Forward error correction in optical communication systems

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Forward Error Correction in Optical Communication Systems

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Thesis

Submitted by XIE Bing

For the partial fulfillment of the requirements for the degree of
Doctor of Philosophy

Supervisor

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School of Electrical and Electronic Engineering

Nanyang Technological University

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Declaration

I hereby declare that this thesis is my own work and has not been submitted in any form for another degree or diploma at any university or any other institute of tertiary education. Information derived from the published and unpublished work of others has been acknowledged in the text and a list of reference is given.

Xie Bing

December, 2008

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Summary

Forward error correction (FEC) technique has been widely used in wireless communication and mass storage applications. In recent years, FEC has also become very important to optical communication because of its powerful error correcting ability. FEC codes provide performance gain by trading off coding gain with coding redundancy and decoding complexity. For transmission bit rate of 10 Gb/s or higher, coding redundancy of less than 25% is desired. Therefore, FEC schemes with moderate coding redundancy and tolerable decoding complexity are desirable and very important for high-speed long-haul optical systems. In this thesis, two FEC augmentation schemes, namely wavelength interleaving and errors-and-erasures decoding (E&ED), are proposed to improve the FEC performance of optical channels subjected to polarization mode dispersion (PMD) and chromatic dispersion (CD). Wavelength interleaving provides frequency diversity, hence reducing the effective error burst length and improving FEC performance without requiring more coding redundancy. In E&ED scheme, an erasure zone is used to collect highly unreliable received symbols called erasure symbols. By using Reed Solomon codes, twice as many erasures as hard-decision decoding (HDD) errors can be corrected by E&ED, hence more errors induced by PMD and CD can be corrected. To understand the achievable improvement of the two proposed techniques, the system models are deduced and verified by simulations, the system requirements are studied, the system performance is analyzed, and the corresponding optical receivers are designed.

Table of Contents

Declaration.....	3
Acknowledgements.....	4
Summary	5
Table of Contents.....	7
List of Figures	10
List of Tables	15
List of Abbreviations	16
Mathematic Notation	18
Chapter 1 Introduction	22
1.1 Motivation	22
1.2 Objectives and Approaches.....	24
1.3 Major Contributions	25
1.4 Thesis Organization.....	26
Chapter 2 Literature Review	28
2.1 Advantages of Fiber Optic Systems	28
2.2 Limitations and Compensation in Fiber Optic Systems.....	28
2.2.1 Power Loss, Noise and Nonlinear Effects	28
2.2.2 Chromatic Dispersion	29
2.2.3 Polarization Mode Dispersion	33
2.3 Forward Error Correction.....	37
2.3.1 Convolutional Codes and Block Codes	39
2.3.2 Reed-Solomon Code.....	40
2.3.3 Interleaving in FEC	42
2.3.4 Decoding in FEC	43
2.4 Development of FEC in Optical Communication System	44
Chapter 3 FEC Performance of PMD Mitigation Using Wavelength Interleaving	48
3.1 Introduction	48
3.2 Modeling of PMD-Impaired Fiber Link.....	49
3.2.1 PMD Characteristic	49

3.2.2 Channel BER in the Presence of PMD	50
3.3 Decoded BER in PMD channel without Wavelength Interleaving.....	56
3.4 Decoded BER in PMD channel with Wavelength Interleaving	57
3.4.1 System Design	57
3.4.2 Two-Wavelength Interleaving System	60
3.4.3 Four-Wavelength Interleaving System	62
3.4.4 Verification of Derived Formulas.....	65
3.5 Discussions.....	67
3.5.1 Average BER.....	67
3.5.2 Outage Probability	71
3.5.3 Simplification of Decoded BER Expressions.....	73
3.5.4 Effect of rise/fall time t_r on Optical System Performance	75
3.6 Chapter Conclusion	77
Chapter 4 FEC Performance of Chromatic Dispersion Mitigation Using Wavelength Interleaving	79
4.1 Introduction	79
4.2 Channel BER Model for CD Impaired Systems	80
4.2.1 Channel BER Modeling.....	80
4.2.2 Verification of Derived Formula	83
4.3 Decoded BER in CD Channel with and without Wavelength Interleaving	88
4.3.1 No wavelength interleaving.....	88
4.3.2 With Wavelength Interleaving.....	89
4.4 Wavelength Separation Requirement.....	90
4.5 Improvement in System Dispersion Tolerance	94
4.6 Chapter Conclusion	97
Chapter 5 Errors-and-Erasures Decoding for Optical Systems	99
5.1 Introduction	99
5.2 Error-and-Erasure Decoding Algorithm.....	100
5.2.1 Binary erasure decoding algorithm.....	100
5.2.2 Nonbinary erasure decoding algorithm	100
5.3 E&ED Scheme in Optical FEC Systems.....	103
5.4 System Model.....	105

5.4.1 Channel Bit Error Rate and Bit Erasure Rate	105
5.4.2 Symbol Error Rate and Symbol Erasure Rate	112
5.4.3 Output BER of E&ED	113
5.5 Optimization of Erasure Zone	117
5.6 E&ED Performance and Analysis	119
5.7 E&ED to mitigate PMD Impairment	121
5.7.1 System Model of PMD Impaired Systems with FEC and E&ED	121
5.7.2 Formulas of FEC Performance Analysis with E&ED and PMD	123
5.7.3 Results and Data Analysis	124
5.8 E&ED to mitigate CD Impairment	128
5.8.1 System Model of CD Impaired Systems with FEC and E&ED	128
5.8.2 Results and Data Analysis	130
5.9 Adaptive Erasure Zone	134
5.10 Chapter Conclusion	139
Chapter 6 Conclusion and Future Work	142
6.1 Conclusion	142
6.2 Suggestions for Future Works	144
Author's Publications	147
References	148

List of Figures

- Figure 2-1. Dispersion slopes of typical single-mode fibers.
- Figure 2-2. Reflectivity and group delay spectra of a 12-channel dispersion slope compensation grating.
- Figure 2-3. Polarization mode dispersion.
- Figure 2-4. A typical simplified optical system with FEC.
- Figure 2-5. Schematic diagram of interleaving and de-interleaving.
- Figure 3-1. Maxwellian probability density function of normalized instantaneous DGD $\Delta\tau$. Normalized mean DGD $\langle\Delta\tau\rangle$ is 0.2.
- Figure 3-2. Eye diagram.
- Figure 3-3. Worst case detected signal value as given by equation (3-6).
- Figure 3-4. An example of DGD spectrum (a) and its corresponding BER (b).
- Figure 3-5. Schematic of an optical FEC system model with two interleaved wavelengths.
- Figure 3-6 (a) FEC frame 1 of λ_1 and FEC frame 2 of λ_2 before de-interleaving, (b) FEC frame after de-interleaving. Symbols transmitted on wavelength λ_1 is denoted by 1, on λ_2 is denoted by 2.
- Figure 3-7. An example of a FEC frame in double-wavelength interleaving system after de-interleaving. (a) FEC frames of different symbol error probability $P1 = P_{s1}$ and $P2 = P_{s2}$ at the input of FEC decoder. (b) The equivalent of (a).
- Figure 3-8. Comparison of simulation result to formula result on decoding SER of (a) double-wavelength interleaving system, function (3-17) and (b) four-wavelength interleaving system, function (3-21) with RS(255,239). P1, P2, P3 and P4 are SERs P_{s1} , P_{s2} , P_{s3} and P_{s4} before FEC decoding.
- Figure 3-9. Receiver Q factor required to achieve a BER of 10^{-12} with and without FEC, of double-wavelength interleaving system, compared to that of

single wavelength system, when mean DGD (normalized by T_b) of the chosen wavelengths are in different value. $\langle t_1 \rangle$ and $\langle t_2 \rangle$ in the figure is normalized mean DGD $\langle \Delta \tau_1 \rangle$ and $\langle \Delta \tau_2 \rangle$, normalized rise/fall time $tr=0.5$, and $p=1$.

Figure 3-10. Receiver Q factor required to achieve a BER of 10^{-12} with and without FEC, of four-wavelength interleaving system, compared to that of single wavelength system, when DGD (normalized by T_b) of the chosen wavelengths are in different value. $\langle t_1 \rangle$ and $\langle t_{2,3,4} \rangle$ in the figure is normalized mean DGD $\langle \Delta \tau_1 \rangle$ and $\langle \Delta \tau_{2,3,4} \rangle$, normalized rise/fall time $tr=0.5$, and $p=1$.

Figure 3-11. Required Q-factor to achieve outage probability of (a) 10^{-3} and (b) 10^{-6} for PMD channels with and without FEC, with and without 2-wavelength interleaving. The two interleaved wavelengths have same normalized mean DGD, $\langle \Delta \tau_1 \rangle = \langle \Delta \tau_2 \rangle$.

Figure 3-12. Comparison between original decoding symbol error functions and the respective approximated ones. Original function is denoted by long (L) function, approximated function is denoted by short (S) function in figure.

Figure 3-13. Required receiver Q factor to achieve a BER of 10^{-12} with and without FEC of double-wavelength interleaved system at different normalized rise/fall time tr . Normalized mean DGD $\langle \Delta \tau_1 \rangle$ changes and $\langle \Delta \tau_2 \rangle$ is fixed to be 0.1, $p=1$.

Figure 4-1 Scheme of pulse (a) without spread and (b) with spread.

Figure 4-2. Schematic of VPI simulation model.

Figure 4-3. Comparison of channel BER model (4-8) and VPI simulation results on output BER vs. Q-factor at different transmission length for 10 Gbit/s system, $\lambda_0=1.55 \mu\text{m}$, $D_{CD}=16 \text{ ps/nm}\cdot\text{km}$, $R=10 \text{ Gb/s}$, $T_b = 100 \text{ ps}$, normalized rise time $tr=0.3$.

Figure 4-4. Comparison of channel BER model (4-8) and VPI simulation results on required Q-factor to achieve an output BER of 10^{-12} as a function of

chromatic dispersion. For 10 Gbit/s system, $\lambda_0=1.55 \mu\text{m}$, $D_{CD}=16$ ps/nm·km, $R=10$ Gb/s, $T_b=100$ ps, normalized rise time $tr=0.3$.

Figure 4-5. Receiver Q factor required to achieve a BER of 10^{-12} with and without FEC of single wavelength system at different normalized pulse spread $\Delta\tau$ caused by chromatic dispersion. Normalized $T_s=0.3, 0.5, 0.7$, and $p=1$.

Figure 4-6. (a) Q-factor required to achieve an output BER 10^{-12} vs. normalized pulse difference $d\tau$ for double-wavelength interleaving system, (b) minimum normalized pulse difference $d\tau_{\min}$ required to get the highest gain in Q-factor for double-wavelength and four-wavelength interleaving systems, vs. normalized pulse spread $\Delta\tau_1$ of the worst channel λ_1 , based on RS(255, 223) code. $p=1, R=10$ Gb/s, $tr=0.3$.

Figure 4-7. Receiver Q factor vs. output BER with RS(255, 223) code in systems without wavelength interleaving compared to that in (a) double-wavelength interleaved systems, and (b) four-wavelength interleaved systems. $\Delta\tau$ in this figure is the mean DGD normalized by T_b , $p=1, R=10$ Gb/s, normalized rise/fall time $tr=0.3$.

Figure 4-8. Receiver Q factor required to achieve an output BER of 10^{-12} without FEC compared to that with RS(255, 223) code in single wavelength, double-wavelength and four-wavelength interleaved systems. $\Delta\tau$ in this figure is the mean DGD normalized by T_b , $p=1, R=10$ Gb/s, normalized rise/fall time $tr=0.3$.

Figure 5-1. Block diagrams of optical communication systems with FEC and hard-decision decoding (HDD) or the proposed errors-and-erasures decoding (E&ED).

Figure 5-2. Proposed erasure zone definition in the eye diagram of an optical channel.

Figure 5-3. Probability density functions of detected signal value of HDD systems.

Figure 5-4. Probability density functions for detected signal value of E&ED systems.

- Figure 5-5. Comparison of function (5-28), (5-38) and simulation results for RS(255, 239) code on decoded SER of E&ED at different input symbol erasure rate, and $P_{S_err}=0$.
- Figure 5-6. Error correction capabilities of RS(255,239) code with E&ED, (a) at different η (eta), (b) at different R_{EZ} . $Q=9.5$ dB, $\Delta I_{Th}=0.3$.
- Figure 5-7. Relationship of output BER and receiver Q-factor without FEC, with FEC based on HDD and with FEC based on E&ED. HDD threshold is 0.3. E&ED is set as: $\eta=4\%$ and $R_{EZ}=p$.
- Figure 5-8. Required Q-factor to get an output BER of 10^{-12} with HDD and with E&ED changing with code rate R , when code length is different at (a). $q=16$, (b). $q=32$, (c). $q=64$, (d). $q=128$, (e). $q=256$.
- Figure 5-9. Required receiver Q factor to achieve an average output BER of 10^{-12} . Mean DGD in this figure is mean DGD normalized by T_b , $p=1$, normalized rise/fall time $tr=0.5$, E&ED is set as: $\eta=4\%$ and $R_{EZ}=p$.
- Figure 5-10. Required Q-factor to achieve outage probability of 10^{-3} or 10^{-6} . Mean DGD in this figure is mean DGD normalized by T_b , $p=1$, normalized rise/fall time $tr=0.5$, E&ED is set as: $\eta=4\%$ and $R_{EZ}=p$.
- Figure 5-11. Receiver Q factor required to achieve an output BER of 10^{-12} with RS(255, 223) using HDD vs. E&ED. Pulse spread is normalized pulse spread by T_b , $p=1$, $R=10$ Gb/s, normalized $tr=0.3$, E&ED is set as: $\eta=5\%$ and $R_{EZ}=p$.
- Figure 5-12. Receiver Q factor vs. output BER with RS(255, 223) code in systems with E&ED compared to that in systems with HDD without wavelength interleaving and with two wavelengths interleaved. $p=1$, $R=10$ Gb/s, normalized rise/fall time $tr=0.3$, E&ED is set as: $R_{EZ}=p$, $\eta=5\%$ for $\Delta\tau_1=0.1$ and $\eta=3\%$ for $\Delta\tau_1=0.4$.
- Figure 5-13. Output BER vs. erasure zone width, at different instantaneous DGD level (normalized by T_b) with (a) RS(255, 239), (b) RS(255, 223) coding. $p=1$, normalized rise/fall time $tr=0.5$.
- Figure 5-14. Schematic diagram of the proposed E&ED receiver with adaptive erasure zone.

Figure 5-15. Comparison of required Q factor value to get an output average BER of 10^{-12} using E&ED with fixed vs. adaptive erasure zone with (a) RS(255, 239) coding, (b) RS(255, 223) coding, average DGD level is normalized by T_b , $p = 1$, normalized rise/fall time $tr=0.5$.

List of Tables

- Table 2-1. Characteristics of three generations of FEC in optical transmission system.
- Table 4-1. CD-induced normalized pulse spread $\Delta\tau$ on difference wavelengths at C-band and C+L-band in 10 Gb/s systems. Central wavelength $\lambda_0=1550$ nm, $D_{CD0}=17.6$ ps/nm/km is dispersion factor at λ_0 , dispersion slop $S=0.08$ ps/nm².km. (a)10 Gb/s system, (b)40 Gb/s system.
- Table 5-1. Optimized η of erasure zone for different M -ary Reed-Solomon codes with different code rate R .

List of Abbreviations

2WI	2-wavelength interleaving
4WI	4-wavelength interleaving
ASE	amplified spontaneous emission
ASIC	application specific integrated circuit
BCH	Bose Chaudhuri and Hocquenghem
CD	chromatic dispersion
CFBG	chirped Fiber Bragg Grating
CG	coding gain
DCF	dispersion compensating fiber
DGD	differential group delay
DPSK	differential phase-shift keying
DQPSK	differential quadrature phase-shift keying
E&ED	errors-and-erasures decoding
EOD	error-only decoding
FEC	forward error correction
FPGA	field programmable gate array
GMD	generalized minimum distance
GVD	group velocity dispersion
HDD	hard-decision decoding
ISI	inter-symbol interference
LDPC	low-density parity-check
NCG	net coding gain

OOK	on-off keying
PCF	photonic crystal fibers
PIN	p-i-n
PMDC	polarization mode dispersion compensator
PMD	polarization mode dispersion
PSP	principal states of polarization
RS	Reed-Solomon
SDD	soft-decision decoding
SDH	synchronous digital hierarchy
SER	symbol error rate
SM	single-mode
SNR	signal-to-noise ratio
SOPMD	second order PMD
SP	sampling point
TPC	Turbo product code
VHDL	very high speed integrated circuit hardware description language
VPI	virtual photonics interface
WDM	wavelength division multiplexing
WER	word error rate

Mathematic Notation

η	the normalized width parameter of the erasure zone
λ	wavelengths
λ_0	central wavelength
$\Delta\tau$	T_b normalized instantaneous DGD
$\langle\Delta\tau\rangle$	T_b normalized mean DGD
ΔI_0	erasure zone at side “0”
ΔI_1	erasure zone at side “1”
ΔI_e	width of the erasure zone
σ_1 and σ_0	the standard deviation of amplitudes of 1 level and 0 level
d	the minimum distance of the nonbinary code
$d\tau$	pulse spread difference normalized by T_b between the interleaved channels
$d\tau_{\min}$	minimum $d\tau$ normalized by T_b
D_{CD}	chromatic dispersion parameter
D_{CD0}	dispersion factor at λ_0
D_{PMD}	average PMD parameter
$\text{erfc}(x)$	complimentary error function
I_{Th0}	amplitude level of erasure zone and at side “0”
I_{Th1}	amplitude level of erasure zone and at side “1”
I_{Th}	is the threshold detected signal level
L	transmission distance
m	prime power

$M(.)$	Maxwellian distribution, probability density function
(n, k, j)	The convolutional code has the n encoder outputs depending on k encoder inputs and j previous input block.
p	rate of the standard deviation of amplitude histogram about logic one-level to that about zero-level
P_b	channel BER
$\overline{P_b}$	statistical average of P_b over the probability density function of DGD
P_{B_ee}	output BER of E&ED
$\overline{P_{B_ee}}$	the statistical average of P_{B_ee} over the probability density function of DGD, $M_{<\Delta\tau>}(\Delta\tau)$
P_{B_era}	input bit erasure probability for E&ED
$\overline{P_{B_era}}$	the statistical average of P_{B_era} over the probability density function of DGD, $M_{<\Delta\tau>}(\Delta\tau)$
P_{B_err}	input bit error probability for E&ED
$\overline{P_{B_err}}$	the statistical average of P_{B_err} over the probability density function of DGD, $M_{<\Delta\tau>}(\Delta\tau)$
P_{B_FEC}	output bit error rate after decoding
$\overline{P_{B_FEC}}$	Average output BER of FEC
$P_{B_FEC_2},$ $P_{B_FEC_4}$	output bit error rate after decoding in 2- and 4-wavelength interleaving systems.
P_{B_RS}	decoding information bit error rate
P_{C_ee}	codeword decoding error probability of E&ED
P_{df}	the probability of a decoding failure when $i = (t+1) \sim d$
P_{dsw}	the average decoded-symbol error probability of a given code word
P_s	symbol error probability before decoding

$P_{s1}, P_{s2}, P_{s3}, P_{s4}$	symbol error probability before decoding in wavelength λ_1 and λ_2
P_{S_cor}	correct symbol probability after the second decision before E&ED
P_{S_ee}	decoded symbol error rate of E&ED
P_{S_era}	symbol erasure probability after the second decision before E&ED
P_{S_err}	symbol error probability after the second decision before E&ED
P_{S_FEC}	output symbol error probability after decoding
$P_{S_FEC_2},$ $P_{S_FEC_4}$	output symbol error probability after decoding in 2- and 4-wavelength interleaving systems.
P_{s_in}	the channel-symbol error probability
P_{S_RS}	decoding information symbol error rate probability
P_{ud}	the probability of an undetected error when $i = (d+1) \sim n$.
P_{W_RS}	decoding information codeword error rate probability
P_{w_in}	the probability that one or more errors occur in a decoded word
$q = 2^m$	dimension of the finite vector space
Q	receiver Q-factor
Q_t	back to back Q-factor
R	code rate
R_{EZ}	width ratio of the erasure zone at “0”side to that at “1”side
$RS(n, k)$	$RS(n, k)$ code has code length of n symbols, and k information symbols.
S	dispersion slop
t	the number of symbol errors a $RS(n, k)$ code can guarantee to correct
T_b	bit period
t_d	time delayed between the two polarization states

tr	rise/fall times normalized by T_b
T_s	the rise/fall time of the received pulse normalized by T_b
T_t	the rise/fall time of the transmitted pulse normalized by T_b

Chapter 1

Introduction

1.1 Motivation

In recent years, the growth in demand for broad-band services has led to much increased activity in research for high-capacity and high-speed (>10 Gb/s) optical systems and networks. Comparing to the optical systems in speed lower than 10Gb/s, high-speed long-haul system has much higher requirement on system performance and lower tolerance on all kinds of system impairment. In optical communication systems, the chromatic dispersion (CD), polarization mode dispersion (PMD), fiber nonlinearities, and amplified spontaneous emission (ASE) noise from the amplifiers are the main sources of impairment to limit the system performance. In fiber optics, researchers designed special filters or decrease launched optical power to mitigate non-linear effects, dispersion compensating fibers (DCFs) and Chirped Fiber Bragg Gratings (CFBGs) for CD compensation, and proposed optical PMD compensators (PMDCs) to deduce PMD impairment. In a single fiber link, each kind of channel impairment described above needs a special method to handle, and most methods are costly. Therefore, a method effective on mitigating multiple impairments will be much desirable.

PMD is one of the most critical channel impairment factors especially in high-speed long-haul optical transmission systems. PMD can generate burst errors lasting for several hours, since PMD variations in a field fiber are mainly driven by environmental changes. Thus the effect of fiber PMD will lead to severe bit error rate (BER) degradation and system outage event in high-speed optical transmission systems. In recent years, several different electrical and optical methods, to mitigate the inter-symbol interference (ISI) induced by these fiber link impairments, have been demonstrated in single wavelength optical systems. Optical methods use compensation to reduce the impairment, while electrical methods use error correction

to improve system performance. Optical PMD compensation [1] has been widely studied, and is found to be very effective. But it has limit in compensating precision, since compensators are only able to add or reduce the same amount of PMD on each wavelength but the severity is much different for different wavelengths. This means that, compensator can only compensate perfectly on one single wavelength. Furthermore, compensators have to be placed in every several hundreds of miles, so that the cost will very high especially in long-haul or ultra-long-haul fiber links. Electric adaptive compensation [2] has also been actively studied, since the integrated electrical equalizers are much cheaper than optical compensators. However, electrical equalizers offer less performance improvement than PMD compensators. Therefore, a method can both provide higher system performance and be less expensive will be promising.

Chromatic dispersion is the most common channel impairment in optical transmission systems. However, in broadband wavelength division multiplexing (WDM) systems, the chromatic dispersion varies on each channel due to the nonzero dispersion slope, which makes it impossible to compensate the entire system using a common chromatic dispersion compensating fiber. Dispersion slope causes the residual dispersion to change over the signal bandwidth, especially on wideband systems, the residual dispersion changes severely over the signal bandwidth and leads to great penalties from the accumulated dispersion slope. Therefore, dispersion slope has to be managed especially for 40 Gb/s and higher bit rate transmission over nonslope-matched fibers [3][4]. A method that can reduce the difference of accumulated dispersion on each wavelength and average the entire system will be welcomed.

After a lot of study and experiments, forward error correction (FEC) technique, which is currently widely used in wireless and mass storage applications, has been proved to be able to help on many of the expectations indicated above. FEC codes can correct errors resulted from different kinds of channel impairments, so they can improve the overall system performance of fiber optic systems; and FEC is able to increase the transmission distance without repeaters, which helps to reduce commercial cost in long-haul systems.

In ITU-T G.975, Reed-Solomon (RS) code (255,239) was recommended for fiber transmission systems, which can provide a coding gain (CG) of 5.8 dB with 7% redundancy. Concatenated codes [5] have been demonstrated to be able to provide higher CG than single RS code, but will have higher redundancy too. For example, a Reed-Solomon code concatenated with a convolutional code has been demonstrated to have about 10 dB of CG, but the required coding redundancy is 113%. T. Mizuochi proposed a block turbo code [6] with 3-bit soft-decision and 25% redundancy, so as to provide a 10 dB of CG at the BER of 10^{-13} . However, Turbo decoding operation requires channel measurement and soft iterative decoding, which are difficult to put into practice for systems operating at the speed of 10 Gb/s or higher. However, high-speed transmission systems do not have unlimited system tolerance of coding redundancy. For optical transmitter of 10 Gb/s or higher, redundancy of less than 25% is highly desirable [6]. Therefore a method to improve FEC performance with moderate coding redundancy and tolerable decoding complexity is desirable and important for high-speed long-haul optical systems.

1.2 Objectives and Approaches

This thesis aims at improving the FEC performance and enhancing the system tolerance of optical transmission systems subjected to burst errors due to PMD and chromatic dispersion. The specific approaches and design objectives are elaborated below:

Objective 1:

- To design a wavelength interleaving system with FEC to provide frequency diversity to mitigate the effect of dispersions (PMD and CD).

Approaches to achieve Objective 1:

- To derive the channel BER model and FEC performance model for FEC systems with wavelength interleaving in optical channels impaired by PMD and CD.

- To analyze the system performance advantage of wavelength interleaving systems over the systems without wavelength interleaving.

Objective 2:

- To design errors-and-erasures decoding (E&ED) schemes to improve system tolerance to burst errors induced by PMD and CD impairments in fiber links.

Approaches to achieve Objective 2:

- To derive mathematic models of E&ED which are applicable for optical transmission systems with on-off keying (OOK), FEC with E&ED, PMD and CD.
- To analyze and benchmark the system performance of E&ED systems with errors-only decoding (EOD) systems.
- To optimize the erasure zone of the E&ED system, and to design a practical optical receiver with adaptive erasure zone

All of the investigations presented in this thesis are based on the assumptions that the errors caused by PMD or CD in different optical wavelengths are independent. This implies that the interleaved wavelengths should be separated by more than the coherence bandwidth.

1.3 Major Contributions

In this thesis, a novel wavelength interleaving scheme is designed to provide frequency diversity to mitigate the long burst error effect of PMD and CD in optical communication systems with nonzero dispersion slop. Based on the FEC decoding error probability of optical transmission system, the analytical FEC decoding error probability of optical transmission system with wavelength interleaving is developed. A channel BER expression with transmission impairment of chromatic dispersion for OOK system, which takes account of the influence of transmitter rise/fall times,

receiver Q-factor and noises, is also derived. These analytical expressions help us to investigate how the final decoded output BER is impacted by each factors in the system, and how much our new method can improve on FEC performance. The improvement of system performance is found to increase with the number of interleaved wavelengths. The minimum wavelength separation to be required for such wavelength-interleaving FEC systems suffering from CD to get the highest coding gain is determined.

Next, E&ED for optical systems to enhance its FEC performance over hard-decision decoding (HDD) has been designed. The corresponding channel BER, bit and symbol erasure probability expressions after E&ED are developed for optical FEC systems with PMD and CD. Furthermore, the erasure zone is optimized in terms of two major parameters: η , normalized width parameter of the erasure zone, and R_{EZ} , width ratio of the erasure zone at each side of “0” and “1”. With these models and a fixed optimized erasure zone, the system tolerance improvement is analyzed. In addition, adaptive erasure zone and its PMD mitigation capability over an optimized fixed erasure zone are also investigated. Then practical optical receiver architecture to implement an adaptive erasure zone is proposed.

1.4 Thesis Organization

The rest of this thesis is organized as follows. Chapter 2 gives a brief introduction to FEC in optical transmission systems. The existing works are reviewed to identify the research issues.

In Chapter 3 and Chapter 4, a novel technique, wavelength interleaving, is proposed and designed to improve FEC performance without adding redundancy. Various types of interference and noises are introduced and their effects have been considered based on Gaussian approximation. Chapter 3 and Chapter 4 are focused on investigating PMD-impaired systems and CD-impaired systems correspondingly. Full system model for an optical transmission system with FEC and wavelength interleaving is

derived and evaluated. Furthermore, the system performance of wavelength interleaving systems are analyzed and compared with that of the common optical systems without wavelength interleaving.

In Chapter 5, an optical transmission system with E&ED is designed and evaluated, whose system performance is analyzed and compared with that of optical systems with errors-only decoding (EOD). Meanwhile, system architecture for optic fiber link with OOK, FEC and E&ED is proposed. The erasure zone is optimized in two key parameters. With these models and optimized erasure zone, improvement of system tolerance on PMD and CD with E&ED is analyzed. Finally, a scheme of optical receiver with an adaptive erasure zone is proposed and designed.

Finally in Chapter 6, this thesis is concluded and some ideas are recommend for future investigation.

Chapter 2

Literature Review

2.1 Advantages of Fiber Optic Systems

Fiber optic technology is currently the best long distance communication method. One of the most important advantages of fibers is that they are able to carry more information and provide much faster data transfer speeds than conventional copper wire. And its material glass costs less than copper. Costs are often the most important consideration in a system. Furthermore, fibers offer a high degree of security and privacy. Because fibers do not radiate the energy, it's difficult for an intruder to detect the signal being transmitted. Besides, the signal carried on optic fiber cannot be disrupted by outside sources, like electricity, rain, humidity, or other things that tend to damage signals in conventional copper wire or in the air. The attenuation of optical fibers is very low compared to other transmission media such as air or copper. This allows optical transmission to cover large distances in a cost effective way.

2.2 Limitations and Compensation in Fiber Optic Systems

However fiber optic systems are not perfect, they have technical and economic limitations. In optical communication systems, attenuation, dispersion, including chromatic dispersion and polarization mode dispersion, fiber nonlinearities, and noises are considered to be the main sources of impairment to limit the system performance. [7]-[10]

2.2.1 Power Loss, Noise and Nonlinear Effects

The attenuation of optical fibers is lower than other transmission media, but in long distance optical transmission systems, power loss is still necessary to be remedied to

satisfy system power budget. A possible remedy is to add an optical amplifier after encoding, but the cost of the system will be increased.

Noise degrades the signal and is always present. There're several different types of noise existing in optical transmission systems. The major causes of signal degradation occurring during reception are thermal noise like in the other communication systems, shot noise because of the particle properties of light, and amplified spontaneous emission (ASE) noise because of the spontaneous photon emissions over large bandwidths in optical amplifiers. Besides, there're also background noise and laser noise present in fiber link.

Signal amplification is necessary to remedy attenuation in fiber links, but is always accompanied by an equal amount of noise amplification and the amplifier contributes additional noise of its own. For this reason, amplification cannot improve the ratio of signal power to noise power without limit. As the received signal power diminishes toward the noise power, the signal becomes less and less discernible. In this way, attenuation ultimately limits the length of a fiber transmission system. Therefore after some transmission distance, the transmitted signal needs to be detected to get the useful data. Then a repeater re-sends out a clean version of the data in an appropriate signal-to-noise ratio (SNR).

Nonlinear effects [8][9] in fibers can also be a problem, mainly for long-distance communications. Among the nonlinear effects, what have been studied extensively includes self-phase modulation, cross-phase modulation, four-wave mixing, stimulated Raman scattering, and stimulated Brillouin scattering. The nonlinearities are dependent on the intensity of the signals and are not significant at low power.

2.2.2 Chromatic Dispersion

Dispersion refers to the effect of pulse distortion caused by different components of the transmitted signal traveling at different velocities in the fiber and arriving at different times at the receiver. The amount of dispersion generally rises with the fiber

length. Several forms of dispersion arise in optical communication systems. In this section, chromatic dispersion (CD) will be discussed and previous investigation on its compensation will be reviewed. Polarization mode dispersion (PMD) has will be discussed in 2.2.3.

CD exists because different frequency components of a pulse and also signals at different wavelengths travel with different group velocities in the fiber, and thus arrive at different times at the receiving end. It causes the light pulses to spread and becomes a critical issue as the transmission bit rates increase beyond 10 Gb/s. CD in single mode fibers limits the data transmission rate in broadband wavelength division multiplexing (WDM) systems.

Figure 2-1. Dispersion slopes of typical single-mode fibers.[14]

Dispersion compensating fiber (DCF) [15] is widely used to mitigate CD. However, a problem with WDM systems is, since the chromatic dispersion varies for each channel due to the nonzero slope of the chromatic dispersion profile, it may not be possible to compensate for the entire system using a common chromatic dispersion compensating fiber. In a single-mode optical fiber, dispersion slope is the rate of change of

dispersion with respect to wavelength. It will change at different dispersion wavelengths. Some typical fibers' dispersion curves and their dispersion slopes are shown in Figure 2-1 [14]. The positive dispersion of installed fibers can be compensated by a DCF with a large dispersion of opposite sign. Figure 2-2 shows the reflectivity and group delay spectra of a 12-channel dispersion slope compensation grating in the upper two graphs, and the dots in the bottom graph gives the dispersion values obtained from a curve fit of the group delay while the solid line represents the target [16].

Figure 2-2. Reflectivity and group delay spectra of a 12-channel dispersion slope compensation grating. [16].

A typical spread of the total chromatic dispersion can be compensated by another stage of chromatic dispersion slope compensation where an appropriate length of the

fiber whose chromatic dispersion slope is opposite to that of the residual chromatic dispersion is used. However, it is difficult to fabricate positive chromatic dispersion fiber with negative slope today, so that this technique can only be used for system employing positive dispersion, positive slope fiber for transmission (and negative dispersion, negative slope fiber for dispersion, and dispersion slope, compensation). Thus, in submarine systems that use negative dispersion, positive slope fiber, dispersion slope compensation using dispersion compensating fiber is not possible.

DCFs are presently the most effective and preferred solution for CD compensation in WDM systems. A disadvantage of DCF is that it adds loss to the system. The added loss has to be compensated by adding erbium doped fiber amplifier to the system, which is very expensive. Although the dispersion is properly compensated for one channel, the residual dispersion that exists at other channels can become significant and has to be compensated for by another DCF after a certain propagation length, which adds more loss [15][17][18].

In recent years, electronic dispersion compensation [75]-[81] has attracted much interest, because electronic signal processing techniques offer great potential to reduce the cost. However the conventional electronic dispersion compensation using direct detection is limited in effectiveness due to the loss of the phase information, and transforming between linear and nonlinear distortion with square-law detection significantly increases the operational complexity. Viterbi equalization is proposed in [77][78] and is verified by Monte-Carlo simulation to be able to outperform the conventional electronic equalizations. It can improve the chromatic dispersion tolerance to 5000 ps/nm, and polarization mode dispersion tolerance to about 160ps at 3 dB optical signal to noise ratio. Electronic dispersion compensation can adaptively compensate the dispersion in fiber links and networks, but it needs feedback. This will seriously add system complexity. Also, rapid dispersion variations caused by thermal drift, vibration, optical network switching, and polarization rotation cannot be compensated.

Recently, a multi-carrier format called coherent optical orthogonal frequency division multiplexing (CO-OFDM) [73][74] is proposed to combat fiber chromatic dispersion, and have been shown to be able to traverse 3000 km of standard single-mode fiber without dispersion compensation. Reference [73] shows that incoherent optical orthogonal frequency division multiplexing (IO-OFDM) provides similar chromatic dispersion tolerance with a simpler detection scheme. Compared to electrical equalization, OFDM could be an attractive technology for adaptive compensation of systems with rapid variations, either environmental or deliberate, such as in optically-switched networks, as it does not require a feedback path with its intrinsic time delay. However, standard OFDM requires a high bias [82] to convert bipolar electrical to unipolar optical signals, which degrades receiver sensitivity by more than 5 dB.

Therefore, an alternative technique with low cost, ease of integration and ability to compensate optical transmission impairments will be useful.

2.2.3 Polarization Mode Dispersion

Scientists and engineers for years have expressed the fear that as optical networking systems get faster and send signals longer distance, major physics-related problems would become a limiting force. The technology for years had been given a free ride as it grew from 140 Mbps to 622 Mbps to 1.244 Gb/s. A problem began to surface in 10 Gb/s systems and threaten 40 Gb/s networking. For the first time, the fiber optic industry was faced with a networking killer. The problem, which itself was not even discovered until the early 1990s, is polarization mode dispersion [11][12]. It can distort signals, render bits inaccurate, and destroy the integrity of a network.

In single-mode fiber, there are not one but two modes of lightwave propagation (traveling on physically the same path). This is due to the fact that light can exist in two orthogonal polarizations. Hence, two signals can be sent without interference on single-mode fibers if their polarizations are orthogonal. In normal single-mode fiber a signal consists of both polarizations. However, polarization states are not maintained

in standard single-mode fibers. During its journey light couples from one polarization to the other randomly.

Birefringence is the name given to the characteristic found in some materials where the light path exhibits a different refractive index to the different polarizations. It can be a source of dispersion but this is usually less than 0.5 ps/nm/km (trivial for most applications). The effect is to cause a circular or elliptical polarization to form as the signal travels along the fiber. Dispersion resulting from the birefringent properties of fiber is called “polarization mode dispersion” [7].

The origin of PMD lies in the fact that different polarizations travel with different group velocities due to the ellipticity of the fiber core. Moreover, the distribution of signal energy over the different states of polarizations changes slowly with time, due to changes in the ambient temperature, vibrations and movements along the fiber. This causes the PMD penalty to vary with time as well. In addition to the fiber itself, PMD can also arise from individual components used in the network.

PMD is qualified in terms of the Differential Group Delay (DGD), which is shown in Figure 2-3. A fiber has a distribution of DGD values, not a single fixed value, over time. The probability of the DGD of a fiber section being a certain value at any particular time follows a Maxwellian distribution. As an approximation, the maximum instantaneous DGD is about 3.2 times the average DGD of a fiber. The details will be shown in section 3.2.1.

As Figure 2-3, the PMD effect which has been discussed so far is a consequence of the fact that the two orthogonal polarization modes in optical fiber travel at slightly different speeds, which leads to a differential time delay (i.e. DGD) between these two modes. DGD causes signal broaden and worsens the eye diagram. However, this differential time delay itself is frequency dependent and varies over the bandwidth of the transmitted pulse. This effect is called second order PMD. Second order PMD is an effect that is similar to chromatic dispersion and thus can lead to pulse spreading,

and cause problems during the decoding of optical pulses at the receiver. In this thesis, I will focus more on first order PMD, i.e. DGD.

Figure 2-3. Polarization mode dispersion.

Polarization mode dispersion can be a major critical channel impairment factor in high-bit-rate long-haul optical transmission systems (≥ 10 Gb/s/channel). At high bit rates, first and high order PMD lead to unacceptable pulse dispersion, inter-symbol interference (ISI), bit-error rate (BER) deterioration, performance variance and system outage.

To mitigate PMD impairments [1][13], various PMD compensation techniques have been proposed and demonstrated, including optical PMD compensation, electronic equalization and passive mitigation techniques [66]-[70]. Optical PMD compensation [1][67] is found to be very effective. But it has limits in its compensating precision, since compensator can only perfectly compensate one single wavelength.

Due to the time varying nature of PMD, it has to be compensated or mitigated by adaptive compensators, therefore all PMD compensation techniques are based on feedback systems. Two common sources of feedback information is the degree of polarization and the electrical power spectrum. The first method utilizes the depolarizing effect that PMD has on signals by measuring and maximizing the degree of polarization in order to suppress PMD-induced broadening of the optical pulse. The second method is potentially more straightforward to implement in a practical system and it is based on the feedback on the electrical power spectrum. However they are not effective when the distortion changes quickly, which requires electric adaptive compensation [67]-[72]. The integrated electrical equalizers are much cheaper than optical compensators. However, electrical equalizers offer less performance improvement than the optical compensators. An optical PMD compensator reverses the PMD induced distortions and an electronic equalizer mitigates the inter-symbol interference. As they work on different mechanisms, combining optical PMD compensator with electronic equalizer could increase the PMD tolerance of a system. Although there are several reports on optical PMD compensators and electronic equalizers alone, there are very few reports on their combined performance [67][71].

In reference [100], Sarkimukka proposes to use wavelength switching in a WDM system, where some clean channels are reserved for use by the channels with severe PMD deterioration, to improve the BER induced by PMD. When the signal on a channel starts to be distorted, it can be redirected to the reserved channels. This method helps to improve PMD tolerance but the number of available channels will be decreased by the reserved channels. Hence, alternative methods to resolve the worst-case PMD without consuming much channel resources will be useful.

CO-OFDM has been mentioned in section 2.2.4 to be capable of CD mitigation [74]. In [72], W. Shieh shows that a CO-OFDM signal at 10 Gbit/s experiences no penalty through first order PMD fiber with a differential group delay (DGD) of 700 ps, and all-order PMD fiber with a mean PMD of 150 ps. This finding is significant because it shows that the CO-OFDM system could provide a solution to the mitigation of PMD

in the existing installed fiber links. Furthermore, the PMD mitigation of CO-OFDM is embedded in the data demodulation module of the receiver without requiring any additional hardware such as polarization controllers and birefringence elements. In [73], it is also shown that IO-OFDM provides similar chromatic dispersion tolerance with a simpler detection scheme, but it does not provide a PMD mitigation capability.

Over the past few years, interest in applying forward error correction (FEC) coding to optical transmission was kindled [22][23]. Highly integrated FEC chips are less costly than optical compensating methods, and, unlike equalization, FEC needs no feedback. Analysis and experiments employing FEC have been reported by several groups, which show that FEC is very powerful to improve the system BER performance deteriorated by all kinds of transmission impairments, including PMD. When PMD is severe, FEC with time-interleaving is able to disperse the burst errors and correct the dispersed errors. In this thesis, a novel wavelength interleaving method is proposed and designed for optical systems with FEC.

2.3 Forward Error Correction

Forward error correction coding is common in today's satellite and digital radio transmission systems, but is traditionally not used in lightwave systems. In most lightwave systems the transmission rates are considered too high for implementation of FEC before 1988 [19]. Now, FEC is one of recent critical technologies that together with optical amplification and dispersion management, drives the push for increased transmission capacity in WDM long-haul fiber-optic links [20]-[26]. Optical transmission systems typically require performance margin against line impairments such as ASE, channel cross talk, nonlinear pulse distortion, fiber aging-induced losses, and PMD. The goal of forward error correction coding is, by adding intelligent redundancy, to increase amplifier spacing or reduced optical power. FEC codes are now standard practice in undersea systems (e.g., the ITU G.975 Reed-Solomon code with 7% overhead), and are starting to be deployed in terrestrial systems [27].

Forward error correction technique is defined as a system of error control for data transmission wherein the receiving device has the capability to detect or correct any character or code block that contains less than a predetermined number of symbols in error, no matter those errors are caused by which impairment. It is accomplished by adding error control bits to each transmitted character or code block, using a predetermined algorithm. Previous analysis and experiments have approved that this character makes FEC able to improve the system performance and system tolerance against all channel impairments in fiber links [28]-[36]. Hence FEC can help to save much cost in fiber links, and becomes one of the most important techniques especially in long haul optical transmission systems.

Figure 2-4. A typical simplified optical system with FEC.

Figure 2-4 shows a simplified optical system with FEC. Before being sent out, data source will be encoded with a particular FEC encoder, and then is transmitted in the fiber link with noise and other system impairments. Finally the received signal will be decoded with the FEC decoder corresponding to the encoder.

FEC codes insert redundancy into the transmitted data stream so that the receiver can detect and possibly correct errors that occur during transmission. In general, to estimate the percentage of FEC overhead, either a redundancy or a code rate R [5] is commonly used. Redundancy is defined as the ratio of the FEC overhead in the encoded data block to the data block size, while code rate R is defined as the ratio of the data block size to the total encoded block size. So, $redundancy = R^{-1} - 1$. A moderate redundancy between 14% and 24% is a typical number for today's FEC used in the 10 Gb/s systems [37].

If the code is well selected, the BER at the output of the decoder is better than that at the output of the demodulator in the original uncoded system. The amount of improvement in BER is usually discussed in terms of the additional transmitter power that is required to obtain the same performance without coding. This difference in power is called the Coding Gain [5]. For short, Coding Gain is usually defined as the difference between SNR values in dB with and without FEC, at the BER of 10^{-12} or other target values. FEC performance benefit is quantified by the comparison of Net Coding Gain (NCG):

$$\text{Net Coding Gain (dB)} = \text{Coding Gain (dB)} + 10\log(R) \quad (2-1)$$

2.3.1 Convolutional Codes and Block Codes

There are two main classes of FEC codes: convolutional codes and block codes.

Convolutional codes (n, k, j) have the n encoder outputs depending on k encoder inputs and j previous input blocks. Convolutional codes are based on encoding the signal with a finite impulse response filter, this encoding does not require the bit stream to be divided into blocks. It can be decoded by the Viterbi algorithm, which is maximum likelihood decoding method that finds the best path through a trellis of states and state transitions. An important property of the Viterbi algorithm is that it is simple to extend it to soft input decoding. In order to limit the required memory length, a survival depth is defined. The survival depth is the maximum number of bits that are used in the computation of the metric before a definite decision is taken about the first bit in that sequence. Thus it puts an upper limit to the delay that is incurred in the decoding. The complexity of the Viterbi algorithm increases with the number of states in the encoder. Therefore, it is not feasible to use codes with a low amount of redundancy, puncturing is used instead. That is bits are deleted at the sender according to a certain pattern and the receiver inputs a metric value that corresponds to on information being available. However, because convolutional codes introduce a memory order j , they are less attractive in high-speed optical transmission systems.

With block codes, data will be divided into blocks of fixed size, and each block is encoded individually. Any block code can be expressed in systematic form, so that the original data remains in an unmodified sequence and the parity symbols are added in the end. In block code, it is usually not necessary to use puncturing in order to achieve a suitable code rate since the complexity of the decoding process is mainly determined by the length of the code. Hence, a suitable code can be chosen also for a channel where the overhead has to be kept low.

Block codes include linear block FEC codes, such as Hamming codes [57][58], Bose Chaudhuri and Hocquenghem (BCH) codes [59][60] and nonlinear block codes, such as Reed-Solomon (RS) codes [38]-[43], which transform a chunk of bits into another chunk of bits in such a way that errors up to some threshold in each block can be detected and corrected. RS codes are unusual in that they correct bursts of errors with very little additional data. Also they permit a designer to trade-off computational power to get more data per block, or an ability to correct larger groups of errors. Hence Reed-Solomon code is a popular error-correction code in optical transmission system.

2.3.2 Reed-Solomon Code

A nonbinary block code consists of a set of fixed-length code words in which the elements of the code words are selected from an alphabet of q symbols, denoted by $\{0, 1, 2, \dots, q-1\}$. Usually, $q = 2^m$, so that m information bits are mapped into one of the q symbols. The length of the nonbinary code word is denoted by n and the number of information symbols encoded into a block of n symbols is denoted by k . The minimum distance of the nonbinary code is denoted by d . A systematic (n, k) block code consists of k information symbols and $(n-k)$ parity check symbols.

Among the various types of nonbinary linear block codes, the RS codes are some of the most important for practical applications. These codes, $RS(n, k)$ codes, are defined by the parameters:

$$\begin{aligned}
 n &= q - 1 = 2^m - 1 \\
 k &= 1, 2, 3, \dots, n-1 \\
 d &= n - k + 1 \\
 t &= \frac{1}{2}(d - 1) = \frac{1}{2}(n - k) \\
 R &= k / n \\
 \text{redundancy}(\%) &= 100 * \frac{(n - k)}{k}
 \end{aligned} \tag{2-2}$$

where t is the number of symbol errors that the FEC code can guarantee to correct, R is code rate.

Let P_w denote the probability that one or more errors occur in a decoded word, including the contributions due both to undetected errors and to a failure to decode resulting from detected but uncorrectable errors. Let P_s denote the channel-symbol error probability. The performance of the hard-decision decoder and erasure decoding may be characterized by the following upper bound on the code word error probability:

$$P_w \leq \sum_{i=t+1}^n \binom{n}{i} (P_s)^i (1 - P_s)^{n-i} \tag{2-3}$$

If the channel symbol errors are independent, the relationship of symbol error probability before and after hard-decision Reed Solomon decoding is approximated by [43]:

$$P_{S_FEC} = \frac{d}{n} \sum_{i=t+1}^d \binom{n}{i} (P_s)^i (1 - P_s)^{n-i} + \frac{1}{n} \sum_{i=d+1}^n i \binom{n}{i} (P_s)^i (1 - P_s)^{n-i} \tag{2-4}$$

The error probability expression function (2-4) is composed by two parts: the second part is the probability of a decoding failure, and the first part is the probability of an undetected error.

Following [43], the output bit error rate P_{B_FEC} and the output symbol error rate P_{S_FEC} after FEC decoding is related by:

$$P_{B_FEC} = \frac{2^{m-1}}{2^m - 1} P_{S_FEC} \tag{2-5}$$

2.3.3 Interleaving in FEC

Historically, interleaving was used in ordering block storage on disk-based storage devices such as the floppy disk and the hard disk. Interleaving is also used in digital data transmission systems to protect data against burst errors. The burst errors corrupt a lot of bits consecutively, so a typical FEC scheme designed for correcting random errors can be overwhelmed because most FEC schemes can only correct a finite number of error bits. To reduce the effect of burst errors, the bits of a number of code words are interleaved before being transmitted. At the receiver, the interleaved bits are put back into their original order by a reverse process called de-interleaving. After proper interleaving and de-interleaving, a burst error from the channel will be broken up into much shorter error patterns in the decoder, hence the decoder can decode the codewords correctly.

Figure 2-5. Schematic diagram of interleaving and de-interleaving.

Figure 2.5 shows the schematic diagram of a simple interleaving method. In the original data stream, bits are re-sequenced before transmission so that the originally adjacent bits are separated by some guaranteed distance in the transmitted stream, and returned to their original order at the receiver. The interleaved bits can be transmitted in time sequence or frequency sequence, which determines whether the interleaving scheme is time interleaving or frequency interleaving. The FEC schemes standardized

in the ITU-T Recommendation G.709 offers some time interleaving, but the interleaving depth may not be sufficient to fully counteract long-fading PMD, or the residual chromatic dispersion in some wavelengths. This is why wavelength interleaving is proposed in this thesis to provide frequency interleaving to further improve the FEC performance on PMD and CD channels.

2.3.4 Decoding in FEC

Decoding is the reverse process of encoding, undoing the encoding so that the original information can be retrieved. Decoders can be divided into two classes. The ones using hard-information, or 2-level quantized information, are said to perform hard-decision decoding (HDD). Their counterpart taking advantage of soft-information, the likelihood of each symbol, are said to perform soft-decision decoding (SDD). These latter decoders are typically more powerful, however their increased error-correction ability will induce increased computational complexity.

In a traditional HDD scheme, the received channel bits are decided according to a single threshold, called hard-detection threshold. Depending on whether the received signal level is above or below the threshold, the demodulator will output a “1” or “0”. Then RS hard-decoder will execute HDD based on the “hard” input. HDD is simple and fast executive. A number of commercial hardware implementations exist, including "off-the-shelf" integrated circuits that encode and decode Reed-Solomon codes. These ICs tend to support a certain amount of programmability, such as RS(255, 239), which has a t below 16 symbols.

On the other hand, SDD receiver takes advantage of side information generated by the receiver bit decision circuitry. Rather than simply assigning a zero or a one to each received noisy binary signal, a more flexible approach is taken through the use of multi-bit (>2-bit) quantization. Commonly, soft-decision decoding can provide an increase in coding gain of 2 to 3 dB over hard-decision decoding. However, it's a trade-off between decoding complexity and decoding performance.

Errors-and-erasures decoding (E&ED) [94]-[99] can be considered as the simplest SDD scheme with only two thresholds and 3 detection levels: “1”, “0” and “Erasure”. But it doesn’t need soft-in and/or soft-out and/or iterative decoding used in SDD, thus has a moderate decoding complexity which is tolerable for optical systems. In E&ED, the receiver uses an additional level “Erasure” to indicate a received signal whose corresponding symbol value is not clearly 1 or 0. The threshold line in HDD expands into an erasure zone in E&ED. If the received signal falls outside the erasure zone, it will be decided to be a “0” or “1” respectively. Otherwise, it will be treated as an erasure bit. The locations of erasures are known, the values of errors and erasures will be determined by the E&ED decoding operation, and used to correct the errors and erasures. For RS codes, FEC with E&ED can correct twice as many erasures as the number of errors. This technique has been investigated in wireless transmission systems for years, and was firstly proposed in optical transmission systems in 2005 [99].

2.4 Development of FEC in Optical Communication System

The initial interest for FEC was for submarine cables where it is important to extend the distances between optical amplifiers and regenerators to reduce the cost. Reed-Solomon codes were therefore applied to add coding gain to the power budget [37]. More recently, optical channel coding has not only been used in submarine networks, but also in other long-haul transmission systems. And there is commercially available hardware for encoding and decoding at bit rates of 10 Gb/s. The capacity of optical transmission systems will be rapidly increased in the near future. For the bit-rate of 10 Gb/s or higher, the following constraints have to be considered: low FEC redundancy (<25%) [6] and low complexity decoder. Under the requirement, FEC coding in optical system has developed through three generations [23][83]-[85].

1. First Generation: Reed-Solomon Code

Reed-Solomon code (255, 239) was recommended as the first generation FEC in ITU-T G.975 for fiber transmission systems, which can provide a coding gain of 5.8 dB with 7% redundancy at output BER of 10^{-12} [44].

2. Second Generation: Concatenated Code

In order to realize capacity beyond 1Tbps, there no doubt that much further improvement in FEC gain will become necessary. Concatenated RS code with iterative decoding provides net coding gains of about 7.5 to 8.2 dB. Specifically, RS(255, 239)+RS(255, 223) [21][22] gives a 2 dB additional coding gain compared to RS(255, 239). Similar investigations have been made on RS(239, 223)+RS(255, 239) [23], RS(248, 232)+RS(144, 128) [24], RS(255, 239)+CSOC (Convolutional Self Orthogonal Code) [25], and RS(247, 239)+RS(255, 247) [26]. Time interleaving, deinterleaving and iterative decoding techniques are used together with these concatenated codes to obtain improved error correction performance. For concatenated coding, a 10% increase in redundancy causes approximately 1 dB coding gain [40]. In [19], a concatenated FEC scheme based on soft decision was proposed which feature a 10 dB net additional gain margin for 10^{-12} BER. However, this solution generates high redundancy (>100%), thus prohibiting implementation at very high bit rate.

3. Third Generation: Turbo Product Code (TPC)

It has been shown that Turbo codes [45][85]-[87] and low-density parity-check (LDPC) codes [88]-[93] are powerful FEC codes which can achieve a performance close to Shannon's theoretical limits. Turbo codes are based on a concatenation of two or more codes associated with an iterative decoding based on soft inputs and soft outputs decoding. Two families of turbo code exist, convolutional turbo code which uses a parallel concatenation of convolutional codes, and turbo product code (TPC) which uses concatenation of block codes. For transmission requiring a low redundancy, it is more convenient to choose TPC. TPCs has the following benefits: excellent performance at high code rates, no puncturing required, low complexity relative to coding gain, lower cost and lower power consumption. TPCs offer significant improvement over concatenated Reed-Solomon/convolutional codes, and

are available as standard products and licensable cores. Akita proposed a third generation FEC employing a TPC based on Bose, Chaudhuri and Hocquenghem (BCH) product codes using soft input iterative decoding [6][20]. The TPC is composed of two constituent BCH codes, BCH(128, 113) and BCH(256, 239), so the TPC has minimum distance 36 and code rate 0.82 (21.3% redundancy). The experiments show that the performance of TPC with 3 bit soft decision is superior by about 0.6 dB to that of the TPC with 2 bit soft decision, and is superior by about 0.4 dB to that of the TPC with hard decision for 10^{-6} BER. From the view point of hardware implementation, they give conclusion that 3 bit soft decision is the optimum solution in terms of quantization.

Table 2-1. Characteristics of three generations of FEC in optical transmission system.

	FEC Scheme	Redundancy	Coding Gain (dB)	Other requirements
	Uncoded	0	0	
1 st Generation	RS(255,239) [44]	6.7%	5.8	
2 nd Generation Concatenated Code	RS(255,239)+ RS(255,239) [37]	13.8%	5.8 + 1.6 = 7.4 5.8 + 2.3 = 8.1 5.8 + 2.5 = 8.3	1 iteration 2 iteration 4 iteration
	RS(255,223) + RS(255,239) [23]	22.0%	5.8 + 2 = 7.8	Interleaving depth of 32 bytes
	BCH(128,113,6) + BCH(128,113,6) [40]	28%	5.8 + 4 = 9.8	With 4 bits for quantization, 1 sign bit and 3 data bits
3 rd Generation TPC and LDPC	BCH(128,113,6) x BCH(256,239,16) [22]	21.3%	5.8 + 4.3 = 10.1	3 bits soft decision, 4 iteration
	LDPC(9252,7967) + RS(992,956)	20.5%	5.8 + 4 = 9.8	2 bits soft-decision, without increasing circuit complexity, no error-floor

Besides turbo codes, low-density parity-check (LDPC) code is another kind of iteratively decodable codes which have generated significant research attention. In references [88]-[92], Djordjevic has shown that iteratively decodable LDPC codes outperform turbo product codes in BER performance. The decoder complexity of these codes is comparable to that of turbo product codes, and significantly lower than

that of serial/parallel concatenated turbo codes. Due to these reasons, LDPC code is an attractive choice for the FEC scheme of a high-speed long-haul optical transmission system. The soft iteratively decodable codes, turbo and LDPC codes, are commonly referred to as the third generation of FEC. The above reviews can be summarized in Table 2-1.

From Table 2-1, it is clear that RS(255, 239) code is an important code even for next-generation high speed optical transmission system. In this thesis, RS(255, 239) code is chosen to be the basic channel code for investigating our proposed methods to offer coding gain improvement on optical FEC systems. In our future work, the other more powerful and more complex codes for example turbo code and LDPC, will be studied.

Chapter 3

FEC Performance of PMD Mitigation Using Wavelength Interleaving

3.1 Introduction

Polarization mode dispersion (PMD) has been a major critical channel impairment factor in high-speed (≥ 10 Gb/s/channel) long-haul optical transmission systems. It causes BER deterioration and random performance variations. Due to the varying nature of PMD, it must be compensated or mitigated with adaptive compensators, therefore all PMD compensation techniques require feedback of channel state information. FEC does not require channel information feedback. The RS(255, 239) code is a popular FEC code used in optical systems. However, although the RS(255, 239) code has some burst error correction capability, and the FEC schemes standardized in the ITU-T recommendation G.709 offers some time interleaving, the interleaving depth may still not be sufficient to counteract long-fading PMD encountered in practice which may typically last for more than an hour [46]-[48]. Thus a method to improve the FEC performance under long-fading PMD is desirable. In this chapter, a novel wavelength interleaving scheme is proposed to provide frequency diversity to mitigate the effect of PMD in optical FEC systems. It will be shown to provide considerable performance gains in terms of PMD tolerance and net coding gain. The rest of this chapter is organized as follows: Section 3.2 presents the optical communication system model with PMD and FEC without wavelength interleaving; Section 3.3 presents the optical communication system model with PMD and FEC with wavelength interleaving; Section 3.4 provides some results and discussions, system performance with and without wavelength interleaving is compared and analyzed in this section; and Section 3.5 gives the conclusion.

3.2 Modeling of PMD-Impaired Fiber Link

3.2.1 PMD Characteristic

In single mode optical fibers, PMD can be characterized by the Differential Group Delay (DGD) between signals transmitting along the two principal states of polarization (PSP). If the DGD is severe, the receiver at some distance L cannot accurately decode the optical pulse, so bit errors will happen. The mean DGD is a property of the fiber. It is constant over time or wavelength, and grows as the square-root of the fiber length. Mean DGD normalized by T_b , $\langle \Delta \tau \rangle$ is defined as:

$$\langle \Delta \tau \rangle = \frac{D_{PMD} \sqrt{L}}{T_b} \quad (3-1)$$

where D_{PMD} is the average PMD parameter in $\text{ps}/\text{km}^{1/2}$. For commercial single mode fibers, the average PMD parameter D_{PMD} is usually less than $0.5 \text{ ps}/\text{km}^{1/2}$ [47].

Figure 3-1. Maxwellian probability density function of normalized instantaneous DGD $\Delta \tau$. Normalized mean DGD $\langle \Delta \tau \rangle$ is 0.2.

In long fibers, the DGD is known to have a Maxwell probability density function [50]:

$$M_{\langle \Delta \tau \rangle}(\Delta \tau) = \frac{32 \cdot \Delta \tau^2}{\pi^2 \langle \Delta \tau \rangle^3} \exp\left(-\frac{4 \cdot \Delta \tau^2}{\pi \langle \Delta \tau \rangle^2}\right) \quad (3-2)$$

An example of Maxwellian probability density function of normalized instantaneous DGD is shown in Figure 3-1.

Coherence bandwidth is a statistical measure of the range of frequencies over which the channel response can be considered "flat", in other words, the approximate maximum bandwidth or frequency interval over which two frequencies of a signal are likely to experience comparable or correlated PMD. PMD coherence bandwidth $\Delta \nu_{\text{PMD}}$ is given as [65]:

$$\Delta \nu_{\text{PMD}} = \frac{1}{8 \langle \Delta \tau \rangle T_b} \quad (3-3)$$

where $\langle \Delta \tau \rangle$ is the normalized mean DGD value. With the typical requirement of mean DGD less than $0.1 T_b$ ($\langle \Delta \tau \rangle < 0.1$), $\Delta \nu_{\text{PMD}} = 50$ GHz for a 40Gbps system. For wavelength channels separated by $\Delta \nu_{\text{PMD}}$ apart, their PMD statistics can be considered independent. This property will be exploited in this thesis to design a wavelength-interleaving scheme to improve the FEC performance of PMD-impaired optical systems.

3.2.2 Channel BER in the Presence of PMD

Assuming that trapezoidal optical pulses with bit period T_b , rise/fall times (normalized by T_b) tr , and infinite extinction ratio are launched into the SM fiber, half of the total power (worst case analysis [49]) will travel along the fiber in each of the principal states of polarization (PSP). A fixed normalized DGD $\Delta \tau$ is assumed to be present between these two equivalent orthogonal channels. The waveform recorded at the optimum sampling point (maximum eye diagram opening) and the eye diagram when $\Delta \tau < 1$ (only the effect of adjacent bits is considered), it can be seen that the main effect of a fixed DGD between the two equivalent channels is to split the I_l level

(received signal one level when normalized DGD is equal to 0) into three sublevels I_1^{sup} , I_1^{med} and I_1^{inf} , and I_0 level (received signal zero level when normalized DGD=0) into I_0^{sup} , I_0^{med} and I_0^{inf} [50]. The sublevels are shown in Figure 3-2 (a). Figure 3-2 (b) shows the definition of σ_1 and σ_0 is the standard deviation of amplitudes of 1 level and 0 level, I_1 , I_0 is the mean value of logic one-level and logic zero-level respectively. Thus, the Q -factor is defined as:

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} \quad (3-4)$$

$$= \frac{I_1}{\sigma_1 + \sigma_0} \quad \text{if } I_0 = 0$$

Assuming the mean value of zero-level is zero ($I_0 = 0$), equation (3-4) front part can be written as latter part.

(a) Sublevels of eye diagram

(b) Simplified eye diagram

Figure 3-2. Eye diagram.

If only low penalties are being studied (as in this work), the approximation $I_l^{sup}=1 \forall \Delta\tau$ will then introduce very little error. A closed form expression can be given for the worst case approximation discussed [50]:

$$I_0^a = I_l(1 - g^a(\Delta\tau, tr)); \quad I_l^a = I_l g^a(\Delta\tau, tr) \quad (3-5)$$

where $a = sup, med, or inf$, and the normalized signal level $g^a(\Delta\tau, tr)$ is obtained from [50]:

$$\begin{aligned} g^{sup}(\Delta\tau, tr) &= 1 \\ g^{med}(\Delta\tau, tr) &= \begin{cases} 1 & , \quad 1 - tr \geq \Delta\tau \\ 1 - \frac{\Delta\tau + tr - 1}{4 \cdot tr} & , \quad 1 + tr \geq \Delta\tau \geq 1 - tr \\ 0.5 & , \quad \Delta\tau \geq 1 + tr \end{cases} \\ g^{inf}(\Delta\tau, tr) &= \begin{cases} 1 & , \quad 1 - tr \geq \Delta\tau \\ 1 - \frac{\Delta\tau + tr - 1}{2 \cdot tr} & , \quad 1 \geq \Delta\tau \geq 1 - tr \\ 0.5 & , \quad \Delta\tau \geq 1 \end{cases} \end{aligned} \quad (3-6)$$

Figure 3-3 presents a sketch of equation (3-6).

Figure 3-3. Worst-case detected signal value as given by equation (3-6).

The channel BER before decoding can be calculated as follows:

$$\begin{aligned} P_b &= P(I_1^{sup})P(0 | I_1^{sup}) + P(I_1^{med})P(0 | I_1^{med}) + P(I_1^{inf})P(0 | I_1^{inf}) \\ &\quad + P(I_0^{sup})P(1 | I_0^{sup}) + P(I_0^{med})P(1 | I_0^{med}) + P(I_0^{inf})P(1 | I_0^{inf}) \end{aligned} \quad (3-7)$$

where $P(I_I^{\text{sup}})=1/8$ is the probability of having a detected signal value I_I^{sup} at the point of the decision circuit and $P(0|I_I^{\text{sup}})$ is the probability of deciding that a “0” has been sent when the detected signal value was I_I^{sup} . The other symbols in (3-7) have similar meaning.

When optical amplifiers are considered and the optical noise bandwidth is much bigger than the bit period, the received noise value follows a Gaussian distribution. With this assumption, the conditional probability in (3-7) can be written as:

$$P(0|I_1^a) = \frac{1}{2} \operatorname{erfc}\left(\frac{I_1^a - I_{Th}}{\sigma_1 \sqrt{2}}\right); P(1|I_0^a) = \frac{1}{2} \operatorname{erfc}\left(\frac{I_{Th} - I_0^a}{\sigma_0 \sqrt{2}}\right) \quad (3-8)$$

where $I_{Th} = \sigma_0 I_1 / (\sigma_0 + \sigma_1)$, is the threshold value at the input of the decision device when $I_0 = 0$ and $\sigma_{0,1}^2$ are the corresponding variance for the received bits “0” and “1” (simple formulas relating these noise variances with the amplifier noise figure and gain, the optical bandwidth and other system parameters can be found in [50]). In function (3-8), $\operatorname{erfc}(x)$ is the complimentary error function:

$$\operatorname{erfc}(x) = 1 - 2 / \sqrt{\pi} \cdot \int_0^x \exp(-\eta^2) d\eta \quad (3-9)$$

Carrying (3-8) into (3-7), using the detected signal values given by function (3-8), and taking into account that $P(I_I^{\text{sup}})=1/8$, $P(I_I^{\text{med}})=1/8$, and $P(I_I^{\text{inf}})=1/8$, the BER in (3-7) can be written as:

$$P_b(Q, p, \Delta\tau, tr) = \frac{1}{8} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) + \frac{1}{8} \operatorname{erfc}\left(\frac{Q \cdot (g^{\text{med}}(1+p) - p)}{\sqrt{2}}\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q \cdot (g^{\text{inf}}(1+p) - p)}{\sqrt{2}}\right) + \frac{1}{8} \operatorname{erfc}\left(\frac{Q \cdot (g^{\text{med}}(1+p) - 1)}{p\sqrt{2}}\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q \cdot (g^{\text{inf}}(1+p) - 1)}{p\sqrt{2}}\right) \quad (3-10)$$

where $Q = I_1 / (\sigma_0 + \sigma_1)$ is the complete receiver Q factor and $p = \sigma_0 / \sigma_1$.

Carrying (3-6) into (3-10), the BER expression P_b can be separated into four sections based on the normalized instantaneous DGD $\Delta\tau$ as below:

1. $0 < \Delta\tau < 1 - tr$

$$P_b(Q, p, \Delta\tau, tr) = \frac{1}{4} \operatorname{erfc}\left(\frac{pQ}{\sqrt{2}}\right) + \frac{1}{4} \operatorname{erfc}\left(\frac{Q}{p\sqrt{2}}\right) \quad (3-11a)$$

2. $1-tr < \Delta\tau < 1$

$$P_b(Q, p, \Delta\tau, tr) = \frac{1}{16} \operatorname{erfc}\left(\frac{pQ}{\sqrt{2}}\right) + \frac{1}{8} \operatorname{erfc}\left(\frac{Q \cdot \left(1 - \frac{\Delta\tau + tr - 1}{4 \cdot tr}\right) (1+p) - 1}{\sqrt{2}}\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q \cdot \left(1 - \frac{\Delta\tau + tr - 1}{2 \cdot tr}\right) (1+p) - 1}{\sqrt{2}}\right) \quad (3-11b)$$

$$+ \frac{1}{16} \operatorname{erfc}\left(\frac{Q}{p\sqrt{2}}\right) + \frac{1}{8} \operatorname{erfc}\left(\frac{Q \cdot \left(1 - \frac{\Delta\tau + tr - 1}{4 \cdot tr}\right) (1+p) - p}{p\sqrt{2}}\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q \cdot \left(1 - \frac{\Delta\tau + tr - 1}{2 \cdot tr}\right) (1+p) - p}{p\sqrt{2}}\right)$$

3. $1 < \Delta\tau < 1+tr$

$$P_b(Q, p, \Delta\tau, tr) = \frac{1}{16} \operatorname{erfc}\left(\frac{pQ}{\sqrt{2}}\right) + \frac{1}{8} \operatorname{erfc}\left(\frac{Q \cdot \left(1 - \frac{\Delta\tau + tr - 1}{4 \cdot tr}\right) (1+p) - 1}{\sqrt{2}}\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q \cdot (p-1)}{2\sqrt{2}}\right) \quad (3-11c)$$

$$+ \frac{1}{16} \operatorname{erfc}\left(\frac{Q}{p\sqrt{2}}\right) + \frac{1}{8} \operatorname{erfc}\left(\frac{Q \cdot \left(1 - \frac{\Delta\tau + tr - 1}{4 \cdot tr}\right) (1+p) - p}{p\sqrt{2}}\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q \cdot (1-p)}{p2\sqrt{2}}\right)$$

4. $1+tr < \Delta\tau$

$$P_b(Q, p, \Delta\tau, tr) = \frac{1}{16} \operatorname{erfc}\left(\frac{pQ}{\sqrt{2}}\right) + \frac{3}{16} \operatorname{erfc}\left(\frac{Q \cdot (p-1)}{2\sqrt{2}}\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q}{p\sqrt{2}}\right) + \frac{3}{16} \operatorname{erfc}\left(\frac{Q \cdot (1-p)}{2\sqrt{2} \cdot p}\right) \quad (3-11d)$$

Figure 3-4 (a) gives an example of a DGD spectrum, and Figure 3-4 (b) gives the BER P_b calculated based on (3-11), $Q=17$ dB, $tr=0.5$, $p=1$, and the instantaneous DGD as shown in Figure 3-4 (a). It shows that the channel BER is mainly affected by instantaneous DGD. When DGD is lower than about 50 ps, PMD is not severe, the channel BER will be lower than 10^{-12} . When DGD is higher than 60 ps, the system performance is fast weakened by PMD, the channel BER will be higher than 10^{-6} . When DGD is higher than 70 ps, system performance is weakened severely by PMD, the channel BER will be higher than 10^{-2} .

Figure 3-4. An example of DGD spectrum (a) and its corresponding BER (b).

3.3 Decoded BER in PMD channel without Wavelength Interleaving

Without FEC, the channel BER $P_b(Q, \Delta\tau)$ of an optical channel with PMD is a function of the Q-factor and DGD expressed in function (3-10). Since the DGD is a random variable, the overall BER of the system without FEC as a function of the normalized DGD and Q-factor should be the statistical average of $P_b(Q, \Delta\tau)$ over the probability density function of normalized DGD (3-2):

$$\overline{P_b}(<\Delta\tau>, Q) = \int_0^\infty M_{<\Delta\tau>}(\Delta\tau) P_b(Q, \Delta\tau) d\Delta\tau \quad (3-12)$$

where $\Delta\tau$ is instantaneous DGD normalized by T_b , and $<\Delta\tau>$ is mean DGD normalized by T_b given in (3-1), $M_{<\Delta\tau>}(\Delta\tau)$ is the probability density function of normalized mean DGD given in (3-2).

The relationship of symbol error probability before and after hard-decision Reed Solomon decoding is approximated by [51]:

$$P_{S_FEC} \approx \frac{d}{n} \sum_{i=d+1}^d \binom{n}{i} (P_s)^i (1-P_s)^{n-i} + \frac{1}{n} \sum_{i=d+1}^n i \binom{n}{i} (P_s)^i (1-P_s)^{n-i} \quad (3-13)$$

where P_s is symbol error probability before decoding while optical transmitter operates on bits in BER $P_b(Q, \Delta\tau)$ by function (3-10).

Because RS code works on symbol error correction, the channel BER should be converted into symbol error rate before FEC, and the symbol error rate after FEC decoding should be converted back to BER values to facilitate system performance evaluation. In a 2^m -ary RS code, a symbol consists of m bits. Only when every bit in a symbol is correct will the symbol be correct, otherwise a symbol error will happen. So the symbol-error rate P_s before FEC decoding is:

$$P_s = 1 - (1 - P_b)^m \quad (3-14)$$

where P_b is the channel BER defined earlier and m is related to the code length n by $n=2^m-1$ for a RS(n, k) code. Following [51], the output bit error rate P_{B_FEC} and the output symbol error rate P_{S_FEC} after FEC decoding is related by:

$$P_{B_FEC} = \frac{2^{m-1}}{2^m - 1} P_{S_FEC} \quad (3-15)$$

If the PMD fading time is very long and no time interleaving is used, the error probability of every bit in a received FEC frame will be the same. Thus, the overall decoded BER of the system can be obtained by averaging P_{B_FEC} of (3-15) throughout all possible states of DGD, as shown below:

$$\overline{P_{B_FEC}} = \int_0^\infty M_{<\Delta\tau>}(\Delta\tau) P_{B_FEC}(P_s(Q, \Delta\tau)) d\Delta\tau \quad (3-16)$$

3.4 Decoded BER in PMD channel with Wavelength Interleaving

3.4.1 System Design

Traditional FEC optical system provides some time interleaving, which is not sufficient when PMD is severe. In this section, a novel FEC scheme with wavelength interleaving is proposed to provide frequency diversity to mitigate the effect of PMD in optical FEC systems. By wavelength interleaving, different symbols from a FEC frame will be distributed and transmitted across different wavelengths; at the receiver, they will be put back into the original order and re-assembled into proper FEC frames before decoding.

In typical optical transmission system, different wavelengths may be in very different PMD levels. The probability that the interleaved wavelengths are simultaneous deteriorated severely by high PMD is much smaller than that only one wavelength is deteriorated by PMD. Thus if wavelength interleaving is applied to the FEC frames,

the effectiveness of the FEC scheme to correct the PMD-induced channel errors is expected to be much enhanced.

Figure 3-5 shows the schematic of wavelength interleaving optical FEC system model. The system model is simplified with the use of only double-wavelength interleaving scheme to illustrate the concept. Synchronous Digital Hierarchy (SDH) in Figure 3-5 provides the capability to send data at multi-gigabit rates over today's single-mode fiber optic links.

Figure 3-5. Schematic of an optical FEC system model with two interleaved wavelengths.

At the transmitter:

The data of the output of SDH is electrical encoded in the same FEC encoding format, shown as the blocks of FEC1 and FEC2. Each FEC-encoded data stream is “demultiplexed” (i.e., split along the symbol boundary) into multiple data sub-streams. Then, alternating symbols from different sub-streams are multiplexed (i.e., merged) and transmitted in an assigned wavelength channel to implement wavelength interleaving. The FEC frames with and without wavelength interleaving are described in details in Figure 3-6. The interleaved FEC frames of multiplexers are converted

from electrical signal into optical signal in different wavelengths λ_1 and λ_2 by Electrical/Optical (E/O) converter. λ_1 and λ_2 are separated by more than coherence bandwidth [63]-[65] of the optical wavelength. Then the optical signals are optically multiplexed and transmitted in single-mode fiber.

At the receiver:

The data from the fiber are optically demultiplexed back to separated channels in wavelength λ_1 and λ_2 , which may experience different DGD states and error probabilities respectively. The optical signals of two wavelengths are converted into electrical signal by the O/E converters separately. The frames before DEMUX are shown in Figure 3-6 (a). Symbols transmitted in λ_1 and λ_2 are denoted by 1 and 2 respectively. These FEC frames are demultiplexed symbol by symbol and these de-interleaved symbols are re-assembled back to their original order before FEC decoding, as shown in Figure 3-6 (b). The de-interleaved FEC 1 frames and FEC 2 frames are then decoded separately.

Figure 3-6 (a) FEC frame 1 of λ_1 and FEC frame 2 of λ_2 before de-interleaving, (b) FEC frame after de-interleaving. Symbols transmitted on wavelength λ_1 is denoted by 1, on λ_2 is denoted by 2.

Figure 3-6 (a) shows the FEC frames transmitted on two separate wavelengths, λ_1 and λ_2 . Due to different PMD levels in λ_1 and λ_2 , FEC frames 1 and 2 bear different channel BER, which is described in Figure 3-7. Either of these FEC frames may be subjected to long error burst. However, if λ_1 and λ_2 are separated by more than the

coherence bandwidth of the optical channel, the probability that they are simultaneously deteriorated severely by PMD will be very small. In our proposed wavelength interleaving system, alternative symbols in each FEC frame are transmitted on different wavelengths with uncorrelated PMD.

3.4.2 Two-Wavelength Interleaving System

Figure 3-7 (a) illustrates the schematic of FEC frames after de-interleaving and before decoding, when signals are transmitted in a double-wavelength interleaving system. n is the number of total symbols, i is the number of errors, $(n-i)$ is the number of correct symbols. In fiber, symbols propagating in the different two wavelengths have different symbol error probability, P_{s1} and P_{s2} . These two wavelengths are separated by more than coherence bandwidth. It can be assumed that, the symbols are independent to each other both in the same FEC frame and among different FEC frames in transmission. With this assumption, Figure 3-7 (a) can be considered to be equal to Figure 3-7 (b).

Figure 3-7 (b) indicates that after de-interleaving, there are n symbols with different symbol error probability in one FEC frame. Because n is odd for RS code, it is assumed there are $(n+1)/2$ symbols in symbol error probability P_{s1} and $(n-1)/2$ symbols in symbol error probability P_{s2} . Assuming that there are i symbol errors in one codeword, j of them use wavelength λ_1 with error probability P_{s1} and the other $(i-j)$ of them use wavelength λ_2 with error probability P_{s2} . Thus in the same codeword, the left $(n-i)$ symbols are correctly received, among which there are $(\frac{n+1}{2} - j)$ correct symbols of wavelength λ_1 occurring in probability $(1-P_{s1})$ and $\frac{n-1}{2} - (i-j)$ correct symbols of wavelength λ_2 in probability $(1-P_{s2})$. If λ_1 and λ_2 are separated by more than the coherence bandwidth, P_{s1} and P_{s2} can be considered to be independent to each other.

Figure 3-7. An example of a FEC frame in double-wavelength interleaving system after de-interleaving. (a) FEC frames of different symbol error probability $P1 = P_{s1}$ and $P2 = P_{s2}$ at the input of FEC decoder. (b) The equivalent of (a).

Using function (3-13), the symbol error probability of double-wavelength interleaving system after FEC decoding can be shown to be:

$$P_{S_FEC_2} \approx \frac{d}{n} \sum_{i=t+1}^d \sum_{j=0}^i \left\{ \binom{\frac{n+1}{2}}{j} (P_{s1})^j (1-P_{s1})^{\frac{n+1}{2}-j} \cdot \binom{\frac{n-1}{2}}{i-j} (P_{s2})^{i-j} (1-P_{s2})^{\frac{n-1}{2}-(i-j)} \right\} + \frac{1}{n} \sum_{i=d+1}^n \sum_{j=\max(0, i-\frac{n-1}{2})}^{\min(i, \frac{n+1}{2})} i \left\{ \binom{\frac{n+1}{2}}{j} (P_{s1})^j (1-P_{s1})^{\frac{n+1}{2}-j} \cdot \binom{\frac{n-1}{2}}{i-j} (P_{s2})^{i-j} (1-P_{s2})^{\frac{n-1}{2}-(i-j)} \right\} \quad (3-17)$$

According to function (3-14), the channel symbol error rates (before decoding) P_{s1} and P_{s2} should be calculated by:

$$\begin{aligned} P_{s1} &= 1 - (1 - P_{b1}(Q, \Delta\tau_1))^m \\ P_{s2} &= 1 - (1 - P_{b2}(Q, \Delta\tau_2))^m \end{aligned} \quad (3-18)$$

where P_b is the instantaneous channel BER defined in (3-10), and m is related to the code length n by $n=2^m-1$ for RS(n, k). $\Delta\tau_1$ and $\Delta\tau_2$ are the DGD of wavelength λ_1 and wavelength λ_2 respectively.

Following function (3-15), the output bit error rate $P_{B_FEC_2}$ of double-wavelength interleaving scheme and the output symbol error rate $P_{S_FEC_2}$ after FEC decoding is related by:

$$P_{B_FEC_2} = \frac{2^{m-1}}{2^m - 1} P_{S_FEC_2} \quad (3-19)$$

After decoding, the average decoded BER of the double-wavelength interleaving system with PMD and FEC should be a double integral of the conditional decoded BER over the probability density functions of $\Delta\tau_1$ and $\Delta\tau_2$, because alternative symbols in each FEC frame are transmitted on wavelengths λ_1 and λ_2 with uncorrelated normalized DGDs $\Delta\tau_1$ and $\Delta\tau_2$. The average decoded BER of the double-wavelength interleaving system is shown to be:

$$\overline{P_{B_FEC_2}} = \int_0^\infty \int_0^\infty M_{<\Delta\tau_1>}(\Delta\tau_1) \cdot M_{<\Delta\tau_2>}(\Delta\tau_2) \cdot P_{B_FEC_2}(P_{b1}(Q, \Delta\tau_1), P_{b2}(Q, \Delta\tau_2)) d\Delta\tau_1 d\Delta\tau_2 \quad (3-20)$$

3.4.3 Four-Wavelength Interleaving System

The four-wavelength interleaving system model is similar to the double-wavelength interleaving system model. Symbols propagating in the four wavelengths have different symbol error probabilities, denoted by P_{s1} , P_{s2} , P_{s3} and P_{s4} separately. After deinterleaving, there are n symbols with different symbol error probability in one FEC frame. Because n is odd, it is assumed for simplicity that there are $(n+1)/4$ symbols each in symbol error probability P_{s1} , P_{s2} , P_{s3} and $(n-3)/4$ symbols in symbol error

probability P_{s4} . If there are i symbol errors in one codeword, among which a symbol errors are assumed to be in probability P_{s1} , b symbol errors in probability P_{s2} , c symbol errors in probability P_{s3} and $(i-a-b-c)$ symbol errors in probability P_{s4} . Thus in a codeword, there're $(n-i)$ correct symbols, among which there are $(n+1)/4-a$ correct symbols of wavelength λ_1 occurring in probability $(1-P_{s1})$, $(n+1)/4-b$ correct symbols of wavelength λ_2 in probability $(1-P_{s2})$, $(n+1)/4-c$ correct symbols of wavelength λ_3 occurring in probability $(1-P_{s3})$ and $(n-3)/4-(i-a-b-c)$ correct symbols of wavelength λ_2 in probability $(1-P_{s4})$. Wavelengths, λ_1 , λ_2 , λ_3 and λ_4 , are separated by more than the coherence bandwidth of the optical channel, hence P_{s1} , P_{s2} , P_{s3} and P_{s4} are independent to each other.

For the four-wavelength interleaved FEC frame, the four wavelengths may experience different normalized DGD states $\Delta\tau_1$, $\Delta\tau_2$, $\Delta\tau_3$ and $\Delta\tau_4$ respectively. According to function (3-13) and (3-17), after decoding, the system symbol error probability with four-wavelength interleaving can be shown to be approximated by:

$$\begin{aligned}
 P_{S_FEC_4} \approx & \frac{d}{n} \sum_{i=t+1}^d \sum_{a=0}^i \sum_{b=0}^{i-a} \sum_{c=0}^{i-a-b} \left\{ \binom{\frac{n+1}{4}}{a} (P_{s1})^a (1-P_{s1})^{\frac{n+1}{4}-a} \right. \\
 & \cdot \binom{\frac{n+1}{4}}{b} (P_{s2})^b (1-P_{s2})^{\frac{n+1}{4}-b} \cdot \binom{\frac{n+1}{4}}{c} (P_{s3})^c (1-P_{s3})^{\frac{n+1}{4}-c} \\
 & \cdot \left. \binom{\frac{n-3}{4}}{i-a-b-c} (P_{s4})^{i-a-b-c} (1-P_{s4})^{\frac{n-3}{4}-(i-a-b-c)} \right\} \\
 & + \frac{1}{n} \sum_{i=d+1}^n \sum_{a=\max(0, i-\frac{3n-1}{4})}^{\min(i, \frac{n+1}{4})} \sum_{b=\max(0, i-a-\frac{n-1}{2})}^{\min(i-a, \frac{n+1}{4})} \sum_{c=\max(0, i-a-b-\frac{n-3}{4})}^{\min(i-a-b, \frac{n+1}{4})} i \left\{ \binom{\frac{n+1}{4}}{a} (P_{s1})^a (1-P_{s1})^{\frac{n+1}{4}-a} \right. \\
 & \cdot \binom{\frac{n+1}{4}}{b} (P_{s2})^b (1-P_{s2})^{\frac{n+1}{4}-b} \cdot \binom{\frac{n+1}{4}}{c} (P_{s3})^c (1-P_{s3})^{\frac{n+1}{4}-c} \\
 & \cdot \left. \binom{\frac{n-3}{4}}{i-a-b-c} (P_{s4})^{i-a-b-c} (1-P_{s4})^{\frac{n-3}{4}-(i-a-b-c)} \right\}
 \end{aligned} \tag{3-21}$$

Similarly to the double-wavelength interleaving system, the function of the system symbol error probability for four-wavelength interleaving system after decoding and the overall BER of the four-channel interleaving system can be derived accordingly. According to equation (3-14), channel symbol error rates (before decoding) P_{s1} , P_{s2} , P_{s3} and P_{s4} should be described as:

$$P_{si} = 1 - (1 - P_{bi}(Q, \Delta\tau_i))^m, \quad i = 1, 2, 3, 4 \quad (3-22)$$

where P_b is the instantaneous channel BER in (3-10), P_s is the system symbol error. $\Delta\tau_1$, $\Delta\tau_2$, $\Delta\tau_3$ and $\Delta\tau_4$ are normalized DGD in wavelength λ_1 , λ_2 , λ_3 and λ_4 respectively.

The corresponding bit error rate of four-wavelength interleaving system is described as:

$$P_{B_FEC_4} = \frac{2^{m-1}}{2^m - 1} P_{S_FEC_4} \quad (3-23)$$

where $P_{B_FEC_4}$ is the output BER of FEC, $P_{S_FEC_4}$ is the output symbol error rate of FEC in four-wavelength interleaving system.

After decoding, the average decoded BER of the four-wavelength interleaving system with PMD and FEC should be a quadruple integral of the conditional decoded BER over the probability density functions of $\Delta\tau_1$, $\Delta\tau_2$, $\Delta\tau_3$ and $\Delta\tau_4$, because alternative symbols in each FEC frame are transmitted on wavelengths λ_1 , λ_2 , λ_3 and λ_4 with uncorrelated DGDs $\Delta\tau_1$, $\Delta\tau_2$, $\Delta\tau_3$ and $\Delta\tau_4$. The average decoded BER of the four-wavelength interleaving system is shown to be:

$$\begin{aligned} \overline{P_{B_FEC_4}} = & \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty M_{<\Delta\tau_1>}(\Delta\tau_1) \cdot M_{<\Delta\tau_2>}(\Delta\tau_2) \cdot M_{<\Delta\tau_3>}(\Delta\tau_3) \cdot M_{<\Delta\tau_4>}(\Delta\tau_4) \\ & \cdot P_{B_FEC_4}(P_{b1}(Q, \Delta\tau_1), P_{b2}(Q, \Delta\tau_2), P_{b3}(Q, \Delta\tau_3), P_{b4}(Q, \Delta\tau_4)) d\Delta\tau_1 d\Delta\tau_2 d\Delta\tau_3 d\Delta\tau_4 \end{aligned} \quad (3-24)$$

3.4.4 Verification of Derived Formulas

To verify the derived expressions of FEC decoded symbol error probability, (3-17) for double-wavelength interleaving systems and (3-21) for four-wavelength interleaving systems, MATLAB simulations are conducted using RS(255, 239) as the FEC code. The channel SERs P_{s1} , P_{s2} , P_{s3} and P_{s4} for the interleaved wavelengths λ_1 , λ_2 , λ_3 and λ_4 respectively are shown in Figure 3-8. In each simulation loop, random channel errors are added to the RS(255, 239) codewords. Then the decoded symbols and original symbols before transmission are compared to find the decoded symbol errors. Once 100 decoded symbol errors have been collected, simulation will stop and the decoded SER is calculated as the symbol error number over the total transmitted symbol number.

Figure 3-8 shows the comparison of simulation result to the derived formulas (3-17) and (3-21) on decoding SER of (a) double-wavelength interleaving system and (b) four-wavelength interleaving system respectively. It shows that the analytical results are very close to the simulation results. When input SER is lower, the results get closer to simulation results. In fiber optical communication systems, the output BER of interest is from 10^{-9} to 10^{-12} . Although the simulation has an output decoding SER range from 10^{-4} to 10^{-7} due to time constraint, it is believed that at output BER of 10^{-9} to 10^{-12} , the formulas of decoding performance (3-17) for double-wavelength interleaving systems and (3-21) on four-wavelength interleaving systems will be accurate.

(a).

(b).

Figure 3-8. Comparison of simulation result to formula result on decoding SER of (a) double-wavelength interleaving system, function (3-17) and (b) four-wavelength interleaving system, function (3-21) with RS(255, 239). P1, P2, P3 and P4 are SERs P_{s1} , P_{s2} , P_{s3} and P_{s4} before FEC decoding.

3.5 Discussions

3.5.1 Average BER

In Figure 3-9 and 3-10, the receiver Q-factor required by the optical system to achieve a BER of 10^{-12} , with and without RS(255, 239) coding and wavelength interleaving, is evaluated as a function of the normalized DGD. For the double-wavelength interleaved system studied in Figure 3-9, the normalized mean DGD $\langle\Delta\tau_2\rangle$ of the second wavelength is fixed at 0.1, 0.17, 0.18 or 0.2, while the normalized mean DGD $\langle\Delta\tau_1\rangle$ of the first wavelength is varied continuously. For four-wavelength interleaving system in Figure 3-10, the normalized mean DGD of the second to fourth wavelengths are similarly set to a few fixed values for ease of presentation.

First, an important feature of the Q-factor, critical point, needs to be defined. Referring to Figure 3-9, it divides the Q-factor plot into two parts. The first part is a horizontal line indicating that AWGN is dominant and PMD effect is negligible. The latter part is a monotonically increasing function of Q value, where PMD effect becomes dominant in the system.

Figure 3-9. Receiver Q factor required to achieve a BER of 10^{-12} with and without FEC, of double-wavelength interleaving system, compared to that of single wavelength system, when mean DGD (normalized by T_b) of the chosen wavelengths are in different value. $\langle t_1 \rangle$ and $\langle t_2 \rangle$ in the figure is normalized mean DGD $\langle \Delta \tau_l \rangle$ and $\langle \Delta \tau_2 \rangle$, normalized rise/fall time $tr=0.5$, and $p=1$.

Figure 3-9 shows the required Q-factor to achieve a BER of 10^{-12} in double-wavelength interleaved system, when the normalized mean DGD of the second wavelength is set to a fixed value $\langle \Delta \tau_2 \rangle$, which sweeps from 0.1 to 0.2, “2WI” means double wavelength interleaving. It is clearly shown that, when $\langle \Delta \tau_2 \rangle$ is lower than 0.17, the curves are all superposed, and the critical point is at $\langle \Delta \tau_1 \rangle = 0.17$. When $\langle \Delta \tau_2 \rangle$ is bigger than 0.17, the critical point increases along the Q-factor curve of $\langle \Delta \tau_2 \rangle$ lower than 0.17, and the horizontal part of the Q-factor curve increases along with the critical point.

As shown in Figure 3-9, in the conventional FEC system without wavelength interleaving, when the normalized mean DGD is less than 0.15, FEC provides a net coding gain of about 6 dB in receiver Q factor. But when the normalized mean DGD is higher than 0.19, FEC without wavelength interleaving no longer provides any net coding gain, as a result of the 7% bit rate expansion due to FEC coding, and uncorrectable error burst due to severe PMD.

Figure 3-10. Receiver Q factor required to achieve a BER of 10^{-12} with and without FEC, of four-wavelength interleaving system, compared to that of single wavelength system, when mean DGD (normalized by T_b) of the chosen wavelengths are in different value. $\langle t_1 \rangle$ and $\langle t_{2,3,4} \rangle$ in the figure is normalized mean DGD $\langle \Delta \tau_l \rangle$ and $\langle \Delta \tau_{2,3,4} \rangle$, normalized rise/fall time $tr=0.5$, and $p=1$.

In contrast to the conventional system, our proposed wavelength-interleaved FEC system can provide coding gain at all PMD levels, since their performance lines never merge with the uncoded performance line, as shown in both Figure 3-9 and 3-11. Defining “tolerable DGD” as the mean DGD value corresponding to the receiver Q-factor of 17 dB typically considered in optical transmission systems, the tolerable DGD of the non-interleaved system is $0.175T_b$, while the tolerable DGD of double-wavelength interleaved system is $0.21T_b$ (Figure 3-9) and that of four-wavelength interleaved system is $0.22T_b$ (Figure 3-10). Hence the tolerable PMD level is extended by 20% and 26% respectively. Since the mean DGD of a fiber link is a function of the square root of the fiber length L as indicated in (3-2), the maximum transmission distance of the optical fiber link is correspondingly increased by about 45% and 60%. This implies large savings in the system cost. Figure 3-9 further shows that the FEC performance of receiver Q-factor as a function of $\langle\Delta\tau_1\rangle$ is virtually invariant when $\langle\Delta\tau_2\rangle$ is lower than 0.17. When $\langle\Delta\tau_2\rangle$ is larger than 0.17, the coding gain at low $\langle\Delta\tau_1\rangle$ region (corresponding to the horizontal portion of the FEC performance lines) will decrease.

Similar performance trends can be observed in Figure 3-10 for the four-wavelength interleaving system too. These observations suggest that the performance of the proposed wavelength-interleaved FEC system is at its best when the mean DGD of any of its wavelength channels does not exceed a certain threshold value, which is observed to be $0.17T_b$ and $0.18T_b$ for the double-wavelength and four-wavelength examples considered in Figure 3-9 and 3-10 respectively. This value is believed to be a function of the interleaved channel parameters and the FEC code used.

As what is analyzed in the beginning of section 3.4, different wavelength may be in very different PMD levels at the same time in common optical transmission system: some may be high, while others may be low. The probability that the interleaved wavelengths are simultaneous deteriorated severely by PMD is much smaller than that only one wavelength is deteriorated by PMD. Thus with wavelength interleaving, the effectiveness of FEC to mitigate PMD effect is enhanced.

3.5.2 Outage Probability

Besides average BER, the outage probability [9] is another important parameter for system performance evaluation. In this thesis, system outage happens when the decoding BER is higher than the system required BER of 10^{-12} . The outage probabilities larger than an instantaneous BER of 10^{-12} are calculated by $P\{P_b(Q, \Delta\tau) > 10^{-12}\}$ for system without FEC, $P\{P_{B_FEC}(Q, \Delta\tau) > 10^{-12}\}$ for FEC systems without wavelength interleaving and $P\{P_{B_FEC_2}(Q, \Delta\tau_1, \Delta\tau_2) > 10^{-12}\}$ for FEC systems with double-wavelength interleaving.

Figure 3-11 shows the required Q-factor for an outage probability of 10^{-3} and 10^{-6} as a function of Q-factor and normalized instantaneous DGD, when two wavelengths with same mean DGD are interleaved. Figure 3-11 shows that the required Q-factor decreases by using double-wavelength interleaving compared to the case of no wavelength interleaving. The wavelength interleaving system performance is improved most when both wavelengths have severe DGD, but when it is too severe, the amount of improvement shrinks back. This is determined by the error correcting capability of the RS code. When PMD is too serious and induces too many errors over the correcting ability of FEC, wavelength interleaving is not going to provide enormous improvement. At the receiver Q-factor of 17 dB for an outage probability of 10^{-6} , the tolerable DGD in Figure 3-11 increases about 14~36% by using double-wavelength interleaving, and the maximum transmission distance of the optical fiber link is correspondingly increased by about 30~85%.

(a)

(b)

Figure 3-11. Required Q-factor to achieve outage probability of (a) 10^{-3} and (b) 10^{-6} for PMD channels with and without FEC, with and without 2-wavelength interleaving. The two interleaved wavelengths have same normalized mean DGD, $\langle \Delta \tau_1 \rangle = \langle \Delta \tau_2 \rangle$. [55]

3.5.3 Simplification of Decoded BER Expressions

In section 3.3, the relationship of SER before and after hard-decision Reed Solomon decoding has been presented in (3-13). The error probability expression (3-13) is composed of two parts: P_{df} denotes the probability of a decoding failure when $(t+1) < i < d$, and P_{ud} denotes the probability of an undetected error when $(d+1) < i < n$, and can be written as:

$$\begin{aligned} P_{S_FEC} &\approx \frac{d}{n} \sum_{i=t+1}^d \binom{n}{i} (P_s)^i (1-P_s)^{n-i} + \frac{1}{n} \sum_{i=d+1}^n i \binom{n}{i} (P_s)^i (1-P_s)^{n-i} \\ &\approx P_{df} + P_{ud} \end{aligned} \quad (3-25)$$

In optical transmission system, the SER after decoding P_{S_FEC} is required to be lower than 10^{-12} , which means that before decoding P_s should be lower than 10^{-2} RS(255, 239) code. When the RS code length n is big, $(P_s)^i$ is small and $(1-P_s)^{n-i}$ is almost 1 in P_{ud} , while $(P_s)^i$ of P_{df} is much bigger than that of P_{ud} . Thus P_{df} is dominant for RS(255, 239), and P_{S_FEC} can be approximated by.

$$P_{S_FEC} \approx \frac{d}{n} \sum_{i=t+1}^d \binom{n}{i} (P_s)^i (1-P_s)^{n-i} \quad (3-26)$$

where P_s is symbol error probability before decoding.

In double-wavelength interleaving system, the SER after hard-decision Reed Solomon decoding is given by function (3-17). For the same reasons explained above, function (3-17) can be similarly simplified into:

$$P_{S_FEC_2} \approx \frac{d}{n} \sum_{i=t+1}^d \sum_{j=0}^i \left\{ \binom{\frac{n+1}{2}}{j} (P_{s1})^j (1-P_{s1})^{\frac{n+1}{2}-j} \cdot \binom{\frac{n-1}{2}}{i-j} (P_{s2})^{i-j} (1-P_{s2})^{\frac{n-1}{2}-(i-j)} \right\} \quad (3-27)$$

where P_{s1} and P_{s2} have been defined in function (3-17). Similarly, the relationship of SER after hard-decision Reed Solomon decoding in four-wavelength interleaving system, can also be simplified into:

$$P_{S_FEC_4} \approx \frac{d}{n} \sum_{i=1}^d \sum_{a=0}^i \sum_{b=0}^{i-a} \sum_{c=0}^{i-a-b} \left\{ \left(\frac{n+1}{4} \right) \binom{n+1}{a} (P_1)^a (1-P_1)^{\frac{n+1}{4}-a} \cdot \left(\frac{n+1}{4} \right) \binom{n+1}{b} (P_2)^b (1-P_2)^{\frac{n+1}{4}-b} \right. \\ \left. \cdot \left(\frac{n+1}{4} \right) \binom{n+1}{c} (P_3)^c (1-P_3)^{\frac{n+1}{4}-c} \cdot \left(\frac{n-3}{4} \right) \binom{n-3}{i-a-b-c} (P_4)^{i-a-b-c} (1-P_4)^{\frac{n-3}{4}-(i-a-b-c)} \right\} \quad (3-28)$$

where P_{s1} , P_{s2} , P_{s3} and P_{s4} have been defined in function (3-18).

Figure 3-12. Comparison between original decoding symbol error functions and the respective approximated ones. Original function is denoted by long (L) function, approximated function is denoted by short (S) function in figure.

The comparison of original decoding symbol error functions and the respective approximated ones are shown in Figure 3-12. It clearly shows that function (3-26) can be used as the decoding symbol error probability function of conventional optic link

instead of (3-13), and function (3-27), (3-28) instead of function (3-17), (3-21) for wavelength interleaving systems. This simplification can save much time on simulation and calculation, and has been used for outage probability calculation discussed in session 3.5.2.

3.5.4 Effect of rise/fall time tr on Optical System Performance

Pulse rise time is defined to be the amount of time for the output voltage to go from 10% of the logic "1" level to 90% of the logic "1" level. Pulse fall time is defined to be the time required for the output voltage to go from 90% of the logic "1" level to 10% of the logic "1" level. They are assumed to be equal, and are collectively called rise/fall time. In this thesis, tr denotes the rise/fall time normalized by T_b . In this section, the system improvement provided by wavelength interleaving at different tr is investigated, in order to understand the effect of pulse shape change on the wavelength interleaving technique. The analysis of four-wavelength interleaving system model costs much more time than that of double-wavelength interleaving system model. Because their trends of system performance are similar, double-wavelength interleaving system is taken as the main investigating object.

Figure 3-13 shows the required receiver Q factor to achieve a BER of 10^{-12} with and without FEC, of double-wavelength interleaving system when the normalized pulse rise/fall time tr varies. When tr is 0.7, the critical point is at $\langle \Delta \tau_I \rangle = 0.13$. When tr is lower, the critical point moves outward. This indicates that, with lower tr , the system earns bigger PMD tolerance. This is because the relative delay of two principle states of polarization increases and lead to higher DGD to weaken the system performance.

Figure 3-13. Required receiver Q factor to achieve a BER of 10^{-12} with and without FEC of double-wavelength interleaved system at different normalized rise/fall time tr . Normalized mean DGD $\langle \Delta \tau_1 \rangle$ changes and $\langle \Delta \tau_2 \rangle$ is fixed to be 0.1, $p=1$.

3.6 Chapter Conclusion

In this chapter, an optical channel impairment called PMD and a class of channel code called FEC code that is capable of detecting and correcting errors caused by PMD in the optical link are studied. FEC has been verified to be able to provide a great performance improvement at all PMD levels and to increase the tolerable PMD level accordingly, when used with sufficient interleaving. However, sufficient time interleaving for long fading PMD is not feasible, because the interleaving length and delay will be too long. Without sufficient time interleaving, FEC won't work in high PMD level. In this chapter, a novel wavelength interleaving scheme is proposed to achieve frequency interleaving to address the limitation of time interleaving in mitigating the effect of PMD in optical transmission system. RS(255, 239) code recommended by ITU-T as the FEC code is selected.

Furthermore, the system performance of the conventional optical transmission system with PMD and FEC is analyzed in details. Based on the FEC decoding error probability function of the conventional optical transmission system without wavelength interleaving, a new error probability function for the system with multiple wavelengths interleaving is derived. Simulation results match well with the derived error probability functions.

Based on RS(255, 239) coding, wavelength interleaving is found to provide a net coding gain in the receiver Q-factor of over 6 dB for double-wavelength and four-wavelength interleaving systems at all tolerable PMD levels. Compared to a similar system without wavelength interleaving, this technique can also extend the tolerable PMD value by more than 20% for double-wavelength and 26% for four-wavelength interleaving systems, based on receiver Q-factor of 17 dB and output BER of 10^{-12} . For an outage probability of 10^{-6} , the net coding gain is about 14~36% by applying double-wavelength interleaving. This brings about a corresponding increase in the maximum transmission distance.

With the findings presented in this chapter, it can be concluded that wavelength interleaving is effective in improving the FEC performance of optical channels impaired by long-fading PMD. This is because wavelength interleaving intersperses the channel errors in different wavelength channels with uncorrelated PMD to reduce the net burst error length in a de-interleaved FEC frame. Clearly, the performance gain of this technique increases with the number of interleaved wavelength channels used. Another benefit of wavelength interleaving is that the multiple wavelengths used in a wavelength interleaved system will require a common Q-factor instead of very different ones in a non-wavelength interleaved system. This decreases inter-channel interference and further meliorates the system.

Chapter 4

FEC Performance of Chromatic Dispersion Mitigation Using Wavelength Interleaving

4.1 Introduction

Chromatic dispersion (CD) causes light pulses to spread, induces inter-symbol interference and becomes a critical transmission impairment factor when transmission bit rates increase to and beyond 10 Gb/s. Using dispersion compensation fiber (DCF) is the most popular solution for compensating CD. However, CD varies for each channel due to the nonzero dispersion slope, which is hard to be perfectly compensated because of manufacture limitation. Nonzero dispersion slope causes the residual dispersion to change over the signal bandwidth. The wider the optical spectrum, the larger the residual dispersion changes over the signal bandwidth, the larger the CD will be induced from the accumulated dispersion slope. As a result, dispersion tolerance becomes smaller as the channel bit rate increases. Therefore, accumulated dispersion slope has to be managed especially for 40 Gb/s and higher bit rate transmission over nonslope-matched fibers [3][4]. Forward error correction technique is another popular method to mitigate chromatic dispersion effects, which provides some time interleaving and correcting ability. In chapter 3, a wavelength interleaving scheme is proposed to provide frequency diversity to mitigate the effect of polarization mode dispersion (PMD) in optical FEC systems. It's reasonable to believe that wavelength interleaving can also help to mitigate the CD problem.

In this chapter, the channel BER expression with CD impairment is derived, and is verified by simulation using the VPI simulator. The wavelength interleaved FEC performance in optical transmission system with chromatic dispersion impairment is

analyzed. The minimum wavelength separation required for such wavelength-interleaving FEC systems to get the highest coding gain is also determined.

4.2 Channel BER Model for CD Impaired Systems

In [53][54], a BER expression for an optical system with chromatic dispersion is given in terms of dispersion index, dispersion coefficient, chip rate, chip duration, transmission length, optical carrier wavelength, SNR, and noises. In this section, a simpler channel BER expression is derived with consideration of chromatic dispersion in optical OOK system. In order to be consistent with the rest of this thesis, it is derived based on the channel BER expression in Chapter 3. It takes account of the influence of transmitter rise/fall times, receiver Q-factor and noises, which help us to investigate how the output BER is affected by these system factors and how much FEC performance the wavelength interleaving can improve on.

4.2.1 Channel BER Modeling

To make the analysis tractable and simple, it is assumed that trapezoidal pulses with bit period T_b and original rise/fall time tr (normalized by T_b) are launched into the single mode fiber. In real optical transmission systems, dispersion and filtering in general would result in variations of the pulse shapes. In this thesis, a pulse impaired by the CD is assumed to maintain a trapezoidal shape after propagating through the optical channel, but its width increases with transmission distance due to dispersion, and the rise/fall time changes accordingly. Hence the pulse spread normalized by T_b can be shown to be:

$$\Delta\tau = D_{CD} \cdot \Delta\lambda \cdot L / T_b \quad (4-1)$$

where D_{CD} is the chromatic dispersion parameter in ps/nm·km, $\Delta\lambda$ is the spectral width of the pulse and L is the fiber length. The pulse spread discussed throughout this thesis is normalized pulse spread $\Delta\tau$.

At the receiver, the rise/fall time of the received pulse (normalized by T_b), T_s , affected by the pulse spread induced by CD, and is changed to:

$$T_s = \sqrt{tr^2 + \Delta\tau^2} \quad (4-2)$$

(a)

(b)

Figure 4-1. Scheme of pulse (a) without spread and (b) with spread.

Assuming no power loss, the area of the transmitted trapezoidal pulse remains constant. The amplitude of level 0 is zero, while the amplitude of level 1 of the received pulses will be reduced to:

$$I_1' = \frac{I_1}{1 + \Delta\tau} \quad (4-3)$$

where I_1 and I_1' are the mean pulse amplitudes of level 1 at transmitter and receiver respectively, as shown in Figure 4-1 (b).

The receiver Q-factor has the following relationship:

$$Q = \frac{I_1'}{\sigma_1 + \sigma_0} \quad (4-4)$$

where σ_1 and σ_0 are the standard deviation of amplitudes of level 1 and level 0 respectively. Here it can be deduced that the back to back Q-factor Q_i and system receiver Q-factor Q has the following relationship:

$$Q = \frac{I_1'}{\sigma_1 + \sigma_0} = \frac{1}{1 + \Delta\tau} \cdot \frac{I_1}{\sigma_1 + \sigma_0} = \frac{1}{1 + \Delta\tau} Q_i \quad (4-5)$$

According to [50] and equation (4-3), the levels $I_{0/1}^{sup/med/inf}$ of received pulses can be modeled by:

$$I_0^a = I_1' (1 - g^a(\Delta\tau, T_s)); \quad I_1^a = I_1' g^a(\Delta\tau, T_s) \quad (4-6)$$

where $a=sup, med, inf$, and

$$\begin{aligned} g^{sup}(\Delta\tau, T_s) &= 1, \\ g^{med}(\Delta\tau, T_s) &= \begin{cases} 1, & 1 - T_s \geq \Delta\tau \\ 1 - \frac{\Delta\tau + T_s - 1}{4T_s}, & 1 + T_s \geq \Delta\tau \geq 1 - T_s \\ 0.5, & \Delta\tau \geq 1 + T_s \end{cases} \\ g^{inf}(\Delta\tau, T_s) &= \begin{cases} 1, & 1 - T_s \geq \Delta\tau \\ 1 - \frac{\Delta\tau + T_s - 1}{2T_s}, & 1 \geq \Delta\tau \geq 1 - T_s \\ 0.5, & \Delta\tau \geq 1 \end{cases} \end{aligned} \quad (4-7a)$$

With equation (4-2), (4-7a) can be written as:

$$\begin{aligned} g^{sup}(\Delta\tau, tr) &= 1 \\ g^{med}(\Delta\tau, tr) &= \begin{cases} 1, & \frac{1 - tr^2}{2} \geq \Delta\tau \\ 1 - \frac{\Delta\tau + \sqrt{tr^2 + \Delta\tau^2} - 1}{4\sqrt{tr^2 + \Delta\tau^2}}, & \Delta\tau \geq \frac{1 - tr^2}{2} \end{cases} \\ g^{inf}(\Delta\tau, tr) &= \begin{cases} 1, & \frac{1 - tr^2}{2} \geq \Delta\tau \\ 1 - \frac{\Delta\tau + \sqrt{tr^2 + \Delta\tau^2} - 1}{2\sqrt{tr^2 + \Delta\tau^2}}, & 1 \geq \Delta\tau \geq \frac{1 - tr^2}{2} \\ 0.5, & \Delta\tau \geq 1 \end{cases} \end{aligned} \quad (4-7b)$$

The channel BER (before decoding) of an optical system has been presented in Chapter 3, as (3-7).

Based on functions (3-7), (4-4) and (4-6), the CD-impaired channel BER expression can be derived to be:

$$\begin{aligned}
 P_b(Q, \Delta\tau) &= P(I_1^{\text{sup}})P(0|I_1^{\text{sup}}) + P(I_1^{\text{med}})P(0|I_1^{\text{med}}) + P(I_1^{\text{inf}})P(0|I_1^{\text{inf}}) \\
 &\quad + P(I_0^{\text{sup}})P(1|I_0^{\text{sup}}) + P(I_0^{\text{med}})P(1|I_0^{\text{med}}) + P(I_0^{\text{inf}})P(1|I_0^{\text{inf}}) \\
 &= \frac{1}{16} \operatorname{erfc}\left(\frac{Q(1-p\cdot\Delta\tau)}{\sqrt{2}(1+\Delta\tau)}\right) + \frac{1}{8} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}(1+\Delta\tau)}[g^{\text{med}}(1+p) - p(1+\Delta\tau)]\right) \\
 &\quad + \frac{1}{16} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}(1+\Delta\tau)}[g^{\text{inf}}(1+p) - p(1+\Delta\tau)]\right) + \frac{1}{16} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right) \\
 &\quad + \frac{1}{8} \operatorname{erfc}\left(\frac{Q}{p\sqrt{2}(1+\Delta\tau)}[g^{\text{med}}(1+p) + p\cdot\Delta\tau - 1]\right) \\
 &\quad + \frac{1}{16} \operatorname{erfc}\left(\frac{Q}{p\sqrt{2}(1+\Delta\tau)}[g^{\text{inf}}(1+p) + p\cdot\Delta\tau - 1]\right)
 \end{aligned} \tag{4-8}$$

where $p = \sigma_0 / \sigma_1$ and g^a ($a = \text{sup, med, inf}$) is defined in function (4-7).

4.2.2 Verification of Derived Formula

Virtual Photonics Interface (VPI) [61][62] is a photonic simulation engine. VPI accelerates the design of new photonic systems including short-range, access, metro and long haul optical transmission systems and allows technology upgrade and component substitution strategies to be developed for existing network plants.

Figure 4-2 shows the VPI schematic of our designed optical transmission simulation system affected only by noise and chromatic dispersion. At the transmitter (marked “Tx” in the Figure 4-2), pulses are generated by an optical pulse source with a narrow spectrum. Before the signals are transmitted by a single mode fiber with only chromatic dispersion considered, additive Gaussian white noise (marked “GWNoise” in the Figure 4-2) is polarized and added into the system. At the end of the fiber,

signals are received by a p-i-n (marked “PIN” in the Figure 4-2) detector, and then synchronized by a Re-clock (marked “Recldeal” in the Figure 4-2). Finally, two estimators (marked “BER” in the Figure 4-2) are placed, one is at the end of the whole link to see output channel BER, the other one is place at the start point of the fiber to watch the transmitter Q-factor.

The Q-factor obtained from the VPI model Q_t is the system transmitter Q-factor required when the output BER at the receiver is 10^{-12} . According to the relationship between the system transmitter Q-factor Q_t and receiver Q-factor Q in function (4-5), $Q(\text{dB}) = Q_t(\text{dB}) - 20\log(1+\Delta\tau)$ can be obtained to compare with the mathematical BER model. In our 10 Gbit/s VPI simulation system, super Gaussian pulses ($m=3$) are transmitted. The super Gaussian pulses are approximated as trapezoidal pulses in our BER analysis. At the central wavelength $\lambda_0=1.55 \mu\text{m}$, the CD parameter is $D_{CD}=16 \text{ ps/nm}\cdot\text{km}$, $R=10 \text{ Gb/s}$, $T_b = 100 \text{ ps}$, normalized rise time $tr=0.3$.

Figure 4-2. Schematic of VPI simulation model.

(a) Transmission length = 20 km

(b) Transmission length = 40 km

(c) Transmission length = 60 km

(d) Transmission length = 80 km

Figure 4-3. Comparison of channel BER model (4-8) and VPI simulation results on output BER vs. Q-factor at different transmission length for 10 Gbit/s system, $\lambda_0=1.55$ μm , $D_{CD}=16$ ps/nm-km, $R=10$ Gb/s, $T_b = 100$ ps, normalized rise time $tr=0.3$.

Figure 4-3 (a-d) show the comparisons of BERs of the channel BER expression model (4-8) and VPI simulation result, when the distance L is 20, 40, 60 and 80 km respectively. The curves of our deduced channel BER model and VPI simulation results match well when the transmission distance is 20, 40 and 60 km. At $L = 80\text{km}$, the required receiver Q has a difference up to 2.5 dB between the math model and VPI when the output BER is higher than 10^{-9} and up to 1 dB when the output BER is $10^{-12} \sim 10^{-9}$, which is around the optical system requirement operating region. Hence the derived channel BER model can represent the system performance fairly accurately.

Figure 4-4 shows the comparison of required Q-factor to achieve an output BER of 10^{-12} as a function of CD-induced pulse spread based on expression (4-8) (the blue curve) and VPI simulation results (the red curve). In Figure 4-4, the two lines closely match to each other, which verify our dispersion-induced channel BER expression (4-8) is reliable even though trapezoidal pulse is used in the analysis to estimate the Gaussian pulse commonly used in practice.

(a)

(b)

Figure 4-4. Comparison of channel BER model (4-8) and VPI simulation results on required Q-factor to achieve an output BER of 10^{-12} as a function of chromatic dispersion. For 10 Gbit/s system, $\lambda_0=1.55 \mu\text{m}$, $D_{CD}=16 \text{ ps/nm}\cdot\text{km}$, $R=10 \text{ Gb/s}$, $T_b=100 \text{ ps}$, normalized rise time $tr=0.3$.

4.3 Decoded BER in CD Channel with and without Wavelength Interleaving

4.3.1 No wavelength interleaving

Figure 4-5 shows the receiver Q factor required to achieve a BER of 10^{-12} with FEC compared to that without FEC of single wavelength system. RS(255, 239) code provides a net coding gain (NCG) of 5.8 dB or more to system performance at all chromatic dispersion levels. It is clearly shown in this figure, system tolerance of dispersion increases with the decreasing of the pulse rise/fall time.

Figure 4-5. Receiver Q factor required to achieve a BER of 10^{-12} with and without FEC of single wavelength system at different normalized pulse spread $\Delta\tau$ caused by chromatic dispersion. Normalized $T_s=0.3, 0.5, 0.7$, and $p=1$.

System performance evaluation models are same to those have been described in Section 3.3. The relationship of SER P_s before and P_{S_FEC} after RS(n, k) hard decoding can be approximated by (3-13). The SER P_s before FEC decoding can be calculated by (3-14) based on the channel BER P_b by (4-8). And output BER P_{B_FEC} and the output SER P_{S_FEC} is related by (3-15).

4.3.2 With Wavelength Interleaving

Due to the nonzero dispersion slope, the amount of CD impairment varies for different wavelengths. With wavelength interleaving, alternative symbols are proposed to transmit in each FEC frame on different wavelengths with different CD-induced pulse spread levels. At the receiver, these interleaved symbols are re-assembled back to their original order before FEC decoding.

For a double-wavelength interleaved FEC frame, the two wavelengths λ_1 and λ_2 experience different dispersion level with different normalized pulse spreading $\Delta\tau_1$ and $\Delta\tau_2$ respectively. Their channel SER (before decoding) can be denoted to be $P_s(Q, \Delta\tau_1)=P_{s1}$ and $P_s(Q, \Delta\tau_2)=P_{s2}$ respectively. Without loss of generality, it can be assumed that $\Delta\tau_1 > \Delta\tau_2$. Then, the output SER after decoding in double-wavelength interleaving systems can be calculated by (3-17) to be $P_{S_FEC_2}(P_s(Q, \Delta\tau_1), P_s(Q, \Delta\tau_2))$.

Similarly, for a four-wavelength interleaved FEC frame, the four wavelengths $\lambda_1, \lambda_2, \lambda_3$ and λ_4 experience different chromatic dispersion with different normalized pulse spreading $\Delta\tau_1, \Delta\tau_2, \Delta\tau_3$ and $\Delta\tau_4$ respectively. Their channel SER (before decoding) can be denoted to be $P_s(Q, \Delta\tau_1)=P_{s1}$, $P_s(Q, \Delta\tau_2)=P_{s2}$, $P_s(Q, \Delta\tau_3)=P_{s3}$ and $P_s(Q, \Delta\tau_4)=P_{s4}$ respectively. The formula of decoder-output SER for four-wavelength interleaving system can be similarly shown to be: $P_{S_FEC_4}(P_s(Q, \Delta\tau_1), P_s(Q, \Delta\tau_2), P_s(Q, \Delta\tau_3), P_s(Q, \Delta\tau_4))$, and calculated by (3-21).

4.4 Wavelength Separation Requirement

In PMD-impaired systems with wavelength interleaving, if the chosen wavelengths are separated by more than the coherence bandwidth of the optical channel, PMD effect will be significantly mitigated [54]. Therefore, a detailed investigation of this topic will be conducted in this section. First of all, λ_1 is assumed to be the wavelength of the worse channel (with higher CD) to be interleaved, while the pulse spread difference normalized by T_b between the interleaved channels is defined to be $d\tau = \Delta\tau_1 - \Delta\tau_2$.

In a two-wavelength interleaving system, different values of $d\tau$ will generally lead to different output BER. Figure 4-6 (a) shows the Q-factor required to achieve an output BER of 10^{-12} using RS(255, 223) code. The dashed line separates the figure into 2 parts: to the left of the dashed line, the required Q decreases with increasing $d\tau$, to the right of it, the required Q remains essentially constant with increasing $d\tau$. Hence the dashed line traces out the locus of $d\tau_{\min}$, normalized minimum pulse spread difference which the interleaved wavelengths need to have pulse spread difference greater than, in order to get the highest BER improvement. The loci of $d\tau_{\min}$ for double-wavelength and four-wavelength interleaved systems based on RS(255, 223) code are plotted in Figure 4-6 (b) as a function of the pulse spread value of the worst channel, $\Delta\tau_1$.

(a).

(b).

Figure 4-6. (a) Q-factor required to achieve an output BER 10^{-12} vs. normalized pulse difference $d\tau$ for double-wavelength interleaving system, (b) minimum normalized pulse difference $d\tau_{\min}$ required to get the highest gain in Q-factor for double-wavelength and four-wavelength interleaving systems, vs. normalized pulse spread $\Delta\tau_1$ of the worst channel λ_1 , based on RS(255, 223) code. $p=1$, $R = 10$ Gb/s, $tr = 0.3$.

Table 4-1. CD-induced normalized pulse spread $\Delta\tau$ on difference wavelengths at C-band and C+L-band in 10 Gb/s systems. Central wavelength $\lambda_0=1550$ nm, $D_{CD0}=17.6$ ps/nm/km is dispersion factor at λ_0 , dispersion slop $S=0.08$ ps/nm².km.

(a) 10 Gb/s system

	C-band, 1530~1562 nm D_{CD} : 16~18.5 ps/nm/km,	C+L-band, 1530~1610 nm D_{CD} : 16~22.4 ps/nm/km,
$L=80$ km	$\Delta\tau = 10\%\sim 13\%$;	$\Delta\tau = 10\%\sim 20\%$;
$L=100$ km	$\Delta\tau = 15\%\sim 20\%$;	$\Delta\tau = 15\%\sim 28\%$;
$L=200$ km	$\Delta\tau = 51\%\sim 65\%$;	$\Delta\tau = 51\%\sim 87\%$;

(b) 40 Gb/s system

	C-band, 1530~1562 nm: D_{CD} : 16~18.5 ps/nm/km,	C+L-band, 1530~1610 nm: D_{CD} : 16~22.4 ps/nm/km,
$L=80$ km	$\Delta\tau = 730\% \sim 850\%$;	$\Delta\tau = 730\% \sim 1020\%$;
$L=100$ km	$\Delta\tau = 910\% \sim 1060\%$;	$\Delta\tau = 910\% \sim 1270\%$;
$L=160$ km	$\Delta\tau = 1450\% \sim 1680\%$;	$\Delta\tau = 1450\% \sim 2030\%$;
$L=200$ km	$\Delta\tau = 18.1\sim 21.0$;	$\Delta\tau = 18.1\sim 25.3$;

Table 4-1. shows that the CD-induced normalized pulse spread $\Delta\tau$ increases with distance and bandwidth. In the system defined here, central wavelength is $\lambda_0=1550$ nm, and the dispersion factor at λ_0 is $D_{CD0}=17.6$ ps/nm/km. Due to the dispersion slope $S=0.08$ ps/nm².km, dispersion factor changes on different wavelengths. In C band (from 1530 to 1562 nm), dispersion factor D_{CD} varies from 16 to 18.5 ps/nm/km. And in the C+L band (from 1530nm to 1610nm), dispersion factor D_{CD} varies from 16 to 22.4 ps/nm/km. With expression (4-1), the dispersion values are obtained and are shown in Table 4-1. It is obvious that either larger transmission distance or higher transmission speed will induce higher dispersion value and bigger difference between wavelengths. In 40 Gb/s fiber optic systems, the pulse spread caused by both

dispersion and dispersion slope has been a fatal problem after 80 km transmission. Therefore, another DCF fiber needs to be added to compensate the dispersion on $\lambda_0=1550$ nm. But the DCF used now usually still has another positive dispersion slope, which will aggravate the pulse spread difference. Wavelength interleaving will average the dispersion accumulated on different wavelengths, to make the system flat and easy to be compensated.

Referring to Figure 4-6(b), if the wavelength separation produces a normalized dispersion difference bigger than 0.1 for double-wavelength interleaving systems or 0.15 for four-wavelength interleaving systems, wavelength interleaving can achieve its biggest impact on improving system performance. Comparing the C band and the C+L band in Table 4-1, since the C+L band has larger range of $\Delta\tau$ values, C+L band is expected to benefit more from the proposed wavelength interleaving system, especially in long distance (>100 km) and very high speed (≥ 40 Gb/s) transmission systems.

4.5 Improvement in System Dispersion Tolerance

WDM system design is based on the worst-case channel. Hence, if the performance of the worst channel can be improved by interleaving them with other better channels in the same system, it means that the whole system is improved.

Figure 4-7 shows the output BER as a function of the receiver Q factor, achieved by different systems (without wavelength interleaving, with two- or four-wavelength interleaving), all employing RS(255, 223) code. Figure 4-7 (a) compares double-wavelength interleaved systems with non interleaved systems. λ_1 and λ_2 denote the channels experiencing higher and lower dispersion without interleaving, “ $\lambda_1+\lambda_2$ ” denotes the interleaved channel. The results show that if λ_1 with $\Delta\tau_1=0.1$ in group A (or $\Delta\tau_1=0.4$ in group B) and λ_2 with $\Delta\tau_2=0$ (or $\Delta\tau_2=0.3$) are interleaved, the performance of double-wavelength interleaved system can be improved by 0.6 dB (or 0.8 dB) over the worse channel (λ_1). Figure 4-7 (b) shows similar comparison for the four-wavelength interleaved systems. Here, λ_2 , λ_3 and λ_4 are assumed to be adjacent channels experiencing lower dispersions, hence λ_2 , λ_3 and λ_4 can be considered to have very similar amount of dispersion, represented by a common $\Delta\tau_{2,3,4}$ value. They are selected to be far from λ_1 , the channel experiencing higher dispersion $\Delta\tau_1$, such that $\Delta\tau_1-\Delta\tau_{2,3,4}$ meets the normalized minimum pulse spread difference $d\tau_{\min}$ shown in Figure 4-6 (b). In this figure, it is similarly observed that if λ_1 with $\Delta\tau_1=0.1$ in group C (or $\Delta\tau_1=0.45$ in group D) and $\lambda_{2,3,4}$ with $\Delta\tau_{2,3,4}=0$ (or $\Delta\tau_{2,3,4}=0.3$) are interleaved, the performance of the four-wavelength interleaved system can be improved by 0.8 dB (or 1.7 dB) over the worst channel (λ_1).

(a). Double-wavelength interleaving system

(b). Four-wavelength interleaving system

Figure 4-7. Receiver Q factor vs. output BER with RS(255, 223) code in systems without wavelength interleaving compared to that in (a) double-wavelength interleaved systems, and (b) four-wavelength interleaved systems. $\Delta\tau$ in this figure is the mean DGD normalized by T_b , $p=1$, $R=10$ Gb/s, normalized rise/fall time $tr=0.3$.

Since WDM (wavelength division multiplexing) system is typically designed based on the worst-case channel, if the performance of the worst channel can be improved by sacrificing some performance of the better channels in the wavelength-interleaved system, it means that the overall system performance is effectively improved by the same amount as the improvement seen by the worst channel.

The results in Figure 4-7 correspond to specific values of $\Delta\tau_1$ and $\Delta\tau_{2,3,4}$. Figure 4-8 shows the performance improvement obtained at BER of 10^{-12} by double- and four-wavelength interleaving systems over the worst-case channels for a larger range of $\Delta\tau_1$ values, assuming that the $\Delta\tau_2$, $\Delta\tau_3$ and $\Delta\tau_4$ values satisfy the normalized minimum pulse spread difference value, $d\tau_{\min}$, shown in Figure 4-6(b). As shown in Figure 4-8, FEC without wavelength interleaving provides a coding gain (CG) of about 7 dB in receiver Q factor, and our proposed FEC scheme with wavelength interleaving can provide an additional coding gain of over 1 dB with double-wavelength interleaving and 2~3 dB with four-wavelength interleaving, especially when the chromatic dispersion is severe. Defining “tolerable dispersion” as the value of the pulse spread at the receiver Q-factor of 17 dB, the tolerable normalized dispersion of the conventional non-interleaved system is 0.39, while that is 0.44 for double-wavelength interleaving system and 0.475 for four-wavelength interleaving system. Hence the tolerable dispersion levels are extended by 13% and 22% respectively. Since the pulse spread caused by CD of a fiber link is a function of the product of fiber length L and dispersion index D as indicated in (1), the maximum transmission distance of the optical fiber link is correspondingly increased by about 13% or 22% for double- or four-wavelength interleaving systems. This implies savings in the system cost.

It is clear from Figure 4-8 that the improvement in tolerable dispersion is more pronounced when the normalized pulse spread is not too severe (less than 0.5). Therefore, it is proposed that any CD compensation performed at the system level should first attempt to reduce the normalized dispersion down to 0.5 or lower, then

rely on wavelength interleaving to obtain more performance gains as indicated in the plots.

Figure 4-8. Receiver Q factor required to achieve an output BER of 10^{-12} without FEC compared to that with RS(255, 223) code in single wavelength, double-wavelength and four-wavelength interleaved systems. $\Delta\tau$ in this figure is the mean DGD normalized by T_b , $p=1$, $R=10$ Gb/s, normalized rise/fall time $tr=0.3$.

4.6 Chapter Conclusion

A channel BER model which takes into account of the influence of CD-induced pulse spread, transmitter rise/fall times, receiver Q-factor, and noises is derived in this chapter. Its reliability has been verified by VPI simulation results. This model does

not need to analyze the eye diagram at the receiver, thus it is more convenient to use than the existing models.

To mitigate the CD effect on optical transmission performance, FEC with wavelength interleaving is proposed and analyzed. The results show that wavelength interleaving can improve the overall system performance by averaging out the performance of a set of channels experiencing higher and lower dispersion. Although the interleaved system performance is worse than the non-interleaved channels experiencing lower dispersion, the overall system is improved because WDM systems are typically designed based on the worst-case channels. Compared to a non-interleaved FEC system, wavelength-interleaved FEC is found to be able to increase the tolerable dispersion of the system by about 13% for double-wavelength interleaving, and 22% for four-wavelength interleaving system, based on receiver Q-factor of 17 dB, RS(255, 223) code and output BER of 10^{-12} . This brings about an increase in the maximum transmission distance, and economic benefit correspondingly.

In the design of the FEC systems with wavelength interleaving, the interleaved wavelengths need to have pulse spread difference greater than the minimum pulse spread difference shown in Figure 4-6 (b), in order to get the highest BER improvement. Once this condition is met, the output BER is determined only by the worst-case wavelength. Investigation shows that wideband systems can be expected to benefit more from the proposed wavelength interleaving system, especially in long distance (>100 km) and very high speed (≥ 40 Gb/s) transmission systems.

Since an optical system typically suffers both CD and PMD impairments (which could also benefit from wavelength-interleaved FEC, as reported in chapter 3), the interleaved wavelengths should be separated by more than the coherence bandwidth and have pulse spread difference greater than the minimum pulse spread difference shown in Figure 4-6 (b), in order for wavelength-interleaved FEC to mitigate both PMD and CD impairments at the same time.

Chapter 5

Errors-and-Erasures Decoding for Optical Systems

5.1 Introduction

A RS code concatenated with a convolutional code [52] has demonstrated about 10 dB of code gain (CG), but the required coding redundancy is 113%. For the bit-rate of 10 Gb/s or higher, redundancy of less than 25% is highly desirable. T. Mizuochi proposed a block turbo code [6] with 3-bit soft-decision and 25% redundancy to provide a 10 dB CG at a BER of 10^{-13} . However, turbo decoding operation requires channel measurement and soft iterative decoding, which are difficult to put into practice at 10 Gb/s and higher speed systems. Therefore a method to improve the FEC performance with moderate coding redundancy and low decoding complexity is desirable.

Errors-and-erasures decoding (E&ED) [104]-[107] has been studied in wireless communication systems for years. E&ED can be considered as the simplest soft-decision decoding (SDD) scheme with only two thresholds and 3 detection levels: “1”, “0” and “Erasure”. But it doesn’t need soft-in soft-out decoder used in Turbo decoders, and it can be used directly with the high-rate RS codes, hence E&ED may be more attractive for high-speed implementation.

In this chapter, it is first verified that the decoding complexity of E&ED is tolerable in optical systems. Then an E&ED scheme is proposed that works in optical systems without increasing coding redundancy and has tolerable decoding complexity, and deduce the BER models of E&ED in optical channels with PMD and CD accordingly. Furthermore, the erasure zone is optimized in two major parameters: 1). η , the normalized width of the erasure zone, 2). R_{EZ} , the width ratio of erasure zone at each side of the “0” and “1” levels. With these models and optimized erasure zone, system

performance improvement with E&ED is analyzed. At the end of this chapter, an optical receiver scheme with adaptive erasure zone is proposed.

5.2 Error-and-Erasure Decoding Algorithm

5.2.1 Binary erasure decoding algorithm

Binary erasure decoding can be performed by making small modifications to the standard decoder. The procedure of binary erasure decoding algorithm is shown as follows [5]:

1. Given a received word $\mathbf{r}$, place zeros in all erased coordinates and decode normally. Label the resulting code word as $\mathbf{c}_0$.
2. Then place ones in all erased coordinates and decode normally. Label the resulting code word as $\mathbf{c}_1$.
3. Compare $\mathbf{c}_0$ and $\mathbf{c}_1$ to $\mathbf{r}$, selecting the one that is closest in Hamming distance to $\mathbf{r}$ as the final decoded output code word.

Now let's show how this algorithm works. Firstly it is assumed that the number of errors t and erasures s caused by the channel satisfies the constraint $(2t+s) < d_{min}$. If assigning zeros to the s erased coordinates generates t_0 errors, the total number of the errors at the input of the decoder will be $t+t_0$. Else if assigning ones to the s erased coordinates generates t_1 errors, the total error number will be $t+t_1 = t+s-t_0$. Either t_0 or $s-t_0$ must be less than or equal to $s/2$. Therefore, in at least one of the decoding operations, the total number of the errors t_{total} will satisfy $2t_{total} \leq 2(t+s/2) < d_{min}$. At least step 1 or step 2 can yield the correct code word.

5.2.2 Nonbinary erasure decoding algorithm

In [5], Stephen B. Wicker has introduced Berlekamp's algorithm, Berlekamp-Massey's algorithm and Euclid's algorithm on nonbinary EOD and E&ED schemes.

In this section, Berlekamp-Massey's algorithm is taken as an example to study the complexity that E&ED adds over EOD.

Suppose that a received word has v errors and f erasures. The errors occur in coordinates $i_1, i_2, \dots, i_v$, while the erasures occur in coordinates $j_1, j_2, \dots, j_f$. In the analysis that follows the coordinates for errors and erasures are designated using the error locators $X_1 = \alpha^{i_1}, X_2 = \alpha^{i_2}, \dots, X_v = \alpha^{i_v}$ and the erasure locators $Y_1 = \alpha^{j_1}, Y_2 = \alpha^{j_2}, \dots, Y_f = \alpha^{j_f}$. The primary difference between the error locators and the erasure locators is that the values are known for the latter at the beginning of the decoding operation. The first of the two tasks for our E&ED operation is to determine the values of the error locators. The second task is to find the values $\{e_{ik}\}$ associated with the error locators and the values $\{f_{jk}\}$ associated with the erasure locators. Therefore, the additional task of E&ED over EOD is to determine the erasure values.

In order to compute the syndrome for the received word, the values have to be inserted at every coordinate where an erasure has been indicated. Obviously, if the value zero is selected for this substitution, the computation will be much simpler. Now the syndrome is a function of error/erasure polynomial instead of error polynomial in EOD:

$$S(x) = \sum_{l=1}^{2t} x^l \left(\sum_{k=1}^v e_{ik} X_k^l + \sum_{k=1}^f f_{jk} Y_k^l \right) \quad (5-1)$$

with which the key equation for E&ED is obtained:

$$\Lambda(x)\Gamma(x)[1 + S(x)] \equiv \Omega(x) \bmod x^{2t+1} \quad (5-2)$$

where $\Lambda(x)$ is the error locator polynomial, $\Omega(x)$ is the magnitude polynomial, and $\Gamma(x)$ is the erasure locator polynomial which is already decided at the beginning of decoding, and is computed using the erasure locators:

$$\Gamma(x) = \prod_{l=1}^f (1 - Y_l x) \quad (5-3)$$

Equation (5-2) can be slightly simplified by combining all information that is known at the beginning of the decoding operation into a single modified syndrome polynomial $\Xi(x)$:

$$1 + \Xi(x) \equiv \Gamma(x)[1 + S(x)] \bmod x^{2t+1} \quad (5-4)$$

Therefore the key equation (5-11) now takes the form:

$$\Lambda(x)[1 + \Xi(x)] \equiv \Omega(x) \bmod x^{2t+1} \quad (5-5)$$

which is having the same format with the key equation for EOD:

$$\Lambda(x)[1 + S(x)] \equiv \Omega(x) \bmod x^{2t+1} \quad (5-6)$$

And can be solved using either one among Berlekamp's algorithm, Berlekamp-Massey's algorithm and Euclid's algorithm.

The procedure of Berlekamp-Massey's algorithm nonbinary erasure decoding is shown as follows:

1. Compute the erasure locator polynomial $\Gamma(x)$ using the erasure information provided by the receiver.
2. Replace the erased coordinates with zeros and compute the syndrome polynomial $S(x)$.
3. Compare the modified syndrome polynomial $l\Xi(x)$ by $\Xi(x) \equiv (\Gamma(x)[1 + S(x)] - 1) \bmod x^{2t+1}$.
4. Apply the Berlekamp-Massey's algorithm to find the connection polynomial $\Lambda(x)$ for the LFSR that generates the modified syndrome coefficients
5. Find the roots of $t_i(x) = \Lambda(x)$, thus determining the error locations.
6. Determine the magnitude of the errors and erasures.

Once the error locator polynomial is known, combine it with the erasure locator polynomial to obtain a single error/erasure locator polynomial $\Psi(x)$.

$$\psi(x) = \Lambda(x)\Gamma(x) \quad (5-7)$$

Taking (5-7) into the key equation (5-2), it is found that it is in the same format as the key equation for EOD now. This means that the subsequent decoding process can be handled by normal EOD algorithm.

A modified version of the Forney algorithm [5] can then be used to compute the error and erasure values.

$$e_{ik} = \frac{-X_k \Omega(X_k^{-1})}{\psi'(X_k^{-1})}, \quad f_{ik} = \frac{-Y_k \Omega(Y_k^{-1})}{\psi'(Y_k^{-1})} \quad (5-8)$$

An error/erasure polynomial is then constructed and subtracted from the received polynomial to obtain the desired code polynomial. Note that, before the correction is performed, the received polynomial should have the same values at the erased positions as used in the computation of the syndromes.

In short, E&ED only requires slight modifications in the HDD decoder, and does not require soft-in soft-out decoding and iterative decoding, which is a must for some SDD. Therefore, the decoding complexity of E&ED is slightly higher than HDD but is much more tolerable in high-speed optical systems than SDD.

5.3 E&ED Scheme in Optical FEC Systems

Figure 5-1 shows the block diagram of optical FEC communication systems with HDD or proposed E&ED. Firstly, data is encoded by Reed-Solomon encoder and modulated by On-Off Keying (OOK) modulator at the transmitter. After fiber transmission, the received signal is demodulated at OOK demodulator to prepare for FEC decoding. In the traditional HDD scheme, the received channel bits are decided according to a single threshold, called hard-detection threshold. Depending on whether the received signal level is above or below the threshold, the demodulator will output a “1” or “0” respectively. Then RS hard-decoder will execute HDD based on the “hard” input.

Figure 5-1. Block diagrams of optical communication systems with FEC and hard-decision decoding (HDD) or the proposed errors-and-erasures decoding (E&ED).

However, in some cases, the received signal level is very near to the detection threshold. Hard detection will force a decision that may be incorrect in this case, while on the other hand, soft-decision receiver uses an expanded selection of quantity levels to report to the decoder the quality of the received bits/symbols. Many soft-decision receivers use 3 bits to quantify the received signal levels into 8 levels. In E&ED, the receiver uses 1 additional level “Erasures” to indicate a received signal whose corresponding symbol value is not clearly 1 or 0. Hence it is less complex than SDD and more attractive for high-speed implementation.

In our proposed E&ED scheme, the hard-detection threshold extends into an “erasure zone”, where hard detection errors are most likely to happen. If the received signal falls outside the erasure zone, it will be decided to be a “0” or “1” respectively.

Otherwise, it will be treated as an erasure bit. Since RS decoder operates on symbols rather than bits, this is followed by another decision device to convert “bit erasure” patterns to “symbol erasure”. Finally, the RS decoder will perform E&ED.

A RS(n, k) code has a minimum distance $d_{min}=(n-k+1)$, and can correctly decode all received words containing t errors and s erasures so long as $(2t+s) \leq d_{min}$. For a RS(255, 239) code, when $s=0$, it is hard-decision decoding can correct 8 symbol errors; when $t=0$, it will do full erasure decoding and can correct 16 symbol erasures. Therefore, it is clear that E&ED is able to greatly improve the correcting capability of RS code over HDD by correcting erasures, when the number of erasures satisfies the restriction of $(2t+s) \leq d_{min}$. However, system performance might be degraded when the erasure zone is chosen too wide that induces more erasures than d_{min} . Hence a properly defined erasure zone is necessary. This issue will be discussed in a later section in this chapter.

5.4 System Model

5.4.1 Channel Bit Error Rate and Bit Erasure Rate

To clarify the bit detection and bit erasure generation before E&ED, which is related to our model of the channel bit error rate and bit erasure rate, I firstly show the proposed erasure zone definition in eye diagram in Figure 5-2.

The erasure zone extends from the HDD threshold to both sides, and is bounded by the amplitude level I_{Th0} and I_{Th1} at side “0” and side “1” respectively. The received data is detected by the two thresholds of erasure zone. If the detected signal value is lower than I_{Th0} , the output is a “0”. Otherwise the detected signal value is higher than I_{Th0} . If it is also higher than I_{Th1} , the output is a “1”. If it is also lower than I_{Th1} , the output is a “bit erasure”. Then all the detected bits, “0”, “1” and “bit erasures”, will be formatted into symbols according to the FEC code format, preparing to be put into E&ED decoder operating decoding.

Figure 5-2. Proposed erasure zone definition in the eye diagram of an optical channel.

To investigate the error correction capability of the proposed FEC system with E&ED, the channel noise's effect on the channel BER are first considered. Channel BER model for the system with E&ED and PMD impairment is derived.

5.4.1.1 General Form

Note that $I_0=0$ when OOK modulation is assumed. Therefore, the HDD threshold $I_{Th} = (\sigma_1 I_0 + \sigma_0 I_1) / (\sigma_0 + \sigma_1)$ is considered to be $I_{Th} = \sigma_0 I_1 / (\sigma_0 + \sigma_1)$ for $I_0=0$. It is the threshold value used by the hard decision detection device, where σ_0^2 and σ_1^2 are the variances corresponding to the received bits “0” and “1”, respectively. The erasure zone extends from the HDD threshold to both sides, and is bounded by the amplitude level I_{Th0} at “0” side and I_{Th1} at “1” side, where I_{Th0} and I_{Th1} are the bounds of the erasure zone on the “0” and “1”.

FEC performance with E&ED greatly depends on precision of the erasure zone, which is determined by two key parameters:

a) η , the normalized width of the erasure zone:

$$\eta = (I_{Th1} - I_{Th0}) / I_1, \quad 0 < \eta < 1 \quad (5-9)$$

b) R_{EZ} , the width ratio of the erasure zone at “0” side to that at “1” side:

$$R_{EZ} = (I_{Th} - I_{Th0}) / (I_{Th1} - I_{Th}) \quad (5-10)$$

Therefore, the width of the erasure zone is:

$$\Delta I_e = \Delta I_1 + \Delta I_0 = (1 + R_{EZ}) \cdot \Delta I_1 = I_{Th1} - I_{Th0} = \eta \cdot I_1 \quad (5-11)$$

where ΔI_1 and ΔI_0 is the erasure zone at side “1” and side “0” respectively.

The two thresholds of E&ED, amplitude level I_{Th0} and I_{Th1} at side “0” and side “1” are respectively defined as:

$$\begin{cases} I_{Th1} = I_{Th} + \Delta I_1 = I_{Th} + \eta \cdot I_1 \cdot \frac{1}{1 + R_{EZ}} \\ I_{Th0} = I_{Th} - \Delta I_0 = I_{Th} - \eta \cdot I_1 \cdot \frac{R_{EZ}}{1 + R_{EZ}} \end{cases} \quad (5-12)$$

To simplify the equations, another variable y should be introduced:

$$R_{EZ} = \frac{\Delta I_0}{\Delta I_1} = \frac{1 + p - y}{y} \quad (5-13)$$

where $0 < y < 1 + p$, ratio $p = \sigma_0 / \sigma_1$.

Therefore the reverse function of (5-12) can be obtained:

$$y = \frac{1 + p}{1 + R_{EZ}} \quad (5-14)$$

According to equation (5-13), equation (5-12) can be written as:

$$\begin{cases} I_{Th1} = I_{Th} + \eta \cdot I_1 \cdot \frac{1}{1 + R_{EZ}} = I_{Th} + \eta \cdot I_1 \cdot \left(\frac{y \sigma_1}{\sigma_1 + \sigma_0} \right) \\ I_{Th0} = I_{Th} - \eta \cdot I_1 \cdot \frac{R_{EZ}}{1 + R_{EZ}} = I_{Th} - \eta \cdot I_1 \cdot \frac{(1 + p - y) \cdot \sigma_1}{\sigma_1 + \sigma_0} \end{cases} \quad (5-15)$$

Figure 5-3. Probability density functions of detected signal value of HDD systems.

Figure 5-4. Probability density functions for detected signal value of E&ED systems.

Assuming the received noise of an optical receiver follows a Gaussian distribution. Figure 5-3 and 5-4 shows the definition of probability density functions for detected signal values of HDD systems and E&ED systems. With this noise distribution, based

on upper equations (5-12) and (5-15), the conditional probabilities of E&ED systems can be shown to be:

$$\begin{aligned}
 P(I < I_{Th0} | 1) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th0}}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th} + \eta \cdot I_1 \cdot \frac{(1+p-y) \cdot \sigma_1}{\sigma_1 + \sigma_0}}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_1 \cdot \left(\frac{\sigma_0}{\sigma_1 + \sigma_0} \right) + \eta \cdot I_1 \cdot \frac{(1+p-y) \cdot \sigma_1}{\sigma_1 + \sigma_0}}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 (\sigma_1 + \sigma_0 - \sigma_0 + \eta \cdot \sigma_1 \cdot (1+p-y))}{\sigma_1 \sqrt{2} (\sigma_1 + \sigma_0)} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q(\sigma_1 + \eta \cdot \sigma_1 \cdot (1+p-y))}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1+p-y)) \right); \tag{5-16a}
 \end{aligned}$$

$$\begin{aligned}
 P(I > I_{Th1} | 0) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th1} - I_0}{\sigma_0 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{(I_{Th} + \frac{y}{1+p} \Delta I_e) - I_0}{\sigma_0 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 \cdot \left(\frac{\sigma_0}{\sigma_1 + \sigma_0} \right) + \eta \cdot I_1 \cdot \frac{y \cdot \sigma_1}{\sigma_1 + \sigma_0} - I_0}{\sigma_0 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 (\sigma_0 + \eta \cdot \sigma_1 \cdot y)}{\sigma_0 \sqrt{2} (\sigma_1 + \sigma_0)} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q(\sigma_0 + \eta \cdot \sigma_1 \cdot y)}{\sigma_0 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot \left(1 + \frac{\eta \cdot y}{p} \right) \right); \tag{5-16b}
 \end{aligned}$$

$$\begin{aligned}
 P(I_{Th0} < I < I_{Th1} | 1) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th1}}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th0}}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th} - \eta \cdot I_1 \cdot \frac{y \cdot \sigma_1}{\sigma_1 + \sigma_0}}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1 + p - y)) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_1 \cdot \left(\frac{\sigma_0}{\sigma_1 + \sigma_0} \right) - \eta \cdot I_1 \cdot \frac{y \cdot \sigma_1}{\sigma_1 + \sigma_0}}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1 + p - y)) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 (\sigma_1 + \sigma_0 - \sigma_0 - \eta \cdot \sigma_1 \cdot y)}{\sigma_1 \sqrt{2} (\sigma_1 + \sigma_0)} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1 + p - y)) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q \sigma_1 - \eta \cdot \sigma_1 \cdot y}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1 + p - y)) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 - \eta \cdot y) \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1 + p - y)) \right); \tag{5-16c}
 \end{aligned}$$

$$\begin{aligned}
 P(I_{Th0} < I < I_{Th1} | 0) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th0} - I_0}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th1} - I_0}{\sigma_0 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th} - \eta \cdot I_1 \cdot \frac{(1 + p - y) \cdot \sigma_1}{\sigma_1 + \sigma_0} - I_0}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot \left(1 + \frac{\eta \cdot y}{p} \right) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 \cdot \left(\frac{\sigma_0}{\sigma_1 + \sigma_0} \right) - \eta \cdot I_1 \cdot \frac{(1 + p - y) \cdot \sigma_1}{\sigma_1 + \sigma_0} - I_0}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot \left(1 + \frac{\eta \cdot y}{p} \right) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 (\sigma_0 - \eta \cdot \sigma_1 \cdot (1 + p - y))}{\sigma_0 \sqrt{2} (\sigma_1 + \sigma_0)} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot \left(1 + \frac{\eta \cdot y}{p} \right) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q \sigma_0 - \eta \cdot \sigma_1 \cdot (1 + p - y)}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot \left(1 + \frac{\eta \cdot y}{p} \right) \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot \left(1 - \eta \cdot \frac{(1 + p - y)}{p} \right) \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot \left(1 + \frac{\eta \cdot y}{p} \right) \right); \tag{5-16d}
 \end{aligned}$$

where $Q = \frac{I_1}{\sigma_1 + \sigma_0}$.

When $I < I_{Th0}$ or $I > I_{Th1}$, the received signal is decided to be 0 or 1; when $I_{Th0} < I < I_{Th1}$, received signal is flagged as a bit erasure. Hence, the channel bit erasure probability P_{B_era} can be written as:

$$P_{B_era} = P(1)P(I_{Th0} < I < I_{Th1} | 1) + P(0)P(I_{Th0} < I < I_{Th1} | 0) \quad (5-17)$$

where $P(0)$ or $P(1)$ is the probability that 0 or 1 was sent. Similarly, the channel BER P_{B_err} can be written as:

$$P_{B_err} = P(0)P(I > I_{Th1} | 0) + P(1)P(I < I_{Th0} | 1) \quad (5-18)$$

Carrying (5-16) into (5-17) and (5-18), and assuming $P(1) = P(0) = 1/2$, P_{B_era} can be estimated by:

$$P_{B_era} = \frac{1}{2} \left[\frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 - \eta \cdot y) \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1 + p - y)) \right) \right] \\ + \frac{1}{2} \left[\frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 - \eta \cdot \frac{(1 + p - y)}{p}) \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \frac{\eta \cdot y}{p}) \right) \right]; \quad (5-19)$$

and P_{B_err} can be estimated by:

$$P_{B_err} = \frac{1}{4} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \eta \cdot (1 + p - y)) \right) + \frac{1}{4} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \frac{\eta \cdot y}{p}) \right) \quad (5-20)$$

5.4.1.2 Simplified Form

Session 5.4.1.1 derives the general form of channel BER and bit erasure rate. In this session, the general form into a special form at $R_{EZ} = p$ will be simplified. It is proved to be the optimum R_{EZ} value in the following section 5.6, when the erasure zone optimizing is discussed.

In practice, σ_0^2 and σ_I^2 are usually not the same in optical systems. Therefore it is reasonable to define $R_{EZ} = \Delta I_0 / \Delta I_I = p = \sigma_0 / \sigma_I$, which leads equation (5-14) to $y = 1$. Hence I_{Th0} and I_{Th1} in equations (5-11) and (5-14) can be simplified to be:

$$\begin{cases} I_{Th1} = I_{Th} + \Delta I_e \cdot 1 / (1 + p) = I_{Th} + \Delta I_1 = I_{Th} + \eta \cdot I_1 \cdot 1 / (1 + p) \\ I_{Th0} = I_{Th} - \Delta I_e \cdot p / (1 + p) = I_{Th} - \Delta I_0 = I_{Th} - \eta \cdot I_1 \cdot p / (1 + p) \end{cases} \quad (5-21)$$

Assuming the received noise of an optical receiver follows a Gaussian distribution. Based on the erasure zone specified in equation (5-21), the conditional probabilities for E&ED systems can be modified to be:

$$P(I < I_{Th0} | 1) = \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th0}}{\sigma_1 \sqrt{2}} \right) = \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (1 + p \cdot \eta)}{\sqrt{2}} \right); \quad (5-22a)$$

$$P(I > I_{Th1} | 0) = \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th1} - I_0}{\sigma_0 \sqrt{2}} \right) = \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \frac{\eta}{p}) \right); \quad (5-22b)$$

$$\begin{aligned} P(I_{Th0} < I < I_{Th1} | 1) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th1}}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th0}}{\sigma_1 \sqrt{2}} \right) \\ &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (1 - \eta)}{\sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (1 + p \cdot \eta)}{\sqrt{2}} \right); \end{aligned} \quad (5-22c)$$

$$\begin{aligned} P(I_{Th0} < I < I_{Th1} | 0) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th0} - I_0}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th1} - I_0}{\sigma_0 \sqrt{2}} \right); \\ &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 - \eta) \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \frac{\eta}{p}) \right); \end{aligned} \quad (5-22d)$$

Carrying (5-22) into (5-17) and (5-18), and assuming $P(1,0)=1/2$, P_{B_era} can be estimated by:

$$P_{B_era} = \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (1 - \eta)}{\sqrt{2}} \right) - \frac{1}{4} \operatorname{erfc} \left(\frac{Q \cdot (1 + p \cdot \eta)}{\sqrt{2}} \right) - \frac{1}{4} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \frac{\eta}{p}) \right) \quad (5-23)$$

and P_{B_err} can be estimated by:

$$P_{B_err} = \frac{1}{4} \operatorname{erfc} \left(\frac{Q \cdot (1 + p \cdot \eta)}{\sqrt{2}} \right) + \frac{1}{4} \operatorname{erfc} \left(\frac{Q}{\sqrt{2}} \cdot (1 + \frac{\eta}{p}) \right) \quad (5-24)$$

5.4.2 Symbol Error Rate and Symbol Erasure Rate

Since the RS decoder operates on symbols rather than bits, the demodulated bits need to be converted to symbols. Similarly the “bit erasures” need to be converted to “symbol erasures” by another decision device before E&ED decoder. A RS(n, k) code can correct $(n-k)/2$ errors or $(n-k)$ erasures. Therefore, a symbol with both bit

erasures and bit errors should preferentially be flagged as a symbol erasure, so that the FEC correcting ability can be best exploited. In this thesis, a symbol erasure is considered to be created when a received symbol has at least a bit erasure. Accordingly, the probability P_{S_era} that a symbol erasure will occur can be estimated as:

$$P_{S_era} = \sum_{s=1}^m \binom{m}{s} (P_{B_era})^s (1 - P_{B_era})^{m-s} \quad (5-25)$$

Only if every bit in a symbol is correct will the symbol be correct, otherwise a symbol error or erasure will happen. The probability P_{S_cor} that the symbol is correctly received can be written to be:

$$P_{S_cor} = (1 - P_{B_err} - P_{B_era})^m \quad (5-26)$$

where m is related to the code length n by $n=(2^m-1)$ as being presented in prior part.

Finally, the symbol-error rate P_{S_err} before FEC decoding is:

$$P_{S_err} = 1 - P_{S_era} - P_{S_cor} \quad (5-27)$$

5.4.3 Output BER of E&ED

In this section, the relationship of the required Q-factor and output BER for a communication system with E&ED is derived.

Formula I:

According to [5], the decoded SER P_{S_ee} of RS(n, k) code for E&ED is:

$$P_{S_ee} \approx \sum_{e=0}^n \binom{n}{e} P_{S_err}^e \sum_{\substack{s=n-k+1-2e \\ s>0}}^{n-e} \frac{e + \frac{s}{2}}{n} \cdot \binom{n-e}{s} P_{S_era}^s (1 - P_{S_err} - P_{S_era})^{n-e-s} \quad (5-28)$$

where e and s are the numbers of erroneous and erased symbols per codeword, P_{S_err} is the channel symbol error probability, and P_{S_era} is the symbol erasure probability.

The corresponding output BER P_{B_ee} of E&ED is related to its decoding SER by [10]:

$$P_{B_ee} = \frac{2^{m-1}}{2^m - 1} P_{S_ee} \approx \frac{1}{2} P_{S_ee} \quad (5-29)$$

since $m=8$ for RS(255, 239).

Formula II:

Equation (5-28) can be used to calculate the decoded SER if the codewords are not long,. If the RS codewords are long with ideal time interleaving, an alternative model for the decoded SER with E&ED from [56] can be considered and compared against function (14) here. Let A_i , $i = 0, 1, \dots, n$ be:

$$A_i = \begin{cases} 0, & \text{if the demodulated code symbol before FEC is correct} \\ 1, & \text{if the demodulated code symbol before FEC is erased} \\ 2, & \text{if the demodulated code symbol before FEC is in error} \end{cases} \quad (5-30)$$

With (5-30), it can be shown that a codeword can be decoded correctly using E&ED when the sum of A_i to all the n symbols in the codeword, $\sum_{i=1}^n A_i$, does not exceed $(n-k)$.

Therefore, the codeword decoding error probability P_{C_ee} can be shown to be:

$$P_{C_ee} = \sum_{j=n-k+1}^n P\left(\sum_{i=1}^n A_i = j\right) \quad (5-31)$$

The well-known Gaussian approximation can be invoked since $\{A_i\}$, $i = 0, 1, \dots, n$ can be assumed to be independent, so that the computation and the subsequent optimizing can be simplified. According to the central limit theorem, the distribution of $\sum_{i=1}^n A_i$ approaches a Gaussian distribution with a mean value [56]:

$$M_c = E\left[\sum_{i=1}^n A_i\right] = n(P_{S_era} + 2P_{S_err}) \quad (5-32)$$

and variance:

$$\begin{aligned}
 \sigma_c^2 &= \text{Var} \left[\sum_{i=1}^n A_i \right] \\
 &= E \left[\left(\sum_{i=1}^n A_i \right)^2 \right] - E^2 \left[\sum_{i=1}^n A_i \right] \\
 &= n \left[1 - P_{S_era} - (1 - P_{S_era} - 2P_{S_err})^2 \right]
 \end{aligned} \tag{5-33}$$

Consequently the codeword decoding error probability P_{C_ee} can be approximated as:

$$\begin{aligned}
 P_{C_ee} &= \int_x^\infty \frac{1}{\sqrt{2\pi} \cdot \sigma_c} \exp \left(-\frac{(t - M_c)^2}{\sigma_c^2} \right) dt \\
 &= Q(X_1) - Q(X_2)
 \end{aligned} \tag{5-34}$$

where $Q(x)$ is the Gaussian Q-function:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp \left(-\frac{t^2}{2} \right) dt \tag{5-35}$$

and

$$X_1 = \frac{\sqrt{n}(1 - R - P_{S_era} - 2P_{S_err})}{\sqrt{(1 - P_{S_era} - (1 - P_{S_era} - 2P_{S_err})^2)}} \tag{5-36}$$

$$X_2 = \frac{\sqrt{n}(1 - P_{S_era} - 2P_{S_err})}{\sqrt{(1 - P_{S_era} - (1 - P_{S_era} - 2P_{S_err})^2)}} \tag{5-37}$$

where $R = k/n$ is the code rate.

The corresponding decoding word error rate (WER) of an error-and-erasure system P_{C_ee} is related to its decoding SER by:

$$P_{S_ee} = 1 - \left(1 - P_{C_ee} \right)^{\frac{1}{n}} \approx \frac{P_{C_ee}}{n} \tag{5-38}$$

And the decoding BER P_{B_ee} of an error-and-erasure decoding FEC system is related to its decoding SER by equation (5-29).

Figure 5-5. Comparison of function (5-28), (5-38) and simulation results for RS(255, 239) code on decoded SER of E&ED at different input symbol erasure rate, and $P_{S_err}=0$.

MATLAB is used to run the simulation of a decoding process by RS(255, 239) code with E&ED. In Figure 5-5, the two decoded SER formulas with E&ED described above, (5-28) and (5-38) are compared with simulation results at different input symbol erasure rate. For simplicity, it is assumed that input error rate is zero $P_{S_err}=0$, which means that it is erasure only decoding in this comparison. The results shows that, when output SER is lower than 10^{-3} , (5-38) matches the simulation result better. In optical systems, output SER of interest is lower than 10^{-9} typically, therefore (5-38) is chosen for further investigation.

5.5 Optimization of Erasure Zone

In section 5.4, it has been indicated that FEC performance with E&ED depends on precision of the erasure zone, which is parameterized by two key factors 1). η , the normalized width parameter of the erasure zone, 2). R_{EZ} , width ratio of the erasure zone at each side of “0” and “1”. In this section, optimizing the erasure zone parameters η and R_{EZ} will be investigated.

Figure 5-6. Error correction capabilities of RS(255, 239) code with E&ED, (a) at different η (eta), (b) at different R_{EZ} . $Q=9.5\text{ dB}$, $\Delta I_{Th}=0.3$.

Figure 5-6(a) shows the error correction capabilities of RS(255, 239) code with E&ED at different η . According to Figure 5-6(a): When the width of erasure zone is zero ($\eta=0\%$), the erasure zone shrinks back to the hard-decision threshold line and E&ED is equivalent to HDD; when η is around 4%, with E&ED, RS(255, 239) can achieve the highest CG; when η is higher than 8%, RS(255, 239) with E&ED has lower error correction ability than the same code with HDD. These observations show that if the erasure zone is too narrow, it will not be able to flag out sufficient erasures;

if the erasure zone is too wide, it will get more than $(n-k)$ erased symbols which exceed the code's error correcting capability.

Figure 5-6(b) shows the error correction capabilities of RS(255, 239) code with E&ED at different R_{EZ} . According to Figure 5-6(b), $R_{EZ}=p$ is optimum for E&ED at the output BER between 10^{-10} and 10^{-13} . This is the reason why $R_{EZ}=p$ is chosen in session 5.4.1.2 simplified form of channel BER expression with E&ED. Therefore, the two key factors of erasure zone of RS(255, 239) is optimized to be: $\eta = 4\%$ and $R_{EZ}=p=0.3/0.7$.

For other M-ary Reed-Solomon codes with different code rates, the two major parameters of optimized erasure zone are similarly found to be: $R_{EZ}=p$, and η as per Table 5-1.

Table 5-1. Optimized η of erasure zone for different M-ary Reed-Solomon codes with different code rate R .

η	$M=16$	$M=32$	$M=64$	$M=128$	$M=256$
$R=0.02$	11%	15%	16%	20%	22%
$R=0.1$	11%	12%	14%	15%	17%
$R=0.2$	9%	11%	13%	13%	13%
$R=0.3$	9%	9%	10%	10%	11%
$R=0.4$	8%	8%	9%	9%	9%
$R=0.5$	7%	7%	7%	8%	7%
$R=0.6$	6%	6%	7%	7%	6%
$R=0.7$	5%	6%	6%	6%	5%
$R=0.8$	4%	5%	5%	5%	5%
$R=0.9$	3%	4%	4%	4%	4%
$R=0.98$	2%	2%	2%	4%	3%

5.6 E&ED Performance and Analysis

Figure 5-7. Relationship of output BER and receiver Q-factor without FEC, with FEC based on HDD and with FEC based on E&ED. HDD threshold is 0.3. E&ED is set as: $\eta = 4\%$ and $R_{EZ} = p$.

Figure 5-7 shows the comparison of relationship of output BER and receiver Q-factor without FEC, with FEC based on HDD and with FEC based on E&ED. Without coding, Q is 17 dB to ensure the output BER lower than 10^{-12} . With HDD, RS(255, 239) can provide 5.8 dB CG. Based on the same RS(255, 239) code, our proposed E&ED can provide about 7.1 dB CG, so the additional CG over HDD is 1.3 dB. Similarly, based on RS(255, 223) code, our proposed E&ED can provide a CG of 9.5 dB at the system-required receiver Q-factor to get an output BER of 10^{-12} . The additional CG over HDD is 1.2 dB under the same conditions.

(a)

(b)

(c)

(d)

(e)

Figure 5-8. Required Q-factor to get an output BER of 10^{-12} with HDD and with E&ED changing with code rate R , when code length is different at (a). $q=16$, (b). $q=32$, (c). $q=64$, (d). $q=128$, (e). $q=256$.

Figure 5-8 gives the plot of required Q-factor to get an output BER of 10^{-12} with HDD and E&ED with changing code rate R , when the code length is different at (a). $q=16$, (b). $q=32$, (c). $q=64$, (d). $q=128$, (e). $q=256$. The figure shows that E&ED provides higher improvement on system performance on shorter codes or code with higher code rate R . However, longer codes have higher correcting capability on burst errors than shorter codes with same code rate, and codes with lower code rate can correct more errors yet add more redundancy to cost system bandwidth. Therefore, to choose an appropriate code, one needs to balance the tradeoff between error correcting ability and system burden. Considering the tolerance of high-speed optical link, the study will be focused on RS(255, 239) and RS(255, 223).

5.7 E&ED to mitigate PMD Impairment

5.7.1 System Model of PMD Impaired Systems with FEC and E&ED

According to previous definition in Section 3.2.2, the levels $I_{0/1}^{sup/med/inf}$ can be modeled by: $I_0^a = I_1(1 - g^a(\Delta\tau, tr))$, $I_1^a = I_1 g^a(\Delta\tau, tr)$, where $a=sup, med, inf$; and the received levels are given by g^a [50] for optical system impaired by PMD. Similarly to the derivation of formula (5-16) in session 5.4.1, the probabilities for the fiber optic system with PMD and E&ED can be obtained in the following ways:

$$\begin{aligned}
 P(I^a < I_{th0} | 1) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{th0}}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - (I_{th} - p \cdot \Delta I_e)}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{(I_1 - I_{th}) \cdot (1 + p \cdot \eta)}{\sigma_1 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (g \cdot (1 + p) - p) \cdot (1 + p \cdot \eta)}{\sqrt{2}} \right);
 \end{aligned} \tag{5-39a}$$

$$\begin{aligned}
 P(I^a > I_{Th1} | 0) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th1} - I_0}{\sigma_0 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{(I_{Th} + \Delta I_e) - I_0}{\sigma_0 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{(I_{Th} - I_0) + \eta \cdot (I_1 - I_{Th})}{\sigma_0 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (g \cdot (1+p) - 1 + \eta)}{p \sqrt{2}} \right);
 \end{aligned} \tag{5-39b}$$

$$\begin{aligned}
 P(I_{Th0} < I^a < I_{Th1} | 1) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th1}}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - I_{Th0}}{\sigma_1 \sqrt{2}} \right) \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - (I_{Th} + \Delta I_e)}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{I_1 - (I_{Th} - p \cdot \Delta I_e)}{\sigma_1 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{(I_1 - I_{Th}) \cdot (1 - \eta)}{\sigma_1 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{(I_1 - I_{Th}) \cdot (1 + p \cdot \eta)}{\sigma_1 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (g \cdot (1+p) - p) \cdot (1 - \eta)}{\sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (g \cdot (1+p) - p) \cdot (1 + p \cdot \eta)}{\sqrt{2}} \right);
 \end{aligned} \tag{5-39c}$$

$$\begin{aligned}
 P(I_{Th0} < I^a < I_{Th1} | 0) &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th0} - I_0}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th1} - I_0}{\sigma_0 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{(I_{Th} - p \cdot \Delta I_e) - I_0}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{(I_{Th} + \Delta I_e) - I_0}{\sigma_0 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{I_{Th} - I_0 - p \cdot \eta \cdot (I_1 - I_{Th})}{\sigma_0 \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{(I_{Th} - I_0) + \eta \cdot (I_1 - I_{Th})}{\sigma_0 \sqrt{2}} \right); \\
 &= \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (g \cdot (1+p) - 1 - p \cdot \eta)}{p \sqrt{2}} \right) - \frac{1}{2} \operatorname{erfc} \left(\frac{Q \cdot (g \cdot (1+p) - 1 + \eta)}{p \sqrt{2}} \right);
 \end{aligned} \tag{5-39d}$$

Taking into account that $P(I_{0/I}^{\sup})=1/8$, $P(I_{0/I}^{\text{med}})=1/4$, $P(I_{0/I}^{\inf})=1/8$, the instantaneous probability P_{B_era} that the received signal is flagged as a bit erasure can be calculated as:

$$\begin{aligned}
 P_{B_era} &= P(I_1^{\sup})P(erasure | I_1^{\sup}) + P(I_1^{\text{med}})P(erasure | I_1^{\text{med}}) + P(I_1^{\inf})P(erasure | I_1^{\inf}) \\
 &\quad + P(I_0^{\sup})P(erasure | I_0^{\sup}) + P(I_0^{\text{med}})P(erasure | I_0^{\text{med}}) + P(I_0^{\inf})P(erasure | I_0^{\inf}) \\
 &= \frac{1}{8}\text{erfc}\left(\frac{Q \cdot (1-\eta)}{\sqrt{2}}\right) - \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (1+p \cdot \eta)}{\sqrt{2}}\right) - \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (p+\eta)}{p\sqrt{2}}\right) \\
 &\quad + \frac{1}{8}\text{erfc}\left(\frac{Q \cdot (g^{\text{med}} \cdot (1+p) - p) \cdot (1-\eta)}{\sqrt{2}}\right) - \frac{1}{8}\text{erfc}\left(\frac{Q \cdot (g^{\text{med}} \cdot (1+p) - p) \cdot (1+p \cdot \eta)}{\sqrt{2}}\right) \\
 &\quad + \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (g^{\inf} \cdot (1+p) - p) \cdot (1-\eta)}{\sqrt{2}}\right) - \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (g^{\inf} \cdot (1+p) - p) \cdot (1+p \cdot \eta)}{\sqrt{2}}\right) \\
 &\quad + \frac{1}{8}\text{erfc}\left(\frac{Q \cdot (g^{\text{med}} \cdot (1+p) - 1 - p \cdot \eta)}{p\sqrt{2}}\right) - \frac{1}{8}\text{erfc}\left(\frac{Q \cdot (g^{\text{med}} \cdot (1+p) - 1 + \eta)}{p\sqrt{2}}\right) \\
 &\quad + \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (g^{\inf} \cdot (1+p) - 1 - p \cdot \eta)}{p\sqrt{2}}\right) - \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (g^{\inf} \cdot (1+p) - 1 + \eta)}{p\sqrt{2}}\right)
 \end{aligned} \tag{5-40}$$

and the channel BER P_{B_err} can be calculated as:

$$\begin{aligned}
 P_{B_err} &= P(I_1^{\sup})P(0 | I_1^{\sup}) + P(I_1^{\text{med}})P(0 | I_1^{\text{med}}) + P(I_1^{\inf})P(0 | I_1^{\inf}) \\
 &\quad + P(I_0^{\sup})P(1 | I_0^{\sup}) + P(I_0^{\text{med}})P(1 | I_0^{\text{med}}) + P(I_0^{\inf})P(1 | I_0^{\inf}) \\
 &= \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (1+p \cdot \eta)}{\sqrt{2}}\right) + \frac{1}{8}\text{erfc}\left(\frac{Q \cdot (g^{\text{med}} \cdot (1+p) - p) \cdot (1+p \cdot \eta)}{\sqrt{2}}\right) \\
 &\quad + \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (g^{\inf} \cdot (1+p) - p) \cdot (1+p \cdot \eta)}{\sqrt{2}}\right) + \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (p+\eta)}{p\sqrt{2}}\right) \\
 &\quad + \frac{1}{8}\text{erfc}\left(\frac{Q \cdot (g^{\text{med}} \cdot (1+p) - 1 + \eta)}{p\sqrt{2}}\right) + \frac{1}{16}\text{erfc}\left(\frac{Q \cdot (g^{\inf} \cdot (1+p) - 1 + \eta)}{p\sqrt{2}}\right)
 \end{aligned} \tag{5-41}$$

5.7.2 Formulas of FEC Performance Analysis with E&ED and PMD

At the receiver, since the DGD is a random variable, the overall channel bit error probability of the optical system without FEC should be the statistical average of $P_{B_err}(Q, \Delta\tau, \eta)$ over the probability density function of DGD, $M_{<\Delta\tau>}(\Delta\tau)$ shown in 3.2.1:

$$\overline{P_{B_err}}(<\Delta\tau>, Q, \eta) = \int_0^\infty M_{<\Delta\tau>}(\Delta\tau) \cdot P_{B_err}(Q, \Delta\tau, \eta) d(\Delta\tau) \tag{5-42}$$

and, the average probability to flag a bit erasure should also be the statistical average of $P_{B_era}(Q, \Delta\tau, \eta)$ over $M_{<\Delta\tau>}(\Delta\tau)$:

$$\overline{P_{B_era}}(<\Delta\tau>, Q, \eta) = \int_0^\infty M_{<\Delta\tau>}(\Delta\tau) \cdot P_{B_era}(Q, \Delta\tau, \eta) d(\Delta\tau) \quad (5-43)$$

where $\Delta\tau$ is the T_b normalized DGD, and $<\Delta\tau>$ is the T_b normalized mean DGD, $M_{<\Delta\tau>}(\Delta\tau)$ is the probability density function of DGD, $P_{B_err}(Q, \Delta\tau, \eta)$ is the instantaneous BER and $P_{B_era}(Q, \Delta\tau, \eta)$ is the instantaneous bit erasure probability.

With ideal time interleaving, each bit in the FEC with E&ED has an independent average BER $\overline{P_{B_err}}(<\Delta\tau>, Q, \eta)$ given by (5-42) and an independent average bit erasure probability $\overline{P_{B_era}}(<\Delta\tau>, Q, \eta)$ given by (5-43). According to the E&ED performance formula (5-29), the resulting BER with FEC and E&ED can be shown to be:

$$P_{B_ee}' = P_{B_ee}(\overline{P_{B_err}}(<\Delta\tau>, Q, \eta), \overline{P_{B_era}}(<\Delta\tau>, Q, \eta)) \quad (5-44)$$

With no interleaving, if the PMD fading time is very long, the error probability of every bit in a received FEC frame will be the same. Thus, the overall decoded BER of the system can be obtained by averaging P_{B_ee} throughout all possible states of DGD, as shown below:

$$\overline{P_{B_ee}} = \int_0^\infty M_{<\Delta\tau>}(\Delta\tau) \cdot P_{B_ee}(P_{S_err}(Q, \Delta\tau, \eta), P_{S_era}(Q, \Delta\tau, \eta)) d(\Delta\tau) \quad (5-45)$$

5.7.3 Results and Data Analysis

The Reed Solomon (255, 239) code is a popular FEC code used in optical system, and was chosen as the FEC code when the wavelength interleaving on PMD mitigation was studied in Chapter 3. To fairly compare the performance of E&ED with that of HDD with wavelength interleaving on PMD mitigation, RS(255, 239) is chosen to be the FEC code in this section as well.

FEC performance with E&ED greatly depends on precision of the erasure zone, which is defined by η and R_{EZ} . Numerical simulation results in section 5.5 show that $\eta=4\%$ and $R_{EZ}=p=\sigma_0/\sigma_1$ is the optimum for RS(255, 239) code. Hence, in this section, $\eta=4\%$ and $R_{EZ}=p$ is chosen. When the received bits “0” and “1” have the same variances, $\sigma_0=\sigma_1$, the HDD threshold I_{Th} will be 0.5 and $R_{EZ}=p=1$.

Figure 5-9. Required receiver Q factor to achieve an average output BER of 10^{-12} . Mean DGD in this figure is mean DGD normalized by T_b , $p=1$, normalized rise/fall time $tr=0.5$, E&ED is set as: $\eta=4\%$ and $R_{EZ}=p$.

5.7.3.1 Average BER

Figure 5-9 shows the comparison of the required receiver Q factor to achieve an average output BER of 10^{-12} for HDD systems with and without double-wavelength interleaving and E&ED system without wavelength interleaving.

Comparing the uncoded performance line (line 1) and the non-interleaved HDD performance line (line 2), the latter provides 5.8 dB improvement at low DGD values. But when the normalized mean DGD is higher than 0.19, FEC without wavelength interleaving no longer provides any net coding gain, as a result of the 7% bit rate expansion due to FEC coding, and uncorrectable error burst due to severe PMD.

Comparing the HDD performance line of double-wavelength interleaved system (line 3) with line 1 and 2, the double-wavelength interleaved system with RS(255, 239) code and HDD can provide over 5.8 dB coding gain at all PMD levels, since their performance lines never merge with the uncoded performance line 1. Here, the two interleaved wavelengths should be separated by more than the coherence bandwidth of the optical channel.

Compare the E&ED performance line (line 4) in single wavelength system with the previous three lines: *a)* the high DGD portion line 4 almost merges with line 3, which shows that E&ED without wavelength interleaving can improve system PMD tolerance by the same value as double-wavelength interleaving. EAED also extends the tolerable PMD level from $0.15T_b$ to $0.172T_b$, by $0.022T_b$ and 15%, and increases the maximum transmission distance of the optical fiber link by about 39%. *b)* At low DGD values, the required receiver Q-factor is 1.7 dB lower when EAED is applied instead of HDD. Therefore, E&ED can provide an adding coding gain of 1.7 dB over HDD, in both single-wavelength systems and wavelength-interleaved systems.

In short, although both double-wavelength interleaving and E&ED can equally extend the tolerable PMD level by 15% over HDD at high DGD and high Q values, wavelength interleaving provide no additional coding gain over HDD, while E&ED

can provide an adding coding gain of 1.7 dB over HDD at low DGD and low Q values. This can be explained as follows: in double-wavelength interleaving systems, the FEC frames transmitted on two separate wavelengths. Noise statistics do not change with wavelengths, therefore, wavelength interleaving can provide no improvement of FEC performance when noise is dominant (low Q values). With E&ED, an optimized erasure zone can flag almost all of the errors as erasures, no matter whether the errors are caused by noise or PMD. This also explains why line 3 and 4 merges into each other when PMD is severe.

Figure 5-10. Required Q-factor to achieve outage probability of 10^{-3} or 10^{-6} . Mean DGD in this figure is mean DGD normalized by T_b , $p=1$, normalized rise/fall time $tr=0.5$, E&ED is set as: $\eta=4\%$ and $R_{EZ}=p$.

5.7.3.2 Outage Probability

Besides average BER, system outage probability is another important parameter for performance evaluation in the systems impaired by PMD. Figure 5-10 shows the required Q-factor for an outage probability of 10^{-3} and 10^{-6} . The outage probabilities for an instantaneously BER larger than 10^{-12} are obtained by $P\{P_e(Q, \Delta\tau) > 10^{-12}\}$ for system without FEC; $P\{P_{B_FEC}(P_e(Q, \Delta\tau)) > 10^{-12}\}$ for system with FEC and HDD; $P\{P_{B_FEC_2}(P_e(Q, \Delta\tau_1), P_e(Q, \Delta\tau_2)) > 10^{-12}\}$ for system with FEC, HDD and double-wavelength interleaving; and $P\{P_{B_ee}(P_{S_era}(Q, \Delta\tau, \eta), P_{S_err}(Q, \Delta\tau, \eta)) > 10^{-12}\}$ for system with FEC and EAED respectively. All the other settings are same as Figure 5-9.

In Figure 5-10, at the receiver Q-factor of 11.2 dB: for an outage probability of 10^{-6} , using E&ED instead of HDD can increase the tolerable DGD from $0.15T_b$ to $0.185T_b$ by 23%, and the maximum transmission distance of the optical fiber link is correspondingly increased by about 48%; for an outage probability of 10^{-3} , using E&ED instead of HDD can increase the tolerable DGD from $0.2T_b$ to $0.25T_b$ by 25%, and the maximum transmission distance of the optical fiber link is correspondingly increased by about 50%.

5.8 E&ED to mitigate CD Impairment

The system performance with FEC and E&ED will be investigated in a system impaired by CD in this section. System model is derived, and system performance improvement with E&ED is analyzed.

5.8.1 System Model of CD Impaired Systems with FEC and E&ED

Similarly to deriving the system model of PMD impaired systems with FEC and E&ED in section 5.7.1, the system model of CD impaired systems with FEC and E&ED can be similarly derived. Based on functions (4-1) to (4-5), the instantaneous

probability P_{B_era} that the received signal is flagged as a bit erasure can be shown to be:

$$\begin{aligned}
 P_{B_era} &= P(I_1^{\sup})P(erasure | I_1^{\sup}) + P(I_1^{\text{med}})P(erasure | I_1^{\text{med}}) + P(I_1^{\inf})P(erasure | I_1^{\inf}) \\
 &\quad + P(I_0^{\sup})P(erasure | I_0^{\sup}) + P(I_0^{\text{med}})P(erasure | I_0^{\text{med}}) + P(I_0^{\inf})P(erasure | I_0^{\inf}) \\
 &= \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [1 - \eta + (p + \eta) \cdot \Delta \tau]}{\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad - \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [1 + p \cdot \eta + p \cdot \Delta \tau \cdot (\eta - 1)]}{\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad + \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot (1 - \eta)}{\sqrt{2}} \right) - \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot (p + \eta)}{p\sqrt{2}} \right) \\
 &\quad + \frac{1}{8} \operatorname{erfc} \left(\frac{Q \cdot [g^{\text{med}} \cdot (p + 1) - (p + \eta) \cdot (1 + \Delta \tau)]}{\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad - \frac{1}{8} \operatorname{erfc} \left(\frac{Q \cdot [g^{\text{med}} \cdot (p + 1) + p \cdot (\eta - 1) \cdot (1 + \Delta \tau)]}{\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad + \frac{1}{8} \operatorname{erfc} \left(\frac{Q \cdot [(g^{\text{med}} - 1) \cdot (p + 1) + p \cdot (1 - \eta) \cdot (1 + \Delta \tau)]}{p\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad - \frac{1}{8} \operatorname{erfc} \left(\frac{Q \cdot [(g^{\text{med}} - 1) \cdot (p + 1) + (p + \eta) \cdot (1 + \Delta \tau)]}{p\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad + \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [g^{\inf} \cdot (p + 1) - (p + \eta) \cdot (1 + \Delta \tau)]}{\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad - \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [g^{\inf} \cdot (p + 1) + p \cdot (\eta - 1) \cdot (1 + \Delta \tau)]}{\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad + \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [(g^{\inf} - 1) \cdot (p + 1) + p \cdot (1 - \eta) \cdot (1 + \Delta \tau)]}{p\sqrt{2}(1 + \Delta \tau)} \right) \\
 &\quad - \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [(g^{\inf} - 1) \cdot (p + 1) + (p + \eta) \cdot (1 + \Delta \tau)]}{p\sqrt{2}(1 + \Delta \tau)} \right)
 \end{aligned} \tag{5-46}$$

and the channel BER P_{B_err} can be shown to be:

$$\begin{aligned}
 P_{B_err} &= P(I_1^{\sup})P(0 | I_1^{\sup}) + P(I_1^{\text{med}})P(0 | I_1^{\text{med}}) + P(I_1^{\text{inf}})P(0 | I_1^{\text{inf}}) \\
 &\quad + P(I_0^{\sup})P(1 | I_0^{\sup}) + P(I_0^{\text{med}})P(1 | I_0^{\text{med}}) + P(I_0^{\text{inf}})P(1 | I_0^{\text{inf}}) \\
 &= \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [p+1 + p \cdot (\eta-1) \cdot (1+\Delta\tau)]}{\sqrt{2}(1+\Delta\tau)} \right) + \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot (p+\eta)}{p\sqrt{2}} \right) \\
 &\quad + \frac{1}{8} \operatorname{erfc} \left(\frac{Q \cdot [g^{\text{med}} \cdot (p+1) + p \cdot (\eta-1) \cdot (1+\Delta\tau)]}{\sqrt{2}(1+\Delta\tau)} \right) \\
 &\quad + \frac{1}{8} \operatorname{erfc} \left(\frac{Q \cdot [(g^{\text{med}}-1) \cdot (p+1) + (p+\eta) \cdot (1+\Delta\tau)]}{p\sqrt{2}(1+\Delta\tau)} \right) \\
 &\quad + \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [g^{\text{inf}} \cdot (p+1) + p \cdot (\eta-1) \cdot (1+\Delta\tau)]}{\sqrt{2}(1+\Delta\tau)} \right) \\
 &\quad + \frac{1}{16} \operatorname{erfc} \left(\frac{Q \cdot [(g^{\text{inf}}-1)(p+1) + (p+\eta)(1+\Delta\tau)]}{p\sqrt{2}(1+\Delta\tau)} \right) \tag{5-47}
 \end{aligned}$$

where $I_{Th} = \sigma_0 I_l / (\sigma_0 + \sigma_l)$, and g^a ($a = \text{sup, med, inf}$) is defined in Chapter 4, function (4-7). The erasure zone parameters are similarly defined as that in section 5.4, $R_{EZ} = p = \sigma_0 / \sigma_l$, and erasure zone I_{Th0} and I_{Thl} is defined in (5-21).

5.8.2 Results and Data Analysis

Figure 5-11. Receiver Q factor required to achieve an output BER of 10^{-12} with RS(255, 223) using HDD vs. E&ED. Pulse spread is normalized by T_b , $p=1$, $R=10$ Gb/s, normalized $tr = 0.3$, E&ED is set as: $\eta = 5\%$ and $R_{EZ} = p$.

Figure 5-11 shows receiver Q factor required to achieve an output BER of 10^{-12} with HDD compared to E&ED in 10 Gb/s RS(255, 223) systems. Similarly to section 5.7.2, $\eta = 5\%$ and $R_{EZ} = p$ is chosen as the setting for the erasure zone in this section. The solid line in Figure 5-11 is obtained from section 4.2 and expression (3-15), and denotes the system with RS(255, 223) and HDD. The points in Figure 5-11 are obtained from (5-46), (5-47) and (5-31) and denote the same system with RS(255, 223) and E&ED. Define “tolerable dispersion” as the value of the pulse spread at the receiver Q-factor of 16 dB which is typically considered in optical transmission systems. At lower Q factor of 10.5 dB, E&ED helps to improve the tolerable normalized pulse spread from 0.05 under HDD to 0.12. Improvement of system dispersion tolerance is $0.07T_b$ and 1.4 times. When the Q factor is 16 dB, E&ED helps

to improve tolerable dispersion from $0.36T_b$ under HDD into $0.42T_b$. Improvement of system dispersion tolerance is $0.06T_b$ and 17%. At higher Q factor of 24 dB, the improvement on tolerable normalized pulse spread that E&ED is able to provide over HDD is $0.1\sim 0.2T_b$ or 2~4%. Therefore, E&ED can provide more improvement when the chromatic dispersion is not too severe to induce too many errors exceeding the error correcting ability of the FEC code used.

The tolerable dispersion level of the E&ED over HDD is improved by 17%. Since the pulse spread caused by CD is a function of the product of fiber length L as indicated in expression (4-5), the maximum transmission distance of the optical fiber link is correspondingly increased. This implies savings on system cost.

Figure 5-12 shows the receiver Q factor vs. output BER with RS(255, 223) code in systems with E&ED compared to that in systems with HDD without wavelength interleaving and with two wavelengths interleaved. Erasure zone parameters are set as $R_{EZ}=p$, $\eta=5\%$ for $\Delta\tau_1=0.1$ and $\eta=3\%$ for $\Delta\tau_1=0.4$. In order to investigate the advantage of E&ED over HDD with and without wavelength interleaving on the CD mitigation, Figure 5-12 is plotted based on the same Condition Groups A and B in Figure 4-7 (a) in Section 4.5. The BER lines for the wavelength λ_1 stand for channels with HDD without interleaving and BER lines “ $\lambda_1+\lambda_2$ ” stand for interleaved channels with HDD from Figure 4-7 (a), then plot new lines λ_1 for E&ED without interleaving. Figure 5-12 shows that when the output BER is higher than 10^{-9} , wavelength requires same or lower system Q-factor to get the same output BER than E&ED. When output BER is lower than 10^{-9} , E&ED surpasses HDD with and without wavelength interleaving.

Figure 5-12. Receiver Q factor vs. output BER with RS(255, 223) code in systems with E&ED compared to that in systems with HDD without wavelength interleaving and with two wavelengths interleaved. $p=1$, $R=10$ Gb/s, normalized rise/fall time $tr=0.3$, E&ED is set as: $R_{EZ}=p$, $\eta=5\%$ for $\Delta\tau_1=0.1$ and $\eta=3\%$ for $\Delta\tau_1=0.4$.

5.9 Adaptive Erasure Zone

In previous sections, E&ED with a fixed erasure zone is investigated, and is proved to outperform HDD. However, the optimum erasure zone changes with the severity of PMD and different Q factor value. In this section, the improvement that adaptive erasure zone can provide over a fixed erasure zone is studied, and a practical optical receiver with adaptive erasure zone for EAED decoder is proposed.

Figure 5-13 gives the output BER (based on Formula II in section 5.4.3) as a function of erasure zone width, at different instantaneous DGD levels with (a) RS(255, 239), (b) RS(255, 223) coding. At each DGD level with a fixed Q factor, the output BER changes with the width of erasure zone. Comparing Figure 5-13 (a) and (b), the erasure zone width affects output BER more on RS(255, 223) than RS(255, 239). When the output BER is minimum, the erasure zone width is the optimized value. When the T_b normalized DGD level is lower than $(1-tr)$, noise is the dominant impairment instead of PMD, and the output BER will not change with DGD. With RS(255, 239) and Q factor of 9.7 dB, the optimized erasure zone width shown in Figure 5-13 (a) is 6%. With RS(255, 223) and Q factor of 8.7 dB, the optimized erasure zone width shown in Figure 5-13 (b) is 7%. Higher instantaneous DGD level will require higher Q factor to get an output BER of 10^{-12} , and the optimized erasure zone width is wider with bigger Q factor. In practice, the instantaneous DGD changes slowly with time. The observations in Figure 5-13 motivate us to explore adjusting the erasure zone adaptively to minimize the E&ED output BER.

(a). RS(255, 239)

(b). RS(255, 223)

Figure 5-13. Output BER vs. erasure zone width, at different instantaneous DGD level (normalized by T_b) with (a) RS(255, 239), (b) RS(255, 223) coding. $p = 1$, normalized rise/fall time $tr = 0.5$.

Figure 5-14. Schematic diagram of the proposed E&ED receiver with adaptive erasure zone.

Therefore, an optical receiver with dual adaptive erasure zone is designed for E&ED decoder. This raises a novel concept of optical receiver designing which optimize its error correcting operation in real time. Figure 5-14 shows the schematic diagram of the decision stage in a novel OOK receiver which has two adaptive thresholds I_{th0} and I_{th1} for the implementation of erasure decoding based on RS(255, 239) code. The front end of the optical receiver is not shown in Figure 5-14. It is usually built up of photo detector, trans-impedance amplifier, optional filter and master amplifier. The XOR gate output “out₂” indicates the erasure bits whose levels are falling in between I_{th0} and I_{th1} . These bits are counted based on “clk” (clock) and cleared on a periodic base of “ctr” (control). The “ctr” period should be long enough to make sure plenty of erasure bits can be collected to get a reliable erasure bit probability. The erasure bits are counted and D/A converted periodically, in order to provide reference for adjustment of the decision levels. Simultaneously, analogue level of those identified erasure bits can be abstracted to avoid losing information of received signal. The pos 0 of the switch is for normal output, whereas the pos 1 is for output of analogue level of erasure bits. Those erasure bits need more special handling when an advanced FEC technique with E&ED is applied, which can refer to section 5.2.2. Because the positions and levels of those erasure bits are detected at the same time, which will greatly reduce the arithmetic burden of the successive circuits for E&ED. Recursive

calculation could possibly be avoided hence it's more practical to a system in the speed of 10 Gb/s or higher.

The adaptive optimization of the erasure zone is performed at the “Erasure Zone Width η adjustment” block in Figure 5-14. First, instantaneous $\Delta\tau$, Q , I_1 , σ_0 , σ_1 are estimated by the receiver using the eye diagram. Then, the two major erasure zone parameters, R_{EZ} (ratio of the erasure zone width at the “0”side to that at the “1”side) and η (total width of the erasure zone), will be set in the following way: R_{EZ} is chosen to be p ; η may be scanned for the optimum value using Formula I. Given the instantaneous $\Delta\tau$, Q , I_1 , σ_0 , σ_1 , the decoded BER in Equation (5-29) will be decided only by η . Hence, we can optimize η to get the lowest decoded BER using (5-29) with P_{B_era} by (5-40) and P_{B_err} by (5-41). Then the erasure zone thresholds can be accordingly set to:

$$\begin{cases} I_{Th1} = I_{Th} + \eta \cdot I_1 \cdot \frac{1}{1+p} \\ I_{Th0} = I_{Th} - \eta \cdot I_1 \cdot \frac{p}{1+p} \end{cases}$$

where $p = \sigma_0 / \sigma_1$, and $I_{Th} = \sigma_0 I_1 / (\sigma_0 + \sigma_1)$.

In contrast to turbo or LPDC decoder, the E&ED decoder does not need to perform complex iterative soft decoding. Hence E&ED is potentially more practical to implement in an optical system of 10Gb/s or higher.

(a). RS(255, 239)

(b). RS(255, 223)

Figure 5-15. Comparison of required Q factor value to get an output average BER of 10^{-12} using E&ED with fixed vs. adaptive erasure zone with (a) RS(255, 239) coding, (b) RS(255, 223) coding, average DGD level is normalized by T_b , $p = 1$, normalized rise/fall time $tr=0.5$.

Figure 5-15 shows the comparison of required Q factor value to get an output average BER of 10^{-12} with E&ED with fix vs. adaptive erasure zone with (a) RS(255, 239) coding (b). RS(255, 223) coding. The fixed erasure zone width is chosen to be 6% for RS(255, 239) code in (a) and 7% for RS(255, 223) code in (b). These values are taken from Figure 5-13. Figure 5-15 (a) shows that adaptive erasure zone can provide 0.4 dB improvement over a fixed erasure zone when mean DGD is serious, after critical point A, and system PMD tolerance at Q factor of 9.7 dB is increased from $0.15T_b$ to $0.16T_b$. Figure 5-15 (b) shows that adaptive erasure zone can provide 0.5 dB improvement over a fixed erasure zone when mean DGD is serious, and system PMD tolerance at Q factor of 8.7 dB is increased from $0.15T_b$ to $0.16T_b$.

However, in systems with very low BER, erasure bits occur less frequently and decision level adjustment will no longer be effective. If such a receiver is followed by FEC, the required input BER is only in the range of $10^{-3} \sim 10^{-4}$ and decision level adjustment should be effective because of higher occurrence of erasure bits.

5.10 Chapter Conclusion

In this chapter, FEC with E&ED is proposed for fiber optic communication systems for the first time. E&ED is the simplest form of soft-decision decoder (SDD) with two decision thresholds and three decision levels: “1”, “0” and “Erasure”. It only requires slight modifications to the hard-decision decoder (HDD), and does not require soft-in soft-out decoding and iterative decoding that are a must for Turbo and LDPC decoders. Therefore, the decoding complexity of E&ED is only slightly higher than HDD but is much more tolerable in high-speed optical systems than Turbo or LDPC decoding. E&ED also outperforms HDD in error correcting ability. This is because a RS(n, k) code can correct $(n-k)/2$ symbol errors with HDD, but can correct $n-k$ symbol errors and erasures with E&ED.

However, the system performance improvement of E&ED depends on correct setting of the erasure zone. To analyze the improvement that E&ED can provide over HDD, a complete set of system models of channel BER, SER and output BER is derived for the E&ED system. Furthermore, the erasure zone is parameterized and optimized via two key parameters: (i) the normalized width parameter of the erasure zone, η , and (ii) the width ratio of the erasure zone at each side of the received “0” and “1” regions. The optimization results show that the width ratio should be set equal to the ratio of the corresponding standard deviations for the “0” and “1” regions (i.e. $R_{EZ} = \Delta I_0 / \Delta I_1 = p = \sigma_0 / \sigma_1$). Furthermore, with an optimized fixed E&ED, 1.7 dB CG improvement over HDD can be achieved with RS(255, 239) code in optic fiber systems impaired by PMD; the tolerable chromatic dispersion level is improved by 17% respectively over HDD with RS(255, 223) code. Taking PMD channel as an example to investigate the advantage of adaptive erasure zone, it is found that an additional 0.4 dB coding gain can be achieved with an adaptive erasure zone over a fixed optimized erasure zone.

Next, an optical receiver with adaptive erasure zone is proposed and designed as an optional scheme. In contrast to turbo or LPDC decoder, the E&ED decoder does not need to perform complex iterative soft decoding. Hence E&ED is potentially more practical to implement in an optical system of 10Gb/s or higher.

The optimized E&ED system can considerably improve the FEC performance without increasing the coding redundancy. This also brings about a corresponding increase on system tolerance of noise and maximum transmission distance, which implies large savings in the system cost. Therefore E&ED appears to be a promising technique for high-speed optical transmission systems.

Between wavelength interleaving and E&ED, it is found that E&ED outperforms wavelength interleaving when the PMD or CD is not severe and noise is dominant, but wavelength interleaving can slightly outperform E&ED when the PMD or CD is

very severe. Therefore they can be jointly applied in an optical system to achieve good system performance at different times.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, a class of error correction code called forward error correction code is studied. Forward error correction (FEC) is capable of providing higher communication performance by detecting and correcting errors caused by two major impairments in the optical link, PMD and CD. In this thesis, two methods, wavelength interleaving and E&ED, are proposed to augment the FEC performance in optical channels impaired by PMD and CD.

In part I of this thesis, FEC with wavelength interleaving is proposed and studied. The system performance of the conventional optical transmission system with PMD and FEC is analyzed in details. Based on the FEC decoding error probability function of conventional optical transmission system (without wavelength interleaving), a new error probability function is derived for the system with multiple wavelength interleaving. The simulation results obtained tally well with the theoretical analysis, which indicates that the derived expressions are reliable.

In systems weakened by PMD, it is found that, double-wavelength interleaving and RS(255, 239) channel coding can extend the PMD tolerance level by over 20% at an output BER of 10^{-12} . Four-wavelength interleaving and RS(255, 239) channel coding can extend the PMD tolerance level by over 26% at an output BER of 10^{-12} . As a result, the maximum transmission distance is increased by about 45% and 60% respectively, which means that the cost of system can be substantially saved. Double-wavelength interleaving can also provide a net coding gain of about 6 dB at all PMD levels. Finally, the results show that the Q-factor is determined by the wavelength which has the highest PMD of all the interleaved wavelengths.

In systems weakened by CD effect, it is similarly found that wavelength interleaving will average out the dispersion effect on different wavelengths, to make the system easier to be compensated. In the design of the FEC systems with wavelength interleaving, the interleaved wavelengths need to have pulse spread difference greater than the minimum pulse spread difference shown in Figure 4-6 (b), in order to get the highest BER improvement. Once this condition is met, the output BER is determined only by the worse-case wavelength (Figure 4-6). Hence wideband systems can be expected to benefit from the proposed wavelength interleaving system, especially in long distance (>100 km) and very high speed (≥ 40 Gb/s) transmission systems. Compared to a non-interleaved FEC system, wavelength-interleaved FEC based on receiver Q-factor of 17 dB, RS(255,223) coding and output BER of 10^{-12} is found to be able to increase the tolerable CD of the system by about 13% for double-wavelength interleaving, and 22% for four-wavelength interleaving system. This brings about an increase in the maximum transmission distance and economic benefit.

With the findings presented in this thesis, it can be concluded that wavelength interleaving scheme is attractive for optical transmission system suffering from long fading PMD and CD. First of all, it provides frequency diversity to mitigate the PMD or CD effect of long burst errors. Next, it can extend PMD/CD tolerance, and improve the system performance at all PMD/CD levels. Third, different wavelengths used will require one common Q-factor instead of multiple different ones to meliorate the whole system.

In Part II of this thesis, chapter 5, FEC with E&ED is proposed, analyzed and optimized. The system models of channel BER (before decoding), SER and output BER (after decoding) are derived. E&ED performance is optimized via two major parameters of the erasure zone: (i) η , the normalized width parameter of the erasure zone, and (ii) R_{EZ} , the width ratio of the erasure zone at each side of the received “0” and “1” regions. Our analysis shows that: with an optimized fixed erasure zone, E&ED can provide 1.7 dB CG improvement over HDD with RS(255, 239) code in optic fiber systems impaired by PMD; while the tolerable chromatic dispersion level is improved by 17% respectively over HDD with RS(255, 223) code. Besides fixed

erasure zone, adaptive erasure zone is also investigated, and it is found an additional 0.4 dB gain can be achieved on PMD mitigation using adaptive erasure decoding over fixed erasure decoding. A practical optical receiver architecture with an adaptive erasure decoding is also designed.

Comparing the two methods, wavelength interleaving and E&ED, which have been proposed in this thesis to improve the FEC performance of high-speed optical impaired by PMD and CD, it is found that E&ED outperforms wavelength interleaving when PMD/CD is not severe and noise is dominant, while wavelength interleaving can slightly outperform E&ED when PMD/CD is very severe. The reason is that the probability that both wavelengths are simultaneously deteriorated severely by PMD/CD is very small. However, noise statistics do not change much with wavelength. Therefore, wavelength interleaving technique can extend the tolerable PMD/CD level but provides no improvement of FEC performance when noise is dominant. On the other hand, E&ED can correct more symbol erasures than errors. The erasure zone covers the most unreliable detection area where the errors are most ambiguous, hence an optimized erasure zone can flag almost all of the errors as erasures, whether the errors are caused by noise or PMD/CD. This is why wavelength interleaving improves the FEC correcting ability on the errors induced by PMD/CD, while E&ED improves the FEC correcting capability on the errors caused by both PMD/CD and noise.

6.2 Suggestions for Future Works

In this section, some suggestions for future works are proposed.

Firstly, the two techniques, E&ED and wavelength interleaving, have been investigated separately in this thesis. In fact, they may be applied to the system at the same time to reap combined benefits. Similarly, the effects of CD and PMD have been considered separately in this thesis, but in practice, both effects may be present

at the same time. It is therefore useful to study the FEC performance improvement provided by both wavelength interleaving and E&ED, considering the co-existence of both of CD and PMD in the optical systems.

Secondly, it will be interesting to study FEC performance of wavelength interleaving and E&ED on nonlinear channel impairments, such as four-wave mixing, self-phase modulation, and cross-phase modulation. Nonlinear effects induce inter-frequency interference problems, so it is reasonable to expect that wavelength interleaving and E&ED is able to help on alleviating these impairments. Similarly, NRZ is considered in this thesis, because the channel BER model without wavelength interleaving used in this thesis is quoted from [50] and it considers only NRZ signal. Since RZ is known to offer higher resistance to PMD than NRZ, it will be of interest to investigate the performance gain that wavelength interleaving and E&ED can give to a RZ optical system with PMD and CD.

Thirdly, Reed-Solomon code is chosen as the FEC scheme in this thesis to study how much FEC performance gain wavelength interleaving and E&ED can give. It is reasonable to believe that these two techniques are also effective to other optical FEC codes such as low-density parity-check (LDPC) codes [88]-[93]. In references [88]-[92], iteratively decodable LDPC codes are shown to outperform turbo product codes in BER performance. The decoder complexity of LDPC codes is comparable to that of turbo product codes, and significantly lower than that of turbo codes. These make LDPC code an attractive choice for high-speed long-haul optical transmission systems. Therefore LDPC is recommended for future study.

The fourth idea is to investigate wavelength interleaving and E&ED in differential phase-shift keying (DPSK) [66][78] and differential quadrature phase-shift keying (DQPSK) [101]-[103] systems instead of OOK ones. In reference [101], Boffi presents a research on time-interleaving of two orthogonally polarized RZ-DQPSK signals and shows that it is effective to mitigate the crosstalk due to non-ideal polarization stabilization. The combination of polarization multiplexing and RZ-DQPSK also shows a high CD tolerance. To investigate how much improvement

wavelength interleaving will provide for a DQPSK FEC system will be an interesting research direction.

Another possible future research direction is to consider wavelength interleaving and E&ED for coherent receiver [108]-[110] in optical communication systems. Multilevel modulation is found to be one of the most effective ways to achieve high spectral efficiency in optical communication systems. However, with the increasing number of modulation level, it becomes more and more difficult to apply optical delay detection because of degraded signal-to-noise (SNR) of the demodulated signal. Coherent receiver can recover multilevel phase-shift keying (M -ary PSK) signals through phase-diversity homodyne detection, and studies have shown that M -ary PSK systems with coherent receivers have higher tolerance to CD and PMD. In reference [109], a total dispersion of up to 4000 ps/nm is compensated effectively through a simple transversal digital filter implemented in our coherent receiver. It's reasonable to believe that FEC with time interleaving and wavelength interleaving will be able to exploit its advantages to improve coherent MPSK performance impaired by PMD or CD. Furthermore, in optical systems with FEC and incoherent receivers, analog-to-digital conversion (ADC) is not required for decoding decisions any more, although it is a basic requirement for the FEC systems. Therefore, the enhanced FEC with wavelength interleaving and E&ED can be considered to be integrated into the front-end of the receiver. To investigate wavelength interleaving and E&ED in optical systems with coherent receiver will be an attractive topic for future research.

Author's Publications

The publications are sorted in chronological order, starting with the most recent one.

Journal Publications

1. B. Xie, Z. Li, Y. L. Guan and C. Lu, "FEC performance of PMD-impaired optical communication system with multiple wavelength interleaving", *IEEE Photonics Technology Letters*, vol: 16, no.3, March 2004, pp 936-938.
2. B. Xie, Y. L. Guan, J. Chen, and C. Lu "Improvement of dispersion tolerance using wavelength-interleaving and forward error correction", *Optics Communications*, 268 (2006), July 2006, pp 226-230.
3. J. Chen, B. Xie, Y. L. Guan, and Y. Wang, "An On-Off-Keying optical receiver with dual thresholds and erasure zone", *IEEE Photonics Technology Letters*, vol.19, Issue 20, October 2007, pp1592-1594.

Conference Publications

1. B. Xie, Y. L. Guan, Z. H. Li, and C. Lu, "FEC performance of optical communication systems with PMD and wavelength interleaving", presented on *ICCCAS2004*, Cheng Du, May 2004.
2. B. Xie, Y. L. Guan, J. Chen, C. Lu and Z. H. Li, "Application of erasure decoding in fiber optical systems with FEC", presented on *LEOS'05 Summer Topical Meetings*, San Diego, CA, 2005, pp 171-172.

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