Subband adaptive algorithms for high-order transversal filters

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Subband Adaptive Algorithms for High-Order Transversal Filters

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School of Electrical & Electronic Engineering

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Kong A. Lee
Singapore, Nov 2005.

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Summary

Adaptive filters have been employed in wide range of applications where signal characteristics are not available, or are time-varying, such as acoustic echo cancellation and adaptive noise cancellation. Despite their widespread use, performance of the adaptive filters is degraded in some applications by two major factors, namely, (i) coloring of the input signal, and (ii) long impulse response of the unknown system. The classical least-mean-square (LMS) algorithm is computationally efficient but its convergence speed deteriorates as the eigenvalue spread of the input autocorrelation matrix increases due to the coloring of the input signal. The convergence problem worsens when the order of the adaptive filter is increased in order to model the unknown system with sufficient accuracy. In contrast, recursive least-squares (RLS) algorithm is adequate in dealing with colored excitation with large spectral dynamic range. Nevertheless, high computational complexity renders the algorithm not applicable in real-time for the case of high-order adaptive filter.

Those problems have motivated various approaches of employing subband and multirate techniques in designing computationally efficient adaptive algorithms with improved convergence performance against high eigenvalue disparity. Conventional subband adaptive filters (SAFs) decompose the input signal and desired response into multiple contiguous spectral bands, decimate the subband signals, process each subband with separate adaptive subfilters, and finally interpolate and recombine the resulting subband signals to obtain a filtered output. Intuitively, faster convergence is possible because the spectral dynamic range is greatly reduced in each subband. Furthermore, the computational burden can be reduced by decimating both the order and adaptation rate of the subfilters. Yet, detailed analysis shows that the convergence

rate of the conventional SAF is limited by aliasing and band-edge effects, in spite of a number of modifications that have been proposed, such as, introducing adaptive cross-filters or spectral gaps between adjacent subfilters, as well as oversampled scheme.

This thesis describes, analyzes, and generalizes a new class of SAFs, called the normalized SAF (NSAF), whereby the adaptive filter is no longer separated into subfilters. Instead, subband signals, which are normalized by their respective subband input variance, are used to adapt the fullband tap weights of a modeling filter. The modeling filter is placed before the decimators, which is different from that of the conventional structure where a set of subfilters are placed after the decimators. In such a configuration, the modeling filter operates on a set of subband input signals at the original sampling rate. The weight adjustment applied on the modeling filter, at each iteration, is a linear combination of normalized subband regressors. Implicitly, the weight-control mechanism is driven by an equalized input spectrum, which is a composite of the normalized version of the contiguous spectral bands of the original spectrum. The equalized spectrum accelerates the convergence due to its reduced spectral dynamic range, while avoiding any possible aliasing and band-edge effects. Moreover, computational reduction can be attained by decimating the adaptation rate of the modeling filter, with the convergence properties remain intact.

The formulation of the new subband weight-control mechanism described above greatly relies on a special case of partial decorrelation, where the subband signals are orthogonal at zero lag. A mathematical proof is provided to confirm that this feature of filter banks can be approximated by cosine-modulating a high stopband attenuation prototype filter. The relation of the subband orthogonality to the convergence performance of the NSAF algorithm is analyzed. A detailed mathematical analysis of

the convergence behavior of the NSAF algorithm is also presented. In particular, stability bounds for the step-size and an expression for the excess MSE are obtained. Finally, the implementation issues of the NSAF algorithm are studied, where the delayless implementations for the NSAF algorithm are proposed.

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List of Abbreviations and Acronyms

AEC	Acoustic Echo Cancellation
AP	Affine Projection
c.f.	confer: compare with
DCT	Discrete Cosine Transform
DFT	Discrete Fourier Transform
DTFT	Discrete Time Fourier Transform
e.g.	exempli gratia: for example
ERLE	Echo Return Loss Enhancement
FBAF	Filter Bank Adaptive Filter
FDAF	Frequency Domain Adaptive Filter
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FSF	Frequency Sampling Filter
IDFT	Inverse Discrete Fourier Transform
i.e.	id est: that is
IFFT	Inverse Fast Fourier Transform
IIR	Infinite Impulse Response
LMS	Least-Mean-Square
LTI	Linear Time Invariant
MSE	Mean-Square Error
NLMS	Normalized Least-Mean-Square
NSAF	Normalized Subband Adaptive Filter
PRA	Partial Rank Algorithm
PQMF	Pseudo-QMF
QMF	Quadrature Mirror Filter
SAF	Subband Adaptive Filter
RLS	Recursive Least-Squares
TDAF	Transform Domain Adaptive Filter
WSS	Wide Sense Stationary
WSAF	Weighted Subband Adaptive Filter

List of Symbols

$\mathbf{A}^{-1}$	Inverse of matrix $\mathbf{A}$
$\mathbf{A}^T$	Transposition of matrix $\mathbf{A}$
$\mathbf{A}^H$	Hermitian Transposition of matrix $\mathbf{A}$
$\kappa(\mathbf{A})$	Condition number of matrix $\mathbf{A}$
$\mathbf{I}_{N \times N}$	The $N \times N$ identity matrix
$\mathbf{0}$	Zero vector or matrix
$\mathbf{\Lambda} = \text{diag}(\cdot)$	Diagonal matrix
$\tilde{\mathbf{E}}(z)$	Paraconjugate of $\mathbf{E}(z)$
$\ \cdot\ $	Euclidean norm
$ \cdot $	Absolute value
$\sum$	Summation
$*$	Complex conjugation
$E\{\cdot\}$	Statistical expectation
$\otimes$	Convolution
$\delta(n)$	Unit sample sequence
$O(\cdot)$	Order of complexity
$\gg$	Much larger than
$\ll$	Much smaller than
$\approx$	Approximately equal to
$\forall$	For all
∞	Infinity
∇	Gradient
$\equiv$	Defined as
$\downarrow N$	N -fold decimation
$\uparrow N$	N -fold interpolation
j	$\sqrt{-1}$
$\delta(n)$	Unit sample sequence
i	Subband index
N	Number of subbands, number of time-domain constraints
M	Length of the adaptive tap-weight vector
μ	Step-size parameter
α	Regularization parameter
$\gamma_{uu}(l)$	Autocorrelation function of the random signal $u(n)$

$\Gamma_{uu}(e^{j\omega})$	Power spectrum of the random signal $u(n)$
$J = E\{e^2(n)\}$	MSE function
$J_{\mathcal{M}}$	Multiband MSE function
$J_{ex}(\infty)$	Steady-state excess MSE

Chapter 1

Introduction

A filter is a system that is designed to extract the desired information contained in its input signal, and a digital filter is the one that processes discrete-time signals. An adaptive algorithm can be easily attached to a digital filter, so that the filter is able to operate satisfactorily in an unknown environment. The adaptive algorithm provides a mechanism for the control of the adjustable parameters that determine the filter characteristics, according to the signal conditions. Digital filters with such a self-designing capability, commonly referred to as adaptive filters [18], [19], [20], [27], [37], [44], [74], [107], [122], [128], find applications in many diverse fields, such as communications, radar, sonar, navigation, seismology, biomedical engineering, and financial engineering.

This thesis is concerned with the derivation and analysis of computationally efficient and effective subband adaptive algorithms for high-order digital filters. High-order adaptive filters are desirable, for example, for canceling the acoustic echo [6],[32],[39]-[42],[60], [109], [116] in hands-free telephone systems, teleconferencing systems, and voice-control systems, among others. The high-order nature of the adaptive filters, together with the highly correlated excitation signal, renders the performances of most adaptive algorithms unsatisfactory. Adaptive algorithms that are effective in dealing with the ill-conditioning problem are available; however, such algorithms are usually computational demanding, thereby rules out their applications for high-order filters.

This chapter begins with a review on the fundamental concepts of adaptive filtering, with emphasis on setting the stage and giving motivation for the development of subband adaptive filtering techniques that will be presented in the forthcoming chapters. The motivation and the main contributions of the thesis are spelled out in Section 1.3 and 1.4, respectively. Finally, a chapter-by-chapter overview is presented in Section 1.5.

1.1 Adaptive Transversal Filters

An adaptive filter is formally defined as a self-designing system that relies on a recursive algorithm for adjusting its free parameters to operate satisfactorily in an unknown environment [43]. An adaptive filter consists of two basic components [44, 58]: (i) a digital filter to perform the desired signal processing, and (ii) an adaptive algorithm to adjust the parameters of the digital filter. Figure 1.1 shows a general structure of adaptive filter. The filtering and adaptation processes work cooperatively with each other. The filtering process computes the output $y(n)$ of the linear filter in response to the input signal $u(n)$, and generates an estimation error $e(n)$ by comparing the output $y(n)$ with the desired response $d(n)$. The estimation

Figure 1.1 Basic elements of an adaptive system.

error $e(n)$ is in turn used by the adaptive algorithm to adjust the free parameters of the digital filter.

The filter structure for the adaptive system of Figure 1.1 takes on many forms. Of special interest in this thesis is the *adaptive transversal filter* of Figure 1.2. The adjustable parameters $\{w_m(n)\}_{m=0}^{M-1}$, indicated by circles with arrows through them, are referred to as the filter tap weights with M indicating the filter length. These tap weights form the elements of an $M \times 1$ tap-weight vector

$$\mathbf{w}(n) \equiv [w_0(n), w_1(n), \dots, w_{M-1}(n)]^T, \quad (1.1)$$

where the superscript T denotes matrix transposition. Correspondingly, the input samples $\{u(n-m)\}_{m=0}^{M-1}$, referred to as the tap inputs, form the elements of an $M \times 1$ tap-input vector

$$\mathbf{u}(n) \equiv [u(n), u(n-1), \dots, u(n-M+1)]^T. \quad (1.2)$$

With these matrix notations, the output signal $y(n)$ of the adaptive transversal filter can be conveniently taken as the inner product of $\mathbf{w}(n)$ and $\mathbf{u}(n)$, as follows

$$y(n) = \mathbf{w}^T(n) \mathbf{u}(n). \quad (1.3)$$

Figure 1.2 An M -tap adaptive transversal filter.

1.1.1 Error performance surface

The error signal $e(n)$ in Figure 1.1 measures the difference between the desired response $d(n)$ and the filter output $y(n)$:

$$e(n) = d(n) - \mathbf{w}^T(n) \mathbf{u}(n). \quad (1.4)$$

The tap-weight vector $\mathbf{w}(n)$ is iteratively updated such that the estimation error $e(n)$ is minimized in some statistical senses. A commonly used performance criterion is the minimization of the mean-square error (MSE), which is defined as the expectation of the square error

$$J \equiv E\{e^2(n)\}. \quad (1.5)$$

For a given weight vector $\mathbf{w}$, with stationary input $u(n)$ and desired response $d(n)$, the MSE is calculated from (1.4) and (1.5) as

$$J = E\{d^2(n)\} - 2\mathbf{p}^T \mathbf{w} + \mathbf{w}^T \mathbf{R} \mathbf{w}, \quad (1.6)$$

where $\mathbf{R} \equiv E\{\mathbf{u}(n)\mathbf{u}^T(n)\}$ is the input autocorrelation matrix, and $\mathbf{p} \equiv E\{\mathbf{u}(n)d(n)\}$ is the correlation vector between the desired response and the tap-input vector. The iteration index n has been dropped from the vector $\mathbf{w}(n)$ for the context where the MSE is treated as a function of the tap weights.

It is clear from (1.6) that the MSE is precisely a quadratic function of the tap weights. That is, when (1.6) is expanded, the tap weights $w_0, w_1, \dots, w_{M-1}$ will appear in first and second degree only. A typical error performance surface is shown in Figure 1.3 for a 2-tap transversal filter. For $M > 2$ the error performance surface is a hyperboloid. The practical virtue of quadratic performance surface is that it is characterized by a single global optimum $\mathbf{w}_o$. The optimum solution $\mathbf{w}_o$ can be

obtained by taking the first derivative of (1.6) with respect to $\mathbf{w}$, and then equating the derivative to zero. The resulting equation is known as the Wiener-Hopf equation:

$$\mathbf{R}\mathbf{w}_o = \mathbf{p}. \quad (1.7)$$

Assuming that $\mathbf{R}$ has an inverse, the optimum tap-weight vector is given by

$$\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p}, \quad (1.8)$$

which corresponds to the minimum MSE of

$$J_{\min} = E\{d^2(n)\} - \mathbf{p}^T \mathbf{w}_o. \quad (1.9)$$

Figure 1.3 A typical error performance surface for a 2-tap adaptive transversal filter.

1.1.2 Adaptive algorithms

An adaptive algorithm is a set of equations used to adjust the tap-weight vector $\mathbf{w}(n)$ such that it is iteratively driven to the optimum solution $\mathbf{w}_o$ that lies at the bottom of the error performance surface. Among various adaptive filtering algorithms, the least-mean-square (LMS) is most widely used because of its simplicity and robustness. The LMS algorithm updates the adaptive tap-weight vector by using the gradient of the instantaneous square error in the following form

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{u}(n) e(n), \quad (1.10)$$

where μ is the step size controlling the stability and the convergence rate. The step size is chosen in the range $0 < \mu < 1/\lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{R}$. Yet, it has been shown in some studies of the LMS algorithm that the stability of the algorithm indeed requires a more stringent condition on the upper limit of μ [27], [44].

A normalized form of the LMS algorithm is often employed. The normalized LMS (NLMS) algorithm can easily be obtained from (1.10) by including an additional normalization term $\mathbf{u}^T(n)\mathbf{u}(n)$ in the following form

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{u}(n) [\mathbf{u}^T(n)\mathbf{u}(n)]^{-1} e(n), \quad (1.11)$$

where the step-size is now bounded in the range $0 < \mu < 2$. The simplest interpretation of (1.11) is that it is basically a way of making the convergence rate independent of signal power by normalizing the input regressor $\mathbf{u}(n)$ with $\mathbf{u}^T(n)\mathbf{u}(n) = M\hat{\gamma}(0)$, where $\hat{\gamma}(0) = \mathbf{u}^T(n)\mathbf{u}(n)/M$ is a running-average estimate of the signal power $\gamma(0) \equiv E\{u^2(n)\}$. From this perspective, no significant difference would be expected between the convergence performance of the LMS and NLMS algorithms for stationary signals. The advantage of the NLMS algorithm only becomes apparent for non-stationary signals like speech, where significantly faster convergence can be achieved for the same level of excess MSE.

The LMS and NLMS algorithms are computationally efficient. Assume that all the signals and tap weights are real. The transversal filter requires M multiplications to produce a sample of $y(n)$ at each iteration n , where M is the number of tap weights used in the adaptive transversal filter. The update equation (1.10) requires $M+1$

multiplications, resulting in a total computational burden of $2M + 1$ multiplications per iteration for the LMS filter. On the other hand, the NLMS algorithm requires an additional $M + 1$ multiplications for the normalization term, giving a total of $3M + 2$ multiplications in one iteration. Since M is generally much larger than the factor of three, the computational complexity of the LMS and NLMS algorithms is $O(M)$.

1.1.3 Spectral dynamic range and condition number

It is well established that the convergence behavior of the LMS filter is directly related to the eigenvalue spread of the correlation matrix $\mathbf{R}$. The eigenvalue spread of the correlation matrix is measured by the condition number, defined as $\kappa(\mathbf{R}) = \lambda_{\max} / \lambda_{\min}$, where $\lambda_{\max}$ and $\lambda_{\min}$ denote the maximum and minimum eigenvalues, respectively. Furthermore, the condition number depends on the spectral distribution of the input signal [27, pp. 97], [74, pp. 124] and can be shown to be bounded from above by the dynamic range of the input spectrum $\Gamma_{uu}(e^{j\omega})$, as follows

$$\kappa(\mathbf{R}) \leq \frac{\max_{\omega} \Gamma_{uu}(e^{j\omega})}{\min_{\omega} \Gamma_{uu}(e^{j\omega})}, \quad (1.12)$$

where $\max_{\omega} \Gamma_{uu}(e^{j\omega})$ and $\min_{\omega} \Gamma_{uu}(e^{j\omega})$ denote the maximum and minimum of the input spectrum, respectively. Ideal conditioning occurs when $\kappa(\mathbf{R}) = 1$ for white excitation signal. The convergence rate of the LMS filter deteriorates as the ratio increases. When the input signal is highly correlated, the correlation matrix $\mathbf{R}$ is ill-conditioned due to the large spectral dynamic range of the input signal; the LMS filter suffers from slow convergence. Such a deficiency of the LMS and NLMS algorithms has been analyzed and proven many times in the literature [18], [19], [21], [27], [44], [107], [122], [128].

1.2 Adaptive Identification of Systems with Long Impulse Response

Response

Figure 1.4 shows an adaptive identification system, where the adaptive filter is placed in parallel with an unknown system. The adaptive filter is used to provide a linear model that represents the best fit to the unknown system. The signal $u(n)$ serves as the input to the unknown system and the adaptive filter, while $\eta(n)$ represents the disturbance occurs within the unknown channel. The technique of adaptive identification has been widely applied in echo cancellation [6], [32], [39]-[42], [60], [109], [116] and active noise control systems [57]. Of particular interest in this thesis is the identification of unknown systems with long impulse response. A typical example that involves the modeling of long impulse responses is the acoustic echo cancellation (AEC). Acoustic echo canceller finds application in hands-free telephone systems, audio or video conference systems, hearing aids, voice-control systems, and many more.

Figure 1.4 Block diagram of adaptive system identification.

1.2.1 Acoustic echo cancellation

The problem of acoustic echo arises wherever a loudspeaker and a microphone are placed such that the microphone picks up the signal radiated by the loudspeaker and its reflections at the borders of the enclosure. Figure 1.5 shows a general acoustic echo cancellation setup for hands-free telephony systems. Speech originated from the far-end is amplified and reproduced by a loudspeaker. The output of the loudspeaker fills the entire room, via many paths of reflection, until they reach and picked-up by the microphone. This acoustically echoed speech is transmitted back to the far-end where it is perceived as echo. The far-end user is annoyed by listening to his (or her) own speech delayed by the round-trip time of the system.

Figure 1.5 Block diagram of an acoustic echo canceller.

The acoustic echo can be compensated electronically by using an adaptive echo canceller, as illustrated in Figure 1.5. The adaptive filter models the acoustic path between the loudspeaker and microphone. The estimated transfer function is used to filter the far-end signal $u(n)$ so as to create an echo replica $y(n)$. The estimated

echo $y(n)$ is then used to cancel the acoustic echo components in the microphone signal $d(n)$, results in the echo-free signal $e(n) = d(n) - y(n)$.

AEC is far more difficult than it might seem to be at first sight. Three major factors that make the AEC a formidable task, and have attracted a great deal of research interest, are summarized as follows:

- (i) The acoustic echo path possesses long impulse response. The characteristics of the acoustic path depend heavily on the physical condition of the enclosure. For a typical office, the duration of the impulse response (i.e., the reverberation time) is about 300 ms. At 8 kHz sampling frequency, an adaptive transversal filter of more than 1,000 taps is necessary in order to model the acoustic path to an acceptable accuracy. High-order adaptive transversal filter is computational demanding. In that respect, infinite impulse response (IIR) filter would be preferable for AEC. However, adaptive IIR filter may cause stability problem that is much difficult to handle. Furthermore, the impulse response of the acoustic path exhibits a highly detailed and irregular shape. Adaptive transversal filter is most suitable since it offers a large number of adjustable tap weights such that sufficient good match can be achieved.
- (ii) The transfer characteristic of the acoustic path is not constant over time and it may change rapidly due to the movement (e.g., the motion of users, doors opened or closed, etc.) and changes in temperature in the enclosure. Hence, an adaptive algorithm has to constantly monitor the excitation and desired signals, and update the tap weights of the modeling filter accordingly. Furthermore, it has to converge quickly to keep up with any rapid changes.

- (iii) The excitation signals involved are highly correlated and non-stationary speech signals. For a speech segment of short duration, its spectral density differs by more than 40 dB at different frequencies.

1.3 Motivation of Thesis

Highly time-variant unknown system with long impulse response coupled with highly correlated excitation signals impose constraints and difficulties on the applications of adaptive filtering algorithms, for example, in acoustic echo cancellation. The LMS and NLMS algorithms are computationally efficient but they suffer from slow convergence when the input signal is highly correlated. The convergence problem worsens when the length of the modeling filter is increased [47], [48] in order to model the unknown system with sufficient accuracy. More involved algorithms like affine projection (AP) and recursive least-squares (RLS) algorithms are adequate in dealing with colored signals. However, with a computational complexity of $O(N^2M)$ and $O(M^2)$, respectively, the AP (of order $N \gg 1$) and RLS algorithms are not economically feasible to be implemented in real-time for high-order transversal filter.

Hence, the goal of this thesis is to derive a computationally efficient adaptive algorithm for high-order transversal filter with improved convergence performance against high eigenvalue disparity. Such an algorithm is attainable by properly exploiting subband and multirate techniques. The motivation for employing subband and multirate techniques in designing adaptive algorithms for high-order transversal filters is twofold:

- (i) A fullband signal can be decomposed into multiple contiguous spectral bands while having them being orthogonal at zero lag [61], [64], [65]. Our approach to the development of computationally efficient subband adaptive filters (SAFs),

which are effective in dealing with colored excitation signals, is based on this central idea. Decomposing a fullband signal into multiple spectral bands, permits the manipulation of the subband signals in such a way that their properties can be exploited, thereby allowing a more effective adaptation. Computational complexity of subband adaptive algorithms can be reduced by exploiting both the orthogonality and multirate nature of the subband signals.

- (ii) Previous attempts to perform adaptive filtering in subbands encounter aliasing and band-edge effects [16],[27],[34]-[36], [55], [111]. These structural problems can be solved by having the modeling transversal filter to operate on the subband signals at the original sampling rate [13], [14], [61], [62], [77], [78], [99]. The fullband tap weights of the modeling filter are adapted, at a decimated rate, using the subband signals that have been normalized by their respective subband input variance.

1.4 Contributions of Thesis

The major contributions of this thesis can be summarized as follows.

- (i) A multiple-constraint optimization criterion based on the *principle of minimal disturbance* is formulated. The deterministic design criterion results in a new subband adaptive filter (SAF) referred to as the normalized SAF (NSAF). Compared to the NLMS filter, the NSAF is able to achieve faster convergence under colored excitation, with almost equivalent computational complexity.
- (ii) A novel correlation-domain formulation for multirate filter banks is proposed. The correlation-domain formulation is used to show that the outputs of a cosine-modulated filter bank are nearly orthogonal at zero lag for arbitrary type of input spectrum. The subband orthogonality of the filter bank plays an important role in the derivation and convergence analysis of the proposed NSAF algorithm.

- (iii) The NSAF algorithm is formulated in both stochastic and deterministic frameworks. From these framework, the inherent decorrelating and least-perturbation properties, as well as the underlying affinity between the NSAF, NLMS, and AP algorithms are analyzed.
- (iv) Convergence behavior of the NSAF algorithm in the mean and mean-square senses is analyzed. A bound for the step-size and an expression for the excess MSE are obtained.
- (v) Implementation issues of the NSAF algorithm are studied. Delayless open-loop and closed-loop implementations for the NSAF algorithm are proposed.

1.5 Thesis Overview

The thesis is organized into seven chapters.

Chapter 1 serves as the introduction to the topic of the thesis. The fundamental concepts of adaptive filtering are reviewed, with emphasis on setting the stage and giving motivation for employing subband and multirate techniques in adaptive filtering.

Chapter 2 reviews the fundamental concepts and design techniques for multirate filter banks. In particular, the theory and various design techniques for cosine-modulated filter banks are studied and summarized. Cosine modulation is an attractive choice for the design and implementation of uniform filter banks with a large number of subbands.

Chapter 3 investigates the second-order characteristics of multirate filter banks. The chapter begins with a novel correlation domain formulation for multirate filter banks. With the new formulation, the necessary condition is defined for a special case of subband orthogonality, where the outputs of the filter bank are orthogonal at zero lag. Most importantly, we show that subband orthogonality (for arbitrary type of input

spectrum) can be obtained by cosine-modulating a lowpass filter with high stopband attenuation. This feature pertaining to the cosine-modulated filter bank plays an important role in the derivation and the analysis of subband adaptive algorithms presented in Chapter 4 and Chapter 5.

Chapter 4 constitutes the most important part of the thesis, where the NSAF algorithm is presented. A unique characteristic of the NSAF is that it uses a set of normalized subband signals to adapt the fullband tap weights of a single modeling filter. The modeling filter is placed before the decimators, which is different from that of the conventional SAF where a set of subfilters are placed after the decimators. In the second part of the chapter, the NSAF algorithm is analyzed from stochastic and deterministic viewpoints in order to develop new insights into its underlying behavior in dealing with colored excitation.

Chapter 5 deals with the stability and performance analysis of the NSAF algorithm. In particular, the performance of the NSAF algorithm is characterized in terms of the stability of the tap-weight adaptation, the speed of convergence, and the estimation accuracy in the steady state after the algorithm has converged. Such a mathematical analysis provides a set of working rules that can be used for its design in practical applications. In this chapter, the architectural beauty of the NSAF becomes more evident for which it enables a tractable convergence analysis, which is generally a formidable task for the case of conventional SAF.

Chapter 6 presents a comparative study of the conventional SAF and the normalized SAF. The aliasing and band-edge effects, which limit the convergence performance of conventional SAFs, are studied in detail. It is shown that the structural problems are annihilated in the NSAF. The concept of delayless implementation for the conventional SAFs and NSAF is also presented. It is shown that conventional SAF

implemented in a delayless closed configuration is able to overcome the aliasing and band-edge effects. Finally, the performances of various delayless SAFs are compared through simulations.

Chapter 7 concludes this thesis with a summary of the major issues. Directions for future research are pointed out as well.

Appendix A gives a listing of adaptive algorithms that can be related to the NSAF algorithm. Appendix B contains the mathematical proof for the phase properties of cosine-modulated filters. Appendix C is concerned with the polyphase implementation of the NSAF algorithm. Appendix D explains and relates the terms paraunitary, lossless, and power complementary.

Chapter 2

Subband Decomposition and Multirate Systems

In some applications, such as subband adaptive filtering [14], [16], [26], [36], [55], [61], [80], [84], [99], [111] and subband coding [28], [73], [114], [125], [127], it is advantageous to decompose the input signal into a set of subband signals prior to application specific processing. Decomposing a signal into multiple spectral bands facilitates the manipulation of the information contained in the accessible subband signals. Each subband signal occupies only a portion of the original frequency band, and consequently can be critically decimated in order to preserve the total effective sampling rate and the total number of samples to be processed. In other words, critical decimation eliminates the redundancy that exists among the subband signals (i.e., their bandwidth is smaller than the original signal), and thus improves the efficiency of the subband processing system. Hence, both subband and multirate techniques [1], [2], [10], [11], [28], [73], [82], [105], [125] are always employed together in designing computationally efficient and effective signal processing systems.

The major objective of this chapter is to review the fundamental concepts, design techniques, and to define various formulations for multirate filter banks. In Section 2.1, the basic theory of multirate signal processing is presented. These concepts are used in Section 2.2 for the analysis of multirate filter banks. Section 2.3 deals with a special class of filter banks, namely, paraunitary filter banks, which is able to perfectly reconstruct the fullband signal after subband decomposition. In Section 2.4,

the underlying affinity between filter bank and block transform is addressed. Finally, design techniques for cosine-modulated filter banks, which will be used extensively in subsequent chapters, are discussed in detail in Section 2.5.

2.1 Multirate Systems

Digital signal processing systems that use more than one sampling rate in the processing of digital signals are referred to as multirate systems [10], [28], [73], [82], [100], [125]. Decimator and interpolator are two basic sampling rate alteration devices employed in a multirate system to achieve different sampling rates at different stages.

A decimator, as depicted in Figure 2.1(a), retains only those samples of $x(n)$ that occur at instants of time equal to multiple of D :

$$x_D(k) = x(kD), \quad (2.1)$$

where D denotes the decimation factor. The decimated sequence $x_D(k)$ has a sampling rate that is D times smaller than that of the input sequence $x(n)$. The input-output relation for a D -fold decimator can be written in the transform domain as

$$X_D(z) = \frac{1}{D} \sum_{l=0}^{D-1} X(z^{1/D} W_D^l), \quad (2.2)$$

where $W_D \equiv e^{-j2\pi/D} = \sqrt[D]{e^{-j2\pi}}$ is the D th root of unity, and $j \equiv \sqrt{-1}$.

On the other hand, the interpolator depicted in Figure 2.1(b) increases the sampling rate of its input by inserting $(I-1)$ zero samples between each pair of input samples according to the relation

$$x_I(k) = \begin{cases} x(k/I), & \text{for } k = 0, \pm I, \pm 2I, \dots, \\ 0, & \text{otherwise,} \end{cases} \quad (2.3)$$

Figure 2.1 Sampling rate alteration devices. (a) Decimation by a factor D . (b) Interpolation by a factor I .

where I denotes the interpolation factor. The z-transform of the input and output of the I -fold interpolator are related via

$$X_I(z) = X(z^I). \quad (2.4)$$

Note that we use the variable n to index the original sequence $x(n)$, and k for the time index of the decimated and interpolated sequences, $x_D(k)$ and $x_I(k)$, which are the outputs of the sampling rate alteration devices.

Decimators and interpolators are linear but shift-variant operators [10], [73], [82], [125]. The linearity of the decimators and interpolators ensures that any memoryless operation, such as addition and multiplication by a constant, will commute with a decimator or an interpolator, as illustrated in Figure 2.2. On the other hand, the cascade of a filter (i.e., a dynamic system with memory) and a sampling rate alteration device is not commutative due to the shift-variant property. Hence, it makes a great difference whether the filter comes before or after the sampling rate alteration device.

Figure 2.2 Linearity of the decimator. Any memoryless operations commute with the decimator.

2.2 Filter Banks

A filter bank is a set of bandpass filters with a common input (i.e., the analysis filter bank) or a summed output (i.e., the synthesis filter bank) [11], [28], [73], [82], [125]. Figure 2.3 shows an N -channel (or N -band) filter bank, where $H_i(z)$ and $F_i(k)$ are the analysis and synthesis filters, respectively, and $i = 0, 1, \dots, N-1$ denotes the subband index. The analysis filter bank partitions an incoming signal $X(z)$ into N subbands with each of the subband signals $X_i(z)$ occupying a portion of the original frequency band; whereas the synthesis filter bank reconstructs a good approximation to the input signal, $Y(z)$, from the subband signals.

A filter bank is referred to as a uniform filter bank if all the subfilters have equal bandwidth with uniformly spaced center frequencies. In an N -band decomposition with equal-bandwidth filters, each of the subband signals $X_i(z)$ would occupy a portion of the original spectral band with a bandwidth of approximately π/N . Since the bandwidth of the subband signals $X_i(z)$ is N times smaller than that of the incoming signal $X(z)$, they can be N -fold decimated to $1/N$ th of the original sampling rate while preserving the original information. A filter bank, in which the

Figure 2.3 An N -channel maximally decimated filter bank. The combined structure of analysis-synthesis system is often referred to as quadrature-mirror filter (QMF) bank.

decimation factor equal to the number of subbands N is referred to as a maximally decimated (or critically subsampled) filter bank [11], [28], [73], [82], [125]. Critical decimation preserves the total effective sampling rate with each of the N decimated subband signals, $X_{i,D}(z)$, having $1/N$ th the original sampling rate, thereby the total number of subband samples is identical to that of the fullband signal $X(z)$. In the synthesis section, the decimated subband signals $X_{i,D}(z)$ are interpolated by the same factor before being combined by the synthesis filter bank. By so doing, the original sampling rate is restored in the reconstructed signal $Y(z)$.

In practice, realizable filters have non-zero transition bandwidth and stopband gain. Therefore, critical subsampling results in aliasing. However, with a proper design, the aliasing components can be cancelled out by the synthesis filters giving rise to an alias-free filter bank. The combined structure of analysis and synthesis filter banks with such a property is generally referred to as quadrature-mirror filter (QMF) bank [7], [54], [82], [102], [125].

2.2.1 Input-output relation

Consider the N -channel filter bank of Figure 2.3. Using (2.2), the decimated version of the subband signal $X_i(z) = H_i(z)X(z)$ can be written as

$$X_{i,D}(z) = \frac{1}{N} \sum_{l=0}^{N-1} H_i(z^{1/N} W_N^l) X(z^{1/N} W_N^l) \text{ for } i = 0, 1, \dots, N-1. \quad (2.5)$$

The decimated subband signals $X_{i,D}(z)$ are interpolated by the same factor N before being combined by the synthesis filter bank to form the output $Y(z)$. In light of (2.4) and (2.5), the output of the synthesis filter $F_i(z)$ is given by

$$\begin{aligned} Y_i(z) &= F_i(z) \left[\frac{1}{N} \sum_{l=0}^{N-1} H_i(z W_N^l) X(z W_N^l) \right] \\ &= \frac{F_i(z)}{N} \left[H_i(z) X(z) + \sum_{l=1}^{N-1} H_i(z W_N^l) X(z W_N^l) \right] \\ &= \frac{F_i(z)}{N} \left[X_i(z) + \sum_{l=1}^{N-1} X_i(z W_N^l) \right]. \end{aligned} \quad (2.6)$$

Clearly, $Y_i(z)$ consists of the component $X_i(z) = H_i(z)X(z)$ and its frequency shifted versions, $X_i(z W_N^l) = H_i(z W_N^l) X(z W_N^l)$ for $l = 1, 2, \dots, N-1$. For $i = 0, 1, \dots, N-1$, it can be noted that the terms $X(z W_N^0), X(z W_N^1), \dots, X(z W_N^{N-1})$ are common to all the synthesis filter outputs $Y_0(z), Y_1(z), \dots, Y_{N-1}(z)$. By grouping these common terms together, the input-output relation of the N -channel QMF bank of Figure 2.3 can be written as

$$\begin{aligned} Y(z) &= \sum_{i=0}^{N-1} Y_i(z) \\ &= \sum_{i=0}^{N-1} \left[\frac{1}{N} \sum_{l=0}^{N-1} F_i(z) H_i(z W_N^l) X(z W_N^l) \right] \\ &= \sum_{l=0}^{N-1} \left[\frac{1}{N} \sum_{i=0}^{N-1} F_i(z) H_i(z W_N^l) \right] X(z W_N^l) \\ &= \sum_{l=0}^{N-1} a_l(z) X(z W_N^l), \end{aligned} \quad (2.7)$$

where

$$a_l(z) = \frac{1}{N} \sum_{i=0}^{N-1} F_i(z) H_i(zW_N^l). \quad (2.8)$$

The term $X_i(zW_N^l)$ is called the l th aliasing component, which can be seen as the direct consequence of sampling rate alteration (i.e., the decimation and interpolation operations, as illustrated in Figure 2.3) on the subband signals $X_i(z)$. The transfer function $a_l(z)$ can be seen as the gain of the l th alias component $X(zW_N^l)$ at the output $Y(z)$. Equation (2.7) signifies that the QMF bank is a time-varying system [125, pp. 226].

The input-output relation in (2.7) can also be written in a more compact form, as follows

$$Y(z) = \frac{1}{N} \mathbf{f}^T(z) [\mathbf{H}^{(m)}(z)]^T \mathbf{x}^{(m)}(z), \quad (2.9)$$

where

$$\mathbf{h}(z) \equiv [H_0(z), H_1(z), \dots, H_{N-1}(z)]^T, \quad (2.10)$$

$$\mathbf{f}^T(z) \equiv [F_0(z), F_1(z), \dots, F_{N-1}(z)], \quad (2.11)$$

$$[\mathbf{H}^{(m)}(z)]^T \equiv [\mathbf{h}(z), \mathbf{h}(zW_N), \dots, \mathbf{h}(zW_N^{N-1})], \text{ and} \quad (2.12)$$

$$\mathbf{x}^{(m)}(z) \equiv [X(z), X(zW_N), \dots, X(zW_N^{N-1})]^T. \quad (2.13)$$

In the above equations, $\mathbf{h}(z)$ denotes the analysis filter bank, $\mathbf{f}^T(z)$ denotes the synthesis filter bank, $\mathbf{H}^{(m)}(z)$ is the modulation matrix of the analysis filter bank, and $\mathbf{x}^{(m)}(z)$ is the modulation vector of the input signal $X(z)$. Notice that the analysis filter bank is a one-input N -output system with the transfer matrix $\mathbf{h}(z)$, whereas the synthesis filter bank is an N -input one-output system with the transfer matrix $\mathbf{f}^T(z)$.

2.2.2 Perfect reconstruction filter banks

Looking back at (2.7), it can be seen that the output of a QMF bank $Y(z)$ is a weighted sum of its input $X(z)$ and the uniformly-shifted versions $X(zW_N^l)$. With a proper design of analysis and synthesis filters in such a way that $a_l(z)=0$ for $l=1,2,\dots,N-1$, the common aliasing components from all the N subbands would cancel each other out. The QMF bank is then referred to as alias-free. To this end, the filter bank reduces to a linear-time-invariant (LTI) system [125, pp. 228] with its input-output relation given by

$$\begin{aligned} Y(z) &= \left[\frac{1}{N} \sum_{i=0}^{N-1} F_i(z) H_i(z) \right] X(z) \\ &= T(z) X(z), \end{aligned} \quad (2.14)$$

where

$$T(z) = a_0(z) = \frac{1}{N} \sum_{i=0}^{N-1} F_i(z) H_i(z) \quad (2.15)$$

is known as the distortion transfer function of the filter bank. The transfer function $T(z)$ is a measure of any possible phase and magnitude distortions of the filter bank output. If $T(z)$ is further restricted to have linear phase and constant magnitude at all frequencies, for which $T(z) = cz^{-\tau}$, we arrive at the so called *perfect reconstruction* (i.e., phase and magnitude preserving in addition to alias-free) filter bank, where the output is just a time-delay copy of the scaled input, i.e., $Y(z) = cX(z)z^{-\tau}$.

As pointed out earlier, a QMF bank is alias-free if the alias components from all the subbands cancel out each other when the interpolated subband signals $Y_i(z)$ are combined to form the output $Y(z)$. Nevertheless, it should be noted that aliasing do exist in the decimated subband signals $X_{i,D}(z)$ of (2.5), even though the QMF bank

is alias-free. Of course, aliasing can be eliminated if the analysis filters are ideal brick-wall filters with zero-transition band and infinite attenuation, which is unachievable with any realizable filter. However, it would be practical enough to assume that the stop-band attenuation of the filters is sufficiently high, thus only some of the aliasing $H_i(zW_N^l)X(zW_N^l)$ that overlap with $H_i(z)X(z)$ are significant.

2.2.3 Polyphase representation

An analysis filter bank can be represented by an $N \times 1$ transfer matrix $\mathbf{h}(z)$ as shown in Section 2.2.1. The set of analysis filters $\mathbf{h}(z)$ can be alternatively expressed through an N -band polyphase decomposition in the following form

$$H_i(z) = \sum_{r=0}^{N-1} E_{ir}(z^N) z^{-r}, \text{ for } i=0,1,\dots,N-1, \quad (2.16)$$

where $E_{ir}(z)$ is the r th polyphase component of the i th analysis filter $H_i(z)$:

$$E_{ir}(z) = \sum_{n=0}^{K-1} h_i(nN+r) z^{-n}. \quad (2.17)$$

In the above equations, the analysis filter $H_i(z)$ is assumed to be causal with impulse response $h_i(n)$ of length $L = KN$, where K denotes the length of the polyphase components. An equivalent matrix representation of the set of equations in (2.16) is given by

$$\mathbf{h}(z) = \mathbf{E}(z^N) \mathbf{e}(z), \quad (2.18)$$

where the $N \times N$ matrix

$$\mathbf{E}(z) \equiv \begin{bmatrix} E_{0,0}(z) & E_{0,1}(z) & \cdots & E_{0,N-1}(z) \\ E_{1,0}(z) & E_{1,1}(z) & \cdots & E_{1,N-1}(z) \\ \vdots & \vdots & \ddots & \vdots \\ E_{N-1,0}(z) & E_{N-1,1}(z) & \cdots & E_{N-1,N-1}(z) \end{bmatrix} \quad (2.19)$$

is called the *Type I polyphase component matrix* for the analysis filter bank $\mathbf{h}(z)$, and

$$\mathbf{e}(z) \equiv [1, z^{-1}, \dots, z^{-N+1}]^T \quad (2.20)$$

represents the delay chain, as depicted in Figure 2.4.

Likewise, the N synthesis filters $\mathbf{f}^T(z)$ can also be represented in polyphase form, as follows

$$F_i(z) = \sum_{r=0}^{N-1} R_{ri}(z^N) z^{-(N-1-r)} \quad \text{for } i=0,1,\dots,N-1, \quad (2.21)$$

where $R_{ri}(z)$ are the *Type II polyphase components* of the filter $F_i(z)$. For a given transfer function $H(z) = \sum_{r=0}^{N-1} E_r(z^N) z^{-r}$, its *Type II polyphase components* are permutation of $E_r(z)$ in the form of $R_r(z) = E_{N-1-r}(z)$. In matrix notation, the set of equations in (2.21) can be written as

$$\mathbf{f}^T(z) = z^{-N+1} \tilde{\mathbf{e}}(z) \mathbf{R}(z^N), \quad (2.22)$$

where

$$\mathbf{R}(z) \equiv \begin{bmatrix} R_{0,0}(z) & R_{0,1}(z) & \cdots & R_{0,N-1}(z) \\ R_{1,0}(z) & R_{1,1}(z) & \cdots & R_{1,N-1}(z) \\ \vdots & \vdots & \ddots & \vdots \\ R_{N-1,0}(z) & R_{N-1,1}(z) & \cdots & R_{N-1,N-1}(z) \end{bmatrix} \quad (2.23)$$

is referred to as the *Type II polyphase component matrix* for the synthesis filter bank $\mathbf{f}^T(z)$, and

$$z^{-N+1} \tilde{\mathbf{e}}(z) = [z^{-N+1}, z^{-N+2}, \dots, 1] \quad (2.24)$$

is the delay chain pertaining to the *Type II polyphase representation*, as depicted in Figure 2.5. In the above equations, $\tilde{\mathbf{e}}(z)$ denotes the *paraconjugate* of $\mathbf{e}(z)$ (see Section 2.3). It should be noted that each row of the matrix $\mathbf{E}(z)$ corresponds to an analysis filter, whereas each column of the matrix $\mathbf{R}(z)$ gives the polyphase components of a synthesis filter.

Figure 2.4 Type I polyphase representation of the analysis filter bank.

Figure 2.5 Type II polyphase representation of the synthesis filter bank.

Figure 2.6(a) shows the polyphase realization of an analysis filter followed by a decimator. The decimator can be moved into the branches by exploiting the linearity property (i.e., the addition and decimation operations are commutative, as explained in Section 2.1). Furthermore, by means of the noble identities [73], [125] the decimators can be moved across the polyphase subfilters leading to the equivalent structure in Figure 2.6(b). The cascade of an interpolator and a synthesis filter can

also be implemented in the polyphase forms, as illustrated in Figure 2.7 (a). Likewise, the equivalent structure depicted in Figure 2.7(b) can be obtained by exploiting the noble identities and the linearity property.

Figure 2.6 An analysis filter $H_i(z)$ followed by a decimator. (a) Type I polyphase realization of the analysis filter. (b) The decimator can be moved into the branches and across the polyphase subfilters.

Figure 2.7 A synthesis filter $F_i(z)$ preceded by an interpolator. (a) Type II polyphase realization of the synthesis filter. (b) The interpolator can be moved into the branches and across the polyphase subfilters.

Now, using the polyphase representation of Figure 2.4 and Figure 2.5 in Figure 2.3, the N -channel QMF bank can equivalently be represented in polyphase form as depicted in Figure 2.8(a). Next, by exploiting the linearity of the decimator and interpolator together with the noble identities (as outlined in Figure 2.6 and Figure 2.7 for the analysis and synthesis filters, respectively), we arrive at the simplified structure of Figure 2.8(b). The set of delays and decimators on the left of Figure 2.8(b) can be regarded as the polyphase representation of a *serial-to-parallel converter*. For a given signal $x(n)$, it gathers blocks of N samples

$$\mathbf{x}(k) = [x(kN), x(kN-1), \dots, x(kN-N+1)]^T.$$

Figure 2.8 N -channel maximally decimated filter bank. (a) Polyphase representation. (b) Rearrangement using noble identities.

On the other hand, the set of delays and interpolators on the right of Figure 2.8(b) is the multirate representation of a *parallel-to-serial* converter which appends blocks of N samples to the output. Thus, for the case where $\mathbf{R}(z)\mathbf{E}(z)=\mathbf{I}$, we obtain a trivial perfect reconstruction filter bank, as depicted in Figure 2.9. The filter bank picks up a block of input samples $\mathbf{x}(k)$ and sends it to the output giving rise to a delayed version of the input, $y(n)=x(n-N+1)$.

Figure 2.9 A trivial perfect reconstruction filter bank.

2.3 Paraunitary Filter Banks

In Figure 2.8, the analysis and synthesis filters of the N -channel maximally decimated filter bank are expressed in terms of the polyphase component matrices, $\mathbf{E}(z)$ and $\mathbf{R}(z)$. A perfect reconstruction system can be obtained by first imposing the *paraunitary* property on $\mathbf{E}(z)$ such that

$$\tilde{\mathbf{E}}(z)\mathbf{E}(z)=\mathbf{I}, \quad (2.25)$$

and then taking the synthesis filters as the time-reversal of the analysis filters in the following form

$$\mathbf{R}(z) = z^{-K+1} \tilde{\mathbf{E}}(z). \quad (2.26)$$

In the above equations, $\mathbf{I}$ denotes the identity matrix, $K-1$ is the order of the polyphase component matrices, and $\tilde{\mathbf{E}}(z) = \mathbf{E}_*^T(z^{-1})$ denotes the *paraconjugate* of the matrix $\mathbf{E}(z)$. Again, we assume that the analysis and synthesis filters are finite impulse response (FIR) filters of length $L = KN$.

The paraunitary condition in (2.25) implies that $\mathbf{E}^{-1}(z) = \tilde{\mathbf{E}}(z)$. To obtain the paraconjugate of $\mathbf{E}(z)$, we first conjugate the coefficients and replace z with z^{-1} for each entry of $\mathbf{E}(z)$, i.e., $\tilde{E}_{ir}(z) = E_{ir,*}(z^{-1})$, and then transpose the resulting matrix, which finally gives $\tilde{\mathbf{E}}(z) = [\tilde{E}_{ri,*}(z^{-1})]$, for $i, r = 0, 1, \dots, N-1$. The polyphase components of the synthesis filter bank can then be obtained according to (2.26) as $R_{ri}(z) = z^{-K+1} \tilde{E}_{ir}(z)$. Since each of the polyphase components is of order $K-1$, the delay z^{-K+1} ensures that the resulting $\mathbf{R}(z)$ is causal. The transfer matrix $\mathbf{f}^T(z)$ of the synthesis filter bank can then be obtained from $\mathbf{R}(z)$ by substituting (2.26) into (2.22) in the following form

$$\begin{aligned} \mathbf{f}^T(z) &= z^{-N+1} \tilde{\mathbf{e}}(z) \left[z^{(-K+1)N} \tilde{\mathbf{E}}(z^N) \right] \\ &= z^{-KN+1} \tilde{\mathbf{e}}(z) \tilde{\mathbf{E}}(z^N) \\ &= z^{-L+1} \tilde{\mathbf{h}}(z), \end{aligned} \quad (2.27)$$

where $\tilde{\mathbf{h}}(z) = \tilde{\mathbf{e}}(z) \tilde{\mathbf{E}}(z^N)$ is the paraconjugate of $\mathbf{h}(z) = \mathbf{E}(z^N) \mathbf{e}(z)$. In light of (2.10), (2.11), and (2.27), the transfer function of the synthesis filters is given by

$$F_i(z) = z^{-L+1} \tilde{H}_i(z), \text{ for } i = 0, 1, \dots, N-1, \quad (2.28)$$

or, equivalently, in the time-domain as

$$f_i(n) = h_i(L-1-n), \text{ for } i = 0, 1, \dots, N-1, \quad (2.29)$$

where the filter coefficients are assumed to be real. That is, the impulse response of

the synthesis filter is equal to the time-reversed impulse response of the corresponding analysis filter.

Equations (2.25) and (2.26) define a sufficient condition for perfect reconstruction [82], [125]. By imposing the paraunitary condition (2.25) on $\mathbf{E}(z)$ and then having $\mathbf{R}(z) = z^{-K+1} \tilde{\mathbf{E}}(z)$, the product $\mathbf{R}(z)\mathbf{E}(z)$ of the matrices reduces to z^{-K+1} . As such, the cascade of $\mathbf{E}(z)$ and $\mathbf{R}(z)$ in Figure 2.8(b) results in a net delay of $K-1$, at the decimated rate. Taking into account the delay inflicted by the trivial perfect reconstruction system (which consists of the delay chains and sampling rate alteration devices, as illustrated in Figure 2.9), the QMF bank introduces an overall delay of $\tau = N-1 + (K-1)N = KN-1 = L-1$ sampling periods on the input signal. To this end, the aliasing has been cancelled, and the distortion transfer function $T(z)$ has been forced to be a delay $z^{-\tau}$.

2.4 Block Transforms

A block transform, such as the discrete Fourier transform (DFT) and discrete cosine transform (DCT), can be regarded as a maximally decimated paraunitary filter bank with a zero-order polyphase component matrix, i.e., $\mathbf{E}(z) = \mathbf{A}^H$, where $\mathbf{A}$ denotes the unitary (or orthogonal) transformation matrices. The fact is well known, see [10], [24], [25], [27], [73] and references therein. Notice that any unitary matrix is (trivially) paraunitary [125, pp. 289]. The basis functions of the transform (i.e., the columns of the matrix $\mathbf{A}$) can be seen as bandpass filters with the length L of their impulse responses equal to the block size N . Alternatively, we may interpret a filter bank as a block transform with memory [73], where the length of the bandpass filters L is larger than the block size N .

The fundamental difference between a block transform and a filter bank is the degree of subband separation (or decorrelation) that can be achieved, and the computational complexity involved. In some applications, we may need the subband signals (or the transformed variables) to be uncorrelated. Strong subband separation can generally be achieved by using filter banks with high-order analysis filters. Thus, increasing the filter length serves as a mean to reduce the correlation between subbands, which comes with a penalty of higher computational complexity.

Block transforms and filter banks are widely employed in adaptive filtering, for examples, in transform domain adaptive filter (TDAF) [3], [4], [24], [25], [53], [75], [87], subband adaptive filter (SAF) [16], [27], [34]-[36], [55], [111], and filter bank adaptive filter [124]. From the underlying affinity between block transforms and filter banks, we readily see that these adaptive filters are closely related to each others. A brief description and comparison of these adaptive filters is tabulated in Appendix A. The table also includes a brief description on the normalized SAF [13], [14], [61], [62], [99], [77], [78], which will be discussed further in Chapter 4.

2.4.1 Filter bank as block transform

An analysis filter bank can be treated as a block transform with memory, whereas the synthesis filter bank can be regarded as the corresponding inverse transform [72], [73]. The idea is illustrated in Figure 2.10, where the analysis and synthesis filter banks are, respectively, represented as $L \times N$ matrices $\mathbf{H} = [\mathbf{h}_0, \mathbf{h}_1, \dots, \mathbf{h}_{N-1}]$ and $\mathbf{F} = [\mathbf{f}_0, \mathbf{f}_1, \dots, \mathbf{f}_{N-1}]$. The columns of the matrices, $\mathbf{h}_i = [h_i(0), h_i(1), \dots, h_i(L-1)]^T$ and $\mathbf{f}_i = [f_i(0), f_i(1), \dots, f_i(L-1)]^T$, hold the coefficients of the analysis and synthesis filters, respectively. Recall that, for paraunitary filter bank, the synthesis filters are taken as the time reversed versions of the corresponding analysis filters.

In Figure 2.10, the delay chain $\mathbf{e}_L(z) = [1, z^{-1}, \dots, z^{-L+1}]$ consists of $L-1$ delay elements. For every N input samples, the input vector

$$\mathbf{x}(kN) \equiv [x(kN), x(kN-1), \dots, x(kN-N+1), x(kN-N), \dots, x(kN-L+1)]^T \quad (2.30)$$

is packed with N new samples and $L-N$ old samples. The input vector of length L is then mapped into a transformed vector $\mathbf{x}_D(k) = \mathbf{H}^T \mathbf{x}(kN)$ of length N . Notice that the elements of the vector $\mathbf{x}_D(k) \equiv [x_{0,D}(k), x_{1,D}(k), \dots, x_{N-1,D}(k)]^T$ are essentially the decimated subband signals.

The input vector $\mathbf{x}(kN)$ has a higher dimension than the transformed vector $\mathbf{x}_D(k)$. Furthermore, there is an overlap of $L-N$ samples between $\mathbf{x}(kN)$ and the consecutive block $\mathbf{x}[(k+1)N]$, which indicates the memory of the transform. Similar overlapping operation applied in the synthesis section. As shown in Figure 2.10, consecutive blocks of inverse transform $\mathbf{F}\mathbf{x}_D(k)$ have to be overlapped and added to synthesize the fullband output through a the delay chain on the right of this figure.

Figure 2.10 Filter bank as block transform with memory.

2.5 Cosine-Modulated Filter Banks

The theory and design of cosine-modulated filter banks have been studied extensively in the past [7], [8], [67], [73], [76], [91], [102], [105], [125]. In an N -channel cosine-modulated filter bank, the analysis and synthesis filters are cosine-modulated version of a lowpass prototype filter $P(z)$ with a cutoff frequency at $\pi/2N$, as depicted in Figure 2.11(a). In particular, the cosine-modulated analysis filters are obtained as

$$H_i(z) = \alpha_i P[zW^{(i+0.5)}] + \alpha_i^* P[zW^{-(i+0.5)}], \text{ for } i = 0, \dots, N-1, \quad (2.31)$$

where W is the $2N$ th root of unity $W = W_{2N} = e^{-j\pi/N}$, α_i is a unit-magnitude constant given by

$$\alpha_i = \exp\left\{j\left[\theta_i - \frac{\pi}{N}(i+0.5)\left(\frac{L-1}{2}\right)\right]\right\}, \quad \theta_i = (-1)^i \frac{\pi}{4}, \quad (2.32)$$

with L denotes the length of the of the prototype filter. The synthesis filters are then obtained by time-reversing the analysis filters, in the form

$$F_i(z) = z^{-L+1} H_i(z^{-1}). \quad (2.33)$$

The design of the whole filter bank thus reduces to the design of the prototype filter.

Perfect (or approximately perfect) reconstruction for the whole analysis-synthesis system can be achieved by optimizing the prototype filter to satisfy a set of predetermined constraints. For example, the flatness constraint [7], [8], [9], [73], [76], [91], [102], [125] and the $2N$ th band constraint [67], [89] lead to approximate perfect reconstruction. For perfect reconstruction [72], [73], each of the N polyphase components of $P(z)$ should be a spectral factor of a halfband filter. Different constraints and optimization procedures have been proposed giving rise to two main classes of cosine-modulated filter banks, namely, (i) pseudo-QMF cosine-modulated banks, and (ii) paraunitary cosine-modulated filter banks, as summarized in Table 2.1.

Table 2.1 Two main classes of cosine-modulated filter banks.

	Pseudo-QMF cosine-modulated filter banks	Paraunitary cosine-modulated filter banks
Aliasing cancellation	Significant aliasing component is cancelled structurally between adjacent bands by selecting appropriate phase factor in the modulating function. Aliasing from distant bands is assumed to be sufficiently attenuated. That is, the stopband attenuation of a given analysis filter in all non-adjacent bands is sufficiently high by imposing the following condition on the prototype filter: $\left P(e^{j\omega}) \right = 0 \text{ for } \omega > \pi/N.$	The polyphase component matrix $\mathbf{E}(z)$ of the filter bank is paraunitary by forcing each of the N polyphase components of the prototype filter to be a spectral factor of a halfband filter. The paraunitary property, together with the time-reversal relationship between the analysis and synthesis filters, ensures a perfect signal reconstruction, i.e., phase preserving, amplitude preserving, and alias-free.
Phase distortion	Phase distortion is avoided with the time-reversal relationship between the analysis and synthesis filters.	
Amplitude distortion	Amplitude distortion is eliminated by having magnitude response of the distortion transfer function, $ T(e^{j\omega}) $, acceptably flat.	
Constraint	<p><i>a. Flatness constraint:</i></p> <p>The prototype filter $P(z)$ fulfills as much as possible</p> $\left P(e^{j\omega}) \right ^2 + \left P(e^{j(\omega-\pi/N)}) \right ^2 = 1, \text{ for } 0 < \omega < \pi/N.$ <p><i>b. 2Nth band constraint:</i></p> <p>The prototype filter $P(z)$ is a spectral factor of a 2Nth band filter $Q(z)$, where</p> $Q(z) = P(z) \tilde{P}(z),$ <p>and</p> $\sum_{l=0}^{2N-1} Q(zW^l) = 1.$ <p>Let $q(n)$ be the non-causal impulse response of the 2Nth band filter $Q(z)$, the 2Nth band constraint can be equivalently written in time domain as</p> $q(n) = p(n) * p(-n),$ <p>and</p> $q(2Nn) = \frac{1}{2N} \delta(n).$	<p><i>Orthogonality constraint:</i></p> <p>Each of the N polyphase components $P_r(z)$ of the prototype filter $P(z) = \sum_{r=0}^{N-1} P_r(z^N) z^{-r}$ is a spectral factor of a halfband filter $Q_r(z)$, where</p> $Q_r(z) = P_r(z) \tilde{P}_r(z),$ <p>and</p> $Q_r(z) + Q_r(-z) = 1.$ <p>Let</p> $q_r(l) = p_r(l) * p_r(-l)$ <p>be the autocorrelation sequence of the rth polyphase component $p_r(l) = p(Nl+r)$ of the prototype filter $p(n)$. For $p_r(l)$ a spectral factor of a halfband filter, its autocorrelation sequence satisfies the following condition</p> $q_r(2l) = \frac{1}{2} \delta(l).$

Figure 2.11 Cosine modulation. (a) Frequency response of a typical prototype filter. (b) Frequency responses of the cosine-modulated analysis filters.

The prototype filter $P(z)$ is generally restricted to be a linear-phase filter with real-valued symmetric impulse response. Even though the prototype filter $p(n)$ has linear phase, cosine-modulation does not result in linear phase analysis and synthesis filters. The proof is given in Appendix B. Nevertheless, the time-reversing property of (2.33) ensures that the cascade of the analysis and synthesis filters $F_i(z)H_i(z)$ for the i th channel, and thus the overall distortion transfer function $T(z)$, has linear phase, as shown below

$$\begin{aligned}
 T(e^{j\omega}) &= \frac{1}{N} \sum_{i=0}^{N-1} F_i(e^{j\omega}) H_i(e^{j\omega}) \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} [e^{-j\omega(L-1)} \tilde{H}_i(e^{j\omega})] H_i(e^{j\omega}) \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} e^{-j\omega(L-1)} [H_i(e^{j\omega})]^* H_i(e^{j\omega}) \\
 &= \frac{e^{-j\omega(L-1)}}{N} \sum_{i=0}^{N-1} |H_i(e^{j\omega})|^2.
 \end{aligned} \tag{2.34}$$

It can be noted from (2.31) that a single cosine-modulated analysis filter $H_i(z)$ consists of two complex-valued filters, $P[zW^{(i+0.5)}]$ and $P[zW^{-(i+0.5)}]$ as illustrated in Figure 2.11(b), which are generated from $P(z)$ by complex modulation. The impulse response of the right-shifted version $P[zW^{(i+0.5)}]$ is the complex conjugate of that pertaining to the left-shifted version $P[zW^{-(i+0.5)}]$. Combination of this conjugate pair of impulse responses yields the analysis filter $H_i(z)$, and the corresponding synthesis filter $F_i(z)$, with real impulse responses, as given below

$$\begin{aligned} h_i(n) &= 2p(n) \cos \left[\frac{\pi}{N}(i+0.5) \left(n - \frac{L-1}{2} \right) + \theta_i \right] \\ f_i(n) &= 2p(n) \cos \left[\frac{\pi}{N}(i+0.5) \left(n - \frac{L-1}{2} \right) - \theta_i \right] \end{aligned} \quad (2.35)$$

From our previous discussion, the practical virtue of cosine-modulated filter banks can be summarized as follows.

- (i) The design of the whole filter bank can be accomplished by designing a single lowpass prototype filter, and cosine-modulating the lowpass prototype to generate the analysis and synthesis filters according to (2.35). Perfect or approximate signal reconstruction for the whole filter bank (analysis/synthesis system) can then be achieved by designing the single prototype filter to satisfy a predefined constraint, as listed in Table 2.1.
- (ii) A cosine-modulated filter bank consists of analysis and synthesis filters with real coefficients where complex arithmetic can be avoided. In real-time subband signal processing system, it would be expedient to avoid complex calculations where the real, imaginary, and cross-product terms must be handled.
- (iii) Fast transform-based implementation is available due the modulated nature of the bandpass filters [56], [71], [72], [73], [125]. For example, the extended

lapped transform (ELT) filter bank can be computed efficiently, with computational complexities that are in the same order of magnitude as those of standard transform [72], [73].

- (iv) Cosine-modulating a lowpass filter with high stopband rejection results in a filter bank that is able to generate subband signals that are nearly orthogonal at zero lag.

The first three advantages of the cosine-modulated filter banks are well known. The orthogonality property, which seems to have gone unnoticed in the literature, will be analyzed in Chapter 3.

2.5.1 Design example

Consider the design of a sixteen-channel cosine-modulated filter bank. The Parks-McClellan algorithm is used to design the prototype filter of length $L = 256$ taps. The stopband edge ω_s of the prototype is fixed at $\pi/16$, whereas its passband edge ω_p is iteratively adjusted to minimize the following objective function

$$\phi = \max_{\omega} \left[|P(e^{j\omega})|^2 + |P(e^{j(\omega-\pi/N)})|^2 - 1 \right], \text{ for } 0 < \omega < \pi/N, \quad (2.36)$$

such that the flatness constraint is approximately satisfied at the end of the optimization [9]. The magnitude responses of all the cosine-modulated filters are depicted in Figure 2.12(a). The transition bands of non-adjacent filters do not overlap, since the stopband edge of the prototype filter is fixed at $\omega_s = \pi/16$. Hence, aliasing from distant bands are sufficiently attenuated. The distortion transfer function $T(e^{j\omega})$ of the whole analysis/synthesis system is plotted in Figure 2.12(b). Clearly, the frequency response is almost constant, and thus allows a nearly perfect reconstruction of the input signal.

Figure 2.12 A 16-channel pseudo-QMF cosine-modulated filter bank. (a) Magnitude responses of all the cosine-modulated filters. (b) Distortion transfer function $T(e^{j\omega})$ for the analysis/synthesis system.

2.6 Conclusions

We have reviewed the theory and design technique for multirate filter banks, which are the fundamental component of subband adaptive filters. In Section 2.1 the fundamental concepts of multirate signal processing was introduced, with emphasis on the properties of decimators and interpolators. The input-output relations of these sampling rate alteration devices were discussed, both in the time-domain and transform-domain. The basic structure of multirate filter banks was reviewed in Section 2.2. In particular, the input-output relation of multirate filter banks was formulated, and the distortion transfer function was defined. Polyphase representation,

which is a convenient form of representing the multirate filter banks, was discussed as well.

In Section 2.3, the concept of paraunitaryness was presented. We showed that perfect reconstruction is achievable by imposing the paraunitary condition on the polyphase component matrix of the filter bank. The paraunitary condition can be exploited to simplify the convergence analysis of subband adaptive filters, as we shall see later in Chapter 5. In Section 2.4, the relationships between filter banks and block transforms are briefly described. In particular, we showed that filter bank can be treated as block transform with memory. Such an interpretation provides a convenient way for formulating the subband adaptive filtering algorithms, as we shall see in Chapter 4 and Chapter 6.

Section 2.5 is devoted to the analysis and design of cosine-modulated filter banks. Cosine modulation is an attractive choice for the design and implementation of uniform filter banks with a large number of subbands. A cosine-modulated filter bank is comprised of filters with real coefficients where complex arithmetic can be avoided. Furthermore, fast transform-based implementation is possible due to the modulated nature of the filters. Beside their low design and implementation cost, cosine-modulated filter banks possess an attractive partial decorrelation feature, as well shall examined closely in Chapter 3.

Chapter 3

Subband Orthogonality of Multirate Filter Banks

A realizable filter has non-zero transition bandwidth and finite stopband attenuation [50], [93], [100]. Moreover, for a filter bank to be lossless (i.e., stable, causal, and paraunitary), no spectral gaps between subbands are allowed. Consequently, spectral overlap between filters is unavoidable in a realizable and lossless filter bank [125], notably those overlaps pertaining to adjacent filters. That is to say that a realizable and lossless filter bank is not able to decompose its input signal into non-overlapping spectral bands (and thus mutually exclusive in their spectral contents), at least when a priori knowledge of the input signal is not available. Consequently, considerable degree of correlation exists among subband signals due to the spectral overlaps of the analysis filters. Subband orthogonality [64], [65] is concerned with the analysis of the correlation that exists between subband signals.

This chapter begins with a novel correlation-domain formulation for multirate filter banks. With the new formulation, a necessary condition is defined for a special case of subband orthogonality, whereby the outputs of a filter bank are orthogonal at zero lag for arbitrary type of input spectrum. In particular, we show that the subband orthogonality can be obtained by cosine-modulating a lowpass filter with high stopband attenuation. This feature pertaining to the cosine-modulated filter banks plays an important role in the derivation and analysis of subband adaptive algorithms in Chapter 4 and Chapter 5.

In the following discussion, a random signal model is assumed for the input signal, and thus, the resulting subband signals are also random in nature. A brief description on the basic concepts and notations of stochastic processes can be found in [27], [44], [58], [74], [93], [100], [120]. Please refer to [96], [119] for a more detailed treatment on the subject.

3.1 Correlation-Domain Formulation

Consider the multirate filter bank of Figure 3.1(a). Let the input signal $u(n)$, for $-\infty < n < \infty$, be a zero-mean random signal; $u_i(n)$ and $u_p(n)$, for $i, p = 0, 1, \dots, N-1$, are two arbitrary outputs of the N -channel filter bank. Assuming that $u(n)$ is wide-sense stationary (WSS), its autocorrelation function depends only on the time difference l (called lag) between samples, which can be defined in the following form

$$\gamma_{uu}(l) \equiv E\{u(n)u(n-l)\}, \quad (3.1)$$

where $E\{\cdot\}$ denotes the expectation operator. By virtue of the linear-time-invariant (LTI) property of the analysis filters, the subband signals are also WSS [93], and are related to the input signal by the convolution sum, as follows

$$u_i(n) = \sum_{r=0}^{\infty} h_i(r)u(n-r) = h_i(n) \otimes u(n), \text{ for } i = 0, 1, \dots, N-1, \quad (3.2)$$

where the operator $\otimes$ denotes the convolution operation. In the literature, the symbol for the convolution sum is $*$. However, as the superscript $*$ is reserved for the complex conjugation operation (as in Section 3.2), we have adopted the symbol $\otimes$ to denote the convolution sum operation. Notice that the lower limit on the convolution sum reflects the causality of the analysis filters $h_i(n)$.

Figure 3.1 Correlation-domain formulation for multirate filter banks. (a) The i th and p th channels of an N -band maximally decimated analysis filter bank. (b) The effect of band-partitioning a random signal can be conveniently described in terms of the effect of the system $q_{ip}(l)$ on the autocorrelation function $\gamma_{uu}(l)$.

The cross-correlation function between two arbitrary subband signals, $u_i(n)$ and $u_p(n)$, in response to the random excitation $u(n)$, is defined as

$$\gamma_{ip}(n, n-l) \equiv E\{u_i(n)u_p(n-l)\}. \quad (3.3)$$

Using (3.1) and (3.2) in the right hand side of (3.3), and after some algebraic manipulation, we obtain

$$\begin{aligned} E\{u_i(n)u_p(n-l)\} &= E\left\{\sum_{r=0}^{\infty} h_i(r)u(n-r)\sum_{m=0}^{\infty} h_p(m)u(n-l-m)\right\} \\ &= \sum_{r=0}^{\infty} h_i(r)\sum_{m=0}^{\infty} h_p(m)E\{u(n-r)u(n-l-m)\} \\ &= \sum_{r=0}^{\infty} h_i(r)\sum_{m=0}^{\infty} h_p(m)E\{u(n)u[n-(l+m-r)]\} \\ &= \sum_{r=0}^{\infty} h_i(r)\sum_{m=0}^{\infty} h_p(m)\gamma_{uu}(l+m-r) \\ &= \sum_{r=0}^{\infty} h_i(r)\sum_{s=-\infty}^{\infty} h_p(r-s)\gamma_{uu}(l-s) \\ &= \sum_{s=-\infty}^{\infty} \gamma_{uu}(l-s)\sum_{r=0}^{\infty} h_i(r)h_p(r-s). \end{aligned} \quad (3.4)$$

Clearly, when $u(n)$ is WSS, the subband signals, $u_i(n)$ and $u_p(n)$, are jointly WSS, i.e., their cross-correlation depends only on the time lag l . Taking a closer look at (3.4), it can be easily seen that the cross-correlation function can also be expressed in terms of the input autocorrelation function $\gamma_{uu}(l)$, in the following form

$$\gamma_{ip}(l) = \sum_{s=-\infty}^{\infty} q_{ip}(s) \gamma_{uu}(l-s) = q_{ip}(l) \otimes \gamma_{uu}(l), \quad (3.5)$$

where

$$q_{ip}(l) \equiv \sum_{r=0}^{\infty} h_i(r) h_p(r-l) = h_i(l) \otimes h_p(-l) \quad (3.6)$$

is the deterministic cross-correlation sequence between the impulse responses of the analysis filters, $h_i(n)$ and $h_p(n)$. It should be emphasized that $q_{ip}(l)$ is the cross-correlation of two finite-energy sequences and should not be confused with the correlation functions of the infinite-energy random processes, $\gamma_{uu}(l)$ and $\gamma_{ip}(l)$.

3.1.1 Critical subsampling

The correlation-domain formulation can be easily extended to include the critical subsampling operation. Let the cross-correlation function between the decimated subband signals, $u_{i,D}(k)$ and $u_{p,D}(k)$, be defined as

$$\gamma_{ip,D}(k, k-l) \equiv E\{u_{i,D}(k) u_{p,D}(k-l)\}. \quad (3.7)$$

The $N:1$ decimators retain only those samples of $u_i(n)$ and $u_p(n)$ that occur at instants of time equal to multiples of N (as explained in Section 2.1). Hence, the decimated subband signals can be written as

$$u_{i,D}(k) = u_i(kN), \text{ and } u_{p,D}(k-l) = u_p[(k-l)N]. \quad (3.8)$$

Substituting (3.8) into (3.7), and knowing that for $u_i(n)$ and $u_p(n)$ being jointly WSS their cross-correlation depends only on the time difference lN , we arrive at

$$\gamma_{ip,D}(l) = E\{u_i(kN)u_p(kN - lN)\} = \gamma_{ip}(lN). \quad (3.9)$$

Equation (3.9) signifies that the cross-correlation function $\gamma_{ip,D}(l)$ of the decimated subband signals is the N -fold decimated version of $\gamma_{ip}(l)$, as illustrated in Figure 3.1(b).

The correlation-domain formulation for multirate filter banks is summarized in Figure 3.1. With this formulation, the effect of filtering a random signal with a filter bank can be conveniently described in terms of the effect of the system on the autocorrelation function $\gamma_{uu}(l)$ of the random signal. Specifically, the cross-correlation function $\gamma_{ip}(l)$ between the subband signals can be determined in terms of the input autocorrelation function $\gamma_{uu}(l)$ and the characteristics of the analysis filters $q_{ip}(l)$. This concept of correlation-domain formulation is well understood in signal processing studies [93], [100], [74], [44]. Here, we have extended the formulation for multirate filter banks, which facilitates the analysis of their second-order characteristics. Of particular interest are those pertaining to cosine-modulated filter banks, as we shall examine closely in Section 3.4.

We have assumed that all filter coefficients and signals are real. The real-valued formulation is not restrictive as the subband algorithm that we are going to derive in Chapter 4 employs real-valued filter banks in order to avoid complex arithmetic. Nevertheless, the correlation-domain formulation can be easily generalized to include complex-valued filter banks and subband signals.

3.2 Cross Spectrum

The cross-correlation function $\gamma_{ip}(l)$ represents the time-domain description of the statistical relation that exists between subband signals. The statistical relation can be represented in the frequency domain by taking the Fourier transform of (3.5), in which we obtain the cross-power spectrum $\Gamma_{ip}(e^{j\omega})$ between the subband signals, in the following form

$$\begin{aligned}\Gamma_{ip}(e^{j\omega}) &= \sum_{l=-\infty}^{\infty} \gamma_{ip}(l) e^{-j\omega l} \\ &= \left[\sum_{l=-\infty}^{\infty} q_{ip}(l) e^{-j\omega l} \right] \left[\sum_{l=-\infty}^{\infty} \gamma_{uu}(l) e^{-j\omega l} \right] \\ &= Q_{ip}(e^{j\omega}) \Gamma_{uu}(e^{j\omega}).\end{aligned}\tag{3.10}$$

where $\Gamma_{uu}(e^{j\omega})$ is the power spectrum of the input signal $u(n)$, and $Q_{ip}(e^{j\omega})$ is the cross-energy spectrum between the two finite-energy impulse responses of the analysis filters. In particular, the cross-energy spectrum $Q_{ip}(e^{j\omega})$ between the i th and p th analysis filters is obtained by taking the Fourier transform of (3.6), as shown below

$$\begin{aligned}Q_{ip}(e^{j\omega}) &\equiv \sum_{l=-\infty}^{\infty} q_{ip}(l) e^{-j\omega l} \\ &= \left[\sum_{l=0}^{\infty} h_i(l) e^{-j\omega l} \right] \left[\sum_{l=-\infty}^0 h_p(-l) e^{-j\omega l} \right] \\ &= \left[\sum_{l=0}^{\infty} h_i(l) e^{-j\omega l} \right] \left[\sum_{l=0}^{\infty} h_p(l) e^{j\omega l} \right] \\ &= \left[\sum_{l=0}^{\infty} h_i(l) e^{-j\omega l} \right] \left[\sum_{l=0}^{\infty} h_p(l) e^{-j\omega l} \right]^* \\ &= H_i(e^{j\omega}) [H_p(e^{j\omega})]^* \\ &= |H_i(e^{j\omega})| |H_p(e^{j\omega})| e^{j\phi_p(\omega)},\end{aligned}\tag{3.11}$$

where $H_i(e^{j\omega})$ and $H_p(e^{j\omega})$ are the frequency responses of the analysis filters, $\phi_{ip}(\omega) = \phi_i(\omega) - \phi_p(\omega)$ is their phase difference, and $*$ denotes the complex conjugation. The cross-correlation function $\gamma_{ip}(l)$ can be recovered from the cross-power spectrum $\Gamma_{ip}(e^{j\omega})$ through the inverse Fourier transform, as follows

$$\begin{aligned}\gamma_{ip}(l) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \Gamma_{ip}(e^{j\omega}) e^{j\omega l} d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_i(e^{j\omega}) [H_p(e^{j\omega})]^* \Gamma_{uu}(e^{j\omega}) e^{j\omega l} d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |H_i(e^{j\omega})| |H_p(e^{j\omega})| \Gamma_{uu}(e^{j\omega}) e^{j[\phi_p(\omega) + \omega l]} d\omega.\end{aligned}\tag{3.12}$$

From (3.12), we can interpret the cross-power spectrum $\Gamma_{ip}(e^{j\omega_0})$ as a measure of the correlation between two subband signals at a given frequency ω_0 , i.e., it can be thought of as correlation coefficient in the frequency domain.

Recall that the subband signals $u_i(n)$ are generated from a common input signal $u(n)$ by means of analysis filters $H_i(z)$ with their passbands occupying contiguous portions of the original spectral band. The resulting subband signals, $u_i(n)$ and $u_p(n)$, are orthogonal if their cross-correlation function $\gamma_{ip}(l)$ is identically zero at all lag l . From (3.12), it follows that the orthogonality of the subband signals can be assured if the filter bank consists of ideal non-overlapping bandpass filters $H_i(e^{j\omega})$, whereby their cross-energy spectrum $Q_{ip}(e^{j\omega})$ is identically zero at all frequencies ω . In the time domain, this condition is equivalent to the case where $q_{ip}(l)$ are identically zero at all lag l , as can be seen from (3.5).

Therefore, the subband signals can be assured to be orthogonal if they occupy non-overlapping portions of the original frequency band. Such a total decorrelation

(or separation) of subbands may be desirable, but it is not achievable in practice because realizable filters have non-zero transition bandwidth and stopband gain. In a realizable and lossless filter bank [125], significant overlap between its adjacent bands is unavoidable. Consequently, considerable degree of correlation exists between adjacent subband signals. Since total decorrelation of subband signals is not possible in practical situation, it will be beneficial to devote to a less stringent condition where the subband signals are orthogonal at zero lag only (i.e., partially decorrelated), as pointed by the author in [61], [62], [64], [65].

3.3 Orthogonality at Zero Lag

Assuming that the analysis filters, $h_i(n)$ and $h_p(n)$, are causal FIR filters of length L . Their cross-correlation $q_{ip}(l)$, as defined in (3.6), is then a non-causal sequence of length $2L-1$:

$$q_{ip}(l) = \begin{cases} h_i(l) \otimes h_p(-l), & |l| \leq L-1, \\ 0, & |l| \geq L. \end{cases} \quad (3.13)$$

Substituting (3.13) into (3.5), the cross-correlation of the subband signals at zero lag, $l=0$, can be written as

$$\gamma_{ip}(0) = \sum_{s=-L+1}^{L-1} q_{ip}(s) \gamma_{uu}(-s). \quad (3.14)$$

The autocorrelation function of a WSS random process is a symmetric function of the time lag l , i.e., $\gamma_{uu}(l) = \gamma_{uu}(-l)$. It can be easily seen from (3.14) that, if the deterministic cross-correlation sequence is anti-symmetric, where $q_{ip}(l) = -q_{ip}(-l)$, we obtain $\gamma_{ip}(0) = 0$ because the sum of the product of the symmetric sequence $\gamma_{uu}(-s)$ and the anti-symmetric sequence $q_{ip}(s)$ is always zero. In other words, the subband signals can be orthogonal at zero lag, i.e., $\gamma_{ip}(0) = E\{u_i(n)u_p(n)\} = 0$ for

$i \neq p$, even though their cross-correlation $\gamma_{ip}(l)$ may be large at $l \neq 0$. This property of filter banks can be obtained by manipulating the relative phase between the analysis filters in such a way that the cross-correlation sequence between their impulse responses is anti-symmetric.

The practical virtue of orthogonality at zero lag can be seen by considering a random vector of subband variables $\mathbf{u}_N(n)$, as shown in Figure 3.2, in the following form

$$\mathbf{u}_N(n) \equiv [u_0(n), u_1(n), \dots, u_{N-1}(n)]^T. \quad (3.15)$$

Clearly, if the subband signals are orthogonal at zero lag, where $\gamma_{ip}(0) = E\{u_i(n)u_p(n)\} = 0$ for $i \neq p$, then the correlation matrix of the random vector $\mathbf{R}_N \equiv E\{\mathbf{u}_N(n)\mathbf{u}_N^T(n)\}$ is diagonal. That is to say that the random vector $\mathbf{u}_N(n)$ consists of uncorrelated components. In the simplest form, if $\mathbf{R}_N$ is diagonal, the system of equations $\mathbf{R}_N \mathbf{x} = \mathbf{y}$ is uncoupled. That is, the components $E\{u_i^2(n)\}x_i = y_i$, for $i = 0, 1, \dots, N-1$, do not depend on each other and thus they can be solved

Figure 3.2 An N -channel analysis filter. The random vector $\mathbf{u}_N(n)$ consists of the random variables $u_0(n)$, $u_1(n)$, ..., $u_{N-1}(n)$.

separately. Here, the variables x_i and y_i are elements of the vectors $\mathbf{x}$ and $\mathbf{y}$, respectively. This feature can be exploited to improve the efficiency of subband processing algorithms that involve explicit usage of the correlation matrix $\mathbf{R}_N$, or its estimates, see, for examples [61], [62], [25]. It should be emphasized that, as a direct consequence of (3.9), the orthogonality of the subband variables [where $E\{u_i(n)u_p(n)\}=0$] implies the orthogonality of the decimated subband variables [where $E\{u_{i,D}(n)u_{p,D}(n)\}=0$].

3.3.1 Paraunitary condition

In filter bank design, it is common that a paraunitary condition being imposed on the analysis filters in order to obtain a perfect reconstruction filter bank. Using the notion of the correlation-domain formulation of Figure 3.1, the paraunitary condition [73, pp. 150-152], [80, pp. 168] for a filter bank is given by

$$q_{ip}(lN) = \sum_{r=0}^{L-1} h_i(r) h_p(r-lN) = \delta(l) \delta(i-p), \quad (3.16)$$

where

$$\delta(l) = \begin{cases} 1, & \text{for } l = 0, \\ 0, & \text{for } l \neq 0, \end{cases} \quad (3.17)$$

denotes the unit sample sequence. Equation (3.16) indicates that the impulse responses of the paraunitary filter bank, $h_i(n)$ for $i = 0, 1, \dots, N-1$, as well as the lN samples shifted versions, $h_i(n-lN)$, are orthogonal (in a deterministic sense). Substituting $\gamma_{uu}(l) = \sigma_u^2 \delta(l)$ into (3.5) for the case where $u(n)$ is a white noise process with variance σ_u^2 , we arrive at

$$\gamma_{ip}(l) = q_{ip}(l) \otimes [\sigma_u^2 \delta(l)] = \sigma_u^2 q_{ip}(l). \quad (3.18)$$

From (3.16) and (3.18), it follows that the subband signals are orthogonal at zero lag, where $\gamma_{ip}(0) = \sigma_u^2 q_{ip}(0) = 0$ for $i \neq p$. Furthermore, the cross-correlation function $\gamma_{ip}(l)$ is characterized by periodic zero-crossing separated by N sampling periods, which is a direct consequence of the paraunitary condition (3.16). However, the result holds if and only if the input signal is white. For non-white excitation where $\gamma_{uu}(l) \neq \sigma_u^2 \delta(l)$, the cross-correlation function $\gamma_{ip}(l)$ is the convolution of the two sequences, $q_{ip}(l)$ and $\gamma_{uu}(l)$, as given in (3.5). The characteristics of $q_{ip}(l)$ induced by the paraunitary condition are generally not preserved in $\gamma_{ip}(l)$ due to the convolution operation. Therefore, the paraunitary condition is not sufficient for the orthogonality of subband signals, except when the input signal is white. For example, the discrete cosine transform (DCT) produces transformed variables that are orthogonal under white excitation because the DCT matrix is orthogonal (or paraunitary [125]). However, under colored excitation, the transformed variables are generally not orthogonal [25]. Orthogonality of a filter bank does not ensure the orthogonality of the subband signals.

The antisymmetry of the deterministic cross-correlation sequence $q_{ip}(l)$ leads to the orthogonality of the subbands. The paraunitary condition (3.16) only ensures that $q_{ip}(l)$ have periodic zero crossing, therefore does not result in the orthogonality at zero lag for arbitrary type of input spectrum. Nevertheless, it should be pointed that when the number of subbands N is sufficiently large and the stopband attenuation is high enough, the spectrum of the input signal could be assumed white over each subband. Under such situation, the paraunitary property imposed on $q_{ip}(l)$ would *approximately* be preserved in $\gamma_{ip}(l)$, thereby, allowing the outputs of the paraunitary

filter bank to be *approximately* orthogonal at zero lag. In the following discussion we shall confine our attention to the subband orthogonality property induced by the antisymmetry of $q_{ip}(l)$, which holds for arbitrary type of input spectrum and arbitrary number of subbands.

3.4 Subband Orthogonality of Cosine-Modulated Filter Banks

In an N -channel cosine-modulated filter bank, the analysis filters are cosine-modulated versions of a lowpass prototype filter $P(z)$ with a cutoff frequency at $\pi/2N$. The prototype filter $P(z)$ is generally restricted to be a linear phase filter with symmetric impulse response [9], [73], [89], [125], [105] so that $p(n) = p(L-1-n)$ and its frequency response can be written as

$$P(e^{j\omega}) = e^{-j\omega(L-1)/2} P_R(\omega), \quad (3.19)$$

where $P_R(\omega)$ is the real-valued zero-phase response (or amplitude response) of the prototype filter. Evaluating (2.31) along the unit circle and use (3.19) in the expression, the frequency response of the cosine-modulated filter $H_i(z)$ can be formulated in terms of the zero-phase response $P_R(\omega)$ in the following form

$$H_i(e^{j\omega}) = e^{-j\omega\left(\frac{L-1}{2}\right)} \left[e^{j\theta_i} U_i(\omega) + e^{-j\theta_i} V_i(\omega) \right], \quad (3.20)$$

where

$$U_i(\omega) = P_R\left[\omega - \frac{\pi}{N}(i+0.5)\right] \text{ and } V_i(\omega) = P_R\left[\omega + \frac{\pi}{N}(i+0.5)\right] \quad (3.21)$$

are the right- and left-shifted versions of the zero-phase response, respectively. Using (3.20) and (3.21) in (3.11), the cross-energy spectrum for the cosine-modulated filters can be written in terms of the shifted zero phase responses, as follows

$$\begin{aligned}
 Q_{ip}(e^{j\omega}) &= H_i(e^{j\omega})[H_p(e^{j\omega})]^* \\
 &= [e^{j\theta_i}U_i(\omega) + e^{-j\theta_i}V_i(\omega)][e^{-j\theta_p}U_p(\omega) + e^{j\theta_p}V_p(\omega)] \\
 &= e^{j(\theta_i-\theta_p)}U_i(\omega)U_p(\omega) + e^{-j(\theta_i-\theta_p)}V_i(\omega)V_p(\omega) \\
 &\quad + e^{j(\theta_i+\theta_p)}U_i(\omega)V_p(\omega) + e^{-j(\theta_i+\theta_p)}V_i(\omega)U_p(\omega).
 \end{aligned} \tag{3.22}$$

Now, consider the cross-energy spectrum between adjacent subbands $Q_{i,i+1}(e^{j\omega})$, where $p = i+1$ and $i = 0, 1, \dots, N-2$. With the phase alignment factor θ_i of the modulation function as indicated in (2.32), it can be easily seen that

$$\begin{aligned}
 \theta_i - \theta_{i+1} &= (-1)^i \frac{\pi}{4} - (-1)(-1)^i \frac{\pi}{4} = (-1)^i \frac{\pi}{2} \quad \text{and} \\
 \theta_i + \theta_{i+1} &= (-1)^i \frac{\pi}{4} + (-1)(-1)^i \frac{\pi}{4} = 0.
 \end{aligned} \tag{3.23}$$

Substituting these results into (3.22), the cross-energy spectrum between adjacent subbands can be written as

$$Q_{i,i+1}(e^{j\omega}) = S_{i,\text{even}}(e^{j\omega}) + jS_{i,\text{odd}}(e^{j\omega}), \tag{3.24}$$

where

$$\begin{aligned}
 S_{i,\text{even}}(e^{j\omega}) &= U_i(\omega)V_{i+1}(\omega) + V_i(\omega)U_{i+1}(\omega) \quad \text{and} \\
 S_{i,\text{odd}}(e^{j\omega}) &= (-1)^i [U_i(\omega)U_{i+1}(\omega) - V_i(\omega)V_{i+1}(\omega)]
 \end{aligned} \tag{3.25}$$

correspond to the symmetric and anti-symmetric components of $q_{i,i+1}(l)$, respectively.

That is, for a real sequence $q_{i,i+1}(l)$, its symmetric component transforms to $S_{i,\text{even}}(e^{j\omega})$, while its anti-symmetric component transforms to $jS_{i,\text{odd}}(e^{j\omega})$.

Figure 3.3(a) shows a typical prototype filter with stopband edge $\omega_s < \pi/N$. It can be noted from Figure 3.3(b) that the transition band of $U_i(\omega)$ extends to the frequency range of the adjacent $U_{i+1}(\omega)$. Similarly, significant overlap between $V_i(\omega)$ and $V_{i+1}(\omega)$ can be observed as well. Both of these overlaps (i.e., their

Figure 3.3 A typical prototype filter and shifted amplitude responses. (a) Amplitude response of the prototype filter. (b) Overlaps of the shifted amplitude responses form a significant amount of cross-energy spectrum.

product) form a significant amount of the anti-symmetric component $S_{i,\text{odd}}(e^{j\omega})$, as indicated by the second equation in (3.25). On the other hand, $U_i(\omega)$ and $V_{i+1}(\omega)$ do not overlap (assuming high stopband attenuation), $V_i(\omega)$ and $U_{i+1}(\omega)$ do not overlap either, which implies that the symmetric component $S_{i,\text{even}}(e^{j\omega})$ is negligible according to the first equation in (3.25).

Figure 3.4 illustrates the situation where the stopband edge ω_s of the prototype filter extends beyond π/N . Notice that, undesirable overlaps which contribute to the symmetric component $S_{i,\text{even}}(e^{j\omega})$ may occur at $i = 0$ and $i = N - 2$, as shown in Figure 3.4(b). However, it is customary to have the stopband edge ω_s of the prototype filter to be less than π/N , where the constraint

$$|P(e^{j\omega})| = 0, \text{ for } \omega > \pi/N, \quad (3.26)$$

is generally imposed on the prototype filter [9], [73], [89], [125], [105]. This constraint ensures that the spectral components from distant (i.e., non-adjacent) subbands is sufficiently attenuated, and at the same time, annihilates the situation illustrated in Figure 3.4. Consequently, the anti-symmetric component $S_{i,\text{odd}}(e^{j\omega})$ always dominates the cross-spectrum $Q_{i,i+1}(e^{j\omega})$. Thus, the cross-correlation sequence $q_{i,i+1}(l)$ can be assumed anti-symmetric, which in turn ensures that the adjacent subband signals of a cosine-modulated filter bank are always orthogonal at zero lag, regardless of the excitation signal.

Figure 3.4 Stopband edge ω_s of the prototype filter extends beyond π/N . (a) Amplitude response of the prototype filter. (b) The circled regions denoted undesirable overlaps that form the symmetric components $S_{0,\text{even}}(e^{j\omega})$ and $S_{N-1,\text{even}}(e^{j\omega})$.

3.5 Simulations

In this section, the second-order characteristics of an eight-channel pseudo-QMF cosine-modulated filter bank are examined. The prototype filter has a finite length of $L=128$ taps with its stopband edge at $\omega_s = \pi/8$, and is designed using the method proposed in [9]. The magnitude responses of all the cosine-modulated analysis filters are plotted in Figure 3.5. Notice that the analysis filters are normalized to have 0 dB in their passband. The excitation to the analysis filter bank is a colored signal generated by filtering a white Gaussian noise with an auto-regressive (AR) model of order $P=15$. The input spectrum, as plotted in Figure 3.6, is given by

$$\Gamma_{uu}(e^{j\omega}) = \frac{\sigma_w^2}{\left|1 + \sum_{l=1}^P a_l e^{-j\omega l}\right|^2}, \quad (3.27)$$

where $\sigma_w^2=1$ is the variance of the innovation process, and a_l are the coefficients of the AR model. The AR coefficients are taken as the linear-prediction-coding (LPC) coefficients of a 30 ms speech segment (sampled at 16 kHz) selected from the TIMIT speech corpus [123].

Figure 3.5 Magnitude responses of all the analysis filters in an eight-channel cosine-modulated filter bank.

Figure 3.6 Power spectrum of a colored noise generated by AR-filtering a white noise with 15 LPC coefficients.

As can be seen from Figure 3.5, spectral overlaps between non-adjacent filters are negligible due to high stopband attenuation. On the other hand, adjacent filters significantly overlap implying that considerable degree of correlation exists between them. The correlation between adjacent filters can be observed from their cross-correlation sequence $q_{i,i+1}(l)$ and the corresponding cross-energy spectrum $|Q_{i,i+1}(e^{j\omega})|$, as shown in Figure 3.7 (a) and (b), respectively. Notice that the cross-correlation sequence $q_{i,i+1}(l)$ is characterized by periodic zero-crossing separated by $N=8$ sampling periods. This characteristic implies that the pseudo-QMF design results in analysis filter bank that nearly satisfies the paraunitary condition (3.16). As mentioned earlier, the paraunitary condition leads to a perfect analysis\synthesis system.

It can be noted from Figure 3.7(b) that the adjacent filters are correlated at the frequencies where their passbands overlap. These frequency components appear in the subband signals, which lead to a high correlation between adjacent subband signals, as illustrated in Figure 3.7(c). From the power spectrum in Figure 3.7(d), it

Figure 3.7 Cross-correlation between adjacent bands for a randomly selected case $i=2$. (a) The deterministic cross-correlation sequence $q_{i,i+1}(l)$ between adjacent filters. (b) The cross-energy spectrum $|Q_{i,i+1}(e^{j\omega})| = |H_i(e^{j\omega})||H_{i+1}(e^{j\omega})|$. The dotted lines represent the magnitude responses $|H_i(e^{j\omega})|^2$ and $|H_{i+1}(e^{j\omega})|^2$. (c) The normalized cross-correlation sequence between adjacent subband signals. (d) The cross-power spectrum.

can be verified that the subband signals are correlated at the frequencies where the responses of the analysis filters overlap. The normalized cross-correlation sequence in Figure 3.7(c) is estimated from the subband samples $\{u_i(n), u_{i+1}(n)\}_{n=0}^{M-1}$, as follows

$$\hat{\gamma}_{i,i+1}(l) = \frac{1}{\sqrt{\sigma_i^2 \sigma_{i+1}^2}} \left[\frac{1}{M-|l|} \sum_{n=0}^{M-1} u_i(n) u_{i+1}(n-l) \right], \quad (3.28)$$

where M is the length of the data record used in the estimation, and σ_i^2 denotes the variance of the subband signal. We use a very long data record for the estimation since the subband signals are WSS. Notice that the purpose of normalization is to have $\hat{\gamma}_{i,i+1}(l) \leq 1$ so that the estimate is independent of signal scaling. We taper the cross-correlation estimate $\hat{\gamma}_{i,i+1}(l)$ with a rectangular window to retain only the cross-correlation coefficients for lags $|l| \leq L$, where $L = 150$ for current case. The magnitude spectrum $|\hat{\Gamma}_{i,i+1}(e^{j\omega})|$, as shown in Figure 3.7(d), is then obtained by taking the Fourier transform of the windowed cross-correlation estimate.

As pointed out earlier, the cross-energy spectrum $Q_{i,i+1}(e^{j\omega})$ can be decomposed into symmetric and anti-symmetric components, $S_{i,\text{even}}(e^{j\omega})$ and $S_{i,\text{odd}}(e^{j\omega})$, as shown in Figure 3.8. It should be emphasized that $S_{i,\text{even}}(e^{j\omega})$ and $S_{i,\text{odd}}(e^{j\omega})$ are real-valued functions of ω . As expected, the symmetric component is vanishing small at all frequency as compared to the anti-symmetric component. That is to say that the cross-energy spectrum $Q_{i,i+1}(e^{j\omega})$ is dominated by $S_{i,\text{odd}}(e^{j\omega})$, and thus the cross-correlation sequence $q_{i,i+1}(l)$ of Figure 3.7(a) can be assumed anti-symmetric. The anti-symmetry of $q_{i,i+1}(l)$ ensures that adjacent subband signals are nearly orthogonal at zero lag, i.e., $\gamma_{i,i+1}(0) \approx 0$, as depicted in Figure 3.7(c) for the colored excitation.

The absolute value of the normalized cross-correlation coefficients, $|\hat{\gamma}_{i,i+1}(0)|$ for $i = 0, 1, \dots, N-2$, are found to be smaller than 0.2×10^{-4} . Note that the cross-correlation may be large for other lags, where $l \neq 0$.

(a)

(b)

Figure 3.8 Decomposition of the cross-energy spectrum $\mathcal{Q}_{i,i+1}(e^{j\omega})$ between two adjacent filters into symmetric and anti-symmetric components, for a randomly selected case $i = 2$. (a) Absolute value of symmetric component $|S_{i,\text{even}}(e^{j\omega})|$. (b) Absolute value of the anti-symmetric component $|S_{i,\text{odd}}(e^{j\omega})|$.

3.6 Conclusions

This chapter investigated the second-order characteristics of multirate filter banks. A novel correlation-domain formulation has been presented. With this formulation, the effect of filtering a random signal with a filter bank can be conveniently described in terms of the effect of the system on the autocorrelation function of the input signal. It was shown that the outputs of a filter bank are orthogonal if they occupy non-overlapping portions of the input spectrum. Such a total separation of subbands may be desirable, but it is not achievable in practice because realizable filters have non-zero transition bandwidth and stopband gain. Furthermore, it may not be necessary in some applications where partial decorrelation is sufficient.

A special case of partial decorrelation was defined, whereby the subband signals are orthogonal at zero lag. It was shown that, if the deterministic cross-correlation sequence is anti-symmetric, the subband signals are orthogonal at zero lag. This feature of filter banks can be obtained by manipulating the relative phase between the analysis filters. In particular, it was found that orthogonality at zero lag can be approximated by cosine-modulating a high stopband attenuation prototype filter. In other words, the outputs of a cosine-modulated filter bank are nearly orthogonal at zero lag, for arbitrary type of input spectrum, as long as the stopband attenuation of the prototype filter is sufficiently high. This feature, which seems to have gone unnoticed in the literature, can be exploited to improve the computational efficiency and effectiveness of subband processing algorithms.

Chapter 4

Subband Adaptive Filtering Using

Constrained Subband Updates

Subband and multirate techniques [10], [11], [28], [73], [82], [105], [125] have been employed in designing computationally efficient adaptive filters with improved convergence performance against high eigenvalue disparity. One application of interest is acoustic echo cancellation [6], [22], [32], [39]-[41], [70], [109], which involves colored excitation and the modeling of a long impulse response. In conventional subband adaptive filter (SAF), each subband adapts a separate subfilter in its own adaptation loop [16], [27], [34]-[36], [55], [111]. The conventional structure can achieve some computational savings; however, detailed analysis shows that its convergence rate is limited by aliasing and band-edge effects [16], [19], [35], [36], [84]. To annihilate these structural problems, a new weight-control mechanism has been adopted in [13], [14], [61], [62], [77], [78], [99], whereby the modeling filter is no longer separated into subfilters. Instead, subband signals, which are normalized by their respective subband input variance, are used to adapt the fullband tap weights of the modeling filter.

This chapter begins with the formulation of a generic subband structure for the new weight-control mechanism described above. Based on the subband structure, the recursive equation of the weight-control mechanism is derived from a deterministic optimization criterion. The adaptive algorithm so devised is referred to as the normalized-SAF (NSAF) algorithm. In the second part, the proposed NSAF algorithm

is analyzed from stochastic and deterministic viewpoints in order to develop new insights into its underlying behavior in dealing with colored excitation. In particular, we show that the NSAF algorithm can be seen as a generalized form of the NLMS algorithm presented in Section 1.1.2.

4.1 Subband Adaptive Weight-Control Mechanism

Figure 4.1 shows a subband structure, where the desired response $d(n)$ and filter output $y(n)$ are partitioned into N subbands by means of analysis filters $H_0(z)$, $H_1(z), \dots, H_{N-1}(z)$. The subband signals, $d_i(n)$ and $y_i(n)$ for $i=0,1,\dots,N-1$, are critically decimated to a lower rate commensurate with their bandwidth. The sampling rate of the decimated sequences is thus N times slower than for the original sequences, thereby preserving the total effective sampling rate and the total number of

Figure 4.1 Subband structure showing the subband desired responses, subband filter outputs, and subband estimation errors, where the subband signals are used to adapt a fullband transversal filter $W(k, z)$.

samples to be processed. Notice that the variable n is used to index the original sequences, and k is used for the time index of the decimated sequences.

Figure 4.2 shows two equivalent structures for the selected portion in Figure 4.1. Assuming that the filter $W(k, z)$ is stationary (i.e., adaptation of its tap weights is frozen), it can be transposed to follow the analysis filter bank as depicted in Figure 4.2(a). Each subband signals occupies only a portion of the original frequency band. Therefore, the subband signals

$$y_i(n) = \sum_{m=0}^{M-1} w_m(k) u_i(n-m), \text{ for } i = 0, 1, \dots, N-1, \quad (4.1)$$

can be regarded as the responses of the transversal filter $W(k, z) = \sum_{m=0}^{M-1} w_m(k) z^{-m}$, of length M , to the input signal $u(n)$ in each of the N contiguous spectral bands. The

Figure 4.2 Two equivalent subband structures. (a) The input signal is split into subband signals before going through the filters $W(k, z)$ and decimators. (b) Sample-based filtering process followed by decimation is equivalent to block-based filtering process at a lower rate.

$N:1$ decimator retains only those samples of $y_i(n)$ that occur at instants of time equal to multiples of N . The decimated subband filter output can then be written as

$$\begin{aligned} y_{i,D}(k) &= y_i(kN) \\ &= \sum_{m=0}^{M-1} w_m(k) u_i(kN - m) \\ &= \mathbf{w}^T(k) \mathbf{u}_i(k), \end{aligned} \quad (4.2)$$

where

$$\begin{aligned} \mathbf{u}_i(k) &= [u_i(kN), u_i(kN-1), \dots, u_i(kN-N+1), \\ &\quad u_i(kN-N), \dots, u_i(kN-M+1)]^T \end{aligned} \quad (4.3)$$

denotes the regression vector for the i th subband, the vector $\mathbf{w}(k) = [w_0(k), w_1(k), \dots, w_{M-1}(k)]^T$ holds the tap weights of the modeling filter $W(k, z)$, and the superscript T denotes matrix transposition. Equations (4.2) and (4.3) indicate that, at every time instant k , each vector $\mathbf{u}_i(k)$ is packed with N new samples and $M-N$ old samples to produce a single sample of $y_{i,D}(k)$. The block-based operation is illustrated in Figure 4.2(b).

By employing the subband structure of Figure 4.2(a) in Figure 4.1, we arrive at the subband structure of Figure 4.3. Notice that the estimation error $e_{i,D}(k)$ measures how far the filter output $y_{i,D}(k)$ is from the desired response $d_{i,D}(k)$ in each of the N spectral bands, at the decimated rate:

$$e_{i,D}(k) = d_{i,D}(k) - \mathbf{w}^T(k) \mathbf{u}_i(k) \quad \text{for } i = 0, 1, \dots, N-1. \quad (4.4)$$

The estimation error in all the N subbands, $\mathbf{e}_D(k) \equiv [e_{0,D}(k), e_{1,D}(k), \dots, e_{N-1,D}(k)]^T$, can be expressed in a compact form as

$$\mathbf{e}_D(k) = \mathbf{d}_D(k) - \mathbf{U}^T(k) \mathbf{w}(k), \quad (4.5)$$

where the $M \times N$ subband data matrix $\mathbf{U}(k)$ and the $N \times 1$ desired response vector $\mathbf{d}_D(k)$ are defined, respectively, as

$$\mathbf{U}(k) \equiv [\mathbf{u}_0(k), \mathbf{u}_1(k), \dots, \mathbf{u}_{N-1}(k)], \text{ and} \quad (4.6)$$

$$\mathbf{d}_D(k) \equiv [d_{0,D}(k), d_{1,D}(k), \dots, d_{N-1,D}(k)]^T. \quad (4.7)$$

In Figure 4.3, the diagonalized N -input N -output system $W(k, z)\mathbf{I}_{N \times N}$ is basically a bank of parallel filters with identical transfer function $W(k, z)$, where $\mathbf{I}_{N \times N}$ denotes the $N \times N$ identity matrix. These N copies of the adaptive transversal filter are placed before the decimators, which is different from that of the conventional SAF [16], [36], [55], [111], where a set of subfilters are placed after the decimators. In such a configuration, the modeling filter (i.e., the adaptive transversal filter from a system identification perspective) $W(k, z)$ operates on the set of bandlimited input

Figure 4.3 A new subband structure for adaptive filtering. Error signals estimated in subbands are used to adapt the fullband tap-weights of the modeling filter $W(k, z)$.

signals, $u_i(n)$, at the original sampling rate, while the fullband tap weights of the filter, $\{w_n(k)\}_{n=0}^{M-1}$, being iteratively updated by means of the subband data $\{\mathbf{U}(k), \mathbf{e}_D(k)\}$ at the decimated rate.

The weights $\{w_n(k)\}_{n=0}^{M-1}$ are considered as fullband because the subband signals $u_i(n)$ actuating the tap-weight adaptation collectively cover contiguous spectral bands of the original frequency bandwidth. The fullband nature of the modeling filter $W(k, z)$ is also evident from Figure 4.1. On the other hand, by decimating the tap-weight adaptation rate, responses of the modeling filter $W(k, z)$ to the N subband input signals, $u_i(n)$ for $i=0, 1, \dots, N-1$, are evaluated at $1/N$ th of the input sampling rate, as indicated by (4.2). Thereby, the computational complexity remains unchanged even though there are N replicas of $W(k, z)$ in the subband structure of Figure 4.3, as opposed to a single $W(k, z)$ in Figure 4.1. An efficient implementation for the transversal filter with a critically decimated output is described in Appendix C.

The SAFs reported in [13], [14], [99] are different approaches to and embellishments of the basic forms depicted in Figure 4.3, which we have proposed earlier in [61], [62]. In [99], the modeling filter $W(k, z)$ is decomposed into polyphase components, whereas in [13], [14], a delayless structure is employed. Nevertheless, the basic concept remains the same: error signals are estimated in subbands, whereas the coefficients that explicitly adapted are the fullband tap weights of a modeling filter. The polyphase implementation of the new SAF structure is presented in Appendix C. We shall look into the delayless implementation in Chapter 6. It should also be emphasized that the adaptive algorithm proposed in [61], [62] is derived from a deterministic criterion based on the *principle of minimal disturbance* [44], [129],

whereas the adaptive algorithms in [13], [14], [99] are derived as stochastic-gradient algorithms minimizing a weighted mean-square-error (MSE) function.

4.2 Multiple-Constraint Optimization Criterion

4.2.1 Principle of minimal disturbance

The *principle of minimal disturbance* [44, pp. 321], [129] states that: from one iteration to the next, coefficients of an adaptive filter should be changed in a minimal manner, subject to a (set of) constraint(s) imposed on the updated filter output. In [44], the principle of minimal disturbance is used to formulate a constraint optimization criterion for deriving the NLMS algorithm.

For the subband structure of Figure 4.3 with the accessible subband quantities $\{\mathbf{U}(k), \mathbf{d}_D(k)\}$, a criterion that would ensure a convergence to the optimum solution $\mathbf{w}_o$ after sufficient number of iterations, is to have the updated weight vector $\mathbf{w}(k+1)$ nulling the a posteriori errors in all the N subbands

$$\begin{aligned}\xi_{0,D}(k) &= \mathbf{w}^T(k+1) \mathbf{u}_0(k) - d_{0,D}(k) = 0, \\ \xi_{1,D}(k) &= \mathbf{w}^T(k+1) \mathbf{u}_1(k) - d_{1,D}(k) = 0, \\ &\vdots \\ \xi_{N-1,D}(k) &= \mathbf{w}^T(k+1) \mathbf{u}_{N-1}(k) - d_{N-1,D}(k) = 0,\end{aligned}\tag{4.8}$$

at each iteration k . On the other hand, by adjusting the tap-weights in a minimal fashion, the effects of the noise $\eta(n)$ on the tap-weight adaptation can be minimized, while driving the modeling filter converges to the optimum $\mathbf{w}_o$. The term $\eta(n)$, as shown in Figure 4.1, accounts for measurement noise, unmodeled dynamics, and other kind of noise within the system.

4.2.2 Constrained subband updates

Based on the *principle of minimal disturbance*, we formulate a multiple-constraint optimization problem as follows:

Minimize the squared Euclidean norm of the change in the tap-weight vector

$$f[\mathbf{w}(k+1)] = \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2, \quad (4.9)$$

subject to the set of N subband constraints imposed on the filter output

$$d_{i,D}(k) = \mathbf{w}^T(k+1)\mathbf{u}_i(k) \text{ for } i = 0, 1, \dots, N-1. \quad (4.10)$$

The constrained optimization problem described above can be converted into one of unconstrained optimization by means of the method of Lagrange multipliers [44], [74]. Specifically, the quadratic function of (4.9) are combined with the multiple constraints in (4.10) to form the Lagrangian function, as follows

$$J[\mathbf{w}(k+1), \lambda'_0, \dots, \lambda'_{N-1}] = f[\mathbf{w}(k+1)] + \sum_{i=0}^{N-1} \lambda'_i [d_{i,D}(k) - \mathbf{w}^T(k+1)\mathbf{u}_i(k)], \quad (4.11)$$

where λ'_i are the Lagrange multipliers pertaining to the multiple constraints. Taking the derivative of the Lagrangian function with respect to the tap-weight vector $\mathbf{w}(k+1)$, and then setting this derivative equal to zero, we get

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{1}{2} \sum_{i=0}^{N-1} \lambda'_i \mathbf{u}_i(k). \quad (4.12)$$

Using (4.12) and (4.4) in (4.10), as follows

$$\begin{aligned} d_{i,D}(k) - \left[\mathbf{w}(k) + \frac{1}{2} \sum_{j=0}^{N-1} \lambda'_j \mathbf{u}_j(k) \right]^T \mathbf{u}_i(k) &= 0 \\ 2[d_{i,D}(k) - \mathbf{w}^T(k)\mathbf{u}_i(k)] - \mathbf{u}_i^T(k) \sum_{j=0}^{N-1} \lambda'_j \mathbf{u}_j(k) &= 0 \\ 2e_{i,D}(k) - \mathbf{u}_i^T(k) \sum_{j=0}^{N-1} \lambda'_j \mathbf{u}_j(k) &= 0 \end{aligned}$$

leads to a linear system of N equations:

$$\mathbf{u}_i^T(k) \sum_{j=0}^{N-1} \lambda'_j \mathbf{u}_j(k) = 2e_{i,D}(k) \text{ for } i = 0, 1, \dots, N-1. \quad (4.13)$$

Using the notations defined in (4.5) and (4.6), the N linear equations in (4.13) can be formulated into a matrix form:

$$\begin{bmatrix} \mathbf{u}_0^T(k) \mathbf{u}_0(k) & \cdots & \mathbf{u}_0^T(k) \mathbf{u}_{N-1}(k) \\ \vdots & \ddots & \vdots \\ \mathbf{u}_{N-1}^T(k) \mathbf{u}_0(k) & \cdots & \mathbf{u}_{N-1}^T(k) \mathbf{u}_{N-1}(k) \end{bmatrix} \begin{bmatrix} \lambda'_0 \\ \vdots \\ \lambda'_{N-1} \end{bmatrix} = 2 \begin{bmatrix} e_{0,D}(k) \\ \vdots \\ e_{N-1,D}(k) \end{bmatrix} \quad (4.14)$$

$$[\mathbf{U}^T(k) \mathbf{U}(k)] \boldsymbol{\lambda} = 2\mathbf{e}_D(k),$$

where $\boldsymbol{\lambda} = [\lambda'_0, \lambda'_1, \dots, \lambda'_{N-1}]^T$ denotes the $N \times 1$ Lagrange vector. Solving the linear system for the Lagrange multipliers, we arrive at

$$\boldsymbol{\lambda} = 2[\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_D(k). \quad (4.15)$$

Assuming that the subband signals are orthogonal at zero lag, the off-diagonal elements of the matrix $\mathbf{U}^T(k) \mathbf{U}(k)$ are negligible (will be described in Section 4.5).

With this diagonal assumption, (4.15) essentially reduces to a simple form

$$\lambda'_i = 2 \frac{e_{i,D}(k)}{\|\mathbf{u}_i(k)\|^2} \text{ for } i = 0, 1, \dots, N-1. \quad (4.16)$$

Combining the results in (4.12) and (4.16), a recursive relation for updating the tap-weight vector can be obtained, as follows

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|^2 + \alpha} e_{i,D}(k), \quad (4.17)$$

where μ is a positive step-size parameter to exercise control over the change in the tap-weight vector, and α is a small positive constant to avoid possible division by zero. The choice of step size is discussed in Chapter 5. Using the notations defined in (4.5) and (4.6), the recursion (4.17) can be expressed more compactly as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \mathbf{U}(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k), \quad (4.18)$$

where

$$\mathbf{\Lambda}(k) = \text{diag}[\mathbf{U}^T(k) \mathbf{U}(k) + \alpha \mathbf{I}] \quad (4.19)$$

denotes a diagonal matrix of the normalization factors, $\lambda_i(k) = \|\mathbf{u}_i(k)\|^2 + \alpha$. The adaptive algorithm so devised is referred to as the normalized SAF (NSAF) algorithm, in order to highlight the presence of the normalization factors. The simplest interpretation of (4.17), or (4.18), is that it is a way of making the fullband tap-weight vector $\mathbf{w}(k)$ converges equally for all spectral bands by normalizing the subband regressors $\mathbf{u}_i(k)$ with $\lambda_i(k) = \|\mathbf{u}_i(k)\|^2 = M \hat{\gamma}_{ii}(0)$, where $\hat{\gamma}_{ii}(0) = \mathbf{u}_i^T(k) \mathbf{u}_i(k) / M$ is the running-average estimate of the subband variance $\gamma_{ii}(0) = E\{u_i^2(n)\}$.

The constrained optimization problem defined above involves N equality constraints. The number of constraints N (i.e., number of subbands) must be smaller than the number of unknowns M (i.e., the length of the adaptive tap-weight vector) for the minimum norm solution to exist. Hence, this requirement sets an upper limit on the number of allowable subbands in the NSAF algorithm. In the application of adaptive echo cancellation, in which the filter dimension M is considerably large ($M = 100 \sim 2000$), the condition is not particularly limiting.

4.2.3 Computational complexity

The NSAF algorithm is summarized in Table 4.1, along with the computational cost of the algorithm in terms of number of multiplications per sampling period T_s . Since the adaptation process is performed at the decimated rate $1/NT_s$, the number of multiplications incurred at each iteration is divided by N . The NSAF algorithm requires $3M+2$ multiplications for the adaptation process, and $(N+2)L$ multiplications for the analysis and synthesis filter banks, which together represent an overall computational complexity of $O(M)+O(NL)$. In Table 4.1, the analysis and synthesis filter banks are represented as $\mathbf{H}$ and $\mathbf{F}$, respectively, and they are implemented as block transforms with memory. The columns of the matrices, $\mathbf{h}_i = [h_i(0), h_i(1), \dots, h_i(L-1)]^T$ and $\mathbf{f}_i = [f_i(0), f_i(1), \dots, f_i(L-1)]^T$, represent the analysis and synthesis filters, respectively. A direct implementation of the filter banks is possible but would require $3NL$ multiplications per sampling period T_s , as shown in [62].

The tap-weight adaptation equation (4.17) is in a simple form comparable to that of the NLMS algorithm (1.11). In the NSAF algorithm, the fullband tap-weight vector $\mathbf{w}(k)$ is adapted by means of N normalized subband regressors $\mathbf{u}_i(k)$, instead of a single fullband regressor $\mathbf{u}(n)$ in the NLMS algorithm. By virtue of critical sub-sampling, the tap-weight vector is adapted at a decimated rate. Thus, the number of multiplications incurred for the tap-weight adaptation is always $3M+2$ for an arbitrary number of subbands N . Compared to the NLMS algorithm, the NSAF algorithm requires additional $(N+2)L$ multiplications for the filter banks. In acoustic echo cancellation applications, M is typically more than a few hundreds taps,

which is much larger than the NL product in most situations. Clearly, the NSAF algorithm exploits the properties of the subband signals to accelerate the convergence, as we shall see in the next section, while remain computationally efficient by means of a multirate structure.

Table 4.1 Summary of the NSAF algorithm.

Computation	Multiplications/ T_s
For $k=0,1,2,\dots$, where $kN=n$,	
<i>Error estimation:</i> $\mathbf{e}_D(k) = \mathbf{d}_D(k) - \mathbf{U}^T(k) \mathbf{w}(k)$	$\frac{N \times M}{N} = M$
<i>Normalization matrix:</i> $\mathbf{\Lambda}(k) = \text{diag}[\mathbf{U}^T(k) \mathbf{U}(k) + \alpha \mathbf{I}]$	$\frac{N \times M}{N} = M$
<i>Tap-weight adaptation:</i> $\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \mathbf{U}(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k)$	$\frac{2N \times NM}{N} = M + 2$
<i>Band-partitioning:</i> $\mathbf{U}_1^T(k) = \mathbf{H}^T \mathbf{A}(kN)$ $\mathbf{d}_D(k) = \mathbf{H}^T \mathbf{d}(kN)$	$\frac{N^2 L + NL}{N} = (N+1)L$
<i>Synthesizing:</i> $\mathbf{e}(kN) = \mathbf{F} \mathbf{e}_D(k)$	$\frac{N \times L}{N} = L$
<i>Parameters:</i>	
M number of adaptive tap weights	
N number of subbands	
μ step-size parameter	
L length of the analysis and synthesis filters	
α small positive constant	
<i>Variables:</i>	
$\mathbf{U}^T(k) = [\mathbf{U}_1^T(k), \mathbf{U}_2^T(k-1)]$	
$\mathbf{U}_2^T(k-1)$ = first $M-N$ columns of $\mathbf{U}^T(k-1)$	
$\mathbf{A}(kN) = [\mathbf{a}(kN), \mathbf{a}(kN-1), \dots, \mathbf{a}(kN-N+1)]$	
$\mathbf{a}(kN) = [u(kN), u(kN-1), \dots, u(kN-L+1)]^T$	
$\mathbf{d}(kN) = [d(kN), d(kN-1), \dots, d(kN-L+1)]^T$	
$\mathbf{H} = [\mathbf{h}_0, \mathbf{h}_1, \dots, \mathbf{h}_{N-1}]$ analysis filter bank	
$\mathbf{F} = [\mathbf{f}_0, \mathbf{f}_1, \dots, \mathbf{f}_{N-1}]$ synthesis filter bank	

4.3 Stochastic Interpretations

In this section, the NSAF algorithm is motivated from two stochastic viewpoints, namely, a diagonalized Newton's method and error signal equalization, which enable us to look at different facets of the algorithm's behavior. More importantly, the NSAF algorithm can be shown to possess some fundamental decorrelating properties that whiten the input samples prior to the tap-weight adaptation.

4.3.1 Stochastic approximation to Newton's method

Newton's method is an iterative optimization procedure, which can be cast for the quadratic case [18], [27], [44], [128] as follows

$$\mathbf{w}(l+1) = \mathbf{w}(l) + \mu[\mathbf{R} + \alpha'\mathbf{I}]^{-1}[\mathbf{p} - \mathbf{R}\mathbf{w}(l)], \quad (4.20)$$

where the correlation matrix $\mathbf{R}$ and cross-correlation vector $\mathbf{p}$ are signal statistics characterizing the mean-square-error (MSE) function $J = E\{e^2(n)\}$ to be minimized; the index l denotes the step in the iterative process, and $\alpha' = \alpha/N$ is the regularization parameter. The regularized Newton's recursion in (4.20) is better known as Levenberg-Marquardt method in the literature [37], [107]. To develop an adaptive algorithm for the subband structure of Figure 4.3 from (4.20), we replace the ensemble averages, $\mathbf{R}$ and $\mathbf{p}$, by their estimates in terms of $\{\mathbf{U}(k), \mathbf{d}_D(k)\}$ as follows:

$$\hat{\mathbf{R}}(k) = \frac{1}{N} \sum_{i=0}^{N-1} \mathbf{u}_i(k) \mathbf{u}_i^T(k) = \frac{1}{N} \mathbf{U}(k) \mathbf{U}^T(k) \text{ and} \quad (4.21)$$

$$\hat{\mathbf{p}}(k) = \frac{1}{N} \sum_{i=0}^{N-1} \mathbf{u}_i(k) d_{i,D}(k) = \frac{1}{N} \mathbf{U}(k) \mathbf{d}_D(k). \quad (4.22)$$

The estimates are unbiased if the filter bank is power complementary, as we shall see later in Section 4.5. Substituting (4.21) and (4.22) into (4.20) for $\mathbf{R}$ and $\mathbf{p}$, and using (4.5) in the expression, we obtain

$$\begin{aligned}\mathbf{w}(k+1) &= \mathbf{w}(k) + \mu \left[\frac{1}{N} \mathbf{U}(k) \mathbf{U}^T(k) + \alpha \mathbf{I} \right]^{-1} \\ &\quad \times \left[\frac{1}{N} \mathbf{U}(k) \mathbf{d}_D(k) - \frac{1}{N} \mathbf{U}(k) \mathbf{U}^T(k) \mathbf{w}(k) \right] \\ &= \mathbf{w}(k) + \mu \left[\mathbf{U}(k) \mathbf{U}^T(k) + N\alpha \mathbf{I} \right]^{-1} \mathbf{U}(k) [\mathbf{d}_D(k) - \mathbf{U}^T(k) \mathbf{w}(k)] \\ &= \mathbf{w}(k) + \mu \left[\mathbf{U}(k) \mathbf{U}^T(k) + \alpha \mathbf{I} \right]^{-1} \mathbf{U}(k) \mathbf{e}_D(k).\end{aligned}\tag{4.23}$$

Note that the iteration index l has been replaced by the time index k so that the weight recursion coincides with the time update of the estimates, $\hat{\mathbf{R}}(k)$ and $\hat{\mathbf{p}}(k)$.

The recursion in (4.23) requires the inversion of the M -by- M matrix $[\mathbf{U}(k) \mathbf{U}^T(k) + \alpha \mathbf{I}]$ at each iteration, which can be simplified by invoking the matrix inversion lemma [44, 107]:

$$[\mathbf{A} + \mathbf{BCD}]^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1} \mathbf{B} [\mathbf{C}^{-1} + \mathbf{D} \mathbf{A}^{-1} \mathbf{B}]^{-1} \mathbf{D} \mathbf{A}^{-1}.\tag{4.24}$$

Let $\mathbf{A} = \alpha \mathbf{I}$, $\mathbf{B} = \mathbf{U}(k)$, $\mathbf{C} = \mathbf{I}$, and $\mathbf{D} = \mathbf{U}^T(k)$, the matrix inversion can be obtained as follows

$$\begin{aligned}[\mathbf{U}(k) \mathbf{U}^T(k) + \alpha \mathbf{I}]^{-1} &= \alpha^{-1} \mathbf{I} - \alpha^{-1} \mathbf{U}(k) [\mathbf{I} + \mathbf{U}^T(k) \alpha^{-1} \mathbf{U}(k)]^{-1} \mathbf{U}^T(k) \alpha^{-1} \mathbf{I} \\ &= \alpha^{-1} \mathbf{I} - \alpha^{-1} \mathbf{U}(k) [\alpha \mathbf{I} + \mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{U}^T(k).\end{aligned}\tag{4.25}$$

Multiply both sides of (4.25) by $\mathbf{U}(k)$ from the right

$$\begin{aligned}&[\mathbf{U}(k) \mathbf{U}^T(k) + \alpha \mathbf{I}]^{-1} \mathbf{U}(k) \\ &= \alpha^{-1} \mathbf{U}(k) - \alpha^{-1} \mathbf{U}(k) [\alpha \mathbf{I} + \mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{U}^T(k) \mathbf{U}(k) \\ &= \alpha^{-1} \mathbf{U}(k) \left\{ \mathbf{I} - [\alpha \mathbf{I} + \mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{U}^T(k) \mathbf{U}(k) \right\} \\ &= \alpha^{-1} \mathbf{U}(k) [\alpha \mathbf{I} + \mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \{ \alpha \mathbf{I} + \mathbf{U}^T(k) \mathbf{U}(k) - \mathbf{U}^T(k) \mathbf{U}(k) \} \\ &= \mathbf{U}(k) [\mathbf{U}^T(k) \mathbf{U}(k) + \alpha \mathbf{I}]^{-1}.\end{aligned}\tag{4.26}$$

Using this result in (4.23) leads to a mathematically equivalent form for the recursive equation

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \mathbf{U}(k) [\mathbf{U}^T(k) \mathbf{U}(k) + \alpha \mathbf{I}]^{-1} \mathbf{e}_D(k). \quad (4.27)$$

The recursion in (4.27) is preferable to that in (4.23) because it involves the inversion of a lower dimension $N \times N$ matrix $[\mathbf{U}^T(k) \mathbf{U}(k) + \alpha \mathbf{I}]$, where $N \ll M$ for high-order adaptive filters. Furthermore, by virtue of the diagonal assumption, the off-diagonal elements of the matrix $[\mathbf{U}^T(k) \mathbf{U}(k) + \alpha \mathbf{I}]$ can be assumed negligible, in which case the tap-weight adaptation equation can ideally be further reduced to a simple form:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \mathbf{u}_i(k) [\mathbf{u}_i^T(k) \mathbf{u}_i(k) + \alpha]^{-1} e_{i,D}(k). \quad (4.28)$$

Interestingly enough, the tap-weight adaptation equation of the NSAF algorithm in (4.17) is exactly identical to (4.28). The difference is that, here, the weight recursion is obtained from a stochastic approach as an approximation to the Newton's method, as summarized in Figure 4.4(a).

It should be emphasized that a power complementary filter bank preserves the second-order statistics of the input signal, $u(n)$, and the desired response, $d(n)$, giving rise to the unbiased estimates $\hat{\mathbf{R}}(k)$ and $\hat{\mathbf{p}}(k)$ in (4.21) and (4.22), respectively. The use of these unbiased estimates in the recursive estimation allows the algorithm to converge (with noise superposed) to the optimal Wiener solution, $\mathbf{w}_o = \mathbf{R}^{-1} \mathbf{p}$.

Figure 4.4 Stochastic interpretations of the NSAF algorithm. (a) Stochastic approximation to Newton's method in minimizing the classical MSE function. (b) Stochastic approximation to a steepest-descent algorithm in minimizing a weighted MSE function.

4.3.2 Weighted criterion

In their papers, de Courville and Duhamel [13], [14], Pradhan and Reddy [99] proposed an innovative weighted MSE function defined as the weighted sum of the mean-squared value of the subband estimation errors, as follows

$$J_\xi = \sum_{i=0}^{N-1} \lambda_i^{-1} E\{e_{i,D}^2(k)\}, \quad (4.29)$$

where λ_i^{-1} denote the weighting factors (i.e., the reciprocal of the normalization factors λ_i defined earlier). Starting from a steepest-descent algorithm minimizing the weighted MSE function, then taking the instantaneous estimates of the ensemble averages that appear in the recursion, and finally by setting the weighting factors λ_i^{-1} to $[\|\mathbf{u}_i(k)\|^2 + \alpha]^{-1}$, the NSAF algorithm in (4.17) can be derived. However, different from that in Section 4.3.1, the current approach formulates the NSAF algorithm as a

stochastic approximation to the steepest-descent method, as summarized in Figure 4.4(b). The weighting factors λ_i^{-1} are proportional to the inverse of the variance of the subband input signals. Hence, the weighted MSE function gives higher weight to the error corresponding to the subband with lower signal power. The weighting scheme can be seen as a decorrelating procedure, as shown below. A brief discussion on the similarities and the differences of the NSAF algorithm and those in [13], [14], and [99] is presented in Section 4.1.

In the following analysis, we assume that all signals are wide-sense stationary (WSS), and the modeling filter $W(k, z)$ is time-invariant (i.e., the tap-weight adaptation is frozen). The estimation error $e_{i,D}(k)$ measures the difference between the desired response $d_{i,D}(k)$ and the filter output $y_{i,D}(k)$ at the decimated rate, as defined in (4.4), and as illustrated in Figure 4.5(a). Since the addition and decimation operations are commutative (as explained in Section 2.1), the estimation error can be alternatively obtained as $e_{i,D}(k) = e_i(kN)$, as shown in Figure 4.5(b), where

$$\begin{aligned} e_i(n) &= d_i(n) - \sum_{m=0}^{M-1} w_m u_i(n-m) \\ &= d_i(n) - w_n \otimes u_i(n) \end{aligned} \quad (4.30)$$

is the subband (i.e., bandlimited) estimation error at the original sampling rate. Notice that the iteration index k has been dropped from the tap weights, $\{w_n(k)\}_{n=0}^{M-1}$, as we assume that the tap-weight adaptation is frozen. By taking the analysis filter into consideration, the subband estimation error $e_i(n)$ in (4.30) can be expressed in terms of a fullband estimation error $e(n)$, as follows

$$\begin{aligned} e_i(n) &= h_i(n) \otimes d(n) - w_n \otimes [h_i(n) \otimes u(n)] \\ &= h_i(n) \otimes [d(n) - w_n \otimes u(n)] \\ &= h_i(n) \otimes e(n), \end{aligned} \quad (4.31)$$

Figure 4.5 Analysis of the weighted MSE function, J_ξ , assuming that all signals and systems are stationary and time-invariant.

where $h_i(n)$ is the impulse response of the i th analysis filter. We have applied the associative, commutative, and distributive law of the LTI system in the previous step. Equation (4.31) implies that the subband estimation errors $e_i(n)$ can be obtained by band-partitioning the fullband error signal $e(n)$, as illustrated in Figure 4.5(c).

Applying the correlation domain formulation (as described in Section 3.1) on (4.31), the autocorrelation function $g_i(l) \equiv E\{e_i(n)e_i(n-l)\}$ of the subband estimation error $e_i(n)$ can be written as

$$g_i(l) = h_i(l) \otimes h_i(-l) \otimes g_e(l), \text{ for } i = 0, 1, \dots, N-1, \quad (4.32)$$

where $g_e(l) \equiv E\{e(n)e(n-l)\}$ denotes the autocorrelation function of the fullband estimation error $e(n)$. Next, define a weighted sum of subband autocorrelation functions $g_i(l)$ in the following form

$$g_\xi(l) \equiv \sum_{i=0}^{N-1} \lambda_i^{-1} g_i(l) = \sum_{i=0}^{N-1} \lambda_i^{-1} E\{e_i(n)e_i(n-l)\}. \quad (4.33)$$

Substituting (4.32) into (4.33), we arrive at

$$\begin{aligned} g_\xi(l) &= \sum_{i=0}^{N-1} \lambda_i^{-1} [h_i(l) \otimes h_i(-l) \otimes g_e(l)] \\ &= \left[\sum_{i=0}^{N-1} \lambda_i^{-1} h_i(l) \otimes h_i(-l) \right] \otimes g_e(l) \\ &= [h_\xi(l) \otimes h_\xi(-l)] \otimes g_e(l). \end{aligned} \quad (4.34)$$

The resulting $h_\xi(n)$ can be regarded as an equalizing filter with the autocorrelation sequence of its impulse response given by

$$h_\xi(n) \otimes h_\xi(-n) = \sum_{i=0}^{N-1} \lambda_i^{-1} [h_i(n) \otimes h_i(-n)], \quad (4.35)$$

or, equivalently in the frequency domain, by taking the Fourier transform of (4.35), as

$$|H_\xi(e^{j\omega})|^2 = \sum_{i=0}^{N-1} \lambda_i^{-1} |H_i(e^{j\omega})|^2, \quad (4.36)$$

where $|H_i(e^{j\omega})|$ denotes the magnitude response of the i th analysis filter. As shown in Figure 4.6(a), the filter $h_\xi(n)$ operates on the fullband error signal $e(n)$ giving rise to a preconditioned error signal, $\xi(n) = h_\xi(n) \otimes e(n)$. The autocorrelation of $\xi(n)$ can be written in the following form

$$\begin{aligned} E\{\xi(n)\xi(n-l)\} &= [h_\xi(l) \otimes h_\xi(-l)] \otimes g_e(l) \\ &= g_\xi(l). \end{aligned} \quad (4.37)$$

Figure 4.6 (a) Minimizing the weighted MSE function is equivalent to minimizing an equalized error function. (b) An equivalent structure obtained from (a) by moving $h_{\xi}(n)$ into the branches and also across the modeling filter $\mathbf{w}$.

Let $l = 0$ in (4.33) and (4.37), we obtain

$$\sum_{i=0}^{N-1} \lambda_i^{-1} E\{e_i^2(n)\} = E\{\xi^2(n)\}. \quad (4.38)$$

Knowing that the variance of the subband signal and its decimated version are equivalent, we have

$$E\{e_{i,D}^2(k)\} = E\{e_i^2(kN)\} \text{ and } E\{\xi_D^2(k)\} = E\{\xi^2(kN)\}, \quad (4.39)$$

where $\xi_D(k) = \xi(kN)$ is the decimated version of the preconditioned error signal $\xi(n)$. Now, reworking (4.29) by using (4.38) and (4.39), we arrive at an equivalent representation for the weighted MSE function, as follows

$$J_{\xi} = \sum_{i=0}^{N-1} \lambda_i^{-1} E \{ e_i^2(kN) \} = E \{ \xi^2(kN) \} = E \{ \xi_D^2(k) \}. \quad (4.40)$$

Clearly, minimizing the weighted MSE function (4.29) is equivalent to minimizing a preconditioned error function (4.40). The idea is illustrated in Figure 4.6(a). Since we assume that the input signal is WSS and the tap-weight adaptation is frozen, the filter $h_{\xi}(n)$ can be moved into the branches and across the modeling filter $W(k, z)$, as shown in Figure 4.6(b). In that respect, the filter $h_{\xi}(n)$ can be regarded as an equalizing filter (i.e., an estimate of the inverse of the coloring filter of the input signal) that flattens the input spectrum $\Gamma_{uu}(e^{j\omega})$ in order to reduce its spectral dynamic range. Using (4.36) in Figure 4.6(b), the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$ can be written in the following form

$$\begin{aligned} \Gamma_{\xi}(e^{j\omega}) &= |H_{\xi}(e^{j\omega})|^2 \Gamma_{uu}(e^{j\omega}) \\ &= \sum_{i=0}^{N-1} \lambda_i^{-1} |H_i(e^{j\omega})|^2 \Gamma_{uu}(e^{j\omega}). \end{aligned} \quad (4.41)$$

For random signals, equalization can be interpreted as whitening, thereby decorrelating the input samples. Thus, the subband decomposition and normalization features in (4.29) essentially form a decorrelation filter $h_{\xi}(n)$ that whitens the input signal before passing it to the adaptive algorithm. The idea is somewhat similar to that of pre-whiten type of adaptive algorithms [6, pp. 50-53], [29], [39], [40], but in our case, the decorrelation filter is embedded in the algorithm forming a self-designing decorrelation mechanism.

4.3.3 Decorrelating properties

From both stochastic interpretations, it follows that the NSAF algorithm possesses some fundamental decorrelating properties. At one end, the decorrelating properties manifest themselves as an orthogonalization matrix, namely, the matrix $[\mathbf{U}(k)\mathbf{U}^T(k) + \alpha\mathbf{I}]^{-1}$ in (4.23), which forces the update direction to point to the minimum $\mathbf{w}_o$ of the underlying error performance surface at each iteration. At the other end, the decorrelation properties appear in a form of decorrelation filter $h_\xi(n)$ that whitens the input prior to tap-weight adaptation. In both cases, the underlying principle remains the same. The NSAF algorithm decorrelates the input signal and follows the steepest-descent path on the preconditioned error-performance surface (4.40).

The effectiveness of the inherent decorrelation filter $h_\xi(n)$ in dealing with colored excitation greatly depends on the characteristics of the analysis filters $h_0(n), \dots, h_{N-1}(n)$, which form the basis in constructing $h_\xi(n) \otimes h_\xi(-n)$, as indicated by (4.35). Taking a closer look at Figure 4.4, it can be easily seen that the steepest-descent-based algorithm translates into a Newton-based algorithm when the following conditions are enforced:

- (i) the analysis filters are power complementary,
- (ii) the diagonal assumption is valid, and
- (iii) the weighting factor $\lambda_i^{-1} = [\|\mathbf{u}_i(k)\|^2 + \alpha]^{-1}$ is employed.

A Newton's method is expected to have better transient behavior (i.e., faster convergence) than a steepest-descent method. Therefore, we may infer that the above requirements define an appropriate basis and weighting factors in forming an efficient decorrelation filter $h_\xi(n)$.

4.4 Underdetermined Least-Squares Solutions

For the case where $\mu = 1$, $\alpha = 0$ (i.e., an un-regularized NSAF algorithm with unit step-size), and the diagonal assumption is valid, i.e., $\mathbf{U}^T(k)\mathbf{U}(k) \approx \mathbf{\Lambda}(k)$, the tap-weight adaptation equation (4.18) becomes

$$\delta \mathbf{w}(k+1) = \mathbf{U}(k) [\mathbf{U}^T(k)\mathbf{U}(k)]^{-1} \mathbf{e}_D(k), \quad (4.42)$$

where $\delta \mathbf{w}(k+1) = \mathbf{w}(k+1) - \mathbf{w}(k)$ denotes the weight adjustment. Since $\mathbf{U}(k) [\mathbf{U}^T(k)\mathbf{U}(k)]^{-1}$ is the pseudoinverse of $\mathbf{U}^T(k)$, the weight adjustment $\delta \mathbf{w}(k+1)$ can be regarded as the minimum-norm solution to the linear system of N equations with M unknowns:

$$\mathbf{U}^T(k) \delta \mathbf{w}(k+1) = \mathbf{e}_D(k). \quad (4.43)$$

Using (4.5) in (4.43) leads to

$$\mathbf{U}^T(k) \mathbf{w}(k+1) = \mathbf{d}_D(k). \quad (4.44)$$

Hence, from a deterministic perspective, the NSAF algorithm can be seen as a recursive estimator that iteratively updates the weight vector in a minimal manner (i.e., squared Euclidean norm of $\delta \mathbf{w}(k+1)$ is minimized) while nulling the *a posteriori* error in all the N subbands, $\xi(k) = \mathbf{d}_D(k) - \mathbf{U}^T(k) \mathbf{w}(k+1)$, at each iteration.

Equations (4.42) and (4.43) define an underdetermined case of least-squares estimation problem where the number of parameters, M , is more than the number of constraints, N . It has been shown in [44], [86], [107] that the NLMS and AP algorithms can also be derived as underdetermined least-squares solutions, as listed in Table 4.2. Compare the minimum-norm solutions in (4.42) and Table 4.2, both AP and NSAF algorithms can be seen as generalized forms of the NLMS algorithm. The AP algorithm generalizes the NLMS along the time-axis using multiple time-domain

Table 4.2 Deterministic interpretations of the NLMS and AP algorithms.

Algorithm	Underdetermined least-squares solution	Complexity
NLMS	$\delta \mathbf{w}(n+1) = \mathbf{u}(n) [\mathbf{u}^T(n) \mathbf{u}(n)]^{-1} e(n),$ where $e(n) = d(n) - \mathbf{u}^T(n) \mathbf{w}(n),$ and $\mathbf{u}(n) \equiv [u(n), u(n-1), \dots, u(n-M+1)]^T.$	$O(M)$
AP	$\delta \mathbf{w}(n+1) = \mathbf{A}(n) [\mathbf{A}^T(n) \mathbf{A}(n)]^{-1} \mathbf{e}(n)$ where $\mathbf{A}(n) \equiv [\mathbf{u}(n), \mathbf{u}(n-1), \dots, \mathbf{u}(n-N+1)],$ $\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{A}^T(n) \mathbf{w}(n),$ and $\mathbf{d}(n) \equiv [d(n), d(n-1), \dots, d(n-N+1)]^T.$	$O(N^2M)$

constraints, whereas the NSAF algorithm generalizes the NLMS along the frequency-axis using multiple subband constraints. These multiple constraints contribute to the decorrelating properties of the algorithm that accelerates its convergence under colored excitation. See [103], [112], [113] and references therein for a description on the decorrelating properties of the AP algorithm.

For the special case of $N=1$, the NSAF algorithm (4.17) reduces to the NLMS algorithm

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \frac{\mathbf{u}(k)}{\|\mathbf{u}(k)\|^2 + \alpha} e(k). \quad (4.45)$$

As illustrated in Figure 4.7, the filter bank reduces to a single filter with its impulse response given by the unit impulse $\delta(n)$ because the band-partitioning and the subsequent signal reconstruction are no longer required. Furthermore, since $k=n$ for $N=1$, the weight adaptation (4.45) is performed at the original sampling rate.

Figure 4.7 The NLMS algorithm can be regarded as the special case of the NSAF algorithm for $N=1$ subband.

4.4.1 Projection interpretation

From a projection interpretation [33], the NSAF algorithm can be seen as a subband variant of the AP algorithm, where subband basis $\mathbf{u}_0(k), \dots, \mathbf{u}_{N-1}(k)$ is used in place of the time-domain basis $\mathbf{u}(n), \dots, \mathbf{u}(n-N+1)$ (i.e., delayed versions of the tap-input vector) in the iterative projection. The projection interpretation of the NSAF algorithm is detailed in Section 5.3.1. The major difference is on the nature of the basis used in the projection. The advantage of the subband basis is due to its orthogonality properties, i.e., the inner product $\mathbf{U}^T(k)\mathbf{U}(k)$ is diagonal, which greatly reduces the computational burden in getting its inverse $[\mathbf{U}^T(k)\mathbf{U}(k)]^{-1}$ for the weight adaptation. The subband basis can be seen as a remedy to the matrix inversion problem encountered in AP algorithm. It can be regarded as a viable alternative to the fast implementation of AP algorithm (FAP). FAP recursively calculates the matrix inversion in which numerical instabilities are introduced, thus requires proper regularization and reinitialization strategy.

4.5 Filter Bank Design Issues

4.5.1 The diagonal assumption

The diagonal assumption suggests that the off-diagonal elements of the matrix

$$\mathbf{U}^T(k)\mathbf{U}(k) = \begin{bmatrix} \mathbf{u}_0^T(k)\mathbf{u}_0(k) & \cdots & \mathbf{u}_0^T(k)\mathbf{u}_{N-1}(k) \\ \vdots & \ddots & \vdots \\ \mathbf{u}_{N-1}^T(k)\mathbf{u}_0(k) & \cdots & \mathbf{u}_{N-1}^T(k)\mathbf{u}_{N-1}(k) \end{bmatrix} \quad (4.46)$$

are negligible, and thus the matrix can be well approximated by its diagonal counterpart which contains only the diagonal elements $\lambda_i(k) = \|\mathbf{u}_i(k)\|^2$. The assumption holds if the outputs of the filter bank, $u_i(n)$ and $u_p(n)$, are orthogonal at zero lag, i.e.,

$$\gamma_{ip}(0) \equiv E\{u_i(n)u_p(n)\} = 0 \text{ for } i \neq p, \quad (4.47)$$

while the filter bank is persistently excited at all frequencies. Such an orthogonality condition can be easily achieved using cosine-modulated filter banks, as proven earlier in Section 3.4.

Assuming that the input signal $u(n)$ is ergodic (and thus WSS), the cross-correlation $\gamma_{ip}(0)$, for $i, p = 0, 1, \dots, N-1$, can be approximated with the time average $\hat{\gamma}_{ip}(0) = \mathbf{u}_i^T(k)\mathbf{u}_p(k)/M$. In the case where $i = p$, the time average $\hat{\gamma}_{ii}(0) = \|\mathbf{u}_i(k)\|^2/M$ is an estimate of the variance of the subband signal $u_i(n)$. Hence under the situation where the filter bank is persistently excited at all frequencies, the orthogonality condition (i.e., *orthogonality at zero lag*) as specified in (4.47) ensures that the off-diagonal elements $\mathbf{u}_i^T(k)\mathbf{u}_p(k) = M\hat{\gamma}_{ip}(0)$ of the matrix $\mathbf{U}^T(k)\mathbf{U}(k)$ are much smaller than its diagonal elements $\|\mathbf{u}_i(k)\|^2 = M\hat{\gamma}_{ii}(0)$. Therefore, the matrix

$\mathbf{U}^T(k)\mathbf{U}(k)$ can be simplified to a diagonal matrix, and thus justifying the diagonal assumption. It should be emphasized that the algorithm (4.17) is recursive in nature, which effectively averages the estimates $\hat{\gamma}_{ip}(0)$ during the course of adaptation. Hence, the length of the adaptive tap-weight vector, M , may not in any means, restricts the validity of the diagonal assumption. That is to say that the diagonal assumption would still hold for lower order adaptive transversal filters.

4.5.2 Power complementary filter bank

From (4.3) and (4.21), the expected value of the instantaneous estimate $\hat{\mathbf{R}}(k)$ can be expressed in an expanded form as:

$$E[\hat{\mathbf{R}}(k)] = \begin{bmatrix} \gamma(0) & \cdots & \gamma(M-1) \\ \vdots & \ddots & \vdots \\ \gamma(-M+1) & \cdots & \gamma(0) \end{bmatrix}, \quad (4.48)$$

with each of its elements, $\gamma(l)$ for $|l| = 0, 1, \dots, M-1$, given by

$$\gamma(l) = \frac{1}{N} \sum_{i=0}^{N-1} E\{u_i(kN)u_i(kN-l)\}. \quad (4.49)$$

Next, consider the subband structure in Figure 4.3. For the N outputs of the filter bank $u_i(n)$, their autocorrelation function $\gamma_{ii}(l) \equiv E\{u_i(n)u_i(n-l)\}$ can be formulated in terms of the input autocorrelation function $\gamma_{uu}(l) \equiv E\{u(n)u(n-l)\}$ as follows:

$$\gamma_{ii}(l) = h_i(l) \otimes h_i(-l) \otimes \gamma_{uu}(l). \quad (4.50)$$

Using (4.50) in (4.49), we arrive at

$$\gamma(l) = \left[\frac{1}{N} \sum_{i=0}^{N-1} h_i(l) \otimes h_i(-l) \right] \otimes \gamma_{uu}(l). \quad (4.51)$$

If the analysis filters are power complementary, i.e.,

$$\frac{1}{N} \sum_{i=0}^{N-1} h_i(l) \otimes h_i(-l) = \delta(l), \quad (4.52)$$

we readily see that (4.51) reduces to $\gamma(l) = \gamma_{uu}(l)$, which implies that $\hat{\mathbf{R}}(k)$ is an unbiased estimator of the ensemble average $\mathbf{R}$. A similar procedure can be used to show that $\hat{\mathbf{p}}(k)$ is an unbiased estimator of $\mathbf{p}$ as well. In (4.52), we assumed that all the analysis filters have been scaled to obtain a magnitude of $\sqrt{N}$ in the frequency response $|H(e^{j\omega})|$.

4.5.3 Number of subbands

The quality of the orthogonalization matrix depends on the accuracy of the instantaneous estimate $\hat{\mathbf{R}}(k)$ in approximating $\mathbf{R}$. Assuming that $\mathbf{U}(k)$ has full column rank, it is clear that $\hat{\mathbf{R}}(k)$ in (4.21) has rank equal to the number of subbands N . In this case, the regularization constant α' added to the diagonal of $\hat{\mathbf{R}}(k)$ prevents the undesired behavior in inverting a rank defective matrix. The rank of $\hat{\mathbf{R}}(k)$ approaches to that of $\mathbf{R}$ (rank M), as the number of subbands N approaches the length of the adaptive filter M . This fact suggests that a more efficient orthogonalization matrix, and thus, a more rapid convergence can be attained with more subbands.

The number of subbands, N , to be used in any specific application depends on the spectral distribution of the input signal. Generally, more subbands are required for input spectrum with more peaks and valleys. For a power spectrum $\Gamma_{uu}(e^{j\omega})$ with N peaks (and valleys), we would typically need N analysis filters in order to decompose the spectrum into N partitions, $|H_i(e^{j\omega})|^2 \Gamma_{uu}(e^{j\omega})$, such that each partition contains a single dominant peak (or valley). These partitions are normalized and recombined to form the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$, as given by (4.41). The dynamic range of $\Gamma_{\xi}(e^{j\omega})$ is reduced because the N peaks (and valleys) have been

aligned to approximately the same level. Notice that, for real-valued input signal and filter coefficients, both the $|H_i(e^{j\omega})|^2$ and $\Gamma_{uu}(e^{j\omega})$ are even and periodic functions with fundamental period 2π . Therefore, only the region $0 \leq \omega \leq \pi$ has to be considered.

For AR random signals, the number of peaks that exist in the power spectrum depends on the order and the location of the poles. Generally, higher order AR model gives more peaks (and valleys as well) in the spectrum. In that sense, the number of subbands, N , can be related to the order of the AR excitation signal. That is, the number of subbands to be used can be determined with some known order-selection criteria that are commonly used in parametric spectrum estimation. For examples, the *Akaike information criterion* (AIC) and the *final prediction error* (FPE) criterion [100].

4.6 Simulations

In this section, the results of several sets of simulations performed in the context of system identification are presented. These simulations are designed in order to further illustrate and to verify the analytical results in the previous sections, and to justify the efficacy of the proposed NSAF algorithm in dealing with colored excitation and high-order identification problem. In particular, the unknown system $\mathbf{w}_0$ to be identified is an actual acoustic response of a room with 300 ms reverberation time. The impulse response was measured at 8 kHz sampling frequency, and then truncated to 1024 samples, as shown in Figure 4.8. Furthermore, three different types of input signals, namely, white Gaussian noise, auto-regressive (AR) random signal, and real speech signal, are used in the simulations.

Figure 4.8 Acoustic response of a room used in the simulations.

In all simulations, the length of the adaptive tap-weight vector $\mathbf{w}(k)$ is fixed at $M = 1024$, which is identical to that of the unknown system $\mathbf{w}_o$. Tap weights of the adaptive filter are initialized to zero. White noise, uncorrelated with the input, is added to the output of the unknown system, giving a 40 dB SNR. The added noise disrupts the adaptive algorithm in matching the tap-weight vector $\mathbf{w}(k)$ to the unknown system $\mathbf{w}_o$, and thus, limits the final steady-state MSE to a value bounded from below by the level of the added noise.

4.6.1 Convergence performance of the NSAF algorithm

In the first set of simulations, the convergence performance of the NSAF algorithm is investigated. The NSAF algorithm is chosen to have $N = 4, 8, 16$, and 32 subbands for each simulation. Pseudo-QMF cosine-modulated filter banks with filter length of $L = 8N$ are used for the analysis and synthesis sections. The length L of the analysis filters increases with N to maintain a 60 dB of stopband attenuation. High stopband attenuation ensures that the frequency responses of adjacent analysis filters do not significantly overlap. Hence, the cross-correlation between non-adjacent subbands can be completely neglected. On the other hand, by virtue of the cosine-modulated nature of the analysis filters, adjacent subband signals are orthogonal at zero lag. Recall that

the diagonal assumption, as stated in Section 4.5.1, is justified if the subband signals are orthogonal at $l = 0$.

Figures 4.9 and 4.10 show the MSE learning curves of the NLMS algorithm and the NSAF algorithm for white noise and AR(2) random signal, respectively. The spectrum of the AR(2) random signal is plotted in Figure 4. 11(a). The MSE learning curves are obtained by ensemble averaging over 200 independent trials, and then smoothed with a 10 points moving average filter. For all cases, the learning rate for both algorithms is chosen to be $\mu = 0.1$, and the regularization parameter α is set to 0.0001. Identical steady-state MSE is achieved for both algorithms, and thus allowing a fair comparison of their transient behavior. From Figure 4.9, it can be observed that the MSE learning curves of the NLMS and the NSAF algorithms overlap when the input signal is white. The reason is that the inherent decorrelating mechanism of the

Figure 4.9 MSE learning curves of the NLMS algorithm and the NSAF algorithm (for $N = 4, 8, 16, 32$ subbands) under white noise excitation. The step-size and regularization factor are set at $\mu = 0.1$ and $\alpha = 0.0001$, respectively.

Figure 4.10 MSE learning curves of the NLMS algorithm and the NSAF algorithm (for $N = 4, 8, 16, 32$ subbands) under colored excitation. The step-size and regularization factor are set at $\mu = 0.1$ and $\alpha = 0.0001$, respectively.

NSAF algorithm has no effect on white noise excitation with a flat spectrum $\Gamma_{uu}(e^{j\omega}) = \sigma_u^2$. It should be noted that, for high-order adaptive filter with $M = 1024$ taps, the signal path delay introduced by the analysis and synthesis filter banks is relatively insignificant.

The advantages of the NSAF algorithm compared to the NLMS algorithm become obvious for colored excitation, as shown in Figure 4.10. It can be observed that the NLMS algorithm exhibits a slow asymptotic convergence after a fast initial convergence, due to the large spectral dynamic range of the input signal. On the other hand, the NSAF exhibits a more consistent and faster convergence rate (i.e., the rate at which the learning curves decay to the steady-state) due to the reduced spectral dynamic range of the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$. Furthermore, with an increased number of subbands, the convergence rate improves considerably. Hence, the NSAF

algorithm is an improvement over the NLMS algorithm. Comparing the NSAF algorithm for different N , it can be noted that the performance improvement obtained by increasing the number of subbands tapers off with higher number of subbands. The number of subbands, N , to be used in any specific applications depends on the ability of the inherent decorrelation filter $|H_\xi(e^{j\omega})|^2$ in getting the inverse of the coloring filter. Generally, more subbands are required for input spectrum with more peaks and valleys.

The NSAF converges faster than the NLMS algorithm mainly due to its inherent decorrelating mechanism that whitens the input signal prior to tap-weight adaptation. Figure 4.11(a) shows the power spectrum of the AR(2) random process, $\Gamma_{uu}(e^{j\omega})$, with a spectral dynamic range of 25.11 dB. The inherent decorrelation filter

Figure 4.11 The inherent decorrelating mechanism of the NSAF algorithm. (a) Power spectrum of the AR(2) random signal used in the simulations. (b) The inherent decorrelating filter (indicated by the thick line). (c) The resulting equalized spectrum.

$|H_{\xi}(e^{j\omega})|^2$ of the NSAF algorithm adjusts accordingly in response to this colored excitation, as shown in Figure 4.11(b). More precisely, it partitions the input spectrum $\Gamma_{uu}(e^{j\omega})$ into $N=4$ overlapping spectral bands, normalizes each subband with its respective subband energy $\lambda_i(k) = \|\mathbf{u}_i(k)\|^2 + \alpha$, and then recombines the normalized partitions to yield the equalized power spectrum $\Gamma_{\xi}(e^{j\omega}) = |H_{\xi}(e^{j\omega})|^2 \Gamma_{uu}(e^{j\omega})$ with a reduced spectral dynamic range of 18.34 dB, as shown in Figure 4.11(c).

Figure 4.12 shows the decorrelation filter for the cases of $N = 8, 16$, and 32 subbands. Clearly, a better match to the inverse of the input spectrum can be obtained by increasing the number of subbands, N . The efficiency of the decorrelation filter

Figure 4.12 The inherent decorrelating filter of the NSAF algorithm with (a) $N = 8$ subbands, (b) $N = 16$ subbands, and (c) $N = 32$ subbands.

can be observed from the resulting equalized spectra, as shown in Figure 4.13. The dynamic range of the equalized spectrum reduces significantly when the number of subbands, N , is increased. For the cases of 8, 16, and 32 subbands, the spectral dynamic range are 12.99 dB, 7.94 dB, and 3.86 dB, respectively.

Figure 4.13 The equalized spectrum for the NSAF algorithm with (a) $N = 8$ subbands, (b) $N = 16$ subbands, and (c) $N = 32$ subbands.

4.6.2 Subband and time-domain constraints

In the second set of simulations, the convergence performance of the NSAF algorithm is compared to that of the NLMS and AP algorithms. Recall that the NLMS algorithm updates the tap weights on the basis of a single input vector, $\mathbf{u}(n)$. The AP algorithm extends the NLMS algorithm by introducing additional $N-1$ past input vectors, $\mathbf{u}(n-1), \dots, \mathbf{u}(n-N+1)$, into the tap-weight adaptation. On the other hand, the NSAF algorithm extends the NLMS algorithm by decomposing the input vector $\mathbf{u}(n)$

into N subband vectors $\mathbf{u}_0(k), \mathbf{u}_1(k), \dots, \mathbf{u}_{N-1}(k)$. It should be emphasized that, in all cases, the coefficients that are explicitly adapted are the fullband tap weights $\{w_n\}_{n=0}^{M-1}$ of a transversal filter. The aim of the simulations is to compare the merit of employing multiple subband constraints to that of time-domain constraints, in terms of convergence performance and computational complexity.

Two highly correlated excitation signals, namely, an AR(10) random signal and a real speech, are used in the simulations. The AR coefficients are obtained through application of the Yule-Walker method to a segment of unit variance speech. The AR(10) random signal exhibits a power spectrum that closely resembles the power spectrum of speech signal, as shown in Figure 4.14 together with the pole-zero plot for the AR model. The NSAF algorithm is chosen to have $N=10$ subbands using

Figure 4.14 (a) Power spectrum of the AR(10) random signal used in the simulations. (b) Pole-zero plot of the AR model.

pseudo-QMF cosine-modulated filter banks with analysis filter length of $L = 8N = 80$. Two variations of sample- and block-based AP algorithm are considered, both with an order of $N = 10$. The block-based AP algorithm is commonly known as partial rank algorithm (PRA) [86], [107]. The step-size and regularization parameter are set to $\mu = 0.1$ and $\alpha = 0.0001$, respectively, such that identical steady-state MSE is achieved by all the algorithms for the stationary excitation.

Figure 4.15 shows the ensemble-average learning curves for the AR(10) random signal. It can be noted that, the convergence speed of the NSAF algorithm is higher than that of the NLMS, and close to that of the AP and PRA. The subband decomposition and adaptation process of the NSAF algorithm altogether represent a computational complexity of $O(M) + O(NL)$. On the other hand, the computational

Figure 4.15 MSE learning curves of the NLMS, PRA, NSAF, and AP algorithms. The step-size and regularization factor are set at $\mu = 0.1$ and $\alpha = 0.0001$, respectively.

complexity of the NLMS, PRA, and AP are $O(M)$, $O(NM)$, and $O(N^2M)$, respectively. For $M = 1024$ and $N = 10$, the NLMS, NSAF, PRA, and AP algorithms, respectively, requires 3073, 4034, 12299, and 122990 multiplications in a single sampling period T_s . Clearly, the NSAF algorithm can achieve almost equivalent convergence rate as the AP algorithm with a computational complexity close to that of the NLMS algorithm.

Figure 4.16 shows the convergence behaviors of the NLMS, PRA, NSAF, AP algorithms on the speech signal. Speech signals are highly correlated and non-stationary. For such excitations, we find that it is convenient to compare the convergence of the adaptive algorithms with the use of the *normalized misalignment* [6], [40], which is defined as the norm of the weight-error, $\|\mathbf{w}(k) - \mathbf{w}_o\|$, normalized by the norm of the optimum weights, $\|\mathbf{w}_o\|$, as follows:

$$\text{normalized misalignment} = 20 \log_{10} \frac{\|\mathbf{w}(k) - \mathbf{w}_o\|}{\|\mathbf{w}_o\|}. \quad (4.53)$$

The performance index measures the distance of the adaptive tap-weight vector, $\mathbf{w}(k)$, from the optimum weight vector, $\mathbf{w}_o$, at each iteration. Notice that, the definition of the *normalized misalignment* does not involve the expectation operator $E\{\cdot\}$, thus no ensemble averaging is required in getting the learning curves. The iteration index k is taken as n for the cases of the NLMS and AP algorithms since the tap-weight adaptation is performed at the input sampling rate. For the cases of NSAF and PRA, the learning curves are plotted as functions of kN . Furthermore, for the sake of simplicity, a noiseless situation is considered, that is, no disturbance is added to output of the unknown system.

Figure 4.16 Normalized misalignment learning curves for the NLMS, PRA (order 10), NSAF (10 subbands), and AP (order 10) algorithms under speech excitation. The step-size is set at $\mu = 0.1$.

From the learning curves, we readily see that the NSAF algorithm performs equally well with the speech signal, as compared to the stationary colored excitation. As can be seen, the NSAF algorithm outperforms the NLMS and PRA. Furthermore, its convergence speed is closest to that of the AP with a far lower computational complexity. These results indicate that, the analytical results obtained by assuming that the input signal is WSS hold for non-stationary input signal as well. The NSAF algorithm is capable to deal with non-stationary excitation mainly due to the running average normalization matrix $\Lambda(k)$ employed in the tap-weight adaptation.

4.7 Conclusions

This chapter described and analyzed a class of SAFs referred to as the normalized SAF (NSAF). The NSAF has a unique weight-control mechanism whereby subband signals, normalized by their respective subband input variance, are used to adapt the fullband tap weights of a modeling filter. The recursive equation of the new weight-control mechanism was derived in Section 4.2 from a multiple-constraint optimization criterion based on the *principle of minimal disturbance*. The NSAF recursion can also be formulated as the minimum-norm solution to an underdetermined least-squares estimation problem, as shown in Section 4.4. Within these deterministic frameworks, the NASF algorithm can be seen as a recursive estimator that iteratively updates the tap-weight vector $\mathbf{w}(k)$ in a minimal manner, while nulling the a posteriori error $\xi_D(k)$ in all the N subbands. Similar least-perturbation properties were also found in the NLMS and AP algorithms. Both the AP and NSAF algorithms can be seen as generalized forms of the NLMS algorithm. The AP algorithm generalizes the NLMS along the time-axis using multiple time-domain constraints, whereas the NSAF algorithm generalizes the NLMS along the frequency-axis using multiple subband constraints.

The multiple subband constraints introduced into the NSAF recursion render its convergence less sensitive to the coloring of the input signal. In Section 4.3, the NSAF algorithm was motivated from two stochastic viewpoints. The NSAF recursion was formulated as an optimization process that approximates

- (i) a steepest-descent method in minimizing the classical MSE function, and
- (ii) a Newton's method in minimizing the weighted MSE function.

From these stochastic viewpoints, the inherent decorrelating properties can be expressed in the form of a decorrelation filter or alternatively as an orthogonalization

matrix. The inherent decorrelating properties of the NSAF algorithm whiten the input signal prior to tap-weight adaptation; thereby accelerate its convergence under colored excitation.

Numerical simulations were performed in the context of system identification in order to justify the theoretical analysis and to examine the convergence behavior of the NSAF algorithm under different conditions. Compared to the NLMS, the NSAF was found to exhibit faster convergence under colored excitation with almost equivalent computational complexity. Furthermore, with an increased number of subbands, the convergence rate increases considerably. The NSAF is insensitive to coloring of the input signal. Simulation results confirmed that the NSAF algorithm can achieve almost equivalent convergence rate as the AP algorithm with computational complexity close to that of the NLMS algorithm. Hence, the NSAF algorithm would be preferable from these aspects, especially for high-order adaptive filters.

Chapter 5

Stability and Performance Analysis

In this chapter, the convergence behavior of the NSAF algorithm [61], [62], [63] is analyzed. In particular, the performance of the NSAF algorithm is characterized in terms of the stability of the tap-weight adaptation, the speed of convergence, and the estimation accuracy in the steady state after the algorithm has converged. Such a mathematical analysis of the NSAF algorithm provides a set of working rules that can be used for its design in practical applications. A unique characteristic of the NSAF algorithm is that it uses a set of normalized subband signals to adapt the fullband tap weights of a modeling filter. The modeling filter is placed before the decimators, which is different from that of the conventional SAF structure where a set of subfilters are placed after the decimators [16], [36], [55], [111]. In what follows, the architectural beauty of such a configuration becomes more evident for which it enables a tractable convergence analysis of the subband adaptive filtering algorithm [61], [63], which is generally a formidable task for the case of conventional SAF.

In the first part of this chapter, we introduce various subband quantities, data model, performance measure, and assumptions for the convergence analysis. The second part of this chapter begins with a mean analysis of the NSAF algorithm. Subsequently, the mean-square performance of the algorithm is analyzed based on an energy conservation relation [104], [106], [107], [108], [110], [131], [132]. Stability bounds for the step size and an expression for the steady-state MSE are determined by manipulating various error quantities in the energy conservation relation.

5.1 Weight Recursion, Data Model, and Assumptions

Before plunging into the details of the statistical analysis, it would be helpful to revisit the NSAF recursion, and to introduce various quantities, data model, and assumptions that will be called upon later in the convergence analysis. With these model and assumptions, the average behavior of the weight recursion can be studied and evaluated in a tractable manner.

5.1.1 The NSAF recursion

The NSAF algorithm is a subband adaptive filtering algorithm in the sense that it partitions the input signal $u(n)$ and the desired response $d(n)$ into multiple spectral bands for tap-weight adaptation, as illustrated in Figure 4.3. However, different from that in the conventional SAF, where each subband adapts a separate subfilter in its own adaptation loop [16], [36], [55], [111], the NSAF algorithm collectively adapts the multiple spectral bands of a single transversal filter $W(k, z) = \sum_{n=0}^{M-1} w_n(k) z^{-n}$ by using the subband signals. The weight adjustment applied on the adaptive tap-weight vector $\mathbf{w}(k) \equiv [w_0(k), w_1(k), \dots, w_{M-1}(k)]^T$ is iteratively calculated via

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \mathbf{U}(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k), \quad (5.1)$$

$$\mathbf{e}_D(k) = \mathbf{d}_D(k) - \mathbf{U}^T(k) \mathbf{w}(k), \text{ and} \quad (5.2)$$

$$\mathbf{\Lambda}(k) = \text{diag}[\mathbf{U}^T(k) \mathbf{U}(k) + \alpha \mathbf{I}], \quad (5.3)$$

where μ and α are the step-size and regularization parameters, respectively. The subband regressor $\mathbf{u}_i(k)$, the $M \times N$ subband data matrix $\mathbf{U}(k)$, and the $N \times 1$ desired response vector $\mathbf{d}_D(k)$ are defined, respectively, as follows

$$\mathbf{u}_i(k) \equiv [u_i(kN), u_i(kN-1), \dots, u_i(kN-M+1)]^T, \quad (5.4)$$

$$\mathbf{U}(k) \equiv [\mathbf{u}_0(k), \mathbf{u}_1(k), \dots, \mathbf{u}_{N-1}(k)], \text{ and} \quad (5.5)$$

$$\mathbf{d}_D(k) \equiv [d_{0,D}(k), d_{1,D}(k), \dots, d_{N-1,D}(k)]^T. \quad (5.6)$$

The subband regressors $\mathbf{u}_i(k)$, as defined in (5.4) for $i = 0, 1, \dots, N-1$, although bandlimited, holds the samples of the subband input signals $u_i(n)$ at the original sampling rate. That is to say that the modeling filter $W(k, z)$ operates on the subband signals at the original sampling rate n , while its tap weights being iteratively updated at the decimated rate k . The filtering operation will not appear as the bottleneck in real-time implementation of the NSAF algorithm since most digital signal processors have very efficient architecture to perform FIR filtering [59].

5.1.2 Linear data model

The physical mechanism for generating the desired response $d(n)$ is described by a linear model [44], [107], [128] of the form

$$d(n) = \mathbf{w}_o^T \mathbf{u}(n) + \eta(n), \quad (5.7)$$

where $\mathbf{w}_o = [w_{o,0}, w_{o,1}, \dots, w_{o,M-1}]^T$ denotes an unknown parameter vector of the model, $\mathbf{u}(n) = [u(n), u(n-1), \dots, u(n-M+1)]^T$ is the input vector (or regressor), and $\eta(n)$ denotes the irreducible additive white noise that is statistically independent of the input signal $u(n)$. Furthermore, it is also assumed that the length M of the adaptive transversal filter is exactly equal to that of the data model.

Figure 5.1(a) shows the linear data model followed by a multirate filter bank. By virtue of the linearity of the analysis filters and the decimators, as depicted in Figure 5.1(b), the desired response vector $\mathbf{d}_D(k)$ of (5.6) that arises from the linear data model of (5.7) can be written as

(a)

(b)

Figure 5.1 Analysis of the linear data model. (a) The desired response arises from a linear data model is decomposed into subbands and decimated. (b) Equivalent structure obtained by virtue of the linearity of the filters and the decimators.

$$\mathbf{d}_D(k) = \mathbf{U}^T(k) \mathbf{w}_o + \boldsymbol{\eta}_D(k). \quad (5.8)$$

In the above equation, $\boldsymbol{\eta}_D(k) \equiv [\eta_{0,D}(k), \eta_{1,D}(k), \dots, \eta_{N-1,D}(k)]^T$ denotes the noise vector that comprised of the critically decimated subband noises

$$\eta_{i,D}(k) \equiv \eta_i(kN) = \sum_{l=0}^{L-1} h_i(l) \eta(kN-l). \quad (5.9)$$

Using (5.8) in (5.2), the estimation error $\mathbf{e}_D(k)$ can be expressed as

$$\begin{aligned} \mathbf{e}_D(k) &= [\mathbf{U}^T(k) \mathbf{w}_o + \boldsymbol{\eta}_D(k)] - \mathbf{U}^T(k) \mathbf{w}(k) \\ &= \mathbf{U}^T(k) [\mathbf{w}_o - \mathbf{w}(k)] + \boldsymbol{\eta}_D(k) \\ &= \mathbf{U}^T(k) \boldsymbol{\varepsilon}(k) + \boldsymbol{\eta}_D(k). \end{aligned} \quad (5.10)$$

From (5.10), it can be noted that the estimation error $\mathbf{e}_D(k)$ consists of two components, namely, the modeling error $\mathbf{U}^T(k) \boldsymbol{\varepsilon}(k)$ and the measurement noise $\boldsymbol{\eta}_D(k)$. The weight-error vector $\boldsymbol{\varepsilon}(k) \equiv \mathbf{w}_o - \mathbf{w}(k)$ measures how far the weight estimate $\mathbf{w}(k)$ is from the true weight vector $\mathbf{w}_o$ to be identified. Hence, the product $\mathbf{U}^T(k) \boldsymbol{\varepsilon}(k)$ represents the undisturbed estimation error that measures how well the adaptive filter output $\mathbf{U}^T(k) \mathbf{w}(k)$ in approximating $\mathbf{U}^T(k) \mathbf{w}_o$, which is the undisturbed portion of $\mathbf{d}_D(k)$.

5.1.3 Paraunitary filter banks

The paraunitary condition for filter banks is defined in the transform domain by (2.25). Imposing the paraunitary condition on the filter banks that band-partition the input signal $u(n)$ and desired response $d(n)$, leads to the following two-fold consequence:

- (i) the filter banks are power complementary;
- (ii) the noise vector $\mathbf{n}_b(k)$ in (5.8), at any particular time instant k , is uncorrelated with the current and all the previous weight estimates, $\mathbf{w}(k), \mathbf{w}(k-1), \mathbf{w}(k-2), \dots$.

The first point is well understood in the literature, for examples, in [73, pp. 124] and [125, pp. 296]. A detailed description on this property is given in Appendix D. The reasoning for the second point, which greatly simplifies the mathematical analysis, goes as follows.

Let the cross-correlation between the impulse responses of the analysis filters, $h_i(n)$ and $h_p(n)$ for $i, p = 0, 1, \dots, N-1$, be defined as

$$q_{ip}(l) \equiv \sum_{n=0}^{L-1} h_i(n) h_p(n-l) = h_i(l) \otimes h_p(-l), \quad (5.11)$$

where L is the length of the analysis filters, and $\otimes$ denotes convolution. In the time domain, the paraunitary condition [73], [81] ensures that the N -fold decimated version of $q_{ip}(l)$ appears in the following form

$$q_{ip}(lN) = \delta(l) \delta(i-p). \quad (5.12)$$

Equations (5.11) and (5.12) indicate that the cross-correlation sequences $q_{ip}(l)$ of a paraunitary filter bank are characterized by periodic zero-crossing separated by N sampling periods. Notice that $q_{ip}(l)$ is a non-causal sequence of length $2L-1$ as we

assume that the analysis filters are causal FIR filters of length L .

Recall that the disturbance $\eta(n)$ in the linear data model (5.7) is assumed to be a white noise process such that its successive samples are uncorrelated, as shown by

$$E\{\eta(n)\eta(n-l)\} = \sigma_\eta^2 \delta(l), \quad (5.13)$$

where σ_η^2 denotes the variance of $\eta(n)$. Applying the correlation-domain formulation (see Section 3.1) on the upper branch of Figure 5.1(b), the correlation function $\gamma_{ip}(l) \equiv E\{\eta_i(n)\eta_i(n-l)\}$ between the subband noises $\eta_i(n)$, for $i = 0, 1, \dots, N-1$, can be expressed as

$$\gamma_{ip}(l) = q_{ip}(l) \otimes [\sigma_\eta^2 \delta(l)] = \sigma_\eta^2 q_{ip}(l). \quad (5.14)$$

Let $\eta_{i,D}(k) = \eta_i(kN)$ be the critically-decimated subband noises. According to the result in Section 3.1.1, the cross-correlation function $\gamma_{ip,D}(l) \equiv E\{\eta_{i,D}(k)\eta_{p,D}(k-l)\}$ between the critically-decimated subband noises is given by the N -fold decimated version of $\gamma_{ip}(l)$ in (5.14) as follows

$$\gamma_{ip,D}(l) = \gamma_{ip}(lN) = \sigma_\eta^2 q_{ip}(lN). \quad (5.15)$$

Using (5.12) in (5.15), it follows that

$$E\{\eta_{i,D}(k)\eta_{p,D}(k-l)\} = \begin{cases} \sigma_\eta^2 \delta(l), & \text{for } i = p, \\ 0 \quad \forall l, & \text{for } i \neq p, \end{cases} \quad (5.16)$$

which implies that the critically decimated subband noises, $\eta_{i,D}(k)$ for $i = 0, 1, \dots, N-1$, are white and are uncorrelated to each other. It should be emphasized that the subband noises, $\eta_i(n)$ for $i = 0, 1, \dots, N-1$, before critical decimation, are generally colored and correlated to each others. As can be seen from (5.11) and (5.14), the correlation function $\gamma_{ip}(l)$ is characterized by the analysis filters, $h_i(n)$ and $h_p(n)$.

Now, by using the result in (5.16), the fact in point (ii) can be easily observed from the update equation of the NSAF algorithm. Substituting (5.10) in (5.1) and setting $k = k - 1$, the current weight estimate $\mathbf{w}(k)$ can be expressed as

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mu \mathbf{U}(k-1) \mathbf{\Lambda}^{-1}(k) [\mathbf{U}^T(k-1) \mathbf{\varepsilon}(k-1) + \mathbf{\eta}_D(k-1)]. \quad (5.17)$$

Clearly, $\mathbf{w}(k)$ is dependent on the past noise vectors $\mathbf{\eta}_D(k-1), \mathbf{\eta}_D(k-2), \dots$, and the past input matrices $\mathbf{U}(k-1), \mathbf{U}(k-2), \dots$. However, the current noise vector $\mathbf{\eta}_D(k)$ is uncorrelated with the past noises as indicated by (5.16). Furthermore, $\mathbf{\eta}_D(k)$ is also assumed to be statistically independent (see Section 5.1.2) of the input matrices, $\mathbf{U}(k)$ for all k . Therefore, $\mathbf{\eta}_D(k)$ is uncorrelated with the current and all the previous weight estimates, $\mathbf{w}(k), \mathbf{w}(k-1), \mathbf{w}(k-2), \dots$.

5.2 Performance Measure

This section introduces a novel multiband MSE function as a performance measure for the NSAF algorithm. By means of the Wiener filter theory, the multiband MSE function is shown to correspond to the classical fullband MSE function. Hence, the multiband MSE function serves as a performance measure that allows a fair comparison of the NSAF algorithm with other adaptive filtering algorithms that rely on the classical MSE function.

5.2.1 Multiband MSE function

Consider an error function defined as the average of the mean-squared value of the subband estimation errors, as follows

$$J_{\mathcal{M}}(\mathbf{w}) \equiv \frac{1}{N} \sum_{i=0}^{N-1} E \{ e_{i,D}^2(k) \} = \frac{1}{N} E \{ \|\mathbf{e}_D(k)\|^2 \}. \quad (5.18)$$

Substituting (5.2) into (5.18), the multiband MSE function $J_{\mathcal{M}}(\mathbf{w})$ can be written in the following expanded form

$$\begin{aligned} J_{\mathcal{M}}(\mathbf{w}) &= \frac{1}{N} E \left\{ \left[\mathbf{d}_D(k) - \mathbf{U}^T(k) \mathbf{w} \right]^T \left[\mathbf{d}_D(k) - \mathbf{U}^T(k) \mathbf{w} \right] \right\} \\ &= \frac{1}{N} E \left\{ \|\mathbf{d}_D(k)\|^2 - 2[\mathbf{U}(k) \mathbf{d}_D(k)]^T \mathbf{w} + \mathbf{w}^T \mathbf{U}(k) \mathbf{U}^T(k) \mathbf{w} \right\} \\ &= \frac{1}{N} \left[E \left\{ \|\mathbf{d}_D(k)\|^2 \right\} - 2E \left\{ \mathbf{U}(k) \mathbf{d}_D(k) \right\}^T \mathbf{w} + \mathbf{w}^T E \left\{ \mathbf{U}(k) \mathbf{U}^T(k) \right\} \mathbf{w} \right]. \end{aligned} \quad (5.19)$$

Now, assume that the filter banks for band-partitioning the input signal $u(n)$ and the desired response $d(n)$ are identical and power complementary. Using the similar procedure described in Section 4.5.2, the second-order moments in (5.19) can be shown to be equivalent to the second-order moments characterizing the classical MSE function

$$J(\mathbf{w}) = E \{ e^2(n) \} = \sigma_d^2 - 2\mathbf{p}^T \mathbf{w} + \mathbf{w}^T \mathbf{R} \mathbf{w}, \quad (5.20)$$

where

$$\begin{aligned} \sigma_d^2 &\equiv E \{ d^2(n) \} = \frac{1}{N} E \left\{ \|\mathbf{d}_D(k)\|^2 \right\}, \\ \mathbf{p} &\equiv E \{ \mathbf{u}(n) d(n) \} = \frac{1}{N} E \left\{ \mathbf{U}(k) \mathbf{d}_D(k) \right\}, \text{ and} \\ \mathbf{R} &\equiv E \{ \mathbf{u}(n) \mathbf{u}^T(n) \} = \frac{1}{N} E \left\{ \mathbf{U}(k) \mathbf{U}^T(k) \right\} \end{aligned} \quad (5.21)$$

are the abovementioned second-order moments pertaining to the MSE functions $J(\mathbf{w})$ and $J_{\mathcal{M}}(\mathbf{w})$.

A power complementary filter bank preserves the second-order moments of its input signal. By virtue of the power complementary property, the multiband MSE function $J_{\mathcal{M}}(\mathbf{w})$ is equivalent to the classical MSE function $J(\mathbf{w})$. Both the MSE functions $J(\mathbf{w})$ and $J_{\mathcal{M}}(\mathbf{w})$ are equivalent in the sense that they are quadratic functions of $\mathbf{w}$ that describe an identical error-performance surface characterized by

the second-order moments $\{\sigma_d^2, \mathbf{p}, \mathbf{R}\}$ of (5.21). In that sense, minimizing the MSE functions in (5.18) and (5.20) with any iterative optimization methods would lead to the same optimal Wiener solution $\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p}$, which is located at the bottom of the error-performance surface.

5.2.2 Excess MSE

The linear data model defined in (5.7), or equivalently in (5.8), describes a stationary environment for generating the desired response. Substituting (5.10) into (5.18), the multiband MSE function under the linear data model assumption can be expressed in the following form

$$\begin{aligned} J_{\mathcal{M}}(k) &= \frac{1}{N} E \left\{ [\mathbf{e}_a(k) + \boldsymbol{\eta}_D(k)]^T [\mathbf{e}_a(k) + \boldsymbol{\eta}_D(k)] \right\} \\ &= \frac{1}{N} E \left\{ \|\boldsymbol{\eta}_D(k)\|^2 + 2\boldsymbol{\eta}_D^T(k) \mathbf{e}_a(k) + \|\mathbf{e}_a(k)\|^2 \right\} \\ &= \frac{1}{N} E \left\{ \|\boldsymbol{\eta}_D(k)\|^2 \right\} + 2 \left[\frac{1}{N} E \left\{ \boldsymbol{\eta}_D^T(k) \mathbf{e}_a(k) \right\} \right] + \frac{1}{N} E \left\{ \|\mathbf{e}_a(k)\|^2 \right\}, \end{aligned} \quad (5.22)$$

where $\mathbf{e}_a(k) \equiv [e_{0,a}(k), e_{1,a}(k), \dots, e_{N-1,a}(k)]^T = \mathbf{U}^T(k) \boldsymbol{\varepsilon}(k)$ denotes the undisturbed estimation error. The cross-term $E \left\{ \boldsymbol{\eta}_D^T(k) \mathbf{e}_a(k) \right\}$ in (5.22) can be eliminated by exploiting the fact that the noise vector $\boldsymbol{\eta}_D(k)$ is uncorrelated with the current tap weights $\mathbf{w}(k)$ due to the paraunitary condition imposed on the filter banks (as explained in Section 5.1.3). As such, the multiband MSE function reduces to the following form

$$J_{\mathcal{M}}(k) = \sigma_{\eta}^2 + \frac{1}{N} E \left\{ \|\mathbf{e}_a(k)\|^2 \right\}, \quad (5.23)$$

where

$$\sigma_{\eta}^2 \equiv E \left\{ \eta^2(n) \right\} = \frac{1}{N} E \left\{ \|\boldsymbol{\eta}_D(k)\|^2 \right\} = \frac{1}{N} \sum_{i=0}^{N-1} E \left\{ \eta_{i,D}(k) \right\} \quad (5.24)$$

is the variance of the additive white noise $\eta(n)$, considering that the analysis filter bank is power complementary (due to the paraunitary condition, as explained in Appendix D).

Perfect adaptation is achieved when the adaptive tap-weight vector $\mathbf{w}(k)$ takes on the optimum value $\mathbf{w}_o$, which gives the *minimum MSE* $J_{\min}$. Under such situation, the weight-error vector $\mathbf{\varepsilon}(k)$, and the undisturbed estimation error $\mathbf{e}_a(k) = \mathbf{U}^T \mathbf{\varepsilon}(k)$ in (5.23) are essentially equal to zero vectors. By substituting the perfect adaptation condition $\mathbf{e}_a(k) = \mathbf{0}$ into (5.23), it follows that the minimum MSE $J_{\min}$ under the linear data model assumption is given by the variance σ_η^2 of the disturbance $\eta(n)$. Nevertheless, due to the stochastic nature of adaptive algorithms, the tap-weight vector computed will never terminate at the bottom $\{J_{\min}, \mathbf{w}_o\}$ of the error performance surface. Instead, it executes a random motion around $\{J_{\min}, \mathbf{w}_o\}$, giving rise to the so-called *steady-state excess MSE* $J_{ex}(\infty)$, which can be written in the following form

$$J_{ex}(\infty) \equiv J(\infty) - J_{\min} = J_{\mathcal{M}}(\infty) - \sigma_\eta^2 = \frac{1}{N} E \left\{ \|\mathbf{e}_a(\infty)\|^2 \right\}. \quad (5.25)$$

In (5.25), it is assumed that the multiband MSE function $J_{\mathcal{M}}$ is identical to the classical MSE function J by virtue of the paraunitary condition of the filter banks. In Section 5.4, we shall use the excess MSE $J_{ex}(\infty)$ to evaluate the steady-state performance of the NSAF algorithm.

5.3 Mean Analysis

In this section, we begin the mean analysis by giving a projection interpretation to the NSAF recursion. The mean behavior of the NSAF algorithm is then observed from the mean behavior of the projection matrix.

5.3.1 Projection interpretation

In the following analysis, it is assumed that the desired response $d(n)$ is derived from a noiseless linear data model, where the disturbance $\eta(n)$ in (5.7) is negligible. Furthermore, we also assume that the regularization parameter α in (5.3) has been set to zero. Manipulating (5.1), (5.2), and (5.8), the tap-weight adaptation equation of the NSAF algorithm can be expressed as

$$\begin{aligned}\mathbf{w}(k+1) &= \mathbf{w}(k) + \mu \mathbf{U}(k) \boldsymbol{\Lambda}^{-1}(k) [\mathbf{U}^T(k) \mathbf{w}_o - \mathbf{U}^T(k) \mathbf{w}(k)] \\ &= \mathbf{w}(k) - \mu \mathbf{U}(k) \boldsymbol{\Lambda}^{-1}(k) \mathbf{U}^T(k) \mathbf{w}(k) + \mu \mathbf{U}(k) \boldsymbol{\Lambda}^{-1}(k) \mathbf{U}^T(k) \mathbf{w}_o \quad (5.26) \\ &= [\mathbf{I} - \mu \mathbf{P}(k)] \mathbf{w}(k) + \mu \mathbf{P}(k) \mathbf{w}_o,\end{aligned}$$

where $\mathbf{P}(k)$ denotes a projection matrix [118] of the form

$$\mathbf{P}(k) \equiv \mathbf{U}(k) \boldsymbol{\Lambda}^{-1}(k) \mathbf{U}^T(k) = \sum_{i=0}^{N-1} \frac{\mathbf{u}_i(k) \mathbf{u}_i^T(k)}{\mathbf{u}_i^T(k) \mathbf{u}_i(k)}. \quad (5.27)$$

Clearly, the projection matrix $\mathbf{P}(k)$ is uniquely determined by the subband data matrix $\mathbf{U}(k)$, which consists of N subband regressors $\mathbf{u}_i(k)$ defined in (5.5). The subband vectors are orthogonal if the diagonal assumption (see Section 4.5.1) is valid. Therefore, the column space of $\mathbf{U}(k)$ is span by an orthogonal basis consisting of $\mathbf{u}_0(k), \mathbf{u}_1(k), \dots, \mathbf{u}_{N-1}(k)$.

The weight recursion in (5.26) can be interpreted as an iterative projection operation. At each iteration, the projection matrix $\mathbf{P}(k)$ projects the true system impulse response $\mathbf{w}_o$ onto the columns space of $\mathbf{U}(k)$, while the old estimate $\mathbf{w}(k)$

is projected onto a subspace that is perpendicular to the columns space of $\mathbf{U}(k)$. The sum of the two projections results in the updated tap-weight vector $\mathbf{w}(k+1)$, at each iteration. The iterative projection operation (5.26) is illustrated in Figure 5.2. Since we assume that subband regressors $\mathbf{u}_0(k), \mathbf{u}_1(k), \dots, \mathbf{u}_{N-1}(k)$ are orthogonal (by virtue of the diagonal assumption), the projection matrix $\mathbf{P}(k)$ can be directly decomposed into N projection matrices $\mathbf{P}_i(k) = \mathbf{u}_i(k) \mathbf{u}_i^T(k) / \mathbf{u}_i^T(k) \mathbf{u}_i(k)$ of rank one. Projection onto the column space of $\mathbf{U}(k)$ is then taken as the sum of the N projections onto the vectors $\mathbf{u}_i(k)$, for $i = 0, 1, \dots, N-1$. Similar concept applies to the complement projection.

Figure 5.2 Projection interpretation of the NSAF recursion. The updated tap-weight vector $\mathbf{w}(k+1)$ is the sum of projections onto two perpendicular subspaces.

The complement projection $[\mathbf{I} - \mu\mathbf{P}(k)]\mathbf{w}(k)$ in (5.26) can be interpreted as a way to discard the old information $\mathbf{P}(k)\mathbf{w}(k)$ [i.e., the component of the old estimate $\mathbf{w}(k)$ that lies in the column space of $\mathbf{U}(k)$] from $\mathbf{w}(k)$. On the other hand, the projection $\mathbf{P}(k)\mathbf{w}_0$ serves as the new information replacing the old information

$\mathbf{P}(k)\mathbf{w}(k)$ that has been discarded from $\mathbf{w}(k)$. The amount of information flow from one iteration to the next is regulated by the step size μ , for which a stability bound can be established, as shown in the next section. For the case where $N = M$, it holds that $\mathbf{P}(k)\mathbf{w}(k) = \mathbf{w}(k)$ and $\mathbf{P}(k)\mathbf{w}_o = \mathbf{w}_o$, because the projection of a vector, of length M , onto the column space of the full rank $M \times M$ matrix $\mathbf{U}(k)$ is equal to the vector itself. If we further assume that $\mu = 1$, then the adaptive tap-weight vector $\mathbf{w}(k)$ converges in one step, as shown below:

$$\begin{aligned}\mathbf{w}(k+1) &= \mathbf{w}(k) - \mathbf{P}(k)\mathbf{w}(k) + \mathbf{P}(k)\mathbf{w}_o \\ &= \mathbf{w}(k) - \mathbf{w}(k) + \mathbf{w}_o \\ &= \mathbf{w}_o.\end{aligned}\tag{5.28}$$

Convergence in one step, though attractive, is not practical since the measurement noise $\mathbf{n}_D(k)$ is not avoidable in real application. Furthermore, for $N = M$, the filter banks would introduce unacceptably long delay, which is generally greater than that introduced by the modeling filter.

5.3.2 Mean behavior

The effectiveness of the NSAF algorithm in dealing with colored excitation signals can be noticed from the mean behavior of the projection matrix $\mathbf{P}(k)$. Subtracting (5.26) from $\mathbf{w}_o$, rearranging terms, taking the expectation of both sides, and making the usual independence assumption [21], [27], [44], [128] [i.e., the tap-weight vector $\mathbf{w}(k)$ is statistically independent of the subband regressors $\mathbf{u}_i(k)$], the mean behavior of the weight-error vector $\boldsymbol{\varepsilon}(k) \equiv \mathbf{w}_o - \mathbf{w}(k)$ can be described in the following form

$$E\{\boldsymbol{\varepsilon}(k+1)\} = [I - \mu\mathbf{R}_w]E\{\boldsymbol{\varepsilon}(k)\},\tag{5.29}$$

where

$$\mathbf{R}_w \equiv E\{\mathbf{P}(k)\} = \sum_{i=0}^{N-1} E\left\{\frac{\mathbf{u}_i(k)\mathbf{u}_i^T(k)}{\mathbf{u}_i^T(k)\mathbf{u}_i(k)}\right\} \quad (5.30)$$

is the mean of the projection matrix $\mathbf{P}(k)$. Clearly, the difference equation (5.29) for the NSAF algorithm is of the same mathematical form with that of the LMS algorithm [21], [27], [44], [128], except for the correlation matrices. Equation (5.29) suggests that the convergence of the NSAF recursion in the mean is driven by M natural modes which are characterized by the eigenvalues of $\mathbf{R}_w$.

Assuming that M is large, the fluctuations in the subband energy $\|\mathbf{u}_i(k)\|^2$ from one iteration to the next would be small enough, thereby, justifying the following approximation

$$\begin{aligned} E\left\{\frac{\mathbf{u}_i(k)\mathbf{u}_i^T(k)}{\|\mathbf{u}_i(k)\|^2}\right\} &\approx \frac{E\{\mathbf{u}_i(k)\mathbf{u}_i^T(k)\}}{E\{\|\mathbf{u}_i(k)\|^2\}} \\ &= \frac{E\{\mathbf{u}_i(k)\mathbf{u}_i^T(k)\}}{M\gamma_{ii}(0)}, \end{aligned} \quad (5.31)$$

where $\gamma_{ii}(0)$ denotes the variance of the subband signal $E\{u_i^2(n)\}$. Designating $\mathbf{R}_i$ as the subband correlation matrix of the form $\mathbf{R}_i = E\{\mathbf{u}_i(k)\mathbf{u}_i^T(k)\}$, the mean of the projection matrix can be seen as a weighted sum of N subband correlation matrices, as follows

$$\mathbf{R}_w \approx \sum_{i=0}^{N-1} \lambda_i^{-1} \mathbf{R}_i, \quad (5.32)$$

where $\lambda_i^{-1} = 1/M\gamma_{ii}(0)$ denote the weighting factors. From (5.31) and (5.32), it is obvious that the weighted correlation matrix $\mathbf{R}_w$ is determined by a weighted correlation function $\gamma_\xi(l)$ of the form

$$\gamma_{\xi}(l) = \left[\sum_{i=0}^{N-1} \lambda_i^{-1} h_i(l) \otimes h_i(-l) \right] \otimes \gamma_{uu}(l), \quad (5.33)$$

or, equivalently in the frequency domain, by an equalized spectrum

$$\Gamma_{\xi}(e^{j\omega}) = \left[\sum_{i=0}^{N-1} \lambda_i^{-1} |H_i(e^{j\omega})|^2 \right] \Gamma_{uu}(e^{j\omega}). \quad (5.34)$$

In the above equations, $h_i(n)$ and $|H_i(e^{j\omega})|$ are the impulse and magnitude responses of the i th analysis filter, respectively; $\gamma_{uu}(l)$ and $\Gamma_{uu}(e^{j\omega})$ are the autocorrelation function and the power spectrum of the input signal $u(n)$, respectively. Notice that the results in (5.33) and (5.34) are consistent with that of (4.41) in Section 4.3.2, where we analyze the inherent decorrelating properties of the NSAF algorithm.

The correlation matrix $\mathbf{R}_w$ is completely determined by $\gamma_{\xi}(l)$. Therefore, the condition number of the weighted correlation matrix $\mathbf{R}_w$ is bounded from above by the dynamic range of the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$:

$$\chi(\mathbf{R}_w) \leq \frac{\max_{\omega} \Gamma_{\xi}(e^{j\omega})}{\min_{\omega} \Gamma_{\xi}(e^{j\omega})}. \quad (5.35)$$

The band-partitioning and normalization features in (5.34) flattens the input spectrum $\Gamma_{uu}(e^{j\omega})$, giving rise to an equalized $\Gamma_{\xi}(e^{j\omega})$ with a reduced spectral dynamic range. The equalized spectrum $\Gamma_{\xi}(e^{j\omega})$ brings down the eigenvalue spread of the correlation matrix $\mathbf{R}_w$, thereby renders the NSAF algorithm less sensitive to the coloring of the input signal.

5.4 Mean-Square Analysis

In this section, stability bounds for the step size and an expression for the excess MSE are derived based on the energy conservation relation proposed by Sayed *et al.* in [104], [106], [107], [110], [131], [132].

5.4.1 Energy conservation relation

The first step in establishing an energy conservation relation for the NSAF algorithm is to represent the weight recursion in terms of weight-error vector $\mathbf{\varepsilon}(k) \equiv \mathbf{w}_o - \mathbf{w}(k)$.

Thus, subtracting (5.1) from $\mathbf{w}_o$, we get

$$\mathbf{\varepsilon}(k+1) = \mathbf{\varepsilon}(k) - \mu \mathbf{U}(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k). \quad (5.36)$$

Let $\mathbf{e}_a(k) = \mathbf{U}^T(k) \mathbf{\varepsilon}(k)$ and $\mathbf{e}_p(k) = \mathbf{U}^T(k) \mathbf{\varepsilon}(k+1)$ denote the *a priori* error and *a posteriori* error, respectively. Multiplying both sides of (5.36) by $\mathbf{U}^T(k)$ from the left, the error quantities $\mathbf{e}_a(k)$ and $\mathbf{e}_p(k)$ can be related via

$$\begin{aligned} \mathbf{U}^T(k) \mathbf{\varepsilon}(k+1) &= \mathbf{U}^T(k) \mathbf{\varepsilon}(k) - \mu \mathbf{U}^T(k) \mathbf{U}(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k) \\ \mathbf{e}_p(k) &= \mathbf{e}_a(k) - \mu [\mathbf{U}^T(k) \mathbf{U}(k)] \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k). \end{aligned} \quad (5.37)$$

The *a priori* error $\mathbf{e}_a(k)$ represents the undisturbed estimation error produced by the current tap-weight vector, whereas the *a posteriori* error $\mathbf{e}_p(k)$ indicates the undisturbed estimation error produced by the updated tap-weight vector. Assuming that the subband data matrix $\mathbf{U}(k)$ has full column rank, and thus the matrix $\mathbf{U}^T(k) \mathbf{U}(k)$ is invertible, (5.37) can be solved for $\mu \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k)$, as follows

$$\begin{aligned} \mu [\mathbf{U}^T(k) \mathbf{U}(k)] \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k) &= \mathbf{e}_a(k) - \mathbf{e}_p(k) \\ \mu \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k) &= [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} [\mathbf{e}_a(k) - \mathbf{e}_p(k)]. \end{aligned}$$

Substituting the result into (5.36), and rearranging terms, the NSAF algorithm can be written in terms of the weight-error error vectors and the undisturbed estimation errors at two successive iterations, $\{\boldsymbol{\varepsilon}(k), \boldsymbol{\varepsilon}(k+1), \mathbf{e}_a(k), \mathbf{e}_p(k)\}$, in the following form

$$\begin{aligned} \boldsymbol{\varepsilon}(k+1) + \mathbf{U}(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_a(k) \\ = \boldsymbol{\varepsilon}(k) + \mathbf{U}(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_p(k). \end{aligned} \quad (5.38)$$

Evaluating the energy (i.e., taking the squared Euclidean norm) of expression on the left-hand side of (5.38), we find that

$$\begin{aligned} \left\| \boldsymbol{\varepsilon}(k+1) + \mathbf{U}(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_a(k) \right\|^2 \\ = \left\{ \begin{aligned} &\left\| \boldsymbol{\varepsilon}(k+1) \right\|^2 + 2\mathbf{e}_p^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_a(k) \\ &+ \mathbf{e}_a^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_a(k) \end{aligned} \right\}. \end{aligned} \quad (5.39)$$

Similarly, the energy of the expression on right-hand side of (5.38) is given by

$$\begin{aligned} \left\| \boldsymbol{\varepsilon}(k) + \mathbf{U}(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_p(k) \right\|^2 \\ = \left\{ \begin{aligned} &\left\| \boldsymbol{\varepsilon}(k) \right\|^2 + 2\mathbf{e}_a^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_p(k) \\ &+ \mathbf{e}_p^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_p(k) \end{aligned} \right\}. \end{aligned} \quad (5.40)$$

By equating the energies of both sides of (5.38), we arrive at the following energy conservation relation:

$$\begin{aligned} \left\| \boldsymbol{\varepsilon}(k+1) \right\|^2 + \mathbf{e}_a^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_a(k) \\ = \left\| \boldsymbol{\varepsilon}(k) \right\|^2 + \mathbf{e}_p^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_p(k). \end{aligned} \quad (5.41)$$

The energy conservation relation in (5.41) shows how the energies of the weight-error vectors at two successive iterations are related to the *weighted* energies of the a priori and a posteriori errors. It has been shown in [107], [110], [131], [132] that similar energy conservation relation can be established for a handful of adaptive filtering algorithms, for examples, LMS, NLMS, APA, and RLS algorithms.

5.4.2 Variance relation

Before proceed further, let $c(k)$ denote the mean-square deviation, defined by

$$c(k) = E\{\|\mathbf{\epsilon}(k)\|^2\}. \quad (5.42)$$

By taking the expectation of both sides of (5.41), the energy relation can be translated into a variance relation in terms of $\{c(k), c(k+1)\}$:

$$\begin{aligned} c(k+1) + E\{\mathbf{e}_a^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_a(k)\} \\ = c(k) + E\{\mathbf{e}_p^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_p(k)\}. \end{aligned} \quad (5.43)$$

Knowing that $\mathbf{e}_p(k)$ is related to $\mathbf{e}_a(k)$ via (5.37), the second term of the expression on the right-hand side of (5.43) can be expanded into the following form

$$\begin{aligned} E\{\mathbf{e}_p^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_p(k)\} \\ = \left\{ \begin{aligned} &E\{\mathbf{e}_a^T(k) [\mathbf{U}^T(k) \mathbf{U}(k)]^{-1} \mathbf{e}_a(k)\} \\ &-2\mu E\{\mathbf{e}_a^T(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_d(k)\} \\ &+\mu^2 E\{\mathbf{e}_d^T(k) \mathbf{\Lambda}^{-1}(k) [\mathbf{U}^T(k) \mathbf{U}(k)] \mathbf{\Lambda}^{-1}(k) \mathbf{e}_d(k)\} \end{aligned} \right\}. \end{aligned} \quad (5.44)$$

Using (5.44) in (5.43), the variance relation can be simplified to

$$\begin{aligned} c(k+1) = c(k) - 2\mu E\{\mathbf{e}_a^T(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_d(k)\} \\ + \mu^2 E\{\mathbf{e}_d^T(k) \mathbf{\Lambda}^{-1}(k) [\mathbf{U}^T(k) \mathbf{U}(k)] \mathbf{\Lambda}^{-1}(k) \mathbf{e}_d(k)\}. \end{aligned} \quad (5.45)$$

Various aspects of the convergence behavior of NSAF algorithm can be analyzed by evaluating the mean-squared values of the error quantities $\{\mathbf{\epsilon}(k), \mathbf{\epsilon}(k+1), \mathbf{e}_a(k), \mathbf{e}_d(k)\}$ that appear in the variance relation (5.45). To this end, it should be noted that no approximation has been used to establish the energy conservation relation (5.41) and the variance relation (5.45) as well. Hence, reliable results for the mean-square performance can be obtained by manipulating the variance relation.

5.4.3 Stability of the NSAF algorithm

Assuming that the diagonal assumption is valid (i.e., the off-diagonal elements of the matrix $\mathbf{U}^T(k)\mathbf{U}(k)$ are negligible), and the regularization parameter α has been set to zero, such that $\mathbf{U}^T(k)\mathbf{U}(k) \approx \mathbf{\Lambda}(k)$, the variance relation of (5.45) can be further simplified to

$$c(k+1) - c(k) \approx \mu^2 E\{\mathbf{e}_D^T(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k)\} - 2\mu E\{\mathbf{e}_a^T(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k)\}. \quad (5.46)$$

The underlying principle of the NSAF algorithm is to have the updated tap-weight vector $\mathbf{w}(k+1)$ as close as possible to the previous weight vector $\mathbf{w}(k)$ subject to a set of subband constraints. For the algorithm to be stable, the mean-square deviation $c(k)$ must decrease monotonically with increasing k , i.e., $c(k+1) < c(k)$ from one iteration to the next. From (5.46), for $c(k+1)$ to be less than $c(k)$, the step size μ has to fulfill the following condition

$$0 < \mu < 2 \left\{ \frac{E\{\mathbf{e}_a^T(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k)\}}{E\{\mathbf{e}_D^T(k) \mathbf{\Lambda}^{-1}(k) \mathbf{e}_D(k)\}} \right\}. \quad (5.47)$$

Now, consider the situation where the disturbance $\mathbf{\eta}_D(k)$ in (5.10) is negligible.

Under the noiseless situation, it is obvious that the estimation error $\mathbf{e}_D(k)$ is equal to the undisturbed error $\mathbf{e}_a(k) = \mathbf{U}^T(k) \mathbf{\epsilon}^T(k)$. Hence, in the absence of disturbance, the necessary and sufficient condition for the convergence in the mean-square sense is that the step-size parameter must satisfy the double inequality

$$0 < \mu < 2. \quad (5.48)$$

5.4.4 Steady-state excess MSE

The steady-state excess MSE $J_{ex}(\infty)$ defined in (5.25) indicates the difference between the MSE $J(\infty)$ produced by an adaptive algorithm operating in steady state and the minimum MSE $J_{\min}$ pertaining to the optimum Wiener solution $\mathbf{w}_o$. The adaptive algorithm is said to operate in steady state [107], [131], [132] if its mean-squared deviation $c(k)$ tends to a finite constant value c as k increases without bound, as follows

$$c(k+1) = c(k) = c < \infty, \text{ as } k \rightarrow \infty. \quad (5.49)$$

It should be emphasized that the step-size parameter μ has to be small enough, as bounded in (5.47) or (5.48), in order to guarantee a convergence to the steady state.

In what follows, an expression for the steady-state excess MSE $J_{ex}(\infty)$ is derived for the NSAF algorithm. For mathematical simplicity, we assume that the filter banks are paraunitary, the diagonal assumption is valid, and the regularization parameter has been set to zero. Furthermore, it is also customary to assume that the desired response arises from the linear data model of (5.8). Substituting (5.10) into (5.46), and by exploiting the fact that the noise vector $\boldsymbol{\eta}_D(k)$ is uncorrelated with the weight-error vector $\boldsymbol{\varepsilon}(k)$ due to the paraunitary filter banks, we arrive at

$$\begin{aligned} c(k+1) - c(k) \approx & (\mu^2 - 2\mu) E\{\mathbf{e}_a^T(k) \boldsymbol{\Lambda}^{-1}(k) \mathbf{e}_a(k)\} \\ & + \mu^2 E\{\boldsymbol{\eta}_D^T(k) \boldsymbol{\Lambda}^{-1}(k) \boldsymbol{\eta}_D(k)\}. \end{aligned} \quad (5.50)$$

Introducing the steady-state condition (5.49) into (5.50), we obtain an equation that describes the steady-state relation between the subband undisturbed error signals $e_{i,a}(k)$ and the subband disturbances $\eta_{i,D}(k)$, as follows

$$E\{\mathbf{e}_a^T(k)\Lambda^{-1}(k)\mathbf{e}_a(k)\} \approx \frac{\mu}{2-\mu} E\{\boldsymbol{\eta}_D^T(k)\Lambda^{-1}(k)\boldsymbol{\eta}_D(k)\}, \text{ as } k \rightarrow \infty. \quad (5.51)$$

By virtue of the diagonal assumption, the normalization matrix $\Lambda^{-1}(k)$ can be assumed diagonal with its diagonal elements given by $1/\|\mathbf{u}_i(k)\|^2$ for the case of zero regularization parameter. Using this result in (5.51), the steady-state variance relation can be equivalently written as

$$\sum_{i=0}^{N-1} E\left\{\frac{e_{i,a}^2(k)}{\|\mathbf{u}_i(k)\|^2}\right\} \approx \frac{\mu}{2-\mu} \sum_{i=0}^{N-1} E\left\{\frac{\eta_{i,D}^2(k)}{\|\mathbf{u}_i(k)\|^2}\right\}, \text{ as } k \rightarrow \infty. \quad (5.52)$$

If we further assume that the subband signals are uncorrelated such that the variance $E\{e_{i,a}^2(k)\}$ of the undisturbed error signal in the i th subband is due only to the variance $E\{\eta_{i,D}^2(k)\}$ of disturbance in the same subband; then we may deduce that the i th terms of the summations on both sides of (5.52) correspond to each other, as follows

$$E\left\{\frac{e_{i,a}^2(k)}{\|\mathbf{u}_i(k)\|^2}\right\} \approx \frac{\mu}{2-\mu} E\left\{\frac{\eta_{i,D}^2(k)}{\|\mathbf{u}_i(k)\|^2}\right\}, \text{ for } i = 0, 1, \dots, N-1, \text{ as } k \rightarrow \infty. \quad (5.53)$$

For high-order filter, where $M \gg 1$, the fluctuation in the subband energy $\|\mathbf{u}_i(k)\|^2$ from one iteration to the next can be assumed to be small enough, thereby, justifying the following assumptions

$$E\left\{\frac{e_{i,a}^2(k)}{\|\mathbf{u}_i(k)\|^2}\right\} \approx \frac{E\{e_{i,a}^2(k)\}}{E\{\|\mathbf{u}_i(k)\|^2\}} \text{ and } E\left\{\frac{\eta_{i,D}^2(k)}{\|\mathbf{u}_i(k)\|^2}\right\} \approx \frac{E\{\eta_{i,D}^2(k)\}}{E\{\|\mathbf{u}_i(k)\|^2\}}. \quad (5.54)$$

Inserting this approximation into (5.53), we arrive at

$$E\{e_{i,a}^2(k)\} \approx \frac{\mu}{2-\mu} E\{\eta_{i,D}^2(k)\}, \text{ for } i = 0, 1, \dots, N-1, \text{ as } k \rightarrow \infty. \quad (5.55)$$

As defined in (5.25), the steady-state excess MSE $J_{ex}(\infty)$ is given by the average of the variance of the subband undisturbed error signals. Using (5.55) in (5.25), the steady-state excess MSE for the NSAF algorithm can be expressed as

$$J_{ex}(\infty) \approx \frac{\mu}{2-\mu} \left[\frac{1}{N} \sum_{i=0}^{N-1} E\{\eta_{i,D}^2(k)\} \right] = \frac{\mu\sigma_\eta^2}{2-\mu}, \quad (5.56)$$

where σ_η^2 denotes the variance of the additive white noise, as defined in (5.24). The expression holds as long as the frequency responses of the analysis filters do not overlap significantly and their stopband attenuation is sufficiently high. Otherwise, the excess MSE would generally be higher than that given in (5.56), because the variance of the undisturbed error signal $E\{e_{i,a}^2(k)\}$ would receive contribution from cross-channel disturbances. Recall that the NLMS algorithm can be seen as special case of the NSAF algorithm with $N=1$. The expression for steady-state excess MSE given by (5.56) is consistent with that of the NLMS algorithm reported in the literature.

From (5.56), it is obvious that the excess MSE $J_{ex}(\infty)$ is independent of the number of subbands N . Hence, the convergence of the NSAF algorithm can be accelerated by increasing the number of subbands N , while maintaining the same level of steady-state excess MSE with the step size remains unchanged. The drawback is that additional delay is introduced into the signal path, since higher order analysis filters are required in order to achieve sufficient stopband attenuation. Signal path delay can be eliminated with a delayless structure, as we shall see later in Chapter 6.

5.5 Simulations

In this section, the validity of the analytical results is demonstrated through simulations, which are performed in the context of system identification. The desired response $d(n)$ is generated by means of the linear data model (5.7). The unknown system $\mathbf{w}_0$ has a length of $M = 1024$ taps, as shown in Figure 4.8, and the additive noise $\eta(n)$ has a variance of $\sigma_\eta^2 = 1 \times 10^{-4}$. Two types of input signals, namely, white Gaussian noise and AR(2) random signal, are used in the simulations. The spectrum of the AR(2) random signal is plotted in Figure 4.11(a). The NSAF algorithm is chosen to have $N = 4, 8, 16$, and 32 subbands, with the regularization parameter α set to 0.0001 for all cases. The length of the adaptive tap-weight vector $\mathbf{w}(k)$ is fixed at $M = 1024$, with the tap weights initialized to zero in all the simulations.

Paraunitary cosine-modulated filter banks [73] with filter length of $L = 8N$ are used for the analysis and synthesis sections, since we impose the paraunitary condition on the analysis filters for mathematical simplicity in Section 5.1.3. Nevertheless, pseudo-QMF cosine-modulated filter banks can also be used. It has been confirmed in [56] that good pseudo-QMF designs can result in analysis filter bank that is *almost* paraunitary.

5.5.1 Mean of the projection matrix

The major motivation of the NSAF algorithm is to improve the convergence rate of the NLMS algorithm under colored excitation. The effectiveness of the NSAF algorithm in dealing with colored excitation can be seen from the mean behavior of the projection matrix $\mathbf{P}(k)$, i.e., the weighted correlation matrix $\mathbf{R}_w$. Table 5.1 shows the condition numbers of $\mathbf{R}_w$ for $N = 1, 4, 8, 16$, and 32 subbands, using

white and colored excitation signals. Recall that the special case of NSAF with $N = 1$ corresponds to the NLMS algorithm, in which the convergence of the algorithm is governed by a normalized correlation matrix of the form $\mathbf{R}_w \approx \mathbf{R}/M\gamma_{uu}(0)$, where $\mathbf{R} \equiv E\{\mathbf{u}(n)\mathbf{u}^T(n)\}$ is the fullband correlation matrix, and $\gamma_{uu}(0) \equiv E\{u^2(n)\}$ denotes the variance of the excitation signal.

The condition numbers of the weighted correlation matrices for white excitation signal are listed in the second column of Table 5.1. The correlation matrices, for $N = 1, 4, 8, 16$, and 32 subbands, are equally well-conditioned with their condition numbers approximately equal to one. This explains why the NLMS and the NSAF algorithms exhibit identical learning curves when the input signal is white, as shown in Figure 4.9. The convergence rate of the NLMS algorithm deteriorates for colored excitation, where the condition number increases tremendously from 1.471 to 414.997 (for $N = 1$). By introducing more constraints into the tap-weight adaptation (i.e., by increasing the number of subbands, N), the condition number of the weighted correlation matrix decreases accordingly, as can be observed from the third column of Table 5.1. Smaller

Table 5.1 Condition number of the weighted correlation matrix $\mathbf{R}_w$ for white and colored excitation signals.

Number of subbands, N	Condition number, $\chi(\mathbf{R}_w)$	
	White	Colored
1	1.471	414.997
4	1.488	68.139
8	1.480	19.518
16	1.488	6.372
32	1.486	2.298

eigenvalue spread gives a faster convergence rate. The fact can be observed from the learning curves in Figure 4.10. From Table 5.1, it can be concluded that, the convergence of algorithm becomes less sensitive to the coloring of the input signal with an increased number of subbands.

5.5.2 Stability bounds

Figures 5.3 and 5.4 show the steady-state MSE $J(\infty)$ of the NSAF algorithm (with $N = 4, 8, 16$, and 32 subbands), plotted as a function of the step-size μ , for white noise and the AR(2) excitation signals, respectively. At each step-size μ that varies from 0.01 to 1.99, the ensemble average learning curve is first obtained by averaging the instantaneous squared error curve over 200 independent trials. For each trial, the NSAF algorithm is iterated until a steady-state is reached. The time-average of the last 15,000 samples of the ensemble average learning curve is then used to calculate the steady-state MSE.

The steady-state MSE curve, treated as a function of the step-size, is bounded from below and from the right by two asymptotes, namely, $J(\infty) = J_{\min} = -40$ dB and $\mu = 2.0$, as illustrated in Figures 5.3 and 5.4. As the step-size approaches the stability bound at $\mu = 2.0$, the steady-state MSE increases tremendously. The algorithm diverges when the step-size μ is outside the stability bounds defined in (5.48). The minimum MSE, $J_{\min}$, is given by the variance of the disturbance, $\sigma_{\eta}^2 = 1 \times 10^{-4}$. Smaller step-size leads to a lower steady-state MSE towards the vicinity of the horizontal asymptote $J(\infty) = J_{\min}$. Nevertheless, the attainable steady-state MSE is always higher than $J_{\min}$, which reflects the stochastic nature of the algorithm.

Figure 5.3 Steady-state MSE of the NSAF algorithm (for $N = 4, 8, 16$, and 32 subbands) under white noise excitation.

Figure 5.4 Steady-state MSE of the NSAF algorithm (for $N = 4, 8, 16$, and 32 subbands) under colored excitation.

5.5.3 Steady-state excess MSE

Figures 5.5 and 5.6 show the experimental and theoretical values of the steady-state excess MSE for the NSAF algorithm at various step-sizes. The theoretical values are calculated by using the expression in (5.56). For Figure 5.5, the NSAF algorithm (for $N = 4, 8, 16$ and 32 subbands) is excited by white Gaussian noise, whereas the results in Figure 5.6 is for the case of the AR(2) random signal. Observe that the expression (5.56) leads to a good fit between the theory and practice over the range of step-size from 0.01 to 1.00, for white and colored excitation signals. Both the theoretical and experimental results also agree that the excess MSE, $J_{ex}(\infty)$, is independent to the number of subbands, N .

Figure 5.5 Steady-state Excess MSE of the NSAF algorithm (for $N = 4, 8, 16$, and 32 subbands) under white excitation.

Figure 5.6 Steady-state Excess MSE of the NSAF algorithm (for $N = 4, 8, 16$, and 32 subbands) under colored excitation.

Comparing the results in Figure 5.5 to those in Figure 5.6, it can be noted that a better fit between theory and practice can be obtained for the case of white excitation. Recall that, we have assumed that the subband signals are uncorrelated in order to arrive at (5.53) from (5.52). Consequently, the experimental values in Figure 5.6 deviate from the values given by the expression (5.56). However, when the input signal is white, all the subband quantities in (5.52) are normalized by the same factor, where

$$E\{\|\mathbf{u}_i(k)\|^2\} = M\sigma_u^2, \text{ for } i = 0, 1, \dots, N-1. \quad (5.57)$$

Therefore, (5.52) directly leads to (5.53) without needing the assumption that the subband signals are uncorrelated, which is generally not true even for the case of white excitation. Consequently, the experimental values in Figure 5.5 present a better match with theoretical values given by (5.56).

5.6 Conclusions

This chapter analyzed and evaluated the convergence behavior of the NSAF algorithm in the mean and mean-square senses. Filter banks are essential elements for subband adaptive filtering. However, the existence of the filter banks in SAFs complicates the convergence analysis of subband adaptive algorithms. In order to make the mathematical analysis tractable, filter banks were not explicitly included in the convergence analysis presented in this chapter. Instead, the properties that the filter bank would impose on the subband signals were identified and taken into consideration. In particular, a novel multiband MSE function was introduced as the performance measure for the NSAF algorithm. By virtue of the paraunitary property of the analysis filter bank, the multiband MSE function was shown to be equivalent to the classical MSE function. Since the multiband MSE function is defined in terms of subband estimation errors, mathematical terms pertaining to the filter banks were avoided in the mathematical analysis.

Mean analysis presented in Section 5.3 showed that the convergence of the algorithm in the mean is governed by a weighted correlation matrix, which can be completely defined by an equalized spectrum. The band-partitioning and normalization features of the NSAF algorithm flatten the input spectrum, giving rise to the equalized spectrum with a reduced spectral dynamic range. As a special case of the NSAF algorithm with $N = 1$ subband, the input spectrum to the NLMS recursion is normalized but not equalized. By increasing the number of subbands N , the NSAF algorithm becomes less sensitive to the coloring of the input signal.

Mean-square performance of the NSAF algorithm was analyzed based on an energy conservation relation. By manipulating various error quantities in the energy relation, stability bounds for the step-size and an expression for the steady-state MSE

are derived. In spite of a number of assumptions that have been introduced, a good match between theoretical and experimental results is confirmed through simulations. In particular, both the theoretical and experimental results agree that the excess MSE is independent to the number of subbands N . Mean analysis revealed that the convergence of the NSAF algorithm can be accelerated by increasing the number of subbands. Therefore, for the case of colored excitation, higher convergence rate can be achieved by increasing the number of subbands, without compromising on the steady-state excess MSE.

Chapter 6

A Comparative Study of Subband Adaptive Filtering Structures

A filter bank partitions its input signal into multiple subbands, occupying contiguous portions of the original frequency band. The subband signals can be processed at a lower rate commensurate with their bandwidths. Subsequently, with a proper design of analysis/synthesis filters that satisfies a certain structural relationship [28], [73], [82], [105], [125], a fullband output signal with minimum distortion can be generated. By so doing, a signal processing task is decomposed into multiple parallel operations, which deal with narrower bandwidths. This feature permits the manipulation of the subband signals in such a way that their properties can be exploited, thereby, allowing a more effective processing. Furthermore, processing the subband signals at the decimated rate would lead to a computationally efficient implementation for the signal processing system. The idea of subband filtering evolved primarily from subband coding [52]. The same idea has been extended to subband adaptive filtering [34], [35], [55], [116]. Since then, a number of adaptive filtering structures based on subband and multirate techniques have been proposed in the literature [13], [14], [46], [61], [62], [79], [80], [85], [99].

This chapter begins with an introduction to the conventional subband adaptive filters (SAFs). Two major structural problems encountered in the conventional structure are investigated, along with a discussion on some of the solutions [35], [36], [130] that have been reported in the literature. In the second part, the normalized SAF

(NSAF) is compared with the conventional SAF, and we show that the structural problems are annihilated in the NSAF. The concept of delayless subband adaptive filtering [85], [121] is presented in the third part of this chapter. Finally, the performances of the delayless SAF and its variants are compared through simulations.

6.1 Conventional Subband Adaptive Filters

Figure 6.1 shows the conventional structure for subband adaptive filtering [27], [34], [35], [55], [111], [116]. The input signal $u(n)$ and desired response $d(n)$ are split into N spectral bands by means of analysis filters $H_0(z), H_1(z), \dots, H_{N-1}(z)$. These subband signals are decimated by the same factor D , to a lower rate, before being processed by separate adaptive subfilter $\bar{W}_i(z)$ in each subband. Each of the adaptive subfilters has its own adaptation loop, where estimation error $e_{i,D}(k)$ is locally evaluated for updating the subband tap-weights in such a way that the subband errors are minimized in some statistical senses. The fullband error signal $e(n)$ is then obtained by interpolating and recombining the outputs of the adaptive subfilters using a synthesis filter bank. Notice that the variable n is used to index the original sequences, and k is used for the time index of the decimated sequences.

The primary motivation for subband adaptive filtering is to reduce the computational complexity and to improve the convergence performance against high eigenvalue disparity [34], [55], [116]. In Figure 6.1, the set of adaptive subfilters $\bar{W}_0(z), \bar{W}_1(z), \dots, \bar{W}_{N-1}(z)$ are used to model the fullband unknown system $W_o(z)$. Computational saving is achieved since the adaptive subfilters can be shorter in length, and operate at a lower rate, than an equivalent fullband adaptive filter. Furthermore, the subband signals, in general, will have a flatter spectrum (i.e., less correlated), and thus faster convergence is possible for gradient-based adaptive algorithms.

Figure 6.1 Conventional subband adaptive filter. A set of adaptive subfilters are used to identify the unknown system.

Consider the situation where the impulse response of the unknown system $W_o(z)$ can be modeled to a sufficient accuracy with a fullband transversal filter $W(z)$ of length M . The subfilters $\bar{W}_i(z)$, as shown in Figure 6.1, operate at a decimated rate D times slower than the fullband rate. Hence, the length of the subfilters can be reduced to $M_s = M/D$, and yet cover the same duration as the fullband filter $W(z)$. Taking all the N subbands into consideration, there are a total number of $M_s \times N = MN/D$ subband tap-weights. With a decimation factor of D , the number of multiplications incurred by the subfilters is thus MN/D^2 in one fullband sampling period. Compared to the fullband filter, which requires M multiplications, the gain factor is

$$\frac{\text{complexity of fullband filter}}{\text{complexity of subfilters}} = \frac{D^2}{N}, \quad (6.1)$$

where $D \leq N$ for a minimum aliasing distortion. The greatest computational saving is achieved for the case of critical subsampling, where $D = N$. Notice that the result in (6.1) does not include the overheads incurred by the filter banks. Moreover, the computations involved in adapting the tap weights are not considered as well. Equation (6.1) serves as a general guideline indicating the computational saving that can be achieved with subband filtering compared with fullband filtering. In [27, pp. 306-307], the computational efficiency of subband filtering was evaluated for the case when complex-valued DFT filter banks are employed.

Gradient-based algorithms (e.g., LMS and NLMS) suffer from slow convergence when the input signal is highly correlated. The slow convergence problem is intimately related to the wide spread of eigenvalues in the input autocorrelation matrix, which is due to the large spectral dynamic range of the input signal [18], [27], [44], [74]. Subband decomposition followed by a critical decimation (where $D = N$) operation, reduces the spectral dynamic range in each subband. Figure 6.2 shows the power spectrum $\Gamma_{uu}(e^{j\omega})$ of an AR(2) random signal, together with the magnitude responses of the analysis filters, $|H_i(e^{j\omega})|$, in a four-channel cosine-modulated filter bank. The analysis filters have a finite length of $L = 64$ taps. Using the notion of the correlation-domain formulation defined in Section 3.1, the power spectrum of the subband signal $u_i(n)$ can be expressed as

$$\Gamma_{ii}(e^{j\omega}) = |H_i(e^{j\omega})|^2 \Gamma_{uu}(e^{j\omega}), \text{ for } i = 0, 1, \dots, N-1. \quad (6.2)$$

The power spectrum $\Gamma_{ii}(e^{j\omega})$ is depicted in Figure 6.3. Notice that the subband signals $u_0(n), u_1(n), \dots, u_{N-1}(n)$ have a smaller bandwidth than the original fullband signal $u(n)$. Decimating the subband signals to a lower rate commensurate with their bandwidth, reduces the spectral dynamic range. The autocorrelation function of the

Figure 6.2 Power spectrum of an AR(2) random signal and the magnitude responses of all the analysis filter in a four-channel pseudo-QMF cosine-modulated filter bank.

decimated subband signal $u_{i,D}(k)$ is the decimated version of the autocorrelation function of the subband signal $u_i(n)$. Hence, the power spectrum $\Gamma_{ii,D}(e^{j\omega})$ of the decimated subband signal $u_{i,D}(k)$, as shown in Figure 6.4, can be obtained from (6.2), as follows

$$\Gamma_{ii,D}(e^{j\omega}) = \frac{1}{N} \sum_{l=0}^{N-1} \Gamma_{ii}(e^{j(\omega-2\pi l)/N}). \quad (6.3)$$

Clearly, the spectrum of the subband signal after critical decimation, $\Gamma_{ii,D}(e^{j\omega})$, is closer to that of a white noise. Intuitively, faster convergence is possible because the spectral dynamic ranges of the subband signals are smaller than that of the fullband signal. Furthermore, the length of the subfilters is shorter with respect to the fullband filter, which further increases the convergence rate [47], [48]. However, detailed analysis shows that conventional structure suffers from aliasing and band-edge effects [16], [19], [35], [36], [84], which limit its convergence performance, as we shall examine closely in the next section.

Figure 6.3 Power spectra of the subband signals before decimation.

Figure 6.4 Power spectra of the subband signals after critical decimation.

6.1.1 Aliasing and band-edge effects

Critical decimation reduces the spectral dynamic range in each subband, thereby accelerates the convergence of the adaptive subfilters. However, aliasing components are introduced into the subbands due to the subsampling operation. The aliased part of the subband spectrum acts as a noise that disturbs the adaptation process, resulting in a high asymptotic level in the reconstructed fullband MSE. Note that aliasing is unavoidable in the individual subbands, even though the aliasing components can be collectively cancelled in the reconstructed signal by the synthesis filter bank with a perfect reconstruction analysis/synthesis system.

In order to reduce the effects of aliasing in the system, oversampled schemes ($D < N$) are often used instead of critically sampled schemes ($D = N$). Notable success has been achieved using oversampled DFT filter bank [6], [16], [26], [42]. The complex-valued subband signals are subsampled at a higher rate than that actually required for their narrower bandwidth. Severe aliasing components contaminating the subband signals are avoided. Nevertheless, oversampling the subband signals gives rise to small eigenvalues in the subband autocorrelation matrices, due to the band edges of the analysis filters [84]. This phenomenon is termed as the band-edge effects, and can be observed from the MSE learning curve as a slow asymptotic convergence, as shown in Section 6.5.

Figure 6.5 depicts the magnitude responses $|H_i(e^{j\omega})|$ of the analysis filters of a four-channel DFT filter bank, together with the power spectrum $\Gamma_{uu}(e^{j\omega})$ of the AR(2) random signal over the range $0 \leq \omega \leq 2\pi$. The prototype filter $P(z)$, of length $L=128$, is designed using the MATLAB *fir1*(127,1/4) routine. The analysis filters $H_i(z)$ are then generated by exponential modulating the prototype $P(z)$ via

Figure 6.5 Power spectrum of an AR(2) random signal and the magnitude responses of all the analysis filters in a four-channel uniform DFT filter bank.

$H_i(z) = P(z e^{-j2\pi i/N})$, for $i = 0, 1, \dots, N-1$. Notice that the analysis filters $H_i(z)$ are (in general) complex, and hence $|H_i(e^{j\omega})|$ does not necessarily exhibit symmetry with respect to $\omega = 0$. For N even, there is a lowpass filter at $i = 0$, and a highpass filter at $i = N/2$; both filters have real coefficients, and are symmetric with respect to $\omega = 0$.

The analysis filters $H_i(z)$ partition the input spectrum $\Gamma_{uu}(e^{j\omega})$ into $N = 4$ spectral bands. For $2\times$ oversampling, the subband signals are decimated with a factor of $D = N/2 = 2$. The resulting subband spectra [using (6.2) and (6.3)] are depicted in Figure 6.6. Notice that the subband spectrum exhibit large dynamic range over the band edges, which cause the subband correlation matrix to be ill-conditioned, thereby degrading the overall convergence rate. Figure 6.7 shows the spectrum of the subband signals for a critically sampled scheme, where $D = N = 4$. Clearly, the band edges are shaved down by the decimation operation. However, the subband signals contains undesirable aliasing components (indicated by the thin lines) that impair the adaptation ability of the adaptive subfilters.

Figure 6.6 Spectra of the subband signals in a $2\times$ oversampled SAF using DFT filter bank. Band edges of the subband spectrum introduce small eigenvalues into the subband autocorrelation matrix.

Figure 6.7 Spectra of the subband signals in a critically sampled SAF using DFT filter bank. Critical decimation shaves down the band-edges, at the same time, distorts the subband signals with aliasing (indicated by the thin lines).

A number of solutions have been proposed to overcome the aliasing effects encountered in the conventional SAF with critical subsampling, as outlined below.

- (i) Using less overlapping analysis filters with spectral gaps between adjacent subbands [130]. This approach tends to introduce spectral holes (located at the edges of the subbands) in the output of the adaptive filter. In acoustic echo cancellation, the error signal contains local speech to be sent to the far-end room. These spectral gaps impair the signal quality (especially when the number of subbands is large), and thus this approach is unacceptable in practice [35], [36].
- (ii) Incorporating adaptive cross-filters between adjacent subbands to compensate for the aliasing effects [35], [36]. The cross-filters adaptively process the input signal from adjacent subbands so as to compensate for the existence of the alias components due to the critical decimation. The adaptive cross-filters promote an accurate identification of the unknown system, as we shall see in Section 6.1.2.
- (iii) Employing a multiband scheme [13], [14], [61], [62], [63], [77], [78], [99]. The aliasing effects can be seen as the direct consequence of substituting the fullband modeling filter with separate subfilters in each subband. In a multiband scheme, the modeling filter is no longer separated into subfilters. Instead, subband signals are used to adapt the fullband tap weights of the modeling filter. We shall discuss this issue in Section 6.2.
- (iv) Employing a delayless close-loop structure. In [46], [85], [92], it has been recognized that the aliasing effects can actually be reduced with a closed-loop structure, which minimizes the fullband MSE instead of individual subband MSE. The capability of the delayless closed-loop structure in eliminating the aliasing effects will be investigated in Section 6.3.

6.1.2 Adaptive cross-filters

From a subband identification perspective [35], [36], [79], [80], a matrix of subfilters $\bar{\mathbf{W}}(z) = [\bar{W}_{i,j}(z)]$, for $i, j = 0, 1, \dots, N-1$, is indeed required in order to model the unknown system $W_o(z)$ correctly with a critically-sampled subband structure. The idea is illustrated in Figure 6.8. The diagonal elements $\bar{W}_{i,i}(z)$ of the model matrix $\bar{\mathbf{W}}(z)$ correspond to the adaptive subfilters $\bar{W}_i(z)$ in Figure 6.1; whereas the off-diagonal elements, $\bar{W}_{i,j}(z)$ for $j \neq i$, are the adaptive cross-filters between subbands. The model matrix $\bar{\mathbf{W}}(z)$ forms the estimates $y_{i,D}(k)$ of the desired response components $d_{i,D}(k)$ from the subband input signals $u_{i,D}(k)$. The adaptive cross-filter $\bar{W}_{i,j}(z)$ channel the signal $u_{j,D}(k)$ from other subband so as to compensate for the aliasing component in $u_{i,D}(k)$ that arises from critical decimation. With such a

Figure 6.8 Adaptive identification in subbands. The unknown system $W_o(z)$ is modeled with a matrix of subfilters $\bar{\mathbf{W}}(z)$ in the subband domain.

configuration, minimizing the subband MSEs $E\{e_{i,D}^2(k)\}$ is equivalent to minimizing the fullband MSE $E\{e^2(n)\}$, and thus leading to an accurate identification of the unknown system.

Elimination of the adaptive cross-filters, $\bar{W}_{i,j}(z)$ for $j \neq i$, hinders the ability to cancel off the aliasing components and to properly model the unknown system. Assuming that the analysis filters are selective enough (such that they overlap only with adjacent filters), inserting adaptive cross-filters between adjacent subbands is sufficient to compensate for the aliasing effects. The adaptive elements $\bar{W}_{i,i}(z)$, $\bar{W}_{i,i-1}(z)$, $\bar{W}_{i,i+1}(z)$ for the i th subbands, can be adapted using a common estimation error $e_{i,D}(k)$, together with their respective subband input signals $u_{i,D}(k)$, $u_{i-1,D}(k)$, and $u_{i+1,D}(k)$. However, on the downside, the cross-filters increase the computational complexity. Furthermore, they fail to converge quickly to the optimal solution, because the subband signals use in the adaptation are correlated. It has been concluded in [36] that the critically sampled scheme with cross-filters does not give better convergence performance than a fullband adaptive filter.

Recently, an alternative approach has been proposed in [98]. In the new subband structure, as shown in Figure 6.9, cross-channel information for the adaptive subfilter $\bar{W}_i(z)$ is derived from adjacent subbands by means of branches composed by the analysis filters $H_i(z)H_j(z)$ and adjacent subfilters $\bar{W}_j(z)$, where $j = i \pm 1$. These branches have a role similar to the adaptive cross-filters of Figure 6.8 in the sense that they provide the information that can be used to compensate for the aliasing component that arises from critical decimation. The major advantage of the new structure of Figure 6.9 is that the number of subfilters to be adapted remains equal to

the number of subbands N . However, it should be noted that similar copies of the adaptive subfilters are found in the branches where the cross-channel information is derived. Some promising results on the convergence performance of the new structure under colored excitation have also been reported in [98].

Figure 6.9 An N -band critically-sampled subband structure proposed in [98].

6.2 Multiband Scheme

Figure 6.10 shows the normalized SAF (NSAF) with the following distinctive features in comparison with the conventional SAF of Figure 6.1.

- (i) A diagonalized model matrix $W(z)\mathbf{I}_{N \times N}$ that is placed before the decimators: The N -input N -output system $W(z)\mathbf{I}_{N \times N}$ is basically a bank of parallel filters, of length M , with identical transfer function $W(z)$, where $\mathbf{I}_{N \times N}$ denotes the $N \times N$ identity matrix. In the conventional SAF of Figure 6.1, a set of subfilters $\bar{W}_0(z)$, $\bar{W}_1(z), \dots, \bar{W}_{N-1}(z)$ are placed after the decimators so that their order $M_s - 1$ can be reduced according to the expression $M_s = M/D$. That is, the conventional SAF employs a set of decimated subfilters $\bar{W}_i(z)$ that are, in fact, unable to perfectly model the unknown system $W_0(z)$, as discussed Section 6.1.2. In contrast, the NSAF uses a fullband (since identical filters are used in all the subbands) transversal filter $W(z)$ to model the unknown system $W_0(z)$. Perfect modeling can be achieved if the length of modeling filter is sufficiently high, and the analysis filters are power complementary.
- (ii) A novel weight-control mechanism: The modeling filter $W(z)$, placed before the decimators, operates on the set of bandlimited input signals $u_i(n)$ at the original sampling rate. The fullband tap-weights of the modeling filter $\{w_n\}_{n=0}^{M-1}$ are iteratively updated by a single adaptive algorithm that takes the subband signals $\{u_i(n), e_{i,D}(k)\}_{i=0}^{N-1}$ as its inputs. In contrast, there are N adaptive filters (usually updated with the LMS or NLMS algorithms) operate separately in each subband of the conventional SAF, as illustrated in Figure 6.1.

Figure 6.10 A multiband scheme for subband identification. Identical fullband modeling filters are placed in all the subbands.

The multiband nature of the NSAF algorithm can be seen from its weight-control mechanism. The weight adjustment applied on the fullband tap weights, at each iteration, is a weighted combination of the subband regressors $\mathbf{u}_i(k)$, as given by (4.17). From the stochastic interpretation presented in Section 4.3.2, the tap-weight adaptation is shown to be driven by an equalized spectrum of the form

$$\Gamma_{\xi}(e^{j\omega}) = \sum_{i=0}^{N-1} \lambda_i^{-1} |H_i(e^{j\omega})|^2 \Gamma_{uu}(e^{j\omega}). \quad (6.4)$$

From (6.2) and (6.4), the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$ can be seen as a linear combination of the normalized subband spectra, $\Gamma_{ii}(e^{j\omega})/\lambda_i$ for $i = 0, 1, \dots, N-1$.

Consider again the AR(2) random signal and the cosine-modulated filter bank of Figure 6.2. The analysis filters partition the input spectrum $\Gamma_{uu}(e^{j\omega})$ into $N=4$ subband, as depicted in Figure 6.3. Now, we proceed further by normalizing each subband spectrum $\Gamma_{ii}(e^{j\omega})$ with a factor of $\lambda_i = \|\mathbf{u}_i(k)\|^2 + \alpha$, as illustrated in Figure

6.11(a) to 6.11(d). Notice that the dotted line in the subfigures represents the subband spectrum $\Gamma_{ii}(e^{j\omega})$ before normalization. Finally, by summing the normalized partitions $\lambda_i^{-1}\Gamma_{ii}(e^{j\omega})$ according to (6.4), we arrive at the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$ of Figure 6.11(e), with a reduced dynamic range with respect to the input spectrum $\Gamma_{uu}(e^{j\omega})$. Clearly, band edges of the subband spectrum $\Gamma_{ii}(e^{j\omega})$ disappear in the summation. Furthermore, aliasing distortion is annihilated as the input signals $u_i(n)$ involved in the tap-weight adaptation are not decimated. Each of the bandlimited

Figure 6.11 The normalized subband spectrum, $\Gamma_{ii}(e^{j\omega})/\lambda_i$, in (a), (b), (c), and (d) are combined to form the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$ in (e).

spectra $\Gamma_{ii}(e^{j\omega})$, which are allowed to overlap with neighboring subbands, is used to adapt a single spectral band of the wideband filter $W(z)$. The effectiveness of the multiband scheme in dealing with the aliasing and band-edge effects can be observed from its learning curve, as we shall see in Section 6.5.

The major motivation of the NSAF algorithm is to improve the convergence rate of the NLMS algorithm with a minimum amount of additional computations [61]. In the multiband scheme, computational reduction is achieved by

- (i) exploiting the orthogonality of the subband signals (i.e., by having the subband signals to be orthogonal at zero lag), and
- (ii) decimating the adaptation rate.

The first point has been discussed extensively in Chapter 4. The rationale behind decimating the adaptation rate is justified by considering the fact that subband signals can be critically decimated, while retaining all the information, assuming that the analysis filter bank is lossless. Updating the tap weights at the decimated rate would not cause loss of performance since similar amount of information is used, provided that the decimation factor is smaller than or equal to the number of subbands.

6.3 Delayless Structure

In the subband structure of Figure 6.1, the fullband error signal $e(n)$ is obtained by interpolating and recombining the subband error signals $e_{i,D}(k)$ using synthesis filters $F_0(z), F_1(z), \dots, F_{N-1}(z)$. The analysis and synthesis filter banks, which are used to derive the subband signals and to reconstruct the fullband error $e(n)$, introduce an undesirable delay into the signal path. In applications, such as acoustic echo cancellation (AEC) and wideband active noise control, the delay is highly undesirable [6], [42], [85]. In AEC applications, the fullband error signal bears the near-end speech to be transmitted to the far-end. The transmission delay has to be limited to quantities that are compatible to, for example, telephone specification [51].

The signal path delay can be eliminated with a delayless structure, as shown in Figure 6.12. The basic idea of delayless subband adaptive filtering has been conceived by Morgan and Thi [85], [121]. Detailed analysis has been reported in [15], [45], [83], [90], [92]. In the following, the general techniques, structures, and features of delayless SAF are presented.

6.3.1 Weight transformation

In the delayless structure, as shown in Figure 6.12, the signal path delay is eliminated by implementing an auxiliary loop for the adaptation process. The adaptive subfilters $\bar{W}_i(z)$ are adapted in the subbands. The updated subband tap-weights are then collectively transformed into a set of fullband tap-weights for the modeling filter $W(z)$, which is located in the signal path. By so doing, the filtering process is implemented in the time domain, while the adaptation process is kept in the subbands.

Figure 6.12 Delayless closed-loop SAF.

Notice that the conventional SAF of Figure 6.1 requires a pair of analysis and synthesis filter banks. In contrast, the delayless structure of Figure 6.12 can be implemented with only analysis filter banks. The error signal $e(n)$ is directly obtained from the fullband filter $W(z)$, instead of synthesizing using the synthesis filters.

Several techniques to perform the weight transformation have been proposed in [45], [46], [79], [80], [85]. The subband-to-fullband weight transformation procedure greatly depends on the characteristics of the analysis filter bank used for the subband decomposition. Each of the weight transformation techniques has some distinctive features that are related to the analysis filter bank, as shown in the following subsections.

6.3.1.1 Frequency sampling method

The original delayless structure, proposed by Morgan and Thi [85], [121] employs non-critically decimated DFT filter banks. Recall that an N -channel DFT filter bank is comprised of N complex-modulated bandpass filters $H_i(z) = P(z e^{-j2\pi i/N})$, where $P(z)$ is the real-valued lowpass prototype filter with a cutoff frequency at π/N . The impulse response coefficients of $H_i(z)$ and $H_{N-i}(z)$, for $i = 1, 2, \dots, N/2 - 1$, are complex conjugate of each other. Hence, for real signals, only $N/2 + 1$ subbands need to be processed. Similar to that in the conventional SAF, the adaptive subfilters $\bar{W}_i(z)$ used in the subbands are of length $M_s = M/D$, where D is the decimation factor.

Since the subband signals are complex, the weights of the adaptive subfilters are complex as well. The function of the weight transformation is to map the complex subband tap-weights into an equivalent set of real-valued fullband tap-weights. The steps for the weight transformation can be summarized as follows.

Step 1: Transform the tap weights of the adaptive subfilters $\bar{\mathbf{w}}_i$ into the DFT-domain by computing the M_s -point FFT of $\bar{\mathbf{w}}_i$, for $i = 0, 1, \dots, N/2$. The vector $\bar{\mathbf{w}}_i = [\bar{w}_{i,0}, \bar{w}_{i,1}, \dots, \bar{w}_{i,M_s-1}]^T$ can be seen as the time-domain representation of the subfilter $\bar{W}_i(z) = \sum_{k=0}^{M_s-1} \bar{w}_{i,k} z^{-k}$. The tap-weight vectors $\bar{\mathbf{w}}_i$ for the first $N/2 + 1$ subbands are transformed by the FFT to obtain M_s DFT coefficients for each subband.

Step 2: The $M_s \times (N/2 + 1)$ DFT coefficients obtained in the previous step are stacked to form (frequency) points $0 \sim (M/2 - 1)$ of an M -element vector. The vector is then completed by setting point $M/2$ equal to zero and using the

complex conjugates of point $1 \sim (M/2-1)$ in reverse order as points $(M/2+1) \sim M-1$. The fullband tap-weights are then obtained as the IFFT of the M element array. The procedure for frequency stacking is illustrated in Table 6.1, for the case of $N=8$ subbands, $M=128$ taps, and $D=N/2$ for $2\times$ oversampling. In [15], [49], the frequency stacking procedure proposed by Morgan and Thi, as described above, is formulated in equivalent mathematical expressions.

Table 6.1 Mapping from subband DFT coefficient to fullband DFT coefficient for 8-subbands with 32 points per subband to 128-point fullband coefficients.

Subband DFT coefficient index	Fullband DFT coefficient index				
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = \frac{N}{2} = 4$
0	0		32		
1	1		33		
2	2		34		
3	3		35		
4	4		36		
5	5		37		
6	6		38		
7	7		39		
8		8		40	
9		9		41	
10		10		42	
11		11		43	
12		12		44	
13		13		45	
14		14		46	
15		15		47	
16		16		48	
17		17		49	
18		18		50	
19		19		51	
20		20		52	
21		21		53	
22		22		54	
23		23		55	
24			24		56
25			25		57
26			26		58
27			27		59
28			28		60
29			29		61
30			30		62
31			31		63

The weight transformation procedure described above can be interpreted from a frequency sampling perspective. The idea of the weight transformation is to first transform the subfilter $\bar{w}_i$ into the DFT-domain (i.e., the frequency responses of the subfilters are sampled along the unit circle), and then to extract the DFT coefficients of the subfilter at the frequencies that appear within the passbands of the subband spectrum $\Gamma_{ii,D}(e^{j\omega})$. Since the subband signals are $2\times$ oversampled, only half of the DFT-coefficients would fall within the passband of the subband spectrum $\Gamma_{ii,D}(e^{j\omega})$. Furthermore, due the decimation with a factor of $D = N/2$, the subband spectra exhibit their passband at $\{0 \sim \pi/2, 3\pi/2 \sim 2\pi\}$ and $\{\pi/2 \sim 3\pi/2\}$ for i even and odd, respectively (see Figure 6.6). Hence, the DFT coefficients that appear at the passband are taken according to the order indicated in Table 6.1. The DFT coefficients extracted from consecutive subbands are then stacked together, forming the DFT coefficients for the fullband filter. Finally, the fullband filter is transformed into the time-domain by taking the inverse DFT.

The weight-adjustment applied on the subfilters $\bar{W}_i(z)$ is a function of the subband signals $\{u_{i,D}(k), e_{i,D}(k)\}$. In the weight transformation described above, each of these subfilters $\bar{W}_i(e^{j\omega})$ is used to form a spectral portion of the fullband filter $W(e^{j\omega})$. Thus, the set of subband signals are used in an indirect manner in updating the fullband tap weights of the modeling filter $W(z)$. Clearly, the weight-control mechanism of the delayless structure of Figure 6.12 is similar to that of the NSAF algorithm in the sense that a set of subband signals are used to adapt the fullband tap weights of the modeling filter $W(z)$.

6.3.1.2 DFT filter bank with fractional delays

Recent development by Merched, Diniz, and Petraglia [19], [79], [80] showed that, weight transformation for critically decimated SAF is possible by using a DFT filter bank with its lowpass prototype $P(z)$ being an N th-band filter. Recall that the polyphase components $E_l(z)$ of a lowpass filter $P(z) = \sum_{l=0}^{N-1} E_l(z^N) z^{-l}$, with a cutoff frequency at π/N , would approximately appear as fractional delays due to their relative phase difference [100, pp. 799]. Furthermore, the l th polyphase component $E_l(z)$ generates a forward time shift of l/N sample with respect to the zeroth polyphase component $E_0(z)$. Now, if the prototype $P(z)$ is an N th-band filter with its last polyphase component given by $E_{N-1}(z) = z^{-D_{\text{int}}}$, we obtain a DFT analysis filter bank with fractional delays, as shown in Figure 6.13. The polyphase components of $P(z)$ take the following form

$$\begin{aligned} E_0(z) &\approx z^{-D_{\text{int}} - (N-1)/N} \\ E_1(z) &\approx z^{-D_{\text{int}} - (N-2)/N} \\ &\vdots \\ E_{N-1}(z) &= z^{-D_{\text{int}}}. \end{aligned} \quad (6.5)$$

Figure 6.13 DFT filter bank with fractional delays. The matrix $\mathbf{D} = [a_{mn}]$ denotes the DFT matrix with elements $a_{mn} = e^{-j2\pi mn/N}$.

Besides the fractional delays, the length of the adaptive subfilters has to be increased by one sample, such that $M_s = M/N + 1$, in order to allow an accurate modeling of the unknown system [19], [80]. The subband tap-weights are mapped into the corresponding fullband tap-weights through the following steps.

Step 1: Compute N -point IFFT on each of the M_s columns of the matrix on the right-hand-side of (6.6), as follows

$$\begin{bmatrix} \mathbf{g}_{0,0} & \mathbf{g}_{0,1} & \cdots & \mathbf{g}_{0,M_s-1} \\ \mathbf{g}_{1,0} & \mathbf{g}_{1,1} & \cdots & \mathbf{g}_{1,M_s-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{g}_{N-1,0} & \mathbf{g}_{N-1,1} & \cdots & \mathbf{g}_{N-1,M_s-1} \end{bmatrix} = \text{IFFT} \left(\begin{bmatrix} \bar{\mathbf{w}}_{0,0} & \bar{\mathbf{w}}_{0,1} & \cdots & \bar{\mathbf{w}}_{0,M_s-1} \\ \bar{\mathbf{w}}_{1,0} & \bar{\mathbf{w}}_{1,1} & \cdots & \bar{\mathbf{w}}_{1,M_s-1} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\mathbf{w}}_{N-1,0} & \bar{\mathbf{w}}_{N-1,1} & \cdots & \bar{\mathbf{w}}_{N-1,M_s-1} \end{bmatrix} \right). \quad (6.6)$$

Each row of the matrix on the right-hand-side of (6.6), i.e., $\bar{\mathbf{w}}_i^T = [\bar{\mathbf{w}}_{i,0}, \bar{\mathbf{w}}_{i,1}, \dots, \bar{\mathbf{w}}_{i,M_s-1}]$, represents the impulse response of the adaptive subfilter $\bar{W}_i(z)$. Each row of the matrix on the left-hand-side, i.e., $\mathbf{g}_i^T = [\mathbf{g}_{i,0}, \mathbf{g}_{i,1}, \dots, \mathbf{g}_{i,M_s-1}]$, represents the impulse response of the fractionally-delayed polyphase component $G'_i(z) = G_i(z)z^{-i/N}$, where $G_0(z), \dots, G_{N-1}(z)$ are the polyphase components of the fullband filter $W(z)$. The IFFT returns real column vectors since the coefficients, $\bar{\mathbf{w}}_{i,k}$ and $\bar{\mathbf{w}}_{N-i,k}$ for $i=1, 2, \dots, N/2$, are conjugate of each other, for $u(n)$ and $d(n)$ real-valued signals.

Step 2: For the first polyphase component, we simply take $G_0(z) = G'_0(z)$. For the subsequent $i=1, 2, \dots, N-1$, the impulse response of $G'_i(z)$ is convolved with the fractional delay $E_{i-1}(z)$, as follows

$$G_i(z)z^{-(D_{\text{int}}+1)} = G'_i(z)E_{i-1}(z), \text{ for } i=1, 2, \dots, N-1. \quad (6.7)$$

Step 3: Discard samples so that the polyphase components $G_i(z)$ of the fullband filter have length $M/N = M_s - 1$. For the first polyphase component $G_0(z)$, we simply discard the last sample. For $G_1(z), G_2(z), \dots, G_{N-1}(z)$ we discard the first $D_{\text{int}} + 1$ samples and retain the next $M_s - 1$ samples. The fullband filter can then be easily constructed from the polyphase components according to the following equation

$$W(z) = \sum_{i=0}^{N-1} G_i(z^N) z^{-i}. \quad (6.8)$$

6.3.2 Minimizing the fullband MSE

The original intention of the delayless structure was to eliminate the signal path delay caused by the analysis and synthesis filter banks, while retaining the computational and convergence speed advantages of subband processing. Later on, it has been recognized that the aliasing effects encountered in critically sampled SAFs [46], [79], [80], [92], and the band-edge effects encountered in oversampled SAFs [85] can actually be reduced with a delayless closed-loop configuration. Such features of the delayless closed-loop structure are intimately related to its capability in minimizing the fullband MSE, instead of the individual subband MSEs.

In the conventional SAF, the subfilters are adapted by separate adaptive algorithms (e.g., LMS and NLMS) in minimizing the MSE in each subband. The subband structure can be seen as a direct form of realizing the adaptive algorithm by dividing it into subbands. Different from the subband structure of Figure 6.1, the delayless structure of Figure 6.12 involves a closed-loop feedback of the fullband error $e(n)$ for subband weight adaptation. Any non-zero frequency components of the error will be fed back to the subband adaptive weight update, and will have the

effect of driving that component to zero. By so doing, the delayless close-loop configuration allows the minimization of the fullband error signal, and guarantees that the modeling filter $W(z)$ converges to the optimal Wiener solution.

The delayless structure can also be implemented in an open-loop configuration, as shown in Figure 6.14. Notice that the estimation error $e_{i,D}(k)$ is derived in each subband as the difference between subband desired response $d_{i,D}(k)$ with the output of the adaptive subfilter $\bar{W}_i(z)$. The fullband error $e(n)$ is not fed back for the subband weight calculation. Therefore, the delayless open-loop SAF inherits the structural problems (i.e., the aliasing and band-edge effects) encountered in the conventional SAF. Since our emphasis in this chapter is on overcoming the structural problems of the conventional SAF, the open-loop configuration of Figure 6.14 will not be considered further in the following discussion.

Figure 6.14 Delayless open-loop SAF.

6.3.3 Computational Requirements

The computational complexities of the delayless closed-loop SAFs by Morgan *et al.* and Merched *et al.* are summarized in Table 6.2. The computational requirements of the algorithms are separated into four parts: (i) filter bank implementation, (ii) adaptive weight update, (iii) fullband filter convolution, and (iv) weight transformation. Detailed breakdown of the computational requirements can be found in [19], [80], [85].

Table 6.2 Computational complexity of delayless closed-loop SAFs in terms of number of real multiplications in one input sample.

Computational requirements	Delayless closed-loop SAF	
	Morgan <i>et al.</i>	Merched <i>et al.</i>
Filter bank implementation	$4\left(\frac{L}{N} + \log_2 N\right)$	$2\left(\frac{L}{N} + \log_2 N\right)$
Adaptive weight update	$\frac{16M}{N}$	$4\left(\frac{M}{N} + 1\right)$
Fullband filter convolution	M	M
Weight transformation	$\left[2\log_2\left(\frac{2M}{N}\right) + \log_2 M\right]J$	$\left[\left(1 + \frac{N}{M}\right)\log_2 N + \frac{L(N-1)}{N^2}\right]J$

The NLMS algorithm is used for the adaptation of the subband filters $\bar{\mathbf{w}}_i(k)$. Since the signals and the filter coefficients in the subbands are complex-valued, the complex form of the NLMS algorithm is employed, as follows

$$\bar{\mathbf{w}}_i(k+1) = \bar{\mathbf{w}}_i(k) + \frac{\mu \bar{\mathbf{u}}_i^*(k)}{\|\bar{\mathbf{u}}_i(k)\|^2 + \alpha} e_{i,D}(k), \text{ for } i = 0, 1, \dots, N-1, \quad (6.9)$$

where $\bar{\mathbf{u}}_i(k) \equiv [u_{i,D}(k), u_{i,D}(k-1), \dots, u_{i,D}(k-M_s+1)]^T$ is the subband regressor for the subfilter $\bar{\mathbf{w}}_i(k)$ of length M_s . Notice that the computation of the weight

transformation involves a variable J which determines how often the weight transformation is performed. In particular, the weight transformation is performed once for every M/J input samples, where M is the length of the fullband modeling filter. The computational complexity can be reduced by using smaller values of J . It has been shown in [85] that the SAFs do not exhibit severe degradation in their performance for J in the range from one to eight.

6.4 The Delayless NSAF algorithm

The NSAF algorithm can easily be implemented in delayless open-loop or closed-loop configurations. For both configurations, no subband-to-fullband weight transformation is required since the NSAF algorithm is able to directly derive the fullband tap weights from the subband signals. The adaptive tap weights updated using the subband signals are directly copied to another fullband modeling filter in order to eliminate the delay inflicted by the filter banks. It should be emphasized that the NSAF algorithm does not suffer from the aliasing and band-edge effects. Hence, it does not require the closed-loop feedback of the fullband error to overcome the structural problems.

6.4.1 Open-loop configuration

Figure 6.15 shows the delayless open-loop implementation of the NSAF algorithm. The steps, parameters, and variables of the open-loop algorithm are described in Table 6.3. Notice that the matrix $\mathbf{H}$ denotes the analysis filter bank with its columns $\mathbf{h}_i = [h_i(0), h_i(1), \dots, h_i(L-1)]^T$ represents the analysis filters. In the open-loop configuration, the fullband error $e(n)$ is not fed back for tap-weight computation. The fullband error $e(n)$ and the subband estimation error $\mathbf{e}_D(k) \equiv [e_{0,D}(k), e_{1,D}(k),$

$\dots, e_{N-1,D}(k)]^T$ are two separate error quantities. In the case of acoustic echo cancellation, $e(n)$ is the near-end signal to be transmitted to the far-end room, while $e_D(k)$ serves as the estimation error in the tap-weight adaptation. As far as the adaptation process is concerned, the fullband filtering process is an unrelated operation that does not in any means affect the convergence behavior of the NSAF algorithm. The convergence behavior of the delayless open-loop NSAF is therefore similar to that of the original NSAF algorithm of Figure 4.3, except for the signal path delay (introduced by the filter banks) that has been eliminated with the delayless structure.

Figure 6.15 Delayless open-loop NSAF algorithm.

The computational cost of the delayless open-loop NSAF algorithm in terms of number of multiplications per sampling period T_s is also summarized in Table 6.3. Notice that the number of multiplications incurred at each iteration is divided by N because the adaptation process is performed at the decimated rate $1/NT_s$. The NSAF algorithm implemented in the delayless open-loop configuration requires $3M + 2$

multiplications for the adaptation process, M multiplications for the fullband filtering process, and $(N+1)L$ multiplications for the two analysis filter banks, which together entail $4M + NL + L + 2$ multiplications in one sampling period. Clearly, the computational complexity of the delayless NSAF is higher than that of the delayless SAFs presented in Section 6.3, which are based on the conventional SAF structure. However, the delayless NSAF would be preferable in terms of convergence performance, as well shall see in Section 6.5.

Table 6.3 Delayless open-loop implementation of the NSAF algorithm.

Computation	Multiplications/ T_s
For $n=0,1,2,\dots$, <i>Filtering and error formation:</i> $e(n)=d(n)-\mathbf{w}^T(k)\mathbf{u}(n)$	M
For $k=0,1,2,\dots$, where $kN=n$, <i>Error estimation:</i> $\mathbf{e}_D(k)=\mathbf{d}_D(k)-\mathbf{U}^T(k)\mathbf{w}(k)$	$\frac{N \times M}{N} = M$
<i>Normalization matrix:</i> $\mathbf{\Lambda}(k)=\mathbf{U}^T(k)\mathbf{U}(k)+\alpha\mathbf{I}$	$\frac{N \times M}{N} = M$
<i>Tap-weight adaptation:</i> $\mathbf{w}(k+1)=\mathbf{w}(k)+\mu\mathbf{U}(k)\mathbf{\Lambda}^{-1}(k)\mathbf{e}_D(k)$	$\frac{2N \times NM}{N} = M+2$
<i>Band-partitioning:</i> $\mathbf{U}_1^T(k)=\mathbf{H}^T\mathbf{\Lambda}(kN)$ $\mathbf{d}_D(k)=\mathbf{H}^T\mathbf{d}(kN)$	$\frac{N^2L+NL}{N}=(N+1)L$
<i>Parameters:</i> M number of adaptive tap weights N number of subbands μ step-size parameter L length of the analysis filters α small positive constant	
<i>Variables:</i> $\mathbf{U}^T(k)=[\mathbf{U}_1^T(k), \mathbf{U}_2^T(k-1)]$ $\mathbf{U}_2^T(k-1)$ = first $M-N$ columns of $\mathbf{U}^T(k-1)$ $\mathbf{\Lambda}(kN)=[\mathbf{a}(kN), \mathbf{a}(kN-1), \dots, \mathbf{a}(kN-N+1)]$ $\mathbf{a}(kN)=[u(kN), u(kN-1), \dots, u(kN-L+1)]^T$ $\mathbf{d}(kN)=[d(kN), d(kN-1), \dots, d(kN-L+1)]^T$ $\mathbf{H}=[\mathbf{h}_0, \mathbf{h}_1, \dots, \mathbf{h}_{N-1}]$ analysis filter bank	

6.4.2 Closed-loop configuration

The delayless closed-loop implementation of the NSAF algorithm is described in Figure 6.16 and in Table 6.4. Different from that in the open-loop configuration, the subband estimation error $\mathbf{e}_D^{\text{cl}}(k) = \mathbf{H}^T \mathbf{e}(kN)$ is derived from the fullband error $\mathbf{e}(kN) \equiv [e(kN), e(kN-1), \dots, e(kN-L+1)]^T$ by means of the analysis filter bank $\mathbf{H}$. The fullband error signal $e(n)$ is generated by taking the difference between the desired response $d(n)$ and the output of the modeling filter $W(z)$. A delay is introduced by the analysis filter bank applied to the fullband error $e(n)$. Furthermore, the adaptation algorithm uses past information from previous estimations $e(kN-1), e(kN-2), \dots, e(kN-L+1)$ that present in the delay line of the analysis filters. Both of these factors limit the convergence rate, and reduce the upper bound of the step-size that can be

Figure 6.16 Delayless closed-loop NSAF algorithm.

Table 6.4 Delayless closed-loop implementation of the NSAF algorithm.

Computation	Multiplications/ T_s
For $n=0,1,2,\dots$, <i>Filtering and error estimation:</i> $e(n)=d(n)-\mathbf{w}^T(k)\mathbf{u}(n)$	M
For $k=0,1,2,\dots$, where $kN=n$ <i>Normalization matrix:</i> $\mathbf{\Lambda}(k)=\mathbf{U}^T(k)\mathbf{U}(k)+\alpha\mathbf{I}$	$\frac{N \times M}{N} = M$
<i>Tap-weight adaptation:</i> $\mathbf{w}(k+1)=\mathbf{w}(k)+\mu\mathbf{U}(k)\mathbf{\Lambda}^{-1}(k)\mathbf{e}_D^{\text{cl}}(k)$	$\frac{2N \times NM}{N} = M+2$
<i>Band-partitioning:</i> $\mathbf{U}_1^T(k)=\mathbf{H}^T\mathbf{A}(kN)$ $\mathbf{e}_D^{\text{cl}}(k)=\mathbf{H}^T\mathbf{e}(kN)$	$\frac{N^2L+NL}{N} = (N+1)L$
<i>Parameters:</i> M number of adaptive tap weights N number of subbands μ step-size parameter L length of the analysis filters α small positive constant	
<i>Variables:</i> $\mathbf{U}^T(k)=[\mathbf{U}_1^T(k), \mathbf{U}_2^T(k-1)]$ $\mathbf{U}_2^T(k-1)$ = first $M-N$ columns of $\mathbf{U}^T(k-1)$ $\mathbf{A}(kN)=[\mathbf{a}(kN), \mathbf{a}(kN-1), \dots, \mathbf{a}(kN-N+1)]$ $\mathbf{a}(kN)=[u(kN), u(kN-1), \dots, u(kN-L+1)]^T$ $\mathbf{e}(kN)=[e(kN), e(kN-1), \dots, e(kN-L+1)]^T$ $\mathbf{H}=[\mathbf{h}_0, \mathbf{h}_1, \dots, \mathbf{h}_{N-1}]$ analysis filter bank	

employed for the closed-loop NSAF algorithm [14]. That is to say that the NSAF algorithm implemented in a delayless closed-loop structure behaves differently from that of the original NSAF algorithm of Figure 4.3.

The delayless closed-loop NSAF algorithm is summarized in Table 6.4. It can be noted that the delayless closed-loop NSAF algorithm involves a total of $3M + NL + L + 2$ multiplications in one sampling period. The delayless closed-loop NSAF involves less computations (about M multiplications/ T_s) compared to the delayless open-loop NSAF. The computational savings are achieved by deriving the

subband estimation error $\mathbf{e}_D^{\text{cl}}(k)$ from fullband error $e(n)$, instead of the subband estimation, where $\mathbf{e}_D(k) = \mathbf{d}_D(k) - \mathbf{U}^T(k)\mathbf{w}(k)$. The price to pay for the computational reduction is the degradation of the convergence rate.

In [13], [14], the delayless closed-loop NSAF is referred to as the weighted SAF (WSAF). In fact, the formulation and the approach taken in deriving the WSAF algorithm are different from that of the delayless closed-loop NSAF algorithm. The WSAF algorithm was derived as a stochastic-gradient algorithm, whereas the NSAF was derived with a deterministic approach based on the *principle of minimal disturbance*. Nevertheless, setting aside the minor discrepancies, both algorithms are identical in the sense that they possess similar structures for filtering and tap-weight adaptation.

6.5 Simulations

Two sets of simulations performed in the context of system identification are presented in this section. The impulse response of the unknown system $W_o(z)$ is depicted in Figure 4.8. In all simulations, the length of the fullband filter is taken as $M = 1024$, which is identical to that of the unknown system.

6.5.1 Aliasing and band-edge effects

In the first set of simulations, a noiseless signal model with white excitation signal is considered. The purpose of these simulations is to investigate the effectiveness of the NSAF algorithm in dealing with the aliasing and band-edge effects. The NSAF is chosen to have $N = 4$ subbands using the pseudo-QMF cosine-modulated filter bank in Figure 6.2. On the other hand, the DFT filter bank in Figure 6.5 is used for the conventional SAF. The design specifications of the filter banks are detailed in Section

6.1.1. The length of the adaptive subfilters $\bar{W}_i(z)$ is set to $M_s = M/N = 256$ for the critically sampled scheme, and $M_s = 2M/N = 512$ for the $2\times$ oversampled scheme of the conventional SAF. The coefficients of the subfilters $\bar{W}_i(z)$ are adapted using the NLMS algorithm (6.9) with $\mu = 0.1$ and $\alpha = 0.0001$. Similar step-size and regularization parameters are used for the NSAF algorithm.

Figure 6.17 shows the MSE learning curves of the NSAF compared to that of the oversampled and critically sampled schemes. These learning curves are obtained by averaging over 200 independent trials, and then smoothed with a 10-point moving average filter. Observed that the MSE learning curve of the critically sampled scheme quickly level off due to the aliasing effects and its inability to properly model the unknown system. The learning curve of the critically sampled scheme is characterized by a high asymptotic level of MSE. The aliasing effects and high asymptotic MSE are

Figure 6.17 MSE learning curves of the conventional SAF and the normalized SAF (NSAF) for $N = 4$ subbands.

eliminated in the oversampled scheme. However, the convergence rate of the oversampled scheme is limited by a slow asymptotic convergence (after a fast initial convergence), which is due to the small eigenvalues generated by the band-edges of the subband spectrum. Clearly, the NSAF algorithm does not suffer from the aliasing and band-edge effects. It converges to the optimal solution with a consistent rate.

6.5.2 Delayless alias-free SAFs

In the second set of simulations, a comparison of three delayless SAFs listed below is demonstrated:

- (i) delayless closed-loop SAF ($2\times$ oversampled) of Morgan *et al.* [85],
- (ii) delayless closed-loop SAF (critically sampled) of Merched *et al.* [79], [80], and
- (iii) delayless open-loop NSAF (critically sampled).

The first and the second delayless SAFs rely on the closed-loop feedback of a fullband error to overcome the band-edge and aliasing effects, respectively. The open-loop NSAF (as summarized in Table 6.3) does not suffer from these structural problems due to the multiband nature of the NSAF algorithm. The multiband scheme of the NSAF algorithm and the delayless closed-loop structure are two of the most promising solutions to the aliasing and band-edge effects. The objective of the simulations is to compare the merit of employing the multiband scheme to that of delayless closed-loop structure, in terms of convergence performance and computational complexity.

Two highly correlated signals, namely, an AR(10) random signal and a real speech, as in Section 4.6.2, are used in the simulations. The unknown system $\mathbf{w}_o$ of length $M=1024$ is shown in Figure 4.8. The delayless SAFs are chosen to have $N=16$ subbands. The length of the adaptive subfilters is set to $M_s = 2M/N = 128$

and $M_s = M/N + 1 = 65$ for the delayless SAFs of Morgan *et al.* and Merched *et al.*, respectively (see Section 6.3.1.1 and 6.3.1.2). The subband tap-weights are collectively transformed into a fullband tap-weight vector $\mathbf{w}(k)$ of length $M = 1024$. The weight transformation is performed for every $M/J = 128$ samples, where $J = 8$ controls the update rate of the fullband tap-weight vector $\mathbf{w}(k)$. Weight transformation is not required for the NSAF algorithm since it is able to directly update the fullband tap-weight vector $\mathbf{w}(k)$ by using the subband signals.

The NSAF uses a paraunitary cosine-modulated filter bank with a filter length of $L = 8N = 128$ taps. On the other hand, the delayless SAFs of Morgan and Merched require a DFT filter bank. The lowpass prototype filter $P(z)$ for the DFT filter bank was designed to have $L = 8N - 1 = 127$ taps using the MATLAB *fir1(L-1, 1/N)* routine. For $L = 127$ an odd number, the MATLAB routine returns a windowed sinc function, which is exactly an N th band filter. Figure 6.18 shows the impulse response $p(n)$ of

Figure 6.18 An N th band prototype filter for the DFT filter bank.

the prototype filter. The lower plot depicts the impulse response $e_{15}(n) = p(nN + 15)$ of the last polyphase component $E_{15}(z) = \sum_n e_{15}(n)z^{-n} = z^{-3}$. With reference to Figure 6.13, the integer part of the delay is therefore $D_{\text{int}} = 3$. This quantity is crucial for the subband-to-fullband weight transformation described in Section 6.3.1.2.

Figure 6.19 shows the ensemble-average learning curves of the delayless SAFs under the excitation of the stationary AR(10) random signal. The learning curve of a fullband NLMS filter is also shown in the figure. Step sizes are chosen such that identical steady-state MSE is achieved for all the adaptive filters. Notice that the SNR at the output of the unknown system is set at 50 dB. Clearly, the delayless NSAF exhibits the faster convergence rate. It can also be noted that all the three delayless

Figure 6.19 MSE learning curves of the fullband NLMS filter ($\mu = 0.10$), delayless closed-loop SAF of Merched *et al.* ($N = 16, \mu = 0.20$), delayless closed-loop SAF of Morgan *et al.* ($N = 16, \mu = 0.17$), and delayless open-loop NSAF ($N = 16, \mu = 0.10$). The regularization parameter is set at $\alpha = 0.0001$.

SAFs outperform the fullband NLMS filter. For $M = 1024$ taps and $N = 16$ subbands, the delayless open-loop NSAF entails 6274 real multiplications per sampling period. The fullband NLMS filter involves 3072 real multiplications per sampling period, whereas the delayless closed-loop SAFs of Morgan and Merched require 2288 and 1400 real multiplications, respectively. The delayless closed-loop SAFs of Morgan and Merched improve the convergence rate of the NLMS algorithm under colored excitation, while achieving some computational savings. Further improvement in the convergence rate is obtained with the NSAF algorithm, but it involves more computations. Nevertheless, it should be emphasized that the computational complexities of the adaptive filters considered herein are of the same order of magnitude.

The efficacy of the NSAF algorithm becomes more apparent for speech signal. Figure 6.20 shows the misalignment learning curves of the delayless SAFs, using the same step sizes μ and number of subbands N as in the previous simulations. For the sake of simplicity no disturbance is added to the output of the unknown system. It can be observed that delayless NSAF performs equally well under the speech excitation signal, with the faster convergence rate among all the adaptive filters. It can also be noted that the delayless closed-loop SAF of Merched *et al.* converges faster than the delayless closed-loop SAF of Morgan *et al.*. This result may be due to the noiseless signal model that has been assumed in the simulations. That is, the delayless closed-loop SAF of Merched *et al.* would perform better under a noiseless situation. Nevertheless, the learning curves of Figure 6.20 shows that both the delayless closed-loop SAFs outperform the fullband NLMS filter.

The convergence of the delayless open-loop NSAF can be accelerated with a larger step size. Highest convergence rate is achieved at $\mu \approx 1.0$. In contrast, the

delayless closed-loop SAFs of Morgan and Merched generally diverge at $\mu \approx 1.0$. For instance, the delayless closed-loop SAFs diverge at $\mu \geq 0.5$, for $N = 16$ subbands and the AR(10) excitation signal. The closed-loop feedback of the fullband error ameliorates the band-edge and aliasing effects, but at the same time, a delay (due to the analysis filter bank that partitions the fullband error signal into subbands) is introduced into the weight update path. This delay reduces the upper bound of the step size μ that can be used, thereby limits the convergence performance of the delayless closed-loop SAFs [12], [38], [68], [69]. Such a deficiency is avoided in the delayless open-loop NSAF.

Figure 6.20 Normalized misalignment learning curves of the fullband NLMS filter ($\mu = 0.10$), delayless closed-loop SAF of Merched *et al.* ($N = 16, \mu = 0.20$), delayless closed-loop SAF of Morgan *et al.* ($N = 16, \mu = 0.17$), and delayless open-loop NSAF ($N = 16, \mu = 0.10$). The regularization parameter is set at $\alpha = 0.0001$.

6.6 Conclusions

In this chapter, various structures for subband adaptive filtering were discussed. It is shown that the conventional SAF can achieve significant computational savings, however its convergence performance is limited by aliasing and band-edge effects. These structural problems were analyzed by using the correlation-domain formulation presented in Chapter 3. In particular, we showed the multiband nature of the NSAF algorithm is effective in eliminating the aliasing and band-edge effects. The weight-control mechanism of the NSAF is driven by an equalized spectrum comprised of normalized version of the band-limited spectrum from each subband. Aliasing distortion is annihilated since the subband signals used for the tap-weight adaptation are band-limited without decimation. Furthermore, band edges of the subband spectrum disappear in the summation that produces the normalized spectrum.

The concept of delayless SAF was presented in Section 6.3. The original intention of the delayless structure was to eliminate the signal path delay caused by the analysis and synthesis filter banks. The performance of two delayless closed-loop SAFs independently proposed by Morgan *et al.* [85] and Merched *et al.* [79], [80] were investigated. It was found that the aliasing and band-edge effects encountered in the conventional SAF can be reduced with a delayless closed-loop implementation. However, the closed-loop feedback of the fullband error introduces a delay into the weight update path. This delay reduces the upper bound of the step size μ , thus limits the convergence performance by the small step size that can be used.

The limitation on step-size is avoided in a delayless open-loop configuration. In that respect, a delayless open-loop configuration coupled with the NSAF algorithm would be a better combination in terms of convergence performance. The advantages of this new combination are: (i) the signal path delay is eliminated by means of an

auxiliary loop with a delayless structure, (ii) the aliasing and band-edge effects are annihilated by virtue of the multiband nature of the NSAF algorithm, and (iii) no extra limitation is imposed on the step-size due to the open-loop configuration. This assertion has been confirmed through simulations.

Chapter 7

Conclusions and Future Works

7.1 Conclusions

This thesis considered the derivation and analysis of new subband adaptive algorithms for high-order transversal filters. Subband and multirate techniques were employed in deriving adaptive filtering algorithms that are computationally efficient and possess improved convergence performances against high spectral dynamic range of the input signal. One application of interest is acoustic echo cancellation (AEC), which involves colored excitation and the modeling of long impulse response. Acoustic echo canceller finds application in hands-free telephone systems, audio or video conference systems, hearing aids, voice-control systems, and many more.

Conventional subband adaptive filter (SAF) decomposes the input signal and desired response into multiple subbands, which are then processed with separate adaptive subfilters. These subfilters are adapted separately in their own adaptation loop. Intuitively, faster convergence is possible because the spectral dynamic range is greatly reduced in each subband. Furthermore, the computational burden can be reduced by decimating both the order and adaptation rate of the subfilters. Yet, detailed analysis showed that the convergence rate of the conventional SAF is limited by aliasing and band-edge effects, as we have seen in Chapter 6. To annihilate these structural problems, an alternative structure for subband adaptive filtering has been adopted in this thesis, whereby the modeling filter is no longer separated into

subfilters. Instead, subband signals, which are normalized by their respective subband input variance, are used to adapt the fullband tap weights of the modeling filter.

The formulation of the new subband weight-control mechanism described above greatly relies on a special case of partial decorrelation, where the subband signals are orthogonal at zero lag. This feature of filter banks can be obtained by manipulating the relative phase between analysis filters, as described in Chapter 3. More precisely, if the deterministic cross-correlation sequence between two analysis filters is anti-symmetric, their outputs are orthogonal at zero lag. In Section 3.4, it was shown that orthogonality at zero lag can be approximated by cosine-modulating a high stopband attenuation prototype filter. That is, the outputs of a cosine-modulated filter bank are nearly orthogonal at zero lag, for arbitrary type of input spectrum, as long as the stopband attenuation of the prototype filter is sufficiently high. This feature pertaining to the filter bank was exploited in improving the computational efficiency and effectiveness of the normalized SAF (NSAF).

The *principle of minimal disturbance* [44, pp. 321] states that: from one iteration to the next, tap weights of an adaptive filter should be changed in a minimal manner, subject to a (set of) constraint(s) imposed on the updated filter output. Based on this principle, the design criterion for the NSAF algorithm was defined as a constrained optimization problem (4.9) involving multiple subbands constraints (4.10) imposed on the updated filter output. By virtue of the deterministic optimization criterion and the subband orthogonality of the filter bank, the resulting NSAF recursion (4.17) appears in a simple form comparable to that of the NLMS algorithm (1.11). The NLMS algorithm updates the tap weights of a transversal filter on the basis of a single input vector (1.2). The NSAF algorithm decomposes the fullband vector (1.2) into multiple subband vectors (4.6), normalizes each subband vector with its energy (i.e., squared

Euclidean norm of that vector), and finally combines the normalized subband vectors to collectively update the fullband tap weights of the transversal filter. The unique structure of the NSAF is depicted in Figure 4.3.

The deterministic formulation is elegant in the sense that it relates the NSAF algorithm to the NLMS and AP algorithms in a direct manner. The attribute that the NSAF algorithm has in common with the NLMS and AP algorithms is that they are manifestations of the *principle of minimal disturbance*. These algorithms can be seen as recursive estimators that iteratively update the tap-weight vector in a minimal manner, while having the updated filter output subject to a single fullband constraint (for the case of NLMS algorithm), or multiple time-domain or subband constraints (for the cases of AP and NSAF algorithms, respectively). In that respect, both the AP and NSAF algorithms can be regarded as generalized forms of the NLMS algorithm. The AP algorithm generalizes the NLMS along the time-axis using multiple time-domain constraints, whereas the NSAF algorithm generalizes the NLMS along the frequency-axis using multiple subband constraints. In particular, for $N=1$ subband, the NSAF algorithm (4.17) reduces to the NLMS algorithm (1.11).

The major motivation of the NSAF algorithm is to improve the convergence rate of the NLMS algorithm with minimum amount of additional computations. The multiple subband constraints introduced into the NSAF recursion contribute to the decorrelating properties of the algorithm. The inherent decorrelating properties of the NSAF algorithm whiten the input signal prior to tap-weight adaptation; thereby accelerate its convergence under colored excitation. With an improved immunity to high eigenvalue disparity, the NSAF algorithm still remains computationally efficient. Computational savings are achieved by (i) having the subband signals to be orthogonal at zero lag, and (ii) updating the tap weights at a decimated rate

commensurate with the bandwidth of the subband signals. As a result, the order of complexity of the NSAF algorithm is $O(M) + O(NL)$, which is equivalent to that of the NLMS algorithm except for the additional term $O(NL)$ that accounts for the filter banks implementation. For high-order adaptive filter, where $M \gg NL$, the NSAF algorithm has almost the same order of complexity with the NLMS algorithm. Compared to the AP algorithm with an order of complexity of $O(N^2M)$, the NSAF algorithm is able to achieve almost equivalent convergence rate with far less computations.

The convergence behavior of the NSAF algorithm was analyzed in the mean and mean-square senses in Chapter 5. The convergence analysis is performed for the weight recursion, without taking the delay introduced by the filter banks into consideration. Mean analysis showed that the convergence of the NSAF algorithm in the mean is governed by a weighted correlation matrix, which can be completely defined by an equalized spectrum $\Gamma_{\xi}(e^{j\omega})$. As a special case of the NSAF algorithm with $N=1$ subband, the input spectrum to the NLMS recursion is normalized but not equalized. For colored excitation signal, the dynamic range of the equalized spectrum $\Gamma_{\xi}(e^{j\omega})$ reduces with an increased number of subbands. Simulation results confirmed that faster convergence can be achieved with an increased number of subbands N , even though the adaptation is performed at a lower rate after critical decimation.

Mean-square performance of the NSAF algorithm was evaluated based on an energy conservation relation. Stability bounds for the step-size and an expression for the steady-state MSE were derived and confirmed through extensive simulations. In the absence of disturbance, the step-size bound for the NSAF algorithm was found to be similar to that of the NLMS algorithm. Both the theoretical and experimental

results agree that the excess MSE is independent to the number of subbands N . Hence, for colored excitation signal, higher convergence rate can be achieved by increasing the number of subbands N , without compromising on the steady-state excess MSE. Nevertheless, additional delay will be introduced into the signal path, since the order of the analysis and synthesis filters has to be increased according to the number of subbands to maintain a reasonable level of stopband attenuation.

The signal path delay due to the analysis and synthesis filter bank can be easily eliminated with a delayless structure. The concept of delayless subband adaptive filtering was presented in Chapter 6. In the delayless SAF, subband tap-weight adaptation process is implemented in an auxiliary loop. The output of the auxiliary loop is a fullband tap-weight vector that can be directly used to filter the input signal and to generate a fullband error signal. By so doing, the filter banks and the associated delay are eliminated from the signal path. In application such as acoustic echo cancellation (AEC), the fullband error signal is the near-end speech to be transmitted to the far-end.

Besides eliminating the signal path delay, a delayless SAF implemented in a closed-loop configuration was also found to be effective in dealing with the aliasing and band-edge effects encountered in the conventional SAF. The performance of two delayless closed-loop SAFs independently proposed by Morgan *et al.* [85] and Merched *et al.* [79], [80] were investigated. It was found that the closed-loop feedback of the fullband error ameliorates the band-edge and aliasing effects, but at the same time, it introduces a delay into the weight update path. This delay reduces the upper bound of the step size μ , thus limits the convergence rate by the small step size that can be used. The limitation on step-size is avoided in a delayless open-loop configuration. In that respect, a delayless open-loop configuration coupled with the

NSAF algorithm would be a better combination in terms of convergence performance. The advantages of this new combination are: (i) the signal path delay is eliminated by means of an auxiliary loop with a delayless structure, (ii) the aliasing and band-edge effects are annihilated by virtue of the multiband nature of the NSAF algorithm, and (iii) no extra limitation is imposed on the step-size as in a closed-loop configuration. This assertion has been backed up through simulations in Section 6.5.

7.2 Future Works

Filter banks are essential elements for SAFs. The analysis filter banks partition the input signal and desired response into subbands, whereas the synthesis filter bank reconstructs a fullband output by interpolating and recombining the subband signals after processing. These filter banks allow an efficient and effective adaptive filtering, but at the same time, they increase the end-to-end delay of the adaptive filter. The signal path delay inflicted by the filter banks can be easily eliminated with a delayless structure, as we have seen in Chapter 6. However, the subband signals for the tap-weight adaptation (in an auxiliary) are delayed by the analysis filter banks even though the fullband filtering is delayless. This delay may be negligible for high-order adaptive filter, where the length of the adaptive tap-weight vector is much longer than that of the analysis filters. Nevertheless, to a certain extent, the delay may have a considerable impact on the tracking performance of the NSAF algorithm for non-stationary signals, as postulated in [111, pp. 15] for the general case of subband adaptive filtering. As such, the tracking performance of the NSAF algorithm under non-stationary signals may be worth analyzing. For this purpose, the energy conservation relation presented in Section 5.4 can be extended by using a first-order random walk model [107], [110], [131], [132].

Cosine-modulated filter banks are able to generate subband signals that are orthogonal at zero lag, as shown through mathematical analysis and simulation in Chapter 3. The NSAF algorithm relies on the subband orthogonality of the cosine-modulated filter banks in order to justify the diagonal assumption, which greatly simplifies the NSAF recursion. The restriction on the type filter banks may limit the applications of the NSAF algorithm. Therefore, it would be of interest to investigate the feasibility of using other type filter banks for the NSAF algorithm. One possible candidate is the frequency sampling filter (FSF) [58 pp. 214-218], which leads to a computationally efficient filter bank implementation. It should be emphasized that, the filter banks to be used for the NSAF algorithm have to fulfill the following conditions: (i) the outputs of the filter bank are orthogonal at zero lag, and (ii) the analysis filters are power complementary. Recall that no synthesis filter bank is required if a delayless structure is employed in implementing the NSAF algorithm.

Aside from the abovementioned refinements, some other possible extensions to the NSAF algorithm could also be identified. First of all, the NSAF algorithm can be extended to multichannel case with additional constraints for channel decorrelation. The resulting algorithm is of particular interest in stereophonic and multichannel AEC [5], [30], [31], [60], [115], [117]. The additional constraints for channel decorrelation are useful in dealing with the non-uniqueness problem encountered in stereophonic and multichannel AEC. Secondly, the NSAF algorithm can be integrated with the subband audio coder in communication systems. The NSAF algorithm can be implemented as the echo cancellation front-end for the communication systems with minimum extra cost on the filter bank implementation, since cosine-modulated filter bank is widely used in subband audio coders [95]. The basic idea of joint filter bank implementation for AEC and audio coding has been reported in [23]. The strategy for

resource sharing between signal processing blocks can also be extended to include spectral subtraction and beamforming techniques for noise reduction in the communication systems.

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Appendix A

Subband and Transform Domain Adaptive Filters

Adaptive filters	Adaptive tap weights	Features	Objectives	Remark
Transform domain adaptive filter (TDAF)	Adaptive tap weights are recursively updated in transform domain	<ul style="list-style-type: none"> An N-by-N orthogonal matrix $\mathbf{A}$ is applied on the input signal. The orthogonal matrix $\mathbf{A}$ is used to diagonalize the input autocorrelation matrix $\mathbf{R} = E\{\mathbf{u}(n)\mathbf{u}^T(n)\}$ of the input signal $\mathbf{u}(n)$. The transformation is effective [24, 25] if $\mathbf{A}^T \mathbf{R} \mathbf{A} \approx \text{a diagonal matrix}.$ <p>Here, it is assumed that the transformation matrix $\mathbf{A}$ is real-valued and the columns of the matrix represent the basis functions of the transform [73, pp 11].</p> Sample based delayless structure. No inverse transform is necessary in deriving the output signal. Convergence improvement is achieved by diagonalizing (i.e., rotate the correlation matrix to the principal axes) and normalizing (i.e., scaling) the correlation matrix as close as possible to an identity matrix. 	<ul style="list-style-type: none"> To improve the convergence rate of gradient-based algorithms when the input signal is colored. There is an increment in the computational load due to the transformation. 	<ul style="list-style-type: none"> The transform size N depends on the desired length M of the time domain adaptive tap-weight vector. For acoustic echo cancellation (AEC) application, large transform size is required. The optimal orthogonal transform is Karhunen-Loève transform (KLT). However, KLT is data dependent and thus it is not applicable when no a priori information of the input is available. Optimal convergence can only be achieved with KLT. It has been concluded in [75] that there are no optimal or near optimal transform except KLT.
Filter bank adaptive filter (FBAF)		<ul style="list-style-type: none"> An N-by-L transformation matrix (i.e. a filter bank) is applied on the input signal. The length L of the basis functions of the transform is larger than the transform size N. Similar structure to that of the TDAF. No inverse transform is necessary in deriving the output signal. Can be seen as an extension to the TDAF, where a set of N sparse filters are applied on the transformed outputs instead of one adaptive tap weight for each of the transformed variables in TDAF. 		<ul style="list-style-type: none"> For TDAF, the transform size N has to be increased for higher order adaptive filter. FBAF offers an alternative, where additional adaptive weights are introduced by extending the single weight to a sparse filter. The FBAF approximates higher-order system with a smaller number of free parameters. Therefore, it can not represent all the higher-order system [124].

Adaptive filters	Tap weight adaptation	Features	Objectives	Remark
Conventional subband adaptive filter (SAF)	Adaptive tap weights are recursively updated in subband domain	<ul style="list-style-type: none"> An N-by-L transformation matrix (i.e. a filter bank, as explained in Section 2.4) is employed, where the length L of the basis functions being larger than the transform size N. The input signal and desired response are partitioned into N subbands. Each subband has its own subfilter and adaptation loop, where subband estimation errors are locally evaluated for tap-weight adaptation. A synthesis filter bank (i.e., an inverse transform) is required in deriving the fullband error signal. Computational complexity is greatly reduced by decimating both the order and adaptation rate of the adaptive subfilters. 	<ul style="list-style-type: none"> To improve the convergence rate of gradient-based algorithms when the input signal is colored. Computational complexity is greatly reduced with the shorter subfilters that operate at a lower rate. 	<ul style="list-style-type: none"> End-to-end delay of the adaptive filter is increased due to the analysis and synthesis filter banks. In a critically-sampled scheme, there is degradation in convergence performance due to aliasing [36]. In an over-sampled scheme, band-edges of the subband spectrum introduce small eigenvalues, which limits the convergence rate [84].
Normalized Subband Adaptive Filter (NSAF)	Adaptive tap weights are recursively updated in time domain	<ul style="list-style-type: none"> An N-by-L transformation matrix is employed, where the length L of the basis functions being larger than the transform size N. The input signal and desired response are partitioned into N subbands. These subband signals are used to adapt a fullband tap-weight vector. The fullband filter operates on the bandlimited input signals at the original sampling rate, while its tap weights being iteratively updated at the decimated rate. A synthesis filter bank (i.e., an inverse transform) is required in deriving the fullband error signal. The NSAF algorithm can be seen as the generalization of the NLMS algorithm, where the input regressor $\mathbf{u}(n)$ is decomposed into N subband regressors. Such a generalization contributes to the decorrelating of the NSAF algorithm. 	<ul style="list-style-type: none"> To improve the convergence rate of the NLMS algorithm with a minimum amount of additional computations [61, 62]. 	<ul style="list-style-type: none"> End-to-end delay of the adaptive filter is increased due to analysis and synthesis filter banks. The NSAF does not suffer from the aliasing and band-edge effects [61, 62].

Appendix B

Cosine Modulation

In an N -channel cosine-modulated filter bank the analysis filters are cosine-modulated versions of a lowpass prototype filter $P(z)$ with a cutoff frequency at $\pi/2N$. The prototype filter $P(z)$ is generally restricted to be a linear phase filter with symmetric impulse response [9, 73, 89, 105, 125], such that

$$P(e^{j\omega}) = e^{-j\omega(L-1)/2} P_R(\omega), \quad (\text{B1})$$

where $P_R(\omega)$ is the real-valued zero-phase response (or amplitude response) of the prototype filter. Evaluating (2.31) along the unit circle and use (B1) in the expression, the frequency response of the cosine-modulated filter $H_i(z)$ can be formulated in terms of the zero-phase response $P_R(\omega)$ in the following form

$$H_i(e^{j\omega}) = e^{-j\omega\left(\frac{L-1}{2}\right)} \left[e^{j\theta_i} U_i(\omega) + e^{-j\theta_i} V_i(\omega) \right], \text{ with } \theta_i = (-1)^i \frac{\pi}{4}, \quad (\text{B2})$$

where

$$U_i(\omega) = P_R\left[\omega - \frac{\pi}{N}(i+0.5)\right] \text{ and } V_i(\omega) = P_R\left[\omega + \frac{\pi}{N}(i+0.5)\right] \quad (\text{B3})$$

are the right- and left-shifted versions of the zero-phase response, respectively.

For a causal real-valued FIR filter $H(z)$ of length L , it has linear phase with symmetric or anti-symmetric impulse, if

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{L-1}{2}\right)} \left(e^{\pm j\frac{\pi}{2}} H_R(\omega) \right), \quad (\text{B4})$$

where $H_R(\omega)$ denotes the real-valued zero-phase response of $H(z)$. Comparing (B1) with (B4), it is obvious that the cosine-modulated filter $H_i(e^{j\omega})$ can not have

linear phase response due to the alignment factor $\theta_i = \pm\pi/4$ that is not equal to $\pm\pi/2$.

Appendix C

Polyphase Implementation of the NSAF Algorithm

In the normalized SAF (NSAF) of Figure 4.3, the diagonalized N -input N -output system $W(k, z)\mathbf{I}_{N \times N}$ is essentially a bank of parallel filters with identical transfer function $W(k, z)$. These N copies of the adaptive transversal filter operate on the subband signals $u_0(n), u_1(n), \dots, u_{N-1}(n)$ at the original sampling rate, while the tap-weight adaptation is performed at a decimated rate. The filter outputs defined in (4.2), which is reproduced here for convenience,

$$y_{i,D}(k) = y_i(kN) = \mathbf{w}^T(k) \mathbf{u}_i(k), \text{ for } i = 0, 1, \dots, N-1, \quad (\text{C1})$$

are evaluated once for every N input samples since the adaptive tap weights are updated at the decimated rate. The realization of a transversal filter followed by a critical decimation operation is illustrated in Figure C1. Notice that there are M

Figure C1 Direct-form realization of an adaptive transversal filter with a decimated output.

decimators embedded within the transversal filter (c.f. Figure 1.2). For every N input samples, the delay line and the decimators gather a block of M samples

$$\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \dots, u_i(kN-N+1), \\ u_i(kN-N), \dots, u_i(kN-M+1)]^T, \quad (\text{C2})$$

which consists of N new samples and $M-N$ old samples for the adaptive tap weights $\mathbf{w}(k) = [w_0(k), w_1(k), \dots, w_{M-1}(k)]^T$.

In [99], Pradhan and Reddy proposed an alternative realization for the transversal filter $W(k, z)$ based on polyphase decomposition. In their proposed structure, the transversal filter $W(k, z)$ is represented in polyphase form

$$W(k, z) = \sum_{r=0}^{N-1} W_r(k, z^N) z^{-r}, \quad (\text{C3})$$

where $W_r(k, z)$ is the r th polyphase component of $W(k, z)$, which is given by

$$W_r(k, z) = \sum_{n=0}^{K-1} w_{nN+r}(k) z^{-n}, \text{ for } r = 0, 1, \dots, N-1. \quad (\text{C4})$$

In the above equations, k is the iteration index for tap-weight adaptation, K denotes the length of the polyphase components, and it is assumed that the adaptive transversal filter is of length $M = NK$ for the sake of simplicity. The polyphase realization is depicted in Figure C2.

The realizations in Figure C1 and C2 are equivalent. Both realizations are computationally efficient, where all the multiplications and additions are performed at the decimated rate. The advantage of the direct-form realization of Figure C1 over the polyphase realization of Figure C2 is that it allows a handy and compact representation of the filtering operation using matrix notation as in (C1).

Figure C2 Polyphase realization of an adaptive transversal filter with a decimated output.

Appendix D

Paraunitary, Lossless, and Power Complementary

An analysis filter bank can be conveniently represented with matrix notation as in (2.10) and (2.18), which is reproduced here for convenience:

$$\begin{aligned}\mathbf{h}(z) &\equiv [H_0(z), H_1(z), \dots, H_{N-1}(z)]^T \\ &= \mathbf{E}(z^N) \mathbf{e}(z),\end{aligned}\tag{D1}$$

where $\mathbf{E}(z)$ is the polyphase component matrix of the filter bank, and $\mathbf{e}(z)$ is the delay chain defined by (2.20). The filter bank $\mathbf{h}(z)$ is paraunitary if the following condition is fulfilled:

$$\tilde{\mathbf{E}}(z) \mathbf{E}(z) = \mathbf{I}.\tag{D2}$$

If the filter bank is causal and stable, in addition to paraunitary, it is referred to as a lossless system [125, pp. 288]. As far as causal FIR filter is concerned, the terms paraunitary and lossless are interchangeable. The relation of the paraunitary condition to perfect reconstruction analysis/synthesis system is explained in Section 2.3.

Apart from the perfect reconstruction property, the paraunitary condition also leads to a power complementary filter bank. The proof goes as follows. Using (D1) in (2.12) leads to

$$\begin{aligned}[\mathbf{H}^{(m)}(z)]^T &= [\mathbf{E}(z^N) \mathbf{e}(z), \mathbf{E}(z^N W_N^N) \mathbf{e}(z W_N), \dots, \\ &\quad \mathbf{E}(z^N W_N^{N(N-1)}) \mathbf{e}(z W_N^{N-1})].\end{aligned}\tag{D3}$$

Knowing that

$$(W_N^l)^N = (e^{-j2\pi l/N})^N = 1, \text{ for } l = 0, 1, \dots, N-1,$$

(D3) can be further reduced to

$$[\mathbf{H}^{(m)}(z)]^T = \mathbf{E}(z^N) [\mathbf{e}(z), \mathbf{e}(z W_N), \dots, \mathbf{e}(z W_N^{N-1})].\tag{D4}$$

Let $\Delta(z) \equiv \text{diag}[1, z^{-1}, \dots, z^{-N+1}]$ denote a diagonal matrix of delay. From (2.20), it follows that

$$\mathbf{e}(zW_N^{-L}) = \Delta(z)[1, W_N^{-L}, \dots, W_N^{-L(N-1)}]^T. \quad (\text{D5})$$

Substituting the result of (D5) into (D4), and then taking the transposition, leads to

$$\mathbf{H}^{(m)}(z) = [\mathbf{E}(z^N)\Delta(z)\mathbf{D}^*]^T = \mathbf{D}^H \Delta(z) \mathbf{E}^T(z^N), \quad (\text{D6})$$

where $\mathbf{D} = [a_{mn}]$ denotes the DFT matrix with elements $a_{mn} = e^{-j2\pi mn/N}$, and the superscript H denotes Hermitian transposition. Taking the product of $\mathbf{H}^{(m)}(z)$ and its paraconjugate

$$\tilde{\mathbf{H}}^{(m)}(z) = \tilde{\mathbf{E}}^T(z^N) \tilde{\Delta}(z) \mathbf{D}, \quad (\text{D7})$$

and then using the facts that $\tilde{\mathbf{E}}(z^N)\mathbf{E}(z^N) = \mathbf{I}$, $\mathbf{D}^H \mathbf{D} = N\mathbf{I}$, and $\Delta(z)\tilde{\Delta}(z) = \mathbf{I}$, we arrive at

$$\begin{aligned} \mathbf{H}^{(m)}(z) \tilde{\mathbf{H}}^{(m)}(z) &= \mathbf{D}^H \Delta(z) [\tilde{\mathbf{E}}(z^N)\mathbf{E}(z^N)]^T \tilde{\Delta}(z) \mathbf{D} \\ &= N\mathbf{I}. \end{aligned} \quad (\text{D8})$$

From (D8), it can be noticed that the first diagonal element of $\mathbf{H}^{(m)}(z) \tilde{\mathbf{H}}^{(m)}(z)$ is given by

$$\sum_{i=0}^{N-1} H_i(z) \tilde{H}_i(z) = N. \quad (\text{D9})$$

Now, evaluating (D9) along the unit circle, we obtain

$$\begin{aligned} \sum_{i=0}^{N-1} [H_i(e^{j\omega})][H_i(e^{j\omega})]^* &= N \\ \frac{1}{N} \sum_{i=0}^{N-1} |H_i(e^{j\omega})|^2 &= 1. \end{aligned} \quad (\text{D10})$$

Equation (D10) indicates that the paraunitary filter bank $\mathbf{h}(z)$ is power complementary.