Design, flight dynamic analysis and hover control of a vertical take-off and landing micro air vehicle

Setiawan, Suhartono

2009

Setiawan, S. (2009). Design, flight dynamic analysis and hover control of a vertical take-off and landing micro air vehicle. Master's thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/19010>

<https://doi.org/10.32657/10356/19010>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**DESIGN, FLIGHT DYNAMIC ANALYSIS AND HOVER
CONTROL OF A VERTICAL TAKE-OFF & LANDING
MICRO AIR VEHICLE**

SUHARTONO SETIAWAN

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

2009

**DESIGN, FLIGHT DYNAMIC ANALYSIS AND HOVER
CONTROL OF A VERTICAL TAKE-OFF & LANDING MICRO AIR VEHICLE**

SUHARTONO SETIAWAN

2009

**DESIGN, FLIGHT DYNAMIC ANALYSIS AND HOVER
CONTROL OF A VERTICAL TAKE-OFF & LANDING
MICRO AIR VEHICLE**

SUHARTONO SETIAWAN

School of Mechanical and Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Master of Engineering

2009

ABSTRACT

A ducted fan Vertical Take-off and Landing (VTOL) Micro Air Vehicle (MAV) is developed. The vehicle is equipped with a wing that can be rotated in the pitch direction (variable incidence). This feature is expected to suppress the altitude loss during transition and to reduce the gust effect in hover. The selected configuration is capable to yield satisfactory aerodynamic performances and satisfy the static stability requirements in the cruise-flight. The variable-incidence wing feature is found to yield a higher lift and a lower drag at high angles of attack flight.

In cruise and small angles of attack, the dynamics of the MAV are similar to those of a conventional aircraft. In hover and high angles of attack, the MAV is inherently unstable. An automatic control system is developed to stabilize the MAV in hover and to maintain the position of the MAV under wind disturbances. The Single Input Single Output (SISO) root locus technique is applied to design the control system. The pitch and yaw attitude controls are used as inner loops to achieve stability. The velocity and position controls are put in the outer loops of the attitude controls. The performance of the controlled vehicle under crosswind disturbances is also examined.

The worthwhile problems are the ones you can really solve or help solve the ones you can really contribute something to. No problem is too small or trivial if we can really do something about it.

Richard Feynman (Nobel Laureate of Physics 1965) on the Letters of Richard Feynman

ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisor Assistant Professor Yongki Go Tiau Hiong for his valuable guidance, practical insights into the problem and assistance throughout this research work.

I would also address my thankfulness to the member of the MAV development team - Billy Nguyen, Zhao Weihua, Sheila Tobing, Adnan Maqsood and Dr. Omar Arief - for their meaningful suggestions and ideas. This thesis would not have been possible without helps from the staffs and research students in the CAD/CAM Laboratory (Yap Chun Wee, Kenny Yong, Kiyohide Wada, Ba Te, etc) and the Main Aircraft Laboratory / Flight Simulation Laboratory. The permission and assistance to use the facilities in those labs have made my life easier.

In this occasion, I would recognize the role of AUN/SEED Net – JICA organization, which has supported me financially during the course of my study.

Lastly, all my respect, wish and appreciation are given to my parents (Mr. and Mrs. Djaenudin Setiawan), brothers, sisters and my nieces in Indonesia. Their support and encouragement have been the main fuel that boosts me to move forward. *The wind of hopes and changes never stop blowing!!!*

TABLE OF CONTENTS

ABSTRACT	ii
ACKNOWLEDGEMENTS	iv
TABLE OF CONTENTS	v
LIST OF FIGURES	ix
LIST OF TABLES	xiv
LIST OF SYMBOLS	xv

Chapter One:

Introduction	1
1.1 Background	1
1.2 Motivation	6
1.3 Objectives and Scope	8
1.4 Organization of the Thesis	10

Chapter Two

The MAV Design Process	11
2.1 Design Process	11
2.2 Configuration Selection	12
2.3 Power Consumption and Weight Estimation	13
2.3.1 Hardware Selection and Initial Weight Estimation	13
2.3.2 Power Consumption Calculation Based on the Momentum Theory	14
2.3.3 Induced Power Characteristics of Ducted Propeller Vehicle	18
2.3.4 Power Consumption at Two Design Points	20
2.3.5 Propulsion System and Power Source Selection	21
2.4 Aerodynamics and Static Stability Formulation	22

TABLE OF CONTENTS

2.4.1	Estimation of Aerodynamics	22
2.4.2	Static Stability Criteria	31
2.5	Thrust Testing	31
2.5.1	Hacker and AXI Motor Performance	33
2.5.2	Performance of Hacker Motor at Maximum Point	34
2.6	Airfoil Selection	36
2.7	Flow Velocity and Angle of Attack at the Tails	38
2.8	Evolution of the Designs and Details of the Final Configuration	41
2.8.1	Evolution of the Designs	41
2.8.2	Details of the Final Configuration	47
2.8.3	Stator Design	49
2.8.4	Advance Feature of the MAV	49
2.9	Aerodynamics Forces and Moments	50
2.9.1	Aerodynamic Forces for the Fixed-Wing Configuration	50
2.9.2	Moment Characteristics for the Fixed-Wing Configuration	52
2.9.3	Effect of CG Variations into the Slopes of Moment Coefficient Curves	54
2.9.4	Aerodynamics of the Movable (Variable Incidence) Wing Configuration	55
2.10	Prototype Development and Testing	57
2.11	Summary of the MAV Design Process	58
 Chapter Three		
Dynamic Analysis of the VTOL MAV		60
3.1	Introduction	60
3.2	References Frames	62
3.3	Equations of Motion	64
3.4	Aerodynamics and Propulsion Models	66
3.5	Trim Points for Cruise-to-Hover Transition	69
3.6	Linearization of the Equations of Motion	74
3.6.1	$0 \leq \theta < 90$	74
3.6.2	$\theta = 90^\circ$ (Hover)	76

TABLE OF CONTENTS

3.7	Stability and Control Derivatives for Cruise-to-Hover Transition	77
3.8	Dynamics of the MAV in Cruise-to-Hover Transition	86
3.8.1	Dynamics of the MAV for $0 \leq \theta < 90$	86
3.8.2	Coupled Response	93
3.8.3	Dynamics of the MAV in Hover	95
3.8.4	Effect of Changing Tail Location on the Dynamics	100
3.8.5	Effect of CG Location Variations in Hover	104
3.9	Summary on the Dynamic Analysis of the VTOL MAV	107

Chapter Four

Design of Automatic Hover Control for the VTOL MAV		109
4.1	Introduction to Automatic Control Design for Micro Air Vehicle	109
4.2	Overview of the Control Strategy and Design Method	112
4.3	Mathematical Models of Actuators and Electrical Motor	113
4.3.1	Servo Actuator Model	113
4.3.2	Electrical Motor Model	114
4.4	Design of the Inner Loop Controllers	114
4.4.1	Pitch Attitude Control	114
4.4.2	Yaw Attitude Control	117
4.5	Outer Loop Controllers	119
4.5.1	$x_{\text{INERTIAL}}(x_I)$ Position Control	119
4.5.2	$y_{\text{INERTIAL}}(y_I)$ Position Control	121
4.5.3	Altitude Control	123
4.5.4	Roll Angle Control	125
4.6	6 DoF Linear Simulation	128
4.7	Summary of the Automatic Hover Control Design	133

Chapter Five

Conclusions	134
5.1 Conclusions	134
5.2 Recommendations for Further Study	135

TABLE OF CONTENTS

REFERENCES	137
APPENDIX A - Brushless Outrunner Motor Technical Data	143
APPENDIX B – Three View Drawings of the VTOL MAV	144

LIST OF FIGURES

Figure 1.1 Wingspan and payload mass of MAV compared with UAV	2
Figure 1.2 Mission scenarios for the MAV	2
Figure 1.3 Endurance, dimension and weight data for various MAV	6
Figure 1.4 Mission Profile of the MAV	8
Figure 2.1 Detail processes in the conceptual design phase	11
Figure 2.2 Method to determine the weight of the MAV	13
Figure 2.3 Flow illustrations of free and ducted propellers in hover	14
Figure 2.4 Figure of Merit of ducted propeller MAV	18
Figure 2.5 Induced power of ducted propeller MAV in hover	18
Figure 2.6 Induced power of ducted propeller MAV in climb	19
Figure 2.7 Induced power of ducted propeller MAV in transition	20
Figure 2.8 Total power consumed by ducted propeller MAV with (a) $D = 0.15$ m and mass 400 grams (b) $D = 0.2$ m and mass 500 grams	21
Figure 2.9 Illustration of wind and body axis coordinate system	27
Figure 2.10 (a) Test bench for thrust testing (b) Illustration of thrust experiment set up	33
Figure 2.11 Current and thrust of the Hacker and AXI motors using half charged battery	33
Figure 2.12 Current and thrust relation of the Hacker motor using fully charged battery	34
Figure 2.13 Battery power and torque of the Hacker motor using fully charged battery	35
Figure 2.14 Efficiency of the Hacker motor	30
Figure 2.15 Lift and pitching moment coefficient curves of low Re airfoils	37
Figure 2.16 Lift coefficient curves of symmetric airfoil at $Re\ 5 \times 10^4$	37
Figure 2.17 Effect of freestream and jet flow from the duct (a) Below α_{crit} (b) above α_{crit}	38

LIST OF FIGURES

Figure 2.18 α_{crit} as a function of tails position from the duct exit point	40
Figure 2.19 Horizontal tails (a) angle of attack and (b) velocity in cruise-to-hover transition with constant freestream velocity $V_{\infty} = 15$ m/s	41
Figure 2.20 Evolution of the configuration: from 1 st to 4 th designs	43
Figure 2.21 Evolution of the configuration: from 5 th to 8 th designs	44
Figure 2.22 Isometric drawing of the MAV with material information	45
Figure 2.23 Three view drawing of the MAV	46
Figure 2.24 Movable or variable incidence wing concepts in transition and hover flights	50
Figure 2.25 Lift coefficient of the MAV with fixed-wing configuration	51
Figure 2.26 Drag coefficient of the MAV with fixed-wing configuration	51
Figure 2.27 Side force coefficients of the MAV with fixed-wing configuration	52
Figure 2.28 Pitching moment coefficient of the MAV for with fixed-wing configuration	53
Figure 2.29 Yawing moment coefficients with fixed-wing configuration	54
Figure 2.30 Rolling moment coefficients with fixed-wing configuration	54
Figure 2.31 Slopes of the pitching moment coefficient curve at cruise for various CG	55
Figure 2.32 Slopes of the yawing moment coefficient curve at cruise for various CG	55
Figure 2.33 Lift coefficients for variable incidence wing configuration	56
Figure 2.34 Drag coefficients variable incidence wing configuration	57
Figure 2.35 Pitching moment coefficients variable incidence wing configuration	57
Figure 2.36 (a) The prototype of the MAV (b) the rig for hover test	58
Figure 3.1 Framework for dynamic analysis and control development	61
Figure 3.2 Coordinate systems used in the flight dynamics analysis	62
Figure 3.3 (a) General Euler angles transformation for $0 \leq \theta < 90$ (b) Vertical Euler angles transformation for $\theta = 90^\circ$	63
Figure 3.4 Procedure to calculate total aircraft forces and moments	67
Figure 3.5 Relationship between the stick deflections and the total thrust	68
Figure 3.6 Relationship between the stick deflections with the engine torque	68

LIST OF FIGURES

Figure 3.7 Convergence history for trim point calculation at $V = 15$ m/s	70
Figure 3.8 Convergence histories for trim point calculation in hover	70
Figure 3.9 Velocity and angle of attack relation in cruise-to-hover transition	73
Figure 3.10 Throttle stick deflection in cruise-to-hover transition	73
Figure 3.11 Horizontal tail and stator angles in cruise-to-hover transition	73
Figure 3.12 Stability derivatives due to u perturbation	81
Figure 3.13 Stability derivatives due to w perturbation	81
Figure 3.14 Stability derivatives due to q perturbation	82
Figure 3.15 Stability derivatives due to v perturbation	82
Figure 3.16 Stability derivatives due to p perturbation	83
Figure 3.17 Stability derivatives due to r perturbation	83
Figure 3.18 Coupling derivatives	84
Figure 3.19 Control derivatives due to δ_{HTP} perturbation	84
Figure 3.20 Control derivatives due to δ_T perturbation	85
Figure 3.21 Control derivatives due to δ_{VTP} perturbation	85
Figure 3.22 Control derivatives due to δ_s perturbation	85
Figure 3.23 Short period and phugoid poles migration in hover-to-cruise transition	88
Figure 3.24 Dutch roll, roll and spiral poles migration in hover-to-cruise transition	89
Figure 3.25 Responses of the MAV at 15 m/s due to horizontal tail step input	93
Figure 3.26 Responses of the MAV at 6 m/s due to horizontal tail step input	94
Figure 3.27 Responses of the MAV at 15 m/s due to 1° stator step input	49
Figure 3.28 Responses of the MAV at 6 m/s due to 1° stator step input	94
Figure 3.29 Poles of the MAV in hover	97
Figure 3.30 Current (A) and modified (B) configurations	100
Figure 3.31 Velocity and angle of attack relation in cruise-to-hover transition	101
Figure 3.32 Throttle stick deflection in cruise-to-hover transition	101
Figure 3.33 Horizontal tail deflections in cruise-to-hover transition	101
Figure 3.34 Longitudinal poles migration of configuration A and B in cruise-to-hover transition	103
Figure 3.35 Lateral-directional poles migration of configuration A and B in cruise-to-hover transition	103

LIST OF FIGURES

Figure 3.36 Longitudinal poles of configuration A and B in hover	104
Figure 3.37 Lateral directional poles of configuration A and B in hover	104
Figure 3.38 Positive and negative CG illustration	105
Figure 3.39 Longitudinal poles of configuration A & B in hover for various CG	106
Figure 3.40 Directional poles of configuration A in hover for various CG	106
Figure 3.41 Directional poles of configuration B in hover for various CG	107
Figure 4.1 SISO subsystems with their controlled variables	112
Figure 4.2 Schematic diagram of (a) pitch rate and (b) pitch attitude controls	114
Figure 4.3 Root locus of the pitch rate controller using proportional gain	115
Figure 4.4 Root locus of the pitch angle control using (a) Proportional (b) PID	116
Figure 4.5 Unit step response of the pitch attitude control	116
Figure 4.6 Block diagram of yaw attitude control	117
Figure 4.7 Root locus of the yaw angle control using (a) Proportional and (b) PID	118
Figure 4.8 Unit step response of the yaw control	119
Figure 4.9 Schematic diagrams for $x_{\text{INERTIAL}}(x_I)$ Control	119
Figure 4.10 Root locus of the w control	120
Figure 4.11 (a) Root locus and (b) unit step response of x_I position control	121
Figure 4.12 Schematic diagrams for $y_{\text{INERTIAL}}(y_I)$ controls	121
Figure 4.13 Root locus of the v velocity controller	122
Figure 4.14 (a) Root locus and (b) unit step response of y_I position controller	123
Figure 4.15 Schematic diagrams of altitude controller	123
Figure 4.16 Root locus of the u controller	124
Figure 4.17 (a) Root locus and (b) unit step response of the altitude controller	125
Figure 4.18 Schematic diagrams for roll angle control	125
Figure 4.19 Root locus of roll rate controller	126
Figure 4.20 (a) Root locus and (b) unit step responses of roll angle controller	127
Figure 4.21 SIMULINK [®] diagram of the linear simulation model	127
Figure 4.22 Position of the MAV in the 3-D space due to the initial position error	128
Figure 4.23 Response of the MAV states due to the initial position error	129
Figure 4.24 Illustration of the wind disturbance introduced to the MAV	130
Figure 4.25 Position of the MAV in the 3-D space as due to the wind blows for 5 seconds	130

LIST OF FIGURES

Figure 4.26 Responses of the MAV states due to the wind blows for 5 seconds	131
Figure 4.27 Magnitude of the random wind	132
Figure 4.28 Position of the MAV in the 3-D space due to the random wind disturbance	132
Figure 4.29 Responses of the MAV states due to the random wind disturbance	133

LIST OF TABLES

Table 2.1 Comparison of four possible MAV configurations	12
Table 2.2 Hardware carried in MAV for Radio Controlled (R/C) and Autonomous modes	14
Table 2.3 Data required in the calculation of induced power for ducted propeller vehicle	17
Table 2.4 Weight distribution of the MAV	48
Table 2.5 Geometrical properties of the MAV	48
Table 3.1 Requirements for trim calculation	69
Table 3.2 Summary of the trim parameters for cruise to hover conversion	72
Table 3.3 Stability derivatives in hover	79
Table 3.4 Control derivatives in hover	80
Table 3.5 Decoupled eigenvalues, eigenstructure, magnitude and phase of the MAV in cruise with $V = 15$ m/s	90
Table 3.6 Decoupled eigenvalues, eigenstructure, magnitude and phase of the MAV with $V = 6$ m/s	91
Table 3.7 Decoupled eigenvalues, eigenstructure, magnitude and phase of the MAV with $V = 1$ m/s	92
Table 3.8 Decoupled eigenvalues, eigenstructure, magnitude and phase of the MAV in hover	98
Table 3.9 Coupled Decoupled eigenvalues, eigenstructure, magnitude and phase of the MAV in hover	99
Table A.1 Brushless Motor Technical Data	143

LIST OF SYMBOLS

Dimensional Symbols

w	: Propeller wake velocity (m/s)
	: Velocity in Z axis of body coordinate system (m/s)
v_i	: Induced velocity across the propeller in climb, In descent and forward flight (m/s)
v_h	: Induced velocity across the propeller in hover (m/s)
T	: Thrust (N)
ρ	: Air density (kg/m ³)
A	: Propeller disk area (m ²)
R	: Radius of the propeller / rotor (m)
P_i	: Induced power in hover (W)
P_h, P_C and P_D	: Power consumption in hover, cruise and descent (W)
V_C and V_D	: Climb and descent speeds (m/s)
V_∞ or V_0	: Freestream velocity or speed in forward flight (m/s)
α	: Propeller and aircraft angles of attack (deg)
β	: Angle of sideslip (deg)
Ω	: Rotation speed of the propeller (rad/s)
b	: Span of the wing (m)
S	: Area of the wing (m ²)
c or $\bar{c}$	: Chord or mean aerodynamics chord of the wing or tails (m)
$\alpha_{C_L=0}$	: Angle of attack at zero lift coefficient of wing or tails (deg)
$\alpha_{C_l=0}$	: Angle of attack at zero lift coefficient of airfoil (deg)
$\alpha_{C_{l_{\max}}}$	: Angle of attack at maximum lift or stall angle (deg)
ε_t	: Twist angle (deg)
S_{wet}	: Wetted area of the aircraft's component (m ²)

LIST OF SYMBOLS

S_{ref}	: Reference area (m ²)
l	: Body length (m)
d	: Diameter of the body (m)
L	: Lift coefficient (N)
D	: Drag coefficient (N)
q	: Dynamics pressure (kg/m-s ²)
S_b	: Projection area of the pod (m ²)
A_p	: Perimeter area of the pod (m ²)
$C_{d_{\alpha=90^\circ}}$	: 2-D drag coefficient at 90° angle of attack
SF	: Side force (N)
X, Y, Z	: Forces in the body coordinate system (N)
$\dot{m}$	: Mass flow rate (kg/s)
l	: Rolling moment (Nm)
M	: Pitching moment (Nm)
N	: Yawing moment (Nm)
x, y and z	: Distances in three dimensional vector from the aircraft CG to the aerodynamic center of each individual component (m)
M_0	: Wing pitching moment at zero angle of attack (Nm)
x_p	: Center of pressure location (m)
x_{cg}, y_{cg} and z_{cg}	: Center of gravity location in three dimensional spaces (m)
α_{crit}	: Critical angle of attack (deg)
V_{Tail}	: Velocity at the tails (m/s)
α_{Tail}	: Tails angle of attack (deg)
V_D	: Duct exit velocity (m/s)
i	: Setting angle of wing or tails (deg)
δ	: Deflection angle of wing or tails (deg)
ϕ, θ, ψ	: Roll, pitch and yaw angles (deg)
ϕ_v, θ_v, ψ_v	: Vertical roll, vertical pitch and vertical yaw angles (deg)
u, v, w	: Velocity vector in body coordinate system (m/s)

LIST OF SYMBOLS

p, q, r	: Angular velocity vector in body coordinate system (rad/s)
$\dot{u}, \dot{v}, \dot{w}$	: Accelerations in the body coordinate system (m/s ²)
I	: Moments and products of inertia (m ⁴)
x, y, z	: Positions in the inertial coordinate system (m)
$\dot{x}, \dot{y}, \dot{z}$	: Velocities in the inertial coordinate system (m/s)
γ_0	: Climb angle (deg)
ω_n	: Undamped natural frequency (rad/s)
T_s	: Time constant (s)
C_{L_α}	: Slope of the lift coefficient curve with respect to angle of attack (/deg or /rad)
C_{l_β}	: Slope of the rolling moment coefficient curve with respect to angle of sideslip (/deg or /rad)
C_{n_β}	: Slope of the yawing moment coefficient curve with respect to angle of sideslip (/deg or /rad)
$X_u = \frac{dX}{du}$	: Derivative of X force with respect to u velocity (N-s/m)
$Z_u = \frac{dZ}{du}$	: Derivative of Z force with respect to u velocity (N-s/m)
$M_u = \frac{dM}{du}$	: Derivative of pitching moment M with respect to u velocity (N-s)
$X_w = \frac{dX}{dw}$	: Derivative of X force with respect to w velocity (N-s/m)
$Z_w = \frac{dZ}{dw}$	: Derivative of Z force with respect to w velocity (N-s/m)
$M_w = \frac{dM}{dw}$	: Derivative of pitching moment M with respect to w velocity (N-s)
$X_q = \frac{dX}{dq}$	: Derivative of X force with respect to q (N-s)
$Z_q = \frac{dZ}{dq}$	: Derivative of Z force with respect to q (N-s)

LIST OF SYMBOLS

$M_q = \frac{dM}{dq}$	: Derivative of pitching moment M with respect to q (Nm-s)
$Y_v = \frac{dY}{dv}$	: Derivative of Y force with respect to v velocity (N-s/m)
$l_v = \frac{dl}{dv}$	: Derivative of rolling moment l with respect to v velocity (N-s)
$N_v = \frac{dN}{dv}$	: Derivative of pitching moment N with respect to v velocity (N-s)
$Y_p = \frac{dY}{dp}$	: Derivative of Y force with respect to p (N-s)
$l_p = \frac{dl}{dp}$	: Derivative of rolling moment l with respect to p (Nm-s)
$N_p = \frac{dN}{dp}$	: Derivative of pitching moment N with respect to p (Nm-s)
$Y_r = \frac{dY}{dr}$	: Derivative of Y force with respect to r (N-s)
$l_r = \frac{dl}{dr}$	: Derivative of rolling moment l with respect to r (Nm-s)
$N_r = \frac{dN}{dr}$	: Derivative of pitching moment N with respect to r (Nm-s)
$X_{\delta_{HTP}} = \frac{dX}{d\delta_{HTP}}$	: Derivative of X force with respect to horizontal tail deflection δ_{HTP} (N/rad)
$Z_{\delta_{HTP}} = \frac{dZ}{d\delta_{HTP}}$	: Derivative of Z force with respect to horizontal tail deflection δ_{HTP} (N/rad))
$M_{\delta_{HTP}} = \frac{dM}{d\delta_{HTP}}$	: Derivative of pitching moment M with respect to horizontal tail deflection δ_{HTP} velocity (Nm/rad)
$Y_{\delta_{VTP}} = \frac{dY}{d\delta_{VTP}}$	: Derivative of Y force with respect to vertical tail deflection δ_{VTP} (N/rad)

LIST OF SYMBOLS

$N_{\delta_{VTP}} = \frac{dN}{d\delta_{VTP}}$	: Derivative of yawing moment N with respect to vertical tail deflection δ_{VTP} (Nm/rad)
$X_{\delta_s} = \frac{dX}{d\delta_s}$	: Derivative of X force with respect to stator deflection δ_s (N/rad)
$l_{\delta_s} = \frac{dl}{d\delta_s}$	: Derivative of rolling moment l with respect to stator deflection δ_s (Nm/rad)
$X_{\delta_T} = \frac{dX}{d\delta_T}$	: Derivative of X force with respect to thrust stick deflection δ_T (N/% Stick Deflection)
x_I	: Position of the MAV in the X_I axis of inertial coordinate system (m)
y_I	: Position of the MAV in the Y_I axis of inertial coordinate system (m)
h	: Altitude (m)
G_a	: Actuator Model (rad/volt)
G_e	: Electrical motor dynamics model (% Stick Deflection/Volt)

Dimensionless Symbols

C_p	: Power and thrust coefficients
C_T	: Power and thrust coefficients
κ	: Propeller correction factor due to losses
σ	: Blade solidity factor
C_{d_0}	: 2-D drag coefficient of wing or propeller or horizontal and vertical tails
η	: Propeller efficiency in hover
λ_h	: Inflow ratio in hover
λ	: Inflow ratio
	: Taper ratio of the wing or tails
μ	: Ratio between forward and rotational speeds of propeller vehicle

LIST OF SYMBOLS

AR	: Aspect Ratio
Re	: Reynolds numbers
M	: Mach numbers
$\Lambda, \Lambda_{c/4}, \Lambda_{0.5c}$	: Swept angle of the wing and tails at quarter or half chords
$C_{L_{\max}}$	: Maximum lift coefficient of wing or tails
$C_{l_{\max}}$	: Maximum lift coefficient of airfoil
C_{D_0}	: Drag coefficient at zero lift coefficient of aircraft's components
C_f	: Frictional component
Q	: Interference factor
FF	: Form factor
t/c	: Thickness-per-chord ratio
$(x/c)_m$	: Location of the maximum thickness point
C_L	: Lift coefficient
C_D	: Drag coefficient
C_{SF}	: Side force coefficient
e	: The wing efficiency factor
$C_{D_{\text{induced}}}$	: Induced drag coefficient
X_B, Y_B, Z_B	: Axis of body coordinate system
X_W, Y_W, Z_W	: Axis of wind coordinate system
C_l	: Rolling moment coefficient
C_m	: Pitching moment coefficient
C_n	: Yawing moment coefficient
Δ	: Small perturbations
ζ	: Damping ratio

Abbreviations

UAV	: Unmanned Aerial Vehicle
MAV	: Micro Air Vehicle
DARPA	: Defense Advanced Research and Projects Agency

LIST OF SYMBOLS

VTOL	: Vertical Take-off and Landing
MEMS	: Micro Electro Mechanical System
ESC	: Electronic Speed Controller
FCC	: Flight Control Computer
IMU	: Inertial Measurement Unit
FM	: Figure of Merit
COTS	: Commercially Off the Shelf
DRO	: Design Requirements and Objectives
CFD	: Computational Fluid Dynamics
R/C	: Radio Controlled
CG	: Center of Gravity
ICE	: Internal Combustion Engine
LiPo	: Lithium Polymer battery
LE	: Leading Edge
STOL	: Short Take-off and Landing
LTI	: Linear Time Invariant
SISO	: Single Input Single Output
MIMO	: Multi Input Multi Output
PI/PID	: Proportional Integral/Proportional Integral and Derivative

Subscripts

Pod	: pod
Duct	: duct
Wing	: wing
Stator	: stator
S	: Stator vane
HTP	: Horizontal tail plane
VTP	: Vertical tail plane
T	: Throttle Stick / Thrust
Mom Drag	: Momentum drag
I	: Inertial Coordinate System
B	: Body Coordinate System
CG	: Center of gravity

LIST OF SYMBOLS

<i>ac</i>	: Aerodynamics center
0	: Reference/trim condition
<i>Max</i>	: Maximum

Chapter One**INTRODUCTION****1.1 Background**

Since the first heavier-than-air flight by Wright brothers in 1903, aircraft has undergone significant evolution in its design. The use of unmanned vehicles to replace manned systems is one of the notables design evolution. Unmanned Aerial Vehicle (UAV) is developed to minimize the risk of human lost, when performing an operation in a hostile area [1]. In the future, its development will be focused on the enhancement of its performance including safety and reliability, autonomous level, as well as reduction of production and operational costs [2, 3]. However, conventional UAV is inappropriate to execute missions in the narrow space environments, like; caves, tunnel and indoor, due to its large sizes. In addition, the deployment of conventional UAV also requires high cost, large numbers of operators and long operator training time.

These drawbacks lead to the development of a smaller size UAV called Micro Air Vehicle (MAV). According to Defense Advanced Research and Projects Agency (DARPA), MAV or μ AV is a small drone with a maximum span, length and height of less than 15 cm [4]. Figure 1.1 shows comparisons between MAV and UAV wingspan and payload mass. The small dimension of the MAV makes the payload mass is limited. Typical payload mass for the MAV is 100 grams only. However, due to unavailability of reliable payload with light weight, the MAV payload can exceed 500 grams. To compensate this, a longer wingspan and more powerful propulsion system are needed.

The MAV is dedicated as an asset of an individual or small group of operators, so it must be fit nicely in a small backpack. With this small size, the mobility of the users will be improved. The small sizes of MAV make it suitable to perform missions in narrow and hostile / hazardous environments. The typical missions that can be executed by a MAV include; intelligence, surveillance and reconnaissance (ISR)

missions in cave, indoor, tunnel and also sensing, tagging and targeting in the destroyed building, as illustrated in Figure 1.2.

Figure 1.1 Typical wingspan of MAV and UAV [5]

Figure 1.2 Mission scenarios for the MAV [4]

The development of MAV was started by feasibility studies conducted at the RAND Corporation [6] and Lincoln Laboratory [5]. The main burdens to realize the MAV concept are the small thrust and efficiency provided by small propulsion system and the availability of reliable micro electronic components. The technology advancement in the area of microchip development allows higher computational load at small power consumption and at low weight. The advancement in the area of micro

manufacturing leads to powerful electrical motor with better efficiency. All these allow massive development of MAV.

Since the studies performed by RAND Corporation and Lincoln Laboratory, many universities and industries have involved in the development of MAV. The developed vehicles mainly incorporated four types of configuration; *the fixed-wing, the flapping-wing, the rotary-wing and the ducted fan configurations.*

Fixed-wing MAV Configuration

The interest to build a fixed-wing MAV is driven mainly due to its longer endurance than the other types. The manufacturing of a fixed-wing MAV is also easier than the counterparts. The fixed-wing MAVs can be categorized into two different types: conventional fixed-wing and flying-wing [7]. A conventional fixed-wing MAV has the same arrangements with the large fixed-wing aircraft. This type of configuration uses high aspect ratio wing concept. A high aspect ratio wing usually gives higher maximum lift coefficient but with relatively low stall angle. A MAV of such type is not compact in dimension [8]. The appeal of a flying wing concept resides on the large proportion of the wing sizes, high stalling angle and compactness in dimensions. The main drawback is on its small lift coefficient. Flying wing configuration have been adapted into *Black Widow* [9] and *Mosquito* [10].

In recent years, the fixed-wing MAV with the capability to fly at very low speed or to perform hover maneuvers is preferred. This is to enhance the overall versatility and functionality of the MAV. Low flying speed MAV can be built by decreasing the wing loading. The decrease in wing loading makes the stall speed lower, so the MAV can fly at a small velocity. The decrease in wing loading is done by increasing the span. However, this option makes the dimensions exceed the requirement [11, 12]. To perform hover maneuver, a high grade motor (Thrust-to-Weight > 1) needs to be mounted on a fixed-wing MAV [13, 14]. This high grade motor is required to counter the weight of the MAV in hover.

Rotary-Wing MAV Configuration

The rotary-wing MAV uses rotor to generate lift and propulsive force. Interest in the development of such configuration arises from its capabilities to perform Vertical Take-off Landing (VTOL), hover and low speed maneuvers. Unfortunately,

the rotary-wing MAV consumes a lot of power for doing these maneuvers, so the endurance is generally much shorter than the fixed-wing MAV for the same power source capacity [15]. This high power consumption relates to high thrust required to keep the rotary wing vehicle float in the air. In addition, most of rotary-wing aircraft are inherently unstable and require an automatic control system to stabilize the platform [11]. The rotary-wing MAV might also endanger people standing close to it due to direct exposure to its rotor.

Another drawback is from its small Figure of Merit. Figure of Merit (FM) is the efficiency of the rotor in hover and represents the ratio between the ideal power and the non-ideal power used to generate thrust. The FM for MAV's rotor is around 40%, as seen in *Micor* MAV [16, 17], while full scale helicopter has FM in the range of 70-80%. This inefficiency can be traced back to the large profile drag losses on the blades and also to the large turbulent associated with the structure of the rotor wake [18].

Flapping-wing MAV Configuration

The superiority of flapping-wing configuration lays on its capability to take-off and landing in a very tight space. The flapping-wing configuration has also been thought as one of the possible solutions to overcome the low aerodynamic efficiency in the low *Reynolds number* (Re) flight, since the flapping motion can induce extra energy to the flow, thus delaying flow separation. However, the flow around a flapping body is highly complex [18, 19]. In addition, the mechanism to generate flapping, plunge and rotational motions of the wings is relatively complex. Keennon & Grasmeyer [20], developed *MicroBat* flapping-wing that can fly for 14 minutes using a lithium polymer (LiPo) battery.

Ducted Fan MAV Configuration

Ducted fan or annular wing or shrouded propeller has been used for many years as the lift augmentation system in VTOL aircraft and as a counter torque devices in modern helicopter. Due to its interesting features like; supplementary safety feature attributed to enclosing the rotating fans in the duct, low noise level, high static thrust and high propulsive efficiency, the ducted fan has also been adapted into UAV and MAV. A higher thrust and propulsive efficiency of a ducted fan vehicle

are obtained from the extra thrust provided by the duct and the suppression of the blade tip loss by the duct's wall [21]. Abrego & Bulaga [22] found that the FM of a ducted fan is around 50%, while it is around 40% for a small-scale propeller.

Another benefit of ducted fan configuration is on its capability to generate aerodynamic forces and moments. The aerodynamic forces perpendicular and parallel to the airflow direction are the lift and drag of ducted fan. The lift of ducted fan vehicle at zero angle of attack is zero, since at this angle of attack the flow distribution is symmetric. At positive angle of attack, the lift is positive also [23]. This lift will assist the ducted fan vehicle during translational flight.

Flatcher [24] examined the aerodynamic characteristics of five ducted fans (annular airfoils) under power-off condition. The results showed that the aerodynamic center of annular airfoil located closes at quarter chord point. At small angles of attack, the lift curve slope of the ducted fan is approximately twice the slope of a straight wing with a similar aspect ratio, while its induced drag is half of the induced drag of an elliptical wing. At a very small aspect ratio, the aerodynamics of annular wing is identical with the aerodynamics of a slender body in revolution.

Under the influence of crosswind, the forces and moments of ducted fan are dominated by the momentum drag and pitch up moment act at the duct's lips. The momentum drag appears as the reaction force used to align the flow with the duct axis. The lift at the inlet is the primary reason for the destabilizing moment [25].

Figure 1.3 shows various MAV configurations with their endurance and weight data. The implementation of the ducted fan concept in unmanned vehicle was started with the development of *iSTAR* ducted fan MAV by *MicroCraft Inc* [26]. The aerodynamic characteristics of *iSTAR* was studied in detail by Guerrero *et al*, [27] who also developed AVID OAV software. AVID OAV codes incorporate semi-empirical equations combined with Flatcher [24] data to predict the aerodynamic properties of any ducted fan UAV. Following the development of *iSTAR*, Schaefer & Baskett [15] developed *GoldenEye* ducted fan UAV. The interesting feature of *GoldenEye* UAV is the presence of wing on both sides of the duct. These wings can be oriented freely about its pitch axis to a specific angle in order to effectively produce lift. The wings also help in increasing gust rejection level and fixing the vehicle position during hover. Avanzini *et al*, [28] developed a ducted fan vehicle with doughnut-shape fuselage for environmental monitoring. *Fantail* is a ducted fan

developed by ST Aerospace in Singapore. The main problem encountered by *Fantail* is altitude loss during the transition maneuver [29].

Figure 1.3 Endurance, dimension and weight data for various MAV [30].

1.2 Motivation

Employing a MAV for accomplishing a mission will strengthen and enhance the mobility and situational awareness of the users. The involvement of MAV might also improve the level of successfulness and the level of safety of any missions. This is because a properly designed MAV can deliver real-time information regarding the area of interest in a very short time. In addition, the MAV can also cover the area uncovered by the conventional UAV. Therefore, the development of MAV is important to answer the needs for aerial vehicle that can be quickly deployed, highly mobile and usable in the limited space environments.

In recent years, the main focus of research in the area of MAV development is more toward the realization of a versatile vehicle. A versatile MAV is the one that is capable of executing VTOL, hover and high speed cruise maneuvers. The aforementioned maneuvers are some important features to support the suitability of

the deployment of the MAV in the environment with limited space. The design and development of this class of vehicle is more challenging, since it involves optimization or trade-off study. This is due to the nature of the vehicle which has a larger flight envelope and each segment of this envelope has significantly different characteristics.

The VTOL concept has been implemented in some state-of-the-art UAV. Nonetheless, the existing VTOL UAV needs to be scaled down in terms of its sizes. The miniaturization of VTOL UAV imposes some difficulties, including: how to select a small-scale propulsion system with high thrust-to-weight ratio and how to handle poor aerodynamics characteristics of lifting surface as a result of low Re flight ($Re : 2 \times 10^5$ to 5×10^4). The added difficulty in miniaturization effort is due to the small space available for the integration of the MAV instrumentations [5].

There are also problems that need to be addressed to improve the overall functionality of the MAV. One of the problems is the design of the controller that can handle the wide operation envelope of the VTOL MAV. The small weight budget available might limit the autopilot hardware that can be carried onboard. In recent years, the use of commercially-off-the-shelf (COTS) components for MAV is preferred in order to reduce the production cost. Another problem is regarding the overshoot or altitude drop encountered by VTOL MAV during cruise-to-hover transition or vice versa. The designed control law must be able to eliminate or reduce this problem to avoid unexpected event during execution of the mission.

This work is motivated by the need to develop a small-size and versatile VTOL MAV. To make the implementation process simpler, the developed MAV is planned to use COTS components. Subsequently, the MAV needs automatic control system to enable it to operate autonomously. This autonomous feature will reduce the work load of the users and should be able to handle all problems encountered during the flight. Prior to the development of the control system, dynamics study must be performed. This is used to capture the important behavior of the MAV in flight due to control inputs or external disturbances. Some of the key development problems, including: manufacturing and testing, flight dynamics analysis and control system development are addressed in this thesis.

1.3 Objectives and Scope

One of the objectives of this research is to design, manufacture and testing a type of MAV that has VTOL and hover capabilities. Following the design stage, the flight dynamics analysis of the designed vehicle in hover-to-cruise transition is carried out. The dynamics analysis will be focused more in hover condition. Based on this dynamics analysis, an automatic hover control system will be developed to enable the MAV operates autonomously in the future. Due to the time limitation, the control design task is focused in hover condition only.

Figure 1.4 Mission Profile of the MAV

The design process follows a systematic procedure and it is guided by the Design Requirement and Objectives (DRO). At the end of the design process, a proof of concept prototype will be produced and used in the testing phase. The DRO for the MAV to be designed is summarized as follow:

1. Capable to perform VTOL, hover and cruise maneuvers

These characteristics are important when performing missions in the environment with limited space available, like; caves, tunnels, inside the buildings and urban environment. Typical mission profile of MAV is illustrated in Figure 1.4.

2. Packable in a small backpack and easy in operation

This requirement will improve the mobility and eliminate the need for comprehensive training of the users.

3. Sufficient endurance and Range

In this initial design process, the performance requirements are more flexible. At this stage, the main objective is to design a flyable VTOL MAV. The performance requirements will be fulfilled later after the main objective is accomplished.

Since this work is an initial work to develop fully autonomous VTOL MAV, complete analysis covering whole aspect of design can not be performed. The time available becomes the main constraint to do this task. Hence, the research work covers the following aspects only:

1. Design, manufacturing and testing of a VTOL MAV

The detail design process and a short description in the manufacturing and testing phases will be covered.

2. Flight dynamics analysis

The flight dynamics analysis will be done for transition flight, with main emphasis on hover flight. The analysis is performed for wing-fixed configuration only.

3. Hover control development of the MAV

The control system here is dedicated mainly to stabilize the system and to maintain the position under the external disturbances and serve as a platform for the dynamic model validation.

1.4 Organization of the Thesis

This thesis is organized into the following structure

- **Chapter I – Introduction**

Chapter one describes the background, motivation and objectives, as well as the scopes of the work.

- **Chapter II – The MAV Design Process**

The design process as well as the rationale and the justification for the design are presented in this part. A brief manufacturing and testing activities will also be presented in this chapter.

- **Chapter III – Dynamics Analysis of the VTOL MAV**

This chapter describes the development of linear models, the dynamics and stability analysis of the MAV in cruise-to-hover transition.

- **Chapter IV – Design of Automatic Hover Control for the VTOL MAV**

Chapter IV explains the development of automatic control system for the corresponding MAV in hover.

- **Chapter V – Conclusions**

The conclusion of research and the planned future work to accomplish are described in this chapter.

- **References**

- **Appendix A – Brushless Outrunner Electrical Motor Data**

- **Appendix B – Geometrical Properties and Technical Drawing of the VTOL MAV**

Chapter Two**THE MAV DESIGN PROCESS****2.1 Design Process**

Aircraft design is a highly iterative process, but it follows a systematic procedure. In general, aircraft design task can be divided into three phases, which are conceptual design, preliminary design and detail design [31, 32, 33]. The conceptual design phase covers all activities intended to answer basic questions regarding configuration arrangement, size, weight and performance. The preliminary design phase includes Computational Fluid Dynamics (CFD) analysis or wind tunnel test to assess aerodynamics configuration, structural stress analysis and control system development. The detail design phase is where each part of the aircraft is comprehensively analyzed and manufactured.

Figure 2.1 Detail processes in the conceptual design phase [31].

The conceptual design phase is the initial stage in the design process. Figure 2.1 presents more detail processes in the conceptual design phase. It is started with a statement on the requirements or new design ideas. Should the promising design has been found, a sketch of the concept needs to be drawn. The sketch should give information about the general arrangement, sizes and weight. More detail analyses are performed, if a prospective concept has been obtained. These analyses include; aerodynamics, stability and performance assessments. Iterations and trade-off might be needed to find a design that match with the requirements. In general, the MAV design process follows the same approach as explained above.

2.2 Configuration Selection

There are four possible configurations that can be adapted as the main configuration of the MAV, including; fixed-wing, rotary-wing, ducted fan and flapping-wing. The configuration is selected based on the comparison shown in Table 2.1. The ducted fan configuration deems to be the one that highly matches with the design requirements. Importantly, the ducted fan offers simplicity in the mechanism involved, compared to flapping-wing or rotary-wing configurations. It is also possible to reduce the size of the ducted fan vehicle to the MAV class with the availability of the micro components in the market. The duct also reduces the possibility of collision between the rotor and the people around it. Therefore, the ducted fan configuration is taken as the main configuration of the MAV.

	Fixed-wing	Rotary-wing	Ducted Fan	Flapping-wing
1. VTOL	No (A few yes)	Yes	Yes	Yes
2. Hover	No(A few yes)	Yes	Yes	Yes
3. Endurance	Long	Short	In the middle	Short
4. Lift Provided by	Wing	rotor	duct and rotor	flapping motion
5. Easy to operate	Yes	No	Yes	Yes
6. Propeller Collision (with obstacles)	Yes	Yes	No	Doesn't use propeller
7. Packable in a briefcase	Easy	Difficult	Easy	Difficult

Table 2.1 Comparison of four possible MAV configurations

2.3 Power Consumption and Weight Estimation

The power consumption data of the MAV can be used as a basis for the selection of the propulsion system and power source. To obtain the power consumption data, an initial weight data of the MAV is needed. This data can be estimated based on the weight of the hardware and the possible payloads carried onboard. The total aircraft weight can be obtained by combining the information of the propulsion, power source, fixed equipment and structural weights. The whole process to find the total weight of MAV is illustrated in Figure 2.2. Iteration might be needed to complete this task.

Figure 2.2 Method to determine the weight of the MAV

2.3.1 Hardware Selection and Initial Weight Estimation

Hardware with a light weight and small dimension is sought, due to the constraint in the total weight and the space available. The weights of the hardware and payloads for Radio Controlled (R/C) and autonomous operation are summarized in Table 2.2. The total MAV weight used for the initial power calculation can be chosen based on the data that the typical hardware weight is approximately 30 – 40 % the total weight for MAV. The total weight of 500 grams is chosen as the initial weight for the power calculation based on the hardware weight given in Table 2.2.

Components		Weights (grams)		Note
		Radio Controlled (R/C) Mode	Autonomous Mode	
Hardware	1. Electronic Speed Controller (ESC)	20	20	ESC with < 18/25 A
	2. Flight Control Computer (FCC)	-	30	-
	3. Servo	20	20	5 Sub Micro Servo
	4. Navigation	25	30	-
	5 Data Link	10	15	-
Payload	1. Camera	15	15	Camera
	2. Transmitter	15	15	-
Total Equipment Weight		105	145	-
Initial Total Weight		500		-

Table 2.2 Hardware carried in MAV for
Radio Controlled (R/C) and Autonomous modes

2.3.2 Power Consumption Calculation Based on the Momentum Theory

The total power consumption of an aircraft using a propeller consists of the induced power and the profile power. The induced power is the power supplied to rotate the propeller and produce the thrust. For aircraft in hover or flying at a very low speed, the induced power is much higher compared with the profile power.

Figure 2.3 Flow illustrations of free and ducted propellers in hover

The induced power can be approximated using the momentum theory [34]. The momentum theory assumes that there is a thin actuator disk where the pressure difference exists between the upper and lower surface of the propeller as a result of the blade rotational motion. The power to generate this blade rotational motion is supplied in the form of the torque. From the flow analysis, the wake area downstream of the free propeller is approximately half of the disk area, as illustrated in Figure 2.3. This implies that the velocity in the wake area (w) is two times of the induced velocity across the propeller (v_h). For ducted propeller case, the wake area is equal to the disk area for the case where the duct diameter equals with the propeller diameter. Thus, the wake velocity also equals to the induced velocity in the propeller plane. The induced velocity and the induced power of the ducted propeller configuration in hover can be calculated using [34]:

$$v_h = \sqrt{\frac{T}{\rho A}} \quad (2.1)$$

$$P_h = T v_h = \frac{1}{2} T \left(\sqrt{\frac{T}{\rho A}} \right) = \frac{T^{\frac{3}{2}}}{\sqrt{4 \rho A}} = \frac{T^{\frac{3}{2}}}{\sqrt{4 \rho \pi R^2}} \quad (2.2)$$

where P_h and T stand for the induced power and thrust respectively. ρ is air density, whereas A is the disk area and R is the propeller radius from the hub.

Equation (2.2) gives an ideal induced power, since it does not account for the effect of non-uniform inflow, tip losses, swirl effect, and the blade profile power used to overcome the drag. The non ideal induced power formula for a rectangular blade with no twist is given by [34]:

$$C_p = \frac{\kappa C_T^{\frac{3}{2}}}{\sqrt{2 \rho A}} + \frac{\sigma C_{d_0}}{8} \quad (2.3)$$

C_p and C_T are the power and thrust coefficients. κ is a correction factor for the non ideal case, while σ is the blade solidity factor. C_{d_0} is 2-D profile drag coefficient. The blade solidity factor is defined as the ratio of propeller area and the disk area.

The κ value for the ducted propeller configuration can only be obtained from the experiment. As a first approximation, the κ value of the free propeller will be used for the FM calculation of the ducted propeller configuration.

Hover efficiency or the FM of the propeller is formulated as [34]:

$$FM = \frac{\text{Ideal Power}}{\text{Induced Power} + \text{Profile Power}} = \frac{\frac{C_T^{\frac{3}{2}}}{\sqrt{2}}}{\kappa \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{\sigma C_{d_0}}{8}} \quad (2.4)$$

The induced velocity and induced power formula for ducted propeller in vertical climbing and descent can be written as [34]:

- Ducted propeller in climb

$$\frac{v_i}{v_h} = -\frac{V_c}{2v_h} \pm \sqrt{\left(\frac{V_c}{2v_h}\right)^2 + 1} \quad (2.5)$$

$$\frac{P_c}{P_h} \approx \frac{V_c}{v_h} + \frac{v_i}{v_h} \approx \frac{V_c}{2v_h} + \sqrt{\left(\frac{V_c}{2v_h}\right)^2 + 1} \text{ for } V_c / v_h \geq 0 \quad (2.6)$$

- Ducted propeller in descent

$$\frac{v_i}{v_h} = -\frac{V_D}{2v_h} \pm \sqrt{\left(\frac{V_D}{2v_h}\right)^2 - 1} \quad (2.7)$$

$$\frac{P_D}{P_h} = \frac{V_D}{v_h} + \frac{v_i}{v_h} \approx \frac{V_D}{2v_h} - \sqrt{\left(\frac{V_D}{2v_h}\right)^2 - 1} \text{ for } V_D / v_h \leq 2 \quad (2.8)$$

Here, v_i refers to the induced velocity at the condition of interest, while v_h is the induced velocity in hover. V_c and V_D are the climbing and descent speeds respectively. The power consumption in hover, climbing and descent are defined as P_h , P_c and P_D respectively.

In forward flight, the rotor must be tilted, so the thrust vector has components in the horizontal and vertical directions. This condition creates highly asymmetrical flow. The rotor equations in the forward flight can be derived using the method derived by Glauert, as explained in Leishman [34], as follows:

$$v_i = \frac{v_h^2}{\sqrt{(V_\infty \cos \alpha)^2 + (V_\infty \sin \alpha + v_i)^2}} \quad (2.9)$$

$$\frac{P_{forward}}{P_h} = \frac{\mu}{\lambda_h} \tan \alpha + \frac{\lambda_h}{\sqrt{\mu^2 + \lambda_h^2}} \quad (2.10)$$

where α indicates the rotor angle of attack and V_∞ is the freestream velocity. μ is the ratio between forward and rotational speeds ($\mu = \frac{V_\infty \cos \alpha}{\Omega R}$). λ is the inflow ratio in the forward flight and λ_h is the inflow ratio in hover. Both parameters are calculated as:

$$\lambda = \frac{(V_\infty \sin \alpha + v_i)}{\Omega R} \quad (2.11)$$

$$\lambda_h = \frac{v_h}{\Omega R} \quad (2.12)$$

Ω is propeller rotational speed. The solution of above Equation (2.9) and (2.10) can be found by implementing the Newton-Rhapson iteration method with the hover induced velocity used as an initial value.

	Value	Note
Mass	200, 300, 400, 500 (grams)	For calculation of induced power in climb and transition, the mass is assumed to be 500 grams
Duct Diameter	15, 20, 25 (cm)	Propeller diameter equals to duct diameter. For calculation of induced power in climb and transition, the propeller/duct diameter is assumed to be 20 cm
Ω	3000 – 8000 (RPM)	Propeller Rotation Speed
κ	1.78	Correction factor for non ideal case [34]
C_{d_0}	0.03	2-D drag coefficient of the propeller
Blade chord	0.03 (m)	The blade is assumed to have rectangular shape
α	0° - 90°	In hover and vertical climb, the propeller angle of attack is zero. In transition, the propeller angle of attack is between 0° and 90°

Table 2.3 Data required in the calculation of induced power
for ducted propeller vehicle

2.3.3 Induced Power Characteristics of Ducted Propeller Vehicle

Table 2.3 presents data used to calculate induced power of a ducted propeller vehicle. The results are shown in the following figures. Figure 2.4 presents the FM of the ducted two-blade, rectangular-shape propeller. The total mass used in the calculation is 500 grams, as mentioned in the previous section. The FM value at the typical MAV operating point is found to be 56.18 %, which compares well with the value obtained from experiment (50 %) by Abrego & Bulaga [22]. If this FM value is taken for the power calculation, the result of this calculation can be assumed to represent the upper bound of the power consumption, since theoretically FM of the ducted propeller is higher than FM of the free propeller.

Figure 2.4 Figure of Merit of ducted propeller MAV

Figure 2.5 Induced power of ducted propeller MAV in hover

Figure 2.5 gives the induced power curves for the ducted propeller in hover for various masses. For the same vehicle mass, enlarging propeller diameter is found to result in a significant reduction in the induced power. This happens because higher propeller diameter promotes higher mass flow into the propeller plane, so higher thrust can be obtained with smaller power consumption.

Figure 2.6 gives the induce power of the ducted propeller with mass of 500 grams in vertical climb. As previously stated, the magnitude of the induced power depends on the velocity across the propeller plane. The increase in the rate of climb allows higher flow mass and higher velocity travel across the propeller plane, and thus the induced power by the propeller also increases.

Figure 2.6 Induced power of ducted propeller MAV in climb

The induced power in transition for the propeller angle of attack range from 5° to 80° is given in Figure 2.7. In this calculation, the mass of the vehicle is assumed to be 500 grams also. The propeller angle of attack is defined as the angle between the horizontal plane and the propeller axis. Below 15° angle of attack, the induced power decreases with the increase in transition velocity until the minimum induce power is achieved. After the point of minimum induced power is passed, the induced power will grow exponentially with the increase of the transition velocity. Above 25° angle of attack, the induced power trend changes considerably, where an increase in the transition velocity causes an exponential increase in the induced power. The trend of

the transition induced power curve at high angles of attack is identical to the climb condition.

Figure 2.7 Induced power of ducted propeller MAV in transition

2.3.4 Power Consumption at Two Design Points

The previous results show the general induced power trends of a ducted propeller vehicle in hover, vertical climb and transition. In this section, the calculation will be performed at specific design points. Two design points are considered in the power consumption calculation of the MAV. The first design points assumes the total weight of 400 grams and propeller diameter of 15 cm, while the second design point, assumes the total weight of 500 grams and the propeller diameter of 20 cm. The data needed for this calculation can be found again in Table 2.3. In the calculation, the vertical climb and transition speeds are taken as 1 m/s. The contribution of the profile power into the total power is neglected, since the MAV flies at a relatively slow velocity. The motor efficiency is estimated to be 70 %, which is 10 % smaller than the manufacturer data. At horizontal flight, the total power consumption is smaller compared to the other conditions. This is because the propulsion system is used to counter the drag only, instead of the weight. The power consumed by the MAV for the two design points at various flight conditions is presented in Figure 2.8. The total power consumed for both configurations are in the range of 70 -78 Watts.

(a)

(b)

Figure 2.8 Total power consumed by ducted propeller MAV with
(a) $D = 0.15$ m and mass 400 grams (b) $D = 0.2$ m and mass 500 grams

2.3.5 Propulsion System and Power Source Selection

The power consumption data obtained in the previous subsection is used to select the propulsion system and the power source. There are two possible options for the propulsion system, including internal combustion engine (ICE) and electrical

motor. In this case, electrical propulsion is chosen because aircraft propelled with an electrical motor has a fixed center of gravity (CG) location, which is important to simplify the stability and control analysis. The electric motor is also more suitable for stealth operation as it has a lower sound intensity level.

Appendix A summarizes the technical data of three different motors available in the market that are considered in this study. These motors can deliver the required power, if a propeller with diameter bigger than 15 cm (6") is used. After comparison, Hacker A20-22L is selected due to its highest maximum power delivered. The efficiency of this motor is also better than the others considered. Thunder Power lithium polymer (LiPo) battery with 1320 mAH capacity and 11.1 V is selected as the main power source. This brand is chosen due to its high energy density and its relatively low weight.

2.4 Aerodynamics and Static Stability Formulation

2.4.1 Estimation of Aerodynamics

The main components of a ducted fan MAV include wing, horizontal and vertical tails, duct and payload compartment. The total aerodynamics of ducted fan MAV is governed by the aerodynamics of its individual component. The following section explains the method to find the aerodynamic characteristics of each component and the method to determine the aerodynamics of aircraft.

The MAV can be equipped with high or low aspect ratio wings. The aerodynamic characteristics of high aspect ratio MAV can be calculated using the semi-empirical equations derived from the lifting line theory. The behavior of the flow for the low aspect ratio wing is different than the high aspect ratio one. The lift of the low aspect ratio wing is affected mainly by the presence of a strong vortex, resulting in nonlinear aerodynamic characteristics [35]. The short span of the low aspect ratio wing makes the vortex created at the tip influence the flow at the center of the wing. This is a primary source of the nonlinearity in the aerodynamics. This nonlinear aerodynamic characteristics are confirmed by Torres & Mueller [36], whom experimentally investigated the aerodynamics of wings with aspect ratio of less than two. The aerodynamic characteristics of low aspect ratio (AR) wings can be approximated using Polhamus & Lamar equations, where the constants necessary for

the calculation are obtained from Torres & Mueller experimental data [36]. Both methods have been proven to be useful for aerodynamics analysis in the conceptual design phase.

The lift coefficient curve of a high aspect ratio wing can be constructed by determining first the lift curve slope, the maximum lift coefficient, the angle of attack at zero lift and the stall angle. All of these parameters can be obtained by solving a set of equation below (Equations (2.13) – (2.17)) [31, 32].

The slope of the lift coefficient curve (C_{L_α}) can be calculated using:

$$C_{L_\alpha} = \frac{2 \pi k AR}{2 + \sqrt{\frac{AR^2(1-M^2)}{k^2} \left[1 + \frac{\tan^2(\Lambda_{0.5})}{1-M^2} \right]} + 4} \quad (2.13)$$

where AR is the wing aspect ratio and it is defined as $AR = \frac{b^2}{S} = \frac{b}{\bar{c}}$, with b and $\bar{c}$ are the span and the mean aerodynamic chord respectively. $\Lambda_{0.5}$ is the sweep angle at 50% chord. M represents the Mach numbers of the flow. For AR of 2 to 4, parameter k in the Equation (2.13) can be found using [31]:

$$k = 1 + \frac{(1.87 - 0.00023\Lambda_{LE})}{100} AR \quad (2.14)$$

Λ_{LE} is the swept angle at leading edge of the wing.

The maximum aircraft lift coefficient ($C_{L_{\max}}$) is approximated from the airfoil maximum lift coefficient ($c_{l_{\max}}$) using the following relationship [31]:

$$C_{L_{\max}} = 0.9c_{l_{\max}} - 0.1 \quad (2.15)$$

The angle of attack at zero lift ($\alpha_{C_L=0}$) and the stall angle ($\alpha_{C_{L_{\max}}}$) can be estimated using the expressions below [31]:

$$\alpha_{C_L=0} = \left(\alpha_{C_l=0} + \frac{\Delta\alpha_0}{\varepsilon_t} \varepsilon_t \right) \quad (2.16)$$

$$\alpha_{C_{L_{\max}}} = \frac{C_{L_{\max}}}{C_{L_\alpha}} + \alpha_{0_L} \quad (2.17)$$

where ε_t is the twist angle and $\alpha_{C_l=0}$ is the airfoil angle at zero lift coefficients. $\Delta\alpha_0$ is the increment of angle of attack.

The lift coefficient (C_L) is calculated using following equation:

$$\begin{cases} C_L = C_{L_\alpha} (\alpha - \alpha_{C_L=0}) & \text{for } \alpha < \alpha_{C_{L_{\max}}} \\ C_L = C_{L_{\max}} & \text{for } \alpha = \alpha_{C_{L_{\max}}} \end{cases} \quad (2.18)$$

The drag prediction of a high aspect ratio wing can be performed by using the component build up or the equivalent skin friction method. Luke & Bowman [37] and Guerrero, *et al* [27] have incorporated the component build-up method to obtain the drag coefficient of the fixed-wing and ducted fan MAV.

The drag coefficient when the lift coefficient is zero (C_{D_0}) can be calculated by using the following formula [31]:

$$C_{D_0} = \frac{C_f (FF) Q S_{wet}}{S_{ref}} \quad (2.19)$$

where C_f is the coefficient of friction. S_{wet} and S_{ref} are the wetted area and the reference area respectively. The wetted area is the area of the surface exposed by the flow. The reference area is the projection area of the wing in the horizontal surface. The parameter Q indicates the interference factor among the components.

The value of C_f for the laminar flow is given as:

$$C_f = \frac{1.328}{\sqrt{Re}} \quad (2.20)$$

Re is the Reynolds number of the flight. The parameter FF in Equation (2.19) indicates the form factor. For the wing, tail, strut and pylon, the form factors are given as:

$$FF = \left[1 + \frac{0.6}{(x/c)_m} \left(\frac{t}{c} \right) + 100 \left(\frac{t}{c} \right)^4 \right] \left[1.34 M^{0.18} (\cos \Lambda)^{0.28} \right] \quad (2.21)$$

while for the fuselage, canopy and external storage, the form factor are given by:

$$FF = 1 + \frac{60}{(l/d)^3} + \left(\frac{1}{400} \frac{l}{d} \right) \text{ or} \quad (2.22)$$
$$FF = \left(1 + \frac{0.35}{(l/d)} \right)$$

(l) is the body length and (d) is the diameter of the body. The parameters $(x/c)_m$ indicate the location of the maximum thickness point and $(\frac{t}{c})$ is the thickness-to-chord ratio of the airfoil.

The total drag can be modeled using a general parabolic equation as seen in Equation (2.23):

$$C_D = C_{D_0} + \frac{1}{\pi \text{AR} e} C_L^2 \quad (2.23)$$

with e is the wing efficiency factor.

For the configuration with multi wings, the total drag can be calculated based on the Munk's formula, as presented in Equation (2.24) [27]:

$$C_D = C_{D_0} + C_{D_{induced}} = C_{D_0} + \left(\begin{array}{l} \frac{C_L^2}{\pi \cdot \text{AR} \cdot e} \text{ for monoplane} \\ \frac{C_L^2}{2\pi \cdot \text{AR} \cdot M} \text{ for biplane} \\ \frac{C_L^2}{6\pi \cdot \text{AR}} \text{ for triplane} \end{array} \right) \quad (2.24)$$

$C_{D_{induced}}$ is the induced drag coefficient.

The lift and drag coefficients (C_L and C_D) for the low aspect ratio wing ($\text{AR} \leq 2$) are calculated using Polhamus & Lamar equations, as presented below [36]:

$$C_L = K_p \sin \alpha \cos^2 \alpha + K_v \cos \alpha \sin^2 \alpha \quad (2.25)$$

$$C_D = C_{D_0} + K_p \sin^2 \alpha \cos \alpha + K_v \sin^3 \alpha \quad (2.26)$$

The value of the parameters K_p , K_v and K are given in Torres & Mueller [36]. C_{D_0} can be calculated using Equation (2.19).

Above the stall angle, the lift and drag of low and high aspect ratio wings can be estimated based on the data presented in the Zimmerman [38, 39] or Moschetta *et al* [40]. This technique is performed due to unavailability of the semi-empirical method to predict the lift and drag of the lifting surface beyond the stall point.

The lift and drag characteristics of the duct might be obtained by interpolating the data from Fletcher [24], as performed by Guerrero *et al* [27]. This data was obtained from the wind tunnel test of ducted fan vehicle with various aspect ratios.

A payload pod or slender body of revolution used as the payload compartment can be analyzed based on the formula given by Allen & Perkins [41]:

$$L_{pod} = q \left[S_b \sin(2\alpha) \cos\left(\frac{\alpha}{2}\right) + C_{d_{\alpha=90^\circ}} A_p \sin^2 \alpha \cos \alpha \right] \quad (2.27)$$

$$D_{pod} = q \left[\left(A C_{d_0} \right) \cos^3 \alpha + S_b \sin(2\alpha) \sin\left(\frac{\alpha}{2}\right) + C_{d_{\alpha=90^\circ}} A_p \sin^3 \alpha \right] \quad (2.28)$$

S_b and A_p are the projection and perimeter area of the pod. $C_{d_{\alpha=90^\circ}}$ is the 2-D drag coefficient at 90° angle of attack. q is the dynamic pressure based on the vehicle velocity. C_{d_0} is the drag coefficient at the zero lift of the 2-D profile.

The total lift and drag of the ducted propeller MAV can be built up by the contribution of each component, as described in the following equations:

$$L = L_{wing} + L_{duct} + L_{pod} + L_{HTP} \quad (2.29)$$

$$D = D_{wing} + D_{pod} + D_{duct} + D_{VTP} + D_{HTP} + D_{stator} + D_{non-lifting-component} \quad (2.30)$$

The subscripts in Equation (2.28) and (2.29) refer to the name of each component. HTP and VTP stand for Horizontal Tail Plane and Vertical Tail Plane. The stator refers to component used to counter the engine torque and to provide roll control. The pod is a component used to contain the components.

The side force (SF) of the ducted propeller MAV comprises side force from the duct, vertical tail and the pod. Due to the symmetric geometry of the payload pod and the duct, the side force due to a sideslip angle equals to the lift from the same value of angle of attack. The total side force can be calculated using the equation below:

$$SF = SF_{duct} + SF_{pod} + SF_{HTP} + SF_{VTP} + SF_{stator} \quad (2.31)$$

The side forces from the wing, horizontal tail and stator are normally insignificant compared to the side forces from the duct, the vertical tail, the payload pod and the stator.

Figure 2.9 Illustration of wind and body axis coordinate system

Figure 2.9 illustrates the wind ($X_W - Y_W - Z_W$) and body ($X_B - Y_B - Z_B$) coordinate systems. The body and wind axes centered at the CG of aircraft. The X axis of body coordinate system (X_B) is pointed to the aircraft's nose, while the X axis of wind coordinate system (X_W) is parallel to the wind direction. The aerodynamic forces are defined in the wind axis. The lift and drag are defined in the opposite directions with the Z_W and X_W axes, while the side force is toward the starboard and perpendicular to Z_W and X_W axes. The orientations of the body axis with respect to the wind axis are defined by the angle of attack (α) and the angle of sideslip (β). The aerodynamic forces in the body axis are found by projecting the forces in the wind axis to the body axis. The forces expressed in the body axis can be found by transforming the aerodynamic forces of each component using following relations [42]:

$$X = -[D \cos \alpha \cos \beta + SF \cos \alpha \sin \beta - L \sin \alpha] \quad (2.32)$$

$$Y = -D \sin \beta + SF \cos \beta \quad (2.33)$$

$$Z = -[D \sin \alpha \cos \beta + SF \sin \alpha \sin \beta + L \cos \alpha] \quad (2.34)$$

with X, Y and Z are the forces on each axis of the body coordinate system respectively. The directions of these forces are aligned with the axes of the body coordinate system.

The lift, drag and side force can be written in the non-dimensional form by dividing the forces with the dynamics pressure (q) and the wing area (S), as given below:

$$C_L = \frac{L}{qS} \quad (2.35)$$

$$C_D = \frac{D}{qS} \quad (2.36)$$

$$C_{SF} = \frac{SF}{qS} \quad (2.37)$$

C_L , C_D and C_{SF} are the coefficients of lift, drag and side force respectively.

If the MAV comprises of wing, pod, duct and tails, the total forces in the body axis are then:

$$X = X_{wing} + X_{duct} + X_{pod} + X_{HTP} + X_{VTP} + X_{stator} \quad (2.37)$$

$$Y = Y_{Momentum Drag} + Y_{wing} + Y_{pod} + Y_{duct} + Y_{VTP} + Y_{HTP} + Y_{stator} \quad (2.38)$$

$$Z = Z_{Momentum Drag} + Z_{wing} + Z_{pod} + Z_{duct} + Z_{VTP} + Z_{HTP} + Z_{stator} \quad (2.39)$$

The subscript momentum drag in above equations corresponds to the reaction forces that are used to align the flow parallel with the duct center line. The momentum drag is formulated as [27]:

$$Z_{Momentum Drag} = -\dot{m} V_\infty \sin \alpha \quad (2.40)$$

$$Y_{Momentum Drag} = -\dot{m} V_\infty \sin \beta \quad (2.41)$$

where $\dot{m} = \rho A V_\infty$ is the air mass flow rate entering the duct inlet.

The pitching (M), rolling (l) and yawing (N) moments can be expressed as:

$$\begin{aligned} M = & (M_{0_{wing}} - Z_{wing} x_{wing} + X_{wing} z_{wing}) + (X_{duct} z_{duct} - Z_{duct} x_{duct}) \\ & + (M_{pod} + X_{pod} z_{pod} - Z_{pod} x_{pod}) + (X_{HTP} z_{HTP} - Z_{HTP} x_{HTP}) + \\ & (X_{VTP} z_{VTP} - Z_{VTP} x_{VTP}) + (X_{stator} z_{stator} - Z_{stator} x_{stator}) \\ & - (Z_{Momentum Drag} x_{Momentum Drag}) - (X_{thrust} z_{thrust}) \end{aligned} \quad (2.42)$$

$$\begin{aligned} l = & (Z_{wing} y_{wing} + l_{wing}) + (-Y_{duct} z_{duct} + Z_{duct} y_{duct}) + (-Y_{pod} z_{pod} + \\ & Z_{pod} y_{pod}) + (Z_{HTP} x_{HTP}) - (Y_{VTP} z_{VTP}) - (Y_{stator} z_{stator}) - \\ & (Z_{Momentum Drag} x_{Momentum Drag}) \end{aligned} \quad (2.43)$$

$$\begin{aligned}
 N = & (Y_{duct}x_{duct} - X_{duct}y_{duct}) - (X_{thrust}y_{thrust}) - (X_{wing}y_{wing} + N_{wing}) \\
 & + (N_{pod}Y_{pod}x_{pod} - X_{pod}y_{pod}) - (X_{HTP}y_{HTP}) - (X_{stator}y_{stator}) \\
 & + (Y_{VTP}x_{VTP} - X_{VTP}y_{VTP}) + (Y_{Momentum Drag}x_{Momentum Drag})
 \end{aligned} \tag{2.44}$$

In Equations (2.42), (2.43) and (2.44), X, Y, Z and L, M, N are the forces and moments in the body coordinate system, while the lower letters x, y and z represent the distance of the aerodynamic center of each component to the center of gravity of the aircraft. The subscripts following those letters explain the name of the components. The abbreviation HTP and VTP stand for horizontal and vertical tail plane. For example, $x_{wing}, y_{wing}, z_{wing}$ explain the distance between the aerodynamic centers of the wing to the center of gravity in three dimensional spaces.

$M_{0_{wing}}$ in the above equation represents the wing pitching moment at zero angle of attack. For high and low aspect ratio wings, these values can be estimated using the equations below [31, 36]:

For high AR wing:

$$M_{0_{wing}} = qS\bar{c} \left(\frac{AR + (\cos \Lambda_{c/4})^2}{AR + (2 \cos \Lambda_{c/4})} \right) \tag{2.45}$$

$\Lambda_{c/4}$ in the above equations is the swept angle of the wing measured at the quarter chord point. $\bar{c}$ is the main aerodynamic chord.

For low AR wing:

$$M_{0_{wing}} = qS \left(x_p K_p \sin \alpha \cos \alpha + x_p K_v \sin^2 \alpha \right) \tag{2.46}$$

x_p is the center of pressure location of the wing measures from the leading edge. The value of this parameter can be found graphically from Torres & Mueller [36]

The pitching moment created by the payload pod can be calculated using [41]:

$$M_{pod} = q \left[(Q - S_b (l - x_m)) \sin(2\alpha) \cos\left(\frac{\alpha}{2}\right) \right] \tag{2.47}$$

With Q is the volume of the pod, while l and x_m are the length and the aerodynamic center location of the pod. Due to symmetry, the yawing moment by the payload pod can be obtained using the same formula above.

The parameter N_{wing} is the wing yawing moment due to the angle of sideslip. The value is mainly determined by the wing swept angle. This parameter can be determined using following formulas [31]:

$$N_{wing} = qSbC_{n_{\beta wing}} \beta \quad (2.48)$$

$$C_{n_{\beta wing}} = C_L^2 \left[\frac{1}{4\pi A} - \left(\frac{\tan \Lambda}{\pi AR(AR + 4 \cos \Lambda)} \right) \left(\cos \Lambda - \frac{AR}{2} - \frac{A^2}{8 \cos \Lambda} + \frac{6(\overline{x_{wing}} - \overline{x_{cg}})}{AR} \sin \Lambda \right) \right] \quad (2.49)$$

$\overline{x_{wing}}$ and $\overline{x_{cg}}$ are the wing aerodynamic center and center of gravity location divided by the mean aerodynamic chord. $C_{n_{\beta wing}}$ is the slope of yawing moment coefficient of the wing.

The wing rolling moment due to angle of sideslip (l_{wing}) can be determined based on formula given in Etkin [43], as shown in the following expressions:

$$C_{l_{\beta wing}} = \left(\frac{C_{l_{\beta wing}}}{C_L} \right) C_L + (C_{l_{\beta}})_\Gamma + C_{l_{\beta wing-fuselage}} \quad (2.50)$$

$$(C_{l_{\beta}})_\Gamma = -\frac{C_{L_\alpha} \Gamma}{4} \left[\frac{2(1+2\lambda)}{3(1+\lambda)} \right] \quad (2.51)$$

$$C_{l_{\beta wing-fuselage}} = -1.2 \frac{Z_f (D_f + W_f) \sqrt{A}}{b^2} \quad (2.52)$$

$$l_{wing} = qSbC_{l_{\beta wing}} \beta \quad (2.53)$$

$C_{l_{\beta wing}}$ is the slope of rolling moment coefficient of the wing, while $C_{l_{\beta wing-fuselage}}$ is the slope of the rolling moment coefficient due to wing-fuselage interaction. The parameters Γ is the dihedral angle of the wing, while Z_f , D_f and W_f are the maximum height, diameter and width of the fuselage. $(C_{l_{\beta}})_\Gamma$ is the rolling moment coefficient due to the dihedral angle. $\frac{C_{l_{\beta w}}}{C_L}$ can be determined graphically in Etkin [43] or Raymer [31].

Similar to the aerodynamics forces, the moments can also be defined in the non-dimensional form, as follows:

$$C_M = \frac{M}{qSc} \quad (2.54)$$

$$C_l = \frac{l}{qSb} \quad (2.55)$$

$$C_N = \frac{N}{qSb} \quad (2.56)$$

where C_M , C_l and C_N are pitching, rolling and yawing moments respectively.

2.4.2 Static Stability Criteria

Examining the static stability characteristics of the aircraft is one of the stages in the conceptual design phase. This is to see whether the designed aircraft is statically stable in a particular flight condition. Statically stable means that the aircraft could provide sufficient restoring moment to counter the disturbances. For the VTOL MAV case, the static stability characteristics are assessed in the cruise flight only, since it is almost impossible to create statically stable aircraft in the whole flight envelop.

In the longitudinal plane, the slope of the pitching moment curve with respect to the angle of attack must be negative in order to satisfy the longitudinal static stability criteria ($C_{M_\alpha} < 0$). The slope of the rolling moment curve with respect to the angles of sideslip must be positive to meet with the lateral static stability criteria ($C_{l_\beta} < 0$). In addition, the directional static stability can be achieved if the slope of yawing moment curve with respect to the angles of sideslip is positive ($C_{N_\beta} > 0$) [44].

2.5 Thrust Testing

The thrust testing is performed to study the performance of various motor-propeller combinations, in term of the amount of thrust generated. In addition, this experiment is also done to obtain the power required by the motor, the power delivered by the battery, the motor efficiency, the motor rotation speed and the torque generated.

The thrust testing is conducted in hover or static condition only. The dynamic testing can not be performed because of the unavailability of the wind tunnel. In this experiment, the thrust produced by the motor-propeller combination is measured for each throttle stick deflection using the force gauge. At the same time, the current and the voltage are measured using the ammeter and voltmeter. The rotation speed of the

propeller is sensed using the rpm sensor. These outputs are used to obtain the following variables:

$$\text{Battery Power} = \text{Voltage (Battery)} \times \text{Current} \quad (2.57)$$

$$\text{Motor Voltage} = \frac{\text{Measured Rotation Speed}}{\text{Ideal Motor Rotation Speed}} \quad (2.58)$$

$$\text{Motor Power} = \text{Voltage (Motor)} \times \text{Current} \quad (2.59)$$

$$\text{Motor Torque} = \frac{\text{Motor Power}}{\text{Measured Rotation Speed}} \quad (2.60)$$

$$\text{Motor Efficiency} = \frac{\text{Motor Power}}{\text{Battery Power}} 100 \% \quad (2.61)$$

The structural test bench and the experimental setup are depicted in Figure 2.10. The test bench is used to attach the gauges and to prevent the motor from slanting. The equipments used in the test are listed below:

1. MecMesin AGP Force Gauge
2. Hacker A20-22L and AXI 2212 motors with X-20 Pro ESC
3. Propeller with sizes: 10" x 4.7", 10" x 7", 8" x 3.8" and 8" x 6"
4. Thunder Power 11.1 V - 1320 mAh battery
5. A set of Futaba transmitter and receiver, ammeter and voltmeter

(a)

Figure 2.10 (a) Test bench for thrust testing (b) Illustration of thrust experiment set up

Figure 2.11 Current and thrust of Hacker and AXI motors using half charged battery

2.5.1 Hacker and AXI Motor Performance

Figure 2.11 presents the current and thrust relations of the Hacker and AXI motors combined with 10" x 4.7" and 8" x 3.8" propellers. For the same propeller dimension, the Hacker and AXI motors draw similar current and produce the same amount of thrust. The only different observed is the operational range of both motors, where the Hacker motor can operate at a higher current and power than the AXI

motor. This might indicate that the Hacker motor could give more thrust than the counterpart. Paired with the 10" x 4.7" propeller, the maximum thrust produced by the Hacker setup is 3.4 N at 5.9 A, while the AXI gives the maximum thrust of 3.1 N at 4.2 A. The maximum thrust of 2.25 N at 4 A and 2 N at 3 A are obtained using 8" x 3.8" propeller, by both the Hacker and AXI motors.

2.5.2 Performance of Hacker Motor at Maximum Point

Further experiment is focused on examining the performance of Hacker motor at the maximum point powered by a fully charged battery. The results of this experiment are presented in Figure 2.12 and Figure 2.13. The maximum thrust obtained using the 10" x 4.7" propeller is 3.95 N, 3.4 N using the 8" x 3.8" propeller. The maximum thrust data from the manufacturer for the 8" x 3.8" or 8" x 6" propellers is 5.72 N, while it is 8 N for 10" x 4.7" propeller. The main reason for this difference is the power source capacity used in the test. The vibration of test bench might also result in the deviation in reading the gauge. If the duct is assumed to provide an extra thrust up to 30 – 50 %, the maximum thrust obtained for the propeller of 25 cm (10") diameter is between 5.135 - 5.97 N and around 4.42 – 5.1 N for the propeller with 20 cm (8") diameter. This data reveals that the propellers with diameter of 20 cm (8") and 25 cm (10") can be used as the propulsion system for a ducted fan with weight up to 500 grams.

Figure 2.12 Current and thrust of the Hacker motor using fully charged battery

Figure 2.13 Battery power and torque of the Hacker motor using fully charged battery

At the maximum thrust, the 10" x 4.7" propeller generates torque 0.072 N-m, whereas the 8" x 3.8" propeller produces a torque of 0.068 N-m. The efficiency of the Hacker motor for the 8" x 3.8" propeller lies in the range of 70-80%, while it is in the range of 50-60% for the 10" x 4.7" propeller. These facts can be seen in Figure 2.14. The different on the efficiencies of both propellers may be caused by the fact that 10" x 4.7" has a higher operating point than the counterpart. It means that the peak efficiency is achieved at higher current. The high operating point is caused by the high pitch angle of the propeller.

Figure 2.14 Efficiency of the Hacker motor

2.6 Airfoil Selection

Aerodynamic characteristics of airfoil at low Re are influenced by the presence of laminar boundary layer on the surface of the airfoil. Compared to turbulent boundary layer, laminar boundary layer is more sensitive to the adverse pressure gradient. The laminar flow will easily separate from the surface if it experiences this adverse pressure condition. The flow separation reduces lift coefficient and at the same time increases drag [45].

The performance of airfoils at low Re is well covered in Selig *et al* [46]. The lift and drag characteristics of 34 airfoils are described on that report. The performance of other low Re airfoils can be assessed using XFOIL software developed by Drela & Youngren [47]. XFOIL is capable of handling various tasks, including viscous (or inviscid) analysis of an existing airfoil, airfoil design and redesign, blending of airfoils and drag polar calculation with fixed or varying Re and Mach numbers. In this design, the 2D sections of the lifting surfaces are selected by comparing the aerodynamic performances of low Re airfoils obtained from the XFOIL. The contour data of each airfoil can be found from Selig [48]. The simulation is performed for Re range from 5×10^4 to 2×10^5 . Some of the results are presented in Figure 2.15.

From the data in Figure 2.15 Clark-Y and Eppler 180 are selected as the candidate for the wing's and duct's airfoils. Clark-Y airfoil has superior lift coefficient and high stall angle compared to others. The main drawback of this airfoil is a large positive pitching moment created at $\alpha = 0^\circ$. Eppler-180 airfoil has a smaller pitching moment value at $\alpha = 0^\circ$, with nearly the same lift characteristics with Clark-Y. Both Eppler-180 and Clark-Y airfoils have a flat lower surface, which are suitable for application as the duct cross section. These two airfoils are considered as the airfoils for the wing and duct of the MAV.

For the tails, a symmetric airfoil is used to eliminate the pitching moment effect at $\alpha = 0^\circ$. The NACA 0018 and 0012 are deemed to be appropriate airfoils for the tail. The lift coefficient of both airfoils at $Re 5 \times 10^4$ is presented in Figure 2.16, where both airfoils exhibit nonlinearity. The NACA 0018 is chosen as the airfoil for the control surfaces, since it produces higher lift.

(a)

(b)

Figure 2.15 (a) Lift and (b) pitching moment coefficient curves of low Re airfoils

Figure 2.16 Lift coefficient curves of symmetric airfoil at $Re\ 5 \times 10^4$

2.7 Flow Velocity and Angle of Attack at the Tails

The velocity and angle of attack at the tails (horizontal tail, vertical tail and stator) are influenced by the freestream flow and the jet flow from the duct. The placement of the tails with respect to duct exit area determines the magnitude of influence of both flows at the tails, as illustrated in Figure 2.17. The freestream flow will start to influence the local flow at the tails at an angle of attack denoted as critical angle of attack (α_{crit}). This angle of attack is a function of the tail's location from the duct exit point. Below α_{crit} , the local angle of attack and velocity at the tails are determined mainly by the jet velocity from the duct. This is because the duct acts as a shield that isolates the tails from the influence of the surrounding flow. Above α_{crit} , they are influenced by both the jet flow from the duct and the freestream flow.

Figure 2.17 Effect of freestream and jet flow from the duct

(a) Below α_{crit} (b) above α_{crit}

α_{crit} can be calculated using following formula:

$$\alpha_{crit} = \tan^{-1} \left(\frac{DC}{BC} \right) \quad (2.62)$$

where BC is the distance of the tail leading edge from the duct exit point and DC is the distance of the tail from the lowest point of exit lip of the duct.

If the gap between the duct exit point and the tail position is small or zero, the control surface saturation can be prevented or delayed. However, the tail moment arm for this configuration is smaller compared with the case where the tail is placed far from the duct exit point. In the current design, the horizontal and vertical tails and the stator are placed approximately 11.5, 10.5 and 1 cm from the duct exit. This gives α_{crit} value of 41° , 43° and 84° respectively.

The velocity and angle of attack at the tails can be determined based on proposed equations below. These are derived based on condition in Figure 2.16 above.

$$\begin{cases} \text{For } \alpha < \alpha_{crit} \\ V_{Tail} = V_D \\ \alpha_{Tail} = \delta_{Tail} \end{cases} \quad (2.63)$$

$$\begin{cases} \text{For } \alpha \geq \alpha_{crit} \\ V_{Tail} = \sqrt{(V_{\infty} \sin \alpha)^2 + (V_D + V_{\infty} \cos \alpha)^2} \\ \alpha_{Tail} = \tan^{-1} \left(\frac{V_{\infty} \sin \alpha}{V_D + V_{\infty} \cos \alpha} \right) + \delta_{Tail} \end{cases} \quad (2.64)$$

where V_{Tail} and α_{Tail} are velocity and local angle of attack at the tails. α is the aircraft angle of attack. The duct exit velocity (V_D) is found using the relation $V_D = v_i + V_{\infty} \cos \alpha$, where v_i is the induced velocity of the ducted propeller. δ_{Tail} is the angle of tail deflection (horizontal tail (δ_{HTP}), vertical tail (δ_{VTP}) and stator (δ_{Stator})).

As an example, Figure 2.18 presents the local angles of attack and velocities at the horizontal tail for constant freestream airspeed of 15 m/s. Two horizontal tail locations are selected; 0.125 and 0.075 m from the duct exit point. For the horizontal tail with leading edge (LE) at 0.125 m, its angle of attack is zero for $\alpha_{aircraft} \leq 36^\circ$. Above this angle of attack, there is a jump in the horizontal tail angle of attack and its local velocity, as given in Figure 2.19. This is because the freestream velocity starts to have influences on the local flow at the horizontal tail. For the horizontal tail LE located at 0.075 m, the jump in the local velocity and angle of attack are delayed until $\alpha_{aircraft} = 84^\circ$. It must be underlined that the calculation is performed using constant freestream airspeed. If the freestream airspeed is varied, the trend will be the same and only the magnitude changes.

Figure 2.18 α_{crit} as a function of tails position from the duct exit point

(a)

Figure 2.19 Horizontal tail (a) angle of attack and (b) velocity in cruise-to-hover transition with constant freestream velocity $V_\infty = 15$ m/s

2.8 Evolution of the Designs and Detailed of Final Configuration

2.8.1 Evolution of the Designs

The process to arrive at the final design involves iterations, where changes or evolutions from one prototype to another are unavoidable. This sub-chapter will briefly discuss the design evolutions. Figures 2.20 and 2.21 present this evolution from the early concept until the final design.

Initially, a 15 cm (6") propeller diameter is used as the main propulsion system in the 1st – 4th designs. Due to the small thrust delivered, a change to the 20 cm (6") and 25 cm (10") propellers is incorporated in 5th – 8th designs. This consideration is made based on the result from the thrust testing.

In the 1st design, the vertical and horizontal tails are located in the propeller slipstream and outside the duct. The drawback of this placement is that the tails might be saturated under a strong crosswind disturbance [25]. The addition of the wing is intended to improve the aerodynamics of the MAV. Nevertheless, under a strong gust

the wing will generate a significant amount of drag. To overcome this problem, the wing is designed to be movable, such that its orientation is not fixed relative to the duct center line. In other words, the incidence angle of the wing can be varied.

In the 2nd design, the tails are placed inside the duct to prevent control saturation. The payloads are stored inside the wing. In the 3rd design the payloads location is shifted under the duct to reduce the actuator's load when the wing is rotated. Due to a poor CG placement and a small tail arm, both configurations can't achieve the static stability requirements in the cruise flight. The 4th design is developed to improve static stability characteristics. It is done by increasing the aspect ratio of the duct, using Eppler-180, instead of Clark-Y airfoil and placed the tails outside the duct. A simple prototype of 4th design has been developed and it has been used for the hover test. However, the thrust delivered by this propeller-duct combination could not lift up the vehicle in hover.

The 10" and 8" propellers are incorporated into the 5th and 6th designs and afterwards, since these propellers can deliver thrust up to the weight of the MAV. Nonetheless, the 5th and 6th designs still give static stability and drag problems. Improvements of the static stability characteristics in cruise are achieved in the 7th design and 8th design. Both designs have similar sizes and only the payload box arrangements differ. In the 8th design, the payloads are stored at a compartment located in front of the propeller and aligned with body X-axis. This makes the CG location lay close to the plane of symmetry. The 8th design gives also much better static stability and aerodynamic characteristics. It allows more flexibility to resize and to shift the payload box. Therefore, the 8th design is chosen as the final configuration of the MAV. The detailed drawings of the 8th design are given in Figure 2.22 and 2.23. Appendix B gives the three view drawings of the MAV.

Figure 2.20 Evolution of the configuration: from 1st to 4th designs

Figure 2.21 Evolution of the configuration: from 5th to 8th designs

Figure 2.22 Isometric drawing of the MAV with material information

Figure 2.23 Three view drawing of the MAV

2.8.2 Details of the Final Configuration

The total MAV's weight is obtained by summing all the component weights. The total weight must be the same or smaller than the initial weight determined in Section 2.3.1. If the total weight exceeds the initial weight, the total power consumed by MAV is increased. If the increase in the power consumption is large enough, the propulsion system might not be able to deliver such an amount of the power anymore. Hence, iterative processes, as illustrated in Figure 2.2, may need to be carried out until any change in the weight or the total power consumption are still in the range of the power delivered by the motor.

The complete geometrical properties and the weight distribution of the designed MAV can be found in Table 2.4 and 2.5. The total weight, including the structural weight, is calculated using the SolidWork[®] software, based on the detailed drawing. The final total weight is 487.25 grams and the total structural weight alone is 180 grams. This low structural weight is achieved by using a combination between of high strength and low weight materials.

Three control surfaces, horizontal tail, vertical tail and stator, are used to control the pitch, yaw and roll angles. These control surfaces are placed in the propeller slipstream. The CG of the MAV is located around 2 mm from the duct lips. This is a direct consequence of placing the heaviest component in front of the duct. The advantage of this placement is the aerodynamics forces from duct and wing create a stabilizing moments. However, this condition requires a relatively large tail force to trim the MAV.

The CG location with respect to duct inlet point:

$$\begin{bmatrix} x_{CG} \\ y_{CG} \\ z_{CG} \end{bmatrix} = \begin{bmatrix} -0.002 \\ 0.0015 \\ 0.0005 \end{bmatrix} \text{m} \quad (2.65)$$

x_{CG} , y_{CG} and z_{CG} here are directed in the same fashion with the body axes coordinate system.

The inertia properties taken at the CG and align with body coordinate system:

$$[I] = \begin{bmatrix} 1541061.29 & -20472.22 & -73513.70 \\ -20472.22 & 4392086.16 & -7884.03 \\ -73513.70 & -7884.03 & 5662769.48 \end{bmatrix} \text{g-mm}^2 \quad (2.66)$$

THE MAV DESIGN PROCESS

		Weight		Weight (g)
		(g)		
Structural Group	Duct	28.83	FCC	28.19
	Wing	11.54	Servo	20
		6.29	Inertial	24.96
	HTP		Measurement Unit (IMU)	
	VTP	6.04	Receiver	10
	Landing Gear	22.29	Subtotal	83.15
	Payload Box	9.3	Engine	62.59
	Engine Support and Wing to Duct Connection	51.56	Propeller	5
	Tail to Landing Gear Connection	3.41	ESC	20
	Stator to Landing Gear Connection	1.37	Battery	80
	Stator	39.21	Subtotal	167.59
	Subtotal	179.84	Data link	26.67
			Camera	30
			Subtotal	56.67
			Total Aircraft Weight	487.25

Table 2.4 Weight distribution of the MAV

Aircraft	Length x Span x Height	0.4 x 0.42 x 0.2 m	Span	0.17 m
Duct	Inner Diameter	0.2 m	Chord at root	0.06 m
	Chord	0.15 m	Taper ratio	1
	Aspect Ratio	1.333	Area	0.0102 m ²
	Airfoil	Eppler-180	Sweep	0°
Wing	Span	0.43 m	i _{ht}	-2°
	Chord at root	0.253 m	Span	0.17 m
	Taper ratio	0.4	Chord at root	0.06 m
	Wing area	0.07601 m ²	Taper ratio	1
	Aspect ratio	2.43	Area	0.0102 m ²
	Wing Sweep	5°	Sweep	0°
	Wing Twist	-2°	i _{vt}	-1°
	Dihedral	0°	Airfoil for HTP, VTP and stator	NACA 0018
	Airfoil	Eppler-180	Propeller	Diameter x Pitch
				8" x 3.8"
Stator	Span 1 stator	0.08 m	Left	i _s
	Chord	0.06 m	Stator	No of stator
	Area	0.0048 m ²	Right	i _s
	AR	1.333	Stator	No of stator

Table 2.5 Geometrical properties of the MAV

2.8.3 Stator Design

The stator is used to counter the rolling motion induced by the motor torque and to provide roll control. It works like an aileron, where the left and right stators are installed with different setting angle. The stator must be installed in the propeller slipstream region, so that it can provide rolling moment even when the vehicle is at rest. The stator is designed based on the requirement to control the rolling motion in hover, vertical flight and transition. Due to the small area of the region under the influence of the propeller slipstream, the stator can only have a maximum dimension of 0.08 m (per side). As a consequence, a cascade stator configuration is required.

The torque value at thrust equals to 4.5 N (neglecting the duct contribution) is approximately 0.0845 Nm. The flow speed at the stator in hover is approximately 10.784 m/s. Three stacks of stator per side that are installed antisymmetrically with a setting angle $\pm 21.079^\circ$ generate rolling moment of -0.0845 Nm. This moment will cancel out the engine torque perfectly. Therefore, a cascade stator with three stacks per side must be installed in the MAV to provide the lateral control particularly in hover.

2.8.4 Advance Feature of the MAV

To enhance the lifting capability of the MAV, the wing is presented on the sides of the duct. This wing has a capability to be moved or rotated, such that its orientation or incidence angle in the pitch direction can be varied. This movable wing helps to increase the versatility of the vehicle in various conditions. In hover and under a strong gust, the position loss can be suppressed by rotating the wing. The wing can also help to reduce the altitude loss and the thrust required during the transitional flight. Figure 2.24 shows the movable or variable incidence wing concept in transition and hover flights.

Figure 2.24 Movable or variable incidence wing concepts
in transition and hover flights

2.9 Aerodynamics Forces and Moments

2.9.1 Aerodynamic Forces for the Fixed-Wing Configuration

The aerodynamic forces and moments presented in this section are determined using the semi-empirical equations explained in the previous section. The term fixed-wing configuration means that the wing setting angle is fixed relative to druct chord line. In the design process, the value of the wing setting angle is chosen as 6° . Figure 2.25 presents the lift coefficient of MAV for the airspeed of 5, 10 and 15 m/s and with the horizontal tail setting angle of 0° and -6° . There are two peaks in the lift coefficient curve. The first peak corresponds to the point in which the lift of the wing is at its maximum condition. The lift coefficient at this point is 1.4 and it happens at $\alpha = 12^\circ$. The second peak appears because there is a significant increment in the horizontal tail angle of attack and hence the horizontal tail lift coefficient. The horizontal tail lift is more dominant compared with the lift from the other components beyond the critical angle of attack.

For the three different speeds and for both horizontal tail setting angles, the lift coefficient value is identical until the point of critical angle of attack. Beyond this point, the lift coefficient of the smaller velocity is higher than the other. This is because the horizontal tail is exposed at the angle of attack closer to the point of maximum lift. The configuration with more negative setting angle of the horizontal tail has a smaller lift coefficient. This is mainly due to the presence of a large down force in the horizontal tail. However, this condition is only significant at angles of attack below the critical angle of attack.

Figure 2.25 Lift coefficients of the MAV with fixed-wing configuration

Figure 2.26 Drag coefficients of the MAV with fixed-wing configuration

The drag coefficients of the MAV for three different velocities and two different horizontal tail setting angles are presented in Figure 2.26. The drag coefficients have the tendency to increase with an increase in the angle of attack of MAV. This is caused by the increase in the drag coefficient of each individual component as a result of flow separation and an increase in the pressure drag. The peak of the drag coefficient occurs at the angle of attack between $60^\circ - 70^\circ$.

The side force coefficients of the MAV for the angles of sideslip from -30° to 30° and with the vertical tail setting angle of 0° are presented in Figure 2.27. The side force coefficients shown are composed of the side force from the duct and from the momentum drag. The side force from the vertical tail is zero since the setting angle and the vertical tail angle of sideslip is zero. The zero vertical tail angle of sideslip comes from the fact that the MAV operates at the angles of sideslip smaller than the critical angle. With this fact, the side force coefficient of the MAV is antisymmetric with respect to the origin and the values are identical to each other.

Figure 2.27 Side force coefficients of the MAV for fixed- wing configuration

2.9.2 Moment Characteristics for the Fixed-Wing Configuration

Figure 2.28 presents the pitching moment coefficient of the MAV with the horizontal tail setting angle of -6° . The pitching moment graphs for airspeed of 15 and 10 m/s and below $\alpha = 14^\circ$ have negative slopes. This means that the longitudinal static stability requirement is fulfilled at high speed cruise flights. On the other hand, the pitching moment curve for the speed of 5 m/s and below $\alpha = 14^\circ$ has a positive slope.

For $14^\circ \leq \alpha \leq 30^\circ$, the pitching moment coefficient curves of for all airspeeds exhibit a positive slope. This is influenced mainly by the reduction of the wing's lift coefficient. For $30^\circ \leq \alpha \leq 50^\circ$, the pitching moment of the MAV decreases sharply into a more negative value due to a large negative pitching moment produced by the horizontal tail and also because of an increase in the drag of the wing. The pitching moment coefficient becomes less negative for $50^\circ \leq \alpha \leq 90^\circ$ due to reduction of pitch down moment by the horizontal tail and duct.

Figure 2.28 Pitching moment coefficient of the MAV for fixed-wing configuration

The yawing and rolling moments of the MAV for the airspeed of 5, 10 and 15 m/s are presented in Figures 2.29 and 2.30. The yawing moment coefficient curves for these velocities have positive slopes, while the rolling moment curves have negative slopes. These suggest that the directional and lateral static stability requirements are met. The duct side force and the momentum drag contribute positively to the stabilizing moment in the directional direction, while the rolling moments from the stators and the wing governs the lateral stability of the aircraft.

Figure 2.29 Yawing moment coefficients for fixed-wing configuration

Figure 2.30 Rolling moment coefficients for fixed-wing configuration

2.9.3 Effect of CG Variations on the Slopes of Moment Coefficient Curves

The effect of the CG location variations along the longitudinal axis into the static stability characteristics of the MAV in cruise will be analyzed. Such variation does not change the lateral static stability coefficient, but it changes the longitudinal and directional static stability characteristics. The MAV becomes more stable statically in the longitudinal direction as the CG shifts further forward, far from the duct lips and the aerodynamic centers of the wing and duct. At a specific CG point behind the aerodynamic centers of the wing and duct, the slope of the pitching moment coefficient curve becomes positive and the MAV is statically unstable in the

longitudinal direction. This effect is shown in Figure 2.31. The same trend can be observed for the yawing moment characteristics. The degree of the static directional stability reduces as the CG shifts backward closer to the aerodynamics center of the wing and duct, as can be seen in Figure 2.32. As the CG shifts behind the duct's lips, the moment due to the momentum drag tends to dominate and result in static instability.

Figure 2.31 Slopes of the pitching moment coefficient curve at cruise for various CG

Figure 2.32 Slopes of the yawing moment coefficient curve at cruise for various CG

2.9.4 Aerodynamics of the Movable (Variable Incidence) Wing Configuration

In this section, the aerodynamic characteristics of the movable or variable incidence wing configuration will be analyzed. The variable incidence wing configuration means that the pitch orientation of the wing can be changed with respect to duct center line. The wing orientation can be expressed using the angle between the

wing chord line and the horizontal plane. In this configuration, the wing orientation can be fixed, while the wing setting angle (the angle between the wing chord line and the duct center line) varies as the aircraft angle of attack increases. The wing orientation is chosen so that the wing never enters to the stall region.

Figures 2.33 and 2.34 present the comparison of lift and drag coefficients of the MAV for various wing orientations. The aerodynamic characteristics calculated here ignore the interference factor between the movable and the fixed parts of the wing. Overall, the lift coefficient for the variable incidence wing configuration is higher than the fixed-wing, as long as the wing setting angle for the corresponding case is higher than the wing setting angle of the fixed-wing case. This situation can be seen for whole angles of attack range. The extra lift from the portion of the wing that never goes into stall is the primary reason for this improvement. This advantage can be increased by enlarging the area of the movable section of the wing.

Another improvement can be observed in the drag coefficient charts. The drag coefficients of the movable wing configuration are smaller than the drag coefficient of the fixed-wing configuration. This is because the pressure drag contribution from the flow separation in the wing can be made small, since only small portions of the wing enters the stall regime. The main drawback of moving the wing is the presence of large pitch down moment at high angles of attack, as seen in Figure 2.35.

Figure 2.33 Lift coefficients for variable incidence wing configuration

Figure 2.34 Drag coefficients for variable incidence wing configuration

Figure 2.35 Pitching moment coefficients for variable incidence wing configuration

2.10 Prototype Development and Testing

The prototype of the designed MAV is developed using a simple fabrication procedure. This crude prototype is currently being used in a hover test under the manual control, as illustrated in Figure 2.36. The hover test is performed due to its simplicity compared to cruise test. In addition, the hover test is also made possible by the availability of the rotary rig to attach the MAV. This rig helps to reduce the degrees of freedom, such that the operator would not encounter difficulties in controlling the vehicle. The hover test is done in the indoor environment.

From the testing, it is found that the design is very promising. The MAV is capable of hovering and performing helicopter-like maneuvers. The vertical and

horizontal tails have shown to be very effective in generating forces to control the attitude of the MAV. In the future, the prototype will be used for cruise test, as well as hover test under the automatic control.

Figure 2.36 (a) The prototype of the MAV (b) the rig for hover test

2.11 Summary of the MAV Design Process

The following summary can be drawn from the MAV design phase:

1. A VTOL MAV with ducted fan configuration has been designed. A single-propeller with diameter 20 cm (8") is used as the main propulsion system. This decision is made after the outcome of the thrust testing. The maximum size of the MAV is less than 50 cm. The horizontal tail, vertical tail and stator are used to control the pitch, yaw and roll respectively and they are placed in the propeller slipstream. The wings are presented on both sides of the duct to improve the lifting capability. This wing is movable, such that its pitch orientation can be changed. This is to suppress the altitude lost in the transition and to improve the gust rejection level in hover.
2. The MAV is statically stable in the longitudinal direction only for $\alpha < 14^\circ$. The slope of the pitching moment curve in this range is $-0.8/\text{rad}$. Above this angle of attack, the MAV tend to show both statically stable and unstable behavior. The MAV is statically stable in the lateral and directional directions, with the slope of yawing and rolling moment coefficients are $0.13/\text{rad}$ and $-0.04/\text{rad}$ respectively.

THE MAV DESIGN PROCESS

3. As the CG shifted far behind the duct lips, the MAV becomes longitudinally and directionally unstable.
4. The lift of the movable (variable incidence) wing configuration is higher than the fixed-wing configuration especially at high angles of attack. In addition, its drag is also smaller than the counterpart.
5. The proof-of-concept prototype is developed and it is used for hover test. Using the rotary rig test, the MAV can show hover and helicopter-like maneuvers.

Chapter Three

DYNAMIC ANALYSIS OF THE VTOL MAV

3.1 Introduction

Flight dynamic analysis concerns with the responses and stability of an aircraft at a particular condition. This analysis is important to understand the dynamics behavior of aircraft and also serves as the basis in the control system development stages. Method to perform dynamic analysis of aircraft can be found in many textbooks, for example Stengel [42] and Etkin & Reid [43].

Aircraft is usually modeled as a rigid body, where its motions can be described using six-degrees-of-freedom nonlinear equations. These nonlinear equations are then linearized at a specific trim / equilibrium point to obtain the linear time invariant model (LTI). This LTI model is used for stability and response analysis. The linearization process is accomplished by using *the small perturbation theory and Taylor expansion*. The linear form of equations of motion then can be written in general state space form:

$$\dot{x} = Ax + Bu \tag{3.1}$$

where A and B are the Jacobian matrices. The element of which contain the first order derivatives of force and moment with respect to perturbation variables. Figure 3.1 summarizes the procedure in the flight dynamic analysis and control system development performed here.

The dynamics of a conventional aircraft in cruise are characterized by the stable phugoid, short period and Dutch roll oscillations, as well as spiral and roll subsidence modes [42, 43, 44]. For V/STOL vehicles, the dynamics may deviate significantly from those of the conventional aircraft. For such vehicles, unstable modes may be present and generally take the form of unstable oscillatory motions and mostly happen in hover and transition maneuvers [49]. On the contrary, the instabilities in rotary-wing aircraft are seen in whole flight envelope. [50] .

Figure 3.1 Framework for dynamic analysis and control development

The fixed-wing MAV dynamics in cruise are similar with the conventional aircraft. The main difference is on the frequency of the oscillatory motions, which is higher for UAVs / MAVs than for the large aircraft [51]. The airspeed variations might move the phugoid mode to the right half plane, while the short period and Dutch roll modes locations are relatively unaffected [52].

The dynamics of autorotating rotor-wing UAV was studied by Lopez & Wells [53]. Unstable phugoid and spiral modes were observed in both high and slow speeds. Nonetheless, these instabilities are slowly diverging at low frequency, so they are easy to handle. A tilt-wing UAV's dynamics is presented in Dickeson *et al* [54]. The poles associated with cruise dynamics follow the pattern of the conventional aircraft. As the wing tilted, some poles migrate to the left half plane and become unstable. These are related to the fact that the lifting surfaces no longer provide lift and the weight is mainly countered by the thrust. In hover, the poles agree with the poles of helicopter in stationary flight. The dynamics of a small-scale rotorcraft UAV are discussed in Mettler *et al* [55]. In hover, the dynamics involve unstable slowly varying motions in both roll and pitch channels.

Avanzini *et al* [56] discussed the dynamics of ducted fan UAV. The longitudinal dynamics are dominated by a pair of unstable complex conjugate poles in

hover and an unstable real pole in high speed forward flight. The lateral dynamics are characterized by an unstable slowly varying Dutch-roll mode in the whole flight envelope.

3.2 Reference Frames

The inertial, body fixed and wind/stability axes are generally used as reference frames in the flight dynamics analysis. This set of frames is used in this analysis as well. In this particular case, an extra coordinate system, the intermediate coordinate system, is employed. The intermediate coordinate system is obtained by rotating the inertial coordinate system through a 90° pitch rotation

Figure 3.2 Coordinate systems used in the flight dynamics analysis

The orientation between inertial and body fixed axes are defined using a set of Euler angle, termed as roll (ϕ), pitch (θ) and yaw (ψ) angles. The transformation from one coordinate system to another can be performed using Euler transformation method. The sequence of this transformation preferred in the aerospace community is yaw-pitch-roll rotations. The drawback of this transformation is the singularity encountered at $\theta = 90^\circ$. The quaternion method can alleviate this problem. However,

quaternion do not offer clear physical insight and considered to be more appropriate in the implementation stage.

In this work, two different Euler transformation methods are incorporated. For $0 \leq \theta < 90^\circ$, the common Euler transformation method is used, while for $\theta = 90^\circ$ the vertical Euler transformation is incorporated. In the vertical Euler transformation, the orientation angles are defined with respect to intermediate coordinate system. The orientation angles for vertical Euler transformations are defined as vertical roll (ϕ_v), vertical pitch (θ_v) and vertical yaw (ψ_v), as illustrated in Figure 3.3a and 3.3b. The rotation sequences for transforming a quantity from the intermediate to body coordinate systems are roll-pitch-yaw [57].

Figure 3.3 (a) General Euler angles transformation for $0 \leq \theta < 90$

(b) Vertical Euler angles transformation for $\theta = 90^\circ$

The transformation matrix from the inertial to body coordinate system for $0 \leq \theta < 90$ is given by:

$$H_I^B = \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ -\cos \phi \sin \psi + \sin \theta \sin \phi \cos \psi & \cos \phi \cos \psi + \sin \theta \sin \phi \sin \psi & \sin \phi \cos \theta \\ \sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi & -\sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi & \cos \phi \cos \theta \end{bmatrix} \quad (3.2)$$

$$H_B^I = [H_I^B]^{-1}$$

The similar transformation using vertical Euler angles are formulated as:

$$H_I^B = \begin{bmatrix} (-\cos \psi_v \sin \theta_v \cos \phi_v) + (\sin \psi_v \sin \phi_v) & (\cos \psi_v \sin \theta_v \sin \phi_v) + (\sin \psi_v \cos \phi_v) & -\cos \psi_v \cos \theta_v \\ (\sin \psi_v \sin \theta_v \cos \phi_v) + (\cos \psi_v \sin \phi_v) & (-\sin \psi_v \sin \theta_v \sin \phi_v) + (\cos \psi_v \cos \phi_v) & \sin \psi_v \cos \theta_v \\ \cos \theta_v \cos \phi_v & -\cos \theta_v \sin \phi_v & -\sin \theta_v \end{bmatrix} \quad (3.3)$$

From the equations above, the relation between the normal and vertical Euler angles can be inferred. The relations can be written as [57]:

$$\begin{aligned} \theta &= \sin^{-1}(\cos \psi_v \cos \theta_v) \\ \phi &= \sin^{-1}\left(\frac{\sin \psi_v \cos \theta_v}{\cos \theta}\right) \end{aligned} \quad (3.4)$$

$$\begin{aligned} \psi &= \sin^{-1}\left(\frac{\cos \psi_v \sin \theta_v \sin \phi_v + \sin \psi_v \cos \phi_v}{\cos \theta}\right) \\ \theta_v &= -\sin^{-1}(\cos \phi \cos \theta) \\ \psi_v &= \sin^{-1}\left(\frac{\sin \phi \cos \theta}{\cos \theta_v}\right) \\ \phi_v &= -\sin^{-1}\left(\frac{-\sin \phi \cos \psi + \cos \phi \sin \psi \sin \theta}{\cos \theta_v}\right) \end{aligned} \quad (3.5)$$

3.3 Equations of Motion

The aircraft equations of motion are derived from the Newton's law of rotational and translational motions. Using the normal Euler angles, the translational equations can be written as [42, 58]:

$$\begin{aligned} \dot{u} &= \frac{X}{m} + \frac{T}{m} - g \sin \phi + (vr - qw) \\ \dot{v} &= \frac{Y}{m} + g \sin \phi \cos \theta + (wp - ur) \\ \dot{w} &= \frac{Z}{m} + g \cos \phi \cos \theta + (uq - pv). \end{aligned} \quad (3.6)$$

where u, v and w are the velocity vectors of the vehicle in the body coordinate system. The body angular velocity vectors are represented using p, q and r notations.

The parameter g in the above equations corresponds to the gravity constant. The notation T represents the thrust.

There are slight modifications in the translational equations if the vertical Euler angles are used as a reference, as given in the following equations:

$$\begin{aligned}\dot{u} &= \frac{X}{m} + T - g \cos \theta_v \cos \psi_v + (vr - qw) \\ \dot{v} &= \frac{Y}{m} + g \cos \theta_v \sin \psi_v + (wp - ur) \\ \dot{w} &= \frac{Z}{m} - g \sin \theta_v + (uq - pv)\end{aligned}\tag{3.7}$$

The rotational equations of motion are given as:

$$\begin{aligned}l + \text{Engine Torque} &= I_{xx} \dot{p} - I_{xz} \dot{r} + (I_{zz} - I_{yy})qr - I_{xz}pq + qh_z' - rh_y' \\ M &= I_{yy} \dot{q} + (I_{xx} - I_{zz})pr + (p^2 - r^2)I_{xz} + rh_x' - ph_z' \\ N &= I_{zz} \dot{r} - I_{xz} \dot{p} + (I_{yy} - I_{xx})qr + I_{xz}rq + ph_y' - qh_x'\end{aligned}\tag{3.8}$$

The notation I represents the moments and products of inertia, whereas the subscripts that following I correspond to the axes where the inertias are calculated. h_x', h_y' and h_z' are the rotor angular momentum in the body axis.

The angular rates ($\dot{\phi}, \dot{\theta}$ and $\dot{\psi}$) and the linear velocities ($\dot{x}, \dot{y}$ and $\dot{z}$) in the inertial coordinate system are found by multiplying Equation (3.2) with the linear and angular rates on the body coordinate system. In more detail, the relationships are written in the following sets of equation:

$$\begin{aligned}\dot{x} &= u(\cos \theta \cos \psi) - v(\cos \phi \sin \psi + \sin \theta \sin \phi \cos \psi) + w(\sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi) \\ \dot{y} &= u(\cos \theta \sin \psi) + v(\cos \phi \cos \psi + \sin \theta \sin \phi \sin \psi) - w(\sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi) \\ \dot{z} &= -u(\sin \theta) + v(\sin \phi \cos \theta) + w(\cos \phi \cos \theta) \\ \dot{\phi} &= p + q(\sin \phi \tan \theta) + r(\cos \phi \tan \theta) \\ \dot{\theta} &= q(\cos \phi) - r(\sin \phi) \\ \dot{\psi} &= p(\sin \phi \sec \theta) + r(\cos \phi \sec \theta)\end{aligned}\tag{3.9}$$

In term of vertical Euler angles, the relationships can be expressed as:

$$\begin{aligned} \dot{x} = & u((- \cos \psi_v \sin \theta_v \cos \phi_v) + (\sin \psi_v \sin \phi_v)) + v((\sin \psi_v \sin \theta_v \cos \phi_v) + (\cos \psi_v \sin \phi_v)) \\ & + w(- \cos \psi_v \cos \theta_v) \end{aligned} \quad (3.11)$$

$$\begin{aligned} \dot{y} = & u((- \cos \psi_v \sin \theta_v \sin \phi_v) + (\sin \psi_v \cos \phi_v)) + v((- \sin \psi_v \sin \theta_v \sin \phi_v) + (\cos \psi_v \cos \phi_v)) \\ & + w(- \cos \theta_v \sin \phi_v) \end{aligned}$$

$$\dot{z} = u(\cos \theta_v \cos \phi_v) + v(\sin \psi_v \cos \theta_v) + w(- \sin \theta_v)$$

$$\dot{\phi}_v = p(\cos \psi_v \sec \theta_v) - q(\sin \psi_v \sec \theta_v)$$

$$\dot{\theta}_v = q(\cos \psi_v) + p(\sin \psi_v) \quad (3.12)$$

$$\dot{\psi}_v = p(\sin \psi_v \tan \theta_v) + r(\cos \psi_v \tan \theta_v)$$

The velocity, angle of attack and angle of sideslip as a function of the velocities in the body coordinate system are given as:

$$V = \sqrt{u^2 + v^2 + w^2} \text{ and } \dot{V} = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{V} \quad (3.13)$$

$$\alpha = \tan^{-1} \frac{w}{u} \text{ and } \dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{u^2 + w^2} \quad (3.14)$$

$$\beta = \sin^{-1} \frac{v}{V} \text{ and } \dot{\beta} = \frac{V\dot{v} - v\dot{V}}{V\sqrt{u^2 + w^2}} \quad (3.15)$$

3.4 Aerodynamics and Propulsion Models

The aerodynamics data obtained from the design process are used in this dynamic analysis and will be used further in the simulation and control system development. The total aerodynamics of aircraft is found by summing the aerodynamic data of individual component and ignoring the interaction effect among the components. Figure 3.4 illustrates the procedure to calculate total aircraft aerodynamic forces and moments.

There are aerodynamic forces and moments generated from changes in the angular velocities and angle of attack. In the first approximation, the aerodynamics due to time varying of angle of attack and angle of sideslip are ignored. On the other hand, the effects of change in angular rates to the aerodynamic forces and moments can't be neglected.

Figure 3.4 Procedure to calculate total aircraft forces and moments

The change in angular rates mainly influences the magnitude of the forces and moments produced by the tails. These extra forces and moments can be determined directly from the fact that the angular rate variations only alter the local angle of attack/angle of sideslip of the tails. Also, the variation of angular rate creates additional aerodynamic forces and moments on the wing. These supplementary forces and moments can be calculated based on the semi-empirical formulas given in Etkin & Reid [43], McCormick [59] or Stengel [42]. For example, the changes in the rolling moment coefficient due to roll and yaw rates (C_{l_p} and C_{l_r}) are calculated using:

$$C_{l_p} = \left(\frac{\beta C_{l_p}}{\kappa} \right) C_{L=0} \left(\frac{\kappa}{\beta} \right) \left(\frac{(C_{l_p})_{\Gamma}}{(C_{l_p})_{\Gamma=0}} \right) + (\Delta C_{l_p})_{\text{drag}} \quad (3.16)$$

$$C_{l_r} = C_L \left(\frac{C_{l_r}}{C_L} \right)_{C_{L=0}} + \left(\frac{\Delta C_{l_r}}{\theta} \right) \theta + \left(\frac{\Delta C_{l_r}}{\Gamma} \right) \Gamma \quad (3.17)$$

The variable Γ is the dihedral angle of the wing. All parameters involve in Equation (3.16) and (3.17) can be obtained graphically from Etkin & Reid [43].

The changes in the yawing moment coefficient due to the roll and yaw rates (C_{n_p} and C_{n_r}) can be determined using:

$$C_{n_r} = \left(\frac{C_{n_r}}{C_L^2} \right) C_L^2 + \left(\frac{C_{n_r}}{C_{D_0}} \right) C_{D_0} \quad (3.18)$$

$$C_{n_p} = -C_{l_p} \tan \alpha - \left[-C_{l_p} \tan \alpha - \left(\frac{C_{n_p}}{C_L} \right)_{C_L=0M} C_L \right] + \left(\frac{\Delta C_{n_p}}{\theta} \right) \theta \quad (3.19)$$

The parameters involve in Equation (3.18) and (3.19) can be obtained graphically also from Etkin & Reid [43].

Figure 3.5 Relationship between the stick deflections and the total thrust

Figure 3.6 Relationship between the stick deflections with the engine torque

A simple propulsion model is developed based on the data from static thrust testing. The data are then extrapolated to predict the value of thrust and torque at a higher current. The thrust and the torque generated by the engine and their relation with throttle stick deflection are depicted in Figures 3.5 and 3.6. In the calculation, the thrust is always equal to the drag. Once the thrust obtained, the throttle stick deflection needed and the torque generated are found from the aforementioned figures.

3.5 Trim Points for Cruise-to-Hover Transition

The trim point is defined as a point where the aircraft's linear and angular accelerations are zero. The trim point can be searched by minimizing the trim cost. The trim cost itself is created from the sum of the square of the linear and angular accelerations. Writing the nonlinear equations and all the necessary aerodynamics and propulsion data in MATLAB[®]. The minimization task can be done in MATLAB[®] using built in command “*fminsearch*” developed based on the *Nelder-Mead simplex algorithm*.

Specified Variables	Varied Variables	Constrained Variables	Free to select Variables
V	$\alpha = \theta_0$ $i_T, i_{HTP}, i_{VTP}, i_S$	$\dot{p}, \dot{q}, \dot{r} = 0$	ψ
		$\dot{u}, \dot{v}, \dot{w} = 0$	h
		$\phi_0 = \gamma_0 = \beta = 0$	

Table 3.1 Requirements for trim calculation

The trim algorithm will search for the value of angle of attack or pitch angle (at a particular velocity) by varying the control vectors. The MAV itself has four control vectors; $\delta_T, \delta_{HTP}, \delta_{VTP}, \delta_{Stator}$. δ variable represents the deflection of each control vector and the subscripts followed the δ stand for throttle stick, horizontal tail, vertical tail and stator. In the trim point calculation, the MAV is assumed always in the wing-level flight ($\phi = 0$), no sideslip ($\beta = 0$) and non-climbing / descent flight ($\gamma = 0$). The heading angle (ψ) and altitude (h) can be freely chosen. Table 3.1 summarizes the requirement for trim point calculation.

Figure 3.7 Convergence history for trim point calculation at $V = 15$ m/s

Figure 3.8 Convergence histories for trim point calculation in hover

Figures 3.7 and 3.8 present convergence histories of the iteration to find the trim points for the aircraft speed of 15 m/s and in hover. *For this calculation, the CG location is assumed to be exactly in the symmetric plane. In addition, the analysis is performed for the fixed-wing configuration only.* For hover, the transformation method and the equations of motion incorporated the vertical Euler angles instead of the normal Euler angles. The complete trim point data in cruise-to-hover transition are presented in Table 3.2 and Figures 3.9 – 3.11.

During the cruise-to-hover transition, the weight is countered by increasing the thrust (pulling the throttle stick) and pitching up the MAV at higher angle. These facts can be observed in Figures 3.9 and 3.10. The increase in angle of attack will add the lift generated by the lifting surfaces. The maximum thrust needed for the transition is between 4.43 – 4.41 N. This value equals to 1.005 times the weight of the MAV. One of the possible reasons for this is because the static thrust data of the propeller is taken instead of the dynamic thrust. In the real condition, the thrust value needed should be higher than the current value, in order to achieve a transition without altitude loss.

The horizontal tail and stator angles to trim the MAV during cruise-to-hover transition are shown in Figure 3.11. The stator angle of deflection increases while doing these maneuvers. This deflection is mainly used to counter the increase in the engine torque, as the thrust delivered by the engine also increases. Unlike the stator, the horizontal tail deflections during the transition do not follow a regular pattern. The influence of freestream flow into the local angle of attack and velocity in the horizontal tail contributes to this random pattern.

The lack of control authority from the horizontal tail is encountered at the speed of 10 m/s. This occurs because the horizontal tail is deflected close to its stall angle. Between the airspeed of 3 m/s and 5 m/s, the angle of attack due to the interference between the freestream flow and the jet flow from the duct are very high. As a result, the horizontal tail operates above its stall angle. To prevent this, the horizontal tail must be deflected at an angle higher than its stall angle, in order to reduce the net local angle of attack.

α (deg)	V (m/s)	δ_{HTP} (deg)	δ_{VTP} (deg)	δ_s left stator (deg)	δ_T (% throttle)	β (deg)	p (rad/s)	q (rad/s)	r (rad/s)	φ (deg)	θ (deg)	ψ (deg)	V_{HTP} (m/s)	α_{HTP} (deg)
1.35	15	-7.79	0	-1.02	0.15	0	0	0	0	0	1.35	0	19.25	0
2.18	14	-9.10	0	-1.15	0.15	0	0	0	0	0	2.18	0	18.26	0
3.20	13	-10.55	0	-1.32	0.16	0	0	0	0	0	3.20	0	17.34	0
4.46	12	-12.15	0	-1.55	0.16	0	0	0	0	0	4.46	0	16.44	0
6.03	11	-13.92	0	-1.84	0.18	0	0	0	0	0	6.03	0	15.56	0
7.97	10	-15.48	0	-2.22	0.19	0	0	0	0	0	7.97	0	14.69	0
27.77	7	-13.40	0	-6.91	0.59	0	0	0	0	0	27.77	0	14.55	0
36.10	6	-11.37	0	-8.90	0.69	0	0	0	0	0	36.10	0	13.92	0
45.57	5	-17.76	0	-11.36	0.76	0	0	0	0	0	45.57	0	17.14	12.03
56.55	4	-17.78	0	-14.18	0.81	0	0	0	0	0	56.55	0	15.02	12.83
67.13	3	-15.61	0	-16.98	0.84	0	0	0	0	0	67.13	0	13.14	12.14
76.52	2	-11.13	0	-19.22	0.85	0	0	0	0	0	76.52	0	11.79	9.49
83.95	1	-5.56	0	-20.62	0.85	0	0	0	0	0	83.95	0	11.02	5.18
90.00	0	0.00	0	-21.08	0.86	0	0	0	0	0	90.00	0	10.78	0.00

Table 3.2 Summary of the trim parameters in cruise-to-hover transition

Figure 3.9 Velocity and angle of attack relation in cruise-to-hover transition

Figure 3.10 Throttle stick deflection in cruise-to-hover transition

Figure 3.11 Horizontal tail and stator angles in cruise-to-hover transition

3.6 Linearization of the Equations of Motion

3.6.1 $0 \leq \theta < 90$

Equations (3.6) and (3.8) can be linearized at a particular trim point by applying *the small perturbation theory*. To perform the linearization tasks, the states of the aircraft are written in term of the trim and perturbation variables, as seen in equations below:

$$\begin{aligned} u &= u_0 + \Delta u & v &= \Delta v & w &= w_0 + \Delta w \\ p &= \Delta p & q &= \Delta q & r &= \Delta r \\ \theta &= \theta_0 + \Delta \theta & \psi &= \Delta \psi & \phi &= \Delta \phi \end{aligned} \quad (3.20)$$

The subscript zero (0) in the above equation represents the variable values in the trim condition and the variables initiate with Δ represent the perturbation. Inserting these variables into Equations (3.6) and (3.8) leads to the following linear form:

$$\begin{aligned} m(\dot{u} + w_0 q) &= X - mg \sin \theta_0 - (mg \cos \theta_0) \theta \\ m(\dot{v} - w_0 p + u_0 r) &= Y - mg \phi \end{aligned} \quad (3.21)$$

$$\begin{aligned} m(\dot{w} - u_0 q) &= Z - mg \cos \theta_0 + (mg \sin \theta_0) \theta \\ I_x \dot{p} - I_{xz} \dot{r} &= l \\ I_y \dot{q} &= M \\ I_x \dot{r} - I_{xz} \dot{p} &= N \end{aligned} \quad (3.22)$$

In deriving the linear form of equations of motion, the contribution from the multiplications of small perturbation parameters and propeller angular momentums are neglected.

The forces and moments in the body axis can also be defined as a function of their value in the trim condition and the deviation after perturbation:

$$\begin{aligned} X &= X_0 + \Delta X & l &= l_0 + \Delta l \\ Y &= Y_0 + \Delta Y & M &= M_0 + \Delta M \\ Z &= Z_0 + \Delta Z & N &= N_0 + \Delta N \end{aligned} \quad (3.23)$$

The final form of the linear equations of motion can be obtained by replacing the forces and moments in Equation (3.21) and (3.22) with Equation (3.23). The forces

and moments need to be expanded with respect to the motion variables using first order *Taylor series*. For example, the axial force X can be expressed as:

$$X = X_0 + \frac{\partial X}{\partial u} \Delta u + \frac{\partial X}{\partial v} \Delta v + \frac{\partial X}{\partial w} \Delta w + \frac{\partial X}{\partial p} \Delta p + \frac{\partial X}{\partial q} \Delta q + \frac{\partial X}{\partial r} \Delta r + \frac{\partial X}{\partial \delta_{HTP}} \delta_{HTP} + \frac{\partial X}{\partial \delta_{VTP}} \delta_{VTP} + \frac{\partial X}{\partial \delta_s} \delta_s + \frac{\partial X}{\partial \delta_T} \delta_T \quad (3.24)$$

$$X = X_0 + X_u \Delta u + X_v \Delta v + X_w \Delta w + X_p \Delta p + X_q \Delta q + X_r \Delta r + X_{\delta_{HTP}} \delta_{HTP} + X_{\delta_{VTP}} \delta_{VTP} + X_{\delta_s} \delta_s + X_{\delta_T} \delta_T \quad (3.25)$$

In Equation (3.24), the higher order terms are neglected. The parameters $X_u, X_v, X_w, X_q, X_p, X_r$ are called as stability derivatives of axial force(X) with respect to the motion variables. The parameters $X_{\delta_{HTP}}, X_{\delta_{VTP}}, X_{\delta_s}, X_{\delta_T}$ are the control derivatives of axial force with respect to the control vectors. The stability and control derivatives of Y, Z forces and l, M, N moments with respect to the motion variables are written in the same form as in Equation (3.25)

The final linear forms of the equations of motion can be written in the state space form, as follows:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \bar{X}_u & \bar{X}_w & \bar{X}_q - w_0 & -g \cos \theta_0 & \bar{X}_v & \bar{X}_p & \bar{X}_r & 0 \\ \bar{Z}_u & \bar{Z}_w & \bar{Z}_q + u_0 & -g \sin \theta_0 & \bar{Z}_v & \bar{Z}_p & \bar{Z}_r & 0 \\ \bar{M}_u & \bar{M}_w & \bar{M}_q & 0 & \bar{M}_v & \bar{M}_p & \bar{M}_r & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \bar{Y}_u & \bar{Y}_w & \bar{Y}_q & 0 & \bar{Y}_v & \bar{Y}_p + w_0 & \bar{Y}_r - u_0 & g \cos \theta_0 \\ (\bar{A}l_u + \bar{B}N_u) & (\bar{A}l_w + \bar{B}N_w) & (\bar{A}l_q + \bar{B}N_q) & 0 & (\bar{A}l_v + \bar{B}N_v) & (\bar{A}l_p + \bar{B}N_p) & (\bar{A}l_r + \bar{B}N_r) & 0 \\ (\bar{C}l_u + \bar{A}N_u) & (\bar{C}l_w + \bar{A}N_w) & (\bar{C}l_q + \bar{A}N_q) & 0 & (\bar{C}l_v + \bar{A}N_v) & (\bar{C}l_p + \bar{A}N_p) & (\bar{C}l_r + \bar{A}N_r) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} \bar{X}_{\delta_T} & \bar{X}_{\delta_{HTP}} & \bar{X}_{\delta_s} & \bar{X}_{\delta_{VTP}} \\ \bar{Z}_{\delta_T} & \bar{Z}_{\delta_{HTP}} & \bar{Z}_{\delta_s} & \bar{Z}_{\delta_{VTP}} \\ \bar{M}_{\delta_T} & \bar{M}_{\delta_{HTP}} & \bar{M}_{\delta_s} & \bar{M}_{\delta_{VTP}} \\ 0 & 0 & 0 & 0 \\ \bar{Y}_{\delta_T} & \bar{Y}_{\delta_{HTP}} & \bar{Y}_{\delta_s} & \bar{Y}_{\delta_{VTP}} \\ \bar{l}_{\delta_T} & \bar{l}_{\delta_{HTP}} & \bar{l}_{\delta_s} & \bar{l}_{\delta_{VTP}} \\ \bar{N}_{\delta_T} & \bar{N}_{\delta_{HTP}} & \bar{N}_{\delta_s} & \bar{N}_{\delta_{VTP}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_T \\ \delta_{HTP} \\ \delta_s \\ \delta_{VTP} \end{bmatrix} \quad (3.26)$$

where $A = \frac{1}{\left(1 - \frac{I_{xz}^2}{I_x I_z}\right)}$, $B = \frac{I_{xz}}{I_x \left(1 - \frac{I_{xz}^2}{I_x I_z}\right)}$ and $C = \frac{I_{xz}}{I_z \left(1 - \frac{I_{xz}^2}{I_x I_z}\right)}$

State matrix in Equation (3.26) contains normalized forces and moments derivatives with respect to the motion variables. $\overline{X_u}, \overline{Y_v}, \overline{Z_w}, \overline{Z_{\delta_{HTP}}}$ are examples of normalized force derivatives, while $\overline{l_p}, \overline{M_w}, \overline{N_v}, \overline{M_{\delta_{HTP}}}$ are examples of normalized moment derivatives. *The normalized forces derivatives are formed by dividing their values with the mass of MAV. The moments of inertia are used to normalize the moment derivatives.*

3.6.2 $\theta = 90^\circ$ (Hover)

The linearization for this case can be performed using the same approach as in the previous section. In hover, the post perturbation states consist of perturbation variables only since the values of the state at the trim point are zero, as can be seen in the following equations:

$$\begin{aligned} u &= \Delta u & v &= \Delta v & w &= \Delta w \\ p &= \Delta p & q &= \Delta q & r &= \Delta r \\ \theta_v &= \Delta \theta & \psi_v &= \Delta \psi & \phi_v &= \Delta \phi \end{aligned} \quad (3.27)$$

The final form of linear equations of motion in hover in the state space form is:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta}_v \\ \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\psi}_v \end{bmatrix} = \begin{bmatrix} \overline{X_u} & \overline{X_w} & \overline{X_q} & -g \sin \theta_{v0} & \overline{X_v} & \overline{X_p} & \overline{X_r} & 0 \\ \overline{Z_u} & \overline{Z_w} & \overline{Z_q} & -g \cos \theta_{v0} & \overline{Z_v} & \overline{Z_p} & \overline{Z_r} & 0 \\ \overline{M_u} & \overline{M_w} & \overline{M_q} & 0 & \overline{M_v} & \overline{M_p} & \overline{M_r} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \overline{Y_u} & \overline{Y_w} & \overline{Y_q} & 0 & \overline{Y_v} & \overline{Y_p} & \overline{Y_r} & g \cos \psi_{v0} \\ (\overline{A l_u} + \overline{B N_u}) & (\overline{A l_w} + \overline{B N_w}) & (\overline{A l_q} + \overline{B N_q}) & 0 & (\overline{A l_v} + \overline{B N_v}) & (\overline{A l_p} + \overline{B N_p}) & (\overline{A l_r} + \overline{B N_r}) & 0 \\ (\overline{C l_u} + \overline{A N_u}) & (\overline{C l_w} + \overline{A N_w}) & (\overline{C l_q} + \overline{A N_q}) & 0 & (\overline{C l_v} + \overline{A N_v}) & (\overline{C l_p} + \overline{A N_p}) & (\overline{C l_r} + \overline{A N_r}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta_v \\ v \\ p \\ r \\ \psi_v \end{bmatrix} + \begin{bmatrix} \overline{X_{\delta_T}} & \overline{X_{\delta_{HTP}}} & \overline{X_{\delta_S}} & \overline{X_{\delta_{VTP}}} \\ \overline{Z_{\delta_T}} & \overline{Z_{\delta_{HTP}}} & \overline{Z_{\delta_S}} & \overline{Z_{\delta_{VTP}}} \\ \overline{M_{\delta_T}} & \overline{M_{\delta_{HTP}}} & \overline{M_{\delta_S}} & \overline{M_{\delta_{VTP}}} \\ 0 & 0 & 0 & 0 \\ \overline{Y_{\delta_T}} & \overline{Y_{\delta_{HTP}}} & \overline{Y_{\delta_S}} & \overline{Y_{\delta_{VTP}}} \\ \overline{l_{\delta_T}} & \overline{l_{\delta_{HTP}}} & \overline{l_{\delta_S}} & \overline{l_{\delta_{VTP}}} \\ \overline{N_{\delta_T}} & \overline{N_{\delta_{HTP}}} & \overline{N_{\delta_S}} & \overline{N_{\delta_{VTP}}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_T \\ \delta_{HTP} \\ \delta_S \\ \delta_{VTP} \end{bmatrix} \quad (3.28)$$

The notations used in Equation (3.28) have the same meaning with the notations used in Equation (3.26).

3.7 Stability and Control Derivatives for Cruise-to-Hover Transition

The stability and control derivatives for each trim point in transition are given in Figures 3.12 – 3.22. To avoid confusion, the hover derivatives are shown separately in Tables 3.3 and 3.4. These derivatives are found using *the finite difference method*. Furthermore, the insignificant derivatives are not presented.

The u derivative arises from the change in the forces and moments due to change in the forward speed of the MAV. The increase in speed will increase the drag, lift and the axial force. Thus X_u derivative has a negative value. At high angle of attack, X_u derivative has a positive value. This happens when the lift generated by the MAV is much higher than the drag. The increase in lift makes Z_u derivative has a negative value. The positive M_u derivative mainly generated from an increase in the pitch up moment from the horizontal tail.

The change in w will increase the angle of attack and also the lift, drag and pitch down moment of the MAV. This causes Z_w and M_w derivatives have negative values. The irregularity of w derivatives in cruise-to-hover transition can be related with the nonlinear shape of the lift and drag curve. They are also influenced by the velocity and angle of attack variations at the horizontal tail and stator.

The primary contributors to q derivatives are the change in the lift and drag in the stator and horizontal tail. The negative value of both Z_q and M_q indicate an increase in the lift and pitch down moment as a positive pitch rate is introduced. The M_q derivative is termed as the pitch damping derivative.

The v derivatives are analogous with w derivatives, where a positive increase in the side velocity will increase the magnitude of sideslip angle. This leads to additional side force and drag on the MAV, as well as the positive yawing moment. The increase in both forces and moment are manifested by negative Y_v and positive N_v derivatives. The l_v derivative is the rolling moment produced by the wing due to sideslip angle. The vertical tail contribution to this derivative is negligible due to symmetric vertical tail configuration.

Introducing a positive roll rate to the MAV will alter the local angle of attack at the wing, the tails and the stator. The rolling and yawing moments derivatives due

to roll rate (l_p & N_p) have negative values. The contributions to these derivatives from the wing become smaller at very small velocity (high angle of attack). A positive yaw rate will change the local flow on the wing and the vertical tail. This results in an increase in the side force and yawing moment on each component above increase. These increments contribute to the positive value of Y_r and N_r . The wing is the primary contribution to the rolling moment derivative due to yaw rate (l_r).

Figure 3.18 present the coupling stability derivatives between the longitudinal and lateral directional modes. The significant coupling derivatives are X_p, l_u, l_w and X_{δ_s} . The stator is the primary contributor to these derivatives. Change in the forward velocity and roll rate will change the local velocity and angle of attack in the stator. As a consequence, the stator will generate forces and moments in both longitudinal and lateral-directional directions. These coupling derivatives are relatively weak in the cruise flight but become significant at high angles of attack and hover.

Figure 3.19 gives control derivatives due to the deflection of horizontal tail. The deflection of horizontal tail creates change in the normal force (Z) and the pitching moment. As the MAV transitioning to hover, the induced flow in the propeller slipstream is higher. This condition makes the normal force and pitching moment due to the horizontal tail deflection significantly increase. Hence, the values of the $Z_{\delta_{HTP}}$ and $M_{\delta_{HTP}}$ are increase also. Figure 3.20 gives side force and yawing moment derivatives due to the vertical tail deflection ($Y_{\delta_{VTP}}$ and $N_{\delta_{VTP}}$). The increase in the derivative values as the MAV transitioning to hover can be related also with the increase of velocity in the propeller slipstream area.

The deflection of throttle stick only affects the axial force, since the thrust line is in the symmetric plane and parallel with CG line. The control derivative of axial force due to throttle stick deflection is given in Figure 2.21. Figure 2.22 presents the control derivatives due to stator deflection. The stator deflection mainly affects the axial force and the rolling moment.

For hover, the stability and control derivatives are given in Table 3.3 – 3.4. Some derivatives significant in cruise and transition flights are very weak in hover, e.g. the cross coupling derivatives between the lateral and directional modes. These

cross coupling derivatives might appear in the linear equations of motion if the product of inertia is large.

There are two significant derivatives that couple the longitudinal and lateral-directional modes in hover. These derivatives are l_w and X_{δ_s} . The l_w derivative appears because a disturbance in w alters the angle of attack of the left and right stator, which leads to a rolling moment generation. The l_w derivative only appears in hover, since in this condition the stator is totally exposed to the freestream flow. An increase in the drag due to stator deflection is the primary reason for the X_{δ_s} derivative.

For analysis of the stability and control derivatives in hover, the MAV system of equations can be divided into smaller subsystems, namely; longitudinal, directional and axial modes. This is valid if l_w and X_{δ_s} are ignored and the product of inertia is assumed small. In the longitudinal mode, w , q , θ_v and δ_{HTP} are the most important variables. The dynamics in the directional mode are described by v , r , ψ_v and δ_{VTP} . The axial dynamics involve u , p , ϕ , δ_s and δ_T only. This situation might reduce the difficulties in the analysis and the development of the automatic control system.

	u	w	q	v	p	r
$\frac{dX}{d}$	-0.004 (N/(m/s))	0.005 (N/(m/s))	-0.0002 (N-s)	0 (N/(m/s))	-0.0004 (N-s)	0 (N-s)
$\frac{dZ}{d}$	0	-1.13 (N/(m/s))	-0.128 (N-s)	0	0	0
$\frac{dM}{d}$	0	-0.139 (N-s)	-0.027 (Nm-s)	0	-0.002 (Nm-s)	0
$\frac{dY}{d}$	0	0	0	-0.627 (N/(m/s))	0	0.0549 (N-s)
$\frac{dl}{d}$	-0.0158 (N-s)	-0.0037 (N-s)	0	0	-0.0021 (Nm-s)	0
$\frac{dN}{d}$	0	0	0	0.0553 (N-s)	0	-0.0141 (Nm-s)

Table 3.3 Stability derivatives in hover

DYNAMIC ANALYSIS OF THE VTOL MAV

	δ_{HTP}	δ_T	δ_{VTP}	δ_S
$\frac{dX}{d}$	0	0.1 (N/% Stick)	0	0.102 (N/rad)
$\frac{dZ}{d}$	-2.279 (N/rad)	0	0	0
$\frac{dM}{d}$	-0.638 (Nm/rad)	0	0	0
$\frac{dY}{d}$	0	0	-2.279 (N/rad)	0
$\frac{dl}{d}$	0	0	0	0.231 (Nm/rad)
$\frac{dN}{d}$	0	0	0.592 (Nm/rad)	0

Table 3.4 Control derivatives in hover

Figure 3.12 Stability derivatives due to u perturbation

Figure 3.13 Stability derivatives due to w perturbation

Figure 3.14 Stability derivatives due to q perturbation

Figure 3.15 Stability derivatives due to v perturbation

Figure 3.16 Stability derivatives due to p perturbation

Figure 3.17 Stability derivatives due to r perturbation

Figure 3.18 Coupling derivatives

Figure 3.19 Control derivatives due to δ_{HTP} perturbation

Figure 3.20 Control derivatives due to δ_{VTP} perturbation

Figure 3.21 Control derivatives due to δ_T perturbation

Figure 3.22 Control derivatives due to δ_s perturbation

3.8 Dynamics of the MAV in Cruise-to-Hover Transition

3.8.1 Dynamics of the MAV for $0 \leq \theta < 90$

The dynamics of the MAV can be inferred from the roots of the characteristic equation of the state space form. For convenient, the linear equations of motion are decoupled into the common longitudinal and lateral-directional state space forms, as seen in Equation (3.29) and (3.30). *To find these roots, the stability derivatives data need to be substituted into above equations.* The same process is done for each trim point. The hover dynamics are presented separately in Section 3.8.3 to avoid confusion.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \overline{X_u} & \overline{X_w} & \overline{X_q} - w_0 & -g \cos \theta_0 \\ \overline{Z_u} & \overline{Z_w} & \overline{Z_q} + u_0 & -g \sin \theta_0 \\ \overline{M_u} & \overline{M_w} & \overline{M_q} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} \overline{X_{\delta_r}} & \overline{X_{\delta_{HTP}}} \\ \overline{Z_{\delta_r}} & \overline{Z_{\delta_{HTP}}} \\ \overline{M_{\delta_r}} & \overline{M_{\delta_{HTP}}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_r \\ \delta_{HTP} \end{bmatrix} \quad (3.29)$$

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \overline{Y_v} & \overline{Y_p} + w_0 & \overline{Y_r} - u_0 & g \cos \theta_0 \\ (\overline{A\overline{l}_v} + \overline{B\overline{N}_v}) & (\overline{A\overline{l}_p} + \overline{B\overline{N}_p}) & (\overline{A\overline{l}_r} + \overline{B\overline{N}_r}) & 0 \\ (\overline{C\overline{l}_v} + \overline{A\overline{N}_v}) & (\overline{C\overline{l}_p} + \overline{A\overline{N}_p}) & (\overline{C\overline{l}_r} + \overline{A\overline{N}_r}) & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} \overline{Y_{\delta_s}} & \overline{Y_{\delta_{VTP}}} \\ \overline{l_{\delta_s}} & \overline{l_{\delta_{VTP}}} \\ \overline{N_{\delta_s}} & \overline{N_{\delta_{VTP}}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_s \\ \delta_{VTP} \end{bmatrix} \quad (3.30)$$

Looking at the eigenstructures data presented in Table 3.5, the longitudinal dynamics of the MAV in cruise are characterized by two pairs of lightly damped poles with eigenvalues of $-7.5 \pm 13.8i$ and $-0.074 \pm 0.83i$. The first pair of eigenvalue corresponds to the short period mode, while the second refers to the phugoid mode. The short period mode is well-damped ($\zeta = 0.48$) with natural frequency of 15.7 rad/s. This mode is associated with the oscillation in the w and q . The phugoid mode is a lightly damped ($\zeta = 0.09$) oscillation with smaller natural frequency ($\omega_n = 0.83$ rad/s). The phugoid mode is expressed largely by the oscillation in the u, q and θ . The short period mode is mainly determined by the $\overline{Z_w}, \overline{M_w}$ and $\overline{M_q}$, while the phugoid mode is affected by the $\overline{X_u}$ and $\overline{Z_u}$.

The spiral, roll and Dutch roll modes mark the dynamics in the lateral-directional degrees of freedom. The roll mode is a fast rolling motion ($T_s = 0.06$ sec), which is determined mainly by the $\overline{l_p}$ derivative. The spiral mode is a slowly

convergence mode ($T_s = 18\text{sec}$) that involve changes in the v , r and ϕ . The side velocity (v) and yaw rate dominate the Dutch roll mode. This mode of motion is lightly damped ($\zeta = 0.38$) with the natural frequency of $\omega_n = 9.27\text{ rad/s}$. The Dutch roll mode is governed by the derivatives Y_v , Y_r , N_v and N_r .

The traces of the eigenvalues for the longitudinal and lateral-directional modes in cruise-to-hover transition are depicted in Figures 3.23 and 3.24. For the longitudinal mode, the short period poles remain as a pair of complex conjugate pole. The natural frequency of the short period mode tends to decrease, while its damping ratio increases as the MAV transitions from the high speed cruise to hover. At the speeds of 6 m/s and 1 m/s, the short period's damping ratios are 0.72 and 0.76 and their natural frequencies are 5.07 and 5.3 rad/s, respectively, as can be seen in Tables 3.6 and 3.7. A significant reduction in the natural frequency can be related to a significant decrease of the M_w and Z_w derivatives. The damping of the short period mode increases in the transitioning flight because the M_q derivative decreases at a smaller rate than the Z_w . At both speeds, the short period oscillations become more visible in the u and θ responses, although the main components are still the w and q .

The stable phugoid poles in cruise migrate to be a pair of unstable poles or a stable and unstable real pole in near hover to hover conditions. At the speed of 6 m/s, the phugoid poles are still stable, and their damping ratio and frequency increase considerably ($\zeta = 0.143$ & $\omega_n = 1.43\text{ rad/s}$). This condition is escalated by the significant decrease of the value of Z_u and the relatively unaltered X_u . For the airspeed of 1 m/s, the phugoid poles transform into an unstable pole and a stable real poles. The stable exponential pole corresponds to heave dynamics and its value is mainly affected by X_u derivative. The unstable exponential pole gives rise to a pitch divergence mode and it is associated with the motion in w , q and θ . This fast divergence mode makes the MAV very hard to be manually controlled without any augmentation system. The pitch divergence mode is affected by the Z_w , M_w and M_q .

During the cruise-to-hover transition, the damping ratios of the Dutch roll mode increases and its natural frequency decreases. The decrease in the natural frequency is mainly affected by the reduction in N_v derivative. Since the yaw

damping derivative (N_r) value decreases slower than the Y_v and N_v derivatives, its damping increases considerably. At the airspeed of 6 m/s, the damping ratio and natural frequency for the Dutch roll mode are 0.598 and 4.37 rad/s respectively, as can be seen in Table 3.6. For the speed of 1 m/s, the Dutch roll poles turn into two real stable poles with 0.57 and 0.54 seconds time constants, as given in Table 3.7. These poles are the primary cause of the yaw subsidence motions and these motions involve mainly the yaw rate r .

The spiral pole's location changes slightly as the airspeed reduces. For the speeds of 6 m/s and 1 m/s, the time constants of this mode are 12.7 and 5.9 seconds, respectively. Y_v and I_p are the parameters that strongly influenced the spiral mode. The roll mode is governed mainly by the roll rate derivative. The roll mode's time constants for above velocities are 0.06 and 0.92 seconds, respectively. The influence of the roll rate derivative on the roll mode at low speed significantly decreases.

Figure 3.23 Short period and phugoid poles migration in hover-to-cruise transition

Figure 3.24 Dutch roll, roll and spiral poles migration in hover-to-cruise transition

Eigenvalues, Eigenstructures, Magnitude and Phases of Each Modes						
Longitudinal			Lateral Directional			
$-7.52 \pm 13.8i$ Short Period $\zeta = 0.4788$ $\omega_n = 15.713 \text{ rad/s}$		$-0.074 \pm 0.83i$ Phugoid $\zeta = 0.088$ $\omega_n = 0.83 \text{ rad/s}$	-16.56 Roll $T_s = 0.0604 \text{sec}$	-0.055 Spiral $T_s = 18.08 \text{sec}$	$-3.507 \pm 8.56i$ Dutch Roll $\zeta = 0.38$ $\omega_n = 9.266 \text{ rad/s}$	
Eigenstructures						
u	$0.01 \pm 0.02i$	0.99	v	-0.054	0.31	0.84
w	0.72	$-0.02 \pm 0.003i$	p	-0.993	-0.05	$-0.26 \pm 0.11i$
q	$0.059 \pm 0.69i$	$0.071 \pm 0.004i$	r	-0.029	0.46	$-0.026 \pm 0.47i$
θ	$0.037 \pm 0.024i$	$-0.012 \pm 0.084i$	ϕ	0.06	0.83	$0.022 \pm 0.022i$
Magnitudes and Phases						
u	$0.023 / 62.6^\circ$	$0.99 / 0^\circ$	v	$0.054 / 180^\circ$	$0.31 / 0^\circ$	$0.84 / 0^\circ$
w	$0.72 / 0^\circ$	$0.015 / 169.7^\circ$	p	$0.996 / 180^\circ$	$0.05 / 180^\circ$	$0.28 / 157.2^\circ$
q	$0.69 / 85.1^\circ$	$0.071 / 3.03^\circ$	r	$0.029 / 180^\circ$	$0.46 / 0^\circ$	$0.47 / -93.1^\circ$
θ	$0.044 / -33.5^\circ$	$0.085 / 98.1^\circ$	ϕ	$0.06 / 0^\circ$	$0.83 / 0^\circ$	$0.03 / 45^\circ$

Table 3.5 Decoupled eigenvalues, eigenstructure, magnitude and phase
of the MAV in cruise with $V = 15 \text{ m/s}$

Eigenvalues, Eigenstructures, Magnitude and Phases of Each Modes					
Longitudinal			Lateral Directional		
-3.65 ± 3.52i	-0.2 ± 1.42i		-6.27	-0.078	-2.61 ± 3.5i
Short Period	Phugoid		Roll	Spiral	Dutch Roll
$\zeta = 0.72$	$\zeta = 0.14$		$T_s = 0.06\text{sec}$	$T_s = 12.7\text{sec}$	$\zeta = 0.598$
$\omega_n = 5.07\text{ rad/s}$	$\omega_n = 1.43\text{ rad/s}$				$\omega_n = 4.37\text{ rad/s}$
Eigenstructures					
<i>u</i>	-0.11 ± 0.304i	0.95	<i>v</i>	-0.25	0.63
<i>w</i>	0.74	0.16 ± 0.06i	<i>p</i>	0.93	-0.04
<i>q</i>	-0.08 ± 0.57i	0.204 ± 0.09i	<i>r</i>	0.24	0.57
θ	0.09 ± 0.07i	-0.08 ± 0.13i	ϕ	-0.15	0.52
Magnitudes and Phases					
<i>u</i>	0.32 / 109.4°	0.95 / 0°	<i>v</i>	0.25 / 180°	0.63 / 0°
<i>w</i>	0.74 / 0°	0.17 / 19°	<i>p</i>	0.93 / 0°	0.04 / 180°
<i>q</i>	0.58 / 98.1°	0.22 / -23.9°	<i>r</i>	0.23 / 0°	0.57 / 0°
θ	0.11 / -38°	0.16 / -122.2°	ϕ	0.15 / 180°	0.52 / 0°

Table 3.6 Decoupled eigenvalues, eigenstructure, magnitude and phase
of the MAV with V = 6 m/s

Eigenvalues, Eigenstructures, Magnitude and Phases of Each Modes								
Longitudinal				Lateral Directional				
-4.01 ± 3.46i Short Period $\zeta = 0.76$ $\omega_n = 5.3$ rad/s	2.71 Pitch Divergence $T_s = -0.4$ sec	-0.08 Heave $T_s = 12.5$ sec	-1.09 Roll $T_s = 0.92$ sec	-0.17 Spiral Subsidence $T_s = 5.88$ sec	-1.75 $T_s = 0.57$ sec	-1.85 $T_s = 0.54$ sec		
					Yaw Subsidence			
Eigenstructures								
u	-0.12 ± 0.08i	-0.34	-0.98	v	0.82	-0.67	0.14	-0.13
w	-0.07 ± 0.36i	-0.59	-0.18	p	-0.32	0.13	-0.27	0.18
q	-0.90	0.69	0	r	0.37	-0.13	0.94	0.97
θ	0.13 ± 0.11i	0.25	0.03	ϕ	0.29	-0.71	0.16	-0.09
Magnitudes and Phases								
u	0.15 / 146°	0.34 / 180°	0.98 / 180°	v	0.68 / 0°	0.67 / 180°	0.14 / 0°	-0.13 / 180°
w	0.37 / -101°	0.59 / 180°	0.18 / 180°	p	0.13 / 180°	0.13 / 0°	-0.27 / 180°	0.18 / 0°
q	0.9 / -180°	0.69 / 0°	0 / 180°	r	0.13 / 0°	0.13 / 180°	0.94 / 0°	0.97 / 0°
θ	0.17 / -40.74°	0.26 / 0°	0.03 / 0°	ϕ	0.71 / 0°	0.73 / 180°	0.16 / 0°	-0.09 / 180°

Table 3.7 Decoupled eigenvalues, eigenstructure, magnitude and phase
of the MAV with $V = 1$ m/s

3.8.2 Coupled Response

Figures 3.25 and 3.26 present the response of MAV due to 1° step input of the horizontal tail at the trimmed speeds of 15 m/s and 6 m/s. The first row of graphs in Figures 3.25 and 3.26 shows the longitudinal response; while the second row of graphs show the lateral-directional response. The lateral-directional response appears as a consequence of the coupling derivatives l_u and N_u . The influence of these coupling derivatives is larger as the velocity reduces. Nonetheless, the lateral-directional responses due to coupling are slow and thus can be suppressed easily. Figures 3.27 and 3.28 give the response of the MAV due to 1° step input of the stator for the same trim speeds. Different with the previous case, the insignificant values of the coupling derivatives $X_v, X_p, X_r, Z_v, Z_p, Z_r, M_v, M_p, M_r$ cause the response of the longitudinal states due to the stator deflection are very small (nearly zero). These facts are true for the speeds of 15 m/s and 6 m/s.

Figure 3.25 Responses of the MAV at 15 m/s due to 1° horizontal tail step input

DYNAMIC ANALYSIS OF THE VTOL MAV

Figure 3.26 Responses of the MAV at 6 m/s due to 1° horizontal tail step input

Figure 3.27 Responses of the MAV at 15 m/s due to 1° stator step input

Figure 3.28 Responses of the MAV at 6 m/s due to 1° stator step input

3.8.3 Dynamics of the MAV in Hover

For the purpose of analyzing the dynamics of the MAV in hover, Equation (2.26) is decoupled into the common longitudinal and lateral-directional components, as follows:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta}_v \end{bmatrix} = \begin{bmatrix} \overline{X_u} & \overline{X_w} & \overline{X_q} & -g \sin \theta_{v0} \\ \overline{Z_u} & \overline{Z_w} & \overline{Z_q} & -g \cos \theta_{v0} \\ \overline{M_u} & \overline{M_w} & \overline{M_q} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta_v \end{bmatrix} + \begin{bmatrix} \overline{X_{\delta_r}} & \overline{X_{\delta_{HTP}}} \\ \overline{Z_{\delta_r}} & \overline{Z_{\delta_{HTP}}} \\ \overline{M_{\delta_r}} & \overline{M_{\delta_{HTP}}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_r \\ \delta_{HTP} \end{bmatrix} \quad (3.31)$$

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\psi}_v \end{bmatrix} = \begin{bmatrix} \overline{Y_v} & \overline{Y_p} & \overline{Y_r} & g \cos(\psi_0) \\ (\overline{A_l} + \overline{B N_v}) & (\overline{A_l} + \overline{B N_p}) & (\overline{A_l} + \overline{B N_r}) & 0 \\ (\overline{C l_v} + \overline{A N_v}) & (\overline{C l_p} + \overline{A N_p}) & (\overline{C l_r} + \overline{A N_r}) & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \psi_v \end{bmatrix} + \begin{bmatrix} \overline{Y_{\delta_s}} & \overline{Y_{\delta_{VTP}}} \\ \overline{l_{\delta_s}} & \overline{l_{\delta_{VTP}}} \\ \overline{N_{\delta_s}} & \overline{N_{\delta_{VTP}}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_s \\ \delta_{VTP} \end{bmatrix} \quad (3.32)$$

As in the previous case, the dynamics can be inferred from the roots of the characteristic equations. To find this, the stability derivatives data in hover must be substituted into Equations (3.29) and (3.30).

Figure 3.29 gives the dynamics of the MAV in hover. Tables 3.8 and 3.9 give the decoupled and coupled eigenstructures of the MAV in hover. In the longitudinal degree of freedom, the associated modes are the short period, heave and pitch divergence modes. The eigenvalues for each mode of the MAV are $-5.03 \pm 4.33i$, -0.01 and 4 respectively, as given in Table 3.8. These are identical with the dynamics at the velocity of 1 m/s. The short period and pitch divergence modes involve the w , q and θ_v variables. These modes are determined by the Z_w , M_w and M_q derivatives. The heave mode is affected mainly by the X_u derivative. For the coupled model, there is a strong influence from the roll rate (p) to the heave mode, but it impacts to the short period mode is negligible, as can be seen in Table 3.9.

The lateral-directional dynamics of the MAV in hover are marked by the pure roll motion, yaw divergence and yaw oscillation motions. The eigenvalues of the corresponding modes are $-1.01, 3.14$ and $-3.16 \pm 3.39i$, as presented in Table 3.8. In this case, the vertical yaw angle becomes a more important parameter than the bank angle. The main components of the yaw divergence and stable yaw oscillation modes are the v , r and ψ_v . The derivatives that affect these modes are Y_v , Y_r and N_r . The

rolling motion is governed mainly by the l_p derivative. For the coupling model, the u velocity has a small effect on the roll mode and negligible effect on the other modes.

For simplification, the dynamics of the MAV in hover can be decoupled into three smaller subsystems, namely axial, longitudinal and directional modes. This idea has also been explained in the previous section. Such decoupling method is more valid when the coupling derivatives, such as, l_w and X_{δ_s} are very weak, as in the case of ducted fan MAV with counter-rotating propellers or with properly designed stator, just like the MAV discussed here.

The state space forms of these decoupled equations of motion are given below:

Longitudinal mode:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta}_v \end{bmatrix} = \begin{bmatrix} Z_w & Z_q & -g \\ M_w & M_q & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta_v \end{bmatrix} + \begin{bmatrix} Z_{\delta_{HTP}} \\ M_{\delta_{HTP}} \\ 0 \end{bmatrix} [\delta_{HTP}] \quad (3.33)$$

Directional mode:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi}_v \end{bmatrix} = \begin{bmatrix} Y_v & Y_r & g \\ N_v & N_r & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi_v \end{bmatrix} + \begin{bmatrix} Y_{\delta_{VTP}} \\ N_{\delta_{VTP}} \\ 0 \end{bmatrix} [\delta_{VTP}] \quad (3.34)$$

Axial mode:

$$\begin{bmatrix} \dot{u} \\ \dot{p} \\ \dot{\phi}_v \end{bmatrix} = \begin{bmatrix} X_u & X_p & 0 \\ l_u & l_p & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ p \\ \phi_v \end{bmatrix} + \begin{bmatrix} X_{\delta_s} & X_{\delta_r} \\ l_{\delta_s} & l_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_s \\ \delta_r \end{bmatrix} \quad (3.35)$$

The hover control design discussed in Chapter 4 is based on these decoupled dynamic models.

Figure 3.29 Poles of the MAV in hover

Eigenvalues, Eigenstructures, Magnitude and Phases of Each Modes							
Longitudinal				Lateral Directional			
	-5.03 ± 4.33i Short Period	4 Pitch Divergence	-0.01 Heave		-1.01 Roll	3.1403 Yaw Divergence	-3.16 ± 3.39i Yaw Oscillation
Eigenstructures							
u	0	0	1	v	0	0.56	0.17 ± 0.43i
w	-0.08 ± 0.23i	-0.38	0	p	1	0.02	-0.04 ± 0.01i
q	-0.96	0.89	0	r	0	0.79	0.87
θ	0.11 ± 0.09i	0.22	0	ψ	0	0.25	0.13 ± 0.14i
Magnitudes and Phases							
					0 / 0°		
u	0 / -42°	0 / 180°	1 / 0°	v	1 / 0°	0.56 / 0°	0.46 / -68°
w	0.24 / 108°	0.38 / 180°	0 / 0°	p	0 /	0.02 / 0°	0.04 / -169°
q	0.96 / 180°	0.89 / 0°	0 / 0°	r	180°	0.79 / 0°	0.87 / 180°
θ	0.14 / 40°	0.22 / 0°	0 / 0°	ψ	0 / 0°	0.25 / 0°	0.19 / 47°

Table 3.8 Decoupled eigenvalues, eigenstructure, magnitude and phase
of the MAV in hover

DYNAMIC ANALYSIS OF THE VTOL MAV

Eigenvalues, Eigenstructures, Magnitude and Phases of Each Modes							
Longitudinal				Lateral Directional			
-5.03 ± 4.33i Short Period		4 Pitch Divergence	-0.15 Heave	-0.87 Roll		3.14 Yaw Divergence	-3.16 ± 3.39i Yaw Subsidence
Eigenstructures							
u	0	0	0.11	u	0.02	0	0
w	0.08 ± 0.23i	-0.37	0	w	0	0	0
q	0.96	0.89	0	q	0	0	0
θ	-0.11 ± 0.09i	0.22	0	θ	0	0	0
v	0	0	0	v	-0	-0.56	-0.17 ± 0.43i
p	0.07 ± 0.03i	0.13	-0.99	p	-0.99	-0.02	0.04 ± 0.01i
r	0	0	0	r	0	-0.79	0.87
ψ	0	0	0	ψ	0	-0.25	-0.13 ± 0.14i
Magnitudes and Phases							
u	0 / 155°	0 / 180°	0.11 / 0°	u	0.02 / 0°	0 / 180°	0 / 122°
w	0.24 / -71°	0.37 / 180°	0 / 180°	w	0 / 180°	0 / 180°	0 / 120°
q	0.96 / 0°	0.89 / 0°	0 / 0°	q	0 / 0°	0 / 0°	0 / 168°
θ	0.14 / 139°	0.22 / 0°	0 / 180°	θ	0 / 180°	0 / 0°	0 / 12°
v	0 / 69°	0 / 0°	0 / 180°	v	0 / 180°	0.56 / 180°	0.46 / 112°
p	0.07 / -24°	0.13 / 0°	0.99 / 180°	p	0.99 / 180°	0.02 / 180°	0.04 / 10.2°
r	0 / -24°	0 / 0°	0 / 0°	r	0 / 0°	0.79 / 180°	0.87 / 0°
ψ	0 / -164°	0 / 0°	0 / 180°	ψ	0 / 180°	0.25 / 180°	0.19 / -133°

Table 3.9 Coupled Decoupled eigenvalues, eigenstructure, magnitude and phase
of the MAV in hover

3.8.4 Effect of Changing Tail Location on the Dynamics

It is important to analyze the effect of moving the tail location closer to the duct into the dynamics of the MAV. This is to see the possibility to reduce the overall size of the MAV. To accomplish this task, the design of current configuration (Configuration A) has been slightly modified. The modified configuration (Configuration B) has a stator inside the duct, while the horizontal and vertical tails placed just outside the duct exit area. Hence, the flow in the stator and the vertical tail will only be affected by the flow from the propeller slipstream. Due to smaller tail moment arm, the horizontal and vertical tail sizes in configuration B are enlarged. This is to make the MAV enable to be trimmed in the cruise-to-hover transition. Figure 3.30 depicts the comparison between configurations A and B.

In cruise-to-hover transition, the trim angle of attack and throttle stick deflection are higher for configuration B than configuration A, as observed in Figures 3.31 and 3.32. This is to compensate for the increase in drag and the reduction of the tail moment arm. The horizontal tail deflections needed to trim configuration B can be seen in Figure 3.33. The control saturation problem can be suppressed in configuration B, because the horizontal tail deflection needed to trim the MAV is always smaller than its stall angle.

Figure 3.30 Current (A) and modified (B) configurations

Figure 3.31 Velocity and angle of attack relation in cruise-to-hover transition

Figure 3.32 Throttle stick deflection in cruise-to-hover transition

Figure 3.33 Horizontal tail deflections in cruise-to-hover transition

The M_w of configuration B is much higher than configuration A since it is trimmed at higher angle of attack for the same airspeed. Due to a shorter tail arm, the M_q of configuration B is slightly lower than its counterpart. This combination results in the short period mode of the configuration B to have smaller damping ratio and higher natural frequency, as can be seen from Figure 3.34. At the angles of attack near

hover, Z_w and M_w of configuration B are decrease considerably, which leads to the decrease in the natural frequency of the short period mode. The phugoid poles of configuration B are located nearly in the same place as configuration A. This can be explained by the relatively the same values of X_u and Z_u in both configurations. At high angle of attack, both configurations exhibit unstable oscillatory and non oscillatory behavior.

The lateral-directional eigenvalues of configurations A and B are presented in Figure 3.35. The roll modes of both configurations are located close to each other. The reason is l_p is unchanged by shifting the stator's location forward. In cruise, the spiral mode of configuration B is unstable. Enlarging the size of the vertical tail increases the value of N_v , but only slightly modifies the yaw damping derivative N_r . At lower velocities, the spiral pole moves to the left half plane. A higher N_v derivative makes the Dutch roll mode of configuration B has a smaller damping ratios and with relatively higher natural frequencies than configuration A.

The placement of the control surfaces inside the duct in configuration B helps to eliminate the coupling derivative l_w . In addition, such placements results in a significant reduction in the values of Z_w, M_w, Y_v and N_v . Figures 3.36 and 3.37 show the poles of both configurations in hover. It can be seen that the decrease of these derivatives makes the oscillation modes to have lower natural frequencies. The divergence poles also lie closer to the origin for configuration B.

From the explanation above, moving the tails forward closer or inside the duct give several advantages. It might eliminate some coupling derivatives. In addition, the aerodynamics of the tails is more certain, since it is mainly determined by the flow from the propeller slipstream and the tails deflection angle only. An increase in the size of the tails is needed if the tails are moved inside the duct. This is to enable the MAV to be trimmed at the cruise flight. Another solution to this problem is to move the CG far above the duct lips, such that the tail moment arms are increased.

DYNAMIC ANALYSIS OF THE VTOL MAV

Figure 3.34 Longitudinal poles migration of configuration A and B
in cruise-to-hover transition

Figure 3.35 Lateral-directional poles migration of configuration A and B
in cruise-to-hover transition

Figure 3.36 Longitudinal poles of configuration A and B in hover

Figure 3.37 Lateral directional poles of configuration A and B in hover

3.8.5 Effect of CG Location Variation in Hover

Inexact arrangement of the payloads makes the CG location is uncertain. Therefore, it is important to analyze the effect of change in CG location on the dynamics of the MAV, especially in hover. The CG of the MAV can be made positive or negative depends on the distribution of the payloads. The positive and negative CG definitions for configuration A and B are illustrated in Figure 3.38. The

I_z and I_y inertias will change as the CG location changes, while I_x stay unchanged. The stability and control derivatives of the longitudinal and directional modes are also affected by this CG shifting.

Figure 3.38 Positive and negative CG illustration

As the CG shifts further aft, the natural frequencies of short period mode for both configurations A and B become only slightly higher. This is because changing the CG will not influence M_w and Z_w considerably. The short period pole locations are not altering very much if the CG is shifted forward/backward. The pitch divergence pole moves closer to the origin as the CG is moved further behind the duct lips. This can be seen in Figure 3.39.

For configuration A, if the CG moves further behind the duct lips, its yaw oscillation modes have higher damping ratio, but with nearly the same natural frequency. At the same time, the yaw divergence pole shifts toward the origin because the value of N_v derivative reduces faster than the yaw N_r . Interestingly, as the CG shifts backward, the stable yaw oscillation mode of configuration B turns into an unstable oscillation mode. At the same time, the yaw divergence mode changes into the yaw subsidence mode. This condition is illustrated in Figures 3.40 and 3.41. Furthermore, the placement of the CG above the duct lips has an advantage. This makes the tail moment arm longer and hence the static stability characteristics are improved.

DYNAMIC ANALYSIS OF THE VTOL MAV

Figure 3.39 Longitudinal poles of configuration A & B in hover for various CG

3.40 Directional poles of configuration A in hover for various CG

Figure 3.41 Directional poles of configuration B in hover for various CG

3.9 Summary on the Dynamic Analysis of the VTOL MAV

The summary from this section are presented as follows:

1. In cruise, the dynamics of the VTOL MAV are characterized by the same modes seen in the large aircraft. They are short period, phugoid, Dutch roll, spiral and roll modes.
2. In the cruise-to-hover transition, the short period poles move closer to the real axis, but with relatively unaffected value of natural frequency. The locations of phugoid poles only alter slightly at small angles of attack. At high angles of attack, the phugoid poles shift to the right, crossing the stability boundary and become unstable.
3. The Dutch roll poles exhibit nearly the same behavior with the short period poles as the MAV undergoing transition maneuver. However, the Dutch roll poles change into two real subsidence poles at high angles of attack. The spiral pole is relatively unchanged, while the roll pole travels closer to origin as the MAV flying at higher angle of attack.

4. The coupling derivatives cause the deflection of the longitudinal control variables induce motion in the lateral-directional states. This coupling is mainly produced by the deflection of the stator.
5. In hover, the dynamics of the longitudinal mode are characterized by the pitch divergence and short period motions. The yaw divergence and the yaw oscillation mark the motion in the directional mode. The axial dynamics are characterized by the heave and roll modes.
6. Ignoring any coupling derivatives, the dynamics of the MAV in hover can be divided into smaller subsystems, namely; longitudinal, directional and axial modes.
7. It is found that shifting the location of the tails closer to the duct exit area and enlarging the size of the tails at the same time do not change the dynamic characteristics in cruise-to-hover transition significantly.
8. For the unmodified configuration (Configuration A) in hover, shifting the CG backward will move the pitch divergence and the yaw divergence poles closer to the origin. On the other hand, the short period and yaw oscillation poles are relatively unchanged.
9. The longitudinal dynamics of the modified configuration (Configuration B) in hover exhibit the same behavior with the configuration A as the CG pulled backward. However, the directional dynamics show different behavior. The yaw oscillation poles change into unstable complex conjugate poles. At the same time, the yaw divergence pole turns into the yaw subsidence pole.

Chapter 4

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV

4.1 Introduction to Automatic Control Design for Micro Air Vehicle

In hover the MAV dynamics are characterized by unstable divergent in both longitudinal and directional modes. These conditions imply that manual control is hardly possible for flying the MAV without any artificial stabilization system and under the manual control. An automatic control is needed to stabilize the platform in this particular condition. Besides for stabilization purpose, this controller is also dedicated to maintain the position of the MAV in hover under the wind disturbances.

Various methods have been implemented to design controllers for MAV, particularly in hover. The difficulty in designing such controllers arises from the fact that the plant is nonlinear. Simplification can be made by assuming that the plant is linear at a particular trim point around hover. This assumption might lead to modeling error, since some of the nonlinearity can't be approximated properly using linear function. However, strong coupling between each mode of motion still presents. The linear model of the plant itself can be determined from the identification method [55, 60] or by linearizing the nonlinear equations using *the small perturbation theory*. *The latter method has been chosen in this work to obtain the linear model.*

Having linear models of the plant, one can implement the classical Single Input Single Output (SISO) methods, such as root locus or Bode techniques, to design controller for the plant. Such methods offer simplicity and are relatively intuitive. Multiple loop SISO controllers are generally used for both regulation and tracking tasks. The controller gains are tuned until the desired performances are achieved. The theory about the classical control design is well covered in Ogata [61] and the example of application of the theory in the aerospace field is given in Blakelock [62], McRuer *et al* [63], Steven & Lewis [58] and Nelson [44].

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Most autopilots for fixed-wing MAV reported in the literature are designed based on the SISO root locus technique [64, 65]. Stone [57, 66] extended the application of the root locus method to designing tail-sitter's altitude and roll controls in hover. Shim [67], Kim & Shim [68] and Lee *et al* [69] employed the root locus method to design controller for small scale rotorcraft UAV in hover. The attitude is stabilized using a proportional controller by feeding back the orientation angles, instead of angular rates. Proportional Derivative (PD) based controllers are used for position keeping in the outer loop channels. Overall, the controller performances are found to be acceptable, despite a decrease in performance when the vehicle hovers under strong crosswind. Mettler *et al* [70] took the same approach and combined the PD controllers with notch filters to improve the robustness of the controller.

Lipera *et al* [26] developed Proportional Integral (PI) controllers to stabilize the *i-STAR* ducted fan UAV in hover. The feedback signals from the angular rates are used, since accurate attitudes are difficult to obtain. The linear model for the control design is derived from a nonlinear six-degree-of-freedom model. Proportional Integral Derivative (PID) based controllers are used to stabilize the attitude dynamics of Bertin Technologies ducted fan [71]. More advance technique, like backstepping, is employed in the latest development of this ducted fan UAV [72, 73]. This approach is used to improve the performance of the controller under strong nonlinearities, uncertainty of parameters and external disturbances.

The Multi Input Multi Output (MIMO) design method offers ability to capture the multiple inputs/outputs nature behavior of the system and includes some coupling derivatives in the calculation. The Linear Quadratic Regulator (LQR) is a famous MIMO control design technique. However, it requires the availability of all aircraft states for feedback. An observer must be used to reconstruct the plant dynamics if some of the states can not be measured. Castillo-Effen *et al* [74] stressed that the advantages of the LQR method is on its capability to give optimum gains, while at the same time guarantees the closed loop stability of the system.

Frank *et al* [75] combined the LQR method with a gain scheduling technique to control a fixed-wing UAV in hover, cruise and transition maneuvers. All states are measured by inertial measurement unit (IMU) and motion capturing sensors. Stone [57, 66] employed the LQR approach to design the controllers for the longitudinal and directional modes of a tail-sitter UAV in hover, vertical flight and transition. Gavrillets

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

et al [76] extended the use of LQR method to control a rotorcraft UAV that can perform aggressive roll maneuver.

The robustness of the MIMO controller, in term of input / output sensitivity, noise sensitivity and disturbance rejection, can be improved further by implementing a robust control (H_∞) technique. Dickeson *et al* [77] selected gain scheduling robust control technique to control tilt-wing UAV in hover and transition. Avanzini *et al* [78] implemented the same approach to control a ducted fan UAV in hover and cruise.

In recent years, the adaptive feedback linearization / dynamic inversion technique has been attempted. Such controller cancels the nonlinearity presence in the system and suppresses the modeling error. The drawbacks of this method are the difficulty in finding the proper function to cancel the nonlinearity of the plant and the problem related with the stability of the system. Spaulding *et al* [79] used nonlinear dynamics inversion technique to control a ducted fan MAV in whole flight envelopes. Knoebel *et al* [80] , Knoebel [81] and Osborne [82] implemented feedback linearization technique to control the tail-sitter VTOL UAV in hover and transition. The same method has also been employed by Bijmens *et al* [83] for a rotorcraft UAV.

Non model based controllers, like: neural network and fuzzy flight controls can also be implemented to design controllers for any flying vehicles [84]. Significant modeling error can be avoided, since this method does not depend on the availability of the dynamics model. The main drawback is that this method does not guarantee the stability of the closed system. Johnson & Kannan [85] and Lower [84] applied this method to control small-scale rotorcrafts UAV. Johnson & Turbe [86] extended the application of this method for controlling a ducted fan UAV.

The controller design task in this work follows a systematic approach. *The root locus technique will be used to design all the controllers.* This method is applicable since the linear SISO models are available, as explained in Chapter Three. The first goal of the controllers is to stabilize the system, since the vehicle dynamics are unstable in hover. Another goal of the controllers is to keep the position of the MAV under the wind disturbance. The attitude controllers will be designed first. The position controllers are developed after the attitude controllers have been designed. For each controller, its structure and gain are selected to achieve satisfactory responses.

4.2 Overview of the Control Strategy and Design Method

Recall from Chapter Three, the linear system of equations of the MAV in hover can be divided into smaller subsystems; longitudinal, directional and axial. This is done by ignoring the coupling derivatives and assuming that the cross product inertias are small. Each subsystem is linear and has a single input only. In the longitudinal mode, the horizontal tail is used to control the pitch attitude, w and the x_I position. x_I is the position of MAV in the X_I axis of inertial coordinate system. The yaw angle, v and y_I position are controlled through the deflection of vertical tail. y_I is the position of MAV in the Y_I axis of inertial coordinate system. In the axial mode, the roll angle is controlled by deflecting the stator. The axial velocity and altitude are controlled separately using throttle stick. These are illustrated in Figure 4.1.

For the attitude controllers, the feedback signals can be obtained by integrating the angular rates data from the gyroscope of the Inertial Measurement Unit (IMU). The control signals for the velocity and position controllers are found from integrating the data obtained from the accelerometers of the IMU. To give more accurate position signal, the position data from the IMU is usually combined with the data from the Global Positioning System (GPS). The altitude data is obtained from the pressure altimeter, while the roll orientation data is found from the magnetometer. In this control design task, all sensors are assumed working perfectly. It means the gains of the controllers are 1.

Figure 4.1 SISO subsystems with their controlled variables

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

The state space forms of the longitudinal, directional and axial subsystems are given in Equations (3.31), (3.32) and (3.33). In the following equations, the corresponding state space forms have been augmented to include the position vectors. The relations are true for the MAV that flies around hover.

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta}_v \\ \dot{x}_I \end{bmatrix} = \begin{bmatrix} Z_w & Z_q & -g & 0 \\ M_w & M_q & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta_v \\ x_I \end{bmatrix} + \begin{bmatrix} Z_{\delta_{HTP}} \\ M_{\delta_{HTP}} \\ 0 \\ 0 \end{bmatrix} [\delta_{HTP}] \quad (4.1)$$

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi}_v \\ \dot{y}_I \end{bmatrix} = \begin{bmatrix} Y_v & Y_r & g & 0 \\ N_v & N_r & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \\ y_I \end{bmatrix} + \begin{bmatrix} Y_{\delta_{VTP}} \\ N_{\delta_{VTP}} \\ 0 \\ 0 \end{bmatrix} [\delta_{VTP}] \quad (4.2)$$

$$\begin{bmatrix} \dot{u} \\ \dot{p} \\ \dot{\phi}_v \\ \dot{h} \end{bmatrix} = \begin{bmatrix} X_u & X_p & 0 & 0 \\ l_u & l_p & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ p \\ \phi_v \\ h \end{bmatrix} + \begin{bmatrix} X_{\delta_s} & X_{\delta_r} \\ l_{\delta_s} & l_{\delta_r} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_s \\ \delta_r \end{bmatrix} \quad (4.3)$$

4.3 Mathematical Models of Actuators and Electrical Motor

4.3.1 Servo Actuator Model

The control surface actuators are modeled based on the identification performed by Salluce [60]. In the identification work, several different types of commercially available servos are analyzed to find their equivalent LTI model. Based on the database provided on [60], the size and weight of the actuators used in the MAV are identical to the *Airtronics 90491* servo. Therefore, the *Airtronics 90491* LTI model will be taken to approximate the dynamics of the currently used actuators. The *Airtronics 90491* is modeled as a second order system that has damping ratio 0.5446 and undamped natural frequency 17.429 rad/s. The transfer function of this actuator is given as:

$$G_a = \frac{303.77}{s^2 + 18.9837s + 303.77} \text{ rad/volt} \quad (4.4)$$

4.3.2 Electrical Motor Model

The dynamics of the electrical motor is modeled using a first order system with the time constant of 0.25 s. This leads to a settling time of around 1 s. Based on the experience in operating the MAV manually, that value of settling time is reasonable for the electrical motor used. Hence the transfer function for this electrical motor is given as:

$$G_E = \frac{4}{s+4} \text{ \% Throttle Stick/volt} \quad (4.5)$$

4.4 Design of the Inner Loop Controllers

4.4.1 Pitch Attitude Control

The pitch attitude is found to be unstable in hover. Therefore, the first controller to be designed is used to stabilize the pitch attitude. The transfer functions from horizontal tail to pitch rate/pitch angle are found by substituting the necessary stability and control derivatives at the hover equilibrium condition into Equation (4.1). They are presented as follows:

$$\frac{q}{\delta_{HTP}} = \frac{-81.8246s(s+1.4)}{(s-3.999)(s^2+10.05s+43.97)} \text{ rad/sec/rad} \quad (4.6)$$

$$\frac{\theta}{\delta_{HTP}} = \frac{-81.8246(s+1.4)}{(s-3.999)(s^2+10.05s+43.97)} \text{ rad/rad} \quad (4.7)$$

Figure 4.2 Schematic diagram of (a) pitch rate and (b) pitch attitude controllers

One of the methods to control the pitch attitude is by feeding back the pitch rate, as illustrated in Figure 4.2a. In this case, a simple proportional feedback of pitch rate can't be used to stabilize the pitch attitude, due to presence of a zero at the origin, which causes a root locus branch from the pitch divergence pole to be trapped in the

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

right half plane, as seen in Figure 4.3. A PI / PID controller can help to achieve the stability, since the integrator will cause the root locus branch to go from the positive real pole moves to the left toward the zero at $s = -1.4$. However, this approach involves a pole cancellation at the origin and also cannot give satisfactory performance.

Another method to stabilize the longitudinal dynamics is by feeding back the pitch angle, which is used here. The schematic diagram of this controller can be found in Figure 4.2b. The root locus plot of the pitch attitude control using a simple proportional control is shown in Figure 4.4a. Unlike in the previous case, the unstable pole moves to the stable zero of the plant. However, the stable complex conjugate poles of the plant move toward the right half plane at the same time. Therefore, the system can not be stabilized using proportional control only.

A PID controller can stabilize the systems and achieve short settling time, as well as zero steady state error. However, the magnitude of the overshoot is relatively significant. This is due to the presence of the zero near the origin. More complex controller schemes might be needed to reduce this overshoot. However, to simplify the system, the previously developed PID controller is used in the pitch attitude controller. The unit step response of this controller is given in Figure 4.5.

$$G_{PID} = 0.0633 \frac{(s+5)(s+9)}{s} \quad (4.8)$$

Figure 4.3 Root locus of the pitch rate controller using proportional gain

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

(a)

(b)

Figure 4.4 Root locus of the pitch angle control using (a) Proportional (b) PID

Figure 4.5 Unit step response of the pitch attitude control

4.4.2 Yaw Attitude Control

In the directional channel, the yaw attitude must be stabilized first before the position control is designed. This is due to the inherent stability of the directional mode, as observed also in the longitudinal mode. Using Equation (4.2) and the stability and control derivatives data given in Chapter Three, the transfer functions from the vertical tail to the yaw rate and the yaw angle are:

$$\frac{r}{\delta_{VTP}} = \frac{73.6214s(s+0.9191)}{(s-3.147)(s^2+6.295s+21.42)} \quad \frac{\text{rad/sec}}{\text{rad}} \quad (4.9)$$

$$\frac{\psi}{\delta_{VTP}} = \frac{73.6214(s+0.9191)}{(s-3.147)(s^2+6.295s+21.42)} \quad \frac{\text{rad}}{\text{rad}} \quad (4.10)$$

Figure 4.6 Block diagram of yaw attitude control

From Equations (4.9) and (4.10), the yaw dynamics are identical to pitch dynamics. Both are dominated by a pair of stable complex conjugate pole and unstable real pole. The main different is the location of each pole, where they are closer to origin than the pitch dynamics. Therefore, a similar control approach as in the pitch attitude control will be used to control the yaw attitude. The yaw attitude is controlled by feeding back the yaw angle to the comparator. The schematic diagram of the yaw attitude controller is illustrated in Figure 4.6.

The root locus of the yaw attitude control using a proportional gain is given in Figure 4.7a. The system can be stabilized but the dominant poles will be located very close to the imaginary axis. In this condition, the system behaves like neutrally stable system. A PID based controller can stabilize the system, achieve zero steady state error and fast settling time, but the response exhibits a high overshoot. This is due to the presence of the zero near origin, as seen in Figure 4.7b. The overshoot can be suppressed by using more complex controller or introducing low pass pre-filter to filter the commanded input. However, these options are considered to add more load to the system. In addition, adding a pre-filter will suppress the high frequency input.

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Hence, the above designed PID controller is used as the yaw attitude controller. The unit step response of the system is presented in Figure 4.8. The settling time is around 5 seconds, the overshoot is 90 % and the steady state error is zero.

$$G_{PID} = 0.121 \frac{(s + 2.5)(s + 4.5)}{s} \quad (4.11)$$

(a)

(b)

Figure 4.7 Root locus of the yaw attitude control using (a) Proportional and (b) PID

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Figure 4.8 Unit step response of the yaw attitude control

4.5 Outer Loop Controllers

4.5.1 $x_{\text{INERTIAL}}(x_I)$ Position Control

The pitch attitude control developed above is used as the inner loop of the x position control as depicted in the block diagram in Figure 4.9. As can be seen from the figure, w feedback is also used to improve the performance.

Figure 4.9 Schematic diagrams $x_{\text{INERTIAL}}(x_I)$ control

The transfer functions from the horizontal tail to the w and the x_I (x inertial position) during hover are obtained from Equation (4.1) as follows:

$$\frac{w}{\delta_{HTP}} = \frac{-5.0652(s-13.12)(s+12.06)}{(s-3.999)(s^2+10.05s+43.97)} \frac{\text{m/s}}{\text{rad}} \quad (4.12)$$

$$\frac{x_I}{\delta_{HTP}} = \frac{-5.0652(s-13.12)(s+12.06)}{s(s-3.999)(s^2+10.05s+43.97)} \frac{\text{m}}{\text{rad}} \quad (4.13)$$

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

The transfer function of the closed loop pitch attitude control is:

$$\frac{\theta}{\theta_{COMM}} = \frac{1537.3759(s+9)(s+5)}{(s^2 + 2.24s + 6.94)(s^2 + 5.46s + 72.36)(s^2 + 17.33s + 197.3)} \quad (4.14)$$

Equation (4.12) needs to be combined with Equation (4.14) to obtain the

$\frac{w}{w_{error}}$ transfer function. Subsequently, both equations and the velocity loop transfer

function are mixed to get the $\frac{x_I}{x_{I_{error}}}$ transfer function

The root locus of the w loop for a proportional gain is given in Figure 4.10. A good response of the w velocity can be found by taking the value of the gain as 0.0547. Notice that there is one non-minimum phase zero at $s = 13.12$. The effect of this zero to the response is small since this zero has very short time constant.

Figure 4.10 Root locus of the w control

The integrator term in $\frac{x_I}{\delta_{HTP}}$ transfer function makes the steady state error for

the x_I position controller is zero. Satisfactory response can be achieved by using proportional gain with value -0.658, as seen in the root locus plot of Figure 4.11a. Although the system exhibits a small overshoot and zero steady state error, the settling time of the system is rather slow at 8 seconds. This problem can be improved by adding a derivative controller. However this is not considered at this point.

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

(a)

(b)

Figure 4.11 (a) Root locus and (b) unit step response of x_I position control

4.5.2 $y_{\text{INERTIAL}}(y_I)$ Position Control

The yaw attitude controller developed earlier is used as the inner loop for the y position (y_I) control, as illustrated in Figure 4.12.

Figure 4.12 Schematic diagrams for $y_{\text{INERTIAL}}(y_I)$ position control

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

According to Equation (4.2), the transfer functions from the vertical tail to v and y_I are:

$$\frac{v}{\delta_{HTP}} = \frac{-5.0652(s-11.94)(s+11.93)}{(s-3.147)(s^2+6.295s+21.42)} \frac{\text{m/s}}{\text{rad}} \quad (4.15)$$

$$\frac{y_I}{\delta_{VTP}} = \frac{-5.0652(s-11.94)(s+11.93)}{s(s-3.147)(s^2+6.295s+21.42)} \frac{\text{m}}{\text{rad}} \quad (4.16)$$

The transfer function of the closed loop yaw attitude control is:

$$\frac{\psi}{\psi_{COMM}} = \frac{2706.0407(s+4.5)(s+2.5)}{(s^2+1.39s+1.78)(s^2+16.18s+94.33)(s^2+4.56s+166.4)} \quad (4.17)$$

The $\frac{v}{v_{error}}$ and $\frac{y_I}{y_{I,error}}$ transfer functions can be found by combining Equation

(4.15) and (4.16) with Equation (4.17).

The root locus of the v loop is given in Figure 4.13. The non-minimum phase zero is located far from the origin, so its effect to the response is small. The gain value chosen for the v loop is -0.0462.

The integrator in the vertical tail to y_I position transfer function helps to eliminate the steady state error of the position response. Under this situation, a simple proportional control can be used to achieve the desired y_I . The root locus of the y_I position controller are given in Figure 4.14a and its unit step response at a gain value of -0.404 is shown in Figure 4.14b. The response shows faster settling time than the x_I controller, since the dominant poles in this case are located farther from the origin.

Figure 4.13 Root locus of the v controller

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

(a)

(b)

Figure 4.14 (a) Root locus and (b) unit step response of y_r position controller

Figure 4.15 Schematic diagrams for the altitude control

4.5.3 Altitude Control

The altitude is controlled using adjustment on the throttle. The schematic diagram of this controller is shown in Figure 4.15.

The transfer function from the throttle to u and h are given below:

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

$$\frac{u}{\delta_T} = \frac{0.2232(s+1.01)}{(s+1.018)(s+0.00154)} \quad \frac{\text{m/s}}{\% \text{ Throttle Deflection}} \quad (4.18)$$

$$\frac{h}{\delta_T} = \frac{0.2232(s+1.01)}{s(s+1.018)(s+0.00154)} \quad \frac{\text{m}}{\% \text{ Throttle Deflection}} \quad (4.19)$$

The u dynamics are exponentially stable and minimum phase. Hence a simple proportional control feedback loop is sufficient to get satisfactory responses. The root locus is given in Figure 4.16.

For the altitude loop, the integrator term in the transfer function eliminates the steady state error. The root locus of this outer loop with proportional control is given in Figure 4.17a. Using a gain of 1, the altitude controller gives a satisfactory response characterized by less than 20 % overshoot and 5 seconds settling time. The unit step response of the altitude controller is given in Figure 4.17b.

Figure 4.16 Root locus of the u controller

(a)

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Figure 4.17 (a) Root locus and (b) unit step response of the altitude controller

4.5.4 Roll Angle Control

The roll angle is used to maintain the roll orientation of the MAV in hover. The roll angle can be controlled using the stator deflection by feeding back the roll rate and the roll angle. The roll rate feedback is placed in the inner loop and it is used to damp the rolling motion. The schematic diagram of the roll angle control is given in Figure 4.18. The roll mode is dynamically stable and non-minimum phase. The presence of the slow roll pole and the non-minimum phase zero near the origin makes the roll rate response very slow. The transfer functions from the stator to the roll rate and roll angle are given as:

$$\frac{p}{\delta_s} = \frac{109.734(s - 0.0059)}{(s + 1.018)(s + 0.0015)} \frac{\text{rad/sec}}{\text{rad}} \quad (4.20)$$

$$\frac{\phi}{\delta_s} = \frac{109.734(s - 0.0059)}{s(s + 1.018)(s + 0.0015)} \frac{\text{rad}}{\text{rad}} \quad (4.21)$$

Figure 4.18 Schematic diagrams for roll control

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

The roll rate control can be designed with positive or negative gains. For a negative gain, the roll and heave poles will meet in between, encircle the origin and terminate at the non-minimum phase zero. The response of such controller has the settling time in the order of hundred seconds, but with a large undershoots. For a positive gain, the roll pole moves toward the non-minimum phase zero. The system is stable at a very low gain and the steady state error is small. However, the settling time is long due to the pole near the origin. Although the settling time is very long, the roll rate control with a positive proportional gain is chosen for the roll rate controller. This is because such controller does not induce undesired undershoot. The root locus of the roll rate controller with a positive gain of 0.0012 is given in Figure 4.19.

The roll angle control is placed in the outer loop of the roll rate control. The presence of the integrator in the $\frac{\phi}{\delta_s}$ transfer function eliminates the steady state error.

The roll angle is controlled using a simple proportional control with the gain value of -0.001. The root locus and unit step response of such controller is given in Figures 4.20a and 4.20b. A dynamic compensation in the form of PD, lead or PID can not help to improve the response significantly due to the presence of roll pole near the origin.

Figure 4.19 Root locus of roll rate controller

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

(a)

(b)

Figure 4.20 (a) Root locus and (b) unit step responses of roll angle controller

Figure 4.21 SIMULINK[®] diagram of the linear simulation model

4.6 6 DoF Linear Simulation

The linear-coupled simulation model is used to test the performances of all the controllers. Saturators are also incorporated to limit the maximum deflection of the tails and the throttle stick deflection. The SIMULINK[®] block diagram of this linear-coupled simulation model is given in Figure 4.21.

In this work, three simulation conditions are considered. In the first simulation, the MAV's desired position is at the coordinate $(0,0,-100)$ m in the inertial coordinate system. An initial position error is introduced; the MAV's initial position is given as $(5,5,-100)$ m. Figures 4.22 and 4.23 give the result of this simulation. The controllers are capable to return the position of the MAV within 15 seconds. The errors in the position are very small. At the end of the simulation, the pitch angle, the yaw angle, the u, v and w values are zero. However, there is a slight change in the value of roll orientation (the yaw orientation in the helicopter notation) at the beginning of the response. After 20 seconds, the roll orientation returns to its initial value.

Figure 4.22 Position of the MAV in the 3-D space due to the initial position error

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Figure 4.23 Response of the MAV states due to the initial position error

In the second simulation, a wind disturbance of 2 m/s blowing from the South-West direction is introduced. In this case, the wind blows for 5 seconds duration only. Initially, the MAV is positioned at the coordinate $(0,0,-100)$ m in the inertial coordinate system. The illustration of this simulation is given in Figure 4.24. The controllers are expected to bring the MAV back to the initial position after the disturbance. The results of the simulation are shown in Figures 4.25 and 4.26.

The positions of the MAV are deviated significantly at the beginning of the simulation. As the time is running, the controllers start to work on bringing the positions of the vehicle back to the initial position. Finally after 15 seconds, the MAV returns to its original position. Examining Figure 4.26, all the states are damped out and the values are nearly zero after 15 seconds.

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Figure 4.24 Illustration of the wind disturbance introduced to the MAV

Figure 4.25 Position of the MAV in the 3-D space as
due to the wind blows for 5 seconds

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Figure 4.26 Responses of the MAV states
due to the wind blows for 5 seconds

For the third simulation case, a random wind disturbance blowing from the South-West direction is exerted to the MAV. The magnitude of this wind can be seen in Figure 4.27. In this case, the simulation is running for 100 seconds. Initially, the MAV is also positioned at the coordinate $(0,0,-100)$ m in the inertial coordinate system.

The response curves inform that the attitude controls can stabilize the MAV very well under the random wind disturbance. In addition, the position of the MAV can be kept within a box of 5×5 m. The altitude controller can maintain the MAV in the air with an altitude error of less than 10 cm. The roll orientation controller also gives satisfactory performance in this simulation. These can be seen in Figures 4.28 and 4.29. In the future, the gains must be reselected in order to reduce the degree of the positional errors. The new gains must be able to keep the MAV in a smaller box, if the same wind disturbances are introduced.

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Figure 4.27 Magnitude of the random wind

Figure 4.28 Position of the MAV in the 3-D space
due to the random wind disturbance

DESIGN OF AUTOMATIC HOVER CONTROL FOR THE VTOL MAV IN HOVER

Figure 4.29 Responses of the MAV states due to the random wind disturbance

4.7 Summary of the Automatic Hover Control Design

The conclusions drawn from the automatic hover control design section are:

1. The SISO root locus technique is used to design the automatic control throughout this research work. Multiple SISO control loops are needed to control all the variables.
2. PID controllers are capable to stabilize the pitch and the yaw attitudes, as well as achieving zero steady state error and short settling time. Both variables are controlled by deflecting the horizontal and the vertical tails respectively.
3. Simple proportional controllers are sufficient to control the x and y positions in the inertial axis, as well as the altitude.
4. The roll orientation is kept with the roll angle control. This controller takes the feedback signal from the roll rate and the roll angle. The presence of a small non-minimum phase zero and the pole near origin result in a very slow response.
5. The controllers have been tested in the linear-coupled model simulation model. The performances of the controllers are acceptable.

Chapter Five**CONCLUSIONS****5.1 Conclusions**

Conclusions that can be drawn from this work are as follows:

From the MAV design:

A VTOL MAV with ducted fan configuration has been designed. The design process follows a systematic approach and it is guided by the design requirements and objectives. The size and the configuration of the final design is selected by considering and trading off several aspects, including the availability of the components in the market, the aerodynamics and the static stability requirements. The maximum dimension of the currently designed MAV is less than 50 cm. This is relatively small compared with the currently available VTOL UAV. A proof-of-concept prototype has been developed and will be used for various testing.

The most interesting feature developed for the MAV is the variable incidence wing on the sides of the duct. This wing improves the aerodynamic performance of the MAV and reduces the power consumption. This is especially useful to eliminate/reduce the altitude loss during the transition and to improve gust/crosswind rejection during hover.

From the dynamic analysis:

Dynamic analyses of the MAV in hover, cruise and hover-to-cruise transition have also been performed. The dynamics of the MAV in the cruise and the small angles of attack flights are identical to those of the conventional aircraft. At high angles of attack and hover flights, the MAV becomes inherently unstable and there is a strong coupling between the longitudinal and lateral-directional degrees of freedom. This coupling is unavoidable in the current design, as it is generated by the stator used to counter the engine torque.

Besides influencing the static stability characteristics in the cruise flight, the CG location also affects the dynamics of the MAV in hover. The CG location

influences the eigenvalues of the system and the type of the unstable mode in hover. If the CG is above the duct lips, the instabilities take the form of divergence with no oscillatory motions. On the other hand, the instabilities will be in the form of divergent oscillations, if the CG is below the duct lips.

From the hover control design:

The main objectives of the control design task are to stabilize the MAV and also to keep the position of the MAV during hover. The controllers are designed based on the linearized MAV equations of motion using the SISO root locus technique. PID types of controllers are shown to be able to achieve the control goals.

The performances of the controllers have been examined using the 6 DoF linear-coupled simulations. It is shown that the controllers can yield satisfactory performance even in the presence of the wind disturbances.

5.2 Recommendations for Further Study

The overall project goal is to develop a VTOL MAV with fully autonomous capability. This research work represents a milestone to achieve that final destination. The future works needed to achieve the goal are as follows:

1. Nonlinear Simulation of the MAV

The designed control should be tested in the nonlinear computer simulation environment to assess the performance of the controller in the nonlinear plant.

2. Hardware in the loop control development

Hardware in the loop setup allows the testing of the controller in the controlled environment without possible catastrophic failure of the prototype.

3. Design of controller for the whole flight envelope

The control system for this case can be developed based on the SISO or MIMO design techniques. The models needed for this task can be obtained from the linearization or parameter identification tasks.

4. Flight testing under the automatic control

After the control design task has been finished, the control algorithm must be applied in the real environment.

5. Improvement of the MAV design

CONCLUSIONS

Improvement to the design will be performed if necessary. This improvement can be judged based on the flight testing data or the result from further detailed analysis.

REFERENCES

- [1] Tsach, S., Peled, A., Penn, D., Keshales, B. & Guedj, R., "*Development Trends For Next Generation UAV Systems*," AIAA Aerospace Conference and Exhibit, Rohnert Park, California, 2007
- [2] Hanon, L., "*Robots on the Battlefield - Are We Ready For Them?*," 3rd AIAA "Unmanned Unlimited" Technical Conference, Workshop and Exhibit, Chicago, Illinois, 2004
- [3] Clough, B. T., "Unmanned Aerial Vehicles: Autonomous Research Challenges, a Researcher's Perspective," *Journal of Aerospace, Computing, Information and Communication*, vol. 2, pp. 327-347, 2005.
- [4] McMichael, J. M., Francis, M. S., "*Micro Air Vehicles - Toward a New Dimension in Flight*," Defence Advanced Research Program Agency - TTO 1996.
- [5] Davis, W. R., Kosicki, B. B., Boroson, D. M. & Kostishack, D. F., "Micro Air Vehicles for Optical Surveillance," *The Lincoln Laboratory Journal*, vol. 9, pp. 197-214, 1996.
- [6] Hundley, R. O., Gritton, E. C., "*Future Technology-Driven Revolutions in Military Operations*," RAND National Defence Research Institutes, Santa Monica, California 1994.
- [7] Burdman, P. K., Guest, M., Nieto, F., Pradeloux, F. & Senabre, J., "*Design of a Micro Air Vehicle (MAV)*," Department of Mechanical Engineering, Linköping University, Linköping 2004.
- [8] Kordes, T., Buschmann, M., Winkler, S. & Schulz, H. W., "*Progress in the Development of the Fully Autonomous MAV "COROLO"*," 2nd AIAA "Unmanned Unlimited" System, Technology and Operations Conference, San Diego, California, 2003
- [9] Grasmeyer, J. M., Keennon, M. T., "*Development of Black Widow Micro Air Vehicle*," 39th AIAA Aerospace Sciences Meeting & Exhibit Reno, NV, 2001
- [10] Abershitz, A., Penn, D., Levy, A., Shapira, A. & Shavit, Z., "*IAI's Micro / Mini UAV Systems - Development Approach*," AIAA Information Technology at Aerospace Conference, Arlington, Virginia, 2005
- [11] Oh, P. Y., Green, W. E., "*CQAR: Closed Quarter Aerial Robot Design for Reconnaissance, Surveillance and Target Acquisition Tasks in Urban Areas*," *International Journal of Computational Intelligence*, vol. 1 pp. 353 - 360, 2004.
- [12] Nicoud, J. D., Zufferey, J. C., "*Toward Indoor Flying Robots*," Didel SA, , Belmont, Switzerland.
- [13] Green, W. E., Oh, P. Y., "*A MAV That Flies Like an Airplane and Hovers Like a Helicopter*," IEEE International Conference on Advanced Intelligent Mechatronics, pp. 693-698, Monterey, California, 2005.
- [14] Marek, P., Smrcek, L., "*Development of DART MAV - Fixed Wing, Hover Capable Micro Aerial Vehicle*," Department of Aerospace Engineering, University of Glasgow, Glasgow 2001.

REFERENCES

-
- [15] Schaefer, C. G., Baskett, L. J., "Goldeneye: The Clandestine UAV," 2nd AIAA "Unmanned Unlimited" System, Technologies and Operations vol. AIAA 2003 - 6634, San Diego, California, 2003.
 - [16] Bohorquez, F., Samuel, P., Sirohi, J., Pines, D. & Rudd, L., "Design, Analysis and Hover Performance of Rotary Wing Micro Air Vehicle," *Journal of American Helicopter Society*, vol. 48 No 2, pp. 80-97, 2003.
 - [17] Bohorquez, F., Samuel, P., Sirohi, J., Pines, D. & Rudd, L., "Hover Performances of Rotor Blade at Low Reynolds Number for Rotary Wing MAV: An Experimental and CFD Study," 21st AIAA Applied Aerodynamics Conference, Orlando, Florida, 2003
 - [18] Ramasamy, M., Lee, T. E., Leishman, J. G., "Flowfield of a Rotating Wing Micro Air Vehicle," *Journal of Aircraft*, vol. 44 No 4, pp. 1236-1244, 2007.
 - [19] Hamilton, H. H., "Navigation and Control Problems for Classes of Micro Air Vehicles," AIAA Atmospheric Flight Mechanics Conference and Exhibit, Keystone, Colorado, 2006
 - [20] Keennon, M. T., Grasmeyer, J. M., "Development of the Black Widow and Microbat MAVs and a Vision of the Future of MAV Design," AIAA/ICAS International Air and Space Symposium and Exposition: The Next 100 Y Dayton, Ohio, 2003
 - [21] Ruzicka, G. C., Strawn, R. C., Meadowcroft, E. T., "Discrete-Blade, Navier-Stokes Computational Fluid Dynamics Analysis of Ducted Fan Flow," *Journal of Aircraft*, vol. 42 No 5, pp. 1109 - 1117, 2005.
 - [22] Abrego, A. I., Bulaga, R. W., "Performance Study of a Ducted Fan System," Aerodynamic, Acoustic, Test and Evaluation Technical Specialist Meeting, San Francisco, California, 2002
 - [23] Parlett, L. P., "Aerodynamic Characteristics of A Small-Scale Shrouded Propeller at Angle of Attack from 0° to 90°," NACA-TN 3547, NACA, Langley, Virginia 1955.
 - [24] Fletcher, H. S., "Experimental Investigation of Lift, Drag and Pitching Moment of Five Annular Airfoils." Langley, Virginia: National Advisory Committee For Aeronautics, 1957.
 - [25] Fleming, J., Jones, T., Gelhausen, P. & Enns, D., "Improving Control System Effectiveness for Ducted Fan VTOL UAV Operating in Crosswinds," 2nd AIAA "Unmanned" Unlimited System, Technology and Operations, San Diego, California, 2003
 - [26] Lipera, L., Colbourne, J. D., C, R. M. & Patangui, P., "The Micro Craft iSTAR Micro Air Vehicle Control System Design and Testing," American Helicopter Society Annual Forum, Washington DC, 2001
 - [27] Guerrero, I., Londenberg, K., Gelhausen, P. & Myklebust, A., "A Powered Lift Aerodynamic Analysis For The Design of Ducted Fan UAVS," 2nd AIAA "Unmanned Unlimited" Systems, Technologies and Operations, San Diego, California, 2003
 - [28] Avanzini, G., D'Angelo, S., Matteis, G. d., "Design and Development of A VTOL Uninhabited Aerial Vehicle," AIDAA XVI Congresso Nazionale Palermo, Italy, 2001
 - [29] Tjin, I., "Challenges in Small VTOL UAV Development - The ST Dynamics Experience," National UAV Seminar, Singapore, 2002
 - [30] Pines, D. J., Bohorquez, F., "Challenges Facing Future Micro Air Vehicle Development," *Journal of Aircraft*, vol. 43, pp. 290-305, 2006.
-

REFERENCES

-
- [31] Raymer, D. P., *Aircraft Design - A Conceptual Approach*, AIAA Education Series, AIAA, Washington, D.C, 1992.
 - [32] Torenbeek, E., *Synthesis of Subsonic Airplane Design* Delft University Press - Martinus Nijhoff Publishers, Delft 1982.
 - [33] Jenkinson, L., Marchman, J., *Aircraft Design Projects for Engineering Student*, Butterworth - Heinemann, Oxford, 2003.
 - [34] Leishman, J. G., *Principles of Helicopter Aerodynamics*, 2nd Edition, Cambridge University Press, Cambridge, 2006.
 - [35] Gursul, I., Taylor, G., Wooding, C. L., "Vortex Flows over Fixed-Wing Micro Air Vehicles," 40th AIAA Aerospace Sciences Meeting & Exhibit, Reno, NV, 2002
 - [36] Torres, G. E., Mueller, T. J., "Low Aspect Ratio Wing Aerodynamics at Low Reynolds Numbers," *Journal of Aircraft*, vol. 42 No 5, pp. 865 - 873, 2004.
 - [37] Luke, M., Bowman, W. J., "Predicting Drag Polar for Fixed-Wing Micro Air Vehicles," 3rd AIAA "Unmanned Unlimited" Technical Conference, Workshop and Exhibit, Chicago Illinois, 2004
 - [38] Zimmerman, C. H., "Characteristics of Clark Y Airfoils of Small Aspect Ratios," NACA TR 431, NACA, Virginia 1933.
 - [39] Zimmerman, C. H., "Aerodynamics Characteristics of Several Airfoils of Low Aspect Ratio," NACA - TN - 539, NACA, Virginia 1935.
 - [40] Moschetta, J.-M., Bataille, B., Thipyopas, C. & Shkarayev, S., "On Fixed-Wing Micro-Air Vehicles with Hovering Capabilities," 46th AIAA Aerospace Science Meeting and Exhibition, Reno, Nevada, 2008
 - [41] Allen, H. J., Perkins, E. W., "Characteristics of Flow Over Inclined Bodies of Revolution," NACA-RM-A50L07, NACA, Washington 1951.
 - [42] Stengel, R. F., *Flight Dynamics*, Princeton University Press, Princeton, New Jersey, 2004.
 - [43] Etkin, B., Reid, L. D., *Dynamics of Flight Stability and Control - 3rd Edition*, John Wiley and Sons, New York, 1996.
 - [44] Nelson, R. C., *Flight Stability and Automatic Control*, McGraw-Hill Book Company, New York, 1989.
 - [45] Gad-el-Hak, M., "Micro Air Vehicles: Can They Be Controlled Better?," *Journal of Aircraft*, vol. 38 No 3, pp. 419-428, 2001.
 - [46] Selig, M. M., Guglielmo, J. J., Broern, A. P. & Gugierre, P., "Experiments on Airfoil at Low Reynolds Numbers," 34th AIAA Aerospace, Science, Meeting and Exhibit, Reno, NV, 1996
 - [47] Drela, M., Youngren, H., "XFOIL 6.92." Cambridge, Massachusetts: MIT, 2001.
 - [48] Selig, M., "UIUC Airfoil Coordinate Database".
 - [49] Franklin, J. A., *Dynamics, Control and Flying Qualities of V/STOL Aircraft*, AIAA Education Series, Washington, DC, 2002.
 - [50] Padfield, G. D., *Helicopter Flight Dynamics - The Theory and Application of Flying Qualities and Simulation Modeling*, 2nd Edition, AIAA Education Series, Washington, DC, 2007.
 - [51] Foster, T. M., Bowman, W. J., "Dynamic Stability and Handling Qualities of Small Unmanned-Aerial Vehicle," 43rd AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada, 2005
-

REFERENCES

-
- [52] Kuo, Z.-S., Soong, C.-Y., Chang, Y.-S., "Dynamics Modeling and Analysis of Whole Wing MAV," 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Material, Honolulu, Hawaii, 2007
- [53] Lopez, C. A., Wells, V. L., "Dynamics and Stability of an Autorotating Rotor/Wing Unmanned Aircraft," *Journal of Guidance Control and Dynamics*, vol. 27 No 2, pp. 259 - 270, 2004.
- [54] Dickeson, J. J., Mix, D. R., Koenig, J. S., Linda, K. M. & Oguzhan Cifdaloz, "*H_∞ Hover-to-Cruise Conversion for a Tilt-Wing Rotorcraft*," 44th IEEE Conference on Decision and Control, and the European Control Conference, Madrid, Spain, 2005
- [55] Mettler, B., Tischler, M. B., Kanada, T., "System Identification Modeling of a Small-Scale Unmanned Rotorcraft for Flight Control Design," *Journal of American Helicopter Society*, vol. 47 Number 1, pp. 50-63, 2001.
- [56] Avanzini, G., d'Angelo, S., Matteis, G. d., "Performance and Stability of Ducted-Fan Uninhabited Aerial Vehicle Model," *Journal of Aircraft*, vol. 40 No 1, pp. 86 - 93, 2003.
- [57] Stone, R. H., "*Modeling and Control of Tandem-Wing Tail-Sitter UAV*," in *Modeling and Control of Mini Flying Machine*, pp. 133-164. London: Springer, 2006.
- [58] Stevens, B. L., Lewis, F. L., *Aircraft Control and Simulation*, John Wiley & Sons, New York, 2003.
- [59] McCormick, B. W., *Aerodynamics, Aeronautics and Flight Mechanics*, 2nd Edition, John Wiley and Sons, New York, 1995.
- [60] Salluce, D. N., "*Comprehensive System Identification of Ducted Fan UAVs*," California Polytechnic State University, California 2004.
- [61] Ogata, K., *Modern Control Engineering*, 3rd Edition, Prentice-Hall International, Inc, New Jersey, 1997.
- [62] Blakelock, J. H., *Automatic Control for Aircraft and Missiles*, 2nd Edition, John Wiley and Sons, Inc, New York, 1991.
- [63] McRuer, D., Ashkenas, I., Graham, D., *Aircraft Dynamics and Automatic Flight Control*, Princeton University Press, Princeton, New Jersey, 1973.
- [64] Lombaerts, T. J. J., Mulder, J. A., Voorsluijs, G. M., "*Design of a Robust Flight Control System for a Mini-UAV*," AIAA Guidance, Navigation and Control Conference and Exhibit, San Fransisco, California, 2005
- [65] Krashanitsa, R., Platinitis, G., Silin, B. & Shkarayev, S., "*Aerodynamics and Control Design for Autonomous Micro Air Vehicles*," AIAA Atmospheric Flight Mechanics Conference and Exhibit, Keystone, Colorado, 2006
- [66] Stone, R. H., "Control and Guidance for a Tail-Sitter Unmanned Air Vehicle," *Field and Service Robotics*, vol. 25, pp. 453 - 464, 2006.
- [67] Shim, H., "*Hierarchical Flight Control System Synthesis for Rotorcraft-based Unmanned Aerial Vehicle*," University of California, Berkeley, California 2000.
- [68] Kim, H. J., Shim, D. H., "A Flight Control System for Aerial Robots: algorithms and experiments," *Control Engineering Practice*, vol. 11, pp. 1389 - 1400, 2003.
- [69] Lee, I.-h., Lee, D.-j., Bang, H., "*A Small Scale Rotor UAV Autonomous Flight Control Design and Verification*," AIAA 2007 Conference and Exhibition, Rohnert Park, California, 2007
-

REFERENCES

-
- [70] Mettler, B., Tischler, M. B., Kanada, T. & Wessner, W., "Attitude Control Optimization for a Small-Scale Unmanned Helicopter," AIAA Guidance, Navigation, Control Conference, Denver, 2000
- [71] Pflimlin, J. M., Binetti, P., Soueres, P., Hamel, T. & Trouchette, D., "Modeling and Attitude Control Analysis of a Ducted-fan Micro Air Vehicle," *Control Engineering Practice*, 2007.
- [72] Pflimlin, J. M., Soueres, P., Hamel, T., "Position Control of a Ducted Fan VTOL UAV in Crosswind," *International Journal of Control*, vol. 80 No 5, pp. 666 - 683, 2006.
- [73] Pflimlin, J. M., Soueres, P., Hamel, T., "Hovering Flight Stabilization in Wind Gusts For Ducted Fan UAV," 43rd IEEE Conference on Decision and Control, pp. 3491-3496, Paradise Island, Bahamas, 2004.
- [74] Castillo-Effen, M., Castillo, C., Moreno, W. & Valavanis, K. P., "Control Fundamentals of Small Miniature Helicopter - A Survey," *Advanced in Unmanned Aerial Vehicle*, pp. 73 - 118, 2007.
- [75] Frank, A., McGrew, J., Valenti, M., Levine, D. & How, J. P., "Hover, Transition and Level Flight Control Design for a Single-Propeller Indoor Airplane," Department of Aeronautics and Astronautics, MIT, Cambridge, Massachusetts 2007.
- [76] Gavrilets, V., Martinos, I., Mettler, B. & Feron, E., "Aggressive Maneuvering Flight Tests of a Miniature Robotic Helicopter," *Experimental Robotics* vol. VIII, pp. 456-465, 2003.
- [77] Dickeson, J. J., Miles, D., Cifdaloz, O., Wells, V. L. & Rodriguez, A. A., "Robust LPV H_∞ Gain-Scheduled Hover-to-Cruise Conversion for a Tilt-Wing Rotorcraft in the Presence of CG Variations," IEEE American Control Conference, pp. 5266-5271, New York City, 2007.
- [78] Avanzini, G., Ciniglio, U., Matteis, G. d., "Full-Envelope Robust Control of a Shrouded-Fan Unmanned Vehicle," *Journal of Guidance Control and Dynamics*, vol. 29 No 2, pp. 435 - 443, 2006.
- [79] Spaulding, C. M., Mansur, M. H., Tischler, M. B., Hess, R. A. & Franklin, J. A., "Nonlinear Inversion Control for a Ducted Fan UAV," AIAA Atmospheric Flight Mechanics Conference and Exhibit, San Francisco, California, 2005
- [80] Knoebel, N. B., Osborne, S. R., Snyder, D. O., "Preliminary Modeling, Control and Trajectory Design for Miniature Autonomous Tail-Sitter," AIAA Guidance, Navigation and Control Conference and Exhibit, Keystone, Colorado, 2006
- [81] Knoebel, N. B., "Adaptive Quaternion Control of A Miniature Tail-Sitter UAV," Brigham Young University, Salt Lake City, Utah 2007.
- [82] Osborne, S. R., "Transition between Hover and Level Flight for a Tail-Sitter UAV," Brigham Young University, Salt Lake City, Utah 2007.
- [83] Bijmens, B., Chu, Q. P., Voorsluijs, G. M. & Mulder, J. A., "Adaptive Feedback Linearization Flight Control for Helicopter UAV," AIAA Guidance, Navigation and Control Conference and Exhibit, San Francisco, California, 2005
- [84] Lower, M., Szlachetko, B., Krol, D., "Fuzzy Flight Control System for Helicopter Intelligence in Hover," IEEE 5th International Conference on Intelligent System Design and Applications, 2005.
-

REFERENCES

- [85] Johnson, E. N., Kannan, S. S., "*Adaptive Flight Control for an Autonomous Helicopter*," AIAA Guidance, Navigation and Control Conference and Exhibition, Monterey, California, 2002
- [86] Johnson, E. N., Turbe, M. A., "Modeling, Control and Flight Testing of A Small Ducted Fan Aircraft," *Journal of Guidance Control and Dynamics*, vol. 29 No 4, pp. 769 - 779, 2006.

Appendix A

BRUSHLESS OUTRUNNER MOTOR TECHNICAL DATA

Table A.1 Brushless Motor Technical Data

	Hacker A20-22L	AXI 2212-26	JH2812
Weight (grams)	57	57	39
Diameter x Length (mm)	27.8 x 32	27.7 x 30	25 x 25
Current (A)	6 – 14	6 – 10	6 -11
Peak Current (A) / Duration (s)	17/15	12/60	16/60
Voltage (V)	7.4 – 11.1 (2 – 3 cells LiPo)	7.4 – 11.1 (2 – 3 cells LiPo)	7.4 – 11.1 (2 – 3 cells LiPo)
Max Power (W)	190	120	160
Resistance (Ohm)	0.089	0.021	-
RPM/V	924	920	1534
Max Efficiency (%)	85	80	82
Shaft Diameter (mm)	3	3.17	3

Hacker A20 – 22L

AXI 2212 – 26

JH2812

Appendix B

Three View Drawings of the VTOL MAV

B.1 Complete three view drawings

B.2 Top View

Nanyang Technological University		
TITLE: Three View and Isometric Drawing of The MAV	DRAWN: TONO	
DWG NO:	SCALE: 1 : 2	DATE : 2006 - 2008
MATERIAL:	PROJ: MAV	ALL DIMENSIONS IN MM

B.3 Left View

B.4 Front View