Controller design for nonlinear micromachined parallel plate actuators

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2009

Nikapanah Mohammad Hossein. (2009). Controller design for nonlinear micromachined parallel plate actuators. Master's thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/19275>

<https://doi.org/10.32657/10356/19275>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**CONTROLLER DESIGN FOR NONLINEAR
MICROMACHINED PARALLEL PLATE ACTUATORS**

**NIKPANAH MOHAMMAD HOSSEIN
SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING
2009**

CONTROLLER DESIGN FOR NONLINEAR MICROMACHINED PARALLEL PLATE ACTUATORS

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Master of Engineering

2009

STATEMENT OF ORIGINALITY

I hereby certify that the work embodied in this thesis is the result of my original research and has not been submitted for a higher degree to any other University or Institution.

07 Sep - 09
Signature and Date
Nikpanah Mohammad Hossein

ACKNOWLEDGEMENTS

First and foremost, I wish to express gratitude to my supervisor, Professor Wang Youyi for his careful guidance, stimulating suggestions, exact insight and profound knowledge in supervising my research project. Without his precise and appropriate instruction at every step during my research, I could not come this far so smoothly for my graduate study.

I am also very thankful to Professor Frank Lewis, the University of Texas at Arlington, TX, USA, for his useful comments on my research work. He imparted his perfect consultations to me.

I am sincerely grateful to Associate Professor Liu Ai Qun for his continuous support. I really enjoyed being in his group and the contributions from his group members played an important role in this research work.

Special thanks to Dr. Wu Jiuhui for his expertise in mathematics, Dr. Zhang Xuming and Dr. Liu Bing for their inspirations in MEMS.

I would like to convey my appreciation to all technicians in Electrical Power Research Lab and Photonics Lab I, for their kind assistance and support.

I would like to express my sincere appreciation to all the individuals, specially my brother, who have contributed to the successful completion of my graduate studies and this research thesis.

Also Many thanks to some of my friends for their help in handling the word process and for the completion of my thesis;

It is necessary to thank my family for their kind support.

Finally I am very grateful for A*STAR for great financial support as well and providing me a nice life experience at SINGAPORE.

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SUMMARY

The use of electrostatic actuation for microelectromechanical systems (MEMS) is attractive because of the high energy densities and large forces available in microscale devices. Nonlinear micromachined electrostatic parallel plate microactuators which work by electrostatic actuation, play an important role in MEMS. Compared with other types of microactuators, these microactuators are the more common and easy to use and it's because of the fact that they generate lower force and consume relatively no electrical power. Low power applications are ideal candidates for electrostatic actuation because electrostatic force is generated by charge distribution not current fellow.

Electrostatic parallel plate microactuators have many applications such as pressure sensors, accelerometers or comb-drive actuators, RF communication components, transmission line switches, reflective diffraction grating, laser cavities, stepper positioners, microrelays, micro shutters, optical routers, tunable capacitors, wavelength division multiplexing (WDM) filters, tunable LED's and tuned laser.

Electrostatic parallel plate microactuators are normally driven by DC voltage source scheme which is called static open-loop voltage control scheme. A major problem in this control strategy is that the voltage source causes a positive feedback in the electrostatic actuation. There is a well-known instability, inherent in the use of electrostatically actuated MEMS device called "snap-through" or "Pull-In". "Pull-In" instability comes from positive feedback which causes the actuator top plate position becomes unstable and collapses at a distance of two-thirds of the zero- bias capacitive gap and it snaps the bottom plate. This prevents application of open loop voltage control scheme for devices that require an analog control of stable position within the entire gap.

The objective of this thesis is to improve the performance of electrostatic parallel plate microactuators. Hence it is necessary to extend the stable and controllable travel range

of the device to full gap and control plate deflection through whole gap to achieve desired switching on-off time. Another issue is that our proposed methods must be easily implementable.

Fist some methods are introduced for this purpose which they have been done in the past by other people. Among of all methods, it is comprehensible that the Closed-Loop Feedback Control method is the best method for this purpose.

One of the previous methods uses Closed-Loop Feedback Control method. This method obtains the objective above except it causes extra ordinary fluctuations due to the control signal function. This drawback makes it impossible to be implemented.

Then switching control scheme is introduced to eliminate extra ordinary fluctuations problem from control scheme above. But on the other hand, this scheme has two major drawbacks, O.S problem and operational limitation for a special range of MEMS device parameter.

Another control scheme called non-negative input design is introduced next to improve the performance of closed-loop control system compared to the first control scheme by elimination of O.S problem. But operational limitation problem still remains.

For developing our control scheme operational range without O.S problem, last control scheme called positive state variable design is described finally. This scheme works for a wider range of MEMS device compared to the former control schemes.

Chapter1. Introduction

1.1 Background

The use of electrostatic actuation for microelectromechanical systems (MEMS) is attractive because of the high energy densities and large forces available in microscale devices [1]. Electrostatic Parallel Plate microactuators which work by electrostatic actuation, play an important role in MEMS. Compared with other types of microactuators, these microactuators are the more common and easy to use and it's because of the fact that they generate lower force and consume relatively no electrical power [2]. Low power applications are ideal candidates for electrostatic actuation because electrostatic force is generated by charge distribution not current fellow [3].

Pressure sensors, accelerometers or comb-drive actuators can be found as electrostatically actuated MEMS devices [4]. Electrostatic actuators find many applications today in several areas [5] such as relays [6-8], optical devices, RF communication components [9- 11] as variable capacitors, transmission line switches, etc. Electrostatic actuators can have many different analog applications [12] such as the reflective diffraction grating [1], applications in laser cavities [13], stepper positioners [14]. They also have digital applications [15] such as microrelays [7], micro shutters [15] and optical routers [10]. Electrostatic actuators are available today as wide variety of devices [16] such as tunable capacitors [9,11], resonant cavity photonic devices as vertical lasers [13], wavelength division multiplexing (WDM) filters [17], or tunable LED's [18]. All of these applications require an analog control of stable positions within the gap [16]. Micromechanical electrostatic actuators have application in optical devices [19] such as a micromechanically tuned laser [13].

One of the difficulties inherent in the use of electrostatically actuated MEMS device is an instability known as “snap-through” or “Pull-In” [4], [20-25] that causes the oppositely charged mechanical members to snap together [4]. At a distance of two-thirds of the zero-bias capacitive gap, the actuator position becomes unstable and collapses [2]. A serious limitation on the use of these devices lies in the well-known “Pull-In” instability which makes the mobile electrode collapse when the deflection exceeds $1/3$ of the initial gap [5]. When the deflection distance is equal to one-third the initial separation distance; the required

voltage stops increasing. “Snap-through” occurs when an increase in the deflection no longer requires an increase in applied voltage [3].

In electrostatic actuation, the interaction of a nonlinear electrostatic force with a linear elastic restoring force causes the “Pull-In” phenomenon [1]. The origin of the “Pull-In” limitation is rooted in the fact that the electrostatic force applied to a mobile electrode by actuator charge is a nonlinear function of the remaining gap [5]. Electrostatic actuation is the application of electrostatic force to a mechanical system. This is the drawback of electrostatic actuation that electrostatic force is a nonlinear function of distance; as the distance between the electrodes decreases, electrostatic force increases exponentially [3].

Nonlinear machined electrostatic microactuators are normally driven by static open-loop voltage control schemes. In other words the constant voltage is directly applied to the MEMS device. A major problem in this control strategy is that the voltage source causes a positive feedback in the electrostatic actuation [2]. The “Pull-In” instability comes from positive feedback. The positive feedback mechanism can be explained as follows. An increase in the constant voltage source results in an increase in the charge and then increase in the electric field and keeps the potential difference between the plates constant. The increase in charge and field in turn increases the electrostatic force, which pushes the plate lower [1]. This increases the capacitance and thus the charge and the electric field because of the applied voltage again. This positive feedback causes the actuator to deflect further [2]. This phenomenon is fundamental to many voltage-controlled electrostatic actuators [1].

Normal voltage drive causes the “Pull-In” instability problem and so it is a limitation of the applicability of this drive mode [16]. When a voltage source drives an electrostatic parallel plate actuator; the well-known “Pull-In” instability limits the range of displacement to 1/3 of the gap. Hence, the “Pull-In” instability is inherent to the voltage drive, which is commonly the used drive. When the voltage source is programmed to higher values than the “Pull-In” value, the source gives as much charge as the system demands [20]. This charge consumption leads to unnecessary high power consumption and the collapse of the movable plate with the fixed electrode [12].

The “Pull-In” instability effectively prohibits stable operation of many devices over their full range of mechanical deflection [4]. This instability limits the stable travel distance of elastically suspended parallel-plate electrostatic microactuators to about 1/3 of the

undeflected gap distance [1]. This is a recognized drawback, limiting the travel range and hence, the application of these devices [5]. The open-loop constant voltage drive is unable to provide stable and controllable deflection beyond 33% of the zero-bias capacitive gap. This prevents application of open loop voltage control scheme for devices that require an analog control of stable position within the entire gap [2]. Because the position control of the movable electrode at any distance across the gap faces with the “Pull-In” instability, it makes the digital operation easier than the analog one [12].

1.2 Objectives

In this research work, we are concerned with stabilization strategies [4]. Driving of electrostatic actuators faces several challenges, both functional and technological, in today MEMS technologies. The driving circuitry has to a) provide a fast response of the actuator movement, b) achieve full gap movement as low voltage as possible for sake of compatibility with conventional electronics, c) consume low power level, mainly for portable systems, d) have high reliability to avoid contacts and e) be able to achieve stable deflections within the gap in analog- tuned applications [16]. A large range of motion for analog positioning can be extremely useful for a wide variety of tuning applications. It is desirable to try to increase the controllable travel range of the electrostatic actuators beyond the usual 1/3 of the gap [1]. The analog position control of the movable electrode of an electrostatic microactuator has become an important issue recently reported in many applications [12]. For analog control of stable position within the entire gap applications, it is desirable to extend the stable and controllable range to the full gap [2]. A method or mechanism to extend that travel range, preferably to the extent of the entire initial gap, is highly desirable, especially for optical applications. Such a method will increase the tunable range of optical devices. This method will also enable the continuous analog control of a positioner instead of the limited on-off behavior that avoids teetering near the threshold of instability [19].

The objective of this research work is to improve the performance of nonlinear micromachined electrostatic microactuators. It is necessary to extend the stable and controllable travel range of the device to full gap and control plate deflection through whole gap to achieve desired switching on-off time. Another issue is that our proposed methods must be easily implementable.

1.3 Contributions

In this research work, three control schemes are proposed: one indirect control scheme and two direct control schemes. They will be designed for the MEMS device modified model (MEMS device model after using direct feedback linearization method). The major difference between indirect scheme and direct schemes is that; in indirect control scheme, the nonlinear control scheme proposed in [2] is a part of closed-loop system but in direct control schemes it is not.

The first control scheme called auxiliary controller design control scheme (indirect control scheme) which consists of two parts; an auxiliary controller and the nonlinear control scheme proposed in [2]. The auxiliary controller design control scheme uses a two-position switch which in each position chooses one of two controllers above according to a threshold value. The auxiliary controller is designed according to state feedback method. This technique eliminates extra ordinary fluctuations problem due to the nonlinear control scheme proposed in [2].

The second control scheme called non-negative input design control scheme (direct control scheme) which is based on state feedback method. It improves the system performance in comparison with the auxiliary controller design control scheme by eliminating overshoot problem from the closed-loop system.

The third control scheme called positive state variable design control scheme (direct control scheme) uses direct state feedback method, which works for a wider range of MEMS devices in comparison with the non-negative input design control scheme.

This research work has resulted in publication of two conference papers on first and second control scheme in ICARCV'08 and ICMAT 2009 conferences respectively. The third control scheme is considered to be published as a journal paper in near future.

1.4 Organization of the Thesis

There are six chapters in this thesis, and they are organized as follows.

Chapter one introduces the background, objectives and contributions of the research.

In Chapter two, the literature review on MEMS control is given. Also device modeling, "Pull-In" point analysis, stability analysis, simulation results and performance

requirements of the existing control methods are discussed. The advantages and drawbacks of the existing control method are also discussed. It is comprehensible that the Closed-Loop Feedback Control method is the best method to avoid the “Pull-In” instability of MEMS electrostatic parallel-plate microactuators and it is the main reason that persuades us to use this method to overcome the “Pull-In” phenomenon. In this chapter, the detailed discussion is given to the method proposed in [2]. From the simulation results, the nonlinear control method proposed [2] causes extra ordinary fluctuations which is due to the control signal function as an input of device. This drawback makes the nonlinear control scheme proposed in [2] difficult to be implemented.

In the next three chapters, the novel closed-loop feedback control schemes are introduced to propose implementable methods for solving the “Pull-In” instability problem. In this research work, system stability is studied; hence the controller is designed for system steady state. As we know, an actuator has a desired movement which is given as a command to the actuator input. The desired output, movement, is acquired when the actuator can follow input command as much as possible. For nonlinear micromachined electrostatic parallel plate microactuator, the input command is the applied voltage between two plates and the output is the plate deflection. Hence the tracking error which is defined by subtraction of the output from the input command, needs to goes to zero when time goes to infinity.

In Chapter three, the auxiliary controller design control scheme is introduced to eliminate extra ordinary fluctuations from the nonlinear control scheme proposed in [2]. The proposed control scheme consists of two parts: an auxiliary controller and the nonlinear feedback control law in [2]. The auxiliary controller design control scheme uses a two-position switch which in each position chooses one of two controllers above according to a threshold value. The auxiliary controller is designed for the auxiliary model which is similar to the MEMS device modified model according to state feedback method. Then a mathematical proof is given to prove it works for the MEMS device modified model. This technique eliminates extra ordinary fluctuations problem due to the nonlinear control scheme proposed in [2]. But on the other hand this scheme has two major drawbacks: overshoot problem and usage limitation of this scheme, for a special range of MEMS device parameter. Also the simulation results show that the stable and controllable whole gap displacement and

output tracking is achieved without any extra ordinary fluctuations but in spite of overshoot in some cases.

In Chapter four, a new control scheme called non-negative input design is proposed. This control method is based on state feedback method. It is designed for the MEMS device modified model and the system performance is improved in comparison with the auxiliary controller design control scheme by eliminating overshoot problem from the closed-loop system. But the usage limitation problem still remains for a special range of MEMS device parameters. In this scheme, one of the state variables from modified state space model of the MEMS device is used as new control input to shape other state variables as desired. The simulation results show that the stable and controllable whole gap displacement and output tracking are achieved without any extra ordinary fluctuations and overshoot.

To use the control scheme for a wider range of nonlinear micromachined electrostatic parallel-plate microactuators, a new control scheme called positive state variable design is described in Chapter five. It is designed for the MEMS device modified model and uses direct state feedback method. In this scheme, a positive state variable is substituted instead of a state variable of the MEMS device modified model. Then the modified model is transformed to a linear state space model. Hence the state feedback method can be applied for controller design. Moreover mathematical proof is done to show that this scheme works for a wider range of MEMS devices in comparison with the former control schemes.

In Chapter six, conclusions and recommendations for future work are provided.

Chapter2. Literature Review

2.1 RF MEMS Switches

The schematic of two kinds of RF switches which are under studies in this research work are shown in Figure 2.1. The one in Figure 2.1(a) is the capacitive shunt switch and the one in Figure 2.1(b) is the DC contact series switch.

Figure 2.1. Schematic of MEMS RF switches. (a) Capacitive shunt switch; (b) DC contact series switch

In a specific frequency domain ($f \ll f_0$), they act as a variable capacitor which has a fixed bottom plate and a top movable plate. When a voltage is applied to the device the top plate moves down toward the bottom plate and when the voltage is gotten back from device the top plate moves back. Actually these are on-off switches and in a specific voltage, the top plate sticks to the bottom one and the switch is on, otherwise the switch is off.

From Figures 2.2 and 2.3, it is seen that the microactuator can be modeled as a parallel plate capacitor consisting of a movable top plate and a fixed bottom plate as shown in Figure 2.4. Assuming stiff electrodes and ignoring fringing fields [4], the capacitance between the electrodes is approximated as $C = \epsilon A/g$ where ϵ is the absolute permittivity of the material between the electrodes, A is the area of the capacitive plates and g is the distance between the electrodes.

Figure 2.2. Off-On states of the capacitive shunt switch

Figure 2.3. Off-On state of the DC contact series switch

When a voltage, V , is placed between the plates, the free electrode is subject to an electrostatic attractive force, neglecting fringing effects [1], approximated by $f_e = \epsilon AV^2/2g^2$ [23]. This voltage can be produced by a voltage source drive or a current source drive. Electrostatic force results from the attraction between unlike charges when a voltage is placed between two electrodes [3]. The nonlinearity of the electrostatic force and its dependency on the remaining gap is clear [5].

Also the mechanical part of the microactuator is modeled as a spring-mass-damper system [23], as shown in Figure 2.4.

The governing equation for the 1-D model according to the Newton's second law is:

$$m\ddot{x} + b\dot{x} + kx = f_e \quad (2.1)$$

where k is the mechanical stiffness, b is the damping factor; m is equivalent mass of the movable plate and x is its deflection.

Figure 2.4. 1- Dimensional model of MEMS electrostatic microactuator

The parameter g_0 is the zero-bias capacitive gap or undeflected gap size which is the maximum gap size. It is known that the actuator charge on the plates, Q , is dependent on the voltage between the electrodes with $Q = CV$. Hence the electrostatic force, f_e , can be written according to the capacitor charge as $f_e = Q^2/2\epsilon A$. The basic one-dimensional (1-D) lumped model, for a parallel plate microactuator, shows how the nonlinear electrostatic force, $f_e = \epsilon AV^2/2g^2$, works against the elastic restoring force, $f_m = kx$, (a linear spring) in steady-state [5]. However the important electric magnitude, controlling the balance of forces, is the electric charge. This charge, Q , can be provided by an electric source, traditionally by a voltage source or recently by a pulse current source [12]. In many electrostatic actuators, the positions of electrodes are controlled by a balance between an electrostatic attractive force and a mechanical restoring force [1].

2.2 Design and Analysis of Microswitch System Stabilization

Digital applications as microswitches or microrelays [10] are generally driven by a single voltage source directly plugged to the electrodes [16]. This is called static open-loop voltage control scheme [2].

Here [4] presents a simple method proposed for stabilization using nothing more than the addition of series capacitance in the DC path of the MEMS Bridge circuit. A capacitor, C_f in series with the device, as shown in Figure 2.5 ([4]), provides stabilizing, negative feedback. If the free electrode begins to snap, the capacitance of the device drops. Since the circuit is a voltage divider, the increase in capacitance of the device results in a decrease in voltage across it. This in turn, causes the electrodes to separate, counteracting the original motion.

Figure 2.5. Device with capacitor feedback

The feedback capacitor stabilizes the device by passively controlling the voltage across the electrodes as a function of the gap size. In this circuit, the voltage across the device is as follows.

$$V = \frac{V_s}{1 + \frac{C}{C_f}} = \frac{V_s}{1 + K \frac{x_0}{x}} \quad (2.2)$$

where $K = C_0/C_f$, $C_0 = \epsilon A/x_0$ (where ϵ is the absolute permittivity of the material between the electrodes, A is the area of the capacitive plates), x_0 is zero-voltage or maximum gap of the device and x is the capacitive gap. The electrostatic force as a function of supply voltage V_s is.

$$f_e = \frac{\epsilon A V_s^2}{2(x + Kx_0)^2} \quad (2.3)$$

For $K = 2$, two circuits shown in Figure 2.6 are considered and it can be shown that they are equivalent. The governing equations on circuit (a) are described as follows.

$$V = \frac{x}{x + 2x_0} V_s \quad (2.4)$$

$$-m\ddot{x} - b\dot{x} + k(x_0 - x) = \frac{\epsilon A V^2}{2x^2} \quad (2.5)$$

Substituting Eq (2.4) in Eq (2.5) gives the complete dynamic equation of circuit (a) by:

$$-m\ddot{x} - b\dot{x} + k(x_0 - x) = \frac{\epsilon A V_s^2}{2(x + 2x_0)^2} \quad (2.6)$$

By defining $u = x + 2x_0$, Eq (2.6) is rewritten according to u as follows:

$$-m\ddot{u} - b\dot{u} + k(3x_0 - u) = \frac{\varepsilon AV^2}{2u^2} \quad (2.7)$$

Figure 2.6. (a) Device with capacitor feedback and (b) Device with extending gap

Eq (2.7) is the same with dynamic equation for the circuit (b), so these two circuits are equivalent.

In order to achieve stable deflections within the gap, a primary idea [19] has been introduced before series capacitor method as follows. The electrostatic “Pull-In” effect means that gap sizes or sacrificial layers would need to be about three times as thick as the target travel distance if a standard parallel-plate, linear restoring force actuator is used [1]. Figure 2.7(a) from [19] shows a schematic of a conventional electrostatically actuated micromechanical actuator. The movable electrode moves under voltage control up to one-third of the initial gap. If actuated beyond that threshold, the movable electrode snaps down onto the bottom electrode. If the goal is to achieve a travel range of g_0 , then the initial gap can be extended to $3g_0$, as shown in Figure 2.7(b) from [19], to have a stable region of up to g_0 .

The series capacitor is a modified method, introduced in [19] instead of using extended initial gap. In fact simpler method to achieve a travel range of g_0 is to use a capacitance in series with the actuator [1]. Because noting to Figure 2.8(a) configuration from [19], is actually two capacitors in series, the additional $2g_0$ gap can be replaced with an equivalent series capacitor as shown in Figure 2.8(b) from [19]. The electric field configuration is maintained; hence, the movable plate can traverse the entire g_0 gap stably.

Figure 2.7. Electrostatic micromechanical actuator (a) Conventional actuator with limited Range of travel (b) Actuator with extended gap and, hence, extended range of travel.

Figure 2.8. Electrostatic micromechanical actuator; (a) Actuator with extended gap and, hence Extended range of travel (b) Actuator with series capacitor that is equivalent to design in (a)

The voltage required to actuate the movable plate increases with the addition of the series capacitor because the total initial effective electrical gap is three times as large as the original. Since actuation voltages scales to the $3/2$ power of the gap, the voltages increase by at least a factor of five. Since the additional gap must be at least twice the original gap for

stable full-gap travel, the equivalent series capacitance must be less than one-half the initial capacitance of the original actuator.

The series feedback capacitance method [4], [26, 27] employs a capacitor in series with the electrostatic actuator to extend the effective electrical gap of the actuator and to provide stabilizing negative feedback. That way, the movable element can travel up to one-third of the new effective gap, which is larger than the entire initial gap of the original actuator.

The original actuator and series capacitor form a voltage divider circuit that provides negative feedback to stabilize the system. As the movable electrode approaches the fixed electrode, the capacitance of the actuator increases, thus decreasing the fraction V_0 of the total applied voltage V_a , which is imposed across the actuator according to:

$$V_0 = \frac{C_1}{C_0 + C_1} V_a \quad (2.8)$$

where C_0 is the variable capacitance of the original actuator and C_1 is the series capacitance. When the movable electrode goes beyond one-third of the original gap, the rapid increase in capacitance provides the negative feedback necessary to stabilize the actuator so it can traverse the entire gap stably. Initially, V_0 increases with V_a , with the rate of increase decreasing as C_0 increases, as indicated by (2.8). Beyond one-third of the initial gap where V_0 equals to the original V_{PI} , V_0 actually starts to decrease since C_0 increases rapidly in this region, thus providing negative feedback. As the movable plate approaches the fixed plate, the capacitance C_0 goes toward infinity and V_0 goes toward zero. The electrostatic force remains finite, however, since the electrostatic force is inversely proportional to the square of the gap, which also goes toward zero. At all times, the mechanical and electrostatic forces are balanced and the system is stable.

2.2.1. Parasitics from Layout

The cross section of a typical electrostatically actuated device fabricated in MUMPs is shown in Figure 2.9(a) from [19]. C_0 and C_1 are the intrinsic or desired device capacitances, whereas C_{P1} , C_{P2} and C_{P3} are parasitics. C_{P2} and C_{P3} are typically large because the dielectric layer is electrically thin. Depending on whether the substrate is left floating or grounded, the parasitic capacitances can be in parallel with either C_0 or C_1 . The configuration of Figure 2.8(b)

can be generalized to the circuit of Figure 2.9(b) from [19] which includes parasitic capacitances.

(a)

(b)

Figure 2.9. (a) Cross section of a typical electrostatically actuated device designed for MUMPs, Large Parasitic capacitances are connected to the pad. (b) Circuit of actuator with series capacitor, Augmented by parasitic capacitances in parallel and in series with the variable capacitor

Here C_1 , includes parasitics formerly in parallel with the desired series capacitance. C_3 is not important to static behavior since it is driven directly by the voltage source, but it will affect dynamics. The expressions describing the static behavior of the actuator in the presence of a series feedback capacitor and parasitic capacitors can now be derived. Let

$$C_0 \propto \frac{1}{g_0 - u}, \quad C_1 \propto \frac{1}{ng_0}, \quad C_2 \propto \frac{1}{mg_0} \quad (2.9)$$

where g_0 is the desired travel range, u is displacement, and m and n are positive constants. The voltage across C_0 is determined by the capacitive voltage divider in Figure 2.9(b) to be

$$V_0 = \frac{C_1}{C_0 + C_1 + C_2} V_a = \frac{m(g_0 - u)}{(m + n)(g_0 - u) + mn g_0} V_a \quad (2.10)$$

The sum of the electrostatic and mechanical forces gives the total equilibrium force on the movable electrode

$$F_t = -ku + \frac{\varepsilon A}{2(g_0 - u)^2} \left[\frac{m(g_0 - u)}{(m + n)(g_0 - u) + mn g_0} V_a \right]^2 = 0 \quad (2.11)$$

Differentiating Eq (2.11) with respect to u to determine the point at which the equilibrium solution becomes unstable gives

$$u_{max} = \frac{g_0}{3} \frac{m + n + mn}{m + n} \quad (2.12)$$

As the maximum stable displacement of the movable electrode as a function of m and n . In the limit as $m \rightarrow +\infty$ (no parasitic capacitor in parallel with C_0)

$$u_{max} \rightarrow \frac{g_0}{3} (1 + n) \quad (2.13)$$

Implying that n should be larger than two for full gap travel ($u_{max} \rightarrow g_0$), as noted previously. As $n \rightarrow +\infty$ (infinitely small series capacitor for maximum feedback and stability)

$$u_{max} \rightarrow \frac{g_0}{3} (1 + m) \quad (2.14)$$

Implying C_2 that must be no larger than $C_0/2$ if full gap travel is to be achieved. Thus, the electrostatic positioner must have well-controlled capacitances and parasitics.

2.2.2. Parasitics from Deformation

Another source of “parasitics” arises from the deformation of a beam in two dimensions. When the beam in Figure 2.9 deforms, the displacement of the center portion is largest whereas the portions near the step-up supports hardly move at all. This two-dimensional (2-D) non-uniform displacement is a subtle, but significant source of parasitic capacitance in parallel with the intrinsic device. The 2-D beam/capacitor can be modeled as the sum of two one-dimensional (1-D) capacitances—a variable capacitor in parallel with a fixed capacitor, as shown in Figure 2.10 from [19]. The total capacitance can be expressed as:

$$C = C_0 + C_2 \propto \frac{1-q}{g_0-u} + \frac{q}{g_0} \quad (2.15)$$

where C_0 and C_2 represent the same elements as in Figure 2.9(b) and q is a proper fraction that increases as u , the center displacement, increases. The larger the value of q causes the more significant effect of the parasitic fixed capacitance.

Figure 2.10. Ideal 2-D beam with non-uniform displacement
And its two-lump equivalent (variable + fixed capacitor)

Figure 2.11(a) from [19] shows the normalized simulated capacitances of several 2-D, 400- μm long, electrostatically actuated beams. These coupled electromechanical simulations were performed in Matlab [28]. The beams are 2- μm thick and are suspended 2- μm above a ground plane—the nominal dimensions of a MUMPS device. The 2-D capacitances are normalized to the capacitance of an ideal 1-D parallel-plate capacitor, whose gap is equal to the distance between the center of the 2-D beam and ground plane. As shown, the capacitances of the 2-D devices are only fractions of the 1-D device as the center of the beam approaches the ground plane. If the length of the fixed bottom electrode under the 2-D beam is reduced, as a percentage of the beam length, the 2-D device approaches 1-D-like behavior because the deformation is more uniform over the more limited center region. q is computed from the capacitance-displacement curves of Figure 2.11(a) using Eq (2.15) and plotted in Figure 2.11(b) from [19] as a function of center displacement u , and parameterized by bottom electrode length. The shorter the bottom electrode results the more 1-D-like the behavior of the system and hence the smaller the value of q . q increases as the displacement increases because the diminishing gap amplifies the lack of flatness of the deformed beam. According to Eq (2.9), the ratio of the variable capacitance to the fixed capacitance

C_0/C_2 is $mg_0/(g_0 - u)$. Equating this with the same ratio obtained from Eq (2.15) gives m in terms of q .

$$m = \frac{1 - q}{q} \quad (2.16)$$

Substituting in Eq (2.14) gives:

$$u_{max} = \frac{g_0}{3q} \quad (2.17)$$

Plotting $q = g_0/3u$ in Figure 2.11(b) gives the maximum q allowed for the desired range of motion u . The displacement at the intersection of this line with the previous $q - u$ curves indicates the maximum achievable stable travel. Beyond that q is too large, i.e., the parasitic is too large for effective stabilization. This assumes that the mechanical restoring force as used in Eq (2.11) is still linear with displacement, which is not true in real life due to stress-stiffening effects. Nonlinear stress stiffening actually increases the range of stable travel, even without capacitive stabilization, to about one-half of the initial gap, up from the one-third of the linear case. Thus, all the 2-D devices shown in Figure 2.11 are stable up to about 1- μm of displacement. Beyond that, capacitive feedback can stabilize the device until the increasing displacement causes q to increase to the limit indicated by the dotted line. For example, for the device with a bottom electrode that is 30% of the upper electrode length, capacitive feedback will allow stable travel up to 1.8- μm or 90% of the 2- μm gap. This, according to Eq (2.14), assumes an infinitely small series feedback capacitor—a larger capacitor will reduce the stable travel range. The parasitics due to the non-uniform deformation of cantilevers is even more severe than for fixed–fixed beams. It is clear that the 2-D-like behavior must be avoided.

Figure 2.11. (a) Capacitance of 400- μm -long beams as a function of displacement; normalized to the Capacitance of a 1-D device with displacement equal to the displacement of the beam center. The Labels indicate the length of the bottom actuating electrode as a fraction of the upper beam length. (b) q -fraction as a function of displacement, computed from the capacitances in (a). Capacitive Feedback is ineffective for displacements with q -fractions above the dotted line.

Another idea described in [3] examines methods of increasing the electrodes travel distance in an electrostatically actuated micromirror can travel before “snap-through”. Here the theoretical basis of the “snap-through” phenomenon is presented and the calculations for the deflection of electrostatically actuated micromirrors with nonlinear flexures are shown. This work describes how nonlinear force flexures can be used to increase the stable deflection distance of an electrostatically-operated micromirror. It uses a multiphase piecewise-linear mechanical flexure to extend travel distance [1]. Also specific nonlinear flexure designs are presented and evaluated in following.

The micromirror is driven by electrostatic actuation. A traditional micromirror is an electrostatic device with linear force flexures. Traditional micromirrors have flexures that provide a restorative mechanical force as a linear function of deflection. After a traditional micromirror has deflected one-third of the initial separation distance between the top and bottom electrode, the mirror's position becomes unstable, and the mirror quickly jumps down to the bottom electrode. In the micromirror shown in Figure 2.12 from [3], the top electrode is the center region of the micromirror supported by the flexures. The bottom electrode is the substrate. The micromirror's bottom electrode is isolated from the top electrode and flexures.

Figure 2.12. An electrostatically actuated piston micromirror with linear force flexures

Eq (2.18) can be used to calculate electrostatic force, F_e , between two parallel plates [29]:

$$F_e = \frac{\epsilon_0 A V^2}{2D^2} \quad (2.18)$$

where A is the common surface area between the plates, D is the separation distance, ϵ_0 is the permittivity of free space, and V is the voltage potential between the plates. Eq (2.18) does not consider the effects of fringing losses or plate surface deformations resulting from the electric field. References [31] and [32] demonstrated that a good model can be developed even if these effects are neglected [30,31]. The flexures for the micromirror pictured in Figure 2.12 can be modeled as cantilevers. For small deflections, a cantilever's force is given by Hooke's law [32]:

$$F_c = kd \quad (2.19)$$

where k is the spring constant, d is the deflection distance, F_c and is the cantilever's force. The flexure spring constant can be calculated by [30] and [33]:

$$k = \left[\frac{Ewt^3}{L^3} + \frac{\sigma(1-\nu)wt}{2L} \right] \quad (2.20)$$

where E is the modulus of elasticity; w is the flexure width, L is the flexure length, t is the flexure thickness, σ is the flexure material stress, and ν is the flexure material Poisson's ratio. The spring constant is determined during design and construction; it is not a function of voltage or deflection.

A nonlinear second order flexure has a restorative force that is proportional to the square of the deflection distance. If the flexure can be designed to provide a higher order restorative force, such as:

$$F_c = kd^r \quad (2.21)$$

where r is order of the flexure's force (i.e. $r = 2$ is a quadratic force flexure). For nonlinear flexures (r not equal to one), the value of the spring constant, k , may not be determined by Eq (2.20); the flexure may not be a simple uniform cross-section cantilever.

Two nonlinear flexure designs are shown in Figure 2.13 from [3]. The truck suspension relies on a main spring for support under most load and driving conditions. If a heavy load or excessively bumpy road is encountered, the axle will be depressed upward so that the auxiliary spring contacts the bearing plates. The auxiliary spring increases the system spring constant, and provides additional restorative force to counter the axle's upward motion [34]. This suspension concept applied to micromirrors is called “auxiliary flexure design”. Another nonlinear flexure design is illustrated by the mattress spring shown in Figure 2.13(b). The mattress spring has two different spring constants, k_1 and k_2 . The spring constant for the outer portion of the spring, k_1 , is substantially less than the spring constant for the inner portion of the spring, k_2 . As the spring is depressed by the weight of the sleeper, the outer spring regions depress first. If additional weight is placed on the spring, the inner portion of the spring begins to compress [35]. Thus the single spring has two different spring constants, and the system's spring constant depends on the distance the spring is depressed. Although micromirror flexures designed with dual spring constants are mechanically quite different than the spring shown in Figure 2.13(b), the micromirror's spring constant is

functionally similar. Micromirrors flexures with two spring constants are called “dual spring constant flexure designs”.

Figure 2.13. Examples of nonlinear flexures

The maximum stable deflection distance for a micromirror can be increased using auxiliary flexures. Figure 2.14 from [3] shows two electrostatically actuated piston micromirrors similar to the micromirror in Figure 2.12 except for flexure modifications. The initial separation between the top electrode and the bottom electrode is $2\mu\text{m}$ for both micromirrors. The micromirror depicted in Figure 2.14(a) uses a conventional flexure at each corner of the micromirror. This flexure is always engaged to resist electrostatic force, and is called the main flexure. In addition, an auxiliary flexure passes under two main flexure overhangs connected to the micromirror's top electrode. These overhangs are similar in function to the bearing plates in Figure 2.13(a). After the micromirror in Figure 2.14(a) is deflected 666.6 nm, it snaps down 84 nm to rest on the auxiliary flexure (total deflection 750 nm). The micromirror can then be deflected an additional 475 nm before “snap-through”. The micromirror in Figure 2.14(a) has a stable deflection region that is 71 % larger than the micromirror in Figure 2.12.

A schematic diagram of the micromirror flexure used in Figure 2.14(b) is presented in Figure 2.15 from [3]. When a voltage is first applied to the micromirror in Figure 2.14(b) the main flexure bends until the auxiliary flexure rests on the auxiliary support post. The micromirror does not exhibit “snap-through” on the main flexure because the auxiliary

flexure adds support before the micromirror is displaced two-thirds of a micron. The tip of the auxiliary flexure contacts the auxiliary support post before “snap-through” because the auxiliary flexure is angled down as a result of the main flexure bending to accommodate electrostatic force. As the applied voltage increases, both the main and auxiliary flexures continue to resist the electrostatic force, until “snap-through” is reached at a displacement of 950 nm. The micromirror in Figure 2.14(b) is limited by “snap-through” because the combined flexure resistance is still a linear function of deflection (after the auxiliary flexure rests on the auxiliary support post). However, the use of auxiliary flexures increased the maximum stable deflection distance by 50%.

Figure 2.14. Electrostatically actuated micromirrors with auxiliary flexures

Figure 2.15. Cross-sectional schematic diagram of a dual support post auxiliary flexure micromirror

Unlike auxiliary flexure designs, dual spring constant flexure designs rely on a single

flexure that provides two or more different spring constants depending on the deflection distance. Ideally the spring force will vary as a nonlinear function of deflection.

In this design, the gratings use dual spring constant flexures instead of using linear force flexures (single spring constant). The slats in linear force flexure design exhibit “snap-through” after the edge of the slat travels one-third of the initial separation distance. The dual spring constant flexure is used to increase the maximum stable deflection distance for the slat's edge as it is tilted towards the substrate. Figure 2.16 from [3] is a schematic diagram of both grating designs; linear force and dual spring constant flexures. Both grating designs use the substrate as the bottom electrode. The slat in Figure 2.16(a) is supported by a linear force torsion flexure. After the edge of the slat opposite the flexure travels approximately two-thirds of a micron, it snaps down to the substrate.

When a voltage is placed between the dual spring constant flexure slat and the substrate, electrostatic force pulls the edge of the slat opposite the flexure towards the substrate. If the voltage is increased, the slat will snap down to rest on the top of the support post. Higher voltages result in the slat rotating on top of the support post, until the edge of the slat eventually rests on the substrate. The slat does not exhibit “snap-through” until the edge of the slat is less than 300 nm above the substrate. The slat flexure exhibits two distinct spring constants: one value before the slat rests on the support post, and a higher value after the slat rests on the support post. The second spring constant is caused by changing the direction and amount of force on the flexure. The flexure bends to counter the electrostatic force before the slat rests on the support post. After the slat rests on the support post, the flexure continues to bend and the edge of the flexure opposite the support post is lifted up.

Figure 2.16. Schematic diagrams of grating designs with single and dual spring constant flexures

The nonlinear force flexure designs can be combined to further increase the stable deflection region. Figure 2.17 from [3] is a picture of a micromirror with double auxiliary flexures. Figure 2.18 from [3] is a cross-sectional view of the dual auxiliary flexure used in Figure 2.17. After the micromirror in Fig.2.17 has been deflected approximately 500 nm, the end of the first auxiliary flexure will rest on the substrate. If the voltage is increased, the first auxiliary flexure will bend until the end of the second auxiliary flexure touches the silicon nitride layer. The second auxiliary flexure will then add support to both the main and first auxiliary flexures.

Figure 2.17. Micromirror with dual auxiliary flexures

Figure 2.18. Cross-sectional schematic diagram of a partially deflected dual auxiliary flexure

In following, a new method is explained and it is chosen to investigate the detailed design of the simplest beam actuator in the hopes of identifying ways to improve the travel range [1]. Two good approaches are identified here, called “leveraged bending” and “strain stiffening”. The leveraged bending and strain-stiffening methods are examined for extending the stable travel range of electrostatic actuators beyond the $1/3$ of the gap due to the “Pull-In” instability limitation for elastically suspended parallel-plate electrostatic actuators. The leveraged bending effect can be used to achieve full gap travel at the cost of increased

actuation voltage. The strain-stiffening effect can be used to minimize actuation voltage for a given travel range. In this work, the use of a “leveraged bending” approach [36] is described to design an electrostatic actuator that can achieve complete gap travel without the “Pull-In” instability. The actuator is used to position mirrors in an analog-tuned reflective diffraction grating known as the polychromator [37-39]. The polychromator is a specific example of analog positioning using electrostatic actuators. The polychromator diffraction grating consists of an array of flat, closely packed rectangular mirrors. Each of these mirrors can be individually positioned vertically in an analog fashion using electrostatic actuation. Surface micromachining was chosen as the fabrication technique for the polychromator grating because it combines the ability to build the array of closely spaced moveable mirrors with the ability to build multilayer electrode and ground interconnects beneath the mirror array. Polychromator design and implementation is a case-study example of the leveraged bending technique. In addition, the use of a nonlinear mechanical strain-stiffening effect to increase the stable travel range of electrostatic actuators is investigated and it is seen that using the polychromator design as an example, the strain-stiffened actuator designs can be optimal for achieving a given travels distance while minimizing actuation voltage.

The idea behind the leveraged bending method is to work around the “Pull-In” instability by applying electrostatic force to only a portion of a structure, then using the rest of the structure as a “lever” to position specific parts of the structure through a large range of motion as shown in Figure 2.19 from [1]. “Leveraged bending” [36] is a simple method and requires the electrostatic force be applied between the fulcrum (typically the anchor) and the segment of the beam that should be deflected [19].

The key is that the electrostatically actuated portions of the structure deflect less than the “Pull-In” limit, while other portions of the structure can move through the entire gap. Figure 2.19(a) shows the leveraged bending technique applied to a simple cantilever. Instead of putting an electrode under the entire beam, a smaller electrode is placed under the beam close to the beam’s support. In this case, the tip of the beam can move the entire gap distance, even if the portion of the beam that is being electrostatically actuated moves a third of the gap or less. The same principle can again be applied to a doubly clamped fixed–fixed beam, shown in Figure 2.19(b), by placing electrodes close to the beam supports and using the characteristic bent shape of the beam to move the center of the beam the entire gap distance

before the “Pull-In” instability occurs over the electrode. The leverage in this case relies on the position of the inflection point being located somewhat beyond the end of the actuating electrode.

A second way to increase the controllable travel range of deformable fixed–fixed beams is to take advantage of the “strain-stiffening” effect. As the beam deflects; the tensile stress in the beam increases due to increasing strain in the beam. This increase in stiffness (a form of geometric nonlinearity) tends to reduce the positive feedback that leads to the “Pull-In” instability, and thus extends the range of stable travel. As with leveraged bending, the effectiveness of the strain- stiffening effect depends on the residual stress in the beam. If a beam is tension-dominated due to residual stress, the strain-stiffening effect is reduced since the increase in stress due to deflection is small compared to the initial residual stress.

Figure 2.19. Using leveraged bending to extend the stable travel range of an Electrostatic actuator, Simple examples of leveraged bending applied To (a) cantilevers and (b) doubly clamped, fixed–fixed beams

Then a new driving mode of electrostatic actuators has been examined [5]. It is based on charge drive rather than on voltage drive. Charge drive is accomplished by a pulsed current source delivering a given amount of charge to the actuator. The alternative proposed in this work consists on a different type of drive, a pulse-current drive, in such a way that the charge delivered to the actuator becomes independent of the remaining gap, and then the nonlinearity is avoided. In this work, this new driving mode is analyzed theoretically, conceptually introduced in [20], that consists in replacing the conventional voltage source by

a pulsed current source (then becoming a charge source). A theoretical analysis is carried out for the simplest case of an ideal charge source.

It is assumed that a simplified one-dimensional geometry of an electrostatic actuator, such as depicted in Figure 2.20 from [5], consisting of a parallel plate capacitor in which the upper electrode, of mass m , can move attracted by an electrostatic force working against a linear restoring force, represented by an elastic constant k , in a medium represented by a linear viscous damping coefficient b . In contrast to a conventional voltage drive, where a constant voltage source is connected to the parallel plate actuator, the drive is performed by means of switched current source (SI), $I_s(T)$ assumed ideal. The following electromechanical and circuit equations can be written as follows.

$$m \frac{d^2 X}{dT^2} + b \frac{dX}{dT} + kX = \frac{Q^2}{2\epsilon A} \quad (2.22)$$

$$\frac{dQ}{dT} = I_s(T) \quad (2.23)$$

For sake of generality of the results, Eqs (2.22) and (2.23) can be normalized by defining the following normalized variables [20].

$$x = \frac{X}{G_0}, \quad t = \omega_0 T, \quad q = \frac{Q}{Q_p} \quad (2.24)$$

$$Q_p = \frac{3}{2} C_0 V_{PI}, \quad V_{PI} = \sqrt{\frac{8kG_0^2}{27C_0}}, \quad C_0 = \frac{\epsilon A}{G_0}, \quad \omega_0 = \sqrt{\frac{k}{m}} \quad (2.25)$$

Then Eqs (2.22) and (2.23) become as follows.

$$\frac{d^2 x}{dt^2} + 2\alpha \frac{dx}{dt} + x = \frac{q^2}{3} \quad (2.26)$$

$$\frac{dq}{dt} = \frac{2}{3} i_s(t) \quad (2.27)$$

with

$$\alpha = \frac{b}{2m\omega_0}, \quad i_s = \frac{I_s}{V_{PI}\omega_0 C_0}, \quad t_s = \omega_0 T_s \quad (2.28)$$

Figure 2.20. Switched current source and 1-D actuator model

The actuator deflection is set by the amount of charge injected by the current source. As shown in Figure 2.20, this is accomplished by a current-pulse of amplitude I_s and length T_s . The key idea is that the independent control of the charge that is delivered to the actuator, by adjusting the parameters of a current source, makes the electrostatic force independent of the remaining gap of the actuator, which was the origin of the nonlinearity and of the instability. If Eq (2.27) is modified to make explicit the electrical charges, after time integration, it is obtained as

$$q = \frac{2}{3} \int_0^t i_s(\tau) d\tau \quad (2.29)$$

The parasitic capacitances, playing a key role in drive circuits, are particularly important in the charge drive mode. One of the main hypotheses underlying the previous analysis concerns the ideality of the current source used in the charge drive. The non-ideality comes from both parasitic capacitive and charge leakage effects not considered in the schematic circuit in Figure 2.20 and it is most important and detrimental effect which can be large and can't be avoided in microelectromechanical devices because of the geometry and layout of structural layers, gaps or contacts. A simple, but more realistic equivalent circuit is depicted in Figure 2.21 from [5] where a lumped parasitic capacitor C_p is connected in parallel with the current source to take into account all parasitic capacitances connected to the same node, either lumped or distributed. The equations governing the dynamics of the parallel plate actuator including the capacitive term as:

$$m \frac{d^2 X}{dT^2} + b \frac{dX}{dT} + kX = \frac{Q^2}{2\epsilon A} \quad (2.30)$$

$$\frac{dQ}{dT} + C_P \frac{dV}{dT} = I_s(T) \quad (2.31)$$

Figure 2.21. Switched current source circuit including parasitic capacitances

After normalization and time integration of Eq (2.31), both Eqs (2.30) and (2.31) can be obtained as

$$\frac{d^2x}{dt^2} + 2\alpha \frac{dx}{dt} + x = \frac{q^2}{3} \quad (2.32)$$

$$q + \sigma q(1 - x) = \frac{2}{3} \int_0^t i_s(\tau) d\tau \quad (2.33)$$

where:

$$\sigma = \frac{C_P}{C_0} \quad (2.34)$$

Eq (2.34) is the parasitic capacitance to the initial gap capacitance ratio. The analysis of the parasitic capacitance model is the following: the parasitic capacitance is first charged at the same time as the actuator itself that is during the current-pulse; the charge, so stored at the parasitic capacitance, is afterwards transferred to the actuator, once the current-pulse of the drive has come to an end. In fact, the parasitic capacitance acts as a parasitic voltage source and this voltage drive mechanism replaces the charge drive mechanism after the source charge pulse has ended. In fact the parasitic capacitances have also to be charged and discharged during the transients. Switched current sources have been used in a large variety of analog and digital circuits [40, 41].

Of course, some sort of leakage will always be there having the effect of discharging in both the actuator and the parasitic capacitances through these leakage paths. As the charge is lost, the position reached by the actuator will also drift towards the initial position. Figure 2.22 from [5] shows the normalized deflection transients for $\sigma = 1$, $\alpha = 1$, $q_0 = (4/3)\sqrt{2}$

and three values of the normalized leakage current i_{LK} , running from zero to 10^{-5} . If the discharge time is long enough, the mobile electrode will return to the initial position for any non-zero value of i_{LK} . In some sense this is similar to the situation in dynamic RAM memories, where the information stored at a given time in a memory position has to be periodically refreshed at a given frequency in order to restore the charge losses at the capacitors.

Leakage currents in state of the art MOS and CMOS devices are of the order of Pico amperes [42].

In order to analyze quantitatively in normalized variables, the effect of the charge leakage, an analysis of the drift has been performed in the total charge stored at equilibrium in the system and estimate the minimum refresh frequency needed.

Figure 2.22. Transients of normalized deflection for four Different values of the total normalized leakage current

Assuming that an initial equilibrium point has been reached at (q_0, x_0) , the total charge stored in the system at a given time after the initial charging can be written as:

$$q_{tot}(t) = q[1 + \sigma(1 - x)] = q_0[1 + \sigma(1 - x_0)] - \frac{2}{3} \int_0^{t_{dis}} i_{LK} dt \quad (2.35)$$

where t_{dis} is the discharge time elapsed after the charging pulse has ended. In relative and percentage, the charge lost can be written as:

$$\Delta q_{tot}(\%) = \frac{q_0[1 + \sigma(1 - x_0)] - q_{tot}(t)}{q_0[1 + \sigma(1 - x_0)]} \times 100 = \frac{2}{3} \frac{i_{LK} t_{dis}}{q_0[1 + \sigma(1 - x_0)]} \times 100 \quad (2.36)$$

Assuming that the normalized refresh frequency, f_{ref} , can be approximated as $1/t_{dis}$, Eq (2.36) becomes a simple relation that illustrates an expected result: for a given actuator and current-pulsed source circuitry, charge losses and its effects may be controlled by selecting an adequate i_{LK}/f_{ref} ratio.

At the end of the charge loss period, the actuator will reach a new stable position x . Then, a normalized deflection loss $\Delta x = (x_0 - x)/x_0$ may be defined. Due to the nonlinearities involved no closed analytical form relation between Δx and $i_{LK} t_{dis}$ can be found. In spite of this Δx can be easily evaluated by numerical means. As an example, plots of the normalized charge and deflection losses against the $i_{LK} t_{dis}$ product are shown in Figure 2.23 from [5], for $\sigma = 1$, $\alpha = 1$, and $q_0 = (4/3)\sqrt{2}$. Figure 2.23 contains useful information for an optimal refresh strategy design. As an example any value of the normalized product, $i_{LK} \times t_{dis}$, less than 2.5×10^{-4} leads to deflection losses less than 0.7%. That is for a leakage current of 2 pA, a minimum refresh frequency of 7.65 kHz is needed to keep deflection losses under 0.7%.

Figure 2.23. Normalized charge and deflection losses vs. normalized Leakage current $\times$ refresh period product

After introducing the charge drive method, in next work [12], two different approaches are introduced to control charge in the actuator by means of current driving.

Both approaches are based on the use of current pulses injecting the required amount of charge to fix the position of the movable plate. In these new approaches, a switched-current source is directly applied to a parallel plate circular microactuator and the injected current pulse defines the charge.

However, first method uses a series capacitance to sense the voltage drop produced by the injected charge [43]. Depending on the sensed voltage more charge is injected, switching on the driving current source, or removed, with the current sink. The second approach operates in open loop configuration [5]. The current source is switched off after injecting the required charge to fix the movable electrode to the desired position.

As pointed out above, the use of current sources instead of the conventional voltage source to drive electrostatic actuators is presented in this work. In this scenario, the voltage across the actuator structure, including parasitic and, eventually, a series capacitor, is not limited by the voltage source but free to evolve as imposed by the injected charge. In this configuration the current is limited by the driving current source. On the contrary, voltage drive operation limits the maximum voltage drop, but does not limit the total current demanded by the actuator as it moves. This duality in the operation of current and voltage drive mode explains the significant differences observed in the steady-state [5] and the transient or dynamic response [20].

Two different driving methods based on current sources driving the actuator are described in following, part A: a) a closed-loop current drive circuitry and b) an open-loop current drive circuitry. The methods are studied and compared with the series-feedback voltage drive, focusing on the stable range of travel and the steady-state voltage developed across the actuator. As stated in [20], the voltage across the actuator, in current drive mode, has a maximum value that it is not necessarily produced at the maximum stable displacement of the electrode. Moreover, this maximum value depends on the inertial and viscous damping terms in the dynamical state equations of the system and, typically, can be 30% higher than the “Pull-In” voltage for critical damping conditions [20]. For this reason, the value of the voltage at the maximum stable displacement point has been chosen as figure of merit to compare the three driving methods.

A. Voltage Drive with Feedback Capacitor

Let us remind here the results presented in [19], for the generic parallel plate geometry depicted in Figure 2.24 from [12], in which a series capacitor is used to stabilize positions after the “Pull-In” distance. The equations governing the dynamics of the parallel plate actuator can be written as follows.

$$m \frac{d^2 X}{dT^2} + b \frac{dX}{dT} + kX = \frac{\epsilon A V_0^2}{2(D_0 - X)^2} \quad (2.37)$$

The total applied voltage V_a is related to the actuator voltage V_0 from the capacitive voltage divider as,

$$V_0 = \frac{C_1}{C_0 + C_1 + C_2} V_a \quad (2.38)$$

Figure 2.24. (a) Cross section of the actuator and (b) equivalent Circuit for the voltage drives with series capacitor

B. Current Drive with Feedback Capacitor

A completely different scenario appears when the voltage source used to drive the actuator in case a) is replaced by a current source. This approach consists on setting up a charge–discharge cycle enabling and disabling in sequence the current source and the current sink [43]. This sequence is established depending on the relative value of the voltage drop across the series capacitor C_1 , (due to injected charge) and the voltage reference used at the input of the comparator, V_{ref} , as depicted in Figure 2.25 from [12]. The operation of the loop keeps the voltage across capacitor C_1 around V_{ref} and indirectly the value of V_{ref} sets the value of the displacement of the movable electrode. The synchronous operation of the control loop is given by a D Flip-Flop connected at the output of the comparator. The output of the flip-flop is the input signal used to control the operation of the current sources. With this configuration the stored charge in the actuator, Q_{total} , can be considered to be independent of the time in steady state.

As can be seen, the operation of this circuit is entirely similar to the operation of a $\sigma - \delta$ converter with the exception that the variables switched at the input are current sources of opposite signs. In fact this current method is a mixed charge–voltage control loop based on a current $\sigma - \delta$ topology. The price to pay is the ringing around the final position produced by the charging and discharging cycles. This ringing can be minimized using a higher clock frequency than the mechanical resonance frequency, and low charging and discharging currents.

Figure 2.25. Block diagram of the proposed current mode drive in closed-loop configuration

Considering one period of charging status, T , leads to calculate the governing equation on the actuator can be expressed as

$$Q = \int_0^t I_{charge}(\tau) d\tau \rightarrow Q_{total} = \int_0^T I_{charge}(\tau) d\tau \quad (2.39)$$

$$Q = C_1 V_1 + C_3 V_3 \quad (2.40)$$

$$C_1 V_1 = C_0 V_0 + C_2 V_0 \quad (2.41)$$

$$V_0 + V_1 = V_3 \quad (2.42)$$

According to Eqs (2.40), (2.41) and (2.42), the actuator voltage can be attained as follows.

$$V_0 = \frac{C_1}{C_3(C_2 + C_1 + C_0) + C_1(C_2 + C_0)} Q \quad (2.43)$$

C. Current Drive without Feedback Capacitor

A particular scenario, derived from case b), is the open loop current drive configuration schematically shown in Figure 2.26 from [12]. The stability and dynamics of the circuit have already been theoretically analyzed by the authors [5] concluding that the system is inherently stable if the parallel parasitic capacitors, C_2 and C_3 , are zero, and subjected to certain conditions if they are not. The proposed switched-current control method in open-loop configuration is based on charge injection by means of current pulses [5]. Controlling their amplitude and duration, a certain amount of charge is injected. The method uses two current sources, one for charging and another one for discharging. The amplitude and the duration of the current pulse define the desired position. First, a charging current pulse injects the designed charge to set the movable electrode in a certain position. After fixing the position, current sources are switched off. Any undesired movement due to the leakage currents, at the output of the current sources or at the actuator, can be counterbalanced resetting the charge periodically by means of a refresh cycle. Hence, the actuator is discharged with the current sink and charged again with the current source. C_1 will not be used and only the parasitic capacitance in parallel with the actuator will limit the maximum displacement.

Figure 2.26. Block diagram of the proposed current mode drive in open-loop configuration

In both, voltage and current drive circuits, a main engineering or design parameter is of special interest. This parameter is the “Pull-In” voltage, V_{PI} , which sets the minimum voltage value required by an actuator to pull in and hence sets the minimum voltage value of the drive.

In [16], the “Pull-In” voltage is calculated and then the results of δ -pulse current drive has been compared to the results achieved by the method using a voltage drive and a series feedback capacitor as described below. This design parameter has a simple analytical mathematical formulation for voltage drive only under simplifying assumptions [10] and it is still very useful as design parameter. In the case of current drive, the voltage across the actuator is not fixed by any voltage source but instead it is free to evolve as current is feed into the actuator. In [1], it was shown that when a constant current is feed into the actuator, the voltage developed across the actuator has a maximum sometime during the switching transient that is greater than the “Pull-In” voltage. Drive circuits of electrostatic actuators have to be able to either provide enough voltage (in voltage drive) to the actuator to operate, or to comply with the voltage developed across the actuator during current or charge drive. So the voltage required becomes a matter of interest. In fact one of the charge drive solutions proposed in [27] uses a capacitor in series with the actuator, needs a voltage several times the value of V_{PI} , hence having limited applicability. The first purpose of this work is to examine the voltage developed across the actuator when driving an actuator with a current drive and find a maximum value for this voltage which could be used as a design parameter.

First the concept of a δ -pulse current drive is introduced and it is used to calculate an analytical expression for the maximum voltage developed across the actuator. Also the validity of achieved results, when more realistic current pulses are used, is examined here.

If the one-dimensional (1-D) geometry of a two parallel plate electrostatic actuator shown in Figure 2.27 from [16] is considered, the moveable upper electrode is attracted to the fixed plate by an electrostatic force working against a linear restoring force, kX , in a medium represented by a linear viscous damping coefficient b . If an ideal current drive is considered first with no parasitic capacitance by setting $C_p = 0$ in Figure 2.27(a), the following normalized electromechanical and circuit differential equations governing the moving plate dynamics can be written as [5]

$$\frac{d^2x}{dt^2} + 2\alpha \frac{dx}{dt} + x = \frac{q^2}{3} \quad (2.44)$$

$$\frac{dq}{dt} = \frac{2}{3} i_s(t) \quad (2.45)$$

The normalized variable are

$$x = \frac{X}{G_0}, \quad t = \omega_0 T, \quad q = \frac{Q}{Q_p}, \quad v = \frac{V}{V_{PI}}, \quad i_s = \frac{I_s}{V_{PI}\omega_0 C_0}, \quad t_s = \omega_0 T_s \quad (2.46)$$

where X is the deflection, Q is the charge stored on the two-plate capacitor, T is the time, m is the mass of the moveable plate, b is the damping coefficient, k is the elastic constant, G_0 is the initial gap, $C_0 = \epsilon A/G_0$ is the capacitance at rest, and

$$Q_p = \frac{3}{2} C_0 V_{PI}, \quad V_{PI} = \sqrt{\frac{8kG_0^2}{27C_0}}, \quad \alpha = \frac{b}{2m\omega_0}, \quad \omega_0 = \sqrt{\frac{k}{m}} \quad (2.47)$$

Additionally, the normalized voltage across the actuator becomes

$$v_{act} = \frac{3}{2} q(1 - x) \quad (2.48)$$

If a δ -pulse current source is now used in Figure 2.27(a), it means $T_s \rightarrow 0$ and $I_s \rightarrow +\infty$, but $I_s T_s = cte$. According to Eq (2.45) the normalized charge, q , is the time integral of the δ -pulse current as follows.

$$q = \int_0^t dq = \frac{2}{3} \int_0^t i_s dt = \frac{2}{3} \int_0^{t_s} i_s dt = \frac{2}{3} i_s t_s = q_s = cte \quad (2.49)$$

This amount of charge is injected into the actuator capacitance before the moveable electrode starts moving. Then Eq (2.44) becomes.

$$\frac{d^2x}{dt^2} + 2\alpha \frac{dx}{dt} + x = \frac{q_s^2}{3} \quad (2.50)$$

As q_s is a constant for a given pulse, Eq (2.50) has a closed analytical solution.

$$x(t) = Ae^{r_1 t} + Be^{r_2 t} + \frac{q_s^2}{3} \quad (2.51)$$

where the coefficients are calculated as follows:

$$A = \frac{q_s^2}{3 \left(\frac{r_1}{r_2} - 1 \right)}, \quad B = \frac{q_s^2}{3 \left(\frac{r_2}{r_1} - 1 \right)}, \quad r_1 = -\alpha + \sqrt{\alpha^2 - 1}, \quad r_2 = -\alpha - \sqrt{\alpha^2 - 1} \quad (2.52)$$

As the voltage waveform developing after the current pulse is the matter of interest, substituting Eqs (2.49) and (2.51) into Eq (2.48), gives the voltage across the actuator as a function of time by:

$$v_{act} = \frac{3}{2} q_s (1 - x) = \frac{3}{2} q_s \left(1 - Ae^{r_1 t} - Be^{r_2 t} - \frac{q_s^2}{3} \right) \quad (2.53)$$

Figure 2.27. 1-D model parameters for a two parallel-plate electrostatic actuator and two alternate Driving schemes: (a) current pulse drive and (b) voltage drive with a series capacitor feedback

The maximum value of the voltage can be easily found according to Eq (2.53), resulting that the maximum is located at $x = 0$. Replacing $x = 0$ in Eq (2.53), the maximum value of the voltage is given by

$$v_{max} = v_{act}(x = 0) = \frac{3}{2} q_s \quad (2.54)$$

If Eq (2.54) is denormalized

$$V_{max} = \frac{Q_s}{C_0} = \frac{I_s T_s}{C_0} \quad (2.55)$$

This rather obvious result says that the maximum value of the voltage developed across the actuator equals the total amount of charge injected to the actuator divided by the initial capacitance, thereby indicating that the capacitor charges instantaneously with charge Q_s and, as the moveable plate has not yet had time to move from the initial position, the voltage at the initial time is simply the ratio of charge and initial capacitance. As can be seen in Eq (2.54), v_{max} is linearly related to the amount of normalized charge injected into the actuator at the beginning of the transient by the δ -pulse current source. Of course, if there are no parasitic capacitances, all points across the gap are stable [5] and the moveable electrode will be displaced completely to the bottom electrode for a given; amount of charge. Beyond that point complex dynamics may occur [44], including chaotic movement and the excess charge may be discharged by leakage resistances or stored at contact capacitance. This result can be easily extended to a more realistic situation: an actuator having a parallel parasitic capacitance C_p as shown in Figure 2.27(a). By defining the capacitance ratio σ as C_p/C_0 , Eq (2.45) becomes

$$q_s = q[1 + \sigma(1 - x)] \quad (2.56)$$

This means that the total amount of charge delivered by the current source, q_s , is distributed between the actuator, q , and the parasitic capacitance and it causes that the actuator charge, q , is not a constant. As in $t = 0$ the moveable electrode is still in its initial position, replacing $x = 0$ in Eq (2.56) the initial charge at the actuator, $q(0)$, is found as a function of q_s . The maximum value of the voltage can be found using Eqs (2.48) and (2.56) as

$$\begin{aligned} q_s &= q[1 + \sigma(1 - x)] = q + \sigma q(1 - x) = \frac{2}{3} \frac{v_{act}}{1 - x} + \frac{2}{3} \sigma v_{act} = \frac{2}{3} v_{act} \left(\frac{1}{1 - x} + \sigma \right) \rightarrow \\ v_{act} &= \frac{3}{2} \frac{q_s}{\left(\frac{1}{1 - x} + \sigma \right)} \rightarrow v_{max} = v_{act}(x = 0) = \frac{3}{2} \frac{q_s}{1 + \sigma} \end{aligned} \quad (2.57)$$

Hence the maximum value of the voltage is located at $t = 0$. Denormalizing Eq (2.57) gives that

$$V_{max} = \frac{Q_s}{C_0 + C_p} = \frac{I_s T_s}{C_0 + C_p} \quad (2.58)$$

Two main conclusions can be drawn from Eq (2.58): I) the value of the maximum voltage is independent of the damping coefficient because it happens at $t = 0$. II) the parasitic capacitance reduces the maximum value of the voltage by the same factor as the charge delivered by the current source, as it could be expected. Of course, the amount of charge delivered to the actuator is smaller due to the charging of the parasitic capacitance. Now the results of our δ -pulse current drive have been compared to the results achieved by the method using a voltage drive and a series feedback capacitor [19] shown in Figure 2.27(b), where C_1 is the series capacitance added to enlarge the stable deflection range of the original actuator, C_2 includes parasitics in parallel with the actuator and C_3 includes parasitics in parallel with the whole device. Obviously, C_3 does not affect the static behavior of the actuator.

In order to make the simulations comparable, the same conditions are obtained for both circuits such as the same deflection values at the equal time. For this purpose, the equilibrium equations of both circuits in Figure 2.27(a) and (b) are:

$$\forall T \geq 0 \rightarrow X_a = X_b \rightarrow Q_a = Q_b \rightarrow V_{act\ a} = V_{act\ b} \rightarrow$$

$$\frac{\frac{Q_s}{\frac{\epsilon A}{G_0 - X_a} + C_p}}{\frac{C_1 V_s}{\frac{\epsilon A}{G_0 - X_b} + C_1 + C_2}} = \frac{C_1 V_s}{\frac{\epsilon A}{G_0 - X_b} + C_1 + C_2} \quad (2.59)$$

According to Eq (2.59), $Q_s = C_1 V_s$ and $C_p = C_1 + C_2$ ensure the same deflection values for both circuits (a) and (b) or equivalency of the circuits. This is an interesting result showing that our current drive method achieves the same actuator dynamics as voltage drive method, requiring at most a voltage across the gap, given by $C_1 V_s / (C_0 + C_1 + C_2)$.

The last method which is proposed in [2] uses of the closed-loop feedback control techniques for system driving in extending the desired operation range of the electrostatic micro- actuators. The feasibility of a robust nonlinear control scheme for electrostatic microactuators using feedback is demonstrated here. To overcome this travel range limitation whilst improving the dynamic performance of the MEMS device is to employ closed-loop feedback control techniques using voltage actuation. Also a resistance in series with the MEMS device is used as shown in Figure 2.28 from [2].

Figure 2.28. Model of open-loop voltage actuation of MEMS device

According to device modeling in the previous part, the complete equation of motion is given by

$$m\ddot{X} + b\dot{X} + kX = \frac{Q^2}{2\epsilon A} \quad (2.60)$$

Applying Kirchoff's Voltage law on the model shown in Figure 2.28, the current through the input resistance is

$$\dot{Q}(t) = \frac{1}{R_s} \left(V_s(t) - \frac{Q(t)g(t)}{\epsilon A} \right) \quad (2.61)$$

Normalization of Variables

For control design purposes, a normalization of variables is performed as [16]

$$x = \frac{X}{G_0}, \quad t = \omega_0 T, \quad q = \frac{Q}{Q_p}, \quad u = \frac{V_s}{V_{PI}}, \quad \tau = C_0 R_s \quad (2.62)$$

where $C_0 = \epsilon A / g_0$ is the capacitance at rest, T is the time and

$$Q_p = \frac{3C_0 V_{PI}}{2}, \quad V_{PI} = \sqrt{\frac{8kG_0^2}{27C_0}}, \quad \omega_0 = \sqrt{\frac{k}{m}}, \quad \alpha = \frac{b}{2m\omega_0} \quad (2.63)$$

Using the above normalization, Eqs (2.60) and (2.61) can be transformed into the normalized differential equations governing the moving plate dynamics as

$$\ddot{x} + 2\alpha\dot{x} + x = \frac{q^2}{3} \quad (2.64)$$

$$\dot{q} = \frac{2}{3\tau} \left[u - \frac{3}{2}q(1-x) \right] \quad (2.65)$$

Imposing the state transformation, $x_1 = x$, $x_2 = \dot{x}$ and $x_3 = q$, the system defined by Eqs (2.64) and (2.65) can be transferred into the state space form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 - 2\alpha x_2 + x_3^2/3 \\ -x_3(1-x_1)/\tau \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2/3\tau \end{bmatrix} u \quad (2.66)$$

This is the open-loop state space model shown in Figure 2.28. Since it is possible and relatively inexpensive to accurately measure the voltage and capacitance across the device, the normalized voltage, y_1 , and normalized charge, y_2 , (deduced from voltage and capacitance measurements) across the actuator are considered as the system outputs. They are thus defined as

$$y_1 = \frac{3x_3(1-x_1)}{2} \quad (2.67)$$

$$y_2 = x_3 \quad (2.68)$$

Model Based Feedback Linearization

For a greater degree of freedom in designing a suitable controller, the system is considered as a single-input single-output (SISO) system with the output redefined as

$$y = 1 - \frac{2y_1}{3y_2} = x_1 \quad (2.69)$$

Using a nonlinear control technique called direct feedback linearization [45] and replacing u with $v - \frac{3}{2}x_1x_3$, the MEMS device model in Eq (2.66) is changed to a modified model, represented by state space model below:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - 2\alpha x_2 + \frac{x_3^2}{3} \\ \dot{x}_3 = -\frac{x_3}{\tau} + \frac{2}{3\tau}v \\ y = x_1 \end{cases} \quad (2.70)$$

The complete SISO electrostatic microactuator system can be generalized in the form

$$\dot{x} = f(x) + g(x)u, \quad y = h(x) \quad (2.71)$$

The control objective is output tracking. To generate a linear input-output relation, it is necessary to differentiate the output function y repeatedly until the input u appears for the first time on the right-hand side and then design u to cancel the nonlinearities in the system

[46]. Differentiating the output function until the input appears on the right-hand side gives that

$$y^{(3)} = L_f^3 h(x) + L_g L_f^2 h(x) u \quad (2.72)$$

where

$$L_f^3 h(x) = 2\alpha x_1 + (4\alpha^2 - 1)x_2 - \frac{2}{3}\left(\alpha + \frac{1}{\tau}\right)x_3^2 + \frac{2}{3\tau}x_1 x_3^2 \quad (2.73)$$

$$L_g L_f^2 h(x) = \frac{4}{9\tau} x_3 \quad (2.74)$$

Here, $L_g h(x)$ stands for the Lie derivative of $h(x)$ along f , $L_f^2 h(x)$ stands for $L_f(L_f h)(x)$ and so on. Thus, with respect to this output, the system has a well-defined relative degree of three (equal to the system order) at each point such that $x_3 \neq 0$. Around any of such points, there is no internal dynamics associated with the input-output linearization and the system can be transformed into a linear and controllable system by means of the feedback control law [46].

$$u = \frac{1}{L_g L_f^2 h(x)} (-L_f^3 h(x) + v^*) \quad (2.75)$$

Substituting Eq (2.75) into Eq (2.72) yields the simple linear relation

$$y^{(3)} = v^* \quad (2.76)$$

To design the tracking controller, it is assumed that a desired trajectory y_r and its derivatives are generated from a proper reference trajectory generator. Letting $e = y - y_r$ be the tracking error, and choosing the new input v as

$$v^* = y_r^{(3)} - k_3 \ddot{e} - k_2 \dot{e} - k_1 e \quad (2.77)$$

The tracking error dynamics is seen to be given by

$$e^{(3)} + k_3 \ddot{e} + k_2 \dot{e} + k_1 e = 0 \quad (2.78)$$

The error function $e(t)$ satisfies a linear differential equation of order three, whose coefficients can be arbitrarily preset. The coefficients $k_i, i = 1, 2, 3$, are chosen such that the characteristic equation associated with Eq (2.78) has all its roots strictly in the left hand plane.

$$A(s) = s^3 + k_3 s^2 + k_2 s + k_1 \quad (2.79)$$

Using the Routh-Hurwitz stability criterion, the coefficients should be selected as

$k_1 > 0, k_3 > 0, k_2 > k_1/k_3$. Therefore, under the effect of an input of the form Eq (2.75), the output of the system tracks the desired output $y_r(t)$ with an error which can be made to converge to zero, as $t \rightarrow +\infty$ with arbitrarily fast exponential decay. The physical control input u is given by

$$u = \frac{9\tau}{4x_3} \left[-2\alpha x_1 - (4\alpha^2 - 1)x_2 - \frac{2}{3\tau} x_1 x_3^2 + \frac{2}{3} \left(\alpha + \frac{1}{\tau} \right) x_3^2 + v^* \right] \quad (2.80)$$

The feedback control input u is undefined at $x_3 = 0$. It means the system is at rest with $(0, 0, 0)$. This is an equilibrium point of the undriven system. Therefore, the controller (2.80) will stabilize the system in all gaps except at the origin, where it is adequate to remove the control input.

2.3 Equilibrium and “Pull-In” Point with Stability Analysis

Based on the method described in [4], at equilibrium condition the free electrode is static and it has no movement. Hence the distance between the electrodes for the device in equilibrium point, x^* , can be found from Eq (2.5). It means the equilibrium position of the plate can be found by equating the electrostatic and mechanical forces as follows [1].

$$k(x_0 - x^*) = \varepsilon A V^2 / 2x^{*2} \quad (2.81)$$

Figure 2.29 from [4] shows dimensionless plots of both sides of Eq (2.81) due to constant voltage for finding the intersections which indicate equilibrium positions.

Figure 2.29. Normalized restoring and electrostatic forces vs. normalized electrode separation

On the electrostatic force curve corresponding to the critical “Pull-In” voltage, V_{pi} , there is exactly one equilibrium point. Below this voltage, the device has two equilibriums above this voltage, the electrostatic force dominates the restoring force, and hence, there is no equilibrium. This holds true for nonlinear restoring forces provided that they remain finite at $x = 0$. Figure 2.30 from [4] is a plot of equilibrium position, x^* , versus voltage, V , from Eq (2.81) for a device subject to a linear restoring force. The plot shows that, as indicated earlier, the system has two equilibriums for voltages less than the critical “Pull-In” voltage.

Figure 2.30. Equilibrium position vs. device voltage

Figure 2.31. Graph showing actuator deflection versus applied voltage

As V increases, the gap distance at first decreases gradually as shown in Figure 2.31 from [1]. However, at a critical voltage value called the “Pull-In” voltage, a bifurcation occurs and the plate suddenly collapses. The maximum stable deflection of the plate before

the “Pull-In” instability is $1/3$ of the undeflected gap [1]. For a device subject to a linear restoring force, it is well known that the maximum voltage for which there is an equilibrium point, “Pull-In” voltage, is $V_{pi} = (2/3\sqrt{3})\gamma$. This can be attained from Eq (2.81) using derivative of V^2 respect to x^* and putting equal to zero. It occurs at $x^* = 2x_0/3$ as it is seen below. ($\gamma = \sqrt{2kx_0^3/\epsilon A}$)

For a nonlinear mechanical system subject to damping forces, $\beta(\dot{x})$, and conservative forces, $k(x)$, the stability of an equilibrium point can be determined from the potential energy of the system. Minima in the potential energy indicate stable equilibrium, while the maxima indicate unstable equilibrium [47]. Figure 2.32 from [4] illustrates this concept. The potential energy of the electromechanical system is as follows.

$$U(x) = -\int_{x_0}^x (k(u) - f_e(u) - f_{ext}) du \quad (2.82)$$

The extremum of $U(x)$ occurs at the equilibrium position, x^* , and is minima of the potential energy if:

$$\left. \frac{d^2U}{dx^2} \right|_{x^*} > 0 \rightarrow \left. \frac{d}{dx} (-k(x) + f_e(x)) \right|_{x^*} > 0 \rightarrow k - \frac{\epsilon AV^2}{x^{*3}} > 0 \quad (2.83)$$

According to Eq (2.81), $\epsilon AV^2 = 2kx^{*2}(x_0 - x^*)$. The stability of the equilibrium point result that $x^* > 2x_0/3$.

Figure 2.32 shows plots of the potential energy for the electromechanical device with a linear spring restoring force and constant voltage. This plot shows that for voltages less than V_{pi} only one equilibrium point is stable; the other is unstable. Furthermore, the device is stable only if it lacks enough energy to overcome the potential barrier marked by the unstable equilibrium.

Thus, under equilibrium conditions, the free electrode can only move one-third of the zero-voltage gap-size using electrostatic actuation. The free electrode cannot come to rest at any point between $0 < x^* < 2x_0/3$.

Figure 2.32. Normalized potential energy vs. separation

The feedback capacitor has an effect that is analogous to shift the electrostatic force curve, and it prevents the force from becoming infinite at zero gap value. Figure 2.33 from [4] shows force plots for the device in series with a feedback capacitor ($K = 2$). The plots are similar to those in Figure 2.29 except that for each value of V_s there is only one equilibrium point. With capacitor feedback, the voltage required to close the gap is $K\gamma$ (for $K \geq 2$) which can be proved using Eq (2.6) with setting $x^* = 0$. For $K = 2$, a factor of $3\sqrt{3}$ greater than the “Pull-In” voltage is needed for the device without feedback.

Figure 2.33. Restoring and electrostatic forces with capacitor feedback ($K = 2$)

According to aforementioned and noting Eq (2.7) and, u^* is stable for the gap sizes between $2x_0$ and $3x_0$ as shown in Figure 2.6. Hence x^* is stable for gap sizes between 0

and x_0 , which means achieving stable full range movement. In the other way, using equation (2.83) to determine stability leads to.

$$\left. \frac{d^2 U}{dx^2} \right|_{x^*} > 0 \rightarrow \left. \frac{d}{dx} (-k(x) + f_e(x)) \right|_{x^*} > 0 \rightarrow k - \frac{\epsilon A V_s^2}{(x^* + K x_0)^3} > 0 \quad (2.84)$$

According to Eq (2.6); $\epsilon A V_s^2 = 2k(x^* + 2x_0)^2(x_0 - x^*)$. The stability of the equilibrium point result that $x^*/x_0 > (2 - K)/3$. Thus for $K \geq 2$ or equivalently $C_f \leq C_0/2$, all operating states are stable. For $K < 2$, an unstable region still exists, but the stable region is extended. The potential energy for the device with the critically sized feedback capacitor ($K = 2$) shows that the system is stable over the entire operating range as shown in Figure 2.34 from [4].

Figure 2.34. Normalized potential energy vs. electrode separation with capacitor feedback ($K = 2$)

In following, the “Pull-In” point analysis of method [3] is discussed. For an electrostatically actuated micromirror in stable deflection (deflection distance less than the “snap-through” threshold) the electrostatic force is balanced by the restorative flexure force. Setting Eq (2.18) equal to the number of flexures, N , times Eq (2.19) results in:

$$\frac{\epsilon_0 A V^2}{2D^2} = NkD \quad (2.85)$$

Solving Eq (2.85) for applied voltage, V , as a function of micro mirror deflection leads to [30, 48]:

$$V = (z_0 - d) \sqrt{\frac{2Nkd}{\epsilon_0 A}} \quad (2.86)$$

where the separation distance, D , in Eq (2.85) has been replaced by:

$$D = (z_0 - d) \quad (2.87)$$

where z_0 is the initial separation distance and d is the micromirror deflection distance. Eq (2.86) calculates the required voltage, V , to deflect the micromirror a distance d . Table 2-1 from [3] is a list of constants used in Eq (2.86) from [49].

For an electrostatically actuated micromirror with nonlinear flexures defined by Eq (2.21), Eq (2.86) is no longer valid and in stable deflection (deflection distance less than the “snap-through” distance), Eq (2.86) becomes:

$$V = (z_0 - d) \sqrt{\frac{2Nkd^r}{\epsilon_0 A}} \quad (2.88)$$

Item	Symbol	Value
Modulus of elasticity [11]	E	169 GPa
Flexure width	w	2.0 μm
Flexure thickness	t	1.5 μm (Poly-2)
Flexure length	L	97.0 μm
Flexure material stress [11]	σ	1.20 MPa
Flexure material Poisson ratio [11]	ν	0.22
Flexure spring constant (calculated)	k	0.4258 N/m
Initial electrode separation	z_0	2.09 μm
Permittivity of free space	ϵ_0	8.854×10^{-14} F/cm
Electrode surface area	A	5,252 μm^2

Table 2-1. Design and material constants used to model an Electrostatically actuated micromirror with linear flexures

Figure 2.35 from [3] is a normalized plot of Eq (2.88) for several values of r . Note that even though the distance before “snap-through” was increased, the “snap-through” phenomenon was not eliminated.

“Snap-through” still occurs because electro- static force and flexure force are not balanced over all of the micromirror's deflection range. A second order flexure does not

exhibit “snap-through” until the micromirror is deflected one-half the initial separation between the top and bottom electrode.

Figure 2.35. Deflection versus voltage plots for electrostatically Actuated micromirrors with nonlinear force flexures

“Snap-through” occurs when the derivative of Eq (2.88) with respect to deflection changes sign, so it is possible to calculate the “snap-through” point for an electrostatically actuated device with any flexure force order. Setting the derivative of Eq (2.88) equal to zero results in the following solution for deflection:

$$d_{snap} = \frac{rZ_0}{r+2} \quad (2.89)$$

where d_{snap} is the deflection distance to “snap-through”, and r is the order of the flexure force. As r grows large, d_{snap} approaches the initial separation distance; however, the voltage verses deflection curve shown in Figure 2.35 becomes sharper. Higher order flexures are capable of traveling a larger distance before “snap-through”. Note that Eq (2.89) is independent of the number of flexures, the spring constant, and electrode surface area.

It should be mentioned that dual spring constant flexure designs allow micromirror

deflection verses voltage profiles similar to those predicted in Figure 2.35.

Now the stability analysis for current drive method [5] is investigated as follows. It is theoretically shown that if the pulse current drive is ideal, any position across the gap is stable. The main reason underlying this result is that the electrostatic force does not depend on the remaining gap of the actuator and hence, the “Pull-In” instability is avoided. These findings are supported on simulations using an analog high level description language to represent the actuator by its differential equations. The important changes in the dynamics of the actuator lead to changes in the conditions required for the stability and, as it is shown, the mobile electrode position is inherently stable for any value of the deflection from zero to the full gap provided that the current source is ideal.

The conditions for a given position of the moveable plate are required to be stable. At equilibrium point both deflection and charge remain constant. Hence their derivatives are equal to zero. According to Eq (2.27), $i_s(t_0)$ is equal to zero, which t_0 is the equilibrium time. It means that t_0 is greater than t_s . Eqs (2.16) and (2.29) lead to the result that the equilibrium under steady-state regime is reached at a point (x_0, q_0) given by:

$$x_0 = \frac{q_0^2}{3}, \quad q_0 = \frac{2}{3} \int_0^{t_0} i_s(\tau) d\tau = \frac{2}{3} \int_0^{t_s} i_s(\tau) d\tau = \frac{2}{3} i_s t_s \quad (2.90)$$

This is an important result showing a unique difference between charge drive and voltage drive, because here the charge delivered to the actuator only depends on the source parameters i_s and t_s which can be independently controlled. This fact is the origin of the stability of this equilibrium point as can be easily demonstrated considering that the stability condition is that the derivative of the total force (electrical minus mechanical) should be negative at the equilibrium point. The electrical force F_e and the mechanical force F_m are given by:

$$F_e = \frac{Q^2}{2C_0 G_0} = \frac{q^2 k G_0}{3}, \quad F_m = kX = kG_0 x \quad (2.91)$$

By normalizing these two forces, the normalized electrostatic f_e and mechanical f_m can be obtained as

$$f_e = \frac{F_e}{kG_0} = \frac{q^2}{3}, \quad f_m = \frac{F_m}{kG_0} = x \quad (2.92)$$

Then the stability condition is attained at $x = x_0$ when $t \rightarrow +\infty$ as

$$\frac{d}{dx} \left(\frac{q^2}{3} - x \right) < 0 \rightarrow \frac{2}{3} q \frac{dq}{dx} - 1 < 0 \rightarrow \frac{2}{3} q_0 \left(\frac{\dot{q}}{\dot{x}} \right)_{t \rightarrow +\infty} - 1 = \frac{2}{3} q_0 \left(\frac{0}{\dot{x}} \right) - 1 = -1 \quad (2.93)$$

The value of the left hand side is always -1 and hence, the condition in Eq (2.93) is satisfied for any value of x_0 provided q_0 is constant, which is precisely our case. This is illustrated in Figure 2.36 from [5], where plots of $f_m = x_0$ and $f_e = q_0^2/3$ against the normalized equilibrium position, x_0 , for three values of the total normalized charge, q_0 , are shown. As can be seen there is only one intersection point between the two curves for each given value of q_s . The fundamental limitation of the voltage driven “Pull-In” based devices to a maximum deflection of 1/3 of the gap is avoided and, theoretically, the moveable electrode can be stabilized anywhere across the gap. Secondly the effect of parasitic capacitances on the stability conditions has also been analyzed. Here a simple analytical stability condition can be written relating the value of the parasitic capacitance to the maximum deflection allowed for a stable equilibrium point.

Figure 2.36. Normalized electrical and mechanical forces vs. normalized Deflection for three different values of the total normalized charge

These results are also supported by simulation. In a steady-state situation, x_0 and q_0 are constant and also $t \rightarrow +\infty$. After making derivatives equal to zero in Eq (2.32), equilibrium point (x_0, q_0) can be attained from Eqs (2.32) and (2.33) as follows.

$$x_0 = \frac{q_0^2}{3}, \quad q_0 + \sigma q_0 \left(1 - \frac{q_0^2}{3}\right) = \frac{2}{3} i_s t_s \quad (2.94)$$

The equilibrium points, if there are any, are the solutions of this equation. As an example, Figure 2.37 from [5] shows a plot of $f_m = x_0$ and $f_e = q_0^2/3$ against the normalized equilibrium position, x_0 , for $i_s t_s = 2\sqrt{2}$ and four values of the capacitance ratio σ . As can be seen, the intersection points can be one, two or none depending on the value of σ . A single equilibrium position is reached at $x = 2/3$ for $\sigma = 1$, while two equilibrium points are reached for $\sigma = 2$ and 10, and no equilibrium points are reached for $\sigma = 1/2$. The question is now, whether all the intersection or equilibrium points are stable. The conditions for stability can be analyzed in the same way as in section above. Let us first notice in Figure 2.38 from [5], the total normalized force $f_e - f_m$ plotted against the normalized value of the deflection for the same values of σ as in Figure 2.37. An important difference is now observed in this case, where the parasitic capacitance is not zero, as compared with the ideal case described in section above.

Figure 2.37. Normalized electrical and mechanical forces vs. normalized deflection for four different values of capacitances

It can be clearly seen that the derivative of the total normalized force $f_e - f_m$ is negative only in part of the travel range, this is particularly clear for the $\sigma = 1, 2$ and 10 plots, there are two regions, one in which the derivative is negative, and hence an equilibrium

point in that region will be stable, and a second region, where the derivative is positive and any equilibrium point in that region will be unstable. It becomes clear that a quantitative condition can be found for an equilibrium point to be stable when $\sigma > 0$. Applying the derivative condition as in section above gives that.

$$\frac{d(f_e - f_m)}{dx} = \frac{d}{dx} \left[\frac{q^2}{3} - x \right]_{x_0} = \frac{d}{dx} \left[\frac{\left(\frac{2}{3} \int_0^t i_s(\tau) d\tau \right)^2}{3(1 + \sigma(1 - x))^2} - x \right]_{x_0} < 0 \quad (2.95)$$

After some algebraic manipulation, Eq (2.95) leads to:

$$\sigma(3x_0 - 1) \leq 1 \quad (2.96)$$

Figure 2.38. Total normalized force vs. normalized deflection for four different values of capacitances

Figure 2.39. Stable and unstable working areas as a Function of the capacitance ratio and the normalized deflection

Eq (2.96) is an important result, consistent with the analysis in [27], quantifying the conditions that the parasitic capacitance value has to fulfill in order to obtain a stable deflection value. Figure 2.39 from [5] shows a plot of Eq (2.96), where it can be seen for example, that for a parasitic capacitance equal to the initial actuator capacitance ($\sigma = 1$), a stable equilibrium can be reached for values of the deflection smaller than $2/3$ of the initial gap, and that for a parasitic capacitance doubling C_0 ($\sigma = 2$), a stable position is possible for deflection as large as half of the initial gap. The plot shows the area where stable working points can be obtained.

As an example of the situation at the limit between stable and unstable zones, let us focus in the plots for $\sigma = 1$ in Figures 2.23 and 2.24. As can be seen, f_m and f_e curves are tangent at $x = 2/3$ which is exactly the solution of Eq (2.96) at the border line.

The analysis of both maximum stable displacement and maximum voltage across the gap for three different methods, described in [12], are surveyed below.

A. Voltage Drive with Feedback Capacitor

Defining D_0 , the initial gap distance, X , the displacement and m, n and r are scaling factors of the constitutive capacitors of the circuit Figure 2.24.

$$C_0 = \frac{\epsilon A}{D_0 - X}, \quad C_1 = \frac{\epsilon A}{n D_0}, \quad C_2 = \frac{\epsilon A}{m D_0}, \quad C_3 = \frac{\epsilon A}{r D_0} \quad (2.97)$$

The steady-state governing equation is

$$kX = \frac{\varepsilon AV_0^2}{2(D_0 - X)^2} \quad (2.98)$$

A substitution of capacitors values from Eq (2.97) into Eq (2.38) results the actuator voltage by

$$V_0 = \left[\frac{m(D_0 - X)}{(m + n + mn)D_0 - (m + n)X} \right] V_a \quad (2.99)$$

Substituting Eq (2.99) in Eq (2.98) gives the equilibrium equation clearly.

$$X[(m + n + mn)D_0 - (m + n)X]^2 = m^2 \frac{\varepsilon AV_a^2}{2k} \quad (2.100)$$

Taking derivative of the left hand side respect to X and equating to zero, results the maximum stable displacement of the movable electrode by

$$X_{max} = \frac{D_0}{3} \frac{m + n + mn}{m + n} \quad (2.101)$$

Substituting this value into Eq (2.100) results the maximum allowable applied voltage for stability condition

$$V_{a \max} = V_{pi} \sqrt{1 + \frac{mn}{m + n}} \left(1 + n + \frac{n}{m} \right) \quad (2.102)$$

where V_{pi} is the “Pull-In” voltage for the voltage drive without feedback capacitors and equal to $V_{pi} = (2\sqrt{2}/3\sqrt{3})D_0\sqrt{kD_0/\varepsilon A}$. Now the purpose is to minimize $V_{a \max}$ consider -ring achieving full travel range, $X_{max} = D_0$, which is found in configurations where parasitic capacitance, C_2 , tends to zero and series capacitor, C_1 , is fixed to the largest value, that is, $m \rightarrow +\infty, n = 2$. Thus the developed voltage responds to

$$V_{a \max} = V_{pi} \sqrt{27} \approx 5.2V_{pi} \quad (2.103)$$

B. Current Drive with Feedback Capacitor

In contrast with the voltage drive configuration, C_3 in Figure 2.25 cannot be neglected in order to study the maximum stable displacement. The voltage drop across C_3 is not restricted by the voltage source, and depending on its value could behave like a voltage source in parallel with the actuator and the series capacitor. In this new configuration the voltage V_0 across the actuator relates to the capacitors and the total charge Q_{total} in the circuit are expressed as

$$V_0 = \frac{C_1}{C_3(C_2 + C_1 + C_0) + C_1(C_2 + C_0)} Q_{total} \quad (2.104)$$

Replacing the capacitors by their values from Eq (2.97) and inserting Eq (2.104) in Eq (2.98) and then taking derivative of the left hand side respect to X and equating to zero, results the maximum stable displacement of the movable electrode by

$$X_{max} = \frac{D_0}{3} \left(1 + \frac{m(n+r)}{m+n+r} \right) \quad (2.105)$$

In the limit when $C_3 = 0$, ($r \rightarrow \infty$), the stability is ensured up to

$$X_{max} = \frac{D_0}{3} (1 + m) \quad (2.106)$$

Hence, for $m \geq 2$ the stable range is fully extended to D_0 . For $C_2 = 0$, ($m \rightarrow \infty$), X_{max} is given by

$$X_{max} = \frac{D_0}{3} (1 + n + r) \quad (2.107)$$

Stability is ensured for any value of n if $r \geq 2$, therefore C_1 can be larger than C_0 making small the voltage drop across the sensing capacitance. On the other hand, taking into account that the developed voltage for the system V_a relates to the voltage drop across the actuator V_0 as

$$V_0 = \frac{C_1}{C_2 + C_1 + C_0} V_a \rightarrow V_a = \frac{C_2 + C_1 + C_0}{C_1} V_0 \quad (2.108)$$

Combining Eqs (2.98), (2.104), (2.105), and (2.108), the maximum allowable applied voltage for stability condition is determined as

$$V_{a \max} = V_{pi} \sqrt{1 + \frac{m(n+r)}{m+n+r}} \times \left[1 + \frac{n(1+m)}{m} - \frac{mr}{2(m+n+r)} \right] \quad (2.109)$$

In contrast to Eq (2.102), a new subtracting term is included reducing the developed voltage. The reduction of the developed voltage is highly dependent on the sensing capacitor C_1 . The larger value of C_1 results the smaller the developed voltage. The ideal situation would be when $n \rightarrow 0$. However in the closed loop configuration n cannot be zero.

C. Current Drive without Feedback Capacitor

In fact, the second implementation shown in Figure 2.26 is a particular case of the previous proposal shown in Figure 2.25 for $n = 0$. Thus, the maximum stable displacement can be obtained from studying the governing equations or, directly, from Eq (2.105)

$$X_{max} = \frac{D_0}{3} \left(1 + \frac{mr}{m+r} \right) \quad (2.110)$$

When parasitic capacitors have the same value, full range of travel is possible for $m, r \geq 4$. For $m \rightarrow +\infty$ or $r \rightarrow +\infty$, full travel range is ensured for $r \geq 2$ or $m \geq 2$, respectively. From Eq (2.110), the developed voltage for $X = X_{max}$ will be given by

$$V_{a \max} = V_{pi} \sqrt{1 + \frac{mr}{m+r}} \times \left[1 - \frac{mr}{2(m+r)} \right] \quad (2.111)$$

For any combination of m and r allowing full stable travel, $X_{max} = D_0$, the developed voltage will be zero, as stated in [20].

The “Pull-In” phenomenon is due to the physical structure of device. It limits the travel distance of the movable plate which is precisely explained in [2] as follows. At equilibrium, the distance $x(t)$ traveled by the movable plate for any applied constant voltage is determined by the elastic stiffness and the magnitude of the applied voltage. The movable plate has no movement and voltage and charge of the device is constant. Hence according to Eqs (2.60) and (2.61), the equilibrium equations of the device can be written as

$$kX = \frac{Q^2}{2\varepsilon A} \quad (2.112)$$

$$\frac{QG}{\varepsilon A} = V_s \quad (2.113)$$

From Eqs (2.112) and (2.113), the applied voltage at equilibrium is

$$V_s = \sqrt{\frac{2k}{\varepsilon A} (G_0 - X)^2 X} \quad (2.114)$$

Figure 2.40 from [2] shows a plot of applied voltage versus normalized plate deflection at equilibrium for a microactuator with plate area $A = 4 \cdot 10^{-8} m^2$, $g_0 = 4 \cdot 10^{-6} m$ and $k = 10 Nm^{-1}$.

Figure 2.40. Normalized plate deflection vs. applied voltage

The plot shows that the system has two equilibrium points for applied voltages less than the “Pull-In” voltage, V_{PI} , and zero beyond. At $X = G_0/3$, the increase in the electrostatic force is greater than the increase in the restoring force resulting in the collapse of the movable plate to the fixed plate position. Taking the derivative of Eq (2.114) with respect to X and setting to zero, the plate deflection at which the “Pull-In” instability occurs is found to be exactly one-third of the zero-bias capacitive gap and the “Pull-In” voltage is found to be

$$V_{PI} = \sqrt{\frac{8kG_0^3}{27\epsilon A}} \quad (2.115)$$

Thus, under equilibrium conditions with constant voltage operation, the movable plate can only move 1/3 of the entire capacitive gap. Violating this restriction results in the plates snapping together which is undesirable for some applications.

2.4 Simulation Results and Discussions

Figure 2.41 from [4] shows the results of MATLAB [50] simulations for an under damped device with capacitor feedback ($K = 2$). In this simulation, the supply voltage was stepped three times moving the electrode 30%, 60%, and 90% of the nominal gap. It illustrates that using of feedback; the device is stable and can be operated in the normally unstable region. It is also shown that the device with capacitor feedback requires larger supply voltages to achieve the same amount of deflection as the device without capacitor feedback. Finally, the simulations show that for larger deflections, the damping increases.

Figure 2.41. Step response for device with capacitor feedback supply voltage is shown for each step

In order to validate current-pulse drive (charge drive) approach [5], time response simulations have been carried for the parallel plate actuator. Some of the results are shown in Figure 2.42 from [5], where plots of the normalized deflection against the normalized time are shown. All transients in Figure 2.42 have been simulated for $\alpha = 5$, corresponding to an overcritical damping situation. As predicted by the above analysis, the deflection stabilizes at values either smaller or larger than $1/3$ of the gap.

Figure 2.42. Transients of normalized deflection for four different values of the total normalized charge

Figure 2.43. Transients of normalized deflection for two different values of the damping coefficient

Figure 2.44. Transients of normalized voltage for two different values of the damping coefficient

Figure 2.43 from [5] shows the same kind of transients for two other values of the coefficient α . It is clearly seen that the mobile electrode also stabilizes, but the transient contains overshoot that fades away at different rates depending on the value of α . Figure 2.44 from [5] shows the corresponding normalized voltage transients, where it can be seen that it exceeds unity, indicating that even if the “Pull-In” voltage is exceeded in a part of the transient the final position of the electrode is stable and far beyond the 1/3 of the initial gap.

In order to validate our analytical results considering effect of parasitic capacitances, simulations of the transient dynamics have also been performed. Some of the results are shown in Figure 2.45 from [5] for $\sigma = 1$ and $\alpha = 1$, where it can be seen that stable positions, even larger than $1/3$ of the gap, are reached for normalized source charge values not exceeding $q_s = (4/3)\sqrt{2}$. Beyond that point, the moveable electrode collapses to the bottom electrode.

The voltage source role of the parasitic capacitance is clearly seen in Figure 2.46 from [5] where the normalized voltage across the actuator is plotted against time for the same cases as in Figure 2.45. An equilibrium voltage value is reached provided the equilibrium point is stable. In contrast, for the cases where the equilibrium point is unstable, the voltage also collapses, meaning that the charge stored at the parasitic capacitance is delivered to the actuator as its capacitance grows as the gap narrows.

Figure 2.45. Transients of normalized deflection for five different values of the total normalized charge

Figure 2.46. Transients of normalized voltage for five different values of the total normalized charge

In following simulation results for δ -pulse current drive [16] is shown. Table 2-2 from [16] shows a comparison of significant results from two driving schemes simulations, as shown in Figure 2.27. It is seen that the values of v_{max} and v_{final} are the same, that is, the transients are equal. It is also seen that the circuit (b) requires larger source voltage values (up to 6 times the “Pull-In” voltage) to achieve the same results obtained with small amounts of charge in the δ -pulse current scheme.

x_0	Voltage Drive			δ -pulse current drive		
	V_s	V_{max}	V_{final}	Q_s	V_{max}	V_{final}
1/3	5.714	1.273	0.991	1.523	1.274	0.992
1/2	6.417	1.430	0.918	1.699	1.431	0.918
2/3	6.702	1.493	0.707	1.784	1.495	0.707
3/4	6.734	1.5	0.567	1.793	1.502	0.570

Table 2-2. Simulation results comparison for two drive schemes shown in Figure 2.27 and four stable Steady-state deflections across the gap, being $x_0 = 3/4$ the maximum theoretical deflection

Figures 2.47 and 2.48 from [16] show a plot of the δ -pulse current drive transients for $q_s = 1.7, \sigma = 4/5$ and three values of the damping coefficient: $\alpha = 0.1$ (underdamped), $\alpha = 1$ (critical damping) and $\alpha = 10$ (overdamped). According to current drive theory, the

steady state deflection corresponding to this value of q_s is $x_0 = 1/2$. As can be seen the maximum value of the voltage is located at the beginning of the transient.

The effect of the total charge injected in the actuator is examined in Figure 2.49 from [16] for $\sigma = 4/5$, where transients for four values of q_s leading to four stable final deflections somewhere in the gap are shown.

Figure 2.47. Normalized voltage transients for δ -pulse current drive

For three values of α and a stable target deflection $x_0 = 1/2$

Figure 2.48. Normalized deflection transients for δ -pulse current drive

For three values of α and a stable target deflection $x_0 = 1/2$

Figure 2.49. Normalized voltage transients for δ -pulse current drive $\alpha = 1$

And four stable final deflections across the gap

In Figure 2.50 from [16] transients for three values of q_s leading to full gap deflection are also shown. As can be seen the maximum value of the voltage is always located at $t = 0$ and takes higher values for increasing values of q_s . It is also shown that increasing values of q_s lead to decreasing values of the “Pull-In” time.

Figure 2.50. Normalized voltage transients for δ -pulse current drive $\alpha = 1$

And three values of q_s leading to full gap deflection

Another interesting result can be obtained if it is noticed that the normalized final voltages, v_{final} , in all transients in Figures 2.48-2.50, and Table 2-2 are constant and only dependent on the value of the steady state deflection, x_0 . Then from Eq (2.48), the steady state voltage can be written according to the steady state deflection, x_0 as

$$v_{final} = v_{act}(t \rightarrow +\infty) = \frac{3}{2} q_0(1 - x_0) = \frac{3}{2} \sqrt{3x_0}(1 - x_0) \quad (2.116)$$

Also from Eqs (2.56) and (2.57), total normalized charge, q_s , and normalized maximum voltage developed are dependent on the values of σ and the steady state deflection, x_0 , as

$$q_s = q_0[1 + \sigma(1 - x_0)] = \sqrt{3x_0}[1 + \sigma(1 - x_0)] \quad (2.117)$$

$$v_{max} = v_{act}(x = 0 \equiv t = 0) = \frac{3}{2} \frac{q_s}{1 + \sigma} = \frac{3}{2} \frac{\sqrt{3x_0}[1 + \sigma(1 - x_0)]}{1 + \sigma} \quad (2.118)$$

The performance of the proposed control scheme [2] is simulated on the MEMS electrostatic microactuator using MATLAB/SIMULINK. The reference signal used in the simulation is selected such that y_r is the desired final equilibrium position and thus $y_r^{(3)} = \ddot{y}_r = \dot{y}_r = 0$.

Note that all units in the presented simulation results are non-dimensional corresponding to the normalized state space representation in Eq (2.66).

For purposes of comparison and demonstration of the effectiveness of the controller, the open loop system is first simulated under constant voltage actuation conditions. In Figure 2.51 from [2], the open loop system is simulated with an applied voltage of $V_S = V_{PI}$.

Figure 2.51. Open-loop system under constant voltage actuation with $V_s = V_{PI}$.

Figure 2.52 from [2] shows results of simulations for the open-loop system under constant voltage actuation with an applied voltage of $V_s = 1.1 V_{PI}$

The responses in Figures 2.51 and 2.52 show that with an applied voltage equal to the “Pull-In” voltage, the system has one equilibrium position at a deflection of $1/3$ the nominal gap and the system has a very slow response time. Also, for an applied voltage greater than the “Pull-In” voltage, the system has no stable equilibrium position and the electrostatic force acting on the movable plate is much greater than the restoring force resulting in its collapse to the stationary plate position.

Figure 2.53 from [2] shows simulation results using the proposed control method for responses corresponding to a deflection of 30%, 60% and 90% of the nominal capacitive gap for a system with a damping ratio $\alpha = 1$.

It is assumed that initially the movable plate is at the equilibrium position $(0, 0, 0)$ which is the rest position of the microactuator. To initiate motion from the rest position, a relatively small voltage is momentarily applied to the system. A saturation block is employed to impose upper and lower bounds on the control input signal.

Figure 2.52. Open-loop system under constant voltage actuation with $V_S = 1.1 V_{PI}$

Figure 2.53. Normalized plate deflection vs. normalized Time for a microactuator with a damping ratio $\alpha = 1$

Figure 2.54 from [2] shows the results of simulations corresponding to a deflection of 30%, 60% and 90% of the normalized gap for three systems with damping ratios, α , equal to 0.1, 0.5 and 2.5.

The time responses in Figures 2.53 and 2.54 show a fast response time with almost negligible overshoot and no oscillations. Deflections beyond 1/3 of the nominal capacitive

gap are stable and controllable. This clearly demonstrates an immense improvement in the dynamic performance of the microactuator system beyond the “Pull-In” instability point. Also, the effect of damping on the system dynamic response is relatively out of merit but more pronounced for larger deflections.

Figure 2.54. Normalized Plate Deflection vs. Normalized Time for Electrostatic microactuators with damping ratio variations

All simulations were performed with the same controller designed using a nominal plant model with a damping ratio $\alpha = 1$. It can thus be inferred from the simulation results that the proposed nonlinear controller approach presented has extremely good parameter variation robustness. Another interesting observation is that the initial control signal spikes due to the initial small value of charge on the movable plate makes the plates of the electrostatic microactuator initially attract each other strongly, speeding up the system response. But the most important issue as a major drawback of this control scheme appears by the simulation result shown in Figure 2.55 for damping ratio $\alpha = 1$ and new trajectory defined by $y_r(t) = u_{-1}(t) - u_{-1}(t - 30)$.

Figure 2.55. Normalized charge (x_3) vs. time for a microactuator with a damping ratio $\alpha = 1$

2.5 Conclusions

Digital applications as microswitches or microrelays [10] are generally driven by a single voltage source directly plugged to the electrodes. One of the main drawbacks of this technique is that it consumes a large amount of power, a large fraction of it being lost in dissipation at the parasitic resistances and in kinetic energy at the impact of both electrodes, hence having a potential for low reliability. Moreover, voltage drive is unable to provide stable deflections beyond 1/3 of the initial gap [16].

In the series capacitor method [4], it is shown that the “Pull-In” instability, which significantly limits the operation of electrostatically actuated devices, can be avoided by the simple addition of a series capacitance in the DC path of the MEMS bridge circuit but in general, the device with feedback requires larger supply voltages to reach any equilibrium point as shown in Figure 2.56 from [4]. The need for higher voltage is a tradeoff for stabilizing the device and increasing the range of motion.

Figure 2.56. Equilibrium position vs. supply voltage feedback devices are critically sized

This technique requires the fabrication of additional structures or electronics. The polychromator requires a high fill factor and dense packing of devices, so there is very little extra room for additional structures to be fabricated alongside the individual actuators. In addition, small feature sizes, large gaps, and close spacing mean that actuators have low capacitance and large parasitic, which make them difficult to sense and control [1].

Nevertheless, this strategy requires higher voltage than the “Pull-In” voltage value to achieve full range of travel. This approach requires a much higher voltage to be applied to the composed device, compared with the voltage required by a direct voltage driving. Also, the performance of control scheme above is seriously affected by the natural damping of the device [12].

The concept of series feedback capacitance method is simple and the resultant device is compact—not any larger than a conventional device—but the challenge is to maintain the simplicity in the face of nonidealities [19].

Extending gap sizes or sacrificial layers to be about three times as thick as the target travel distance is not very satisfactory because it requires a large gap that can be difficult to fabricate [19].

Thick sacrificial layers are a disadvantage in a surface micromachined process because they lead to long deposition times and increased topography for subsequent processing. In addition, thick sacrificial layers can reduce the ability to control variations in layer thickness and can make it more difficult to pattern and etch small features such as posts

and anchors. Also For the polychromator, a small gap distance is especially attractive because it means less electrostatic crosstalk due to fringing fields from neighboring actuators [1].

Using of leveraged-bending and strain-stiffening [1] leads to following conclusions. The Leveraged-bending is very sensitive to the residual tensile stress in the beam. When the beam has a high tensile stress, it acts like a rubber band. When force is applied near the beam supports, the middle of the beam remains flat, reducing the lever effect that increases travel as shown in Figure 2.57 from [1]. Quantitative limits on allowable stress levels. The leveraged-bending effect can be used to maximize the travel range of an electrostatic actuator for a given gap size, achieving up to full-gap travel at the cost of increased actuation voltage. Reducing gap size, however, can be very important in order to reduce the thickness of sacrificial layers in fabrication processes. Thinner sacrificial layers are quicker and easier to deposit, control, and pattern. The strain-stiffening effect is generally more effective with longer, thinner structures, and may be used to minimize actuation voltage for a given travel range. The polychromator optical constraints on the dense packing, small size, and large number of actuators limit the use of alternative techniques for increasing the travel range of electrostatic actuators.

Figure 2.57. Using leveraged bending to extend the stable travel range of an electrostatic actuator

For fixed-fixed beams, tensile residual stress can limit the leveraged bending effect.

Leveraged bending [36] is a simpler method in comparison with the series feedback capacitance [4], and [26, 27], but mechanical advantage and increased device area are traded off for the increased range of motion [19].

It [3] is shown that using nonlinear flexures, the distance before “snap-through” was increased but the “snap-through” phenomenon was not eliminated. An additional benefit of using a higher order force flexure is shown in Figure 2.35; a large segment of the micromirror deflection range for second and third order flexure force is closer to a linear

function of applied voltage than deflection with linear flexure force. Micromirrors with continuously adjustable deflection distances may be difficult to control for high order flexure forces.

Linear force flexures provide a restorative force that is a linear function of deflection. Electrostatically actuated micromirrors supported by linear force flexures have a stable deflection region that is only a third of the initial separation distance. Nonlinear force flexures can increase the stable deflection region and increase the useful range of motion. Two categories of nonlinear flexures have been demonstrated: auxiliary flexures and dual spring constant flexures. These nonlinear flexures can be used to approximate higher order flexure spring constants. The nonlinear force flexure designs can be combined to further increase the stable deflection region.

In addition to an increased useful range of motion, nonlinear flexures provide a fabrication benefit as well; micromirrors with nonlinear flexures tend to suffer stiction to the substrate much less than micromirrors with linear flexures. Also, micromirrors with auxiliary flexures may have higher optical power ratings than micromirrors with traditional flexures because additional flexures assist in conducting absorbed optical power out of the micromirror [51]. Future research is directed towards developing nonlinear flexures that closely resemble the higher order flexures predicted by Figure 2.35. New designs are focusing on flexures with non-uniform cross-sections.

Current drive method [5] appears to have several advantages as follows. Some of them were analyzed theoretically in [20], where it was shown that the energy consumed by an electrostatic actuator in a switching event is drastically reduced as compared with a voltage drive. The main result shown in [20] is that a totally different dynamics is created by current drive in such a way that the actuator movement does not accelerate sensibly at the end of the deflection and does not claim for a large spike of current. It is demonstrated that a new procedure to drive electrostatic actuators based on pulsed-current drive can avoid the well-known “Pull-In” instability. Stable positions anywhere across the gap can be reached if the ideal charge drive is used.

In following, the non-ideality of the current drive method [5] is considered and it is seen that the using the non-ideal current source decreases the performance of this method. It means that considering the non-ideality results the values of the parasitic capacitances have

been identified as a key parameter governing the stability of equilibrium points. The parasitic voltage drive mechanism competition with the charge drive (the parasitic capacitances charge and discharge) results in a stable travel range reduction of the actuator position which will not be able to reach the entire gap. Not all the positions across the gap are stable positions, but that a certain condition has to be satisfied between the ratio of the parasitic capacitance to the initial actuator capacitance, σ , and the deflection of the mobile electrode at equilibrium point, as seen in Eq (2.96), in order to assure a stable position. It has been concluded that the larger the parasitic capacitance is, the smaller is the stable travel range of the actuator and at the limit of an infinitely large parasitic capacitance, a maximum travel range of only 1/3 of the gap can be reached. This means that the worst case in charge drive mode equals the best case in voltage drive mode. According to stability condition result in the precise value of the parasitic capacitance sets the limit of a stable travel range across the gap. The asymptotic value of 1/3 for large values of σ obviously tells us that any value of parasitic capacitance can be tolerated, but leading to stable positions provided the deflection remains smaller than 1/3 of the gap, consistent with the well-known results for a voltage drive circuit [24].

Leakage currents are the second origin of non-ideality of the charge drive mode described here. It has been shown that leakage currents tend to discharge both, parasitic and actuator capacitance and hence for time long enough, the stable position of the beam drifts towards its initial position. To prevent this behavior, a comparison with other charge controlled devices and circuits such as CCDs and dynamic memories are made and the concept of charge refresh is introduced. A trade-off between relative loss of the charge (position drift of the equilibrium point) and the minimum refresh frequency has been formulated.

The current driving mode has been considered as an interesting alternative because it reduces the required power and the kinetic energy [20], extends the stable deflection range beyond the 1/3 barrier, and does not slow the speed of the switching event, provided that the amount of charge delivered is properly sized [5]. On the other hand, the current drive suffers from the parasitic capacitance effects, which, if they are sufficiently large, convert the current drive in a conventional voltage drive where the voltage source is the parasitic capacitance itself. Leakage currents may also be a problem in designing practical circuits [16].

In order to compare each approach in [12], the obtained equations for the developed voltage, as seen in Eqs (2.102), (2.109) and (2.111), have been evaluated in this section, for

combinations of m , n and r , for a given maximum stable displacement of $0.9D_0$ and D_0 . First, the value of m is fixed, and afterwards, n and r are calculated from Eqs (2.101), (2.105), and (2.110) to achieve the desired value for X_{max} . As can be seen, from the results summarized in Table 2-3 from [12], Case A, requires the highest applied voltage. The voltage value extends from $5.2V_{pi}$ up to $+\infty$. Case B, with a fix value of $n = 1/5$ for the series capacitor, develops a voltage drop between $0.76V_{pi}$ and $0.4V_{pi}$, showing better results than the previous method. However, the best results are obtained with Case C, with a developed voltage of $0.25V_{pi}$ for a maximum displacement of $0.9D_0$ and zero voltage for combinations of m and r allowing full range of travel.

In following the voltage required to achieve full travel range for an electrostatic actuator connected to a series capacitor has been studied. It has been shown that, in the best case, full travel range can be achieved for an applied voltage, approximately, five times the “Pull-In” value, specifically, $5.2V_{pi}$. Then two different methods, based on current driving, have been theoretically analyzed showing that full range travel can be achieved, with smaller developed voltage values than the required in the voltage drive approach.

As predicted by simulations Approach B, the closed-loop configuration shown in Figure 2.25, can become unstable after several cycles of operation. In approach C, the open-loop configuration shown in Figure 2.26, the position of the electrode is fixed every refreshing cycle without stability problems and the position shift due to the leakage currents is eliminated by a refresh cycle.

Finally some important differences in the dynamics of the actuator, using a current source drive rather than voltage source drive, were found showing that the delivery of a controlled amount of charge to the actuator is a potential solution to extend the travel range beyond the “Pull-In” point, without actuator geometry changes [36], large voltage penalties [4], [19], and complex circuit implementations [27].

x_{max}/D_0	m	Voltage drive		Closed loop current drive		Open loop current drive	
		n	V_d/V_{pi}	r	V_d/V_{pi} ($n=0.2$)	r	V_d/V_{pi}
0.9	$\rightarrow \infty$	1.7	4.44	1.5	0.74	1.7	0.25
1	$\rightarrow \infty$	2	5.2	1.8	0.52	2	0
0.9	10	2.05	5.35	1.85	0.74	2.05	0.25
1	10	2.5	6.5	2.3	0.52	2.5	0
0.9	5	2.58	6.73	2.38	0.75	2.58	0.25
1	5	3.33	8.65	3.13	0.52	3.33	0
0.9	2	11.33	29.57	11.13	0.76	11.33	0.25
1	2	$\rightarrow \infty$	$\rightarrow \infty$	$\rightarrow \infty$	0.52	$\rightarrow \infty$	0

Table 2-3. Comparison of the methods to extend the stable range of travel

Several conclusions may be drawn from the calculations shown in δ -pulse current drive [16]. It is seen that a finite value of the damping coefficient does not affect the maximum value of the voltage developed across the actuator, but more parasitic capacitance, a realistic shape of δ -pulse current, decreases it directly. Then it is shown that the actuator dynamics using a δ -pulse current is the same as the dynamics resulting from a voltage drive with a capacitor in series. The δ -pulse current drive requires much less voltage than voltage drive. The maximum voltage developed across the actuator for a δ -pulse current drive is proportional to the injected charge.

The significant limitations of the mentioned control schemes above have caused using of the closed-loop feedback control techniques for system driving in extending the desired operation range of the electrostatic microactuators [2]. This work primarily focused on the development of a nonlinear tracking control system that stabilizes electrostatic microactuators and increased the range of stable and controllable motion to the entire capacitive gap under suitable operation conditions and ensuring the desired dynamic performance is discussed and analyzed. An input-output feedback linearization approach to nonlinear state feedback controller design is illustrated. The numerical simulations demonstrate explicitly that the proposed control scheme has extremely good tracking performance. The effect of the natural damping of the mechanical system on the

microactuator dynamics is also appreciably diminished with this control strategy. It is shown that with normalized deflection as output, the driven system is feedback linearizable with relative degree three (equal to the system order) in the region of interest. Simulation results show that the proposed nonlinear control scheme not only has good tracking ability, but also has extremely good parameter variation robustness. But the simulation result in Figure 2.55 shows a major drawback in this control strategy which makes it difficult to be implemented because of the controller structure. The control signal, input of device, contains a term of $1/x_3$, in which x_3 is the normalized charge across the plates. From controller structure it is obvious that when x_3 goes to zero, the control input signal will become infinity this causes extra ordinary fluctuations inside of normalized charge curve.

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -z_1 - 2\alpha z_2 + \frac{pz_3}{3} \\ \dot{z}_3 = -\frac{z_3}{\tau} + \frac{2}{3\tau} v_z \\ y_z = z_1 \end{cases} \quad (3.1)$$

If $0 \leq |x_3| \leq p$ and p is small enough then auxiliary model (3.1) will be similar to modified model (2.70). For more similarity, the initial conditions of the auxiliary model are defined as same as the modified model initial conditions.

3.3 Auxiliary Controller Design for Auxiliary Model

The purpose is to design an auxiliary controller for the device modified model (2.70). Because the model (2.70) is a nonlinear model and controller design for nonlinear models is more difficult than the controller design for the linear system, a linear model called auxiliary model (3.1) is defined. As it is seen in model (3.1), v_z is the control input for the auxiliary model and so called auxiliary controller. In this part, v_z is designed for the model (3.1) to comply output tracking and stability of closed loop system. In the next part this controller design, v_z , is used for the modified model (2.70) by setting $v = v_z$ in the model (2.70) and a analytical proof is given which shows that $v = v_z$ complies all requirements of the modified model (2.70).

For auxiliary model, error matrix is defined below:

$$\begin{cases} \beta_1 = \beta = z_1 - y_r \\ \beta_2 = \dot{\beta} = z_2 \\ \beta_3 = \ddot{\beta} = -z_1 - 2\alpha z_2 + \frac{pz_3}{3} \end{cases} \quad (3.2)$$

The control objective is output tracking. To generate a linear input-error relation, the error function is repeatedly differentiated until the control signal v_z appears for the first time on the right-hand side and then design v_z to achieve the desired closed-loop system performance [46].

From Eqs (3.1) and (3.2), it results that

$$\beta^{(3)} = -\dot{z}_1 - 2\alpha \dot{z}_2 + \frac{p\dot{z}_3}{3} = -\frac{1}{\tau} y_r - \frac{1}{\tau} \beta_1 - \left(1 + \frac{2\alpha}{\tau}\right) \beta_2 - \left(2\alpha + \frac{1}{\tau}\right) \beta_3 + \frac{2p}{9\tau} v_z \quad (3.3)$$

It is open-loop description of the auxiliary model. Because this system is an LTI system, so control signal is designed according to states feedback theory and it can be written as follows [52].

$$v_z = \frac{b_0}{p} y_r - \frac{b_1}{p} z_1 - \frac{b_2}{p} z_2 - b_3 z_3 \quad (3.4)$$

It can be rewritten according to the system errors.

$$p v_z = a_0 y_r - a_1 \beta_1 - a_2 \beta_2 - a_3 \beta_3 \quad (3.5)$$

Substituting this description of v_z in Eq (3.3), it results that:

$$\beta_4 = \beta^{(3)} = k_0 y_r - k_1 \beta_1 - k_2 \beta_2 - k_3 \beta_3 \quad (3.6)$$

Eq (3.6) is closed-loop description of the auxiliary system. If $k_0 = 0$ it gives:

$$\beta^{(3)} = -k_1 \beta - k_2 \dot{\beta} - k_3 \ddot{\beta} \quad (3.7)$$

The coefficients are chosen such that all roots, of polynomial, $s^3 + k_3 s^2 + k_2 s + k_1 = 0$ are placed in the left handed plane [52]. Exerting the initial condition in Eq (3.2), error function can be easily calculated in time domain and then from Eq (3.7), it is derived that:

$$z_1 = y_r + A_1 e^{-s_1 t} + A_2 e^{-s_2 t} + A_3 e^{-s_3 t} \quad (3.8)$$

$$\frac{p z_3}{3} = y_r + B_1 e^{-s_1 t} + B_2 e^{-s_2 t} + B_3 e^{-s_3 t} \quad (3.9)$$

3.4 Applying Auxiliary Controller to Microactuator Model

Now the control signal v_z is applied to the modified model (2.70), which it means $v = v_z$. It results that $x_3 = z_3$ because the third equations of both models (2.70) and (3.1) and also their initial conditions are exactly the same. Noting controller design in Eq (3.4), an important issue is how the control signal v can be rewritten according to variables of the modified model. For this purpose it is needed to attain the state variables of the auxiliary model according to the state variables of the modified model as

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2\alpha \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(\frac{p x_3 - x_3^2}{3} \right) \quad (3.10)$$

where

$$\begin{cases} \Delta x_1 = z_1 - x_1, & \Delta x_1(0) = 0 \\ \Delta x_2 = z_2 - x_2, & \Delta x_2(0) = 0 \end{cases} \quad (3.11)$$

Then according to controller design, $v = v_z$, and equation (3.4) control signal v is attained as

$$v = \frac{b_0}{p} y_r - \frac{b_1}{p} (x_1 + \Delta x_1) - \frac{b_2}{p} (x_2 + \Delta x_2) - b_3 x_3 = \frac{b_0}{p} y_r - \frac{b_1}{p} x_1 - \frac{b_2}{p} x_2 - b_3 x_3 - \frac{b_1}{p} \Delta x_1 - \frac{b_2}{p} \Delta x_2 \quad (3.12)$$

The device is studied under steady state condition, so reference input y_r will be in form of step function with final value of k . The objective is to calculate output signal x_1 using control signal in Eq (3.12). For better understanding of this method a numerical example is given which holds the general proof. For instance suppose that $s_1 = 4, s_2 = 5, s_3 = 7$ and $y_r = k$, then Eq (3.9) results that

$$\frac{x_3^2}{3} = \frac{3}{p^2} (k^2 + B_1^2 e^{-8t} + B_2^2 e^{-10t} + B_3^2 e^{-14t}) + \frac{6k}{p^2} (B_1 e^{-4t} + B_2 e^{-5t} + B_3 e^{-7t}) + \frac{6}{p^2} (B_1 B_2 e^{-9t} + B_1 B_3 e^{-11t} + B_2 B_3 e^{-12t}) \quad (3.13)$$

For aforementioned root selection, the term $x_3^2/3$ consists of nine different exponential functions. According to the first and second equations of the modified model, output signal x_1 can be attained as

$$\ddot{x}_1 + 2\alpha \dot{x}_1 + x_1 = \frac{x_3^2}{3} \quad (3.14)$$

The variable x_1 can be separated into sum of two terms. The term x_1^h is all possible homogeneous responses and the term x_1^p is a particular private response for the differential equation in Eq (3.14). The term x_1^h can be calculated from differential equation below

$$\ddot{x}_1^h + 2\alpha \dot{x}_1^h + x_1^h = 0 \quad (3.15)$$

The analytical response of differential equation above is given as

$$\begin{cases} x_1^h = D_1 e^{-t} + D_2 t e^{-t} & \alpha = 1 \\ x_1^h = D_1 e^{q_1 t} + D_2 t e^{q_2 t} & \alpha \neq 1 \end{cases} \quad (3.16)$$

where q_1 and q_2 are roots of $q^2 + 2\alpha q + 1 = 0$. To find, coefficients D_1 and D_2 , it is needed to exert the initial conditions into differential equation. For values of α less than one; q_1, q_2, D_1 and D_2 are complex numbers. Also the term x_1^p can be calculated from Eqs (3.13) and (3.14) as

$$x_1^p = C_0 + C_1 e^{-4t} + C_2 e^{-5t} + C_3 e^{-7t} + C_4 e^{-8t} + C_5 e^{-9t} + C_6 e^{-10t} + C_7 e^{-11t}$$

$$+C_8e^{-12t}+C_9e^{-14t} \quad (3.17)$$

The coefficients C_i can be attained with substituting Eq (3.17) and (3.13) in Eq (3.14) which results C_0 equal to $3k^2/p^2$. If a new variable called δ is defined by $3k/p^2$, then from Eqs (3.9), (3.16) and (3.17) state variables x_1 and x_3 can be rewritten in the form

$$x_3 = \delta p + \sum \text{exp functions 1}, \quad x_1 = x_1^p + x_1^h = \delta k + \sum \text{exp functions 2} \quad (3.18)$$

The output tracking condition results that $\delta k = k$, which is possible only for non-negative values of k and hence the reference input takes non-negative values. If k equals to zero then the values of δk and δp will be zero and both signals converge to zero and output tracking objective is achieved. So the parameter p can be chosen arbitrary. If k is positive, for confirming the output tracking variable δ must be equal to one. Hence the threshold value is set $p = \sqrt{3k}$.

The coefficients in Eq (3.5) are calculated from Eqs (3.3), (3.5) and (3.6) as follow

$$\begin{cases} a_0 = \frac{9\tau}{2}k_0 + \frac{9}{2} \\ a_1 = \frac{9\tau}{2}k_1 - \frac{9}{2} \\ a_2 = \frac{9\tau}{2}k_2 - \frac{9\tau}{2} - 9\alpha \\ a_3 = \frac{9\tau}{2}k_3 - 9\alpha\tau - \frac{9}{2} \end{cases} \quad (3.19)$$

Finally the controller coefficients are calculated from Eqs (3.2), (3.4) and (3.5) as follow

$$\begin{cases} b_0 = a_0 + a_1 \\ b_1 = a_1 - a_3 \\ b_2 = a_2 - 2\alpha a_3 \\ b_3 = \frac{a_3}{3} \end{cases} \quad (3.20)$$

3.5 Simulation Results and Discussions

Figures 3.2 to 3.9 show the simulation results of auxiliary controller design control scheme for three different values of α (from 0.2 to 2). It is seen that the stable and controllable travel range of the device to full gap is achieved in normalized deflection curve. Also there are no extra ordinary fluctuations in normalized charge curve. But for α values less than one, the overshooting is occurred in the normalized deflection curve.

All related equations to the simulation results are named as follow:

Equation (2.66) is the MEMS device model. Equation (2.70) is the modified model which is the same as MEMS device model with a feedback linearization inside. Equation (2.76) is v^* definition. Equation (2.78) is the tracking error dynamic. Equation (2.80) is the controller function for the MEMS device model. Equation (3.10) is deviation of the auxiliary model from the modified model. Equation (3.12) is the auxiliary controller function for the modified model.

Figure 3.2. Normalized deflection (x_1) Vs. time, $\alpha = 2$, auxiliary controller design control scheme

Figure 3.3. Normalized charge (x_3) Vs. time, $\alpha = 2$, auxiliary controller design control scheme

Figure 3.4. Normalized deflection (x_1) Vs. time, $\alpha = 1$, auxiliary controller design control scheme

Figure 3.5. Normalized charge (x_3) Vs. time, $\alpha = 1$, auxiliary controller design control scheme

Figure 3.6. Normalized deflection (x_1) Vs. time, $\alpha = 0.2$, auxiliary controller design control scheme

Figure 3.7. Normalized charge (x_3) Vs. time, $\alpha = 0.2$, auxiliary controller design control scheme

3.6 Conclusions

An auxiliary controller design control scheme is proposed in this chapter. It is clear that the stable and controllable whole gap displacement, output tracking condition in the normalized deflection curve and elimination of extra ordinary fluctuation in the normalized charge curve are achieved by using this control scheme. In simulation, three different values of α (from 0.2 to 2) are considered. If α takes very small values, near to zero, or very big values, much greater than one, the homogeneous response will converge to zero very slowly and especially for α equals to zero, it will never converge. Hence in these two cases, there is usage limitation of this method. Moreover for α values less than one, the overshooting is occurred in the normalized deflection curve. Two direct control schemes will be proposed in Chapters 4 and 5. They are designed for the modified model (2.70).

Chapter4. Non-Negative Input Design Control Scheme

In this chapter, a direct control scheme will be applied to the MEMS actuator system for the controller design to improve our first control scheme results by elimination of overshoot problem.

4.1 New Control Input Definition

Looking at the first and second equations of the modified model (2.70), it is found that the term $x_3^2/3$ plays the role of control signal, control input, and can make the desired shape of x_1 and x_2 curves.

In modified model, v is a new control signal; input. Now looking at the third equation shows that the term v only depends to the term x_3 and the term x_3 is so. So control signal v can be found from state variable x_3 . The control strategy is explained as follows.

If let the term $x_3^2/3$ is set to be u^* , then first and second equations of modified model become as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - 2\alpha x_2 + u^* \end{cases} \quad (4.1)$$

4.2 New Control Input Design

As it is described above, u^* is a control signal, control input, for the second order linear state space model above and using state feedback theory let u^* be [52]

$$u^* = k_0 y_r - k_1 x_1 - k_2 x_2 \quad (4.2)$$

Our analysis is for stability condition, so reference input y_r will be in form of step functions. From Eqs (4.1) and (4.2), the closed loop system is described as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a_1 x_1 - a_2 x_2 + k_0 y_r \end{cases} \quad (4.3)$$

First the purpose is to set controller gains in Eq (4.2) in order to obtain desired output, x_1 , curve followed by system stability and then calculating the control signal v according to u^* , by using the third equation of the modified model is explained. In fact Eq (4.1) is not a usual second order linear system because control signal u^* must always be non negative and this imposes some limitations which must be considered. For attaining the controller gains, it is easier to transfer to the Laplace domain. Controller design is somehow to reach exact

output tracking and stability of the closed loop system. It is assumed that the initial values of the state variables x_1 and x_2 are both zero. According to Eq (4.3) it results that

$$\begin{cases} sX_1(s) = X_2(s) \\ sX_2(s) = -a_1X_1(s) - a_2X_2(s) + k_0Y_r(s) \end{cases} \quad (4.4)$$

Then $X_1(s)$ and $X_2(s)$ are calculated from Eq (4.4) as

$$Y = X_1 = \frac{k_0}{s^2 + a_2s + a_1} Y_r, \quad X_2 = \frac{k_0s}{s^2 + a_2s + a_1} Y_r \quad (4.5)$$

The tracking error is defined by $e = y_r - y$. It is assumed that y_r is a step function with final value k . Output tracking means that after a time, tracking error goes to zero.

$$\lim_{t \rightarrow +\infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} s \frac{k}{s} \left(1 - \frac{k_0}{s^2 + a_2s + a_1} \right) = k \left(1 - \frac{k_0}{a_1} \right) = 0 \quad (4.6)$$

Eq (4.6) results $k_0 = a_1$. Now gains a_1 and a_2 must be designed to locate the real part of the poles of the closed loop system in Eq (4.4) in the left hand side plane [52]. To satisfy this condition, it is enough to have

$$\begin{cases} 0 < a_1 \\ 0 < a_2 \end{cases} \quad (4.7)$$

4.3 Analysis of Non-Negative Control Input Constraint

The control signal u^* must be non-negative. For satisfying the condition that the control signal u^* must be non-negative, it is necessary for u^* to be calculated in time domain. The Laplace transform of u^* is calculated easily according to Eqs (4.2), (4.5) and (4.6)

$$Laplace(u^*) = a_1Y_r - k_1X_1 - k_2X_2 = a_1Y_r \left(1 - \frac{k_2s + k_1}{s^2 + a_2s + a_1} \right) \quad (4.8)$$

Then Laplace inverse will be used to attain u^* in time domain:

$$u^* = a_1y_r(t) - a_1y_r(t) * Laplace^{-1} \left(\frac{k_2s + k_1}{s^2 + a_2s + a_1} \right) \quad (4.9)$$

It is necessary to have a partial fraction description of the second order polynomial; $s^2 + a_2s + a_1$. This polynomial is called the characteristic polynomial of the closed loop system. Suppose its roots to be s_1 and s_2 . A parameter called Δ is defined by $a_2^2 - 4a_1$. As it is known a_1 and a_2 are always positive numbers. Hence Δ is a real number but it is included three different cases such as positive, zero, negative. According to each case, the partial fraction description of the characteristic polynomial will be different from the other two

cases which are completely explained below. For calculating u^* in time domain, it is needed to rewrite the characteristic polynomial in terms of its roots for each case.

4.3.1 Δ Is Positive

In this case, according to Eq (4.7), the characteristic polynomial has two different real roots which are negative as

$$s^2 + a_2s + a_1 = (s + s_1)(s + s_2) \quad (4.10)$$

Suppose s_2 is bigger than s_1 . According to Eqs (4.1), (4.2), (4.3) and (4.10), it gives that

$$\left\{ \begin{array}{l} 0 < s_1 < s_2 \\ s_1s_2 = a_1 = 1 + k_1, \quad s_1 + s_2 = 2\alpha + k_2 \end{array} \right\} \quad (4.11)$$

According to Eqs (4.10) and (4.11), the fraction $(k_2s + k_1)/(s^2 + a_2s + a_1)$ can be decomposed as

$$\left\{ \begin{array}{l} \frac{k_2s + k_1}{s^2 + a_2s + a_1} = \frac{A_1}{s + s_1} + \frac{A_2}{s + s_2} \\ A_1 = -\frac{s_1^2 - 2\alpha s_1 + 1}{s_2 - s_1}, \quad A_2 = \frac{s_2^2 - 2\alpha s_2 + 1}{s_2 - s_1} \end{array} \right\} \quad (4.12)$$

For analyzing steady state of the system, the reference input must be in the form of step functions, so the general shape of the reference input y_r is shown below which $u_{-1}(t)$ is unit step function

$$y_r(t) = M_2u_{-1}(t) - M_1u_{-1}(t - T), \quad 0 < T, \quad 0 \leq M_1 \leq M_2 \quad (4.13)$$

According to Convolution definition, it is derived that

$$u_{-1}(t) * e^{-st} = \frac{1 - e^{-st}}{s} u_{-1}(t), \quad u_{-1}(t - T) * e^{-st} = \frac{1 - e^{-s(t-T)}}{s} u_{-1}(t - T) \quad (4.14)$$

Also from Laplace transformation definition, it is concluded that

$$\text{Laplace}^{-1} \left(\frac{1}{s + a} \right) = e^{-at} u_{-1}(t) \quad (4.15)$$

According to Eqs (4.9), (4.12), (4.13), (4.14) and (4.15), the term u^* is calculated as

$$\frac{u^*(t)}{a_1} = y_r(t) - y_r(t) * (A_1 e^{-s_1 t} + A_2 e^{-s_2 t}) = M_2 f(t) - M_1 f(t - T) \geq 0 \quad (4.16)$$

where:

$$f(t) = \left[1 - A_1 \left(\frac{1 - e^{-s_1 t}}{s_1} \right) - A_2 \left(\frac{1 - e^{-s_2 t}}{s_2} \right) \right] u_{-1}(t) \quad (4.17)$$

To satisfy the condition that the control signal u^* must be non-negative for all $t \geq 0$, it is easier to consider t in two intervals as

$$(1): \text{if } 0 \leq t < T \Rightarrow f(t - T) = 0 \Rightarrow 0 \leq \frac{u^*(t)}{a_1} = M_2 f(t) \Rightarrow 0 \leq f(t) \quad (4.18)$$

Since T is arbitrary so it results that

$$\forall 0 \leq t \Rightarrow 0 \leq f(t) \quad (4.19)$$

$$(2): \text{if } T < t \Rightarrow 0 \leq M_2(f(t) - f(t - T)) \leq \frac{u^*(t)}{a_1} \Rightarrow f(t - T) \leq f(t) \quad (4.20)$$

Since T is arbitrary and according to Eq (4.20) it is derived that $f(t)$ must be a rising function. If so, Eq (4.19) is also satisfied because $f(0) = 1$. So the function $f(t)$ is differentiated with respect to t , to find out under which condition, $f(t)$ is always a rising function.

$$0 \leq \dot{f}(t) = -(A_1 e^{-s_1 t} + A_2 e^{-s_2 t}) \Rightarrow A_1 e^{(s_2 - s_1)t} + A_2 \leq 0 \quad (4.21)$$

The condition $s_2 > s_1$, results that $1 \leq e^{(s_2 - s_1)t} \leq +\infty$. Then condition in Eq (4.21) can be satisfied only if $A_1 \leq 0$ and it can be analyzed into two cases as follow

Case (1): $A_1 = 0$ and $A_2 \leq 0$:

Using Eq (4.12) it results that

$$A_1 = 0 \Rightarrow \{1 \leq \alpha\} \wedge \begin{cases} s_1 = \alpha - \sqrt{\alpha^2 - 1} \\ \vee \\ s_1 = \alpha + \sqrt{\alpha^2 - 1} \end{cases} \quad (4.22)$$

$$A_2 \leq 0 \Rightarrow \{1 \leq \alpha\} \wedge \{\alpha - \sqrt{\alpha^2 - 1} \leq s_2 \leq \alpha + \sqrt{\alpha^2 - 1}\} \quad (4.23)$$

Therefore

$$\{1 < \alpha\} \wedge \{s_1 = \alpha - \sqrt{\alpha^2 - 1}\} \wedge \{\alpha - \sqrt{\alpha^2 - 1} < s_2 \leq \alpha + \sqrt{\alpha^2 - 1}\} \quad (4.24)$$

Case (2): $A_1 < 0$ and $A_1 + A_2 \leq 0$:

Using Eq (4.12) it results that

$$A_1 < 0 \Rightarrow \{0 \leq \alpha < 1\} \vee \begin{cases} 1 \leq \alpha \\ \wedge \\ s_1 < \alpha - \sqrt{\alpha^2 - 1} \\ \wedge \\ 0 < s_1 \end{cases} \vee \begin{cases} 1 \leq \alpha \\ \wedge \\ \alpha + \sqrt{\alpha^2 - 1} < s_1 \end{cases} \quad (4.25)$$

$$A_1 + A_2 \leq 0 \Rightarrow \{0 < \alpha\} \wedge \begin{cases} s_1 + s_2 \leq 2\alpha \\ \wedge \\ 0 < s_1 < s_2 \end{cases} \quad (4.26)$$

Therefore

$$\left\{ \begin{array}{c} 0 < \alpha < 1 \\ \wedge \\ s_1 + s_2 \leq 2\alpha \\ \wedge \\ 0 < s_1 < s_2 \end{array} \right\} \vee \left\{ \begin{array}{c} 1 \leq \alpha \\ \wedge \\ s_1 < \alpha - \sqrt{\alpha^2 - 1} \\ \wedge \\ s_1 + s_2 \leq 2\alpha \\ \wedge \\ 0 < s_1 < s_2 \end{array} \right\} \quad (4.27)$$

It is necessary that one of Eqs (4.24) and (4.27) is satisfied to ensure that the condition that the control signal u^* must be non-negative is satisfied. According to Eqs (4.5) and (4.10), s_1 and s_2 are closed loop poles. From the linear control, it is known that the location of the closed loop poles in the S-plane determines the speed of the system time response. If they locate nearer to the imaginary axis, their real part is important, the system time response will be slower [52]. From the conditions in Eqs (4.24) and (4.27), it is clear that if α goes to infinity or zero then s_1 goes to zero which mean it comes toward zero-real axis. Hence for very small or very big values of α , the system time response is slow. For the case of α equal to zero, the condition in Eq (4.21) cannot be satisfied because the conditions in Eqs (4.24) and (4.27) do not contain this case, so this approach does not work for α equal to zero.

4.3.2 Δ Is Zero

In this case, the characteristic polynomial has a real double root which is negative. Hence it can be written as

$$s^2 + a_2s + a_1 = (s + s_0)^2 \quad (4.28)$$

According to Eqs (4.1), (4.2), (4.3) and (4.28)

$$\left\{ \begin{array}{c} 0 < s_0 \\ s_0^2 = a_1 = 1 + k_1, \quad 2s_0 = a_2 = 2\alpha + k_2 \end{array} \right\} \quad (4.29)$$

According to Eqs (4.28) and (4.29), the fraction $(k_2s + k_1)/(s^2 + a_2s + a_1)$ can be decomposed as

$$\left\{ \begin{array}{l} \frac{k_2s + k_1}{s^2 + a_2s + a_1} = \frac{k_2(s + s_0) + k_1 - k_2s_0}{(s + s_0)^2} = \frac{k_2}{s + s_0} + \frac{k_1 - k_2s_0}{(s + s_0)^2} \\ k_2 = 2s_0 - 2\alpha, \quad k_1 - k_2s_0 = -(s_0^2 - 2\alpha s_0 + 1) \end{array} \right\} \quad (4.30)$$

According to the Convolution and definition, it is derived that

$$u_{-1}(t) * te^{-st} = \frac{1 - (1 + st)e^{-st}}{s^2} u_{-1}(t), \rightarrow$$

$$\rightarrow u_{-1}(t-T) * te^{-st} = \frac{1 - (1 + s(t-T))e^{-s(t-T)}}{s^2} u_{-1}(t-T) \quad (4.31)$$

Also from Laplace transformation definition, it is concluded that

$$\text{Laplace}^{-1}\left(\frac{1}{(s+a)^2}\right) = te^{-at}u_{-1}(t) \quad (4.32)$$

According to Eqs (4.9), (4.13), (4.14), (4.30), (4.31) and (4.32), u^* is calculated as

$$\begin{aligned} \frac{u^*(t)}{a_1} &= y_r(t) - y_r(t) * (k_2 e^{-s_0 t} + (k_1 - k_2 s_0) t e^{-s_0 t}) \Rightarrow \\ &\rightarrow M_2 f(t) - M_1 f(t-T) \geq 0 \end{aligned} \quad (4.33)$$

where:

$$f(t) = \left[1 - k_2 \left(\frac{1 - e^{-s_0 t}}{s_0} \right) - (k_1 - k_2 s_0) \left(\frac{1 - (1 + s_0 t) e^{-s_0 t}}{s_0^2} \right) \right] u_{-1}(t) \quad (4.34)$$

To satisfy the condition that the control signal u^* must be non-negative for all $t \geq 0$, it is easier to consider t in two intervals as

$$(1): \text{if } 0 \leq t < T \Rightarrow f(t-T) = 0 \Rightarrow 0 \leq \frac{u^*(t)}{a_1} = M_2 f(t) \Rightarrow 0 \leq f(t) \quad (4.35)$$

Since T is arbitrary, so:

$$\forall 0 \leq t \Rightarrow 0 \leq f(t) \quad (4.36)$$

$$(2): \text{if } T < t \Rightarrow 0 \leq M_2(f(t) - f(t-T)) \leq \frac{u^*(t)}{a_1} \Rightarrow f(t-T) \leq f(t) \quad (4.37)$$

Since T is arbitrary and according to the condition in Eq (4.37), it is derived that $f(t)$ must be a rising function. If so, the condition in Eq (4.36) is also satisfied because $f(0) = 1$. So the function $f(t)$ is differentiated with respect to t , to find out under which condition, $f(t)$ is always a rising function.

$$0 \leq \dot{f}(t) = -(k_2 e^{-s_0 t} + (k_1 - k_2 s_0) t e^{-s_0 t}) \Rightarrow k_2 + (k_1 - k_2 s_0) t \leq 0 \quad (4.38)$$

Because $0 \leq t < +\infty$, satisfying the condition in Eq (4.38) results that $k_2 \leq 0$ and also $(k_1 - k_2 s_0) t \leq 0$. According to Eq (4.29) it is derived that

$$k_2 \leq 0 \Rightarrow \{0 < \alpha\} \wedge \{0 < s_0 \leq \alpha\} \quad (4.39)$$

$$k_1 - k_2 s_0 \leq 0 \Rightarrow \{0 \leq \alpha \leq 1\} \vee \left\{ \begin{array}{c} 1 < \alpha \\ \wedge \\ s_0 \leq \alpha - \sqrt{\alpha^2 - 1} \\ \wedge \\ 0 < s_0 \end{array} \right\} \vee \left\{ \begin{array}{c} 1 < \alpha \\ \wedge \\ \alpha + \sqrt{\alpha^2 - 1} \leq s_0 \end{array} \right\} \quad (4.40)$$

Therefore

$$\left\{ \begin{array}{c} 0 < \alpha \leq 1 \\ \wedge \\ 0 < s_0 \leq \alpha \end{array} \right\} \vee \left\{ \begin{array}{c} 1 < \alpha \\ \wedge \\ 0 < s_0 \leq \alpha - \sqrt{\alpha^2 - 1} \end{array} \right\} \quad (4.41)$$

It is needed that the condition in Eq (4.41) is satisfied to ensure the condition that the control signal u^* must be non-negative is satisfied. According to Eqs (4.5) and (4.28), s_0 is the closed loop pole of the system. From the linear control, it is known that the location of the closed loop poles in the S-plane determines the speed of the system time response. If they locate nearer to the imaginary axis, their real part is important, the system time response will be slower [52]. From condition in Eq (4.41), it is clear that if α goes to infinity or zero then s_0 goes to zero which mean it comes toward zero-real axis. Hence for very small or very big values of α , the system time response is slow. For the case of α equal to zero, the condition in Eq (4.41) cannot be satisfied because the condition in Eq (4.39) does not contain this case, so this approach does not work for α equal to zero.

4.3.2 Δ Is Negative

In this case the characteristic polynomial has two complex conjugate roots with negative real part. Hence it can be written in the form of

$$s^2 + a_2s + a_1 = (s + s_1 + js_2)(s + s_1 - js_2) \quad (4.42)$$

According to Eqs (4.1), (4.2), (4.3) and (4.42), it is derived that

$$\left\{ \begin{array}{c} 0 < s_1, \quad 0 < s_2 \\ s_1^2 + s_2^2 = a_1 = 1 + k_1, \quad 2s_1 = a_2 = 2\alpha + k_2 \end{array} \right\} \quad (4.43)$$

According to Eqs (4.42) and (4.43), the fraction $(k_2s + k_1)/(s^2 + a_2s + a_1)$ can be decomposed as

$$\left\{ \begin{array}{c} \frac{k_2s + k_1}{s^2 + a_2s + a_1} = \frac{A_1}{s + s_1 + js_2} + \frac{A_2}{s + s_1 - js_2} \\ P_1 = A_1 + A_2, \quad P_2 = j(A_2 - A_1) \\ P_1 = 2s_1 - 2\alpha, \quad P_1s_1 + P_2s_2 = s_1^2 + s_2^2 - 1 \end{array} \right\} \quad (4.44)$$

From Eqs (4.15) and (4.44), it is concluded that

$$\text{Laplace}^{-1} \left(\frac{k_2s + k_1}{s^2 + a_2s + a_1} \right) = P_1 e^{-s_1 t} \cos(s_2 t) + P_2 e^{-s_1 t} \sin(s_2 t) \quad (4.45)$$

According to the Convolution definition, it is derived that

$$\begin{aligned}
 u_{-1}(t) * e^{-s_1 t} \cos(s_2 t) &= \frac{1}{a_1} [\cos(\theta) - e^{-s_1 t} \cos(s_2 t + \theta)] u_{-1}(t), \rightarrow \\
 \rightarrow u_{-1}(t - T) * e^{-s_1 t} \cos(s_2 t) &= \frac{1}{a_1} [\cos(\theta) - e^{-s_1(t-T)} \cos(s_2(t - T) + \theta)] u_{-1}(t) \rightarrow \\
 \rightarrow u_{-1}(t) * e^{-s_1 t} \sin(s_2 t) &= \frac{1}{a_1} [\sin(\theta) - e^{-s_1 t} \sin(s_2 t + \theta)] u_{-1}(t), \rightarrow \\
 \rightarrow u_{-1}(t - T) * e^{-s_1 t} \sin(s_2 t) &= \frac{1}{a_1} [\sin(\theta) - e^{-s_1(t-T)} \sin(s_2(t - T) + \theta)] u_{-1}(t) \rightarrow \\
 \rightarrow \cos(\theta) &= \frac{s_1}{\sqrt{s_1^2 + s_2^2}}, \quad \sin(\theta) = \frac{s_2}{\sqrt{s_1^2 + s_2^2}}, \quad a_1 = \sqrt{s_1^2 + s_2^2} \quad (4.46)
 \end{aligned}$$

According to Eqs (4.9), (4.13), (4.45) and (4.46), u^* is calculated as

$$\begin{aligned}
 \frac{u^*(t)}{a_1} &= y_r(t) - y_r(t) * (P_1 e^{-s_1 t} \cos(s_2 t) + P_2 e^{-s_1 t} \sin(s_2 t)) \Rightarrow \\
 \rightarrow M_2 f(t) - M_1 f(t - T) &\geq 0 \quad (4.47)
 \end{aligned}$$

where:

$$\begin{aligned}
 f(t) &= \left(1 - \frac{P_1}{a_1} [\cos(\theta) - e^{-s_1 t} \cos(s_2 t + \theta)] \right) \rightarrow \\
 \rightarrow -\frac{P_2}{a_1} [\sin(\theta) - e^{-s_1 t} \sin(s_2 t + \theta)] & u_{-1}(t) \quad (4.48)
 \end{aligned}$$

To satisfy the condition that the control signal u^* must be non-negative for all $t \geq 0$, it is easier to consider t in two intervals as

$$(1) \text{ if } 0 \leq t < T \Rightarrow f(t - T) = 0 \Rightarrow 0 \leq \frac{u^*(t)}{a_1} = M_2 f(t) \Rightarrow 0 \leq f(t) \quad (4.49)$$

Since T is arbitrary, so

$$\forall 0 \leq t \Rightarrow 0 \leq f(t) \quad (4.50)$$

$$(2) \text{ if } T < t \Rightarrow 0 \leq M_2(f(t) - f(t - T)) \leq \frac{u^*(t)}{a_1} \Rightarrow f(t - T) \leq f(t) \quad (4.51)$$

Since T is arbitrary and according to the condition in Eq (4.51), it is derived that $f(t)$ must be a rising function. If so, the condition in Eq (4.50) is also satisfied because $f(0) = 1$. So the function $f(t)$ is differentiated with respect to t , to find out under which condition, $f(t)$ is always a rising function.

$$\begin{aligned}
 0 \leq \dot{f}(t) &= -(P_1 e^{-s_1 t} \cos(s_2 t) + P_2 e^{-s_1 t} \sin(s_2 t)) \Rightarrow \\
 \rightarrow P_1 \cos(s_2 t) + P_2 \sin(s_2 t) &\leq 0 \quad (4.52)
 \end{aligned}$$

The condition in Eq (4.52) is satisfied if and only if $P_1 = P_2 = 0$. According to Eq (4.44) it is derived that

$$P_1 = 0 \Rightarrow \{0 < \alpha\} \wedge \{s_1 = \alpha\} \quad (4.53)$$

From Eqs (4.44) and (4.53)

$$P_1 = P_2 = 0 \Rightarrow \{\alpha < 1\} \wedge \{s_2 = \sqrt{1 - \alpha^2}\} \quad (4.54)$$

Therefore

$$\{0 < \alpha < 1\} \wedge \{s_1 = \alpha\} \wedge \{s_2 = \sqrt{1 - \alpha^2}\} \quad (4.55)$$

It is needed that the condition in Eq (4.55) is satisfied to ensure the condition that the control signal u^* must be non-negative is satisfied. According to the Eqs (4.5) and (4.42), s_1 is the real part of the closed loop complex conjugate poles. According to [52], the real part of imaginary poles determines the speed of the system time response. From the condition in Eq (4.55) if α goes to zero; then s_1 goes to zero. So for small values of α , near to zero, the time response is slow. For the case α equal to zero, the condition in Eq (4.53) cannot be satisfied, so this approach does not work for α equal to zero.

4.4 Controller Design

Now it is the time to explain how to design the control input v . Using Eq (4.2) it is derived that

$$u^* = \frac{x_3^2}{3} \Rightarrow x_3 = \pm\sqrt{3u^*} = \pm\sqrt{3(a_1y_r - k_1x_1 - k_2x_2)} \quad (4.56)$$

Here the positive case is chosen arbitrary as

$$x_3 = \sqrt{3(a_1y_r - k_1x_1 - k_2x_2)} \quad (4.57)$$

The idea is that the third equation of modified model (2.70) is used to obtain control input v .

$$\dot{x}_3 = -\frac{1}{\tau}x_3 + \frac{2}{3\tau}v = \frac{-3x_3 + 2v}{3\tau} \quad (4.58)$$

In fact the modified model (2.70) consists of two linear systems which are series. The first one is a first order system which is defined by Eq (4.58) with v as input and x_3 as output and the second one is a second order system defined by Eq (4.1) with u^* as input and x_1 as

output. The output of first system, x_3 makes the input of the second system, u^* , by $u^* = x_3^2/3$ and hence they are series.

τ is the time constant of the first system and is very small in the order of 10^{-6} . Hence the dynamic, transient response speed, of the first system is very fast. The second system is a typical second order LTI system and as it is obvious, its dynamic, transient response speed, is usual. Hence the dynamic of the first system can be ignored than the dynamic of the second system and so the first system can be approximated by an equivalent zero order system. The equivalent system is given by the equation of $-3x_3 + 2v/3\tau = 0$, which gives $x_3 = 2v/3$. Hence the approximation (4.49) is a suitable approximation with a high accuracy.

From Eq (4.57), the design of control input v is as follow

$$-\frac{1}{\tau}x_3 + \frac{2}{3\tau}v = 0 \Rightarrow v = \frac{3}{2}\sqrt{3(a_1y_r - k_1x_1 - k_2x_2)} \quad (4.59)$$

Finally control input u design is given by:

$$u = v - \frac{3}{2}x_1x_3 = \frac{3}{2}\sqrt{3(a_1y_r - k_1x_1 - k_2x_2)} - \frac{3}{2}x_1x_3 \quad (4.60)$$

In following tables, the closed-loop roots selection according to Eqs (4.24), (4.27), (4.41) and (4.55) has been summarized to reach the possible maximum speed.

Δ Is Positive:

Case (1):

$1 < \alpha$	
$s_1 = \alpha - \sqrt{\alpha^2 - 1}$	$s_2 = \alpha + \sqrt{\alpha^2 - 1}$

Table 4-1. Characteristic polynomial roots for case (1) and $\Delta > 0$

Case (2):

$0 < \alpha < 1$	
$s_1 = \alpha - \Delta\alpha$	$s_2 = \alpha + \Delta\alpha$

Table 4-2. Characteristic polynomial roots for case (2) and $\Delta > 0$

$1 \leq \alpha$	
$s_1 = \alpha - \sqrt{\alpha^2 - 1} - \Delta\alpha$	$s_2 = \alpha + \sqrt{\alpha^2 - 1} + \Delta\alpha$

Table 4-3. Characteristic polynomial roots for case (2) and $\Delta > 0$

Δ Is Zero:

$0 < \alpha \leq 1$	$s_0 = \alpha$
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Table 4-4. Characteristic polynomial root for $\Delta = 0$

$1 < \alpha$	$s_0 = \alpha - \sqrt{\alpha^2 - 1}$
--------------	--------------------------------------

Table 4-5. Characteristic polynomial root for $\Delta = 0$

Δ Is Negative:

$0 < \alpha < 1$	
$s_1 = \alpha$	$s_2 = \sqrt{1 - \alpha^2}$

Table 4-6. Characteristic polynomial real and imaginary part for $\Delta < 0$

To reach possible maximum speed for $0 < \alpha \leq 1$ among tables above, table 4-4 and for $1 < \alpha$ among tables above, table 4-1 has been used for the closed-loop roots selection.

The coefficients in Eq (4.3) are calculated from Eqs (4.10), (4.28) and (4.42) as follow

Δ Is Positive:

$$\begin{cases} a_1 = s_1 s_2 \\ a_2 = s_1 + s_2 \end{cases} \quad (4.61)$$

Δ Is Zero:

$$\begin{cases} a_1 = s_0^2 \\ a_2 = 2s_0 \end{cases} \quad (4.62)$$

Δ Is Negative:

$$\begin{cases} a_1 = s_1^2 + s_2^2 \\ a_2 = 2s_1 \end{cases} \quad (4.63)$$

Finally the controller coefficients in Eq (4.2) are calculated from Eqs (4.1), (4.2), (4.3), (4.61), (4.62) and (4.63) as follow

$$\begin{cases} k_0 = a_1 \\ k_1 = a_1 - 1 \\ k_2 = a_2 - 2\alpha \end{cases} \quad (4.59)$$

4.5 Simulation Results and Discussions

Figures 4.1-4.3 show the simulation results of non-negative input design control scheme for three different values of α (from 0.1 to 5). It can be seen that the stable and controllable travel range of the device to full gap is achieved without the overshooting. Also extra ordinary fluctuations do not appear in normalized charge curve because of the controller function in Eq (4.60).

All related equations to the simulation results are named as follow:

Eq (2.70) is the modified model. Eq (4.2) is u^* design. Eq (4.56) is u^* definition. Eq (4.57) is x_3 selection. Eq (4.59) is the controller function for the modified model (Equivalently Eq (4.60) can be used as the controller function for the MEMS device model). Eq (4.59) is the controller coefficients design.

Figure 4.1. Normalized deflection (x_1) Vs. time, $\alpha = 0.1$, non-negative input design control scheme

Figure 4.2. Normalized deflection (x_1) Vs. time, $\alpha = 1$, non-negative input design control scheme

Figure 4.3. Normalized deflection (x_1) Vs. time, $\alpha = 5$, non-negative input design control scheme

4.6 Conclusions

A non-negative input design control scheme is proposed in this chapter. It is clear that the stable and controllable whole gap displacement, output tracking condition in the normalized deflection curve are achieved by using the proposed control scheme. In

simulation, three different values of α (from 0.1 to 5) are considered. From conclusions of each three cases, it is clear that if α goes to infinity or zero then real part of the closed loop pole goes to zero which mean it comes toward zero-real axis. Hence for very small values of α , near to zero, or for very large values of α , much greater than one, the system time responses are slow. For the case of α equal to zero, this approach does not work too.

The advantage of this control scheme comparing to the previous scheme in Chapter 3 is elimination of the overshooting in the normalized deflection curve. A new direct control scheme will be proposed in Chapter 5.

Chapter 5. Positive State Variable Design Control Scheme

Finally another direct control scheme is described to design a new controller for developing our control scheme work space. Then mathematical proof is done to prove that this scheme works for a wider range of MEMS devices in comparison with the former control schemes. In this method, the system is a linear state space model but some state variables are positive [53].

5.1 State Variable Substitution

Consider the third equation of modified model (2.70) and multiply both sides by x_3 , it results that

$$x_3 \dot{x}_3 = -\frac{1}{\tau} x_3^2 + \frac{2}{3\tau} v x_3 \Rightarrow (\dot{x}_3^2) = -\frac{2}{\tau} x_3^2 + \frac{4}{3\tau} v x_3 \quad (5.1)$$

A new positive state variable is defined by $x_4 = x_3^2$. It is seen that always state variable x_4 is non-negative. Also a new control input needs is defined by $w = v x_3$. According to these changes, the modified model is rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - 2\alpha x_2 + \frac{1}{3} x_4 \\ \dot{x}_4 = -\frac{2}{\tau} x_4 + \frac{4}{3\tau} w \\ y = x_1 \end{cases} \quad (5.2)$$

Note that the state space model in Eq (5.2) seems to represent a linear model. However in reality there is a nonlinearity hidden inside the system and it is that state variable x_4 is non-negative. Now the new control input w will be designed to achieve the desired closed loop of system in Eq (5.2). Then control input v is attained from multiplying $1/x_3$ by control input w . Here x_3 equal to zero makes problem because $1/x_3$ goes to infinity and then control input v cannot be achievable from control input w . In order to avoid this problem it is needed to ensure that for all non-negative t , the state variable x_3 must not be zero. Consequently, the state variable x_4 , will be always non-negative for all non-negative t . Since one of the state variables of the model in Eq (5.2) is always positive, this method is called the positive state variable design.

$$\begin{cases} x_3 \neq 0 \Rightarrow x_4 \neq 0 \\ 0 \leq x_4 \end{cases} \Rightarrow 0 < x_4 \quad (5.3)$$

5.2 Controller Design

Because state space model in Eq (5.2) is a linear system, the control input w is designed according to state feedback theory as [52]

$$\begin{cases} w = k_0 y_r - k_1 x_1 - k_2 x_2 - k_4 x_4 \\ x_4 = x_3^2 \end{cases} \quad (5.4)$$

Our analysis is for stability condition, so reference input y_r will be in form of step functions. From Eq (5.4), control input v is given by

$$w = v x_3 \Rightarrow v = \frac{1}{x_3} (k_0 y_r - k_1 x_1 - k_2 x_2 - k_4 x_4) \quad (5.5)$$

Finally the control input u design is expressed as

$$u = v - \frac{3}{2} x_1 x_3 = \frac{1}{x_3} (k_0 y_r - k_1 x_1 - k_2 x_2 - k_4 x_4) - \frac{3}{2} x_1 x_3 \quad (5.6)$$

The closed loop system of state space model is obtained from Eqs (5.2) and (5.4) as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - 2\alpha x_2 + \frac{1}{3} x_4 \\ \dot{x}_4 = a_0 y_r - a_1 x_1 - a_2 x_2 - a_4 x_4 \\ y = x_1 \end{cases} \quad (5.7)$$

The closed loop system can be rewritten in general matrix form

$$\dot{x} = Ax + By_r, \quad y = Cx \quad (5.8)$$

According to Eqs (5.7) and (5.8) matrixes A , B and C are attained as

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -2\alpha & 1/3 \\ -a_1 & -a_2 & -a_4 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ a_0 \end{pmatrix}, \quad C = (1 \quad 0 \quad 0) \quad (5.9)$$

In contrast to the previous design, it is assumed that the initial conditions are all zero except x_3 and it is considered $x_3(0) = l$. For determining the desired closed loop poles, it is easier to transfer to the frequency domain, as before. The Laplace transformation of Eq (5.8) is

$$X(s) = (sI - A)^{-1} (BY_r(s) + D), \quad Y(s) = CX(s), \quad D = \begin{pmatrix} 0 \\ 0 \\ l^2 \end{pmatrix} \quad (5.10)$$

From Eqs (5.9) and (5.10), the characteristic polynomial of the closed loop system in Eq (5.7) is given by the formula

$$\Delta(s) = |sI - A| = s^3 + (2\alpha + a_4)s^2 + \left(2\alpha a_4 + \frac{1}{3}a_2 + 1\right)s + \left(a_4 + \frac{1}{3}a_1\right) \quad (5.11)$$

Our focus is on the state variables x_1 and x_4 . They can be attained using Eq (5.10) as

$$Y = X_1(s) = \frac{a_0 Y_r(s) + l^2}{3\Delta(s)}, \quad X_4(s) = \frac{(s^2 + 2\alpha s + 1)}{\Delta(s)}(a_0 Y_r(s) + l^2) \quad (5.12)$$

The tracking error is defined by $e = y_r - y$. It is assumed that y_r is a step function with final value k . For designing of the desired state variable x_1 , the tracking error is needed to be zero to ensure output tracking.

$$\begin{aligned} \lim_{s \rightarrow 0} sE(s) &= \lim_{s \rightarrow 0} s \left(Y_r(s) - \frac{a_0 Y_r(s) + l^2}{3\Delta(s)} \right) = \lim_{s \rightarrow 0} s \left(Y_r(s) - \frac{a_0 Y_r(s)}{3\Delta(s)} \right) \Rightarrow \\ &\rightarrow \lim_{s \rightarrow 0} s \frac{k}{s} \left(1 - \frac{a_0}{3\Delta(s)} \right) = k \left(1 - \frac{a_0}{3\Delta(0)} \right) \Rightarrow a_4 + \frac{1}{3}a_1 = \frac{1}{3}a_0 \end{aligned} \quad (5.13)$$

The desired closed loop system in Eq (5.7) needs to be stable as well and it depends to the characteristic polynomial roots which are closed loop poles. It is needed that all roots to locate in left hand of imaginary axis. As like as previous control scheme, it can be analyzed into several cases. One of these cases is that $\Delta(s)$ has three real different roots which are negative. Hence it can be written as

$$\Delta(s) = (s + s_0)(s + s_1)(s + s_2), \quad 0 < s_0 < s_1 < s_2 \quad (5.14)$$

If all roots are negative, the closed loop system in Eq (5.7) will be stable. Notice that according to Eqs (5.11) and (5.14), the coefficients a_1 , a_2 and a_4 can be attained from the roots above. Also from Eq (5.13) and (5.14), a_0 is equal to $3s_0s_1s_2$.

5.3 Analysis of Positive State Variable Constraint

Now our focus is to design the state variable x_4 to satisfy the condition in Eq (5.3). The state variable x_4 needs to be calculated in time domain because this condition is in time domain. Here the Laplace inverse transformation is used to calculate state variable x_4 in time domain. According to Eq (5.12)

$$x_4(t) = a_0 y_r(t) * Laplace^{-1} \left(\frac{s^2 + 2\alpha s + 1}{\Delta(s)} \right) + l^2 Laplace^{-1} \left(\frac{s^2 + 2\alpha s + 1}{\Delta(s)} \right) \quad (5.15)$$

Because the reference input is in the form of step functions as shown in Eq (4.13), hence $y_r(t)$ is always non-negative. The idea to satisfy the condition in Eq (5.15) is that

$$\begin{aligned} \text{if } g(t) = \text{Laplace}^{-1} \left(\frac{s^2 + 2\alpha s + 1}{\Delta(s)} \right) \geq 0 \Rightarrow \forall t > 0: y_r(t) * g(t) > 0 \Rightarrow \forall t > 0: \rightarrow \\ \rightarrow x_4(t) = a_0 y_r(t) * g(t) + l^2 g(t) > 0 \end{aligned} \quad (5.16)$$

Also it is needed that at time $t = 0$, x_4 to be positive. From assumption $x_3(0) = l$, it is derived that $x_4(0) = l^2$ and so $x_4(0)$ is positive. For calculating $g(t)$ in time domain, the partial fraction description of $\Delta(s)$ is used. From Eq (5.14), it is concluded that

$$\left\{ \begin{aligned} \frac{s^2 + 2\alpha s + 1}{\Delta(s)} &= \frac{A_0}{s + s_0} + \frac{A_1}{s + s_1} + \frac{A_2}{s + s_2} \\ A_0 &= \frac{s_0^2 - 2\alpha s_0 + 1}{(s_2 - s_0)(s_1 - s_0)}, A_1 = -\frac{s_1^2 - 2\alpha s_1 + 1}{(s_2 - s_1)(s_1 - s_0)}, A_2 = \frac{s_2^2 - 2\alpha s_2 + 1}{(s_2 - s_0)(s_2 - s_1)} \end{aligned} \right\} \quad (5.17)$$

According to Eqs (4.15), (5.16) and (5.17), it results that

$$0 \leq g(t) = A_0 e^{-s_0 t} + A_1 e^{-s_1 t} + A_2 e^{-s_2 t} \quad (5.18)$$

It is equivalent with

$$0 \leq f(t) = A_0 e^{(s_2 - s_0)t} + A_1 e^{(s_2 - s_1)t} + A_2 \quad (5.19)$$

Because $s_2 - s_1 < s_2 - s_0$, the condition in Eq (5.19) is satisfied only if $0 \leq A_0$ and it can be analyzed into five cases as follows. In the case of $A_0 = 0$: $0 \leq A_1$.

Case (1): $A_0 = A_1 = 0$ and $0 \leq A_2$:

Using Eq (5.17), it results that

$$A_0 = A_1 = 0 \Rightarrow \{1 < \alpha\} \wedge \{s_0 = \alpha - \sqrt{\alpha^2 - 1}\} \wedge \{s_1 = \alpha + \sqrt{\alpha^2 - 1}\} \quad (5.20)$$

$$0 \leq A_2 \Rightarrow \{0 \leq \alpha \leq 1\} \vee \left\{ \begin{aligned} 1 < \alpha \\ \wedge \\ s_2 \leq \alpha - \sqrt{\alpha^2 - 1} \end{aligned} \right\} \vee \left\{ \begin{aligned} 1 < \alpha \\ \wedge \\ \alpha + \sqrt{\alpha^2 - 1} \leq s_2 \end{aligned} \right\} \quad (5.21)$$

Therefore

$$\{1 < \alpha\} \wedge \{s_0 = \alpha - \sqrt{\alpha^2 - 1}\} \wedge \{s_1 = \alpha + \sqrt{\alpha^2 - 1}\} \wedge \{\alpha + \sqrt{\alpha^2 - 1} < s_2\} \quad (5.22)$$

Case (2): $A_0 = 0$, $0 < A_1$ and $0 \leq A_1 + A_2$:

Using Eq (5.17), it results that

$$A_0 = 0 \Rightarrow \left\{ \begin{aligned} 1 \leq \alpha \\ \wedge \\ s_0 = \alpha - \sqrt{\alpha^2 - 1} \end{aligned} \right\} \vee \left\{ \begin{aligned} 1 \leq \alpha \\ \wedge \\ s_0 = \alpha + \sqrt{\alpha^2 - 1} \end{aligned} \right\} \quad (5.23)$$

$$0 < A_1 \Rightarrow \{1 < \alpha\} \wedge \{\alpha - \sqrt{\alpha^2 - 1} < s_1 < \alpha + \sqrt{\alpha^2 - 1}\} \quad (5.24)$$

$$\text{if } \alpha + \sqrt{\alpha^2 - 1} \leq s_2 \Rightarrow 0 \leq A_2 \quad (5.25)$$

Therefore

$$\begin{aligned} &\{1 < \alpha\} \wedge \{s_0 = \alpha - \sqrt{\alpha^2 - 1}\} \wedge \{\alpha - \sqrt{\alpha^2 - 1} < s_1 < \alpha + \sqrt{\alpha^2 - 1}\} \rightarrow \\ &\rightarrow \wedge \{\alpha + \sqrt{\alpha^2 - 1} \leq s_2\} \end{aligned} \quad (5.26)$$

Case (3): $1 \leq \alpha$ and $s_0 < \alpha - \sqrt{\alpha^2 - 1}$:

Using Eq (5.17), it results that

$$\{1 \leq \alpha\} \wedge \{s_0 < \alpha - \sqrt{\alpha^2 - 1}\} \Rightarrow 0 < A_0 \quad (5.27)$$

Here s_1 and s_2 are chosen such that to have $0 \leq A_1$ and $0 \leq A_2$, then all coefficients of the function $f(t)$ are non-negative, so the condition in Eq (5.19) is satisfied. For satisfying condition above, it is enough to consider

$$\{\alpha - \sqrt{\alpha^2 - 1} \leq s_1 \leq \alpha + \sqrt{\alpha^2 - 1}\} \wedge \{\alpha + \sqrt{\alpha^2 - 1} < s_2\} \quad (5.28)$$

Therefore

$$\begin{aligned} &\{1 \leq \alpha\} \wedge \{s_0 < \alpha - \sqrt{\alpha^2 - 1}\} \wedge \{\alpha - \sqrt{\alpha^2 - 1} \leq s_1 \leq \alpha + \sqrt{\alpha^2 - 1}\} \rightarrow \\ &\rightarrow \wedge \{\alpha + \sqrt{\alpha^2 - 1} < s_2\} \end{aligned} \quad (5.29)$$

Case (4): $1 \leq \alpha$ and $\alpha + \sqrt{\alpha^2 - 1} < s_0$:

Using Eq (5.17), it results that

$$\{1 \leq \alpha\} \wedge \{\alpha + \sqrt{\alpha^2 - 1} < s_0\} \Rightarrow \{\alpha + \sqrt{\alpha^2 - 1} < s_0 < s_1 < s_2\} \quad (5.30)$$

It results that

$$\{\alpha + \sqrt{\alpha^2 - 1} < s_0 < s_1 < s_2\} \Rightarrow \{0 < A_0\} \wedge \{A_1 < 0\} \wedge \{0 < A_2\} \quad (5.31)$$

Case (5): $0 \leq \alpha < 1$:

Using Eq (5.17), it results that

$$\{0 \leq \alpha < 1\} \Rightarrow \{0 < A_0\} \wedge \{A_1 < 0\} \wedge \{0 < A_2\} \quad (5.32)$$

In Cases (4) and (5) as it is seen in Eqs (5.31) and (5.32), the coefficients signs of function $f(t)$ are the same and satisfying the condition in Eq (5.19) for these two cases is not as simple as Cases (1), (2) and (3). It is needed to find the minimum of the function $f(t)$ using derivative with respect to t , then find the minimum value of the function $f(t)$.

If the minimum of the function $f(t)$ is non-negative then it will be non-negative itself for all non-negative values of t .

$$\dot{f}(t) = e^{(s_2-s_1)t} \left(A_0(s_2 - s_0)e^{(s_1-s_0)t} + A_1(s_2 - s_1) \right) \quad (5.33)$$

Because $A_1 < 0 < A_0$, then $0 < A_0(s_2 - s_0)$ and $A_1(s_2 - s_1) < 0$. For determining when $\dot{f}(t)$ is positive or negative, it is needed to calculate the term

$$A_0(s_2 - s_0) + A_1(s_2 - s_1) = 2\alpha - (s_0 + s_1) \quad (5.34)$$

It can be analyzed under two conditions

$$(I): s_0 + s_1 \leq 2\alpha \Rightarrow 0 < \dot{f}(t) \Rightarrow f(0) \leq f(t) \quad (5.35)$$

$$(II): 2\alpha < s_0 + s_1 \Rightarrow \exists t^*: \{ \dot{f}(t^*) = 0 \} \wedge \left\{ \begin{array}{l} 0 \leq t < t^* \\ \dot{f}(t) < 0 \end{array} \right\} \rightarrow \\ \rightarrow \wedge \left\{ \begin{array}{l} t^* < t \\ 0 < \dot{f}(t) \end{array} \right\} \Rightarrow f(t^*) \leq f(t) \quad (5.36)$$

In Case (4) it is obvious that the condition (II) from Eq (5.35) is satisfied. First it is needed to calculate $\dot{f}(t^*)$ using Eq (5.17) by defining $\rho(s) = s^2 - 2\alpha s + 1$

$$\dot{f}(t^*) = 0 \Rightarrow e^{(s_1-s_0)t^*} = \frac{-A_1(s_2 - s_1)}{A_0(s_2 - s_0)} = \frac{\rho(s_1)}{\rho(s_0)} \Rightarrow t^* = \frac{\ln\left(\frac{\rho(s_1)}{\rho(s_0)}\right)}{s_1 - s_0} \quad (5.37)$$

Then according to condition (II) from Eq (5.35), it is needed to calculate $f(t^*)$ as

$$\begin{aligned} f(t^*) &= e^{(s_2-s_1)t^*} \left(A_0 e^{(s_1-s_0)t^*} + A_1 \right) + A_2 \Rightarrow \\ &\rightarrow e^{(s_2-s_1)t^*} \left(\frac{\rho(s_0)}{(s_2 - s_0)(s_1 - s_0)} \times \frac{\rho(s_1)}{\rho(s_0)} - \frac{\rho(s_1)}{(s_2 - s_1)(s_1 - s_0)} \right) + A_2 \Rightarrow \\ &\rightarrow e^{(s_2-s_1)t^*} \left(\frac{\rho(s_1)}{(s_2 - s_0)(s_1 - s_0)} - \frac{\rho(s_1)}{(s_2 - s_1)(s_1 - s_0)} \right) + A_2 \Rightarrow \\ &\rightarrow \frac{-e^{(s_2-s_1)t^*} \rho(s_1) + \rho(s_2)}{(s_2 - s_0)(s_2 - s_1)} \end{aligned} \quad (5.38)$$

To satisfy the condition in Eq (5.19), it is needed to set $f(t^*)$ to be non-negative

$$0 \leq f(t^*) \Rightarrow e^{(s_2-s_1)t^*} \leq \frac{\rho(s_2)}{\rho(s_1)} \Rightarrow t^* \leq \frac{\ln\left(\frac{\rho(s_2)}{\rho(s_1)}\right)}{s_2 - s_1} \Rightarrow \frac{\ln\left(\frac{\rho(s_1)}{\rho(s_0)}\right)}{s_1 - s_0} \leq \frac{\ln\left(\frac{\rho(s_2)}{\rho(s_1)}\right)}{s_2 - s_1} \quad (5.39)$$

But the inequality in Eq (5.39) is impossible to be satisfied. A theoretical proof is given to prove our claim. A function $h(x)$ is defined by $\ln(\rho(x))$ and it is claimed that it is a concave function.

This can be easily proved by calculating the second derivative of $h(x)$ as

$$\ddot{h}(x) = \frac{-2((x - \alpha)^2 + (\alpha^2 - 1))}{(\rho(x))^2} \quad (5.40)$$

In Case (4), since $1 \leq \alpha$, so it results that $\ddot{h}(x)$ is always negative and it proves our claim. The property of the concave functions is that

$$\forall 0 \leq \beta \leq 1 \Rightarrow (1 - \beta)h(s_0) + \beta h(s_2) \leq h((1 - \beta)s_0 + \beta s_2) \quad (5.41)$$

Because $s_0 < s_1 < s_2$, there is a variable $0 \leq \beta \leq 1$ such that $s_1 = (1 - \beta)s_0 + \beta s_2$, then it is derived that

$$\frac{s_1 - s_0}{s_2 - s_1} = \frac{\beta}{1 - \beta} \quad (5.42)$$

According to the definitions above, the inequality in Eq (5.39) is equivalent with

$$h(s_1) \leq (1 - \beta)h(s_0) + \beta h(s_2) \quad (5.43)$$

The inequality in Eq (5.43) is against the property of concave function mentioned in Eq (5.41) which is not true. All aforementioned shows that case (4) cannot be satisfied at all. Finally with assuming correctness of case (5), it is proved that the condition (II) from Eq (5.35) can be satisfied. Here to satisfy the condition in Eq (5.19), it is enough to condition in Eq (5.43) is satisfied. Hence function $h(x)$ must be a convex function and the function $\ddot{h}(x)$ must be non-negative as well.

$$0 \leq \ddot{h}(x) \Rightarrow \left\{ \alpha - \sqrt{1 - \alpha^2} \leq x \leq \alpha + \sqrt{1 - \alpha^2} \right\} \quad (5.44)$$

According to condition (II) from Eq (5.35), s_0 , s_1 and s_2 can be chosen such that the condition in Eq (5.44), is held too.

Therefore

$$\begin{aligned} &\{0 \leq \alpha < 1\} \wedge \{2\alpha < s_0 + s_1\} \rightarrow \\ &\rightarrow \wedge \left\{ \alpha - \sqrt{1 - \alpha^2} \leq s_0 < s_1 < s_2 \leq \alpha + \sqrt{1 - \alpha^2} \right\} \end{aligned} \quad (5.45)$$

According to conclusion in Eq (5.45), the condition in Eq (5.43) is satisfied and also the inequality in Eq (5.39). All aforementioned shows that case (5) can be satisfied.

Finally it is needed that one of the conditions in Eqs (5.22), (5.26), (5.29) and (5.45) is satisfied to ensure the condition in Eq (5.19) is satisfied. According to Eqs (5.12) and (5.14), s_0 , s_1 and s_2 are closed loop poles. Similar to the previous mentioned, the location of closed loop poles determines the speed of system time responses [52]. Here they are assumed a real numbers and if they are located nearer to the imaginary axis, the system time response

will be slower. From the conclusions (5.22), (5.26) and (5.29), if α goes to infinity; then s_0 goes to zero. So for large values of α , the system time response is slow. As it is proved, this scheme works for very small values of α , near to zero and for α equal to zero, the system time response is fast. It shows this control scheme is a better design in comparison with the previous control schemes.

In following tables, the closed-loop roots selection according to Eqs (5.22), (5.26), (5.29) and (5.45) has been summarized to reach the possible maximum speed.

Case (1): $0 < \Delta\alpha$

$1 < \alpha$		
$s_0 = \alpha - \sqrt{\alpha^2 - 1}$	$s_1 = \alpha + \sqrt{\alpha^2 - 1}$	$s_2 = \alpha + \sqrt{\alpha^2 - 1} + \Delta\alpha$

Table 5-1. Characteristic polynomial roots for case (1)

Case (2): $0 < \Delta\alpha < 2(\sqrt{\alpha^2 - 1})$

$1 < \alpha$		
$s_0 = \alpha - \sqrt{\alpha^2 - 1}$	$s_1 = \alpha + \sqrt{\alpha^2 - 1} - \Delta\alpha$	$s_2 = \alpha + \sqrt{\alpha^2 - 1}$

Table 5-2. Characteristic polynomial roots for case (2)

Case (3): $0 < \Delta\alpha$

$1 \leq \alpha$		
$s_0 = \alpha - \sqrt{\alpha^2 - 1} - \Delta\alpha$	$s_1 = \alpha - \sqrt{\alpha^2 - 1}$	$s_2 = \alpha + \sqrt{\alpha^2 - 1} + \Delta\alpha$

Table 5-3. Characteristic polynomial roots for case (3)

Case (5): $0 < \alpha < 2(\sqrt{1 - \alpha^2})/3$

$0 \leq \alpha < 1$		
$s_0 = \alpha + \sqrt{1 - \alpha^2} - 2\Delta\alpha$	$s_1 = \alpha + \sqrt{1 - \alpha^2} - \Delta\alpha$	$s_2 = \alpha + \sqrt{1 - \alpha^2}$

Table 5-4. Characteristic polynomial roots for case (5)

To reach possible maximum speed for $1 < \alpha$ among tables above, table 5-1 has been used for the closed-loop roots selection. It is obvious that for $0 \leq \alpha < 1$ table 5-4 and for $\alpha = 1$ table 5-3 has been used for the closed-loop roots selection.

If the definition (5.46) is done, the coefficients in Eq (5.7) will be calculated from Eqs (5.11) and (5.14) as follow

$$\begin{cases} p_1 = s_0 + s_1 + s_2 \\ p_2 = s_0 s_1 + s_1 s_2 + s_2 s_0 \\ p_3 = s_0 s_1 s_2 \end{cases} \quad (5.46)$$

$$\begin{cases} a_0 = 3p_3 \\ a_1 = 3(p_3 - p_1 + 2\alpha) \\ a_2 = 3(p_2 - 2\alpha p_1 + 4\alpha^2 - 1) \\ a_4 = p_1 - 2\alpha \end{cases} \quad (5.47)$$

Finally the controller coefficients in Eq (5.4) are calculated from Eqs (5.2), (5.4), (5.7) and (5.47) as follow

$$\begin{cases} k_0 = \frac{3\tau}{4} a_0 \\ k_1 = \frac{3\tau}{4} a_1 \\ k_2 = \frac{3\tau}{4} a_2 \\ k_4 = \frac{3\tau}{4} a_4 - \frac{3}{2} \end{cases} \quad (5.48)$$

5.4 Simulation Results and Discussions

Figures 5.1- 5.6 show the simulation results of positive state variable design control scheme for three different values of α (from 0 to 5). It is seen that the stable and controllable travel range of the device to full gap is achieved without the overshooting. As simulation results show the extra ordinary fluctuations do not appear in normalized charge curve because the new state variable x_4 is considered to be always positive even at $t = 0$, and then the zero over zero problem does not occur in the control input v function in Eq (5.5).

All required equations for the simulation results are named as follow:

Eq (2.70) is the modified model. Eq (5.4) is w design and x_4 definition. Eq (5.5) is w definition and the controller function for the modified model (Equivalently Eq (5.6) can be used as controller function for the MEMS device model). Eq (5.48) is the controller coefficients design.

Figure 5.1. Normalized deflection (x_1) Vs. time, $\alpha = 0$, positive state variable design control scheme

Figure 5.2. Normalized charge (x_3) Vs. time, $\alpha = 0$, positive state variable design control scheme

Figure 5.3. Normalized deflection (x_1) Vs. time, $\alpha = 1$, positive state variable design control scheme

Figure 5.4. Normalized charge (x_3) Vs. time, $\alpha = 1$, positive state variable design control scheme

Figure 5.5. Normalized deflection (x_1) Vs. time, $\alpha = 5$, positive state variable design control scheme

Figure 5.6. Normalized charge (x_3) Vs. time, $\alpha = 5$, positive state variable design control scheme

5.5 Conclusions

A positive state variable design control scheme is proposed in this chapter. It is clear that the stable and controllable whole gap displacement, output tracking condition without overshooting problem in the normalized deflection curve are achieved by using this control

scheme. In the simulations, three different values of α (from 0 to 5) are considered. From simulation results, it is clear that when α is equal to zero, the system time responses are quite fast.

Hence positive state variable design is able to be applied to a wider range of nonlinear micromachined electrostatic parallel plate microactuators because as simulation results show, this scheme works for α equal to zero.

The advantage of this scheme comparing to the previous schemes is that for α values less than one, the system time response is much faster in comparison with previous control schemes, shown in Chapters 3 and 4.

Chapter 6. Conclusions and Recommendations

6.1 Conclusions

There are many methods that can be used to design a controller for nonlinear micromachined parallel plate actuators and RF switches. For actuators, the most important matter to be considered is that the output follows its input as much as possible.

It is necessary to reduce the overshooting and system time response as much as possible and also tracking error is set to be zero. Also, the design must be able to be implemented for fabricated MEMS device.

Among all methods, have been done in the past by other people, to improve the performance of MEMS electrostatic microactuator systems, it is comprehensible that the Closed-Loop Feedback Control method is the best method for this purpose. This is the main reason which persuades us to use this method to overcome the “Pull-In” phenomenon.

One of previous methods uses Closed-Loop Feedback Control method, so it overcomes the “Pull-In” phenomenon. But as simulation result shows, there are extraordinary fluctuations due to its control signal function. This drawback prevents of using this method for real time implementation.

First auxiliary controller design control scheme is introduced to eliminate extraordinary fluctuations problem from control scheme above. It may seem this technique solves all problems but there are other problems inside of this technique as simulation results show. The auxiliary controller design control scheme has two major drawbacks, the overshooting problem and usage limitation of this technique, for a special range of MEMS device parameter.

Another control scheme called non-negative input design is introduced next to improve our first control scheme results by elimination of the overshooting problem. But usage limitation problem still remains for a special range of MEMS device parameter. Also the simulation results justify these claims.

For developing our control scheme work space, last control scheme called positive state variable design is described finally. Then mathematical proof proves that this scheme works for a wider range of MEMS devices in comparison with the former control schemes.

6.2 Recommendations

1. Implementation of real time tests which needs a chip of parallel-plate microactuators and D-Space card. In fact, many experiments need to be done on the MEMS device to achieve its real model. The state space equation model (3.1) that has been used in this research is an approximation of the real model. The controllers must be implemented on the real device through the D-Space cards which can provide electrical command for real device. The MEMS device is very small so connecting D-Space cards to the MEMS device is another important issue.
2. In order to make the control signal as the device input, the D-Space needs to take some signals from the device. These signals are the state variables of the system, so the value of all variable states such as plate movement, velocity and the charge is necessary. The voltage and capacitance across the device can be easily and accurately measured at a relatively low cost. From these measurements, the charge across the device and the deflection of the movable plate are easily attainable. But the measurement of the velocity is very costly and difficult. A reduced order nonlinear state observer must be constructed to provide a reliable estimate of the velocity for feedback. A reduced order state observer that reliably and accurately estimates the device velocity with arbitrarily fast error dynamics must be constructed to confront this problem.
3. Robust control technique [54] will be applied to develop new control scheme for the nonlinear micromachined actuator system.

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