Robust coordinated multiple-axis motion control systems and applications

Xiao, Yong

2006

Xiao, Y. (2006). Robust coordinated multiple-axis motion control systems and applications. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/3444>

<https://doi.org/10.32657/10356/3444>

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ROBUST COORDINATED MULTIPLE-AXIS MOTION
CONTROL SYSTEMS AND APPLICATIONS

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NANYANG TECHNOLOGICAL UNIVERSITY

2006

Robust Coordinated Multiple-axis Motion Control Systems And Applications

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A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2006

Acknowledgements

I would first like to thank my supervisor, Dr. Zhu Kuanyi, for all the advice he has dispensed over the years. During the course of this study he always took an active interest in my work and helped me to channel my sometimes very scattered thoughts in the right direction. Thank you, Dr. Zhu, for all your guidance and support.

I then take this chance to express my gratitude to Dr. Ling Keck Voon for his enthusiastic encouragement and introducing me to many applications. A very special thank goes to Mr. Liao Hwee Choo for sharing his industrial experience with me.

Thanks are also dedicated to my colleagues and technicians at Process Instrumentation Laboratory and staff members of the School of Electrical & Electronic Engineering at Nanyang Technological University for their invaluable support. I would also like to acknowledge the Nanyang Technological University and the Singapore Institute of Manufacturing Technology, for providing the financial support and the research facilities.

Last but definitely not least, I want to thank my parents, brother, sister, wife and all the others who love and care for me through my long educational journey. Without their steadfast love, support, sacrifice and understanding, I could never have achieved this goal.

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Summary

Motion control is an essential component of modern manufacturing machinery, and highly competitive markets encourage manufacturing industry to realize improved performance through greater machine flexibility and productivity. A challenging problem in recent motion control field is that the motion of multiple axes or motors must be controlled in a coordinated manner to perform highly accurate operation, such as plotting, laminating and laser cutting.

In this thesis, new cross-coupling model predictive control (MPC) algorithms for multiple axes motion systems are proposed. This is to ensure that the multiple motion axes are synchronized while they track their desired trajectories. This is realized by minimizing the modified quadratic performance indices where the synchronization errors of the multiple motion axes are incorporated. In particular, a robust synchronization algorithm based on MPC technique in the transfer-function description is first developed. The resulting control law generates optimal cross-coupling control action and in consequence, improved synchronization performance can be achieved. Robustness analysis of the proposed control system is also investigated thoroughly. Computer simulations on an electro-hydraulic synchronous motion system and a positioning system with two permanent magnet linear motors (PMLMs) demonstrate the effectiveness of this method.

Then, an extended synchronization algorithm with reference models follows. It is shown that by introducing the reference model in the algorithm pole-zero cancellation occurs, and the system will behave asymptotically as the reference model provided that the canceled poles are stable. An application of this algorithm to an electro-hydraulic process control is presented.

Stability of our MPC system is investigated in the powerful framework of linear quadratic Gaussian (LQG) optimal control, and stability properties relating to this system are discussed thoroughly. For this, a synchronization controller in the state space is developed by augmenting the original system with a synchronization observer, where the synchronization errors across each axis and the other two axes (but not all other axes), without loss of generality, are considered. One advantage of this control strategy is that various control requirements can be met by setting the output transition matrices accordingly. Simulation results on a magnetic bearing-based flywheel energy storage system (FESS) show that this approach is very effective to recover the unstable system in the presence of disturbances.

Finally, an experimental test of the cross-coupling MPC algorithm on the positioning platform with two PMLMs is accomplished. The proposed algorithm is written in C language within the dSPACE control and development environment and embedded into an S-function block of MATLAB/Simulink, so that the experiment can be easily realized. The experimental results have demonstrated that the synchronized MPC algorithm can render better performance in terms of robustness and synchronization than the conventional proportional-integral-differential (PID) controller.

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Chapter 1

Introduction

In this chapter, we will introduce the idea of coordinated motion control, the goal of which is to improve the overall performance of a multi-axis motion system in terms of robustness and synchronization. First of all, the motivation behind this thesis will be given. Previous research on relevant topics will then be reviewed, and main contributions of the thesis will be summarized. Finally, the organization of the thesis will be outlined.

1.1 Motivation

Motion control systems for machining, laser cutting, rolling and other manufacturing applications are required to provide accurate multi-dimensional tracking or multi-axis synchronization, and highly competitive markets encourage manufacturing industry to realize improved performance through greater machine flexibility and productivity. For example, one fundamental problem is that the motion of multiple axes or motors of a machining center must be coordinated to perform specific operations, as poor coordination of the relevant motion axes may result in products with inferior dimensional accuracy. On the other hand, the performance of the control system may be degraded

by many factors, such as mismatched dynamics, disturbances, system parameter variations due to the environmental changes and system nonlinearities, if they are not taken into consideration in the design of motion controllers. It is also recognized that increasing the plant size and power may not be the best solution because of increased inertia as well as increased cost and power consumption. Hence, the challenge is to develop robust motion controllers to improve the synchronization performance of multi-axis motion systems, which will be addressed in this thesis.

1.2 Background and Previous Work

1.2.1 Coordinated Motion Control

The term “axis of motion” refers to one degree of freedom, or forward and backward motion along one direction. It may be linear or rotary motion, and may take the form of a conveyer belt, a rotary knife, or many other types. When two or more axes of motion are involved in a single machine, that machine is employing multi-axis motion. In industrial manufacturing systems, one often encounters situations in which two or more motion axes must be coordinated. The most familiar examples of multi-axis motion systems requiring synchronization are the X-Y plotter and laser cutter, in which there exist two axes, the X direction and the Y direction. Each may move independently of each other, but if a two-dimensional figure is to be drawn or to be cut accurately, their motion must be coordinated. For the case of a rolling machine, however, different rollers are required to be operating at coordinated speeds, so that high accuracy of products can be obtained even if disturbances and model uncertainties are present.

In reality, motion systems may be dynamically coupled among the subsystems due to operational requirements and/or physical limits. For instance, a magnetic bearing-

based flywheel energy storage system is dynamically coupled [1]. Although the contactless nature of magnetic bearings brings up many advantages over the conventional bearings, one of the challenging problems is to synchronize the rotor of the magnetic bearing-based system in the radial direction, so that the gyroscopic effect due to outside disturbances and plant uncertainties can be suppressed. In practice, there are also many situations where the motion systems are completely uncoupled or their couplings are quite weak and therefore, their effects can be neglected. In this case, all subsystems are considered dynamically independent, even though they must operate synchronously. One example of such systems can be found in an electro-hydraulic synchronous structure, where the controlled object is a mechanical punch driven by two independent symmetric cylinders and the two cylinders must move synchronously and smoothly for good quality of products and the lifetime of the machine. It is considered that this system exhibits a very weak dynamic coupling when both cylinders operate in certain range. Indeed, the need for multi-axis synchronization arises whenever the axes must move together to perform coordinated operations. It is believed that a properly designed synchronization controller might improve the synchronization performance regardless of the actual system, which exhibits dynamic coupling in nature.

On the other hand, motion control has evolved from traditional mechanical control systems to electronic servo control systems, to the latest coordinated motion and sequential control platforms [2]. This new approach allows communication across single architecture, eliminating multiple control complexities that serve as barriers to efficiency [3]. As for today's complex processes, tighter coordination between motion and sequential control is required. Figure 1.1 depicts an offset printing control system, whereby ink is spread on a metal plate with etched images, then transferred to an intermediary surface such as a rubber blanket, and finally applied to paper by pressing

the paper against the intermediary surface. However, individual servo motors of the color print stations must be precisely synchronized for increasing accuracy, printing speed and flexibility of adding or removing print stations with minimum downtime.

Figure 1.1: Offset printing control systems

In reality, coordination, performance and maintenance challenges of such a system can arise from:

- mounting and wiring multiple controls.
- separate programming packages for the sequential and motion control, with the requirement to manage and maintain multiple programs.
- redundant I/O or network interfaces to link the sequential and motion controllers.
- complex synchronization logic in the sequential and motion controllers.
- switching between multiple programming packages, which may be tied in via multiple network connections, to monitor or diagnose malfunctions.

As mentioned before, the fundamental problem in motion control area is that the motion of multiple axes or motors must be coordinated to perform specific operations. In

most multi-axis motion systems, however, the controller is usually designed for each axis independently, which results in a collection of decoupled single-input single-output (SISO) control systems. This decoupled design may be preferable if performance degradation in one axis should not affect the performance of other axes. Otherwise, decoupling may not be appropriate. In the case of a differential-drive robot, for example, two independent wheels driven by two motors should move synchronously for some operations [4]. In conventional control systems, any error caused by disturbances in one loop is corrected only by this loop, while the other loop may carry on its own correction. This, due to lacking communication, may cause unacceptable synchronization errors in the resultant path. Therefore, if a motion axis is significantly perturbed, a cross-coupling control sharing the feedback information of the axes becomes necessary.

The idea of cross-coupling control was first introduced by Koren *et al.* in [5,6]. Kulkarni and Srinivasan [7–9] improved the two-dimensional tracking performance of two-axis Cartesian systems by introducing cross-coupled control to a conventional uncoupled design, where the cross-coupling terms are functions of the instantaneous slope of the desired trajectory. Bulter *et al.* [10,11] approached motion coordination from the point view of trajectory planning. Chiu [12] proposed three different coordination algorithms based on linear quadratic optimal control, Lyapunov synthesis and formulation in the task (local) coordinate frame. Coordinated motion control subject to actuator saturation was also investigated by Niu [13]. A cross-coupling motion controller for mobile robots was proposed by Feng *et al.* [14] to adjust the moving path accurately by compensating the orientation error. Sun *et al.* [15] applied the cross-coupling concept into multi-robot coordinated assembly tasks. Analysis and design of integrated control for multi-axis motion system were presented by Yeh *et al.* [16,17]. Other synchronization approaches, such as fuzzy logic coupling controller [18] and neuro-controller for synchro-

nization [19], can also be found in the literature. However, the principal shortcoming of the existing control techniques is that they cannot explicitly incorporate plant model uncertainties in the formulation of the control system, so that there is not a general algorithm that is suitable for the stable control of the majority of real processes with multiple motion axes, especially, when high precision is a major requirement.

1.2.2 Model Predictive Control

Model predictive control (MPC) based on a quadratic performance index is regarded as the advanced control technique – that is, more advanced than the proportional-integral-differential (PID) control – to have had a significant and widespread impact on industrial processes, and is an effective strategy. MPC refers to a class of computer control algorithms that utilize an explicit process model to predict the future response of a plant. At each control interval an MPC algorithm attempts to optimize future plant behavior by computing a sequence of future manipulated variables. The first input in the optimal sequence is then sent into the plant, and the entire calculation is repeated at subsequent control intervals. In the formulation of MPC, the prediction and control horizons, and the weighting factor in the performance index are key parameters as they determine the dynamics of the closed-loop system.

Since the concept of predictive control was introduced simultaneously by Richalet [20] and Cuttler [21] in the late seventies, the theory of predictive control has been well developed nearly in all aspects, such as stability, nonlinearity, and robustness [22–25]. As an open methodology, there are many ways to design a predictive controller within the framework of predictive control. As a result, over ten different predictive controllers, each with different properties, have been reported. Well-known predictive controllers are generalized predictive control (GPC, [26–28]), dynamic matrix control

(DMC, [21]), extended prediction self-adaptive control (EPSAC, [29]), predictive functional control (PFC, [30]), extended horizon adaptive control (EHAC, [31]) and unified predictive control (UPC, [32]). Amongst the most popular predictive control strategy is GPC. Its popularity is partly due to its control acumen to deal with the multivariable constrained control problems, as the actual control objectives and the operating constraints can be represented explicitly in the optimization function. Furthermore, it can be used to control a wide variety of processes, such as nonminimum phase and unstable processes, without the designer having to take much special precautions. Especially in high-performance applications such as the control of flexible systems GPC can achieve better robustness of either dead-time or model order to prior choices, such as minimum-variance control [33] and generalized minimum-variance control [34–36].

Several recent publications provided a good introduction to practical issues associated with MPC technology. Rawlings [37] provided an excellent introductory tutorial aimed at control practitioners. A survey of industrial MPC technology based on linear models was presented by Qin *et al.* [38], summarizing applications through 1995. A review of industrial MPC applications using nonlinear models was also presented in [39]. Brempt *et al.* [40] developed a new model predictive control technology that enables a high performance in the chemical process industry with low development and implementation costs. Froisy [41] and Kulhavy *et al.* [42] described industrial MPC practice and future developments from the vendor's viewpoint. Development of MPC technology within operating companies were also reported by Young *et al.* [43]. A survey of MPC technology in Japan provides a wealth of information on application issues from the point of view of MPC users [44]. As for the current status of commercial MPC technology, the companies surveyed and their products are given in [45].

1.3 Objectives

MPC has traditionally been applied to plants where the dynamics are slow enough to permit a sampling rate amenable to optimal input computations between samples, e.g. chemical process plants. These systems are usually also governed by strict constraints on states, inputs and/or combinations of both. With the advent of faster modern computers, it has become possible to extend the MPC approach to systems governed by faster dynamics that warrants this type of solution, such as food processing, automotive, and aerospace applications. On the other hand, interest in the multi-axis coordinated control problem has increased in recent years where joint autonomous tasks are of interest, e.g. micro-satellite formations for communications and imaging and unmanned air vehicle (UAV) cooperation in adversarial environments for defense purposes. Dunbai [46] extended MPC to a coordinated multi-vehicle realm to achieve a formation and perform joint maneuvers in formation. Chen [47] proposed a cross-coupled predictive controller for electro-hydraulic synchronous motion systems, and investigated the robustness of the algorithm. Zhu *et al.* [48] presented a cross-coupling design of generalized predictive control based on a reference model.

However, most of the synchronization efforts so far have focused on multi-axis synchronization with the same desired trajectory. There exists an increasing demand of developing new control algorithms that can address coordination of multiple motion axes with different commands and can render more satisfactory synchronization performance or higher tracking accuracy of the desired contour. Therefore, the control algorithms developed in the following chapters are based on minimization of the modified performance indices, in which the synchronization errors and their differences are incorporated, so that improved control performance can be obtained.

1.4 Major Contributions

Major contributions of this thesis are summarized below.

- A robust synchronization MPC algorithm is developed. In this algorithm, both external disturbances and unmodelled dynamics are considered. A new quadratic performance index incorporating the synchronization errors of the multiple motion axes is introduced, so that cross-coupling control action is generated and improved synchronization performance can be obtained. Also, robustness of the proposed control system is analyzed.
- A synchronized MPC system with reference models is also proposed. It is shown that such an algorithm provides a combined feedback-feedforward control resulting in pole-zero cancelation of poles which do not correspond to the reference model. Hence, the system behaves asymptotically as the reference model provided the canceled poles are stable. Also, it can reduce the synchronization deviations of processes with multiple motion axes effectively.
- Stability of the control scheme in the state space is investigated thoroughly, and a synchronization control algorithm with constraints is formulated.
- An application of the proposed algorithms to a positioning system with two motors is completed.

1.5 Organization of the Thesis

The remainder of this thesis is organized as follows. In Chapter 2, robust synchronization MPC system is proposed. A new quadratic performance index incorporating the synchronization errors of the multiple motion axes is introduced. Optimal synchronization control algorithm minimizing the performance index is then derived. Robustness

the proposed algorithm is analyzed, followed by computer simulations on an electro-hydraulic synchronous motion system and a positioning system with two permanent magnet linear motors (PMLMs).

In Chapter 3, MPC algorithm with reference models is introduced. The synchronization performance with respect to the weighting factors for the synchronization errors is studied. An application to an electro-hydraulic process with two motion axes is given.

Chapter 4 is concerned with formulation and stability analysis of the synchronization control system in the state space. A synchronization estimator is introduced to the original system so that the tracking control can be simplified into a regulation one. Stability of the control scheme is investigated and stability results are established. A synchronization control algorithm with constraints is also developed. Simulations on a positioning system and a magnetic bearing-based FESS in the presence of disturbances are carried out.

In Chapter 5, the proposed algorithm is tested on a positioning system. Modeling of a positioning system is presented. The system parameters are then estimated using Recursive Least Squares (RLS) method. The control algorithm is written in C language and embedded into a Simulink block within the dSpace development and rapid prototyping control system. Comparison of the real-time synchronization performance of the synchronized MPC algorithm with the PID controller is accomplished.

Conclusions and recommendations for future research are given in Chapter 6.

Chapter 2

Robust Synchronization MPC Algorithm

In this chapter, a robust synchronization MPC algorithm for multi-axis coordinated motion systems is developed. First, the standard MPC algorithm is briefly reviewed. Then, the synchronization strategy and the closed-loop control structure are presented. This is followed by robustness analysis and simulation studies on an electro-hydraulic synchronous system and a positioning system with two PMLMs. Finally, guidelines on the choice of the control parameters are given, and conclusion of this chapter is drawn.

2.1 Process Model and MPC

Consider an uncoupled multi-axis motion system described by:

$$A_i(q^{-1})y_i(k) = B_i(q^{-1})u_i(k-1) + C_i(q^{-1})\xi_i(k)/\Delta, \quad i = 1, 2, \dots, n \quad (2.1)$$

where k is the normalized discrete time, i denotes the i th motion axis, $y_i(k)$ and $u_i(k)$ are the output and the control input of the process respectively, $\xi_i(k)$ represents system

disturbance and unmodeled dynamics, Δ is the differencing operator, and $A_i(q^{-1})$, $B_i(q^{-1})$ and $C_i(q^{-1})$ are polynomials in the backward shift operator q^{-1} :

$$A_i(q^{-1}) = 1 + a_{i1}q^{-1} + a_{i2}q^{-2} + \cdots + a_{in_A}q^{-n_{A_i}} \quad (2.2)$$

$$B_i(q^{-1}) = 1 + b_{i1}q^{-1} + b_{i2}q^{-2} + \cdots + b_{in_B}q^{-n_{B_i}} \quad (2.3)$$

$$C_i(q^{-1}) = 1 + c_{i1}q^{-1} + c_{i2}q^{-2} + \cdots + c_{in_C}q^{-n_{C_i}} \quad (2.4)$$

where n_{A_i} , n_{B_i} and n_{C_i} are degrees of polynomials $A_i(q^{-1})$, $B_i(q^{-1})$ and $C_i(q^{-1})$ respectively. Throughout the remainder of the thesis, the explicit arguments q^{-1} will be dropped when no confusion arises, and if there is no special statement, $i = 1, 2, \dots, n$ will be regarded as the default value.

Because in predictive controllers, minimization of a performance index yields the predictive control law, the choice of the performance index is of paramount importance. Design objectives such as overshoot, rise time, settling time and damping ratio can easily be understood and specified. However, it is difficult to minimize performance indexes based on such objectives, because the relationship between the controller parameters and these criteria is in general highly nonlinear. This is why mathematically convenient performance index is often used. Probably the simplest one for (2.1) is:

$$I_p = \sum_{i=1}^n \left\{ \sum_{j=1}^{H_p} (w_i(k+j) - \hat{y}_i(k+j|k))^2 + \lambda_i \sum_{j=1}^{H_c} \Delta \hat{u}_i^2(k+j-1|k) \right\} \quad (2.5)$$

where $w_i(k+j)$ is the reference signal, H_p and H_c are the prediction and control horizons, respectively, λ_i is a non-negative weighting factor for the control input. Note that if $\lambda_i = 0$, then minimization of (2.5) yields a minimum-variance controller which minimizes the tracking error between the predicted process output and the ref-

erence trajectory. Unfortunately, such a controller applied to a process with badly situated zeros results in a badly damped or even unstable control system. Hence, The weighting factor with $\lambda_i > 0$ is introduced to make a trade-off between two conflicting objectives: the minimization of the tracking error and the minimization of the control input. Increasing the weighting factor makes the control input variance more important in the performance index, and the controller is less active. However, the tracking of the trajectory becomes less important resulting in a slower process output.

By ignoring the future disturbance the future output $\hat{y}_i(k+j|k)$ can be expressed as

$$\hat{y}_i(k+j|k) = G_{ij}\Delta\hat{u}_i(k+j-1|k) + o_{ij}(k), \quad j = 1, 2, \dots, H_p \quad (2.6)$$

where

$$o_{ij}(k) = \frac{F_{ij}}{C_i}y_i(k) + \frac{H_{ij}}{C_i}\Delta u_i(k-1), \quad (2.7)$$

and polynomials F_{ij} , G_{ij} and H_{ij} satisfy the following Diophantine equations:

$$C_i = \Delta A_i E_{ij} + q^{-j} F_{ij} \quad (2.8)$$

$$B_i E_{ij} = C_i G_{ij} + q^{-j} H_{ij} \quad (2.9)$$

with $G_{ij} = G_{ij-1} + g_{ij}q^{-j+1}$, and the degrees of E_{ij} , F_{ij} , G_{ij} and H_{ij} given by

$$\begin{cases} n_{E_{ij}} = n_{G_{ij}} = j - 1 \\ n_{F_{ij}} = \max(n_{A_i}, n_{C_i} - j) \\ n_{H_{ij}} = \max(n_{B_i}, n_{C_i}) - 1 \end{cases} \quad (2.10)$$

Representing the j -step-ahead predictor (2.6) in a matrix notation yields

$$\hat{\mathbf{Y}}_i = \mathbf{G}_i \hat{\mathbf{U}}_i + \mathbf{O}_i \quad (2.11)$$

where

$$\hat{\mathbf{Y}}_i = [\hat{y}_i(k+1|k), \hat{y}_i(k+2|k), \dots, \hat{y}_i(k+H_p|k)]^T \quad (2.12)$$

$$\hat{\mathbf{U}}_i = [\Delta \hat{u}_i(k|k), \Delta \hat{u}_i(k+1|k), \dots, \Delta \hat{u}_i(k+H_c-1|k)]^T \quad (2.13)$$

$$\mathbf{O}_i = [o_{i1}(k), o_{i2}(k), \dots, o_{iH_p}(k)]^T \quad (2.14)$$

$$\mathbf{G}_i = \begin{bmatrix} g_{i0} & 0 & \cdots & 0 \\ g_{i1} & g_{i0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ g_{iH_c-1} & g_{iH_c-2} & \ddots & g_{i0} \\ \vdots & \ddots & \ddots & \vdots \\ g_{iH_p-1} & g_{iH_p-2} & \cdots & g_{iH_p-H_c} \end{bmatrix}. \quad (2.15)$$

The performance index (2.5) can now be rewritten as

$$I_p = \sum_{i=1}^n \left\{ (\mathbf{W}_i - \hat{\mathbf{Y}}_i)^T (\mathbf{W}_i - \hat{\mathbf{Y}}_i) + \lambda_i \hat{\mathbf{U}}_i^T \hat{\mathbf{U}}_i \right\} \quad (2.16)$$

where

$$\mathbf{W}_i = [w_i(k+1), w_i(k+2), \dots, w_i(k+H_p)]^T. \quad (2.17)$$

2.2 Synchronization Strategy

In the coordinated motion control, both the tracking and synchronization performance amongst multiple axes are specified. However, if the axes follow different trajectories, i.e., $w_i(k) \neq w_j(k)$, a scalar of $\frac{w_i(k)}{w_j(k)}$ should be introduced to indicate the relationship between the i th axis and the j th axis accurately.

Definition 2.1. *The synchronization error across multiple axes is defined as*

$$\phi_{ij}(k) = \alpha_{ij}(y_i(k) - \rho_{ij}y_j(k)) \quad (2.18)$$

where α_{ij} and ρ_{ij} are the coupling and synchronization factors between the i th axis and the j th axis, respectively.

Note that $\alpha_{ij} = 0$ means that the i th axis and the j th axis are not coupled. Moreover, even if $\phi_{ij}(k) = 0$ at an instant time, the overall system may not be synchronized perfectly, due to $\Delta\phi_{ij}(k+1) \neq 0$, where $\Delta\phi_{ij}(k+1) = \phi_{ij}(k+1) - \phi_{ij}(k)$. Hence, the following definition is introduced.

Definition 2.2. *The difference between the consecutive synchronization errors is derived by*

$$\psi_{ij}(k) = (1 - q^{-1})\phi_{ij}(k). \quad (2.19)$$

Obviously, if $\phi_{ij}(k) = 0$ and $\psi_{ij}(k) = 0$ for $i, j = 1, \dots, n$ hold, then perfect synchronization performance can be achieved.

Denoting

$$\hat{\Phi}_{ij} = [\hat{\phi}_{ij}(k+1|k), \hat{\phi}_{ij}(k+2|k), \dots, \hat{\phi}_{ij}(k+H_p|k)]^T \quad (2.20)$$

$$\hat{\Psi}_{ij} = [\hat{\psi}_{ij}(k+1|k), \hat{\psi}_{ij}(k+2|k), \dots, \hat{\psi}_{ij}(k+H_p|k)]^T. \quad (2.21)$$

and setting the first element of $\hat{\Psi}_{ij}$ as zero, it can be obtained that

$$\hat{\Psi}_{ij} = \mathbf{L}\hat{\Phi}_{ij} \quad (2.22)$$

$$\hat{\Phi}_{ij} = \alpha_{ij}(\hat{\mathbf{Y}}_i - \rho_{ij}\hat{\mathbf{Y}}_j) \quad (2.23)$$

where

$$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \quad (2.24)$$

Now for satisfactory synchronization performance (2.16) can be modified as

$$\begin{aligned} I_{mp} &= I_p + \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^n \hat{\Phi}_{ij}^T \hat{\Phi}_{ij} + \eta \sum_{i=1}^{n-1} \sum_{j=i+1}^n \hat{\Psi}_{ij}^T \hat{\Psi}_{ij} \\ &= I_p + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \hat{\Phi}_{ij}^T \Lambda \hat{\Phi}_{ij} \end{aligned} \quad (2.25)$$

where I_{mp} is the modified performance index, I_p is defined in (2.16), $\Lambda = \nu \mathbf{I} + \eta \mathbf{L}^T \mathbf{L}$, where ν and η are non-negative weighting factors used to compensate the synchronization error and its difference, and $\mathbf{I}$ is an identity matrix with suitable dimension.

2.3 Synchronization Control Algorithm

Denote a matrix $\mathbf{X}$ as $[\mathbf{X}_{ij}]$, where $\mathbf{X}_{ij}$ is the submatrix. Then, minimizing the modified performance index (2.25) with respect to $\hat{\mathbf{U}}_i$ results in

$$\begin{bmatrix} \hat{\mathbf{U}}_1 \\ \hat{\mathbf{U}}_2 \\ \vdots \\ \hat{\mathbf{U}}_n \end{bmatrix} = \Xi_1^{-1} \Xi_2 \begin{bmatrix} \mathbf{W}_1 - \mathbf{O}_1 - \Lambda \sum_{m=2}^n \alpha_{1m}^2 (\mathbf{O}_1 - \rho_{1m} \mathbf{O}_m) \\ \vdots \\ \mathbf{W}_i - \mathbf{O}_i - \Lambda \sum_{m=i+1}^n \alpha_{im}^2 (\mathbf{O}_i - \rho_{im} \mathbf{O}_m) \\ \vdots \\ \mathbf{W}_n - \mathbf{O}_n \end{bmatrix}$$

$$+ \left[\begin{array}{c} 0 \\ \vdots \\ \Lambda \sum_{m=1}^{i-1} \alpha_{mi}^2 \rho_{mi} (\mathbf{O}_m - \rho_{mi} \mathbf{O}_i) \\ \vdots \\ \Lambda \sum_{m=1}^{n-1} \alpha_{mn}^2 \rho_{mn} (\mathbf{O}_m - \rho_{mn} \mathbf{O}_n) \end{array} \right] \quad (2.26)$$

where

$$\Xi_1 = [\Xi_{ij}], \quad \Xi_{ij} = \begin{cases} -\alpha_{ij}^2 \rho_{ij} \mathbf{G}_i^T \Lambda \mathbf{G}_j & \text{if } i < j \\ \mathbf{G}_i^T \left[\mathbf{I} + \Lambda \left(\sum_{m=1}^{i-1} \alpha_{mi}^2 \rho_{mi}^2 + \sum_{m=i+1}^n \alpha_{im}^2 \right) \right] \mathbf{G}_i + \lambda_i \mathbf{I} & \text{if } i = j \\ -\alpha_{ji}^2 \rho_{ji} \mathbf{G}_j^T \Lambda \mathbf{G}_i & \text{if } i > j \end{cases} \quad (2.27)$$

$$\Xi_2 = \text{diag}(\mathbf{G}_1^T, \mathbf{G}_2^T, \dots, \mathbf{G}_n^T). \quad (2.28)$$

In practice, only the first components of $\hat{\mathbf{U}}_i$ are applied at time k and they will be recomputed at time $k+1$. Hence, the following optimal synchronization algorithm is obtained,

$$\begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \\ \vdots \\ \Delta u_n(k) \end{bmatrix} = \mathbf{\Omega}^{-1} \left(\mathbf{\Pi} \begin{bmatrix} w_1(k) \\ w_2(k) \\ \vdots \\ w_n(k) \end{bmatrix} - \mathbf{\Gamma} \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_n(k) \end{bmatrix} \right) \quad (2.29)$$

where

$$\mathbf{\Omega} = [\mathbf{\Omega}_{ij}], \quad \mathbf{\Omega}_{ij} = \begin{cases} 1 + q^{-1} \{ v_{ii} + (\sum_{m=1}^{i-1} \alpha_{mi}^2 \rho_{mi}^2 (v_{ii} - v_{im}) \\ + \sum_{m=i+1}^n \alpha_{im}^2 (v_{ii} - v_{im})) \Lambda \} \frac{\mathbf{H}_i}{C_i}, & \text{if } i = j \\ q^{-1} \{ v_{ij} + (\sum_{m=1}^{j-1} \alpha_{mj}^2 \rho_{mj}^2 (v_{ij} - v_{im}) \\ + \sum_{m=j+1}^n \alpha_{jm}^2 (v_{ij} - v_{im})) \Lambda \} \frac{\mathbf{H}_j}{C_j}, & \text{if } i \neq j \end{cases} \quad (2.30)$$

2.3 Synchronization Control Algorithm

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$$\mathbf{\Gamma} = [\mathbf{\Gamma}_{ij}], \quad \mathbf{\Gamma}_{ij} = q^{-1} \left\{ v_{ij} + \left(\sum_{m=1}^{j-1} \alpha_{mj}^2 \rho_{mj}^2 (v_{ij} - v_{im}) + \sum_{m=j+1}^n \alpha_{jm}^2 (v_{ij} - v_{im}) \right) \Lambda \right\} \frac{\mathbf{F}_j}{C_j} \quad (2.31)$$

$$\mathbf{H}_i = [H_{i1}, H_{i2}, \dots, H_{iH_p}]^T \quad (2.32)$$

$$\mathbf{F}_i = [F_{i1}, F_{i1}, \dots, F_{iH_p}]^T, \quad (2.33)$$

and $\mathbf{\Pi} = [\Pi_{ij}]$, where Π_{ij} is the sum of the elements of v_{ij} , and $(v_{i1}, v_{i2}, \dots, v_{in})$ represents the $((i-1)H_c + 1)$ th row of $\mathbf{\Xi}_1^{-1} \mathbf{\Xi}_2$.

Substituting (2.29) into (2.1) results in the following feedback control system, as illustrated in Figure 2.1:

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_n(k) \end{bmatrix} = \mathbf{\Theta}^{-1} \begin{bmatrix} q^{-1}B_1 & 0 & \dots & 0 \\ 0 & q^{-1}B_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q^{-1}B_n \end{bmatrix} \times \mathbf{\Omega}^{-1} \mathbf{\Pi} \begin{bmatrix} w_1(k) \\ w_2(k) \\ \vdots \\ w_n(k) \end{bmatrix} + \mathbf{\Theta}^{-1} \begin{bmatrix} C_1 \xi_1(k) \\ C_2 \xi_2(k) \\ \vdots \\ C_n \xi_n(k) \end{bmatrix}, \quad (2.34)$$

where

$$\mathbf{\Theta} = \begin{bmatrix} A_1 \Delta & 0 & \dots & 0 \\ 0 & A_2 \Delta & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & A_n \Delta \end{bmatrix} + \begin{bmatrix} q^{-1}B_1 & 0 & \dots & 0 \\ 0 & q^{-1}B_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q^{-1}B_n \end{bmatrix} \times \mathbf{\Omega}^{-1} \mathbf{\Gamma}. \quad (2.35)$$

Properties of the robust synchronization MPC algorithm are summarized below.

- By appropriately selecting factors α_{ij} and ρ_{ij} ($i = 1, \dots, n-1$, $j = i+1, \dots, n$) many kinds of coordinated motions can be synchronized. For example, if all axes

Figure 2.1: Block diagram of the robust synchronization control system

in the system are required to be synchronized to follow the same trajectory, i.e. $w_1 = w_2 = \dots = w_n$, then $\alpha_{ij} = 1$ and $\rho_{ij} = 1$ can be set. In general, α_{ij} can be set as either 1 or 0, where value 0 indicates that the axes are not coupled. But if $\rho_{ij} = 0$, then $\alpha_{ij} = 0$. For some complex systems α_{ij} can also be set as any non-negative value. ρ_{ij} is determined by the ratio of the reference signals $w_i(k)$ and $w_j(k)$.

- For sequential motion control systems, it is important to synchronize an axis with the next one, which can be realized by setting $\alpha_{(i-1)i} = \alpha_{i(i+1)} = 1$ and $\alpha_{ij} = 0$ for $|j - i| > 2$. In this case, Ξ_1 becomes a tri-diagonal matrix, and (2.26) becomes

$$\begin{aligned}
\begin{bmatrix} \hat{\mathbf{U}}_1 \\ \hat{\mathbf{U}}_2 \\ \vdots \\ \hat{\mathbf{U}}_n \end{bmatrix} &= \mathbf{\Xi}_1^{-1} \mathbf{\Xi}_2 \left(\begin{bmatrix} \mathbf{W}_1 - \mathbf{O}_1 - \mathbf{\Lambda}(\mathbf{O}_1 - \rho_{12}\mathbf{O}_2) \\ \vdots \\ \mathbf{W}_i - \mathbf{O}_i - \mathbf{\Lambda}(\mathbf{O}_i - \rho_{i,i+1}\mathbf{O}_{i+1}) \\ \vdots \\ \mathbf{W}_n - \mathbf{O}_n \end{bmatrix} \right. \\
&\quad \left. + \begin{bmatrix} \mathbf{0} \\ \vdots \\ \rho_{i-1,i}\mathbf{\Lambda}(\mathbf{O}_{i-1} - \rho_{i-1,i}\mathbf{O}_i) \\ \vdots \\ \rho_{n-1,n}\mathbf{\Lambda}(\mathbf{O}_{n-1} - \rho_{n-1,n}\mathbf{O}_n) \end{bmatrix} \right) \quad (2.36)
\end{aligned}$$

where

$$\mathbf{\Xi}_1 = [\mathbf{\Xi}_{ij}], \quad \mathbf{\Xi}_{ij} = \begin{cases} \mathbf{G}_i^T [\mathbf{I} + \mathbf{\Lambda}(\rho_{i-1,i}^2 + 1)] \mathbf{G}_i + \lambda_i \mathbf{I} & \text{if } i = j \\ -\rho_{ij} \mathbf{G}_i^T \mathbf{\Lambda} \mathbf{G}_j & \text{if } i = j - 1 \\ -\rho_{ji} \mathbf{G}_j^T \mathbf{\Lambda} \mathbf{G}_i & \text{if } i = j + 1 \\ \mathbf{0} & \text{if } |i - j| > 2 \end{cases}$$

- In the control algorithm, if $\nu = 0$ and $\eta = 0$, then $\mathbf{\Lambda} = \mathbf{0}$ and $\mathbf{\Xi}_1$ becomes a diagonal matrix. This means that no cross-coupling control action exists across the multiple axes and each axis is controlled independently. It will be shown that introduction of the parameters ν and η can improve synchronization performance.
- Although the proposed algorithm is based on multiple SISO subsystems, where $\hat{\mathbf{Y}}_i$ is independent of $\hat{\mathbf{U}}_j$ for $i \neq j$, it can be easily extended to MIMO systems through modification of the predictor (2.11) accordingly [56, 57].

Note that the influence of the control parameters on the stability of the closed-loop system will not be studied explicitly in this chapter. However, an easier way to analyze the closed-loop stability in the state-space description will be presented in Chapter 4. Moreover, a thorough analysis of H_p , H_c , λ , ν and $C_i(q^{-1})$ on the stability and performance can also be found in [32,47]. Hence, the focus of next section is robustness analysis of the proposed control algorithm.

2.4 Robustness Analysis

The structure of MPC enables to handle many difficult control problems. Several tuning guidelines have been proposed for a stable process of MPC [34, 36, 49]. In spite of these advantages, MPC controllers employ an estimated model of the process relating past outputs to past inputs, and the algorithms for calculating the control signal are based upon this model. Typically the estimated model is unable exactly to match the true process, so the control calculation generally involves a reduced-order model [60]. This reflects a general lack of robustness analysis in such a control scheme. Some publications have provided the insight to bridge the gap between the robustness guidelines and closed-loop stability of predictive control, based on the uncertainty description [51, 52]. Principally based on the criterion of Nyquist, bounds on the magnitude of the unmodeled dynamics giving a sufficient condition for stability of the closed loop have been obtained for a controller expressed in the form of a pole-placement algorithm [53, 54]. In this section the key robustness results are based on the small gain theorem, which provides a sufficient condition for guaranteed stability of a closed-loop system. First of all, the terms ‘performance’ and ‘robustness’ are defined.

2.4.1 Performance

Two types of performance can be distinguished: servo and regulator performance [32]. For example, the servo behavior of the closed-loop system (2.34) is determined by the first part of the right side, and the regulator behavior is determined by the second part. Normally, the regulator behavior can be measured by applying $\xi_i(t)$ to a system while the reference trajectory is set as zero. Then, one has the two criteria: the process output variance and the control input variance, which are given by

$$\varsigma_{iyN}^2 = \frac{1}{N+1} \sum_{k=0}^N y_i(k)^2 \quad (2.37)$$

$$\varsigma_{iuN}^2 = \frac{1}{N+1} \sum_{k=0}^N u_i(k)^2 \quad (2.38)$$

Meantime, the servo behavior of a closed-loop system is evaluated through a unit step test (hence, $w(k+1) = w(k+2) = \dots = w(k+H_p) = 1$), as illustrated in Figure 2.2. The criteria: rise time t_{rk} , peak time t_{pk} , settling time t_{sk} , overshoot $\mathcal{O}$ and steady-state error ε_{ss} , are defined as

$$t_{rk} = T_s \min_k \{k / y(kT_s) \geq 0.9\} \quad (2.39)$$

$$t_{pk} = T_s \arg \max_k \{y(kT_s)\} \quad (2.40)$$

$$t_{sk} = T_s \min_k \{k / |y(kT_s) - y_{ss}| < 0.02 \mid y_{ss} \mid\} \quad (2.41)$$

$$\mathcal{O} = (y(t_p) - y_{ss}) \times 100[\%] \quad (2.42)$$

$$\varepsilon_{ss} = |1 - y_{ss}| \times 100[\%] \quad (2.43)$$

where y_{ss} is the steady-state value of $y(t)$.

Figure 2.2: Servo performance criterion

2.4.2 Robustness

A closed-loop system is robust if it maintains certain properties in spite of process variations. Two major properties are stability and performance, and two forms of robustness can be distinguished: stability robustness and performance robustness. Stability robustness means that the closed-loop system remains stable in spite of process changes. Performance robustness ensures that the performance of the closed-loop system, as defined in the preceding subsection, remains within specified bounds if the true process is different from its model [32].

Stability robustness

The most common specifications of system uncertainty is ‘norm-bounded’. It is as-

sumed that a nominal model of system (2.1) is given by,

$$\hat{\mathbf{P}}(q^{-1}) = \begin{bmatrix} \frac{q^{-1}\hat{B}_1}{\hat{A}_1} & 0 & \cdots & 0 \\ 0 & \frac{q^{-1}\hat{B}_2}{\hat{A}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{q^{-1}\hat{B}_n}{\hat{A}_n} \end{bmatrix} \quad (2.44)$$

The real system $\mathbf{P}(q^{-1})$ is given by a description such as:

$$\mathbf{P}(q^{-1}) = \hat{\mathbf{P}}(q^{-1}) + \mathfrak{d}_A, \text{ for additive model uncertainties} \quad (2.45)$$

$$\mathbf{P}(q^{-1}) = \hat{\mathbf{P}}(q^{-1})[\mathbf{I} + \mathfrak{d}_M], \text{ for multiplicative model uncertainties} \quad (2.46)$$

where $\mathfrak{d}_A$ and $\mathfrak{d}_M$ are stable bounded operators, and $\mathbf{P}(q^{-1})$ is often normalized in such a way that $\|\mathfrak{d}\| \leq 1$. Of course one does not know exactly what $\mathfrak{d}$ is. Various assumptions can be made about the nature of $\mathfrak{d}$: nonlinear, linear time-varying, linear parameter-varying and linear time-invariant being the most common ones. Also various norms can be used; the most commonly used one is the ‘H-infinity’ norm $\|\mathfrak{d}\|_\infty$, which is defined even for nonlinear systems, as the worst-case ‘energy gain’ of an operator.

A stable linear feedback controller $\mathbf{K}(q^{-1})$ designed for the nominal system $\hat{\mathbf{P}}(q^{-1})$ will certainly give a stable closed loop if the system is exactly the same as the model $\hat{\mathbf{P}}(q^{-1})$. But it will not necessarily be stable when used with the real system $\mathbf{P}(q^{-1})$. It is therefore important to have some way of testing whether it will remain stable for all possible systems allowed by the uncertainty description. Suppose that the uncertainty about the system is represented by the additive model (2.45). Then the feedback combination of the real system with the controller can be drawn as Figure 2.3.

Figure 2.3: Combination of system, controller and additive uncertainty

The uncertainty $\mathbf{d}_A$ is in parallel with the nominal system model $\hat{\mathbf{P}}(q^{-1})$, but the figure has been drawn to show it as being in feedback around the closed loop formed by the controller $\mathbf{K}(q^{-1})$ and $\hat{\mathbf{P}}(q^{-1})$ (which is enclosed by the broken line). The transfer function (matrix) of the block within the broken line can be shown to be $\mathbf{K}(q^{-1})\mathbf{S}(q^{-1})$, where $\mathbf{S}(q^{-1}) = [\mathbf{I} + \hat{\mathbf{P}}(q^{-1})\mathbf{K}(q^{-1})]^{-1}$ is the sensitivity function, and this transfer function is of course stable, by design. Thus the ‘loop gain’ transfer function is $\mathbf{K}\mathbf{S}\mathbf{d}_A$. It then follows from *the small-gain theorem* that the feedback combination of this system with the uncertainty block $\mathbf{d}_A$ will remain stable if

$$\bar{\sigma}[\mathbf{K}(e^{j\omega T_s})\mathbf{S}(e^{j\omega T_s})]\|\mathbf{d}_A\|_{\infty} < 1 \quad (2.47)$$

where $\bar{\sigma}[\cdot]$ denotes the largest singular value.

Inequality (2.47) can be tested for a given design, to check whether robust stability is guaranteed for any system within the set allowed by the uncertainty description. When tuning a controller, one can try to influence the frequency response properties in such a way as to make (2.47) hold. Note that (2.47) is only a sufficient condition for robust stability; if it is not satisfied, robust stability may nevertheless have been obtained.

Other uncertainty descriptions lead to other tests for robust stability. For example, the ‘input multiplicative’ uncertainty model (2.46) leads to the following sufficient condition for robust stability:

$$\bar{\sigma}[\mathbf{K}(e^{j\omega T_s})\hat{\mathbf{P}}(e^{j\omega T_s})(\mathbf{I} + \mathbf{K}(e^{j\omega T_s})\hat{\mathbf{P}}(e^{j\omega T_s})^{-1})]\|\mathfrak{d}_M\|_\infty < 1 \quad (2.48)$$

Note that for multivariable systems, $\mathbf{K}\hat{\mathbf{P}} \neq \hat{\mathbf{P}}\mathbf{K}$, so that the transfer function which appears in (2.47) is not the same as the complementary sensitivity.

Descriptions such as those given in (2.45) and (2.46), in which the only specification on the uncertainty $\mathfrak{d}$ is a norm-bounded, are known as *unstructured uncertainty description*. In practice much more specific information is available about the way in which the system behavior can vary. For example, it may be known that specific physical parameters, such as mass or resistance, can vary within known limits. In such cases a norm-bounded description can still be used, but the operator $\mathfrak{d}$ then is restricted to have a fixed structure. It can usually be arranged that this structure is that of a block-diagonal transfer function, with specified size of the blocks. The advantages of doing this is that robust stability tests can then be used which are less conservative than singular-value tests such as (2.48). This is due to the fact that the singular-value tests check whether any perturbation $\mathfrak{d}$ of the permitted ‘size’ (as measured by the norm) could destabilize the closed loop, even a perturbation whose structure does not correspond to any possible parameter variation. A more refined test, which checks only perturbations of the permitted size and structure is the *structured singular value*, also commonly referred to as μ . In principle, a necessary and sufficient test of robust stability can be obtained by a further refinement, the real structured singular value, or ‘real- μ ’. In practice, it is known that such a value can be extremely difficult to

compute (its computation is known to be ‘NP-hard’), and one has to be content with upper and lower bounds for it. Consequently, there is usually some conservativeness in the robust stability robustness test, even if a ‘real- μ ’ is employed.

Performance robustness

According to Kwakernaak [55], performance robustness can be defined as the sensitivity of the performance of the closed-loop system to process changes. The less sensitive the performance of the closed-loop system is with respect to process changes, the larger is the performance robustness. In this chapter, the performance robustness is not examined explicitly, in contrast to the stability robustness. However, there is a close relationship between both forms of robustness because the sum of the sensitivity and the complementary sensitivity function is equal to 1 for all frequency. It means that a large stability robustness will result in a large regulator performance robustness, a small servo-performance robustness and a small regulator performance, and vice versa [32].

2.4.3 Robust Analysis with Synchronization Compensation

The tuning procedure of the proposed controller (2.29) consists of two steps. The first step is concerned with choosing performance index weights and horizons such as to guarantee nominal stability (i.e. closed-loop stability, assuming that the linear plant model is perfect). The second step is to introduce the synchronization factors ν and η so as to improve the synchronization performance against plant/model mismatch. In practice, it is considered that a larger value of ν might improve stability robustness of the closed-loop control system, which will be verified later by the simulations. Next, we analyze the robustness of the proposed control algorithm.

For simplicity, a two-axis synchronous motion system is considered, with models $P_i =$

$\frac{q^{-1}B_i(q^{-1})}{A_i(q^{-1})}$, $i = 1, 2$. It is also assumed that the two axes are desired to track the same trajectory, i.e. $w(k) = w_1(k) = w_2(k)$, $\alpha_{12} = 1$ and $\rho_{12} = 1$ can be set. Define $\mathcal{Z}$ as an operator that converts a vector into a polynomial, i.e. $\mathcal{Z}_{v1} = \mathcal{Z}\{v_1^T\}$, and denote $\Xi_1^{-1}\Xi_2$ in (2.26) as

$$\Xi_1^{-1}\Xi_2 = \begin{bmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{bmatrix}. \quad (2.49)$$

Controller (2.29) can then be decomposed into

$$\begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \end{bmatrix} = \begin{bmatrix} \mathcal{Z}_{v1} \\ \mathcal{Z}_{v2} \end{bmatrix} w(k) - \begin{bmatrix} \mathcal{Z}_{h1} & \mathcal{Z}_{r1} \\ \mathcal{Z}_{r2} & \mathcal{Z}_{h2} \end{bmatrix} \begin{bmatrix} \frac{\Delta u_1(k-1)}{C_1} \\ \frac{\Delta u_2(k-1)}{C_2} \end{bmatrix} - \begin{bmatrix} \mathcal{Z}_{f1} & \mathcal{Z}_{s1} \\ \mathcal{Z}_{s2} & \mathcal{Z}_{f2} \end{bmatrix} \begin{bmatrix} \frac{y_1(k)}{C_1} \\ \frac{y_2(k)}{C_2} \end{bmatrix} \quad (2.50)$$

where

$$\begin{aligned} \mathbf{e}_1^T &= [1, 0, \dots, 0] & \deg\{\mathbf{e}_1^T\} &= 1 \times H_c \\ \mathbf{v}_1^T &= \mathbf{e}_1^T(\beta_1 + \beta_2) & \deg\{\mathbf{v}_1^T\} &= 1 \times H_p \\ \mathbf{x}_1^T &= \beta_1(\mathbf{I} + \Lambda) - \beta_2\Lambda & \deg\{\mathbf{x}_1^T\} &= H_c \times H_p \\ \mathbf{x}_2^T &= \beta_2(\mathbf{I} + \Lambda) - \beta_1\Lambda & \deg\{\mathbf{x}_2^T\} &= H_c \times H_p \\ \mathbf{h}_1^T &= \mathbf{e}_1^T \mathbf{x}_1^T \mathbf{H}_1 & \deg\{\mathbf{h}_1^T\} &= 1 \times n_{B_1} \\ \mathbf{f}_1^T &= \mathbf{e}_1^T \mathbf{x}_1^T \mathbf{F}_1 & \deg\{\mathbf{f}_1^T\} &= 1 \times (n_{A_1} + 1) \\ \mathbf{r}_1^T &= \mathbf{e}_1^T \mathbf{x}_2^T \mathbf{H}_2 & \deg\{\mathbf{r}_1^T\} &= 1 \times n_{B_2} \\ \mathbf{s}_1^T &= \mathbf{e}_1^T \mathbf{x}_2^T \mathbf{F}_2 & \deg\{\mathbf{s}_1^T\} &= 1 \times (n_{A_2} + 1) \\ \mathbf{v}_2^T &= \mathbf{e}_1^T(\beta_3 + \beta_4) & \deg\{\mathbf{v}_2^T\} &= 1 \times H_p \\ \mathbf{x}_3^T &= \beta_4(\mathbf{I} + \Lambda) - \beta_3\Lambda & \deg\{\mathbf{x}_3^T\} &= H_c \times H_p \\ \mathbf{x}_4^T &= \beta_3(\mathbf{I} + \Lambda) - \beta_4\Lambda & \deg\{\mathbf{x}_4^T\} &= H_c \times H_p \end{aligned}$$

$$\begin{aligned}
\mathbf{h}_2^T &= \mathbf{e}_1^T \mathbf{x}_3^T \mathbf{H}_2 & \deg\{\mathbf{h}_2^T\} &= 1 \times n_{B_2} \\
\mathbf{f}_2^T &= \mathbf{e}_1^T \mathbf{x}_3^T \mathbf{F}_2 & \deg\{\mathbf{f}_2^T\} &= 1 \times (n_{A_2} + 1) \\
\mathbf{r}_2^T &= \mathbf{e}_1^T \mathbf{x}_4^T \mathbf{H}_1 & \deg\{\mathbf{r}_2^T\} &= 1 \times n_{B_1} \\
\mathbf{s}_2^T &= \mathbf{e}_1^T \mathbf{x}_4^T \mathbf{F}_1 & \deg\{\mathbf{s}_2^T\} &= 1 \times (n_{A_1} + 1)
\end{aligned}$$

Note that if $\nu = 0$, then $\beta_2 = 0$ and $\beta_3 = 0$ in (2.49) can be obtained. This results in $\mathbf{x}_2^T = 0$ and $\mathbf{x}_4^T = 0$. It then follows that $\mathbf{r}_i^T = 0$ and $\mathbf{s}_i^T = 0$. Hence, the system becomes two independent subsystems, and the cross-coupling control action across the motion axes or motors are absent. Generally, in order to achieve satisfactory synchronization performance, characteristics for each drive should be identical or very similar to each other. This means in the case of perfect modeling, $\hat{P}_1 = \hat{P}_2$ is desired. If $C_2 = C_1$ is also set, then the MPC control parameters in (2.50) for each drive are symmetrical. Equivalently, (2.50) can be rewritten as

$$\begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} w(k) - \begin{bmatrix} S_1 + \iota_1 & -\iota_1 \\ -\iota_2 & S_2 + \iota_2 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} \quad (2.51)$$

where

$$\begin{aligned}
R_1 &= C_2(C_1 C_2 + C_1 \mathcal{Z}_{h2} q^{-1} + C_2 \mathcal{Z}_{h1} q^{-1} + \mathcal{Z}_{h1} \mathcal{Z}_{h2} q^{-2} - \mathcal{Z}_{r1} \mathcal{Z}_{r2} q^{-2}) \\
T_1 &= C_1 C_2 (\mathcal{Z}_{v1} C_2 + \mathcal{Z}_{v1} \mathcal{Z}_{h2} q^{-1} - \mathcal{Z}_{r1} \mathcal{Z}_{v2} q^{-1}) \\
S_1 &= C_2 (\mathcal{Z}_{f1} C_2 + \mathcal{Z}_{f1} \mathcal{Z}_{h2} q^{-1} - \mathcal{Z}_{r1} \mathcal{Z}_{s2} q^{-1}) - \iota_1 \\
\iota_1 &= C_1 (\mathcal{Z}_{r1} \mathcal{Z}_{f2} q^{-1} - \mathcal{Z}_{s1} C_2 - \mathcal{Z}_{s1} \mathcal{Z}_{h2} q^{-1}) \\
R_2 &= C_1 (C_1 C_2 + C_1 \mathcal{Z}_{h2} q^{-1} + C_2 \mathcal{Z}_{h1} q^{-1} + \mathcal{Z}_{h1} \mathcal{Z}_{h2} q^{-2} - \mathcal{Z}_{r1} \mathcal{Z}_{r2} q^{-2}) \\
T_2 &= C_1 C_2 (\mathcal{Z}_{v2} C_1 + \mathcal{Z}_{v2} \mathcal{Z}_{h1} q^{-1} - \mathcal{Z}_{r2} \mathcal{Z}_{v1} q^{-1}) \\
S_2 &= C_1 (P_{f2} C_1 + \mathcal{Z}_{f2} \mathcal{Z}_{h1} q^{-1} - \mathcal{Z}_{r2} \mathcal{Z}_{s1} q^{-1}) - \iota_2 \\
\iota_2 &= C_2 (\mathcal{Z}_{r2} \mathcal{Z}_{f1} q^{-1} - \mathcal{Z}_{s2} C_1 - \mathcal{Z}_{s2} \mathcal{Z}_{h1} q^{-1})
\end{aligned}$$

Then, the following closed-loop control system can be obtained:

$$\begin{bmatrix} \Delta y_1(k) \\ \Delta y_2(k) \end{bmatrix} = \begin{bmatrix} \varsigma_{1s} \\ \varsigma_{2s} \end{bmatrix} w(k) - \begin{bmatrix} \varsigma_{1r} & \varsigma_{1c} \\ \varsigma_{2c} & \varsigma_{2r} \end{bmatrix} \begin{bmatrix} \xi_1(k) \\ \xi_2(k) \end{bmatrix} \quad (2.52)$$

where

$$\begin{cases} \varsigma_{1s} = \frac{(\Delta R_2 T_1 + P_2(S_2 + \iota_2)T_1 + \iota_1 P_2 T_2)P_1}{1 + \vartheta} \\ \varsigma_{1r} = \frac{(\Delta R_2 + P_2(S_2 + \iota_2))R_1 C_1 / A_1}{1 + \vartheta} \\ \varsigma_{1c} = \frac{\iota_1 P_1 C_2 R_2 / A_2}{1 + \vartheta} \end{cases} \quad (2.53)$$

$$\begin{cases} \varsigma_{2s} = \frac{(\Delta R_1 T_2 + P_1(S_1 + \iota_1)T_2 + \iota_2 P_1 T_1)P_2}{1 + \vartheta} \\ \varsigma_{2r} = \frac{(\Delta R_1 + P_1(S_1 + \iota_1))R_2 C_2 / A_1}{1 + \vartheta} \\ \varsigma_{2c} = \frac{\iota_2 P_2 C_1 R_1 / A_2}{1 + \vartheta} \end{cases} \quad (2.54)$$

where the loop transfer function ϑ is given by

$$\vartheta = (\Delta R_1 + P_1(S_1 + \iota_1))(\Delta R_2 + P_2(S_2 + \iota_2)) - \iota_1 \iota_2 P_1 P_2 - 1, \quad (2.55)$$

Note that $1 + \vartheta = 0$ is the characteristic equation of the two-axis synchronous control system, and the stability of the process is guaranteed if all roots of $1 + \vartheta = 0$ are located in the unit circle. Clearly, (2.52) also shows that the parameter ι is necessary to stabilize the closed-loop control system while disturbances are present. Furthermore, the selection of the user-defined filter $C_i(q^{-1})$ is very critical for performance and robustness.

In the case of additive uncertainties with $P_i = \hat{P}_i + \mathfrak{d}_{iA}$, $i = 1, 2$, the actual characteristic

equation becomes

$$(\Delta R_1 + (\hat{P}_1 + \mathfrak{d}_{1A})(S_1 + \iota_1))(\Delta R_2 + (\hat{P}_2 + \mathfrak{d}_{2A})(S_2 + \iota_2)) - \iota_1 \iota_2 (\hat{P}_1 + \mathfrak{d}_{1A})(\hat{P}_2 + \mathfrak{d}_{2A}) = 0. \quad (2.56)$$

Rewrite (2.56) as

$$1 + \mathfrak{d}_{1A} \vartheta_{1A} = 0 \quad (2.57)$$

where

$$\vartheta_{1A} = \frac{(S_1 + \iota_1)(\Delta R_2 + (\hat{P}_2 + \mathfrak{d}_{2A})(S_2 + \iota_2)) - \iota_1 \iota_2 (\hat{P}_2 + \mathfrak{d}_{2A})}{(\Delta R_1 + \hat{P}_1(S_1 + \iota_1))(\Delta R_2 + (\hat{P}_2 + \mathfrak{d}_{2A})(S_2 + \iota_2)) - \iota_1 \iota_2 (\hat{P}_2 + \mathfrak{d}_{2A})}. \quad (2.58)$$

Then, the stability of the closed-loop system is guaranteed if the nominal system and the process are stable and if the following condition is satisfied:

$$|\mathfrak{d}_{1A}| \cdot |\vartheta_{1A}| < 1, \quad \forall \omega \geq 0 \quad (2.59)$$

By making the second part of (2.59) small for a certain frequency, the first part may be large without violating the stability condition. Hence, for those frequency where $|\vartheta_{1A}|$ is small, large deviations between process and model are possible without making the closed-loop system unstable; the stability robustness is large.

Likewise, the actual characteristic function for the case of multiplicative uncertainties with $P_i = \hat{P}_i(1 + \mathfrak{d}_{iM})$ is given by

$$1 + \mathfrak{d}_{1M} \vartheta_{1M} = 0 \quad (2.60)$$

where

$$\vartheta_{1M} = \frac{\hat{P}_1[(S_1 + \iota_1)(\Delta R_2 + \hat{P}_2(1 + \mathfrak{d}_{2A})(S_2 + \iota_2)) - \iota_1 \iota_2 \hat{P}_2(1 + \mathfrak{d}_{2A})]}{(\Delta R_1 + \hat{P}_1(S_1 + \iota_1))(\Delta R_2 + \hat{P}_2(1 + \mathfrak{d}_{2A})(S_2 + \iota_2)) - \iota_1 \iota_2 \hat{P}_1 \hat{P}_2(1 + \mathfrak{d}_{2A})}. \quad (2.61)$$

Hence, the stability condition of the synchronous system is obtained:

$$|\mathfrak{d}_{1M}| \cdot |\vartheta_{1M}| < 1, \quad \forall \omega \geq 0 \quad (2.62)$$

In next section, simulation studies on stability boundaries and tracking performance with different control parameters will be presented.

2.5 Simulation Studies

2.5.1 Electro-Hydraulic Synchronous System

Figure 2.4 shows a typical electro-hydraulic synchronous structure [74], which consists of two symmetrical cylinders driven by independent electro-hydraulic servo-valves. The controlled object is that the mechanic punch along with a heavy slide loaded on two symmetric cylinders. As the performance of this system may be degraded by many factors, such as mismatched dynamics and disturbances, it requires that

- The two symmetric cylinders must move synchronously and smoothly for good quality of products and for the lifetime of the machine;
- The punch must keep in parallel to the worktable all the time;
- The punch driven by the cylinders can be loaded to any position accurately and rapidly.

The characteristics of the power amplifier can effectively compensate the nonlinear behavior such as dead zone, clearance, backlash, and hysteresis in the hydraulic system.

Figure 2.4: Schematic representation of an electro-hydraulic control system

Also, the transfer function of the servo-valve can be approximated as a proportional gain, because the dynamic behavior of the process is mainly determined by the cylinder which is characterized by an integral plus an second order system. Hence, the transfer function of each subsystem is described by

$$G_p(s) = \frac{K_a K_v K_c}{s \left(\frac{s^2}{\omega_h^2} + \frac{2\zeta_h s}{\omega_h} + 1 \right)} \quad (2.63)$$

where K_a , K_v , K_c are gains of amplifier, servo-valve and cylinder, respectively. Natural frequency is $\omega_h = \sqrt{\frac{4\beta A_t^2}{V_t m}}$, and damping coefficient is $\zeta_h = \frac{K_c m \omega_h}{1 A_t^2}$, and m is the load mass, A_t is the cylinder area, β is the elasticity modulus and V_t is the compression cubage.

From the system property one has $n = 2$, $\alpha_{12} = 1$ and $\rho_{12} = 1$. Then, with a sampling period of 0.08 seconds the discrete-time models are given by

$$A_i y_i(k) = B_i u_i(k-1) + C_i \xi_i(k), \quad i = 1, 2 \quad (2.64)$$

where $y_i(k)$ is the position of the cylinder, $u_i(k)$ is the control input, and

$$\begin{cases} A_i = 1 + 0.4006q^{-1} - 0.8699q^{-2} - 0.5307q^{-3} \\ B_i = 0.4453 + 0.8739q^{-1} + 0.309q^{-2} \\ C_i = 1 - \tau q^{-1}, \tau \text{ is an unknown parameter} \end{cases} \quad (2.65)$$

The stability of this control system is easy to ensure in MPC and is guaranteed in such controllers as proposed by Clarke and Scattolini [60], Mosca and Zhang [61]. With that stability of the nominal closed-loop system, one can now introduce the cross-coupling control action ($\nu \neq 0$ and $\eta \neq 0$), and moves on to the stability of the real closed-loop system, i.e. the problem of robustness.

Case 1 *Influence of prediction and control horizons on stability robustness*

The prediction and control horizons are closely related to the stability of the closed-loop system. In the case of additive uncertainties with $\mathfrak{d}_{2A} = 0, 0.5$, the peak value of gain $|\vartheta_{1A}|$ in (2.59) against variation of prediction horizon is illustrated in Figure 2.5, while $H_c = 1$, $\lambda = 3$ and $C = 1 - 0.8q^{-1}$ are set. Also, if $\nu = 0$ and $\eta = 0$ are chosen, the cross-coupling control action between the two cylinders are then absent. It can be seen that a larger prediction horizon results in a smaller gain, and after a certain value the gain remains almost unchanged. Hence, by choosing a large enough prediction horizon, the stability robustness of the control system can be improved. Figure 2.6 shows the gain evolution when the control horizon is varying. Clearly, a smaller control horizon

H_c may enhance the stability robustness of the control system.

Figure 2.5: Peak value of gain $|v_{1A}|$ against H_p

Figure 2.6: Peak value of gain $|v_{1A}|$ against H_c

If the closed-loop system is stable, the prediction horizon can be chosen equal to $\text{int}(t_{s,\infty}/T_s)$, where $t_{s,\infty}$ is the settling time of the step response of the closed-loop system for $H_p \rightarrow \infty$, and $\text{int}(\cdot)$ is a function that converts a real value to an integer. If the process is stable and $H_c > 1$, a rule of thumb on how to choose the prediction horizon is : $H_p = \text{int}(t_s(5\%)/T_s)$, where $t_s(5\%)$ is the 5% settling time of the process step response. As a result, the closed-loop system is hardly sensitive to changes in H_p . Further, H_p depends on the sampling period. A short sampling period results in a larger prediction horizon. However, if, as a rule of thumb, the sampling period is chosen 10 – 20 times smaller than the settling time of the closed-loop system's step response, the typical range for H_p is 1-20.

On the other hand, if the process is stable but has badly situated zeros, then H_c must be chosen $\leq n_A + n_C$. The smaller is H_c , the slower is the closed-loop system's step response and the larger is its robustness. If the process is badly damped or unstable, H_c should be chosen > 1 because mean-level controllers cannot be used to control unstable process. Further, as a rule of thumb, H_p can be chosen according to $H_p = \text{int}(2\omega_s/\omega_b)$, where ω_s is the sampling frequency and ω_b is the bandwidth of the process. The resulting robustness is quite small compared to the robustness that can be obtained by controlling stable processes. Obviously, in controlling an unstable process an accurate model is required.

Case 2 *Influence of the filter $C_i(q^{-1})$ on stability robustness*

In this case, the control parameters $H_p = 10$, $H_c = 4$, $\lambda = 3$ have been chosen. Then, the variation of the peak value of $|\vartheta_{1A}|$ in (2.59) is shown in Figure 2.7, while the filter pole τ is varying and $\mathfrak{d}_{2A} = 0, 0.5$ is considered. It can be seen that $\tau = \tau_1$ gives a larger robustness bound than $\tau = \tau_2$ if the bandwidth of the low-pass filter $1/\tau_1$ is

larger than of $1/\tau_2$ and they have equal high frequency roll-off. However, the selection of $\tau \approx 0.8$ is most suitable to guarantee a satisfactory response performance.

Figure 2.7: Peak value of gain $|g_{1A}|$ against τ

The design objective via $C_i(q^{-1})$ is to ensure a prescribed high frequency roll-off of the transfer function S_i/C_i in (2.51). Hence, it is desirable for the polynomial S_i to be independent of C_i , which is not generally possible. A criterion proposed by Yoon *et al.* [59] allows the feedback dynamics to be specified to enhance robustness by choosing the polynomial. In addition, robust design through $C_i(q^{-1})$ is related to an H_∞ -optimal control scheme using the so-called Q -parameterization, which is also seen to result in robustness comparable with that obtained from the H_∞ method.

Case 3 Influence of control input weighting factor λ on stability robustness

The stability robustness bounds shown in Figure 2.8 is obtained by varying λ , while $H_p = 10$, $H_c = 4$ and $C = 1 - 0.8q^{-1}$ are set. Similarly, the additive uncertainties with $\mathfrak{d}_{2A} = 0, \pm 0.5$ are considered. Figure 2.8 shows that a higher λ can improve the

stability robustness of the control system.

Figure 2.8: Peak value of gain $|\vartheta_{1A}|$ against λ

In general, increasing the weighting factor λ makes the closed-loop system slower. If $\lambda \rightarrow \infty$, feedback disappears and the poles of the process appear as closed-loop poles. Hence, in this case the closed-loop system is stable if the process is stable and hence the stability robustness of the closed-loop system is ensured. Of course, this is not a realistic situation because the controller output is zero. However, this special choice of λ shows that an increase in robustness and a decrease in servo and regulator performance can be expected if λ is increasing beyond a certain value.

Case 4 Influence of synchronization weighting factors ν and η on stability robustness

The synchronization factor ν is introduced to compensate the synchronization error across the multiple axes or motors. In fact, an increased ν can slightly decrease stability robustness bound of the control system, as shown in Figure 2.9. When η is further introduced to minimize the difference of the consecutive synchronization errors, the

stability robustness bound is further decreased, as illustrated in Figure 2.10. Fortunately, we can increase prediction horizon and control weighting factor so that the stability of the control system is maintained while the synchronization performance can be improved.

Figure 2.9: Peak value of gain $|\vartheta_{1A}|$ against ν

Case 5 Synchronization performance while ν and η are varying

The actual model parameters and the control settings for the synchronous system (3.32) are given as follows:

$$\text{Settings : } H_p = 10, H_c = 4, \lambda_i = 3, C_i(q^{-1}) = 1 - 0.8q^{-1}$$

$$\text{process : } A_1(q^{-1}) = 1 + 0.2q^{-1} - 0.8256q^{-2} - 0.3744q^{-3}$$

$$B_1(q^{-1}) = 0.3453 + 1.1739q^{-1} + 0.0593q^{-2}$$

$$A_2(q^{-1}) = 0.6q^{-1} - 0.9276q^{-2} - 0.6724q^{-3}$$

$$B_2(q^{-1}) = 0.6453 + 0.9739q^{-1} + 0.709q^{-2}$$

Figure 2.10: Peak value of gain $|g_{1A}|$ against η

Figure 2.11 demonstrates the tracking responses of the two cylinders when the controller based on the nominal model (3.32) is applied. Clearly, the two cylinders behave quite differently, and evident synchronization error can be observed, due to model uncertainties and step disturbances existing. However, Figures 2.12 - 2.14 obtained by introducing variant μ and η show that larger synchronization weighting factors can improve the synchronization performance more effectively.

Figure 2.11: Tracking responses when $\nu = 0$ and $\eta = 0$ Figure 2.12: Tracking responses when $\nu = 5$ and $\eta = 0$

Figure 2.13: Tracking responses when $\nu = 5$ and $\eta = 5$ Figure 2.14: Comparison of synchronization performance for variant ν and η

2.5.2 Positioning System with Permanent Magnet Linear Motors

Consider a positioning system with two permanent magnet linear motors (PMLMs), as shown in Figure 2.15. For higher positioning accuracy the two motors are required to travel in a coordinated manner. The dynamics of the two PMLMs with evident ripple forces can approximately be described by (see Chapter 5 for details)

$$\begin{aligned}\ddot{y}_1 = & -20.6532\dot{y}_1(t) + 11.0055u_1(t - 0.0016) + 1.4178 \sin\left(\frac{2\pi}{0.0744}y_1\right) \\ & - 0.3125 \cos\left(\frac{2\pi}{0.0744}y_1\right)\end{aligned}\quad (2.66)$$

$$\begin{aligned}\ddot{y}_2 = & -5.9165\dot{y}_2(t) + 5.0484u_2(t - 0.0008) + 0.6762 \sin\left(\frac{2\pi}{0.051}y_2\right) \\ & - 0.1914 \cos\left(\frac{2\pi}{0.051}y_2\right).\end{aligned}\quad (2.67)$$

With a sampling interval $T_s = 0.0008$ seconds, system models used to derive the synchronized MPC algorithm are given by

$$A_i(q^{-1})y_i(k) = q^{-d_i}B(q^{-1})u_i(k-1) + \xi_i(k) \quad (2.68)$$

where

$$\left\{ \begin{array}{l} A_1(q^{-1}) = 1.0 - 1.9836q^{-1} + 0.9836q^{-2} \\ B_1(q^{-1}) = 3.5024 + 3.4832q^{-1} \\ A_2(q^{-1}) = 1.0000 - 1.9953q^{-1} + 0.9953q^{-2} \\ B_2(q^{-1}) = 1.6129 + 1.6104q^{-1} \end{array} \right. \quad (2.69)$$

and $d_1 = 2$, $d_2 = 1$, k denotes the discrete time, and $\xi_i(k)$ represents system disturbance due to friction force, ripple force, load force, and so on.

Figure 2.15: A positioning control system

In this simulation, the two PMLMs are desired to track different sinusoidal waveforms, as shown in Figure 2.16. The parameters of the robust synchronization MPC controller are selected as: $H_p = 10$, $H_c = 4$, $\lambda_i = 1650$, $\nu = 20$, $\eta = 10$, $\alpha_{12} = 1$, $\rho_{12} = 0.75$, $C_i = 1 - 0.8q^{-1}$. For comparison, the following PID controllers are proposed:

$$u_1(t) = K_{p1}e_1(t) + K_{i1} \int e_1(t)dt + K_{d1} \frac{d(e_1(t))}{dt} + K_{c1}\phi_{12}(t) \quad (2.70)$$

$$u_2(t) = K_{p2}e_2(t) + K_{i2} \int e_2(t)dt + K_{d2} \frac{d(e_2(t))}{dt} - K_{c2}\phi_{12}(t)/\rho_{12} \quad (2.71)$$

where $e_i(t)$ for $i = 1, 2$ denote the tracking errors, i.e., $e_i(t) = w_i(t) - y_i(t)$, K_{ci} is the cross-coupling gain or filter used to compensate the synchronization error, and K_{pi} , K_{ii} , K_{di} are the parameters of the PID controller for each motor. For the different desired

trajectories here, $K_{p1} = 0.007$, $K_{i1} = 1.0$, $K_{d1} = 0.00005$, $K_{c1} = -0.001$, $K_{p2} = 0.02$, $K_{i2} = 0.8$, $K_{d2} = 0.00006$ and $K_{c2} = -0.001$ are set optimally.

Figure 2.16: Desired position trajectories

Simulation results shown in Figures 2.17 - 2.20 and Table 2.1 are obtained by applying the proposed controllers into motors (2.66) and (2.67) with ripple forces. Figures 2.17 and 2.18 show the results with the PID controller, and evident synchronization error in this case can be observed. Figures 2.19 and 2.20 show the results under the synchronized controller with cross-coupling control action, due to $\nu \neq 0$ and $\eta \neq 0$, and the improved synchronization performance can be obtained.

Figure 2.17: Synchronization error for PID controller

Figure 2.18: Control inputs for PID controller

Figure 2.19: Synchronization error for robust synchronization controller

Figure 2.20: Control inputs for robust synchronization controller

In practice, however, real system may vary with loading or operating conditions. For instance, the subsystems (2.66) and (2.66) are disturbed as:

$$\begin{aligned}\ddot{y}_1 = & -18.4408\dot{y}_1(t) + 7.8081u_1(t - 0.0016) + 0.7821 \sin\left(\frac{2\pi}{0.0744}y_1\right) \\ & -1.5527 \cos\left(\frac{2\pi}{0.0744}y_1\right) + 0.1\end{aligned}\quad (2.72)$$

$$\begin{aligned}\ddot{y}_2 = & -6.2005\dot{y}_2(t) + 4.5395u_2(t - 0.0008) + 0.4255 \sin\left(\frac{2\pi}{0.051}y_2\right) \\ & -0.1279 \cos\left(\frac{2\pi}{0.051}y_2\right) - 0.2\end{aligned}\quad (2.73)$$

where step disturbances 0.1 and -0.2 occur after 2 seconds. Also, the initial positions of the motors may commonly be different. In this case, simulation results for the previous controllers are illustrated in Figures 2.21-2.24. Clearly, the synchronized MPC algorithm is more robust and can provide better performance in terms of the synchronization error and its variance than the PID controller.

Figure 2.21: Synchronization error for PID controller with model uncertainties

Figure 2.22: Control inputs for PID controller with model uncertainties

Figure 2.23: Synchronization error for robust synchronization controller with model uncertainties

Figure 2.24: Control inputs for robust synchronization controller with model uncertainties

Table 2.1: Comparison of the synchronization performance in simulation

Controller	$\max(\text{syn.})(\mu m)$	$\text{var}(\text{syn.})(\mu m^2)$
RSC	3.1368	0.8196
PID	3.8531	1.0344
RSCW	19.5551	2.5614
PIDW	26.8295	3.5335

$\max(\text{sync.})$ – maximum synchronization error

$\text{var}(\text{sync.})$ – variance of the synchronization error

RSC – robust synchronization controller

RSCW – robust synchronization controller with model uncertainties

PID – PID controller

PIDW – PID controller with model uncertainties

2.6 Guidelines on Selection of Control Parameters

The real power of the MPC approach lies in the assumptions made about the future control actions. Hence, guidelines on selection of the control parameters are listed below.

- H_p is the maximum output horizon. If the system has an initially negative-going non-minimum phase response, H_p should be chosen so that the later positive-going output samples are included in the cost: in discrete-time this implies that H_p exceeds the degree of $B_i(q^{-1})$. In practice, a value corresponding more closely to the rise-time of the system is suggested.

- H_c is the control horizon. For a simple system (e.g. open-loop stable though with possible dead-time and non-minimum phase) the value of 1 gives generally acceptable control. Increasing H_c makes the control and the output response more active until a stage is reached, and any further increase in H_c makes little difference. An increased value of H_c is more appropriate for complex systems, as it is found that good control can be achieved if H_c is at least equal to the number of unstable or badly-damped poles. The use of $H_c < H_p$ moreover significantly reduces the computational burden.
- λ_i is the control input weighting factor. Increasing the weighting factor makes the control input variance more important in the criterion function, so that minimization of the criterion function may result in a less active control input as well as a slower process output. In general, there is a constraint about λ_i for the limitation of actuators. On the other hand, although the effect of λ_i on the closed-loop system is clear it is hard to choose λ_i such that the system behaves as desired because λ_i depends on the process and must usually be determined by simulations in combination with the well-known and often-used trial-and-error method.
- ν is the weighting factor for compensating the synchronization error. Increasing ν can improve the synchronization performance, but after a certain value, the synchronization performance will remain unchanged [47,48].
- η is the weighting factor for compensating the synchronization error. With an adequate ν and an increased η the synchronization error can be effectively reduced [58]. However, if the system oscillates due to the introduction of η , an increase in λ_i can overcome this problem.

- Factors α and ρ can be any non-negative value, but in general, α is assumed to be either 1 or 0, and ρ is determined by the corresponding reference signals, i.e. $w_i = \rho_{ij}w_j$.
- $C_i(q^{-1})$ is the observer or prefiltering polynomial for the robustness of the closed-loop system. A criterion proposed by Yoon *et al.* [59] allows the feedback dynamics to be specified to enhance robustness by choosing the polynomial. In addition, robust design through $C_i(q^{-1})$ is related to an H_∞ -optimal control scheme using the so-called Q -parameterization, which is also seen to result in robustness comparable with that obtained from the H_∞ method.

2.7 Conclusion

This chapter presents a robust synchronization MPC algorithm for multi-axis coordinated motion systems. Simulation results have shown that with adequately adjusting the control parameters improved synchronization performance can be obtained. In practice, the selection of the control parameters can be done as follows: first, appropriate H_p , H_c and λ_i are selected so that the control system provides a stable and satisfactory tracking response and then, factors ν and η are introduced to reduce the synchronization errors caused by mismatched dynamics, system disturbances, model uncertainties, and so on. However, if feedforward compensators for the frictional and ripple forces are integrated into the proposed control system, then the synchronization performance might be further improved [62, 63]. In the derivation of the control algorithm, it is assumed that the parameters of the process are known and fixed. Otherwise, an identification algorithm would be necessary to estimate system parameters. In the next chapter, a cross-coupling model reference control algorithm will be presented.

Chapter 3

Cross-Coupling Model Reference Control Algorithm

In this chapter, an extended MPC algorithm with reference models is developed so that multiple axes of the system can synchronously follow the outputs of the reference model. Model reference control is first reviewed. Then, the synchronization MPC algorithm with reference models is developed, followed by the closed-loop structure and synchronization properties of the control system. Simulation studies on an electro-hydraulic synchronous control system with two axes are also carried out. Finally, conclusion of this chapter is given.

3.1 Model Reference Control

As an important control strategy, model reference control was originally developed at Massachusetts Institute of Technology (MIT) by Whitaker [64] in the late 1950s, and has been used in different forms by many researchers. Nijmeijer *et al.* [65] and Goh [66] studied model reference control for non-linear systems, while Huang *et al.* [67] and Chien *et al.* [68] considered model reference control for linear time-varying systems.

Applications of model reference control to practical plants were also investigated by Perez [69] and Aguirre *et al.* [70]. As far as robust model reference control of multivariable, time-invariant linear systems is concerned, and Chen *et al.* [71] proposed a minmax approach, in which an observer-based controller is employed, and external disturbance decoupling is also tackled. Sun *et al.* [72] and Duan *et al.* [73] considered model reference control of multivariable linear systems against modeling errors or parameter uncertainties.

3.2 Cross-Coupling MPC Algorithm

Consider an uncoupled process with multiple motion axes described by:

$$A_i y_i(k) = B_i u_i(k-1) + \xi_i(k), i = 1, 2, \dots, n \quad (3.1)$$

Note that model parameters here are the same as that of the CARIMA model given in the preceding chapter except the disturbance description.

Definition 3.1. *The desired behaviour of the controlled process is described by the following equations,*

$$A_{im} y_{im}(k) = B_i Z_i w_i(k-1), \quad (3.2)$$

$$A_{im} u_{im}(k) = A_i Z_i w_i(k) \quad (3.3)$$

where $w_i(k)$ is the reference signal for the process output, A_{im} is a monic polynomial corresponding to the desired dynamics and polynomial Z_i is used to achieve a unity gain of the desired model, i.e. $A_{im}(1) = B_i(1)Z_i(1)$.

This definition gives the common reference model and (3.3) computes the control input, which should be applied to the process to force the output to track the reference signal

as closely as possible. Then, we give the following definition.

Definition 3.2. *The errors between the process and the reference model are defined by*

$$\begin{aligned}\epsilon_{iy}(k) &= y_i(k) - y_{im}(k) \\ &= y_i(k) - \frac{B_i Z_i}{A_{im}} w_i(k-1),\end{aligned}\tag{3.4}$$

$$\begin{aligned}\epsilon_{iu}(k) &= u_i(k) - u_{im}(k) \\ &= u_i(k) - \frac{A_i Z_i}{A_{im}} w_i(k).\end{aligned}\tag{3.5}$$

Hence, an alternative to the original model (3.1) can be obtained:

$$A_i \epsilon_{iy}(k) = B_i \epsilon_{iu}(k-1) + \xi_i(k).\tag{3.6}$$

Note that the output and control input of the process model here denote the errors between the reference model and actual process.

By applying the following Diophantine equations:

$$\begin{cases} 1 = A_i E_{ij} + q^{-k} F_{ij} \\ B_i E_{ij} = G_{ij} + q^{-j} H_{ij} \end{cases}, \text{ for } j = 1, 2, \dots, H_p\tag{3.7}$$

we have the predictor over the prediction horizon H_p and the control horizon H_c :

$$\hat{\mathbf{Y}}_i = \mathbf{G}_i \hat{\mathbf{U}}_i + \mathbf{O}_i\tag{3.8}$$

where

$$\hat{\mathbf{Y}}_i = [\hat{\epsilon}_{iy}(k+1|k), \hat{\epsilon}_{iy}(k+2|k), \dots, \hat{\epsilon}_{iy}(k+H_p|k)]^T\tag{3.9}$$

$$\hat{\mathbf{U}}_i = [\hat{\epsilon}_{iu}(k|k), \hat{\epsilon}_{iu}(k+1|k), \dots, \hat{\epsilon}_{iu}(k+H_c-1|k)]^T \quad (3.10)$$

$$\mathbf{G}_i = \begin{bmatrix} g_{i0} & 0 & \cdots & 0 \\ g_{i1} & g_{i0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ g_{iH_c-1} & g_{iH_c-2} & \ddots & g_{i0} \\ \vdots & \ddots & \ddots & \vdots \\ g_{iH_p-1} & g_{iH_p-2} & \cdots & g_{iH_p-H_c} \end{bmatrix} \quad (3.11)$$

$$\mathbf{O}_i = [o_{i1}(k), o_{i2}(k), \dots, o_{iH_p}(k)]^T, \quad o_{ij}(k) = F_{ij}\epsilon_{iy}(k) + H_{ij}\epsilon_{iu}(k-1). \quad (3.12)$$

According to the synchronization strategy discussed in Chapter 2, the synchronization error between the outputs $\epsilon_{iy}(k)$ and $\epsilon_{jy}(k)$ and its difference are described in the following definition.

Definition 3.3. For the modified process (3.6), the synchronization error and its difference are given by

$$\phi_{ij}(k) = \alpha_{ij}(\epsilon_{iy}(k) - \rho_{ij}\epsilon_{jy}(k)) \quad (3.13)$$

$$\psi_{ij}(k) = (1 - q^{-1})\phi_{ij}(k) \quad (3.14)$$

where α_{ij} and ρ_{ij} are the coupling and synchronization factors.

From (3.9), (3.13) and (3.14), it can be obtained that

$$\hat{\Psi}_{ij} = \mathbf{L} \hat{\Phi}_{ij} \quad (3.15)$$

$$\hat{\Phi}_{ij} = \alpha_{ij}(\hat{\mathbf{Y}}_i - \rho_{ij}\hat{\mathbf{Y}}_j) \quad (3.16)$$

where $\mathbf{L}$ is given in (2.24) and

$$\hat{\Phi}_{ij} = [\hat{\phi}_{ij}(k+1|k), \hat{\phi}_{ij}(k+2|k), \dots, \hat{\phi}_{ij}(k+H_p|k)]^T \quad (3.17)$$

$$\hat{\Psi}_{ij} = [0, \hat{\psi}_{ij}(k+2|k), \dots, \hat{\psi}_{ij}(k+H_p|k)]. \quad (3.18)$$

In order to achieve a synchronization control, the following performance index is introduced,

$$\begin{aligned} I_{mp} &= \sum_{i=1}^n \left\{ \hat{\mathbf{Y}}_i^T \hat{\mathbf{Y}}_i + \lambda_i \hat{\mathbf{U}}_i^T \hat{\mathbf{U}}_i \right\} + \nu \sum_{i=1}^{n-1} \sum_{j=i+1}^n \hat{\Psi}_{ij}^T \hat{\Psi}_{ij} + \eta \sum_{i=1}^{n-1} \sum_{j=i+1}^n \hat{\Phi}_{ij}^T \hat{\Phi}_{ij} \\ &= \sum_{i=1}^n \left\{ \hat{\mathbf{Y}}_i^T \hat{\mathbf{Y}}_i + \lambda_i \hat{\mathbf{U}}_i^T \hat{\mathbf{U}}_i \right\} + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \hat{\Phi}_{ij}^T \Lambda \hat{\Phi}_{ij} \end{aligned} \quad (3.19)$$

where $\Lambda = \nu \mathbf{I} + \eta \mathbf{L}^T \mathbf{L}$, ν and η are the synchronization weighting factors.

Minimizing the performance index (3.19) with respect to $\hat{\mathbf{U}}_i$ results in

$$\begin{aligned} \begin{bmatrix} \hat{\mathbf{U}}_1 \\ \hat{\mathbf{U}}_2 \\ \vdots \\ \hat{\mathbf{U}}_n \end{bmatrix} &= \Xi_1^{-1} \Xi_2 \left(\begin{bmatrix} -\mathbf{O}_1 - \Lambda \sum_{m=2}^n \alpha_{1m}^2 (\mathbf{O}_1 - \rho_{1m} \mathbf{O}_m) \\ \vdots \\ -\mathbf{O}_i - \Lambda \sum_{m=i+1}^n \alpha_{im}^2 (\mathbf{O}_i - \rho_{im} \mathbf{O}_m) \\ \vdots \\ -\mathbf{O}_n \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} \mathbf{0} \\ \vdots \\ \Lambda \sum_{m=1}^{i-1} \alpha_{mi}^2 \rho_{mi} (\mathbf{O}_m - \rho_{mi} \mathbf{O}_i) \\ \vdots \\ \Lambda \sum_{m=1}^{n-1} \alpha_{mn}^2 \rho_{mn} (\mathbf{O}_m - \rho_{mn} \mathbf{O}_n) \end{bmatrix} \right) \end{aligned} \quad (3.20)$$

where Ξ_1 and Ξ_2 are given in (2.27) and (2.28), respectively.

Let $(v_{i1} \ v_{i2} \ \dots \ v_{in})$ be the $((i-1)H_c + 1)$ th row of $\Xi_1^{-1} \Xi_2$. The cross-coupling model reference control law of the multi-axis motion system is then given by

$$\begin{bmatrix} \epsilon_{1u}(k) \\ \epsilon_{2u}(k) \\ \vdots \\ \epsilon_{nu}(k) \end{bmatrix} = -\mathbf{\Omega}^{-1}\mathbf{\Gamma} \begin{bmatrix} \epsilon_{1y}(k) \\ \epsilon_{2y}(k) \\ \vdots \\ \epsilon_{ny}(k) \end{bmatrix} \quad (3.21)$$

or

$$\begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_n(k) \end{bmatrix} = \begin{bmatrix} u_{1m}(k) \\ u_{2m}(k) \\ \vdots \\ u_{nm}(k) \end{bmatrix} - \mathbf{\Omega}^{-1}\mathbf{\Gamma} \begin{bmatrix} \epsilon_{1y}(k) \\ \epsilon_{2y}(k) \\ \vdots \\ \epsilon_{ny}(k) \end{bmatrix} \quad (3.22)$$

where $\mathbf{\Omega}$ and $\mathbf{\Gamma}$ are given in (2.30) and (2.31), respectively, and

$$\mathbf{H}_i = [H_{i1}, H_{i2}, \dots, H_{iH_p}]^T \quad (3.23)$$

$$\mathbf{F}_i = [F_{i1}, F_{i2}, \dots, F_{iH_p}]^T. \quad (3.24)$$

Clearly, the following property holds.

Theorem 3.1. *For this synchronization MPC algorithm with reference models, if*

- $\lambda_i = 0$
- $H_p = H_c$

then the control algorithm (3.21) is uncoupled, i.e.

$$\begin{bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \vdots \\ \hat{U}_n \end{bmatrix} = \begin{bmatrix} \mathbf{G}_1^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{G}_n^{-1} \end{bmatrix} \begin{bmatrix} -\mathbf{O}_1 \\ -\mathbf{O}_2 \\ \vdots \\ -\mathbf{O}_n \end{bmatrix}. \quad (3.25)$$

Proof. From (3.20) we have

$$\begin{aligned}
 & \begin{bmatrix} -\mathbf{O}_1 - \Lambda \sum_{m=2}^n \alpha_{1m}^2 (\mathbf{O}_1 - \rho_{1m} \mathbf{O}_m) \\ \vdots \\ -\mathbf{O}_i - \Lambda \sum_{m=i+1}^n \alpha_{im}^2 (\mathbf{O}_i - \rho_{im} \mathbf{O}_m) \\ \vdots \\ -\mathbf{O}_n \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \vdots \\ \Lambda \sum_{m=1}^{i-1} \alpha_{mi}^2 \rho_{mi} (\mathbf{O}_m - \rho_{mi} \mathbf{O}_i) \\ \vdots \\ \Lambda \sum_{m=1}^{n-1} \alpha_{mn}^2 \rho_{mn} (\mathbf{O}_m - \rho_{mn} \mathbf{O}_n) \end{bmatrix} \\
 &= \mathbf{\Delta} \begin{bmatrix} -\mathbf{O}_1 \\ -\mathbf{O}_2 \\ \vdots \\ -\mathbf{O}_n \end{bmatrix} \tag{3.26}
 \end{aligned}$$

where

$$\mathbf{\Delta} = [\mathbf{\Delta}_{ij}], \quad \mathbf{\Delta}_{ij} = \begin{cases} -\alpha_{ij}^2 \rho_{ij} \Lambda & \text{if } i < j \\ \mathbf{I} + \Lambda \left(\sum_{m=1}^{i-1} \alpha_{mi}^2 \rho_{mi}^2 + \sum_{m=i+1}^n \alpha_{im}^2 \right) & \text{if } i = j \\ -\alpha_{ji}^2 \rho_{ji} \Lambda & \text{if } i > j \end{cases} . \tag{3.27}$$

Similarly, for $\lambda_i = 0$, it has

$$\mathbf{\Xi}_1 = \mathbf{\Xi}_2 \mathbf{\Delta} \mathbf{\Xi}_2^T. \tag{3.28}$$

Since $H_p = H_c$, and $\mathbf{G}_i$ and $\mathbf{\Xi}_2$ are invertible, substitution of (3.26) and (3.27) into (3.20) yields

$$\begin{bmatrix} \hat{\mathbf{U}}_1 \\ \hat{\mathbf{U}}_2 \\ \vdots \\ \hat{\mathbf{U}}_n \end{bmatrix} = \mathbf{\Xi}_1^{-1} \mathbf{\Xi}_2 \mathbf{\Delta} \begin{bmatrix} -\mathbf{O}_1 \\ -\mathbf{O}_2 \\ \vdots \\ -\mathbf{O}_n \end{bmatrix}$$

$$\begin{aligned}
&= (\Xi_2^T)^{-1} \Delta^{-1} \Xi_2^{-1} \Xi_2 \Delta \begin{bmatrix} -\mathbf{O}_1 \\ -\mathbf{O}_2 \\ \vdots \\ -\mathbf{O}_n \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{G}_1^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{G}_n^{-1} \end{bmatrix} \begin{bmatrix} -\mathbf{O}_1 \\ -\mathbf{O}_2 \\ \vdots \\ -\mathbf{O}_n \end{bmatrix}.
\end{aligned}$$

The proof is completed.

In practice, λ_i is necessary to obtain a stable and smooth system response. Hence, substituting the control law (3.21) into (3.6) results in

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_n(k) \end{bmatrix} = \begin{bmatrix} y_{1m}(k) \\ y_{2m}(k) \\ \vdots \\ y_{nm}(k) \end{bmatrix} + \Theta^{-1} \begin{bmatrix} \xi_1(k) \\ \xi_2(k) \\ \vdots \\ \xi_n(k) \end{bmatrix} \quad (3.29)$$

where

$$\Theta = \begin{bmatrix} A_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & A_n \end{bmatrix} + \begin{bmatrix} q^{-1}B_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & q^{-1}B_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & q^{-1}B_n \end{bmatrix} \times \Omega^{-1} \Gamma \quad (3.30)$$

(3.29) clearly shows that the closed-loop system behaves asymptotically as the desired model (3.2) whatever the process parameters and the weighting factors in the

performance index may be, provided that the synchronization error approaches zero. However, it must be pointed out that this property is exclusively related to the feedforward term in the controller and holds whatever the transfer function of the feedback controller may be. Since the control system consists of feedforward, feedback and cross coupling control actions, the control system developed above can provide good performance in terms of tracking and synchronization and can also exhibit a dynamic behavior as the reference model introduced. In addition, the synchronization control action introduced in an optimal way may effectively reduce the deviations of processes with multiple motion axes by adjusting the parameters ν and η adequately. Figure 3.1 illustrates the block diagram of the cross-coupling model reference control system, and the synchronization performance between two motion axes can be evaluated as follows,

$$y_i(k) - \rho_{ij}y_j(k) = y_{im}(k) - \rho_{ij}y_{jm}(k) + [0, \dots, 1, \dots, -\rho_{ij}, \dots, 0]\Theta^{-1} \begin{bmatrix} \xi_1(k) \\ \xi_2(k) \\ \vdots \\ \xi_n(k) \end{bmatrix}. \quad (3.31)$$

3.3 Computer Simulation

Consider the electro-hydraulic synchronous system, as shown in Figure 2.4. The discrete-time models can be described by

$$A_i y_i(k) = B_i u_i(k-1) + \xi_i(k), \quad i = 1, 2 \quad (3.32)$$

Figure 3.1: Block diagram of the cross-coupling model reference control system

where

$$\begin{cases} A_i = 1 + 0.4006q^{-1} - 0.8699q^{-2} - 0.5307q^{-3} \\ B_i = 0.4453 + 0.8739q^{-1} + 0.309q^{-2} \end{cases} \quad (3.33)$$

Case 1: The influence of λ_i , ν and η on the synchronization performance

As $A_1 = A_2$ and $B_1 = B_2$ for this system, Θ^{-1} is a symmetric matrix with equivalent diagonal elements. The synchronization performance can then be specified by $|y_1(k) - y_2(k)|$ as follows:

$$\begin{aligned} |y_1(k) - y_2(k)| &= \left| [1 \ -1] \Theta^{-1} \begin{bmatrix} \xi_1(k) \\ \xi_2(k) \end{bmatrix} \right| \\ &= \left| [1 \ -1] \Theta^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xi_1(k) + [1 \ -1] \Theta^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xi_2(k) \right| \\ &= \left| [1 \ -1] \Theta^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xi_1(k) + [1 \ -1] \Theta^{-1} \begin{bmatrix} -1 \\ 0 \end{bmatrix} \xi_2(k) \right| \end{aligned}$$

$$\begin{aligned}
&= \left| [1 \ -1] \Theta^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} (\xi_1(k) - \xi_2(k)) \right| \\
&\leq \sigma |\xi_1(k) - \xi_2(k)|
\end{aligned}$$

where $\sigma = \left| [1 \ -1] \Theta^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right|$. This manipulation means that minimization of σ may result in reduced synchronization error. Figures 3.2 and 3.3 show the magnitude of σ with respect to various ν and η , while $\alpha_{12} = 1$, $\beta_{12} = 1$, $A_{im} = 1 - 0.5q^{-1}$, $H_p = 10$ and $H_c = 4$ are set. Clearly, with appropriate weighting factors, an optimal synchronization performance can be realized.

Figure 3.2: The magnitude of σ for $\lambda = 1$

Figure 3.3: The magnitude of σ for $\lambda = 3$ **Case 2:** *The influence of ν and η on the tracking responses*

In this simulation, parameter uncertainties are introduced so that the two cylinders behave quite differently, and $\alpha_{12} = 1$, $\beta_{12} = 1$, $A_{im} = 1 - 0.5q^{-1}$, $H_p = 10$, $H_c = 4$ and $\lambda_i = 3$ are chosen. Figures 3.4 - 3.6 show the tracking responses when various ν and η are selected. Figure 3.4 shows the tracking responses without compensation of the synchronization errors. It can be seen that the two outputs can follow the same square wave. However, the synchronization error between the two cylinders can be clearly seen. Figure 3.5 shows the result for the same settings but $\nu = 5$. Note that the synchronization error is reduced as compared to the previous case. Figure 3.6 shows the result when η is also introduced, such as $\eta = 5$, and the synchronization performance can be further improved. Comparison of the synchronization performance for the three cases is illustrated in Figure 3.7.

3.3 Computer Simulation

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Figure 3.4: Tracking responses when $\nu = 0$ and $\eta = 0$

Figure 3.5: Tracking responses when $\nu = 5$ and $\eta = 0$

Figure 3.6: Tracking responses when $\nu = 5$ and $\eta = 5$

Figure 3.7: Comparison of the synchronization performance

3.4 Conclusion

This chapter presents a synchronized MPC algorithm with reference models so that the input-output dynamics mainly depends on the selected reference model. However, the closed-loop system has other poles related to the weighting factors and the performance index. These poles appear in disturbance output transfer function, but in the input-output transfer function they may be cancelled by zeroes, due to the feedforward action. As these poles may be unstable, a careful stability analysis must be undertaken in any case. In fact, this property holds for any kind of feedback controller combined with a suitable feedforward action. In the next chapter, a stabilizing synchronization control algorithm in the state space and its stability will be investigated.

Chapter 4

Stabilizing Synchronization Control Systems

MPC can be recast as a special case of Linear Quadratic Gaussian (LQG) optimal control. As such, one way to analyze the stability of the proposed MPC systems is to embed them into the powerful framework of LQG [75]. This chapter derives the synchronization controller in the state space. Stability of the proposed controller is thoroughly analyzed, and constrained problem is discussed. Finally, simulation results and conclusion follow.

4.1 Introduction

Predictive control using the receding horizon idea is a feedback control policy. There is therefore a risk that the resulting control system might be unstable. Even though the performance of the plant is being optimized over the prediction horizon, this optimization does not care about what happens beyond the prediction horizon.

There are now many ways of ensuring that the nominal closed loop, with the assumption

that the model is perfect, is stable, such as terminal constraints ensuring stability [76], infinite horizons [77, 78], using the Youla parameterization [79, 80]. If a predictive controller is designed off-line, however, nominal stability of the closed loop is then hardly an issue. It is quite easy to obtain stability by tuning the parameters in the problem formulation, and is very easy to check stability of the designed system. Current applications are invariably implemented on stable plants, and the dynamic performance demanded from predictive controllers is typically not very challenging. However, as predictive control has more other applications, this situation may change.

4.2 Problem Formulation

4.2.1 Plant Model

A coordinated motion system can be represented by the multiple subsystems and the dynamic couplings among them, as shown in Figure 4.1.

Figure 4.1: A multi-axis coordinated motion system

The discrete-time state equation of such a motion system with n axes is given by

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_d\mathbf{d}_x(k) \quad (4.1)$$

where $\mathbf{x}(k) \in \mathcal{R}^{rn}$ and $\mathbf{u}(k) \in \mathcal{R}^n$ are the state and control input vectors respectively, i.e.,

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \\ \vdots \\ \mathbf{x}_n(k) \end{bmatrix}, \mathbf{u}(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_n(k) \end{bmatrix}, \quad (4.2)$$

and r denotes the system order of each axis, $\mathbf{A}$ and $\mathbf{B}$ are system matrices given in the following form:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1n} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{n1} & \mathbf{A}_{n2} & \cdots & \mathbf{A}_{nn} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1n} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{n1} & \mathbf{B}_{n2} & \cdots & \mathbf{B}_{nn} \end{bmatrix}, \quad (4.3)$$

$\mathbf{d}(k)$ is the system disturbance, $\mathbf{B}_d$ is the disturbance matrix, and the following assumption is made.

Assumption 4.1. *For the multiple motion axes system,*

- $[\mathbf{A}, \mathbf{B}]$ is controllable;
- $\mathbf{x}_i \in \mathcal{R}^r$, where $\mathbf{x}_i$ is the state variable of the i th motion axis;
- The output of each motion axis only depends on its own state.

Note that $\mathbf{A}_{ii}$ and $\mathbf{B}_{ii}$ here represent system matrices of the i th motion axis, and $\mathbf{A}_{ij}$ and $\mathbf{B}_{ij}$, for $i \neq j$, denote the coupling matrices between the i th and j th axes. There-

fore, if $\mathbf{A}_{ij} = 0$ and $\mathbf{B}_{ij} = 0$, multiple axes of the open loop system are independent, and (4.1) is equivalent to (2.1). Figure 4.2 shows the closed-loop control structure of such a coordinated motion system with desired trajectory.

Figure 4.2: The closed-loop structure of the motion control system

4.2.2 Synchronization Estimator

Definition 4.1. The tracking error of the i th axis with respect to a desired trajectory, $w_i(k)$, is given by

$$e_i(k) = w_i(k) - \mathbf{T}_i \mathbf{x}_i(k). \quad (4.4)$$

where $\mathbf{T}_i$ is the output transition matrix of the i th axis.

Note that $\mathbf{T}_i$ is such a matrix that different control requirements can be realized by selecting appropriate $\mathbf{T}_i$. For example, if full states with position, velocity and acceleration can be measured and the motion position is to be controlled, $\mathbf{T}_i = [1, 0, 0]$ can then be set. Otherwise, $\mathbf{T}_i = [0, 1, 0]$ can be set for velocity control.

Definition 4.2. The synchronization error and its difference are defined by

$$\phi_{ij}(k) = \alpha_{ij}(\mathbf{T}_i \mathbf{x}_i(k) - \rho_{ij} \mathbf{T}_j \mathbf{x}_j(k)) \quad (4.5)$$

$$\psi_{ij}(k) = \phi_{ij}(k) - \phi_{ij}(k-1) \quad (4.6)$$

where α_{ij} is the non-negative coupling factor, ρ_{ij} is the synchronization factor.

From Definition 4.1 and 4.2, the following result can be obtained.

Theorem 4.1. *If a stable controller is applied to the system with state equation (4.1), then the synchronization error $\phi_{ij}(k)$ converges to zero asymptotically, i.e.*

$$\lim_{k \rightarrow \infty} \phi_{ij}(k) = 0. \quad (4.7)$$

Proof. Given the initial state $\mathbf{x}(0)$, the tracking error at time k becomes

$$\begin{aligned} e_i(k) &= w_i(k) - \mathbf{T}_i \mathbf{x}_i(k) \\ &= w_i(k) - [\mathbf{0}, \dots, \mathbf{T}_i, \dots, \mathbf{0}] \mathbf{x}(k) \\ &= w_i(k) - [\mathbf{0}, \dots, \mathbf{T}_i, \dots, \mathbf{0}] (\mathbf{A}^t \mathbf{x}(0) + \mathbf{A}^{t-1} \mathbf{B} \mathbf{u}(0) + \dots + \mathbf{B} \mathbf{u}(k-1)). \end{aligned}$$

As $[\mathbf{A}, \mathbf{B}]$ is controllable, it is possible to find a sequence of $(\mathbf{u}(0), \dots, \mathbf{u}(k-1))$ such that $\lim_{t \rightarrow \infty} e_i(k) = w_i(\infty) - \mathbf{T}_i \mathbf{x}_i(\infty) = 0$ holds. It has then,

$$\begin{aligned} \lim_{t \rightarrow \infty} \phi_{ij}(k) &= \alpha_{ij} \lim_{k \rightarrow \infty} (\mathbf{T}_i \mathbf{x}_i(k) - \rho_{ij} \mathbf{T}_j \mathbf{x}_j(k)) \\ &= \alpha_{ij} (\mathbf{T}_i \mathbf{x}_i(\infty) - \rho_{ij} \mathbf{T}_j \mathbf{x}_j(\infty)) \\ &= \alpha_{ij} (w_i(\infty) - \rho_{ij} w_j(\infty)) \\ &= 0. \end{aligned}$$

The proof is completed.

To reduce undesired coupling action, in some cases, coordinated relationships among n axes can be divided into n pairs with $(\phi_{12}(k) = 0, \psi_{12}(k) = 0)$, $(\phi_{23}(k) = 0, \psi_{23}(k) =$

0), ..., $(\phi_{n1}(k) = 0, \psi_{n1}(k) = 0)$, which can be realized by setting the corresponding coupling factors as zeros. Hence, integrating (4.5) and (4.6) respectively results in the following definition.

Definition 4.3. *The synchronization observer, as shown in Figure 4.3, is described by*

$$\boldsymbol{\delta}(k) = \mathbf{L}\mathbf{x}(k) \quad (4.8)$$

$$\boldsymbol{\varepsilon}(k) = \boldsymbol{\delta}(k) - \boldsymbol{\delta}(k-1) \quad (4.9)$$

where

$$\boldsymbol{\delta}(k) = \begin{bmatrix} \phi_{12}(k) \\ \phi_{23}(k) \\ \vdots \\ \phi_{n1}(k) \end{bmatrix}, \boldsymbol{\varepsilon}(k) = \begin{bmatrix} \psi_{12}(k) \\ \psi_{23}(k) \\ \vdots \\ \psi_{n1}(k) \end{bmatrix}, \mathbf{L} = \begin{bmatrix} \alpha_{12}\mathbf{T}_1 & -\alpha_{12}\rho_{12}\mathbf{T}_2 & \cdots & \mathbf{0} \\ \mathbf{0} & \alpha_{23}\mathbf{T}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha_{n1}\rho_{n1}\mathbf{T}_1 & \mathbf{0} & \cdots & \alpha_{n1}\mathbf{T}_n \end{bmatrix}.$$

4.2.3 Augmented System Model and Synchronization

Introducing a new state vector $\boldsymbol{\chi}(k) = [\mathbf{x}(k)^T, \boldsymbol{\delta}(k)^T, \boldsymbol{\varepsilon}(k)^T]^T$ and an output transition matrix $\boldsymbol{\mathcal{T}} = \text{diag}(\mathbf{T}_1, \dots, \mathbf{T}_n, \mathbf{I})$ with $\mathbf{I}$ denoting the $n \times n$ identity matrix, model (4.1) can be augmented as follows:

$$\boldsymbol{\chi}(k+1) = \tilde{\mathbf{A}}\boldsymbol{\chi}(k) + \tilde{\mathbf{B}}\mathbf{u}(k) \quad (4.10)$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}\mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}\mathbf{A} & -\mathbf{I} & \mathbf{0} \end{bmatrix}, \tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{L}\mathbf{B} \\ \mathbf{L}\mathbf{B} \end{bmatrix}. \quad (4.11)$$

Figure 4.3: The observer of the synchronization errors

Clearly, the following result can be obtained.

Corollary 4.1. *Consider the multi-axis motion system (4.1) with the synchronization measurements (4.5) and (4.6). $[\tilde{\mathbf{A}}, \tilde{\mathbf{B}}]$ is stabilizable if Assumption 4.1 is true.*

For the synchronization errors and their differences to converge, a new trajectory for the augmented model should be defined as $\boldsymbol{\gamma}(k) = [\mathbf{w}^T(k), \underbrace{0, \dots, 0}_{2n}]^T$, where $\mathbf{w}(k) = [w_1(k), \dots, w_n(k)]^T$ and the rest are set by zeros. A new cost function is also introduced as:

$$V(k) = \sum_{i=1}^{H_p} \|\boldsymbol{\gamma}(k+i) - \mathcal{T}\hat{\boldsymbol{\chi}}(k+i|k)\|_{\mathbf{Q}_i}^2 + \sum_{i=1}^{H_c} \|\hat{\mathbf{u}}(k+i-1|k)\|_{\mathbf{R}_i}^2 \quad (4.12)$$

where $\hat{\boldsymbol{\chi}}(k+i|k)$ is the future state vector, $\hat{\mathbf{u}}(k+i-1|k)$ is the future control input vector, $\mathbf{Q}_i$ and $\mathbf{R}_i$ are time-varying weighting matrices given by

$$\mathbf{Q}_i = \text{diag}(\mathbf{I}_{rn \times rn}, \nu_i \mathbf{I}, \eta_i \mathbf{I}) \quad (4.13)$$

$$\mathbf{R}_i = \lambda_i \mathbf{I} \quad (4.14)$$

where $\mathbf{I}_{rn \times rn}$ denotes the $rn \times rn$ identity matrix, λ_i is a positive weighting factor used to adjust the control action, ν_i and η_i are non-negative synchronization weighting factors for $\hat{\delta}(k+i|k)$ and $\hat{\varepsilon}(k+i|k)$, respectively.

Rewrite the cost function (4.12) as

$$\begin{aligned} V(k) = & \|\gamma(k+H_p) - \mathcal{T}\hat{\chi}(k+H_p|k)\|_{\mathbf{Q}_{H_p}}^2 \\ & + \sum_{j=0}^{H_p-1} \{ \|\gamma(k+H_p-1-j) - \mathcal{T}\hat{\chi}(k+H_p-1-j|k)\|_{\tilde{\mathbf{Q}}_j}^2 \\ & + \|\hat{\mathbf{u}}(k+H_p-1-j|k)\|_{\tilde{\mathbf{R}}_j}^2 \} \end{aligned} \quad (4.15)$$

where

$$\tilde{\mathbf{Q}}_j = \begin{cases} \mathbf{Q}_{H_p-1-j} & \text{if } j = 0, 1, \dots, H_p-2 \\ \mathbf{0} & \text{if } j = H_p-1 \end{cases} \quad (4.16)$$

$$\tilde{\mathbf{R}}_j = \begin{cases} \infty \mathbf{I} & \text{if } j = 0, 1, \dots, H_p-1-H_c \\ \mathbf{R}_{H_p-j} & \text{if } j = H_p-H_c, \dots, H_p-1 \end{cases}. \quad (4.17)$$

Since $\gamma(k)$ is an externally prescribed signal, an additional state equation of

$$\gamma(k+1) = \tilde{\mathbf{A}}_d \gamma(k) \quad (4.18)$$

may be incorporated, where

$$\tilde{\mathbf{A}}_d = \begin{bmatrix} \mathbf{A}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \quad (4.19)$$

Note that $\mathbf{A}_d = \mathbf{I}$ is chosen for tracking constant reference signals.

By applying the following state equation:

$$\check{\chi}(k+1) = \check{\mathbf{A}}\check{\chi}(k) + \check{\mathbf{B}}\mathbf{u}(k), \quad (4.20)$$

where

$$\check{\chi}(k) = \begin{bmatrix} \chi(k) \\ \gamma(k) \end{bmatrix}, \quad \check{\mathbf{A}} = \begin{bmatrix} \tilde{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{A}}_d \end{bmatrix}, \quad \check{\mathbf{B}} = \begin{bmatrix} \tilde{\mathbf{B}} \\ \mathbf{0} \end{bmatrix}, \quad (4.21)$$

the cost function (4.15) can be recast as

$$\begin{aligned} V(k) = & \|\check{\chi}(k+H_p|k)\|_{\check{\mathbf{P}}_0}^2 + \sum_{j=0}^{H_p-1} \{ \|\check{\chi}(k+H_p-1-j|k)\|_{\check{\mathbf{Q}}_j}^2 \\ & + \|\hat{\mathbf{u}}(k+H_p-1-j|k)\|_{\check{\mathbf{R}}_j}^2 \}, \end{aligned} \quad (4.22)$$

where

$$\check{\mathbf{P}}_0 = \begin{bmatrix} \mathcal{T}^T \mathbf{Q}_{H_p} \mathcal{T} & -\mathcal{T}^T \mathbf{Q}_{H_p} \\ -\mathbf{Q}_{H_p} \mathcal{T} & \mathbf{Q}_{H_p} \end{bmatrix}, \quad \check{\mathbf{Q}}_j = \begin{bmatrix} \mathcal{T}^T \tilde{\mathbf{Q}}_j \mathcal{T} & -\mathcal{T}^T \tilde{\mathbf{Q}}_j \\ -\tilde{\mathbf{Q}}_j \mathcal{T} & \tilde{\mathbf{Q}}_j \end{bmatrix}. \quad (4.23)$$

Note that the additional state $\gamma(k)$ in (4.20) is not reachable from the control input $\mathbf{u}(k)$. However, they are stabilizable.

4.3 Generalized Synchronization Controller

Minimizing (4.22) with respect to $\hat{\mathbf{u}}(k+H_p-1-j|k)$ results in

$$\hat{\mathbf{u}}(k+H_p-1-j|k) = -\check{\mathbf{K}}_j \check{\chi}(k+H_p-1-j|k), \quad \text{for } j = 0, 1, \dots, H_p-1 \quad (4.24)$$

where

$$\check{\mathbf{K}}_j = \left(\check{\mathbf{B}}^T \check{\mathbf{P}}_j \check{\mathbf{B}} + \check{\mathbf{R}}_j \right)^{-1} \check{\mathbf{B}}^T \check{\mathbf{P}}_j \check{\mathbf{A}}, \quad (4.25)$$

and $\check{\mathbf{P}}_j$ is obtained by the following recursive Riccati Difference Equation (RDE),

$$\check{\mathbf{P}}_{j+1} = \check{\mathbf{A}}^T \check{\mathbf{P}}_j \check{\mathbf{A}} - \check{\mathbf{A}}^T \check{\mathbf{P}}_j \check{\mathbf{B}} \left(\check{\mathbf{B}}^T \check{\mathbf{P}}_j \check{\mathbf{B}} + \check{\mathbf{R}}_j \right)^{-1} \check{\mathbf{B}}^T \check{\mathbf{P}}_j \check{\mathbf{A}} + \check{\mathbf{Q}}_j \quad (4.26)$$

from the initial condition $\check{\mathbf{P}}_0$.

If $\check{\mathbf{P}}$ in (4.26) is partitioned as

$$\check{\mathbf{P}} = \begin{bmatrix} \mathbf{P} & \mathbf{\Pi} \\ \mathbf{\Pi}^T & \check{\mathbf{P}}^{22} \end{bmatrix} \quad (4.27)$$

where $\mathbf{P}$ is a $(r+2)n \times (r+2)n$ matrix, then the control gain matrix $\check{\mathbf{K}}$ in (4.24) only depends upon $\mathbf{P}$ and $\mathbf{\Pi}$, i.e.,

$$\check{\mathbf{K}}_j = \left(\tilde{\mathbf{B}}^T \mathbf{P}_j \tilde{\mathbf{B}} + \tilde{\mathbf{R}}_j \right)^{-1} \begin{bmatrix} \tilde{\mathbf{B}}^T \mathbf{P}_j \tilde{\mathbf{A}} & \tilde{\mathbf{B}}^T \mathbf{\Pi}_j \tilde{\mathbf{A}}_d \end{bmatrix}. \quad (4.28)$$

In practice, referring to the receding horizon strategy, only the control action with $j = H_p - 1$ is applied. Hence, the optimal solution of the LQG approach is given by

$$\mathbf{u}(t)_{opt} = - \left(\tilde{\mathbf{B}}^T \mathbf{P}_{H_p-1} \tilde{\mathbf{B}} + \tilde{\mathbf{R}}_{H_p-1} \right)^{-1} \tilde{\mathbf{B}}^T \left(\mathbf{P}_{H_p-1} \tilde{\mathbf{A}} \boldsymbol{\chi}(t) + \mathbf{\Pi}_{H_p-1} \tilde{\mathbf{A}}_d \boldsymbol{\gamma}(t) \right) \quad (4.29)$$

where $\mathbf{P}_{H_p-1}$ is obtained from the RDE,

$$\mathbf{P}_{j+1} = \tilde{\mathbf{A}}^T \mathbf{P}_j \tilde{\mathbf{A}} - \tilde{\mathbf{A}}^T \mathbf{P}_j \tilde{\mathbf{B}} \left(\tilde{\mathbf{B}}^T \mathbf{P}_j \tilde{\mathbf{B}} + \tilde{\mathbf{R}}_j \right)^{-1} \tilde{\mathbf{B}}^T \mathbf{P}_j \tilde{\mathbf{A}} + \mathcal{T}^T \tilde{\mathbf{Q}}_j \mathcal{T}, \quad (4.30)$$

with initial condition $\mathbf{P}_0 = \mathcal{T}^T \mathbf{Q}_{H_p} \mathcal{T}$, and $\mathbf{\Pi}_{H_p-1}$ from the Lyapunov equation,

$$\mathbf{\Pi}_{j+1} = \left(\tilde{\mathbf{A}} - \tilde{\mathbf{B}} \left(\tilde{\mathbf{B}}^T \mathbf{P}_j \tilde{\mathbf{B}} + \tilde{\mathbf{R}}_j \right)^{-1} \tilde{\mathbf{B}}^T \mathbf{P}_j \tilde{\mathbf{A}} \right)^T \mathbf{\Pi}_j \tilde{\mathbf{A}}_d - \mathcal{T}^T \tilde{\mathbf{Q}}_j \quad (4.31)$$

with $\mathbf{\Pi}_0 = -\mathcal{T}^T \mathbf{Q}_{H_p}$.

Under the following assumption, Theorem 4.2 is obtained [81].

Assumption 4.2. Suppose that λ_i , ν_i and η_i are time-invariant except $\eta_1 = 0$, and are denoted as λ , ν and η , respectively.

Theorem 4.2. Consider equations (4.30) and (4.31) with constant weighting matrices $\mathbf{Q}$ and $\mathbf{R}$, and infinite prediction and control horizons.

If

- i) $\mathbf{P}_0$ is nonnegative definite,
- ii) $\mathbf{R}$ is positive definite,
- iii) $[\tilde{\mathbf{A}}, \tilde{\mathbf{B}}]$ is stabilizable,
- iv) $[\tilde{\mathbf{A}}, \mathbf{Q}^{1/2} \mathcal{T}]$ is observable,

then, the sequences $\mathbf{P}_{j+1}$ and $\mathbf{\Pi}_{j+1}$ will converge to steady-state values $\mathbf{P}_\infty$ and $\mathbf{\Pi}_\infty$ respectively.

Note that the conditions i) and ii) in Theorem 4.2 can be satisfied by selecting weighting factors λ , ν and η appropriately, condition iii) is given in Corollary 4.1 and condition iv) is true for the observability of the synchronization errors (4.8) and (4.9). Clearly, this method may result in considerable computation burden for a large number of the motion axes, as $\mathbf{P}_j$ is a $(r+2)n \times (r+2)n$ matrix. For this, a simplified controller is presented in the next section.

4.4 Controller Simplification

Since $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{A}}_d$ in the state equation (4.20) are actually sparse matrices, $\mathbf{P}_{H_p-1}$ and $\mathbf{\Pi}_{H_p-1}$ can be decomposed, and the following version of the synchronization controller is resulted.

Theorem 4.3. *Consider system (4.1) with the synchronization measurements (4.8) and (4.9). Under Assumption 4.2 and*

$$\mathbf{\Xi} = \mathbf{P}^{11} + \mathbf{P}^{12}\mathbf{L} + (\mathbf{P}^{12}\mathbf{L})^T + \mathbf{L}^T\mathbf{P}^{22}\mathbf{L} + \mathbf{L}^T\mathbf{P}^{33}\mathbf{L} \quad (4.32)$$

$$\mathbf{\Delta} = \mathbf{\Pi}^{11} + \mathbf{L}^T\mathbf{\Pi}^{21} \quad (4.33)$$

$$\mathbf{F} = \mathbf{L}^T\mathbf{P}^{33} \quad (4.34)$$

a simplified synchronization controller is then given by

$$\mathbf{u}(k)_{opt} = -(\mathbf{B}^T\mathbf{\Xi}_{H_p-1}\mathbf{B} + \lambda\mathbf{I})^{-1}\mathbf{B}^T(\mathbf{\Xi}_{H_p-1}\mathbf{A}\mathbf{x}(k) + \mathbf{\Delta}_{H_p-1}\mathbf{A}_d\mathbf{w}(k)) \quad (4.35)$$

where $\mathbf{\Xi}_{H_p-1}$ and $\mathbf{\Delta}_{H_p-1}$ are obtained by iterating the following equations:

$$\mathbf{P}_{j+1}^{11} = \mathbf{A}^T\mathbf{\Xi}_j\mathbf{A} - \mathbf{A}^T\mathbf{\Xi}_j\mathbf{B}(\mathbf{B}^T\mathbf{\Xi}_j\mathbf{B} + \tilde{\mathbf{R}}_j)^{-1}\mathbf{B}^T\mathbf{\Xi}_j\mathbf{A} + \mathbf{P}_0^{11} \quad (4.36)$$

$$\mathbf{\Pi}_{j+1}^{11} = \mathbf{A}^T\mathbf{\Delta}_j\mathbf{A}_d - \mathbf{A}^T\mathbf{\Xi}_j\mathbf{B}(\mathbf{B}^T\mathbf{\Xi}_j\mathbf{B} + \tilde{\mathbf{R}}_j)^{-1}\mathbf{B}^T\mathbf{\Delta}_j\mathbf{A}_d + \mathbf{\Pi}_0^{11} \quad (4.37)$$

$$\mathbf{P}_{j+1}^{12} = -\mathbf{A}^T\mathbf{F}_j + \mathbf{A}^T\mathbf{\Xi}_j\mathbf{B}(\mathbf{B}^T\mathbf{\Xi}_j\mathbf{B} + \tilde{\mathbf{R}}_j)^{-1}\mathbf{B}^T\mathbf{F}_j \quad (4.38)$$

$$\mathbf{P}_{j+1}^{22} = \mathbf{P}_j^{33} - \mathbf{F}_j^T\mathbf{B}(\mathbf{B}^T\mathbf{\Xi}_j\mathbf{B} + \tilde{\mathbf{R}}_j)^{-1}\mathbf{B}^T\mathbf{F}_j + \nu\mathbf{I} \quad (4.39)$$

$$\mathbf{\Pi}_{j+1}^{21} = \mathbf{F}_j^T\mathbf{B}(\mathbf{B}^T\mathbf{\Xi}_j\mathbf{B} + \tilde{\mathbf{R}}_j)^{-1}\mathbf{B}^T\mathbf{\Delta}_j\mathbf{A}_d \quad (4.40)$$

$$\mathbf{P}_{j+1}^{33} = \eta_{H_p-1-j}\mathbf{I} \quad (4.41)$$

with the initial conditions: $\mathbf{P}_0^{11} = \text{diag}(\mathbf{T}_1^T \mathbf{T}_1, \dots, \mathbf{T}_n^T \mathbf{T}_n)$, $\mathbf{P}_0^{22} = \nu \mathbf{I}$, $\mathbf{P}_0^{33} = \eta \mathbf{I}$, $\mathbf{P}_0^{12} = \mathbf{0}$, $\mathbf{\Pi}_0^{11} = \text{diag}(-\mathbf{T}_1^T, \dots, -\mathbf{T}_n^T)$, $\mathbf{\Pi}_0^{21} = \mathbf{0}$.

Proof. Partition $\mathbf{P}$ in (4.30) as

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}^{11} & \mathbf{P}^{12} & \mathbf{P}^{13} \\ \mathbf{P}^{12T} & \mathbf{P}^{22} & \mathbf{P}^{23} \\ \mathbf{P}^{13T} & \mathbf{P}^{23T} & \mathbf{P}^{33} \end{bmatrix} \quad (4.42)$$

where $\mathbf{P}^{11}$ is a $rn \times rn$ matrix, $\mathbf{P}^{22}$ and $\mathbf{P}^{33}$ are $n \times n$ matrices, and define

$$\mathbf{F} = \mathbf{P}^{13} + \mathbf{L}^T (\mathbf{P}^{23} + \mathbf{P}^{33}) \quad (4.43)$$

$$\mathbf{\Xi} = \mathbf{P}^{11} + \mathbf{P}^{12}\mathbf{L} + (\mathbf{P}^{12}\mathbf{L})^T + \mathbf{P}^{13}\mathbf{L} + (\mathbf{P}^{13}\mathbf{L})^T + \mathbf{L}^T (\mathbf{P}^{22} + \mathbf{P}^{23} + (\mathbf{P}^{23})^T + \mathbf{P}^{33}) \mathbf{L}. \quad (4.44)$$

Substituting $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$ and $\mathcal{T}$ into (4.30) results in

$$\begin{aligned} \mathbf{P}_{j+1} &= \begin{bmatrix} \mathbf{A}^T \mathbf{\Xi}_j \mathbf{A} & -\mathbf{A}^T \mathbf{F}_j & \mathbf{0} \\ -\mathbf{F}_j^T \mathbf{A} & \mathbf{P}_j^{33} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{A}^T \mathbf{\Xi}_j \mathbf{B} \\ -\mathbf{F}_j^T \mathbf{B} \\ \mathbf{0} \end{bmatrix} \left(\mathbf{B}^T \mathbf{\Xi}_j \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \\ &\quad \times \begin{bmatrix} \mathbf{B}^T \mathbf{\Xi}_j \mathbf{A}, & -\mathbf{B}^T \mathbf{F}_j, & \mathbf{0} \end{bmatrix} + \text{diag}(\mathbf{T}_1^T \mathbf{T}_1, \dots, \mathbf{T}_n^T \mathbf{T}_n, \nu \mathbf{I}, \eta_{H_p-1-j} \mathbf{I}), \end{aligned}$$

which is equivalent to the following equations

$$\mathbf{P}_{j+1}^{11} = \mathbf{A}^T \mathbf{\Xi}_j \mathbf{A} - \mathbf{A}^T \mathbf{\Xi}_j \mathbf{B} \left(\mathbf{B}^T \mathbf{\Xi}_j \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \mathbf{B}^T \mathbf{\Xi}_j \mathbf{A} + \text{diag}(\mathbf{T}_1^T \mathbf{T}_1, \dots, \mathbf{T}_n^T \mathbf{T}_n) \quad (4.45)$$

$$\mathbf{P}_{j+1}^{12} = -\mathbf{A}^T \mathbf{F}_j + \mathbf{A}^T \mathbf{\Xi}_j \mathbf{B} \left(\mathbf{B}^T \mathbf{\Xi}_j \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \mathbf{B}^T \mathbf{F}_j \quad (4.46)$$

$$\mathbf{P}_{j+1}^{22} = \mathbf{P}_j^{33} - \mathbf{F}_j^T \mathbf{B} \left(\mathbf{B}^T \mathbf{\Xi}_j \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \mathbf{B}^T \mathbf{F}_j + \nu \mathbf{I} \quad (4.47)$$

$$\mathbf{P}_{j+1}^{33} = \eta_{H_p-1-j} \mathbf{I}, \quad (4.48)$$

with $\mathbf{P}_{j+1}^{13} = \mathbf{0}$, $\mathbf{P}_{j+1}^{23} = \mathbf{0}$, $F = \mathbf{L}^T \mathbf{P}^{33}$ and $\Xi = \mathbf{P}^{11} + \mathbf{P}^{12} \mathbf{L} + (\mathbf{P}^{12} \mathbf{L})^T + \mathbf{L}^T (\mathbf{P}^{22} + \mathbf{P}^{33}) \mathbf{L}$. Similarly, if

$$\mathbf{\Pi} = \begin{bmatrix} \mathbf{\Pi}^{11} & \mathbf{\Pi}^{12} & \mathbf{\Pi}^{13} \\ \mathbf{\Pi}^{21} & \mathbf{\Pi}^{22} & \mathbf{\Pi}^{23} \\ \mathbf{\Pi}^{31} & \mathbf{\Pi}^{32} & \mathbf{\Pi}^{33} \end{bmatrix}, \quad \Delta = \mathbf{\Pi}^{11} + \mathbf{L}^T \mathbf{\Pi}^{21} + \mathbf{L}^T \mathbf{\Pi}^{31} \quad (4.49)$$

Equation (4.31) can then be decomposed into

$$\mathbf{\Pi}_{j+1}^{11} = \mathbf{A}^T \Delta_j \mathbf{A}^d - \mathbf{A}^T \Xi_j \mathbf{B} \left(\mathbf{B}^T \Xi_j \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \mathbf{B}^T \Delta_j \mathbf{A}^d + \text{diag}(-\mathbf{T}_1^T, \dots, -\mathbf{T}_n^T) \quad (4.50)$$

$$\mathbf{\Pi}_{j+1}^{21} = -\mathbf{\Pi}_j^{31} \mathbf{A}^d + F_j^T \mathbf{B} \left(\mathbf{B}^T \Xi_j \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \mathbf{B}^T \Delta_j \mathbf{A}^d \quad (4.51)$$

$$\mathbf{\Pi}_{j+1}^{22} = -\nu \mathbf{I} \quad (4.52)$$

$$\mathbf{\Pi}_{j+1}^{33} = -\eta_{H_p-1-j} \mathbf{I}. \quad (4.53)$$

Hence, the optimal control input (4.29) becomes

$$\begin{aligned} \mathbf{u}(k)_{opt} &= - \left(\mathbf{B}^T \Xi_{H_p-1} \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \left[\mathbf{B}^T \Xi_{H_p-1} \mathbf{A} \quad - \mathbf{B}^T F_{H_p-1} \quad \mathbf{0} \right] \begin{bmatrix} \mathbf{x}(k) \\ \boldsymbol{\delta}(k) \\ \boldsymbol{\varepsilon}(k) \end{bmatrix} \\ &\quad - \left(\mathbf{B}^T \Xi_{H_p-1} \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \left[\mathbf{B}^T \Delta_{H_p-1} \mathbf{A}_d \quad \mathbf{0} \quad \mathbf{0} \right] \begin{bmatrix} \mathbf{w}(k) \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ &= - \left(\mathbf{B}^T \Xi_{H_p-1} \mathbf{B} + \tilde{\mathbf{R}}_j \right)^{-1} \mathbf{B}^T \left((\Xi_{H_p-1} \mathbf{A} - F_{H_p-1} \mathbf{L}) \mathbf{x}(t) + \Delta_{H_p-1} \mathbf{A}_d \mathbf{w}(k) \right). \end{aligned}$$

The proof can thus be completed by letting $F_{H_p-1} = \mathbf{0}$ due to $\eta_1 = 0$.

In practice, application procedure of the synchronization control algorithm can be expressed as follows:

Step 1. Construct system model (4.1) and specify the desired trajectory $\mathbf{w}(k)$.

Step 2. Measure the state vector $\mathbf{x}(k)$.

Step 3. Iterate (4.36)-(4.41) to get matrices Ξ_{H_p-1} and Δ_{H_p-1} .

Step 4. Calculate the control law using (4.35).

Step 5. At the next time instant, repeat the above steps if an adaptive control is necessary. Otherwise, step 3 is ignored.

4.5 Stability Analysis

Indeed, as receding horizon LQ control is a feedback strategy effected through the application of a stationery feedback law applied prospectively over an infinite interval, questions of stability naturally arise while solutions are sometimes slow to emerge. One of the major drawbacks of MPC control is that the closed-loop stability is not guaranteed, and that attempts at producing stability result for MPC on the basics of its explicit input-output description have been remarkably unsuccessful, usually necessitating the abandonment of a specific control performance.

Having established the exact state-space LQ synchronization control equivalent of MPC, one is now in a position to bring to bear the powerful tools and properties of Riccati equations on the solution of these stability questions for receding horizon LQ controllers in general, and for MPC in particular. This is the starting point of the

analysis developed in this section, where one will use recently developed properties of the solutions of the Riccati equations to enforce closed-loop stability of receding horizon controllers. The reason why MPC in its generic formulation may produce unstable closed loops whatever the games played with the finite horizons H_p and H_c will then become transparent, and so will the necessity, for guaranteed closed loop stability, to resort to the more classical state-space LQ formulation with appropriate choices of the weighting matrices. Of course, in a nonadaptive implementation, the closed loop stability of any designed controller can always be verified before actually closing the loop on the plant, and nominal stability margins can be computed as well. But in adaptive implementation the advantage of using controller designs with guaranteed nominal stability margins becomes self-evident.

Somewhat unsurprisingly perhaps, the results will show that receding horizon controllers can be designed with guaranteed asymptotic closed loop stability provided their cost function weighting matrices can be chosen in such a way that the problem becomes equivalent to an infinite horizon LQ problem with different (but related) weightings. Hence, we begin this section by studying the closed loop stability of receding horizon LQ controllers by the use of monotonicity of solutions of Riccati Difference Equations (RDE). However, although the generalized synchronization controller (4.35) can easily be applied easily, it is difficult to analyze its stability. For this purpose, we consider $\gamma(k) = 0$ and in this case, it has

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \tag{4.54}$$

where $\mathbf{y} = (y_1, \dots, y_n)^T$ and $\mathbf{C} = \text{diag}(\mathbf{T}_1, \dots, \mathbf{T}_n)$.

Cost function (4.12) then becomes

$$\begin{aligned}
 V(k) = & \sum_{i=1}^{H_p} \hat{\mathbf{y}}^T(k+i|k) \hat{\mathbf{y}}(k+i|k) + \lambda \sum_{i=1}^{H_c} \hat{\mathbf{u}}^T(k+i-1|k) \hat{\mathbf{u}}(k+i-1|k) \\
 & + \nu \sum_{i=1}^{H_p} \hat{\boldsymbol{\delta}}^T(k+i|k) \hat{\boldsymbol{\delta}}(k+i|k) + \eta \sum_{i=2}^{H_p} \hat{\boldsymbol{\varepsilon}}^T(k+i|k) \hat{\boldsymbol{\varepsilon}}(k+i|k) \quad (4.55)
 \end{aligned}$$

where $\hat{\mathbf{y}}(k+i|k)$ is the future output vector.

Rewrite (4.55) as

$$V(k) = \sum_{j=0}^{H_p-1} \ell(\hat{\mathbf{x}}(k+H_p-1+j|k), \hat{\mathbf{u}}(k+H_p-1+j|k)) + \|\hat{\mathbf{x}}(k+H_p|k)\|_{\mathbf{P}_0}^2 \quad (4.56)$$

where

$$\begin{aligned}
 \ell(\hat{\mathbf{x}}, \hat{\mathbf{u}}) = & \|\hat{\mathbf{x}}(k+H_p-1+j|k)\|_{\mathbf{Q}_j}^2 + \|\hat{\mathbf{u}}(k+H_p-1+j|k)\|_{\mathbf{R}_j}^2 \\
 & + 2\hat{\mathbf{u}}(k+H_p-1+j|k)^T \mathbf{M}_j \hat{\mathbf{x}}(k+H_p-1+j|k) \quad (4.57)
 \end{aligned}$$

$$\mathbf{P}_0 = \mathbf{C}^T (\mathbf{I} + \nu \mathbf{L}^T \mathbf{L}) \mathbf{C} \quad (4.58)$$

$$\mathbf{Q}_j = \begin{cases} \mathbf{C}^T (\mathbf{I} + \nu \mathbf{L}^T \mathbf{L}) \mathbf{C} + \eta (\mathbf{A} - \mathbf{I})^T \mathbf{C}^T \mathbf{L}^T \mathbf{L} \mathbf{C} (\mathbf{A} - \mathbf{I}) & \text{if } j = 0, 1, \dots, H_p - 2 \\ \mathbf{0} & \text{if } j = H_p - 1 \end{cases} \quad (4.59)$$

$$\mathbf{R}_j = \begin{cases} \infty \mathbf{I}, & \text{if } j = 0, 1, \dots, H_p - H_c - 1 \\ \lambda \mathbf{I} + \eta \mathbf{B}^T \mathbf{C}^T \mathbf{L}^T \mathbf{L} \mathbf{C} \mathbf{B}, & \text{if } j = H_p - H_c, \dots, H_p - 2 \\ \lambda \mathbf{I}, & \text{if } j = H_p - 1 \end{cases} \quad (4.60)$$

$$\mathbf{M}_j = \begin{cases} \eta (\mathbf{A} - \mathbf{I})^T \mathbf{C}^T \mathbf{L}^T \mathbf{L} \mathbf{C} \mathbf{B}, & \text{if } j = 0, 1, \dots, H_p - 2 \\ \mathbf{0}, & \text{if } j = H_p - 1 \end{cases} \quad (4.61)$$

Minimization of the cost function (4.56) results in:

$$\mathbf{u}(k) = -\mathbf{K}_{H_p-1}\mathbf{x}(k) \quad (4.62)$$

where

$$\mathbf{K}_{H_p-1} = (\mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{B} + \lambda \mathbf{I})^{-1} \mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{A} \quad (4.63)$$

with $\mathbf{P}_{H_p-1}$ obtained by the following recursive RDE,

$$\mathbf{P}_{j+1} = \mathbf{A}^T \mathbf{P}_j \mathbf{A} - (\mathbf{B}^T \mathbf{P}_j \mathbf{A} + \mathbf{M}_j^T)^T (\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)^{-1} (\mathbf{B}^T \mathbf{P}_j \mathbf{A} + \mathbf{M}_j^T) + \mathbf{Q}_j \quad (4.64)$$

with the initial condition $\mathbf{P}_0$.

Note that stability of the control system can sometimes be guaranteed with finite horizons, even if there is no explicit terminal constraint. The finite horizon predictive control problem is associated with a time-varying RDE which is related to the optimal value of the cost function. The Fake Algebraic Riccati Technique (FART) [81] is used to replace this equation with an Algebraic (time-invariant) Riccati Equation (ARE) and if this equation has properties analogous to the (real) ARE of infinite-horizon control, stability can then be inferred. For this, monotonicity of the RDE (4.64) is first discussed.

Theorem 4.4. *Consider $\mathbf{P}_j$, the solution of the RDE (4.64), with $\mathbf{P}_0$, $\mathbf{Q}_j$, $\mathbf{R}_j$ and $\mathbf{M}_j$ given in (4.58)-(4.61), then for all $j \geq 0$, $\mathbf{P}_j$ is a symmetric nonnegative definite matrix.*

Proof. Rewrite (4.64) as

$$\mathbf{P}_{j+1} = \mathbf{A}^T \mathbf{P}_j \mathbf{A} - \mathbf{K}_j^T (\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j) \mathbf{K}_j + \mathbf{Q}_j \quad (4.65)$$

where

$$\mathbf{K}_j = (\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)^{-1} (\mathbf{B}^T \mathbf{P}_j \mathbf{A} + \mathbf{M}_j^T). \quad (4.66)$$

Clearly, $\mathbf{P}_{j+1}$ is symmetric. The proof is completed if it can be shown that $\mathbf{P}_{j+1}$ is nonnegative definite. For this purpose, (4.65) is rewritten as

$$\begin{aligned} \mathbf{P}_{j+1} &= \mathbf{A}^T \mathbf{P}_j \mathbf{A} - (\mathbf{A}^T \mathbf{P}_j \mathbf{B} + \mathbf{M}_j) (\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)^{-1} (\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j) \mathbf{K}_j + \mathbf{Q}_j \\ &= \mathbf{A}^T \mathbf{P}_j (\mathbf{A} - \mathbf{B} \mathbf{K}_j) - \mathbf{M}_j \mathbf{K}_j + \mathbf{Q}_j. \end{aligned} \quad (4.67)$$

Premultiplying both sides of (4.66) by $\mathbf{K}_j^T (\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)$ yields

$$\mathbf{K}_j^T \mathbf{R}_j \mathbf{K}_j = \mathbf{K}_j^T \mathbf{B}^T \mathbf{P}_j (\mathbf{A} - \mathbf{B} \mathbf{K}_j) + \mathbf{K}_j^T \mathbf{M}_j^T. \quad (4.68)$$

By subtracting (4.68) from (4.67), we have

$$\mathbf{P}_{j+1} = (\mathbf{A} - \mathbf{B} \mathbf{K}_j)^T \mathbf{P}_j (\mathbf{A} - \mathbf{B} \mathbf{K}_j) + \mathbf{K}_j^T \mathbf{R}_j \mathbf{K}_j - \mathbf{M}_j \mathbf{K}_j - \mathbf{K}_j^T \mathbf{M}_j^T + \mathbf{Q}_j. \quad (4.69)$$

Assume that the term $\ell(\mathbf{x}, \mathbf{u})$ is nonnegative, i.e.,

$$\|\mathbf{x}\|_{\mathbf{Q}}^2 + 2\mathbf{u}^T \mathbf{M} \mathbf{x} + \|\mathbf{u}\|_{\mathbf{R}}^2 \geq 0. \quad (4.70)$$

Then, since $\mathbf{R}$ is nonsingular, (4.70) is equivalent to:

$$\mathbf{Q} - \mathbf{M} \mathbf{R}^{-1} \mathbf{M}^T \geq 0. \quad (4.71)$$

By virtue of above results,

$$\begin{aligned} \mathbf{K}_j^T \mathbf{R}_j \mathbf{K}_j - \mathbf{M}_j^T \mathbf{K}_j - \mathbf{K}_j^T \mathbf{M}_j + \mathbf{Q}_j &\geq \mathbf{K}_j^T \mathbf{R}_j \mathbf{K}_j - \mathbf{M}_j^T \mathbf{K}_j - \mathbf{K}_j^T \mathbf{M}_j + \mathbf{M}_j^T \mathbf{R}_j^{-1} \mathbf{M}_j \\ &= (\mathbf{R}_j^{-1} \mathbf{M}_j^T - \mathbf{K}_j)^T \mathbf{R}_j (\mathbf{R}_j^{-1} \mathbf{M}_j^T - \mathbf{K}_j). \end{aligned} \quad (4.72)$$

From (4.69), (4.72) and the nonnegative definiteness of $\mathbf{P}_j$, it can be seen that $\mathbf{P}_{j+1}$ is a lower-bounded matrix by the sum of two nonnegative definite matrices. In other words, $\mathbf{P}_{j+1}$ is nonnegative definite. Hence, by induction, $\mathbf{P}_j$ is a symmetric nonnegative definite matrix for all $j \geq 0$.

In particular, beyond the control horizon where $\mathbf{R}_j = \infty$, $\mathbf{Q}_j$ becomes a constant matrix denoted as $\mathbf{Q}$. In this case, the following equation is obtained,

$$\mathbf{P}_{j+1} = \mathbf{A}^T \mathbf{P}_j \mathbf{A} + \mathbf{Q} \quad (4.73)$$

and

$$\mathbf{P}_j = (\mathbf{A}^j)^T \mathbf{P}_0 \mathbf{A}^j + \sum_{i=0}^{j-1} (\mathbf{A}^i)^T \mathbf{Q} \mathbf{A}^i. \quad (4.74)$$

It can be seen that $\mathbf{P}_j$ is monotonically nondecreasing. However, for stable $\mathbf{A}$, $\mathbf{P}_j$ will converge to a constant matrix as $j \rightarrow \infty$, which indicates that with infinite $\mathbf{R}_j$ and finite $\mathbf{Q}$, the control inputs will always be zeroes and questions of stabilizability are not sensible. Nevertheless, this may yield a useful procedure to develop a monotonically nonincreasing $\mathbf{P}_j$ with finite $\mathbf{R}_j$. The following lemma gives the property between solutions of the RDE.

Lemma 4.1. [86] Let $\mathbf{P}_j^1$ and $\mathbf{P}_j^2$ be the solutions of the RDE (4.64) with $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{R}_j$ but possibly different $\mathbf{Q}_j$, i.e., $\mathbf{Q}_j^1$ and $\mathbf{Q}_j^2$. Then, the matrix

$$\bar{\mathbf{P}}_j = \mathbf{P}_j^2 - \mathbf{P}_j^1 \quad (4.75)$$

satisfies

$$\bar{\mathbf{P}}_{j+1} = \bar{\mathbf{A}}_j^{1T} \mathbf{P}_j \bar{\mathbf{A}}_j^1 - \left(\bar{\mathbf{A}}_j^{1T} \mathbf{P}_j \mathbf{B} + \mathbf{M}_j \right) \left(\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \bar{\mathbf{R}}_j \right)^{-1} \left(\mathbf{B}^T \mathbf{P}_j \bar{\mathbf{A}}_j^1 + \mathbf{M}_j^T \right) + \bar{\mathbf{Q}}_j \quad (4.76)$$

where

$$\bar{\mathbf{A}}_j^1 = \mathbf{A} - \mathbf{B} \left(\mathbf{B}^T \mathbf{P}_j^1 \mathbf{B} + \mathbf{R}_j \right)^{-1} \left(\mathbf{B}^T \mathbf{P}_j^1 \mathbf{A} + \mathbf{M}_j^T \right) \quad (4.77)$$

$$\bar{\mathbf{Q}}_j = \mathbf{Q}_j^2 - \mathbf{Q}_j^1 \quad (4.78)$$

$$\bar{\mathbf{R}}_j = \mathbf{B}^T \mathbf{P}_j^1 \mathbf{B} + \mathbf{R}_j. \quad (4.79)$$

This lemma represents a simplification of the results of Nishimura [87] and [88] and is a discrete-time counterpart to those of [89]. Directly from Lemma 4.1, we have the following results, the first of which extends the continuous-time observations of Kailath [90].

Theorem 4.5. [88] If $\mathbf{P}_j$ converges after the first $H_p - H_c$ iterations, then for all $j \geq H_p - H_c$, $\mathbf{P}_j$ is always monotonically nonincreasing, i.e.,

$$\mathbf{P}_{j+1} \leq \mathbf{P}_j, \text{ for } j \geq H_p - H_c. \quad (4.80)$$

Proof. Denote the steady-state value of $\mathbf{P}_j$ for $j < H_p - H_c$ as

$$\mathbf{P}_0^\infty = \mathbf{A}^T \mathbf{P}_0^\infty \mathbf{A} + \mathbf{Q}. \quad (4.81)$$

With the initial condition $\mathbf{P}_0^\infty$ for (4.64), it has

$$\begin{aligned}\mathbf{P}_1^\infty &= \mathbf{A}^T \mathbf{P}_0^\infty \mathbf{A} - (\mathbf{B}^T \mathbf{P}_0^\infty \mathbf{A} + \mathbf{M}_j^T)^T (\mathbf{B}^T \mathbf{P}_0^\infty \mathbf{B} + \mathbf{R}_j)^{-1} (\mathbf{B}^T \mathbf{P}_0^\infty \mathbf{A} + \mathbf{M}_j^T) + \mathbf{Q}_j \\ &\leq \mathbf{A}^T \mathbf{P}_0^\infty \mathbf{A} + \mathbf{Q}_j \\ &= \mathbf{P}_0^\infty.\end{aligned}$$

If $\mathbf{P}_j^1 = \mathbf{P}_1^\infty$, $\mathbf{P}_j^2 = \mathbf{P}_0^\infty$ and $\mathbf{Q}_j^1 = \mathbf{Q}_j^2$ in Lemma 4.2, then, from (4.76) $\bar{\mathbf{P}}$ is nonnegative definitely after time j , i.e.

$$\mathbf{P}_{j+i+1} \leq \mathbf{P}_{j+i}, \text{ for all } i \geq 0. \quad (4.82)$$

Therefore, $\mathbf{P}_{j+i}$ is monotonically nonincreasing (The proof is completed).

Recall the stability property of the Algebraic Riccati Equation (ARE) [91].

Lemma 4.2. *Consider the following ARE with an infinite horizon linear quadratic control,*

$$\mathbf{P} = \mathbf{A}^T \mathbf{P} \mathbf{A} - \mathbf{A}^T \mathbf{P} \mathbf{B} (\mathbf{B}^T \mathbf{P} \mathbf{B} + \mathbf{R})^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A} + \mathbf{Q} \quad (4.83)$$

where

- $[\mathbf{A}, \mathbf{B}]$ is stabilizable,
- $[\mathbf{A}, \mathbf{Q}^{1/2}]$ is detectable,
- $\mathbf{Q} \geq \mathbf{0}$ and $\mathbf{R} > \mathbf{0}$.

Then

- there exists a unique, maximal, non-negative definite symmetric solution $\bar{\mathbf{P}}$.

- $\bar{\mathbf{P}}$ is a unique stabilizing solution, i.e.

$$\mathbf{A} - \mathbf{B}(\mathbf{B}^T \bar{\mathbf{P}} \mathbf{B} + \mathbf{R})^{-1} \mathbf{B}^T \bar{\mathbf{P}} \mathbf{A} \quad (4.84)$$

has all its eigenvalues strictly within the unit circle.

Rewrite (4.83) as

$$\mathbf{P} = (\mathbf{A} - \mathbf{BK})^T \mathbf{P} (\mathbf{A} - \mathbf{BK}) + \mathbf{K}^T \mathbf{R} \mathbf{K} + \mathbf{Q}. \quad (4.85)$$

In order to connect the RDE (4.64) with the ARE (4.83), we use the Fake Algebraic Riccati Technique (FART) as:

$$\mathbf{P}_j = (\mathbf{A} - \mathbf{BK}_j)^T \mathbf{P}_j (\mathbf{A} - \mathbf{BK}_j) + \mathbf{K}_j^T \mathbf{R}_j \mathbf{K}_j + \bar{\mathbf{Q}}_j \quad (4.86)$$

where

$$\begin{aligned} \bar{\mathbf{Q}}_j = & \mathbf{Q}_j - \mathbf{M}_j(\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)^{-1} \mathbf{B}^T \mathbf{P}_j \mathbf{A} - \mathbf{A}^T \mathbf{P}_j \mathbf{B}(\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)^{-1} \mathbf{M}_j^T \tilde{\mathbf{Q}}_j \\ & - 2\mathbf{M}_j(\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)^{-1} \mathbf{M}_j^T - (\mathbf{P}_{j+1} - \mathbf{P}_j) \end{aligned} \quad (4.87)$$

Clearly, while we have not altered the RDE in viewing it as a masquerading ARE, we do have the immediate result following from Lemma 4.2 and (4.86).

Corollary 4.2. Consider (4.86), or (4.87) defining the matrix $\bar{\mathbf{Q}}_j$. If

- $[\mathbf{A}, \mathbf{B}]$ is stabilizable,
- $[\mathbf{A}, \bar{\mathbf{Q}}_j^{1/2}]$ is detectable,
- $\bar{\mathbf{Q}}_j \geq \mathbf{0}$, $\mathbf{R}_j > \mathbf{0}$,

then $\mathbf{P}_j$ is stabilizing, i.e. the closed-loop transition matrix

$$\bar{\mathbf{A}}_j = \mathbf{A} - \mathbf{B}(\mathbf{B}^T \mathbf{P}_j \mathbf{B} + \mathbf{R}_j)^{-1} (\mathbf{B}^T \mathbf{P}_j \mathbf{A} + \mathbf{M}_j^T) \quad (4.88)$$

has all its eigenvalues strictly within the unit circle.

This corollary forms the centerpiece of our developments to translate stability properties from the ARE arena to the RDE. Then, regarding the receding horizon strategy, only the solution of $\mathbf{P}_j$ with $j = H_p - 1$ will be applied. This leads to

$$\mathbf{P}_{H_p-1} = \mathbf{A}^T \mathbf{P}_{H_p-1} \mathbf{A} - \mathbf{A}^T \mathbf{P}_{H_p-1} \mathbf{B} (\mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{B} + \lambda \mathbf{I})^{-1} \mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{A} + \bar{\mathbf{Q}}_{H_p-1} \quad (4.89)$$

where

$$\bar{\mathbf{Q}}_{H_p-1} = \mathbf{P}_{H_p-1} - \mathbf{P}_{H_p}. \quad (4.90)$$

Based on (4.89) and (4.90), the stability result of our control system can be formulated in the following theorem.

Theorem 4.6. Consider (4.89) with the weighting matrix $\bar{\mathbf{Q}}_{H_p-1}$ given in (4.90). If

- $[\mathbf{A}, \mathbf{B}]$ is stabilizable,
- $[\mathbf{A}, \bar{\mathbf{Q}}_{H_p-1}^{1/2}]$ is detectable,
- $\mathbf{P}_{H_p-1}$ is nonincreasing, $\lambda > 0$,

then the proposed control system (4.62) is stabilizing, i.e. the closed-loop transition matrix

$$\bar{\mathbf{A}}_{H_p-1} = \mathbf{A} - \mathbf{B}(\mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{B} + \lambda \mathbf{I})^{-1} \mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{A} \quad (4.91)$$

has all its eigenvalues strictly within the unit circle.

Proof. The proof is completed by setting $j = H_p - 1$ in Corollary 4.2.

It can be seen from the above theorem that the prediction horizon H_p is a key parameter for stability, and increasing H_p is always favorable. This was the main motivation of extension of the one-step-ahead control to long range predictive control.

4.6 Constrained Control System

Now we will deal with the case when constraints are present. Assume that the following constraints are applied:

$$\underline{\mathbf{u}} \leq \mathbf{u}(k) \leq \bar{\mathbf{u}}, \quad \underline{\mathbf{x}} \leq \mathbf{x}(k) \leq \bar{\mathbf{x}} \quad (4.92)$$

where $\underline{\mathbf{u}}$ and $\bar{\mathbf{u}}$ denote the lower and upper limits of the control input $\mathbf{u}(k)$, and $\underline{\mathbf{x}}$ and $\bar{\mathbf{x}}$ denote the lower and upper limits of the state vector $\mathbf{x}(k)$.

Rewrite (4.92) as

$$\mathbf{E}\mathbf{u}(k) \leq \mathbf{e}, \quad \mathbf{F}\mathbf{x}(k) \leq \mathbf{f} \quad (4.93)$$

where

$$\mathbf{E} = \begin{bmatrix} \mathbf{I}_{n \times n} \\ -\mathbf{I}_{n \times n} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \mathbf{I}_{rn \times rn} \\ -\mathbf{I}_{rn \times rn} \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} \bar{\mathbf{u}} \\ -\underline{\mathbf{u}} \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} \bar{\mathbf{x}} \\ -\underline{\mathbf{x}} \end{bmatrix}. \quad (4.94)$$

The following form of constraints is then derived [24]:

$$\tilde{\mathbf{E}} \begin{bmatrix} \hat{\mathbf{u}}(k|k) \\ \vdots \\ \hat{\mathbf{u}}(k + H_c - 1|k) \end{bmatrix} \leq \tilde{\mathbf{e}}, \quad \tilde{\mathbf{F}} \begin{bmatrix} \hat{\mathbf{x}}(k + 1|k) \\ \vdots \\ \hat{\mathbf{x}}(k + H_p|k) \end{bmatrix} \leq \tilde{\mathbf{f}} \quad (4.95)$$

where

$$\tilde{\mathbf{E}} = \text{diag}(\overbrace{\mathbf{E}, \dots, \mathbf{E}}^{H_c \text{ blocks}}), \quad \tilde{\mathbf{e}} = (\overbrace{\mathbf{e}^T, \dots, \mathbf{e}^T}^{H_c \text{ blocks}})^T, \quad \tilde{\mathbf{F}} = \text{diag}(\overbrace{\mathbf{F}, \dots, \mathbf{F}}^{H_p \text{ blocks}}), \quad \tilde{\mathbf{f}} = (\overbrace{\mathbf{f}^T, \dots, \mathbf{f}^T}^{H_p \text{ blocks}})^T. \quad (4.96)$$

Note that (4.1) can be recursively represented as

$$\begin{aligned} \hat{\mathbf{x}}(k+1|k) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\hat{\mathbf{u}}(k|k) \\ \hat{\mathbf{x}}(k+2|k) &= \mathbf{A}^2\mathbf{x}(k) + \mathbf{A}\mathbf{B}\hat{\mathbf{u}}(k|k) + \mathbf{B}\hat{\mathbf{u}}(k+1|k) \\ &\vdots \\ \hat{\mathbf{x}}(k+H_p|k) &= \mathbf{A}^{H_p}\mathbf{x}(k) + \mathbf{A}^{H_p-1}\mathbf{B}\hat{\mathbf{u}}(k|k) + \dots + \mathbf{A}^{H_p-H_c}\mathbf{B}\hat{\mathbf{u}}(k+H_c-1|k) \end{aligned}$$

which can be rearranged in the matrix-vector form:

$$\boldsymbol{\zeta}(k) = \boldsymbol{\Omega}\mathbf{x}(k) + \boldsymbol{\Gamma}\mathbf{u}(k) \quad (4.97)$$

where

$$\boldsymbol{\zeta}(k) = \begin{bmatrix} \hat{\mathbf{x}}(k+1|k) \\ \vdots \\ \hat{\mathbf{x}}(k+H_p|k) \end{bmatrix}, \quad \boldsymbol{\Omega} = \begin{bmatrix} \mathbf{A} \\ \vdots \\ \mathbf{A}^{H_p} \end{bmatrix}, \quad \mathbf{u}(k) = \begin{bmatrix} \hat{\mathbf{u}}(k|k) \\ \vdots \\ \hat{\mathbf{u}}(k+H_c-1|k) \end{bmatrix},$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{B} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{A}^{H_c-1}\mathbf{B} & \dots & \mathbf{B} \\ \vdots & \vdots & \vdots \\ \mathbf{A}^{H_p-1}\mathbf{B} & \dots & \mathbf{A}^{H_p-H_c}\mathbf{B} \end{bmatrix}.$$

Therefore, constraints (4.95) can be assembled into the single inequality:

$$\begin{bmatrix} \tilde{\mathbf{E}} \\ \tilde{\mathbf{F}}\boldsymbol{\Gamma} \end{bmatrix} \mathbf{u}(k) \leq \begin{bmatrix} \tilde{\mathbf{e}} \\ \tilde{\mathbf{f}} - \tilde{\mathbf{F}}\boldsymbol{\Omega}\mathbf{x}(k) \end{bmatrix}. \quad (4.98)$$

(Replace $\mathbf{x}(k)$ by $\hat{\mathbf{x}}(k)$ if an observer is used.)

Similarly, iterating (4.10) H_p times results in

$$\tilde{\boldsymbol{\zeta}}(k) = \tilde{\boldsymbol{\Omega}}\boldsymbol{\chi}(k) + \tilde{\boldsymbol{\Gamma}}\mathbf{u}(k) \quad (4.99)$$

where

$$\tilde{\boldsymbol{\zeta}}(k) = \begin{bmatrix} \hat{\boldsymbol{\chi}}(k+1|k) \\ \vdots \\ \hat{\boldsymbol{\chi}}(k+H_p|k) \end{bmatrix}, \quad \tilde{\boldsymbol{\Omega}} = \begin{bmatrix} \tilde{\mathbf{A}} \\ \vdots \\ \tilde{\mathbf{A}}^{H_p} \end{bmatrix}, \quad \tilde{\boldsymbol{\Gamma}} = \begin{bmatrix} \tilde{\mathbf{B}} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{A}}^{H_c-1}\tilde{\mathbf{B}} & \cdots & \tilde{\mathbf{B}} \\ \vdots & \vdots & \vdots \\ \tilde{\mathbf{A}}^{H_p-1}\tilde{\mathbf{B}} & \cdots & \tilde{\mathbf{A}}^{H_p-H_c}\tilde{\mathbf{B}} \end{bmatrix}.$$

Since

$$\tilde{\boldsymbol{\gamma}}(k) = \begin{bmatrix} \gamma(k+1) \\ \vdots \\ \gamma(k+H_p) \end{bmatrix},$$

$$\tilde{\boldsymbol{\mathcal{T}}} = \text{diag}(\overbrace{\boldsymbol{\mathcal{T}}, \dots, \boldsymbol{\mathcal{T}}}^{H_p \text{ blocks}}),$$

$$\boldsymbol{\mathcal{Q}} = \text{diag}(\mathbf{Q}_1, \dots, \mathbf{Q}_{H_p}),$$

$$\boldsymbol{\mathcal{R}} = \text{diag}(\mathbf{R}_1, \dots, \mathbf{R}_{H_c})$$

the cost function (4.12) can be rewritten as:

$$\begin{aligned}
V(k) &= (\tilde{\gamma}(k) - \tilde{\mathcal{T}}\tilde{\zeta}(k))^T \mathcal{Q}(\tilde{\gamma}(k) - \tilde{\mathcal{T}}\tilde{\zeta}(k)) + \mathbf{u}(k)^T \mathcal{R}\mathbf{u}(k) \\
&= (\tilde{\gamma}(k) - \tilde{\mathcal{T}}\tilde{\Omega}\chi(k) - \tilde{\mathcal{T}}\tilde{\Gamma}\mathbf{u}(k))^T \mathcal{Q}(\tilde{\gamma}(k) - \tilde{\mathcal{T}}\tilde{\Omega}\chi(k) - \tilde{\mathcal{T}}\tilde{\Gamma}\mathbf{u}(k)) \\
&\quad + \mathbf{u}(k)^T \mathcal{R}\mathbf{u}(k) \\
&= (\tilde{\gamma}(k) - \tilde{\mathcal{T}}\tilde{\Omega}\chi(k))^T \mathcal{Q}(\tilde{\gamma}(k) - \tilde{\mathcal{T}}\tilde{\Omega}\chi(k)) - 2\mathbf{u}(k)^T \tilde{\Omega}^T \tilde{\mathcal{T}}^T \mathcal{Q}(\tilde{\gamma}(k) \\
&\quad - \tilde{\mathcal{T}}\tilde{\Omega}\chi(k)) + \mathbf{u}(k)^T (\tilde{\Omega}^T \tilde{\mathcal{T}}^T \mathcal{Q} \tilde{\mathcal{T}} \tilde{\Omega} + \mathcal{R})\mathbf{u}(k) \\
&= \mathbf{u}(k)^T \mathcal{H}\mathbf{u}(k) + \mathbf{u}(k)^T \mathcal{G} + \text{const.}
\end{aligned} \tag{4.100}$$

where

$$\mathcal{H} = \tilde{\Omega}^T \tilde{\mathcal{T}}^T \mathcal{Q} \tilde{\mathcal{T}} \tilde{\Omega} + \mathcal{R} \tag{4.101}$$

$$\mathcal{G} = -2\tilde{\Omega}^T \tilde{\mathcal{T}}^T \mathcal{Q}(\tilde{\gamma}(k) - \tilde{\mathcal{T}}\tilde{\Omega}\chi(k)). \tag{4.102}$$

Here the cost function $V(k)$ is still the same as that of the unconstrained optimization problem. So we have to solve the following optimization problem:

$$V(k) = \mathbf{u}(k)^T \mathcal{H}\mathbf{u}(k) + \mathbf{u}(k)^T \mathcal{G} \tag{4.103}$$

subject to the inequality constraint given by (4.98).

Assuming that the optimal solution for $\mathbf{u}(k)_{opt}$ is obtained, then we only use part of $\mathbf{u}(k)_{opt}$ to give

$$\mathbf{u}(k)_{opt} = \text{diag}(\mathbf{I}, \overbrace{\mathbf{0}, \dots, \mathbf{0}}^{(H_c-1)\text{blocks}}) \mathbf{u}(k)_{opt}. \tag{4.104}$$

Clearly, this is a standard optimization problem known as the *Quadratic Programming* or (*QP*) problem, and algorithms for its solution, such as *Active Set* methods [82] and

the more recent *Interior Point* methods [83–85], are available. The main issue with constrained optimization is that it may be infeasible. To avoid this dilemma, various approaches have been suggested:

- avoid hard constraints on the output of the plant;
- actively manage the constraint definition at each k ;
- actively manage the horizons at each k ;
- use non-standard solution algorithms.

4.7 Case Study – Tracking Performance

Consider the positioning system presented in Chapter 2. With $\mathbf{x}(t) = [y_1, \dot{y}_1, y_2, \dot{y}_2]^T$ and $\mathbf{u}(t) = [u_1, u_2]^T$, the discrete state-space description of (2.68) is given by

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \quad (4.105)$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 0.00079343 & 0 & 0 \\ 0 & 0.98361 & 0 & 0 \\ 0 & 0 & 1 & 0.00079811 \\ 0 & 0 & 0 & 0.99528 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.0000035024 & 0 \\ 0.0087321 & 0 \\ 0 & 0.0000016129 \\ 0 & 0.0040292 \end{bmatrix}.$$

In the study, the two PMLMs follow an asymptotical square wave, as shown in Figure 4.4. The control parameters of (4.35) are set as: $H_p = 10$, $H_c = 4$, $\lambda = 0.02$, $\nu = 10$, $\eta = 5$, $\alpha_{12} = 1$, $\rho_{12} = 1$, $\mathbf{T}_1 = \mathbf{T}_2 = [1.0, 0]$. Figures 4.5 and 4.6 show the results under the PID controllers (2.70) and (2.71) with $K_{p1} = 0.06$, $K_{i1} = 0.8$, $K_{d1} = 0.001$,

Figure 4.4: Desired position trajectory

$K_{c1} = -0.0002$, $K_{p2} = 0.05$, $K_{i2} = 2$, $K_{d2} = 0.001$ and $K_{c2} = -0.0002$, and maximum synchronization error of $5.5714\mu m$ can be observed. Figures 4.7 and 4.8 show the results under the generalized synchronization controller (4.35) with $\nu = 0$ and $\eta = 0$. Obviously, the synchronization performance in this case is comparable. When $\nu \neq 0$ and $\eta \neq 0$ are selected, however, the maximum synchronization error can be reduced to $1.1706\mu m$, as shown in Figures 4.9 and 4.10.

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Figure 4.5: Synchronization error for PID controller

Figure 4.6: Control inputs for PID controller

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Figure 4.7: Synchronization error for generalized synchronization controller with $\nu = 0$ and $\eta = 0$

Figure 4.8: Control inputs for generalized synchronization controller with $\nu = 0$ and $\eta = 0$

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Figure 4.9: Synchronization error for generalized synchronization controller with $\nu = 10$ and $\eta = 5$

Figure 4.10: Control inputs for generalized synchronization controller with $\nu = 10$ and $\eta = 5$

4.8 Case Study – Disturbance Rejection

In modern power industries, an exciting alternative to traditional battery systems is the magnetic bearing-based flywheel energy storage system (FESS), due to high energy efficiency, low wear, absence of lubrication and mechanical maintenance, and wide range of work temperature [92–97]. However, the spinning rotor under a magnetic suspension may experience two kinds of whirl modes. The conical whirl mode gives rise to the gyroscopic forces to twist the rotor, thereby severely affecting stability of the rotor if not properly controlled [98]. The translatory whirl mode constrains the rotor to synchronous motion in the radial directions so as to avoid the gyroscopic rotation, which can has been extensively used in industry [99].

4.8.1 System Dynamics

Figure 4.11 shows basic features of a compact and efficient FESS with integrated magnetic bearings [100]. The radial magnetic bearing which consists of eight pairs of electromagnets is constructed around the circumference of hollow center, as shown in Figure 4.12. The motor and generator with disk-type geometry are combined into a single electric machine. The rotor is sandwiched between two disk-type stators, and each of the upper and lower stators carries a set of three-phase copper windings to be fed with sinusoidal currents. Furthermore, both axial faces of the rotor contain rare-earth permanent magnets embedded beneath the surfaces. A combination of active and passive magnetic bearings allows the rotor to spin and remain in magnetic levitation. Based on the well-known Maxwell's law, the total magnetic forces of one set of the magnetic bearings along x -axis and y -axis can be modeled as

$$f_x = (f_2 + f_3 - f_6 - f_7) \sin \beta + (f_1 + f_4 - f_5 - f_8) \cos \beta$$

Figure 4.11: Basic parts of a magnetic bearing-based FESS

Figure 4.12: The rotor of the magnetic bearing-based FESS

$$f_y = (f_1 + f_8 - f_4 - f_5) \sin \beta + (f_2 + f_7 - f_3 - f_6) \cos \beta$$

where β is a constant angle corresponding to the structure of electromagnets, and

$$f_1 = G \left(\frac{I_0 + i_y}{h_0 - x \cos \beta - y \sin \beta} \right)^2, \quad f_2 = G \left(\frac{I_0 + i_x}{h_0 - x \sin \beta - y \cos \beta} \right)^2,$$

$$f_3 = G \left(\frac{I_0 + i_x}{h_0 - x \sin \beta + y \cos \beta} \right)^2, \quad f_4 = G \left(\frac{I_0 - i_y}{h_0 - x \cos \beta + y \sin \beta} \right)^2,$$

$$f_5 = G \left(\frac{I_0 - i_y}{h_0 + x \cos \beta + y \sin \beta} \right)^2, \quad f_6 = G \left(\frac{I_0 - i_x}{h_0 + x \sin \beta + y \cos \beta} \right)^2,$$

$$f_7 = G \left(\frac{I_0 - i_x}{h_0 + x \sin \beta - y \cos \beta} \right)^2, \quad f_8 = G \left(\frac{I_0 + i_y}{h_0 + x \cos \beta - y \sin \beta} \right)^2,$$

where I_0 is the bias current, i_x and i_y are the control currents near x -axis and y -axis, respectively, h_0 is the nominal air gap at equilibrium, G is an electromagnet constant given by $G = \frac{1}{4}\mu_0 A_g N^2$, μ_0 is the air permeability, A_g is the cross-sectional area of air gap, and N is the number of turns of the winding circuit.

The magnetic forces vary nonlinearly with the control currents and displacements of the rotor. At equilibrium with $x = 0$ and $y = 0$ they can be linearized as

$$\begin{bmatrix} f_{xa} \\ f_{xb} \\ f_{ya} \\ f_{yb} \end{bmatrix} \approx K_p \begin{bmatrix} x_a \\ x_b \\ y_a \\ y_b \end{bmatrix} + K_c \begin{bmatrix} i_{xa} \\ i_{xb} \\ i_{ya} \\ i_{yb} \end{bmatrix} \quad (4.106)$$

where $K_p = \frac{8GI_0^2}{h_0^3}$ and $K_c = \frac{8GI_0 \sin \beta}{h_0^2}$.

Let x_c and y_c denote the displacements of the mass center of the rotor in the x and y directions, and θ and φ the roll angles of rotation about x -axis and y -axis, respectively. The motion of the rotor can then be described by

$$\begin{cases} m\ddot{x}_c = f_{xa} + f_{xb} \\ m\ddot{y}_c = f_{ya} + f_{yb} \\ J_x\ddot{\theta} = f_{yb}l_b - f_{ya}l_a - J_z\omega\dot{\varphi} \\ J_y\ddot{\varphi} = f_{xa}l_a - f_{xb}l_b + J_z\omega\dot{\theta} \end{cases} \quad (4.107)$$

subject to the following relations:

$$x_c = \frac{l_b}{l_a + l_b}x_a + \frac{l_a}{l_a + l_b}x_b, \quad y_c = \frac{l_b}{l_a + l_b}y_a + \frac{l_a}{l_a + l_b}y_b \quad (4.108)$$

$$\theta = \frac{1}{l_a + l_b}(y_b - y_a), \quad \varphi = \frac{1}{l_a + l_b}(x_a - x_b). \quad (4.109)$$

where m is the mass of the rotor, J_x , J_y and J_z are the moments of inertia about x -axis, y -axis and z -axis respectively, and ω is the spinning rate about z -axis.

If the state variable $\mathbf{x} = [x_a, \dot{x}_a, x_b, \dot{x}_b, y_a, \dot{y}_a, y_b, \dot{y}_b]^T$, the forcing vector $\mathbf{u} = [i_{xa}, i_{xb}, i_{ya}, i_{yb}]^T$, the output vector $\mathbf{y} = [x_a, x_b, y_a, y_b]^T$ and the output transition matrices $\mathbf{T}_1 = \mathbf{T}_2 = \mathbf{T}_3 = \mathbf{T}_4 = [1, 0]$ are given, a linearized version of model (4.107) is obtained as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_c \mathbf{x} + \mathbf{B}_c \mathbf{u} + \mathbf{B}_x \mathbf{d}_x \\ \mathbf{y} = \mathbf{C}_c \mathbf{x} + \mathbf{D}_y \mathbf{d}_y \end{cases} \quad (4.110)$$

where $\mathbf{d}_x$ and $\mathbf{d}_y$ denote model uncertainties or system disturbances with appropriate matrices $\mathbf{B}_x$ and $\mathbf{D}_y$,

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{c21} & 0 & A_{c23} & 0 & 0 & A_{c26} & 0 & A_{c28} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ A_{c41} & 0 & A_{c43} & 0 & 0 & A_{c46} & 0 & A_{c48} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & A_{c62} & 0 & A_{c64} & A_{c65} & 0 & A_{c67} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & A_{c82} & 0 & A_{c84} & A_{c85} & 0 & A_{c87} & 0 \end{bmatrix}, \quad (4.111)$$

with

$$\begin{aligned}
 A_{c21} &= \left(\frac{1}{m} + \frac{l_a^2}{J_y} \right) K_p, \quad A_{c23} = \left(\frac{1}{m} - \frac{l_a l_b}{J_y} \right) K_p, \quad A_{c26} = -\frac{J_z \omega l_a}{J_y(l_a + l_b)}, \quad A_{c28} = \frac{J_z \omega l_a}{J_y(l_a + l_b)}, \\
 A_{c41} &= \left(\frac{1}{m} - \frac{l_a l_b}{J_y} \right) K_p, \quad A_{c43} = \left(\frac{1}{m} + \frac{l_b^2}{J_y} \right) K_p, \quad A_{c46} = \frac{J_z \omega l_b}{J_y(l_a + l_b)}, \quad A_{c48} = -\frac{J_z \omega l_b}{J_y(l_a + l_b)}, \\
 A_{c62} &= \frac{J_z \omega l_a}{J_x(l_a + l_b)}, \quad A_{c64} = -\frac{J_z \omega l_a}{J_x(l_a + l_b)}, \quad A_{c65} = \left(\frac{1}{m} + \frac{l_a^2}{J_x} \right) K_p, \quad A_{c67} = \left(\frac{1}{m} - \frac{l_a l_b}{J_x} \right) K_p, \\
 A_{c82} &= -\frac{J_z \omega l_b}{J_x(l_a + l_b)}, \quad A_{c84} = \frac{J_z \omega l_b}{J_x(l_a + l_b)}, \quad A_{c85} = \left(\frac{1}{m} - \frac{l_a l_b}{J_x} \right) K_p, \quad A_{c87} = \left(\frac{1}{m} + \frac{l_b^2}{J_x} \right) K_p,
 \end{aligned}$$

$$\mathbf{B}_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \left(\frac{1}{m} + \frac{l_a^2}{J_y} \right) K_c & \left(\frac{1}{m} - \frac{l_a l_b}{J_y} \right) K_c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \left(\frac{1}{m} - \frac{l_a l_b}{J_y} \right) K_c & \left(\frac{1}{m} + \frac{l_b^2}{J_y} \right) K_c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{m} + \frac{l_a^2}{J_x} \right) K_c & \left(\frac{1}{m} - \frac{l_a l_b}{J_x} \right) K_c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{m} - \frac{l_a l_b}{J_x} \right) K_c & \left(\frac{1}{m} + \frac{l_b^2}{J_x} \right) K_c \end{bmatrix} \quad (4.112)$$

$$\mathbf{C}_c = \text{diag}(\mathbf{T}_1, \mathbf{T}_2, \dots, \mathbf{T}_4). \quad (4.113)$$

Model (4.110) is then capable of simulating the rotor dynamics in conjunction with the actively controlled magnetic bearings. During a closed-loop control phase, the position and rate of the shaft are constantly monitored by contactless sensors, and are processed in a controller, so that a control current to the coils of electromagnets which attract or repel the shaft is amplified and fed back.

4.8.2 Simulation Results

The parameters of this system are summarized in Table 4.1, and the coupling and synchronization factors are set as: $\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{41} = 1$ and $\rho_{12} = \rho_{23} = \rho_{34} = \rho_{41} = 1$. As the eigenvalues of $\mathbf{A}_c$ are: $\pm 2.0353i$, $\pm 10.4i$, ± 149.3 and ± 149.3 respectively, the open-loop system is unstable. If the sampling period $T_s = 0.008$ seconds, $H_p = 6$, $H_c = 3$, $\lambda = 0.01$, $\nu = 10$ and $\eta = 5$ are selected to solve the RDE (4.64) from $\mathbf{P}_0 = \mathbf{C}^T \mathbf{C} + \nu \mathbf{C}^T \mathbf{L}^T \mathbf{L} \mathbf{C}$, then all of the eigenvalues of the closed-loop transition matrix $\bar{\mathbf{A}}_{H_p-1}$ are within the unit circle, which are: $-0.7824 \pm 0.5554i$, -0.3791 , 0.4812 , 0.3784 , $0.1275 \pm 0.1954i$ and -0.0266 respectively. Figure 4.13 shows the detailed evolutions of $\mathbf{P}_j(1,1)$ and the control gain $\mathbf{K}_j(1,1)$ with respect to the iteration step, where $(1,1)$ represents the indexes of the corresponding elements. It can be seen that the convergence of $\mathbf{P}_j$ to the steady-state value may have occurred and therefore the stability of the control system would be guaranteed. In other words, for this flywheel system $H_p = 6$ and $H_c = 3$ are suitable enough to ensure the nominal stability of the control scheme.

In this simulation, some force disturbances are assumed to act on the bearings of the rotor, which are $1N$, $-1N$, $1N$ and $-1N$ on x_a -axis, x_b -axis, y_a -axis and y_b -axis at time 0.2, 0.6, 1.0, 1.4 seconds respectively, with the duration of 0.2 seconds each time. Figures 4.14–4.19 show the numerical results of the control algorithm for the two cases: with $\nu = 0$, $\eta = 0$; with $\nu = 10$, $\eta = 5$. Clearly, with no cross-coupling control action due to $\nu = 0$, $\eta = 0$, evident synchronization errors and a conical whirl mode during the transient responses are resulted. However, when $\nu = 10$ and $\eta = 5$ are introduced, the synchronization performance can be improved significantly, especially in terms of the rolling angles, as shown in Figure 4.18. Hence, with control parameters selected adequately the improved synchronization performance as well as guaranteed stability

Table 4.1: Model parameters of the magnetic bearing-based FESS

<u>Disk:</u>		
Mass	$m = 11.7$	kg
Radial Moment-of-Inertia	$J_x = J_y = 0.16272$	$kg - m^2$
Axial Moment-of-Inertia	$J_z = 0.0110962$	$kg - m^2$
Spinning Speed	$\omega = 30,000$	rpm
Location to Bearings a	$l_a = 0.115$	m
Location to Bearings b	$l_b = 0.115$	m
Angle between axis x and f_1	$\beta = 67.5$	deg
<u>Magnetic Bearings:</u>		
Magnetic Constant	$G = 2.879 \times 10^{-7}$	$N - m^2/A^2$
Air Gap	$h_0 = 0.00025$	m
Nominal Current	$I_0 = 1.33$	A

of the FESS might be obtained, and in consequence, the whirling rotor in the presence of disturbances would be more easily stabilized near the nominal position.

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Figure 4.13: Evolutions of $\mathbf{P}_j(1, 1)$ and $\mathbf{K}_j(1, 1)$

Figure 4.14: Radial displacements of the rotor near bearing x_a

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Figure 4.15: Radial displacements of the rotor near bearing x_b Figure 4.16: Control currents of bearing x_a used to reject outside disturbances

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Figure 4.17: Control currents of bearing x_b used to reject outside disturbances

Figure 4.18: Mass center of the rotor

Figure 4.19: Rolling angles of the rotor

4.9 Conclusion

In this chapter, a generalized synchronization controller in the state space has been presented. To achieve better synchronization performance while maintaining good tracking response, a new LQG optimal control problem based on a synchronization observer is formulated. By solving the RDEs the optimal synchronization control algorithm is generated. However, as this algorithm based on receding horizon is a feedback control, there is a risk that the resulting control system might be unstable. Hence, stability of the control system is analyzed, and properties on stability are presented. Without loss of generality, constrained synchronization control is also formulated. Finally, case study on the magnetic bearing-based FESS is presented. In the next chapter, robust synchronization control algorithms will be applied to a real coordinated motion system.

Chapter 5

Real Applications

In this chapter, experiments will be carried out to verify the effectiveness of the proposed control algorithms. First, the background of modeling and identification is introduced. Mathematical model of the permanent magnet linear motor for this experiment is then built. Parameter estimation using the Recursive Least Squares method and system validation are performed. Finally, experimental results are given.

5.1 Introduction

A model is a very useful and compact way to summarize the knowledge about a system. The system model can sometimes be obtained from first principles of physics. It is more difficult to get the model of the disturbances, which is equally important. These models then often have to be obtained from experiments. In general, a system cannot be characterized by one mathematical model, but by a hierarchy of models ranging from detailed and complex simulation models to very simple models, which are easy to manipulate analytically. The simple models are used for exploratory purposes and to obtain the gross features of the system behavior. The complicated models are used for a detailed check of the performance of the control system, however, it will take a

long time to develop.

System identification is the experimental approach to system modeling with its roots in standard statistical techniques. Many of the basic routines have direct interpretations as well-known statistical methods such as Least Squares and Maximum likelihood. The control community took an active part in developing and applying these basic techniques to dynamic systems right after the birth of “modern control theory” in the early 1960s. Maximum likelihood estimation was applied to difference equations (ARMAX models) in [101], and thereafter a wide range of estimation techniques and model parameterizations flourished. Now the area is mature with established and well-understood techniques [45].

5.2 Modeling of a Positioning System

Prior to the invention of the linear motor, the only way to produce linear motion was to use either hydraulic or pneumatic cylinders or to use a rotary electric motor, translating the rotary motion into linear motion using lead screws or belts and pulleys. Each of these methods has their advantages and disadvantages. Pneumatic and hydraulic cylinders are capable of generating high forces but fail to provide the accuracy required in many modern processes. Rotary motors connected to lead screws are capable of generating high forces but only at very low speeds. And with each of these devices there are wearing parts that not only require maintenance but also reduce the accuracy of the system as the parts wear.

Linear electric motors drive loads directly. They do not need transmission devices such as hydraulics or pneumatics or mechanical gearing and as a result they do not exhibit the compromises mentioned above. Among the electric motor drives, permanent magnet linear motors (PMLMs) are probably the most naturally linked to applications involving high speed and high precision motion control. In particular, the PMLM is designed by cutting and unrolling their rotary counterparts, literally similar to the imaginary process of cutting a conventional motor rotary armature and rotary stator along a radial plane and unroll to lay it out flat, as shown in Figure 5.1. Other benefits of the PMLM include the high force density achievable, low thermal losses and the simplicity in mechanical structure. Also, the PMLM requires no indirect coupling mechanisms as in gear boxes, chains and screws coupling. This greatly reduces the effects of contact-type nonlinearities and disturbances such as backlash and frictional forces, especially when they are used with aerostatic or magnetic bearings [102].

Figure 5.1: Schematic of unrolling a rotary motor

The positioning system, as shown in Figure 2.15, was manufactured by Linear Drive, UK. The two linear motors here have travel lengths of 300mm and are equipped with Renishaw optical encoders with the effective resolution of $1\mu\text{m}$. The dynamics of the PMLM can be viewed as comprising of two components: a dominantly linear motor, and an uncertain and non-linear remnant. The mechanical and electrical dynamics of

the PMLM can then be expressed as

$$\begin{cases} m\ddot{y} + V_a\dot{y} + F_d = F_m \\ K_e\dot{y} + L_a\frac{dI_a}{dt} + R_aI_a = K_u u(t) \\ F_m = K_f I_a \end{cases} \quad (5.1)$$

where $y(t)$ denotes the motor position, $u(t)$ denotes the time-varying motor terminal voltage, I_a denotes armature current, V_a is viscosity constant, R_a denotes armature resistance, L_a denotes armature inductance, K_f denotes the amount of force produced by the motor, K_e is the back electromagnetic fields (EMF) voltage, K_u denotes power amplifier gain, m is the moving thrust block mass, and F_d denotes system disturbances, including model uncertainties, ripple forces, frictional forces, loads and so on.

Figure 5.2 depicts the block diagram of the motor, including explicitly exogenous disturbances present, such as the ripple force, load force and friction force. Since the electrical time constant is typically much smaller than the mechanical one, the delay due to electrical transient response can be ignored. In this case, the mathematical model of the voltage-controllable linear DC motor can be approximately described by the simplified differential equation:

$$m\ddot{y} = -\frac{K_f K_e + R_a V_a}{R_a} \dot{y} + \frac{K_f K_u}{R_a} u(t) - F_d. \quad (5.2)$$

By letting

$$a = -\frac{K_f K_e + R_a V_a}{R_a m}, \quad b = \frac{K_f K_u}{R_a m}, \quad f_r(t) = \frac{F_d}{m} \quad (5.3)$$

(5.2) becomes

$$\ddot{y}_i = a_i \dot{y}_i(t) + b_i u_i(t) - f_{ri}(t) \quad (5.4)$$

Figure 5.2: Block diagram of a PMLM

where $i = 1, 2$ denotes the corresponding motors.

In fact, the ripple force is the prominent component of nonlinear effects associated with the PMLM, due to the magnetic structure and other physical imperfections. Figures 5.3 and 5.4 show the step responses of this system, and Figures 5.5 and 5.6 show the corresponding velocity-position characteristics. Interesting observations may be inferred from the responses with more step sizes. First, the ripple periodicity is independent of the step size (*i.e.*, independent of the velocity). However, it exhibits a fixed relationship with respect to the position. Secondly, the ripple amplitude is dependent on both position and velocity of the motor. At a higher velocity, the ripple amplitude decreases compared to the case when the PMLM is run at a low velocity and the higher dosage of ripple effects is experienced. Therefore, a first-order model of the force ripple can be described by

$$F_{ripple}(y) = A(y) \sin(\omega y + \varphi) \quad (5.5)$$

where $A(y)$ and φ are the magnitude and phase of the ripple force, respectively. Of course, higher harmonics of the ripple force may be included in higher order models.

Figure 5.3: Step response of Motor 1

Figure 5.4: Step response of Motor 2

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Figure 5.5: Velocity versus position for Motor 1

Figure 5.6: Velocity versus position for Motor 2

5.3 Parameters Identification

5.3.1 Off-line Analysis

By denoting $v_i(t) = \dot{y}_i(t)$ the step response of model (5.4) is described by

$$v_i(t) = [S_i D_i (1 - e^{-(t-\iota_i)/\tau_i}) - f_{ri}(t)] u(t), \text{ for } t \geq 0 \quad (5.6)$$

where $u(t)$ is a unit step function, S_i represents the final value of the step input, ι_i is the time delay, $D_i = -\frac{b_i}{a_i}$ and $\tau_i = -\frac{1}{a_i}$, so that in the steady state, $v_i(\infty) = S_i D_i - f_{ri}(\infty)$. Then, taking velocity of the motor observed at time t_k as $v_i(k)$ yields

$$v_i(k) + f_{ri}(k) = (v_i(\infty) + f_{ri}(\infty)) (1 - e^{-(t_k - \iota_i)/\tau_i}). \quad (5.7)$$

If $f_{ri}(\infty)$ is assumed to be a constant for different step inputs, a straightforward technique for estimating the unknown parameters of model (5.4) then follows.

- *Steady-State Gains, D_i and $f_{ri}(\infty)$*

Given the ultimate value $v_i(\infty)$ and the step signal S_i , the following relations can be achieved:

$$v_i^1(\infty) = S_i^1 D_i - f_{ri}(\infty) \quad (5.8)$$

$$v_i^2(\infty) = S_i^2 D_i - f_{ri}(\infty). \quad (5.9)$$

Solving the above equations results in

$$\begin{bmatrix} D_i \\ f_{ri}(\infty) \end{bmatrix} = \begin{bmatrix} S_i^1 & -1 \\ S_i^2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} v_i^1(\infty) \\ v_i^2(\infty) \end{bmatrix}. \quad (5.10)$$

Obviously, D_i and $f_{ri}(\infty)$ can be obtained if two different step inputs are applied.

- Time Constant τ_i and Time Delay ι_i

The solution is nonlinear in the parameter τ_i , but it can be rearranged as

$$1 - \frac{v_i(k) + f_{ri}(k)}{v_i(\infty) + f_{ri}(\infty)} = e^{-(t_k - \iota_i)/\tau_i}. \quad (5.11)$$

Then, it has

$$p_i(k) = \frac{\iota_i}{\tau_i} - \frac{t_k}{\tau_i} \quad (5.12)$$

where $p_i(k) = \ln \left(1 - \frac{v_i(k) + f_{ri}(k)}{v_i(\infty) + f_{ri}(\infty)} \right)$.

Thus, a plot of the transformation of the step-response data against time t should be a straight line with a slope of $-1/\tau_i$, and an intercept on the ordinate of ι_i/τ_i .

Also, this straight line will meet the t -axis at point $t = \iota_i$. Therefore, these parameters may be estimated graphically.

In this experiment, we have selected $S_1^1 = 2$, $S_1^2 = 1.5$, $S_2^1 = 1$, $S_2^2 = 0.8$, $v_1^1(\infty) = 1.058$, $v_1^2(\infty) = 0.752$, $v_2^1(\infty) = 0.755$, and $v_2^2(\infty) = 0.565$. Thus,

$$\begin{bmatrix} D_1 \\ f_{r1}(\infty) \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1.5 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1.058 \\ 0.752 \end{bmatrix} = \begin{bmatrix} 0.612 \\ 0.166 \end{bmatrix} \quad (5.13)$$

$$\begin{bmatrix} D_2 \\ f_{r2}(\infty) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0.8 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0.755 \\ 0.565 \end{bmatrix} = \begin{bmatrix} 0.95 \\ 0.195 \end{bmatrix}. \quad (5.14)$$

Suppose $f_{ri}(k) \approx f_{ri}(\infty)$. By applying linear regression on these datum with $S_1^1 = 2$ and $S_2^1 = 1$ we obtain: $\tau_1 = 0.04375$, $\iota_1 = 0.0016$, $\tau_2 = 0.1455$, $\iota_2 = 0.0008$, as shown in Figure 5.7. As a result, $\hat{a}_1 = -22.857$, $\hat{b}_1 = 13.988$, $\hat{a}_2 = -6.873$ and $\hat{b}_2 = 6.529$,

and the estimated transfer functions of the two PMLMs are given by

$$\hat{G}_1(s) = \frac{13.988}{s(s + 22.857)} e^{-0.0016s} \quad (5.15)$$

$$\hat{G}_2(s) = \frac{6.529}{s(s + 6.873)} e^{-0.0008s} \quad (5.16)$$

where s represents the Laplace variable.

Note that this method can only be suitable for the ideal system with constant disturbances. However, for our system with varying ripple force, it may result in large estimation error. For this purpose, the following method is developed.

Figure 5.7: $\ln \left(1 - \frac{v_i(k) + f_{ri}(k)}{v_i(\infty) + f_{ri}(\infty)} \right)$ against time t_k

5.3.2 On-line Identification

From Figures 5.5 and 5.6 we can measure the frequencies of the ripple forces: $\omega_1 = 2\pi/0.0744$ and $\omega_2 = 2\pi/0.051$. By ignoring friction force and load force model (5.4) can be simplified as

$$\ddot{y}_i = a_i \dot{y}_i(t) + b_i u_i(t - \iota_i) - A_{ri} \sin(\omega_i y_i) - B_{ri} \cos(\omega_i y_i) \quad (5.17)$$

where $\iota_1 = 0.0016$ seconds and $\iota_2 = 0.0008$ seconds (from previous analysis).

In order to avoid the pure differentiation of the velocity $v_i(t)$, a stable filter of H_f is introduced to (5.17), e.g. $\dot{v}_{if} = (sH_f)v_i(t)$, so that

$$\ddot{y}_{fi} = a_i \dot{y}_{fi}(t) + b_i u_{fi} - A_{ri} H_f \sin(\omega_i y_i) - B_{ri} H_f \cos(\omega_i y_i) \quad (5.18)$$

where $\ddot{y}_{fi} = H_f \dot{y}_i(t)$, $u_{fi} = H_f u_i(t - \iota_i)$.

With the vector of the unknown parameters $\hat{\theta}_i = [\hat{a}_i, \hat{b}_i, \hat{A}_{ri}, \hat{B}_{ri}]^T$ and the vector of the regression variables $\varphi_f = [\dot{x}_{fi}, u_{fi}, -H_f \sin(\omega_i y_i), -H_f \cos(\omega_i y_i)]^T$, the identification algorithm using Recursive Least Squares (RLS) follows:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \mathbf{K}(t) \left(\dot{v}_{if} - \varphi_f^T(t) \hat{\theta}(t-1) \right) \quad (5.19)$$

$$\mathbf{K}(t) = \mathbf{P}(t-1) \varphi_f(t) \left(1 + \varphi_f^T(t) \mathbf{P}(t-1) \varphi_f(t) \right)^{-1} \quad (5.20)$$

$$\mathbf{P}(t) = (\mathbf{I} - \mathbf{K}(t) \varphi_f^T(t)) \mathbf{P}(t-1). \quad (5.21)$$

If filter H_f is chosen as $\frac{1}{s+20}$, the evolutions of the estimated parameters are illustrated in Figures 5.8 - 5.18, and the final results are summarized in Table 5.1.

Table 5.1: Identified parameters of the positioning system

Motor	Input(V)	a_i	b_i	A_{ri}	B_{ri}
$i = 1$	2	-20.6532	11.0055	-1.4178	0.3125
	1.5	-20.5025	10.3055	-0.4539	1.6098
	1	-18.4408	7.8081	-0.7821	1.5527
$i = 2$	1	-5.9165	5.0484	-0.6762	0.1914
	0.8	-6.2005	4.5395	-0.4255	-0.1279
	0.5	-7.2209	3.3094	-0.1564	-0.1825

Figure 5.8: Evolutions of a_1 and b_1 with step input $2V$

Figure 5.9: Evolutions of A_{r1} and B_{r1} with step input 2V Figure 5.10: Evolutions of a_1 and b_1 with step input 1.5V

Figure 5.11: Evolutions of A_{r1} and B_{r1} with step input 1.5V Figure 5.12: Evolutions of a_1 and b_1 with step input 1V

Figure 5.13: Evolutions of A_{r1} and B_{r1} with step input 1V Figure 5.14: Evolutions of a_2 and b_2 with step input 1V

Figure 5.15: Evolutions of A_{r2} and B_{r2} with step input 1V Figure 5.16: Evolutions of a_2 and b_2 with step input 0.8V

Figure 5.17: Evolutions of A_{r2} and B_{r2} with step input 0.8V Figure 5.18: Evolutions of a_2 and b_2 with step input 0.5V

Figure 5.19: Evolutions of A_{r2} and B_{r2} with step input $0.5V$

5.4 Model Validation

Since the friction force and load force are not considered in the preceding identification algorithm, the estimated parameters cannot keep constant for different test inputs. In fact, the disturbance would relatively be less evident for larger input. It means that the estimated parameters with larger step input could be more accurate. Hence, we choose the estimated models of the two PMLMs as

$$\ddot{y}_1 = -20.6532\dot{y}_1(t) + 11.0055u_1(t - 0.0016) + 1.4178 \sin\left(\frac{2\pi}{0.0744}y_1\right) - 0.3125 \cos\left(\frac{2\pi}{0.0744}y_1\right) \quad (5.22)$$

$$\ddot{y}_2 = -5.9165\dot{y}_2(t) + 5.0484u_2(t - 0.0008) + 0.6762 \sin\left(\frac{2\pi}{0.051}y_2\right) - 0.1914 \cos\left(\frac{2\pi}{0.051}y_2\right). \quad (5.23)$$

Consider the starting N datum with $u_1 = 2V$ and $u_2 = 1V$ and define the modelling

error as $E_{mi}(k) = y_i(k) - \hat{y}_i(k)$, for $i = 1, 2$ and $k = 1, 2, \dots, N$. The mean-square error (MSE), the correlation factor (CF) and autocorrelation (AC) of the modelling error given by

$$MSE_i = \frac{1}{N} \sum_{k=1}^N [y_i(k) - \hat{y}_i(k)]^2, \quad (5.24)$$

$$CF_i = \frac{\sum_{k=1}^N [y_i(k) - \bar{y}_i][y_i(k) - \bar{\hat{y}}_i]}{\sqrt{\sum_{k=1}^N [y_i(k) - \bar{y}_i]^2} \sqrt{\sum_{k=1}^N [\hat{y}_i(k) - \bar{\hat{y}}_i]^2}}, \quad (5.25)$$

and

$$AC_i(h) = \frac{\sum_{k=1}^{N-h} [E_{mi}(k) - \bar{E}_{mi}][E_{mi}(k+h) - \bar{E}_{mi}]}{\sum_{k=1}^N [E_{mi}(k) - \bar{E}_{mi}]^2}, \quad (5.26)$$

are used for model validation, with time lag h ($h = 0, 1, \dots$). Figures 5.20 and 5.21 shows the modelling error and its autocorrelation using models (5.22) and (5.23) with $N = 500$, and numerical results are summarized in Table 5.2.

Figure 5.20: Comparison of measured and predicted outputs

Figure 5.21: Autocorrelation of modelling error

Table 5.2: Numerical results of model validation

Index	Best value	Motor 1	Motor 2
MSE	0	1.9501×10^{-6}	3.4777×10^{-7}
CF	1	0.9999	0.9998

5.5 Experimental Setup

The dSPACE control development and rapid prototyping system is adopted for the positioning system. The dSPACE integrates the entire development cycle seamlessly into a single environment, so that individual development stages between simulation and test can be run repeatedly without readjustment. The basic steps for this experiment are illustrated in Figure 5.22, and detailed descriptions are given below.

Figure 5.22: Block diagram of experimental setup

The ControlDesk, dSPACE well-established experiment software, provides all the functions to control and monitor experiments and make the development of controllers more efficient. The ControlDesk uses a lot of standard Windows techniques and also other features not provided by standard Windows applications. For example, the Platform Manager in the ControlDesk provides the functions to handle the platforms installed in the PC or expansion box, and the applications assigned to these platforms. Before handling applications, the following operations should be performed.

- Specify the connection mode that corresponds to our installation.
- Register multiprocessor systems containing DS1003 and DS1004 DSP Boards.
- If the system contains more than one platform, a working board should be selected.
- To download and start handcoded applications with the ControlDesk, the System Description file (SDF) has to be created manually.
- If the application requires a firmware version higher than the one currently installed, an automatic or manual update must be performed.

Moreover, the ControlDesk Standard provides a set of powerful instruments, which are designed to monitor and/or control variables interactively and to display data captures. Instruments can be arranged freely in one or more windows, called layout windows. Any set of instruments arranged in a layout window and connected to the corresponding variables is called a virtual instrument panel. To trigger the data transfer or execution of an action, we can use events, which can be the standard system events such as system polling or specific instrument events such as WriteData. We can also build connections between layouts and instruments. These instruments are not used to transfer data

but to control the display of layouts. Then, the instrument panel can be activated by switching to the Animation mode.

5.6 Results and Discussions

In this experiment, the MPC controller (2.29) and the PID controllers (2.70) and (2.71) presented in Chapter 2 are tested to track the desired trajectories, as shown in Figure 2.16. The program diagram of the synchronization MPC algorithm is depicted in Figure 5.23.

Since the mechanical structure and other components in the system have inherent unmodeled high-frequency dynamics that should not be excited, and load and frictional forces are unknown, some of the control parameters are adjusted as: $\nu = 10$, $\eta = 5$, $K_{i1} = 0.6$ and $K_{i2} = 0.5$. For safety, constraints on control inputs are applied, i.e., $-1V \leq u_i(t) \leq 1V$. Figures 5.24-5.25 show the experimental results for the robust synchronization controller and for the PID controller, respectively, and the comparison of the synchronization performance is given in Table 5.3. It can be observed that, under the PID controller, maximum synchronization error of $11\mu m$ is obtained. However, for the robust synchronization controller with the cross-coupling control action, maximum synchronization error can be reduced to $8\mu m$.

In reality, the dynamics and disturbances of each motor may vary due to changes in operational conditions or loadings. To verify robustness of the proposed synchronized control scheme, a fixture is mounted on the motors which creates model uncertainties and disturbances, as shown in Figure 5.26. In this experiment, we desire the two motors coupled by the fixture to track the same sinusoidal wave $w_2(t)$ in Figure 2.16.

Figure 5.23: Program diagram of robust synchronization controller

Figure 5.24: System responses for robust synchronization controller

Figures 5.27 and 5.28 show the experimental results with the previous control parameters. Clearly, the robust synchronized controller is relatively insensitive to model

Figure 5.25: System responses for PID controller

uncertainties and provides much better synchronization performance than the PID controller.

Figure 5.26: The positioning system with a coupled fixture

Table 5.3: Comparison of the synchronization performance in experiment

Controller	$\max(\text{syn.})(\mu\text{m})$	$\text{var}(\text{syn.})(\mu\text{m}^2)$
RSC	8	8.3650
PID	11	11.8071
RSCW	12	12.3871
PIDW	32	23.6503

$\max(\text{sync.})$ – maximum synchronization error

$\text{var}(\text{sync.})$ – variance of the synchronization error

RSC – robust synchronization controller

PID – PID controller

RSCW – robust synchronization controller with model uncertainties

PIDW – PID controller with model uncertainties

Figure 5.27: System responses for robust synchronization controller with model uncertainties

Figure 5.28: System responses for PID controller with model uncertainties

5.7 Conclusion

The robust synchronization MPC system has been implemented to the positioning system with two PMLMs through Matlab/Simulink and the dSpace control system. The mathematical model of the positioning system is built from the principle of physics. Then, parameters identification using RLS is carried out. A filter is introduced into the RLS algorithm so that the pure differentiation of the velocity can be avoided, which is sometimes difficult to be measured. The experimental results have shown that the improved performance of the synchronization MPC control system can be obtained, even if model uncertainties, outside disturbances are present. In practice, such a platform can be used for wafer fabrication and inspection, ultra high vacuum systems for chemical analysis and thickness measurement, vacuum motion system for interferometric autofocus, and laser surgeries, where the motion axes perpendicular to each other are mounted.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

In this thesis, new cross-coupling MPC algorithms have been developed to achieve satisfactory synchronization performance for a class of multi-axis motion systems in the presence of model uncertainties and external disturbances. In light of coordinated motion control, the proposed control systems are based on optimization of modified quadratic performance indices in which the synchronization errors are embedded in a natural way. With adequately adjusting control parameters the resulting control systems can provide improved synchronization performance. The stability of the control systems is investigated based on RDEs, and guidelines for choice of the control parameters are also addressed. The proposed algorithms have been applied to several processes, and the results show that they are effective and suitable for coordinated motion control systems. A more detailed summary is given below.

- Modeling and identification of typical coordinated motion systems have been presented. They are positioning systems, electro-hydraulic synchronous motion systems and magnetic bearing-based flywheel energy storage systems. With on-line

identification, an adaptive synchronization control algorithm may be obtained.

- A definition about synchronization errors of multiple motion axes is given. It is to introduce the cross-coupling action into the control system.
- A robust synchronization MPC algorithm has been developed [103]. In the formulation, both external disturbances and model uncertainties are included. A new quadratic performance index incorporating the synchronization errors and their differences across the multiple motion axes is introduced so that the resulting control law generates cross-coupling control action and in consequence, the improved tracking and synchronization performance can be obtained. The key for the control system is to ensure that each motor tracks its desired trajectory while synchronizing motion with others.
- A synchronization MPC algorithm with reference models has also been proposed [104]. It is shown that this control algorithm provides a combined feedback-feedforward control resulting in pole-zero cancellation of poles which do not correspond to the reference models. Hence, the system behaves asymptotically as the reference models provided the canceled poles are stable. In addition, the cross-coupling control action introduced in an optimal way will effectively reduce the synchronization deviations of the multiple motion axes.
- A generalized synchronization control algorithm in the state space has been developed [105, 106]. In this scheme a synchronization observer is introduced for the synchronization errors between the closest axes, so that on-line computational burden can be reduced. An augmented system is then formulated and the synchronization control is generated by solving the standard RDEs. A simplified version of the control system is achieved. In fact, this control system can be used

to control MIMO systems, and can easily realize position and velocity control by assigning the output transition matrix accordingly.

- Stability analysis of the control algorithm is investigated thoroughly, and properties of the stability are also given.
- A constrained synchronization control and possible solutions have been presented.
- Implementation of the proposed algorithms to a real system has been completed. For the sake of demonstration, only the algorithm proposed in Chapter 2 is applied to the positioning system, and experimental results have shown that it is very effective to improve the synchronization performance in the presence of evident ripple forces and unmodeled system dynamics.

6.2 Recommendations for Future Research

Although the synchronization MPC strategies are shown to work effectively, future research can continue in many directions:

- On the selection of the control parameters, further research needs to be done to find a suboptimal controller in terms of control performance in a systematic way. A possible approach is through convex optimization by casting the problem into an appropriate formulation.
- A disturbance observer and a feedforward controller may be used to reduce the effect of the mismatched dynamics on the performance, such as the ripple, friction and load forces.
- Synchronization control subject to actuator saturation should be studied in the future, since nearly all physical systems are subject to some types of constraints.

In this research, such constraints actually become inactive by appropriately adjusting the weighting factor λ_i .

- A combination of MPC and PID methods as well as trajectory planning technique may be considered for further improvement of the synchronization performance.
- The proposed framework of coordinated motion control should be further verified experimentally. While the positioning system with two PMLMs was utilized for convenience, such a system may not be the best bed for all coordinated control systems. For example, when the spinning speed of the rotor in the magnetic bearing-based FESS is changed significantly, an adaptive version of the control algorithm should be considered.

Author's Publications

Journals

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