Design and fabrication of diffractive optical elements for micro-beam shaping

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Design and Fabrication of Diffractive Optical Elements for Micro-beam Shaping

Tao Shaohua

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2005

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other university of institution.

11/4/2005
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Date

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Tao Shaohua

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SUMMARY

Diffractive optical elements (DOEs) have been being used widely in the areas such as, optical imaging and displaying, information security, data storage and processing, beam shaping, and optical tweezing, etc., because the devices are more flexible, more efficient, and more compact than the conventional refractive or reflective optical elements. Studies on DOEs have to cover design, fabrication, and applications as the three aspects are closely integrated and related. In this thesis, various micro-beam shapings using DOEs have been extensively studied.

First, the author investigated and introduced three methods of design different kinds of DOEs. The hybrid algorithm combining the Gerchberg-Saxton algorithm and the simulated annealing algorithm is quick and efficient at getting a phase retrieval solution; the holography interference method that employs a digital-recording technique to generate a DOE is easily implemented; and the way of generating DOE based on an analytic expression is highly efficient but is restricted to unavailable analytic solutions. Therefore, based on specified applications of DOEs, we can select one most effective method from these methods to design the DOEs efficiently.

In addition to investigation of DOE designs, the author proposed two special algorithms for balancing reconstruction efficiency in design and implementation of fabrication. After many computations and comparisons, one effective and practical quantization method has been found to prevent decreasing of diffraction efficiency from a DOE design in an iterative Fourier transform algorithm (IFTA). As we know, polarization-selective DOEs that apply a form-birefringence or a material-birefringence in the sub-structures are useful in optical switch and optical storage, but their fabrication by using current technologies is difficult. Thus, the author proposed an optimization approach to simplifying the structures of such DOEs and retaining the contrast ratio as well.

To realize the DOE designs, the author introduced two micro-fabrication techniques: the first one is using a laser printer and a mask aligner to fabricate a large-sized DOE. The experimental results demonstrated that the fabrication was in good agreement with the design. The second one is using an e-beam direct writing machine, a high-energy beam-sensitive glass, and a mask aligner to fabricate a multi-phase-step DOE which has been designed with an iterative algorithm, and the beam shaping results verified that the fabricated DOE does

have multilevel structures; furthermore, the author proposed a novel technique using a polydimethylsiloxan (PDMS) material to replicate a multilevel DOE in a large amount and at a fast speed. Moreover, employment of a spatial light modulator for the replacement of fabricated DOEs was also investigated.

On the basis of the design and the fabrication, several novel laser beams, e.g. fractional-order Bessel beams (FBBs), and interference patterns of fractional-order Bessel beams, have been generated by using DOEs. It is the first time that such a beam (FBB) that possesses multiple diffraction properties such as, non-diffraction, self-reconstruction, helical-phase, and asymmetrical transverse intensity, has been found. Furthermore, the interference patterns and the generation of FBBs have also been examined. These novel laser beams provide potential applications in optical micro-manipulations.

Moreover, the author utilized an effective and simple technique generating an array of propagation-invariant beams. A composite hologram which consists of many smaller holograms can be employed to reconstruct N by N array of Bessel beams, optical bottle beams, self-imaging beam, or vortex beams. The simulations and experiments have demonstrated generation of arrays of Bessel beams and optical bottle beams. In addition to that, the shape of the arrays can be customized into any geometrical patterns such as a triangle, a cross, a square, and etc.

Finally, the author designed and fabricated a Dammann grating to split a beam into multiple beams. On the other hand, the grating was also applied for a five-beam addition. The proposed beam-addition system is simple and compact, and the preliminary experiments showed that the coupling efficiency has achieved over 52%. The coupling efficiency could be higher if a recommended configuration is implemented.

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CHAPTER 1 INTRODUCTION

1.1 Motivation

Since the hologram concept was first proposed by Gabor in 1947 [1] and the first laser in the world was invented by Maiman in 1960 [2], developments of holography have been greatly influenced by new technologies and new applications. Today, holograms are not limited to the product of 3-D imaging generated by interference, but are extended to a larger category that apply diffraction theory to obtain a specified beam distribution using phase modulation. Diffractive optical elements (DOEs) are devices where the working principle is based on diffraction theory rather than conventional geometric optics. Unlike conventional refractive or reflecting optical elements such as lenses or mirrors, DOEs not only fulfill the functions of conventional optical elements, but also have many additional advantages. DOEs, which consist of computer-generated holograms (CGHs), diffractive phase plates, volume holograms, diffraction gratings, kino-forms, zone-plates, and so on, are mainly based on a surface-relief pattern that modulates an optical wave's phase to control the propagating wave-front's behavior. DOEs can be used for splitting, combining, reflecting, or re-forming light beams and are well-known for excellent flexibility, high efficiency, and high accuracy in the control of light wave-fronts. Design methods for different beam shaping projects can be varied. DOEs can be categorized mainly into four groups, such as (a) DOEs with regular structures or obtained by using analytical phase functions; (b) DOEs generated by interferences; (c) DOEs generated using computer algorithms; and (d) DOEs generated by other means. Moreover, in each above category the design means can be classified further. Applications of DOEs have been employed in areas such as optical imaging and displaying, beam shaping, optical interconnections, optical data processing, optical trapping, information security, and etc.

DOEs' extending applications and capabilities have resulted in more intensive interests in their studies in recent years. For example, gratings can be used in optical fiber communications [3]; CGHs has been integrated in optical trapping systems [4]; Fresnel zone plates are used for focusing the lights in microscope systems, optical writing systems, and optical imaging systems [5-6]; DOEs are also applied in laser resonators to tailor the output laser beam directly [7]. Furthermore, investigation of DOEs has also enhanced developments

in related areas such as micro-fabrication technology, beam property studies, and the interaction between beam and particles, etc.

Bessel beams [8] are well-known for non-diffraction or propagation-invariance; as a result, they have been used as optical tweezers in particle trapping and guiding [9]. Moreover, as higher-order Bessel beams demonstrate helical phase structure in propagating, they also have orbit angular momentum (OAM) which enables micro-particles to rotate [10]. Currently, as Bessel beams are the main optical tools used in the new generation of optical tweezers system, studies on the Bessel beams and relevant laser beams are also becoming hot topics in optical manipulation fields.

Besides research on the applications of DOEs, studies on the design and fabrication technology are also very important to the development of DOEs; for example, when pixel size of a DOE is comparable to or small than the working wavelength, the DOE demonstrate a new function such as polarization-selection, which can be utilized in optical switch or in information storage [11-12].

With these understandings, the author explores research on DOEs' design, fabrication, testing, and applications. Furthermore, the author employs DOEs to generate some novel laser beams, which have potential applications in optical trapping and data processing, and investigates properties of the novel beams extensively.

1.2 Objectives

Generally, research on DOEs covers three aspects, i.e. design, fabrication, and applications. Since the three aspects are closely integrated in the DOE's developments, any improvement made from one of the three aspects would affect the developments of the other two aspects.

The main objectives of this thesis are as follows:

Develop a set of algorithms for designing various DOEs; investigate some practical problems in DOE's design and fabrication, and work out corresponding solutions to the specified examples such as phase quantization and DOE multiplexing.

Employ micro-fabrication techniques to realize designs of DOEs and to improve performances of DOEs.

Utilize DOEs to generate some novel laser beams for potential applications in optical trapping, and study their properties as well.

Generate arrays of propagation-invariant beams and interfering patterns of these beams for potential applications in optical trapping.

Implement applications on optical beam-addition using DOEs to obtain a higher-power laser beam.

1.3 Major contributions of the thesis

In this study, a hybrid algorithm for the design of DOEs has been developed. The hybrid algorithm retains the advantages of the Gerchberg-Saxton algorithm (G-S) and the simulated annealing algorithm (SA) while restraining the disadvantages. Thus, the hybrid algorithm is more efficient, flexible, and applicable to both the near and the far diffraction-fields. Furthermore, on the basis of the iterative-Fourier transform algorithm (IFTA), an effective and practical quantization method has been found to improve the performance of quantized DOEs. This technique can increase reconstruction efficiency of the DOEs in the fabrication. Moreover, an optimization algorithm has been developed in multiplexing two polarization-selective DOEs; the algorithm can significantly simplify structures of the multiplexed DOE while retaining its diffraction efficiency.

Fabrication technology is a key factor in fulfilling the design target of DOEs. The author has found a new technique for generating a multi-level DOE (MDOE) by using e-beam direct writing and UV lithography. In addition, a novel technique by use of a polydimethylsiloxane (PDMS) material has been employed to replicate multi-level DOEs. This novel fabrication technique cannot only improve reconstruction efficiency of DOEs, but also simplify the fabrication process.

Using DOEs, the author has produced a family of novel laser beams that are named as fractional-order Bessel beams (FBBs). The beams have been proven to be nondiffracting like the zero-order Bessel beam and the higher-order Bessel beams, and the fractional-order Bessel beams are also with helical phase in propagation. Moreover, the fractional-order Bessel beams

intrinsically have an opening slit in their concentric intensity distributions during the propagation in free-space. In this thesis, the fractional-order Bessel beams have also been proven to be stable and self-reconstructive in the free-space propagation although they are asymmetrical in transverse intensity. It is the first time that such optical beams have been generated, and have been found to possess multiple unique properties. Furthermore, the author studied a series of interference patterns of the fractional-order Bessel beams, and worked out a means to generate various interference patterns of the beams. As the interference patterns have different characteristics and distributions depending on the combinations of the parameters of the fractional-order Bessel beams involved, they provide some special tools for applications in optical trapping.

The author first used one single DOE illuminated by one beam to generate an array of propagation-invariant beams. The array can comprise Bessel beams, optical bottle beams, self-imaging beams, and so on. In addition, shape of the array can be customized by arranging the combinations of the smaller holograms in the DOE.

Finally, the author designed, fabricated, and tested a Dammann grating for beam addition. The grating has been successfully used to coherently combine five laser beams into one. The proposed beam addition method is simple and efficient.

1.4 Organization of the thesis

The thesis is organized in eight chapters: in Chapter 1, a brief introduction is given to present the motivation, objectives, and major contributions in this thesis.

In Chapter 2, an overall introduction in DOEs' design is presented first, followed by a description of DOE design using digital holography, and then an algorithm integrated with G-S algorithm and SA algorithm is introduced. Finally, a DOE design using analytic formulas is investigated.

In Chapter 3, an effective and practical quantization method is proposed first, and then an optimization algorithm multiplexing two polarization-selective DOEs is presented.

In Chapter 4, the author describes a contact printing method for fabricating a binary DOE, which converts a Gaussian beam into a Bessel beam, followed by an introduction to

fabricating a multilevel DOE, and finally describes the soft replication technique in generating multi-level DOEs.

In Chapter 5, an overall introduction to the fractional-order Bessel beams is given first, and then generations of a fractional-order Bessel beam are presented, after that, properties of the fractional-order Bessel beam are investigated, furthermore, interference patterns of the fractional-order Bessel beams are also studied.

In Chapter 6, method of using one single DOE to generate an array of propagation-invariant beams has been realized.

In Chapter 7, five laser beam addition using a Dammann grating is implemented.

In Chapter 8, conclusions are drawn and recommendations for further research are provided.

Moreover, to illustrate the design process of DOEs more clearly, the author attaches some program codes, which are created and run under a Matlab environment, in the appendix; in addition, a data conversion program for laser writing system and e-beam writing system and analysis of diffraction efficiency of holograms are also appended.

CHAPTER 2 PRINCIPLES OF DIFFRACTIVE OPTICAL ELEMENTS (DOES) DESIGN

2.1 Introduction

Beam shaping, which re-forms one laser beam profile into a desired form on the basis of diffraction, is required in a variety of applications. The key devices used in the transformation are mainly refractive optical elements or DOEs. The DOEs are more widely utilized for laser beam shaping because of their compact size, good performance, and high flexibility. DOEs are usually fabricated as phase-only or amplitude-only to modulate optical light. The phase-only DOEs, which modulate optical beam's phase, are preferred in beam shaping as they have higher efficiency. The performance of a DOE's reconstruction is often judged by the reconstruction efficiency, which is defined as the energy ratio of the reconstructed image to the desired image.

As generation of DOEs can be classified into many categories, such as iterative algorithm, optimization method, holographic interference, and simulation of phase or amplitude distribution of an analytic formula, it is our task to select the most efficient and simplest method to design a DOE. Three methods of generating DOEs are investigated in this Chapter, and these methods will be employed in designing DOEs in the following Chapters.

When an object beam and one reference beam interfere, the interference pattern can be recorded as a hologram, which will reconstruct the object light when the hologram is illuminated by the reference light [13]. Although this hologram can faithfully reconstruct the object light's amplitude and phase, the reconstruction efficiency is low due to many diffraction terms accompanied. Implementing interference between an object and a reference is also troublesome when the object is unavailable. Thus, digital holography will be introduced in Section 2.2 to solve that problem. Using this method, we can reconstruct arbitrary complex-amplitude distribution without implementing the interference in real situations.

DOEs that are obtained by running an iterative algorithm can produce any intensity distribution in the diffraction plane. This method is well known phase retrieval problems,

which searches phase distributions to satisfy the constraints of the input and the output intensity distributions [14]. Numerous approaches to the phase retrieval problem have been proposed, and the most commonly used is the iterative Fourier transform algorithm (IFTA) [15-21]. The IFTA, which is not only simple in implementation but also efficient in handling large amounts of data, was first developed by Gerchberg and Saxton in 1972 [15], and has been modified and improved by a number of authors, e.g., Fienup [16], Wyrowski and Bryngdahl [20], *et al.* However, the algorithm is prone to stagnation in local minimum or to pre-convergence, thus the resultant DOE would produce many noisy speckles in the reconstruction. To improve reconstruction efficiency of DOEs, many pure and hybrid optimization methods have been proposed; the pure optimization methods include a simulated-annealing [22-26], a Genetic algorithm [27-29], a conjugate-gradient algorithm [30], a Monte Carlo method [31-32], and etc. [33]; hybrid optimization methods are the combinations of two or more pure optimization algorithms [34]. When we choose algorithm for a DOE design, reconstruction efficiency of the DOE and running time of the program must be considered first. For this reason one hybrid algorithm comprising the IFTA and the simulated-annealing algorithm will be developed in Section 2.3.

As distributions of some optical beams (such as the Bessel beams) are the solutions to some defined equations, the solutions can be obtained by solving diffraction integrals or ray tracing equations analytically. It is sensible for us to apply the analytic solutions for a DOE to reconstruct the optical beams inversely, that is to say, if analytic solutions are known to us already, generating such DOEs that simulate the analytic solutions will be very easy. As this kind of DOEs is very efficient and accurate in reconstructing object beams, it will be introduced in Section 2.4 and applied to generate novel laser beams in the following chapters as well.

2.2 DOE design using digital holography

Holography records interference patterns of light waves onto light sensitive film (or CCD). When the film is developed and re-exposed to the reference light, it re-creates all the points of light that originally came from the object. The material recorded with interference patterns is named as hologram. The term hologram was from the Greek words holos, meaning "whole," and gram, meaning "message." Holography dates from 1947 [1, 13], when British scientist Dennis Gabor developed the theory of holography while working to improve the resolution of an electron microscope. In 1971 Dennis Gabor was awarded the Nobel Prize in Physics for his discovery of holography in 1947. In 1962 Leith and Upatnieks [35] invented an "off-axis"

technique borrowed from their work in the development of side-reading radar. Their result was the first laser transmission hologram of 3-D objects (a toy train and bird). These transmission holograms produced images with clarity and realistic depth but required laser light to view the holographic image.

Holography's unique ability to record and reconstruct optical information makes it a valuable tool for industry, science, business, and education. Applications in areas such as optical imaging, displaying, information security, and etc. have been employed as well.

Conventional configurations for holographic recording and reconstruction are schematically shown in Figure 2-1 [36].

Figure 2-1. (a) Recording of a hologram; (b) reconstruction of the hologram

Assume the object light wave's complex amplitude is $O(x,y)\exp[i\alpha(x,y)]$ and the reference light's is $R(x,y)\exp[i\beta(x,y)]$, then the interference pattern on the recording film is written as coherent addition of the two light waves, i.e.,

$$A(x,y) = O(x,y)\exp[i\cdot\alpha(x,y)] + R(x,y)\exp[i\cdot\beta(x,y)] \quad (2-16)$$

The beam intensity on the recording film is written as

$$\begin{aligned} I(x,y) &= |O(x,y)\exp[i\cdot\alpha(x,y)] + R(x,y)\exp[i\cdot\beta(x,y)]|^2 \\ &= |O(x,y)|^2 + |R(x,y)|^2 + O(x,y)\exp[i\cdot\alpha(x,y)] \cdot R(x,y)\exp[-i\cdot\beta(x,y)] + \\ &\quad O(x,y)\exp[-i\cdot\alpha(x,y)] \cdot R(x,y)\exp[i\cdot\beta(x,y)] \end{aligned} \quad (2-17)$$

There are four terms in Equation (2-17). When the intensity is recorded linearly, a hologram is formed. Thus, the reconstruction process of the hologram can be mathematically denoted by

$$\begin{aligned} U(x,y) &= t \cdot I \cdot R(x,y)\exp[i\cdot\beta(x,y)] \\ &= t\{|O(x,y)|^2 \cdot R(x,y)\exp[i\cdot\beta(x,y)] + |R(x,y)|^2 \cdot R(x,y)\exp[i\cdot\beta(x,y)] \\ &\quad + O(x,y)\exp[i\cdot\alpha(x,y)] \cdot R^2(x,y) + \\ &\quad O(x,y)\exp[-i\cdot\alpha(x,y)] \cdot R^2(x,y)\exp[i\cdot 2\beta(x,y)]\} \end{aligned} \quad (2-18)$$

where t is a constant related to the recording material. Hence, the object (the third term) and the conjugate object (the fourth term) are reconstructed, respectively.

However, it can be seen from Equation (2-16) that the object light wave must take part in the optical recording process so that the hologram can be produced, that is to say, typical holography cannot be used to reconstruct a wave that is unavailable for the initial interference; furthermore, the set-up requires critical stability because movement as small as a quarter wavelength of light during exposures of a few minutes or even seconds can completely spoil a hologram. Hence, a new kind of holography technology named digital holography has been invented. Digital holography uses computer-generated holograms for optical reconstruction, or numerically reconstructs holograms that are optically recorded with a CCD camera. For example, suppose that an object's complex amplitude is given as $E(\rho,\varphi) = J_1(\alpha\rho)\exp(in\varphi)$,

where α is a constant, and J_1 is the first-order Bessel function of the first kind, thus, the hologram generated by interference with a plane wave is shown in Figure 2-2.

Figure 2-2. Hologram generated by computer from simulating holography recording.

It should be noted that reconstruction efficiency of the hologram generated by holography is low as the reconstruction light comprises four components shown in Equation (2-18) and only one of the terms is useful. The detailed calculation of the reconstruction efficiency can be found in Appendix C.

Using the technology of digital holography, we can optically reconstruct any object in high fidelity if the object's complex amplitude is known to us beforehand. Moreover, we can easily generate such holograms with numeric recording. Some applications that utilize digital holography for generating novel laser beams will be presented in detail in Chapter 5.

2.3 DOE design with numerical algorithms

2.3.1 Gerchberg-Saxton algorithm (G-S)

A beam shaping system shown in Figure 2-3 composes of an incident light in the input plane, a DOE in the input plane, and a desired distribution in the output plane. The system is assumed to be illuminated by an incident light which is monochromatic, and intensity or phase distributions of both the input light and the output light are known to us. When the incident light in the input plane reaches the output plane through a specified DOE, the desired distribution will be formed. For simplicity, the DOE here is referred to as phase-only, and the desired output distribution is an intensity profile with arbitrary phase. The arbitrary phase gives freedom to the intensity-intensity constraint and does not affect the intensity distribution

in the output plane. The incident beam is assumed to be propagated in the free-space. The main problem in beam shaping is to find an optimized phase distribution for the DOE.

Figure 2-3. Schematic diagram of beam shaping with DOE.

The DOEs are usually designed with an iterative algorithm based on the G-S algorithm, which is depicted as follows [15]: the process begins with an incident beam of given amplitude $A_1(u, v)$ and random initial phase $\theta_1(u, v)$. Firstly, the complex amplitude of the incident beam is transformed (T) into an image with complex amplitude $AA_{2n}(x, y)\exp[i\theta_{2n}(x, y)]$, where n is the number of iterations. Then the intensity-intensity constraint is applied, i.e., $AA_{2n}(x, y)\exp[i\theta_{2n}(x, y)]$ is replaced with $A_2(x, y)\exp[i\theta_{2n}(x, y)]$, after that, the new complex amplitude $A_2(x, y)\exp[i\theta_{2n}(x, y)]$ will be inversely transformed (T^{-1}) into $AA_{1n}(u, v)\exp[i\theta_{1n}(u, v)]$. Similarly, the constraint is applied again, i.e., $AA_{1n}(u, v)\exp[i\theta_{1n}(u, v)]$ is replaced with $A_1(x, y)\exp[i\theta_{1n}(x, y)]$. The iteration is repeated until the number of the iterations reaches to the set number N . Finally, the sought phase distribution for the DOE is phase $\theta_{1N}(u, v)$ at iteration N . The flow chart of the G-S algorithm is shown in Figure 2-4.

Figure 2-4. Block diagram of the G-S algorithm.

The G-S algorithm is very simple and it is quick to obtain a solution to this beam shaping problem. However, the initial random phase and the phase freedom in the image plane introduce speckle noise in the reconstructed image. Furthermore, the algorithm tends to stagnate, thus the resultant DOE usually has low reconstruction efficiency.

Diffraction efficiency is a very important parameter to determine the performance of a DOE. The diffraction efficiency is the ratio of the intensity of the desired order to the total intensity of the incident beam. When pixel size of DOEs is much larger than the wavelength, the DOEs are considered thin, so their diffraction efficiency can be estimated based on the scalar diffraction theory. Roughly, DOEs with continuous phase profile can have diffraction theory as high as 100%, and DOEs with binary phases have the maximum theoretical diffraction efficiency of 40.5%. A diffraction efficiency list [37] is shown in Table 2-1 for DOEs with different quantized phases, and detailed calculation is attached as Appendix C.

Table 2-1. The maximum theoretic diffraction efficiency for different DOEs

Quantization level	2	4	6	8	10	16	20	32	∞
Diffraction efficiency	40.5 %	81.1 %	91.2 %	95.0 %	96.8 %	98.7 %	99.2 %	99.7 %	100 %

2.3.2 Simulated Annealing algorithm (SA)

Simulated annealing (SA) algorithm is a computational optimization method that is analogous to the annealing of metals. The system to be optimized consists of a variable and a cost function representing the system configuration. Using the probability process with an appropriately controlled temperature parameter that is employed in the Boltzman probability distribution, we can find with simulated annealing the global minimum of the cost function that corresponds to the optimum condition of the system [38]. In this algorithm, the variable is the phase value of the DOE, and the cost function is related to the performance of the DOE's reconstruction. It is obvious that if the reconstruction efficiency is higher, the DOE's phase will be closer to the optimum.

The cost function can be mean-square error (MSE) or root-mean-square (RMS) error. Both the MSE and the RMS have a trade-off with reconstruction efficiency of DOEs. The MSE is defined as.

$$MSE = \sum_{x,y} |I_0(x,y) - \alpha \cdot I(x,y)|^2 \quad (2-1)$$

where $I_0(x,y)$ is the intensity of the desired image and α is a scale factor given by

$$\alpha = \frac{\iint I_0(x,y) dx dy}{\iint I(x,y) dx dy} \quad (2-2)$$

$I(x,y)$ can be obtained by the transformation of the incident beam with the phase of the DOE. Alternatively we can adopt the RMS as the cost function. The RMS is defined as

$$RMS = \sqrt{\sum_{x,y} |I_0(x,y) - \alpha \cdot I(x,y)|^2} \quad (2-3)$$

Thus the cost function E can be represented by either MSE or RMS. The transformation function is dependent on the exact diffraction system, for example, in a Fourier transform system, the $I(x,y)$ can be written as

$$I(x,y) = |FT[U(x,y) \cdot \exp(i \cdot \theta)]|^2 \quad (2-4)$$

where $U(x,y)$ represents the complex distribution of the incident beam. θ is the DOE's phase distributions ranging between 0 and 2π . In practice, DOE's phases are quantized because of the resolution limitations of recording devices and fabrication tools. θ_q is represented by the quantization function below,

$$\theta_q = \text{floor}\left[\frac{\theta \cdot Level}{2\pi}\right] \cdot \frac{2\pi}{Level}, n=0, 1, 2, \dots, L-1 \quad (2-5)$$

where $Level$ is the number of quantization level, floor is a function that rounds the element to the nearest integer towards minus infinity.

The SA algorithm can be described as follows. A random phase distribution of DOE is given first, and the temperature is set to a relatively high value to determine a large range of the perturbation probability. The convergence condition depends on the initial temperature and the cooling rate. If the initial temperature is low or is decreased at too fast a rate, the system could become trapped in a local minimum [38]. The cost function is written on the basis of actual diffraction system. After the initialization, a first cost function value with the initial phase is calculated, after which, one pixel of the DOE is randomly selected and its current phase value is replaced with any of the other quantization levels. As a result, a new cost function E_n is calculated again. After that, a variation $\Delta E = E_n - E_i$ is calculated, if the variation is less than zero the phase replacement is accepted; otherwise, acceptance or rejection of the phase replacement is determined by the probability of the Boltzman distribution, which is given as

$$P(\Delta E) = \exp\left(-\frac{\Delta E}{T}\right) \quad (2-6)$$

where T is the temperature of the simulated annealing. The concept “simulated annealing” originated from the annealing process of a metal, in which the melt is slowly cooled to be re-crystallized at the lowest temperature from a disordered state at initial high temperature. If the initial temperature of the metal is too low or the cooling is too quick the system may become quenched forming defects or freezing out in metastable states (i.e. trapped in a local minimum energy state); but if the initial temperature is too high or the decrement of temperature is too small, the cooling time may be very long. Thus, to avoid the metal system trapped in local minimum energy state (quenching) and the annealing time unbearable we choose suitable annealing parameters, such as initial temperature, iterations to be performed at each temperature, and decrement of temperature at each cooling step, to control the annealing process. The process of searching optimum phase of a DOE is like the annealing process of a metal, and the DOE with the highest reconstruction efficiency is analogous to the metal with the global minimum energy state.

The above pixel replacement procedures will be repeated till the variation falls below the set value. Then the temperature is decreased at a rate of

$$T = \frac{T_0}{1+t} \quad (2-7)$$

where T_0 is the initial temperature and t is the iteration number. After every decrease in temperature, the pixel phase replacement iteration will continue again until the value of the cost function has converged to a minimum value. Then all the iterations are terminated and the phase with the minimum cost function value is saved as the final distribution of the DOE. A typical SA algorithm is schematically shown in Figure 2-5 [38].

Figure 2-5. Flow chart of the SA algorithm.

The SA algorithm takes advantages of strong search ability and global optimization, but its running time is very noticeable. As there are many sets of loop in the algorithm, the time consumption will be huge when the amount of data is large. For example, it may take several days to get an optimized 256×256 -pixel phase distribution.

2.3.3 Hybrid algorithm (G-S and SA)

The G-S is quick at getting a phase distribution although the resultant DOE has low reconstruction efficiency, and the SA can optimize a phase distribution globally although it is time-consuming. As both the G-S algorithm and the SA algorithm have advantages and disadvantages in designing a DOE, it is understandable to combine them so that the hybrid algorithm can complement each other and restrain their disadvantages. After the combination, the phase obtained by the G-S is used as the initial phase, which will be optimized further by the SA. Hence, the hybrid algorithm is able to get a phase distribution faster and with higher reconstruction efficiency. The block diagram of the hybrid algorithm is presented in Figure 2-6.

Figure 2-6. Block diagram of the hybrid algorithm.

The hybrid algorithm applies the SA's temperature probability to accept or reject a pixel phase change if the related cost function value is not less than that of the previous phase distribution. However, this probability control may result in the cost function values being unstable or trapped in local minimum, and the process could be time-consuming due to the temperature probability control. Thus, to append the problem, we can remove the temperature probability

control and replace it with an error-reduction control, where the acceptance or rejection of a phase change is only determined by the variation of the cost function values, i.e., if the variation $\Delta E = E_n - E_{n-1}$ is not greater than zero, the change will be accepted; otherwise, the change will be rejected. With this error-reduction control, the cost function values will not fluctuate too much, as a result the running time of the program will be reduced.

Moreover, in the hybrid algorithm, the light beam's propagation processes between the input plane and the output plane are related to transform functions; in practice the transformation, which can be Fourier transform or Fresnel diffraction integral, depends on the exact propagation process of the light beam. Thus, we can obtain phase distributions of DOEs in the near field or the far field by selecting corresponding transform-functions in the hybrid algorithm.

2.3.4 Example: re-forming Gaussian beam to flat-top beam

An example demonstrating the usage of the hybrid algorithm is presented. A Flat-top beam is one whose intensity distribution in the target area is uniform. It is required in many applications such as material processing, laser projection printing, optical data processing, and optical gauging [39-40]. Numerous ways have been proposed to obtain a uniform light using refractive optical components or diffractive elements, such as, Shealy *et al.* [41] designed an afocal optical system, made of two aspherical lenses, that converts a Gaussian beam into a uniform beam with high reconstruction efficiency and light uniformity; however, the system is rather complicated to produce. Veldkamp [42] described a method that used diffraction gratings to obtain uniform far-field intensity but strong side-lobes appeared in his results. Han *et al.* [43] used a pair of holograms for getting a uniform near-field phase and intensity distribution in a circular region; the first hologram redistributes energy of the beam to a rectangular profile, while the second one subsequently collimates the beam; however, precise alignment between the two holograms in the optical system is strictly necessary. Ido and Mendlovic [44] suggested using a binary phase DOE to merge +1st and -1st diffraction orders for creating a flat-top; this method is stable and efficient but inflexible in generating a flat-top beam at a given area. The G-S algorithm has problems referring to the convergence especially for diffraction limited sizes, as the light uniformity is restricted in a given area. Moreover, most of the DOE reshaping methods have used Fourier lens to reconstruct the uniform beam at the far-field. A transfer function approach to designing DOE will be employed to yield a flat-top beam in the near-field, thus the Fourier lens is removed from the optical system, and the flat-top beam can be reconstructed at a given distance.

For the hybrid algorithm, only numeric solutions to the phase retrieval problem can be found, so we have to use discrete functions to represent the continuous functions; the sampling rule follows the Whittaker-Shannon sampling theorem. When the DOE and image planes are arbitrarily close, the exact form for the propagation of an optical disturbance must be used. A treatment of scalar diffraction theory by Harvey [45] is provided. The method provides an alternative to the 'angular plane wave spectrum' treatment [46], which has been developed for the description of light propagation in the Fresnel regime [45-47]. The transfer function in Fourier space is described as [47-48],

$$H(u, v, z) = \exp[i \cdot 2\pi z \cdot (1 - u^2 - v^2)^{1/2}] \quad (2-8)$$

where u and v are the spatial frequencies in the x and y directions, respectively, and z , the distance between the incident and image planes, is treated as a parameter. If $f(x, y)$ is the optical disturbance at $z=0$, then the optical disturbance $g(x, y)$ in the plane at $z>0$ is given by

$$g(x, y, z) = FT^{-1}\{FT[f(x, y)]H(u, v, z)\} \quad (2-9)$$

where FT and FT^{-1} denote the Fourier transform and the inverse Fourier transform, respectively. Note that all physical coordinates are scaled by the illuminating wavelength λ so that all the quantities are dimensionless, that is, x should be understood as x/λ , u as λu , etc.

Assume that the incident beam is a Gaussian profile beam with a waist width of w_0 written by

$$f(x, y) = \exp\left(-\frac{x^2 + y^2}{w_0^2}\right) \quad (2-10)$$

The DOE is placed in the incident plane of the Gaussian beam, thus the expression after the DOE is written as

$$f(x, y) = \exp\left(-\frac{x^2 + y^2}{w_0^2}\right) \cdot \exp(i \cdot \theta) \quad (2-11)$$

where θ is the phase distribution of the DOE. The flat-top beam's modulus is expressed by a super Gaussian beam as is shown in Equation (2-12).

$$g(x, y) = \exp\left(-\frac{x^{16} + y^{16}}{w^{16}}\right) \quad (2-12)$$

where w is the width of the flat-top area of the beam. After sampling the continuous functions as discrete, we can run the hybrid algorithm to yield a phase distribution of DOE. The intensity distributions of the input beam (the Gaussian beam), and the desired flat-top beam (the super Gaussian beam) are shown in Figure 2-7 and Figure 2-8, respectively.

Figure 2-7. Incident Gaussian profile beam.

Figure 2-8. Desired flat-top beam.

In the G-S iteration, the propagation processes between the input plane and the output plane are written as

$$g_k(m, n) = FT^{-1}\{FT\{|f(m, n)|\exp[i \cdot \theta_k(m, n)]\} \cdot \exp(i \cdot 2\pi \cdot z) \cdot [1 - (md_1)^2 - (nd_2)^2]\},$$

$$\theta_k = \text{angle}[f_{k-1}(m, n)]$$

$$f_k(m,n) = FT^{-1}\{FT\{|g(m,n)|\exp[i \cdot \alpha_k(m,n)]\} \cdot \exp\{(-i \cdot 2\pi \cdot z) \cdot [1 - (md_1)^2 - (nd_2)^2]\}\},$$

$$\alpha_k = \text{angle}[g_k(m,n)], \quad d_{1,2} = \lambda / D_{1,2}, \quad m = 1,2,\dots,M, \quad n = 1,2,\dots,N$$

(2-13)

where $D_{1,2}$ is the width of the DOE. The iteration starts with DOE's initial phase distribution (a M by N matrix with random phase values). The number of the G-S iterations is 100. The sampling points in the program of the hybrid algorithm are 128 by 128 with each pixel size of 100 μm by 100 μm ; thus the size of the DOE is 12.8 mm by 12.8 mm. The waist width of the Gaussian beam is 12.8 mm, and the super Gaussian beam's is 0.64 mm. The reconstruction distance z of the DOE is set as 4000 mm. To facilitate the fabrication, we set phase step of the DOE as binary. The 3D and 2D reconstruction images after the G-S iterations are shown in Figure 2-9 and Figure 2-10, respectively. It is clearly seen in Figure 2-7 that the non-uniformity (which is defined later) in the desired flat-top area is very noticeable.

Figure 2-9. 3D reconstruction after the G-S.

Figure 2-10. 2D reconstruction after the G-S.

The SA part of the hybrid algorithm is run 2000 iterations for the temperature cooling and 100 iterations for the phase-change; the cost function in the this part is defined as,

$$Cf = \frac{\sum_{m,n} (|g_k(m,n)| - |g(m,n)|)^2}{\sum_{m,n} |g(m,n)|^2},$$

$$g_k(m,n) = FT^{-1}\{FT\{|f(m,n)|\exp[i \cdot \theta_k(m,n)]\} \cdot \exp(i \cdot 2\pi \cdot z) \cdot [1 - (md_1)^2 - (nd_2)^2]\}$$

(2-14)

Variation of the cost function values between the adjacent two phase-change iterations can be calculated based on this equation too.

The curve of the value of the cost function in the SA part is plotted in Figure 2-11. The optimization process decreases the value of the cost functions from ~0.09 to ~0.015.

Figure 2-11. Plot of the cost function in SA.

The final phase distribution of the DOE is shown in Figure 2-12, where the black denotes phase value 0 and the white denotes π .

Figure 2-12. Phase distribution of the DOE after SA optimization.

The 2-D and 3-D reconstruction images of the DOE are shown in Figure 2-13 and Figure 2-14, respectively.

Figure 2-13. 3D reconstruction after the SA.

Figure 2-14. 2D reconstruction after the SA.

The non-uniformity of the reconstruction image is calculated as,

$$U = \frac{\sum_{a,b} (|g_k(a,b)| - |\bar{g}_k|)^2}{\sum_{a,b} |g(a,b)|^2} \quad (2-15)$$

where (a, b) is the point in the area of the desired flat-top; $\bar{g}_k$ is the average value of all the points within the area of the desired flat-top. The computed non-uniformity is ~6.5%. For the comparison of the results without SA and with SA, close-up 3D reconstruction images including the flat-top area are shown in Figure 2-15 and Figure 2-16, respectively.

Figure 2-15. 3D reconstruction image without SA.

Figure 2-16. 3D reconstruction image with SA.

It can be seen in Figures 2-15 and 2-16 that the final result of the hybrid algorithm is much improved from that of a pure G-S algorithm.

2.4 DOE design based on analytic solutions

Besides the DOE designs mentioned above, there exists one kind of DOEs whose transmission can be derived from its desired reconstruction distribution, which can be written in an analytic expression. The reconstruction of such kind of DOEs is in excellent fidelity to the desired object, and the diffraction efficiency of the DOEs can also be very high. For example, if the DOE has continuous phase, the diffraction efficiency can be 100%. Even for a 4-level phase hologram, its diffraction efficiency can be over 80%. Distribution of a lens-array DOE can be calculated from phase expression of every single Fourier lens, where the Fourier lens can be represented by the expression: $\exp[-i\frac{k(x^2 + y^2)}{2f}]$, f is the focal length of lens, and k is the wave vector. Fresnel zone plates can also be designed by an analytic solution as follows [49]:

$$r_m = (2m\lambda f)^{1/2}, \Phi_r = \pi r^2 / (\lambda f) \quad (2-19)$$

where f = focal length, λ = wavelength, Φ_r = phase variation, r_m = radius of m th zone, m = ordinal zone number. Sample of such a Fresnel zone plate is shown in Figure 2-17 [50].

Figure 2-17. A sample of Fresnel zone plate.

A beam that is known as an optical doughnut or optical vortex, with phase singularities, has received great interest in recent years; such a beam has been widely applied in optical trapping and studied because it exhibits orbital angular momentum (OAM) [51-53]. As we know, when a beam has one phase singularity or more, the beam will form one optical intensity doughnut or more in free-space propagation. Each dark spot has a topological charge that represents the number of 2π accumulated when the phase gradient is integrated around it. The phase near a singularity has a helical structure, while the intensity at the singularity must be zero. With these understandings, we can generate a DOE, which will be reconstructed as a doughnut beam, using the following transmission:

$$t(\theta) = \exp(i \cdot n\theta) \quad (2-20)$$

where θ is a polar angular coordinate in the hologram plane; n is an integer.

Furthermore, the Bessel beam, which is well known to be non-diffracting [8] i.e., the transverse complex amplitude at propagation distance $z=0$ is the same as that at subsequent propagation distance z , can also be generated following an analytic expression. The distribution of the Bessel beam in cylindrical coordinates is defined by

$$A(\rho) = J_0(k_r \cdot \rho) \quad (2-21)$$

where J_0 is the zero-order Bessel function of the first kind, k_r is the radial wave vector, and $\rho = (x^2 + y^2)^{1/2}$. The practical Bessel beams are nondiffracting only within a limited propagation distance because of its finite size of the beam. The nondiffracting distance can be calculated by,

$$Z_{\max} = k \cdot a / k_r \quad (2-22)$$

where k is the wave vector of the beam, a is the radius of the beam.

Let us suppose that a plane wave illuminates a circular DOE of radius D ; the phase distribution of the DOE is characterized by [54-55],

$$t(r) = \exp(-i \cdot 2\pi \cdot r / r_0) \quad (2-23)$$

where r_0 is a constant related to the k_r . The electric field in an arbitrary plane $z=\text{constant}$ behind the finite-aperture hologram can be calculated by a Fresnel integral,

$$E(\rho, \varphi, z) = \frac{\exp(ikz)}{ikz} \exp\left(\frac{ik\rho^2}{2z}\right) \int_0^{D/2} \int_0^{2\pi} t(r) \exp\left(\frac{ikr^2}{2z}\right) \exp\left[\frac{-ikr\rho \cos(\theta - \varphi)}{z}\right] 2\pi r dr d\theta \quad (2-24)$$

Here ρ and φ are coordinates in the observation plane; r and θ are coordinates in the hologram plane. Thus, we obtain the resultant integral of Equation (2-24) as [55],

$$E(\rho, \varphi, z) = C \cdot J_0\left(\frac{2\pi\rho}{r_0}\right) \quad (2-25)$$

where C is a constant. This output represents a zero-order Bessel function profile exactly. Therefore, if a DOE is given a distribution of Equation (2-23), it reconstructs a zero-order Bessel beam at plane z . Similarly, we can use a DOE with the following formulae to reconstruct a higher-order Bessel beam [55]:

$$t(r, \theta) = \exp(-i \cdot 2\pi \cdot r / r_0) \cdot \exp(i \cdot n\theta) \quad (2-26)$$

The electric field that is formed at a distance z from the plane of the hologram is given by

$$E(\rho, \varphi, z) = C \cdot J_n\left(\frac{2\pi\rho}{r_0}\right) \exp(i \cdot n\varphi) \quad (2-27)$$

where J_n represents a n th-order Bessel function of the first kind. Furthermore, apart from the fact that the density of the Bessel beam rings can be adjusted by the parameter r_0 , the center of the Bessel beam can be relocated in off-axis by adding a carrier frequency into Equation (2-23) or Equation (2-26); the carrier frequency is written as

$$t(r) = \exp(-i \cdot 2\pi \cdot r / r_0) \quad (2-28)$$

where v is an adjustable constant, $r \sin \theta$ represents the shifted distance of the Bessel beam center along y axis in the Cartesian coordinates.

Moreover, since we can generate the zero-order Bessel beam and the higher-order Bessel beams by using transmission of Equation (2-23) or Equation (2-26) correspondingly, the interference of the Bessel beams can also be generated by coherently adding the **respective** formula. For example, interference pattern of a zero-order Bessel beam and a first-order one can be generated using the following transmission

$$t(r, \theta) = \exp(-i \cdot 2\pi \cdot r / r_0) + \exp(-i \cdot 2\pi \cdot r / r_0) \cdot \exp(i \cdot \theta) \quad (2-29)$$

Thus, it is not difficult for us to generate various interference patterns of the Bessel beams by applying the same rules.

Although advantages of the DOEs generated from an analytic solution are obvious, application of this method is restricted to only specified wave re-forming problems.

2.5 Conclusion

In conclusion, three methods of DOE design have been investigated. The method of applying iteration algorithm to design DOEs is flexible, but the reconstruction efficiency of the DOE is dependent on the quality of the selected algorithm. DOEs can be numerically recorded or reconstructed by digital holography easily, but the reconstruction efficiency is low owing to the multiple reconstruction terms. The DOE that is generated from an analytic solution has high reconstruction efficiency and good re-forming result, but the method cannot be applied to an arbitrary beam shaping. Hence, we may choose one most effective method to design DOEs in the following chapters.

CHAPTER 3 EFFECTIVE OPTIMIZATIONS TECHNIQUES FOR SPECIFIED DOE DESIGN

3.1 Introduction

During the design of DOEs, many concerns in terms of diffraction efficiency, time consumption of the design, fabrication difficulty, implementation of reconstruction set-up, and etc., have to be taken care of. To balance the performance of a DOE and limitations of fabrication tools, some optimization methods may be adopted to get a better combination of these factors which affect the reconstruction of the DOE. Thus, in this Chapter two specific methods are presented to facilitate fabrication process or to improve performance of a DOE. First, an effective quantization method applied in iterative-Fourier transform algorithm is proposed; then an optimization method is introduced to simplify structures of a multiplexing DOE of polarization-selection. Both of the optimizations take care of practical fabrication predicaments while retaining the reconstruction efficiency as high as possible.

3.2 Practical phase quantization technique in Iterative Fourier Transform Algorithm

3.2.1 Quantization methods

Computer-generated holograms (CGHs) comprise a class of mainly phase-only DOEs, which are especially useful in re-forming the wavefront of incident light. Numerous applications of CGHs have been studied and reported in the past ten years. With the development of microscale and nanoscale fabrication technologies it is foreseeable that CGHs will find more promising applications in an increasing variety of research fields. Various established algorithms, for example, the iterative Fourier transform algorithm (IFTA) and Gerchberg-Saxton (G-S) algorithm, simulated annealing (SA), and a genetic algorithm (GA), are being used in the design of CGHs. The evolution of the optimization algorithms has been driven mainly by the demand for high fidelity of reconstruction with high diffraction efficiency in applications.

An important criterion in measuring the performance of a CGH is the diffraction efficiency, which is defined as the ratio of the desired energy to the total energy of the CGH reconstruction. There have been many optimization approaches to keeping the diffraction efficiency of a CGH as close to the theoretical limit as possible. For example, optimization algorithms such as simulated annealing, a genetic algorithm, and the conjugate-gradient algorithm have been used. Some revisions of the algorithms have been also widely studied; e.g. Liu and Taghizadeh [56] modified the constraining functions to improve the Gercheberg-Saxton algorithm. Zhai *et al.* [57] improved the quality of binary images obtained from the IFTA by amplitude adjustment. Improvements of the fabrication technologies can also prevent loss or distortion of the phase information, which could potentially affect the CGH's diffraction efficiency.

Although continuous phase elements have the highest diffraction efficiency, CGHs are usually quantized during their fabrication process because it is difficult in practice to create CGHs with continuous phase profile due to limitations in the fabrication procedure. By comparison, binary CGHs have the structures that are simplest to fabricate and thus are widely used. However, their low number of quantization levels will cause much of the actual phase information to be lost or skewed. Hence there is a finely balanced trade-off between retaining phase information and reducing the number of phase levels. As in most kinds of CGM fabrication, quantization is inevitably needed, and the resultant loss in diffraction efficiency that could result from using an inappropriate quantization method could be significant. Therefore selection of the best quantization method will be important in keeping the diffraction efficiency of a CGH close to its theoretical limitation.

Quantization methods in iterative algorithms have been studied by Wyrowski [58] and Mait [59], with specific emphasis on the methodology of the quantization technique in terms of selection of phase values. In general, previous research has explained how quantization was used for improvement of diffraction efficiency. However, it has come to the author's attention that optimization of a CGH is also affected by the actual implementation of quantization during the optimization. That is, it is the author's intention to demonstrate some numerical experiments to show where quantization should be implemented in the optimization.

Without loss of generality, in this section approaches to implementing quantization by using an IFTA will be investigated. First, flow chart of the IFTA is shown in Figure 3-1; FT and FT^{-1} represent the Fourier transform and the inverse Fourier transform, respectively.

Figure 3-1. Block diagram of the IFTA, where 'x' represents possible quantization position.

The process begins with amplitude $A_1(u, v)$ of the given incident beam and random initial phase $\theta_1(u, v)$. Firstly, the complex amplitude of the incident beam is Fourier transformed into an image $AA_{2n}(x, y)\exp[i\theta_{2n}(x, y)]$, where n is the number of iterations. The constraint in the reconstruction plane is applied by keeping the newly computed phase $\theta_{2n}(x, y)$ and replacing the amplitude $AA_{2n}(x, y)$ with the desired amplitude $A_2(x, y)$. Then the new complex amplitude $A_2(x, y)\exp[i\theta_{2n}(x, y)]$ is inversely Fourier transformed into $AA_{1n}(u, v)\exp[i\theta_{1n}(u, v)]$. Similarly, the constraint on the incident plane is applied, i.e. amplitude of the beam in the incident plane is always set as $A_1(u, v)$, so the new complex amplitude comprises the amplitude $A_1(u, v)$ and the computed phase $\exp[i\theta_{1n}(u, v)]$. After that, the iteration is repeated until the number of iterations reaches N . The final phase $\theta_{1N}(u, v)$ is the solution to the desired phase element.

There are four possible conditions in which a quantized phase distribution $\theta_{1N}(u, v)$ can be incorporated into the algorithm:

- (1) when $\theta_{1n}(u, v)$ and $\theta_{2n}(x, y)$ are both quantized in the FT and FT^{-1} iterations,
- (2) when $\theta_{1n}(u, v)$ is quantized while $\theta_{2n}(x, y)$ is continuous in the FT and FT^{-1} iterations,
- (3) when $\theta_{1n}(u, v)$ and $\theta_{2n}(x, y)$ are both continuous in the FT and FT^{-1} iterations but $\theta_{1n}(u, v)$ is quantized after the iterations are terminated, and

(4) when $\theta_{2n}(x, y)$ is quantized while $\theta_{1n}(u, v)$ is continuous in the FT and FT^{-1} iterations but $\theta_{1n}(u, v)$ is quantized after the iterations are terminated.

As was mentioned above, having more quantization levels will result in reduced distortion of information, and the influence of the quantization will decrease correspondingly. However, for the purpose of this investigation these methods of implementing a quantized phased distribution to binary and to eight-level phase elements will be applied to study how each influences diffraction efficiency.

3.2.2 Comparisons of different quantization methods

The letter E, with 128×128 pixels as shown in Figure 3-2, was selected as the pattern for the target of the output; the letter was located in the upper left-hand quadrant of the output. We assumed that the amplitude of the incident beam is a plane wave.

Figure 3-2. Desired pattern of the IFTA with 128 by 128 sampling points.

We computed the root-mean-square (RMS) error of the reconstructed image of the computed phase for all iterations in the algorithm. Normally the IFTA will converge after ~ 20 iterations. To ensure that the best achievable solution for the algorithm is obtained, we run it for 50 iterations for each CGH, and the phase distribution for the lowest RMS error obtained within the 50 iterations is kept for the following comparisons. The RMS is defined as follows:

$$RMS = \sqrt{\sum_{k,l}^{K,L} |AA_2(k,l) - A_2(k,l)|^2} / (K \times L) \quad (3-1)$$

where $AA_2(k,l)$ and $A_2(k,l)$ denote the reconstructed image of the quantized CGH and the desired image, respectively, and both of them are normalized to 1. $K \times L$ represents the

number of total points of the matrix. From Equation (3-1) it is easy to understand that there is a trade-off between the RMS and the diffraction efficiency of a CGH.

The algorithm was run for 10 CGHs (each with 50 iterations) for the four methods of implementing a binary quantization described above. The related quantization procedure in the program is described by

$$\begin{aligned} \theta &\in [0, 2\pi) \\ \theta_q &= \text{floor}[\theta \cdot \text{Level} / (2\pi)] \cdot 2\pi / \text{Level} \end{aligned} \quad (3-2)$$

where θ refers to the continuous phase distribution and is normalized into a range from 0 to 2π . *Level* is the quantization level of the CGH. *floor* is a Matlab function that rounds the elements to the nearest integers towards minus infinity, and θ_q is the obtained quantized phase distribution.

Figure 3-3 shows the RMS results of the four quantization methods; the data are sorted in order. Methods 1-4 represent the corresponding quantization approaches.

Figure 3-3. RMS results of binary quantization with 128 x 128 sampling points.

It can be seen from Figure 3-3 that method 2 is the one which has the lowest RMS errors. Table 3-1 also shows numerical comparisons of the RMS error results.

Table 3-1. Calculated RMS results with different binary quantization methods.

Comparison parameters	Method 1	Method 2	Method 3	Method 4
Mean RMS	0.1103	0.0707	0.0802	0.1078
Relative Mean RMS	1.5601	1.0000	1.1344	1.5248
Lowest RMS	0.1021	0.0652	0.0749	0.0953
Relative lowest RMS	1.5660	1.0000	1.1488	1.4617
Highest RMS	0.1179	0.0741	0.0867	0.1215
Relative Highest RMS	1.5911	1.0000	1.1700	1.6397

In Table 3-1 the Mean RMS is the mean RMS of the 10 CGHs obtained with the same quantization method used in the algorithm; similarly, we can understand the meanings of the Lowest RMS and the Highest RMS from the respective definitions. The relative Mean RMS is the respective Mean RMSs divided by the Mean RMS of method 2; similarly, other relative ones are obtained by dividing the one of method 2 correspondingly. From Table 3-1 it can be seen that the second quantization method, i.e. $\theta_{1n}(u, v)$ is quantized, whereas $\theta_{2n}(x, y)$ is continuous in the FT and FT^{-1} iterations and has the lowest values of the Mean RMS error, the Lowest RMS error, and the Highest RMS error. Compared with the other three quantization methods, the second method can decrease the mean RMS error by as much as 56.01%, 13.44% and 52.48%, respectively, for remarkable improvement of the reconstruction quality of the CGHs. For the Lowest RMS errors, quantization method 2 can give the lowest relative value, 1.000; others are 1.566, 1.1488 and 1.4617, respectively. The RMS errors were also computed for eight-level CGHs under the same circumstances. The results are shown in Figure 3-4.

Figure 3-4. RMS results of the eight-level quantization.

It can be seen from Figure 3-4 that quantization methods 2, 3, and 4 produce similar RMS errors in reconstruction owing to the increase of the quantization level, which shows the quantization method is not as important as that of binary CGHs. However, the difference between method 1 and other methods is still huge; therefore in practice we should not adopt method 1 in the CGH design. Results of a detailed comparison are listed in Table 3-2.

Table 3-2. Calculated RMS results with different 8-level quantization methods.

Comparison parameters	Method 1	Method 2	Method 3	Method 4
Mean RMS	0.0862	0.0555	0.0579	0.583
Relative Mean RMS	1.5532	1.0000	1.0432	1.0505
Lowest RMS	0.0803	0.0536	0.0546	0.0539
Relative lowest RMS	1.4981	1.0000	1.0187	1.0056
Highest RMS	0.0955	0.0599	0.0608	0.0618
Relative Highest RMS	1.5943	1.0000	1.0150	1.0317

From Table 3-2 it can be seen that method 2 still gives the lowest values of the mean RMS, the lowest RMS, and the highest RMS.

Comparing the results of Table 3-1 and Table 3-2, one can see that the second quantization approach in the algorithm always gives the best results; it improves the mean RMS by at least

13% for the binary and 4% for the eight-level CGHs. It also improves the Lowest RWS by at least 14% for the binary. While there may be slight fluctuations in the RMS errors when different target patterns of the output are adopted, more calculations will show that method 2 is still the best among the four schemes used for quantization, especially binary quantization.

To further illustrate the effects of the quantization methods on the diffraction efficiencies of the CGHs, one comparison, with sampling points of 256 by 256, is used as an example. In this case the desired output pattern is three characters, NTU, in the upper left-hand quadrant of the output plane, as is shown in Figure 3-5. The results are shown in Figure 3-6, where each method has also been implemented 10 times to yield 10 CGHs and the number of iteration loops in the algorithm is 50, too.

Figure 3-5. Desired output of the IFTA with 256 by 256 sampling points.

Figure 3-6. RMS results of binary quantization with 256 by 256 sampling points.

It can be seen from Figure 3-6 that method 2 still gives the best results when an output pattern and a number of sampling points different from those in Figure 3-2 are chosen.

To summarize, an effective method for implementing phase quantization in an iterative algorithm has been found. It is advisable to adopt quantization method 2 [when $\theta_{1n}(u, v)$ is quantized and $\theta_{2n}(x, y)$ is continuous in the FT and FT^l iterations] or to avoid adopting quantization method 1 [when $\theta_{1n}(u, v)$ and $\theta_{2n}(x, y)$ are both quantized in the FT and FT^l iterations] in an iteration algorithm. With the application of this method, the CGHs can retain the highest diffraction efficiency; as shown above, their mean RMS errors for binary quantization are lower by at least 13% than those of other methods. The proposed method could provide an opportunity to further improve the design of a CGH with well-known optimization algorithms.

3.3 Optimization of multiplexing of polarization-selective DOEs

3.3.1 Theory of multiplexing of DOEs

Conventional phase-only DOEs that their feature size is much greater than the incident wavelength are normally independent of light polarization states. However, when the feature size of pixels of a DOE is comparable with or smaller than the incident wavelength, the DOE would demonstrate a phenomenon of polarization-selection, which displays reconstruction patterns depending on the polarization state of the light incident to the optical element. Such a DOE is known as polarization-selective DOE (PSDOE). It is obvious that a subwavelength-sized DOE is also a PSDOE. With design feasibility and developments of micro-fabrication and nano-fabrication technologies, PSDOEs have been a topic for some years [60-71]. The creation of the PSDOEs can base on the birefringence of materials or the form-birefringence of subwavelength-sized structures in an isotropic substrate [66-67]. Fabrication of PSDOEs can base on form-birefringence (subwavelength-sized) or material-birefringence. However, it is more practical for us to fabricate PSDOEs with subwavelength size, because birefringent materials are very difficult to be arranged in a DOE with the desired orientations of optical axis of numerous tiny pixels. Since reconstructions of a PSDOE can be two different images for orthogonal polarization lights, it is possible for us to multiplex two totally different images into one PSDOE, that is to say, phase delays of two conventional DOEs could be physically encoded into one PSDOE. Thus, one PSDOE can replace two conventional DOEs.

However, fabrication of PSDOEs is challenging, because subwavelength sized structures or birefringent materials are a must in the fabrication to achieve polarization-selection effect. To implement the multiplexing, we need four phase combinations for a PSDOE for TE and TM polarizations, i.e. $(0, 0)$, $(0, \pi)$, $(\pi, 0)$, and (π, π) , in parenthesis the first entries are the phase delays related to a TE incident light, and the second entries are related to a TM incident light. Generally, we have to use four different types of structures to realize the four combinations. For the case of phase combination of $(0, \pi)$ or $(\pi, 0)$, if one pixel demonstrates such phase delays when orthogonal polarized light is incident, the pixel must be made of subwavelength sized structure or birefringent material. As binary subwavelength sized horizontal/vertical gratings are the simplest form-birefringent structures, they have been widely adopted in realizing phase combinations of $(0, \pi)$ and $(\pi, 0)$. The other two combinations of $(0, 0)$ and (π, π) could be implemented by multilevel depths as reported in Ref [63] or by using other subwavelength sized structures as proposed in Refs [63-66]; however, these implementations will increase complexity in fabrication. If we can reduce phase combinations of $(0, 0)$ and (π, π) into one combination, either $(0, 0)$ or (π, π) , fabrication of PSDOE will be significantly simpler. Thus, an alternative way of avoiding multilevel structures or more than two subwavelength structures in a PSDOE fabrication will be proposed in this section; combination of $(0, 0)$ or (π, π) will be removed from a PSDOE through an optimization process. Thus the PSDOE will have only three structures while its polarization-selection effect remains.

As fabrication of subwavelength sized structure is more difficult than that of the super wavelength structures, an advantage of this optimization method is that the total number of the required subwavelength structures in a form-birefringent PSDOE can be reduced significantly. Similarly, a multilevel material-birefringent PSDOE can be simplified following the same optimization rules. Furthermore, the multiplexing contrast ratio of PSDOE can be taken into account in the optimization process simultaneously.

Let us consider two conventional DOEs as input functions in the polarization multiplexing. As an example, both the conventional DOEs comprise 128×128 pixels; phase value of the each pixel is either 0 or π . Without loss of generality, the desired reconstructions of the conventional DOEs are considered as a flat-top beam and a Bessel beam as shown in Figure 3-7, respectively.

Figure 3-7. Desired reconstructed patterns for the two conventional DOEs, (a) a flat-top beam; (b) a Bessel beam.

Phase distributions of the two conventional DOEs have been designed using the G-S algorithm. As a result, the two DOEs will reconstruct a flat-top beam and a Bessel beam, respectively. The phase distributions of the two DOEs are shown in Figure 3-8 (a) and (b), respectively. The black and the white dots in Figure 3-8 represent phase value 0 and π , respectively.

Figure 3-8. Conventional DOEs' phase distributions. (a) The DOE for the flat-top beam; (b) the DOE for the Bessel beam.

Reconstructed images of the two DOEs are shown in Figures 3-9 (a) and (b), respectively. Resulting from binary phase-only structures, double reconstructed images which conjugate each other are appeared in the ± 1 st orders of each DOE.

Figure 3-9. Reconstructed images by two DOEs, respectively. (a) flat-top beams;
(b) Bessel beams.

When two DOEs are incorporated into a single PSDOE, the resultant PSDOE has four combinations of the phase values, which are listed in Table 3-3.

Table 3-3. Multiplexing of two conventional DOEs into one PSDOE.

Combinations	DOE A	DOE B	PSDOE
1	0	0	0, 0
2	0	π	0, π
3	π	0	π , 0
4	π	π	π , π

In the Table, the possible phase value for each pixel in two conventional DOEs **and** possible phase values for each polarization-selective pixel of a PSDOE are listed. Because the two conventional DOEs are binary phase-only, the possible phase value for each pixel is 0 or π , and because a PSDOE is polarization-selective, each pixel of the PSDOE can have two different phase values. When a PSDOE is used to represent two conventional DOEs, it must contain information of the two conventional DOEs, i.e. one pixel of PSDOE represents the phase combination of pixel in DOE A and pixel in DOE B. It can be seen in Table 3-3 that a PSDOE should have four basic pixels to realize the phase combinations of the two conventional DOEs. As discussed above, we will replace one combination with one of the other three combinations. As an example, we let combination 4, i.e. (π, π) , be replaced with other three combinations.

3.3.2 Optimization approach

The optimization approach involves two steps: replacement process and random optimization process. The first step of the optimization approach is replacing combination state (π, π) with one of the other three combination states $(0, 0)$, $(0, \pi)$, and $(\pi, 0)$, which are illustrated in Table 3-3. In this step we search and then replace combination 4, pixel by pixel. Therefore, after this step all the pixels of combination 4 would be replaced with the other three combination states.

During this replacement process, we have to determine which of the three phase combinations is used to replace combination state 4. Criterion of the replacement here is chosen as reconstruction efficiency, which will be calculated after every possible replacement with one of the other three combinations. Thus, in the process merit costs of the three possible replacements have been computed. The merit function for the replacement is described by Equation (3-3) as follows:

$$Cost_{te,tm} = \sum_{m,n} \left| Int_{TErec, TMrec}(m,n) - Int_{TEdes, TMdes}(m,n) \right|^2 / \sum_{m,n} \left| Int_{TEdes, TMdes}(m,n) \right|^2 \quad (3-3)$$

where $Int_{TErec, TMrec}$ represents the reconstructed image intensity of the PSDOE by the TE or TM incident light, and $Int_{TEdes, TMdes}$ represents the desired reconstructed image intensity of the corresponding DOE. The replacement rules should be governed by the following criteria: (1) the PSDOE has the minimum merit cost with either TE or TM polarization, or (2) the PSDOE has the minimum sum of merit costs for TE and TM polarizations. In addition to the natural differences of the original images in TE and TM polarizations, the reconstructed images might introduce further discrepancies in terms of total intensities and image sizes. With such embedded variations in the cost values of TE and TM polarizations, a weighting factor can be appended onto the initial polarization results. The weighting factors are calculated as the total intensity of the two images divided by the intensity of respective image. For the case shown in this section, the weighting factor of 2.0107 and 1.9894 have been used for TE and TM polarizations, respectively, and the reconstruction results are shown in Figure 3-10. Alternatively, if the replacement process takes too much time, we can replace combination state 4 randomly with any of the other three states during the replacement process but the subsequent computations may take a longer time.

After optimized by the replacement process, the PSDOE has only three combination states. The reconstructed images of the PSDOE are shown in Figure 3-10 (a) and (b) with respect to TE and TM polarizations. Although the desired reconstructed patterns (Flat-tops in Figure 3-10 (a) and the Bessels in Figure 3-10 (b)) are clear, the undesired patterns (the Bessels in Figure 3-10 (a) and Flat-tops in Figure 3-10 (b)) are also discernable. Hence, we need to further optimize the PSDOE to suppress the undesired patterns.

Figure 3-10. Reconstructed images of the PSDOE after combination 4's has been replaced. (a) Reconstructed flat-top beams; (b) reconstructed Bessel beams.

In the second step, we will separately optimize the PSDOE using a random search for TE and TM polarizations, respectively, i.e., the phase combination of one randomly chosen pixel of the PSDOE will be changed randomly into any of other two phase combinations. The detailed optimization procedures for TE and TM polarizations are shown in Figure 3-11.

(a)

(b)

Figure 3-11. Flow chart of the second-step optimization for, (a) TE polarization;
(b) TM polarization.

This step is similar to the simulated annealing algorithm, but the criterion of acceptance or rejection is different. There is no probability control with temperature variations in this step. After every random change of phase combinations, if the merit value of the PSDOE does not increase, the change will be accepted otherwise it will be rejected. After the optimization process for the TE optimization is terminated, the optimization process for the TM polarization will be continued. In this step, to ensure that combination 4 will not appear again after the random pixel change, we restrict the changes that happened among the three combinations only, that is to say, we only select combination (0,0) and $(\pi,0)$ for the TE polarization, and combination (0,0) and $(0,\pi)$ for the TM polarization. As a result, there is no crosstalk between the optimization processes for the TE and the TM polarization states. After the two polarizations have been optimized respectively, combination 4 will not appear again. To increase the contrast ratio and diffraction efficiency of the PSDOE, the second step can be repeated for many times.

The criterion for a pixel change in the second step is also based on the merit functions, which are similar to Equation (3-3); during the computations, if the cost value does not increase, the program accepts the phase change; otherwise, the pixel keeps its original value.

The optimization process for the TE part and the TM part has been executed respectively, and the non-uniformities of TE's images and TM's images comparing with the desired patterns are plotted in Figure 3-12 (a) and (b), respectively. The non-uniformity E_N is similar to Equation 2-15, defined as:

$$E_N = \frac{\sum_{M,N} |I_{calculated}(i,j) - I_{desired}(i,j)|}{\sum_{M,N} I_{desired}(i,j)} \quad (3-4)$$

where $I_{calculated}$ is the intensity of the calculated image and $I_{desired}$ is the intensity of the desired image, i and j are the respective row and column of the matrixes, M and N are the total number of the i and j .

Figure 3-12. Non-uniformities versus iterations, (a) for TE reconstructed flat-top beams; (b) for TM reconstructed Bessel beams.

In the optimizations of Figure 3-11, the computations took 1000 iterations within the outer loop and 2000 iterations within the inner loop. It can be seen from Figure 3-12 that after about 400 outer iterations, the non-uniformities have been nearly unchanged. To further decrease the non-uniformities, we can repeatedly run the TE and TM optimization process as shown in Figure 3-11, because any phase change in TE or TM optimization process in Figure 3-11 will affect the overall layout of the combination states. The reconstructed images of the final PSDOE after the second optimization step are shown in Figure 3-13 (a) and (b). It can be seen from Figure 3-13 that the desired pattern is much clearer than the ones after the first step, and the noise level has also been lowered substantially. Furthermore, the quality of reconstructed images after the second optimization process has also been improved significantly.

Figure 3-13. Reconstructed images after the second optimization step, (a) reconstructed flat-top beams; (b) reconstructed Bessel beams.

The contrast ratio is 28 in Figure 3-13. The first-order diffraction efficiency equals ~30%, and the theoretical maximum efficiency of the first-order for a binary phase grating is 40.5%. Since the PSDOE is a combination of two binary-phase DOEs, the maximum theoretical diffraction efficiency of the PSDOE with orthogonal polarization lights will be the same as a single binary DOE. The root-mean-square errors in Figure 3-10 (a) and (b) reconstructed by the conventional DOEs are 0.0772 for the flat-tops and 0.0516 for the Bessels, respectively. After the first optimization step, the RMS errors of the PSDOE in Figure 3-10 (a) and (b) are increased to 0.0909 and 0.0571, respectively, however, the RMS errors of the final PSDOE are reduced half after the second optimization step; the RMS errors in Figure 3-13 (a) and (b) are 0.0398 and 0.0229, respectively.

The 128×128 -pixel PSDOE, comprising only the first three combination states of Table 3-3, is shown in Figure 3-14 (a).

Figure 3-14. Phase pattern of the optimized PSDOE. (a) The whole PSDOE; (b) the first 20×20 pixels of the PSDOE.

A close-up image of the first 20×20 pixels of the PSDOE is shown in Figure 3-14 (b). The blank pixel stands for $(0, 0)$, the horizontal for $(0, \pi)$, and the vertical grooves for $(\pi, 0)$, respectively. Therefore, the PSDOE clearly reconstructs two different patterns with only three phase combinations.

Thus, two conventional DOEs have been multiplexed into one polarization-selective DOE (PSDOE) with only three phase combinations. The fourth phase combination is replaced with one of the other three combinations through an optimization process. The optimization involved two main steps: the replacement of the fourth combination with one of the other three phase combinations, and further optimization of the TE and TM phase distributions

separately. The resultant PSDOE can reconstruct two clear patterns with respective orthogonal polarization lights. Furthermore, we can increase the diffraction efficiency and contrast ratio of the PSDOE by repeating the second optimization step. Therefore, the PSDOE can replace two conventional DOEs with simpler phase structures. Similarly, for other binary multiplexing optical devices, this method can also be applied to eliminate the most complicated pixel in all the four combinations, and design and fabrication of the resultant multiplexing device will be simpler.

3.4 Conclusion

Two optimization methods have been proposed in this Chapter. First, different quantization methods at different steps of the iterative Fourier transform algorithm (IFTA) with calculated root-mean-square (RMS) errors of the DOE reconstruction have been implemented. As a result the most effective quantization approach has been identified with the RMS; value decreasing by at least 13% for a binary DOE which retains the maximum diffraction efficiency in the mean time; second, an optimization method in reducing a PSDOE's four phase combinations into three has also been investigated. The PSDOE's first-order diffraction efficiency is 30% and the contrast ratio is 28 after the optimization.

CHAPTER 4 FABRICATION AND DYNAMIC IMPLEMENTATION OF DOE

4.1 Introduction

When the design of DOE has been finalized, we need to realize the design by utilizing some fabrication techniques. Fabrication technology is a very important factor that finally determines performance of DOEs; thus during the stages of design and fabrication, diffraction efficiency of the devices always has to be taken into account. In fabrication, diffraction efficiency of DOEs is mainly affected by the accuracy and resolution of the fabrication tools; for example, a two-level, Fourier-transformed DOE has a maximum reconstruction efficiency of only 40.5% but a 16-level DOE can direct over 99% of the incident light into the first order. Thus, improvement in fabrication tools and techniques means that performance of a DOE could be improved or a novel design of DOE can be realized. Currently, many types of equipment, which include laser direct writing system, e-beam direct-writing lithography system, contact printing system, and so on, have been applied in fabricating DOEs; many techniques such as reactive ion etching, plasma etching, wet etching, and mask exposure have also been utilized to replicate or transfer original patterns to other substrates. With developments in nano-technology, DOEs can be achieved with higher resolution; fabrication of novel DOEs such as subwavelength-sized DOEs or polarization-selective DOEs is also feasible. Furthermore, applications of new materials in fabrication not only enhance performances of DOEs but also simplify the process of fabrication.

In this Chapter, some conventional micro-fabrication techniques will be introduced first; and then a detailed fabrication process of binary DOE, which re-forms a Gaussian profile beam into a Bessel beam, will be presented; then a four-level DOE will be fabricated with e-beam direct writing system and UV exposure system; finally, the four-level DOE will be replicated using a PDMS master mask.

4.2 Fabrication of binary DOE

4.2.1 Comparison of fabrication techniques

During the fabrication process, DOEs are always quantized into many phase levels which correspond to respective substrate depths or heights. Binary DOEs are the most frequently fabricated ones owing to economic combinations of time, resolution, and equipment costs for fabrications. Some devices such as laser direct writing systems and e-beam lithography systems have great freedom and high resolution in generating arbitrary DOE patterns, but their times are huge when the size of DOE is large. In the contrast, some equipments fabrication has high speed but low resolution in writing a pattern; it would be more suitable for fabricating the kind of DOE where the feature size is comparatively large. Suleski and O'Shea have demonstrated a technique that used a single grey-scale mask, which has been generated by a commercial slide imagers, to lithographically transfer patterns onto a photoresist [72-73]. Currently, most commercial laser printers can reach or exceed a resolution of 1200 dpi. Thus, when the pixel size of a DOE is comparable to the resolution of a printer, the pattern of the DOE is best printed by a printer to reduce time and cost. Therefore, a technique combining laser printing and UV exposure to fabricate a binary DOE of large pixel size is exhibited in this section.

DOEs have been used for re-forming plane waves into a Bessel beam [54-55]; however, most laser modes from laser resonators are the Gaussian modes or others, and not a plane wave. As the hybrid algorithm introduced in Chapter 2 has high flexibility in converting a given incident wave into an arbitrary wave, it will be employed here to design a DOE which re-forms a Gaussian profile beam into a Bessel beam. Simulating light propagation in the G-S iterations and the cost functions in the simulated-annealing part are based on the angular plane wave spectrum theory. As we know, a binary DOE designed in the far-field diffraction regime always reconstructs twin images: one image of the first diffraction-order and one image of the minus first diffraction-order. However, a binary DOE designed in the Fresnel regime reconstructs only one image. Thus, the DOE designed in the Fresnel regime eliminates possible overlapping of the twin images reconstructed by a binary DOE that is designed in the far field.

In the fabrication process, the designed DOE pattern will be first printed onto a slide by a laser printer; then the black-white slide is used as mask for UV exposure under a mask aligner;

finally the pattern is developed on a sol-gel film. The experiments of reconstruction of the fabricated DOE will be implemented as well.

4.2.2 Design of DOE for Bessel beam shaping

To illustrate the design process clearly, the hybrid algorithm of the GS and the SA is described again as follows: first an initial random phase is given to the incident Gaussian beam, and the modulus of the beam is retained, then the angular wave spectrum is applied simulating light propagations in the G-S iterations; after the iterations, the computed phase of the incident beam is kept as the initial value for the SA optimization; phase of the DOE in the SA algorithm is optimized using the same propagation function as that in the G-S algorithm.

The DOE has been designed with the following parameters: the pixel size is $50\text{ }\mu\text{m}$ by $50\text{ }\mu\text{m}$; the sampling grid is 256 by 256; the reconstruction distance is 2000 mm; the loop number in G-S is 30; the temperature loop in SA is 2000; the loop of phase change of random pixel in SA is 200. After running the algorithm, the obtained phase distribution of the DOE is shown in Figure 4-1, where the black parts represent phase value of π and the white parts represent 2π .

Figure 4-1. Phase pattern (black: π , white: 2π) of the designed DOE.

Cross-sections of the DOE reconstruction and of the ideal Bessel beam are shown in Figure 4-2, where the continuous line represents the one reconstructed by the DOE, and the dashed line represents the desired Bessel beam.

Figure 4-2. 2D plots of the reconstructed (continuous line) Bessel beam and the ideal (dashed line) Bessel beam.

It can be seen from Figure 4-2 that the reconstructed Bessel beam is consistent with the desired Bessel beam.

4.2.3 Fabrication process

Fabrication process of the DOE mainly comprises three steps: preparation of sol-gel film, pattern printing onto slide, and pattern transferring with UV exposure. The sol-gel process is a versatile solution process for making ceramic and glass materials. In general, the sol-gel process involves the transition of a system from a liquid "sol" (mostly colloidal) into a solid "gel" phase; detailed process of sol-gel making can be found in many references [74-85]. The preparation procedures of sol-gel film are listed as follows:

- a. Rinse a quartz or glass substrate with acetone in an ultra-sound wave tank for three minutes.
- b. Clean the substrate in the ultra-sound wave tank with de-ion water.
- c. Dry the substrate with a nitrogen blower.
- d. Use a syringe to take in about 1-2ml sol-gel fluid.
- e. Load a 0.1 μm filter on the syringe.

- f. Put the substrate on a spin-coater and fasten the substrate in vacuum.
- g. Squeeze some sol-gel fluid onto the substrate.
- h. Set speed and duration of the spin-coater. The speed is 1500 rpm, and the duration is 35 seconds.
- i. Run the spin-coater.
- j. Bake the coated substrate on a hot plate at temperature 90 °C for 10 minutes.

As was mentioned above, the pixel size of the designed DOE is 50 μm by 50 μm , so we can use a laser printer with a resolution of 1200 dpi or more to print the pattern shown in Figure 4-1 onto a slide, which is transparent for the wavelength of 365 nm. However, the ink of the print is non-transparent to the wavelength. The refractive index of the sol-gel film is 1.51 for the 25% (mol ratio) Ti doped and increased to 1.55 at 50% doping concentration [74]. The illumination wavelength for the DOE was 633 nm, the expected step depth of the sol-gel film is $\sim 0.6 \mu\text{m}$ to achieve a phase difference of π . The exact phase depth can be obtained by adjusting the speed and the duration of a spin coater.

A Q-2001CT mask aligner (Quintel Corporation) with wavelength 365 nm and irradiance 15 mw/cm^2 is used for transferring the slide pattern to the sol-gel film. The film is exposed for 10 minutes under the UV light first, and then is developed with an ethanol solution for seven seconds; after that, the unexposed part of the sol-gel film is removed using de-ionized water, and finally the sol-gel film is baked for 35 minutes at temperature of 160 °C. Figure 4-3 schematically demonstrates the UV exposure arrangement with the slide mask. Flow of the DOE fabrication is schematically shown in Figure 4-4 [74].

Figure 4-3. Schematic diagram of pattern transferring.

Figure 4-4. Flow chart of the DOE fabrication.

Since the overall size of the DOE is too large for a microscope viewing, only part of the sol-gel DOE under a microscope is shown in Figure 4-5. The step depth of the DOE has been measured by a surface profiler. The measurement is shown in Figure 4-6, where the depth of the step is $\sim 0.6 \mu\text{m}$.

Figure 4-5. Part of the sol-gel DOE under a microscope.

Figure 4-6. DOE's step depth measured by a surface profiler.

4.2.4 Experimental results

The schematic experimental setup is shown in Figure 4-7. A Gaussian beam from a He-Ne laser is expanded by a pinhole and an objective lens, and then the expanded beam illuminates the DOE, after that the intensity profile of the reconstructed beam is recorded by a beam analyzer, which is placed at different distances along the beam propagation axis. The distances are measured from the plane of the DOE.

Figure 4-7. Schematic configuration of the DOE reconstruction.

Cross-sections of intensity profiles of the reconstructed beam at propagation distances of one meter, two meters, and three meters are shown in Figure 4-8, respectively. It can be seen from Figure 4-8 that the beam reconstructed by the DOE is roughly invariant during the free-space propagation.

Figure 4-8. Cross-sections of the reconstructed beam at propagation distances of,

(a) 1 meter; (b) 2 meter; (c) 3 meter.

Plot of the radii of the n th bright rings of the reconstructed Bessel beam and the plot of that of the ideal Bessel beam, at reconstruction distance of one meter, are shown in Figure 4-9, where the horizontal axis represents the ordinal number of the bright rings, and the vertical axis represents radii of the bright rings.

Figure 4-9. Radii of the n th bright rings of both the reconstructed Bessel beam and the ideal Bessel beam at propagation distance of one meter, where *denotes the ideal Bessel beam, and + denotes the reconstructed Bessel beam.

Figure 4-9 shows that the ideal Bessel beam and the reconstructed beam have the same intensity profiles, which verifies that the reconstructed beam is very close to the designed Bessel beam.

Hence, a sol-gel DOE which is used to re-form a Gaussian beam to a Bessel beam has been fabricated by using a slide mask and a UV mask aligner. The experimental results of the DOE reconstruction agree well with the simulations.

4.3 Fabrication of multilevel DOEs

4.3.1 Problems existed in fabrication of MDOEs

The design of DOEs using iteration algorithms has been discussed in Chapter 2, and fabrication of these DOEs will be studied further in this section. Currently, it is feasible to fabricate these DOEs applying many micro-fabrication technologies such as laser direct writing (LDW) or e-beam lithography (EBL) techniques with high accuracy. In this section, all the DOEs are referred to the ones that are generated by iterative algorithms. As phase distributions of these DOEs are yielded from phase-retrieval algorithms, the resultant structures of the DOEs are irregular, and seem randomly distributed; thus the fabrication process of the DOEs will be much different from that of some micro-optical elements with regular surface-relief. Owing to resolution limitations in the fabrication tools, phase of the DOEs have to be quantized although the quantized structure only approximates to the target structure of design. Hence, the DOEs are sometimes categorized by their phase structures; binary DOEs refer to phase elements that have only two steps of phase values, and multilevel DOEs (MDOEs) are the ones with more than two phase steps. Apparently, binary DOEs are the simplest to be fabricated; on the other hand, the fabrication of MDOEs is more complicated because MDOEs have multiple phase steps. Pixel alignment and height control are the main problems during the fabrication process of MDOEs, as heights of adjacent pixels of MDOEs are discrete and vary irregularly. Some fabrication techniques which can produce micro-lens or gratings [78-85] may not be applicable for fabricating MDOEs anymore. For example, the method of using interference plane waves has been used for producing a grating [84-85] but the method is not applicable for generating irregular patterns; the reflow technique has been used for micro-lens fabrication [86-87] but it is not suitable for MDOE fabrication either. Although fabricating a MDOE is more complicated than fabricating a binary DOE, MDOEs are still preferred to binary DOEs owing to the higher diffraction efficiency. Many methods such as multiple exposures or etchings with binary-transmission masks, direct writing systems (UV laser, electron beam, and diamond turning), grey-scale mask projection system, and contact printing technique, etc., have been employed to generate MDOEs. Among these methods, direct writing technique is most flexible in writing a prototype of MDOE patterns, but the method using a grey-mask to transfer a pattern is the most efficient. To yield

higher diffraction efficiency, MDOEs are usually designed with microscale or nanoscale pixels. E-beam direct writing system has the highest resolution in fabricating a MDOE as the size of the e-beam spot can be as small as tens of nanometer. Although direct-writing techniques take advantages of variable and controllable spot sizes, usually the time consumption is noticeable as the writing spot has to expose or etch substrate pixel by pixel. For instance, fabricating one piece of binary DOE with 256 by 256 pixels, where each pixel size is 10 μm by 10 μm , a laser direct writing system may take over 10 hours, and e-beam lithography system may take several hours to finish the writing. Therefore, reducing time consumption in mass production of MDOEs is also very important.

Thus, a technique consisting of e-beam direct writing with a grey-scale mask will be utilized for MDOE fabrication, and then a new technique using a polydimethylsiloxane (PDMS) material [88] for mass replicating the MDOE on sol-gel substrates will be presented. During the process, the e-beam direct writing machine is employed to write grey-scale pattern on a high-energy beam-sensitive (HEBS) glass, which turns dark instantaneously upon exposure to an electron beam. Unlike halftone masks such as chrome masks that rely on varying pinhole density, HEBS glass is capable of resolution to molecular dimensions without graininess. The darkness of HEBS glass is controlled by the writing electron beam; the more the electron dosage, which is the multiplication of electron beam current and dwell time, the darker the glass gets [89-90]. Although the PDMS is a fluid originally, when it is spun on surface relief of a master mask and is heated, it will become elastic and hydrophobic and can be peeled off from the master mask [83]. The newly generated PDMS is then used as the new master mask for mass replication. Many copies of PDMS patterns can be generated in one time, and consequently each PDMS pattern can be employed to replicate many copies; thus, a large amount of MDOEs could be replicated in a very short time. The PDMS replication technology has been applied in mass production of micro-lenses [83-84] but to the author's knowledge, the PDMS has never been applied for MDOE (the type generated from iteration algorithms) fabrication. In the experiment, the material of the end-product of the MDOE is sol-gel glass. Advantages of using this material lie in its excellent optical properties, low-cost, and simple fabrication process. Details of the preparations for sol-gel film and photoresist film can be found in Refs [74-75, 92-94]. After a HEBS mask has been produced by an e-beam writing system, we transfer the pattern of the low-level MDOE on a photoresist AZ5214 first, and then mould the AZ5214 pattern on a PDMS film; finally we can use the PDMS MDOE as master mask to replicate sol-gel DOEs. The reason of using photoresist as an intermediate medium between the HEBS mask and the PDMS master mask is that photoresist can be etched with high aspect ratio and high fidelity.

4.3.2 Fabrication process of MDOE

Before the e-beam writing process, the phase pattern of a MDOE has been previously obtained by an iteration algorithm. As the e-beam machine accepts CIF and GDSII file formats only, the computed phase data or phase pattern have to be saved as CIF or GDSII file format. Some commercial software such as LEDIT does not support importing such phase data or image files directly from external sources, and drawing phase patterns using the software is bothersome as one MDOE of 128 by 128 contains over ten thousand pixels. Therefore, a program (modified on a program of FYP report 6072,2001) has been coded to convert phase data or an image pattern to a CIF file. Although the designed MDOE has multilevel phase steps, and each phase step of the MDOE corresponds to one layer of fabrication, it should be noted that all the layers must be combined into one file in which one layer corresponds to one phase step; the reason is that the e-beam machine resets after each file writing, and the reset may result in pixel misalignment among the files. The curve between depths of photoresist AZ5214 and e-beam doses is then characterized after design of the MDOE. Characteristics of depths of the AZ5214 versus e-beam doses were measured as shown in Figure 4-10.

Figure 4-10. Measured resist depth versus the electron dose for AZ5214.

To obtain the expected multilevel phase delays on sol-gel film, the step depths of the MDOE can be calculated by employing the refractive index of sol-gel glass. On the basis of the obtained phase depths, corresponding e-beam doses have been calculated using the characteristics curve. The refractive indices of the AZ5214 material and the sol-gel are 1.6 and 1.50, respectively. Then an e-beam direct writing system (LEO982) is employed to generate a grey-scale pattern on a HEBS glass. The HEBS mask has been written in one step as different layers of the MDOE have been combined into one file. The Q-2001CT mask aligner (Quintel Corporation) with wavelength 365 nm and irradiance $15 \text{ mW}/\text{cm}^2$ is used to transfer the grey-scale pattern to AZ5214 film. The exposure duration is 12 seconds and the development

duration is 20 seconds. The AZ5214 MDOE is then baked at 120°C for five minutes. The PDMS solution is spun onto the AZ5214 MDOE and baked at temperature 100°C for one hour subsequently. After that, the PDMS film is peeled off from the AZ5214 MDOE while the desired surface relief has been retained by the PDMS film. Then the PDMS replica is pressed onto a soft sol-gel film, at the time the PDMS replica and the sol-gel film are ensured to be well in contact. Finally, both of them are exposed under UV light for 10 minutes so that the sol-gel is completely polymerized and stabilized. After the UV curing, the PDMS replica is peeled off from the sol-gel MDOE. Thus, the PDMS can be applied to other sol-gel films again. In the replication process, the PDMS replica has been found to be very stable and keeps high fidelity to the master pattern. The main procedures of the fabrication and replication are shown in Figure 4-1 1.

Figure 4-1 1. Block diagram of the MDOE's fabrication and replication.

4.3.3 Reconstruction experiments

The MDOE was designed with an iterative Fourier transform algorithm, and the desired output is a zero-order Bessel beam, which is positioned in the lower right-hand quadrant of the

reconstruction plane. The designed MDOE comprising 200 by 200 pixels is shown in Figure 4-12 (a); simulated Fourier-transformed reconstruction of the MDOE is shown in Figure 4-12 (b).

Figure 4-12. (a) Four-level phase pattern of designed MDOE; (b) reconstruction of the designed MDOE.

For the case of the applied sol-gel glass, four phase depths are calculated as $[(x-1)\lambda]/[X(n-1)]$, where x is of $(1, 2, 3... X)$, X is the number of the MDOE's total phase levels, and n is the refractive index of the sol-gel. The pixel size of the MDOE is $8\text{ }\mu\text{m}$ by $8\text{ }\mu\text{m}$. The doses of the e-beam machine for the four grey-scale layers of the HEBS are $4.9118 \times 20\text{ }\mu\text{c}/\text{cm}^2$, $7.697 \times 20\text{ }\mu\text{c}/\text{cm}^2$, $10.2102 \times 20\text{ }\mu\text{c}/\text{cm}^2$, and $12.6153 \times 20\text{ }\mu\text{c}/\text{cm}^2$, respectively. The desired phase depths for the four phase steps are $0\text{ }\mu\text{m}$, $0.316\text{ }\mu\text{m}$, $0.633\text{ }\mu\text{m}$, and $0.949\text{ }\mu\text{m}$ if the wavelength of illuminating light to the sol-gel MDOE is $0.633\text{ }\mu\text{m}$.

The fabricated sol-gel MDOE is shown in Figure 4-13 (a), where different gray scales under a microscopy represent different phase-heights. Part of Figure 4-13 (a) is zoomed in and shown in Figure 4-13 (b).

Figure 4-13. Microscopic picture of the sol-gel MDOE with pixel size of $8\text{ }\mu\text{m}$ by $8\text{ }\mu\text{m}$. (a) 10 magnifications; (b) zoomed in part of (a).

Depths of the sol-gel MDOE are also measured by a surface profiler. The profile of the measured part is shown in Figure 4-14, where four notations are given to demonstrate the four phase steps. It is shown that the MDOE profile has four distinct steps, which are measured as $0\text{ }\mu\text{m}$, $0.57\text{ }\mu\text{m}$, $0.68\text{ }\mu\text{m}$, and $0.81\text{ }\mu\text{m}$ respectively. These depths are roughly close to the desired ones except for the second phase depth, which the phase difference is attributed to the incorrect depth of the AZ5214 MDOE.

Figure 4-14. Measured phase depths of the sol-gel MDOE. The annotated numbers highlight four different heights of the pixels.

Experimental reconstruction by the sol-gel MCGH is shown in Figure 4-15. It can be seen from Figure 4-15 that there is only one single image that is located around the zero-order spot. The reconstruction phenomenon of the MDOE proves that the hologram is multilevel structured [94].

Figure 4-15. Experimental reconstruction of the sol-gel MDOE.

In summary, a technique using e-beam direct writing and grey-scale HEBS glass has been employed in generating a multilevel DOE; a novel replication technology that uses PDMS master mask replicating the MDOE with high fidelity has also been demonstrated. The experimental results and the simulated results of the MDOE reconstruction agree well. The PDMS replica can be repeatedly used for one-step replication of MDOEs.

4.4 Spatial light modulator

4.4.1 What is SLM?

Laser beams have unique light properties which differ from those of conventional beams; laser wavefronts can satisfy a variety of specified applications in different research and industry fields, for instance, a flat-top beam can be used for laser machining and processing, the Bessel beams have been used for measuring and data processing, doughnut beams are used for

trapping, etc. However, each customized beam wave-front needs at least one specified DOE, i.e. each DOE only aims at one unique output mode when the incident beam is given first, thus one fabricated DOE cannot be adapted for alternative beam shaping choices. As was introduced in the above sections that DOEs can be realized by micro-fabrication technologies, but the resolution, time, and costs in the fabrication are still the main concerns to us. Phase-only DOEs have higher diffraction efficiency than the amplitude-only ones, and those with more quantized phase levels have higher diffraction efficiency than the ones with fewer quantized phase levels. However, DOEs are usually quantized into fewer than 32 phase steps because of the limitations of the current fabrication technologies and costs of those equipments. Furthermore, fabricating a multilevel DOE normally takes at least several hours or days. With these understandings, a new device--spatial light modulator (SLM) has been introduced recently in simulating a real DOE to re-form light or to display images [95-96]. The SLM is an electrically-addressable phase/Intensity modulator which is capable of handling VGA signal inputs. The phase shifts imposed on each pixel of the SLM is determined through a computer interface, and they are controlled and refreshed by a PC program. The SLM can act as a DOE with multilevel phase steps more than 128, in addition to that, the response time of the SLM is very short (only milliseconds or shorter) after desired DOE patterns are loaded into the SLM; patterns in the SLM can be reconstructed in high fidelity and with high reconstruction efficiency. Furthermore, the loaded patterns controlled by a PC can be displayed one by one by the SLM with high speed, thus its fast image refresh rate can be used for dynamic control of DOE reconstructions.

The SLM takes advantages of its multilevel phase/amplitude steps, high efficiency, high fidelity, low cost, fast response, and dynamic control in imaging, thus, it is a good replacement for fabricated DOEs, which may be difficult to fabricate or the fabrications are too expensive and time consuming. Designed DOE patterns can also be tested by the SLMs instantly in a large amount.

In the following, characteristics of a reflectance SLM produced by Boulder Nonlinear Systems (BNS), Inc. [97] will be studied in detail, and this kind of SLM has been frequently used in beam shaping in the following Chapters.

4.4.2 Phase-type SLM

Currently many commercial SLMs are available for beam shaping and imaging [97-99]; one representative kind of these SLMs is from BNS. There are two different versions of 512 x

512 SLM from the BNS, one is a 15 micron pitch device, which has a maximum LCD screen size of 7.68 mm x 7.68 mm, and another is a 7 micron pitch device with maximum LCD screen size of 3.584 mm x 3.584 mm. As there is a 15 micron pitch SLM in the Laboratory one (in Photonics Research Center, NTU) and the device has been frequently utilized by the author, its detailed descriptions and manipulations to the device will be given in this section. The SLM is filled with a Nematic Liquid Crystal (NLC) to modulate visible light in pure amplitude, pure phase, or coupled amplitude and phase, with a very high frame rate. Nematic devices are good for displays (good amplitude uniformity, higher yield) or for applications where a large phase-only modulation depth is needed (2π or greater). The NLC molecules usually lie parallel to the surface of the device giving the maximum retardance due to the birefringence of the Liquid Crystal (LC). When an electric field is applied, the molecules tilt parallel to the electric field. As the voltage is increased the index of refraction along the extraordinary axis, and hence the birefringence, is effectively decreased causing a reduction in the retardance of the device. When the correct input polarization state is selected, the device will act as if its refractive index were electronically modulated. Figure 4-16 shows the cross section of the SLM, where a polarized light enters the device from the top, passes through the cover glass, transparent electrode, and liquid crystal layer, and then is reflected off the shiny pixel electrodes, and returns on the same path finally. The optical head of a 512 x 512-pixel SLM is shown in Figure 4-17 [97]. The optical head includes a LCD screen, wires, adjustment controller, and electrical parts.

Figure 4-16. Side view of the SLM LCD screen.

Figure 4-17. Optical head of the SLM.

The SLM uses computer PCI-bus interface for loading data from the computer memory or disk. The technical parameters of the SLM are given in Table 4-1 [97].

Table 4-1. Technical parameters of the SLM.

Number of Pixels:	262,144 (512 x 512)
Array Size:	7.68 mm x 7.68 mm
Pixel Pitch:	15 μ m
Flat Fill factor:	83%
Zero-order Diffraction Efficiency	65%
Electrical Addressing	7 bits
Optical Flatness	$\lambda/4$ or better
Optical Modulation	variable retardation change
Full frame Loading	164.16 μ s
Device Configuration	Reflective
Maximum Usable Frame Rate	30-150 Hz, Depends on phase stroke, wavelength, and temperature
Contrast	200:1 zero-order monochromatic light
Driver Interfaces	PCI-bus computer slot; 128-bit high-speed data port loads full frame in 164.16 μ s
Optical Response (10%-90%)	1-20 ms rise, 2-30 ms fall Depends on phase stroke, wavelength, and temperature
Driver Memory	1024 frames of SDRAM

Since the applied electric field on the SLM will result in a change in the index of refraction in one axis (n_e) of the liquid crystal, a linearly polarized beam parallel to the n_e axis will undergo a pure phase modulation, and any other polarization states will result in a phase and amplitude coupled modulation; thus, we can use the following configuration as shown in Figure 4-18 [97] to yield a pure phase modulation.

Figure 4-18. Configuration of the SLM optical setup.

Note that when the SLM is diffracted, the diffraction will be accompanied by many speckle noises and images of higher diffraction orders because the filling factor of the SLM pixels is only 83%, not the maximum, 100%. The SLM with unfilled pixels results in many diffraction orders, which reduce the energy of the desired diffraction order.

4.4.3 PC interface and dynamic control

Phases of the SLM are controlled by a gray-scale image with maximum 256 gray scales. Each gray scale of the image file is supposed to be a phase step. Total pixel number of the loaded image pattern cannot exceed 512 by 512. The optical head of the SLM is connected to a DAC board, which is connected to a PCI card plugged into a standard PC. A software named as "BLINK [97]" is used to demonstrate the capabilities of the SLM. The software not only controls the output that is displayed on the SLM, but also allows the user to specify a sequence of images to be displayed. The loaded image file can be generated by an iterative algorithm or by a capturing tool. Each of the 512 by 512 elements of the SLM can be programmed to 256 discrete voltage states, as the voltage of the pixel changes, so does the retardance of the liquid crystal layer producing a phase shift. However, since the phase modulation of the SLM does not respond to the pixel voltage linearly, the relationship between the phase modulation and the gray scales must be taken into account when displaying

images and utilizing the SLM. The typical phase response to gray scales of the loaded image file is shown in Figure 4-19 [97]. It can be seen in Figure 4-19 that the responses in the range of (0-127) are the same as the range of (255-128). Thus, although the SLM device accepts 256 gray levels, the gray scales of the image file must be in the range of (0-127) or (255-128) to produce corresponding phases.

Figure 4-19. Phase response of the SLM to the loaded gray-scales.

The software “BLINK” converts gray scales of the loaded images into phase scales through the SLM electronic devices. The software can store 1024 image frames preliminarily, thus, the 1024 frames can be displayed in sequence or in random. This means that DOE phase patterns can be displayed either statically or dynamically with a rate depending on the SLM technology used (from 10 Hz to several tens of KHz).

The SLM can be conveniently employed for static or dynamic beam shaping. In the following a designed hologram loaded onto the SLM and the experimental reconstruction are present. The phase distribution of a hologram is first calculated by a holographic method and is shown in Figure 4-20 (a), where the gray scales (ranging from 0-255) represent the phase values of the hologram. However, the pattern should be converted to a BMP file while the gray-scales of the image should be based on the curve of Figure 4-19. Then the figure can be loaded onto the SLM. The figure loaded in the SLM is shown in Figure 4-20(b). It is designed to reconstruct an interference pattern of a first-order Bessel beam and a 4th-order Bessel beam.

Figure 4-20. (a) The phase distribution of a hologram whose gray scaling range is from 0-255, (b) the corresponding pattern loaded onto the SLM.

The reconstruction of the hologram is captured by a CCD camera and shown in Figure 4-21, where a higher-order image of the hologram is also reconstructed in the figure owing to the nature of the hologram.

Figure 4-21. Captured reconstruction of the interfering hologram with the SLM.

The captured image demonstrates that the SLM can reconstruct the hologram efficiently and in high fidelity.

It can be seen that the SLM has many advantages for beam shaping. The 512 x 512-pixel SLM, including the technical parameters, working principle, and optical configuration, has been introduced, and the PC interface and the controlling of the SLM have also been presented. In the following Chapters the SLM will be used for many kinds of beam shaping.

4.5 Conclusion

In conclusion, fabrication processes of DOEs have been investigated. First, a binary DOE for shaping a Gaussian beam into a Bessel beam has been fabricated using a contact printing method and has been tested; second, a MDOE has been produced by combining e-beam direct writing and contact exposure, and a novel technique using PDMS has been developed in replicating MDOEs; finally, an SLM utilized as a DOE in beam shaping has been introduced.

CHAPTER 5 NOVEL LASER BEAMS: LASER MICRO-BEAM SHAPING USING DOES

5.1 Introduction

As we know, for a monochromatic field in a homogeneous and isotropic medium, the Helmholtz wave equation can be written as,

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2})E(r,t) = 0 \quad (5-1)$$

where c is the velocity of the light. In the case of circular cylindrical coordinates, the solution to the scalar Helmholtz equation is given by [8,10],

$$E_n(r, \varphi, z) = A \exp(ik_z z) J_n(k_r r) \exp(in\varphi) \quad (5-2)$$

which gives an exact expression of an n th-order Bessel beam. In the Equation (5-2), J_n is the n th-order Bessel function of the first kind, and n is an integer. k_z and k_r are the longitudinal and radial components of the free-space wave vector, respectively, and $k = 2\pi/\lambda = (k_z^2 + k_r^2)^{1/2}$. φ is the azimuthal phase. The Bessel beams, which were first reported by J. Durnin and his colleagues in 1987 [8, 100], have one feature (diffraction-free) that is strikingly different from corresponding feature of the well-know Gaussian mode of the paraxial equation. Diffraction-free means that the complex amplitude $\Psi(x, y, z, t)$ has the property of [8]

$$\psi(x, y, z, t) = \exp[-i(wt - \beta z)] \psi(x, y, z = 0, t = 0) \quad (5-3)$$

The simplest diffraction-free beam solution to Equation (5-3) is $J_0(\alpha\rho)$, $\rho = (x^2 + y^2)^{1/2}$, α and β are real constants that satisfy $\alpha^2 + \beta^2 = (w/c)^2$, and c is the speed of light. In real situations, the Bessel beams generated by DOEs only approximates to be diffraction-free owing to their definite size. There has been much research on the Bessel beam's generation, propagation, applications, and etc. The zero-order Bessel beam can be generated by a circular

slit [8], an axicon [101-102], or a hologram [54-55]; higher-order Bessel beams can be generated by a helix doughnut beam and axicon, or a hologram.

Ashkin *et al.* first demonstrated the optical trapping of micron-sized spheres using a single light beam focused tightly with a microscope objective lens in 1980s [103-104]. This optical tweezer was based on a Gaussian beam, which has a high-intensity gradient to confine particle to the beam centre. Although the light incorporated in the first optical tweezers was a Gaussian beam, the Gaussian beam has two limitations [106]: 1, because the Gaussian beam is a tightly focused beam to act as the optical trap, the more tight the laser is focused, the more rapidly it diverges. This means that the force exerted by the beam drops off very quickly as the laser moves away from the trapping region. 2, the beam behind an obstruction will bear little resemblance to the original due to effects such as diffraction, hence the beam cannot trap particles in multiple planes. Since the optical tweezers were first demonstrated at Bell Labs (Murray Hill, NJ) in 1986 [103], many kinds of laser beams have been employed for optical micromanipulations. For example, optical vortex beams were intensively studied. This is due to the fact that optical vortex beams possess helical phase wave front with respect to their topological charges about the beam axis. Furthermore, the optical vortex beam, which contains phase term of $\exp(in\theta)$, possesses an orbital angular momentum that can be observed through their mechanical torque on microparticles [51, 105]. This mechanical effect has offered optical micromanipulation with an additional degree of freedom. H. He *et al.* experimentally trapped reflective and absorptive particles in the dark central spot of a focused Laguerre-Gaussian (LG) beam with charge 3 singularity in 1990s [51]. The LG laser beam exerts an orbital angular momentum on particles and induces them into rotation.

In recent years, Bessel beams have been also received much interest in the area of optical micromanipulation. Since Bessel beams have been proven diffraction-free and self-reconstructive [9, 107], they are an ideal tool to compensate the shortages of the Gaussian beams in optical micromanipulations. Apart from the properties mentioned above, higher-order Bessel beams possess an extra characteristic, i.e., helical phase wave front [10]. This extra property allows the beams to be able to transfer their orbital angular momenta to the microparticles like an LG beam, so the higher-order Bessel beams can also trap low-index particles. The Bessel beams' characteristics, such as self-reconstruction and nondiffraction, have been made use of to trap multiple micro particles separated at a distance in different planes [9]. Furthermore, when Rhodes *et al.* [108] reported an experimental study of guiding a low velocity intense source (LVIS) of cold atoms along co-propagating and oblique blue-detuned LG light beams, they predicted that the Bessel beams can offer extended advantages

on atom guiding due to their non-diffracting nature. The Bessel beams are considered as the new-generation tools for optical trapping and guiding.

The above mentioned Bessel beams [8] are well known as integer-order Bessel beams (IBB), where their orders of the first kind of the Bessel function are integers and their phase singularity charges have the same integers. However, a family of new beams, which are named as fractional-order Bessel beam (FBB), with fractional order of the Bessel function of the first kind and the fractional charges of azimuthal phase, will be investigated for the first time. The FBBs are not only different from the IBBs by the choice of integer or fractional orders, but also provide some unique properties in addition to the IBBs family. It is predictable that the FBBs cannot only be used for optical trapping, but also be applied for studying properties of the light. In Rhodes et al's experiment, they placed an obstruction into the trap region of a LG beam to create a hole in the guide in its circumference, which will increase the efficiency of atoms coupled into an oblique guide; however, integer-order Bessel beams do not have a natural opening in the guide to facilitate the flowing of the atoms into trapping and guiding. The FBBs would take advantage of their natural openings in their concentric rings.

In this Chapter, after the definition of the FBBs, the important properties of the FBBs in terms of controllable opening slit, non-diffraction propagation distance, and the helical phase distribution with fractional nature will be investigated in detail. Taking full advantages of the non-diffraction and the orbital angular momentum of the IBBs, the so-called fractional-order Bessel beams have demonstrated an additional property that provides a controllable natural opening slit across the concentric rings; the size and the status of the opening are fully controlled by adjusting the fractional orders of the FBBs. Note that the artificial opening slit was intentionally generated by inserting an obstacle in Ref [108], but the proposed fractional-order Bessel beams are believed to offer excellent opportunities in micro-manipulating applications because of the above advantages. As SLMs can be refreshed as dynamic holograms to display holographic patterns, thus they will be employed to produce different orders of the FBBs. Propagation dynamics and dynamic trapping ability of the FBBs generated from an SLM will be presented; theoretical and experimental results of the generated FBBs will be discussed too. In this Chapter, to investigate the self-reconstruction property of an FBB for three-dimensional optical trapping applications, the author employs a numerical analysis based on angular spectrum approach to demonstrate the self-reconstruction effect and verify it with experimental results.

5.2 Fractional-order Bessel beams (FBBs)

5.2.1 Definition of the FBB

In 1994, Beijersbergen et al. [109] investigated the fractional LG modes by means of a spiral phase plate. In the experiment, they found that the phase singularity of half integer appears next to the center of the beam; upon integer order introduced, the phase singularity shifts back to the center of the beam. Sonja Franke-Arnold *et al.* [110] suggested that the resulting beam from the fractional number is unstable upon propagation.

As we know, the Bessel beam's amplitude and phase distribution on the plane at $z=0$ mm can be expressed by

$$E_{\pm n}(\rho, \phi) = J_n(\alpha\rho) \exp(\pm in\phi) \quad (5-4)$$

where J_n is the n th Bessel function of the first kind, n is a positive integer. α is a coefficient, ρ is the transverse spatial coordinate, and ϕ is an azimuthal phase. However, if we set the order n as a fractional number, the complex distribution of Equation (5-4) will result in a different distribution, which is referred to as a fractional-order Bessel beam (FBB). According to Equation (5-4), the intensity patterns of the FBBs of any order are concentric rings at the emergent plane. The beam propagation in free-space can be calculated by the angular spectrum of plane waves [46],

$$A(x, y, z) = FT^{-1} \{ FT[A(x, y, z - \Delta z)] \exp(-ik_z \Delta z) \} \quad (5-5)$$

where FT and FT^{-1} denote the Fourier transform and the inverse Fourier transform, respectively. $k_z = (k^2 - k_x^2 - k_y^2)^{1/2} = 2\pi(1/\lambda^2 - v_x^2 - v_y^2)^{1/2}$, v_x and v_y are spatial frequencies, and Δz is the propagation distance. $A(x, y, z)$ is the complex-amplitude distribution of the beam to be propagated.

If we change the value n , the resultant Bessel beam will possess different intensity distribution. Higher-order Bessel beams, for integer values of n , will have a dark circular spot in the center while maintaining the outer concentric rings, but the fractional orders of Bessel beams between two adjacent integer orders will have an opening slit with varying widths. For the sake of clearer observation, a much higher order Bessel beam $n=4.5$ is chosen to

demonstrate its properties. The intensity profile and phase distribution of the beam on plane $z=0$ mm are shown in Figure 5-1. The size of the beam is approximately 3.84 mm by 3.84 mm. The parameter α in J_n is given as 13/mm.

Figure 5-1. FBB at $z=0$ mm, $n=4.5$. (a) The intensity; (b) the phase.

For the comparison, the intensity and phase distributions of an IBB of $n=4$ are shown in Figure 5-2. The figures are obtained with the same parameters as that of the FBB shown in Figure 5-1 except for the order.

Figure 5-2. IBB at $z=0$ mm, $n=4$. (a) The intensity; (b) the phase.

It can be seen from Figures 5-1 and 5-2 that both of the intensity profiles of the FBBs and the IBBs are very similar except for the sizes of the central dark spots. Their intensity rings are fully closed without any opening. However, the phase distributions of them are much different. The fourth-order Bessel beam has eight arc sections distributed evenly from the inner to the outermost. On the contrary, the fractional-order beam has one fractional arc section in addition to eight arc sections. In the central part of the phase patterns, there are four

arms distributed evenly in the IBB's and there are four arms and one incomplete arm in the FBB's.

The intensity differences between them can be clearly observed as the beams are propagating in the free-space. We can calculate out their distributions at a propagation distance z by using the angular spectrum wave theory. The FBB and the IBB on the plane $z=300$ mm are shown in Figure 5-3 (a) and (b), respectively. The outer rings in Figure 5-3 appear rather square due to the tailored square beam profile in the simulation.

Figure 5-3. Diffraction intensity distributions of, (a) FBB of $n=4.5$ at $z=300$ mm; (b) IBB of $n=4$ at $z=300$ mm.

In Figure 5-3, the FBB has radial gaps in its concentric intensity rings, and all the rings of the IBB are fully closed. However, the sizes of the intensity rings of both are nearly unchanged, and the further simulations and experiments have demonstrated that both the beams are diffraction-free within this distance.

5.2.2 Opening slit of the FBB

First, we choose n from 1.1, 1.2... 2.0 to demonstrate the intensity pattern evolution of the FBBs. The size of the beam is 3.84 mm by 3.84 mm. The beam is propagated and reconstructed at $z=300$ mm, and the diffraction intensity patterns have been calculated by Equation (5-5). The intensity patterns are shown in Figure 5-4.

Figure 5-4. FFBs of $n=1.1\sim 2.0$ at $z=300$ mm.

It can be seen that an obvious difference among the intensity patterns in Figure 5-4 is the variation of the opening slits. When the order n is set closer to an integer, i.e., 1 or 2, the opening slit become thinner; in the contrary, if n is set as in the middle of two adjacent integers, i.e., 1.5, the size of the opening slit is the maximum. Generally, the opening slit variations demonstrate the same fashion for other orders of the FFBs. Without loss of generality, we choose a high order Bessel beam, e.g. order $n=4.5$, to study the variations of the opening slit in propagation. The beam size is also the same as 3.84 mm by 3.84 mm.

When the FBB with order 4.5 propagates along the z axis, patterns on the planes at different propagation distances can be calculated by Equation (5-5). The yielded intensity patterns are shown in Figure 5-5.

Figure 5-5. FBB at $z=1$ mm~100 mm, $n=4.5$.

The propagation distance of the beam ranges from $z=1$ mm to $z=100$ mm. Figure 5-5 shows that the opening slit is developing in propagation. Detailed images of the opening slits are simultaneously shown as the close-up pictures in Figure 5-5. As was mentioned above, at $z=0$ mm plane, the rings of the patterns are closed; however, when the beam propagates, it can be seen from Figure 5-5 that an opening gap is appeared on each ring. With the increase of the propagation distance, the openings become wider and wider. At the distance around $z=100$ mm, the size of the openings has reached the maximum.

5.2.3 Diffraction-free propagation

If the FBB continues to propagate, the diffraction property of the FBBs can be examined by diffraction patterns shown in Figure 5-6, at different propagation distances from $z=120$ mm to $z=300$ mm. It can be seen in Figure 5-6 that sizes of the rings along the propagation distances are roughly unchanged, and the opening slits are well maintained in the propagation too; hence the FBB is diffraction-free within a definite range of propagation distance.

Figure 5-6. Propagation of the FBB from $z=120$ mm to $z=300$ mm.

However, if the beam propagates further, the openings become distorted and blurred. The propagation patterns from $z=0.3$ m to $z=1$ m are also shown in Figure 5-7. The orientation of the opening gap of the innermost ring rotates almost 90 degrees in distance of 1 m. Although the opening gaps from the outer rings to the inner rings at the distance 1 m have almost disappeared, the size of the innermost ring remains unchanged.

Figure 5-7. FBB at $z=0.3\text{ m}\sim 1\text{ m}$, $n=4.5$.

For the IBBs, the valid diffraction-free distance can be estimated by Equation 2-22 or $z_{max}\approx 1.6D/(\alpha\lambda)$, where D is the beam size, α is the parameter in the Bessel function and $\alpha = k_r/(2\pi)$. λ is the working wavelength. For example, if the size of an IBB is 3.84 mm by 3.84 mm, and the parameter α in J_n is about 13/mm, the diffraction-free length is ~ 700 mm. In Figures 5-5 ~ 5-7 the same parameters for the FBB with order of 4.5 have been set, it can be seen from the figures that the diffraction-free distance of the FBB is slightly shorter than that of the IBB. However, in an optical trapping system, the predicted 300 mm non-diffraction distance is long enough for applications both in 2D and 3D micro-manipulation experiments.

5.2.4 Helical phase wavefront

If the helical phase is removed from Equation (5-4), i.e., the beam's distribution is a pure Bessel function, the central dark spot of the beam will be evolved into a bright spot in propagating; furthermore, there is no opening slit at all. Therefore, it is obvious that the fractional phase results in the openings in the intensity rings of the FBBs.

The phase distributions of the FBB and the IBB are again shown in Figure 5-8. Although the intensity rings of the FBB at plane $z=0$ mm are fully closed, the phase distribution is not regular. The beam's phase has eight full arc sections and one more fractional arc section. In the central part of the phase distribution, four clear arms and one dim arm appear. The dim arm and the fractional arc are caused by the fractional order of the phase, which affects the intensity distributions in propagating as well.

Figure 5-8. Phase distributions at $z=0$ mm. (a) The IBB of $n=4$; (b) the FBB of $n=4.5$.

To illustrate revolving effect of the helical phase in propagation, phase distributions of the FBB of $n=4.5$ in one wavelength distance are shown in Figure 5-9. The distance is from $100 \text{ mm} + \lambda/5$ to $100 \text{ mm} + \lambda$, and the interval is $\lambda/5$ in every phase pattern.

Figure 5-9. Phase distributions of the FBB, $n=4.5$, $z=100 \text{ mm} + \lambda/5 \sim z=100 \text{ mm} + 5\lambda/5$.

Figure 5-9 shows that the phase wave-front of the FBB is revolving in propagating. The outer part of the phase has nine arc sections, but one of the sections is distorted seriously owing to the fractional phase. There are four and half arms in the central part of the phase distributions; however, the half arm is unstable and will be getting distorted or even disappeared in further propagation.

From a quantum mechanics perspective, the total angular momentum per photon for paraxial beams is related to the order of the azimuthal phase. Furthermore, for all kinds of beams possessing a helical phase term, they all have angular momenta [111]. Since the phase distribution of the FBB is of a helical wave-front nature, it possesses orbital angular momentum, too. Furthermore, since the order of an FBB can be set fractional, the orbital angular momentum of an FBB can be finely adjusted by selecting fractional orders.

In summary, properties of the fractional-order Bessel beams such as non-diffraction and helical phase in propagation have been demonstrated. Moreover, it is revealed that the FBBs provide a controllable opening slit across the diffraction pattern, and can generate fractional orbital angular momenta. These additional unique properties offer great potential for FBBs in applications such as optical trapping and atoms guiding.

5.3 Dynamic generation of FBBs using SLM

5.3.1 Generation of FBB using SLM

To generate IBBs with an SLM, the phase pattern on the SLM is a transmission function of the form as [55, 112]

$$T(r) = \exp(in\theta) \exp(-i2\pi ux) \exp(-i2\pi r/r_0) \quad (5-6)$$

The phase function in Equation (5-6) has three terms: the first is the phase singularity of charge n associated with the azimuthal phase, for example, $n=0, 1, 2 \dots n$ denote the ordinal of the zero, first, second... nth Bessel beams, correspondingly; the second is an adjustable phase factor that represents angle of the propagation axis relative to the normal for the diffraction-free Bessel beam, and the spatial frequency u is related to the angle α by which the propagation axis is rotated as $\alpha = \lambda u = \varepsilon \lambda / (N\Delta)$, where N is the number of sampling points of the hologram, λ is the illumination wavelength, Δ is the pixel size, and ε is an adjustable parameter with a maximum value of $N/2$; the third term creates a beam with a zero-order Bessel beam with width staying constant over a distance of $Z_{max} = Dr_0/(2\lambda)$ or $kD/(2k_r)$, where Δ is the width of the beam, r_0 is an adjustable constant parameter with $r_0 = 2\pi/k_r$. When the whole phase function is encoded as a hologram, the continuous function must be sampled into discrete. For example, $r^2 = (s^2 + t^2)\Delta^2$, where s and t are integers identifying each pixel, in this case, $r_0 = q\Delta$, where q is an adjustable parameter, and the center of this term can be shifted to $(x=x_c, y=y_c)$, thus $r^2 = (x-x_c)^2 + (y-y_c)^2$. Hologram patterns of both the 4th-order IBB and the 4.5th-order FBB are shown in Figure 5-10 (a) and (b), respectively.

Figure 5-10. (a) Hologram pattern for the Bessel Beam of $n=4$; (b) Hologram pattern for the fractional-order Bessel beam of $n=4.5$; simulation result of the central spot of the beam generated from the hologram of, (c) intensity profile reconstructed by the hologram (a), where $n=4$; (d) intensity profile reconstructed by the hologram (b), where $n=4.5$.

To simulate reconstructions of the holograms in Figure 5-10 (a) and (b), Equation 5-5 is used to simulate their propagation. The resultant simulations of the intensity patterns are shown in Figure 5-10 (c) and (d), respectively. The simulation result of the FBB shows a clear opening at the center spot. This opening is clearly resulted from the fractional phase variation of the beam.

Furthermore, by displaying phases of beams of $n=4$ and of 4.5 , respectively shown in Figure 5-11 (a) and (b), we can observe that the fraction in the phase azimuthal variation contributing an additional half-fringe instead of the usual full fringe of an integer phase azimuthal variation. Since there is a phase azimuthal variation, there will be an orbital angular momentum presented both in the FBB and the IBB.

Figure 5-11. Phase of the Bessel beam's central spot generated from the hologram of, (a) $n=4$; (b) $n=4.5$.

Figure 5-12. The set-up to observe the FBB generated from the SLM. A is a He-Ne laser operating at 632.8 nm; B is a non-polarized beam-splitter; C is an SLM; D is a SONY IRIS color CCD; E is a variable polarizing plate.

Figure 5-12 shows the set-up to observe the intensity distribution of the FBB of order 4.5. In this set-up, a He-Ne laser is directed onto the 512×512 SLM, which contains the gray-scale pattern of the hologram. The reconstructed beam is reflected from the SLM to the beam-splitter, which directs the reconstructed beam onto the CCD. In this configuration, the probability of the output image being distorted due to the angle of reflection from the SLM is minimized.

In order to show that the FBB is propagation-invariant, a series of images have been taken for a distance of 50 cm shown in Figure 5-13 (a-f). In both simulation and experimental results, which are observed for a distance from 1.0 m to 1.5 m, the central spot of the FBB remains unchanged and stable through the propagation distance. The intensity pattern from the

simulation result in Figure 5-13 (a-c) is consistent with the experimental results shown in Figure 5-13 (d-f).

Figure 5-13. A series of intensity patterns in the FBB of $n=4.5$, observed from the set-up shown in Figure 5-12; (a-c) simulation results of the intensity pattern at a distance of 1.0, 1.2, 1.5 m, respectively; (d-f) the actual experimental results at the same intervals.

5.3.2 Orbital angular momentum of the FBB

Within the paraxial limit, any electromagnetic field ψ with azimuthal dependence of the form $\exp(in\theta)$ possesses density of both orbital and spin angular momentum, and each photon carries $n\hbar$ of orbital angular momentum, thus the FBB could still possess orbital angular momentum. For simplicity, the total orbital angular momentum in a linear polarized beam per second is given by [111],

$$\Gamma_z = \frac{P}{2\pi\nu} n \quad (5-7)$$

where P is the laser power, n is the azimuthal mode index and ν is the frequency of light. Hence, from the experimental and simulation results seen in Figures 5-11 and 5-13, it can be shown that the FBB is propagation-invariant and stable, which will deem it as a hollow optical beam with a natural opening, with the presence of orbital angular momentum.

In the present optical trapping, the fractional beam will be advantageous for low index particles. Since the LG beam or the IBB, which possesses a full-doughnut shaped intensity pattern at the center spot, repel the low index particles away from the dark central spot due to

radiation pressure, it will be difficult to trap and rotate small low index particles within the central dark spot easily. Furthermore, the full-doughnut shaped intensity pattern also pose difficulty for atomic trapping, as researchers have had to insert an obstruction to create a horse-shoe intensity pattern in order to increase its coupling efficiency of atoms into the hollow optical guide [108].

The FBB is a possible solution to the problem, as it possesses a clear opening slit shown in Figure 5-13. Furthermore, if we change the FBB hologram on the SLM to an IBB's, the opening slit can easily be closed. It can be observed from Figure 5-14 that the closing of the opening slit as the FBB hologram changes from fractional 4.5 to an integer 5. This dynamic changing of the opening and closing of the slit will allow a possible selective process of the micro-particles.

Figure 5-14. (a-f) A series of intensity patterns in the Bessel beam from $n = 4.5$, 4.6, 4.7, 4.8, 4.9, 5, observed at a distance of 1.5m.

Thus, the fractional-order Bessel beam is demonstrated as stable upon a propagation distance of around 50 cm while maintaining an orbital angular momentum due to its azimuthal phase variation. Since orbital angular momentum exhibits both intrinsically and extrinsically [114] with respect to the beam axis, an FBB will be ideal to investigate the behavior of orbital angular momentum at the opening of the beam. Besides that, the FBB generated from an SLM can be used for dynamic optical trapping. As small low index particles and absorptive micro particles can be trapped and rotated into the central dark spot of the beam. In atomic optics, such a beam with the dynamic control of SLM will allow more coupling efficiency in atom guiding without using an additional external obstruction to create the opening slit [108].

5.4 Holographic generation of the FBBs

It is known that axicon and hologram are two common devices that can be used to produce Bessel beams. With an axicon, when a plane wave passes through a conical lens the beam will be transformed into a zero-order Bessel beam. Similarly, shining a higher order LG beam through the axicon then transforms the beam into a higher-order Bessel beam. The nondiffracting distance of the Bessel beam is determined to a large extent by the tip angle of the axicon and by the beam size of the incident light shining onto the axicon. Such a method of generating Bessel beams has high optical efficiency; however, an axicon lacks flexibility and requires stringent alignment. Furthermore, to produce a higher-order Bessel beam it is necessary to generate a corresponding-order LG beam as an incident wave-front for the axicon.

Turunen et al. [54] reported a holographic method for generating Bessel beams. A hologram can generate the same phase values as the phase retardation produced by the axicon with which to reconstruct a zero-order Bessel beam. Similarly, a hologram whose phase simulates the superposition of the axicon phase distribution and a higher-order helical phase distribution can also generate a higher-order Bessel beam [55, 112]. The holographic method gives much flexibility in generating Bessel beams with various beam coefficients and orders, and the system is simple and compact.

However, the Bessel beams described above are assumed to have integer orders; i.e., the amplitude of the beam is an integer order of the Bessel function of the first kind, and the helical phase order is also set as the same integer. A holographic method, which has been introduced in Chapter 2, will be employed to generate the FBBs and to investigate the unique properties of FBBs.

Computer-generated holograms (CGHs), which are produced by some phase-retrieval algorithms, can modulate the phases of the incident beams to yield predefined amplitude or phase distributions. However, it is difficult for a single computer-generated hologram obtained by a phase-retrieval algorithm to customize a reconstructed beam in amplitude and phase simultaneously.

In Section 5.3 the dynamic generation of FBBs by a spatial light modulator (SLM) has been investigated [115]. The FBBs were generated by use of a simulated axicon with a fractional helical phase. In this section, a method with which to utilize the amplitude of a fractional-

order Bessel function coupled with the appropriate fractional order of a helical phase such that the resultant FBB has high purity, is proposed. An interference method is used to generate the FBB. Such a method is simple and cost-effective and has been widely applied to generate holograms [116]. With this method we reconstruct a pure FBB, i.e., amplitude and phase with the same order. The experimental results demonstrate that the quality of the FBB intensity patterns produced in this way is better than that of patterns generated by the axicon-approximation method.

To produce an FBB experimentally, we cause interference between a reference plane wave and a desired FBB to generate a hologram. The hologram's phase data are then loaded in an SLM. Thus, when a reference plane wave is incident upon the SLM, the object is reconstructed. To verify the simulation result, we continue to use the FBB ($n=4.5$) and a plane wave to generate the hologram. To separate the reconstructed images of different orders, we use an oblique plane wave $\{p = \exp[i2\pi(x/d)\delta]\}$, where δ is an adjustable constant that was set as 121 here. x is the position of the sampling point along x axis, and d is the pixel width of the SLM.) to be the reference light. The resultant hologram is shown in Figure 5-15.

Figure 5-15. Hologram produced by the interference of the FBB of $n=4.5$. The gray scales represent different phase values. The phase is quantized into 32 levels.

The phase data of the hologram are subsequently saved as a gray-scale image file. The different gray scales of the image correspond to different phase values. We then use an SLM to reconstruct the hologram. The SLM from the BNS is a nematic type working on reflection and can have more than 64 phase levels. The schematic of the reconstruction system is shown in Figure 5-16.

Figure 5-16. Schematic of the experimental setup.

Figure 5-16 shows a He-Ne laser beam of wavelength 632.8 nm expanded by a collimation system and then projected onto a SLM screen. The loaded hologram data in the SLM have 512 by 512 sampling points, and each pixel size of the SLM is 15 μm by 15 μm . The hologram has been quantized to 32 levels. The reflected light is received by a Bemstar-1500 CCD beam profiler. The reconstructed patterns at different distances from the SLM are measured by the beam profiler. The intensity profiles of the beam are shown in Figure 5-17.

Figure 5-17. Reconstructed FBB intensity patterns at different distances from the SLM, $n=4.5$. (a) $z=0.3$ m; (b) $z=0.5$ m; (c) $z=0.6$ m; (d) $z=1$ m.

The overall sizes of the patterns in Figure 5-17 are approximately 3.84 mm by 3.84 mm. The intensity profiles observed in the form of an elliptical shape in the figures are due to the aspect ratio of the CCD camera. The FBB is diffraction free along the propagation distance between $z=0.3$ to $z=0.5$ m, as shown in Figures 5-17 (a) and (b); the distance is considered long enough for a micro-manipulation application. The IBB's non-diffracting distance with the same beam parameters is ~ 0.7 m. Beyond the non-diffraction range the size of the innermost ring is almost unchanged at $z=0.6$ m and $z=1.0$ m in Figures 5-17 (c) and (d), but the orientation of the openings rotates with the beam's propagation. Such a rotation of the optical beam's intensity has been attributed to the helical wavefront of the beam [117-118].

To further illustrate the existence of helical phase of the FBBs we employ the method of interference between two FBBs of opposite helicity such that we could observe their phases. Hence, two different diffracted orders (the first and the minus first of the hologram) of the 4.5th-order Bessel beam were redirected by a Michelson interferometer to interfere on the plane at a distance $z=1$ m from the SLM plane. Simulations of the interferences of the IBBs and the FBBs are presented in Figure 5-18 (a) and (b), respectively.

Figure 5-18. Interference patterns at $z=1$ m. (a) Simulation of interference, $n=4.5$ and $n=-4.5$; (b) simulation of interference, $n=4$ and $n=-4$.

In the experiment the interference fringes were recorded by a CCD camera; they are shown in Figure 5-19 (a). The interference fringes from $n=4$ through $n=-4$ are shown in Figure 5-19 (b).

It can be seen from Figures 5-18 and 5-19 that both interference patterns have forklike fringes, which confirm an existence of the helical azimuthal phase variations in FBBs and IBBs. For the IBBs it was proven that the helical phase induced an orbital angular momentum. This could provide an analogy with the FBB. In Figures 5-18 (a) and (b), the brightest rings in both patterns are composed of many petals, but one of the petals in the FBB caused by the FBB's

fractional helical wavefront is not observed clearly in the figure because of the fractional-order Bessel-function amplitude of the beam.

Figure 5-19. Interference patterns at $z=1$ m. (a) Experiment of interference, $n=4.5$ and $n=-4.5$; (b) experiment of interference, $n=4$ and $n=-4$.

A clear and simple method of obtaining an FBB by using a single optical element has been shown, and the experimental generation of the FBB has been demonstrated to be in good accordance with the simulation results.

The main use of the FBB can best be described as its use as a dark optical trap for confining neutral atoms [119]. Rhodes *et al.* [108] have shown that the obstructed guide, which creates artificial openings, increases coupling efficiency. They also mentioned that higher coupling efficiency into an obstructed LG beam and a higher-order Bessel beam can be achieved, for instance, by dropping a magneto-optic trap cloud under gravity into an oblique guide.

Hence it is not difficult for us to see the FBB's potential usefulness for trapping neutral atoms. The openings from the FBB create a clear opening in a hollow beam with a nondiffracting distance that is not easily attainable by other means. Hence, this fractional-order Bessel beam can be a great alternative to the obstructed guide of Rhodes *et al.*

5.5 Self-reconstruction of FBB

5.5.1 Simulations on self-reconstruction

Many researchers have studied self-reconstruction phenomenon of the zero-order Bessel beam. An interesting shadowing property of the zero-order Bessel beam demonstrated by

MacDonald *et al.* [120] is that on blocking the intense central spot the propagating ring pattern acts to recover the central spot at a short distance behind the obstruction. The self-reconstruction of the zero-order Bessel beam in nonlinear medium was also examined [121]. The self-reconstruction of the zero-order Bessel beam in free-space has been fully demonstrated and explained by Bouchal *et al.* in 1998 [107]. Furthermore, Garces-Chavez *et al.* utilized such a property to simultaneously micro-manipulate particles in multiple planes [9]. However, the zero-order Bessel beam is different from higher-order Bessel beams in some aspects, for example, a higher-order Bessel beam has helical phase wavefront, and as a result the higher-order Bessel beam has a dark spot in the center of the intensity rings but the zero-order Bessel beam has a central bright spot in the contrary.

On the basis of the investigations on the Bessel beams with an integer order and a fractional order as were mentioned above, we know that the FBBs have an additional unique property, i.e., there exists an opening slit across the concentric intensity rings and the opening slit has been demonstrated stable over a reasonable working distance in propagation. As we know, the IBBs are symmetrical in transversal amplitude and phase along the propagation; among them, the zero-order Bessel beam has been proven as self-reconstructive. However, since the transversal distributions of the FBB are asymmetrical (as shown in Refs. 109, 111) when it is propagating in the free-space, it is interesting to explore existence of self-reconstruction property of the FBBs. Similar to the higher-order IBBs, the FBBs also possess helical phase wavefronts with fractional values in the azimuthal phase distribution. Therefore, if the FBBs do have self-reconstruction property, with properties such as opening slit and self-reconstruction after propagating through an obstacle, it will be an additional advantage for their potential applications.

Without loss of generality, in simulations the order of the FBB is again set as 4.5, and the beam size is 3.84 mm by 3.84 mm. A sampling grid of 256 by 256 is chosen, thus the pixel size of the point is 15 μm by 15 μm . k_r is set as 13/mm. The beam propagation in the free-space can be described by the angular spectrum of plane waves.

Note that the FBBs have fully-closed circular concentric intensity rings in the emergent plane ($z=0$ mm). Figure 5-20 (a) shows an example of the intensity profile of an FBB with the order of 4.5 at the emergent plane ($z=0$ mm) and Figure 5-20 (b) shows the intensity profile at a propagation distance of $z=300$ mm. The values of propagation distance z are measured from the emergent plane. The illuminating wavelength is 0.633 μm .

Figure 5-20. (a) Intensity pattern of an FBB of order of 4.5 at $z=0\text{mm}$; (b) intensity pattern of the FBB at $z=300\text{ mm}$.

Self-reconstruction is a unique property for some light beams. It is known that the zero-order Bessel beam has such a property. However, as shown in Figures 5-20, the intensity profiles of the FBBs are much different from that of an IBB in the propagation; it is interesting to explore whether the FBB is self-reconstructive. Numerical simulations have been implemented in a Matlab code. For the comparison with other well-known beams, we also examine propagations of a Gaussian beam and an IBB of order 4 when the beams are propagated through the same obstacles as tested in the FBB case. In the simulation the width of the waist of the Gaussian beam is set as half of the beam size of the Bessel beams.

The propagation of a beam “with obstructs” can be schematically described in Figure 5-21, where x axis and y axis represent the cross-section plane of the beam, and z axis is the propagation direction. In Figure 5-21, two obstacles are placed in the plane perpendicular to the z direction, and beam self-reconstructions at recording planes with different propagation distances can be simulated from Equation (5-8).

Figure 5-21. Schematic diagram of the obstruction and the propagation of a light beam.

First, simulation results of the Gaussian beam's propagation are presented in Figure 5-22. Figure 5-22 (a) shows the Gaussian beam's cross-section at $z=65$ mm, and Figure 5-22 (b) shows the cross-section at $z=190$ mm, without going through an obstacle. Correspondingly, in Figure 5-22(c), cross-section of the Gaussian beam at $z=65$ mm is shown with two obstacles shown as two black squares in the figure. The two obstacles have the same size of $240\text{ }\mu\text{m}$ by $240\text{ }\mu\text{m}$. In Figure 5-22 (d), beam profile of the obstructed beam at $z=190$ mm is also presented. In Figure 5-22, the orthogonal coordinates denote sampling positions of the cross-sections of the beam in propagation, and gray-scales of the cross-sections represent the intensity magnitudes of the beam.

Figure 5-22. (a) The cross-section of the Gaussian beam without any obstacle at $z=65$ mm; (b) the Gaussian beam's cross-section at $z=190$ mm; (c) the Gaussian beam is blocked by two obstacles at $z=65$ mm; (d) the Gaussian beam's propagation after the blocks at $z=190$ mm.

Propagations of an IBB of $n=4$ are also simulated and shown in Figure 5-23, in which the beam parameters are used the same as the FBB's parameters (except for the n) that were adopted above. Cross-sections of the beam at different propagation distances with and without obstacles are shown in Figure 5-23.

Figure 5-23. (a) The cross-section of the fourth-order Bessel beam without any obstacle at $z=65$ mm; (b) the Bessel beam's cross-section at $z=190$ mm; (c) the fourth-order Bessel beam is blocked by two obstacles at $z=65$ mm; (d) the Bessel beam's propagation after the blocks at $z=190$ mm.

Propagations of the FBB of $n=4.5$ are shown in Figure 5-24, under the same circumstances as the IBB's and the Gaussian's.

Figure 5-24. (a) The cross-section of an FBB with order of 4.5 without any obstacle at $z=65$ mm; (b) the FBB's cross-section at $z=190$ mm; (c) the FBB is blocked by two obstacles at $z=65$ mm; (d) the FBB's propagation after the blocks at $z=190$ mm.

It can be seen from Figures 5-22~ 5-24 that both the IBB and the FBB can self-reconstruct after a propagation distance, but the Gaussian beam experiences serious distortion and cannot recover itself at all. To further illustrate the full propagation procedures of the beams before and behind the two blocks, a couple of 3D and 2D plots sampled by a line vertically passing through the center of the intensity cross-section of the beams as shown in Figures 5-22 ~5-24 are also presented. The 3D and 2D views of the Gaussian beam's cross-section shown with one of the obstacles are presented in Figure 5-25. The propagation distance ranges from $z=40$ mm to $z=190$ mm.

Figure 5-25. 3D views of the Gaussian beam's intensity versus the propagation distance. One of the obstacles is positioned at a propagation distance of 25 mm in the figure. (a) A view towards the z direction; (b) a planar view.

In Figure 5-25 (a), the horizontal axis represents the sampling points ranging from 1 to 256, in which the unit size is $15 \mu\text{m}$ per pixel; the vertical axis represents the propagation distance in 30 sampling points with an interval of 5 mm. In Figure 5-25 (b), the horizontal axis denotes the propagation distance, and the vertical axis represents the sampling points. Simulations of the IBB and the FBB in the free-space propagation are exhibited in Figure 5-26 and Figure 5-27, respectively.

Figure 5-26. 3D views of the IBB's intensity versus the propagation distance. One of the obstacles is positioned at a propagation distance of 25 mm in the figure. (a) A view towards the z direction; (b) a planar view.

Figure 5-27. 3D views of the FBB's intensity versus the propagation distance. One of the obstacles is positioned at a propagation distance of 25 mm in the figure. (a) A view towards the z direction; (b) a planar view.

It can be seen clearly from Figures 5-25~ 5-27 that the bright rings of the FBB and the IBB are able to regenerate themselves after the obstacles; however, the Gaussian beam does not have such a property, as the obstacles have changed the shape of the beam destructively.

To further verify the FBB's self-reconstruction, we also compute the FBB's phase evolutions with and without obstacles. The same obstacles and the beam parameters in the above simulations have been adopted. Snapshots of the FBB's phases with and without obstruction are shown in Figure 5-28, where the gray-scales represent phase values ranging from 0 to 2π . Phases of the FBB, which are blocked by the two obstacles and are propagated thereafter, are shown in Figures 5-28 (a-d) for propagation distances at $z = 65$ mm, 90 mm, 140 mm, and 190 mm, respectively. The undisturbed phases of the FBB at $z = 65$ mm, 90 mm, 140 mm, and 190

mm are shown in Figures 5-28 (e-h), respectively. It is observed from Figure 5-28 (d) and Figure 5-28 (h) that both the phase distributions of the FBB with and without obstruction are almost identical at last, that is to say, the FBB's phase is also self-reconstructive.

Figure 5-28. The snapshots of the FBB's phase with and without obstruction. (a-d) With the obstacles, the phase cross-sections of the FBB at $z=65$ mm, 90 mm, 140 mm, and 190 mm, respectively; (e-f) without obstruction, the phase cross-sections of the FBB at $z=65$ mm, 90 mm, 140 mm, and 190 mm, respectively.

The full propagations of the unobstructed and obstructed beams from $z=40$ to $z=190$ mm are presented in Figures 5-29 (a-b), respectively. In the figures the horizontal axis represents the propagation distance, and the vertical axis represents the phase vertically bisecting through the center of the phase cross-section at each sampled propagation distance. The phase of the obstructed beam, shown in Figure 5-29 (b), is distorted initially; however, after a propagation distance of ~ 100 mm, the disturbed phase recovered to normal.

Figure 5-29. Phase evolutions of the FBB without and with obstructions. (a) Without obstruction, the phases at the propagation distance that is ranging from $z=65$ to $z=190$ mm; (b) with the obstructions, the phases at the propagation distance that is ranging from $z=65$ to $z=190$ mm.

5.5.2 Experiments on self-reconstruction

The FBB of $n=4.5$ has also been experimentally generated by use of the interference method [122], with which a computer-generated hologram was fabricated by e-beam lithography. The size of the hologram is 7.68 mm by 7.68 mm, and k_r of the desired FBB is given as 13/mm. A beam profiler was used to record the FBB's intensity at different propagation distances. Experimental self-reconstruction of the FBB through an obstacle has been implemented, and the experimental results are presented in Figure 5-30. The particle has an elliptical shape with a size of 1.5 mm in long axis and 1.1 mm in short axis, and it is shown in Figure 5-30(a). In the experiment, the particle was placed 200 mm behind the hologram, and the particle's image was captured at 20 mm behind the particle plane.

Figure 5-30. Experimental results of, (a) the particle used in the experiment; it is positioned 200 mm behind the hologram; (b-f) the captured intensity profiles of the obstructed FBB at a distance of $z= 300$ mm, 500 mm, 550 mm, 600 mm and 700 mm, respectively from the hologram.

The measured intensity profiles of the FBB at distances of $z=300$ mm, 350 mm, 500 mm, 550 mm, 600 mm, and 700 mm are shown in Figures 5-30 (b-f), respectively. The interference of other rings is observed in the figures due to the overlapping of the images of the higher diffraction orders of the hologram. The experimental results in Figures 5-30 (b-f) also demonstrate that the FBB can self-reconstruct from a distortion.

5.5.3 Discussions

In the simulations, several factors are found affecting the self-reconstruction effect of the FBB. First, it needs a shorter distance for the beam to recover from a smaller obstacle; if the overall transverse dimension of the obstacles is small, less light will be blocked, as a result the beam will be recovered more easily. Second, since the light energy of the FBB is not distributed evenly, location of an obstacle would affect the beam's recovery too; for example, an obstacle located on the brighter rings of the FBBs is more difficult to be self-reconstructive because that the beam loses more energy when the brighter rings are blocked. Since the ability of the self-reconstruction is based on the fact that the energy on different rings is exchangeable, more blocked light means that the beam needs to transfer more energy from other rings to compensate the loss. In the simulations (for which the results are not presented here), if the obstacles are located in the dark area in the center or between the intensity rings,

they have little influence on the beam's propagation. Thus in the applications of guiding or trapping, numerous particles could be positioned in the central dark area of the beam and the beam's distribution will not be affected at all. Bouchal *et al.* [107] gave an expression $Z_{\max} \approx dk/(2\alpha)$ to estimate the recovery length behind the obstacle; in the expression d denotes the transverse dimension of the obstacle. Therefore, in our case in the simulations, $Z_{\max}$ for an IBB is calculated as about 90 mm. From Figures 5-15 ~5-15 it can be observed that the $Z_{\max}$ for an FBB is roughly the same as that of the IBB. Thus the expression is also applicable to an FBB. However, in the expression, the $Z_{\max}$ excludes the influence of the location of the obstacles.

Hence, we have verified the self-reconstruction ability of an FBB of order $n=4.5$, as an example. The beam can overcome some blocks and recover itself from distortions after a propagation distance, and the self-reconstruction length is dependent on the location and the size of obstacles, and the beam parameters. As a comparison, reconstructions of a Gaussian beam and an IBB with an order of $n=4$ are also presented. It is shown that the IBB also has the self-reconstruction ability but the Gaussian beam does not have.

To the author's knowledge, it is the first time that an FBB with asymmetric distributions has been demonstrated possessing the stable self-reconstructive property in the free-space propagations. The FBB provides a useful tool for potential applications in optical trapping.

5.6 Interference of FBBs

5.6.1 Interference pattern of FBBs

Studies on coherent addition of Bessel beams and behavior of the interference have been reported in the past years [123], such as interference of Bessel beams of different radial wave vectors and the same order [124], or interference of different orders and the same radial wave vector [125]. The Bessel beams, which take part in the interference, are diffraction-free and self-reconstructive, and the angular spectrum of each Bessel beam consists of single radial frequency or radial wave vector, which is related to the propagation constant--longitudinal wave vector. Inheriting the diffraction properties from the respective Bessel beams, interference patterns of the Bessel beams of different radial wave vectors and the same order can demonstrate a Talbot-effect or self-imaging effect [126] because the interference is a coherent superposition of the non-diffracting modes [127-128]. The self-imaging effect

represents the spatial analog of the mode-locking realized in the temporal domain; due to the interference of the modes, the transverse intensity profile of the interference re-appears periodically, and vanishes after the nondiffracting distance of the Bessel beams [129-130]. Interference patterns of the Bessel beams provide a means for the three-dimensional confinement of particles [119, 130]. Furthermore, as higher-order Bessel beams have helical phase wave-front, the interference patterns of these Bessel beams may also possess orbital angular momentum, which now is dependent on the superposition of the respective helical phases of the Bessel beams. Applications of interference patterns of the Bessel beams on optical trapping have been implemented, such as the interference patterns used for controlled rotation of microscopic particles in optical tweezers and rotators [131].

As was introduced above, the FBBs have the amplitude of fractional-order Bessel function and the helical phase of fractional-order; the beams have many commons with the IBBs. However, the FBBs are distinctly different from the IBBs in the transverse distributions of both the amplitude and the phase in the free-space propagation. Thus, study on the interference patterns of FBBs will extends new areas on study of light property and applications of optical trapping.

Because there are many possible interference combinations in terms of the radial wave vectors and the orders, many novel and interesting phenomena on the interferences of the FBBs have never been investigated so far. Thus, various interferences of FBBs by varying beams' parameters will be investigated.

In the past, several techniques have been introduced to obtain interference patterns of Bessel beams: 1, using double or multiple annular slits to generate interference patterns of the zero-order Bessel beams of different radial wave vectors [132]; 2, using combination of LG beams and axicons to yield a superposition of higher-order Bessel beams [133-134]; 3, using holograms [135]. However, the first two have obvious disadvantages; for example, the angular-slits method has very low efficiency and can produce only interference patterns of the zero-order Bessel beams; the axicon method is inflexible and complicated, and due to the radial wave vector filtering effect of the axicon, different axicons have to be applied to generate Bessel beams of different radial wave vectors, moreover, the corresponding-orders of LG beams have to be produced beforehand such that the LG beams can be converted into higher-order Bessel beams by the axicons. On the contrary, the hologram method is efficient and flexible for generation of various Bessel beams; furthermore, with the help of the spatial light modulator (SLM), dynamic interference patterns can also be obtained easily. Thus, the

method of using holograms to generate various interference patterns of FBBs with variable parameters, such as the beam sizes, the radial wave vectors, and the orders, will be investigated in this section too.

Interference patterns of Bessel beams can be written in an analytic form by

$$A(\rho, \phi) = J_{n_1}(2\pi \cdot \rho / r_{01}) \exp(i \cdot n_1 \cdot \phi) + J_{n_2}(2\pi \cdot \rho / r_{02}) \exp(i \cdot n_2 \cdot \phi) \quad (5-8)$$

where ρ and ϕ are the coordinates in the interference plane. J_{n_1} and J_{n_2} represent the Bessel functions of the first kind, n_1 and n_2 are the orders of the Bessel functions of the first kind. k_{r1} and k_{r2} are the radial wave vectors of the respective Bessel beams, and $k_r = 2\pi / r_0$, $k = 2\pi / \lambda$, $k^2 = k_r^2 + k_z^2$. Based on Equation (5-S), we can obtain various interference patterns if we select parameter combinations of the radial wave vectors and the orders. Note that the radial wave vector is proportional to the density of the transverse rings of a Bessel beam, so the value of k_r should not be too large or too small.

In the simulations, the sampling grid is 256 by 256 with pixel size of 15 μm by 15 μm , and the wavelength of the incident light is 633 nm. The propagations in the free-space can be calculated by the angular spectrum wave theory. Some examples with various combinations of the beam parameters have been given in the following:

Case 1 is the interference of FBBs with the same radial wave vector and different orders, where $n_1=0$, $n_2=4.5$, and $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $k_{r2}=0.2/15 \mu\text{m}^{-1}$.

Figure 5-31. Transverse intensity distribution of $n_1=0$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=4.5$,

$k_{r2}=0.2/15 \mu\text{m}^{-1}$, at, (a) $z=20 \text{ mm}$; (b) $z=310 \text{ mm}$.

The axial propagations along the z direction are shown in Figure 5-32, where Figure 5-32(a) samples the line vertically bisecting the transverse distributions of every 20 mm distances from $z=30$ mm to 610 mm, and Figure 5-32(b) samples the transverse intensity horizontally. The number of the sampling points along the propagation is 30 with interval of 20 mm.

Figure 5-32. Axial intensity distribution of $n_1=0$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=4.5$, $k_{r2}=0.2/15 \mu\text{m}^{-1}$ from $z=30$ mm to $z=610$ mm with interval of 20 mm, (a) vertical line through the center of the transverse intensity; (b) horizontal line through the center of the transverse intensity.

It can be seen from Figures 5-31 and 5-32 that the parts excluding the region around the opening slit are stable and diffraction-free, and the intensity patterns show that the petal number is the addition/subtraction of the orders.

Case 2 is the interference of the beams of $n_1=n_2=4.5$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, and $k_{r2}=0.4/15 \mu\text{m}^{-1}$. The interference pattern is repeated periodically along the propagation distance.

Figure 5-33. Transverse intensity distribution of $n_1=4.5$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=4.5$, $k_{r2}=0.4/15 \mu\text{m}^{-1}$, at different distances.

Full propagations of the horizontal and the vertical lines of the transverse intensity patterns are shown in Figure 5-34, respectively.

Figure 5-34. Axial intensity distribution of $n_1=4.5$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=4.5$, $k_{r2}=0.4/15 \mu\text{m}^{-1}$ from $z=30$ mm to $z=610$ mm with interval of 20 mm, (a) vertical line through the center of the transverse intensity patterns; (b) horizontal line through the center of the transverse intensity patterns.

It can be seen from Figures 5-33 and 5-34 that the superposition of the FBBs demonstrates a clear self-imaging effect. As was mentioned above, the self-imaging effect is a superposition of non-diffracting beams.

Case 3 is that the interference of beams with $k_{r1} = 0.2/15 \mu\text{m}^{-1}$, $k_{r2} = 0.4/15 \mu\text{m}^{-1}$, $n_1 = 1.5$, and $n_2 = 4.5$. The transverse intensity patterns at $z = 20 \text{ mm}$ and $z = 310 \text{ mm}$ are shown in Figures 5-35 (a) and (b), respectively.

Figure 5-35. Transverse intensity distribution of $n_1 = 1.5$, $k_{r1} = 0.2/15 \mu\text{m}^{-1}$, $n_2 = 4.5$, $k_{r2} = 0.4/15 \mu\text{m}^{-1}$, at, (a) $z = 20 \text{ mm}$; (b) $z = 310 \text{ mm}$.

The interference patterns show that the beam size is nearly unchanged, and the petal number in the transverse intensity is the subtraction of the orders of the Bessel beams. Full propagations of the horizontal line and the vertical line of the transverse intensity patterns are shown in Figures 5-36 (a) and (b), respectively:

Figure 5-36. Axial intensity distribution of $n_1 = 1.5$, $k_{r1} = 0.2/15 \mu\text{m}^{-1}$, $n_2 = 4.5$, $k_{r2} = 0.4/15 \mu\text{m}^{-1}$ from $z = 30 \text{ mm}$ to $z = 10 \text{ mm}$ with interval of 20 mm , samplings of (a) vertical line; (b) horizontal line.

The self-imaging effect can be observed both in Figure 5-35 and Figure 5-36, where the patterns are the superposition of an IBB and an FBB.

Case 4 is the interference of FBBs with $\mathbf{k}_{r1} = \mathbf{k}_{r2} = 0.2/15 \mu\text{m}^{-1}$, $n_1=4.5$, and $n_2=-4.5$. The transverse patterns at $z=30 \text{ mm}$ and $z=310 \text{ mm}$ are shown in Figure 5-37.

Figure 5-37. Transverse intensity distribution of $n_1=4.5$, $\mathbf{k}_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=-4.5$, $\mathbf{k}_{r2}=0.2/15 \mu\text{m}^{-1}$, at, (a) $z=20 \text{ mm}$; (b) $z=310 \text{ mm}$.

The interference patterns show that part of the interference beam is non-diffracting, and the petal number in the transverse intensity is the addition of the orders of the Bessel beams. As a comparison an interference of the IBB is implemented while the propagation parameters are adopted the same as those of the FBBs. The parameters of the two IBBs are $\mathbf{k}_{r1} = \mathbf{k}_{r2} = 0.2/15 \mu\text{m}^{-1}$, $n_1=4$, and $n_2=-5$. The transverse patterns at $z=30 \text{ mm}$ and $z=310 \text{ mm}$ are shown in Figure 5-38, respectively.

Figure 5-38. Transverse intensity distribution of $n_1=4$, $\mathbf{k}_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=-5$, $\mathbf{k}_{r2}=0.2/15 \mu\text{m}^{-1}$, at, (a) $z=20 \text{ mm}$; (b) $z=310 \text{ mm}$.

The propagations of the FBBs along z direction are shown in Figure 5-39.

Figure 5-39. Axial intensity distribution of $n_1=4.5$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=-4.5$, $k_{r2}=0.2/15 \mu\text{m}^{-1}$ from $z=30 \text{ mm}$ to $z=610 \text{ mm}$ with interval of 20 mm , in, (a) vertical line through the transverse center; (b) horizontal line through the transverse center.

From Figures 5-37 and 5-39 it can be seen that the petal number in the transverse intensity is the result of the subtraction of two orders of the beams, and the intensity in vertical direction is diffraction-free, but that in the horizontal direction is unstable owing to the opening slit of the beams. It is interesting that part of the intensity pattern is diffraction-free while another part is unstable, and both the parts can be coexisted in one beam. It also can be observed from Figure 5-38 that the petals are very stable in the interference of IBBs.

Case 5 shows the interference of FBB and IBB with $k_{r1}=k_{r2}=0.2/15 \mu\text{m}^{-1}$, $n_1=4.5$, and $n_2=4$. The transverse intensity patterns at $z=20 \text{ mm}$ and $z=310 \text{ mm}$ are shown in Figures 5-40 (a) and (b), respectively.

Figure 5-40. Transverse intensity distribution of, $n_1=4.5$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=4$, $k_{r2}=0.2/15 \mu\text{m}^{-1}$, at, (a) $z=20 \text{ mm}$; (b) $z=310 \text{ mm}$.

The corresponding full propagations are shown in Figure 5-41 (a) and (b), respectively.

Figure 5-41. Axial intensity distribution of $n_1=4.5$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, $n_2=4$, $k_{r2}=0.2/15 \mu\text{m}^{-1}$ from $z=30 \text{ mm}$ to $z=610 \text{ mm}$ with interval of 20 mm , sampling of (a) vertical line; (b) horizontal line.

From this case we can see that the central intensity of the beam is stable but the opening slit is still unstable as the opening slit has disappeared after a propagation distance.

Thus, interference patterns of the FBBs follow the same rule **as** those of the interferences of the IBBs. Such patterns can be easily understood as the interference is the pattern in reminiscence to the conventional pattern of IBBs, while the additional slit opening, which tends to be unstable beyond nondiffracting distance, causes the vanishing of the additional vortex and the bright spot as the beam propagates in the free space. However, beam size of the interference patterns has almost unchanged within the nondiffracting propagation distance although the unstable opening slit has resulted in the interference patterns partially unstable, and the overall interference patterns of the FBBs are still stable and diffraction-free.

5.6.2 Generation of interference patterns of FBBs

As was mentioned before, one single Bessel beam can be generated by use of the following expression when the incident beam of the hologram is a uniform light:

$$T_n(r, \theta) = \exp(in\theta) \exp(-i2\pi r/r_0) \quad (5-9)$$

where θ and r are coordinates of the hologram plane, r_0 is an adjustable scaling factor, and n represents the order of the Bessel beam to be created. For the FBBs, they also can be generated by this equation.

To generate interference patterns of Bessel beams, transmission of the hologram can be written by

$$T(r, \theta) = \exp(i \cdot n_1 \cdot \theta) \exp(-i \cdot 2\pi \cdot r / r_{01}) + \exp(i \cdot n_1 \cdot \theta) \exp(-i \cdot 2\pi \cdot r / r_{02}) \quad (5-10)$$

It can be seen that Equation (5-10) is a linear combination of the two holograms of Equation (5-9), and the transmission of the hologram in Equation (5-10) comprises both amplitude and phase

However, although the hologram method is efficient and flexible in generating Bessel beams, holograms are difficult to fabricate with complex-amplitude. Most fabricated holograms are phase-only or amplitude-only such that they can not modulate optical wave's amplitude and phase simultaneously. Even though we could use an amplitude-only hologram and a phase-only hologram together to modulate light's complex-amplitude, stringent alignments between them have to be satisfied. Because Equation (5-10) can not be represented by a phase-only or amplitude-only hologram, we try to use uniform amplitude to replace the amplitude and use a SLM to simulate the phase of Equation (5-10). The phase extracted from Equation (5-10) is saved as an image file to control a SLM through a PC interface. The detailed coding technique on the SLM can be found in Refs [115, 122].

Holograms need a reconstruction distance to obtain target images, and the reconstruction distance can be roughly estimated by the expression of $Z_{max} = kR / (\lambda k_r)$, where R is the radius of the hologram, λ is the illumination wavelength, and k_r is the radial wave vector. For the interference of two Bessel beams, the reconstruction distance is the smaller of the two Z_{max} s. In the following generations of interference patterns, $r_0 = 2\pi / k_r$, and the sampling grid is 256×256 .

The first example is the one that corresponds to the above simulation-case 2, where $n_1 = n_2 = 4.5$, $k_{r1} = 0.2/15 \mu\text{m}^{-1}$, and $k_{r2} = 0.4/15 \mu\text{m}^{-1}$. Snapshots of the interference patterns at different reconstruction distances are shown in Figure 5-42.

Figure 5-42. Reconstruction patterns of a hologram, where $n_1=n_2=4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.4$.

It can be seen from Figure 5-42 that the central spot of the interference patterns is varying from large to small and to large again. Full propagations in vertical sampling and horizontal sampling of the transverse intensity are shown in Figures 5-43 (a) and (b), respectively.

Figure 5-43. Axial intensity distribution of $n_1=4.5$, $n_2=4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.4$ from $z=500\text{mm}$ to $z=1080\text{mm}$ with interval of 20mm , in, (a) vertical line sampling; (b) horizontal line sampling.

The hologram is shown in Figure 5-44, where the gray-scales correspond to the phase values of the transmission. The experimental configuration here for generation of interference patterns was adopted the same as those in Figures 5-12 and 5-16.

Figure 5-44. Hologram to reconstruct interference of beams of $n_1=4.5$, $n_2=4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.4$.

The reconstructed images of the hologram were captured by use of a CCD beam profiler. The images are shown in Figure 5-45.

Figure 5-45. Interference patterns ($n_1=4.5$, $n_2=4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.4$) captured by a CCD beam profiler at different reconstruction distances along the z axis. (a) $z=900$ mm, (b) $z=1000$ mm, (c) $z=1100$ mm.

The pictures were captured at distance from 900 mm to 1100 mm with an interval of 100 mm. It can be seen from Figure 5-45 that the interference patterns show a clear opening slit and a self-imaging effect; however, the interference patterns are much deformed because of misalignment among the incident light, the SLM, and the CCD camera. The aberrations from the SLM are very critical to the image quality of the generated patterns [18], and the

quantization process to the hologram results in shorter propagation distances of the reconstructed beam.

The second example is the one that corresponds to simulation case 4, where $n_1=4.5$, $n_2=-4.5$, $k_{r1}=0.2/15 \mu\text{m}^{-1}$, and $k_{r2}=0.2/15 \mu\text{m}^{-1}$. Snapshots of the interference patterns at different propagation distances are shown in Figure 5-46.

Figure 5-46. Reconstruction patterns of a hologram, where $n_1=4.5$, $n_2=-4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.2$.

Full propagations of the beam in vertical sampling and horizontal sampling are shown in Figures 5-47 (a) and (b), respectively.

Figure 5-47. Axial intensity distribution of $n_1=4.5$, $n_2=-4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.2$ from $z=500 \text{ mm}$ to $z=1080 \text{ mm}$ with interval of 20 mm , (a) sampling of the vertical lines; (b) sampling of the horizontal lines.

The propagation in the nondiffracting distance shows that the beam is roughly unchanged, and the patterns are similar to the simulations above. The hologram loaded onto the SLM is shown in Figure 5-48.

Figure 5-48. Hologram reconstructing interference of beams of $n_1=4.5$, $n_2=-4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.2$.

The experimental results of this case are shown in Figure 5-49, where only the central part of the interference patterns is presented for a clearer comparison to the simulation results. The measurement distances in Figure 5-49 (a-b) are 1000 mm and 1200 mm , respectively.

Figure 5-49. Interference patterns ($n_1=4.5$, $n_2=-4.5$, $r_{01}=2\pi \cdot 15 \mu\text{m}/0.2$, and $r_{02}=2\pi \cdot 15 \mu\text{m}/0.2$) captured by a CCD beam profiler.

There are 8 bright segments and a blurring segment in the center of the interference patterns. The variation of the interference segments in Figure 5-49 is the same as that shown in Figure 5-46.

It should be noted that although the above generation method can reconstruct the desired optical beams approximately, the reconstructed intensity variations can occur owing to consideration of only the phase of the complex sum of the interference patterns. The intensity variation behavior is complex but these variations did not hinder in the reconstruction experiments [132]. As a complementary solution, the interference hologram introduced previously can be applied to generate the interference patterns of the optical beams. The advantage of this method lies in its high fidelity to the desired beam but it is also well known for the low reconstruction efficiency.

Such a fractional phase with its influence in the intensity profiles both in single beam and in interference beam is novel; as a result this will directly influence the capabilities of the beams when used as trapping beam in the optical manipulation. The hologram's reconstructions show that similar patterns to the above simulations have been obtained by use of the phase term of Equation (5-10).

In summary, various interference patterns of the FBBs with different combinations of beam parameters have been demonstrated in simulation and analyzed subsequently; a method has been introduced to create phase-only holograms so as to produce interference patterns with variable parameters of Bessel beams.

5.7 Conclusion

A fractional-order Bessel beam (FBB) has been generated and investigated. The FBB can be generated by using an analytic formula which is similar to the generation of higher-order Bessel beams, or can be generated by using holographic interference. Furthermore, the FBB can be generated and controlled by use of refreshed gray-scale patterns which are displayed in a SLM. The FBBs are diffraction-free in a characteristic distance and possess helical phase in propagation; the transverse intensity of the FBB is asymmetric in propagation. The simulations and experiments have shown that the FBB is self-reconstructive after obstruction. To the author's knowledge, it is the first time that a beam with asymmetric intensity has been found stable and self-reconstructive; this phenomenon would be instructive to further study the properties of light. With the developments in optical trapping, more and more laser beams with specified structures in transverse intensity or axial intensity have been generated such as interference of Bessel beams for three-dimensional trapping; correspondingly, the interferences of FBBs have been investigated in this Chapter too. The study showed that the interference patterns of FBBs are similar to the patterns of corresponding integer-order Bessel beams, but many novel phenomena are also existed in those of FBBs. Furthermore, a hologram method has been found to generate any interference patterns of FBBs with adjustable beam-parameter combinations.

CHAPTER 6 GENERATION OF AN ARRAY OF PROPAGATION-INVARIANT BEAMS USING COMPOSITE HOLOGRAM

6.1 Introduction

Propagation invariant fields maintain constant transverse intensity distribution for finite distance like Bessel and Mathieu beams [133] or regenerate their transverse intensity distribution at regular intervals along the propagation direction (self-imaged beam) [134]. With the development in the field of micro-beam shaping and optical tweezers the importance of an array or multiple trapping spots using a single laser source were realized. To trap an array of microparticles simultaneously and manipulate them individually, there is a need of dynamic control of the shape, size, intensity, and position of each trapping spot. Hence, plethora of work [135-142] were reported in the static and dynamic generation of an array of optical beams, although the dynamic generation of an array of propagation invariant beams has not been convincingly covered. In this Chapter the author make a dynamic holographic generation of an array of various propagation invariant beams using a spatial light modulator (SLM).

There are various ways to generate an array of diffracting beams like LG and Gaussian beams using commercial available $N \times N$ diffractive array generators [MEMS Optical Inc., Huntsville AL]. Another technique time shared trapping employing the scanning of a single beam through a sequence of discrete locations creates a time-shared trapping pattern [137]. Computer-generated holograms using the iterative algorithm were also used to dynamically generate an array of LG or Gaussian beams [137-138, 141], such techniques required an SLM to impose phase modulation onto the input beam's wave-front. The generalized phase contrast (GPC) was also explored in detail for the diffracting beams [139, 141]. However, each of the above techniques has a limit for the dynamic generation of propagation invariant beams owing to the need to conserve the wave-vector distribution of the propagation invariant beams. A propagation invariant beam contains a single radial frequency in the angular spectrum, which in the geometrical interpretation represents as a superposition of the plane waves whose wave-vectors are arranged in a cone shape [142-143]. It is necessary for a propagation invariant beam to maintain its wave-vector distribution to possess nondiffracting propagation. Hence conventionally used diffractive array generators cannot be used to generate an array of

propagation invariant beams, as the divergence property of the grating will spoil the wave-vector distribution of the propagation invariant beams and their non-diffracting and self-reconstructive properties. On the other hand, the critical issue with CGHs using iterative algorithms for the generation of an array of propagation invariant beams is that the kernel of the iterative algorithm can only approximate the phase and hence the intensity distribution of each nondiffracting spots in the array. The generalized phase contrast method transforms a phase pattern into a corresponding intensity pattern, but to generate a pattern with the conserved phase or wave-vector distribution by such technique will be cumbersome. Hence dynamic generation of an array of various propagation invariant beams like a Bessel beam and a self-imaged beam is still an open question. Generation of an array of propagation invariant beams is recently reported by Z. Bouchal [142-144] by radial sampling of $N \times N$ intensity spots in the Fourier domain to impart the wave-vector distribution, which is necessary for the nondiffracting propagation. Such techniques suffer a limit due to low efficiency by the annular slit and can only generate an array of nondiffracting beams of similar radial wave-vector (k_r), which is determined by radius of the annular slit [8]. Furthermore, an array of propagation invariant beams was reported by using a volume hologram [145] and by microfabrication of an array of axicon [146]; such techniques are static and complex in nature and are difficult to generate a customized array of nondiffracting beams. In this Chapter a simple and effective method, in which a computer-generated composite hologram consisting of $N \times N$ holograms reconstructs the corresponding array of propagation invariant beams, is proposed. The composite hologram encodes the actual phase to individual beam spot in the array as each individual hologram can reconstruct the ideal propagation invariant beam, contrary to the iterative algorithm approach, which approximates phase of each beam spot in an array and hence reconstructs the approximated beam array. Furthermore, by arranging different phase on individual hologram in a composite hologram approach, propagation invariant beams with different radial wave-vectors and azimuthal indices can easily be obtained. A composite hologram can also be customized such that it can generate an irregular array consisting of propagation invariant beams.

6.2 A composite hologram and simulations

A Bessel beam can be generated by a hologram with a phase transmission again given as:

$$T_n(r, \theta) = \exp(in\theta) \exp(-i2\pi r/r_0) \quad (6-1)$$

where n represents the order of a Bessel beam, and r_0 is an adjustable constant parameter. For a single Bessel beam, the non-diffraction distance can be estimated by the following expression,

$$Z_{\max} = dk/k_r \quad (6-2)$$

where d is the radius of the Bessel beam, k_r is the radial wave-vector of the beam, and k is the wave-vector with $k = 2\pi/\lambda$. The relationship between the adjustable parameter r_0 and k_r is $k_r = 2\pi/r_0$. To generate an array of $N \times N$ Bessel beams, we use a composite hologram approach where $N \times N$ holograms with the transmission distribution of Equation (6-1) are arranged in an array. The schematic arrangement of a composite hologram is shown in Figure 6-1. When a light is incident on a composite hologram, the hologram regenerates the array pattern determined by individual hologram's location and phase distribution.

Figure 6-1. Schematic arrangement of a composite hologram approach to generate an array of propagation invariant beams.

Figure 6-2(a) shows a hologram to generate a single zero-order Bessel beam and a composite hologram embedding 3×3 array of such holograms is shown in Figure 6-2(b). In Figure 6-2(a), the parameters used are $r_0 = 188 \mu\text{m}$ and size of the hologram is 2.55 mm by 2.55 mm. Thus the non-diffraction distance of the reconstructed Bessel beam is calculated as 380 mm. We used the angular plane wave theory to simulate the composite hologram in reconstruction and propagation to verify its effectiveness. Total size of the composite hologram is 7.68 mm by 7.68 mm with each pixel size of $15 \mu\text{m}$ by $15 \mu\text{m}$, and r_0 each smaller hologram in the array is 188 μm . Size of each individual hologram in the array is 2.55 mm by 2.55 mm.

Figure 6-2. (a) Hologram to generate a zero-order Bessel beam, (b) composite hologram to generate an array of 3 x 3 zero-order Bessel beams.

Figure 6-3 (a-d) shows the transverse intensity profile of a Bessel beam, which is reconstructed at different propagation distances with the hologram located in the center of the composite hologram. The snapshots are taken at $z = Z_{\max}/4$, $Z_{\max}/2$, $3Z_{\max}/4$, and $Z_{\max}$, respectively, where $Z_{\max}$ ($=380$ mm) is the maximum non-diffraction distance. Figure 6-3 (e-h) shows a full-view of transverse intensity distribution of the array of Bessel beams generated by the composite hologram at the same intervals.

Figure 6-3. (a-d) Propagation of the Bessel beam located in the center of the array (e-h) propagation of the full Bessel beam array.

The calculated intensity errors between the beam generated by a single hologram and each beam generated by the composite hologram are less than 10% within the non-diffraction distance. The intensity error is calculated based on the following formula:

$$E_{error} = \sum_{i,j}^{N,N} |Com\_Int_{i,j} - Ideal\_Int| / (N \cdot N \cdot Ideal\_Int) \quad (6-3)$$

where $Com\_Int_{i,j}$ stands for intensity of the Bessel beam reconstructed by the (i, j) hologram (where i and j stand for the respective column and row of the hologram in the array) in the composite hologram, and $Ideal\_Int$ stands for the Bessel beam reconstructed by a single hologram. Figure 6-4 shows a plot of the transverse intensity versus propagation distance of the beams generated by the composite hologram. The horizontal numbers represent the propagation distances from $Z_{max}/20$ to Z_{max} , and the vertical ones represent the transverse intensity of the Bessel beam array. It is observed that the Bessel beams in the array are well maintained within the non-diffraction distance and the interference among them is not strong.

Figure 6-4. Plot of the Bessel beam array versus propagation distance, where the horizontal numbers stand for the propagation distance from $z=Z_{max}/20$ to $z=Z_{max}$, and the vertical numbers represent the pixels of the bisecting line of the Bessel beam array.

We further generate an array of higher-order Bessel beams carrying optical vortex through the composite hologram approach. A hologram that generates a 4th order Bessel beam is shown in Figure 6-5(a) and a composite hologram to generate a 3 x 3 array of 4th order Bessel beams is shown in Figure 6-5(b).

Figure 6-5. (a) A hologram to generate a single 4th-order Bessel beam, (b) a composite hologram to generate a 3 x 3 array of 4th-order Bessel beams.

Figure 6-6 shows the transverse intensity distribution of the higher order Bessel beams reconstructed by the composite hologram, and the propagation distance is from $Z_{\max}/4$ to $Z_{\max}$.

Figure 6-6. A 3 x 3 array of higher order Bessel beams using a composite hologram approach, reconstructed Bessel beams at propagation distance of

(a) $Z_{\max}/4$, (b) $Z_{\max}/2$, (c) $3Z_{\max}/4$, (d) $Z_{\max}$.

6.3 Experimental generation of Bessel beam array

We experimentally generated the arrays of various propagation invariant beams using the composite hologram approach through a spatial light modulator, which has 512 by 512 pixels and size of each pixel is $15\ \mu\text{m}$ by $15\ \mu\text{m}$. Figure 6-7(a) shows the composite hologram to generate an 8 x 8 array of Bessel beams and the captured image at a propagation distance of 50 mm from SLM is shown in Figure 6-7(b). Only part of the image was captured owing to the limited view field of the CCD camera.

Figure 6-7. (a) A composite hologram used in the experiment to generate a 8 x 8 array of Bessel beams, (b) part of the experimentally generated Bessel beam array at 50 mm.

We also generated a 3 x 3 array of 4th order Bessel beams, part of which is shown in Figure 6-8.

Figure 6-8. Experimentally generated 3 x 3 array of 4th-order Bessel beam using the composite hologram of Figure 6-5(b), (a) at 50 mm, (b) at 70 mm.

In addition to the generation of a regular array of $N \times N$ propagation invariant beams, using this method we can also generate an array of irregular pattern of propagation invariant beams. The individual holograms in the composite holograms can be arranged into any desired patterns and the beams will reconstruct based on the position of the individual hologram in the composite hologram. For example, several irregular patterns of composite holograms are shown in Figure 6-9 (a1-d1) and part of the reconstructed beams is correspondingly demonstrated in Figure 6-9 (a2-d2) with the propagation distance of 70 mm.

Figure 6-9. Composite holograms are used to generate an array of propagation invariant beams of irregular patterns. (a1-d1) show various composite holograms of irregular pattern, (a2-d2) show the parts of the experimentally reconstructed array of the irregular patterns of Bessel beams at distance of 70 mm.

For a closer observation of beam quality in the arrays, zoomed-in pictures of the captured Bessel beams [in Figures 6-9 (a1) and (a2)] are also shown in Figures 6-10 (a) and (b) with propagation distance of 50 mm and 70 mm, respectively.

Figure 6-10. (a) Part of the reconstructed Bessel beams in Figure 6-9 (a2) at 50 mm, (d) part of the reconstructed Bessel beams at 70 mm.

In addition to the flexibility of generating any irregular patterns, the composite hologram approach can be effectively used to generate different propagation invariant beams. To illustrate the simplicity and effectiveness of such composite holograms we generated an array of self-imaged optical bottle beams and zero-order Bessel beams. Such self-imaged optical bottle beams are propagation invariant beams, which periodically generate bottles (low

intensity points) and high intensity point along the propagation axis [147]. The array is arranged in such a manner, that the optical bottle beams and the zero-order Bessel beams are placed alternatively. Using the composite hologram approach we can easily vary the radial wave-vector (k_r) of each individual hologram in the array so that location of the bottles and high intensity points can be dynamically varied along the propagation axis. As an example, we generated such a 4 x 4 array of optical beams and manipulated the radial wave-vectors of individual hologram to generate alternative maximum and minimum intensity points as shown in Figure 6-11. Figure 6-11(a) shows a composite hologram to generate a 4 x 4 array of self-imaged optical bottle beams and zero-order Bessel beams, and Figure 6-11(b) shows the simulation result.

Figure 6-11. (a) Composite hologram to generate an array comprising optical bottle beams and zero-order Bessel beams, (b) simulated reconstruction of the composite hologram.

Figure 6-12 exhibits the reconstruction and propagation of the composite hologram experimentally. In the figure we can observe intensity variations in the centers of the optical bottle beams, which are surrounded by the zero-order Bessel beams.

Figure 6-12. Experimental generation of array of self-imaged optical bottle beams and Bessel beams at different distances, (a) bottle (null intensity point) appears in the optical bottle beams which are surrounded by the zero-order Bessel beams, (b) all are bright spots, (c) bottle again appears in the optical bottle beams, which are surrounded by the zero-order Bessel beams.

Generation of such patterns can be effectively used in optical tweezers to periodically trap various microparticles based on refractive indices. Both low and high index particles can be periodically trapped in such an array. Such patterns trapping periodically an array of microparticles of different refractive indices could be used to form multilayer photonic crystals such as photonic bandgap materials [148]. Advantage of using self-imaged optical bottle beam array over an array of vortices for the generation of photonic crystals is that such self-imaged optical bottle beam does not carry any OAM as compared to the vortex beams carrying OAM of $n\hbar$ per photon (n is the charge number of the vortex). Hence while forming the lattice structure for the photonics crystals, OAM carried by the vortex beam will be transferred to the microparticle, such transfer of the momentum will not be desirable for the stable array.

6.4 Conclusion

In conclusion, an effective and simple method for generating an array of propagation invariant beams has been proposed and verified. Such technique of composite holograms besides being dynamic provides full control to vary all the parameters of the propagation invariant beams like radial wave vector and azimuthal index, and at the same time it conserves the wave-vector distribution necessarily for the invariant beams. Experimental results show that such method can effectively reconstruct an array of Bessel beams and self-imaged beams with a single composite hologram. It is further observed that the reconstructed Bessel beams array have little interference within the nondiffracting distance. Similarly, this method can be applied to generate other arrays of optical beams such as LG beams, interfering Bessel beams, and etc. Array of these optical beams can be used in the optical manipulations too; for example, we can arrange a composite hologram to form a fluid chamber or a structure of photonics crystal.

CHAPTER 7 LASER MICRO-BEAM ADDITION USING DOES

7.1 Introduction

One kind of DOEs, which splits one incident laser beam into many equal-intensity beams or combines many equal-intensity laser beams into one, are important parts of many image multiplication, data processing, and fiber optical communication systems [149]. Many laser sources are limited in practical output power by the limitations of their designs or physical mechanisms. Thus, it is necessary to combine multiple laser beams with low power into one with higher power for some applications; such a way is named as beam addition. Beam-addition can be divided into two main categories: one is coherent beam addition? another is incoherent beam addition. For the incoherent beam addition, we can use some geometrical optical elements such as lens, mirrors, etc. to focus the beams, but it is difficult to combine many bunches of incident beams into one beam with higher power and small divergence; for the coherent beam addition, the most frequently used method is using a resonator, in which light is resonated and enhanced, to combine beams, but the system is complicated and instable because the DOEs have to be incorporated into the resonator of a laser. Coherent beam addition is ideal for achieving small beam divergence and high power densities; DOEs offer a means of diffractively controlling and correcting optical wavefronts, and can be effectively used for coherent beam addition. Gratings are of DOEs because they re-form lights on the basis of diffraction; they can be effectively used for some specified beam shaping problems as they usually have structures which are exact or approximate analytic solutions to the defined problems. The ideal of using binary grating to achieve multiple equal-intensity beams was first presented by Dammann and Gortler in 1971 [149], and Leger and Veldkamp in Lincoln Laboratory (MIT, USA) employed Dammann gratings experimentally to combine laser beams coherently in 1986 [150-151]. In Leger and Veldkamp's experiments, an array of GaAlAs laser diode was used; the system is composed of laser diode array, collimating lens, grating, Fourier lens, aperture stop, output mirror, and auxiliary lens. The front facet of the diode array is antireflection coated, thus the lasing cavity extends between the rear laser facet and the output mirror. In the cavity, only the zero-order beam of the grating will be enhanced to lase. Besides for gratings, micro-lenses have been utilized to form a laser resonator thus to combine laser diode arrays or other coherent laser beams into a stronger beam [152-154].

In this Chapter a simple method of using Dammann grating to combine multiple beams is proposed; the Dammann grating, which can combine five laser beams into one, will be designed, fabricated, and tested.

7.2 Theory of beam addition

Since one Dammann grating can be designed to diffract one laser beam into N equal-intensity beams, the same grating can also be used to combine the N beams into one inversely. Assume one period of a Dammann grating is shown in Figure 7-1.

Figure 7-1. One period of Dammann grating.

In Figure 7-1, the values of $a_1, b_1, a_2, b_2, \dots, a_L, b_L$, and 1 on the x axis represent the coordinates of the transverse (side view) of the grating period, where the dark areas represent the un-etched parts and the bright areas represent the etched parts. The first point and the last point of the grating period are $a_1=0$ and 1, respectively. The refractive indices of the un-etched parts and the etched parts are n and 1, respectively.

The diffraction orders can be obtained by the following formulas [155], for an incident plane wave on the grating. The complex amplitude of the zero-order is:

$$T_p(0) = [2T_a(0) - 1] \sin \theta + i \cos \theta \quad (7-1)$$

where $(\pi/2+\theta)$ and $(\pi/2-\theta)$ are the phase steps of the grating respectively. And

$$T_a(0) = \sum_{i=1}^L (b_i - a_i) \quad (7-2)$$

Other diffraction orders are:

$$T_p(m) = 2 \sin \theta T_a(m) \quad (7-3)$$

where,

$$\begin{aligned} T_a(m) &= \frac{1}{2\pi m} [T_{aR}(m) + iT_{aI}(m)] \\ T_{aR}(m) &= \sum_{i=1}^L [\sin(2\pi m b_i) - \sin(2\pi m a_i)] \\ T_{aI}(m) &= \sum_{i=1}^L [\cos(2\pi m b_i) - \cos(2\pi m a_i)] \end{aligned} \quad (7-4)$$

Based on the above Equations (7-1) ~ (7-4), these are solvable to yield a set of coordinates of a_i and b_i of a grating to diffract N equal-intensity diffraction orders. Because of the complexity of the equations, optimization algorithms such as simulated-annealing algorithm, Genetic algorithm, and etc. can be used in the calculation. When θ is 90° and the phase step of the grating is n , the intensity of the designed diffraction orders will be the highest.

Performance of beam-addition is evaluated using other equations, one of which is obtained by expressing the transmittance of the grating as a superposition of plane waves corresponding to the diffraction orders of the grating:

$$t(x) = \sum_{n=-\infty}^{\infty} a_n \exp(j\phi_n) \exp(jn\alpha x) \quad (7-5)$$

where a_n and $\exp(j\phi_n)$ are the magnitude and the phase of the n th plane-wave component and α is proportional to the sine of the angle between diffraction orders. The grating is illuminated with N laser beams at angles corresponding to those of the diffraction orders of the grating, and the N laser beams have unit amplitude and phases which are the complex conjugates of the original diffraction orders. The interference pattern of these beams in the plane of the grating is given by,

$$E(x) = \sum_{m=-[N/2]}^{[N/2]} \exp(-j\phi_m) \exp(-jm\alpha x) \quad (7-6)$$

The amplitude of the light after passing through the grating is given by the product of Equations (7-5) and (7-6). The cross terms of the result represent all the plane waves leaving the grating. The on-axis plane waves correspond to the terms where $m=n$. The intensity in the zero diffraction order of the grating is given by the square of the coherent sum of these on-axis plane waves:

$$I = \left| \sum_{n=-[N/2]}^{[N/2]} a_n \right|^2 \quad (7-7)$$

On the basis of the above, a Dammann grating that gives an output of equal-intensity beam has been designed; the design procedure is laid out in the algorithm as follows:

Assume that intensities of the diffraction orders $0, \pm 1, \pm 2$ are equal.

Based on the assumption, three equations are obtained, **i.e.**

$$\begin{aligned} |T_p(0)| &= |T_p(\pm 1)|, \\ |T_p(0)| &= |T_p(\pm 2)|, \\ |T_p(\pm 1)| &= |T_p(\pm 2)|. \end{aligned} \quad (7-8)$$

With the above three equations, we can solve the equations numerically to yield three solutions:

- a. First, we select three unknown variables b_1, a_2, b_2 in the equations.
- b. Put the $a_1=0, b_1, a_2, b_2$ into Equations (7-1) ~ (7-4) and (7-8) and then use numeric methods to solve Equation (7-8).
- c. Finally, we obtain values of the b_1, a_2, b_2 , **i.e.**, structure of the grating.

Considering our current fabrication equipment's resolutions and ability, we only find a_1, b_1 's precision to sub micrometer scale with a period of 50 micrometers. Our solutions to Equation (7-8) are $a_1=0, b_1=0.264, a_2=0.58, b_2=0.62$. The calculated amplitudes of the five diffraction orders are:

Table 7-1. Diffraction orders of the designed grating.

Diffraction order	Complex amplitude
0	$-0.3920 + 0.000\cdot j$
1	$0.2525 - 0.2994\cdot j$
-1	$0.2525 + 0.2994\cdot j$
2	$-0.0034 - 0.3911\cdot j$
-2	$0.0034 + 0.3911\cdot j$

The phase states of the five diffraction orders are 180° , -50° , -90° , 50° , 90° , respectively. The intensities of them are: $I(0)=0.154$, $I(1)=0.153$, $I(-1)=0.153$, $I(2)=0.153$, $I(-2)=0.153$, respectively. Thus, the splitting efficiency can be calculated as:

$$\eta = \frac{I(0) + I(1) + I(-1) + I(2) + I(-2)}{1} = 76.65\%$$

In the above expression, the total intensity of the incident beam is regarded as I , $I(0)$, $I(1)$, $I(-1)$, $I(2)$, and $I(-2)$ are the total intensity of respective split diffraction orders. Only 76.65% energy of the incident beam has been allocated to the split five beams, the remained 23.35% energy is consumed by the other higher orders and reflections of the grating. The principle of beam addition can be schematically described as follows: when one laser beam is incident on the grating, the beam will split into N equal-intensity beams with different phase states. If another N beams identical to the N beams shown in Figure 7-2 is reverse-incident on the grating, they will be combined into one much stronger beam as shown in Figure 7-2.

Figure 7-2. N beams incident to the grating.

Aside from the consideration of phase states for the N output beams from the grating, the N beams also have different diffraction angles. The diffraction angles of different orders can be calculated by the well-known grating formula,

$$d \sin \theta = n \lambda \quad (7-9)$$

where d stands for the period of the grating, n is the diffraction order, λ is the illuminating wavelength, and θ is the diffraction angle of the corresponding diffraction order. Taking the above principle, we use the set-up shown as Figure 7-3 to combine five incident beams.

Figure 7-3. Schematic diagram of beam addition set-up.

In Figure 7-3, a He-Ne laser beam is first spatial filtered by a 25 μm pin hole before being expanded and collimated by a microscope system. The collimated light can be regarded as a plane wave. When the plane wave passes through five identical apertures in one line, it will be tailored into five parallel light sources. Then the five beams are focused into one point on the grating. If the incident beams satisfy the condition of the beam addition as described above, the zero-order of the output beam will contain most of the energy.

7.3 Fabrication of Dammann grating

For the grating structure designed by using the above equations ($a_1=0$, $b_1=0.264$, $a_2=0.58$, $b_2=0.62$) a period of 50 μm has to be chosen because the resolution limit of the available micro-fabrication equipments is about 1 μm . The exact grating structure is, $a_1=0$, $b_1=13$ μm , $a_2=29$ μm , $b_2=31$ μm . The chromium-plate mask of the grating, shown in Figure 7-4, has been directly written by an e-beam writing system.

Figure 7-4. Grating mask with period of $50\text{ }\mu\text{m}$; the whites and blacks represent un-etched parts and etched parts, respectively.

By use of Equation (7-9) the total number of diffraction orders is calculated to be about 150! It can be seen that a larger period will result in the desired five orders having lower energy.

The material of the grating is photoresist AZ5214, the refractive index is 1.49. The fabrication procedures of the phase grating are listed below:

- a. Clean the elements such as glass substrate and mask; prepare photoresist solution, developer; Switch on the mask aligner and hot plate to warm up.
- b. Spin coat AZ5214 on the glass substrate with a spin coater, the rotation speed is 2000 rpm, and the duration is 35 seconds.
- c. Pre-bake the AZ5214 film on the hot plate for 5 minutes at $90\text{ }^{\circ}\text{C}$.
- d. Expose the sample using the mask aligner with the UV light of 365 nm. The duration is 10seconds.
- e. Develop the sample for 14seconds. Then clean it with de-ionized water.
- f. Post-bake the sample for half hour at $90\text{ }^{\circ}\text{C}$.
- g. Measure the profiles of the grating.

The Q-200 1 CT mask aligner (Quintel Corporation) with wavelength 365 nm and irradiance 15 mw/cm^2 was used to transfer the pattern to the photoresist film. A phase grating obtained by use of the above procedures is shown in Figure 7-5,

Figure 7-5. Photoresist AZ5214 grating. The period is 50 μm .

The grating's profiles was measured using a surface profiler. The depths of the grating are shown in Figure 7-6.

Figure 7-6. Profiles of the AZ5214 grating.

It can be seen that the depths of the phase grating steps are approximately 1.8 μm , which the phase difference of the step is about π . The narrower portions of the grating are about 2 μm , but they are observed to be always shorter than the wider grating portions in Figure 7-6 because the narrower parts are too fragile and result from its top being cut off by the surface profiler's stylus.

The phase grating was tested with a single 633 nm incident beam, the main five diffraction orders are shown in Figure 7-7. It can be seen from Figure 7-7 that the five diffraction orders have the same intensity levels.

Figure 7-7. Diffraction orders of the phase grating.

To obtain five equal-intensity beams in the experiment, five identical apertures are used to restrict the size of the incident beams. The diameter of the collimated plane wave is about 15 mm and the focal length of the convex lens is 200 mm, thus the distance between the apertures is calculated as 2.52 mm. Therefore the diameter of the apertures should not exceed 1 mm and the diameter of the apertures is set to be 500 μm . In practice it is difficult to make the five apertures with defined diameters and distances by hand, consequently they have to be fabricated with a laser direct writing system. The procedures of fabricating the apertures can be depicted in Figure 7-8.

Figure 7-8. Flow chart of the fabrication of the apertures.

The beam profiles behind the apertures for a single incident plane wave are measured using a surface profiler; the pattern is shown in Figure 7-9.

Figure 7-9. Beam profiles behind the apertures.

It is observed that the profiles are nearly identical though the capture angle of the camera slightly affected their appearances.

7.4 Beam addition Experiment

The schematic diagram of the setup has already been depicted in Figure 7-3. The actual diagram for the experimental setup is now shown below in Figure 7-10.

Figure 7-10. Experimental setup of the beam addition.

The set-up consists of a He-Ne laser, spatial filter with a pinhole (diameter 25 μm), collimation lens ($f=100\text{ mm}$), five apertures (interval 2.52 mm, diameter 0.5 mm), focus lens ($f=200\text{ mm}$), grating, diaphragm (to filter unnecessary light), and power meter. The power meter has a resolution up to nano-watt scale. As has been verified in the author's previous work on double beam addition, interference of the incident lights behind a grating is the process of energy transfer. Stronger interference will cause the zero-order diffraction beam to be more intense. In the experiment, the most important tasks are to adjust the distance between the grating and the focus lens and to adjust the angle between the grating plane and the laser rays. Thus a five-axis adjustable device with resolution in micrometer scale is used to control the position of the grating accurately. The diffraction orders of the output from the grating are shown in Figure 7-11.

Figure 7-11. Diffraction orders of the grating with five beam incident.

In Figure 7-11 the white color of the beam stands for the saturation of the power meter, hence it can be seen that the central circular spot has the highest intensity. Other diffraction orders are much weaker than the diffraction zero-order. The strong intensity strips perpendicular to the diffraction spots are due to the interference of the residue lights outside the five apertures. A set of measured data from the five-beam addition is given in Table 7-2.

Table 7-2. The experimental results of the five-beam addition.

Measurement times	Power of the input beams (μw)	Power of the zero-order beam of the grating (μw)	Coupling efficiency
1	0.443	1.189	52.13%
2	0.460		
3	0.510		
4	0.434		
5	0.434		

Assume that the incident beam for both a double-beam and five-beam additions have the same energy, a comparison between the both cases is made in Table 7-3:

Table 7-3. Comparison between a double-beam addition and a five-beam addition.

Beam addition	Input power	Coupling efficiency	Output power
2 beams	1w, 1w	70%	1.4w
5 beams	1w, 1w, 1w, 1w, 1w	50%	2.4w

It is clear in Table 7-3 that the five-beam addition will provide a much higher intensity for the resultant output beam even though the coupling efficiency is lower than that of the double-beam addition.

7.5 Discussions and conclusion

As was mentioned previously, the output beams from the Dammann grating have defined fixed phase states. Therefore, when multiple beams incident to the grating inversely, they must have the exact same fixed phase relationships to each other. Using Equations (7-5) and (7-6), it is possible to calculate the coupling efficiency of the beam addition for the case where

the phase conditions are not satisfied. In the calculations, the coupling efficiency was found to have decreased to 30% due to the mismatch in phases. In actual experiments, the measured coupling efficiency has reached over 52% because the phase relationships were partially satisfied by micro-adjusting the grating. However, it can be foreseen that the phase relationships can hardly be fully satisfied by only adjusting the grating. We propose a solution to resolve the phase problems as shown in Figure 7-12.

Figure 7-12. Proposed set-up for the beam addition, where a phase plate is inserted.

The configuration in Figure 7-12 is inserted with a phase plate, where the phase plate has the predefined five phase states mentioned above. The schematic structure of the phase plate is shown in Figure 7-13.

Figure 7-13. Schematic structure for the phase plate to be inserted in the configuration.

Besides the phase mismatch condition, a grating period that is too large (the current grating period is $50\text{ }\mu\text{m}$) will also result in a low coupling efficiency. A Dammann grating with

smaller period will decrease the strength of the higher diffraction orders and thus the five main diffraction orders will be enhanced. The upper limit of the theoretical coupling efficiency of the designed grating is below 77%, but it is predicted that the theoretical coupling efficiency of an optimized grating can reach 87% [153]. Thus, we can further increase the theoretical limit of the diffraction efficiency by optimizing the grating structures using algorithms such as simulated annealing algorithm or genetic algorithm.

In summary, a Dammann grating's design and optimization procedures have been investigated. It is capable of splitting a single beam into five main beams with a splitting efficiency of about 76%. By applying micro-fabrication techniques to make the grating and five micron-scale apertures, the author has done the experiments using the grating to combine five separate beams; the measured coupling efficiency is about 52%, with which we can obtain one much stronger laser beam with five incident beams. The coupling efficiency can be further increased by inserting a phase plate with the predefined phases.

CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

In this thesis, the author has investigated DOE's design, fabrication, and a wide range of applications, and the conclusions are listed as follows:

First, three ways of design DOEs have been introduced in detail. The author proposed to use holographic interference to generate a DOE; the method is simple and the reconstruction is faithful to the desired. The developed hybrid algorithm is faster and more efficient than the single G-S algorithm and the simulated annealing algorithm; an example of re-forming a Gaussian beam to a flat-top beam has been given in detail, and the simulations showed that the hybrid algorithm has obtained better beam-shaping results than any of the two algorithms. Moreover, a class of DOEs found can be generated by applying analytic solutions to solve the defined problems; this kind of DOEs is high efficient and can reconstruct expected images accurately. The study on the designs has facilitated the subsequent research.

Second, two optimization algorithms for specified DOE problems have been proposed. The author compared four different quantization methods in an iterative Fourier transform algorithm and found a method that can effectively quantize a DOE with the highest diffraction efficiency, and this effective quantization method is useful in practical design and fabrication of DOEs. Subwavelength-sized DOEs (polarization-selective DOEs) have potential applications as optical switch and optical storage medium but these DOEs are difficult to fabricate due to their small feature size. The author developed an optimization algorithm in reducing the phase combinations of a polarization-selective DOE (PSDOE) so as to simplify structures of the PSDOE. The simulations showed that the optimization algorithm can remove one of the four structures from the PSDOE and can keep the reconstruction efficiency of the resultant PSDOE as high as possible.

Third, on the basis of the designs, some DOEs for practical applications have been fabricated using a series of micro-fabrication tools. When the feature size and the overall size of a DOE are large, it is recommended to fabricate the DOE using a direct printing and exposure technique. The author presented such a DOE, which has been fabricated by a laser printer and

a UV mask aligner, to re-form a Gaussian beam into a Bessel beam. The reconstructed Bessel beam was a very close match with the desired one. In addition, a multilevel DOE, which has irregular phase distributions and was generated by a computer, has been fabricated with an e-beam direct writing system, a high-energy beam sensitive glass, and a UV mask aligner. The reconstruction experiment which reconstructed a Bessel beam demonstrated that the fabricated DOE possessed multilevel phase structures. Furthermore, a novel technique was proposed to replicate multilevel DOEs by use of polydimethylsiloxan (PDMS). This new technique can be applied to quickly replicate a large amount of DOEs. Besides that, a spatial light modulator used for both static and dynamic micro-beam shapings has also been investigated.

By implementing the design and fabrication of DOEs, the author generated a novel laser beam which was named as fractional-order Bessel beam. The fractional-order Bessel beam maintains the beam size and possesses helical phase in propagation; it has a natural opening slit in its transverse intensity profiles during the propagation. The beam has been generated by different methods and its diffraction properties have been studied extensively. This novel beam could be used for dynamic manipulation of micro-particles by use of a spatial light modulator. Furthermore, it is the first time that the asymmetric laser beam has been found self-reconstructive. The simulations and experiments showed that the beam can recover itself from a distortion. Moreover, interferences of fractional-order Bessel beams have also been studied in simulations and experimentally, and the simulation results demonstrated that the interferences roughly follows the general rules of those of the interferences of corresponding integer-order Bessel beams although some instabilities and opening slits have been observed. An efficient method of generating the interference patterns of the fractional-order Bessel beams has been presented.

Next, the author presented an efficient and simple technique, i.e. using a composite hologram to split one beam into an array of optical beams. This composite hologram approach can reconstruct arrays of Bessel beams, optical bottle beams, self-imaging beams, vortex beams, and so on. To the author's knowledge it is the first time array of propagation-invariant beams has been generated with one single optical element. Furthermore, shape of the array of propagation-invariant beams can be customized by arranging the positions of the smaller holograms in the composite hologram.

Finally, a Dammann grating has been designed, fabricated, and tested for five-beam addition. The author proposed a new configuration to combine five incident laser beams coherently into

one stronger beam. The experiments demonstrated that the configuration can effectively combine incident beams. The implementation of the set-up is also simple.

8.2 Recommendations for further research

On the basis of the results obtained in this work, recommendations for future research work comprise the following cases: (1) improvement on the fabrication of multilevel DOE; (2) fractional-order Bessel beams for optical trapping; (3) improvement on coupling efficiency of the five-beam addition.

(1). Improvement on the fabrication of multilevel DOE: the experiments have shown that the fabricated DOE possesses multilevel phase structures, but the diffraction efficiency is low because there are many higher diffraction orders and speckle noises. The ways of improving the performance of the DOE can be realized by, (a) characterize the gray-scales of high-energy beam sensitive glass versus the depths of the photoresist accurately; the main reason that resulted in the low diffraction efficiency is the incorrect depths of the DOE, so if we can fabricate a DOE with correct phase structures, the reconstruction efficiency would be increased remarkably; (b) increase the filling factor of each pixel of the DOE by using higher-resolution direct-writing equipment, because the incomplete filling of pixel will result in many diffraction orders that will share a definite amount of energy too.

(2). Fractional-order Bessel beams for optical trapping: the fractional-order Bessel beam has been extensively studied and demonstrated, and these novel properties that the beam has would be an advantage for optical trapping applications. It is predictable that the applications would provide a more flexible, more efficient, and more specified tool for micro-manipulations.

(3). Improvement on coupling efficiency of the five-beam addition: the Damman grating has been successfully employed for the beam addition; however, the difference between the experimental and the theoretical coupling efficiencies is still large. The low coupling efficiency is mainly due to the phase mismatching. For future work, a phase plate using micro-fabrication techniques could be inserted in the set-up. To increase coupling efficiency of the Damann grating, we can decrease its period in order to eliminate the higher diffraction orders; furthermore, we can explore the use of various optimization methods to restructure the grating in order to get a higher theoretical coupling efficiency.

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Appendix A

Hybrid algorithm: G-S algorithm and SA algorithm

```
%%%%%%%%%%
%%%%% A modified G-S and SA algorithm,
%%%%%
%%%%%%%%%% Matlab version

close all;
clear all;
clc;

% cd 'C:\Documents and Settings\eshtao\Desktop';
RR=0;
smm=1000;
dt=[];
Fresnel=1;
for m=1:1
NUM=128;
RATE=2;
lambda=0.6328e-6;
size_x=3.84e-3;
size_y=3.84e-3;
period=size_x/NUM;
LH=1500e-3;
step1=-NUM/2:NUM/2-1;
step2=-NUM/2:NUM/2-1;
[x,y]=meshgrid(step1,step2);
hpz=exp(i*2*pi*LH*sqrt(1-(lambda*x/size_x).^2-(lambda*y/size_y).^2)/lambda);
ihpz=exp(-i*2*pi*LH*sqrt(1-(lambda*x/size_x).^2-(lambda*y/size_y).^2)/lambda);

GS=exp(-(x.^2+y.^2)/64.^2);
FL=exp(-(x.^16+y.^16)/16.^16);
% FL=imread('bird.bmp'); %import target pattern from external source.
```

```

% FL=double(FL)+1e-3; % increase freedom of phase
FL=(1e-4+double(FL))./max(max(double(FL+1e-4)));
phase1=rand(NUM,NUM).*2.*pi;
Mcost=[];
dMSE=1000;
FL2=(abs(FL).^2-min(min(abs(FL).^2))./(max(max(abs(FL).^2))-min(min(abs(FL).^2)));

for kk=1:50

    if Fresnel==1
        G2=ifft2(fftshift((fftshift(fft2((GS.*exp(i*phase1))))).*hpz));
    else
        G2=(fft2(GS.*exp(i*phase1)));
    end
    phase2=angle(G2);

    % phase2=phase2.*(phase2>=0)+(phase2+2*pi).*(phase2<0);
    % phase2=rem(phase2,2*pi);
    % phase2=floor(phase2.*RATE/2/pi)*2*pi/RATE;

    nG2=(abs(G2).^2-min(min(abs(G2).^2))./(max(max(abs(G2).^2))-min(min(abs(G2).^2)));

    MSE=sum(sum(abs(nG2-FL2).^2))/sum(sum(FL2).^2); % the MSE expression is
    dependent on the exact % image pattern.
    if MSE<=dMSE
        ph1=phase1;
        dMSE=MSE;
    end

    if Fresnel==1
        G1=ifft2(fftshift((fftshift(fft2((FL.*exp(i*phase2))))).*ihpz));

```

```

else
    G1=(ifft2(FL.*exp(i*phase2)));
end
    phase1=angle(G1);
    phase1=phase1.*(phase1>=0)+(phase1+2*pi).*(phase1<0);
    phase1=rem(phase1,2*pi);
    phase1=floor(phase1.*RATE/2/pi)*2*pi/RATE;
    Mcost=[Mcost MSE];

end

%   ph1=ph1.*(ph1>=0)+(ph1+2*pi).*(ph1<0);
%   ph1=rem(ph1,2*pi);
%   ph1=floor(ph1.*RATE/2/pi)*2*pi/RATE;

dt=[dt dMSE];
if dMSE<=smm
    ph=ph1;    % keeps the phase of the smallest MSE
    smm=dMSE;
end

end

saflag1=1; % Simulated annealing iterations
if saflag1==1

    tep0=-2*1e-2*smm/log(0.5);
    tloop=100;
    TZ=2;

```

```

newphase0=ph;
phase0=sign(newphase0./pi-0.5);
ideal0=FL2;
deltacost0=200;
deltacost=[];
phase1=phase0;
pp=[];
for tt=1:tloop
    tepend=tep0/(1+tt);
    for touzi=1:TZ
        m=floor(rand(1).*NUM+1);
        n=floor(rand(1).*NUM+1);
        phase1(m,n)=-phase0(m,n);

    if Fresnel==1
        recons0=ifft2(fftshift((fftshift(fft2((GS.*exp(i*(phase1+1)/2*pi))))).*hpz));
    else
        recons0=(fft2(GS.*exp(i*(phase1+1)/2*pi)));
    end

    reconsscale=(abs(recons0)).^2;
    reconsscale=(reconsscale-min(min(reconsscale)))/(max(max(reconsscale))-min(min(reconsscale)));
    delta0=sum(sum(abs(ideal0-reconsscale).^2))/sum(sum(abs(ideal0).^2));

    deltaend=delta0-deltacost0;
    prob=exp(-deltaend/tepend);
    pp=[pp prob];
    if deltaend <=0 % | (deltaend <0 & rand(1)<prob) % Boltzman probability control
        deltacost0=delta0;
        costvalue=delta0;
        phase0=phase1;
    else
        phase1=phase0;
    end
end

```

```
end
deltacost=[deltacost deltacost0];
end

phiuz=(phase0+1)/2.*pi;

figure(99)
plot(deltacost);
title('cost value of SA');

end% saglag1


if Fresnel==1
out=ifft2(fftshift((fftshift(fft2((GS.*exp(i*phiuz))))).*hpz));
else
out=(fft2(GS.*exp(i*phiuz)));
end
out1=abs(out).^2;
out1=(out1-min(min(out1)))/(max(max(out1))-min(min(out1)));


figure(1)
image(nG2*255);
title('GS Output');
colormap gray;
axis square;


figure(2)
plot(Mcost);
title('MSE');


figure(3)
image(out1*255);
```



```
title('SA Output');  
colormap gray;  
axis square;
```

```
figure(4)  
subplot(2,1,1)  
mesh(nG2);  
title('GS Output');  
subplot(2,1,2)  
mesh(out1);  
title('SA Output');
```

```
save tt; % save the workspace to a file "tt"
```

```
% imwrite(uint8(phiuz/max(max(ph))*255),'FSbird.bmp'); % save the final phase
```

Appendix B

Data format conversion: from matrix data or gray-scale image to file with CIF or GDSII format

```
clear all;
clc;
% cd 'C:\Documents and Settings\eshtao\Desktop'

%holo=load('cgh00');
A1=imread('FSbird.bmp');
A1=double(A1)/max(max(double(A1)));
A1=double(double(A1)==1); % make A1 normalized to 1
ZOOM=10;
Xlength=ZOOM;%10 is the pixel size
Ylength=ZOOM;%10 is the pixel size
Scale=1000;%1000 is the default scaling factor
% Xlength and Ylength are the dimensions of the pixel along x and y
% directions, respectively. The unit is in micron.
fnum=101;
while(fnum<=356)
    filename= sprintf('FSB2%d.cif',fnum); % filename restricted to 4 letters.

    files((fnum-100),1:11)=filename;
    fnum = fnum +1;
end
% filename1='c:\my documents\tsh\';
%files=strcat(filename1,files);
%files=char(files);
fid=zeros(256,1);%%(256,1) is default

% Please replace Filename with a more specified strings,
% for example:
%      c:\mask\mask.cif
```

```
count=1;
fid(count,1)=fopen(files(count,1:11),'wt');
fprintf(fid(count,1),'DS 1 1 10;\n');
fprintf(fid(count,1),'9 Cell0;\n\n');
fprintf(fid(count,1),'L CPG;\n\n'); % This sentence used for LEDIT for conversion from CIF
to GDSII.
```

```
[NumofRow,NumofCol]=size(A1);
for k=1:NumofRow
    for j=1:NumofCol
        remainder = rem(k,2000);% 32 is a parameter for files length

        if remainder==0
            if j==1
                count=count+1;
                fid(count,1)=fopen(files(count,1:11),'wt');
                fprintf(fid(count,1),'DS 1 1 10;\n');

                % Ignore this, just some kind of description for CIF file
                fprintf(fid(count,1),'9 Cell0;\n\n');
            end
        end

        if A1(k,j)==1 %j,k
            xj=floor((j-1)*(Xlength*Scale));
            yk=floor((k-1)*(Ylength*Scale));

            x1j= floor(xj+(Scale*ZOOM));
            y1k= floor(yk+(Scale*ZOOM));

            fprintf(fid(count,1),'P');
            fprintf(fid(count,1),' %d,%d',xj,yk);
            fprintf(fid(count,1),' %d,%d',x1j,yk);
            fprintf(fid(count,1),' %d,%d',x1j,y1k);
```

```
fprintf(fid(count,1),' %d,%d;\n',xj,y1k);
end
remainder = rem(k,32);% the parameter such as 32 is an adjustable parameter to
determine a file's %size
    if remainder==0
        if j==1
            fprintf(fid((count-1),1),'DF;\n');
                fprintf(fid((count-1),1),'E');
                fclose(fid((count-1),1));
        end
    end
end
end
fprintf(fid(count,1),'DF;\n');
fprintf(fid(count,1),'E');
fclose(fid(count,1));

% Modified
```

Appendix C

Computation of diffraction efficiency of a hologram

Holograms can be classified in many ways depending on their method of recording, method of reconstruction, method of generation etc. For example, Holograms can be grouped as thin or thick, super-wavelength-sized or subwavelength-sized, amplitude modulated or phase modulated, and so on. A hologram is considered thin or thick depending on the distance between interference fringes recorded on the film, the wavelength of light being used, and the density of particles in the emulsion of the film plate. The Q parameter is used to distinguish between the thin or thick holograms, as defined by [Appl. Opt. 35,6227-6230 (1996)]

$$Q = \frac{2\pi\lambda d}{n\Lambda^2} \quad (1)$$

where λ is the wavelength of the illuminating light, d is the thickness of the emulsion layer, n is the refractive index of the emulsion, and Λ is the spacing between the recorded fringes. A hologram is considered thick (such as volume hologram) if $Q \geq 10$ and thin if $Q \leq 1$. Holograms with Q between 1 and 10 can be thin sometimes or thick at other times. For computer generated holograms, their spacing periods are quite long, the Q value is much smaller than 1, thus they are thin holograms. However, for the subwavelength sized holograms, their Q value can be larger than 10, and their diffraction efficiency is computed with vector diffraction theory such as rigorous coupled-wave analysis. We will calculate diffraction efficiencies of different holograms which are considered as thin. Two types of holograms, i.e. amplitude and phase, will be computed as follows [Ref. 37].

The periodic transmittance function $T(x)$ can be expressed by the Fourier series as

$$T(x) = \sum_{-N}^N A_n \exp\left(\frac{i \cdot 2\pi n x}{\Lambda}\right) \quad (2)$$

$$A_n = \frac{1}{\Lambda} \int_0^{\Lambda} T(x) \exp\left(\frac{-i \cdot 2\pi n x}{\Lambda}\right) dx \quad (3)$$

where A_n represents the amplitude of each diffraction order. A normal incident wave with unit amplitude is assumed in the above formulae, and the first order is the desired order for the output of a hologram, thus $|A_{+1}|^2$ is the diffraction efficiency of the hologram.

A. Quantized phase-only hologram

The transmittance function of a quantized phase-only hologram is expressed by,

$$T(x) = \exp(im\alpha), m = N \text{ mod}(x, \Lambda) / \Lambda \quad (4)$$

where N is the quantization level, $\text{mod}()$ is a function that takes the integer part out of the division of x over Λ , and α is the phase step of the quantization, normally $\alpha = 2\pi / N$. We substitute equation (4) into (3) and get,

$$\begin{aligned} A_n &= \frac{1}{\Lambda} \sum_{m=0}^{N-1} \int_m^{(m+1)\Lambda/N} \exp(im\alpha) \cdot \exp\left(\frac{-i \cdot 2n\pi x}{\Lambda}\right) \\ &= \left[\exp\left(\frac{-i \cdot 2n\pi}{N}\right) - 1\right] \cdot \frac{i}{2n\pi} \cdot \frac{1 - \exp[iN(\alpha - 2n\pi / N)]}{1 - \exp[i(\alpha - 2n\pi / N)]} \end{aligned} \quad (5)$$

Thus, the efficiency is given by,

$$|A_{+1}|^2 = \frac{\sin^2(\pi / N)}{(\pi / N)^2} \cdot \frac{\sin^2[N(\alpha - 2\pi / N) / 2]}{N^2 \cdot \sin^2[(\alpha - 2\pi / N) / 2]} \quad (6)$$

Using equation (6), we can obtain diffraction efficiencies for phase-only holograms with quantization level N . For example, a binary phase hologram's diffraction efficiency ($N=2$) is 40.5%, a four-level phase hologram is 81.1%, an eight-level's is 95.0%, a 16-level's is 98.7%, and a 32-level's is 99.7%! Thus, for a hologram with continuous phase, the N is infinite, so the diffraction efficiency can be 100%.

B. Binary amplitude hologram

Because a binary amplitude hologram is used more often than any other kinds of amplitude holograms in this thesis, we compute the efficiency of a binary amplitude hologram only. The transmittance of a binary amplitude grating can be written as,

(7)

where t is the transmittance of the dark part of the grating, normally $t=0$.

Similarly, we substitute equation (7) into (3) and get,

$$T(x) = \begin{cases} 1, & 0 < x < \Lambda/2 \\ t, & \Lambda > x > \Lambda/2 \end{cases} \quad (8)$$

Thus, the diffraction efficiency of a binary amplitude hologram is computed as,

$$|A_{+1}|^2 = \frac{1}{\pi^2} \quad (9)$$

so the calculated maximum efficiency is 10.1%.

Note that, although the maximum diffraction efficiency for different holograms can be estimated based on the above equations, reconstruction efficiency for different holograms, especially for computer generated holograms used for beam shaping, varies as it is also dependent on the beam shaping algorithm.