{l} \mu_{\Delta_r}(G_{RRCs11}(e^{j\omega})) < 1, \quad \text{and} \\ \max_{\|\Delta_r\|_\infty \leq 1} \mu_{z^{-N+L}}(\mathcal{F}_u(G_{RRCs}(e^{j\omega})), \Delta_r) < 1 \end{array} \right. & \quad (5.16) \end{aligned}$$

From the first part of Equation (5.16),

$$\mu_{\Delta_r}(G_{RRCS11}(e^{j\omega})) < 1 \Leftrightarrow \|T_N W_r\|_\infty < 1$$

It means original feedback control system with multiplicative uncertainties is robustly stable, i.e., the roots of $1 + K(z)P_\Delta(z) = 0$ are inside the unit circle.

On the other hand, by the definition of LFT, the second part of Equation (5.16) leads to

$$\begin{aligned} \max_{\|\Delta_r\|_\infty \leq 1} \mu_{z^{-N+L}}(\mathcal{F}_u(G_{RRCS}(e^{j\omega})), \Delta_r) &< 1 \\ \Leftrightarrow \|Q(z)z^{-N+L}[1 - K_g z^L \Phi(z)T_\Delta(z)]\|_\infty &< 1 \end{aligned}$$

Here is the condition in (5.7). This completes the proof.

Remark 6 Theorem 9 is the pervasive stability condition for repetitive control system. However, it is not convenient to use such criterion to design repetitive module because of the existence of uncertainties. Theorem 10 supplies tractable criterion for the design of repetitive control system.

Equation (5.14) implies $\|G_{RSRLC}\|_\infty < 1$. Note that $\|G_{RSRLC11}\|_\infty < 1$ means robust stability of original feedback control system and it is the counterpart of condition 1 in Theorem 9. $\|G_{RSRLC22}\|_\infty < 1$ is exactly the condition (5.7) in Theorem 9. However, $\|G_{RSRLC11}\|_\infty < 1$ and $\|G_{RSRLC22}\|_\infty < 1$ are only necessary for $\|G_{RSRLC}\|_\infty < 1$ holds. Hence, we have the following remark:

Remark 7 According to Theorem 10, the robust stability of original feedback system plus repetitive stability condition (5.7) does not automatically guarantee robust stability of overall repetitive control system.

In summary, the design procedure of the robust stable repetitive control scheme is as follows:

- (1) Design a feedback controller K to achieve robust stability of system without repetitive module, i.e., $\|TW_r\|_\infty < 1$.
- (2) Design the repetitive controller, i.e., determine the parameters: K_g , L and $Q(z)$.
- (3) Check whether the condition (5.14) is satisfied. If not, go back to step 2 and redesign the repetitive controller.

5.4 Robust Performance Repetitive Control

To achieve overall system robust performance, the feedback controller needs to be designed in the existence of repetitive module. Since the robust stable repetitive control scheme has determined the repetitive module, the overall system robust performance can be achieved by redesigning the feedback controller K_P to replace the original K in Figure 5.4.

The robust performance repetitive control system is shown in Figure 5.6, where the repetitive module has been determined and K_P is the feedback controller to be designed. In addition, $\Delta_f \in \mathcal{RH}_\infty$ with $\|\Delta_f\|_\infty \leq 1$ and the performance objective is specified by W_f :

$$\|W_f(e^{j\omega})S(e^{j\omega})\|_\infty \leq 1 \quad (5.17)$$

According to H_∞ optimal control theory [146], the order of controller is equal to the order of augmented plant. Because of the big delay N , the augmented plant including repetitive control module will have very high order. In order to reduce the order of controller, a fictitious uncertainty Δ_k is introduced to replace the big

Figure 5.6: Robust Performance Repetitive Control System

delay z^{-N+L} in the design. Suppose $\Delta_k \in \mathcal{RH}_\infty$ and $\|\Delta_k\|_\infty \leq 1$, it is obvious that z^{-N+L} is an admissible element of Δ_k , i.e., $z^{-N+L} \in \Delta_k$.

Define

$$G_{RPRLC} =$$

$$\begin{bmatrix} -T_P W_r & -K_P S_P W_r & -K_P S_P W_r \\ K_g P Q S_P & Q(z^{-L} - K_g T_P) & K_g Q S_P \\ P S_P W_f & -T_P W_f & S_P W_f \end{bmatrix} \quad (5.18)$$

with

$$S_P = \frac{1}{1 + K_P P} \quad T_P = 1 - S_P$$

and

$$\Delta_{RPRLC} = \begin{bmatrix} \Delta_r & 0 & 0 \\ 0 & \Delta_k & 0 \\ 0 & 0 & \Delta_f \end{bmatrix} \quad (5.19)$$

Then the robust performance of system can be described by the following theorem.

Theorem 12 *The repetitive control system in Figure 5.6 is robustly stable and robust performance specified by (5.17) is achieved for all $\Delta_r \in \mathcal{RH}_\infty$ with $\|\Delta_r\|_\infty \leq$*

1, if

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RPRLC}}(G_{RPRLC}(e^{j\omega})) < 1, \quad (5.20)$$

Proof 5 Since $z^{-N+L} \in \Delta_k$, we have

$$\bar{\Delta}_{RPRLC} = \begin{bmatrix} \Delta_r & 0 & 0 \\ 0 & z^{-N+L} & 0 \\ 0 & 0 & \Delta_f \end{bmatrix} \in \Delta_{RPRLC}$$

then,

$$\begin{aligned} \sup_{\omega \in \mathbb{R}} \mu_{\bar{\Delta}_{RPRLC}}(G_{RPRLC}(e^{j\omega})) \\ \leq \sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RPRLC}}(G_{RPRLC}(e^{j\omega})) \end{aligned}$$

Hence if

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RPRLC}}(G_{RPRLC}(e^{j\omega})) < 1$$

the overall system robust performance can be achieved.

Hence, the design procedure of robust performance repetitive control is as follows:

- (1) Design a robust stable repetitive control system following the previous procedures;
- (2) Redesign the feedback control K_P to satisfy Equation (5.20) and replace original controller K . If it results in no feasible controller in this step, choose different performance index W_f . If there is always no solution with a reasonable W_f , go back to step 1 of robust stability design and choose a different W_r .

5.5 Experiment Results

Follow the design procedure of robust stable repetitive control scheme, the H_∞ controller is designed by μ -synthesis. It can stabilize the system since $\|T_N W_r\|_\infty = 0.499 < 1$ (Step 1 of robust stable repetitive control). Figure 5.7 shows the design result of robust stable repetitive control with linear phase lead $L = 1$ (Step 2 of robust stable repetitive control). The compensated phase response is always limited within $\pm\pi/2$. Hence from Equation (5.10) the cutoff frequency of $Q(z)$ can be set to be $\omega_c = 0.9$ (in normalized frequency), which is compared with $L = 0$ and $\omega_c = 0.3$. From previous discussion, wider bandwidth of $Q(z)$ ensures better tracking accuracy. Figure 5.7 also shows the curve of $2\cos(\omega L + \theta_T(\omega))/M_T(\omega)$ for $L = 1$ and $L = 0$. For $L = 1$, the minimum value is 2, which is the largest admissible K_g . Here we choose $K_g = 1.0$. For $L = 0$, K_g can only be chosen as 0.5, which results in slower convergence speed. The resulting $\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RRCS}}(G_{RRCS}(e^{j\omega})) = 0.683 < 1$, hence, the repetitive control system is robustly stable.

Figure 5.7: Robust Stable Repetitive Control with Linear Phase Lead

Figure 5.8 shows the design results of robust performance repetitive control, where W_f is the desired performance index in (5.17). The deep notch in sensitivity function means repetitive control module's extra suppression to periodic disturbances. The line with circle marks are from the RRC method. It shows that our proposed method offers better periodic disturbance rejection performance at high frequency than RRC, which will be further illustrated in the experimental results. The resulting $\sup_{\omega \in \mathbb{R}} \mu_{\Delta_{RRCP}}(G_{RRCP}(e^{j\omega})) = 0.996 < 1$ and the robust performance is satisfied.

Figure 5.8: Sensitivity Function of Repetitive System Comparison

Figure 5.9(a) and 5.9(c) show the steady state output voltage and load current waveforms comparison with rectifier load. Less waveform distortions can be found in RRCLPL controlled voltage. Figure 5.9(b) and 5.9(d) show the harmonics spectrums of output voltages. The total harmonic distortion (THD) with only feedback controller is 10.11%. The THD with RRC is 4.84% while with RRCLPL is 1.20%. It shows that repetitive control is an effective approach for harmonics cancelation and it is clear that the RRCLPL controlled voltage contains less harmonics than RRC scheme. The transient response of tracking error convergence with RRC and

(a) RRC Controlled Output Voltage and Load Current

(b) RRC Controlled Output Voltage Harmonics Spectrum

(c) RRLC Controlled Output Voltage and Load Current

(d) RRLC Controlled Output Voltage Harmonics Spectrum

Figure 5.9: Steady state response comparison with rectifier load

RRCLPL are shown in Figure 5.10. The peak of error has been forced to 2V with RRC and to 1V with RRCLPL. Since RRCLPL admits larger learning gain, the transient state lasts only 0.2s, which compared with 0.6s in RRC controlled output. It is clear that the proposed RRCLPL method shows better tracking performance over RRC scheme.

5.6 Conclusion

A robust repetitive control scheme is addressed in this chapter. Linear phase lead compensation, which originally comes from iterative learning controller design, has been introduced to compensate the phase lag of feedback system and extend the cutoff frequency of the Q filter. More periodic disturbance harmonics can

(a) RRC controlled tracking error history

(b) RRLC controlled tracking error history

Figure 5.10: Tracking error histories comparison with rectifier load

be eliminated to get better performance. Robust stability and performance of the proposed scheme are analyzed and design method is discussed. Experimental results verify the effectiveness of the proposed scheme.

Chapter 6

Fractional Delay Based Robust Repetitive Control with Application to PWM DC/AC Inverter

In repetitive control system, the period of exogenous signals must be the integer number of sampling points. However, it can not be always satisfied in real applications. In this chapter, a systematic approach for non-integer delay repetitive control system with fixed sampling rate is proposed. The proposed fractional delay based repetitive control scheme employs the design techniques in digital signal processing theory and two different implementation structures are presented. One is easy to design and the other has optimized performance. Application of the proposed method to PWM DC/AC inverter systems is studied to illustrate the design procedure and performance. Experimental results demonstrate the effectiveness of the proposed approach.

6.1 Introduction

In repetitive control system, the period of exogenous signals must be the integer number of sampling points. In real applications, however, it can not be always satisfied. Moreover, in many applications, the periodic signal changes dynamically in period so the period can never always be integer. For the non-integer delay cases, there are two simple approaches to handle it: 1) Adjust the sampling rate of repetitive control system. This method is easy but not practical in general, since it requires to redesign the whole control system. 2) Round to the nearest integer. It is the most conventional method and easy to implement, but with limited performance. On the other hand, [126, 114] studied the similar phenomena and proposed different algorithms to reject uncertain periodic signals.

In this chapter, a Fractional Delay based Repetitive Control (FD-RC) scheme is proposed. FD-RC supplies a systematic approach for non-integer delay case in repetitive control system with fixed sampling rate. This method is based on the fractional delay filter design [63, 145], which is a technique for bandlimited interpolation between samples, in digital signal processing theory. Two different realizations are introduced in this chapter, one realization is Lagrange Interpolator method, which can be considered as the comprehensive version of linear interpolation method; the other is least square FIR optimized approximation method. The advantage of Lagrange Interpolator method lies in easy explicit formulas for the coefficients and good approximations at low frequency range. The least square FIR method is, in principle, with the smallest least square error in the defined approximation band. The repetitive controller with higher order interpolation is proposed in [139], and the performance and stability issues are further discussed in [51]. A similar optimal interpolation approach was also proposed for repetitive control in [144] with application to repetitive acoustic noise cancelation.

A possible application of proposed FD-RC could be auto-tuning or adaptive repetitive control system, i.e., the period of signal is changing with time. The online esti-

mation of period could be non-integer number of sampling points. Some results of adaptive repetitive control scheme or similar topics were reported in [3, 39, 127, 44].

6.2 Period Sensitivity Analysis

Figure 6.1 shows the block diagram of conventional plug-in repetitive control system. Where $P(z)$ is the nominal plant model. The periodic signal generator

Figure 6.1: Block Diagram of Repetitive Control System

consists of $Q(z)$, z^{-N+L} and z^{-L} , where N is an integer and denotes the period of disturbance. L is the linear phase lead. Assume $N \gg L$. $Q(z)$ is a unity gain low-pass filter. K_g is learning gain and $\Phi(z)$ is a compensator. K_g , L , $\Phi(z)$ and $Q(z)$ are parameters to be determined during the design of a repetitive control system. The sensitivity function can be easily obtained,

$$\begin{aligned}
 S_{rep}(z) &= \frac{E(z)}{D(z)} \\
 &= \frac{1 - Q(z)z^{-N}}{1 - Q(z)z^{-N}[1 - K_g z^L T(z)\Phi(z)]} \\
 &= \frac{1 - Q(z)z^{-N}}{1 - Q(z)z^{-N}\Gamma(z)}
 \end{aligned} \tag{6.1}$$

where $T(z) = K(z)P(z)/(1 + K(z)P(z))$.

The repetitive control requires the exact knowledge of the period-time of the external signals. The tracking performance is highly dependent on the exactness of

period-time N . The relationship can be illustrated from the the *period sensitivity function* $S_p(e^{j\omega})$, which is defined as follows,

$$\begin{aligned} S_p(e^{j\omega}) &= \lim_{\Delta N \rightarrow 0} \frac{\Delta S_{rep}(e^{j\omega})/S_{rep}(e^{j\omega})}{\Delta N/N} \\ &\approx \frac{2\pi Q(e^{j\omega})e^{-jN\omega}[1 - \Gamma(e^{j\omega})]}{[1 - Q(e^{j\omega})e^{-jN\omega}\Gamma(e^{j\omega})][1 - Q(e^{j\omega})e^{-jN\omega}]} \end{aligned} \quad (6.2)$$

Period sensitivity function $S_p(e^{j\omega})$ evaluates the sensitivity of $S_{rep}(e^{j\omega})$ to the perturbation in N . It is easy to find that if $\omega = (2k+1)\pi/N$, $S_p(e^{j\omega}) = 0$, which means that small change of N will not effect S_{rep} at this frequency point. If $\omega = 2k\pi/N$, $S_p(e^{j\omega}) = \infty$. It means that S_{rep} is rather sensitive to N at the neighborhood of $\omega = 2k\pi/N$, i.e., small perturbation of N will effect the tracking performance dramatically. Where $k = 0, 1, \dots, N/2 - 1$ for N even or $k = 0, 1, \dots, (N-1)/2$ for N odd. Numerical examples show that 1% variation of N will degrade the error suppression ratio from $-\infty$ dB to -20 dB at fundamental frequency, i.e., $k = 1$. It can be even worse with k increasing.

In real applications, the period $N = F/F_s$ can not always be integer, where F is the fundamental frequency of reference signal and F_s is the sampling frequency, and it is often rounded to the nearest integer. However, from the pervious analysis, the performance is sacrificed.

6.3 Fractional Delay based Repetitive Control

In this chapter, the Fractional Delay based Repetitive Control (FD-RC) scheme, which is based on the fractional delay filter design theory in digital signal processing [63], is proposed for both integer and non-integer period applications. In this scheme, a fractional delay low-pass FIR filter $Q_{FD}(z)$ is introduced to replace the original $Q(z)$ filter in Figure 6.1 and the performance with non-integer period

is improved. Two different $Q_{FD}(z)$ design methods are introduced: Lagrange Interpolator method and least square method. Lagrange Interpolator method has simple implicit formula and is easy to implement, but with limited performance. Least square method has the smallest least square error in the defined approximation band and hence has better performance.

6.3.1 Problem Formulation

Define N as the integer part of D and d as the fractional part, i.e.,

$$D = N + d = \lfloor D \rfloor + d \quad (6.3)$$

where $0 \leq d < 1$. Here we need only consider the fractional part d . The transfer function of an ideal delay element may be written as

$$H_i(z) = z^{-d} \quad (6.4)$$

The magnitude response of (6.4) is unity for all frequencies, while the phase response is linear with a slope of $-d$. The corresponding impulse response is obtained via inverse discrete-time Fourier transform,

$$h_i(n) = \frac{\sin[\pi(n-d)]}{\pi(n-d)} = \text{sinc}(n-d) \quad (6.5)$$

Unfortunately, the ideal impulse response is not only infinitely long but also non-causal, which makes it impossible to implement it in real-time applications. A finite-order FIR filter can be used to approximate the ideal response in Equation (6.3), i.e.,

$$z^{-d} \approx H(z) = \sum_{n=0}^N h(n)z^{-n} \quad (6.6)$$

where N is the order of filter. It is now desired to determine the coefficients $h(n)$ such that the chosen norm of the frequency-domain error function

$$E(e^{j\omega}) = H(e^{j\omega}) - H_i(e^{j\omega}) \quad (6.7)$$

is minimized.

6.3.2 Lagrange Interpolator Method

The error function can be made maximally flat at a certain frequency, typically at $\omega_0 = 0$, so that the approximation is at its best close to this frequency. This means that the derivatives of frequency-domain error function shown in Equation (6.7) are set to zero at this point, that is

$$\left. \frac{d^n E(e^{j\omega})}{d\omega^n} \right|_{\omega=\omega_0} = 0 \text{ for } n = 0, 1, \dots, N \quad (6.8)$$

The solution of Equation (6.8) is equal to the classical Lagrange interpolation formula and given in an explicit form as

$$h_{li}(n) = \prod_{k=0, k \neq n}^N \frac{D - k}{n - k} \text{ for } n = 0, 1, \dots, N \quad (6.9)$$

The case $N = 1$ corresponds to linear interpolation between two samples. In this case the two coefficients are

$$h(0) = 1 - D, \quad h(1) = D$$

Hence, in this realization,

$$Q_{FD}^{LI}(z) = H_{LI}(z)Q(z) \quad (6.10)$$

where $H_{LI}(z)$ is the z-transformed $h_{li}(n)$ and $Q(z)$ is the zero-phase low-pass filter.

Lagrange interpolation method is easy to implement. However, we may find that it is far from the ideal all-pass filter since $H_{LI}(e^{j\omega}) = 1$ only if $\omega = 0$. It means that the sensitivity function (6.1) is not zero even at low frequency range. Hence, Lagrange interpolation is with limited performance.

6.3.3 Least Square Method

In this method, we combine the design of fractional delay filter z^{-d} and the low-pass filter $Q(z)$ together, i.e.,

$$H_{id}(z) = z^{-d}Q(z) \quad (6.11)$$

where $Q(z)$ is zero-phase low-pass filter with bandwidth ω_p . Then $H_{id}(z)$ is a complex-valued function that specifies both the magnitude and the phase responses as

$$\begin{cases} \|H_{id}(e^{j\omega})\| = 1, & \text{for } \omega \leq \omega_p \\ \|H_{id}(e^{j\omega})\| = 0, & \text{for } \omega > \omega_p \\ \arg\{H_{id}(e^{j\omega})\} = -d\omega, & \text{for } \omega \leq \omega_p \end{cases} \quad (6.12)$$

where $0 \leq \omega_p \leq \pi$.

It is now desired to determine the coefficient h_{ls} such that the L_2 -norm of the frequency domain error function $E'(e^{j\omega})$

$$\begin{aligned} E'(e^{j\omega}) &= \frac{1}{\pi} \int_0^{\omega_p} |Q_{FD}(e^{j\omega}) - H_{id}(e^{j\omega})|^2 d\omega \\ &\quad + \frac{1}{\pi} \int_{\omega_p}^{\pi} |Q_{FD}(e^{j\omega})|^2 d\omega \end{aligned} \quad (6.13)$$

is minimized. To formulate the solution in a compact form, introduce vector notation as

$$\mathbf{h}_{ls} = [h_{ls}(0) \ h_{ls}(1) \cdots h_{ls}(M)] \quad (6.14)$$

$$\mathbf{e} = [1 \ e^{-j\omega} \cdots e^{-jM\omega}] \quad (6.15)$$

$$\mathbf{C} = Re\{\mathbf{e}\mathbf{e}^H\} \quad (6.16)$$

Then Equation (6.13) is rewritten as

$$\begin{aligned} E'(e^{j\omega}) &= \frac{1}{\pi} \int_0^{\omega_p} [\mathbf{h}_{ls}\mathbf{e} - H_{id}(e^{j\omega})][\mathbf{h}_{ls}\mathbf{e} - H_{id}(e^{j\omega})]^* d\omega \\ &\quad + \frac{1}{\pi} \int_{\omega_p}^{\pi} [\mathbf{h}_{ls}\mathbf{e}][\mathbf{h}_{ls}\mathbf{e}]^* d\omega \\ &= \frac{1}{\pi} \int_0^{\omega_p} [\mathbf{h}_{ls}^T \mathbf{C} \mathbf{h}_{ls} - 2\mathbf{h}_{ls}^T Re\{H_{id}(e^{j\omega})e^*\} \\ &\quad + |H_{id}(e^{j\omega})|^2] d\omega + \frac{1}{\pi} \int_{\omega_p}^{\pi} [\mathbf{h}_{ls}^T \mathbf{C} \mathbf{h}_{ls}] d\omega \\ &= \mathbf{h}_{ls}^T \mathbf{P} \mathbf{h}_{ls} - 2\mathbf{h}_{ls}^T \mathbf{P}_1 + \mathbf{P}_0 \end{aligned} \quad (6.17)$$

where

$$\begin{aligned} \mathbf{P} &= \frac{1}{\pi} \int_0^{\pi} \mathbf{C} d\omega \\ \mathbf{P}_1 &= \frac{1}{\pi} \int_0^{\omega_p} [Re\{H_{id}(e^{j\omega})\}\mathbf{c} - Im\{H_{id}(e^{j\omega})\}\mathbf{s}] d\omega \\ \mathbf{P}_0 &= \frac{1}{\pi} \int_0^{\pi} |H_{id}(e^{j\omega})|^2 d\omega \\ \mathbf{c} &= [1 \ \cos(\omega) \cdots \cos(M\omega)]^T \\ \mathbf{s} &= [0 \ \sin(\omega) \cdots \sin(M\omega)]^T \end{aligned}$$

The error function in Equation (6.17) is quadratic with a unique minimum-error solution. It can be derived by setting the derivative with respect to $\mathbf{h}_{ls}$ to zero, then we have

$$\mathbf{h}_{ls} = \mathbf{P}^{-1}\mathbf{P}_1 \quad (6.18)$$

Here is the optimal solution for the least square error function (6.13). Hence, the least square realization of $Q_{FD}(z)$ is as follows,

$$Q_{FD}^{LS}(z) = H_{LS}(z) \quad (6.19)$$

where $H_{LS}(z)$ is the z-transformed $\mathbf{h}_{ls}$

Compared with Equation (6.10), Least square realization has better performance at low frequency range. Although the matrix inversion in (6.18) requires lots of computations, it can be calculated off-line. In addition, H_{LS} itself is a low-pass filter and does not need cascade additional low-pass filter $Q(z)$, which helps to reduce the computational cost of repetitive controller.

6.4 Application to CVCF PWM DC/AC Inverters

The effectiveness of the proposed FD-RC scheme is illustrated by the application to constant-voltage-constant-frequency pulse-width modulated (CVCF PWM) DC/AC inverter systems.

The system is designed to track an ideal sine wave with 60Hz frequency and the sampling rate is 0.1ms. Hence, a non-integer delay $500/3$ is required. In conventional RC scheme, it will be rounded to the nearest integer, i.e., $N = 167$. Other repetitive controller parameters are chosen as follows: $L = 2$, $\Phi(z) = 1$

and $Q = (z + 8 + z^{-1})/10$. On the other hand, in our proposed FD-RC scheme, $N = 166$, $d = 2/3$, $\omega_p = 1.6\pi$ and a 4th order fractional delay low-pass FIR filter $Q_{FD}(z)$ will be designed. The feedback controller design follows the Three-Step Robust Repetitive Controller (TSRRC) design procedure introduced in Chapter 4. When Q_{FD} is designed by Lagrange Interpolation method, we have

$$\begin{aligned} Q_{FD}^{LI}(z) &= H_{LI}(z)Q(z) \\ &= -0.0111 - 0.0333z^{-1} + 0.4889z^{-2} + 0.5000z^{-3} \\ &\quad + 0.0556z^{-4} \end{aligned} \tag{6.20}$$

When Q_{FD} is designed by Least square method, we have

$$\begin{aligned} Q_{FD}^{LS}(z) &= H_{LS}(z) \\ &= 0.0292 - 0.1518z^{-1} + 0.4681z^{-2} + 0.6839z^{-3} \\ &\quad - 0.0294z^{-4} \end{aligned} \tag{6.21}$$

The comparison of these two realizations is shown in Figure 6.2. It shows that least square realization has better approximation at low frequency, which coincides with the theoretical analysis in section II.

Plug (6.20) and (6.21) into Equation(6.1), we have the closed-loop sensitivity function. Figure 6.3 shows the sensitivity function comparison between conventional RC and Least square FD-RC. Conventional RC (dashed line) places the first notch at 59.88Hz and the attenuation ratio at 60Hz is about -60dB; In the sensitivity function of Least square FD-RC (Solid line), the first notch appears exactly at 60Hz and $S_{rep}(e^{j\omega}) \approx 0$ with $\omega = 120\pi$. This figure shows the effectiveness of proposed fractional delay based repetitive controller with least square realization. Figure 6.4 shows the comparison between least square and Lagrange Interpola-

Figure 6.2: Q_{FD}^{LI} and Q_{FD}^{LS} Comparison

tion implementations. Both of the two implementations have notches at exactly 60Hz and the harmonics, which meet our design objective. However, least square method has better performance since it has deeper notches, which is corresponding to better disturbance rejection performance at 60Hz and harmonics. The better performance of least square implementation comes from the better approximation at low frequency, as shown in Figure 6.2.

The experiment setup of the PWM inverter system consists of DSPACE (DS1102) DSP development toolkits, H-bridge IGBT switches inverter, oscilloscope and etc.

Figure 6.5(a), 6.6(a) and 6.7(a) show the steady state output voltage and load current waveforms comparison with rectifier load. Less waveform distortions can be found in FD-RC controlled voltage. Figure 6.5(b), 6.6(b) and 6.7(b) show the harmonics spectrums of output voltages. The THD with conventional RC is 9.28% while with LS FD-RC is 1.52% and with LI FD-RC is 1.67%. It is clear that the FD-RC controlled voltage contains less harmonics than conventional RC scheme. The LS FD-RC is a little bit better than LI FD-RC. The transient response of

Figure 6.3: Sensitivity Function Comparison

Figure 6.4: Sensitivity Function Comparison

Figure 6.5: Conventional RC Steady State Response with Rectifier Load

Figure 6.6: Least Square FD-RC Steady State Response with Rectifier Load

tracking error convergence with conventional RC and FD-RC are shown in Figure 6.8 6.9 and 6.10, respectively. The peak of error has been forced to 8V with conventional RC and to 1V with proposed FD-RC. Since FD-RC admits larger learning gain, the transient state lasts less than 0.1s, as compared with 0.3s in conventional RC controlled output. It is clear that the proposed FD-RC method shows much better tracking performance over the conventional scheme.

Figure 6.7: Lagrange Interpolation FD-RC Steady State Response with Rectifier Load

Figure 6.8: Conventional RC controlled tracking error history

Figure 6.9: LS RC controlled tracking error history

Figure 6.10: LI RC controlled tracking error history

6.5 Conclusion

In this chapter, we have shown that the newly proposed fractional delay based repetitive control scheme for both integer and non-integer delay cases. By using fractional delay filter design theory in digital signal processing, two different structures are introduced and the performances are compared. The Lagrange Interpolation method is easy to design and implement whereas Least square method has better performance. Experiments of PWM inverter system are provided to demonstrate the effectiveness of the proposed method.

Chapter 7

Conclusion and Recommendations

This chapter presents conclusions of the thesis and leave some open questions for future work.

7.1 Conclusions

In this thesis, a general framework of robust repetitive learning control was constructed, and was developed into practical methodology by applying it to CVCF PWM inverter systems.

In the first part, which consists of Chapters 2 and 3, some backgrounds of this dissertation was introduced, including H_∞ robust control theory and modelling of PWM inverters. In Chapter 3, load changes were modelled as plant uncertainties by Linear Fractional Transformation and hence less conservative uncertainty bounds could be generated.

In the second part, including Chapters 4 and 5, an iterative and consequential design sequence of robust repetitive controller was first introduced to improve periodic disturbance rejection performance. Based on the same scheme, robust

repetitive learning controller, which employs ILC design methodologies to achieve better performance, was introduced. Linear phase lead compensation was employed in the repetitive controller to render higher bandwidth of controller and larger learning gain. Overall system robustness performance was also guaranteed.

In the last part of dissertation, i.e. Chapter 6, another issue of repetitive controller design was discussed: non-integer delay repetitive control system with fixed sampling rate. A systematic fractional delay based repetitive control scheme were proposed and two different implementation structures were presented. Much better performance could be achieved by just small modifications of conventional repetitive controllers.

As a conclusion, this dissertation succeeded in constructing a general framework of robust repetitive controller design. Some practical issues of implementation were also discussed and could be applied to many other applications.

7.2 Recommendations for further research

The research in robust repetitive learning control presented in this thesis is just some preliminary results. There are still much space for extensions and improvements. Some possible research topics are suggested as follows:

7.2.1 RRLC based on LMI

Linear matrix inequalities (LMIs) have emerged as a powerful formulation and design technique for a variety of linear control problems [4]. We may design the robust repetitive learning controller via LMI approach. Compared with μ theory, the advantages of LMI approach can be concluded as following:

- (1) Although one cannot usually find an *analytic solution* to one's system or

control theory problem, if one can express it as a solution to a LMI, since there are efficient algorithms to solve it, one can consider that problem solved for all practical purposes.

- (2) By LMI approach, one can synthesis controllers with a mix of time and frequency domain specifications ranging from H_2 and H_∞ performance to regional pole placement, asymptotic tracking or regulation, and settling time or saturation constraints. This is a multi-objective control problem.
- (3) For the multi-objective problem, it has been proposed to specify the closed-loop objectives in terms of a common Lyapunov function [56, 2]. As a benefit, the same technique has proved valuable in arriving at design procedures.
- (4) It can deal with optimization problems of not only linear time-invariant system, but also the time varying system with uncertainties.
- (5) The optimization problems are treated purely in the discrete-time domain to avoid any performance degradation involved with continuous-time to discrete-time conversion.

LMI approach guarantees the desired specifications at the expense of conservatism. This conservatism comes from the Lyapunov functions which guarantee stability for arbitrarily fast time variations.

7.2.2 Robust repetitive learning control for MIMO systems

Repetitive control is originally developed for single-input-single-output (SISO) linear time-invariant (LTI) system. In real applications, however, we may need to control multi-input multi-output (MIMO) system with periodic disturbances. One important application is hard disk driver (HDD) with second-stage actuator. With the aim to increase storage capacity of HDD in particular the track-per-inch (TPI)

value, dual-stage actuated HDD systems have been of interest for many years [28, 38, 89, 86, 109, 140]. Dual-stage actuator systems, comprising a voice-coil-motor (VCM) actuator for coarse actuation and a high bandwidth micro-actuator, have been an attractive alternative to the commercially used single VCM actuator system [60] since they allow for higher servo bandwidth and more accurate positioning of the read/write head. With this technique, the servo bandwidth of HDD can be as large as 2 kHz comparing to 600-900 Hz for a typical bandwidth under single-stage actuation with a VCM. Because of these advantages, its adoption is widely considered inevitable although the dual-stage actuator technique is not yet mature for mass production, and, its cost is still high [1].

Several design methods have been proposed for dual-stage servo design, which can be roughly categorized into two major categories. The first includes those methodologies that utilize decoupled or sequential SISO classical frequency shaping design techniques, such as the master-slave mode [61], the PQ method [108], the parallel design method [109], and the sensitivity transfer functions decoupling or decoupled master-slave method [31, 76, 89], etc. The second includes those methodologies that explicitly account for coupling effects between the VCM and MA actuators and utilize multi-variable optimal control design techniques such as LQG, LQG/LTR, H_∞ , or μ synthesis [46, 38, 122].

Although repetitive control has been successfully used in HDD servo control systems [14, 74, 87, 94], there is new challenges for dual-stage systems because of coupled dynamics. Figure 7.1 shows the control diagram of dual-stage servo control system for HDD [117]. This is a Single-In-Dual-Out (SIDO) controller design problem. The second objective is to design the SIDO robust repetitive controllers for dual-stage servo system.

In the dual-stage servo control system, micro-actuator controller should be designed to reject high frequency disturbances while VCM controller should be designed to reject low frequency disturbances. We use repetitive control in dual-stage

Figure 7.1: Dual-stage Servo Control System Structure

HDD follow this idea. Figure 7.2 shows the control scheme of SIDO repetitive controller. $F_1(z)$ and $F_2(z)$ are low-pass and high-pass filter respectively. With these two filters we can divide periodic disturbances into two parts: disturbances at low frequency is cancelled by VCM and high frequency disturbances is reduced by micro-actuator.

Figure 7.2: Block Diagram of SIDO Repetitive Controller

7.2.3 Adaptive repetitive control with application to PWM inverters

In Chapter 6, a fractional delay based repetitive control (FD-RC) is proposed. A possible application of proposed FD-RC could be auto-tuning or adaptive repetitive control system, i.e., the period of signal is changing with time. The online estimation of period could be non-integer number of sampling points. Some results of adaptive repetitive control scheme or similar topics were reported in [3, 39, 127, 44].

The repetitive control with Adaptive Notch Filter (ANF) will be another promising method of adaptive repetitive control. Figure 7.3 shows the block diagram of adaptive repetitive control system with ANF approach. A minimal parameter Infinite Impulse Response (IIR) ANF was first introduced in [91, 115], which is proved to be an efficient structure for frequency estimation and sinusoidal disturbance cancellation. With the same structure, Adaptive Comb Filter [92] is proposed for harmonic signal enhancement. The direct frequency estimation approaches [12, 71] of ANF are also proposed to achieve faster convergence speed and less computational costs.

Figure 7.3: Block Diagram of Adaptive Repetitive Control with ANF approach

Author's Publication

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