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Modeling of Short-Pulse Child-Langmuir Law

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SUMMARY

This report concentrates on the development of 1D short-pulse space-charge-limiting (SCL) models for the classical, weakly relativistic and quantum regimes. Here, we are interested in the electron flows in the planar configuration at the space-charge limit. The effects of the finite pulse length of a SCL electron pulse emitted from a planar diode for a wide range of applied voltage, gap spacing, and pulse duration have been investigated. The results from the analytical short-pulse models in the classical and relativistic regimes are consistent with particle-in-cell (PIC) simulations. The transit time of the electron flows across a gap in various regimes (classical, relativistic, quantum) has also been formulated in analytical form. In addition, 1D short-pulse models of classical and relativistic SCL law in a drift space is developed. And a self-consistent model of transition from field emission to short-pulse classical CL law is also presented.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Although most of consumer electronics are based on solid-state devices, vacuum electronics is still evolving with inventions that take advantage of new materials, electromagnetic structures, and approaches to achieve record levels of performance and reliability. With the recent advances of micro-fabrication techniques, vacuum devices can also be integrated on the same platform together with other vacuum or solid-state devices. More importantly, vacuum devices are inherently more electrically rugged, much more resistant to nuclear electromagnetic pulses and electrostatic discharge, higher radiation hardness, faster (higher mobility), able to provide higher power output, exhibit greater reliability for high radio-frequency (RF) generation as compared to the solid-state equivalent of comparable size. As new requirements emerge, vacuum electronics will continue to be the enabling technology for entire classes of high-power high-frequency applications in the most demanding specifications in both military and commercial systems, such as space and avionics electronics, which are constantly exposed to a high level of cosmic

1.1 Motivation

radiation, radar requiring very high power output and satellite communication systems. Besides, broadcasting, microwave ovens (for industrial and home use), and even the forthcoming flat panel display technologies like carbon nanotube based field emission displays and surface-conduction emitter displays, are heavily dependent on vacuum devices for reliable performance at high power and high efficiency. In addition to a wide variety of military and commercial applications requiring high power at high frequency, vacuum electronic devices are used in scientific research areas such as high-energy particle accelerators and plasma heating for controlled thermonuclear fusion; they are also widely used in many medical systems, such as compact RF accelerators and hypothermia applicators. Only vacuum electronic devices meet many of these demanding requirements.

In many high power applications, in order to extract as much current as possible, therefore to maximize the power output from the vacuum devices, the devices are operating in high current regime that space-charge effect will become important. As a result, the space-charge-limited (SCL) current, which is the maximum possible current that can be transported across a vacuum gap, is a crucial factor to be considered during the design of vacuum electronics. Moreover, SCL flow has been an area of active research in the development of non-neutral plasma physics, high current diodes, high power microwave sources, vacuum microelectronics, and sheath physics.

With the rapid progress in nanoscale fabrication, nanoscale vacuum devices, such as field emission nano-diodes and nano-triodes, compact high-power-microwave (HPM) devices like nano-klystron for microwave and terahertz applications can be fabricated. In these nanoscale devices, quantum effects, including electron tunneling and many electron exchange-correlation interactions, can not be ignored during the investigation of space-charge-limited current flows. On the other hand, as the requirement for high power output in vacuum electronic devices such as virtual cathode oscillators, magnetrons, traveling wave tubes (TWT), SuperReltrons, and

1.2 Objectives

electron injectors, relativistic diodes are usually used to accelerate the electron beam, in which the electrons are accelerated to MV range, and the relativistic effects have to be included in the analysis.

Instead of continuous (steady-state) electron beam, electron pulses (or bunches) are used widely in order to extract higher current injection. However, only classical short-pulse SCL current is studied extensively. A better understanding of electron short-pulse in the quantum and relativistic regime is of our immediate interest.

Besides the acceleration diode used in vacuum microelectronics and even nanoelectronics devices, another important component is the drift space, in which there is no net electric field inside the gap between the two electrodes. The behavior of SCL electron flows in a drift space would be different from that in a diode, regarding to the potential profile, the location of the virtual cathode formation, and electron gap transit time, and so on. It would be interesting to study the SCL electron flows in a drift space, under various regimes at short-pulse condition.

In addition, for most of vacuum electronics devices, the electrons employed are extracted from electron emitters. One of popular electron sources is field emission or Fowler-Nordheim (FN) law based cathode. For short-pulse electron beams, a self-consistent transition model from Fowler-Nordheim emission to SCL emission is greatly required.

1.2 Objectives

The models of short-pulse SCL current are still remaining at the very initial stage. Since the pioneering work of Child and Langmuir [2, 3], the 1D steady-state classical Child-Langmuir (CL) law was extended to multi-dimensional models [4, 5, 6, 7] and quantum regime [8, 9, 10, 11]. Besides, when the pulse length of the electron beam is less than the gap transit time, the steady state CL law is no longer valid, and a

1.3 Organization of the Report

classical short-pulse CL law has been derived [12]. However, this short-pulse model is only applicable to the classical SCL flows merely in classical regime, where the gap size is relatively large and the applied gap voltage is well below the relativistic situation. It is only recently that the short-pulse CL law has been extended into multi-dimensional scenario [13]. The quantum effects will become significant in a nanoscale gap, or the relativistic effects are important with gap voltage larger than the electron rest mass. In this context, it is of interests to develop a simple 1D model of ultrashort-pulse CL law to account for the essential quantum and relativistic effects over a wide range of applied voltage, gap spacing, and pulse duration. Besides, it is also straightforward to extend the existing short-pulse model to SCL flows in a drift space and to include the emission mechanics in the cathode.

1.3 Organization of the Report

In chapter 2, a brief review of the classical one-dimensional (1D) Child-Langmuir (CL) law or the SCL current for a planar diode will be presented. The CL law gives us the maximum current that can be transmitted across a vacuum gap in the planar geometry. Recent developments and extensions to the classical 1D SCL law to include finite initial velocity, bipolar, short-pulse, relativistic and quantum effects are also emphasized. Previous work on the transition from field emission to space-charge regime are also discussed in this chapter. The 1D SCL theory of a short-pulse planar diode is then discussed in detail in chapter 3. In chapter 4, the quantum extension of the classical CL law is comprehensively explained. Using the equivalent diode approximation, we will present a self-consistent quantum short-pulse CL law that encounters quantum effects at short-pulse limit. In chapter 5, the relativistic effects in the SCL electrons flow will be briefly reviewed. We will present a self-consistent relativistic short-pulse CL law. Particle-in-cell simulations are also performed to

1.3 Organization of the Report

verify the 1D short-pulse relativistic model. The short-pulse models for the SCL flows in classical and relativistic drift spaces will be introduced in chapter 6. Chapter 7 will briefly present a simple transition model from field emission to short-pulse classical CL law. Finally, we will summarize and conclude the thesis in chapter 8. Some suggestions for future work will also be discussed.

CHAPTER 2

LITERATURE SURVEY

In recent years, continuously growing attention has been drawn to the space-charge-limited (SCL) current transport across a gap with an applied voltage, which has been an area of active research in non-neutral plasma physics, sheath physics, RF and microwave vacuum electronics. Space charge is defined as the excess of electrons or ions in a given volume. In vacuum electronics, space-charge effect is due to the charge repulsion of the electrons in an accelerating gap. When there is no current injection, the potential profile inside the gap varies linearly. However, when the significant current is injected, due to the space-charge effect within the vacuum gap, the potential profile is redistributed, and the magnitude of the electric field near to the cathode will be suppressed. When the emission current exceeds a maximum allowed limit known as space-charge limit, the electric field near to the cathode approaches zero (and negative value) causing the emitted electrons to reflect back to the cathode. This process in turn reduces the charge density near to the cathode, which limits the further suppression of the electric field, and thus a maximum current is allowed. This kind of behaviour occurs at a position near to the cathode (as illustrated in Fig. 2.1), which is known as the virtual cathode formation. At steady-state, the electric field

2.1 Classical Regime

near to the cathode is stabilized at a value closed to zero, and the steady state current density reaches the space-charge limited value. In many applications, the onset of the virtual cathode need to be avoided. Therefore, it would be a great necessity to predict the SCL current for carrier flows under various external conditions.

2.1 Classical Regime

The space-charge-limited (SCL) law is first introduced by C. D. Child in 1911 during his study of the discharge from hot calcium oxide (CaO), where the impact of the dominant positive ions become significant to limit the amount of total current flowing across the discharge tube [2]. Two years later, Irving Langmuir independently derived the same formula in 1913 for his study in the effect of space charge on thermionic current flow from a heated carbon or metal filament in high vacuum [3]. The model which predicts the maximum allowed current density that can be transported across a vacuum gap with a gap spacing D and gap voltage V_g at steady state condition, is so commonly known as Child-Langmuir (CL) law.

The classical CL law is essentially an one-dimensional (1D) planar model for non-relativistic electron flow across a vacuum gap with zero emission (or initial) velocity, given by

$$J_{CL} = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{V_g^{3/2}}{D^2}, \quad (2.1)$$

where e and m are the charge and mass of the electron, respectively, ϵ_0 is the free space permittivity, and D and V_g are the gap spacing and applied voltage across the gap, respectively. In the derivation, there is no perturbation of flows caused by other particles like resident gases, and the carrier flow is only along the gap direction from cathode to anode, any other directions of flows are ignored. Many authors tried to re-investigate the CL law by using other approaches, like Lagrangian flow description [14, 15], and vacuum capacitance method [16]. Note the scaling of $V^{3/2}/D^2$ remains

2.1 Classical Regime

the same for all the derivations as described in the original CL law. Other than planar configuration, Langmuir and Blodgett gave a corresponding SCL model in a cylindrical diode, know as Langmuir-Blodgett (LB) law [17].

Figure 2.1: Potential profile in a gap spacing D with gap voltage V_g under various current conditions. The solid line denotes the steady-state SCL potential for the classical case and, the dashed lines denote the potential profiles when the instantaneous current is higher (bottom) or lower (top) than steady-state SCL value.

Recently, the 1D planar CL law was also extended to the 2D models. In 1996, the 1D classical CL law is generalized into a 2D model via particle-in-cell (PIC) simulation by Luginsland *et al.* [4], which shows that the 2D CL law is a monotonically decreasing function of W/D , where D is the gap separation and W is the width of the finite stripe. Lau later established, for the first time, an analytical model for the 2D CL law, for both finite planar strip and the circular emission patch, in 2001 [5]. Other recent developments on multi-dimensional CL law include the works of Rokhlenko *et al.* who reported an approximated analytic theory for 2D non-uniform model of a 2D thin electron beam [18], the investigation on the perveance of a planar diode with a finite

2.2 Relativistic Regime

diameter by Saveliev *et al.* [19], and the beam-edge effect of a 2D space-charge-limited emission was also discussed [6, 20]. An universal 3D model had been also developed for uniform emission model [21].

2.2 Relativistic Regime

When the particles (usually electrons) are moving across the vacuum gap in a speed of comparable to the speed of light, the relativistic effect can not be ignored to describe the charged particle's dynamics. The steady-state CL law in relativistic regime was initially studied by several authors, [22, 23, 24], and they illustrated the relativistic CL law by using series expansion. Besides, Jory *et al.* formulated the relativistic CL law in the form of standard elliptic integrals [25]. Nation *et al.* derived the limiting current for a relativistic foilless diode under the influence of a strong magnetic field [26, 27]. To understand the physics and diode properties of a relativistic rod-pinch diode, Cooperstein *et al.* conducted both the theoretical and experimental studies in the weakly-pinch regime and the magnetically limited regime [28, 29].

Most of the theoretical treatment of the relativistic SCL flow reported are restricted by ignoring self-magnetic field of the electron beam. The our knowledge, only two models include the influence of the self-magnetic field generated by the relativistic beam, which are models of the parapotential flow [27] and the focused flow [30, 31]. For the focused flow model reported by Goldstein *et al.*, the framework was basically built on Maxwell equations and without considering ion-pinch effects. A comprehensive overview of the parapotential flow and the focused flow concept was illustrated by Miller [27]. Recently, Lopez *et al.* studied the relativistic SCL current in a crossed-field planar diode including the self-electric and magnetic fields, which proposed a set of universal curves to obtain the maximum current density at various gap voltages and the model had a smooth transition to the classical results [32].

2.3 Quantum Regime

A review of the relativistic derivation of CL law will be presented in Chapter 4.

2.3 Quantum Regime

With the rapid progress in nanoscale fabrication, nano-size gaps between metallic electrodes have been successfully fabricated [33, 34, 35]. Nano-diodes and nano-triodes [33, 34], with gap spacing on the order of $D < 100$ nm, which can operate at low voltage $V_g < 40$ V, had been fabricated. Using nano-scale emitters, high current ($> 1 \mu A$ per tip) can be obtained under low voltage at room temperature, which corresponds to a local electron current density as high as $10^8 A/cm^2$ transporting across a nano-gap. It is only recently that the classical space-charge-limited (SCL) flow theory is extended to the quantum regime [8, 9, 10, 11]. Lau *et al.* developed the basic framework to calculate the quantum extension of the classical CL law [10]. Subsequently, a more detailed study was undertaken by Ang *et al.* [8], who included the electron exchange-correlation effects and extended the basic planar model to the cylindrical and spherical geometries, and 2D models. The classical CL scaling for the planar diodes, where the SCL current density scales to $3/2$'s power of gap voltage ($V_g^{3/2}$) and the inverse squared power of gap spacing (D^{-2}), are no longer valid in the quantum regime, instead, a new quantum scaling of $V_g^{1/2}$ and D^{-4} in the deep quantum regime are indicated [8, 11]. In addition, a review of the developments of the quantum SCL flow theory was reported recently [9].

In Chapter 5, a review of the quantum CL law formulation will be given in detail.

2.4 Short Pulse SCL Flow

When the pulse length (τ) of the electron beam is less than the gap transit time, the steady-state (long pulse) CL law is no longer valid. This relatively new area

2.4 Short Pulse SCL Flow

of interest in short-pulse SCL flow commence with the rapid improvements in photocathode technology within the last decades. Initially, free electron lasers (FEL) usually operated in continuous mode, which used thermionic cathode. Nowadays, the thermionic cathodes have been replaced by photocathode guns which are operating in pulsed mode [36, 37], as photocathode guns have shown to be an excellent electron source and are able to emit high currents (> 100 mA) with low emittances and small energy spread, and high brightness. Electron emission from traditional photocathodes are normally source-limited and the amount of emitted current or charges is limited by the quantum efficiency of the materials. However, with great progress in photocathode research, photo-injectors using novel cathodes such as niobium photocathode [38], carbon or ceramic-coated metal cathodes [39], and needle-like cathodes [40], have shown improvements in its quantum efficiency and robustness, in order to produce short bunches of electron beam with high charge density on the order of nC per bunch. Improvements in ultrafast laser technology [41] have also enabled photocathodes to operate in the space-charge-limited (SCL) regime generating very high charges (several nC) in each pulse.

The classical SCL laws such as CL law and LB law, which were derived based on steady-state condition (or long pulse limit) do not hold for the short pulse SCL emission like high current photocathodes. Recently, experiments showed that the traditional long-pulse CL law can be enhanced by a large factor at the short-pulse limit. The 1D planar short-pulse SCL law had also been derived as a function of the pulse length of the charged particle beam and it has been confirmed with particle-in-cell (PIC) simulation [12]. Recently, the classical short-pulse SCL laws (including CL law and LB law) had also been extended to multidimensional models in both planar and cylindrical geometries [42]. However, all the existing models are restricted in the classical regime, where the relativistic effects as well as the quantum effects are not

2.5 Other Studies on SCL Flows

considered. New short pulse SCL models in the relativistic and quantum regime will be one of the contribution of this thesis [43].

The basic classical short pulse SCL flow model will be reviewed in Chapter 3.

2.5 Other Studies on SCL Flows

Besides the developments and improvements mentioned above, the SCL flows are also investigated in other scopes, including finite initial velocity injection, bipolar SCL flows, and transition from source-limited flow (especially field emission) to SCL flow, just to name a few.

Fry [44] and Langmuir [45] investigated the contributions of the finite velocities of thermally emitted charged particles on the potential distribution and thermionic current of a SCL diode. Recently, Akimov *et al.* generalized the classical CL law to nonzero emission velocities using the accurate Lagrangian flow approach [15]. Finite emission energy in the quantum regime was also discussed by Ang *et al.* [8].

Classical bipolar SCL flow was first discussed by Langmuir in 1929 [46]. Several decades later, Howes expanded the classical bipolar CL law to include the mono-energetic initial velocities of the electrons and positive ions in a monotonic potential field [47, 48]. The Lagrangian formulation to describe the ion dynamical effects for a two component plasma (electrons and ions) diode was developed by Schamel *et al.* [49]. Bipolar SCL flow in a relativistic diode [50, 51] and quantum diode [52] were also discussed recently.

The transition from field emission or Fowler-Nordheim (FN) regime to the space-charge or CL regime was first shown by Barbour *et al.* in 1953 [53]. Subsequently, Lau *et al.* reviewed the transition problem to determine a universal curve (independent of cathode material) linking the FN and CL regimes for a planar diode [54]. The inclusion of ion effects for a planar field-emission-limited diode was also investigated

2.5 Other Studies on SCL Flows

by Lin [55]. The transition problem and the response of the emission model to strong electric field near the transition point is also studied recently by particle-in-cell (PIC) simulation [56].

CHAPTER 3

CLASSICAL SHORT-PULSE CL LAW

3.1 Introduction

In high power devices, space charge is the fundamental limiting factor on the amount of current that can be transported from the cathode to anode. The space charge effect is predicted by the Child-Langmuir (CL) law, as introduced in Chapter 2. With the rapid progress in the development of photocathode technology and the ultrafast laser technology, the pulsed mode operation becoming more and more popular instead of continuous operation mode. In such a pulsed mode, the pulse length of the electron beam can be less than the gap transit time, and the steady-state (long pulse) CL law is no longer valid. In this context, 1D short-pulse CL law was derived by Valfells *et al.* recently [12].

In this chapter, we review the derivation of the classical long pulse CL law as done by Child and Langmuir about one century ago. Moreover, using the methodology proposed by Valfells *et al.* [12], the formulation of the classical short-pulse CL law will be explained in details.

3.2 Classical Steady-State CL Law

3.2 Classical Steady-State CL Law

Consider a 1D planar gap with a grounded cathode, and a gap voltage of V_g is applied to the anode, at a spacing of D . An uniform electron beam (of zero initial velocity and current density J) is injected normally into the vacuum gap from the cathode. From Poisson's equation and conservation of energy, we have

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\epsilon_0}, \quad (3.1)$$

$$\frac{1}{2}mv^2 = e\phi, \quad (3.2)$$

where e and m are the charge and mass of the electron, respectively, ϵ_0 is the free space permittivity, ρ is the electron charge density, and ϕ and v are the potential and velocity of the electrons in the x direction in the gap, respectively. Using the current density of $J = -\rho v$, we have

$$\frac{d^2\phi}{dx^2} = \frac{J}{\epsilon_0} \sqrt{\frac{m}{2e\phi}}. \quad (3.3)$$

Multiplying both side of the above equation by $d\phi/dx$, the resulted equation is integrated to obtain

$$\left(\frac{d\phi}{dx}\right)^2 - \left(\frac{d\phi}{dx}\right)^2\bigg|_{x=0} = \frac{4J}{\epsilon_0} \sqrt{\frac{m}{2e}} \sqrt{\phi}, \quad (3.4)$$

In order to obtain the maximum current density that can be transported across the gap, the electric field at the cathode is zero ($\frac{d\phi}{dx}|_0 = 0$), which can be integrated to obtain the following classical CL law with the boundary conditions:

$$J = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{V_g^{3/2}}{D^2} \equiv J_{CL}. \quad (3.5)$$

Using Gauss's law, the anode's electric field $E_a = \frac{d\phi}{dx}|_{x=D}$ is

$$E_a = \frac{J_{CL} T_{CL}}{\epsilon_0}, \quad (3.6)$$

3.3 Classical Short-Pulse CL Law

where T_{CL} is the electron transit time across the gap D . Using Eqs. (3.4)-(3.6), we can express T_{CL} as

$$T_{CL} \equiv \frac{3D}{\sqrt{2eV_g/m}}. \quad (3.7)$$

3.3 Classical Short-Pulse CL Law

We now turn our attention to the problem of determining the SCL current density for virtual cathode formation in an 1D planar diode at the short-pulse condition, where the pulse width is shorter than the gap transit time T_{CL} . We will approach this problem by using two different approximations: single sheet model and equivalent diode approximation, as shown by Valfells *et al.* [12].

3.3.1 Single Sheet Model

As before, the emission velocity of the electrons is assumed to be negligible, and the injection current is uniform. We will treat the electron pulse as a single sheet. Prior to the injection of the electron sheet, the electric field in the gap is simply given by V_g/D . Since the emission velocity is negligible, virtual cathode formation occurs when the electric field drops to zero behind the sheet of charge. The drop in the field strength is given by $\Delta E = \sigma/\epsilon_0$ where σ is the charge density of the electron sheet. The condition for virtual cathode formation is $\sigma = \epsilon_0 V_g/D$. The assumption that the short pulse may be treated as a single sheet can be expressed as $\sigma = J\tau$, where J is the injection current density and τ is the pulse duration ($\tau \leq T_{CL}$). From these considerations, we obtain the following simple formula for the maximum current density at the short-pulse limit, J_{SCL} , allowed before the formation of virtual cathode as a function of τ :

$$J_{SCL} = \frac{\epsilon_0 V_g}{D\tau}, \quad (3.8)$$

3.3 Classical Short-Pulse CL Law

By defining the vacuum transit time T_V as the time required for a single electron to cross a unloaded gap, which is $T_V = 2D/\sqrt{2eV_g/m}$. From Eq.(3.7), it is clear that $T_{CL} = \frac{3}{2}T_V$. Thus we may rewrite the short-pulse SCL current J_{SCL} as a function of normalized pulse length (X_V or X_{CL}):

$$J_{SCL} = \frac{9J_{CL}}{8X_V} = \frac{3J_{CL}}{4X_{CL}}, \quad (3.9)$$

where $X_V = \tau/T_V$, $X_{CL} = \tau/T_{CL}$, and J_{CL} is the classical Child-Langmuir limiting current density.

3.3.2 Equivalent Diode Approximation

The single sheet model can be improved by treating the pulse with a finite pulse length. It is assumed that over a time period $0 < t < \tau$ a continuous current, of density J , is injected into the gap. At the time the pulse extends into the anode-cathode gap with $x = \zeta$, as shown in Fig. 3.1. We denote the potential at the front of the beam as ϕ_ζ , and the electric field at the front of the beam as E_ζ .

To obtain the SCL current density $J = J_{SCL}$ at the short-pulse limit, we assume that the SCL emission occurs in an equivalent diode with a gap spacing of ζ having a potential difference of ϕ_ζ , and the electric field at the cathode goes to zero. In other words the density profile in the beam is the same as it would be in the steady-state 1D diode model with an equivalent gap spacing ζ and potential ϕ_ζ and the current density is given by the Child-Langmuir law, expressed as

$$J_{SCL} = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{\phi_\zeta^{3/2}}{\zeta^2} = J_{CL} \frac{\bar{\phi}_\zeta^{3/2}}{\bar{\zeta}^2}, \quad (3.10)$$

where $\bar{\zeta} = \zeta/D$, and $\bar{\phi}_\zeta = \phi_\zeta/V_g$. Under this condition, the electric field at $x = \zeta$ is continuous, which has the normalized form of $\bar{E}_\zeta = E_\zeta/(V_g/D)$:

$$\bar{E}_\zeta = -\frac{4\bar{\phi}_\zeta}{3\bar{\zeta}} = -\frac{\bar{\phi}_\zeta - 1}{\bar{\zeta} - 1}. \quad (3.11)$$

3.3 Classical Short-Pulse CL Law

Figure 3.1: A 1D planar diode of gap spacing D with a short pulse propagating a distance $x = \zeta$ into the vacuum gap having a potential of ϕ_ζ at the interface of the electron pulse and vacuum. The solid line denotes the potential profile distributed in the gap, the electric field at the interface of electron pulse and vacuum is continuous.

3.4 Summary

Similar to Eq. (3.6), the electric field can also be expressed as

$$E_\zeta = -\frac{J_{SCL}\tau}{\epsilon_0}, \quad (3.12)$$

After normalization, we obtain a relationship between $\bar{\phi}_\zeta$, $\bar{\zeta}$, and the normalized transit time X_{CL} , given by

$$X_{CL}\bar{\phi}_\zeta^{1/2} = \bar{\zeta}. \quad (3.13)$$

Thus, by solving Eqs.(3.10), (3.11), and (3.13), we obtain the normalized 1D short-pulse CL law as a function of normalized pulse length, which is

$$\Gamma_{SCL} \equiv \frac{J_{SCL}}{J_{CL}} = \frac{2}{X_{CL}^3} \left(1 - \sqrt{1 - \frac{3}{4}X_{CL}^2} \right). \quad (3.14)$$

The normalized pulse width $\bar{\zeta}$ and pulse front potential $\bar{\phi}_\zeta$ can also be expressed as

$$\bar{\zeta} = 2 \left(1 - \sqrt{1 - \frac{3}{4}X_{CL}^2} \right), \quad (3.15)$$

$$\bar{\phi}_\zeta^{1/2} = \frac{2}{X_{CL}} \left(1 - \sqrt{1 - \frac{3}{4}X_{CL}^2} \right). \quad (3.16)$$

At $X_{CL}=1$, we have $\Gamma_{SCL}=1$, where the 1D short-pulse CL law recovers back to the 1D classical long-pulse CL law as expected. At small $X_{CL} \ll 1$, $\Gamma_{SCL} = \frac{3}{4}X_{CL}^{-1}$, $\bar{\phi}_\zeta^{1/2} = \frac{3}{4}X_{CL}$, and $\bar{\zeta} = \frac{3}{4}X_{CL}^2$. Figure 3.2 shows the particle-in-cell (PIC) simulation results of the normalized short-pulse SCL current density as a function of the normalized pulse length, in comparison with the two analytical models (single sheet and equivalent diode approximation) [12]. Clearly the agreement between the approximate theory and the simulation is very good, especially at shorter X_{CL} .

3.4 Summary

In summary, the derivation of the 1D classical steady-state and short-pulse CL law had been reviewed. The steady-state CL law scales according to 3/2's power of gap

3.4 Summary

Figure 3.2: Normalized SCL density, J/J_{CL} as a function of the normalized pulse length, X_{CL} . Comparison of two approximate models [dashed line for single sheet model, solid line for equivalent diode approximation with XPDP1 simulation (circles for voltage=100 V, spacing=10 cm; crosses for voltage=100 V, spacing=5 cm; squares for voltage=1 kV, spacing=2.5 cm)]. [from Ref. 9]

3.4 Summary

voltage ($V_g^{3/2}$) and the inverse squared power of gap spacing (D^{-2}). At the short-pulse limit, of which the pulse length τ is less than the electron transit time across the gap, the maximum allowed current density that can be transported through the gap is enhanced by a large factor compared to the steady-state (long pulse) case. At $X_{CL} \ll 1$, the short-pulse CL law is proportional to the inverse power of the normalized pulse length (X_{CL}^{-1}). At $X_{CL} = 1$, the classical short-pulse CL law recovers the classical long-pulse CL law.

CHAPTER 4

SHORT-PULSE RELATIVISTIC CL LAW

4.1 Introduction

In high-power vacuum electron devices such as virtual cathode oscillators [57], SuperReltrons [58], and electron injectors [59], relativistic diodes are usually used to accelerate the electron beam to high energy. Jory and Trivelpiece [25] considered the problem of current-limited emission in a one-dimensional planar diode including the relativistic corrections in the equation of motion (of the electrons). They obtained an exact relativistic solution without self-magnetic field effects for the one-dimensional planar diode. Recently, laser-driven short-pulse electron beam is becoming crucial component in photoinjectors for free electron lasers (FEL) [60]. In 2005, a new particle acceleration mechanism by using laser radiation to accelerate relativistic electrons in a semi-infinite vacuum was demonstrated [61]. In order to study the relativistic SCL flows under short-pulse condition, the existing classical short-pulse SCL model has to be reformulated by including the relativistic effects, which becomes significant when the electron bunches are accelerated to MV range inside the gap. After a review of

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the steady-state relativistic CL law [25], we will present a new short-pulse relativistic CL law.

4.2 Steady-State Relativistic CL Law

To analyze the flow of electrons in a relativistic planar diode with gap spacing D and gap voltage V_g , we solve Poissons equation coupled with the law of energy conservation:

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\epsilon_0}, \quad (4.1)$$

$$e\phi = (\gamma - 1)mc^2, \quad (4.2)$$

where e and m are the charge and mass of the electron, respectively, c is the speed of light in free space, ϵ_0 is the free space permittivity, ρ is the charge density, ϕ is the electric potential, and $\gamma = 1 + e\phi/mc^2$ is the relativistic factor. The current density of electrons is $J = -\rho v$, where v is the electron velocity in the x direction. Using Eqs. (4.1) and (4.2), we have

$$\frac{d^2\gamma}{dx^2} = \frac{eJ}{\epsilon_0 mc^3} \frac{\gamma}{(\gamma^2 - 1)^{1/2}}. \quad (4.3)$$

after an integration, which gives

$$\frac{d\gamma}{dx} = \left(\frac{2eJ}{\epsilon_0 mc^3} \right)^{1/2} (\gamma^2 - 1)^{1/4}. \quad (4.4)$$

By using the boundary conditions at both electrodes, the above equation can be solved to obtain

$$J_R = \frac{\epsilon_0 mc^3}{2e} \frac{G^2(\gamma_0)}{D^2}, \quad (4.5)$$

where $G(u) = \int_1^u (r^2 - 1)^{-1/4} dr$ is an integral of dummy variable u , $\gamma_0 = 1 + eV_g/mc^2 = 1 + U$ and $U = eV_g/mc^2$ is the normalized potential, which is the ratio of potential energy eV_g to the electron rest mass energy mc^2 . Equation (4.5) gives the maximum SCL current density that can be transported across the gap from cathode

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to anode in a relativistic planar diode, under the steady-state (long pulse) condition. Note in the derivation, we had assumed that the electric field at the cathode is zero at the SCL condition. Figure 4.1 shows the normalized relativistic SCL current density over the classical steady-state CL law J_{CL} as a function of the normalized gap potential U . When the gap potential is well below the electron rest mass energy ($U \ll 1$), the relativistic effects are negligible and the model gives the same current as the classical CL law ($J_R/J_{CL} = 1$). However, when the gap potential is comparable or larger than the electron rest mass energy, the relativistic effects become important to suppress the SCL current to a value lower than J_{CL} .

Figure 4.1: The normalized relativistic SCL current density J_R/J_{CL} as a function of the normalized gap potential $U = eV_g/mc^2$.

Using Gauss's law, the anode electric field is

$$E_a = \frac{J_R T_{RCL}}{\epsilon_0}, \quad (4.6)$$

4.3 Short-Pulse CL Law in Relativistic Regime

where $E_a = \frac{d\phi}{dx}|_{x=D}$ is the electric field at anode, and T_{RCL} is the transit time across the gap for the relativistic SCL electron flows. Solving Eqs. (4.4)-(4.6), we can express T_{RCL} as

$$T_{RCL} \equiv \frac{2D}{c} \frac{(\gamma_0^2 - 1)^{1/4}}{G(\gamma_0)}, \quad (4.7)$$

By defining $\bar{\phi} = \phi/V_g$, and $\bar{x} = x/D$, the normalized electric field $a_0 = \frac{d\bar{\phi}}{d\bar{x}}|_{x=D}$, can be described as

$$a_0 = \frac{G(\gamma_0)(\gamma_0^2 - 1)^{1/4}}{\gamma_0 - 1}. \quad (4.8)$$

4.3 Short-Pulse CL Law in Relativistic Regime

While the 1D steady-state relativistic CL law was first derived decades ago [25], the short-pulse effects of the relativistic CL law has never been studied. Here, we develop a simple 1D short-pulse relativistic CL law without including the effects of self-magnetic field (as it will require a 2D model)[43]. Using the same methodology used in the classical SCL model, we solve the equivalent relativistic SCL model to obtain the short-pulse relativistic CL law. At the short pulse limit, we assume that the SCL emission occurs in an equivalent diode with a gap spacing ζ at a potential difference of ϕ_ζ , and the related short-pulse relativistic CL law is

$$J_{SRCL} = \frac{\epsilon_0 mc^3}{2e} \frac{G^2(\gamma_\zeta)}{\zeta^2}, \quad (4.9)$$

where $\gamma_\zeta = 1 + e\phi_\zeta/mc^2$ is the relativistic factor of electrons at the equivalent anode (beam-vacuum interface) due to the pulse front potential of ϕ_ζ . Using Eq. (4.8), the normalized electric field a_ζ at the equivalent anode is

$$a_\zeta = \frac{G(\gamma_\zeta)(\gamma_\zeta^2 - 1)^{1/4}}{\gamma_\zeta - 1}, \quad (4.10)$$

which is equivalent to a_0 in Eq. (4.8) if we set $\phi_\zeta = V_g$ and $\zeta = D$. Based on Gauss's law, a_ζ can also be expressed as

$$a_\zeta \frac{\phi_\zeta}{\zeta} = \frac{J_{SRCL}\tau}{\epsilon_0}, \quad (4.11)$$

4.3 Short-Pulse CL Law in Relativistic Regime

where τ is the pulse length. Using Eq. (4.9)-(4.11), we have

$$\frac{\bar{\zeta} \bar{\phi}_\zeta^{1/4}}{X_{CL}} = \frac{G(\gamma_\zeta)}{G(\gamma_0)} \left(\frac{1 + \gamma_0}{1 + \gamma_\zeta} \right)^{1/4}, \quad (4.12)$$

where $\bar{\zeta} = \zeta/D$ and $\bar{\phi}_\zeta = \phi_\zeta/V_g$, and $X_{CL} = \tau/T_{RCL} \leq 1$ is the normalized pulse length. Under this condition, the electric field at $x = \zeta$ is continuous, which has the normalized form of $\bar{E}_\zeta = E_\zeta/(V_g/D)$:

$$\bar{E}_\zeta = -a_\zeta \frac{\bar{\phi}_\zeta}{\bar{\zeta}} = -\frac{\bar{\phi}_\zeta - 1}{\bar{\zeta} - 1}. \quad (4.13)$$

Thus, the normalized short-pulse relativistic CL law (in terms of the classical CL law) is

$$\Gamma_R \equiv \frac{J_{SRCL}}{J_{CL}} = \frac{9}{8\sqrt{2}} \frac{G^2(\gamma_\zeta)}{(\gamma_0 - 1)^{3/2} \bar{\zeta}^2}, \quad (4.14)$$

which recovers the classical limit at $U \ll 1$.

The following algorithm has been used to determine the normalized limiting current density Γ_R for given normalized potential U and pulse length X_{CL} . The value of relativistic SCL current density J_{SRCL} is immediately known from $J_{SRCL} = \Gamma_R J_{CL}$ if the gap spacing D is given, where J_{CL} is the classical Child-Langmuir limiting current. There are many possible ways of solving the problem numerically. Our numerical steps are summarized as follows to obtain Γ_R (or J_{SRCL}) as a function of X_{CL} and U :

1. For a given U , we calculate $G(\gamma_0)$;
2. For a given X_{CL} , we guess initial $\bar{\phi}_\zeta$, e.g. using the expressions for classical short-pulse CL law in Eq. (3.16);
3. Using this value of $\bar{\phi}_\zeta$, we obtain $\gamma_\zeta = 1 + U\bar{\phi}_\zeta$;
4. Obtain $\bar{\zeta}$ from Eq. (4.12);
5. Calculate a_ζ from Eq. (4.10);

4.3 Short-Pulse CL Law in Relativistic Regime

6. Using the obtained ζ and a_ζ , update $\bar{\phi}_\zeta$ in Eq. (4.13);
7. Repeat steps 3-6 until convergence occurs;
8. Obtain Γ_R from Eq. (4.14);
9. Repeat steps 2-8 for a new X_{CL} ;
10. Repeat steps 1-9 for a new U .

Figure 4.2 shows the calculated Γ_R as a function of X_{CL} up to $U = 100$, where the classical limits ($U \ll 1$) are plotted in dashed lines. For a given U , Γ_R increases with decreasing value of X_{CL} , which also scales as X_{CL}^{-1} at small X_{CL} . From the figure, we see that for $U \geq 1$, Γ_R decreases as U increases for a given X_{CL} , due to the relativistic effects. When $U \ll 1$, Γ_R is independent of U , the relativistic effects can be ignored. Thus, the relativistic short-pulse CL law recovers back to the classical short-pulse CL law.

A 2D PIC simulation code called MAGIC2D [62] has been used to verify the relativistic short-pulse CL law as shown in Fig. 4.2(b). The simulations were performed by using the over-injection method in a planar diode with a gap separation of 1 cm and an electrode length of 20 cm, to confirm the limit of the 1D model. The finite pulse τ of current density J is generated using two step functions where the first step function turns on the emission at t_{rise} (rise time of the external circuit from zero to V_g) and the second one turns off the emission after $t_{rise} + \tau$. The injected current density J is increased until the formation of a virtual cathode, which causes the reflection of electrons to the cathode. The comparison shows good agreements up to $U = 10$, and the errors are within 5%. At $U > 10$, the self-magnetic field of the relativistic electron flows becomes important that our 1D short-pulse CL model is no longer valid. The calculated results are also plotted in terms of the long-pulse

4.3 Short-Pulse CL Law in Relativistic Regime

Figure 4.2: The normalized short-pulse relativistic SCL current density in terms of classical CL law (Γ_R), as a function of (a) $X_{CL} = \tau/T_{RCL}$ for $U = 1$ to 100 (top to bottom), and (b) U for $X_{CL} = 0.01$ to 1 (top to bottom). The symbols in (b) are the PIC simulation results. (c) The enhancement of the short-pulse relativistic SCL current density over the long-pulse limit $J_{SRCL}[X_{CL}]/J_R[1]$ as a function of X_{CL} for $U = 1$ to 100 (top to bottom). The dashed lines are the short-pulse classical CL law.

4.4 Summary

relativistic CL law $J_{RCL}[X_{CL} = 1]$ in Fig. 4.2(c), which shows clearly that the relativistic effects at $U \geq 1$ will decrease the enhancement due to short-pulse effects.

4.4 Summary

In summary, with a review of the 1D relativistic steady-state CL law, a new 1D short-pulse model of relativistic Child-Langmuir law has been developed [43]. Relativistic effects can not be ignored when the applied voltage is comparable to the electron rest mass, which decreases the enhancement of short-pulse CL law as compared to classical model. It is found that the enhancement of the short-pulse CL law over the long-pulse CL law is proportional to the inverse power of the normalized pulse length in relativistic regime, which is the same as in the classical regime, in the limit of very short pulse length with respect to the gap transit time. The smooth transition from the relativistic model to the classical limits is demonstrated. A 2D PIC simulation code, MAGIC2D has been used to verify the relativistic short-pulse CL law. The comparison shows good agreement in the weakly relativistic region, where the applied voltage is comparable to the electron rest mass energy. However, in the deep relativistic regime, where the applied voltage is much larger than the electron rest mass, the self-magnetic field of the relativistic electron flows becomes important and our 1D short-pulse relativistic model is no longer valid. A 2D model including the effects of self-magnetic field at much larger $U > 10$ would be much more complicated, which is beyond of our current scope.

CHAPTER 5

SHORT-PULSE QUANTUM CL LAW

5.1 Introduction

Laser-driven short-pulse electron beam is a crucial component in various areas, such as photoinjectors for free electron lasers (FEL) [60], laser acceleration of relativistic electrons in free space [61], ultrafast electron microscopy [63], picosecond cathodoluminescence [64], and femtosecond electron diffraction [65]. Using a low power femtosecond laser irradiation on a sharp tungsten tip (down to 2 nm in diameter), ultrashort electron pulse from 70 fs to less than 1 fs duration was produced by using optical field emission at an optical electric field of >1 GV/m [66, 67]. The local current intensity of such ultrashort electron pulse is about 15 kA/cm^2 or more [67]. For traditional laser-driven photocathodes used in FEL, the electron currents of 0.1 to 1 nC bunches in 10 – 50 ps with applied dc fields of 10-50 MV/m are produced through photoemission by using low work-function coated metal surface [68]. In general, photoemission based cathodes are normally operated in source-limited regime. To extract as much current as possible from a photocathode, space charge effects will become important at high current operation [12]. It is only recently

5.1 Introduction

that the classical space-charge-limited (SCL) flow theory is extended to the quantum regime [8, 9, 10, 11]. Lau *et al.* developed the basic framework to calculate the quantum extension of the classical CL law [10]. Subsequently, a more completed study was undertaken by Ang *et al.*, who included the electron exchange-correlation effects and extended the basic planar model to the cylindrical and spherical geometries. The classical CL scaling for the planar diodes, where the SCL current density scales to $3/2$'s power of gap voltage ($V_g^{3/2}$) and the inverse squared power of gap spacing (D^{-2}), are no longer valid in the quantum regime, instead, a new quantum scaling of $V_g^{1/2}$ and D^{-4} in the deep quantum regime have been proposed [8, 11].

When the pulse length (τ) of the electron beam is less than the gap transit time, the classical short-pulse CL law [12] will not be valid at ultrashort time scale (like 1 ps or less), for which the size of the electron pulse is comparable to the electron de Broglie wavelength (near to the cathode). As an example, consider a femtosecond laser irradiation on a sharp emitter tip with a local dc field of $E = 0.2$ GV/m [67], the emitted shortpulse electron beam (without space charge effects) is estimated to have a characteristic length of $\delta x \sim \frac{e}{m} E \delta t^2 \sim 2$ nm with a pulse length of $\delta t = 7$ fs. For a typical photoinjector at a dc field of 10 MV/m, the characteristic length is about $\delta x \sim 18$ nm if a 100 fs electron bunch is assumed. Since the length scale is in nanometer regime, quantum effects of the electron transport near to the cathode are important [8, 9]. Thus, the 1D classical short-pulse CL law introduced in Chapter 3 requires a complete revision to include quantum effects. In this context, we speculate that the recent advances in ultrafast electron bunches [66, 67] may be operated in high current regime, where space charge effects cannot be ignored. It is of interests to develop a simple 1D model of ultrashort-pulse CL law to account for the essential quantum effects over a wide range of applied voltage, gap spacing, and pulse duration.

5.2 Steady-State Quantum CL Law

This chapter will give a review on the steady-state quantum CL law model developed recently [8, 10]. More importantly, based on the steady-state quantum CL law, a new short-pulse quantum CL law will be presented thereafter.

5.2 Steady-State Quantum CL Law

Consider a gap with spacing D and voltage V_g , electrons with zero initial velocity are injected normally into the gap from cathode to anode. Using mean field theory, the 1D time-independent Schrödinger equation, the Poisson equation and charge conservation relation are solved to obtain the maximum current density J that can be transported under the SCL condition. The respective 1D time-independent Schrödinger equation, and the Poisson equation are

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = (eV - V_{xc} \times E_H)\psi, \quad (5.1)$$

$$\frac{d^2V}{dx^2} = \frac{e\psi\psi^*}{\epsilon_0}, \quad (5.2)$$

where e and m are the charge and mass of the electron, respectively, ϵ_0 is the free space permittivity, ψ is the complex electron wave function, and V_{xc} is the electron exchange-correlation potential of the transmitted current (in terms of the Hartree energy E_H), which has important effect on the behavior of electrons in such a many-body system [11]. The physical origin of the exchange-correlation potential is due to the Pauli exclusion principle in quantum mechanics [69], which is calculated by using Kohn-Sham density functional theory (DFT) (under random phase approximation) [70]. Take note that in Eq. (5.1) we used zero energy for the electron wave functions, which is based on our assumption that most of the electrons are emitted near Fermi energy level. Charge conservation requires that the current density J be constant for all x , which is given by

$$J = e \frac{i\hbar}{2m} (\psi\psi'^* - \psi^*\psi'), \quad (5.3)$$

5.2 Steady-State Quantum CL Law

where the prime denotes the derivatives with respect to x .

For simplicity, we introduce several normalized parameters: $\bar{x} = x/D$, $\phi = V/V_g$, $\lambda = D/\lambda_0$ is the normalized gap spacing, $\phi_g = eV_g/E_H$ is the normalized gap voltage, $q^2 = n/n_0 = |\psi|^2/n_0$ is the normalized electron density, and $\mu = J/J_{CL}$ is the normalized electron current density. The normalized scales $\lambda_0 = \sqrt{\hbar^2/2emV_g}$ is the electron de Broglie wavelength at V_g , $n_0 = 2\epsilon_0 V_g/3eD^2$ is the electron density scale, $E_H = e^2/4\pi\epsilon_0 a_0 = 27.2\text{eV}$ is the Hartree energy, $a_0 = 4\pi\epsilon_0 \hbar^2/me^2 = 0.0529\text{nm}$ is the Bohr radius, and J_{CL} is the classical long-pulse SCL current density.

In terms of the normalized parameters, the 1D time-independent Schrödinger equation, the Poisson equation and charge conservation relation can be formulated as

$$q'' + \lambda^2(\phi - \frac{V_{xc}}{\phi_g} - \frac{4\mu^2}{9q^4})q = 0, \quad (5.4)$$

$$\phi'' = \frac{2}{3}q^2, \quad (5.5)$$

where the prime denotes the derivatives with respect to $\bar{x}$. The boundary conditions [8, 10], for Eqs.(5.4) and (5.5) are

$$q(1) = \sqrt{2\mu/3}, \quad (5.6)$$

$$q'(1) = 0, \quad (5.7)$$

$$\phi(0) = 0, \quad (5.8)$$

$$\phi(1) = 1, \quad (5.9)$$

The boundary conditions (5.8) and (5.9) specify the potential imposed on the gap, whereas (5.6) and (5.7) follow from the requirement that the solution at $\bar{x} = 1$ matches a (preassigned) transmitted wave $q_t(\bar{x})$. The normalized electric field at the cathode and anode are, respectively, $\phi'(0)$ and $\phi'(1)$, which are numerically calculated for a given μ . With the boundary conditions, we determine the SCL current density through the a maximum value of μ , which is defined as μ_Q . For $\mu \leq \mu_Q$, solutions to

5.2 Steady-State Quantum CL Law

Eqs. (5.4) and (5.5) subject to the boundary conditions can always be found. In fact, when $\mu \leq \mu_Q$, we can find two solutions of Eqs. (5.4) and (5.5) satisfying the given boundary conditions, for specified λ , ϕ_g , and μ . At $\mu = \mu_Q$, the two solutions merge. But for $\mu > \mu_Q$, no solution for Eqs. (5.4) and (5.5) exists. The following steps are taken to solve the equation numerically:

1. For given λ and ϕ_g , we guess a small value of μ , typically set $\mu_1 = 1$;
2. Since we only know the value of wave function at the anode ($\bar{x} = 1$), we integrate Eqs.(5.4) and (5.5) backward, from $\bar{x} = 1$ to $\bar{x} = 0$. To perform the integration, we use (5.6), (5.7), and (5.9), by assuming a value for $\phi'(1)$ as the initial condition at $\bar{x} = 1$. The value $\phi'(1)$ is adjusted so that condition (5.8) is satisfied at $\bar{x} = 0$. As mentioned earlier, two solutions can be found, the corresponding $\phi'(1)$ for the two solutions are denoted as Φ_1 and Φ_2 ($\Phi_1 < \Phi_2$). For $\phi'(1)$ in the range of $[\Phi_1, \Phi_2]$, we can find a minimum value of $\phi(0)$, denoted as ϕ_{m1} ;
3. Guess another larger value of μ , repeat step (2) till the minimum value of $\phi(0)$ is larger than 0, i.e. $\phi_{m2} > 0$, the corresponding μ value is denoted as μ_2 ;
4. Using the bisection method for μ in the range of $[\mu_1, \mu_2]$ (ϕ_m in $[\phi_{m1}, \phi_{m2}]$), to obtain the maximum allowed value of μ , i.e. μ_Q , at which $|\phi_m - 0| < \epsilon$, where ϵ is the numerical error tolerance, e.g. $\epsilon = 10^{-9}$;
5. Repeat steps 1-4 for a new set of λ and ϕ_g .

Once μ_Q is known for a given gap with separation D and applied voltage V_g , the steady-state quantum SCL current density J_Q can be found immediately as

$$\mu_Q \equiv \frac{J_Q}{J_{CL}}, \quad (5.10)$$

5.2 Steady-State Quantum CL Law

Figure 5.1: The normalized steady-state quantum CL law μ_Q as a function of λ for various $\phi_g = 10^{-2}$ to 10^2 (solid lines: top to bottom), $\phi_g \gg 1$ (dashed line), and classical limit (dash-dotted line) [from Ref. 7].

5.2 Steady-State Quantum CL Law

Figure 5.1 [8] shows the calculated μ_Q as a function of λ for various values of ϕ_g . For a given ϕ_g , μ_Q increases with decreasing values of λ (quantum regime) and exceeds the classically predicted values when λ is around or less than 10. The value of μ_Q is roughly proportional to λ^{-2} at small λ , and this indicates that $J_Q \propto D^{-4}$ and $V_g^{1/2}$ is the correct scaling in the quantum regime for small D , as compared to the classical scaling of $J_{CL} \propto D^{-2}$ and $V_g^{3/2}$. From the figure, we also see that the enhancement of μ_Q is more dominant at small ϕ_g due to electron exchange-correlation interaction.

Comparison to the classical and relativistic steady-state CL law, the electric field at the cathode is no longer zero, instead, it has a finite negative value, due to the effect of electron tunneling through the space charge field in the quantum gap. From Gauss's law, we have

$$E_a - E_c = \frac{d\phi}{dx}|_{x=D} - \frac{d\phi}{dx}|_{x=0} = \frac{J_Q T_{QCL}}{\epsilon_0}, \quad (5.11)$$

where E_a and E_c are the electric field at anode and cathode respectively, and T_{QCL} is the transit time across the whole gap D for the quantum SCL electron flows. The normalized electric fields at anode a_Q and at cathode c_Q are defined as

$$a_Q = \phi'(1) = \frac{d\bar{\phi}}{d\bar{x}}|_{\bar{x}=1} = \frac{D}{V_g} E_a, \quad (5.12)$$

$$c_Q = \phi'(0) = \frac{d\bar{\phi}}{d\bar{x}}|_{\bar{x}=0} = \frac{D}{V_g} E_c, \quad (5.13)$$

With Eqs. (5.10)-(5.13), T_{QCL} can be presented in the following form:

$$T_{QCL} \equiv \frac{3}{4} \frac{a_Q - c_Q}{\mu_Q} T_{CL}, \quad (5.14)$$

where T_{CL} is the gap transit time of the 1D classical CL law, and μ_Q , a_Q and c_Q are calculated by using Eqs. (5.4) and (5.5) numerically.

In this formulation, we have assumed a slowly varying electron density in the gap, where the electron exchange-correlation potential depends only on the local electron

5.3 Short-Pulse CL Law in Quantum Regime

density (using LDA). This assumption is valid since the spatial variation of electron density in our calculation is on the order of gap spacing which is larger than the atomic scale. This model has also ignored the electron emission mechanisms in the vicinity of the cathode's surface.

5.3 Short-Pulse CL Law in Quantum Regime

In this section, we speculate that the recent advances in ultrafast electron bunches may be operated in high current regime, where space charge effects cannot be ignored. For simplicity, we consider a 1D planar diode of gap spacing D , with a grounded cathode, and a dc potential of V_g is applied to the anode. A sheet of uniform electron beam of a current density J with a finite pulse length τ is injected normally into the vacuum gap from the cathode. Here, we assume unlimited electron current can be supplied and ignore the process of the electron emission at the cathode (either photoemission or optical field emission). (For the cases that the electron emission from the cathode can not be neglected, we may include the emission mechanism in our formulation and solve with the space charge effect self-consistently, an example of this treatment can be found in Chapter 7 where we include the effect of field emission. However, this is out of scope in the present chapter, as we are interested on the maximum current emission limited by space charge effects). Since the pulse length τ is shorter than the gap transit time, the electron pulse can only extend to a distance of ζ , which is smaller than the gap spacing D . At the beam-vacuum interface (or beam front) of $x = \zeta$, the electric potential is defined as ϕ_ζ with a continuous electric field. In this 1D model, we calculate the maximum current density that can be transported (before the formation of virtual cathode) as a function of V_g , D , and τ .

5.3 Short-Pulse CL Law in Quantum Regime

To obtain the short-pulse quantum SCL current density $J = J_{SQCL}$, the beam propagation region of $0 \leq x \leq \zeta$ (during the pulse length τ), which has a potential difference of ϕ_ζ , is considered as an equivalent quantum diode based thin-sheet model, and J_{SQCL} is formulated as

$$J_{SQCL} \equiv \mu_\zeta \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{\phi_\zeta^{3/2}}{\zeta^2} = \frac{a_\zeta - c_\zeta}{\tau} \frac{\phi_\zeta}{\zeta}. \quad (5.15)$$

Here, μ_ζ is the quantum enhancement factor, a_ζ and c_ζ are, respectively, the normalized electric field at the beam front ($x = \zeta$), and at the cathode ($x = 0$), which are calculated by solving the 1D time-independent Schrödinger equation, the Poisson equation, and charge conservation relation in a mean field model [8], i.e. the model of steady-state quantum CL law as introduced in previous section [Eqs.(5.4) and (5.5)]. Here, the parameter a_ζ is related to ζ , ϕ_ζ , D , and V_g by the continuous electric field at the beam front, which is

$$a_\zeta \frac{\phi_\zeta}{\zeta} = \frac{V_g - \phi_\zeta}{D - \zeta}. \quad (5.16)$$

By setting $\phi_\zeta = V_g$ and $\zeta = D$ in Eq. (5.15), the gap transit time in the quantum regime is found as $T_{QCL} = (3/4)[(a_0 - c_0)/\mu_0]T_{CL}$, which is exactly the same as in Eq. (5.14). The parameters: (μ_0, a_0, c_0) are the corresponding values of $(\mu_\zeta, a_\zeta, c_\zeta)$ calculated by using Eqs.(5.4) and (5.5) at the long-pulse limit of $\tau = T_{QCL}$.

By normalizing the pulse length τ with the transit time T_{QCL} , Eq. (5.15) gives

$$\frac{\bar{\zeta}}{X_{CL} \bar{\phi}_\zeta^{1/2}} = \frac{\mu_\zeta a_0 - c_0}{\mu_0 a_\zeta - c_\zeta}, \quad (5.17)$$

$$\Gamma_Q \equiv \frac{J_{SQCL}}{J_{CL}} = \mu_\zeta \frac{\bar{\phi}_\zeta^{3/2}}{\bar{\zeta}^2}, \quad (5.18)$$

where $X_{CL} = \tau/T_{QCL} \leq 1$ is the normalized pulse length, $\bar{\phi}_\zeta = \phi_\zeta/V_g$ is the normalized potential at the beam front, $\bar{\zeta} = \zeta/D$ is the normalized position of the beam front, and Γ_Q is the normalized short-pulse quantum CL law in terms of the 1D classical steady-state CL law.

5.3 Short-Pulse CL Law in Quantum Regime

To determine Γ_Q as a function of D , V_g , and τ , we first calculate μ_0 , a_0 , c_0 at the long-pulse limit ($X_{CL} = 1$) from Eqs. (5.4) and (5.5). For a given value of $X_{CL} < 1$, Eqs. (5.4), (5.5), (5.16) and (5.17) are solved numerically for μ_ζ , a_ζ , c_ζ , ζ , and ϕ_ζ , to obtain a maximum value of Γ_Q defined in Eq. (5.18), for which no solutions can be found at other values larger than this maximum value. When quantum effects are negligible [i.e., $\lambda \gg 1$ in Eq. (5.4)], the solutions are $a_\zeta = a_0 = 4/3$, $c_\zeta = c_0 = 0$, $\mu_\zeta = \mu_0 = 1$, $T_{QCL} = T_{CL}$, $\bar{\phi}_\zeta^{1/2} = 2(1 - \sqrt{1 - 3X_{CL}^2/4})/X_{CL}$, $\bar{\zeta} = 2(1 - \sqrt{1 - 3X_{CL}^2/4})$, and Eq. (5.18) recovers the classical short-pulse CL law, as shown Chapter 3, in Eq. (3.14).

The algorithms used for solving the short-pulse CL law in quantum regime are summarized in the following steps:

1. For a given ϕ_g and λ , we calculate μ_0 , a_0 , c_0 at the steady-state condition [solving Eqs. (5.4) and (5.5) by the algorithm stated in section 5.2];
2. For a given X_{CL} , we guess initial $\bar{\zeta}$, $\bar{\phi}_\zeta$ values, e.g. using the expressions for classical short-pulse CL law [Eqs. (3.15) and (3.16)];
3. Start an initial value of μ_ζ , like $\mu_\zeta = 1$;
4. Obtain ϕ_ζ and ζ from $\phi_\zeta = \bar{\phi}_\zeta \phi_g$, $\zeta = \lambda \bar{\zeta} \sqrt{\bar{\phi}_\zeta}$. The obtained ϕ_ζ and ζ are the potential difference and spacing of the equivalent (short-pulse) quantum diode, can be denoted as $\phi_{g\zeta} = \phi_\zeta$ and $\lambda_\zeta = \zeta$, respectively;
5. Using μ_ζ , $\phi_{g\zeta}$ and λ_ζ as the inputs for μ , ϕ_g and λ in Eqs. (5.4) and (5.5), calculate the corresponding a_ζ and c_ζ . For $\mu \leq \mu_Q$, we can find two solutions of Eqs. (5.4) and (5.5) satisfying the given boundary conditions, for specified inputs λ , ϕ_g , and μ . For various reasons, we argue that the one with higher potential energy is inaccessible. We choose only the solution with lower potential energy;

5.3 Short-Pulse CL Law in Quantum Regime

6. Update $\bar{\phi}_\zeta$ by using Eq. (5.16);
7. Update $\bar{\zeta}$ by using Eq. (5.17);
8. Repeat steps 4-7 till convergence occurs;
9. Obtain Γ from Eq. (5.18);
10. Slowly increase (the increasing step size determines the precision of the final result, keep the step size as small as possible) μ_ζ and repeat steps 4-9 after every increment of μ_ζ , till a maximum value of μ_ζ is reached.
11. From the obtained values of Γ , we find the maximum value, i.e. Γ_Q , which is the maximum current density that can be possibly transported across the gap for a given set of ϕ_g , λ and X_{CL} ;
12. Repeat steps 2-11 for a new X_{CL} ;
13. Repeat steps 1-12 for a new set of ϕ_g and λ .

To illustrate the importance of quantum effects at the short-pulse CL law, we first consider the quantum effects of an ultrashort-pulse electron beam ($X_{CL} \ll 1$) in a classical (large) gap of $V_g = 1$ to 10 kV, and $D = 0.1$ to 1 cm. In this case, the quantum effects are not significant at the long-pulse limit ($X_{CL} \approx 1$). In Fig. 5.2, the normalized short-pulse quantum CL law (Γ_Q), and the pulse width (ζ [nm]) are plotted as a function of $X_{CL} = 0.001$ to 0.01, which corresponds to a pulse length of about $\tau = 50$ fs to 16 ps, depending on the values of V_g and D . Here, the values are chosen to have a dc applied electric field in the range of 0.1 to 10 MV/m, which is typical for photoinjectors and field emission based cathodes. It is clear that the classical short-pulse CL law [dashed lines in Fig. 5.2(a)] is no longer valid when the pulse width ζ is in nanometer scale, and the pulse length is less than 1 ps.

5.3 Short-Pulse CL Law in Quantum Regime

Figure 5.2: The normalized short-pulse quantum CL law in terms of the classical CL law (Γ_Q), and the pulse width of the electron sheet (ζ [nm]), as a function of $X_{CL} = \tau/T_{CL}$ in a large gap for (a) $D = 1\text{ cm}$ and $V_g = 1\text{ kV}$, and (b) $D = 1\text{ mm}$ at $V_g = 1$ and 10 kV . The dashed lines in (a) are the short-pulse classical CL law.

5.3 Short-Pulse CL Law in Quantum Regime

Figure 5.3: The normalized short-pulse quantum CL law in terms of the classical CL law (Γ_Q) as a function of $X_{CL} = \tau/T_{CL}$ in a nano gap for (a) $D = 10\text{ nm}$ at $V_g = 1$ to 100 V (top to bottom), and (b) $V_g = 10\text{ V}$ at $D = 1$ to 100 nm (top to bottom). The dashed lines are the short-pulse classical CL law.

5.4 Summary

Similar quantum effects may also occur in a nanogap with low voltage, where the quantum effects are important for finite values of $X_{CL} \leq 1$, even at the long-pulse limit. In Fig. 5.3, Γ_Q is plotted as a function of $X_{CL} = 0.01$ to 1 for $V_g = 1$ to 100 V, and $D = 1$ to 100 nm. In comparison to the classical results (dashed lines), the quantum effects are more significant at small values of X_{CL} , V_g , and D . At $X_{CL} < 0.1$, both normalized quantum and classical short-pulse CL law increase to the inverse power of the normalized pulse length ($\Gamma_Q \propto X_{CL}^{-1}$), but the quantum model is enhanced by a large factor due to the effects of electron tunneling through the potential barrier including space charge field, and exchange-correlation potential.

5.4 Summary

In conclusion, a 1D short-pulse model of quantum Child-Langmuir law has been developed. It is found that the enhancement of the short-pulse CL law over the long-pulse CL law is proportional to the inverse power of the normalized pulse length in all regimes (classical, quantum and relativistic), when the pulse length is much smaller than the gap transit time. Quantum effects are important when the pulse length is ultrashort (like < 1 ps) when the size of electron pulse is comparable to the electron de Broglie wavelength, and thus the classical CL current density is increased by a large factor due to electron tunneling through the space charge field. The transition from the quantum models to the classical limits is demonstrated.

It is worth to note that the simple short-pulse quantum CL law presented here is strictly a 1D model, where other practical issues in a photoinjector design are ignored, such as complicated electrode geometry and external focusing magnetic fields. However, the scaling laws developed here may be used as a first estimate in the design or even as an emission algorithm in the traditional gun codes for the development of femtosecond laser-driven high current photocathodes in FEL and other applications,

5.4 Summary

where the quantum effects of such high current ultrashort electron bunches become important.

CHAPTER 6

SHORT-PULSE SCL FLOWS IN A DRIFT SPACE

6.1 Introduction

Laser-driven photocathode guns are excellent electron sources to produce low emittance electron beam for free electron lasers (FEL) [60]. Recently, photoinjectors using novel cathodes [38, 71, 40] have shown to be able to produce short electron bunches with high charge density on the order of nC per bunch. At such high current operation, space charge effects will become important [12], especially at the early stages of beam acceleration at an electron beam source and the following drifting. The region of drift space in high power microwave device design usually degrades the beam emittance and need to be designed with care. When the pulse length (τ) of the electron beam is less than the gap transit time, the steady-state (long pulse) model is no longer valid and a classical short-pulse CL law has been derived [12]. However, most of short pulse work to date only focus on the accelerating stage [12, 42, 43], where a simple diode is used to accelerate the space-charge-limited (SCL) electron beam. Thus, it is of interests to develop an analytical model to predict the maximum

6.2 Classical SCL Flows in Drift Space

SCL current density at short pulse condition in the drift space. Here, a model is presented, which endeavors to address this requirement.

6.2 Classical SCL Flows in Drift Space

6.2.1 Classical Steady-State SCL Flows in Drift Space [1]

We assume that we have two infinite plane-parallel electrodes separated by a distance D and held at the same potential V_g . The electrons are assumed to be mono-energetic. The emission velocity of the electrons is fixed at $v_0 = \sqrt{2eV_g/m}$, where e and m are the electron charge and mass respectively. Solving Poisson's equation coupled with the law of energy conservation, one can easily get

$$\frac{d^2\phi(x)}{dx^2} = \frac{J}{\epsilon_0 \sqrt{2e\phi_x/m}}, \quad (6.1)$$

where $\phi(x)$ is the potential in the charge filled region. Multiplying by $2(d\phi/dx)$ on both sides, Eq. (6.1) is integrated to give

$$E_x = \frac{d\phi(x)}{dx} = \pm \left[4 \frac{J}{\epsilon_0} \sqrt{\frac{m}{2e}} (\sqrt{\phi(x)} - \sqrt{\phi_m}) \right]^{1/2}, \quad (6.2)$$

where E_x is the electric field and ϕ_m is the minimum potential inside the gap. For $\phi_m > 0$, the above equation can be integrated to give

$$(\sqrt{\bar{\phi}} - \sqrt{\bar{\phi}_m})(\sqrt{\bar{\phi}} + 2\sqrt{\bar{\phi}_m})^2 = \alpha(\bar{\zeta} - \frac{1}{2})^2, \quad (6.3)$$

where $\bar{\phi} = \phi/V_g$, $\bar{\phi}_m = \phi_m/V_g$, $\bar{\zeta} = x/D$, and $\alpha = J/J_{CL}$, J_{CL} is the classical long-pulse Child-Langmuir Law. The value of the minimum potential for a given current is found by evaluating Eq. (6.3) at $x = 0$, giving

$$4\bar{\phi}_m^{3/2} - 3\bar{\phi}_m^{1/2} + \left(\frac{\alpha}{4} - 1\right) = 0. \quad (6.4)$$

By setting $d\alpha/d\phi_m = 0$, we can find the maximum current density is $\alpha = 8$ with its corresponding $\bar{\phi}_m = 0.25$. The potential minimum is at the center of the gap with a symmetric profile.

6.2 Classical SCL Flows in Drift Space

6.2.2 Classical Short-Pulse SCL Flows in Drift Space

When the pulse length (τ) of the electron beam is less than the gap transit time, the steady-state (long pulse) model is no longer valid. Short-pulse models of SCL flow in a drift space are therefore introduced.

Figure 6.1: A 1D drift space of gap spacing D in (a) single sheet model with a single electron sheet, and (b) equivalent drift space model with a short pulse having a width of ζ and potential of ϕ_ζ at the both interfaces of the electron pulse and vacuum, propagating inside the vacuum gap. The electric fields at the interfaces of electron pulse and vacuum are continuous. The solid line denotes the potential profile distributed in the gap.

First, we consider a very simple single-sheet model for approximating the short-pulse electrons flows in a drift space. We will treat the electron pulse as a single

6.2 Classical SCL Flows in Drift Space

sheet, as shown in Fig. 6.1 (a). Prior to the injection of the electron sheet, the electric field in the gap is zero, since both electrodes are held at the same voltage V_g . As the charge sheet enters the gap, a potential drop would be induced at the charge sheet position x , due to negative charging of electrons. The electric fields in the vacuum region at the front and back of the charge sheet, are $(V_g - V_x)/(D - x)$ and $(V_g - V_x)/x$, respectively. Thus, the electron sheet will experience deceleration before it reaches the midpoint of the gap $x = D/2$, followed by acceleration when it passes over the midpoint. The velocity of the electron sheet reaches its minimum v_m at $x = D/2$, where the electric fields on both sides of the electron sheet will be equal but in opposite direction. The reflection of charge (i.e. the virtual cathode formation) starts to occur when v_m approaching zero. Due to the conservation of energy, the potential at this point is also zero. The change in the field strength on each side is given by $\Delta E = \sigma/2\epsilon_0$, where σ is the charge density of the electron sheet. Therefore the condition for the initiation of charge reflection is $\sigma = 4\epsilon_0 V_g/D$. The single sheet can then be expressed as $\sigma = J\tau_p$, where J is the uniform injection current and τ_p is the pulse length. From these considerations, the maximum allowed current density J_M before the reflection of charge is in the following form:

$$J_M = \frac{4\epsilon_0 V_g}{D\tau_p}, \quad (6.5)$$

The long-pulse (steady state) drift space transit time, T_d , is the time required for electrons to cross the whole gap, is $T_d = 3D\sqrt{m/8eV_g}$. Thus the equation for maximum allowed current density can be rewritten in a normalized form as

$$\mu = \frac{J_M}{J_d} = \frac{3}{4X_T}, \quad (6.6)$$

where $X_T = \tau_p/T_d$ is normalized pulse length, and J_d is the classic long-pulse drift space limiting current density, which is 8 times of the classical Child-Langmuir (CL) limiting current density [1]. Interestingly the maximum allowed current density in drift space is in the same form as that for the classical short-pulse CL law [12].

6.2 Classical SCL Flows in Drift Space

Compared to single sheet model, a more precise model using the equivalent drift space approximation is also investigated. In this case, we follow the same assumptions as those stated in the single sheet model, except that the pulse will be treated as a pulse of finite length, instead of a single sheet, as shown in Fig. 6.1 (b). The total charge density entered the gap during pulse injection duration τ_p is $\sigma = J\tau_p$. As the electron pulse with length ζ propagates forward, the potential at the pulse front ϕ_a and at the pulse end ϕ_c would gradually increase and decrease, respectively. When the potential at pulse end drops and becomes equal to that at the pulse front, $\phi_a = \phi_c = \phi_\zeta$, from Eq. (6.2), the magnitude of electric field at the pulse front E_a and pulse end E_c are equal as well.

The electric fields are continuous at both beam-vacuum interfaces, thus we have

$$E_c = \frac{V_g - \phi_c}{0 - x_c}, \quad (6.7)$$

$$E_a = \frac{V_g - \phi_a}{D - x_c - \zeta}, \quad (6.8)$$

where x_c is the pulse end position. When $E_c = E_a$, the pulse is located at the center of the gap.

To obtain the short-pulse space-charge-limited (SCL) current in a drift space, we may assume the pulse charge filled region as an equivalent drift space with gap spacing of ζ and with gap potential of ϕ_ζ , which yields the maximum allowed current density J_M as

$$J_M = 8J_{CL}(\zeta, \phi_\zeta) = \frac{32}{9}\epsilon_0\sqrt{\frac{2e}{m}}\frac{\phi_\zeta^{3/2}}{\zeta^2}, \quad (6.9)$$

The current density can also be expressed as

$$J_M = \frac{\epsilon_0}{\tau_d}(E_a - E_c) = \frac{2\epsilon_0}{\tau_d}E_a, \quad (6.10)$$

where τ_d is the electron transit time across this equivalent drift space. Combining with Eq.(6.2) and Eqs. (6.7)-(6.10), we obtain

$$\mu(X_d) = \frac{2}{X_d^3}\left(1 - \sqrt{1 - \frac{3}{4}X_d^2}\right), \quad (6.11)$$

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$$\bar{\phi}_\zeta^{1/2}(X_d) = \frac{2}{X_d} \left(1 - \sqrt{1 - \frac{3}{4}X_d^2} \right), \quad (6.12)$$

$$\bar{\zeta}(X_d) = 2 \left(1 - \sqrt{1 - \frac{3}{4}X_d^2} \right), \quad (6.13)$$

where $\mu = J_M/J_d$, $\bar{\phi}_\zeta = \phi_\zeta/V_g$, $\bar{\zeta} = \zeta/D$, and $X_d = \tau_d/T_d$, which is the normalized transit time across this equivalent drift space. At $X_d = 1$, $\bar{\phi}_\zeta = \bar{\zeta} = \mu = 1$, the short-pulse drift-space limiting current recovers the long-pulse drift-space limiting current.

However, the equivalent drift space transit time τ_d may not be the same as the electron injection time τ_p , due to the pulse expansion along its propagation. Because in a drift space the electrons are usually injected with high kinetic energy, the space charge effect near the injection electrode (cathode) may be negligible, especially for a very short pulse. Thus, during beam injection time τ_p , the electrons can be considered moving with constant velocity of v_0 . For any gap D with voltage V at both electrodes, the electron transit time with space charge effect t_d and without space charge effect t_p are

$$t_d = 3D\sqrt{\frac{m}{8eV}}, \text{ and} \quad (6.14)$$

$$t_p = D\sqrt{\frac{m}{2eV}}. \quad (6.15)$$

Thus, we may establish an approximate relation between pulse equivalent drift space transit time τ_d and pulse injection time τ_p , by comparing t_d and t_p , which will give $\tau_d = 1.5\tau_p$. Normalizing in terms of the steady-state drift space transit time T_d , we have

$$X_d = \tau_d/T_d = \frac{3}{2}\tau_p/T_d = \frac{3}{2}X_T. \quad (6.16)$$

Substituting Eq. (6.16) into (6.11), yields the maximum allowed current in terms of normalized injection time $X_T = \tau_p/T_d$:

$$\mu(X_T) = \frac{16}{27X_T^3} \left(1 - \sqrt{1 - \frac{27}{16}X_T^2} \right), \quad (6.17)$$

6.2 Classical SCL Flows in Drift Space

Figure 6.2: Normalized space-charge-limited (SCL) current density $\mu = J_M/J_d$, as a function of the normalized pulse length, $X_T(X_d)$. Comparison of two approximate models [dashed line for single sheet model, solid line (bottom) for equivalent drift space approximation]. The model for drift space approximation without considering pulse expansion effect is also plotted for comparison (top), which is in the form of $\mu(X_d)$. The symbols denote the OOPIC Pro simulation (circles for $V_g = 1\text{kV}$, $D = 1\text{cm}$; crosses for $V_g = 100\text{kV}$, $D = 1\text{cm}$; and squares for $V_g = 1\text{kV}$, $D = 10\text{cm}$). When X_T is large (> 0.1), $X_T \sim X_d$, the simulation fits to the $\mu(X_d)$ as well as the single sheet model; when X_T is small (< 0.1), the simulation follows the equivalent drift space model, $\mu(X_T)$.

6.3 Relativistic SCL Flows in Drift Space

which is only valid for $X_T < 4/(3\sqrt{3}) \simeq 0.77$.

PIC simulation code OOPIC Pro [72] has been used to verify the analytical results for short-pulse space-charge-limited (SCL) current density in a 1D classical drift space as shown in Fig. 6.2. The simulations were performed by using the over-injection method in a planar drift space with a gap separation much smaller than the electrode length. A finite pulse τ of current density J is injected into the gap, and the value of J is increased until the formation of a virtual cathode, which causes the reflection of electrons to the cathode. The comparison shows quite good agreements, especially at shorter pulses. Basically, the pulse lengths can be divided into two regimes. When the pulse is large, $X_T > 0.1$, the pulse expansion during its propagation is relatively small, so the pulse injection time τ_p (or X_T) is approximately the same as the equivalent drift space transit time τ_d (or X_d), the simulation results match the single sheet model, as well as the 1D model for short-pulse classical CL law for a diode [12], which has the same form as $\mu(X_d)$, as shown in Eq. (6.11). When the pulse is small, $X_T < 0.1$, the simulation agrees better with the equivalent drift space model $\mu(X_T)$, as in Eq. (6.17). It is clear that the enhancement (in terms of the steady state SCL current density in drift space) is proportional to X_T^{-1} for small pulse length.

6.3 Relativistic SCL Flows in Drift Space

6.3.1 Relativistic Steady-State SCL Flows in Drift Space

It is obvious that if the electron beam voltage is higher than the electron rest mass, like $U = eV_g/mc^2 \geq 1$, relativistic effects of the electron flows can not be ignored. To analyze the flow of electrons in a relativistic drift space with gap spacing D and electron injection voltage V_g , we solve Poissons equation coupled with the law of

6.3 Relativistic SCL Flows in Drift Space

energy conservation to obtain:

$$\frac{d^2\gamma}{dx^2} = \frac{eJ}{\epsilon_0 mc^3} \frac{\gamma}{(\gamma^2 - 1)^{1/2}}, \quad (6.18)$$

where $\gamma = 1 + e\phi/mc^2$ is the relativistic factor. This equation can be integrated to give

$$\frac{d\gamma}{dx} = \pm \left(\frac{2eJ}{\epsilon_0 mc^3} \sqrt{\gamma^2 - 1} - \sqrt{\gamma_m^2 - 1} \right)^{1/2}, \quad (6.19)$$

where $\gamma_m = 1 + e\phi_m/mc^2$ is the relativistic factor at the minimum potential ϕ_m inside the gap. Let x_m be the location of the minimum potential ϕ_m . It can be shown easily that $x_m = D/2$. Integrating Eq. (6.19) from x_m to $x = D$, we have

$$\left(\frac{eJ}{2\epsilon_0 mc^3} \right)^{1/2} D = g(\gamma_0), \quad (6.20)$$

where $g(\gamma_0) = \int_{\gamma_m}^{\gamma_0} (\sqrt{\gamma^2 - 1} - \sqrt{\gamma_m^2 - 1})^{-1/2} d\gamma$. Thus, the maximum current density J_R can be found as

$$J_R \equiv \frac{2\epsilon_0 mc^3}{e} \frac{G^2(\gamma_0)}{D^2}, \quad (6.21)$$

where $G(\gamma_0)$ is the maximum value of $g(\gamma_0)$, and it is found by scanning $g(\gamma_0)$ from $\gamma_m = 1$ till $\gamma_m = \gamma_0$.

From Gauss's law, we have

$$E_a = E_c = \frac{J_R T_{RD}}{2\epsilon_0}, \quad (6.22)$$

where $E_a = \frac{d\phi}{dx}|_{x=D}$ and $E_c = \frac{d\phi}{dx}|_{x=0}$ are the electric field at anode and cathode, respectively, and T_{RD} is the transit time across the gap D for the relativistic SCL electron flows. Solving Eqs. (6.19), (6.21) and (6.22), we can express T_{RD} as

$$T_{RD} \equiv \frac{2D}{c} \frac{(\sqrt{\gamma_0^2 - 1} - \sqrt{\gamma_m^2 - 1})^{1/2}}{G(\gamma_0)}. \quad (6.23)$$

By defining $\bar{\phi} = \phi/V_g$, and $\bar{x} = x/D$, the normalized electric field $a_0 = \frac{d\bar{\phi}}{d\bar{x}}|_{\bar{x}=D}$, can be described as

$$a_0 = \frac{2}{\gamma_0 - 1} G(\gamma_0) \left(\sqrt{\gamma_0^2 - 1} - \sqrt{\gamma_m^2 - 1} \right)^{1/2}. \quad (6.24)$$

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6.3.2 Relativistic Short-Pulse SCL Flows in Drift Space

Based on the primary work of steady-state relativistic SCL flow in a drift space, a simple 1D short-pulse relativistic SCL law in a drift space is developed, without including the effects of self-magnetic field (as it will require a 2D model). We solve the equivalent relativistic SCL model in the charge-filled region to obtain the short-pulse relativistic SCL law in a drift space, which is

$$J_{RM} \equiv \frac{2\epsilon_0 mc^3}{e} \frac{G^2(\gamma_\zeta)}{\zeta^2} = \frac{2\epsilon_0}{\tau_{rd}} a_\zeta \frac{\phi_\zeta}{\zeta}, \quad (6.25)$$

$$a_\zeta = \frac{2G(\gamma_\zeta)}{\gamma_\zeta - 1} \left(\sqrt{\gamma_\zeta^2 - 1} - \sqrt{\gamma_{m\zeta}^2 - 1} \right)^{1/2}, \quad (6.26)$$

where, a_ζ and $\gamma_\zeta = 1 + e\phi_\zeta/mc^2$ are, respectively, the normalized electric field and the relativistic factor at the beam-vacuum interfaces with a potential of ϕ_ζ , and ζ is the pulse width, $\gamma_{m\zeta} = 1 + e\phi_{m\zeta}/mc^2$ is the relativistic factor at the location of potential minimum of $\phi_{m\zeta}$ inside the charge filled region. With the normalized transit time across the equivalent relativistic drift space $X_d = \tau_{rd}/T_{RD} \leq 1$, Eq. (6.25) gives

$$\frac{\bar{\zeta}}{X_d} = \frac{G(\gamma_\zeta)}{G(\gamma_0)} \left(\frac{\sqrt{\gamma_0^2 - 1} - \sqrt{\gamma_m^2 - 1}}{\sqrt{\gamma_\zeta^2 - 1} - \sqrt{\gamma_{m\zeta}^2 - 1}} \right)^{1/2}, \quad (6.27)$$

The normalized short-pulse relativistic drift space SCL law (in terms of the classical long-pulse drift space limiting current density) is

$$\mu_R \equiv \frac{J_{RM}}{J_d} = \frac{9}{16\sqrt{2}} \frac{G^2(\gamma_\zeta)}{(\gamma_0 - 1)^{3/2} \bar{\zeta}^2}, \quad (6.28)$$

which recovers the classical limit at $U \ll 1$. Thus, by using Eqs. (6.26) and (6.27), and the relation of electric field continuity at the beam-vacuum interface [Eqs. (6.7) and (6.8)], we can calculate $\mu_R(X_d)$ as a function of X_d and U . Similar to the classical case, we establish a relationship between the relativistic pulse equivalent drift space transit time τ_{rd} and the pulse injection time τ_{rp} as

$$b(\gamma_0) = \frac{\tau_{rd}}{\tau_{rp}} = \frac{2\sqrt{\gamma_0^2 - 1}}{\gamma_0 G(\gamma_0)} \left(\sqrt{\gamma_0^2 - 1} - \sqrt{\gamma_m^2 - 1} \right)^{1/2}, \quad (6.29)$$

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Thus, we can easily obtain the normalized limiting current $\mu_R(X_T)$ as a function of normalized pulse injection time $X_T = \tau_{rp}/T_{RD}$ and U .

Figure 6.3 shows the calculated μ_R as a function of normalized pulse length up to $U = 100$. For a given U , μ_R increases with the decreasing value of X_T , which also scales as X_T^{-1} at small X_T . It shows clearly that the relativistic effects at $U \geq 1$ will decrease the enhancement of the short-pulse effects. Also, for a given pulse length, the pulse expansion effect decreases as U increases, $\mu_R(X_T)$ and $\mu_R(X_d)$ will merge together for $U > 10$, and the variation of electron's velocity, which is approaching speed of light, is not sensitive to space charge effect.

To check our calculation results, the transit time of a single particle computed by the PIC code (symbols) is compared with the theoretical calculations (lines) in both electrostatic (ES) and electromagnetic (EM) mode at different settings as shown in Fig. 6.4. It is found that the transit time from the PIC code is not accurate in the ES mode at relativistic energy, which is equal to the transit time at non-relativistic energy (see cross and circle symbols in Fig. 6.4). Compare with the calculations, they are only valid up to $U \approx 0.1$. The EM results from PIC code are however accurate (see inverted triangle symbols in Fig. 6.4). This comparison suggests that the particles are not properly *pushed* in the PIC code that we used in the ES mode. In order for us to verify our calculations (based on a simple electrostatic model) by using PIC simulation in the ES mode, we have used the following two methods in changing the value of the electron mass in the input file to account for the relativistic effect in the transit time. First, we assume the free electron mass is enhanced by the relativistic factor of $\gamma = 1 + U$. In doing so, the transit time of the single particle becomes more accurate (but still somehow smaller) as compared to the calculated results (see triangle symbols in Fig. 6.4). For further improvement, we adjust the electron mass in order to have the accurate transit time at each value of U (see diamond symbols in Fig. 6.4). With these modifications, the PIC simulations in the ES mode are able

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Figure 6.3: The normalized short-pulse relativistic SCL current density in drift space μ_R , as a function of (a) the normalized pulse injection duration $X_{rT} = \tau_{rp}/T_{rd}$ for $U = 0.01$ to 100 (top to bottom), and (b) the normalized electron beam voltage U for $X_{rT} = 0.001$ to 1 (long pulse). The solid and dashed lines represent, respectively, the results with and without pulse expansion. The symbols in (b) illustrate the PIC simulation results in the electrostatic mode: cross is with no adjustment, triangle is with γm_e , and diamond is with adjustment to have the same transit time to the theoretical value.

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to confirm our calculations of the relativistic short pulse SCL electron flow in drift space as shown in Fig. 6.3b.

Figure 6.4: The comparison between the transit time of a single particle in a 1 cm gap obtained from PIC simulation (symbols) and calculations (lines) as a function of $U = 0.001$ to 1000.

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In conclusion, 1D short-pulse models of classical and relativistic SCL law in a drift space have been developed. It is found that the enhancement of the short-pulse limiting current density over the long-pulse SCL law is proportional to the inverse power of the normalized pulse length in both classical and relativistic regimes, when the pulse length is much smaller than the gap transit time. The transition from the relativistic models to the classical limits is demonstrated.

We should note that the simple short-pulse SCL models for drift space presented here are strictly a 1D models, in which other practical issues in a hotoinjector design

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are ignored, such as complicated electrode geometry and external focusing magnetic fields. However, the scaling laws developed here may be used as a first estimate in the design for the development of short-pulse laser-driven high current photocathodes in FEL and other applications.

CHAPTER 7

TRANSITION FROM FIELD EMISSION TO SHORT-PULSE CLASSICAL CL LAW

7.1 Introduction

Vacuum microelectronics offers exciting possibilities in miniature coherent radiation sources [73], flat panel displays [74], and ultrafast logic circuits [75]. In vacuum electronic devices, diodes are usually used to accelerate the electron beams. In previous chapters, we deal with ideal diodes. We assumed there are unlimited amount of electrons that can be provided to our diodes. However, this may not be the case when we use practical cathodes or emitters to supply electrons. Thus, it is crucial to consider the electron emission process. We will focus on field emission from cold cathode only, which is a quantum mechanical tunneling of electrons through the potential barrier from the surface of the metal, and is described by the well-known Fowler-Nordheim equation [53, 76, 77, 78].

Field emission process is self-regulating. As the surface electric field increases, the emission current increases, leading to a charge buildup which in turn limits the

7.2 Formulation

surface electric field. In fact, when this happens, the space charge effects become important and we expect that the voltage-current ($V - I$) characteristics converges into the Child-Langmuir law [2]. The long-pulse (steady-state) transition problem has been studied by several authors [54, 55, 56]. However, under short-pulse condition where the injection electron pulse length (τ) is less than the electron gap transit time, the transition model between FN law and short-pulse classical CL law need to be reviewed carefully.

7.2 Formulation

In the one dimensional model, the cathode is located at $x = 0$ and the anode at $x = D$, held at voltage V_g with respect to the cathode. We assume that the electrons are emitted from the cathode with negligible initial velocity. And the electron pulse is assumed to extend from cathode to a distance of ζ inside the vacuum gap. Applying Poisson's equation and conservation of energy in the charge-filled region, we can easily derive

$$\frac{d^2\phi}{dx^2} = \frac{J}{\epsilon_0} \sqrt{\frac{m}{2e\phi}}, \quad (7.1)$$

where e and m are the charge and mass of the electron, respectively, ϵ_0 is the free space permittivity, and ϕ is the potential of the electrons in the x direction in the gap.

Multiplying both side of the above equation by $d\phi/dx$, the resulted equation is integrated to get

$$\frac{d\phi}{dx} = \left(\frac{4J}{\epsilon_0} \sqrt{\frac{m}{2e}} \sqrt{\phi} + E_S^2 \right)^{1/2}, \quad (7.2)$$

where $E_S = d\phi/dx|_{x=0}$ is the surface electric field at cathode. Based on Eq. (7.2), we can express the electric field at the beam-vacuum interface (equivalent anode) in the

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normalized form as

$$a_\zeta = \left. \frac{d\bar{\phi}}{d\bar{x}} \right|_{x=\zeta} = \sqrt{\frac{16}{9}\mu_\zeta + c_\zeta^2}, \quad (7.3)$$

where $\bar{\phi} = \phi/\phi_\zeta$, $\bar{x} = x/\zeta$, $c_\zeta = E_S\zeta/\phi_\zeta$ is the normalized cathode surface electric field, and $\mu_\zeta = J/J_{CL}[\zeta, \phi_\zeta]$ is the current density normalized over the steady-state classical Child-Langmuir law $J_{CL}[\zeta, \phi_\zeta]$ in the charge-filled equivalent diode.

By integrating Eq. (7.2) from cathode $x = 0$ to the equivalent anode $x = \zeta$, we could express the current density J as

$$J = \frac{\epsilon_0}{4D^2} \sqrt{\frac{2e}{m}} \left[\int_0^{\phi_\zeta} (\phi^{1/2} + \frac{\epsilon_0}{4J} \sqrt{\frac{2e}{m}} E_S^2)^{-1/2} d\phi \right]^2, \quad (7.4)$$

After normalizing over CL law in the equivalent diode, Eq. (7.4) becomes

$$\mu_\zeta = \frac{J}{J_{CL}[\zeta, \phi_\zeta]} = \frac{9}{16} \left[\int_0^1 (\sqrt{\bar{\phi}} + \frac{9}{16} \frac{c_\zeta^2}{\mu_\zeta})^{-1/2} d\bar{\phi} \right]^2, \quad (7.5)$$

For electrons extracted from field emission, the current density would obey the Fowler-Nordheim equation [53, 76, 77, 78],

$$J = \frac{AE_S^2}{\psi t(y)^2} \exp\left(\frac{-Bv(y)\psi^{3/2}}{E_S}\right), \quad (7.6)$$

where A and B are the Fowler-Nordheim coefficients, and ψ is the effective work function assumed to be a constant. $t(y)$ and $v(y)$ were given by Ref [79] as shown below:

$$t(y)^2 = 1.1, \quad (7.7)$$

$$v(y) = 0.95 - y^2, \quad (7.8)$$

$$y = 3.79 \times 10^{-5} E_S^{1/2} / \psi, \quad (7.9)$$

where y is the Schottky lowering of the effective work function barrier. The current density J is normalized over classical CL law in the equivalent diode as

$$\mu_\zeta = \frac{J}{J_{CL}[\zeta, \phi_\zeta]} = \frac{9Ac_\zeta^2\phi_\zeta^{1/2}}{4\epsilon_0\psi t(y)^2} \sqrt{\frac{m}{2e}} \exp\left(\frac{-Bv(y)\psi^{3/2}\zeta}{c_\zeta\phi_\zeta}\right), \quad (7.10)$$

7.2 Formulation

Using Gauss's Law and the electric field continuity at the beam-vacuum interface [Eq. (5.15)-(5.16)], we can obtain

$$\frac{\bar{\zeta}}{X_T \bar{\phi}_\zeta^{1/2}} = \frac{\mu_\zeta a_0 - c_0}{\mu_0 a_\zeta - c_\zeta}, \quad (7.11)$$

$$\bar{\phi}_\zeta = \frac{\bar{\zeta}}{a_\zeta + (1 - a_\zeta) \bar{\zeta}}, \quad (7.12)$$

$$\Gamma_T \equiv \frac{J_{Trans}}{J_{CL}[D, V_g]} = \mu_\zeta \frac{\bar{\phi}_\zeta^{3/2}}{\bar{\zeta}^2}, \quad (7.13)$$

where $\bar{\phi}_\zeta = \phi_\zeta/V_g$ is the normalized potential at the beam front, $\bar{\zeta} = \zeta/D$ is the normalized position of the beam front, and Γ_T is the normalized maximum allowed current density from field emission under short-pulse classical space-charge-limited (SCL) condition, in terms of the 1D classical steady-state CL law, and $X_T = \tau/T_0 \leq 1$ is the normalized pulse length, in which the gap transit time $T_0 = (3/4)[(a_0 - c_0)/\mu_0]T_{CL}$ is found by setting $\phi_\zeta = V_g$ and $\zeta = D$ in Eq. (5.15), and (μ_0, a_0, c_0) are the corresponding values of $(\mu_\zeta, a_\zeta, c_\zeta)$ calculated by using Eqs.(7.5) and (7.10) at the long-pulse limit of $\tau = T_0$.

One example of the iteration routines used to solve Γ_T is shown as follows:

1. For a given V_g and D , we guess a initial value for μ_0 ,
2. Set $\phi_\zeta = V_g$, $D = \zeta$, and $\mu_\zeta = \mu_0$,
3. From Eq. (7.10), we get c_ζ ,
4. Using c_ζ with Eq. (7.5) to calculate μ_ζ ,
5. Repeat steps 3-4 till converge occurs, so far we could obtain (μ_0, a_0, c_0) ,
6. For a given X_T , we guess $\bar{\zeta}$, $\bar{\phi}_\zeta$ using the classical formulas Eqs. (3.15) and (3.16), therefore to get ζ and ϕ_ζ ,
7. Use step 5 to get $(\mu_\zeta, a_\zeta, c_\zeta)$,

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8. Insert $(\mu_\zeta, a_\zeta, c_\zeta)$ into Eq. (7.12) to get new $\bar{\phi}_\zeta$,
9. Insert the obtained $\bar{\phi}_\zeta$ into Eq. (7.11) to get new $\bar{\zeta}$,
10. Repeat steps 7-9 till converge occurs,
11. Calculate Γ_T from Eq. (7.13).

Figure 7.1: Maximum allowed current density J_{Trans} in a diode with $D = 1 \mu m$ and $\psi = 2 eV$, as a function of applied gap voltage V_g for $X_T = \tau/T_0 = 0.001$ to 1 (top to bottom). Dotted line is FN relation, and dashed lines are classical short-pulse CL law.

7.3 Results

To illustrate the transition from the source-limited (field emission) condition to SCL (space-charge-limited) condition at small electric field ($< 1 V/nm$), we choose an example of diode with gap separation $D = 1 \mu m$ and cathode with a work function of 2.0 eV. Figure 7.1 shows the calculated maximum allowed current density J_{Trans}

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Figure 7.2: Normalized maximum allowed current density (a) $\Gamma_T = J_{\text{Trans}}[X_T]/J_{\text{CL}}[1]$, (b) $\Gamma_{T1} = J_{\text{Trans}}[X_T]/J_{\text{Trans}}[1]$ as a function of normalized pulse length X_T for $V_g = 4.0$ to 1.5kV (top to bottom). Dashed line is the classical short-pulse CL law.

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as a function of applied gap voltage under various short pulse durations. At low gap voltage, the curves asymptotically approaches the FN equation, which indicates that the diode is operating at source-limited condition. While as the gap voltage increases, the maximum current density gradually follows the SCL predictions. Furthermore, the short-pulse effects will enhance the value of limiting current, more significantly at shorter pulse length. The effects of short pulse are further clarified in Fig. 7.2. At low gap voltage, the short-pulse enhancement will become less than the classical short-pulse CL law, which is proportional to X_T^{-1} . At the same normalized pulse length X_T , the short-pulse effects will decrease when the applied gap voltage decreases. When the applied gap voltage is very low, around the cut-off value, there will be little enhancement of the maximum current density from short-pulse effects.

7.4 Summary

In conclusion, a simple model of transition from field emission to short-pulse classical CL law has been presented. It is found that the enhancement of the short-pulse effects on the limiting current density decreases when the applied gap voltage decreases. The established model gives an immediate determination of the importance of space charge effects under any pulse length, and work function.

However, the simple analysis given omits many physically important aspects, such as the geometrical effects [80], the dependence of FN coefficients A and B on the electric field [76, 80], and the time-dependence model of FN law. These are important subjects but are beyond the scope of this work.

CHAPTER 8

CONCLUSIONS AND FUTURE WORKS**8.1 Conclusions**

This report concentrates on the development of 1D short-pulse space-charge-limiting (SCL) model for the classical, weakly relativistic and quantum regimes. Here, we are interested in the SCL electron flow in the planar configuration. The effects of the finite pulse length of a SCL electron pulse emitted from a planar diode over a wide range of applied voltages, gap spacings, and pulse durations have been investigated. The results from the analytical short-pulse model in the classical and relativistic regimes are consistent with particle-in-cell (PIC) simulations. The transit time of the electron flows across a gap in various regimes (classical, relativistic, quantum) has also been formulated in analytical form.

Firstly, we reviewed the well-known classical steady-state and short-pulse CL law. The steady-state CL law scales according to $3/2$'s power of gap voltage ($V_g^{3/2}$) and the inverse squared power of gap spacing (D^{-2}). At the short-pulse limit, where the pulse length τ is less than the electron transit time across the gap, the maximum allowed current density that can be transported through the gap is enhanced by a

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large factor compared to the steady-state (long pulse) case. At $X_{CL} \ll 1$, the short-pulse CL law increase to the inverse power of the normalized pulse length (X_{CL}^{-1}). At $X_{CL} = 1$, the classical short-pulse CL law recovers the classical long-pulse CL law.

Next, with a review of the 1D relativistic steady-state CL law, we developed a new 1D short-pulse model of relativistic Child-Langmuir law, for SCL flows in the relativistic regime. Relativistic effects become important when the voltage difference across the gap is comparable to the electron rest mass, which decreases the enhancement of short-pulse CL law. It is found that the enhancement of the short-pulse CL law over the long-pulse CL law is proportional to the inverse power of the normalized pulse length in relativistic regime, when the pulse length is much smaller than the gap transit time. The smooth transition from the relativistic model to the classical limits is demonstrated. Besides, A 2D PIC simulation code, MAGIC2D has been used to verify the relativistic short-pulse CL law. The comparison shows good agreement in the weakly relativistic region, where the applied voltage is comparable to the electron rest mass energy. However, in the deep relativistic regime, where the applied voltage is much larger than the electron rest mass, the self-magnetic field of the relativistic electron flows becomes important and our 1D short-pulse relativistic model is no longer valid.

Furthermore, we developed a 1D short-pulse model of quantum Child-Langmuir law. Quantum effects are important when the pulse length is ultrashort (like < 1 ps) when the size of electron pulse is comparable to the electron de Broglie wavelength, and thus the classical CL current density is increased by a large factor due to electron tunneling through the space charge field and the exchange-correlation effects. It is found that the enhancement of the short-pulse CL law over the long-pulse CL law is also proportional to the inverse power of the normalized pulse length in quantum regime, when the pulse length is much smaller than the gap transit time. The transition from the quantum models to the classical limits is demonstrated.

8.2 Planning for Further Research

Based on the above findings, we conclude that the enhancement of the short-pulse CL law over the long-pulse CL law is proportional to the inverse power of the normalized pulse length in all regimes (classical, relativistic, and quantum), when the pulse length is much smaller than the gap transit time. During the formulation, we assume unlimited electron current can be supplied and ignore the process of the electron emission at the cathode (either photoemission or optical field emission). Also, we assume that the initial energy of the injected electrons are zero in all the cases. It is worth to note that the simple short-pulse CL law models presented are strictly 1D models, where other practical issues in a photoinjector design are ignored, such as complicated electrode geometry and external focusing magnetic fields. However, the scaling laws developed here may be used as a first estimate in the design or even as an emission algorithm in the traditional gun codes for the development of femtosecond laser-driven high current photocathodes in FEL and other applications.

In addition, 1D short-pulse models of classical and relativistic SCL law in a drift space have been developed. It is found that the inverse power scaling law still holds in both cases.

Finally, a simple model of transition from field emission to short-pulse classical CL law is presented. It is found that the enhancement of the short-pulse effects on the limiting current density decreases when the applied gap voltage decreases. The established model gives an immediate determination of the importance of space charge effects under any pulse length.

8.2 Planning for Further Research

Our research focus on the investigation of short-pulse space-charge-limited flows in the classical, weakly relativistic and quantum regimes. So far, we developed the simple 1D planar short-pulse SCL models in all the three regimes, extended the short-pulse

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models in drift space, and included the field emission process. However, there are some issues have not been studied. Some possible extensions from the topics of this dissertation are given as follows:

- Our models presented in this report are strictly 1D model, where practical issue like complicated electrode geometry in a photoinjector design is ignored. In our following research work, these electrode geometry effects can be included in the developed 1D model. It is of interest to extend our developed 1D short-pulse relativistic and quantum models into 2D short-pulse models, where the cathode and anode are with finite emission area.
- It is also possible but not so easy to extend our short-pulse relativistic SCL model to include the effects of self-magnetic field of the relativistic electron flows, which become important especially at deep relativistic regime where the applied voltage is much larger than the electron rest mass.
- In our transition model, we only focus on field emission. While other emission mechanics such as thermionic emission, photoemission, and optical field emission could also be included in the development of the SCL models. And the transition from source-limited emission to space-charge-limited emission in relativistic and quantum regimes under short-pulse condition may also be studied in later research.
- The single short-pulse consideration is sufficient in most practical scenarios as the frequency of the typical laser that is used to excite the photocathodes are usually not high enough to create more than one pulse in the acceleration gap at the same time. However as the development of high repetition rate ultrafast lasers become mature, it is totally possible to have multiple electron pulses flowing across the acceleration gap. Thus, multi-short-pulse models may be required.

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- In addition to the unipolar flows we considered so far, there are also bipolar flows in vacuum devices where both negative electrons and positive ions presented, for example, in a realistic high power vacuum microwave (HPM) device using the explosive field emission cathode. Our short-pulse models can also be extended to the case of bipolar flows.

AUTHOR'S PUBLICATIONS

Journal Papers

1. L. K. Ang and P. Zhang, "Ultrashort-pulse Child-Langmuir law in the quantum and relativistic regimes", *Phys. Rev. Lett.*, **98**:164802 (2007).
2. P. Zhang, W. S. Koh, L. K. Ang and S. H. Chen, "Short-pulse space-charge-limited electron flows in a drift space", *Phys. Plasmas*, **15**:063105 (2008).

Conference Papers

1. P. Zhang and L. K. Ang, "Ultrashort electron pulse at space-charge-limited condition", *2007 IEEE Pulsed Power and Plasma Science Conference, Albuquerque, New Mexico, USA*, 5E8 (2007) [supported by IEEE student travel grant](oral)
2. P. Zhang, W. S. Koh and L. K. Ang, "Space-charge-limited electron flow in a drift space", *2007 IEEE Pulsed Power and Plasma Science Conference, Albuquerque, New Mexico, USA*, 3P36 (2007) [supported by IEEE student travel grant](poster)

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