Analysis and design of networked control systems

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**ANALYSIS AND DESIGN OF NETWORKED
CONTROL SYSTEMS**

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ANALYSIS AND DESIGN OF NETWORKED CONTROL SYSTEMS

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SUMMARY

In Networked Control Systems (NCSs), the communication between different components, is established through a shared digital network. This kind of communication, results in problems such as network induced transmission delays, packet losses, bandwidth limitations, multiple packet transmission, etc., which may affect the performance of the system or even lead to instability. Therefore, in designing controller for such systems these unfavorable effects should be taken into consideration.

In this thesis, the focus is on control over networked systems with time delays or packet dropouts. Our goal is to first model NCSs with delays and packet dropouts, develop tools for stability analysis and control synthesis, and then validate the results through numerical examples and simulations and/or practical systems implementation.

First, NCSs with two separate uncertain time varying delays in sensor and controller channels are investigated. Uncertainties of the networked control system will be characterized by polytopic uncertain parameters using Jordan canonical form. Markov jump linear systems are then applied to model the time delay of the sensor channel. Control synthesis of the system is considered by using a Lyapunov-Krasovskii approach, and a mode-dependent static output feedback controller is designed which changes for each different subinterval of delays. The results are given in the form of

Bilinear Matrix Inequalities (BMIs) which can be solved using methods and algorithms available in the literature. Subsequently, through some condition on output matrix, Linear Matrix Inequalities (LMIs) are derived for static output feedback controller. Further extensions for state feedback and dynamic output feedback controllers design are presented as well.

Next, NCSs with packet dropouts are discussed. A compensator is proposed to be located at the actuator side to generate control commands whenever there's a communication outage in the system. The compensator listens to the signals received from the controller, updates its states and sends the signal to actuator in normal cases. However, in the case of outages, it uses its states and produces the control command. The model of the NCS with controller and compensator is defined using Markov jump linear systems and by employing H_2 control, sufficient conditions for stability of the system are given. Then, the results are verified through a numerical example.

Finally, inverted pendulum is considered as an application for evaluating the static output feedback controller designed with the consideration of possible transmission delays.

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SYMBOLS AND ACRONYMS

SYMBOLS	Description
$\ \cdot\ $	Any norm.
$*$	Symmetric terms of a symmetric matrix.
$\circ^T$	Matrix transposition.
$\circ^*$	Conjugate transpose.
1	Indicator function.
Γ	N -sequence of system matrices.
$\theta(k)$	Markov state at time k .
λ_i	i^{th} eigenvalue.
$E(\cdot)$	Expected value of a random variable.
$\text{diag}\{\cdot\}$	Block diagonal matrix.
I, I_n	Identity matrix, $n \times n$ identity matrix.
$0_{m \times n}$	Matrix with zero elements of size $m \times n$.
$P > 0$ (≥ 0)	Positive definite (Semi-definite) matrix.
Λ	Transition probability matrix.
π_{ij}	Probability of transition from mode i to mode j .
$\mathbf{Q}(k)$	Second moment of $x(k)$.

$Q_i(k)$	Second moment of $x(k)$ at mode i .
$r_\sigma(\bullet)$	Spectral radius.
$\text{tr}(\bullet)$	Trace operator.
N	Number of operation modes.
$\mathbb{N}$	Set of operation modes.
$\mathbb{N}$	Set of nonnegative integers.
$\mathbf{B}(\mathfrak{R}^n, \mathfrak{R}^m), \mathbf{B}(\mathfrak{R}^n)$	Norm bounded linear space of $m \times n, (n \times n)$ real matrices.
$H^{n,m}$	Linear space.
$\mathfrak{R}^n$	n -dimensional real Euclidean space.

ACRONYMS	Description
i.i.d.	Independent and Identically Distributed.
LMI	Linear Matrix Inequality.
MJLS	Markov Jump Linear System.
MSS	Mean Square Stability.
NCS	Networked Control System.
BMI	Bi-linear Matrix Inequality.
WSN	Wireless Sensor Network.

Chapter 1

INTRODUCTION

1.1 Background

During the past decades, control engineering has evolved gradually. Figure 1.1 indicates the process of control systems evolution [1]. The year 1932 is taken as the birth date of modern feedback control, then in early 1950 digital control concepts started being widely discussed [1], and computer controlled systems started to emerge as well. At the beginning stage, the potential of using digital computers as control system components was limited, since the computers at that time were too big, consumed too much power, and were not highly reliable. The early implementation of computers in control systems operated in supervisory mode, either as an operator guide or as a set point control. In 1983 an internal program to evaluate the concept of using a data network in automobile control applications was commenced by Bosch

GmbH, this became the start of Networked Control Systems (NCSs) technology according to [1].

Figure 1.1 Control systems evolution [1].

1.2 Motivations

Computer controlled systems became popular with the development of computer technologies. One group of the computer controlled systems modes, which are well-received recently, are Networked Control Systems (NCSs). Along by the advances in communication, control and computation technologies, networked control systems, and their implementation have drawn considerable interest. Since these modern control systems use a shared digital communication network to establish connections between the controller and each sensor or actuator point, they are positioned at the crossing line of control and communication theories. Control theories study connected dynamical systems under the assumption of ideal channels, while communication

theories study the information transmission over imperfect channels. Networked Control Systems are in fact a combination of these two frameworks.

In contrast with NCSs, traditional architecture of control systems, which has been implemented in industry for decades, consists of point to point connections through wires. A control system with a point to point wiring configuration is shown in Figure 1.2. As the complexity and scale of a system increase, the burden of wiring causes these traditional control systems to no longer meet the new requirements such as quick and easy installation, and low cost.

Figure 1.2 A control system with a point to point wiring configuration [2].

Networked control systems, however, don't have the problems of their traditional architectures. As shown in Figure 1.3, a typical networked control system is established by the use of four basic elements along with the main plant. These four

components are sensors, actuators, controller, and communication network. The use of a multipurpose communication network to connect the distributed elements together, leads the modern networked control systems to offer a number of advantages over their traditional predecessors, including better efficiency, higher reliability and flexibility, simpler installation, and less maintenance costs. In addition, an NCS offers the ability to locate the controller for various processes at convenient locations that are easily accessible.

Figure 1.3 Networked control system [2].

The advantages of using networked control systems have resulted in a great devotion of research area. These systems have also become an enabling technology for many applications, such as mobile sensor networks, automated highway systems, remote medical or health care, military and industrial procedures. For example, in deep sea explorations or nuclear plants a controller can be placed in an easy-to-access office environment whereas sensors and actuators can be placed at the required locations.

This control architecture allows the operator to take corrective measures almost instantaneously [3]. Networked control systems can also be found in modern office buildings, where security and environmental functions are monitored and controlled through special purpose networks [2].

It should be mentioned that besides all the significant benefits of NCSs, there are also some inevitable disadvantages which make their analysis much harder than the traditional control systems. Energy management, delay and packet loss controlling, multiple packets transmissions are some of the challenges that researchers face in control synthesis of NCSs. The most important feature of NCSs, which causes these challenges, is that the control and feedback signals are exchanged among the system's components through a network in the form of information packages. Placing the shared communication network in the control loop makes the analysis and design of NCSs complex. With the existence of network congestion, imperfect channel, and limited bandwidth of shared networks, the packages may be lost or reach the destination with time delay. Energy consumption during the transmission of packages is also another issue in wireless networked control systems which needs to be addressed.

To make the disadvantages of NCSs clear, consider an information transmission process through NCSs which involves sampling a continuous time signal, encoding its samples into digital format, sending the packages through network, and decoding data

at the receiver side. This procedure will take some time which causes the controller or actuator to receive the information with delay. The overall delay can be highly variable due to both network access and transmission delay which can alter with network conditions. Thus, sampling and time delay can easily deteriorate the control performance or even completely destroy the stability of the system.

On the other hand, the bandwidth limitations that depend on the communication protocol, the network equipment, etc, that restrict the accuracy and availability of data, must be taken into consideration as an important factor of control performance degradation. Packet dropouts, resulting from transmission errors in physical network links, buffer overflow, and sometimes long transmission delays, affect the stability of NCSs as well.

As it can be observed, there are many issues that can enormously affect the stability and the control performance of NCSs. These challenges complicate the application of standard results of conventional control theory. The previous methods must be re-evaluated for their applicability to modern NCSs. Therefore, a major trend has been to design and analyze the network-based systems, in order to present methods to deal with the problems arising from the use of network.

Two of the abovementioned issues of the networked control systems which have been considered by many researchers through the past few years, are delay and packet

losses. Regarding these problems, which form the basic motivation of this study, there exist a variety of proposed approaches for modeling, stabilization and control of networked systems in literature.

Considering the available approaches to model and analyze delay and uncertainty in a NCS, the Markov jump linear systems approach is one of the most convenient and effective ones. Discrete time Markovian jump linear systems are a special class of hybrid systems with two components: the modes and the states. The modes take their value within a finite set of integers where each mode denotes a set of physical state of the system. Due to different cases that may occur in systems with uncertainty, Markov jump linear systems with different modes can be applied efficiently to describe the system. For this reason, Markovian jump linear systems are being used increasingly in networked control system research field.

On the other hand, one of the common methods which many of the researchers consider for stabilizing of Networked Control Systems with time delays is to define Lyapunov functions. Using Lyapunov approach to prove the stability of NCSs provides a better analysis tool and also a better insight into the design of controllers for these systems.

On the subject of packet dropout, as well as time delay, different techniques have been introduced. Retransmission of packets is one solution which in most cases is not

efficient. Observer design, derivation of estimator and quantized control are other ways of facing missing measurements in networked control systems. Additionally, compensators can be considered in order to compensate for the lost information packets.

Finally, it should be mentioned that networked control systems are always affected with noises and disturbances. Therefore, it is necessary to minimize their influences on the systems. H_2 and H_∞ are powerful tools that can reduce noise and disturbance actions.

Motivated by these observations, in this thesis we discuss control perspective of networked systems, aiming to address some basic issues associated with them. Specifically, the two issues, i.e. time delays and packet dropouts, are considered. For modeling and stabilization purposes, control approaches such as Markov jump linear systems, Lyapunov functional, and H_2 control are exploited.

1.3 Objectives

The main objectives of this thesis are the following:

- Investigate the problems arising as a result of using shared digital communication network in control systems.
- Model networked control systems with time delays and uncertainties using available approaches such as Markov jump linear systems.
- Design a static output feedback controller to stabilize the networked system with uncertainty.
- Extend the results further to design a dynamic output feedback controller for the modeled NCS.
- Apply the results to an Inverted Pendulum.
- Consider packet dropouts in NCSs and model the system as a Markov jump linear system.
- Design a compensator for the missing measurements.

1.4 Major Contributions

In this study, the focus is mainly on stabilization and control of network based systems with special emphasis on transmission time delays and packet dropouts. The main

contributions of this thesis consist of control over networked system with uncertain time delays and compensation for packet dropouts in networked control systems.

1.4.1 Control over Networked Systems with Uncertain Time Delays

A networked control system with bounded random time delays is taken into consideration. Using Jordan canonical form and Markov jump linear systems, the uncertainty of the NCS and the time delays are modeled. Using Lyapunov-Krasovskii method mode-dependent output feedback controllers are designed to stabilize the system. Moreover, extensions to the design of other forms of controllers are presented. In order to validate the proposed control design, simulations and experimental results are carried out.

1.4.2 Compensation for Packet Dropouts in NCSs

In this study networked control systems with packet dropouts are investigated. A compensator is considered to be co-located with the actuator node at the plant side to produce compensating commands when there are outages in network. Stability and H_2 performance of the closed loop system are studied by modeling the network controlled system as a Markov jump linear system.

1.5 Thesis Outline

The rest of the thesis is organized as follows:

Chapter 2 includes literature review. It presents an introduction to networked control systems, stabilization, and controller design methods. First, basic issues of networked control systems such as time delays, packet dropouts, bandwidth limitation, and etc are discussed and then, existing modeling and control approaches for dealing with these issues are reviewed.

Chapter 3 provides the basic concepts that the study is based on. Markov jump linear systems and H_2 control problem are introduced in details.

In Chapter 4, networked control systems with uncertain time delays are investigated. Detailed modeling of the NCS with bounded random time varying delays as well as the stabilization of the system via mode-dependent static output feedback controller are given. The results of the proposed method are in the form of Bilinear Matrix Inequalities. Simulations are also provided to verify the efficiency.

In Chapter 5, we propose a new method to face the effects of packet losses in networked control systems. A compensator is introduced at the actuator side that produces control commands at the time when outages happen. Markov jump linear

systems and H_2 control are exploited for designing the compensator. Simulation results are also given.

Chapter 6 mainly discusses the application of the proposed networked control to inverted pendulum and the evaluation of the control performance. A brief introduction to the inverted pendulum, its mathematical model, and system identification is presented. Experimental results are given.

Finally, Chapter 7 concludes the thesis and provides some possible directions and recommendations for further research.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, we mainly consider the research issues in networked control systems and survey the related literature. First, networked control systems are discussed and then, the basic issues of these systems and the previous related studies are addressed. Modeling approaches and control design algorithms for NCSs which have been developed by previous researchers are presented as well. Finally, a brief survey of existing works on Markov jump linear systems closes the chapter.

2.2 Networked Control Systems

Networked Control Systems (NCSs) have recently become a focus area of research community due to their increasing theoretical and practical usage. Networked Control Systems consist of common basic elements such as sensors, actuators, controllers, and the plant which are connected together through a shared communication network. A digital communication network is the physical medium that provides the connection between the transmitter and the receiver [4]. As the medium is shared by other applications, data transmission is possibly constrained by limitations such as fading, channel bandwidth, delay and packet dropouts [1]. In addition, the efficiency with which information from a given source can be represented, and the rate at which information can be transmitted reliably are other issues of a digital communication channel that affect the dynamic behavior of the control system [5].

The use of a multipurpose shared network to connect spatially distributed elements results in flexible architectures, ease of system diagnosis, and generally reduces installation and maintenance costs [6]. As a result, NCSs have been utilized in a broad range of applications. On the other hand, there are also some undesirable effects which may influence the stability and the performance of NCSs. The stability problem of NCSs has been investigated in [7-10]. Network-induced delay, packet dropouts, bandwidth and packet size constraints are among the most important problems caused by shared communication network. These issues which have been addressed in [7-10],

introduce new challenges. Therefore, as Murray et al. [11] believe, the control over networks is one of the main future directions [6].

Figure 2.1 shows a general shared networked control system. As it can be observed, each plant transmits information through network. A continuous signal from the plant is first sampled by the sensors before proceeding to the controller. Then the samples are encoded into digital format by the encoder. These digital information packages will be sent through the network and will be received at the other side. Again a digital control policy will be produced by the controller, sent through the network, decoded by the decoder at the plant side, and finally delivered to the actuators to be applied to the main process [6].

Figure 2.1 A general shared networked control system [6].

Through the above mentioned process, some problems occur, due to the existence of shared network, which should be taken into consideration in modeling and stabilization of NCSs. The first problem is the time varying transmission period. Because of the nondeterministic factors during a transmission process, equal distance sampling cannot be guaranteed [12]. Another problem is the network scheduling when there is a set of plants connecting to the network. In this case, if no scheduling has been considered, concurrent transmissions of packages may take place, which result in delayed or even missing data.

Multiple packet transmission, due to the bandwidth and packet size constraints of the network, is another problem as part or all of them may not arrive at their destination [12]. More other issues that are specifically of concern are network-induced delays and packet dropouts. These can emerge as a result of network congestion, bandwidth variation, etc. The next section will discuss about the network problems and review previous works done.

2.3 Fundamental Issues in NCSs

Basic problems in Networked Control Systems consist of [13]:

2.3.1 Time Delays

Communication delays arise in the data exchanges between the components of the network. These time delays can be constant, time varying or random depending on the network and scheduling policy of the system. Time delays in an NCS include important components such as preprocessing time, waiting time, transmission time, post-processing time, etc [14]. A control system with time delays in its feedback loop is shown in Figure 2.2. As it can be seen in the figure, time delays can happen in sensor-controller channel or controller-actuator channel or both.

Figure 2.2 A control system with time delays in its feedback loop [2].

Sensor to controller delay and controller to actuator delay have different natures. Sensor to controller delay can be known when the controller uses the sensor's data to generate the control signal, provided the sensor and controller clocks are synchronized and the message is time-stamped. However, the controller to actuator

delay is different in that the controller does not know how long it will take the control signal to reach the actuator [12].

Wittenmark et al. [15] and Cloosterman et al. [16] showed that a stable system would become unstable if the characteristics of the time delay change. As mentioned earlier, time delays are often a combination of network access delay and the transmission delay. The first one is the time it takes for a shared network to accept the data, and the second one is the time during which data are in transit inside the network [6]. The overall delay can be highly variable as the two parts depend highly on variable network conditions [6].

2.3.2 Data Packet Dropout

Packet dropouts are other challenges that researchers face in NCSs field in contrast with the traditional control systems. Generally packet dropouts are caused by several reasons, such as transmission errors in physical network links, buffer overflows due to congestion, and long transmission delays [6]. The stabilization problem of linear NCSs with limited packet losses has been taken into consideration in [17] and [18], where NCSs were modeled as switched systems. Xiong and Lam [18] have considered two types of package loss, one is the i.i.d. packet loss process and the other is the Markovian packet loss.

2.3.3 Bandwidth Limitation

It is well known that generally communication networks can only carry a finite amount of information at a time. Therefore, the data packets, containing the plant or controller information, are usually transmitted through different limited channels in given sequences. This limitation results in significant constraints on the operation of NCSs.

In 1997 Wong and Brockett considered a state estimation problem involving finite communication capacity constraints [19]. In their work the observations are coded and transmitted over a digital communication channel with finite capacity. The existence of stable and asymptotically convergent coder-estimator schemes was confirmed under conditions related to the communication data rate and the rate of change of the underlying dynamics [19]. They also continued their work by considering a new class of feedback control problem in 1999 [20].

Zhang and Hristu [21] studied the stabilization problem of networked control systems with multiple sensors and actuators which communicate with the controller through a limited access shared communication network. They present an algorithm providing a complete and straightforward method for simultaneously determining stabilizing gains and communication policies. This algorithm avoids the limitations and complexities which exist in other proposed models [21].

To end the section, it must be noted that there are other issues and problems in the field of Networked Control Systems, especially if the elements of the system are connected through a wireless communication network. Power limitation is one of these problems. Wireless Sensor Networks (WSNs) are those systems which have a large number of sensors with limited power resources, thus minimization of the energy consumption plays an essential role in wireless sensor network.

2.4 Modeling and Control of NCSs

In this thesis, the main focus of the study is on the two main issues of the networked control systems including time delays and packet dropouts. Therefore, in this section we will address the previous related works, methods, and approaches regarding the modeling, stabilization and control of networked control systems with time delays and NCSs with packet losses.

2.4.1 Networked Control Systems with Time Delays

In NCSs, because of the variable network conditions, the time delays can be variant, random and uncertain. In literature there are many different approaches for modeling, analysis, and control synthesis of NCSs with network delays. Overviews of these approaches are available in [22] and [23]. Tipsuwan and Chow presented the recently developed control methodologies which handle the effect of delay in

NCSs in [22]. They also discussed NCSs structures and network delays including characteristics and effects. Yang in [23] proposed a simple framework and some general formulations for the study of NCSs. Besides, [23] also reviewed the impact of NCSs on traditional large-scale system control methodologies with a related application.

Most of the proposed methods in literature use discretization of the continuous-time plants for modeling and analysis of NCSs with delay, as in [24-32]. Although there are also some continuous-time NCSs available in [9], [33] and the references therein. It is more natural to analyze an NCS from the discrete point of view; since in a typical NCS operation, physical signals are sampled and then transmitted on a network medium after a short delay [34].

Hetel et al [26] used the Taylor series approach to describe the discrete time system for NCSs with unknown time-varying delays. In [26] the model of NCS with delay is approximated using a Lyapunov-based technique and Taylor series, and then Linear Matrix Inequalities (LMIs) are derived for stabilization and controller design of the system. The mentioned procedure is iterative, i.e. the order of the Taylor series increases until-if-ever a feasible controller is found. The LMIs which are proposed in [30] are sufficient conditions for the stability of the system, thus the iterative procedure of [26] is avoided.

Throughout the years, different types of time delays have been taken into consideration by different researchers. Constant and periodic time-delays for discrete time NCSs were studied in [25] by Halevi and Ray. They assumed a single time delay for sensor-controller and controller-actuator and a single time skew between sensor and controller sampling instants, and derived a closed loop model for the NCS using augmented states [25]. Åström and Wittenmark in [24] and Zhang et al. [10] discussed the stability analysis of NCSs with constant time delays which are either smaller or larger than the sampling interval, while Nilsson in [28] considered NCSs with time-varying delays that are upper-bounded by sampling time, through stochastic methods.

In 2004, Yang et al. [32] considered NCSs with varying time-delays larger than a sampling interval. The effect of multiple control inputs during one sampling time and vacancy sampling has been also described [32]. In 2006 Pan et al. [29] proposed a method for the stabilization and performance control of NCSs in a way that uncertain time-varying delays are divided into a constant and a time-varying part which could be larger than one sampling interval. Therefore the proposed model consists of a set of constant and uncertain matrices. Cloosterman et al. [16] have proposed a method in which the uncertainty of NCSs is contrived into a state-space system model, and sufficient conditions for stabilizing and control synthesis of NCSs are derived in terms of LMIs.

In [35] Nilsson and Wittenmark studied a discrete time model with a single sensor-controller delay and a single controller-actuator delay and designed a stochastic controller. Then, in [28] Nilsson considered the case with multiple delays in sensor and controller channel; however, it was assumed that the total maximum network delay is less than one sampling interval. Branicky, Phillips, and Zhang [10] discussed a case where sensor and controller channel delays were lumped together; assuming that the total delay is less than one sampling time as well as Nilsson's work.

Many of the methods in existing literature consider the total network induced delay which is the summation of the sensor channel delay and the controller channel delay as in [16] and [36], although, there are some methods in which the two mentioned delays are taken into consideration separately. As in [37], where Huang and Nguang investigated the stabilization problem of uncertain networked control systems with two separate random delays. The uncertainty caused by random delays is modeled into a norm-bounded uncertainty where controller design is done by using Markov chain processes. Regarding norm-bounded uncertainty they introduced two separate Markov processes for the two delays and by using Lyapunov-Razumikhin method they proposed a mode-dependent state feedback controller. Existence of the controller was presented in terms of BMIs.

In 2005 Zhang et al. [21] proposed a new method for stabilization of networked control systems with random delays. They considered the problem by defining two Markov modes for two network channels delays separately. Then, a Markov mode-dependent controller was designed and the results were given in the form of linear matrix inequalities. In fact the final conditions are in the form of LMIs with some inversion constraints. The authors stated that their results were nonconvex and suggested to use an iterative LMI approach called as the cone complementarities linearization.

Gao et al. in [38] presented a solution to the output feedback stabilization problem for discrete time systems with time varying delay in the state. First, a stability condition is given in their work which depends on the minimum and maximum delay bounds, and then through some LMIs the problems of stabilization by static and dynamic output feedback controllers are solved. As the given conditions are not in the form of strict LMIs, the authors proposed a cone complementary linearization procedure for solving the nonconvex feasibility problem. Finally, the results were extended to discrete time delay systems with norm bounded uncertain parameters.

Boukas, Liu, and Shi considered the stochastic stability and stabilization problem of NCSs modeled as Markov jump systems with time delay under dynamical output feedback control command in [39]. The stochastic and exponential stability of the

system was established via LMI-based sufficient conditions. In addition, a sufficient and necessary condition for the MJS to be stabilizable in the sense of mean square quadratic stability was proposed.

Furthermore, it is also possible to compensate known time delays which occur in the feedback loop of networked control systems. If the delays of the sensor channel and controller channel are known to the controller, then it can easily estimate the plant states based on the time when control commands arrive at the actuator, and calculate the control signals using its estimation. An example of delay compensator is given by Zhang et al. [10] for networked control systems having fixed transmission delays.

2.4.2 Networked Control Systems with Packet Dropouts

Generally, packet dropouts happen in networked control systems when there are node failures or message collisions [40]. In most of the network protocols, if the packages are not received at the destination, there is a limited time for retransmission of packages. However, after this time has elapsed, the packets are dropped. In real time control systems, this is actually advantageous to transmit a new packet rather than an old and un-transmitted message [40].

Sinopoli et al. studied the effect of packet loss on the stability of Kalman filter in [41]. They presented a Kalman filter scheme for MIMO systems in which the arrival of output information is modeled as a Bernoulli process [2]. In his other work [42] a Linear Quadratic Gaussian (LQG) optimal control problem is considered. In this discrete time setting, the packet loss may take place between the sensor nodes and the estimator/controller and also between the controller and the actuator. It has been shown that in the case of TCP, the separation principle remains valid. A class of NCSs is also considered in [43] in which packet dropout is considered in communication links between components of the system. It presents the design for optimal LQG controllers by using a separation principle.

In [44] NCSs with packet dropouts and multiple packet transmissions are modeled through describing the history of packet dropouts by different independent Markov chains and sufficient conditions for the stochastic stability of the system are provided in the form of Linear Matrix inequalities. Zhang et al. [10] considered a NCS in which the controller is directly connected to the plant. Then, they discussed that the states which are transmitted through the network, might be lost due to packet dropout effect of the NCSs. Thus, using the stability analysis for asynchronous dynamical systems, the authors investigated the maximum packet dropping rate under which the overall system is stable. By assuming that the controller and plant are linked directly, Seiler and Sengupta analyzed the stability of a linear NCS, based on the stability of Markovian jump linear systems, in the

presence of packet losses [45]. In 2009, Henriksson, Sandberg, and Johansson proposed a new method to compensate outages in networked control systems. They suggested a compensator co-located at the actuator side to produce a replacement command, based on the history of past control commands [46].

It should be noted that most of the works done before, consider a one-way time delay and packet loss, or they consider a two-way network-induced delay or two way packet loss alone. However, in real NCSs there are network-induced delays and packet dropout simultaneously in both the feedback and feedforward path. An available method which considers both network-induced delay and package losses can be found in [28]. To compensate the concurrent time delays larger than one sampling interval and successive data packet losses in both the control forward and the sensor feedback paths, the stochastic optimal estimator in [28] is extended to be a multi-step-ahead state estimator.

2.5 Markov Jump Linear Systems

During the past decade, linear systems with Markovian jumping parameters have drawn an increasing attention. This class of systems is referred to as Markov Jump Linear (MJL) systems. Markov Jump Linear systems provide an appropriate model for the plants whose structures are subject to random abrupt changes [47]. These changes which can be the results of, for instance, abrupt environment disturbances, random

component failures, changes of the operating point of a linearized model of a nonlinear system and etc. are called uncertainties.

Uncertainty is an inseparable part of Networked Control Systems due to the complexity of systems, external disturbances, and measurement errors. Hence, Yue, Han and Lam addressed networked control systems with uncertainties in [48]. Considering both network induced delays and packet dropouts, they have studied robust H_∞ approach based on a Lyapunov-Krasovskii functional method.

Discrete time Markovian jump linear systems are hybrid systems with their state vector comprising of two components. One is a discrete part being known as Markov chain mode, and the other one, a continuous part being known as Markov state. Each Markov mode in a process takes its value within a finite set of integer numbers and it determines a particular physical state of the system.

There are many people who considered control problems related to discrete time Markovian jump linear systems [49-53]. Some of them like Boukas and Liu in 2001 [51], Boukas and Shi in 1998 [52], Costa and Marques in 1998 [53], in particular, studied the uncertain discrete time MJL systems, robust stability and control of systems with norm-bounded uncertainty.

Kawka and Alleyne in 2006 [54] used a stochastic two state Markov model to examine a packet-based control strategy, in order to improve the stability and performance of a networked control system with transmission losses. By packet-based control they signify an environment that there is no penalty for sending additional information in each transmission up to the fixed minimum packet size.

Considering the previous works done, it is obvious that Markov Jump Linear systems act efficiently in modeling networked control systems with random uncertainties. Therefore, in this thesis we utilize the MJL systems approach for the modeling of networked control systems.

Chapter 3

MARKOV JUMP LINEAR SYSTEMS

3.1 Introduction

One of the main issues in control systems is their capability of maintaining an acceptable behaviour and meeting some performance requirements even in the presence of abrupt changes in the system dynamics. These changes can be due, for instance, to abrupt environmental disturbances, component failures or repairs, changes in subsystems interconnections, abrupt changes in the operation point for a non-linear plant, etc [55].

Control systems are often designed based on the mathematical model of the process to be controlled. This model should be able to describe the behavior of the process accurately. Many processes are possible to be well expressed by time invariant linear

models. However, there are a large number of systems whose structures are subject to uncertain changes and require a more complex approach [55]. For example, consider a dynamical system that in a certain moment is well defined by the model G_1 . Then, after a certain amount of time, due to abrupt changes, the process dynamics convert to a new model G_2 . More generally, it can be imagined that the system is subject to a series of possible qualitative changes that make it switch, over time, among a countable set of models, e.g., $\{G_1, G_2, \dots, G_N\}$. Each of these models can be associated to an operation mode of the system, and it is said that the system jumps from one mode to another or that there are transitions between them [55].

The jumps evolve stochastically according to a Markov chain, that is, given that at a certain instant k the system lies in mode i , the jump probability for each of the other modes, and also the probability of remaining in mode i are known. The Markov state (or operation mode), which is denoted by $\theta(k)$, is known at each time k as well. The process models at all operation modes are assumed to be discrete-time, time-invariant linear models. Hence, a coherent body of theory can be constructed. It is also possible to develop the basic concepts of control and filtering and present controller and filter design procedures for this class of systems, known in the literature as Discrete-Time Markov Jump Linear Systems (MJLS) [55].

A discrete time homogeneous Markov chain process is shown by $\theta(k)$. At each time k , the jump variable $\theta(k)$ takes its values within a finite set $\mathbb{N} = \{1, 2, \dots, N\}$, and the stationary transition probability matrix is given by $\Lambda = [\pi_{ij}]$, where

$$\pi_{ij} := P\{\theta_{l+1} = j \mid \theta_l = i\} \quad (3.1)$$

with $\pi_{ij} \geq 0$, $\forall i, j$, $i, j \in \mathbb{N}$, $P\{\cdot\}$ showing the probability, and $\sum_{j=1}^N \pi_{ij} = 1$, for

$i, j \in \mathbb{N}$. And, the initial distribution for $\theta(0)$ is denoted by $\nu = \{\nu_1, \dots, \nu_N\}$.

3.2 Stability of Markov Jump Linear Systems (MJLS)

In control system design problems, stability of the system is one of the most important requirements. In this section, stability definitions and conditions for Markov jump linear systems are presented. It is shown that mean square stability for MJLS is guaranteed in terms of the spectral radius of an augmented matrix being less than one, or in terms of the existence of a positive-definite solution for a set of coupled Lyapunov equations [55].

Consider the following difference equations

$$x(k+1) = f(x(k)) \quad (3.2)$$

$$x(k+1) = Ax(k) \quad (3.3)$$

with $k \in \{0, 1, 2, \dots\}$, $x(k) \in \mathfrak{R}^n$, $f: \mathfrak{R}^n \rightarrow \mathfrak{R}^n$ and A with appropriate dimension. A point $x_e \in \mathfrak{R}^n$ is called an equilibrium point of system (3.2), if $f(x_e) = x_e$. In particular, $x_e = 0$ is an equilibrium point of system (3.3) [55].

The following definitions and lemmas are the preliminaries which form the basis of our study:

Definition 3.1 (Lyapunov Stability) [55]. An equilibrium point x_e is said to be stable in the sense of Lyapunov if for each $\varepsilon > 0$ there exists $\delta_\varepsilon > 0$ such that $\|x(k) - x_e\| \leq \varepsilon$ for all $k \geq 0$ whenever $\|x(0) - x_e\| \leq \delta_\varepsilon$.

Definition 3.2 (Asymptotic Stability) [55]. An equilibrium point is said to be asymptotically stable if it is stable in the sense of Lyapunov and there exists $\delta > 0$ such that whenever $\|x(0) - x_e\| \leq \delta$ we have that $x(k) \rightarrow x_e$ as k increases. It is globally asymptotically stable if it is asymptotically stable and $x(k) \rightarrow x_e$ as k increases for any $x(0)$ in the state space.

Definition 3.3 (Lyapunov Function) [55]. Let x_e be an equilibrium point for system (3.2). A positive function $\varphi: \Gamma \rightarrow \mathfrak{R}$, where Γ is such that $x_e \in \Gamma \subseteq \mathfrak{R}^n$, is said to be a Lyapunov function for system (3.2) and equilibrium point x_e if

1. $\varphi(\cdot)$ is continuous,

2. $\varphi(x_e) < \varphi(x)$ for every $x \in \Gamma$ such that $x \neq x_e$,
3. $\Delta\varphi(x) = \varphi(f(x)) - \varphi(x) \leq 0$ for all $x \in \Gamma$.

Lemma 3.1 [56]. If there exists a Lyapunov function $\varphi(x)$ for system (3.2) and x_e , then the equilibrium point is stable in the sense of Lyapunov. Moreover, if $\Delta\varphi(x) < 0$ for all $x \neq x_e$, then it is asymptotically stable. Furthermore if φ is defined on the entire state space and $\varphi(x)$ goes to infinity as any component of x gets arbitrarily large in magnitude then the equilibrium point x_e is globally asymptotically stable.

Lemma 3.2 [57]. The followings are equivalent,

1. $x=0$ is the only globally asymptotically stable equilibrium point for system (3.3).
2. $r_\sigma(A) < 1$.
3. For any $S > 0$, there exists a unique $V > 0$ such that

$$V - A^*VA = S \quad (3.4)$$

4. For some $V > 0$, we have

$$V - A^*VA > 0. \quad (3.5)$$

Next, we introduce some of the operators which will be used throughout the study.

Consider the following homogeneous Markov jump linear system,

$$G = \begin{cases} x(k+1) = \Gamma_{\theta(k)}x(k) \\ x(0) = x_0, \theta(0) = \theta_0 \end{cases} \quad (3.6)$$

where $\Gamma = (\Gamma_1, \dots, \Gamma_N) \in \mathbb{H}^n$. $x(k)$ in (3.6) is not a Markov process, but the joint process of $x(k), \theta(k)$ is. Now defining the indicator function as,

$$1_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{otherwise} \end{cases} \quad (3.7)$$

we have,

$$\mu(k) = \mathbb{E}(x(k)) = \sum_{i=1}^N \mathbb{E}\left(x(k) 1_{\{\theta(k)=i\}}\right), \quad (3.8)$$

and,

$$\mathbf{Q}(k) = \mathbb{E}\left(x(k)x^*(k)\right) = \sum_{i=1}^N \mathbb{E}\left(x(k)x^*(k) 1_{\{\theta(k)=i\}}\right). \quad (3.9)$$

Moreover, for $V = (V_1, \dots, V_N) \in \mathbb{H}^n$ and $i, j \in \mathbb{N}$, a linear operator as below,

$$\mathcal{T}_j(V) \triangleq \sum_{i=1}^N \pi_{ij} \Gamma_i V_i \Gamma_i^* \in \mathbf{B}(\mathfrak{R}^n), \quad (3.10)$$

is defined [55].

3.3 Mean Square Stability (MSS)

3.3.1 Mean Square Stability

Stability of a Markov jump linear system in the mean square sense means that

$\mathbb{E}\left(x(k)x^*(k)\right)$ converges, when k goes to infinity. See the following definition:

Definition 3.4 [55]. A Markov jump linear system defined in (3.6), is Mean Square Stable (MSS), if for any initial condition x_0 and θ_0 there exist $\mu \in \mathfrak{R}^n$ and $\mathbf{Q} \in \mathbf{B}(\mathfrak{R}^n)$ (independent of initial conditions) such that

1. $\|\mu(k) - \mu\| \rightarrow 0$ as $k \rightarrow \infty$,
2. $\|\mathbf{Q}(k) - \mathbf{Q}\| \rightarrow 0$ as $k \rightarrow \infty$.

Consequently, we proceed to the following lemma, a proof of which can be found in [55].

Lemma 3.3 [55]. The following assertions are equivalent,

1. System model (3.6) is MSS.
2. For any $S \in \mathbf{H}^n$, $S > 0$, there exist a unique $V \in \mathbf{H}^n$, $V > 0$, such that

$$V - \mathcal{T}(V) = S. \quad (3.11)$$

3. For some $V \in \mathbf{H}^n$, $V > 0$, we have

$$V - \mathcal{T}(V) > 0 \quad (3.12)$$

4. For some $\beta \geq 1$, $0 < \zeta < 1$, we have for all x_0 and θ_0 ,

$$\mathbb{E}(\|x(k)\|^2) \leq \beta \zeta^k \|x_0\|_2^2, \quad k = 0, 1, \dots \quad (3.13)$$

5. For all x_0 and θ_0

$$\sum_{k=0}^{\infty} \mathbb{E} \left(\|x(k)\|^2 \right) < \infty. \quad (3.14)$$

3.3.2 Mean Square Stabilizability

Considering another version of the system (3.6), defined as,

$$\mathbf{G} = \begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) \\ y(k) = C_{\theta(k)}x(k) \\ x(0) = x_0, \theta(0) = \theta_0 \end{cases} \quad (3.15)$$

we proceed to the following definitions and tests related to mean square stabilizability.

Definition 3.5 (Mean Square Stabilizability) [55]. In the system (3.15), let $A = (A_1, \dots, A_N) \in \mathbb{H}^n$, $B = (B_1, \dots, B_N) \in \mathbb{H}^{m,n}$. The pair (A, B) is mean square stabilizable if there is $F = (F_1, \dots, F_N) \in \mathbb{H}^{n,m}$ such that when $u(k) = F_{\theta(k)}x(k)$, system (3.15) is MSS. In this case, F is said to stabilize the pair (A, B) .

(Mean Square Stabilizability test) [55]. The pair (A, B) is mean square stabilizable if and only if there are $W_1 = (W_{11}, \dots, W_{N1}) \in \mathbb{H}^n$, $W_1 > 0$, $W_2 = (W_{12}, \dots, W_{N2}) \in \mathbb{H}^{m,n}$, $W_3 = (W_{13}, \dots, W_{N3}) \in \mathbb{H}^m$, $W_3 > 0$, such that for all $j \in \mathbb{N}$,

$$\sum_{i=1}^N \pi_{ij} \left(A_i W_{i1} A_i^* + B_i W_{i2}^* A_i^* + A_i W_{i2} B_i^* + B_i W_{i3} B_i^* \right) - W_{j1} < 0 \quad (3.16)$$

$$\begin{bmatrix} W_{j1} & W_{j2} \\ W_{j2}^* & W_{j3} \end{bmatrix} \geq 0 \quad (3.17)$$

$$W_{j1} > 0 \quad (3.18)$$

For necessity part of the test, if we assume that the pair (A, B) is mean square stabilizable, then, there exists $F \in \mathbb{H}^{n,m}$ such that the closed loop system of (3.15) with $\Gamma_i = A_i + B_i F_i$ for $i \in \mathbb{N}$ is MSS. From Lemma 3.3 there is $X = (X_1, \dots, X_N) > 0$ in $\mathbb{H}^n$ such that

$$-X_j + \sum_{i=1}^N \pi_{ij} (A_i + B_i F_i) X_i (A_i + B_i F_i)^* < 0, \quad (3.19)$$

for all $j \in \mathbb{N}$. Now taking $W_{j1} = X_j$, $W_{j2} = W_{j1} F_j^*$ and $W_{j3} = W_{j2}^* W_{j1}^{-1} W_{j2}$ for $j \in \mathbb{N}$, it can be verified from (3.19) that (3.16)-(3.18) are satisfied and hence, the set defined below is not an empty set,

$$\begin{aligned} \phi \triangleq & \left\{ W_1 \in \mathbb{H}^n, W_2 \in \mathbb{H}^{m,n}, W_3 \in \mathbb{H}^m; \right. \\ & \left. W_{j1}, W_{j2}, W_{j3} \text{ satisfy (3.15)–(3.17) for all } j \in \mathbb{N} \right\}. \end{aligned} \quad (3.20)$$

For sufficiency part, assume that the set ϕ is not empty, therefore there are $W_1 \in \mathbb{H}^n$, $W_1 > 0$, $W_2 \in \mathbb{H}^{m,n}$, and $W_3 \in \mathbb{H}^m$, $W_3 > 0$ satisfying (3.16)-(3.18). Now, let $F = (F_1, \dots, F_N) \in \mathbb{H}^{n,m}$ be given by $F_j = W_{j2}^* W_{j1}^{-1}$ for $j \in \mathbb{N}$, thus,

$$-W_{j1} + \sum_{i=1}^N \pi_{ij} (A_i + B_i F_i) W_{i1} (A_i + B_i F_i)^*$$

$$\begin{aligned}
&= -W_{j1} + \sum_{i=1}^N \pi_{ij} \left(A_i + B_i W_{i2}^* W_{i1}^{-1} \right) W_{i1} \left(A_i + B_i W_{i2}^* W_{i1}^{-1} \right)^* \\
&= -W_{j1} + \sum_{i=1}^N \pi_{ij} \left(A_i W_{i1} A_i^* + B_i W_{i2}^* A_i^* + A_i W_{i2} B_i^* + B_i W_{i2}^* W_{i1}^{-1} W_{i2} B_i^* \right) \\
&\leq -W_{j1} + \sum_{i=1}^N \pi_{ij} \left(A_i W_{i1} A_i^* + B_i W_{i2}^* A_i^* + A_i W_{i2} B_i^* + B_i W_{i3} B_i^* \right) < 0
\end{aligned}$$

for all $j \in \mathbb{N}$. From Lemma 3.3 we have that F stabilizes (A, B) in the mean square sense. Therefore, the pair (A, B) is mean square stabilizable.

The stability criteria and tests mentioned above are relevant to the case when no noise has been taken into consideration. However, in the case of noise existence, it is necessary to discuss and consider its effect on the system performance. When the state equation includes an additive wide sense white noise sequence, in addition to stabilizability, it is desired to minimize the influence of the noise on the output of the system. One of the efficient approaches is H_2 -norm control which will be introduced in the next section.

3.4 H_2 Control Problem

H_2 control problem is often brought forth for discussion when the system contains an additive wide sense white noise sequence as in system G below,

$$G = \begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) + G_{\theta(k)}w(k) \\ z(k) = C_{\theta(k)}x(k) + D_{\theta(k)}u(k) \\ x(0) = x_0, \theta(0) = \theta_0 \end{cases} \quad (3.21)$$

In this situation, it is desirable to minimize the limit value of the expected value of a quadratic cost among all closed loop mean square stabilizing dynamic Markov jump controllers defined by $u(k) = F_{\theta(k)}x(k)$, $F = (F_1, \dots, F_N) \in \mathcal{F}$, where $\mathcal{F}$ is the set of stabilizing controllers. To fulfill the minimization, H_2 control is taken into consideration. In the following, the definition of H_2 -norm for the closed loop system G_{cl} , which represents system G with the control command given above, is presented.

Definition 3.6 According to [55], the H_2 -norm of the closed loop system G_{cl} expressed as,

$$G_{cl} = \begin{cases} x(k+1) = \Gamma_{\theta(k)}x(k) + G_{\theta(k)}w(k) \\ z(k) = \Psi_{\theta(k)}x(k) \end{cases} \quad (3.22)$$

when $x(0) = 0$, with the input sequence given by $(w(0), w(1), \dots)$, $w(0) = e_s$, $w(k) = 0$ for $k > 0$, where $\{e_1, \dots, e_r\}$ forms a basis for $\mathfrak{R}^r$, is denoted by $\|G_{cl}\|_2$ and is defined as,

$$\|G_{cl}\|_2^2 = \sum_{s=1}^r \|z_s\|_2^2 \quad (3.23)$$

where,

$$\|z_s\|_2^2 = \sum_{k=1}^{\infty} E\left(\|z_s(k)\|^2\right). \quad (3.24)$$

The H_2 -norm can be computed from the unique solution of the discrete coupled Grammians of observability and controllability, as there is a direct relation between them. For any $F \in \mathbb{F}$, let $O(F) = (O_1(F), \dots, O_N(F)) \in \mathbb{H}^n$, $O_i(F) > 0$ for $i \in \mathbb{N}$, be defined as,

$$O_i(F) \triangleq C_i^* C_i + F_i^* D_i^* D_i F_i, \quad (3.25)$$

and let $S_o(F) = (S_{o_1}(F), \dots, S_{o_N}(F)) \in \mathbb{H}^n$, $S_{o_i}(F) > 0$ for $i \in \mathbb{N}$ and $S_c(F) = (S_{c_1}(F), \dots, S_{c_N}(F)) \in \mathbb{H}^n$, $S_{c_i}(F) > 0$ for $i \in \mathbb{N}$ be the unique solutions of the observability and controllability Grammians respectively. Then, the connection between the H_2 -norm and the Grammians is established by,

$$\|G_{cl}\|_2^2 = \sum_{i=1}^N \text{tr} \left[\begin{bmatrix} C_j^* C_j & 0 \\ 0 & D_j^* D_j \end{bmatrix} \begin{bmatrix} S_{c_j}(F) & S_{c_j}(F) F_j^* \\ F_j S_{c_j}(F) & F_j S_{c_j}(F) F_j^* \end{bmatrix} \right], \quad (3.26)$$

the proof of which is given in [55]. An alternative way of computing the H_2 -norm of system G_{cl} can be considered as follows. Define:

$$Q_i(k) = E\left(x(k) x^*(k) 1_{\{\theta(k)=i\}}\right), \quad i \in \mathbb{N}, \quad (3.27)$$

recalling that for a wide sense white noise sequence, $E(w(k)) = 0$ and $E(w(k) w^*(k)) = I$, we have,

$$Q_j(k+1) = \sum_{i=1}^N \pi_{ij} \left[(A_i + B_i F_i) Q_i(k) (A_i + B_i F_i)^* + v_i G_i G_i^* \right]. \quad (3.28)$$

Additionally, as the closed loop system is MSS, then $Q(k) \xrightarrow{k \uparrow \infty} Q$, where $Q = (Q_1, \dots, Q_N)$ is the unique solution of the controllability Grammian. i.e. $Q = S_c(F)$ [55]. Thus,

$$\begin{aligned} E\left(\|z(k)\|^2\right) &= E\left(\text{tr}\left(z(k)z(k)^*\right)\right) \\ &= E\left(\text{tr}\left((C(\theta_k)x(k) + D(\theta_k)u(k))(C(\theta_k)x(k) + D(\theta_k)u(k))^*\right)\right) \\ &= \sum_{i=1}^N \text{tr} \left(\begin{bmatrix} C_i^* C_i & 0 \\ 0 & D_i^* D_i \end{bmatrix} \begin{bmatrix} Q_i(k) & Q_i(k) F_i^* \\ F_i Q_i(k) & F_i Q_i(k) F_i^* \end{bmatrix} \right) \\ &\xrightarrow{k \uparrow \infty} \sum_{i=1}^N \text{tr} \left(\begin{bmatrix} C_i^* C_i & 0 \\ 0 & D_i^* D_i \end{bmatrix} \begin{bmatrix} Q_i & Q_i F_i^* \\ F_i Q_i & F_i Q_i F_i^* \end{bmatrix} \right) = \|G_{cl}\|_2^2. \end{aligned} \quad (3.29)$$

Respecting numerical solutions for stabilizability and H_2 control problems, Linear Matrix Inequality (LMI) approach is a very efficient and reliable one. The main results of this thesis are expressed in the terms of LMIs. Thus, the next section is allocated to the introduction of this technique.

3.5 Linear Matrix Inequality (LMI)

Linear Matrix Inequalities have always been one of the researchers' interests, due to the fact that, the LMI approach offers an effective scheme for analysis or synthesis

problems in terms of convex optimization which can be solved efficiently [58]. The basic idea is to obtain a convex programming problem which is either in the form of LMIs or may be converted to one, and then solve the optimization problem instead of the original one [55]. Some of the advantages regarding the usage of LMI technique include [55]:

1. Existence of the efficient algorithms for solving LMIs. It is usually simple to get a numerical solution for the problems that are expressed in the LMI form.
2. With the current knowledge on MJLS area, some of the control problems concerning these systems can only be solved via LMI approximations.

Definition 3.7 [55]. A linear matrix inequality (LMI) is any constraint that can be written or converted to

$$H(x) = H_0 + x_1 H_1 + x_2 H_2 + \dots + x_m H_m < 0, \quad (3.30)$$

where x_i are the variables and the hermitian matrices $H_i \in \mathbf{B}(\mathfrak{R}^n)$ for $i = 1, \dots, m$ are known. The equation (3.30) is a strict LMI, the non-strict LMIs have the form $H(x) \leq 0$.

To convert the control problems which are mainly in the form of quadratic equations into larger dimension linear ones the Schur complements is used. See the following lemma.

Lemma 3.4 [59]. Consider an hermitian matrix Q such that,

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^* & Q_{22} \end{bmatrix},$$

then, $Q > 0$ if and only if

$$\begin{cases} Q_{22} > 0 \\ Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}^* > 0 \end{cases}$$

or,

$$\begin{cases} Q_{11} > 0 \\ Q_{22} - Q_{12}^*Q_{11}^{-1}Q_{12} > 0 \end{cases}.$$

Next, we proceed to the modeling and control of networked control systems with Markov jump parameters using the basic theorems and definitions introduced in this chapter.

Chapter 4

NETWORKED CONTROL SYSTEMS WITH UNCERTAIN DELAYS

4.1 Modeling of NCSs with Time Delays

Networked Control Systems are systems whose components or subsystems communicate with each other through network. They usually contain a continuous time physical plant and a discrete time controller with different sampling frequencies. Data transmission in these systems is confronted by some problems such as time delays. Time delays can be of different characteristics (constant or random time-varying) due to network and scheduling policy of the system.

4.1.1 Constant Network-induced Delays

Considering network-induced time delays to be constant for all transmissions is the simplest existing model. To analyze the system, the mean value or the worst case delay is used. This modeling can be appropriate when the sampling interval is much larger than the time delay. However, it definitely degrades the stability and performance of the system, if this is not the case.

4.1.2 Random Time-varying Network-induced Delays

There exist several sources which cause network delays to be random and time-varying:

- Varying time that is needed for the data to access the network.
- Transmission errors which result in retransmission of data.
- Collisions which can be avoided if data is transmitted with a random wait.

To model these random delays, their probabilistic distribution is taken into account. Sometimes, the probability distribution function of the delays can be estimated from the delay measurements. Thus, it becomes easy to get a model of the delays. However, in most of the times, Markov chain process is used, since there usually exists temporal correlations of delays. Markov process consists of the states and the modes. Markov chain modes indicate different possible states of the system. The probability of transition between these states is given by the transition

matrix. Each element of this matrix is the probability of transition between two modes. In our network model, each mode of the Markov chain postulates the probability distribution for sensor channel delay.

4.2 Mathematical Model of NCS with Delay

A typical Networked Control System is shown in Figure 4.1. The system is formed of a continuous time physical plant sampled with a fixed sampling rate and a discrete time controller which receives information from the plant only at sampling time instants [16].

Figure 4.1 A typical networked control system.

It is assumed that the sensor is clock-driven (sampling at time instant kh , $k \in \mathbb{N}$ with $h > 0$, the sampling interval), and the controller and actuator are event-driven. Further, the network clock is considered synchronized and there are no packet losses

in the network. As the bandwidth of the network is limited, there are two delays in the system. These two delays are shown by τ_k as the sensor channel delay and η_k as the controller channel delay. The continuous time plant is described as:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ u(t) = u(kh) \\ y(t) = Cx(t - \tau_k) \end{cases} \quad t \in [kh + \eta_k, (k+1)h + \eta_{k+1}] \quad (4.1)$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^q$ is the measured output received by the controller, A, B, C are constant matrices with appropriate dimensions, and h is the sampling period. η_k and τ_k are respectively the delays from the controller to the actuator and from the sensor to the controller. We assume that $\eta_k \in [\eta_{\min}, \eta_{\max}]$ with $0 \leq \eta_{\min} \leq \eta_{\max} < h$, i.e. the delay from the controller to the actuator is less than one sampling period. Then we define $x(k) := x(t_k)$, where $t_k = kh$. It is easy to obtain the discrete time representation of the system at the sampling instants as:

$$x(k+1) = e^{Ah}x(k) + \int_0^{h-\eta_k} e^{As}dsBu(k) + \int_{h-\eta_k}^h e^{As}dsBu(k-1) \quad (4.2)$$

It should be noted that (4.2) is only valid at the sampling instants [16].

On the other hand, the delay τ_k related to the sensor channel is known to the controller, i.e. the measurement is time-stamped. It is reasonable to assume that the

delay τ_k is bounded, i.e., $\tau_k \in [\tau_{\min}, \tau_{\max}]$, with $0 \leq \tau_{\min} \leq \tau_{\max}$. Defining

$z(k) = [x^T(k) \ u^T(k-1)]^T$, we have the corresponding state space representation:

$$z(k+1) = \begin{bmatrix} e^{Ah} & \int_{h-\eta_k}^h e^{As} ds B \\ 0_{m \times n} & 0_{m \times m} \end{bmatrix} z(k) + \begin{bmatrix} \int_0^{h-\eta_k} e^{As} ds B \\ I_m \end{bmatrix} u(k). \quad (4.3)$$

One way to solve the existing integrals of the augmented system is to consider Jordan canonical decomposition which is used in this study. Using this approach, the time variation in the delays can be represented as a combination of uncertainty functions.

Let the Jordan canonical decomposition of A be [36]:

$$A = QJQ^{-1} \quad (4.4)$$

where J is the Jordan canonical form, and Q is formed by the generalized eigenvectors of A . There would be p Jordan blocks if $p \in \mathbb{N}$ is considered as the number of distinct eigenvalues. These blocks also consist of $\mu_i > 1$ different Jordan blocks, with $\mu_i \in \mathbb{N}$ the geometric multiplicity of the i^{th} eigenvalue:

$J_i = \text{diag}\{J_{i,1}, J_{i,2}, \dots, J_{i,\mu_i}\}$, and $i = \{1, 2, \dots, p\}$. Hence, the NCS in (4.3) would

become [36]:

$$z(k+1) = \begin{bmatrix} Qe^{Jh}Q^{-1} & \int_{h-\eta_k}^h Qe^{Js} ds Q^{-1}B \\ 0_{m \times n} & 0_{m \times m} \end{bmatrix} z(k) + \begin{bmatrix} \int_0^{h-\eta_k} Qe^{Js} ds Q^{-1}B \\ I_m \end{bmatrix} u(k). \quad (4.5)$$

Defining the maximum size of the Jordan blocks that represents the i^{th} eigenvalue as:

$$l_i = \max_{\tilde{j}=\{1,2,\dots,\mu_i\}} \left(\dim J_{i,\tilde{j}} \right), \quad (4.6)$$

it follows that [36],

$$e^{Js} = \sum_{i=1}^p \sum_{j=0}^{l_i-1} \frac{s^j}{j!} e^{\lambda_i s} S_{i,j}, \quad (4.7)$$

where $S_{i,j}$ is an appropriate matrix of the same dimension as J .

Remark 4.1 There are three cases in solving the integrals in (4.5), corresponding to non-zero eigenvalues, zero eigenvalues, and complex eigenvalues.

As an example, we consider the first case in Remark 4.1. Combining (4.5) and (4.7) gives:

$$z(k+1) = F_0 z(k) + \sum_{i=1}^p \sum_{j=0}^{l_i-1} \alpha_j(\eta_k) F_j z(k) + G_0 u(k) + \sum_{i=1}^p \sum_{j=0}^{l_i-1} \alpha_j(\eta_k) G_j u(k) \quad (4.8)$$

where

$$\alpha_j(\eta_k) = \frac{(h - \eta_k)^j}{j!} e^{\lambda_i(h - \eta_k)} \quad (4.9)$$

and,

$$\begin{aligned} F_0 &= \begin{bmatrix} Qe^{Jh}Q^{-1} & QJ^{-1}e^{Jh}Q^{-1}B \\ 0_{m \times n} & 0_{m \times m} \end{bmatrix} & G_0 &= \begin{bmatrix} -QJ^{-1}Q^{-1}B \\ I_m \end{bmatrix} \\ F_j &= \begin{bmatrix} 0_{n \times n} & -QJ^{-1}S_{i,j}Q^{-1}B \\ 0_{m \times n} & 0_{m \times m} \end{bmatrix} & G_j &= \begin{bmatrix} QJ^{-1}S_{i,j}Q^{-1}B \\ 0_{m \times m} \end{bmatrix} \end{aligned} \quad (4.10)$$

Since η_k is uncertain, $\alpha_j(\eta_k)$ is an uncertain parameter. Hence our state space representation includes uncertainty. In order to model this uncertainty into the system, we use upper and lower limits as follows. Let

$$\bar{\alpha}_j = \max_{\eta \in [\eta_{\min}, \eta_{\max}]} \alpha_j(\eta_k) \quad \underline{\alpha}_j = \min_{\eta \in [\eta_{\min}, \eta_{\max}]} \alpha_j(\eta_k) \quad (4.11)$$

be the maximum and minimum possible values for $\alpha_j(\eta_k)$, and noting that the total number of different uncertain parameters is

$$v = \sum_{i=1}^p l_i, \quad (4.12)$$

where l_i is defined in (4.6) and p is the number of distinct eigenvalues of A . Then, the networked control system can be formulated as an uncertain system with polytopic uncertainty [36]:

$$z(k+1) = \tilde{A}(\eta_k) z(k) + \tilde{B}(\eta_k) u(k) \quad (4.13)$$

where

$$\tilde{A}(\eta_k) = \left(\bar{F}_0 + \sum_{j=1}^v \gamma_j \bar{F}_j \right) \quad \tilde{B}(\eta_k) = \left(\bar{G}_0 + \sum_{j=1}^v \gamma_j \bar{G}_j \right) \quad (4.14)$$

and

$$\begin{aligned} \bar{F}_0 &= F_0 + \sum_{j=1}^v \underline{\alpha}_j F_j & \bar{F}_j &= (\bar{\alpha}_j - \underline{\alpha}_j) F_j \\ \bar{G}_0 &= G_0 + \sum_{j=1}^v \underline{\alpha}_j G_j & \bar{G}_j &= (\bar{\alpha}_j - \underline{\alpha}_j) G_j \end{aligned} \quad (4.15)$$

$$\gamma_j \in (0,1) \quad j \in \{1,2,\dots,v\} \quad (4.16)$$

Hence, the uncertainty is now modeled into the state space representation of the system. Next, we introduce a discrete time homogeneous Markov chain process $\theta(k)$ to model the delay in sensor channel. At each time k , a jump variable $\theta(k)$ takes its values within a finite set $\mathbb{N} = \{1,2,\dots,N\}$, where each mode represents a particular subinterval of a possible delay range $[\tau_{\min}, \tau_{\max}]$, and the stationary transition probability matrix is given by $\Lambda = [\pi_{ij}]$, where

$$\pi_{ij} := P\{\theta_{l+1} = j | \theta_l = i\} \quad (4.17)$$

with $\pi_{ij} \geq 0$, $\forall i, j$, $i, j \in \mathbb{N}$, $P\{\cdot\}$ showing the probability, and $\sum_{j=1}^N \pi_{ij} = 1$, for $i, j \in \mathbb{N}$. Note that a Markov model is adopted due to the correlation of network congestion which causes the delay. The output y can now be written as:

$$y(k) = \tilde{C}z(k - \tau_{\theta(k)}) \quad \tilde{C} = [C \quad 0_{q \times m}] \quad (4.18)$$

where $\tau_{\theta(k)}$ indicates the delay related to the mode $\theta(k)$. In this way, the mathematical model of discrete time networked control system is described in (4.13)-(4.16) and (4.18).

It is worth noting that most of the previous works consider total delay of sensor channel and the controller channel as a single delay which is uncertain. Whereas, it is

obvious that the sensor channel delay is known to the controller, which should be taken into consideration in controller design.

4.3 Controller Design

Communication delays in a data network usually vary from one transmission to another. Therefore, the standard methods which are used for time-invariant systems are not sufficient for analyzing and control design of the systems with varying delays. For these systems, modern control theories like robust control and optimal control have been applied to NCSs to further improve the control performance. Robust control approach is the one used in this work. Here, we state two lemmas which will be utilized in controller synthesis.

Considering the definition of Mean square stability presented in chapter 3, we now provide the following definition:

Definition 4.1 Discrete time system defined as:

$$x(k+1) = \Gamma_{\theta(k)}(\eta_k)x(k)$$

where $\theta(k)$ is a Markov process and η_k is an uncertain deterministic parameter, is robustly mean square stable if it is MSS for all admissible η_k .

Lemma 4.1 [60]. The discrete time system:

$$x(k+1) = A_{\theta(k)}x(k) \quad (4.19)$$

with $\theta(k)$ the discrete time Markov chain process with transition probability matrix in (4.17), is robustly mean square stable (MSS) if there exist matrices $P_i > 0$, $i = 1, \dots, N$ such that the following inequality is satisfied:

$$A_i^T \left(\sum_{j=1}^N \pi_{ij} P_j \right) A_i - P_i < 0 \quad \forall i \in \mathbb{N}. \quad (4.20)$$

Lemma 4.2 [61]. In Lemma 4.1, (4.20) is equivalent to the following inequality for some $P_i > 0$, $i = 1, \dots, N$, Γ_{ij} , Ψ_j :

$$\begin{bmatrix} -P_i + A_i^T \Gamma_{ij} + \Gamma_{ij}^T A_i & -\Gamma_{ij}^T + A_i^T \Psi_j \\ * & P_j - (\Psi_j + \Psi_j^T) \end{bmatrix} < 0 \quad \forall (i, j) \in \mathbb{N}. \quad (4.21)$$

The objective of control synthesis in this work is to design a controller in such a way that it guarantees the stability of networked control system with uncertain time delays. To stabilize the closed loop NCS we consider three different control schemes. First static output feedback controller with two different approaches are proposed. Then, for further extension, state feedback controller, and finally dynamic output feedback controller are designed.

4.4 Static Output Feedback Controller Design

For static output feedback controller the first approach is for the case when no condition is applied to the system output matrix, and the second one is considered when the mentioned matrix is full rank. While the former results in a set of BMIs, the latter ends in LMIs which is more efficient in numerical computations.

4.4.1 Static Output Feedback Controller

In a networked control system, as the output is measured and transmitted through a delayed network, the controller receives the state information with delay. This is taken into consideration in the definition of output in (4.1). A mode-dependent static output feedback controller is described as:

$$u(k) = K_{\theta(k)} y(k), \quad \theta(k) \in \{1, 2, \dots, N\} \quad (4.22)$$

with $K_{\theta(k)} \in \Re^{m \times q}$.

As it is mentioned in the previous section, $\theta(k)$ is a Markov mode parameter determining the subinterval the delay value lies in. By taking into consideration $\theta(k)$, a mode-dependent controller gain can be designed. To this end, set the control law as:

$$u(k) = K_{\theta(k)} \tilde{C}z(k - \tau_{\theta(k)}). \quad (4.23)$$

The closed loop system of (4.13) with the controller is now described by

$$z(k+1) = \tilde{A}(\eta_k)z(k) + \tilde{B}(\eta_k)K_{\theta(k)}\tilde{C}z(k - \tau_{\theta(k)}) \quad (4.24)$$

In order to design the controller gain, we consider a mode-dependent Lyapunov-Krasovskii functional which results in a set of bilinear matrix inequalities (BMIs). Bilinear matrix inequalities are much harder to solve than Linear Matrix Inequalities (LMIs). The LMI approach offers an effective scheme for analysis or synthesis problems in terms of convex optimization which can be solved efficiently [58]. However, there are interesting control problems which are so difficult or even impossible to be formulated in the form of LMIs. Hence, we have to express them within the more general bilinear matrix inequalities (BMIs) framework.

The following theorems are instrumental, leading to a design method in terms of BMIs.

Theorem 4.1 The closed loop system given by (4.24) is robustly MSS if there exist matrices $P_{\theta(k)} > 0$, $\theta(k) \in \mathbb{N}$, $W > 0$, $R > 0$, U , T and $K_{\theta(k)}$ for $\theta(k) \in \mathbb{N}$ such that:

$$\begin{bmatrix} U & T \\ T^T & W \end{bmatrix} \geq 0, \quad (4.25)$$

$$\Xi = \begin{bmatrix} \zeta_{11} & \zeta_{12} \\ * & \zeta_{22} \end{bmatrix} < 0, \quad (4.26)$$

where $*$ denotes entry which can be deduced from the symmetry of the matrix,

$$\begin{aligned}
\zeta_{11} &= \tilde{A}^T \bar{P}_{\theta(k)} \tilde{A} - P_{\theta(k)} + N U + T + T^T + N (\tilde{A} - I)^T W (\tilde{A} - I) + N R \\
\zeta_{12} &= \tilde{A}^T \bar{P}_{\theta(k)} (\tilde{B} K_{\theta(k)} \tilde{C}) + N (\tilde{A} - I)^T W (\tilde{B} K_{\theta(k)} \tilde{C}) - T \\
\zeta_{22} &= (\tilde{B} K_{\theta(k)} \tilde{C})^T (\bar{P}_{\theta(k)} + N W) (\tilde{B} K_{\theta(k)} \tilde{C}) - R,
\end{aligned} \tag{4.27}$$

and

$$\bar{P}_{\theta(k)} = \sum_{j=1}^N \pi_{\theta(k)j} P_j \quad \text{for } j \in \mathbb{N}. \tag{4.28}$$

Proof. To guarantee the stability of (4.24), we choose the Lyapunov candidate functional:

$$\begin{aligned}
V(z(k), k) &= V_1(z(k), k) + V_2(z(k), k) + V_3(z(k), k) + V_4(z(k), k), \\
V_1(z(k), k) &= z^T(k) P_{\theta(k)} z(k), \\
V_2(z(k), k) &= \sum_{l=k-\tau_{\theta(k)}}^{k-1} z^T(l) R z(l), \\
V_3(z(k), k) &= \sum_{l=-N}^{-1} \sum_{m=k+l}^{k-1} \delta^T(m) W \delta(m), \\
V_4(z(k), k) &= \sum_{l=-N+2}^0 \sum_{m=k+l-1}^{k-1} z^T(m) R z(m),
\end{aligned} \tag{4.29}$$

and define,

$$\begin{cases} \delta(l) = z(l+1) - z(l) \\ \Omega = z(k) - z(k - \tau_{\theta(k)}) \end{cases} \tag{4.30}$$

Now, to investigate the stability, we evaluate the first order forward difference of the Lyapunov candidate functional.

$$\begin{aligned}
\Delta V_1 &= \mathbb{E} \left[V_1(z(k+1), k+1) | \theta(k) \right] - V_1(z(k), k) \\
&= z^T(k) \left[\left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right) - P_{\theta(k)} \right] z(k) \\
&\quad - z^T(k) \left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{B}K_{\theta(k)} \tilde{C} \right) \Omega \\
&\quad - \Omega^T \left(\tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right) z(k) + \Omega^T \left(\tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{B}K_{\theta(k)} \tilde{C} \right) \Omega.
\end{aligned}$$

By taking (4.25) into consideration, ΔV_1 can be rewritten as:

$$\begin{aligned}
\Delta V_1 &\leq z^T(k) \left[\left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right) - P_{\theta(k)} \right] z(k) + Nz^T(k) Uz(k) \\
&\quad + \sum_{l=k-N}^{k-1} \delta^T(l) W \delta(l) + \Omega^T \left(\tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{B}K_{\theta(k)} \tilde{C} \right) \Omega \\
&\quad + z^T(k) \left[T + T^T - \left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{B}K_{\theta(k)} \tilde{C} \right) - \right. \\
&\quad \left. \left(\tilde{B}K_{\theta(k)} \tilde{C} \right)^T \bar{P}_{\theta(k)} \left(\tilde{A} + \tilde{B}K_{\theta(k)} \tilde{C} \right) \right] \Omega. \tag{4.31}
\end{aligned}$$

We continue the same process for V_2 , V_3 , and V_4 as follows:

$$\begin{aligned}
\Delta V_2 &= \mathbb{E} \left[V_2(z(k+1), k+1) | \theta(k) \right] - V_2(z(k), k) \\
&= \pi_{\theta(k)\theta(k)} \left[\sum_{l=k-\tau_{\theta(k)}+1}^k - \sum_{l=k-\tau_{\theta(k)}}^{k-1} \right] z^T(l) R z(l)
\end{aligned}$$

$$\begin{aligned}
& + \sum_{j \neq \theta(k)} \pi_{\theta(k)j} \left[\sum_{l=k-\tau_j+1}^k - \sum_{l=k-\tau_{\theta(k)}}^{k-1} \right] z^T(l) R z(l) \\
& = \pi_{\theta(k)\theta(k)} \left(z^T(k) R z(k) - z^T(k - \tau_{\theta(k)}) R z(k - \tau_{\theta(k)}) \right) \\
& + \sum_{j \neq \theta(k)} \pi_{\theta(k)j} \left[\sum_{l=k-\tau_j+1}^k - \sum_{l=k-\tau_{\theta(k)}+1}^{k-1} \right] z^T(l) R z(l) - \sum_{j \neq \theta(k)} \pi_{\theta(k)j} z^T(k - \tau_{\theta(k)}) R z(k - \tau_{\theta(k)}) \\
& = z^T(k) R z(k) - z^T(k - \tau_{\theta(k)}) R z(k - \tau_{\theta(k)}) \\
& + \sum_{j \neq \theta(k)} \pi_{\theta(k)j} \left[\sum_{l=k-\tau_j+1}^{k-1} - \sum_{l=k-\tau_{\theta(k)}+1}^{k-1} \right] z^T(l) R z(l).
\end{aligned}$$

Note that,

$$\sum_{l=k-\tau_j+1}^{k-1} = \sum_{l=k-\tau_j+1}^{k-\tau_{\theta(k)}} + \sum_{l=k-\tau_{\theta(k)}+1}^{k-1} \quad \text{and} \quad \sum_{l=k-\tau_j+1}^{k-\tau_{\theta(k)}} z^T(l) R z(l) \leq \sum_{l=k-N+1}^{k-1} z^T(l) R z(l).$$

Hence,

$$\begin{aligned}
\Delta V_2 & \leq z^T(k) R z(k) - z^T(k - \tau_{\theta(k)}) R z(k - \tau_{\theta(k)}) \\
& + \sum_{j \neq \theta(k)} \pi_{\theta(k)j} \sum_{l=k-N+1}^{k-1} z^T(l) R z(l).
\end{aligned} \tag{4.32}$$

Next using $\sum_{j \neq \theta(k)} \pi_{\theta(k)j} = 1 - \pi_{\theta(k)\theta(k)} \leq 1 - \underline{\pi}$, with $\underline{\pi} = \min \{ \pi_{\theta(k)\theta(k)}, \theta(k) \in \mathbb{N} \}$,

we obtain that

$$\Delta V_2 \leq z^T(k) R z(k) - z^T(k - \tau_{\theta(k)}) R z(k - \tau_{\theta(k)}) + (1 - \underline{\pi}) \sum_{l=k-N+1}^{k-1} z^T(l) R z(l). \quad (4.33)$$

It is clear that

$$(1 - \underline{\pi}) \sum_{l=k-N+1}^{k-1} z^T(l) R z(l) \leq \sum_{l=k-N+1}^{k-1} z^T(l) R z(l),$$

hence,

$$\Delta V_2 \leq z^T(k) R z(k) - z^T(k - \tau_{\theta(k)}) R z(k - \tau_{\theta(k)}) + \sum_{l=k-N+1}^{k-1} z^T(l) R z(l). \quad (4.34)$$

Next, considering ΔV_3 , we have:

$$\begin{aligned} \Delta V_3 &= E[V_3(z(k+1), k+1) | \theta(k)] - V_3(z(k), k) \\ &= \sum_{l=-N}^{-1} \left[\sum_{m=k+l+1}^k - \sum_{m=k+l}^{k-1} \right] \delta^T(m) W \delta(m) = \sum_{l=-N}^{-1} [\delta^T(k) W \delta(k) - \delta^T(k+l) W \delta(k+l)] \\ &= N \left[(\tilde{A} - I) z(k) + (\tilde{B} K_{\theta(k)} \tilde{C}) z(k - \tau_{\theta(k)}) \right]^T W \left[(\tilde{A} - I) z(k) + (\tilde{B} K_{\theta(k)} \tilde{C}) z(k - \tau_{\theta(k)}) \right] \\ &\quad - \sum_{l=k-N}^{k-1} \delta^T(l) W \delta(l). \end{aligned} \quad (4.35)$$

Further,

$$\begin{aligned} \Delta V_4 &= E[V_4(z(k+1), k+1) | \theta(k)] - V_4(z(k), k) \\ &= \sum_{l=-N+2}^0 \left[\sum_{m=k+l}^k - \sum_{m=k+l-1}^{k-1} \right] z^T(m) R z(m) = \sum_{l=-N+2}^0 [z^T(k) R z(k) - z^T(k+l-1) R z(k+l-1)] \\ &= (N-1) z^T(k) R z(k) - \sum_{l=k-N+1}^{k-1} z^T(l) R z(l). \end{aligned} \quad (4.36)$$

Since

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4$$

by combining (4.31), (4.34)-(4.36), $\forall \theta(k) \in \mathbb{N}$, we obtain

$$\Delta V \leq \begin{bmatrix} z(k) \\ z(k - \tau_{\theta(k)}) \end{bmatrix}^T \Xi \begin{bmatrix} z(k) \\ z(k - \tau_{\theta(k)}) \end{bmatrix} < 0 \quad \forall \theta(k) \in \mathbb{N}.$$

where Ξ is given in (4.26), (4.27). This implies the robust MSS of the system. ■

Theorem 4.1 presents a sufficient condition for (4.24) to be robustly mean square stable for a given control law $K_{\theta(k)}$. Observe that (4.25)-(4.27) are neither linear nor bilinear in $K_{\theta(k)}$ and other unknown variables. Thus, it poses a major difficulty in controller design. In the following theorem, we shall apply Lemma 4.2 to matrix Ξ in order to obtain a parameterized bilinear matrix inequality (BMI), which is an equivalent condition for (4.26).

Theorem 4.2 The system given by (4.24) is robustly MSS if there exist matrices $K_{\theta(k)}$, $P_{\theta(k)}$ for $\theta(k) \in \mathbb{N}$, $R > 0$, $W > 0$, U , T , $\Phi \in \mathbb{R}^{(n+m) \times (n+m)}$, and real scaling parameters ε_l , $l \in \{1, 2, 3, 4\}$ such that

$$\begin{bmatrix} U & T \\ T^T & W \end{bmatrix} \geq 0, \quad (4.37)$$

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ * & \beta_{22} & \beta_{23} & \beta_{24} \\ * & * & \beta_{33} & 0 \\ * & * & * & \beta_{44} \end{bmatrix} < 0, \quad (4.38)$$

where,

$$\begin{aligned} \beta_{11} &= -P_{\theta(k)} + N(U + R) + T + T^T + \varepsilon_1 \tilde{A}_r^T \Phi + \varepsilon_1 \Phi^T \tilde{A}_r \\ \beta_{12} &= -T + \varepsilon_2 (\tilde{A}_r - I)^T + \varepsilon_1 \Phi^T (\tilde{B}_r K_{\theta(k)} \tilde{C}) \\ \beta_{13} &= \varepsilon_3 \tilde{A}_r^T \Phi - \varepsilon_1 \Phi^T \quad \beta_{14} = \varepsilon_4 (\tilde{A}_r - I)^T \\ \beta_{22} &= -R + \varepsilon_2 (\tilde{B}_r K_{\theta(k)} \tilde{C})^T + \varepsilon_2 (\tilde{B}_r K_{\theta(k)} \tilde{C}) \\ \beta_{23} &= \varepsilon_3 (\tilde{B}_r K_{\theta(k)} \tilde{C})^T \Phi \quad \beta_{24} = \varepsilon_4 (\tilde{B}_r K_{\theta(k)} \tilde{C})^T - \varepsilon_2 I \\ \beta_{33} &= \bar{P}_{\theta(k)} - \varepsilon_3 (\Phi + \Phi^T) \quad \beta_{44} = NW - 2\varepsilon_4 I \\ r &= \{1, 2, \dots, 2^v\} \quad \theta(k) \in \mathbb{N} \end{aligned} \quad (4.39)$$

Proof. The matrix Ξ can be rewritten as:

$$\begin{aligned} & \begin{bmatrix} -P_{\theta(k)} + NU + T + T^T + NR & -T \\ * & -R \end{bmatrix} \\ & + \begin{bmatrix} \tilde{A}^T & (\tilde{A} - I)^T \\ (\tilde{B}K_{\theta(k)}\tilde{C})^T & (\tilde{B}K_{\theta(k)}\tilde{C})^T \end{bmatrix} \begin{bmatrix} \bar{P}_{\theta(k)} & 0 \\ 0 & NW \end{bmatrix} \begin{bmatrix} \tilde{A} & (\tilde{B}K_{\theta(k)}\tilde{C}) \\ (\tilde{A} - I) & (\tilde{B}K_{\theta(k)}\tilde{C}) \end{bmatrix} < 0 \end{aligned} \quad (4.40)$$

Utilizing Lemma 4.2 and by choosing a special form for matrices Γ_{ij} and Ψ_j as,

$$\Gamma_{ij} = \begin{bmatrix} \varepsilon_1 \Phi & 0 \\ 0 & \varepsilon_2 I_{(n+m)} \end{bmatrix} \quad \Psi_j = \begin{bmatrix} \varepsilon_3 \Phi & 0 \\ 0 & \varepsilon_4 I_{(n+m)} \end{bmatrix}, \quad (4.41)$$

the result follows immediately from the convexity with respect to r in $\tilde{A}$, $\tilde{B}$. ■

Remark 4.2 Observe that for given ε_l , $l \in \{1, 2, 3, 4\}$, (4.38) and (4.39) represent a set of BMIs which can be solved with existing algorithms proposed in literature [37], [62]. Hence, Theorem 4.2 gives a parameterized BMI condition for solving the mode dependent static output feedback control problem.

4.4.2 An LMI Approach to Static Output Feedback Controller

In this sub-section, the case when the system output matrix $\tilde{C}$ is full row rank is considered. When $\tilde{C}$ is full row rank, there exists a nonsingular matrix $\tilde{T}$ such that $\tilde{C}\tilde{T}^{-1} = \begin{bmatrix} I_q & 0 \end{bmatrix}$. Hence the new state-space representation of the system (4.1) is given by:

$$\bar{A}(\eta_k) = \tilde{T}\tilde{A}(\eta_k)\tilde{T}^{-1}, \quad \bar{B}(\eta_k) = \tilde{T}\tilde{B}(\eta_k), \quad \text{and} \quad \bar{C} = \tilde{C}\tilde{T}^{-1} = \begin{bmatrix} I_q & 0 \end{bmatrix}. \quad (4.42)$$

It is clear that the closed loop system given in (4.24) is robust MSS if and only if

$$\bar{z}(k+1) = \bar{A}(\eta_k)\bar{z}(k) + \bar{B}(\eta_k)K_{\theta(k)}\bar{C}\bar{z}(k - \tau_{\theta(k)}) \quad (4.43)$$

is robust MSS.

In order to design the controller gain, first the following lemma is presented.

Lemma 4.3 Given matrices $W_1 > 0$, $Q_{\theta(k)} > 0$ and a scalar $\mathcal{G}$, it holds that,

$$Q_{\theta(k)} W_1^{-1} Q_{\theta(k)} \geq 2\mathcal{G}Q_{\theta(k)} - \mathcal{G}^2 W_1$$

and specifically by setting $\mathcal{G} = 1$,

$$Q_{\theta(k)} W_1^{-1} Q_{\theta(k)} \geq 2Q_{\theta(k)} - W_1. \quad (4.44)$$

Theorem 4.3 The system given by (4.43) is robustly MSS if there exist matrices

$K_{1\theta(k)}$, $Q_{\theta(k)}$ for $\theta(k) \in \mathbb{N}$, $R_{1\theta(k)} > 0$, $W_1 > 0$, $U_{1\theta(k)}$, and $T_{1\theta(k)}$ such that:

$$\begin{bmatrix} U_{1\theta(k)} & T_{1\theta(k)} \\ T_{1\theta(k)}^T & 2Q_{\theta(k)} - W_1 \end{bmatrix} \geq 0, \quad (4.45)$$

$$\begin{bmatrix} -\Upsilon & 0 & \omega_{\theta(k)}^T \bar{A} Q_{\theta(k)} & \omega_{\theta(k)}^T \bar{B} K_{1\theta(k)} \\ * & -\frac{1}{N} W_1 & (\bar{A} - I) Q_{\theta(k)} & \bar{B} K_{1\theta(k)} \\ * & * & \Pi_{\theta(k)} & -T_{1\theta(k)} \\ * & * & * & -R_{1\theta(k)} \end{bmatrix} < 0, \quad (4.46)$$

where,

$$\Pi_{\theta(k)} = -Q_{\theta(k)} + N U_{1\theta(k)} + T_{1\theta(k)} + T_{1\theta(k)}^T + N R_{1\theta(k)},$$

$$\Upsilon = \Psi^{-1} = \text{diag}\{Q_1, Q_2, \dots, Q_N\}, \text{ and } W_1 = W^{-1}. \quad (4.47)$$

Moreover, the controller gain $K_{\theta(k)}$ is given as,

$$K_{\theta(k)} = K_{1\theta(k)} \begin{bmatrix} Q_{11\theta(k)} \\ Q_{12\theta(k)} \end{bmatrix} \left(Q_{11\theta(k)}^2 + Q_{12\theta(k)}^2 \right)^{-1} \quad (4.48)$$

with $Q_{\theta(k)}$ partitioned as $Q_{\theta(k)} = \begin{bmatrix} Q_{11\theta(k)} & Q_{12\theta(k)} \\ Q_{12\theta(k)} & Q_{22\theta(k)} \end{bmatrix}$.

Proof. Following the same procedure as in the proof of Theorem 4.1, and utilizing its result, we have,

$$\Delta V \leq \begin{bmatrix} z(k) \\ z(k - \tau_{\theta(k)}) \end{bmatrix}^T \Xi \begin{bmatrix} z(k) \\ z(k - \tau_{\theta(k)}) \end{bmatrix} < 0 \quad \forall \theta(k) \in \mathbb{N}.$$

According to the definition of $\bar{P}_{\theta(k)}$ in (4.28), we can write it as,

$$\bar{P}_{\theta(k)} = \sum_{j=1}^N \pi_{\theta(k)j} P_j = \omega_{\theta(k)} \Psi \omega_{\theta(k)}^T$$

$$\omega_{\theta(k)} = \begin{bmatrix} \sqrt{\pi_{\theta(k)1}} I & \sqrt{\pi_{\theta(k)2}} I & \dots & \sqrt{\pi_{\theta(k)N}} I \end{bmatrix}, \quad \Psi = \text{diag}\{P_1, P_2, \dots, P_N\}. \quad (4.49)$$

Now, rewriting the matrix inequality $\Xi < 0$ as,

$$\begin{aligned} \Xi = & \begin{bmatrix} -P_{\theta(k)} + NU + T + T^T + NR & -T \\ * & -R \end{bmatrix} \\ & + \begin{bmatrix} \bar{A}^T \omega_{\theta(k)} & (\bar{A} - I)^T \\ (\bar{B}K_{\theta(k)}\bar{C})^T \omega_{\theta(k)} & (\bar{B}K_{\theta(k)}\bar{C})^T \end{bmatrix} \begin{bmatrix} \Psi & 0 \\ 0 & NW \end{bmatrix} \begin{bmatrix} \omega_{\theta(k)}^T \bar{A} & \omega_{\theta(k)}^T (\bar{B}K_{\theta(k)}\bar{C}) \\ (\bar{A} - I) & (\bar{B}K_{\theta(k)}\bar{C}) \end{bmatrix} < 0, \end{aligned}$$

we can convert it to the following form using Schur complements,

$$\begin{bmatrix} -\Psi^{-1} & 0 & \omega_{\theta(k)}^T \bar{A} & \omega_{\theta(k)}^T \bar{B}K_{\theta(k)}\bar{C} \\ * & -\frac{1}{N}W^{-1} & (\bar{A} - I) & \bar{B}K_{\theta(k)}\bar{C} \\ * & * & -P_{\theta(k)} + NU + T + T^T + NR & -T \\ * & * & * & -R \end{bmatrix} < 0. \quad (4.50)$$

Considering the above condition and also

$$\begin{bmatrix} U & T \\ T^T & W \end{bmatrix} \geq 0, \quad (4.51)$$

define,

$$Q_{\theta(k)} = P_{\theta(k)}^{-1}, \quad \forall \theta(k) \in \mathbb{N}, \quad Y = \Psi^{-1} = \text{diag}\{Q_1, Q_2, \dots, Q_N\}, \quad \text{and } W_1 = W^{-1}. \quad (4.52)$$

Pre- and post-multiplying the matrix (4.50) by $\text{diag}\{I, I, Q_{\theta(k)}, Q_{\theta(k)}\}$ and its transpose respectively, and also pre- and post-multiplying the other condition (4.51) by $\text{diag}\{Q_{\theta(k)}, Q_{\theta(k)}\}$ and its transpose and using Lemma 4.3, we get the final results and thus the theorem is proved. ■

Remark 4.3 It should be mentioned that, in the above proof, as $\bar{C} = \begin{bmatrix} I_q & 0 \end{bmatrix}$,

therefore $\bar{C}Q_{\theta(k)} = \begin{bmatrix} Q_{11\theta(k)} & Q_{12\theta(k)} \end{bmatrix}$, and $K_{1\theta(k)} = K_{\theta(k)} \begin{bmatrix} Q_{11\theta(k)} & Q_{12\theta(k)} \end{bmatrix}$.

Furthermore, the matrices $U_{1\theta(k)}$, $T_{1\theta(k)}$, $R_{1\theta(k)}$ are,

$$U_{1\theta(k)} = Q_{\theta(k)} U Q_{\theta(k)}, \quad T_{1\theta(k)} = Q_{\theta(k)} T Q_{\theta(k)}, \quad R_{1\theta(k)} = Q_{\theta(k)} R Q_{\theta(k)}. \quad (4.53)$$

Remark 4.4 Notice that the conditions (4.45)-(4.47), contrary to the conditions given in (4.37)-(4.39), are a set of LMIs which can be solved easily using MATLAB robust control toolbox.

4.5 State Feedback Controller Design

A mode dependent state feedback controller is defined by:

$$u(k) = K_{\theta(k)} z(k - \tau_{\theta(k)}) \quad (4.54)$$

with $K_{\theta(k)} \in \mathbb{R}^{m \times n}$, $\theta(k)$ indicating the mode of the system and $\tau_{\theta(k)}$ presenting the delay related to that mode. The closed loop system is then,

$$z(k+1) = \tilde{A}(\eta_k) z(k) + \tilde{B}(\eta_k) K_{\theta(k)} z(k - \tau_{\theta(k)}). \quad (4.55)$$

Theorem 4.4 The system (4.55) is robustly MSS if there exist matrices $K_{1\theta(k)}$, $Q_{\theta(k)}$

for $\theta(k) \in \mathbb{N}$, $R_{1\theta(k)} > 0$, $W_1 > 0$, $U_{1\theta(k)}$, and $T_{1\theta(k)}$ such that:

$$\begin{bmatrix} U_{1\theta(k)} & T_{1\theta(k)} \\ T_{1\theta(k)}^T & 2Q_{\theta(k)} - W_1 \end{bmatrix} \geq 0, \quad (4.56)$$

$$\begin{bmatrix} -\Upsilon & 0 & \omega_{\theta(k)}^T \tilde{A} Q_{\theta(k)} & \omega_{\theta(k)}^T \tilde{B} K_{1\theta(k)} \\ * & -\frac{1}{N} W_1 & (\tilde{A} - I) Q_{\theta(k)} & \tilde{B} K_{1\theta(k)} \\ * & * & \Pi_{\theta(k)} & -T_{1\theta(k)} \\ * & * & * & -R_{1\theta(k)} \end{bmatrix} < 0, \quad (4.57)$$

where,

$$\Pi_{\theta(k)} = -Q_{\theta(k)} + N U_{1\theta(k)} + T_{1\theta(k)} + T_{1\theta(k)}^T + N R_{1\theta(k)},$$

$$\Upsilon = \Psi^{-1} = \text{diag}\{Q_1, Q_2, \dots, Q_N\}, \text{ and } W_1 = W^{-1}. \quad (4.58)$$

Moreover, the controller gain $K_{\theta(k)}$ is given as,

$$K_{\theta(k)} = K_{1\theta(k)} Q_{\theta(k)}^{-1}. \quad (4.59)$$

Proof. The proof is similar to that of Theorem 4.3. ■

4.6 Dynamic Output Feedback Controller Design

A mode dependent dynamic output feedback controller is,

$$\begin{cases} x_c(k+1) = A_{c\theta(k)}x_c(k) + B_{c\theta(k)}y(k) \\ u(k) = C_{c\theta(k)}x_c(k) + D_{c\theta(k)}y(k) \end{cases} \quad (4.60)$$

with $A_{c\theta(k)}$, $B_{c\theta(k)}$, $C_{c\theta(k)}$, and $D_{c\theta(k)}$ having proper dimensions. By defining

$$\zeta(k) = \begin{bmatrix} z^T(k) & x_c^T(k) \end{bmatrix}^T, \quad (4.61)$$

the closed loop system is described by

$$\zeta(k+1) = A_{t\theta(k)}\zeta(k) + A_{d_t\theta(k)}\zeta(k - \tau_{\theta(k)}), \quad (4.62)$$

where

$$A_{t\theta(k)} = \begin{bmatrix} \tilde{A} & \tilde{B}C_{c\theta(k)} \\ 0 & A_{c\theta(k)} \end{bmatrix}, \quad A_{d_t\theta(k)} = \begin{bmatrix} \tilde{B}D_{c\theta(k)}\tilde{C} & 0 \\ B_{c\theta(k)}\tilde{C} & 0 \end{bmatrix}. \quad (4.63)$$

Theorem 4.5 The closed loop system defined in (4.62)-(4.63) is robustly MSS if there exist matrices $P_{\theta(k)} > 0$, $Q_{\theta(k)} > 0$, $\theta(k) \in \mathbb{N}$, $\mathbb{N} = \{1, 2, \dots, N\}$, $W > 0$, $M > 0$,

$R > 0$, U , T and $L_{\theta(k)}$ (to be defined in (4.66)) such that the following optimization

problem satisfies the criterion $\left[\text{tr}(\Psi\Upsilon) + \text{tr}(WM) \right] = 2(n+m)(N+1)$,

$$\min \left[\text{tr}(\Psi\Upsilon) + \text{tr}(WM) \right]$$

subject to:

$$\begin{bmatrix} U & T \\ T^T & W \end{bmatrix} \geq 0, \quad \begin{bmatrix} \Psi & I \\ I & \Upsilon \end{bmatrix} \geq 0, \quad \begin{bmatrix} W & I \\ I & M \end{bmatrix} \geq 0 \quad (4.64)$$

$$\begin{bmatrix} -\Upsilon & 0 & \omega_{\theta(k)}^T (\hat{A} + EL_{\theta(k)}H_1) & \omega_{\theta(k)}^T (EL_{\theta(k)}H_2) \\ * & -\frac{1}{N}M & (\hat{A} + EL_{\theta(k)}H_1 - I) & EL_{\theta(k)}H_2 \\ * & * & -P_{\theta(k)} + NU + T + T^T + NR & -T \\ * & * & * & -R \end{bmatrix} < 0, \quad (4.65)$$

with

$$\hat{A} = \begin{bmatrix} \tilde{A} & 0 \\ 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} \tilde{B} & 0 \\ 0 & I \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \quad H_2 = \begin{bmatrix} \tilde{C} & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$L_{\theta(k)} = \begin{bmatrix} D_{c\theta(k)} & C_{c\theta(k)} \\ B_{c\theta(k)} & A_{c\theta(k)} \end{bmatrix}. \quad (4.66)$$

Proof. The proof is similar to that of Theorem 4.3 using cone complementarity linearization algorithm. ■

Subsequently, the derived results are verified through numerical example and simulations.

4.7 Numerical Example

This section provides a numerical example to verify the effectiveness of the proposed approach.

Example 1. Consider a networked system described as,

$$\dot{x}(t) = Ax(t) + Bu(t)$$

with

$$A = \begin{bmatrix} 1.2 & -0.5 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \\ 0.01 \end{bmatrix}. \quad (4.67)$$

Due to the existence of network, the system information will experience uncertain time delays before reaching the controller and/or actuator. This uncertainty can be modeled into a state space representation using equations (4.2)-(4.18). Then by defining the sensor channel delay, which is known to the controller, as a Markov process, state, static output, and dynamic output feedback controllers are designed. Considering $h = 0.15$ second and $\eta_k \in [0.02, 0.15]$, the system state space matrices can be computed. Then, let the sensor channel delay belong to either $\tau_k \in [0, 0.1]$ ($\theta(k) = 1$), or $\tau_k \in [0.1, 0.15]$ ($\theta(k) = 2$), in which $\theta(k)$ is the Markov process with transition probability matrix $\begin{bmatrix} 0.5 & 0.5 \\ 0.7 & 0.3 \end{bmatrix}$. Therefore if the sensor channel delay is within the first interval, controller K_1 , or else K_2 is applied to the system.

4.7.1 Static Output Feedback Controller (BMI Approach)

For the static output feedback controller design, the output is defined as

$$y(t) = [35 \quad 25 \quad 150]x(t - \tau_k), \quad (4.68)$$

Applying Theorem 4.2, the controller gains are computed,

$$K_{\theta(k)} = \begin{cases} -2.2197 & \text{for } \theta(k) = 1 \\ -3.4399 & \text{for } \theta(k) = 2 \end{cases} \quad (4.69)$$

The state trajectories of the closed loop system for the initial condition $x(0) = [0.5 \quad 0.1 \quad -0.8]^T$, and the control input are shown in Figure 4.2 and Figure 4.3, respectively.

Figure 4.2 State trajectories of the closed loop system.

Figure 4.3 Control input.

4.7.2 Static Output Feedback Controller (LMI Approach)

For the same system under the same conditions, by applying the LMI design method given in Theorem 4.3, the controller gains are obtained as:

$$K_{\theta(k)} = \begin{cases} -0.6538 & \text{for } \theta(k) = 1 \\ -1.8499 & \text{for } \theta(k) = 2 \end{cases} \quad (4.70)$$

The state trajectories and control input are given in Figure 4.4 and Figure 4.5, respectively.

Figure 4.4 State trajectories of the closed loop system.

Figure 4.5 Control input.

4.7.3 State Feedback Controller

The controller gains designed for the same system using the conditions obtained in Theorem 4.4, are

$$K_{\theta(k)} = \begin{cases} [-36.852 & -26.112 & -86.769 & -1.485] & \text{for } \theta(k)=1 \\ [-43.819 & -30.383 & -103.512 & -2.196] & \text{for } \theta(k)=2 \end{cases} \quad (4.71)$$

The state trajectories and control input are indicated in Figure 4.6 and Figure 4.7, respectively.

Figure 4.6 State trajectories of the closed loop system.

Figure 4.7 Control input.

4.7.4 Dynamic Output Feedback Controller

For the dynamic output feedback case, the simulation is much harder than the previous cases and needs more time as the optimization problem is required to be done iteratively. The iterations stop when the criterion $[\text{tr}(\Psi\Upsilon) + \text{tr}(WM)] - 2(n+m)(N+1) \leq \varepsilon$, where ε is a pre-defined sufficient small positive scalar, is satisfied. The controller state space matrices are obtained as follows and the state trajectories of the controller and the closed loop system are shown in Figure 4.8, and Figure 4.9, respectively.

$$\text{for } \theta(k)=1: \begin{cases} A_{c1} = \begin{bmatrix} 0.3573 & 8.5667 & 14.0043 & 8.4694 \\ -1.953 & -0.1006 & 3.7268 & 1.5217 \\ 0.3886 & 23.4649 & 18.8156 & 16.2986 \\ 2.8549 & -23.2842 & -26.3316 & -19.3522 \end{bmatrix} \\ B_{c1} = \begin{bmatrix} 0.024 \\ 0.781 \\ -0.519 \\ -0.813 \end{bmatrix} \end{cases} \quad C_{c1} = [-0.1164 \quad 0.2837 \quad -0.6127 \quad -0.8165] \quad (4.72)$$

$$\text{for } \theta(k)=2: \begin{cases} A_{c2} = \begin{bmatrix} 0.3573 & 8.5667 & 14.0043 & 8.4694 \\ -1.953 & -0.1006 & 3.7268 & 1.5217 \\ 0.3886 & 23.4649 & 18.8156 & 16.2986 \\ 2.8549 & -23.2842 & -26.3316 & -19.3522 \end{bmatrix} \\ B_{c2} = \begin{bmatrix} 0.024 \\ 0.781 \\ -0.519 \\ -0.813 \end{bmatrix} \end{cases} \quad C_{c2} = [-0.1164 \quad 0.2837 \quad -0.6127 \quad -0.8165] \quad (4.73)$$

Figure 4.8 State trajectories of the controller.

Figure 4.9 State trajectories of the closed loop system.

4.8 Conclusion

In this chapter, a new control design for uncertain networked control systems with bounded random time delays was proposed. Contrary to the previous methods, which considered both the delays in sensor and actuator channels as uncertainty, the proposed method considered the two delays separately that led to less conservative results. In our approach, the methods such as Jordan canonical form, and the Markov parameters were used to model the NCSs with time delays. Sufficient conditions in terms of BMIs and LMIs were given to guarantee the stability of the system with mode-dependent static output feedback controller, using the Lyapunov-Krasovskii

method. Mode dependent state feedback and dynamic output feedback controllers were designed for the modeled NCS, as well. Validating our results was carried out by simulation studies for a numerical example.

Chapter 5

COMPENSATION FOR OUTAGES IN NETWORKED CONTROL SYSTEMS

5.1 Background

As discussed in the previous chapters, a networked control system is a system whose elements are connected and controlled through a shared digital network. Consequently, there are many advantages arising from the use of the network which result in a broad range of applications of these systems. However, besides the benefits, there exist some disadvantages which are particularly apparent when wireless networks are employed in the control systems. One of the important challenges is network-induced delay that was discussed in chapter 4, and another one is packet dropout which is considered in this chapter. Packet dropout is a direct effect of

network traffic congestions and limited network reliability and can degrade the control performance of the NCSs as the controller and/or actuator needs to make decisions with incomplete information [63]. Hence, modeling and control of networked systems with package losses are of importance. Therefore, we provide a compensation method in this chapter to counteract packet dropout effects on system performance.

The idea of this chapter, as well as some previous works [46], [63], is to design a compensator which acts when the packages are not received at their destination. In our study, the compensator is a filter that is co-located with the actuator node at the plant side and suggests a replacement command in the event of packet dropouts. Some motivations and applications are to be mentioned.

5.2 Compensation Algorithms

First, in order to understand the idea of compensation algorithms better, consider the control setup in Figure 5.1. The signals and transfer function of the system are then defined. $d(k)$ here indicates the disturbance entering the system. $u_c(k)$ and $\hat{u}(k)$ are the commands produced by the controller and compensator respectively.

Figure 5.1 NCS with Outage Compensator [46].

The final control action applied to the main plant is decided by the switching mechanism in the compensator. It follows the form below,

$$u(k) = \begin{cases} u_c(k) & \text{Control command is received} \\ \hat{u}(k) & \text{Control command is lost} \end{cases} \quad (5.1)$$

System and controller outputs are given by [46],

$$y(k) = P(q)(u(k) + d(k)) = \frac{B(q)}{A(q)}(u(k) + d(k)) \quad (5.2)$$

$$u_c(k) = C(q)(r(k) - y(k)) = \frac{S(q)}{R(q)}(r(k) - y(k)), \quad (5.3)$$

where $B(q)$, $A(q)$, $S(q)$, and $R(q)$ are polynomials of appropriate degree and q here denotes the one step forward shift operator.

5.2.1 Zero Compensator

The simplest version of outage compensators is zero compensator. In the event of packet dropout, this compensator applies zero signal to the system.

$$\hat{u}(k) = 0. \quad (5.4)$$

5.2.2 Hold Compensator

Another version is hold compensator that keeps the last received control command value when packages are lost.

$$\hat{u}(k) = \hat{u}(k-1). \quad (5.5)$$

5.2.3 General Predictive Outage Compensator

General Predictive Outage Compensator (POC) introduced by [46] is mainly a generalization of common compensation algorithms including zero, and constant holding compensation. This POC monitors the control signal and uses a signal model to extrapolate the signal in the event of a communication outage [46].

The POC listens to the control signal sent from the controller. If the signal is received the POC passes the control signal to the actuator and updates its own internal states using the received signal. In the case that no control action is received, the POC uses its internal model to extrapolate the control signal based on the signal model and previous received data [46]. In this sub-section, detailed procedure which is based on [46] is described.

The control signal estimate in the general predictive outage compensator is,

$$\hat{u}(k) = \frac{E(q)}{F(q)} \hat{d}(k) \quad (5.6)$$

where $E(q)$, and $F(q)$ are appropriate dimensioned polynomials and $\hat{d}(k)$ is an estimate of $d(k)$. Now, for the sake of simplicity and without loss of generality, $r(k) = 0$ is assumed. Thus, the equations (5.2) and (5.3) can be rewritten as,

$$(A(q)R(q) + B(q)S(q))y(k) = B(q)R(q)d(k) \quad (5.7)$$

$$(A(q)R(q) + B(q)S(q))u_c(k) = -B(q)S(q)d(k) \quad (5.8)$$

$$\Rightarrow u_c(k) = -\frac{B(q)S(q)}{A(q)R(q) + B(q)S(q)} d(k) = \frac{E(q)}{F(q)} d(k), \quad (5.9)$$

for the case when communication is working and the control signal is received.

However, when the communication is lost we have,

$$y(k) = P(q)(\hat{u}(k) + d(k)) \quad (5.10)$$

$$\Rightarrow F(q)A(q)y(k) = B(q)E(q)\hat{d}(k) + B(q)F(q)d(k). \quad (5.11)$$

During the time that POC listens to the received control signal, it uses the signal to filter out an estimate of the disturbance as below,

$$\hat{d}(k) = \frac{F(q)}{E(q)} u_c(k), \quad (5.12)$$

$$\Rightarrow q^{-n_F} E(q) \hat{d}(k) = q^{-n_F} F(q) u_c(k), \quad (5.13)$$

$$\begin{aligned} \Rightarrow e_0 \hat{d}(k + (n_E - n_F)) &= u_c(k) + \dots + f_{n_F} u_c(k - n_F) \\ &\quad - e_1 \hat{d}(k - n_F - 1 + n_E) - \dots - e_{n_E} \hat{d}(k - n_F). \end{aligned} \quad (5.14)$$

Therefore, at the time of communication outage, the compensator holds the last known disturbance estimate $\hat{d}(k)$, which is indicated by $\bar{d}$, and generates the control command by,

$$\hat{u}(k) = \frac{E(q)}{F(q)} \bar{d} \quad (5.15)$$

$$\Rightarrow q^{-n_F} F(q) \hat{u}(k) = q^{-n_F} E(q) \bar{d} \quad (5.16)$$

$$\Rightarrow \hat{u}(k) = (e_0 + \dots + e_{n_E}) \bar{d} - f_1 \hat{u}(k-1) - \dots - f_{n_F} \hat{u}(k - n_F). \quad (5.17)$$

According to [46], then, the nominal control signal and the POC model is redefined in the following way,

$$u_c(k) = \frac{q^{n_F-1}}{F(q)} v(k), \quad \left(q^{n_F-1} v(k) = E(q) d(k) \right) \quad (5.18)$$

$$\hat{u}(k) = \frac{q^{n_F-1}}{F(q)} \hat{v}(k), \quad \left(q^{n_F-1} \hat{v}(k) = E(q) \hat{d}(k) \right). \quad (5.19)$$

The signal $v(k)$ in above equations is introduced as an innovation signal that contains the terms related to disturbance. The signal $\hat{v}(k)$ has a similar interpretation for POC generated control command [46].

Finally, the generated control signal by the compensator can be shown as,

$$\hat{u}(k+1) = -f_1 \hat{u}(k) - \dots - f_{n_F} \hat{u}(k - n_F + 1) + \hat{v}(k), \quad (5.20)$$

where f_i are the coefficients of $F(q)$.

5.3 Proposed Algorithm

As mentioned earlier, there are many applications where a plant must be controlled by a remote controller which connects to the plant by a communication network. Assuming that the actuator has some computing power, a method to reduce the effect of packet dropouts on the performance of NCSs is to employ compensators. The proposed compensator in this study is a filter which is co-located with the actuator node at the plant side. If the information packets are received at the actuator, they are passed to the plant; else the filter generates a replacing control command and sends it to the actuator to be applied to the process. Hence, the mechanism is a switching one with two cases that can be modeled as a Markov jump linear system with two modes.

The control law which is applied to the system is either the one received from the controller in less than one sampling interval or the one that is produced by the filter for compensation. Therefore, we have:

$$u(k) = \begin{cases} u_c(k) & \text{no packet loss} \\ u_f(k) & \text{packet loss} \end{cases} \quad (5.21)$$

with $u_c(k)$ as the command produced by the controller and $u_f(k)$ the filter produced control command. Now the two potential cases of the closed loop system can be

modeled into a Markov jump linear system with two modes. We introduce a discrete time homogenous Markov chain process $\{\theta(k)\}$ which takes its values within the set $S_1 = \{1, 2\}$, and its stationary transition probability matrix is given by

$$\Lambda = [\pi_{ij}] = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix},$$

$$\pi_{ij} := \mathbb{P}\{\theta_{l+1} = j \mid \theta_l = i\}, \pi_{ij} \geq 0, \forall i, j, i, j \in \mathbb{N}, \quad (5.22)$$

where p and q , are respectively called the failure and recovery rate, $p, q > 0$, and they are both in $(0, 1)$. Moreover, arrival and loss of the packets, which are assumed to be known to the filter, are indicated by $\theta(k) = 2$ and $\theta(k) = 1$ respectively. The initial distribution for $\theta(0)$ is denoted by $v = \{v_1, \dots, v_N\}$.

The plant dynamical description is given by

$$\begin{cases} x(k+1) = Ax(k) + B_1 u(k) + B_2 w(k) \\ z(k) = Cx(k) \end{cases} \quad (5.23)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $z(k) \in \mathbb{R}^p$ is the output signal, and $w(k) \in \mathbb{R}^g$ is the process noise which is assumed to be white with normal probability distribution; i.e. $\mathbb{P}\{w\} \sim \mathcal{N}(0, Q)$. In practice the process noise covariance $Q \geq 0$ might change with each step or measurement, however here we assume that it's constant.

The controller considered for stabilization of the plant is a mode independent state feedback controller defined by

$$u_c(k) = K_c x(k), \quad (5.24)$$

and the filter is described by

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + B_1 u(k) + K(u(k) - u_f(k)) \\ u_f(k) = K_f \hat{x}(k) \end{cases} \quad (5.25)$$

Now, the closed loop system in each mode can be defined as follows,

First Mode: In this case $\theta(k) = 1$ meaning that the packet is lost and $u(k) = u_f(k)$:

$$\begin{cases} x(k+1) = Ax(k) + B_1 K_f \hat{x}(k) + B_2 w(k) \\ \hat{x}(k+1) = A\hat{x}(k) + B_1 K_f \hat{x}(k) \\ z(k) = Cx(k) \end{cases} \quad (5.26)$$

Defining,
$$\zeta(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T, \quad (5.27)$$

the closed loop system is written as

$$\begin{cases} \zeta(k+1) = \begin{bmatrix} A & B_1 K_f \\ 0 & A + B_1 K_f \end{bmatrix} \zeta(k) + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} w(k) \\ z(k) = \begin{bmatrix} C & 0 \end{bmatrix} \zeta(k) \end{cases} \quad (5.28)$$

Second Mode: In this case $\theta(k) = 2$ meaning that the packet is received and

$$u(k) = u_c(k):$$

$$\begin{cases} x(k+1) = (A + B_1 K_c) x(k) + B_2 w(k) \\ \hat{x}(k+1) = A \hat{x}(k) + B_1 K_c x(k) + K (K_c x(k) - K_f \hat{x}(k)) \\ z(k) = C x(k) \end{cases} \quad (5.29)$$

$$\Rightarrow \begin{cases} \zeta(k+1) = \begin{bmatrix} A + B_1 K_c & 0 \\ (B_1 + K) K_c & A - K K_f \end{bmatrix} \zeta(k) + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} w(k) \\ z(k) = [C \quad 0] \zeta(k) \end{cases} \quad (5.30)$$

Finally, the closed loop systems (5.28) and (5.30) can be expressed in terms of Markov jump parameter $\theta(k) \in \{1, 2\}$ as follows:

$$\Sigma: \begin{cases} \zeta(k+1) = \hat{A}_{\theta(k)} \zeta(k) + G_{\theta(k)} w(k) \\ z(k) = \hat{C}_{\theta(k)} \zeta(k) \end{cases} \quad (5.31)$$

where

$$\begin{aligned} \hat{A}_{\theta(k)} &= \begin{bmatrix} A + (\theta(k) - 1) B_1 K_c & (2 - \theta(k)) B_1 K_f \\ (\theta(k) - 1) (B_1 + K) K_c & A - (\theta(k) - 1) K K_f + (2 - \theta(k)) B_1 K_f \end{bmatrix} \\ G_{\theta(k)} &= \begin{bmatrix} B_2 \\ 0 \end{bmatrix}, \quad \hat{C}_{\theta(k)} = [C \quad 0] \end{aligned} \quad (5.32)$$

Now, having the dynamical model of the system, the controller and filter gains can be designed. Remembering the stability definitions and mean square stabilizability test discussed in chapter 3, we fulfill the control and filter design in the following. The H_2 -control problem is considered as well, due to the existence of the process noise.

Lemma 5.1 [55]. Define a set,

$$\Psi = \left\{ W = (W_1, \dots, W_N); \text{for } j \in \mathbb{N}, W_j = \begin{bmatrix} W_{j1} & W_{j2} \\ W_{j2}^* & W_{j3} \end{bmatrix} \geq 0, W_{j1} > 0, \right. \\ \left. 0 \leq W_{j1} - \sum_{i=1}^N \pi_{ij} \left(A_i W_{i1} A_i^* + B_i W_{i2}^* A_i^* + A_i W_{i2} B_i^* + B_i W_{i3} B_i^* + v_i G_i G_i^* \right) \right\}, \quad (5.33)$$

and

$$\lambda(W) = \sum_{i=1}^N \text{tr} \left(\begin{bmatrix} C_i^* C_i & 0 \\ 0 & D_i^* D_i \end{bmatrix} \begin{bmatrix} W_{i1} & W_{i2} \\ W_{i2}^* & W_{i3} \end{bmatrix} \right). \quad (5.34)$$

If there exists $W = \{W_1, \dots, W_N\} \in \Psi$ such that

$$\lambda(W) = \min \{ \lambda(V); V \in \Psi \}, \quad (5.35)$$

then for a set of stabilizing controllers as $F = (F_1, \dots, F_N)$ with $F_i = W_{i2}^* W_{i1}^{-1}$, $i \in \mathbb{N}$,

we have that the system defined below,

$$G_{cl} = \begin{cases} x(k+1) = A_{\theta(k)} x(k) + B_{\theta(k)} F_{\theta(k)} x(k) + G_{\theta(k)} w(k) \\ z(k) = C_{\theta(k)} x(k) + D_{\theta(k)} F_{\theta(k)} x(k) \\ x(0) = x_0, \theta(0) = \theta_0 \end{cases} \quad (5.36)$$

with known x_0 and θ_0 , is MSS and

$$\|G_{cl}\|_2^2 \leq \lambda(W). \quad (5.37)$$

Theorem 5.1 The system defined by (5.31) and (5.32) is MSS and $\|\Sigma\|_2^2 \leq \lambda(W)$, with

$$\lambda(W) = \sum_{i=1}^2 \text{tr} \left(\hat{C}_i W_i \hat{C}_i^* \right) \text{ if there exists } W = (W_1, W_2) \in \hat{\Psi}, \text{ where}$$

$$\hat{\Psi} = \left\{ W = (W_1, W_2); \text{ for } j \in S_1, W_j = \begin{bmatrix} W_{j1} & W_{j2} \\ W_{j2}^* & W_{j3} \end{bmatrix} > 0, W_{j1} > 0, \right.$$

$$\left. 0 < W_j - \sum_{i=1}^2 \pi_{ij} \left(\hat{A}_i W_i \hat{A}_i^* + v_i G_i G_i^* \right) \right\} \quad (5.38)$$

$$\text{such that,} \quad \lambda(W) = \min \left\{ \lambda(V); V \in \hat{\Psi} \right\}. \quad (5.39)$$

Proof. The theorem follows directly from Lemma 5.1. ■

Rewriting the stability condition of Theorem 5.1 using Schur complements results in,

$$\begin{bmatrix} W_j & \hat{A}_1 & \hat{A}_2 & G \\ * & \frac{1}{\pi_{1j}} W_1^{-1} & 0 & 0 \\ * & * & \frac{1}{\pi_{2j}} W_2^{-1} & 0 \\ * & * & * & \frac{1}{\pi_{1j} v_1 + \pi_{2j} v_2} I \end{bmatrix} > 0, \text{ for } j \in S_1 \quad (5.40)$$

where $G_1 = G_2 = G$. Now, considering $M_j = W_j^{-1}$ for $j \in S_1$, and pre and post-

multiplying (5.40) with $\text{diag}\{M_j, I, I, I\}$ and its transpose respectively, we get

$$\begin{bmatrix} M_j & M_j \hat{A}_1 & M_j \hat{A}_2 & M_j G \\ * & \frac{1}{\pi_{1j}} M_1 & 0 & 0 \\ * & * & \frac{1}{\pi_{2j}} M_2 & 0 \\ * & * & * & \frac{1}{\pi_{1j} v_1 + \pi_{2j} v_2} I \end{bmatrix} > 0, \text{ for } j \in S_1. \quad (5.41)$$

Remark 5.1 Since $\hat{A}_1, \hat{A}_2$ include unknown controller parameters, the condition (5.41) is not an LMI. In order to be able to obtain the controller and filter gains, we consider a possible solution for the problem:

1. Take the filter-produced control command gain to be equal to the controller gain, i.e., $K_f = K_c$.

2. Define $M_1 = \begin{bmatrix} M_{11} & 0 \\ 0 & M_{13} \end{bmatrix}$ and $M_2 = \begin{bmatrix} M_{21} & 0 \\ 0 & M_{13} \end{bmatrix}$. Then,

$$\Rightarrow M_1 \hat{A}_1 = \begin{bmatrix} M_{11} A & M_{11} B_1 K_c \\ 0 & M_{13} A + M_{13} B_1 K_c \end{bmatrix}, \quad M_1 \hat{A}_2 = \begin{bmatrix} M_{11} A + M_{11} B_1 K_c & 0 \\ M_{13} B_1 K_c + N K_c & M_{13} A - N K_c \end{bmatrix}$$

$$M_2 \hat{A}_1 = \begin{bmatrix} M_{21} A & M_{21} B_1 K_c \\ 0 & M_{13} A + M_{13} B_1 K_c \end{bmatrix}, \quad M_2 \hat{A}_2 = \begin{bmatrix} M_{21} A + M_{21} B_1 K_c & 0 \\ M_{13} B_1 K_c + N K_c & M_{13} A - N K_c \end{bmatrix}.$$

where $N = M_{13} K$. Now, the algorithm below leads to a solution to the problem.

Step 1: Under the ideal channel, we find an initial value for K_c . In this case we have,

$$\begin{cases} x(k+1) = (A + B_1 K_c) x(k) + B_2 w(k) \\ z(k) = C x(k) \end{cases} \quad (5.42)$$

For this system, the optimal H_2 controller gain can be easily obtained as follows,

$$\min \text{trace}(T), \text{ subject to: } \begin{cases} \begin{bmatrix} T & CP \\ * & P \end{bmatrix} > 0 \\ \begin{bmatrix} P & AP + B_1 Z & B_2 \\ * & P & 0 \\ * & * & I \end{bmatrix} > 0 \end{cases} \quad (5.43)$$

where $Z = K_c P$. Solving the optimization problem (5.43), we get an initial value for

K_c using $K_c = ZP^{-1}$.

Step 2: Having the derived initial value for K_c , the filter gain $K = M_{13}^{-1}N$ can be found by solving the optimization problem:

$$\min: \sum_{j=1}^2 \text{tr}(\tilde{W}_j), \text{ subject to: } \begin{bmatrix} \tilde{W}_j & \tilde{C}_j \\ * & M_j \end{bmatrix} > 0, \begin{bmatrix} M_j & M_j \hat{A}_1 & M_j \hat{A}_2 & M_j G \\ * & \frac{1}{\pi_{1j}} M_1 & 0 & 0 \\ * & * & \frac{1}{\pi_{2j}} M_2 & 0 \\ * & * & * & \frac{1}{\pi_{1j} v_1 + \pi_{2j} v_2} I \end{bmatrix} > 0, \text{ for } j \in S_1, \quad (5.44)$$

Step 3: In this step using the derived value for K , M_1 and M_2 we repeat solving the LMIs (5.44) to find a new value for K_c .

Repeat the last two steps to obtain the converged values for controller and filter gains.

5.4 Numerical Example

Consider a networked system given by

$$\begin{cases} x(k+1) = Ax(k) + B_1u(k) + B_2w(k) \\ z(k) = Cx(k) \end{cases}$$

where,

$$A = \begin{bmatrix} 1.1972 & -0.0885 & -0.1530 & 0.1621 \\ 0 & 1.1618 & 0 & -0.1477 \\ 0 & 0 & 0.8607 & 0.0014 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0085 \\ -0.0141 \\ 0 \\ 1 \end{bmatrix} \quad (5.45)$$

$$B_2 = B_1 \quad C = [1 \quad 0 \quad -1 \quad 1]$$

with $p = 0.2$, $q = 0.7$, $v_1 = 0.4$, and $v_2 = 0.6$. First, assuming that no packet dropouts occur in the channels, we design an H_2 -controller which stabilizes the system and minimizes the noise effect. This controller is calculated using (5.43) as follows,

$$K = [-5.2565 \quad -1.1352 \quad 2.4546 \quad -0.5151]. \quad (5.46)$$

Figure 5.2 shows the output response of the system with designed controller.

Next, assuming that there are random packet losses in the controller channel, which are governed by a Markov process, we study the performance of zero, hold, and general POC compensators and compare their performances with the proposed method.

To investigate the performance of previous compensators, the NCS with the earlier designed controller is applied to channels with Markov packet dropouts under zero, hold, and general POC compensation algorithms and the results are compared with the proposed approach. Following the algorithm explained in Remark 5.1, the controller and the filter gain for the presented method are calculated as

$$K_c = [-5.2918 \quad -1.1542 \quad 1.4180 \quad -0.515], K = \begin{bmatrix} -0.3601 \\ 0.4404 \\ -0.0017 \\ 0 \end{bmatrix}. \quad (5.47)$$

The output response for each compensation schemes is indicated in Figure 5.3.

Figure 5.2 Output response.

Figure 5.3 Output responses.

According to the Figure 5.3, zero and hold compensators give a similar performance, while the general POC produces a better result. However, it can be observed that in the presence of noise with random packet dropouts, the proposed method offers the best performance.

5.5 Conclusion

In this chapter, a new method for compensation of packet dropouts in networked control systems was proposed. The compensation procedure was to locate a filter with

simple computational power, with the actuator at the plant side which generates a control command when no signal is received. Markov jump linear system and H_2 -control approaches were employed to model the NCSs with packet loss. An algorithm was then proposed for designing the controller and the filter gains, and finally, simulations were accomplished to verify the results. The simulations showed that the performance of the proposed filter for noisy systems with random packet dropouts is more efficient.

Chapter 6

APPLICATION TO INVERTED PENDULUM

6.1 Introduction

Inverted Pendulum (IP) system is among the most difficult and challenging problems in the area of control systems. It is a highly nonlinear, multivariable and inherently unstable dynamic system that can be treated to be linear, without much error, for quite a wide range of variation. Hence, it is employed most of times to verify the performance and effectiveness of control techniques. One of the IP system types is rotary inverted pendulum indicated in Figure 6.1 that has a pendulum attached to a rotary arm, which requires a continuous correction mechanism to stay upright. This correction mechanism can be fulfilled through designing a controller. Thus, it's essential to model the IP and linearize the model for the operating range.

Figure 6.1 Rotary inverted pendulum.

The rotary inverted pendulum used in this study is KRi PP-300 which consists of a short arm that rotates in the horizontal plane, driven by a motor mounted vertically. A pendulum rod is fixed at one end of the arm, moving in the vertical plane.

6.2 Mathematical Modeling

The nonlinear dynamical model of the inverted pendulum in the upright position is as below [64],

$$\begin{bmatrix} J_0 + mL^2 + ml^2 \sin^2 \beta & -mLl \cos \beta \\ -mLl \cos \beta & J_1 + ml^2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix} +$$

$$\begin{bmatrix} C_0 + \frac{K_t K_b}{R_a} + \frac{1}{2} ml^2 \dot{\beta} \sin 2\beta & mLl \dot{\beta} \sin \beta + \frac{1}{2} ml^2 \dot{\alpha} \sin 2\beta \\ -\frac{1}{2} ml^2 \dot{\alpha} \sin 2\beta & C_1 \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} +$$

$$\begin{bmatrix} 0 \\ -mgl \sin \beta \end{bmatrix} = \begin{bmatrix} \frac{K_t K_u}{R_a} \\ 0 \end{bmatrix} u \quad (6.1)$$

where m is the effective mass of the pendulum, l the length of the rod, α the arm angle about the z-axis, taken from the positive x-axis, β the pendulum angle, taken from the upward vertical axis, J_0 the moment of inertia of the arm, J_1 the moment of inertia of the pendulum, C_0 the viscous friction co-efficient of arm, C_1 the viscous friction co-efficient of pendulum, L the physical distance between pivot of pendulum and axis of rotation of arm, K_t the motor torque constant, K_b the motor back EMF constant, K_u the motor driver amplifier gain, and R_a the motor armature coil resistance.

The model given in (6.1) can also be described with the pendulum in the downward position. However, the focus in our study is on the upward model.

Next, we consider the linearization of the given model by assigning that

$$\dot{\alpha} = 0, \beta = 0, \text{ and } \dot{\beta} = 0 \quad (6.2)$$

which leads to,

$$\begin{bmatrix} J_0 + mL^2 & -mLl \\ -mLl & J_1 + ml^2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix} + \begin{bmatrix} C_0 + \frac{K_t K_b}{R_a} & 0 \\ 0 & C_1 \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -mgl \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \frac{K_t K_u}{R_a} \\ 0 \end{bmatrix} u. \quad (6.3)$$

To write the model in a state space form, define

$$a = J_0 + mL^2, \quad b = ml^2, \quad c = mL, \quad d = C_0 + \frac{K_t K_b}{R_a}$$

$$e = \frac{K_t K_u}{R_a}, \quad f = J_1 + ml^2, \quad h = mgl \quad (6.4)$$

which leads to,

$$\dot{x} = Ax + Bu$$

where,

$$x = \begin{bmatrix} \dot{\alpha} \\ \beta \\ \dot{\beta} \end{bmatrix}, \quad A = \frac{1}{af - c^2} \begin{bmatrix} -df & ch & -cC_1 \\ 0 & 0 & af - c^2 \\ -cd & ah & -aC_1 \end{bmatrix}, \quad B = \frac{1}{af - c^2} \begin{bmatrix} ef \\ 0 \\ ce \end{bmatrix}. \quad (6.5)$$

6.3 System Identification

For identification, the system is excited with a known test signal and the output response data are collected. It has been shown that certain patterns of test signals produce consistently good results. For example, step signal that drops to zero after a finite duration provides good excitation for the transient response and allows the system to exhibit an unforced oscillation response. Based on this procedure, the derived model is [64],

$$\dot{x} = \begin{bmatrix} -1.3705 & 3.8557 & -0.01242 \\ 0 & 0 & 1 \\ -0.9367 & 51.5747 & -0.1661 \end{bmatrix} x + \begin{bmatrix} 0.5721 \\ 0 \\ 0.3910 \end{bmatrix} u. \quad (6.6)$$

On the other hand, when working with real system, in addition to the pendulum, the encoder and PWM generator are also involved in the collected data sets. Therefore, their model should also be considered for the control design process. The overall model, which is obtained using the same procedure as described above, is,

$$\dot{x} = \begin{bmatrix} -4.218 & 2.661 & 1.604 \\ 0.9107 & -25.9 & -15.61 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0.257 \\ -0.05549 \\ 0 \end{bmatrix} u. \quad (6.7)$$

The PWM input, pendulum angle and arm speed output data collected for identification are shown in the following figure.

Figure 6.2 Input and output data collected for identification.

6.4 Controller Design

Based on the identified model in the previous section, we design a static output feedback controller for the inverted pendulum using the approach provided in chapter 4. The conditions considered here, are: $h = 0.1$ the sampling interval, $\eta_k \in [0.01, 0.1]$ the controller channel delay, $\tau_k \in [0, 0.15]$ the sensor channel delay. Two Markov modes for this delay are defined, $\tau_k \in [0, 0.1]$ ($\theta(k) = 1$) and $\tau_k \in [0.1, 0.15]$ ($\theta(k) = 2$). The resulting controller gains are as follows,

$$K_{\theta(k)} = \begin{cases} -1.185 & \text{for } \theta(k) = 1 \\ -1.196 & \text{for } \theta(k) = 2 \end{cases} \quad (6.8)$$

The designed controller and state trajectories are shown in Figure 6.3 and Figure 6.4, respectively. For experimental results, through LABVIEW, we consider purposely generated delays as defined above and apply to the inverted pendulum. Then, using a PIC Microchip as data transmission device and a PC as the controller with the designed gain, we control the pendulum through LABVIEW. The experimental data can be seen in Figure 6.5.

Figure 6.3 Designed controller output.

Figure 6.4 State trajectories of the pendulum.

Figure 6.5 Experimental data.

6.5 Conclusion

In this chapter, inverted pendulum was introduced with its mathematical modeling. Then, identification of a KRi-PP 300 rotary inverted pendulum was fulfilled. Based on the approach derived in chapter 4, a mode dependent static output feedback controller was designed and tested on the pendulum. As it can be observed, the pendulum with delay in sensor and controller channels is stabilized with the designed controller.

Chapter 7

CONCLUSION AND RECOMMENDATION

7.1 Conclusions

In this thesis, we basically studied networked control systems and the related problems which complicate the stability analysis and control synthesis. Among these challenges, time delays and packet dropouts are two important ones that could deteriorate the performance of the control system and even lead to instability. Therefore, modeling and control synthesis of the networked control systems with time delays were taken into consideration, as well as, compensation of networked control systems with packet losses. Moreover, inverted pendulum was employed to verify the performance of the designed controller.

Following a survey on networked control systems in chapter 2, and a brief introduction to fundamental concepts which form the basis of our study in chapter 3, the major contributions were discussed in detail in two parts: chapters 4 and 5. Experiments were carried out on a practical system in the last part of the thesis in chapter 6.

In the first part, a new control design for uncertain networked control systems with bounded random time delays was proposed. Using Jordan canonical form, the networked control system was formulated as a mode dependent polytopic uncertain system. Then the Markov parameter was employed to model the known time varying delay in the sensor channel and the unknown delay in the control channel was converted to parameter uncertainties in the system matrices. Sufficient conditions in terms of BMIs were given to guarantee the stability of the system with mode-dependent static output feedback controller, using the Lyapunov-Krasovskii method. Next, further extensions were developed to obtain LMI conditions for static output feedback controller design. Mode dependent state feedback and dynamic output feedback controllers were designed for the modeled NCS, as well. Validating our results was carried out by simulation studies for a numerical example.

The substantial distinction of the proposed procedure presented in the first part is that it considered the two delays of the sensor channel and the controller channel

separately. Compared with existing methods which treat both the delays in sensor and actuator channels as uncertainty, the proposed method is less conservative.

In the second part, a new method for compensation of packet dropouts in networked control systems was proposed. The compensation procedure was to locate a filter with the actuator at the plant side which generates a control command when no signal is received. In the case when the control signal is received by the compensator, it just passes the signal to the actuator and updates its own internal states. To model the NCS with packet loss, a Markov jump linear system approach was employed. Using mean square stability analysis and H_2 -control, sufficient conditions were given for the stability of the closed loop system. An algorithm was then proposed for designing the controller and the filter gains, and finally, simulations were accomplished to verify the results. The simulations showed that the performance of the proposed filter for noisy systems with random packet dropouts is efficient.

In the last part of the thesis, inverted pendulum was introduced with its mathematical modeling. Then, identification of a KRi-PP 300 rotary inverted pendulum was fulfilled. Based on the approach derived in chapter 4, a mode dependent static output feedback controller was designed and tested on the pendulum.

7.2 Recommendations for Further Research

Regarding the results presented in this thesis, further study on the following issues would improve the performance of networked control systems.

- Extend the studies in chapter 4 by considering the controller channel network induced time delay of larger than one sampling interval. This allows a more general case of networked control systems with delays although it requires more complex analysis and computation.
- The result of dynamic output feedback controller design, due to the use of cone complementarity linearization, is a local solution to the problem and not a global one. The solution is highly dependent on the initial values. Less conservative and more efficient methods are worthy of further investigations.
- On the subject of packet dropouts, considering a lossy network which has more than one user, therefore the scheduling of the networked control system is required which is another interesting issue. Different methodologies can be studied for modeling, stabilization, and/or compensation of the system.
- In our study, we took delay and packet loss into consideration separately. However, in real networked control systems, these two can occur concurrently. Therefore, the stability analysis and control synthesis of NCSs with both delays and packet dropouts are one of the important and interesting challenges.

AUTHOR'S PUBLICATION

S. Salehi, L. Xie, W. Cai, "Robust Controller Design for Networked Control Systems with Uncertain Time Delays", in *Proc. of the 10th International Conference on Control, Automation, Robotics and Vision*, Hanoi, Vietnam, pp. 1407-1412, Dec. 2008.

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