Creation of a sensor to measure strain of highly deformable solids

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Creation of a Sensor to Measure Strain of Highly Deformable Solids

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School of Mechanical and Aerospace Engineering

Thesis

***Creation of a Sensor to Measure
Strain of Highly Deformable Solids***

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Under supervision of:

A/P Nader Vahdati

Abstract

Abstract

In recent years, industrial application of rubber components has been increased. One of the important issues in industry is strain measurement of soft materials. Health and usage monitoring of aerospace components in flight has been lately getting a lot of attention by aerospace companies. For instance, in the helicopter industry, efforts are underway to monitor vibration and stress levels, in real time, of most helicopter components made of metal and composites using accelerometers and metal foil strain gauges. On most helicopters and fixed wing airplanes, there are other flight critical components that are not made of metal and composites, but made of rubber such as rubber bearings, rubber lead-lag dampers and rubber engine mounts which also require stress monitoring. However, metal foil strain gauges cannot be used on the surface of these rubber components for strain measurement since rubber flexes too much and most strain gauges are incapable of handling such large strains.

In many applications, the allowable deflection of the structure defines the safe operational limit; therefore, it is of great interest to have a structurally monitoring system capable of real time strain measurement. In fact, the purpose of continuously monitoring the distribution of strain in crucial structures is to better understand the behavior of the structure in extreme circumstances or otherwise anomalous conditions, whereby remedial action can be performed when required.

Abstract

A new sensor for measuring strain of highly deformable solids is proposed. This sensor is capable of measuring the strain of rubber or plastic component on which the sensor is bonded, in real time.

First, the literature is looked over in order to investigate the latest state of the art in large strain measurement. Second, behavior of cylindrical rubber blocks, which are the primary part of the proposed sensor, under shear loadings are studied and then the accuracy of the new sensor and viability of its miniaturization is examined using Abaqus software; afterwards, experimental justification are shown; eventually, conclusions are drawn and suggestions for the future graduate students are presented.

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List of Symbols

G'	Apparent shear modulus
I	Area moment of inertia of Aluminium plate
K	Bulk modulus
d^*	Critical displacement
A_p	Cross sectional area of Aluminium plate
L	Deformed length
I_i	Deviatoric invariant
D	Diameter
d	Displacement
E	Engineering strain
J^{el}	Elastic volume ratio
F	Force
h	Height
ICD	Initial center distance (of cylindrical rubber blocks)
L_0	Initial length
PZT	Lead zirconate titanate (a ceramic) that shows piezoelectric effect
E^l	Logarithmic strain
M	Moment
ν	Poisson's ratio
a	Radius

List of Symbols

S	Shape factor
G	Shear modulus
σ	Stress
λ	Stretch ratio
W	Strain energy function
J_{th}	Thermal volume ratio
ε_{th}	Thermal expansion
ε_{th}	Thermal expansion
t	Thickness of Aluminium plate
J	Total volume ratio
b	Width of Aluminium plate
E	Young's modulus
E_{Al}	Young's modulus of the Aluminium plate

Chapter 1: Introduction

The strain measurement has been a matter of much interest in engineering for many years. Unique properties of rubber have made it to have stupendous applications like sealing elements and elastomeric isolators, etc. In order to take full advantage of rubber components, the displacement and deformation of rubber materials are studied. In laboratories, instead of testing different conceivable loadings that material may encounter, standard tests are performed on the samples which are prepared beforehand and the results of these limited tests are used for predicting the behavior of material in different loading situations. With the intention of meeting this need, a wide variety of measuring devices comprised of mechanical, electrical, and optical devices have been used throughout the industry. Few of these devices, however, are able to measure relatively large strains that are commonly encountered in the rubber industry. Handling materials which experience small deflections, such as steel and concrete, thereby predicting material behavior by knowing a restricted number of material constants such as Young's modulus and bulk modulus, is more familiar for engineers than soft materials. Since rubber-like materials experience large deformations because of their good elasticity performance, mechanical behavior of these materials is substantially different from those of metals. Hence properties of such materials are often not well understood. In addition, unexpected and unwanted properties are imposed to the material when fillers such as carbon black are added to natural and synthetic elastomers as diluents so as to reduce manufacturing costs. In fact, the

Chapter 1

complications arise from experiencing large deformations and highly nonlinear behavior of rubber-like materials.

Above mentioned large deformation sometimes should be controlled in order to take a preventive action before the onset of catastrophic failures. For instance, in the rotary wing applications, elastomeric bearings and dampers (See Fig.1 and 2) are used on the main and tail rotors to control vibration and provide motion accommodation. The main and tail rotors pitch, flap, and move in many different directions, and the elastomeric components accommodate these motions. Since most of these motions are large, the rubber flexes to a large extent. In order to be assured that the mentioned flight critical components do not suddenly fail in flight, it is of great interest to monitor rubber strains in real time, just like other flight critical components.

While accelerometers and external displacement devices cater for rigorous reading of global structure performance, local structural performance is measured directly by conventional strain gauge device, whereby reliable measurement can be met up to $\pm 3\%$ strain; however, for rubber-like components, since the strains are generally large, no metal foil strain gauge can withstand such large strains. Therefore, there is a need to develop a sensor that can measure rubber strains of not only flight critical rubber components, but also any component which experiences large deformations. In this research, a new sensor is proposed capable of measuring large strains both statically and dynamically.

Chapter 1

Figure 1 Elastomeric lead-lag dampers on the main rotor of Apache AH-64D helicopter

Figure 2 An elastomeric lead-lag damper

1.2 Objective

The current research project serves as a study with the goal of measuring strain of highly deformable solids which experience large elongations when a small stress is applied to them.

Therefore, the objective of the current work is:

“To develop a new sensor capable of measuring large strains”.

Chapter 1

1.3 Scope of the research

In this project, a new method of measuring rubber strains is developed. The scope of the project includes:

- ✓ To design a new sensor capable of measuring strain of highly deformable solids.
- ✓ To formulate equations for the sensor.
- ✓ To verify the theoretical results with Abaqus software generated results.
- ✓ To build a prototype of the strain measuring device.
- ✓ To collect experimental data to justify the accuracy of the proposed sensor.

1.4 Thesis organization

The organization of the later chapters is reviewed herein:

Chapter 2

The literature survey is presented in this chapter wherein different methods of surface strain measurement of highly deformable solids are reviewed.

Chapter 3

In this chapter, after studying some important concepts of rubber-like materials, previous works in the field of shear loading of cylindrical rubber blocks are reviewed and afterwards shear strain of cylindrical rubber blocks, which are the primary parts of the proposed sensor, is studied by employing finite element software, Abaqus; the Abaqus shear strain results are then compared with theoretical results; the strain

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behavior of the new sensor, off the host structure and on the host structure are studied in the next part; afterwards feasibility of miniaturization of the sensor is investigated through finite element modeling.

Chapter 4

In this chapter, first uniaxial tension test data, of a set of purchased cylindrical rubber blocks and rubber sheet, is curve-fitted whence their hyperelastic material coefficients were calculated; afterwards, experimental results obtained from testing the built prototype are compared with corresponding theoretical results and finite element results and afterwards, some comments regarding experimental procedure are presented. Finally, a practical example as an application for the sensor is described.

Chapter 5

The main achievements of the project are concluded in this chapter. Furthermore, some suggestions for future graduate students and author's contribution are presented.

Chapter 2: Literature review

2.1 Review of the methods used for the measurement of large strains

A comprehensive literature survey was conducted to find the prior art in regard to sensors capable of measuring large strains, which can be classified into three main categories:

a) Whitney mercury-in-rubber strain gauge:

In 1953 [1], the blood flow in extremities was measured in a non-invasive way using loops of narrow fine-bore silicone rubber tube, filled with mercury (called Whitney mercury-in-rubber strain gauge which is shown in Fig. 3) with Copper electrodes inserted at each end of the tube. Leads from the two copper electrodes enable the gauge to form one arm of a Wheatstone bridge. Expansion of the limbs results in stretching of the tube wrapped around the limbs; whereupon its length and diameter change and as a result, resistance of the mercury thread filling the tube, changes, which is measured by Wheatstone bridge. This device, even though was never used to measure rubber surface strains, is effective in measuring soft material surface strains. However there are disadvantages, such as thermal sensitivity, long term instability, and mercury toxicity and being capable of only measuring change of volumes with almost circular cross sections. Due to these disadvantages, the mercury strain gauge sensors never was tried to monitor rubber surface strains.

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Figure 3 Constructional details of two-strand mercury-in-rubber gauge. (a) Lateral view of gauge mounting viewed along axis of limb to which gauge is fitted. (b) Plane view of gauge mounting. (c) Plan view of gauge mounting partly dismantled to show electrical connexions. The plug in each end of the mercury-filled gauge rubber (i) is secured in a hole bored in the terminal block (ii) by means of a set screw (iii). The two terminal blocks are each secured to the curved celluloid base (v) by means of a countersunk screw (iv), the ends of the screws protruding above the blocks providing attachments for the Perspex bridge (vi). Each block also carries a curved guide rail (vii) for the sliding Perspex block (viii). A semicircular groove in (viii) accommodates the looped end of the gauge rubber, the rubber being kept in the groove by means of the hinged flap. The adjustment screw has a hinged attachment to the sliding block and passes through the Perspex bridge, any backlash of the adjusting nut (ix) being absorbed by the compression spring mounted on the screw between the bridge and the sliding block. The copper winding for thermal compensation (x) is mounted on the celluloid base, and the three electrical leads from the winding and the two terminal blocks are taken from the gauge by the three-core flexible lead (xi). [1]

b) Optical methods:

In 1977 [2], a method for determining the complete surface strain field on a rubber structure was developed. The technique consists of photographically depositing a set of grid lines on the surface of a photo-elastic coating. The coating is bonded to the surface of the object of interest. Photo-elastic data together with grid photographs

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provide sufficient information for determining the complete surface strain state at all points on the coating. However, this technique is only pertinent to strain measurement in the range from 1% to 10% and not very effective in measuring dynamic strains. When using this method, special care has to be taken to apply an evenly distributed pressure to the soft coating during bonding so as to avoid residual stress. They must be accounted for when reducing the experimental data. Also, there exists a possibility of the coating altering the stiffness of the part being studied, if the coating is too thick or material is very stiff. In 1996 [3] another contact free optical method using CCD-camera and image analysis techniques within the MATLAB image processing toolbox was proposed. Experimental setup for this method is shown in Fig. 4:

Figure 4 Experimental setup for image analysis techniques [3]

This technique is able to automatically track the motions of the dots and compute their centroid coordinates in a sequence of images. The method is a full field strain measurement technique and can be used for tiny scales like small specimens or a small area of a specimen, like a fracture process area. As it is shown in Fig. 5, rubber specimen bonded to steel plate is stretched in x-direction. A constructed finite element mesh is used for investigating displacement of dots before and after loading.

Chapter 3

Figure 5 Deformed membrane and mesh at maximum displacement. The strain in the x-direction is also displayed for the deformed membrane [3]

In 1998 [4] a servo-controlled CCD camera and a PC-based frame grabber was used to keep track of minuscule, relatively undeformable polystyrene globules positions, directly deposited on the surface of the elastomer, as it can be seen in Fig. 6. In this method, it is intended to monitor the local deformation in a relatively small moving area containing small globules. The globule positions are then used to determine the surface deformation characteristics. To anticipate large deformation measurements of rubber and rubber-like materials, the present system is accommodated with a CCD camera whose position is servo controlled.

Figure 6 (a) The image of four globules (by 10X enlargement), (b) schematics side view of the globule attachment [4]

In 2002 [5], using Moiré interferometry method, observation of deformation in small areas without changing its structure was investigated. It utilizes the interference between gratings to produce fringes that clearly stand out and create a pattern that

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indicate the amount of strain that is experienced by the specimen. These fringes are then illuminated and magnified by laser beam so that the conventional photographic devices may be used to record the pattern. This method requires precise specimen fabrication. Moiré also requires a smooth, planar surface to obtain accurate results. The rubber specimen has to be polished before the fringe pattern can be made. In order to prevent over heating which will change the properties of the rubber, frozen rubber is used to polish instead. However, freezing of the rubber would have an effect on the rigidity and decreases the Poisson effect of the rubber. This may result in a concave shaped polished surface. All these will decrease the accuracy of the moiré fringe analysis. Thus, although the moiré method is a high sensitivity measuring method, it requires high precise in the specimen fabrication. Surface of high optical quality has to be fabricated to allow diffraction grating. Deformation of a fringe after 10 minutes is shown in Fig 7.

Figure 7 Fringe picture at time 0 and time =10 minutes [5]

In 2004 [6], using 3D image correlation software and a calibrated pair of video cameras, a full-field dynamic displacement and strain measurement was proposed. In this method, deformation of a random or regular pattern of spots with good contrast applied to the surface of the test object is studied. The deformation of the structure under various load conditions is recorded by the CCD cameras and evaluated. High

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speed fracture of a rubber specimen under rapid tensile loading analysed by this method is shown in Fig. 8.

Figure 8 Sequence of captured images showing large deformations from rapid tensile loading [6]

In 2004 [7], a new method of template matching which is an image analysis was proposed whereby strain field is determined by tracking pixel blocks (material points) from one image to another. The output of this matching technique results in horizontal and vertical components of the displacement vector compared with undeformed state. Material point tracking of a rubber specimen subjected to shear deformation is shown in Fig 9.

Figure 9 Visualization of displacement field (shear deformation): (a) undeformed; and (b) deformed [7]

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From the above optical methods, there are indeed several methods which can be used to find the strain experienced by soft materials. Non-contact methods like Moiré techniques and methods using camera for tracking the deformation of the surfaces spotted with marks, which are very accurate in measuring the strain, study the changes in the patterns to determine the strain experienced by the material. Thus they needed special care to prepare the specimen so that clear images can be formed. These methods of studying the grating images can be time consuming too. Opaque material does not support some of the methods. On the other hand, optical methods reviewed are suitable only for testing the specimens in the laboratories in order to find the mechanical behavior of materials; virtually all of the aforementioned methods are incapable of real time strain measurement whereby data indicative of structural performance are supplied and lack the quality of occupying small room.

Furthermore, disadvantages of optical methods are very obvious and not always applicable in special cases like aerospace applications particularly in rotary wing applications where the elastomeric part is rotating with the rotor and the only way to keep track of grid spots is to move the CCD camera with the same rotor speed. It is also very expensive since it requires machine vision technology to monitor the motion of grid lines or spots.

c) Fiber-optic:

In 2007 [8], a surface-mounted hybrid silica/polymer optical fiber sensors for large strain measurement was proposed. In this method, two aligned silica optical fibers were mounted on the surface of the host structure and a polymer fiber core as a

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sensing element is constructed between them. Whilst optical light is transmitted from one optical fiber to the other one through the polymer sensing element, transmission properties of the sensing element are altered, if strain is experienced. Therefore, strain measurement is stemmed from monitoring the changes in optical power of the transmitted light. A hybrid silica/polymer optical fiber sensor is shown in Fig. 10. Light weight, small size, flexible hardness and immune from electromagnetic interference are the advantages of this sensor which make it suitable for space applications [12, 13] and health monitoring of civil structures [9-11]. The drawback of this method which makes it unsuitable for large strain measurement is that the maximum strain that can be measured by this method is only 5%.

Figure 10 A micrographic image of a hybrid silica/polymer optical fiber strain sensor [8]

2.2 Summary

In this chapter, a literature survey on previous methods of large strain measurement was reviewed. These methods were classified into three main categories as shown in Table below and their pros and cons are shown:

Chapter 3

Summary table of different large strain measurement methods

Strain measurement method	Pros	Cons	Measurement range
Whitney mercury-in-rubber strain gauge	Small size, acts similar to metal strain gages	1) Thermal sensitivity, 2) long term instability 3) mercury toxicity 4) only capable of measuring change of volumes with almost circular cross sections.	Large
Optical methods	Accurate because of using camera	1) Special preparation of the specimen to get clear images 2) time consuming 3) Opaque material does not support some of the methods 3) suitable only for testing the specimens and finding mechanical properties in the laboratories 5) not being small	Large
Fiber-optic	1) Light weight, 2) small size 3) flexible hardness 4) immune from electromagnetic interference	Unsuitable for large strain measurement	Maximum 5% strain

The summary table indicates many drawbacks (cons) for the three existing large strain measurement techniques. In all, a new method of measuring strain on rubber material has to be proposed so that it does not have all the above drawbacks, and it is suitable for use in isolated or dangerous areas. It should not be bulky and should be able to measure high strains related to working condition of rubber components in a simple and accurate manner. More importantly, it should be able to measure static strains as well as dynamic strains.

Chapter 3

Chapter 3: Background Theory, introducing the sensor

3.0 Introduction

As it was mentioned in chapter one, our objective is to propose a new sensor which is capable of measuring strain of highly deformable solids like rubber, plastic, etc.

In measurement and instrumentation science, it is a well known fact that any time a sensor is added to a system to measure a parameter; we are changing the behavior of that system (Heisenberg's Uncertainty Principle). For instance, when an accelerometer is added to a cantilever beam to measure its natural frequency and free end motion, the mass of the accelerometer alters the beam's mass; consequently, it's natural frequency and its free end motion. Therefore, to minimize the impact of the accelerometer mass on the beam, light weight accelerometers are used. With metal parts, when we bond a strain gauge or a PZT patch on the surface of the metal, we are causing increase in the bending stiffness of the beam. However if the strain or PZT gauge is kept thin, the sensor will have little impact on the beam.

To measure surface strain of elastomeric parts, our focus in this research is on sensors that are directly bonded to the elastomeric part. However since elastomeric parts are in general soft and flexible, the sensor which is bonded to the host rubber component should be softer so as to faithfully reflect the strains experienced by the host structure. As there is a large selection of polymers with different mechanical properties, opting for a suitable elastomeric part bonded to host structure will not be a difficult task.

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One of the possible ways to achieve the mentioned goals is to utilize intermediate rubber blocks with one end bonded to specimen whose strain is being measured and their other ends are bonded to an Aluminum plate. Cross sectional area of the rubber blocks can be circular or rectangular. Fig. 11 and Fig. 12 indicate a thin metal (like Aluminum) plate with two rubber pieces bonded to its bottom side towards the edges of the Aluminum plate. On the Aluminum plate's top side or its bottom side or both, one or more metal foil strain gauges can be bonded.

Figure 11 Rubber strain gauge sensor, design 1

Figure 12 Rubber strain gauge sensor, design 2

The sensor comprising cylindrical rubber blocks is thoroughly scrutinized in this chapter.

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When the host structure is stretched, the rubber blocks deform and transfer the force and moment from host structure to the Aluminum plate and strain of the Aluminum plate is shown by strain gauge soldered onto it. Fig. 13 shows sensor bonded on surface of a rubber sheet, before and after loading.

Figure 13 (a) Sensor bonded on a rubber sheet before loading (b) after loading

In chapter 3, we divide our studies in few main parts:

First, from section 3.1 to section 3.3, we study some important concepts of rubber material models in general in order to get familiar with their constitutive equations and determine which material model to use for our finite element modeling. Afterwards, we study the shear strain behavior of cylindrical rubber blocks in section 3.4 in order to extract the equations required for design of cylindrical rubber blocks and the sensor. Finally, finite element software, Abaqus, is used to compare results obtained from finite element analysis versus analytical closed-form equations:

1. Finite element of cylindrical rubber blocks, (in section 3.5) is conducted in order to study their shear behavior.

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2. Finite element of cylindrical rubber blocks, with one end bonded to Aluminum plate and the other unbounded end uniaxially loaded, (in section 3.6.1), is conducted.
3. Finite element of the sensor bonded to host structure (rubber sheet) loaded uniaxially, (in section 3.6.3.1), will be conducted.
4. Investigating the feasibility of miniaturization of the sensor through finite element (in section 3.6.4).

We need to do the above steps separately, because the behavior of cylindrical rubber blocks, bonded in one ended and displaced at the other end, is known for us from literature, but their behavior bonded to an Aluminum plate and also bonded to a soft host structure are unknown.

3.1 Some important concepts

In this part important concepts required for present work are illustrated.

3.1.1 Stretch ratio

For a test specimen stretched uniaxially, stretch ratio, λ known as relative elongation is defined as

$$\lambda = \frac{L}{L_0} \quad (4.1)$$

where L is deformed length and L_0 is initial length of the test specimen.

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3.1.2 True strain

True strain or logarithmic strain, E^l of the test specimen under tension is defined as

$$E^l = \ln\left(\frac{L}{L_0}\right) \quad (4.2)$$

while engineering strain, ε is defined as ratio of difference in length over initial length given by

$$\varepsilon = \frac{\Delta L}{L_0} \quad (4.3)$$

The relationship between stretch ratio and strain of the test specimen is given by

$$\varepsilon = \lambda - 1 \quad (4.4)$$

3.1.3 Shape factor

An interesting property of rubber blocks, which makes them versatile, is that they can be soft or stiff depending on their shape (for instance with the same cross sectional area but different heights), while in the case of steel blocks, the extension per unit thickness is not changed for tall or short blocks. This is related to the shape factor (S) of the block which is defined as “the loaded area of a block divided by the total area free to bulge” [12].

For instance shape factor for a cylindrical block with height of h and diameter of D , equates to

$$S = \frac{D}{4h} \quad (4.5)$$

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The smaller the numerical value of the shape factor, the softer will be the rubber block.

3.1.4 Static modulus

Static modulus, as a measure of stiffness for a material, is commonly defined as stress for a given strain:

$$\text{Static modulus} = \frac{\text{Stress}}{\text{Strain}} \quad (4.6)$$

This value differs when material encounter dynamic movement, such as in a tire rotating against the road.

Young's modulus is usually defined as the gradient of the curve of stress-strain in the linear region. This definition is only good for materials for which stress is directly proportional to strain (Hooke's law) up to the elastic limit. However for rubber-like materials, the gradient of the curves changes as it is deformed. As strain is increased, the stress-strain gradient changes which indicates that material has a lower tensile modulus and gradually becomes softer. For a nonlinear stress/strain behavior, it is necessary to specify the strain at which the modulus is evaluated [13]. Young's modulus changes with temperature and strain rate. The value of Young's modulus decreases as temperature increases, and increases as strain rate increases.

3.1.5 Relationship between Young's modulus and shear modulus for incompressible materials

The relationship between Young's modulus (E), shear modulus (G) and Poisson's ratio ν (which is the ratio of normal strain to lateral strain) in classical mechanics is given by [12]

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$$E = 2G(1 + \nu) \quad (4.7)$$

Rubber is nearly incompressible and if rubber was fully incompressible, the Poisson's ratio of the rubber would be 0.5.

Therefore, for incompressible rubber, it is found from Eq. 3.7 that

$$E = 3G \quad (4.8)$$

This relationship is applicable to unfilled rubbers [13].

In reality, rubber is slightly compressible, with a large, but finite bulk modulus [14]. Because of the fact that the shear modulus of the rubber is relatively low, about a thousand times smaller than its bulk modulus, the change in volume of the material under a kind of loading is negligible compared to its change in shape. Therefore, the rubber is usually assumed to be incompressible.

3.1.6 Vulcanization

Uncured natural rubber is sticky and can easily deform when warms up, and is brittle when it become solid. Therefore, it cannot be used in industrial application unless it is cured. Natural rubber is composed of long polymer chains which result in independent motion of them towards each other. A chemical process, whereby polymer molecules are linked to one another by atomic bridges, such as sulfur atoms or carbon to carbon bonds, is called vulcanization. With the addition of heat and the addition of sulfur or other curatives, aforementioned curing process makes springy rubber molecules become cross-linked to a greater or lesser extent, whereupon the bulk material becomes harder, much more durable and also more resistant to chemical attack with a higher melting point [15]. It also makes the surface of the material smoother and

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prevents it from sticking to metal or plastic chemical catalysts. Cross-linking caused by vulcanization results in reversible change of the deformed shape.

3.2 Material behavior of rubber

The most important concept related to rubber is nonlinearity which means its deformation is not proportional to the load applied. Rubber also is one of the incompressible or nearly compressible materials. It means that it cannot be compressed significantly under hydrostatic loading and its volume does not change substantially with stress.

3.2.1 Stress-strain curve for rubber-like materials

For a material like rubber, which is elastic in nature, the stress-strain relations are nonlinear [16]. A typical set of three stress-strain curves for rubber-like material are shown in Fig. 14.

Figure 14 Stress vs. strain curve for rubber [17]

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Depending on the amount of filler from which, the rubber specimen is composed of and the maximum stretch applied, different strain-stress curves for loading and unloading of rubber specimens, can be obtained.

Fig. 15(a) [18] shows only the loading curve which is re-traced precisely on unloading only for natural rubber and many synthetic rubbers in case when strain is small and therefore, the materials are modeled as perfectly elastic. Fig. 15(b) [18] shows the typical simple tension behavior of a specimen of natural rubber filled by 1% by volume of carbon black particles. From its virgin state the material is loaded to a stretch ratio $\lambda = 3$, unloaded, then loaded again to the same stretch ratio in several loading-unloading cycles until a repeatable pattern is established. Since the filler content is small the behavior approximates that of an unfilled rubber. In this case there is little difference between the loading and unloading curves, particularly after the initial loading.

Figure 15 Simple tension data, with nominal stress t vs. stretch λ up to $\lambda = 1.5, 2, 2.5$: (a) data of Treloar [19], units of t , $Kg.cm^{-2}$; (b) from Dorfmann and Ogden [20], units of t , $N.mm^{-2}$ [18]

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Fig. 16 shows the corresponding picture for filled natural rubber with (a) 20 % and (b) 60 % by volume of carbon black filler. Here, the residual strain, which is also evident in Fig. 15 (b), is much more noticeable.

Figure 16 Cycles of nominal stress, t (units $N.mm^{-2}$) vs. stretch λ up to $\lambda = 1.5, 2, 2.5$: rubber compound with (a) 20% and (b) 60% by volume of carbon black filler [18]

3.3 Theory of hyperelasticity

There are two types of nonlinearity related to soft materials. First, material nonlinearity which means the relationship between stress and strain of the material is nonlinear. Second, geometric nonlinearity related to very large deformations. When rubber-like materials experience the above two nonlinearities, they are modeled as hyperelastic (Green elastic) materials [21] characterised with different types of strain energy (density) functions. A strain energy density function is a model describing a hyperelastic material. The function relates the energy stored in an elastic material, and thus the stress-strain relationship, only to the three strain (elongation) components shown by Rivlin [22], thus disregarding the deformation history, heat dissipation,

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stress relaxation etc. The assumption of being elastic and isotropic is intrinsic to the strain energy function (usually denoted by W). Derivation of strain energy function with respect to strain will result in corresponding stress component.

We need to know different energy density functions in order to choose among them for the finite element of our models and also for experimental purpose when material is tested. Examples of strain energy density functions are concisely introduced:

i. Ogden model

Ogden [22] proposed that a strain energy function can be expanded in the form of

$$W = \sum_{i=1}^N \frac{2G_i}{\alpha_i^2} (\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3) + \sum_{i=1}^N \frac{1}{D_i} (J_{el} - 1)^{2i} \quad (4.9)$$

Where $\bar{\lambda}_i$ are the deviatoric principal stretches, $\bar{\lambda}_i = J^{-\frac{1}{3}} \lambda_i$, J^{el} is the elastic volume ratio (current volume/reference volume), λ_i are the principal stretches, N is a material parameter, and G_i , α_i and D_i are temperature-dependent material parameters. The initial shear modulus and for the Ogden model is given by

$$G_0 = \sum_{i=1}^N G_i \quad (4.10)$$

and bulk modulus is given by

$$K_0 = \frac{2}{D_1} \quad (4.11)$$

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Neo-Hookean model and Mooney-Rivlin model can be obtained for special choices of G_i and α_i .

ii. Neo-Hookean model

One of the models applicable for plastics and rubber-like materials is the Neo-Hookean solid model derived from molecular theory by Treloar [23, 24] which is an extension of Hooke's law for the case of large deformations. The strain energy for this model is:

$$W = C_{10}(\bar{I}_1 - 3) + \frac{1}{D_1}(J_{el} - 1)^2 \quad (4.12)$$

where W is the strain energy, I_1 is the first deviatoric invariant defined as

$$\bar{I}_1 = \bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2 \quad (4.13)$$

where $\bar{\lambda}_i = J^{-1/3}\lambda_i$, J is the total volume ratio, J_{el} is the elastic volume ratio defined in "thermal expansion", λ_i are the principal stretches, C_{10} and D_1 are constants. The initial shear modulus is given by

$$G_0 = 2C_{10} \quad (4.14)$$

and initial bulk modulus is given by

$$K_0 = \frac{2}{D_1} \quad (4.15)$$

In simple shear, Neo-Hookean solid shows linear dependence of shear stresses upon shear deformation [25].

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iii. Mooney-Rivlin Model

Mooney-Rivlin solid is an extension of the Neo-Hookean solid model derived originally by Mooney [26], where the strain energy potential, W is given by

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + \frac{1}{d}(J_{el} - 1)^2 \quad (4.16)$$

where $\bar{I}_1$ and $\bar{I}_2$ are the first and the second invariant of deviatoric strain invariants

$$\begin{aligned} \bar{I}_1 &= \bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2 \\ \bar{I}_2 &= \bar{\lambda}_1^{(-2)} + \bar{\lambda}_2^{(-2)} + \bar{\lambda}_3^{(-2)} \end{aligned} \quad (4.17)$$

where $\bar{\lambda}_i = J^{-1/3} \lambda_i$, J is the total volume ratio, J_{el} is the elastic volume ratio defined in “thermal expansion”, λ_i are the principal stretches, and C_{10} , C_{01} and D_1 are constants.

The initial shear modulus is given by

$$G_0 = 2(C_{10} + C_{01}) \quad (4.18)$$

and initial bulk modulus is given by

$$K_0 = \frac{2}{D_1} \quad (4.19)$$

Only hyperelastic material model can experience isotropic thermal expansion. The total volume ratio, J is related to the thermal volume ratio, J_{th} with the elastic volume ratio, J_{el} :

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$$J_{el} = \frac{J}{J_{th}} \quad (4.20)$$

where J_{th} is given by

$$J_{th} = (1 + \varepsilon_{th})^3 \quad (4.21)$$

where ε_{th} is the linear thermal expansion strain obtained from the temperature and the isotropic thermal expansion coefficient.

iv. Arruda-Boyce model

Arruda and Boyce [27] proposed a constitutive relation based on long-chained molecules defined as

$$W = G \left[\frac{1}{2}(\bar{I}_1 - 3) + \frac{1}{20\lambda_m^2}(\bar{I}_1 - 9) + \frac{11}{1050\lambda_m^4}(\bar{I}_1 - 27) + \frac{19}{7000\lambda_m^6}(\bar{I}_1 - 81) + \frac{519}{673750\lambda_m^8}(\bar{I}_1 - 243) \right] + \frac{1}{D} \left(\frac{J_{el}^2 - 1}{2} - \ln J_{el} \right) \quad (4.22)$$

Where G, λ_m and D are temperature-dependent material parameters, $\bar{I}_1$ is the first deviatoric strain invariant and J_{el} , is the elastic volume ratio. The initial shear modulus, G_0 is related to G with the expression

$$G_0 = G \left(1 + \frac{3}{5\lambda_m^2} + \frac{99}{175\lambda_m^4} + \frac{513}{875\lambda_m^6} + \frac{42039}{67375\lambda_m^8} \right) \quad (4.23)$$

A typical value of λ_m is 7, for which $G_0 = 1.0125G$ and the initial bulk modulus is given by

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$$K_0 = \frac{2}{D} \quad (4.24)$$

v. Marlow form

The form of the Marlow strain energy potential is

$$W = W_{dev}(I_1) + W_{vol}(J_{el}) \quad (4.25)$$

where W_{dev} and W_{vol} are deviatoric and volumetric parts of the strain energy potential

W ; I_1 and J_{el} are defined as before.

vi. Polynomial model

Treloar [25] suggested that strain energy referred to as polynomial model can be expressed as

$$W = \sum_{i+j=1}^N C_{ij} (\bar{I}_1 - 3)^i (\bar{I}_2 - 3)^j + \sum_{i=1}^N \frac{1}{D_i} (J_{el} - 1)^{2i} \quad (4.26)$$

where C_{ij} and D_i are temperature-dependent material parameters and the other coefficients are defined as before and the initial shear modulus and bulk modulus are the same as ones obtained for Mooney-Rivlin model. In fact, some of the material models like Mooney-Rivlin; Neo-Hookean models are obtained for special choices of C_{ij} .

3.4 Elastic rubber blocks

Rubber blocks are structural components that have widely been used for multitudinous industrial applications acting as frictional dampers, hysteretic dampers and springs. The shape and the loading direction of these components vary depending

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on the application. Therefore, in order to meet these requirements for a particular condition, rubber blocks should have specified chemical mixtures, mechanical properties and shapes. For our purpose, cylindrical rubber blocks were chosen and required modelings were performed in order to understand the behavior of cylindrical rubber blocks under shear for an optimized design.

3.4.1 Shear behavior of cylindrical rubber blocks

Rivlin and Saunders [28] showed that to produce a simple shear in a rubber layer, traction forces must be applied to its free surfaces, otherwise the loading status will be a combination of simple shear and bending. He showed this fact experimentally which is shown in Fig 17.

Figure 17 Deformation in a shear mounting (a) No load. (b).small load (c) Large load [28]

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It is clear from Fig. 17 that grids on the surface of the cylindrical rubber block is experiencing both shear and bending.

Rivlin [29] showed that in order to have a simple shear case, surface tractions must be applied; otherwise, special attention should be paid to the bending or the results will be incorrect.

Rivlin and Saunders [28] utilized the superposition concept for the case of simple shear in which the free surfaces are not loaded, as it is shown in Fig. 18:

Figure 18 Shearing and bending of a typical shear mounting [28]

Southwell [30] showed that the displacement due to bending, δ_B is

$$\delta_B = \frac{Fh^3}{12EAk^2} \quad (4.27)$$

where F is the applied load, h is the height of the block, A is the cross sectional area, E is the Young's modulus for the rubber, and k is the radius of gyration (that is equal to $\frac{a}{2}$ where a is the radius of the cross section).

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The share of the shearing in displacing the upper end will be given by [28]

$$\delta_s = \frac{Fh}{AG} \quad (4.28)$$

where G is the true modulus of rigidity of the material (since rubber is considered incompressible, $G = \frac{E}{3}$).

Finally, the total displacement is given by

$$\delta = \delta_B + \delta_s = \frac{Fh}{AG} \left(1 + \frac{h^2}{36k^2}\right) \quad (4.29)$$

Eq. 3.29 is comparable with corresponding simple shear case whence the displacement of upper end is given by

$$\delta = \frac{Fh}{AG'} \quad (4.30)$$

Therefore, G' which is called the “apparent shear modulus” is given by

$$G' = G \left(\frac{1}{1 + h^2/36k^2} \right) \quad (4.31)$$

or

$$G' = G \left(\frac{1}{1 + \frac{1}{9}(h/a)^2} \right) \quad (4.32)$$

Apparent shear displacement is a case in which one end is fixed and the other end is displaced in parallel to the fixed end, in its own plane.

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Gent and Meinecke [31] critically reconsidered the work of Rivlin and Saunders [28] and studied bending of a block more rigorously and showed that, as it is shown in Fig. 19, the required couple M for producing a rotation θ in one end of a bonded layer with respect to the other is given by

$$M = f_b A k^2 E \theta / h \quad (4.33)$$

Where k is the radius of gyration of the cross section and f_b is a constant related to the preventing effect of the bonded surfaces and is a combination of two constants:

$$f_b = f_{b1} + f_{b2} \quad (4.34)$$

First, f_{b1} which is the result of simple extension or compression of the block elements and the corresponding component of the couple is M_1 .

Second, f_{b2} which arises from the shear displacement required for restoring points in the bonded surfaces to their virgin position in these planes and the corresponding component of the couple is M_2 . For a cylindrical block with radius of r and height

of h , f_{b1} and f_{b2} are 1 and $\frac{r^2}{6h^2}$, respectively.

Figure 19 Cross section of a block subjected to bending [31]

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Gent and Meinecke [31] like Rivlin and Saunders [28] considered the apparent shear displacement as a combination of simple shear and bending and presented a new formulation for the ratio of true apparent shear modulus $G(=\frac{E}{3})$ to the apparent shear modulus G' which is given by

$$G' = G \left(\frac{1}{1 + \frac{1}{9f_b} (h/a)^2} \right) \quad (4.35)$$

They add numerical factor, f_b to the formulation of Rivlin and Saunders [28] and showed that the term f_{b2} , corresponding to M_2 was neglected. For a cylindrical rubber block with radius of a and height of h , f_b equates to

$$f_b = 1 + \frac{a^2}{6h^2} \quad (4.36)$$

It can be found that for small heights, deviation of Eq. 3.32 is considerable from Eq. 3.35.

Horton [32] studied circular-section bonded rubber blocks under three different loading conditions: simple bending, cantilever loading and apparent shear, which briefly is illustrated here and some of the equations are derived again in order to use the new feature of them in present work:

A rectangular Cartesian coordinate system (x, y, z) is defined for a rubber block of right-circular cross-section with radius a and axial height h with an origin O at the centre of the lower end face in which Oz is along the axis of the block, as shown in

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Fig. 20. For a general point p within the block, relationship between the cylindrical polar coordinates (r, θ, z) and Cartesian coordinates is defined by equations

$$x = r \cos \theta, y = r \sin \theta, z = z \quad (4.37)$$

Figure 20 Undeformed cross-section of the block through the $y = 0$ plane [32]

The rubber block is bonded at $z = 0$ and $z = h$ by bonding to two rigid end plates. As it is shown in Fig. 20 the general loading is a shear force with magnitude of F at $z = h$ in the negative direction of Ox together with a moment of magnitude $M - Fh$ in the negative direction of the y -axis. The general loading will result in tilting the face at $z = h$ through an angle of rotation ϕ about the y -axis and the end face at $z = h$, laterally experience a small displacement d in the negative Ox direction.

For mentioned general loading, displacement components suggested in the form of [32]

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$$u_r = U \cos \theta, u_\theta = V \sin \theta, u_z = \alpha r \cos \theta \quad (4.38)$$

Where the rotation of the $z = \text{constant}$ planes, α is function of z only, with the assumption that plane right-circular sections remain plane whilst rotating and U and V are only functions of r and z .

Rubber is assumed to be isotropic, homogenous and incompressible. After applying classical linear theory of elasticity for the small displacement gradients of general deformation, general solution is derived and after applying boundary conditions, the angle of rotation, ϕ is given by

$$\phi = \frac{8h}{7G\pi a^4} \left(M - \frac{Fh}{2} \right) \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2} \right) \quad (4.39)$$

and the total lateral displacement of the bonded end, d at $z = h$ is given by

$$d = \frac{Fh}{G\pi a^2} + \frac{1}{147G\pi a^2} \left[Fh + 42 \left(\frac{h}{a} \right)^2 + \left(M - \frac{Fh}{3} \right) - \omega h \left(M \tanh \frac{\omega h}{2} + Fh \operatorname{cosec} \omega h \right) \right] - kh \quad (4.40)$$

where the value of the unknown constant, k depends upon the values chosen for F and M . G being shear modulus and ω is given by

$$\omega^2 = \frac{84}{a^2} \quad (4.41)$$

Horton [32] postulate new formulation for cantilever loading and apparent shear loading illustrated respectively:

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3.4.1.1 Cantilever loading:

The term “cantilever loading” is related to the case when $M = Fh$. Therefore, free end of the block is laterally displaced and is allowed to rotate. Therefore, there will be only a shear force at the end face at $z = h$ and according to general solution, Eq. 3.39, the rotation of the face at $z = h$ is given by

$$\phi = \frac{4F}{7G\pi a^4} \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2}\right) \quad (4.42)$$

and its lateral displacement is given by

$$d = \frac{Fh}{G\pi a^2} \left[1 + \frac{8}{21} \left(\frac{h}{a}\right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2}\right)\right] \quad (4.43)$$

A block under cantilever loading is shown in Fig. 21.

Figure 21 A cylindrical rubber block under cantilever loading

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3.4.1.2 Apparent shear loading

As it was illustrated before, when the end face at $z = h$ is not allowed to rotate and is displaced horizontally in its plane (it means that $\phi = 0$), the loading is called apparent shear. A cylindrical block under apparent shear loading is shown in Fig. 22.

Figure 22 Cylindrical rubber block under apparent shear loading

According to Eq. 3.39, if $M = \frac{Fh}{2}$, the rotation (ϕ) of end face at $z = h$ will be zero

and total displacement of bonded face at $z = h$ in apparent shear loading is given by

$$d_b = \frac{Fh}{G\pi a^2} \left[1 + \frac{2}{21} \left(\frac{h}{a} \right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2} \right) \right] \quad (4.44)$$

where πa^2 is the cross sectional area of the block.

It is known from strength of material that for a layer with a thickness of t and a cross sectional area of A , bonded in one end and other end displaced with amount of d , shear stress, τ is given by

$$\tau = \frac{F}{A} = G\gamma \quad (4.45)$$

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where G is the shear modulus of material and γ is the shear strain defined as

$$\gamma = \frac{d}{t} \quad (4.46)$$

Substituting Eq. 3.44 into Eq. 3.43, one can derive

$$d = \frac{Ft}{GA} \quad (4.47)$$

Drawing an analogy between Eq. 3.44 and Eq. 3.47, it is found that the ratio, $\frac{G}{G'}$, of the true shear modulus, G , to the apparent shear modulus, G' , can be written in the form of [32]

$$\frac{G}{G'} = 1 + \frac{2}{21} \left(\frac{h}{a}\right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2}\right) \quad (4.48)$$

Thus, the displacement of the end face, d is given by

$$d = \frac{Fh}{G'\pi a^2} \quad (4.49)$$

Therefore, required force for applying apparent shear displacement d is given by

$$F = \frac{Gd\pi a^2}{h \left(1 + \frac{2}{21} \left(\frac{h}{a}\right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2}\right)\right)} \quad (4.50)$$

and corresponding moment preventing free end from rotation is given by

$$M = \frac{Fh}{2} \quad (4.51)$$

Eq. 3.48, for $\frac{h^2}{a^2} < 5.42$ is closely approximated by [32]

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$$\left(\frac{G}{G'}\right)^{approx} = 1 + \frac{h^2}{36k^2[1.4 + \frac{1}{6}(\frac{a}{h})^2]} \quad (4.52)$$

where k is the radius of the gyration of the cross sectional area.

Eq. 3.52 is comparable with proposed equations by Rivlin and Saunders [28] (Eq. 3.32) and Gent and Meinecke [31] (Eq. 3.35).

Finally, in order to study rupture of a cylindrical rubber blocks in apparent shear loading, it is found that maximum values of stresses occur at the bonded ends $z = 0$ and $z = h$ given by [32]

$$\sigma = \sigma_{rr} = \sigma_{\theta\theta} = \sigma_{zz} = \pm \frac{6F}{\pi a^2} \left(\frac{h}{a}\right) \left(\frac{r}{a}\right) \left(1 - \frac{r^2}{a^2}\right) \cos \theta \quad (4.53)$$

Maximum stress occurs at $r = a/\sqrt{3}$ expressed as [32]

$$\sigma_{\max} = \frac{4F}{\sqrt{3}\pi a^2} \left(\frac{h}{a}\right) \quad (4.54)$$

Gent and Lindley [33] showed that failure of a bonded rubber cylinder subjected to a loading state of hydrostatic tension occurs owing to internal rupture when the magnitude of the tension reaches a critical value of between $0.783E$ to $0.833E$

$\left(= \frac{5G}{2}\right)$ in which E and G are Young's modulus and shear modulus of the material,

respectively. More recently, Gent and Meinecke [31] suggested $\frac{9G}{4}$, whilst Muhr [34]

proposed $\frac{5G}{2}$.

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Using Eq. 3.50 and 3.54 for a critical value at which rupture occurs chosen to be

$\sigma_{\max} = \frac{5G}{2}$, one can find the critical value $\left(\frac{d}{a}\right)^*$ of the scaled lateral deflection [32] of the bonded end $z = h$ given by [32]

$$\left(\frac{d}{a}\right)^* = \frac{5\sqrt{3}}{8} \left[1 + \frac{2}{21} \left(\frac{h}{a}\right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2} \right) \right] \quad (4.55)$$

Critical displacement is considered as a measure for choosing proper dimensions of blocks for a selected displacement.

Substituting Eq. 3.32 and 3.35 into Eq. 3.49, one can find corresponding equations for force, using Rivlin's work [28] and Gent's work [31], respectively:

$$F = \frac{Gd\pi a^2}{h \left(1 + \frac{1}{9} (h/a)^2 \right)} \quad (4.56)$$

$$F = \frac{Gd\pi a^2}{h \left(1 + \frac{1}{9(1 + \frac{a^2}{6h^2})} (h/a)^2 \right)} \quad (4.57)$$

In section 3.6, it will be introduced that apparent shear loading is suitable for studying the behavior of our sensor.

3.5 Finite element modeling of cylindrical rubber block

In order to justify the formulations of Horton [32], Gent [31] and Rivlin [28] and suitability of employing them in present work, multitudinous cylindrical rubber blocks with different dimensions were modeled separately using Abaqus software version 6.7 before considering the whole structure of the sensor.

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3.5.1 Choosing proper energy density function

Whereas for uniaxial extension or simple shear, material response can be completely modeled by a strain energy density function of the form $W(I_1)$ [35], the cylindrical rubber blocks are assumed to consist of nearly incompressible Neo-Hookean elastic material with a Poisson's ratio of 0.4997. A value of 0.22 MPa was assigned to C_{10} , corresponding to a Young's modulus E of 1.37 MPa (200 Psi) for the cylindrical rubber blocks (according to Eqs.3.8 and 3.14, $E = 6C_{10}$).

3.5.2 Opting for a suitable bulk modulus

It is recommended that for linear finite element methods of rubber-like materials which encounter huge compression loading, the bulk modulus be assumed to be 1400 MPa, if the bulk modulus is unknown [36, 37]. The influence of bulk modulus, when the rubber is in "compression loading" is profound and if it is not considered, the compression modulus of the material will be highly overestimated [38].

In practice, for the other states of loading, the rubber material is considered completely incompressible (Poisson's ratio=0.5) or slightly compressible (Poisson's ratio between 0.49 and 0.499) [39] and the bulk modulus is assumed to be very large that can be approximated by linear relationship between Young's modulus and bulk modulus is given by

$$E = 3K(1 - 2\nu) \quad (4.58)$$

where E is the Young's modulus, K is the bulk modulus, and ν is the Poisson's ratio of the material.

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3.5.3 Mesh element and boundary conditions

The element, C3D20R (a 20-node quadratic brick) which is in the family of three dimensional stress (3D Stress) elements was chosen as meshing element. As for boundary condition, the cylindrical rubber block was modeled with one end fixed as it is shown in Fig. 23 and the other end was displaced horizontally and prevented from rotating in its own plane (apparent shear loading). The reason for considering only apparent shear loading is described in section 3.6.

Figure 23 Boundary condition of cylindrical rubber block

3.5.4 Results of finite element modeling of cylindrical rubber block

Results obtained from Abaqus and for apparent shear modeling of cylindrical blocks with dimensions for diameter, D and height, h of the block, critical displacement, d^* (according to Eq. 3.55) and displacement of the free end, d together with forces calculated using Eqs.50, 56 and 57, are shown in Table 1. Forces are in Newton (N) and all the spatial dimensions are in meters (m). The displacements applied to the blocks were set to be less than critical displacement. Shape factors, S of the blocks are in the range of 0.071-0.25. The definition of error shown in Table 1 is as follows:

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$$Error\% = \left| \frac{(Force_{Abaqus}) - (Force_{Equation})}{Force_{Abaqus}} \times 100 \right| \quad (4.59)$$

Table 1 Comparison of the force values for apparent shear loading of cylindrical block

D	h	S	d^*	d	Force- Abaqus	Force- Horton	Force- Gent	Force- Rivlin	Least Error (%)
0.004	0.014	0.071	0.011	0.003	0.2128	0.2241	0.1926	0.1920	5.36 (Horton)
0.002	0.006	0.083	0.0046	0.0015	0.08	0.083	0.0724	0.0722	4.32 (Horton)
0.003	0.008	0.093	0.0058	0.003	0.3373	0.3386	0.2941	0.2928	0.37 (Horton)
0.005	0.013	0.096	0.0093	0.003	0.57	0.60	0.522	0.520	4.5 (Horton)
0.007	0.016	0.1	0.010	0.0045	1.65	1.71	1.50	1.49	4.14 (Horton)
0.007	0.016	0.1	0.010	0.0035	1.27	1.33	1.17	1.16	5.23 (Horton)
0.008	0.02	0.1	0.014	0.004	1.34	1.41	1.23	1.22	5.22 (Horton)
0.007	0.012	0.14	0.0077	0.0045	3.16	3.23	2.89	2.87	2.48 (Horton)
0.007	0.011	0.159	0.0071	0.0028	2.30	2.40	2.16	2.14	4.5 (Horton)
0.003	0.004	0.187	0.0026	0.002	0.9669	1.001	0.9167	0.9074	3.59 (Horton)
0.003	0.0038	0.197	0.0025	0.0015	0.78	0.82	0.76	0.75	3.6 (Gent)
0.003	0.00365	0.205	0.0024	0.0015	0.84	0.88	0.81	0.80	3.1 (Gent)

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Table 1 (continued) Comparison of the force values for apparent shear loading of cylindrical block

D	h	S	d^*	d	Force- Abaqus	Force- Horton	Force- Gent	Force- Rivlin	Least Error (%)
0.003	0.0035	0.21	0.0023	0.0019	1.14	1.20	1.11	1.09	2.69 (Gent)
0.005	0.0053	0.23	0.0037	0.003	3.52	3.69	3.44	3.40	2.04 (Gent)
0.005	0.005	0.25	0.0036	0.002	2.56	2.69	2.53	2.49	1.15(Gent)
0.006	0.006	0.25	0.0043	0.003	4.60	4.85	4.55	4.49	1.12 (Gent)
0.007	0.007	0.25	0.005	0.005	8.98	9.43	8.85	8.74	1.37 (Gent)

3.5.5 Judging the results

Comparison of forces obtained from Horton's, Rivlin's and Gent's equations (Eqs.3.50, 3.56 and 3.57 respectively) for the first row (with minimum S of 0.071) and last row (with maximum S of 0.25) of the Table. 1 is shown in Figs. 24 and 25. It is clear from these two figures that difference between force results for smaller shape factors is distinctly higher than difference between force results for larger shape factors. As it is apparent from Table 1, the least error for shape factors (S) in the range of 0.19-0.25 is derived from Gent's equation (Eq.3.57) while for shape factors (S) less than 0.19 Horton's equation (Eq. 3.50) is a better equation to use.

As the height of a cylindrical rubber block decreases and its diameter increases (shape factors increases), the block becomes stiffer. Since our purpose is to design a non-invasive small sensor, whose stiffness is less than the specimen on which the sensor

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will be stuck to, blocks with high shape factors are not considered for modeling and we use only Horton's equations (Eq. 50 and Eq. 51) for our purpose.

Figure 24 Comparison between forces required for apparent shear loading of a cylindrical rubber block with shape factor of 0.071

Figure 25 Comparison between forces required for apparent shear loading of a cylindrical rubber block with shape factor of 0.25

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3.6 Relationship between strain of host structure and strain of the proposed sensor

Circular elastomeric layers utilized as seismic isolators [40, 41], possess very large ratio of vertical stiffness to the horizontal stiffness, thus alleviating vertical loads experienced by buildings. Although using elastomeric isolators as energy dissipation device is common, however they have not been employed as a component of a sensor like cylindrical rubber blocks used in our sensor yet.

It is believed that when a soft layer is bonded to a metal, the metal reinforcement is treated as completely rigid [41]. For shear of rubber blocks, authors assume that one end of the block is completely fixed, and the other end is displaced.

When the surface of the host rubber specimen is stretched; cylindrical rubber blocks bonded to this surface follow the displacement of the host rubber structure. The state of loading in the cylindrical rubber blocks is akin to apparent shear. When the faces bonded to soft material are displaced horizontally, the faces bonded to the Aluminum plate come down vertically and rotate. When the cylindrical rubber blocks go in shear, the Aluminum plate is bent and the strain of the upper or lower surface of the metal is measured with a strain gauge. For small center distance of rubber blocks, the displacements and rotations of the faces bonded to the Aluminum plate are almost negligible and the displacements and rotations of the faces bonded to soft material would be small and in the direction of displacement of the soft material; therefore the state of loading can be approximated by apparent shear loading rather than cantilever loading. Therefore, we claim that for a displacement with magnitude of d applied to

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free end of the cylindrical rubber blocks, the reaction force and moment are calculated from Eqs. 3.50 and 3.51.

An Aluminum plate with width of b , thickness of t and length of L is shown in Fig. 26.

Figure 26 An Aluminum plate with width of b , length of L and thickness of t (to which two cylindrical rubber block are bonded)

Free body diagram of this plate is shown in Fig. 27. In this diagram, force and moment imposed on plate from cylindrical rubber blocks which are under apparent shear loading, are shown.

Figure 27 Free body diagram for a plate when the cylindrical rubber blocks bonded to it are under apparent shear loading

One can find the stress on the top side of the Aluminum plate, σ_{top} which is given by

(see appendix A)

$$\sigma_{top} = \frac{F}{A_p} - \frac{Mt}{2I} \quad (4.60)$$

where A_p is the cross sectional area of the Aluminum plate given by

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$$A_p = b \times t \quad (4.61)$$

And I is the area moment of inertia as follows

$$I = \frac{1}{12} A_p t^2 \quad (4.62)$$

Substituting F and M (from Horton's equations, Eq. 3.50 and Eq. 3.51) into Eq. 3.60,

one can find σ_{top} given by

$$\sigma_{top} = \left(\frac{G d \pi a^2}{h b t \left(1 + \frac{2}{21} \left(\frac{h}{a} \right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2} \right) \right)} \right) \left(1 - \frac{3h}{t} \right) \quad (4.63)$$

Thus, strain of the Aluminum plate, regarding Hooke's law equates to

$$\varepsilon_{Al-plate} = \frac{\sigma_{top}}{E_{Al}} = \left(\frac{G d \pi a^2}{E_{Al} h b t \left(1 + \frac{2}{21} \left(\frac{h}{a} \right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2} \right) \right)} \right) \left(1 - \frac{3h}{t} \right) \quad (4.64)$$

where E_{Al} is the Young's modulus of the Aluminum plate and ω is given by Eq. 3.41.

Important feature of Eq. 3.64 is the linear relationship between strain of metal,

$\varepsilon_{Al-plate}$ and displacement of the host structure, d (or in other words the strain of the

host structure). On the other hand, having $\varepsilon_{Al-plate}$, one can find force and moment

required for deformation of the host structure; this fact can be profitable in

characterising the uniaxial properties of the skin which is briefly introduced in

“suggestion for future students”, in chapter 5.

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3.6.1 Justification with finite element modeling

In order to check the effect of sticking rubber blocks to the Aluminum plate which is bent due to reaction forces transferred from stretched rubber blocks, two set of data are compared: first, magnitude of reaction force and moment at the free end of the cylindrical rubber blocks obtained from modeling of Fig. 28 (RF_{sensor} , RM_{sensor}) and second, reaction force and moment at free end of a cylindrical rubber block with the same dimension and clamped boundary condition and apparent shear loading (like Fig. 22) in the other end ($RF_{clamped}$, $RM_{clamped}$). The result of finite element modeling for fix center distance of 30 mm and different dimensions for diameter (D) and height (h) of the cylindrical rubber blocks, width (b) of the Aluminum plate and displacement of the free end of the blocks (d) are shown in Table. 2. The thickness (t) of the Aluminum plate was set to 1 mm for all the models. Magnitude of the strain at the top of the Aluminum plate obtained from Abaqus, ε_{Abaqus} and the strain calculated from Eq. 3.64, $\varepsilon_{equation}$ together with percentage of error between mentioned strains are also shown in Table. 2. Spatial dimensions, forces and moments are in meter (m), Newton (N) and Newton. meter ($N.m$), respectively.

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Figure 28 The modeled sensor before and after apparent shear loading

Mesh element, material properties of rubber blocks and boundary conditions are introduced below:

The element, C3D20R (a 20-node quadratic brick) which is in the family of three dimensional stress (3D Stress) elements was chosen as meshing element. The cylindrical rubber blocks are assumed to consist of nearly incompressible Neo-Hookean elastic material with a Poisson's ratio of 0.4997. A value of 0.22 MPa was assigned to C_{10} , corresponding to a Young's modulus E of 1.37 MPa (200 Psi) for the cylindrical rubber blocks. As for boundary condition, the free end of cylindrical rubber blocks are displaced horizontally and prevented from rotating in their own plane ($UR_y = 0$), known as apparent shear loading. The other ends of cylindrical rubber blocks are fixed to the Aluminum plate.

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Table 2 Comparison of finite element results between two cases: first, sensor composed of two cylindrical rubber blocks bonded to Aluminum plate subjected to apparent shear loading (similar to Fig. 28) and second, a cylindrical rubber block under apparent shear loading (similar to Fig. 22)

D	h	b	d	RF_{sensor}	RM_{sensor}	$RF_{clamped}$	$RM_{clamped}$	ε_{Abaqus}	$\varepsilon_{equation}$	Error (%)
0.009	0.016	0.0133	0.004	3.27	-0.026	3.43	-0.0274	-1.7e-4	-1.75e-4	3.3
0.007	0.012	0.011	0.0045	3.12	-.018	3.23	-0.019	-1.48e-4	-1.49e-4	0.9
0.008	0.011	0.011	-0.0012	-1.43	-0.008	-1.51	-0.0083	6.5e-5	6.38e-5	1.7
0.005	0.011	0.01	0.0012	0.0034	-0.0018	0.0035	-0.0019	-1.7e-5	-1.66e-5	2.4
0.003	0.0035	0.006	0.0014	0.85	-0.0014	0.88	-0.0015	-2.46e-5	-2.02e-5	18.5
0.003	0.004	0.006	0.0018	0.87	-.0017	0.9	-0.0018	-2.8e-5	-2.4e-5	15
0.003	0.007	0.006	0.003	0.469	-0.0015	0.467	-0.0016	-2.44e-5	-2.26e-5	7.3
0.005	0.005	0.01	0.0018	2.31	-0.0057	2.42	-0.00606	-5.54e-5	-4.92e-5	11.2
0.005	0.0053	0.01	0.0028	2.35	-0.0061	2.46	-0.0065	-5.9e-5	-5.31e-5	9.9

From comparing force and moments obtained, it is clear that sticking the one end of rubber blocks to the flexible Aluminum plate and applying apparent shear loading to the other ends, gives acceptable results without compromising the original apparent shear loading results in which the boundary condition of the faces bonded to the Aluminum plate is clamped.

The negative sign for the displacement in the third row of Table 2 is related to a loading whereby free ends of the cylindrical rubber blocks are displaced toward each other. The implication is that proposed sensor can measure strain of components experiencing compression load.

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3.6.2 Effect of reinforcement on the shear stiffness of an elastic layer

As it was mentioned in introduction of this chapter, cylindrical rubber blocks of our sensor are stuck to the surface of the host structure. The host structure is like reinforcement for the rubber blocks. We want to know the effect of this reinforcement on the shear stiffness of the rubber blocks.

Hsiang-Chuan Tsai [41] posited that for an elastomer layer bonded between plates, reinforcing plates constrain the elastomer from lateral expansion and provide a high compression and bending stiffness, however reinforcing plates have no effect on the shear stiffness of the elastomer layer. Seval Pinarbasi [42] also postulated that for layers bonded to metal in one end and soft material in the other end, lateral stiffness of the layer is not significantly influenced comparing with the case, when it is bonded to metal at one end and the other end is displaced laterally without being reinforced. This is an important property considering that albeit the resistance of a reinforced elastic layers to compression and bending can be increased; its flexibility in shear is not compromised.

3.6.3 Sensor bonded to a host structure

In this part, we study the behavior of the sensor if its free ends of its cylindrical rubber blocks are bonded to a rubber sheet as host structure.

A long rectangular isotropic rubber sheet which is under tension is shown in Fig. 29. As it can be seen, strain is completely uniform in the distance sufficiently far from the place where force is applied.

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Figure 29 Uniform strains through a rectangular rubber sheet under tension

Since strain is uniform, distribution of displacement in the direction of stretching is linear as it is shown in Fig. 30.

Figure 30 Linear distribution of displacement for a rectangular rubber sheet under tension

If the sensor is bonded to a rubber sheet which experiences tension, the interface between cylindrical rubber blocks and rubber sheet is not identical to the case when it is displaced in absence of bond. Since two rubber-like materials are bonded to each other, the rubber blocks follow the displacement of the rubber sheet and the circular

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sections in interface will be somehow elliptical when the rubber sheet is stretched, as it is shown in Fig. 31 and Fig. 32.

Figure 31 Rubber sheet at the bottom, Aluminum plate at the top and circular cross section of the blocks on the rubber sheet and Aluminum plate before loading

Figure 32 Rubber sheet at the bottom (with strain of 0.287), Aluminum plate at the top and elliptical cross section of the blocks on the rubber sheet after loading

As it is shown in Fig. 33, when a rectangular rubber sheet with length of L is elongated by a displacement of m in each side (which totally amounts to $2m$), the distance, u , between center of the rubber blocks bonded to rubber sheet will be displaced with amount of $2d$ (m , u and d are constants). Since the rubber sheet strain is uniform, the relationship between m , L , d and u is given by

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$$\frac{2m}{L} = \frac{2d}{u} \quad (4.65)$$

or

$$d = \frac{um}{L} \quad (4.66)$$

where u is the initial center distance of the cylindrical rubber blocks.

Figure 33 Rectangular rubber sheet in Tension with sensor bonded to it

The equation for strain of Aluminum plate (Eq. 3.64) was derived in section 3.6.

Strain of the Aluminum plate, $\epsilon_{Al-plate}$, is measured by the metal strain gauge which is bonded on the Aluminum plate, with the help of a Wheatstone bridge as shown in Fig.

34. Input voltage of the bridge is e_i ; its output voltage is e_o and its gauge factor is R .

Figure 34 Wheatstone bridge used for the metal foil strain gauge.

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Therefore, the strain of the Aluminum plate, ε_{plate} is given by [43]

$$\varepsilon_{Al-plate} = \frac{4 e_o}{R e_i} \quad (4.67)$$

Comparing Eq. 3.64 and 3.67, the displacement of the center of the cylindrical rubber block, d is given by

$$d = \left(\frac{4hbtE_{Al}}{G\pi a^2(1 - \frac{3h}{t})} \right) \left(1 + \frac{2}{21} \left(\frac{h}{a} \right)^2 \left(1 - \frac{2}{\omega h} \tanh \frac{\omega h}{2} \right) \right) \left(\frac{e_o}{R e_i} \right) \quad (4.68)$$

Therefore, local logarithmic rubber strain is given by

$$\varepsilon_{rubber} = \ln((u + 2d) / u) \quad (4.69)$$

where u is the initial center distance (ICD) between the centers of the two cylindrical rubber blocks.

3.6.3.1 Finite element modeling of the sensor bonded to a rubber sheet

In order to justify the functionality of the proposed sensor, Abaqus version 6.7 software was employed to develop the sensor model bonded to a rubber sheet which experiences tension. Fig. 35 shows strain distribution in the deformed shape of the proposed sensor bonded to a rubber sheet. A rectangular rubber sheet with length of 0.1 m, width of (b) 0.015 m and thickness (t) of 0.002 m together with two cylindrical rubber blocks with diameter (D) in the range of 0.005 – 0.007 m, height (h) in the range of 0.009 – 0.016m, critical displacement of d^* and an Aluminum plate with a length of 0.045 m, width (b) in the range of 0.010 – 0.012 m and thickness of 0.001 m were modeled. The initial center distance (ICD) of the

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cylindrical rubber blocks was set to 0.03 m. The length of 0.005 m, in each side of the rubber sheet, was used for pulling the rubber sheet. Since for uniaxial extension or simple shear, material response can be completely modeled by a strain energy density function of the form $W(I_1)$ [35], rectangular rubber sheet and cylindrical rubber blocks were assumed to consist of a Neo-Hookean elastic material, and almost incompressible with a Poisson's ratio of 0.4997. A value of 0.68 MPa was assigned to the elastic constant C_{10} , corresponding to a Young's modulus E of 4.13 MPa (600 Psi) for the rubber sheet and 0.22 MPa was assigned to C_{10} , corresponding to a Young's modulus, E of 1.37 MPa (200 Psi) for cylindrical rubber blocks. The Aluminum plate was assumed to have Young's modulus of 69 GPA and a Poisson's ratio of 0.33. The rectangular rubber sheet is displaced uniaxially at both ends with a value of 0.015 m corresponding to a strain of 0.287. The element, C3D20R (a 20-node quadratic brick) which is in the family of three dimensional stress (3D Stress) elements was chosen as meshing element.

Figure 35 Strain distribution of the deformed model of a rectangular rubber sheet together with sensor bonded on it

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Comparison of strain in the area, in between the cylindrical rubber blocks ($\varepsilon_{between}$) and the strain in the area outside of the cylindrical rubber blocks, ($\varepsilon_{outside}$), are utilized as a benchmark for assessing the non-destructive effect of cylindrical rubber blocks on the uniformity of the strain of the rubber sheet. As explained earlier, a good sensor is one that does not alter strain distribution of the host structure.

Comparison of finite element results and analytical results are shown in Table 3. In the last column, error determined by comparing strain of the Aluminum plate, obtained from Abaqus modeling ($\varepsilon_{(Al-plate)-Abaqus}$) with strain obtained from Eq. 3.69, $\varepsilon_{(Al-plate)-equation}$ is shown. All the spatial dimensions are in meters (m).

Table 3 Comparison of finite element results and analytical results for a sensor bonded on a rubber sheet experiencing strain of 0.287

D	h	b	d^*	$\varepsilon_{between}$ (%)	$\varepsilon_{outside}$ (%)	$\varepsilon_{(Al-plate)-Abaqus}$	$\varepsilon_{(Al-plate)-equation}$	Error (%)
0.007	0.009	0.012	0.0059	26	30	-1.95e-4	-1.6e-4	22.4
0.007	0.010	0.012	0.0065	26.5	30.4	-1.8e-4	-1.54e-4	17.1
0.007	0.011	0.012	0.0098	26.9	30	-1.65e-4	-1.43e-4	15.9
0.007	0.012	0.012	0.0077	27.2	30	-1.52e-5	-1.33e-5	14.4
0.007	0.013	0.012	0.0084	27	30	-1.39e-4	-1.22e-4	14.4
0.007	0.015	0.012	0.010	28	29.7	-1.17e-4	-1.04e-4	13.3
0.007	0.016	0.012	0.0109	28	29.6	-1.08e-4	-9.6e-5	12.9
0.006	0.012	0.01	0.0079	27.7	29.6	-1.12e-4	-1.02e-4	10.3
0.006	0.013	0.012	0.0087	27.7	29	-8.5e-5	-7.7e-5	9.42
0.006	0.014	0.01	0.0096	28	29.4	-9.26e-5	-8.53e-5	8.6
0.005	0.009	0.01	0.0058	27.5	29.6	-8.74e-5	-8.2e-5	6.69
0.005	0.011	0.01	0.0074	28	29	-6.91e-5	-6.56e-5	5.37

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The important parameters for designing the proposed sensor are firstly: diameter, height, initial center distance and material properties of the cylindrical rubber blocks and secondly: width and thickness of the Aluminum plate. The rubber blocks with smaller heights and larger radii are stiffer and therefore their destructive effect on the strain field of the host structure will be higher. Also, rubber blocks with larger Young's modulus, are stiffer.

As it is apparent from Table 3, by decreasing the stiffness of the rubber blocks (decreasing the radius and increasing the height), the error is reduced. For example, it can be seen from that last row that a sensor composed of rubber blocks with radius of 0.0025 m, height of 0.011 m, initial center distance of 0.03 m, Aluminum plate with width of 0.010 m and thickness of 0.001 m can measure strain of 0.287 with a mere error of 5.37%. When the cylindrical rubber blocks get stiff, it causes the strain distribution of the host structure to be non-uniform and high percentage of error occurs.

Besides, although the amount of force and moment at the section where rubber blocks are bonded to the host structure, are not large (for example for a rubber block with diameter of 0.004 m and height of 0.012 m, the force and moment for making an apparent shear displacement of 0.003 m at free end of the cylindrical rubber block, are 0.32 N and $0.00188 \frac{N}{m}$ obtained from simulation, respectively), when the thickness of the host structure gets thin, it flexes due to these forces and moments. Therefore, the thickness of the host structure needs to be considered for designing our proposed sensor.

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As for the Aluminum plate, its dimensions should be chosen such that strain of the plate can be measured by a strain gauge. If the thickness of the Aluminum plate chosen is too thick, the Aluminum plate cannot bend, so the strain gauge may not show any strains. The center distance of the cylindrical rubber blocks specifies the length of the Aluminum plate.

3.6.4 Miniaturization of the sensor

In order to investigate the possibility of miniaturizing the proposed sensor, finite element modeling of sensor with smaller diameters, heights and initial center distance (ICD) for cylindrical rubber blocks and also smaller thickness and width for Aluminum plate was conducted. The same as section 3.6.3.1, a rectangular rubber sheet with length of 0.1m, width (b) of 0.015 m and thickness (t) of 0.002 m was modeled as the host structure and a length of 0.005 m, in each side of the rubber sheet, was used for pulling the rubber sheet with an amount of 0.015 m corresponding to rubber sheet logarithmic strain of the 0.287. Rectangular rubber sheet and cylindrical rubber blocks were assumed to consist of a Neo-Hookean elastic material, and almost incompressible with a Poisson's ratio of 0.4997. A value of 0.68 MPa was assigned to the elastic constant C_{10} , corresponding to a Young's modulus E of 4.13 MPa (600 Psi) for the rubber sheet and 0.22 MPa was assigned to C_{10} , corresponding to a Young's modulus, E of 1.3 MPa (200 Psi) for cylindrical rubber blocks. The diameter (D), height (h) and shape factor (S) (in range of 0.062-0.125) of the cylindrical rubber blocks are shown in Table 4. The Aluminum plate was assumed to have Young's modulus of 69 GPa and a Poisson's ratio of 0.33. Abaqus finite element software programme is used for modeling and the element, C3D20R (a 20-

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node quadratic brick) was chosen as meshing element. Comparison of finite element results and analytical results are shown in Table 4. In the last column, error determined by comparing strain of the Aluminum plate, obtained from Abaqus modeling ($\varepsilon_{(Al-plate)-Abaqus}$) and strain obtained from Eq. 3.69, $\varepsilon_{(Al-plate)-equation}$ is shown.

All the spatial dimensions are in meter (m).

Table 4 Finite element results of miniaturizing the sensor

D	h	S	b	t	ICD	$\varepsilon_{(Al-plate)-Abaqus}$	$\varepsilon_{(Al-plate)-equation}$	Error (%)
0.004	0.011	0.091	0.008	0.001	0.025	-3.35e-5	-3.36e-5	0.5
0.035	0.008	0.109	0.006	0.0009	0.025	-5.5e-5	-5.48e-5	3.6
0.035	0.008	0.109	0.006	0.0009	0.02	-4.5e-5	-4.38e-5	2.5
0.0035	0.007	0.125	0.005	0.0009	0.016	-5.23e-5	-4.96e-5	5.7
0.003	0.0085	0.088	0.005	0.0009	0.015	-2.29e-5	-3.13e-5	6.7
0.003	0.007	0.107	0.005	0.0009	0.015	-2.98e-5	-2.80e-5	5.9
0.002	0.0055	0.091	0.004	0.0008	0.01	-1.13e-5	-1.03e-5	8.6
0.0015	0.0045	0.083	0.003	0.00075	0.008	-7.05e-6	-6.12e-6	13.1
0.0015	0.0043	0.087	0.003	0.00075	0.008	-7.6e-6	-6.55e-6	13.8
0.0015	0.004	0.094	0.003	0.00075	0.008	-8.2e-6	-7.2e-6	11.3
0.001	0.0028	0.089	0.002	0.00075	0.006	-4.14e-6	-3.28e-6	20
0.001	0.003	0.083	0.002	0.00075	0.006	-3.69e-6	-2.97e-6	19
0.001	0.004	0.062	0.002	0.0007	0.006	-2.54e-6	-2.17e-6	14

It can be inferred from Table 4 that the proposed sensor can be miniaturized with smaller design parameters like initial center distance (ICD), dimensions of cylindrical rubber blocks and Aluminum plate, in order to have acceptable error. For example, for a sensor composed of cylindrical rubber blocks with diameter of 1 mm, and initial center distance of 6 mm, one can decrease the error of measurement from 20% to 14%

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by decreasing the shape factor from 0.089 to 0.062. In order to have a sense of the size for miniaturized sensor, in Fig. 36 two finite element models with the same host structure but different sensors have been shown: one with diameter of 0.007 m, height of 12 mm, initial center distance of 0.030 m for cylindrical rubber blocks, width and thickness of 0.012 m and 0.001 m, respectively for Aluminum plate and the other one composed of cylindrical rubber blocks with diameter of 0.001 m, height of 0.004 m, initial center distance of 0.006 m and Aluminum plate with width and thickness of 0.002 m and 0.0007 m respectively.

Figure 36 (a) sensor composed of cylindrical rubber blocks with diameter of 0.007 m, height of 0.012 m, initial center distance of 0.030 m and Aluminum plate with width and thickness of 0.012 m and 0.001 m, respectively, bonded on the host structure. (b) sensor composed of cylindrical rubber blocks with diameter of 0.001 m, height of 0.004 m, initial center distance of 0.006 m and Aluminum plate with width and thickness of 0.002 m and 0.0007 m, respectively, bonded on the host structure

3.7 What happens if the Young's modulus of the cylindrical rubber blocks and rubber sheet are almost the same?

As it was mentioned in design requirements of the sensor, the sensor should be softer than the host structure. Therefore in previous sections, the Young's modulus of the cylindrical rubber blocks was set almost three times smaller than Young's modulus of the rubber sheet. In this section, we want to see the effect of having almost the same Young's modulus for cylindrical rubber blocks as rubber sheet. For that purpose, six sets of models from Table 3 and 4 were randomly chosen. A rectangular rubber sheet with length of 0.1 m, width of (b) 0.015 m and thickness (t) of 0.002 m together with two cylindrical rubber blocks were modeled. The diameter (D), height (h), shape factor (S), initial center distance (ICD) of the cylindrical rubber blocks together with width (b) and thickness (t) of the Aluminum plate are shown in Table 5. The length of 0.005 m, in each side of the rubber sheet, was used for pulling the rubber sheet. Rectangular rubber sheet and cylindrical rubber blocks were assumed to consist of a Neo-Hookean elastic material, and almost incompressible with a Poisson's ratio of 0.4997. A value of 0.23 MPa was assigned to the elastic constant C_{10} , corresponding to a Young's modulus E of 1.38 MPa for the rubber sheet and 0.229822 MPa was assigned to C_{10} , corresponding to a Young's modulus, E of 1.378 MPa for cylindrical rubber blocks. The Aluminum plate was assumed to have Young's modulus of 69GPa and a Poisson's ratio of 0.33. The rectangular rubber sheet is displaced uniaxially at both ends with a value of 0.015 m corresponding to a strain of 0.287. The element, C3D20R (a 20-node quadratic brick) which is in the family of three dimensional stress (3D Stress) elements was chosen as the meshing element.

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Comparison of finite element results and analytical results are shown in Table 5. In the last column, error determined by comparing strain of the Aluminum plate, obtained from Abaqus modeling ($\varepsilon_{(Al-plate)-Abaqus}$) with strain obtained from Eq. 3.69, $\varepsilon_{(Al-plate)-equation}$ is shown. All the spatial dimensions are in meters (m).

Table 5 Comparison of finite element results and analytical results for a sensor bonded on a rubber sheet experiencing strain of 0.287 (cylindrical rubber blocks and rubber sheet have almost the same Young's modulus)

D	h	S	b	t	ICD	$\varepsilon_{(Al-plate)-Abaqus}$	$\varepsilon_{(Al-plate)-equation}$	Error (%)
0.004	0.011	0.09	0.008	0.001	0.025	-2.95e-5	-3.36e-5	14
0.003	0.007	0.107143	0.005	0.0009	0.015	-2.64e-5	-2.80e-5	6.19
0.0035	0.007	0.125	0.005	0.0009	0.016	-4.53e-5	-4.96e-5	9.6
0.006	0.013	0.115	0.012	0.001	0.03	-6.57e-5	-8.5e-5	23
0.005	0.011	0.1136	0.01	0.001	0.03	-5.63e-5	-6.56e-5	23
0.006	0.014	0.107	0.01	0.001	0.03	-7.24e-5	-9.26e-5	28

Comparing errors in Table 5 with their corresponding errors in Table 3 and 4, it can be seen that when the cylindrical rubber blocks have almost the same Young's modulus as the rubber sheet, the error of measurement is increased. This fact is more distinct for sensors containing cylindrical rubber blocks with larger shape factors.

Chapter 4: experimental justification and example of practical application

4.1 Introduction

In this chapter, at first the procedure to find material properties of a set of cylindrical rubber blocks and also rubber sheet specimen is illustrated and afterwards by means of experimental results, the accuracy of the sensor is justified. Comparison of experimental results, finite element results and theoretical results are presented graphically.

4.2 Material characterization

In general, In order to measure the stress-strain response of hyperelastic materials under simple states of loading like uniaxial, biaxial and planar loading, standard characterisation tests are utilized. Test data obtained from mentioned tests, are adjusted and fitted to hyperelastic models with curve-fitting tools available in Abaqus software. The following section provides material characterisation.

4.2.1 Uniaxial tension test of cylindrical rubber block

One set of rubber cylinders was purchased. The diameters of the cylindrical rubber blocks were 6.85 mm and their heights were 29 mm. Rubber cylinders were subjected to uniaxial tension test. The stress-strain curve related to this uniaxial test is shown in Fig. 37. Engineering stress was calculated by dividing the force, recorded by the Instron microtensile test machine, by initial cross sectional area and engineering strain was also calculated by dividing extension, recorded by tensile test machine, by initial

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gauge length. The stress-strain characterisation data points are provided in Table 6 of Appendix B.

Figure 37 uniaxial test data for a cylindrical rubber with diameter of 6.85 mm

4.2.2 Curve-fitting

The coefficients whereby a hyperelastic model is defined, thus defining properties of a rubber-like material, are not directly measured, however rather fitted to experimental characterisation data like uniaxial tension test, equibiaxial test and planar test data. The discussion herein is devoted to curve-fitting of test data. The Abaqus curve fitting module generates coefficients for a chosen hyperelastic model. For instance, one can import only uniaxial test data or together with equibiaxial and planar test data, whereupon coefficients of selected hyperelastic models for counterparts of uniaxial, equibiaxial and planar loading are obtained [44].

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4.2.2.1 General curve-fitting in Abaqus for cylindrical rubber blocks

Uniaxial characterization data (stress-strain data points) is imported to Abaqus version 6.7 and fitted to different hyperelastic models so as to find the best curve-fit. It was observed that all the models guaranteed the stability limit of the material. Curve-fitted experimental data for a cylindrical rubber block with a diameter of 6.85 mm is shown in Fig. 38.

Figure 38 curve-fitting of uniaxial test data for a cylindrical rubber with diameter of 6.85 mm. It is clear from Fig. 37 that up to strain of 5 or 6% most of the hyperelastic models match test data well but beyond 6% the second order Ogden model (Ogden_N2), fits the experimental data points the best. The coefficients of Ogden_N2 model are: $\alpha_1 = -8.39287$, $\alpha_2 = -16.0933$, $D_1 = 0$, $D_2 = 0$, $G_1 = -1728556$ and $G_2 = 3410639$.

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4.2.3 Uniaxial test of rubber sheet

Specimen of rubber sheet (host structure) in shape of dog bone which is shown in Fig.

39 was also subjected to uniaxial tension.

Figure 39 Rubber sheet specimen in shape of dog bone used for uniaxial test

The uniaxial test data for a cylindrical rubber block with a diameter of 6.85 mm is shown in Fig. 40.

Figure 40 Stress-strain curve of rubber sheet subjected to uniaxial tension test

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The curve-fitted experimental data is shown in Fig. 41.

Figure 41 curve-fitting of uniaxial test data for rubber sheet specimen

It was seen that the only stable curve fitted data, is second order Ogden model (Ogden_N2) for which the coefficients are: $\alpha_1 = 7.63391321$, $\alpha_2 = -25$, $D_1 = 0$, $D_2 = 0$, $G_1 = -3706397.48$ and $G_2 = 7539485.11$. Experimental data according to uniaxial test of rubber sheet specimen are provided in Table 7 of appendix B.

4.3 Testing the sensor without the rubber sheet (as host structure)

In order to investigate the accuracy of the sensor, cylindrical rubber blocks were bonded to a thin Aluminum plate and with their other end bonded to small piece of metals in order to applying uniaxial displacement. A strain gauge with gauge factor of 2.14 was soldered to the top surface of the thin Aluminum plate. The apparatuses used for our experiments are shown in Fig. 42.

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Figure 42 Apparatuses used for uniaxial test of the sensor

As it is shown in Fig. 42, the free end of the lower cylindrical rubber block is fixed to the bottom fixture of the test machine and the free end of the upper cylindrical rubber block is displaced outwards (tensile test, shown in Fig. 43 (a)) or inwards (compression test, shown in Fig. 43 (b)).

Figure 43 (a) Sensor under outward displacement (b) Sensor under inward displacement

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Two sets of numbers must be read simultaneously: the extension of the moving end of the upper cylindrical rubber block which is acquired from microtensile test machine and strain of the Aluminum plate which is read from strain-meter to which soldered strain gauge is connected via a wire. Therefore, in order to have accurate reading, changes of the said values were recorded, courtesy of a camera. Afterwards, the captured footage was monitored so as to record output of the microtensile test machine which is extension of the moving end of the cylindrical rubber blocks and the corresponding strain values of the Aluminum plate obtained from strain-meter. Strain at the top surface of the Aluminum plate (obtained from Eq. 3.64), was also provided in order to compare test results with theoretical equations. All the above mentioned data are provided in appendix B.

4.3.1 Testing of the sensor without the rubber sheet subjected to outward movement

Comparison of strain of the Aluminum plate obtained from tensile test (strain meter) and Aluminum plate strain obtained from Eq. 3.64 together with strain of the Aluminum plate obtained from Abaqus modeling versus tensile strain (outward movement) of free end of the cylindrical rubber block with diameter of 6.85 mm and height of 18.5 mm are shown in Fig. 44. Target point is strain of 14.8 % of the moving end of the block. The Young's modulus, E of the cylindrical rubber blocks used in equations (Eq. 3.64) is 4 MPa (corresponding to shear modulus, G of 1.3 MPa) which is approximation of test data according to Fig. 37. Abaqus modeling like section 3.6.1 was performed and material properties of the cylindrical rubber blocks obtained in sections 4.2.2.1 was used for the material modeling. The results of Abaqus modeling are also shown in Fig. 44. The final error between Abaqus modeling and

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experimental data is 13.6% and the final error between analytical equation and experimental data is 9.2%. Corresponding experimental data and equations data are provided in Table 8 (see appendix B).

Figure 44 Comparison of experimental results, equation results and Abaqus modeling results for outward movement of sensor (like Fig 42 (a)) composed of blocks with diameter of 6.85 mm

4.3.2 Testing of the sensor without rubber sheet subjected to inward movement

Comparison of strain of the Aluminum plate obtained from tensile test, and strain obtained from Eq. 3.64 together with strain of the Aluminum plate obtained from Abaqus modeling versus compression strain (inward movement) of free end of the cylindrical rubber block with diameter of 6.85 mm and height of 18.5 mm are shown in Fig. 45. Target point is strain of -15.1% of the moving end of the block. The

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Young's modulus, E of the cylindrical rubber blocks used in equations (Eq. 3.64) is 4 MPa (corresponding to shear modulus, G of 1.3 MPa) which is approximation of test data according to Fig. 37. Abaqus modeling similar to section 3.6.1 was conducted and material properties of the cylindrical rubber blocks obtained in sections 4.2.2.1 were used for the material modeling. The results of Abaqus modeling are also shown in Fig. 45. The final error between Abaqus modeling and experimental data is 12.5% and the final error between equation results and experimental data is 5.7%. Corresponding experimental data and equations data are provided in Table 9 (see appendix B).

Figure 45 Comparison of experimental results, equation results and Abaqus modeling results for inward movement of sensor (like Fig 42 (b)) composed of blocks with diameter of 6.85 mm

It can be deduced from the Figs. 44 and 45 that proposed sensor is capable of measuring large strains with good accuracy; the relationship between strain of the

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Aluminum plate read from strain-meter and strain of the moving end of the rubber cylinders (or in other words strain of the host structure) is linear; this fact, proves the said linear relationship in Eq. 3.64.

4.4 Testing the sensor bonded to the rubber sheet (on the host structure)

As it can be seen in Figs. 37 and 40, the cylindrical rubber blocks and rubber sheet (in higher strains) have Young's modulus, E of almost 4 MPa and 5 MPa, respectively. It means that rubber blocks are a bit softer than rubber sheet but not much. It was shown in chapter 3 (section 3.7) that for rubber blocks and rubber sheet with almost same Young's modulus, we can design our sensor by choosing design parameters in a way that the errors are small. So for our experimental work we need to carefully choose the dimensions of the cylindrical rubber, so the following dimensions were used:

Cylindrical rubber blocks with diameter of 6.85 mm and height of 18.5 mm (with shape factor (S) of 0.0925), as it is shown in Fig. 46, were bonded from one side to Aluminum plate on which a strain gauge with gauge factor of 2.14 was soldered and from other side were bonded to a rectangular rubber sheet (as host structure).

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Figure 46 Sensor bonded to rubber sheet from one side and from other side bonded to Aluminum plate on which a strain gauge is soldered

The width and thickness of the Aluminum plate were 11 mm and 1 mm, respectively and the width, thickness and length of the rubber sheet were 10 mm, 1 mm and 62.5 mm, respectively. 5 mm metal sheets were glued to each side of the host rubber sheet to prevent the rubber sheet from sliding in the test machine. Initial center distance (ICD) of cylindrical rubber blocks was 30.1 mm. Comparison of experimental results, equation results and Abaqus modeling results are shown in Fig. 47. Target point is strain of 14.1 % of the rubber sheet. The Young's modulus, E of the cylindrical rubber blocks used in equations (Eq. 3.64) is 4 MPa which is approximation of test data according to Fig. 37. Abaqus modeling was conducted and material properties of the cylindrical rubber blocks and rubber sheet obtained in sections 4.2.2.1 and 4.2.3 were used for the material modeling. The final error between Abaqus modeling and

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experimental data is 19.4% and the final error between equation results and experimental data is 11.2%. This mere error arises because of inaccuracy of material properties, inaccuracy of the strain-meter and microtensile test machine. Corresponding experimental data and analytical results are provided in Table 10 (see appendix B).

Figure 47 Comparison of experimental results, equation results and Abaqus modeling results for sensor bonded on the rubber sheet subjected to uniaxial tension

4.5 Some comments regarding experimental procedure

- I. Experiments illustrated above were done more than one time and each time the target strain of the moving end was increased. It was observed that the corresponding values for each experiment were analogous to each other; it can

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be inferred from these observations that if the host structure does not encounter permanent strain, the sensor monitors the strain of the host structure rigorously. Although the permanent strain which remains in the host structure can be measured by the sensor, however this permanent strain alters the initial shape of the rubber blocks. Therefore, if it is desired to measure the strain of the host structure for a new loading, the new sensor can be bonded to the surface of the host structure or the sensor can be separated out and after returning to initial shape can be bonded again. In this case, strong adhesive should be used for bonding the end of the rubber blocks to the Aluminum plate but the other end can be bonded to the host structure using double sided tapes for the sake of easy separation. It is important to mention that use of double sided tapes is okay in the lab but impractical in the field.

Another approach to handle permanent strain is to zero the Wheatstone bridge, if permanent strain occurs.

- II. The cylindrical rubber blocks should be vulcanized; otherwise the blocks will not have desirable resilience to return to their initial shape after encountering large strains.
- III. Using a very accurate strain-meter is suggested, otherwise data will be error-prone. The small fluctuations in the experimental results are because of the fact that the strain-meter is not as accurate as microtensile test machine. The implication is that the microtensile test machine records the displacement of the moving end of the cylindrical rubber blocks which changes in the order of

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micro meter (10^{-6} m); however strain-meter records the change in strain of the Aluminum plate. While the displacement of the moving end of the rubber block is occurring, there is some times when the strain-meter does not show any changes in the strain of the Aluminum plate, since the change of strain is not large enough to be recorded by the strain-meter. Therefore, using a more accurate strain-meter is preferable. Basically, a better resolution strain-meter is needed.

- IV. For a given Aluminum metal strain, it is not necessary to use Horton equation to derive the host structure strain. The relationship between aluminum metal strain and induced strain can be found and once this relationship experimentally is found, it can be used to find host structure strain when needed.

4.6 A practical example

Numerous studies and experimental fatigue data collected on rubber components indicate that rubber does not tolerate tension well [36, 45]. Rubber components have longer fatigue lives if rubber stays in compression and rarely sees tension.

The purpose of the rubber strain gage sensor, proposed in this thesis, can be monitoring the state of surface strain of rubber components and make sure that rubber stays in compression. The sensor should not only monitor dynamic strains, but also static strains since static strains can cause tension in the rubber.

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To illustrate how static and dynamic strains can occur in the field, the following industrial application is described. Fig. 48 shows one locomotive body mount used underneath the locomotive body.

Figure 48 Locomotive body mount (sandwich mount) (courtesy of Lord Corporation)

The locomotive body is supported by several rubber mounts similar to the rubber sandwich mount design of Fig. 48. The rubber sandwich mounts will be mostly under compression due to the weight of the locomotive body. Due to this static compression, the rubber layers experience static strains. Since rubber layers are constrained, temperature change and other environmental factors can increase or decrease the static strains of the rubber layers.

As the locomotive accelerates or decelerates, the locomotive body moves horizontally (fore-aft motion) causing the sandwich mounts to shear. As a result of this dynamic shear, the rubber layers experience dynamic strains.

In addition to the fore-aft dynamic motion, the locomotive body can also have dynamic motion in the vertical, and the lateral directions. These simultaneous loads

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can create multi-axes rubber surface strains. Therefore, in order to know the complete surface strain of rubber layers, multiple rubber strain gauge sensors can be used to pick up the rubber static and dynamic strains in multiple directions.

Chapter 5: Conclusions

In this chapter, conclusions of the research are reviewed:

A review of the prior arts of measuring strain of highly deformable solids indicated that there is no viable strain measuring device for highly deformable solids. In this research, a new method for surface strain measurement of highly deformable solids was introduced.

The following conclusions were drawn from the present work:

- Successful measurement of common range of measuring large strains stems from thorough choice of design parameters of the sensor like dimensions of the cylindrical rubber blocks and their initial center distance, dimensions of the Aluminum plate and also material properties of the cylindrical rubber blocks.
- Both tension and compression strain of the host structure can be captured by the proposed new sensor.
- Viability of miniaturization of the sensor until the shape factor of 0.062 for cylindrical rubber blocks was investigated by means of finite element modeling.
- The accuracy of the proposed sensor up to almost 15 % strain of the host structure was justified through finite element modeling and experiment and it was shown that analytical results and finite element results match the experimental results with good accuracy.

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5.1 Suggestions for future graduate students

The author, during the first year, also has worked on the following topics, but due to the fact that there is only one year left to complete the project, the following topics are left for the future students to continue:

1. Utilization of rectangular rubber blocks instead of cylindrical rubber blocks for the proposed sensor can be studied.
2. The effect of temperature on the sensor should be studied.
3. Other designs like purely metallic sensor (it means that instead of rubber blocks in our sensor, a flexible metal is used) can be considered for the purpose of large strain measurement.
4. One of the other fields, that our proposed sensor can be used is in “uniaxial in-vivo measurement of the mechanical properties of skin” [46-52]. One of the common devices used in uniaxial in-vivo test is extensometer whereby the force-displacement diagram of the skin subjected to uniaxial test is measured. A typical uniaxial extensometer applies displacement to the surface of the skin using two pads stuck to the skin by means of double-sided tapes. The displacement is measured using a linear variable differential transformer sensor (LVDT) [52] or a transducer [53] and the force needed for deformation is measured in different ways such as: with a magnet situated inside a coil [53], with a load cell placed between the movable pad and apparatus body [50] or by means of a load cell stuck to the movable pad [51]. Three of these apparatuses are shown in the following figures:

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Figure 49 Section through the apparatus used for testing the skin in-vivo: A: fixed arm; B: movable arm; C: LVDT transducer; D: lead screw; E: connection for flexible drive [48]

Figure 50 In-vivo apparatus for recording the recoil of stretched skin after removal of a uniaxial deforming force [54]

Figure 51 a schematic drawing of extensometer: A: fixed pad; B: movable pad (attached to the load cell); C: shield pad (moves simultaneously with pad B); D: load cell; E: lead screw; F: carriage and G: servo motor [51]

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Instead of using LVDT, our sensor can be mounted on the extensometer of Fig. 50 in order to measure displacement of the movable pad and the force required. Therefore, there will not be any need to use one device for measuring displacement and one for measuring force. Schematic figure of mentioned design, drawn in Inventor software, is shown in Fig. 52. Future graduate students should consider the viability of our proposed sensor for measurement of skin properties, since skin experiences large strains just like rubber.

Figure 52 (a): 1: our proposed sensor, 2: push and pull solenoid or any other actuator that produces push and pull movements, 3: movable pad, 4: fixed pad; (b): three dimensional view of the extensometer

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5.2 Author's contributions

The author's contributions are:

- ✓ A new sensor for strain measurement of highly deformable solids has been proposed.
- ✓ The measurement procedure and theoretical equations for the proposed designed of the sensor has been documented.
- ✓ The accuracy of the proposed sensor was investigated experimentally and the error between experimental results and analytical results is only 11.2 %.

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Appendix A

Appendix A

A.1 Beam theory background

The theoretical analysis of the deflection of an isotropic beam subjected to out-of-plane loading is well illustrated in mechanics texts [55] and it will be summarized concisely here. Consider a simply supported beam subjected to a symmetric loading of shear force, F and moment M at each side. The theory of bending gives

$$\sigma = \frac{F}{A} \pm \frac{My}{I} \quad (5.1)$$

where σ is the in-plane longitudinal stress, y is the vertical distance from the middle of the cross sectional area, E is the Young's modulus of the beam material and I is the second moment of area of the cross-section of the beam.

A.2 Simple shear and pure shear

There are two type of shear strain loading:

- a. Simple shear strain which can be rendered by movement of planes parallel to a given plane through a distance proportional to their distance from the said plane (in other words simple shear is apparent shear illustrated in the chapter 3).
- The amount of the shear strain, γ is related to the angle φ by

$$\gamma = \tan \varphi \quad (5.2)$$

and the shear stress, t_{xy} is given by

$$t_{xy} = G\gamma \quad (5.3)$$

where G is the shear modulus of the material.

Appendix A

- b. Pure shear strain or planar tension (compression) corresponds to extension in three perpendicular directions, without rotation of the principal axes of strain.

A simple shear is thus equivalent to a pure shear together with a rotation. The state of pure shear can be represented by applying principal stresses t_1 and

t_2 only, with $t_3 = 0$.

The states of simple shear strain and pure shear strain are shown in Fig. 53:

Figure 53 (a) Simple shear; (b) pure shear [25].

Appendix B

Appendix B: Experimental data

B.1 Introduction

In the following tables, characterisation data points for uniaxial tensile test of specimens together with data obtained from testing the sensor are provided.

B.2 Characterization data

In this section, characterization data of cylindrical rubber block and rubber sheet are presented:

B.2.1 Uniaxial test data for cylindrical rubber of diameter 6.85 mm

The uniaxial test data for cylindrical rubber of diameter 6.85 mm are is provided in Table 6. Gauge length for this test is 29 mm.

Table 6: Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0	0	0	0
0.006277	5.28E-05	1432.827	0.000216
0.014748	0.000101	2747.91	0.000509
0.022956	0.000157	4272.118	0.000792
0.031414	0.000207	5609.035	0.001083
0.039426	0.000262	7105.101	0.00136
0.047556	0.00031	8405.143	0.00164
0.056316	0.00038	10303.61	0.001942

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.0648	0.000435	11806.31	0.002234
0.073009	0.000487	13210.51	0.002518
0.081283	0.000546	14824.8	0.002803
0.089636	0.000588	15962.95	0.003091
0.098369	0.000645	17500.09	0.003392
0.106854	0.000676	18344.04	0.003685
0.115272	0.000731	19838.65	0.003975
0.12331	0.00078	21167.32	0.004252
0.131572	0.000826	22420.78	0.004537
0.140567	0.000882	23939.97	0.004847
0.148907	0.000919	24927.05	0.005135
0.15709	0.000987	26793.82	0.005417
0.165324	0.001024	27782.35	0.005701
0.173809	0.001087	29499.03	0.005993
0.182358	0.001124	30498.56	0.006288
0.19079	0.001172	31801.19	0.006579
0.199248	0.001209	32803.31	0.006871
0.207273	0.001258	34131.82	0.007147
0.215508	0.001302	35340.16	0.007431
0.224412	0.001353	36717.35	0.007738
0.232752	0.001402	38033.56	0.008026
0.240869	0.001456	39511.84	0.008306
0.249274	0.001506	40868.16	0.008596
0.25764	0.001543	41881.12	0.008884
0.266059	0.001592	43203.32	0.009174

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.274385	0.00164	44493.82	0.009462
0.282738	0.001674	45410.38	0.00975
0.290685	0.001722	46733.39	0.010024
0.298986	0.001752	47549.03	0.01031
0.307994	0.001814	49233.19	0.01062
0.316308	0.001841	49945.64	0.010907
0.324294	0.001896	51459.5	0.011183
0.332594	0.001951	52934.06	0.011469
0.340973	0.001991	54038.07	0.011758
0.349497	0.002032	55131.57	0.012052
0.357811	0.00206	55888.5	0.012338
0.366321	0.002121	57544.37	0.012632
0.374307	0.002167	58797.83	0.012907
0.38245	0.002201	59718.11	0.013188
0.391223	0.002243	60864.34	0.01349
0.399536	0.002289	62110.52	0.013777
0.40764	0.002331	63239.29	0.014057
0.415993	0.00239	64858.43	0.014345
0.424438	0.002412	65462.84	0.014636
0.432778	0.002476	67189.71	0.014923
0.441118	0.002518	68314.43	0.015211
0.449562	0.002538	68858.02	0.015502
0.457574	0.002578	69949.26	0.015778
0.465731	0.002615	70960.12	0.01606
0.474674	0.002675	72596.09	0.016368

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.48283	0.00272	73801.67	0.016649
0.491144	0.002765	75025.86	0.016936
0.499536	0.0028	75984.96	0.017225
0.507824	0.00285	77342.42	0.017511
0.51615	0.002878	78097.89	0.017798
0.524516	0.002935	79641.02	0.018087
0.532896	0.002964	80428.36	0.018376
0.540777	0.002997	81324.87	0.018647
0.549012	0.003051	82789.23	0.018931
0.557955	0.003093	83929.48	0.01924
0.566216	0.003151	85514.34	0.019525
0.574307	0.003163	85838.29	0.019804
0.582725	0.003196	86717.5	0.020094
0.591052	0.003247	88118.62	0.020381
0.599497	0.003293	89358.17	0.020672
0.607784	0.003334	90475.61	0.020958
0.616098	0.003365	91313.9	0.021245
0.623992	0.003398	92203.94	0.021517
0.632371	0.003449	93586.78	0.021806
0.641288	0.003505	95100.48	0.022113
0.649457	0.003526	95669.96	0.022395
0.657614	0.003568	96817.97	0.022676
0.666006	0.003606	97851.62	0.022966
0.674451	0.003659	99276.36	0.023257
0.682895	0.003693	100213	0.023548

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.691235	0.003731	101235	0.023836
0.699641	0.003775	102435.2	0.024126
0.707679	0.00382	103642	0.024403
0.715888	0.003848	104419	0.024686
0.724726	0.003878	105239.6	0.024991
0.732987	0.003926	106518.3	0.025275
0.740986	0.003971	107755.4	0.025551
0.74947	0.004008	108748.5	0.025844
0.757954	0.004042	109686.4	0.026136
0.766137	0.004098	111187.5	0.026419
0.77466	0.004114	111625.9	0.026712
0.782895	0.00416	112875.7	0.026996
0.790829	0.004193	113781.7	0.02727
0.799156	0.004247	115249	0.027557
0.807954	0.00429	116413.5	0.02786
0.816189	0.004318	117180.3	0.028144
0.824241	0.004348	117971.4	0.028422
0.832751	0.004386	119002.6	0.028716
0.841052	0.004441	120495.9	0.029002
0.849457	0.00447	121303.6	0.029292
0.857915	0.00452	122663.2	0.029583
0.86636	0.004557	123648.5	0.029874
0.874319	0.004589	124516.4	0.030149
0.882318	0.004628	125569	0.030425
0.891327	0.004652	126244.9	0.030735

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.899614	0.004704	127655.4	0.031021
0.907587	0.004748	128832.2	0.031296
0.915993	0.004771	129447.9	0.031586
0.924332	0.004828	131000.1	0.031874
0.932685	0.004853	131699.1	0.032162
0.941222	0.004896	132849.2	0.032456
0.949614	0.00494	134050	0.032745
0.957561	0.004973	134937.4	0.033019
0.966071	0.005015	136074.4	0.033313
0.974765	0.005063	137396	0.033613
0.982974	0.005084	137942	0.033896
0.991065	0.005123	139007.8	0.034175
0.999562	0.005152	139795	0.034468
1.007889	0.005176	140449.9	0.034755
1.01611	0.00525	142465.6	0.035038
1.024608	0.005291	143560.9	0.035331
1.032974	0.005309	144064.9	0.03562
1.040789	0.005326	144511.4	0.035889
1.049129	0.005366	145608.8	0.036177
1.058033	0.00542	147084.3	0.036484
1.066307	0.005464	148256	0.036769
1.074424	0.005484	148819.8	0.037049
1.082856	0.005546	150498.1	0.03734
1.091209	0.005579	151380.1	0.037628
1.099614	0.005583	151483.6	0.037918

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.108046	0.005635	152911.9	0.038208
1.116189	0.005691	154415.2	0.038489
1.124057	0.005706	154834.6	0.038761
1.132672	0.005763	156383.1	0.039058
1.141314	0.005787	157036.5	0.039356
1.149588	0.005849	158724.5	0.039641
1.157679	0.005878	159498.3	0.03992
1.166005	0.005923	160733.3	0.040207
1.174398	0.005942	161222.7	0.040496
1.182724	0.005974	162097.6	0.040784
1.19134	0.006015	163214.7	0.041081
1.199627	0.006057	164369.3	0.041366
1.207547	0.006086	165144.2	0.04164
1.216123	0.006121	166088.4	0.041935
1.224765	0.006165	167273.6	0.042233
1.233052	0.006213	168579.8	0.042519
1.241117	0.006247	169502	0.042797
1.249522	0.006268	170086.9	0.043087
1.257888	0.006303	171036.4	0.043375
1.266163	0.006319	171470.5	0.043661
1.274594	0.006379	173097	0.043952
1.282842	0.006434	174594.8	0.044236
1.290763	0.006447	174935.9	0.044509
1.299325	0.006487	176019.2	0.044804
1.308046	0.006529	177166.7	0.045105

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.316228	0.006569	178240.8	0.045387
1.324502	0.006586	178706.8	0.045672
1.332777	0.006636	180075.6	0.045958
1.34109	0.006664	180833.2	0.046244
1.349522	0.006686	181431.9	0.046535
1.357901	0.00676	183440.7	0.046824
1.366228	0.006797	184449	0.047111
1.37428	0.006811	184825.8	0.047389
1.382514	0.006832	185399	0.047673
1.391274	0.006881	186712.6	0.047975
1.399404	0.006914	187604.8	0.048255
1.407508	0.006972	189186.2	0.048535
1.41594	0.006992	189740.7	0.048826
1.424293	0.007028	190694.3	0.049114
1.432711	0.007082	192158.8	0.049404
1.441169	0.007093	192478.4	0.049695
1.449548	0.007121	193214.6	0.049984
1.457495	0.007163	194379.9	0.050258
1.466044	0.007205	195499.5	0.050553
1.474699	0.007248	196684.3	0.050852
1.482777	0.007271	197309.1	0.05113
1.490972	0.007323	198712.8	0.051413
1.499273	0.007341	199203.2	0.051699
1.507639	0.007384	200370.7	0.051988
1.515939	0.007427	201539.7	0.052274

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.524542	0.007458	202366.5	0.05257
1.532881	0.0075	203512.1	0.052858
1.540697	0.007521	204068.8	0.053127
1.549024	0.007566	205299.3	0.053415
1.557927	0.007595	206091.8	0.053722
1.566123	0.007641	207348.3	0.054004
1.574174	0.007652	207643	0.054282
1.582711	0.007706	209108.5	0.054576
1.591169	0.007747	210224.6	0.054868
1.599456	0.007773	210914.3	0.055154
1.608072	0.007824	212311	0.055451
1.616189	0.007843	212806	0.055731
1.624083	0.007866	213449.3	0.056003
1.632527	0.007907	214566.9	0.056294
1.641352	0.007933	215269.7	0.056598
1.649509	0.007998	217013.2	0.05688
1.657573	0.00802	217626.8	0.057158
1.666057	0.008069	218955.7	0.05745
1.674437	0.008106	219948.9	0.057739
1.682658	0.008114	220174.7	0.058023
1.691352	0.008172	221748.4	0.058322
1.69964	0.008183	222048.6	0.058608
1.707534	0.008236	223478.5	0.05888
1.716123	0.008243	223678.6	0.059177
1.724804	0.008311	225512.2	0.059476

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.732986	0.008331	226066.3	0.059758
1.740959	0.008374	227238.2	0.060033
1.749404	0.008399	227913	0.060324
1.757822	0.008428	228704.4	0.060615
1.766018	0.008469	229802.2	0.060897
1.774489	0.008507	230834.6	0.061189
1.782724	0.008557	232189.2	0.061473
1.790657	0.00856	232268.4	0.061747
1.799089	0.008618	233846.1	0.062038
1.80798	0.008667	235172.4	0.062344
1.816201	0.008673	235350.5	0.062628
1.824292	0.008702	236132.3	0.062907
1.832698	0.008752	237475.8	0.063196
1.840972	0.008779	238204.3	0.063482
1.849495	0.008817	239246.7	0.063776
1.857875	0.008852	240197.4	0.064065
1.866293	0.008899	241468.6	0.064355
1.874266	0.008905	241645.4	0.06463
1.882448	0.008971	243428.9	0.064912
1.891195	0.008996	244099	0.065214
1.899535	0.00903	245037.4	0.065501
1.907586	0.009048	245515.5	0.065779
1.916018	0.009104	247046	0.06607
1.924463	0.00913	247730	0.066361
1.932737	0.009163	248645.4	0.066646

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.941287	0.00921	249915.5	0.066941
1.949613	0.009225	250327.6	0.067228
1.95752	0.009263	251347.7	0.067501
1.965703	0.009284	251907.3	0.067783
1.974581	0.009345	253562.8	0.068089
1.982816	0.009365	254108.4	0.068373
1.99109	0.009393	254880.7	0.068658
1.999207	0.009418	255547.2	0.068938
2.00752	0.009459	256662.7	0.069225
2.015926	0.009518	258258.2	0.069515
2.024305	0.009513	258132.7	0.069804
2.032724	0.009575	259810.4	0.070094
2.040618	0.009591	260239.5	0.070366
2.048722	0.009629	261282.5	0.070646
2.057743	0.009674	262500.6	0.070957
2.065965	0.009698	263158.2	0.07124
2.074108	0.009744	264413.9	0.071521
2.082592	0.009779	265364.3	0.071814
2.090841	0.0098	265930.7	0.072098
2.099351	0.009826	266637.7	0.072391
2.107809	0.009879	268064.8	0.072683
2.116096	0.009908	268864.3	0.072969
2.124082	0.009936	269611.2	0.073244
2.132383	0.009977	270729.6	0.07353
2.14126	0.010009	271602.2	0.073837

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.149403	0.010045	272570.8	0.074117
2.157599	0.010067	273169.4	0.0744
2.165991	0.010087	273703.1	0.074689
2.174292	0.010153	275488.7	0.074976
2.18275	0.010163	275777.1	0.075267
2.191365	0.010215	277195.3	0.075564
2.199718	0.010229	277568.6	0.075852
2.207822	0.010276	278845.4	0.076132
2.216031	0.010294	279335.1	0.076415
2.224908	0.010337	280481.8	0.076721
2.233091	0.010368	281334.2	0.077003
2.241103	0.01039	281945.6	0.077279
2.249574	0.010443	283364	0.077572
2.258018	0.010478	284327	0.077863
2.266424	0.010503	285008.1	0.078153
2.27479	0.010549	286239.5	0.078441
2.283012	0.010551	286299.4	0.078725
2.290985	0.010607	287816.6	0.078999
2.299233	0.010654	289082.4	0.079284
2.308228	0.010671	289555.4	0.079594
2.316463	0.010694	290190.6	0.079878
2.324528	0.010733	291241.7	0.080156
2.332802	0.010773	292331.5	0.080441
2.341142	0.010795	292921.7	0.080729
2.349508	0.010827	293792.5	0.081018

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.357848	0.010879	295193.9	0.081305
2.366266	0.010905	295895.4	0.081595
2.374265	0.010937	296764.4	0.081871
2.382343	0.01097	297682.1	0.08215
2.391076	0.011007	298666.2	0.082451
2.399416	0.011044	299668.4	0.082738
2.407454	0.011066	300281.8	0.083016
2.415585	0.011088	300864.6	0.083296
2.424016	0.011138	302230.4	0.083587
2.432487	0.011168	303038.1	0.083879
2.440984	0.011195	303777.8	0.084172
2.449311	0.01122	304447.2	0.084459
2.457376	0.011255	305412.1	0.084737
2.465558	0.011288	306291.3	0.085019
2.474488	0.011316	307059.6	0.085327
2.482527	0.011342	307756.5	0.085604
2.490945	0.011376	308691.5	0.085895
2.499193	0.011404	309444.1	0.086179
2.507599	0.011451	310728.6	0.086469
2.51603	0.011488	311717	0.08676
2.524541	0.01151	312335.9	0.087053
2.532894	0.011558	313621.2	0.087341
2.540775	0.011581	314250.4	0.087613
2.549075	0.011631	315595.3	0.087899
2.558149	0.011644	315959.6	0.088212

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.566306	0.011679	316911.1	0.088493
2.574449	0.011719	317981.5	0.088774
2.582854	0.011733	318366.1	0.089064
2.591142	0.011763	319188.9	0.08935
2.599678	0.011815	320587.6	0.089644
2.608254	0.011847	321470	0.08994
2.616319	0.011888	322567.4	0.090218
2.624278	0.011906	323056.2	0.090492
2.63271	0.011933	323788	0.090783
2.641496	0.011971	324832.2	0.091086
2.649678	0.011983	325155	0.091368
2.657743	0.012032	326491.9	0.091646
2.666227	0.012065	327378.7	0.091939
2.674541	0.012116	328771.6	0.092226
2.682802	0.012134	329263.3	0.09251
2.69143	0.012168	330164.5	0.092808
2.699757	0.012199	331011.5	0.093095
2.707625	0.012221	331621.1	0.093366
2.716017	0.012259	332655.7	0.093656
2.724763	0.01227	332956.4	0.093957
2.732854	0.012328	334518.9	0.094236
2.740906	0.012346	335001.4	0.094514
2.74956	0.0124	336484.8	0.094812
2.757913	0.012396	336354	0.0951
2.766266	0.012454	337947.9	0.095388

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.77479	0.012468	338305.1	0.095682
2.783051	0.012525	339857.1	0.095967
2.790958	0.012536	340171.8	0.09624
2.799429	0.012564	340925.5	0.096532
2.808149	0.012613	342242.5	0.096833
2.816319	0.012635	342844.8	0.097114
2.824462	0.012675	343929.6	0.097395
2.83288	0.012687	344268.8	0.097686
2.841207	0.012735	345559.1	0.097973
2.849534	0.012754	346074.7	0.09826
2.858031	0.012774	346618	0.098553
2.866214	0.012814	347703.2	0.098835
2.874291	0.012856	348838.2	0.099113
2.882356	0.012892	349823.3	0.099392
2.891259	0.012907	350233.8	0.099699
2.899534	0.012943	351210.7	0.099984
2.907506	0.012977	352119.3	0.100259
2.915925	0.01301	353030.2	0.100549
2.924304	0.013022	353354	0.100838
2.932657	0.013073	354732.8	0.101126
2.941325	0.013114	355851.5	0.101425
2.949534	0.013133	356370.9	0.101708
2.957467	0.013163	357186	0.101982
2.965807	0.013192	357976.3	0.102269
2.974527	0.013215	358585.1	0.10257

Appendix B

Table 6 (continued): Uniaxial tensile test characterization data for cylindrical rubber block with diameter of 6.85 mm

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.98271	0.013268	360032.1	0.102852
2.990813	0.01327	360079.2	0.103131
2.999403	0.013301	360920.4	0.103428
3.007637	0.013352	362305	0.103712
3.015951	0.013378	363005.3	0.103998
3.02458	0.013392	363380.2	0.104296
3.032906	0.013433	364515.2	0.104583
3.040787	0.013452	365009.3	0.104855
3.049337	0.013477	365684.2	0.10515
3.058018	0.013507	366518.4	0.105449

Appendix B

B.2.2 Uniaxial test data for rubber sheet

The uniaxial test data for rubber dog bone specimen with gauge length of 18.5 mm, width of 5mm and thickness of 1 mm is provided in Table 7.

Table 7 Uniaxial tensile test characterization data for rubber sheet specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0	0	0	0
0.006989	0.000136	27292.97	0.000378
0.01546	0.000189	37810.8	0.000836
0.024062	0.000213	42628.05	0.001301
0.032009	0.000262	52430.63	0.00173
0.040441	0.000284	56722.16	0.002186
0.048807	0.000313	62580.11	0.002638
0.056661	0.000338	67530.87	0.003063
0.065395	0.000374	74806.21	0.003535
0.073984	0.000411	82277.06	0.003999
0.082061	0.000443	88534.36	0.004436
0.090427	0.000449	89831.35	0.004888
0.098898	0.000464	92852.12	0.005346
0.107422	0.000504	100710.4	0.005807
0.115801	0.000503	100501.8	0.00626
0.124167	0.000555	110993.4	0.006712
0.132101	0.000569	113711.4	0.007141
0.140309	0.000564	112860.2	0.007584
0.149567	0.000602	120360.9	0.008085
0.157841	0.00062	124098.1	0.008532

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.16588	0.000636	127129.6	0.008966
0.174298	0.000664	132850.4	0.009422
0.182664	0.000696	139188.8	0.009874
0.190978	0.000712	142464.6	0.010323
0.199738	0.000716	143243.1	0.010797
0.208117	0.000762	152462.7	0.01125
0.216103	0.000772	154451.1	0.011681
0.22422	0.0008	160020.6	0.01212
0.233202	0.000803	160664.3	0.012606
0.241594	0.000815	162985.3	0.013059
0.249515	0.000858	171575.5	0.013487
0.258064	0.000868	173509.1	0.013949
0.26647	0.000883	176632.4	0.014404
0.27477	0.000891	178110.6	0.014852
0.283176	0.000937	187311.2	0.015307
0.291555	0.000954	190796.9	0.01576
0.299501	0.000969	193750.9	0.016189
0.307828	0.000987	197340.3	0.016639
0.316693	0.001001	200274	0.017119
0.324954	0.001009	201801.1	0.017565
0.333071	0.001053	210582	0.018004
0.341424	0.001041	208225.3	0.018455
0.349751	0.001081	216149.1	0.018905
0.358248	0.001082	216405.4	0.019365
0.366601	0.001081	216189.6	0.019816

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.375098	0.001122	224436.5	0.020276
0.382992	0.001117	223313.6	0.020702
0.391358	0.001154	230712.9	0.021154
0.400144	0.001156	231176.6	0.021629
0.408326	0.001163	232555.9	0.022072
0.416352	0.001189	237758.2	0.022505
0.424783	0.001217	243392	0.022961
0.433162	0.00124	248053.1	0.023414
0.441424	0.001239	247849.2	0.023861
0.450065	0.001258	251543.5	0.024328
0.458431	0.001249	249840	0.02478
0.466286	0.001289	257731.7	0.025205
0.474836	0.001289	257852.1	0.025667
0.483634	0.001315	262960.2	0.026142
0.49183	0.001329	265789	0.026585
0.499934	0.001356	271122.5	0.027023
0.508248	0.001352	270372.6	0.027473
0.516601	0.00139	278073.5	0.027924
0.524862	0.001382	276447.5	0.028371
0.533582	0.001408	281626	0.028842
0.541883	0.001404	280770.1	0.029291
0.549763	0.001437	287438.6	0.029717
0.558274	0.001452	290418.9	0.030177
0.56702	0.001466	293297.8	0.03065
0.575229	0.001489	297768.1	0.031093

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.583346	0.00148	296036	0.031532
0.59166	0.00152	304063.6	0.031982
0.599934	0.001539	307755.5	0.032429
0.608339	0.001529	305879.1	0.032883
0.616771	0.001541	308150.1	0.033339
0.625032	0.001549	309827.3	0.033786
0.633044	0.001588	317605.7	0.034219
0.64141	0.001602	320495.4	0.034671
0.650288	0.001617	323337.3	0.035151
0.658484	0.001598	319699	0.035594
0.666496	0.001622	324387.6	0.036027
0.674914	0.001648	329595.8	0.036482
0.683254	0.001642	328362	0.036933
0.691555	0.001663	332586.8	0.037381
0.700104	0.001694	338766.6	0.037843
0.708287	0.001685	336952.2	0.038286
0.716351	0.001713	342545.5	0.038722
0.724586	0.001744	348795.7	0.039167
0.733542	0.001726	345288.5	0.039651
0.741856	0.001737	347406.9	0.0401
0.74996	0.001781	356229.5	0.040538
0.75847	0.001763	352629.4	0.040998
0.766823	0.001772	354397.3	0.04145
0.77515	0.001784	356754.1	0.0419
0.783621	0.001798	359685.4	0.042358

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.791882	0.001817	363445.3	0.042804
0.799907	0.001841	368138.6	0.043238
0.8083	0.001855	370978.1	0.043692
0.816902	0.001861	372221.5	0.044157
0.825189	0.001879	375795.4	0.044605
0.833398	0.001914	382877.6	0.045049
0.841817	0.001937	387460	0.045504
0.850196	0.001916	383278.1	0.045957
0.858483	0.001923	384645.5	0.046405
0.866954	0.001925	385067.5	0.046862
0.875307	0.00194	388063.2	0.047314
0.883254	0.001952	390312.7	0.047743
0.89162	0.001966	393204.7	0.048196
0.900235	0.001994	398880.2	0.048661
0.908365	0.001998	399545.4	0.049101
0.916443	0.002017	403319.6	0.049537
0.924573	0.002025	405008.8	0.049977
0.932978	0.002041	408231	0.050431
0.941292	0.002053	410608.1	0.050881
0.949802	0.002076	415252.4	0.051341
0.958077	0.002076	415197.6	0.051788
0.966089	0.002067	413456	0.052221
0.97452	0.002097	419490.3	0.052677
0.983227	0.002097	419457	0.053147
0.991462	0.002122	424377.9	0.053593

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
0.999606	0.002129	425809.6	0.054033
1.007932	0.002164	432735.7	0.054483
1.016194	0.002145	428959.1	0.054929
1.024573	0.002138	427618	0.055382
1.033149	0.002168	433608.3	0.055846
1.041384	0.002208	441581	0.056291
1.04937	0.002209	441818.2	0.056723
1.057854	0.002213	442596.7	0.057181
1.066666	0.002218	443512.2	0.057658
1.07494	0.002254	450866.2	0.058105
1.083175	0.002243	448622.7	0.05855
1.091646	0.002252	450426.3	0.059008
1.099973	0.002246	449103.1	0.059458
1.108247	0.002277	455311.5	0.059905
1.116915	0.002295	459092.9	0.060374
1.125189	0.002302	460484	0.060821
1.13307	0.002311	462150.6	0.061247
1.141502	0.002318	463500	0.061703
1.15034	0.002351	470204.4	0.062181
1.158588	0.002353	470513.1	0.062626
1.166495	0.00237	473980.9	0.063054
1.17494	0.002372	474318.3	0.06351
1.183385	0.002369	473808.1	0.063967
1.191567	0.002388	477597.7	0.064409
1.200235	0.00241	482040.6	0.064878

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.208509	0.002403	480638.7	0.065325
1.216495	0.002426	485106.7	0.065756
1.224796	0.002452	490372.2	0.066205
1.233621	0.002429	485798.1	0.066682
1.241908	0.002452	490357.9	0.06713
1.249855	0.002455	491030.2	0.06756
1.258273	0.002466	493168.8	0.068015
1.266692	0.00247	494089.1	0.06847
1.274927	0.002483	496504.3	0.068915
1.283463	0.002495	498954.1	0.069376
1.291724	0.002523	504542.6	0.069823
1.299553	0.00252	503975.2	0.070246
1.30784	0.002515	502994.1	0.070694
1.316862	0.002565	513057.7	0.071182
1.325032	0.002548	509631.6	0.071623
1.333162	0.002559	511705.9	0.072063
1.341633	0.00256	512092.1	0.072521
1.350143	0.002597	519461.6	0.072981
1.358299	0.002592	518350.6	0.073422
1.36681	0.002598	519698.9	0.073882
1.375202	0.002617	523387.2	0.074335
1.383188	0.002616	523109.4	0.074767
1.391318	0.002661	532162.2	0.075206
1.40013	0.002639	527781.2	0.075683
1.408325	0.002638	527596.5	0.076126

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.416377	0.002683	536631.3	0.076561
1.424795	0.002681	536210.5	0.077016
1.433188	0.002669	533773.9	0.07747
1.441462	0.002688	537600.5	0.077917
1.450038	0.002695	538924.9	0.07838
1.458443	0.002709	541782.4	0.078835
1.466272	0.002726	545289.5	0.079258
1.474546	0.002745	548963.5	0.079705
1.483555	0.002738	547567.6	0.080192
1.491777	0.00274	547978.9	0.080637
1.499815	0.002771	554283.9	0.081071
1.508168	0.002768	553669.9	0.081523
1.516521	0.00278	555999.3	0.081974
1.524874	0.002777	555465.2	0.082426
1.533253	0.002793	558538.4	0.082879
1.541685	0.002816	563259.1	0.083334
1.549579	0.002827	565357.2	0.083761
1.557932	0.002835	567017.8	0.084213
1.566993	0.002864	572868.6	0.084702
1.57511	0.002836	567191.8	0.085141
1.583253	0.002856	571111.4	0.085581
1.591593	0.002859	571827.9	0.086032
1.600077	0.002857	571308.1	0.086491
1.608404	0.002895	578903	0.086941
1.616783	0.002885	577092.2	0.087394

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.625267	0.002912	582480.4	0.087852
1.633279	0.002923	584504.6	0.088285
1.641278	0.002937	587395.4	0.088718
1.6503	0.002933	586582.4	0.089205
1.658469	0.002957	591447.4	0.089647
1.666482	0.002947	589306.4	0.09008
1.674756	0.002977	595490.9	0.090527
1.683319	0.002978	595612.5	0.09099
1.691632	0.00298	596059.6	0.09144
1.699985	0.002984	596877.3	0.091891
1.708456	0.002973	594644.5	0.092349
1.716416	0.002998	599540.5	0.092779
1.724585	0.00302	603926.2	0.093221
1.733476	0.003012	602368.1	0.093701
1.741895	0.00303	605974.2	0.094156
1.749946	0.00305	609901	0.094592
1.75822	0.003065	612930.1	0.095039
1.766744	0.003061	612274.4	0.0955
1.775149	0.003087	617458.8	0.095954
1.783279	0.003069	613814.6	0.096393
1.79179	0.003086	617194.2	0.096853
1.799657	0.003088	617601.9	0.097279
1.807971	0.003125	624982.1	0.097728
1.816927	0.003114	622743.4	0.098212
1.825241	0.003139	627740.6	0.098662

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Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
1.833253	0.003152	630432.4	0.099095
1.841698	0.003153	630687.5	0.099551
1.850116	0.003161	632294.4	0.100006
1.858286	0.003162	632321.8	0.100448
1.86673	0.003189	637886.5	0.100904
1.87511	0.003184	636862.5	0.101357
1.883187	0.003175	634959.9	0.101794
1.891422	0.003197	639436.2	0.102239
1.900142	0.003224	644848.3	0.10271
1.908391	0.003209	641831.2	0.103156
1.916376	0.003222	644426.3	0.103588
1.924611	0.003219	643739.7	0.104033
1.933004	0.003251	650155.5	0.104487
1.94137	0.003244	648897.9	0.104939
1.94992	0.003264	652774.6	0.105401
1.958391	0.003267	653328.9	0.105859
1.966298	0.003279	655763.1	0.106286
1.974402	0.003303	660615	0.106724
1.983489	0.003288	657634.7	0.107216
1.991671	0.003305	661000	0.107658
1.99967	0.003312	662344.7	0.10809
2.008036	0.003319	663708.4	0.108543
2.016494	0.003321	664147.1	0.109
2.024887	0.003339	667845	0.109453
2.033266	0.003369	673826.9	0.109906

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.041698	0.003366	673131.9	0.110362
2.049631	0.003371	674110.7	0.110791
2.057866	0.00336	672053.1	0.111236
2.066901	0.003396	679148.4	0.111724
2.074992	0.003394	678715.7	0.112162
2.083161	0.003393	678517.8	0.112603
2.091737	0.003412	682383.8	0.113067
2.100103	0.003423	684680.9	0.113519
2.108469	0.003423	684647.6	0.113971
2.116809	0.003435	686917.3	0.114422
2.125227	0.003438	687544.3	0.114877
2.133108	0.003438	687540.8	0.115303
2.141265	0.003446	689296.7	0.115744
2.150418	0.003464	692794.3	0.116239
2.158613	0.003466	693236.6	0.116682
2.166691	0.003473	694547.9	0.117118
2.175149	0.003489	697718.9	0.117576
2.183462	0.0035	699962.4	0.118025
2.191711	0.00353	705969.3	0.118471
2.200273	0.003512	702315.6	0.118934
2.2086	0.003512	702446.7	0.119384
2.216442	0.003516	703182.2	0.119808
2.224677	0.003537	707316.4	0.120253
2.233725	0.003558	711691.4	0.120742
2.242012	0.003555	711058.4	0.12119

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.249985	0.003578	715665.8	0.121621
2.258548	0.003585	717066.5	0.122084
2.266953	0.00359	718025	0.122538
2.275136	0.003598	719697.5	0.12298
2.283633	0.003605	721094.6	0.12344
2.291842	0.003593	718691.3	0.123883
2.299762	0.003613	722568	0.124311
2.308115	0.003631	726236.1	0.124763
2.316979	0.00366	732003.5	0.125242
2.32524	0.003647	729466.7	0.125689
2.333239	0.003666	733192	0.126121
2.341815	0.003674	734849	0.126585
2.349985	0.00365	729903	0.127026
2.358233	0.003661	732284.8	0.127472
2.366861	0.003678	735588.1	0.127938
2.375057	0.003692	738322.7	0.128381
2.382885	0.003714	742765.7	0.128805
2.391238	0.003707	741428.1	0.129256
2.400194	0.003725	744944.8	0.12974
2.408403	0.003711	742288.8	0.130184
2.416389	0.003736	747163.3	0.130616
2.424794	0.003733	746523.1	0.13107
2.4332	0.003742	748404.3	0.131524
2.441474	0.003734	746897.5	0.131972
2.450181	0.003768	753645.9	0.132442

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.458403	0.003782	756474.7	0.132887
2.466323	0.003782	756304.3	0.133315
2.474716	0.003766	753148.8	0.133768
2.48358	0.003807	761491.1	0.134248
2.491802	0.00383	765968.6	0.134692
2.499853	0.003825	765047.1	0.135127
2.508233	0.003804	760706.7	0.13558
2.516704	0.003831	766178.4	0.136038
2.524939	0.00384	767948.6	0.136483
2.533528	0.003836	767161.8	0.136947
2.541776	0.00385	769964.5	0.137393
2.549591	0.003852	770312.5	0.137816
2.558246	0.003873	774606.5	0.138284
2.566979	0.003869	773806.6	0.138756
2.575148	0.003861	772258	0.139197
2.583318	0.003884	776720	0.139639
2.59171	0.003889	777825.1	0.140092
2.600063	0.003907	781317.9	0.140544
2.608298	0.003915	782922.5	0.140989
2.616795	0.003921	784174.2	0.141448
2.625161	0.003929	785834.8	0.141901
2.633095	0.003937	787402.4	0.142329
2.641474	0.003957	791326.8	0.142782
2.650299	0.003966	793107.7	0.143259
2.658455	0.00397	794084.1	0.1437

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.666546	0.003973	794583.6	0.144138
2.674978	0.003966	793170.9	0.144593
2.683252	0.003985	797051.2	0.145041
2.69171	0.003999	799803.7	0.145498
2.700168	0.00401	802059.2	0.145955
2.708364	0.00401	801953.1	0.146398
2.716323	0.004019	803735.3	0.146828
2.724689	0.004011	802153.3	0.14728
2.733659	0.004024	804858.2	0.147765
2.741972	0.004036	807256.7	0.148215
2.750089	0.004067	813474.7	0.148653
2.758482	0.004048	809584.9	0.149107
2.76673	0.004055	810973.6	0.149553
2.775109	0.004068	813674.9	0.150006
2.783501	0.004072	814462.9	0.15046
2.791684	0.004078	815538.2	0.150902
2.79963	0.004086	817121.3	0.151331
2.808141	0.004105	821015.8	0.151791
2.816926	0.0041	820052.6	0.152266
2.825135	0.004105	821048	0.15271
2.833422	0.004124	824760.2	0.153158
2.841854	0.004118	823699.2	0.153614
2.850115	0.004148	829550	0.15406
2.858495	0.00415	829967.3	0.154513
2.866821	0.004148	829578.6	0.154963

Appendix B

Table 7 (continued): Uniaxial tensile test characterization data for rubber test specimen

Extension (mm)	Load (KN)	Stress (Pa)	Strain
2.875122	0.004163	832592.2	0.155412
2.883095	0.00418	836039.8	0.155843
2.891277	0.00417	833942.9	0.156285
2.900325	0.004195	839012.9	0.156774
2.908416	0.004213	842545	0.157212
2.916507	0.004198	839541	0.157649
2.92486	0.004193	838524.1	0.158101
2.933344	0.004204	840749.7	0.158559
2.941697	0.004238	847562.6	0.159011
2.95022	0.004252	850454.6	0.159471
2.958324	0.004254	850898	0.159909
2.966152	0.004251	850201.8	0.160333

Appendix B

B.3 Data obtained from testing the sensor off the rubber sheet (without host structure)

Data obtained from testing the sensor bonded to Aluminum plate from one side and displaced freely (without bonding to host structure) are provided here.

B.3.1 Experimental results for testing sensor composed of blocks with diameter of 6.85 mm moving outwards

Data from testing sensor in which the free ends of its cylindrical rubber blocks are displaced outwards (tensile test) are provided in Table 8; gauge factor of the strain gauge soldered onto the Aluminum plate is 2.14; the diameter and height of the rubber blocks are 6.85 mm and 18.5 mm, respectively; the center distance of the rubber blocks is 28.85 mm; the target point is strain of 0.148; the width and thickness of the Aluminum sheet are 11 mm and 1 mm, respectively. The strain of the moving end of cylindrical rubber block amounts to division of extension by initial center distance of the blocks. Strain of the Aluminum plate obtained from strain-meter, Abaqus modeling and Eq. 3.64 are -110 (e-6), -120.16 (e-6) and -125 (e-6), respectively, corresponding to strain of 0.148 of displaced end of the cylindrical rubber block.

Table 8 Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
0.04	0.001385522	-0.5	-1.04036166
0.0493	0.00170738	-1.5	-1.282245745

Appendix B

Table 8 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
0.0584	0.002022217	-2	-1.518928023
0.0676	0.002340413	-3	-1.758211205
0.0955	0.003304759	-3.5	-2.483863462
0.1232	0.004261272	-4.5	-3.204313912
0.1588	0.005489239	-5	-4.130235789
0.1771	0.006119883	-5.5	-4.606201248
0.1949	0.006732916	-6.5	-5.069162186
0.2398	0.008277604	-7	-6.236968149
0.2585	0.008920235	-7.5	-6.723337225
0.2856	0.009850801	-8.5	-7.428182249
0.3133	0.010801077	-9	-8.148632699
0.3411	0.011753875	-10	-8.871684052
0.3593	0.012377159	-10.5	-9.345048607
0.387	0.013325037	-11.5	-10.06549906
0.4333	0.014907395	-12	-11.26971768
0.448	0.015409261	-12.5	-11.65205059
0.4593	0.015794879	-13.5	-11.94595276
0.4687	0.016115545	-14	-12.19043775
0.4779	0.016429288	-14.5	-12.42972093
0.5235	0.017982914	-15.5	-13.61573322
0.5326	0.018292669	-16	-13.8524155
0.5694	0.019544328	-17	-14.80954822
0.597	0.020482044	-17.5	-15.52739777
0.6146	0.02107955	-18	-15.9851569

Appendix B

Table 8 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
0.66	0.022619196	-19	-17.16596738
0.6785	0.023245906	-19.5	-17.64713465
0.6963	0.023848532	-18.11009559	-22.83717463
0.7046	0.024129407	-18.32597063	-23.10939716
0.7516	0.025718421	-19.54839558	-24.65089825
0.7788	0.026636868	-20.25584151	-25.54300101
0.7981	0.027288049	-20.75781601	-26.17600039
0.834	0.028498187	-21.6915406	-27.35344484
0.8608	0.029400623	-22.38858291	-28.23242844
0.888	0.030315696	-23.09602884	-29.1245312
0.9234	0.031505384	-24.01674891	-30.2855767
0.9329	0.03182441	-24.26383481	-30.5971567
0.9696	0.033055902	-25.21836663	-31.80083947
0.9874	0.033652647	-25.68132757	-32.38464201
1.0425	0.03549762	-27.11442575	-34.19180605
1.0606	0.03610294	-27.5851894	-34.78544796
1.0786	0.036704552	-28.05335215	-35.37581008
1.1246	0.038240363	-29.24976806	-36.88451327
1.1422	0.038827355	-29.70752719	-37.46175623
1.1321	0.038490544	-29.44483587	-37.13049748
1.2062	0.04095897	-31.37210584	-39.56082154
1.2157	0.041274994	-31.61919174	-39.87240155
1.2588	0.042707495	-32.74018143	-41.28599084
1.2859	0.043607159	-33.44502645	-42.17481381

Appendix B

Table 8 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
1.3141	0.044542483	-34.17848142	-43.09971446
1.3414	0.045447123	-34.88852825	-43.99509701
1.3685	0.046344327	-37	-35.59337328
1.3965	0.047270482	-38	-36.32162644
1.4146	0.04786872	-38.5	-36.79239009
1.4688	0.049657989	-39	-38.20208014
1.4862	0.050231726	-40	-38.65463746
1.5056	0.050871022	-40.5	-39.15921287
1.5423	0.052079294	-41.5	-40.11374469
1.5606	0.052681239	-42	-40.58971015
1.6075	0.054222276	-42.5	-41.80953419
1.6255	0.054813089	-43.5	-42.27769694
1.6438	0.055413391	-44	-42.7536624
1.6711	0.056308255	-45	-43.46370923
1.6895	0.056910935	-45.5	-43.9422756
1.7349	0.05839643	-46	-45.12308608
1.7706	0.059562992	-47	-46.05160886
1.789	0.060163714	-47.5	-46.53017523
1.8168	0.061070643	-48.5	-47.25322658
1.8719	0.062865762	-49	-48.68632476
1.9355	0.064933807	-50	-50.3404998
1.944	0.065209873	-50.5	-50.56157666
1.9631	0.065829931	-51	-51.05834935
1.991	0.066734981	-52	-51.78400161

Appendix B

Table 8 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
2.0364	0.068205965	-52.5	-52.96481209
2.047	0.068549099	-53	-53.24050793
2.102	0.070327624	-54	-54.67100521
2.111	0.070618355	-54.5	-54.90508658
2.1384	0.071502948	-55.5	-55.61773432
2.1473	0.071790111	-56	-55.84921479
2.2132	0.073913846	-56.5	-57.56321063
2.2222	0.074203536	-57.5	-57.797292
2.2483	0.075043162	-58	-58.47612798
2.2763	0.075943128	-59	-59.20438114
2.3038	0.076826235	-59.5	-59.91962978
2.3761	0.079144291	-60	-61.80008348
2.3853	0.079438873	-61	-62.03936667
2.3942	0.079723766	-61.5	-62.27084713
2.4403	0.081198152	-62.5	-63.46986395
2.4584	0.081776439	-63	-63.9406276
2.5225	0.08382172	-64	-65.60780716
2.5313	0.084102181	-64.5	-65.83668672
2.5592	0.084990851	-65	-66.56233898
2.5951	0.086133175	-66	-67.49606357
2.6319	0.087302785	-66.5	-68.4531963
2.6499	0.087874378	-67	-68.92135904
2.7038	0.089584033	-68	-70.32324638
2.7134	0.089888229	-68.5	-70.57293318

Appendix B

Table 8 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
2.7407	0.090752781	-69.5	-71.28298001
2.7855	0.092169915	-70	-72.44818507
2.8589	0.094487406	-70.5	-74.35724872
2.8678	0.094768045	-71.5	-74.58872918
2.8942	0.095600039	-72	-75.27536788
2.9413	0.097082674	-73	-76.50039373
2.9598	0.097664425	-73.5	-76.981561
2.9788	0.098261547	-74	-77.47573279
3.0317	0.099922185	-75	-78.85161108
3.0506	0.100514825	-75.5	-79.34318197
3.1239	0.102809952	-76.5	-81.24964471
3.1334	0.103107025	-77	-81.4967306
3.2054	0.105355663	-77.5	-83.36938159
3.2147	0.105645744	-78.5	-83.61126568
3.2327	0.106206951	-79	-84.07942842
3.2783	0.107627269	-80	-85.26544072
3.2975	0.108224694	-80.5	-85.76481431
3.3064	0.108501505	-81	-85.99629478
3.3155	0.108784457	-82	-86.23297706
3.3895	0.111082416	-82.5	-88.15764613
3.4169	0.111931944	-83.5	-88.87029387
3.4269	0.112241811	-84	-89.13038428
3.4451	0.112805523	-85	-89.60374884
3.5095	0.114797648	-85.5	-91.27873111

Appendix B

Table 8 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
3.5279	0.115366098	-86	-91.75729747
3.5645	0.116495861	-87	-92.70922839
3.5831	0.117069513	-87.5	-93.19299656
3.619	0.118175795	-88	-94.12672115
3.6544	0.119265472	-89	-95.04744122
3.6728	0.119831389	-89.5	-95.52600758
3.7105	0.120989904	-90.5	-96.50654845
3.7469	0.122107199	-91	-97.45327756
3.8014	0.123777741	-91.5	-98.87077032
3.8105	0.124056404	-92.5	-99.1074526
3.8379	0.124894986	-93	-99.82010033
3.8824	0.12625542	-94	-100.9775027
3.9105	0.127113528	-94.5	-101.7083567
3.947	0.128227055	-95	-102.6576868
3.9836	0.129342388	-96	-103.6096177
4.0021	0.129905677	-96.5	-104.0907849
4.0283	0.130702873	-97.5	-104.7722218
4.0645	0.131803297	-98	-105.7137491
4.1287	0.133751906	-98.5	-107.3835296
4.1373	0.134012646	-99.5	-107.6072074
4.1742	0.135130633	-100	-108.566941
4.2125	0.136289717	-100.5	-109.5630873
4.2383	0.137069753	-101.5	-110.2341205
4.256	0.137604542	-102	-110.6944806

Appendix B

Table 8 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing outward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
4.3019	0.138990037	-103	-111.8882956
4.329	0.139807153	-103.5	-112.5931406
4.3562	0.140626613	-104.5	-113.3005865
4.418	0.142485981	-105	-114.9079453
4.428	0.142786525	-105.5	-115.1680357
4.4376	0.143074962	-106.5	-115.4177225
4.4827	0.144428904	-107	-116.5907303
4.5203	0.14555629	-108	-117.5686702
4.5376	0.146074581	-108.5	-118.0186267
4.5656	0.146912864	-109	-118.7468798
4.6203	0.148548485	-110	-120.1695744

B.3.2 Experimental results for testing sensor composed of blocks with diameter of 6.85 mm moving inwards

Data from testing sensor in which the free ends of the cylindrical rubber blocks move inwards (compression test) are provided in Table 9; gauge factor of the strain gauge soldered onto the Aluminum plate is 2.14; the diameter and height of the rubber blocks are 6.85 mm and 18.5 mm, respectively; the center distance of the rubber blocks is 28.85 mm; the target point is strain of -0.151; the width and thickness of the Aluminum plate are 11 mm and 1 mm, respectively. Strain of the Aluminum plate obtained from strain-meter, Abaqus modeling and Eq. 3.64 are 99.5 (e-6), 112 (e-6) and 105.2 (e-6), respectively, corresponding to strain of -0.151 of displaced end of the cylindrical rubber block.

Appendix B

Table 9: Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing inward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
-0.1205	-0.00419	3.5	3.134089
-0.1383	-0.00481	4	3.59705
-0.157	-0.00546	4.5	4.08342
-0.1852	-0.00644	5.5	4.816874
-0.2043	-0.00711	6	5.313647
-0.2219	-0.00772	6.6	5.771406
-0.2314	-0.00805	7.5	6.018492
-0.2674	-0.00931	8	6.954818
-0.2863	-0.00997	8.5	7.446389
-0.3134	-0.01092	9.5	8.151234
-0.3405	-0.01187	10	8.856079
-0.3677	-0.01283	10.5	9.563525
-0.3769	-0.01315	11.5	9.802808
-0.4229	-0.01477	12	10.99922
-0.442	-0.01544	12.5	11.496
-0.4875	-0.01704	13.5	12.67941
-0.4968	-0.01737	14	12.92129
-0.5241	-0.01833	14.5	13.63134
-0.5504	-0.01926	15.5	14.31538
-0.578	-0.02024	16	15.03323
-0.6045	-0.02118	16.6	15.72247
-0.6321	-0.02215	17.5	16.44032
-0.6507	-0.02281	18	16.92408
-0.6867	-0.02409	18.5	17.86041
-0.7231	-0.02538	19.5	18.80714

Appendix B

Table 9 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing inward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
-0.7415	-0.02604	20.5	19.2857
-0.7873	-0.02767	21.5	20.47692
-0.8145	-0.02864	22	21.18436
-0.8319	-0.02926	22.5	21.63692
-0.8684	-0.03056	23.5	22.58625
-0.9034	-0.03181	24	23.49657
-0.913	-0.03216	24.5	23.74625
-0.9397	-0.03311	25.5	24.4407
-0.9674	-0.03411	26	25.16115
-0.994	-0.03506	26.5	25.85299
-1.0034	-0.0354	27.5	26.09747
-1.0393	-0.03669	28	27.0312
-1.0759	-0.03801	28.5	27.98313
-1.1029	-0.03898	29.5	28.68537
-1.1209	-0.03963	30	29.15353
-1.1575	-0.04095	30.5	30.10547
-1.1757	-0.04161	31.5	30.57883
-1.2209	-0.04324	32	31.75444
-1.239	-0.0439	32.5	32.2252
-1.2754	-0.04521	33.5	33.17193
-1.2849	-0.04556	34	33.41902
-1.3208	-0.04686	34.5	34.35274
-1.3482	-0.04786	35.5	35.06539
-1.3655	-0.04849	36	35.51535

Appendix B

Table 9 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing inward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
-1.3926	-0.04947	36.5	36.22019
-1.4385	-0.05115	37.5	37.41401
-1.4497	-0.05156	38	37.70531
-1.4847	-0.05283	38.5	38.61562
-1.5201	-0.05413	39.5	39.53634
-1.5563	-0.05545	40	40.47787
-1.5753	-0.05615	40.5	40.97204
-1.5937	-0.05683	41.5	41.45061
-1.6392	-0.0585	42	42.63402
-1.6575	-0.05917	42.5	43.10999
-1.676	-0.05985	43.5	43.59115
-1.6946	-0.06053	44	44.07492
-1.7307	-0.06186	44.5	45.01385
-1.7763	-0.06355	45.5	46.19986
-1.7942	-0.06421	46	46.66542
-1.8217	-0.06523	46.5	47.38067
-1.8308	-0.06556	47.5	47.61735
-1.8675	-0.06692	48	48.57188
-1.894	-0.0679	48.5	49.26112
-1.9032	-0.06825	49.5	49.50041
-1.9493	-0.06996	50	50.69942
-1.9769	-0.07098	50.5	51.41727
-2.0029	-0.07195	51.5	52.09351
-2.0397	-0.07332	52	53.05064

Appendix B

Table 9 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing inward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
-2.058	-0.07401	52.5	53.52661
-2.112	-0.07602	53.5	54.9311
-2.1302	-0.07671	54	55.40446
-2.149	-0.07741	54.5	55.89343
-2.1852	-0.07877	55.5	56.83496
-2.2134	-0.07982	56	57.56841
-2.2496	-0.08118	56.5	58.50994
-2.2842	-0.08249	57.5	59.40985
-2.2931	-0.08282	58	59.64133
-2.3207	-0.08386	58.5	60.35918
-2.3742	-0.08588	59.5	61.75067
-2.3929	-0.08659	60	62.23704
-2.402	-0.08693	60.5	62.47372
-2.4381	-0.0883	61.5	63.41264
-2.4834	-0.09001	62	64.59085
-2.5015	-0.0907	62.5	65.06162
-2.5286	-0.09173	63.5	65.76646
-2.5651	-0.09312	64	66.71579
-2.6016	-0.0945	64.5	67.66512
-2.6198	-0.0952	65.5	68.13849
-2.6462	-0.09621	66	68.82513
-2.6557	-0.09657	66.5	69.07221
-2.6742	-0.09727	67.5	69.55338
-2.6928	-0.09799	68	70.03715

Appendix B

Table 9 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing inward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
-2.7295	-0.09939	68.5	70.99168
-2.7467	-0.10005	69.5	71.43903
-2.793	-0.10182	70	72.64325
-2.8286	-0.10319	70.5	73.56917
-2.8471	-0.1039	71.5	74.05034
-2.8925	-0.10565	72	75.23115
-2.9284	-0.10703	72.5	76.16488
-2.9376	-0.10739	73.5	76.40416
-2.9467	-0.10774	74	76.64084
-2.965	-0.10845	74.5	77.11681
-3.0014	-0.10985	75.5	78.06354
-3.0382	-0.11128	76	79.02067
-3.0657	-0.11234	76.5	79.73592
-3.0937	-0.11343	77.5	80.46417
-3.1114	-0.11412	78	80.92453
-3.1655	-0.11622	78.5	82.33162
-3.193	-0.11729	79.5	83.04687
-3.2112	-0.118	80	83.52023
-3.2297	-0.11872	80.5	84.0014
-3.2835	-0.12083	81.5	85.40069
-3.3112	-0.12191	82	86.12114
-3.3383	-0.12297	82.5	86.82598
-3.3754	-0.12443	83.5	87.79092
-3.3934	-0.12513	84	88.25908

Appendix B

Table 9 (continued): Data obtained from testing sensor composed of cylindrical rubber blocks with diameter of 6.85 mm experiencing inward displacement

Extension (mm)	Strain of displaced end of the cylindrical rubber block	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
-3.4299	-0.12657	84.5	89.20841
-3.4562	-0.1276	85.5	89.89245
-3.498	-0.12925	86	90.97963
-3.5109	-0.12976	86.5	91.31514
-3.5379	-0.13083	87.5	92.01739
-3.5653	-0.13191	88	92.73004
-3.602	-0.13336	88.5	93.68457
-3.6289	-0.13443	89.5	94.38421
-3.6588	-0.13562	90	95.16188
-3.683	-0.13658	90.5	95.7913
-3.72	-0.13805	91.5	96.75363
-3.7566	-0.13951	92	97.70557
-3.7918	-0.14091	92.5	98.62108
-3.8189	-0.14199	93.5	99.32593
-3.8375	-0.14273	94	99.8097
-3.8739	-0.14419	94.5	100.7564
-3.9187	-0.14599	95.5	101.9216
-3.9372	-0.14673	96	102.4028
-3.9736	-0.14819	96.5	103.3495
-3.9828	-0.14856	97.5	103.5888
-4.01	-0.14965	98	104.2963
-4.0289	-0.15042	98.5	104.7878
-4.046	-0.15111	99.5	105.2326

Appendix B

B.4 Data obtained from testing the sensor bonded to the rubber sheet (on the host structure)

Cylindrical rubber blocks with diameter of 6.85 mm and height of 18.5 mm were bonded from one side to Aluminum plate with width of 11 mm and thickness of 1 mm, and from other side to rubber sheet with width of 10 mm and thickness of 1 mm and length of 62.5 mm in each side of the rubber sheet was glued to two small pieces of metal (as shown in Fig. 45) in order to prevent the rubber sheet from sliding. Initial Center distance (ICD) of the cylindrical rubber blocks were 30.1 mm. Gauge factor of the strain gauge soldered onto the Aluminum plate is 2.14. Target point is rubber sheet strain of 0.141. Extensions of the rubber sheet recorded by microtensile machine together with strain of the Aluminum plate shown by strain-meter are provided in Table 10. Strain of the Aluminum plate obtained from strain-meter, Abaqus modeling and Eq. 3.64 are -107 (e-6) , -127.8 (e-6) and -119.07 (e-6) , respectively, corresponding to strain of 0.141 of displaced end of the cylindrical rubber block.

Table 10 Data obtained from testing the sensor bonded to the rubber sheet

Extension (mm)	True strain of the rubber sheet	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
0	0	0	0
0.0331	0.00063	-1	-0.49358
0.142	0.002701	-2	-2.11748
0.216	0.004106	-3	-3.22096
0.307	0.005831	-4	-4.57794
0.37	0.007023	-5	-5.51738
0.42	0.007968	-6	-6.26298

Appendix B

Table 10 (continued) Data obtained from testing the sensor bonded to the rubber sheet

Extension (mm)	True strain of the rubber sheet	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
0.52	0.009856	-7	-7.75416
0.56	0.01061	-8	-8.35064
0.64	0.012117	-9	-9.54358
0.751	0.014203	-10	-11.1988
0.889	0.016792	-11	-13.2566
0.992	0.018719	-12	-14.7926
0.9972	0.018816	-13	-14.8701
1.058	0.019952	-14	-15.7767
1.141	0.021501	-15	-17.0144
1.195	0.022507	-16	-17.8197
1.2782	0.024055	-17	-19.0603
1.323	0.024888	-18	-19.7284
1.412	0.02654	-19	-21.0555
1.47	0.027615	-20	-21.9204
1.586	0.029762	-21	-23.6502
1.605	0.030113	-22	-23.9335
1.679	0.03148	-23	-25.037
1.761	0.032993	-24	-26.2598
1.835	0.034355	-25	-27.3632
1.89	0.035367	-26	-28.1834
2.001	0.037406	-27	-29.8386
2.045	0.038213	-28	-30.4947
2.091	0.039056	-29	-31.1807
2.174	0.040575	-30	-32.4184
2.284	0.042585	-31	-34.0587

Appendix B

Table 10 (continued) Data obtained from testing the sensor bonded to the rubber sheet

Extension (mm)	True strain of the rubber sheet	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
2.341	0.043625	-32	-34.9086
2.422	0.045101	-33	-36.1165
2.495	0.046429	-34	-37.2051
2.55	0.047429	-35	-38.0252
2.605	0.048427	-36	-38.8454
2.741	0.050892	-37	-40.8734
2.78	0.051598	-38	-41.4549
2.858	0.053008	-39	-42.6181
2.942	0.054524	-40	-43.8707
3.017	0.055876	-41	-44.9891
3.097	0.057316	-42	-46.182
3.17	0.058628	-43	-47.2706
3.269	0.060405	-44	-48.7468
3.306	0.061068	-45	-49.2986
3.398	0.062715	-46	-50.6705
3.47	0.064003	-47	-51.7441
3.551	0.065449	-48	-52.952
3.61	0.066501	-49	-53.8318
3.68	0.067748	-50	-54.8756
3.763	0.069224	-51	-56.1133
3.845	0.07068	-52	-57.3361
3.9	0.071656	-53	-58.1562
3.97	0.072896	-54	-59.2
4.03	0.073958	-55	-60.0948
4.11	0.075372	-56	-61.2877

Appendix B

Table 10 (continued): Data obtained from testing the sensor bonded to the rubber sheet

Extension (mm)	True strain of the rubber sheet	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
4.22	0.077314	-57	-62.928
4.276	0.078301	-58	-63.7631
4.34	0.079427	-59	-64.7174
4.413	0.080711	-60	-65.806
4.468	0.081677	-61	-66.6261
4.578	0.083606	-62	-68.2665
4.623	0.084394	-63	-68.9375
4.726	0.086195	-64	-70.4734
4.772	0.086999	-65	-71.1594
4.845	0.088272	-66	-72.2479
4.881	0.0889	-67	-72.7847
5.029	0.091476	-68	-74.9917
5.093	0.092588	-69	-75.9461
5.149	0.09356	-70	-76.7811
5.267	0.095605	-71	-78.5407
5.422	0.098284	-72	-80.8521
5.468	0.099078	-73	-81.538
5.532	0.100181	-74	-82.4924
5.595	0.101266	-75	-83.4318
5.667	0.102505	-76	-84.5055
5.723	0.103467	-77	-85.3405
5.814	0.105029	-78	-86.6975
5.869	0.105972	-79	-87.5177
5.944	0.107256	-80	-88.636
6.025	0.108641	-81	-89.8439

Appendix B

Table 10 (continued) Data obtained from testing the sensor bonded to the rubber sheet

Extension (mm)	True strain of the rubber sheet	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
6.11	0.110092	-82	-91.1114
6.137	0.110553	-83	-91.514
6.21	0.111797	-84	-92.6026
6.273	0.112869	-85	-93.542
6.349	0.114162	-86	-94.6753
6.404	0.115096	-87	-95.4955
6.513	0.116945	-88	-97.1209
6.567	0.117859	-89	-97.9261
6.632	0.118959	-90	-98.8954
6.715	0.120362	-91	-100.133
6.843	0.122521	-92	-102.042
6.879	0.123127	-93	-102.579
6.981	0.124844	-94	-104.1
7.025	0.125583	-95	-104.756
7.09	0.126675	-96	-105.725
7.16	0.127849	-97	-106.769
7.278	0.129825	-98	-108.528
7.324	0.130594	-99	-109.214
7.389	0.13168	-100	-110.184
7.534	0.134098	-101	-112.346
7.673	0.136411	-102	-114.419
7.729	0.137341	-103	-115.254
7.774	0.138088	-104	-115.925
7.784	0.138254	-105	-116.074
7.919	0.14049	-106	-118.087

Appendix B

Table 10 (continued) Data obtained from testing the sensor bonded to the rubber sheet

Extension (mm)	True strain of the rubber sheet	Strain from strain-meter (e-6)	Strain from Eq. 3.64 (e-6)
7.985	0.141582	-107	-119.071