On equalizer tap and antenna selection for UWB and MIMO systems with linear MMSE receivers

Lin, Zhiwei

2006

Lin, Z. W. (2006). On equalizer tap and antenna selection for UWB and MIMO systems with linear MMSE receivers. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/2484>

<https://doi.org/10.32657/10356/2484>

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**ON EQUALIZER TAP AND ANTENNA SELECTION
FOR UWB AND MIMO SYSTEMS WITH
LINEAR MMSE RECEIVERS**

LIN ZHIWEI

SCHOOL OF COMPUTER ENGINEERING

2006

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for UWB and MIMO Systems with
Linear MMSE Receivers**

Lin Zhiwei

School of Computer Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

2006

Acknowledgements

I wish to express my sincere gratitude to my advisor, Professor A.B. Premkumar, for his guidance, advice and support during my Ph.D. studies in NTU. He is always patient, gives me encouragement and confidence. I have benefited very much from his research motivation, intuition and technical insight.

I would like to thank Professor A.S. Madhukumar for everything he has done for me. This has included research guidance, helpful discussions, inspiration and encouragement.

I would like to thank Professor Edmund Lai for his help and guidance in research methodology.

I wish to thank for the continuous support from Center of Multimedia and Network Technology (CEMNET) throughout my Ph.D. program.

Lastly, I would like to thank my parents, my wife and family for their love, support and encouragement over the years.

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List of Abbreviations

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BGA	Backward Greedy Algorithm
BPSK	Binary Phase Shift Keying
CDF	Cumulative Density Function
CDMA	Code Division Multiple Access
CG	Conjugate Gradients
CIR	Channel Impulse Response
CSI	Channel State Information
DSSS	Direct Sequence Spread Spectrum
DFE	Decision Feedback Equalizer
DL	Diagonal Loading
ES	Exhaustive Search
EVD	Eigen Value Decomposition
GA	Greedy Algorithm
i.i.d.	independent and identically distributed
ICI	Inter Chip Interference
ISI	Inter Symbol Interference
LMS	Least Mean Square
LNR	Loading-to-Noise Ratio
LS	Least Squares
MAI	Multiple Access Interference
MIMO	Multiple Input Multiple Output
MMSE	Minimum Mean Square Error
MSE	Mean Square Error

MUD	Multiuser Detection
MRC	Maximum Ratio Combiner
ML	Maximum Likelihood
MLD	Maximum Likelihood Detector
MWF	Multistage Wiener Filter
MVDR	Minimum Variance Distortionless Response
MP	Matching Pursuit
MN	Minimum Norm
NLOS	Non-Line-Of-Sight
NBI	Narrow Band Interference
OFDM	Orthogonal Frequency Division Multiplexing
ORLS	Order Recursive Least Squares
PAM	Pulse Amplitude Modulation
PPM	Pulse Position Modulation
PAPR	Peak-to-Average Power Ratio
PDP	Power Delay Profile
PC	Principal Components
QC	Quadratic Constraint
RMS	Root Mean Square
RLS	Recursive Least Squares
SNR	Signal-to-Noise Ratio
SIR	Signal-to-Interference Ratio
SINR	Signal-to-Interference-Plus-Noise Ratio
SVD	Singular Value Decomposition
SER	Symbol Error Rate
SDMA	Space Division Multiple Access
THSS	Time-Hopping Spread Spectrum
THLS	Thresholded Least Squares
UWB	Ultra-Wideband
WPAN	Wireless Personal Area Network
WLAN	Wireless Local Area Network
ZF	Zero Forcing

Abstract

The research work in this dissertation addresses reduced complexity equalization technique for multipath channels with large delay spread. This has been one of the key challenges for high data rate impulse radio based Ultra-Wideband (UWB) communication systems using low cost receiver design. Our research focus is on developing efficient and effective tap selection techniques for non-uniformly spaced Minimum Mean Squared Error (MMSE) equalization and applying it to UWB systems to achieve high performance with reduced receiver complexity.

In this dissertation, a new class of tap selection techniques based on Order Recursive Least Squares (ORLS) and Matching Pursuit (MP) algorithms is proposed for UWB multipath channel equalization in the presence of Inter Symbol Interference (ISI) and unknown co-channel interference. The proposed tap selection techniques are directly implemented based on training symbol sequence under Least Squares (LS) criterion without the need for explicit channel estimation. In addition, Diagonal Loading (DL) technique is incorporated into the tap selection and MMSE equalization processes to insure system robustness given limited number of training symbols in the case of practical implementation. The proposed tap selection based equalizer is shown to outperform the conventional uniformly spaced linear equalizer and is with reduced computational complexity. The LS based technique proposed for equalizer tap selection is then extended to receive antenna selection for high data rate UWB systems with multiple receive antenna elements to achieve improved performance in the presence of Narrow Band Interference (NBI).

Based on the LS criterion, antenna selection techniques for Multiple Input Multiple Output (MIMO) systems are also explored in our research work. Novel transmit and receive antenna selection techniques are proposed for MIMO spatial multiplexing systems with linear receivers under LS criterion. Unlike conventional approaches,

the proposed method directly implements the antenna selection algorithms based on training symbol sequence from the desired transmitter without the need for Channel State Information (CSI). In order to obtain CSI, blind channel estimation may be required in the case when unknown co-channel interference is presented. The proposed method avoids the complexity for blind channel estimation and is able to retain the diversity order of a full complexity system in the presence of unknown co-channel interference. DL technique is also incorporated into the proposed antenna selection process and is shown to achieve improved performance under finite training sample support. By incorporating the standard fast Backward Greedy Algorithm (BGA), it is shown that the proposed receive antenna selection algorithm can be practically implemented with reasonable computational complexity. Further, a joint transmit and receive antenna selection technique is discussed and shown to achieve improved performance when compared with either single side transmit or single side receive antenna selection technique.

Chapter 1

Introduction

This chapter provides a brief introduction to Ultra-Wideband (UWB) radio for its unique advantages, the potential for high data rate wireless communication, current status and challenges. Our research motivation and objective are then described. Followed by the summary of our research contributions.

1.1 Historical Perspective of UWB

Ultra-wideband technology has been around for more than three decades [78]. It was firstly developed for US military to be mainly used for radar-based applications. Much of the early work in UWB was particularly in the area of impulse-based technology, namely, impulse radio. In recent days, UWB systems have received significant research interest and industry attention [84, 23]. Because of its wideband nature, UWB promises to deliver very high data rate for wireless transmission. It can also be used for accurate ranging.

In February of 2002, the FCC (Federal Communications Commission) amended its Part 15 rules to include the operation of UWB devices without a license [21]. The FCC defines UWB signals as gigahertz radio having a fractional bandwidth equal to or greater than 0.20, that is, $(f_H - f_L)/f_C > 20\%$, where f_H , f_L and f_C denote the highest, the lowest and the central frequency of the given UWB signal spectrum respectively, or a 10dB equivalent bandwidth equal to or greater than 500MHz. The FCC ruling allocates the entire spectrum from 3.1 to 10.6GHz for UWB. It also

limits the transmitted power of UWB systems to an Effective Isotropic Radiated Power (EIRP) of -41.25dBm/MHz , in order to insure that UWB systems do not cause harmful interference to other radio systems (e.g. 802.11a WLAN) that fall in the frequency range of the UWB bandwidth. Figure 1.1 illustrates the FCC defined spectral mask for UWB indoor commercial systems [84].

Figure 1.1: FCC spectral mask for UWB indoor commercial systems

UWB radio has the advantages of high capacity, fine time resolution, multipath immunity, low power consumption due to low duty cycle, low probability of detection, low interference with existing narrowband radios, suitability for CMOS implementation and low cost due to its simple RF architecture. These make UWB an attractive alternative to narrowband and conventional wideband technologies. The IEEE 802.15.3a [2] standard committee is currently evaluating UWB as a new physical layer design for very high data rate Wireless Personal Area Network (WPAN) systems ¹. For example, a Direct Sequence (DS) based UWB method [22] has been proposed for

¹On January 19, 2006, IEEE 802.15.3a task group (TG3a) voted to withdraw the January 2003 project authorization request that initiated the development of high data rate UWB standards. In response to this, the UWB Forum [4] and WiMedia Alliance [5] announced that industry will continue to grow the UWB market. The TG3a's most commendable achievement is the consolidation of 23 UWB PHY specifications into two proposals: Direct Sequence-UWB (DS-UWB), supported by the UWB Forum, and MultiBand Orthogonal Frequency Division Multiplexing (MB-OFDM) UWB, supported by the WiMedia Alliance.

scaling the data rate from 55Mbps to 1320Mbps for either lower or higher operation band. On the other hand, a multi-band multicarrier Orthogonal Frequency Division Multiplexing (OFDM) based UWB method [9] has been proposed for scaling the data rate from 110Mbps at 10m distance to 480Mbps at 2m distance.

The growing demand for wireless data capability in portable devices is driving the development of UWB technology as an alternative for the short-range wireless standards like Bluetooth [1]. UWB is also projected to be a promising candidate to coexist with Wireless Local Area Network (WLAN) [6].

Shannon-Hartley theorem specifies the maximum channel capacity for an Additive White Gaussian Noise (AWGN) channel as

$$C = B \log_2(1 + S/N) \quad (1.1.1)$$

It is observed that the upper bound on the capacity of an AWGN channel grows linearly with the total available bandwidth. Due to its ultra-wide bandwidth, a UWB system appears to have huge potential for supporting future high-capacity wireless communication systems.

Impulse Radio vs. Multi-band UWB:

Impulse radio technique was the initially proposed approach for UWB. It involves the use of very short duration baseband pulses, often in the sub-nanosecond range, with a bandwidth of several Gigahertz. Data is modulated on pulses by various methods such as Pulse Position Modulation (PPM) or Pulse Amplitude Modulation (PAM). Multiple access capability can also be supported by spread spectrum techniques such as Time-Hopping Spread Spectrum (THSS) or Direct Sequence Spread Spectrum (DSSS). FCC ruling that specified a UWB signal could be any signal that occupied a minimum 10dB bandwidth of 500MHz has revolutionized the design of multi-band UWB systems [9, 70]. In these systems, the entire bandwidth from 3.1GHz to 10.6GHz has been divided into several subbands, each with a bandwidth of 500MHz or larger.

The design of baseband transmitted impulse radio systems does encounter some critical challenges. In order to coexist with other existing narrowband or wideband

systems, notch filters are required to mitigate the interference. However, such filters introduce distortion in the transmitted signal waveforms. The generation of the baseband signal by pulse shaping to fit into the FCC spectral mask is not an easy task. In contrast, a multi-band UWB scheme can simply avoid transmitting in the frequency bands where the other systems like IEEE 802.11a may exist. It avoids using notch filters and also eases the requirement for pulse shaping filters. These reasons have made industries to consider a multi-band UWB implementation [9, 22] instead of the initially proposed, carrier-less, baseband transmitted impulse radio. In a multi-band UWB system, the information on each sub-band can be modulated using either single-carrier (e.g. CDMA) or multi-carrier (e.g. OFDM) based techniques.

CDMA based UWB vs. OFDM based UWB:

Based on CDMA techniques, a single-carrier, multi-band UWB system [22, 70] transmits information by modulating data on narrow pulses. It has scaleable complexity and is capable of running multiple simultaneous independent overlapping networks. It has the advantage of multipath robustness and simple transmitter design, but requires multiple finger RAKE receivers to capture the transmitted signal energy in dense multipath in an indoor environment. High Pulse Repetition Frequency (PRF) is necessary to support high data rate. This makes the system vulnerable to Inter Symbol Interference (ISI) and timing jitter. For coherent detection, fast acquisition is also required to minimize the preamble overhead.

On the other hand, a multi-carrier, multi-band UWB system [9] has been proposed that uses OFDM technique to transmit information in each of the sub-bands. OFDM technique [79] is well known to be spectrally efficient for wideband, high data rate wireless communications. It effectively converts a high rate data stream into a set of lower rate data streams to be transmitted in parallel by multiple orthogonal carriers (cyclic prefix is introduced to maintain the orthogonality in the presence of ISI and inter-carrier-interference). Equivalently, this converts a frequency-selective fading channel into a set of parallel flat-fading channels. OFDM is a proven technology (e.g. IEEE 802.11a/g is based on OFDM technique) that is robust to multipath

ISI. Moreover, it has the advantage of improved spectral flexibility and worldwide compliance since it can easily place deep notches in its transmit spectrum to protect sensitive services or comply with new regulations in other countries. However, a multi-band OFDM system relies on frequency diversity and Forward Error Correction (FEC) to overcome the effect of Rayleigh fading. Moreover, it requires FFT/IFFT for transceiver design that is power consuming and may result in higher Peak-to-Average Power Ratio (PAPR) for the transmitted signal in time domain (PAPR is a critical assessing parameter for CMOS implementation). Furthermore, timing drift and phase noise become critical challenge that may degrade the system performance significantly.

1.2 Motivation and Objective

From the discussion for CDMA based UWB vs. multi-carrier OFDM based UWB given in the previous section, it is understood that each side has its own pros and cons.

Although there are various challenges in exploiting the capacity of UWB to meet the demands of future wireless data communication and in designing high performance low complexity UWB systems for either CDMA or multi-band OFDM based schemes. In this thesis, we focus only on impulse based UWB systems for high data rate transmission in the presence of severe ISI and Multiple Access Interference (MAI). Since impulse radio has been the focus at the time when we just started our research in early 2003. Nevertheless, the research method proposed in this thesis can be applied to single carrier DS-CDMA based UWB systems with multiple operational band [22, 70]. On the other hand, impulse radio UWB has also become a promising candidate for low rate communication and ranging/location in WPAN proposed by IEEE 802.15.4a task group [3] targeted for applications such as in wireless sensor networks. We expect the research method proposed in this thesis can be extended to the lower rate UWB communication systems in the presence of MAI as well.

Considering high data rate impulse radio based UWB systems, channel response

may span multiple symbol durations. Conventional RAKE receiver suffers from performance degradation due to severe ISI and MAI [52]. A Minimum Mean Squared Error (MMSE) multiuser detection [81] based receiver is superior for ISI and MAI mitigation. But this requires large number of training symbols and involves high computational complexity due to the long delay spread in UWB channel. A key challenge is to develop a high performance equalizer given limited training sample support with manageable complexity. This motivate us to pursuit a method using MMSE detection with reduced complexity for efficient UWB multipath channel equalization in the presence severe ISI and/or MAI.

Since multipath channel estimation and timing synchronization also pose some of the key challenges in UWB systems [90] especially in the presence of interference. Equalization methods that do not rely overly on the accurate channel estimation would be preferred. We consider MMSE receiver with over sampling and large observation window that improves energy capturing as well as avoids the requirement of accurate timing synchronization (i.e., the detection of time of arrival). Least Squares (LS) estimation is then utilized to obtain equalizer coefficients based on training samples that avoids explicit channel estimation. Adaptive filter algorithms such as Least Mean Square (LMS) or Recursive Least Squares (RLS) can be applied as well. Moreover, tap selection based non-uniformly spaced equalizer is adopted to achieve high performance with reduced complexity. Further, receive antenna selection technique that is able to achieve improved performance but retains hardware implementation simplicity is investigated for high data rate low power UWB systems.

The objective of this research work is to propose an efficient MMSE based multipath channel equalization technique with reduced complexity for high data rate UWB communication systems in the presence of ISI and MAI. Further, as an extension of the work, we also aim at developing an effective antenna selection technique for Multiple Input Multiple Output (MIMO) systems in the presence of unknown co-channel interference.

1.3 Contributions

The contributions in the research work presented in this dissertation are two fold.

Firstly, a new class of tap selection techniques is proposed for UWB long delay spread multipath channel equalization. Compared with the conventional RAKE receiver, the proposed tap selection based MMSE receiver achieves superior performance in the presence of severe ISI and MAI. Compared with the conventional uniformly spaced MMSE equalization method, this tap selection based non-uniformly spaced MMSE equalization method is able to achieve reduced computational complexity and improved system performance under limited training symbol support. On the other hand, compared with existing tap selection methods that require Channel Impulse Response (CIR), the proposed LS based tap selection technique is directly implemented based on training symbol sequence without the need for explicit channel estimation. As a result, it avoids the difficulty of accurate channel estimation in the presence of unknown co-channel interference. Moreover, Diagonal Loading (DL) technique is incorporated into the tap selection and MMSE equalization process to insure the system robustness given limited training symbol support in the case of practical implementation.

Secondly, as an extension of the proposed tap selection technique, LS based transmit antenna and receive antenna selection techniques for MIMO spatial multiplexing systems with linear MMSE receivers are proposed. Unlike conventional antenna selection approaches that require Channel State Information (CSI) which may not be easy to obtain, this proposed technique is directly implemented based on training sample sequence without the need for explicit channel estimation. As a result, it is able to retain the diversity order of a full complexity system in the presence of unknown co-channel interference. Moreover, practical implementation is made possible by incorporating fast Backward Greedy Algorithm (BGA) into the proposed receive antenna selection technique.

1.4 Outline of Thesis

This dissertation is organized as follows

In chapter 1, following the brief introduction to UWB technology, our research objective is described and research contributions are summarized.

In chapter 2, UWB multipath channel characterization and performance analysis with conventional RAKE receivers are briefly described. In order to achieve improved interference mitigation capability, linear MMSE receiver techniques are introduced which form the required background for discussions in the following chapters.

In chapter 3, a simplified matrix representation for UWB MIMO systems is formulated and the performance for THSS/DSSS based UWB systems with linear MMSE receivers is evaluated. This contribution has been published in [10] in the author's publication list.

In chapter 4, assuming CIR is known at receiver, the performance for tap selection based MMSE equalization technique is evaluated for UWB indoor multipath channels. This contribution has been published in [4,9] in the author's publication list.

The result from chapter 4 is shown to be promising. However, channel estimation becomes a pre-requisite procedure. Since UWB multipath channel estimation poses a challenge in the presence of unknown co-channel interference, in chapter 5, a new class of tap selection techniques is proposed for UWB multipath channel equalization under LS criterion. These techniques are directly implemented based on training symbol sequence. Explicit channel estimation is avoided. This contribution has been published in [3,6,7,8] in the author's publication list.

In chapter 6, assuming CSI at receiver end, fast transmit and receive antenna selection techniques are proposed under MMSE criterion for MIMO spatial multiplexing systems with linear receivers.

Similarly, obtaining accurate channel estimation for MIMO systems in the presence of unknown co-channel interference may not be an easy task. In chapter 7, novel transmit and receive antenna selection techniques are proposed for MIMO spatial multiplexing systems with linear MMSE receivers under LS criterion, without the need

for CSI. As a result, it avoids channel estimation and is able to retain the diversity order of a full complexity system in the presence of unknown co-channel interference. This contribution has been published in [1,2,5] in the author's publication list.

Further, we extend the LS based receive antenna selection technique to UWB system to improve its performance by mitigating the channel shadowing effect.

Lastly, the research findings are summarized and the conclusions are drawn in chapter 8.

Chapter 2

UWB Receivers: An Overview

Channel characterization and modelling are essential for Ultra-Wideband (UWB) receiver design and for its performance evaluation. In this chapter, UWB indoor channel characterization and IEEE 802.15.3a recommendations for UWB indoor channel modelling are briefly described. Impulse radio based UWB system model is then introduced. The multiple access performance for UWB systems with conventional RAKE receivers is discussed. Minimum Mean Squared Error (MMSE) based linear Multiuser Detection (MUD) technique is also introduced that is more effective in mitigating Inter Symbol Interference (ISI) and Multiple Access Interference (MAI). In addition, as a rank reduced MMSE equalizer, Multistage Wiener Filter (MWF) is described which requires lower computational complexity and less training samples.

2.1 UWB Multipath Channel Modelling

Unlike a conventional radio channel, a UWB indoor channel exhibits very fine time resolution and resolvable multipath characteristics due to its ultra-wide bandwidth. UWB communication systems can benefit from their dense multipath channels for temporal diversity with appropriate receiver design to capture the multipath energy and mitigate ISI, MAI, as well as Narrow Band Interference (NBI). Successful system design relies on the accurate channel modelling.

2.1.1 UWB Indoor Channel Characterization

Pioneering works on the UWB indoor channel measurement and characterization have been done in Ultra-Lab in University of Southern California (USC) [72, 85, 86, 87, 16]. These works show that UWB signal is robust against multipath fading and multipath energy capture is critical to the receiver performance. The studies in [87] also show that the number of dominant specular multipath components for an indoor UWB propagation is much larger than 5 and typically less than 50 in a typical modern laboratory and office building. This indicates that an effective RAKE receiver may be required to employ tens of taps to insure energy capturing and benefit from the multipath diversity.

Figure 2.1: An ideal received monocycle signal

Figure 2.1 shows an ideal received Gaussian monocycle signal initially proposed for impulse radio [87]. It can be represented mathematically as

$$w_{rec}(t + 0.35) = (1 - 4\pi(t/\tau_m)^2) \exp(-2\pi(t/\tau_m)^2), \quad \tau_m = 0.2877. \quad (2.1.1)$$

The actual received monocycle signal measured at 1 meter distance as shown in [87] exhibits some distortion in waveform due to the antenna differentiating effect and the multipath channel fading effect. This should be taken into account for the

correlator's template design for conventional RAKE receivers. Resolvable multipath components will facilitate the energy acquisition and diversity combining for robust performance even under extremely low SNR as studied in [87, 17].

2.1.2 IEEE 802.15.3a Channel Model Recommendation

2.1.2.1 Statistical Channel Modelling

Multipath fading [66] refers to the transmitted signal reaching the receiver via different paths due to reflection, refraction and scattering. In narrow band transmission, the multipath medium causes fluctuations in the received signal envelope and phase. In wide band transmission, multipath effect will produce a series of delayed and attenuated echoes for each transmitted pulse. In order to reduce the performance degradation due to multipath fading in a typical indoor radio propagation channel, accurate channel characterization and modelling is critical for receiver design.

Assuming a quasi-static channel, the general model of Channel Impulse Response (CIR) for time-invariant indoor propagation channel is given by [40],

$$h(t) = \sum_{k=0}^{n_L-1} \alpha_k \delta(t - \tau_k) e^{j\theta_k} \quad (2.1.2)$$

To fully characterize the multipath indoor channel by CIR as given in (2.1.2) [40], the number of multipath components n_L , statistical distributions of path amplitudes $\{\alpha_k\}$ and arrival times $\{\tau_k\}$, as well as distribution of path phases $\{\theta_k\}$, should be considered. The distribution of the arrival time sequence is well modelled by Poisson model or its variations. The distribution of path amplitudes is empirically characterized by distribution functions such as Rayleigh distribution. The distribution of path phases is usually considered as uniform distribution.

Root Mean Square (RMS) delay spread [66] is considered as the most important single parameter that characterizes a multipath channel. It is defined as the square root of the second central moment of a Power Delay Profile (PDP), i.e.,

$$\tau_{rms} \triangleq \sqrt{\tau^2 - (\bar{\tau})^2} \quad (2.1.3)$$

where $\overline{\tau^n}$, $n = 1, 2$, is defined as

$$\overline{\tau^n} \triangleq \frac{\sum_k \tau_k^n \alpha_k^2}{\sum_k \alpha_k^2} \quad (2.1.4)$$

RMS delay spread serves as an indication of potential performance degradation caused by ISI. The mean values of τ_{rms} for conventional narrow-band/wide-band channels are around $20 \sim 30$ nsec with $5 \sim 30m$ antenna separation [40].

2.1.2.2 IEEE 802.15.3a Channel Model Recommendation

The goal of the channel model is to capture both the path loss¹ and multipath characteristics of typical environments where IEEE 802.15.3a (WPAN) devices are expected to operate. The model should be relatively simple to use in order to allow PHY designers to use the model in a timely manner to evaluate the performance of their PHY in typical operational environments. In addition, it should be reflective of actual channel measurements. Since it may be difficult for a single model to reflect all of the possible channel environments and characteristics, the IEEE 802.15.3a working group chose different sets of modelling parameters to match the following primary characteristics of the multipath channel, i.e., (1) RMS delay spread (2) power decay profile (3) number of multipath components (defined as the number of multipath arrivals that are within 10 dB of the peak multipath arrival) to the channel measurements in four types of operational environments as given in Table 2.1.

As the clustering of the multipath arrivals was observed in the channel measurements, a statistical channel model based on Saleh-Valenzuela (S-V) model [71] was proposed since the experimental comparisons showed that the S-V model was able to best fit the measured channel characteristics. In addition, the Rayleigh and log-normal amplitude distribution was compared with measurement data, and the results showed that the lognormal distribution best fit the characteristics of the measurement data. Moreover, independent fading is assumed for each cluster as well as each ray within the cluster. Therefore, the final model proposed was the S-V model with a independent lognormal fading distribution on the amplitudes.

¹Please refer to [25] for the path loss model in detail.

Thus, the multipath CIR for UWB indoor propagation channel is modelled by

$$h_i(t) = X_i \sum_{l=0}^L \sum_{k=0}^K \alpha_{k,l}^{(i)} \delta(t - T_l^{(i)} - \tau_{k,l}^{(i)}) \quad (2.1.5)$$

where i refers to the i -th channel realization, X_i represents the log-normal distribution for shadowing, $\alpha_{k,l}^{(i)}$ is the amplitude of multipath components, $T_l^{(i)}$ is the delay of the l -th cluster and $\tau_{k,l}^{(i)}$ is the delay of the k -th ray of the l -th cluster. In fact, S-V model given in (2.1.5) is a special case of the generic model given in (2.1.2). The distribution of cluster arrival time and the ray arrival time are given as follows.

The Poisson arrival process for the 1st ray of each cluster is given by

$$p(T_l|T_{l-1}) = \Lambda \exp[-\Lambda(T_l - T_{l-1})], \quad l > 0 \quad (2.1.6)$$

The Poisson arrival process for the rays within each cluster is given by

$$p(\tau_{k,l}|\tau_{(k-1),l}) = \lambda \exp[-\lambda(\tau_{k,l} - \tau_{(k-1),l})], \quad k > 0 \quad (2.1.7)$$

where Λ is the cluster arrival rate and λ is the ray arrival rate within each cluster.

Unlike the complex baseband model utilized for narrowband systems to capture its channel behavior of amplitude and phase of carrier frequency independently, a real valued channel modelling at RF is considered for UWB systems where the phase information can be easily accounted for by introducing delays for multipath components in UWB channel. The amplitude of multipath components are defined as

$$\alpha_{k,l} = p_{k,l} \xi_l \beta_{k,l} \quad (2.1.8)$$

where $p_{k,l}$ is equiprobable as ± 1 to account for possible signal inversion due to the reflections, ξ_l reflects the fading associated with the l -th cluster and $\beta_{k,l}$ corresponds to the fading associated with the k -th ray of the l -th cluster. In addition, $\xi_l \beta_{k,l}$ is lognormally distributed as

$$20 \log(\xi_l \beta_{k,l}) \propto \mathcal{N}(\mu_{k,l}, \sigma_1^2 + \sigma_2^2), \quad \text{or} \quad (2.1.9)$$

$$\xi_l \beta_{k,l} = 10^{(\mu_{k,l} + n_1 + n_2)/20}$$

where $n_1 \propto \mathcal{N}(0, \sigma_1^2)$ and $n_2 \propto \mathcal{N}(0, \sigma_2^2)$ are independent and correspond to the fading on each cluster and ray respectively.

The expectation of the power delay profile agrees with the exponential decay law as given by

$$E[|\xi_l \beta_{k,l}|^2] = \Omega_0 \exp(-T_l/\Gamma) \exp(-\tau_{k,l}/\gamma) \quad (2.1.10)$$

where Ω_0 is the mean energy of the first path of the first cluster, Γ is the cluster decay factor and γ is the ray decay factor. Further, $\mu_{k,l}$ is given by

$$\mu_{kl} = \frac{10 \ln(\Omega_0) - 10T_l/\Gamma - 10\tau_{kl}/\gamma}{\ln 10} - \frac{(\sigma_1^2 + \sigma_2^2) \ln(10)}{20} \quad (2.1.11)$$

Since the log-normal shadowing of the total multipath energy is captured by the term X_i , the total energy contained in the terms $\{\alpha_{k,l}^i\}$ is normalized to unity for each realization. This shadowing term is characterized by $20\log(X_i) \propto \mathcal{N}(0, \sigma_x^2)$.

Finally, the recommendation provides four sets of parameters to match main channel characteristics for four types of measurement environments. That is defined as four channel models (CM1-4) as given in Table 2.1.

Characteristics	CM1	CM2	CM3	CM4
Mean excess delay τ_m (nsec)	5.0	9.94	15.9	30.1
RMS delay spread τ_{rms} (nsec)	5	8	15	25
NP_{10dB}	12.5	15.3	24.9	41.2
Distance	LOS(0-4m)	NLOS(0-4m)	NLOS(4-10m)	NLOS

Table 2.1: IEEE802.15.3a channel model characteristics

As the above model is a modified Saleh-Valenzuela model where Rayleigh fading distribution that is usually applied to conventional narrowband/wideband channel modelling has been replaced by lognormal distribution. This indicates that fading phenomenon in a UWB channel is not as severe as that in a conventional Rayleigh fading channel.

Based on IEEE 802.15.3a channel model recommendation, Figure 2.2 illustrates the realizations of four typical CIR profiles with a sampling duration of $0.167nsec$ for CM1 to CM4 respectively.

Figure 2.2: Typical channel impulse response for UWB multipath channel realizations

2.2 UWB System Models

In this section, system models for impulse radio based UWB is introduced. The performance of conventional RAKE receiver is discussed for UWB multipath channel. An impulse radio model proposed in [72, 85] for multiple access communications uses Time Hopping Spread Spectrum (THSS) technique together with Pulse Position Modulation (PPM). The transmitted signal is represented as

$$s_{tr}^{(k)}(t) = \sum_{j=-\infty}^{\infty} w_{tr}(t - jT_f - c_j^{(k)}T_c - \delta d_{\lfloor j/N_s \rfloor}^{(k)}) \quad (2.2.1)$$

where k refers to the k -th transmitter, T_f denotes the pulse repetition time (or symbol duration), $\{c_j^{(k)}\}$ is the time-hopping sequence for the k -th transmitter where $0 \leq c_j^{(k)} < N_h$, T_c denotes the duration of addressable time delay bin for time-hopping where $N_h T_c \leq T_f$ and N_h denotes possible number of hopping positions contained within a symbol duration T_f , δ is the modulation index for PPM that can be chosen to optimize the performance, $\{d_j^{(k)}\}_{j=-\infty}^{\infty}$ denotes data sequence for the k -th transmitter where $d_l^{(k)} \in \{0, 1\}$. Considering over sampled modulation with N_s monocycles per symbol, we then have symbol duration $T_s = N_s T_f$ and symbol rate $R_s = 1/(N_s T_f)$. The term $w_{tr}(t)$ represents the transmitted monocycle waveform. A typical ideal received monocycle at the output of the receiver antenna (considering the effect of antenna differentiating and free space propagation) was shown in Figure 2.1.

For simplicity, $N_s = 1$ is assumed for high data rate transmission, i.e. only one monocycle per symbol is transmitted. In general, the impulse radio based on Direct Sequence Spread Spectrum (DSSS) as well as THSS technique with binary data modulation scheme by bipolar or antipodal signaling (BPSK) [24, 26] is defined as follows. For the k -th transmitter, the transmitted signal is given as, for THSS-PPM:

$$s_{tr}^{(k)}(t) = \sum_{j=-\infty}^{\infty} w_{tr}(t - jT_f - c_j^{(k)}T_c - \delta d_j^{(k)}) , \quad d_j^{(k)} \in \{0, 1\} \quad (2.2.2)$$

for THSS-BPSK:

$$s_{tr}^{(k)}(t) = \sum_{j=-\infty}^{\infty} d_j^{(k)} w_{tr}(t - jT_f - c_j^{(k)}T_c) , \quad d_j^{(k)} \in \{\pm 1\} \quad (2.2.3)$$

for DSSS-BPSK:

$$\begin{aligned} s_{tr}^{(k)}(t) &= \sum_{j=-\infty}^{\infty} d_j^{(k)} w'_{tr}(t - jT_f), \quad d_j^{(k)} \in \{\pm 1\} \\ w'_{tr}^{(k)}(t) &= \frac{1}{\sqrt{n_s}} \sum_{i=1}^{n_s} c_i^{(k)} w_{tr}(t - iT_c) \end{aligned} \quad (2.2.4)$$

In the case of THSS transmission, pseudorandom time hopping is applied to eliminate catastrophic collisions due to multiple access transmitters. Each transmitter is assigned a distinctive time-hopping sequence of codes, i.e., $c^{(k)} = \{c_i^{(k)}\}$, $i = 1, \dots, n_h$. These codes are periodic with the period of n_h , i.e., $c_{(l)}^{(k)} = c_{[l/n_h]+1}^{(k)}$. Where we assume $n_h \leq N_h$. Otherwise, the chance of code collision may increase. Then the entire waveform $\sum_{j=-\infty}^{\infty} w_{tr}(t - jT_f - c_j^{(k)}T_c)$ is periodic with the period of $T_p = n_h T_f$.

In the case of DSSS transmission, pseudorandom spreading sequence is multiplied with the antipodal pulse stream. Usually quasi-orthogonal codes are chosen for different transmitters to facilitate multiple access transmissions. The spreading codes are periodic with the period of n_s , i.e., $c^{(k)} = \{c_i^{(k)} \in \pm 1\}$, $i = 1, \dots, n_s$. For simplicity, assume $n_s T_c = T_f$, that is, the same spreading code is repeated in each symbol interval. On the other hand, some DSSS systems such as Code Division Multiple Access (CDMA) cellular phone systems operate with much longer spreading codes for coding security.

In terms of binary PPM data modulation, both orthogonal or overlapped PPM can be applied by choosing the appropriate modulation index parameter δ . M-ary data modulation scheme may be applied in similar way to impulse radio with THSS or DSSS transmission scheme. For example, an impulse radio by THSS with M-ary PPM or an impulse radio by DSSS with M-ary Pulse Amplitude Modulation (PAM) may be considered.

When n_{user} transmitters are active for multiple access transmission, the received signal at the output of the receiver antenna can be modelled as

$$r(t) = \sum_{k=1}^{n_{user}} A^{(k)} s_{rec}^{(k)}(t - \tau_k) + n(t) \quad (2.2.5)$$

where $A^{(k)}$ denotes the received signal amplitude from the k -th transmitter and $n(t)$ is the Addictive White Gaussian Noise (AWGN) signal with zero mean and standard deviation of σ_n . The antenna and propagation channel modify the shape of the transmitted monocycle $w_{tr}(t)$ to $w_{rec}(t)$. Hence the transmitted signal $s_{tr}(t)$ is modified to $s_{rec}(t)$ at receiver side. Without loss of generality, the 1st transmitter is assumed as the desired transmitter for all the following discussions.

2.3 Conventional RAKE Receivers

2.3.1 AWGN Channel

Firstly, signal detection in AWGN channel is considered. Assuming the MAI as zero mean Gaussian random process, the optimal receiver is simply a pulse correlator as discussed in [74]. The waveform template $v(t)$ for the optimal correlator can be expressed as follows.

For THSS-PPM systems

$$v(t) = w_{rec}(t) - w_{rec}(t - \delta) \quad (2.3.1)$$

For THSS-BPSK systems

$$v(t) = w_{rec1}(t) - w_{rec2}(t) = 2w_{rec}(t) \quad (2.3.2)$$

where $w_{rec1}(t) = -w_{rec2}(t) = w_{rec}(t)$. For both THSS-PPM and THSS-BPSK systems, the correlation template at the j -th symbol (bit) duration is then given by

$$v_{bit}(t) = v(t - jT_f - c_j^{(1)}T_c - \tau_1) \quad (2.3.3)$$

For DSSS-BPSK systems, the optimal correlation template can be written as

$$v(t) = w_{rec1}(t) - w_{rec2}(t) = 2w_{rec}(t) \quad (2.3.4)$$

$$w'_{rec}(t) = \frac{1}{\sqrt{n_s}} \sum_{i=1}^{n_s} c_i^{(1)} w_{rec}(t - iT_c) \quad (2.3.5)$$

$$v'(t) = w'_{rec1}(t) - w'_{rec2}(t) = 2w'_{rec}(t) = \frac{2}{\sqrt{n_s}} \sum_{i=1}^{n_s} c_i^{(1)} w_{rec}(t - iT_c) \quad (2.3.6)$$

$$v_{bit}(t) = v'(t - jT_f - \tau_1) = \frac{2}{\sqrt{n_s}} \sum_{i=1}^{n_s} c_i^{(1)} v(t - jT_f - iT_c - \tau_1) \quad (2.3.7)$$

The test statistic for the desired transmitter is then given by

$$T^{(1)}(j) = \int_{jT_f}^{(j+1)T_f} r(t) v_{bit}(t) dt \quad (2.3.8)$$

2.3.2 Multipath Channel

Considering a practical UWB indoor dense multipath channel and applying a general tapped-delay-line channel model given in (2.1.2) with minimum *bin* width of $\Delta\tau$, disregarding phase delay, that is,

$$h(t) = \sum_{p=0}^{n_p} \alpha_p \delta(t - (p-1)\Delta\tau - \tau_k) \quad (2.3.9)$$

the received signal can be written as

$$r(t) = h(t) * s(t) + n(t) \quad (2.3.10)$$

$$r(t) = \sum_{k=1}^{n_{user}} \sum_{p=1}^{n_p} \alpha_p^{(k)} A^{(k)} s_{rec}^{(k)}(t - (p-1)\Delta\tau - \tau_k) + n(t) \quad (2.3.11)$$

where, $\{\alpha_p^{(k)}, p = 1, \dots, n_p\}$ denotes the CIR components for the k -th transmitter and τ_k represents its channel delay. It can be referred to S-V model given in (2.1.5) for detail, where the S-V model is just a special case of the generic model given in (2.3.9).

In order to capture the transmitted signal energy in resolvable multipath, a multiple correlator based RAKE receiver with a Maximum Ratio Combiner (MRC) is introduced [66], where each correlator is coherent with a selected multipath component. The test statistic for the desired transmitter is given by

$$T^{(1)}(j) = \sum_{p=1}^{n_p} \alpha_p^{(1)} \int_{jT_f + (p-1)\Delta\tau}^{(j+1)T_f + (p-1)\Delta\tau} r(t) v_{bit}(t - (p-1)\Delta\tau) dt \quad (2.3.12)$$

A RAKE receiver with MRC maximizes the SNR. It is an optimal receiver for multipath channels only in the absence of interference such as ISI and MAI [14].

2.3.3 BER Performance Analysis

With the test statistic given in (2.3.8) and (2.3.12), multiple access performance for UWB systems can be analyzed as follows by assuming Gaussian approximation for ISI and MAI.

For an AWGN channel, we rewrite (2.2.5) as

$$r(t) = A^{(1)} s_{rec}^{(1)}(t - \tau_1) + n_{tot}(t) \quad (2.3.13)$$

where

$$n_{tot}(t) = \sum_{k=2}^{n_{user}} A^{(k)} s_{rec}^{(k)}(t - \tau_k) + n(t) \quad (2.3.14)$$

is assumed to be a zero mean Gaussian random process. The Bit Error Rate (BER) of the impulse radio system can be expressed as follows [86],

$$P_b(n_{user}) = Q(\sqrt{SNR(n_{user})}) \quad (2.3.15)$$

where

$$SNR(n_{user}) = \frac{(A^{(1)})^2 R_{vw}^2(0)}{\sigma_{rec}^2 + \sigma_a^2 \sum_{k=2}^{n_{user}} (A^{(k)})^2} \quad (2.3.16)$$

$$R_{vw}(\tau) = \int_{-\infty}^{\infty} v(t)w(t - \tau)dt \quad (2.3.17)$$

$$\sigma_a^2 = T_f^{-1} \int_{-\infty}^{\infty} R_{vw}^2(\tau)d\tau \quad (2.3.18)$$

$$\sigma_{rec}^2 = E \left\{ \left[\int_{-\infty}^{\infty} n(t)v(t)dt \right]^2 \right\} = \sigma_n^2 \int_{-\infty}^{\infty} v^2(t)dt = \frac{N_0}{2} \int_{-\infty}^{\infty} v^2(t)dt \quad (2.3.19)$$

In the case when only the desired transmitter 1 is active, the SNR can be written as

$$SNR(1) = \frac{(A^{(1)})^2 R_{vw}^2(0)}{\sigma_{rec}^2} = \frac{(A^{(1)})^2 R_{vw}^2(0)}{\frac{N_0}{2} \int_{-\infty}^{\infty} v^2(t)dt} \quad (2.3.20)$$

In the case when n_{user} transmitters are active, the SNR degradation due to MAI can be derived as

$$\begin{aligned} SNR(n_{user}) &= \left\{ SNR^{-1}(1) + \frac{\sigma_a^2}{R_{vw}^2(0)} \sum_{k=2}^{n_{user}} \left(\frac{A_k}{A_1} \right)^2 \right\}^{-1} \\ &= \left\{ SNR^{-1}(1) + \frac{\int_{-\infty}^{\infty} R_{vw}^2(\tau)d\tau}{T_f R_{vw}^2(0)} \sum_{k=2}^{n_{user}} \left(\frac{A_k}{A_1} \right)^2 \right\}^{-1} \end{aligned} \quad (2.3.21)$$

The performance analysis given in (2.3.20) and (2.3.21) is valid for impulse radio under AWGN channel only. In the case of multipath channel propagation, based on a tapped-delay-line channel model given in (2.3.9) and a RAKE receiver with MRC given in (2.3.12), BER performance can be written as follows [77].

$$SNR(1) = \frac{E_s}{\sigma_n^2} = \frac{A_1^2 \left(\sum_{p=1}^{n_p} (\alpha_p^{(1)})^2 \right)^2 R_{vw}^2(0)}{\left(\sum_{p=1}^{n_p} (\alpha_p^{(1)})^2 \right) \frac{N_0}{2} \int_{-\infty}^{\infty} v^2(t) dt} = \frac{A_1^2 \left(\sum_{p=1}^{n_p} (\alpha_p^{(1)})^2 \right) R_{vw}^2(0)}{\frac{N_0}{2} \int_{-\infty}^{\infty} v^2(t) dt} \quad (2.3.22)$$

$$SNR(n_{user}) = \left\{ SNR^{-1}(1) + \frac{\frac{1}{T_f} \left(\sum_{k=2}^{n_{user}} \left(\frac{A_k}{A_1} \right)^2 \sum_{p=1}^{n_p} \sum_{m=1}^{n_{Lk}} (\alpha_p^{(1)} \alpha_m^{(k)})^2 + \sum_{p=1}^{n_p} \sum_{m=1, m \neq p}^{n_{L1}} (\alpha_p^{(1)} \alpha_m^{(1)})^2 \right) \int_{-\infty}^{\infty} R_{vw}^2(\tau) d\tau}{\left(\sum_{p=1}^{n_p} (\alpha_p^{(1)})^2 \right)^2 R_{vw}^2(0)} \right\}^{-1} \quad (2.3.23)$$

The SNR degradation due to MAI and ISI terms in (2.3.23) are approximately modelled by Gaussian distributions. When the number of multipath is set to one, equation (2.3.22) and (2.3.23) reduce to AWGN case.

The above performance analysis is derived based on UWB THSS as given in [86, 77]. For the case of UWB DSSS, equation (2.3.22) and (2.3.23) are still valid by replacing the waveform correlation function $R_{vw}(\tau)$ with

$$R_{v'w'}^{(k)}(\tau) = \int_{-\infty}^{\infty} v'(t) w'^{(k)}(t - \tau) dt \quad (2.3.24)$$

where $w'^{(k)}(t)$ and $v'(t)$ are defined in (2.2.4) and (2.3.6) respectively.

It has been shown that impulse radio based UWB is capable of supporting large number of multiple access users in an ideal power-controlled AWGN environment [86]. However, performance degradation due to ISI and MAI does affect multiple access capability significantly when a practical multipath channel is considered for evaluation [77]. Performance analysis given in (2.3.23) by applying Gaussian approximation for ISI and MAI is also shown to be over optimistic by simulation.

2.4 Linear MMSE Receivers

In a frequency selective fading channel, multipath propagation often lengthens the transmission time required for the baseband signal and smears the signal by ISI. For large multipath time delay spread, timing error and ISI are dominant factors which cause BER performance degradation under high symbol rate transmission.

In multiple access environment, the received signals from multiple access users are no longer orthogonal due to any of the following reasons [61]; quasi-orthogonal spreading codes being used, asynchronous transmission, imperfect symbol synchronization, or multipath propagation with large delay spread which may introduce interpath interference as well as ISI. In all these cases, MAI is inevitable. MAI will cause BER performance degradation and result in severe near-far problem for a matched filter or correlator based receiver.

In view of these factors, multiuser detection techniques [81] have been introduced which effectively improve receiver BER performance and near-far resistance by suppressing the ISI as well as MAI [54]. A Multiple Input Multiple Output (MIMO) system which incorporates space-time processing techniques to exploit the spatial diversity as well as temporal diversity is able to achieve improved performance.

For high data rate impulse radio based multiple access UWB systems, channel response may span over multiple symbol durations. Conventional RAKE receiver suffers from performance degradation due to severe ISI and MAI. An MMSE multiuser detection [81] based receiver is superior for ISI and MAI mitigation. In [75], multiple access performance has been discussed for THSS and DSSS UWB based on MMSE detection under AWGN channel assumption. An adaptive MMSE receiver utilizing Least Mean Square (LMS) or Recursive Least Squares (RLS) algorithm has been applied to Direct Sequence Code Division Multiple Access (DS-CDMA) based UWB. This has shown superior multiple access performance when compared with a conventional RAKE receiver at higher data transmission rate of 100Mbps [52]. The following section reviews the linear MMSE receiver techniques.

2.4.1 Discrete Time Models

As discussed in [61], consider a wireless communication system with multiple receive antennas in the presence of ISI. A discrete-time block signal model for consecutive samples of transmitted signals can be represented as

$$\mathbf{Y}(t_k) = \mathbf{H}\mathbf{S}(t_k) + \mathbf{N}(t_k) \quad (2.4.1)$$

where matrix $\mathbf{Y}$ ($n_R \times n_L$) denotes the output of block signals from the receive antenna array, n_R is the number of receive antennas and n_L is the number of samples within the observation window. The matrix $\mathbf{S}$ ($n_{ISI} \times n_L$) denotes the transmitted binary block signals where n_{ISI} is the number of symbol periods within the range of multipath time delay spread. The matrix $\mathbf{H}$ ($n_R \times n_{ISI}$) denotes the generalized channel transmission matrix in the presence of ISI and $\mathbf{N}$ ($n_R \times n_L$) denotes the matrix of AWGN signal. This model can be further generalized to take into account the multiple access transmitters (n_{user}) and multiple transmit antennas (n_T) by defining $\mathbf{S}$ with dimension of $n_{ISI}n_{user}n_T \times n_L$ and $\mathbf{H}$ with dimension of $n_R \times n_{ISI}n_{user}n_T$. Therefore, a discrete-time MIMO system model considering ISI and MAI can be represented as given in equation (2.4.1).

It is straight forward to rewrite the output block signal matrix into a signal vector by defining $\mathbf{y} = \text{vec}(\mathbf{Y}(k)) = (y_1^{(1)}, \dots, y_{n_R}^{(1)}, \dots, y_1^{(n_L)}, \dots, y_{n_R}^{(n_L)})^T$. It is also convenient to convert the input block signal matrix $\mathbf{S}(k)$ into a signal vector $\mathbf{s}$ in a similar way. The channel transmission matrix $\mathbf{H}$ is then required to be reconfigured as a block Toeplitz matrix accordingly. Therefore, we simply introduce a standard MIMO representation as a generalized system model given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (2.4.2)$$

Based on the generalized discrete-time models given in (2.4.2), a tapped-delay-line based linear receiver structure for MIMO systems with multipath channels is illustrated in Figure 2.3. UWB systems with multipath channels in the presence of ISI and MAI can be represented by this generalized MIMO model representation as well. The detailed modelling and performance evaluation for UWB systems with linear MMSE receivers will be discussed in the following chapter.

Figure 2.3: A tapped-delay-line based linear receiver structure for MIMO systems with multipath channels

2.4.2 Linear Receivers: ZF and MMSE

An optimal multiuser detector performs Maximum Likelihood Sequence Detection (MLSD) [64] which estimates the data sequence that is most likely to have been sent given the received signal samples. A real-time forward dynamic programming algorithm known as Viterbi algorithm can be carried out to search for the best path in a trellis diagram [81]. Assuming there are n_{user} equivalent transmitters, for binary case, the total number of states will be $2^{n_{user}}$ and the computational complexity for exhaustive search is on the order of $O(2^{n_{user}})$. In the case of long multipath delay spread, the number of states in the trellis becomes large since ISI spans over many symbol periods and will behave as equivalent transmitters in system model. The search time will increase exponentially. Instead of using MLSD in the case of large number of transmitters in the presence of ISI and MAI, linear receivers have been introduced to fit into the large gaps in performance and complexity between the conventional single-user matched filter receiver and the optimal maximum likelihood detector. Linear decorrelating detector and MMSE detector are frequently applied.

2.4.2.1 Decorrelating/ZF Detector

Considering a generalized system model in (2.4.2), it is natural to choose a linear detector in matrix form $\mathbf{W}$, to be able to recover the transmitted symbols free of error in the absence of additive noise by defining,

$$\mathbf{W} = \mathbf{H}^{-1} \quad (2.4.3)$$

where $\mathbf{W}$ is simply a Zero Forcing (ZF) equalizer. Since $\mathbf{H}$ may be singular, a more generalized form is a Moore-Penrose pseudo-inverse as given in [81],

$$\mathbf{W} = \mathbf{H}^\dagger = \begin{cases} (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H, & \text{for overdetermined system} \\ \mathbf{H}^H (\mathbf{H} \mathbf{H}^H)^{-1}, & \text{for underdetermined system} \end{cases} \quad (2.4.4)$$

where superscript H denotes conjugate transpose. The detection of the transmitted binary symbols is then given by

$$\hat{\mathbf{s}} = \text{sgn}(\mathbf{H}^\dagger \mathbf{y}) \quad (2.4.5)$$

The decorrelating detector is a suboptimal detector since it enhances the noise while completely eliminating the MAI. Assuming $E[\mathbf{nn}^H] = \sigma_n^2 \mathbf{I}$, the BER performance for the 1^{st} transmitter is simply given by

$$P_b^{(1)} = Q\left(\frac{1}{\sigma_n} \frac{(\mathbf{H}^\dagger \mathbf{H})_{11}}{\sqrt{(\mathbf{H}^\dagger (\mathbf{H}^\dagger)^H)_{11}}}\right) \quad (2.4.6)$$

It is shown in [81] that a decorrelating detector is indeed optimal in terms of MAI elimination and achieves the maximum near-far resistance.

2.4.2.2 MMSE Detector

In order to reduce noise enhancement made by ZF detector, a linear MMSE detector is introduced by choosing vector $\mathbf{W}$ to minimize the mean square error between the detected symbols and the desired symbols, that is,

$$\mathbf{W} = \arg \min_{\mathbf{W}} \left\{ E \|\mathbf{W}^H \mathbf{y} - \mathbf{s}\|^2 \right\} \quad (2.4.7)$$

Following from the orthogonality principle, the solution for MMSE detection is given by a Wiener filter [41], that is,

$$\mathbf{W} = \{E(\mathbf{y}\mathbf{y}^H)\}^{-1} E(\mathbf{y}\mathbf{s}^H) \quad (2.4.8)$$

where superscript H denotes conjugate transpose. The detector output is then given by

$$\hat{\mathbf{s}} = \text{sgn}(\mathbf{W}^H \mathbf{y}) \quad (2.4.9)$$

The MMSE detector is an optimal linear detector which suppresses the MAI as well as AWGN. It will reduce to a conventional matched filter based receiver when no interference exists. Moreover, it will converge to a decorrelating or ZF detector when SNR goes to infinity. Assuming $E[\mathbf{nn}^H] = \sigma_n^2 \mathbf{I}$, the BER performance for the desired transmitter, namely the 1^{st} transmitter, is given by [81],

$$P_b^{(1)} = 2^{1-n_{user}} \sum_{s_2, \dots, s_{n_{user}} \in \{\pm 1\}^{n_{user}-1}} Q \left(\frac{1}{\sigma_n} \frac{(\mathbf{W}^H \mathbf{H})_{11}}{\sqrt{(\mathbf{W}^H \mathbf{W})_{11}}} \left(1 + \sum_{k=2}^{n_{user}} \beta_k s_k \right) \right) \quad (2.4.10)$$

where,

$$\begin{aligned} B_k &= (\mathbf{W}^H \mathbf{H})_{1k} \\ \beta_k &= B_k / B_1 \end{aligned} \quad (2.4.11)$$

It is shown in [54] and [81] that the linear MMSE detector is near-far resistant and offers superior MAI and ISI suppression. Moreover, equation (2.4.8) for Wiener filter can be solved by adaptive filter methods such as LMS and RLS algorithms [41]. These adaptive algorithms provide a practical solution for the receiver to be adapted to the unknown channels in interference environment.

2.4.3 Least Squares Estimation

The method of Least Squares (LS) [41] may be viewed as an alternative to Wiener filter theory. The Wiener filter given in (2.4.8) is derived from ensemble averages. It is optimum in a statistical sense assuming the underlying environment be wide-sense stationary. On the other hand, the method of LS is deterministic in approach. It involves the use of time averages and depends on the number of samples used in the computation. The method of LS is a batch-processing approach. The filter is adapted to the operational environment by repeating the computation on a block-by-block basis.

Define an observation data matrix

$$\mathbf{A} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{n_{tap}}] \quad (2.4.12)$$

where $\mathbf{u}_j = [u_j(1), u_j(1), \dots, u_j(n_{sample})]^T$, n_{tap} denotes the number of taps for the linear filter and n_{sample} denotes the number of samples used for LS estimation. The estimation output of the desired response $\mathbf{d}$ can be written as

$$\hat{\mathbf{d}} = \mathbf{A}\mathbf{w} \quad (2.4.13)$$

where $\mathbf{w}$ denotes the filter coefficient vector. The criterion is to minimize the LS estimation error given by

$$\mathcal{E}_{LS} = \|\mathbf{d} - \hat{\mathbf{d}}\|^2 = \|\mathbf{d} - \mathbf{A}\hat{\mathbf{w}}\|^2 \quad (2.4.14)$$

Following from the principle of orthogonality, the normal equation for LS estimation can be written as [41]

$$\mathbf{A}^H \mathbf{A} \mathbf{w} = \mathbf{A}^H \mathbf{d} \quad (2.4.15)$$

Assuming that the inverse matrix $(\mathbf{A}^H \mathbf{A})^{-1}$ exists, the LS estimation of the filter $\mathbf{w}$ is given by

$$\hat{\mathbf{w}} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{d} \quad (2.4.16)$$

The LS estimation error is then given as

$$\mathcal{E}_{LS} = \mathbf{d}^H \mathbf{d} - \mathbf{d}^H \mathbf{P} \mathbf{d} \quad (2.4.17)$$

where matrix $\mathbf{P}$ denotes the projection operator which is defined as

$$\mathbf{P} = \mathbf{A}(\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \quad (2.4.18)$$

and has the properties of $\mathbf{P}^H \mathbf{P} = \mathbf{P} = \mathbf{P}^H$.

2.4.4 Decision Feedback Equalization

Decision Feedback Equalization (DFE) [81] is simply applying Successive Interference Cancellation (SIC) technique in the process of equalization. It is initially used in

single user channels subject to ISI, wherein previous decision output will be used as transmitted data to cancel ISI. It is a nonlinear approach which improves the system performance when combined with adaptive linear MMSE detector [7]. V-BLAST algorithm [27, 28, 89, 33, 57, 29] is another effective way for combining SIC with linear detection techniques to achieve improved performance. It bridges the performance gap between linear MMSE detection and optimal ML detection. The disadvantage of SIC is that the performance gain will be limited by error propagation.

2.5 Multistage Wiener Filter: Rank Reduced MMSE Receiver

As described in the previous section, the linear receiver such as ZF or MMSE detector that requires matrix inversion can still be quite demanding in computation when it is applied to channel transmission matrix with larger dimension. Hence it is necessary to look for more efficient algorithm with further reduced computational complexity. Multistage Wiener Filter (MWF) [34] is a rank reduced MMSE equalizer that requires lower computational complexity and less training sample support.

A MWF is obtained by projecting the received observation samples onto a much lower dimensional subspace. It was originally proposed for the Minimum Variance Distortionless Response (MVDR) adaptive array beamforming [34]. The MWF has been applied to the DS-CDMA systems and shown to achieve near full-rank performance with fewer training samples and much reduced filter rank [45]. Unlike other subspace based rank reduced methods such as Principal Components (PC) and Cross Spectral (CS) method [45], the MWF does not rely on an explicit estimate of the signal subspace, but rather generates a set of basis vectors by means of a successive refinement procedure. It requires neither matrix inversion nor eigen-decomposition of the sample covariance matrix which is computationally expensive (on the order of $\mathcal{O}(N^3)$ where N is the dimension of sample vectors) and is required by full rank or other rank reduced filter techniques.

The MWF is a stage by stage implementation of the classical Wiener filter that

minimizes Mean Square Error (MSE). It is simply the solution of Wiener-Hopf equation by employing the Krylov subspace ($\mathcal{K}_n(\mathbf{R}_y, \mathbf{h}) = \text{span}\{\mathbf{h}, \mathbf{R}_y \mathbf{h}, \mathbf{R}_y^2 \mathbf{h}, \dots, \mathbf{R}_y^{n-1} \mathbf{h}\}$) [44] [47] as the basis for the resulting rank reducing transformation where $\mathbf{h}$ represents the steering vector, $\mathbf{R}_y$ denotes the sample covariance matrix and n is the desired number of stages.

In addition to using the Krylov subspace, the transformation matrix can be formed from an orthonormal set of basis ($\mathbf{T} = \text{span}\{\mathbf{h}, \mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_{n-1}\}$) which is generated by means of a successive refinement procedure [34]. This transformation is shown to be equivalent to the span of the Krylov subspace [44] but avoids the sample covariance matrix estimation which could cost $\mathcal{O}(N^2 K)$ where K denotes the number of samples.

Figure 2.4: A Multistage Wiener Filter (MWF) structure

A fast implementation algorithm for the MWF has been proposed in [69]. It has a scalable structure shown in Figure 2.4. Firstly, the observation data vectors are projected onto the steering vector to form an initial estimate of the desired signal,

$$d_0(k) = \mathbf{h}^H \mathbf{y}(k) \quad (2.5.1)$$

Consider a square blocking matrix given by

$$\mathbf{B} = \mathbf{I} - \mathbf{h}\mathbf{h}^H \quad (2.5.2)$$

the residue of the signal can then be written as

$$\mathbf{y}_0(k) = (\mathbf{I} - \mathbf{h}\mathbf{h}^H) \mathbf{y}(k) = \mathbf{y}(k) - \mathbf{h}d_0(k) \quad (2.5.3)$$

Given $d_0(k)$ and $\mathbf{y}_0(k)$ as inputs, the processing continues with a recursion of adaptive stages. At the stage i , we have the averaged correlation vector given by

$$\mathbf{r}_{i-1} = \frac{1}{K} \sum_{k=1}^K \mathbf{y}_{i-1}(k) d_{i-1}^*(k) \quad (2.5.4)$$

where superscript $*$ denotes the conjugate operation. Its magnitude and direction are given respectively as

$$\delta_i = \|\mathbf{r}_{i-1}\| = (\mathbf{r}_{i-1}^H \mathbf{r}_{i-1})^{1/2} \quad (2.5.5)$$

and

$$\mathbf{h}_i = \mathbf{r}_{i-1} / \|\mathbf{r}_{i-1}\| \quad (2.5.6)$$

The projections of the data signal and its residue can then be computed by

$$d_i(k) = \mathbf{h}_i^H \mathbf{y}_{i-1}(k) \quad (2.5.7)$$

$$\mathbf{y}_i(k) = (\mathbf{I} - \mathbf{h}_i \mathbf{h}_i^H) \mathbf{y}_{i-1}(k) = \mathbf{y}_{i-1}(k) - \mathbf{h}_i d_i(k) \quad (2.5.8)$$

The recursive analysis process may be terminated at the stage n where $n \leq N-1$. After the analysis procedure, ε_{n-1} is set as d_{n-1} and the recursive process is followed for synthesis. For each stage i ($n-1 \geq i \geq 0$), we have

$$\xi_i = \frac{1}{K} \sum_{k=1}^K |\varepsilon_i(k)|^2 \quad (2.5.9)$$

$$w_i = \delta_i / \xi_i \quad (2.5.10)$$

$$\varepsilon_{i-1}(k) = d_{i-1}(k) - w_i \varepsilon_i(k) \quad (2.5.11)$$

The ultimate result is the estimation output $\varepsilon_0(k)$ that can be expressed as

$$\varepsilon_0(k) = \mathbf{w}_{MWF}^H \mathbf{y}(k) \quad (2.5.12)$$

where $\mathbf{w}_{MWF}$ is the equivalent weight vector for the MWF and it can be written as

$$\mathbf{w}_{MWF} = \mathbf{h} - w_1 \mathbf{h}_1 + w_1 w_2 \mathbf{h}_2 - w_1 w_2 w_3 \mathbf{h}_3 + \cdots \quad (2.5.13)$$

The computational complexity for this fast MWF implementation is on the order of $\mathcal{O}(nNK)$.

2.6 Summary

In this chapter, firstly, the UWB indoor channel characterization and IEEE 802.15.3a channel model recommendation are introduced. The multiple access performance for impulse radio based UWB systems with conventional RAKE receivers is then described. Further, linear equalization technique is introduced which is capable of effectively mitigating ISI and MAI in multipath channel and multiple access environment.

Chapter 3

Performance Evaluation for UWB Systems with Linear MMSE Receivers

For multiple access communication, Ultra-Wideband (UWB) systems may use either Time Hopping Spread Spectrum (THSS) or Direct Sequence Spread Spectrum (DSSS) methods [72][86][26]. UWB multiple access performance has been analyzed with conventional matched filter or RAKE receivers under ideal AWGN transmission channel assumption [72][86], or assuming multipath UWB channels by approximating Inter Symbol Interference (ISI) and Multiple Access Interference (MAI) components as zero mean Gaussian random process [77].

In this chapter, we contribute to formulate a generalized matrix representation for UWB Multiple Input Multiple Output (MIMO) systems in the presence of ISI and MAI. Based on this formulation, the Bit Error Rate (BER) performance for THSS and DSSS based UWB systems with linear Minimum Mean Squared Error (MMSE) receiver is evaluated. Further, a simplified UWB MIMO channel model is assumed to evaluate the system performance under different MIMO channel spatial correlation assumptions.

3.1 Discrete Time System Formulation for UWB

In this chapter, impulse radio based UWB systems using THSS and DSSS with bipolar (or BPSK) data modulation are considered. Let $d_n^{(i)} \in \{\pm 1\}$ be the transmitted data

bit stream and $w(t)$ be the transmitted pulse waveform. The transmitted signal for the i -th transmitter is represented as follows.

For THSS-BPSK system,

$$s^{(i)}(t) = \sum_{n=-\infty}^{\infty} d_n^{(i)} w(t - nT_f - c_{(n)}^{(i)} T_c) \quad (3.1.1)$$

where T_f and T_c denote symbol period and chip period respectively. The term $\{c_{(n)}^{(i)}\}$ is the pseudorandom time-hopping sequence for which the elements are chosen from a finite set $\{0, 1, \dots, n_h - 1\}$ and where we assume $n_h = N_h = T_f/T_c$ for simplicity. For DSSS-BPSK system,

$$s^{(i)}(t) = \frac{1}{\sqrt{n_s}} \sum_{n=-\infty}^{\infty} \sum_{l=1}^{n_s} d_n^{(i)} c_l^{(i)} w(t - nT_f - lT_c) \quad (3.1.2)$$

where $\{c_l^{(i)} \in \pm 1\} (l = 1, \dots, n_s)$ is the spreading sequence and n_s is the spreading factor.

Applying a tapped-delay-line based multipath channel model, the received signal is given as

$$r(t) = \sum_{i=1}^{n_{user}} \sum_{p=1}^{n_L} \alpha_p^{(i)} s^{(i)}(t - (p-1)\Delta\tau - \tau_i) + n(t) \quad (3.1.3)$$

where, $\{\alpha_p^{(i)}, p = 1, \dots, n_L\}$ denotes the Channel Impulse Response (CIR) and τ_i is the channel delay for the i -th transmitter. $\Delta\tau$ denotes the minimum resolvable *bin* width of the multipath channel and $n(t)$ is the AWGN signal with $\mathcal{N}(0, \sigma_n^2)$.

Extending the model given in (3.1.3) into a generic form for UWB MIMO systems by assuming a tapped-delay-line based wideband MIMO channel model

$$\mathbf{H}(t) = \sum_{p=1}^{n_L} \mathbf{H}_p \delta(t - (p-1)\Delta\tau) \quad (3.1.4)$$

The MIMO system model is then given by

$$\mathbf{r}(t) = \sum_{p=1}^{n_L} \mathbf{H}_p \mathbf{s}(t - (p-1)\Delta\tau) + \mathbf{n}(t) \quad (3.1.5)$$

where, $\mathbf{s}(t) = [s_1(t), \dots, s_{n_T}(t)]^T$ denotes the transmitted signal vector and $\mathbf{r}(t) = [r_1(t), \dots, r_{n_R}(t)]^T$ denotes the received signal vector. The matrix $\mathbf{H}_p$ represents the

p -th tap channel transmission matrix. The number of transmit and receive antennas are n_T and n_R respectively. For simplicity, the MAI is not given explicitly in (3.1.5). Nevertheless, the MAI transmitters can be accounted for in (3.1.5) as extra transmit antennas for further analysis.

To represent discrete time models for UWB THSS and DSSS systems in a consistent form, both systems are assumed to be using the same chip duration T_c and with the same spreading gain, i.e., $n_h = n_s = T_f/T_c$. For the n -th symbol duration, a spreading sequence matrix $\mathbf{C}(n)$ for either THSS or DSSS system can be defined as

$$\mathbf{C}(n) = \begin{bmatrix} \mathbf{C}_1(n) \\ \vdots \\ \mathbf{C}_{n_s}(n) \end{bmatrix}_{n_s n_T \times n_T} \quad (3.1.6)$$

and

$$\mathbf{C}_q(n) = \begin{bmatrix} c_q^{(1)}(n) & & \\ & \ddots & \\ & & c_q^{(n_T)}(n) \end{bmatrix}_{n_T \times n_T} \quad (3.1.7)$$

where for THSS-BPSK system, $c_q^{(i)}(n) \in \{0, 1\}$, $q = 1, \dots, n_s$ is the time hopping equivalent spreading sequence for the i -th transmitter and can be defined as

$$c_q^{(i)}(n) = \begin{cases} 1, & \text{if } q = \text{time hopping chip position at symbol } n \\ 0, & \text{otherwise} \end{cases} \quad (3.1.8)$$

For DSSS-BPSK system, $c_q^{(i)}(n) \in \{\pm 1/\sqrt{n_s}\}$ is simply the normalized spreading sequence for the i -th transmitter.

For simplicity, chip synchronous transmission for all transmitters is assumed. In the case of chip asynchronous transmission as discussed in [55] for CDMA systems, the effect of chip asynchronism in the worst case may result in an equivalent 3dB SNR loss for a chip rate sampling system. Using over sampling (faster than chip rate) will alleviate the SNR loss until it becomes negligible as sampling rate increases.

In order to derive a discrete time model under over sampling condition, assume that the observed signal at receiver side as a superposition of signals arriving from different paths within the given minimum resolvable *bins* (it is assumed as the sampling

duration). Further, the ideal unit-amplitude impulse is assumed for the transmitted pulse wave form $w(t)$. In practice, a generalized CIR ($h(t) = c(t) * w(t)$) can be considered to incorporate the effect due to the pulse wave form. Applying sampling at time $t = nT_f + (p-1)\Delta\tau$ for the n -th symbol duration in the p -th multipath component, the resulting chip based transmit symbol vector $\mathbf{x}'(n)$ after spreading can be expressed as

$$\mathbf{x}'(n) = \mathbf{C}(n)\mathbf{x}(n) = \begin{bmatrix} \mathbf{x}^{(1)}(n) \\ \vdots \\ \mathbf{x}^{(n_s)}(n) \end{bmatrix}_{n_s n_T \times 1} \quad (3.1.9)$$

and

$$\mathbf{x}(n) = \begin{bmatrix} x_1(n) \\ \vdots \\ x_{n_T}(n) \end{bmatrix}_{n_T \times 1} \quad (3.1.10)$$

$$\mathbf{x}^{(q)}(n) = \begin{bmatrix} c_q^{(1)}(n)x_1(n) \\ \vdots \\ c_q^{(n_T)}(n)x_{n_T}(n) \end{bmatrix}_{n_T \times 1} \quad (3.1.11)$$

where $x_i(n) = d_i(n)$ is the transmitted data symbol from the i -th transmitter at the n -th symbol duration.

For simplicity, assume that the sampling rate is an integer multiple of the symbol repetition rate, that is, let $n_\tau = T_f/\Delta\tau$ be an integer which denotes the number of samples within a symbol duration. The number of symbols affected by ISI is given as $n_{ISI} = n_L/n_\tau$, where assuming channel length n_L is an integer multiple of n_τ obtained by truncating the very low power tail of CIR. Let $y_j^{(p)}$ denote the receive signal sample from the p -th tap of the j -th receive antenna and $m_j^{(p)}$ denote the corresponding AWGN term. The discrete time model for UWB system in the

presence of ISI can be obtained as follows,

$$\begin{aligned}
 \mathbf{y}(n) &= \sum_{k=n-n_{ISI}+1}^n \mathbf{H}_k^n \mathbf{x}'(k) + \mathbf{m}(n) \\
 &= \sum_{k=n-n_{ISI}+1}^n \mathbf{H}_k^n \mathbf{C}(k) \mathbf{x}(k) + \mathbf{m}(n) \\
 &= \sum_{k=n-n_{ISI}+1}^n \mathbf{H}_k^n \mathbf{x}(k) + \mathbf{m}(n)
 \end{aligned} \tag{3.1.12}$$

where the over sampled received signal vector is given by

$$\mathbf{y}(n) = \begin{bmatrix} \mathbf{y}^{(1)}(n) \\ \vdots \\ \mathbf{y}^{(n_\tau)}(n) \end{bmatrix}_{n_R n_\tau \times 1} \tag{3.1.13}$$

$$\mathbf{y}^{(p)}(n) = \begin{bmatrix} y_1^{(p)}(n) \\ \vdots \\ y_{n_R}^{(p)}(n) \end{bmatrix}_{n_R \times 1} \tag{3.1.14}$$

Let $\mathbf{H}_k^n$ be the channel transmission matrix for transmission from the k -th transmitting symbol to the n -th receiving symbol duration. It can be defined as

$$\mathbf{H}_k^n = \mathbf{H}_k'^n \mathbf{C}(k) \tag{3.1.15}$$

where transmission matrix $\mathbf{H}_k'^n$ corresponds to the chip based transmit vector $\mathbf{x}'(k)$ given in (3.1.9) and can be expressed as

$$\mathbf{H}_k'^n = \begin{bmatrix} \mathbf{H}_k^n(1, 1) & \cdots & \mathbf{H}_k^n(1, n_s) \\ \vdots & \vdots & \vdots \\ \mathbf{H}_k^n(n_\tau, 1) & \cdots & \mathbf{H}_k^n(n_\tau, n_s) \end{bmatrix}_{n_R n_\tau \times n_T n_s} \tag{3.1.16}$$

and

$$\mathbf{H}_k^n(p, q) = \begin{bmatrix} \alpha_{1,1}^{(p,q)}(n, k) & \cdots & \alpha_{1,n_T}^{(p,q)}(n, k) \\ \vdots & \vdots & \vdots \\ \alpha_{n_R,1}^{(p,q)}(n, k) & \cdots & \alpha_{n_R,n_T}^{(p,q)}(n, k) \end{bmatrix}_{n_R \times n_T} \tag{3.1.17}$$

Let $D^{(p,q)}(n, k)$ be the overall transmission delay between the p -th sample at the n -th receiving symbol duration and the q -th chip position at the k -th transmitting symbol duration, we have

$$D^{(p,q)}(n, k) = p - (q - 1)n_\tau^c + (n - k)n_\tau \tag{3.1.18}$$

where $n_\tau^c = T_c/\Delta\tau$ and $n_\tau = T_f/\Delta\tau$. It can be concluded that,

$$\alpha_{j,i}^{(p,q)}(n,k) = \begin{cases} \alpha_{j,i}^{(D^{(p,q)}(n,k))}, & \text{if } 1 \leq D^{(p,q)}(n,k) \leq n_L \\ 0, & \text{otherwise} \end{cases} \quad (3.1.19)$$

where, $\alpha_{j,i}^{(D)}$ is the D -th tap of the multipath impulse response components for the channel from the i -th transmitter to the j -th receiver.

Consider sampling over n_{sym} symbol durations ($n_{sym} \leq n_{ISI}$) to acquire enough multipath energy and diversity to detect the transmitted symbols at the n -th symbol duration. It implies that the signal is detected after a delay of n_{sym} symbol durations with a total observation window of length $n'_L = n_{sym}n_\tau$ taps where $n'_L \leq n_L$. A discrete time model for UWB MIMO systems extended from (4.2.3), can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{m} \quad (3.1.20)$$

and

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}(n) \\ \vdots \\ \mathbf{y}(n + n_{sym} - 1) \end{bmatrix}_{N_R \times 1} \quad (3.1.21)$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}(n - n_{ISI} + 1) \\ \vdots \\ \mathbf{x}(n) \\ \vdots \\ \mathbf{x}(n + n_{sym} - 1) \end{bmatrix}_{N_T \times 1} \quad (3.1.22)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{n-n_{ISI}+1}^n & \cdots & \mathbf{H}_n^n & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \mathbf{H}_{n+n_{sym}-n_{ISI}}^{n+n_{sym}-1} & \cdots & \mathbf{H}_{n+n_{sym}-1}^{n+n_{sym}-1} \end{bmatrix}_{N_R \times N_T} \quad (3.1.23)$$

where we define $N_T = n_T(n_{ISI} + n_{sym} - 1)$ and $N_R = n_R n_\tau n_{sym} = n_R n'_L$.

Assuming the achievable diversity order for base-band transmitted UWB systems as half of the corresponding pass-band MIMO systems (this will be discussed in section 3.3), the order of diversity for MMSE detection can be approximated as $(N_R - N_T + 1)/2$. This indicates the diversity benefit due to the dense resolvable multipath components in UWB channels.

3.2 A Simplified UWB MIMO Channel Modelling

3.2.1 Wideband MIMO Channel Modelling

As shown in [63, 91, 92], a wideband MIMO channel model for Non-Line-of-Sight (NLOS) indoor MIMO channel can be represented by combining a Single Input Single Output (SISO) tapped-delay-line based wideband model and a narrowband MIMO channel model with correlation matrix as shown below

$$\mathbf{H}(t) = \sum_{l=1}^{n_L} (\mathbf{R}_{Rx}^l)^{1/2} \mathbf{G}_l ((\mathbf{R}_{Tx}^l)^{1/2})^T \delta(t - (l-1)\Delta\tau) \quad (3.2.1)$$

where $\mathbf{G}$ is a matrix with independent and identically distributed (i.i.d.) zero mean complex Gaussian elements. The distribution for the power of each element in $\mathbf{G}$ is modelled similarly as the power delay profile for a SISO channel, that is, it complies with exponential decay law. In addition, $\mathbf{R}_{Rx}$ and $\mathbf{R}_{Tx}$ are defined as the channel covariance matrices at the receiver and transmitter side respectively. Based on the assumption that the spatial correlation between receiver antennas is independent of the transmitter antennas and vice versa, the overall MIMO channel covariance matrix can be well approximated by the Kronecker product as follows,

$$\mathbf{R}_H = \mathbf{R}_{Tx} \otimes \mathbf{R}_{Rx}, \quad (3.2.2)$$

where

$$\mathbf{R}_{Tx} = E[(\mathbf{h}_i^H \mathbf{h}_i)^T], \text{ for } i = 1, \dots, n_R \quad (3.2.3)$$

and $\mathbf{h}_i$ is the i -th row of $\mathbf{H}$.

$$\mathbf{R}_{Rx} = E[\mathbf{h}^j (\mathbf{h}^j)^H], \text{ for } j = 1, \dots, n_T \quad (3.2.4)$$

and $\mathbf{h}^j$ is the j -th column of $\mathbf{H}$.

$$\mathbf{R}_H = E[\text{vec}(\mathbf{H})(\text{vec}(\mathbf{H}))^H] \quad (3.2.5)$$

where

$$\text{vec}(\mathbf{H}) = [(\mathbf{h}^1)^T, (\mathbf{h}^2)^T, \dots, (\mathbf{h}^{n_T})^T]^T. \quad (3.2.6)$$

For simplification, the tap index l is omitted in above formulas.

3.2.2 UWB MIMO Channel Modelling

Based on IEEE 802.15.3a UWB channel model recommendation [25] which is a modified Saleh-Valenzuela model as described in section 2.1.2 and a wideband MIMO channel model as given in (3.2.1), a simplified UWB MIMO channel model is proposed as follows.

Consider the Kronecker product approximation for the channel covariance matrix as given in (3.2.2). For UWB MIMO case, assume that $\mathbf{G}_{k,l} = \{\alpha_{j,i}^{k,l}\}$ denotes a $n_R \times n_T$ UWB MIMO channel coefficient matrix for the (k,l) -th tap where its real value elements are i.i.d. zero mean random variables and their magnitudes are with lognormal distributions as described in section 2.1.2. Following the procedure given in [73], approximately we may assume that

$$\mathbf{H}_{k,l} \sim (\mathbf{R}_{Rx}^{k,l})^{1/2} \mathbf{G}_{k,l} ((\mathbf{R}_{Tx}^{k,l})^{1/2})^T \quad (3.2.7)$$

where ‘ $\sim$ ’ denotes its two sides have the same statistical distribution and we assume that a lognormal sum can be approximated by a lognormal random variable based on some widely used approximations [10]. Thus, the UWB MIMO channel can be approximately modelled as

$$\mathbf{H}(t) = \sum_{l=0}^L \sum_{k=0}^K (\mathbf{R}_{Rx}^{k,l})^{1/2} \mathbf{G}_{k,l} ((\mathbf{R}_{Tx}^{k,l})^{1/2})^T \delta(t - T_l - \tau_{k,l}) \quad (3.2.8)$$

Assuming equal correlation between any two adjacent antennas, covariance matrices $\mathbf{R}_{Rx}^{k,l}$ and $\mathbf{R}_{Tx}^{k,l}$ will be in Toeplitz form. For simplicity, assuming equally correlated antenna arrays at either transmitter or receiver side, the covariance matrices for transmitter and receiver can be represented by two single parameters r_{Rx} and r_{Tx} respectively as

$$\mathbf{R}_{Rx}^{k,l} = \begin{bmatrix} 1 & \cdots & r_{Rx} \\ \vdots & \ddots & \vdots \\ r_{Rx} & \cdots & 1 \end{bmatrix} \quad (3.2.9)$$

$$\mathbf{R}_{Tx}^{k,l} = \begin{bmatrix} 1 & \cdots & r_{Tx} \\ \vdots & \ddots & \vdots \\ r_{Tx} & \cdots & 1 \end{bmatrix} \quad (3.2.10)$$

For extreme cases, the correlation coefficient $r_{Rx} = r_{Tx} = 0$ represents uncorrelated MIMO channel and $r_{Rx} = r_{Tx} = 1$ represents completely correlated MIMO channel. According to the studies for the correlation properties of MIMO channels for indoor pico cell scenarios (2.05GHz UMTS band) discussed in [49], the chance of achieving a low correlation coefficient $r < 0.5$ is very high (about 90%) as long as the antenna array elements are properly separated by distances on the order of the wavelength. For UWB case, the wavelength is extremely short, so the proper antenna separation for low correlation should be viable.

3.2.3 UWB MIMO Channel Capacity

As described in [91, 59, 60], the wideband MIMO channel capacity can be estimated by (3.2.11) when the transmission power is equally allocated to each transmitter antenna element and for each frequency sub-band.

$$C(\text{bits/sec}) = \int_W \log_2 \det(\mathbf{I}_{n_R} + \frac{\rho}{n_T} \mathbf{H}(f) \mathbf{H}^H(f)) df \quad (3.2.11)$$

Figure 3.1: UWB MIMO channel capacity (CM2, $SNR = 20dB$)

where MIMO frequency response $\mathbf{H}(f)$ is obtained from (3.2.8) by FFT and then normalized as given in (3.2.12) to insure that ρ represents the average SNR seen at each receiver antenna over the entire bandwidth.

$$\sum_{i=1}^{n_T} \sum_{j=1}^{n_R} \int_W E(|\mathbf{H}_{ji}(f)|^2) df = n_T n_R W. \quad (3.2.12)$$

Figure 3.1 illustrates the Cumulative Density Function (CDF) curves for capacity based on 100 UWB MIMO channel realizations obtained by using (3.2.8) under various correlation assumptions for CM2. Numerical integration is carried out across each small frequency span to obtain the channel capacity for a given bandwidth (where we set $BW = 3.1 \sim 4.9GHz$ referring to the first band group given in IEEE proposal [9]). It is noticed that the range of variations in capacity curves for UWB channels is much narrower than that for narrowband or conventional wideband radio channels as shown in [91]. This is due to the peculiar property of its ultra-wide bandwidth for UWB that insures high reliability for data transmission.

3.3 Diversity for UWB Multipath Channels

In this section, theoretical analysis for the diversity of UWB multipath channels is discussed. In order to obtain a closed form representation for BER performance evaluation, we assume i.i.d. Rayleigh fading model instead of S-V model based UWB channel model. But this assumption is only restricted in this section. More accurate semi-analytic performance evaluation using S-V model based UWB channel model will be given in section 3.4.

As shown in [64, Sec 14.4], assuming BPSK signaling and a multipath channel with L components which are i.i.d. with Rayleigh fading distributions, the BER performance can be written as

$$P_b = \left[\frac{1}{2}(1 - \mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1 + \mu)\right]^k \quad (3.3.1)$$

$$\mu = \sqrt{\frac{\bar{\gamma}_c}{1 + \bar{\gamma}_c}} \quad (3.3.2)$$

$$\bar{\gamma}_c = \bar{\gamma}_b / L$$

where $\bar{\gamma}_c$ is the average SNR per path and $\bar{\gamma}_b$ is the average bit SNR.

In the case of impulse radio based UWB systems, assuming baseband transmission without carrier, its channel is modelled by real-valued impulse response components with a tapped-delay-line model in contrast to complex-valued channel gain for Rayleigh fading channel. In order to obtain a similar closed form representation as given in (3.3.1), we assume that all L multipath components for UWB channel are i.i.d. Gaussian random variables. Then, BER performance for UWB channel with L multipath diversity can be derived as follows.

Assume BPSK signal constellation and in the absence of interference, the received signal can be represented as

$$r(t) = \sum_{k=1}^L \alpha_k s(t - t_k) + n(t) \quad (3.3.3)$$

The output signal by RAKE MRC can be expressed as

$$T = E \sum_{k=1}^L \alpha_k^2 + \sum_{k=1}^L \alpha_k \int_0^T n_k(t) s_k(t) dt \quad (3.3.4)$$

For a fixed set of multipath components $\{\alpha_k\}$, the output signal T is a Gaussian random variable with mean and variance given by

$$E(T) = \mathcal{E} \sum_{k=1}^L \alpha_k^2 \quad (3.3.5)$$

$$\sigma_T^2 = \frac{N_0}{2} \mathcal{E} \sum_{k=1}^L \alpha_k^2, \quad (3.3.6)$$

We then have the BER performance given as

$$P_b(\gamma_b) = Q(\sqrt{2\gamma_b}) \quad (3.3.7)$$

$$\gamma_b = \frac{(E(T))^2}{2\sigma_T^2} = \frac{\mathcal{E}}{N_0} \sum_{k=1}^L \alpha_k^2 \quad (3.3.8)$$

For $L = 1$, $\gamma_b = \gamma_1 = \frac{\mathcal{E}}{N_0} \alpha_1^2$ is with a chi-square distribution of one degree of freedom (in contrast to a chi-square distribution of two degree of freedom for Rayleigh fading distribution of $\{\alpha_k\}$). The characteristic function of γ_1 is given by

$$\psi_{\gamma_1}(jv) = \frac{1}{(1 - j2v\bar{\gamma}_c)^{1/2}} \quad (3.3.9)$$

where $\bar{\gamma}_c = \frac{\mathcal{E}}{N_0} E(\alpha_1^2)$.

For $L > 1$, assume that $\bar{\gamma}_c = \frac{\mathcal{E}}{N_0} E(\alpha_k^2)$, $k = 1, \dots, L$ to be identical for all L multipath components. The characteristic function for $\gamma_b = \sum_{k=1}^L \gamma_k$ is given by

$$\psi_{\gamma_b}(jv) = \frac{1}{(1 - j2v\bar{\gamma}_c)^{L/2}} \quad (3.3.10)$$

This is the characteristic function of a chi-square distribution with L degrees of freedom. The BER can be obtained by averaging the conditional probability over the channel fading statistic, that is

$$P_b = \int_0^\infty P_b(\gamma_b) p(\gamma_b) d\gamma_b \quad (3.3.11)$$

For even number of L , we have a closed form BER representation similar to that given in [64, Sec 14.4]. That is,

$$P_b = \left[\frac{1}{2}(1 - \mu)\right]^{L/2} \sum_{k=0}^{L/2-1} \binom{L/2-1+k}{k} \left[\frac{1}{2}(1 + \mu)\right]^k \quad (3.3.12)$$

$$\mu = \sqrt{\frac{2\bar{\gamma}_c}{1 + 2\bar{\gamma}_c}} \quad (3.3.13)$$

$$\bar{\gamma}_c = \bar{\gamma}_b/L$$

This is exactly half of the diversity order in comparison with that of a multipath channel with L Rayleigh fading components. For UWB systems with n_R receive antennas, total number of diversity paths can be expressed as $N_R = n_R \times L$. Thus, the BER performance becomes,

$$P_b = \left[\frac{1}{2}(1 - \mu)\right]^{N_R/2} \sum_{k=0}^{N_R/2-1} \binom{N_R/2-1+k}{k} \left[\frac{1}{2}(1 + \mu)\right]^k \quad (3.3.14)$$

$$\mu = \sqrt{\frac{2\bar{\gamma}_c}{1 + 2\bar{\gamma}_c}} \quad (3.3.15)$$

$$\bar{\gamma}_c = \bar{\gamma}_b/L$$

where $\bar{\gamma}_b$ represents the bit SNR per receiver antenna.

Figure 3.2 illustrates the theoretic BER performance obtained by using (3.3.14) for UWB systems with different number of diversity paths under the assumption of

Figure 3.2: Analytic BER performance evaluation for UWB multipath channel with diversity combining (BER vs. $\bar{\gamma}_b$)

Gaussian i.i.d. distribution for its multipath components. The performance curve in AWGN channel provides a lower bound. As discussed in [87], UWB channel is usually characterized with tens to hundreds of multipath components. These dense resolvable multipath components contribute to the system performance significantly and it may approach the performance under ideal AWGN channel as long as the multipath energy is captured appropriately.

In general, it is known that for a MIMO communication system with N_R receive antennas and N_T transmit antennas under i.i.d. Rayleigh fading, a Maximum Likelihood Detector (MLD) will achieve a diversity order of N_R and a linear MMSE detector will achieve a diversity order of $N_R - N_T + 1$ [64, Sec 14.7].

From the above discussion for UWB multipath diversity, it is noticed that for UWB multipath channel under i.i.d. Gaussian distribution assumption, a linear MMSE detector will approximately achieve a diversity order of $(N_R - N_T + 1)/2$. Considering the large number of paths, namely, $N_R = n_R \times n_L$ for UWB MIMO channel due to its

dense resolvable multipath components (n_L), the order of diversity can be achieved with linear MMSE receivers is very high.

3.4 Semi-analytic Performance Evaluation for UWB Systems with Linear MMSE Receivers

The BER performance of UWB systems can be obtained based on MMSE detection criterion as follows.

$$\mathbf{V} = \arg \min_{\mathbf{V} \in R^{N_T \times N_R}} \{E \|\mathbf{V}\mathbf{y} - \mathbf{x}\|^2\} \quad (3.4.1)$$

As given in (2.4.8), the solution for $\mathbf{V}$ is given by a Wiener filter, that is,

$$\mathbf{V}^T = \{E(\mathbf{y}\mathbf{y}^T)\}^{-1} E(\mathbf{y}\mathbf{x}^T) \quad (3.4.2)$$

From system model given in (3.1.20), i.e., $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{m}$, we obtained,

$$\begin{aligned} \mathbf{V} &= \mathbf{H}^T (\mathbf{H}\mathbf{H}^T + \mathbf{R}_m \mathbf{I}_{N_R})^{-1} \\ &= (\mathbf{H}^T \mathbf{H} + \mathbf{R}_m \mathbf{I}_{N_T})^{-1} \mathbf{H}^T \end{aligned} \quad (3.4.3)$$

where $\mathbf{R}_m = \sigma_m^2 \mathbf{I}$ denotes the covariance matrix for AWGN vector $\mathbf{m}$, $\mathbf{I}$ denotes the identity matrix and superscript T denotes transpose operation. Let $\mathbf{x} = [x_1, \dots, x_{N_T}]^T$. The detector output for the i -th element x_i can be written as

$$\hat{x}_i = (\mathbf{V}\mathbf{y})_i = (\mathbf{V}\mathbf{H}\mathbf{x} + \mathbf{V}\mathbf{m})_i = B_i x_i + I_{ISI} + \tilde{m} \quad (3.4.4)$$

where I_{ISI} denotes the residual ISI term after interference suppression by MMSE detection and

$$B_k = (\mathbf{V}\mathbf{H})_{ik} \quad (3.4.5)$$

$$I_{ISI} = \sum_{k=1, k \neq i}^{N_T} B_k x_k \quad (3.4.6)$$

$$\tilde{m} = \mathcal{N}(0, \sigma_m^2) \quad (3.4.7)$$

$$\sigma_{\tilde{m}}^2 = (\mathbf{V}\mathbf{V}^T)_{ii} \sigma_m^2 \quad (3.4.8)$$

Gaussian approximation can be applied to the residual ISI term I_{ISI} . This approximation has been proven to be accurate since MMSE detector can suppress ISI and

MAI effectively [81]. Thus, we obtain the Signal-to-Interference-Plus-Noise Ratio (SINR) at the decision point given as

$$SINR^{(i)} = \frac{B_i^2}{\sum_{k=1, k \neq i}^{N_T} B_k^2 + \sigma_m^2} \quad (3.4.9)$$

$$P_b^{(i)} = Q(\sqrt{SINR^{(i)}}) \quad (3.4.10)$$

In practice, the detection performance can be further improved by applying decision feedback cancellation for previously transmitted data symbols that have already been detected, that is, symbol $k = n - n_{ISI} + 1, \dots, n - 1$.

3.5 Performance Evaluation: Numerical Results

In this section, the UWB system performance is discussed by numerical analysis using (3.4.9) and (3.4.10). The results obtained by the semi-analytic method given in section 3.4 have shown to be consistent with the corresponding simulation results shown in [81]. Based on the IEEE 802.15.3a UWB channel model recommendation introduced in section 2.1.2, totally 100 realizations of UWB channel impulse response are generated for each channel model (CM1-CM4). These channel profiles are combined with the simplified MIMO channel correlation matrices as given in (3.2.9) and (3.2.10) to generate the UWB MIMO channel profiles as given by (3.2.8) in section 3.2.2.

3.5.1 MMSE Receiver vs. RAKE Receiver

In this section, the BER performance for impulse based UWB systems with linear MMSE receivers is evaluated and compared with RAKE receivers with MRC based on following experimental settings. The comparison results show that the MMSE receiver significantly outperforms the RAKE receiver with MRC in the presence of severe ISI and MAI.

3.5.1.1 Experimental Settings

The numerical results shown in this section are based on UWB channel realizations for CM3, which is based on Non-Line-Of-Sight (NLOS) (4-10m) channel setting and with a RMS delay spread of $\tau_{rms} = 15 \text{ nsec}$. The UWB THSS systems with BPSK modulation is assumed, which is based on the system model given in (2.2.3).

The experiments are carried out to evaluate the BER performance for UWB systems under different settings given as follows

1. Performance evaluation in the presence of MAI
 - lower symbol transmission rate ($R_s = 5.85 \text{ Mbps}$), without ISI;
 - heavy MAI load with perfect power control for MAI, that is, equal transmission power is assumed for all multiple access transmitters ($SIR = 0 \text{ dB}$).
2. Performance evaluation for near-far resistance
 - heavy MAI load with higher transmission power for MAI transmitters ($SIR = -10 \text{ dB}$).
3. Performance evaluation in the presence of MAI under various symbol transmission rates
 - various symbol transmission rates are set as ($R_s = 5.85/46.8/93.6 \text{ Mbps}$), higher R_s indicates the transmission is in the presence of ISI;
 - heavy MAI load with perfect power control ($SIR = 0 \text{ dB}$).
4. Performance evaluation in the presence of MAI with multiple receive antennas for spatial diversity gain
 - higher symbol transmission rate is set at ($R_s = 93.6 \text{ Mbps}$);
 - heavy MAI load with perfect power control ($SIR = 0 \text{ dB}$);
 - UWB Single Input Multiple Output (SIMO) setting: $n_T = 1$, $n_R = 2$.

The parameter settings for UWB channel realizations are: sampling duration $\Delta\tau = 0.167 \text{ nsec}$; maximum multipath length in samples $n_L = 1024$.

The observation window length for both MMSE and RAKE receiver is set as 256 *samples*, which is shown to effectively capture about 90% of transmitted signal energy from UWB multipath channel realizations for CM3. Pseudo random codes are applied for time-hopping and code length is set as $n_h = 64$.

In addition, Gaussian pulse waveform is assumed as given in (2.1.1), i.e.,

$$w(t) = (1 - 4\pi(t/\tau_m)^2) \exp(-2\pi(t/\tau_m)^2), \text{ where } \tau_m = 0.167 \text{ nsec}.$$

We have assumed THSS UWB with BPSK modulation. Settings for matched filter based RAKE receiver as given in (2.3.23) are obtained as $v(t) = w(t)$, $R_{vw}^2(0) = E_w^2$, $\int_{-\infty}^{\infty} v^2(t)dt = E_w$, and $\int_{-\infty}^{\infty} R_{vw}^2(\tau)d\tau = 0.086E_w^2$.

3.5.1.2 Numerical Results

Performance evaluation in the presence of MAI

In this case, lower symbol transmission rate ($R_s = 5.85 \text{ Mbps}$) is assumed so that the multiple access performance for UWB systems is evaluated in the absence of ISI.

Figure 3.3 illustrates the BER performance comparison for the MMSE receiver and the RAKE receiver with MRC in the presence of MAI, where all MAI transmitters are assumed to have the same transmission power as the desired transmitter. It is observed that the BER performance for MMSE receiver under heavily loaded MAI ($n_{MAI} = 95$) exhibits only about 1dB degradation in terms of SNR at BER=1.0e-5 when compared to that by optimal RAKE receiver under single transmitter case ($n_{MAI} = 0$). On the other hand, the BER performance obtained by RAKE receiver under heavily loaded MAI ($n_{MAI} = 95$) suffers from much worse degradation.

Performance evaluation for near-far resistance

Figure 3.4 illustrates the BER performance for MMSE receiver under the near-far setting. The Signal-to-Interference Ratio (SIR) is set as -10dB for all the MAI transmitters, that is, all the MAI transmitters are assumed to have 10 times more power as that of desired transmitter. It is observed that the RAKE receiver suffers

Figure 3.3: BER performance comparison for MMSE receiver and RAKE receiver with MRC ($R_s = 5.85Mbps$, $n_{MAI} = 95$, $SIR = 0dB$)

Figure 3.4: BER performance comparison for MMSE receiver and RAKE receiver with MRC ($R_s = 5.85Mbps$, $n_{MAI} = 31/95$, $SIR = -10dB$)

from severe performance degradation due to the near-far problem and exhibits performance floor under moderate MAI load. On the other hand, the performance of the MMSE receiver is almost unaffected when compared with the case under ideal power control as given in Figure 3.3.

Performance evaluation in the presence of MAI under various symbol transmission rates

In this case, symbol transmission rate is increased from $R_s = 5.8\text{Mbps}$ to $R_s = 46.8\text{Mbps}$ and $R_s = 93.6\text{Mbps}$, to evaluate the multiple access performance for UWB systems in presence of severe ISI.

Figure 3.5: BER performance comparison for MMSE receiver and RAKE receiver with MRC ($n_{MAI} = 31$, $SIR = 0\text{dB}$, $R_s = 5.85/46.8/93.6\text{Mbps}$)

Figure 3.5 illustrates the performance comparison for MMSE receiver and RAKE receiver. The performance degradation due to ISI becomes more and more severe with the increased symbol rate. All the MAI transmitters are assumed to have the same transmission power as the desired transmitter. It is observed that the BER performance for the MMSE receiver degrades much more gracefully than that for the

RAKE receiver with increased symbol transmission rate. When R_s is increased from $5.8Mbps$ (w/o ISI) to $46.8Mbps$, the performance degradation for the MMSE receiver is about $2.5dB$ in terms of SNR at $BER=1.0e-4$. However, for the RAKE receiver, the performance degradation is severe and it leads to a performance floor quickly.

Performance evaluation in the presence of MAI with multiple receive antennas for spatial diversity gain

In order to improve the multiple access performance for UWB systems at high symbol transmission rate, i.e., in the presence of severe ISI, multiple receiver antennas ($n_T = 1, n_R = 2$) are utilized to benefit from the spatial diversity gain.

Figure 3.6: BER performance comparison for MMSE receiver and RAKE receiver with MRC for SIMO UWB systems ($n_T = 1, n_R = 2, n_{MAI} = 31, SIR = 0dB, R_s = 46.8/93.6Mbps$)

Figure 3.6 illustrates the performance comparison for MMSE receiver and RAKE receiver in the presence of severe ISI and MAI. It is observed that significant performance improvement is achieved for UWB systems from the diversity gain by adding one more receiving antenna when compared with the performance obtained for SISO

UWB systems given in Figure 3.5. The diversity gained from multiple receiver antennas is able to support higher symbol transmission rate or more multiple access transmitters. On the other hand, MMSE receiver consistently outperforms RAKE receiver for UWB SISO systems as well as for UWB SIMO systems.

3.5.2 MMSE Receivers: THSS vs. DSSS

In this section, the BER performance for UWB MIMO spatial multiplexing systems with linear MMSE receiver is analyzed based on the simplified MIMO channel modelling proposed in section 3.2.2. The BER performance comparison between THSS based UWB systems and DSSS based UWB systems is discussed. The comparison shows that a DSSS based UWB system is more vulnerable to severe ISI than a THSS based UWB system under very high symbol transmission rate. It is also observed that a UWB MIMO system ($n_R = 2$; $n_T = 2$) with THSS is capable of delivering promising performance for very high data rate transmission ($2 \times 748.50 Mbps$). A UWB MIMO system ($n_R = 2$; $n_T = 8$) with either THSS or DSSS is also capable of very high data rate transmission ($8 \times 187.13 Mbps$) by dividing the high rate data stream into multiple data streams corresponding to multiple transmit antennas in parallel under moderate symbol transmission rate.

3.5.2.1 Experimental Settings

A total of 100 independent UWB MIMO channel realizations are generated from CM1 to CM4 respectively. The sampling duration is set as 0.167 nsec . The size of observation window is chosen from 64 to 256 samples which effectively captures the signal energy from UWB multipath channels. Pseudo random codes are applied for both THSS and DSSS. The THSS/DSSS based UWB systems with BPSK modulation is assumed, which is based on the system models given in (2.2.3) and (2.2.4).

The following experiments are carried out to evaluate the BER performance for THSS/DSSS based UWB systems with linear MMSE receivers under different settings given as

1. Performance evaluation for THSS/DSSS based UWB systems at various symbol transmission rates
 - $R_s = 93.56/374.25/748.50 Mbps$;
 - for CM2 ($\tau_{rms} = 8 \text{ nsec}$);
2. Performance evaluation for THSS/DSSS based UWB MIMO systems under different channel correlation assumptions
 - at very high transmission rate ($R_s = 748.50 Mbps$), that is, in the presence of severe ISI;
 - under different channel correlation assumptions ($r_{R_x} = r_{T_x} = 0.0/0.5/0.7/0.9/1.0$);
 - for CM1 ($\tau_{rms} = 5 \text{ nsec}$);
3. Performance evaluation for THSS/DSSS based UWB MIMO systems with various number of transmit antennas
 - at moderately high symbol transmission rate ($R_s = 187.13 Mbps$), or
 - at very high symbol transmission rate ($R_s = 748.5 Mbps$);
 - under moderate channel correlation assumption ($r_{R_x} = r_{T_x} = 0.5$);
 - for CM1/CM2/CM3/CM4 ($\tau_{rms} = 5/8/15/25 \text{ nsec}$).
4. Performance evaluation for THSS/DSSS based UWB MIMO systems with equal number of transmit and receive antennas
 - for CM1 ($\tau_{rms} = 5 \text{ nsec}$);
 - increasing n_T and n_R at the same time.
5. Performance evaluation for THSS based UWB systems with or without Successive Interference Cancellation (SIC)
 - at very high symbol transmission rate ($R_s = 748.5 Mbps$);

- for CM2 ($\tau_{rms} = 8 \text{ nsec}$);
- with SIC to cancel the ISI effect using previously detected symbols.

3.5.2.2 Numerical Results

Performance evaluation for THSS/DSSS based UWB systems at various symbol transmission rates

The BER performance for THSS and DSSS based UWB systems (SISO, $n_T = 1$, $n_R = 1$) at various symbol rates $R_s = 93.56/374.25/748.50 \text{ Mbps}$ are evaluated for CM2 (0-4m, NLOS) [25]. From Figure 3.7 it is observed that a DSSS based UWB system suffers from more performance degradation than a THSS based UWB system when evaluated at very high symbol transmission rate. This is due to the existence of Inter Chip Interference (ICI) in addition to ISI for DSSS based systems. However, the performance difference between THSS and DSSS based UWB systems is not significant under moderate symbol transmission rate.

Figure 3.7: BER performance comparison for THSS and DSSS based UWB systems (SISO, $n_T = 1$, $n_R = 1$)

Performance evaluation for THSS/DSSS based UWB systems under different channel correlation assumptions

Figure 3.8: BER performance evaluation for THSS based UWB MIMO systems ($n_T = 2$, $n_R = 2$, $R_s = 748.50 \text{ Mbps}$)

The BER performance for THSS and DSSS based UWB MIMO systems ($n_T = 2$, $n_R = 2$) is evaluated under different MIMO channel correlation assumptions ($r = 0.0/0.5/0.7/0.9/1.0$). Symbol transmission rate is set as $R_s = 748.50 \text{ Mbps}$ and systems are evaluated based on CM1 (0-4m, LOS) [25]. The BER curves for THSS and DSSS based systems are illustrated in Figure 3.8 and Figure 3.9 respectively. It is observed that under moderate channel correlation assumption ($0 \leq r \leq 0.7$), the performance degradation is minimal for both THSS and DSSS based systems. In addition, DSSS based systems suffer from more performance degradation due to ICI in addition to ISI. However, in the case when MIMO channels are highly or fully correlated ($0.9 \leq r \leq 1.0$), DSSS based systems seem to have a better chance than THSS based systems. As we apply different pseudorandom spreading sequences to different transmitting data streams, the multiple antenna transmission can be considered as

Figure 3.9: BER performance evaluation for DSSS based UWB MIMO systems ($n_T = 2$, $n_R = 2$, $R_s = 748.50 \text{ Mbps}$)

an uplink multiple access transmission with different channels that may be correlated or uncorrelated. On the other hand, the multiple antenna transmission can be considered as a downlink multiple access transmission with the same channel (or highly similar channels) when these channels are highly or fully correlated. This indicates that DSSS based UWB may provide better multiple access capability for downlink case than THSS based UWB under very high data rate transmission (i.e., in the presence of severe ISI). It may be explained as that the DS spreading may provide better multiple access capability than the TH spreading when systems are heavily loaded. Since the performance for DS spreading relies on the correlation properties of the sequences but the performance for TH spreading relies on the collision avoidance. However, when the systems are heavily loaded (under severe ISI and MAI), the collision is hard to avoid. More study is required to justify this observation. For uplink multiple access transmission, different multipath channels will act like spreading with different sequences for THSS based systems. In this case, ICI will dominate and affect

the performance of DSSS based systems. Nevertheless, both methods will perform similarly under moderate data rate transmission as noticed from Figure 3.7.

Performance evaluation for THSS/DSSS based UWB MIMO systems with various number of transmit antennas

Figure 3.10: BER performance for UWB MIMO Systems for CM1-4 ($R_s = 748.50\text{Mbps}$, $n_T = 2$, $n_R = 2$, $\text{corr} = 0.5$, $n_{\text{tap}} = 128$)

The BER performance for THSS and DSSS based UWB MIMO systems is evaluated for different channel model settings (CM1: LOS 0-4m; CM2: NLOS 0-4m; CM3: NLOS 4-10m; CM4: NLOS beyond 10m) [25]. Firstly, symbol rate is set at $R_s = 748.50\text{Mbps}$ and MIMO configuration as $n_T = 2$, $n_R = 2$. From Figure 3.10, it is observed that such a high transmission rate works better for THSS based UWB systems with short range transmission (CM1, CM2). On the other hand, for THSS based systems with longer range transmission (CM3, CM4) or DSSS based UWB systems, the performance degradation due to ISI will be significant and higher diversity order needs to be exploited. Secondly, setting symbol transmission rate as $R_s = 187.1\text{Mbps}$ and MIMO configuration as $n_T = 8$, $n_R = 2$, from Figure 3.11 it is

Figure 3.11: BER performance for UWB MIMO Systems for CM1-4 ($R_s = 187.10\text{Mbps}$, $n_T = 8$, $n_R = 2$, $\text{corr} = 0.5$, $n_{\text{tap}} = 128$)

observed that under moderate symbol rate, both THSS and DSSS based systems are capable of delivering very high data rate by employing multiple transmit antennas.

Performance evaluation for THSS/DSSS based UWB MIMO systems with equal number of transmit and receive antennas

From Figure 3.12, it is observed that increasing the number of transmit and receive antennas at the same time improves the BER performance for UWB MIMO systems. Assume that the diversity order is given as $(N_R - N_T + 1)/2 = [n_R n_\tau n_{\text{sym}} - n_T(n_{\text{ISI}} + n_{\text{sym}} - 1) + 1]/2$. The performance improvement can be achieved as N_R grows faster than N_T if we choose $n'_L = n_\tau n_{\text{sym}} > (n_{\text{ISI}} + n_{\text{sym}} - 1)$. In contrast, for narrowband MIMO systems, the diversity order with linear MMSE receiver will always equal to one assuming the same number for transmit and receive antennas.

Performance evaluation for THSS based UWB systems with/without Successive Interference Cancellation (SIC)

Figure 3.13 illustrates the performance comparison for THSS based UWB systems

Figure 3.12: BER performance for THSS based UWB MIMO systems (CM1, $n_T = n_R = 1/2/3/4$, $R_s = 748.50Mbps$, $n_{tap} = 64$)

Figure 3.13: BER performance evaluation by semi-analytic and simulation for THSS based UWB systems with/without SIC (CM2, $R_s = 748.50Mbps$, $n_{tap} = 80$)

with and without SIC where SIC is applied to cancel the ISI effect using previously detected symbols. It is observed that SIC helps to improve the system performance for linear MMSE receiver. On the other hand, the results obtained by semi-analytic method given in (3.4.9) and (3.4.10) is shown to be consistent with the results obtained by simulation. The effect of error propagation is also noticed on the simulation curve with SIC.

3.6 Summary

In this chapter, a matrix representation for impulse radio based UWB MIMO systems with THSS and DSSS methods is formulated. This formulation takes ISI and MAI into account for high data rate multiple access UWB transmission in an indoor environment. A simplified UWB MIMO channel model is proposed based on the IEEE 802.15.3a channel model recommendation [25] and the covariance matrix representation for MIMO spatial correlation [73][91]. The BER performance for UWB systems with linear MMSE receivers is evaluated and compared to RAKE receivers with MRC. The comparison results show that the MMSE receiver significantly outperforms the RAKE receiver in the presence of severe ISI and MAI. Also, performance comparison is carried out for THSS and DSSS based UWB MIMO systems based on MMSE detection. Results show that THSS based systems are capable of delivering promising performance up to very high symbol transmission rate. On the other hand, DSSS based systems are more vulnerable to high symbol rate transmission due to the existence of severe ICI in addition to ISI.

Chapter 4

Tap Selection Based UWB Multipath Channel Equalization

In chapter 3, Minimum Mean Squared Error (MMSE) receiver has been shown to outperform conventional RAKE receiver significantly for high data rate Ultra-Wideband (UWB) systems in the presence of severe Inter Symbol Interference (ISI) and Multiple Access Interference (MAI). In this chapter, a tap selection based non-uniformly spaced MMSE equalization technique is proposed for UWB systems with reduced complexity. This technique is shown to outperform the conventional uniformly spaced equalizer significantly with the same number of taps. Also, the impact of channel estimation error on the performance of tap selection based Decision Feedback Equalization (DFE) is analyzed. This channel estimation based DFE is shown to perform well with small amount of training symbols. It can then be adapted effectively using Least Mean Square (LMS) algorithm for improved performance.

4.1 Introduction

For high data rate impulse radio based UWB systems, channel response may span over multiple symbol durations. Conventional RAKE receiver suffers from performance degradation due to severe ISI. An MMSE equalization based receiver is superior for ISI mitigation. But this requires large number of training symbols and involves large computational complexity due to the long delay spread channel in UWB. A key challenge is to develop a high performance equalizer given limited training sample

support with manageable complexity.

The MMSE receiver has been considered for a Direct Sequence Spread Spectrum (DSSS) based UWB system and has been shown to have superior interference mitigation performance over RAKE receiver in [52]. The analysis and evaluation in chapter 3 have shown that either Time Hopping Spread Spectrum (THSS) or DSSS based UWB systems with linear MMSE receivers is capable of delivering promising performance for very high data rate transmission. In [52], a large number of training symbols is assumed for an equalizer with large number of taps using adaptive algorithms such as Least Mean Square (LMS) or Recursive Least Squares (RLS). To reduce the complexity, suboptimal receivers with various hybrid RAKE and MMSE combining schemes have been proposed for UWB [53] [31] that inevitably result in performance degradation in the presence of severe interference. A time-reversal based MMSE equalizer is also proposed for UWB [76] which is similar to pre-RAKE structure. It assumes channel knowledge and pre-processing is done at the transmitter to reduce the complexity at the receiver. However, obtaining multipath channel estimation at transmitter is not an easy task and it will pose a problem in this type of pre-equalization or pre-RAKE based technique for UWB channels.

Non-uniformly spaced equalizer has been discussed for sparse multipath channels such as high-definition television test channel [51], “typical urban” and “hilly terrain” profiles [8]. In [51], simple design rules for allocating non-uniformly spaced taps are developed based on the tapped-delay-line channel assumption. In [8], tap selection is based on the assumption of uncorrelated ISI. In these works, various intuitive tap selection techniques are discussed based on sparse channel delay profiles and only very limited number of taps are selected for reduced complexity. These prior works motivate us to consider tap selection based equalization for UWB channels. However, the UWB indoor channel is not as sparse as those channels discussed in the prior works. Therefore, it is important to have instructive performance analysis in order to evaluate the effectiveness for using tap selection based equalization for UWB systems.

4.2 Performance Evaluation for Tap Selection based UWB Multipath Channel Equalization

4.2.1 UWB System Model and MMSE Detection

As discussed in section 3.1, a discrete time UWB system model can be formulated using matrix representation as given in (3.1.20). For simplicity, we assume single transmitter case without adding spreading sequence in following discussions. This simplified model is also able to accommodate co-channel multiple access interference transmitters due to the self spreading multipath channel characteristics in UWB. A similar mathematical analysis can be carried out for systems modelled with spreading sequence by either THSS or DSSS as given in section 3.1.

Consider an impulse based UWB system with BPSK modulation. Let $d(n) \in \{\pm 1\}$ be the data bit stream and $w(t)$ be the pulse waveform. The transmitted signal is represented as follows.

$$s(t) = \sum_{n=-\infty}^{\infty} d(n)w(t - nT_f) = x(t) * w(t) \quad (4.2.1)$$

where $x(t) = \sum_{n=-\infty}^{\infty} d(n)\delta(t - nT_f)$ and T_f is the symbol duration. Applying a tapped-delay-line based multipath channel model $c(t) = \sum_{p=1}^{n_L} \alpha_p \delta(t - (p-1)\Delta\tau - \tau)$, where n_L denotes the channel length in samples, $\Delta\tau$ is the sampling duration and τ is the channel delay, the received signal is given by

$$r(t) = c(t) * s(t) + n(t) = h(t) * x(t) + n(t) \quad (4.2.2)$$

where $h(t) = c(t) * w(t)$ denotes the generalized Channel Impulse Response (CIR) and $n(t)$ represents the AWGN signal.

For simplicity, assume that the sampling rate is an integer multiple of the symbol repetition rate, that is, let $n_\tau = T_f/\Delta\tau$ be an integer. The number of symbols affected by ISI is given as $n_{ISI} = n_L/n_\tau$, where it is assumed that n_L is an integer multiple of n_τ and this may be done by truncating the tail end of channel delay profiles. Then, we have

$$\mathbf{y}(n) = \sum_{k=n-n_{ISI}+1}^n \mathbf{h}_k^n x(k) + \mathbf{m}(n) \quad (4.2.3)$$

where $x(k) = d(k)$ is the transmitted symbol at the k -th symbol duration. The vector $\mathbf{y}(n) = [y^{(1)}(n), \dots, y^{(n_\tau)}(n)]^T$ denotes the over sampled received signal vector and $\mathbf{m}(n)$ is the corresponding AWGN signal vector. The vector $\mathbf{h}_k^n$ denotes the channel transmission matrix from the k -th transmitting symbol to the n -th receiving symbol duration and is expressed as

$$\mathbf{h}_k^n = \mathbf{h}(n - k) = \begin{bmatrix} h^{(1)}(n, k) & \dots & h^{(n_\tau)}(n, k) \end{bmatrix}_{1 \times n_\tau}^T \quad (4.2.4)$$

Let $h^{(p)}(n, k) = h_q$, where $q = p + (n - k)n_\tau$ and h_q is the q -th path of the sampled generalized CIR $h(t)$.

To acquire more multipath energy and diversity, $x(n)$ is detected after a delay of n_{sym} symbol duration. From (4.2.3), a UWB system model can then be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{m} \quad (4.2.5)$$

where $\mathbf{y} = [\mathbf{y}^T(n), \dots, \mathbf{y}^T(n + n_{sym} - 1)]^T$,

$\mathbf{x} = [x(n - n_{ISI} + 1), \dots, x(n), \dots, x(n + n_{sym} - 1)]^T$ and the channel transmission matrix in block Toeplitz form is represented as

$$\mathbf{H} = \begin{pmatrix} \mathbf{h}(n_{ISI} - 1) & \dots & \mathbf{h}(0) & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \mathbf{h}(n_{ISI} - 1) & \dots & \mathbf{h}(0) \end{pmatrix}_{N_R \times N_T} \quad (4.2.6)$$

where $N_R = n_\tau n_{sym}$, $N_T = n_{ISI} + n_{sym} - 1$.

Considering Successive Interference Cancellation (SIC) for the previously detected symbols ($\mathbf{x}'$), (4.2.5) can be re-written as

$$\mathbf{y}' = \begin{pmatrix} \mathbf{H}' & \mathbf{H} \end{pmatrix} \begin{pmatrix} \mathbf{x}' \\ \mathbf{x} \end{pmatrix} + \mathbf{m} \quad (4.2.7)$$

$$\mathbf{y} = \mathbf{y}' - \mathbf{H}'\mathbf{x}' = \mathbf{H}\mathbf{x} + \mathbf{m} \quad (4.2.8)$$

where $\mathbf{x} = [x(n), \dots, x(n + n_{sym} - 1)]^T$ and $\mathbf{x}' = [x(n - n_{ISI} + 1), \dots, x(n - 1)]^T$.

Given UWB system model in (4.2.8), assume that the desired symbol for detection is the *first* element in vector $\mathbf{x}$. From the MMSE detection given in (3.4.3), the Minimum Mean Squared Error (MMSE) can be derived as follows,

$$MMSE^{(1)} = \{E[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T]\}_{1,1} = \{(\mathbf{H}^T \mathbf{R}_m^{-1} \mathbf{H} + \mathbf{I}_{N_T})^{-1}\}_{1,1} \quad (4.2.9)$$

where the subscript (i, j) in $\{\mathbf{A}\}_{i,j}$ denotes the element from the i -th row and j -th column of the matrix $\mathbf{A}$.

In the presence of AWGN signal vector $\mathbf{m}$ with covariance of $\mathbf{R}_m = \sigma_m^2 \mathbf{I}$, the Signal-to-Interference-Plus-Noise Ratio (SINR) at the decision point for unbiased MMSE detection is given by [18]

$$SINR^{(1)} = \frac{1}{MMSE^{(1)}} - 1 = \frac{1}{\sigma_m^2 \{(\mathbf{H}^T \mathbf{H} + \sigma_m^2 \mathbf{I}_{N_T})^{-1}\}_{1,1}} - 1 \quad (4.2.10)$$

Thus, the Bit Error Rate (BER) performance can be evaluated by

$$P_b^{(1)} = Q(\sqrt{SINR^{(1)}}) \quad (4.2.11)$$

4.2.2 Greedy Algorithm Based Tap Selection Technique

Under MMSE detection criterion, tap selection can be considered as forming a transmission matrix $\mathbf{H}_S$ by choosing a subset of rows from $\mathbf{H}$ to maximize the $SINR_{\mathbf{H}_S}^{(1)}$ as given in (4.2.10) or equivalently to minimize the $MMSE_{\mathbf{H}_S}^{(1)}$ as given in (4.2.9).

Let n_L be the total number of taps within the observation window for over sampled observation signal and n_S be the number of selected taps for non-uniformly spaced equalization. For selecting n_S taps optimally out of total n_L taps, an exhaustive search should be made which requires the MMSE detection performance to be evaluated for $\binom{n_L}{n_S}$ possible combination of the tap subsets. This is not feasible for large n_L . So a Greedy Algorithm (GA) based method is proposed for the performance evaluation for tap selection based equalization.

From (4.2.10), maximizing the $SINR_{\mathbf{H}_S}^{(1)}$ is equivalent to minimizing an objective function defined as

$$\{\mathbf{A}_S\}_{1,1} = \{(\mathbf{H}_S^T \mathbf{H}_S + \sigma_m^2 \mathbf{I}_{N_T})^{-1}\}_{1,1} \quad (4.2.12)$$

Suppose that at the $(n-1)$ -th step, a selected transmission matrix with $(n-1)$ rows is denoted as $\mathbf{H}_{n-1}$. Define,

$$\mathbf{A}_{n-1} = (\mathbf{H}_{n-1}^T \mathbf{H}_{n-1} + \sigma_m^2 \mathbf{I}_{N_T})^{-1} \quad (4.2.13)$$

To obtain $\mathbf{H}_n$ at the n -th step, one more row denoted as $\mathbf{h}_s$ will be selected and appended to the matrix $\mathbf{H}_{n-1}$. Also notice that the rows in the transmission matrix

may be re-ordered. We then have,

$$\mathbf{H}_n^T \mathbf{H}_n = \begin{pmatrix} \mathbf{H}_{n-1}^T & \mathbf{h}_s^T \end{pmatrix} \begin{pmatrix} \mathbf{H}_{n-1} \\ \mathbf{h}_s \end{pmatrix} = \mathbf{H}_{n-1}^T \mathbf{H}_{n-1} + \mathbf{h}_s^T \mathbf{h}_s \quad (4.2.14)$$

$$\mathbf{A}_n = (\mathbf{H}_n^T \mathbf{H}_n + \sigma_m^2 \mathbf{I}_{N_T})^{-1} = (\mathbf{A}_{n-1}^{-1} + \mathbf{h}_s^T \mathbf{h}_s)^{-1} \quad (4.2.15)$$

Applying matrix inversion lemma [35], i.e., $(\mathbf{A} + \mathbf{U}\mathbf{V}^T)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{U}(\mathbf{I} + \mathbf{V}^T \mathbf{A}^{-1} \mathbf{U})^{-1} \mathbf{V}^T \mathbf{A}^{-1}$, an iterative formula is obtained with reduced computation cost for matrix inversion as follows

$$\mathbf{A}_n = \mathbf{A}_{n-1} - \mathbf{A}_{n-1} \mathbf{h}_s^T (1 + \mathbf{h}_s \mathbf{A}_{n-1} \mathbf{h}_s^T)^{-1} \mathbf{h}_s \mathbf{A}_{n-1} \quad (4.2.16)$$

where the computationally intensive part comes from the vector and matrix multiplication term of $\mathbf{A}_{n-1} \mathbf{h}_s^T$. This can be effectively reduced by defining $\mathbf{b}_{n-1} = \mathbf{A}_{n-1} \mathbf{h}_s^T$ and by obtaining an iterative formula for updating $\mathbf{b}_n$ as follows

$$\begin{aligned} \mathbf{b}_n &= \mathbf{A}_n \mathbf{h}_s^T \\ &= \mathbf{A}_{n-1} \mathbf{h}_s^T - \mathbf{A}_{n-1} \mathbf{h}_s^T (1 + \mathbf{h}_s \mathbf{A}_{n-1} \mathbf{h}_s^T)^{-1} \mathbf{h}_s \mathbf{A}_{n-1} \mathbf{h}_s^T \\ &= \mathbf{b}_{n-1} - \mathbf{b}_{n-1} (1 + \mathbf{h}_s \mathbf{b}_{n-1})^{-1} \mathbf{h}_s \mathbf{b}_{n-1} \end{aligned} \quad (4.2.17)$$

Thus, the computational cost ($\mathcal{O}(n^2)$) for computing $\mathbf{A}_{n-1} \mathbf{h}_s^T$ is reduced to ($\mathcal{O}(n)$) for computing $\mathbf{h}_s \mathbf{b}_{n-1}$.

Therefore, a GA based incremental iterative algorithm is developed for tap selection assuming CIR as follows

1. Start from $n = 0$, set $\mathbf{A}_0 = \frac{1}{\sigma_m^2} \mathbf{I}_{N_T}$
2. At step n , using (4.2.16), one more row $\mathbf{h}_s$ is selected by choosing $s = \arg \min_k \left\{ \{\mathbf{A}_n\}_{1,1} \right\} = \arg \min_k \left\{ \{(\mathbf{A}_{n-1}^{-1} + \mathbf{h}_k^T \mathbf{h}_k)^{-1}\}_{1,1} \right\}$, where k denotes the unselected row indices for $\mathbf{H}$
3. Set $n = n + 1$, continue looping to select the taps until $n = n_S$.

4.2.3 Performance Evaluation

Based on the IEEE 802.15.3a UWB channel model recommendation [25], the performance evaluation using (4.2.11) for tap selection based MMSE equalization is illustrated in Figure 4.1. The channel model CM2 is assumed with a sampling duration of $0.167nsec$ and total number of samples $n_L = 512$. It is shown that GA based incremental iterative tap selection method is able to achieve near optimal performance for tap selection. Since the BER performance curve by tap selection with merely $n_S = 64$ taps is already quite close to the optimal MMSE detection performance obtained with full complexity equalizer by applying all 512 taps. It is also noticed that tap selection based MMSE equalization system significantly outperforms the conventional uniformly spaced MMSE equalization system with either 64 equally spaced taps over the entire channel length or the 64 earliest taps, as shown in Figure 4.1.

Figure 4.1: Tap selection based MMSE equalization performance evaluation (CM2, $R_s = 375Mbps$)

An intuitive way to make the tap selection is to simply choose n strongest paths from CIR. It is observed in Figure 4.1 that the performance gap between greedy

Figure 4.2: Tap selection based MMSE equalization performance evaluation (CM4, $R_s = 375Mbps$)

algorithm based tap selection and the strongest path based tap selection is limited under a single transmitter scenario. However, the performance gap is expected to become larger with increased SNR . Similar performance curves for tap selection are observed for channel model CM4 as illustrated in Figure 4.2 where more number of taps (n_S) is required for effective interference mitigation due its longer multipath channel that will result in more severe ISI.

When MAI is present, the system model from (4.2.5) is then written as follows (this in fact has the same form as (4.2.5))

$$\mathbf{y} = \mathbf{H}_{(1)}\mathbf{x}_{(1)} + \mathbf{H}_{(MAI)}\mathbf{x}_{(MAI)} + \mathbf{m} = \mathbf{H}\mathbf{x} + \mathbf{m} \quad (4.2.18)$$

where $\mathbf{H} = [\mathbf{H}_{(1)} \mathbf{H}_{(MAI)}]$ and $\mathbf{x} = [\mathbf{x}_{(1)}^T \mathbf{x}_{(MAI)}^T]^T$, subscript (1) indicates the desired transmitter and (MAI) denotes the terms corresponding to the MAI transmitters.

In the presence of both ISI ($n_{ISI} = 8$) and MAI ($n_{MAI} = 0 - 3$), the BER performance for tap selection based MMSE equalization is illustrated in Figure 4.3, where the symbol rate for UWB transmission is set at $93.56Mbps$. It is observed that

the performance gap between *Strongest Paths* based simple tap selection method and GA based near optimal tap selection method becomes larger with increased number of MAI transmitters, as well as with increased SNR level. This can be interpreted as follows.

Figure 4.3: Tap selection based MMSE equalization performance evaluation in the presence of MAI (CM2, $R_s = 93.56 Mbps$, $n_S = 32$)

From system model given in (4.2.18), the $SINR$ for tap selection based MMSE detection can be written in another form as described in [81],

$$SINR_{\mathbf{H}_S}^{(1)} = \frac{B_1^2}{\sigma_{I_{residual}^{(ISI)}}^2 + \sigma_{I_{residual}^{(MAI)}}^2 + \|\mathbf{f}_{mmse}\|^2 \sigma_m^2} \quad (4.2.19)$$

where $\mathbf{f}_{mmse}$ is the equalization filter, $B_1 = (\mathbf{f}_{mmse}^T \mathbf{H}_S)_1$ and B_1^2 represents the multipath energy capture for the desired symbol. Gaussian approximation is applied to the terms $I_{residual}^{(ISI)}$ and $I_{residual}^{(MAI)}$ which represent the residual ISI and MAI respectively. The *Strongest Paths* based tap selection is optimal only in the absence of ISI and MAI, in view of Maximum Ratio Combining (MRC) theory. In the presence of ISI

and MAI, GA based tap selection method carefully selects the taps one by one by trying to minimize the denominator term $\sigma_{I_{residual}^{(ISI)}}^2 + \sigma_{I_{residual}^{(MAI)}}^2$ and maximize the numerator term B_1^2 at the same time. In contrast, the *Strongest Paths* based tap selection is less capable of ISI and MAI mitigation compared with GA based method. The effect of ISI is less severe than MAI due to the exponentially decaying power law in UWB channel, assuming equal transmission power for all the transmitters. So the performance gap shown in Figure 4.3 becomes larger with increased number of MAI transmitters. On the other hand, in the case of higher SNR (lower σ_m^2), the term $\sigma_{I_{residual}^{(ISI)}}^2 + \sigma_{I_{residual}^{(MAI)}}^2$ dominates the interference power in the denominator in (4.2.19). Thus, increased SNR also results in larger performance gap in Figure 4.3.

4.3 UWB Multipath Channel Estimation

Perfect CIR has been assumed for tap selection based UWB receiver in the above analysis. In practice, CIR needs to be estimated before tap selection can be made as discussed in section 4.2.2. Imperfect channel estimation inevitably results in performance degradation. It is therefore important to discuss the receiver performance based on the channel estimation techniques.

In practice, training symbols can be inserted inside the data bursts to estimate the CIR for multipath channels. Given the UWB system model in (4.2.5) for single transmitter scenario, an alternative representation is

$$\mathbf{X}\mathbf{h} + \mathbf{m} = \mathbf{y} \quad (4.3.1)$$

where $\mathbf{h}$ denotes the generalized CIR and $\mathbf{m}$ represents the AWGN signal vector. Assuming the length of transmitted training symbols as $(n_{tr} = n_x n_{ISI})$, where n_x is a scale multiplier, the observation window will span over total $(n_x n_L)$ received signal

samples. A Toeplitz matrix that consists of training symbols can be represented as

$$\mathbf{X} = \begin{pmatrix} x(n) & 0 & \cdots & 0 & x(n-1) & 0 & \cdots \\ 0 & x(n) & 0 & \cdots & 0 & x(n-1) & \cdots \\ \vdots & & & \ddots & \vdots & & \\ x(n+1) & 0 & \cdots & 0 & x(n) & 0 & \cdots \\ \vdots & & & & \vdots & & \\ x(n+n_{ISI}-1) & 0 & \cdots & \cdots & x(n+n_{ISI}-2) & 0 & \cdots \\ \vdots & & & & \vdots & & \vdots \end{pmatrix}_{(n_x n_L) \times n_L} \quad (4.3.2)$$

4.3.1 Least Squares Based Channel Estimation

From (4.3.1), Least Squares (LS) [41] based CIR estimation can be obtained as

$$\hat{\mathbf{h}} = \mathbf{X}^\dagger \mathbf{y} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \quad (4.3.3)$$

Let $\mathbf{h} = \hat{\mathbf{h}} + \Delta \mathbf{h}$, where $\mathbf{h}$ denotes the true CIR and $\Delta \mathbf{h}$ is the channel estimation error. We then have

$$\Delta \mathbf{h} = \mathbf{h} - \mathbf{X}^\dagger (\mathbf{X} \mathbf{h} + \mathbf{m}) = \mathbf{X}^\dagger \mathbf{m} \quad (4.3.4)$$

Assume $E[\mathbf{m}] = 0$ and $E[\mathbf{m} \mathbf{m}^T] = \sigma_m^2 \mathbf{I}$. The mean and covariance of the channel estimation $\hat{\mathbf{h}}$ can be derived as

$$E[\hat{\mathbf{h}}] = E[\mathbf{X}^\dagger \mathbf{y}] = E[\mathbf{X}^\dagger (\mathbf{X} \mathbf{h} + \mathbf{m})] = \mathbf{h} \quad (4.3.5)$$

$$\text{cov}(\hat{\mathbf{h}}) = E[\Delta \mathbf{h} \Delta \mathbf{h}^T] = E[(\mathbf{X}^\dagger \mathbf{m})(\mathbf{X}^\dagger \mathbf{m})^T] = \sigma_m^2 (\mathbf{X}^T \mathbf{X})^{-1} \quad (4.3.6)$$

Suppose that the input symbols are uncorrelated, i.e., $E[x(i)x(j)] = \delta_{ij}$. In practice, an m-sequence can be applied to approximate the uncorrelated training symbol sequence. From (4.3.2) we obtain

$$\mathbf{X}^T \mathbf{X} = \left[\sum_k E[x_{k,i} x_{k,j}] \right] = n_x n_{ISI} \mathbf{I}_{n_L} = n_{tr} \mathbf{I}_{n_L} \quad (4.3.7)$$

Then

$$\hat{\mathbf{h}} = \frac{1}{n_{tr}} \mathbf{X}^T \mathbf{y} \quad (4.3.8)$$

This in fact represents the correlation based channel estimation. From (4.3.6), the covariance matrix for LS channel estimation is given by

$$\text{cov}(\hat{\mathbf{h}}) = \frac{\sigma_m^2}{n_{tr}} \mathbf{I}_{n_L} \quad (4.3.9)$$

Let $\hat{\mathbf{y}} = \mathbf{X}\hat{\mathbf{h}}$. The Mean Square Error (MSE) per sample for LS estimation is given by

$$MSE_{LS} = \frac{1}{n_x n_L} E[\|\mathbf{y} - \hat{\mathbf{y}}\|^2] = (1 + \frac{1}{n_x}) \sigma_m^2 \quad (4.3.10)$$

where using (4.3.9), we have $E[\|\Delta\mathbf{h}\|^2] = \text{trace}\{\text{cov}(\hat{\mathbf{h}})\} = \frac{n_L \sigma_m^2}{n_x n_{LSI}}$. From (4.3.10), it is noticed that increasing the number of training symbols ($n_{tr} = n_x n_{LSI}$) will reduce the LS channel estimation error. But it will be lower bounded by σ_m^2 . Tradeoff should be made since the computational cost involved in LS estimation will increase and data transmission throughput will decrease when we increase n_{tr} .

4.3.2 Thresholded Least Squares Based Channel Estimation

LS based channel estimation assumes that all the taps have nonzero values. This in fact affects channel estimation accuracy and hence channel equalization performance in the presence of sparse CIR. Based on IEEE 802.15.3a channel model recommendation, UWB indoor channel with long delay spread can be considered to be sparse due to its clustering effect at very high sampling rate ($\Delta\tau = 0.167ns$). By thresholding the LS channel estimation for CIR [19], the MSE of channel estimation can be effectively reduced and this improves the performance for channel estimation based equalization. This is described as follows.

From (4.3.9), it does not make sense to estimate a path h_k when the energy of the path is very low ($\|h_k\|^2 < \frac{\sigma_m^2}{n_{tr}}$). By simply thresholding those paths with very low energy to zero, the variance for path estimation will be limited by $\sigma_{\hat{h}}^2 \leq \frac{\sigma_m^2}{n_{tr}}$. From (4.3.10), MSE for channel estimation will then be bounded by $MSE_{THLS} \leq (1 + \frac{1}{n_x}) \sigma_m^2$. Thus, Thresholded Least Squares (THLS) does help to improve channel estimation performance. As LS channel estimation involves large computational complexity ($\mathcal{O}(n_L^3)$) due to matrix inversion as given in (4.3.3). Matching Pursuit

based channel estimation method which is computationally more efficient is discussed in section 4.3.3.

4.3.3 Matching Pursuit Based Channel Estimation

Matching Pursuit (MP) [56] is a greedy algorithm for subset selection, which progressively refines the signal approximation with an iterative procedure. It has been effectively applied to sparsely distributed multipath channel estimation such as “hilly terrain” delay profile [20]. In our experiments, MP algorithm is applied to channel estimation for UWB indoor channels due to its lower complexity when compared with THLS based method. As discussed in [20], the detailed process for using MP algorithm for channel estimation is introduced below.

From (4.3.1), ignoring the noise vector, the signal model is given as $\mathbf{X}\mathbf{h} = \mathbf{y}$. Let $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{n_L}]$, then the sparse channel path detection can be simply stated as finding the subset of column vectors in $\mathbf{X}$, that are best aligned with the observation signal vector $\mathbf{y}$. Set initial vector $\mathbf{y}_0 = \mathbf{y}$. Then, at the p -th step, the iterative algorithm is given by

$$k_p = \arg \max_l \frac{|\mathbf{x}_l^T \mathbf{y}_{p-1}|^2}{\|\mathbf{x}_l\|^2} \quad (4.3.11)$$

The k_p -th path value is given by the vector projection as

$$\hat{h}_{k_p} = \frac{\mathbf{x}_{k_p}^T \mathbf{y}_{p-1}}{\|\mathbf{x}_{k_p}\|^2} \quad (4.3.12)$$

Then, the residual vector is updated as given by

$$\mathbf{y}_p = \mathbf{y}_{p-1} - \hat{h}_{k_p} \mathbf{x}_{k_p} \quad (4.3.13)$$

The loop is repeated until a specified number of paths are detected or the residual becomes sufficiently small. The rest of the paths are assumed to be zero. Moreover, the residual $\mathbf{y}_p$ represents the approximation error for signal vector $\mathbf{y}$ at the p -th iteration.

Our numerical simulation shows that MP achieves slightly better channel estimation and equalization performance as compared to THLS, especially given limited number of training symbols. This is due to that MP algorithm estimates the channel

path one by one. It starts the estimation for the strongest path, which is supposed to be the most reliable estimation. Then, the residual signal is computed and utilized to continue the estimation process for the next strongest path. MP algorithm involves less computational complexity ($\mathcal{O}(n_L^2 n_S)$, $n_S \ll n_L$) as compared with LS estimation. Assuming nonzero paths have been detected by MP algorithm, the LS algorithm can then be applied to re-estimate these nonzero paths.

4.4 Channel Estimation Based Adaptive Decision Feedback Equalizer

From (4.2.8), to formulate the channel estimation error, let $\mathbf{H} = \hat{\mathbf{H}} + \Delta\mathbf{H}$ and $\mathbf{H}' = \hat{\mathbf{H}}' + \Delta\mathbf{H}'$ where $\Delta\mathbf{H}$ and $\Delta\mathbf{H}'$ represent the matrices due to estimation error. For simplicity, the subscript S used in $\mathbf{H}_S$ to denote the tap subset selection is dropped. Then, we have

$$\mathbf{y} = \mathbf{y}' - \hat{\mathbf{H}}'\mathbf{x}' = \hat{\mathbf{H}}\mathbf{x} + \Delta\mathbf{H}\mathbf{x} + \Delta\mathbf{H}'\mathbf{x}' + \mathbf{m} = \hat{\mathbf{H}}\mathbf{x} + \mathbf{w} \quad (4.4.1)$$

where define $\mathbf{w} = \mathbf{m}_{\Delta Hx} + \mathbf{m}_{\Delta H'x'} + \mathbf{m}$, $\mathbf{m}_{\Delta Hx} = \Delta\mathbf{H}\mathbf{x}$ and $\mathbf{m}_{\Delta H'x'} = \Delta\mathbf{H}'\mathbf{x}'$. Assume Gaussianity for interference signal $\mathbf{m}_{\Delta Hx}$ and $\mathbf{m}_{\Delta H'x'}$ which are due to channel estimation error. Then, the covariance matrix for $\mathbf{w}$ can be represented as

$$\mathbf{R}_w = \text{cov}(\mathbf{w}) = \mathbf{R}_{\Delta H} + \mathbf{R}_{\Delta H'} + \sigma_m^2 \mathbf{I} \quad (4.4.2)$$

where we have assumed $E[\mathbf{x}\mathbf{x}^T] = E[\mathbf{x}'\mathbf{x}'^T] = \mathbf{I}$ and $E[\mathbf{x}'\mathbf{x}^T] = \mathbf{0}$.

From (4.2.6), $\mathbf{R}_{\Delta H}$ and $\mathbf{R}_{\Delta H'}$ can be evaluated as follows

$$(\mathbf{R}_{\Delta H})_{i,j} = E \left[\sum_k \Delta\mathbf{H}(i,k) \Delta\mathbf{H}(j,k) \right] = \sum_k E [\Delta h(i') \Delta h(j')] \quad (4.4.3)$$

where $E [\Delta h(i') \Delta h(j')]$ can be obtained from the corresponding element of the covariance matrix of channel estimation as given in (4.3.9). Based on the evaluation using (4.4.3), it has been observed that the interference contributions from $\mathbf{R}_{\Delta H}$ and $\mathbf{R}_{\Delta H'}$ can be many times less than the diagonal elements in AWGN term $\sigma_m^2 \mathbf{I}$, as long as reasonably enough number of training symbols is provided for channel estimation under single transmitter scenario in the absence of co-channel interference.

From the orthogonality principle [41], let $\hat{x}(n) = \mathbf{f}_{est}^T \mathbf{y}$, the MMSE equalizer based on channel estimation is obtained as

$$\mathbf{f}_{est}^T = \{\hat{\mathbf{H}}^T(\hat{\mathbf{H}}\hat{\mathbf{H}}^T + \mathbf{R}_w)^{-1}\}_1 \approx \{\hat{\mathbf{H}}^T(\hat{\mathbf{H}}\hat{\mathbf{H}}^T + \sigma_m^2 \mathbf{I})^{-1}\}_1 \quad (4.4.4)$$

From (4.4.1) we obtain

$$\hat{x}(n) = \mathbf{f}_{est}^T (\mathbf{H}\mathbf{x} + \Delta\mathbf{H}'\mathbf{x}' + \mathbf{m}) = B_1 x(n) + I_{ISI} + I'_{ISI} + \tilde{m} \quad (4.4.5)$$

where $B_1 = (\mathbf{f}_{est}^T \mathbf{H})_1$, $\sigma_m^2 = \|\mathbf{f}_{est}^T\|^2 \sigma_n^2$. The residual ISI term after equalization is given as $I_{ISI} = \sum_{k=2}^{n_{sym}} B_k x(n+k-1)$, $B_k = (\mathbf{f}_{est}^T \mathbf{H})_k$ for $k = 1, \dots, n_{sym}$. The residual ISI term due to channel estimation error $\Delta\mathbf{H}'$ which affects SIC is given as $I'_{ISI} = \sum_{k=1}^{n_{ISI}-1} B'_k x(n-n_{ISI}+k)$, $B'_k = (\mathbf{f}_{est}^T \Delta\mathbf{H}')_k$ for $k = 1, \dots, n_{ISI}-1$.

Applying Gaussian approximation to the residual interference terms, the *SINR* for MMSE based DFE is given by

$$SINR_{est}^{(1)} = \frac{B_i^2}{\sum_{k=2}^{N_T} B_k^2 + \sum_{k=1}^{N'_T} B'_k{}^2 + \sigma_m^2} \quad (4.4.6)$$

Thus, the *BER* performance is obtained by

$$P_{b,est}^{(1)} = Q(\sqrt{SINR_{est}^{(1)}}) \quad (4.4.7)$$

4.5 Simulation Results and Discussions

In this section, the BER performance for tap selection based DFE is evaluated. Simulation result is obtained to show the effectiveness of tap selection based equalization for UWB channels.

For simplicity, CM2 (0-4m, NLOS) [25] is applied with sampling duration of $0.167ns$. Only single transmitter is assumed and the symbol rate for UWB BPSK transmission is set at $375MHz$, in the presence of severe ISI ($n_{ISI} = 32$). UWB channel estimation is obtained by MP algorithm with ($n_{tr} = 2 \times n_{ISI} = 64$) training symbols. In the absence of MAI, only ($n_S = 32$) strongest paths are selected out of total $n_L = 512$ samples of the estimated CIR. This is applied to tap selection based equalization to obtain all the simulation curves depicted in Figure 4.4. A

DFE (32 feedforward *taps* + 31 feedbackward *taps*) is obtained based on the channel estimation (denoted as *Ch-Est-DFE*), where the feedforward filter is estimated by (4.4.4) and the feedbackward filter is then obtained by $\mathbf{f}_{back}^T = -\mathbf{f}_{est}^T \hat{\mathbf{H}}'$. LMS algorithm is then applied to adapt the DFE with additional 128 training symbols (denoted as *Ch-Est+LMS-DFE*). In practice, decision directed adaptation by LMS can be implemented without the need for additional training symbols. The result is also compared with a DFE (32 + 31 *taps*) directly trained from the same amount of ($64 + 128 = 192$) training symbols by Recursive Least Squares (RLS) algorithm (denoted as *RLS-DFE*).

Figure 4.4: Channel estimation based DFE performance evaluation (CM2, $R_s = 375\text{Mbps}$, $n_S = 32$)

From Figure 4.4, it is observed that

1. The performance curve obtained using (4.4.6) consistently agrees with the simulation curve obtained by *Ch-Est-DFE* using (4.4.4).
2. Tap selection based *Ch-Est-DFE* is much more efficient than a direct RLS or LMS based DFE without tap selection. It requires much less training symbols

to estimate the DFE with reasonably good quick start performance. This is especially important for UWB channels with large delay spread which implies long training sequence is required for LMS or even RLS based adaptive equalizer implementation using all taps.

3. The simple LMS algorithm can be effectively applied to adapt the DFE after a quick start by channel estimation. Performance of *Ch-Est+LMS-DFE* can be improved by decision directed adaption.
4. Given limited number of training symbols, a *Ch-Est+LMS-DFE* performs slightly better than a *RLS-DFE*.
5. The performance gap between the simulation curve using *Ch-Est+LMS-DFE* and the analysis curve using DFE by assuming perfect CIR is within $1.5dB$ in terms of SNR for BER at 10^{-5} .

4.6 Summary

This chapter evaluates the tap selection based MMSE equalization performance for high data rate UWB system in the presence of severe ISI using greedy method. Based on this evaluation, a non-uniformly spaced MMSE equalizer that significantly outperforms the conventional uniformly spaced equalizer is developed for UWB channels. With a small portion of the total CIR samples chosen by appropriate tap selection methods, the complexity of the equalizer is greatly reduced with limited performance degradation when compared to that obtained by optimal MMSE detection using all taps. This indicates that the tap selection based equalization technique is a promising method for high performance UWB receiver design with reduced complexity. In addition, the impact of channel estimation error on the performance of tap selection based DFE is evaluated under single transmitter scenario. The analysis and simulation results show that a channel estimation based DFE with tap selection is able to achieve a reasonably good quick start performance given limited number of training

symbols. LMS algorithm can then be effectively applied to adapt the DFE for improved performance. Under the case of single transmitter, *Strongest Paths* based tap selection method is shown to work reasonably well for high data rate UWB channel equalization. It has limited performance degradation compared to the near optimal tap selection by GA based method. However, in the presence of MAI transmitters, larger performance degradation is observed when using this simple *Strongest paths* based tap selection method. This degradation also accelerates with increased SNR. Further, accurate multipath channel estimation for UWB becomes a difficult task in the presence of unknown interference. Therefore, it is necessary to develop a new tap selection algorithm which is able to achieve good performance in the presence of unknown co-channel interference.

Chapter 5

Least Squares Based Tap Selection Techniques

In chapter 4, tap selection based technique has been discussed for Ultra-Wideband (UWB) multipath channel equalization. Higher sampling rate is considered to benefit from the high diversity order of UWB multipath channel. Rank reduced equalization is achieved by tap selection. A Greedy Algorithm (GA) based tap selection method is developed and shown to achieve near optimal performance assuming perfect Channel Impulse Response (CIR). In practice, multipath channels need to be estimated before tap selection can be made on the estimated CIR. Accurate channel estimation for multipath channels with large delay spread such as in UWB radio is not an easy task in the presence of unknown co-channel interference. Channel estimation error will affect tap selection and system performance. Thus, in this chapter, a new class of Least Squares (LS) based tap selection techniques is proposed for UWB multipath channel equalization. These tap selection techniques are directly implemented based on training sequence without the need for explicit channel estimation.

5.1 Least Squares Method

5.1.1 UWB System Model and MMSE Detection

As discussed in section 4.2.1, a matrix representation for UWB system model in the presence of Inter Symbol Interference (ISI) and Multiple Access Interference (MAI) is given in following.

Let the signal for transmitter k be $s^{(k)}(t) = \sum_{n=-\infty}^{\infty} x^{(k)}(n)w(t - nT_f)$, where $x^{(k)}(n) \in \{\pm 1\}$ is the bipolar data bit stream, $w(t)$ is the pulse waveform and T_f is the symbol duration. Applying a tapped-delay-line channel model $c^{(k)}(t) = \sum_{p=1}^{n_L} \alpha_p^{(k)} \delta(t - (p-1)\Delta\tau - \tau_k)$, where $\alpha_p^{(k)}$ denotes the sampling channel impulse response, $\Delta\tau$ is the sampling duration, τ_k is the channel delay and n_L is the channel length in samples. For simplicity, assume that the sampling rate is an integer multiple of the symbol repetition rate, that is, let $n_\tau = T_f/\Delta\tau$ be an integer. The number of symbols affected by ISI is given as $n_{ISI} = n_L/n_\tau$, where assume that n_L is an integer multiple of n_τ obtained by truncating the tail end of CIR. A UWB system model can be represented as follows in the presence of ISI and MAI.

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{H}^{(MAI)}\mathbf{x}^{(MAI)} + \mathbf{m} \quad (5.1.1)$$

where $\mathbf{H}$ is the channel transmission matrix in block Toeplitz form that is constructed from the sampled generalized CIR ($h(t) = c(t)*w(t)$), $\mathbf{y}$ is the received sampling signal vector and $\mathbf{m}$ is the corresponding AWGN vector with covariance of $\sigma_m^2 \mathbf{I}$. The superscript (MAI) denotes the term corresponding to MAI transmitters. For simplicity, self spreading multipath channels of UWB are utilized for co-channel multiple access interference mitigation without adding extra spreading sequence. Strict channel synchronization is not necessary since the tap selection technique will automatically choose right paths for the desired symbol equalization.

Without loss of generality, assume that the desired symbol $x(n)$ is the first element in $\mathbf{x}$. From the orthogonality principle, the Minimum Mean Squared Error (MMSE) for linear MMSE detection can be derived as [81],

$$MMSE = \sigma_m^2 \left\{ ([\mathbf{H} \ \mathbf{H}^{(MAI)}]^T [\mathbf{H} \ \mathbf{H}^{(MAI)}] + \sigma_m^2 \mathbf{I}_{N_T})^{-1} \right\}_{1,1} \quad (5.1.2)$$

Similarly as discussed in section 3.4, the estimated symbol is given by

$$\hat{x}(n) = \mathbf{f}_{MMSE}^T \mathbf{y} = B_1 x(n) + I_{residual}^{(ISI)} + I_{residual}^{(MAI)} + \mathbf{f}_{MMSE}^T \mathbf{m} \quad (5.1.3)$$

where $\mathbf{f}_{MMSE}$ denotes the equalization filter and $B_1 = (\mathbf{f}_{MMSE}^T \mathbf{H})_1$. For the residual ISI and MAI, it can be shown that $E[I_{residual}^{(ISI)}] = E[I_{residual}^{(MAI)}] = 0$. Given the transmitted symbol as $x(n)$, the mean and covariance for estimated $\hat{x}(n)$ can be obtained

as

$$E[\hat{x}(n)] = B_1 x(n) \quad (5.1.4)$$

$$\sigma_{\hat{x}(n)}^2 = \sigma_{I_{residual}^{(ISI)}}^2 + \sigma_{I_{residual}^{(MAI)}}^2 + \|\mathbf{f}_{MMSE}\|^2 \sigma_m^2 \quad (5.1.5)$$

where σ^2 denotes the variance. The variance of the detected signal will be lower bounded by the AWGN term, that is,

$$\sigma_{\hat{x}(n)}^2 \geq \|\mathbf{f}_{MMSE}\|^2 \sigma_m^2 \quad (5.1.6)$$

5.1.2 Least Squares Estimation with Diagonal Loading

Assume that the receiver does not have the knowledge of the MAI transmitters. Blind estimation for multipath channels ($\mathbf{H}$ and $\mathbf{H}^{(MAI)}$) is a complicated task. However, with training symbol sequence $\mathbf{x}_{tr} = [x(1), \dots, x(n_{tr})]^T$ for the desired transmitter, the system model for Least Squares (LS) estimation [41] of equalization filter can be formulated as

$$\mathbf{x}_{tr} = \mathbf{Y}_{tr} \mathbf{f} \quad (5.1.7)$$

where the block Toeplitz matrix $\mathbf{Y}_{tr}$ consists of samples of received signal. It is represented by

$$\mathbf{Y}_{tr} = \begin{pmatrix} y^{(1)}(1) & y^{(2)}(1) & \cdots & y^{(n_\tau)}(1) & y^{(1)}(2) & \cdots & y^{(n_\tau)}(n_{ISI}) \\ y^{(1)}(2) & y^{(2)}(2) & \cdots & y^{(n_\tau)}(2) & y^{(1)}(3) & \cdots & y^{(n_\tau)}(n_{ISI} + 1) \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ y^{(1)}(n_{tr}) & y^{(2)}(n_{tr}) & \cdots & y^{(n_\tau)}(n_{tr}) & y^{(1)}(n_{tr} + 1) & \cdots & y^{(n_\tau)}(n_{tr} + n_{ISI} - 1) \end{pmatrix}_{n_{tr} \times n_L} \quad (5.1.8)$$

where $\{y^{(p)}(k), p = 1, \dots, n_\tau\}$ denotes n_τ received signal samples for the k -th symbol period, n_{tr} is the number of training symbols (considering the block Toeplitz form for $\mathbf{Y}_{tr}$, the actual number of training symbols required is $n_{tr} + n_{ISI}$).

Assuming $n_{tr} \geq n_L$, the LS estimation [41] for the equalization filter $\hat{\mathbf{f}}$ is given by

$$\hat{\mathbf{f}} = (\mathbf{Y}_{tr}^T \mathbf{Y}_{tr})^{-1} \mathbf{Y}_{tr}^T \mathbf{x}_{tr} \quad (5.1.9)$$

Let $\hat{\mathbf{x}} = \mathbf{Y}_{tr} \hat{\mathbf{f}}$. The LS estimation error can be derived as

$$\mathcal{E}_{LS} = \|\mathbf{x}_{tr} - \hat{\mathbf{x}}\|^2 = \mathbf{x}_{tr}^T \mathbf{x}_{tr} - \mathbf{x}_{tr}^T \mathbf{P} \mathbf{x}_{tr} \quad (5.1.10)$$

where $\mathbf{P}$ is the projection matrix given by

$$\mathbf{P} = \mathbf{Y}_{tr}(\mathbf{Y}_{tr}^T \mathbf{Y}_{tr})^{-1} \mathbf{Y}_{tr}^T \quad (5.1.11)$$

Given limited number of training symbols as in the case of practical implementation, LS estimation error $\mathcal{E}_{LS}$ which is the sample Mean Square Error (MSE) may fall below the MMSE as given in (5.1.2). For the observation signal contaminated with AWGN, further reducing the sample MSE ($\mathcal{E}_{LS}$) within the training set does not help and may even degrade the overall system performance. From (5.1.6), it is observed that the quadratic term $\|\hat{\mathbf{f}}\|^2$ plays an important role in minimizing the detection error. This suggests the use of Quadratic Constraint (QC), i.e., $\|\mathbf{f}\|^2 = f_0$, for minimizing the MSE, where f_0 is a constraining value. Thus, we have

$$\min_{\mathbf{f}} \|\mathbf{x}_{tr} - \hat{\mathbf{x}}\|^2 \quad \text{subject to} \quad \|\hat{\mathbf{f}}\|^2 = f_0 \quad (5.1.12)$$

where $\hat{\mathbf{x}} = \mathbf{Y}_{tr} \hat{\mathbf{f}}$. Using the method of Lagrange multipliers [41], the new cost function for error minimization under QC is given by

$$\mathcal{E}_{LS}^{(QC)} = (\mathbf{x}_{tr} - \mathbf{Y}_{tr} \hat{\mathbf{f}})^T (\mathbf{x}_{tr} - \mathbf{Y}_{tr} \hat{\mathbf{f}}) + \lambda (\hat{\mathbf{f}}^T \hat{\mathbf{f}} - f_0) \quad (5.1.13)$$

where λ is a Lagrange multiplier. It can be solved for $\hat{\mathbf{f}}$ as follows

$$\hat{\mathbf{f}}^{(QC)} = (\mathbf{Y}_{tr}^T \mathbf{Y}_{tr} + \lambda \mathbf{I})^{-1} \mathbf{Y}_{tr}^T \mathbf{x}_{tr} \quad (5.1.14)$$

This in fact can be regarded as introducing Diagonal Loading (DL) [15] technique which has been a popular tool for adding robustness to adaptive array beamforming into LS estimation process.

The DL technique can be interpreted as to artificially introduce a higher fixed noise floor that has the effect of eliminating the influence of poorly estimated small eigenvalues and associated eigenvectors for the Wiener filter. Let $\lambda = n_{tr} \sigma_L^2$, where σ_L^2 denotes the loading level. Defining Loading-to-Noise Ratio (LNR) as

$$LNR = \frac{\sigma_L^2}{\sigma_m^2} \quad (5.1.15)$$

where LNR may be determined empirically, the diagonally loaded Wiener filter $\hat{\mathbf{f}}^{(DL)}$ can be written as

$$\begin{aligned}\hat{\mathbf{f}}^{(DL)} &= (\mathbf{Y}_{tr}^T \mathbf{Y}_{tr} + n_{tr} \sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_{tr}^T \mathbf{x}_{tr} \\ &= \tilde{\mathbf{R}}_{\mathbf{Y}}^{-1} \hat{\mathbf{R}}_{\mathbf{Yx}}\end{aligned}\quad (5.1.16)$$

where sample covariance matrix is defined as

$$\hat{\mathbf{R}}_{\mathbf{Y}} = \frac{1}{n_{tr}} \mathbf{Y}_{tr}^T \mathbf{Y}_{tr} \quad (5.1.17)$$

and correlation matrix is defined as

$$\hat{\mathbf{R}}_{\mathbf{Yx}} = \frac{1}{n_{tr}} \mathbf{Y}_{tr}^T \mathbf{x}_{tr} \quad (5.1.18)$$

DL is equivalent to diagonally loading into the sample covariance matrix by setting

$$\tilde{\mathbf{R}}_{\mathbf{Y}} = \hat{\mathbf{R}}_{\mathbf{Y}} + \sigma_L^2 \mathbf{I} \quad (5.1.19)$$

In (5.1.13), discarding the last term λf_0 which is a constant, the cost function with DL becomes,

$$\mathcal{E}_{LS}^{(DL)} = \mathbf{x}_{tr}^T \mathbf{x}_{tr} - \mathbf{x}_{tr}^T \mathbf{P}^{(DL)} \mathbf{x}_{tr} \quad (5.1.20)$$

where diagonally loaded projection matrix $\mathbf{P}^{(DL)}$ is given by

$$\mathbf{P}^{(DL)} = \mathbf{Y}_{tr} (\mathbf{Y}_{tr}^T \mathbf{Y}_{tr} + n_{tr} \sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_{tr}^T \quad (5.1.21)$$

5.1.3 Minimum Norm Solution with Diagonal Loading

Given very limited number of training symbols, i.e., assuming $n_{tr} < n_L$, the optimum solution to LS estimation problem is the Minimum Norm (MN) solution [41] which is given by

$$\hat{\mathbf{f}}_{MN} = \mathbf{Y}_{tr}^T (\mathbf{Y}_{tr} \mathbf{Y}_{tr}^T)^{-1} \mathbf{x}_{tr} \quad (5.1.22)$$

However, LS estimation with DL as given in (5.3.18) is still valid with nonzero loading level ($\sigma_L^2 > 0$). It can be shown (5.3.18) approaches the Minimum Norm (MN) solution as given in (5.1.22) when σ_L approaches zero. By choosing an appropriate

σ_L , (5.3.18) is expected to outperform (5.1.22) given limited training symbol support. An alternative representation for MN solution with DL is then proposed as

$$\hat{\mathbf{f}}_{MN}^{(DL)} = \mathbf{Y}_{tr}^T (\mathbf{Y}_{tr} \mathbf{Y}_{tr}^T + n_{tr} \sigma_L^2 \mathbf{I}_{n_{tr}})^{-1} \mathbf{x}_{tr} \quad (5.1.23)$$

We assume Singular Value Decomposition (SVD) for $\mathbf{Y}_{tr} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$ where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, $\mathbf{\Sigma}$ is a diagonal matrix with singular values of $\{\sigma_i\}$. The Eigen Value Decomposition (EVD) for $\mathbf{Y}_{tr}^T \mathbf{Y}_{tr}$ is given by $\mathbf{Y}_{tr}^T \mathbf{Y}_{tr} = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^T$, where $\mathbf{\Lambda} = \mathbf{\Sigma}^2$ is a diagonal matrix with eigenvalues of $\{\lambda_i\} = \{\sigma_i^2\}$. With DL, we have $\mathbf{Y}_{tr}^T \mathbf{Y}_{tr} + n_{tr} \sigma_L^2 \mathbf{I} = \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^T$, where $\tilde{\mathbf{\Lambda}}$ is a diagonal matrix with eigenvalues of $\{\tilde{\lambda}_i\} = \{\sigma_i^2 + n_{tr} \sigma_L^2\}$ and the corresponding eigenvectors remain the same. The LS solution with DL as given in (5.3.18) can then be represented as

$$\hat{\mathbf{f}}_{LS}^{(DL)} = \mathbf{V} \tilde{\mathbf{\Lambda}}^{-1} \mathbf{\Sigma} \mathbf{U}^T \mathbf{x}_{tr} = \mathbf{V} \mathbf{\Gamma} \mathbf{U}^T \mathbf{x}_{tr} \quad (5.1.24)$$

where $\mathbf{\Gamma}$ is a diagonal matrix with diagonal elements of $\{\frac{\sigma_i}{\sigma_i^2 + n_{tr} \sigma_L^2}\}$. Similarly, the MN solution with DL as given in (5.1.23) can be written as

$$\hat{\mathbf{f}}_{MN}^{(DL)} = \mathbf{V} \mathbf{\Sigma} \tilde{\mathbf{\Lambda}}^{-1} \mathbf{U}^T \mathbf{x}_{tr} = \mathbf{V} \mathbf{\Gamma} \mathbf{U}^T \mathbf{x}_{tr} \quad (5.1.25)$$

Hence, (5.1.23) is shown to be equivalent to (5.3.18) with SVD but with reduced computational complexity when $n_{tr} < n_L$, that is, $\mathcal{O}((n_L + n_{tr}/3)n_{tr}^2)$ for $\mathbf{f}_{MN}^{(DL)}$ versus $\mathcal{O}((n_{tr} + n_L/3)n_L^2)$ for $\mathbf{f}_{LS}^{(DL)}$ [35].

5.2 Multistage Wiener Filter with Diagonal Loading

As introduced in section 2.5, the Multistage Wiener Filter (MWF) [34] is a stage by stage implementation of the classical Wiener filter that minimizes MSE. It is simply the solution of Wiener-Hopf equation by employing the Krylov subspace [44] [47] given as

$$\mathcal{K}_{n_L}(\mathbf{R}_Y, \mathbf{r}_{Yx}) = \text{span}\{\mathbf{r}_{Yx}, \mathbf{R}_Y \mathbf{r}_{Yx}, \dots, \mathbf{R}_Y^{n_L-1} \mathbf{r}_{Yx}\} \quad (5.2.1)$$

The MWF is shown to produce equivalent solutions as the method of Conjugate Gradients (CG) [82] [43], since both algorithms minimize the same cost function in the Krylov subspace.

In the following, we show both LS solution ($n_{tr} \geq n_L$) and MN solution ($n_{tr} < n_L$) lie in the same Krylov space. Hence when $n_{tr} \geq n_L$, MWF and CG method converge to LS solution. When $n_{tr} < n_L$, MWF and CG method converge to MN solution. For LS solution in the case of $n_{tr} \geq n_L$, we have

$$\mathbf{Y}_{tr} \mathbf{f}_{LS} = \mathbf{x}_{tr} \quad (5.2.2)$$

The normal equation is,

$$\mathbf{R}_Y \mathbf{f}_{LS} = \mathbf{r}_{Yx} \quad (5.2.3)$$

where $\mathbf{R}_Y = \frac{1}{n_{tr}} \mathbf{Y}_{tr}^T \mathbf{Y}_{tr}$ is a symmetric matrix of $n_L \times n_L$ and $\mathbf{r}_{Yx} = \frac{1}{n_{tr}} \mathbf{Y}_{tr}^T \mathbf{x}_{tr}$.

The solution for $\mathbf{f}_{LS}$ in (5.2.3) lies in the Krylov subspace [35], i.e., $\mathcal{K}_{n_L}(\mathbf{R}_Y, \mathbf{r}_{Yx})$ given in (5.2.1).

For MN solution in the case of $n_{tr} < n_L$, we have

$$\mathbf{Y}_{tr} \mathbf{f}_{MN} = \mathbf{x}_{tr} \quad (5.2.4)$$

Let $\mathbf{f}_{MN} = \mathbf{Y}_{tr}^T \mathbf{g}$, the normal equation is,

$$\mathbf{A} \mathbf{g} = \mathbf{x}_{tr} \quad (5.2.5)$$

where $\mathbf{A} = \mathbf{Y}_{tr} \mathbf{Y}_{tr}^T$ is a symmetric matrix of $n_{tr} \times n_{tr}$. The solution for $\mathbf{g}$ in (5.2.5) lies in the Krylov subspace [35]

$$\mathcal{K}_{n_{tr}}(\mathbf{A}, \mathbf{x}) = \text{span}\{\mathbf{x}, \mathbf{A}\mathbf{x}, \dots, \mathbf{A}^{n_{tr}-1}\mathbf{x}\} \quad (5.2.6)$$

Then the MN solution for $\mathbf{f}_{MN}$ lies in the subspace,

$$\begin{aligned} \mathbf{Y}_{tr}^T \mathcal{K}_{n_{tr}}(\mathbf{A}, \mathbf{x}) &= \mathbf{Y}_{tr}^T \text{span}\{\mathbf{x}, \mathbf{A}\mathbf{x}, \dots, \mathbf{A}^{n_{tr}-1}\mathbf{x}\} \\ &= \mathcal{K}_{n_{tr}}(\mathbf{R}_Y, \mathbf{r}_{Yx}) \end{aligned} \quad (5.2.7)$$

where $\mathbf{R}_Y$ is rank deficient in the case of $n_{tr} < n_L$. So the Krylov space $\mathcal{K}_{n_{tr}}(\mathbf{R}_Y, \mathbf{r}_{Yx})$ is of dimension n_{tr} rather than n_L .

In practice, rank reduction can be accomplished by truncating the multistage decomposition in Krylov subspace at the desired number of stages, i.e., n_{stage} . It is noticed that only a small number of n_{stage} is required to achieve the full-rank

performance since the set of basis vectors given in the span of the Krylov subspace $\mathcal{K}_{n_L}(\mathbf{R}_Y, \mathbf{r}_{Yx})$ may become nearly linearly dependent for $n_{stage} \ll n_L$ [45].

In addition to using the rank-reduced Krylov subspace $\mathcal{K}_{n_{stage}}(\mathbf{R}_Y, \mathbf{r}_{Yx})$, the transformation matrix can be formed from an orthonormal set of basis ($\mathbf{T} = \text{span}\{\mathbf{h}, \mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_{n_{stage}-1}\}$) which is generated by means of a successive refinement procedure [34]. This transformation is shown to be equivalent to the span of the Krylov subspace [44] but avoids the sample covariance matrix estimation by $\hat{\mathbf{R}}_Y = \frac{1}{n_{tr}} \mathbf{Y}_{tr}^T \mathbf{Y}_{tr}$ which could cost $(\mathcal{O}(n_L^2 n_{tr}))$. The fast algorithm for MWF implementation is introduced in section 2.5 as given in [69] and the computational complexity is on the order of $(\mathcal{O}(n_{stage} n_{tr} n_L))$.

In practical implementation, the performance of MWF can be sensitive to the rank n_{stage} [45]. DL technique has been incorporated into MWF and shown to be an effective “soft stop” across a broad region in rank where MWF is applied to the Minimum Variance Distortionless Response (MVDR) beamforming for rank-reduced adaptive array processing [43]. In our simulation, MWF with DL is shown to achieve improved performance for UWB multipath channel equalization under limited training sample support where the rank of MWF is simply chosen as a fixed number $n_{stage} = 8$.

5.3 Least Squares Based Tap Selection Techniques

Tap selection can be utilized to effectively reduce the requirement for number of training symbols for LS estimation to $n_{tr} \geq n_S$, where n_S denotes the number of taps to be selected and $n_S \ll n_L$. Based on system model given in (5.1.7), making tap selection can be considered as forming an observation sample matrix $\mathbf{Y}_S$ by selecting a subset of columns from $\mathbf{Y}_{tr}$ which corresponds to a subset of selected taps. The criterion is to maximize the projection power given by

$$\|\mathbf{P}_S \mathbf{x}_{tr}\|^2 = \mathbf{x}_{tr}^T \mathbf{P}_S \mathbf{x}_{tr} \quad (5.3.1)$$

or equivalently to minimize the LS estimation error given as

$$\mathcal{E}_S = \mathbf{x}_{tr}^T \mathbf{x}_{tr} - \mathbf{x}_{tr}^T \mathbf{P}_S \mathbf{x}_{tr} \quad (5.3.2)$$

where the projection matrix is represented by

$$\mathbf{P}_S = \mathbf{Y}_S(\mathbf{Y}_S^T \mathbf{Y}_S)^{-1} \mathbf{Y}_S^T \quad (5.3.3)$$

For optimal selection of n_S taps out of a total of n_L samples, an exhaustive search is needed and this requires $\mathbf{P}_S$ to be evaluated for $\binom{n_L}{n_S}$ possible combinations of the tap subsets. This is not feasible for large n_L . The locally optimal tap selection may be achieved by applying Greedy Algorithm (GA) to incrementally select taps one by one with minimized $\mathcal{E}_S$.

Given limited training sample support, DL is incorporated into LS based tap selection process. The criterion for tap selection is then to minimize the cost function

$$\mathcal{E}_S^{(DL)} = \mathbf{x}_{tr}^T \mathbf{x}_{tr} - \mathbf{x}_{tr}^T \mathbf{P}_S^{(DL)} \mathbf{x}_{tr} \quad (5.3.4)$$

or equivalently to maximize the projection term $\mathbf{x}_{tr}^T \mathbf{P}_S^{(DL)} \mathbf{x}_{tr}$, where diagonally loaded projection matrix $\mathbf{P}^{(DL)}$ is given by

$$\mathbf{P}_S^{(DL)} = \mathbf{Y}_S(\mathbf{Y}_S^T \mathbf{Y}_S + n_{tr} \sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_S^T \quad (5.3.5)$$

DL has the effect of eliminating the influence of poorly estimated small eigenvalues and associated eigenvectors in the covariance matrix of $\mathbf{Y}_S$. This can be interpreted as follows.

Assume Singular Value Decomposition (SVD) for matrix $\mathbf{Y}_S$ as $\mathbf{Y}_S = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$ where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{\Sigma}$ is a diagonal matrix with singular values of $\{\sigma_i\}$. We have $\mathbf{Y}_S^T \mathbf{Y}_S + n_{tr} \sigma_L^2 \mathbf{I} = \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^T$, where $\tilde{\mathbf{\Lambda}}$ is a diagonal matrix with eigenvalues of $\{\tilde{\lambda}_i\} = \{\sigma_i^2 + n_{tr} \sigma_L^2\}$. From (5.3.5), we obtain,

$$\mathbf{x}_{tr}^T \mathbf{P}_S^{(DL)} \mathbf{x}_{tr} = \sum_i^{n_S} \frac{\sigma_i^2}{\sigma_i^2 + n_{tr} \sigma_L^2} |\mathbf{x}_{tr}^T \mathbf{u}_i|^2 \quad (5.3.6)$$

where $\mathbf{u}_i$ are the eigenvectors for $\mathbf{Y}_S \mathbf{Y}_S^T$. In the case of $\sigma_i^2 \gg n_{tr} \sigma_L^2$, the effect of DL can be ignored. In the case of $\sigma_i^2 \ll n_{tr} \sigma_L^2$, eigenvectors $\mathbf{u}_i$ associated with the poorly estimated small eigenvalue σ_i^2 is essentially discarded. Thus, DL improves robustness for tap selection under finite training sample support.

Although $\mathbf{P}_S^{(DL)}$ no longer represents an orthogonal projection matrix, the estimation by $\hat{\mathbf{x}} = \mathbf{Y}_S \hat{\mathbf{f}}^{(DL)} = \mathbf{P}_S^{(DL)} \mathbf{x}_{tr}$ may still be considered as an orthogonal projection onto a virtual subspace by the introduction of an artificial noise matrix. Choosing an appropriate loading level for σ_L^2 (usually is a few dBs higher than σ_m^2) is important for the algorithm to converge to a reasonably good result. It is easy to show, excess loading ($\sigma_L^2 \rightarrow \infty$) will result in convergence to *Strongest Paths* based tap selection method which will be introduced in section 5.3.3.

5.3.1 Order Recursive Least Squares based Tap Selection Method

The computational complexity for GA based tap selection process by computing $\mathbf{P}_S^{(DL)}$ directly is on the order of $\mathcal{O}(\frac{1}{4}n_S^4 n_L + n_S^3 n_{tr} n_L)$ and this is impractical. The fast implementation can be solved by the iterative procedure similar to the Order Recursive Least Squares (ORLS) algorithm given in [48]. Following the procedure, the recursive formulas for incremental ORLS based tap selection method applied to LS estimation model given in (5.1.7) are obtained as follows.

Assuming at the n -th iteration, one more column $\mathbf{y}_k$ from $\mathbf{Y}_{tr}$ is chosen to form $\mathbf{Y}_n = [\mathbf{Y}_{n-1}, \mathbf{y}_k]$. The LS estimation error with DL is given by

$$\mathcal{E}_n^{(DL)} = \mathcal{E}_{n-1}^{(DL)} - \frac{(\mathbf{y}_k^T (\mathbf{I} - \mathbf{P}_{n-1}^{(DL)}) \mathbf{x}_{tr})^2}{\mathbf{y}_k^T (\mathbf{I} - \mathbf{P}_{n-1}^{(DL)}) \mathbf{y}_k + \lambda} \quad (5.3.7)$$

where λ is the DL term given by $\lambda = n_{tr} \sigma_L^2$. Minimizing $\mathcal{E}_n^{(DL)}$ is equivalent to choosing k and maximizing the following term,

$$s_n = \arg \max_k \frac{(\mathbf{y}_k^T (\mathbf{I} - \mathbf{P}_{n-1}^{(DL)}) \mathbf{x}_{tr})^2}{\mathbf{y}_k^T (\mathbf{I} - \mathbf{P}_{n-1}^{(DL)}) \mathbf{y}_k + \lambda} \quad (5.3.8)$$

Then, the ORLS formula for updating $\mathbf{P}_n^{(DL)}$ for the next iteration can be derived as

$$\mathbf{P}_n^{(DL)} = \mathbf{P}_{n-1}^{(DL)} + \frac{(\mathbf{I} - \mathbf{P}_{n-1}^{(DL)}) \mathbf{y}_{s_n} \mathbf{y}_{s_n}^T (\mathbf{I} - \mathbf{P}_{n-1}^{(DL)})}{\mathbf{y}_{s_n}^T (\mathbf{I} - \mathbf{P}_{n-1}^{(DL)}) \mathbf{y}_{s_n} + \lambda} \quad (5.3.9)$$

where $\mathbf{P}_n^{(DL)} = \mathbf{Y}_n (\mathbf{Y}_n^T \mathbf{Y}_n + \lambda \mathbf{I})^{-1} \mathbf{Y}_n^T$ is a $n_{tr} \times n_{tr}$ symmetric matrix.

For each iteration, to select column $\mathbf{y}_{s_n}$, (5.3.8) needs to be evaluated for all $(n_L - n + 1)$ remaining columns $\{\mathbf{y}_k\}$. Directly computing for (5.3.8) will cost $\mathcal{O}(n_{tr}^2)$ mainly due to the computation for the term $\mathbf{y}_k^{(n)}$ which is defined as

$$\mathbf{y}_k^{(n)} \triangleq (\mathbf{I} - \mathbf{P}_n^{(DL)})\mathbf{y}_k \quad (5.3.10)$$

The computational complexity for the selection algorithm is about $\mathcal{O}(n_S n_L n_{tr}^2)$. This can be further reduced by finding out the recursive formulas for updating $\mathbf{y}_k^{(n)}$ as follows

$$\begin{aligned} \mathbf{y}_k^{(n)} &\triangleq (\mathbf{I} - \mathbf{P}_n^{(DL)})\mathbf{y}_k \\ &= (\mathbf{I} - \mathbf{P}_{n-1}^{(DL)})\mathbf{y}_k - \mathbf{q}_n \mathbf{q}_n^T \mathbf{y}_k \\ &= \mathbf{y}_k^{(n-1)} - (\mathbf{q}_n^T \mathbf{y}_k) \mathbf{q}_n \end{aligned} \quad (5.3.11)$$

where

$$\mathbf{q}_n \triangleq \frac{\mathbf{y}_{s_n}^{(n-1)}}{[\mathbf{y}_{s_n}^T \mathbf{y}_{s_n}^{(n-1)} + \lambda]^{1/2}} \quad (5.3.12)$$

From (5.3.9), we have

$$\mathbf{P}_n^{(DL)} = \mathbf{P}_{n-1}^{(DL)} + \mathbf{q}_n \mathbf{q}_n^T \quad (5.3.13)$$

We define

$$x_k^{(n)} \triangleq \mathbf{x}_{tr}^T \mathbf{y}_k^{(n)} = \mathbf{x}_{tr}^T \mathbf{y}_k^{(n-1)} - \mathbf{x}_{tr}^T \mathbf{q}_n \mathbf{q}_n^T \mathbf{y}_k = x_k^{(n-1)} - (\mathbf{q}_n^T \mathbf{y}_k)(\mathbf{x}_{tr}^T \mathbf{q}_n) \quad (5.3.14)$$

and

$$z_k^{(n)} \triangleq \mathbf{y}_k^T \mathbf{y}_k^{(n)} = \mathbf{y}_k^T \mathbf{y}_k^{(n-1)} - \mathbf{y}_k^T \mathbf{q}_n \mathbf{q}_n^T \mathbf{y}_k = z_k^{(n-1)} - (\mathbf{q}_n^T \mathbf{y}_k)^2 \quad (5.3.15)$$

Therefore, the iterative tap selection method based on ORLS can be summarized as follows.

ORLS Based Tap Selection Method:

- Initialize by setting $n = 0$, $\mathbf{Y}_0 = []$ and $\mathbf{P}_0 = 0$, then we have

$$\mathbf{y}_k^{(0)} = \mathbf{y}_k, \quad x_k^{(0)} = \mathbf{x}_{tr}^T \mathbf{y}_k \quad \text{and} \quad z_k^{(0)} = \|\mathbf{y}_k\|^2 \quad \text{for } k = 1, \dots, n_L$$

- Set $n = n + 1$.

Let $\mathbf{Y}_n = [\mathbf{Y}_{n-1} \quad \mathbf{y}_{s_n}]$. This is obtained by adding one more column $\mathbf{y}_{s_n}$ which is selected by $s_n = \arg \max_k \frac{(x_k^{(n)})^2}{z_k^{(n)} + \lambda}$,

where k denotes the unselected column index for $\mathbf{Y}_{tr}$

- Compute $\mathbf{q}_n = \frac{\mathbf{y}_{s_n}^{(n-1)}}{[z_{s_n}^{(n-1)} + \lambda]^{1/2}}$.
For each k , update $\mathbf{y}_k^{(n)}$, $x_k^{(n)}$ and $z_k^{(n)}$ using (5.3.11), (5.3.14) and (5.3.15)
- Continue the iteration for tap selection until $n = n_S$.

The computation of square root $(z_{s_n}^{(n-1)} + \lambda)^{1/2}$ in (5.3.12) for updating $\mathbf{q}_n$ is not necessary since the denominator becomes $(z_{s_n}^{(n-1)} + \lambda)$ without taking square root for updating $\mathbf{y}_k^{(n)}$, $x_k^{(n)}$ and $z_k^{(n)}$. The total computational complexity for *ORLS Based Tap Selection Method* is reduced to about $\{4n_S n_{tr} n_L\}$ in flops.

5.3.2 Matching Pursuit based Tap Selection Method

The above tap selection problem can also be considered as a subset selection problem as discussed in [19]. This subset selection problem can be solved by a Matching Pursuit (MP) algorithm [56]. MP is a fast algorithm that progressively refines the signal approximation with an iterative procedure. In the following, a fast tap selection method based on MP algorithm is proposed where DL is integrated into MP algorithm by the introduction of QC.

For LS estimation model given in (5.1.7), MP algorithm is applied to iteratively select a subset of columns one by one which correspond to a subset of taps. Assume that at the n -th iteration, the criterion for tap selection is to choose a column $\mathbf{y}_k$ for minimizing MSE under QC given as $\|\mathbf{f}\|^2 = g_0$, where g_0 is a constraining value. The signal estimation is given as $\hat{\mathbf{x}}^{(n)} = \mathbf{y}_k f_n$ where f_n is the n -th selected tap coefficient for filter vector $\mathbf{f}$. Thus, the estimation problem becomes

$$\min_{f_n} \|\mathbf{x}_{res}^{(n-1)} - \hat{\mathbf{x}}^{(n)}\|^2 \text{ subject to } \|\mathbf{f}\|^2 = g_0 \quad (5.3.16)$$

where $\mathbf{x}_{res}^{(n-1)}$ denotes the residual signal vector obtained from the previous iteration and $\hat{\mathbf{x}}^{(n)}$ is the signal vector to be estimated at current iteration by choosing column $\mathbf{y}_k$. As in MP algorithm, filter coefficients $\{f_k, k = 1, \dots, n_S\}$ are estimated one by one separately in each iteration. At the n -th iteration, QC can be rewritten as $f_n^2 = g_0 - \sum_{k=1}^{n-1} f_k^2 = g_n$, where g_n is the constraining value for f_n^2 . Using method of

Lagrange multipliers [41], the new cost function for minimization is given by

$$\mathcal{E}_n^{(DL)} = (\mathbf{x}_{res}^{(n-1)} - \mathbf{y}_k f_n)^T (\mathbf{x}_{res}^{(n-1)} - \mathbf{y}_k f_n) + \lambda (f_n^2 - g_n) \quad (5.3.17)$$

where λ is a Lagrange multiplier. This can be solved for f_n as follows,

$$\hat{f}_n^{(DL)} = \frac{\mathbf{y}_k^T \mathbf{x}_{res}^{(n-1)}}{\|\mathbf{y}_k\|^2 + \lambda} \quad (5.3.18)$$

Let $\lambda = n_{tr} \sigma_L^2$, where σ_L^2 is defined as the loading level which can be chosen empirically. Incorporating QC can be regarded as introducing DL technique [15] into MP based tap selection algorithm.

The DL technique mitigates the mismatch problem due to limited number of training symbols. This can be interpreted as artificially introducing a higher fixed noise floor (σ_L^2) that has the effect of preventing the algorithm from selecting the insignificant column $\mathbf{y}_k$ when $\|\mathbf{y}_k\|^2 \ll n_{tr} \sigma_L^2$. Substituting (5.3.18) into (5.3.17) and discarding the last term (λg_n) which is a constant, the cost function becomes,

$$\mathcal{E}_n^{(DL)} = (\mathbf{x}_{res}^{(n-1)})^T \mathbf{x}_{res}^{(n-1)} - (\mathbf{x}_{res}^{(n-1)})^T \mathbf{P}_{\mathbf{y}_k}^{(DL)} \mathbf{x}_{res}^{(n-1)} \quad (5.3.19)$$

where $\mathbf{P}_{\mathbf{y}_k}^{(DL)}$ is given by

$$\mathbf{P}_{\mathbf{y}_k}^{(DL)} = \frac{\mathbf{y}_k \mathbf{y}_k^T}{\|\mathbf{y}_k\|^2 + n_{tr} \sigma_L^2} \quad (5.3.20)$$

Thus, MP based tap selection method with DL is detailed as follows.

MP Based Tap Selection Method:

- Initialize by setting $n = 0$ and $\mathbf{x}_{res}^{(0)} = \mathbf{x}_{tr}$
- Set $n = n + 1$. Select a new column $\mathbf{y}_s$ by $s = \arg \max_k (\mathbf{x}_{res}^{(n-1)})^T \mathbf{P}_{\mathbf{y}_k}^{(DL)} \mathbf{x}_{res}^{(n-1)}$, where k denotes the unselected column index for $\mathbf{Y}_{tr}$,
the ‘projection matrix’ for column vector $\mathbf{y}_k$ is defined as $\mathbf{P}_{\mathbf{y}_k}^{(DL)} = \frac{\mathbf{y}_k \mathbf{y}_k^T}{\|\mathbf{y}_k\|^2 + n_{tr} \sigma_L^2}$
and the ‘projection power’ of vector $\mathbf{x}_{res}^{(n-1)}$ is given as
 $(\mathbf{x}_{res}^{(n-1)})^T \mathbf{P}_{\mathbf{y}_k}^{(DL)} \mathbf{x}_{res}^{(n-1)} = \frac{|\mathbf{y}_k^T \mathbf{x}_{res}^{(n-1)}|^2}{\|\mathbf{y}_k\|^2 + n_{tr} \sigma_L^2}$
- Compute and update the residue $\mathbf{x}_{res}^{(n)} = \mathbf{x}_{res}^{(n-1)} - \mathbf{P}_{\mathbf{y}_s}^{(DL)} \mathbf{x}_{res}^{(n-1)}$,
then continue the iteration for tap selection until $n = n_S$.

The computational complexity of *MP Based Tap Selection Method* is about $\{2n_s n_{tr} n_L\}$ in flops. Comparing *MP Based Tap Selection Method* with *ORLS Based Tap Selection Method*, it is observed that at each iteration MP algorithm selects a new vector by successively projecting the residual signal onto the subspace of each column vector $\mathbf{y}_k$ and determining the best match rather than successively projecting the original signal onto the entire selected subspace of matrix $\mathbf{Y}_n$. Hence MP algorithm saves the computational cost but with some performance tradeoff.

Intuitively, the introduction of the fixed noise floor σ_L^2 in $\mathbf{P}_{\mathbf{y}_k}^{(DL)}$ has the effect of preventing the algorithm from selecting the insignificant column $\mathbf{y}_k$ when $\|\mathbf{y}_k\|^2 \ll n_{tr} \sigma_L^2$. Due to the introduction of DL, matrix $\mathbf{P}_{\mathbf{y}_k}^{(DL)}$ no longer represents an orthogonal projection onto the subspace of vector $\mathbf{y}_k$. But it may still be considered as an orthogonal projection onto the subspace of a virtual vector $\mathbf{y}'_k = \mathbf{y}_k + \mathbf{m}_k$ assuming the introduction of an artificial noise vector $\mathbf{m}_k$. The reselection of basis [19] should be avoided in the DL modified MP algorithm due to the introduction of virtual basis. The convergence of the MP algorithm is shown [56] to depend on the correlation between the residue ($\mathbf{x}_{res}$) and the basis ($\mathbf{y}'_k$). This can be insured by choosing an appropriate loading level for σ_L^2 . Similarly, it can be shown that excess loading ($\sigma_L^2 \rightarrow \infty$) will result in convergence to *Strongest Paths* based tap selection method.

5.3.3 Strongest Paths and Strongest Projections based Tap Selection Methods

An intuitive way for selecting taps can be carried out by simply choosing a subset with the *Strongest Projections* based on the projection power given by

$$\{\|\mathbf{P}_{\mathbf{y}_k} \mathbf{x}_{tr}\|^2 = \mathbf{x}_{tr}^T \mathbf{P}_{\mathbf{y}_k} \mathbf{x}_{tr} = \frac{|\mathbf{y}_k^T \mathbf{x}_{tr}|^2}{\|\mathbf{y}_k\|^2}, k = 1, \dots, n_L\} \quad (5.3.21)$$

This is similar to the *Strongest Paths* based tap selection method assuming correlation output is used as CIR estimation and choosing a subset with the *Strongest Paths* based on the estimated CIR given by

$$\{|\mathbf{y}_k^T \mathbf{x}_{tr}|^2, k = 1, \dots, n_L\} \quad (5.3.22)$$

The computational cost for this *Strongest Projections* or *Strongest Paths* based simple tap selection method is on the order of ($\mathcal{O}(n_{tr}n_L)$). The additional cost for selecting n_S strongest taps out of n_L is no more than ($\mathcal{O}(n_S n_L)$). However, there is extra cost of $\mathcal{O}(n_L)$ division operation for the *Strongest Projections* based method.

5.3.4 Heuristic Matching Pursuit based Tap Selection Method

Our numerical analysis shows that only a small portion of taps selected by either *Strongest Projections* or *Strongest Paths* based simple tap selection method is different from the taps selected by *MP Based Tap Selection Method*. Thus, a fast heuristic tap selection method can be developed by firstly choosing a large portion of taps using *Strongest Projections* or *Strongest Paths* based simple tap selection method and then choosing the rest of taps, i.e., assuming n_{depth} taps, using *MP Based Tap Selection Method*. This heuristic MP based tap selection method incorporated with DL technique for improved robustness given limited number of training symbols is detailed as follows.

Heuristic MP Based Tap Selection Method:

- Initialize by setting $n = 0$ and $\mathbf{x}_{res}^{(0)} = \mathbf{x}_{tr}$
- Pre-selection step:
 Select an initial subset of $(n_S - n_{depth})$ taps based on the *Strongest Projections* of $\mathbf{x}_{tr}^T \mathbf{P}_{\mathbf{y}_s}^{(DL)} \mathbf{x}_{tr}$ and update the residue by $\mathbf{x}_{res}^{(n)} = \mathbf{x}_{res}^{(n-1)} - \mathbf{P}_{\mathbf{y}_s}^{(DL)} \mathbf{x}_{res}^{(n-1)}$ for all these pre-selected columns $\{\mathbf{y}_s\}$.
 Set $n = n_S - n_{depth}$
- MP selection step:
 Set $n = n + 1$.
 Select a new column $\mathbf{y}_s$ by $s = \arg \max_k (\mathbf{x}_{res}^{(n-1)})^T \mathbf{P}_{\mathbf{y}_k}^{(DL)} \mathbf{x}_{res}^{(n-1)}$ and update the residue by $\mathbf{x}_{res}^{(n)} = \mathbf{x}_{res}^{(n-1)} - \mathbf{P}_{\mathbf{y}_s}^{(DL)} \mathbf{x}_{res}^{(n-1)}$.
 Continue the MP iteration until $n = n_S$.

The computational complexity of *Heuristic MP Based Tap Selection Method* is about $\{2(1 + n_{depth})n_{tr}n_L\}$ flops. The numerical analysis shows that for larger n_S , *Heuristic MP Based Tap Selection Method* is capable of making a fast selection by choosing $n_{depth} \ll n_S$ with limited performance tradeoff when compared with *MP Based Tap Selection Method*.

5.4 Simulation Results and Discussions

Numerical results are obtained from 100 UWB channel realizations for CM2 based on IEEE 802.15.3a channel model recommendation [25]. The sampling duration is set as $\Delta\tau = 0.167ns$ and symbol rate is assumed at $R_s = \frac{1}{T_f} = 93.56Mbps$ to let $n_\tau = T_f/\Delta\tau$ be an integer. Then, we have the channel length as $n_L = 512$ samples and number of symbols affected by ISI as $n_{ISI} = 8$. Suppose that there are $n_{MAI} = 3$ MAI transmitters present with equal transmission power as the desired transmitter. The number of effective transmitters is defined as the number of significant eigenvalues associated with the sample covariance matrices. It is around $n_{eff} = 30 \sim 50$ obtained by eigenspectra analysis as shown in Figure 5.1. For simplicity, we take an approximation by $n_{eff} \approx (1 + n_{MAI})n_{ISI} = 32$. For effective interference mitigation, the number of taps to be selected for linear MMSE equalization should satisfy $n_S \geq n_{eff}$. For reasonable comparison, the equalizer coefficients for all the simulations with different tap selection based methods are estimated by LS estimation which has the computational complexity of $(n_{tr} + \frac{1}{3}n_S)n_S^2$ in flops [35]. In practical implementation, RLS with the computational complexity on the order of $\mathcal{O}(n_S^2n_{tr})$ can be applied as well.

5.4.1 Obtaining Appropriate Loading-to-Noise Ratio (LNR) for Diagonal Loading

The DL technique is also equivalent to Tikhonov regularization which is a useful method for solving linear discrete ill-posed problems [39]. A matrix is either ill-conditioned or well-conditioned characterized by its condition number or eigenspread

Figure 5.1: Eigenspectra for sample covariance matrices ($\mathbf{R}_Y$) (CM2, $R_s = 93.56 \text{ Mbps}$, $n_{MAI} = 3$, $n_L = 512$)

($\lambda_{max}/\lambda_{min}$) [35]. Figure 5.1 illustrates the eigenspectra of sample covariance matrices $\mathbf{R}_Y = \frac{1}{n_{tr}} \mathbf{Y}_{tr}^T \mathbf{Y}_{tr}$ for a CM2 channel with different number of training symbols. It is observed that $\mathbf{R}_Y$ has the largest eigenspread or becomes worstly ill-conditioned when $n_{tr} \rightarrow n_L = 512$. In the case when $n_{tr} > n_L$, increasing n_{tr} will result in well-conditioned $\mathbf{R}_Y$ with smaller eigenspread which approaches the ideal eigenspectrum obtained from true CIR when $n_{tr} \rightarrow \infty$. On the other hand, when $n_{tr} < n_L$, decreasing n_{tr} will result in rank-deficient $\mathbf{R}_Y$ but with smaller eigenspread for non-zero eigenvalues (or smaller eigenspread for matrix $\mathbf{Y}_{tr} \mathbf{Y}_{tr}^T$). Therefore, DL technique is most effective for performance improvement when $n_{tr} \rightarrow n_L$. It becomes less effective when either n_{tr} is much larger or much smaller than n_L . But smaller n_{tr} will result in poor performance due to rank deficiency. This will be observed in Figure 5.3 and 5.5.

The appropriate loading level LNR can be decided by L-Curve analysis as described in [39]. The corner (or the knee) which corresponds to the favourable LNR value can be found from the plot of L-curve with different LNR settings. Figure

Figure 5.2: L-Curve analysis (CM2, $R_s = 93.56\text{Mbps}$, $n_{MAI} = 3$, $n_L = 512$, $n_{tr} = 512$)

5.2(a) illustrates the L-Curve (Sample MSE vs. squared filter vector norm $\|\mathbf{f}\|^2$) obtained for a CM2 channel with $n_{tr} = 512$ training symbols. It can be found that the corner is corresponding to an appropriate setting for $LNR = 1\text{dB}$. From Figure 5.2(b), it is observed that increasing LNR to the range of $LNR = 0 \sim 6\text{dB}$, the sample MSE only increases gradually. The squared filter vector norm $\|\mathbf{f}\|^2$ decreases monotonically and results in a lower AWGN bound given in (5.1.6). Increasing LNR beyond this range will result in rapid increasing of the sample MSE and hence the performance degradation. Thus, $LNR = 0 \sim 6\text{dB}$ provides as an appropriate setting based on the L-Curve analysis.

We found it is also convenient to choose the LNR empirically from the BER simulation curves. Figure 5.3 illustrates the BER performance of MN solution versus the LNR given different number of training symbols. It has been understood that the sample covariance matrix $\mathbf{R}_Y$ has the largest eigenspread or becomes worstly ill-conditioned when n_{tr} approaches $n_L = 512$ as illustrated and discussed in Figure 5.1. It can be observed in Figure 5.3 that in the case of very low LNR ($LNR < -11\text{dB}$) the BER curve with $n_{tr} = 512$ performs even worse than that with $n_{tr} = 256/128$. This is due to the reason that very low LNR is not able to depress the incorrectly estimated small eigenvalues for $\mathbf{R}_Y$ (as illustrate in Figure 5.1) which do not help but degrade

Figure 5.3: BER vs. Loading-to-Noise Ratio (LNR) for Minimum Norm (MN) Solution (CM2, $R_s = 93.56 \text{ Mbps}$, $n_{MAI} = 3$, $n_L = 512$, $E_b/N_0 = 10 \text{ dB}$)

the system performance. Hence applying appropriate diagonal loading is critical in the case when $\mathbf{R}_Y$ is ill-conditioned. It is also observed that $LNR = 0 \sim 6 \text{ dB}$ serves as a good choice. Remarkable performance improvement is obtained when $n_{tr} = n_L = 512$. But almost no performance improvement can be achieved when $n_{tr} = 128$. Overloading ($LNR > 6 \text{ dB}$) should be avoided since it will degrade the performance significantly. The appropriate range for LNR setting obtained from BER simulation curves is consistent with the result obtained by the L-Curve analysis.

5.4.2 Equalization Performance for Different Tap Selection Methods: BER vs. SNR

Figure 5.4 illustrates the BER performance of MMSE equalization for UWB channel CM2 by using different tap selection methods described in section 5.3. The computational cost for tap selection and estimation of equalizer coefficients is tabulated in Table 5.1.

From Figure 5.4, the following observations are made

Figure 5.4: BER Performance comparison for different tap selection methods (CM2, $R_s = 93.56 \text{ Mbps}$, $n_{MAI} = 3$, $n_S = 32$, $n_{tr} = 64$)

Tap Selection Methods	Computational Cost
	$n_L = 512$, $n_S = 32$, $n_{tr} = 64$
<i>Full RAKE</i>	$2n_L n_{tr}$
<i>MN Solution</i>	$(n_L + n_{tr}/3)n_{tr}^2$
<i>ORLS + LS</i>	$4n_S n_{tr} n_L + (n_{tr} + n_S/3)n_S^2$
<i>MP + LS</i>	$2n_S n_{tr} n_L + (n_{tr} + n_S/3)n_S^2$
<i>Heuristic MP + LS</i>	$2(n_{depth} + 1)n_{tr} n_L + (n_{tr} + n_S/3)n_S^2$, $n_{depth} = 6$
<i>Strongest Projections + LS</i>	$(2n_L n_{tr} + n_L n_S) + (n_{tr} + n_S/3)n_S^2$
<i>Strongest Paths + LS</i>	$(2n_L n_{tr} + n_L n_S) + (n_{tr} + n_S/3)n_S^2$

Table 5.1: Computational cost for different tap selection methods with MMSE equalization

- *ORLS* based tap selection method achieves better performance but with higher computational complexity compared with *MP* based method. In addition, without DL, either *ORLS* or *MP* algorithm performs poorly due to the covariance matrix mismatch given limited number of training symbols. This demonstrates the importance and effectiveness for introducing DL technique into LS based tap selection methods given limited number of training symbols in practical implementation.
- Comparing with *MP* based tap selection method, *Heuristic MP* ($n_{depth} \ll n_S$) based tap selection method exhibits only very limited performance degradation but benefits from greatly reduced computational complexity. Increasing n_{depth} towards n_S , *Heuristic MP* based method will approach *MP* based method. Thus, the tradeoff between performance and complexity can be made easily by applying *Heuristic MP* based method, especially when the required number of taps to be selected (n_S) is larger.
- The *Strongest Projections* based method performs slightly better than the *Strongest Paths* based method. Since the normalization factor $\frac{1}{\|\mathbf{y}_k\|^2}$ utilized in the *Strongest Projections* based method benefits the selection of some critical CIR samples with lower power ($\|\mathbf{y}_k\|^2$) and improves the performance in the case of higher SNR. Both methods are simple and have similar complexity that is low.
- By using *ORLS/MP/Heuristic MP* based tap selection method, the performance improvement over the *Strongest Projections/Strongest Paths* based simple tap selection method becomes noticeable under higher SNR level ($SNR \geq 10dB$ in the test case). Large performance improvement is expected with increased SNR, since the capability of tap selection methods for ISI and MAI mitigation becomes more important in the case of higher SNR. For lower SNR, the *Strongest Projections/Strongest Paths* based method may be preferred due to its lowest complexity.
- *RAKE* receiver exhibits performance floor and breaks down in the presence of

severe ISI and MAI, although it has the lowest computational complexity.

- *MN Solution* performs similarly to the simple *Strongest Projections/Strongest Paths* based tap selection method given very limited number of training symbol support ($n_{tr} = 64$), but with much higher complexity.

5.4.3 Equalization Performance for Different Tap Selection Methods: *BER* vs. Number of Training Symbols (n_{tr})

(a) BER of MN Solution

(b) BER of Strongest Paths/ORLS + LS

(c) BER of MP/Heuristic MP + LS

(d) BER of Tap Selection + LS + DL

Figure 5.5: BER Performance comparison for different tap selection technique based MMSE equalization with/without Diagonal Loading (DL) (CM2, $R_s = 93.56\text{Mbps}$, $n_{MAI} = 3$, $n_S = 32$, $E_b/N_0 = 10\text{dB}$, $LNR = 0\text{dB}$)

Figure 5.5 and Figure 5.6 illustrate the BER simulation curves and computational complexity respectively for different tap selection methods versus the number

Figure 5.6: Computational complexity (in flops) comparison for different tap selection method based equalization processes (CM2, $R_s = 93.56 \text{ Mbps}$, $n_{MAI} = 3$, $n_s = 32$)

of training symbols. It is observed that,

- For MN solution shown in Figure 5.5(a), DL becomes more and more effective for performance improvement when $n_{tr} \rightarrow n_L = 512$.
- For tap selection based MMSE detection shown in Figure 5.5(b) and 5.5(c), DL is more effective for performance improvement when $n_{tr} \rightarrow n_s = 32$ and becomes less effective with the increased number of training symbols (n_{tr}). But the overall performance consistently improves with the increased n_{tr} .
- In Figure 5.5(d), tap selection based methods (*ORLS/MP/Heuristic MP/Strongest Paths*) with *LS + DL* outperform *MN + DL* given very limited number of training symbols ($n_{tr} \leq 128$ in the test case). With increased n_{tr} , *MP/Heuristic MP* with *LS + DL* exhibits limited performance degradation compared to *MN + DL*. Moreover, *ORLS + LS + DL* consistently achieves the best performance.
- Figure 5.6 compares the computational complexity for different equalization

methods. With the increased number of training symbols (n_{tr}), both *ORLS + LS* and *MP + LS* have lower computational complexity compared with *MN Solution*. *Strongest Paths + LS* and *Heuristic MP + LS* have much reduced computational complexity but still achieve reasonably good performance.

5.4.4 Equalization Performance for MWF and Tap Selection based MWF

Figure 5.7: Performance comparison for Minimum Norm (MN) solution and MWF with/without Diagonal Loading (DL) versus number of training symbols (CM2, $n_L = 512$, $R_s = 93.56 \text{ Mbps}$, $n_{MAI} = 3$, $E_b/N_0 = 10 \text{ dB}$, $LNR = 0 \text{ dB}$)

In section 5.2 we saw that the MWF will converge to MN solution in the case when $n_{tr} < n_L$. Figure 5.7 illustrates the BER simulation curves for MN solution with DL and MWF with DL by various amount of training symbols. It is observed that *MWF + DL* performs almost the same as *MN Solution + DL*, since MWF approaches MN solution which lies in the same Krylov subspace when $n_{tr} < n_L$. However, MWF is with reduced complexity compared to MN solution. Comparing *MWF + DL* to *MWF without DL*, it is observed that DL serves as a “soft stop” to improve the

performance of MWF where the rank is simply fixed at $n_{stage} = 8$. Furthermore, the overall equalization performance is consistently improved with the increasing n_{tr} , but the performance improvement becomes saturated after $n_{tr} > 256$.

Considering tap selection based equalization technique in the presence of severe ISI and MAI, number of taps required to be selected can still be large ($n_S \geq n_{eff}$) in order to insure the diversity order for interference mitigation. The complexity for tap selection based MMSE equalization can be further reduced by introducing tap selection based MWF.

Figure 5.8(a) and 5.8(b) illustrate the BER performance comparison for different tap selection methods with MMSE equalization and MWF respectively. It is observed that tap selection based MWF with DL performs similarly as tap selection based MMSE equalization (by LS estimation) with DL, since MWF approaches LS estimation which lies in the same Krylov subspace. As shown by the numerical simulation, the number of stages required for tap selection based MWF ($n_{stage} = 6$) is less than that required for a standard MWF ($n_{stage} = 8$). Improved performance is obtained for tap selection based methods compared to full complexity MMSE equalizer or MWF given very limited number of training symbols (when $n_{tr} \ll n_L$), since tap selection prevents the unreliable path estimation from being used for LS estimation or MWF subspace estimation. Only slight performance degradation is observed for tap selection based methods compared to full complexity systems when n_{tr} approaches n_L . However, tap selection based methods have much reduced computational complexity. Furthermore, DL technique is critical for improving the equalization performance when n_{tr} approaches n_L .

The computational cost for tap selection based MWF is tabulated in Table 5.2. Figure 5.9 illustrate the computational complexity comparison for different tap selection based MMSE equalizer and MWF versus the number of training symbols. It is observed that with more number of taps to be selected ($n_S = 64$), tap selection based MWF is with reduced complexity compared with tap selection based MMSE equalizer with LS estimation.

(a) BER of tap selection methods + MMSE

(b) BER of tap selection methods + MWF

Figure 5.8: Performance comparison for different tap selection methods with MMSE /MWF ($n_S = 64$, $E_b/N_0 = 10dB$, $LNR = 0dB$)

Tap Selection Methods	Computational Cost
	$n_L = 512, n_S = 32, n_{tr} = 64$
<i>Full RAKE</i>	$2n_L n_{tr}$
<i>MWF</i>	$6n_L n_{tr} n_{stage}, n_{stage} = 8$
<i>Heuristic MP + MWF</i>	$2(n_{depth} + 1)n_{tr} n_L + 6n_S n_{tr} n_{stage}, n_{stage} = 6, n_{depth} = 6$
<i>Strongest Paths + MWF</i>	$(2n_L n_{tr} + n_L n_S) + 6n_S n_{tr} n_{stage}, n_{stage} = 6$

Table 5.2: Computational cost for different tap selection methods with MWF

Figure 5.9: Computational complexity (in flops) comparison for different tap selection method based equalization processes (CM2, $R_s = 93.56 \text{ Mbps}$, $n_{MAI} = 3$, $n_S = 64$)

5.5 Summary

In this chapter, a class of tap selection techniques is proposed for rank reduced UWB multipath channel equalization in the presence of ISI and unknown MAI. These tap selection techniques are directly implemented based on training symbol sequence under LS criterion, without the need for explicit channel estimation. Simulation results show that the simple *Strongest Paths/Strongest Projections* based method is desirable under lower SNR level due to its low complexity. *ORLS/MP* based tap selection methods achieve improved performance with increased SNR. In addition, *Heuristic MP* based tap selection method is developed for fast tap selection implementation with only limited performance tradeoff. DL technique is incorporated into the tap selection and equalization processes to insure robustness given limited number of training symbols in the case of practical implementation. The proposed tap selection based equalizer is shown to outperform the conventional uniformly spaced linear equalizer with reduced complexity.

Chapter 6

Antenna Selection in MIMO Systems

Based on Minimum Mean Squared Error (MMSE) criterion, our research work is extended to the topic of antenna selection in Multiple Input Multiple Output (MIMO) systems in chapter 6 and chapter 7,

6.1 Introduction

MIMO wireless communication channels using multiple transmit and receive antennas can be exploited to improve capacity and reliability in fading environments [62]. MIMO spatial multiplexing systems are able to send parallel independent data streams and achieve overall system capacities approximately linearly scaled up with the minimum of the number of transmit and receive antennas [28, 88]. In practical implementation, the main limitation in increasing the number of antennas is not the number of antenna elements, but is related to the high cost of corresponding RF chains for these antennas. Antenna selection technique in MIMO systems reduces the hardware cost and computational complexity while retaining the diversity degree of full complexity systems [58]. Performance analysis and evaluation for Hybrid Selection/Maximum-Ratio-Combining (H-S/MRC), Hybrid Selection/Maximum-Ratio-Transmission (H-S/MRT) and Hybrid Selection/MIMO (H-S/MIMO) have been summarized in detail in [58] for improved diversity and capacity for MIMO systems.

MIMO spatial multiplexing systems using linear receivers, either Zero Forcing (ZF)

or Minimum Mean Squared Error (MMSE) detectors, are practically important due to their low complexity advantage. Compared with an optimal Maximum Likelihood (ML) detector, the use of linear receiver incurs loss of diversity which is critical in independent identically distributed (i.i.d.) Rayleigh fading channels. Antenna selection is a promising method which is capable of achieving high diversity order with low complexity implementation for MIMO systems with linear receivers.

The core idea behind antenna selection is to optimally select a small number of best antennas from a larger set of available antennas. By utilizing low-cost RF switches, only a limited number of RF chains is adaptively switched to a subset of selected antennas which can be identified during the training phase by probing all antennas. The only mechanism for a truly optimal selection is an Exhaustive Search (ES) of all possible combinations based on the criteria such as maximizing the capacity or minimizing the error rate [58]. However, the computational complexity required for such an optimal selection grows exponentially with the total number of available antennas. Various antenna subset selection algorithms have been summarized in [58]. The simplest method is based on the maximization of the power of received signals which is referred as power based selection. However, power based method is not optimal and breaks down when MIMO spatial multiplexing is applied for high capacity. Several transmit antenna selection methods have been proposed in [36] and [42]. For receive antenna selection, a promising approach was proposed by Gorokhov [38] which finds a near optimal subset selection based on the channel capacity maximization.

In this chapter, a fast transmit and/or receive antenna selection technique is developed for MIMO spatial multiplexing systems with low complexity linear receivers under MMSE criterion. This technique is shown to approach the optimal antenna selection performance by ES but with much reduced complexity. In addition, joint transmit and receive antenna selection technique is discussed and shown to achieve improved performance in comparison with the sequential transmit and receive antenna selection process.

6.1.1 MIMO System Model

A MIMO system with n_T transmit antennas and n_R receive antennas under Rayleigh fading channels can be represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (6.1.1)$$

where $\mathbf{y} = [y_1, y_2, \dots, y_{n_R}]^T$, $\mathbf{x} = [x_1, x_2, \dots, x_{n_T}]^T$ and $\mathbf{n} = [n_1, n_2, \dots, n_{n_R}]^T$ denote the received signal sample vector, the transmitted signal symbol vector, and the corresponding AWGN sample vector respectively. $\mathbf{H}$ represents an $n_R \times n_T$ MIMO transmission matrix for the desired MIMO transmitters.

The quasi-static fading channels for different antennas are assumed to be independent identically distributed (i.i.d.) Rayleigh fading. Therefore the entries of $\mathbf{H}$ are modelled as i.i.d. zero-mean, circularly symmetric complex Gaussian random variables with unit variance, i.e., $\mathcal{CN}(0, 1)$. In addition, the AWGN vector $\mathbf{n}$ is assumed to be i.i.d. with covariance of $\sigma_n^2 \mathbf{I}_{n_R}$ and modelled as $\mathcal{CN}(0, \sigma_n)$.

Applying linear MMSE detection to MIMO spatial multiplexing system given in (6.1.1), the diversity order achievable is $(n_R - n_T + 1)$ [64]. This indicates that the minimum number of receive antenna required is $n_R \geq n_T$. In the case when applying antenna selection to choose $n_T^{(s)}$ antenna elements out of n_T transmit antennas and $n_R^{(s)}$ antenna elements out of n_R receive antennas, it is required that $n_R^{(s)} \geq n_T^{(s)}$.

6.1.2 Performance of Antenna Selection

The performance for MIMO systems with antenna selection has been summarized in detail in [58]. For either diversity systems or spatial multiplexing systems, making antenna selection is equivalent to choosing a sub-channel-matrix to maximize the effective SNR or the channel capacity respectively. For spatial multiplexing systems, the capacity is linearly proportional to $\min(n_R^{(s)}, n_T^{(s)})$. Any increase of n_R and/or n_T while keeping the $\min(n_R^{(s)}, n_T^{(s)})$ fixed only increases the system diversity. It has been shown in [58] that antenna selection retains the diversity degree compared to the full-complexity system, for both linear diversity systems with complete channel knowledge and space-time coded systems.

In following discussion, we focus on the antenna selection algorithms for MIMO spatial multiplexing systems with linear MMSE receivers.

6.2 Fast Antenna Selection Algorithms Assuming Channel State Information

We assume that the Channel State Information (CSI) is known only at the receiver end. Using this information, either transmit or receive antenna subset can be selected based on criterion of minimizing the MMSE or maximizing the minimum Signal to Interference and Noise Ratio (SINR) stream. Both transmit and receive antenna selection processes are to be performed at the receiver end. For transmit antenna selection, the selected subset of transmit antennas is required to be fed back to the transmitter end with a low rate feedback channel.

6.2.1 Antenna Selection Criteria

Given MIMO system model in (6.1.1) and linear MMSE detection as $\hat{\mathbf{x}} = \mathbf{F}\mathbf{y}$, the Mean Square Error (MSE) can be calculated by

$$\mathcal{E}_{MSE} = \text{trace}\{E[(\mathbf{x} - \mathbf{F}\mathbf{y})(\mathbf{x} - \mathbf{F}\mathbf{y})^H]\} \quad (6.2.1)$$

Assume $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_{n_T}$, $E\{\mathbf{n}\mathbf{n}^H\} = \mathbf{R}_n$ and $E\{\mathbf{x}\mathbf{n}^H\} = \mathbf{0}$. Then, we have

$$\mathcal{E}_{MSE} = \text{trace}\{\mathbf{I}_{n_T} - \mathbf{F}\mathbf{H} - \mathbf{H}^H\mathbf{F}^H + \mathbf{F}(\mathbf{H}\mathbf{H}^H + \mathbf{R}_n)\mathbf{F}^H\} \quad (6.2.2)$$

By setting $\frac{\partial \mathcal{E}_{MSE}}{\partial \mathbf{F}} = 0$, we obtain,

$$\begin{aligned} \mathbf{F} &= \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \mathbf{R}_n)^{-1} \\ &= (\mathbf{I}_{n_T} + \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H})^{-1}\mathbf{H}^H\mathbf{R}_n^{-1} \end{aligned} \quad (6.2.3)$$

and

$$\hat{\mathbf{x}}_{MMSE} = (\mathbf{I}_{n_T} + \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H})^{-1}\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{y} \quad (6.2.4)$$

Then, we have

$$\begin{aligned} \mathcal{E}_{MMSE} &= \text{trace}\{E[(\mathbf{x} - \hat{\mathbf{x}}_{MMSE})(\mathbf{x} - \hat{\mathbf{x}}_{MMSE})^H]\} \\ &= \text{trace}\{(\mathbf{I}_{n_T} + \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H})^{-1}\} \end{aligned} \quad (6.2.5)$$

Assuming uncorrelated AWGN vector with $\mathbf{R}_n = \sigma_n^2 \mathbf{I}_{n_R}$, we obtain

$$\mathcal{E}_{MMSE} = \text{trace}\left\{\left(\mathbf{I}_{n_T} + \frac{1}{\sigma_n^2} \mathbf{H}^H \mathbf{H}\right)^{-1}\right\} \quad (6.2.6)$$

The SINR for MMSE detection for the k -th stream is given by [18]

$$\begin{aligned} SINR_k^{(MMSE)} &= \frac{1}{MMSE_k} - 1 \\ &= \frac{1}{\sigma_n^2 \{(\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{n_T})^{-1}\}_{k,k}} - 1 \end{aligned} \quad (6.2.7)$$

The transmit and/or receive antenna selection can be considered as forming a submatrix $\mathbf{H}_s$ by choosing the appropriate columns and/or rows of the channel matrix $\mathbf{H}$. Based on MMSE detection, there are two criteria that can be applied to antenna selection as described below.

Suppose that joint MMSE detection assumes joint encoding of transmit streams and MMSE combining of these streams at receiver end prior to decoding [38], *Criterion 1* is assumed to minimize the MMSE as given in (6.2.6), i.e.,

$$\begin{aligned} \mathcal{E}_{MMSE}\{\mathbf{H}_s\} &= \text{trace}\left\{\left(\mathbf{I}_{n_T} + \frac{1}{\sigma_n^2} \mathbf{H}_s^H \mathbf{H}_s\right)^{-1}\right\} \\ &= \sum_{i=1}^{n_T} \lambda_i \left\{ \left(\mathbf{I}_{n_T} + \frac{1}{\sigma_n^2} \mathbf{H}_s^H \mathbf{H}_s\right)^{-1} \right\} \\ &= \sum_{i=1}^{n_T} \frac{1}{\lambda_i \{\mathbf{H}_s^H \mathbf{H}_s\} + \sigma_n^2} \end{aligned} \quad (6.2.8)$$

where $\lambda_i\{\mathbf{H}_s^H \mathbf{H}_s\}$ denotes the i -th eigenvalue of $\mathbf{H}_s^H \mathbf{H}_s$.

In terms of minimizing the vector symbol error rate as described in [42], *Criterion 2* is assumed to maximize the $\min_k SINR_k^{(MMSE)}$. Similarly, following the process in [42], we have

$$\begin{aligned} \min_k SINR_k^{(MMSE)} &= \frac{1}{\sigma_n^2 \max_k \{(\mathbf{H}_s^H \mathbf{H}_s + \sigma_n^2 \mathbf{I}_{n_T})^{-1}\}_{k,k}} - 1 \end{aligned} \quad (6.2.9)$$

Following from the Rayleigh-Ritz theorem [46], we have

$$\begin{aligned}
& \max_k \{(\mathbf{H}_s^H \mathbf{H}_s + \sigma_n^2 \mathbf{I}_{n_T})^{-1}\}_{k,k} \\
&= \max_k \mathbf{e}_k^H (\mathbf{H}_s^H \mathbf{H}_s + \sigma_n^2 \mathbf{I}_{n_T})^{-1} \mathbf{e}_k \\
&\leq \max_{\mathbf{a}^H \mathbf{a} = 1} \mathbf{a}_k^H (\mathbf{H}_s^H \mathbf{H}_s + \sigma_n^2 \mathbf{I}_{n_T})^{-1} \mathbf{a}_k \\
&= \lambda_{\max}\{(\mathbf{H}_s^H \mathbf{H}_s + \sigma_n^2 \mathbf{I}_{n_T})^{-1}\} \\
&= \frac{1}{\lambda_{\min}\{\mathbf{H}_s^H \mathbf{H}_s\} + \sigma_n^2}
\end{aligned} \tag{6.2.10}$$

where $\mathbf{e}_k$ is the k -th column of the identity matrix $\mathbf{I}_{n_T}$. The representation of $\lambda_{\max}\{(\mathbf{H}_s^H \mathbf{H}_s + \sigma_n^2 \mathbf{I}_{n_T})^{-1}\}$ denotes the maximum eigenvalue of matrix $(\mathbf{H}_s^H \mathbf{H}_s + \sigma_n^2 \mathbf{I}_{n_T})^{-1}$ and $\lambda_{\min}\{\mathbf{H}_s^H \mathbf{H}_s\}$ denotes the minimum eigenvalue of matrix $(\mathbf{H}_s^H \mathbf{H}_s)$. Hence, *Criterion 2* approximately maximizes the minimum eigenvalue $\lambda_{\min}\{\mathbf{H}_s^H \mathbf{H}_s\}$.

Instead of using the ES and computing the $\mathcal{E}_{MMSE}\{\mathbf{H}_s\}$ given in (6.2.8) or $\lambda_{\min}\{\mathbf{H}_s^H \mathbf{H}_s\}$ for each possible combination of the submatrix $\mathbf{H}_s$, fast algorithms utilizing Greedy Algorithm (GA) are proposed for reduced computational complexity but with only limited performance degradation. For simplicity, only *Criterion 1* that minimizes the MMSE is applied in the following algorithms. *Criterion 2* that maximizes the SINR of the stream with minimum SINR can be applied as well by directly computing (6.2.9) rather than (6.2.10).

6.2.2 Fast Algorithm for Receive Antenna Selection

With matrix inversion lemma given in [41], the matrix inversion can be computed utilizing iterative algorithm as follows.

$$\begin{aligned}
\mathbf{A}_{n\pm 1} &= (\mathbf{A}_n^{-1} \pm \mathbf{a}_n \mathbf{a}_n^H)^{-1} \\
&= \mathbf{A}_n - \mathbf{A}_n \mathbf{a}_n (\pm 1 + \mathbf{a}_n^H \mathbf{A}_n \mathbf{a}_n)^{-1} \mathbf{a}_n^H \mathbf{A}_n
\end{aligned} \tag{6.2.11}$$

where, the computational intensive part comes from the vector and matrix multiplication term of $\mathbf{A}_n \mathbf{a}_n$. This can be effectively reduced by defining $\mathbf{b}_n = \mathbf{A}_n \mathbf{a}_n$ and

obtaining the iterative formula for updating $\mathbf{b}_{n\pm 1}$ as follows,

$$\begin{aligned}\mathbf{b}_{n\pm 1} &= \mathbf{A}_{n\pm 1}\mathbf{a}_n \\ &= \mathbf{A}_n\mathbf{a}_n - \mathbf{A}_n\mathbf{a}_n(\pm 1 + \mathbf{a}_n^H \mathbf{A}_n \mathbf{a}_n)^{-1} \mathbf{a}_n^H \mathbf{A}_n \mathbf{a}_n \\ &= \mathbf{b}_n - \mathbf{b}_n(\pm 1 + \mathbf{a}_n^H \mathbf{b}_n)^{-1} \mathbf{a}_n^H \mathbf{b}_n\end{aligned}\tag{6.2.12}$$

Therefore, the computational cost ($\mathcal{O}(n^2)$) for computing $\mathbf{A}_n\mathbf{a}_n$ is reduced to ($\mathcal{O}(n)$) for computing $\mathbf{a}_n^H \mathbf{b}_n$ with some extra storage requirement for $\mathbf{b}_n$.

Incremental GA can be applied for receive antenna selection to choose $n_R^{(s)}$ rows out of channel transmission matrix $\mathbf{H}_{n_R \times n_T}$ one by one. Similarly, decremental GA can be applied to remove $(n_R - n_R^{(s)})$ rows from $\mathbf{H}$ one by one. Let $\mathbf{H}_{n+1} = \begin{pmatrix} \mathbf{H}_n \\ \mathbf{h}_k^H \end{pmatrix}$, we have

$$(\mathbf{H}_{n+1}^H \mathbf{H}_{n+1} + \sigma_n^2 \mathbf{I}) = (\mathbf{H}_n^H \mathbf{H}_n + \sigma_n^2 \mathbf{I}) + \mathbf{h}_k \mathbf{h}_k^H \tag{6.2.13}$$

or

$$(\mathbf{H}_n^H \mathbf{H}_n + \sigma_n^2 \mathbf{I}) = (\mathbf{H}_{n+1}^H \mathbf{H}_{n+1} + \sigma_n^2 \mathbf{I}) - \mathbf{h}_k \mathbf{h}_k^H \tag{6.2.14}$$

Given as an example, the decremental iterative algorithm for receive antenna selection by minimizing the MMSE is detailed as follows.

Decremental GA Based Receive Antenna Selection (Decremental GA)

- Initialize by setting $n = n_R$ and $\mathbf{H}_{n_R} = \mathbf{H}$

- Set $n = n - 1$.

Determine $\mathbf{H}_n = (\mathbf{H}_{n+1})_{\bar{k}}$ by choosing $s = \arg \max_k \text{trace}\{\mathbf{A}_n\}$

where $(\mathbf{H}_{n+1})_{\bar{k}}$ denotes $\mathbf{H}_{n+1}$ with the k -th row $(\mathbf{h}_k^H)$ removed,

$k \in$ all remaining row indices for $\mathbf{H}_{n+1}$ and

$$\mathbf{A}_n = (\mathbf{I}_{n_T} + \frac{1}{\sigma_n^2} \mathbf{H}_n^H \mathbf{H}_n)^{-1}$$

The matrix inversion term $\mathbf{A}_n$ can be computed iteratively by utilizing (6.2.14), (6.2.11) and (6.2.12)

- Continue the iteration until $n = n_R^{(s)}$

The computational complexity of the above algorithm is on the order of $\mathcal{O}(n_T n_R (n_R - n_R^{(s)}))$. Based on the numerical simulation that assumes perfect CSI, decremental algorithm is shown to approach the optimal antenna selection performance by ES. However, the incremental algorithm results in slightly more performance degradation.

6.2.3 Fast Algorithm for Transmit Antenna Selection

Consider the antenna selection criteria given in (6.2.8) and (6.2.10) in section 6.2.1. Both criteria have symmetry representations with respect to $\mathbf{H}_s$ and $\mathbf{H}_s^H$. This indicates that the transmit and receive antenna selection is reciprocal of each other. So the transmit antenna selection can be implemented with the similar algorithm for receive antenna selection assuming its Hermitian transpose of the channel transmission matrix $\mathbf{H}^H$ instead of $\mathbf{H}$. Similar observation has been pointed out in [37] which assumes the criterion for antenna selection to maximize the MIMO channel capacity given by

$$\begin{aligned} C(\mathbf{H}_s) &= \log_2 \det(\mathbf{I}_{n_T^{(s)}} + \frac{E_s}{N_0} \mathbf{H}_s^H \mathbf{H}_s) \\ &= \log_2 \det(\mathbf{I}_{n_R^{(s)}} + \frac{E_s}{N_0} \mathbf{H}_s \mathbf{H}_s^H) \end{aligned} \quad (6.2.15)$$

6.2.4 Joint Transmit and Receive Antenna Selection

In the case when both transmit and receive antennas are to be selected, the simplest method is the sequential selection procedure, i.e., select a transmit antenna subset first, then select a receive antenna subset, or vice versa. A joint transmit and receive antenna selection method by GA where incremental algorithm is applied to transmit antenna selection and decremental algorithm is applied to receive antenna selection is proposed as follows. This joint selection method is shown to achieve improved performance compared to the sequential selection method by numerical simulation.

Joint Transmit and Receive Antenna Selection

- Initialize by setting $n = 0$, $\mathbf{H}_0 = []$

- Set $n = n + 1$
- For each $k \in$ all the unselected column indices for $\mathbf{H}$
 Let $n_T^{(n,k)} = n$, $\mathbf{H}^{(n,k)} = \mathbf{H}_n = [\mathbf{H}_{n-1}, \mathbf{h}_k]$
 where superscript (n, k) is referred to the k -th trial of the n -th iteration.
 Call *Decremental GA* given in section 6.2.2 to select $n_R^{(s)}$ elements (denoted as $Rx_selected^{(n,k)}$) out of total n_R receive antennas
- Choose $s = \arg \max_k trace(\mathbf{A}_n)$,
 where $\mathbf{A}_n = (\mathbf{I}_{n_T} + \frac{1}{\sigma_n^2} \mathbf{H}_n^H \mathbf{H}_n)^{-1}$ has been computed at the last iteration of *Decremental GA*
- Set $Tx_selected^{(n)} = [Tx_selected^{(n-1)}, s]$ and $Rx_selected^{(n)} = Rx_selected^{(n,s)}$
- Continue the iteration until $n = n_T^{(s)}$.

The *Joint Transmit and Receive Antenna Selection* procedure has computational complexity on the order of $\mathcal{O}(n_T(n_T^{(s)})^2 n_R(n_R - n_R^{(s)}))$. This is much lower than the ES based transmit and receive antenna selection procedure that has computational complexity on the order of $\mathcal{O}((n_T^{(s)})^2 n_R^{(s)}) \times \binom{n_T}{n_T^{(s)}} \times \binom{n_R}{n_R^{(s)}}$. The simple sequential transmit and receive antenna selection has the lowest computational complexity on the order of $\mathcal{O}(n_T n_R n_T^{(s)}) + \mathcal{O}(n_T^{(s)} n_R(n_R - n_R^{(s)}))$.

Suppose that the total number of transmit and receive antennas, i.e., $n_{total} = n_T + n_R$, is kept unchanged. Equally distributing the antennas to transmitter and receiver side will result in maximum number of possible transmission channels for antenna selection, i.e., $n_T \times n_R \geq \frac{1}{4} n_{total}^2$ where equality holds when $n_T = n_R = \frac{1}{2} n_{total}$. Assuming these $n_T \times n_R$ channels are i.i.d. Raleigh fading, higher order of diversity or better performance is expected by implementing joint antenna selection at both transmitter and receiver end.

6.3 Simulation Results and Discussions

In this section, performance of the proposed fast antenna selection methods is evaluated via Monte Carlo simulations. The transmitted symbols are modulated using 16-QAM constellation and assumed to be i.i.d. for all n_T desired transmitters. System performance is measured in terms of Symbol Error Rate (SER) for a frame of 128 symbols for each transmitter and averaged over 5000 Rayleigh fading MIMO channel realizations. Assuming CSI ($\mathbf{H}$) is known at the receiver end, the proposed method selects $n_R^{(s)}$ elements out of total n_R receive antennas and/or $n_T^{(s)}$ elements out of total n_T transmit antennas.

Figure 6.1: Performance comparison for receive antenna selection with different n_R (assuming CSI)

Figure 6.1 illustrates the SER performance evaluation of the proposed *Decremental GA Based Receive Antenna Selection* method with different total number of receive antennas (n_R). The SER curves with antenna selection ($n_R^{(s)}/n_R = 4/6 \sim 12$) are shown to outperform the simple system without antenna selection ($n_T = 4, n_R = 4$)

significantly. The SER curve with antenna selection ($n_R^{(s)}/n_R = 4/12$) is shown to approach the diversity order of a full complexity system ($n_T = 4, n_R = 12$), where the diversity order is understood as the slope of the SER curves. Compared with ES based antenna selection method ($n_R^{(s)}/n_R = 4/12, ES$), *Decremental GA Based Receive Antenna Selection* method ($n_R^{(s)}/n_R = 4/12$) exhibits only very limited performance degradation but significantly benefits from reduced computational complexity.

Figure 6.2: Performance comparison for joint transmit and receive antenna selection with different n_T and n_R (assuming CSI)

Figure 6.2 illustrates the SER performance evaluation for the proposed *Joint Transmit and Receive Antenna Selection* method. It is observed that better performance is obtained by implementing joint antenna selection at both transmitter and receiver end compared with the performance by single receive antenna selection as shown in Figure 6.1, assuming that total number of transmit and receive antennas is kept unchanged ($n_{total} = n_T + n_R = 16$). This has been discussed in

section 6.2.4. If more antennas are available for selection, higher performance improvement is expected for *Joint Transmit and Receive Antenna Selection* method as compared with the performance by simple sequential transmit and receive antenna selection procedure (curves shown in dashed line). Compared with ES based method ($n_T^{(s)}/n_T = n_R^{(s)}/n_R = 4/8$, *ES*), GA based method ($n_T^{(s)}/n_T = n_R^{(s)}/n_R = 4/8$, *Joint*) exhibits only limited performance degradation.

6.4 Summary

In this chapter, fast transmit and receive antenna selection techniques based on GA are proposed under MMSE criterion for MIMO spatial multiplexing systems with linear receivers. It is shown that transmit and receive antenna selections are reciprocal under MMSE criterion. Further, joint transmit and receive antenna selection technique is discussed and shown to achieve improved performance as compared with either single transmit or single receive antenna selection when the total number of available antennas is the same.

So far, the antenna selection techniques are discussed assuming perfect CSI at receiver. In chapter 7, a novel antenna selection technique is proposed based on training sample sequence without the need for CSI.

Chapter 7

Least Squares Based Antenna Selection for MIMO Systems

As discussed in chapter 6, most of the prior works for antenna selection assume perfect Channel State Information (CSI) at the receiver end. In practical implementation, channel estimation becomes a pre-requisite for antenna selection procedure. Accurate channel estimation is not an easy task in the presence of unknown co-channel interference. Inaccurate channel estimation will inevitably affect antenna selection performance.

In this chapter, novel transmit and receive antenna selection techniques based on Least Squares (LS) criterion are proposed for Multiple Input Multiple Output (MIMO) spatial multiplexing systems with linear MMSE receivers. Unlike conventional approaches, the proposed methods directly implement the antenna selection algorithms based on training sample sequence without the need for explicit channel estimation. As a result, the proposed methods are shown to retain the diversity benefit for antenna selection in MIMO systems in the presence of unknown Multiple Access Interference (MAI) which has not been discussed before to the best of our knowledge. Further, LS based receive antenna selection technique is extended to UWB systems with multiple receive antenna elements to achieve improved system performance under low power transmission by mitigating the channel shadowing effect.

In following sections, LS based antenna selection algorithms are discussed for receive antenna selection, transmit antenna selection, as well as joint transmit and

receive antenna selection. Numerical simulation results are obtained to show the performance benefit by the proposed antenna selection algorithms in the presence of unknown co-channel interference.

7.1 Least Squares Based Receive Antenna Selection

7.1.1 MIMO System Model and Least Squares Estimation

As discussed in section 6.1.1, a MIMO system with n_T transmit antennas and n_R receive antennas in the presence of n_{MAI} MAI transmitters is represented by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{H}^{(MAI)}\mathbf{x}^{(MAI)} + \mathbf{n} \quad (7.1.1)$$

where $\mathbf{y} = [y_1, y_2, \dots, y_{n_R}]^T$, $\mathbf{x} = [x_1, x_2, \dots, x_{n_T}]^T$, $\mathbf{x}^{(MAI)} = [x_1^{(MAI)}, x_2^{(MAI)}, \dots, x_{n_{MAI}}^{(MAI)}]^T$ and $\mathbf{n} = [n_1, n_2, \dots, n_{n_R}]^T$ denote the received signal sample vector, the transmitted signal symbol vector, the transmitted MAI signal symbol vector, and the corresponding AWGN sample vector respectively. $\mathbf{H}$ represents an $n_R \times n_T$ channel matrix for the desired MIMO transmitters and $\mathbf{H}^{(MAI)}$ is $n_R \times n_{MAI}$ for MAI transmitters. For simplicity, MAI transmitters with Space Division Multiple Access (SDMA) are considered without using the spreading sequence. These MAI transmitters are assumed to be unknown at the receiver end. In practice, other sources of unknown interference can be considered as well.

The quasi-static fading channels for different antennas are assumed to be i.i.d. distributed Rayleigh fading. Therefore, the entries of $\mathbf{H}$ and $\mathbf{H}^{(MAI)}$ are modelled as i.i.d. zero-mean, circularly symmetric complex Gaussian random variables with unit variance, i.e., $\mathcal{CN}(0, 1)$. In addition, the AWGN vector $\mathbf{n}$ is assumed to be i.i.d. with covariance of $\sigma_n^2 \mathbf{I}_{n_R}$ and modelled as $\mathcal{CN}(0, \sigma_n)$.

Applying linear detection to (7.5.1), the diversity order achievable is $(n_R - (n_T + n_{MAI}) + 1)$ [64]. This indicates that the minimum number of receive antenna required is $n_R \geq n_T + n_{MAI}$. In the case of choosing $n_R^{(s)}$ elements out of n_R receive antennas and $n_T^{(s)}$ elements out of n_T transmit antennas, the requirement becomes $n_R^{(s)} \geq n_T^{(s)} + n_{MAI}$.

Assuming that the receiver does not have the knowledge of the MAI transmitters, blind channel estimation in the presence of unknown interferences is a hard task. However, with training symbol vector sequence $\mathbf{X}_{tr} = [\mathbf{x}(1), \mathbf{x}(2), \dots, \mathbf{x}(n_{tr})]^T$ for the desired MIMO transmitters where $\mathbf{x}(k) = [x_1(k), x_2(k), \dots, x_{n_T}(k)]^T$ and n_{tr} denotes the number of training symbols at each transmitter, the system model for Least Squares (LS) estimation [41] can be formulated as

$$\mathbf{X}_{tr} = \mathbf{Y}_{tr} \mathbf{F} \quad (7.1.2)$$

where $\mathbf{Y}_{tr} = [\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(n_{tr})]^T$ and $\mathbf{y}(k) = [y_1(k), y_2(k), \dots, y_{n_R}(k)]^T$ is the training sample vector sequence observed at receiver end. Matrix $\mathbf{F} = [\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_{n_T}]$ where $\mathbf{f}_i = [f_{1i}, f_{2i}, \dots, f_{n_R i}]^T$ denotes the linear equalization filter banks.

7.1.2 Receive Antenna Selection for Overdetermined Systems

Suppose that $n_{tr} \geq n_R$, equation (7.1.2) represents an overdetermined system. The LS estimation can be extended to solve (7.1.2) for multiple dimension input vector sequence. Under LS criterion to minimize the estimation error

$$\mathcal{E}_{LS} = \|\mathbf{X}_{tr} - \mathbf{Y}_{tr} \mathbf{F}\|_F^2 = \text{trace}\{(\mathbf{X}_{tr} - \mathbf{Y}_{tr} \mathbf{F})(\mathbf{X}_{tr} - \mathbf{Y}_{tr} \mathbf{F})^H\} \quad (7.1.3)$$

where $\|\cdot\|_F$ denotes the Frobenius norm [35] and $\text{trace}(\cdot)$ denotes the trace of a square matrix (the sum of the diagonal elements). The LS estimation for the filter banks is given by

$$\hat{\mathbf{F}} = (\mathbf{Y}_{tr}^H \mathbf{Y}_{tr})^{-1} \mathbf{Y}_{tr}^H \mathbf{X}_{tr} \quad (7.1.4)$$

Let $\hat{\mathbf{X}} = \mathbf{Y}_{tr} \hat{\mathbf{F}}$. The LS estimation error can be derived as

$$\mathcal{E}_{LS} = \|\mathbf{X}_{tr} - \hat{\mathbf{X}}\|_F^2 = \text{trace}(\mathbf{X}_{tr}^H \mathbf{X}_{tr}) - \text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}} \mathbf{X}_{tr}) \quad (7.1.5)$$

where $\mathbf{P}_{\mathbf{Y}}$ is the projection matrix [41] given by

$$\mathbf{P}_{\mathbf{Y}} = \mathbf{Y}_{tr} (\mathbf{Y}_{tr}^H \mathbf{Y}_{tr})^{-1} \mathbf{Y}_{tr}^H \quad (7.1.6)$$

Based on linear system model given in (7.1.2), receive antenna selection can be considered as forming an observation sample matrix $\mathbf{Y}_S$ by selecting a subset of

columns from $\mathbf{Y}_{tr}$ which corresponds to a subset of receive antennas. The criterion is to maximize the projection power given as

$$\|\mathbf{P}_{\mathbf{Y}_S} \mathbf{X}_{tr}\|_F^2 = \text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_S}^H \mathbf{P}_{\mathbf{Y}_S} \mathbf{X}_{tr}) = \text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_S} \mathbf{X}_{tr}) \quad (7.1.7)$$

or equivalently to minimize the LS estimation error given by

$$\mathcal{E}_{\mathbf{Y}_S} = \text{trace}(\mathbf{X}_{tr}^H \mathbf{X}_{tr}) - \text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_S} \mathbf{X}_{tr}) \quad (7.1.8)$$

where

$$\mathbf{P}_{\mathbf{Y}_S} = \mathbf{Y}_S (\mathbf{Y}_S^H \mathbf{Y}_S)^{-1} \mathbf{Y}_S^H \quad (7.1.9)$$

For optimal selection of $n_R^{(s)}$ antennas out of a total of n_R receive antennas, an Exhaustive Search (ES) is needed and this requires $\mathbf{P}_{\mathbf{Y}_S}$ to be evaluated for $\binom{n_R}{n_R^{(s)}}$ possible combinations of the antenna subsets. This may not be feasible for large n_R . The locally optimal antenna subset selection may be achieved by applying Greedy Algorithm (GA) to select antennas one by one with minimized $\mathcal{E}_{\mathbf{Y}_S}$. The GA based iterative antenna selection procedure has been shown to achieve near optimal performance as compared with the optimal selection by ES in [38] where perfect CSI is assumed at receiver end. In view of the order of diversity achievable by MIMO systems with linear receivers, decremental selection approach is preferred in the case of ($n_T > 1$) since the minimum order of diversity will not be met for the first ($n_T - 1$) receive antennas selection steps if incremental selection approach is applied.

Therefore, the proposed receive antenna selection method utilizing Backward Greedy Algorithm (BGA) under LS criterion is detailed as follows.

LS BGA Based Receive Antenna Selection (LS BGA):

- Initialize by setting $n = n_R$ and $\mathbf{Y}_{n_R} = \mathbf{Y}_{tr} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{n_R}]$

- Set $n = n - 1$.

Determine $\mathbf{Y}_n = (\mathbf{Y}_{n+1})_{\bar{k}}$ by choosing $s = \arg \max_k \text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_n} \mathbf{X}_{tr})$, where $(\mathbf{Y}_{n+1})_{\bar{k}}$ denotes $\mathbf{Y}_{n+1}$ with the k -th column removed,

$\mathbf{P}_{\mathbf{Y}_n} = \mathbf{Y}_n (\mathbf{Y}_n^H \mathbf{Y}_n)^{-1} \mathbf{Y}_n^H$ and $k \in$ all remaining column indices for $\mathbf{Y}_{n+1}$

- Continue the iteration until $n = n_R^{(s)}$.

7.1.3 Receive Antenna Selection for Underdetermined Systems

In the case when only very limited training samples are available, i.e., $n_{tr} < n_R$, equation (7.1.2) represents an underdetermined system. The LS estimation error $\mathcal{E}_{\mathbf{Y}_S}$ will become zero as long as $\mathbf{Y}_S$ is a fat matrix where assuming $\mathbf{Y}_S$ has full row rank. The covariance matrix $\mathbf{Y}_S^H \mathbf{Y}_S$ becomes singular and *LS BGA* based method breaks down. Nevertheless, from (7.5.1) and (7.1.4), the estimation result for linear detection is given as

$$\hat{\mathbf{x}} = \hat{\mathbf{F}}^T \mathbf{y} = \hat{\mathbf{F}}^T \mathbf{H} \mathbf{x} + \hat{\mathbf{F}}^T \mathbf{H}^{(MAI)} \mathbf{x}^{(MAI)} + \hat{\mathbf{F}}^T \mathbf{n} \quad (7.1.10)$$

The last term on the right hand side of (7.1.10) that represents the signal estimation error due to AWGN signal has the variance of $\|\hat{\mathbf{F}}\|_F^2 \sigma_n^2$. Considering the effect of possible noise amplification by $\|\hat{\mathbf{F}}\|_F^2$, a minimum Frobenius norm solution can be applied to this underdetermined system, i.e.,

$$\min_{\mathbf{F}} \|\mathbf{F}\|_F^2 \text{ subject to } \mathbf{Y}_S \mathbf{F} = \mathbf{X}_{tr} \quad (7.1.11)$$

Using the method of Lagrange multipliers [41], the solution for $\mathbf{F}$ is simply the pseudoinverse [41] given as follows

$$\hat{\mathbf{F}} = \mathbf{Y}_S^H (\mathbf{Y}_S \mathbf{Y}_S^H)^{-1} \mathbf{X}_{tr} \quad (7.1.12)$$

Therefore, an iterative decremental approach can be applied to remove the columns from $\mathbf{Y}_{tr}$ one by one for minimized Frobenius norm until the remaining number of columns in $\mathbf{Y}_n$ equals to n_{tr} . Then, *LS BGA* based method can be applied to continue the decremental selecting procedure to obtain the final subset of selected antennas assuming $n_R^{(s)} < n_{tr}$. The detailed procedure for this Minimum Norm (MN) based receive antenna selection method utilizing BGA is described as follows.

MN BGA Based Receive Antenna Selection Algorithm (MN + LS BGA) (assuming $n_S \leq n_{tr} < n_R$)

- Initialize by setting $n = n_R$ and $\mathbf{Y}_{n_R} = \mathbf{Y}_{tr} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{n_R}]$

- Set $n = n - 1$.
Determine $\mathbf{Y}_n = (\mathbf{Y}_{n+1})_{\bar{k}}$ by choosing $s = \arg \min_k \|\hat{\mathbf{F}}_n\|_F^2$,
where $(\mathbf{Y}_{n+1})_{\bar{k}}$ denotes $\mathbf{Y}_{n+1}$ with the k -th column removed,
 $\hat{\mathbf{F}}_n = \mathbf{Y}_n^H (\mathbf{Y}_n \mathbf{Y}_n^H)^{-1} \mathbf{X}_{tr}$ and $k \in$ all remaining column indices for $\mathbf{Y}_{n+1}$
- Continue the iteration until $n = n_{tr}$ (if $n_R^{(s)} = n_{tr}$ then stop here)
- Start from $n = n_{tr}$, apply *LS BGA Based Receive Antenna Selection* method to continue the decremental selection until $n = n_R^{(s)}$.

7.1.4 Receive Antenna Selection with Diagonal Loading

In the case of practical implementation, the mismatch of sample covariance matrix under finite training sample support may result in inaccurate estimation for filter bank coefficients. From (7.1.10), inaccurately estimated large coefficient matrix norm $\hat{\mathbf{F}}$ will amplify the AWGN term with variance of $\|\hat{\mathbf{F}}\|_F^2 \sigma_n^2$ and degrade the system performance. This suggests the use of Quadratic Constraint (QC), i.e., $\|\mathbf{F}\|_F^2 = f_0$ where f_0 is a constraining value, for minimizing LS estimation error during the antenna selection process. Then, we have

$$\min_{\mathbf{F}} \|\mathbf{X}_{tr} - \hat{\mathbf{X}}\|_F^2 \text{ subject to } \|\mathbf{F}\|_F^2 = f_0 \quad (7.1.13)$$

where $\hat{\mathbf{X}} = \mathbf{Y}_S \hat{\mathbf{F}}$. Using the method of Lagrange multipliers [41], the new cost function for minimization under QC is given by

$$\mathcal{E}_{\mathbf{Y}_S}^{(QC)} = \text{trace}\{(\mathbf{X}_{tr} - \mathbf{Y}_S \mathbf{F})(\mathbf{X}_{tr} - \mathbf{Y}_S \mathbf{F})^H\} + \lambda (\text{trace}(\mathbf{F} \mathbf{F}^H) - f_0) \quad (7.1.14)$$

where λ is a Lagrange multiplier. This can be solved for $\mathbf{F}$ as follows,

$$\hat{\mathbf{F}}^{(QC)} = (\mathbf{Y}_S^H \mathbf{Y}_S + \lambda \mathbf{I})^{-1} \mathbf{Y}_S^H \mathbf{X}_{tr} \quad (7.1.15)$$

As discussed in section 5.1.2, this can be regarded as introducing Diagonal Loading (DL) [15] [80] technique into LS based antenna selection process.

Let $\lambda = n_{tr}\sigma_L^2$, where σ_L^2 denotes the loading level. Defining Loading-to-Noise Ratio (LNR) as

$$LNR = \frac{\sigma_L^2}{\sigma_n^2} \quad (7.1.16)$$

the diagonally loaded Wiener filter $\hat{\mathbf{F}}^{(DL)}$ can be written as

$$\hat{\mathbf{F}}^{(DL)} = (\mathbf{Y}_S^H \mathbf{Y}_S + n_{tr}\sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_S^H \mathbf{X}_{tr} = \tilde{\mathbf{R}}_{\mathbf{Y}}^{-1} \hat{\mathbf{R}}_{\mathbf{YX}} \quad (7.1.17)$$

where sample covariance matrix for $\mathbf{Y}_S$ is defined as $\hat{\mathbf{R}}_{\mathbf{Y}} = \frac{1}{n_{tr}} \mathbf{Y}_S^H \mathbf{Y}_S$ and cross-covariance matrix for $\mathbf{Y}_S$ and $\mathbf{X}_{tr}$ is defined as $\hat{\mathbf{R}}_{\mathbf{YX}} = \frac{1}{n_{tr}} \mathbf{Y}_S^H \mathbf{X}_{tr}$. Therefore, DL is equivalent to diagonally loading into the sample covariance matrix by setting $\tilde{\mathbf{R}}_{\mathbf{Y}} = \hat{\mathbf{R}}_{\mathbf{Y}} + \sigma_L^2 \mathbf{I}$.

In (7.1.14), discarding the last term λf_0 which is a constant, the cost function using DL becomes,

$$\mathcal{E}_{\mathbf{Y}_S}^{(DL)} = \text{trace}(\mathbf{X}_{tr}^H \mathbf{X}_{tr}) - \text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_S}^{(DL)} \mathbf{X}_{tr}) \quad (7.1.18)$$

where $\mathbf{P}_{\mathbf{Y}_S}^{(DL)}$ is given by

$$\mathbf{P}_{\mathbf{Y}_S}^{(DL)} = \mathbf{Y}_S (\mathbf{Y}_S^H \mathbf{Y}_S + n_{tr}\sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_S^H \quad (7.1.19)$$

For simplicity, LNR can be determined empirically. Choosing an appropriate loading level for σ_L^2 (usually is a few *dBs* higher than σ_n^2) is important for the algorithm to converge to a reasonably good result and to mitigate the sample covariance matrix mismatch problem. Over loading will result in unacceptable performance.

With the introduction of DL, the criterion for receive antenna selection is to minimize $\mathcal{E}_{\mathbf{Y}_S}^{(DL)}$ given in (7.1.18), or equivalently to maximize the projection term $\text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_S}^{(DL)} \mathbf{X}_{tr})$. DL has the effect of eliminating the influence of poorly estimated small eigenvalues and associated eigenvectors for the covariance matrix of $\mathbf{Y}_S$. This can be interpreted as follows.

Assume Singular Value Decomposition (SVD) for $\mathbf{Y}_S = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H$ where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{\Sigma}$ is a diagonal matrix with singular values of $\{\sigma_i\}$. The Eigen Value Decomposition (EVD) for $\mathbf{Y}_S^H \mathbf{Y}_S$ is given by $\mathbf{Y}_S^H \mathbf{Y}_S = \mathbf{V} \mathbf{\Lambda} \mathbf{V}^H$, where $\mathbf{\Lambda} = \mathbf{\Sigma}^2$ is a diagonal matrix with eigenvalues of $\{\lambda_i\} = \{\sigma_i^2\}$. With the

introduction of DL, we have $\mathbf{Y}_S^H \mathbf{Y}_S + n_{tr} \sigma_L^2 \mathbf{I} = \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^H$, where $\tilde{\mathbf{\Lambda}}$ is a diagonal matrix with eigenvalues of $\{\tilde{\lambda}_i\} = \{\sigma_i^2 + n_{tr} \sigma_L^2\}$ and the corresponding eigenvectors remain the same. From (7.1.19), we can then obtain,

$$\text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_S}^{(DL)} \mathbf{X}_{tr}) = \sum_i^{n_S} \frac{\sigma_i^2}{\sigma_i^2 + n_{tr} \sigma_L^2} \|\mathbf{X}_{tr}^H \mathbf{u}_i\|^2 \quad (7.1.20)$$

where $\mathbf{u}_i$ is the eigenvectors for $\mathbf{Y}_S \mathbf{Y}_S^H$. In the case when $\sigma_i^2 \gg n_{tr} \sigma_L^2$, the effect of DL can be ignored. In the case when $\sigma_i^2 \ll n_{tr} \sigma_L^2$, the eigenvector $\mathbf{u}_i$ associated with the poorly estimated small eigenvalue σ_i^2 is essentially discarded. Thus, DL improves robustness for LS based antenna selection under finite training sample support.

The detailed receive antenna selection algorithm utilizing LS BGA with DL is described as follows.

LS BGA Based Receive Antenna Selection with DL (LS BGA + DL):

- Initialize by setting $n = n_R$ and $\mathbf{Y}_{n_R} = \mathbf{Y}_{tr} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{n_R}]$

- Set $n = n - 1$.

Determine $\mathbf{Y}_n = (\mathbf{Y}_{n+1})_{\bar{k}}$ by choosing $s = \arg \max_k \text{trace}(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_n}^{(DL)} \mathbf{X}_{tr})$,

where $(\mathbf{Y}_{n+1})_{\bar{k}}$ denotes $\mathbf{Y}_{n+1}$ with the k -th column removed,

$$\mathbf{P}_{\mathbf{Y}_n}^{(DL)} = \mathbf{Y}_n (\mathbf{Y}_n^H \mathbf{Y}_n + n_{tr} \sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_n^H$$

and $k \in$ all remaining column indices for $\mathbf{Y}_{n+1}$

- Continue the iteration until $n = n_R^{(s)}$.

Therefore, *LS BGA Based Receive Antenna Selection* becomes a special case of *LS BGA Based Receive Antenna Selection with DL* when setting $\sigma_L = 0$ (or equivalently by setting $LNR = -Inf(dB)$).

Considering (7.1.17) which is the LS estimation with DL to system $\mathbf{Y}_S \mathbf{F} = \mathbf{X}_{tr}$, it is noticed that the diagonally loaded term $n_{tr} \sigma_L^2 \mathbf{I}$ does help to ease the condition of $n_{tr} \geq n_R$ for an overdetermined system which is originally required for *LS BGA* based method to insure (7.1.4) to be a valid representation. With any nonzero loading level ($\sigma_L^2 > 0$), the solution given in (7.1.17) remains to be valid for an underdetermined system when $n_{tr} < n_R$ since the matrix $(\mathbf{Y}_S^H \mathbf{Y}_S + n_{tr} \sigma_L^2 \mathbf{I})$ will no longer become

singular. In fact, with SVD $\mathbf{Y}_S = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, the LS estimation with DL as given in (7.1.17) can be represented as

$$\hat{\mathbf{F}}_{LS}^{(DL)} = \mathbf{V}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{\Sigma}\mathbf{U}^H\mathbf{X}_{tr} = \mathbf{V}\mathbf{\Gamma}\mathbf{U}^H\mathbf{X}_{tr} \quad (7.1.21)$$

where $\mathbf{\Gamma}$ is a diagonal matrix with diagonal elements of $\{\frac{\sigma_i}{\sigma_i^2 + n_{tr}\sigma_L^2}\}$. In addition, the minimum Frobenius norm solution as given in (7.1.12) can be written as

$$\hat{\mathbf{F}}_{MN} = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^H\mathbf{X}_{tr} \quad (7.1.22)$$

From (7.1.21) and (7.1.22), it is observed that with a slight loading level ($\sigma_L \rightarrow 0$), we have $\mathbf{\Gamma} \rightarrow \mathbf{\Sigma}^{-1} = \text{diag}\{\sigma_i^{-1}\}$ and solution $\hat{\mathbf{F}}_{LS}^{(DL)}$ will approach $\hat{\mathbf{F}}_{MN}$. In other words, for underdetermined system ($n_{tr} < n_R$), ($LS\ BGA + DL$) is expected to produce the same result as ($MN + LS\ BGA$) when setting $\sigma_L \rightarrow 0$. Moreover, as shown in (7.1.20), by choosing an appropriate loading level σ_L , ($LS\ BGA + DL$) is expected to outperform ($MN + LS\ BGA$) given very limited training sample support ($n_{tr} < n_R$).

7.1.5 Fast BGA for Least Squares based Receive Antenna Selection

The computational complexity for ($LS\ BGA + DL$) based receive antenna method by direct computing is still a concern due to the computational requirement for $\mathbf{P}_{\mathbf{Y}_n}^{(DL)}$ in each iteration. Fortunately, an efficient implementation of the standard BGA for sparse signal reconstruction has been proposed in [68]. This fast BGA can be extended to the case of ($LS\ BGA + DL$) based receive antenna selection algorithm for multiple dimension input sequence $\mathbf{X}_{tr}$ and with the DL insertion. Following the procedure in [68], let $\mathbf{Y} = [\mathbf{Y}_{\bar{k}} \ \mathbf{y}_k] \mathbf{\Pi}$ where $\mathbf{Y}_{\bar{k}}$ denotes $\mathbf{Y}$ with the k -th column $\mathbf{y}_k$ removed and $\mathbf{\Pi}$ is a corresponding symmetric permutation matrix [35] which satisfies $\mathbf{\Pi}^2 = \mathbf{I}$. For simplicity, the iteration subscript n for $\mathbf{Y}_n$ is dropped. We define the matrix partition as follows.

$$\begin{aligned} \mathbf{A} &\triangleq (\mathbf{Y}^H\mathbf{Y} + n_{tr}\sigma_L^2\mathbf{I})^{-1} = \mathbf{\Pi} \begin{pmatrix} \mathbf{Y}_{\bar{k}}^H\mathbf{Y}_{\bar{k}} + n_{tr}\sigma_L^2\mathbf{I} & \mathbf{Y}_{\bar{k}}^H\mathbf{y}_k \\ \mathbf{y}_k^H\mathbf{Y}_{\bar{k}} & \mathbf{y}_k^H\mathbf{y}_k + n_{tr}\sigma_L^2 \end{pmatrix}^{-1} \mathbf{\Pi} \\ &\triangleq \mathbf{\Pi} \begin{pmatrix} \mathbf{G}_k & \mathbf{g}_k \\ \mathbf{g}_k^H & \gamma_k \end{pmatrix} \mathbf{\Pi} \end{aligned} \quad (7.1.23)$$

$$\begin{aligned}\mathbf{B} &\triangleq (\mathbf{Y}^H \mathbf{Y} + n_{tr} \sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}^H \mathbf{X}_{tr} = \Pi \left(\mathbf{G}_k \mathbf{Y}_{\bar{k}}^H + \mathbf{g}_k \mathbf{y}_k^H \right) \mathbf{X}_{tr} \\ &\triangleq \Pi \left(\mathbf{D}_k^H \right) \mathbf{X}_{tr}\end{aligned}\quad (7.1.24)$$

The recursive formula for updating the projection term $trace(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_n}^{(DL)} \mathbf{X}_{tr})$ can then be derived as follows by utilizing the partitioned matrix inversion formula [48].

$$trace(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}_{\bar{k}}}^{(DL)} \mathbf{X}_{tr}) = trace(\mathbf{X}_{tr}^H \mathbf{P}_{\mathbf{Y}}^{(DL)} \mathbf{X}_{tr}) - \frac{1}{\gamma_k} \|\mathbf{d}_k^H \mathbf{X}_{tr}\|^2 \quad (7.1.25)$$

where γ_k is the k -th diagonal element of the matrix $\mathbf{A}$ and $\mathbf{d}_k^H \mathbf{X}_{tr}$ is the k -th row of the solution matrix $\mathbf{B}$. At each iteration, (7.1.25) needs to be evaluated for all remaining column indices k . Maximizing (7.1.25) is equivalent to minimizing the term $\frac{1}{\gamma_k} \|\mathbf{d}_k^H \mathbf{X}_{tr}\|^2$.

After choosing the k -th column to be removed, the recursive formulas for updating the matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$(\mathbf{Y}_{\bar{k}}^H \mathbf{Y}_{\bar{k}} + n_{tr} \sigma_L^2 \mathbf{I})^{-1} = \mathbf{G}_k - \frac{1}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H \quad (7.1.26)$$

$$(\mathbf{Y}_{\bar{k}}^H \mathbf{Y}_{\bar{k}} + n_{tr} \sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_{\bar{k}}^H \mathbf{X}_{tr} = (\mathbf{D}_k^H \mathbf{X}_{tr}) - \mathbf{g}_k (\mathbf{y}_k^H \mathbf{X}_{tr}) - \frac{1}{\gamma_k} \mathbf{g}_k \mathbf{g}_k^H (\mathbf{Y}_{\bar{k}}^H \mathbf{X}_{tr}) \quad (7.1.27)$$

where $\mathbf{G}_k$ is the matrix $\mathbf{A}$ with both the k -th row and the k -th column removed, vector $\mathbf{g}_k$ is the k -th column of the matrix $\mathbf{A}$ with the k -th element removed and the matrix term $(\mathbf{D}_k^H \mathbf{X}_{tr})$ is the matrix $\mathbf{B}$ with the k -th row removed. The matrix product $(\mathbf{Y}_{\bar{k}}^H \mathbf{X}_{tr})$ needs to be computed only once as

$$\mathbf{C} \triangleq \mathbf{Y}_{tr}^H \mathbf{X}_{tr} \quad (7.1.28)$$

at the beginning of the iteration and then updated by eliminating the k -th row of the matrix $\mathbf{C}$ at each iteration. The row vector term $(\mathbf{y}_k^H \mathbf{X}_{tr})$ is just this eliminated row.

The fast algorithm needs to be initialized by computing the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ at the beginning of the iteration. This can be implemented by using Cholesky factorization [35] and forward substitution to solve the triangular system [35]. The computational complexity for initializing $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ is approximately $(n_{tr} + \frac{1}{3}n_R)n_R^2 +$

$2(n_{tr} + n_R)n_R n_T + 2n_R^3$ flops where flop denotes a floating point operation. Furthermore, the highest order computation for updating (7.1.26) and (7.1.27) is approximately $2n^2 + 3nn_T + n_{tr}n_T$ flops at the n -th iteration. By summing these operations over from $n = (n_R - 1)$ to $n_R^{(s)}$ and combining with operations needed for initialization, we obtain a total of $\{n_{tr}n_R^2 + 3n_R^3 - \frac{2}{3}(n_R^{(s)})^3 + (3n_{tr}n_R - n_{tr}n_R^{(s)} + \frac{7}{2}n_R^2 - \frac{3}{2}(n_R^{(s)})^2)n_T\}$ flops required for the fast BGA based receive antenna selection algorithm. This complexity is comparable with that required in fast antenna selection algorithm proposed in [32] but without the need for explicit channel estimation.

7.2 Least Squares Based Transmit Antenna Selection

Assuming that the transmitter does not have the CSI as well as the receiver, transmit antenna selection algorithm can be implemented at the receiver end based on the received training samples. The information of selected subset of transmit antennas is then fed back to the transmitter through a low rate feedback channel.

To facilitate the transmit antenna selection based on training sample sequence, suppose that the training symbol sequence for each individual transmit antenna is transmitted one stream at a time separately. To insure a constant total transmitting power as $P_{total} = n_T^{(s)}P$ where P denotes the default transmitting power for each transmitter, we need to raise the transmitting power for the separately transmitted training sequence by $n_T^{(s)}$ times. Thus, the received training samples from the i -th transmitted training symbol stream can be written as

$$\mathbf{y}^{(i)} = \mathbf{h}_i(\sqrt{n_T^{(s)}}x_i) + \mathbf{I}_i^{(MAI)} + \mathbf{n}_i \quad (7.2.1)$$

where $\mathbf{y}^{(i)} = [y_1^{(i)}, y_2^{(i)}, \dots, y_{n_R}^{(i)}]^T$, $i = 1, \dots, n_T$. The vector $\mathbf{h}_i$ denotes the i -th column of MIMO channel matrix $\mathbf{H}$, vector $\mathbf{I}_i^{(MAI)}$ and $\mathbf{n}_i$ denote the MAI term and AWGN term respectively which correspond to the transmitting of the i -th training symbol stream from the i -th transmit antenna.

Similarly, as the LS estimation model formulated in (7.1.2), with n_{tr} received training samples, the LS estimation for the i -th transmitted stream can then be

formulated as

$$\mathbf{Y}_{tr}^{(i)} \mathbf{f}_i = \mathbf{x}_i \quad (7.2.2)$$

where $\mathbf{Y}_{tr}^{(i)} = [\mathbf{y}^{(i)}(1), \mathbf{y}^{(i)}(2), \dots, \mathbf{y}^{(i)}(n_{tr})]^T$ is an $n_{tr} \times n_R$ matrix and $\mathbf{x}_i = [x_i(1), x_i(2), \dots, x_i(n_{tr})]^T$ is an $n_{tr} \times 1$ vector.

Thus, the received sample vector $\mathbf{y}_S$ for combined transmission from any transmit antenna subset denoted as $\{i_1, i_2, \dots, i_n\}$ can be simulated by

$$\mathbf{y}_S \triangleq \frac{1}{\sqrt{n_T^{(s)}}} \sum_{i=\{i_1, i_2, \dots, i_n\}} \mathbf{y}^{(i)} = \mathbf{H}_S \mathbf{x}_S + \frac{1}{\sqrt{n_T^{(s)}}} \sum_i \mathbf{I}_i^{MAI} + \frac{1}{\sqrt{n_T^{(s)}}} \sum_i \mathbf{n}_i \quad (7.2.3)$$

where $\mathbf{H}_S = [\mathbf{h}_{i_1} \dots \mathbf{h}_{i_n}]$ and $\mathbf{x}_S = [x_{i_1} \dots x_{i_n}]^T$ represent the corresponding selected channel sub-matrix and the transmitted signal vector subset respectively. Assume i.i.d. for MAI terms $\{\mathbf{I}_i^{MAI}\}$ and for AWGN terms $\{\mathbf{n}_i\}$ respectively. It is easy to show that when $n = n_T^{(s)}$, the MAI term $(\frac{1}{\sqrt{n_T^{(s)}}} \sum_i \mathbf{I}_i^{MAI})$ has the same covariance as that of its individual term $\mathbf{I}_i^{MAI}$, and AWGN term $(\frac{1}{\sqrt{n_T^{(s)}}} \sum_i \mathbf{n}_i)$ has the same covariance as that of its individual terms $\mathbf{n}_i$ respectively.

Assuming that n_{tr} training samples are obtained, the system model for LS estimation can then be written as

$$\mathbf{Y}_S \mathbf{F}_S = \mathbf{X}_S \quad (7.2.4)$$

where $\mathbf{Y}_S = \sum_{i=\{i_1, i_2, \dots, i_n\}} \mathbf{Y}_{tr}^{(i)}$ is an $n_{tr} \times n_R$ received sample matrix and $\mathbf{X}_S = [\mathbf{x}_{i_1} \dots \mathbf{x}_{i_n}]$ is an $n_{tr} \times n$ transmitted symbols matrix.

For optimum transmit antenna subset selection based on (7.2.4), ES can be employed to minimize the LS estimation error or equivalently to maximize the diagonally loaded projection term $trace\{\mathbf{X}_S^H \mathbf{P}_{\mathbf{Y}_S}^{(DL)} \mathbf{X}_S\}$ where $\mathbf{P}_{\mathbf{Y}_S}^{(DL)}$ is defined in (7.1.19). For larger number of transmit antennas (n_T), GA can be applied to the iterative transmit antenna selection procedure. An incremental selection approach is adopted for transmit antenna subset selection which is opposite to the decremental selection approach applied to receive antenna subset selection in order to meet the minimum diversity order requirement for linear MMSE detection.

Thus, the proposed LS based transmit antenna selection with GA is detailed as follows.

LS Based Transmit Antenna Selection (Tx LS GA):

- Initialize by setting $n = 0$, $\mathbf{X}_0 = []$ and $\mathbf{Y}_0 = []$
- Set $n = n + 1$, find out $\mathbf{X}_n = [\mathbf{X}_{n-1}, \mathbf{x}_k]$ and $\mathbf{Y}_n = \mathbf{Y}_{n-1} + \mathbf{Y}_{tr}^{(k)}$
by choosing $s = \arg \max_k \text{trace}(\mathbf{X}_n^H \mathbf{P}_{\mathbf{Y}_n}^{(DL)} \mathbf{X}_n)$,
where $\mathbf{x}_k$ is the k -th column of $\mathbf{X}_{tr}$,
 $\mathbf{P}_{\mathbf{Y}_n}^{(DL)} = \mathbf{Y}_n (\mathbf{Y}_n^H \mathbf{Y}_n + n_{tr} \sigma_L^2 \mathbf{I})^{-1} \mathbf{Y}_n^H$ and
 $k \in$ all the unselected column indices for $\mathbf{X}_{tr}$
- Continue the iteration until $n = n_T^{(s)}$.

The computational complexity for this (Tx LS GA) based method is approximately on the order of $\mathcal{O}((n_R^3 + n_R^2 n_{tr} + n_R^2 n_T^{(s)} + n_R n_{tr} n_T^{(s)}) n_T n_T^{(s)})$.

7.3 Joint Transmit and Receive Antenna Selection

As discussed in section 6.2.4, in the case when both transmit and receive antennas are to be selected, the simplest method is the sequential selection procedure, i.e., select a transmit antenna subset first, then select a receive antenna subset, or vice versa. In this section, a joint transmit and receive antenna selection method by GA is proposed as follows based on LS criterion.

LS Based Joint Transmit and Receive Antenna Selection (Joint Tx and Rx Selection):

- Obtain a set of training sample matrices $\{\mathbf{Y}_{tr}^{(k)} \mid k = 1, \dots, n_T\}$ by transmitting the training symbol sequence one stream at a time for each individual transmit antennas separately as described in section 7.2
- Initialize by setting $n = 0$, $\mathbf{X}_0 = []$ and $\mathbf{Y}_0 = []$
- Set $n = n + 1$
- For each $k \in$ all the unselected column indices for $\mathbf{X}_{tr}$
Let $n_T^{(n,k)} = n$, $\mathbf{X}_{tr}^{(n,k)} = \mathbf{X}_n = [\mathbf{X}_{n-1}, \mathbf{x}_k]$ and $\mathbf{Y}_{tr}^{(n,k)} = \mathbf{Y}_n = \mathbf{Y}_{n-1} + \mathbf{Y}_{tr}^{(k)}$

Call (*LS BGA + DL*) based method to select $n_R^{(s)}$ elements out of total n_R receive antennas, which is denoted as $Rx_selected(n, k)$

- Select the transmit antenna element by choosing $s = \arg \max_k \text{trace}(\mathbf{X}_n^H \mathbf{P}_{\mathbf{Y}_n}^{(DL)} \mathbf{X}_n)$, where the term $\text{trace}(\mathbf{X}_n^H \mathbf{P}_{\mathbf{Y}_n}^{(DL)} \mathbf{X}_n)$ has been computed at the last iteration of (*LS BGA + DL*) based method
- Set the selected transmit antenna subset at the n -th iteration as $Tx_selected(n) = [Tx_selected(n-1), s]$ and the corresponding selected receive antenna subset as $Rx_selected(n) = Rx_selected(n, s)$
- Continue the iteration until $n = n_T^{(s)}$.

This *LS Based Joint Transmit and Receive Antenna Selection* method has higher computational complexity than that required for the sequential transmit and receive antenna selection method. But both are approximately on the same order of $\mathcal{O}((n_R^3 + n_R^2 n_{tr} + n_R^2 n_T^{(s)} + n_R n_{tr} n_T^{(s)}) n_T n_T^{(s)})$, when fast BGA [68] is incorporated into (*LS BGA + DL*) algorithm.

As discussed in section 6.2.4, equally distributing the antennas to both transmitter and receiver side will result in maximum number of possible transmission channels for antenna selection. Thus, higher order of diversity or better performance is expected by implementing joint antenna selection at both transmitter and receiver end in comparison with either single transmit or receive antenna selection when the total number of available antennas is assumed to be the same.

7.4 Simulation Results and Discussions

In this section, the performance of proposed LS based antenna selection methods is evaluated via Monte Carlo simulations. It is also compared with Gorokhov's JMMSE method (*Gorokhov JMMSE*) [38] for linear receivers and its advantage in the presence of unknown co-channel interference is shown. The transmitted symbols are modulated using 16-QAM constellation and assumed to be i.i.d. for all n_T desired transmitters

as well as n_{MAI} MAI transmitters. System performance is measured in terms of Symbol Error Rate (SER) with a frame of 128 symbols and averaged over 5000 MIMO channel realizations. The received training sample matrix $\mathbf{Y}_{tr}$ or $\mathbf{Y}_{tr}^{(i)}$ consists of the received training samples obtained from all n_R receivers using RF switches. For receive antenna selection by (*LS BGA*), all n_T desired MIMO transmitters as well as n_{MAI} MAI transmitters (when $n_{MAI} > 0$) will contribute to $\mathbf{Y}_{tr}$. For transmit antenna selection by (*Tx LS GA*) or LS based joint transmit and receive antenna selection by (*Joint Tx and Rx Selection*), only the i -th desired MIMO transmitter and all the n_{MAI} MAI transmitters will contribute to $\mathbf{Y}_{tr}^{(i)}$. The transmitted symbol matrix $\mathbf{X}_{tr}$ consists of only the training symbols from n_T desired MIMO transmitters since the MAI transmitters are assumed to be unknown at the receiver end. The proposed method selects $n_R^{(s)}$ elements out of total n_R receive antennas and/or $n_T^{(s)}$ elements out of total n_T transmit antennas directly based on n_{tr} training samples. For Gorokhov's JMMSE method that requires CSI, the Maximum Likelihood (ML) channel estimation [64] is used to obtain the estimated CSI (let $\mathbf{Y}_{tr}^T = \mathbf{H}\mathbf{X}_{tr}^T + \mathbf{N}$, then ML estimation is given by $\hat{\mathbf{H}} = \mathbf{Y}_{tr}^T(\mathbf{X}_{tr}^T)^H(\mathbf{X}_{tr}^T(\mathbf{X}_{tr}^T)^H)^{-1}$ where the unknown MAI transmission is assumed as part of the Gaussian noise in matrix $\mathbf{N}$). However, this estimated $\hat{\mathbf{H}}$ is only applied to the simulation for (*Gorokhov JMMSE*). After antenna selection process, the equalizer coefficients for linear MMSE receiver are directly computed by LS estimation based on training samples (without the need for CSI) to obtain the SER for all the simulation curves. In practice, Recursive Least Squares (RLS) based adaptive filter algorithm can be applied as well.

7.4.1 Least Squares based Receive Antenna Selection

Figure 7.1(a) and 7.1(b) illustrate the SER performance comparison for different receive antenna selection methods in the absence of MAI and in the presence of MAI respectively. For reasonable comparison, set $n_R^{(s)} = 4$ for $n_{MAI} = 0$ in Figure 7.1(a) and $n_R^{(s)} = 6$ for $n_{MAI} = 2$ in Figure 7.1(b) to meet the minimum requirement for diversity order for both cases. By comparing *LS BGA + DL* to *LS BGA w/o DL*, it is observed that DL technique does help to improve the antenna selection performance

Figure 7.1: Performance comparison for different receive antenna selection methods (a) in the absence of interference and (b) in the presence of unknown MAI ($LNR = 5dB$)

under finite training sample support. When compared with ES based method ($LS\ ES + DL$), GA based method ($LS\ BGA + DL$) exhibits only limited performance degradation but significantly benefit from reduced computational complexity. By comparing LS based method ($LS\ ES + DL$) with CSI based method ($Perfect\ CSI + ES$) which assumes perfect CSI for antenna selection, only very limited performance degradation is observed. Moreover, the SER curves by antenna selection with ($LS\ BGA$, $n_S/n_R = 4/16$) in Figure 7.1(a) and ($LS\ BGA$, $n_S/n_R = 6/16$) in Figure 7.1(b) are shown to outperform the system without antenna selection ($w/o\ Selection$, $n_R = 4$ in Figure 7.1(a) and $w/o\ Selection$, $n_R = 6$ in Figure 7.1(b)) significantly. Furthermore, the SER curves for antenna selection are shown to approach the diversity order of a full complexity system utilizing all antennas ($w/o\ Selection$, $n_R = 16$) in both Figure 7.1(a) and 7.1(b), where the diversity order is understood as the slope of the SER curves. On the other hand, *Gorokhov JMMSE* method performs similarly as the proposed LS based method ($LS\ BGA$) in the absence of interference as shown in Figure 7.1(a). However, *Gorokhov JMMSE* method performs poorly in the presence of unknown MAI as shown in Figure 7.1(b) due to the inaccurate channel estimation by assuming unknown MAI transmission as part of the Gaussian noise. This indicates that for CSI based antenna selection method, blind channel estimation (for both $\mathbf{H}$ and $\mathbf{H}^{MAI}$) is required in the presence of unknown interference. But blind estimation is usually a complicated task and exhibits slow convergence. In contrast, the proposed LS based antenna selection method avoids explicit channel estimation and retains the diversity benefit by antenna selection in the presence of unknown co-channel interference.

Figure 7.2 illustrates the SER performance comparison for different antenna selection methods versus the number of training symbols ($n_{tr} = 8 \sim 40$) at a certain SNR ($E_s/N_0 = 16dB$). It is observed that the performance curve obtained by ($LS\ BGA + DL$, $LNR = -30dB$) does overlap with the curve obtained by ($MN + LS\ BGA$) without DL when system is underdetermined ($n_{tr} < n_R$). This confirms that ($LS\ BGA + DL$) will approach ($MN + LS\ BGA$) when loading level σ_L approaches zero. With appropriate loading, ($LS\ BGA + DL$, $LNR = 5dB$) achieves noticeable

performance improvement over ($MN + LS\ BGA$, when $n_{tr} < n_R$) and ($LS\ BGA$, when $n_{tr} \geq n_R$) without DL. The proposed antenna selection methods outperform ($Gorokhov\ JMMSE$) method and the system ($w/o\ Selection$, $n_R = 6$) significantly. Furthermore, with the increased amount of training samples, the performance improvement of the proposed LS based methods becomes saturated. A reasonably good performance can be achieved with $n_{tr} = 2 \times n_R = 32$ using the rule-of-thumb for the training sample requirement [67].

Figure 7.2: Performance comparison for different antenna selection methods, SER versus number of training symbols n_{tr} ($n_R^{(s)}/n_R = 6/16$, $n_T = 4$, $n_{MAI} = 2$, $n_{tr} = 8 \sim 40$ and $E_s/N_0 = 16dB$)

Figure 7.3 illustrates the SER performance versus loading level (LNR in dB) for DL technique given that $n_{tr} = 16/24/32$ for the proposed ($LS\ BGA + DL$) method assuming that $n_R^{(s)}/n_R = 6/16$, $n_{MAI} = 2$ and $E_s/N_0 = 16dB$. It is observed that DL helps to improve the performance especially in the case when very limited training sample support ($n_{tr}=16$) is given. Based on the observation, the loading level for DL can be chosen empirically as $LNR = 0 \sim 5dB$. Over loading will degrade the

performance significantly and hence should be avoided.

Figure 7.3: Performance simulation curves for the proposed *LS BGA + DL* with different number of training symbols, *SER* versus *LNR*(dB) ($n_R^{(s)}/n_R = 6/16$, $n_T = 4$, $n_{MAI} = 2$, $n_{tr} = 16/24/32$ and $E_s/N_0 = 16dB$)

Figure 7.4 shows the SER performance for different antenna selection methods versus the number of receive antennas to be selected ($n_R^{(s)}$) assuming $n_R = 16$, $n_{MAI} = 2$, $n_{tr} = 32$ and $E_s/N_0 = 12dB$. It is observed that for various $n_R^{(s)}$, the proposed method (*LS BGA + DL*, $LNR=5dB$) consistently outperforms (*Gorokhov JMMSE*) method in the presence of unknown MAI transmitters. With antenna selection, the proposed method (*LS BGA + DL*, $LNR=5dB$, $n_R^{(s)} = 10 \sim 14$) may even outperform the full complexity system utilizing all $n_R = 16$ antennas (*w/o Selection*, $n_R = n_R^{(s)} = 16$) given finite training sample support ($n_{tr} = 32$).

Figure 7.5(a) illustrates the SER performance evaluation for the proposed LS based receive antenna selection method (*LS BGA + DL*) with different total number of n_R . The SER curves with antenna selection are shown to outperform the simple system without antenna selection ($n_R^{(s)} = n_R = 4$) significantly. The performance improvement achieved by antenna selection is shown to be saturated when more and more receive antennas are assumed to be available (i.e., a larger n_R is assumed).

Figure 7.4: Performance comparison for different antenna selection methods, SER versus number of receive antennas to be selected $n_R^{(s)}$ ($n_R = 16$, $n_T = 4$, $n_{MAI} = 2$, $n_{tr} = 32$ and $E_s/N_0 = 12dB$)

Similar phenomenon is observed in Figure 7.5(b) in the presence of unknown MAI ($n_{MAI} = 2$ MAI transmitters are presented with equal transmission power as the desired transmitters) where we set $n_R^{(s)} = 6$ in order to mitigate the MAI and meet the minimum requirement for diversity order with linear MMSE receiver.

7.4.2 Least Squares based Transmit Antenna Selection

Figure 7.6 illustrates the SER performance evaluation for the proposed LS based transmit antenna selection method (Tx LS GS) with different total number of n_T . Compared to the SER curves by receive antenna selection shown in Figure 7.5, similar performance curves are observed in Figure 7.6 but with some performance degradation. This is due to that incremental GA being adopted for transmit antenna selection and decremental GA being applied to receive antenna selection. In general, decremental GA performs better than incremental GA since decremental method does more joint consideration when removing antenna elements one by one at each iteration. However, decremental GS is not suitable for the proposed LS based transmit

(a) $n_{MAI} = 0$, $n_T = 4$, $n_R^{(s)} = 4$, $n_{tr} = 32$

(b) $n_{MAI} = 2$, $n_T = 4$, $n_R^{(s)} = 6$, $n_{tr} = 32$

Figure 7.5: Performance comparison for LS based receive antenna selection with different n_R (a) in the absence of MAI and (b) in the presence of MAI

(a) $n_{MAI} = 0, n_T^{(s)} = 4, n_R = 4, n_{tr} = 32$

(b) $n_{MAI} = 2, n_T^{(s)} = 4, n_R = 6, n_{tr} = 32$

Figure 7.6: Performance comparison for LS based transmit antenna selection with different n_T (a) in the absence of MAI and (b) in the presence of MAI

(a) $n_{MAI} = 0$, $n_T^{(s)} = 4$, $n_R^{(s)} = 4$, $n_{tr} = 32$ (b) $n_{MAI} = 2$, $n_T^{(s)} = 4$, $n_R^{(s)} = 6$, $n_{tr} = 32$

Figure 7.7: Performance comparison for LS based transmit and receive antenna selection with different n_T and n_R (a) in the absence of MAI and (b) in the presence of MAI

antenna selection method since the minimum requirement for diversity order with linear receivers may not be met.

7.4.3 Least Squares based Joint Transmit and Receive Antenna Selection

Figure 7.8: Performance comparison for transmit and receive antenna selection, SER versus number of training symbols n_{tr} ($n_{MAI} = 2$, $n_{tr} = 8 \sim 40$ and $E_s/N_0 = 16dB$)

Figure 7.7(a) and 7.7(b) illustrate the SER performance evaluation for the proposed LS based joint transmit and receive antenna selection method (*Joint Tx and Rx Selection*) in the absence of interference and in the presence of unknown MAI respectively. It is observed that better performance is achieved by implementing joint antenna selection at both transmitter and receiver side when compared with that obtained by single transmit or single receive antenna selection as shown in Figure 7.6 and 7.5, under the assumption that total number of transmit and receive antennas is kept unchanged. This has been discussed in section 6.2.4. The more the antennas

available for selection, the better the performance improvement for joint transmit and receive antenna selection when compared with the simple sequential transmit and receive antenna selection procedure.

Figure 7.8 shows the SER performance for different antenna selection methods versus the number of training symbols ($n_{tr} = 8 \sim 40$) at a certain SNR ($E_s/N_0 = 16dB$). Assume that the total number of transmit and receive antennas is the same. It is observed that single receive antenna selection (*Rx Selection*) achieves better performance than single transmit antenna selection (*Tx Selection*). In addition, joint transmit and receive antenna selection (*Joint Tx and Rx Selection*) achieves better performance than sequential transmit and receive antenna selection (*Tx and Rx Selection*). Both *Joint Tx and Rx Selection* and sequential *Tx and Rx Selection* methods outperform the single *Tx Selection* or single *Rx Selection* method. With the increased number of training symbols, the performance improvement for the antenna selection methods become saturated. A reasonably good performance can be achieved given limited training sample support ($n_{tr} \geq 20$) [67].

7.5 Least Squares Based Receive Antenna Selection for UWB Systems

In this section, LS based receive antenna selection technique is extended to receive antenna selection for UWB systems with multiple receive antenna elements to achieve improved system performance under low power transmission while maintaining lower hardware complexity.

In practice, UWB systems are required to co-exist with other narrow band systems. Due to its very low transmission power, UWB systems may suffer from severe performance degradation due to strong Narrow Band Interference (NBI). Various methods such as MMSE equalization [52], MMSE RAKE receiver [11], and multiple receive antenna selection technique to benefit from the “interference diversity” [12] have been proposed for NBI mitigation in UWB systems.

Antenna selection technique for multiple antenna systems in Rayleigh fading channels reduces the hardware cost and computational complexity while retaining the diversity degree of full complexity systems [58]. In frequency-selective fading channels, however, the effectiveness of antenna selection is considerably reduced. In the case of UWB indoor channels with dense resolvable multipath components which lead to high diversity degree, the additional diversity obtained from applying antenna selection may become negligible. On the other hand, UWB indoor channels exhibit shadowing effect [25] which would affect energy capturing and detection performance at the receiver end. Antenna selection technique can be applied to UWB multipath channels to benefit from energy capturing and hence achieve improved performance.

In this section, a LS based receive antenna selection technique is proposed for high data rate UWB systems with linear MMSE receiver in the presence of unknown NBI. This technique is differentiated from the receive antenna selection technique proposed in [12] which benefits from the “interference diversity”. In [12], matched filter or RAKE receiver is assumed for UWB systems which is only optimal in the absence of interference. For high data rate UWB system in the presence of ISI, MAI, or strong NBI, conventional receiver such as matched filter or RAKE receiver will suffer from severe performance degradation or even break down [52]. The simple antenna selection techniques such as power based selection [58] or interference power based selection [12] is no longer applicable when linear MMSE receiver is adopted to mitigate the strong interference. In this case, the proposed LS based receive antenna selection technique is shown to achieve noticeable performance improvement by numerical simulation.

7.5.1 UWB System Model

As discussed in section 5.1.1, a UWB system model can be represented as follows in the presence of ISI and unknown interference.

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{I}^{(unk)} + \mathbf{m} \quad (7.5.1)$$

where $\mathbf{H}$ is the channel transmission matrix in block Toeplitz form, $\mathbf{y}$ is the received

sampling signal vector, $\mathbf{I}^{(unk)}$ denotes the unknown interference vector which can be MAI and/or NBI, and $\mathbf{m}$ is the corresponding AWGN vector with covariance of $\sigma_m^2 \mathbf{I}$. For simplicity, only the self spreading multipath channels of UWB are assumed in the system model, spreading sequence based system can be considered as well. In addition, strict channel synchronization is not necessary due to a large observation window spans over multiple symbol periods is applied. MMSE receiver will properly combine these received samples for the desired symbol equalization.

For UWB indoor propagation, shadowing of multipath channel at the receiver end is modelled by log-normal distribution X_i as given in [25], i.e.,

$$h_i(t) = X_i \sum_{l=0}^L \sum_{k=0}^K \alpha_{k,l}^i \delta(t - T_l^i - \tau_{k,l}^i) \quad (7.5.2)$$

This can be considered as a special case of the tapped-delay-line channel model for system modelling as discussed in section 5.1.1.

A single carrier BPSK modulated NBI is assumed as given in [26]

$$I(t) = \sqrt{2P_I} \cos(\omega_0 t + \theta) \sum_{k=-\infty}^{\infty} g_k v(t - kT_I - \tau_I) \quad (7.5.3)$$

where $\omega_0 = 2\pi f_0$, f_0 is the carrier frequency and θ is its random phase. The term $g_k \in \{\pm 1\}$ denotes the BPSK symbols, $v(t)$ is the baseband signal waveform, T_I is the symbol period and τ_I is the random delay uniformly distributed in $[0 T_I]$. An OFDM interference can be considered similarly as discussed in [52].

7.5.2 Least Squares Based Receive Antenna Selection

As discussed in section 5.1, with the training sequence from the desired transmitter, LS estimation can be utilized to estimate the equalizer filter for MMSE detection in the presence of unknown interference. DL technique can be incorporated into LS estimation process to insure the robustness given limited number of training symbols.

Suppose that there are n_R receive antennas available and the received training sample matrices $\{\mathbf{Y}_{tr}^{(j)} | j = 1, \dots, n_R\}$ are obtained with a RF switch at receiver end. Where $\mathbf{Y}_{tr}^{(j)}$ is defined as in (5.1.8) and j is the index for the j -th antenna. For simplicity, assume that only one antenna will be selected out of n_R receive antennas

for the data transmission. In the presence of unknown interference, LS based criterion can be applied to effectively select the best receive antenna for the optimal performance with linear MMSE detection. From (5.3.19), the criterion for choosing the j -th antenna is to minimize the LS estimation error or equivalently to maximize the projection term based on the training sample matrices $\{\mathbf{Y}_{tr}^{(j)}\}$. This is represented as

$$s = \arg \max_j \mathbf{x}_{tr}^T \mathbf{P}_j^{(DL)} \mathbf{x}_{tr} \quad (7.5.4)$$

where the diagonally loaded projection matrix is given by

$$\mathbf{P}_j^{(DL)} = \mathbf{Y}_{tr}^{(j)} ((\mathbf{Y}_{tr}^{(j)})^T \mathbf{Y}_{tr}^{(j)} + n_{tr} \sigma_L^2 \mathbf{I})^{-1} (\mathbf{Y}_{tr}^{(j)})^T \quad (7.5.5)$$

7.5.3 Simulation Results and Discussions

The numerical results are obtained based on 100 UWB channel realizations for CM2 [25]. Independent channel realizations are assumed for multiple receive antennas. The sampling duration is set as $\Delta\tau = 0.667ns$ and symbol transmission rate is set at $R_s = 1/T_f = 93.56MHz$ (making $n_\tau = T_f/\Delta\tau$ an integer). We have channel length as $n_L = 128$ and number of symbols affected by ISI as $n_{ISI} = 8$. The number of training symbols is set as $n_{tr} = 256$. We also assume that a single carrier BPSK modulated NBI is presented with carrier frequency of $5.75GHz$ and bandwidth of $16MHz$ (e.g., a wideband CDMA signal).

Figure 7.9 illustrates the BER performance comparison for UWB MMSE detection with multiple receive antenna selection based on LS criterion given in (7.5.4) in the absence of NBI. It is observed that about $2dB$ performance improvement in terms of SNR at $BER = 10^{-3}$ is obtained by selecting one out of two receive antennas (*1 of 2, LS select, $n_R = 2$*) compared with that without antenna selection (*1 Antenna, $n_R = 1$*). About $3dB$ performance improvement is obtained by selecting one out of three receive antennas (*1 of 3, LS select, $n_R = 3$*) compared with that without antenna selection. The antenna selection with power based simple selection method as referred in [58] (*1 of 2, Power select/1 of 3, Power select*) performs almost as good as the proposed LS based selection method (*1 of 2, LS select/1 of 3, LS select*) in the

Figure 7.9: Receive antenna selection performance in the absence of NBI (CM2, $R_s = 93.56 \text{ MHz}$, $n_{tr} = 256$, $LNR = 0 \text{ dB}$)

absence of unknown interference. Remarkable performance improvement has been achieved when LS estimation with DL (*1 Antenna*) is compared with LS estimation without DL (*(1 Antenna, w/o DL)*), as well as when LS based antenna selection with DL (*1 of 3, LS select*) is compared with LS based antenna selection without DL (*1 of 3, LS select, w/o DL*). This demonstrates the importance and effectiveness for introducing DL into LS based antenna selection method given limited number of training symbols ($n_{tr} = 256$).

Figure 7.10 illustrates the BER performance comparison for UWB MMSE detection with multiple receive antenna selection in the presence of strong NBI which is set to $SIR = -20 \text{ dB}$. As observed in Figure 7.9, about 2 dB performance improvement in terms of SNR at $BER = 10^{-3}$ is obtained by selecting one antenna out of two receive antennas (*1 of 2, LS select, NBI, $n_R = 2$*) compared with that without antenna selection (*1 Antenna, NBI, $n_R = 1$*). About 3 dB performance improvement is obtained by selecting one antenna out of three receive antennas (*1 of 3, LS select,*

Figure 7.10: Receive antenna selection performance in the presence of strong NBI (CM2, $R_s = 93.56 \text{ MHz}$, $n_{tr} = 256$, NBI $SIR = -20 \text{ dB}$, $LNR = 0 \text{ dB}$)

NBI, $n_R = 3$) compared with that without antenna selection. Similar performance improvement has been achieved when one compares LS estimation with DL (*1 Antenna, NBI*) with LS estimation without DL (*(1 Antenna, NBI, w/o DL)*), as well as when one compares LS based antenna selection with DL (*1 of 3, LS select, NBI*) with LS based antenna selection without DL (*1 of 3, LS select, NBI, w/o DL*), in the presence of NBI. However, the antenna selection with power based simple selection method as referred in [58] (*1 of 2, Power select, NBI/1 of 3, Power select, NBI*) breaks down in the presence of strong NBI ($SIR = -20 \text{ dB}$) and performs just as the case without antenna selection (*1 Antenna, NBI (SIR=-20dB)*).

7.6 Summary

In this chapter, novel transmit and receive antenna selection techniques are proposed for MIMO spatial multiplexing systems with linear MMSE receivers. They are directly implemented based on training sample sequence under LS criterion. These

techniques avoid explicit channel estimation process and retain the diversity order of a full complexity system in the presence of unknown co-channel interference. DL technique is incorporated into the proposed antenna selection process to achieve improved performance under finite training sample support. Practical implementation with manageable complexity is made possible by extending the standard fast BGA into the proposed receive antenna selection algorithm. Simulation results show that the proposed antenna selection technique outperforms the channel estimation based antenna selection technique in the presence of unknown MAI. Joint transmit and receive antenna selection is also discussed and shown to achieve improved performance compared with single transmit or single receive antenna selection when the total number of available antennas is the same. Furthermore, LS based receive antenna selection technique is extended to UWB systems with multiple receive antenna elements in the presence of ISI and NBI. This is shown to be an effective method to achieve improved performance for low power UWB systems by mitigating the channel shadowing effect in the presence of NBI.

Chapter 8

Conclusions and Future Work

This dissertation presents the research work done on multipath channel equalization techniques for impulse radio based Ultra-Wideband (UWB) communication systems. This has been one of the key challenges for high data rate UWB systems with low cost receiver design. The research work focuses on developing an efficient and effective tap selection techniques for non-uniformly spaced Minimum Mean Squared Error (MMSE) equalization and applying it to high performance UWB communication systems for reduced receiver complexity. Also, under MMSE criterion, the research focus is extended to antenna selection technique for Multiple Input Multiple Output (MIMO) systems which is a promising technique for achieving high diversity order in system performance with low complexity in hardware implementation. In this chapter, the research findings are summarized and the conclusions are drawn based on the studies and results presented in the previous chapters. The possible extension for future work is also discussed.

Firstly, a simple matrix representation for impulse radio based UWB MIMO systems is formulated in chapter 3. The formulation takes Inter Symbol Interference (ISI) and Multiple Access Interference (MAI) into account and facilitates the performance evaluation for UWB systems. The performance comparison shows that the MMSE receiver significantly outperforms the RAKE receiver with MRC in the presence of severe ISI and MAI. With linear MMSE receivers, Time Hopping Spread Spectrum (THSS) based UWB MIMO systems are capable of delivering promising performance up to very high symbol transmission rate. The Direct Sequence Spread Spectrum

(DSSS) based UWB MIMO systems are more vulnerable to high symbol rate transmission due to the existence of severe Inter Chip Interference (ICI) in addition to ISI.

Our research work emphasizes linear MMSE equalization based UWB receiver design with reduced complexity. Apart from well known rank reduced MMSE equalization technique, i.e., Multistage Wiener Filter (MWF), tap selection based non-uniformly spaced MMSE equalization technique is considered in our work for its simplicity and effectiveness. In chapter 4, the performance of MMSE equalization is evaluated by utilizing greedy algorithm for tap selection in UWB multipath channels. The evaluation results show that the complexity of the non-uniformly spaced MMSE equalizer can be significantly reduced by choosing only a small portion of the channel impulse response samples with appropriate tap selection methods. The tap selection will only result in limited performance degradation when compared to that obtained with full complexity MMSE equalizer using all taps. This indicates that the tap selection based equalization technique is a promising method for high performance UWB receiver design with reduced complexity. It is also observed that the simple *Strongest Paths* based tap selection method may work reasonably well for high data rate UWB channel equalization in the absence of interference. However, this simple method will result in larger performance degradation in the presence of MAI. In the case when co-channel UWB interference is presented as unknown MAI, accurate multipath channel estimation for UWB becomes a difficult task. Therefore, it is necessary to develop an efficient and effective tap selection method in the presence of unknown interference which has been discussed in chapter 5.

In chapter 5, a new class of tap selection techniques based on Order Recursive Least Squares (ORLS) and Matching Pursuit (MP) algorithms are proposed for UWB multipath channel equalization in the presence of ISI and unknown MAI. These tap selection techniques are directly implemented based on training symbol sequence under Least Squares (LS) criterion without the need for explicit channel estimation. *Heuristic MP* based fast tap selection algorithm is also developed to facilitate the

tradeoff between performance and computational complexity. It is found that incorporating Diagonal Loading (DL) technique into tap selection and MMSE equalization processes is important to insure robustness for algorithms given limited number of training symbols in the case of practical implementation. The proposed tap selection based equalizer is shown to outperform the conventional uniformly spaced linear equalizer with reduced computational complexity. Furthermore, LS based receive antenna selection technique is discussed for high data rate UWB systems with multiple receive antenna elements in the presence of Narrow Band Interference (NBI). This technique is shown to be an effective method to achieve improved performance with simplified hardware implementation by antenna selection for low power UWB transmission.

The problem of antenna selection in MIMO systems can be mathematically formulated on similar lines as the tap selection for multipath channel equalization. Our research focus is then extended to antenna selection techniques for MIMO systems. In prior works, the antenna selection technique has been shown to obtain spatial diversity benefit with simplified hardware implementation by utilizing multiple antennas with low cost RF switches.

In chapter 6, assuming Channel State Information (CSI) at receiver end, fast transmit and receive antenna selection techniques based on Greedy Algorithm (GA) are proposed under MMSE criterion for MIMO spatial multiplexing systems with linear receivers. A joint transmit and receive antenna selection technique is discussed and shown to achieve improved performance in comparison with either single transmit or single receive antenna selection when the total number of available antennas is the same.

In practical implementation, obtaining accurate CSI may not be an easy task for MIMO systems in the presence of unknown co-channel interference. In chapter 7, novel transmit and receive antenna selection techniques are proposed for MIMO spatial multiplexing systems with linear receivers under LS criterion. Unlike conventional approaches, the proposed method directly implements the antenna selection algorithms based on training sample sequence without the need for CSI. As a result,

it avoids explicit channel estimation and retains the diversity order of a full complexity system in the presence of unknown co-channel interference. DL technique is incorporated into the proposed antenna selection process and shown to achieve improved performance under finite training sample support. Practical implementation with manageable computational complexity is made possible by extending the standard fast Backward Greedy Algorithm (BGA) into the proposed antenna selection algorithms.

In addition to spatial multiplexing systems, antenna selection can be applied to space-time coded systems as well. In order to achieve the diversity benefit by antenna selection in the presence of unknown co-channel interference, the proposed techniques may be extended to the case when space time coding is utilized for transmit diversity. Moreover, it may be possible to develop combined tap selection and antenna selection based receiver technique for wideband or UWB MIMO systems to achieve reasonably good performance with reduced complexity. Further, how to choose the optimal number of taps or antennas to be selected given limited number of training samples remains as an interesting problem, especially when the number of effective transmitters or the number of equivalent MAI transmitters in the system are unknown. These topics remain as the possible areas for future work.

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[Journal Papers]

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