Blind source separation algorithms for mitigating co-channel interference and noise in wireless communication systems

Ajay Kumaraswamy Kattepur

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**BLIND SOURCE SEPARATION ALGORITHMS FOR
MITIGATING CO-CHANNEL INTERFERENCE AND NOISE
IN WIRELESS COMMUNICATION SYSTEMS**

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

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“The heights by great men reached and kept were not attained by sudden flight, but they, while their companions slept, were toiling upward in the night.” - Henry Wadsworth Longfellow.

Abstract

Blind Source Separation (BSS) algorithms have been applied with considerable success in a variety of fields. In case of communication signals, these BSS algorithms can be applied for blind beamforming and channel estimation. In this thesis, these BSS algorithms are employed to solve the co-channel interference problem in wireless communication systems.

Algorithms such as Joint Approximate Diagonalization of Eigen-matrices (JADE) and Analytical Constant Modulus Algorithm (ACMA) can handle source separation in the complex domain. We propose a modified version of the JADE source separation algorithm based on the alternating row updating of the un-mixing matrix. This proposed algorithm is called the Alternating Row Diagonalization - Joint Approximate Diagonalization of Eigen-matrices (ARD-JADE). Simulation results show improvement in the Bit Error Rate (BER) and the output Signal to Interference plus Noise ratio (SINR) by application of these source separation algorithms for various SNR levels.

Algorithms developed for application in communication systems must not only have the capability of working in the complex domain, but also have relatively low computational complexity. We propose a Fast Fourier Transform (FFT) based algorithm called Feedback Independent Component Analysis (FEBICA) that is able to blindly separate complex modulated digital signals. By applying this algorithm to communication signals,

it is observed that it has the advantages of SINR gain improvement as well as low computational complexity. The performance of the FEBICA algorithm has also been compared with conventional algorithms like JADE, ACMA and FastICA in order to evaluate its relative performance. The FEBICA is shown to be efficient in both signal to interference plus noise ratio (SINR) improvement as well as computational complexity.

Blind estimation techniques for determining the number of sources are examined. Further analysis of the effect of Doppler Shifts on source separation algorithms is also performed. It is seen that by introducing a slight Doppler shifts in the mixing model, superior source separation performance can be obtained, especially in the ill-conditioned case. The performance improvements due to application of these Doppler shifts have been discussed in detail. Extension of this technique for source tracking is also shown.

Another aspect to consider is the fidelity of the BSS demodulated output (in terms of SINR) specially in mobile telephony, tele-conferencing and Voice over Internet Protocol (VoIP) applications. Using speech signals as an example, we show that by using the diversity provided by an overdetermined case, superior output may be produced. By combining this overdetermined BSS algorithm with some novel noise reduction techniques, the capacity to separate sources even under low signal to noise ratio conditions is shown to be possible. Thus, optimal spatio-temporal filtering is possible by combining the output of conventional BSS with noise reduction techniques like the Minimum Distortion Noise Reduction (MDNR) algorithm.

Other problems of source separation like single channel BSS, reduction of reverberation and widely separated sensors have also been examined. These are promising topics and have the potential for many applications in the communication domain. When sources are widely spaced, there are increased delay complications seen at the sensors. These

have been examined by parallel factor analysis method to resolve individual sources after mixing and delay. The relationship between optimal sensor positioning and single-channel source separation is also shown.

Thus, with the advantages of improvements in bit error rate, output SINR and computational complexity, these source separation algorithms can be applied to communication systems for mitigating co-channel interference and noise by blind beamforming.

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List of Symbols

$\mathbf{x}_1(t)$	Observed mixed signal vector at the first sensor at time t	3
$\mathbf{s}_1(t)$	Original signal vector from the first source at time t	3
$\mathbf{a}_{ij}$	Mixing coefficients in the mixing matrix	3
$p(m_1, m_2)$	Joint probability density function (pdf) of m_1 and m_2	3
$\mathbf{E}\{m\}$	Expectation value for random variable m	4
$\text{kurt}(y)$	Kurtosis of random variable y	5
$\mathbf{H}$	Entropy	6
$\mathbf{J}$	Negentropy	6
$\mathbf{I}$	Mutual information	7
$\mathbf{X}(t)$	Observed row-wise mixed signal matrix at time t	8
$\mathbf{A}$	Instantaneous mixing matrix	8
$\mathbf{S}(t)$	Row-wise source signal matrix at time t	8
$\mathbf{N}(t)$	Additional white Gaussian noise	8
$\mathbf{W}$	Unmixing matrix	8
$\mathbf{Y}(t)$	Estimated matrix of separated sources	8
$\mathbf{E}\{\mathbf{X}\mathbf{X}^T\}$	Covariance matrix	9
σ	Noise variance	18
$\mathbf{Z}(t)$	Whitened mixed matrix	18
$\mathbf{Q}_z$	Corresponding fourth order cumulants of whitened matrix	18
SVD()	Singular value decomposition function	19
$G(\mathbf{W}\mathbf{X})$	Updating contrast function	21
η	Adaptation step size for optimization	22
$\mathbf{L}$	Log-likelihood function	24
$\mathbf{W}_{fb}$	Feedback weight matrix	44

γ	Fast Fourier transform (FFT)	45
μ	Row-wise mean of a matrix	45
ξ	Output of the non-linear transformation	45
ι	Iteration count	46
$\Delta \mathbf{W}_\iota$	Weight update increment with every iteration	46
Ψ	Mean square error	49
Ω	Cost function to be minimized	50
AIC	Akaike information theoretic criterion	61
GLE	Gerschgorin Likelihood Estimator	62
$\mathbf{Z}(\tau, \kappa)$	Short time discrete Fourier transform (STDFT)	63
$\theta_{(\mathbf{Z}(\tau, \kappa))}$	Phase information in STDFT domain	64
$\mathbf{h}_m$	Temporal filter coefficients	77
$\mathbf{Q}_m$	Spatio-temporal prediction matrix	78
$\text{SINR}_{\text{hybrid}}$	Signal to interference plus noise ratio of hybrid model	82
e_{interf}	Deformation of the sources due to interference of unwanted sources	85
e_{noise}	Allowed deformation of the perturbing noise	85
e_{artif}	Deformations induced due to the separation process	85
$\downarrow \phi$	Moves each element of matrix ϕ rows down	97
$\rightarrow \tau$	Moves each element in the matrix τ columns to the right	97

Chapter 1

Introduction

1.1 Blind Source Separation (BSS)

Blind Source Separation (BSS) algorithms are a category of signal processing techniques that have been applied in various application areas including analysis of EEG data, magnetic resonance imaging, speech signal processing and so on [1]. In case of blind source separation algorithms, it is assumed that minimal *a priori* information is available. An offshoot of blind source separation is the process of Independent Component Analysis (ICA) where the signals are extracted based on the assumption that the source signals are statistically independent [2]. The goal of ICA is to recover independent sources given only sensor observations that are unknown linear mixtures of the unknown independent source signals. In contrast to correlation-based transformations such as Principal Component Analysis (PCA) [3], ICA not only decorrelates the signals (based on second-order statistics) but also reduces higher-order statistical dependencies, attempting to make the output separated signals as independent as possible.

1.1.1 Overview of BSS Approaches

According to [4] the source separation algorithms can be grouped into four fundamental approaches as given below. All these belong to the category of unsupervised learning algorithms. These algorithms try to discover a feature underlying a data set and use this for extracting useful representations of the data.

- Cost function that exploits the measure of statistical independence, non-Gaussianity or sparseness of the signals. For this, higher order statistics are used to solve the source separation problem.
- If the sources have non-vanishing temporal correlations (correlation of the sensor's observations with respect to the time domain), the second order statistics can be used. This relaxes the condition of statistical independence of the sources.
- Using the non-stationary property of the sources leading to second order statistics.
- Exploiting the time, frequency or the space-time-frequency diversity. Usually, the components are interpreted as being localized, sparse (having few non-zero elements) and structured (showing independence in T-F domain) signals in the time-frequency plane.

Consider the following two sensor outputs:

$$\mathbf{x}_1(t) = \mathbf{a}_{11}\mathbf{s}_1 + \mathbf{a}_{12}\mathbf{s}_2 \quad (1.1)$$

$$\mathbf{x}_2(t) = \mathbf{a}_{21}\mathbf{s}_1 + \mathbf{a}_{22}\mathbf{s}_2 \quad (1.2)$$

After receiving the signals $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$, the objective of ICA is to determine the original signals $\mathbf{s}_1(t)$ and $\mathbf{s}_2(t)$. This is called the cocktail party problem. If the parameters $\mathbf{a}_{ij}$ are known, this can be solved by a variety of classical techniques. However, having no prior information about the coefficients $\mathbf{a}_{ij}$, information about the statistical properties of the original signals will be needed. By assuming that the original signals are *statistically independent*, the process of independent component analysis can estimate the source signals that are very close to their original counterparts [1]. It must also be assumed that the signals have *non-Gaussian* distributions.

To define the concept of independence, considering two scalar-valued random variables m_1 and m_2 , they are said to be independent if information on the value of m_1 does not provide any information regarding the value of m_2 and vice versa. If we denote the joint probability density function (pdf) of m_1 and m_2 as $p(m_1, m_2)$ and the marginal pdf of random variables m_1 and m_2 as follows:

$$p_1(m_1) = \int p(m_1, m_2) dm_2 \quad (1.3)$$

$$p_2(m_2) = \int p(m_1, m_2) dm_1 \quad (1.4)$$

Then, m_1 and m_2 are independent only when the joint pdf is factorisable in the following way:

$$p(m_1, m_2) = p_1(m_1)p_2(m_2) \quad (1.5)$$

This definition can be extended to a number n of random variables where the joint density must be a product of n terms. A weaker form of independence is uncorrelatedness. If variables are independent, they are uncorrelated, but not vice versa. The variables are

said to be uncorrelated if their mutual co-variance is zero:

$$\mathbf{E}\{m_1, m_2\} - \mathbf{E}\{m_1\}\mathbf{E}\{m_2\} = 0 \quad (1.6)$$

where $\mathbf{E}\{m\}$ refers to the expectation value for the random variable m .

Another fundamental restriction in ICA is that the independent components should be non-Gaussian. If the two variables m_1 and m_2 are Gaussian, uncorrelated and of unit variance, the joint pdf is completely symmetric.

$$p(m_1, m_2) = \frac{1}{2\pi} \exp\left(-\frac{m_1^2 + m_2^2}{2}\right) \quad (1.7)$$

So, no information about the directions of the columns of the mixing matrix may be

Figure 1.1: Distribution of two independent Gaussian variables [1].

extracted based on the distribution (Fig. 1.1) making it impossible to estimate the independent components. In case of Gaussian variables, the ICA model can be estimated up to an orthogonal transformation. If only one of the original sources is Gaussian, it is still possible to separate the mixtures based on diversity offered by the other non-Gaussian sources [1].

1.1.2 Cost Functions for BSS

Intuitively speaking, the key to estimating the ICA model is non-Gaussianity. The fundamental principle used by most BSS algorithms is finding transformations that maximize the non-Gaussianity of the signals. The commonly used measures of non-Gaussianity are:

- **Kurtosis** [1] [5] is a classical measure of the non-Gaussianity based on the fourth order cumulants. For a random variable y , the kurtosis $\text{kurt}(y)$ can be given by:

$$\text{kurt}(y) = \mathbf{E} \{y^4\} - 3(\mathbf{E} \{y^2\})^2 \quad (1.8)$$

A Gaussian distribution (which sometimes is referred to as normal distribution) has a normalized kurtosis equal to zero (Mesokurtic). For most non-Gaussian variables, kurtosis is nonzero. Since Gaussian distribution have normalized kurtosis equal to zero, it can be used as a reference point to know distributions that are below Gaussian distribution (sub-Gaussian, they have negative kurtosis called Platykurtic) and those that are above Gaussian distribution (super-Gaussian, these have positive kurtosis called Leptokurtic). A leptokurtic distribution has a more acute shape around zero, which implies a higher probability than a Gaussian distribution near the mean; and a “long tail” which entails a higher probability than a Gaussian distribution at the extreme values. A good example of a leptokurtic distribution is the Laplacian distribution. A platykurtic distribution is less acute around the mean, this implies a lower probability than Gaussian distribution near the mean and a “small tail”, that is, lower probability than a Gaussian distribution at the extreme values. A typical example of a platykurtic distribution is the Uniform Distribution. Kurtosis is simplistic in nature both computationally and theoretically leading to

its wide use as a measure of non-Gaussianity. The main problem is that kurtosis can be very sensitive to outliers. Its value may depend on only a few observations in the tails of the distribution, which can be erroneous or irrelevant observations. In other words, kurtosis is not a robust measure of non-Gaussianity.

- **Negentropy** [1] [5] is based on the information-theoretic quantity of differential entropy and is another useful measure of non-Gaussianity. The more “random” a variable, the larger is its entropy. A Gaussian variable has the largest entropy among all random variables of equal variance and thus, can be used as a measure of the non-Gaussianity. The entropy $\mathbf{H}$ and the negentropy $\mathbf{J}$ of a random variable y are given by [1]

$$\mathbf{H}(y) = - \sum_i P(y = a_i) \log P(y = a_i) \quad (1.9)$$

$$\mathbf{J}(y) = \mathbf{H}(y_{gauss}) - \mathbf{H}(y) \quad (1.10)$$

where $P(y)$ represents the probability mass function, a_i are all the possible values of y and y_{gauss} is a Gaussian random variable with the same covariance matrix as y . As the Gaussian variable has the highest entropy, the negentropy of a variable y is always non-negative or zero (for the case when y has a Gaussian distribution). The drawback of this robust technique is the high computational complexity. A few approximations of negentropy can be used to overcome the computational load.

- **Approximations of Negentropy** [1] [5] have been used to make up for the difficult estimation of theoretical negentropy. An approximation developed by [8] is based

on the maximum entropy principle:

$$\mathbf{J}(y) \approx \sum_{i=1}^P k_i [\mathbf{E}\{G_i(y)\} - \mathbf{E}\{G_i(\nu)\}]^2 \quad (1.11)$$

where k_i is a positive constant, ν is a Gaussian random variable of zero mean and unit covariance, y is a random variable with zero mean and unit covariance and G_i are nonquadratic functions. Taking $G(y) = y^4$, we can obtain a kurtosis-based approximation. By choosing G that does not grow too fast, the estimates of negentropy are quite accurate. These approximations are a good compromise between kurtosis and negentropy. They are conceptually simple, low computational time and quite robust.

Another cost function, inspired by information theory is minimization of **Mutual Information** [6]. The mutual information $\mathbf{I}$ between m (scalar) random variables, $y_i, i = 1 \dots m$ with respect to the desired random variable y is follows:

$$\mathbf{I}(y_1, y_2, \dots, y_m) = \sum_{i=1}^m \mathbf{H}(y_i) - \mathbf{H}(y) \quad (1.12)$$

Mutual information is a natural measure of the dependence between random variables. Finding an invertible unmixing transformation that minimizes the mutual information is roughly equivalent to finding directions in which the negentropy is maximized. More precisely, it is roughly equivalent to finding 1-D subspaces such that the projections in those subspaces have maximum negentropy. ICA estimation by minimization of mutual information is equivalent to maximizing the sum of non-Gaussianities of the estimates, when the estimates are constrained to be uncorrelated. Thus, maximization of mutual

information gives another justification of the more heuristic idea of finding maximally non-Gaussian directions.

1.1.3 Source Separation Models

The two schemes for BSS models involve either *instantaneous* or *convolutive* mixtures. In the case of instantaneous mixtures, the source separation problem in the time domain is described by

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t) \quad (1.13)$$

Here, $\mathbf{X}(t)$ is the observed mixed signal matrix (of row-wise signals at time t), $\mathbf{A}$ is the mixing matrix, $\mathbf{S}(t)$ is the source signal matrix and $\mathbf{N}(t)$ is the additional noise. The objective of any blind source separation algorithm is to estimate an un-mixing matrix $\mathbf{W}$ such that the resulting signal $\mathbf{Y}(t)$ is a close estimate of the original source signal $\mathbf{S}(t)$.

$$\mathbf{Y}(t) = \mathbf{W}\mathbf{X}(t) \quad (1.14)$$

In the case of convolutive mixtures, the observed signals are assumed to be combinations of delayed and filtered versions of the independent components. The convolutive mixing process can be given by:

$$\mathbf{x}(t) = A * \mathbf{s}(t) \quad (1.15)$$

where $*$ denotes the convolution operator, $\mathbf{x}(t)$ is the mixed signal observation vector due to the convolutive mixing process A and $\mathbf{s}(t)$ is the separated component source vector. As, in the instantaneous case, the convolutive mixture model also makes use of certain assumptions about the independent components such as approximate distributions and

statistics.

Consider the instantaneous mixing model of source separation where $\mathbf{X}$ is the observed data, $\mathbf{A}$ is the mixing matrix and $\mathbf{S}$ is the matrix of independent components that are to be estimated:

$$\mathbf{X} = \mathbf{A}\mathbf{S} \quad (1.16)$$

These basic steps involved in source separation are shown in Fig. 1.2. The first pre-processing step is to center the data. This is done by removing the mean vector making $\mathbf{X}$ a zero mean variable so as to simplify the processing of the independent variables $\mathbf{S}$. After the estimation of the components, the mean can be added back to the centered estimates of $\mathbf{S}$.

The whitening of data is an important pre-processing step in a variety of BSS methods. By whitening (before the application of source separation), we transform the observed data linearly so as to decorrelate the sensor outputs and normalize their variance to unity. A zero mean random vector $\mathbf{z} = [\mathbf{z}_1 \cdots \mathbf{z}_N]^T$ is said to be white if its elements $\mathbf{z}_i$ are uncorrelated and have unit variances. In other words, to whiten, we seek a linear transformation matrix $\mathbf{V}$ of the data $\mathbf{X}$ such that the transformed data $\mathbf{z}$ is white. Perhaps the most popular method for achieving this is through eigenvalue decomposition of the covariance matrix $\mathbf{E}\{\mathbf{X}\mathbf{X}^T\} = \mathbf{F}\mathbf{D}\mathbf{F}^T$, where $\mathbf{F}$ is the orthogonal matrix (inner product is zero or perpendicular to each other) of eigenvectors of $\mathbf{E}\{\mathbf{X}\mathbf{X}^T\}$ and $\mathbf{D}$ is the diagonal matrix with the corresponding eigenvalues along the diagonal. Whitening is performed as [5] [6]:

$$\mathbf{Z} = \mathbf{V}\mathbf{X} = \mathbf{F}\mathbf{D}^{-\frac{1}{2}}\mathbf{F}^T\mathbf{X} \quad (1.17)$$

It must be noted that this whitening matrix is by no means unique. The whitening process

is simply a linear change of coordinate of the mixed data. The usefulness of this lies in the fact that we can now restrict our search for mixing matrices to orthogonal matrices. After this, the preprocessed data is passed through the BSS algorithm that then separates the components. For noisy data some filtering can be desirable as a post processing step.

By finding non-Gaussian projection pursuit directions, ICA is able to recover the original sources which are statistically independent. Projection pursuit [9] is a technique developed in statistics for finding useful projections of multidimensional data. Such projections can then be used for optimal visualization of the data. It has been argued by [9] that the Gaussian distribution is the least interesting one, and that the most interesting directions are those that show the least Gaussian distribution. This is exactly what is done in the ICA model. Thus, in the general formulation, ICA can be considered a variant of projection pursuit. Assuming that those dimensions of the space that are not spanned by the independent components are filled by Gaussian noise, we see that computing the non-Gaussian projection pursuit directions, we effectively estimate the independent components. When all the non-Gaussian directions have been found, all the independent components have been estimated. Such a procedure can be interpreted as a hybrid of projection pursuit and ICA [1].

Intuitively, the ICA rotates the whitened matrix back to the original by minimizing the Gaussianity of the data. This property comes from the central limit theorem which states that any linear mixture of two independent random variables is more Gaussian than the original variables. Since ICA separates sources by maximizing their non-Gaussianity, perfect Gaussian sources cannot be separated. Even when the sources are not independent, ICA can find a space where they are maximally independent. Further analysis of the separability and uniqueness of the BSS output has been studied in detail in [10], [11].

Figure 1.2: Basic processes involved during Source Separation.

This shows that the separation is indeed unique when certain criterion are met in the source mixtures.

Though there are a variety of BSS algorithms available, for a particular set of data, only a specific category of algorithms will achieve optimal separation capability. Thus, judicious choice of appropriate cost functions, updating techniques and post processing is essential for good separation performance.

1.1.4 Limitations of ICA

In the ICA model, the following limitations hold:

- The variances (energies) of the independent components cannot be extracted. The reason is that as neither the original signals nor the mixing matrices are known, the magnitudes of the independent components will have to be fixed. In most cases, the assumption is unit variance: $\mathbf{E} \{\mathbf{s}_i^2\} = 1$. However, this still leaves a slight ambiguity in the sign which is mostly insignificant in most applications. This scaling problem can be overcome by normalizing the data.
- The order of the independent components cannot be determined (Permutation prob-

lem). During the separation procedure, any of the independent components may be set as the first one with subsequent separated sources occupying the next positions. In most cases, back correlation (measure of the departure of two random variables from independence) with the observed signals can be used to predict the approximate order of the components. Some direction of arrival constraints can be also applied to match the separated data with the true sources.

1.2 Motivation

Multiple antenna arrays have been considered for accommodating increasing number of users in mobile communication systems. An antenna array is useful for servicing multiple, spatially-separated users in a common cell on a common frequency (co-channel) by adaptively amplifying the signal for each user with rejecting interference from other users. An antenna array can reject interference from adjacent cells by increasing the SINR of the received signals. By the spatial reuse of allocated frequency slots using antenna arrays, the system capacity can be increased substantially [12]. These properties can lead to many advantages including (i) more users per cell (ii) re-use of frequencies in adjacent cells (iii) smaller reuse distances (iv) lower power levels for transmission. In order to utilize these features, algorithms that estimate spatial signatures of each of the received signals must be used.

Array processing techniques are required at the base station in order to ensure optimal communication system performance. Techniques developed for solving co-channel interference in communication signals may be categorized into two groups: those that use spatial or array calibration information and those that are blind beamforming tech-

niques. Blind beamforming techniques rely on some underlying assumptions about the properties of the signals and do not use training sequences or array calibration techniques. When the channel is unknown and the source is not accessible so that the receiver can be trained, the estimation is “blind”. In case of blind source separation, the goal is to estimate multiple sources in a multi-user environment. As the calibration of each array element is cumbersome and time consuming, it is advantageous to use blind beam forming techniques [13].

The main focus of this thesis is to propose blind source separation techniques to solve the co-channel interference (crosstalk) problems in such antenna array based wireless communication systems. Co-channel interference is observed at each of the receivers in an antenna array with multiple users using the same frequency band. By applying BSS techniques, it is possible to extract the independent components and hence mitigate the effect of the independent components. Such a system has been studied in the specific case of GSM applications in [14]. By application of these blind beamforming techniques for GSM systems, the training sequence length can be reduced which leads to improved throughput and usage of the spectrum for data transmission. Loss and distortion of data frames due to excessive interference is mitigated at the receiver due to application of these BSS algorithms. The effect of noise under low SNR conditions can also be reduced specially when applied to mobile telephony and VoIP systems.

1.3 Contributions of Thesis

The main contributions of this thesis include:

1. Analysis of the assumptions and principles behind Blind Source Separation (BSS).

2. Identifying an appropriate cost function for application to the problem of separating communication signals.
3. Detailed review of available BSS algorithms that can handle complex domain data.
4. Proposing a new algorithm for separating complex domain signals, which can provide better bit error rate improvements than the conventional JADE algorithm using alternating unmixing updates.
5. Deriving weight update equations based on the mutual information criterion for application to separating modulated signals.
6. Proposing a new algorithm for separating complex domain signals that can provide dual advantages of higher output signal to interference plus noise ratio and lower computational complexity.
7. Studying the effect of Doppler shifts when applied to ill-conditioned mixing conditions of BSS and extending this to source tracking.
8. Combining over determined BSS with minimum distortion noise reduction techniques in order to produce a high fidelity speech output for low SNR cases.
9. Investigation of parallel-factor analysis and single-channel source separation techniques.

1.4 Outline

In Chapter 2 , we introduce various standard algorithms that can handle source separation of modulated communication signals. We also provide an overview of the global system

of mobile (GSM) which is the standard wireless communication platform used in most mobile systems.

Chapter 3 explains our proposed ARD-JADE algorithm and its relative advantages in terms of Bit Error Rate (BER) and output Signal-to-Interference plus Noise ratio (SINR) improvement.

A computationally less intensive, FFT based algorithm is developed in Chapter 4. Derivation from a mutual information perspective and demonstration of its efficacy using varied parameters are also included.

The effect of Doppler shifts on the source separation problem is demonstrated in Chapter 5. Blind estimation techniques for evaluation the number of sources are also incorporated.

Chapter 6 examines the fidelity output of BSS algorithms with respect to noise. Combined with noise reduction techniques, superior SINR performance with overdetermined BSS is demonstrated. Preliminary work on reduction of reverberation is also examined.

Parallel factor Analysis used for widely separated sensors and single-channel source separation using NMF are included in Chapter 7.

The conclusions and future research directions are given in Chapter 8. Appendices provide a overview of a couple of other algorithms that can prove useful for future incorporation. Glossary of technical terms are also incorporated.

Chapter 2

Source Separation for Communication Systems

This chapter gives a brief overview of the BSS algorithms that have been developed previously for communication systems. The algorithmic steps and the separation parameters employed by these algorithms are discussed. Further, an introduction to Gaussian Minimum Shift Keying (GMSK) modulation schemes are provided that will be used for simulations in the next few chapters.

2.1 State of the Art

In wireless communication systems, co-channel interference may be prevented by applying Medium Access Control (MAC) protocols or other random access protocols [15]. By applying cross layer designs, signal processing techniques like blind source separation can be employed along with MAC protocols to improve the performance. As the source separation is applied on digitally modulated signals, it is imperative that the processing is

performed in the complex domain. Separation of complex valued linear signal mixtures is important for co-channel interference mitigation for wireless communications [16]. Fewer algorithms for separating complex signal mixtures have been described in the literature and the algorithms designed for real valued data are unable to effectively separate these mixtures.

A number of algorithms that have been used for blind source separation include FastICA [17], Infomax [18], JADE [19] [20] and RobustICA [21]. These make use of second or fourth order statistics to estimate the unmixing matrix. They differ based on the complexity of the mixing process model and whether the data is processed iteratively or in batches. While most of the conventional algorithms like FastICA and Infomax may be applied to real signals, algorithms like Joint Approximate Diagonalization of Eigen-matrices (JADE) and Analytical Constant Modulus Algorithm (ACMA) [22] can handle source separation in complex domain. Modified versions of FastICA [23] and Infomax [24] have been proposed to extend these algorithms to handle complex valued signals.

2.1.1 Joint Approximate Diagonalization of Eigen-matrices

The Joint Approximate Diagonalization of Eigen-matrices (JADE) algorithm proposed by Cardoso [19] is well suited for processing of digitally modulated signals. It makes use of Jacobi optimization techniques to extract the source signals. The joint diagonalization of cumulant matrices allows processing of fourth order cumulant sets with the efficiency similar to eigen-based techniques. Based on the instantaneous mixing model give in (1.13), and assuming that the noise is additive, normally distributed and independent from the source signals, the covariance matrix $\mathbf{R}_\mathbf{x}$ is developed based on the observed mixed signals

given by

$$\mathbf{R}_{\mathbf{X}} \stackrel{def}{=} \mathbf{E} \{ \mathbf{X}(t) \mathbf{X}(t)^* \} \quad (2.1)$$

where $\mathbf{X}(t)$ is the observed signal matrix (with row-wise sensor observations) and $*$ represents the complex conjugate. Based on the second order statistics and eigendecomposition of $\mathbf{R}_{\mathbf{X}}$, estimate the n largest eigenvalues $\mu_1, \mu_2, \dots, \mu_n$ and the corresponding eigenvectors $h_1, h_2, \dots, h_n$. Then, the whitening matrix $\hat{w}$ based on the noise variance σ is given by

$$\hat{w} = [(\mu_1 - \sigma)^{-1/2} h_1, \dots, (\mu_n - \sigma)^{-1/2} h_n]^H \quad (2.2)$$

where H represents the Hermitian operator for complex matrices. This provides us with the whitened process $\mathbf{Z}(t)$ and the corresponding sample fourth order cumulants $\mathbf{Q}_{\mathbf{z}}$ given by [20]

$$\mathbf{Z}(t) = \hat{w} \mathbf{X}(t) \quad (2.3)$$

$$\mathbf{Q}_{\mathbf{z}} \stackrel{def}{=} \{Cum(\mathbf{z}_i, \mathbf{z}_j^*, \mathbf{z}_k, \mathbf{z}_l^*) | i \leq 1, j, k, l \leq d\} \quad (2.4)$$

where $\mathbf{z}$ refers to the d dimensional complex-valued row-vectors of the matrix $\mathbf{Z}(t)$. The computation of the n most significant eigenvalues of this matrix is extracted and ordered by decreasing order of magnitude to obtain the eigen-set $\wp$ of $\mathbf{Q}_{\mathbf{z}}$ given by [20]

$$\wp \stackrel{def}{=} \{\lambda_r \mathbf{Z}_r | 1 \leq r \leq n\} \quad (2.5)$$

where r represents the index of the n most significant eigenvalues of the matrix $\mathbf{Z}(t)$. Now, jointly diagonalize the set $\wp$ by a unitary matrix $\mathbf{G}$ as shown in (2.7). This is performed by the Jacobi technique for diagonalization [25] performed by successive Givens rotations.

This is done until the resulting set is approximately diagonal. The resulting estimate of the mixing matrix $\hat{\mathbf{A}}$ is then given by the following where $\hat{w}^\circ$ is given by [20]

$$\hat{w}^\circ = [(\mu_1 - \sigma)^{1/2}h_1, \dots, (\mu_n - \sigma)^{1/2}h_n] \quad (2.6)$$

$$\hat{\mathbf{A}} = \hat{w}^\circ \mathbf{G} \quad (2.7)$$

The above JADE algorithm has been shown to perform with considerable success in case of digitally modulated signals as well as EEG samples [26].

2.1.2 Analytical Constant Modulus Algorithm

This is a class of constant modulus BSS algorithms, where other than the independence of the signals, another additional criterion is that the signals should have a constant modulus. The ACMA algorithm was proposed by Van der Veen *et al.* [22] and can be applied to digitally modulated signals with a constant amplitude. Based on the instantaneous mixing model shown in (1.13), the ACMA algorithm is described. This CM algorithm, though not iterative, provides certain advantages over other BSS techniques including robustness in presence of noise and lack of dependence on initialization. However, it is prone to high computational loads for long sequence data.

First, compute the singular value decomposition $SVD()$ of $\mathbf{X}(t)$ providing the unitary matrices $\mathbf{U}$ and $\mathbf{V}$ containing the singular vectors and Σ is the real diagonal matrix with non-negative entries.

$$SVD(\mathbf{X}) = \mathbf{U}\Sigma\mathbf{V} \quad (2.8)$$

Now, estimate the rank of $\mathbf{X}$ from Σ which provides the number of sources. Assuming the

variable $d = \text{rank}(\mathbf{X})$, define $\hat{\mathbf{V}}$ as the first d rows of $\mathbf{V}$. Construct the matrix $\mathbf{P}$ based on $\hat{\mathbf{V}}$ given by the following [22]:

$$\hat{\mathbf{V}} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \cdots \ \mathbf{v}_n] \quad (2.9)$$

$$\mathbf{P} = [\text{vec}(\mathbf{v}_1 \mathbf{v}_1^*) \cdots \text{vec}(\mathbf{v}_n \mathbf{v}_n^*)]^T \quad (2.10)$$

where T defines the matrix transpose and $\text{vec}(\mathbf{Y})$ for a $n \times n$ matrix is given by

$$\text{vec}(\mathbf{Y}) = \begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \cdots & \mathbf{Y}_{21} & \cdots & \mathbf{Y}_{nn} \end{bmatrix}^T \quad (2.11)$$

Performing Householder transformation on the matrix $\mathbf{P}$, generate the variable $\hat{\mathbf{P}}$ [22].

$$\begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p} \end{bmatrix} = \mathbf{Q}\mathbf{P} \quad (2.12)$$

Generate the SVD of $\hat{\mathbf{P}}$ to obtain the diagonal matrix $\Sigma_{\mathbf{p}}$.

$$\text{SVD}(\hat{\mathbf{P}}) = \mathbf{U}_{\mathbf{p}} \Sigma_{\mathbf{p}} \mathbf{V}_{\mathbf{p}}^* \quad (2.13)$$

The dimension of kernel $\hat{\mathbf{P}}$ is equated to variable δ which provides the number of constant modulus signals. Then, we equate $[\mathbf{y}_1 \ \mathbf{y}_2 \ \cdots \ \mathbf{y}_\delta]$ to the last δ columns of $\mathbf{V}_{\mathbf{p}}$.

Simultaneously diagonalize the vectors row wise and produce it into a Kronecker structure $\hat{\mathbf{Y}}_k$ based on the following [22]:

$$\mathbf{Y}_1 = \text{vec}^{-1}(\mathbf{y}_1), \cdots, \mathbf{Y}_\delta = \text{vec}^{-1}(\mathbf{y}_\delta) \quad (2.14)$$

$$\hat{\mathbf{Y}}_k = \alpha_{k1} \mathbf{Y}_1 + \cdots + \alpha_{k\delta} \mathbf{Y}_\delta \quad (2.15)$$

where the $vec^{-1}()$ operation is given by:

$$vec^{-1}(\mathbf{y}) = \begin{bmatrix} (\mathbf{y})_1 & (\mathbf{y})_2 & \cdots & (\mathbf{y})_n \\ (\mathbf{y})_{n-1} & (\mathbf{y})_{n-2} & \cdots & (\mathbf{y})_{2n} \\ \vdots & & \ddots & \vdots \\ (\mathbf{y})_{n^2-n-1} & \cdots & & (\mathbf{y})_{n^2} \end{bmatrix} \quad (2.16)$$

The generalized eigenvalue problem may also be formulated as a Schur decomposition problem with quadratic iterative convergence. Based on the matrices $\hat{\mathbf{Y}}_k$ recover the original signals using [22]:

$$\hat{\mathbf{Y}}_k \approx \mathbf{w}_k^* \mathbf{w}_k \quad (2.17)$$

$$\mathbf{s}_k = \mathbf{w}_k \hat{\mathbf{V}}_k \quad (2.18)$$

The vectors $\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_\delta$ are the rows of $\mathbf{S}$, the separated source signals.

2.1.3 Fast Independent Component Analysis

The FastICA algorithm [17] makes use of an efficient learning rule to maximize the non-Gaussianity of the projection. It is among the most commonly used algorithms for optimal search of the unmixing matrix $\mathbf{W}$ that is updated based on a nonlinear contrast function. The optimization techniques like gradient search or Newton optimization are used for updating the contrast function $G(\mathbf{W}\mathbf{X})$, where $\mathbf{X}$ is the observed matrix of the mixed source signals. The function $G()$ can be any non-quadratic function which must be chosen to provide efficient updates [17]. We observe $\mathbf{X} = \mathbf{A}\mathbf{S}$ and want to find $\mathbf{Y} = \mathbf{W}\mathbf{X} \approx \mathbf{S}$.

The general form of the gradient search (2.19) and Newton optimization (2.20) techniques for updating the unmixing matrix $\mathbf{W}$ is given by:

$$\mathbf{W}_{n+1} = \mathbf{W}_n + \eta g(\mathbf{W}_n \mathbf{X}) \quad (2.19)$$

$$\mathbf{W}_{n+1} = \mathbf{W}_n - \eta \frac{g(\mathbf{W}_n \mathbf{X})}{g'(\mathbf{W}_n \mathbf{X})} \quad (2.20)$$

where $g(\mathbf{W}_n \mathbf{X})$ and $g'(\mathbf{W}_n \mathbf{X})$ are the first and second derivatives of the contrast function and η is an adaptation step size.

The commonly used fixed point FastICA method makes use batch processing of the observed data such that at each step, one row vector $\mathbf{w}$ of the unmixing matrix $\mathbf{W}$ can be estimated. The optimization of the objective function $G(\mathbf{w}^T \mathbf{x})$ ($\mathbf{x}$ refers to each row with fixed sensor observation of the mixture $\mathbf{X}$) is subject to the constraint $\mathbf{E}[(\mathbf{w}^T \mathbf{x})(\mathbf{w}^T \mathbf{x})^T] = 1$. This constraint basically assumes that the components are independent and hence produce unit-covariance. The solution is given based on the variable ϕ as [17]:

$$\mathbf{E}[\mathbf{x}g(\mathbf{w}^T \mathbf{x})] - \phi \mathbf{w} = 0 \quad (2.21)$$

At the optimal value $\mathbf{w}_0$ with $\|\mathbf{w}_0\| = 1$ yields:

$$\phi = \mathbf{E}[\mathbf{w}_0^T \mathbf{x}g(\mathbf{w}_0^T \mathbf{x})] \quad (2.22)$$

To perform a Newton type optimization, the Jacobian matrix of (2.21) is given by:

$$\zeta(\mathbf{w}) = \mathbf{E}[\mathbf{x}\mathbf{x}^T g'(\mathbf{w}^T \mathbf{x})] - \phi \mathbf{I} \quad (2.23)$$

where $\zeta(\mathbf{w})$ is the Jacobian function, $g'(\mathbf{w}^T \mathbf{x})$ is the second derivative of the contrast function and I is an identity matrix. If we assume that the observed data has been whitened prior to FastICA such that $\mathbf{E}(\mathbf{x}\mathbf{x}^T) = I$, then:

$$\mathbf{E}[\mathbf{x}\mathbf{x}^T g'(\mathbf{w}^T \mathbf{x})] \approx \mathbf{E}[\mathbf{x}\mathbf{x}^T] \mathbf{E}[g'(\mathbf{w}^T \mathbf{x})] = \mathbf{E}[g'(\mathbf{w}^T \mathbf{x})] \quad (2.24)$$

The unmixing process can then be optimized based on the Newton optimization method with the update for the n^{th} iteration given as

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \eta [(\mathbf{E}[\mathbf{x}g(\mathbf{w}_n^T \mathbf{x})] - \varphi \mathbf{w}_n) / \mathbf{E}[g'(\mathbf{w}_n^T \mathbf{x})] - \phi] \quad (2.25)$$

$$\mathbf{w}_{n+1} = \mathbf{w}_{n+1} / \|\mathbf{w}_{n+1}\| \quad (2.26)$$

where $\mathbf{w}_{n+1}$ is the new estimated value for every n^{th} iteration and φ is the step size.

As shown by [1], the FastICA algorithm can also be compared to the stochastic gradient method for maximizing likelihood like the infomax method [18]. However, the convergence of FastICA is cubic or quadratic, which is much faster than the linearly converging gradient descent methods. It can also be used to estimate both sub-Gaussian and super-Gaussian independent components. Though not capable of efficiently processing complex domain data, it performs well for non-reverberant mixtures of audio and speech signals.

2.1.4 Information Maximization

A very popular approach for estimating the ICA model is maximum likelihood estimation based on the log-likelihood function $\mathbf{L}$:

$$\mathbf{L} = \sum_{t=1}^T \sum_{i=1}^n \log f_i(\mathbf{w}_i^T \mathbf{x}(t)) + T \log |\det \mathbf{W}| \quad (2.27)$$

where $\mathbf{W}$ is the unmixing matrix, f_i is the density function of the original signals and $\mathbf{x}(t)$ are the observed mixed signals. The log-likelihood function comes from the classic rule for linearly transforming random variables and their densities.

Infomax [18] is based on maximizing the output entropy (or information flow) $\mathbf{H}$ of a neural network with non-linear outputs.

$$\mathbf{L}_2 = \mathbf{H}(g_1(\mathbf{w}_1^T \mathbf{x}), \dots, g_n(\mathbf{w}_n^T \mathbf{x})) \quad (2.28)$$

where the g_i are some non-linear scalar functions, and the $\mathbf{w}_i$ are the weight vectors of the neurons. One then wants to maximize the entropy of the outputs $\mathbf{L}_2$. Tan-sigmoid functions and other such non-linear scalar function may be used in infomax. Then, standard optimization methods like gradient descent may be used to iteratively update the unmixing matrix.

2.2 Gaussian Minimum Shift Keying

The Global System for Mobile (GSM) is one of the most popular 2G technologies [27]. It supports eight time slotted users for each 200 kHz radio channels and uses Frequency Division Duplexing (FDD) combined with Time Division Multiple Access (TDMA). The

basic structure of the GSM burst structure is shown in Fig. 2.1 and consists of 114 data bits and 26 bits of training sequence.

Figure 2.1: Typical Structure of a GSM burst.

Figure 2.2: The implementation of GMSK by quadrature baseband method [27] shown in (a) and an example of the modulated output shown in (b).

The modulation scheme used by GSM is the 0.3 GMSK (Gaussian Minimum Shift Keying). Gaussian minimum shift keying is a continuous-phase frequency-shift keying modulation scheme. It is similar to standard minimum-shift keying (MSK); however the digital data stream is first shaped with a Gaussian filter before being applied to a frequency modulator. This has the advantage of reducing sideband power, which in turn reduces out-of-band interference between signal carriers in adjacent frequency channels. However, the Gaussian filter increases the modulation memory in the system and causes inter symbol interference, making it more difficult to discriminate between different transmitted data values and requiring more complex channel equalization algorithms such as an adaptive

equalizer at the receiver. GMSK has high spectral efficiency, but it needs a higher power level than QPSK, for instance, in order to transmit the same amount of data reliably. The implementation and modulated signal constellation is shown in Fig. 2.2.

Such GMSK signals, when used for transmission with antenna array communications are susceptible to co-channel interference and degradation. By applying source separation and blind beamforming, it is possible to improve the bit error rate (BER) and output signal-to-interference plus noise ratio (SINR) at the receivers without resorting to multiplexing techniques. By applying such blind beamforming techniques, it is also possible to reduce the number of training bits shown in Fig. 2.1. As this occupies approximately 18% of the whole frame, such a reduction will improve the efficient use of the spectrum.

Chapter 3

Alternating Row Diagonalization - Joint Approximate Diagonalization of Eigen-matrices (ARD-JADE)

This chapter provides an overview of the eigen-based ARD-JADE algorithm. By employing alternating updates of the unmixing matrix, it is shown to work better than the JADE algorithm for complex modulated signals. The system model and an analysis of the steps involved are provided. Comparison between the ARD-JADE and JADE based on output BER and SINR are included. Some of the limitations of the ARD-JADE algorithm are also discussed.

3.1 System Model for ARD-JADE

Consider a system of a three element antenna array receiving three co-channel communication signals. Fig. 3.1 shows a typical system model for the co-channel interference

Figure 3.1: System model based on three co-channel GSM signals.

problem. For the proper working of these systems, it is essential that the receiver is able to separate the sources.

In order to develop a realistic model, noise is added to each of the source signals. The blind source separation algorithms are applied at the receiver end in order to estimate the source signals. This process must be implemented before demodulating the received signals. The estimated signals are then compared with the original signals.

In order to compare the performance of the blind source separation algorithms, GMSK modulated signals are generated using the simulator developed by [28]. The signals are then distorted with additional white Gaussian noise but with no Doppler shift. For a realistic model of the mixing matrix, the locations and transmit directions of the sources and sensors have been incorporated. Consider the Fig. 3.2, where the location of the sources are given by $(\mathbf{P}_x, \mathbf{P}_y, \mathbf{P}_z)$ based on half wavelength separation. The horizontal and vertical angular deviations with respect to line of sight are given by (θ_H, θ_V) .

The mixing matrix may then be expressed by the following in terms of the column

Figure 3.2: Array channel mixing model used for developing the mixing model for the case of two interfering sources S_1 and S_2 with respect to sensor R_1 .

vectors $\mathbf{P}_x$, $\mathbf{P}_y$ and $\mathbf{P}_z$:

$$\mathbf{P} = [\mathbf{P}_x \quad \mathbf{P}_y \quad \mathbf{P}_z] \quad (3.1)$$

$$k = [\cos(\theta_H) \cos(\theta_V) \quad \sin(\theta_H) \cos(\theta_V) \quad \sin(\theta_V)] \quad (3.2)$$

$$\mathbf{A} = e^{j\pi \mathbf{P} k'} \quad (3.3)$$

where $\mathbf{A}$ represents the mixing matrix with contributions from locations and direction vectors of the sources and k' represents the transpose of the vector k .

Now, the objective of applying statistical methods such as ICA is to extract the sources without any prior training. In such cases, the permutation problem comes into play after the source separation is performed. As most blind separation algorithms receive mixtures of signals, it is difficult to estimate which of the three estimated signals are to be matched with the respective source locations. In case of few users, correlation of the estimated signals with the observed mixed signals can be used as a solution. However, with a large

number of sources, the correlation process is extremely time consuming and may not be feasible. A promising technique is to make use of a reference signal which can then be used to estimate the appropriate match of sources. In the technique presented in [29], the use of clustering based on the spatial distribution of sources is described. This is possible if the sources are assumed to be sparse in the domain used for separation.

3.2 Algorithmic Steps for ARD-JADE

The JADE algorithm proposed by Cardoso [19] is well suited for processing of digitally modulated signals. The above JADE algorithm has been shown to perform with considerable success in case of digitally modulated signals. However, under low SNR conditions of less than 10 dB the JADE algorithm has been shown to have inferior performance when compared to other algorithms such as non stationary source separation using simultaneous digitalization (NSS-SD) [30].

In order to improve its performance in separating digitally modulated signals, we combine the JADE algorithm with the Alternating Row Diagonalization (ARD) algorithm [31]. The ARD algorithm has been originally applied to a real valued mixing matrix. Over JADE, the ARD algorithm tends to have better convergence rates especially for shorter data lengths as shown in [31].

The proposed Alternating Row Diagonalization - Joint Approximate Diagonalization of Eigen-matrices (ARD-JADE) algorithm is described in the following steps. It makes use of eigen-based alternative update of the unmixing matrix. Using such eigen based techniques, it is possible to separate complex valued data with accuracy.

1. Initialize a square matrix $\mathbf{W}$ to be complex values. This is an initial estimate of the

unmixing matrix produced by either the JADE or estimation of signal parameters via rotational invariance techniques (ESPIRIT) [32] algorithm. The matrix may be expressed in the form of vectors given by:

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_N \end{bmatrix}^T \quad (3.4)$$

where T represents the transpose of the matrix. In case of ARD-JADE, the initialization controls the accuracy of the convergence in the complex domain to a large extent and hence, the choice of the initial matrix $\mathbf{W}$ must be taken judiciously. We assume here the scenario when the number of sources is equal to the number of sensors and hence $\mathbf{W}$ is initialized as a square matrix. This is a prior assumption of this algorithm that along with independent sources, the setting is critically determined (the number of observed mixtures corresponds to the number of sources). Such a prior assumption of knowledge of the number of sources is used by JADE, FastICA and ACMA algorithms as well.

2. In order to avoid a trivial solution, initialize by the normalization factors $\mathbf{q}_i$ such that:

$$\|\mathbf{w}_i/\mathbf{q}_i\| = 1 \quad (3.5)$$

3. The matrix $\mathbf{Q}$ for updating each i th column is defined to be the matrix $\mathbf{W}$ with the i th column missing:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_{i-1} & \mathbf{w}_{i+1} & \cdots & \mathbf{w}_N \end{bmatrix}^T \quad (3.6)$$

4. An iterative process of alternatively updating the unmixing matrix is performed.

Based on the original observed matrix $\mathbf{X}$ having K rows, we define:

$$\mathbf{M} = \sum_{k=1}^K \mathbf{X}_k \mathbf{Q} \mathbf{Q}^T \mathbf{X}_k^T \quad (3.7)$$

For every value of $\mathbf{M}$, associated with a particular $\mathbf{Q}$ (i th iteration), the unit eigenvector u associated with the least eigenvalue of $\mathbf{M}$ is extracted. In [31], the matrix $\mathbf{X}$ has been defined to be symmetric in nature. For the case of complex signals, $\mathbf{X}$ is treated a non-symmetric matrix with its rows corresponding to each of the observed signals.

5. The unmixing matrix is iteratively updated such that the optimum value is:

$$\mathbf{w}_i^{opt} = \mathbf{q}_i u \quad (3.8)$$

This causes the updated unmixing matrix at the end of each iteration to be

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_{i-1} & \mathbf{q}_i u & \mathbf{w}_{i+1} & \cdots & \mathbf{w}_N \end{bmatrix}^T \quad (3.9)$$

So, at the end of the iterative process, the unmixing matrix is finally updated.

6. The minimizing cost function in this case can be expressed as:

$$\rho = 2(\mathbf{q}_i)^2 \left(\frac{\mathbf{w}_i}{\mathbf{q}_i} \right)^T \left[\sum_{k=1}^K \mathbf{X}_k \mathbf{Q} \mathbf{Q}^T \mathbf{X}_k^T \right] \left(\frac{\mathbf{w}_i}{\mathbf{q}_i} \right) \quad (3.10)$$

3.3 Results for ARD-JADE

Figure 3.3: Signal Constellations of the signals (A) Original undistorted signal (B) Signal after mixing and adding noise (C) Signal received after using JADE (D) Observed signal after applying ARD-JADE.

For mixture of three source signals, the outputs have been passed through the JADE and the ARD-JADE algorithms. Fig. 3.3 shows the signal constellations obtained for a GMSK modulated signal during the various stages of transmission. It is observed that the ARD-JADE produces a signal constellation that is very close to the original undistorted signal. In this example, the SNR is set to 30 dB and number of iterations to 100.

The next stage is to compare the performance of the algorithms based on the bit error rates (BERs). A sample mixing matrix is generated using the source position vector model is shown in (3.11), (3.12), (3.13). These are for a critically determined mixtures of two, three and four sources. The two, three and four sources are uniformly and linearly spaced with 5 units half-wavelength separation between them. The source signal direction

Figure 3.4: The cross correlation values obtained for various combinations of original signals S_1, S_2, S_3 with the observed mixed signals X_1, X_2, X_3 .

vector is set to 15 degree angle separation.

$$\hat{A}_2 = \begin{bmatrix} -0.4956 - 0.8685i & 0.4440 + 0.8960i \\ 0.4956 - 0.8685i & 0.9399 - 0.3414i \end{bmatrix} \quad (3.11)$$

$$\hat{A}_3 = \begin{bmatrix} -0.8609 - 0.5087i & -0.2663 + 0.9639i & 0.9529 + 0.3033i \\ -0.0000 - 1.0000i & 0.9282 + 0.3720i & -0.3033 + 0.9529i \\ 0.8609 - 0.5087i & 0.4731 - 0.8810i & -0.9529 - 0.3033i \end{bmatrix} \quad (3.12)$$

Figure 3.5: Bit Error Rate improvement for a mixture of two, three and four GSM signals.

$$\hat{A}_4 = \begin{bmatrix} -0.9999 - 0.0151i & -0.8410 + 0.5411i & -0.8882 + 0.4594i & -0.9907 + 0.1358i \\ -0.4956 - 0.8685i & 0.4440 + 0.8960i & -0.4594 - 0.8882i & 0.7055 + 0.7087i \\ 0.4956 - 0.8685i & 0.9399 - 0.3414i & 0.8882 - 0.4594i & 0.1404 - 0.9901i \\ 0.9999 - 0.0151i & -0.2346 - 0.9721i & 0.4594 + 0.8882i & -0.8747 + 0.4847i \end{bmatrix} \quad (3.13)$$

In the simulations, the BER is expressed as the ratio of number of erroneous bits received to the total number of bits transmitted. As the number of sources is not too many, correlation of the separated signals with the mixed signals is used to overcome the permutation problem. An application of this cross correlation is shown in Fig. 3.4, where the signals with the highest cross correlation values are chosen.

Figure 3.6: The Signal to Interference plus Noise Ratio improvement after application of the ARD-JADE algorithm.

As seen from Fig. 3.5, at SNR levels of greater than 10 dB, the ARD-JADE outperforms the JADE algorithm and is able to reduce the bit error rate significantly. This consistent improvement in performance is seen for critically determined mixtures of two, three and four sources. The deterioration in performance (increased error rate) with increase in the number of sources is seen in most BSS algorithms as demonstrated in [33]. For the critically determined case, more sensors do not signify improved performance. Each sensor has to separate more components from the mixture it observes, when the number of sources are increased. In all the cases, less than 100 iterations of the ARD-JADE algorithm provided accurate convergence to the cost function minima.

The reasons for improvements over the JADE algorithm are:

- **Eigenvalue Estimation** - In case of the JADE algorithm, approximate diagonalization of the eigenvectors are performed. Though this is computationally efficient,

Figure 3.7: Bit Error Rate improvement for a mixture of three GSM signals with increase in the number of sensors.

there is still some lack of accuracy in the estimation of the unmixing matrix. Moreover, JADE ignores the non-significant eigenvalues in the observed signal. By performing computationally intensive tasks of extracting the lowest eigenvalues, the ARD-JADE is able to provide a more accurate estimate of the unmixing matrix. This becomes critical in the case of communication systems where a slight perturbation in the estimation can lead to failure in decoding, which in turn reflects in high bit error rates.

- **Noise Estimation** - The noise estimation in JADE consists of whitening the observed signals, which is essentially based on second order statistics. Non-minimum phase signals and certain types of phase coupling (associated with nonlinearities)

Figure 3.8: Separation performance of ARD-JADE on real field data (a) One of the observed Mixed signal (b) Separated Signal 1 (c) Separated Signal 2.

cannot be correctly identified by second order techniques. When Gaussian noise is employed, second order statistics are more prone to distortion than higher order statistics. This failure of JADE is demonstrated in [34]. By employing estimates of higher order statistics, the ARD-JADE is in a better position to estimate and counter the effect of Gaussian noise.

The number of iterations required in order to minimize the cost function in (3.10) vary depending on factors such as singularity of the mixing process, noise levels, length of the signals and effective initial estimates. For successful convergence, the number of iterations should be set such that a local minima is observed while minimizing the cost function. If this is not observed, the number of iterations should be increased for improved separation performance. Further details on global and local convergence analysis of source separation algorithms may be found in [35].

The use of source separation algorithms has a positive effect on the output signal to interference plus noise ratio as demonstrated in Fig. 3.6. Without the BSS algorithms, the output SINR would be significantly lower than the input SINR. However, the exploitation of the BSS algorithms improves the quality of the signal raising the output SINR well above the input SNR. This is another significant parameter to consider when incorporating source separation for communication systems and shows the output efficacy of the algorithm. Combining Figs. 3.5 and 3.6 we can make some deductions. For example, we can expect a 50 dB output SINR and 10^{-5} output BER for a critically determined mixture of 2 sources at input SNR 20 dB. So, a combination of these figures can provide an indication of the performance bounds under different settings of input SNR and the number of sources.

As shown in Fig. 3.7, an increase in the number of sensors (overdetermined condition) does not provide any significant increase in the output BERs. This is because, the ARD-JADE algorithm does not have an parameter to extract the diversity obtained by increasing the number of sensors. Indeed, this spatial diversity can be used to increase the noise reduction capability of certain algorithms as we will demonstrate in Section 6.

Further proof of the accurate separation performance is shown in Fig. 3.8 when applied to real field data. The I-Q data of CFSK modulated signals produced by two sources was sampled at 25 kHz and received by two receivers. The SNR value was recorded to be in the range of 30-40 dB. Plotted in the time-frequency domain, the separation performance is shown to be quite accurate with the two individual sources recognized successfully.

Chapter 4

Feedback Independent Component Analysis (FEBICA)

This chapter describes the FEBICA algorithm that originates from minimizing mutual information between the mixed signals. The feedback of complex weights is described along with the constraints for effectively minimizing the cost function employed. Comparison with other algorithms for output SINR and computational complexity are provided. An example for application on real world data is also included.

4.1 FEBICA from a Mutual Information Perspective

The BSS algorithms of Chapter 3 should, in addition to processing complex signals, also have the added capacity to process the data in real time. In order to satisfy this, an iterative FFT based algorithm is proposed uses a feedback architecture and called the Feedback Independent Component Analysis Algorithm (FEBICA). The FEBICA algorithm belongs to the class of algorithms that make use of information maximization. By

using a nonlinear function with updated weight vectors, this type of algorithms rely on the mutual information providing a simple learning rule. The performance of the FEBICA algorithm has been demonstrated in terms of computational complexity as well as signal-to-interference plus noise ratio (SINR) improvements. The FEBICA algorithm provides a good tradeoff between improved SINR performance and lower computational complexity. Other algorithms deteriorate in either separation performance on the real time processing capability [36].

The well known measures of BSS performance include kurtosis, negentropy and mutual independence of source signals [1]. We exploit the mutual independence of sources as used in [18]. The mutual independence of functions based on joint probability density functions is given by:

$$f_s(\mathbf{s}) = \prod_{i=1}^n f_i(\mathbf{s}_i) \quad (4.1)$$

where $f_i(\mathbf{s}_i)$ is the pdf of signal $\mathbf{s}_i$. Based on signal entropy $\mathbf{H}$, the mutual information $\mathbf{I}$ is given by:

$$\mathbf{I}(\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_n) = \sum_{i=1}^n \mathbf{H}(\mathbf{s}_i) - \mathbf{H}(\mathbf{s}) \quad (4.2)$$

$$\mathbf{H}(\mathbf{s}_i) = - \int f_i(\mathbf{s}_i) \ln f_i(\mathbf{s}_i) d\mathbf{s}_i \quad (4.3)$$

The objective of BSS is to find an unmixing matrix $\mathbf{W}$, such that $\mathbf{I}(s_i, s_j) = 0$. Traditionally, higher order statistics (HOS) and associated nonlinearities may be used to produce mutual independence [2]. For observed signals $\mathbf{Y} = \mathbf{W}\mathbf{X}$, let the transformed function (due to nonlinearities) be given by:

$$\mathbf{z}_i = g_i(\mathbf{y}_i) \quad (4.4)$$

FEBICA from a Mutual Information Perspective

For a single variable $\mathbf{z}$, the $\mathbf{H}(\mathbf{z})$ is maximized when non-linearity $g(\cdot)$ is a cumulative density function. In other words, $\mathbf{H}(\mathbf{z})$ is maximized when $\mathbf{z}$ has uniform distribution [18]:

$$f_{\mathbf{z}}(\mathbf{z}) = \frac{f_{\mathbf{y}}(\mathbf{y})}{d\mathbf{z}/d\mathbf{y}} \quad (4.5)$$

This is of uniform distribution when:

$$\frac{d\mathbf{z}}{d\mathbf{y}} = f_{\mathbf{y}}(\mathbf{y}) \quad (4.6)$$

For n variables, this may be extended using the Jacobian form [18]:

$$f_{\mathbf{z}}(\mathbf{z}) = \frac{f_{\mathbf{y}}(\mathbf{y})}{|\mathbf{J}|} \quad (4.7)$$

$$\mathbf{J} = \det \begin{bmatrix} \frac{dz_1}{dy_1} & \dots & \frac{dz_1}{dy_n} \\ \vdots & \ddots & \vdots \\ \frac{dz_n}{dy_1} & \dots & \frac{dz_n}{dy_n} \end{bmatrix} \quad (4.8)$$

The output joint entropy is then given by:

$$\begin{aligned} \mathbf{H}(\mathbf{z}) &= -\mathbf{E}\{\ln f_{\mathbf{z}}(\mathbf{z})\} = -\mathbf{E}\{\ln f_{\mathbf{x}}(\mathbf{x})\} \\ &\quad + \mathbf{E}\{\ln |\mathbf{J}|\} = \mathbf{H}(\mathbf{x}) + \mathbf{E}\{\ln |\mathbf{J}|\} \end{aligned} \quad (4.9)$$

For maximizing $\mathbf{H}(\mathbf{z})$, the updating weights for the unmixing matrix are given by [18]:

$$\Delta \mathbf{W} \propto \frac{d\mathbf{H}(\mathbf{z})}{d\mathbf{W}} = \mathbf{E}\left\{\frac{d \ln |\mathbf{J}|}{d\mathbf{W}}\right\} \cong \frac{d}{d\mathbf{W}} \ln |\mathbf{J}| \quad (4.10)$$

Using the definition of the Jacobian:

$$\mathbf{J} = \det(\mathbf{W}) \prod_{i=1}^n \left| \frac{d\mathbf{z}_i}{d\mathbf{y}_i} \right| \quad (4.11)$$

$$\Delta \mathbf{W} \propto \frac{d}{d\mathbf{W}} \ln |\det(\mathbf{W})| + \frac{d}{d\mathbf{W}} \ln \prod_{i=1}^n \left| \frac{d\mathbf{z}_i}{d\mathbf{y}_i} \right| \quad (4.12)$$

It can be proved that [18]:

$$\frac{d}{d\mathbf{W}} \ln |\det(\mathbf{W})| = |\mathbf{W}^T|^{-1} \quad (4.13)$$

$$\frac{d}{d\mathbf{W}} \ln \prod_{i=1}^n \left| \frac{d\mathbf{z}_i}{d\mathbf{y}_i} \right| = \frac{d}{d\mathbf{W}} \prod_{i=1}^n \ln \left| \frac{d\mathbf{z}_i}{d\mathbf{y}_i} \right| = f(\mathbf{y}) \mathbf{x}^T \quad (4.14)$$

Substituting this into (4.12), we get [18]:

$$\Delta \mathbf{W} \propto |\mathbf{W}^T|^{-1} + f(\mathbf{y}) \mathbf{x}^T \quad (4.15)$$

$$\mathbf{W}_{(k+1)} = \mathbf{W}_{(k)} + \eta(k) [(\mathbf{W}^T)^{-1} + f(\mathbf{y}) \mathbf{x}^T] \quad (4.16)$$

Multiplying this learning rule by $\mathbf{W}^T \mathbf{W}$, we get the simplified expression [18]:

$$\mathbf{W}_{(k+1)} = \mathbf{W}_{(k)} + \eta(k) [I + f(\mathbf{y}) \mathbf{y}^T] \mathbf{W} \quad (4.17)$$

Making use of the Global gradient update rule, we can write the above expression as:

$$\mathbf{W}(t+1) = \mathbf{W}(t) + \eta(t) [I - f(\mathbf{y}(t)) g^T(\mathbf{y}(t))] \mathbf{W}(t) \quad (4.18)$$

Thus, making use of the criterion for mutual independence, the general expression

for serial updating of weight vectors is shown. The crux of the problem is to use a cost function for measuring independence and an appropriate optimization criterion for updating the weights. The feedback architecture used in the proposed FEBICA algorithm is an extension of the adaptive neural network approach used by [37] in order to separate a mixture of odor sources. Originally used to estimate the olfactory (sense of smell) perception of odors in animals, it has now been modified to update complex domain unmixing matrices. This has been incorporated with weight updates and a gradient ascent learning rule for the application to separation of sources as previously used by [38]. An extension of the infomax algorithm to handle complex fMRI data has been proposed in [24]. A key contribution is proposing a source separation in the complex domain which has not been done for this category of adaptive neural network algorithms applied to communication systems. This type of formulation involves using complex weights to update the unmixing matrix using a non-linear boundary condition. Such kind of algorithms can then be directly applied to complex modulated data for effective source separation.

The proposed Feedback Independent Component Analysis (FEBICA) algorithm is described in the following steps with the feedback procedure in steps 4 and 5:

1. Initialize complex weights $\mathbf{W}$ of the FEBICA algorithm. This can either be done randomly or by using the unmixing matrix estimate from the JADE or ACMA algorithms. The feedback weights $\mathbf{W}_{fb}$ are then initialized by the off-diagonal elements of the weights $\mathbf{W}$ given by:

$$\mathbf{W}_{fb}(n, m) = \mathbf{W}(n, m) \quad \forall n \neq m \quad (4.19)$$

where n and m represent the row and columns of the matrices respectively. The complex weights will eventually converge to the desired unmixing matrix while the feedbacks weights are used to control the convergence rates.

2. The Fast Fourier Transform (FFT) γ and the row-wise mean μ of the observed signal matrix $\mathbf{X}$ (with M columns) are given by:

$$\gamma_{k,m} = \sum_{n=0}^{N-1} \mathbf{x}_{n,m} e^{-j\frac{2\pi}{N}nk} \quad k = 0, \dots, N-1. \quad (4.20)$$

$$\mu_m = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}_{n,m} \quad (4.21)$$

where N is the length of the complex sequence in each row of the observed matrix $\mathbf{X}$ given by $\mathbf{x}_1, \dots, \mathbf{x}_{M-1}$.

3. Based on (4.18) we introduce a nonlinear function $g(\cdot)$ which is a suitably chosen odd non-linearity, providing stability in the process of separation. The non-linear function should be judiciously selected to deal with the super-Gaussian, sub-Gaussian, stationary and non-stationary signals. A popular choice is a sigmoidal shaped function. This nonlinearity in the function also creates a narrow boundary condition that is responsible for distinguishing various independent components.

$$\xi(\gamma_{k,m}, \beta_k, \delta_m) = e^{-\beta_k/\delta_m} \cdot \gamma_{k,m} \quad (4.22)$$

where β and δ are constants and ξ is the output of the nonlinear function. A plot of the non-linear function is shown in Fig. 4.1.

4. Based on the weights $\mathbf{W}$ and feedback weights $\mathbf{W}_{fb}$, we further define a variable ψ

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Figure 4.1: The behavior of the non-linear operator for various settings of parameters β and δ .

which is updated iteratively. This is based on minimizing the mutual information between the original signals. As the fundamental assumption of ICA is independence of sources, the mutual information must tend to zero as the separation progresses.

$$\psi_{\iota} = \mathbf{W}_{\iota} \gamma - (\mathbf{W}_{fb})_{\iota} \xi \quad (4.23)$$

Here, ι represents the iteration count for the updating process.

5. The iterative process of updating the weights is described in terms of the following equations where η is the scaling factor. This is a gradient ascent method of updating the weights by minimizing the mutual information between the signals based on a nonlinear gradient.

$$\Delta \mathbf{W}_{\iota} = \eta \left(1 + \frac{\xi''}{\xi'} \psi_{\iota} \right) \mathbf{W}_{\iota} \quad (4.24)$$

$$\mathbf{W}_{\iota} = \mathbf{W}_{\iota-1} + \Delta \mathbf{W}_{\iota} \quad (4.25)$$

where $\Delta \mathbf{W}_{\iota}$ refers to the increments of the weight updates as in (4.10) and ξ' refers to the derivative of the nonlinear function defined in (4.22). Here, we also update

the feedback weights $\mathbf{W}_{fb\iota}$ with the new value of $\mathbf{W}_\iota$ as shown in (4.19).

6. Steps 4 and 5 are repeated till convergence. After the weights converge with the values of ψ , the independent vectors are obtained as the rows of $\mathbf{Y}$:

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mu \quad (4.26)$$

where, $\mathbf{Y}$ represents the estimated original signals by the FEBICA algorithm and μ is the mean obtained in (4.21).

The described FEBICA algorithm is succently represented in Fig. 4.2 to show the interaction of various parameters for weight update along with corresponding equation numbers.

4.2 Analysis of FEBICA Algorithm

In the FEBICA algorithm, by creating the narrow nonlinear boundary ξ , the probability of finding a single independent vector within that reduced space increases. Furthermore, the feedback architecture reduces the mutual information between the mixed components which is updated based on a nonlinear gradient. This is a stochastic gradient ascent algorithm which tries to maximize the sum of fourth order cumulants. The performance criterion is expressed as:

$$J(\mathbf{W}) = \sum_{i=1}^n E\{f(\mathbf{y}(i))\} \quad (4.27)$$

where the function $f(t)$ represents the objective function used in the algorithm and $\mathbf{y}(i)$ are the unmixed signals. The objective function is chosen to be of the form $f(t) = \ln[\cosh(t)]$ which on differentiation provides the non-linearity of the form $g(t) = f'(t) = \tanh(t)$. In

Figure 4.2: Parameters and operations involved in the FEBICA algorithm along with corresponding equation numbers.

the learning rule given in (4.18), the non-linearity $g(t)$ will dominate and the learning rule will converge to a separating matrix $\mathbf{W}$.

In case of FEBICA, Fourier transform is used for the process of separating the signals via mutual independence. This Fourier domain application to BSS has been used in [39] and [40]. As shown in [41], by using a Fourier basis, advantages such as compact representations, higher convergence rate and lower mean squared error may be achieved. Consider the following Fourier basis function:

$$q_n[\mathbf{y}(k)] = e^{jn\omega\mathbf{y}(k)} \quad (4.28)$$

where ω is the system frequency. The gradient ascent rule used in (4.24) can be written as:

$$w_n(k+1) = w_n(k) + \eta \Delta w_n^q(k) \quad (4.29)$$

$$\Delta w^q(k) = \frac{1}{N} \sum_{j=0}^N e(j) q_n[\mathbf{y}(k)] \quad (4.30)$$

$$e(j) = \mathbf{y}(j+1) - \hat{\mathbf{y}}(j+1) \quad (4.31)$$

where $\hat{\mathbf{y}}(j+1)$ is the estimated unmixed signal with every j^{th} iteration. As shown in [41], the mean square error Ψ with each iteration would drop as:

$$\Psi = \frac{1}{N} \sum_{j=0}^N [\mathbf{y}(j+1) - \hat{\mathbf{y}}(j+1)]^2 \quad (4.32)$$

$$E_r = \frac{\left\| \sqrt{\Psi(k)} - \sqrt{\Psi(k-1)} \right\|}{\left\| \sqrt{\Psi(k)} \right\|} \quad (4.33)$$

Thus, by controlling the relative error E_r , a better accuracy measure for the source sepa-

ration problem can be achieved. The Fourier basis is also computationally less intensive which is useful in the case of online source separation. These, along with other comparative results, are discussed in the next section.

For the fast convergence of the FEBICA algorithm, we need to set the scaling factor η such that cost function Ω is minimized with every iteration.

$$\Omega = \eta \left(1 + \frac{\xi''}{\xi'} (\mathbf{W}_i \gamma - (\mathbf{W}_{fb})_i \xi) \right) \quad (4.34)$$

where, (4.34) holds true only when the scaling factor η satisfies the condition given in (4.36). Thus, this is the condition for convergence of the FEBICA algorithm.

$$\chi = \frac{1}{1 + e^\gamma} (1 + 2\xi e^{-\gamma}) (\mathbf{W} \gamma - \mathbf{W}_{fb} \xi) \quad (4.35)$$

$$0 < \eta \leq \min(\chi) \quad (4.36)$$

Thus, the FEBICA algorithm converges when the scaling factor is less than or equal to the minima of the function χ . For a single iteration with a single source, the convergence of the function Ω is shown in Fig. 4.3. The value of η which satisfies (4.37) will produce faster convergence.

$$\log_{10}(\Omega) < 1 \quad (4.37)$$

4.3 Results for FEBICA

In order to compare the performance of the blind source separation algorithms, they are applied to communication signals. As an example, Gaussian Minimum Shift Keying

Figure 4.3: Convergence of the FEBICA algorithm for one iteration with various settings of scaling factor η .

(GMSK) modulated signals are generated using the simulator developed by [28]. Sensor signals are generated using a realistic mixing model developed in Section 3.1. In this model, the mixing process embeds the source directions in the mixing matrix. The setup consists of 3-sensor ULA (uniform linear array) X_1, X_2, X_3 with half-wavelength spacing and 3 sources positioned as shown in Fig. 4.4. The sources S_1 and S_3 are situated 5 half-wavelengths away from S_2 making an angle of 15 degrees from the normal to the array axis. The sensor signals are then distorted with additional white Gaussian noise but with no Doppler shift. The observed mixed signals are then separated using the JADE, ACMA and FEBICA algorithms.

Comparison with the widely used Second Order Blind Identification (SOBI) algorithm [42] is shown in Fig. 4.5. The SOBI algorithm makes use of joint unitary diagonalization and is supposed to separate complex valued instantaneous mixtures. For a mixture of three GMSK modulated signals at input SNR of 20 dB, the performance of the FEBICA algorithm is shown to be out perform than the SOBI algorithm. The SOBI algorithm is unable to separate the mixture and retain the signal constellation. However, the FEBICA algorithm separates the signals and produces a unit circular constellation seen in the

Figure 4.4: Configuration of the sources S_1 , S_2 and S_3 with respect to the sensor array used to generate the mixing process.

original signals.

We next compare the signal to interference plus noise ratio (SINR) improvement provided by various source separation algorithms. The two situations considered are cases when the sources are far apart (non-singular mixing matrix) and when the sources are close to each other (nearly singular mixing matrix). For a set of 3 sources, $\mathbf{A}_1$ is an example of a non-singular mixing matrix while $\mathbf{A}_2$ is an example of a nearly singular mixing matrix.

$$\mathbf{A}_1 = \begin{bmatrix} -0.7458 + 0.6661i, -0.9794 + 0.2019i, -1.0000 + 0.0000i \\ 0.0000 + 1.0000i, -0.2291 + 0.9734i, -1.0000 + 0.0000i \\ 0.7458 + 0.6661i, 0.7865 + 0.6176i, -1.0000 - 0.0000i \end{bmatrix} \quad (4.38)$$

Figure 4.5: Comparative performance of the FEBICA and the SOBI algorithm. The reason for the small values seen in the index of the SOBI algorithm is the uncertainty in the amplitude and energies inherent in BSS algorithms as discussed in Section 1.1.4.

$$\mathbf{A}_2 = \begin{bmatrix} 0.1483 - 0.9889i, 0.5158 - 0.8567i, -1.0000 + 0.0000i \\ 0.1213 - 0.9926i, 0.4936 - 0.8697i, -1.0000 + 0.0000i \\ 0.0943 - 0.9955i, 0.4711 - 0.8821i, -1.0000 + 0.0000i \end{bmatrix} \quad (4.39)$$

The eigenvalues and eigenvectors of $\mathbf{A}_1$ are given by diagonals of D_1 and column vectors of V_1 .

$$D_1 = \begin{bmatrix} -1.4723 + 2.2598i & 0 & 0 \\ 0 & -0.4771 - 0.7272i & 0 \\ 0 & 0 & -0.0255 + 0.1070i \end{bmatrix} \quad (4.40)$$

Figure 4.6: The SINR improvement for the various algorithms for different SNR settings with 10 source signals.

$$V_1 = \begin{bmatrix} 0.4657 + 0.3984i & -0.1929 - 0.5905i & 0.7544 \\ 0.6171 & 0.1260 - 0.0864i & -0.6158 + 0.0988i \\ 0.3688 - 0.3281i & 0.7686 & 0.0398 + 0.2007i \end{bmatrix} \quad (4.41)$$

Similarly, the eigenvalues and eigenvectors of $\mathbf{A}_2$ are given by diagonals of D_2 and column vectors of V_2 . We notice the lack of distinct eigenvalues in the nearly singular case.

$$D_2 = \begin{bmatrix} -0.3770 - 1.8210i & 0 & 0 \\ 0 & 0.0189 - 0.0376i & 0 \\ 0 & 0 & 0.0000 + 0.0000i \end{bmatrix} \quad (4.42)$$

Figure 4.7: The SINR improvement with increase in the number of sources for an input SNR of 20 dB.

$$V_2 = \begin{bmatrix} 0.5685 + 0.0272i & -0.1173 + 0.4501i & -0.6334 - 0.2646i \\ 0.5772 + 0.0137i & 0.3375 + 0.2312i & 0.7261 \\ 0.5854 & 0.7851 & 0.0190 - 0.0350i \end{bmatrix} \quad (4.43)$$

As illustrated in Fig. 4.6, for the case of non-singular sources, the FEBICA algorithm improves with increase in SINR and outperforms the JADE algorithm. The performance of all three BSS algorithms drops when the mixing matrix is nearly singular. However, we see that the FEBICA and ACMA algorithms perform comparatively well. Without the BSS algorithms, the output SINR would be significantly lower than the input SINR. However, the implementation of the BSS algorithms improves the quality of the signal pushing the output SINR well above the input SNR. This, along with output BER and computational complexity, is another significant performance index to consider when incorporating source separation for communication systems. The ACMA algorithm per-

Figure 4.8: The Floating Point values of various BSS algorithms for increasing signal length and number of sources.

forms the best for non-singular sources due to the computationally intensive processing schemes employed. However, it deteriorates in performance for nearly singular sources.

Furthermore, the effect of increasing the number of sources on the BSS algorithms is compared. As seen from Fig. 4.7, the improvement provided by the FEBICA algorithms for modulated signals of data length 1480 samples and an input SNR of 25 dB, is consistent over a wide range of the number of sources. While the JADE algorithm provides nearly optimal performance for less than 4 sources, the algorithms performance deteriorates drastically with increase in the number of sources. By making use of gradient ascent techniques for optimizing the unmixing weights, the FEBICA algorithm is more stable over a range of sources and does not deteriorate in performance. The ACMA algorithm is also quite stable over the range of sources. However, it is computationally more intensive as it is shown in the following results.

Table 4.1: Floating Point operations involved with FEBICA with corresponding equation numbers

Equation	Flop Count
$\gamma_{k,m} = \sum_{n=0}^{N-1} \mathbf{x}_{n,m} e^{-j \frac{2\pi}{N} nk} \quad (4.20)$	$m(2^k \log(2^k)) \quad \forall 2^k \geq n$
$\mu_m = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}_{n,m} \quad (4.21)$	mn
$\xi(\gamma_{k,m}, \beta_k, \delta_m) = e^{-\beta_k/\delta_m} \cdot \gamma_{k,m} \quad (4.22)$	mn
$\psi_\iota = \mathbf{W}_\iota \gamma - (\mathbf{W}_{fb})_\iota \xi \quad (4.23)$	$\text{Idm}^2 n$
$\Delta \mathbf{W}_\iota = \eta \left(1 + \frac{\xi''}{\xi} \psi_\iota \right) \mathbf{W}_\iota \quad (4.24)$	$\text{Idm}^2 n$
FEBICA	$2m[n(1 + \text{Idm}) + 2^{k-1} \log(2^k)]$

Table 4.1 shows the floating point operations involved with the various algorithmic steps of our FEBICA algorithm. The computation is determined based on the sizes of the matrices $\mathbf{X}$ ($m \times n$), $\mathbf{U}$ ($d \times m$) and the number of iterations for convergence $\mathbf{I}$. The 2^k term is introduced here since FFT is performed on multiples of 2. The computational complexity of ACMA and JADE algorithms have been investigated by [22] and [43], respectively.

Fig. 4.8 represents the floating point operations associated with various BSS algorithms. We observe that the FEBICA algorithm substantially outperforms the ACMA algorithm when the number of sources is more than six. It performs comparably well with the JADE algorithm especially for a large number of sources. Comparing Figs. 4.6 and 4.8, we see that the FEBICA algorithm can provide higher SINR improvement for the GMSK mixtures while maintaining low computational complexity. The computational complexity of the FEBICA depends on the number of corresponding iterations which in turn relates to the optimal scaling factor η . Hence, for efficient performance of the FEBICA algorithm, the scaling factor should satisfy the threshold specified in (4.37).

The results for the application of the FEBICA algorithm on real field data is shown

Figure 4.9: The separation performance of JADE and FEBICA when applied to real field data: (a) Observed mixed signal constellation (b) Signal constellation after JADE (c) Signal constellation after FEBICA (d) Relative phase distribution of sources s_1 and s_2 (e) Relative phase distribution after JADE (f) Relative phase distribution after FEBICA.

in Fig. 4.9. The I-Q data of CFSK modulated signals produced by two sources was sampled at 25 kHz and received by two receivers. The data was sent out in bursts with 80% of the time frame occupied by data and 20% left for synchronization. The SNR value was recorded to be in the range of 30-40 dB. As shown in Fig. 4.9, the JADE and FEBICA algorithms restore the signal constellations of the received data to that of a typical GMSK signal. The relative phase between the two separated signals is also randomized. This shows that the signals have been made more independent than the received data which follows from the fundamental assumptions of BSS. Further illustration of the good separation performance of FEBICA is seen in the time-frequency plots of the mixed and separated outputs, as shown in Fig. 4.10.

Figure 4.10: The separation performance of FEBICA when applied to real field data: (a) Observed mixed signal in TF domain (b) One of the separated signal (c) The other separated signal.

Chapter 5

Doppler Aided BSS Algorithms

When overdetermined situations are considered, calculating the number of sources (blindly) is required to perform the separation process. This is because we can no longer assume the number of observations correspond to the number of sources. This chapter first introduces techniques for blindly estimating the number of sources. Then, the Doppler Aided BSS model is introduced which is applied for cases when the sources are moving and close to each other. The BER and SINR improvements provided by this model are described along with techniques to track the sources to aid in effective source separation.

5.1 Estimating the Number of Sources

Most BSS algorithms require the specification of the number of sources for accurate separation. However, if the number of sources is unknown or changing, the problem of estimating the number of sources becomes critical in accurate source separation. In case of Doppler aided BSS, as the observations are converted to the overdetermined case, accurate blind estimation of the number of sources is required. This is because the number of

sources being equal to the number of sensors assumption (used in the critically determined case) is no longer valid.

One of the simplest techniques is to estimate the auto covariance matrix of the observed mixed signals and use the most significant eigenvalues as the estimate of the number of sources. However, the accuracy of this technique is dependent on the threshold set. It is also not reliable in case of low SNR conditions.

An application of the Akaike information theoretic criterion (AIC) [45] is useful as it does not require a subjective threshold and determines the number of users based on the value for which the criterion is minimized. In the AIC approach, the number of signals $\hat{D}$, is determined as the value of $d \in 0, 1, \dots, M-1$ which minimizes the following criterion [45]:

$$AIC(d) = -\log \left\{ \frac{\prod_{i=d+1}^M \lambda_i^{\frac{1}{(M-d)}}}{\frac{1}{M-d} \sum_{i=d+1}^M \lambda_i} \right\}^{(M-d)N} + d(2M - d) \quad (5.1)$$

where λ_i are the eigenvalues in the covariance matrix, N is the number of independent snapshots used to compute the covariance matrix and M is the number of elements in the array. The first term in the above equation is based on the log-likelihood function while the second term is the penalty factor added by the AIC criterion.

5.1.1 Gerschgorin Radii

As shown by [46], the number of sources may be estimated by the use of Gerschgorin Radii of a unitary transformed input covariance matrix. The Gerschgorin theorem on eigenvalues of a matrix provides a method for estimating the location of eigenvalues from

the values of the matrix elements. For a $M \times M$ matrix $\mathbf{A} = \mathbf{a}_{ij}$, Gerschgorin proved that all of the eigenvalues of the matrix are contained in the union of M discs $\mathbf{O}_i$, $i = 1, \dots, M$. These discs are centered at $\mathbf{a}_{ii}$, and have radii, called Gerschgorin radii $\mathbf{r}_i$, equal to the sum of the magnitudes of all elements of the i^{th} row vector, excluding the i^{th} element.

$$\mathbf{r}_i = \sum_{j=1, j \neq i}^M |\mathbf{a}_{ij}| \quad (5.2)$$

The eigenvalues of a matrix are located within the Gerschgorin disks which represent the collection of points in the complex plane whose distance to $\mathbf{a}_{ii}$ is, at most, $\mathbf{r}_i$. As shown by [46], at low SNR conditions, the eigenvalues of the input covariance matrix are spread across a large range. Through a unitary transformation which preserves the eigenvalues, the overlap in the Gerschgorin disks can be reduced and used for effective source number detection. This is done by rotating the covariance matrix so that the Gerschgorin disks can be formed into two distinct signal and noise constellations. The source collection with larger Gerschgorin radii will contain exactly M largest signal eigenvalues, while the noise collection with small Gerschgorin radii will contain the remaining noise eigenvalues. Once this is done, the number of sources can be estimated by the classification of the disks.

The new estimator function incorporating Gerschgorin radii information into the log-likelihood function is called the Gerschgorin Likelihood Estimator (GLE) and is given by [46]:

$$GLE(d) \approx -N(M-1-k) \log \left[\frac{\left(\prod_{i=d+1}^{M-1} \lambda'_i \right)^{\frac{1}{M-d-1}}}{\frac{1}{M-d-1} \sum_{i=d+1}^{M-1} \lambda'_i} \right] - N \log \left(\mathbf{r}_{MM} - \sum_{i=1}^d \frac{\mathbf{r}_i^2}{\lambda'_i} \right) \quad (5.3)$$

where N is the number of data snapshots. Both AIC and GLE are based on the assumption of Gaussian and spatially white noise.

5.1.2 Time-Frequency Methods

A commonly used approach specially in convolutive separation of audio and speech mixtures is to make use of time-frequency representations [47]. The short time discrete Fourier transform (STDFT) on the observed signal produces an output of the form:

$$\mathbf{Z}(\tau, \kappa) = \sum_{n=-\infty}^{\infty} \mathbf{x}[n] \mathbf{p}[n - \tau] e^{-j\kappa n} \quad (5.4)$$

where $\mathbf{Z}(\tau, \kappa)$ is the STDFT of the observed mixed signal $\mathbf{x}[n]$ and $\mathbf{p}[n]$ is the window function. By using this kind of transformation, the sources are seen to be much more sparse than in the purely time domain. As shown by [48], the information in the STDFT domain can be used to estimate the number of sources blindly. The transformation can be presented as:

$$|\mathbf{Z}(\tau, \kappa)| = \left| \sum_{j=1}^d \mathbf{a}_j \mathbf{s}_j(\tau, \kappa) \right| \quad (5.5)$$

where d represents the number of sources in the mixture and the observation vectors are mixtures of all the sources $\mathbf{s}_j$ determined by the mixing parameters $\mathbf{a}_j$. When the sources are sparse, the likelihood of them being disjoint in a certain time-frequency $(\tau - \kappa)$ bin increases and may belong to a single source.

Let the observation vectors from $\mathbf{Z}(\tau, \kappa)$ and $\mathbf{Z}(\tau + 1, \kappa)$ belong to two consecutive time frames of the same frequency bin κ . If the observation vectors are a mixture of two

sources, then [48]

$$|\mathbf{Z}(\tau, \kappa)| = |\mathbf{a}_{j1}\mathbf{s}_{j1}(\tau, \kappa) + \mathbf{a}_{j2}\mathbf{s}_{j2}(\tau, \kappa)| \quad (5.6)$$

Defining the phase information contained in the real ($\mathbf{Z}_r$) and imaginary ($\mathbf{Z}_i$) parts of the above, we have [48]:

$$\theta_{(\mathbf{Z}(\tau, \kappa))} = \tan^{-1} \left(\frac{\mathbf{Z}_i(\tau, \kappa)}{\mathbf{Z}_r(\tau, \kappa)} \right) \quad (5.7)$$

The $\theta_{(\mathbf{Z}(\tau, \kappa))}$ and $\theta_{(\mathbf{Z}(\tau+1, \kappa))}$ from two consecutive time frames will not be the same unless $\mathbf{s}_{j1}$ and $\mathbf{s}_{j2}$ remain constant along both frames. Since this is nearly impossible, observation vectors with constant phases along two consecutive time-frequency bins will belong to a single source. The single sources may be selected based on a appropriately small threshold Θ as:

$$|\theta_{\mathbf{Z}(\tau, \kappa)} - \theta_{\mathbf{Z}(\tau+1, \kappa)}| < \Theta \quad (5.8)$$

In most BSS algorithms, the assumption is that the number of sources are known. However, if the sources are not stationary and can transmit intermittently, estimating the number of sources becomes critical. By using the above technique, the source separation can be performed without having prior information about the number of sources.

5.2 Doppler Model for BSS

The performance of these source separation algorithms is very sensitive to the singularity of the mixing matrix and the noise levels. A nearly singular mixing matrix physically corresponds to the sources being very close to each other. The performance of source separation algorithms deteriorates significantly as the mixing matrix nears singularity. By exploiting the inherent Doppler shifts observed in non-stationary sources, the separation

performance in even a nearly singular mixing condition can be improved in terms of bit error rate (BER) and output signal to interference plus noise ratio (SINR).

The effect of Doppler on source separation has been discussed in [44], where an estimation algorithm is proposed to minimize the effect of Doppler shifts in source separation. The proposed mixing model is developed to a more general case which can be applied to generic BSS algorithms like JADE and ACMA. We further exploit the diversity provided by these Doppler shifts to improve the source separation performance. If a constant Doppler shift is assumed, this technique can be used to track the source locations as well.

We incorporate the two cases of stationary sensors and movement of sensors in order to develop an accurate model for the effect of Doppler shifts in BSS. As shown by [49], the effect of Doppler shifts can be incorporated into the mixing matrix as follows. Consider a case when there are N sources and M sensors ($M \geq N$), with the sensors having p tapped delay lines spaced D delay units apart. Assuming the N sources have the same center frequency ω_c . The output of the h^{th} delay unit of the i^{th} sensor may be given by:

$$\begin{aligned} \mathbf{x}_i(t - hT) = & \sum_{k=1}^N \mathbf{a}_{ik} \mathbf{s}_k(t) e^{-j\omega_c \Gamma_{ik} - j\omega_c hD} e^{-j\omega_k \Gamma_{ik} - j\omega_k hD} \\ & + \mathbf{n}_i(t - hD) \end{aligned} \quad (5.9)$$

where $i = 1 \dots M$, $h = 0 \dots (p - 1)$, ω_k is the constant Doppler frequency associated with the k^{th} source, Γ_{ik} is the propagation delay between the i^{th} sensor and the k^{th} source and $\mathbf{a}_{ik}$ is the complex impulse response of the i^{th} sensor and the k^{th} source.

Assuming there are simultaneous snapshots of the signals at times $t_1, t_2, \dots, t_G$ yielding

G snapshots. This can be written in the simplified form:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (5.10)$$

$$\mathbf{x}^T(t) = [\mathbf{x}_1^T(t) \quad \mathbf{x}_2^T(t) \quad \dots \quad \mathbf{x}_M^T(t)] \quad (5.11)$$

where $\mathbf{A}$ is given in (5.12). This model is specific to instantaneous mixtures with time delays considered. The above mixing matrix incorporates both the effects of co-channel interference as well as Doppler shifts. This provides an additional Doppler dimension that can prove to be advantageous specially when the sources are placed close to each other. While conventional blind-beamforming techniques might fail under such resolutions, such a Doppler modified model can still provide optimal separation performance.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_M \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{11}e^{-j\omega_1\tau_{11}}e^{-j\omega_c\tau_{11}} & \dots & \mathbf{a}_{1N}e^{-j\omega_N\tau_{1N}}e^{-j\omega_c\tau_{1N}} \\ \vdots & \ddots & \vdots \\ \mathbf{a}_{M1}e^{-j\omega_1\tau_{M1}-j\omega_1(p-1)D}e^{-j\omega_c\tau_{M1}-j\omega_c(p-1)D} & \dots & \mathbf{a}_{MN}e^{-j\omega_N\tau_{MN}-j\omega_N(p-1)D}e^{-j\omega_c\tau_{MN}-j\omega_c(p-1)D} \end{bmatrix} \quad (5.12)$$

5.3 Frequency Estimation

After using the Doppler-aided BSS separation technique to improve the separation performance, it is possible to further extend frequency estimation techniques like ESPRIT or MUSIC [50] in order to track the sources.

When the signals of interest are narrowband, that means that the message component $\mathbf{m}$ changes very slowly with time and can be considered to be constant.

$$\mathbf{s}[m] = \sum_{k=1}^K \mathbf{m}_k e^{j\omega_k m} + \varepsilon[m], \quad m = 0, \dots, M-1 \quad (5.13)$$

where $\mathbf{m}_k$ is the message of the k -th signal and ω_k represents the frequency component to be estimated.

As shown by [50], this is a typical frequency estimation problem which can be solved using a *space-time* FIR filter $\mathbf{h}_M$. This has an optimal solution given by:

$$\mathbf{h}_{M,k} = \frac{\mathbf{R}_x^{-1} e^{j\omega_k}}{(e^{j\omega_k})^H \mathbf{R}_x (e^{j\omega_k})} \quad (5.14)$$

where $\mathbf{R}_x$ is the autocorrelation matrix. This can then be extended to develop the pseudo-spectrum as [50]:

$$\hat{s}(e^{j\omega_k}) = \frac{1}{(e^{j\omega_k})^H \mathbf{R}_x^{-1} (e^{j\omega_k})}, \quad k = 1, \dots, K \quad (5.15)$$

which can provide an accurate estimate of the frequency components (Doppler frequencies) present in the sources.

By this, the blind source separation can be performed at particular time instances while source tracking at other instances can be employed to estimate the unmixing matrices.

By assuming constant Doppler shifts, this technique can be used in place of performing source separation on every frame of data.

5.4 Results for Doppler Aided BSS

In order to compare the performance of the blind source separation algorithms, they are applied to communication signals. As an example, Gaussian Minimum Shift Keying (GMSK) modulated signals are generated using the simulator developed by [28]. The signals are then distorted with additional white Gaussian noise but with no Doppler shift. The observed mixed signals are then separated using the JADE and ACMA to test the effect of Doppler shifts. Estimating the number of sources with the time-frequency technique using threshold $\Theta = 0.05$ degrees is found to be optimal in most cases. As we develop a larger number of delayed and Doppler aided observations, estimating the number of sources is critical to make the process blind. This is because, we convert a critically determined case to an overdetermined case. If there is no prior information about the number of sources being equal to the number of observations, the number of sources should be determined before we attempt the overdetermined separation problem.

A plot of the narrow-band ambiguity function is shown in Fig. 5.1. After applying Doppler shifts, additional diversity is seen in the output of the Doppler-delay ambiguity plots which can be used for source separation.

The improvement provided due to the application of Doppler shift is shown in Fig. 5.2. Before applying the Doppler shift matrix, the mixing matrix is ill-conditioned (rep-

Figure 5.1: The narrow-band ambiguity function plots (a) Prior to application of Doppler shifts (b) After application of Doppler shifts (c) A zoomed version of (b) showing the additional diversity.

resenting closely spaced source position vectors) and may be given by:

$$\mathbf{A}' = \begin{bmatrix} -0.7611 - 0.6486i & -0.1007 - 0.9949i \\ -0.6661 - 0.7458i & 0.0278 - 0.9996i \end{bmatrix} \quad (5.16)$$

After applying the Doppler shifts by the mixing model (5.9), with $D = 0.6$ seconds and utilizing additional diversity of five delayed version of the observed mixtures, the source separation is performed. As we can see from Fig. 5.2, the Doppler shift input into the

system helps improve the performance of the JADE and ACMA algorithms. Though unable to separate the sources before in the ill-conditioned case, by application of slight Doppler shifts (≈ 60 Hz in the 1.8 GHz range) into the system improves the performance considerably.

Figure 5.2: Improvement due to the application of Doppler shifts: (a) Original GSMK source (b) Mixture of GSMK sources at SNR 20 dB and ill-conditioned mixing matrix (c) Separation performance of JADE without Doppler (d) Separation performance of ACMA without Doppler (e) Separation performance of JADE after Doppler shift (f) Separation performance of ACMA after Doppler shift.

Results for improvements in terms of the output bit error rate and signal to interference plus noise ratio are shown in Figs. 5.3 and 5.4, respectively. Improved performance for both algorithms is seen after Doppler aided source separation. Once again, combining the two graphs can provide additional insights. For example, post Doppler ACMA can give less than 10^{-3} output BER and approximately 25 dB output SINR for an input SNR of 15 dB for critically determined mixtures of two sources. This is an improvement over

the pre Doppler ACMA that can provide only 0.03 output BER and 15 dB output SINR for the same settings. Comparing the time-frequency distributions, the incorporation

Figure 5.3: Bit Error Rate improvements provided due to Doppler aided diversity in BSS.

of a Doppler shift considerably improves the separation performance. Applying it to a set of three GMSK modulated signals as shown in Fig. 5.5, we see that the separation performance of the ACMA algorithm is drastically improved when the sources are in motion (Doppler shift). By making use of this technique, it is possible to further extend frequency estimation techniques like ESPRIT or MUSIC [50] in order to track the sources. By this, the blind source separation can be performed at particular time instances while source tracking at other instances may be employed to estimate the unmixing matrices. An example of tracking the frequencies of three sources over several time instances is shown in Fig. 5.6. It is assumed here that each of the sources have different Doppler shifts with the rate of change of Doppler being constant. By using such frequency tracking techniques,

Figure 5.4: Output SINR improvements provided due to Doppler aided diversity in BSS. The sources are constrained to move at a constant velocity of 10 m/s with respect to each other.

the computational burden of running BSS algorithms for every iteration can be reduced. For instance, only once every few iterations, the BSS algorithms may be run to check for any updates / changes in the rate of change of Doppler in the system. For the rest of the iterations, keeping some parameters like source velocity and sensor position constant, the frequency estimation process can be employed to provide approximate parameters for updating the unmixing matrix for source separation.

An example of this dual - BSS and frequency tracking technique is shown in Fig. 5.7. While conventional BSS techniques perform source separation for each data packet at time t_1 to t_n , the hybrid approach uses the frequency estimation information. Assuming constant rate of change of Doppler shifts, the estimated frequencies after time instance t_1 can be used to track the sources. The time difference between instances t_1 and t_2

Figure 5.5: Time-frequency plots for various stages of source separation (a) Original GMSK source (b) Mixture of three GMSK sources at 20 dB SNR without Doppler shift (c) Mixture of three GMSK sources at 20 dB SNR after Doppler shift (d) Performance of ACMA on pre-Doppler mixture (e) Performance of ACMA on post-Doppler mixture.

can be used to update the unmixing matrix parameters (Doppler shifts), thus effectively providing the updated unmixing matrix for that time instance. Hence, the computational burden to performing source separation can be reduced.

Results for Doppler Aided BSS

Figure 5.6: Frequency estimation of Doppler separated signals using ESPRIT.

Figure 5.7: Comparison of standard BSS and the frequency estimation tracking aided BSS.

Chapter 6

High Fidelity Source Separation

High fidelity or hi-fi reproduction is a term used to refer to high-quality reproduction of sound that is very faithful to the original performance. Ideally, high-fidelity equipment has a minimal amount of noise and distortion and an accurate frequency response.

To obtain satisfactory results using conventional BSS techniques, the noise is usually neglected or high input signal-to-interference plus noise ratio (SINR) is considered. For separation of speech / audio signals, the noise is always present at the sensors. Reduction of noise especially in low signal-to-noise ratio (SNR) conditions is crucial for accurate reconstruction especially in teleconferencing and voice over internet protocol (VoIP) applications. The source separation of speech and audio signals in noisy environments have been studied in [51]. However, these techniques do not make use of spatial diversity of the sensors. The aim of this section is to study the electronic reproduction of sound after blind source separation (especially from broadcast or recorded sources) to check for minimal distortion.

We propose a hybrid configuration for two-stage source separation and noise reduction scheme under over-determined setting by exploiting the spatial diversity. By combining

the commonly used source separation techniques like FastICA and Infomax, with the minimum distortion noise reduction (MDNR) algorithm [52], we have shown the improvement in terms of the output SINR and signal-to-artifact ratio (SAR). Unlike other beamforming algorithms, the MDNR algorithm does not require the direction-of-arrival (DOA) information which would have restricted the position of the sources and sensors. The simulation results show that the MDNR algorithm provides better output when compared to the conventional Delay-and-Sum (DAS) beamforming.

6.1 Performance bounds of BSS algorithms

The FastICA algorithm [17] as discussed in section 2.1.3 makes use of an efficient learning rule to maximize the non-Gaussianity of the projection. It is among the most commonly used algorithms for optimal search of the unmixing matrix $\mathbf{W}$ that is updated based on a nonlinear contrast function. The optimization techniques like gradient search or Newton optimization are used for updating the contrast function $G(\mathbf{W}\mathbf{X})$, where $\mathbf{X}$ is the observed matrix of the mixed source signals.

The performance bounds of noisy linear ICA has been studied in [53, 54]. The optimal solution in the case of noisy ICA is close to the minimum mean square error (MMSE) solution given by:

$$\mathbf{W}^{MMSE} = \mathbf{A}^T (\mathbf{A}\mathbf{A}^T + \sigma^2 \mathbf{I})^{-1} \quad (6.1)$$

where σ^2 is the noise variance. This leads to the minimum attainable signal to interference plus noise ratio for the k^{th} estimated signal characterized by [53, 54]:

$$\Psi = (I + \sigma^2 (\mathbf{A}^T \mathbf{A})^{-1})^{-1} \quad (6.2)$$

$$\min \text{SINR}_k = \frac{\Psi_{kk}^2}{\sum_{i \neq k}^d \Psi_{ki}^2 + \sigma^2 \sum_{i=1}^d (\Psi \mathbf{A}^{-1})_{ki}^2} \quad (6.3)$$

where i and k represent the rows and columns of the observed mixed signals, respectively and d refers to the total number of signals observed in the mixture. This shows that the bound is dependent only on the mixing matrix $\mathbf{A}$ and the noise variance σ^2 .

6.2 Minimum Distortion Noise Reduction

The MDNR algorithm proposed in [52] addresses the problem of estimating *one* source signal given the received signals at the microphone array. Let $\{\mathbf{y}_1(k), \dots, \mathbf{y}_N(k)\}$ be the discretized received signals of L samples. By exploiting the spatio-temporal diversity, the source signal of m -th sensor at k -th sample $x_m(k)$ can be obtained by passing the received signals at N sensors (of which there are L samples) through N temporal filters of length L

$$\hat{x}_m(k) = \mathbf{h}_m^T \mathbf{y}(k) = \mathbf{h}_m^T \mathbf{x}(k) + \mathbf{h}_m^T \mathbf{v}(k) \quad (6.4)$$

where $\mathbf{h}_m = [\mathbf{h}_{1m}^T, \dots, \mathbf{h}_{Nm}^T]^T$, $\mathbf{h}_{nm}$ is the column vector of L coefficients of the temporal filter for the n^{th} received signal. $\mathbf{y}(k) = [\mathbf{y}_1^T(k), \dots, \mathbf{y}_N^T(k)]^T$, $\mathbf{x}(k) = [\mathbf{x}_1^T(k), \dots, \mathbf{x}_N^T(k)]^T$, and $\mathbf{v}(k) = [\mathbf{v}_1^T(k), \dots, \mathbf{v}_N^T(k)]^T$ are the received signal, clean signal and noise signal column vectors, respectively. Notice that we have grouped the signal term and noise term separately.

Using this form shown in (6.4), the task of the estimator is to find $\mathbf{h}_m$ by minimizing the mean-square-error due to the noise term under the constraints that the error due to

the signal term $(\mathbf{h}_m^T \mathbf{x}(k) - x_m(k))$ is zero. That is, by solving the following optimization

$$\mathbf{h}_{m,o} = \arg \min_{\mathbf{h}_m} \mathbf{h}_m^T \mathbf{R}_{vv} \mathbf{h}_m \quad \text{s.t.} \quad \mathbf{Q}_m \mathbf{h}_m = \mathbf{u}_1 \quad (6.5)$$

where $\mathbf{Q}_m = [\mathbf{Q}_{1m}^T, \dots, \mathbf{Q}_{Nm}^T]$ is the spatial-temporal prediction matrix, which relates the signal at one microphone to others: $\mathbf{x}_n(k) = \mathbf{Q}_{nm} \mathbf{x}_m(k)$. $\mathbf{R}_{vv} = E[\mathbf{v}(k) \mathbf{v}^T(k)]$. $\mathbf{u}_1 = [1, 0, \dots, 0]^T$.

Solving (6.5) using Lagrangian multiplier method, the optimum $\mathbf{h}_m$ can be computed given the spatial-temporal prediction matrix. Instead of using the true $\mathbf{Q}_m$, which is usually unknown, an estimate can be obtained easily as [52]

$$\mathbf{Q}_{nm,o} = (\mathbf{R}_{y_n, y_m} - \mathbf{R}_{v_n, v_m})(\mathbf{R}_{y_m, y_m} - \mathbf{R}_{v_m, v_m})^{-1} \quad (6.6)$$

where $\mathbf{R}_{v_n, v_m} = E[\mathbf{v}_n(k) \mathbf{v}_m(k)]$. The same definition applies similarly to $\mathbf{R}_{y_n, y_m}$. Therefore, the final expression of $\mathbf{h}_{m,o}$ is obtained by solving (6.5) and substituting (6.6) into the solution [52]

$$\mathbf{h}_{m,o} = \mathbf{R}_{vv}^{-1} \mathbf{Q}_{m,o}^T [\mathbf{Q}_{m,o} \mathbf{R}_{vv}^{-1} \mathbf{Q}_{m,o}^T]^{-1} \mathbf{u}_1 \quad (6.7)$$

where $\mathbf{Q}_{m,o}$ is arranged the same way as $\mathbf{Q}_m$. Here $\mathbf{Q}_m$ is the ideal spatial-temporal prediction matrix while $\mathbf{Q}_{m,o}$ is an estimate based on (6.6).

It is stated in [52] that the worst-case performance of the MDNR algorithm will be that of the delay-and-sum beamforming [55] which is the case when only spatial diversity can be exploited for noise reduction. In this case, the noise power will be reduced by a factor of $1/N$ while the signal power remains unchanged. Given that the signal and noise power are σ_s^2 and σ_n^2 , respectively. The worst-case output SINR for the MDNR algorithm

can be expressed as

$$\text{SINR}_{\text{mdnr}} = \frac{N\sigma_s^2}{\sigma_n^2} \quad (6.8)$$

6.3 System Model for High Fidelity BSS

The problem of separating speech sources, or the typical cocktail party problem, has been investigated in previous literature [2]. Both convolutive and instantaneous mixtures for separating speech sources have also been studied. However, the efficient separation performance is limited to the case when either the noise is ignored or the input SNR is high. Gaussian noise causes deterioration of second order cumulants which the source separation algorithms can depend on. In case of FastICA, the assumptions regarding the covariance of the observed signals can be distorted especially under low SNR conditions. The mixing matrix can also become ill-conditioned leading to poor separation and denoising capabilities.

In order to improve both the noise reduction and separation performance, we propose a hybrid approach. With limited pre-processing, the observed noisy data is passed through the blind source separation algorithm. By making use of the overdetermined condition when the number of sensors is more than the number of sources, the diversity in each of the separated outputs is used for noise reduction. The minimum distortion noise reduction algorithm makes use of the outputs from these multiple channels to achieve noise reduction.

Unlike conventional preprocessing/post processing noise reduction techniques used by common source separation algorithms, this scheme exploits the spatial diversity of the sensor locations for multiple channel noise reduction. Under low input SNR, other

noise reduction schemes may distort the speech signal output. By exploiting the spatial diversity of the BSS algorithms and applying the MDNR technique, high fidelity in the speech output is ensured which is advantageous in its application to low SNR conditions. As illustrated in Fig. 6.1, three speech source signals $S_1(t)$, $S_2(t)$ and $S_3(t)$ are received

Figure 6.1: Scenario used for testing the proposed algorithm.

by an array of 5 sensors after passing through a channel mixing matrix. Because of the over-determined condition, we form 3 sub-arrays and perform source separation for each sub-array. This will provide the spatial diversity required for multiple channel noise reduction. The next stage is to utilize the output of the BSS algorithm $\hat{S}_{11}(t)$, $\hat{S}_{12}(t)$ and $\hat{S}_{13}(t)$ (from the first, second and third sub-arrays respectively) as the input to the MDNR algorithm. This multiple channel noise reduction process, in turn, provides a high fidelity

output $\hat{S}_1(t)$ of the target source signal $S_1(t)$. This procedure can be further repeated for the sources $S_2(t)$ and $S_3(t)$ to similarly obtain high fidelity outputs $\hat{S}_2(t)$ and $\hat{S}_3(t)$, respectively.

A problem with most BSS algorithms like FastICA is the ordering of sources (permutation problem). In our technique, as we make use of outputs from each sub-array, this ordering is critical to provide accurate input to the MDNR stage. The correlation between the separated signals is used to solve this. As seen from Fig. 6.2, the highest correlation values r_1 and r_2 are used as the basis for matching the separated outputs.

Figure 6.2: Example of using correlation to solve the permutation problem. The solid lines indicate highest correlation matching the separated output of each sub-array to a particular source.

6.4 SINR Improvement Bounds

The performance of the proposed hybrid approach can be evaluated in terms of the overall SINR output. Let S_k denote the desired speech signal to be separated. Given that there are d speech signals, the rest of the speech signals (S_i where $i \neq k$ and $i = 1, 2, \dots, d$) are considered as interferences.

The minimum attainable output SINR of noisy linear ICA has been given in (6.3). The expression contains three different terms for the desired signal, interferences and noise

power:

$$\sigma_s^2 = \Psi_{kk}^2 \quad (6.9)$$

$$\sigma_i^2 = \sum_{i \neq k}^d \Psi_{ki}^2 \quad (6.10)$$

$$\sigma_n^2 = \sigma^2 \sum_{i=1}^d (\Psi \mathbf{A}^{-1})_{ki}^2 \quad (6.11)$$

From Fig. 6.1, it can be seen that the output of the noisy linear ICA is also the input of the MDNR algorithm. Therefore, the output of the hybrid approach $\text{SINR}_{\text{hybrid}}$ can be expressed as

$$\text{SINR}_{\text{hybrid},\min} \leq \text{SINR}_{\text{hybrid}} \leq \text{SINR}_{\text{hybrid},\max} \quad (6.12)$$

where $\{\text{SINR}_{\text{hybrid},\min}, \text{SINR}_{\text{hybrid},\max}\}$ are the minimum and maximum attainable SINR output which can be written as

$$\text{SINR}_{\text{hybrid},\min} = \frac{\Psi_{kk}^2}{\sum_{i \neq k}^d \Psi_{ki}^2 + \frac{\sigma^2}{N_{\text{sub}}} \sum_{i=1}^d (\Psi \mathbf{A}^{-1})_{ki}^2} \quad (6.13)$$

$$\text{SINR}_{\text{hybrid},\max} = \frac{\Psi_{kk}^2}{\frac{\sigma^2}{N_{\text{sub}}} \sum_{i=1}^d (\Psi \mathbf{A}^{-1})_{ki}^2} \quad (6.14)$$

where $N_{\text{sub}} = N - d + 1$ is the number of subarrays formed after the BSS and N is the total sensors used. Notice that the above inequality is used to express the output SINR of the proposed hybrid approach, because the MDNR algorithm is not formulated for suppressing the interferences. Thus, the expression for $\text{SINR}_{\text{hybrid},\min}$ relates to only reduction of noise by the MDNR algorithm with no interference suppression by BSS. The $\text{SINR}_{\text{hybrid},\max}$ is achieved when the BSS technique has a perfect signal separation with improved noise suppression by the MDNR algorithm.

As compared to using a standard direct approach by just applying the BSS technique, the hybrid approach offers additional noise reduction capability. This is reflected on the noise power expression at the output of the hybrid approach. It is clear that the noise power has been reduced to $(1/N)$ fraction of the noise power at the BSS intermediate output. Applying the standard direct BSS approach does not effectively exploit the extra sensor outputs as the performance is similar to the critically determined case. By effectively using the overdetermined criterion, the hybrid approach offers additional SINR improvement by reducing the output noise power. This produces a significantly better output than the direct BSS approach.

6.5 Noise Reduction Performance

Based on the scenario shown in Fig. 6.1 the speech sources are mixed based with no reverberation considered. The array processing toolbox developed by [57] is used for the mixing process based on the position of the speech sources and the distribution of sensors (uniform linear array). The sources and sensors are placed at distances of 1 and 0.1 meters apart, respectively (Note that the geometry of the array can be made arbitrary). The source separation of the observed mixed signals at the sensor array is performed based on the FastICA algorithm. The permutation problem is seen in all the BSS algorithms, specially those that operate in the frequency domain. In this application, back correlation with respect to the input signal is used as the solution to the permutation problem. That is, the highest correlated BSS output with respect to the input signal is used as the corresponding estimate of that particular input signal. Further denoising of the separated output based on the sub-array structure is achieved by either the minimum distortion

noise reduction (MDNR) or the delay-and-sum (DAS) beamforming algorithms.

Fig. 6.3 shows the performance of the algorithm when applied to three noisy mixed speech signals. The outputs of the BSS algorithm with input SNR of -10 dB is input to both the MDNR and the DAS algorithms. The MDNR algorithm is able to successfully recover the denoised version of the original signal. The output of the proposed algorithm

Figure 6.3: Example of using the algorithm to separate and denoise the signal (a) Original signal (b) Separated signal before denoising (c) Estimated signal after MDNR (d) Estimated signal after DAS. Audio samples for this may be found in [56].

has been tested based on the output SINR and SAR (Signal to Artifact Ratio) improvements using the toolbox developed by [58]. The separation performance is computed for each estimated source $\hat{\mathbf{s}}_j$ and compared with the true source $\mathbf{s}_j$. The first step is to decompose the estimated unmixed signal as shown.

$$\hat{\mathbf{s}}_j = \mathbf{s}_{target} + e_{interf} + e_{noise} + e_{artif} \quad (6.15)$$

where $\mathbf{s}_{target}$ is a version of $\mathbf{s}_j$ modified by an allowable distortion, e_{interf} is an allowed deformation of the sources which accounts for the interferences of the unwanted sources, e_{noise} is an allowed deformation of the perturbing noise and e_{artif} is an artifact term that corresponds to artifacts of the separation algorithm such as musical noise or to deformations induced by the separation algorithm that are not allowed.

The next step is to compute energy ratios to evaluate the relative amount of each of the four terms in (6.15) either on the whole signal duration or on local frames. The computation of the SINR and SAR follows from the equations given below.

$$SINR = 10 \log_{10} \frac{\|\mathbf{s}_{target}\|^2}{\|e_{interf} + e_{noise}\|^2} \quad (6.16)$$

$$SAR = 10 \log_{10} \frac{\|\mathbf{s}_{target} + e_{interf} + e_{noise}\|^2}{\|e_{artif}\|^2} \quad (6.17)$$

While SINR is a measure of the separation performance, SAR measures the distortions caused by the source separation algorithm on the signals of interest.

As shown in Fig. 6.4, the MDNR algorithm provides better SINR output when compared to the DAS beamforming, specially at low input SNR. The cases for two and three mixed sources have also been considered. At higher SNR, the noise suppression performance of both DAS and MDNR techniques converge. Due to this, we notice an overlap of the graphs at high SNR, specially for the case of three sources. This is because, at higher SNR, the performance is dependent mainly on the separation performance. A reference to the minimum attainable SINR is provided for a mixture of three sources.

As seen in Fig. 6.5, the SAR improvements for two, three and four sources are considerable especially under low input SNR. Low input SNR is the main region of operation of

this algorithm and successfully demonstrates the superior performance over conventional BSS techniques. This shows that both source separation and noise reduction have been successfully incorporated assuring the high fidelity of the output speech signals. As expected, the SAR improvements is maximum for two and three sources cases and decreases for the case of four sources. This is due to the loss of additional diversity that more number of sensors would provide for noise reduction. The mean values of SINR and SAR have been used in all the above cases with approximations based on the toolbox in [58].

Figure 6.4: Output SINR for various settings of input SINR based on a mixture of two and three sources.

6.6 Reduction of Slight Reverberation

The effect of reverberatory environments in speech signal source separation is an aspect of significant research. In case of reverberatory environments, as beamforming is not from

Figure 6.5: Output SAR for various input settings based on a mixture of two, three and four sources.

a single direction, the ability to accurately separate sources is reduced considerably.

However, in mildly reverberant conditions, it is still possible to extract the sources up to a certain extent. The fidelity of the separated output comes into question here once again. By utilizing the MDNR algorithm, the model shown in Fig. 6.6 is developed. An experiment done using two speech signals mixed in a room simulation environment [57] with low reverberant conditions (wall absorption coefficients of 0.7 in a $10 \times 10 \times 10$ room) yielded the results shown in Fig. 6.7. The separation was done using a modified version of the Infomax algorithm [18] with ability to handle convolutive mixtures. The example shows that the output of the MDNR can reduce the effect of reverberation. However, the performance is significantly restricted by the separation performance of the BSS algorithm

Reduction of Slight Reverberation

Figure 6.6: The model for reducing the effect of reverberation in a two source mixture.

and will not work under high reverberant conditions.

Figure 6.7: Example for demonstrating the effect of MDNR on reverberation (a),(b) Original Speech Signals (c),(d) Mixed Signals in a mildly reverberant environment (e),(f) Separated Signals with the reverberation effects highlighted (g),(h) MDNR output with higher fidelity and lowered effect of reverberation.

Chapter 7

Other Aspects of Source Separation for Communication Systems

Discussed in this chapter are two techniques which are useful for special settings. The Parallel Factor Analysis (PARAFAC) model can be used for situations where the sensors are widely spaced and the instantaneous model is no longer approximate. Single channel convolutive source separation can be used to separate real valued signals from single channel convolutive recordings.

7.1 Widely Separated Sensors

In most source separation problems, it is assumed that the sensors are placed in an array close to each other. When the sensors are placed far apart, time-delays are introduced which can be problematic for most instantaneous source separation techniques. The method described by [64] makes use of a complex-valued Parallel Factor Analysis (PARAFAC) model [65] in order to separate delayed versions of sinusoids observed at

widely spaced sensors.

7.1.1 PARAFAC

The three-way PARAFAC technique is characterized by the following model [65]:

$$\mathbf{x}_{ijk} = \sum_{r=1}^R \mathbf{a}_{ir} \mathbf{b}_{jr} \mathbf{c}_{kr} + \varepsilon_{ijk} \quad (7.1)$$

where $i = 1, \dots, I, j = 1, \dots, J, k = 1, \dots, K$ with an associated least squares model [65]:

$$\min_{\mathbf{A}, \mathbf{B}, \mathbf{C}} \sum_{ijk} \left\| \mathbf{x}_{ijk} - \sum_r \mathbf{a}_{ir} \mathbf{b}_{jr} \mathbf{c}_{kr} \right\|^2 \quad (7.2)$$

where $\mathbf{A} = (\mathbf{a}_1 \dots \mathbf{a}_R)$, $\mathbf{B} = (\mathbf{b}_1 \dots \mathbf{b}_R)$, $\mathbf{C} = (\mathbf{c}_1 \dots \mathbf{c}_R)$ denote the I, J, K matrices containing the R different loading factors as column vectors. The three-way PARAFAC does provide a unique decomposition when $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are of full rank and there are proportional changes in the relative contribution from one factor to another in all three domains so that no two factors in any domain are collinear. The PARAFAC model has been extended in [64] to incorporate any delays that may occur due to widely spaced sensors. In the simulations, time delays of greater than 1 second are assumed to be a threshold for using this widely spaced sensor model.

7.1.2 Results for PARAFAC

This technique when extended to GMSK signals with delays of 2.7 seconds between the sensors also provided good separation as shown in Fig. 7.3. An implementation of the algorithm proposed by [64] was tested on a mixture of three sinusoids at different

Figure 7.1: Performance of the PARAFAC model (a),(b),(c) Original Sinusoidal Signals (d),(e),(f) Delayed and Mixed versions (g),(h),(i) Estimated outputs.

Figure 7.2: Pseudo spectrum Estimate via MUSIC (a),(c),(e) Original Sinusoids (b),(d),(f) Estimated Outputs.

Figure 7.3: Performance of the PARAFAC model for GMSK signals (a),(b) Original Signals (c),(d) Delayed and Mixed versions (e),(f) Estimated outputs.

Figure 7.4: Performance of the PARAFAC model for two GMSK signals (a) Zero Doppler shifts with varying delays (b) Zero delays with varying Doppler shifts.

frequencies with delays of 1.7 seconds between sensors (Fig. 7.1). The estimated separated sinusoids were found to be quite accurate when examined with a MUSIC pseudo-spectrum as shown in Fig. 7.2.

This technique was examined for separation of two GMSK sources with widely separated sensors at an input SNR of 20 dB. As shown in 7.4, the introduction of delays and large Doppler shifts considerably improve the output performance. The Doppler shift parameter D is similar to the parameter used in Doppler-aided BSS techniques. As seen in Fig. 7.4, the change in delays (zero Doppler shifts) causes significant SINR improvements. Changes in the Doppler shifts (zero Delay) produces improvements in the range of approximately 0.05 dB. Clearly, the output SINR is influenced by the delay samples rather than the Doppler shift parameters. Thus, by exploiting this diversity, further improvement in separation with widely placed sensors is possible.

7.2 Single Channel Convolutional Source Separation

In certain applications, the sensors can be positioned in only one optimal position. Medical applications, mobile telephony and wireless car telephones are a few examples where only one microphone is generally used. In such cases, BSS algorithms can be used to reduce the effect of unwanted interference and noise. If the delayed and scaled versions of the mixtures are incorporated, the model is convolutional. Observations can be considered as the combinations of the unknown filtered versions of the source signals. Under the assumption of anechoic recording condition, the mixing process can be formulated as:

$$x(k) = \sum_{j=1}^N w_j b_j(k - l_j) + v(k) \quad (7.3)$$

where $b_j(k)$, $x(k)$ and $v(k)$ denote respectively the j^{th} source signal, the observed signal and the noise captured by the sensor at time instance k . The attenuation w_j and the delay l_j of the j^{th} source to the sensor would be determined by the physical position of the source relative to the sensor.

7.2.1 Failure Analysis of BSS Algorithms

Algorithms typically used for BSS like JADE [19], FastICA [17], ACMA [22] and the time-frequency techniques like LI-TIFROM [61], have many applications in multi-channel source separation. These include the separation of communication, speech and audio signals. However, when applied to single channel convolutional mixtures, the above mentioned algorithms fail due to various reasons as shown in Table 7.1. Therefore, more specific BSS algorithms are required to separate the single channel convolutional mixtures. The non-negative matrix factorization (NMF) algorithm satisfying these criteria, thus becomes a suitable choice for this particular application.

Algorithm	Failure Analysis
JADE	Unable to handle under-determined mixtures. Unable to handle convolutional mixtures.
FastICA	Unable to handle single source mixtures. Deteriorated performance for convolutional mixtures.
ACMA	Unable to handle under-determined mixtures. Requires additional constant modulus criterion. Deteriorated performance for audio & speech mixtures.
LI-TIFROM	Unable to handle convolutional mixtures. Requires strict sparsity criterion.

Table 7.1: Failure Analysis of Common BSS algorithms.

7.2.2 Non-negative Matrix Factorization

The non-negative matrix factorization technique introduced by [59], is able to produce useful representations of real world data and can be applied to the problem of single channel source separation. The non-negative constraints usually required for these class of algorithms are relaxed by making use of standard ICA algorithms to zero-mean the observed data. However, particular emphasis should be given to the independence and sparsity of the observed data as discussed in [4].

Based on the observed single channel data $\mathbf{x}$, the NMF decomposes it into two basis matrices $\mathbf{A}$ and $\mathbf{S}$. This provides a reduced representation of the original data where each feature is a linear combination of the original attribute set. The NMF has low computational complexity and unlike time-frequency techniques, it is able to deal with both dense and sparse data sets.

The NMF algorithm may be described in the following steps [59]:

1. Initialize the elements of $\mathbf{A}$ and $\mathbf{S}$ to random non-negative values. Normalize each column of $\mathbf{A}$ to unit 2-norm.
2. Update the matrix $\mathbf{A}$ by either least squares or Kullback-Leibler Divergence(KLD) as shown:

$$\mathbf{A} \leftarrow \mathbf{A} \cdot \frac{\mathbf{x}\mathbf{S}^T}{\mathbf{A}\mathbf{S}\mathbf{S}^T} \quad (7.4)$$

$$\mathbf{A} \leftarrow \mathbf{A} \cdot \frac{\frac{\mathbf{x}}{\mathbf{A}\mathbf{S}}\mathbf{S}^T}{1 \cdot \mathbf{S}} \quad (7.5)$$

where ‘ $\cdot$ ’ is the element-wise multiplication operator and ‘ $-$ ’ is the element-wise division operator. $\mathbf{A}$ values below an assigned threshold ϵ are approximated to be zero. Normalize each column of $\mathbf{A}$ to unit norm.

3. Update matrix $\mathbf{S}$ similarly as in step (2).

$$\mathbf{S} \leftarrow \mathbf{S} \cdot \frac{\mathbf{A}^T \mathbf{x}}{\mathbf{A}^T \mathbf{A} \mathbf{S}} \quad (7.6)$$

$$\mathbf{S} \leftarrow \mathbf{S} \cdot \frac{\mathbf{A}^T \frac{\mathbf{x}}{\mathbf{A} \mathbf{S}}}{\mathbf{A} \cdot 1} \quad (7.7)$$

4. Iterate steps (2) and (3) till convergence is achieved.

The technique proposed by [60] is based on 2D deconvolution and non-negative matrix factorization (NMF). In order to successfully separate convolutional mixtures, the NMF model is extended to the 2-dimensional case incorporating the time τ and pitch (fundamental frequencies) ϕ of the signal.

$$\mathbf{x} = \sum_{\tau} \sum_{\phi} \mathbf{A}_{\tau}^{\downarrow \phi} \mathbf{S}_{\phi}^{\rightarrow \tau} \quad (7.8)$$

where $\downarrow \phi$ represents the downward shift operator which moves each element of matrix ϕ rows down and $\rightarrow \tau$ denotes the right shift operator which moves each element in the matrix τ columns to the right. The least squares and KLD approach for updating $\mathbf{A}$ and $\mathbf{S}$ are then applied to separate the convolutional mixtures.

7.2.3 Results for NMF

The problem of single channel source separation was considered for a single channel mixture of speech signals. This has specific applications to hands-free mobile devices where there are interfering speech signals other than the desired signal. As shown in Fig. 7.5, the separation performance is quite good for the speech signals. Though inferior to multi-channel BSS techniques, it offers the capacity to reduce constraints on the number of

Figure 7.5: Performance of the NMF on speech signals. (a),(b) Original signals (c) Mixed single channel mixture (e),(f) Separated outputs.

sensors required considerably. Another aspect to examine is the accurate positioning of this single source sensor. The scenario presented in Fig. 7.6 was used. For each symmetric location of the sensor in the (x, y) plane, the received power of the single channel mixture was captured. This was then separated using the NMF algorithm and the average SIR improvement was measured. As shown in Fig. 7.6, two cases are analyzed including placing the sensor away from both the sources (Case A) and in between the sources (Case B). The separation performance of NMF which was convolutively mixed based on the specification in [62] was tested based on the signal to interference ratio (SIR) improvement. This is given by:

$$SIR = 10 \log_{10} \frac{\|\mathbf{s}_{target}\|^2}{\|\mathbf{e}_{interf}\|^2} \quad (7.9)$$

Figure 7.6: Scenario used for modeling single channel source separation.

where $\mathbf{s}_{target}$ is the target signal and e_{interf} is an allowed deformation of the sources which accounts for the interferences of the unwanted sources. As seen in Figs. 7.7 and 7.8, the SIR improvements are superior when the sensor position is far away from both the sources. Similarly, the SIR improvements are superior midway between both the sources. When compared to the received power in each case, a power level of less than -5 dB indicates optimal separation performance. Thus, there are factors such as received power levels and efficient sensor positioning for providing a good separation performance in the case of single channel mixtures. In this case, it is assumed that there is good signal to noise ratio during the recording.

Figure 7.7: Received power and SIR improvement for a mixture two speech signals for Case A.

Figure 7.8: Received power and SIR improvement for a mixture two speech signals for Case B.

Chapter 8

Conclusions and Recommendations

8.1 Conclusion

The main focus of our research is to propose Blind Source Separation algorithms for mitigating co-channel interference and noise in communication systems. For digitally modulated signals, the algorithms should have the capability of processing data in the complex domain, have low complexity and able to provide BER and output SINR improvements.

The basic steps and assumptions of most source separation algorithms have been shown. The motivation for their application in blind beam forming has also been investigated. The well known JADE, ACMA, FastICA and INFOMAX algorithms proved to be advantageous for the specific application of this thesis and have been discussed in detail.

The ARD-JADE source separation algorithms in the complex domain proved to be advantageous when applied to digitally modulated signals. The signals that have been distorted due to co-channel interference and white noise are retrieved with significantly less distortion after applying the algorithm. The ARD-JADE algorithm is shown to outperform the conventional JADE when the SNR is greater than 10 dB. The performance

of the method has been analyzed in terms of BER and output SINR improvements.

The FEBICA scheme performs well in complex domain with low computational complexity and good SINR improvements. When applied to GMSK modulated signals, the FEBICA algorithm is shown to out-perform the JADE and ACMA algorithms based on SINR improvement. Furthermore, it is shown to have low complexity for both increasing signal length as well as number of sources. This makes the algorithm suitable for practical applications in wireless receivers.

When the sources are closely placed, the blind beamforming algorithms are not able to separate them due to restrictions of resolution in a particular domain and singularity of the observed mixtures. By exploiting Doppler shifts in such systems, this problem may be overcome. Combined with blind techniques for estimating the number of sources, the Doppler aided source separation models are shown to perform with increased accuracy. The tracking of the sources is also shown feasible using frequency estimation techniques like MUSIC and ESPRIT. Another aspect looked into is the fidelity of the BSS outputs for speech and audio case. By utilizing the diversity provided by overdetermined BSS, the noise level may be significantly reduced even in the case of deteriorating SNR. Using the MDNR algorithm, the efficacy of this process is shown based on output SINR. These will prove useful at the signal separation procedures for VoIP and teleconferencing applications. An extension of overdetermined mixtures to the reduction of reverberation, post-BSS has also been examined.

The use of non-negative matrix factorization for single channel source separation of speech sounds have been separated with promising results. Parallel Factor Analysis is shown to be useful for source separation in case of widely spaced sensors where delays are prominent. A comparative analysis summarizing the properties of the algorithms

Algorithms	Mixing Model	Number of sensors / sources	Input Signals	Performance indicators	Limitations
ARD - JADE	Instantaneous	Critically determined	Complex Valued	Improved output BER / SINR	Works well for few sources / sensors
FEBICA	Instantaneous	Critically determined	Complex valued	Improved output SINR Lower complexity Stable over large number of sources / sensors	Works better for nearly singular mixing processes
Doppler Aided BSS	Instantaneous	Critically determined	Complex valued	Improved output BER / SINR Works well for closely placed sources	Needs change of Doppler to be constant
High Fidelity BSS	Instantaneous	Over determined	Real valued	Improved output SINR / SAR Reduction of slight reverberation	More sensors than sources needed
PARAFAC	Instantaneous	Critically determined	Complex Valued	Improved SINR for widely spaced sensor positions	Needs joint Doppler – delays to work
Single Channel BSS	Convolutional	Under determined	Real valued	Improved SIR for various sensor positions for single channel case	Sensor position plays a prominent role

Figure 8.1: Comparison of source separation algorithms.

introduced in shown in Fig. 8.1. It explains the mixing models, source/sensor numbers, performance indicators and limitations of these algorithms. Based on the setting and performance indicators, the algorithms should be judiciously chosen to obtain the best source separation performance. As a rule of thumb, for better BER performance use ARD-JADE while for better computational complexity (real time implementation) use FEBICA. For closely placed (moving) sensors use Doppler aided BSS while for widely placed sensors, PARAFAC is optimal. High Fidelity BSS can be used to reduce noise in overdetermined conditions while single channel BSS can be used for undetermined cases.

Thus, various aspects of source separation for wireless communications have been studied in this thesis. The efficacy of these algorithms in interference and noise mitigation have been successfully demonstrated.

8.2 Future work

A brief list of the possible future research directions is provided below.

- i) **Underdetermined case** : As of now, the number of sensors has been assumed to be greater than or equal to the number of sources. The underdetermined case will assume that the number of sensors is less than the number of sources. Though not essential for communication systems, this is a core area of research for audio and speech BSS. The time-frequency techniques like LI-TIFROM are a starting point for this research direction.
- ii) **Permutation Problem** : For all source separation applications, be it in the time or frequency domain, there are ambiguities in the ordering of the sources. Though back-correlation and direction of arrival estimates are currently in use, more robust techniques

are needed to counter the permutation in the source order.

iii) **Wideband applications** : The assumption used for source separation in communication systems is that the sources are narrowband in nature. If a wideband approach is used, there will be a series of unmixing matrices required for various frequency values. This is due to the fact that phase changes and delays act differently on varying frequencies. Frequency normalization with a reference frequency should then be employed for the unmixing process.

iv) **Non-linear BSS** : General direction of BSS research makes use of linear models for source separation. However, in some complex channels, non-linear models come into the picture. For these, additional constraints and assumptions are needed. This is still a developing research area with a lot of potential for real-life applications.

iv) **Further implementation** : The use of the source separation techniques may also be applied to Code Division Multiple Access (CDMA) and Global Positioning Systems (GPS). Implemented at the base station receivers, this can help in mitigating co-channel interference and optimizing the system performance.

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Appendix A: Adaptive Source Separation

The Equivalent Adaptive Separation via Independence (EASI) algorithm [63] makes use of a family of serial updating algorithms that can converge quickly to the optimal unmixing matrix. Based on a set of non-linear function $g(\cdot)$ and the unmixing output $\mathbf{y}$, the serial updates of weights $\mathbf{W}$ can be given by [63]:

$$\mathbf{W}_{t+1} = \mathbf{W}_t - \lambda_t [\mathbf{y}_t \mathbf{y}_t^T - I + g(\mathbf{y}_t) \mathbf{y}_t^T - \mathbf{y}_t g(\mathbf{y}_t)^T] \mathbf{W}_t \quad (8.1)$$

where λ_t represents the adaptation steps.

When applied to a set of QAM signals, the separation criterion may be determined by the modulus of the global system C_t given by [63]:

$$C_t = |\mathbf{W}_t \mathbf{A}| \quad (8.2)$$

where $\mathbf{A}$ is the mixing matrix. Ideally, this should have a value of one (for an identity output) or zero (permutation). As seen in Fig. 8.2, the global unmixing coefficient converges to the estimated values within a few iterations. The non linearity used in

Figure 8.2: Convergence performance of the EASI algorithm for QAM signals.

this case is a cubic form $g(y)=|y^2|y$. Such an adaptive algorithm is useful specially in the case of dynamically changing mixing scenarios.

Appendix B: Time-Frequency

Techniques for BSS

As proposed by [61], the following Time-Frequency method is useful, specially when the sparsity of the sources can be ensured:

- In this technique, BSS is seen as the successive source cancellations considering linear combinations of observations

$$\mathbf{y}(t) = \mathbf{x}_m(t) - c\mathbf{x}_n(t) \quad m \neq n \quad (8.3)$$

- Assumption is that for each source $\mathbf{s}_i$, there exists some adjacent TF windows (t, ω) where only $\mathbf{s}_i$ occurs. It first computes Short-Time Fourier Transform (STFT) of the mixed sources $\mathbf{x}_i$ given by:

$$\mathbf{Z}_i(t, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{x}_i(\tau) h(\tau - t) e^{-j\omega\tau} d\tau \quad (8.4)$$

- Detection stage: it finds Time-Frequency (TF) zones where a single source is active (called single-source zones). Compute the ratio as follows for $i \neq 1$ for a

single couple $(\mathbf{Z}_1, \mathbf{Z}_i)$:

$$\alpha(t_j, \omega_k) = \frac{\mathbf{Z}_1(t_j, \omega_k)}{\mathbf{Z}_i(t_j, \omega_k)} \quad (8.5)$$

Select M successive time windows available for the series of windows Γ_q and calculate the variance $\text{var}[\alpha](\Gamma_q, \omega_k)$.

- Identification stage: it estimates a column of the unknown mixing matrix in each of the above zones and only keeps the most distant columns in terms of separability. The values of $\text{var}[\alpha](\Gamma_q, \omega_k)$ are sorted in ascending order and only those values below a user defined threshold are retained. For a two source case, calculate the ratio $c_{i,j}$ as shown [61]:

$$c_1 = \frac{\mathbf{a}_{11}}{\mathbf{a}_{21}} \quad c_2 = \frac{\mathbf{a}_{12}}{\mathbf{a}_{22}} \quad (8.6)$$

The columns of values corresponding to the TF areas chosen give us Q columns of the estimated matrix as [61]:

$$\tilde{A} = \begin{bmatrix} 1 & \cdots & 1 \\ 1/c_{12} & \cdots & 1/c_{Q2} \\ \vdots & & \vdots \\ 1/c_{1N} & \cdots & 1/c_{QN} \end{bmatrix} \quad (8.7)$$

- Reconstruction stage: If $Q = N$ use the inverse of this square matrix $\tilde{A}$ to achieve a global inversion up to a scale factor.

Glossary of Technical Terms

1. **ACMA** - Analytical Constant Modulus Algorithm which makes use of the constant modulus property of the source signals for blind source separation [22].
2. **Akaike Information Criterion** - Is a measure of the goodness of fit of an estimated statistical model [45].
3. **ARD** - Proposed by [31], the Altering Row Diagonalization algorithm alternates between updates of individual demixing matrix rows to solving the eigenvector problem for enhanced source separation.
4. **BER** - Bit Error Rate which is given by the number of erroneous bits received divided by the total number of bits transmitted.
5. **BSS** - Blind source separation is the separation of a set of signals from a set of mixed signals, with minimal prior information and assumptions about the source signals and the mixing process.
6. **Computational complexity** - Measure of the amounts of computational resources required for the execution of algorithms.
7. **Correlation** - Often measured as a correlation coefficient, indicates the strength and direction of a linear relationship between two random variables. In general statistical usage, correlation refers to the departure of two random variables from independence [67].
8. **Covariance** - Is a measure of how much two variables change together. If the variables vary above their expected value simultaneously, they are said to exhibit positive covariance. On the other hand, if one of them tends to be above its expected value when the other variable is below its expected value, they exhibit negative covariance [67].
9. **Critically determined system** - When the number of sources N is equal to the number of sensors M ($N = M$).
10. **Eigenvalue, Eigenvector** - An eigenvalue and eigenvector of a square matrix A are, respectively, a scalar λ and a nonzero vector ν that satisfy the equation $A\nu = \lambda\nu$.

11. **FastICA** - Proposed by [17] it uses maximization of non-Gaussianity for separation.
12. **FEBICA** - Feedback ICA which uses the feedback architecture to reduce mutual information between sources.
13. **Gerschgorin theorem** - The Gerschgorin theorem on eigenvalues of a matrix provides a method for estimating the location of eigenvalues from the values of the matrix elements.
14. **GMSK** - Gaussian minimum shift keying is a continuous-phase frequency-shift keying modulation scheme used in Global System for Mobile communications (GSM) systems [27].
15. **Hermitian matrix** - A square matrix with complex entries which is equal to its own conjugate transpose.
16. **High Fidelity** - High quality reproduction of sound that is very faithful to the original performance.
17. **ICA** - Independent Component Analysis [1].
18. **Independence** - Two scalar-valued random variables m_1 and m_2 , they are said to be independent if information on the value of m_1 does not provide any information regarding the value of m_2 and vice versa [67].
19. **Infomax** - Information Maximization algorithm proposed by [18] that uses entropy maximization in order to separated sources.
20. **Jacobian Matrix** - Matrix of all first-order partial derivatives of a vector-valued function.
21. **JADE** - Joint Approximate Diagonalization of Eigenvectors proposed by [19] uses efficient joint cumulant diagonalization to separated the mixed signals.
22. **Kurtosis** - A classical measure of the non-Gaussianity based on the fourth order cumulants.
23. **LI-TIFROM** - Proposed by [61], it is a time-frequency sparsity based approach for source separation.
24. **MDNR** - Minimum Distortion Noise Reduction [52].
25. **MMSE** - Minimum Mean Square Error estimator [67].
26. **Mutual Information** - Is a natural measure of the dependence between random variables.
27. **Negentropy** - Negentropy is used as a measure of distance to normality. Negentropy is always nonnegative, is invariant by any linear invertible change of coordinates, and vanishes if and only if the signal is Gaussian [1].
28. **NMF** - Non-negative matrix factorization [59].

29. **Orthogonal** - Two vectors are said to be orthogonal if their inner product is zero and they are perpendicular to each other.
30. **Orthonormal** - Vectors of unit length and orthogonal to each other.
31. **Overdetermined system** - When the number of sources N is less than the number of sensors M ($N < M$).
32. **PARAFAC** - Parallel Factor Analysis [65].
33. **PCA** - Principal Component Analysis involves a mathematical procedure that transforms a number of correlated variables into a number of uncorrelated variables called principal components [3].
34. **Probability density function (PDF)** - Is defined as the derivative of the cumulative distribution function. The cumulative distribution function describes the probability that a variable $\mathbf{X}$ takes on a value less than or equal to a number x ($P(\mathbf{X} \leq x)$) [67].
35. **Projection Pursuit** - Projections from high- to low-dimensional space that reveal the most details about the structure of the data set.
36. **SAR** - The signal-to-artifact ratio that measures the distortion caused on the retrieved signal due to source separation.
37. **Singular Matrix** - A square matrix without an inverse.
38. **SIR** - The signal-to-interference ratio is the ratio between the average received modulated carrier power and the average received co-channel interference power (cross-talk) from other transmitters than the desired signal.
39. **SOBI** - Second Order Blind Identification (SOBI) algorithm [42] the uses joint unitary diagonalization for the process of source separation.
40. **Sparse Matrix** - A matrix that allows special techniques to take advantage of the large number of zero elements contained in it.
41. **Temporal correlations** - Correlation of the sensor's observations with respect to the time domain.
42. **Uncorrelated** - Two random variables are said to be uncorrelated if their covariance is zero. However, uncorrelated behavior does not guarantee independent variables.
43. **Underdetermined system** - When the number of sources N is greater than the number of sensors M ($N > M$).
44. **Whitening** - The whitening process removes any correlations in the data and is used as a pre-processing step for noisy ICA [1].

Author's Publications

- A. K. Kattepur, F. Sattar and C. M. S. See, “A Fast Fourier Transform based Blind Source Separation Algorithm for Wireless Communication”, *Elsevier Signal Processing Journal*, 2009(submitted - under review).
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