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Modeling Structural Changes in Simultaneous Equation System

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SUMMARY

The fact that structural change and dynamics are the inherent facets of economic life implies the crucial importance of detecting and modeling structural change. However, current literature has mainly focused on testing the structural change of known timing rather on estimating the breakdate as an unknown parameter. Among the few studies on structural change estimation, sequential method has been suggested, the estimates of which are not guaranteed to be identical to those obtained by global optimization. Meanwhile, no attempt has been seen to address this issue in the context of Simultaneous Equation Model (SEM). It is the purpose of this dissertation to fill part of the above gap.

This dissertation provides a comprehensive treatment of the issues related to the estimation of linear models with multiple structural changes, to the detection of the presence of these unknown structural breaks and to the determination of the number of changes present. A systematical and operational Recursive Segmentation Method (RSM) is proposed in which an algorithm is rediscovered that is based on the principle of dynamic programming and allows global minimizers to be obtained by a number of sums of squared residuals rather than an exhaustive grid search. Along with this principle, a more efficient stopping criterion to partition the sample is adopted which is based on model selection rather than sequential testing thus proving to be more appealing to us. This dissertation has two major contributions. First, it propounds the estimation of all the structural breakpoints concurrently which, in comparison with the

sequential 'one-at-a-time' estimation, guarantees global optimization of the target function. Computational simulation is further conducted where estimating breakpoints concurrently signifies the correct separation of data sample. Second, the dissertation investigates structural change analysis in SEM which enables us to find the possible switching points and to formalize the way the switching point could be traced. In particular, the RSM is generalized to SEM that takes into account both limited information method and system method, which has not been studied in current literature. The single equation estimation of structural changes is able to detect the structural instability triggered from individual equation, which is not necessary from the entire simultaneous equation system; whereas system estimation, or full-information method, allows detecting structural changes across the entire system.

The proposed method is employed to study the U.S. real interest rate and Singapore's private housing market. The application to the U.S. interest rate exemplifies the merit of estimating all the structural break points concurrently, compared with sequential method, by identifying the structural changes with a better approximation of sample and closer fit to the historical events. For Singapore's housing market, a SEM is established and structural change is estimated in this context, for the first time, without prior knowledge or information of the pattern and timing of possible structural shifts. The structural break points detected are proved to be consistent with policy changes as well as external shocks to the model, and segmented or piecewise regression models after taking into consideration the structural changes, manifest better goodness-of-fit in estimation and improved accuracy in forecast.

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CHAPTER 1 INTRODUCTION

Before an econometric model can be used to draw inference about economic phenomena, it is of great importance to assess the adequacy of its specification. To test structural change, or parameter constancy as the simplest case, is particularly appropriate for dynamic econometric models as policy prescriptions might be quite different with the presence of structural changes. Failure to take into account structural changes for economic model, given their presence, may lead to incorrect policy implications and predictions; conversely, proper treatment of structural changes can be useful in uncovering the underlying factors that fostered the changes, in identifying misspecification, and in analyzing the effect of a policy change.

Many practical situations involve a function whose form cannot be uniformly well approximated by the leading terms of a single Taylor expansion. In these circumstances, a segmented model may be able to explain a real phenomenon and therefore has much of the simplicity of the classical linear methodology and more flexibility. It may be regarded as a piecewise linear approximation deriving from different Taylor expansions in different subdomains.

There are some situations in which segmentation regression can be of particular interest in time series analysis. For instance, in forecasting time series, one may want to consider basing one's forecasts only on the most recent version of a time series model rather than on a model built from the entire series. The motivation here is a possibility that the recent past contains more information about the immediate future

than the distant past and that one can discriminate between the recent and distant past by using the fit of time series model under consideration.

Simultaneous equation model has been widely used in econometric literatures; nevertheless there are only a few results available on analyzing and testing the stability of structural coefficients and structural changes. Andrews and Fair (1988) discussed the problem in a general setting, while more concrete situations are studied in Lo and Newey (1985) and Erilat (1983). The former work extended Chow's (1960) tests to simultaneous equations and proposed a simple Wald test, composed of two-stage least-squares (2SLS) estimator and the estimate of its covariance matrix. Erilat (1983) advocated an exact F test for the cases when there are inadequate degrees of freedom. It also constituted an extension of Gile's (1981) result, where CUSUM and CUSUM of squares tests for parameter stability in a single structural equation are developed. However, current literature only examines the structural change of one single equation belonging to a simultaneous equation model rather than considering structural breaks across the entire system. Meanwhile, it is a common drawback of most of the tests mentioned above to assume that the switching point is known a priori, which is often not the case in applied research. As stated by Honda (1992), some preliminary efforts to find the possible switching point is needed and it would be better if we could formalize the way the switching point could be traced. Treating the date of structural change—the breakdate—as an unknown parameter, the issue of interest becomes how to estimate the breakdate.

The single-equation counterpart on this agenda has been discussed by several authors (Chong, 1995 Bai, 1997). In many applications, the number of structural breaks as well as the locations of break points are taken as unknown parameters and a theory of least squares estimation has been developed. Operationally, the whole sample is split at each possible breakpoint, other parameters are estimated by OLS, and the sum of squared errors (SSE) is calculated. The least squares breakpoint estimation is the value that minimizes the full sample SSE. Independently in an early study, Huang, Liu and Zhang (1985) considered multiple structural changes in a linear model estimated by least-squares and proposed an information criterion for the selection of the number of changes. Their study discussed the problem in a more general framework, where the form of the model is not fixed between various segments. However, there has not been a corresponding development of tests and estimation method for structural changes in simultaneous equation models. The purpose of this thesis is to try to close this gap a bit—to investigate the issue in simultaneous equations context and to demonstrate how analogous estimation can be constructed in simultaneous equation model estimated by common estimator like 2SLS and 3SLS.

Our analysis aims to present a comprehensive treatment of issues related to the estimation of linear models with multiple structural changes, to the detection of the presence of these changes occurring at unknown timing and to the determination of the number of changes present. Aside from generalizing the structural change estimating from single equation to simultaneous equations system, this thesis also attempts to contribute to the literature in the following senses. We apply a

simultaneous estimation of all the changing points at one time, rather than the commonly used sequential method of detecting them one by one. Along with this principle, we adopt a different stopping criterion in partitioning the sample; it is based on ideas of model selection rather than sequential testing and seems more appealing to us. The importance of this has been pointed out by Bai and Perron (1998) stating that the relative merits of different methods to select the number of structural change appears to be an important one among the topics to be investigated. In particular, we will reexamine U.S. real interest rate to illustrate the use of Information Criterion for simultaneous estimation in comparison with the results derived from sequential method. An application to Singapore's property price and in particular, the structural change of private housing market will be studied as well.

With the above-mentioned objectives, we consider in this thesis the case of possible multiple switching of the parameters at unknown sample points and investigate the simultaneous estimation of multiple structural changing points along with the regression coefficients within subdomains. The inferential problem confronting us involves three parts: the specification of the number of segments in the model, I ; the detection of the change point $\{i_s\}$, or the boundaries of intervals over which each of the model pieces applies; and the estimation of the model parameters within each segment. Summing the residual sums of squares for the various intervals yields an overall index of the quality of fit of the segmented model. With I fixed, the $\{i_s\}$ may be estimated by minimizing this index. Further minimization of the index to estimate I will be based on information criterion for model selection problem.

The outline of the thesis is as follows. Chapter 2 reviews relevant literatures on the areas of structural changes, the use of information criteria in model selection as well as simultaneous equation model. Based on the comparison of pros and cons of currently available methodologies, Chapter 3 suggests a Recursive Segmentation Method (RSM) which is able to correctly detect and estimate the existence and the timing of unknown changing points in simultaneous equations. This method provides a systematic and operational approach that can accurately detect structural changing points without any prior information or knowledge of the pattern and timing of possible structural shifts. The algorithm presented here is based on the principle of dynamic programming and allows global minimizers to be obtained using a number of sums of squared residuals rather than an exhaustive grid search. The use of information criterion is also incorporated in this method in determining the number of changes. Our method, in addition, is applicable to simultaneous equation model and detailed discussion is given in Chapter 4. Chapter 5 reports some empirical applications to illustrate the merits of the proposed method. Concluding remarks are provided in Chapter 6.

CHAPTER 2 LITERATURE REVIEWS

Econometric models are made up of assumptions which never exactly match reality. Among the most contested ones is the requirement that the coefficients of an econometric model remain stable over time. Recent decades have seen numerous attempts to test for it or to model possible structural change when it can no longer be ignored. Structural change has become both a major concern and the most challenging field of econometrics as economists are often troubled by external shocks while searching for the optimal model to obtain insights of economic systems. This chapter reviews the literature relating to structural changes and the use of information criteria. Recent work on structural changes in simultaneous equation model will be discussed as well.

2.1 The Econometrics of Structural Change—A Review

In order to capture the economic fundamentals, economists have created numerous models, which use abstract mathematical expressions to reveal and summarize the relationship between the economic variables. In most of the models, especially those dealing with economic development and macro-economic stability, structural changes are always assumed to be existing in the models' framework. In the past decades, the econometrics of structural change looks for systematic methods to identify structural breaks and the most important contributions to this literature fall into the following two areas: 1) Tests for a structural break; 2) Estimation of the timing of a structural

break. These two innovations have dramatically altered the face of applied time series econometrics (Hansen, 2001). We will discuss the two topics in turn.

2.1.1 Testing for Structural Change of Known Timing

Both the econometrics and statistics literatures contain a large amount of work on issues related to structural change. Among them, the classical F test for the equality of two sets of coefficients in linear regression models is commonly referred to by economists as the Chow test, after the seminal paper by Chow (1960). His famous testing procedure splits the sample into two subperiods, estimates the parameters for each subperiod, and then tests the equality of the two sets of parameters using a classic F statistic. This test is designed to test the null hypothesis of constant parameters against an alternative of a one-time shift in the parameters at some known time. Chow test was popular for many years and was extended to cover most econometric models of interest. There are several simple ways to calculate this test statistic, each of which involves splitting the sample at the hypothesized time of structural change. One method is to compare the estimates obtained from each subsample using the appropriate covariance matrix. A second method is to calculate the estimates using just the data from one subsample, and compare this with the estimates using the full sample. These methods are essentially equivalent and easy to apply in practice.

An important assumption made in using the Chow test is that the disturbance variance is the same in both regressions. If the restricted model, if this is not true, is

heteroscedastic, then the results for the classical regression model no longer apply. That is to say, like a conventional F tests, Chow test is (in general) valid only under the rather strong assumption of homoscedasticity. This assumption may be particularly implausible when one is testing the equality of two sets of regression parameters, since if the parameter vector differs between two regimes the variance may well be different as well. As analyzed by Schmidt and Sickles (1977) and Toyoda and Ohtani (1986), it is quite likely that the actual probability of type I error will be smaller than the significance level we have chosen. There are direct ways to deal with this problem. Assuming that we can estimate both the separate regressions, we can examine our estimates of the disturbance variances. With these in hand, for instance, we can test for significant differences. Without any significant difference, we can proceed as described. If, however, there is evidence to suggest that the variances actually are different, then there are still simple and appropriate ways to carry out the analysis by explicitly estimating the model, accounting for the heteroscedasticity. If the sample is reasonably large, then we have a test that is valid whether or not the disturbance variances are the same. Suppose that $\hat{\theta}_1$ and $\hat{\theta}_2$ are two normally distributed estimators of a parameter based on independent samples, with covariance matrices V_1 and V_2 . Then under the null hypothesis that the two estimators have the same expected value, $\hat{\theta}_1 - \hat{\theta}_2$ has mean 0 and variance $V_1 + V_2$. Thus the Wald statistic, $W = (\hat{\theta}_1 - \hat{\theta}_2)'(V_1 + V_2)^{-1}(\hat{\theta}_1 - \hat{\theta}_2)$, has a chi-squared distribution with K degrees of freedom. A test that the difference between the parameters is zero can be based on this

statistic. It is straightforward to apply this to the test of common parameter vectors in regressions. Large values of the statistic lead us to reject the hypothesis.

Gujarati (1970) proposed the dummy variables tests for equality sets of coefficients in the linear regression, which tries to insert dummy variables, representing exogenous "shocks", into the regression equation and assess the significance of such "shocks" by conducting general T-test on the significance of the coefficients of the dummy variables. This method also requires the knowledge of the location of the changing points so as to assign the correct value for the dummies.

H. Tsurumi is one of the first econometricians who introduced the Bayesian approach into the research of structural change in the regression models. In 1977, he first introduced a Bayesian test of a parameter shift in regression models and an application. In 1982, Tsurumi extended the test by using maximum likelihood technique to analyze the gradual switching regressions.

Hsu (1952) played a prominent role in the issue of the robustness to the standard assumptions in structural change models, which had not been studied extensively. Using the exponential power class of distributions of the error terms of a linear model with one change, he developed a complete posterior analysis and demonstrated the serious consequences of wrongly assumed normality for the data by analyzing an investment risk model of stock return price.

Dufour and Kiviet (1996) suggested several finite-sample tests of the parameter constancy for a linear regression model with one lagged dependent variable and independent normal disturbances.

2.1.2 Testing for Structural Change of Unknown Timing

Chow test is simple to apply, and the distribution theory is well developed. The test is crippled, however, by the need to specify a priori the time of the structural change that occurs under the alternative hypothesis. The problem is that the date of structural change is not defined (has no meaning) under the null hypothesis, and standard testing theory is not applicable. A researcher has several options open. First, the timing can be selected in an arbitrary way, such as at the sample midpoint. This solution effectively eliminates the dating question, but is ad hoc and would not be expected to have particularly good power against many alternatives of interest. This selection of convenient time periods hardly qualifies as serious analysis. Second, the data and regression residuals can be plotted for indications of structural changes. If this is done, the Chow test can be misleading, as the candidate breakdate is endogenous—it is correlated with the data—and the test is likely to indicate a break falsely when none in fact exists. Since the results can be highly sensitive to these arbitrary choices, it is hardly an example of sound scientific practice.

The test of structural change described in 2.1.1 assume that the process underlying the data is stable up to a known transition point, at which it makes a discrete change to a

new, but thereafter stable, structure. In many other settings, however, the change to a new regime might be more gradual and less obvious. The econometrics literature has recently witnessed an upsurge of interest in extending procedures to various models with an unknown change point. In this section, we will examine tests that are based on the idea that a regime change might take place slowly, and at an unknown point in time, or that the regime underlying the observed data might simply not be stable at all.

Hansen's (1992) test of model stability was based on a cumulative sum of the least squares residuals. From the least squares normal equations, we have

$$X'e = \sum_{t=1}^T x_t e_t = 0,$$

and, in addition, $\sum_{t=1}^T (e_t^2 - \frac{e'e}{n}) = 0$ by construction. Let the vector f_t be the

$(K+1) \times 1$ t^{th} observation in this pair of sums. Then, $\sum_{t=1}^T f_t = 0$. Let $s_t = \sum_{r=1}^t f_r$, so

$s_T = 0$. Finally, let $F = 1/T \sum_{t=1}^T f_t f_t'$ and $S = \sum_{t=1}^T s_t s_t'$. The test statistic can be

computed simply as $H = \text{tr}(F^{-1}S)$. Large values of H give evidence against the hypothesis of model stability. The logic of Hansen's test is that if the model is stable through the T periods, then the cumulative sums in S will not differ greatly from those in F . Note that the statistic involves both the regression and the variance. The distribution theory underlying this nonstandard test statistic is much more complicated than the computation. Hansen provides asymptotic critical values for the test of model constancy which vary with the number of coefficients in the models.

Recognizing the need for tests that reveal model instability of general form, Brown, Durbin and Evans (1975) proposed the CUSUM test (BDE), which was fairly widely programmed and used in the late 1970s and early 1980s. They first introduced the concept of recursive residuals, which were defined to be uncorrelated with zero mean and constant variance. The technique is appropriate for time-series data and might be used if one is uncertain about when a structural change might have taken place. The null hypothesis is that the coefficient vector is the same in every period; the alternative is simply that it is not. The test is quite general in that it does not require a prior specification of when the structural change takes place. The cost, however, is that the power of the test is rather limited compared with that of the Chow test. Theoretical investigations eventually revealed that the CUSUM test is essentially a test to detect instability in the intercept alone (see Kramer, Ploberger and Alt, 1988). Another test proposed from a similar motivation is the CUSUM of square test. This test, however, has poor asymptotic power against instability in the regression coefficients (Ploberger and Kramer, 1990). Instead, the CUSUM of squares tests can be viewed as a test for detecting instability in the variance of the regression error.

Casting a doubt on the power of the BDE test in detecting this kind of instability that parameter changes are insignificant in the short run, but become significant over a longer span of time, Hatanaka (1983) proposed a confidence interval on the predicted value from a dynamic equation.

Besides these two, tests for structural change for every breakpoint could also be calculated, and the largest test statistic examined. This idea—and solution—goes back to Quandt (1960), who proposed looking for the worst-case Chow statistic, the breakdate where the test is largest. This is Quandt's statistic. When the breakdate is unknown a priori, the chi-square critical values are inappropriate. The distributional theory has been given recently in Andrews (1990) and Hansen (1992).

W. Ploberger, W. Kramer and K. Karl (1989) developed a new test for the constancy of regression coefficients in linear models, and derived the limiting null distribution of the test statistic. The test does not require that possible change points be known. They showed that the test has non-trivial power against many local alternatives, and it compares favorably to both CUSUM and CUSUM-square tests. The null distribution and local power of an OLS-based version of the CUSUM test were discussed by Ploberger and Kramer (1996) and they showed this can be also applied to trending data.

Recent contributions also include work of Andrews (1993) who considered sup Wald, Likelihood Ratio and Lagrange Multiplier tests. Weighted versions of these tests satisfying some asymptotic optimality criterion were discussed in Andrews and Ploberger (1994). Andrews, Lee and Ploberger (1996) determined a class of finite sample tests for the existence of a changing point at unknown time in a normal linear multiple regression model with known variance. The change point tests they introduced can be used to test the null hypothesis of parameter constancy against the

alternative of multiple parameter shifts at unknown times or, more generally, against parameter changes of a less specific nature. The Quandt-Andrews and Andrews-Ploberger family of statistics have essentially replaced the Chow statistic in recent econometric practice.

Literature addressing tests for multiple structural changes also included Bai and Perron (1998). Their method is sequential, starting by testing for a single structural break. If the test rejects the null hypothesis that there is no structural break, the sample is split in two (based on the breakdate estimate presented in the next section) and the test is reapplied to each subsample. This sequence continues until each subsample test fails to find evidence of a break.

2.1.3 Estimating the Timing of Structural Change

In many applications, it is useful to know when the structural change occurred. Treating the date of structural change—the breakdate—as an unknown parameter, the issues are how to estimate the breakdate and how to obtain confidence intervals for the breakdate. While it may seem unlikely that a structural break could be immediate and might seem more reasonable to allow a structural change to take a period of time to take effect, we most often focus on the simple case of an immediate structural change for the sake of simplicity and parsimony.

An obvious candidate for a breakdate estimate is the date that yields the largest value of the Chow test sequence. It turns out that this is known to be a good estimate only in one special case—in linear regressions when the Chow test is constructed with the "homoskedastic" form of the covariance matrix.

In regression models, an appropriate method to estimate the parameters—including the breakdate—is least squares. Operationally, the sample is split at each possible breakdate, the other parameters estimated by ordinary least squares and the sum of squared errors calculated and stored. The least squares breakdate estimate is the date that minimizes the full-sample sum of square errors, or equivalently, minimizes the residual variance.

A theory of least squares estimation has been developed in a sequence of papers by Bai (1997), who derived the asymptotic distribution of the breakdate estimation and showed how to construct confidence intervals for the breakdate. These confidence intervals are easy to calculate and hence are very useful in applications, as they indicate the degree of estimation accuracy. Bai, Lumsdaine and Stock (1998) extended this analysis to multiple time series with simultaneous structural breaks. They showed that using multiple time series improves estimation precision. Bai and Perron (1998) discussed the general case of a partial structural changes model where not all parameters are subject to shifts. Both fixed and shrinking magnitudes of shifts were studied and they obtain the rates of convergence for the estimated break fractions.

Chong (1995) and Bai (1997) showed how to estimate multiple breakdates sequentially. The key insight is that when there are multiple structural breaks, the sum of squared errors (as a function of breakdate) can have a local minimum near each breakdate. Thus, the global minimum can be used as a breakdate estimator, and the other local minima can be viewed as candidate breakdate estimators. The sample is then split at the breakdate estimate, and analysis continues on the subsamples. Bai (1997) showed that important improvements are obtained by iterative refinements: reestimation of breakdates based on refined samples. Liu, Wu and Zidek (1997) also considered multiple shifts in a linear model estimated by least squares. They studied the rate of convergence of the estimated break dates, as well as the consistency of a modified Schwarz model selection criterion to determine the number of breaks.

In an early and independent study, Huang, Liu and Zhang (1985) considered the cases where the parameters of the model may vary between various unknown connected subsets, and the functional form of the model is not fixed. Moreover, their method is capable of detecting the changing points, estimating the regression lines and performing significance test without prior information about the changing points and functional form of the changes. It is because of these merits that we are going to base our analysis mainly on this method and more details will be discussed in Chapter 3

2.2 Information Criteria

Full reality cannot be included in a model; thus we seek a good model to approximate effects or factors supported by the empirical data. The selection of an appropriate

approximating model is critical to statistical inference from many types of empirical data. Information theory (see Guiasu, 1977; Cover and Thomas, 1991) and, in particular, the Kullback-Leibler (K-L) "information" form the deep theoretical basis for data-based model selection (Kullback and Leibler, 1951). Akaike (1973) found a simple relationship between expected Kullback-Leibler information and Fisher's maximized log-likelihood function. This relationship leads to a simple, effective, and very general methodology for selecting a parsimonious model for the analysis of empirical data.

2.2.1 Introduction

In most statistical analysis it is taken for granted that the family of the probability distribution functions (p.d.f.) may be correctly specified on a priori grounds. Uncertainty exists, therefore, only with reference to the values of parameters involved in the specified family of probability distribution functions. In practice, however, we are seldom in such an ideal situation; that is, we are more or less uncertain about the family to which the true p.d.f. might belong. It may be very likely that the true distribution is in fact too complicated to be represented by a simple mathematical function such as those given in ordinary textbooks.

In practice we approximate the true distribution by one of the alternative p.d.f.'s. Needless to say, we try to choose the most adequate p.d.f. with due thought to a priori considerations. A p.d.f. specified by a convenient mathematical function is usually termed a model. For further analysis a postulated model is identified at least

tentatively with the true distribution. To put it differently, in the process of conventional statistical analysis a sharp distinction is seldom drawn between the postulated model and the true distribution.

In recent years, however, more and more emphasis has been placed on the problem of model identification, that is, how to identify the model when it cannot be completely specified from a priori knowledge. The basic attitude toward the problem of how to propose and analyze statistical criteria for model identification in regression analysis is to recognize the fact that a certain amount of discrepancy inevitably exists between the true distribution and the model. The best we can do in trying to cope with this sort of situation is to identify the most adequate model relatively among a given set of alternatives. The adequacy of a model needs to be quantified by defining a suitable measure of the difference of the model from the unknown true distribution.

It is expected intuitively that a more complicated model will provide a better approximation to reality. But, on the contrary, in most practical situations a less complicated model is likely to be preferred if we wish to pursue the accuracy of estimation. This consideration leads us naturally to the so-called principle of parsimony. That is, more parsimonious use of parameters should be pursued so as to raise the accuracy of estimates for unknown parameters in a model. In general, closeness to the true distribution is incompatible with parsimony of parameters. These two criteria form a trade-off: if one pursues one of the criteria, the other must be necessarily sacrificed. The multiple correlation coefficient adjusted for the degrees of

freedom may be the most commonly used statistic that incorporates the two incompatible criteria into a single statistic.

Akaike (1973) proposed a more general as well as more widely applicable statistic that ingeniously incorporates the above two criteria. As it is based on the Kullback-Leibler Information Criterion, Akaike's statistic is called the Akaike Information Criterion and is abbreviated as the AIC. What is proposing here is not an estimation procedure but a procedure for model identification. More precisely, in the present context we aim to develop a procedure for identifying the most adequate model from a given set of alternatives rather than estimating unknown parameters involved in a given true model.

2.2.2 The Kullback-Leibler Information

Well over a century ago measures were derived for assessing the "distance" between two models or probability distributions. Most relevant here is Boltzmann's concept of generalized entropy in physics and thermodynamics (see Akaike, 1983, for a brief review). Shannon (1948) employed entropy in his famous treatise on communication theory. Kullback and Leibler (1951) derived an information measure that happened to be the negative of Boltzmann's entropy: now referred to as the Kullback-Leibler information. The motivation for Kullback and Leibler's work was to provide a rigorous definition of "information" in relation to Fisher's "sufficient statistics". The

K-L information has also been called the K-L discrepancy, divergence and number -- these terms are synonyms, we tend to use information in the material to follow.

The Kullback-Leibler information can be conceptualized as a directed "difference" between two models, say f and g (Kullback, 1959). Strictly speaking this is a measure of discrepancy; it is not a simple distance because the measure from f to g is not the same as the measure from g to f – it is directed or oriented. The K-L information is perhaps the most fundamental of all information measures in the sense of being derived from minimal assumptions and its additivity property. The K-L information between models is a fundamental quantity in science and information theory (Akaike, 1983) and is the logical basis for model selection as defined by Akaike. In the heuristics given here, we will assume the models in question are continuous probability distributions denoted as f and g . It is useful to think of f as full reality and let it have (conceptually) an infinite number of parameters. This "crutch" of infinite dimensionality at least keeps the concept of reality even though it is in some unattainable perspective.

Let g be the approximating model being compared to (measured against) f . We use x to denote the data being modeled and θ to denote the parameters in the approximating model g . We use $g(x)$ as an approximating model, whose parameters must be estimated from these data (in fact, we will make this explicit using the notation $g(x|\theta)$, read as the approximating model g for data x given the parameters θ). If the parameters of the model have been estimated, using ML or LS methods, we will

denote this by $g(x|\hat{\theta})$. Generally, in any real world problem, the model $g(x|\theta)$ is a function of sample data (often multivariate) and the number of parameters (θ) in g might often be of high dimensionality. Finally, we will want to consider a set of approximating models as candidates for the representation of the data; this set of models is denoted as $\{g_i(x|\theta), i=1, \dots, R\}$. It is critical that this set of models be defined prior to probing examination of the data (i.e., no data dredging).

The K-L information between the models f and g is defined for continuous functions as the (usually multi-dimensional) integral

$$I(f, g) = \int f(x) \log\left(\frac{f(x)}{g(x|\theta)}\right) dx,$$

where $\log$ denotes the natural logarithm. Kullback and Leibler (1951) developed this quantity from "information theory", thus they used the notation $I(f, g)$; $I(f, g)$ is the "information" lost when g is used to approximate f :

Of course, we seek an approximating model that loses as little information as possible; this is equivalent to minimizing $I(f, g)$ over g . f is considered to be given (fixed) and only g varies over a space of models indexed by θ . An equivalent interpretation of minimizing $I(f, g)$ is finding an approximating model that is the "shortest discrepancy" away from truth. We will use both interpretations as both seem useful.

The material above makes it obvious that both f and g (and their parameters) must be known to compute the K-L information between these two models. We see that this requirement is diminished as $I(f, g)$ can be written equivalently as

$$I(f, g) = \int f(x) \log(f(x)) dx - \int f(x) \log(g(x|\theta)) dx.$$

Note, each of the two terms on the right of the above expression is a statistical expectation with respect to f (truth). Thus, the K-L information (above) can be expressed as a difference between two expectations,

$$I(f, g) = E_f[\log(f(x))] - E_f[\log(g(x|\theta))],$$

each with respect to the true distribution f . This last expression provides easy insights into the derivation of AIC. The important point is that the K-L information $I(f, g)$ is a measure of the directed "discrepancy" between the probability models f and g .

The first expectation $E_f[\log(f(x))]$ is a constant that depends only on the unknown true distribution and it is clearly not known (i.e., we do not know $f(x)$ in actual data analysis.) Therefore, treating this unknown term as a constant, only a measure of relative, directed difference is possible (Bozdogan, 1987; Kapur and Kesavan, 1992) in practice. Clearly, if one computed the second expectation, $E_f[\log(g(x|\theta))]$, one could estimate $I(f, g)$, up to a constant (namely $E_f[\log(f(x))]$),

$$I(f, g) = \text{Constant} - E_f[\log(g(x|\theta))],$$

$$\text{or } I(f, g) - \text{Constant} = -E_f[\log(g(x|\theta))].$$

The term $(I(f, g) - \text{Constant})$ is a relative, directed difference between the two model f and g , if one could compute or estimate $E_f[\log(g(x|\theta))]$. Thus, $E_f[\log(g(x|\theta))]$ becomes the quantity of interest.

In data analysis the model parameters must be estimated and there is usually substantial uncertainty in this estimation. Models based on estimated parameters, hence on $\hat{\theta}$ not θ , represent a major distinction from the case where model parameters would be known. This distinction affects how we must use K-L information as a basis for model selection. The difference between having θ (we do not) and having $\hat{\theta}$ (we do) is quite important and basically causes us to change our model selection criterion to that of minimizing expected estimated K-L information rather than minimizing known K-L information (over the set of *a priori* models considered).

Thus, we use the concept of selecting a model based on minimizing the estimated Kullback-Leibler information

$$\hat{I}(f, g) = \int f(x) \log\left(\frac{f(x)}{g(x|\hat{\theta}(y))}\right) dx.$$

Consequently, we can determine a method to select the model g_i that on average minimizes, over the set of models $g_1, \dots, g_R$, a very relevant expected K-L information.

The K-L information can be made smaller by adding more structure, via parameters, to the approximating model g . However, when the added parameters are estimated (rather than being known or "given"), further uncertainty is added to the estimation of

the relative K-L information. Thus, for a fixed sample size, the addition of estimated parameters to a poor fitting model $\hat{g}_i$, thus getting a different model, $\hat{g}_j$, will allow the expanded fitted model to be closer to f . However, at some point, the addition of still more estimated parameters (to what may already be a suitable fitted model) will have the opposite effect and the estimate of the relative K-L information will increase. This fact represents the trade-off between bias and variance, or the trade-off between under-fitting and over-fitting the sample data, which is fundamental to the Principle of Parsimony.

2.2.3 Akaike Information Criteria and Some Modifications

Akaike (1973) showed that the maximized log-likelihood is biased upward as an estimator of the model selection criterion. Second, he showed that under certain conditions (these conditions are important, but quite technical), that this bias is approximately equal to K , the number of estimable parameters in the approximating model. Thus, an approximately unbiased estimator of the relative, expected K-L information is $\log(\zeta(\hat{\theta})) - K$. This result is equivalent to

$$\log(\zeta(\hat{\theta})) - K = \text{Constant} - \hat{E}_\theta[\hat{I}(f, g)].$$

Akaike's finding of a relation between the relative K-L information and the maximized log-likelihood has allowed major practical and theoretical advances in model selection and the analysis of complex data sets (Stone, 1982; Bozdogan, 1987;

deLeeuw, 1992). Akaike then defined "*an information criterion*" by multiplying by -2 ("taking historical reasons into account") to get

$$AIC = -2 \log(\zeta(\hat{\theta}|y)) + 2K,$$

where K is the number of independent parameters. For example, for the most familiar model in econometrics $Y_i = x_i' \beta + \varepsilon_i$, x_i 's and β s are p-vectors, if we assume that the errors are distributed as $N(0, \sigma^2)$, AIC can be expressed as

$$AIC(\kappa) = n \log(S_\kappa / n) + 2k,$$

where S_κ denotes the residual sum of squares under the model M_κ .

This has become known as "*Akaike's Information Criterion*" or *AIC*. Here it is important to note that AIC has a strong theoretical underpinning, based on information theory and Kullback-Leibler information within a realistic data analysis philosophy that no model is true, rather truth as f is far more complex than any model used. Akaike's inferential breakthrough was realizing that a predictive expectation version of the log-likelihood could (as one approach) be used to estimate the relative expected K-L information between the approximating model and the true generating mechanism. Thus, rather than having a simple measure of the directed difference between two models (i.e., the K-L information), one has instead an estimate of the expected relative, directed information between the fitted model and the unknown true mechanism (perhaps of infinite dimension) which actually generated the observed data. Because the expectation of the logarithm of $f(x)$ drops out as a constant, independent of the data, AIC is defined without specific reference to a "true model" (Akaike, 1983).

Thus, one should select the model that yields the smallest value of AIC because this model is estimated to be "closest" to the unknown reality that generated the data, from among the candidate models considered. This seems a very natural, simple concept; select the fitted approximating model that is estimated, on average, to be closest to the unknown f . Perhaps none of the models in the set are good, but AIC attempts to select the best approximating model of those in the candidate set. Thus, every effort must be made to assure that the set of models is well founded.

Akaike derived an estimator of the K-L information quantity; however, AIC may perform poorly if there are too many parameters in relation to the size of the sample (Sugiura, 1978). Sugiura derived a second order variant of AIC that he called c-AIC, or CAI, where the penalty function takes the form of $2N(m_k + 1) / (N - m_k - 2)$. Here m_k is the corresponding number of independent variables.

Hurvich and Tsai (1989) further studied this small-sample (second order) bias adjustment which led to a criterion that is called AIC c.

$$AIC_c = -2 \log(\zeta(\hat{\theta})) + 2K \left(\frac{n}{n - K - 1} \right),$$

where the penalty term is multiplied by the correction factor $n/(n - K - 1)$. This can be rewritten as

$$AIC_c = -2 \log(\zeta(\hat{\theta})) + 2K + \frac{2K(K + 1)}{n - K - 1},$$

or equivalently,

$$AIC_c = AIC + \frac{2K(K+1)}{n-K-1},$$

where n is sample size and K denotes the number of parameters in the model. AIC_c merely has an additional bias correction term. If n is large with respect to K , then the second order correction is negligible and AIC should perform well.

Bhansali and Downham (1977) analyzed the effect of modifying $2K$ in the definition of AIC and suggested the simultaneous use of some variants of AIC such as AIC3 and AIC4, where AIC_α denotes AIC with $2K$ replaced by αK ($\alpha > 0$). This suggestion was directly based on the result of an analysis of the frequency distribution of correct order determination when the process is really of finite order.

Schwarz (1978), based on the "dimension consistent" criteria, derived the so termed SBC for Bayesian Information Criterion. It goes simply as

$$SBC = -2 \log(\zeta(\hat{\theta}|y)) + K \times \log(n).$$

SBC was derived in a fully Bayesian context with prior probability $1/M$ on each of M models and very vague priors on the parameters θ in each model in the set. SBC has been widely used in several applied fields.

Both AIC and SBC has been widely used for inference about model selection in econometric literature. In fact using any standard software automatically we get the values for both of them. However, it has been proved that these procedures, especially AIC, may give inconsistent or inefficient estimates in underspecified or overspecified

models. For example, if the assumed family contains the true density, then AIC is asymptotically unbiased but if it does not contain then AIC can be asymptotically biased. As an alternative TIC (Takeuchi Information Criterion) is becoming increasingly popular in information theoretic approaches.

According to Rao and Wu (2001), the model selection criterion of TIC can be expressed in the following form:

$$TIC(\theta) = -2 \log(\max_{\theta} \text{Likelihood}) + 2 \text{Tr}\{\hat{J}(G)^{-1} \hat{K}(G)^{-1}\},$$

where $\hat{J}(G)$ and $\hat{K}(G)$ are consistent estimates of $J(G)$ and $K(G)$ which are the expectation of outer product and inner product expression respectively under the true distribution.

Basically, AIC was derived under the assumptions that the estimation is by MLE and the more importantly the parametric family of distributions includes the true model. But TIC is an alternative when the candidate model is not close to the true model. When the true model is included in the set of candidate models TIC becomes AIC.

2.3 Simultaneous Equations Model

The literature on simultaneous equations models is enormous and continually expanding. An extensive use of simultaneous equation models has been made in the econometric models built by several econometricians. Even constructing a complete and current bibliography is a considerable undertaking. Three surveys in the

Handbook of Econometrics, Vol. 1 (Griliches and Intrilligator, 1983) provided a good overview of the literature, including one by Hsiao on identification, one by Hausman on specification and estimation, and one by Phillips on small-sample properties of estimators. Other extensive sources included work by Judge, Griffiths, Hill, Liitkepohl and Lee (1985) and Fomby, Hill and Johnson (1984). The work by Schmidt (1976) contained many useful theorems and results.

Simultaneous equation model has been widely used in econometric literatures; nevertheless there are only a few results available on analyzing and testing the stability of the coefficients in each structural equation. Andrews and Fair (1988) discussed the problem in a general setting, while more concrete situations are studied in Lo and Newey (1985) and Erhat (1983). The former work extended Chow's (1960) tests to simultaneous equations and proposed a simple Wald test, composed of two-stage least-squares (2SLS) estimator and the estimate of its covariance matrix. Erhat (1983) advocated an exact F test for the cases when there are inadequate degrees of freedom. It also constituted an extension of Gile's (1981) result, where CUSUM and CUSUM of squares tests for parameter stability in a single structural equation were developed. The case of deficient observation was discussed by Honda (1992) who considered how to test the stability of coefficient parameters in a single structural equation with insufficient observations in the second sample. Relevant literatures also included Dufour, Ghysels and Hall (1994) and Goldfeld and Quandt (1976b). However, it is a common drawback of most of the tests mentioned above that the switching point is assumed to be known a priori, which is often not the case in applied

research. Current literature, in addition, only examines the structural change of one single equation belonging to a simultaneous equation model rather than considering the structural breaks across the entire system.

2.4 Concluding Remarks

Structural change is pervasive in economic time series relationships. Government interventions and policy changes at designated times can influence both economic structure and market structure, and it can be quite perilous to ignore them. Inferences about economic relationships can go astray, forecasts can be inaccurate, and policy recommendations can be misleading or worse (Hansen, 2001). The literature review shows that the new methods developed in the past few years are useful aids in econometric model specification, analysis and evaluation.

From what have been reviewed above, we find that literature contains a large amount of work on issues related to structural changes, most of which is specially designed for the case of a single change (See Andrews, 1993; Andrews and Ploberger, 1994). In comparison, the literature addressing issue of multiple structural changes is relatively sparse. Recent contributions included the comprehensive treatment of Andrews, Lee and Ploberger (1996) and Garcia and Person (1996). Yet most of existing work only focuses on the problem of testing rather than estimation for structural changes. Recent developments of estimating structural changing points included Chong (1995) and Bai and Person (1998) who showed how to estimate multiple breakdates sequentially. Flowever, the sequential estimates are not guaranteed to be identical to those obtained

by global optimization in that sequential method starts from two part separation and the first break point will be fixed as given once it is detected, which will all but leave the rest of optimization subject to this constraint. That means further break points can be only estimated based upon the condition that the first structural break occur at the estimated point. Results from this procedure may not be in line with the true data generation function and are not guaranteed to be identical with global optimization.

In an independent study, Liu, Wu and Zidek (1997) considered multiple shifts in a linear model estimated by least squares and propound a modified Schwarz model selection criterion to determine the number of breaks. An early study by Huang, Liu and Zhang (1985) considered multiple structural changes in a linear model estimated by least-squares and proposed an information criterion for the selection of the number of changes. In this thesis, we will address the issue under less restrictive assumption and in a more general framework where different forms of information criteria will be examined and other estimating criteria other than OLS are allowed.

Literature on analyzing and testing the stability) of coefficients in simultaneous equations included Gile (1981), Erlat (1983), Lo and Neway (1985) and Andrews and Fair (1988). But no comprehensive treatment has been seen to detect unknown changing points without any prior information and no results available to estimate structural changes other than testing. More importantly, currently literature on simultaneous model only looks at single equation among the whole system, which neglects information contained in other equations. Intuition would surely suggest that

full information, or systems methods of estimation, is asymptotically better than limited information methods and brings efficiency gain in this respect.

Therefore what we plan to do in this thesis is first to propound the estimation of all the structural breakpoints concurrently which, in comparison with the sequential 'one-at-a-time' estimation, guarantees global optimization of the target function. Computational simulation is further conducted where estimating breakpoints concurrently signifies the correct separation of data sample. Second, the dissertation investigates structural change analysis in SEM which enables us to find the possible switching points and to formalize the way the switching point could be traced. In particular, the RSM we proposed is generalized to SEM that takes into account both limited information method and system method, which has not been studied in current literature. The single equation estimation of structural changes is able to detect the structural instability triggered from individual equation, which is not necessary from the entire simultaneous equation system; whereas system estimation, or full-information method, allows detecting structural changes across the entire system. Our methodology, to be presented in the following chapter, considers the more general cases and enjoys the advantages of simplicity and flexibility.

CHAPTER 3 RECURSIVE SEGMENTATION METHOD

In this chapter we present a Recursive Segmentation Method (RSM) which is able to correctly detect and estimate the existence and the timing of unknown structural changing points. This method provides a systematic and operational approach that can accurately detect structural changing points without any prior information or knowledge of the pattern and timing of possible structural shifts. The method is based on the principle of dynamic programming and allows global minimizers to be obtained using a number of sums of squared residuals rather than an exhaustive grid calculation.

The basis of the method, for specialized cases, is documented by Fisher (1958) and Guthery (1974). However, thorough treatment and description in the econometrics literature seems still sparse. We hope to fill a small part of that breach in this chapter where the main ideas are rediscovered. Main procedures of the method are depicted in Appendix I.

3.1 Procedures of Recursive Segmentation Method

The structure of a model normally refers to the functional form, variables and parameters included in the model and their domains and the probability distribution of the random disturbances of the model.

We now give a more precise description of the problem addressed in this thesis. For single equation model, if we use y to represent the dependent variable, x^T for the

vectors of independent variables, $\tilde{\theta}$ for the vectors of parameters and $\tilde{\varepsilon}$ for the vectors of random disturbances. The structure of the model can be represented mathematically as:

$$g(y, x^T, \tilde{\theta}, \tilde{\varepsilon}) = 0. \quad (3.1.1)$$

Assume i is a deterministic variable associated with the socio-economic system(s) described by the model. When i varies in different domains, if anyone of the corresponding factors of the model, i.e. the functional form, variables and parameters, and the probability distribution of the random disturbances also changes, the model's structure change is said to be changed with respect to variable i and i is called the index variable.

Accordingly, if an econometric model based on a given set of data reflects the characteristics of structure change, these types of models are called the structural changing model. Mathematically, the structural changing model corresponding to (3.1.1) can be written as:

$$\begin{aligned} g_1(y(i), x^T_1(i), \tilde{\theta}_1, \tilde{\varepsilon}_1(i)) &= 0 \quad \text{when} \quad i \in I_1, \\ g_2(y(i), x^T_2(i), \tilde{\theta}_2, \tilde{\varepsilon}_2(i)) &= 0 \quad \text{when} \quad i \in I_2, \\ &\vdots \\ g_j(y(i), x^T_j(i), \tilde{\theta}_j, \tilde{\varepsilon}_j(i)) &= 0 \quad \text{when} \quad i \in I_j. \end{aligned} \quad (3.1.2)$$

Here $I_1 \cup I_2 \cup \dots \cup I_j = I$ (I is the domain of i in the set of samples.) Usually, we may decompose the domain of index variable into non-overlapping intervals

$I_s \cap \mathbf{I}_t = \emptyset (s \neq t)$. The intervals correspond to different regression models. $x_s^T(i)$ is the independent variable of different dimensions, corresponding to different $\mathbf{I}_s$. The index variable i can be either continuous or discrete. Here, $\tilde{\theta}$ and $\tilde{\varepsilon}$ are the vectors of parameters and random disturbances respectively, where $\tilde{\varepsilon}$ are independent and identically distributed random variables having mean 0 and common variance σ^2 . It is assumed that $\tilde{\varepsilon}$ is independent of $x_s^T(i)$. The $\tilde{\varepsilon}$ need not be i.i.d.; they could have different distributions from one modeling interval to another. We adopt the assumption for the sake of convenience and simplicity. Except in unusual situations, the model parameters as well as the number of structural break must be estimated.

Suppose $\{Z_i\}$ is the set of vectors of samples, and we have divided the set $\{Z_i\}$ into l segments: $\{Z_{i_1}, Z_{i_1+1}, \dots, Z_{i_2-1}\}, \{Z_{i_2}, Z_{i_2+1}, \dots, Z_{i_3-1}\}, \dots, \{Z_{i_l}, Z_{i_l+1}, \dots, Z_N\}$. Here, $1 = i_1 < i_2 < \dots < i_l < N$ are the subscripts of the first data point of each segment. We denote such segmentation as $P(N, l) = \{i_1, \dots, i_l\}$.

We define target function as the statistics that describe the goodness-of-fit of the model using certain estimation criteria. In order to differentiate between goodness-of-fit of individual segment and that of the whole model, the value of the target function within a segment is called the diameter, denoted as d , while target function refers specifically to the overall goodness-of-fit of the model over the range of the whole sample. We denote it as e . Obviously, e is a function of the diameter of every segment. To illustrate the segmentation procedure, the most commonly used criterion in

regression, ordinary least square (OLS), is used in this section. The residual sum of squares for error would provide an indication of goodness of fit. This gives specific form to the target function and diameters and consistency and asymptotic normality of estimates would thus be gained within intervals. Other methods, like maximum likelihood and minimax criteria may also be applied.

Assume the functional form of the model is $y_t = f(x_t^T, \beta) + \varepsilon_t$, using OLS, the diameter of the segment from point i_s to $i_{s+1}-1$ is defines as:

$$d(i_s, i_{s+1}-1) = \min_{\beta^{(s)}} \sum_{t=i_s}^{i_{s+1}-1} [(y_t - f(x_t, \beta^{(s)}))]^2 \quad (s=1, 2, \dots, l). \quad (3.1.3)$$

And e is defined as:

$$e[p(N, l)] = \min_{\beta^{(s)}} \left\{ \sum_{s=1}^l \sum_{t=i_s}^{i_{s+1}-1} [(y_t - f(x_t, \beta^{(s)}))]^2 \right\} \quad (s=1, 2, \dots, l). \quad (3.1.4)$$

When $\beta^{(s)}$ ($s=1, 2, \dots, l$) are all different, from (3.1.4) we have:

$$e[p(N, l)] = \sum_{s=1}^l \left\{ \min_{\beta^{(s)}} \sum_{t=i_s}^{i_{s+1}-1} [(y_t - f(x_t, \beta^{(s)}))]^2 \right\} = \sum_{s=1}^l d(i_s, i_{s+1}-1). \quad (3.1.5)$$

Equation (3.1.5) shows that by the construction of the method, the target function $e[p(N, l)]$ can be decomposed into the sum of individual diameters. This is to say, given l , the overall optima with respect to β can be achieved by optimizing each segments (since the segments are independent from each other). The ultimate goal is to obtain the optimal segmentation: $\tilde{p}(n, l) = \{\tilde{i}_1, \tilde{i}_2, \dots, \tilde{i}_l\}$, which minimizes the target function:

$$e[\tilde{p}(N, l)] = \min_{p \in p(N, l)} e[p(N, l)]. \quad (3.1.6)$$

Here $p(N, l)$ is the set of all possible l segments having $\begin{bmatrix} N \\ l-1 \end{bmatrix}$ number of elements.

Because the target function is the additive function of diameters, it is obvious that $e[p(N, l)]$ satisfies the separability condition in a multi-stage decision-making problem in dynamic programming. Thus, instead of an exhaustive grid search, we use the technique of backward recursive optimization, the following relationship can be derived:

$$\begin{aligned} e[\tilde{p}(n, s)] &= \min_{s \leq j_s < n} \{e[\tilde{p}(j_s - 1, s - 1)] + d(j_s, n)\} \\ &= e[\tilde{p}(\tilde{j}_s - 1, s - 1)] + d(\tilde{j}_s, n) \quad (1 < s < n \leq N) \end{aligned} \quad (3.1.7)$$

In (3.1.7), $\tilde{p}(j_s - 1, s - 1)$ represents the optimal $s - 1$ segmentation of first $j_s - 1$ observations, while $\tilde{j}_s$ is the corresponding value of j_s that minimizes the value of (3.1.7), denoted as $g(n, s)$.

Especially when $s=2$, (3.1.7) becomes:

$$\begin{aligned} e[\tilde{p}(n, 2)] &= \min_{2 \leq j_2 < n} \{d(1, j_2 - 1) + d(j_2, n)\} \\ &= d(1, \tilde{j}_2 - 1) + d(\tilde{j}_2, n) \quad (2 < n \leq N) \end{aligned} \quad (3.1.8)$$

Equations (3.1.7) and (3.1.8) are the main devices to obtain optimal segmentation.

Using (3.1.7) recursively, we can obtain all $e[\tilde{p}(n, l')]$, $l' = 3, 4, \dots, (l' < n < N)$ and the corresponding $g(n, l')$; For any given l , ($l = 1, 2, \dots, [N/m]$) (N is the sample size while m is the number of independent variables), check $\tilde{i}_l = g(N, l)$, which is the subscript of the first data point of the l^{th} segment. Then, from

$$\tilde{i}_s = g(\tilde{i}_{s+1} - 1, s), (s = 1, 2, \dots, l). \quad (3.1.9)$$

We obtain all the subscripts of the first data points of the rest segments,

$\tilde{l}_{l-1}, \tilde{l}_{l-2}, \dots, \tilde{l}_2$. Thus the optimal segmentation is derived as:

$$\tilde{p}(N, l) = \{\tilde{l}_1, \tilde{l}_2, \dots, \tilde{l}_l\}. \quad (3.1.10)$$

The critical step of the RSM method is the recursive equation (3.1.7). That is why it is so termed '*recursive segmentation method*'. By using this backward recursive optimization in dynamic programming we avoid the exhaustive grid search in detecting and finding out the structural change points.

3.2 Determining Optimal Number of Segments

By using recursive classification, we can obtain different recursive segmentations, given different number of segments¹. If and only if $l = l^0$ (l^0 is the true number of subdomain), the optimal l segmentation can optimally fit in the given data and reflect the structural changing characteristics of the model. In practice we may not have such information of the exact number of changes or the number of segments. Another standard problem is that an improvement in the objective function is always possible by allowing more breaks. Therefore, in determining optimal l^0 , we have to take both the goodness-of-fit and the efficiency of the model into consideration. Information criterion which is derived from maximizing the posterior likelihood in a model

¹ Actually we do not consider literally "all" possible breakdates. We cannot consider breakdates too close to the beginning or the end of the sample, as there are not enough observation to identify the subsample parameters.

selection paradigm and enjoys widespread use in model identification provides a natural baseline.

In the multiple change point model, it has been found that sequential estimation is consistent to estimate the model without treating all break points concurrently. The basic logic is to estimate the break point using the whole sample data and then to divide the sample into two sub-samples at the estimated break point which allows the greatest reduction in the sum of squared residuals. Estimate an additional break whenever the sub-sample fails the parameter constancy test. This step is repeated until all the sub-domains do not reject the null hypothesis of no structural changes. Although it yields consistent estimates of the break points, the estimates are not guaranteed to be identical to those obtained by global minimization.

Here we will discuss how to determine the number of breaks using model selection criteria, and in particular, information criterion. From what has been introduced in Chapter 2 about the literature in this area, we have known that the general form of Information Criterion (IC) is:

$$IC_s = -2\ln[L(M_s)] + P(m_s) \quad (3.2.1)$$

where M_s represents the s^{th} model, m is the corresponding number of independent variables. $L(M_s)$ is the value of the maximum likelihood function of the model, which takes into account of the goodness-of-fit of the model. $P(m_s)$ is the penalty function, which is an increasing function with respect to m_s and penalizes the index when the model's efficiency decreases. Thus, the RSM method should choose the one with

smallest IC value. By using computer simulation, the investigation of the penalty function with different values of N , m and the variance σ^2 suggests that the AIC function by Akaike, SBC of Schwarz and CAI of Sugiura are all appropriate.

Based on the results obtained in section 3.1, for different number l , we have found the optimal segmentations. Now the determination of l^0 will be obtained according to the IC criteria, i.e., the one which allows the greatest reduction in the IC value:

$$l^0 = \arg \min IC(l^s, i_1, i_2, \dots, i_{l^s}).$$

Another more intuitive way of determining l , as introduced by Huang, Liu and Zhang (1985), is through graphical method, i.e., to plot the value of target function against number of breaks l .

Figure 1 Using Diagram to Determine The Real l

When the slope of the line converges, the corresponding l value is chosen. In Figure 1, the appropriate l can be 3 or 4.

3.3 Combining Similar Segments and Test of Significance

After determining number of segments l^* , we should try to test the significance of the structural changes, and combine the segments that have similar structures. This step not only simplifies the model, increases the model's degree of freedom, but also helps to enhance the effectiveness of the estimation of the coefficients and the accuracy of the information provided by the regression model. Particularly, it helps strengthen the model's predicting power in the case whereby the last segment of the model has few data points.

Figure 2 Illustration of Combining Procedure

In Figure 2, it is presented a simple linear relationship between dependent variable Y and explanatory variable X, which is subjected to some occasional shift:., G_i is the data for each segment, I_l refers to the data set after the combination of G_i and G_j , which as depicted in the figure, share similar structure.

In the process of determining optimal number of segments, it is ensured that the structural differences are significant between all the joint segments. Thus, we should focus on the structural differences between disjoint segments. Relative Akaike Information Criteria (AIC) Test can be used for this purpose.

Suppose a structural changing model M_1 with three segments identified

$$\begin{aligned} M_1 : y_t &= f_1(x_t^1, \gamma_1) + \varepsilon_t^1 & t \in I_1, \\ y_t &= f_2(x_t^2, \gamma_2) + \varepsilon_t^2 & t \in I_2, \\ y_t &= f_3(x_t^3, \gamma_3) + \varepsilon_t^3 & t \in I_3, \end{aligned} \quad (3.3.1)$$

where $x^{(i)}$ and γ_i are the vectors of explanatory variables and parameters. And the combined model can be represented as:

$$M_2 : \begin{cases} y_t = f_1(x_t^{(1)}, \gamma_1) + \varepsilon_t^{(1)} & t \in I_1 \cup I_3 \\ y_t = f_2(x_t^{(2)}, \gamma_2) + \varepsilon_t^{(2)} & t \in I_2 \end{cases} \quad (3.3.2)$$

From (3.2.1), we can compute the IC values of M_1 and M_2 respectively. By referring to their IC, we will be able to tell the "optimal" model from the rest. That model should have the smaller IC, indicating a better approximation of the real data. If we denote $\Delta IC = IC_1 - IC_2$, when we have $\Delta IC > 0$, the structural difference between the first and the third segment is considered to be not "significant" and we should choose M_2 , i.e., the combined model. When $\Delta IC < 0$, that implies that the structure is significantly different and there is no gain by combining the 1st and 3rd segment. The relative AIC test procedure can only provide the description of the relative effectiveness of "segmented model" and "the combined model" under certain context. The selection using such ideology is called "relative" testing procedures, in order to

distinguish with ordinary test procedures. This "relative" testing procedure has its practical meaning in that it can combine the segments with similar structure and efficiency is gained in this respect.

3.4 Comparison of Recursive Segmentation Method with Sequential Testing

In this section, simulation studies are used to assess the performance of the procedure proposed in the preceding section. We are particularly interested to look into the capability of our method of correctly identifying all the multiple structural changing points and to demonstrate the circumstances where our method outperforms the commonly used sequential testing in detecting structural breaks.

Sequential estimation starts from looking for the first break point; once it is identified, the sample will be split into two subsamples separated at this first estimated break point. Each subsample may be then partitioned further at the break point which allows the greatest reduction in the sum of squared residuals. This process will be continued until all the break points, if the number is given, are selected. If the total number of structural change or the total number of segments is not known a priori, which is more likely to be of particular relevance in practice, sequential method will suggest the following treatment. Again start by estimating a model with one break (two segments). Then perform parameter-constancy tests for every subsample, adding a break to a subsample whenever the test is rejected. This process is repeated by increasing the

number of subsample sequentially until each subset of the sample does not reject the null hypothesis of no structural changes.

Sequential method of testing and estimating of multiple structural changes yields consistent estimates of the break points. However, the estimates are not guaranteed to be identical to those obtained by global optimization. The basic idea behind is to optimize a target function constructed under certain estimating criterion and the difference between our method and the sequential method may origin from the way that target function is optimized, with or without any constraint attached. The fact that sequential method starts from two part separation and that the first break point will be fixed as given once it is detected will all but leave the rest of optimization subject to this constraint. That means further break points can be only estimated based upon the condition that the first structural break occur at the estimated point. Results from this procedure may not be in line with the true data generation function and are not guaranteed to be identical with global optimization. While our proposed method using Recursive Segmentation procedure and information criteria will yield optimal estimation of the break point consistent with global minimization (or maximization) of the duly defined target function. By detecting all those multiple structural changing points concurrently, our method effectively avoids the risk from sequential testing. The following simulation results verify the above assertion.

We design 3 model with two structural changes, i.e., three segments and sample size is chosen to be 10 for each subsample. The basic form of equation is

$$y_t = aX_t + \varepsilon_t,$$

where X sample was randomly generated from a uniform (0, 10) distribution. The error terms were randomly generated from a normal distribution with mean 0 and variance 2. The values of parameter for each subsample are chosen to be $a_1 = 2, a_2 = 0.5, a_3 = -1$. Therefore under this setting, the Y sample was created exactly as the equations given below,

$$\begin{aligned} y_t &= a_1 X_t + \varepsilon_t & 1 \leq t \leq 10, \\ y_t &= a_2 X_t + \varepsilon_t & 11 \leq t \leq 20, \\ y_t &= a_3 X_t + \varepsilon_t & 21 \leq t \leq 30. \end{aligned}$$

The above simulation and the procedures of RSM are realized by commands written in Statistical Analysis System (SAS) program. Programming details are provided in Appendix II.

We first, after generating the whole sample data, use sequential method to detect structural break points. Starting from two-segment separation, we find surprisingly the suggested breakpoint (estimated from the greatest reduction of sum of squared residual) is not any of the designed threshold, but the sample's middle point (the 15th data point). Hold this as given, we then search for another break point and the results suggest another threshold to be the 25th data point.

On contrast, we apply our recursive segmentation method and the above mentioned algorithm is able to detect those two structural changes concurrently. And the SAS program correctly indicates that the structural changes occur at the 10th and 20th data points, which is consistent with our initial simulation design. Results from sequential

method and our RS method are compared in the following figure; where we see evidences strongly support our method.

Figure 3 Comparison of Sequential Method and Recursive Segmentation Method

Since OLS is used as the estimation criterion, we can take a further look at the full sample sum of squared residual, after segmentation, resulted from both methods. According to sequential method, the whole sample data is suggested to be divided into three parts at the estimated structural break points, the 15th and 25th point. Based on this kind of separation, we estimate each subsample and sum up the SSE from each subset and obtain the SSE for the whole sample (30 observations) which is 213.341. Compared to this, results from our recursive segmentation method suggests two structural breaks at the 10th and the 20th data points. Under this separation, we calculate the whole sample SSE after taking into account the estimated structural change and we are able to reach a much smaller value for SSE, which is 43.776. This provides good evidence that our method suggests a better segmented model with a higher goodness-of-fit.

CHAPTER 4 RECURSIVE SEGMENTATION METHOD IN SIMULTANEOUS EQUATION MODEL

The procedure of Recursive Segmentation is applicable, as will be shown in this chapter, to simultaneous equation model. So are the determination of number of structural break and the relative tests of significance of structural changes, as presented earlier in previous chapter.

The standard linear simultaneous equations model is considered first, where all identities are assumed to have been substituted out of the system of equations:

$$YB + Z\Gamma = U$$

where Y is the $N \times M$ matrix of jointly dependent variables, Z is the $N \times K$ matrix of predetermined variables, and U is a $N \times M$ matrix of the structural disturbances of the system. Thus the model consists of M equations and N observations. We have assumed B is nonsingular, $rk(Z) = K$, and that all equations satisfy the rank condition for identification. Lastly, the orthogonality condition applies between the predetermined variables and structural errors, and the second order moment matrices of the current predetermined and endogenous variables are assumed to have nonsingular probability limits. The structural errors are assumed to be independent and identically distributed. Then structural change is said to be present within the range of the index i if

$$Y(i)B_1 + Z(i)\Gamma_1 = U_1(i) \quad i \in I_1$$

$$Y(i)B_2 + Z(i)\Gamma_2 = U_2(i) \quad i \in I_2$$

$$Y(i)B_l + Z(i)\Gamma_l = U_l(i) \quad i \in I_l \quad (4.1)$$

Here $I_1 \cup I_2 \cup \dots \cup I_l = I$ (I is the domain of i in the set of samples). Again, we have

$I_s \cap I_t = \emptyset \quad (s \neq t)$ The index variable i , in time series data, corresponds to time or observation.

Now with the index or partitioning variable identified, the inferential problem confronting us involves three parts: (1) the specification of the number of segments in the model, l ; (2) the detection of the change point $\{i\}$, or the boundaries of intervals over which each of the model pieces applies; (3) the estimation of the model parameters within each subdomain. If l and the $\{i_s\}$ were specified, step 3 would simply consist of applying the classical theory, interval by interval. Summing the residual sums of squares for the various intervals yields an overall index of the quality of fit of the segmented model. With l fixed, the $\{i_s\}$ may be estimated by minimizing this index. Further minimization of the index to estimate l will be based on information criteria for model selection problem.

In estimating the appropriate sample separation in simultaneous equation system, there are two approaches to analyze the timing and form of structural changes, either to estimate equation by equation individually using a limited information estimator, or globally consider joint estimation of the entire system. In this section, we first deal

with the estimation of structural changes in one equation embedded in a system of simultaneous equations. We have parallel, more complex results for system methods of detecting structural changes. The corresponding discussion follows.

4.1 Single Equation: Limited Information Method

We shall, without loss of any generality, consider the first equation in the system of simultaneous equations and write it as

$$y_1 = Y_1\beta_1 + X_1\gamma_1 + u_1, \quad (4.2)$$

where y_1 and u_1 are $(N \times 1)$, $u_1 \sim N(0, \sigma^2 I)$, Y_1 is $(N \times g_1)$, X_1 is $(N \times m_1)$ and the elements of β_1, γ_1 identified.

The reduced form corresponding to (4.2) is

$$Y_1 = Z\Pi_1 + V_1, \quad (4.3)$$

where $Y_1 = (y_1, Y_1)$, $Z = (X_1, X_2)$, $\Pi_1 = (\pi_1, \Pi_1)$ and $V_1 = (v_1, V_1)$. Z is the $(N \times G)$ matrix of non-stochastic exogenous variables in the complete system, Π_1 is a $G \times (m_1 + 1)$ matrix of reduced form coefficients, and V_1 is a $G \times (m_1 + 1)$ matrix of reduced form disturbances whose rows are assumed to be normally and independently distributed with zero mean and covariance Ω_1 .

Now comparing (4.2) and (4.3), we have $u_1 = v_1 - V_1\beta_1 = V_1\beta_1^0$ where $\beta_1^0 = (1, -\beta_1)'$.

Thus, one may estimate u_1 by utilizing appropriate estimators for V_1 and β_1^0 . Then V_1

may be estimated by applying OLS to (1.3) to yield $\hat{V}_1 = Y_1 - Z\hat{\Pi}_1$, given that V_1 is

reduced form coefficient and OLS will give consistent estimation. Meanwhile, β_1^0 will be estimated from (4.2) using 2SLS. Thus, the appropriate estimator of u_1 would then be $u_1^* = \hat{V}_1 \cdot \hat{\beta}_1^0$.

Since we know that $\hat{V}_1 = M_Z Y_1$, where $M_Z = \mathbf{I} - Z(Z'Z)^{-1}Z'$, it follows that u_1^* may be obtained directly as the residual vector of the unrestricted OLS regress of $y_1 - Y_1 \hat{\beta}_1^0$ on Z , that is, regressing $y^* = Y_1 \hat{\beta}_1^0$ on Z . Denoting the $G \times 1$ coefficient vector of said regression by δ and u_1^* would be expressed alternatively as

$$u_1^* = y^* - Z\delta. \quad (4.4)$$

As shown in Harvey and Phillips (1980), conditional on $\hat{\beta}_1^0$, u_1^* has the same distributional property as the OLS residuals from the general linear regression model with well-behaved disturbances.

Now we proceed to the structural change analysis. Suppose we have structural change model in the form of (4.1) and have divided the entire sample set into l segments, and let $1 = i_1 < i_2 < \dots < i_l < N$ be the subscripts of the first data points of each segment. We denote such segmentation with N observation of l segments as $P(N, l) = \{i_1, \dots, i_l\}$.

Following the above definition and using OLS, the diameter of the segment from point i_s to $i_{s+1} - 1$, out of N observations, is defines as:

$$d(i_s, i_{s+1} - 1) = \min_{\delta} \sum_{t=i_s}^{i_{s+1}-1} (y_t^* - Z_t \delta_t)^2$$

$$= \sum_{t=i_s}^{i_{s+1}-1} (y_t^* - Z_t \hat{\delta}_t)^2 = \sum_{t=i_s}^{i_{s+1}-1} (u_t^*)^2 \quad s = 1, 2, \dots, l \quad (4.5)$$

where y_t^* is the t^{th} element of y^* matrix. And e is defined as:

$$e[p(N, l)] = \min_{\delta^{(s)}} \left\{ \sum_{s=1}^l \sum_{t=i_s}^{i_{s+1}-1} (y_t^* - Z_t \delta_t)^2 \right\} \quad s = 1, 2, \dots, l, \quad (4.6)$$

that when $\delta^{(s)}$ ($s=1, 2, \dots, l$) are all different, from (4.6) we have:

$$e[p(N, l)] = \sum_{s=1}^l \sum_{t=i_s}^{i_{s+1}-1} (y_t^* - Z_t \hat{\delta}_t)^2 = \sum_{s=1}^l d(i_s, i_{s+1} - 1) \quad (4.7)$$

Here $\hat{\delta}$ denotes the resulting estimates based on the given 1 partition $(i_1, i_2, \dots, i_l)$.

Substituting these estimates in the objective function and denoting the resulting sum of squared residuals as $SSE(i_1, i_2, \dots, i_l)$, the estimated break points $\{\tilde{i}_1, \tilde{i}_2, \dots, \tilde{i}_l\}$ can be alternatively expressed as $\{\tilde{i}_1, \tilde{i}_2, \dots, \tilde{i}_l\} = \arg \min_{i_1, i_2, \dots, i_l} SSE(i_1, i_2, \dots, i_l)$.

The same kind of logic to determine the optimal number of segments applies here for simultaneous equation case. Again, the determination of l^0 will be the one that minimize IC criteria, i.e., $l^0 = \arg \min IC(l^s, i_1, i_2, \dots, i_{l^s})$.

We allow different equation to change their structural at different timing and structural break from any equation among the system will cause the entire simultaneous equation model to change. On this account, the total number of structural change occurring to the whole system will be the multiple product of the number of breaks from individual

equation. This limited information method to estimate and detect structural changes can be applied to each equation in the system one by one, and therefore allow us to examine whether the fluctuation of one equation is transitory to others or the spill-over effect of structural breaks across equations in simultaneous equation system.

4.2 Full Information Method of Estimation

Structural change analysis using limited information method allows each equation among the system to react differently to external shocks, or the structural breaks. That means individual equation might change its structure at different timing. However, intuition would surely suggest that full information, or systems methods of estimation are asymptotically better than limited information methods which estimate the system one equation at a time, and therefore neglect information contained in other equations. Apparently system method brings efficiency gains in this respect. Now we examine how to detect the structural instability using the technique of joint estimation of the entire system of equations.

The stated discussion about structural changes detection and, in particular, the RSM method is also applicable to this circumstance. We may formulate the full system as

$$Y = Z\delta + U,$$

where $E(U) = 0$ and $E[UU'] = \Sigma \otimes I$. In line with the principle of system methods, the technique of three-stage least square is used here for joint estimation of the entire system globally. Thus the 3SLS estimator is

$$\hat{\delta}_{3SLS} = [\hat{Z}'(\Sigma^{-1} \otimes I)\hat{Z}]^{-1} \hat{Z}'(\Sigma^{-1} \otimes I)Y, \quad (4.8)$$

where $\hat{Z}$ is the IV estimator for 2SLS.

Again, the model is assumed to have 1–1 structural changes occurred in the whole sample period, i.e., l subsamples. Following the definition of diameter and target function stated in section 3.1, we have, after a choice of a normalization rule,

$$d(i_s, i_{s+1} - 1) = \sum_{h=1}^M d_h(i_s, i_{s+1} - 1), \quad (4.9)$$

where $d_h(i_s, i_{s+1} - 1)$ is the diameter of the h^{th} equation, one of M equations for the entire system, for the individual segment starting from i_s to $i_{s+1} - 1$. $d(i_s, i_{s+1} - 1)$ is the summation of all the diameters throughout the system for a particular segment.

Given the structural changes in the form of $P(N, l) = \{i_1, i_2, \dots, i_l\}$, we have

$$e[p(N, l)] = \sum_{s=1}^l d(i_s, i_{s+1} - 1) = \sum_{s=1}^l \sum_{h=1}^M d_h(i_s, i_{s+1} - 1). \quad (4.10)$$

Those corresponding diameters can be calculated from 3SLS estimators according to

(4.8). Similarly, we have the optimum of target function

as $e[\tilde{p}(N, l)] = \min_{p \in P(N, l)} e[p(N, l)]$. Again, the estimated break points will be

$$\{\tilde{i}_1, \tilde{i}_2, \dots, \tilde{i}_l\} = \arg \min_{i_1, i_2, \dots, i_l} SSE(i_1, i_2, \dots, i_l).$$

Since 3SLS estimation embeds the principle of GLS, different weight is therefore given to each equation according to the weighting matrix.

It is obvious that the technique of backward recursive optimization and the dynamic programming procedure are applicable and we can apply RSM method to detect the structural changes without grid search calculation. The use of full information or system method in model estimation fully utilizes the cross-equation correlations of the disturbances. In so doing, the structural change analysis is conducted with regard to the whole simultaneous equation system instead of with each equation at one time. The structural changes occurred are therefore assumed to affect the whole system, with all equations included, at the same timing.

Simulation studies have been conducted and the results show that RSM method is able to correctly detect the structural break for simultaneous equation models, both for an individual equation and for the entire system.²

² As pointed out by an examiner, it is possible to directly apply the RSM method to detect structural changes on each equation of the reduced form first, then to apply structural equation estimation method to estimate structural models based on estimated breakpoints and segmentation. This involves a combination of the method mentioned in the thesis about estimating structural changes using both limited and system method. Efficiency gain in estimation will therefore be brought about for models which are exactly identified.

CHAPTER 5 EMPIRICAL APPLICATIONS

In this chapter, we discuss two empirical applications of the method and the procedure proposed in this thesis. The first reevaluates some findings of Garcia and Perron (1996) and Bai and Perron (2003) on U.S. ex post real interest series. The second analyzes in more details Singapore's property price and the structural change of private housing market using simultaneous equations model.

5.1 U.S. Real Interest Rate

In this section, we analyze the U.S. ex-post real interest series and apply the RSM method to detect the structural changes concurrently.

The common assertion amongst literatures is that the real interest rate is not constant and this nonstationarity of the real interest rate has important implications, both for the policy determination and for some topics central to financial theory. Garcia and Perron (1996) considered the time series behavior of the U.S. ex-post real interest rate, which is the difference between the nominal interest rate and the inflation rate. Their Markov switching model indicated a statistical description which three states were allowed, say, from 1961-1973, 1973-1981 and 1981-1986. Bai and Perron (2003) reevaluated the time series properties using a sequential method and suggested a model with three breaks (1966:4, 1972:3, and 1980:3), or, four states. Our goal is to reconsider the time path of the ex-post real interest rate that allows nonstationarity in the form of infrequent changes in mean and variance.

To make our point as unambiguous as possible, we use the same data set as previous studies, which are quarterly series from 1961:1 to 1986:3. The real interest rate is constructed from the U.S. 90-days Treasury bill rate for the nominal interest rate and a quarterly inflation rate series constructed from the U.S. CPI non-seasonally adjusted. A graph of the data is presented in Figure 4.

Figure 4 Real Interest Rate

From the stationarity test, we found the series unstable. To allow for an arbitrary number of changes occurring at unknown times and to find those changing points concurrently, we use RSM method. Since the issue of interest is the presence of structural change, in the mean and variance of the series, we regress the interest rate on a constant regressor, as modeled in Bai and Perron (2003). The analysis of data using the recursive segmentation method is implemented by the program written in SAS. In Figure 5, we plot the whole sample sum of squared errors as a function of the number of breaks, i.e., the value for $e[\tilde{p}(103, l)]$ and corresponding l .

Figure 5 Sum of Squared Error VS No. of Breaks

As what can be seen in Figure 5, value of $e[\tilde{p}(103, l)]$ reduces dramatically, as the number of segments increases. Typically, its value starts to converge to 0 at point $l=3$ and $l=4$. Information criteria test is used here to obtain the optimal number of segments. Recall that the Schwarz Criterion (SBC) is defined by

$$SC(l) = \ln\{e[\tilde{p}(103, l)]/(n-l)\} + 2m_s \ln(N)/N.$$

The minimum value of SC_l is reported at $l=4$. Thus, the optimal number of segments is 4, given the form of the criterion. The program implemented by SAS indicates that the 1st, 48th, 77th and the 83rd data points are the starting points of each segment. The corresponding period is the second quarter of 1961, the fourth quarter of 1972, the first quarter of 1980 and the third quarter of 1981. This indicates that there are three significant structural changes in the whole sample period and suggests segmenting the data set into four sub-samples for further investigation. This can be simply described by the following figure.

Figure 6 Optimal Segmentation

Now we look at the four parts separately. The individual regression shows that the series is in four persistent states over the sample: during 1961-1972, the series has a mean of 1.358%; during 1972-1979, the mean is negative (-1.88%); during 1980-1981, the series has a mean of 0.88%, and finally, from third quarter of 1981 to the end of sample, the mean of the series is close to 5.84%. Our results show that the mean of the real interest rate series is subject to three occasional shifts and the endogenously determined shifts in the level of the series occur at the third quarter of 1972, in the end of 1979 and in the middle of 1981. This characterization of the real interest rate corresponds to the sudden jump in oil prices around 1972. The dating of the second break is attributed to a restrictive monetary policy starting from the last quarter of 1979 and the last shift is in line with the rise of the federal budget deficit in 1981. Relevant IC test is done after segmentation and no segment combination is suggested.

To further justify our findings, we examine the overall goodness-of-fit of the segmented model. RSM suggests a separation of whole sample data which yields an overall SSE value of 398.194, while segmentation following sequential method gives a whole sample SSE of 433.287. Given the documented facts that our method

optimizes the target function by detecting the changing points that minimizes the global SSE without any constraint, we conclude in favor of the presence of three breaks at the dates given above, which generate a better approximating model of the real data.

There are some who believe that the worth of a model should be based upon its ability to predict or forecast, hence we examine the out of sample performance of our segmentation. From the following figures we can see that the most recent regime of our segmentation leads to more accurate forecasts and the resulted prediction is closer to real data than that from sequential method.

Figure 7 Comparison of Forecast Value

Various tests and sensitivity analysis are conducted to justify the number of segments specified and to examine the general robustness of the chosen specification, i.e., the presence of three structural changes and the dates of their occurrence. For the results of Chow Test, the p-value for the null hypothesis of no structural breaks is less than 1% indicating a strong rejection, and the test statistics at the estimated break points are rather significant. These facts strongly support the results obtained above using the RSM methodology.

The above analysis indicates that shocks to the real interest rate series may be temporary in nature and that the average value which it reverts to is subject to occasional shifts led by important structural events. Our application shows that the ex-post real interest rate is a random process around a mean which exhibits infrequent but important shifts. Our results partly reconfirm Garcia and Perron (1996) and Bai and

Perron (2003)'s finding about the existence of structural break for the series. Difference is observed with respect to the dates of these structural breaks. Our segmented model assures the global optimization of the target function and hence enjoys a better approximation of the real data.

5.2 Structural Changes in Singapore Private Housing Market

Booms and slumps in housing prices have attracted the attention of both the general public and academic economists ever since. From the academic point of view, the ready availability of time-series data and the important policy implications of high and volatile prices have meant that empirical modeling of housing prices has been both a fertile and a challenging area.

5.2.1 Introduction

Buying a house is the largest investment made by most households in almost every country, while owning a home today constitutes the largest single source of personal wealth. Residential real estate cycles, therefore, can have a profound influence on personal wealth as well as on the general economy, particularly in a country with high home-ownership. These facts have motivated a series of housing price studies for a few decades.

In Singapore there are two segments in residential housing market, the private housing market and the HDB³ resale housing market. The main difference between the two is that the HDB resale housing market is, to some extent, regulated and subsidized, while the private housing market receives limited government intervention although the prices in both markets are determined by the market forces. The public housing sector⁴ is mostly providing housing services to the lower- and middle-income groups while the private housing sector is accommodating the middle- to high-income groups as well as a large portion of one million foreigners that account for 25% of the total population.

According to the 2000 Census of Population in Singapore, 92.3 % of Singapore resident households are owner-occupiers, either owning a private home in the private housing market or owning a public housing flat. Of the remaining 7.7 % of households, most are in the lowest income-group, living in heavily subsidized public rental flats. The private rental housing market mainly accommodates the foreign expatriates or students. Therefore, the housing choices for a Singapore resident household are either becoming a private home-owner, or to become a public home-owner (Tu, 1999). More specifically, a Singapore resident household can access home-ownership in three ways: --- by buying a private housing unit from the private housing market at a market price against a home loan with market mortgage interest rates; the buyers are then free to enter or exit from the market;

³ HDB, Housing Development Board, is a statutory board of the Ministry of National Development, considered to be the national housing agency.

⁴ Public housing refers to housing provided by public authorities or municipalities, while private housing refers to housing built by private developers.

--- by buying a new public housing unit at a heavily subsidized price against a home loan with the subsidized interest rates; however, very strict entrance and exit regulations apply to buyers; or

--- by buying a resale public housing unit from the public resale market at a market price against a home loan with the subsidized interest rates; the buyers are only subject to limited entrance and exit regulations.

In Singapore, the public housing market is the dominant form of housing and there is an active secondary market for public housing. More than 80% of the population lives in public housing provided by the government through the Housing Development Board. Public housing is heavily subsidized when purchased directly from HDB, but buyers are permitted to sell their flats in the open market after a time-bar of 5 years. According to the definition given by Ong and Sing (2002), the public resale market is an actively traded public owner-occupier housing market that is second to the heavily regulated new public owner-occupier housing sector. In which the new public housing units are sold at a heavily subsidized price. In the second quarter of 1998, a total of 14357 resale transactions took place in contrast to only 1543 private (non-public) residential property transaction.

Traditionally, the private housing in Singapore has catered to the housing demands of those groups that were not eligible to buy subsidized flats directly from the government and those high-income households which did not meet HDB's income ceiling requirements. As a matter of government policy, the income ceiling is

frequently reviewed to make sure every nine of ten Singaporeans are eligible to enjoy the benefits of public housing. Along with this reasoning, the target market for private sector developers would be those top income groups in Singapore. Beyond these Singaporean natives, foreign investors and the expatriate community form another group of players in the private housing market.

Private housing market operates in a laissez-faire economic system, where private housing prices are mainly determined by a function of the demand and supply in the market. (Sing, Tsai and Chen, 2004) This segment of the market is dominated by few major private developers. The government's intervention through the sale of leasehold private residential lands program and government linked property companies also help indirectly to cushion unnecessary price inflation. The private housing market is not parallel to the public resale housing market. However, compared with the public housing units, the private housing units have much higher housing prices with better designs, quality of finishes and fully-equipped recreational facilities (see Table 1). Getting into the private housing market is therefore viewed as the upper end of Singapore owner-occupiers' housing career. Ownership of private residential property is well regarded as a social status, and a dream of HDB dwellers and those who have not owned a house. A variety of housing forms, with the hierarchical structure from apartment, condominiums, terrace, semi-detached house to detached housing, is made available by private developers to meet different preference and aspiration of potential buyers.

Table 1 Characteristics of Private and Public Housing Market in Singapore

Housing Style	Median Area (square Metres)	Median Price (S\$)	Median Price per square metre (S\$)
Private Housing			
<i>Housing with land</i>			
Detached/Bungalow	1314.75	4,927,479	3747.845
Semi-detached	340.10	1,440,098	4234.337
Terrace	208.18	1,052,364	5055.068
<i>Housing without land</i>			
Apartment	133.79	803,168	6003.199
Condominium	125.46	743,830	5928.822
Resale public housing flats			
Executive flat	130	403,400	3103.077
Five-room	110	321,500	2922.727
Four-room	90	232,800	2586.667
New public housing flats			
Executive flat	125	360,050	2880.40
Five-room	110	228,683	2078.94
Four-room	90	137,000	1522.22

Note: S\$1=US\$ 0.584 on 30 August 2004 (Source: OUB Bank Singapore)

Prices of resale public housing flats are taken from the HDB website (www.hdb.gov.sg).

Although being relatively small, the private housing market has a significant impact on the Singapore economy. According to the estimation given by Phang (2001), the ratio of gross housing wealth in the private housing sector to GDP is 1.48, while the

same ratio in the public housing sector is 1.38. This implies that the fluctuation of private housing prices could have important implications for the national wealth holding. Moreover, the private housing sector provides not only high-quality housing units, but also an exclusive lifestyle. Therefore, becoming a private home-owner has become a national phenomenon, attracting a significant proportion of public home-owners to upgrade to private housing. The public housing subsidies therefore leak out into the private housing sector through such upward mobility and its social economic impacts are significant.

Although Singapore has a relatively free economy, its housing market is far from being perfect. It is strongly dominated by the public sector, in the forms of both direct provision and control of major housing stock, and regulating the eligibility criteria, housing finance, prices, rentals and transaction costs. It has been found that certain, but not all, public housing policies can generate significant impacts on both the private and public housing prices. The impacts of the public housing policies on the private housing prices are profound, albeit indirect. As pointed out by Phang, Wong, Tay and Amy (1995), the effect of government intervention on the public housing could filter into the private housing market. Changes in the supply, expected price, finance and eligibility criteria of public housing will influence the private property market significantly. These policy distortions have resulted in remarkable structural changes in the private property market over time.

The free-market operation of the private market implies that the market is more responsive and susceptible to shocks in economics, and price correction should be less sticky vis-a-vis public housing market. In recent years, Singapore's residential property market, especially the private housing market has been suffering from irregular price fluctuations. This has caused much public concern about the affordability of private housing. Thus the study of the structure of Singapore private housing market and the behavior of the market is of great importance in controlling real estate inflation. Interest in this sector stems from the fact that it is subject to the full rigour of market forces, in sharp contrast to the established public housing market where state-administered social pricing prevails mainly through subsidies and loans to the HDB. It also helps to ensure the affordability of housing, reduce volatility and improve the allocative efficiency of land resources.

5.2.2 Literatures

The literature on modeling of housing prices is very extensive, especially in the developed housing markets in the UK and North America. Many empirical housing models have been developed based mainly on the stock-flow adjustment or the classical Hendry's neo-classical frameworks. In Hendry's theory of equilibrium demand and supply functions, he derived the price of existing houses as a function of personal disposable income, rental rate, interest rate, stock of mortgage, tax rate, and number of families. Dicks (1990) extended Hendry's model for prices of new housing

in the UK. Hsieh (1990) further separated housing demand into service and investment demand in a study of Taiwan's housing market.

Following the traditional two-equation stock-flow model of the residential market, the demand is typically stated as a function of the real price of housing, the user cost of financing that price, the alternative cost of renting as well as demographic characteristics and real permanent income. In supply side, construction is always assumed to depend on housing prices, factor costs and various interest rates (DiPasquale and Wheaton, 1994). Empirical practices show that the functional forms and lags used tend to be largely data determined (Muellbauer and Murphy, 1997).

The stock-flow approach posits that the housing market will clear through prices that equate demand with the existing stock of housing. Supply is often taken to be exogenous as it is determined by the decisions of housing producers in prior periods. Such a specification fails to include supply-side features (Muellbauer and Murphy, 1997) and ignores the relationship between housing stock and land market conditions (DiPasquale and Wheaton, 1994). Taking housing stock as fixed will lead to a short run fluctuation in which price are completely demand-driven. However, as shown in many studies (Peng and Wheaton, 1994; Rosen and Smith, 1983), the effect of a demand shock on prices depends on the state of supply. This is particularly relevant for the Singapore market where land sales program is potentially useful mechanism for bringing the housing market into steady-state equilibrium (Lum, 2002).

Despite the small market share of the private residential property market in Singapore, research, however has been concentrated on this sector of the market. In the city-state of Singapore, the studies focusing on modeling private residential housing market dynamics contain only limited theoretical structure. Empirical analysis includes the impacts of government policies on private housing prices (Phang and Wong, 1997) and the inflation-hedging characteristics of private housing prices (Cheng and Sing, 2000).

Ho and Cuervo (1999) and Tu (2001) incorporated an error correction term in the cointegration models to adjust for the short-term non-stationary variations. Ho and Tay (1993) developed a system of six simultaneous equations for the supply of and demand for private residential properties in Singapore in a two-stage least squares process. Other studies included Ong and Sing (2002) on price discovery between private and public housing market using a Granger causality error-correction model and Sing (2001) on the dynamics of the condominium market in Singapore.

The free-market operation of the private market implies that the market is more responsive and susceptible to shocks in economics. But few have been seen to study the structural change of long run behavior of private housing market in Singapore, especially in structural equation model. This provides the basic rational for this empirical work.

5.2.3 The Model

As indicated in Ho and Cuervo (1999), "structural demand and supply" model would have been more appropriate compared with the VECM (Vector Error Correction Model) if the objective of the study were to establish causal relationships for structural analysis; to determine elasticities and multipliers for policy analysis; and to make forecasts for planning purposes. Because of these merits of system analysis, we will in this study look at the demand and supply of private housing market using simultaneous equation model. Our model extends the analysis in previous literature by proposing a new approach to examine the time series path of private housing market in structural model. This allows us to disentangle supply-side factors from demand-side influence, and in particular, to study the structural breaks in housing market behaviors over time.

Based on the literature and private housing market dynamics reviewed in the previous section, several important macro-economic determinants of private housing prices are identified and tabulated in Table 2.

Table 2 Private Housing Market Determinants

Demand Side Factors	Supply Side Factors
Private housing prices	Private housing stock
National income	Private housing price
Interest rate (lending rate)	Basic materials costs
Public housing prices	Labor costs

Consumer Price index	Banks loan rate to developers ⁵
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The Demand Model

Private housing prices (RPPI)

In order to obtain an aggregate measurement of price level in the private residential property market, the private Residential Property Price Index (RPPI⁶) is used. The RPPI data from first quarter of 1990 to the first quarter of 2004 is collected from REALIS⁷—Real Estate Information System. Figure 8 shows the fluctuation of the Private RPPI from 1990 to 2004.

⁵ As pointed out correctly by an examiner, the banks loan rate to developers should be an important explanatory variable for supply equation for housing market, as a measurement of construction financing cost. However, due to the data availability, we use prime lending rate as a proxy instead, which is found to be highly correlated with other bank loan rates and is proven in our model to be significant in explaining the price movement for the given time period.

⁶ The Residential Property Price Index is computed for all residential transactions on a quarterly basis. It should be differentiated from the Property Price Index that is an agglomeration of residential, commercial and industrial property sales.

⁷ This database is provided by Urban Redevelopment Authority (URA), the national planning authority of Singapore which is entrusted with the responsibility of planning the physical development and optimizing the scarce land resource in Singapore. The URA provides comprehensive and up-to-date data and information of the real estate market to improve the market's efficiency and transparency. The private residential property price indices published by the URA are transaction based indices compiled from caveats lodged with the Land Registry.

Figure 8 PRRI Index (1990Q1-2004Q1,1998Q4=100)*Public housing prices (HDB)*

Prices of public housing units are used as the benchmark of the price of private housing market. The Resale Price Index of HDB Flat⁸ was used as a proxy. This HDB variable is supposed to capture the price level of public housing. This data is obtained from the website of Housing Development Board, where 1998Q4 is adopted as the base period with index at 100. The pricing of HDB flats is largely determined by the statutory board and is considered a policy decision, bearing in mind that affordability is the main thrust of public housing here, although it does take into account prevailing property market conditions.

The intermarket mobility between public and private market occurs as the income of the population increases and preferences change. In land-scarce Singapore ownership of private residential property confers social status. Since most of the population is housed in public flats, the aspiration of many Singaporeans is to upgrade to private property. It is estimated that upgraders constitute about 60% of annual demand for private housing (Ong, 1999). Moreover, public housing is regarded as one of "investment of a large part of long term savings" (Chua, 2000), meaning that public housing is a source of wealth. Appreciation in the values of public flats enhances the affordability of flat-owners to upgrade. Upgraders, defined as those who upgrade from

⁸ The HDB Resale Price Index is based on the transactions of public Housing Development Board flats on the resale market. In other words, resale transactions are open-market transactions that occur subsequent to the initial sale, which is heavily subsidized by the government.

public to private housing, typically rely on the capital appreciation of their flats to enable them to purchase private properties (Ong, 1999).

By this kind of reasoning, the rising public resale price directly increased the accessibility of public home-owners to upgrade to private housing, which will transfer the public housing subsidies to private housing. That is to say, changes in the prices of public flats would have the tendency to increase the potential demand for private housing and hence boost the prices of private housing. As the value of a public flat falls, so does the affordability of the upgraders. This would cause the prices of private dwellings to fall as well. So public housing price is an important determinant in demand for private housing. This upgrading effect exceeds the effect of being substitute for private housing. On this account, the HDB resale price is expected to be positively related to the demand for private housing.

Empirical evidence has shown that the two owner-occupier housing markets are strongly related with a significant bi-directional Granger causality (Ong and Sing, 2002). The demand for the private housing market is not only driven by economic forces but also by the dynamics of the public resale market (Phang and Wong, 1997).

The time series plot of RPPI and the average price of public housing—the resale price of HDB flat—shows positive correlation between RPPI and average public housing price. Thus the strong predicting power of average price of public housing to RPPI level is anticipated.

Both RPPI and HDB resale price indexes are compiled based on transactions and do not suffer from the smoothing biases in appraisal price series.

National income (GDP)

An earlier Ministry of Trade and Industry's article (2001) has shown that private residential property prices in Singapore are fundamentally driven by economic growth, which captures both the improvement in household purchasing power as well as population growth. Phang, Wong, Tay and Amy (1995) also suggested that the fundamental of the private property market is determined by factors of the macro-economic environment. Both owner occupiers and investors are sensitive to the changes in the economic environment, which strongly affect their household income, lending rate and the construction activities.

It is well accepted that as an economy expands, national income rises and the outcome should be a greater demand for housing space. For a given level of housing space, rents must therefore rise if the demand to use space is to be equal to the available space. Therefore higher rents then lead to greater house prices, which in turn generate a higher stock of space associated with new market equilibrium. In housing literature demand for housing has been widely studies, and it has been shown that income is an important determinant of residential price movements, which in turn depend on the economic wellbeing of the country. (Ong and Teck, 1996).

Meanwhile, Singapore's housing finance system allows the would-be private home buyers to use their monthly Central Provident Fund ⁹ (CPF) contribution to pay off their mortgage debts. The contribution rates are adjustable and are positively related to medium to long term economic performance. This positive relationship implies that macroeconomic performance may directly affect the would-be home-buyers's housing affordability.

Ong and Sing (2002) provided evidence that real GDP is a significant variable reflecting the impact of long-run economic performance on the housing market. From the third quarter of 1986 until end of 1996, the growth of Singapore economy has been strong. The growth in household income and their CPF boosted the private housing market. Conversely, the poor economic performance in 1996 and 1997 has resulted in a dramatic fall in the prices of private properties. Therefore, GDP value is chosen as one of the potential key factors determining private housing prices, with a positive relationship expected. The trend of private housing prices in Singapore and the real GDP index is presented in Figure 9. The associated changes on a quarter-to-quarter basis for residential private price index (RPPI) and real GDP are presented in Figure 10. An expected consequence of continued income growth over the study period is an increase in the demand for private housing both for consumption and as a means of wealth accumulation.

⁹ CPF is the Singaporean's social security system, mainly providing pension schemes and medical care schemes. It is mandatory for both the employee and the employer to contribute monthly a certain fraction of the employees' salary to the fund to take care of the retirement, homeownership, and healthcare needs of the members. The CPF Board was set up to administer and preserve the value of the savings of its members. The CPF enables easy home-ownership through two popular schemes-the Public Housing Scheme for HDB flats and the Residential Properties Scheme for all housing properties built on freehold land or with a lease of at least 60 years remaining.

Figure 9 Trend of Private Housing Prices (RPPI) and Real GDP (1998Q4=100)

Figure 10 Real GDP Change VS RPPI Change (1990Q2-2004Q1)

Interest rate (PLR)

It has been suggested by economic theory that interest rates and house prices be inversely related. Generally, lower interest rates tend to increase housing demand, and therefore pushing up housing prices. However, this effect is softened by a similar

increase in the supply of housing in response to higher house prices and lower construction financing costs result from reduced interest rates. Thus, interest rates influence house prices through the demand for, and supply of private housing. We use PLR (Prime Lending Rate) in our model, which is the average of nominal bank lending rate, serves as the measure of the cost of housing finance or the cost of borrowing. Figure 11 shows that the capital appreciation of private housing in recent years may have been due in part to the availability of cheaper loans.

Figure 11 Real Housing Price and Prime Lending Rates

Other variables

It is shown from housing economics literature that wage, as a representative of the average real household income, could be an important factor affecting housing prices. Yet, wage rate per employee may not be a significant determinant in explaining private housing prices in Singapore. Private housing market in Singapore attracts

either foreigners or local residents from middle or upper-middle income groups, whose incomes are not available in time-series format. Measurement bias would exist if simply using the average income for all employees. Therefore, household income is not included in our model for private housing demand.

Expectation is normally considered an important factor in housing market, especially the demand side of the market. Expectation factor could be captured by stock exchange price index where real assets might be included in the institutions' investment portfolio to diversify risk and share prices would vary directly with property prices. It is therefore important to take into consideration the interaction between real estate space and capital markets. However, for the given time span and the data frequency I choose, expectation which was measured by stock exchange all share price index, happened to be not significant in private price determination and therefore is not included in our model. This could partly because of the time period chosen and frequency of the data.

Finally, demographic variable like household formation, which is often used in housing study of UK and US, is not included. The reason is that about 86% of the population is absorbed by the public housing sector in Singapore, while private housing sector acts as the upper end of the home-owner's housing career. Therefore, new household formation is not expected to be significant in explaining private housing prices movements.

The Supply Model

In contrast to the demand side, housing supply is necessarily specified in terms of the flow of new investment. In the market for new construction, the supply of new housing units can be expected to increase in response to positive production signals provided by rising prices and/or declining costs. This could be supported by the following Figure 12, in the Singapore context, which plots the number of private housing units which are under the building commencement stage with respect to RPPI and BMC (basic materials costs) on a real basis from 1991 to 2004.

Figure 12 Supply in Pipelines, RPPI and BMC

Profit-maximizing firms will have a positive supply response to selling prices for structures and a negative response to their own costs of production (wages and materials costs). We use index of supply of private residential units in the pipelines¹⁰

¹⁰ This comprises statistics on the supply of uncompleted private residential units in the pipeline. This supply in the pipeline covers all developments under construction as well those on which construction

as well as price and cost variables. Total housing stock is also included in the supply function and a negative sign is expected reflecting the responsiveness of new housing construction to housing stock. Given other factors unchanged, available urban land becomes scarce as the total housing stock increases. Higher negative responsiveness of new housing construction to the total housing stock would be an indication of a slow-down in new housing construction with respect to the level of housing stock, especially in a highly urbanized city state like Singapore.

Figure 13 Supply VS Stock

The above figure shows the influence of current housing stock on new supply of private residential units in pipelines.

5.2.4 Empirical Analysis

(1) Data Collection

have not coninenced. Developments on which construction have not commenced comprised those with written permission, provisional permission and those submitted to the Competent Authority and are under consideration for planning approval. and planned land sales by the government. The data are obtained from a combination of administrative records from the Development Control Division, URA and the Building and Construction Authority and field surveys to update the construction status.

Table 3 List of Empirical Variables and Their Sources

Notation	Variable Description	Source
<i>a. endogenous variable</i>		
Quantity	Private housing quantity (demanded and supplied) represented by the index of supply of private residential units in the pipelines	REALS
RPPI	Private Residential Property Price Index	REALS
<i>b. exogenous variable</i>		
PLR	Prime leading rate, the average of nominal bank lending rate	IFS ¹¹
GDP	Rate of the GDP growth, used to estimate the changes in the income level	TRENDS ¹²
HDB	Resale Price Index of HDB Flat was used as a proxy	HDB website
Stock	Stocks of completed private housing represented by available private residential units	REALS
BMC	Basic material costs index (base year 1985)	TRENDS
LC	Labor costs index	TRENDS
CPI	Consumer Price Index	IFS

¹¹ IFS--International Financial Statistics (IFS) is the International Monetary Fund's principal statistical publication and is the standard source for all aspects of international and domestic finance.

¹² The Time Series Retrieval and Dissemination (TREND) database maintained by the Department of Statistics in Singapore is used to construct the time-series for the variables identified in the model. All the variables are in their quarterly series. The TRENDS database is the national repository of macroeconomic variables and sector-specific variables for the Singapore economy. The reliability and integrity of this database, which is maintained and updated by the Ministry's DOS, are beyond any measure of doubt.

(2) Preferred Specification

As discussed earlier, we have two endogenous variables—price and quantity—and two equations determining them in the form of supply and demand equations. The error terms are likely to be correlated across equations as well, given the tight relationship between variables. Therefore we use three stage least squares instrumental variables estimator to avoid statistical problems involved with using endogenous explanators.

The standard Augmented Dickey-Fuller test suggested by Said and Dickey (1984) and Phillips-Perron test proposed by Phillips and Perron (1988) are first conducted to test the stationarity of time series data. The results are summarized in Table 4.

Table 4 Result of Unit Roots Tests

Variables	ADF t-statistics		PP adj. t-statistics	
	Level	1 st order difference	Level	1 st order difference
BMC	-2.00	-4.439*	-1.548	-4.389*
CPI	-2.197	-4.669*	-1.869	-4.605*
GDP	-0.806	-13.068*	-0.761	-31.890*
HDB	-0.001	-2.613*	0.457	-2.729*
LC	0.604	-4.111*	0.765	-15.570*
RPPI	-0.296	-2.663*	0.055	-2.779*
PLR	-0.978	-5.298*	-0.612	-5.152*
Q	-1.448	-6.825*	-1.898	-20.599*

STOCK	-1.626	-7.439*	-1.647	-7.439*
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* Significant at the 1% level

From the stationarity test, we found all the series non-stationary in level. Rather than applying the commonly used error correction model, we use RSM method to study the structure changes of private housing market. The analysis of data using RSM method is again implemented by the program written in SAS.

Using the whole sample data from 1991Q1 to 2004Q1, we have the simultaneous equations model for private housing market. All series are transformed to logarithmic form for the usual statistical reasons, and hence the variable coefficients estimate the percent change in quantity for a 1 percent change in the variable. Regression results and parameters estimation are shown in Table 5.

Table 5 Structural Model Estimation for Private Housing Market in Singapore
(using whole sample data)

<i>Regression Model:</i>	Demand Model	Supply Model
<i>Variables:</i>	Regression Coefficients	
Constant	-0.077 (0.082)	-0.003 (-0.008)
PLR	-0.158 (0.501)	-0.128 (0.148)
HDB	2.347 (2.609)	
GDP (one period lag)	0.126 (0.314)	

RPPI	-3.797 (4.999)	0.749 (0.196)
CPI	22.839 (25.056)	
BMC		-0.182 (0.966)
STOCK		-0.017 (0.115)

Note: values in the parentheses are standard errors for the coefficients

RSM method is then applied to estimate the structural break during the data time period. Figure 14 shows the corresponding value for e and l . Here we apply the system methods of detecting structural changes, i.e., we examine the structural instability globally, using the technique of joint estimation of the entire system of equations.

Figure 14 e VS l

As can be seen from Figure 14, value of target function reduces dramatically, as the number of segments increases. Typically, its value starts to converge to 0 at point $l=2$. Akaike's Information Criterion (AIC) and Schwarz's Bayesian Criterion (SBC) are useful for comparing models with different numbers of parameters and the model with the smallest value of AIC or SBC is considered the best. In particular, since our sample size is relatively small, the AIC, SBC and CAI of Sugiura are all appropriate in choosing the number of structural breaks. These statistics are used here to determine the correct number of segments.

$$CAI_l = N \ln e[\tilde{p}(N, l)] + 2N(ml + 2)/(N - ml - 3),$$

$$AIC_l = N \ln(e[\tilde{p}(N, l)]/N) + 2mlN,$$

$$SBC_l = N \ln(e[\tilde{p}(N, l)]/N) + 2ml \ln N.$$

The results of the selection procedure is summarized in Table 6

Table 6 IC Test for value of l

l	Target Function	CAI	AIC	SBC
4	0.122129	1126.428	-45.6583	-58.5252
3	0.14006	194.462	-64.624	-74.2742
2	0.195652	58.35993	-78.88	-85.3134
1	1.409757	43.4442	-57.4797	-60.6964

From the table we can see that the minimum value of AIC and SBC are reported at $l=2$. Nevertheless, CAI reports ambiguous results in that CAI of both one break and none break are very similar while CAI value for three or four breaks are very large in magnitude. Should there be one structural break during the sample period, the program implemented by SAS indicates that the 31st data point is the structural break point by using RSM method. To further clarify this point, various tests and sensitivity analysis are conducted to justify the number of segments specified and to examine the general robustness of the model specification. The CUSUM and CUSUM of squares tests are applied to examine the stability of the coefficients. The test statistics were beyond the pair of 5-percent critical values for both tests indicating the instability of the coefficient and hence favor the significance of the stated structural change occurred at the above-mentioned date.

In view of this, we conclude that the optimal number of segments is 2, given the result from above tests. The corresponding periods are the first quarter of 1991 and the second quarter of 1999. This indicates one significant structural change during the whole sample period and suggests segmenting the data set into two sub-samples for further investigation. This can be simply illustrated by the following figure.

Figure 15 Segmentation of Whole Sample Data

Now we look at two parts separately. We have individual model whose estimation results are summarized in the following tables:

**Table 7 Structural Model Estimation for Private Housing Market in Singapore
(for the first segment)**

<i>Regression Model:</i>	Demand Model	Supply Model
<i>Variables:</i>	Regression Coefficients	
Constant	-0.026 (0.021)	0.008 (0.019)
PLR	-0.570* (0.201)	-0.081 (0.177)
HDB	0.322 (0.595)	
HDB(one period lag)	0.156 (0.178)	
GDP	0.283** (0.134)	
GDP (one period lag)	0.291** (0.148)	
RPPI	-0.262 (0.353)	0.834* (0.315)
RPPI (one period lag)	0.679* (0.387)	
CPI	4.047 (4.633)	-1.250 (4.410)
BMC		-0.429 (1.378)
STOCK		-0.017 (0.137)
LC (two periods lag)		-0.283* (0.113)

Note: values in the parentheses are standard errors for the coefficients

*(**) Denotes coefficient is significant at 1% (10%) level

**Table 8 Structural Model Estimation for Private Housing Market in Singapore
(for the second segment)**

<i>Regression Model:</i>	Demand Model	Supply Model
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<i>Variables:</i>	Regression Coefficients	
Constant	-0.024** (0.010)	0.025** (0.012)
PLR (one period lag)	-0.108 (0.422)	-0.128 (0.148)
HDB	1.030** (0.476)	
GDP (one period lag)	0.255 (0.272)	
GDP (two period lag)	0.055 (0.254)	
RPPI	-0.081 (0.642)	0.412** (0.161)
RPPI (one period lag)	-0.126 (0.450)	
RPPI (two period lag)		0.173 (0.129)
CPI (two period lag)	3.390** (2.124)	
CPI	1.065 (1.962)	
Quantity demanded (two period lag)	(-0.225) (0.217)	
BMC		-0.326 (1.040)
LC		-0.051 (0.120)
LC (two period lag)		-0.131 (0.112)
STOCK		-3.565* (1.161)

*(**) Denotes coefficient is significant at 1% (10%) level

After taking into consideration of structural change, each individual segment achieves much better goodness-of-fit (higher adjusted R^2 value). Moreover, from the view of forecasting power, we find that the segmented model outperforms the whole sample model in term of prediction power. The model's efficiency is tested by dynamic simulation involving prediction and simulation under 'PROC SIMLIN' of the SAS

software.¹³ The graphical plot of predicted and actual values of the endogenous variable, RPPI, against time is reproduced in the following figures. The figures show simulation results, where the simulation using the second segment indicate a forecast much more closer to the real data than the prediction from whole sample data. This is firstly due to the occurrence of structural change during the sample period, resulting the poor prediction performance out of an unstable series from the whole sample. Another reason behind is that the most recent past contains more information about the immediate future than the distant past and on this account, most recent regime may lead to better forecasts.

Figure 16 Simulation Result Using Whole Sample Data

¹³ The SIMLIN procedure reads the coefficients for a set of linear structural difference equations (usually from a data set produced by PROC SYSLIN), computes the reduced form, and uses the reduced form equations to generate predicted and residual values for the endogenous variables.

Figure 17 Simulation Result Using Data from Second Segment

5.2.5 Discussion and Policy Implication

Singapore private residential property market is driven primarily by market demand and supply, although it is subjected to prudential government regulations and, to some extent, competition from public housing. It is important to model economic forces and market factors that drive the private housing market in Singapore, from the perspective of policy makers, developers as well as investors. That will help to improve the judgment of the market dynamics and thus to ensure a more effective implementation of housing policy.

Singapore private housing market has undergone cycles of boom and bust over the last twenty years. In early 1990's, the relaxation of the HDB rules, which allowed HDB

flat dwellers who have lived in their flats for more than three years to invest in private residential properties, had stimulated demand for private housing. Later on, the government announced, in April 1993, one policy on HDB Resale flat valuation and further liberalization and another one on relaxation of Mortgage Loan Financing Scheme in May of the same year. These further expanded demand and explained the sharp increase in the prices.

In the following boom years of 1994-96, prices in the residential markets more than doubled, driven by strong income growth, bullish stock market performance, ease of obtaining financing through banks (banks and finance companies could provide financing for up to 100% of the value of the property for loan periods of up to 35 years), and property speculation (which was common not only among Singaporeans but also foreign buyers as well). What was most troubling about the property market fever was the sharp increase in housing loans, particularly the easing of financing terms. A survey of banks conducted in the housing loan market found that 55% of housing loans provided more than 80% of financing in 1995. Such high level of financing did not provide banks and finance companies sufficient cushion to absorb losses in the event of a decline in property values and the inability of borrowers to service their loans. This could affect the quality of loan portfolios and undermine the soundness of Singapore's financial system. This latest escalation in price of private housing had sustained until 15 May 1996 when some anti-speculative measures were imposed. The government announced the measurements to curb speculation in the property market and to prick the property price bubble which was mainly supported by

speculators and investors, rather than genuine homeowners. The launch of Executive Condominiums¹⁴ also set the benchmark of the private housing prices at a relatively low level.

Moreover, Singapore economy was badly affected by the global recession in the electronic sector in the fourth quarter of 1996, which resulted in several downward adjustments in the growth projection in the year. The transaction volume and the take-up rate of new private property fell dramatically. The RPPI index fell 1.9% and 2.7% in the third and fourth quarter of 1996. Subsequently, the Asian financial crisis and a recession in 1998 further weakened the property market, as prices bottomed out in the fourth quarter of 1998.

As discovered by our model, the private housing market experienced a significant structural change in year 1999. Prices of residential properties rose 11.4% in the 2nd Quarter 1999, compared with 4.4% in the previous quarter. Prices of landed properties rose 13.9% compared with 4.3% in the previous quarter, among which prices of semi-detached 15.7%, detached houses and terrace houses 13.4% and 13.1% respectively. Prices of non-landed properties rose 9.4%, compared with 4.5% in the previous quarter. Of this, prices of condominiums rose 9.0% while those of apartments increased by 10.5%. The number of private residential units under construction decreased by 6.6% to 30,455 units as at the end of 2nd Quarter 1999. The number of uncompleted private residential units with sale licenses and building plan approvals

¹⁴ Executive condominium (EC) is a hybrid housing class that is created in the mid 1996 to meet the “sandwiched” class of young professionals, and also to stabilize the overheating private housing prices. The EC sites are sold by the government at discounts to make ECs more affordable.

declined 4.8 %. A total of 1,360 new private residential units were launched for sale in the 2nd Quarter 1999, 5.1% lower than the 1,433 units launched in the 1st quarter. During the 2nd quarter, 2,723 new private residential units were sold by developers, 17.8% lower than the 3,313 units sold in the 1st quarter 1999.

From our segmented model we notice that, for demand side, price of private housing is significant, and reversely related, to housing demand in both two subsample periods. While the sign for price with one time lag change from positive for the first segment to negative for second segment. This could partly be explained by the decreasing demand for speculation purpose. GDP, together with its lag terms, remain to be significant and positively associate with demand in two subdomain of data. The coefficient of PLR is negative for both segments. The negative coefficient may be due to less demand for private housing as a result of a higher cost of borrowing money.

As shown from supply model, we find that, supply of private housing is conversely related to current housing stock, basic material cost and labor cost. Positive relationship is found between supply and price level. Especially for the 2nd segment, price with two periods lag becomes significant in determining housing supply. Another worthnoting fact is the big jump of coefficient of stock in supply model, indicating an increasing responsiveness of new housing supply to the current stock.

As has been demonstrated by our model, the private housing market is sensitive to changes in the public housing market with a higher correlation coefficient. On this

account, it should be realized by policy makers that the measures directed at the public housing sector may have increasingly significant implications for private housing price movements. This dynamics of these two markets normally reinforces each other and this calls for a more integrated approach to study the housing market as a whole.

The private housing market is expected to turnaround in late 2004, yet caution continues to reign. From demand side, well-located and reasonably priced projects continue to draw crowds to the showflats. However, potential home buyers and upgraders have been more prudent with their buys as the government move to encourage a more flexible wage system and in the light of CPF cuts. Uncertainty to the incomes of potential home owners is thus introduced. From the supply side, investment market is getting active with some developer restocking their residential landbank. Another positive fact is the number of unsold units in projects decreased. Some firmer signs of pick-up of the price are shown from these sale activities. Currently the mood in the private residential property market continues to be cautious. Buyers remain concerned in the light of the CPF cuts and ongoing restructuring of the economy.

In this chapter, we have estimated structural models for housing supply and demand for Singapore private housing market that fit the data reasonably well for the chosen time periods. The RSM regression model is established as well. It is found that the RSM method is able to detect the structural changes in the market accurately, without any prior information about the changing points or the timing of the external shocks.

The changing points indicated by the RSM method are consistent with the timing of the policy change and economics shocks. Our model reconfirms the findings by Lum (2002), that demand and supply macro-variables are found to be significant determinant for private housing prices over the long run. The land sale program and the liberalization of public housing market are proven to be effective short-run policy tools adopted by government in stabilizing the private housing market.

CHAPTER 6 CONCLUSION

Both econometrics and statistics literatures contain a vast amount of work on issues associated with structural change and plenty of practical advantages arise from the estimation and inference of structural change model. Amongst the others, it first helps to identify the events that may foster the structural breaks. By comparing the estimated structural break date of the model with the effective date of a policy change (or policy implementation), we can examine the effectiveness of policy. Second, it is useful in the field of forecasting. Basically, using the most recent regime may lead to better forecasts. The motivation here is a possibility that the recent past contains more information about the immediate future than the distant past and that one can discriminate between the recent and distant past by using the fit of time series model under consideration.

In this dissertation, Chapter 1 provides an introduction, highlighting the motivation and objectives of this study. The second chapter reviews relevant literatures on structural change analysis and the application to simulation equation models. We find that literature contains a large amount of work on issues related to structural changes, most of which is specially designed for the case of a single change, while literature addressing issue of multiple structural changes is relatively sparse. Moreover, most of existing work only focuses on the problem of testing rather than estimation for structural changes. For structural break estimation, the commonly used sequential estimation of the break points is not guaranteed to be identical to those obtained by global minimization. Besides the limitation of sequential testing, we realize that

literature on analyzing and testing the stability of coefficients in simultaneous equations is quite limited. In the context of simultaneous equation model, no comprehensive treatment has been seen to detect unknown structural changing points without any prior information and no results available to consider structural changes across the whole system. These facts provide the rational for this study.

The interest of this study lies in the situations where piecewise linear regression generates a better approximation, given the occurrence of structural changes within the sample period. In Chapter 3 we present a comprehensive treatment of issues related to the estimation of linear models with multiple structural changes, to the detection of the presence of multiple structural changes and to the determination of the number of changes. It will further allow us to state facts about the number of segments present in the horizon covered, the magnitude of the mean and variance in each subsample, the nature of the dynamics in the error component, and the timing of the changes in regime.

In particular, the proposed Recursive Segmentation Method is able to correctly detect and estimate the existence and the timing of unknown changing points. This method provides a systematic and operational approach that can accurately detect structural changing points without any prior information or knowledge of the pattern and timing of possible structural shifts. The method is based on the principle of dynamic programming and the use of recursive regression allows global minimizers to be

obtained using a number of sums of squared residuals rather than an exhaustive grid search.

The common practice in the multiple change point model to determine the number of break is to use sequential method. The basic logic is to estimate an additional break whenever the sub-sample fails the parameter constancy test. This step is repeated until all the sub-domains do not reject the null hypothesis of no structural changes. Although it yields consistent estimates of the break points, the estimates are not guaranteed to be identical to those obtained by global minimization. In our method we apply instead model selection criteria to determine the number of breaks. By doing so, we can find out the global minimization and optimize the target function without any constraint. Simulation studies show that our method correctly can identify the break points for the segmented model and the number of pieces, in small-to-moderate sample.

It is worthy noting that our method is a good complementary to the traditional way of identifying structural change, rather than to replace it. Indeed one of the main objectives of the work is to investigate a new method which is complementary to the traditional way of identifying structural change. Structural break detected by our method can be used to examine whether a subjectively guessed structural change led by historical event is statistically significant or not. The opposite holds true in that the result derived from our method can be proved valid if it is consistent with the major changes in policy or macro economic environment.

Chapter 4 further generalizes our RSM method to simultaneous equation model. The single equation estimation of structural changes enables us to detect the structural instability in individual equation, which is not necessary that from the entire simultaneous system. Since the changing points estimated among different equations may differ to some extent, this provides a new angle to explain the spillover effect of some policy implementation. Meanwhile, allowance is made to adapt different estimating criterion for different equation and the flexibility, in this respect, is gained. Another obvious practical consideration of estimating equation-wide instead of system-wide is the computational simplicity of the single equation methods. But the current state of available software has all but eliminates this advantage.

Although the system methods are asymptotically better, they are risky at propagating any specification error in one structural model throughout the system. But obtaining a unique structural changing point estimator helps in consistency interpretation and enjoys the simplicity in coping with structural changes at the same timing throughout the system.

Recursive Segmentation Method is employed in Chapter 5 to examine the time-series behavior of the US ex-post real interest rate and the structural dynamic; of Singapore's private housing market. The application to the U.S. interest rate exemplifies the merit of estimating all the structural break points concurrently, compared with sequential method, by identifying the structural changes with a better

approximation of sample and closer fit to the historical events. For Singapore's housing market, a SEM is established and structural change is estimated in this context, for the first time, without prior knowledge or information of the pattern and timing of possible structural shifts. The structural break points detected are proved to be consistent with policy changes as well as external shocks to the model, and segmented or piecewise regression models after taking into consideration the structural changes, manifest better goodness-of-fit in estimation and improved accuracy in forecast.

This dissertation discusses issues on modeling structural changes and the generalization of the proposed method to Simultaneous Equation Model. At the same time it has produced several open questions. To name a few, our analysis does not cover the case where there is deficient observation for the last segment. Second, it may also be interesting to make inquiries about the structural change detection for nonlinear regression model. More importantly, showing statistical properties or probability distributions of break points would be a sound way of justifying the merits and usefulness of RSM method. For the empirical work particularly, as suggested by one of the examiners, the use of banks loan rate to developers might bring important improvement to the model, which can be a good explanatory variable for supply equation for housing market as a measurement of construction financing cost. Besides, better proxy may be tried to measure expectation factor in demand model, say, share price index for selective companies if not an overall index; or moving average of property price. Meanwhile, the dynamics of private and public housing markets

normally reinforces each other and this calls for a more integrated approach to study the housing market as a whole.

APPENDIX

I. Procedures of the Recursive Segmentation Method

1---Optimal segmentation
2---Combining and model simplification
3---Model selection

Appendix II—SAS Program

```
options ls=80;
data a;

seed1=1;
seed2=2;
seed3=3;
seed4=4;
size=10;
a1=2;

do i=1 to size;
x=10*ranuni(seed1);
epsilon1=(sqrt(2))*rannor(seed3);
y1=a1*x+epsilon1;
output;
end;
data a; set a;

data b;

seed1=1;
seed2=2;
seed3=3;
seed4=4;
size=10;
a2=0.5;

do i=1 to size;
x=10*ranuni(seed1);
epsilon2=(sqrt(2))*rannor(seed3);
y1=a2*x+epsilon2;
output;
end;
data b; set b;

data c;

seed1=1;
seed2=2;
seed3=3;
seed4=4;
size=10;
a3=-1;
```

```

do i=1 to size;
x=10*ranuni(seed1);
epsilon3=(sqrt(2))*rannor(seed3);
y1=a3*x+epsilon3;
output;
end;
data c; set c;

data d; set a b c;
data d; set d;
data d; set d; keep y1 - x;
data d; set d;t=_n_;
data d; set d;

proc reg;
model y1=x;
output out=est r=uhat predicted=yhat;
run;
data est; set est;

```

```

data sse;

```

```

%macro apart;
%do i=1 %to 26;
%do j=&i+4 %to 30;

```

```

data abc&i&j; set d; if &i<=t<=&j;

```

```

data abc&i&j;set abc&i&j;
proc reg data=abc&i&j;
model y1=x;
output out=abc&i&j r=uhat;
data abc&i&j;set abc&i&j;
data abc&i&j;
set abc&i&j;
uhatsq=uhat**2;

```

```

data abc&i&j;
set abc&i&j;

```

```

data abc&i&j;

```

```

set abc&i&j;
keep uhatsq;

data abc&i&j;
set abc&i&j;
proc summary;
var uhatsq;
output out=sse&i&j sum=sse;
data sse&i&j; set sse&i&j;keep sse;
data sse&i&j; set sse&i&j;

drange=&i*10000+&j;
data sse&i&j;
set sse&i&j;

data sse; set sse sse&i&j;

data sse; set sse;

%end;
%end;
dm log 'clear' sim-single euqation.sas continue;
%mend;
%apart;

run;

```

```

data diameter; set sse;
data diameter; set diameter;

data diameter; set diameter;
t=_n_;
data diameter; set diameter;

data diameter; set diameter; e=sse;
data diameter; set diameter;drop sse;

data diameter; set diameter;

```

```

if t>1;
data diameter; set diameter;
t=t-1;
data diameter; set diameter;

data e; set diameter; if t<27;
data e; set e;

data diameter; set diameter;
d=e;
data diameter; set diameter;drop e;
data diameter; set diameter;

data e; set e;
erange=(t+4)*10000+1;
data e; set e;

%macro ee;
data ed;
%do k=10 %to 30;
%do m=2 %to &k/5;
%do q=&m-1 %to &m-1,
data ed; set ed;

ee=100000000000000;
data ed; set ed;

%do n=&q*5 %to &k-5;
%do p=&n+1 %to &n+1;

data d1; set diameter; if drange=&p*10000+&k;
data e1; set e; if erange=&n*10000+&q;
data e1; set e1; keep e;data e1; set e1;
data d1; set d1; keep d;data d1; set d1;

data ed; merge ed d1 e1;
data ed; set ed;

e&n=d+e;

if e&n<ee then N=&n;
if e&n<ee then ee=e&n;

```

```
erange=&k*10000+&m;  
data ed;set ed;  
  
data ed; set ed; drop e d e&n;  
data ed; set ed;  
%end;  
  
e=ee;  
data ed; set ed; drop ee;  
data ed; set ed;  
  
data e; set e;  
data e; set ed e;  
data e; set e;  
  
%end;  
dm log 'clear' ee.sas continue;  
  
%end;  
%end;  
%mend;  
%ee;
```

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